# Using structure to improve a noisy PPI network

(now semi-coherent!)

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#### Abstract

Motivation: Protein-protein interaction (PPI) networks are often derived from noisy high-throughput experimental methods such as protein complex co-immunoprecipitation and yeast two-hybrid screening. Consequently, PPI networks are typically both are cumbersome and suffer from low accuracy.

Methods: Here we present a novel method (StructNA) for both reducing the size of PPI networks and improving their accuracy by finding homologs in the network whose interactions are consistent. Homologs are identified in a species-independent way using protein structural information. The method takes as input a single network. It identifies homologs, then merges fully homologous proteins and assigns scores to interactions that are consistent among homologs. It considers both degree of homology and similarity of network topology for both the merging and scoring.

**Results:** The method was able to correctly identify conserved interactions both across species and within individual species in benchmark sets derived from the Database of Interacting Proteins. On average, this resulted in a majority of interaction probabilities in the network being modified, and a reduction of network size between 5% and 10%.

**Availability:** The algorithm was implemented in Java and is licensed under the Apache License version 2.0. It's probably available for download somewhere.

## Introduction

Protein-protein interaction (PPI) networks are often derived from high-throughput experimental assays such as protein complex co-immunoprecipitation [KB08], Tandem affinity purification [RG99], and yeast two-hybrid screening [FS89], or by computational techniques. These techniques are used because they are cost-effective, but they frequently lead to the inclusion of false interactions and missing interactions, and they are unable to accurately determine the probability of interactions.

PPI networks are modeled as undirected graphs in which vertices denote proteins and edges denote protein—protein interactions. Edge weights are used to describe the probability or certainty of an interaction. Psuedographs (which permit self-loops) allow for intramolecular interactions, and hypergraphs allow for multi-molecular interactions.

A number of algorithms have been developed to decrease noise in PPIs by identifying homologs. These algorithms all solve a specific formulation of a network alignment problem. The objective of this class of problems is, given two networks (n networks in the case of a multiple network alignment), to define a

mapping between vertices in the networks that maximizes their overlap. Here, "overlap" can be given a number of definitions, but it usually incorperates both extent of homology between the aligned proteins and similarity of the network topology in the neighborhood of those proteins. Network alignment algorithms generally assume that homology is one-to-one in order to make the problem more tractable.

There are a number of limitations inherent to these approaches. First, homology is not one-to-one in real biological systems. Consequently, network alignment algorithms fail to identify many homology relationships that are biological and should be used to inform interactions. Second, these algorithms cannot identify or use homology relationships between proteins within the same species. Finally, all current network alignment algorithms rely on sequence alignments to identify homologous pairs. Since the data describes physical interactions between proteins, sequence alignment is ill-suited for deciding homology.

Therefore, we propose an algorithm for improving PPI networks that differs from network alignment algorithms in three ways:

- 1. It does not require that homology is one-to-one.
- 2. It can identify homology relationships within the same species.
- 3. It is based on structural information rather than sequence information, making it more suitable for use with PPIs.

The method uses the Structural Classification of Proteins (SCOP) [MA95] and structural alignment based on Combinatorial Extension [SI98] to introduce edges in the graph that indicate homology. It then uses this information to update the probability of interactions that are shared between homologous proteins. Finally, it collapses degenerate vertices via edge contraction, where degeneracy is decided by the presence of a clique whose members share interactions. Clique-finding is performed naively via the Bron–Kerbosch algorithm, which was found to be sufficiently fast biologically real networks.

# Methods

#### **Definitions**

To be merged into other sections.

Let  $G = (V, Int(G) \cup Hom(G))$  be a network whose vertices represent domains.

Let Int(G) is a set of edges indicating interactions.

Let Int(v) for any vertex  $v \in Int(G)$  is the set of edges in Int(G) that are incident to v. Also define Hom(v) in the same way.

Hom(G) is a set of edges indicating homology (similarity) between vertices.

An edge  $e \in Int(G)$  is associated with a probability Prob(e).

An edge  $(a, b) \in Hom(G)$  is associated with a probability denoted S((a, b)). Any edge with score  $S \ge \tau$  is defined as significant.

Let  $\rho(a, b)$  denote the score of the most specific homology relation r identified between nodes a and b. For example, if a and b are from the same SCOP fold but not the same SCOP superfamily,  $\rho(a, b)$  will be given by this relation.

Let  $\alpha(a,b)$  denote the weighted score of an alignment that has been performed between vertices a and b. Let  $\iota(v)$  denote the number of edges (v,v') in Int(G),  $\forall v \in V$ , and let  $\eta(v)$  denote the number of edges (v,v') in Hom(G),  $\forall v \in V$ .

