14.3 The partial derivatives can be evaluated,

$$\frac{\partial f}{\partial x} = -3x + 2.25y$$
$$\frac{\partial f}{\partial y} = 2.25x - 4y + 1.75$$

These can be set to zero to generate the following simultaneous equations

$$3x - 2.25y = 0$$
$$-2.25x + 4y = 1.75$$

which can be solved for $x = 0.567568$ and $y = 0.756757$, which is the optimal solution.

14.4 The partial derivatives can be evaluated at the initial guesses, $x = 1$ and $y = 1$,

$$\frac{\partial f}{\partial x} = -3x + 2.25y = -3(1) + 2.25(1) = -0.75$$
$$\frac{\partial f}{\partial y} = 2.25x - 4y + 1.75 = 2.25(1) - 4(1) + 1.75 = 0$$

Therefore, the search direction is –0.75**i**.

$$f(1 - 0.75h,\ 1) = 0.5 + 0.5625h - 0.84375h^2$$

This can be differentiated and set equal to zero and solved for $h^* = 0.33333$. Therefore, the result for the first iteration is $x = 1 - 0.75(0.3333) = 0.75$ and $y = 1 + 0(0.3333) = 1$.

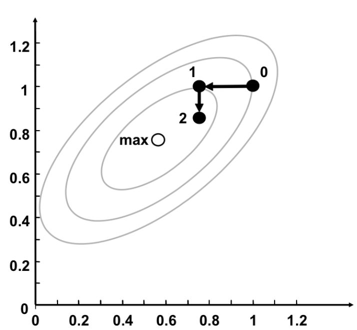
For the second iteration, the partial derivatives can be evaluated as,

$$\frac{\partial f}{\partial x} = -3(0.75) + 2.25(1) = 0$$
$$\frac{\partial f}{\partial y} = 2.25(0.75) - 4(1) + 1.75 = -0.5625$$

Therefore, the search direction is –0.5625**j**.

$$f(0.75,\ 1 - 0.5625h) = 0.59375 + 0.316406h - 0.63281h^2$$

This can be differentiated and set equal to zero and solved for $h^* = 0.25$. Therefore, the result for the second iteration is $x = 0.75 + 0(0.25) = 0.75$ and $y = 1 + (-0.5625)0.25 = 0.859375$.



14.5 (a)

$$\nabla f = \begin{Bmatrix} 2y^2 + 3ye^{xy} \\ 4xy + 3xe^{xy} \end{Bmatrix} \qquad H = \begin{bmatrix} 3y^2e^{xy} & 4y + 3xye^{xy} + 3e^{xy} \\ 4y + 3xye^{xy} + 3e^{xy} & 4x + 3x^2e^{xy} \end{bmatrix}$$

(b)

$$\nabla f = \begin{Bmatrix} 2x \\ 2y \\ 4z \end{Bmatrix} \qquad H = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

(c)

$$\nabla f = \begin{Bmatrix} \dfrac{2x + 2y}{x^2 + 2xy + 3y^2} \\ \dfrac{2x + 6y}{x^2 + 2xy + 3y^2} \end{Bmatrix} \qquad H = \frac{\begin{bmatrix} -2x^2 - 4xy + 2y^2 & -2x^2 - 12xy - 6y^2 \\ -2x^2 - 12xy - 6y^2 & 2x^2 - 12xy - 18y^2 \end{bmatrix}}{\left(x^2 + 2xy + 3y^2\right)^2}$$

14.7 The partial derivatives can be evaluated at the initial guesses, $x = 0$ and $y = 0$,

$$\frac{\partial f}{\partial x} = 4 + 2x - 8x^3 + 2y = 4 + 2(0) - 8(0)^3 + 2(0) = 4$$

$$\frac{\partial f}{\partial y} = 2 + 2x - 6y = 2 + 2(0) - 6(0) = 2$$

$f(0 + 4h, 0 + 2h) = 20h + 20h^2 - 512h^4$

$g'(h) = 20 + 40h - 2048h^3$

The root of this equation can be determined by bisection. Using initial guesses of $h = 0$ and 1 yields a root of $h^* = 0.24390$ after 13 iterations with $\varepsilon_a = 0.05\%$. Therefore,

$x = 0 + 4(0.24390) = 0.976074$

$y = 0 + 2(0.24390) = 0.488037$

14.10 The following code implements the grid search algorithm in VBA:

```
Option Explicit

Sub GridSearch()
Dim nx As Long, ny As Long
Dim xmin As Double, xmax As Double, ymin As Double, ymax As Double
Dim maxf As Double, maxx As Double, maxy As Double
xmin = -2: xmax = 2: ymin = 1: ymax = 3
nx = 1000
ny = 1000
Call GridSrch(nx, ny, xmin, xmax, ymin, ymax, maxy, maxx, maxf)
MsgBox maxf
MsgBox maxx
```

```
MsgBox maxy
End Sub
Sub GridSrch(nx, ny, xmin, xmax, ymin, ymax, maxy, maxx, maxf)
Dim i As Long, j As Long
Dim x As Double, y As Double, fn As Double
Dim xinc As Double, yinc As Double
xinc = (xmax - xmin) / nx
yinc = (ymax - ymin) / ny
maxf = -1000000000#
x = xmin
For i = 0 To nx
  y = ymin
  For j = 0 To ny
    fn = f(x, y)
    If fn > maxf Then
      maxf = fn
      maxx = x
      maxy = y
    End If
    y = y + yinc
  Next j
  x = x + xinc
Next i
End Sub
Function f(x, y)
f = y - x - 2 * x ^ 2 - 2 * x * y - y ^ 2
End Function
```