

MATH 250: USEFUL BOOLEAN IDENTITIES

Here are some basic identities.

- (1) $P \wedge Q = Q \wedge P$
- (2) $P \vee Q = Q \vee P$
- (3) $(P \wedge Q) \wedge R = P \wedge (Q \wedge R) = P \wedge Q \wedge R$
- (4) $(P \vee Q) \vee R = P \vee (Q \vee R) = P \vee Q \vee R$

Here are some less basic identities.

- (1) $\neg(P \wedge Q) = \neg P \vee \neg Q$
- (2) $\neg(P \vee Q) = \neg P \wedge \neg Q$
- (3) $\neg(\neg P) = P$
- (4) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$
- (5) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$
- (6) $\neg(P \Rightarrow Q) = P \wedge \neg Q$
- (7) $\neg(\forall x, P(x)) = \exists x \text{ such that } \neg P(x)$
- (8) $\neg(\exists x \text{ such that } P(x)) = \forall x, \neg P(x)$

We can combine these to negate more complicated statements

- (1) $\neg(P \Rightarrow (Q \vee R)) =$
 $P \wedge \neg(Q \vee R) =$
 $P \wedge \neg Q \wedge \neg R$

- (2) If $1 = 0$ or $2 + 2 = 5$, then the sky is blue and kittens are cute.
If $(P \text{ or } Q)$ then $(R \text{ and } T)$

Its negation:

$(P \text{ or } Q)$ and not $(R \text{ and } T)$
 $(1 = 0 \text{ or } 2 + 2 = 5)$ and $(\text{the sky is not blue or kittens are not cute})$

- (3) Let's negate $\neg Q \Rightarrow \neg P$.

$$\begin{aligned}\neg(\neg Q \Rightarrow \neg P) \\ \neg Q \wedge \neg(\neg P) \\ \neg Q \wedge P\end{aligned}$$

This last example is called the contrapositive, and is a useful proof technique! (Try it on your homework.)

- (1) $(P \Rightarrow Q) = (\neg Q \Rightarrow \neg P)$ because they have the same negation.