
2013/03/17 17 : 42

The Influence of Thermal and Mechanical Boundary Conditions on the Dynamics of Evaporating Liquid Films

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Illustrative example

Slowly evaporating liquid film in zero gravity “wrinkles” and eventually “ruptures”

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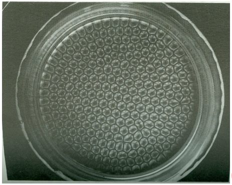
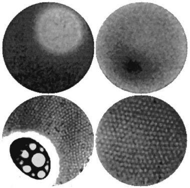
Ubiquity of liquid films

- Paints, Lubricants, time delay coatings of tablets, heat exchangers, pesticide films on leaves and produce, drying of tear films, lava flows, ice sheet flows, crystal growth.
- Coatings for rust prevention cost $100 billion a year (worldwide) per Cheremisinoff[1]. Revenue in the US alone in 2007 was $34 Billion [1].
- Cooling of server towers cost $36 billion in 2007[2–4].
- Concepts applicable to liquid films are also used for improved oil recovery.
- Low gravity life support systems and thermal control.

Problem duality



(Short wave)

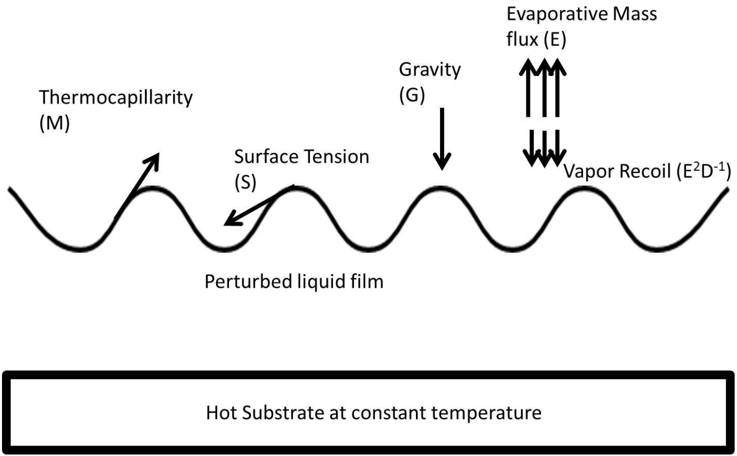


(SW to LW)

Prominent events

1. Short wavelength instablities: Rayleigh (1916), Pearson ('58), Scriven et al. ('64), Koschmeider
2. Long wavelength instablities/LWT: Benney ('64), Scriven et al. ('79), Burelbach et al. ('88), Krishnamoorthy('97), Oron et al. ('95,2005), Vanhook ('97)
3. LWT leads to a "film evolution equation"

Forces on an evaporating liquid film



[slide 10]

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Steps in developing the evaporating film evolution equation

1. The Navier-Stokes' equations (NSE) are non-dimensionalized with h_0 and h_0^2/ν (viscous time scale)
2. Interfacial jump conditions (IJC) are modified to take into account:
   2.1 Effect of evaporative momentum change on normal stress balance
   2.2 Effect of thermocapillarity on shear stress balance
   2.3 Latent heat absorption is included in energy balance
   2.4 Vapor side dynamics decoupled from liquid side
3. Long wave scaling parameter is introduced to rescale NSE and IJC
4. Expression for velocities at the interface is substituted into kinematic condition // Evolution equation

Modified evolution equation

$$h_T + \underbrace{S\nabla\cdot(h^3\nabla\nabla^2 h)}_{\text{Surface tension}} - \underbrace{\frac{Ga}{3}\nabla\cdot(h^3\nabla h)}_{\text{Gravity}}$$

$$+\underbrace{\nabla\cdot\left[\left(\frac{E^2}{D}\frac{h^3}{(\eta_1 h+\eta_2)^3} + \eta_1\eta_2\frac{M}{\mathrm{Pr}}\frac{h^2}{(\eta_1 h+\eta_2)^2}\right)\nabla h\right]}_{\text{Vapor recoil and Thermocapillarity}}$$

$$\underbrace{\frac{5Ra}{48\mathrm{Pr}}\nabla\cdot\left[\frac{K^2}{(h+K)^2}h^4\nabla h + h^4\nabla h\right]}_{\text{Buoyancy}} + \underbrace{\frac{E}{\eta_1 h+\eta_2}}_{\text{Evaporation}} = 0$$

