What is Bayes risk? What is a Bayes rule?

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What is minimax risk?

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What is mean squared error and how does it decompose?

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Definition. *Given a loss function l and a decision procedure δ and a set of probability distributions P_θ parameterized by θ with a distribution P' on the θ the Bayes risk is*

$$r(\delta) = \mathbb{E}[l(\delta(X), \theta)]$$

where the expectation is taken over P' and P_θ.

Definition. *A Bayes rule is a decision procedure δ^* with*

$$r(\delta^*) = \min_\delta r(\delta)$$

Definition. *Given a loss function l and a decision procedure δ and a set of probability distributions P_θ, the max risk is*

$$R(\delta) = \sup_\theta \mathbb{E}[l(\delta(X), \theta)]$$

Definition. *The minimax procedure is a procedure δ^* with*

$$R(\delta^*) = \min_\delta r(\delta)$$

Definition. *The mean squared error is the risk under the quadratic loss function:*

$$R(\delta(X), \theta) = \mathbb{E}[(\delta(X) - \theta)^2] = Bias(\delta)^2 + Var(\delta)$$

What are one parameter exponential families?

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What is a risk set?

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What is an admissable strategy set?

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Definition. *Probability densities with the form*

$$p(x, \theta) = h(x) \exp\{\eta(\theta)T(x) - B(\theta)\}$$

Example. *Normal, Binomial, Poisson, exponential.*

Definition. *The risk set is the set of risk vectors for every decision rule*

$$S = \{(R(\theta, \delta))_{\theta \in \Theta} \mid \delta \in \mathcal{D}\}$$

where $\mathcal{D}$ is the set of decision rules and $R(\theta, \delta)$ is the risk for the particular value of θ.

Remark. *The risk set is a finite set of points in $\mathbb{R}^{|\Theta|}$ if the action space for the decision rules is finite. If stochastic decisions are allowed, the risk set becomes the convex hull of this finite set. For different priors π, the risk for a stochastic strategy is $\langle \pi, (R(\delta, \theta)) \rangle$, hence the level sets of Bayes risk are $\langle \pi, \cdot \rangle = c$. The Bayes estimator is the solution to this linear programming problem over a convex hull and hence is either on a line of the boundary or on a corner. Hence, deterministic strategies do well enough for Bayes risk.*

Remark. *For minimax strategies, the problem, geometrically, is about finding the smallest square that fits below the convex hull (bounded by non-negative weights on each of the θ dimensions).*

Definition.

What is the relationship between minimax risk and Bayes risk?

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When are stochastic decision an improvement on deterministic ones?

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Theorem.

Definition.