
Predmet: AUTOMATI I FORMALNI JEZICI

DOMAĆA ZADAĆA br. 02

PREZIME I IME STUDENTA

Broj indeksa: ____19413____

Lista urađenih zadataka:

1	2	3	4	5	6	7	8	9	10	11	12	13
6/6	3/3	3/3	4/4	2/2	2/3	3/3	2/2	1/1	1/1	1/1	1/1	3/3

Napomena: Oznaka x označava koji dio zadataka je urađen. Naprimjer, ako su u prvom zadatku urađeni podzadaci 1.a, 1.b i 1.c, tada za prvi zadatak treba napisati $x=3$, tj. za prvi zadatak treba napisati 3/6 jer su urađena 3 podzadatka od ukupno 6 podzadataka.

AFJ- Zadaća 2

1. Odredi KNG za jezike:

a) L je komplement jezika $L = \{a^n b^n \mid n \geq 0\}$

1° $\#a > \#b$

2° $\#a < \#b$

3° bilo koji niz simbola

oblika $(aUb)^*b(aUb)^*a(aUb)^*$

1° $A \rightarrow aAb|aA|a$

2° $B \rightarrow aBb|Bb|b$

3° $C \rightarrow XbXaX$

4° $X \rightarrow aX|bX| \epsilon$

Unija \Rightarrow

$P: \begin{cases} S \rightarrow A|B|C \\ A \rightarrow aAb|aA|a \\ B \rightarrow aBb|Bb|b \\ C \rightarrow XbXaX \\ X \rightarrow aX|bX| \epsilon \end{cases}$

$G = (N, \Sigma, P, S)$

$N = \{S, A, B, C, X\}$
 $\Sigma = \{a, b\}$

b) $L = \{a^n b^m \mid n \leq m+2\}$

$P: \begin{cases} S \rightarrow AAB \\ A \rightarrow a| \epsilon \\ B \rightarrow ABb| \epsilon \end{cases}$

$G = (N, \Sigma, P, S)$

$N = \{S, A, B\}$
 $\Sigma = \{a, b\}$

c) $L = \{w \in \{a, b\}^* \mid n_a(w) = n_b(w)\}$

$P: \begin{cases} S \rightarrow aSb|bSa|SS| \epsilon \end{cases}$

$G = (N, \Sigma, P, S)$

$N = \{S\}$
 $\Sigma = \{a, b\}$

d) $L = \{w \in \{a, b\}^* \mid n_a(w) \neq n_b(w)\}$

$P: \begin{cases} S \rightarrow A|B \\ A \rightarrow CaA|CaC \\ B \rightarrow CbB|CbC \\ C \rightarrow aCb|bCa| \epsilon \end{cases}$

$G = (N, \Sigma, P, S)$

$N = \{S, A, B, C\}$
 $\Sigma = \{a, b\}$

e) $L = \{w \in \{a, b\}^* \mid n_a(w) = 2n_b(w) + 4\}$

$P: \begin{cases} S \rightarrow AaAaAaA \\ A \rightarrow AaAaAbA \\ A \rightarrow AaAbAaA \\ A \rightarrow AbAaAaA \\ A \rightarrow \epsilon \end{cases}$

$G = (N, \Sigma, P, S)$

$N = \{S, A\}$
 $\Sigma = \{a, b\}$

f) $L = \emptyset$

$G = (N, \Sigma, P, S)$

$P: \begin{cases} S \rightarrow aS|S \end{cases}$

$N = \{S\}$
 $\Sigma = \{a\}$

2.

a) KNG $G_1 = (\{S, A, B\}, \{a, b\}, P_1, S)$ sa $P_1: \begin{cases} S \rightarrow aAb|bAb \\ A \rightarrow BaAb \\ B \rightarrow aAb|aAb \end{cases}$ transformiši u G_2 sa Chomskyjevim norm.oblikom.

