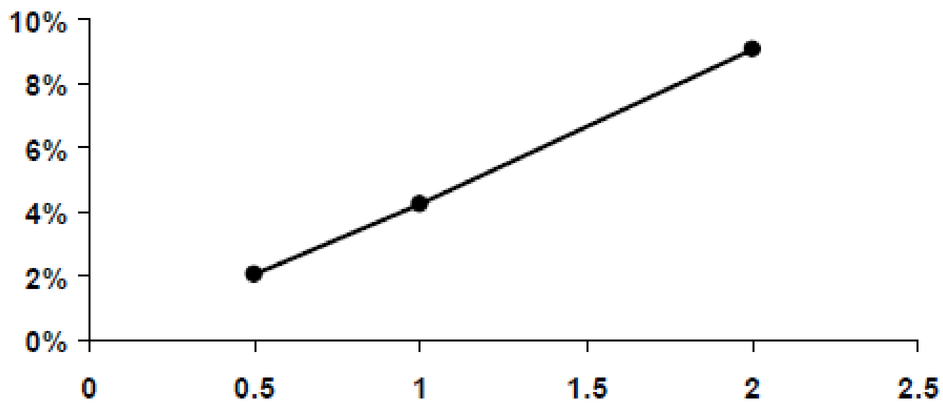
1.2 At $t = 8$ s, the analytical solution is 41.137 (Example 1.1). The relative error can be calculated with

$$\text{absolute relative error} = \left|\frac{\text{analytical} - \text{numerical}}{\text{analytical}}\right| \times 100\%$$

The numerical results are:

step	$v(8)$	absolute relative error
2	44.8700	9.074%
1	42.8931	4.268%
0.5	41.9901	2.073%

The error versus step size can then be plotted as



Thus, halving the step size approximately halves the error.

1.6 (*a*) This is a transient computation. For the period ending June 1:

Balance = Previous Balance + Deposits – Withdrawals + Interest
Balance = 1522.33 + 220.13 – 327.26 + 0.01(1522.33) = 1430.42

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	Deposit	Withdrawal	Interest	Balance
1-May				$1,522.33
	$220.13	$327.26	$15.22	
1-Jun				$1,430.42
	$216.80	$378.51	$14.30	
1-Jul				$1,283.02
	$450.35	$106.80	$12.83	
1-Aug				$1,639.40
	$127.31	$350.61	$16.39	
1-Sep				$1,432.49

(*b*) $\dfrac{dB}{dt} = D(t) - W(t) - iB$

(c) for $t = 0$ to 0.5:

$$\frac{dB}{dt} = 220.13 - 327.26 + 0.01(1522.33) = -91.91$$

$$B(0.5) = 1522.33 - 91.91(0.5) = 1476.38$$

for $t = 0.5$ to 1:

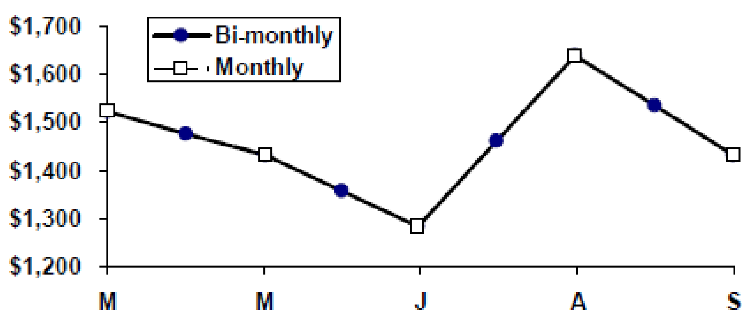
$$\frac{dB}{dt} = 220.13 - 327.260 + 0.01(1476.38) = -92.37$$

$$B(0.5) = 1476.38 - 92.37(0.5) = 1430.19$$

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	Deposit	Withdrawal	Interest	*dB/dt*	Balance
1-May	$220.13	$327.26	$15.22	-$91.91	$1,522.33
16-May	$220.13	$327.26	$14.76	-$92.37	$1,476.38
1-Jun	$216.80	$378.51	$14.30	-$147.41	$1,430.19
16-Jun	$216.80	$378.51	$13.56	-$148.15	$1,356.49
1-Jul	$450.35	$106.80	$12.82	$356.37	$1,282.42
16-Jul	$450.35	$106.80	$14.61	$358.16	$1,460.60
1-Aug	$127.31	$350.61	$16.40	-$206.90	$1,639.68
16-Aug	$127.31	$350.61	$15.36	-$207.94	$1,536.23
1-Sep					$1,432.26

(d) As in the plot below, the results of the two approaches are very close.



