Simulating Exponential Distribution

Sunday, October 26, 2014

Task

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponential with rate parameter $\lambda = 0.2$. You should:

1. Show where the distribution is centered at and compare it to the theoretical center of the distribution.
2. Show how variable it is and compare it to the theoretical variance of the distribution.
3. Show that the distribution is approximately normal.
4. Evaluate the coverage of the confidence interval for $1/\lambda$: $\bar{X} \pm 1.96 \frac{S}{\sqrt{n}}$

Simulation

```
nosim <- 1000
n <- 40
lambda <- 0.2
set.seed(1234)
sim <- matrix(rexp(nosim * n, rate = lambda), nosim)
sim.means <- apply(sim, 1, mean)
```

Theoretical mean of the distribution

The theoretical center of the distributon: $E[X] = 1/\lambda$

```
1 / lambda
```

```
## [1] 5
```

The real center of the distribution is:

```
mean(sim.means)
```

```
## [1] 4.974
```

Theoretical variance of the distribution

The theoretical variance of the distribution: $Var[X] = 1/\lambda^2 n$

```
1 / (lambda ^ 2 * n)
```

```
## [1] 0.625
```

The real variance of the distribution is:

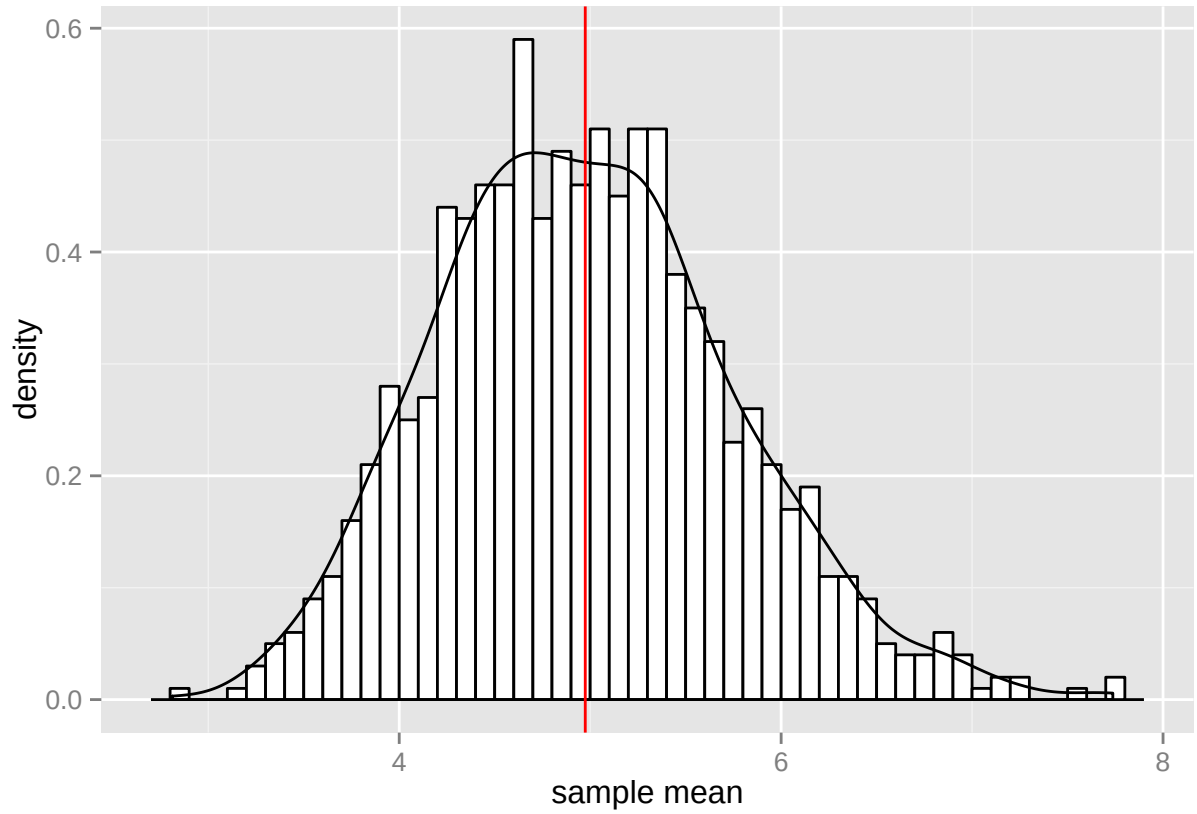
```r
var(sim.means)
```

```
## [1] 0.595
```

Distribution

Distributin of the samale means is approximately normal.

```r
library(ggplot2)
df <- data.frame(no <- c(1:nosim), x = sim.means)
g <- ggplot(df, aes(x = x))
g <- g + geom_histogram(aes(y = ..density..)
              , binwidth = .1
              , color = "black", fill = "white")
g <- g + geom_density()
g <- g + xlab("sample mean")
g <- g + geom_vline(aes(xintercept = mean(x)), color = "red", size = .5)
g
```



Confidence interval

Evaluating the coverage of the confidence interval for $1/\lambda$: $\bar{X} \pm 1.96\frac{S}{\sqrt{n}}, S = 1/\lambda\sqrt{n}$

Theoretical confidence interval:

$$\frac{1}{\lambda}(1 \pm \frac{1.96}{n})$$

```
(1 / lambda) * (1 + c(-1, 1) * 1.96 / n)
```

```
## [1] 4.755 5.245
```

Real confidence interval:

```
mean(sim.means) + c(-1,1) * 1.96 * sd(sim.means) / sqrt(n)
```

```
## [1] 4.735 5.213
```