

1. Say whether the following is true or false and support your answer by a proof. $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$

PROOF The statement is false. The smallest $m, n \in \mathbb{N}$ that we can try are 1 and 1. Substituting these in the above equation: $3 \times 1 + 5 \times 1 = 8 < 12$. Let us try the next possible value for m , which is 2: $3 \times 2 + 5 \times 1 = 11 < 12$. For $m = 3, n = 1$, and $m = 1, n = 2$, and all other combinations, $3m + 5n > 12$. Therefore $\neg[(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)]$. Or, $(\forall m \in \mathbb{N})(\forall n \in \mathbb{N})(3m + 5n \neq 12)$. Hence, the proof. ■