Predmet: AUTOMATI I FORMALNI JEZICI

DOMAĆA ZADAĆA br. 02

PREZIME I IME STUDENTA

Broj indeksa: ____19413_____

Lista urađenih zadataka:

1	2	3	4	5	6	7	8	9	10	11	12	13
6/6	3/3	3/3	4/4	2/2	2/3	3/3	2/2	1/1	1/1	1/1	1/1	3/3

Napomena: Oznaka x označava koji dio zadataka je urađen. Naprimjer, ako su u prvom zadatku urađeni podzadaci 1.a, 1.b i 1.c, tada za prvi zadatak treba napisati x=3, tj. za prvi zadatak treba napisati 3/6 jer su urađena 3 podzadatka od ukupno 6 podzadataka.

AFJ - Zadaća 2

1. Odredi KNG za jezike:

a) L je komplement jezika $L=\{a^n b^n \mid n\geq 0\}$ $\quad G=(N,\Sigma,P,S)$

1° $\#a > \#b$
2° $\#a < \#b$
3° bilo koji niz simbola oblika $(aUb)^*b(aUb)^*a(aUb)^*$

1° $A\to aAb\,|\,aAa$
2° $B\to aBb\,|\,Bb\,|\,b$
3° $C\to XbXaX$
4° $X\to aX\,|\,bX\,|\,\varepsilon$

Unija $\Rightarrow$ $P:\begin{cases} S\to A\,|\,B\,|\,C \\ A\to aAb\,|\,aAa \\ B\to aBb\,|\,Bb\,|\,b \\ C\to XbXaX \\ X\to aX\,|\,bX\,|\,\varepsilon \end{cases}$ $\quad N=\{S,A,B,C,X\}$, $\Sigma=\{a,b\}$

b) $L=\{a^n b^m \mid n\leq m+2\}$ $\quad G=(N,\Sigma,P,S)$

$P:\begin{cases} S\to AAB \\ A\to a\,|\,\varepsilon \\ B\to ABb\,|\,\varepsilon \end{cases}$ $\quad N=\{S,A,B\}$, $\Sigma=\{a,b\}$

c) $L=\{w\in\{a,b\}^* \mid n_a(w)=n_b(w)\}$ $\quad G=(N,\Sigma,P,S)$

$P:\{S\to aSbSaSS\,|\,\varepsilon$ $\quad N=\{S\}$, $\Sigma=\{a,b\}$

d) $L=\{w\in\{a,b\}^* \mid n_a(w)\neq n_b(w)\}$ $\quad G=(N,\Sigma,P,S)$

$P:\begin{cases} S\to A\,|\,B \\ A\to CaA\,|\,CaC \\ B\to CbB\,|\,CbC \\ C\to aCbC\,|\,bCaC\,|\,\varepsilon \end{cases}$ $\quad N=\{S,A,B,C\}$, $\Sigma=\{a,b\}$

e) $L=\{w\in\{a,b\}^* \mid n_a(w)=2n_b(w)+4\}$ $\quad G=(N,\Sigma,P,S)$

$P:\begin{cases} S\to AaAaAaAaA \\ A\to AaAaAbA \\ A\to AaAbAaA \\ A\to AbAaAaA \\ A\to\varepsilon \end{cases}$ $\quad N=\{S,A\}$, $\Sigma=\{a,b\}$

f) $L=\emptyset$ $\quad G=(N,\Sigma,P,S)$

$P:\{S\to aS\,|\,S$ $\quad N=\{S\}$, $\Sigma=\{a\}$

2.

a) KNG $G_1=(\{S,A,B\},\{a,b\},P_1,S)$ sa $P_1:\begin{cases} S\to aA\,|\,bB\,|\,b \\ A\to BaaA\,|\,ba \\ B\to bAA\,|\,ab \end{cases}$ transformiši u G_2 sa Chomskyjevim norm. oblikom.

