### 1 Credit Card Data

## 1.1 data cleaning and extraction of small subset

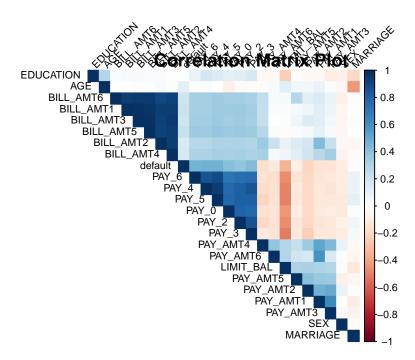
```
set.seed(123)
file_path_credit <- "default-credit-card.csv"</pre>
file_path_credit_names <- "names-default-credit-card.csv"</pre>
data_credit <- read.csv(file_path_credit)</pre>
data_credit_names <- read.csv(file_path_credit_names)</pre>
credit_short_colnames <- colnames(data_credit)</pre>
credit_long_colnames <- colnames(data_credit_names)</pre>
colnames(data_credit) <- credit_long_colnames</pre>
data_credit_unclean=data_credit
data_credit <- data_credit %>%
  filter(PAY_0 != -2) %>% filter(PAY_0 != 0) %>%
  filter(PAY_2 != -2) %>% filter(PAY_2 != 0) %>%
  filter(PAY_3 != -2) %>% filter(PAY_3 != 0) %>%
  filter(PAY_4 != -2) %>% filter(PAY_4 != 0) %>%
  filter(PAY_5 != -2) %>% filter(PAY_5 != 0) %>%
  filter(PAY_6 != -2) %>% filter(PAY_6 != 0) %>%
  filter(EDUCATION > 0) %>% filter(EDUCATION < 4)</pre>
data_credit_edu <-function(edu,def){data_credit %>% filter(EDUCATION == edu) %>% filter(default == 0
combined_df <- rbind(data_credit_edu(1,0), data_credit_edu(1,1),</pre>
                      data_credit_edu(2,0), data_credit_edu(2,1),
                      data_credit_edu(3,0), data_credit_edu(3,1))
write.csv(combined_df, file = "credit_data_education_short.csv")
data_credit_full = data_credit
data_credit = combined_df
```

#### 1.2 correlations in subset

```
colnames(data_credit) <- credit_long_colnames
df = data_credit
correlations <- sapply(df[, -which(names(df) == "default")], function(x) biserial.cor(x, df$default]
list_cor = sort(round(abs(correlations),3))
correlations_df <- data.frame(
    Variable = names(list_cor),
    Correlation = as.vector(list_cor)
)
#correlations_df %>% filter(Correlation > 0)
```

```
# Calculate the correlation matrix
cor_matrix <- cor(data_credit[, -1])

# Create a correlation matrix plot
corrplot(cor_matrix, method = "color", type = "upper", order = "hclust", tl.col = "black", tl.srt = title("Correlation Matrix Plot")</pre>
```



```
correlations <- cor(data_credit[, 'default'], data_credit[, -which(names(data_credit) == 'default')]
round(correlations,3)</pre>
```

```
ID LIMIT_BAL
                        SEX EDUCATION MARRIAGE AGE PAY_0 PAY_2 PAY_3 PAY_4
              -0.344 - 0.104
                                       -0.015 0.12 0.428 0.393 0.432 0.47
    PAY_5 PAY_6 BILL_AMT1 BILL_AMT2 BILL_AMT3 BILL_AMT4 BILL_AMT5 BILL_AMT6
[1,] 0.475 0.466
                    0.213
                              0.164
                                        0.204
                                                   0.18
                                                            0.223
    PAY_AMT1 PAY_AMT2 PAY_AMT3 PAY_AMT4 PAY_AMT5 PAY_AMT6
              -0.218
                       -0.147
[1,]
                               -0.179
                                         -0.086
```

