Recurrence quantification analysis: Nonlinear wave dynamics in the Kuramoto-Sivashinsky equation, Response of the Tri-pendulum and FlanSea WEC and Extreme Events

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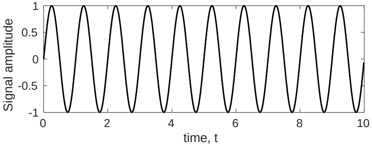
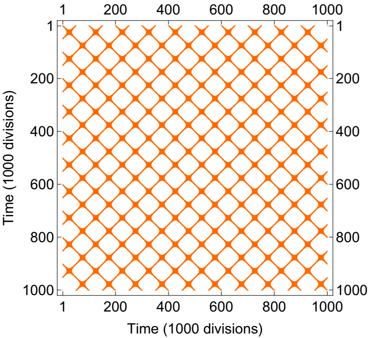
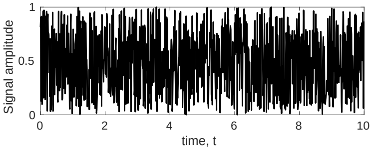


(1) Sine wave

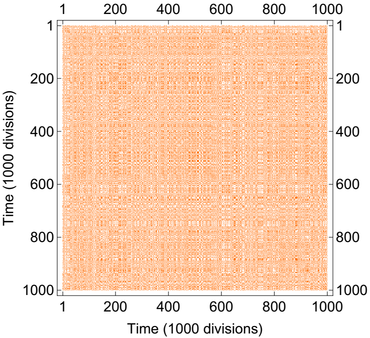


(2) Recurrence plot of Sine wave

Figure I: Recurrence plot for periodic signal/system. Orderly signal leads to an orderly tesselation in the recurrence plot. Frequency of the signal is matched by the criss-cross patterns in the recurrence plot.



(1) Gaussian noise



(2) Recurrence plot of Gaussian noise

Figure II: Recurrence plot for noisy system. Disordered, noisy signal leads to disordered recurrence plot. No discernible features exist in the recurrence plot.

Transitions captured via recurrence plots

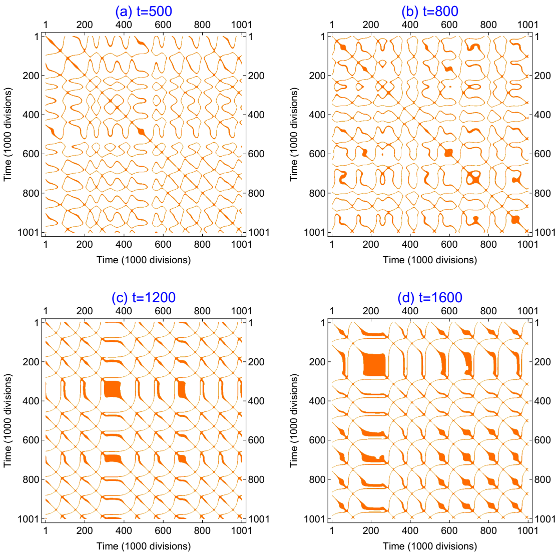


Figure 3: Nonlinear transitions in the Kuramoto-Sivashinsky wave dynamics.

Transitions captured via recurrence plots

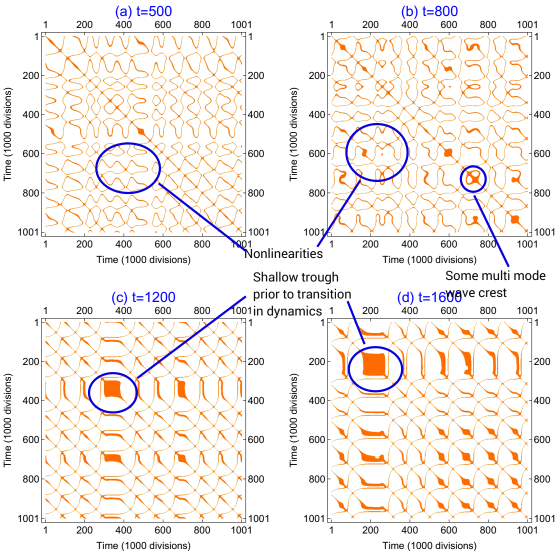


Figure 4: Nonlinear transitions in the Kuramoto-Sivashinsky wave dynamics. Various event signatures.

