MATH 3110 - Spring 2023, Learning Activity 10

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Group member : Dylan Mumm Table number: 14

Team Leader needs to scan and upload it on Gradescope "Learning Activity 10" and add all group members to the submission (due 2/08).

1. (2 points) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix such that $ad = bc$. Is A invertible? Explain your answer. Mark the right answer by **X**.

A is invertible	A is not invertible
	X

Explain your answer:

Since $ad = bc$, this means $ad - bc = 0$. However, for a matrix to be invertible, $\det(A) \neq 0$ or $ad - bc \neq 0$.

2. (3 points) Explain why the matrix $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$ is an elementary matrix.

Your answer:

E is an elementary matrix because it is one ~~step from~~ row operation from becoming the identity matrix, I_3. The row operation to get it to I_3 would be $R_2 + 5R_3 \to R_2$.

3. (3 points) Let A, B and P be $n \times n$ matrices such that P is invertible and $A = PBP^{-1}$. Solve for B in terms of A.

Computation:

$A = PBP^{-1}$

$P^{-1}A = P^{-1}PBP^{-1}$

$P^{-1}A = I_n BP^{-1}$

$P^{-1}A = BP^{-1}$

$P^{-1}AP = BP^{-1}P$

$P^{-1}AP = BI_n$

$\boxed{P^{-1}AP = B}$

$B = P^{-1}AP$

4. (4 points) Compute the inverse of the matrix $M = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 6 & -1 \\ 0 & -4 & 1 \end{bmatrix}$ by using **elementary row operations**. All row operations must be shown.

$1 + 4(-\frac{1}{6}) = 1 - \frac{2}{3} = \frac{1}{3}$

Computation:

M

$$\begin{bmatrix} 2 & 4 & 1 \\ 0 & 6 & -1 \\ 0 & -4 & 1 \end{bmatrix} \xrightarrow{\frac{1}{6}R_2 \to R_2} \begin{bmatrix} 2 & 4 & 1 \\ 0 & 1 & -\frac{1}{6} \\ 0 & -4 & 1 \end{bmatrix} \xrightarrow[R_3+4R_2 \to R_3]{} \begin{bmatrix} 2 & 4 & 1 \\ 0 & 1 & -\frac{1}{6} \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$\xrightarrow{3R_3 \to R_3} \begin{bmatrix} 2 & 4 & 1 \\ 0 & 1 & -\frac{1}{6} \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[\substack{R_2+\frac{1}{6}R_3 \to R_2 \\ R_1 - R_3 \to R_1}]{} \begin{bmatrix} 2 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\cancel{\ } R_1 - 4R_2 \to R_1} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_3$$

$$\xrightarrow{\frac{1}{6}R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3+4R_2 \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & \frac{2}{3} & 1 \end{bmatrix} \xrightarrow{3R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

$-2 - 4(\frac{1}{2}) = -2 - 2 = -4$

$$\xrightarrow[\substack{R_2+\frac{1}{6}R_3 \to R_2 \\ R_1 - R_3 \to R_1}]{} \begin{bmatrix} 1 & -2 & -3 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 3 \end{bmatrix} \xrightarrow{R_1 - 4R_2 \to R_1} \begin{bmatrix} 1 & -4 & -5 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 3 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} \frac{1}{2} & -2 & -\frac{5}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 3 \end{bmatrix}$$

$$\boxed{M^{-1} = \begin{bmatrix} \frac{1}{2} & -2 & -\frac{5}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 3 \end{bmatrix}}$$