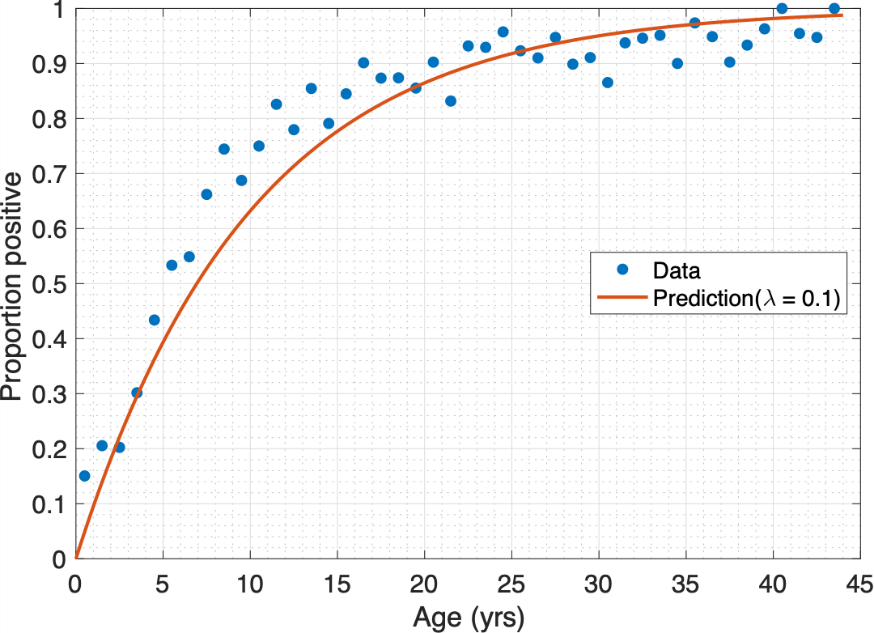
**9-2 Model fitting using MLE and Bayesian inference**

**PART I: Fitting the catalytic model to seroprevalence data to estimate the force of infection using MLE**

Ignoring the maternal antibodies, fit the catalytic model to the UK data (“seroprevalence\_uk”) to estimate the force of infection using MLE in which Binomial distribution is assumed for the data.

1. Assume the initial value for the force infection in the UK to be 0.1. Do you think the true value for the force infection in the UK was greater or smaller than that currently assumed? What is the current value for negative log-likelihood?

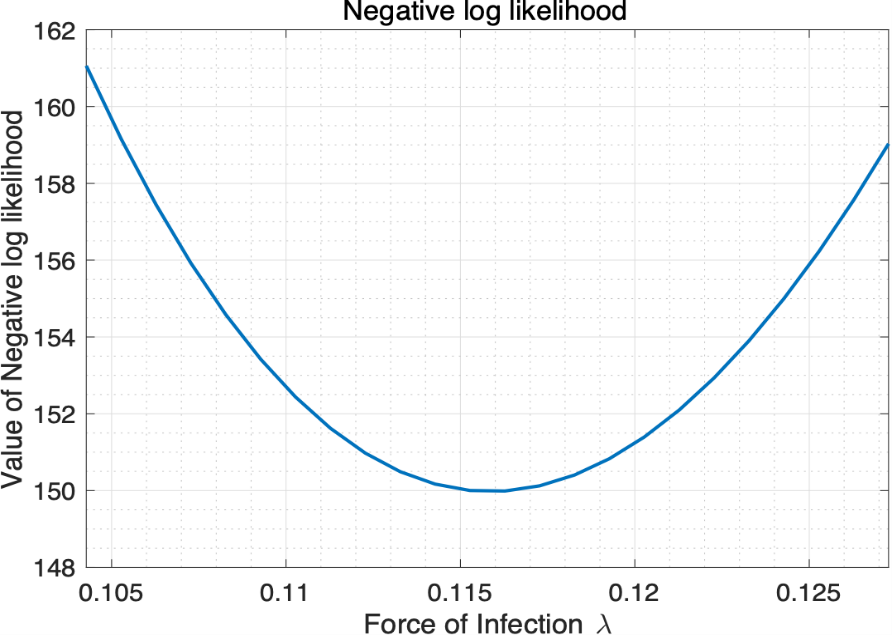
* Plot for a data and prediction plot when force of infection is 0.1



* It seems like under-estimation, so the true for the force of infection in the UK is greater than 0.1
* Current value for negative log-likelihood : 171.480347

1. What is the best-fitting value for the force of infection and the current value for negative log-likelihood? Plot a graph of negative log-likelihood and estimate the 95% confidence interval.

* Plot for negative log-likelihood



* Best-fitting value of force of infection : 0.115859
* 95% confidence interval : 149.975487
* Current value of negative log-likelihood : [0.110912 0.121005]

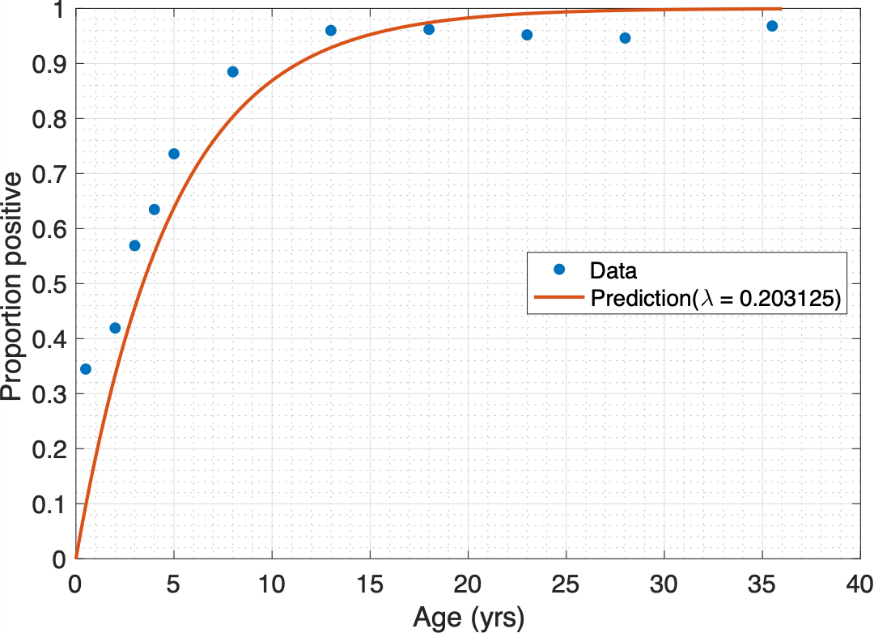
1. Calculate the average age at infection, the R0 (assuming that the life expectancy is 60 years) and herd immunity threshold with 95% confidence interval.

* Average age of infection
  + Value : 8.631153
  + Confidence interval : [8.264116 9.016150]
* R0
  + Value : 6.951563
  + Confidence interval : [6.654725 7.260305]
* Herd immunity threshold
  + Value : 0.856147
  + Confidence interval : [0.849731 0.862265]

Fit the catalytic model to China data (“seroprevalence \_china”) to estimate the force of infection:

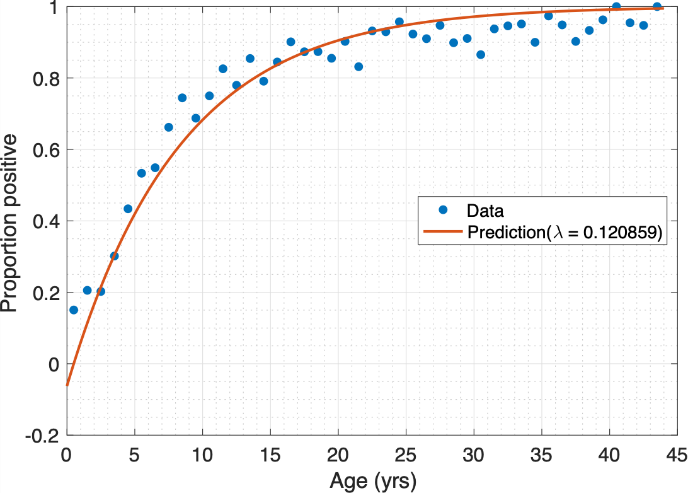
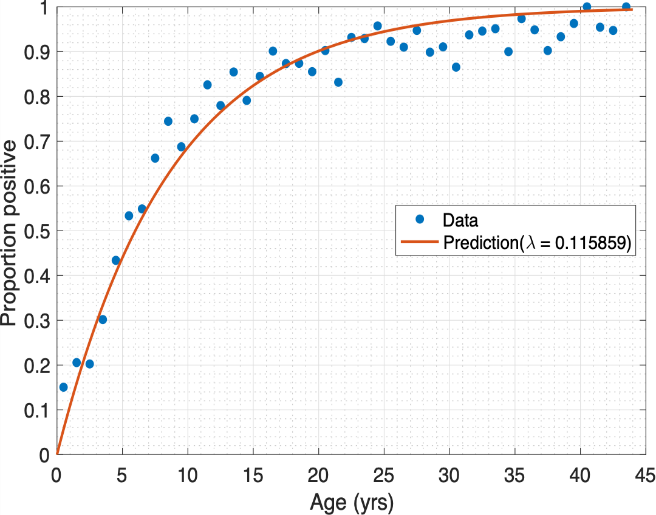
1. Calculate the best-fitting force of infection, the average age at infection, the R0 (assuming that the life expectancy is the same as that in the UK) and herd immunity threshold with 95% confidence interval.

