Mathematical Epidemiology

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Overview

Basic Theories (21.09 - 22.02)
- Compartmental Models
- Others

COVID-19 Model (22.01)
- Model
- Data
- Result

Mathematical Analysis (22.03 - 22.04)

Compartmental Models

Deterministic Models

Difference Equation Models

- SEIR model
- Time step size

Differential Equation Models

- SEIR model

Natural Dynamics

- Epidemic cycles

Data Fitting

Analyze Data

- Force of infection
- Average age of infection
  - Age-independent model
  - Age-dependent model
- Proportion susceptible
- Basic reproduction number
  - Herd immunity

Methods

- Least square method (LSM)
- Maximum likelihood estimation (MLE)

Heterogeneous Mixing

- WAIFW matrix
- Next generation matrix

Stochastic Models

DTMC

- SIS model
- SIR model

CTMC

- SIS model
- SIR model

SDE

- SIS model
- SIR model

Others

Others

- Economic Evaluation
  - Cost-effectiveness analysis
  - Cost-benefit analysis
- Sensitivity Analysis
  - Grid search
  - Random sampling
- Agent-Based Model (ABM)

Model

Diagram of Model

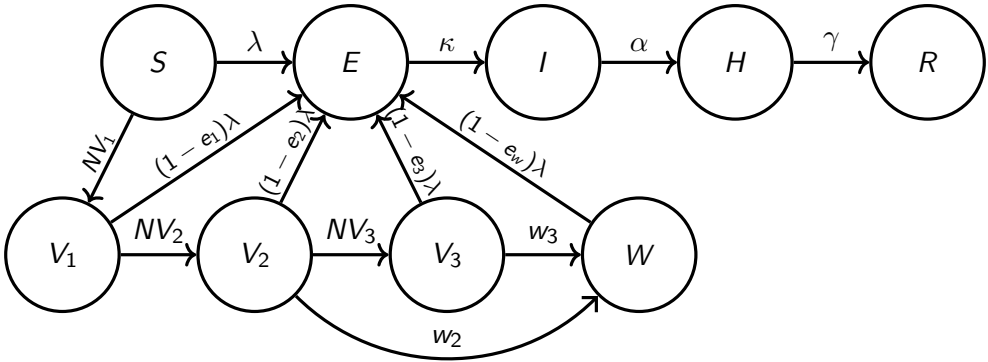

Figure: Diagram of Model

$\lambda(t) = (p_\alpha(t) + p_\delta(t) \times \delta + p_o(t) \times o) \times \beta \times SD(t) \times C \times I(t)$, force of infection

Compartments

Notation	Description
S	Number of susceptible individuals
E	Number of exposed individuals
I	Number of infectious individuals
H	Number of hospitalized individuals
R	Number of recovered (or removed) individuals
V_i	Number of i-th dose vaccinated individuals
W	Number of vaccine waning individuals

Table: Definitions of states

Parameters

Parameter	Description	Value
λ	Force of infection	formula
$1/\kappa$	Average pre-infectious period	4
$1/\alpha$	Average infectious period	4
γ	Recovery rate	1/14
NV_i	Number of new vaccinated individuals	statistic data
e_j	Vaccine efficacy of j-th dose	data
e_w	Vaccine efficacy for W	0.335
$1/w_i$	Vaccine waning period of V_i	data

Table: Definition of parameters

Parameters

Parameter	Description	Value	Reference
$p_*(t)$	Proportion of each variant	data	[1]
δ	Relative infectivity of delta variant	2.9033	model fitting
o	Relative infectivity of omicron variant	0.6614	model fitting
β	Proportionality of transmission rate	0.0425	model fitting
$SD(t)$	Effect of social distancing	40%	model fitting
C	Contact matrix	data	survey

