Do Public Asset Purchases Foster Liquidity?*

Daniel Neuhann[†] Michael Sockin[‡]

December 29, 2025

Abstract

We analyze how public asset purchases affect liquidity and risk sharing in imperfectly competitive financial markets. Even "neutral" policies, such as budget-balanced trading of risk-free debt, affect liquidity and equilibrium trading efficiency. We show that public debt purchases *worsen* liquidity and risk sharing by increasing price impact, while debt *sales* improve risk sharing but distort intertemporal trade. "Expansionary" quantitative easing policies can thus reduce trading efficiency. We also derive quantity-based trading rules for optimal public portfolio management. These call for policy to equalize weighted measures of public and private price impact.

*This paper was previously titled "When Should Governments Buy Assets?" First version: December 2022. We thank Zach Bethune, Eduardo Davila, Keshav Dogra, Thomas Eisenbach, Ana Fostel, Anton Korinek, Wenhao Li, Konstantin Milbradt, Frederic Mishkin, Erwan Quintin (discussant), Jacob Sagi, Lukas Schmid, Victoria Vanasco (discussant), Laura Veldkamp and seminar participants at Imperial College London, Federal Reserve Bank of New York, FIRS, MFA, Rice University, UNC Kenan-Flager, University of Southern California Marshall and University of Virginia for helpful comments.

†University of Texas at Austin. daniel.neuhann@mccombs.utexas.edu

‡University of Texas at Austin. michael.sockin@mccombs.utexas.edu

1 Introduction

Central banks around the world now routinely buy and sell financial securities to achieve a variety of policy objectives, including altering rates of return, supporting market liquidity, or managing reserve portfolios. Such interventions were initially justified as necessary to combat financial crises or to circumvent constraints on short-term policy rates. However, in recent years they were often employed even in "normal times" where markets were not acutely impaired. What are the effects of government interventions on market liquidity and trading efficiency in such circumstances? How do these effects vary with the size of public portfolios? How should optimal public trading strategies be designed?

We address these questions using a framework in which market liquidity (i.e., the equilibrium price impact of a marginal trade) and trading efficiency (i.e., the extent to which gains from trade are realized) endogenously respond to government interventions. We show that even purportedly "neutral" policies – such as purchases of risk-free debt – harm liquidity and risk sharing, while asset sales improve liquidity and risk sharing, albeit potentially at the cost of distorting intertemporal smoothing. This means that standard expansionary policies can have unintended consequences for market functioning and induce inefficient risk management. We also derive implications for optimal policy, showing that it is optimal to align weighted averages of private and public price impact.

The basic mechanism underlying our results is that imperfect liquidity – due to imperfect competition in financial markets – affords investors with market power the ability to extract price concessions by rationing trades.[1] In equilibrium, this manifests as unrealized gains from trade across time and states, which we measure using both cross-sectional misalignment in marginal valuations and welfare wedges from Davila and Schaab (2024). We then show that the intensity of these distortions is proportional to the gross quantity of assets traded, because higher quantities increase private incentives to distort prices. Hence, policy interventions which affect traded quantities but are otherwise neutral – say, exchanging assets for cash – can improve market functioning.

We characterize this new transmission mechanism for a variety of endowment processes and policy tools. To isolate the mechanism from other frictions, we remove

[1] Wallen and Stein (2023) and Pelizzon, Subrahmanyam, and Tomio (2022) show that public trading can crowd out private liquidity provision in bond markets. Pinter, Siriwardane, and Walker (2024) and Breckenfelder, Collin-Dufresne, and Corradin (2024) find that imperfect competition further reduces liquidity.

all distortions other than imperfect liquidity: markets are complete and integrated, and the government fully backs its interventions with non-distortionary taxes and transfers. As in standard macroeconomic models of risk sharing, we also assume investor preferences feature convex marginal utility (e.g., constant relative risk aversion). This leads to a tractable and flexible setting in which price impact functions can be solved in closed form, equilibrium is invariant to the precise set of assets available to trade, and we can accommodate arbitrary sources of gains from trade between investors.

We begin our analysis with a canonical setting in which the government buys or sells only risk-free debt, and all gains from trade between private investors stem from intra-temporal risk sharing across states of the world. We measure liquidity using the equilibrium price impact of marginal trade. Despite the fact that risk-free bond payoffs are orthogonal to all gains from trade between investors, we find that government purchases of risk-free debt generically reduce liquidity and worsen risk sharing. Hence "expansionary" policies which lower the risk-free rate also induce inefficient risk taking.

