Incentive Provision and Revenue-Based Executive Compensation in Oligopolistic Markets

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Abstract

Executive compensation contracts often reward managers based on performance measures other than profits, with sales revenue being a prominent example. I study the cost of incentive provision under contracts based on profits and revenue, and find that more revenue-oriented contracts are more expensive. Such contracts could be a profit-enhancing tool for shareholders in oligopolistic markets, a phenomenon known as strategic delegation. I analyze the interplay between strategic delegation and incentive provision. Under price competition the two reinforce each other, whereas under quantity competition they clash. Results depend nontrivially on whether the manager cuts costs or increases demand.

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1 Introduction

The design of executive compensation contracts is a central issue in corporate finance. One of the most prevalent ways to think about executive compensation is to treat it as a solution of a principal-agent problem between the shareholders (the principal) and the manager (the agent) (Bebchuk and Fried (2003)). An appropriate compensation scheme induces the manager, whose input and knowledge are unobserved, to work and create value for the shareholders. As the value of the company is a discounted sum of future profits, the compensation of the CEO should be tied to the profit of the company—this reasoning is natural, and is adopted both in theoretical studies of principal-agent problems and in real world implementations of compensation contracts. Hall and Liebman (1998) document that between 1980 and 1994 compensation based on stock options, which rewards the manager if the value of the company appreciates, became increasingly more prevalent, and that it was the biggest driver of CEO compensation over that period. Among 171 firms sampled in Murphy (1999) 91% used at least one measure of accounting profit to determine the size of managerial bonuses.

However, managers are not compensated based on profit alone. Murphy (1999) also finds that 75 out of 125 industrial companies use more than one performance measure, 21 of which use revenue. Twenty five companies compensated their CEOs based on their "individual performance", meaning that the executive received compensation if she was able to reach a certain pre-specified goal. More recently, Bennett et al. (2017) finds that 33% of CEO grants are contingent upon reaching a sales goal. Under such contracts managers are not incentivized to exactly maximize firm's profit (or value), even if they are compensated with stocks for meeting the performance goals. Huang et al. (2015) documents that in the period from 1993 to 2007 the number of firms that use revenue as a performance measure in CEO compensation has more than doubled, with 41.06% of firms resorting to it in 2006. This motivates the question: what happens to the cost of incentive provision when the manager is not compensated purely based on profit?

Specifically, consider a principal-agent model in which the profit of the firm depends on the unobserved binary effort of the manager. Suppose both parties are risk-neutral, the manager is subject to a limited liability constraint, and the shareholders offer a linear contract that consists of a fixed wage and some share of the profit—a bonus. This is a standard problem discussed, for example, in Laffont and Martimort (2009). The solution is to pay the manager a zero wage and give her the smallest possible share of the profit that is consistent with incentive compatibility. Suppose now that, instead of getting a fraction of the profit, the manager receives a fraction of a different objective—weighted average of profits and sales. Both the share of this objective needed to ensure effort and the size of the objective depend on the weights, and together they determine how costly it is to implement high effort. This mapping from the weights to the cost of incentive provision is the main object of interest in this paper. Given the central role incentive provision plays in the study of executive compensation, and all the deviations from profit maximization documented above, this question is potentially of a first-order importance.

Economic theory puts restrictions on the way profit and revenue, or, more fundamentally, revenue and costs, are jointly determined. I study a model of market competition in which revenue and costs are endogenous objects. By doing so I potentially sacrifice the generality of the model from the perspective of agency theory in favor of a more focused study of CEO compensations. Importantly, it has been known since at least the seminal papers of Fershtman (1985), Sklivas (1987), and Fershtman and Judd (1987) that in oligopolistic markets shareholders can benefit from tying the compensation of their manager to sales revenue, a phenomenon knows as strategic delegation. Such a manager has incentives to produce more and act, essentially, as a Stackelberg leader, which forces the competitor to produce less—an outcome that benefits the shareholders. My analysis of the costs of incentive provision under revenue-based* contracts is agnostic about the reasons to tie the compensation to revenue. A model of oligopolistic competition provides a reason, and as such, it's a convenient setting to study the trade-offs that revenue-based compensation involves. Last, but not least,

*By revenue-based I mean contracts that pay based on revenue in addition to profit.

an explicit competition model highlights important differences between different goals of managerial effort: I show that investment in cost cutting is fundamentally different from investment aimed at increasing demand.

Another way to look at this paper is that I introduce agency problem to the model of strategic delegation of Fershtman and Judd (1987) (FJ hereafter). To the best of my knowledge, it is the first study of the interplay between strategic delegation and incentive provision, an important step towards a more realistic and testable theory of strategic use of executive compensation contracts. Bloomfield (2018) provides the first empirical test of whether firms use CEO contracts as a competition tool. One of the challenges in that paper is to rule out the possibility that higher weight on revenue does not arise from some underlying agency problem, which requires a theory of the sort that I develop in this paper.

I find that, in general, tying managerial compensation to revenue makes it more expensive to incentivize the manager, i.e. the cost of incentive provision is increasing in the weight placed on sales. The relationship could be non-monotonic when there is strong complementarity in profit between the levels of marginal cost and demand. It's especially expensive to incentivize a manager with a revenue-based contract to cut costs, as opposed to boosting demand. Intuitively, the manager with a high weight on revenue cares less about costs, and therefore she needs to be payed a lot to exert effort to cut costs. I also find that under quantity competition the shareholders and the manager would always agree on whether the effort should be directed to cost cutting or demand boosting.

FJ show that under quantity competition the strategic delegation motive pushes the shareholders to offer revenue-based contracts. My result above implies that the incentive provision and strategic delegation clash in that case. The trade-off could be so severe that, in fact, the shareholders would offer a contract that penalizes the manager for revenue (relative to profit), a direct reversal of the result in FJ. On the other hand, under price competition strategic delegation and incentive provision reinforce each other, both pushing the contract to penalize the manager for revenue.

In the special case when there is no market competition, my model collapses to a principal-agent problem with limited liability on the manager's side and contracts that are linear in profit and revenue. Hölmstrom (1979) shows that contracting on an additional signal that is informative about effort is beneficial for the principal. In line with this result I find that the contract will place a non-zero weight on revenue, necessarily leading the manager away from profit maximization. Despite the fact that Holmström's "informativeness principle" has been known for almost 40 years, the natural application to contracting on revenue and profits seems to have gone unstudied, as well as the resulting deviation from profit maximization. Baker (1992) considers a setting in which the principal cannot contract on the value of her objective (profit) and studies linear contracting on related performance measures (revenue). That paper emphasizes the importance of the statistical relationship between the objective and the performance measure. As I employ a model of market competition, in my paper this relationship is explicitly a function of economic primitives.

By bringing together a model of market competition and a model of executive compensation I also contribute to a diverse literature that studies the market consequences of incentive contracts. One question that the literature has asked is whether competition reduces managerial slack (Hart (1983), Schmidt (1997), Raith (2003)). The relationship of interest is the pay-to-performance sensitivity of the managerial contract as a function of the strength of competition. A different strand of literature studies the effect of common ownership of competing firms on competition and incentives of the managers. Anton, Florian, Gine, and Schmalz (2016) shows that common ownership creates incentives for the shareholders to curb managerial effort in order to soften competition. In these two literatures, just as in my paper, the shareholders anticipate the impact that the incentive contracts they offer will have on the effort choice of their manager and, as a result, on the outcome of the competition. However, unlike my paper, managerial compensation is restricted to be based on profit, so revenue-based compensation and strategic delegation are both absent. Consequently, in my model the contract has an impact on how the manager competes *after* the realization of the random state of the firm she is trying to improve. As a comparative statics exercise I study the impact of increased competition on the cost of incentive provision, which connects my paper with the first literature. My model could be extended in an obvious way to study strategic delegation under common ownership, but it is outside the scope of the present study.

The rest of this paper is organized as follows. Section 2 outlines the benchmark model. Section 3 considers three deviations from the benchmark model: price competition, endogenous choice of project the manager works on, and general equilibrium in contracts. Section 4 concludes. Appendix B contains the proofs.

2 Benchmark Model

2.1 Setup

Two firms, $i = 1, 2$, sell two imperfectly substitutable goods to a mass 1 of identical consumers. A representative consumer has preferences defined over 3-tuples (q_1, q_2, z) given by

$$U(q_1, q_2, z) = d_1 q_1 - \frac{1}{2} q_1^2 + d_2 q_2 - \frac{1}{2} q_2^2 - \gamma q_1 q_2 + z,$$

where q_i is consumption of good $i \in \{1, 2\}$, z is money d_i is a taste parameter for good i that will be a demand shifter for that good, γ measures the degree of substitutability of the two goods—positive values indicate that the goods are substitutes, and negative values indicate that the goods are complements. For the consumer's problem to be well-behaved we need to assume that $\gamma \in (-1, 1)$ and that $d_i > \gamma d_{-i}$ for $i = 1, 2$. For the purposes of my analysis I also assume that $\gamma \geq 0$; the case of $\gamma = 0$ means that firm i is a monopolist in its market. Assuming additionally that income is high enough, we can derive the indirect demand system for goods $i = 1, 2$:

$$P_i(q_i, q_{-i}; d_i) = d_i - q_i - \gamma q_{-i}, \tag{1}$$

These preferences are discussed in greater detail, for example, in Singh and Vives (1984).

A firm is represented by a principal-agent pair. The principal of firm i, whom I will oftentimes refer to as the shareholders, owns the profit of the firm. Each firm is run by an agent (manager), who is compensated according to the contract offered by the shareholders. After being hired, the manager can exert effort to probabilistically boost her firm's state. The state consists of the level of the demand shifter d_i and the level of the constant marginal costs c_i. I assume that each firm starts in the status quo "low" state $\underline{\omega}_i = \{\underline{d}_i, \bar{c}_i\}$, which then could be improved either to $\bar{\omega}_i^d = \{\bar{d}_i, \bar{c}_i\}$ or $\bar{\omega}_i^c = \{\underline{d}_i, \underline{c}_i\}$, with $\bar{d}_i > \underline{d}_i$ and $\bar{c}_i > \underline{c}_i$. In the benchmark model I assume that the manager doesn't have a choice of which project among these two to pursue, but I relax this assumption in section 3.2. After the states are realized the managers observe each other's contracts and realizations of the states, and compete in the market by setting quantities. The case of price competition is analyzed in section 3.1. In what follows, I will be studying the behavior of firm 1 holding the behavior of firm 2 fixed, so for expositional purposes I assume that manager of firm 2 exerts no effort, her contract is fixed, and her only role is to compete in the goods's market in the final stage of the game. I study full equilibrium in contracts in section 3.3. I also omit the i indices when possible, writing all formulas for firm 1.

I assume that at the final stage of the game the shareholders observe their firm's revenue and total costs and that they can contract on these values. Effort of the manager is not contractible, which gives rise to a moral hazard problem. These assumptions reflect the idea that the shareholders meet infrequently and learn about the performance of the firm from aggregate statistics. I restrict attention to linear contracts. This allows me to have a parameter that explicitly quantifies the weight put on revenue. The dependence of cost of incentive provision on this weight, as well as the choice of the weight by the shareholders, are the main objects of interest. It also allows me to contrast my results with the ones obtained in FJ. The compensation schemes I consider take the form $W = \beta_1 TR - \beta_2 TC + w$, where TR and TC are, respectively, firm's total revenue and total costs, w is a fixed salary, and betas are scalar weights that constitute the contract[†]. It is safe to assume that $\beta_1 \neq 0$, which allows me to decompose the contract as in FJ:

$$W = \kappa(TR - (1-\alpha)TC) + w, \tag{2}$$

where $\kappa = \beta_1$ and $(1-\alpha) = \beta_2/\beta_1$. I will refer to the equilibrium value of the term $O \equiv TR - (1-\alpha)TC = \alpha TR + (1-\alpha)\pi = \pi + \alpha TC$ as the *objective* of the manager. It is the value that the manager maximizes during the competition stage, once effort is exerted and cost of effort is sunk. κ is the fraction of the objective that the manager actually receives as a bonus; the level of κ matters when the manager decides whether to exert effort or not. Notice that, when $\alpha = 0$, the manager is compensated based on the profit of her firm. When $\alpha > 0$, she is excessively rewarded for revenue as opposed to profit, or, equivalently, she doesn't internalize some fraction of costs. Contracts of this form are my model of the real world contracts that emphasize revenue in addition to profit. I will refer to a manager with a positive level of α as *aggressive*, because such a manager will have extra incentives to produce more. When $\alpha < 0$ the manager overinternalizes the costs, or puts a negative weight on revenue outside of profit, which leads her to produce less. I will refer to such manager as *conservative*.

