MATH 3110 - Spring 2023, Learning Activity 25

Team Leader: [illegible] Group member: Bode Hooker

Group member: [illegible] Table number: 14

Team Leader needs to scan and upload it on Gradescope "Learning Activity 25" and add all group members to the submission (due 4/03).

1. (4 points) Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in $\mathbb{R}^n$. For each of the following expressions, determine if it is a scalar, a vector, or not defined (cannot be preformed). Mark by **X** the right answer.

$\frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{w}}\vec{w} = \frac{u_1v_1+u_2v_2+\dots+u_nv_n}{u_1w_1+u_2w_2+\dots+u_nw_n}\vec{w}$

$\left(\frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{w}}\right)$ evaluates to a scalar this is defined if $\vec{u}\cdot\vec{w} \neq 0$ & a scalar times a vector ($\vec{w}$) is just a vector.

	a scalar	a vector	not defined
$\frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{w}}\vec{w}$		X	
$\frac{\vec{v}}{\vec{u}\cdot\vec{w}}$		X	
$\left((\vec{u}+\vec{w})\cdot\vec{v}\right)+\vec{w}$			X
$\frac{\vec{v}}{\vec{u}\cdot\vec{w}} \cdot \frac{\vec{u}\cdot\vec{u}}{\vec{w}\cdot\vec{v}}\vec{u}$	X		

$\frac{\vec{v}}{\vec{u}\cdot\vec{w}} = \left(\frac{1}{\vec{u}\cdot\vec{w}}\right)\vec{v}$ evaluates to scalar

$((\vec{u}+\vec{w})\cdot\vec{v})+\vec{w}$ evaluates to some vector $\vec{\lambda}$ so $(\vec{\lambda}\cdot\vec{v})+\vec{w}$ evaluates to scalar, but scalar + vector is not defined.

$\left(\frac{1}{\vec{u}\cdot\vec{w}}\right)\vec{v} \cdot \left(\frac{\|\vec{u}\|}{\vec{w}\cdot\vec{v}}\right)\vec{u}$ scalar, vector, scalar, vector so just $\vec{u}\cdot\vec{r}$ which evaluates to a scalar.

2. Let $\vec{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$ be vectors in $\mathbb{R}^3$.

(a) (3 points) Compute the distance between $\vec{w}$ and $\vec{v}$.

Computation:

$\text{dist}(\vec{w},\vec{v}) = \|\vec{u}-\vec{v}\| = \sqrt{(\vec{u}-\vec{v}) + (\vec{u}-\vec{v})}$
$= \sqrt{(3-6)^2+(-1+2)^2+(-5-3)^2}$
$= \sqrt{(-3)^2+(1)^2+(-8)^2}$
$= \sqrt{9+1+64} = \sqrt{74}$

$\boxed{\text{dist}(\vec{w},\vec{v}) = \sqrt{74}}$

(b) (3 points) Find a unit vector $\vec{u}$ in the direction of $\vec{v}$.

Computation:

$\|\vec{v}\|^2 = \vec{v} \cdot \vec{v} =$ [illegible]

$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}} =$ [illegible]

$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} =$ [illegible]

$\|\vec{u}\|^2 = \vec{u} \cdot \vec{u} = \left(\frac{6}{7}\right)^2 + \left(\frac{-2}{7}\right)^2 + \left(\frac{3}{7}\right)^2$

$= \frac{36}{49} + \frac{4}{49} + \frac{9}{49} = 1$

$$\boxed{\vec{u} = \begin{bmatrix} 6/7 \\ -2/7 \\ 3/7 \end{bmatrix}}$$

3. (5 points) Let $\vec{c}_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$, $\vec{c}_2 = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ and $\vec{z} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$ be vectors in $\mathbb{R}^3$. Let $W = \text{Span}(\vec{c}_1, \vec{c}_2)$. Show that $\vec{z}$ is orthogonal to W (i.e. $\vec{z}$ is in $W^\perp$).

Your answer:

$Z \cdot C_1 = 0 \quad \& \quad z \cdot c_2 = 0$

$$\begin{bmatrix} 2 & 5 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} = (6) + (-10) + (4) = \underline{0}$$

$$\begin{array}{cc} & C_1 \\ \begin{bmatrix} 2 & 5 & 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \end{array} = (2)(-1) + (0) + (2)(1) = -2 + 2 = \underline{0}$$

4. (3 points) Let $\vec{b}_1, \vec{b}_2$ and $\vec{z}$ be vectors in $\mathbb{R}^n$ such that $\vec{b}_1$ and $\vec{z}$ are orthogonal and $\vec{b}_2$ and $\vec{z}$ are also orthogonal. Show that $\vec{w} = 2\vec{b}_1 - \vec{b}_2$ and $\vec{z}$ are orthogonal.

Your answer:

If $\vec{z} \cdot \vec{b}_1 = 0$ & $\vec{z} \cdot \vec{b}_2 = 0$, then $\vec{z} \cdot \vec{w} = \vec{z} \cdot (2\vec{b}_1 - \vec{b}_2)$

$= \vec{z} \cdot 2\vec{b}_1 - \vec{z} \cdot \vec{b}_2 =$

$= \vec{z} \cdot 2\vec{b}_1 - 0 = 2(\vec{z} \cdot \vec{b}_1) = 2(0)$

$= 0$

Since $\vec{z} \cdot \vec{v} = 0$, $\vec{z}$ & $\vec{w}$ are orthogonal