Consumer Reviews and Product Discounts:
Evidence From Video Games

Dmitry Sorokin

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1 Introduction

2 Literature Review

This paper contributes to several closely related literatures in economics and marketing. The oldest of these is the literature investigating the impact of online consumer reviews on sales. An extensive review of the studies focusing on this topic and their meta-analysis is provided, for example, by Floyd et al. [2014]. In a seminal contribution in this literature, Chevalier and Mayzlin [2006] uses a difference-in-differences approach, exploiting the differences in reviews for the same books across Amazon.com and Barnesandnoble.com, to argue that better reviews causally improve sales. A similar approach applied to the same market was recently used by Reimers and Waldfogel [2020], who reaffirm the importance of reviews for sales of books and quantify the welfare implications of the informational content of consumer reviews. Zhu and Zhang [2010] exploit differences between two different video games consoles rather than websites, and find that review ratings matter only for less popular games.

For identification, the difference-in-differences approach relies on the existence of several comparable platforms, and assumes that the unobserved platform-specific tastes for those

platforms are fixed. Another approach to identification exploits the rounding of the review data that platforms use to produce simple visual review labels. For example, Yelp.com, a popular restaurant comparison service, assigns 4 stars to restaurants that have an average review score between 4 and 4.24 (out of 5), but 4.5 stars to restaurants with an average score between 4.25 and 4.74. Anderson and Magruder [2012] and Luca [2016] develop a regression discontinuity approach exploiting this rounding to show that an additional half-star on Yelp.com increases restaurants' traffic and revenue. Sorokin and Stevens [2020] applies a similar approach to the video games market and also finds that better review labels increase sales, albeit raising some concerns about the validity of the approach. In the absence of direct sales data, Sorokin and Stevens [2020] uses a particular regression specification that extracts the information on sales from the video games' usage data. The present paper supplements the panel methods in Sorokin and Stevens [2020] by offering a structural way to estimate the effect of reviews on sales in the absence of direct sales data.

With benefits of having good reviews come the incentives to influence the reviews in order to reap those benefits. There are several ways firms can affect their online reviews. One is review fraud. Mayzlin et al. [2014] provides evidence that small hotel owners leave negative reviews for their competitors, and good reviews for themselves. Luca and Zervas [2016] provide similar evidence for restaurants on Yelp. A more honest way for a firm to influence reviews is by assuming an active approach to review management, and to publicly respond to reviews published online. Gu and Ye [2014] provide evidence that consumers who had left a negative review about their hotel experience in the past were more likely to leave a positive review of the same hotel after a future stay if their previous review was responded to by the management. Xie et al. [2017] shows that lengthy in-depth managerial responses to negative reviews are associated with better financial performance in the future. This literature has also been concerned with how consumers change their reviewing behavior when managers start responding to reviews, essentially moving from studying individual responses to describing equilibrium outcomes. In somewhat contradictory findings, Proserpio

and Zervas [2017] argues that hotels responding to bad reviews experience an increase in both the valence and the volume of their reviews, attributing it to consumers' reluctance to leave short indefensible reviews in a situation when they can be responded to, while Chevalier et al. [2018] finds the opposite effect in the same market, arguing that managerial responses encourage negative review activity, because consumers get a signal that the firm is "listening" to them, and are then stimulated to leave negative reviews that they deem more influential.

Direct ways of managing online reputation discussed above, however, are not the only avenues through which businesses can affect their reviews. Reviews are left by customers, and the happier they are, the better reviews they should leave, at least in principle. Another strand of literature is studying whether different kinds of promotions have an effect on firms' online reputation. In a seminal study, Byers et al. [2012] documents that restaurants receive substantially worse reviews after running a Groupon price promotion. They present evidence that customers attracted by the promotion are different from regular customers, and that they are more likely to be trying out a new restaurant or cuisine while using the Groupon voucher, which can explain the reasons behind their dissatisfaction. Thus, the paper cautions against running price promotions in an attempt to poach new customers and improve the online reputation. Li [2016] confirms the finding in the same setting, but finds that restaurants with fewer and worse reviews can actually benefit from participating in a Groupon promotion. Zhu et al. [2019] shows that customers who received a discount leave more positive, albeit less informative, reviews. In an experimental study on Ebay, Cabral and Li [2015] also find that offering a rebate for leaving (any) review significantly decreased the likelihood of negative feedback, and they provide evidence that this is due to reciprocity on the side of the clients, rather than the differences between customers buying with a rebate and without it.

The present study contributes to this literature by providing novel evidence from a new market. I show that customers buying computer games are significantly more likely to leave positive reviews during discounts. More importantly, I go beyond the descriptive nature of the other papers in the literature, and show that firms leverage this fact to improve their reviews

by giving discounts, thus providing evidence of a novel reputation management behavior. A study similar in spirit to mine is that of Zegners [2017], who argues that less-known authors of eBooks on a crowded online marketplace are more likely to give their books out for free in order to build reputation and escape the "zero reviews trap". However, this study also finds that books are more likely to solicit negative feedback when sold for free, which somewhat undermines the strategy. Similar to Byers et al. [2012], Zegners [2017] argues that purchasers of free books differ from the purchasers of regular books in their observable characteristics, and explains the negativity of the reviews by the selection effect.

3   Institutional Setting and Data

3.1   Steam Marketplace

Steam is arguably the largest online marketplace for selling computer games in the world. Founded in 2003, in 2013 it was responsible for about 75% of PC games sold online globally [Cliff, 2013], and in 2017 it has earned around US$4.3 billion, with an estimated market share of 18% in the entire market for PC games [PC Games, 2017]. Given an ongoing unprecedented growth in the size of the video games' market [$43.4 billion in 2018, about the size of the U.S. film industry, Minotti, 2019] and the increasing share of online sales in this market [83% in 2018, compared to 20% in 2009, Entertainment Software Association, 2019], Steam is a major player in an increasingly more important industry. Thus, this marketplace is not merely a laboratory for studies hoping to extrapolate the findings to other more well-known platforms, but is an important digital market to be studied in its own right.

Any user who has a copy of a game on Steam can leave a review for that game. Starting from late 2016, by default, Steam only uses reviews left by customers who have *purchased* the game in its review score calculation, excluding reviews left by customers who had obtained a key to activate the game through other channels (directly from the developer, or by purchasing the code from a different retailer). This distinguishes Steam from some other platforms

Table 1: The mapping between the review score and review bins on Steam

Review Bin	N. of Reviews	Score
No Score	[0, 10)	-
Negative	Any	[0, 40)
Mixed	Any	[40, 70)
Mostly Positive	Any	[70, 80)
Positive	[0, 50)	[80, 100)
Very Positive	More than 50	[80, 100)
Overwhelmingly Positive	More than 500	[95, 100]

studied in the literature, notably Yelp and TripAdvisor, where any user can leave a review (and, to a certain extent, Amazon, where non-verified users can also leave reviews). This gives me confidence in the authenticity of most reviews. This confidence is further backed by the importance of Steam to game developers and publishers, and Steam's history of monitoring the platform for fraudulent activity and excluding unscrupulous actors. With such high stakes, and a non-negligible risk of being caught, there are good reasons to expect the developers to behave scrupulously.

A review consists of a binary grade, "thumbs up" or "thumbs down", and review's text. A *review score* is defined as the fraction of positive reviews among all reviews a game has, provided it has at least 10 reviews. Based on the score, Steam assigns *review labels*, or bins, to each game. These labels[1] and the mapping rules are summarized in Table 1. For example, a game with 7 positive and 2 negative reviews would have neither a review score, nor a label, but an extra positive review would promote it to the "Positive" bin.