Underlying this result is a sorting property of optimal portfolio choice: in complete markets, investors sell claims on a given state of the world if and only if they are relatively wealthy in that state. Unless there is perfect risk sharing, the marginal value of consumption is thus steeper for buyers than for sellers. Since marginal utility measures the private cost of strategic rationing, sellers ration more aggressively than buyers, creating endogenous scarcity and excessively high asset prices in all state-contingent claims. We dub this effect "strategic scarcity." Public purchases of risk-free debt – i.e., an equal-weighted portfolio of all Arrow securities – amplify this scarcity and worsen risk sharing.

This risk-taking channel of asset market interventions is consistent with empirical evidence. For example, Pinter and Walker (2023) show that markets for interest rate risk are concentrated, that many financial institutions do not fully hedge interest rate risks in derivatives markets, and that there is less hedging during periods of expansionary monetary policy. Similarly, prominent policymakers have raised concerns that public asset purchases may hamper market functioning in normal times (Bernanke, 2012; Coeurè, 2015; Logan and Bindseil, 2019). More constructively, however, we also find that public sales of risk-free debt improve market functioning: they lower price impact and improve risk sharing even without injecting any resources.

Having characterized the case of pure risk sharing, we introduce intertemporal gains from trade. Public sales of risk-free debt now have distributional consequences: they benefit borrowers at the expense of savers. This creates a countervailing force whereby asset sales can improve intra-temporal risk sharing but distort intertemporal smoothing, and the cost of intertemporal distortions are increasing in the size of the intervention. We use this result to derive an "absorption capacity" for asset market interventions beyond which large interventions reduce welfare even if they improve risk sharing.

Although our model does not capture several important channels of monetary-fiscal policy, it is well-suited for analyzing how governments can optimally use trading rules to improve liquidity and trading efficiency. To this end, we derive optimal utilitarian tax-and-trading schemes and show that public portfolios should be chosen to equalize weighted averages of public and private price impact.[2] When the government is restricted to trading risk-free debt, these weights are given by marginal-utility averages of investor asset positions and demand elasticities. We also show how these insights extend to public interventions in arbitrary assets. This is useful for understanding recent policies across different asset classes, such as corporate bonds, equities, and foreign exchange (e.g., Amador, Bianchi, Bocola, and Perri, 2020). When the government faces a constraint that it must trade a certain dollar amount–for example, to achieve a given quantitative easing or tightening objective–we further establish a *pecking order* that indicates which assets can be traded with the smallest distortions. This allows our model to speak to the broader asset market interventions that have been used in recent years.

We conclude with a numerical illustration based on 2014-2017 Eurozone Quantitative Easing program. Through the lens of our model, we find that the program lowered sovereign debt yields with only a modest loss in trading efficiency.

Related literature. We contribute to an important literature studying public asset purchases and other market interventions. Often, quantitative easing and large-scale asset purchases are studied in the context of financial crises with fire sales. A recurring theme in this literature is that asset purchases can improve market liquidity when prices are depressed because of forced sales by market participants(e.g., Davila and Korinek, 2018). We instead focus on a setting without financial constraints, and show that asset purchases may instead hamper it. More broadly, previous literature has considered the

[2]This utilitarian objective does not consider investors outside the model, such as households and firms that may be ultimately affected by such large-scale asset purchase and sale programs. Hence, we view this objective mainly as a convenient way of aggregating payoffs across agents in the model.

determinants and effects of public debt issuance. To the best of our knowledge, ours is the first analysis of such policy interventions in imperfectly competitive markets. To focus on this issue, we abstract from other known determinants of debt issuance, such as incomplete markets (e.g., Angeletos, 2002; Bewley, 1986), convenience yields (e.g., Choi, Kirpalani, and Perez, 2022; He, Krishnamurthy, and Milbradt, 2019), collateral scarcity (e.g., Angeletos, Collard, and Dellas, 2023), credit rationing (e.g., Holmstrom and Tirole, 1998), and liquidity traps (e.g., Caramp and Singh, 2023). We also remove other frictions known to influence the effects of asset purchases, such as market segmentation, (e.g., Gertler and Karadi, 2013; Vayanos and Vila, 2021) and asymmetric information (e.g., Wang, 2023).

Our paper also relates to a literature that studies market concentration in financial markets, and bond markets in particular (Eisenschmidt, Ma, and Zhang, 2022; Huber, 2023; Wallen, 2020; Wang, 2018). In this context, we analyze policy interventions and show that "expansionary" policies generate poor risk sharing. This result has some similarity with the "reach for yield" channel whereby low interest rates induce financial institutions to substitute toward risky assets with high expected returns. However, in our setting risk taking reflects inefficient diversification, not higher expected returns. The two mechanisms also have different origins. Reach for yield is driven by portfolio restrictions, high leverage or moral hazard (Martinez-Miera and Repullo, 2017). Ours instead depends on market structure and asset quantities traded, not the level of interest rates.