[CNF: $A \rightarrow BC$ ili $A \rightarrow a$]

$G = (N, \Sigma, P, S) \rightarrow G' = (N', \Sigma, P', S)$

- 1° Ne izbacujemo negenerativne
- 2° Ne izbacujemo nedohvatljive
- 3° Ne izbacujemo ϵ -produkcije
- 4° Ne izbacujemo jedinične ($A \rightarrow B$)

NEMA
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Dalje Koraci:

1) U skup N' idu svi neter.symb. iz N , u skup P' idu sve produkcije oblika $A \rightarrow BC$ ili $A \rightarrow a$.

$N' = \{S, A, B\}$ $P' = \{S \rightarrow b\}$

2.) Desna strana će imati isključivo term. ili isključivo neterm. symb. Proširujemo $C_a: C_b$ (a) (b)

~~$P': \begin{cases} S \rightarrow CaA|CbB \\ A \rightarrow BCa|CBc \\ B \rightarrow CaA|CbB \end{cases}$~~

$\begin{cases} S \rightarrow CaA \\ S \rightarrow CbB \\ S \rightarrow b \\ A \rightarrow BCaCa \\ A \rightarrow CbCa \\ B \rightarrow CaAACb \\ B \rightarrow CaCb \\ Ca \rightarrow a \\ Cb \rightarrow b \end{cases}$

3.) Proširujemo sa D_1, E_1, F_1 ($CaCa$) (ACb) (AE_1)

jer sljedeće produkcije nisu oblika $A \rightarrow BC$:

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Finalno:

$P_2: \begin{cases} S \rightarrow CaA \\ S \rightarrow CbB \\ S \rightarrow b \\ A \rightarrow BD_1 \\ A \rightarrow CbCa \\ B \rightarrow CbF_1 \\ B \rightarrow CaCb \\ Ca \rightarrow a \\ Cb \rightarrow b \\ D_1 \rightarrow CaCa \\ E_1 \rightarrow ACb \\ F_1 \rightarrow AE_1 \end{cases}$

$G_2 = (N_2, \Sigma_2, P_2, S)$

$N_2 = \{S, A, B, C_a, C_b, D_1, E_1, F_1\}$

$\Sigma_2 = \Sigma = \{a, b\}$

b) Iz G_1 generiši 2 riječi: P' :

$S \rightarrow b$ (riječ "b")

$S \rightarrow aA \rightarrow aba$ (riječ "aba")

c) sa G_2 generiši te iste riječi:

$S \rightarrow b$ (riječ "b")

$S \rightarrow CaA \rightarrow aA \rightarrow aCbCa \rightarrow abCa \rightarrow aba$ (riječ "aba")

5. PA ~~Mu~~ $Mu = (\{z_0, z_1, z_2, z_3\}, \{a, b\}, \{z_0, z_1, z_2, z_3\}, \emptyset)$ praznim stekom

- 1) $\delta_N(z_0, a, z_0) = \{z_0, a, z_0\}$ a) Za Mu konstruiši ekv. KNG G .
 2) $\delta_N(z_0, a, a) = \{z_0, aa\}$ Iz G izbaci beskorisne symb.

3) $\delta_N(z_0, b, a) = \{z_1, \epsilon\}$

4) $\delta_N(z_1, \epsilon, a) = \{z_2, \epsilon\}$

5) $\delta_N(z_2, b, a) = \{z_1, \epsilon\}$

6) $\delta_N(z_1, \epsilon, z_0) = \{z_3, \epsilon\}$

$$S \rightarrow [z_0, z_0, z_0] \mid [z_0, z_0, z_1] \mid [z_0, z_0, z_2] \mid [z_0, z_0, z_3]$$

