

Math 250: Number Theory  
Instructor: David Zureick-Brown (“DZB”)

**All assignments**

Last updated: January 31, 2024

Gradescope code: ZWK583

**Show all work for full credit!**

*Proofs should be written in full sentences whenever possible.*

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Assignment 1: Introduction to course; squares, triangles, and Pythagorean triples.

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Due by 11:25am, eastern, on Thursday, Feb 08

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**Suggested readings for this problem set:** Chapters 1, 2, 3. (Chapter 4 is bonus content.)

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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**Assignment:** due Thursday, Feb 08, 11:25am, via Gradescope (ZWK583):

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- For each of the lists of numbers, (a) find the next three numbers and (b) find a formula for the  $n$ th term in the sequence. Describe the sequence in plain English too, if possible.
  - 7, 14, 21, 28, 35, ...
  - 1, 4, 7, 10, 13, ...
  - 1, 8, 27, 64, 125, ...
  - 2, 4, 8, 16, 32, 64, ...
  - 11, 20, 29, 38, 47, ...
- Try adding up the first few odd numbers and see if the numbers you get satisfy some sort of pattern. Once you find the pattern, express it as a formula. Give a geometric verification that your formula is correct.
- The consecutive odd numbers 3, 5, and 7 are all primes. Are there infinitely many such “prime triplets”? That is, are there infinitely many prime numbers  $p$  such that  $p + 2$  and  $p + 4$  are also primes?
- It is generally believed that infinitely many primes have the form  $N^2 + 1$ , although no one knows for sure.
  - Do you think that there are infinitely many primes of the form  $N^2 - 1$ ?
  - Do you think that there are infinitely many primes of the form  $N^2 - 2$ ?
  - How about of the form  $N^2 - 3$ ? How about  $N^2 - 4$ ?
  - Which values of  $a$  do you think give infinitely many primes of the form  $N^2 - a$ ?

Hint: work out several examples by hand, or with a computer (try using the code

```
{n : n in [1..100] | IsPrime(n^2+1)};
```

at the site <http://magma.maths.usyd.edu.au/calc/>)

- A natural number is called **perfect** if it is equal to the sum of its “proper” divisors (“proper” means smaller). For example,  $6 = 1 + 2 + 3$  so 6 is a perfect number. Find the next perfect number after 6 on your own, then look up the next few perfect numbers after that. Is there a general pattern to these numbers?
- Recall that  $(a, b, c) \in \mathbb{Z}^3$  is a **Pythagorean triple** if each of  $a$ ,  $b$  and  $c$  are positive integers and  $a^2 + b^2 = c^2$ .
  - Do there exist any Pythagorean triples such that  $c = 1$ ? Prove your answer.
  - Do there exist any Pythagorean triples such that  $a = 1$ ? Prove your answer.
- Suppose that  $(a, b, c)$  is a Pythagorean triple such that  $a$  is prime. Prove that  $c = b + 1$ .

8. (a) Use the lines through the point  $(1, 1)$  to describe all the points on the circle  $x^2 + y^2 = 2$  whose coordinates are rational numbers.
- (b) What goes wrong if you try to apply the same procedure to find all the points on the circle  $x^2 + y^2 = 3$  with rational coordinates?



IN PROGRESS! Check back later for the final assignment.



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Assignment 2: TBA

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*Due by 11:25am, eastern, on Thursday, Feb 15*

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***Suggested readings for this problem set:*** Chapters 5 and 6

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Feb 15, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





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Assignment 3: TBA

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*Due by 11:25am, eastern, on Thursday, Feb 22*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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**Assignment:** due Thursday, Feb 22, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





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Assignment 4: TBA

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*Due by 11:25am, eastern, on Thursday, Feb 29*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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**Assignment:** due Thursday, Feb 29, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





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Assignment 5: TBA

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*Due by 11:25am, eastern, on Thursday, Mar 07*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Mar 07, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





IN PROGRESS! Check back later for the final assignment.



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Assignment 6: TBA

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*Due by 11:25am, eastern, on Thursday, Mar 14*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Mar 14, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.







