TOPOLOGY OPTIMIZATION USING Dr. SIGMUND’S 88 LINES MATLAB CODE

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Abstract:

In this study, Topology optimization (TO) is introduced, and the algorithm is implemented based on Dr. Sigmund's 88 lines MATLAB code for minimizing the compliance of a statistically loaded structure at its equilibrium state with respect to its topology. Also, a passive element is added to the structure to make it suitable for adding another elements or pipes to the already existing structure.

Keywords:

Topology Optimization, Sigmund's 88 lines MATLAB code

Acknowledgements:

I thank my Professor, Dr. Yi Ren for his continuous guidance and support to work on this project and to finish this project successfully.

I thank Dr. Ole Sigmund for developing this code for us to learn how to implement topology optimization in various engineering applications.

Contents:

1. Introduction

What is Topology Optimization:

Topology Optimization (TO) is a shape optimization method that uses algorithmic models to optimize material layout within a user-defined space for a given set of loads, conditions, and constraints. TO maximizes the performance and efficiency of the design by removing redundant material from areas that do not need to carry significant loads to reduce weight or solve design challenges like reducing resonance or thermal stress.

Designs produced with topology optimization often include free forms and intricate shapes that are complex or impossible to manufacture with traditional production methods. However, TO designs are a perfect match for additive manufacturing processes that have more forgiving design rules and can easily reproduce complex shapes without additional costs.

Advantages of TO:

- ✓ Less expensive because of lower packaging & transportation costs and also heavy machinery isn't necessary for assembly lines.
- ✓ Solves design challenges such as changes due to resonance and thermal stress.
- ✓ Saves time and reduces environmental impact.
- ✓ Eliminates the errors.

Applications:

Aerospace: To improve the layout design for airframe structures, such as stiffener ribs or brackets for aircraft.

Automotive:

In the automotive industry, topology optimization balances the desirability of lightweight parts for fuel efficiency and power with the stability and strength of a body that can withstand torque and impact.

Besides mass savings, topology optimization can also improve passenger safety by defining the way a structure collapses during an accident.

Medical:

TO tools can also optimize the designs of biodegradable scaffolds for tissue engineering, porous implants, and lightweight orthopedics. Nanotechnology applications—such as cell manipulation, surgery, micro fluids, and optical systems—also use topology optimization.

2. Problem formulation and Boundary Conditions

The design domain, the boundary conditions, and the external load for the beam are shown in below figure.

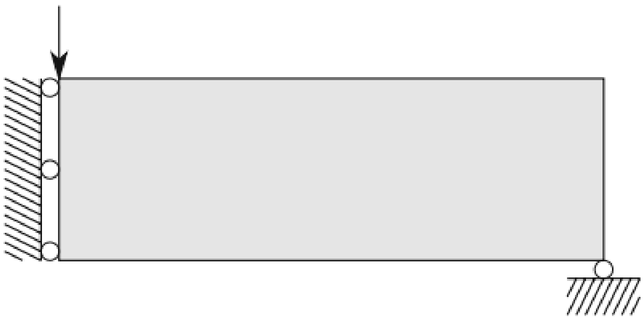


Fig. 1 The design domain, boundary conditions, and external load for the optimization of a symmetric MBB beam

The objective of the optimization problem is to determine the optimal material distribution on the beam with respect to minimum compliance and constant total amount of material.

The mathematical formulation of the optimization problem is as follows:

$$
\begin{aligned}
\min_{\mathbf{x}} : \quad & c(\mathbf{x}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} E_e(x_e)\mathbf{u}_e^{\mathrm{T}}\mathbf{k}_0\mathbf{u}_e \\
\text{subject to :} \quad & V(\mathbf{x})/V_0 = f \\
& \mathbf{K}\mathbf{U} = \mathbf{F} \\
& \mathbf{0} \leq \mathbf{x} \leq \mathbf{1}
\end{aligned}
$$

Where, c is the compliance

U is global displacement vector

F is global Force vector

K is global Stiffness matrix

k_0 is the element stiffness matrix of an element with unit young's modulus

x is the vector of design variables (i.e., The element densities)