Steam's homepage welcomes customers with a selection of featured games, that could include new games, popular games that are currently on a discount, games that have recently released a major update, etc. [Steamworks Documentation, 2020b]. A customer could either click on the games already presented to her, or browse one of the categories that Steam offers. Available categories are based on games' characteristics such as performance (e.g., "Top Sellers", "New Releases"), genre (e.g, "Racing", "Anime", "Simulation"), or technical

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[1]There are labels worse than "Negative", but they are rare, so I bin all of them into "Negative"

characteristics (VR or controller support). Games within a category are organized into a list, with an example session presented in Figure 1. From that list the user can get some basic information about each game, such as its name, price, and if the game is currently on a discount. Importantly, if the user hovers her cursor over a game, she could further see the review label of the game and the number of reviews the game has. I use this feature of the Steam's store to rationalize why later in the analysis I choose to focus on the review labels as my main independent review variables, rather than, say, the review score or the review texts. Granted, informative review texts could be important for purchase decisions [Chevalier and Mayzlin, 2006], but on Steam, in order to gain access to such information, a customer should be willing to click on the game in the first place. Thus, I conjecture that games with better review labels will, other things equal, attract more customers, and lead to more sales. Crucially, according to Steam's documentation, all review labels better than "Negative" have a very similar contribution to visibility on the platform, which means, for example, that games are not ordered by their review labels when the customer is browsing different categories [2019]. Therefore, any effect of the review labels on, say sales, should come through the perceived quality differences between different bins, rather than some mechanical visibility differences. As an illustration, note that the first game in the list depicted in Figure 1 has "Mixed" reviews.
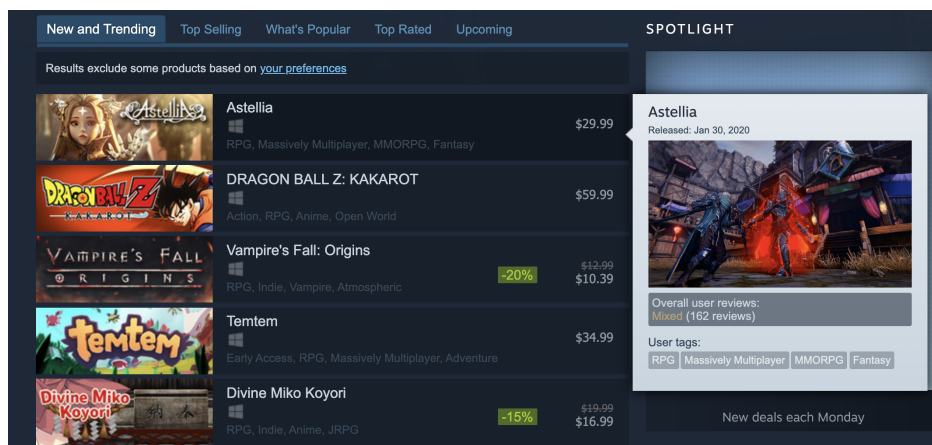

Figure 1: Example Browsing Session on Steam

3.2 Data

Valve Corporation, the owner of Steam, is notoriously secretive about its data and algorithms. Given the importance of the information on the performance of different games for game developers and publishers, the community has responded to this secrecy by establishing projects that monitor Steam in real time and extract information that could be relevant for parties interested in Steam. The majority of data used in this project comes from one such project called "Steam Database", or "SteamDB".

Descriptive information on any game, that I will refer to as "static", such as its title, developer, release date, etc., are readily available from Steam, and this information is also present on SteamDB. More importantly, Steam Database offers a daily time series of prices and player activity for each game on Steam. In principle, the pricing information could be obtained by a repeated scraping of the store. However, the player activity information is not trivial to obtain, as Steam does not directly reveal such data, let alone the data on sales of different titles. The creators of SteamDB have declined to explain their collection method, but their reputation in the gaming community most certainly implies that they are doing the best they can. The resulting variable that SteamDB offers, that I will refer to as the *player count*, measures the maximum number of concurrent player for each game on a daily basis. In other words, if on a particular day 10 people play the game every hour, except noon, when 13 people play the game simultaneously, then the recorded player count for this game on that day will be 13. The price and the player count time series are the major "dynamic" variables used in the analysis. The last dynamic variable, namely the evolution of the review score for each game, is obtained by scraping the reviews directly from Steam, and reverse engineering the path of the review score.

Steam is home to more than 25,000 games that differ in their genres, prices, sales, release dates, frequency of updates, and other characteristics. Inevitably, an appropriate sample should be selected for the analysis. Sorokin and Stevens [2020] studies the causal effect of reviews on sales on Steam, and I mirror closely the sample selection procedure used in that
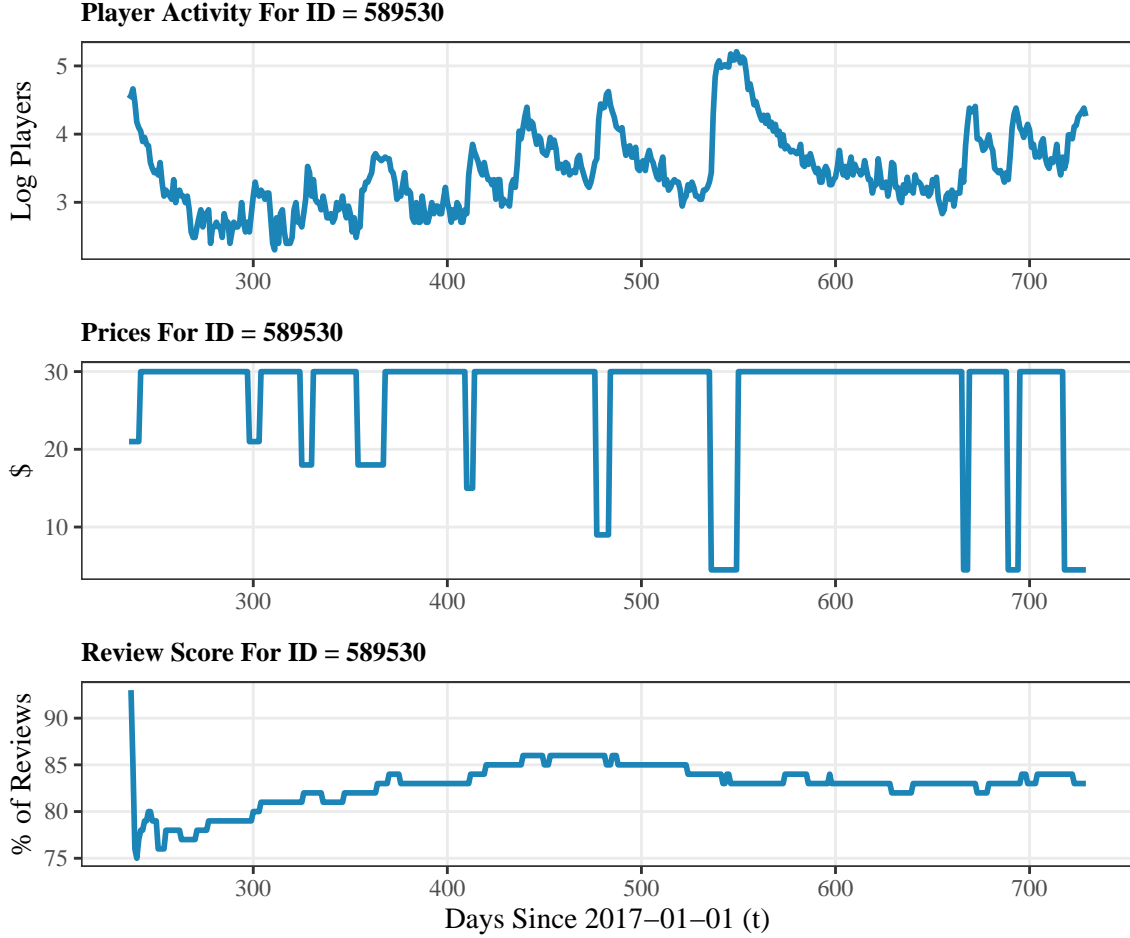
Figure 2: An Example Game in the Sample: Player Count, Price, and Review Score Histories.

paper. First, Steam has made some significant changes to its review system in late 2016, and one of the effects was the removal of the a big number of reviews from the review score calculation. While it could be an intervention that is worthy of an independent study, in practice combining the data from before and after the intervention led to unsound results, so I focus on the two year period from January 2017 to January 2019, keeping the review system as stable as possible. In particular, only games that were released in this time period make it to the sample.

Second, online multiplayer games and games that update frequently are dropped from the sample, as these are products whose quality is changing a lot over time[2]. Unobserved

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[2]Games on Steam have blogs that allow them to share news about updates with the audience. Publishing news about an update is voluntary, so it is possible that some games that continue to update after the release still made it to the sample.

quality would present a big obstacle to the identification of the effect of reviews on pricing decisions or sales, because, say, a game that has just issued a major update might both upgrade to a better review label (because of the positive reception of the new content) and give a big discount to rekindle the interest of the players, but the discounting decision would not be influenced by the review transition. For the same reasons, I also drop free games and games that are released in a beta-version (the so called "Early Access" program), as these products are likely to be adding new content, and the update history is not a perfect way to monitor the updating activity. Finally, I drop few games with a player count of less than four on their median day, as these games are simply very small, and the quality of their player count data is questionable.

3.3   Sample Description

The final sample includes r info[, .N] games which I observe for r panel[, max(age), by = ID][, round(mean(V1))] days each, on average. The main variables in the analysis are the aforementioned dynamic variables: player count, price, and review score history. An example observation from the sample is given in Figure 2. For the majority of the games in the sample, the player count is the highest around the release date, and then it quickly fades away and oscillates around a smaller level. The player count also jumps when a discount is given, reflecting the influx of new players. Games differ a lot in their sizes, as is evident from Figure 3.

