18.5 First, order the points so that they are as close to and as centered about the unknown as possible

$x_0 = 2.5$ $f(x_0) = 14$
$x_1 = 3.2$ $f(x_1) = 15$
$x_2 = 2$ $f(x_2) = 8$
$x_3 = 4$ $f(x_3) = 8$
$x_4 = 1.6$ $f(x_4) = 2$

Next, the divided differences can be computed and displayed in the format of Fig. 18.5,

i	x_i	$f(x_i)$	$f[x_{i+1},x_i]$	$f[x_{i+2},x_{i+1},x_i]$	$f[x_{i+3},x_{i+2},x_{i+1},x_i]$	$f[x_{i+4},x_{i+3},x_{i+2},x_{i+1},x_i]$
0	2.5	14	1.428571	-8.809524	1.011905	1.847718
1	3.2	15	5.833333	-7.291667	-0.651042	
2	2	8	0	-6.25		
3	4	8	2.5			
4	1.6	2				

The first through third-order interpolations can then be implemented as

$$f_1(2.8) = 14 + 1.428571(2.8 - 2.5) = 14.428571$$
$$f_2(2.8) = 14 + 1.428571(2.8 - 2.5) - 8.809524(2.8 - 2.5)(2.8 - 3.2) = 15.485714$$
$$f_3(2.8) = 14 + 1.428571(2.8 - 2.5) - 8.809524(2.8 - 2.5)(2.8 - 3.2)$$
$$+ 1.011905(2.8 - 2.5)(2.8 - 3.2)(2.8 - 2.) = 15.388571$$

The error estimates for the first and second-order predictions can be computed with Eq. 18.19 as

$$R_1 = 15.485714 - 14.428571 = 1.057143$$
$$R_2 = 15.388571 - 15.485714 = -0.097143$$

The error for the third-order prediction can be computed with Eq. 18.18 as

$$R_3 = 1.847718(2.8 - 2.5)(2.8 - 3.2)(2.8 - 2)(2.8 - 4) = 0.212857$$

18.6 First, order the points so that they are as close to and as centered about the unknown as possible

$x_0 = 3 \quad f(x_0) = 19$
$x_1 = 5 \quad f(x_1) = 99$
$x_2 = 2 \quad f(x_2) = 6$
$x_3 = 7 \quad f(x_3) = 291$
$x_4 = 1 \quad f(x_4) = 3$

Next, the divided differences can be computed and displayed in the format of Fig. 18.5,

i	x_i	$f(x_i)$	$f[x_{i+1},x_i]$	$f[x_{i+2},x_{i+1},x_i]$	$f[x_{i+3},x_{i+2},x_{i+1},x_i]$	$f[x_{i+4},x_{i+3},x_{i+2},x_{i+1},x_i]$
0	3	19	40	9	1	0
1	5	99	31	13	1	
2	2	6	57	9		
3	7	291	48			
4	1	3				

The first through fourth-order interpolations can then be implemented as

$$f_1(4) = 19 + 40(4-3) = 59$$
$$f_2(4) = 59 + 9(4-3)(4-5) = 50$$
$$f_3(4) = 50 \ + 1(4-3)(4-5)(4-2) = 48$$
$$f_4(4) = 48 + 0(4-3)(4-5)(4-2)(4-7) = 48$$

Clearly this data was generated with a cubic polynomial since the difference between the 4th and the 3rd-order versions is zero.