Both the firm and the manager are risk-neutral. However, the manager is subject to the limited liability constraint $w \geq 0$. Without this assumption incentivizing effort would be costless for the firm.

I solve the model using backward induction. First, we need to understand what happens in the competition stage, once the contract is offered and effort is exerted. Anticipating the value of her objective in the competition stage, the manager will decide whether to exert effort or not—that's step 2. Finally, the shareholders are able to foresee all that, and they offer the contract that maximizes their payoff.

[†]A contract of the form $W = \tilde{\beta}_1 TR + \tilde{\beta}_2 \pi + w$ is easily decomposed into this form

2.1.1 Competition Stage

In the last stage of the game the state $\omega = (d, c)$ has already realized, and both managers seek to maximize their objectives by choosing quantity. Holding the choice of production by the competitor fixed at the level q_2, the manager is solving

$$\max_{q \geq 0} \quad \kappa(P(q, q_2; d) - (1-\alpha)c)q + w \tag{3}$$

Even though the manager is not maximizing the firm's profit, we can still treat her as a profit maximizer, but with marginal cost $\tilde{c} = (1-\alpha)c$ instead of c. I will refer to $\tilde{c}$ as *perceived* marginal cost. Viewed from this angle the competition subgame is simply a standard Cournot competition game with differentiated goods. It is readily checked that equilibrium production is given by

$$q(\alpha, \omega) = \frac{2(d-\tilde{c}) - \gamma(d_2 - \tilde{c}_2)}{4-\gamma^2}, \tag{4}$$

assuming that both firms produce positive amounts. This assumption is not without a caveat, because the shareholders are in charge of the perception of their marginal cost. If they drive it low enough, it could very well be the case that the competitor will not produce. However, I abstract away from concerns of this sort and assume that both firms produce positive quantities. In line with my definition of aggression and conservatism we see that higher levels of α imply lower perceived marginal cost, and therefore lead to higher production in equilibrium. Equilibrium price is given by $p(\alpha, \omega) = \frac{1}{4-\gamma^2}(2d + (2-\gamma^2)\tilde{c} - \gamma(d_2 - \tilde{c}_2))$, and the equilibrium value of the objective of the manager is

$$O(\alpha, \omega) = \left(\frac{2(d-\tilde{c}) - \gamma(d_2 - \tilde{c}_2)}{4-\gamma^2}\right)^2 \tag{5}$$

This value is simply the profit of the Cournot competitor with marginal cost $\tilde{c}$. The shareholders of the firm, however, are interested in the actual profit of the firm, the one that takes into account the real marginal cost. Recall from (2) that $O = \pi + \alpha TC$, and therefore shareholders' profit from competition is defined as $\pi(\alpha, \omega) := O(\alpha, \omega) - \alpha c q(\alpha, \omega)$ and takes the form

$$\pi(\alpha, \omega) = \frac{[2(d - \tilde{c}) - \gamma(d_2 - \tilde{c}_2)][2(d - c) - \alpha(2 - \gamma^2)c - \gamma(d_2 - \tilde{c}_2)]}{(4 - \gamma^2)^2} \tag{6}$$

Here, unlike in the objective of the manager, the contract α appears independently of the perceived marginal cost $\tilde{c}$. This function is a non-standard object, so it's helpful to establish several properties.

Lemma 2.1. *The functions $\pi(\alpha, \omega)$ and $\alpha^*(\omega) := \operatorname*{argmax}_\alpha \pi(\alpha, \omega)$ satisfy the following properties:*

1. *$\pi(\alpha, \omega)$ is concave in α for any ω.*
2. *$\alpha^*(\omega) > 0$ for any ω, unless $\gamma = 0$.*
3. *$\alpha^*(\bar{\omega}^m) > \alpha^*(\underline{\omega})$ for $m \in \{d, c\}$.*

[Figure 1 goes around here]

The fact that firm's profit from competition is maximized when $\alpha > 0$ is one of the main results and insights in Fershtman and Judd (1987). When firms compete in quantities it pays off to make your manager more aggressive, because, when facing a manager who is committed to overproduction, the competitor's manager optimally decreases her production. This story is very similar in spirit to Stackelberg leadership. A Stackelberg leader has an ability to commit to her output, and she chooses to produce more than a Cournot competitor, while the follower gives in and produces less. More specifically, a Stackelberg leader chooses the profit maximizing point on her competitor's best-response curve. By altering the incentives of the manager through the contract the shareholders do precisely that. They shift their manager's best-response curve and are able to pick any point on the competitor's best-response curve; clearly, they end up inducing the Stackelberg outcome. When $\gamma = 0$, the firm is a monopolist in the market, so there are no strategic concerns—optimal α is zero.

The last finding of the lemma says that the owners are more willing to distort costs when the state of the firm is high. Intuitively, a Stackelberg leader in quantity increases quantity by more than the Cournot competitor does when costs go down or demand goes up, as is readily checked from the corresponding expressions:

$$q^{Stackelberg} = \frac{2(d-c) - \gamma(d_2 - \tilde{c}_2)}{4 - 2\gamma^2} \text{ vs. } q(\alpha, \omega) = \frac{2(d-\tilde{c}) - \gamma(d_2 - \tilde{c}_2)}{4 - \gamma^2}$$

The shareholders are able to induce the Stackelberg outcome only by influencing the Cournot equilibrium. Therefore, when costs go down or demand goes up, α should increase to compensate for the gap.

2.1.2 Choice of Effort

Realizing that the level of the state will affect the value of her objective in the competition stage, the manager considers exerting effort to maximize the expected value of her compensation. I assume that effort is binary, so that it's either exerted at a cost $\psi > 0$ and delivers high state with probability $r > 0$, or the manager shirks at no cost and the state remains low. The tractability that this assumption brings is two-fold. First, it makes algebra easier. Second, perhaps more importantly, it allows for a meaningful comparison of different models. With a continuous effort, in general, the shareholders would like to implement marginally different levels of effort when the manager's project is to increase demand as opposed to cutting costs, or when competition is in prices rather than quantities. By shutting this channel down I am able to compare costs of inducing effort across all these variations of the model.

Given the contract (α, κ, w) the manager wants to exert effort when

$$\kappa \mathbb{E}\left[O(\alpha, \omega^m)\right] + w - \psi \geq \kappa O(\alpha, \underline{\omega}) + w. \tag{7}$$

ω^m is a random variable that takes two values, $\underline{\omega}$ and $\bar{\omega}^m$; the superscript $m \in \{d, c\}$ is a reminder that the object is project-specific. Rewrite $\mathbb{E}\left[O(\alpha, \omega^m)\right] = O(\alpha, \underline{\omega}) + r[O(\alpha, \bar{\omega}^m) -$

$O(\alpha, \underline{\omega})]$ and define $\Delta O^m(\alpha) := O(\alpha, \bar{\omega}^m) - O(\alpha, \underline{\omega})$. Then (7) simplifies into

$$r\kappa \Delta O^m(\alpha) \geq \psi \tag{8}$$

This simply says that the manager is willing to exert effort when the expected personal gain from doing so exceeds the cost. This relationship will be examined in greater detail when I introduce the shareholders' problem.

2.1.3 Choice of Contract

The shareholders of the firm choose a contract that maximizes the expected value of their profit, subject to incentive compatibility, limited liability, and individual rationality constraints on the side of their manager.

The manager's outside option is assumed to be zero. It's readily checked that to incentivize low effort no payment needs to be made to the manager, so in this case the shareholders simply choose $\alpha = \alpha^*(\underline{\omega})$, the maximizer of $\pi(\alpha, \underline{\omega})$. This choice of α is motivated purely by the strategic delegation motive.

Now suppose that the shareholders would like to implement high effort. Their problem then takes the following form:

$$\begin{aligned} \max_{\alpha,\kappa,w} \quad & \mathbb{E}\left[\pi(\alpha, \omega^m)\right] - \kappa \mathbb{E}\left[O(\alpha, \omega^m)\right] - w \\ \text{s.t.} \quad & \kappa \Delta O^m(\alpha) \geq \psi / r \qquad && \text{(IC)} \\ & \kappa \mathbb{E}\left[O(\alpha, \omega^m)\right] + w - \psi \geq 0 \qquad && \text{(IR)} \\ & w \geq 0 \qquad && \text{(LL)} \end{aligned} \tag{9}$$

For a fixed level of α this problem is a standard incentive provision problem with limited liability. When $\gamma = 0$, the agent is a monopolist and there is no strategic delegation motive. What is left is a moral hazard problem, where the principal can contract linearly on revenue and cost. Even though in this case $\pi(\alpha, \omega)$ is always maximized at $\alpha = 0$, I will show later that the optimal choice of α is not zero, as the level of α also has implications for the cost of incentive provision. When $\gamma > 0$, there is some interaction between strategic delegation and incentive provision which I will investigate in greater detail.

Recall that before simplification the IC constraint reads as

$$\kappa\mathbb{E}\left[O(\alpha,\omega^m)\right] + w - \psi \geq \kappa O(\alpha,\underline{\omega}) + w.$$

Under limited liability $\kappa O(\alpha,\underline{\omega}) + w \geq 0$, as the equilibrium value of the objective $O(\alpha,\underline{\omega})$ can't be negative (the manager can always produce nothing and secure a payoff of zero). Therefore, the IR constraint is satisfied as long as the IC is. This also implies that $w^* = 0$. As $\Delta O^m(\alpha) > 0$, we can solve for κ and plug it back to the objective function. The final[‡] version of the problem that the shareholders solve is

$$\max_{\alpha} \ \mathbb{E}\left[\pi(\alpha,\omega^m)\right] - \frac{\psi}{r}\frac{O(\alpha,\underline{\omega})}{\Delta O^m(\alpha)} - \psi \tag{10}$$

The program of the shareholders has two terms in it. Expected profit from competition, $\mathbb{E}\left[\pi(\alpha,\omega^m)\right]$, is the "benefit term". By varying the degree of aggressiveness of the manager the shareholders could benefit from strategic delegation of the firm's operations to the manager. The term $\lambda^m(\alpha) := \psi O(\alpha,\underline{\omega})/r\Delta O^m(\alpha) - \psi$, on the other hand, is the "cost term". Depending on how aggressive the shareholders choose their manager to be, the size of the compensation that they need to provide will vary. Understanding this cost term—the cost of incentive provision—and the trade-off between incentive provision and strategic delegation (the benefit term) is the main focus of this paper.

The presentation of the shareholders' problem is completed with the condition that needs to be satisfied for the shareholders to be willing to implement high effort:

$$\max_{\alpha}\left\{\mathbb{E}\left[\pi(\alpha,\omega^m)\right] - \frac{\psi}{r}\frac{O(\alpha,\underline{\omega})}{\Delta O^m(\alpha)} - \psi\right\} \geq \pi(\alpha^*(\underline{\omega}),\underline{\omega}) \tag{11}$$

[‡]For expositional purposes I abstract from the fact that if $\alpha > 1$ the manager can produce an infinite amount and get an infinite return. One could assume that the manager is compensated based on $TC(q_{sold})$, so that producing more than the quantity demanded at zero prices is not profitable. In the cost cutting case IC doesn't hold for $\alpha > 1$, so these values are not relevant.

2.2 Cost of Incentive Provision

In this subsection I analyze the cost of incentivizing the manager to exert effort $\lambda^m(\alpha)$ when her compensation is tied to revenue. I derive properties of this cost, provide comparative statics results, show how my analysis extends to non-linear demand specifications, and highlight the differences between the projects the manager could be pursuing: increasing demand and cutting costs.

Understanding the cost of incentive provision separate from the problem of the shareholders is valuable because the benefit term in (10) doesn't have to be pure profit from competition. In real world the shareholders might not internalize the effect of strategic delegation, or might have concerns other than pure profit maximization, such as tax incentives or profit of other firms in the industry. As long as the manager is rational and maximizes her objective, the shareholders would have to confront this cost.