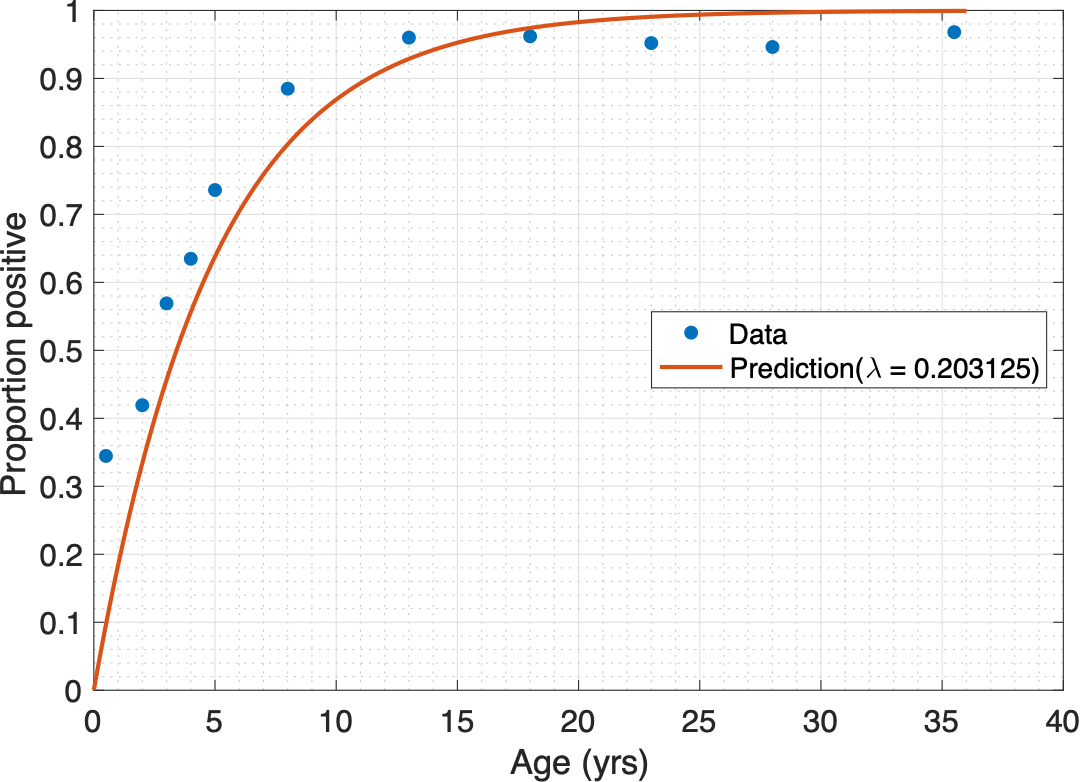
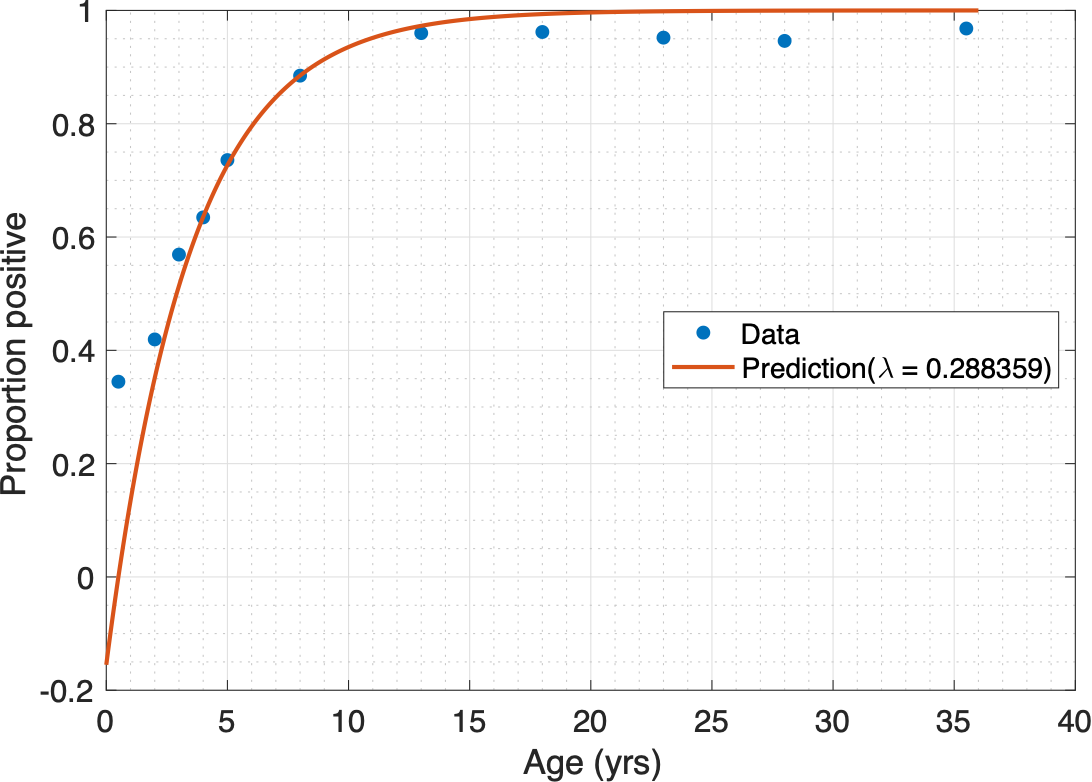
* Plot for a data and prediction plot using best-fitted value



* Force of infection
  + Value : 0.203125
  + Confidence interval : [0.198355 0.208008]
* Average age of infection
  + Value : 4.923077
  + Confidence interval : [4.807499 5.041474]
* R0
  + Value : 12.187500
  + Confidence interval : [11.901282 12.480501]
* Herd immunity threshold
  + Value : 0.917949
  + Confidence interval : [0.915975 0.919875]

1. Modify the expression for the prevalence of previous infection at each age assuming that individuals are immune for the first 6 months of life and are then susceptible. Refit the model to estimate the force of infection in the UK and China and plot a graph of model predictions and observed data.

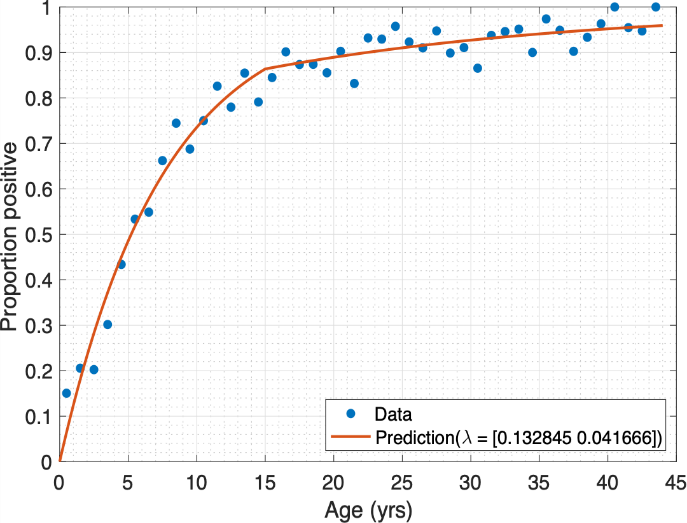
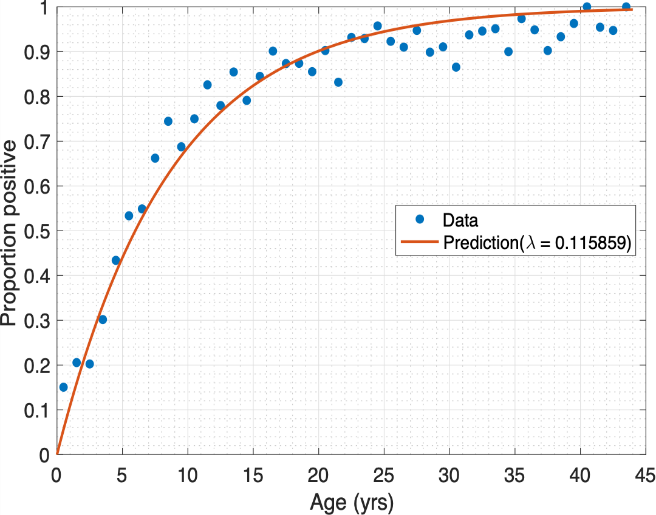
* UK
  + Force of infection
    - Value : 0.120859
    - Confidence interval : [0.115602 0.126258]
* China



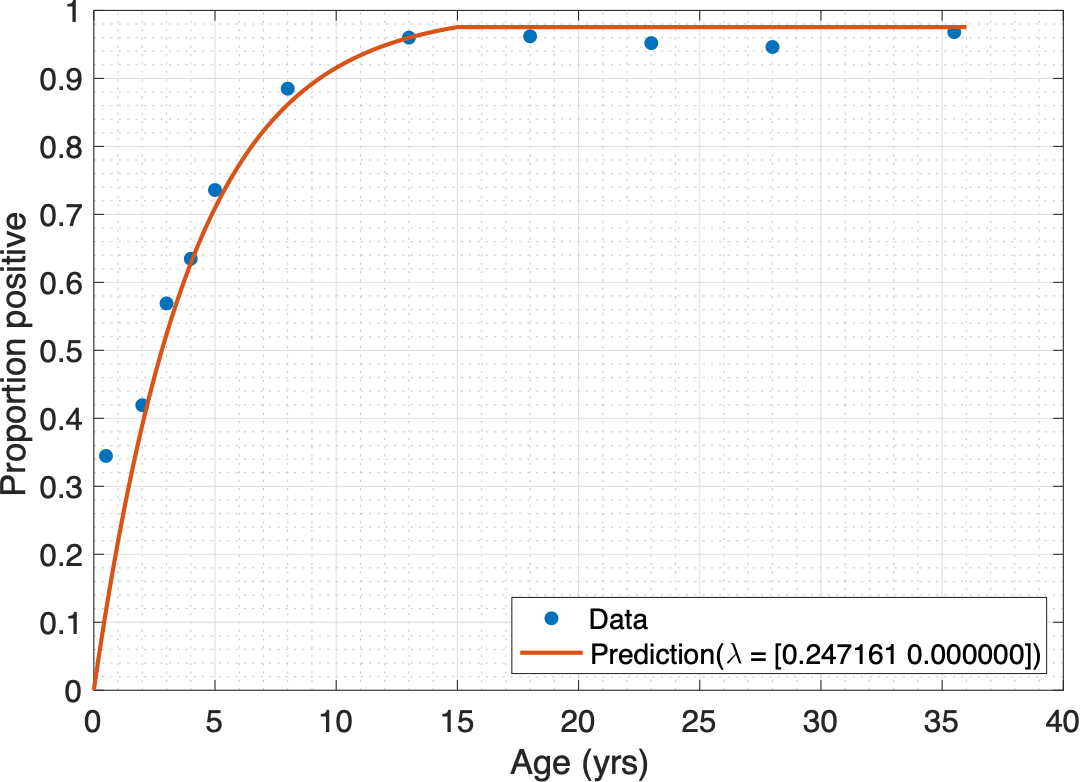
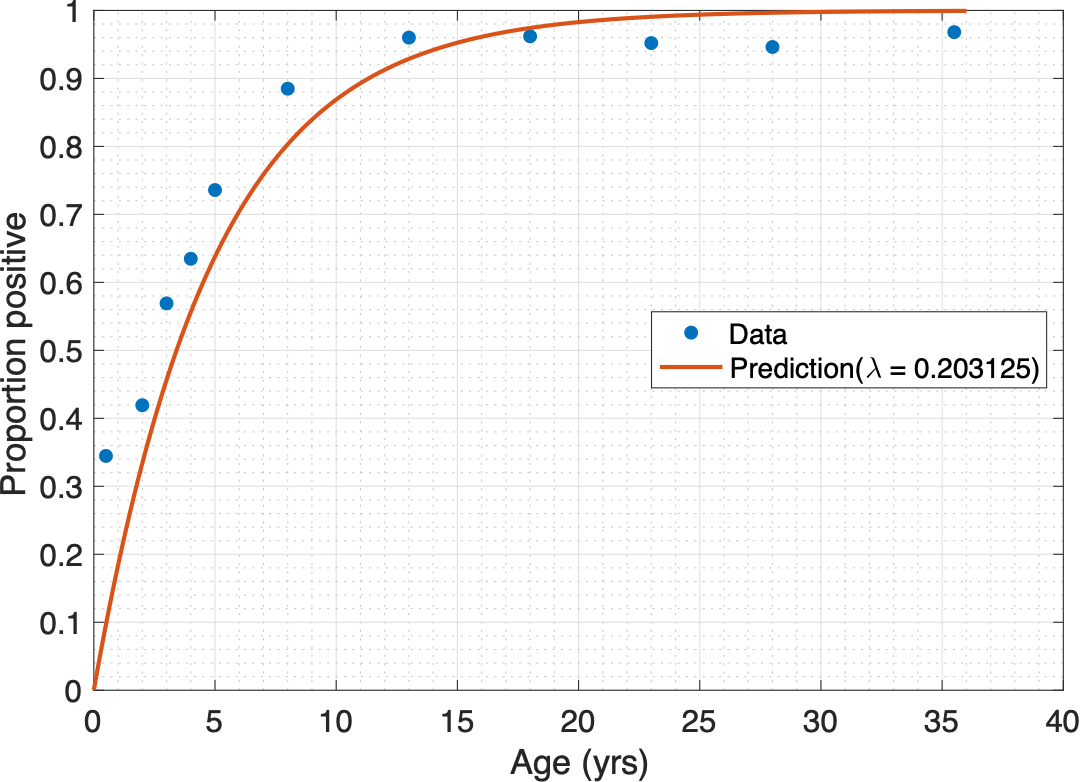
* + Force of infection
    - Value : 0.288359
    - Confidence interval : [0.280009 0.296993]

1. Estimate the age-specific forces of infection and 95% confidence interval using 2 age groups for the UK and China. Compare the graphs of model predictions and observed data using constant force of infection and age-specific forces of infection.