Finally, our work contributes to rich literature on imperfectly competitive financial markets. Using similar equilibrium concepts as ours, Eisenbach and Phelan (2022) study fire sales externalities, Kacperczyk, Nosal, and Sundaresan (2021) investigates the impact of large institutional investors on asset price informativeness. Neuhann and Sockin (2024) use a similar model to study capital misallocation in a production economy, while Neuhann, Sefidgaran, and Sockin (2025) ask how market power varies with the span of assets investors can trade. Basak (1997) and Basak and Pavlova (2004) examine dynamic asset pricing with a monopolistic non-price-taking agent in an Arrow-Debreu economy.

2 Model

There are two dates, $t = \{1, 2\}$. Uncertainty is represented by a set of states of the world $\mathcal{Z} \equiv \{1, 2, \ldots Z\}$, one of which realizes at date 2. The probability of generic state $z \in \mathcal{Z}$ is

$\pi(z) \in (0, 1)$, and all agents share common beliefs.

Market participants. There are two classes of agents: a continuum of competitive agents with mass m_f called the *competitive fringe* that takes prices as given, and a discrete number of *strategic agents* who are large relative to the economy and internalize their impact on prices in financial markets. The presence of a competitive fringe can represent, for instance, retail investors and smaller institutional investors. There is also a government that can buy or sell risk-free debt, but is constrained to balance its budget at each date.

There are N types of strategic agents, indexed by $i \in \{1, 2 \ldots, N\}$, where an agent's type determines her income process. Within each type, there exist $1/\mu$ agents, each of whom has mass μ. For an individual strategic agent j of type i, μ determines her price impact because it measures her size relative to the economy. In the aggregate, μ proxies for market concentration. The competitive equilibrium corresponds to the limit $\mu \to 0$.

Preferences. Strategic agents share common preferences over consumption at both dates. These are represented by the utility index $u(c)$ that is $\mathcal{C}^2$, strictly increasing, strictly concave, homothetic, and satisfies the Inada condition. Marginal utility $u'(c)$ is further assumed to be strictly convex. Risk aversion captures the notion that even large financial institutions can exhibit limited risk-bearing capacity under a variety of frictions, such as capital and risk management constraints. The fringe has quasi-linear preferences: linear in consumption at date 1 and risk-averse at date 2. Its date-2 utility function, $u_f(c)$, satisfies the same properties as that of strategic agents. Although a price-taking fringe is essential for our results, quasi-linearity of its preferences is not.[3]

Income and consumption. The fringe receives initial wealth w_f and state-contingent endowment $y_f(z) > 0$. A strategic agent j of type i receives initial endowment μw_i at date 1, and state-contingent endowment $\mu y_i(z) > 0$ in state z. The total initial endowment and state-contingent income of agents of type i are consequently also w_i and $y_i(z)$, respectively, and the aggregate endowment of all strategic agents is $Y(z) = \sum_i y_i(z)$. These income processes can be interpreted in multiple ways. One interpretation is that they represent the operational cash flow exposures of institutional investors. Another is that they represent the payoffs of asset portfolios that were in place before the government intervenes. Risk sharing needs could then represent the outcome of shocks to the expected

[3] Earlier versions considered a fringe with the same risk-averse preferences at date 0 as at date 1. Although this yields a richer price impact function than with quasi-linearity because of wealth effects, it complicates the analysis without qualitatively adding to our insights on the role of government trading.

payoffs of these portfolios. In the context of insurance companies and pension funds, these could reflect not only differences in existing asset exposures, but also in net cash flows from premiums less payouts to insurees or defined benefit pensioners.

Aggregate resource constraints are as follows. Let $c_{1,j,i}$ and $c_{2,j,i}(z)$ denote consumption of agent j of type i at date 1 and in state z, respectively, and similarly with c_{1f} and $c_{2,f}$ for the fringe. Aggregating within types gives $c_{1,i} = \sum_{j=1}^{1/\mu} \mu c_{1,j,i}$ and $c_{2,i}(z) = \sum_{j=1}^{1/\mu} \mu c_{2,j,i}(z)$. The aggregate resource constraints are

$$\sum_{i=1}^{N} c_{1,i} + m_f c_{1,f} = \sum_{i=1}^{N} w_i + w_f, \tag{1}$$

$$\sum_{i=1}^{N} c_{2,i}(z) + m_f c_{2,f}(z) = Y(z) + m_f y_f(z). \tag{2}$$

Financial markets. Financial markets are complete and open at date 1. The traded assets are the full set of Arrow securities; i.e., there are Z securities such that security z pays one unit of the numeraire in state z and zero otherwise. We show below that equilibrium allocations are invariant to the precise security menu, holding fixed the asset span. As such, it is without loss of generality to focus on trading in Arrow securities only. The risk-free rate is the inverse of the sum of all Arrow security prices,

