1 Counting.

Exercise 1. How many choices are possible for ...

1. one letter from A-Z and one digit from 0-9?
2. a function from $\{A, \ldots, Z\}$ to $\{0, \ldots, 9\}$?
3. a 7-place license plate of the form digit-letter-letter-letter-digit-digit-digit?
4. a license plate as above, where you cannot use the same letter or digit twice?
5. a 7-letter password, where you cannot use the same letter twice in a row?
6. a password of 7 letters/digits, with at least one letter and at least one digit?
7. an arrangement of ten different math books on the shelf?
8. a team of five basketball players, from a group of twelve players?

Solution 1.

1. $\binom{26}{1}, \binom{10}{1}$
2. 10^{26}
3. $10^4 26^3$
4. $\frac{10!}{6!}\frac{26!}{23!}$
5. $26 \cdot 25^6$
6. $36^7 - 26^7 - 10^7$
7. $10!$
8. $\binom{12}{5}$

2 Uniform Probability Spaces.

Sometimes we assume that all outcomes in a sample space Ω are **equally likely**.
In this case, for every event $A \subseteq \Omega$,

$$P(A) \;=\; \frac{|A|}{|\Omega|} \;.$$

Exercise 2. 1. You roll two dice. What is the probability that they add up to 7?

2. You roll six dice. What is the probability that all numbers 1,2,3,4,5,6 appear?
3. You toss 8 coins. What is the probability of 3 heads and 5 tails?
4. You roll ten dice. What is the probability that 6 appears exactly 5 times?
5. What are the odds of winning the Powerball: guessing 5 numbers from $\{1, \ldots, 69\}$ and another one in $\{1, \ldots, 26\}$.
6. A deck of cards contains 52 different cards, 4 of which are aces. You deal the cards to 4 players, 13 cards to each one. What is the probability that each player gets one ace?

Solution 2. 1. $\frac{6}{36}$

2. $\frac{6!}{6^6}$
3. $\frac{\binom{8}{3}}{2^8}$
4. $\frac{\binom{10}{5}5^5}{6^{10}}$
5. $\frac{1}{69^5 \cdot 26}$
6. $\frac{\binom{13}{1}^4 48!4!}{52!}$