

1. Say whether the following is true or false and support your answer by a proof.  $(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12)$

PROOF The statement is false. The smallest  $m, n \in \mathcal{N}$  that we can try are 1 and 1.

- Substituting these in the above equation:  $3 * 1 + 5 * 1 = 8 < 12$ .
- Let us try the next possible value for  $m$ , which is 2:  $3 * 2 + 5 * 1 = 11 < 12$ .
- For  $m = 3, n = 1$ , and  $m = 1, n = 2$ , and all other combinations of  $m$  and  $n$ ,  $3m + 5n > 12$ .
- Therefore  $\neg[(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12)]$ .
- Or,  $(\forall m \in \mathcal{N})(\forall n \in \mathcal{N})(3m + 5n \neq 12)$ .

This completes the proof. ■