

MATH 220 HANDOUT 1 - LOGIC

A **statement** is a sentence for which ‘true or false’ is meaningful.

1. Which of these are **statements**?

- (1) Today it is raining.
- (2) What is your name?
- (3) Every student in this class is a math major.
- (4) $2 + 2 = 5$.
- (5) $x + 1 > 0$.
- (6) $x^2 + 1 > 0$.
- (7) If it is raining, then I will wear my raincoat.
- (8) Give me that.
- (9) This sentence is false.
- (10) If x is a real number, then $x^2 > 0$.

2. Which of these are true?

- (1) (T or F) Every student in this class is a math major and a human being.
- (2) (T or F) Every student in this class is a math major or a human being.
- (3) (T or F) $2 + 2 = 5$ or $1 > 0$.
- (4) (T or F) If x is a real number, then $x^2 \geq 0$.
- (5) (T or F) If x is a complex number, then $x^2 \geq 0$.

3. Write the negations of the following.

- (1) $2 + 2 = 5$
- (2) $1 > 0$.
- (3) $2 + 2 = 5$ or $1 > 0$.
- (4) Every student in this class is a math major.
- (5) Every student in this class is a math major or a human being.
- (6) If x is a real number, then $x^2 > 0$.

4. Prove the following using truth tables.

- (1) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$,
- (2) $(P \vee Q) \vee R = P \vee (Q \vee R)$. (We thus write $P \vee Q \vee R$ for both.)
- (3) $\neg(P \vee Q) = \neg P \wedge \neg Q$,
- (4) $\neg(P \wedge Q) =$ (make a guess similar to problem 3),
- (5) $\neg(\neg P) = P$.

5. In exercise 6, you may use the following variants of exercise 4.

- (1) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$,
- (2) $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$. (We thus write $P \wedge Q \wedge R$ for both.)
- (3) $P \vee Q = Q \vee P$.
- (4) $P \wedge Q = Q \wedge P$.

6. Prove or disprove the following *without* using truth tables.

- (1) $\neg(P \wedge \neg Q) = \neg P \vee Q$.
- (2) $P \vee ((Q \wedge R) \wedge S) = (P \wedge Q) \vee (P \wedge R) \vee (P \wedge S)$.
- (3) $P \vee (Q \wedge R) \wedge S = (P \vee Q) \wedge (P \vee R) \wedge (P \vee S)$.

7. Write the negations of the following implications.

- (1) If n is even, then n^2 is even.
- (2) If $1 = 0$, then $2 + 2 = 5$.
- (3) If there is free coffee, then DZB will drink it
- (4) If $1 = 0$ and $2 + 2 = 5$, then the sky is blue and kittens are popular on youtube
- (5) If x and y are real numbers such that $xy = 0$, then $x = 0$ or $y = 0$.

8. Which of these are true?

- (1) (T or F) For all $x \in \mathbf{Z}$, x is divisible by 2.
- (2) (T or F) There exists an $x \in \mathbf{Z}$ such that x is divisible by 2.
- (3) (T or F) For all $x \in \mathbf{R}$, if $x \neq 0$, then there exists a $y \in \mathbf{R}$ such that $xy = 1$.
- (4) (T or F) For all $x \in \mathbf{R}$, there exists a $y \in \mathbf{R}$ such that $xy = 1$.

9. Write the negations of the following.

- (1) For all $x \in \mathbf{Z}$, x is divisible by 2.
- (2) There exists an $x \in \mathbf{Z}$ such that x is divisible by 2.
- (3) $\neg(\forall x, P(x))$,
- (4) $\neg(\exists x \text{ s.t. } Q(x))$
- (5) $\forall x, (P(x) \wedge Q(x))$.
- (6) If $\exists x \in \mathbf{R}$ such that $2x = 1$, then for all y , $y^2 < 0$.
- (7) For all $x \in \mathbf{R}$, there exists a $y \in \mathbf{R}$ such that $xy = 1$.

10. Write the converse and contrapositive of the statements from problem 7.

Here are some basic identities.

- (1) $P \wedge Q = Q \wedge P$
- (2) $P \vee Q = Q \vee P$
- (3) $(P \wedge Q) \wedge R = P \wedge (Q \wedge R) = P \wedge Q \wedge R$
- (4) $(P \vee Q) \vee R = P \vee (Q \vee R) = P \vee Q \vee R$

Here are some useful identities.

- (1) $\neg(P \wedge Q) = \neg P \vee \neg Q$
- (2) $\neg(P \vee Q) = \neg P \wedge \neg Q$
- (3) $\neg(\neg P) = P$
- (4) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$
- (5) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$
- (6) $\neg(P \Rightarrow Q) = P \wedge \neg Q$
- (7) $\neg(\forall x, P(x)) = \exists x \text{ such that } \neg P(x)$
- (8) $\neg(\exists x \text{ such that } P(x)) = \forall x, \neg P(x)$

We can combine these to negate more complicated statements

- (1) $\neg(P \Rightarrow (Q \vee R)) =$
 $P \wedge \neg(Q \vee R) =$
 $P \wedge \neg Q \wedge \neg R$

- (2) If $1 = 0$ and $2 + 2 = 5$, then the sky is blue and kittens are cute
 If (P and Q) then (R and T)

Its negation:

(P and Q) and not (R and T)
 $(1 = 0 \text{ and } 2 + 2 = 5) \text{ and } (\text{the sky is not blue or kittens are not cute})$

- (3) $\neg Q \Rightarrow \neg P$
 $\neg(\neg Q \Rightarrow \neg P)$
 $\neg Q \wedge \neg(\neg P)$
 $\neg Q \wedge P$

This last example is called the **contrapositive**, and is a useful proof technique! (Try it on your homework.)

$(P \Rightarrow Q) = (\neg Q \Rightarrow \neg P)$ because they have the same negation.