Objective of work

- What are recurrence plots (RPs) and what is recurrence quantification analysis (RQA)?
- Demonstration of RP/RQA :
  - Kuramoto-Sivashinky nonlinear wave PDE and transition dynamics
  - Devices interacting with waves:
    - Point mass/buoy in Brownian ocean (Stochastic oscillator)
    - Tri-pendulum type PTO device
    - FlanSea Point absober type PTO device
    - Control of deterministic chaos using RQA
  - Supplementary: Extreme wave events and quantification using RP/RQA

The recurrence plot matrix [4], $R_{i,j}$ for a signal (time-series data) is calculated using the equation, **for a given window of time**:

$$R_{i,j} = \Theta(\epsilon_i - \|\vec{x}_i - \vec{x}_j\|) \tag{1}$$

When a dynamical system roughly revisits a previous state (within ϵ), the recurrence matrix, which is a square matrix, is populated with '1' at that time instance.

The $R_{i,j}$ binary, square matrix provides a visually appealing explanation of dynamical evolution of a system. When coupled with recurrence quantification measures, it allows for the study of simultaneous interplay of dynamical forces or interactions between sub-systems.

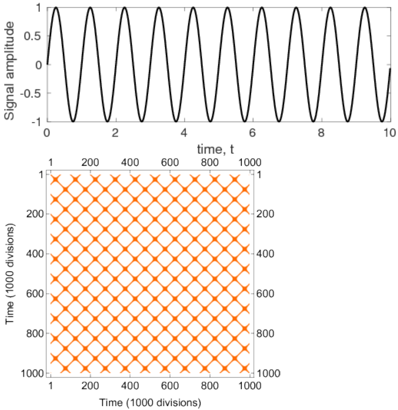
Recurrence Quantification Analysis (RQA)

When RPs are coupled with recurrence quantification analysis (RQA) [2, 5] measures, a quantitative "signature" for the system under consideration and it's transitions can be documented.

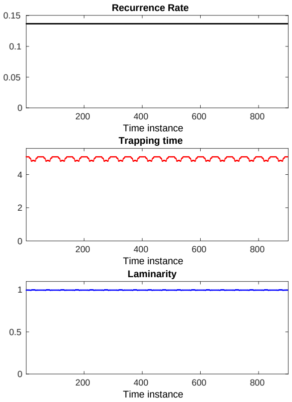
RQA measures considered in this study:

- Recurrence rate: rate of recurrence of current time-window or epoch of phenomena in comparison to previous dynamics
- Laminarity: predictability and linearity of dynamics
- Trapping time: how long an event persists or how long a system stays trapped in a certain state.

RQA: Sine wave, a deterministic signal



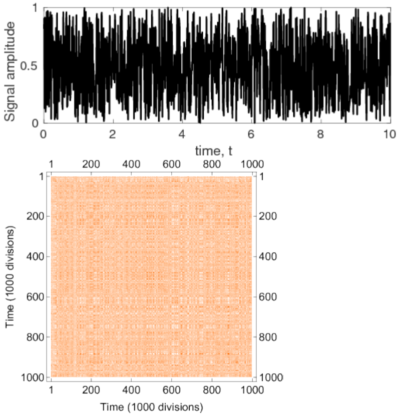
(a) Sine wave



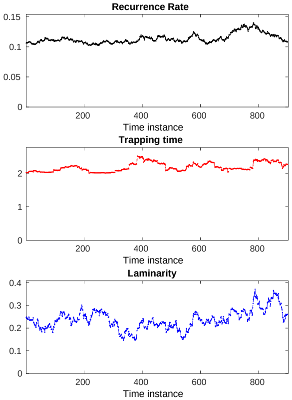
(b) Recurrence quantification of Sine wave

Figure 5: RQA for a periodic system shows constant rate of recurrence, high laminarity and periodic trapping time.

RQA: Gaussian noise, not deterministic



(a) Gaussian noise



(b) Recurrence quantification of Gaussian noise.

Figure 6: RQA for Gaussian noise shows fluctuating rate of recurrence and trapping time, low and fluctuating laminarity.