18.7

First order:
$x_0 = 3 \quad f(x_0) = 19$
$x_1 = 5 \quad f(x_1) = 99$

$$f_1(10) = \frac{4-5}{3-5}19 + \frac{4-3}{5-3}99 = 59$$

Second order:
$x_0 = 3 \quad f(x_0) = 19$
$x_1 = 5 \quad f(x_1) = 99$
$x_2 = 2 \quad f(x_2) = 6$

$$f_2(10) = \frac{(4-5)(4-2)}{(3-5)(3-2)}19 + \frac{(4-3)(4-2)}{(5-3)(5-2)}99 \ + \frac{(4-3)(4-5)}{(2-3)(2-5)}6 = 50$$

Third order:
$x_0 = 3 \quad f(x_0) = 19$
$x_1 = 5 \quad f(x_1) = 99$
$x_2 = 2 \quad f(x_2) = 6$
$x_3 = 7 \quad f(x_3) = 291$

$$f_3(10) = \frac{(4-5)(4-2)(4-7)}{(3-5)(3-2)(3-7)}19 + \frac{(4-3)(4-2)(4-7)}{(5-3)(5-2)(5-7)}99$$
$$+ \frac{(4-3)(4-5)(4-7)}{(2-3)(2-5)(2-7)}6 + \frac{(4-3)(4-5)(4-2)}{(7-3)(7-5)(7-2)}291 = 48$$

18.13 For the present problem, we have five data points and $n = 4$ intervals. Therefore, $3(4) = 12$ unknowns must be determined. Equations 18.29 and 18.30 yield $2(4) - 2 = 6$ conditions

$$
\begin{aligned}
&4a_1 + 2b_1 + c_1 = 8 \\
&4a_2 + 2b_2 + c_2 = 8 \\
&6.25a_2 + 2.5b_2 + c_2 = 14 \\
&6.25a_3 + 2.5b_3 + c_3 = 14 \\
&10.24a_3 + 3.2b_3 + c_3 = 15 \\
&10.24a_4 + 3.2b_4 + c_4 = 15
\end{aligned}
$$

Passing the first and last functions through the initial and final values adds 2 more

$$
\begin{aligned}
&2.56a_1 + 1.6b_1 + c_1 = 2 \\
&16a_4 + 4b_4 + c_4 = 8
\end{aligned}
$$

Continuity of derivatives creates an additional $4 - 1 = 3$.

$$
\begin{aligned}
&4a_1 + b_1 = 4a_2 + b_2 \\
&5a_2 + b_2 = 5a_3 + b_3 \\
&6.4a_3 + b_3 = 6.4a_4 + b_4
\end{aligned}
$$

Finally, Eq. 18.34 specifies that $a_1 = 0$. Thus, the problem reduces to solving 11 simultaneous equations for 11 unknown coefficients,

$$
\begin{bmatrix}
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 6.25 & 2.5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 6.25 & 2.5 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 10.24 & 3.2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.24 & 3.2 & 1 \\
1.6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 4 & 1 \\
1 & 0 & -4 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 1 & 0 & -5 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 6.4 & 1 & 0 & -6.4 & -1 & 0
\end{bmatrix}
\begin{Bmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \\ a_4 \\ b_4 \\ c_4 \end{Bmatrix}
=
\begin{Bmatrix} 8 \\ 8 \\ 14 \\ 14 \\ 15 \\ 15 \\ 2 \\ 8 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
$$

which can be solved for

$b_1 = 15$ $\quad c_1 = -22$

$a_2 = -6$ $\quad b_2 = 39$ $\quad c_2 = -46$

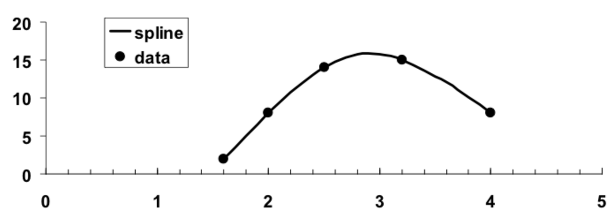
$a_3 = -10.816327$ $\quad b_3 = 63.081633$ $\quad c_3 = -76.102041$

$a_4 = -3.258929$ $\quad b_4 = 14.714286$ $\quad c_4 = 1.285714$

The predictions can be made as

$$
\begin{aligned}
&f(3.4) = -3.258929(3.4)^2 + 14.714286(3.4) + 1.285714 = 13.64107 \\
&f(2.2) = -6(2.2)^2 + 39(2.2) - 46 = 10.76
\end{aligned}
$$

