# Determinent

Let A be n x n matrix, (A = (aij)ij)

Definition:

* det(a) = formula
  + Aij = obtained by deleting rows I and col j of matrix A

Cofactors of A

* Cij = formula

Cofactor expansion across row i

* Det(A) = ai1ci1 + ai2ci2 + … + aincin

Cofactor expansion across col j

* Det(A) = a1jc1j + a2jc2j + … + anjcnj

Equivalent to transpose

* Det(A) = det(AT)

Properties

* If [A] Ri -> Ri+cRj [B], det(A) = det(B)
  + Row summation thing doesn’t change determinant
* If [A] Ri -> Ri+Rj [B], det(A) = -det(B)
  + Swapping row flips sign of determinant
* If [A] Ri -> Ri+cRi [B], c(det(A)) = det(B)
  + Scaling row does change determinant
* If [A] REF -> [V] has no use of row scaling
  + Det(A) = (-1)r det(V)
  + R: # of row interchainging
  + V: in REF (triangular matrix)
    - Det(V) = product of the main design of entries of V
* Det(AB) = det(A) det(B) = det(B) det(A) = det(BA)
  + AB != BA
* A-1 exists (det(A) != 0)
  + Det(A A-1) = det(In) = 1
  + Det(A)det(A-1) = 1 =>
    - Det(A-1) = [1 / det(A)]

Inverse formula

* If det(A) != 0
  + A-1 = [1/det(A)] Adj A, Adj A = [C11 C21 … Cn1, C12 C22 … Cn2, …, C1n C2n Cnn]

# Vector Spaces

(V, +, \*) a vector space

* A subset H of V is a subspace of V if….
  + **0** vector is in H
  + If **u and v** are in H, than u+v is in H
  + If u is in H and c is in R, then cu is in H
* Any substet W = Span(v1, v2, … , vp) where v1 v2 vp are in V is a substapce of V
* Let A be an mxn matrix [vectors a1, a2, … , an]
  + Nul(A) = { vector x in Rn | Ax = 0} is a subspace of Rn}
  + Col(a) = Span(a1, a2, …., an) is a subspace of Rm
  + Row(A) = span(rows of A) is a subspace of Rn

T: Rn -> Rm a linear transformation with standard matrix A

* T(x) = Ax
* Ker(T) = { x in Rn | T(x) = 0}
  + {x in Rn | Ax = 0 } = Nul(A)
* Im(T) = {T(x) | x in Rn} = Col(A)

# Bases

V a vector space

* A basis for V is a spanning set of V which is linearly independent
* If x = c1v1 + c2v2 + cpvp is the parametric vector form of solution of Ax=0
  + {v1, v2, …, vp} is basis for Nul(A)
* The pivot columns of A form a basis for Col(A)
* The non-zero rows of REF of A form a basis for Row(A)

# Spanning Set Theorem

V is a vector space and H a subspace of V

* H = Span(v1, v2, …, vp)

If Vi = c1v1 + c2v2 + … + c(i-1)v(i-1) + c(i+1)v(i+1) + …. + cpvp, then

* H = Span(V1, v2,…, v(i-1), v(i+1), vp)

# Coordinate Systems

Let B = {b1, b2, …., bn} be a basis for V

* For any u in V, there exists a unique set of scalars c1, c2, cn such that
  + U = c1b1 + c2b2 + … + cnbn
  + [u]b = [c1,, c2, …,, cn]
    - B-coordinate vector of u

Change of Basis

* Her notes were entirely untinteligble

Things?

* Dim(V) = # vectors in a basis for V
* Dim(Nul(A)) = # free variables in Ax = 0
* Dim(Col(A) = #pivot columns of A
* Dim(row(A)) = #pivots in A

Let A be m x n matrix

* Rank(A) = dim(Col(A)) = dim(Row(A))
  + Row(A) = col(AT)
* Dim(Col(At) = dim(Row(A) = rank(A)
* Rank(A) + dim(Nul(A)) = n = #columns of A
* Rank(A) <= min(m, n)