* UK



* + Force of infection
    - Value : [0.132845, 0.041666]
    - Confidence interval : [0.124470 0.141629] and [0.023139 0.061888]
* China



* + Force of infection
    - Value : [0.247161, 7.9058e-11]
    - Confidence interval : [0.240283 0.254252] and [0 2.188769e-3]

**PART Ⅱ: Fitting transmission model to prevalence data to estimate the transmission rate using MLE**

Set up the SEIR model of the transmission dynamics of measles as follows:

Population 100000 people

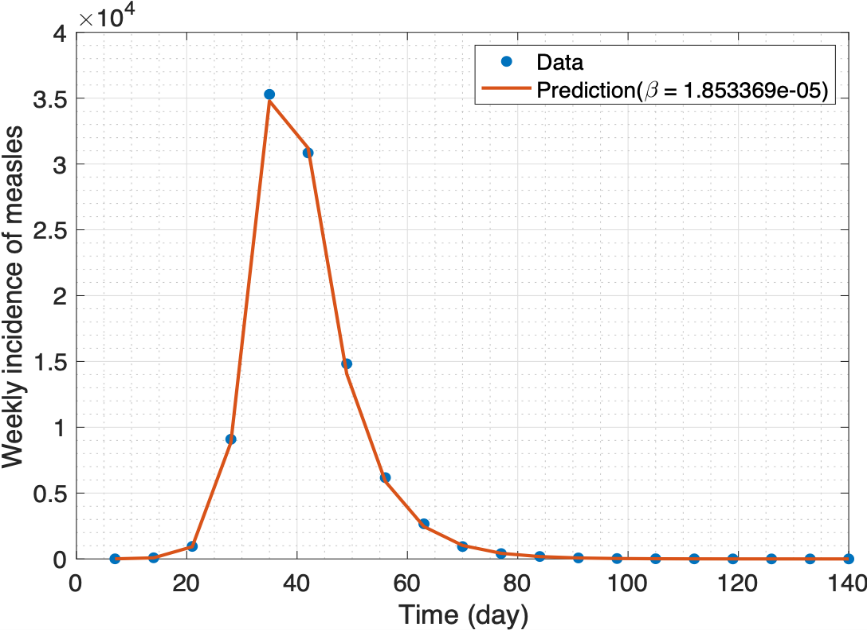
Pre-infectious period 8 days

Infectious period 7 days

Initial values (S,E,I,R)=(99999,0,1,0)

Fit the SEIR model to prevalence data to estimate the transmission rate using MLE in which Poisson distribution is assumed for the data (“prevalence\_measles”):

1. What is the best-fitting value for the transmission rate and the 95% confidence interval? Plot a graph of model predictions and observed data.



* Best-fitting value of force of infection : 1.853369e-05
* 95% confidence interval : [1.849278e-05 1.857518e-05]

1. Calculate R0 and herd immunity threshold with 95% confidence interval.

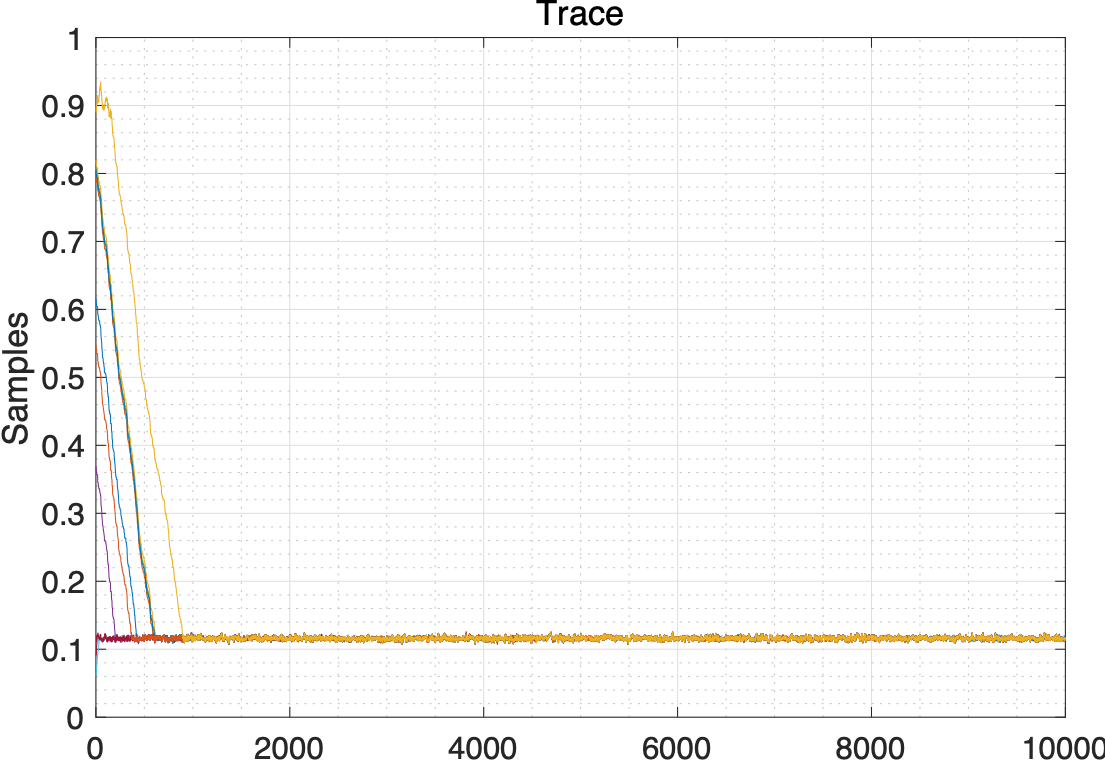
* R0
  + Value : 12.973454
  + Confidence interval : [12.944819 13.002493]
* Herd immunity threshold
  + Value : 0.922920
  + Confidence interval : [0.922749 0.923092]

**PART Ⅲ: Fitting the catalytic model to seroprevalence data to estimate the force of infection using Bayesian inference**

Ignoring the maternal antibodies, fit the catalytic model to the UK data (“seroprevalence\_uk”) to estimate the force of infection using MLE in which Binomial distribution is assumed for the data.

1. Assuming the uniform prior, find the distribution of the force of infection using Metropolis-Hastings algorithm to implement MCMC. Graph the trace plot, distribution and autocorrelation for the force of infection using different initial values and number of iterations.

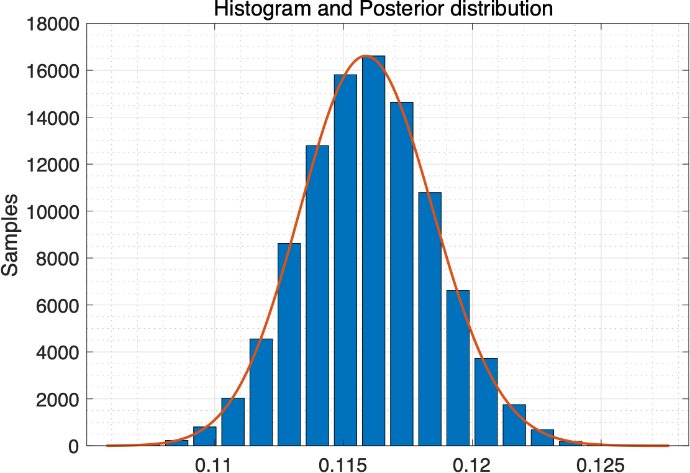
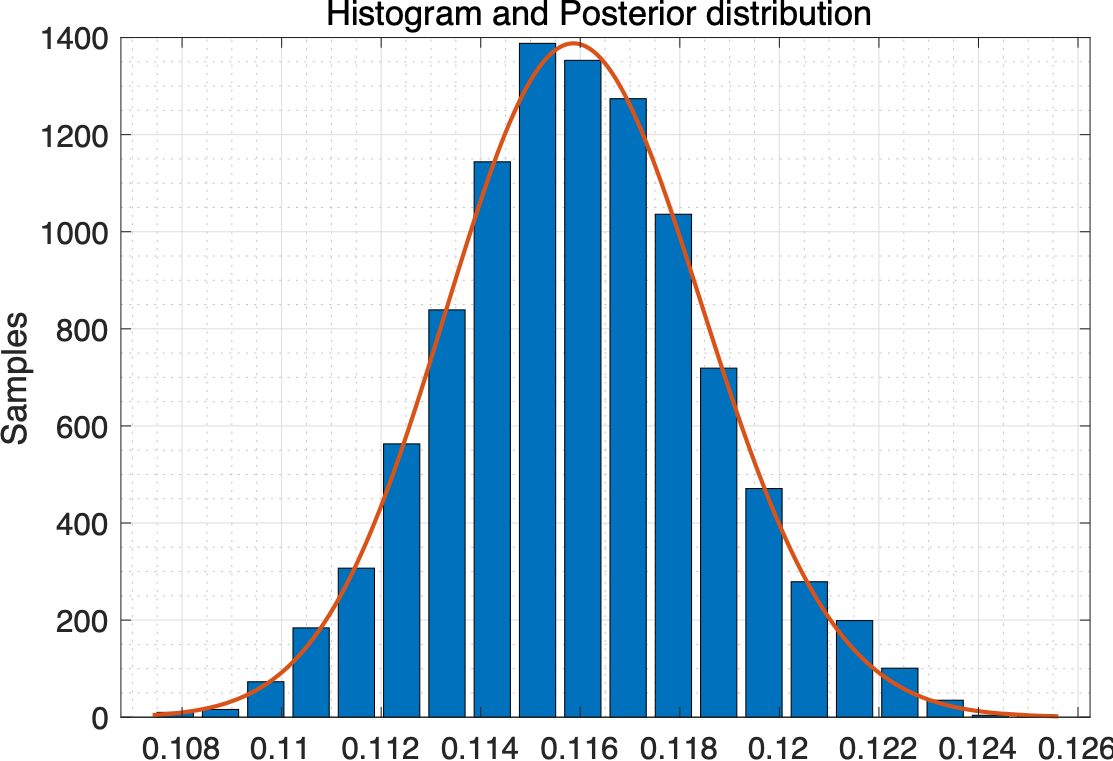
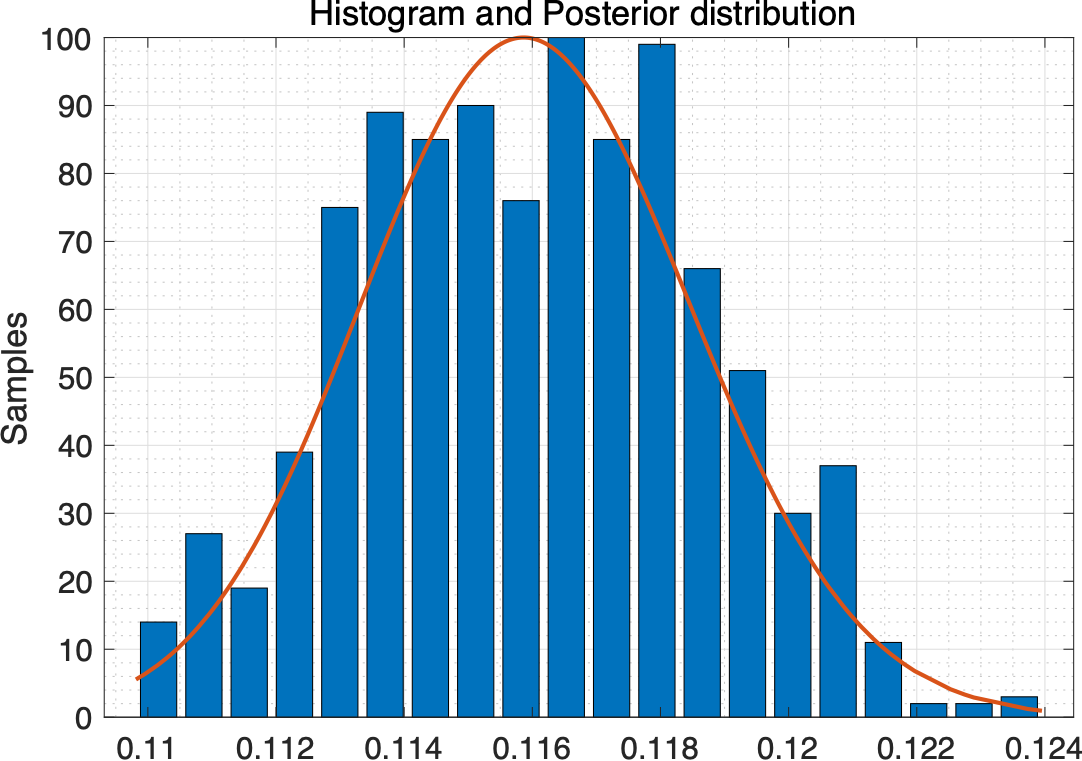
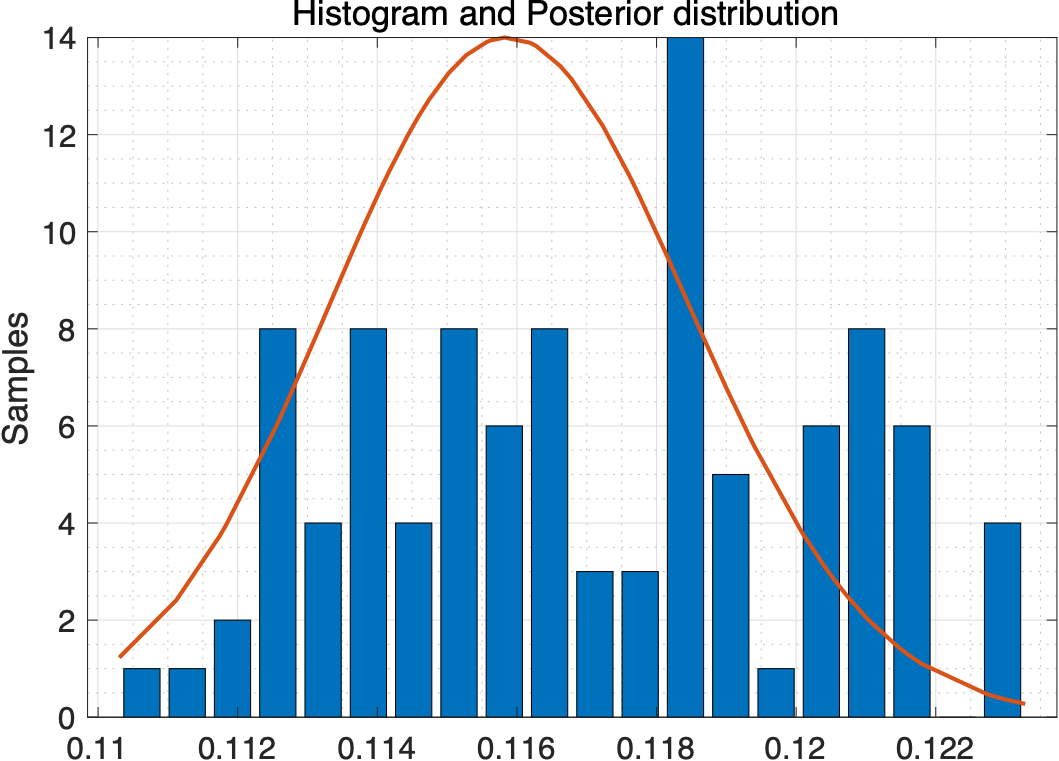
* For different initial values - Number of samples : 10,000 and no burn-in