CNF: $A\to BC$ ili $A\to a$

$G=(N,\Sigma,P,S) \longrightarrow G'=(N',\Sigma,P',S)$

1° Ne izbacujemo negenerativne
2° Ne izbacujemo nedohvatljive
3° Ne izbacujemo ε-produkcije
4° Ne izbacujemo jedinične $(A\to B)$

} NEMA

Dalje koraci:

1) U skup N' idu svi neter. symb. iz N, u skup P' idu sve produkcije oblika $A\to BC$ ili $A\to a$.

$N'=\{S,A,B\}$ $\quad P'=\{S\to b\}$

2.) Desna strana će imati isključivo term. ili isključivo neterm. symb. Proširujemo C_a i C_b $\quad$ (a) (b)

~~P': $S\to CaA\,|\,CbB\,|\,b$, $A\to BCaCa\,|\,CbCa$, $B\to CbAACb\,|$~~

P': $S\to C_aA$, $S\to C_bB$, $S\to b$, $A\to BC_aC_a$, $A\to C_bC_a$, $B\to C_bAAC_b$, $B\to C_aC_b$, $C_a\to a$, $C_b\to b$

3.) Proširujemo sa D_1, E_1, F_1 $\quad$ (C_aC_a) (AC_a) (AE_1)

jer sljedeće produkcije nisu oblika $A\to BC$: $B\to C_bAAC_b$, $B\to C_aC_b$

NEISPRAVNE

Finalno: P_2: $S\to C_aA$, $S\to C_bB$, $S\to b$, $A\to BD_1$, $A\to C_bC_a$, $B\to C_bF_1$, $B\to C_aC_b$, $C_a\to a$, $C_b\to b$, $D_1\to C_aC_a$, $E_1\to AC_b$, $F_1\to AE_1$

$G_2=(N_2,\Sigma_2,P_2,S)$ $\quad N_2=\{S,A,B,C_a,C_b,D_1,E_1,F_1\}$, $\Sigma_2=\Sigma=\{a,b\}$

b) Iz G_1 generiši 2 riječi:

$S\to b$ (riječ "b")

$S\to aA\to aba$ (riječ "aba")

c) sa G_2 generiši te iste riječi:

$S\to b$ (riječ "b")

$S\to C_aA\to aA\to aC_bC_a\to abC_a\to aba$ (riječ "aba")

3. Svojstvom napuhavanja dokaži da jezici nisu kontekstno-neovisni:

a) $L = \{a^n b^{2n} c^{3n} \mid n \geq 0\}$

$W = UVXYZ$
1.) $|VY| > 0$
2.) $|VXY| \leq n$
3.) $UV^i XY^i Z \in L \mid i \geq 0$

npr: $n=1$ $\quad w = abbccc$ $\quad VXY = b$

$U=a$, $V=b$, $X=\varepsilon$, $Y=\varepsilon$, $Z=bccc$

$|VY|>0$ ✓ $\quad |VXY| \leq n$ ✓

npr: $i=2$

$UV^2XY^2Z \in L$, $\quad abbbccc \notin L$

KONTRADIKCIJA

b) $L = \{a^n b^n a^n b^n \mid n \geq 0\}$

$W = UVXYZ$
1.) $|VY| > 0$
2.) $|VXY| \leq n$
3.) $UV^i XY^i Z \in L \mid i \geq 0$

npr: $n=2$ $\quad w = aabbaabb$ $\quad VXY = ab$

$U=a$, $V=a$, $X=\varepsilon$, $Y=b$, $Z=baabb$

$|VY|>0$ ✓ $\quad |VXY| \leq n$ ✓

npr: $i=2$

$UV^2XY^2Z \in L$

$aaabbbaabb \notin L$

KONTRADIKCIJA

c) $L = \{a^n \# a^{2n} \# a^{3n} \mid n \geq 0\}$

$W = UVXYZ$
1.) $|VY| > 0$
2.) $|VXY| \leq n$
3.) $UV^i XY^i Z \in L \mid i \geq 0$