2.2.1 Properties of $\lambda^m(\alpha)$

The cost of incentivizing the agent to exert high effort directed at project $m \in \{d, c\}$ is given by

$$\lambda^m(\alpha) = \frac{\psi}{r} \frac{O(\alpha, \underline{\omega})}{\Delta O^m(\alpha)} + \psi \tag{12}$$

Both $O(\alpha, \underline{\omega})$ and $\Delta O^m(\alpha)$ are positive, so it's evident that the manager is compensated above her costs of exerting effort ψ. The source of rent is the limited liability constraint that prevents the principal from charging the value of the position upfront from the manager. This rent amounts to a fraction $\kappa^m(\alpha) = \psi(r\Delta O^m(\alpha))^{-1}$ of the objective in the low state $O(\alpha, \underline{\omega})$. The fraction $\kappa^m(\alpha)$ of the objective that the manager pockets is determined by the IC constraint of the manager. With the help of lemma 2.2 proposition 2.1 establishes the properties of the cost function λ^m.

Lemma 2.2. *$\Delta O^d(\alpha)$ is increasing in α. $\Delta O^c(\alpha)$ has an inverse U-shape and is decreasing in α for $\alpha \geq \hat{\alpha}$ if and only if $TC(\hat{\alpha}, \underline{\omega}) \geq TC(\hat{\alpha}, \bar{\omega}^c)$.*

The first part of lemma 2.2 says that, as perceived marginal cost goes down, the gain from increasing demand goes up, because having high demand is especially beneficial when marginal cost is low. One would expect that, when perceived marginal cost goes down, the gain from cutting marginal cost should go down as well, but the second part of lemma 2.2 shows that this is not entirely true. The reason is that, if perceived marginal cost is very high, the firm is not producing much, so the difference in profit (before and after cutting costs) could be less than it is when the perceived cost is lower. A necessary and sufficient condition for $\Delta O^c(\alpha)$ to be decreasing after a certain value $\hat{\alpha}$ is that equilibrium total cost must go down when marginal cost goes down from $(1-\hat{\alpha})\bar{c}$ to $(1-\hat{\alpha})\underline{c}$. I will focus on the case when $\Delta O^c(\alpha)$ is decreasing both because the logic for that is more appealing and because it makes for an interesting contrast to the increasing $\Delta O^d(\alpha)$ function.

Proposition 2.1. *$\lambda^m(\alpha)$ is increasing in α and is convex for any $m \in \{d, c\}$.*

Increasing $\lambda^m(\alpha)$ means that it is more expensive for the shareholders to incentivize high effort when they want their manager to be aggressive. The most straightforward way to see the intuition is by examining the case of cost cutting when $\Delta O^c(\alpha)$ is a decreasing function. Suppose that the shareholders would like to make their manager more aggressive. This means that the manager perceives her marginal costs to be low, and, hence, in the competition stage she is maximizing the objective $TR - (1-\alpha)TC$, which, for any level of output, is mechanically higher when aggressiveness α is high. The shareholders have to pay the manager a fraction of this objective as a compensation. However, if they want the manager to exert high effort, they can't pay her a small share of this objective, because the IC constraint requires $\kappa^c(\alpha) = \psi(r\Delta O^c(\alpha))^{-1}$, which is large when $O^c(\alpha)$ is small, as would be the case under the assumption that $O^c(\alpha)$ is decreasing. The reason why the share paid to the manager should be high is that, when costs are deflated, the manager doesn't have strong incentives to cut the costs, and therefore a higher share of the objective must be promised to her. As a result, making the manager more aggressive leads to the shareholders paying her a higher share of a higher objective. Hence, the cost of creating incentives is high.

Now consider an increasing $\Delta O^m(\alpha)$ function, as is always the case when the manager's effort boosts demand. The lemma shows that the share paid to an aggressive manager doesn't become low enough to counteract the fact that the manager is compensated based on a bigger objective, and as a result the cost of incentive provision is still increasing in aggressiveness. When the manager perceives her marginal cost as low, she has strong incentives to increase demand to take advantage of the low costs, and that is why the fraction of the compensation paid to her goes down. It seems intuitive that the stronger is the complementarity between low marginal cost and high demand, the stronger would the incentive to exert effort be for a fixed level of α. The following proposition formalizes this result for an arbitrary demand system.

Proposition 2.2. *Let $x \in \{-c, d\}$ denote the component of the state that the manager could increase. $\Delta O^m(\alpha)$ is increasing in α if and only if $O(\alpha, x)$ is supermodular, and is decreasing iff $O(\alpha, x)$ is submodular. Moreover, $\lambda^m(\alpha)$ is increasing if and only if $O(\alpha, x)$ is log-submodular, and is decreasing iff $O(\alpha, x)$ is log-supermodular.*

This proposition says that, if there is complementarity between low marginal cost and high level of the state, then a more aggressive manager will have stronger incentives to exert effort to increase that state. If the complementarity is very strong ($O(\alpha, x)$ is not just supermodular, but also so is its log), then these incentives would also be very strong, and the shareholders would need to pay little to incentivize the manager ($\lambda(\alpha)$ is decreasing in α). However, if the complementarity between the two is moderate, incentivizing a more aggressive manager to exert effort would be more expensive. This is the case under linear demand. It's important to note that $\lambda^m(\alpha)$ can't be decreasing everywhere in a reasonable range of α, because it is close to zero when α is such that $O(\alpha, \underline{\omega})$ is close to zero, so it must be increasing at least somewhere. This analysis gives me confidence that the results obtained further continue to hold outside of the linear demand model presented in this paper.

2.2.2 Demand vs. Costs

The analysis above suggests that the nature of the project that the manager is pursuing has consequences for the cost of incentive provision. Even though right now the project that the manager can pursue is exogenous, the juxtaposition of cutting costs and boosting demand is still instructive. The following lemma provides a clean comparison of the costs across the projects.

Lemma 2.3. *Assume that $\Delta O^c(0) = \Delta O^d(0)$ and that $\Delta O^c(\alpha)$ is decreasing for $\alpha \geq 0$. Then, $\lambda^d(\alpha) < \lambda^c(\alpha)$ for $\alpha > 0$, and $\lambda^d(\alpha) > \lambda^c(\alpha)$ for $\alpha < 0$.*

[Figure 2 goes around here]

The numerator in $O(\alpha, \underline{\omega})/\Delta O^m(\alpha)$ is the same across the projects, so the comparison depends purely on the denominators. Recall that $O(0, \omega)$ is simply equilibrium profit of a Cournot competitor with marginal cost c, so the assumption $\Delta O^c(0) = \Delta O^d(0)$ means that, were the firm run by its owner rather than the manager, he would be indifferent between increasing demand or cutting cost—the two projects have a commensurate effect on profit. However, once the management is delegated to an aggressive manager, the choice of the project starts to have consequences. A more aggressive manager perceives costs to be lower, and therefore it's costly to incentivize high effort if the goal is to cut costs. On the other hand, a conservative manager with inflated costs has weak incentives to boost demand, as she can't take full advantage of it with high marginal cost. Cutting costs is more attractive for such manager.

2.2.3 Comparative Statics

In this section I analyze how the cost of incentive provision λ^m responds to changes in other variables in the model. In particular, I analyze the effect of the degree of product differentiation γ, cost of effort ψ, probability of success r, sizes of demand improvement $\bar{d}$ and lowered cost $\underline{c}$, as well as cost and demand shifter of the competitor $\tilde{c}_2$ and d_2.

Recall that the cost is given by $\lambda^m(\alpha) = \psi O(\alpha, \underline{\omega})/r\Delta O^m(\alpha) + \psi$. An increase in ψ has the most straightforward effect on the cost of incentive provision: the manager gets fully reimbursed for the cost of effort ψ, so the curve shifts up for any value of α. Moreover, higher ψ increases the slope of the cost curve, as the shareholders need to offer a higher fraction κ to incentivize high effort. Similarly, a decrease in the probability of success r increases the slope, as manager needs to be offered a higher κ.

Suppose that the manager's project is to increase demand ($m = d$). $\bar{d}$ only enters the ΔO term, so higher level of $\bar{d}$ decreases the cost of effort, as the manager has stronger incentives to exert effort for the same contract when the payoff from doing so goes up. Divide the numerator and the denominator by $O(\alpha, \underline{d})$ to see that higher $\underline{d}$ increases the cost of incentive provision, as the manager has less incentives to exert effort when the low state demand increases. In the same way I find that, when the project is to cut costs, lower $\underline{c}$ and higher $\bar{c}$ drive $\lambda^c(\alpha)$ down. Similarly unsurprising is that, under both projects, the cost of incentive provision is higher when the state of the competitor is higher (see Appendix B). This implies that higher α_2 set by the competitor drives up the cost of providing incentives for firm 1.

An interesting question is whether smaller degree of product differentiation (higher γ) decreases the cost of incentive provision. The relationship between the degree of market competition and managerial effort is known to be ambiguous (Schmidt (1997), Raith (2003)). On the one hand, higher competition makes it less attractive to fall behind the competitor, which creates incentives to exert effort, but on the other hand higher competition depresses profits in the market, which disincentives the manager to exert effort. In this model I find that the latter effect is not strong enough, as under quantity competition the extreme case of perfect substitutes doesn't fully eliminate the profits of the competing firms. Hence, higher γ decreases the cost of incentive provision for any α.

2.3 Choice of Contract

In this section the problem of the shareholders (10) is analyzed. I compare the choice of contract in my model with the contracts in Fershtman and Judd (1987) and in the standard principal-agent framework, which are the natural benchmarks that the solution of my model should be contrasted against.

2.3.1 Shareholders' Problem

FJ analyze a pure delegation problem in the absence of moral hazard, and, among other things, are interested in

$$\alpha_{FJ} := \operatorname*{argmax}_{\alpha} \mathbb{E}\left[\pi(\alpha, \omega^m)\right] \tag{13}$$

One of their findings is that, under quantity competition, $\alpha_{FJ} > 0$.

Standard principal-agent problem with limited liability analyzes problem (10) for α fixed at zero, so the benchmark level in this case is $\alpha_{PA} := 0$. The corresponding value of the sensitivity of the manager's compensation to her objective is then $\kappa^m_{PA} = \kappa^m(0) = \psi(r\Delta O^m(0))^{-1}$. The relation between $\kappa(\alpha)^m$ and κ^m_{PA} depends on the sign of α and the choice of project m in a straightforward way, so characterization of α is sufficient to obtain such comparisons.

Proposition 2.3. *Solution α^* of the shareholders' problem (10) is smaller than α_{FJ}.*

Proof. Problem (10) is a concave problem, and the derivative of the objective function at α_{FJ} is $\mathbb{E}\left[\frac{\partial}{\partial\alpha}\pi(\alpha_{FJ}, \omega^m)\right] - \frac{\partial}{\partial\alpha}\lambda^m(\alpha_{FJ}) = -\frac{\partial}{\partial\alpha}\lambda^m(\alpha_{FJ}) < 0$. □

[Figure 4 goes around here]

Proposition (2.3) shows that the goal of providing incentives and exploiting strategic delegation contradict each other when firms compete in quantity. Strategic delegation is calling for the manager to be aggressive, so that she sells more and acts like a Stackelberg leader. However, it's expensive to incentivize high effort when the manager is aggressive, and therefore incentive provision requires the manager to be more conservative. Which force will dominate depends on the parameters, but the manager definitely won't be as aggressive as she would be in FJ. In fact, section 3.3 provides an example in which the solution α^* is negative, which is never possible in FJ. This also implies that no sharp comparison between my model and the standard principal-agent model could be made, as α^* could have any sign.

An interesting corollary of proposition (2.3) is obtained for the special case of my model when $\gamma = 0$. In that case $\alpha_{FJ} = 0$, as the strategic delegation motive is absent when the agent is a monopolist, and profit from competition is maximized when the manager's objective is to maximize profit. The corollary then is that the principal will choose $\alpha^* < 0$ and will write a contract that skews the preferences of the manager away from profit maximization. The entire source of deviation from profit maximization in this example is moral hazard—a novel finding, to the best of my knowledge.