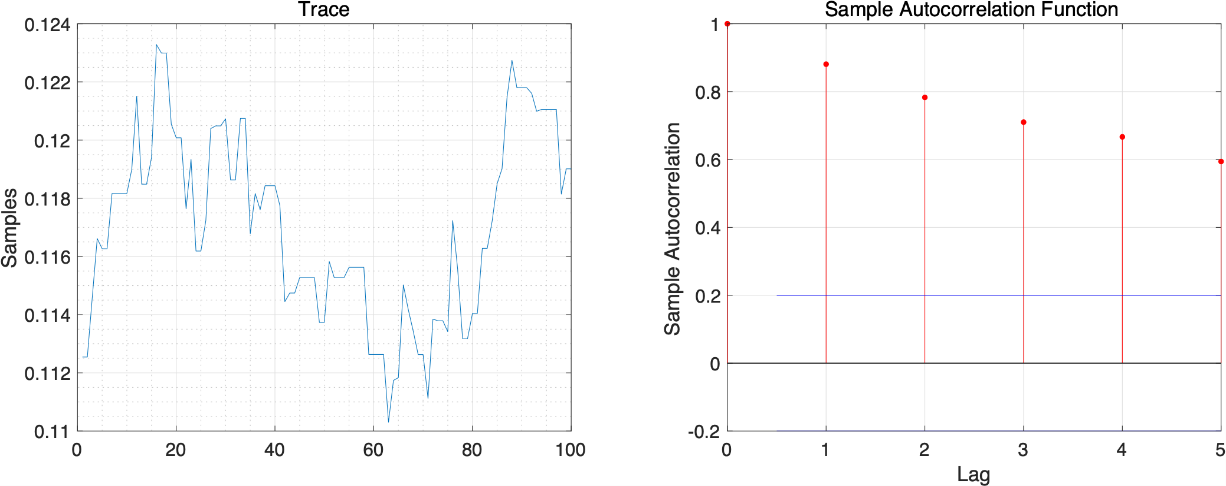
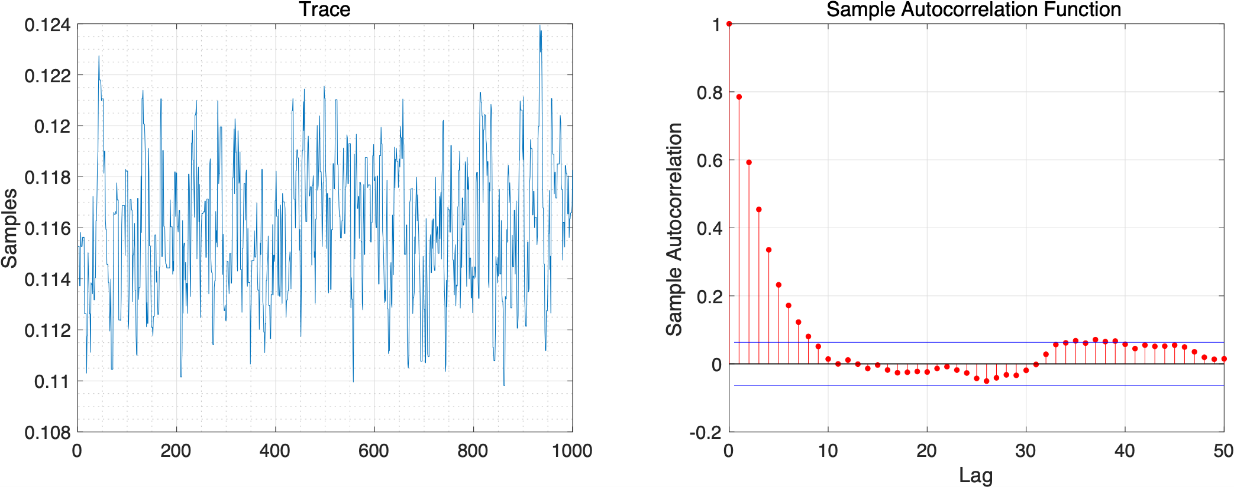
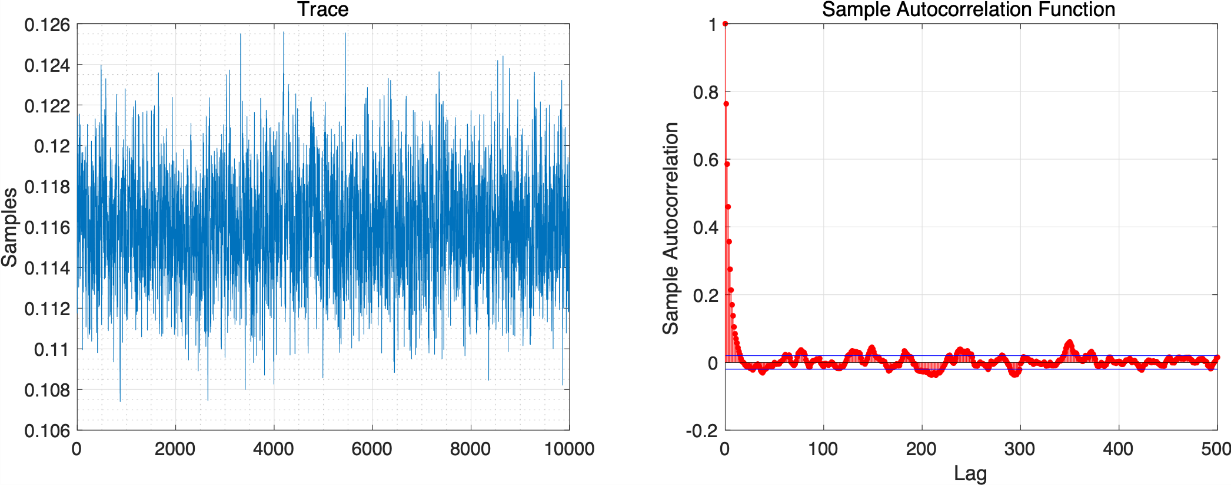
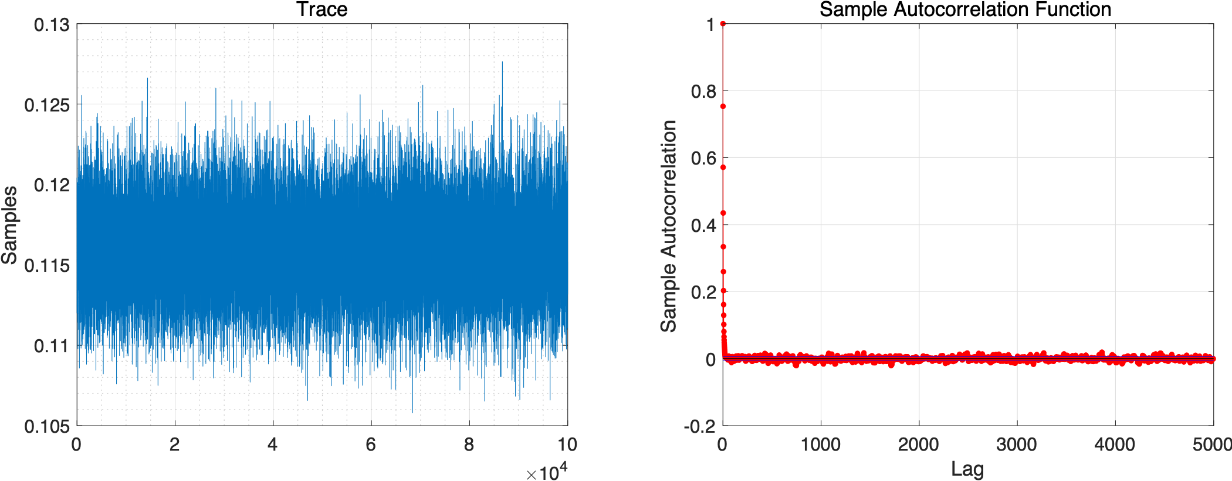
* For different number of iterations –