1. $\eta_1 = \mathrm{Bi}$ and $\eta_2 = 1$ for non-evaporating liquid films (which have $E = 0$)
2. $\eta_1 = 1$ and $\eta_2 = K$ for evaporating liquid films (which have a non-zero E)
3. Disjoining pressure term neglected

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Assumptions made in derivation

1. Liquid film is Newtonian, non-draining and horizontal on a heated substrate
2. Constant temperature boundary condition on the substrate
3. Surface tension is a monotonically decreasing function of temperature
4. Film thickness $\ll$ lateral dimension: **NO** short wavelength/Pearson's instability appear
5. Long wave scaling results in advection term in the NSE to be neglected
6. Vapor side dynamics ignored (vapor evacuated constantly)

Implications

1. Evolution equation can be used for lubrication theory/LWT type problems
2. Can be used for slow evaporation cases
3. Solution of one non-linear PDE is tantamount to solution of coupled set of Navier Stokes' equations for lubrication type problems
4. Selective application of different mechanisms is possible
5. Boundary conditions can be swiftly changed between periodic/*mirror*$\leftrightarrows$Free/*constant slope*$\leftrightarrows$Pinned/*pivot point*
6. Overall a good design tool with small error. Has been compared with CFD[5–7] and experiments[8,9]

[slide 6]

Fastest growing wavelength, $\lambda_{\max}$, for an evaporating liquid film

$$\lambda_{\max} = 2\pi/q_{max}$$

Where,

$$q_{\max} = \frac{1}{\sqrt{2}}\left[-Bo + \delta(h+K)^{-3} + \frac{m}{h}(h+K)^{-2} + Rh\left\{\left(\frac{K}{h+K}\right)^2 - 1\right\}\right]^{1/2}$$

Here $Bo = G/S, \delta = E^2/DS$, $m = MK/\mathrm{Pr}S$ and $R = 5Ra/48\mathrm{Pr}S$

- Derived using linear stability theory on our modified evolution equation
- Describes that wavelength of perturbation that grows the fastest for a certain set of conditions
- Evaporation dynamics has an effect on $\lambda_{\max}$
- Zero gravity evaporation analysis is now possible

Fluid properties used in literature - the "Mathematical fluid"!

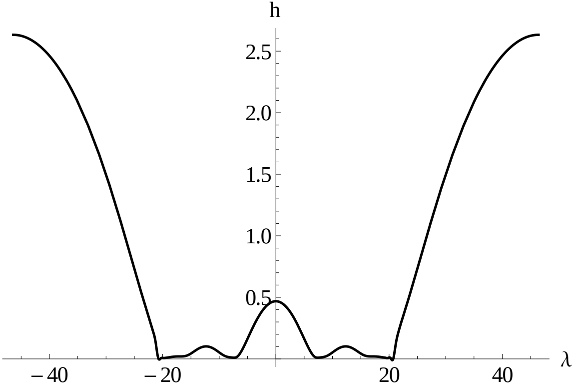
- The "mathematical fluid" is set of properties representing a Newtonian fluid
- It is of similar character as a low centistoke Silicone oil
- The popular[5,10,11] properties are shown in the table
- For a non-evaporating film, it leads to **maximum growth rate** of perturbations

Non-dim. number	Magnitude
Ga	1/3
S	100
M	35.1
Pr	7.02
Bi	1.0

Overview of study

- Effect of domain size and perturbation wavenumber:
  1. Non evaporating mathematical fluid, in non-zero and zero gravity
  2. Slowly Evaporating math fluid in non-zero and zero gravity
- Effect of curvature // Creating structures to meet design specifications
- Evaporating dicholoromethane (DCM) liquid film in zero gravity // Real fluid dynamics
- Evaporating DCM liquid film in zero gravity **with** uniform random disturbance as initial condition // noisy excitation
- Free boundary conditions // realistic BCs

