Review of block 1: The basic methods and dynamics of infectious diseases

Solutions to exercises

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Introduction to Infectious Disease Modelling and its Applications

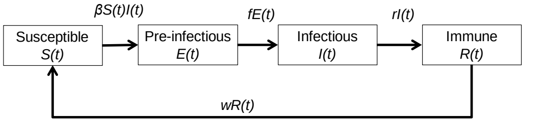
LSHTM

20th June 2018

Public Health England

LONDON SCHOOL of HYGIENE & TROPICAL MEDICINE

Q1. The following is a diagram of the transmission dynamics of *Mycoplasma pneumoniae*. Which of the following statements is correct?



a) The model will predict that the total population size will decrease over time

b) The model will predict that the total population size will increase over time.

c) The model will predict that the total population size will remain unchanged over time.

d) The rate of change in the number of Immune individuals is given by the following equation: $\frac{dR(t)}{dt} = r - w$

e) The rate of change in the number of Immune individuals is given by the following equation: $\frac{dR(t)}{dt} = rI(t) - wR(t)$

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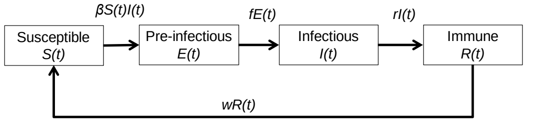

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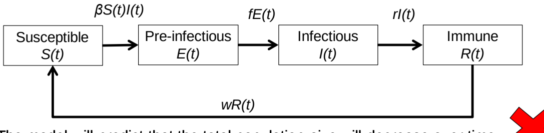


a) The model will predict that the total population size will decrease over time

b) The model will predict that the total population size will increase over time.

c) The model will predict that the total population size will remain unchanged over time.

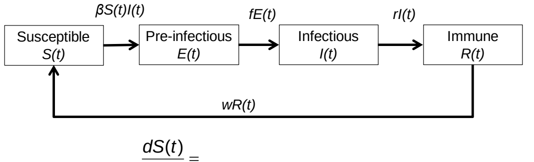
Answer

There are no arrows for births, deaths or migration in the model diagram

∴ No one enters the population and no one leaves it

∴ The population size must remain constant over time

Write down the differential equations corresponding to the following model:



$$\frac{dS(t)}{dt} =$$
$$\frac{dE(t)}{dt} =$$
$$\frac{dI(t)}{dt} =$$
$$\frac{dR(t)}{dt} =$$

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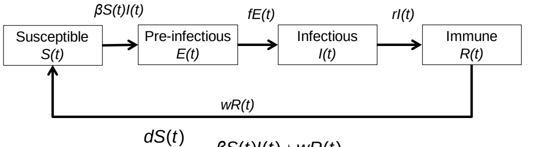

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) + wR(t)$$
$$\frac{dE(t)}{dt} = \beta S(t)I(t) - fE(t)$$
$$\frac{dI(t)}{dt} = fE(t) - rI(t)$$
$$\frac{dR(t)}{dt} = rI(t) - wR(t)$$

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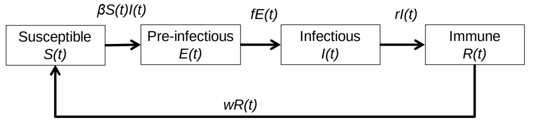

a) The model will predict that the total population size will decrease over time

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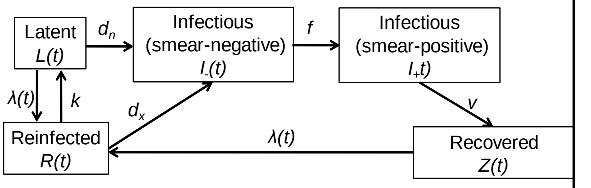
d) The rate of change in the number of Immune individuals is given by the following equation: $\frac{dR(t)}{dt} = r - w$

e) The rate of change in the number of Immune individuals is given by the following equation: $\frac{dR(t)}{dt} = rI(t) - wR(t)$

Q2. The following is a simplified diagram of the transmission dynamics of *M tuberculosis* in a closed high transmission setting.



Smear-negative and smear-positive individuals are both infectious; however, smear-negative individuals are 25% less infectious than are smear-positive individuals.

The *per capita* rate at which smear-positive individuals come into effective contact with others is denoted by the symbol *ß*.