N is the number of elements used to discretize the design domain

V(x) and V0 are the material volume and design domain volume, respectively, and

f is the prescribed volume fraction

2.1. Optimality criteria method

The optimization problem is solved by using a standard optimality criteria method. A heuristic updating scheme is as follows:

$$x_e^{\text{new}} = \begin{cases} \max(0, x_e - m) & \text{if } x_e B_e^{\eta} \leq \max(0, x_e - m) \\ \min(1, x_e + m) & \text{if } x_e B_e^{\eta} \geq \min(1, x_e - m) \\ x_e B_e^{\eta} & \text{otherwise} \end{cases} \quad (3)$$

where m is a positive move limit

η (= 1/2) is a numerical damping coefficient, and

B_e is obtained from the optimality condition

$$B_e = \frac{-\frac{\partial c}{\partial x_e}}{\lambda \frac{\partial V}{\partial x_e}}$$

where λ is the Lagrangian multiplier and it must be chosen in such a way that the volume constraint is satisfied.

The appropriate value for λ can be found by means of a bisection algorithm.

The sensitivities of the objective function c and the material volume V with respect to the element densities x_e is given by:

$$\frac{\partial c}{\partial x_e} = -p x_e^{p-1} (E_0 - E_{\min}) \mathbf{u}_e^{\mathrm{T}} \mathbf{k}_0 \mathbf{u} \quad (5)$$

$$\frac{\partial V}{\partial x_e} = 1 \quad (6)$$

Equation (6) assumes that each element has unit volume.

2.2. Filtering

To ensure existence of solutions to the topology optimization problem and to avoid the formation of checker-board patterns (Díaz and Sigmund 1995; Jog and Haber 1996; Sigmund and Petersson 1998), some restriction on the design must be imposed. A common approach is the application of a filter to either the sensitivities or the densities.

In addition to the sensitivity filter (Sigmund 1994, 1997), which is already implemented in the 99-line code, the new 88-line code also includes density filtering (Bruns and Tortorelli 2001; Bourdin 2001).

The sensitivity filter modifies the sensitivities $\partial c/\partial x_e$ as follows:

$$\widehat{\frac{\partial c}{\partial x_e}} = \frac{1}{\max(\gamma, x_e)\sum_{i\in N_e} H_{ei}} \sum_{i\in N_e} H_{ei} x_i \frac{\partial c}{\partial x_i} \qquad (7)$$

Where, N_e is the set of elements i for which the center-to-center distance Δ (e, i) to element e is smaller than the filter radius r_{min} and H_{ei} is a weight factor which is defined as:

$$H_{ei} = \max(0, r_{\min} - \Delta(e, i))$$

γ (=10^{-3}) is a small positive number introduced newly in 88-line code in order to avoid division by zero.

The density filter transforms the original densities x_e as follows:

$$\tilde{x}_e = \frac{1}{\sum_{i\in N_e} H_{ei}} \sum_{i\in N_e} H_{ei} x_i$$

So, the sensitivities with respect to design variables x_j are obtained by:

$$\frac{\partial \psi}{\partial x_j} = \sum_{e\in N_j} \frac{\partial \psi}{\partial \tilde{x}_e}\frac{\partial \tilde{x}_e}{\partial x_j} = \sum_{e\in N_j} \frac{1}{\sum_{i\in N_e} H_{ei}} H_{je} \frac{\partial \psi}{\partial \tilde{x}_e} \qquad (10)$$

Where, the function ψ represents either the objective function or the material volume V.

