(1)

Feedback 2

A. On continuity

I. In the following exercises we want to study continuity and classify different discontinuities. The exercises given are functions of one real variable.

131. $f(x) = \frac{1}{\sqrt{x}}$. Since we do not have to study continuity at a given point, we study the continuity in general of the function.

Dom f require that $\begin{cases} x \neq 0 \\ x \geqslant 0 \end{cases}$

These two conditions together give Dom f $=]0, +\infty[$

. Thus our function is continuous on its domain

. Discontinuities occur to the right say ~~at~~ 0^+ and $\lim_{x \to 0^+} \frac{1}{\sqrt{x}} = +\infty$, thus we have infinite discontinuity to the right.

132. $f(x) = \frac{2}{x^2+1}$

Dom f $=]-\infty, +\infty[$ | defined for all values of x

The function is continuous on its domain

133. $f(x) = \frac{x}{x^2 - x}$

Dom f: $x^2 - x \neq 0$ / $x(x-1) \neq 0$

Domain $= \{ x \mid x \neq 0, \text{ and } x \neq +1 \}$

or Dom f $=]-\infty, 0[\cup]0, 1[\cup]1, +\infty[$

The function is continuous on its domain. But

. $\lim_{x \to 0} \frac{x}{x^2 - x} = \frac{0}{0}$

$= \lim_{x \to 0} \frac{x}{x(x-1)} = \lim_{x \to 0} \frac{1}{x-1} = -1$

So we have removable discontinuity at $x = 0$.

. $\lim_{x \to 1} \frac{x}{x^2 - x} = \lim_{x \to 0} \frac{x}{x} \cdot \lim_{x \to 1} \frac{1}{x-1}$

$= \lim_{x \to 1} \frac{1}{x-1}$

However $\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$ and

$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$

Thus we have infinite discontinuity at $x = 1$

134 – transcendental

135. – "

136. $f(x) = \frac{|x-2|}{x-2}$

Dom f $= \{$all values except $2\}$

or $]-\infty, 2[\cup]2, +\infty[$

Observe that

$\lim_{x \to 2} \frac{|x-2|}{x-2}$ Exist

$\Rightarrow \begin{cases} \lim_{x \to 2^-} \frac{-(x-2)}{x-2} = -1 \\ \lim_{x \to 2^+} \frac{x-2}{x-2} = 1 \end{cases}$

$\Rightarrow$ We have jump discontinuity at $x = 2$.

137. $f(x) = \tan 2x = \dfrac{\sin 2x}{\cos 2x}$

Dom f requires that $\cos 2x \neq 0$

$2x \neq \pm \pi/2 + 2k\pi$

$x \neq \pm \dfrac{\pi}{4} + k\pi.$

Dom $f = \mathbb{R} \setminus \{\pm \frac{\pi}{4} + k\pi\}$

Our function is continuous on its domain

Since $\lim\limits_{x\to \frac{\pi}{4}} \tan 2x = \infty$ we have an infinite discontinuity at $x = \pi/4$

138. $f(t) = \dfrac{t+3}{t^2+5t+6}$

Domain requires that $t^2+5t+6 \neq 0$

$(t+2)(t+3) \neq 0$

Dom $f = \{t \mid t \neq -2 \text{ and } t \neq -3\}$

. Our function is continuous on its domain.

But $\lim\limits_{t\to -3} \dfrac{t+3}{t^2+5t+6} = \lim\limits_{t\to -3} \dfrac{1}{t+2} = -1$

We have removable discontinuity at $t = -3$.

But $\lim\limits_{t\to -2} \dfrac{t+3}{t^2+5t+6}$

$= \lim\limits_{t\to -2} \dfrac{t+3}{t+3} \cdot \dfrac{1}{t+2}$

$= \lim\limits_{t\to -2} 1 \cdot \lim\limits_{t\to -2} \dfrac{1}{t+2}$

$= \begin{cases} \lim\limits_{t\to -2^-} \dfrac{1}{t+2} = -\infty \\ \lim\limits_{t\to -2^+} \dfrac{1}{t+2} = +\infty \end{cases}$

$\Rightarrow$ Infinite discontinuity at $t = -2$.

II Continuity at a point and classification.

139 $f(x) = \dfrac{2x^2 - 5x + 3}{x-1}$ at $x = 1$.