Validation - Comparison with 2D simulations



When $L = \lambda_{\text{max}}$ and M=35.1, Pr=7.02, S=100, Ga=1/3, rupture time $T_{\text{Rup}} = 1280$ is within 5% of CFD simulations[5]

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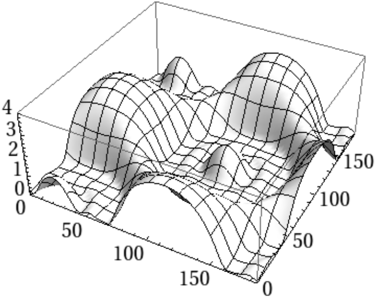
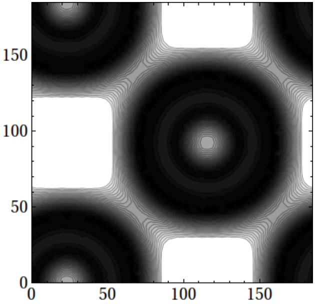
Validation - Comparison with 3D simulations





With $\lambda = 2\lambda_{\text{max}}$, the rupture time is revealed to be 2023.0. This is within 5% of that published by Oron et al.[11]

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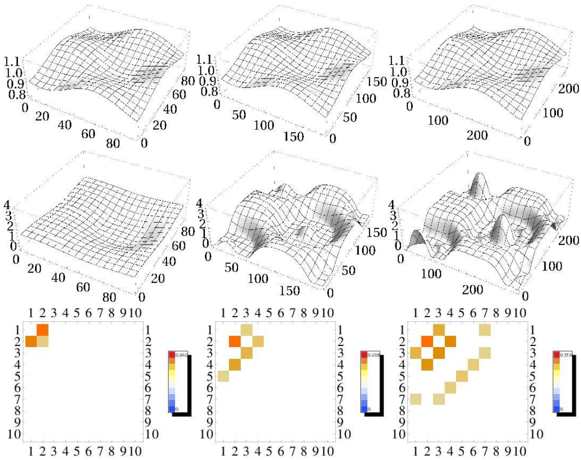
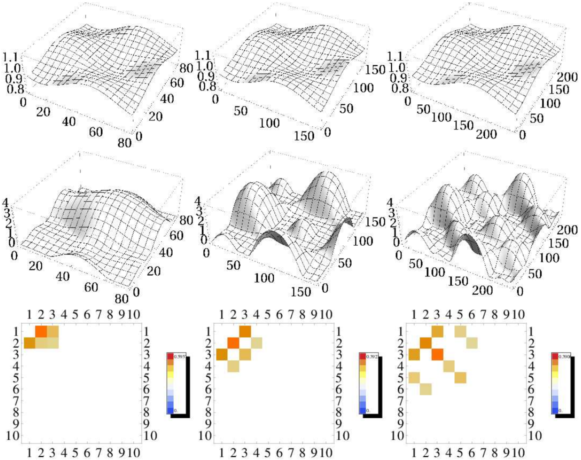
Non evaporating math fluid, **Ga=0.333**,S=100,M=35.1,Pr=7.02,E=0,R=0, k=1

