Exercises on page 229-231

JED HERMAN Book

3.1 calculus (On differentiation)

I. Given different function, we need to use the formula

$$m_{sec} = \frac{f(x) - f(a)}{x - a}$$

or

$$m_{sec} = \frac{f(a+h) - f(a)}{h}$$

to find the slope of our secant line

1. $f(x) = 4x+7$, $x_1 = 2$ and $x_2 = 5$

Since we are given two points our formula changes to

$$m_{sec} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

for our function we have that

$$M_{sec} = \frac{f(5) - f(2)}{5-2} = \frac{27 - 15}{3} = 4$$

Continue on other exercises using this approach! Say Do 2-10 in the same way.

II. For next exercises we need to find the tangent's slope using

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

and find the equation of our tangent.

11. $f(x) = 3-4x$, $a = 2$

a) $$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 2} \frac{(3-4x) - (3-8)}{x-2}$$

$$= \lim_{x \to 2} \frac{3-4x+5}{x-2}$$

$$= \lim_{x \to 2} \frac{8-4x}{x-2} = \lim_{x \to 2} \frac{4(2-x)}{x-2}$$

take the negative sign to get

$$\lim_{x \to 2} \frac{-4(x-2)}{x-2} = -4$$

b. The tangent has the form

$y = mx + n$, with $m = m_{tan} = -4$

But We need a line with slope $m = -4$, passing through the point $(a, f(a))$ as we saw.

Now $(a, f(a)) = (2, f(2)) = (2, -5)$

Thus $y = mx + n$

$y = -4x + n$;

Through $(2,-5)$ we get

$-5 = -4(2) + n$

$-5 + 8 = n \Rightarrow n = 3$

Finally our tangent is

$T \equiv y = -4x + 3.$

⚠ continue with different examples but remember that you should use different techniques for evaluating limits such as

"FACTORIZE AND SIMPLIFY"

"USE CONJUGATE"

"SIMPLIFY COMPLEX FRACTION"

For instance on 16. we have

$$m_{tan} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x+8} - 3}{x - 1}$$

We should use conjugate of $\sqrt{x+8} - 3$ which is $\sqrt{x+8} + 3$.

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So continue from 12 - 20

III. In the exercise we need the derivative of $f(x)$ at a point. Say given $f(x)$, we need $f'(a)$.

21. $f(x) = 5x+4$, $\quad a = -1$

$$f'(-1) = \lim_{x\to -1} \frac{f(x) - f(-1)}{x-(-1)}$$

$$= \lim_{x\to -1} \frac{5x+4-(-1)}{x+1}$$

$$= \lim_{x\to -1} \frac{5x+5}{x+1} = \lim_{x\to -1} \frac{5(x+1)}{x+1} = 5$$

Continue in this way and evaluate different derivatives at different given points.

Remark

. On exercise II, somewhere you can find that $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = 0$. In this case, your tangent line is a horizontal line of equation $y = n$ or $y = f(a)$.

. You should also find that $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = DNE$

This means that you have a vertical tangent at a say the tangent has equation $x = a$.

. On Exercise III if $f'(a) = 0$, then the derivative vanishes at a. As a consequence we have a horizontal tangent there at $x = a$.

. On the other hand if $f'(a)$ DNE, then the derivative at that point DNE. (We shall see what is it in this case).

IV. In this exercise we only need to use different values found in the table and write the formula of our

$$m_{sec} = \frac{f(x)-f(a)}{x-a}$$

Say: if $x = 1.1$,

$f(x) = f(1.1)$

and from $P(1, 8)$ you take $a = 1$. So fill different columns OK.

x	slope
1.1	(i) $\frac{f(1.1)-f(1)}{1.1-1}$
1.01	(ii) $\frac{f(1.01)-f(1)}{1.1-1}$
⋮	
1.000001	vi $\frac{f(1.000001)-f(1)}{1.000001-1}$

use $f(x) = x^2+3x+4$, and $f(1) = 8$. and fill the table continue w... other exercises on this path

V. Given $y = s(t)$ (a function of t), t in seconds. answer a), b) and c) for given $y = s(t)$.

35. $s(t) = \frac{1}{3}t + 5$

Recall that

a. $V_{ave} = \frac{s(t) - s(a)}{t - a}$

Now we need from $t = 2$ up to $t = 2+h$. That is

$v_{ave} = \frac{s(2+h) - s(2)}{2+h-2}$

We need

$v_{ave} = \frac{s(t) - s(a)}{t - a}$ or

$v_{ave} = \frac{s(a+h) - s(a)}{h}$

Since They are the same

Thus

a) $V_{ave} = \frac{[\frac{1}{3}(2+h) + 5] - [\frac{1}{3}\cdot 2 + 5]}{2+h-2}$

$= \frac{\frac{2}{3} + \frac{h}{3} + 5 - \frac{2}{3} - 5}{h}$

$= \frac{h/3}{h} = \frac{h}{3} \cdot \frac{1}{h} = \frac{1}{3}$

b. V_{ave} for given h.

