

MATH 220 HANDOUT 9 - FUNCTIONS AND IMAGES

(1) Give an example of a function with each of the following domain/codomains.

(a) $\mathbf{Z} \rightarrow \mathbf{Z} \times \mathbf{Z}$.

(b) $\mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z}$.

(c) $\mathbf{Z} \rightarrow \mathbf{Z}_{>0}$.

(d) $\mathbf{R} \rightarrow \mathbf{Z}$.

(e) $P(\mathbf{R}) \rightarrow P(\mathbf{Z})$.

(f) $\mathbf{Z} \rightarrow P(\mathbf{Z})$.

(g) $P(\mathbf{Z}) \rightarrow \mathbf{Z}$.

(2) Draw a picture of two different functions from $\{1, 2, 3\} \rightarrow \{4, 5\}$.

Images

- (3) Let $f: A \rightarrow B$ be a function. Finish the following sentence: an element $b \in B$ is not in the image of f if \dots
- (4) Compute the image of the following functions:
- (a) $g: \mathbf{Z} \rightarrow \mathbf{Z}$, where $g(x) = 2n + 1$;
 - (b) $g: \mathbf{R} \rightarrow \mathbf{R}$, where $g(x) = 2x + 3$;
 - (c) $f: \mathbf{R} \rightarrow \mathbf{R}$, where $f(x) = -x^2 + 1$;
 - (d) $\cos: \mathbf{R} \rightarrow \mathbf{R}$;
 - (e) $\tan^{-1}: \mathbf{R} \rightarrow \mathbf{R}$;
- (5) Let $f: A \rightarrow B$ be a function and let $X, Y \subseteq A$. Prove or disprove each of the following:
- (a) $X \subseteq Y \Rightarrow f(X) \subseteq f(Y)$.
 - (b) $X \subseteq Y \Leftarrow f(X) \subseteq f(Y)$.
 - (c) $f(X \cup Y) \subseteq f(X) \cup f(Y)$.
 - (d) $f(X \cup Y) \supset f(X) \cup f(Y)$.
 - (e) $f(X \cap Y) \subseteq f(X) \cap f(Y)$.
 - (f) $f(X \cap Y) \supset f(X) \cap f(Y)$.
 - (g) (HW) $f(X) - f(Y) \subseteq f(X - Y)$.
 - (h) (HW) $f(X) - f(Y) \subseteq f(X - Y)$.