Conclusions on RP/RQA

- Recurrence plots/RP:
  - Binary matrix ('1' and '0' are entries)
  - Suitable qualitative measure to gauge periodicity or deterministic behaviour.
- Recurrence quantification analysis/RQA:
  - Multiple measures available: Recurrence rate and laminarity are primarily used in this study
  - Quantitative data to augment the qualitative nature of RP

RQA Parameters: establishing a threshold

- Values of RQA parameters should first be established using either an empirical or heuristic set of RQA windowing parameters.
- Once a set of RQA windowing parameters have been established, these may be used as a reference while making future measurements of the system in question.
- RQA Parameters:
  - Cut-off distance, ϵ: 0.2 for numerical systems considered here, 3.0 m for wave events captured by buoy 42040
  - RQA window length: 100 time steps
  - Norming operation: Euclidean norm

Establishing RQA signature for dynamical transitions in the Kuramoto-Sivashinsky PDE

$$\delta_i(t)\frac{\partial^3 u(t,x)}{\partial x^3} + \frac{\partial^4 u(t,x)}{\partial x^4} + \frac{\partial^2 u(t,x)}{\partial x^2} + \frac{\partial u(t,x)}{\partial t} + u(t,x)\frac{\partial u(t,x)}{\partial x} = 0$$

Recurrence Quantification Analysis of the Kuramoto-Sivashinky (KS) equation

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$$\textcolor{blue}{\delta_i(t)}\underbrace{\frac{\partial^3 u(t,x)}{\partial x^3}}_{\text{dispersive}} + \underbrace{\frac{\partial^4 u(t,x)}{\partial x^4}}_{\text{stabilizing}} + \underbrace{\frac{\partial^2 u(t,x)}{\partial x^2}}_{\text{destabilizing energy prod.}} + \frac{\partial u(t,x)}{\partial t} + \underbrace{u(t,x)\frac{\partial u(t,x)}{\partial x}}_{\text{Transfer of energy from LW to SW}} = 0$$

$$u(0,x) = e^{-(x-a)^2} + e^{-(a+x)^2},\ u(t,-x_0) = u(t,x_0),\ a \in \mathbb{R} \tag{2}$$

Where the dispersion parameter is given as,

$$\textcolor{blue}{\delta_i(t)} = \begin{cases} 0.0 & t \leq 500 \\ 0.15 & 500 < t \leq 1000 \\ 1.0 & 1000 < t \leq 1500 \\ -1.0 & t > 1500 \end{cases}$$

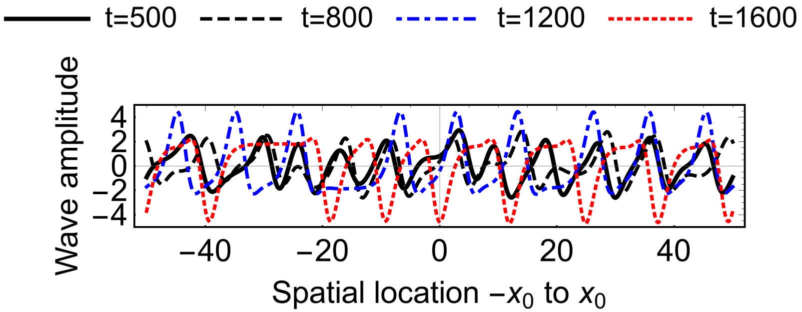

Figure 7: KS wave: transitional dynamics [3] describing the formation of dissipative structures at t≥1200, through the introduction of step changes in δ_i. At $t \geq 1200$, **soliton ($\approx$ sech, tanh functions of time) like dissipative structures** emerge (blue dashed line).

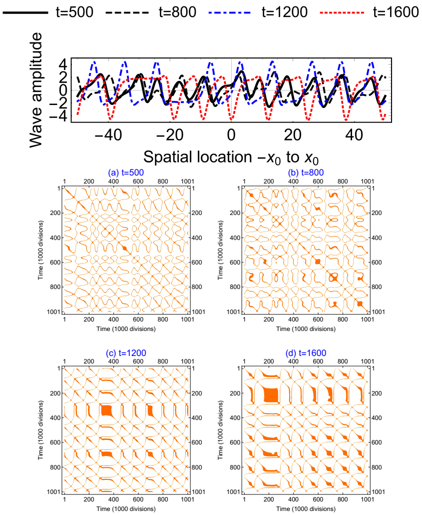

Figure 8: KS wave: transitional dynamics describing the formation of dissipative structures at t≥1200, through the introduction of step changes in δ_i.

