ON BAD CIRCUIT MODELLING

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ABSTRACT

Traditional circuit modelling methods typically assume ideal circuit components. Real world audio circuits exhibit variations in behavior due to non-ideal factors including component tolerances, operating temperature, and aging. We present a brief discussion of each of these non-ideal factors for resistors, capacitors, and operational amplifiers (op-amps), and show how they each individually affect the behavior of a circuit model. We present a models of Sallen-Key lowpass filter, and diode clipper circuits that incorporates all of the non-ideal factors together.

1. INTRODUCTION

Audio effect circuits and circuit models are a vital part of modern audio signal processing. Circuit modelling in particular has seen a rise in popularity in recent years, particularly in the form of audio plugin-ins that model circuits from vintage audio effects, amplifiers, and synthesizers. Many engineers and musicians prefer these software emulations over the original hardware units because of the lower cost, portability, and convenience. However, some users have noticed that the software emulations do not model the unit-to-unit variation in these effects. For example, if two engineers buy the same hardware compressor unit, the resulting hardware units will sound similar, but not identical, due to minor variations in the components that comprise each unit. Modern circuit models do not attempt to emulate this unit-to-unit variation, nor do they consider the non-ideal conditions that create this variation.

In this writing, we examine these non-ideal conditions, and show how existing modelling methods can be expanded to include this behavior. In §2 we examine the effects of component tolerances of resistors and capacitors. §3 discusses the effects of aging capacitors, resistors, and op-amps. Temperature considerations for op-amps are discussed in §4. In §5, we discuss modelling capacitor leakage by including a parallel resistor. Finally, in §6 we show how the above factors can be implemented into existing circuit models using nodal analysis and wave digital filters as examples.

2. COMPONENT TOLERANCES

All resistors and capacitors are labelled with both a component value (e.g. $1k\Omega$ resistor, $1\mu F$ capacitor), and a tolerance rating (e.g. $\pm 5\%$), as shown in fig. 1. We propose adjusting the component values used in a circuit model to a random value within the tolerance range of the component.

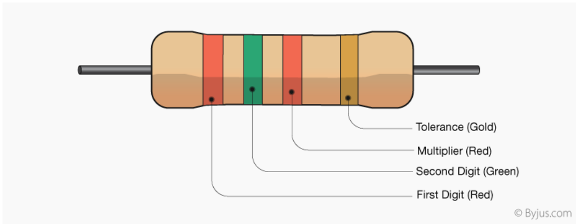

Figure 1: *Resistor labelling. Adapted from* `https://byjus.com/physics/resistor-colour-codes/`.

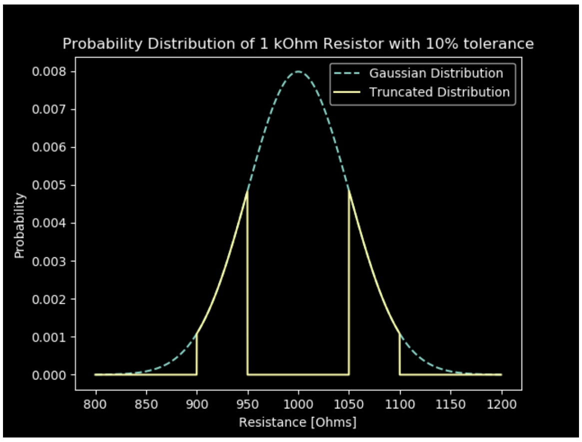

Figure 2: *Probability distribution for the value of a* $10k\Omega$ *resistor with* $\pm 10\%$ *tolerance. Note the "truncated Gaussian" distribution.*

When a manufacturer makes a batch of resistors or capacitors the component values of the batch follows a roughly Gaussian distribution centered at the ideal component value. The manufacturer then extracts the worst components that can still satisfy a certain tolerance rating and sells them at that rating [1]. For example, if a manufacturer sells resistors at $\pm 5\%$ and $\pm 10\%$ tolerance the $\pm 10\%$ components will be distributed in a sort of "truncated Gaussian" distribution, comprised of the original Gaussian distribution truncated between 5 and 10% (see fig. 2). To show how component tolerances can affect the behavior of an audio effect circuit, fig. 3 shows the frequency responses of 1000 Sallen-Key lowpass filters made with components that have $\pm 10\%$ tolerance ratings.