npr: $n=3$ $\quad w = aaa\#aaaaaa\#aaaaaaaaa$ $\quad VXY = a\#a$

$U=aa$, $V=a$, $X=\#$, $Y=a$, $Z=aaaaa\#aaaaaaaaa$

$|VY|>0$ ✓ $\quad |VXY| \leq n$ ✓

npr $i=0$

$UV^0XY^0Z \in L$

$aa\#aaaaa\#aaaaaaaaa$

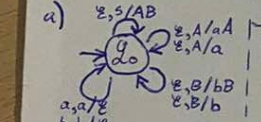
$aa \to$ duplo manje od aaaaa ✗

$aa \to$ tro duplo manje od 3 ✗ sekv

KONTRADIKCIJA

4. Za KNG $G_1 = (\{S, A, B\}, \{a, b\}, P_1, S)$ gdje je P_1: $S \to AB$, $A \to aAla$, $B \to bBlb$ konstruiši PA praznim stekom.

a)



b) slijed za aabbbb i aaabb

Stanje	Nepročitano	Stek	Prelaz
q_0	aabbbb	S	$\delta(q_0, \varepsilon, S) = \{q_0, \}$
q_0	abbbb		$\delta(q_0, a$
q_0			$\delta(q_0, b$
q_0			$\delta(q_0, b$
q_0			$\delta(q_0, b$
q_0			$\delta(q_0, b$

Stanje	Nepročitano	Stek	Prelaz
q_0	aaabb	S	$\delta(q_0, a, S)$
q_0	aabb		$\delta(q_0, a$
q_0			$\delta(q_0, a$
q_0			$\delta(q_0, b$
q_0			$\delta(q_0, b$

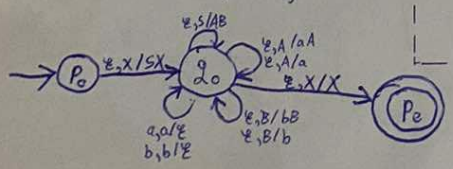
b) slijedovi

$(q_0, aabbbb, S) \vdash (q_0, aabbbb, AB)$
$\vdash (q_0, aabbbb, aAB)$
$\vdash (q_0, abbbb, AB)$
$\vdash (q_0, abbbb, aB)$
$\vdash (q_0, bbbb, B)$
$\vdash (q_0, bbbb, bB)$
$\vdash (q_0, bbb, B)$
$\vdash (q_0, bbb, bB)$
$\vdash (q_0, bb, B)$
$\vdash (q_0, bb, bB)$
$\vdash (q_0, b, B)$
$\vdash (q_0, b, b)$
$\vdash (q_0, \varepsilon, \varepsilon)$

$(q_0, aaabb, S) \vdash (q_0, aaabb, AB)$
$\vdash (q_0, aaabb, aAB)$
$\vdash (q_0, aabb, AB)$
$\vdash (q_0, aabb, aAB)$
$\vdash (q_0, abb, AB)$
$\vdash (q_0, abb, aB)$
$\vdash (q_0, bb, B)$
$\vdash (q_0, bb, bB)$
$\vdash (q_0, b, B)$
$\vdash (q_0, b, b)$
$\vdash (q_0, \varepsilon, \varepsilon)$

c) PA iz a) prebaci u prihv. stanje

Dodat ćemo 2 nova stanja P_0 i P_e, samim tim i novi poč. symb. X.



