MATH 3110 - Spring 2023,

Learning Activity 16

1. Is the vector $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ in Col(A) where $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$? Justify your answer.

Yes, $\vec{w} \in$ Col(A)	No, $\vec{w} \notin$ Col(A)
X	

Justification/computation: Note that $\vec{w}$ is in Col(A) if it is a linear combination of the columns of A or if $A\vec{x} = \vec{w}$ is consistent. Reduce the augmented matrix to REF (their REF is not necessarily the same as mine).

$$\left[\begin{array}{ccc|c} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{array}\right] \xrightarrow{REF} \left[\begin{array}{ccc|c} -8 & -2 & -9 & 2 \\ 0 & 5/2 & 5/4 & 5/2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So $A\vec{x} = \vec{w}$ is consistent and $\vec{w}$ is in Col(A).

One can directly write $\vec{w}$ as a linear combination of the columns of A. For example (by fully solving the above equation), we have (this is optional)

$$\vec{w} = -\frac{1}{2}\vec{a}_1 + \vec{a}_2 + 0\vec{a}_3$$

2. Let $A = \begin{bmatrix} 0 & 0 & 2 & 2 \\ 1 & -6 & 4 & 1 \end{bmatrix}$.

(a) Find a spanning set of Nul(A). Computation: Write the parametric vector form of the solutions of $A\vec{x} = \vec{0}$ and collect the vectors to form a spanning set. We have

$$\left[\begin{array}{cccc|c} 0 & 0 & 2 & 2 & 0 \\ 1 & -6 & 4 & 1 & 0 \end{array}\right] \xrightarrow{REF} \left[\begin{array}{cccc|c} 1 & -6 & 4 & 1 & 0 \\ 0 & 0 & 2 & 2 & 0 \end{array}\right] \Rightarrow \begin{array}{ll} x_1 & = 6x_2 + 3x_4 \\ x_2 & = x_2 \\ x_3 & = -x_4 \\ x_4 & = x_4 \end{array} \Rightarrow \vec{x} = \begin{bmatrix} 6x_2 + 3x_4 \\ x_2 \\ -x_4 \\ x_4 \end{bmatrix}$$

$$\vec{x} = x_2 \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

It follows that Nul(A) is spanned by $\left\{ \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

$$\boxed{\text{Nul}(A) = \text{Span}\left(\begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right)}$$

(b) Is the vector $\vec{v} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ in Nul(A)? Justify your answer.

Yes, $\vec{v} \in$ Nul(A)	No, $\vec{v} \notin$ Nul(A)
X	

Justification/computation: There are two methods to show it

- Compute $A\vec{v}$ and it should be that $A\vec{v} = \vec{0}$.
- From the previous question Nul$(A) = $ Span$\left(\begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right)$ so $\vec{v}$ is in Nul(A).