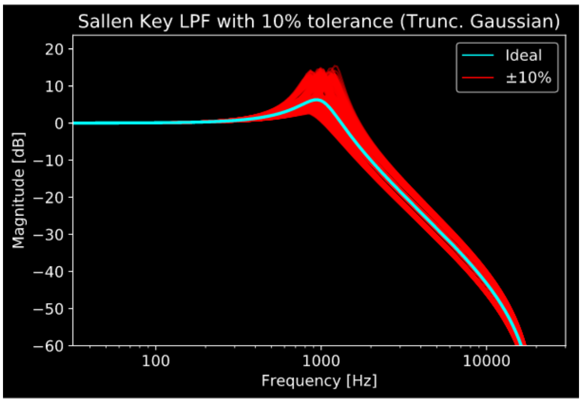

Figure 3: *Frequency response of Sallen-Key lowpass filters made with components with* $\pm 10\%$ *tolerance.*

Figure 4: $1k\Omega$ *Resistor aging over time.*

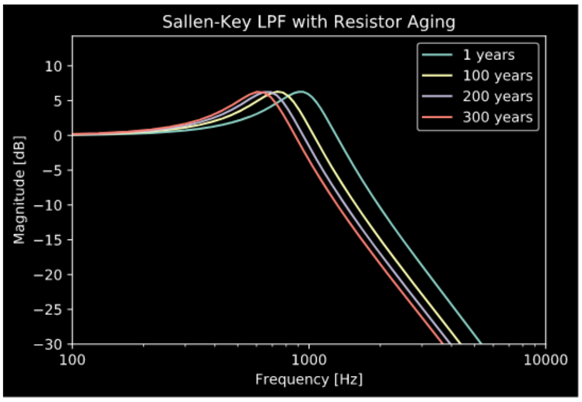

Figure 5: *Sallen-Key LPF with resistor aging.*

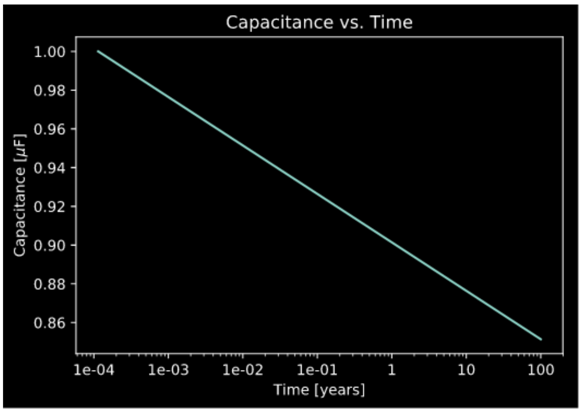

Figure 6: *Capacitor aging over time.*

3. COMPONENT AGING

Another non-ideal factor that can affect the behavior of a circuit is the aging of the components. This factor can be especially important when examining vintage audio circuits.

3.1. Resistor Aging

As a resistor grows old, its resistance tends to increase. For a typical thin-film resistor, the age dependence is described by the following equation [2].

$$\frac{\Delta R}{R} = (1.51 \times 10^{12})\, t^{0.61}\, e^{-15{,}087/T} \tag{1}$$

where t is the length of time the resistor has been used (in hours), and T is the operating temperature in Kelvins. In fig. 4, we show a $1k\Omega$ resistor operating at 400 K running over a period of 100 years. While the resistance change may seem small, these small changes compound over all the resistors in the circuit. Figure 5 shows the frequency response of a Sallen-Key lowpass filter with resistor aging, again with an operating temperature of 400 K.

3.2. Capacitor Aging

For audio circuits, it is typical to use class II X7R capacitors, due to their minimal amount of voltage dependence, meaning the resulting circuit will have a minimal Total Harmonic Distortion (THD). For this class of capacitor, the capacitance decreases by $\sim 2.5\%$ per decade hour [3]. Figure 6 shows the capacitance of a $1\mu F$ capacitor aging over a period of 100 years, and fig. 7 shows the frequency response of a Sallen-Key lowpass filter with capacitor aging.