$L = \lambda_{\max}$ $\quad L = 2\lambda_{\max}$ $\quad L = 3\lambda_{\max}$



When $n/k = 1$ maximizing wavelength is applied

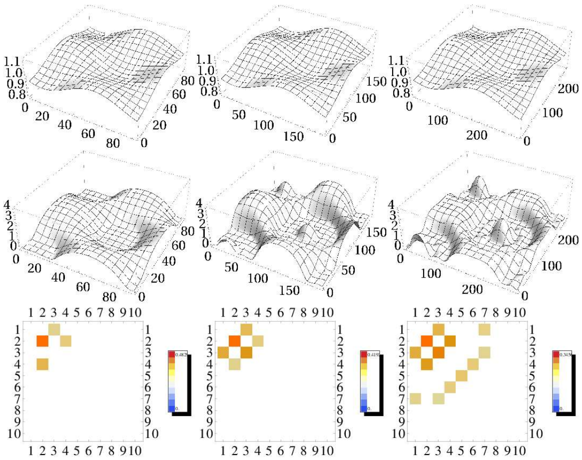
$L = \lambda_{max}$ $L = 2\lambda_{max}$ $L = 3\lambda_{max}$



When $n/\mathrm{k} = 1$ maximizing wavelength is applied

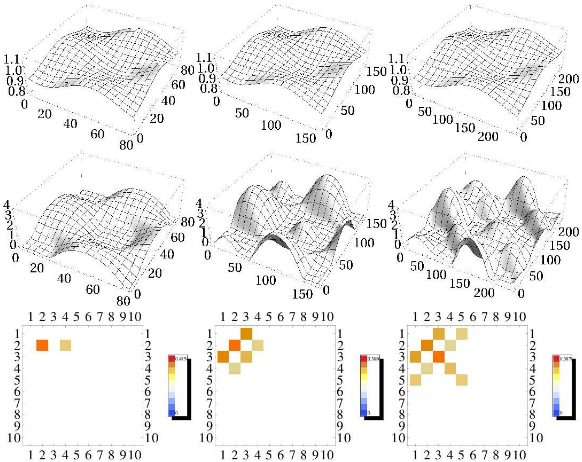
Evaporating math fluid, **Ga=0.333**,S=100,M=35.1,Pr=7.02,E=0.0001,R=0. k=1

$L = \lambda_{\max}$ $\quad L = 2\lambda_{\max}$ $\quad L = 3\lambda_{\max}$



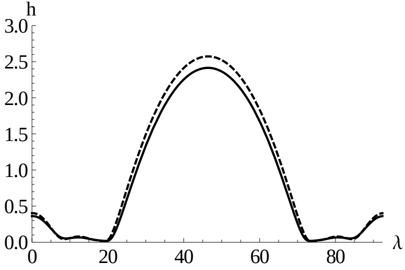
When $n/k = 1$ maximizing wavelength is applied

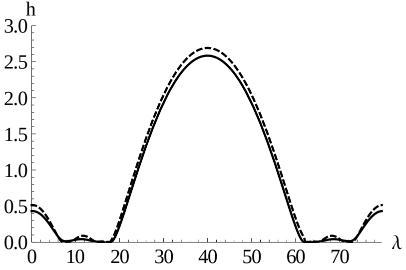
$L = \lambda_{\mathrm{max}}$ $L = 2\lambda_{\mathrm{max}}$ $L = 3\lambda_{\mathrm{max}}$



When $n/\mathrm{k} = 1$ maximizing wavelength is applied

Effect of vapor recoil (Math fluid)





$Ga = 0.333$
No evap (solid), $T_{rup} = 2568$
Slow evap (dotted), $T_{rup} = 1806$

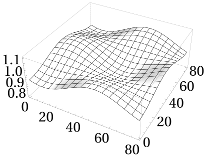
$Ga = 0.0$
No Evap (solid), $T_{rup} = 1987$
Slow evap (dotted), $T_{rup} = 1236$

Summary: Non-evaporating/evaporating mathematical fluids

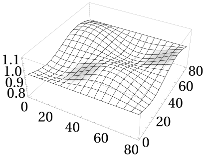
1. As the domain size increases, the complexity of structures increases
2. Effect of vapor recoil on thermocapillary structures is observed close to rupture in regular and zero gravity.
3. Gravity has significant impact on film dynamics.