Prelaz iz P_0 u q_0 je uvijek šablonski.
Prelaz iz svih stanja $\neq P_0$ u P_e je također šablonski.

d) slijedovi

$(P_0, aabbbb, X) \vdash (q_0, aabbbb, SX)$
$\vdash (q_0, aabbbb, ABX)$
$\vdash (q_0, aabbbb, aABX)$
$\vdash (q_0, abbbb, ABX)$
$\vdash (q_0, abbbb, aBX)$
$\vdash (q_0, bbbb, BX)$
$\vdash (q_0, bbbb, bBX)$
$\vdash (q_0, bbb, BX)$
$\vdash (q_0, bbb, bBX)$
$\vdash (q_0, bb, BX)$
$\vdash (q_0, bb, bBX)$
$\vdash (q_0, b, BX)$
$\vdash (q_0, b, bX)$
$\vdash (q_0, \varepsilon, X)$
$\vdash (P_e, \varepsilon, X)$

$(P_0, aaabb, X) \vdash (q_0, aaabb, SX)$
$\vdash (q_0, aaabb, ABX)$
$\vdash (q_0, aaabb, aABX)$
$\vdash (q_0, aabb, ABX)$
$\vdash (q_0, aabb, aABX)$
$\vdash (q_0, abb, ABX)$
$\vdash (q_0, abb, aBX)$
$\vdash (q_0, bb, BX)$
$\vdash (q_0, bb, bBX)$
$\vdash (q_0, b, BX)$
$\vdash (q_0, b, bX)$
$\vdash (q_0, \varepsilon, X)$
$\vdash (P_e, \varepsilon, X)$

5. PA $M_N = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{a, Z_0\}, \delta_N, q_0, Z_0, \emptyset)$ praznim stekom

1) $\delta_N(q_0, a, Z_0) = \{(q_0, aZ_0)\}$
2) $\delta_N(q_0, a, a) = \{(q_0, aa)\}$
3) $\delta_N(q_0, b, a) = \{(q_1, \varepsilon)\}$
4) $\delta_N(q_1, \varepsilon, a) = \{(q_2, \varepsilon)\}$
5) $\delta_N(q_2, b, a) = \{(q_1, \varepsilon)\}$
6) $\delta_N(q_2, \varepsilon, Z_0) = \{(q_3, \varepsilon)\}$

M_N prihvata $L(M_N) = \{a^{2n} b^n \mid n \geq 1\}$

a) Za M_N konstruiši ekv. KNG G.
Iz G izbaci beskorisne symb.

$S \to [q_0, Z_0, q_0] \mid [q_0, Z_0, q_1] \mid [q_0, Z_0, q_2] \mid [q_0, Z_0, q_3]$

$\delta_N(q_0, a, Z_0) = \{(q_0, aZ_0)\}$:

$[q_0, Z_0, q_0] \to a[q_0, a, q_0][q_0, Z_0, q_0] \mid a[q_0, a, q_1][q_1, Z_0, q_0]$
$a[q_0, a, q_2][q_2, Z_0, q_0] \mid a[q_0, a, q_3][q_3, Z_0, q_0]$

$[q_0, Z_0, q_1] \to a[q_0, a, q_0][q_0, Z_0, q_1] \mid a[q_0, a, q_1][q_1, Z_0, q_1]$
$a[q_0, a, q_2][q_2, Z_0, q_1] \mid a[q_0, a, q_3][q_3, Z_0, q_1]$

$[q_0, Z_0, q_2] \to a[q_0, a, q_0][q_0, Z_0, q_2] \mid a[q_0, a, q_1][q_1, Z_0, q_2]$
$a[q_0, a, q_2][q_2, Z_0, q_2] \mid a[q_0, a, q_3][q_3, Z_0, q_2]$

$[q_0, Z_0, q_3] \to a[q_0, a, q_0][q_0, Z_0, q_3] \mid a[q_0, a, q_1][q_1, Z_0, q_3]$
$a[q_0, a, q_2][q_2, Z_0, q_3] \mid a[q_0, a, q_3][q_3, Z_0, q_3]$

$\delta_N(q_0, a, a) = \{(q_0, aa)\}$:

$[q_0, a, q_0] \to a[q_0, a, q_0][q_0, a, q_0] \mid a[q_0, a, q_1][q_1, a, q_0]$
$a[q_0, a, q_2][q_2, a, q_0] \mid a[q_0, a, q_3][q_3, a, q_0]$

