1. Use Cramer's rule to solve the following system of equations:

$$\begin{aligned} -5x_1 + 2x_2 &= 9 \\ 3x_1 - x_2 &= -4 \end{aligned}$$

Computation: (all formulas must be displayed)

We have $A = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$, $A_1(\vec{b}) = \begin{bmatrix} 9 & 2 \\ -4 & -1 \end{bmatrix}$ and $A_2(\vec{b}) = \begin{bmatrix} -5 & 9 \\ 3 & -4 \end{bmatrix}$. So the entries of the solution $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ are given by

$$x_1 = \frac{\det A_1(\vec{b})}{\det A} = \frac{(9)(-1)-(-4)(2)}{(-5)(-1)-(3)(2)} = \frac{-1}{-1} = 1 \quad \text{and} \quad x_2 = \frac{\det A_2(\vec{b})}{\det A} = \frac{(-5)(-4)-(3)(9)}{(-5)(-1)-(3)(2)} = \frac{-7}{-1} = 7$$

$$\boxed{\vec{x} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}}$$

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with standard matrix A such that $\det(A) = 8$. Calculate the area of the parallelogram $R = T(S)$ where S is the parallelogram formed by the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$.

Computation: We have

$$\text{Area}(R) = |\det A| \text{Area}(S)$$

Since S is the parallelogram formed by $\vec{v}_1$ and $\vec{v}_2$

$$\text{Area}(S) = |\det[\vec{v}_1 \ \vec{v}_2]| = |\det[\vec{v}_2 \ \vec{v}_1]| = |(1)(1) - (3)(5)| = 14$$

So

$$\text{Area}(R) = (8)(14) = 112$$

$$\boxed{\text{Area}(R) = 112}$$

3. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 4 & -2 \end{bmatrix}$ be a 3×3 matrix.

(a) Use **elementary row operations** to calculate $\det(A)$.

Computation: all row operations must be displayed.

Note that their REF is not necessarily the same as mine, but the determinant should be the same.

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 4 & -2 \end{bmatrix} \xrightarrow{REF} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 5 & -3 \\ 0 & 0 & \frac{14}{5} \end{bmatrix}$$

So

$$\det A = (1)(5)(\frac{14}{5}) = 14$$

$$\boxed{\det(A) = 14}$$

(b) Use the **Inverse Formula** to find the inverse A^{-1} of A.

Computation: the formala and the computation of each cofactor C_{ij} must be displayed.

We have

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

From the previous question, we have $\det A = 14$.

Recall that $C_{ij} = (-1)^{i+j} \det A_{ij}$

$$C_{11} = \begin{vmatrix} -1 & 1 \\ 4 & -2 \end{vmatrix} = -2, \quad C_{21} = -\begin{vmatrix} 1 & 3 \\ 4 & -2 \end{vmatrix} = 14, \quad C_{31} = \begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} = 4$$

$$C_{12} = -\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = 3, \quad C_{22} = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7, \quad C_{32} = -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 1$$

$$C_{13} = \begin{vmatrix} 1 & -1 \\ 1 & 4 \end{vmatrix} = 5, \quad C_{23} = -\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} = -7, \quad C_{33} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$$

So

$$A^{-1} = \frac{1}{14} \begin{bmatrix} -2 & 14 & 4 \\ 3 & -7 & 1 \\ 5 & -7 & -3 \end{bmatrix} = \begin{bmatrix} -1/7 & 1 & 2/7 \\ 3/14 & -1/2 & 1/14 \\ 5/14 & -1/2 & -3/14 \end{bmatrix}$$