4. Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

PROOF We know that any number that is odd can be expressed in the form $2p + 1$, where $p \in \mathcal{Z}$.

- Consider $4n + 1$: We can simplify this as $2(2n) + 1$. Since n is an integer, $2n$ is an integer - call it p. Therefore, $4n + 1 = 2p + 1$. This shows that $4n + 1$ is an odd number.
- Consider $4n + 3$: We can simplify this as $4n + 2 + 1 = 2(2n + 1) + 1$. $(2n + 1)$ is an integer - call it q. Therefore, $4n + 3 = 2q + 1$. This shows that $4n + 3$ is also an odd number.

Hence, any odd natural number can be expressed either in the form $4n + 1$ or in the form $4n + 3$. This completes the proof. ■