8. Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof The sequence $\{a_n\}_{n=1}^{\infty}$ tends to the limit L, so by the definition of limit: $(\forall\epsilon > 0)(\exists n)(\forall m \geq n)(|a_m - L| < \epsilon/M)$, where $m, n \in \mathcal{R}$. Let $\epsilon > 0$ be given.

1. Simplifying the limit definition mentioned above: $(\exists p)(\forall m \geq p)(M|a_m - L| < \epsilon)$.
2. Simplifying the expression in (1) further: $(\exists p)(\forall m \geq p)(|Ma_m - ML| < \epsilon)$.
3. The expression in (2) is the definition of limit for the sequence $\{Ma_n\}_{n=1}^{\infty}$, and it is evident that it tends to the limit ML.

This completes the proof. ■