As the goal of the paper is to understand if firms use pricing tools in order to improve their online reputation, a detailed overview of pricing and review variables is important. A representative game in the sample never changes its price, but instead occasionally goes on discounts, slowly increasing their magnitude as the game ages. There are r panel[discount==F, diff(price), by = ID][V1>0, .N] incidents of price change in the data, compared to r panel[discNew==T, .N] discounts. Steam has a number of rules that regulate price promotions on its platform. A game can have a launch discount, but, otherwise, it has
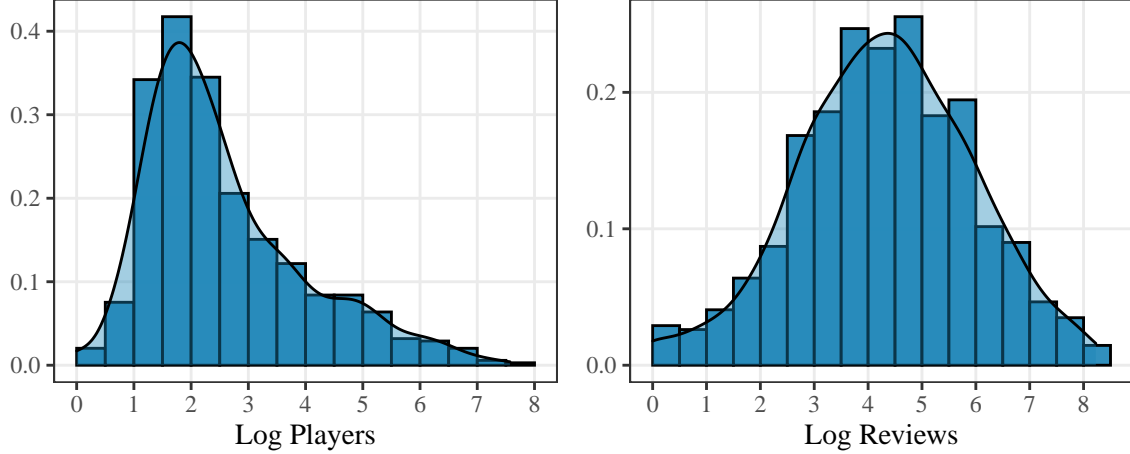
Figure 3: Distributions of the Player Count on, and the Number of Reviews Accumulated by, the Age of 180 Days.

to wait for two months since the release before changing its price or giving a discount. A game can not go on discounts too frequently, and has to wait between four to six weeks after a price promotion to be able to run a new one. The duration of a custom discount is restricted to be at least one full day, and at most two weeks. Besides these custom discounts, that are fully managed by the firms, Steam has a series of curated discounts, when the platform invites selected titles to go on a discount. As Steamworks Documentation [2020a] explains it, "while there aren't strict rules, as a base guideline we tend to focus on the top 10-20% selling games on Steam that are positively reviewed and have otherwise proven to be successful". Curated promotions are featured prominently on Steam's main page, and, arguably, lead to more visibility and sales for the participating titles than custom discounts, which also contribute to visibility, but typically do not get the front-page promotional slots. An important type of curated discounts are the so-called "Seasonal Sales"–big platform-wide events that take place about four times a year around major holidays. Figure 4 shows that the biggest sales take place in Winter (Christmas and New Year) and Summer (July 4), but there are also significant discounts in the Fall (Halloween and Thanksgiving). Around r round(panel[discSeason==T & discNew == T, .N]/panel[discNew==T, .N]*100)% of discounts in the sample go live during a Seasonal Sale. Thus, firms have significant agency when it comes to running price

promotions, but platform regulations and platform-wide discounts are important determinants of firms' decision to discount.
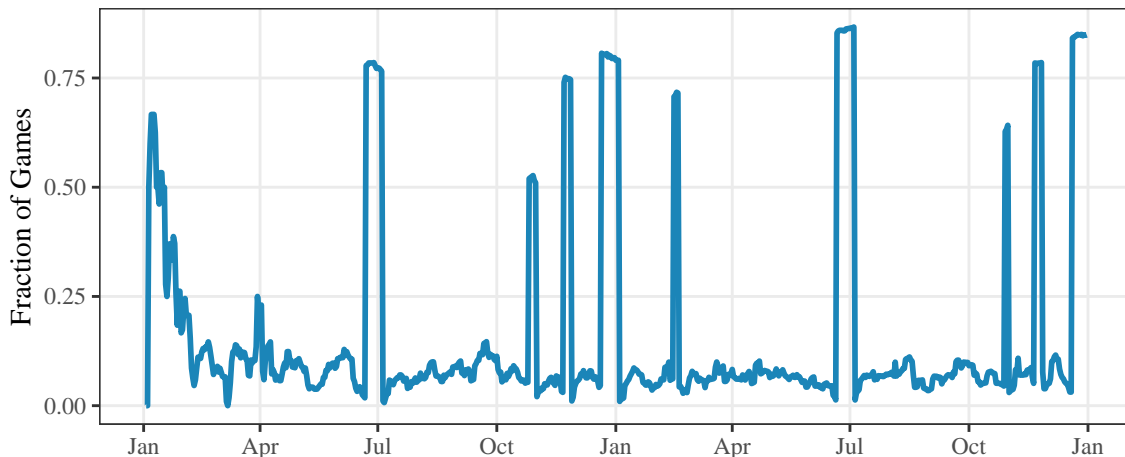

Figure 4: Fraction of Games in the Sample Running a Promotion.

The path of the review score of a typical game is quite different from its price history, in that the review score tends to settle quickly. Recall that the review score is just the fraction of the positive reviews among all reviews, so the law of large number implies that this ratio crystallizes as more reviews flow in. Second plot in Figure 5 shows that games tend to receive half of the reviews they have at the age of 180 days in their first month after the release. On average across games, the standard deviation of the review score (out of one hundred) is just r round(panel[, sd(score, na.rm=T), by = ID][, mean(V1, na.rm = T)], 2), with a split of r round(panel[age <= 30, sd(score, na.rm=T), by = ID][, mean(V1, na.rm = T)], 2) in the first thirty days and r round(panel[age > 30, sd(score, na.rm=T), by = ID][, mean(V1, na.rm = T)], 2) after that. The score is only assigned once the game reaches 10 reviews, and once can see that 10 reviews is enough on average keep the score stable. The distribution of games by bins at the age of 180 days, depicted in Figure 5, is, then, representative of the situation at other ages.

Despite the relative rigidity of the review score, there is enough transitions between the review bins to make the analysis of pricing decisions by firms around such transitions possible. Table 2 describes the transitions between review bins in the sample. The number
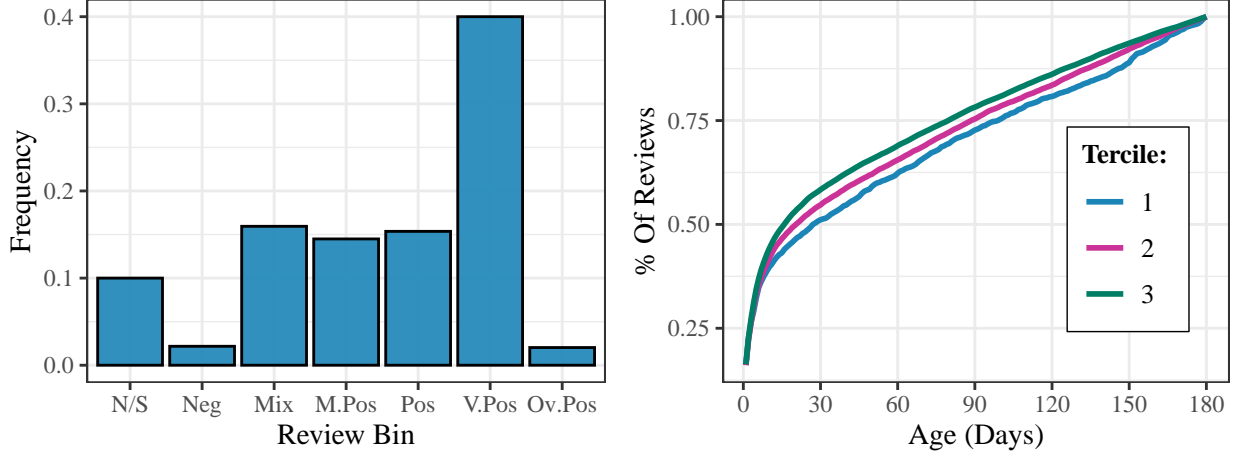
Figure 5: Distribution of Games by Review Bins and the CDF's of Review Arrival Times by Quintiles at 180 Days.

