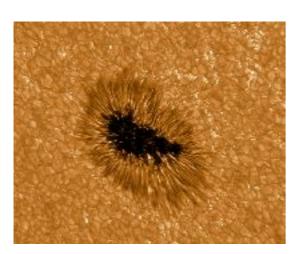
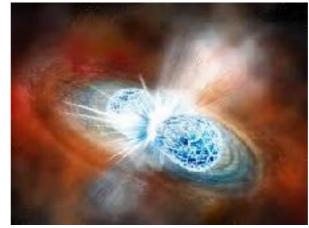
# Lecture course: Computational Fluid Dynamics











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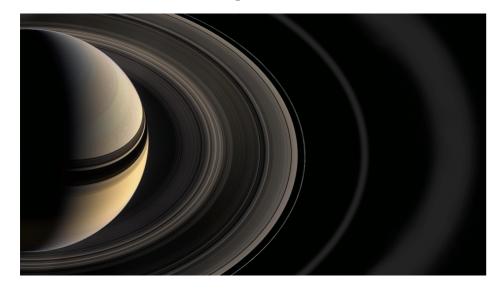
Department of Physics, University of Guelph



Lecture 1: Preliminaries

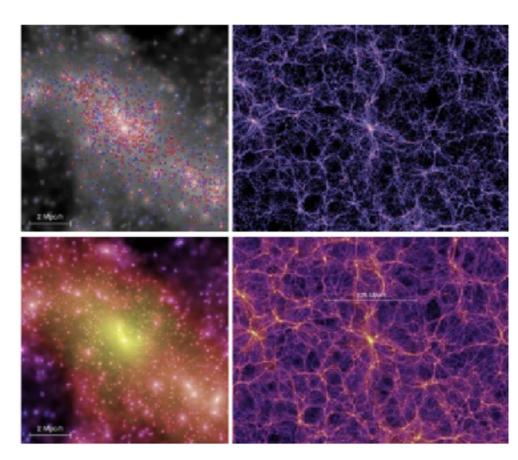
## Astrophysical 'fluids': ubiquitous in the Universe on all scales

Dust rings of Saturn



Credit: NASA/JPL-Caltech/Space Science Institute

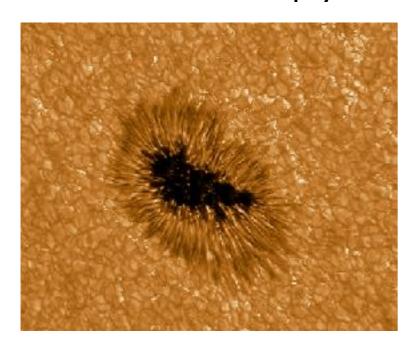
#### Matter distribution of the Universe



Credit: Springel et al. 2005, Nature, 435, 629

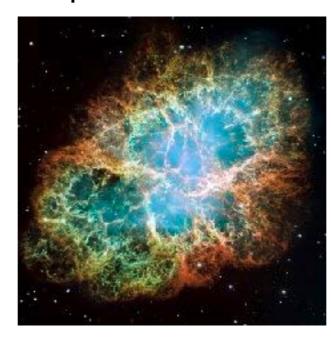
## Astrophysical 'fluids': a few more examples...

Solar & stellar astrophysics



Credit: Leibniz Institute for Solar Physics (KIS)

#### Supernova remnants



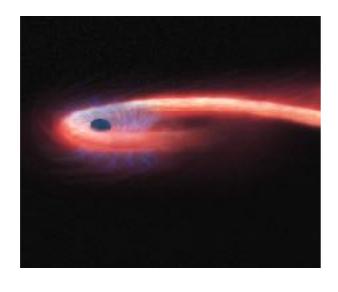
Credit: NASA, STScI, ESA

#### Interstellar medium



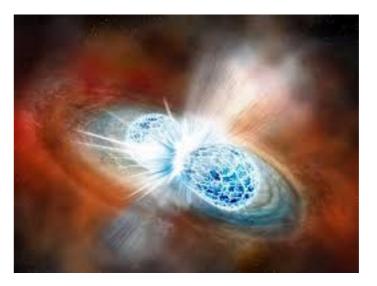
Credit: NASA

#### Tidal disruption events



Credit: NASA / CXC / M. Weiss

#### Neutron-star mergers



Credit: Robin Dienel, Carnegie Institution for Science

## Astrophysical fluid dynamics

- Astrophysical fluid flows central to understanding our universe
  - equations of gas/plasma dynamics (hydrodynamics)
     (+ gravity and microphysics: chemistry, nuclear & atomic physics, EM fields, etc.)
- Receive information about the universe based on EM radiation that is reprocessed as it travels toward us
  - equations of radiation transport
- We now also receive information via gravitational waves and highenergy particles
  - multi-messenger astrophysics

## Interpretation of observational data

Theoretical interpretation of observational data (stages of refinement):

### Analytical estimates

("back of the envelope" calculations)



(Semi-)analytic calculations

(using simplifications/approximations)



Full-scale numerical simulations



experiment 'substitute'