2.1

```
IF x < 100 THEN
  IF x < 50 THEN
    x = 0
  ELSE
    x = 75
  END IF
ELSE
  DO
    IF x ≤ 500 EXIT
    x = x - 50
  END DO
ENDIF
```

2.9 A MATLAB M-file can be written to solve this problem as

```
function annualpayment(P, i, n)
nn = 1:n;
A = P*i*(1+i).^nn./((1+i).^nn-1);
y = [nn;A];
fprintf('\n  year   annual payment\n');
fprintf('%5d %14.2f\n',y);
```

This function can be used to evaluate the test case,

```
>> annualpayment(55000,0.066,5)

  year   annual payment
    1        58630.00
    2        30251.49
    3        20804.86
    4        16091.17
    5        13270.64
```

3.6 For the first series, after 20 terms are summed, the result is

	A	B	C	D	E	F	G	H
1	x	5						
2	n	n!	x^n/n!	Sign	Series	True Value	et (%)	ea(%)
3	0	1	1	1	1.000000E+00	6.737947E-03	14741.32	
4	1	1	5	-1	-4.000000E+00	6.737947E-03	59465.26	125.0000000
5	2	2	12.5	1	8.500000E+00	6.737947E-03	126051.2	147.0588235
6	3	6	20.83333	-1	-1.233333E+01	6.737947E-03	183142.9	168.9189189
7	4	24	26.04167	1	1.370833E+01	6.737947E-03	203349.7	189.9696049
8	5	120	26.04167	-1	-1.233333E+01	6.737947E-03	183142.9	211.1486486
9	6	720	21.70139	1	9.368056E+00	6.737947E-03	138934.3	231.6530764
10	7	5040	15.50099	-1	-6.132937E+00	6.737947E-03	91120.85	252.7499191
11	8	40320	9.68812	1	3.555184E+00	6.737947E-03	52663.6	272.5068890
12	9	362880	5.382289	-1	-1.827105E+00	6.737947E-03	27216.65	294.5801032
13	10	3628800	2.691144	1	8.640391E-01	6.737947E-03	12723.48	311.4609662
14	11	39916800	1.223247	-1	-3.592084E-01	6.737947E-03	5431.125	340.5397724
15	12	4.79E+08	0.509686	1	1.504780E-01	6.737947E-03	2133.292	338.7115011
16	13	6.23E+09	0.196033	-1	-4.555520E-02	6.737947E-03	776.0992	430.3202153
17	14	8.72E+10	0.070012	1	2.445667E-02	6.737947E-03	262.9692	286.2690254
18	15	1.31E+12	0.023337	-1	1.119380E-03	6.737947E-03	83.38693	2084.8412684
19	16	2.09E+13	0.007293	1	8.412283E-03	6.737947E-03	24.84936	86.6935085
20	17	3.56E+14	0.002145	-1	6.267312E-03	6.737947E-03	6.984846	34.2247480
21	18	6.4E+15	0.000596	1	6.863137E-03	6.737947E-03	1.857988	8.6815321
22	19	1.22E+17	0.000157	-1	6.706341E-03	6.737947E-03	0.469074	2.3380286
23	20	2.43E+18	3.92E-05	1	6.745540E-03	6.737947E-03	0.112692	0.5811105

The result oscillates at first. By $n = 20$ (21 terms), it is starting to converge on the true value. However, the relative error is still a substantial 0.11%. If carried out further to $n = 27$, the series eventually converges to within 7 significant digits.

In contrast the second series converges much faster. It attains 6 significant digits by $n = 20$ with a percent relative error of 8.1×10^{-6}%.