Number of samples : 100, 1,000, 10,000, 100,000 and burn-in : first 5%

* + Posterior distribution

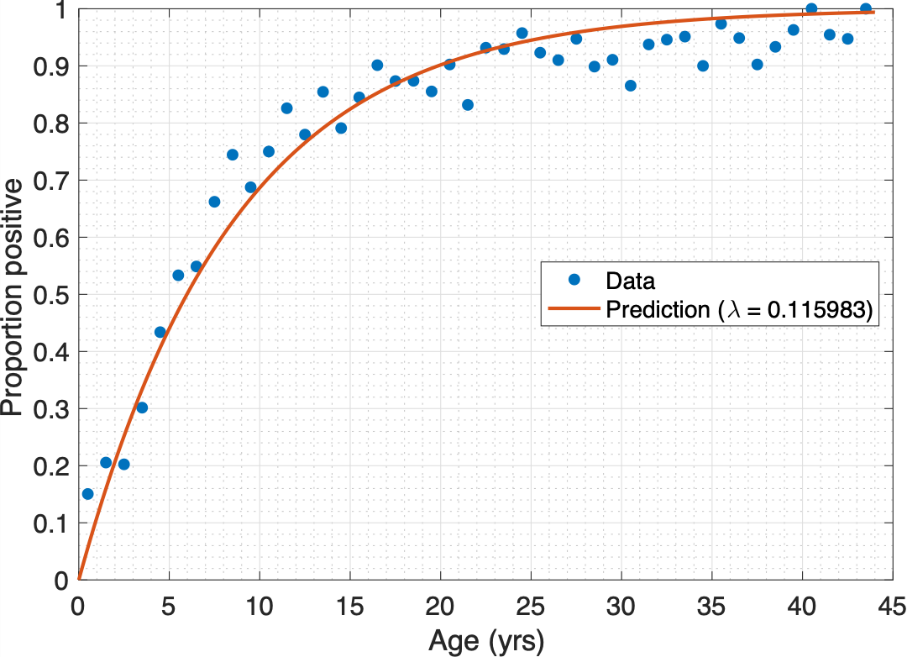


* + Trace and Autocorrelation



1. Calculate the best-fitting force of infection, the average age at infection, the R0 (assuming that the life expectancy is 60 years) and herd immunity threshold with 95% credible interval.

* Plot for a data and prediction plot using best-fitted value

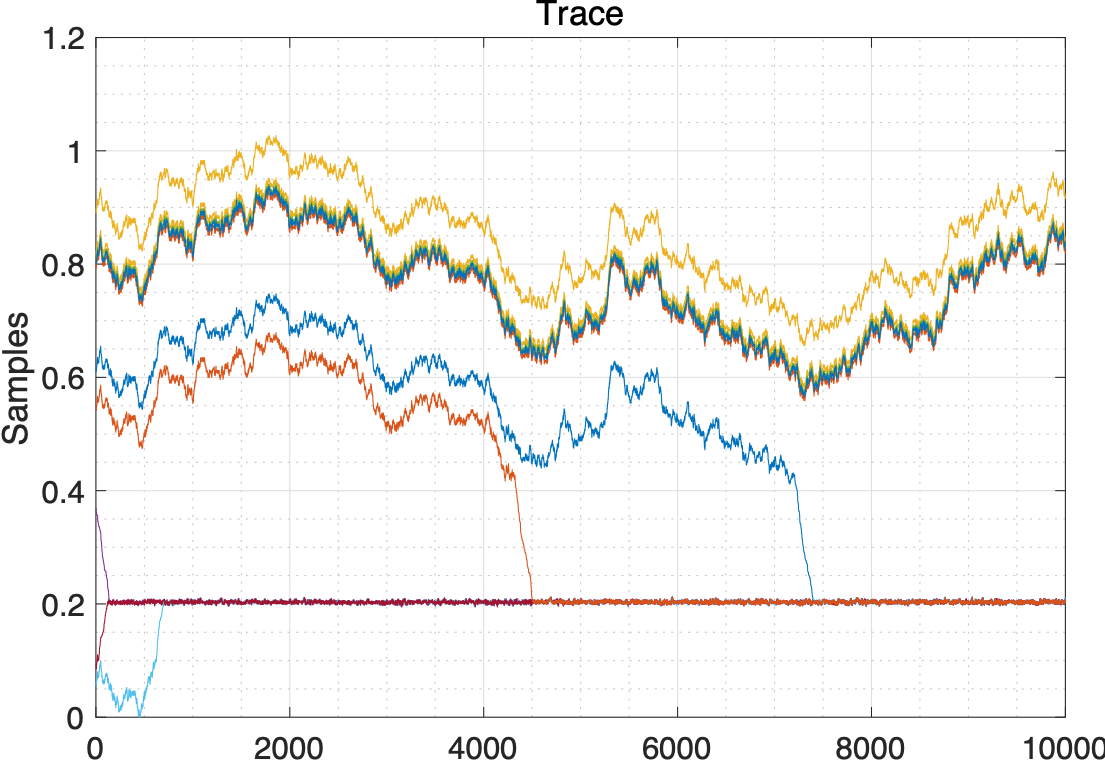


* Force of infection
  + Value : 0.115983
  + Confidence interval : [0.110966 0.121159]
* Average age of infection
  + Value : 8.621953
  + Confidence interval : [8.253598 9.011800]
* R0
  + Value : 6.958981
  + Confidence interval : [6.657937 7.269557]
* Herd immunity threshold
  + Value : 0.856301
  + Confidence interval : [0.849803 0.862440]

Fit the catalytic model to China data (“seroprevalence \_china”) to estimate the force of infection:

1. Graph the trace plot, distribution and autocorrelation for the force of infection.

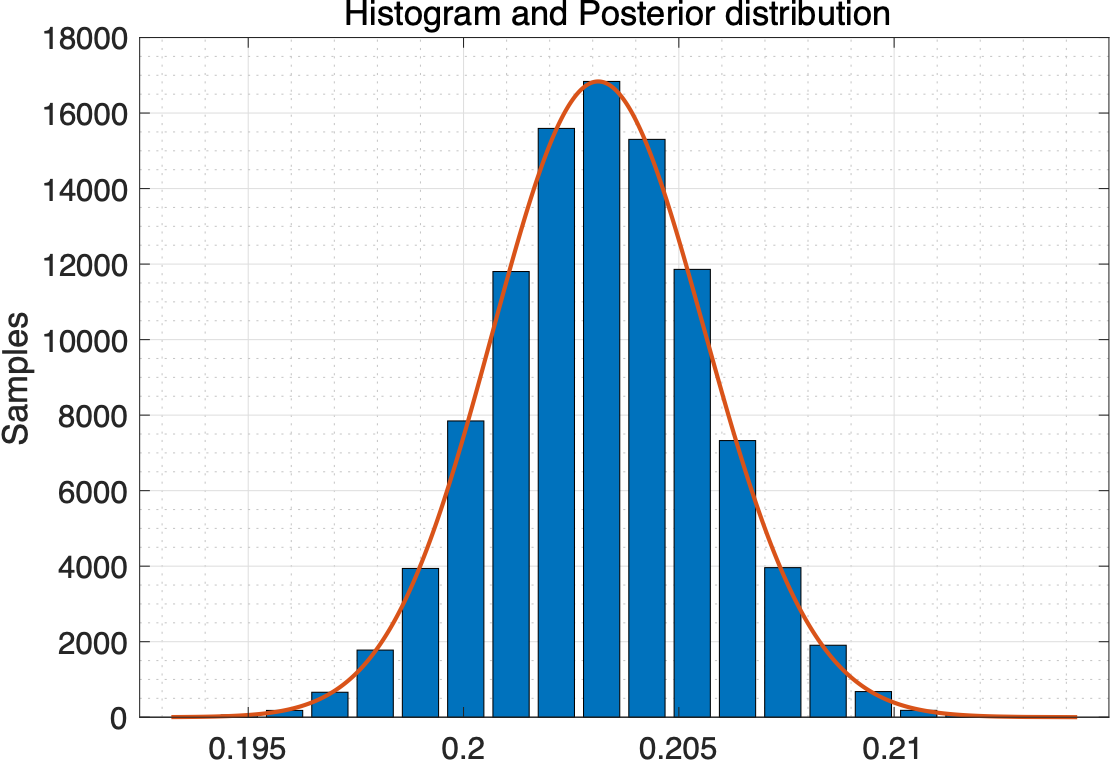
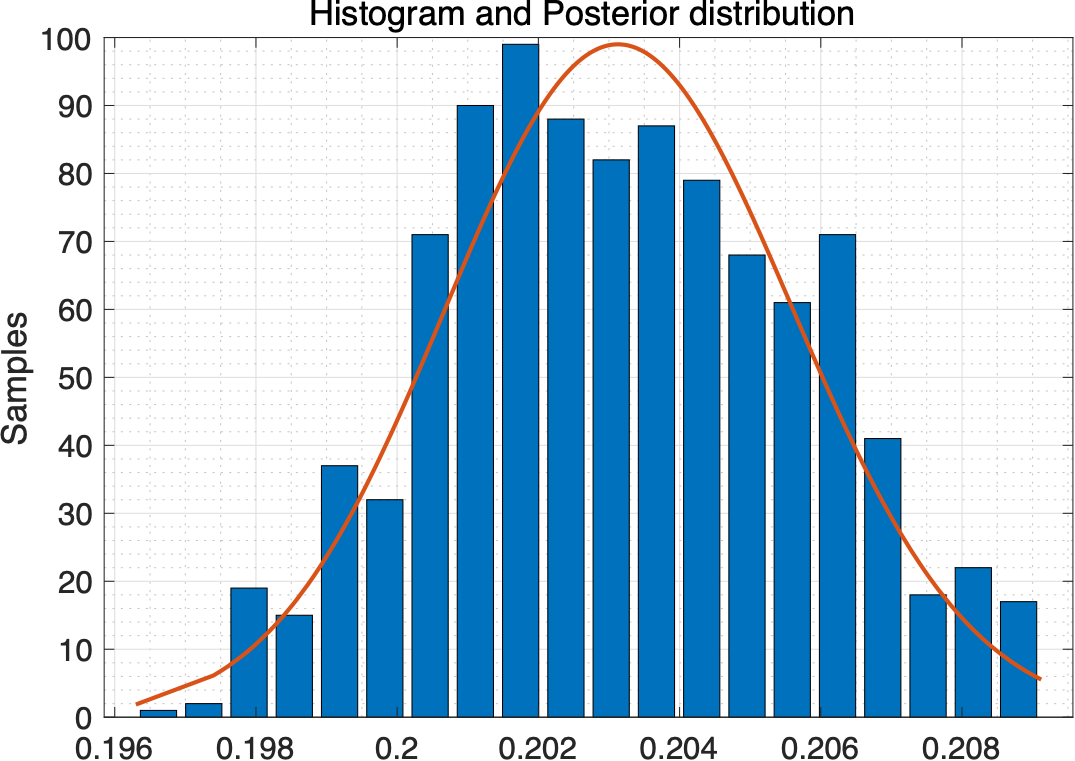
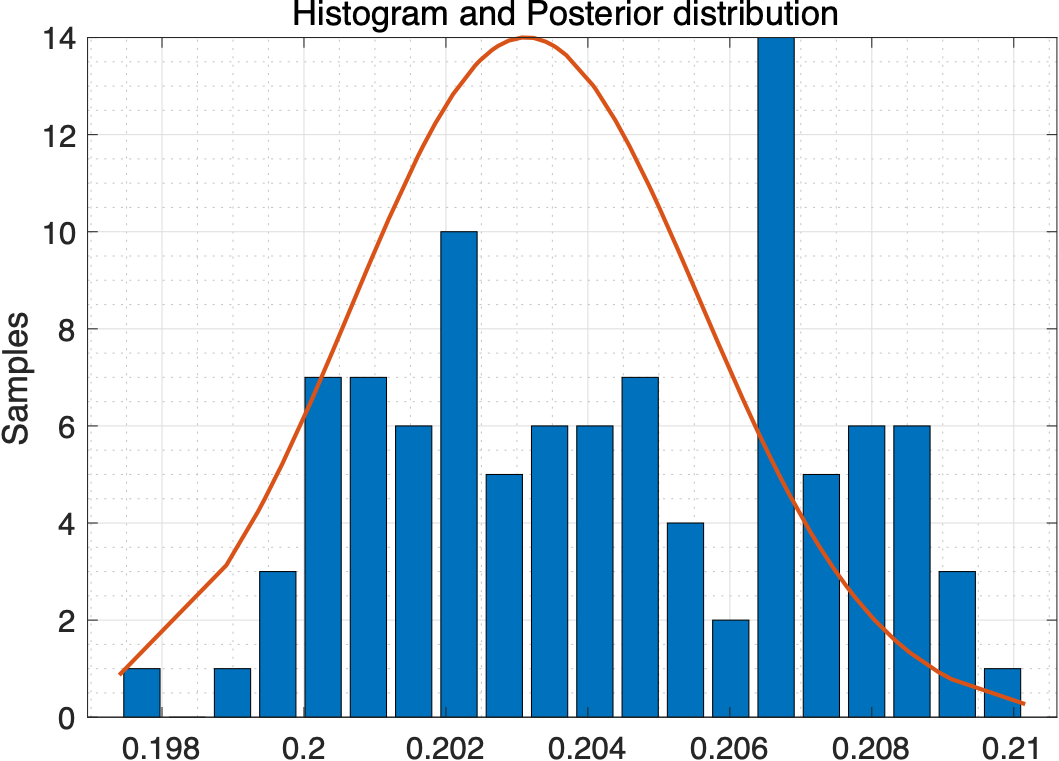
* For different initial values - Number of samples : 10,000 and no burn-in