Table 2: Transition Probability And Count Matrices

		Probabilities							Counts			
Neg	Mix	M. Pos	Pos	V. Pos	Ov. Pos		Neg	Mix	M. Pos	Pos	V. Pos	Ov. Pos
0	100	0	0	0	0	Negative	0	58	0	0	0	0
24	0	76	0	0	0	Mixed	60	0	189	0	0	0
0	43	0	25	33	0	M. Positive	0	245	0	141	188	0
0	0	40	0	60	0	Positive	0	2	207	0	310	0
0	0	84	0	0	16	V. Positive	0	0	220	0	0	43
0	0	0	0	100	0	Ov. Positive	0	0	0	0	25	0

of unique games that have changed review labels in the data is r transitions[, unique(ID)] %>% length, and the number of transitions is r transitions[, .N]. Sometimes games switch their bins briefly, and go back soon after. The number of transitions that led to the game spending at least 7 days in the new bin is r transitions[(is.na(dur) | dur >= 7), .N]. Table 3 describes the state of the games in the two weeks before their transitions. Discounting does take place before the transitions, games transition at very different ages, but tend to have only a limited number of reviews by the time their review bin changes.

4 Descriptive Evidence

In this section I present evidence that firms use discounting to manage their online reputation, in particular, by running price promotions when they are close to improving their review label.

Table 3: Average Discount, Age, and Review Count Two Weeks Before A Transition

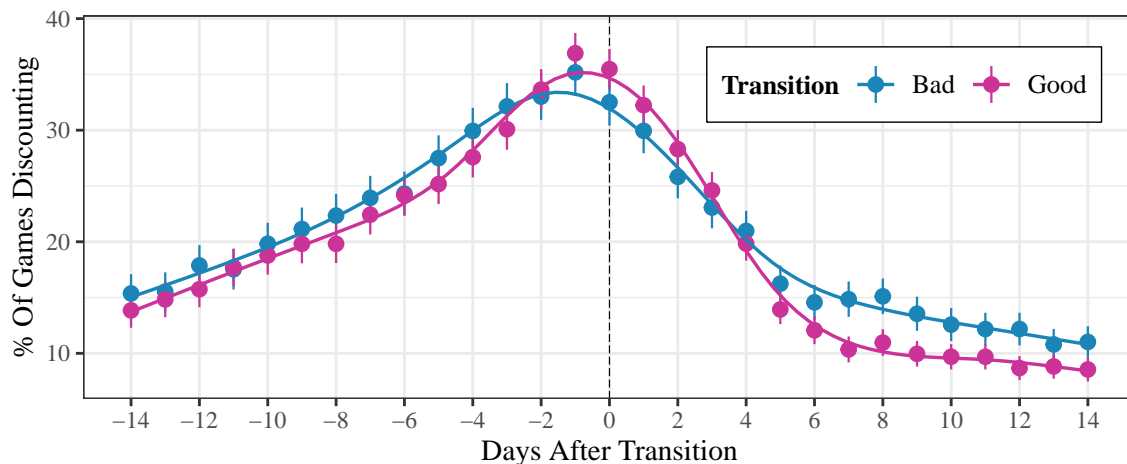| Statistic	N	Mean	St. Dev.	Pctl(25)	Pctl(75)
Mean Discount (%)	1,225	10	14	0	15.9
Age	1,225	142	157	8	226
Reviews	1,225	130	343	26	81

Figure 6: Discounting By Games Around The Transition.

For now, let us resort to a very simplistic story of why a firm would be interested in doing that. Assume that discounts increase the inflow customers, and that more customers means more reviews. Also, take for granted, for now, that better review labels improve sales. Then, if a firm is one positive review away from upgrading its review label, it might be tempted to speed things up by giving a discount. Similarly, a product that is close to sliding into a worse review bin, could be tempted to run a price promotion, in order to avoid a slump in its reviews. I will address this story in more detail in the discussion part of this section, but for now I turn attention to simply establishing that this sort of behavior indeed takes place in the sample.

The departing point of the analysis is Figure 6, which shows that transitions between review labels are very often preceded by discounts. The graph shows that two weeks prior to a transition only 15% of games were on a discount, a number close to the sample average, but that this number more than doubles to around 35% one day

before the label change. Regression analysis controlling for review label, number of reviews, age, day of the week and week time effects, as well as time since the previous discount, confirms that the association depicted in Figure 6 is robust (see Table 5 in the Appendix). A game is approximately r round(reg1$coefficients[2]*14*100) pp more likely to be on a discount on the day of the transition than it is two weeks prior to it, and it is r round(-reg1$coefficients[3]*14*100) pp less likely to be on a discount two weeks following the transition. Given that the probability for a game to be on a discount on a random day is r round(panel[, mean((discount > 0), na.rm=T)]*100)% in the sample, these effects amount to r round(100*(reg1$coefficients[2]/panel[, mean((discount > 0), na.rm=T)]))% and r round(100*(-reg1$coefficients[3]/panel[, mean((discount > 0), na.rm=T)]))% changes in the daily probability of a discount, which is quite sizable.

Of course, this finding simply shows a correlation between firms' discounting behavior and review transitions. The same pattern could emerge if the causality between the two variables is reversed, i.e. the discounts cause transitions, and not the other way around. Imagine a hypothetical world in which games are only bought on discounts. In this world any action in the data would be preceded (and, to some extent, caused) by a discount. In other words, firms could be giving discounts for reasons unrelated to reviews, but transitions sometimes would follow as a result. As a matter of fact, suppose that the story I am after is true, and that firms try to achieve transitions via price promotions. In that case, discounts *have* to be able to aid transitioning, making the reverse causality inherent in this setting. The analysis behind Figure 6 remains imperfect for yet another reason. A discount given when a firm is close to a positive transition is not guaranteed to lead to a transition. Similarly, a firm trying to avoid a negative transition by giving a discount might succeed. In both cases, the behavior that we are interested in goes undetected if one only studies transitions that took place in the data.

The solution to both problems that I suggest is to study *potential* transitional situations instead of the realized ones. First, it solves the reverse causality problem. A discount given

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today can cause a transition tomorrow, but a decision to give a discount can not cause the proximity to a transition that chronologically precedes it. Second, it clearly solves the selection issue explained above, when the analysis considers solely the firms that transitioned.

With the focus now on potential transitions, the crucial question is how to define proximity to a transition. One approach would be to use the raw review count, and to say that a game is close to upgrading its review label when it needs five (ten, twenty ...) more positive reviews to transition. However, given the heterogeneity in the popularity of different games (see Figure 3), five extra reviews could be nothing for a very popular game, and hard to acquire for a small game. For this reason I use a different approach, that measures proximity by the expected number of days that a game has to wait to accumulate the reviews necessary for a transition. In particular, for every game-date pair I first measure how many positive reviews that game needs at the moment to upgrade its review bin, and how many negative reviews that game needs to degrade its review bin. In the next step, for each game I calculate the average speed of review arrivals, by simply dividing the number of positive (negative) reviews as of the last day in the sample by the age of the game on that day. Knowing the speed of positive and negative review arrivals, and the number of reviews necessary for a transition, I then define the proximity to a positive (negative) transition to be the expected number of days needed for the game to accumulate the necessary number of positive (negative) reviews, assuming that it does not receive any negative (positive) reviews during that time.

To illustrate this definition, consider an example game with 19 positive reviews and 6 negative reviews at some moment in time. This game has a review score of 76%, and the "Mostly Positive" review label. It needs 5 additional positive reviews to secure a score of 80%, the threshold that would earn it the "Positive" label. Similarly, 3 new negative reviews would be sufficient for the game to slide into the "Mixed" review category, as the score would become $19/(25 + 3) \times 100\% = 67\%$, which is less than the 70% required for the "Mostly Positive" bin. If this game ends up having 80 positive and 20 negative reviews at the age of 200 days, its last day in the sample, then, on average, it was receiving 0.4 positive and

0.1 negative reviews per day. Thus, for a positive transition it requires $5/0.4 = 12.5$ days of only good reviews arriving at this rate. Similarly, for a negative transition it requires $3/0.1 = 30$ days of only bad reviews arriving at this rate. So, for this game I set its proximity to a potential positive transition to be 12.5 days, and its proximity to a potential negative transition to be 30 days.