Which of the following statements is correct?

a) The rate of change in the total number of infectious individuals ($I(t) = I_+(t) + I_-(t)$) is given by the following equation:

$$\frac{dI(t)}{dt} = d_n L(t) + d_x R(t) - vI_+(t)$$

b) The rate of change in the total number of infectious individuals ($I(t) = I_+(t) + I_-(t)$) is given by the following equation:

$$\frac{dI(t)}{dt} = d_n L(t) + d_x R(t) - fI_-(t) - vI_+(t)$$

c) There are no errors in the following equation:

$$\frac{dL(t)}{dt} = -\lambda(t)R(t) + kL(t) - d_n L(t)$$

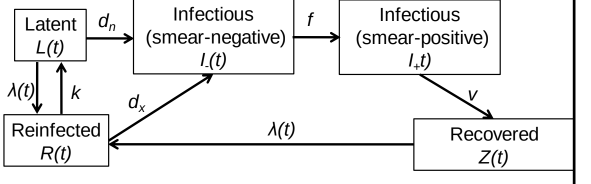
d) The force of infection is given by the following equation:

$$\lambda(t) = \beta I(t)$$

e) The force of infection is given by the following equation:

$$\lambda(t) = \beta I_+(t) + 0.25\beta I_-(t)$$

Q2, a, b



$$\frac{dI_-(t)}{dt} = d_n L(t) + d_x R(t) - fI_-(t)$$

$$\frac{dI_+(t)}{dt} = fI_-(t) - vI_+(t)$$

$$\frac{dI_-(t)}{dt} + \frac{dI_+(t)}{dt} = d_n L(t) + d_x R(t) - \cancel{fI_-(t)} + \cancel{fI_-(t)} - vI_+(t)$$

$$= d_n L(t) + d_x R(t) - vI_+(t)$$

Which of the following statements is correct?

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$$\frac{dL(t)}{dt} = -\lambda(t)R(t) + kL(t) - d_n L(t)$$

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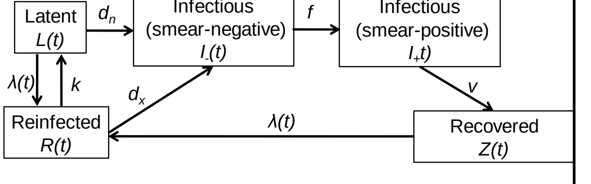
$$\lambda(t) = \beta I_+(t) + 0.25\beta I_-(t)$$

Q2 Is the following statement correct?

Q2 c) There are no errors in the following equation:

$$\frac{dL(t)}{dt} = -\lambda(t)R(t) + kL(t) - d_n L(t)$$



$$\frac{dL(t)}{dt} = -\lambda(t)L(t) + kR(t) - d_n L(t)$$

Q2 Are either of the following equations correct:

d) $\lambda(t) = \beta I(t)$

e) $\lambda(t) = \beta I_{+}(t) + 0.25\beta I_{-}(t)$

Note:

Smear-negative and smear-positive individuals are both infectious; however, smear-negative individuals are 25% less infectious than are smear-positive individuals.

=> Per capita rate at which smear-negatives effectively contact others = 0.75×per capita rate at which smear-positives contact others, so:

$$\lambda(t) = \beta I_{+}(t) + 0.75\beta I_{-}(t)$$

The *per capita* rate at which smear-positive individuals come into effective contact with others is denoted by the symbol *ß*.

So…statements d & e are incorrect

Which of the following statements is correct?

a) The rate of change in the total number of infectious individuals ($I(t) = I_{+}(t) + I_{-}(t)$) is given by the following equation:

$$\frac{dI(t)}{dt} = d_n L(t) + d_x R(t) - vI_{+}(t)$$

b) The rate of change in the total number of infectious individuals ($I(t) = I_{+}(t) + I_{-}(t)$) is given by the following equation:

$$\frac{dI(t)}{dt} = d_n L(t) + d_x R(t) - fI_{-}(t) - vI_{+}(t)$$

c) There are no errors in the following equation:

$$\frac{dL(t)}{dt} = -\lambda(t)R(t) + kL(t) - d_n L(t)$$

d) The force of infection is given by the following equation:

$$\lambda(t) = \beta I_{+}(t) + 0.25\beta I_{-}(t)$$

e) The force of infection is given by the following equation:

$$\lambda(t) = \beta I(t)$$

NB It is often useful to draw the model corresponding to a set of differential equations.