Effect of curvature: Slowly evaporating math fluid in zero gravity for varying domain sizes or varying amplitude of perturbation

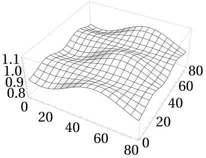
Various initial conditions, $1 - 0.05(\cos[2\pi x/L] + \sin[2\pi x/L])\cos[2\pi y/L]$



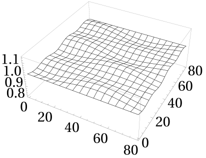
[Cosine]



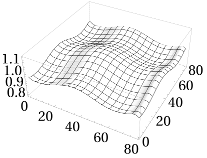
[Sine]



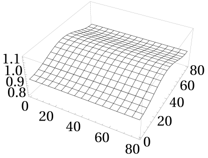
[Cosine-squared]



[Cosine$\times$Sine]



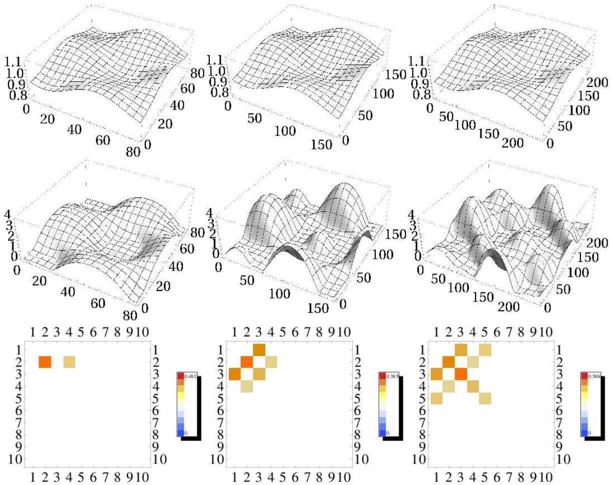
[Cosine-rotated]



[Cosine-y]

Effect of initial conditions with E=0.0001, Ga=0.0

$L = \lambda_{\text{max}}$ $\quad L = 2\lambda_{\text{max}}$ $\quad L = 3\lambda_{\text{max}}$



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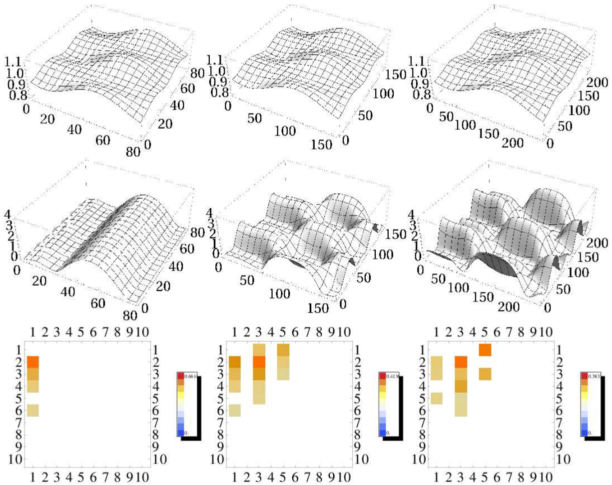
$1 - 0.05(\cos[2\pi x/L] + \sin[2\pi x/L])\cos[2\pi y/L]$

Effect of initial conditions with E=0.0001, Ga=0.0

$L = \lambda_{\max}$ $\quad L = 2\lambda_{\max}$ $\quad L = 3\lambda_{\max}$



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$$1 - 0.05(\cos[2\pi x/L] + \sin[2\pi x/L])(\cos[2\pi y/L])^2$$

Curvature effect: comparison of terms

Comparison of surface tension/ST $\nabla\cdot(h^3\nabla\nabla^2 h)$ and thermocapillarity/MT $\nabla\cdot\left[\left(\frac{h^2}{(h+K)^2}\right)\nabla h\right]$ terms

	λ_{max}	$2\lambda_{max}$	$3\lambda_{max}$
ST, cosine	6.0×10^{-6}	4.0×10^{-7}	8.0×10^{-8}
MT, cosine	1.0×10^{-5}	3.0×10^{-6}	2.0×10^{-6}
ST, cosine-sq	1.0×10^{-5}	8.0×10^{-7}	1.5×10^{-7}
MT, cosine-sq	1.5×10^{-5}	4.0×10^{-6}	2.0×10^{-6}

