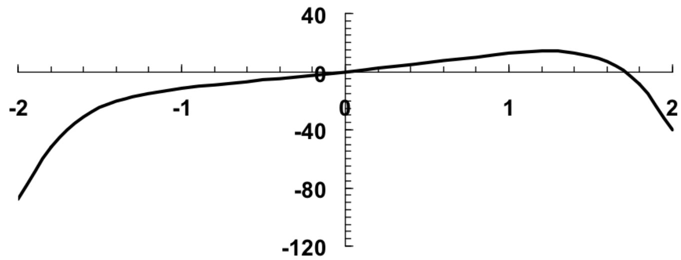
13.2 (a) The function can be plotted



(b) The function can be differentiated twice to give

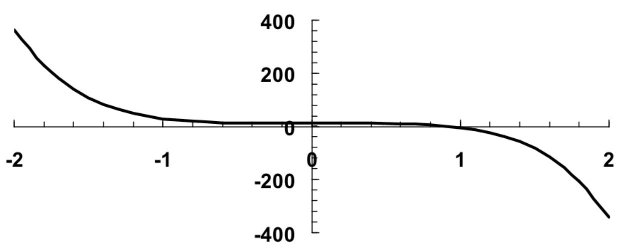
$$f''(x) = -45x^4 - 24x^2$$

Thus, the second derivative will always be negative and hence the function is concave for all values of x.

(c) Differentiating the function and setting the result equal to zero results in the following roots problem to locate the maximum

$$f'(x) = 0 = -9x^5 - 8x^3 + 12$$

A plot of this function can be developed



A technique such as bisection can be employed to determine the root. Here are the first few iterations:

iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	$f(x_l)\times f(x_r)$	ε_a
1	0.00000	2.00000	1.00000	12	-5	-60.0000	
2	0.00000	1.00000	0.50000	12	10.71875	128.6250	100.00%
3	0.50000	1.00000	0.75000	10.71875	6.489258	69.5567	33.33%
4	0.75000	1.00000	0.87500	6.489258	2.024445	13.1371	14.29%
5	0.87500	1.00000	0.93750	2.024445	-1.10956	-2.2463	6.67%

The approach can be continued to yield a result of $x = 0.91692$.

13.3 First, the golden ratio can be used to create the interior points,

$$d = \frac{\sqrt{5}-1}{2}(2-0) = 1.2361$$
$$x_1 = 0 + 1.2361 = 1.2361$$
$$x_2 = 2 - 1.2361 = 0.7639$$

The function can be evaluated at the interior points

$$f(x_2) = f(0.7639) = 8.1879$$
$$f(x_1) = f(1.2361) = 4.8142$$

Because $f(x_2) > f(x_1)$, the maximum is in the interval defined by x_l, x_2, and x_1.where x_2 is the optimum. The error at this point can be computed as

$$\varepsilon_a = (1-0.61803)\left|\frac{2-0}{0.7639}\right| \times 100\% = 100\%$$

For the second iteration, $x_l = 0$ and $x_u = 1.2361$. The former x_2 value becomes the new x_1, that is, $x_1 = 0.7639$ and $f(x_1) = 8.1879$. The new values of d and x_2 can be computed as

$$d = \frac{\sqrt{5}-1}{2}(1.2361-0) = 0.7639$$
$$x_2 = 1.2361 - 0.7639 = 0.4721$$

The function evaluation at $f(x_2) = 5.5496$. Since this value is less than the function value at x_1, the maximum is in the interval prescribed by x_2, x_1 and x_u. The process can be repeated and all three iterations summarized as

i	x_l	$f(x_l)$	x_2	$f(x_2)$	x_1	$f(x_1)$	x_u	$f(x_u)$	d	x_{opt}	ε_a
1	0.0000	0.0000	0.7639	8.1879	1.2361	4.8142	2.0000	-104.0000	1.2361	0.7639	100.00%
2	0.0000	0.0000	0.4721	5.5496	0.7639	8.1879	1.2361	4.8142	0.7639	0.7639	61.80%
3	0.4721	5.5496	0.7639	8.1879	0.9443	8.6778	1.2361	4.8142	0.4721	0.9443	30.90%