$$r_f = \left(\sum_{z \in \mathcal{Z}} q(z) \right)^{-1}. \tag{3}$$

Let $a_{j,i}(z) \in \mathbb{R}$ denote the position of agent j of type i in claim z, where $a_{j,i}(z) < 0$ denotes a sale. Aggregating within and across types yields $a_i(z) \equiv \sum_{j=1}^{1/\mu} \mu a_{j,i}(z)$ and $A(z) \equiv \sum_{i=1}^{N} a_i(z)$. The fringe's and the government's positions in security z are $a_f(z)$ and $a_G(z)$, respectively. Market clearing in the market for claim z requires:

$$A(z) + a_G(z) + m_f a_f(z) = 0. \tag{4}$$

Finally, define $\mathbf{A}$ to be the $(N+2) \times Z$ matrix summarizing portfolios choices of all agents and the government. The equilibrium price function of asset z is denoted $Q(\mathbf{A}, z)$. In contrast, the perceived pricing functional used by agent j of type i to forecast her influence on the price of security z is $\tilde{Q}_{i,j}(\mathbf{A}, z)$.

Government. The government can buy or sell Arrow assets at date 1 subject to budget balance at each date. It maintains budget balance through uniform lump sum transfers τ_1 and $\tau_2(z)$ at dates 1 and 2 to all agents that can be positive or negative.[4] This imposes the budget constraints:

$$\left(N+m_f\right)\tau_1+\sum_{z\in\mathcal{Z}}Q(\mathbf{A},z)a_G(z)=0, \tag{5}$$

$$\left(N+m_f\right)\tau_2(z)+a_G(z)=0. \tag{6}$$

Our complete-markets setting with Arrow securities allows us to flexibly capture a variety of financial market interventions. For example, if we want to impose the restriction that the government trades only risk-free debt, we need only impose that it trades equal quantities of each Arrow security, $a_G(z)=a_g$ for all $z\in\mathcal{Z}$. Other assets can be constructed from Arrow securities in the analogous fashion. We interpret asset sales as the unwinding of previous asset purchases by the government, or as the sale of synthetic assets whose cash flows the government replicates through taxes at date 2.

Although we maintain a general formulation of taxes and transfers throughout our analysis, a natural interpretation of these tax-and-trade schemes is that the government exchanges long-term financial assets (i.e., the Arrow securities) for cash or reserves held by financial institutions. As will become clear, these exchanges can have equilibrium consequences because exchanging cash for financial assets may induce financial institutions to alter their asset portfolios even when their budget set is unchanged.

Decision problems and equilibrium concept. The government is a Stackelberg leader and sets its tax and trading policies first. Conditional on these policies, we search for a *Cournot-Walras* equilibrium in which the competitive fringe takes asset prices as given and strategic agents place limit orders while taking into account their price impact.[5]

A *strategy* $\sigma_{j,i}$ for strategic agent j of type i consists of asset positions and consump-

[4]In Appendix B, we relax the assumption that government trading is budget-balanced to study the additional effect of shifting aggregate resources across dates. Unfunded asset purchases and sales confound the impact of government trading on liquidity with that on the risk-bearing capacity of financial markets.

[5]This equilibrium concept differs from the Equilibrium-in-demand-schedules approach of Kyle (1989). An advantage of our concept is its much greater tractability, which allows us to study complete markets and incorporate rich heterogeneity across investors and trading needs. This ensures that government policy does not operate by completing markets and allows us to discuss distributional consequences of market interventions. Neuhann and Sockin (2024) provide a detailed comparison of the two concepts.

tion, $\sigma_{j,i} = \{\{a_{j,i}(z)\}_{z\in\mathcal{Z}}, c_{1,j,i}, c_{2,j,i}\}$. The decision problem is

$$\begin{aligned} U_{j,i} = \max_{\sigma_{j,i}} \quad & u\left(c_{1,j,i}\right) + \sum_{z\in\mathcal{Z}} \pi\left(z\right) u\left(c_{2,j,i}\left(z\right)\right) \\ \text{s.t.} \quad & \mu c_{1,j,i} = \mu w_i - \mu\tau_1 - \sum_{z\in\mathcal{Z}} \tilde{Q}_{i,j}(\mathbf{A}, z)\mu a_{j,i}\left(z\right), \\ & \mu c_{2,j,i}\left(z\right) = \mu y_i\left(z\right) + \mu a_{j,i}\left(z\right) - \mu\tau_2\left(z\right). \end{aligned} \tag{7}$$

We define preferences and controls in this manner recognizing that the consumption of strategic agent j of type i is actually $\mu c_{1,j,i}$ and $\mu c_{2,j,i}\left(z\right)$ at dates 1 and 2, respectively, and similarly with optimal asset holdings, $\mu a_{j,i}\left(z\right)$. Given homothetic utility, however, optimal policies are invariant to defining a strategic agent's preferences over $\mu c_{t,j,i}$.