3 Extensions

Now that the benchmark model is introduced, we can investigate several deviations from it. Fershtman and Judd (1987) shows that the nature of strategic delegation in oligopolistic markets depends on whether competition is in prices or quantities, therefore it's interesting to consider the case of price competition. Also, in the benchmark model I assume that the choice of project that the manager can pursue is exogenous—in this section I will show that my model predicts a certain degree of unanimity between the shareholders and the manager when it comes to the choice of project. Finally, I show that my results survive when the shareholders of both firms get to offer contracts and incentivize their managers.

3.1 Price Competition

The contracting problem (10) that the shareholders solve remains exactly the same in the price competition case, because the derivations didn't rely on the specific features of $O(\alpha, \omega)$ and $\pi(\alpha, \omega)$. However, the analysis of the problem depends on the properties of these two functions, which we will now derive.

Equation (1) describes the indirect demand system of the representative consumer. Inverting this system yields the following demand curve for firm i:

$$D_i(p_i, p_{-i}; d_i, d_{-i}) = \frac{d_i - \gamma(d_{-i} - p_{-i}) - p_i}{1 - \gamma^2} \tag{14}$$

Similarly to the quantity competition case, in the price competition case the manager of firm i takes the price of her competitor as given and maximizes the profit of her firm, assuming that the marginal cost is $\tilde{c}_i = (1 - \alpha_i)c_i$. Omitting the i index and focusing on firm 1, we get that in the Nash (Bertrand) equilibrium of the competition subgame the firm will be charging the price

$$p(\alpha, \omega) = \frac{(2 - \gamma^2)d + 2\tilde{c} - \gamma(d_2 - \tilde{c}_2)}{4 - \gamma^2}, \tag{15}$$

and the equilibrium production of that firm will be

$$q(\alpha, \omega) = \frac{(2 - \gamma^2)(d - \tilde{c}) - \gamma(d_2 - \tilde{c}_2)}{(4 - \gamma^2)(1 - \gamma^2)} \tag{16}$$

These formulas are similar, but not identical to the ones obtained in the quantity competition case. As before, the key objects are the equilibrium value of the objective of the manager

$$O(\alpha, \omega) = \frac{[(2 - \gamma^2)(d - \tilde{c}) - \gamma(d_2 - \tilde{c}_2)]^2}{(4 - \gamma^2)^2(1 - \gamma^2)} \tag{17}$$

and the shareholders' profit from competition

$$\pi(\alpha, \omega) = \frac{[(2 - \gamma^2)(d - \tilde{c}) - \gamma(d_2 - \tilde{c}_2)][(2 - \gamma^2)(d - c) - \gamma(d_2 - \tilde{c}_2) - 2c\alpha]}{(\gamma^2 - 4)^2(1 - \gamma^2)} \tag{18}$$

Some properties of the profit function are summarized in the following lemma.

Lemma 3.1. *The functions $\pi(\alpha, \omega)$ and $\alpha^*(\omega) := \operatorname*{argmax}_{\alpha} \pi(\alpha, \omega)$ satisfy the following properties:*

1. *$\pi(\alpha, \omega)$ is concave in α for any ω.*

2. $\alpha^*(\omega) < 0$ *for any* ω.

3. $\alpha^*(\bar{\omega}^m) < \alpha^*(\underline{\omega})$ *for* $m \in \{d, c\}$.

The key difference between the profits from competition in the quantity and price competition cases is in property 2. Under quantity competition optimal strategic delegation involves making the manager more aggressive, so that she acts as a Stackelberg leader. In the price competition case, on the other hand, the behavior of the leader is conservative. When a firm gets to set its price first, it pays off to set a high price (roughly corresponding to *low* quantity in the quantity competition), so that the follower also sets a high price and the firm could earn a higher profit. Hence, the shareholders trying to implement the leadership outcome will make their manager more conservative and inflate the perception of marginal cost (by choosing $\alpha < 0$). Property 3 says that, when the state of the firm goes up, it pays off to make the manager more conservative, for the same reason it was profitable to make the manager more aggressive under quantity competition.

Now we turn to the cost of incentive provision $\lambda^m(\alpha) = O(\alpha, \underline{\omega})/\Delta O^m(\alpha)$. It turns out that the properties of this function don't change at all under price competition.

Proposition 3.1. $\Delta O^d(\alpha)$ *is increasing in* α. $\Delta O^c(\alpha)$ *has an inverse U-shape.* $\lambda^m(\alpha)$ *is increasing and convex in* α *for any* $m \in \{c, d\}$.

The intuition and the proof are identical to the quantity competition case. A conservative manager perceives her cost to be high, which makes the value of her baseline objective $O(\alpha, \underline{\omega})$ low. She also has weak incentives to exert effort to increase demand, because it's hard to take advantage of high demand under high cost. To incentivize her the shareholders pay her a high fraction $\kappa(\alpha)$ of her (small) objective as a bonus. However, because the complementarity between the level of marginal cost and the demand shifter is not strong enough under linear demand, this fraction is not high enough, and the shareholders end up paying less to incentivize high effort when the manager is conservative.

Now we are in the position to describe the solution of the shareholders' problem (10) under price competition. Recall that $\alpha_{FJ} := \text{argmax}_\alpha \mathbb{E}\left[\pi(\alpha, \omega^m)\right]$ is the choice of contract when effort is exerted but doesn't have to be incentivized.

Proposition 3.2. *Solution α^* of the shareholders' problem (10) is smaller than α_{FJ}.*

[Figure 4 goes around here]

The proof is obvious, for example, from figure 4. The conceptual difference between price competition and quantity competition is that both the strategic delegation and the incentive provision motives go hand in hand in this case. It pays off to have a conservative manager both because she will be charging higher prices and because it's cheaper to incentivize a conservative manager. Unlike the quantity competition case, where there was a clash of the two goals, there is no reversal of FJ predictions in the price competition case—the contract α will always be negative, just as it is in FJ.

3.2 Choice of Project

Throughout the paper it was assumed that the project that the manager is undertaking—cost reduction or demand boosting—is exogenously determined, and the goal of the shareholders was to simply induce high effort, preferably still benefitting from strategic delegation. We also saw that the choice of project matters for the cost of incentive provision, so it is desirable to endogenize the choice of project. If the preferences of the shareholders and the manager are not aligned, then the shareholders should also consider incentivizing the choice of their preferred project when they sign the contract. In this section I show that this is not necessary, as there will be a consensus among the shareholders and the manager when it comes to the project choice.

I consider the following modification of the original game. The shareholders offer a contract (α, κ). The manager picks a project, and then chooses whether to invest her binary effort in that project. Next, the state of the firm realizes and the manager moves to the competition stage. To make the problem interesting I maintain the assumption that $\Delta O^c(0) = \Delta O^d(0)$, which means that if the manager's objective is to maximize the profit of the firm (as is indicated by $\alpha = 0$), both the shareholders and her are indifferent between the two projects. In the linear model this condition means that if the demand project increases the demand shifter by δ from $\underline{d}$ to $\bar{d} = \underline{d} + \delta$, then the cost cutting project cuts marginal cost by the same amount from $\bar{c}$ to $\underline{c} = \bar{c} - \delta$. I also assume that $\Delta O^c(\alpha)$ is decreasing for positive levels of α (see lemma 2.1).

The choice of project for the manager is straightforward. Her gain from pursuing project m is $\kappa r \Delta O^m(\alpha) - \psi$, and therefore she will choose the one that brings the highest gain. If her contract puts a positive weight on revenue ($\alpha > 0$), then $\Delta O^d(\alpha) > \Delta O^c(\alpha)$, and the manager prefers to increase demand rather than to cut costs. If $\alpha < 0$, then $\Delta O^d(\alpha) < \Delta O^c(\alpha)$ (lemma 2.3), and therefore the manager prefers to cut costs.

Why do the shareholders care about the choice of project? The problem of the shareholders is to maximize $\Pi^m(\alpha) = \mathbb{E}\left[\pi(\alpha, \omega^m)\right] - \lambda^m(\alpha)$. The choice of project affects both the profit from competition and the cost of incentive provision. From 2.3 we know that for positive values of α we have $\lambda^d(\alpha) < \lambda^c(\alpha)$, and for negative ones we have $\lambda^d(\alpha) > \lambda^c(\alpha)$. From the perspective of costs one project is better than the other one depending on the sign of α, exactly in the same manner as it is for the manager.

Comparison of the benefit terms is more subtle. Recall that $\mathbb{E}\left[\pi(\alpha, \omega^m)\right] = (1 - r)\pi(\alpha, \underline{\omega}) + r\pi(\alpha, \bar{\omega}^m)$, where the first term—the profit under the status quo—is independent of the project choice. The following lemma is key to comparing the profits from competition under different projects. The proof is illuminating of the reasons behind the result.

Lemma 3.2. *Functions $\pi(\cdot, \bar{\omega}^d)$ and $\pi(\cdot, \bar{\omega}^c)$ have the same maximum value. The maximizer of the former $\alpha^*(\bar{\omega}^d)$ is smaller than the maximizer of the latter $\alpha(\bar{\omega}^c)$.*

Proof. From $\Delta O^c(0) = \Delta O^d(0)$ we have $\Delta c = \Delta d = \delta$ and $\pi(0, \bar{\omega}^c) = \pi(0, \bar{\omega}^d)$. In the linear model the problem of the Stackelberg leader is a function of $d - c$ only. Under both projects this difference is the same and is equal to $\bar{d} - \bar{c} = \underline{d} - \underline{c} = \underline{d} - \bar{c} + \delta$. Hence, the Stackelberg profits and quantities are the same. As we know, $\alpha^*(\omega)$ implements the Stackelberg outcome; hence, $\max_\alpha \pi(\alpha, \bar{\omega}^d) = \max_\alpha \pi(\alpha, \bar{\omega}^c)$.

Quantities in the Cournot equilibrium in the presence of contracts depend on $d - \tilde{c}$. Equality of outputs under demand and cost projects simplifies to

$$\bar{d} - (1 - \alpha^*(\bar{\omega}^d))\bar{c} = \underline{d} - (1 - \alpha^*(\bar{\omega}^c))\underline{c}.$$

As $\bar{d} - \bar{c} = \underline{d} - \underline{c}$, this implies $\alpha^*(\bar{\omega}^d)\bar{c} = \alpha^*(\bar{\omega}^c)\underline{c}$ and $\alpha^*(\bar{\omega}^d) < \alpha^*(\bar{\omega}^c)$. $\square$

[Figure 5 goes around here]

The lemma essentially says that, were the success of the managerial effort certain, the shareholders wouldn't be concerned about the benefit part of the choice of the project. On the other hand, when success is not guaranteed, the choice of the project does have an impact on the profit from competition, as is evident from figure 5. The shareholders need to deflate costs more when the project is to cut costs, because, mechanically, the same percentage reduction of a smaller cost is not enough to achieve the Stackelberg outcome. However, if the manager doesn't succeed, she will be competing with the same deflation factor α under higher marginal cost, which would make her too aggressive from the shareholders' point of view. Therefore, for positive α we get that in terms of benefits the demand project is better—precisely the comparison on the cost side as well. For negative α we have unambiguously $\pi(\alpha, \omega^d) < \pi(\alpha, \omega^c)$, and, therefore, under negative α the shareholders prefer cost cutting to demand boosting. Clearly, α is the choice variable for the shareholders, so we need to be more rigorous, but it's already evident that the shareholders and the manager prefer the same projects under different signs of α.

Proposition 3.3. *Let $\alpha_m^* := \operatorname{argmax}_\alpha \Pi^m(\alpha)$. Assume that $\Delta O^c(\alpha)$ is decreasing for $\alpha \geq 0$. Then, if $\alpha_c^* > 0$, then both the shareholders and the manager prefer to direct effort to demand boosting rather than cost cutting. If $\alpha_d^* < 0$, then both prefer to cut costs. If neither is true, both still agree on the choice of project.*

The logic behind the proposition is as follows. The shareholders compare the α's they would offer under cost cutting and under demand boosting. If both are positive, then they prefer to increase demand, and so does the manager. If both are negative, both parties prefer to cut costs. The only case that's left is when $\alpha_d^* > 0$ and $\alpha_c^* < 0$, but in this case, whichever project they prefer, the manager will comply as well.

