Kinetics and Reactor Design HW6

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Assigned: March 23, 2023
Due: March 30, 2023

1 Problem Statement

1.1 P9-16$_B$ (b, c)

P9-16$_B$ The production of a product P from a particular gram-negative bacteria follows the Monod growth law

$$r_g = \frac{\mu_{max} C_s C_c}{K_S + C_s}$$

with $\mu_{max} = 1\ h^{-1}$, $K_S = 0.25\ g/dm^3$, and $Y_{c/s} = 0.5\ g/g$.

(a) The reaction is to be carried out in a batch reactor with the initial cell concentration of $C_{c0} = 0.1\ g/dm^3$ and substrate concentration of $C_{s0} = 20\ g/dm^3$.

$$C_c = C_{c0} + Y_{c/s}(C_{s0} - C_s)$$

Plot r_g, $-r_s$, $-r_c$, C_s, and C_c as a function of time.

(b) The reaction is now to be carried out in a CSTR with $C_{s0} = 20\ g/dm^3$ and $C_{c0} = 0$. What is the dilution rate at which wash-out occurs?

(c) For the conditions in part **(b)**, what is the dilution rate that will give the maximum product rate (g/h) if $Y_{p/c} = 0.15\ g/g$? What are the concentrations C_c, C_s, C_p, and $-r_s$ at this value of D?

Figure 1

1.2 P9-18$_B$ (a - e)

P9-18$_B$ The bacteria X-II can be described by a simple Monod equation with $\mu_{max} = 0.8\ h^{-1}$ and $K_S = 4\ g/dm^3$, $Y_{p/c} = 0.2\ g/g$, and $Y_{s/c} = 2\ g/g$. The process is carried out in a CSTR in which the feed rate is 1000 dm³/h at a substrate concentration of 10 g/dm³.

(a) What size fermentor is needed to achieve 90% conversion of the substrate? What is the exiting cell concentration?

(b) How would your answer to **(a)** change if all the cells were filtered out and returned to the feed stream?

(c) Consider now two 5000-dm³ CSTRs connected in series. What are the exiting concentrations C_s, C_c, and C_p from each of the reactors?

(d) Determine, if possible, the volumetric flow rate at which wash-out occurs and also the flow rate at which the cell production rate ($C_c\ \upsilon_0$) in grams per day is a maximum.

Figure 2

(e) Suppose you could use the two 5000-dm³ reactors as batch reactors that take two hours to empty, clean, and fill. What would your production rate be in (grams per day) if your initial cell concentration is 0.5 g/dm³? How many 500-dm³ batch reactors would you need to match the CSTR production rate?

Figure 3

2 Problem Solution

2.1 P9-16$_B$ (b, c)

b) The equation:

$$D = \frac{Y_{C/S}\mu_{max}C_{S0}}{K_M + C_{S0}} \tag{1}$$

We have all these values given and can evaluate D to be 0.494 hr^{-1}.
c) The equation:

$$D_{max} = Y_{C/S}\mu_{max}\left(1 - \sqrt{\frac{K_M}{K_M + C_{S0}}}\right) \tag{2}$$

We have all these values - D$_{max}$ = 0.44 hr^{-1}.
Equation for C$_c$:

$$C_c = \left[\frac{Y_{C/S}(K_M + C_{S0})}{Y_{C/S}\mu_{max} - D_{max}}\right] \times \left[\frac{Y_{C/S}\mu_{max}C_{S0}}{K_M + C_{S0}} - D_{max}\right] \tag{3}$$

C$_c$ = 9.08 g/cm^3.

$$C_s = \frac{D_{max}K_M}{Y_{C/S}\mu_{max} - D_{max}} \tag{4}$$

C$_s$ = 1.83 g/cm^3.
C$_p$ = C$_s$ $\times$ $Y_{P/C}$ = 1.362 g/cm^3.
-r$_s$ = $\frac{\mu_{max}C_cC_s}{K_M+C_s}$ = 7.99 g/(dm^3 $\times$ hr)

2.2 P9-18$_B$ (a - e)

a) What we know: CSTR, X = 90, Monod eqn. values. $r_g = \mu C_s$, $\mu = \frac{\mu_{max} C_s}{K_M + C_s}$.

$$DC_c = r_g \tag{5}$$

$$D(C_{S0} - C_S) = -r_s \tag{6}$$

$$-r_s = Y_{S/C} r_g \tag{7}$$

$$D = \frac{v_0}{V} \tag{8}$$

$$C_S = C_{S0}(1 - X) = 10g/dm^3(0.1) = 1g/dm^3 \tag{9}$$

$$C_c = Y_{C/S}(C_{S0} - C_S) = 0.5(9)g/dm^3 = 4.5g/dm^3 \tag{10}$$

$$V = \left[\frac{\mu_{max} C_s}{(K_M + C_s) v_0}\right]^{-1} \tag{11}$$

V = 6250 dm^3.

b) How would my answer to (a) change if cells were filtered out and returned to feed stream? The derivative of C_c w.r.t time would become r_g - exponential growth (assuming no cell death). This would stop working at some point because the reactor would be full of cells and more cannot enter.

c) Two 5000 dm^3 CSTRs in series. Exiting concentrations from each reactor?

