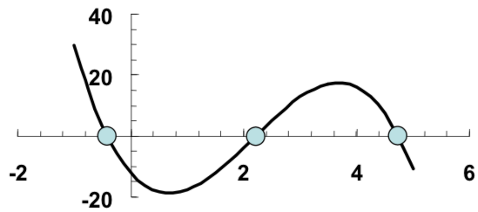
5.4 (a) The graph indicates that roots are located at about –0.4, 2.25 and 4.7.



(b) Using bisection, the first iteration is

$$x_r = \frac{-1+0}{2} = -0.5$$

$$f(-1)f(-0.5) = 29.75(3.34375) = 99.47656$$

Therefore, the root is in the second interval and the lower guess is redefined as $x_l = -0.5$. The second iteration is

$$x_r = \frac{-0.5+0}{2} = -0.25 \qquad \varepsilon_a = \left|\frac{-0.25-(-0.5)}{-0.25}\right| 100\% = 100\%$$

$$f(-0.5)f(-0.25) = 3.34375(-5.5820313) = -18.66492$$

Consequently, the root is in the first interval and the upper guess is redefined as $x_u = -0.25$. All the iterations are displayed in the following table:

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_a
1	–1	29.75	0	–12	–0.5	3.34375	
2	–0.5	3.34375	0	–12	–0.25	–5.5820313	100.00%
3	–0.5	3.34375	–0.25	–5.5820313	–0.375	–1.4487305	33.33%
4	–0.5	3.34375	–0.375	–1.4487305	–0.4375	0.8630981	14.29%
5	–0.4375	0.863098	–0.375	–1.4487305	–0.40625	–0.3136673	7.69%
6	–0.4375	0.863098	–0.40625	–0.3136673	–0.421875	0.2694712	3.70%
7	–0.42188	0.269471	–0.40625	–0.3136673	–0.414063	–0.0234052	1.89%
8	–0.42188	0.269471	–0.41406	–0.0234052	–0.417969	0.1227057	0.93%

Thus, after eight iterations, we obtain a root estimate of **–0.417969** with an approximate error of 0.93%, which is below the stopping criterion of 1%.

(c) Using false position, the first iteration is

$$x_r = 0 - \frac{-12(-1-0)}{29.75-(-12)} = -0.287425$$

$$f(-1)f(-0.287425) = 29.75(-4.4117349) = -131.2491$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = -0.287425$. The second iteration is

$$x_r = -0.287425 - \frac{-4.4117349(-1-(-0.287425))}{29.75-(-4.4117349)} = -0.3794489$$

$$\varepsilon_a = \left|\frac{-0.3794489-(-0.2874251)}{-0.3794489}\right| 100\% = 24.25\%$$

$$f(-1)f(-0.3794489) = 29.75(-1.2896639) = -38.3675$$

Consequently, the root is in the first interval and the upper guess is redefined as $x_u = -0.379449$. All the iterations are displayed in the following table:

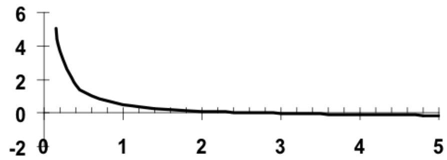
i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_a
1	–1	29.75	0	–12	–0.287425	–4.4117349	
2	–1	29.75	–0.28743	–4.4117349	–0.379449	–1.2896639	24.25%
3	–1	29.75	–0.37945	–1.2896639	–0.405232	–0.3512929	6.36%
4	–1	29.75	–0.40523	–0.3512929	–0.412173	–0.0938358	1.68%
5	–1	29.75	–0.41217	–0.0938358	–0.414022	–0.0249338	0.45%

Therefore, after five iterations we obtain a root estimate of **–0.414022** with an approximate error of 0.45%, which is below the stopping criterion of 1%.

