

8. Prove (from the definition of a limit of a sequence) that if the sequence  $\{a_n\}_{n=1}^{\infty}$  tends to limit  $L$  as  $n \rightarrow \infty$ , then for any fixed number  $M > 0$ , the sequence  $\{Ma_n\}_{n=1}^{\infty}$  tends to the limit  $ML$ .

PROOF Let  $\epsilon > 0$  be given. The sequence  $\{a_n\}_{n=1}^{\infty}$  tends to the limit  $L$ , so, we can find a  $p$  such that for  $m \geq p$ :  $|a_m - L| < \epsilon/M$ , where  $m, p \in \mathcal{N}$ .

1. Simplifying the limit definition mentioned above:  $M|a_m - L| < \epsilon$ .
2. Simplifying the expression in (1) further:  $|Ma_m - ML| < \epsilon$ .
3. The expression in (2) is the definition of limit for the sequence  $\{Ma_n\}_{n=1}^{\infty}$ , and it is evident that it tends to the limit  $ML$ .

This completes the proof. ■