3.3 Equilibrium in Contracts

So far in the analysis it was assumed that the shareholders of the second firm weren't strategic. My analysis was conditional on a fixed choice of α_2, not necessarily zero, and no effort on behalf of the competing manager. The results I obtained highlight the trade-offs that the shareholders and the manager face, and there is no reason for these trade-offs to disappear when the competitors get to respond. In this section I show that equilibrium in contracts indeed doesn't undermine my results.

The timing of the full-scale game is as follows. Shareholders of both firms simultaneously propose contracts to their managers. Managers privately observe their contracts and decide whether to accept or reject. If a manager accepts, she decides on the effort level. Both managers work on the same project $m \in \{c, d\}$. Afterwards, the states of both firms realize, contracts become public, and managers compete in the market by setting quantity. By keeping the contracts private until the competition stage I deny the shareholders the ability to influence the effort of the competitor's manager. Clearly, in equilibrium the manager knows which contract is offered to the competing manager and acts accordingly, but should there be a deviation, she would not observe it. This rules out an additional layer of complexity in the problem that is already complicated enough. I focus on the equilibrium of this game in which both firms incentivize their managers to exert effort.

The major novelty in the analysis is that, when the manager of firm 2 exerts effort, the state of firm 2 becomes random, and therefore there is an extra source of uncertainty in the problem of firm 1. In the competition stage the states $\omega_i = (d_i, c_i)$ are known, and my analysis remains the same. In particular, managers play the Cournot equilibrium with perceived marginal costs $(\tilde{c}_1, \tilde{c}_2)$ and the associated value of the objective $O_i(\alpha_i, \alpha_{-i}; \omega_i, \omega_{-i})$.

In the benchmark model the manager of firm 1 exerts high effort when $\kappa r \Delta O^m(\alpha) \geq \psi$. Now that the competing manager also exerts effort, the gain to the objective from increasing the state becomes random, and therefore the manager takes into account the expected value of the gain: $\kappa r \mathbb{E}\left[\Delta O^m(\alpha_1, \alpha_2; \omega_2)\right]$. By the virtue of the linear setting the gain to objective is linear in the state of the competitor, so the manager of firm 1 exerts effort if

$$\kappa r \Delta O^m(\alpha_1, \alpha_2; \mathbb{E}\left[\omega_2\right]) \geq \psi.$$

At this point only the expected value of the competitor's state matters, so my results from the partial equilibrium analysis apply, as the analysis was valid for arbitrary values of d_2 and $\tilde{c}_2$. In particular, the monotonicity results of lemma 2.2 for $\Delta O^m(\alpha)$ remain true.

The shareholders of firm 1 take the high effort inducing contract α_2 of the competitor as given and choose their contract α_1 to maximize the expected profit from competition less the payment to the manager. After repeating all the steps leading to (10) we get a similar expression for the objective function

$$\max_{\alpha_1} \ \mathbb{E}\left[\pi(\alpha_1, \alpha_2; \omega_1^m, \omega_2^m)\right] - \frac{\psi}{r} \frac{\mathbb{E}\left[O(\alpha_1, \alpha_2; \underline{\omega}, \omega_2)\right]}{\Delta O^m(\alpha_1, \alpha_2, \mathbb{E}\left[\omega_2\right])} - \psi$$

This is the exact same problem as before, except for the expectation in the numerator of the cost of incentive provision. Notably, as profit from competition is a concave function of α_1, the expected profit is concave as well. The cost of incentive provision is different from the partial equilibrium case, as it is not simply an expectation of the object introduced earlier. However, the same properties of this function remain true, albeit the math becomes substantially harder.

Proposition 3.4. $\lambda^m(\alpha_1, \alpha_2) := \dfrac{\psi}{r} \dfrac{\mathbb{E}\left[O(\alpha_1, \alpha_2; \underline{\omega}, \omega_2)\right]}{\Delta O^m(\alpha_1, \alpha_2, \mathbb{E}\left[\omega_2\right])} + \psi$ *is increasing in* α_1 *and is convex in* α_1 *for* $m \in \{d, c\}$.

Objective function $\Pi_i^m(\alpha_i, \alpha_{-i}) := \mathbb{E}\left[\pi(\alpha_i, \alpha_{-i}; \omega_1^m, \omega_2^m)\right] - \lambda^m(\alpha_i, \alpha_{-i})$ of firm i is therefore concave for any value of α_{-i}, and has a unique interior maximizer, as extreme values of

α_1 lead to non-positive profit. In fact, we can compactify the action space of both firms, as firms would never inflate their costs beyond the level that brings zero profit to a monopolist in the market, and would never deflate them below a certain threshold, either the one at which they can satisfy the entire market at a zero price in the demand boosting case, or some value of α close to 1 in the cost cutting case (it becomes impossible to satisfy the IC constraint in the cost cutting case as $\alpha \to 1$). This informal argument suggests that equilibrium always exists by the Glicksberg theorem (Theorem 1.2 in Fudenberg and Tirole (1991)). To formalize this argument one would need to extend the definitions in my paper to cases when the firm chooses to produce zero under low state and, possibly, a positive amount under high state. This extra rigor wouldn't contribute to the analysis, so I omit it.

My partial equilibrium analysis of the quantity competition game suggested that the original result in FJ, namely that managers will be aggressive in equilibrium, may no longer hold when managers need to be incentivized. I now show by example that, in fact, that suggestion is true in the full equilibrium of my game. I also use this example to contrast the partial equilibrium solution to the full equilibrium of the game.

The example goes like this. The fundamental structure is symmetric across firms. The low state is $\underline{\omega} = (\underline{d} = 1, \bar{c} = 1/5)$. Both managers can exert effort at cost $\psi = 1/100$ to increase demand and move the state of their firm to $\bar{\omega}^d = (\bar{d} = 6/5, \bar{c} = 1/5)$ with probability $r = 3/4$. In partial equilibrium firm 2 is run by a profit maximizing manager who exerts no effort, and the shareholders of firm 1 maximize their payoff by solving (10). In full game both shareholders set contracts that induce high effort for their managers. The Nash equilibrium is a pair $(\alpha_1^{NE}, \alpha_2^{NE})$ such that α_i^{NE} maximizes $\Pi_i^d(\alpha_i, \alpha_{-i}^{NE})$ for for both i. I solve for the symmetric Nash equilibrium. I also check that it is not profitable for firms to implement the zero effort contract.

In partial equilibrium firm 1 choses $\alpha_1^* = -0.525$ and makes a profit of 0.194. The competitor's profit is 0.142. The zero effort contract $\alpha = 0.66$ brings the profit of 0.156 to firm 1 and 0.146 to firm 2. Notice that the only motive for distorting the perception of cost in the zero effort contract is strategic delegation, so α must be positive in that case. In the full game under the same value of the parameters in the symmetric equilibrium the shareholders of both firms choose $\alpha_1^{NE} = \alpha_2^{NE} = -0.544$ and make a profit of 0.183 each. First, we see that negative α survives in full equilibrium. Second, firm 1 is worse off now that firm 2 gets to respond. Interestingly, firm 1 makes their manager more conservative in full equilibrium than it does in partial equilibrium. This result goes against the comparative statics in FJ, as in that model the best response function for the shareholders is downward sloping, and, if the competitor chooses to inflate his costs, the firm would respond by deflating its cost. Here, however, the competitor is also inducing high effort by the means of the contract, which increases their state, so the shareholders respond by making the manager less aggressive.

The reason why the shareholders choose to have conservative managers under quantity competition is that it's cheaper to incentivize a conservative manager to exert effort. Consider a lower value of the cost of effort $\psi = 1/400$, under which incentivizing effort is cheaper. Under the new parametrization in full equilibrium the value of α becomes positive and equal to 0.432—the strategic delegation motive takes over. One can easily perform other comparative statics exercises numerically, but they are outside of the research agenda of this paper.

The results in the extensions of the benchmark model also survive in the full equilibrium of the game. Recall that under price competition I found that there was no ambiguity in the sign of optimal α: both the strategic delegation and the incentive provision motives push the shareholders to set a negative α, irrespective of the choice of the competitor. This means that best-response value of α_i is always negative for any value of α_{-i}, and, therefore, the equilibrium values of α are going to be negative as well.

The result that under quantity competition the shareholders and the manager will always agree on the choice of project also extends naturally. As I discussed before, $\mathbb{E}\left[\Delta O^m(\alpha, \omega_2)\right] = \Delta O^m(\alpha, \mathbb{E}\left[\omega_2\right])$, and therefore the behavior of the manager remains the same. Ensuring that the two projects are commensurate, $\Delta O^c(0, \mathbb{E}\left[\omega_2\right]) = \Delta O^d(0, \mathbb{E}\left[\omega_2\right])$, still requires $\Delta d = \Delta c$, and therefore the relation between λ^d and λ^c follows lemma 2.3. Finally, as $\pi(\alpha_1, \alpha_2, \omega_1, \omega_2)$ is quadratic in ω_2, it follows that $\mathbb{E}_{\omega_2}[\pi(\alpha_1, \alpha_2, \omega_1, \omega_2)] = \pi(\alpha_1, \alpha_2, \omega_1, \mathbb{E}[\omega_2]) + const(\gamma)\mathrm{Var}(\omega_2)$, and therefore lemma 3.2 applies to $\mathbb{E}_{\omega_2}[\pi(\alpha_1, \alpha_2, \omega_1^m, \omega_2)]$, $m \in \{c, d\}$. Redefining $\Pi^m(\alpha_1, \alpha_2) = \mathbb{E}_{\omega_1^m}[\mathbb{E}_{\omega_2}[\pi^m]] - \lambda^m$, we can apply proposition 3.3. The interpretation in the full equilibrium setting would be, for example, as follows. Suppose that in a symmetric equilibrium of the game in which both managers boost demand both firms choose positive α's. Then this arrangement is still an equilibrium in the extended game in which each manager has an option to choose the cost project instead.

4 Conclusion

In this paper I analyze the cost of incentive provision for a manager whose compensation is based on both revenue and profit. I write down a model of oligopolistic competition to put structure on the joint distribution of the two. I find that it is always more expensive to incentivize a more revenue-oriented manager to exert effort. This result relies on the degree of complementarity between lower costs and the state the manager is working to improve. It's more expensive to incentivize an aggressive manager to cut costs rather than to increase demand, and the other way around for a conservative manager.

In oligopolistic settings tying managerial compensation to revenue could be a rational profit enhancing choice for the shareholders, a phenomenon known as strategic delegation. When I study the choice of contract by the shareholders, the monotonicity of the cost of incentive provision always pushes the shareholders to put a lower weight on revenue. Therefore, the goal of incentive provision clashes with the delegation motive under quantity competition, and reinforces it under price competition. The clash under quantity competition could be strong enough that the shareholders could offer a conservative contract—a reversal of the result in FJ. In the case in which there is no benefit from strategic delegation my model becomes a principal-agent problem with limited liability and linear contracting on revenue and profits, and the optimal contract always puts a negative weight on revenue. Deviation from profit maximization arises purely due to moral hazard on the part of the manager. I also find that there is no disagreement between the shareholders and the manager about the project the manager should concentrate on.

There are several potential avenues for future research. One is to replace the assumption of limited liability for the manager with that of risk-aversion. Revenue and costs have different variance profiles, and this could add an interesting dimension to the study of costs of incentive provision. A higher demand shifter leads to higher price, higher output, and higher revenue, but a lower marginal cost doesn't necessarily lead to lower total cost, so investment in demand could have higher variance. Another interesting route is to move away from linear contracts to contracts based on performance goals, which is a salient feature of the real world contracts.

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A Pictures

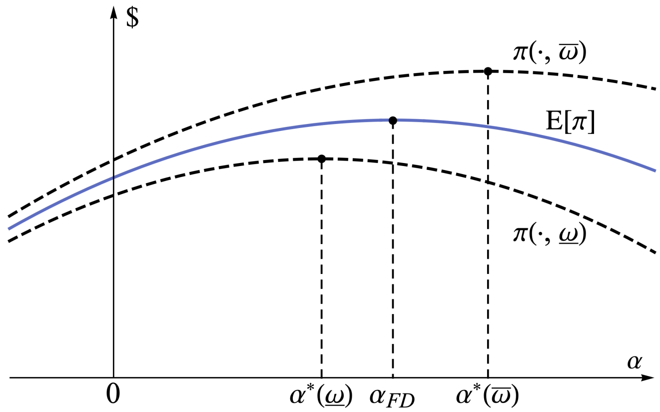

Figure 1: Implications of lemma 2.1: relative position of profits from competition under high and low states, their weighted average $\mathbb{E}\left[\pi(\alpha, \omega^m)\right]$, and their maximizers α^*.

