MATH 3110 - Spring 2023, Learning Activity 26

Team Leader: Bode Hoofer   Group member: Dylan Mumm

Group member: ______   Table number: 14

Team Leader needs to scan and upload it on Gradescope "Learning Activity 26" and add all group members to the submission (due 4/10).

1. Let $S = \{\vec{u}, \vec{v}\}$ where $\vec{u} = \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix}$.

(a) (3 points) Is the set S orthogonal? Justify your answer.

S is orthogonal	S is not orthogonal
X	

Because the dot product of every vector times every other vector is 0, then the set S is orthogonal

Justify your answer:

$$\vec{u}\cdot\vec{v} = \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}\cdot\begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix} = \left(-\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right) + \left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right) + \left(\tfrac{2}{3}\right)(0)$$
$$= \cancel{\left(-\tfrac{2}{9}\right)} + \cancel{\left(\tfrac{2}{9}\right)} + 0 = 0$$

(b) (3 points) Explain why the set S is not orthonormal.

Your answer:

For S to be orthonormal, $\|\vec{u}\|$ and $\|\vec{v}\|$ must be exactly 1.

$$\|\vec{u}\| = \sqrt{\left(-\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right) + \left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) + \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)} = \sqrt{\tfrac{4}{9} + \tfrac{1}{9} + \tfrac{4}{9}} = \sqrt{\tfrac{9}{9}} = \sqrt{1} = \boxed{1}$$

$$\|\vec{v}\| = \sqrt{\left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) + \left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right) + (0)(0)} = \sqrt{\tfrac{1}{9} + \tfrac{4}{9} + 0} = \sqrt{\tfrac{5}{9}} = \tfrac{\sqrt{5}}{3} \neq 1$$

Since $\|\vec{v}\| \neq$ [1], then S is not orthon[ormal]

(c) (3 points) Normalize the vectors in S to produce an orthonormal set S'.

Computation

$\|\vec{u}\| = 1$ $\qquad \vec{u}' = \dfrac{1}{\|\vec{u}\|}\vec{u} = \dfrac{1}{1}\vec{u} = \vec{u} \longrightarrow$ already normalized

$\|\vec{v}\| = \dfrac{\sqrt{5}}{3}$ $\qquad \vec{v} = \dfrac{1}{\|\vec{v}\|}\vec{v} = \dfrac{1}{\sqrt{5}/3}\begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1/3}{\sqrt{5}/3} \\ \frac{2/3}{\sqrt{5}/3} \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{5}/5 \\ \frac{2\sqrt{5}}{5} \\ 0 \end{bmatrix}$

$$S' = \left\{ \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} \sqrt{5}/5 \\ 2\sqrt{5}/5 \\ 0 \end{bmatrix} \right\}$$

2. (5 points) Find the orthogonal projection $\hat{y}$ of $\vec{y} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$ onto $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Computation:

$$\hat{y} = \text{proj}_L(\vec{y}) = \frac{\vec{y}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\vec{u} = \frac{\begin{bmatrix} -3 \\ 9 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 2 \end{bmatrix}}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{(-3+18)}{(1+4)}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{15}{5}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

let $L = \text{span}(\vec{u})$

$$= 3\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\boxed{\hat{y} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}}$$

3. (3 points) Show that if U and V are $n \times n$ orthogonal matrices then UV is an orthogonal matrix (Hint: show that $(UV)^{-1} = (UV)^T$).

Your answer: If U and V are orthogonal, then $U^{-1} = U^T$ and $V^{-1} = V^T$

~~So~~ ~~~~ In other words, U and V are invertible

$$(UV)^{-1} = V^{-1}U^{-1} = V^TU^T = (UV)^T$$

Given V and U are orthogonal

Since, $(UV)^{-1} = (UV)^T$, UV is an orthogonal matrix.