Math 375: Representation Theory
Instructor: David Zureick-Brown ("DZB")

All assignments

Last updated: February 6, 2025

Gradescope code: J7PV4B

Show all work for full credit!
Proofs should be written in full sentences whenever possible.

Gradescope Instructions for submitting work in Math 375

You will be using the online Gradescope progam to submit your homework and exams. These instructions tell you how to sign up initially, and how to submit your written work.

Signing up for Gradescope the first time.

If you haven't used Gradescope for an **Amherst College** course before:

- Go to `http://www.gradescope.com`, click on "Sign up for free" (which may auto-scroll you to the bottom of the page), and select Sign up as [a] "Student".
- In the signup box:
  - Use the course entry code **J7PV4B**
  - Use your full name
  - Use your **Amherst College email** address. Or, if you are a Five-College student, use your email address from your own school.
  - Leave the "Student ID" entry blank.
- You will probably get an email asking to set a password for your account, so check your `amherst.edu` email inbox. (Or your email inbox through your own school, for Five-College students.)

Adding Math 375 to Gradescope.

If you **have** used Gradescope for an Amherst course before, and so you already have an account through your `amherst.edu` email, you still need to add Math 375, so:

- Go to `http://www.gradescope.com` and log in.
- Go to your Account Dashboard (click the Gradescope logo at upper left), and click "Add Course" at bottom right.
- Use the course code **J7PV4B**

(submission instructions on next page)

Submitting written work

First write it out on paper as you would normally. Then **scan it** to create a PDF. One method for scanning is the smartphone app **DropBox**. It makes nice clear scans, and it saves them directly into a folder so that you can have all your assignments in one place. **CamScanner** is another free scanning App, and there are others, too. **Gradescope** now has its own scanning app. You can also use a printer/scanner if you prefer.

Please be kind to our dear graders and make sure your submission is legible !

In particular, please leave some spacing between separate problems.

If you have a tablet computer, you may write your work there (instead of on paper) and save it as a PDF.

Some of you may know the math formatting package LaTeX and may want to use it in Math 375. That's fine, too; if so, you may write up your work in LaTeX and save the resulting PDF.

In short, any method is fine as long as it creates a legible **PDF** file and **NOT a photo**.

For example, if you use the DropBox app, then in your created *Math 375 Homework* Dropbox folder, you can select create (+) at the bottom of the screen and click the *Scan Document* option. Snap a shot of the first page of your homework, and then click [+] to snap shots of any subsequent pages. Do **not** use the *Take Photo* option.

After you have scanned/saved your work as a PDF, submit it on Gradescope as follows:

- Go to `http://www.gradescope.com` and log in.
- Select the course "Math 375, Spring 2025" and the appropriate assignment.
- Select "submit pdf" to submit your work in PDF format. Browse to find your PDF and upload.
- Now it is time to **tag** your problems. This is an **important step**, where you are telling Gradescope which problems are on which page(s).

  For each problem, select the pages of your submission where your written solution appears.

  I think the easiest thing to do is to click on the page of **your** homework upload where you wrote the given problem, and then click on the assigned problem listed. Repeat for each problem.

  You must tag the problems or else you will not get credit for your work.

  Gradescope will give you a warning when you go to submit your assignment if you have not selected the pages correctly. If you tag a problem incorrectly, you can fix it by clicking "More" and "Reselect Pages".
- Click Save or Submit.

After your assignment is graded, you will be able to see your score on the written problems, along with comments, on Gradescope. You should receive an email notifying you when each homework set is graded.

Assignment 1: Introduction to the course; review of groups and linear algebra

Due by 9:55am, eastern, on Tuesday, February 4

- ***Suggested readings for this problem set:*** **Chapter 1**
- Syllabus: https://dmzb.github.io/teaching/2025Spring375/syllabus-math-375-S25.pdf
- Gradescope instructions (previous page)

All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, February 4, 9:55am, via Gradescope (J7PV4B):

1. (a) If $N \trianglelefteq G$, $H \leq G$, then $NH \leq G$. (I.e., if N is a normal subgroup of G and H is any subgroup of G, then NH is also a subgroup of G.)
   (b) Is NH normal? Prove or disprove.[1]

2. Let G be a group. A bijective function $\psi : G \to G$ is called an *automorphism* if

$$(\forall a\,,\, b \in G) \quad \psi(ab) = \psi(a)\psi(b)$$

   holds. (I.e., an automorphism is a bijective homomorphism.)

   The set of all automorphisms is a group with respect to composition; it is called the *automorphism group* of G and is denoted by $\mathrm{Aut}(G)$.

   For any $g \in G$ let $c_g : G \to G$ be defined by $c_g(x) := gxg^{-1}$, and let $\mathrm{Inn}(G) = \{c_g \mid g \in G\}$.[2]

   Let $\phi_G : G \to \mathrm{Aut}\, G$ be the map given by $g \mapsto c_g$.[3].

   (a) Prove that $\mathrm{Inn}(G)$ is a normal subgroup of $\mathrm{Aut}(G)$.
   (b) Give an example of a group G such that $\mathrm{Inn}(G) \neq \mathrm{Aut}(G)$. (No proof necessary.)
   (c) Give an example of a nontrivial group G such that $\mathrm{Inn}(G) = \mathrm{Aut}(G)$. (No proof necessary.)
   (d) Describe the kernel and image of ϕ_G. What can you deduce about this from the first isomorphism theorem?

3. (a) What is the center $Z(S_n)$ of the symmetric group? Prove that your answer is correct.
   (b) Let $c = (1,2)(3,4) \in S_n$, $n > 4$. Determine $|\mathrm{C}_G(c)|$.
   (c) Let $d = (1,2,\ldots,n) \in S_n$. Determine $|\mathrm{C}_G(d)|$.