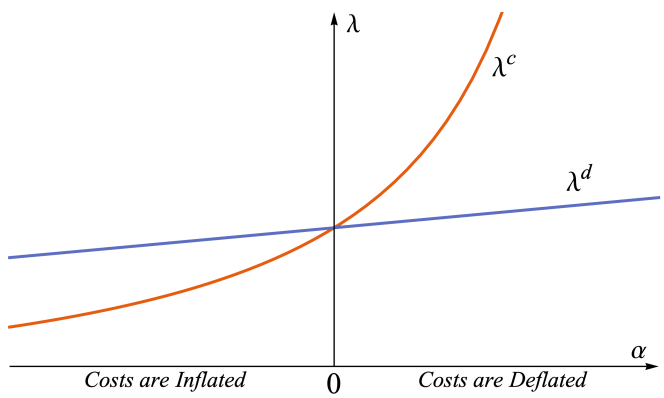

Figure 2: Costs of incentive provision under different projects: cutting marginal cost and boosting demand.

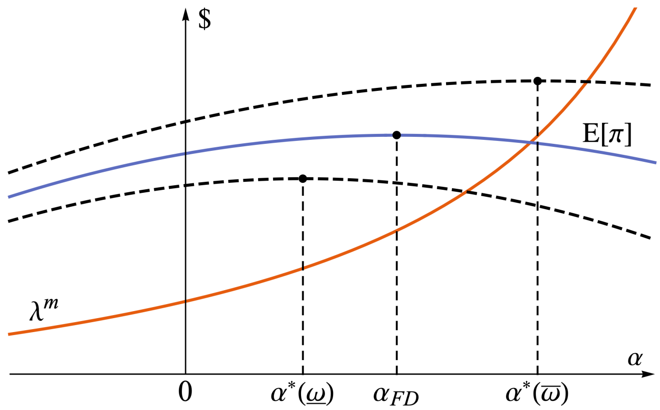

Figure 3: Graphical representation of the shareholders' problem: expected profit from competition $\mathbb{E}\left[\pi(\alpha,\omega^m)\right]$ vs. the cost of incentive provision $\lambda^m(\alpha)$

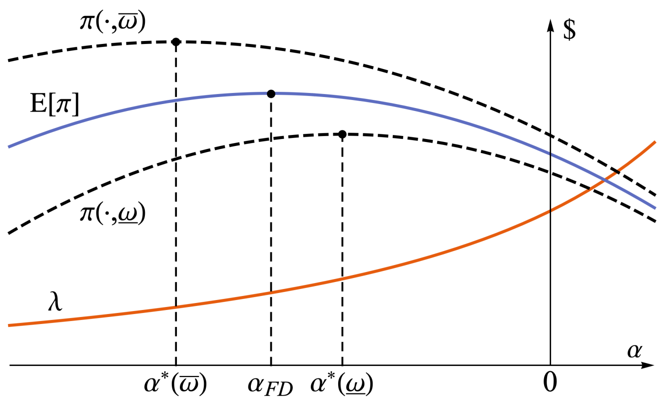

Figure 4: Graphical representation of the shareholders' problem under price competition: expected profit from competition $\mathbb{E}\left[\pi(\alpha,\omega^m)\right]$ vs. the cost of incentive provision $\lambda^m(\alpha)$

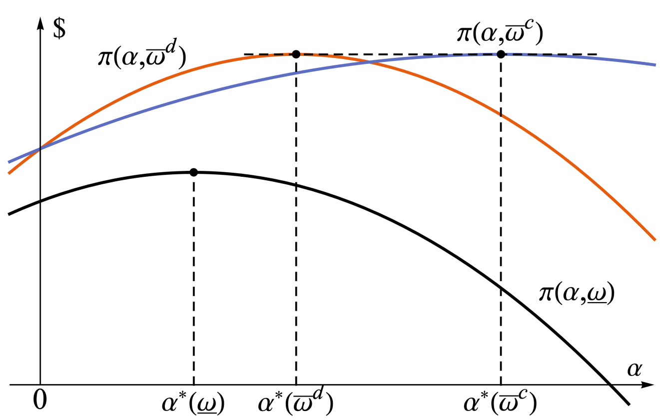

Figure 5: The relationship between $\pi(\alpha,\bar{\omega}^d)$, $\pi(\alpha,\bar{\omega}^c)$, and $\pi(\alpha,\underline{\omega})$ according to lemma 3.2

B Proofs

Proof of Lemma 2.1.

1. $\pi(\alpha, \omega)$ is a parabola in α with the coefficient on α^2 being negative for any $c > 0$.

2. The expression for the maximizer is

$$\alpha^*(\omega) = \frac{\gamma^2(2(d-c) - \gamma(d_2 - \tilde{c}_2))}{4c(2-\gamma^2)}.$$

   $\alpha^*(\omega) > 0 \iff 2(d-c) - \gamma(d_2 - \tilde{c}_2) > 0 \iff q(0, \omega) > 0$, so, as long as in the Cournot equilibrium with a profit maximizing manager both firms produce positive outputs, the maximizer will be positive.

3. $\alpha^*(\omega)$ is clearly increasing in d. As for the c, rewrite the expression as

$$\alpha^*(\omega) = \frac{\gamma^2(2d - \gamma(d_2 - \tilde{c}_2))}{4c(2-\gamma^2)} - \frac{\gamma^2}{2(2-\gamma^2)}$$

   to see that it's decreasing in c. Therefore $\alpha^*(\bar{\omega}_c) > \alpha^*(\underline{\omega})$, as costs are smaller in the first case.

□

Proof of Lemma 2.2.

$$\Delta O^d(\alpha) = \frac{4(\bar{d} - \underline{d})(\bar{d} + \underline{d} - 2(1-\alpha)c - \gamma(d - \tilde{c}_2))}{(4-\gamma^2)^2}$$

is a linear function of α with a positive slope coefficient, so it is increasing.

Now, the expression for $\Delta O^c(\alpha)$ is a quadratic parabola in α:

$$\Delta O^c(\alpha) = \frac{4(1-\alpha)(\bar{c} - \underline{c})(2\underline{d} - (1-\alpha)(\bar{c} + \underline{c}) - \gamma(d_2 - \tilde{c}_2))}{(4-\gamma^2)^2}$$

Without extra assumptions on the parameters, this function could be both increasing and decreasing in the relevant range of α. The maximizer of $\Delta O^c(\alpha)$ is given by $-(2\bar{c}+2\underline{c})^{-1}(2\underline{d} - 2(\bar{c} + \underline{c}) - \gamma(d_2 - c_2))$, which is to the left of some threshold $\hat{\alpha}$ whenever

$$2\underline{d} - 2(\bar{c} + \underline{c}) - \gamma(d_2 - c_2) > -\hat{\alpha}2(\bar{c} + \underline{c}) \iff 2\underline{d} - 2(\tilde{\bar{c}} + \tilde{\underline{c}}) - \gamma(d_2 - c_2) \geq 0,$$

where $\tilde{c} = (1-\hat{\alpha})c$. This condition is equivalent to the condition that, the reduction of marginal cost from $\tilde{\bar{c}}$ to $\tilde{\underline{c}}$ leads to lower total cost in equilibrium: $\tilde{\bar{c}}q(\hat{\alpha}, \underline{w}) > \tilde{\underline{c}}q(\hat{\alpha}, \bar{\omega}^c)$, which simplifies to

$$(\tilde{\bar{c}}-\underline{\tilde{c}})(2\underline{d}-2(\tilde{\bar{c}}+\underline{\tilde{c}})-\gamma(d_2-c_2))\geq 0 \Longleftrightarrow 2\underline{d}-2(\tilde{\bar{c}}+\underline{\tilde{c}})-\gamma(d_2-c_2)\geq 0$$

□

Proof of Proposition 2.1. The expression for $O(\alpha,\underline{\omega})/\Delta O^d(\alpha)$ is

$$\frac{r}{\psi}(\lambda^d(\alpha)-\psi)=\frac{(2\underline{d}-2(1-\alpha)c-\gamma(d_2-\tilde{c}_2))^2}{4(\bar{d}-\underline{d})(\bar{d}+\underline{d}-2(1-\alpha)c-\gamma(d-\tilde{c}_2))}$$

Observe that, as $\bar{d}>\underline{d}$, this function's structure is basically $(A+x)^2/(A+B+x)$, $A,B>0$. Rewrite

$$\frac{(A+x)^2}{(A+B+x)}=\frac{(A+B+x-B)^2}{(A+B+x)}=x+\frac{B^2}{A+B+x}+A-B$$

The function $x+\frac{B^2}{A+B+x}$ is convex and is increasing for $x\geq A$, which is equivalent to $q(0,\underline{\omega})\geq 0$.

Now let's turn to the case of cost cutting. Despite the fact that both the denominator and the numerator in $O(\alpha,\underline{\omega})/\Delta O^c(\alpha)$ could be increasing, the cost of incentive provision is still an increasing function:

$$\frac{O(\alpha,\underline{\omega}^m)}{\Delta O^c(\alpha)}=\frac{(2d-2(1-\alpha)\bar{c}-\gamma(d_2-\tilde{c}_2))^2}{4(1-\alpha)(\bar{c}-\underline{c})(2d-(1-\alpha)(\bar{c}+\underline{c})-\gamma(d_2-\tilde{c}_2))}$$

Let $x=(1-\alpha)>0$, $A=2d-\gamma(d_2-\tilde{c}_2)$, and observe that $A-2\bar{c}x\propto q(\alpha,\bar{c})$ should be positive, so that $x\leq A/2\bar{c}$. We can rewrite the expression above as

$$\frac{(A-2\bar{c}x)^2}{4\Delta cx(A-(\bar{c}+\underline{c})x)}=\frac{([A-(\bar{c}+\underline{c})x]-\Delta cx)^2}{4\Delta cx(A-(\bar{c}+\underline{c})x)}$$

Expand the numerator and divide term by term to get

$$\frac{A-(\bar{c}+\underline{c})x}{4\Delta cx}+\frac{\Delta cx}{4(A-(\bar{c}+\underline{c})x)}-\frac{1}{2}$$

Both of the hyperbolas are convex in the range $x\in[0,A/2\bar{c}]$, so the function is convex. The first hyperbola is decreasing, and the second one is increasing. The sum has a U-shape, reaching the minimum precisely at $x=A/\bar{2}c$, implying that in our range of x the function is decreasing in x. Therefore, it is increasing in α. □

Proof of Proposition 2.2. A function of two variables $O(\alpha,x)$ is said to be supermodular if for any $\alpha_2\geq\alpha_1$ and $x_2\geq x_1$ we have $O(\alpha_2,x_1)+O(\alpha_1,x_2)\geq O(\alpha_1,x_1)+O(\alpha_2,x_2)$. The function is called submodular when the inequality is reversed. Supermodularity condition could be rewritten as

$$\Delta O(\alpha_2) = O(\alpha_2, x_2) - O(\alpha_2, x_1) \geq O(\alpha_1, x_2) - O(\alpha_1, x_1) = \Delta O(\alpha_1),$$

so $\Delta O^m(\alpha)$ is increasing iff $O(\alpha, x)$ is supermodular. By the same logic, $\Delta O^m(\alpha)$ is decreasing iff $O(\alpha, x)$ is submodular.