. $f(1) = \dfrac{2-5+3}{1-1} = \dfrac{0}{0}$

thus $f(1)$ does not exist

$\Rightarrow f(x)$ is not continuous at $x = 1$

. Classification.

$\lim\limits_{x\to 1} \dfrac{2x^2-5x+3}{x-1} = \lim\limits_{x\to 1} \dfrac{(x-1)(2x-3)}{x-1}$

$= \lim\limits_{x\to 1} (2x-3) = -1$

Since the limit exist we have a removable discontinuity.

147. We are given a piecewise defined function

$f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+2}, & x \neq -2 \\ k, & x = -2 \end{cases}$

. Observe that $f(-2) = \dfrac{4-6+2}{2-2} = \dfrac{0}{0}$

So $f(x)$ is not continuous at $x = -2$.

But $\lim\limits_{x\to -2} f(x) = \lim\limits_{x\to -2} \dfrac{(x+1)(x+2)}{x+2}$

$= \lim\limits_{x\to -2} (x+1) = -1$

This means that at $x = -2$

$f(x) = -1$

$\Rightarrow k$ must be equal to -1

for $k = -1$, our function is continuous

1.48. Is a transcendantal exponential function. Carr it

(3)

149. $f(x) = \begin{cases} \sqrt{kx}, & 0 \le x \le 3 \\ x+1, & 3 < x \le 10 \end{cases}$

The point with a problem is $x = 3$ since it is a break point.

. $f(3) = \sqrt{k.3} = \sqrt{3k}$ and it exists for $k \geq 0$ or say $k \in [0, +\infty[$.

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \sqrt{kx} = \sqrt{3k}, k \geq 0$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+1) = 4$

<u>Discussion</u>: Since the limit must exist we have that

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$ i.e

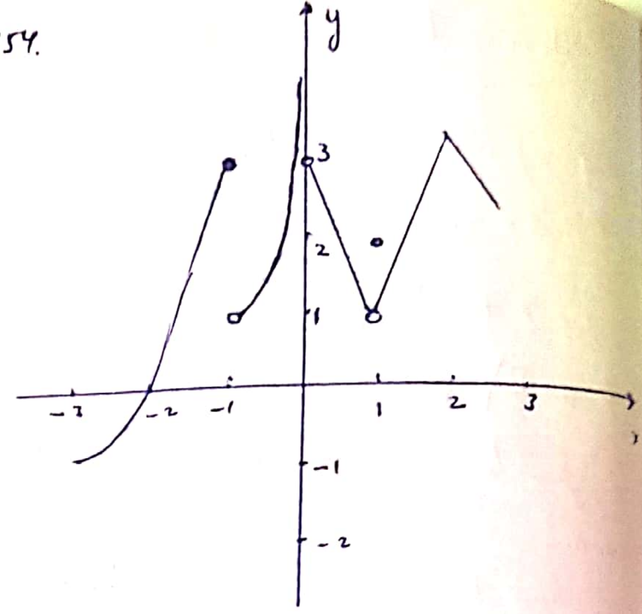
$\sqrt{3k} = 4$

$\Leftrightarrow 3k = 16 \to k = \frac{16}{3}$.

So for $k = \frac{16}{3}$ our function is continuous at $x = 3$

<u>NB</u>: This is a way of defining a discontinuous function at a point so that now it becomes continuous at that point.

154.



a. Observe that at $x = -1$, $x = 0$ and $x = 1$ our function is not continuous

b. at $x = -1$ $f(-1)$ does not exist and $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$

~~But~~

. at $x = 0$, again $f(0)$ does not exist and $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

. at $x = +1$ $f(1)$ exist

But $\lim_{x \to 1} f(x) = 1 \neq f(1) = 2$.

c. at $x = -1$ we have a jump discontinuity since $\lim_{x \to -1^-} f(x) = 3$ and $\lim_{x \to -1^+} f(x) = 1$

$\Rightarrow$ They are finite but different

at $x = 0$, $\lim_{x \to 0^-} f(x) = +\infty$ and $\lim_{x \to 0^+} f(x) = 3$ } but not both finite $\Rightarrow$ infinite discontinuity at $x = 1$

