Rigid Cohomology for Algebraic Stacks

David Zureick-Brown

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Slides available at http://www.mathcs.emory.edu/~dzb/slides/

Develop a theory of **Rigid Cohomology for Algebraic Stacks**

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Develop a theory of **Rigid Cohomology for Algebraic Stacks**; i.e.,

(Coefficients) – define some notion of overconvergent isocrystal; Rigid Cohomology for Algebraic Stacks

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- (Cohomology) define some notion of cohomology of an overconvergent isocrystal;

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- (Cohomology) define some notion of cohomology of an overconvergent isocrystal;
- Construct variants (e.g., cohomology supported in a closed subscheme);

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- Construct variants (e.g., cohomology supported in a closed subscheme);
- ▶ Weil formalism (e.g., excision, Gysin, trace formulas).

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Geometric Langlands for $GL_n(\mathbb{F}_p(C))$:

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Geometric Langlands for $GL_n(\mathbb{F}_p(C))$:

Lafforgue constructs a 'compactified moduli stack of shtukas' X (actually a compactification of a stratification of a moduli stack of shtukas). Rigid Cohomology for Algebraic Stacks

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Geometric Langlands for $GL_n(\mathbb{F}_p(C))$:

- Lafforgue constructs a 'compactified moduli stack of shtukas' X (actually a compactification of a stratification of a moduli stack of shtukas).
- ► The ℓ-adic étale cohomology of étale sheaves on X realize a Langlands correspondence between certain Galois and automorphic representations.

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Geometric Langlands for $GL_n(\mathbb{F}_p(C))$:

• $\ell = p$ is bad for étale cohomology.

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Geometric Langlands for $GL_n(\mathbb{F}_p(C))$:

- $\ell = p$ is bad for étale cohomology.
- $ightharpoonup \mathcal{X}$ is a singular, separated Artin stack, so crystalline cohomology won't work.

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Geometric Langlands for $GL_n(\mathbb{F}_p(C))$:

- $\ell = p$ is bad for étale cohomology.
- X is a singular, separated Artin stack, so crystalline cohomology won't work.
- Generalizing rigid cohomology to Artin stacks would extend Lafforgue's work to the $\ell = p$ case.

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Other Applications

Applications:

- ▶ Geometric Langlands for $GL_n(\mathbb{F}_p(C))$;
- Logarithmic rigid cohomology and crystalline fundamental groups;

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Other Applications

Applications:

- ▶ Geometric Langlands for $GL_n(\mathbb{F}_p(C))$;
- Logarithmic rigid cohomology and crystalline fundamental groups;
- Arithmetic Statistics Cohen-Lenstra heuristics for p-divisible groups.

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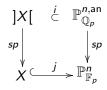
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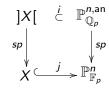
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$$H^{i}_{rig}(X) := H^{i}\left(]X[, i^{-1}\Omega^{\bullet}_{\mathbb{P}^{n,an}_{\mathbb{Q}_{p}}}\right)$$

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$$\begin{array}{ccc}
|X| & \stackrel{i}{\subset} & \mathbb{P}_{\mathbb{Q}_p}^{n,\mathsf{an}} \\
|sp| & & & \downarrow sp \\
X & \stackrel{j}{\longrightarrow} & \mathbb{P}_{\mathbb{F}_p}^n
\end{array}$$

$$H^{i}_{rig}(X) := H^{i}\left(]X[, i^{-1}\Omega^{\bullet}_{\mathbb{P}^{n,an}_{\mathbb{Q}_{p}}}\right)$$

$$\mathsf{Isoc}\,X := \left\{ (M, \nabla) \in \mathsf{MIC}\ W \right\} / \sim$$

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$$\begin{array}{ccc} X[& \stackrel{i}{\subset} & \mathbb{P}^{n,\mathsf{an}}_{\mathbb{Q}_p} \\ & & & \downarrow sp \\ X & \stackrel{j}{\longrightarrow} & \mathbb{P}^n_{\mathbb{F}_p} \end{array}$$

$$H^{i}_{\mathrm{rig}}(X) := H^{i}\left(]X[,i^{-1}\Omega^{ullet}_{\mathbb{P}^{n,\mathrm{an}}_{\mathbb{Q}_{p}}}
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 $\operatorname{Isoc}^{\dagger} X := \{(M, \nabla) \in \operatorname{Isoc} X \text{ s.t. } \nabla \text{ is 'overconvergent' } \}$

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(OK to replace \mathbb{P}^n with a formal scheme which is smooth and proper over Spf \mathbb{Z}_p .)

