Landau-Ginzburg semantics of linear logic

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July 26, 2015

1 Introduction

In this paper we show that there is a natural connection between moduli spaces and logic, using the example of Landau-Ginzburg models. The bicategory of Landau-Ginzburg models $\mathcal{LG}$ has as objects isolated hypersurface singularities, and as 1-morphisms matrix factorisations of differences of potentials. In [**?**] this bicategory was studied in connection with topological field theory with defects and shown to have various good properties, and in [**?**] it was shown how to make composition in this bicategory constructive. In this paper we extend this work by adding *moduli spaces* of matrix factorisations, and showing how the resulting bicategory hosts a model (called a semantics) of linear logic.

- Landau-Ginzburg models
- Algorithmically computable
- Example of a self-composition of rank 1 endo-MFs. This includes P_S's so is already plenty complicated.
- Linear logic
- Semantics of linear logic
- Moduli spaces, simple example
- Bundle over moduli space of self-compositions
- This is the denotation of the proof 2.
- Things get complicated: $\mathbf{int}_{\mathbf{int}_x}$

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