Finally, here is a plot of the data along with the quadratic spline,



18.14 For the first interior knot

$x_0 = 1$ $\quad f(x_0) = 3$
$x_1 = 2$ $\quad f(x_1) = 6$

$x_2 = 3$ $\quad f(x_2) = 19$

$$(2-1)f''(1) + 2(3-1)f''(2) + (3-2)f''(3) = \frac{6}{3-2}(19-6) + \frac{6}{2-1}(3-6)$$

Because of the natural spline condition, $f''(1) = 0$, and the equation reduces to

$$4f''(2) + f''(3) = 60$$

Equations can be written for the remaining interior knots and the results assembled in matrix form as

$$\begin{bmatrix} 4 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 0 & 2 & 8 & 2 \\ 0 & 0 & 2 & 6 \end{bmatrix} \begin{Bmatrix} f''(2) \\ f''(3) \\ f''(5) \\ f''(7) \end{Bmatrix} = \begin{Bmatrix} 60 \\ 162 \\ 336 \\ 342 \end{Bmatrix}$$

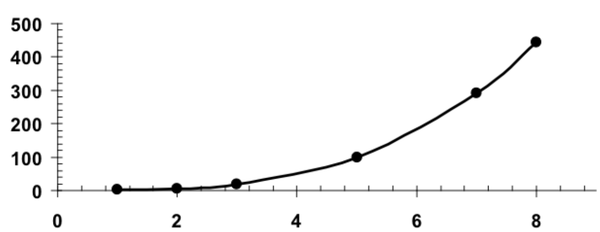
which can be solved for

$$f''(2) = 10.84716$$
$$f''(3) = 16.61135$$
$$f''(5) = 25.74236$$
$$f''(7) = 48.41921$$

These values can be used in conjunction with Eq. 18.36 to yield the following interpolating splines for each interval,

$$f_1(x) = 1.80786(x-1)^3 + 3(2-x) + 4.19214(x-1)$$
$$f_2(x) = 1.80786(3-x)^3 + 2.768559(x-2)^3 + 4.19214(3-x) + 16.23144(x-2)$$
$$f_3(x) = 1.384279(5-x)^3 + 2.145197(x-3)^3 + 3.962882(5-x) + 40.91921(x-3)$$
$$f_4(x) = 2.145197(7-x)^3 + 4.034934(x-5)^3 + 40.91921(7-x) + 129.3603(x-5)$$
$$f_5(x) = 8.069869(8-x)^3 + 282.9301(8-x) + 444(x-7)$$

The interpolating splines can be used to make predictions along the interval. The results are shown in the following plot.



(a) The interpolating equations can be used to determine

$f_3(4) = 48.41157$
$f_2(2.5) = 10.78384$

(b)

$$f_2(3) = 1.80786(3-3)^3 + 2.768559(3-2)^3 + 4.19214(3-3) + 16.23144(3-2) = 19$$
$$f_3(3) = 1.384279(5-3)^3 + 2.145197(3-3)^3 + 3.962882(5-3) + 40.91921(3-3) = 19$$

18.16 The points to be fit are

$x_0 = 1$ $f(x_0) = 3$
$x_1 = 2$ $f(x_1) = 6$
$x_2 = 3$ $f(x_2) = 19$
$x_3 = 5$ $f(x_3) = 99$

Using Eq. 18.26 the following simultaneous equations can be generated

$$a_0 + a_1 + a_2 + a_3 = 3$$
$$a_0 + 2a_1 + 4a_2 + 8a_3 = 6$$
$$a_0 + 3a_1 + 9a_2 + 27a_3 = 19$$
$$a_0 + 5a_1 + 25a_2 + 125a_3 = 99$$

These can be solved for $a_0 = 4$, $a_1 = -1$, $a_2 = -1$, and $a_3 = 1$. Therefore, the interpolating polynomial is

$$f(x) = 4 - x - x^2 + x^3$$