3. MATLAB implementation

```
%%%% Modified by Max Yi Ren (ASU) %%%%%%%%%%%%%%%%%%%%%%%
%%%% AN 88 LINE TOPOLOGY OPTIMIZATION CODE Nov, 2010 %%%%
function top88(nelx,nely,volfrac,penal,rmin,ft)
```

```
nelx=150;
nely=100;
volfrac=0.5;
penal=3;
rmin=5;
ft = 1;
```

MATERIAL PROPERTIES

```
E0 = 1;
Emin = 1e-9;
nu = 0.3;
```

3.1. PREPARE FINITE ELEMENT ANALYSIS

```
A11 = [12 3 -6 -3; 3 12 3 0; -6 3 12 -3; -3 0 -3 12];
A12 = [-6 -3 0 3; -3 -6 -3 -6; 0 -3 -6 3; 3 -6 3 -6];
B11 = [-4 3 -2 9; 3 -4 -9 4; -2 -9 -4 -3; 9 4 -3 -4];
B12 = [ 2 -3 4 -9; -3 2 9 -2; 4 9 2 3; -9 -2 3 2];
KE = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]);
nodenrs = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx);
edofVec = reshape(2*nodenrs(1:end-1,1:end-1)+1,nelx*nely,1);
edofMat = repmat(edofVec,1,8)+repmat([0 1 2*nely+[2 3 0 1] -2 -1],nelx*nely,1);
iK = reshape(kron(edofMat,ones(8,1))',64*nelx*nely,1);
jK = reshape(kron(edofMat,ones(1,8))',64*nelx*nely,1);
% DEFINE LOADS AND SUPPORTS (HALF MBB-BEAM)
F = sparse(2,1,-1,2*(nely+1)*(nelx+1),1);
U = zeros(2*(nely+1)*(nelx+1),1);
fixeddofs = union((1:2:2*(nely+1)),(2*(nelx+1)*(nely+1)));
alldofs = (1:2*(nely+1)*(nelx+1));
freedofs = setdiff(alldofs,fixeddofs);
```

3.2. PREPARE FILTER

```
iH = ones(nelx*nely*(2*(ceil(rmin)-1)+1)^2,1);
jH = ones(size(iH));
sH = zeros(size(iH));
k = 0;
for i1 = 1:nelx
 for j1 = 1:nely
 e1 = (i1-1)*nely+j1;
 for i2 = max(i1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-1),nelx)
 for j2 = max(j1-(ceil(rmin)-1),1):min(j1+(ceil(rmin)-1),nely)
 e2 = (i2-1)*nely+j2;
```

```
 k = k+1;
 iH(k) = e1;
 jH(k) = e2;
 sH(k) = max(0,rmin-sqrt((i1-i2)^2+(j1-j2)^2));
 end
 end
 end
end
H = sparse(iH,jH,sH);
Hs = sum(H,2);
%INITIALIZE ITERATION
x = repmat(volfrac,nely,nelx);
xPhys = x;
loop = 0;
change = 1;
% START ITERATION
while change > 0.01
```

```
loop = loop + 1;
```

3.1.1. FE-ANALYSIS

```
sK= reshape(KE(:)*(Emin+xPhys(:)'.^penal*(E0-Emin)),64*nelx*nely,1);
 K = sparse(iK,jK,sK); K = (K+K')/2;
 U(freedofs) = K(freedofs,freedofs)\F(freedofs);
```

3.3. OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS

```
ce = reshape(sum((U(edofMat)*KE).*U(edofMat),2),nely,nelx); % element-wise strain energy
 c = sum(sum((Emin+xPhys.^penal*(E0-Emin)).*ce)); % total strain energy
 dc = -penal*(E0-Emin)*xPhys.^(penal-1).*ce; % design sensitivity
 dv = ones(nely,nelx);
```

3.2.1 FILTERING/MODIFICATION OF SENSITIVITIES

3.2.1.1. Heaviside projection filter

```
ft=heaviside(x);
 if ft == 2
 dc(:) = H*(x(:).*dc(:))./Hs./max(1e-3,x(:));
 elseif ft == 3
 dc(:) = H*(dc(:)./Hs);
 dv(:) = H*(dv(:)./Hs);
 end
```