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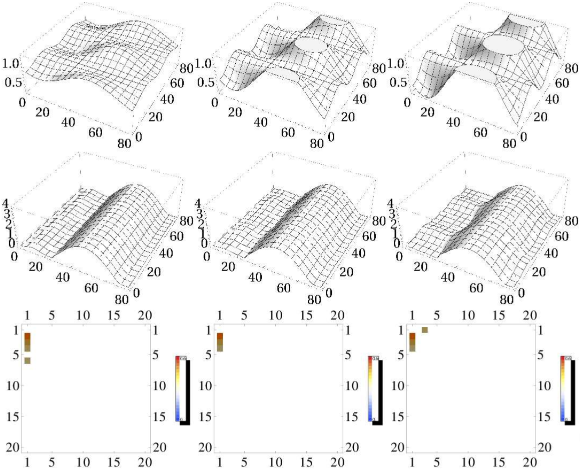
Summary of curvature variation study

1. Changing the domain size and/or initial condition begets a change in curvature.
2. The film dynamics and final structure are heavily influenced by the shape and hence curvature of the initial condition.
3. Thermocapillarity always stronger than surface tension for our cases.
4. Prediction/design of final structure is possible, based on initial curvature.

Effect of increasing the amplitude (% of h_0) for a slowly evaporating math fluid in zero g, $L = \lambda_{\max}$

$L = \lambda_{\max}$	$L = \lambda_{\max}$	$L = \lambda_{\max}$
$A = 15\% h_0$	$A = 35\% h_0$	$A = 50\% h_0$



Summary of amplitude variation study

1. With the mathematical fluid, for a smaller domain size of $L = \lambda_{\text{max}}$, as the amplitude of perturbation increased, the film ruptures sooner.
2. With greater amplitudes of initial perturbations, portions of the film are pushed closer or farther away from the hot subtrate.

Evaporating 2.35 mm Dichloromethane liquid film with Ga=0.0 and Ga with g= $9.81 m/s^2$

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Film parameters for DCM

Comparison of parameters

	DCM ($g = 9.81/g = 0.0m/s^2$)	Math fluid (regular/zero g)
Ga	4.11×10^6 / 0	0.333 / 0
S	143×10^3	100
M	122×10^3	35.1
Pr	3.9	7.02
ϵ	6.01×10^{-8}	0
δ	5.19×10^{-7}	0
M/S	0.85	0.35

