

CS111 Introduction to Computer Science

Recitation 1 (peer leader version)

Exercise 1: Weighing Coins

a. Assume that you have 8 coins, and you know that 7 are ‘okay’ but one is ‘bad’. You know that the bad coin has a different weight than the good coins, but you don’t know whether it is heavier or lighter.

Construct an algorithm to find out which is the bad coin using just 3 weighings on a balance scale. (Hint: Find a way to determine that half of the coins are ‘okay’ with just 1 weighing.) Write your algorithm in the form of a flowchart.

b. Now do the same thing assuming that you have 9 coins, one of which is bad. (Still use just 3 weighings to find the bad coin.)

c. And now for a real challenge, do the same thing assuming that you have 13 coins.

Answer:

With 8 coins: Coins are labeled A through H.

Weigh 1: coins A and B vs. coins C and D.

If these weigh equally, then the ‘bad’ coin is in the group of E through H. Now we take the group of four where the ‘bad’ coin is – let’s label these 1 through 4.

Weigh 2: coin 1 vs. coin 2

Weigh 3: coin 2 vs. coin 3

If:

Weigh 2	Weigh 3	Bad coin is
Equal	Equal	Coin 4
Equal	Unequal	Coin 3
Unequal	Equal	Coin 1
Unequal	Unequal	Coin 2

With 9 coins: Coins are labeled A through I

Note: Keep track of which groups are heavier/lighter!!!

Weigh 1: A, B, and C vs. D, E, and F

Weigh 2: D, E, and F vs. G, H, and I

If:

Weigh 1	Weigh 2	Bad group is
Equal	Equal	impossible
Equal	Unequal	G, H, and I
Unequal	Equal	A, B, and C
Unequal	Unequal	D, E, and F

By keeping track of which groups were heavier/lighter, you know now if the bad group – and therefore the bad coin – is heavier or lighter than the good coin. Now take the bad group, whose members we will label 1, 2, and 3.

Weigh 3: coin 1 vs. coin 2

If even, then the bad coin is 3.

If uneven, you know if the bad coin is heavier or lighter, and therefore also know which coin is the bad one (for example: if you know the bad coin is heavier, and coin 1 is heavier than coin 2, coin 1 is the bad coin)

With 13 coins: This gets a bit complicated in some spots, so be prepared. Coins are labeled A – M.

Weigh 1: A-D vs. E-H

If these are even: (you now know A-H are good coins)

Bad coin is in I – M.
Weigh 2: A-C vs. I-K
If even: (you now know that A-K are good coins)
 Weigh 3: A vs. L
 If even, M is bad.
 If uneven, L is bad.
If uneven: you know the bad coin is in I-K, and you know if it is heavier or lighter (is I-K heavier or lighter than A-C?)
 Weigh 3: I vs. J
 If even, K is bad.
 If uneven, you know if the bad coin is heavier or lighter. (For example, if A-C is lighter than I-K, than the heavier of I and J is the bad coin.)
If uneven: (You know I – M are good coins) Make note of which side is the heavy coins and which side are the light ones! Now we will label L1–L4, and H1–H4.
Take 3 of the good coins, and one of the heavy coins on one side. On the other side, take one of the light coins, and the remaining 3 heavy coins. For example, it can look like this:
 Weigh 2: H4, I, J, K vs. L1, H1, H2, H3.
There are three possible outcomes:
 They balance: The bad coin must be one of the 3 light coins (now labeled coins 1, 2, and 3) not weighed. Now you also know the bad coin is light.
 Weigh 3: 1 vs. 2
 If even, 3 is the bad coin.
 If uneven, the lighter one is the bad coin.
 The heavy side is still heavy: The bad coin must be one of the 3 heavy coins (now labeled coins 1, 2, and 3) that were left on the heavy side. You now also know the bad coin is heavy.
 Weigh 3: 1 vs. 2
 If even, 3 is the bad coin.
 If uneven, the heavier one is the bad coin.
 The heavy side is light: The bad coin is one of the two coins that switched sides [in the example above, H4, which was put on the ‘light’ (now heavy) side, and L1, which was put on the ‘heavy’ (now light)].
 Weigh 3: H4 vs. I
 If even, L1 is the bad coin. ; If uneven, H4 is the bad coin.

Exercise 2: Multiplication flowchart

As a group try and brainstorm ways to perform multiplication without access to an explicit multiplication operation (**SUCCESSIVE ADDITION!!!!**). Together, create a flowchart and then pseudocode showing all of these variables, decisions, and loops on the board.

Exercise 3: ‘Guess my number’ flowchart

The game: Each person picks a number in some range (say, between 1 and 100). The other person tries to guess this number. Whoever gets it right in the fewest guesses wins.

You will be grouped into pairs, and play this game for a few minutes. Try to come up with a winning strategy.

We will then meet as an entire group, pick the best algorithm, and write it out in the form of a flowchart.

The best strategy is binary search. If the students do not come up with this themselves, write up their algorithms as flowcharts first. Then introduce the idea of guessing in the middle and going up or down, and have them write a flowchart for that as well.