3.3.1. OPTIMALITY CRITERIA UPDATE OF DESIGN VARIABLES AND PHYSICAL DENSITIES

```
l1 = 0; l2 = 1e9; move=0.2;
```

3.3.2. Passive Element

```
passive = zeros(nely,nelx);
for i = 1:nelx
    for j = 1:nely
        if sqrt((j-nely/2)^2+(i-nelx/3)^2) < nely/3
        passive(j,i) = 1;
    end
end
end

 while (l2-l1)/(l1+l2) > 1e-3
 lmid = 0.5*(l2+l1);
 xnew = max(0,max(x-move,min(1,min(x+move,x.*sqrt(-dc./dv/lmid)))));
 if ft == 1
 xPhys = xnew;
 elseif ft==2
 xPhys(:) = (H*xnew(:))./Hs;
 end
 xPhys(passive==1) = 0;
 xPhys(passive==2) = 1;
 if sum(xPhys(:)) > volfrac*nelx*nely, l1 = lmid;
 else
 l2 = lmid;
 end
 end
 change = max(abs(xnew(:)-x(:)));
 x = xnew;
```

4. PRINT RESULTS

```
fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f ch.:%7.3f\n',loop,c, ...
 mean(xPhys(:)),change);
```

```
 It.:    1 Obj.:   267.2958 Vol.:  0.500 ch.:  0.200

 It.:    2 Obj.:   163.7187 Vol.:  0.500 ch.:  0.200

 It.:    3 Obj.:    91.4011 Vol.:  0.500 ch.:  0.200

 It.:    4 Obj.:    69.7513 Vol.:  0.500 ch.:  0.200

 It.:    5 Obj.:    64.1504 Vol.:  0.500 ch.:  0.200

 It.:    6 Obj.:    61.8320 Vol.:  0.500 ch.:  0.200

 It.:    7 Obj.:    61.8320 Vol.:  0.500 ch.:  0.200

 It.:    8 Obj.:    61.8320 Vol.:  0.500 ch.:  0.200

 It.:    9 Obj.:    61.8320 Vol.:  0.500 ch.:  0.200

 It.:   10 Obj.:    61.8320 Vol.:  0.500 ch.:  0.200
```

```
It.:   11 Obj.:    61.8320 Vol.:  0.500 ch.:  0.000
```

PLOT DENSITIES

```
image1 = colormap(gray); imagesc(1-xPhys); caxis([0 1]); axis equal; axis off; drawnow;
% Save as png with a resolution of 150 pixels per inch
vwObj = VideoWriter('myfile');
open(vwObj);
f.cdata = image1;
f.colormap = jet(256);
% The colormap will be applied before writing the data to the MPEG4 file
writeVide;o(vwObj, f);
close(vwObj);

end

end
```

4. Results

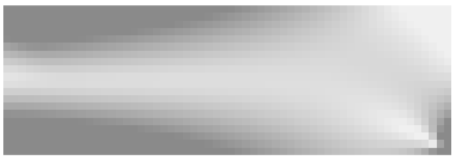
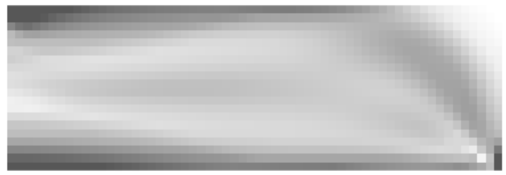
4.1. Without Passive Element:

The below mentioned values are changed in the code:

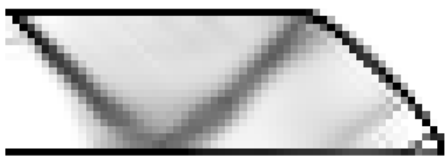
```
nelx=60;
nely=20;
volfrac=0.26;
penal=3;
rmin=1.5;
ft = 1;
```

