8. Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

PROOF If a sequence $\{a_n\}_{n=1}^{\infty}$ tends to the limit L, then $(\forall \epsilon > 0)(\exists n)(\forall m \geq n)(|a_m - L| < \epsilon)$, where $m, n \in \mathcal{R}$. Consider the sequence $\{Ma_n\}_{n=1}^{\infty}$:

1. Assume that $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML as $n \to \infty$ (for some arbitrary M and L). From the limit definition mentioned above: $(\forall \epsilon > 0)(\exists p)(\forall m \geq p)(|Ma_m - ML| < \epsilon)$. Let $\epsilon > 0$ be given.
2. Simplifying the expression in (1): $(\forall \epsilon > 0)(\exists p)(\forall m \geq p)(M|a_m - L| < \epsilon)$ ($M > 0$ is a common factor).
3. Choose a p so large that $|a_p - L| > M/\epsilon$. Since $m \geq p$, $|a_m - L| \geq |a_p - L| > M/\epsilon$. Or, $|a_m - L| < \epsilon/M$, which can be simplified to: $M|a_m - L| < \epsilon$, which agrees with the limit definition in (2).

Hence, our assumption is correct: the sequence $\{Ma_n\}_{n=1}^{\infty}$tends to the limit ML as $n \to \infty$. This completes the proof. ■