H9.00004 : Thermocapillarity Driven Instabilities in Thin Liquid Layers Subject to Long-wave Analysis

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Objective

Overall objective

To study the effect of thermal (evaporation rates, effect of zero/micro gravity) and mechanical (periodic, free and pinned) boundary conditions on the dynamics of non-evaporating and evaporating liquid films through the augmented liquid film evolution equation.

Objective within the purview of this presentation

- Evaporating liquid films in zero gravity
- Effect of initial conditions
- Dichloromethane liquid film in zero gravity

Non-linear evolution equation

$$
\begin{aligned}
&h_T + \underbrace{\frac{E}{(h+K)}}_{\text{Evaporation}} + \underbrace{S(h^3 h_{XXX})_X}_{\text{surface tension}} - \underbrace{\frac{Ga}{3}(h^3 h_X)_X}_{\text{Gravity}} + \\
&\underbrace{E^2 D^{-1}\left[\frac{h^3 h_X}{(h+K)^3}\right]_X}_{\text{vapor recoil}} + \underbrace{KMPr^{-1}\left[\frac{h^2 h_X}{(h+K)^2}\right]_X}_{\text{thermocapillarity}} + \\
&\underbrace{\frac{5Ra}{48\mathrm{Pr}}\left[\frac{K^2}{(h+K)^2} h^4 h_X + h^4 h_X\right]}_{\text{buoyancy driven instabilities}} = 0 \qquad (1)
\end{aligned}
$$

Fastest growing wavelength, λ_{max}

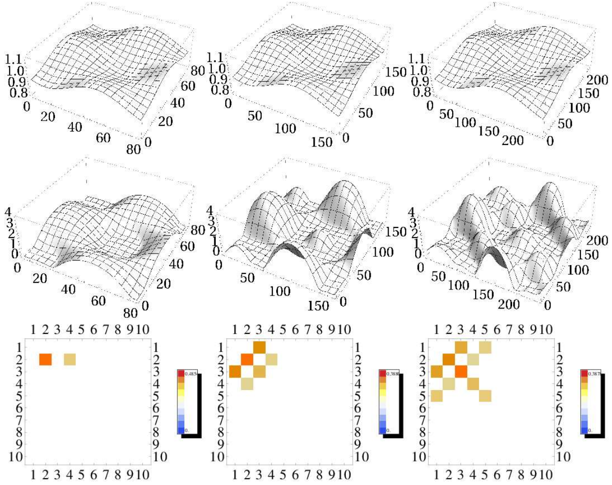
- Derived using linear stability theory on the evolution equation
- Describes that wavelength of perturbation that grows the fastest for a certain set of conditions

$$\lambda_{max} = 2\pi/\mathrm{q}_{max}$$

$$q_{\mathrm{max}} = \left[\frac{-Bo + \delta(h+K)^{-3} + mh^{-1}(Bih+K)^{-2}) + R\{\frac{K}{(h+K)^2} - 1\}h}{2}\right]^{\frac{1}{2}}$$

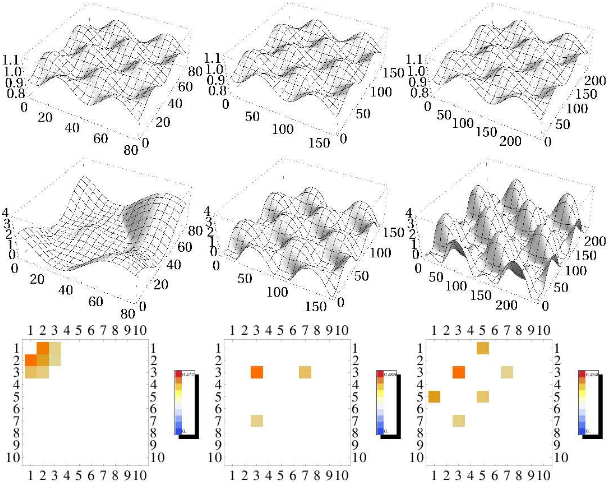
Here, Bo $= G/S, \delta = E^2/DS$, $m = MK/PrS$, $\epsilon = E/S$, $R = Ra/(\mathrm{Pr}S)$ and $\bar{h} = -K + \sqrt{(K+1)^2 - 2\epsilon t}$. Here, $\bar{h}$ is the basic X-independent state and is a thinning layer.