As the example above shows, it is impossible to consider proximity to a positive transition without taking into account the proximity to a negative transition, at least for moderately sized games. If firms' strategies for these types of transitions are different, then studying such transitions separately could attenuate the effect of the proximity to, say, a positive transition, on the discount probability: a firm that is close to upgrading its review bin might not give a discount not because it is a bad way to accomplish that transition, but because that firm is at the same time close to downgrading its review bin, and might prefer to exercise prudence. For this reason, I define two "treatment" variables of interest. Game i at date t is said to be close to a positive transition, $T_{it}^{+} = 1$, if its proximity to a positive transition is less than or equal to r 2*band days. Similarly, game i at date t is said to be close to a negative transition, $T_{it}^{-} = 1$, if its proximity to a positive transition is less than or equal to r 2*band days. The r 2*band cutoff is inspired by the maximum duration of custom discounts on Steam, and the patterns of discounting around successful transitions, depicted in Figure 6. To estimate the effect of being close to a review transition on the discounting behavior, I then estimate the following model:

$$disc_{it} = \beta^{+}T_{it}^{+} + \beta^{-}T_{it}^{-} + X_{it}\beta + f_i + \tau_t + \varepsilon_{it}, \tag{1}$$

$disc_{it} = \mathbb{1}\{Discount_{it} > 0\}$ is a dummy measuring if the game is on a discount or not, X_{it} is a set of control variables that includes log proximities to positive and negative transitions, log review count, review bin dummies, and age; f_i is a set of game-level fixed effects, and τ_t is a set of day of the week effects and sample week time effects. These time effects are especially important to include in the regressions with discounting variables on the left hand side, because Steam's curated discounts all start on predetermined days of the week, and

seasonal sales affect a big number of games at the same time, as depicted in Figure 4. We expect β^+ to be positive, but, when it comes to β^-, any sign could be rationalized. A positive sign on β^- would serve as evidence that firms use discounting in order to escape from slumps in their reviews bins, while a negative sign would indicate that, on the contrary, firms on the verge of a bad transition become more prudent with their discounts, trying to not instigate extra purchasing/reviewing activity.

Table 4: Discounts Close to Potential Transitions

	Dependent variable:		
	Discount Probability		
	Full	1 W. to Tr.	Simple Tr.
Close to Pos. Transition	0.010***	0.009***	0.007***
	(0.002)	(0.002)	(0.003)
Close to Neg. Transition	−0.008***	−0.002	−0.007***
	(0.002)	(0.002)	(0.003)
Log Days to Pos. Tr.	0.005***	0.004***	0.007***
	(0.001)	(0.001)	(0.001)
Log Days to Neg Tr.	−0.003***	−0.002***	−0.001
	(0.001)	(0.001)	(0.001)
Log Reviews	−0.003***	−0.002***	−0.003***
	(0.001)	(0.001)	(0.001)
Mostly Positive	−0.008***	−0.009***	−0.007***
	(0.002)	(0.002)	(0.002)
Positive	−0.012***	−0.013***	
	(0.002)	(0.002)	
Very Positive	−0.008***	−0.007***	
	(0.002)	(0.002)	
Age	0.0001***	0.0001***	0.0001***
	(0.00000)	(0.00000)	(0.00001)
Age ≤ 14	0.050***	0.049***	0.018*
	(0.005)	(0.005)	(0.009)
Time Effects: Weekdays, Week	✓	✓	✓
Game Effects	✓	✓	✓
Poly(t W/O Discount, $d = 2$)	✓	✓	✓
Observations	295,228	295,228	113,901
R^2	0.172	0.172	0.173

Note:	*p<0.1; **p<0.05; ***p<0.01

Notice that the inclusion of both the distance to a potential positive, and a potential negative, transition in the regression drops all observations from the lowest ("Negative") and the highest ("Overwhelmingly Positive") review bins, as for games in those bins only one way of review transition is possible. However, given that the proportion of such observations in the sample is quite small, this is not a big concern. The analysis also excludes the observations with a "No Review Score" label, as I do not want to take a stance on what constitutes an improvement or a deterioration of the review score for such games. The results of estimating Equation 1 are presented in Table 4. The first column is the full specification that uses all observations in the sample (except the omissions just mentioned). The second column uses a more stringent definition of proximity to a transition, requiring a game to be r band, rather than r 2*band, days away from a potential transition to be counted as being "close to a transition". The third column uses only transitions between "Mixed", "Mostly Positive", and "Positive" labels, as these are the transitions that rely on the review score only, while "Very Positive" and "Overwhelmingly Positive" require both a certain review score, and a certain number of reviews.[3]. Sorokin and Stevens [2020] exclusively uses these simple review bins in their regression discontinuity analysis, precisely in order to avoid the complications that arise from double thresholds required for "Very Positive" and "Overwhelmingly Positive".

The results in Table 4 unequivocally support the hypothesis that proximity to a review bin upgrade increases firms' willingness to run a price promotion. Measured against the r round(panel[, mean((discount > 0), na.rm=T)]*100)% probability for a random game-day pair from the sample to have a discount\footnote{The corresponding number for the games in the treatment group is r round(100*panel[ID %in% panel[pos.treatment == 1, unique(ID)], mean((discount > 0), na.rm = T)]).}, the effects constitute a r round(100*(reg3$coefficients[1]/panel[, mean((discount > 0), na.rm=T)]))-r round(100*(reg1$coefficients[1]/panel[, mean((discount > 0), na.rm=T)]))% increase in the daily probability of a discount. The uncertainty about the sign and the significance of the

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[3]Technically, a game improving its review score from 79% to 80% can transition to both "Positive" and "Very Positive", so the results in the third column bundle together these types of transitions.

proximity to a potential negative transition is resolved in favor of the prudent approach on the firms' side: the results suggest that firms under a risk of deteriorating their review label are r -round(100*(reg3\$coefficients[2]/panel[, mean((discount > 0), na.rm=T)]))-r -round(100*(reg1\$coefficients[2]/panel[, mean((discount > 0), na.rm=T)]))% less likely to go on a discount, albeit the effect is only present in two out of three specifications.

4.1 Discussion

It is not unreasonable to say that model 1 is able to identify the causal effect of the proximity to a review transition on discounting behavior. As the sample was selected to include only products with the most stable quality and the lowest number of game updates, the path of the review score is relatively exogenous for the firms. Taking its current review score as given, a firm can decide whether to run a price promotion or not, which makes the main independent variables in my analysis exogenous. On top of that, model 1 includes a rich set of controls and fixed effects. Some threats to identification still remain, of course. One is unobserved marketing interventions preceding discounts. If advertisements improves the review score, then, by definition, advertisement would be correlated with the treatment variable, moving games a bit closer to transitions. If a discount then follows, as a part of the general marketing plan, rather than to leverage the new proximity to a transition, the positive association between discounting and positive transitions would be spurious. Note, however, that it would be more natural to expect firms to roll out all marketing interventions at the same time (i.e., running a price promotion while the game is advertised). In that case, the threat to identification vanishes.

I would now like to step back and look at the story behind the behavior that I am documenting. Several gaps are not filled yet. First, I have not presented evidence that discounting meaningfully increases sales and leads to more reviews. Neither have I presented evidence that better review bins increase sales. Perhaps, more importantly, I have not discussed why firms should expect customers buying during a discount to leave better reviews

than customers paying the regular price. If there is no difference between the two groups, then giving a discount when the game is close to a transition is, at best, a gamble. In fact, there is some empirical evidence that customers who choose to go to a restaurant because of a promotion tend to leave worse reviews [Byers et al., 2012, Li, 2016], even though the effect could be positive in certain cases [Li, 2016, Zhu et al., 2019]. Theoretically, this effect is ambiguous. Consumers who buy when the price is low could be a worse match for the product and, thus, leave worse reviews. At the same time, such customers get a higher utility from paying less, which, together with a desire to reciprocate, can lead them to leave better reviews. See, for example, Acemoglu et al. [2019].

The results I presented in this section, therefore, lead to the following hypotheses. I show that firms give discounts to avoid bad transitions and to facilitate the good ones; thus, it has to be the case that the reviews left on a discount tend to be more positive than the ones left off a discount. Second, if that is indeed the case, then the firms that get especially positive reviews during price promotions should be more likely to try to use discounting to manipulate their reviews. Heterogeneity between the firms is crucial for the second hypothesis. In order to test these hypotheses, and to provide evidence on the effect of discounting on sales and reviewing activity, as well as the effect of reviews on sales, I now turn to formulation and estimation of a structural model demand, reviewing and gaming activity on Steam.

5 Empirical Model

5.1 Setup

Consider game i that is observed on a daily basis. On day t the game sells one copy to each of the B_{it} short-lived buyers that arrive on that day, a number that is unobserved by the econometrician. Define *active players* of this game, A_{it}, to be the customers who have already purchased the game and are still playing it, either because they have not yet completed it or are not bored with it yet. This number has an empirical counterpart in the face of the player

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count variable, observable to the econometrician. The game loses E_{it} players on day t, which is also not observed. I assume that, once a player stops playing the game, she never returns to it again. It is easy to see then that A_{it} follows the following process:

$$A_{it} = A_{it-1} + B_{it} - E_{it} \tag{2}$$

Both the arrival of buyers and the exit of players are not observed, so some assumptions need to be made about them in order to make progress.