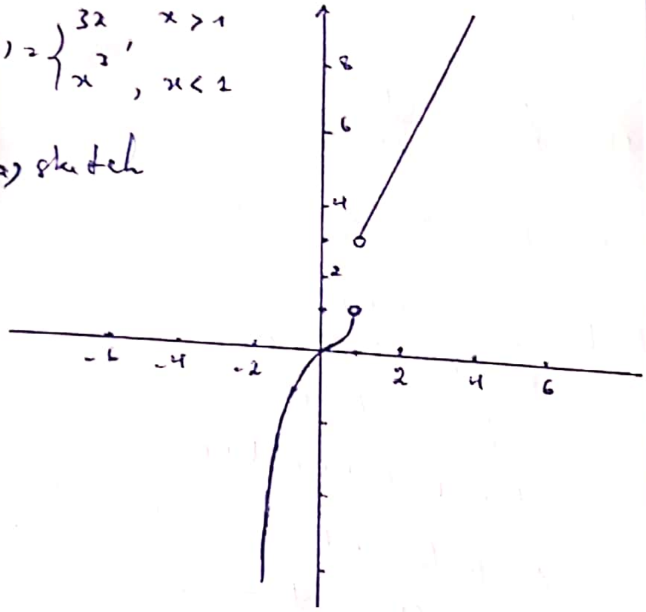
$\lim_{x \to 1} f(x) = 2$ but $\neq f(1) = 2$

$\Rightarrow$ Removable discontinuity.

155.

$$f(x) = \begin{cases} 3x, & x > 1 \\ x^3, & x < 1 \end{cases}$$

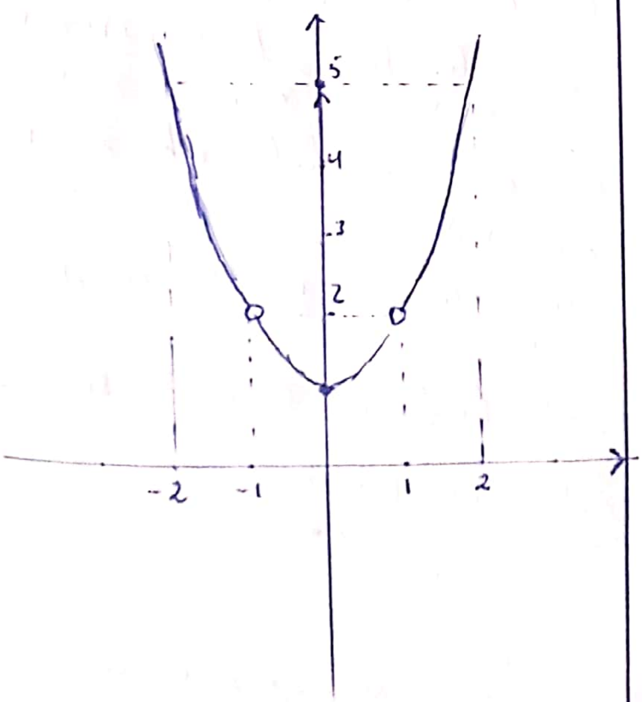
a) sketch



b. We can not define $f(1)$ since we have a jump discontinuity

156. $f(x) = \dfrac{x^4 - 1}{x^2 - 1}$, if $x \neq \pm 1$

x	-2	0	2
$f(x)$	5	1	5



b. Yes. These are found by calculating limits in ± 1 since from the graph we can see that they exist or simply since we have a removable discontinuity

Thus $\lim_{x \to -1} \dfrac{x^4 - 1}{x^2 - 1} = 2 = k_1$

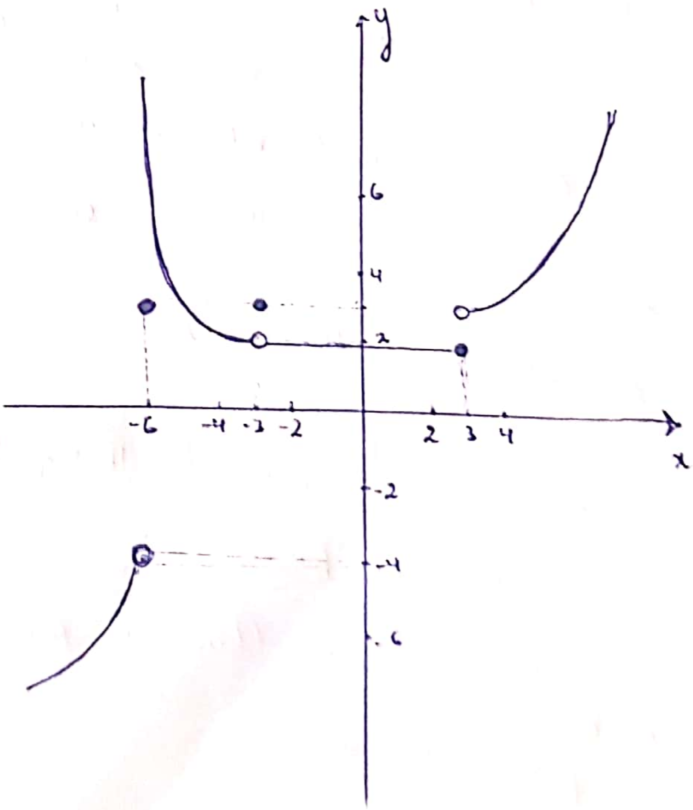
and $\lim_{x \to 1} \dfrac{x^4 - 1}{x^2 - 1} = 2 = k_2$.