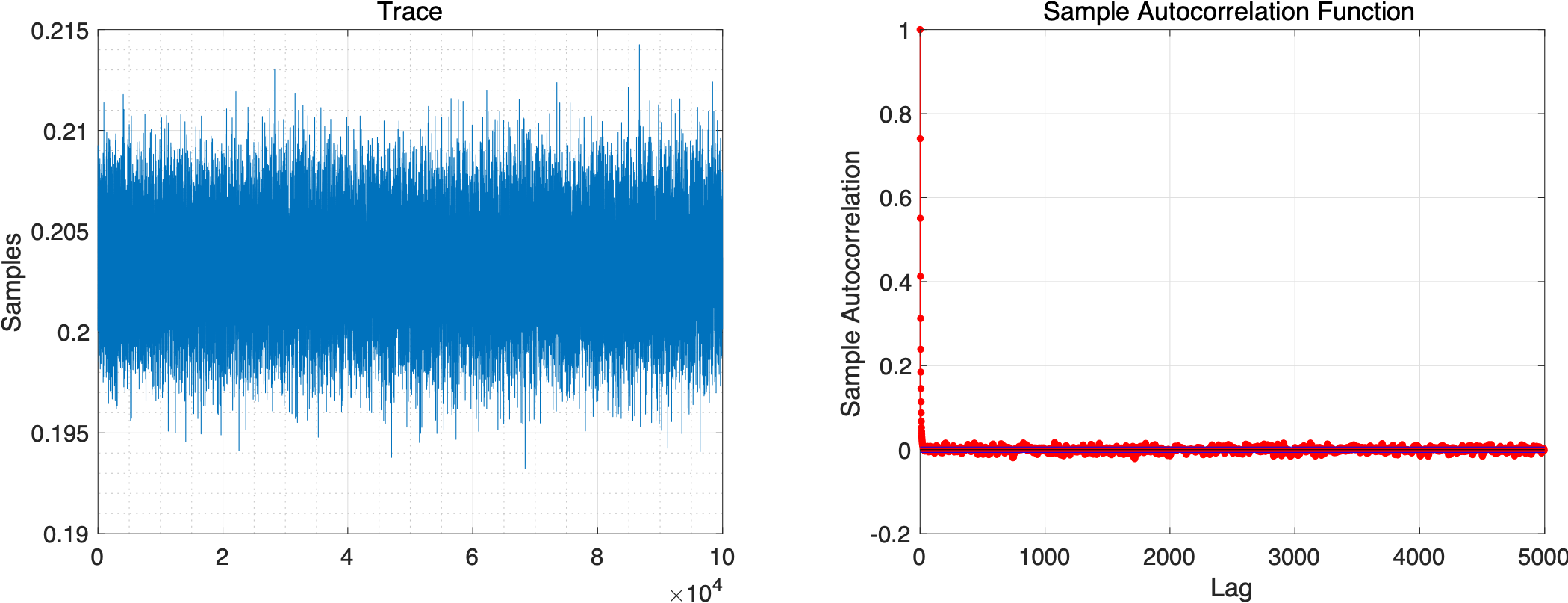
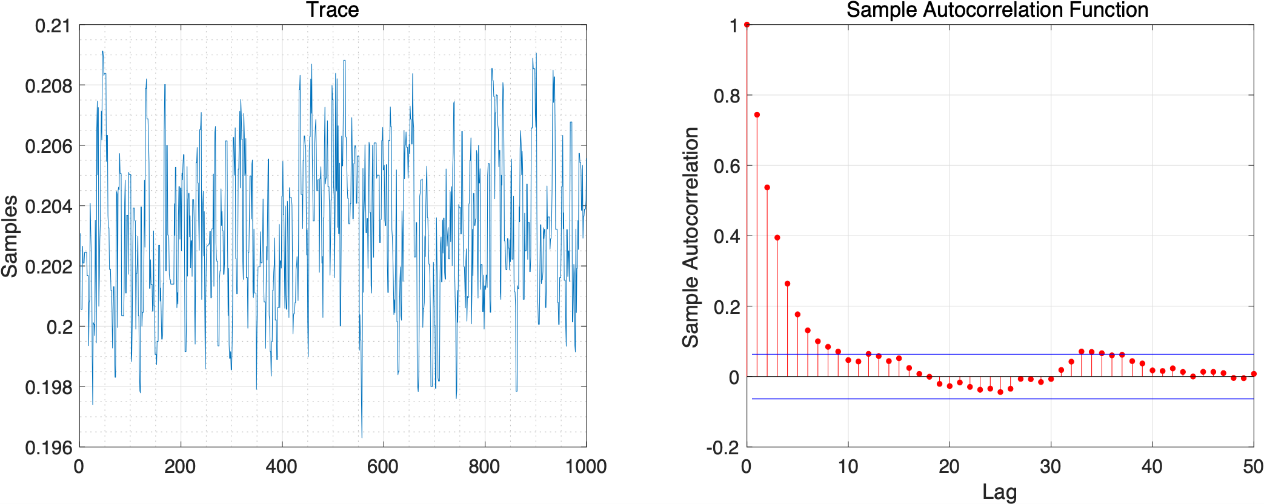
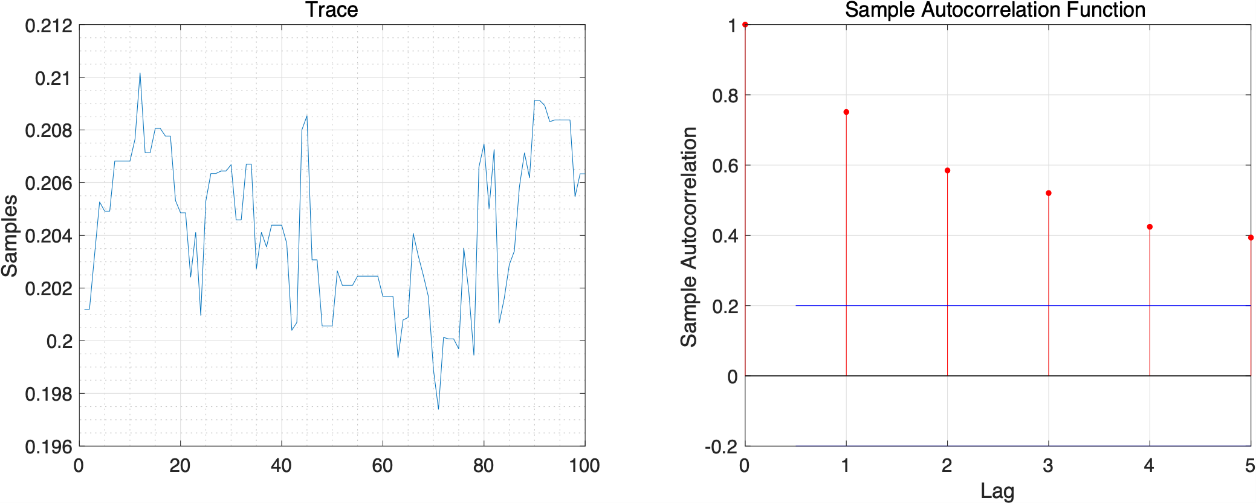
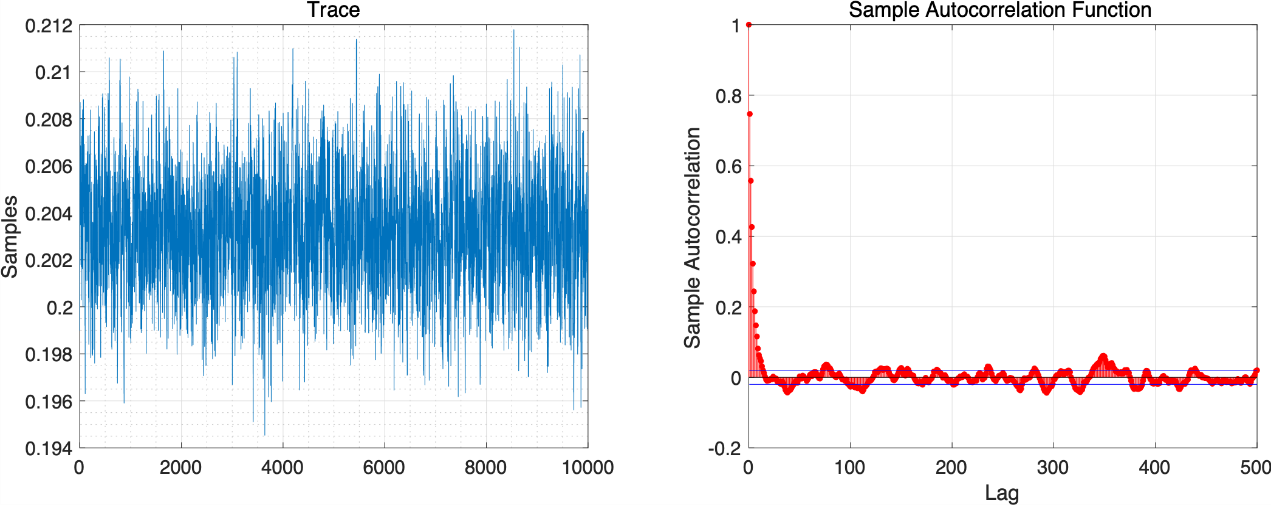
* For different number of iterations –

Number of samples : 100, 1,000, 10,000, 100,000 and burn-in : first 5%

* + Posterior distribution



* + Trace and Autocorreation



1. Calculate the best-fitting force of infection, the average age at infection, the R0 (assuming that the life expectancy is the same as that in the UK) and herd immunity threshold with 95% credible interval.

