

Let (X, \mathcal{T}) be a topological space and $Y \subset X$. The induced (or relative) topology of Y in X, denoted \mathcal{T}_Y , are all subsets of Y which are the intersection of Y with some open set in \mathcal{T} , that is,

$$\mathcal{T}_Y = \{ H \subset Y \mid H = G \cap Y \text{ for some } G \in \mathcal{T} \}.$$

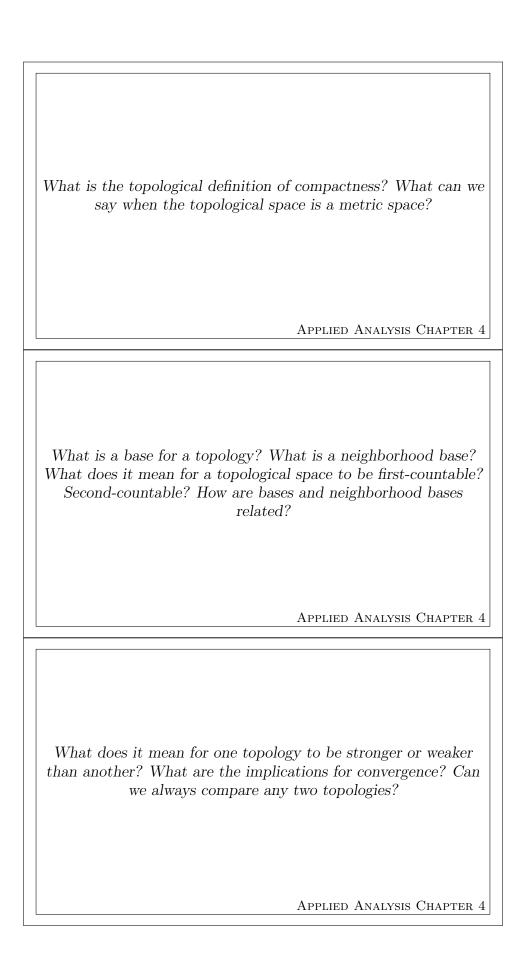
A subset V of X is a topological neighborhood of a point x if there is an open set G such that $x \in G \subset V$.

A topology \mathcal{T} on X is called Hausdorff if for every $x \neq y$ there are neighborhoods V_x or x and V_y of y such that $V_x \cap V_y = \emptyset$. That is, every pair of distinct points has a pair of nonintersecting neighborhoods.

A sequence x_n converges to a point $x \in X$ if for every neighborhood V of x there is a number N such that $x_n \in V$ for every $n \geq N$.

A function $f: X \to Y$ is continuous at $x \in X$ if for every neighborhood W of f(x) there is a neighborhood V of x such that $f(V) \subset W$.

A function $f: X \to Y$ is a homeomorphism if it is bijective and both f and f^{-1} are continuous. If there is a homeomorphism between X and Y, we say X and Y are homeomorphic. Homeomorphic topological spaces are indistinguishable in the sense that $G \in \mathcal{T}_X \iff f(G) \in \mathcal{T}_Y$, and $(x_n) \to x \in X \iff (f(x_n)) \to f(x) \in Y$.



A subset K of a topological space X is compact if every open cover of K contains a finite open subcover.

In metric spaces, compactness is equivalent to sequential compactness, which is that every sequence has a convergent subsequence.

A subset \mathcal{B} of a topology \mathcal{T} is a base for \mathcal{T} if for every $G \in \mathcal{T}$ there is a collection of sets $B_{\alpha} \in \mathcal{B}$ such that $G = \bigcup_{\alpha} B_{\alpha}$. That is, every open set is the union of basis elements.

A collection \mathcal{N} of neighborhoods of a point $x \in X$ is a neighborhood base for x if for each neighborhood V of x, there is a neighborhood $W \in \mathcal{N}$ with $W \subset V$. That is, every neighborhood contains a neighborhood base element.

A topological space is first-countable if every $x \in X$ has a countable neighborhood base. A topological space is second-countable if it has a countable base

A collection of sets \mathcal{B} is a base if and only if it contains a neighborhood base for every $x \in X$.

Let \mathcal{T}_1 and \mathcal{T}_2 be two topologies on the same space X. Then \mathcal{T}_1 is stronger (sometimes referred to as finer) than \mathcal{T}_2 if $\mathcal{T}_2 \subset \mathcal{T}_1$, that is, \mathcal{T}_1 has more open sets. We can also say \mathcal{T}_2 is weaker (or coarser) than \mathcal{T}_1 .

If a sequence converges with respect to a topology, than it converges with respect to any weaker topology.

It is not always possible to compare topologies because it is possible that \mathcal{T}_1 and \mathcal{T}_2 are two topologies on X but $\mathcal{T}_1 \not\subset \mathcal{T}_2$ and $\mathcal{T}_2 \not\subset \mathcal{T}_1$.

What is the relationship between topological comparisons and continuity?
Applied Analysis Chapter 4