Assumption 5.1 *B_{it} is Poisson with arrival rate $\lambda_{it} = \lambda_i(1 + x'_{it}\beta)$, where x_{it} is a vector of observable characteristics of the game, and λ_i and β are parameters. E_{it} follows a binomial distribution $B(A_{it-1}, 1 - \psi_i)$, where ψ_i is a parameter.*

The rationale behind assumption (5.1) is simple. Consumers arrive every day according to the game specific arrival rate λ_i, but that rate can go up or down depending on the values of observable characteristics x_{it} of the game that affect demand: price, reviews, age of the game, or seasonal factors. The mapping between these variables and the number of copies sold is, of course, nothing else but the demand curve for game i. A change of 0.01 in the index $x'_{it}\beta$ means that quantity demanded of game i goes up by one percent. This multiplicative structure is necessary, because it will allow me to estimate the price sensitivity and the effect of reviews on sales in relative terms[4]. Buyers of the game become active players, and are subject to a fixed daily risk of $1 - \psi_i$ of abandoning the game. Given that the number of active players "flipping" this coin at the end of day $t - 1$ is A_{it-1}, this process gives rise to the binomial distribution for the number of exiters.

Every buyer of game i is a potential reviewer. I assume that a buyer who buys the game at $t - k$ leaves a positive (negative) review for the game on day t with probability r_{it}^+ (r_{it}^-), and no review otherwise. For the reasons I will explain later, I will refer to r_{it}^+ as the *like rate*, and to r_{it}^- as the *dislike rate*. The focus of the analysis is on the difference between the reviews left on and off a discount. To that end, I parametrize the like and dislike rates to

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[4]See the Identification section for more details

depend on the discounting behavior of the firm as follows.

Assumption 5.2 *The like rate is a linear function of discounts: $r_{it}^{+} = \rho_{0i}^{+} + \rho_{1i}^{+} disc_{it-k}$. The dislike rate is a linear function of discounts: $r_{it}^{-} = \rho_{0i}^{-} + \rho_{1i}^{-} disc_{it-k}$.*

Notice, that a price change affects the reviewing behavior in two ways. First, a lower prices increases the inflow of buyers through a higher λ_{it}. This leads to more reviews being left. My parametrization allows to further study if the reviewers who bought the game on a discount are more or less likely to leave positive and negative reviews. I use a dummy variable $disc_{it-k}$ to measure discounting because this will allow me to derive a closed-form estimator for the parameters of interest.

5.2 Identification

5.2.1 Demand Parameters

The prediction of A_{it} that follows from (2) takes the form of

$$\mathbb{E}\left[A_{it} \mid A_{it-1}, x_{it}\right] = A_{it-1} + \lambda_i(1 + x_{it}'\beta) - (1 - \psi_i)A_{it-1} = \psi_i A_{it-1} + \lambda_i(1 + x_{it}'\beta), \quad (3)$$

so the model implies the following regression equation

$$A_{it} = \psi_i A_{it-1} + \lambda_i(1 + x_{it}'\beta) + u_{it}, \quad (4)$$

with $\mathbb{E}\left[u_i \mid A_{it-1}, x_{it}\right] = 0$. This model is non-linear in parameters, because λ_i is not known and enters the model multiplying β, the parameters common to all games. If not for this commonality in β, estimation of (4) would be straightforwardly achieved by opening the parentheses and estimating

$$A_{it} = \lambda_i + \psi_i A_{it-1} + x_{it}'\beta_i + u_{it} \quad (5)$$

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using OLS on a game-by-game basis. This commonality is essential, however, because one game in the sample typically does not exhibit enough variation in review labels to be able to identify the effect of upgrading the review tier on sales.

The fact that $x'_{it}\beta$ multiplies the game fixed effect λ_i allows me to estimate the dependence of quantity sold on price and reviews in relative terms, i.e., to obtain the elasticities. Normally, a log transformation is used to achieve that. Indeed, Sorokin and Stevens [2020] is able to estimate these elasticities with a within-estimator, using the following model

$$\log A_{it} = \tilde{\lambda}_i + \tilde{\psi}_i \log A_{it-1} + x'_{it}\tilde{\beta} + u_{it}. \tag{6}$$

This specification "controls" for the (log) number of continuing players in order to overcome the non-availability of direct sales data; however, it is the numbers of active players and buyers that are additive, not their logs. Thus, the structural model I formulate confronts the estimation problem in a more "heads-up" way, and in that regard offers an improvement over Sorokin and Stevens [2020]. I will contrast the results obtained by the two approaches in the results section, comparing a more detailed structural approach with a less precise, but a more easily implementable, log regression approach.

Another important reason for insisting on estimating the relative effects stems from the limitations of my data. Recall that the player count variable measures only the *maximum concurrent* number of active players every day. This implies that the estimated values of λ_{it} would take into account only those new buyers who contribute to gaming activity during the "rush hour". An implicit assumption in my analysis is that new buyers of the game all choose their gaming time following the same game-specific distribution. I use this assumption to say that, if the number of new active players during rush hour goes up by 1%, then sales go up by 1% across all types of players, not only among the "rush hour ones".

Assumption $\mathbb{E}\left[u_i \mid A_{it-1}, x_{it}\right] = 0$ holds by definition for Model 4, and guarantees identification of all parameters, as long as there is sufficient variation in the data. Of course, this

is only true as long as equation 4 is the right model of the data generating process. A threat to identification would come from unobserved demand shifters $\tilde{x}_{it}$ that are correlated with the observed factors x_{it} (omitted variable bias). For instance, an advertising intervention that is coupled with a price promotion would increase demand, but the entire effect would be attributed to the observed change in price. My approach is vulnerable to such events, as long as one is interested in getting the causal estimate of the discount elasticity of demand.

However, I argue that (4) is an adequate specification, if the goal is to estimate the causal effect of reviews on sales, or to get a predictive model of demand. First and foremost, the reviews are not a choice variable of a firm, and rather serve as a state variable that a firm takes as given every day. Of course, there are various things that a firm can do, that can, in a non-guaranteed fashion, affect this variable. But, as long as the major tools that a firm has access to are controlled for in the regression, the exogenous variation in the review variables is sufficient to identify the causal effect of reviews on sales.

One important way in which I control for various tools that firms have at their disposal is through sample selection. Sample selection rules out the possibility of an omitted variable bias stemming from a number of variables that could collectively be referred to as "changing quality". The games in the sample are single player games, and thus are not subject to time-varying network externalities or frequent quality updates, that could be correlated with reviews, depths of discounts, and the player count. Second, game specific effect λ_i controls for all time-invariant characteristics of game i that determine average sales: initial marketing budget, extraneous popularity of the game's plot or setting, etc. Similarly, time effects control for important within-week seasonality in gaming patterns and for the extensive platform-wide sales. Third, note that Equation (4) allows each game to have its own continuation probability ψ_i, which could prove crucial in eliciting sales from the player count data. Two games could have the same observed median daily player count—say, ten people—but very different sales levels, if game one offers a lot of replayability and is played by the same ten people over and over again, and game two is played by new ten people every day. Intuitively, ψ_i is identified

by the rate of decay of the player count when it is far from its (slowly-changing) trend or average. Game release and discounts offer such events, when the player count spikes briefly due to all the new players who just bought the game; see example game in Figure 2.

5.2.2 Price Elasticity of Demand

To close off the discussion of identification of the demand parameters in Equation (4), I would like to elaborate on the identification of the price elasticity parameter. As I mentioned before, at the very least, including the price in the regression controls for unobserved marketing interventions that are coupled with discounts. Estimating the true price elasticity of demand is not important for the questions addressed in this paper, as I am not trying to prescribe the sizes of the discounts that firms should be giving to have a meaningful chance of affecting their review labels when they are close to a review transition. Arguably, a 100% discount is a powerful enough option to make this strategy viable, at least in principle. However, should the price coefficient be of primary interest, I would like to list some further factors that have an impact on the identification of thereof.