$[q_0, a, q_1] \to a[q_0, a, q_0][q_0, a, q_1] \mid a[q_0, a, q_1][q_1, a, q_1]$
$a[q_0, a, q_2][q_2, a, q_1] \mid a[q_0, a, q_3][q_3, a, q_1]$

$[q_0, a, q_2] \to a[q_0, a, q_0][q_0, a, q_2] \mid a[q_0, a, q_1][q_1, a, q_2]$
$a[q_0, a, q_2][q_2, a, q_2] \mid a[q_0, a, q_3][q_3, a, q_2]$

$[q_0, a, q_3] \to a[q_0, a, q_0][q_0, a, q_3] \mid a[q_0, a, q_1][q_1, a, q_3]$
$a[q_0, a, q_2][q_2, a, q_3] \mid a[q_0, a, q_3][q_3, a, q_3]$

$\delta_N(q_0, b, a) = \{(q_1, \varepsilon)\}$: $[q_0, a, q_1] \to b$
$\delta_N(q_1, \varepsilon, a) = \{(q_2, \varepsilon)\}$: $[q_1, a, q_2] \to \varepsilon$
$\delta_N(q_2, b, a) = \{(q_1, \varepsilon)\}$: $[q_2, a, q_1] \to b$
$\delta_N(q_2, \varepsilon, Z_0) = \{(q_3, \varepsilon)\}$: $[q_2, Z_0, q_3] \to \varepsilon$

generativni: $\{[q_0, a, q_1], [q_1, a, q_2], [q_2, a, q_1], [q_2, Z_0, q_3], [q_0, a, q_2], [q_0, Z_0, q_3], S\}$ -OSTALI VAN
dohvatljivi: $\{\text{svi\_generativni}, b, \varepsilon\}$ -NIŠTA VAN

P:
$S \to [q_0, Z_0, q_3]$
$[q_0, Z_0, q_3] \to a[q_0, a, q_2][q_2, Z_0, q_3]$
$[q_0, a, q_2] \to a[q_0, a, q_1][q_1, a, q_2]$
$[q_0, a, q_1] \to a[q_0, a, q_2][q_2, a, q_1]$
$[q_0, a, q_1] \to b[q_1, a, q_2] \to \varepsilon$
$[q_2, a, q_1] \to b[q_2, Z_0, q_3] \to \varepsilon$

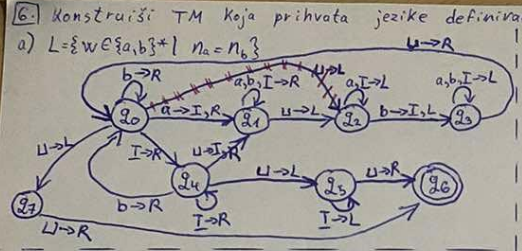
b) Sa dobivenom G pod a) napiši postupak generisanja riječi "aab" i "aaaabb"

aab: $S \to [q_0, Z_0, q_3] \to a[q_0, a, q_2][q_2, Z_0, q_3] \to aa[q_0, a, q_1][q_1, a, q_2][q_2, Z_0, q_3]$
$\to aab\varepsilon\varepsilon \to aab$

aaaabb: $S \to [q_0, Z_0, q_3] \to a[q_0, a, q_2][q_2, Z_0, q_3] \to aa[q_0, a, q_1][q_1, a, q_2][q_2, Z_0, q_3]$
$\to aaa[q_0, a, q_2][q_2, a, q_1][q_1, a, q_2][q_2, Z_0, q_3] \to aaaa[q_0, a, q_1][q_1, a, q_2][q_2, a, q_1][q_1, a, q_2][q_2, Z_0, q_3]$
$\to aaaab[q_1, a, q_2][q_2, a, q_1][q_1, a, q_2][q_2, Z_0, q_3] \to aaaab\varepsilon b\varepsilon\varepsilon \to aaaabb$

