MATH 3110 - Spring 2023,

Learning Activity 11

1. Is it possible for 4×4 matrix A to be invertible when its column do not span $\mathbb{R}^4$? Why or why not?

Yes, A can be invertible	No, A must be non-invertible
	X

Explain: any statement similar to one of the following statements is correct:

- By the Invertible matrix theorem, A is invertible if and only if the columns of A span $\mathbb{R}^4$.
- If the columns of A do not span $\mathbb{R}^4$, then A does not have a pivot in each row. So A has less than 4 pivots, and then A is not invertible.

2. If A is a 5×5 matrix and the equation $A\vec{x} = \vec{b}$ is consistent for every $\vec{b}$ in $\mathbb{R}^5$, is it possible for some $\vec{b}$, the equation $A\vec{x} = \vec{b}$ has more than one solution? Why or why not?

Yes, $A\vec{x} = \vec{b}$ can have infinitely many solutions	No, $A\vec{x} = \vec{b}$ has only a unique solution
	X

Explain: any statement similar to the following statement is correct

- Since $A\vec{x} = \vec{b}$ is consistent for every $\vec{b}$ in $\mathbb{R}^5$, it follows that the columns of A span $\mathbb{R}^5$. So by the Invertible matrix theorem A is invertible and A has 5 pivots. Thus, every variable in $A\vec{x} = \vec{b}$ is a pivot variable. So there is no free variable and the solution is unique.

3. An $m \times n$ matrix is an upper triangular matrix if its entries below the main diagonal are 0's. When is a square upper triangular matrix invertible? Justify your answer.

Your answer: Any condition similar to the following is correct:

A square upper triangular matrix is invertible when each main diagonal entry is different from 0 (in this case, it will have n pivots so it is invertible).

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 8x_2 \\ -2x_1 + 7x_2 \end{bmatrix}$

(a) Show that T is invertible.

Computation/explanation: Recall that T is invertible if and only if its standard matrix is invertible. The standard matrix of T is given by $A = \begin{bmatrix} 2 & -8 \\ -2 & 7 \end{bmatrix}$. Since $\det(A) = (2)(7) - (-2)(-8) = -2 \neq 0$, A is invertible. It follows that T is invertible.

(b) Compute $T^{-1}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right)$.

Computation: Recall that the standard matrix of the inverse T^{-1} of T is A^{-1} with A the standard