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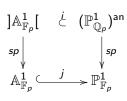
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$$\Gamma\left(i^{-1}\Omega^{\bullet}_{(\mathbb{P}^{1}_{\mathbb{Q}_{p}})^{\mathrm{an}}}\right) = 0 \to \mathbb{Q}_{p}\{x\}^{\dagger} \xrightarrow{d} \mathbb{Q}_{p}\{x\}^{\dagger} dx \to 0$$

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$$\begin{split}]\mathbb{A}^1_{\mathbb{F}_p}[& \stackrel{i}{\subset} (\mathbb{P}^1_{\mathbb{Q}_p})^{\mathrm{an}} \\ s_p \Big| & \quad \downarrow s_p \\ \mathbb{A}^1_{\mathbb{F}_p} \stackrel{j}{\subset} & \mathbb{P}^1_{\mathbb{F}_p} \end{split}$$

$$\Gamma\left(i^{-1}\Omega^{\bullet}_{(\mathbb{P}^1_{\mathbb{Q}_p})^{\mathrm{an}}}\right) = 0 \to \mathbb{Q}_p\{x\}^{\dagger} \xrightarrow{d} \mathbb{Q}_p\{x\}^{\dagger} dx \to 0$$

$$\mathbb{Q}_p\{x\}^{\dagger} \subset \mathbb{Q}_p[|x|],$$

d(f(x)) := f'(x)dx

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$$\begin{array}{ccc}
 &]\mathbb{A}^1_{\mathbb{F}_p}[& \stackrel{.}{\leftarrow} & (\mathbb{P}^1_{\mathbb{Q}_p})^{\mathrm{an}} \\
 & & \downarrow^{sp} & \downarrow^{sp} \\
 & \mathbb{A}^1_{\mathbb{F}_p} & \stackrel{.}{\longrightarrow} \mathbb{P}^1_{\mathbb{F}_p}
\end{array}$$

$$\Gamma\left(i^{-1}\Omega^{\bullet}_{(\mathbb{P}^1_{\mathbb{Q}_p})^{\mathrm{an}}}\right) = 0 \to \mathbb{Q}_p\{x\}^{\dagger} \xrightarrow{d} \mathbb{Q}_p\{x\}^{\dagger} dx \to 0$$

$$\sum p^n x^{p^n} \notin \mathbb{Q}_p\{x\}^{\dagger} \subset \mathbb{Q}_p[|x|],$$

$$d(f(x)) := f'(x) dx$$

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$$\begin{array}{cccc}
 & \mathbb{A}^{1}_{\mathbb{F}_{\rho}} [& \stackrel{i}{\leftarrow} & (\mathbb{P}^{1}_{\mathbb{Q}_{\rho}})^{\operatorname{ar}} \\
 & & \downarrow^{sp} & & \downarrow^{sp} \\
 & \mathbb{A}^{1}_{\mathbb{F}_{\rho}} & \stackrel{j}{\longrightarrow} & \mathbb{P}^{1}_{\mathbb{F}_{\rho}}
\end{array}$$

$$\Gamma\left(i^{-1}\Omega^{\bullet}_{(\mathbb{P}^{1}_{\mathbb{Q}_{p}})^{\mathrm{an}}}\right) = 0 \to \mathbb{Q}_{p}\{x\}^{\dagger} \xrightarrow{d} \mathbb{Q}_{p}\{x\}^{\dagger} dx \to 0$$
$$\sum_{p} p^{n} x^{p^{n}} \notin \mathbb{Q}_{p}\{x\}^{\dagger} \subset \mathbb{Q}_{p}[|x|],$$

$$d(f(x)) := f'(x)dx$$

$$H^{i}_{\mathrm{rig}}(\mathbb{A}^{1}_{\mathbb{F}_{p}}) = \begin{cases} \mathbb{Q}_{p}, & \text{if } i = 0 \\ 0, & \text{if } i \geq 1 \end{cases}$$

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▶ **Independence** of choices is a theorem (Berthelot).

