Exponents and Logarithms

Exponents

Exponential functions are functions with the variable in the exponent of the expression.

The Base

The exponential function $f(x)$ with base b in simplest form:

$$f(x) = b^x \quad or \quad y = b^x$$

The base b must be a **positive constant** and x is any real number. Note that while the base can technically be 1, that would be meaningless because 1 raised to any power is always 1. This implies the following are ***not* exponential functions**:

$f(x) = x^2$	$g(x) = 1^x$	$h(x) = x^x$
x is the base	base of 1	x is both base and exponent

Common bases are 10 (what we are used to dealing with), 2 (called binary and used in computer science), and the natural base, e, or Euler's number or sometimes Napier's constant.

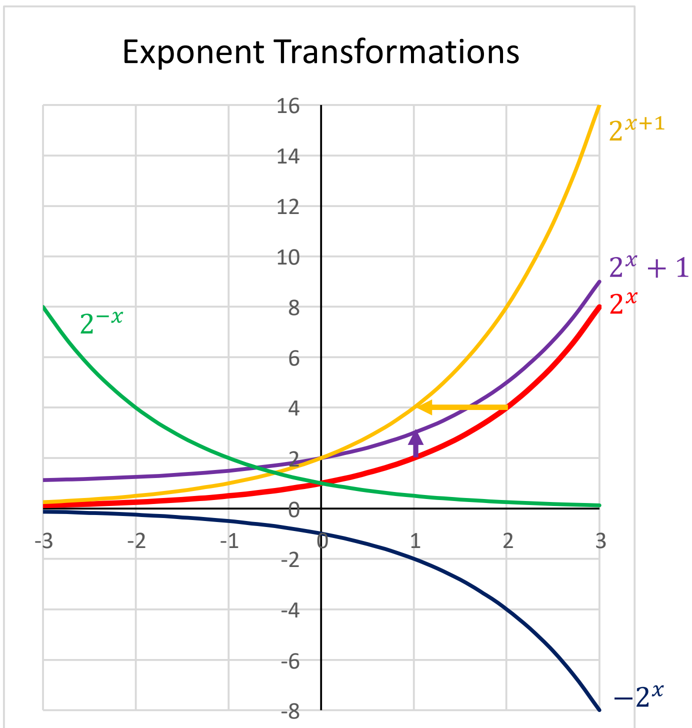
> Note that "Euler" is a German name, Leonhard Euler (1707–1783), a Swiss mathematician and scientist, and is pronounced OY-ler, as "eu" in German is pronounced "OY."

e = 2.71828182845904523536028747135266249775724709369995...

Notations

No different than when the variable x was the base. Note the Greek upper case letter Pi (Π) is an operation analogous to Sigma (Σ) but the operation is multiplication ("product") instead of addition ("sum").

b^x	is equivalent to	$\prod_1^x b$
b^{-x}	is equivalent to	$\frac{1}{b^x}$
$b^{1/x}$	is equivalent to	$\sqrt[x]{b}$



Graphs and Transformations

b^x	Basic form. y-intercept is at 1 and is asymptotic to the x-axis when moving toward $x = -\infty$
$b^x \pm c$	Vertical translation, shift up (+) or down (–) by c
$b^{x \pm c}$	Horizontal translation, shift left (+) or right (–) by c
b^{-x} $-b^x$	Reflection about the y-axis Reflection about the x-axis

Note the shift up and shift left by one unit and the reflections about the x-axis and y-axis.

Logarithms

Inverse Functions

An inverse function is the reverse, that is, undoes, the action of a function.

For a function *f*(*x*), you plug in a value for *x* and you get *y*. Imagine the function *f* as a “black box” that you cannot see inside. You enter a value for *x* on the left and the function’s corresponding value *y* comes out on the right. Move from left to right.

$$\boxed{x} \longrightarrow \boxed{\text{Function } f} \longrightarrow \boxed{y}$$

For the inverse function, you plug in the value for *y* and you get the corresponding *x*. as if you are moving now from right to left.

$$\boxed{x} \longleftarrow \boxed{\text{Inverse Function } f} \longleftarrow \boxed{y}$$

Note that inverse functions are undefined in cases that would result in division by zero.

Logarithm as Inverse of Exponent

A logarithm is an inverse function of an exponent. For some base *b* and exponent *x*:

$$y = b^x \quad \text{then} \quad x = \log_b y$$

Note the notation of the logarithm function: “log” followed by the base as a subscript. Note that many times when using a logarithmic base of 10 (referred to as common log), the 10 subscript may be omitted. When the base is Euler’s number, *e*, the logarithm function is referred to as the natural logarithm, or just “natural log,” and uses “ln” without a subscript instead of “$\log_e$.”

Examples

Base 10: $10^3 = 1{,}000$ therefore $\log_{10} 1000 = 3$

Base 2: $2^4 = 16$ therefore $\log_2 16 = 4$

Note this is the basis for using hexadecimal (base 16) numbers in computers as a shorthand for binary, where each value represents a set of four bits.

Natural Logarithm, base *e*: $e^2 = 7.389$ therefore $\ln 7.389 = 2$

Graph

Note how the graphs of the logarithm and exponent are symmetrical about the 45° line.