[1] For a disproof, give a counterexample
[2] Verify on your own that c_g is an automorphism (i.e., don't submit a proof that c_g is an automorphism).
[3] Verify on your own that ϕ_G is a homomorphism

Assignment 2: Simple groups

Suggested readings for this problem set: **Chapter 1; Chapter 3**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, February 11, 9:55am, via Gradescope (J7PV4B):

1. Let G be a group.

   (a) Show that $C_G(Z(G)) = G$ and $N_G(Z(G)) = G$.

   (b) Let $H \subset G$ be a subgroup. Suppose that H has cardinality 2. Prove that $N_G(H) = C_G(H)$. Deduce that if H is normal, then it is also central. (Central means that H is a subgroup of $Z(G)$.)

   (c) Let N be a proper normal subgroup of G. Show that G/N is simple if and only if N is a maximal normal subgroup of G.

   (d) Assume that $G/Z(G)$ is cyclic. Prove that G is abelian.

   (Click here for a hint.)

2. Let G be the set of upper triangular 3×3 matrices over the field $\mathbb{F}_3$, whose diagonal elements are 1.

$$G := \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \text{ s.t. } a, b, c \in \mathbb{F}_3 \right\}$$

   Verify on your own that G is a group with respect to matrix multiplication.

   (a) Verify that every nonidentity element of G is of order 3.

   (b) Calculate the center of the group G.

   (c) What is the Jordan–Holder series of G?

3. Let G be a group. The subgroup $D = \{(g, g) \mid g \in G\}$ of $G \times G$ is called the *diagonal subgroup* of G.

   (a) Prove that D is a normal subgroup in $G \times G$ if and only if G is abelian

   (b) A subgroup M of a group H is called a *maximal subgroup* if $M \lneqq G$ and there is no subgroup K satisfying $M \lneqq K \lneqq H$. Prove that D is a maximal subgroup of $G \times G$ if and only if G is a simple group, i.e. it has no proper normal subgroup.

 IN PROGRESS! Check back later for the final assignment. 

Assignment 3: Group actions

Suggested readings for this problem set: **Chapter 3; Chapter 1, Section 1.6**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, February 18, 9:55am, via Gradescope (J7PV4B):

1. Compute the center of $\mathrm{GL}_3(F)$.
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

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Assignment 4: Linear representations; Maschke's Theorem

Due by 9:55am, eastern, on Tuesday, February 25

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, February 25, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

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Assignment 5: Schur's Lemma; Character Theory

Due by 9:55am, eastern, on Tuesday, March 4

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, March 4, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

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Assignment 6: Character Theory; orthogonality Relations

Due by 9:55am, eastern, on Tuesday, March 11

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, March 11, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

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Midterm study guide

In person oral exam, Thursday, March 6.

- This will be a 10-20 minute oral exam.
- The intent is that everyone will get an A.
- I plan to ask a few very simple questions (e.g., “what is the definition of a representation?”, or “what is an example of a character?”) and will ask you to do one problem at the board from a list of problems that I give you ahead of time.
- The list of problems will mostly be problems from homework, class, or some theorems and propositions.
- The only thing you need to do to prepare is to keep up with the course (i.e., do the homework every week, and make sure that you understand the content being presented in class).
- The exam will cover all of the material leading up to the exam date, with the exception of the most recent lecture. Once we are closer to the date, I will post more specific details.

The week before the exam, I will post a sign up sheet for 20 minute timeslots.

IN PROGRESS! Check back later for the final assignment.

 IN PROGRESS! Check back later for the final assignment. 

Assignment 7: Orthogonality relations; character Tables

Due by 9:55am, eastern, on Tuesday, April 1

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, April 1, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

 IN PROGRESS! Check back later for the final assignment. 

Assignment 8: Character table for S_n and other examples

Due by 9:55am, eastern, on Tuesday, April 8

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, April 8, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

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Assignment 9: Functorality and duality

Due by 9:55am, eastern, on Tuesday,

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, April 15, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

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Assignment 10: Characters of products; Symmetric and Exterior Powers; Character Ring. Frobenius Groups

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Due by 9:55am, eastern, on Tuesday, 10

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, April 29, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

 IN PROGRESS! Check back later for the final assignment. 

Assignment 11: Burnside's Theorem

Due by 9:55am, eastern, on Tuesday, May 6

Suggested readings for this problem set: **TBA**
All readings are from Robinson, *A Course in the Theory of Groups*.

Assignment: due Tuesday, May 6, 9:55am, via Gradescope (J7PV4B):

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1. TBA
2. TBA
3. TBA
4. TBA
5. TBA
6. TBA
7. TBA
8. TBA

IN PROGRESS! Check back later for the final assignment.

Final exam (oral) study guide

- The **last day of class** is Tuesday, May 6.
- This will be a 10-20 minute oral exam.
- The intent is that everyone will get an A.
- I plan to ask a few very simple questions (e.g., “what is the definition of a representation?”, or “what is an example of a character?”) and will ask you to do one problem at the board from a list of problems that I give you ahead of time.
- The list of problems will mostly be problems from homework, class, or some theorems and propositions.
- The only thing you need to do to prepare is to keep up with the course (i.e., do the homework every week, and make sure that you understand the content being presented in class).
- The exam will be comprehensive. Once we are closer to the date, I will post more specific details.

The week before the exam, I will post a sign up sheet for 20 minute timeslots.

IN PROGRESS! Check back later for the final assignment.

Hints

2.1. There is no “trick” to this problem; you can do it by “writing out what everything means”.