A function $O(\alpha, x)$ is said to be log-supermodular (log-submodular) if the $\log O(\alpha, x)$ is supermodular (submodular). This is equivalent to

$$O(\alpha_2, x_1)O(\alpha_1, x_2) \geq O(\alpha_1, x_1)O(\alpha_2, x_2)$$

Subtract $O(\alpha_1, x_1)O(\alpha_2, x_1)$ from both sides to get

$$O(\alpha_2, x_1)[O(\alpha_1, x_2) - O(\alpha_1, x_1)] \geq O(\alpha_1, x_1)[O(\alpha_2, x_2) - O(\alpha_2, x_1)],$$

which becomes

$$\lambda(\alpha_2) = \frac{O(\alpha_2, x_1)}{O(\alpha_2, x_2) - O(\alpha_2, x_1)} \geq \frac{O(\alpha_1, x_1)}{O(\alpha_1, x_2) - O(\alpha_1, x_1)} = \lambda(\alpha_1)$$

Above we've assumed that $O(\alpha, x_2) - O(\alpha, x_1) > 0$ for any α, which is the case in the model. □

Proof of Lemma 2.3. We need to show that $\Delta O^c(\alpha) > \Delta O^d(\alpha) \iff \alpha < 0$. By assumption, the two functions cross at zero, and $\Delta O^c(\alpha)$ is decreasing for positive α. As $\Delta O^d(\alpha)$ is an increasing function, the comparison for $\alpha > 0$ is immediate. $\Delta O^c(\alpha)$ is a parabola, so there is exactly one solution of $\Delta O^c(\alpha) = \Delta O^d(\alpha)$ other from 0. That solution is $\alpha = [2\underline{c} - 2\underline{d} + \gamma(d_2 - \tilde{c}_2)]/(\bar{c} + \underline{c})$, and it is less than the smallest α consistent with positive production in the low state: $[2\bar{c} - 2\underline{d} + \gamma(d_2 - \tilde{c}_2)]/2\bar{c}$, as $2\underline{d} + 2\bar{c} - \gamma(d_2 - \tilde{c}_2) > 2\underline{d} - 2\bar{c} - \gamma(d_2 - \tilde{c}_2) \propto q(0, \underline{\omega}) > 0$. □

Comparative Statics Results. We are interested in the sign of $\partial \lambda^m(\alpha)/\partial \omega_2$. Recall that

$$O(\alpha, \omega; \omega_2) = \max_q (d - q - \gamma q_2(\alpha, \omega; \omega_2) - \tilde{c})q,$$

where $q_2(\alpha, \omega; \omega_2)$ is the equilibrium production of the competitor. By the envelope theorem

$$\frac{\partial O(\alpha, \omega; \omega_2)}{\partial \omega_2} = -\gamma \frac{\partial q_2(\alpha, \omega; \omega_2)}{\partial \omega_2} q(\alpha, \omega; \omega_2) < 0$$

The higher is the state of the competitor, the higher is their output and the smaller is the compensation of the manager. Note that the derivative is constant in the linear model; denote it by $a := -\gamma \frac{\partial q_2(\alpha,\omega;\omega_2)}{\partial \omega_2} < 0$. It follows that

$$\frac{\partial \Delta O^m(\alpha;\omega_2)}{\partial \omega_2} = aq(\alpha,\bar{\omega}^m;\omega_2) - aq(\alpha,\underline{\omega};\omega_2) = a\Delta q^m(\alpha;\omega_2)$$

Now, suppressing the dependence on ω_2 and α for expositional purposes, the derivative of interest becomes

$$\frac{r}{\psi}\frac{\partial \lambda^m(\alpha)}{\partial \omega_2} = \frac{\partial}{\partial \omega_2}\left(\frac{O(\alpha,\underline{\omega})}{\Delta O^m(\alpha)}\right) = \frac{aq(\underline{\omega})\Delta O^m - O(\underline{\omega})a\Delta q^m}{(\Delta O^m)^2} = \frac{aq(\underline{\omega})}{\Delta O^m} - \frac{aO(\underline{\omega})}{\Delta O^m}\frac{\Delta q^m}{\Delta O^m}$$

Now, using the fact that $[q(\alpha,\omega)]^2 = O(\alpha,\omega)$, and denoting $O(\alpha,\underline{\omega})/\Delta O(\alpha)$ by $\tilde{\lambda}(\alpha,\omega)$ (as it is just an affine transformation of λ), we get

$$\frac{r}{\psi}\frac{\partial \lambda^m(\alpha)}{\partial \omega_2} = \frac{a\tilde{\lambda}}{q(\underline{\omega})} - \frac{a\tilde{\lambda}}{q(\bar{\omega}) + q(\underline{\omega})} = \frac{a\tilde{\lambda}q(\bar{\omega})}{q(\underline{\omega})(q(\bar{\omega}) + q(\underline{\omega}))} < 0,$$

as $a < 0$ and the rest is positive. Higher state of the competitor increases the cost of incentive provision irrespective of the project m.

Now we turn to the effect of higher γ on the cost of incentive provision $\lambda^m(\alpha)$. When $m = d$ we have

$$\lambda^d(\alpha) = \frac{(2(\underline{d} - \tilde{c}) - \gamma(d_2 - c_2))^2}{4(\bar{d} - \underline{d})(\bar{d} + \underline{d} - 2\tilde{c} - \gamma(d_2 - c_2))}$$

Because equilibrium production is assumed to be positive in both states we get

$$\begin{cases} 2(\underline{d} - \tilde{c}) - \gamma(d_2 - c_2) \geq 0 \\ 2(\bar{d} - \tilde{c}) - \gamma(d_2 - c_2) \geq 0, \end{cases}$$

which implies that the denominator $\bar{d} + \underline{d} - 2\tilde{c} - \gamma(d_2 - c_2)$ is positive for any γ that is admissible, as well as is decreasing in γ. A remark is needed here: for a fixed set of parameters not all $\gamma \in (0,1)$ are feasible, as, for instance, the restriction on the demand function implies $d \geq \gamma d_2$ and $d_2 \geq \gamma d$. As $\bar{d} > \underline{d}$, we are analyzing the expression of the form $(a - b\gamma)^2/(a + \Delta - b\gamma)$, which is decreasing, being a product of two decreasing functions:

$$\frac{(a - b\gamma)^2}{(a + \Delta - b\gamma)} = \frac{(a - b\gamma)}{(a + \Delta - b\gamma)} \times (a - b\gamma) = \left(1 - \frac{\Delta}{a + \Delta - b\gamma}\right)(a - b\gamma)$$

When the project is to cut costs the expression for λ^c is

$$\lambda^c(\alpha) = \frac{(2d - 2\bar{\tilde{c}} - \gamma(d_2 - c_2))^2}{4(\bar{\tilde{c}} - \underline{\tilde{c}})(2d - \bar{\tilde{c}} - \underline{\tilde{c}} - \gamma(d_2 - c_2))}$$

Again, because production levels should be positive under both levels of marginal costs, we get that the denominator is positive and decreasing in γ. A similar argument then establishes that $\lambda(\alpha)$ is decreasing in γ. $\square$

Proof of Lemma 3.1.

1. $\pi(\alpha, \omega)$ is a parabola in α with the coefficient on α^2 being negative for any $c > 0$.

2. The expression for the maximizer is

$$\alpha^*(\omega) = -\frac{\gamma^2((2-\gamma^2)(d-c) - \gamma(d_2 - \tilde{c}_2))}{(2-\gamma^2)4c}$$

   $\alpha^*(\omega) < 0 \iff (2-\gamma^2)(d-c) - \gamma(d_2 - \tilde{c}_2) > 0 \iff q(0, \omega) > 0$, so, as long as in the Bertrand equilibrium with a profit maximizing manager both firms produce positive outputs, the maximizer will be negative.

3. $\alpha^*(\omega)$ is clearly decreasing in d. As for the c, rewrite the expression as

$$\alpha^*(\omega) = -\frac{\gamma^2((2-\gamma^2)d - \gamma(d_2 - \tilde{c}_2))}{(2-\gamma^2)4c} + \frac{\gamma^2}{4}$$

   to see that it's increasing in c. Therefore $\alpha^*(\bar{\omega}_c) < \alpha^*(\underline{\omega})$, as costs are smaller in the first case.

$\square$

Proof of Proposition 3.1. $\Delta O^d(\alpha)$ is still a linear function with a positive slope coefficient, and $\Delta O^c(\alpha)$ is a concave parabola. The expression for $O(\alpha, \underline{\omega})/\Delta O^d(\alpha)$ is

$$\frac{O(\alpha, \underline{\omega})}{\Delta O^d(\alpha, \underline{\omega})} = \frac{[(2-\gamma^2)(\underline{d} - (1-\alpha)c) - \gamma(d_2 - \tilde{c}_2)]^2}{(2-\gamma^2)\Delta d[(2-\gamma^2)(\bar{d} + \underline{d} - 2(1-\alpha)c) - 2\gamma(d_2 - \tilde{c}_2)]}$$

Taking $(2-\gamma^2)/2$ outside of the brackets in the numerator and $(2-\gamma^2)$ in the denominator delivers the same functional form as in 2.1 with $\frac{2\gamma}{(2-\gamma^2)}(d_2 - \tilde{c}_2)$ in the place of $\gamma(d_2 - \tilde{c}_2)$, so the monotonicity and convexity proofs remain the same.

In the cost cutting case we have

$$\frac{O(\alpha, \underline{\omega})}{\Delta O^c(\alpha, \underline{\omega})} = \frac{[(2-\gamma^2)(d - (1-\alpha)\bar{c}) - \gamma(d_2 - \tilde{c}_2)]^2}{(2-\gamma^2)(1-\alpha)\Delta c[(2-\gamma^2)(d - (1-\alpha)(\bar{c} + \underline{c})) - 2\gamma(d_2 - \tilde{c}_2)]}$$

Exactly the same trick delivers the functional form studied in 2.1. $\square$

Proof of Proposition 3.3. Recall that $\lambda^c(\alpha)$ is steeper at $\alpha = 0$ than $\lambda^d(\alpha)$, and $\pi(\alpha, \bar{\omega}^d)$ is steeper at zero than $\pi(\alpha, \bar{\omega}^c)$ (parabolas + 3.2), and therefore $\frac{\partial}{\partial \alpha}\Pi^d(0) > \frac{\partial}{\partial \alpha}\Pi^c(0)$. Π^m

is a concave function for any m, and, hence, $\frac{\partial}{\partial\alpha}\Pi^c(0) > 0$ implies that both α_d^* and α_c^* are positive, $0 > \frac{\partial}{\partial\alpha}\Pi^d(0)$ implies that both are negative, and $\frac{\partial}{\partial\alpha}\Pi^d(0) > 0 > \frac{\partial}{\partial\alpha}\Pi^c(0)$ implies that $\alpha_d^* > 0 > \alpha_c^*$.

Suppose that $\alpha_d^* > 0 > \alpha_c^*$. Then, if $\Pi^d(\alpha_d^*) > \Pi^c(\alpha_c^*)$, then the shareholders prefer the demand project and want to have $\alpha_d^* > 0$. Under positive α the manager is happy to boost demand. If $\Pi^d(\alpha_d^*) < \Pi^c(\alpha_c^*)$, then the shareholders prefer the cost project and want to offer $\alpha_c^* < 0$. Under negative α the manager is happy to cut costs.

Now suppose that both α_d^* and α_c^* are positive. We now show that the demand project can bring higher expected profit from competition than the cost project at some at $\alpha < \alpha_c^*$. The proof is simply the formalization of what is almost obvious from figure 5. The expected profit from competition under the demand project is $\pi^d(\alpha) := (1-r)\pi(\alpha, \underline{\omega}) + r\pi(\alpha, \bar{\omega}^d)$ and the expected profit from competition under the cost project is $\pi^c(\alpha) := (1-r)\pi(\alpha, \underline{\omega}) + r\pi(\alpha, \bar{\omega}^c)$. By lemmas 2.1 and 3.2 we have $\alpha^*(\underline{\omega}) < \alpha^*(\bar{\omega}^d) < \alpha^*(\bar{\omega}^c)$. Therefore, for $\alpha \in [0, \alpha^*(\underline{\omega})]$ all three functions $\pi(\alpha, \underline{\omega}), \pi(\alpha, \bar{\omega}^d), \pi(\alpha, \bar{\omega}^c)$ are increasing. As $\max_\alpha \pi(\alpha, \bar{\omega}^d) = \max_\alpha \pi(\alpha, \bar{\omega}^c)$ and both functions are quadratic parabolas, it is the case that for $\alpha \in (0, \alpha^*(\bar{\omega}^d)]$ we have $\pi(\alpha, \bar{\omega}^d) > \pi(\alpha, \bar{\omega}^c)$.

