What is a contraction mapping?

What is a fixed point of a map T? What is the relationship between contraction mappings and fixed points?

How is the Contraction Mapping Theorem applicable to Fredholm Integral Operators?

Let $T : (X, d) \to (X, d)$ be a map on a metrix space X. We say T is a contraction mapping (or just contraction) if there is a small constant $c \in [0, 1)$ such that

$$d(T(x), T(y)) \le cd(x, y) \qquad \forall x, y \in X.$$

Contractions maps points closer together.

Contractions are uniformly continuous.

A point $x \in X$ is called a fixed point of T if $Tx = x$.

Every contraction mapping has a unique fixed point.

Define a map $T : C([a, b] \to C([a, b])$ by $Tf = g + Kf$ where

$$(Kf)(x) = \int_a^b k(x, y)f(y)\mathrm{d}y$$

for some $k \in C([a, b]^2)$ with "max row sum" less than 1, i.e. $\max_{a \le x \le b} \left\{ \int_a^b |k(x, y)|\mathrm{d}y \right\} < 1$. Then for any $g \in C([a, b])$, there is a unique $f \in C([a, b])$ such that $(I - Kj)f = g$.

This is shown by proving T is a contraction mapping, which shows that there is a unique $f \in C([a, b])$ such that $Tf = f$.