Exercise:

Draw the diagram of the model implicit in the following equations:

$$\frac{dS}{dt} = -\lambda(t)S(t) + r_1 E(t)$$
$$\frac{dE}{dt} = \lambda(t)S(t) - r_1 E(t) - r_2 E(t) + r_3 R(t)$$
$$\frac{dR}{dt} = -r_3 R(t) + r_2 E(t)$$

$$\frac{dS}{dt} = -\lambda(t)S(t) + r_1 E(t)$$
$$\frac{dE}{dt} = \lambda(t)S(t) - r_1 E(t) - r_2 E(t) + r_3 R(t)$$
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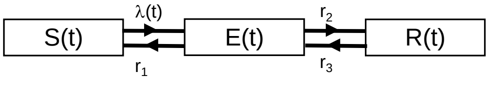

Q3. The inter-epidemic period for rubella in some populations was roughly 4 years before the introduction of vaccination. The value for R_0 for rubella was about 7. Which one of the following statements is correct?

a) The introduction of rubella vaccination among newborns was likely to lead to a reduction in the inter-epidemic period.

b) The introduction of rubella vaccination among newborns was likely to lead to an increase in the inter-epidemic period.

c) When the incidence of rubella was at a peak, on average, 25% of the population was likely to be susceptible.

d) When the incidence of rubella was increasing, the proportion of the population that was susceptible was less than 14%

e) The inter-epidemic period of rubella was likely to be less than that for measles, for which the value for R_0 was about 13

Q3. The inter-epidemic period for rubella in some populations was roughly 4 years before the introduction of vaccination. The value for R_0 for rubella was about 7. Are the following statements correct?

a) The introduction of rubella vaccination among newborns was likely to lead to a reduction in the inter-epidemic period.

b) The introduction of rubella vaccination among newborns was likely to lead to an increase in the inter-epidemic period.

Answer:
For the incidence to start to increase, the proportion susceptible must be above a certain value ($=1/R_0$)

If newborns are vaccinated, it takes longer for the proportion susceptible to reach the required threshold than if no newborns are vaccinated

$\therefore$ b) is correct

Q3. The inter-epidemic period for rubella in some populations was roughly 4 years before the introduction of vaccination. The value for R_0 for rubella was about 7. Is the following statement correct?

c) When the incidence of rubella was at a peak, on average, 25% of the population was likely to be susceptible.

Answer:

When the incidence is at a peak, R_n=1, so R_n=R_0×s=1,

Rearranging the equation R_0×s=1, we see that s=1/R_0 at the peak in incidence

But, for rubella, R_0=7, so s=1/7 ≈0.14 at the peak in incidence

So, statement c) is incorrect.

Q2. The inter-epidemic period for rubella in some populations was roughly 4 years before the introduction of vaccination. The value for R_0 for rubella was about 7. Which of the following statements is correct?

d) When the incidence of rubella was increasing, the proportion of the population that was susceptible was less than 14%

Answer:

When the incidence is increasing, R_n>1, so R_n=R_0×s>1,

Rearranging the equation R_0×s>1, we see that s>1/R_0 when the incidence is increasing

But, for rubella, R_0=7, so s>1/7 ≈014 when the incidence is increasing

So, statement d) is incorrect.

Q2. The inter-epidemic period for rubella in some populations was roughly 4 years before the introduction of vaccination. The value for R_0 for rubella was about 7. Is the following statement correct?

e) The inter-epidemic period of rubella was likely to be less than that for measles, for which the value for R_0 was about 13.

Answer: For an epidemic to occur, s>1/R_0.

For measles, R_0=13, so s>1/13 ≈0.07 for an epidemic to occur

For rubella, R_0=7, so s>1/7 ≈0.14 for an epidemic to occur

It is easier to accumulate sufficient newborns for s to be above 0.07 than it is for s to reach above 0.14.

∴The time until the next epidemic for measles (for which R_0=13) will be less than that for rubella (for which R_0=7)

∴So, e) is incorrect

Q2. The inter-epidemic period for rubella in some populations was roughly 4 years before the introduction of vaccination. The value for R_0 for rubella was about 7. Which one of the following statements is correct?

a) The introduction of rubella vaccination among newborns was likely to lead to a reduction in the inter-epidemic period.

b) The introduction of rubella vaccination among newborns was likely to lead to an increase in the inter-epidemic period.

c) When the incidence of rubella was at a peak, on average, 25% of the population was susceptible was likely to be susceptible.

d) When the incidence of rubella was increasing, the proportion of the population that was susceptible was less than 14%

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