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## Midterm 1 study guide

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*In class on Thursday,*

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**Content:** The questions will all be either

1. homework problems,
2. suggested problems,
3. problems we worked in class, or
4. minor variations of one of these.

Problems with very long proofs or that involved some unusual trick will not be on the exam.

You are allowed to use any previous problem from class or from the homework (e.g., “additivity of divisibility” or “the 2 out of 3 rule”) on the exam without reproving it, unless otherwise noted on the exam. (E.g., if I ask you to prove “additivity of divisibility” on the exam, you will need to prove this using only the definition of divisibility, and I will remind you of this in the statement of the problem.)

A typical exam will have one or two questions from each week of the course. You can expect problems about following:

- TBA

For definitions, I want a definition, in prose (complete sentences), and I want “just” the definition, and not any additional facts about the definition. (E.g., if you give the definition of rational, do not include that a rational number can be written in reduced form; that is a fact about rational numbers not part of the definition of rational.)





IN PROGRESS! Check back later for the final assignment.



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Assignment 7: TBA

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*Due by 11:25am, eastern, on Thursday, Mar 28*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Mar 28, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





IN PROGRESS! Check back later for the final assignment.



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Assignment 8: TBA

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*Due by 11:25am, eastern, on Thursday, Apr 04*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Apr 04, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





IN PROGRESS! Check back later for the final assignment.



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Assignment 9: TBA

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*Due by 11:25am, eastern, on Thursday, Apr 11*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Apr 11, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





IN PROGRESS! Check back later for the final assignment.



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Assignment 10: TBA

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*Due by 11:25am, eastern, on Thursday, Apr 18*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Apr 18, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





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## Midterm 2 study guide

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*In class on Thursday,*

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**Content:** The questions will all be either

1. homework problems,
2. suggested problems,
3. problems we worked in class, or
4. minor variations of one of these.

Problems with very long proofs or that involved some unusual trick will not be on the exam.

You are allowed to use any previous problem from class or from the homework (e.g., “additivity of divisibility” or “the 2 out of 3 rule”) on the exam without reproving it, unless otherwise noted on the exam. (E.g., if I ask you to prove “additivity of divisibility” on the exam, you will need to prove this using only the definition of divisibility, and I will remind you of this in the statement of the problem.)

A typical exam will have one or two questions from each week of the course. You can expect problems about following:

- TBA

For definitions, I want a definition, in prose (complete sentences), and I want “just” the definition, and not any additional facts about the definition. (E.g., if you give the definition of rational, do not include that a rational number can be written in reduced form; that is a fact about rational numbers not part of the definition of rational.)





IN PROGRESS! Check back later for the final assignment.



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Assignment 11: TBA

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*Due by 11:25am, eastern, on Thursday, Apr 25*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, Apr 25, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.





IN PROGRESS! Check back later for the final assignment.



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Assignment 12: TBA

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*Due by 11:25am, eastern, on Thursday, May 02*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, May 02, 11:25am, via Gradescope (ZWK583):

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IN PROGRESS! Check back later for the final assignment.







IN PROGRESS! Check back later for the final assignment.



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Assignment 13: TBA

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*Due ???*

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***Suggested readings for this problem set:*** TBA

All readings are from Silverman, *A Friendly Introduction to Number Theory*.

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***Assignment:*** due Thursday, May 07, 11:25am, via Gradescope (ZWK583):

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1. Which integers are one more than a square and one less than a cube?



IN PROGRESS! Check back later for the final assignment.





IN PROGRESS! Check back later for the final assignment.



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Final exam study guide

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**Final exam** is **May ???, ???pm**, in SMUD 014.

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The **last day of class** is Tuesday, May 7.

There will be **office hours** on before the exam. I will send out a survey to find a time that works for everyone who is planning to attend.

The final exam will be comprehensive.

The exam will be, roughly 8-10 questions, with multiple parts. Some questions will be “prove or disprove”. For disproofs, please write out a counterexample as your disproof.

A typical exam will have roughly one or two questions from each week of the course. You can expect a subset of the following:

- TBA



IN PROGRESS! Check back later for the final assignment.