Let 
$$\iota^* = \max_{v \in V} \{\iota(v)\}, \ \eta^* = \max_{v \in V} \{\eta(v)\}.$$

#### Overview

The algorithm consists of three major steps.

Given an interaction network G = (V, Int(G)), build a network  $G' = (V, Int(G') \cup Hom(G'))$  by using any available information (e.g. homology databases or alignment) to add edges to Hom(G') and to increase the weights of those edges. At the end of this process, remove any edge from Hom(G') with weight less than  $\tau$ .

Second, generate a network G'' = (V, Int(G'')) by using edges in Hom(G') to update edge weights (probabilities) in Int(G). Use both the probability of the shared intereaction and the probability of the homology for both interaction participants in this process.

Remove any edge from Hom(G') with weight less than  $\zeta$ . Generate a network  $G''' = (V', Int(G''')) \cup Hom(G''')$  by merging nodes contained in cliques of (V, Hom(G'')) in which every vertex in the clique is associated with the same set of neighbors in (V, Int(G'')).

### Algorithm

Pre- and post-processing steps are not included in the formal description but are listed below. StructNetAlign $(G, \beta, \tau, \zeta, \xi)$ :

- 1. Parse network.
- 2. For every pair of vertices (a, b), add an edge (a, b) to Hom(G), with label  $\beta \cdot \rho(a, b)$ .
- 3. For every pair of vertices (a, b) not joined by an edge in Hom(G), align a against b and add an edge to Hom(G), with label  $\alpha(a, b)$ .
- 4. For every edge  $e \in G$  such that  $S(e) < \tau$ , remove e.
- 5. For every edge  $(u, v) \in Int(G)$ :
  - (a) Let  $I = \emptyset$ , and let  $Q_a = 0, X_a = Y_a = 0 \ \forall \ a \in V$ .
  - (b) Run breadth-first search on (V, Hom(G)) from u and v concurrently to generate vertex labelings X and Y, respectively. Stop after reaching depth  $\xi$ .
  - (c) For both R = X and R = Y, when visiting a vertex b via edge e:
    - i. If  $R_b = 0$ , let  $R_b := 1$ .
    - ii. Let  $Q := Q + \log S(e)$ .
    - iii. Let  $R_b := R_b exp(Q)$ .
  - (d) For every edge  $(x, y) \in Int(G)$  such  $X_x \neq 0$  and  $Y_y \neq 0$ , let  $Prob((u, v)) := Prob((u, v)) + 1 X_x Y_y Prob((x, y))$ .
- 6. Discard every vertex v such that  $\iota(v) = 0$  and remove every edge incident to v.
- 7. For every edge  $e \in G$  such that  $S(e) < \zeta$ , remove e.
- 8. Run the Bron-Kerbosch algorithm on  $G_i$  to enumerate all maximal cliques  $C = \{c_1 \dots c_x\}$ , for clique sizes  $\gamma(C_i) > 1 \ \forall j$ .
- 9. Partition C into sets  $D = d_1 \dots d_z$  such that  $\forall v \in d_i, u \in d_j, Int(v) \neq Int(u), \forall i, j, and <math>Int(v) = Int(u) \ \forall u, v \in d_i \ \forall i.$
- 10. For each  $d \in D$ , merge by edge contraction every vertex in d, modifying the reference graph G.
- 11. Output the network (V, Int(G)).

#### Initial scoring of interactions

#### Defining homology by relations

A typical input network consists of a list interactions occurring between two polypeptide chains, where the chains are described by identifiers for UniProt [Con12]. In cases where the Protein Data Bank [BH02] contains at least one structure for that chain, a PDB Id and is found for that UniProt Id. From that, a SCOP [MA95] domain corresponding to the PDB Id and chain are found. This selection process is problematic for multi-chain domains, which are ignored for this step. Scores ( $\rho$ ) are assigned by using the most specific SCOP relationship found.

### Defining homology by alignment

In cases where homology is not established by SCOP relationships, homology can be derived either by structural alignment using Combinatorial Extension [SI98], or by Needleman–Wunsch [NB70] global sequence alignment.

#### Using homology to score interactions

Define 
$$R_{i,j}^{(u,v)} = Prob(i,j) \left(1 - Q_{u,i} - Q_{v,j} + Q_{u,i} \cdot Q_{v,j}\right)$$
  
Define  $Q_{a,b} = \prod_{\text{paths } \pi \text{ from a to b}} \left(1 - \prod_{k} S(\pi_k)\right)$ 

Then we want to update the probability of an interaction (u, v) with:

$$Prob((u,v))' = Prob((u,v)) + 1 - \prod_{i,j} (1 - R_{i,j}^{(u,v)})$$

We approximate the above by taking only the shortest path rather than all paths.