5.7 (a) $(0.8 - 0.3x) = 0$

$$x = \frac{0.8}{0.3} = 2.666667$$

(b) The graph of the function indicates a root between $x = 2$ and 3. Note that the shape of the curve suggests that it may be ill-suited for solution with the false-position method (refer to Fig. 5.14)



(c) Using false position, the first iteration is

$$x_r = 3 - \frac{-0.03333(1-3)}{0.5-(-0.03333)} = 2.875$$

$$\varepsilon_t = \left|\frac{2.66667 - 2.875}{2.66667}\right| 100\% = 7.81\%$$

$$f(1)f(2.875) = 0.5(-0.02174) = -0.01087$$

Therefore, the root is in the first interval and the upper guess is redefined as $x_u = 2.875$. The second iteration is

$$x_r = 2.875 - \frac{-0.03333(1-2.875)}{0.5-(-0.03333)} = 2.79688$$

$$\varepsilon_a = \left|\frac{2.79688 - 2.875}{2.79688}\right| 100\% = 2.793\%$$

$$\varepsilon_t = \left|\frac{2.66667 - 2.79688}{2.66667}\right| 100\% = 4.88\%$$

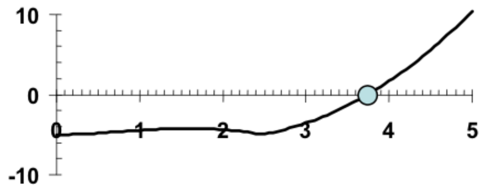
$$f(1)f(2.79688) = 0.5(-0.01397) = -0.00698$$

Consequently, the root is in the first interval and the upper guess is redefined as $x_u = 2.79688$. All the iterations are displayed in the following table:

i	x_l	x_u	$f(x_l)$	$f(x_u)$	x_r	$f(x_r)$	$f(x_l) \times f(x_r)$	ε_a	ε_t
1	1	3.00000	0.5	-0.03333	2.87500	-0.02174	-0.01087		7.81%
2	1	2.87500	0.5	-0.02174	2.79688	-0.01397	-0.00698	2.793%	4.88%
3	1	2.79688	0.5	-0.01397	2.74805	-0.00888	-0.00444	1.777%	3.05%

Therefore, after three iterations we obtain a root estimate of **2.74805** with an approximate error of 1.777%. Note that the true error is greater than the approximate error. This is not good because it means that we could stop the computation based on the erroneous assumption that the true error is at least as good as the approximate error. This is due to the slow convergence that results from the function's shape.

5.9 A graph of the function indicates a positive real root at approximately $x = 3.7$.



Using false position, the first iteration is

$$x_r = 5 - \frac{10.43182(0-5)}{-5-10.43182} = 1.62003$$
$$f(0)f(1.62003) = -5(-4.22944) = 21.147$$

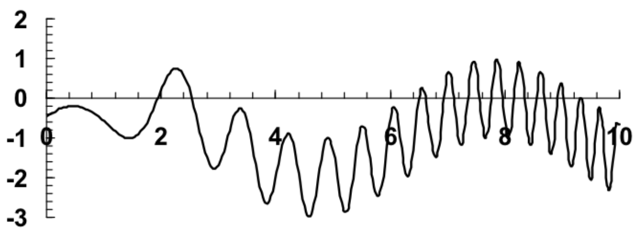
Therefore, the root is in the second interval and the lower guess is redefined as $x_l = 1.62003$. The remaining iterations are summarized below

i	x_l	x_u	$f(x_l)$	$f(x_u)$	x_r	$f(x_r)$	$f(x_l)\times f(x_r)$	ε_a
1	0	5.00000	-5	10.43182	1.62003	-4.22944	21.14722	
2	1.62003	5.00000	-4.2294	10.43182	2.59507	-4.72984	20.00459	37.573%
3	2.59507	5.00000	-4.7298	10.43182	3.34532	-2.14219	10.13219	22.427%
4	3.34532	5.00000	-2.1422	10.43182	3.62722	-0.69027	1.47869	7.772%
5	3.62722	5.00000	-0.6903	10.43182	3.71242	-0.19700	0.13598	2.295%
6	3.71242	5.00000	-0.197	10.43182	3.73628	-0.05424	0.01069	0.639%