Mu prihvata $L(Mu) = \{a^n b^n \mid n \geq 1\}$

~~Primer~~ $\delta_N(z_0, a, z_0) = \{z_0, a, z_0\}$: $[z_0, z_0, z_0] \rightarrow a[z_0, a, z_0][z_0, z_0, z_0] \mid a[z_0, a, z_1][z_1, z_0, z_0]$

$$a[z_0, a, z_2][z_2, z_0, z_0] \mid a[z_0, a, z_3][z_3, z_0, z_0]$$

$$[z_0, z_0, z_1] \rightarrow a[z_0, a, z_0][z_0, z_0, z_1] \mid a[z_0, a, z_1][z_1, z_0, z_1]$$

$$a[z_0, a, z_2][z_2, z_0, z_1] \mid a[z_0, a, z_3][z_3, z_0, z_1]$$

$$[z_0, z_0, z_2] \rightarrow a[z_0, a, z_0][z_0, z_0, z_2] \mid a[z_0, a, z_1][z_1, z_0, z_2]$$

$$a[z_0, a, z_2][z_2, z_0, z_2] \mid a[z_0, a, z_3][z_3, z_0, z_2]$$

$$[z_0, z_0, z_3] \rightarrow a[z_0, a, z_0][z_0, z_0, z_3] \mid a[z_0, a, z_1][z_1, z_0, z_3]$$

$$a[z_0, a, z_2][z_2, z_0, z_3] \mid a[z_0, a, z_3][z_3, z_0, z_3]$$

$\delta_N(z_0, a, a) = \{z_0, aa\}$: $[z_0, a, z_0] \rightarrow a[z_0, a, z_0][z_0, a, z_0] \mid a[z_0, a, z_1][z_1, a, z_0]$

$$a[z_0, a, z_2][z_2, a, z_0] \mid a[z_0, a, z_3][z_3, a, z_0]$$

$$[z_0, a, z_1] \rightarrow a[z_0, a, z_0][z_0, a, z_1] \mid a[z_0, a, z_1][z_1, a, z_1]$$

$$a[z_0, a, z_2][z_2, a, z_1] \mid a[z_0, a, z_3][z_3, a, z_1]$$

$$[z_0, a, z_2] \rightarrow a[z_0, a, z_0][z_0, a, z_2] \mid a[z_0, a, z_1][z_1, a, z_2]$$

$$a[z_0, a, z_2][z_2, a, z_2] \mid a[z_0, a, z_3][z_3, a, z_2]$$

$$[z_0, a, z_3] \rightarrow a[z_0, a, z_0][z_0, a, z_3] \mid a[z_0, a, z_1][z_1, a, z_3]$$

$$a[z_0, a, z_2][z_2, a, z_3] \mid a[z_0, a, z_3][z_3, a, z_3]$$

$\delta_N(z_0, b, a) = \{z_1, \epsilon\}$: $[z_0, a, z_1] \rightarrow b$

$\delta_N(z_1, \epsilon, a) = \{z_2, \epsilon\}$: $[z_1, a, z_2] \rightarrow \epsilon$

$\delta_N(z_2, b, a) = \{z_1, \epsilon\}$: $[z_2, a, z_1] \rightarrow b$

$\delta_N(z_1, \epsilon, z_0) = \{z_3, \epsilon\}$: $[z_1, z_0, z_3] \rightarrow \epsilon$

generativni: $\{[z_0, a, z_1], [z_1, a, z_2], [z_2, a, z_1], [z_1, z_0, z_3], [z_0, a, z_2], [z_0, z_0, z_3], S\}$ - OSTALI VAN

dohvatljivi: $\{ \text{svi generativni}, b, \epsilon \}$ - NIŠTA VAN

P: $S \rightarrow [z_0, z_0, z_3]$

$$[z_0, z_0, z_3] \rightarrow a[z_0, a, z_2][z_2, z_0, z_3]$$

$$[z_0, a, z_2] \rightarrow a[z_0, a, z_1][z_1, a, z_2]$$

$$[z_0, a, z_1] \rightarrow a[z_0, a, z_2][z_2, a, z_1]$$

$$[z_0, a, z_1] \rightarrow b[z_1, a, z_2] \rightarrow \epsilon$$

$$[z_2, a, z_1] \rightarrow b[z_2, z_0, z_3] \rightarrow \epsilon$$

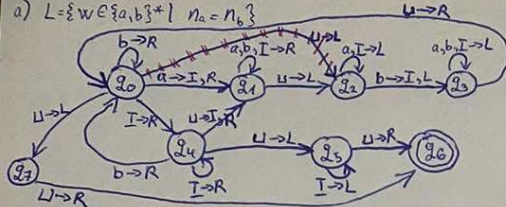
b) Sa dobivenom G pod a) napiši postupak generisanja riječi "aab" i "aaaabb"

