4. Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

PROOF We know from the Division Theorem that any natural number, m, can be expressed in the form $4n + r$, where $n, r \in \mathcal{N}$and $0 \leq r \leq 3$. So, any natural number can be expressed in one of the following forms: $4n, 4n + 1, 4n + 2$, or, $4n + 3$. $4n$ and $4n + 2$ are clearly even.

Hence, any odd natural number can be expressed either in the form $4n + 1$ or in the form $4n + 3$. This completes the proof. ■