Title: Game Tree Searching by Min/Max Approximation

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Summary and Results:

The goal of the paper is to present another technique for searching game trees. The technique relies on the approximation of the min and max functions. Central to the paper is the concept of generalized means or power means and how its values can provide a good approximation to the min and max functions.

The proposed technique uses a single static evaluator using iterative heuristics to expand the tree one step at a time with the game assumed as nonpathological—meaning the increase in depth does not adversely affect the accuracy. The proposed method also uses a penalty-based approach where more penalty weight is given against bad moves.

The game used for the experiments is Connect-Four and the techniques were given a set amount of resource for the move computation for each turn. The approach suggested is said to produce better results than simple minimax search with alpha-beta pruning for the same number of calls but only when CPU time is not limited. When time was the determining factor minimax search with alpha-beta pruning was still the superior approach. In the experiment, alpha-beta pruning had 3500 calls to the move operator per second versus the 800 calls to the move operator per second of the min/max approximation approach.

The paper discusses some interesting points about penalty-based schemes. Penalty-based schemes tend to consume more resources, particularly memory, since it is an iterative scheme requiring keeping the tree in memory, it goes back and forth between the root and leaves and may tend to waste time examining bad or non-optimal moves. Penalty-based schemes are less directed towards selecting the best move and more towards improving the estimate.

To conclude, the suggested approach outplays minimax with alpha-beta pruning if the limit is the number of calls but will lose when time bound since there is a higher overhead with the approximation technique.