	A	B	C	D	E	F	G	H
1	x	5						
2	n	n!	x^n/n!	Series	1/Series	True Value	et (%)	ea(%)
3	0	1	1	1.000000E+00	1.000000E+00	6.737947E-03	1.47E+04	
4	1	1	5	6.000000E+00	1.666667E-01	6.737947E-03	2.37E+03	5.00E+02
5	2	2	12.5	1.850000E+01	5.405405E-02	6.737947E-03	7.02E+02	2.08E+02
6	3	6	20.83333	3.933333E+01	2.542373E-02	6.737947E-03	2.77E+02	1.13E+02
7	4	24	26.04167	6.537500E+01	1.529637E-02	6.737947E-03	1.27E+02	6.62E+01
8	5	120	26.04167	9.141667E+01	1.093892E-02	6.737947E-03	6.23E+01	3.98E+01
9	6	720	21.70139	1.131181E+02	8.840322E-03	6.737947E-03	3.12E+01	2.37E+01
10	7	5040	15.50099	1.286190E+02	7.774898E-03	6.737947E-03	1.54E+01	1.37E+01
11	8	40320	9.68812	1.383072E+02	7.230283E-03	6.737947E-03	7.31E+00	7.53E+00
12	9	362880	5.382289	1.436895E+02	6.959453E-03	6.737947E-03	3.29E+00	3.89E+00
13	10	3628800	2.691144	1.463806E+02	6.831506E-03	6.737947E-03	1.39E+00	1.87E+00
14	11	39916800	1.223247	1.476038E+02	6.774891E-03	6.737947E-03	5.48E-01	8.36E-01
15	12	4.79E+08	0.509686	1.481135E+02	6.751577E-03	6.737947E-03	2.02E-01	3.45E-01
16	13	6.23E+09	0.196033	1.483096E+02	6.742653E-03	6.737947E-03	6.98E-02	1.32E-01
17	14	8.72E+10	0.070012	1.483796E+02	6.739472E-03	6.737947E-03	2.26E-02	4.72E-02
18	15	1.31E+12	0.023337	1.484029E+02	6.738412E-03	6.737947E-03	6.90E-03	1.57E-02
19	16	2.09E+13	0.007293	1.484102E+02	6.738081E-03	6.737947E-03	1.99E-03	4.91E-03
20	17	3.56E+14	0.002145	1.484124E+02	6.737983E-03	6.737947E-03	5.42E-04	1.45E-03
21	18	6.4E+15	0.000596	1.484130E+02	6.737956E-03	6.737947E-03	1.40E-04	4.01E-04
22	19	1.22E+17	0.000157	1.484131E+02	6.737949E-03	6.737947E-03	3.45E-05	1.06E-04
23	20	2.43E+18	3.92E-05	1.484131E+02	6.737948E-03	6.737947E-03	8.11E-06	2.64E-05

4.2 Use the stopping criterion: $\varepsilon_s = 0.5\times10^{2-2}\% = 0.5\%$

True value: $\cos(\pi/3) = 0.5$

<u>zero order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 1$$

$$\varepsilon_t = \left|\frac{0.5-1}{0.5}\right| \times 100\% = 100\%$$

<u>first order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 1 - \frac{(\pi/3)^2}{2} = 0.451689$$

$$\varepsilon_t = 9.66\% \qquad \varepsilon_a = \left|\frac{0.451689-1}{0.451689}\right| \times 100\% = 121.4\%$$

<u>second order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 0.451689 + \frac{(\pi/3)^4}{24} = 0.501796$$

$$\varepsilon_t = 0.359\% \qquad \varepsilon_a = \left|\frac{0.501796-0.451689}{0.501796}\right| \times 100\% = 9.986\%$$

<u>third order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 0.501796 - \frac{(\pi/3)^6}{720} = 0.499965$$

$$\varepsilon_t = 0.00709\% \qquad \varepsilon_a = \left|\frac{0.499965-0.501796}{0.499965}\right| \times 100\% = 0.366\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

4.5 True value: $f(3) = 554$.

<u>zero order:</u>

$$f(3) = f(1) = -62 \qquad \varepsilon_t = \left|\frac{554-(-62)}{554}\right| \times 100\% = 111.191\%$$

<u>first order:</u>

$$f(3) = -62 + f'(1)(3-1) = -62 + 70(2) = 78 \qquad \varepsilon_t = 85.921\%$$

<u>second order:</u>

$$f(3) = 78 + \frac{f''(1)}{2}(3-1)^2 = 78 + \frac{138}{2}4 = 354 \qquad \varepsilon_t = 36.101\%$$

<u>third order:</u>

$$f(3) = 354 + \frac{f^{(3)}(1)}{6}(3-1)^3 = 354 + \frac{150}{6}8 = 554 \qquad \varepsilon_t = 0\%$$

Thus, the third-order result is perfect because the original function is a third-order polynomial.