MAE 598 Design Optimization - HW-5 - SQP ALGORITHM

```
%%%%%%%%%%%%%% Sequential Quadratic Programming Implementation with
 BFGS %%%%%%%%%%%%%%%%
%%%%%%%%%%%%%% By Max Yi Ren and Emrah Bayrak %%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
function solution = mysqp(f, df, g, dg, x0, opt)
    % Set initial conditions

    x = x0; % Set current solution to the initial guess

    % Initialize a structure to record search process
    solution = struct('x',[]);
    solution.x = [solution.x, x]; % save current solution to
 solution.x

    % Initialization of the Hessian matrix
    W = eye(numel(x));             % Start with an identity Hessian
 matrix
    % Initialization of the Lagrange multipliers
    mu_old = zeros(size(g(x)));    % Start with zero Lagrange
 multiplier estimates
    % Initialization of the weights in merit function
    w = zeros(size(g(x)));         % Start with zero weights

    % Set the termination criterion
    gnorm = norm(df(x) + mu_old'*dg(x)); % norm of Largangian gradient

    while gnorm>opt.eps % if not terminated

        % Implement QP problem and solve
        if strcmp(opt.alg, 'myqp')
            % Solve the QP subproblem to find s and mu (using your own
 method)
            [s, mu_new] = solveqp(x, W, df, g, dg);
        else
            % Solve the QP subproblem to find s and mu (using MATLAB's
 solver)
            qpalg = optimset('Algorithm', 'active-
set', 'Display', 'off');
            [s,~,~,~,lambda] = quadprog(W,[df(x)]', dg(x), -g(x), [],
 [], [], [], x, qpalg);
            mu_new = lambda.ineqlin;
        end

        % opt.linesearch switches line search on or off.
        % You can first set the variable "a" to different constant
 values and see how it
        % affects the convergence.
        if opt.linesearch
```

```
            [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w);
        else
            a = 0.1;
        end

        % Update the current solution using the step
        dx = a*s;                % Step for x
        x = x + dx;              % Update x using the step

        % Update Hessian using BFGS. Use equations (7.36), (7.73) and
 (7.74)
        % Compute y_k
        y_k = [df(x) + mu_new'*dg(x) - df(x-dx) - mu_new'*dg(x-dx)]';
        % Compute theta
        if dx'*y_k >= 0.2*dx'*W*dx
            theta = 1;
        else
            theta = (0.8*dx'*W*dx)/(dx'*W*dx-dx'*y_k);
        end
        % Compute  dg_k
        dg_k = theta*y_k + (1-theta)*W*dx;
        % Compute new Hessian
        W = W + (dg_k*dg_k')/(dg_k'*dx) - ((W*dx)*(W*dx)')/(dx'*W*dx);

        % Update termination criterion:
      gnorm = norm(df(x) + mu_new'*dg(x)); % norm of Largangian
 gradient
      mu_old = mu_new;

        % save current solution to solution.x
      solution.x = [solution.x, x];
    end
end
```