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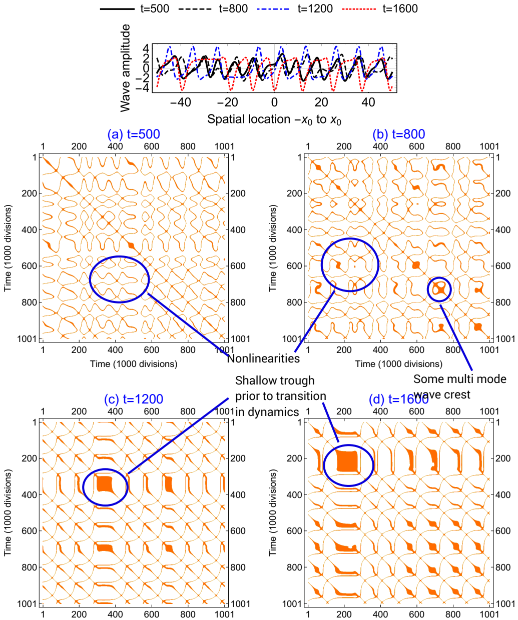

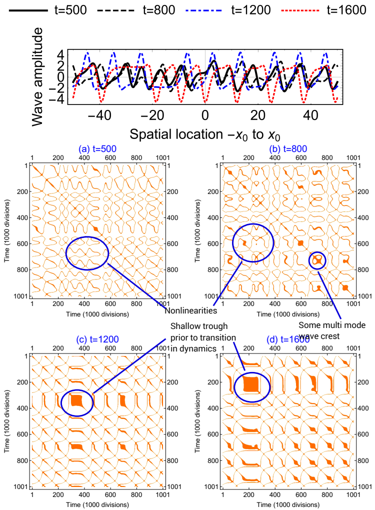

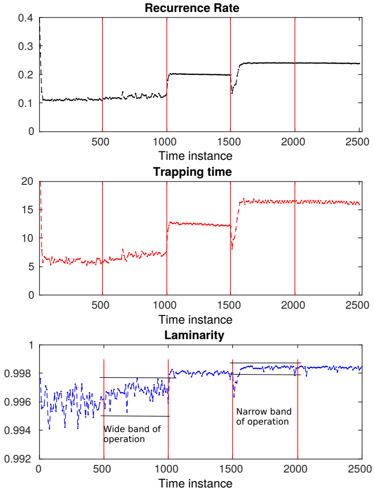

KS wave: transitional dynamics observed through RQA parameters. Change in dynamical behaviour leads to a "jump" of RQA params. **Regular soliton-like structures show narrow band of high laminarity** of operations suggesting deterministic behaviour.

Observations: detail

- the RQA that indicators such as recurrence rate, trapping time and laminarity suffer a marked change:
  - just before the appearance of spatially homogeneous structures or
  - transition from one type of wave structure to another that may show a change in amplitude
- As the value of $|\delta_i|$ is increased,
  - larger amplitude structures arise with an increasing value of laminarity along with a more narrow band of operation as noticeable for $t > 1000$
  - There is also an increase in the recurrence rate as stable structures form.
  - It is observed that all RQA measures first have a decline in their value once $|\delta_i|$ is changed. The author speculates based on the numerical stiffness of the KS equation, that this is a response of the system to adjust to a new state as defined by the new $|\delta_i|$.

Key Fact

RQA parameters suffer changes in their values and band of operation, as different dynamical states are achieved by the KS equation. Could a change in RQA parameter be used to trigger a control force?

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The Logistic map, viz., $x_{n+1} = \mu x_n (1 - x_n)$ is used since it is a prominent problem in deterministic chaos. For the purpose of this presentation, the logistic equation is used as a surrogate model for some system level behaviour of WECs.

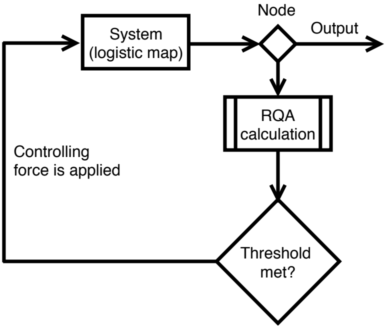

Figure 9: Control strategy: At a time instance when recurrence rate exceeds a threshold value of 0.95, a stabilizing control is triggered and the system is "nudged" along a different trajectory.