A *strategy* σ_f for the competitive fringe consists of asset positions and consumption, $\sigma_f = \{\{a_f(z)\}_{z\in\mathcal{Z}}, c_{1,f}, c_{2,f}\}$. Because it takes prices as given, it chooses its portfolio subject to equilibrium prices, $\{Q(\mathbf{A}, z)\}_{z\in\mathcal{Z}}$. The fringe's decision problem is

$$\begin{aligned} U_f = \max_{\sigma_f} \quad & c_{1f} + \sum_z \pi(z) u(c_{2,f}(z)) \\ \text{s.t.} \quad & c_{1f} = w_f - \tau_1 - \sum_z Q(\mathbf{A}, z) a_f(z), \\ & c_{2,f}\left(z\right) = y_f(z) - \tau_2\left(z\right) + a_f(z). \end{aligned} \tag{8}$$

Our equilibrium concept is Cournot-Walras equilibrium.

Definition 1 (Cournot-Walras Equilibrium) *Fixing a strategy of the government, a Cournot-Walras equilibrium consists of a strategy $\sigma_{j,i}$ for each strategic agent, a strategy σ_f for the competitive fringe, and pricing functions $Q(\mathbf{A}, z)$ for all $z \in \mathcal{Z}$ such that:*

1. *Fringe optimization: σ_f solves decision problem* (8) *given* $\{Q(\mathbf{A}, z)\}_{z\in\mathcal{Z}}$
2. *Strategic agent optimization: For each agent j of type i, $\sigma_{j,i}$ solves decision problem* (7) *given* (i) *other agents' strategies* $\{\sigma_{-j,i}, \sigma_f\}$ *and perceived pricing functions* $\{\tilde{Q}_{j,i}(\mathbf{A}, z)\}_{z\in\mathcal{Z}}$.
3. *Market-clearing: Each market clears with zero excess demand according to* (4).
4. *Consistency: all agents have rational expectations, which requires for strategic agents that* $\tilde{Q}_{j,i}(\mathbf{A}, z) = Q(\mathbf{A}, z)\ \forall\ i, j$ *and* z.

The competitive fringe mediates strategic interactions. Each strategic agent takes the asset positions of other strategic agents as given but internalizes how her own demand impacts equilibrium asset prices by altering the marginal utility of the fringe. Strategic agents thus indirectly influence each other by altering prevailing prices and price impact.

3 Equilibrium

We begin by establishing several properties of equilibrium that will be useful for our analysis. The first step is deriving the pricing functional that determines an investors' influence on prices. We can solve for this object by observing that the competitive fringe optimally aligns its state-contingent marginal utility with the associated Arrow security price. Asset prices are thus pinned down by the fringe's consumption process, and each strategic agent can infer her price impact from the change in the fringe's marginal utility that is induced by a change in her portfolio. This is shown in Lemma 1, which also establishes equilibrium existence and uniqueness.

Lemma 1 (Prices and Price Impact) *There exists a unique equilibrium in which the price of the Arrow security referencing state z is*

$$Q(\mathbf{A}, z) = q(z) \equiv \pi(z) u_f' \left(c_{2,f}(z)\right). \tag{9}$$

The price impact of strategic agent i satisfies

$$\frac{\partial Q_{j,i}(\mathbf{A}, z)}{\partial a_i(z)} = \frac{\mu}{m_f} q'(z) \qquad \text{where} \qquad q'(z) \equiv \frac{\partial q(z)}{\partial A(z)} = -\pi(z) u_f'' \left(c_{2,f}(z)\right) > 0, \tag{10}$$

and price impact $q'(z)$ is increasing in the price level and increasing and convex in strategic agent demand. The law of one price holds, and equilibrium consumption allocations are invariant to the presence of a risk-free or redundant assets.

Lemma 1 implies that the law of one price holds, so that consumption allocations are invariant to the particular asset menu. As such, we can model public interventions in any given asset by constructing the asset as a bundle of Arrow securities. Moreover, each individual agent's price impact scales with her mass μ, and price impact vanishes in the limit $\mu \to 0$. Hence we can nest the benchmark equilibrium with perfect competition. Since the competitive equilibrium achieves the first best, deviations from this benchmark can be used to measure the efficiency consequences of market power. Finally, under convex marginal utility price impact is high (liquidity is low) when fringe consumption is low. As such, price impact is higher for more expensive assets.