13.4 First, the function values at the initial values can be evaluated

$$f(x_0) = f(0) = 0$$
$$f(x_1) = f(1) = 8.5$$
$$f(x_2) = f(2) = -104$$

and substituted into Eq. (13.7) to give,

$$x_3 = \frac{0(1^2 - 2^2) + 8.5(2^2 - 0^2) + (-104)(0^2 - 1^2)}{2(0)(1-2) + 2(8.5)(2-0) + 2(-104)(0-1)} = 0.570248$$

which has a function value of $f(0.570248) = 6.5799$. Because the function value for the new point is lower than for the intermediate point (x_1) and the new x value is to the left of the intermediate point, the lower guess (x_0) is discarded. Therefore, for the next iteration,

$$f(x_0) = f(0.570248) = 6.6799$$
$$f(x_1) = f(1) = 8.5$$
$$f(x_2) = f(2) = -104$$

which can be substituted into Eq. (13.7) to give $x_3 = 0.812431$, which has a function value of $f(0.812431) = 8.446523$. At this point, an approximate error can be computed as

$$\varepsilon_a = \left|\frac{0.81243 - 0.570248}{0.81243}\right| \times 100\% = 29.81\%$$

The process can be repeated, with the results tabulated below:

i	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	x_3	$f(x_3)$	ε_a
1	0.00000	0.00000	1.00000	8.50000	2.0000	-104	0.57025	6.57991	
2	0.57025	6.57991	1.00000	8.50000	2.0000	-104	0.81243	8.44652	29.81%
3	0.81243	8.44652	1.00000	8.50000	2.0000	-104	0.90772	8.69575	10.50%

Thus, after 3 iterations, the result is converging on the true value of $f(x) = 8.69793$ at $x = 0.91692$.

13.5 The first and second derivatives of the function can be evaluated as

$$f'(x) = -9x^5 - 8x^3 + 12$$
$$f''(x) = -45x^4 - 24x^2$$

which can be substituted into Eq. (13.8) to give

$$x_{i+1} = x_i - \frac{-9x_i^5 - 8x_i^3 + 12}{-45x_i^4 - 24x_i^2}$$

Substituting the initial guess yields

$$x_{i+1} = 2 - \frac{-9(2^5) - 8(2^3) + 12}{-45(2^4) - 24(2^2)} = 2 - \frac{-340}{-816} = 1.583333$$

which has a function value of –17.2029. The second iteration gives

$$x_{i+1} = 1.583333 - \frac{-9(1.583333^5) - 8(1.583333^3) + 12}{-45(1.583333^4) - 24(1.583333^2)} = 1.583333 - \frac{-109.313}{-342.981} = 1.26462$$

which has a function value of 3.924617. At this point, an approximate error can be computed as

$$\varepsilon_a = \left|\frac{1.26462 - 1.583333}{1.26462}\right| \times 100\% = 26.316\%$$

The process can be repeated, with the results tabulated below:

i	x	$f(x)$	$f'(x)$	$f''(x)$	ε_a
0	2	-104	-340	-816	
1	1.583333	-17.2029	-109.313	-342.981	26.316%
2	1.26462	3.924617	-33.2898	-153.476	25.202%
3	1.047716	8.178616	-8.56281	-80.5683	20.703%

Thus, within five iterations, the result is converging on the true value of $f(x) = 8.69793$ at $x = 0.91692$.

13.6 (a) First, the golden ratio can be used to create the interior points,

$$d = \frac{\sqrt{5}-1}{2}(4-(-2)) = 3.7082$$
$$x_1 = -2 + 3.7082 = 1.7082$$
$$x_2 = 4 - 3.7082 = 0.2918$$

The function can be evaluated at the interior points

$$f(x_2) = f(0.2918) = 1.04156$$
$$f(x_1) = f(1.7082) = 5.00750$$

Because $f(x_1) > f(x_2)$, the maximum is in the interval defined by x_2, x_1 and x_u where x_1 is the optimum. The error at this point can be computed as

$$\varepsilon_a = (1-0.61803)\left|\frac{4-(-2)}{1.7082}\right| \times 100\% = 134.16\%$$

