

The standard gluon-to-gluon splitting function at LO reads:

$$P_{gg}(x) = 4C_A \left[ \frac{1}{(1-x)_+} + \frac{1}{x} - 2 + x - x^2 \right] + \frac{11C_A - 4N_f T_R}{3} \delta(1-x) \quad (1)$$

where we have used our definition of the expansion parameter  $\alpha_s/4\pi$ . Using the SMEFT approach the same splitting function gets an additional contribution:

$$P_{gg}(x) \rightarrow P_{gg}(x) + \Delta P_{gg}(x, \mu), \quad (2)$$

where:

$$\Delta P_{gg}(x, \mu) = -\frac{4C_A}{g_s(\mu)} \left[ \frac{1}{(1-x)_+} + \frac{1}{x} - 2 + \delta(1-x) \right] \frac{C_G}{\Lambda^2} \mu^2. \quad (3)$$

In eq. (3) the ratio  $C_G/\Lambda^2$  is a fixed parameter while  $g_s(\mu) = \sqrt{4\pi\alpha_s(\mu)}$ . In addition, the factor  $\mu^2$  that appears in eq. (3) can be related to  $\alpha_s$  by inverting the function  $\alpha_s(\mu)$  such that  $\mu^2 \equiv \mu^2(\alpha_s)$ . Finally, we can write:

$$\Delta P_{gg}(x, \mu) = -\frac{\mu^2(\alpha_s)}{\sqrt{4\pi\alpha_s}} 4C_A \left[ \frac{1}{(1-x)_+} + \frac{1}{x} - 2 + \delta(1-x) \right] \underbrace{\frac{C_G}{\Lambda^2}}_{K_{\text{EFT}}}, \quad (4)$$

so that:

$$\frac{\alpha_s}{4\pi} P_{gg}(x, \mu) \rightarrow \frac{\alpha_s}{4\pi} P_{gg}(x, \mu) - \sqrt{\frac{\alpha_s}{4\pi}} \frac{\mu^2(\alpha_s)}{4\pi} 4C_A \left[ \frac{1}{(1-x)_+} + \frac{1}{x} - 2 + \delta(1-x) \right] K_{\text{EFT}} \quad (5)$$