

8. Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \rightarrow \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML .

PROOF Let $\epsilon > 0$ be given. The sequence $\{a_n\}_{n=1}^{\infty}$ tends to the limit L , so, we can find a p such that for $m \geq p$: $|a_m - L| < \epsilon/M$, where $m, p \in \mathcal{R}$.

1. Simplifying the limit definition mentioned above: $M|a_m - L| < \epsilon$.
2. Simplifying the expression in (1) further: $|Ma_m - ML| < \epsilon$.
3. The expression in (2) is the definition of limit for the sequence $\{Ma_n\}_{n=1}^{\infty}$, and it is evident that it tends to the limit ML .

This completes the proof. ■