## 1.3 Stan model $PAY_0 + PAY_2$

```
seed = 123
  )
  end.time <- Sys.time()</pre>
  time.taken <- round(end.time - start.time,2)</pre>
  time.taken
Time difference of 0.01 secs
  predictions <- predict(fit1, newdata = data_credit, type = "response")</pre>
  binary_predictions <- ifelse(predictions[,1] > 0.5, 1, 0)
  conf_matrix <- confusionMatrix(factor(binary_predictions), factor(data_credit$default))</pre>
  conf_matrix
Confusion Matrix and Statistics
          Reference
Prediction 0 1
         0 94 36
         1 26 84
               Accuracy : 0.7417
                 95% CI: (0.6814, 0.7958)
    No Information Rate: 0.5
    P-Value [Acc > NIR] : 1.785e-14
                  Kappa: 0.4833
Mcnemar's Test P-Value: 0.253
            Sensitivity: 0.7833
            Specificity: 0.7000
         Pos Pred Value: 0.7231
         Neg Pred Value: 0.7636
             Prevalence: 0.5000
         Detection Rate: 0.3917
   Detection Prevalence: 0.5417
      Balanced Accuracy: 0.7417
       'Positive' Class: 0
  start.time <- Sys.time()</pre>
  batch_size = 1000;
  start = 1;
  binary_predictions_unclean = c();
  predictions_batch_list = c();
  for (i in 1:30) {
    if (i\%10 == 0) {
      print(i);
    end = start + batch_size - 1;
```

```
predictions_batch <- predict(fit1, newdata = data_credit_unclean[start:end,],</pre>
                     type = "response", allow_new_levels = TRUE)
    predictions_batch_list = c(predictions_batch_list, predictions_batch)
    binary_predictions_batch <- ifelse(predictions_batch[,1] > 0.5, 1, 0)
    binary_predictions_unclean = c(binary_predictions_unclean, binary_predictions_batch);
  }
[1] 10
[1] 20
[1] 30
  end.time <- Sys.time()</pre>
  time.taken <- round(end.time - start.time,2)</pre>
  time.taken
Time difference of 41.83 secs
  conf_matrix <- confusionMatrix(factor(binary_predictions_unclean), reference = factor(data_credit_unclean)</pre>
  conf_matrix
Confusion Matrix and Statistics
          Reference
Prediction 0
         0 20637 3452
         1 2727 3184
               Accuracy: 0.794
                 95% CI: (0.7894, 0.7986)
    No Information Rate: 0.7788
    P-Value [Acc > NIR] : 7.634e-11
                  Kappa: 0.3779
 Mcnemar's Test P-Value : < 2.2e-16
            Sensitivity: 0.8833
            Specificity: 0.4798
         Pos Pred Value: 0.8567
         Neg Pred Value : 0.5387
             Prevalence: 0.7788
         Detection Rate: 0.6879
```

'Positive' Class : 0

Detection Prevalence: 0.8030 Balanced Accuracy: 0.6815

#### 1.4 Stan model EDUCATION Hierarchical

```
start.time <- Sys.time()</pre>
  fit2 <- brm(default ~ PAY_0 + PAY_2 + (1 | EDUCATION), # short dataset
               data = data_credit,
               refresh = 0,
               prior=c(
                   prior(normal(0,100), class="Intercept"),
                   prior(normal(0,100), class="b"),
                   prior(exponential(.02), class="sd")
               ),
               family = bernoulli(),
               file = "model2_education_small_data_simple",
               backend = "cmdstanr",
               iter = 5000,
               warmup = 2500,
               seed = 123
  )
  end.time <- Sys.time()</pre>
  time.taken <- round(end.time - start.time,2)</pre>
  time.taken
Time difference of 0.02 secs
  predictions <- predict(fit2, newdata = data_credit, type = "response", allow_new_levels = TRUE)</pre>
  binary_predictions <- ifelse(predictions[,1] > 0.5, 1, 0)
  conf_matrix <- confusionMatrix(factor(binary_predictions), factor(data_credit$default))</pre>
  conf_matrix
Confusion Matrix and Statistics
          Reference
Prediction 0 1
         0 95 37
         1 25 83
               Accuracy : 0.7417
                 95% CI: (0.6814, 0.7958)
    No Information Rate: 0.5
    P-Value [Acc > NIR] : 1.785e-14
                  Kappa: 0.4833
 Mcnemar's Test P-Value: 0.1624
            Sensitivity: 0.7917
            Specificity: 0.6917
         Pos Pred Value: 0.7197
         Neg Pred Value: 0.7685
             Prevalence: 0.5000
         Detection Rate: 0.3958
   Detection Prevalence: 0.5500
```

```
Balanced Accuracy: 0.7417
       'Positive' Class: 0
  start.time <- Sys.time()</pre>
  batch_size = 1000;
  start = 1;
  binary_predictions_unclean = c();
  predictions_batch_list = c();
  for (i in 1:30) {
    if (i\%10 == 0) {
      print(i);
    end = start + batch_size - 1;
    predictions_batch <- predict(fit2, newdata = data_credit_unclean[start:end,],</pre>
                     type = "response", allow_new_levels = TRUE)
    predictions_batch_list = c(predictions_batch_list, predictions_batch)
    start = end + 1
    binary_predictions_batch <- ifelse(predictions_batch[,1] > 0.5, 1, 0)
    binary_predictions_unclean = c(binary_predictions_unclean, binary_predictions_batch);
  }
[1] 10
[1] 20
[1] 30
  end.time <- Sys.time()</pre>
  time.taken <- round(end.time - start.time,2)</pre>
  time.taken
Time difference of 1.64 mins
  confusionMatrix(factor(binary_predictions_unclean), reference = factor(data_credit_unclean$default)
Confusion Matrix and Statistics
          Reference
Prediction
             0
         0 20409 3372
         1 2955 3264
               Accuracy : 0.7891
                 95% CI: (0.7844, 0.7937)
    No Information Rate: 0.7788
    P-Value [Acc > NIR] : 8.003e-06
                  Kappa : 0.3738
Mcnemar's Test P-Value: 1.696e-07
```

