5. Prove that for any integer n, at least one of the integers $n, n+2, n+4$ is divisible by 3.

PROOF If a number is divisble by 3, it can be expressed in the form $3p$, where $p \in \mathcal{Z}$. Consider an arbitrary integer n.

- If n is divisible by 3, the statement is true and the proof is complete. If n is not divisible by 3, it can be expressed in one of two forms: $3p+1$ or $3p+2$, where $p \in \mathcal{Z}$.
- If n is of the form $3p+1$, $n+2$ simplifies to $(3p+1)+2 = 3(p+1)$. $(p+1)$ is an integer, therefore, $n+2$ is divisible by 3 if n is of the form $3p+1$.
- If n is of the form $3p+2$, $n+4$ simplifies to $(3p+2)+4 = 3(p+2)$. $(p+2)$ is an integer, therefore, $n+4$ is divisible by 3 if n is of the form $3p+2$.

Hence, for any integer n, at least one of $n, n+2$, or $n+4$ is divisible by 3. This completes the proof. $\blacksquare$