21.1 (a) Analytical solution:

$$\int_0^{\pi/2} (6+3\cos x)\; dx = \left[6x+3\sin x\right]_0^{\pi/2} = 6(\pi/2)+3\sin(\pi/2)-0 = 12.42478$$

(b) Trapezoidal rule (n = 1):

$$I = (1.570796-0)\frac{9+6}{2} = 11.78097 \qquad \varepsilon_t = \left|\frac{12.42478-11.78097}{12.42478}\right| \times 100\% = 5.182\%$$

(c) Trapezoidal rule (n = 2):

$$I = (1.570796-0)\frac{9+2(8.12132)+6}{4} = 12.26896 \qquad \varepsilon_t = 1.254\%$$

Trapezoidal rule (n = 4):

$$I = (1.570796-0)\frac{9+2(8.771639+8.12132+7.14805)+6}{8} = 12.38613 \qquad \varepsilon_t = 0.311\%$$

(d) Simpson's 1/3 rule:

$$I = (1.570796-0)\frac{9+4(8.12132)+6}{6} = 12.43162 \qquad \varepsilon_t = 0.055\%$$

(e) Simpson's rule (n = 4):

$$I = (1.570796-0)\frac{9+4(8.771639+7.14805)+2(8.12132)+6}{12} = 12.42518 \qquad \varepsilon_t = 0.0032\%$$

(f) Simpson's 3/8 rule:

$$I = (1.570796-0)\frac{9+3(8.598076+7.5)+6}{8} = 12.42779 \qquad \varepsilon_t = 0.024\%$$

(g) Simpson's rules (n = 5):

$$\begin{aligned} I &= (0.628319-0)\frac{9+4(8.85317)+8.427051}{6} \\ &\quad + (1.570796-0.628319)\frac{8.427051+3(7.763356+6.927051)+6}{8} \\ &= 5.533364+6.891665 = 12.42503 \qquad \varepsilon_t = 0.002\% \end{aligned}$$

21.2 (a) Analytical solution:

$$\int_0^3 \left(1-e^{-2x}\right) dx = \left[x+0.5e^{-2x}\right]_0^3 = 3+0.5e^{-2(3)} - 0 - 0.5e^{-2(0)} = 2.501239$$

(b) Trapezoidal rule (n = 1):

$$I = (3-0)\frac{0+0.997521}{2} = 1.496282$$

$$\varepsilon_t = \left|\frac{2.501239-1.496282}{2.501239}\right| \times 100\% = 40.18\%$$

(c) Trapezoidal rule (n = 2):

$$I = (3-0)\frac{0+2(0.950213)+0.997521}{4} = 2.17346 \qquad \varepsilon_t = 13.10\%$$

Trapezoidal rule (n = 4):

$$I = (3-0)\frac{0+2(0.77687+0.950213+0.988891)+0.997521}{8} = 2.411051 \qquad \varepsilon_t = 3.61\%$$

(d) Simpson's 1/3 rule:

$$I = (3-0)\frac{0+4(0.950213)+0.997521}{6} = 2.399186 \qquad \varepsilon_t = 4.08\%$$

(e) Simpson's rule (n = 4):

$$I = (3-0)\frac{0+4(0.77687+0.988891)+2(0.950213)+0.997521}{12} = 2.490248 \qquad \varepsilon_t = 0.44\%$$

(f) Simpson's 3/8 rule:

$$I = (3-0)\frac{0+3(0.864665+0.981684)+0.997521}{8} = 2.451213 \qquad \varepsilon_t = 2.00\%$$

(g) Simpson's rules (n = 5):

$$I = (1.2-0)\frac{0+4(0.698806)+0.909282}{6}$$
$$+(3-1.2)\frac{0.909282+3(0.972676+0.99177)+0.997521}{8}$$
$$= 0.740901+1.755032 = 2.495933 \qquad \varepsilon_t = 0.21\%$$

21.4 Analytical solution:

$$\int_1^2 (x+2/x)^2 \, dx = \left[\frac{x^3}{3} + 4x - \frac{4}{x}\right]_1^2 = 8.33333$$

Trapezoidal rule (n = 1):

$$(2-1)\frac{9+9}{2} = 9$$

The results are summarized below:

n	Integral	ε_t
1	9	8.00%
2	8.513889	2.17%
3	8.415185	0.98%
4	8.379725	0.56%

21.10
(a) Trapezoidal rule (n = 5):

$$I = (0.5-0)\frac{1+2(8+4+3.5+5)+1}{10} = 2.15$$

(b) Simpson's rules (n = 5):

$$I = (0.2-0)\frac{1+4(8)+4}{6} + (0.5-0.2)\frac{4+3(3.5+5)+1}{8} = 1.233333 + 1.14375 = 2.377083$$

21.11
(a) Trapezoidal rule (n = 6):

$$I = (10-(-2))\frac{35+2(5-10+2+5+3)+20}{12} = 65$$

(b) Simpson's rules (n = 6):

$$I = (10-(-2))\frac{35+4(5+2+3)+2(-10+5)+20}{18} = 56.66667$$