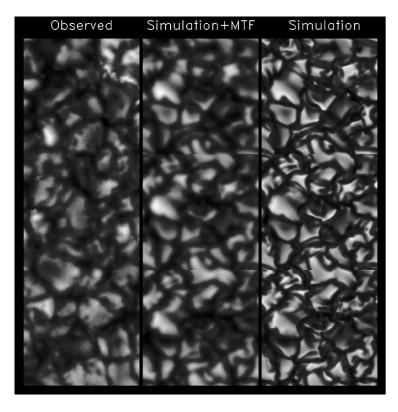




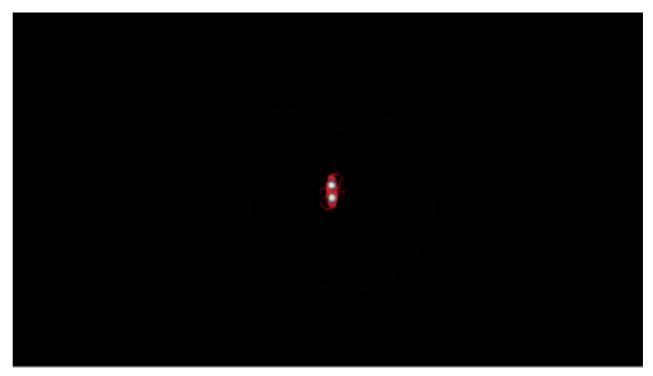
Credit: NASA

## Simulations can achieve amazing things...

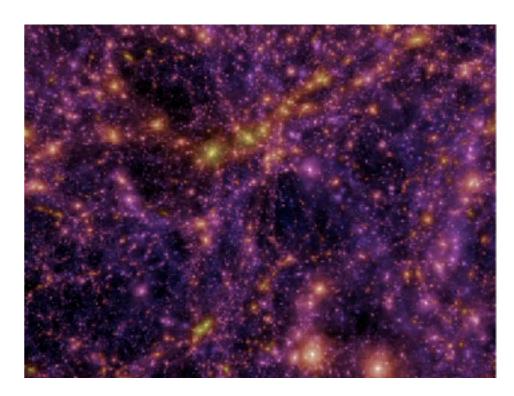
## Solar granulation (radiation hydrodynamics)



Credit: Stein+ 1998, ApJ 499, 914



Movie credit: Ciolfi+, Siegel 2017, PRD 95, 063016



Credit: Springel et al. 2005, Nature, 435, 629

## Interpretation of observational data

Theoretical interpretation of observational data (stages of refinement):

## Analytical estimates

("back of the envelope" calculations)



(Semi-)analytic calculations

(using simplifications/approximations)



Full-scale numerical simulations

'healthy skepticism'

(still approximate,
numerical effects)



experiment 'substitute'





Credit: NASA

## Expectations of expectations

reduce complexity and provide stability to social systems (Luhmann)

...so let's talk about expectations:

## Course contents and 'mechanics'

- For course outline and policies, please see:
  - https://www.physics.uoguelph.ca/course-outlines/special-topics-astrophysics-numericalhydrodynamics-phys7900
- Course website (repository with course material):
   <a href="https://github.com/dmsiegel/computational-fluid-dynamics-course.git">https://github.com/dmsiegel/computational-fluid-dynamics-course.git</a>
- 'Interpolation' between applied mathematics, physics, astrophysics 'blackboard' lectures, hand-written lecture notes
- Homework (30% final grade):

Problem sets with analytical problems

Project components with hands-on implementations:

- finite difference schemes and explore their properties
- an 'exact' Riemann solver for the Euler Equation
- a Godunov-type scheme for Newtonian hydrodynamics
- Exam (70% final grade): date & time TBD

## Course contents and 'mechanics'

#### Contents

- 1. Basic Notions of Partial Differential Equations
- 2. Basic Equations of Computational Fluid Dynamics
- 3. Finite Difference Methods for PDEs
- 4. Properties of conservation laws (theoretical background)
- 5. Riemann problem for the Euler Equations
- 6. Numerical schemes for conservation laws
- 7. Approximate Riemann solvers
- 8. Source terms and higher dimensions
- 9. Outlook: a primer on discontinuous Galerkin methods

Very general, directly relevant for Einstein's equations as well!

## Course contents and 'mechanics'

#### References

This course draws from all the below listed recommended literature

#### Numerical Methods:

- E. Toro: Riemann Solvers and Numerical Methods for Fluid Dynamics (Springer, 3rd edition, 2009)
- R. Leveque: Finite Volume Methods for Hyperbolic Problems (Cambridge Univ. Press, Cambridge Texts in Applied Mathematics, 2002)

#### More mathematically inclined literature:

- D. Kröner: Numerical Schemes for Conservation Laws (Wiley, 1997)
- L. Evans: Partial Differential Equations (Graduate Studies in Mathematics, American Mathematical Society, 2nd edition, 2010)

#### Other useful literature:

- A. Anile: Relativistic fluids and magneto-fluids (Cambridge Univ. Press, 1990)
- P. Bodenheimer, G. Laughlin, M. Rozyczka, H. Yorke: Numerical Methods in Astrophysics (Taylor & Francis, 2007)
- R. Leveque: Finite Difference Methods for Ordinary and Partial Differential Equations (SIAM, 2007)