Lotka-Volterra in Matlab

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In this example document, we want to solve the Lotka-Volterra ODE system given by:

$$\frac{dx}{dt} = \alpha x - \beta xy \tag{1}$$
$$\frac{dy}{dt} = \delta xy - \gamma y. \tag{2}$$

We will solve this problem numerically in Matlab.
Since we have a system of equations, we want to define it in *vector form*:

$$\begin{matrix} \frac{dx}{dt} = \alpha x - \beta xy \\ \frac{dy}{dt} = \delta xy - \gamma y \end{matrix} \equiv \frac{d}{dt} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = F(X), \tag{3}$$

where

$$F(X) = \begin{bmatrix} \alpha x - \beta xy \\ \delta xy - \gamma y \end{bmatrix}. \tag{4}$$

This now is the derivative function we will define for Matlab. First we set our parameters:

```
In [1]: alpha = 2/3;
        beta  = 4/3;
        gamma = 1;
        delta = 1;
```

Next, we define the function handle. Notice that since $X = \begin{bmatrix} x & y \end{bmatrix}^T$, we have `x=X(1)` and `y=X(2)`.

```
In [2]: F = @(t,X) [alpha*X(1)-beta*X(1)*X(2); delta*X(1)*X(2)-gamma*X(2)];
```

We can now pick our initial conditions and the time frame we want to integrate over:

```
In [3]: tspan = [0 25];
        X0 = [0.8; 0.4];
```

Now we're ready to solve the ODE:

```
In [4]: [t, X] = ode45(F, tspan, X0);
```

Now we can plot the solutions.

In [5]:
```
plot(t,X(:,1),t,X(:,2))
xlabel('Time')
ylabel('Population')
title('Lotka-Volterra System')
legend('x(t)','y(t)')
```