Government policies as asset endowments. To understand how public trading affects private portfolio choice, it is useful to interpret public trading as endowing investors with an *inventory* of assets that they may keep or sell. To see this, observe that, by government budget balance, a policy $\{a_g(z)\}$ induces the per-capita tax and transfer process

$$\tau_1 = \frac{1}{N+m_f}\sum_{z\in\mathcal{Z}} q\left(z\right) a_G(z) \qquad \text{and} \qquad \tau_2\left(z\right) = -\frac{1}{N+m_f} a_G(z). \tag{11}$$

Since taxes and transfers are state contingent, they can be interpreted as influencing the *net* holdings of a given Arrow security conditional on the agents' portfolio choices. In particular, we can write type i's *net asset position* in state z as the sum of private trades and the state-contingent transfer induced by policy,

$$\hat{a}_{j,i}\left(z\right) \equiv a_{j,i}\left(z\right) + \frac{a_G(z)}{N+m_f}, \tag{12}$$

Any policy can thus be interpreted as changing the baseline consumption process *prior* to private trading while leaving investor budget sets unchanged. By market-clearing, the consumption process for all investors, and thus prices, is then fully determined by endowments and *net* asset positions. Specifically, we have

$$c_{1i} = w_i - \sum_{z\in\mathcal{Z}} q\left(z\right) \hat{a}_{j,i}\left(z\right) \tag{13}$$

$$c_{2i}(z) = y_i\left(z\right) + \hat{a}_{j,i}\left(z\right) \tag{14}$$

$$q\left(z\right) = \pi\left(z\right) u_f'\left(y_f\left(z\right) - \frac{1}{m_f}\sum_{i=1}^{N} \hat{a}_{j,i}\left(z\right)\right). \tag{15}$$

This observation allows us to establish a benchmark with Ricardian equivalence: the interventions we consider have no effect under perfect competition because investors can simply undo them by trading in financial markets.

Proposition 1 (Ricardian Equivalence under Perfect Competition) *In the competitive equilibrium, government asset purchases and sales do not affect consumption allocations or asset* *prices. Risk sharing is perfect and no gains from trade are lost for any government policy.*

In contrast, Ricardian equivalence fails under imperfect liquidity: when trading is costly, investors do not fully undo their position with the government. This allows policy to influence equilibrium even in complete and integrated markets.

Optimal portfolios. To describe optimal asset positions, define the *state price* of strategic agent j of type i in state z as the state-specific marginal rate of substitution

$$\Lambda_{j,i}(z) \equiv \frac{\pi(z)\, u'\left(c_{2,j,i}(z)\right)}{u'\left(c_{1,j,i}\right)}.$$

We then have the following characterization of the optimal portfolio.

Lemma 2 (Optimal Portfolio) *At an optimum, investor j's asset positions $\{a_{j,i}(z)\}_{\{z\in\mathcal{Z}\}}$ satisfy the first-order necessary conditions*

$$\Lambda_{j,i}(z) - q(z) = \frac{\mu}{m_f} q'(z) \underbrace{\left(\hat{a}_{j,i}(z) - a_G(z)/(N+m_f)\right)}_{=a_{j,i}(z)}. \tag{16}$$

The optimal portfolio rule addresses a standard risk-return tradeoff but is modulated by strategic distortions from price impact. The left-hand side of (16) is the difference between marginal valuations and asset prices, which is zero in competitive markets. Notably, this difference depends only on net asset positions. The right-hand side captures the distortions from price impact, whereby investors ration trades to capture price improvements. This distortion depends on gross quantities because total trading volumes determine the costs and benefits of price impact. This distinction between gross and net quantities allows the government to influence equilibrium even through budget-balanced policies.

Since the optimal portfolio rule is the same for all investors of a given type, we restrict attention to equilibria in which all investors within type choose the same strategies.

3.1 Measuring lost gains from trade

We want to assess how government asset market interventions affect the efficiency of trade in equilibrium. We now develop two classes of trading efficiency measures.

Dispersion in state prices. In our model, strategic agents ration quantities to distort prices. Hence price impact leads to unrealized gains from trade across states and time, which we can measure using the cross-sectional dispersion in state prices. To measure lost gains from trade due to risk sharing, we define the *risk sharing gap* in state z as

$$\Gamma_{RS}(z) \equiv \sqrt{Var^i\left[\Lambda_i(z)\right]}, \tag{17}$$

where Var^i denotes the cross-sectional variance across strategic types. This measure captures the misalignment of marginal valuations within any given state. To measure distortions in trade across time, we define the *intertemporal smoothing gap* as the dispersion in the sum of state prices,

$$\Gamma_{IS} \equiv \sqrt{Var^i\left[\sum_{z\in\mathcal{Z}} \Lambda_i(z)\right]} \tag{18}$$

This measure reflects lost gains from intertemporal trade because the sum of state prices reflects an agent's implied risk-free rate; dispersion in implied risk-free rates thus reflect misaligned intertemporal valuations.