3.2.1. Capacitor Failure

Capacitor aging again makes a minor difference to the overall behavior of a circuit, however older circuit tend to have a larger issue with capacitor failure. For the class of capacitor considered here, the expected lifetime is approximately:

$$L = (5000 \text{ hours})\, 2^{(373-T)/10} \tag{2}$$

While there is some fluctuation in actual failure times, capacitor lifetime tends to follow a normal distribution [4]. When a capacitor fails, its capacitance tends to increase to at least an order of magnitude above its ideal value, and tends to have a large dependence on the voltage across its terminals, resulting in a much greater amount of THD.

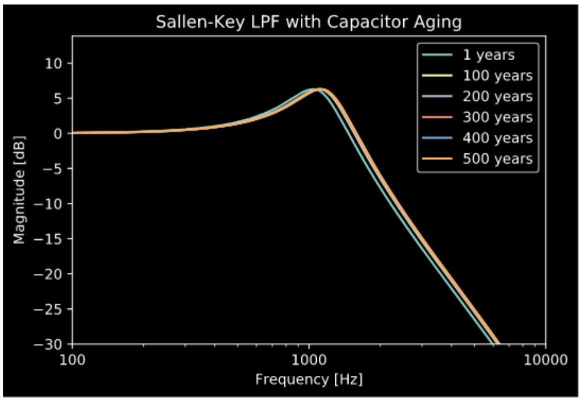

Figure 7: *Sallen-Key LPF with capacitor aging.*

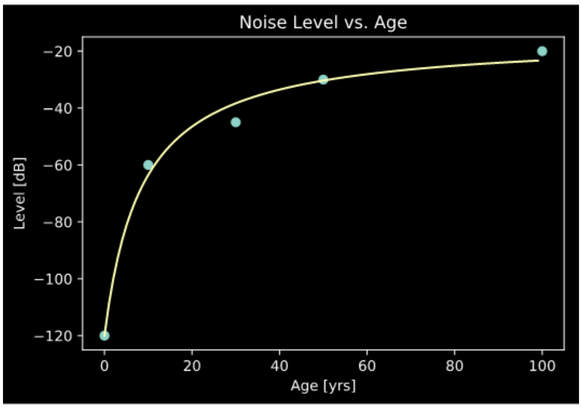

Figure 8: *Op-amp noise over time.*

3.3. Op-Amp Aging

As op-amps age, they are subjected to shorts, and large currents across their terminals, which causes them to develop a noise characteristic, and lose some of their bandwidth [5]. The noise power increases with age, as shown in fig. 8.

From our own measurements, we have found that as an op-amp ages and approaches failure, it starts to exhibit a distortion characteristic similar to the canonical "dropout" or "dead-zone" static nonlinearity, shown in fig. 9.

4. OP-AMP TEMPERATURE

The operating temperature at which a circuit is used can also have an affect on the circuit's overall sound. In particular, op-amps tend to have a relatively strong temperature dependence, particularly in relation to the op-amp's bandwidth. [6] describes a NASA study examining the temperature dependent behavior of the Analog Devices OP181 op-amp, though their results can be scaled to apply to most other op-amp's with similar construction. Figure 10 shows the frequency response of the OP181 at various temperatures.

We propose scaling the data from [6] to apply to the widely used Texas Instruments LM741 op-amp, and model the frequency response using a first-order lowpass filter, with cutoff frequency f_c

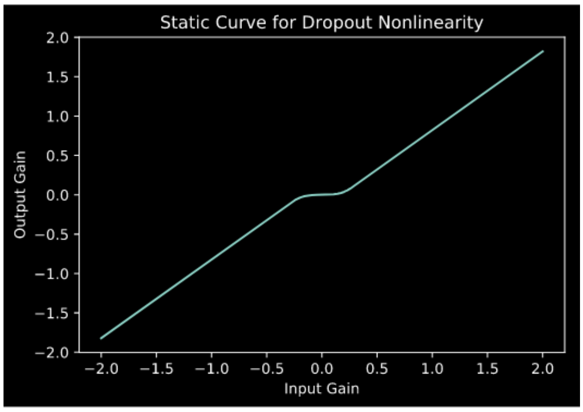

Figure 9: *Dropout nonlinearity.*

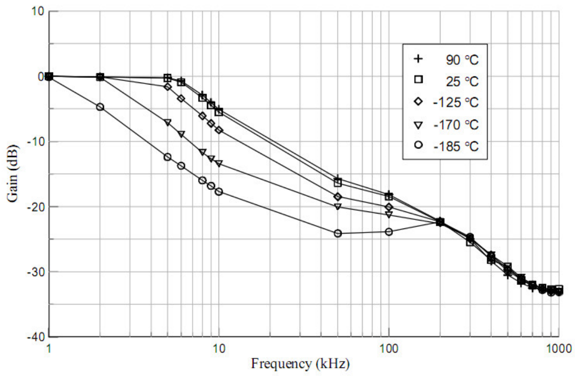

Figure 10: *Op-amp gain vs. frequency at various temperatures. Adapted from [6].*