# Merging equivalent domains

Unfortunately, Max-Clique is NP-Equivalent and APX-Hard. The best known polynomial-time approximation algorithm has an error bound of  $\epsilon = O\left(\frac{n}{\log^2(n)}\right)$ . This makes Max-Clique intractable according to worst-case time-complexity. However, Max-Clique is solvable in practice for small n because, unlike many other NP-Hard problems, naive algorithms for Max-Clique do not require numerically intensive steps. The Bron-Kerbosch algorithm requires only incrementation and trivial set unions and subtractions. Consequentally, it has been found to perform well enough in practice for reasonable n such as those found in real biological networks.

Moreover, we can bound the time-complexity of a naive algorithm (such as Bron–Kerbosch) in terms of the maximum number of homology edges of any vertex in G. Let  $\gamma$  be the maximal clique size in Hom(G). Then  $\gamma^* < \eta^*$ .

We can readily solve MAX-CLIQUE(G) in  $\mathcal{O}(|V|^{\eta^*}(\eta^*)^3)$  time by enumerating all  $|V|^{\gamma}$  subgraphs of size  $\gamma$  for every  $\gamma = 1, 2, ..., \eta^*$ .

Our goal is to identify degenerate sets of vertices. To do this, we require subgraphs with edges in Int(G'), and homology relations between the two subgraphs that result in a clique in (V, Hom(G)). A subset of Hom(G'') defines an isomorphism between two subgraphs of (V', Int(G'')). Given a clique C, we

need to find which vertices in C share interactions. This results in a partitioning of C into disjoint induced subgraphs D. We can then merge every vertex in each  $D_i$ .

We call a subset  $S \subseteq C$  for some clique  $C \in (V, Hom(G'))$  degenerate if  $\forall u, v \in S \ Int(u) = Int(v)$ . If C is a maximal clique and  $\not\exists S_1 \subseteq C$ ,  $S_1 \supset S$  for some degenerate set  $S_1$ , we say S is a maximal degenerate set.

We first identify all maximal cliques on (V, Hom(G')) using the Max-Clique algorithm by Bron and Kerbosch, which has been found in practice to be faster than algorithms with superior worst-case time complexity.

We then identify maximal degenerate sets by associating to each vertex of a clique C a string that uniquely identifies its interactions, then partitioning C.

If a vertex  $v \in V$  belongs to two distinct maximal degenerate sets  $S_1$  and  $S_2$ , we choose whether to include v in  $S_1$  or  $S_2$  by the following:

- i) If  $|S_1| > |S_2|$ , choose  $S_1$
- ii) If  $|S_1| = |S_2|$ , choose the set whose sorted vertex labels form the lexicographically smaller string Finally, we perform edge contraction on each degenerate set with respect to both (V, Int(G')) and (V, Hom(G')). The vertex  $v_0$  with the lexicographically smallest label for each degenerate set is chosen as the representative, and the labels of the other vertices are included as metadata in  $v_0$ .

### Initial scoring of interactions

# Implementation

# Results

Overview

Accuracy

Speed

# Appendix 1. Formal proofs

# References

- [BH02] Bhat TN Bluhm WF Bourne PE Burkhardt K Feng Z Gilliland GL Iype L Jain S Fagan P Marvin J Padilla D Ravichandran V Schneider B Thanki N Weissig H Westbrook JD Zardecki C Berman HM, Battistuz T. The protein data bank. *Acta, Crystallogr. D. Biol. Crystallogr.*, 58:899–907, May 2002.
- [Con12] The UniProt Consortium. Reorganizing the protein space at the universal protein resource (uniprot). Nucleic Acids Res., 40, 2012.

- [FS89] Song O Fields S. A novel genetic system to detect protein-protein interactions. *Nature*, 340:245–246, July 1989.
- [KB08] Perr M Kaboord B. Isolation of proteins and protein complexes by immunoprecipitation. *Methods Mol. Bio.*, 424:349–364, 2008.
- [MA95] Hubbard TJP Chothia C Murzin AG, Brenner SE. Scop: a structural classification of proteins database for the investigation of sequences and structures. *Journal of Molecular Biology*, 247:536–540, 1995.
- [NB70] Wunsch C Needleman B. A general method applicable to the search for similarities in the amino acid sequence of two proteins. *Journal of Molecular Biology*, 48:443–53, 1970.
- [RG99] Rutz B Wilm M Mann M Séraphin B Rigaut G, Shevchenko A. A generic protein purification method for protein complex characterization and proteome exploration. *Nature Biotechnology*, 17:1030–1032, October 1999.
- [SI98] Bourne PE Shindyalov IN. Scop: a structural classification of proteins database for the investigation of sequences and structures. *Protein Eng*, 11:739–47, 1998.