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QP subproblem

The following code solves the QP subproblem using active set strategy

```
function [s, mu0] = solveqp(x, W, df, g, dg)
    % Implement an Active-Set strategy to solve the QP problem given
 by
    % min     (1/2)*s'*W*s + c'*s
    % s.t.    A*s-b <= 0
    %
    % where As-b is the linearized active contraint set

    % Strategy should be as follows:
    % 1-) Start with empty working-set
    % 2-) Solve the problem using the working-set
    % 3-) Check the constraints and Lagrange multipliers
    % 4-) If all constraints are staisfied and Lagrange multipliers
 are positive, terminate!
    % 5-) If some Lagrange multipliers are negative or zero, find the
 most negative one
    %     and remove it from the active set
    % 6-) If some constraints are violated, add the most violated one
 to the working set
    % 7-) Go to step 2


    % Compute c in the QP problem formulation
    c = [df(x)]';

    % Compute A in the QP problem formulation
    A0 = dg(x);

    % Compute b in the QP problem formulation
    b0 = -g(x);

    % Initialize variables for active-set strategy
    stop = 0;           % Start with stop = 0
    % Start with empty working-set
    A = [];         % A for empty working-set
    b = [];         % b for empty working-set
    % Indices of the constraints in the working-set
    active = [];    % Indices for empty-working set

    while ~stop  % Continue until stop = 1
        % Initialize all mu as zero and update the mu in the working
 set
        mu0 = zeros(size(g(x)));

        % Extact A corresponding to the working-set
        A = A0(active,:);
        % Extract b corresponding to the working-set
        b = b0(active);
```

```
        % Solve the QP problem given A and b
        [s, mu] = solve_activeset(x, W, c, A, b);
        % Round mu to prevent numerical errors (Keep this)
        mu = round(mu*1e12)/1e12;

        % Update mu values for the working-set using the solved mu
values
        mu0(active) = mu;

        % Calculate the constraint values using the solved s values
        gcheck = A0*s-b0;

        % Round constraint values to prevent numerical errors (Keep
this)
        gcheck = round(gcheck*1e12)/1e12;

        % Variable to check if all mu values make sense.
        mucheck = 0;        % Initially set to 0

        % Indices of the constraints to be added to the working set
        Iadd = [];              % Initialize as empty vector
        % Indices of the constraints to be added to the working set
        Iremove = [];           % Initialize as empty vector

        % Check mu values and set mucheck to 1 when they make sense
        if (numel(mu) == 0)
            % When there no mu values in the set
            mucheck = 1;         % OK
        elseif min(mu) > 0
            % When all mu values in the set positive
            mucheck = 1;         % OK
        else
            % When some of the mu are negative
            % Find the most negaitve mu and remove it from acitve set
            [~,Iremove] = min(mu);  % Use Iremove to remove the
constraint
        end

        % Check if constraints are satisfied
        if max(gcheck) <= 0
            % If all constraints are satisfied
            if mucheck == 1
                % If all mu values are OK, terminate by setting stop =
1
                stop = 1;
            end
        else
            % If some constraints are violated
            % Find the most violated one and add it to the working set
            [~,Iadd] = max(gcheck); % Use Iadd to add the constraint
        end
        % Remove the index Iremove from the working-set
        active = setdiff(active, active(Iremove));
```

```
        % Add the index Iadd to the working-set
        active = [active, Iadd];

        % Make sure there are no duplications in the working-set (Keep
 this)
        active = unique(active);
    end
end
```

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Armijo line search

The following code performs line search on the merit function

```
function [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w_old)
    t = 0.1; % scale factor on current gradient: [0.01, 0.3]
    b = 0.8; % scale factor on backtracking: [0.1, 0.8]
    a = 1; % maximum step length

    D = s;                  % direction for x

    % Calculate weights in the merit function using eaution (7.77)
    w = max(abs(mu_old), 0.5*(w_old+abs(mu_old)));
    % terminate if line search takes too long
    count = 0;
    while count<100
        % Calculate phi(alpha) using merit function in (7.76)
        phi_a = f(x + a*D) + w'*abs(min(0, -g(x+a*D)));

        % Caluclate psi(alpha) in the line search using phi(alpha)
        phi0 = f(x) + w'*abs(min(0, -g(x)));        % phi(0)
        dphi0 = df(x)*D + w'*((dg(x)*D).*(g(x)>0)); % phi'(0)
        psi_a = phi0 +  t*a*dphi0;                  % psi(alpha)
        % stop if condition satisfied
        if phi_a<psi_a;
            break;
        else
            % backtracking
            a = a*b;
            count = count + 1;
        end
    end
end
```

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```
function [s, mu] = solve_activeset(x, W, c, A, b)
    % Given an active set, solve QP

    % Create the linear set of equations given in equation (7.79)
    M = [W, A'; A, zeros(size(A,1))];
    U = [-c; b];
    sol = M\U;          % Solve for s and mu

    s = sol(1:numel(x));                % Extract s from the solution
    mu = sol(numel(x)+1:numel(sol));    % Extract mu from the solution

end
```

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Optimization Problem

```
clear;
close all;
% Here we specify the objective function by giving the function handle
 to a
% variable, for example:
f = @(x)(x(1)^2+(x(2)-3)^2); % replace with your objective function
% In the same way, we also provide the gradient of the
% objective:
df = @(x)([2*x(1),2*x(2)-6]); % replace accordingly

g = @(x)([x(2)^2-2*x(1);(x(2)-1)^2+5*x(1)-15]);
dg = @(x)([-2 2*x(2);5 2*x(2)-2]);

% Note that explicit gradient and Hessian information is only
 optional.
% However, providing these information to the search algorithm will
 save
% computational cost from finite difference calculations for them.