First, I believe that standard concerns about the endogeneity of prices are not directly applicable in my context. The reason is the high frequency nature of the data and the stickiness of posted prices. Using the classic notation, imagine that a discount is given on date t, and we observe a quantity-price pair (Q_t, P_t), both of which are different from their yesterday's counterparts (Q_{t-1}, P_{t-1}). The standard endogeneity concern is that firm's demand is subject to shocks, and that the firm would change its price precisely when those shocks occur. The two points then, roughly speaking, would belong to different demand curves, and one can not identify the slope of the demand curve[5]. In my setting, however, this would require the firms to systematically give discounts *exactly on the days* of the demand shocks, which requires possessing a level of insight into one's demand condition that is unrealistic, especially for small independent studios. While a discount for a racing game on

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[5]Assuming, as is usually done, that demand shifters lead to parallel shifts in demand

the day of a major F-1 race is not implausible, should the demand for the game go up with a lag of as little as one day, then both (Q_t, P_t) and (Q_{t-1}, P_{t-1}) would belong to the same demand curve, and, therefore, identification would not be threatened.

My second point is that, even though unobserved marketing interventions that are coupled with price promotions, would, undoubtedly, be an issue, this problem could be addressed with some extra data collection. One way to proceed would be to study if such coupled promotions in fact do take place. In particular, one could collect data on YouTube queries mentioning the games in the sample, and use spikes in such queries as a proxy for unobserved marketing campaigns.

A deeper problem for identifying the price elasticity of demand lies in defining precisely the elasticity of interest. A video game is a durable good, and some consumers could be purchasing it strategically, thinking about the probability and depth of discounts that they could get in the near future. A discount that is announced a week in advance will appear to have a strong effect on sales, but only because consumers would abstain from purchasing the game throughout that week. For studying counterfactual price policies one would need to specify a more complicated model of forward-looking consumers, and estimate the fundamentals of their behavior. A related problem is the importance of salience in a marketplace that has many thousands of products. Any price elasticity estimated relies on Steam keeping its algorithms unchanged. A game can give a 99% discount, but the quantity demanded will not change much if that promotion happens to not be reflected in Steam's system. This has implications, for instance, for picking good instruments for prices. Curated discounts created and managed by Steam are suggested to firms, and not chosen by them. In principle, such price changes could be exogenous to daily demand conditions. However, they could hardly be used as instruments for the price, because they also bring an immense boost in visibility, by the virtue of occupying the prime store main page space.

5.2.3 Review Parameters

The part of the model describing the reviewing behavior of the buyers is more starightforward. Intuitively, the propensity to leave a review on and off a discount (the like and dislike rates r_{it}^+, r_{it}^-) are identified by the differences in reviews left on and off a discount. In fact, this intuition becomes a rigorous proof, as wil be shown shortly.

Following assumption 5.1, the arrival of buyers for game i on day t follows a Poisson distribution with arrival rate $\lambda_{it} = \lambda_i(1 + x_{it}'\beta)$. I also assumed that, k days later, each consumer leaves a positive review with probability r_{it}^+, a negative review with probability r_{it}^-, and no review otherwise. Then, the following proposition is true:

Proposition 5.1 *The number of good reviews G_{it} for game i on day t is distributed Poisson with rate $r_{it}^+\lambda_{it-k}$. The number of bad reviews B_{it} for game i on day t is distributed Poisson with rate $r_{it}^-\lambda_{it-k}$. Moreover, G_{it} and B_{it} are independent.*

Proposition 5.1 allows one to easily write down the likelihood for positive and negative reviews separately. I will use the positive reviews as the leading example here, but all the findings automatically translate to the negative reviews case as well. For every game I observe the history of the review arrivals $\{(g_{it}, b_{it})\}_{t=1}^{T_i}$. Since $\mathbb{P}(G_{it} = g_{it}) = \frac{(r_{it}^+\lambda_{it-k})^{g_{it}}}{g_{it}!}e^{-r_{it-k}^+\lambda_{it-k}}$, the log-likelihood of the history of likes is given by

$$\ell(g_i; r_{it}^+, \lambda_{it-k}) = \sum_{t \geq k} g_{it} \log r_{it}^+\lambda_{it-k} - r_{it}^+\lambda_{it-k} - \log g_{it}! \tag{7}$$

I treat λ_{it} as known, because parameters λ_i and β that determine it can be estimated consistently for the majority of games in the sample, given the long time dimension of the panel. For now I also take the lag k between the time when the user buys the game and leaves a review for the game to be known. The parameters of interest are $(\rho_{0i}^+, \rho_{1i}^+)$, which parametrize the like rate as $r_{it}^+ = \rho_{0i}^+ + \rho_{1i}^+ disc_{it-k}$. The likelihood is concave in the parameters, so the parameters are identified as the maximizer of the likelihood. Restricting attention to a binary discount variable $disc_{it}$ allows me to derive a closed-form estimator for $(\rho_{0i}^+, \rho_{1i}^+)$.

During a discount the like rate of game i is $\rho_{0i}^+ + \rho_{1i}^+$, which is consistently estimated as

$$\hat{\rho}_{0i}^+ + \hat{\rho}_{1i}^+ = \frac{\sum_{t \geq k} disc_{it-k} g_{it}}{\sum_{t \geq k} disc_{it-k} \lambda_{it-k}}, \tag{8}$$

As we can see, the ML-estimator of the like rate during a price promotion is just the ratio of the number of good reviews left on a discount over the expected arrival on that day. Similarly, the like rate off a discount can be estimated by

$$\hat{\rho}_{0i}^+ = \frac{\sum_{t \geq k} (1 - disc_{it-k}) g_{it}}{\sum_{t \geq k} (1 - disc_{it-k}) \lambda_{it-k}}, \tag{9}$$

with the interpretation that the like rate in the absence of a discount is just a ratio of the reviews left outside of a discount and the expected number of customers outside a discount.

Note that one can use these estimators to estimate the *average review score* on and off a discount, i.e., whether users are relatively more or less likely to leave a good review during a discount. For example, without a price promotion, one obtains

$$\frac{\sum_{t \geq k} (1 - disc_{it-k}) g_{it}}{\sum_{t \geq k} (1 - disc_{it-k}) g_{it} + \sum_{t \geq k} (1 - disc_{it-k}) b_{it}} = \frac{\hat{\rho}_{0i}^+}{\hat{\rho}_{0i}^+ + \hat{\rho}_{0i}^-} \tag{10}$$

It is intuitively clear that the relative propensity to leave a good, rather than a bad, review should be easily estimable by comparing the ratios of good to bad reviews, without any knowledge of the quantity of games sold. This intuition is fully reflected in Equation (10), where the arrival rates λ_{it-k} have disappeared entirely.

The analysis above is complicated by two facts. First, for each game I observe the exact review count, but my demand estimates only apply to rush hours. If, say, every player leaves a review, and every day there are two new players who play in the morning, and three new players who play in the evening, my data would register 3 players leaving 5 reviews on a daily basis. This is the reason why I refer to the values of (r_{it}^+, r_{it}^-) as rates, rather than probabilities (which they are in the model). Propositon 5.1, stating that the number of good

reviews on day t for game i is a Poisson random variable with rate $r_{it}^+ \lambda_{it-k}$, could be treated as an assumption, rather than a result. In that case, nothing constrains the r_{it}^+ parameter to be less than 1, and the identification argument goes through in the exactly the same way.

Another problem stems from the fact that the purchase date behind a review is not available to the researcher. For that reason, the model features an additional parameter k, the lag between purchasing the game and leaving a review, that I assumed to be known. To estimate that parameter I leverage the fact that spikes in player activity on the first day of a sale represent new users, and study the review response during the following week in order to uncover the modal lag for leaving a review (see Table 6 in the Appendix). This number turns out to be one day. Even though time to posting a review has a non-degenerate distribution, modelling it in a more nuanced way would significantly contribute to complexity: the likelihood of receiving a poisitive review on day 100 would depend on the entire history of buyer arrivals prior to that date. Therefore, I proceed using $\hat{k} = 1$.