Since $V_{ave} = \frac{1}{3}$ (constant) and doesn't depend on h it remains constant for different h.

c. Since $V_{ave} = \frac{1}{3}$ (constant) and

$V_{instan} = \frac{V(a+h) - V(a)}{dh}$

We have

$V_{inst} = \frac{V(a+h) - V(a)}{h} = \frac{\frac{1}{3} - \frac{1}{3}}{h} = 0.$

Thus V_{inst} is zero everywhere.

36. $s(t) = t^2 - 2t$.

a. $V_{ave} = \frac{s(a+h) - s(a)}{h}$

or $a+h = 2+h$, $a = 2$

$\Rightarrow V_{ave} = \frac{s(2+h) - s(2)}{2+h-2}$

$= \frac{[(2+h)^2 - 2(2+h)] - [2^2 - 4]}{h}$

$= \frac{4 + 4h + h^2 - 4 - 2h}{h}$

$= \frac{h^2 + 2h}{h} = \frac{h(h+2)}{h} = h+2$

b. V_{ave} for different h.

. $h = 0.1$

$V_{ave} = 0.1 + 2 = 2.1$

. $h = 0.01$, $V_{ave} = 0.01 + 2 = 2.01$

? $h = 0.001$, $V_{ave} = 0.001 + 2 = 2.001$

;

c. $V_{instant} = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$

$= \lim_{h \to 0} (2+h) = 2.$

Continue in the same way:

VI. Use definition of derivative (the m_{tan} at a) to show that the derivative DNE.

41. $f(x) = x^{1/3}$ and $x = 0$

Now:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{(a+h)^{1/3} - (a)^{1/3}}{h}$$

since $x = a = 0$ we have

$$f'(0) = \lim_{h \to 0} \frac{(0+h)^{1/3} - 0}{h}$$

$$= \lim_{h \to 0} h^{-2/3} = \lim_{h \to 0} \frac{1}{h^{2/3}}$$

Observe that

$$\left.\begin{array}{l} \lim_{h \to 0^-} \frac{1}{h^{2/3}} = +\infty \\ \lim_{h \to 0^+} \frac{1}{h^{2/3}} = +\infty \end{array}\right\} \Rightarrow \text{infinite } \cancel{DNE}$$

42. The same as here above

$x = a = 0$, $f(x) = x^{2/3}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\Rightarrow f'(0) = \lim_{h \to 0} \frac{(0+h)^{2/3} - (0)^{2/3}}{h}$$

$$= \lim_{h \to 0} \frac{h^{2/3}}{h} = \lim_{h \to 0} h^{-1/3}$$

$$= \lim_{h \to 0} \frac{1}{h^{1/3}}$$

$$\left.\begin{array}{l} \lim_{h \to 0^-} \frac{1}{h^{1/3}} = -\infty \\ \lim_{h \to 0^+} \frac{1}{h^{1/3}} = +\infty \end{array}\right\} \Rightarrow DNE.$$

43. $f(x) = \begin{cases} 1, & x < 1 \\ x, & x \geq 1 \end{cases}$, $a = x = 1$

We need that $f'(1)$ to exist

OR simply

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(a)}{x - a}$$

OR

$$f'(1) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

since they are the same

Use the first one:

$$f'(x) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

But from the definition of our function we have that

$f(1) = 1$ since

$f(x) = x$ for $x \geq 1$ $([1, +\infty[$

Now: we will have two limits say,

$$\left.\begin{array}{l} \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} \\ \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} \end{array}\right\} \text{since we have 2 different intervals}$$

Now

$$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{1-1}{x-1} = 0$$

$$\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{x-1}{x-1} = 1$$

So $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} \Rightarrow \lim_{x\to 1}$ DNE

44. $f(x) = \frac{|x|}{x}$, $x = a = 0$

We proceed as in 43 by recalling that

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Now

$$\frac{|x|}{x} = \begin{cases} -\frac{x}{x}, & x < 0 \\ \frac{x}{x}, & x \geq 0 \end{cases}$$

$$= \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

We even conclude that

$\lim_{x\to 0^-} f(x) = -1$ and
$\lim_{x\to 0^+} f(x) = 1$ $\Rightarrow$ DNE.

45. Given $s(t) = 8t^2 - \frac{1}{16}t^3$

a) $V_{ave} = \frac{s(t)-s(a)}{t-a}$

But we have an intervall $[a,b]$ with given $s(t)$.
$[t_1, t_2]$

Now $V_{ave} = \frac{s(t_2)-s(t_1)}{t_2-t_1}$

4 different intervals

b.

$$V_{inst} = \lim_{t\to a} \frac{s(t)-s(a)}{t-a}$$

But as you can see from a we will have a table showing what is going on as $t \to 4$

or 4 | 4.1, 4.01, 4.001, 4.0001 — So we evaluated Our limit using a table

46. The same as 45 but we changed only values

The Remaining Exercises require that we have a program to plot different situations.

- In REB book S4, Have a look at Task 9.7 page 185 and Task 9.6, page 184 Task 9.4 page 178 and Task 9.3 page 174. Ask questions so that we can fix and Establish a link between what we've done and different settings in the material. You shall find some challenging Exercises that require some reasoning, but we saw different things in common so Ask Questions to get help.