10. Give an example of a family of intervals $A_n, n = 1, 2, ...,$ such that $A_{n+1} \subset A_n$ for all n and $\cap_{n=1}^{\infty} A_n$ consists of a single real number. Prove that your example has the stated property.

PROOF Consider the family of closed intervals: $A_n = [0, 1/n]$ on the real line, where $n \in \mathcal{N}$. Then, A_1 is the interval $[0, 1]$, A_2 is the interval $[0, 1/2]$, and so on.

- Therefore, A_{n+1} will be the interval $[0, 1/n + 1]$.
- Clearly, $A_2 \subset A_1, A_3 \subset A_2, ..., A_{n+1} \subset A_n$ since all the elements in each of these intervals also belong to the preceding interval.
- As $n \to \infty$, it is evident that the limit of $1/n \to 0$. Therefore, as $n \to \infty$, the interval A_n approaches $[0, 0]$.
- We know that a closed interval, $[a, b] = \{x | a \leq x \leq b\}$. So, $[0, 0] = \{x | 0 \leq x \leq 0\}$, which is a set with a single real number: $\{0\}$.
- Therefore, it is clear that $\bigcap\limits_{n=1}^{\infty} A_n = \{0\}$ (since 0 is the only element that belongs to all the sets).

Hence, the family of intervals $A_n = [0, 1/n], n = 1, 2, ...,$ satisfies the above property. This completes the proof. ■