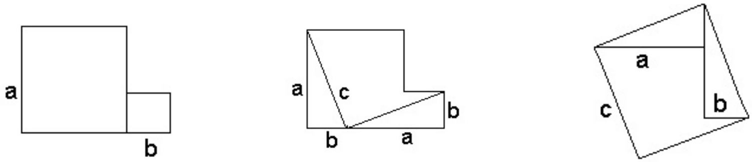
Mathematics Research at Emory

David Zureick-Brown

Slides available at `http://www.math.emory.edu/~dzb/slides/`

November 4, 2022

$$\boxed{a^2 + b^2 = c^2}$$



Research at Emory

Areas

1. Algebra, Geometry, and Number Theory
2. Analysis
3. Discrete Math/Combinatorics
4. Applied and Computational math

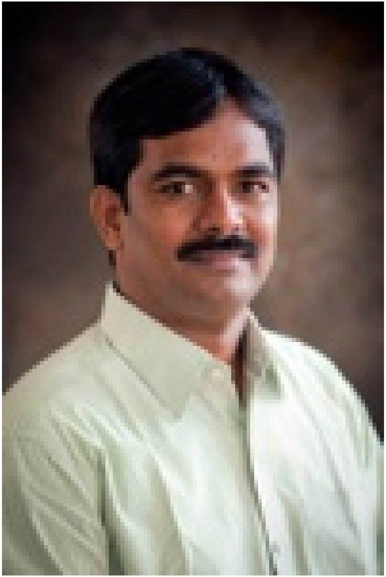
Info

1. Target: 7 incoming graduate students each year
2. 15-20 undergraduates involved in research
3. Unique graduate courses (Stacks, 4 separate combinatorics graduate courses)
4. Research Seminars
5. Student Seminars
6. Conferences (e.g., the Georgia Algebraic Geometry Symposium)

Parimala

1. Quadratic forms
2. Galois cohomology
3. Algebraic groups

Suresh Venapally



1. Quadratic forms
2. Galois cohomology

Brooke Ullery (new!)

1. classical algebraic geometry
2. commutative algebra
3. linear series
4. vector bundles

David Zureick-Brown (DZB)

1. Number Theory
2. Arithmetic Geometry
3. Algebraic Geometry

1. p-adic Cohomology
2. Galois Representations
3. Arithmetic of Varieties
4. arithmetic statistics

http://www.math.emory.edu/site/codes/

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computational-and-data-enabled-science

<codes>

@ Emory

The <CODES>@Emory research group conducts cutting-edge computational and applied mathematics research using mathematically rigorous model- and data-driven approaches. Our research focuses on areas such as the mathematics of deep learning, data assimilation, and uncertainty quantification, with a particular focus on applications in impactful areas of medicine (cardiac modeling, medical imaging), the weather and environment (hurricane storm surge modeling), and disease outbreak modeling. Common threads in these areas are their mathematical foundations, most importantly differential equations, optimization, linear algebra, and advanced techniques from computational science, such as parallel and distributed computing. Please explore our website to learn more about our fantastic group, and our research and educational activities!

Fermat's Last Theorem

Theorem (Wiles et. al)

The only solutions to the equation

$$x^n + y^n = z^n, n \geq 3$$

are multiples of the triples

$$(0,0,0), \quad (\pm 1, \mp 1, 0), \quad \pm(1,0,1), \quad (0, \pm 1, \pm 1).$$

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Fermat Surfaces

Conjecture

The only solutions to the equation

$$x^n + y^n = z^n + w^n, n \geq 5$$

satisfy $xyzw = 0$ or lie on the lines 'lines' $x = \pm y$, $z = \pm w$ (and permutations).

Fermat-like equations

Theorem (Poonen, Schaefer, Stoll)

The coprime integer solutions to $x^2 + y^3 = z^7$ are the 16 triples

$$(\pm 1, -1, 0), \quad (\pm 1, 0, 1), \quad \pm(0, 1, 1),$$

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$$(\pm 71, -17, 2), (\pm 2213459, 1414, 65), \quad (\pm 15312283, 9262, 113),$$
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Generalized Fermat Equations

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What are the solutions to the equation $x^a + y^b = z^c$*?*

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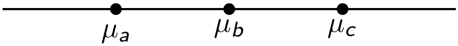
Theorem (Darmon and Granville)

Fix $a, b, c \geq 2$*. Then the equation* $x^a + y^b = z^c$ *has only finitely many coprime integer solutions iff* $\chi = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - 1 \leq 0$*.*



Known Solutions to $x^a + y^b = z^c$

The 'known' solutions with

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} < 1$$

are the following:

$$1^p + 2^3 = 3^2$$

$$2^5 + 7^2 = 3^4,\ 7^3 + 13^2 = 2^9,\ 2^7 + 17^3 = 71^2,\ 3^5 + 11^4 = 122^2$$

$$17^7 + 76271^3 = 21063928^2,\ 1414^3 + 2213459^2 = 65^7$$

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Problem (Beal's conjecture)

These are all solutions with $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} - 1 < 0$.

Generalized Fermat Equations – Known Solutions

Conjecture (Beal, Granville, Tijdeman-Zagier)

This is a complete list of coprime non-zero solutions such that $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} - 1 < 0$.

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...or even for a counterexample.

Examples of Generalized Fermat Equations

Theorem (Poonen, Schaefer, Stoll)

The coprime integer solutions to $x^2 + y^3 = z^7$ *are the* 16 *triples*

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$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} - 1 = 0$$

Examples of Generalized Fermat Equations

Theorem (Darmon, Merel)

Any pairwise coprime solution to the equation

$$x^n + y^n = z^2, n > 4$$

satisfies $xyz = 0$.

$$\frac{1}{n} + \frac{1}{n} + \frac{1}{2} - 1 = \frac{2}{n} - \frac{1}{2} < 0$$

Other applications of the modular method

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Theorem (Bugeaud, Mignotte, Siksek 2006)

The only Fibonacci numbers that are perfect powers are

$$F_0 = 0,\ F_1 = F_2 = 1,\ F_6 = 8,\ F_{12} = 144.$$