G=0.0, S=100, M=35.1, Pr=7.02, E=0.0001, R=0



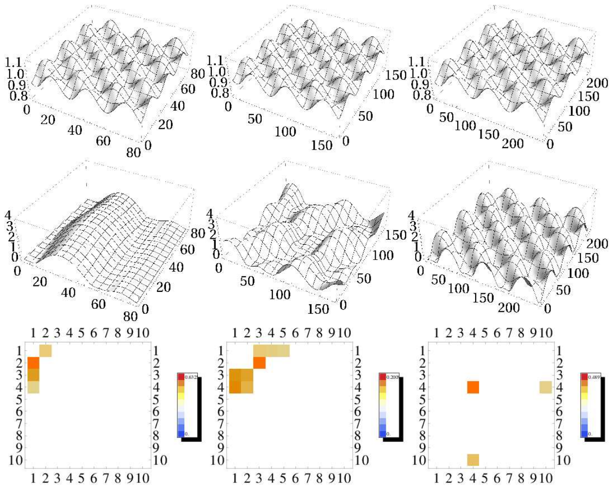
Perturbation avenumber (k/L)=1

G=0.0, S=100, M=35.1, Pr=7.02, E=0.0001, R=0



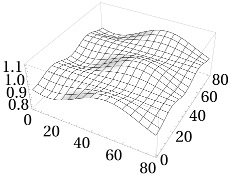
Perturbation wavenumber (k/L)=2

G=0.0, S=100, M=35.1, Pr=7.02, E=0.0001, R=0

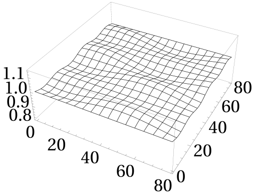


Perturbation wavenumber (k/L)=3

Effect of initial conditions: Slowly evaporating liquid film in zero gravity for varying domain sizes

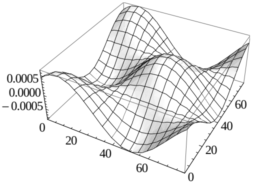
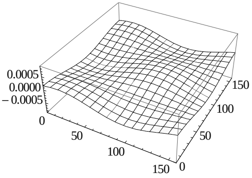
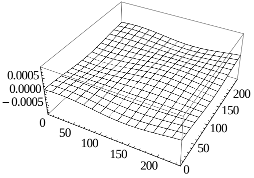


$1\text{-}0.05(\cos[2\pi x/L] + \sin[2\pi x/L])(\cos[2\pi y/L])^2$

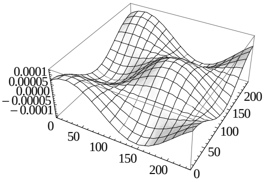
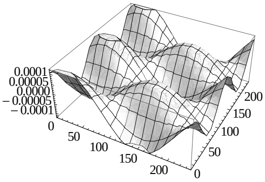
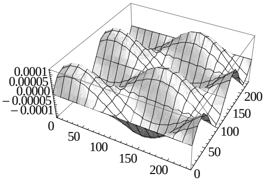


$1 - 0.05(\cos[2\pi x/L] + \sin[2\pi x/L])\cos[2\pi y/L]\sin[2\pi y/L]$

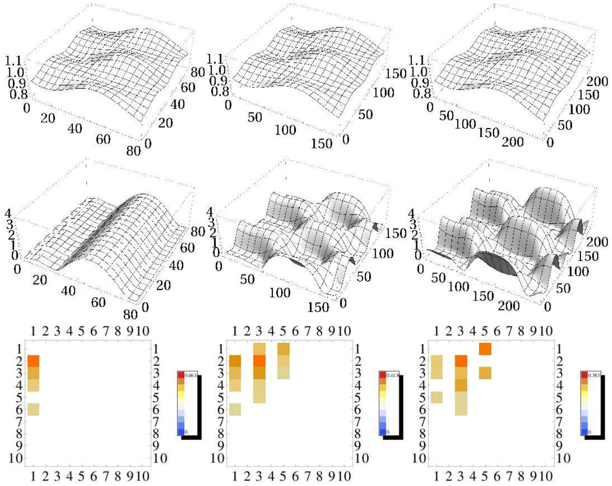
Mean curvature variation with change in domain size $(1 - 0.05(\cos[2\pi x/L] + \sin[2\pi x/L])\cos[2\pi y/L])$

$L = \lambda_{\max}$	$L = 2\lambda_{\max}$	$L = 3\lambda_{\max}$
		
$\kappa_{\max} = 8.9 \times 10^{-4}$	$\kappa_{\max} = 2.3 \times 10^{-4}$	$\kappa_{\max} = 9.01 \times 10^{-5}$

Mean curvature variation from change in perturbation

cos, $L = \lambda_{\max}$	$\cos^2$, $L = \lambda_{\max}$	cos sin, $L = \lambda_{\max}$
		
$\kappa_{\max} = 8.9 \times 10^{-4}$	$\kappa_{\max} = 1.33 \times 10^{-3}$	$\kappa_{\max} = 1.11 \times 10^{-3}$