dependent on the operating temperature. We determined that the relationship between temperature and f_c follows the form of a "binding" function, as follows:

$$f_c(T) = \frac{30.96T}{T + 290.48} \tag{3}$$

where T is the operating temperature in Kelvin. Figure 11 shows the data from the OP181 (adapted from [6]) and the scaled data from the LM741, as well as the binding functions that best fit the data. In fig. 12, we show the frequency response of a Sallen-Key lowpass filter with an LM741 op-amp at various operating temperatures.

5. CAPACITOR LEAKAGE

Beyond imperfections due to temperature, aging, and tolerance, capacitors tend to exhibit "leakage", which can be modelled as a resistor in parallel with the capacitor (RL in fig. 14). Electrolytic capacitors of the type discussed here, typically exhibit a leakage current of $\sim 20nA$ per μF, although that value can increase with the age of the component. In our wave digital filter diode clipper example we introduce a leaky capacitor using the model described above, however we chose not to include capacitor leakage in our nodal analysis example, since including this component would al-

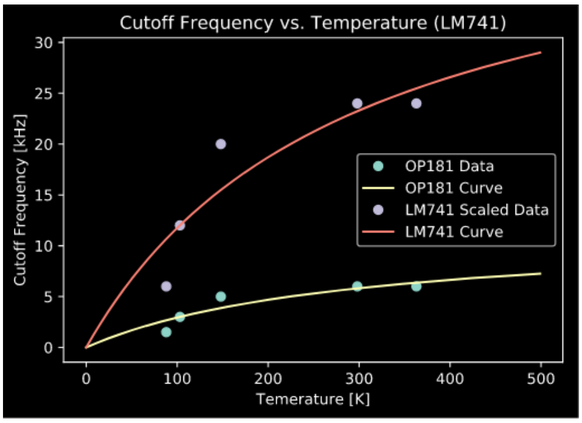

Figure 11: *Cutoff frequency vs. temperature for OP181 and LM741 op-amps, including scaled data.*

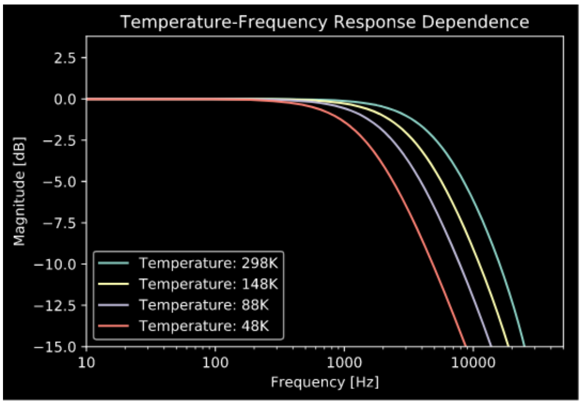

Figure 12: *Frequency response of a Sallen-Key LPF with various op-amp temperatures.*

ter the structure of the circuit, requiring nodal analysis to be entirely recomputed.

Note that other capacitor imperfections can be similarly modelled using traditional circuit elements, including dissipation (RS in fig. 14), and soakage (L); not to mention similar resistor imperfections including internal capacitance and inductance [7]. These factors are ignored in this writing due to their relative insignificance at audio frequencies.

