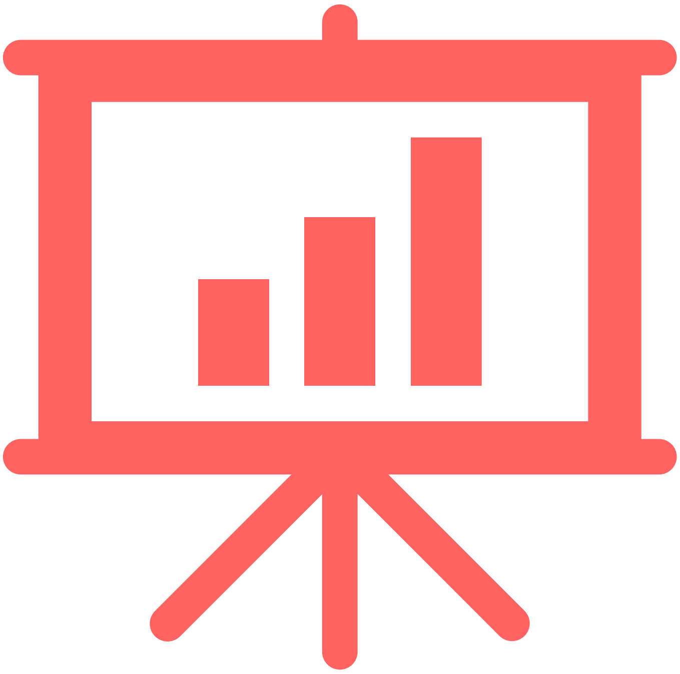

SIT718 Real World Analytics

Assessment 3 | Linear Programming Models

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Question 1

a)

A linear programming model is suitable for this case study as the optimisation problem under investigation is able to be expressed as a minimisation function, there are only two decision variables being considered and both the objective function and all constraint functions can be plotted along linear coordinate planes (Taha 2017).

b)

Decision Variables

Let x_1= # of Litres of Product A used in production of beverage
Let x_2= # of Litres of Product B used in production of beverage

Total cost = cost of existing Product A ($4 p/L) + cost of existing Product B ($10 p/L)

Objective function

To minimise total cost of producing the beverage:

$$\min Z = 4x_1 + 10x_2$$

Constraint satisfaction

$$\frac{0.06x_1 + 0.04x_2}{x_1 + x_2} \geq 0.045$$

**C1: Orange constraint ≥ 4.5%*
(at least 4.5L of Orange p/ 100L)

$$\downarrow$$

$$0.06x_1 + 0.04x_2 \geq 0.045(x_1 + x_2)$$

$$\frac{0.04x_1 + 0.08x_2}{x_1 + x_2} \geq 0.05$$

**C2: Mango constraint ≥ 5%*
(at least 5L of Mango p/100L)

$$\downarrow$$

$$0.04x_1 + 0.08x_2 \geq 0.05(x_1 + x_2)$$

$$0.03x_1 + 0.07x_2 \leq 0.06(x_1 + x_2)$$

**C3: Lime constraint ≤ 6%*
(no more than 6L of Lime p/100L)

$$x_1 + x_2 \geq 100$$

**C4: Demand constraint*
(at least 100L of beverage produced a week)

$$x_1 \geq 0$$

**C5: Sign restriction*

$$x_2 \geq 0$$

**C6: Sign restriction*

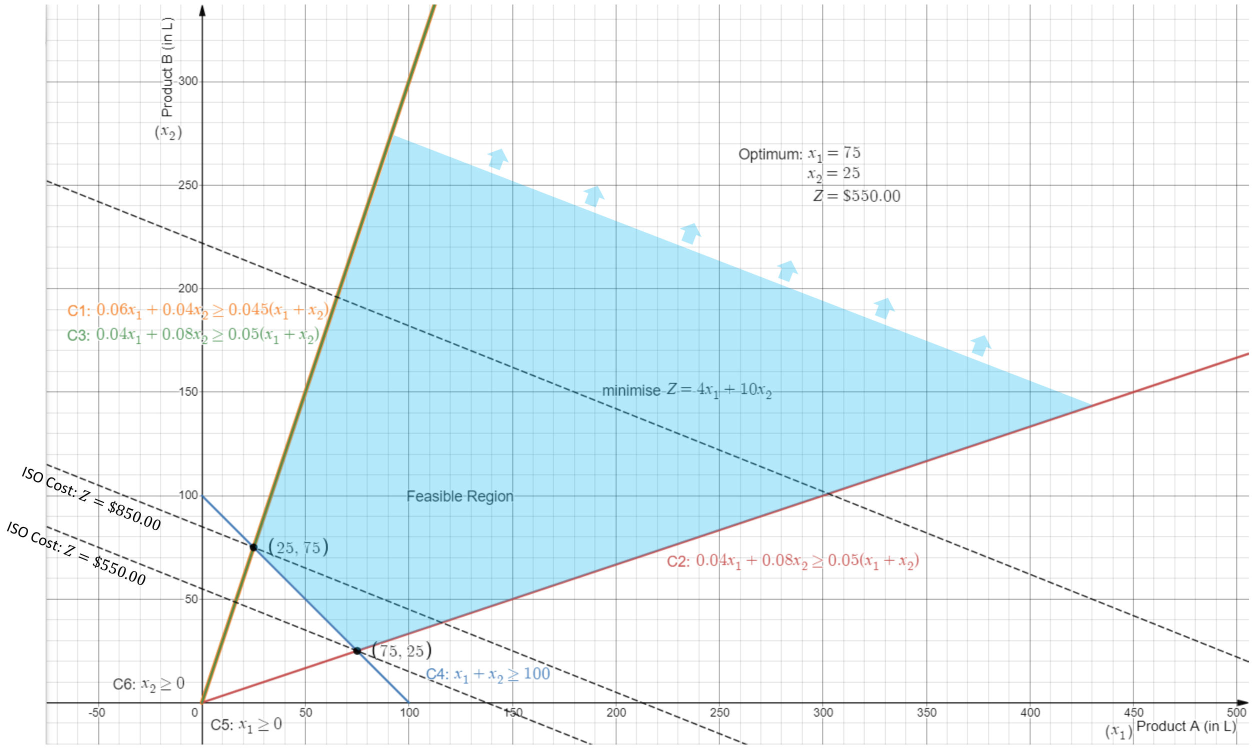
Food Factory LP Model

$$\min Z = 4x_1 + 10x_2$$

subject to

$0.06x_1 + 0.04x_2 \geq 0.045(x_1 + x_2)$	**C1*
$0.04x_1 + 0.08x_2 \geq 0.05(x_1 + x_2)$	**C2*
$0.03x_1 + 0.07x_2 \leq 0.06(x_1 + x_2)$	**C3*
$x_1 + x_2 \geq 100$	**C4*
$x_1,\ x_2 \geq 0$	**C5 & C6*

c)



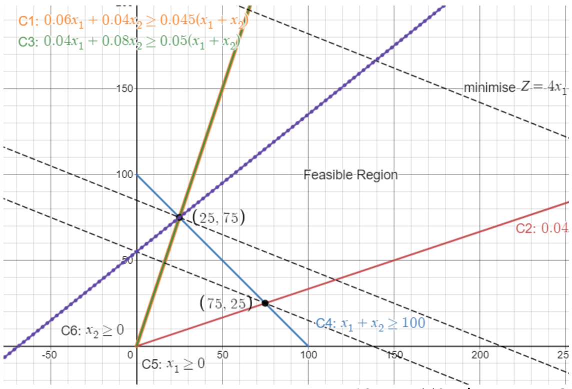
Minimal cost to produce the final product adhering to all constraints is \$550.00 (Z). The optimal solution is achieved through mixing 75 litres of Product A (x_1) and 25 litres of Product B (x_2).

d)