$L = n\lambda_{\text{max}}$, where $n = 1, 2, 3$ E=0.0001, G=0.0, S=100, M=35.1, Pr=7.02, R=0



$$1 - 0.05(\cos[2\pi x/L] + \sin[2\pi x/L])(\cos[2\pi y/L])^2$$

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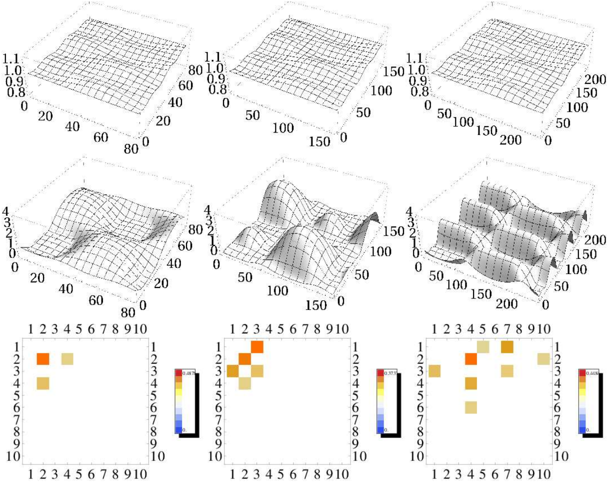
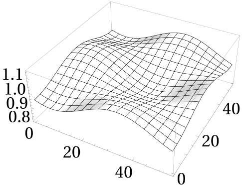


$$1 - 0.05(\cos[2\pi x/L] + \sin[2\pi x/L])\cos[2\pi y/L]\sin[2\pi y/L]$$

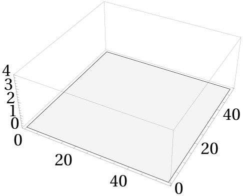
Evaporating 2.35 mm thick Dichloromethane liquid film

	$g = 9.81 m/s^2$ or $g = 0.0 m/s^2$	Math fluid
G	$4.11 \times 10^6/0$	$0.333/0$
S	143×10^3	100
M	122×10^3	35.1
Pr	3.9	7.02
ϵ	6.01×10^{-8}	0
δ	5.19×10^{-7}	0

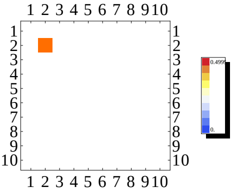
Terrestrial gravity, film depletion in $\sim$ 57seconds



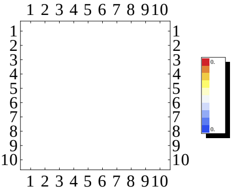
Initial condition



Rupture

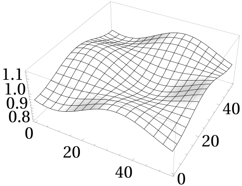


DFT, Initial condition

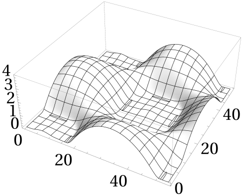


DFT, rupture

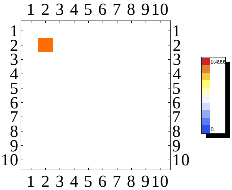
Zero gravity, film rupture in $\sim$ 4 seconds



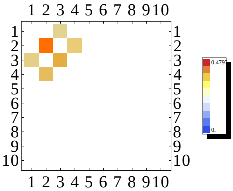
Initial condition



Rupture



DFT, Initial condition



DFT, rupture

Conclusions

- For evaporating liquid films in zero gravity, as the domain size increases, the complexity of structures increases
- When perturbations with fractional λ_{max} are applied, secondary structures (fractional wavelengths) appear
- Curvature of the initial condition affects film evolution
- For DCM films, long wave theory is applicable when $h_0 \leq 100\mu m$ in terrestrial gravity while it is applicable for $h_0 \sim 2.5mm$ in zero gravity

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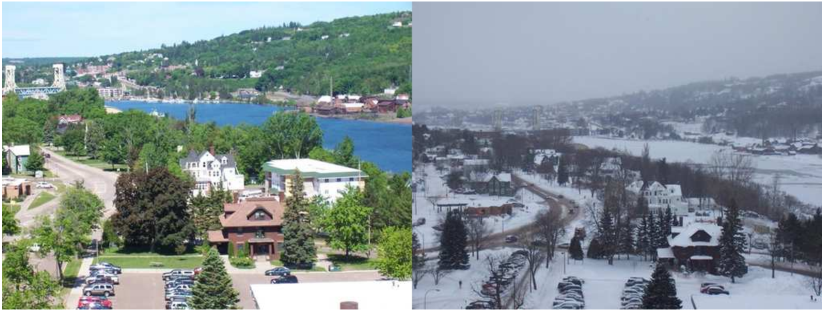
- Dr. Robert W. Kolkka, Dr. Alan Struthers and Dr. Jeffrey S. Allen from Michigan Technological University.



Questions? Comments?

Brief Bibliography