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- ▶ **Independence** of choices is a theorem (Berthelot).
- ▶ **Functorality** is another theorem (Berthelot).

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- Independence of choices is a theorem (Berthelot).
- ▶ Functorality is another theorem (Berthelot).
- ► Hard to prove results about **relative** rigid cohomolgy (e.g., coherence is still an open problem).

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- Independence of choices is a theorem (Berthelot).
- ► **Functorality** is another theorem (Berthelot).
- Hard to prove results about relative rigid cohomolgy (e.g., coherence is still an open problem).
- How to define for a scheme which isn't quasi-projective?

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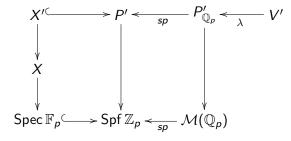
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Le Stum's overconvergent site: $AN^{\dagger}(X)$

▶ **Objects**: (X', V') =



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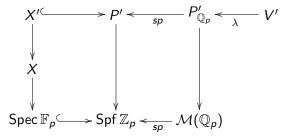
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Le Stum's overconvergent site: $AN^{\dagger}(X)$

▶ Objects: (*X'*, *V'*) =



▶ A morphism $(X', V') \rightarrow (X'', V'')$ is a triple of morphisms compatible with the diagram.

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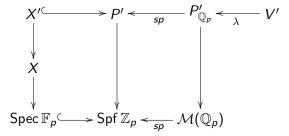
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Le Stum's overconvergent site: $AN^{\dagger}(X)$

▶ Objects: (*X'*, *V'*) =



- A morphism (X', V') → (X", V") is a triple of morphisms compatible with the diagram.
- ▶ $\{(X, V_i) \rightarrow (X', V')\}$ is a **covering** if $V = \cup V_i$ is an open covering of topological spaces.

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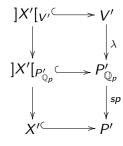
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Strict Neighborhoods

▶ The **tube** of $(X' \hookrightarrow P', P'_{\mathbb{Q}_p} \stackrel{\lambda}{\leftarrow} V')$ is $\lambda^{-1}(]X'[_{P'_{\mathbb{Q}_p}})$.



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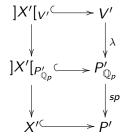
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Strict Neighborhoods

▶ The **tube** of $(X' \hookrightarrow P', P'_{\mathbb{Q}_p} \stackrel{\lambda}{\leftarrow} V')$ is $\lambda^{-1}(]X'[_{P'_{\mathbb{Q}_p}})$.



A morphism $(X', V') \rightarrow (X'', V'')$ induces a morphism $]X'[_{V'} \rightarrow]X''[_{V''}.$

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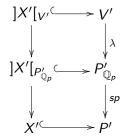
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Strict Neighborhoods

▶ The **tube** of $(X' \hookrightarrow P', P'_{\mathbb{Q}_p} \stackrel{\lambda}{\leftarrow} V')$ is $\lambda^{-1}(]X'[_{P'_{\mathbb{Q}_p}})$.



- A morphism $(X', V') \rightarrow (X'', V'')$ induces a morphism $]X'[_{V'} \rightarrow]X''[_{V''}.$
- ▶ We declare $(X', W') \rightarrow (X', V')$ to be an isomorphism if the induced map on tubes $]X'[_{W'} \rightarrow]X'[_{V'}$ is an isomorphism.

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Structure sheaf \mathcal{O}_X^{\dagger}

- $i:]X'[_{V'} \hookrightarrow V'.$
- $\qquad \qquad \quad \bullet \ \, \mathcal{O}_X^\dagger(X',V') := \Gamma(]X'[_{V'},i^{-1}\mathcal{O}_{V'}).$

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Le Stum's main theorem

Theorem

(i)
$$H^i(AN^{\dagger}X, \mathcal{O}_X^{\dagger}) \cong H^i_{rig}(X)$$
.