The final result, $x_r = 3.73628$, can be checked by substituting it into the original function to yield a near-zero result,

$$f(3.73628) = (3.73628)^2 \left|\cos\sqrt{3.73628}\right| - 5 = -0.05424$$

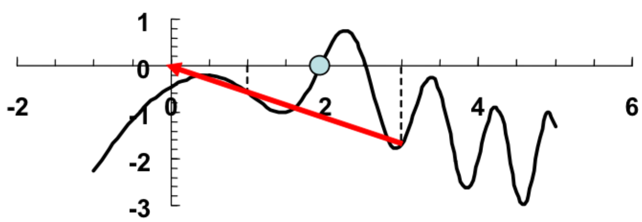
6.7 A graph of the function indicates that the first positive root occurs at about 1.9. However, the plot also indicates that there are many other positive roots.



(a) For initial guesses of $x_{i-1} = 1.0$ and $x_i = 3.0$, four iterations of the secant method yields

i	x_{i-1}	$f(x_{i-1})$	x_i	$f(x_i)$	ε_a
0	1	-0.57468	**3**	-1.697951521	
1	3	-1.69795	**-0.02321**	-0.483363437	13023.081%
2	-0.02321	-0.48336	**-1.22635**	-2.744750012	98.107%
3	-1.22635	-2.74475	**0.233951**	-0.274717273	624.189%
4	0.233951	-0.27472	**0.396366**	-0.211940326	40.976%

The result jumps to a negative value due to the poor choice of initial guesses as illustrated in the following plot:

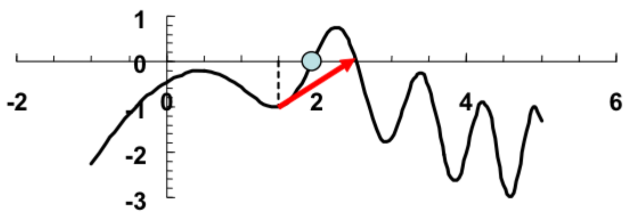


Thereafter, it seems to be converging slowly towards the lowest positive root. However, if the iterations are continued, the technique again runs into trouble when the near-zero slope at 0.5 is approached. At that point, the solution shoots far from the lowest root with the result that it eventually converges to a root at 177.26!

(b) For initial guesses of $x_{i-1} = 1.5$ and $x_i = 2.5$, four iterations of the secant method yields

i	x_{i-1}	$f(x_{i-1})$	x_i	$f(x_i)$	ε_a
0	1.5	-0.99663	**2.5**	0.1663963	
1	2.5	0.166396	**2.356929**	0.6698423	6.070%
2	2.356929	0.669842	**2.547287**	-0.0828279	7.473%
3	2.547287	-0.08283	**2.526339**	0.0314711	0.829%
4	2.526339	0.031471	**2.532107**	0.0005701	0.228%

For these guesses, the result jumps to the vicinity of the second lowest root at 2.5 as illustrated in the following plot:

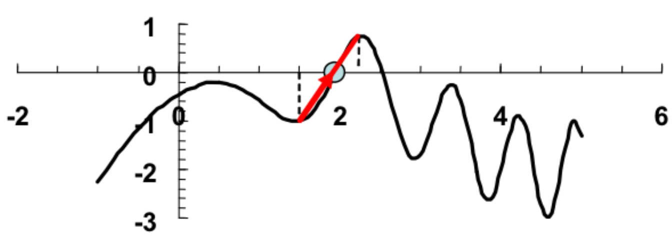


Thereafter, although the two guesses bracket the lowest root, the way that the secant method sequences the iteration results in the technique missing the lowest root.