If $\alpha_c^* \leq \alpha^*(\bar{\omega}^d)$, then $\pi^d(\alpha_c^*) > \pi^c(\alpha_c^*)$, as in that range of α $\pi(\alpha, \bar{\omega}^d) > \pi(\alpha, \bar{\omega}^c)$. Therefore, by switching the project from cutting cost to increasing demand and keeping the same contract the shareholders would earn a higher profit from competition. Suppose now that $\alpha_c^* > \alpha^*(\bar{\omega}^d)$. In this case shareholders would earn a higher profit from competition if they switch the project to demand boosting and set $\alpha = \alpha^*(\omega^d)$. As $\alpha^*(\underline{\omega}) < \alpha^*(\omega^d) < \alpha_c^*$, we would have that $(1-r)\pi(\alpha, \underline{\omega})$ goes up and $r\pi(\alpha^*(\omega^d), \bar{\omega}^d) \geq r\pi(\alpha, \bar{\omega}^c)$ for any α. In both cases we found a way for the shareholders to earn higher profit from competition by (weakly) lowering their α and switching to a different project. For positive levels of α we have $\lambda^d(\alpha) < \lambda^c(\alpha)$. Moreover, as λ^m is increasing for both $m \in \{d, c\}$, switching the projects and weakly decreasing α leads to lower costs of incentive provision. Therefore, as the benefits are higher and the costs are lower under the demand project, the shareholders would always prefer that. Under the positive α they would like to offer the manager wants to boost demand as well.

The last case is when α_d^* and α_c^* are both negative. As $\pi^d(\alpha)$ and $\pi^c(\alpha)$ are quadratic parabolas, they have only 2 intersection points, 0 and some positive value (corollary of 3.2). Therefore, for $\alpha < 0$ we have $\pi^d(\alpha) < \pi^c(\alpha)$. We also have $\lambda^d(\alpha) > \lambda^c(\alpha)$. Clearly, the shareholders want to cut costs. Under the negative α they would like to offer the manager wants to cut costs as well. $\square$

Proof of proposition 3.4. From the partial equilibrium analysis we know that

$$\frac{O(\alpha_1, \alpha_2; \underline{\omega}, \omega_2)}{\Delta O^m(\alpha_1, \alpha_2; \omega_2)}$$

is a convex function for any level of ω_2. This means that the second derivative with respect to α_1 is positive for any ω_2:

$$f(\omega_2, O(\omega_2)) := \frac{\Delta O^2 O'' - \Delta O \left(2O'\Delta O' + O\Delta O''\right) + 2O\Delta O'^2}{\Delta O^3} > 0$$

The arguments are omitted for expositional convenience. By the dependence of the auxiliary function f on O I mean only the dependence on the pure O function, rather than all its derivatives and ΔO. The reason for this will become clear shortly. In particular, for a random ω_2 we have $f(\mathbb{E}\left[\omega_2\right], O(\mathbb{E}\left[\omega_2\right])) > 0$, as $\mathbb{E}\left[\omega_2\right]$ is just one of the possible values of ω_2.

We now need to show that the second derivative of $\mathbb{E}\left[O\right]/\mathbb{E}\left[\Delta O^m\right]$ is positive, where expectation is with respect to ω_2. As $O \propto (d_1 - \tilde{c}_1 - A(d_2 - \tilde{c}_2))^2$, then O'_{α_1}, O''_{α_1} and ΔO are all linear in d_2 and $\tilde{c}_2$. Therefore, the corresponding second derivative for the ratio of expectations

$$\frac{(\mathbb{E}\left[\Delta O\right])^2 \mathbb{E}\left[O\right]'' - \mathbb{E}\left[\Delta O\right]\left(2\mathbb{E}\left[O\right]' \mathbb{E}\left[\Delta O\right]' + \mathbb{E}\left[O\right]\mathbb{E}\left[\Delta O\right]''\right) + 2\mathbb{E}\left[O\right](\mathbb{E}\left[\Delta O\right]')^2}{\mathbb{E}\left[\Delta O\right]^3}$$

could actually be written in terms of functions evaluated at the expected value, rather than the expectations of those function: $\mathbb{E}\left[\Delta O(\omega_2)\right] = \Delta O(\mathbb{E}\left[\omega_2\right])$, $\mathbb{E}\left[O'(\omega_2)\right] = O'(\mathbb{E}\left[\omega_2\right])$, etc. The only term that doesn't yield itself to this substitution is clearly $O(\omega_2)$ itself. Therefore, the derivative of interest collapses to $f(\mathbb{E}\left[\omega_2\right], \mathbb{E}\left[O(\omega_2)\right])$. Now I show that

$$f(\mathbb{E}\left[\omega_2\right], \mathbb{E}\left[O(\omega_2)\right]) \geq f(\mathbb{E}\left[\omega_2\right], O(\mathbb{E}\left[\omega_2\right])),$$

which implies that the derivative of interest is positive, as the right side of this inequality is positive. I collect the terms of f that have $\mathbb{E}\left[O\right]$ to get

$$f(\mathbb{E}\left[\omega_2\right], \mathbb{E}\left[O(\omega_2)\right]) = \mathbb{E}\left[O(\omega_2)\right] \frac{2[\Delta O'(\mathbb{E}\left[\omega_2\right])]^2 - \Delta O(\mathbb{E}\left[\omega_2\right])\Delta O''(\mathbb{E}\left[\omega_2\right])}{[\Delta O(\mathbb{E}\left[\omega_2\right])]^3} + \ldots$$

Depending on the project of the manager, the term ΔO^m is either linear or a concave parabola in α_1, so $\Delta O''(\mathbb{E}\left[\omega_2\right]) \leq 0$. This means that the entire expression in front of $\mathbb{E}\left[O(\omega_2)\right]$ is positive. Finally, because $O(\omega_2)$ is a convex function, we have $\mathbb{E}\left[O(\omega_2)\right] \geq O(\mathbb{E}\left[\omega_2\right])$, so $f(\mathbb{E}\left[\omega_2\right], \mathbb{E}\left[O(\omega_2)\right]) \geq f(\mathbb{E}\left[\omega_2\right], O(\mathbb{E}\left[\omega_2\right]))$, which completes the proof that $\lambda^m(\alpha_1, \alpha_2)$ is convex in α_1 for any m.

Now we proceed to proving monotonicity. Proofs are specific to the choice of project. Consider the case when managers boost demand first. Expand the expectation in the numerator of O to get

$$\lambda^d(\alpha_1,\alpha_2) = \frac{\psi}{r}\frac{rO(\alpha_1,\alpha_2;\underline{\omega},\bar{\omega}_2^d)}{\Delta O^d(\alpha_1,\alpha_2,\mathbb{E}\left[\omega_2\right])} + \frac{\psi}{r}\frac{(1-r)O(\alpha_1,\alpha_2;\underline{\omega},\underline{\omega_2})}{\Delta O^d(\alpha_1,\alpha_2,\mathbb{E}\left[\omega_2\right])} + \psi$$

We will show that both summands are increasing functions of α. For the summand that has $d_2 \in \{\bar{d},\underline{d}\}$ in the numerator rewrite $\mathbb{E}\left[d_2\right]$ in the denominator as $d_2 + a\Delta d$, where, respectively, $a \in \{r, -(1-r)\}$. Now, the expression we get is

$$\frac{O(\alpha_1,\alpha_2;\underline{\omega},d_2)}{\Delta O^d(\alpha_1,\alpha_2,\mathbb{E}\left[d_2\right])} = \frac{(2\underline{d}-2(1-\alpha_1)c-\gamma(d_2-\tilde{c}_2))^2}{4(\bar{d}-\underline{d})(\bar{d}+\underline{d}-2(1-\alpha_1)c-\gamma(d_2+a\Delta d-\tilde{c}_2))}$$

Let $A = 2\underline{d} - 2c - \gamma(d_2 - \tilde{c}_2)$ and $x = 2\alpha_1 c$. Then the numerator is $(A+x)^2$, and the denominator is $\propto (A+B+x)$, where $B = \Delta d - \gamma a \Delta d = (1-\gamma a)\Delta d$. If $B > 0$, then the proof for proposition 2.1 applies. It is indeed positive, as for any $a \in \{r, -(1-r)\}$, we have $1 - \gamma a > 0$. Therefore, both terms in $\lambda^m(\alpha_1,\alpha_2)$ are increasing, and so is the sum.

Now we turn to the cost cutting case. We would like to show that

$$\frac{\mathbb{E}\left[O(\alpha_1,\alpha_2,\bar{c},c_2)\right]}{\mathbb{E}\left[\Delta O^c(\alpha_1,\alpha_2,c_2)\right]}$$

is increasing in α_1. Expand $\Delta O^c = O(\alpha_1,\alpha_2,\underline{c},c_2) - O(\alpha_1,\alpha_2,\bar{c},c_2)$ and divide both sides by the numerator. The entire expression depends on the ratio $\mathbb{E}\left[O(\alpha_1,\alpha_2,\underline{c},c_2)\right]/\mathbb{E}\left[O(\alpha_1,\alpha_2,\bar{c},c_2)\right]$; if it is increasing in α, then so is the original function. This expression is

$$\frac{\mathbb{E}\left[(2d-2\underline{\tilde{c}}-\gamma(d-\tilde{c}_2))^2\right]}{\mathbb{E}\left[(2d-2\bar{\tilde{c}}-\gamma(d-\tilde{c}_2))^2\right]} = \frac{(2d-\gamma(d-\mathbb{E}\left[\tilde{c}_2\right])-2\underline{\tilde{c}})^2+\gamma^2\mathrm{Var}(\tilde{c}_2)}{(2d-\gamma(d-\mathbb{E}\left[\tilde{c}_2\right])-2\bar{\tilde{c}})^2+\gamma^2\mathrm{Var}(\tilde{c}_2)}$$

Let $A = d - \gamma/2(d - \mathbb{E}\left[\tilde{c}_2\right]) > 0$, $B = \gamma^2/4\mathrm{Var}(\tilde{c}_2) > 0$, $x = (1-\alpha_1)$. Then this expression becomes

$$f(x) = \frac{(A-\underline{c}x)^2+B}{(A-\bar{c}x)^2+B}$$

Because we restrict attention only to cases in which production of firm 1 is positive under any realization of uncertainty, it must be positive when its cost is high and the competitor's cost is low; therefore, we only consider

$$x \le \frac{d-\gamma/2(d-\underline{\tilde{c}})}{\bar{c}} \le \frac{d-\gamma/2(d-\mathbb{E}\left[\tilde{c}\right])}{\bar{c}} = A/\bar{c}$$

Differentiate with respect to x to get

$$f'(x) = \frac{2A\bar{c}\underline{c}(\bar{c}-\underline{c})x^2 - 2(A^2+B)(\bar{c}-\underline{c})(\bar{c}+\underline{c})x + 2A(A^2+B)(\bar{c}-\underline{c})}{\left((A-\bar{c}x)^2+B\right)^2}$$

We now show that the numerator is always positive in our range. The numerator is a convex parabola in x, with the minimum attained at

$$x^* = \frac{(A^2+B)(\bar{c}+\underline{c})}{2A\bar{c}\underline{c}}$$

Notice that $x^* > A/\bar{c}$, as

$$\frac{(A^2+B)(\bar{c}+\underline{c})}{2A\bar{c}\underline{c}} > \frac{A}{\bar{c}} \iff A^2\Delta c + B(\bar{c}+\underline{c}) > 0$$

Hence, the numerator is decreasing in the entire range. If the numerator is positive at the largest value of x in our range, then the function is increasing everywhere in the range. I go back from the auxiliary notation of A and B, calculate $\mathrm{Var}(\tilde{c}_2) = r(1-r)(1-\alpha_2)^2\Delta c^2$, and plug $x\bar{c} = d - \gamma/2(d-\tilde{\underline{c}})$ into $f'(x)$ to get

$$\frac{\gamma(2-\gamma)d(1-\alpha_2)(\underline{c}+2\bar{c}(1-r)) + \gamma^2(1-r)(1-\alpha_2)^2\bar{c}(\bar{c}+\underline{c}) + (2-\gamma)^2d^2}{\gamma^3(1-r)(1-\alpha_2)^3\bar{c}\Delta c/4}$$

Assuming that $(1-\alpha_2) > 0$, so that the perception of marginal cost by the competitor is positive, every term in the expression above is positive. $\square$