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Le Stum's main theorem

Theorem

- (i) $H^i(AN^{\dagger}X, \mathcal{O}_X^{\dagger}) \cong H^i_{rig}(X)$.
- (ii) $\operatorname{\mathsf{Coh}} \mathcal{O}_X^\dagger \cong \operatorname{\mathsf{Isoc}}^\dagger X$

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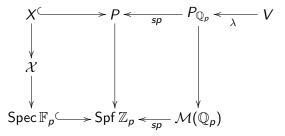
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Let $\mathcal X$ be a stack and define $\mathsf{AN}^\dagger(\mathcal X)$ and $\mathcal O_X^\dagger$ the same way.

▶ **Objects**: (X, V) =



- ▶ A morphism $(X, V) \rightarrow (X', V')$ is a triple of morphisms compatible with the diagram.
- ▶ $\{(X_i, V_i) \rightarrow (X, V)\}$ is a **covering** if $V = \cup V_i$ is an open covering of topological spaces.

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Define:

- $ightharpoonup \mathcal{O}_{\mathcal{X}}^{\dagger}$ as before: $(X,V)\mapsto \Gamma(i^{-1}\mathcal{O}_{V});$
- ▶ $\operatorname{Isoc}^{\dagger}(\mathcal{X}) := \operatorname{Coh} \mathcal{O}_{\mathcal{X}}^{\dagger};$
- $\qquad \qquad H^{i}_{\mathsf{rig}}(\mathcal{X}) := H^{i}(\mathcal{X}_{\mathsf{AN}^{\dagger}}, \mathcal{O}^{\dagger}_{\mathcal{X}}).$

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Sample Theorems: Finiteness and agreement

Theorem (ZB.)

- (i) $H_{\text{rig}}^{i}(\mathcal{X})$ is finite dimensional.
- (ii) $H^i_{rig}(\mathcal{X}) \otimes \mathbb{C}$ agrees with the étale cohomology of \mathcal{X} .
- (iii) $H^i_{rig}(\mathcal{X})$ agrees with the crystalline cohomology of \mathcal{X} when \mathcal{X} is smooth and proper.

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- ▶ Given a closed substack $\mathcal{Z} \subset \mathcal{X}$ with open complement \mathcal{U} , I can define functors $H^i_{\text{rig},\mathcal{Z}}(\mathcal{X})$.
- ▶ **Idea**: use the very general notions of open and closed immersion of **topoi** (of SGA4).

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Cohomology supported in a closed substack: Theorems

Theorem (ZB.)

- (i) $H^{i}_{rig,\mathcal{Z}}(\mathcal{X})$ agrees with Berthelot's construction when \mathcal{X} is a scheme.
- (ii) (Excision) There is a long exact sequence

$$\cdots H^i_{\mathsf{rig},\mathcal{Z}}(\mathcal{X}) o H^i_{\mathsf{rig}}(\mathcal{X}) o H^i(\mathcal{U}) o H^{i+1}_{\mathsf{rig},\mathcal{Z}}(\mathcal{X}) \cdots$$

(iii) (Gysin) When $(\mathcal{X}, \mathcal{Z})$ is a smooth pair, there is an isomorphism

$$H^{i}_{\mathsf{rig},\mathcal{Z}}(\mathcal{X}) \cong H^{i-2d}(\mathcal{Z})$$

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Main tool: cohomological descent

Theorem (ZB.)

Cohomological descent holds on the overconvergent site with respect to smooth, flat, and étale hypercovers.

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Main tool: cohomological descent

Theorem (ZB.)

Cohomological descent holds on the overconvergent site with respect to smooth, flat, and étale hypercovers.

Special case: let $X \to \mathcal{X}$ be a smooth cover. Then there is a spectral sequence

$$H^i_{rig}(X_j) \Rightarrow H^{i+j}_{rig}(\mathcal{X})$$

where X_j is the j+1 fold fiber product $X \times_{\mathcal{X}} \cdots \times_{\mathcal{X}} X$.

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Main tool: cohomological descent

Theorem (ZB.)

Cohomological descent holds on the overconvergent site with respect to smooth, flat, and étale hypercovers.

Special case: let $X \to \mathcal{X}$ be a smooth cover. Then there is a spectral sequence

$$H^i_{rig}(X_j) \Rightarrow H^{i+j}_{rig}(\mathcal{X})$$

where X_j is the j+1 fold fiber product $X \times_{\mathcal{X}} \cdots \times_{\mathcal{X}} X$.

Previous proof for rigid cohomology is \sim 200 pages; mine is \sim 20.

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