3. Say whether the following is true or false and support your answer by a proof: For any integer n, the number $n^2 + n + 1$ is odd.

PROOF Assume the statement is true. We know that any odd number can be expressed in the form 2q + 1, and any even number can be expressed in the form 2q, where q $\in \mathcal{Z}$. Consider three cases: 1. n is odd, 2. n is even, and 3. n is zero.

- Case 1: If n is odd, $n^2 + n + 1 = (2q + 1)^2 + (2q + 1) + 1$. Simplifying the RHS: $(2q + 1)((2q + 1) + 1) + 1$ (factoring out $(2q + 1)$ from the first two terms). Further simplifying, RHS $= 2(2q + 1)(q + 1) + 1$ (factoring out 2 from the second term). $(2q + 1)(q + 1)$ is an integer - let it be s. Therefore, $n^2 + n + 1 = 2s + 1$. Therefore, $n^2 + n + 1$ is odd when n is odd.

- Case 2: If n is even, $n^2 + n + 1 = (2q)^2 + 2q + 1$. Simplifying the RHS: $2q(2q + 1) + 1$. $q(2q + 1)$ is an integer - let it be s. Therefore, $n^2 + n + 1 = 2s + 1$. This shows that $n^2 + n + 1$ is odd when n is even.

- Case3: If n is zero, $n^2 + n + 1$ reduces to 1, which is odd.

Therefore, for any integer n, $n^2 + n + 1$ is odd. This completes the proof. ■