5.3   Estimation and Results

5.3.1   Demand Parameters

5.3.2   Review Parameters

A Additional Tables and Figures

A.1 Discounts Around Transitions

Table 5: Discounts In The Days Around Transition

	Dependent variable:
	Discount Probability
Days to Transition	0.006***
	(0.0005)
Days After Transition	−0.011***
	(0.0005)
No Score	0.181***
	(0.026)
Negative	0.004
	(0.010)
Mostly Positive	−0.001
	(0.005)
Positive	0.032***
	(0.007)
Very Positive	0.001
	(0.006)
Ov. Positive	0.004
	(0.013)
Log Reviews	−0.011***
	(0.002)
Age	0.0003***
	(0.00001)
Const	0.872***
	(0.014)
Time Effects	Weekdays, Week
Game Effects	×
Polynomial(t W/O Discount)	$d = 2$
Observations	32,412
R^2	0.397
Note:	*p<0.1; **p<0.05; ***p<0.01

A.2 Review Lag

Table 6 presents the results of estimating

$$NewReviews_{it} = \sum_{l=0}^{7} \mathbb{1}(\text{Disc. } l \text{ days ago}) + reviewBin_{it} + age_{it} + young_{it} + f_i + \tau_t + u_{it}$$

Table 6: Spikes in Reviews After a Discount

	Dependent variable:
	New Reviews
Discount 0 Days Ago	1.107***
	(0.123)
Discount 1 Day Ago	3.308***
	(0.277)
Discount 2 Days Ago	1.397***
	(0.139)
Discount 3 Days Ago	0.748***
	(0.110)
Discount 4 Days Ago	0.612***
	(0.096)
Discount 5 Days Ago	0.397***
	(0.104)
Discount 6 Days Ago	0.036
	(0.086)
Discount 7 Days Ago	−0.160**
	(0.073)
Time Effects	Weekdays, Week
Game Effects	✓
Observations	355,983
R^2	0.054
Note:	*p<0.1; **p<0.05; ***p<0.01

B Mathematical Appendix

B.1 Proofs

Proposition 5.1. Let the total number of reviewers during a day, R, be distributed Poisson with rate λ, $R \sim P(\cdot; \lambda)$. A reviewer leaves a good review with probability π, and a bad review with probability $1 - \pi$. Then, the numbers of good and bad reviews, G and B, are independent Poisson random variables with rates $\pi\lambda$ and $(1 - \pi)\lambda$.

In the main text I have a Poisson arrival of customers, and only a fraction of them become reviewers, but the same proof goes through with that modification as well. The independence between the numbers of good and bad reviews is the surprising part of the proposition, and it is better highlighted in the form presented here.

Proof: We start by showing that G is Poisson.

$$\mathbb{P}\left(G = g\right) = \sum_{n=g}^{\infty} \mathbb{P}\left(G = g \mid R = n\right) \mathbb{P}\left(R = n\right) = \sum_{n=g}^{\infty} \frac{n!}{g!(n-g)!} \pi^g (1 - \pi)^{n-g} \frac{\lambda^n}{n!} e^{-\lambda}$$

We leave only the parts that depend on n within the sum:

$$e^{-\lambda} \frac{\pi^g}{g!} \sum_{n=g}^{\infty} \frac{(1 - \pi)^{n-g} \lambda^n}{(n-g)!} = e^{-\lambda} \frac{\pi^g \lambda^g}{g!} \sum_{n=g}^{\infty} \frac{(1 - \pi)^{n-g} \lambda^{n-g}}{(n-g)!} = e^{-\pi\lambda} \frac{\pi^g \lambda^g}{g!} \sum_{k=0}^{\infty} \frac{[(1 - \pi)\lambda]^k}{k!} e^{-(1-\pi)\lambda}$$

The sum we have is just $\sum_{k=0}^{\infty} P(k; \lambda) = 1$, so the answer is

$$\mathbb{P}\left(G = g\right) = \frac{[\pi\lambda]^g}{g!} e^{-\pi\lambda} = P(g; \pi\lambda)$$

Now the independence part. We are interested in $\mathbb{P}\left(G = g, B = b\right)$, which could be written as

$$\mathbb{P}\left(G = g, B = b\right) = \mathbb{P}\left(G = g, B = b \mid R = g + b\right) \mathbb{P}\left(R = g + b\right) = \frac{(g + b)!}{g!b!} \pi^g (1 - \pi)^b \frac{\lambda^{g+b}}{(g + b)!} e^{-\lambda}$$

Simply write $e^{-\lambda} = e^{-\pi\lambda}e^{-(1-\pi)\lambda}$, and collect the terms with g and with b to get

$$\mathbb{P}\left(G = g, B = b\right) = \frac{\pi^g \lambda^g}{g!}e^{-\pi\lambda}\frac{(1-\pi)^b\lambda^b}{b!}e^{-(1-\pi)\lambda} = P(g; \pi\lambda)P(b; (1-\pi)\lambda)$$

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B.2   NLLS estimator of β

In this appendix I develop the estimator for the parameters of the model

$$y_{it} = \psi_i y_{it-1} + \lambda_i(1 + x_{it}'\beta) + u_{it}, \tag{11}$$

This model has $2n$ i-specific parameters (λ_i, ψ_i) and $\dim(\beta)$ parameters that are shared by all entities in the panel. Estimation is via Non-linear Least Squares. The F.O.C. of the problem could be reduced so as to concentrate out all (λ_i, ψ_i) parameters. Thus, estimation of $2n + \dim(\beta)$ parameters reduces to solving a system of non-linear equations for $\dim(\beta)$ parameters (or solving a NLLS problem of that dimension).

The estimator is defined as the minimizer of the sum of squared errors:

$$\hat{\theta} := \operatorname*{argmin}_{\lambda_i, \psi_i, \beta} \sum_{i=1}^{n} \sum_{t=2}^{T_i} (y_{it} - \psi_i y_{it-1} - \lambda_i(1 + x_{it}'\beta))^2 \tag{12}$$

The F.O.C. are

$$\psi_i : \sum_{t=2}^{T_i} (y_{it} - \psi_i y_{it-1} - \lambda_i(1 + x_{it}'\beta))y_{it-1} \qquad = 0 \tag{13}$$

$$\lambda_i : \sum_{t=2}^{T_i} (y_{it} - \psi_i y_{it-1} - \lambda_i(1 + x_{it}'\beta))(1 + x_{it}'\beta) = 0 \tag{14}$$

$$\beta : \sum_{i=1}^{n} \sum_{t=2}^{T_i} (y_{it} - \psi_i y_{it-1} - \lambda_i(1 + x_{it}'\beta))\lambda_i x_{it} \quad = 0 \tag{15}$$

Conditional on β, the F.O.C. for (ψ_i, λ_i) are F.O.C.'s of an OLS problem of the form

$$\min_{\psi_i, \lambda_i} \sum_{t=2}^{T_i} (y_{it} - \psi_i y_{it-1} - \lambda_i z_{it})^2,$$

where $z_{it} = (1 + x_{it}'\beta)$. Defining, in the standard way, $\tilde{\mathbf{Z}}_i(\beta)$ to be the matrix with row t given by $[z_{it+1}, y_{it}]$, and y_i to be the vector of y_{it} observations, we get that the values of $\hat{\lambda}_i, \hat{\psi}_i$

that solve (13)-(14) are given by

$$\begin{bmatrix} \hat{\lambda}_i \\ \hat{\psi}_i \end{bmatrix} (\beta) = (\tilde{\mathbf{Z}}'_i(\beta)\tilde{\mathbf{Z}}_i(\beta))^{-1}\tilde{\mathbf{Z}}'_i(\beta)y_i \tag{16}$$

The estimator $\hat{\beta}$ of β is now obtained as a solution to

$$\sum_{i=1}^{n}\sum_{t=2}^{T_i}(y_{it} - \psi_i(\beta)y_{it-1} - \lambda_i(\beta)(1 + x'_{it}\beta))\lambda_i(\beta)x_{it} = 0 \tag{17}$$

In practice I solve the concentrated minimization problem using (17) as the gradient.

Supplying an analytic expression for the Hessian matrix has proven to significantly expedite and improve convergence. The concentrated out sum of squared errors is $SSE(\beta) = SSE(\beta, \lambda(\beta), \psi(\beta))$, and while the expression for the gradient simplifies to $\partial SSE/\partial\beta$ due to the envelope theorem, the expression for the Hessian is more complex:

$$\frac{dSSE}{d^2\beta} = \frac{\partial SSE}{\partial^2\beta} + \sum_i \frac{\partial SSE}{\partial\lambda_i\partial\beta}\left(\frac{d\lambda_i}{d\beta}\right)' + \sum_i \frac{\partial SSE}{\partial\psi_i\partial\beta}\left(\frac{d\psi_i}{d\beta}\right)' \tag{18}$$

The expressions for $\frac{d\lambda_i}{d\beta}$ and $\frac{d\psi_i}{d\beta}$ are obtained by applying the inverse function theorem to (16), and the remaining derivatives could be obtained directly.

The asymptotic covariance matrix for $\hat{\theta}$ in a nonlinear LS problem $\sum_i(y_i - m(x_i, \theta))^2$ is estimated using the sample analog of

$$\mathbf{V}_\theta = (\mathbb{E}\left[m_{\theta i}m'_{\theta i}\right])^{-1}\mathbb{E}\left[m_{\theta i}m'_{\theta i}e_i^2\right](\mathbb{E}\left[m'_{\theta i}m_{\theta i}\right])^{-1}, \tag{19}$$

where $m_{\theta i} = \frac{\partial}{\partial\theta}m(x_i, \theta_0)$, $e_i = y_i - m(x_i, \theta_0)$ [Hanses, 2020, p.751]. More details could be found in the code.

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