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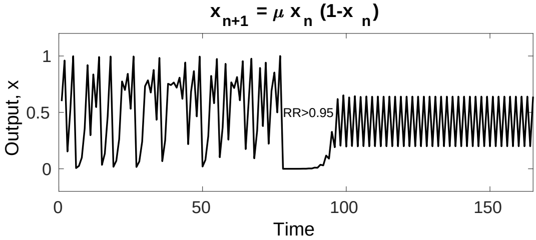
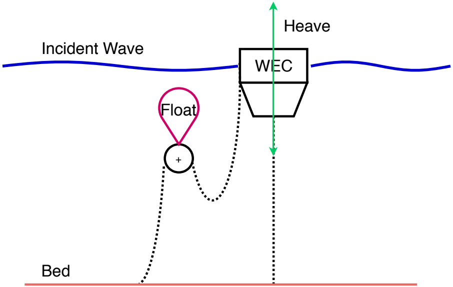


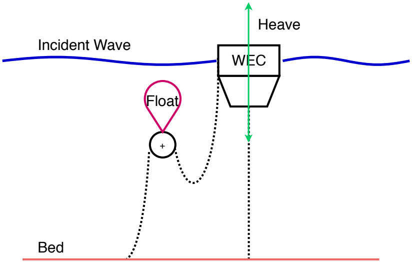
Figure 10: Control strategy: At a time instance when recurrence rate exceeds a threshold value of 0.95, a stabilizing control is triggered and the system is "nudged" along a different trajectory.

RQA of FlanSea Point Absorber type WEC



- For the purpose of illustration of RQA on WEC, the FlanSea WEC [1] is considered (`http://www.flansea.eu/`).
- This point-absorber type WEC is incident upon by a Grestner type Trochoidal wave and it's heave response $(z, abs(\dot{z}))$along with the RP is shown.

FlanSea: Equation of Motion



$$m\,\ddot{z} = -f_{\text{Weight}} + f_{\text{Buoyancy}} + f_{\text{Downward drag}} + \tag{3}$$

$$f_{\text{Reactive force of machinery}} + f_{\text{Grestner wave excitation}} \tag{4}$$

Radiation damping and added mass terms have not been taken into account in this preliminary investigation but will be included in future work. Other incident wave types such as that from a Pierson-Moskowitz spectrum and the modulated KS wave have been experimented with.

RP of FlanSea WEC in Grestner waves

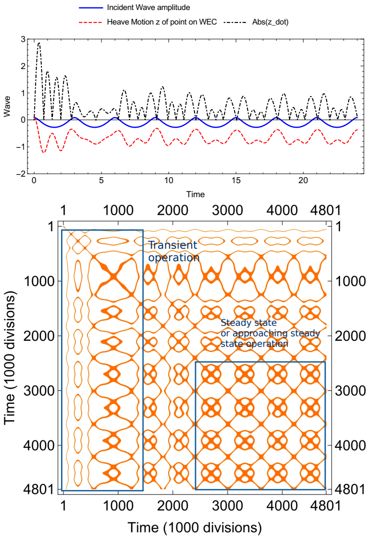
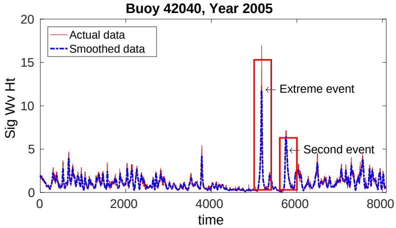


Figure 11: Cut-off distance, $\epsilon = 0.05$. Transient to steady-state is described by a change in RP structure (and RQA).

Supplementary: Extreme wave events and RQA. The Hurricane Katrina.

NDBC Buoy 42040, 2005 data, Extreme event



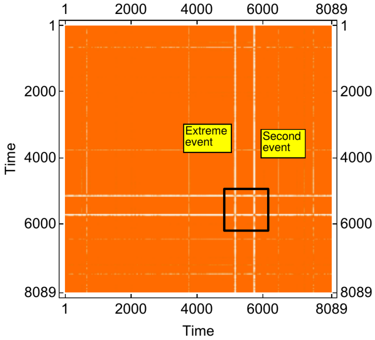
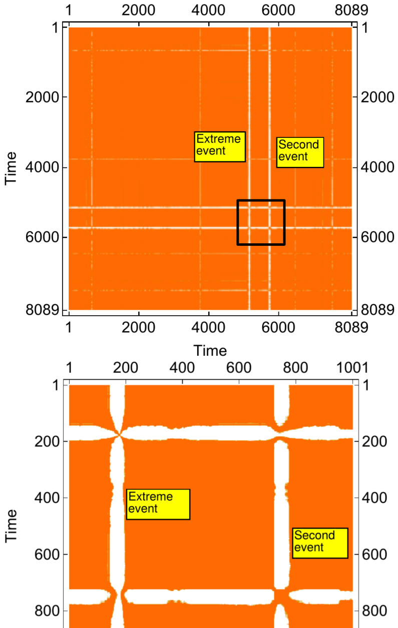


Figure 12: Moving-average smoothing applied to the significant wave height (in meter) time-series buoy data for 2005, the RP for the time-series and the magnified view in the of the extreme event.

