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MATH 220, Mathematical Reasoning and Proof  
MWF 1 - 1:50  
IN PROGRESS

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All assignments  
Last updated: September 8, 2023  
Gradescope code: 7DVWGG

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## Contents

1 (due Sep 15): Introduction to course. Mathematical reasoning. Logic.	2
2 (due Sep 22): “Direct” proofs and divisibility problems.	4
3 (due Sep 29): Proof by contradiction. Unsolvability. Irrationality.	5
4 (due Oct 06): Induction.	6
5 (due Oct 13): Set theory. Basic operations. Proofs with sets.	7
(October 13) Midterm	8
6 (due Oct 20): More sets. DeMorgan’s laws. Cartesian Products. Power sets.	9
7 (due Oct 27): Introduction to functions; images and surjectivity.	10
8 (due Nov 03): Inverse Image (or “Preimage”).	11
9 (due Nov 10): Injectivity.	12
10 (due Nov 17): Composition of functions.	13
(November 17) Midterm	14
11 (due Dec 01): Inverse functions.	15
12 (due Dec 08): Relations	16
13 (due Dec 13): Binary Operations and Bijections/Countability	17
Final Exam	18

# Assignment 1

**Topics:** Introduction to the course. Mathematical reasoning. Logic.

**Reading:** 1.1, 1.2, 1.5, 2.1, 2.2

**Suggested problems** (for extra practice; do not hand in): [Handout 1](#)

**Assignment 1, due Friday, Sep 15, via Gradescope :**

1. Write the negation of each of the following statements.
  - (a) All triangles are isosceles.
  - (b) Every door in the building was locked.
  - (c) Some even numbers are multiples of three.
  - (d) Every real number is less than 100.
  - (e) Every integer is positive or negative.
  - (f) If  $f$  is a polynomial function, then  $f$  is continuous at 0.
  - (g) If  $x^2 > 0$ , then  $x > 0$ .
  - (h) There exists a  $y \in \mathbf{R}$  such that  $xy = 1$ .
  - (i)  $(2 > 1)$  and  $(\forall x, x^2 > 0)$
  - (j)  $\forall \epsilon > 0, \exists \delta > 0$  such that if  $|x| < \delta$ , then  $|f(x)| < \epsilon$ .
2. Write the converse, contrapositive, and negation of each of the following implications.
  - (a) If a quadrilateral is a rectangle, then it has two pairs of parallel sides.
  - (b)  $(P \wedge \neg Q) \Rightarrow R$
  - (c)  $P \Rightarrow (R \Rightarrow \forall x, Q(x))$
3. Let  $P$  and  $Q$  be statements. Write the truth table for
  - (a)  $(\neg P) \vee Q$
  - (b)  $(P \wedge (\neg Q)) \Rightarrow Q$
4. Are the statements  $(P \vee Q) \wedge R$  and  $P \vee (Q \wedge R)$  equivalent? If so, give a proof. If not, explain why by giving a counterexample.
5. Let  $P$  and  $Q$  be statements.
  - (a) Prove that  $\neg(P \Rightarrow Q)$  is equivalent to  $P \wedge \neg Q$ .
  - (b) Prove that  $\neg(P \Rightarrow Q)$  is *not* equivalent to  $\neg P \wedge Q$ .
  - (c) Give an example of statements  $P$  and  $Q$  such that  $\neg P \Rightarrow \neg Q$  is true and  $\neg(P \Rightarrow Q)$  is false.
6. Suppose that  $n$  is an even integer, and let  $m$  be any integer. Prove that  $nm$  is even.

7. Suppose that  $n$  is an odd integer. Prove that  $n^2$  is an odd integer. (Hint: an integer  $n$  is odd if and only if there exists an integer  $k$  such that  $n = 2k + 1$ .)
8. Prove that if  $n^2$  is even, then  $n$  is even. (Hint: page 67.)