The process can be repeated and all the iterations summarized as

i	x_l	$f(x_l)$	x_2	$f(x_2)$	x_1	$f(x_1)$	x_u	$f(x_u)$	d	x_{opt}	ε_a
1	-2.0000	-29.6000	0.2918	1.0416	1.7082	5.0075	4.0000	-12.8000	3.7082	1.7082	134.16%
2	0.2918	1.0416	1.7082	5.0075	2.5836	5.6474	4.0000	-12.8000	2.2918	2.5836	54.82%
3	1.7082	5.0075	2.5836	5.6474	3.1246	2.9361	4.0000	-12.8000	1.4164	2.5836	33.88%
4	1.7082	5.0075	2.2492	5.8672	2.5836	5.6474	3.1246	2.9361	0.8754	2.2492	24.05%
5	1.7082	5.0075	2.0426	5.6648	2.2492	5.8672	2.5836	5.6474	0.5410	2.2492	14.87%
6	2.0426	5.6648	2.2492	5.8672	2.3769	5.8770	2.5836	5.6474	0.3344	2.3769	8.69%
7	2.2492	5.8672	2.3769	5.8770	2.4559	5.8287	2.5836	5.6474	0.2067	2.3769	5.37%
8	2.2492	5.8672	2.3282	5.8853	2.3769	5.8770	2.4559	5.8287	0.1277	2.3282	3.39%
9	2.2492	5.8672	2.2980	5.8828	2.3282	5.8853	2.3769	5.8770	0.0789	2.3282	2.10%
10	2.2980	5.8828	2.3282	5.8853	2.3468	5.8840	2.3769	5.8770	0.0488	2.3282	1.30%
11	2.2980	5.8828	2.3166	5.8850	2.3282	5.8853	2.3468	5.8840	0.0301	2.3282	0.80%

(b) First, the function values at the initial values can be evaluated

$$f(x_0) = f(1.75) = 5.1051$$
$$f(x_1) = f(2) = 5.6$$
$$f(x_2) = f(2.5) = 5.7813$$

and substituted into Eq. (13.7) to give,

$$x_3 = \frac{5.1051(2^2-2.5^2)+5.6(2.5^2-1.75^2)+5.7813(1.75^2-2^2)}{2(5.1051)(2-2.5)+2(5.6)(2.5-1.75)+2(5.7813)(1.75-2)} = 2.3341$$

Second iteration:

$$f(x_0) = f(2) = 5.6$$
$$f(x_1) = f(2.5) = 5.7813$$
$$f(x_2) = f(2.3341) = 5.8852$$

which can be substituted into Eq. (13.7) to give $x_3 = 2.3112$, which has a function value of $f(2.3112) = 5.8846$. At this point, an approximate error can be computed as

$$\varepsilon_a = \left|\frac{2.3112-2.3341}{2.3112}\right| \times 100\% = 0.99\%$$

The process can be repeated, with the results tabulated below:

i	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	x_3	$f(x_3)$	ε_a
1	1.7500	5.1051	2.0000	5.6000	2.5000	5.7813	2.3341	5.8852	
2	2.0000	5.6000	2.5000	5.7813	2.3341	5.8852	2.3112	5.8846	0.99%
3	2.5000	5.7813	2.3341	5.8852	2.3112	5.8846	2.3260	5.8853	0.64%
4	2.3341	5.8852	2.3112	5.8846	2.3260	5.8853	2.3263	5.8853	0.01%

Thus, after 4 iterations, the result is converging rapidly on the true value of $f(x) = 5.8853$ at $x = 2.3263$.

(c) The first and second derivatives of the function can be evaluated as

$$f'(x) = 4 - 3.6x + 3.6x^2 - 1.2x^3$$
$$f''(x) = -3.6 + 7.2x - 3.6x^2$$

which can be substituted into Eq. (13.8) to give

$$x_{i+1} = x_i - \frac{4-3.6x_i+3.6x_i^2-1.2x_i^3}{-3.6+7.2x_i-3.6x_i^2} = 3 - \frac{-6.8}{-14.4} = 2.5278$$

which has a function value of 5.7434. The second iteration gives 2.3517, which has a function value of 5.8833. At this point, an approximate error can be computed as ε_a = 18.681%. The process can be repeated, with the results tabulated below:

i	*x*	*f*(*x*)	*f'*(*x*)	*f''*(*x*)	ε_a
0	3.0000	3.9000	-6.8000	-14.4000	
1	2.5278	5.7434	-1.4792	-8.4028	18.681%
2	2.3517	5.8833	-0.1639	-6.5779	7.485%
3	2.3268	5.8853	-0.0030	-6.3377	1.071%
4	2.3264	5.8853	0.0000	-6.3332	0.020%

Thus, within four iterations, the result is converging on the true value of $f(x)$ = 5.8853 at x = 2.3264.