

1. If a 6×3 matrix A has rank 3, find $\dim(\text{Nul}(A))$, $\dim(\text{Col}(A))$ and $\text{rank}(A^T)$.

Computation: We have $\dim(\text{Nul}(A)) + \text{rank}(A) = 3$ (number of columns) so $\dim(\text{Nul}(A)) = 0$ and $\text{rank}(A^T) = \text{rank}(A) = 3$

$\dim(\text{Nul}(A)) = 0$	$\dim(\text{Col}(A)) = 3$	$\text{rank}(A^T) = 3$
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2. Let B be a 8×7 matrix.

- (a) Could B have a 2-dimensional null space? Justify your answer.

Yes	No
X	

Explain your answer: If $\dim(\text{Nul}(A)) = 2$ then $\text{rank}(A) = \dim(\text{Col}(A)) = 7 - 2 = 5$ which is possible since it's less than 7 (the largest possible rank).

- (b) What is the largest possible rank of B ? Justify your answer.

largest possible rank = 7

Explain your answer: the largest possible rank is $\min(8, 7) = 7$.

3. Assume that the matrix A is row equivalent to the matrix B , given below.

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) Determine $\text{rank}(A)$ and $\dim(\text{Nul}(A))$.

$\text{rank}(A) = 3$	$\dim(\text{Nul}(A)) = 2$
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- (b) Find bases for $\text{Col}(A)$, $\text{Row}(A)$ and $\text{Nul}(A)$.

Computation: **For the basis for $\text{Nul}(A)$** , write the solutions of $A\vec{x} = \vec{0}$ in parametric vector form and collect the vectors. Using the row echelon form B , we have

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ 0 \\ 3/2 \\ 1 \\ 0 \end{bmatrix}, \quad x_2, x_4 \in \mathbb{R}$$

Basis for $\text{Col}(A)$	Basis for $\text{Row}(A)$	Basis for $\text{Nul}(A)$
<p>the pivot columns of A (1st, 3rd, and 5th column)</p> $\mathcal{B}_{\text{Col}(A)} = \left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ -10 \\ -3 \\ 0 \end{bmatrix} \right\}$	<p>the nonzero rows of the row echelon form B of A (1st, 2nd, and 3rd row of B) $\mathcal{B}_{\text{Row}(A)} = \{(1, -3, 0, 5, -7), (0, 0, 2, -3, 8), (0, 0, 0, 0, 5)\}$</p>	$\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 3/2 \\ 1 \\ 0 \end{bmatrix} \right\}$