Computational Fluid Dynamics

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Preface and recommended literature

These lecture notes have been prepared for a new graduate course on compuational fluid dynamics in the Guelph–Waterloo Institute for Physics (GWIP), the joint graduate school of the Universities of Guelph and Waterloo, as well as Perimeter Institute for Theoretical Physics (Fall 2020). Many textbooks helped me compose these lectures and lecture notes. Some suggested textbooks include:

Numerical Methods

- Toro (2009): *Riemann Solvers and Numerical Methods for Fluid Dynamics* (Springer, 3rd edition, 2009)

- Leveque (2002): *Finite Volume Methods for Hyperbolic Problems* (Cambridge Univ. Press, Cambridge Texts in Applied Mathematics, 2002)

Mathematically inclined literature:

- Kröner (1997): *Numerical Schemes for Conservation Laws* (Wiley, 1997)

- Evans (2010): *Partial Differential Equations* (Graduate Studies in Mathematics, American Mathematical Society, 2nd edition, 2010)

Other recommended literature

- Bodenheimer et al. (2006): *Numerical Methods in Astrophysics* (Taylor & Francis, 2007)

- Anile (1989): *Relativistic fluids and magneto-fluids* (Cambridge Univ. Press, 1990)

- Rieutord (2015): *Fluid Dynamics* (Springer, 2015)

- LeVeque (2007): *Finite Difference Methods for Ordinary and Partial Differential Equations* (SIAM, 2007)

Chapter 1

Basic Notions of Partial Differential Equations

1.1 PDEs of 2nd order

Recommended reading: Evans (2010) Chap. 1, Chap. 6.1, 7.1.1, 7.2.1, Strauss (2007) Chap. 1.6, most introductory books on PDEs.

1.2 PDEs of 1st order

Recommended reading: Evans (2010) Chap. 7.3.1, 11.1, Toro (2009) Chap. 2.

1.3 Some properties of 1st order hyperbolic systems

Recommended reading: Toro (2009) Chap. 2, Leveque (2002) Chap. 2.9–2.11, 3.1–3.6, 11.2.

1.3.1 Characteristics

1.3.2 Domain of dependence and range of influence

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Chapter 2

Basic Equations of Computational Fluid Dynamics

Recommended reading: Bodenheimer et al. (2006) Chap. 1, Rieutord (2015) Chap. 11, Lifshitz & Pitaevskii (1981) Chap. 1, Anile (1989) Chap. 2.

2.1 Continuous media and the Boltzmann equation

2.2 From the Boltzmann equation to the Euler equations

2.3 Navier-Stokes equations

2.4 Magnetohydrodynamics

2.5 Radiation transfer

2.6 Relativistic Hydrodynamics

Recommended reading: Gourgoulhon (2012) Chaps. 4 & 6, Gourgoulhon (2006), Baumgarte & Shapiro (2010) Chap. 5, Alcubierre (2008) Chap. 2.2 & 7.

2.7 Relativistic radiation transfer

Recommended reading: Thorne (1981), Shibata et al. (2011), Straumann (2013) Sec. 3.11.

Chapter 3

Finite Difference methods for PDEs

Recommended reading: LeVeque (2007) Chap. 1, 9, 10, Bodenheimer et al. (2006) Chap. 2, Choptuik (2006) Sec. 1., Kröner (1997) Chap. 2.4, Toro (2009) Chap. 5.1.

3.1 Basic notions of discretization

3.2 Finite difference approximations

3.2.1   Partial derivatives & differential operators

3.2.2   Sample discretizations

3.3 Consistency, stability, convergence

3.4 Stability analysis and the CFL condition

3.5 Diffusion and dispersion

3.6 Error analysis and convergence

Bibliography

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Bodenheimer, P., Laughlin, G. P., Rozyczka, M., & Yorke, H. W. 2006, Numerical Methods in Astrophysics : An Introduction, Series in Astronomy and Astrophysics (CRC Press Taylor & Francis)

Choptuik, M. W. 2006, Lectures for VII Mexican School on Gravitation and Mathematical Physics: Relativistic Astrophysics and Numerical Relativity Numerical Analysis for Numerical Relativists, http://laplace.physics.ubc.ca/People/matt/Teaching/06Mexico/mexico06.pdf

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Shibata, M., Kiuchi, K., Sekiguchi, Y., & Suwa, Y. 2011, Progress of Theoretical Physics, 125, 1255

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