* Plot for a data and prediction plot using best-fitted value



* Force of infection
  + Value : 0.203196
  + Confidence interval : [0.198385 0.208067]
* Average age of infection
  + Value : 4.921352
  + Confidence interval : [4.806145 5.040710]
* R0
  + Value : 12.191772
  + Confidence interval : [11.903086 12.484017]
* Herd immunity threshold
  + Value : 0.917977
  + Confidence interval : [0.915988 0.919898]

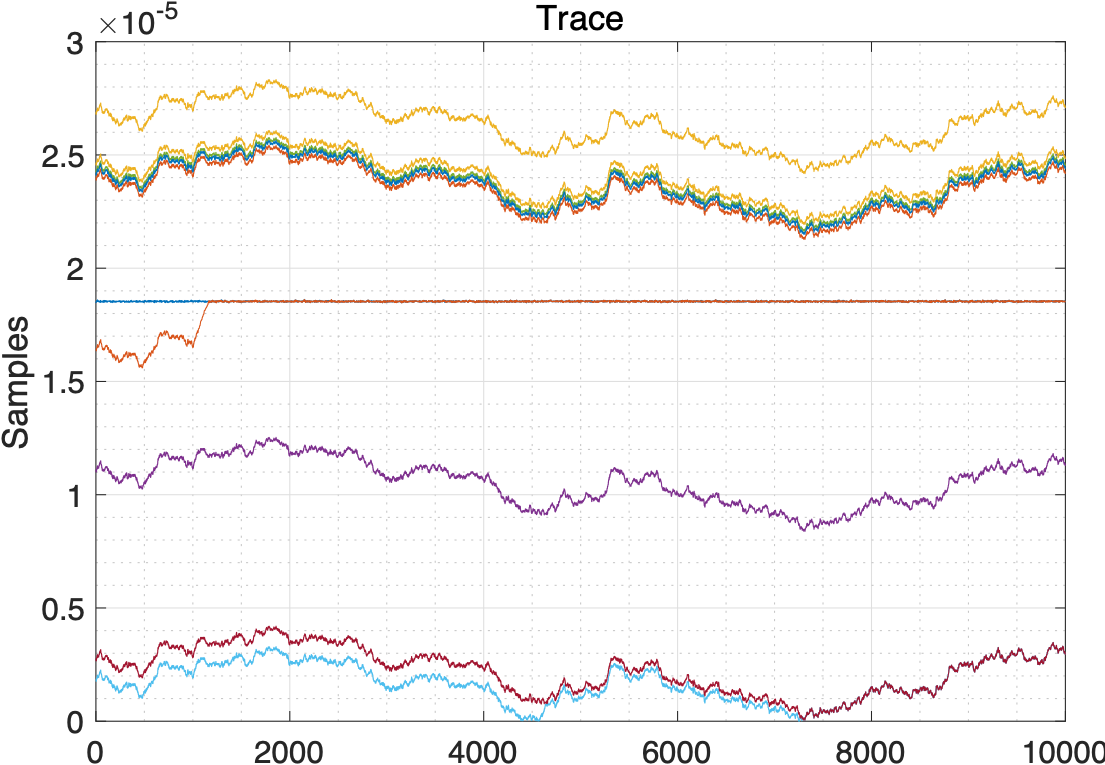
**PART Ⅳ: Fitting transmission model to prevalence data to estimate the transmission rate using Bayesian inference**

Set up the SEIR model of the transmission dynamics of measles as in PART Ⅱ:

Fit the SEIR model to prevalence data to estimate the transmission rate using Bayesian in which Poisson distribution is assumed for the data (“prevalence\_measles”):

1. Graph the trace plot, distribution and autocorrelation for the force of infection.

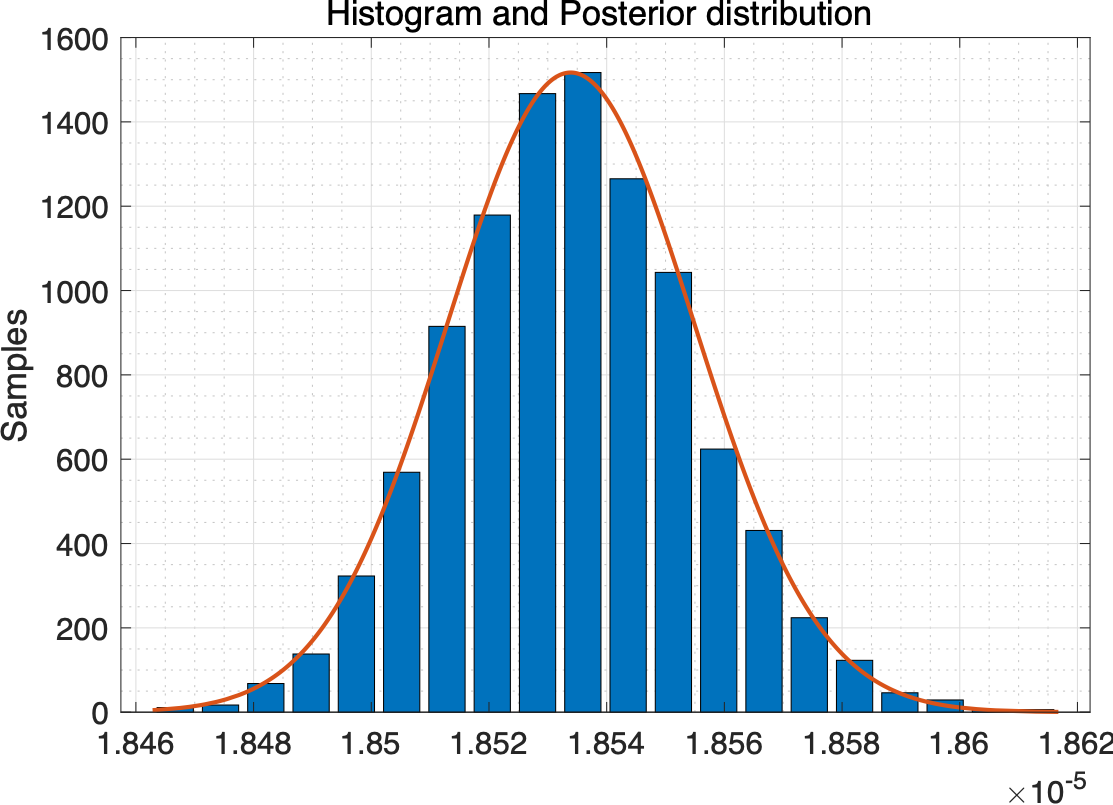
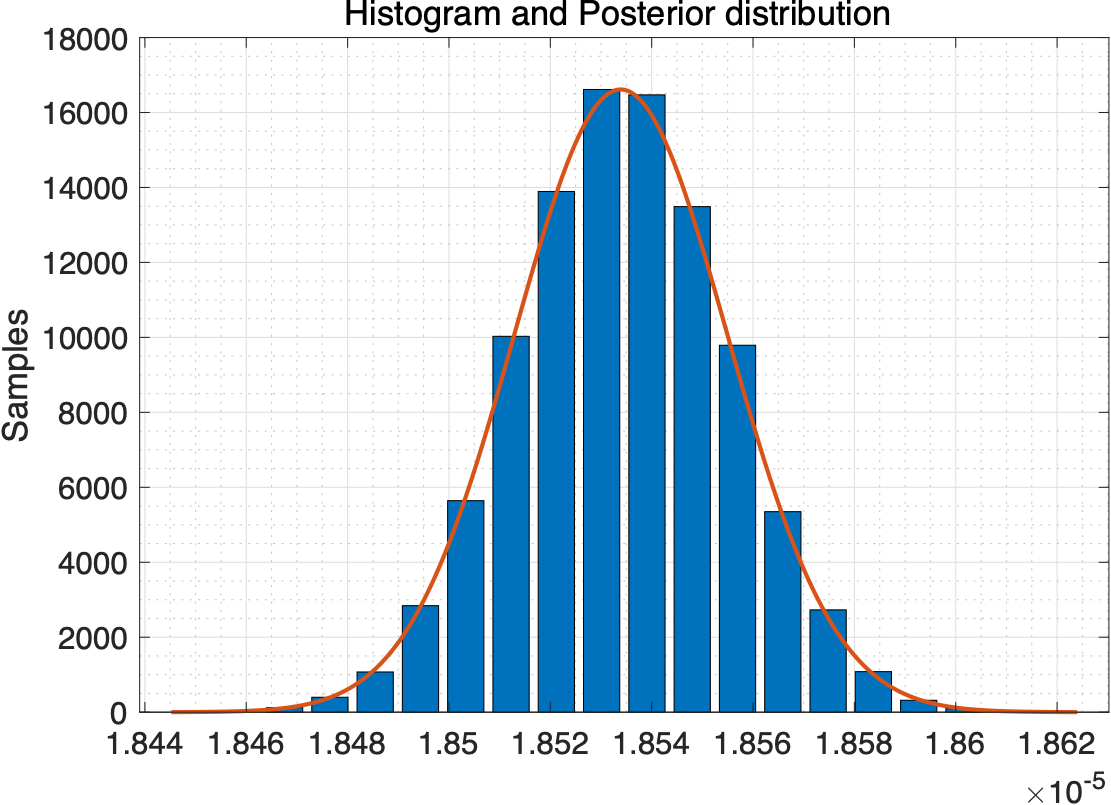
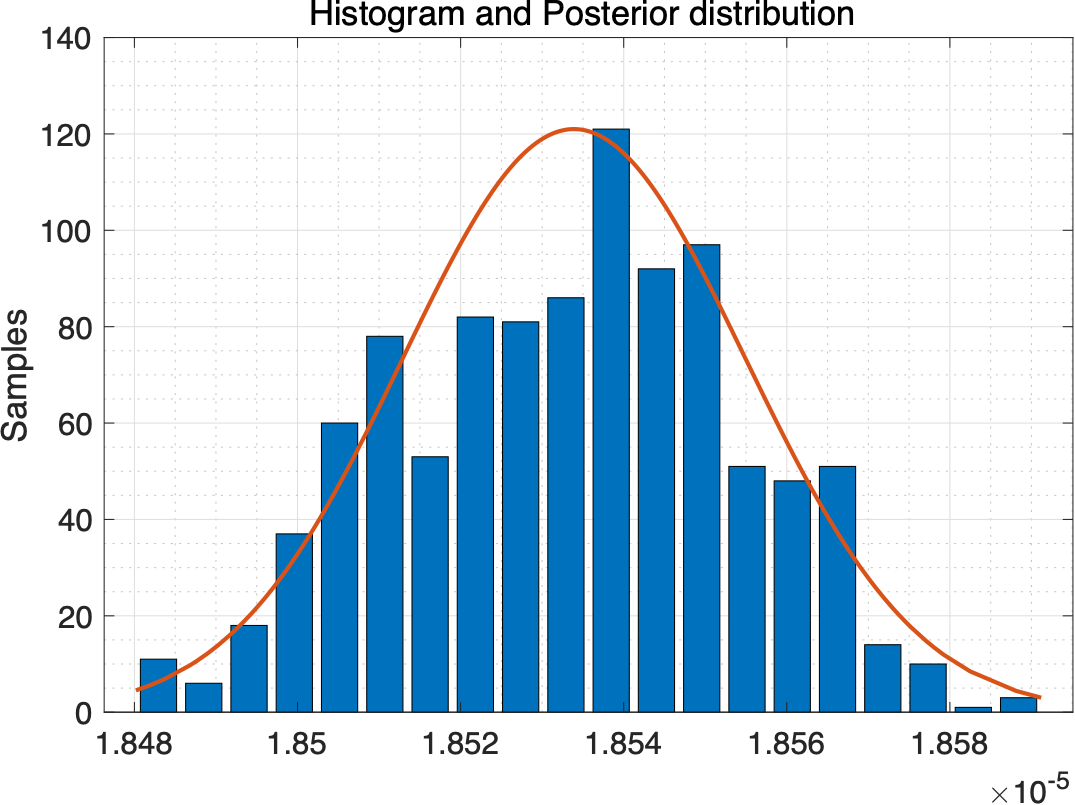
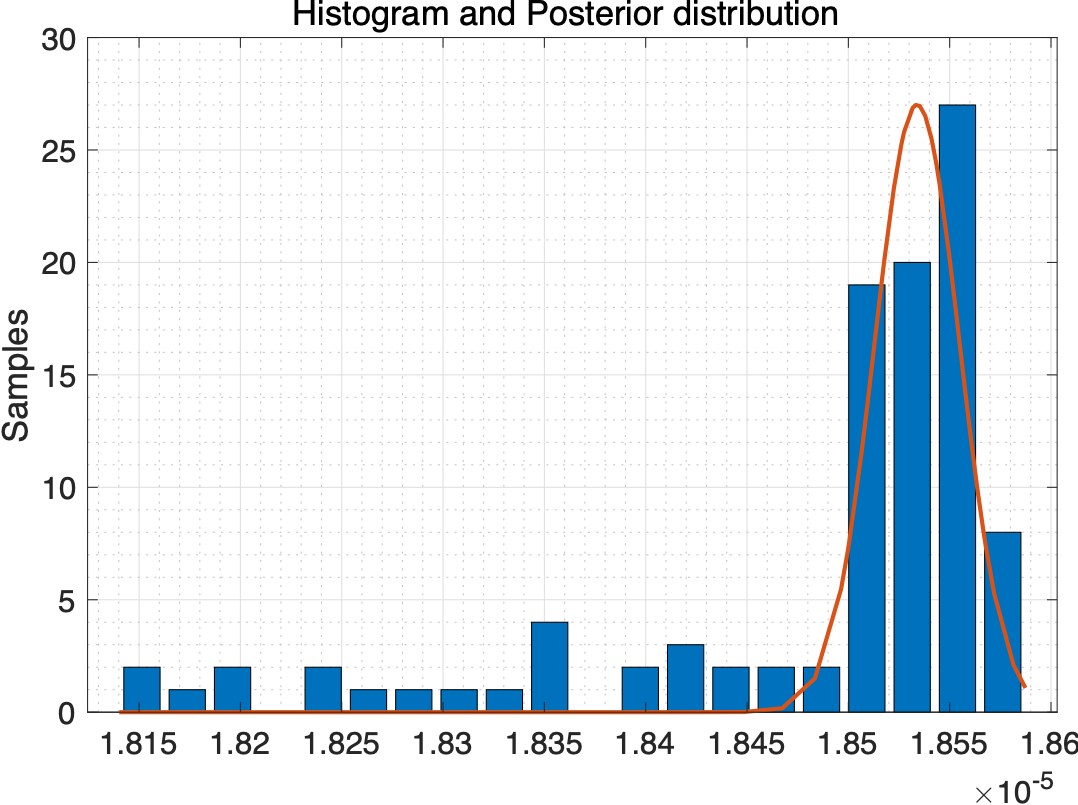
* For different initial values - Number of samples : 10,000 and no burn-in



