7. Prove that for any natural number n, $2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$.

PROOF By mathematical induction:

1. For $n = 1$, the identity reduces to $2^1 = 2^{1+1} - 2$, which simplifies to $2 = 2$. The identity is true for $n = 1$.
2. Assume the identity is true for n. Therefore, $2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$.
3. Add 2^{n+1} to the LHS: $(2 + 2^2 + 2^3 + ... + 2^n) + 2^{n+1}$. But, we know $2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$ from (2).
4. Simplifying the LHS in (3): $(2^{n+1} - 2) + 2^{n+1} = 2 * 2^{n+1} - 2 = 2^{n+2} - 2$, which is the result for $n + 1$.

Hence, by the principle of mathematical induction, the identity is true. ∎