Table: Parameters of λ formula

Data

Daily New Cases Data

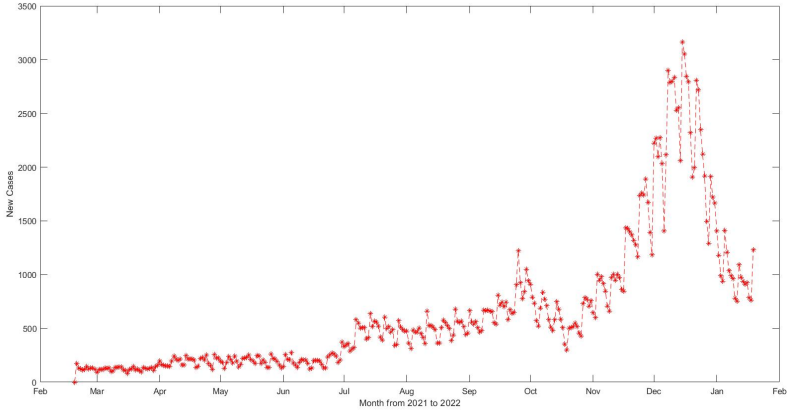

Figure: Daily new cases data

Daily Vaccinated Data

Give NV_1, NV_2, NV_3

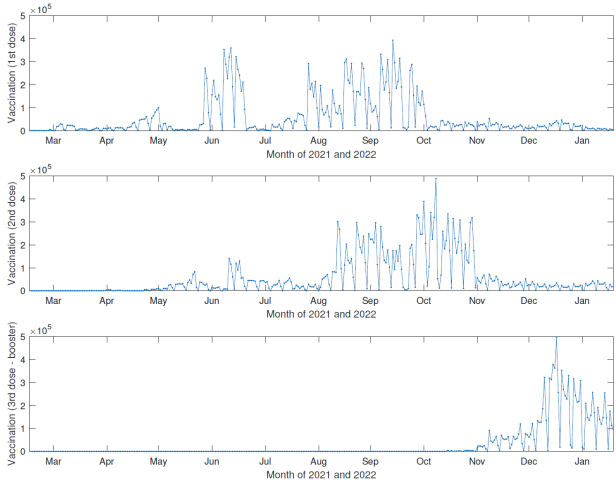

Figure: Daily vaccinated data

Efficacy of Vaccines for Different Variants

	Dose	AstraZeneca	Pfizer	Moderna
α variant	1st	48.7%	47.5%	74.5%
	2nd	74.5%	93.7%	86.3%
	3rd	74.5%	93.7%	86.3%
δ variant	1st	30.0%	35.6%	55.6%
	2nd	67%	88%	79.8%
	3rd	(Pfizer booster) 93.8%	93.7%	94%
o variant	1st	0%	36.1%	20%
	2nd	5.9%	88%	42.8%
	3rd	(Pfizer booster) 71.4%	75.5%	67.7%

Table: Efficacy of Vaccines for Different Variants

Proportion of Variants

Give p_α, p_δ, p_o

yy/mm/w	α	δ	o	yy/mm/w	α	δ	o	yy/mm/w	α	δ	o
21/06/1	97.6	2.4	0	21/08/3	10.4	89.6	0	21/11/2	0.1	99.9	0
21/06/2	98.6	1.4	0	21/08/4	5.7	94.3	0	21/11/3	0	100	0
21/06/3	97.5	2.5	0	21/09/1	1.5	98.5	0	21/11/4	0.1	99.9	0
21/06/4	96.7	3.3	0	21/09/2	1.8	98.2	0	21/12/1	0	99.8	0.2
21/06/5	90.1	9.9	0	21/09/3	0.5	99.5	0	21/12/2	0	98.9	1.1
21/07/1	76.7	23.3	0	21/09/4	0.5	99.5	0	21/12/3	0	98.3	1.7
21/07/2	66.1	33.9	0	21/10/1	0.2	99.8	0	21/12/5	0	98.2	1.8
21/07/3	52.0	48.0	0	21/10/2	0	100	0	21/12/6	0	96.0	4.0
21/07/4	38.5	61.5	0	21/10/3	0.2	99.8	0	22/01/1	0	87.5	12.5
21/08/1	26.9	73.1	0	21/10/4	0.1	99.9	0	22/01/2	0	73.3	26.7
21/08/2	14.7	85.3	0	21/11/1	0	100	0	22/01/3	0	49.7	50.3

Table: Proportion of Variants (%)

After 22/01/4, o variant took 97%.