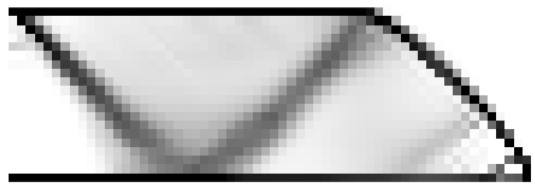
```
It.:    1 Obj.:  7161.9115 Vol.:  0.260 ch.:  0.200
It.:    2 Obj.:  3927.4154 Vol.:  0.260 ch.:  0.200
It.:    3 Obj.:  2511.7836 Vol.:  0.260 ch.:  0.200
It.:    4 Obj.:  1876.5459 Vol.:  0.260 ch.:  0.200
It.:    5 Obj.:  1488.7916 Vol.:  0.260 ch.:  0.200
It.:    6 Obj.:  1248.8078 Vol.:  0.260 ch.:  0.200
It.:    7 Obj.:  1097.5088 Vol.:  0.260 ch.:  0.200
It.:    8 Obj.:   961.3632 Vol.:  0.260 ch.:  0.200
It.:    9 Obj.:   826.1684 Vol.:  0.260 ch.:  0.200
It.:   10 Obj.:   697.1953 Vol.:  0.260 ch.:  0.200
It.:   11 Obj.:   697.1953 Vol.:  0.260 ch.:  0.200
It.:   12 Obj.:   697.1953 Vol.:  0.260 ch.:  0.200
It.:   13 Obj.:   697.1953 Vol.:  0.260 ch.:  0.200
It.:   14 Obj.:   697.1953 Vol.:  0.260 ch.:  0.200
It.:   15 Obj.:   697.1953 Vol.:  0.260 ch.:  0.000
```

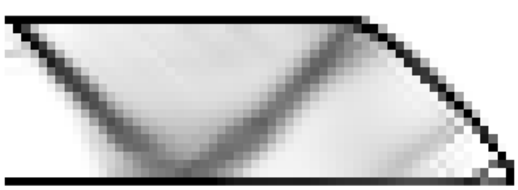
The below list of figures shows the flow of how optimized design of the beam without passive element is obtained.

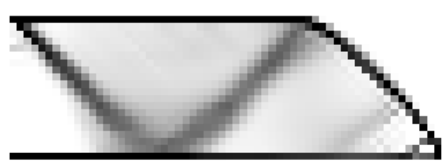





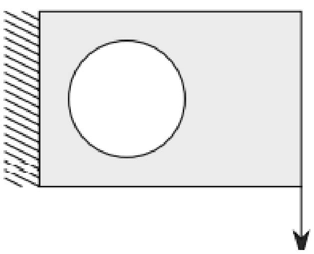




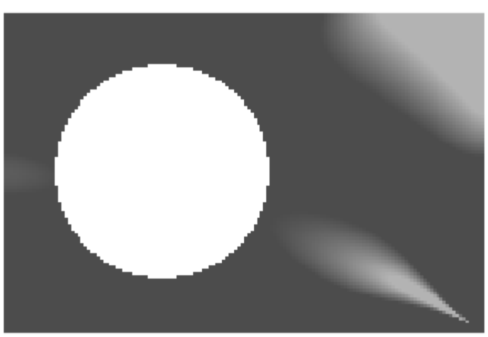


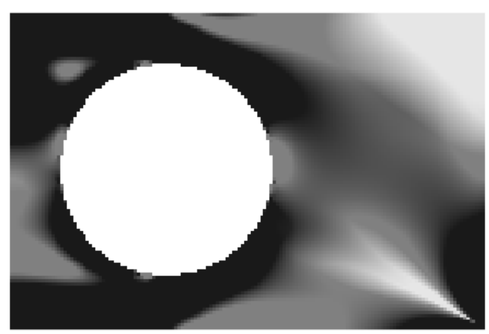


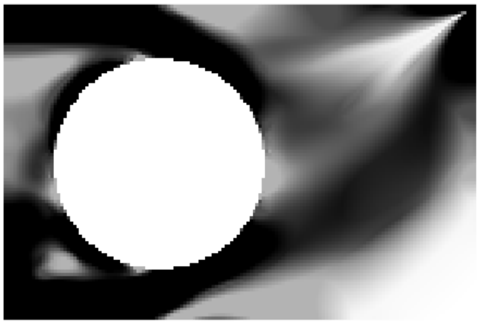
4.2. With Passive Element:

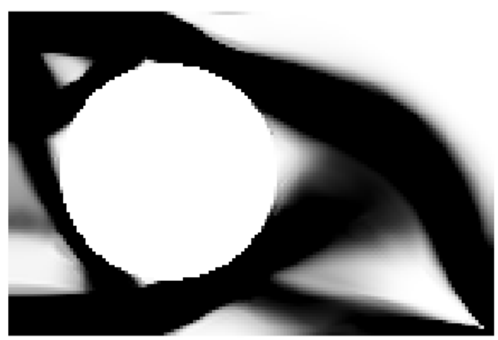


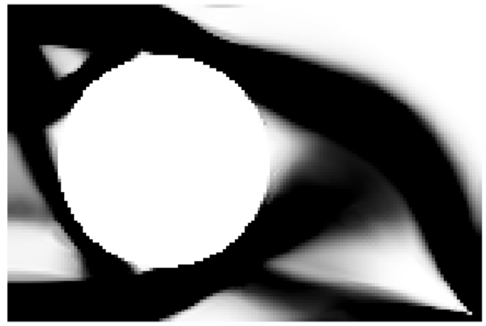
The below list of figures shows the flow of how optimized design of the beam with passive element added to it is obtained.











5. Conclusion

This study shows how topology optimization can be implemented using Dr. Sigmund's 88-line MATLAB code on a beam with and without passive element. Through this study, it is observed that 88-line code has more computational efficiency than that of 99-line code (see appendix). An improvement in speed of implementing the TO algorithm is observed between 88-line code and 99-line code.

6. References

1. Topology Optimization tutorial by Yi (Max) Ren
2. https://github.com/DesignInformaticsLab/DesignOptimization2021Fall/blob/292523767a03eb6974ab9e0058731ed5dae5f0f5/Project/Project%203%20topology%20optimization.ipynb
3. Efficient topology optimization in MATLAB using 88 lines of code by Erik Andreassen, Anders Clausen, Mattias Schevenels, Boyan S. Lazarov and Ole Sigmund
4. A 99 line topology optimization code written in MATLAB by Ole Sigmund.