6. IMPLEMENTATION

6.1. Sallen-Key Lowpass Filter

In the process of this study, we have been using a Sallen-Key lowpass filter [8] (see fig. 13) as an example circuit for examining the effects of the non-ideal factors described in the above sections. The Sallen-Key LPF circuit is useful for testing, because it lends itself easily to simple nodal analysis, assuming that the op-amp in the circuit provides strictly linear gain (there is already existing literature that examines models of the circuit where this assumption is relaxed, e.g. [9]). Specifically, the transfer function of this

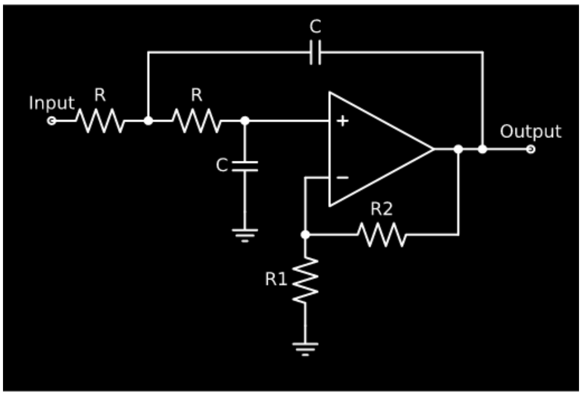

Figure 13: *Sallen-Key LPF circuit.*

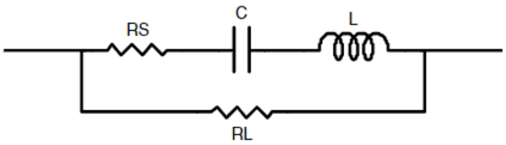

Figure 14: *Modelling capacitor leakage, dissipation, and soakage using circuit elements. Adapted from [7].*

circuit can be written as

$$H(s) = \frac{1}{\left(\frac{s}{2\pi f_c}\right)^2 + \frac{1}{Q}\left(\frac{s}{2\pi f_c}\right) + 1} \tag{4}$$

where

$$f_c = \frac{1}{2\pi RC}, \quad Q = \frac{1}{2 - \frac{R_2}{R_1}} \tag{5}$$

Figures 3, 5, 7 and 12 show the frequency responses of the Sallen-Key lowpass filter circuit for each of the non-ideal factors discussed above.

6.2. WDF Diode Clipper

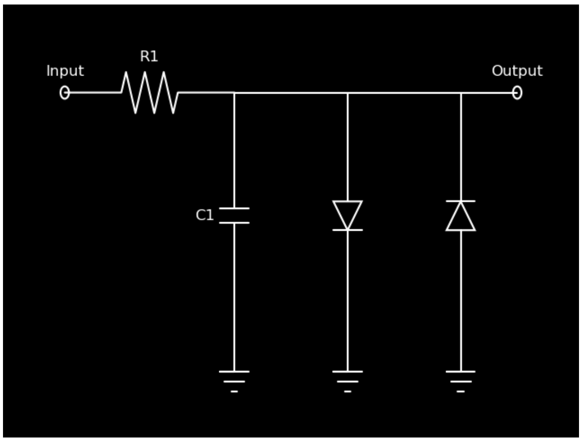

Figure 15: *Diode clipper circuit.*

As a second example circuit, we chose to implement a diode clipper using the Wave Digital Filter (WDF) formulation. Originally, developed by Alfred Fettweis [10] in the 1980's, and expanded on in recent years by Yeh [11], Werner [12], and others [13, 14], WDFs provide a particularly well suited circuit modelling method for incorporating non-ideal factors, due to their highly modular nature. We implemented a wave digital emulation of a simple diode clipper circuit (see fig. 15), based on the formulations derived in [15, 16], including all non-ideal factors described above.

6.3. Software Implementation

The Sallen-Key lowpass filter and diode clipper models have been implemented as audio plugins (VST/AU) using the JUCE/C++ framework. In each plugin, the user can control various parameters related to the component tolerances, age, and temperature, to hear how these factors effect the overall sound of the circuit model, and make real-time comparisons. The plugins and their source code are freely available on GitHub [1].

7. CONCLUSION

In this paper we have discussed non-ideal circuit factors, including component tolerance, age, and temperature, and shown how these factors can be implemented into existing circuits modelled with nodal analysis and wave digital techniques.

The main limitation in this line of research has been the lack of reliable and rigorous sources discussing non-ideal circuit factors. Each of the factors discussed here have been verified experimentally by the author to the greatest possible extent, however future research could include more rigorous testing of component tolerance distributions, age dependence, and more. Another future line of research involves the possibility of including random factors in consumer audio software, perhaps by seeding a random number generator based on the time at which the software is installed, so that each user has a copy of the software that sounds slightly unique.

8. ACKNOWLEDGMENTS

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