Numeros Complejos [Ok]

sea $z \in \mathbb{C}, a, b \in \mathbb{Z}$:

$z = a + bi$

$Re(z) = a$
$Im(z) = b$

- **conjugado** : $\bar{z} = a - bi$
- **modulo** : $|z| = \sqrt{a^2 + b^2}$
- **argumento**: $arg(z) = \theta \Leftrightarrow 0 \leq \theta \leq 2\pi$
- **inverso** : $z^{-1} = \frac{\bar{z}}{|z|^2}$

Argumento :

sea $\theta = Arg(z) \Rightarrow 0 \leq \theta \leq 2\pi$

$a = Re(z), b = Im(z)$

- $cos\theta = \frac{a}{|z|} \Rightarrow \theta = cos^{-1}(\frac{a}{|z|})$
- $sen\theta = \frac{b}{|z|} \Rightarrow \theta = sen^{-1}(\frac{b}{|z|})$

Sea $w, z \in \mathbb{C}$:

- $Arg(z^n) = n * Arg(z)$
- $Arg(z.w) = Arg(z) + Arg(w)$

Operaciones

sea $z_\alpha, w_\theta \in \mathbb{C}, a, b \in \mathbb{Z}$

$z = a + bi$
$w = c + di$

- **suma** : $z + w = Re(z) + Re(z) + i(Im(z) + Im(w))$
- **resta** : $z - w = Re(z) - Re(z) - i(Im(z) - Im(w))$
- **division** : $\frac{z}{w} = \frac{|z|}{|w|}(cos(\alpha - \theta) + isen(\alpha - \theta))$
- **producto** : $z * w = |z| * |w|(cos(\alpha + \theta) + isen(\alpha + \theta))$

Forma de Moivre

sea $z = |z|e^{\theta i}$

- **division** : $\frac{z}{w} = \frac{|z|}{|w|}|z|e^{(\alpha - \theta)i}$
- **producto** : $z * w = |z||w|e^{(\alpha + \theta)i}$
- **potencia** : $z^n = |z|^n e^{n(\theta)i}$

Igualdad de Complejos:

sea $w_\theta, z_\alpha \in \mathbb{C}, z = W$

$$\Leftrightarrow \begin{cases} |z| = |w| \\ arg(z) = arg(w) \end{cases} . \Leftrightarrow \begin{cases} |z| = |w| \\ \alpha = \theta \end{cases}$$

Raices n-esimas:

sea $z_\alpha \in \mathbb{C}$ y sea w_θ una **raiz n-esima** de z:

busco $w_\theta \in \mathbb{C} : w^n = z$

$$\Leftrightarrow \begin{cases} |w|^n = |z| \\ n * arg(w) = arg(z) \end{cases} \Leftrightarrow \begin{cases} |w| = \sqrt[n]{|z|} \\ \theta = \frac{\alpha + 2k\pi}{n} \end{cases}$$

$\Rightarrow w = w_k = \sqrt[n]{|z|}e^{\frac{\theta + 2k\pi}{n}}\ con\ k \in \{0, .., n-1\}$

Raices de la unidad:

sea w_k una **raiz n-esima** de la unidad:

- $w_k = e^{\frac{2k\pi}{n}}\ ,\ \ k \in \{1, 2, ...n-1\}$

sea $w \in G_n \Rightarrow w^n = 1$

- $w^k = w^{r_n(k)}, k \in \mathbb{Z}$

nota: observar que w_k es lo mismo que w^k y que si $w \in G_n$ entonces es una raiz n-esima de unidad!

Grupo G_n

Sea $w \in G_n \Rightarrow w^n = 1$

- $w \in G_n \Rightarrow w^m = w^{r_n m}$
- $w \in G_n \Rightarrow |w| = 1$
- $w \in G_n \Rightarrow \bar{w} \in G_n \wedge \bar{w} = w^{-1}$
- $w \in G_n \Rightarrow \bar{w} = w^{-1}$

Sean $n, m \in \mathbb{N}$,

- $n|m \Rightarrow G_n \subset G_m$.
- $G_n \cap G_m = G_{(m:n)}$.
- $G_n \subset G_m \Leftrightarrow n|m$

Propiedades importantes

sea $z \in \mathbb{C}$:

- $i^2 = -1$
- $z.\bar{z} = |z|^2$
- $\bar{z} = z \Leftrightarrow z \in \mathbb{R}$
- $z + \bar{z} = 2Re(z)$
- $z - \bar{z} = 2Im(z)i$
- $|Re(z)| \leq |z|$
- $|Im(z)| \leq |z|$

sea $w, z \in \mathbb{C}$:

- $\overline{z + w} = \bar{z} + \bar{w}$
- $\overline{z.w} = \bar{z}.\bar{w}$
- $si\ z \neq 0 \Rightarrow \overline{z^{-1}} = \bar{z}^{-1}$
- $si\ z \neq 0 \Rightarrow \overline{z^k} = \bar{z}^k, \forall k \in \mathbb{Z}$
- $|z + w| \leq |z| + |w|$
- $|z.w| \leq |z|.|w|$
- $si\ z \neq 0 \Rightarrow |z^{-1}| = |z|^{-1}$
- $si\ z \neq 0 \Rightarrow |z^k| = |z|^k, \forall k \in \mathbb{Z}$

Tabla de sen & cos

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sen α	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos α	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$