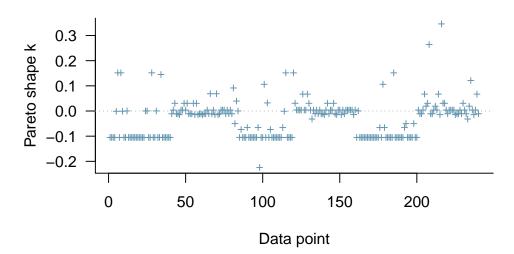
Sensitivity: 0.8735 Specificity: 0.4919 Pos Pred Value: 0.8582 Neg Pred Value: 0.5248 Prevalence: 0.7788 Detection Rate: 0.6803 Detection Prevalence: 0.7927 Balanced Accuracy: 0.6827

'Positive' Class: 0

### 1.5 Model comparison (e.g. with LOO-CV).

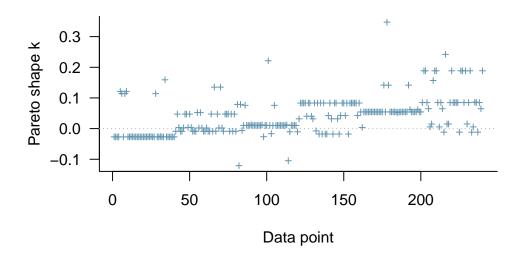
```
11 = loo(fit1)
  12 = loo(fit2)
  loo_compare(11,12)
     elpd_diff se_diff
fit1 0.0
                0.0
fit2 -0.4
                1.0
  plot(11, label_points = TRUE, main = 'PSIS diagnostic plot for pooled model')
```

# PSIS diagnostic plot for pooled model



```
plot(12, label_points = TRUE, main = 'PSIS diagnostic plot for hierarchical model')
```

## PSIS diagnostic plot for hierarchical model



## 1.6 Convergence diagnostic

```
summarize_draws(fit1)
```

```
# A tibble: 5 x 10
  variable
                   mean
                          median
                                         sd
                                               mad
                                                          q5
                                                                       rhat ess bulk
  <chr>
                                      <dbl> <dbl>
                                                      <dbl>
                                                                <dbl> <dbl>
                                                                                <dbl>
                  <dbl>
                            <dbl>
1 b_Intercept
                 -0.213
                          -0.212 0.150
                                             0.151 -4.58e-1
                                                              2.94e-2
                                                                        1.00
                                                                                2592.
2 b_PAY_0
                  0.521
                           0.518 0.154
                                             0.157 2.79e-1
                                                              7.81e-1
                                                                        1.00
                                                                                2058.
3 b_PAY_2
                                                                                2075.
                  0.165
                            0.165 0.131
                                             0.131 -5.02e-2
                                                              3.85e-1
                                                                        1.00
4 lprior
                -16.6
                         -16.6
                                  0.0000398 0
                                                   -1.66e+1 -1.66e+1
                                                                        1.00
                                                                                2418.
               -158.
                        -158.
                                  1.21
                                             0.981 -1.61e+2 -1.57e+2
                                                                       1.00
                                                                                1876.
5 lp__
 i 1 more variable: ess_tail <dbl>
```

summarize\_draws(fit2)

```
# A tibble: 9 x 10
  variable
                           median
                                       sd
                                                                  q95
                                                                       rhat ess_bulk
                    mean
                                               mad
                                                         q5
  <chr>
                   <dbl>
                             <dbl>
                                    <dbl>
                                             <dbl>
                                                      <dbl>
                                                                <dbl> <dbl>
                                                                                <dbl>
1 b_Intercept
               -2.49e-1 -2.25e-1 0.364
                                          0.269
                                                   -9.08e-1
                                                                0.293
                                                                       1.00
                                                                                1622.
2 b PAY 0
                 5.51e-1
                          5.48e-1 0.157
                                          0.156
                                                    2.95e-1
                                                                0.811
                                                                       1.00
                                                                                3162.
3 b_PAY_2
                          1.65e-1 0.130
                                                   -4.99e-2
                                                                0.380
                                                                       1.00
                                                                                3102.
                 1.64e-1
                                          0.130
4 sd_EDUCATIO~
                5.73e-1
                          3.90e-1 0.605
                                          0.368
                                                    4.01e-2
                                                                1.69
                                                                       1.01
                                                                                1170.
5 r_EDUCATION~
                2.08e-1
                          1.25e-1 0.392
                                          0.258
                                                   -3.01e-1
                                                                0.982
                                                                       1.01
                                                                                1601.
6 r_EDUCATION~ -1.61e-1 -1.16e-1 0.384
                                          0.256
                                                   -8.30e-1
                                                                0.407
                                                                       1.00
                                                                                1859.
7 r EDUCATION~
                2.89e-2 5.26e-3 0.377
                                          0.235
                                                   -5.62e-1
                                                                0.699
                                                                       1.00
                                                                                1774.
8 lprior
                -2.05e+1 -2.05e+1 0.0121 0.00741 -2.05e+1
                                                              -20.5
                                                                       1.01
                                                                                1167.
                -1.67e+2 -1.66e+2 2.33
                                          2.26
                                                   -1.71e+2 -163.
                                                                       1.01
                                                                                1550.
# i 1 more variable: ess_tail <dbl>
```