Problem №1 *(Cost Inefficiency in Cournot)* Consider a Cournot competition model with market demand $P(Q) = 1 - Q$. Two firms have different cost functions: $TC_1(q_1) = c_1 q_1$ and $TC_2(q_2) = c_2 q_2$, where $c_1 < c_2$.

(a) *Explain why total welfare in Cournot equilibrium is* $W^C = \frac{1}{2}Q^2 + q_1^2 + q_2^2$*, where* q_i *is equilibrium output of firm* i *and* $Q = q_1 + q_2$*.*

(b) *Imagine that the total output produced in the Cournot equilibrium could be allocated across firms in a different way. Which allocation would be socially optimal?*

(c) *Suppose that the firms form a cartel. What would the ideal output of that cartel be? How is production allocated across firms?*

(d) *Explain why the ideal cartel allocation would be hard to implement in the presence of antitrust scrutiny.*

(e) *Explain why total welfare under cartel is* $W^M = \frac{3}{2}(Q^m)^2$*, where* Q^m *is the cartel output.*

(f) *Are there values of* c_1 *and* c_2 *such that consumer welfare goes up as a result of cartelisation? Interpret.*

(g) *Suppose that* $c_1 = 0$*. Are there values of* c_2 *such that social welfare goes up as a result of cartelisation? Interpret.*

Problem №2 *(Preemption through acquisition)* Two firms (A and B) producing a homogeneous product compete in prices in accordance with the Bertrand model. The market demand is $D(p) = 10 - p$. The total cost function of firm A is $TC_A(q) = 7q$; the total cost function of firm B is $TC_B(q) = 6q$. Consider a hypothetical new technology that allows cheaper production with the total cost function of $TC(q) = 4q$.

(a) *How much is firm A willing to pay to develop the new technology?*

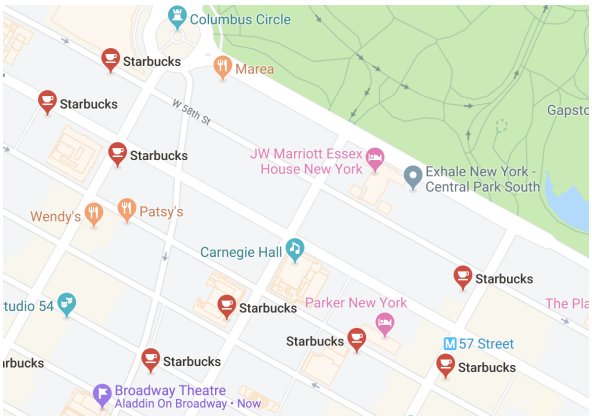
(b) *How much is firm B willing to pay to develop the new technology?*

(c) *If the probability of discovery is proportional to the willingness-to-pay of the company, which firm is more likely to develop the invention?*

(d) *An inventor managed to develop the new technology, and decided to auction exclusive rights for it. The winner of the auction can adopt the new technology and no other firm is allowed to use it. Which firm is going to win the auction, and at what price?*

(e) *Will the outcome of the auction above be socially optimal?*

Problem №3 *(Market Cannibalization)* Consider a street of length 1. There is currently one coffee shop, S, located at point $x = 2/5$. A large mass $M = 1$ of consumers* are uniformly distributed along the street. Each consumer needs one cup of coffee a day, and they all value it at $\bar{v} = \$4$. Transportation costs are quadratic: getting from point x to point y costs $t(x-y)^2$, where $t = 5$. Marginal cost of one cup of coffee is $c = \$1$.



(a) *Suppose that the price of coffee is 0 at the coffee shop—who is buying coffee? (draw the interval and shade all the customers who are buying)*

(b) *Conduct the following experiment: the price starts crawling up—at which price would the shading start to look different? Call it r_1. The price keeps crawling up—what is another price at which the picture starts to look qualitatively different? Call it r_2.*

(c) *Based on the analysis above, one can show that the demand that the coffee shop is facing is given by the function below. Find A, B, C (and r_1, r_2, if you still haven't).*

$$D^m(p) = \begin{cases} A & p \le r_1 \\ B + \sqrt{\frac{4-p}{5}} & p \in (r_1, r_2] \\ C\sqrt{\frac{4-p}{5}} & p \in (r_2, 4] \end{cases}$$