Vaccine Efficacy of j-th Dose

Give e_1, e_2, e_3

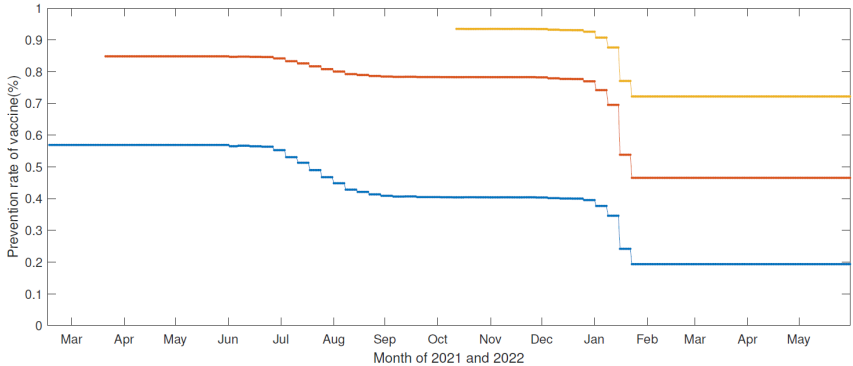

Figure: Vaccine efficacy of j-th dose

Calculated weighted sums by the data of proportion of variants.

Waning Vaccine Efficacy

Give w_1, w_2, and e_w

Dose	Period(Days)	Preventive Effect
2nd	210	22%
3rd	120	45%

Table: Average waning period of vaccine

- People who got first-doses of COVID-19 vaccine will get the second-dose in three to four weeks. Since the first-dose vaccine waned for 7 weeks, we did not assume the waning happens in V_1.
- $w_1 = 1/210$, $w_2 = 1/120$, and $e_w = 0.335$

Social Distance

Social Distance $SD(t)$

- 1 level increase: transmission rate decreases 32%.
- 1 level decrease: transmission rate increases 40%.

Date	Level	Transmission Rate
02.15 - 06.30	2	
07.01 - 07.11	1	$\beta \times 1.4$
07.12 - 10.31	4	$\beta \times 1.4 \times (0.68)^3$
11.01 - 12.17	3	$\beta \times (1.4)^2 \times (0.68)^3$
12.18 - 01.28	2	$\beta \times (1.4)^2 \times (0.68)^4$

Table: 2021.02.15 - Social distance level and change of transmission rate

Result

Best Fitting Result

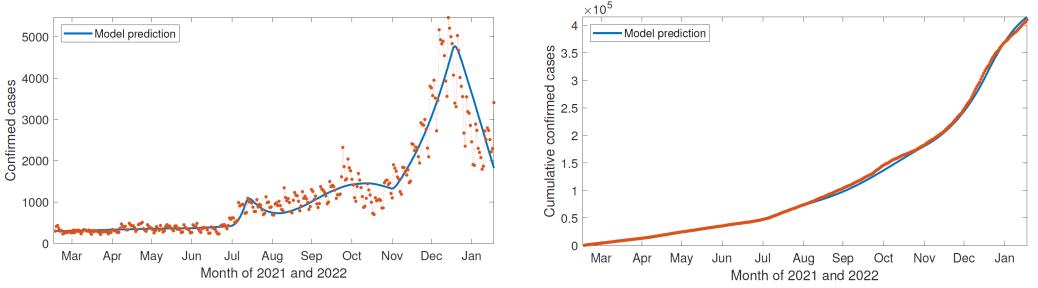

Figure: Model result

Mathematical Analysis

Compartmental Models

- SIR Model
- SEIR Model
- SITR Model
- SEQIJR Model

Equilibria

- Disease-Free Equilibrium
- Endemic Equilibrium

Basic Reproduction Number of

- SEIR Model
- SEITR Model
- Vaccination Model

References

Centers for Disease Control and Prevention (2021) *Variants and Genomic Surveillance for SARS-CoV-2*,Centers for Disease Control and Prevention, https://www.cdc.gov/coronavirus/2019-ncov/variants/

Central Disease Control Headquarters (2021) *Cumulative Cases Statistic Data for COVID-19*, Website Data, https://www.seoul.go.kr/coronaV/coronaStatus.do

Korea Disease Control and Prevention Agency (2021) *Daily Updates of Vaccination Status* Website Data, https://ncv.kdca.go.kr/vaccineStatus.es?mid=a11710000000

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