Thus the function is defined as

$$f(x) = \begin{cases} \dfrac{x^4 - 1}{x^2 - 1}, & \text{if } x \neq -1, 1 \\ 2, & \text{if } x = -1, 1. \end{cases}$$

is continuous for all x.

157. Answers may vary. But take the following example.



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B. Asymptotes

Ex 1.31. $f(x) = \frac{1}{\sqrt{x}}$, $Dom f =]0, +\infty[$

. We have a $VA \equiv x = 0$

. $\lim_{x \to +\infty} f(x)$ for HA

$\lim_{x \to +\infty} \frac{1}{\sqrt{x}} = 0 \Rightarrow HA \equiv y = 0$

No SA since HA exist

132. $f(x) = \frac{2}{x^2+2}$, $Dom f =]-\infty, +\infty[$

. No VA

. $\lim_{x \to \pm\infty} \frac{2}{x^2+2} = 0$, $\Rightarrow HA \equiv y = 0$

. No SA since HA exist

133. $f(x) = \frac{x}{x^2-x}$; $Dom f = \mathbb{R} \setminus \{1, 0\}$

. We have that $VA \equiv x = 1$

since $\lim_{x \to 0} \frac{x}{x^2-x} = \lim_{x \to 0} \frac{1}{x-1} = -1$

We have a hole at $x = 0$

. $\lim_{x \to \pm\infty} \frac{x}{x^2-x} = 0 \Rightarrow HA \equiv y = 0$

. No SA since HA exist

136. $f(x) = \frac{|x-2|}{x-2}$, $Dom f = \mathbb{R} \setminus \{2\}$

. But observe that

$\lim_{x \to 2} \frac{|x-2|}{x-2}$ exist, then we have

a hole at $x = 2$

$\left.\begin{array}{l} \lim_{x \to -\infty} \frac{|x-2|}{x-2} = -1 \\ \lim_{x \to +\infty} \frac{|x-2|}{x-1} = 1 \end{array}\right\}$ and parallel half Horizontal lines

137.

137. $f(x) = \tan 2x$

. $Dom f = \mathbb{R} \setminus \{\pm \frac{\pi}{4} + k\pi\}$

. We have Vertical asymptote $x = \frac{\pi}{4}$ and $x = -\frac{\pi}{4}$ for $k = 0$

138. $f(t) = \frac{t+3}{t^2+5t+6}$

$Dom f = \mathbb{R} \setminus \{-3, -2\}$.

But: $\lim_{t \to -3} \frac{t+3}{t^2+5t+6}$

$= \lim_{t \to -3} \frac{t+3}{(t+3)(t+2)}$

$\Rightarrow$ A hole at $t = -3$

and a $VA \equiv t = -2$

. $\lim_{t \to \infty} \frac{t+3}{t^2+5t+6} = 0 \Rightarrow HA \equiv y = 0$

since HA exist

. No SA since HA exist

139. $f(x) = \frac{2x^2-5x+3}{x-1}$

$Dom f = \mathbb{R} \setminus \{1\}$

. Since $\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{(2x-3)(x-1)}{x-1}$

We have a hole at $x = 1$

No VA.

. $\lim_{x \to \pm\infty} f(x) = \pm\infty \Rightarrow$ No HA

. SA. Perform long division

$$\begin{array}{r|l} 2x^2 - 5x + 3 & x - 1 \\ \cline{2-2} -2x^2 + 2x & 2x - 3 \\ \hline -3x + 3 & \\ 3x - 3 & \\ \hline 0 & \end{array}$$

Then line $y = 2x - 3$ is an S.A

THX!!!

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