MATH 220, Mathematical Reasoning and Proof
MWF 1 - 1:50
IN PROGRESS

All assignments

Last updated: September 11, 2023

Gradescope code: 7DVWGG

Contents

Assignment 1

Topics: Introduction to the course. Mathematical reasoning. Logic.

Reading: 1.1, 1.2, 1.5, 2.1, 2.2

Suggested problems (for extra practice; do not hand in): Handout 1

Assignment 1, due Friday, Sep 15, via Gradescope :

1. Write the negation of each of the following statements.
   (a) All triangles are isosceles.
   (b) Every door in the building was locked.
   (c) Some even numbers are multiples of three.
   (d) Every real number is less than 100.
   (e) Every integer is positive or negative.
   (f) If f is a polynomial function, then f is continuous at 0.
   (g) If $x^2 > 0$, then $x > 0$.
   (h) There exists a $y \in \mathbf{R}$ such that $xy = 1$.
   (i) $(2 > 1)$ and $(\forall x, x^2 > 0)$
   (j) $\forall \epsilon > 0$, $\exists \delta > 0$ such that if $|x| < \delta$, then $|f(x)| < \epsilon$.

2. Write the converse, contrapositive, and negation of each of the following implications.
   (a) If a quadrilateral is a rectangle, then it has two pairs of parallel sides.
   (b) $(P \wedge \neg Q) \Rightarrow R$
   (c) $P \Rightarrow (R \Rightarrow \forall x, Q(x))$

3. Let P and Q be statements. Write the truth table for
   (a) $(\neg P) \vee Q$
   (b) $(P \wedge (\neg Q)) \Rightarrow Q$

4. Are the statements $(P \vee Q) \wedge R$ and $P \vee (Q \wedge R)$ equivalent? If so, give a proof. If not, explain why by giving a counterexample.

5. Let P and Q be statements.
   (a) Prove that $\neg(P \Rightarrow Q)$ is equivalent to $P \wedge \neg Q$.
   (b) Prove that $\neg(P \Rightarrow Q)$ is *not* equivalent to $\neg P \wedge Q$.
   (c) Give an example of statements P and Q such that $\neg P \Rightarrow \neg Q$ is true and $\neg(P \Rightarrow Q)$ is false.

6. Suppose that n is an even integer, and let m be any integer. Prove that nm is even.

7. Suppose that n is an odd integer. Prove that n^2 is an odd integer. (Hint: an integer n is odd if and only if there exists an integer k such that $n = 2k + 1$.)

8. Prove that if n^2 is even, then n is even. (Hint: page 67.)

Assignment 2

Topics: "Direct" proofs, proof by cases, and divisibility problems.

Reading:

- 3.3, from Definition 3.3.6
- 6.4, just Definition 6.4.1
- see Index to find definitions like prime, etc

Suggested problems (do not hand in)

1. Handout 2

Assignment, due Friday, Sep 22, via Gradescope:

1. Suppose that $a \mid b$. Prove that for all $n \in \mathbb{Z}_{>0}$, $a^n \mid b^n$.
2. Suppose that there exists an integer $n \in \mathbb{Z}_{>0}$ such that $a \mid b^n$. Is it true that $a \mid b$? Prove or disprove your answer. (For a disproof, please give a counterexample that demonstrates that the statement is false.)
3. Prove that for all $a \in \mathbb{Z}$ and for $n \in \mathbb{Z}_{\geq 0}$, $a - 1$ divides $a^n - 1$.
4. Prove that for all integers n, n and $n + 1$ have no common divisors other than ± 1.
5. Prove that if x is an integer, then $x^2 + 2$ is not divisible by 4. (Hint: there are two cases: x is even, x is odd. Also, feel free to use basic facts about even or odd, e.g., "odd + odd = even", without additional proof.)
6. Prove that the product of three consecutive integers is divisible by 6. (It suffices to prove that it is divisible by 2 and 3 separately.)
7. Show that for all integers a and b,
$$a^2 b^2 (a^2 - b^2)$$
is divisible by 12. (It suffices to prove that it is divisible by 4 and 3 separately.)
8. Find all positive integers n such that $n^2 - 1$ is prime. Prove that your answer is correct.