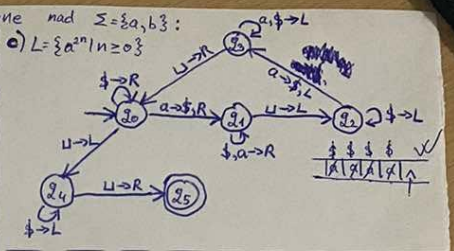
6. Konstruiši TM koja prihvata jezike definirane nad $\Sigma=\{a,b\}$:

a) $L=\{w\in\{a,b\}^* \mid n_a=n_b\}$



(Na desno križaj „a", na lijevo „b")

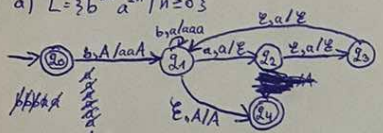
c) $L=\{a^{2^n} \mid n\geq 0\}$



b) X

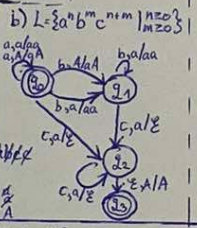
7. Konstruiši PA sa prihv. stanjem nad $\Sigma=\{a,b,c\}$

a) $L=\{b^{3n}a^{2n} \mid n\geq 0\}$

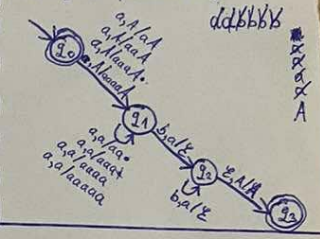


Za "b" dodaj 2 "a", Za "a" skidaj 3 "a". $b^{3n}a^{2n}$ → NZS(3,2)=6 → 6:3=2, 6:2=3