(c) For initial guesses of $x_{i-1} = 1.5$ and $x_i = 2.25$, four iterations of the secant method yields

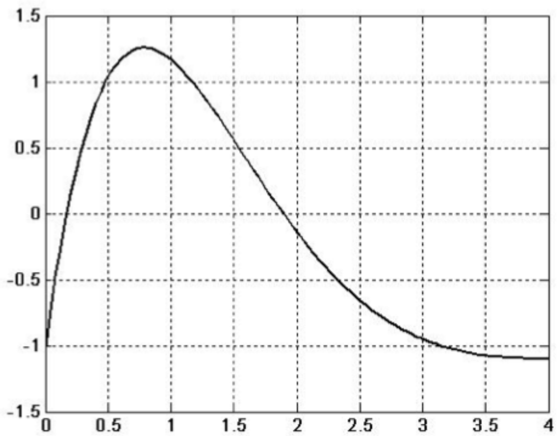
i	x_{i-1}	$f(x_{i-1})$	x_i	$f(x_i)$	ε_a
0	1.5	-0.996635	**2.25**	0.753821	
1	2.25	0.753821	**1.927018**	-0.061769	16.761%
2	1.927018	-0.061769	**1.951479**	0.024147	1.253%
3	1.951479	0.024147	**1.944604**	-0.000014	0.354%
4	1.944604	-0.000014	**1.944608**	0.000000	0.000%

For this case, the secant method converges rapidly on the lowest root at 1.9446 as illustrated in the following plot:



6.10 **(a)** MATLAB can be used to generate a plot

```
>> x = linspace(0,4);
>> y = 7*sin(x).*exp(-x)-1;
>> plot(x,y)
>> grid
```



The lowest positive root seems to be at approximately 0.2.

(b) The formula for Newton-Raphson is

$$x_{i+1} = x_i - \frac{7\sin(x_i)e^{-x_i} - 1}{7e^{-x_i}(\cos(x_i) - \sin(x_i))}$$

Using an initial guess of 3.5, the first iteration yields

$$x_1 = 0.3 - \frac{7\sin(0.3)e^{-0.3} - 1}{7e^{-0.3}(\cos(0.3) - \sin(0.3))} = 0.3 - \frac{0.532487}{3.421627} = 0.144376$$

$$|\varepsilon_a| = \left|\frac{0.144376 - 0.3}{0.144376}\right| \times 100\% = 107.8\%$$

Second iteration:

$$x_2 = 0.144376 - \frac{7\sin(0.144376)e^{-0.144376} - 1}{7e^{-0.144376}(\cos(0.144376) - \sin(0.144376))} = 0.144376 - \frac{-0.12827}{5.124168} = 0.169409$$

$$|\varepsilon_a| = \left|\frac{0.169409 - 0.144376}{0.169409}\right| \times 100\% = 14.776\%$$

Third iteration:

$$x_1 = 0.169409 - \frac{7\sin(0.169409)e^{-0.169409} - 1}{7e^{-0.169409}(\cos(0.169409) - \sin(0.169409))} = 0.169409 - \frac{-0.00372}{4.828278} = 0.170179$$

$$|\varepsilon_a| = \left|\frac{0.170179 - 0.169409}{0.170179}\right| \times 100\% = 0.453\%$$

(c) For the secant method, the first iteration:

$x_{-1} = 0.5$ $f(x_{-1}) = 1.035504$
$x_0 = 0.4$ $f(x_0) = 0.827244$

$$x_1 = 0.4 - \frac{0.827244(0.5 - 0.3)}{1.035504 - 0.827244} = 0.002782$$

$$|\varepsilon_a| = \left|\frac{0.002782 - 0.4}{0.002782}\right| \times 100\% = 14,278\%$$

Second iteration:
$x_0 = 0.4$ $f(x_0) = 0.827244$
$x_1 = 0.002782$ $f(x_1) = -0.98058$

$$x_2 = 0.002782 - \frac{-0.98058(0.4 - 0.002782)}{0.827244 - (-0.98058)} = 0.218237$$

$$|\varepsilon_a| = \left|\frac{0.218237 - 0.002782}{0.218237}\right| \times 100\% = 98.725\%$$

Third iteration:

$x_1 = 0.002782$ $f(x_1) = -0.98058$
$x_2 = 0.218237$ $f(x_2) = 0.218411$

$$x_3 = 0.218237 - \frac{0.218411(0.002782 - 0.218237)}{-0.98058 - 0.218411} = 0.178989$$

$$|\varepsilon_a| = \left|\frac{0.178989 - 0.218237}{0.178989}\right| \times 100\% = 21.93\%$$

(d) For the modified secant method, the first iteration:

$x_0 = 0.3$ $f(x_0) = 0.532487$

$x_0 + \delta x_0 = 0.303 \quad f(x_0 + \delta x_0) = 0.542708$

$$x_1 = 0.3 - \frac{0.01(0.3)0.532487}{0.542708 - 0.532487} = 0.143698$$

$$|\varepsilon_a| = \left|\frac{0.143698 - 0.3}{0.143698}\right| \times 100\% = 108.8\%$$

Second iteration:

$x_1 = 0.143698 \qquad f(x_1) = -0.13175$

$x_1 + \delta x_1 = 0.145135 \qquad f(x_1 + \delta x_1) = -0.12439$

$$x_2 = 0.143698 - \frac{0.02(0.143698)(-0.13175)}{-0.12439 - (-0.13175)} = 0.169412$$

$$|\varepsilon_a| = \left|\frac{0.169412 - 0.143698}{0.169412}\right| \times 100\% = 15.18\%$$

Third iteration:

$x_2 = 0.169412 \qquad f(x_2) = -0.00371$

$x_2 + \delta x_2 = 0.171106 \qquad f(x_2 + \delta x_2) = 0.004456$

$$x_3 = 0.169412 - \frac{0.02(0.169412)(-0.00371)}{0.004456 - (-0.00371)} = 0.170181$$

$$|\varepsilon_a| = \left|\frac{0.170181 - 0.169412}{0.170181}\right| \times 100\% = 0.452\%$$

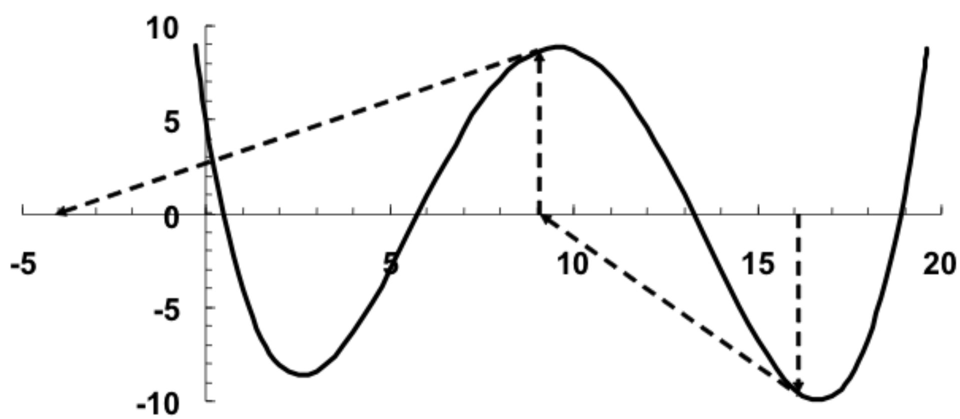
6.17 The formula for Newton-Raphson is

$$x_{i+1} = x_i - \frac{0.0074x_i^4 - 0.284x_i^3 + 3.355x_i^2 - 12.183x_i + 5}{0.0296x_i^3 - 0.852x_i^2 + 6.71x_i - 12.183}$$