NDBC Buoy 42040, 2005 data, Extreme event



NDBC Buoy 42040, 2005 data, “regular operation”

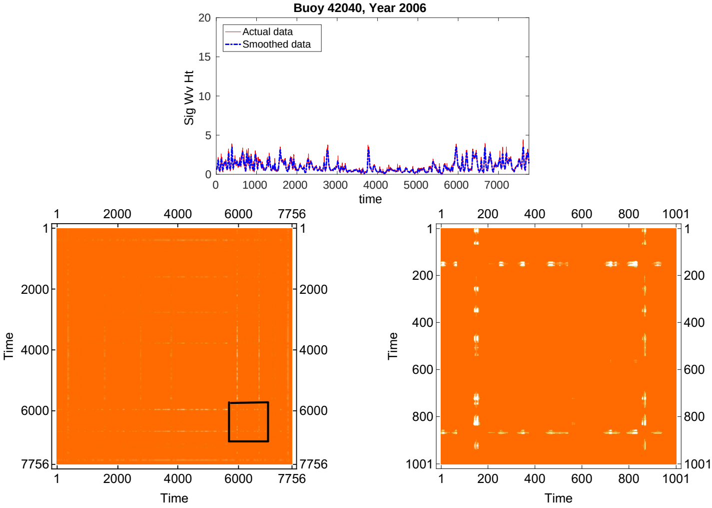


Figure 14: Moving-average smoothing applied to the significant wave height (in meter) time-series buoy data for 2006, the RP for the time-series and the magnified view in the of the boxed region.

RQA Extreme event vs Regular operation

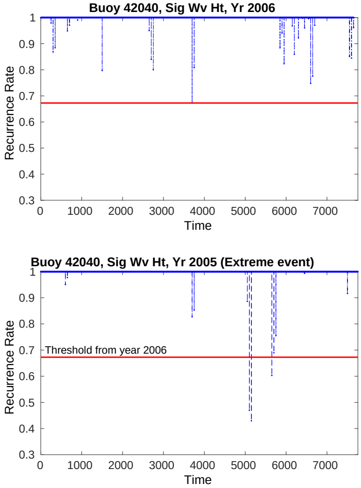


Figure 15: Recurrence rate comparison with between significant wave height data recorded in 2006 with that recorded in 2005 (year of extreme event). $\epsilon = 3.0$, RQA window size of 500 data points, Euclidean norm utilization.

Conclusions

- RP/RQA can be used to gauge/understand periodicity (or otherwise) in the response of a system.
- RP/RQA signature can be established for the regular behaviour of a system and may be used to measure any future divergence from regular behaviour.
- Change in an RQA param may be used to trigger a control force to "steady" a system.

Future work

- Full complement of forces for WEC needs to be used and RQA params for control triggering.
- Cross recurrence plots and joint recurrence plots can be applied to convolution of two signals.

Partial list of References

[1] K. De Koker, G. Crevecoeur, B. Meersman, M. Vantorre, and L. Vandevelde. A power take-off and control strategy in a test wave energy converter for a moderate wave climate. In *International Conference on Renewable Energies and Power Quality (ICREPQâĂŹ16)*. European Association for the Development of Renewable Energies, Environment and Power Quality (EA4EPQ), 2016.

[2] J.-P. Eckmann, S. O. Kamphorst, and D. Ruelle. Recurrence plots of dynamical systems. *EPL (Europhysics Letters)*, 4(9):973, 1987.

[3] N. A. Kudryashov, P. N. Ryabov, and B. A. Petrov. Dissipative structures of the kuramoto–sivashinsky equation. *Automatic Control and Computer Sciences*, 49 (7):508–513, 2015.

[4] N. Marwan, M. C. Romano, M. Thiel, and J. Kurths. Recurrence plots for the analysis of complex systems. *Physics reports*, 438(5):237–329, 2007.

[5] J. P. Zbilut, N. Thomasson, and C. L. Webber. Recurrence quantification analysis as a tool for nonlinear exploration of nonstationary cardiac signals. *Medical Engineering and Physics*, 24(1):53–60, 2002.

Questions? Comments? Thoughts?

Houghton county lift bridge. Noted as one of 1000 places to see before you die in Schultz, Patricia. *1,000 places to see before you die*. Workman Publishing, 2011.