
MATH 220, Mathematical Reasoning and Proof
MWF 1 - 1:50
IN PROGRESS

All assignments
Last updated: September 12, 2023
Gradescope code: 7DVWGG

Contents

1 (due Sep 15): Introduction to course. Mathematical reasoning. Logic.	2
2 (due Sep 22): “Direct” proofs and divisibility problems.	4

Assignment 1

Topics: Introduction to the course. Mathematical reasoning. Logic.

Reading: 1.1, 1.2, 1.5, 2.1, 2.2

Suggested problems (for extra practice; do not hand in): [Handout 1](#)

Assignment 1, due Friday, Sep 15, via Gradescope :

1. Write the negation of each of the following statements.
 - (a) All triangles are isosceles.
 - (b) Every door in the building was locked.
 - (c) Some even numbers are multiples of three.
 - (d) Every real number is less than 100.
 - (e) Every integer is positive or negative.
 - (f) If f is a polynomial function, then f is continuous at 0.
 - (g) If $x^2 > 0$, then $x > 0$.
 - (h) There exists a $y \in \mathbf{R}$ such that $xy = 1$.
 - (i) $(2 > 1)$ and $(\forall x, x^2 > 0)$
 - (j) $\forall \epsilon > 0, \exists \delta > 0$ such that if $|x| < \delta$, then $|f(x)| < \epsilon$.
2. Write the converse, contrapositive, and negation of each of the following implications.
 - (a) If a quadrilateral is a rectangle, then it has two pairs of parallel sides.
 - (b) $(P \wedge \neg Q) \Rightarrow R$
 - (c) $P \Rightarrow (R \Rightarrow \forall x, Q(x))$
3. Let P and Q be statements. Write the truth table for
 - (a) $(\neg P) \vee Q$
 - (b) $(P \wedge (\neg Q)) \Rightarrow Q$
4. Are the statements $(P \vee Q) \wedge R$ and $P \vee (Q \wedge R)$ equivalent? If so, give a proof. If not, explain why by giving a counterexample.

(Two statement forms are equivalent if they have the same truth tables, and here, a counterexample simply means some choice of truth values for P, Q , and R such that the two statement forms give different outputs.)
5. Let P and Q be statements.
 - (a) Prove that $\neg(P \Rightarrow Q)$ is equivalent to $P \wedge \neg Q$.
 - (b) Prove that $\neg(P \Rightarrow Q)$ is *not* equivalent to $\neg P \wedge Q$.

(c) Give an example of statements P and Q such that $\neg P \Rightarrow \neg Q$ is true and $\neg(P \Rightarrow Q)$ is false.

6. Suppose that n is an even integer, and let m be any integer. Prove that nm is even.
7. Suppose that n is an odd integer. Prove that n^2 is an odd integer. (Hint: an integer n is odd if and only if there exists an integer k such that $n = 2k + 1$.)
8. Prove that if n^2 is even, then n is even. (Hint: page 67.)

Assignment 2

Topics: “Direct” proofs, proof by cases, and divisibility problems.

Reading:

- 3.3, from Definition 3.3.6
- 6.4, just Definition 6.4.1
- see Index to find definitions like prime, etc

Suggested problems (do not hand in)

1. [Handout 2](#)

Assignment, due Friday, Sep 22, via Gradescope:

1. Suppose that $a \mid b$. Prove that for all $n \in \mathbb{Z}_{>0}$, $a^n \mid b^n$.
2. Suppose that there exists an integer $n \in \mathbb{Z}_{>0}$ such that $a \mid b^n$. Is it true that $a \mid b$? Prove or disprove your answer. (For a disproof, please give a counterexample that demonstrates that the statement is false.)
3. Prove that for all $a \in \mathbb{Z}$ and for $n \in \mathbb{Z}_{\geq 0}$, $a - 1$ divides $a^n - 1$.
4. Prove that for all integers n , n and $n + 1$ have no common divisors other than ± 1 .
5. Prove that if x is an integer, then $x^2 + 2$ is not divisible by 4. (Hint: there are two cases: x is even, x is odd. Also, feel free to use basic facts about even or odd, e.g., “odd + odd = even”, without additional proof.)
6. Prove that the product of three consecutive integers is divisible by 6. (It suffices to prove that it is divisible by 2 and 3 separately.)
7. Show that for all integers a and b ,
$$a^2b^2(a^2 - b^2)$$
is divisible by 12. (It suffices to prove that it is divisible by 4 and 3 separately.)
8. Find all positive integers n such that $n^2 - 1$ is prime. Prove that your answer is correct.