Our measures of state-price dispersion have two main advantages: they are economically intuitive and they can be written directly in terms of observable data, namely prices and quantities. To see this, use optimality condition (16) to substitute for unobservable state prices, our measures can be equivalently stated as

$$\Gamma_{RS}(z) = \frac{\mu}{m_f} q'(z) \sqrt{Var^i\left[\hat{a}_i(z)\right]}, \tag{19}$$

and

$$\Gamma_{IS} = \sqrt{Var^i\left[\sum_{z\in\mathcal{Z}} \frac{\mu}{m_f} q'(z)\, \hat{a}_i(z)\right]}, \tag{20}$$

As such, lost gains from trade are determined by price impact, which deters trading, and dispersion in quantities, which reflects the available gains from trade. Under perfect competition ($\mu = 0$), all gains from trade are realized and $\Gamma_{RS} = \Gamma_{IS} = 0$; however, in the Cournot-Walras equilibrium, they are strictly positive.

Welfare-based efficiency measures. While tractable and intuitive, our measures of state-price dispersion are not derived from a social welfare criterion and do not account for the competitive fringe. To address these shortcomings, we also consider the efficiency wedges of Davila and Schaab (2024), who decompose the welfare impact of a change in policy into risk-sharing, intertemporal-sharing, and aggregate efficiency wedges. We start from the utilitarian welfare criterion

$$W = \sum_{i=1}^{N} u\left(c_{1,j,i}\right) + \sum_{z \in \mathcal{Z}} \pi\left(z\right) u\left(c_{2,j,i}\right) + m_f \left(\sum_{z \in \mathcal{Z}} \pi\left(z\right) u\left(c_{2,f}\right) - c_{1,f} \right) \tag{21}$$

Denote by $\mathbb{E}^* \left[\cdot\right]$ and $Cov^* \left[\cdot\right]$ the cross-sectional mean and covariance among all agents. Lemma 3 then specializes the Davila and Schaab (2024) wedges to our endowment economy in which the policy experiment is a marginal change in some policy x. We disregard the aggregate efficiency wedge because it equals zero in our endowment economy.

Lemma 3 (Welfare Measures) *The risk-sharing and intertemporal sharing wedges of Davila and Schaab (2024) in our setting are given by:*

$$\Xi_{RS} = \sum_{z \in \mathcal{Z}} \omega_2 E^* \left[\omega_{2,i}\left(z\right) \frac{d\hat{a}_i\left(z\right)}{dx} \right] - \omega_1 E^* \left[\omega_{1,i} \frac{d\left(q\left(z\right)\hat{a}_i\left(z\right)\right)}{dx} \right], \tag{22}$$

and

$$\Xi_{IS} = \sum_{z \in \mathcal{Z}} Cov^* \left[\omega_{2,i}, \omega_{2,i}\left(z\right) \frac{d\hat{a}_i\left(z\right)}{dx} \right] - E^* \left[\omega_{1,i} \frac{d\left(q\left(z\right)\hat{a}_i\left(z\right)\right)}{dx} \right], \tag{23}$$

respectively, where weights ω_t, $\omega_{t,i}$, and $\omega_{t,i}\left(z\right)$ are given by Equations (A.22)-(A.25) and $\frac{da_i(z)}{dx}$ by Equation (A.57) in the appendix. The aggregate efficiency wedge is identically zero.

Compared to these welfare wedges, our dispersion-based measures are more tractable for theoretical analysis and are independent of the specific choice of policy. Moreover, it will turn out that both measures behave quite similarly in our setting. Where applicable, we thus discuss efficiency with respect to both sets of measures.

3.2 Imperfect Competition and Asset Prices without Interventions

Before turning to policy, it is useful to understand how imperfect competition alone affects asset prices in the absence of interventions. To isolate the effects of market power, we study the *strategic limit*, defined as the limit of a sequence of economies in which the mass of the competitive fringe tends to zero but the mass of strategic agents *relative* to fringe converges to a constant. Prices are then fully determined by interactions between strategic investors, but price impact remains well-defined even with an infinitesimal fringe. Neuhann and Sockin (2024) contains a formal description of the strategic limit.

Definition 2 (Strategic Limit) *The strategic limit is the limit of a sequence of economies in which $\mu, m_f \to 0$ and $\mu / m_f \to \kappa$ for some constant $\kappa > 0$.*

For intuition, we consider an economy with ex-ante identical investors who trade only to share risks. This allows us to characterize the effects of market power in the absence of distributional considerations, and yields particularly stark insights. A simple setting that satisfies these conditions is an economy with two types and two states, with each type receiving high income in one state and low income in the other.