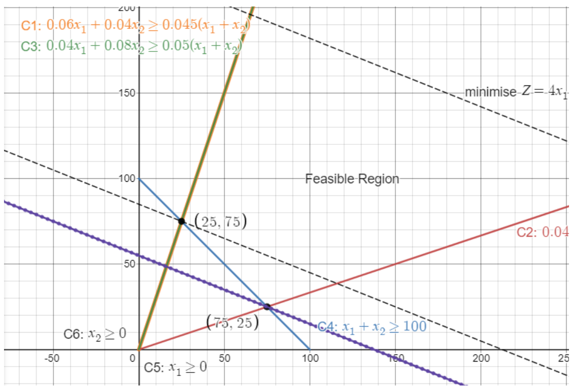
Graphical method

$$4x_1 + 10x_2 = 550$$
$$\downarrow$$
$$cx_1 + 10x_2 = 550$$
$$\downarrow$$
$$-8 \leq c \leq 4$$
$$\downarrow$$
$$0 \leq c \leq 4$$

It is possible to adjust the cost of Product A to any value between \$0 and \$4 without affecting the optimal solution found through the graphical LP sensitivity analysis model.



$cx_1 + 10x_2 = 550$ when $c = -8$



$cx_1 + 10x_2 = 550$ when $c = 4$

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Question 2

a)

Decision Variables

Let x_{ij} be amount of materials (i) used to produce products (j), for i = 1, 2, 3; j = 1, 2, 3.

Number of units of each type of material used:

Cotton:	$x_{11}+x_{12}+x_{13}$
Wool:	$x_{21}+x_{22}+x_{23}$
Silk:	$x_{31}+x_{32}+x_{33}$

Number of units of each type of product produced:

Spring:	$x_{11}+x_{21}+x_{31}$
Autumn:	$x_{12}+x_{22}+x_{32}$
Winter:	$x_{13}+x_{23}+x_{33}$

Revenue from product sales:

$$60(x_{11}+x_{21}+x_{31}) + 55(x_{12}+x_{22}+x_{32}) + 60(x_{13}+x_{23}+x_{33})$$

Cost of materials used in producing product:

$$30(x_{11}+x_{12}+x_{13}) + 45(x_{21}+x_{22}+x_{33}) + 50(x_{31}+x_{32}+x_{33})$$

Production costs:

$$5(x_{11}+x_{21}+x_{31}) + 2(x_{12}+x_{22}+x_{32}) + 5(x_{13}+x_{23}+x_{33})$$

Profit = revenue – material costs – production costs

$$Z = 25x_{11}+23x_{12}+ 25x_{13}+10x_{21}+ 8x_{22}+10x_{23}+ 5x_{31}+3x_{32}+ 5x_{33}$$

Objective function

To maximise profit.

$$\text{Max } Z = 25x_{11}+23x_{12}+ 25x_{13}+10x_{21}+ 8x_{22}+10x_{23}+ 5x_{31}+3x_{32}+ 5x_{33}$$

Constraint satisfaction

Demand constraints

$$x_{11}+x_{21}+x_{31} \le 3200$$ * C1: Maximal demand (in tons) for Spring

$$x_{12}+x_{22}+x_{32} \le 3500$$ * C2: Maximal demand (in tons) for Autumn

$$x_{13}+x_{23}+x_{33} \le 3800$$ * C3: Maximal demand (in tons) for Winter

min. Cotton proportion constraints

$$\frac{x_{11}}{x_{11}+x_{21}+x_{31}} \ge 0.55$$ * C4: min Cotton proportion of 55% in Spring

$$\downarrow$$

$$x_{11} \ge 0.55(x_{11}+x_{21}+x_{31})$$

$$\downarrow$$

$$0.45x_{11} - 0.55x_{21} - 0.55x_{31} \ge 0$$

$0.55x_{12} - 0.45x_{22} - 0.45x_{32} \geq 0$ * C5: min Cotton proportion of 45% in Autumn

$0.7x_{13} - 0.3x_{23} - 0.3x_{33} \geq 0$ * C6: min Cotton proportion of 30% in Winter

min. Wool proportion constraints

$-0.3x_{11} + 0.7x_{21} - 0.3x_{31} \geq 0$ * C7: min Wool proportion of 30% in Spring

$-0.4x_{12} + 0.6x_{22} - 0.4x_{32} \geq 0$ * C8: min Wool proportion of 40% in Autumn

$-0.5x_{13} + 0.5x_{23} - 0.5x_{33} \geq 0$ * C9: min Wool proportion of 50% in Winter

Non-negative constraints

$x_{ij} \geq 0 \ \ (i = 1, 2, 3 \ \ j = 1, 2, 3)$ * C10: Non-negative constraints

b)

Refer to R code.

