# Determinent

Let A be n x n matrix, ($A = (a_{ij})_{ij}$)

Definition:

- det(a) = formula
  - $A_{ij}$ = obtained by deleting rows I and col j of matrix A

Cofactors of A

- $C_{ij}$ = formula

Cofactor expansion across row i

- Det(A) = $a_{i1}c_{i1} + a_{i2}c_{i2} + \ldots + a_{in}c_{in}$

Cofactor expansion across col j

- Det(A) = $a_{1j}c_{1j} + a_{2j}c_{2j} + \ldots + a_{nj}c_{nj}$

Equivalent to transpose

- Det(A) = det($A^T$)

Properties

- If [A] $R_i$ -> $R_i$+c$R_j$ [B], det(A) = det(B)
  - Row summation thing doesn't change determinant
- If [A] $R_i$ -> $R_i$+$R_j$ [B], det(A) = -det(B)
  - Swapping row flips sign of determinant
- If [A] $R_i$ -> $R_i$+c$R_i$ [B], c(det(A)) = det(B)
  - Scaling row does change determinant
- If [A] REF -> [V] has no use of row scaling
  - Det(A) = $(-1)^r$ det(V)
  - R: # of row interchainging
  - V: in REF (triangular matrix)
    - Det(V) = product of the main design of entries of V
- Det(AB) = det(A) det(B) = det(B) det(A) = det(BA)
  - AB != BA
- $A^{-1}$ exists (det(A) != 0)
  - Det(A $A^{-1}$) = det($I_n$) = 1
  - Det(A)det($A^{-1}$) = 1 =>
    - Det($A^{-1}$) = [1 / det(A)]

Inverse formula

- If det(A) != 0
  - $A^{-1}$ = [1/det(A)] Adj A, Adj A = [$C_{11}$ $C_{21}$ … $C_{n1}$, $C_{12}$ $C_{22}$ … $C_{n2}$, …, $C_{1n}$ $C_{2n}$ $C_{nn}$]

## Vector Spaces
(V, +, *) a vector space

- A subset H of V is a subspace of V if….
  - **0** vector is in H
  - If **u and v** are in H, than u+v is in H
  - If u is in H and c is in R, then cu is in H
- Any substet W = Span($v_1$, $v_2$, … , $v_p$) where $v_1$ $v_2$ $v_p$ are in V is a substapce of V
- Let A be an mxn matrix [vectors $a_1$, $a_2$, … , $a_n$]
  - Nul(A) = { vector x in $R^n$ | Ax = 0} is a subspace of $R^n$}
  - Col(a) = Span($a_1$, $a_2$, …., $a_n$) is a subspace of $R^m$
  - Row(A) = span(rows of A) is a subspace of $R^n$

T: $R^n$ -> $R^m$ a linear transformation with standard matrix A

- T(x) = Ax
- Ker(T) = { x in $R^n$ | T(x) = 0}
  - {x in $R^n$ | Ax = 0 } = Nul(A)
- Im(T) = {T(x) | x in $R^n$} = Col(A)

## Bases
V a vector space

- A basis for V is a spanning set of V which is linearly independent
- If x = c1v1 + c2v2 + cpvp is the parametric vector form of solution of Ax=0
  - {v1, v2, …, vp} is basis for Nul(A)
- The pivot columns of A form a basis for Col(A)
- The non-zero rows of REF of A form a basis for Row(A)

## Spanning Set Theorem
V is a vector space and H a subspace of V

- H = Span(v1, v2, …, vp)

If Vi = c1v1 + c2v2 + … + c(i-1)v(i-1) + c(i+1)v(i+1) + …. + cpvp, then

- H = Span(V1, v2,…, v(i-1), v(i+1), vp)

## Coordinate Systems
Let B = {b1, b2, …., bn} be a basis for V

- For any u in V, there exists a unique set of scalars c1, c2, cn such that
  - U = c1b1 + c2b2 + … + cnbn
  - $[u]_b$ = [c1,, c2, …,, cn]
    - B-coordinate vector of u

Change of Basis

- Her notes were entirely untinteligble

Things?

- Dim(V) = # vectors in a basis for V
- Dim(Nul(A)) = # free variables in Ax = 0
- Dim(Col(A) = #pivot columns of A
- Dim(row(A)) = #pivots in A

Let A be m x n matrix

- Rank(A) = dim(Col(A)) = dim(Row(A))
    - Row(A) = col($A^T$)
- Dim(Col(At) = dim(Row(A) = rank(A)
- Rank(A) + dim(Nul(A)) = n = #columns of A
- Rank(A) <= min(m, n)