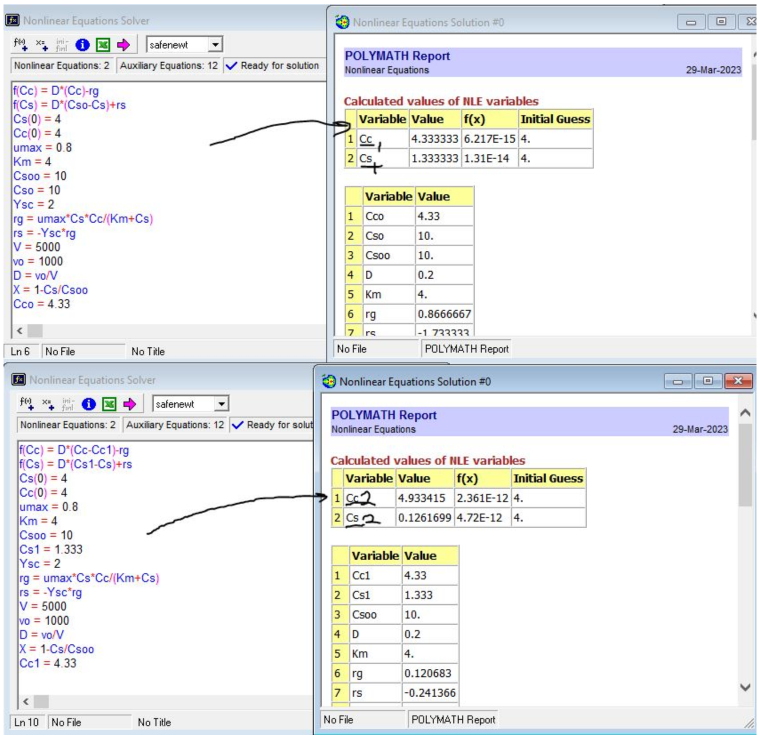

Figure 4: 2 CSTRs in Series - nonlinear Polymath Solution

Not included in image: $C_{P1} = Y_{P/C}C_{C1} = 0.9866$ g/dm^3. C_{C2} and C_{S2} are displayed, have same units.

d) Washout rate volumetric flow rate, also $D_{maxProd}$. For dilution: $C_C = 0$, so $D_{max} = \frac{C_{S0}\mu_{max}}{K_M + C_{S0}} = 0.57$ hr^{-1}.

$$D_{maxProd} = \mu_{max}\left(1 - \sqrt{\frac{K_M}{K_M + C_{S0}}}\right) \tag{12}$$

Thus $D_{maxProd} = 0.37$ hr^{-1}. Production rate then is $C_C 2 v_0$ = 118,392 g/day.

e) Two 5000 dm^3 reactors as batch reactors, 2 hours to empty, clean, fill them. $C_C0 = 0.5$ g/dm^3. Production rate in g/day? How many 500 dm^3 batch reactors would you need to match CSTR production rate (I calculated this in (d)). I'm setting final time for reaction stop at 6 hours - I compared 4 and 6 hours and got a slightly larger number at 6 hours (also both fit neatly into 24 hours).

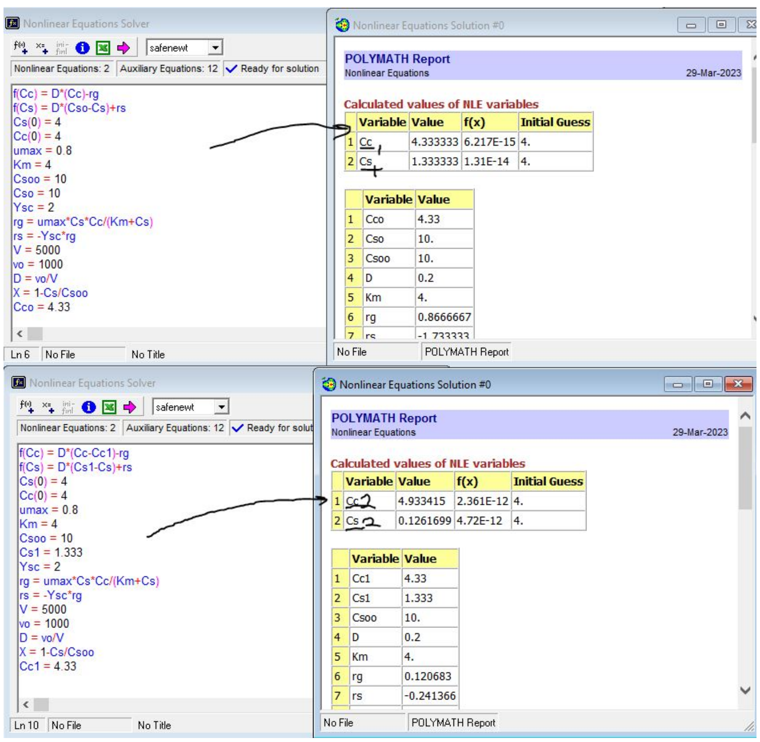

Figure 5: 2 Batch reactors - ODE Polymath Solution for C_C

5000 dm^3 times 5.43 g/dm^3 times 2 reactors times (24 hours/(6 hours to run + 2 to clean) = 162,900 g/day. For 500 dm^3 batches to match CSTR reactor (118,392 g/day), would need (118,392/16,290) reactors, or around 7.25 500 dm^3 batch reactors which run on 6 + 2 hour cycles.