(d) *The price that is optimal for the cafe to charge (the monopoly price p^m) is one of the 3 kink points of the demand curve: $\{r_1, r_2, 4\}$. Which one? Find the corresponding profit.*

(e) *There is a vacant location at $x = 0.6$ that is suitable for another cafe. If the owner of S opens a second store there, the demand for her product would be as below. Find $\tilde{A}, \tilde{B}, \tilde{r}_1, \tilde{r}_2$.*

$$D_2^m(p) = \begin{cases} \tilde{A} & p \le \tilde{r}_1 \\ \tilde{B} + 2\sqrt{\frac{4-p}{5}} & p \in (\tilde{r}_1, \tilde{r}_2] \end{cases}$$

*$M = 1$ thousand, say, but please don't multiply or divide by 1 thousand.

(f) *With two stores open, the new monopoly price is $\tilde{r}_1$. What is the new monopoly profit? Would the owner pay a rent of* \$0.45[†] *to open the store?*

(g) *Suppose now that a competitor, D, opens a cafe at $x = 0.6$. Find equilibrium prices and profits. Would the owner of D pay the rent of* \$0.45 *to open a cafe?*

(h) *Suppose that the owner of S learns that D is thinking about opening a cafe at $x = 0.6$. Would they now find it profitable to open a second cafe there?*

Problem №4 *(Merger waves)* Consider an industry where firms compete by setting output levels (Cournot). Market demand is given by $D(p) = 150 - p$, marginal cost is constant and equal to 50, and fixed cost is 150 (the same for all firms). Suppose that if two firms merge, then the resulting firm will have the same fixed cost and the same marginal cost as any of the merging firms.

(a) *Show that profit per firm is 961, 475, 250 when the number of firms is, respectively, 2, 3, 4.*

(b) *Suppose that initially there are four firms. Show that a merger between Firms 1 and 2 is unprofitable.*

(c) *Suppose that Firms 3 and 4 decide to merge, forming Firm 3&4. Show that now a merger between Firms 1 and 2 is profitable.*

Problem №5 Consider the following edited excerpt from an article published in the The Economist on May 30, 2019:

> On May 27th FCA [Fiat Chrysler Automobiles], an Italian-American firm, said it was seeking a merger with its French counterpart Renault. If a deal goes ahead, it will create an automotive colossus... The grand total of 15m cars [jointly sold by the two firms] would leave everyone in the dust.
>
> Sergio Marchionne, FCA's charismatic boss who died last year, had called for consolidation of the mass market, where slender profits are partly the result of duplicated investment in similar technologies, such as engines, that do little to differentiate brands.
>
> On paper, FCA and Renault look like perfect partners. They plug gaps in each other's businesses both geographically and in terms of products. FCA's strength and profits come from America; Renault's from Europe. The French firm's cheap

[†]Again, thousands of dollars a day, but don't use thousands

> models and EV [electric vehicles] know-how complement FCA's pickups and up-market brands such as Alfa Romeo and Maserati.
>
> If size at the top of the industry moves from 10m to 15m cars a year, will others seek to follow? Ford and VW are in a partnership that could grow closer. PSA, which makes Peugeots and Citroëns, is open to offers.

In the following questions you are expected to uses models and concepts developed in class. The goal is to apply those models to better understand the forces at work that are described in the article.

(a) *Why did Mr. Marchionne worry about firms' investments in technologies that do little to differentiate brands?*

(b) *How would you rationalize the geographic differences in levels of profits and (arguably) market shares among FCA and Renault in the Cournot model?*

(c) *What would the model in (b) predict about the allocation of output across markets if the merger goes through—would FCA start selling more or less in Europe?*

(d) *Would the merged firm keep the output at 15m cars a year? Would the competitors be willing to increase or decrease their production?*

(e) *In the last paragraph, why could the competitors be inclined to merge if the merger described in the article goes through?*

(f) *Based on the facts provided in the article, why could an antitrust authority be worried about this merger? How could the merging firms defend their case in the eyes of the regulator?*