22.2 The integral can be evaluated analytically as,

$$I = \int_1^2 \left(x + \frac{1}{x}\right)^2 dx = \int_1^2 x^2 + 2 + x^{-2}\, dx$$

$$I = \left[\frac{x^3}{3} + 2x - \frac{1}{x}\right]_1^2 = \frac{2^3}{3} + 2(2) - \frac{1}{2} - \frac{1^3}{3} - 2(1) + \frac{1}{1} = 4.8333$$

The tableau depicting the implementation of Romberg integration to $\varepsilon_s = 0.5\%$ is

iteration→	**1**	**2**	**3**
ε_t →	6.0345%	0.0958%	0.0028%
ε_a →		1.4833%	0.0058%
1	5.12500000	4.83796296	4.83347014
2	4.90972222	4.83375094	
4	4.85274376		

Thus, the result is 4.83347014.

22.3

	1	**2**	**3**
n	ε_a →	**7.9715%**	**0.0997%**
1	1.34376994	1.97282684	1.94183605
2	1.81556261	1.94377297	
4	1.91172038		

22.9 (a)

$$\int_2^\infty \frac{dx}{x(x+2)} = \int_0^{0.5} \frac{1}{t^2}(t)\frac{1}{1/t+2}dt = \int_0^{0.5} \frac{1}{1+2t}dt$$

We can use 8 applications of the extended midpoint rule.

$$\frac{1}{16}(0.941176+0.842105+0.761905+0.695652+0.64+0.592593+0.551724+0.516129)=0.34633$$

This result is close to the analytical solution

$$\int_2^\infty \frac{dx}{x(x+2)} = \left[0.5\ln\left(\frac{x}{x+2}\right)\right]_2^\infty = 0.5\ln\left(\frac{\infty}{\infty+2}\right) - 0.5\ln\left(\frac{2}{2+2}\right) = 0.346574$$

(b)

$$\int_0^\infty e^{-y}\sin^2 y\,dy = \int_0^2 e^{-y}\sin^2 y\,dy + \int_2^\infty e^{-y}\sin^2 y\,dy$$

For the first part, we can use 4 applications of Simpson's 1/3 rule

$$I = (2-0)\frac{0+4(0.048+0.219+0.258+0.168)+2(0.139+0.26+0.222)+0.112}{24} = 0.344115$$

For the second part,

$$\int_2^\infty e^{-y}\sin^2 y\,dy = \int_0^{1/2} \frac{1}{t^2}e^{-1/t}\sin^2(1/t)\,dt$$

We can use the extended midpoint rule with $h = 1/8$.

$$I = \frac{1}{8}(0+0.0908+0.00142+0.303) = 0.0494$$

The total integral is

$$I = 0.344115 + 0.0494 = 0.393523$$

This result is close to the analytical solution of 0.4.

(c)

$$\int_0^\infty \frac{1}{(1+y^2)(1+y^2/2)}\,dy = \int_0^2 \frac{1}{(1+y^2)(1+y^2/2)}\,dy + \int_2^\infty \frac{1}{(1+y^2)(1+y^2/2)}\,dy$$

For the first part, we can use Simpson's 1/3 rule

$$(2-0)\frac{1+4(0.9127+0.4995+0.2191+0.0972)+2(0.7111+0.3333+0.1448)+0.0667}{24} = 0.863262$$

For the second part,

$$\int_2^\infty \frac{1}{(1+y^2)(1+y^2/2)}\,dy = \int_0^{1/2} \frac{1}{t^2(1+(1/t)^2)(1+1/(2t^2))}\,dt$$

We can use the extended midpoint rule with $h = 1/8$.

$$I = \frac{1}{8}(0.007722+0.063462+0.148861+0.232361) = 0.056551$$

The total integral is

$$I = 0.863262 + 0.056551 = 0.919813$$

This result is close to the analytical solution of 0.920151.

(d)

$$\int_{-2}^\infty ye^{-y}\,dy = \int_{-2}^2 ye^{-y}\,dy + \int_2^\infty ye^{-y}\,dy$$

For the first part, we can use 4 applications of Simpson's 1/3 rule

$$I = (2-(-2))\frac{-14.78+4(-6.72-0.824+0.303+0.335)+2(-2.72+0+0.368)+0.2707}{24} = -7.807$$

For the second part,

$$\int_2^\infty ye^{-y}\,dy = \int_0^{1/2} \frac{1}{t^3}e^{-1/t}\,dt$$

We can use the extended midpoint rule with $h = 1/8$.

$$I = \frac{1}{8}(0.000461+0.732418+1.335696+1.214487) = 0.410383$$

The total integral is

$$I = -7.80733 + 0.410383 = -7.39695$$

(e)

$$\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\ dx = \int_0^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\ dx + \int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\ dx$$

For the first part, we can use 4 applications of Simpson's 1/3 rule

$$I = (2-0)\frac{0.399 + 4(0.387 + 0.301 + 0.183 + 0.086) + 2(0.352 + 0.242 + 0.130) + 0.054}{24} = 0.47725$$

For the second part,

$$\int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\ dx = \int_0^{1/2} \frac{1}{\sqrt{2\pi}} \frac{1}{t^2} e^{-1/(2t^2)}\ dt$$

We can use the extended midpoint rule with $h = 1/8$.

$$I = \frac{1}{8}(0 + 0 + 0.024413063 + 0.152922154) = 0.02217$$

The total integral is

$$I = 0.47725 + 0.02217 = 0.499415$$

This is close to the exact value of 0.5.

22.14 Change of variable:

$$x = \frac{1.5+0}{2} + \frac{1.5-0}{2} x_d = 0.75 + 0.75 x_d$$
$$dx = \frac{1.5-0}{2} dx_d = 0.75 dx_d$$

$$I = \int_{-1}^{1} \frac{1.5}{\sqrt{\pi}} e^{-(0.75+0.75x_d)^2} \, dx_d$$

Therefore, the transformed function is

$$f(x_d) = \frac{1.5}{\sqrt{\pi}} e^{-(0.75+0.75x_d)^2}$$

Two-point formula:

$$I = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) = 0.765382 + 0.208792 = 0.974173$$
$$\varepsilon_t = \left|\frac{0.966105 - 0.974173}{0.966105}\right| \times 100\% = 0.835\%$$

23.1

	x	$f(x)$
x_{i-2}	0.261799388	0.965925826
x_{i-1}	0.523598776	0.866025404
x_i	0.785398163	0.707106781
x_{i+1}	1.047197551	0.5
x_{i+2}	1.308996939	0.258819045

true = $-\sin(\pi/4) = -0.70710678$

The results are summarized as

	first-order	second-order
Forward	–0.79108963	–0.72601275
	–11.877%	–2.674%
Backward	–0.60702442	–0.71974088
	14.154%	–1.787%
Centered	–0.69905703	–0.70699696
	1.138%	0.016%

23.4 The true value is $-\sin(\pi/4) = -0.70710678$.

$$D(\pi/3) = \frac{-0.25882 - 0.965926}{2(1.047198)} = -0.58477$$

$$D(\pi/6) = \frac{0.258819 - 0.965926}{2(0.523599)} = -0.67524$$

$$D = \frac{4}{3}(-0.67524) - \frac{1}{3}(-0.58477) = -0.70539$$