6. A classic unsolved problem in number theory asks if there are infinitely many pairs of 'twin primes', pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.

PROOF A number, n, is said to be prime if it is greater than divisible only by itself and 1. Or, it can not be expressed in the form $n = pq$, where $p, q \in \mathcal{Z}$. Let n be a prime number. Then, a prime triplet would be $n, n+2, n+4$.

1. If $n = 3$, then the prime triplet is $3, 5, 7$.
2. If $n > 3$, there can not be a prime triplet because at least one of $n, n+2, n+4$, where $n \in \mathcal{Z}$ is divisible by 3. The proof for this is as follows:

- If a number is divisble by 3, it can be expressed in the form $3p$, where $p \in \mathcal{Z}$. Consider an arbitrary integer s.
- If s is divisible by 3, the statement is true and the proof is complete. If s is not divisible by 3, it can be expressed in one of two forms: $3p + 1$ or $3p + 2$, where $p \in \mathcal{Z}$.
- If s is of the form $3p + 1$, $s + 2$ simplifies to $(3p + 1) + 2 = 3(p + 1)$. $(p + 1)$ is an integer, therefore, $s + 2$ is divisible by 3 if s is of the form $3p + 1$.
- If s is of the form $3p + 2$, $s + 4$ simplifies to $(3p + 2) + 4 = 3(p + 2)$. $(p + 2)$ is an integer, therefore, $s + 4$ is divisible by 3 if s is of the form $3p + 2$.
- Hence, for any integer s, at least one of $s, s + 2$, or $s + 4$ is divisible by 3.

Therefore, there is no prime triplet other than 3, 5, and 7. This completes the proof. ■