Texas A&M University
Department of Mechanical Engineering

MEEN 655: Design of Nonlinear Control Systems
Homework 6

April 4, 2022

1. **Stabilization with a Lyapunov function:** Consider a nonlinear system:

$$
\begin{aligned}
\dot{x}_1 &= x_2 - x_1^3 + x_1 x_2^2 - 2x_2 v \\
\dot{x}_2 &= x_1^3 + x_1 v
\end{aligned}
$$

   Using the Lyapunov function, $V(x_1, x_2) = x_1^2 + x_2^2$, find a stabilizing state-feedback controller for the above nonlinear system. What can say about the state-feedback stabilizability of

$$
\begin{aligned}
\dot{x}_1 &= x_2 - x_1^3 + x_1 x_2^2 - 2x_2 x_3 \\
\dot{x}_2 &= x_1^3 + x_1 x_3 \\
\dot{x}_3 &= -5x_3 + u.
\end{aligned}
$$

2. **Backstepping Design:** Suppose $\theta_1, \theta_2 > 0$ are unknown constants/parameters for the following nonlinear system:

$$\dot{x}_1 = x_2 + \theta_1 x_1^2, \quad \dot{x}_2 = \theta_1 x_1^2 + \theta_2 u, \quad y = x_1.$$

   Design an adaptive backstepping control law that regulates the system to $(x_1, x_2) = (0, 0)$.

3. **S-procedure for Lure Systems:** Consider a SISO Lure' nonlinear system as shown below: Suppose ϕ is a sector bounded nonlinearity satisfying

$$(\phi(y) - ky)(\phi(y) + ky) \leq 0, \quad \forall y.$$

   - Using the S-procedure, show that the existence of a $P = P^T \succ 0$ satisfying the following Linear Matrix Inequality (LMI)is sufficient for ensuring absolute stability of the Lure system:

$$\begin{pmatrix} A^T P + PA + C^T C & PB \\ B^T P & -\frac{1}{k^2} I \end{pmatrix} \prec 0.$$

     Which transfer function must be SPR in this case?

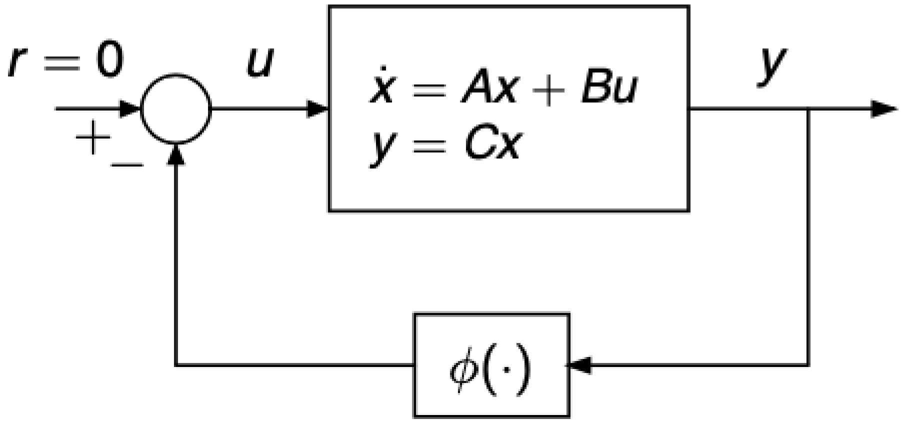

Figure 1: Lure System

- Using the S-procedure, what is the corresponding LMI if one were to consider the Liapunov function:

$$V(x) = x^T P x + \gamma_+ \int_0^y (\phi(\zeta) + k\zeta) d\zeta + \gamma_- \int_0^y (k\zeta - \phi(\zeta))?$$

(Why is this a Liapunov function and for what values of γ_+ and γ_- would this be true?) Which transfer function must be SPR?