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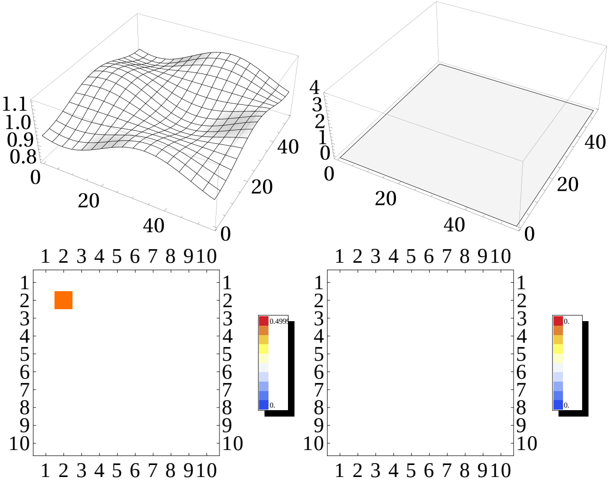
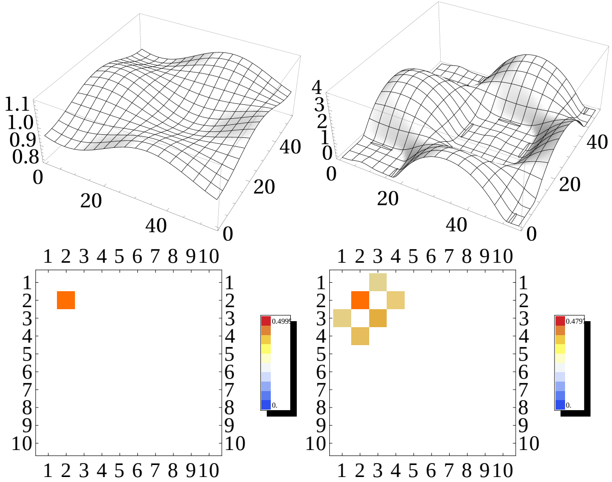
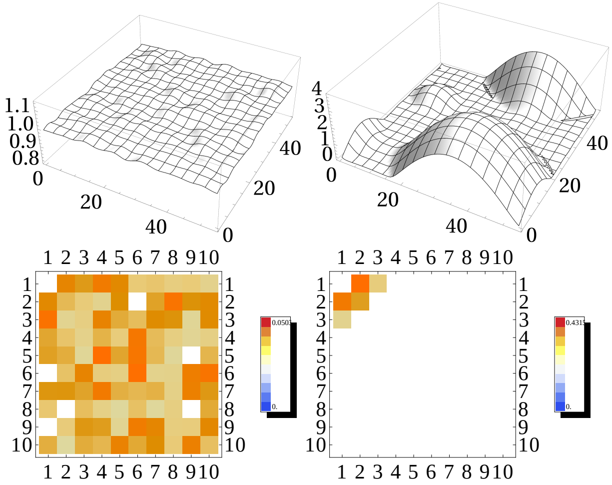
DCM film evaporating in Earth's gravity, $t_{\text{rup}} \sim 60$ seconds



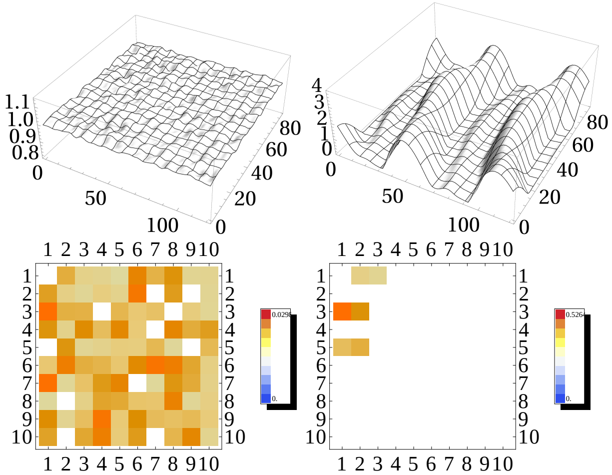
DCM film evaporating in zero gravity, $t_{rup} \sim 4$ seconds



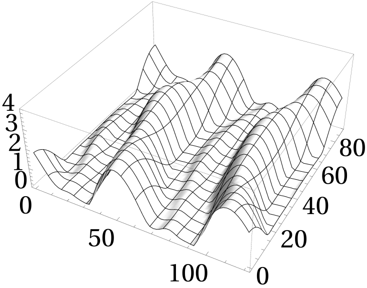
DCM film evaporating in Zero gravity, uniform random perturbation, square domain, whole number side dimension

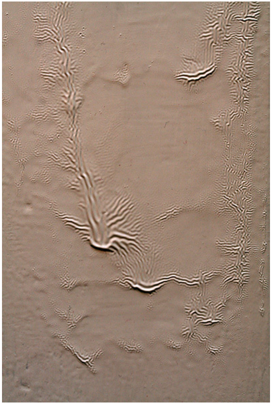


DCM film evaporating in Zero gravity, uniform random perturbation, rectangular domain, non-whole number length and breadth