Optimal profit is $191,600.

This is achieved by manufacturing:
3200 tons of Spring product; using 2240 tons cotton, 960 tons of wool and 0 tons of silk.
3500 tons of Autumn product; using 2100 tons of cotton, 1400 tons of wool and 0 tons of silk.
3800 tons of Winter product; using 1900 tons of cotton, 1900 tons of wool and 0 tons of silk.

Question 3

A_i $= i$th strategy (pure) for Player 1
B_j $= j$th strategy (pure) for Player 2
v_{ij} $=$ Payoff for player 1 if they select Strategy A_i and if Player 2 selects Strategy B_j
Payoff to Player 1 = - Payoff to Player 2

a)

Player 2

			B_1	B_2	B_3	B_4	B_5	B_6			
			RWB	RBW	WRB	WBR	BRW	BWR	s_i		U
Player 1	A_1	RWB	0	0	0	60	-60	0	s_1	-60	
	A_2	RBW	0	0	60	0	0	-60	s_2	-60	
	A_3	WRB	0	-60	0	0	0	60	s_3	-60	-60
	A_4	WBR	-60	0	0	0	60	0	s_4	-60	
	A_5	BRW	60	0	0	-60	0	0	s_5	-60	
	A_6	BWR	0	60	-60	0	0	0	s_6	-60	
			t_1	t_2	t_3	t_4	t_5	t_6			
	t_j		60	60	60	60	60	60			
	L					60					

No saddle point is applicable for this game as there is no equilibrium nor optimal solution as the value for A_i does not equal B_j, as $L > U$.

b)

Player 1's game

$$\max z = v$$

s.t.

$$\begin{aligned}
& v - (-60A_4 + 60A_5) \leq 0 \\
& v - (-60A_3 + 60A_6) \leq 0 \\
& v - (60A_2 - 60A_6) \leq 0 \\
& v - (60A_1 - 60A_5) \leq 0 \\
& v - (-60A_1 + 60A_4) \leq 0 \\
& v - (-60A_2 + 60A_3) \leq 0 \\
& A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 1 \\
& A_i \geq 0, \forall i = 1, 2, 3, 4, 5, 6.
\end{aligned}$$

v u.r.s (unrestricted sign)

Player 2's game

$$\min w = v$$

s.t.

$$\begin{aligned}
& v - (60B_4 - 60B_5) \geq 0 \\
& v - (60B_3 - 60B_6) \geq 0 \\
& v - (-60B_2 + 60B_6) \geq 0 \\
& v - (-60B_1 + 60B_5) \geq 0 \\
& v - (60B_1 - 60B_4) \geq 0 \\
& v - (60B_2 - 60B_3) \geq 0 \\
& B_1 + B_2 + B_3 + B_4 + B_5 + B_6 = 1 \\
& B_i \geq 0, \forall j = 1, 2, 3, 4, 5, 6.
\end{aligned}$$

v u.r.s (unrestricted sign)

c)

Refer to R code.

d)

There is no winner or loser in the game under consideration as the game is fair and free from any bias towards one player winning within the game mechanics itself. Player One's optimal strategy for this game is to play a combination of strategies RWB (A_1), WBR (A_4), and BRW (A_5) equally, 33.33% of the time each. Player Two's optimal strategy for this game is to also play the same combination of strategies RWB (B_1), WBR (B_4), and BRW (B_5) equally, 33.33% of the time each.