b) $L=\{a^n b^m c^{n+m} \mid n\geq 0, m\geq 0\}$

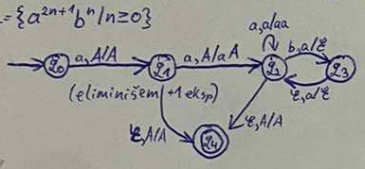


c) $L=\{a^n b^m \mid n\leq m\leq 4n\}$

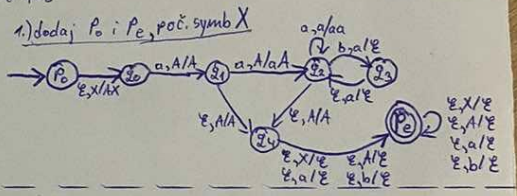


8. Konstruiši PA sa praznim stekom nad $\Sigma=\{a,b\}$

a) $L=\{a^{2n+1}b^n \mid n\geq 0\}$

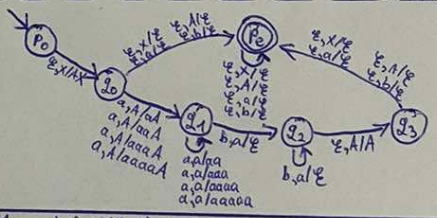


1.) dodaj P_0 i P_e, poč. symb X

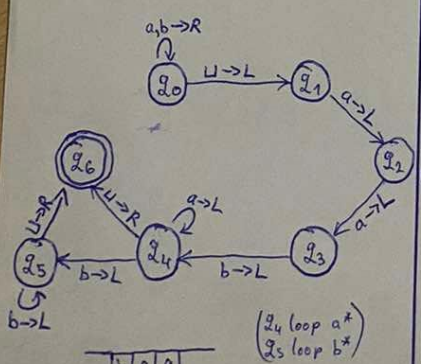


b) $L=\{a^n b^m \mid n\leq m\leq 4n\}$

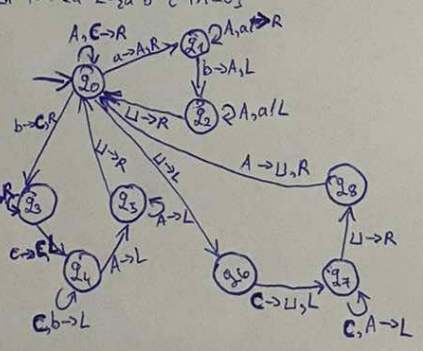
-||- isto kao 7. c) -||- ⟹



9. Konstruiši TM za $L=L(b^*a^*baa)$



10. TM za $L=\{a^n b^{2n} c^n \mid n\geq 0\}$

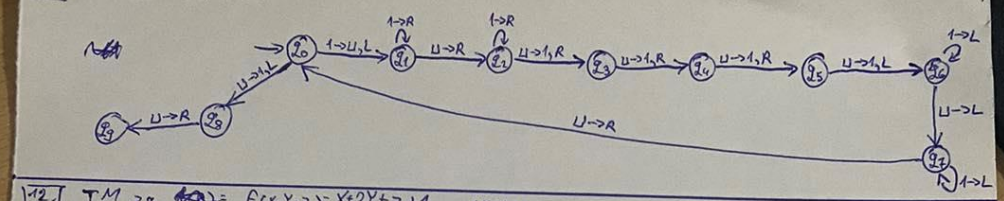


(sve „a" u „A" ; ½ "b" u "A")

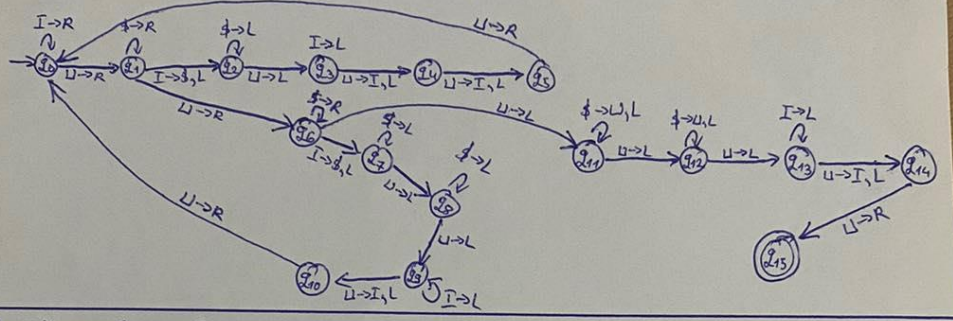
(½ "b" u "C" i "c" u "C")

(briši sa kraja, pa sa početka)

11. TM za $f(n) = 4n+1$ unarno

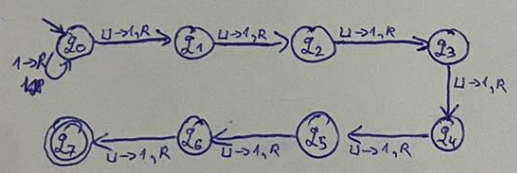


12. TM za ~~f(x)~~ $= f(x,y,z) = x+2y+z+1$ unarno

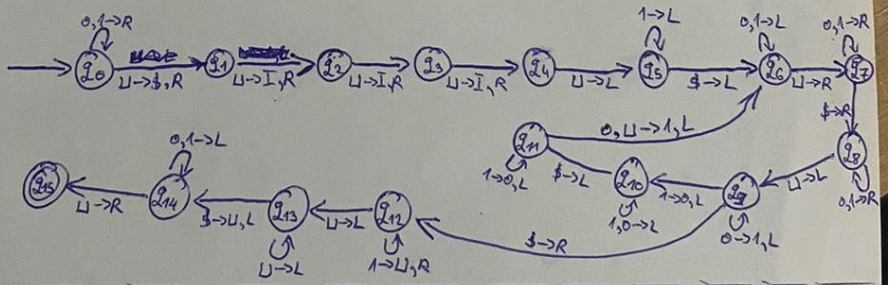


13. TM za $f(n) = n+7$

a) unarno



b) binarno



c) decimalno