Paint wrinkling

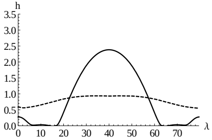




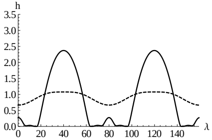
Summary of DCM studies

1. DCM Film stable to long wave instabilities in Earth's gravity
2. DCM film destabilizes via LW instabilities in zero gravity
3. Square domain with noisy initial condition in zero gravity leads to fastest growing wavelength
4. Rectangular domain with noisy initial condition with non-whole number length and breadth, in zero-g leads to cascade of structures

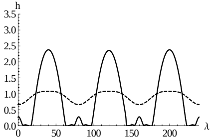
Effect of boundary conditions: periodic vs free / Evap. math fluid, non-zero gravity



$L = \lambda_{\max}$



$L = 2\lambda_{\max}$

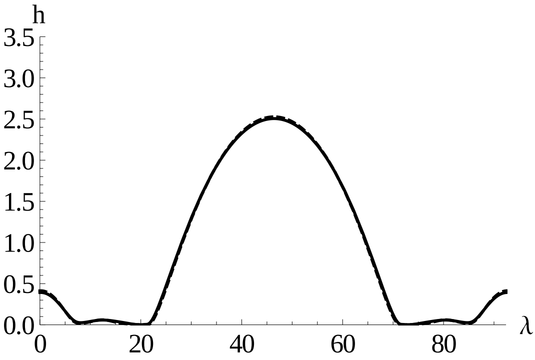


$L = 3\lambda_{\max}$

Summary: Effect of boundary condition change

1. Free boundary conditions have a stabilizing effect on thermocapillary structures
2. Free boundary conditions deccelerate time to rupture

Effect of buoyancy/Evap. math fluid, non-zero gravity



Thick line: Slowly evaporating mathematica fluid (E=0.0001) without the buoyancy term. Dashed line: Full evolution equation with $R = 10^{-4}$. $L = \lambda_{max}$. $T_{\text{Rup, without R}} = 1781$, $T_{\text{Rup, with R}} = 1607$

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Conclusions: Technological impact of research/broader impact

1. **Coatings technology:**
   - Evolution equation can be used to study stability of coatings
   - Design and control of thermocapillary structures with appropriate initial conditions and boundary conditions
2. **Microgravity/zero gravity sciences:**
   - Density stratification in liquids in zero gravity can be studied.
3. **Other:**
   - Evaporative cooling technologies.
   - Wetting conditions on leaves/nature inspired design.
   - Study of foams.

Conclusions // Evolution equation and parametric sweep

- The evolution equation as proposed by Burelbach et al.[10] and Williams and Davis[12] is modified to include bouyancy driven destabilization
- The rich parameter and boundary condition space is explored:
  - The effect of increasing the domain size for a constant perturbation and percentage-of-initial-thickness amplitude is to induce the formation of more complicated secondary and tertiary structures
  - In zero gravity thermocapillary structures grow quicker
  - Vapor recoil has an accelerating effect on film dynamics and amplification of rupture structures

Conclusions // Effect of curvature

- The effect of initial conditions is explored:
  - Six different initial conditions were explored
  - These ICs induced multi/mixed mode perturbations
  - A change in the initial condition changed the curvature hence either strengthening or weakening thermocapillarity
- The effect of curvature:
  - Changing the domain size or amplitude (% of h_0) had an effect on curvature.
  - Changing the domain size (constant amplitude) changed the strength of thermocapillarity and the eventual structures at rupture.
  - Increasing the amplitude (domain size constant) decreased the time required for the depressions to touch the substrate.

Conclusions // Real fluid

- Evaporating dicholoromethane liquid film:
  - A 2.35 mm thick DCM film was "evaporated" in zero gravity and in Earth gravity
  - Total film destabilization and formation of thermocapillary structures was seen in zero gravity via long wave effects
  - No long wave instabilities are observed when evaporating in Earth's gravity
  - Random non-uniform perturbations were applied to a square domain - fastest growing wavelength appeared
  - Random non-uniform perturbations were applied to rectangular domain - fastest growing wavelength appeared

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- Dr. Allan Struthers and Dr. Jeffrey S. Allen.
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