Question 4

A = Ava, B = Bob, C = Chloe
(Voter, Vote)

a)

(A,A)

	(C,A)	(C,B)	(C,C)
(B,A)	Ava	Ava	Ava
(B,B)	Ava	Bob	Ava
(B,C)	Ava	Ava	Chloe

(A,B)

	(C,A)	(C,B)	(C,C)
(B,A)	Ava	Bob	Bob
(B,B)	Bob	Bob	Bob
(B,C)	Bob	Bob	Chloe

(A,C)

	(C,A)	(C,B)	(C,C)
(B,A)	Ava	Chloe	Chloe
(B,B)	Chloe	Bob	Chloe
(B,C)	Chloe	Chloe	Chloe

b)

(A,A)

	(C,A)	(C,B)	(C,C)
(B,A)	(1, 0, 2)	(1, 1, 2)	(1, 2, 2)
(B,B)	(2, 0, 2)	(2, 1, 2)	(2, 2, 2)
(B,C)	(0, 0, 2)	(0, 1, 2)	(0, 2, 2)

(A,B)

	(C,A)	(C,B)	(C,C)
(B,A)	(1, 0, 0)	(1, 1, 0)	(1, 2, 0)
(B,B)	(2, 0, 0)	(2, 1, 0)	(2, 2, 0)
(B,C)	(0, 0, 0)	(0, 1, 0)	(0, 2, 0)

(A,C)

	(C,A)	(C,B)	(C,C)
(B,A)	(1, 0, 1)	(1, 1, 1)	(1, 2, 1)
(B,B)	(2, 0, 1)	(2, 1, 1)	(2, 2, 1)
(B,C)	(0, 0, 1)	(0, 1, 1)	(0, 2, 1)

c)

(A,A)

	(C,A)	(C,B)	(C,C)
(B,A)	(1, 0, 2)	(1, 1, 2)	(1, 2, 2)
(B,B)	(2, 0, 2)	(2, 1, 2)	(2, 2, 2)
(B,C)	(0, 0, 2)	(0, 1, 2)	(0, 2, 2)

(A,B)

	(C,A)	(C,B)	(C,C)
(B,A)	(1, 0, 0)	(1, 1, 0)	(1, 2, 0)
(B,B)	(2, 0, 0)	(2, 1, 0)	(2, 2, 0)
(B,C)	(0, 0, 0)	(0, 1, 0)	(0, 2, 0)

(A,C)

	(C,A)	(C,B)	(C,C)
(B,A)	(1, 0, 1)	(1, 1, 1)	(1, 2, 1)
(B,B)	(2, 0, 1)	(2, 1, 1)	(2, 2, 1)
(B,C)	(0, 0, 1)	(0, 1, 1)	(0, 2, 1)

Best strategy for Ava: (A,A) The matrix with the highest value in the third position.
Best strategy for Bob: (B,B) The row with the highest value in the first position.
Best strategy for Chloe: (C,C) The column with the highest value in the second position.

A singular Nash equilibrium exists for this game and it is for each voter to cast their vote for themselves as President (A,A; B,B; C,C). Assuming all involved to be rational decision-makers, if any candidate were to change their strategy from a self-vote their payoff would decrease when no considerations are made to their opposition's strategy.

d)

A singular Pareto optimality exists for this game and it is the same found as the Nash equilibrium, all voters should cast their own vote for themselves in the Pareto Optimal solution. This is due to there being no alternative outcome that improves all players situation, or payoff, whilst simultaneously not negatively influencing any other.

e)

The extra power afforded to Ava absolutely benefits her ability to influence the candidate selected as president. Ava's additional power is only enacted in cases of a deadlocked vote (each candidate receiving a single vote), 6 of the 27 total outcomes, and her extra power improves the probability of her selection ultimately being selected president from 66.67% or 15 of the 27 total outcomes (if a draw is considered a viable outcome), to 77.78% or 21 of the 27 total outcomes (an 11.11%

increase), with her vote serving to break any impasse. Her extra power improves her own chances to be elected as president by 7.4%, the two outcomes whereby either both Bob and Chloe casts their votes for themselves or in the situation that they both choose to cast their votes for the other candidate who is not Ava. One of these 2 outcomes (all candidates casting their vote for themselves) can be considered both the Nash equilibrium and Pareto optimality strategy so in actuality this probability for Ava's power to influence the selected president would be even higher if all players are optimising their strategies through the adoption of game theory methodology.

References

Taha H (2017) *Operations Research an Introduction, EBook, Global Edition*, Pearson Education, Limited, Harlow, UK.