aab: $S \rightarrow [z_0, z_0, z_3] \rightarrow a[z_0, a, z_2][z_2, z_0, z_3] \rightarrow aa[z_0, a, z_1][z_1, a, z_2][z_2, z_0, z_3] \rightarrow aab \epsilon \epsilon \rightarrow aab$

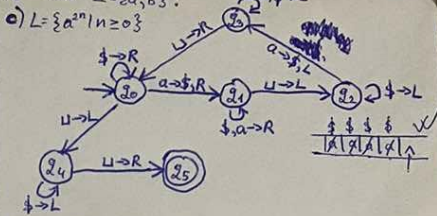
aaaabb: $S \rightarrow [z_0, z_0, z_3] \rightarrow a[z_0, a, z_2][z_2, z_0, z_3] \rightarrow aa[z_0, a, z_1][z_1, a, z_2][z_2, z_0, z_3]$

$$\rightarrow aaa[z_0, a, z_1][z_1, a, z_2][z_2, a, z_1][z_1, a, z_2][z_2, z_0, z_3] \rightarrow aaaab[z_1, a, z_2][z_2, a, z_1][z_1, a, z_2][z_2, z_0, z_3] \rightarrow aaaab \epsilon b \epsilon \epsilon \rightarrow aaaabb$$

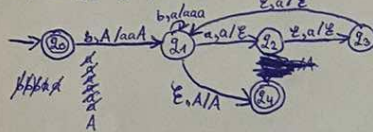
a) $L = \{w \in \{a, b\}^* \mid n_a = n_b\}$



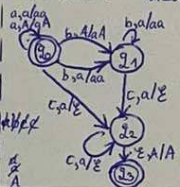
c) $L = \{a^{2^n} \mid n \geq 0\}$



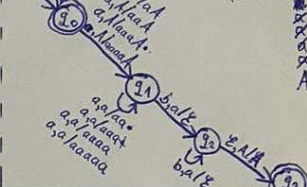
a) $L = \{b^{3n}a^{2n} \mid n \geq 0\}$



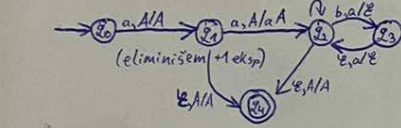
Za "b" dodaj 2 "a". Za "a" skidaj 3 "a". $b^3 a^{2n} \rightarrow \frac{NZS6h}{6^{13}=2}$
 $6^{12}=3$

$$b) L = \{a^n b^m c^{n+m} \mid n \geq 0, m \geq 0\}$$


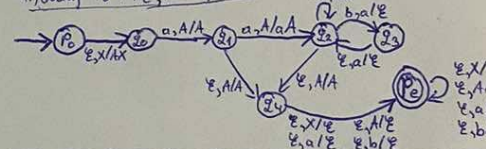
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841,



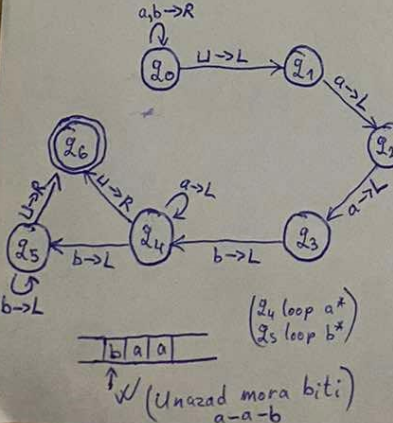
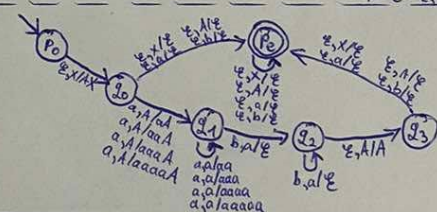
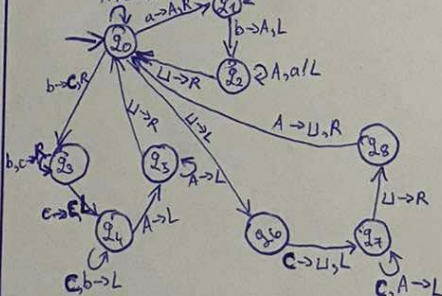
a) $L = \{a^{2n+1}b^n \mid n \geq 0\}$