7. Appendix – Dr. Sigmund's 99-line code

```
%%%% A 99 LINE TOPOLOGY OPTIMIZATION CODE BY OLE SIGMUND, OCTOBER 1999
 %%%
function P3_99lines(nelx,nely,volfrac,penal,rmin)
% INITIALIZE
nelx=60;
nely=20;
% nelz = 4;
volfrac=0.3;
penal=3;
rmin=1.5;
x(1:nely,1:nelx) = volfrac;
loop = 0;
change = 1.;
% START ITERATION
while change > 0.01
loop = loop + 1;
xold = x;
% FE-ANALYSIS
[U]=FE(nelx,nely,x,penal);
% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
[KE] = lk;
c = 0.;
for ely = 1:nely
    for elx = 1:nelx
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)* elx +ely;
        Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2;
 2*n1+1;2*n1+2],1);
        c = c + x(ely,elx)^penal*Ue'*KE*Ue;
        dc(ely,elx) = -penal*x(ely,elx)^(penal-1)*Ue'*KE*Ue;
    end
end
% FILTERING OF SENSITIVITIES
[dc] = check(nelx,nely,rmin,x,dc);
% DESIGN UPDATE BY THE OPTIMALITY CRITERIA METHOD
[x] = OC(nelx,nely,x,volfrac,dc);
% PRINT RESULTS
change = max(max(abs(x-xold)));
fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f ch.:%7.3f\n',loop,c, ...
 sum(sum(x))/(nelx*nely),change);
% disp(['It.: 'sprintf('%4i',loop)' Obj.: 'sprintf('%10.4f',c)' ...
%     'Vol.: ' sprintf('%6.3f',sum(sum(x))/(nelx*nely)) ...
% 'ch.: ' sprintf('%6.3f',change)]);
% PLOT DENSITIES
colormap(gray); imagesc(-x); axis equal; axis tight;
 axis off;pause(1e-6);
end
end
%%%%%%%%%% OPTIMALITY CRITERIA UPDATE %%%%%%%%%
function [xnew]=OC(nelx,nely,x,volfrac,dc)
```

```
l1 = 0; l2 = 100000; move = 0.2;
while (l2-l1 > 1e-4)
lmid = 0.5*(l2+l1);
xnew = max(0.001,max(x-move,min(1.,min(x+move,x.*sqrt(-dc./lmid)))));
if sum(sum(xnew)) - volfrac*nelx*nely > 0
    l1 = lmid;
else
    l2 = lmid;
end
end
end
%%%%%%%%%% MESH-INDEPENDENCY FILTER %%%%%%%%%%
function [dcn]=check(nelx,nely,rmin,x,dc)
dcn=zeros(nely,nelx);
for i = 1:nelx
    for j = 1:nely
        sum=0.0;
        for k = max(i-round(rmin),1):min(i+round(rmin),nelx)
            for l = max(j-round(rmin),1):min(j+round(rmin), nely)
                fac = rmin-sqrt((i-k)^2+(j-l)^2);
                sum = sum+max(0,fac);
                dcn(j,i) = dcn(j,i) + max(0,fac)*x(l,k)*dc(l,k);
            end
        end
        dcn(j,i) = dcn(j,i)/(x(j,i)*sum);
    end
end
end
%%%%%%%%%% FE-ANALYSIS %%%%%%%%%%%
function [U]=FE(nelx,nely,x,penal)
        [KE] = lk;
        K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
        F = sparse(2*(nely+1)*(nelx+1),1); U = sparse(2*(nely+1)*(nelx
+1),1);
        for ely = 1:nely
            for elx = 1:nelx
                n1 = (nely+1)*(elx-1)+ely;
                n2 = (nely+1)* elx +ely;
                edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2;
 2*n2+1;2*n2+2;2*n1+1; 2*n1+2];
                K(edof,edof) = K(edof,edof) + x(ely,elx)^penal*KE;
            end
        end
% DEFINE LOADSAND SUPPORTS(HALF MBB-BEAM)
F(2,1) = -1;
fixeddofs = union([1:2:2*(nely+1)],[2*(nelx+1)*(nely+1)]);
alldofs = [1:2*(nely+1)*(nelx+1)];
freedofs = setdiff(alldofs,fixeddofs);
% SOLVING
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
U(fixeddofs,:)= 0;
end
%%%%%%%%%% ELEMENT STIFFNESS MATRIX %%%%%%%
function [KE]=lk
```

```
    E = 1.;
    nu = 0.3;
    k=[ 1/2-nu/6 1/8+nu/8 -1/4-nu/12 -1/8+3*nu/8 ...
        -1/4+nu/12 -1/8-nu/8 nu/6 1/8-3*nu/8];
    KE = E/(1-nu^2)*...
        [ k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)
        k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3)
        k(3) k(8) k(1) k(6) k(7) k(4) k(5) k(2)
        k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)
        k(5) k(6) k(7) k(8) k(1) k(2) k(3) k(4)
        k(6) k(5) k(4) k(3) k(2) k(1) k(8) k(7)
        k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)
        k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];
end
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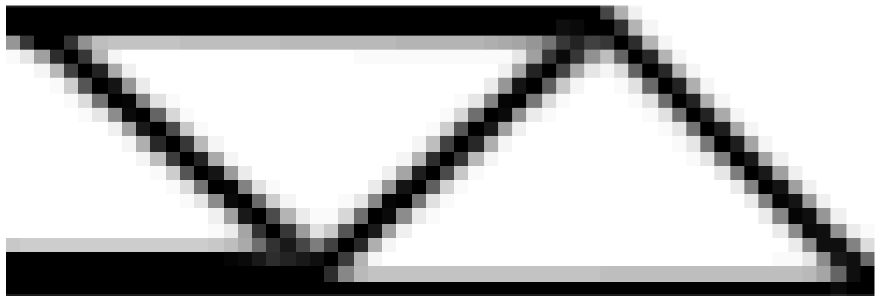

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