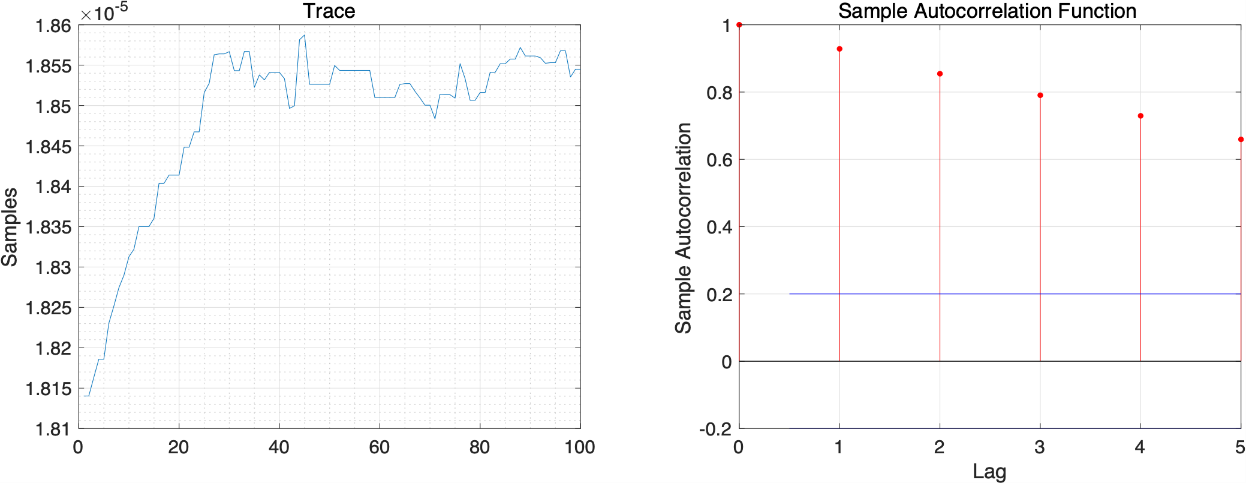
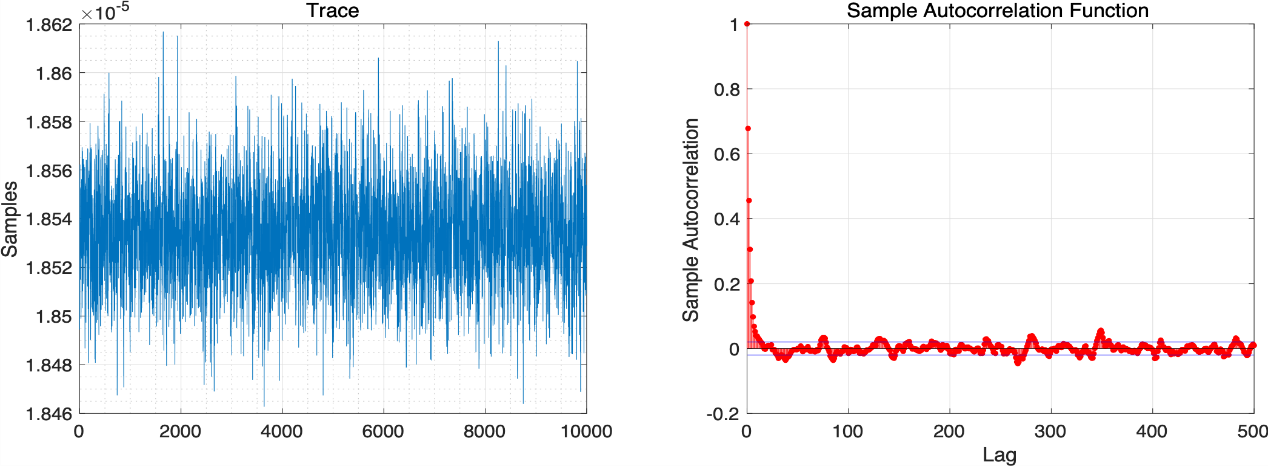
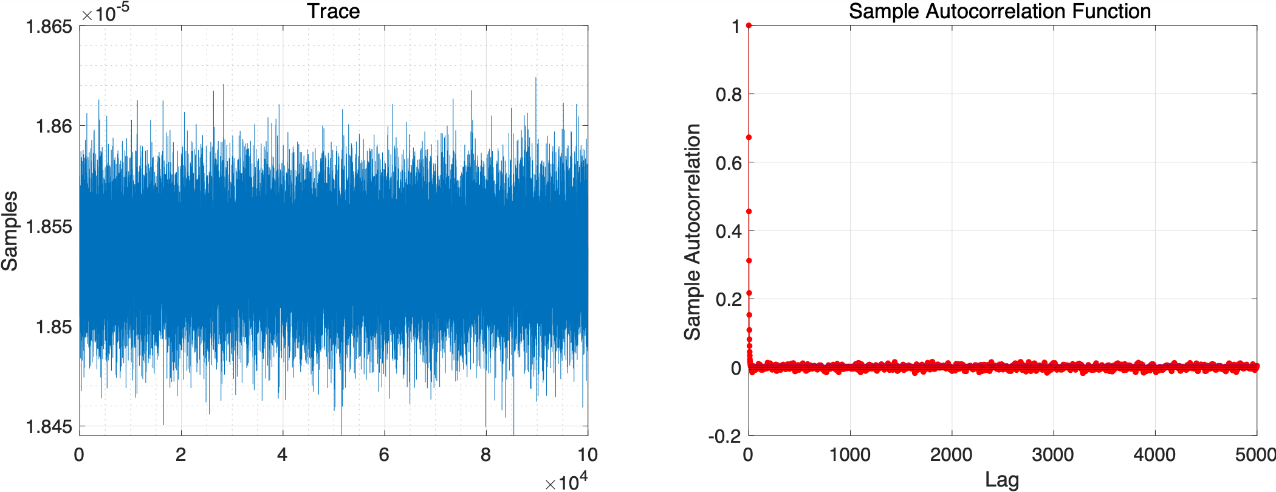
* For different number of iterations –

Number of samples : 100, 1,000, 10,000, 100,000 and burn-in : first 5%

* + Posterior distribution

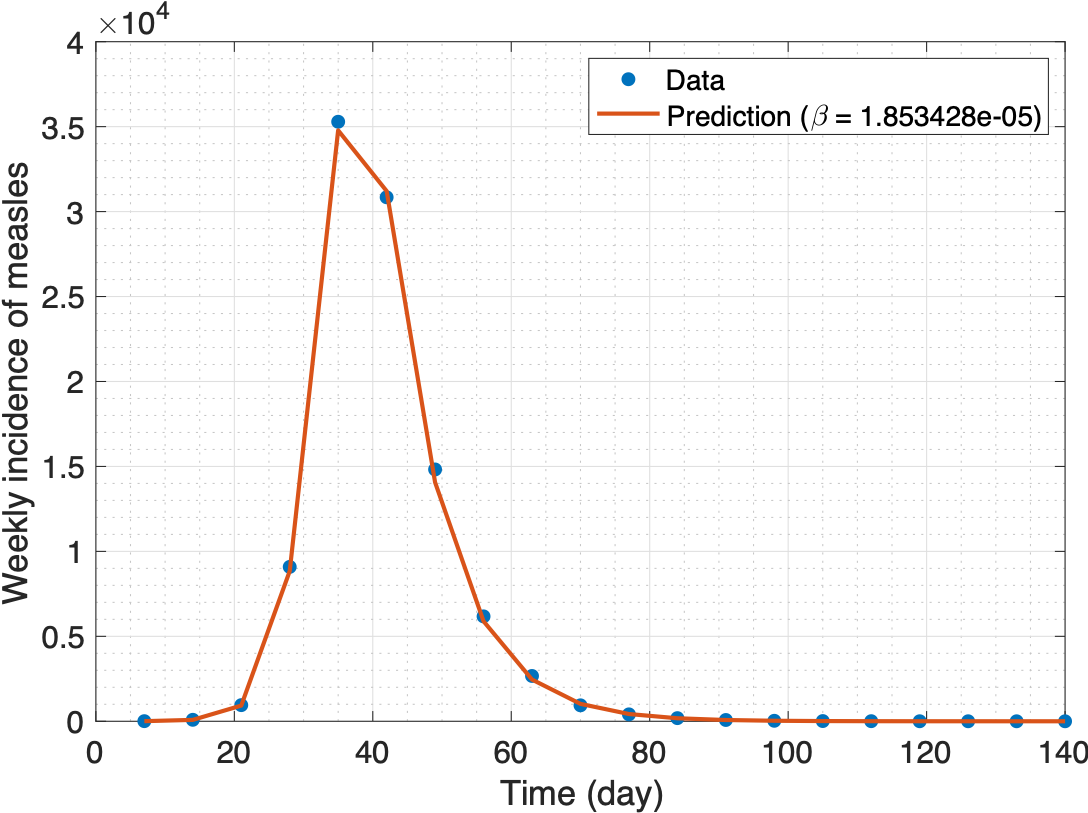


* + Trace and Autocorrelation



1. Calculate the best-fitting transmission rate, R0 and herd immunity threshold with 95% credible interval.

* Plot for a data and prediction plot using best-fitted value



* Force of infection
  + Value : 1.853428e-05
  + Confidence interval : [1.849349e-05 1.857508e-05]
* R0
  + Value : 12.973866
  + Confidence interval : [12.945317 13.002429]
* Herd immunity threshold
  + Value : 0.922922
  + Confidence interval : [0.922752 0.923091]