1.) dodaj p_o i p_p , poč. symb X

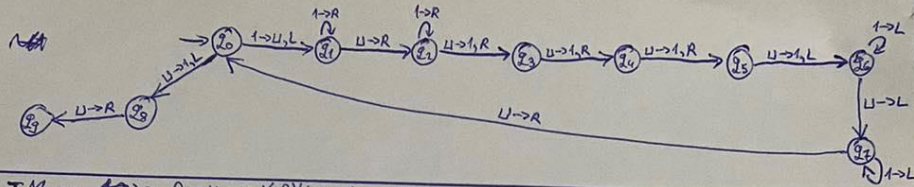


-||- isto kao 7.c) -||- \Rightarrow

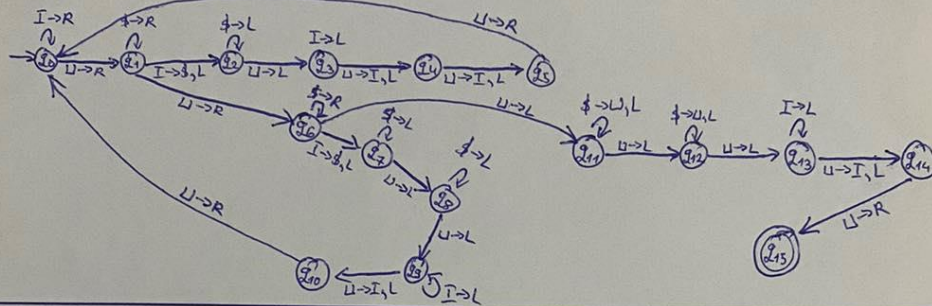

$$A, C \rightarrow R \quad \text{and} \quad A, a \rightarrow R$$


(sve "a" i "A"; $\frac{1}{2}$ "b" i "B"
($\frac{1}{2}$ "b" i "C"; "c" i "C")
(briši sa kraja, pa sa početku)

11. TM za $F(n) = 4n + 1$ unarno

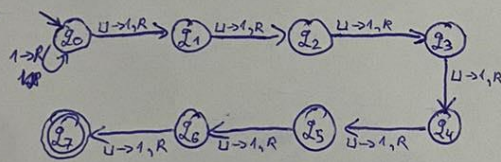


12. TM za $F(x, y, z) = x + 2y + z + 1$ unarno

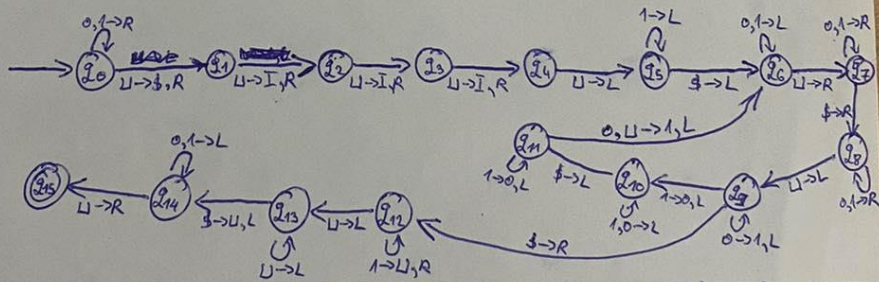


13. TM za $F(n) = n + 7$

a) unarno



b) binarno



c) decimalno

