Navier-Stokes Notes

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1 Introduction

The Navier-Stokes equations according to Jos Stam are used to model fluid states by their velocities in a velocity field, and the change in density through the velocity field.

$$\frac{\partial u}{\partial t} = -(u\nabla)u + v\nabla^2 u + f$$

A variant of the incompressible Navier-stokes equation, omitting the pressure gradient term $-\frac{1}{\rho}\nabla\rho$; which is the projection step in the simulation.

This is the change in the velocity field (array u) over time t

(u∇)u is the advection term, which is nonlinear here. Responsible for fluid motion

v∇^2u is the diffusion term for velocity (Also known as viscosity). Responsible for fluid resistance

f is the external forces; or in this case, the input from the mouse!

$$\frac{\partial \rho}{\partial t} = -(u\nabla)\rho + \kappa^2\rho + S$$

A variant of the advection-diffusion equation

This is the change in density over time

-(u∇)ρ is the advection term; or how fluids move in the density field

$\kappa^2\rho$ is the diffusion term for density; or how density "diffuses" across the field (Jos Stam mentions 2 ways of handling diffusion)

S is the source term; or where the density originates

2 Conceptual

- Indices of vector based fluid simulators generally leave consideration for a single array index in all directions (ie: index 1 to index N - 1)
- In the JavaScript implementation, function calls do not store references, so it is important to note that modified parameters are to be returned and updated.

3 Functions

- addSource(dimension N, array x, array s, step dt)

  Iterates over the array and updates the entries in x according to its respective matching index entry in array s (source)

- diffuse(dimension N, int b, array x, array x0, float diffusion term, step dt)

  Uses the Gauss-Seidel relaxation method. $x_{i,j}^{k+1} = \frac{x0_{i,j} + a(x_{i-1,j}^{k+1} + x_{i+1,j}^{k} + x_{i,j-1}^{k+1} + x_{i,j+1}^{k})}{1+4a}$
  Where a is the scaled coefficient, according to dt and diff.

- setbnd(dimension N, int b, array x)

  Iterates over the input array x (representing density, velocity), and given an input integer b (handles case by case bounds), updates the boundary entries of the array.
  b === 1 or b === 2 for reflection at boundaries, b === 0 for clamping at edges

- advect(dimension N, int b, array d, array d0, array u, array v, float dt)

  Iterates over all entries within the inner bounds, and estimates each velocity using the bilinear interpolation method.

4 Works Cited

`https://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations`

`https://uw.pressbooks.pub/ocean285/chapter/advection-diffusion-equation/`

`https://graphics.cs.cmu.edu/nsp/course/15-464/Fall09/papers/StamFluidforGames.pdf`