% % Specify algorithm
opt.alg = 'matlabqp'; % 'myqp' or 'matlabqp'

% Turn on or off line search. You could turn on line search once other
% parts of the program are debugged.
opt.linesearch = true; % false or true

% Set the tolerance to be used as a termination criterion:
opt.eps = 1e-3;

% Set the initial guess:
x0 = [1;1];

% Feasibility check for the initial point.
if max(g(x0)>0)
    errordlg('Infeasible intial point! You need to start from a
 feasible one!');
    return
end
% Run optimization using SQP algorithm
% Run your implementation of SQP algorithm. See mysqp.m
solution = mysqp(f, df, g, dg, x0, opt);
```

Report

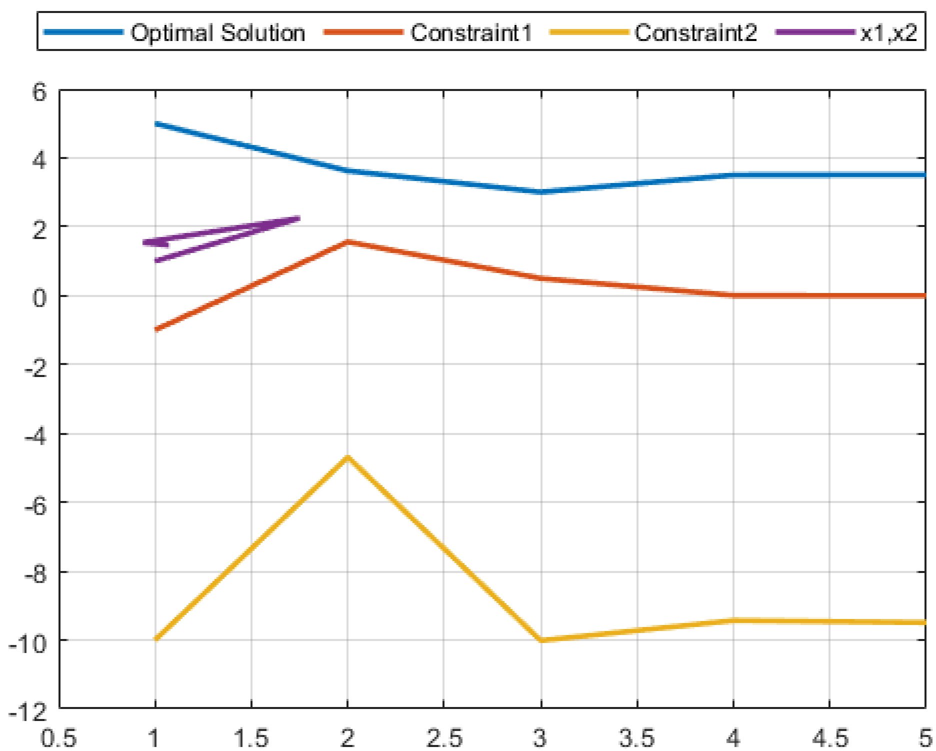
```
%report(solution,f,g);
N = length(solution.x);
for n = 1:N
    optS(n) = f(solution.x(:, n));  % x1, x2 values
    C = g(solution.x(:, n));  % Constaint values
    C_1(n) = C(1);  % g1 constraint
```

```
    C_2(n) = C(2);  % g2 constraint
end
% Optimal values for x1 and x2
fprintf("x1 = %f\n",solution.x(1,end))
fprintf("x2 = %f\n",solution.x(2, end));
% Optimal Solution f(x1,x2)
fprintf("Optimal Solution f(x1, x2) = %f\n",optS(end));
% Constraints, g1 and g2
fprintf("g1(x1, x2) = %f\n",C_1(end));
fprintf("g2(x1, x2) = %f\n",C_2(end));

% Plots
niter = 1:N; % No: of iterations for plotting

figure(1)
plot(niter, optS,niter, C_1,niter,
 C_2,solution.x(1, :),solution.x(2,:),'LineWidth',2)
grid on
legend('Optimal
 Solution','Constraint1','Constraint2','x1,x2','Location','northoutside','Orientat
```

```
x1 = 1.060417
x2 = 1.456336
Optimal Solution f(x1, x2) = 3.507384
g1(x1, x2) = 0.000080
g2(x1, x2) = -9.489673
```



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