

**MATH 220 HANDOUT 12 - COMPOSITIONS AND  
INJECTIVITY/SURJECTIVITY**

- (1) Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be the function  $f(x) = \frac{1}{1+x^2}$  and let  $g: \mathbf{R} \rightarrow \mathbf{R}$  be the function  $g(x) = e^x$ .
- (a) What is  $g \circ f(0)$ ?
  - (b) What is  $f \circ g(0)$ ?
  - (c) Give a formula for  $f \circ g$  and  $g \circ f$ .
- (2) Let  $f: \mathbf{R} \rightarrow \mathbf{Z}$  be the function  $f(x) = \lfloor x \rfloor$  (i.e., round  $x$  down to the nearest integer) and let  $g: \mathbf{Z} \rightarrow \mathbf{Z}$  be the function  $g(n) =$  ‘the number of distinct prime factors of  $n$ ’. (So  $g(0) = g(1) = 0$ ,  $g(4) = 1$ ,  $g(6) = 2$ )
- (a) What is  $g \circ f(\pi)$ ?
  - (b) What is  $g \circ f(91.1023124)$ ?
  - (c) Is  $g \circ f$  injective? Surjective?
- (3) Let  $f: \mathbf{Z} \rightarrow P(\mathbf{Z})$  be the function  $f(n) = \{n\}$  and let  $g: P(\mathbf{Z}) \rightarrow P(\mathbf{Z})$  be the function  $g(S) = S \cap \{1\}$ .
- (a) What is  $g \circ f(0)$ ?
  - (b) What is  $g \circ f(1)$ ?
  - (c) Give a formula for  $g \circ f$ .

- (4) Let  $f: A \rightarrow B$  and  $g: B \rightarrow C$  be functions. Prove or disprove each of the following:
- (a) If  $f$  and  $g$  are injections, then  $g \circ f$  is an injection.
  - (b) If  $f$  and  $g$  are surjections, then  $g \circ f$  is a surjection.
  - (c) If  $f$  and  $g$  are bijections, then  $g \circ f$  is a bijection.
  - (d) If  $g \circ f$  is an injection, then  $f$  and  $g$  are injections.
  - (e) If  $g \circ f$  is a surjection, then  $f$  and  $g$  are surjections.
  - (f) If  $g \circ f$  is a bijection, then  $f$  and  $g$  are bijections.
  - (g) If  $g \circ f$  is an injection, then  $f$  is an injection.
  - (h) (HW) If  $g \circ f$  is an injection, then  $g$  is an injection.
  - (i) (HW) If  $g \circ f$  is a surjection, then  $f$  is a surjection.
  - (j) (HW) If  $g \circ f$  is a surjection, then  $g$  is a surjection.
  - (k) If  $g \circ f$  is a bijection, then  $f$  is a bijection.
  - (l) If  $g \circ f$  is a bijection, then  $g$  is a bijection.
  - (m) If  $g \circ f$  is an injection and  $g$  is a bijection, then  $f$  is an injection.
- (5) Let  $f: A \rightarrow B$  be a function. Let  $X, Y \subset A$  and let  $W, V \subseteq B$ . Each of the following statements are false as stated. Which become true if we assume that  $f$  is injective or surjective? In each case ( $f$  is injective, or  $f$  is surjective), prove your assertion or give a counterexample.
- (a)  $X \subseteq Y \Leftrightarrow f(X) \subseteq f(Y)$ .
  - (b) (HW)  $f(X \cap Y) \subseteq f(X) \cap f(Y)$ .
  - (c)  $f(X) - f(Y) \supseteq f(X - Y)$ .
  - (d)  $X \supseteq f^{-1}(f(X))$ .
  - (e) (HW)  $W \subseteq f(f^{-1}(W))$ .
  - (f)  $V \subseteq W \Leftrightarrow f^{-1}(V) \subseteq f^{-1}(W)$ .