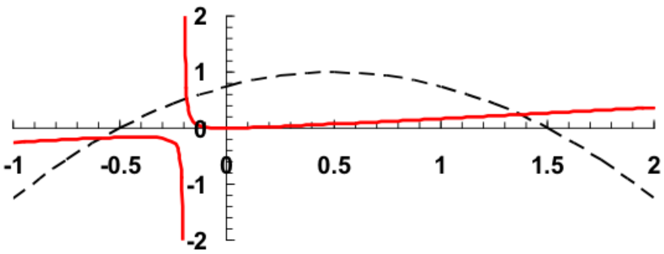
Using an initial guess of 16.15, the iterations proceed as

iteration	x_i	$f(x_i)$	$f'(x_i)$	ε_a
0	16.15	−9.57445	−1.35368	
1	9.077102	8.678763	0.662596	77.920%
2	−4.02101	128.6318	−54.864	325.742%
3	−1.67645	36.24995	−25.966	139.852%
4	−0.2804	8.686147	−14.1321	497.887%
5	0.334244	1.292213	−10.0343	183.890%
6	0.463023	0.050416	−9.25584	27.813%
7	0.46847	8.81E−05	−9.22351	1.163%
8	0.46848	2.7E−10	−9.22345	0.002%

As depicted below, the iterations involve regions of the curve that have flat slopes. Hence, the solution is cast far from the roots in the vicinity of the original guess.



6.22 The functions can be plotted (y versus x). The plot indicates that there are three real roots at about (–0.6, –0.18), (–0.19, 0.6), and (1.37, 0.24).



(a) There are numerous ways to set this problem up as a fixed-point iteration. One way that converges is to solve the first equation for x and the second for y,

$$x = \sqrt{x + 0.75 - y}$$

$$y = \frac{x^2}{1 + 5x}$$

Using initial values of $x = y = 1.2$, the first iteration can be computed as:

$$x = \sqrt{1.2 + 0.75 - 1.2} = 0.866025$$

$$y = \frac{(0.866025)^2}{1 + 5(0.866025)} = 0.14071$$

Second iteration

$$x = \sqrt{0.866025 + 0.75 - 0.14071} = 1.214626$$

$$y = \frac{(1.214626)^2}{1 + 5(1.214626)} = 0.20858$$

Third iteration

$$x = \sqrt{1.214626 + 0.75 - 0.20858} = 1.325159$$

$$y = \frac{(1.325159)^2}{1+5(1.325159)} = 0.230277$$

Thus, the computation is converging on the root at x = 1.372065 and y = 0.239502.

Note that some other configurations are convergent and others are divergent. This exercise is intended to illustrate that although it may sometimes work, fixed-point iteration does not represent a practical general-purpose approach for solving systems of nonlinear equations.

(b) The equations to be solved are

$$u(x, y) = -x^2 + x + 0.75 - y$$
$$v(x, y) = x^2 - y - 5xy$$

The partial derivatives can be computed and evaluated at the initial guesses (x = 1.2, y = 1.2) as

$$\frac{\partial u}{\partial x} = -2x + 1 = -1.4 \qquad \frac{\partial u}{\partial y} = -1$$
$$\frac{\partial v}{\partial x} = 2x - 5y = -3.6 \qquad \frac{\partial v}{\partial y} = -1 - 5x = -7$$

The determinant of the Jacobian can be computed as

$$-1.4(-7) - (-1)(-3.6) = 6.2$$

The values of the function at the initial guesses can be computed as

$$u(1.2, 1.2) = -(1.2)^2 + 1.2 + 0.75 - 1.2 = -0.69$$
$$v(1.2, 1.2) = (1.2)^2 - 1.2 - 5(1.2)(1.2) = -6.96$$

These values can be substituted into Eq. (6.24) to give

$$x = 1.2 - \frac{-0.69(-7) - (-6.96)(-1)}{6.2} = 1.543548$$
$$y = 1.2 - \frac{-6.96(-1.4) - (-0.69)(-3.6)}{6.2} = 0.0290325$$

The remaining iterations are summarized below:

i	x_i	y_i	ε_a
0	**1.2**	**1.2**	
1	**1.543548**	**0.0290325**	4033%
2	**1.394123**	**0.2228721**	86.97%
3	**1.372455**	**0.2392925**	6.86%
4	**1.372066**	**0.2395019**	0.0874%
5	**1.372065**	**0.2395019**	1.87×10^{-5}%