Definition 3 (Pure Risk Sharing Economy) *A pure risk sharing economy is such that all strategic types are ex-ante symmetric: they face the same decision problem up to a relabeling of states.*

The next result shows that in the strategic limit of a pure risk sharing economy, market power inflates *all* Arrow security prices compared to the benchmark competitive equilibrium (where $\mu = 0$). That is, even though all investors strategically ration quantities when buying *and* selling–suggesting that although traded quantities fall, prices may rise *or* fall– in equilibrium the net distortion is such that all prices rise. Said differently, the sell side of any given Arrow security exerts more market power than the buy side, and consumption becomes more expensive state-by-state.

Proposition 2 (Strategic Scarcity in Imperfectly Competitive Markets) *In the strategic limit of the pure risk sharing economy, all Arrow asset prices, $q(z)$, are higher than in the competitive equilibrium (where $\mu = 0$) for all $z \in \mathcal{Z}$. As a consequence, the risk-free rate, r_f, is strictly lower.*

The underlying mechanism is central to understanding the effects of policy. When risk sharing is imperfect, sellers of a given Arrow security (that is, those investors who have a relatively high endowment in the underlying state) consume more in the associated state than buyers. Under convex marginal utility, sellers therefore endogenously face a flatter marginal utility curve than buyers. Since marginal utility determines the cost of quantity distortions, sellers optimally ration quantities more severely than buyers. As such, prices must increase. Because this argument holds state by state, the price of every Arrow security rises. We refer to this mechanism as "strategic scarcity."

4 Positive Effects of Government Bond Trading

We now turn to the positive effects of government trading in asset markets. We begin with a natural benchmark in which the government trades only risk-free debt, $a_G(z) = a_g$. This policy is widespread in practice and purportedly neutral: the government does not take a distorted position in any single security, and all taxes and transfers are symmetric across all investors. Nevertheless, we show that it affects risk sharing, intertemporal smoothing, and asset prices. Later on, we consider more general forms of intervention.

4.1 Prices and Liquidity

We now establish that the government can affect equilibrium prices and liquidity even with fully funded bond purchases in complete markets. For intuition, we sum optimality condition (16) across strategic agents and impose market-clearing. This shows that prices satisfy a distorted consumption-based relation

$$q(z) = \frac{1}{N}\sum_i \Lambda_i(z) + \frac{\mu}{m_f} q'(z) m_f a_f(z) + \frac{\mu}{m_f} q'(z) a_g. \tag{24}$$

Prices are equal to the cross-sectional average of state prices, plus two wedges corresponding to the marginal utility of the fringe (which determines price impact) and the size of the government intervention. This suggests that government trading has intuitive effects: prices fall when the government sells, and rise when it buys.

Specifically, Proposition 3 shows that the government can improve liquidity without injecting or withdrawing resources. Because price impact deters investors from fully undoing their asset position with the government, public bond sales effectively induce an outward shift in the net supply curve of assets at date one. This alleviates the "strategic scarcity" induced by market power (Proposition 2), thereby raising the risk-free rate and lowering price impact. Conversely, "expansionary" policies that reduce the risk-free rate also worsen price impact.

Proposition 3 (Prices and Liquidity with Public Bond Trading) *In the Cournot-Walras equilibrium, government purchases of risk-free bonds affect prices as follows:*

(i) public bond purchases raise asset prices and price impact, $q(z)$ and $q'(z)$, respectively, for all states z, and thus lower the risk-free rate.

(ii) public bond sales lower asset prices and price impact, $q(z)$ and $q'(z)$, respectively, for all states z, and thus raise the risk-free rate.

4.2 Risk Sharing

Next we analyze how public trading in risk-free bonds affects risk sharing efficiency, as measured by state-price dispersion $\Gamma_{RS}(z)$. To isolate effects on risk sharing, we study the strategic limit of a pure risk sharing economy from Section 3.2. In line with our results on liquidity, Proposition 4 shows that public bond purchases worsen risk sharing, but public bond sales improve risk sharing. Since bond purchases lower the risk-free rate, "expansionary policies" are thus associated with inefficient risk sharing.

Proposition 4 (Effects of Public Bond Trading on Risk Sharing) *In the strategic limit of a pure risk sharing economy, the dispersion of marginal valuations $\Gamma_{RS}(z)$, is increasing in public bond purchases for all z, while public bond sales reduce $\Gamma_{RS}(z)$ for all z.*

Perhaps surprisingly, this result obtains even though the policy pertains to an asset, namely risk-free debt, whose payoffs are orthogonal to the gains from trade. The reason is that sellers strategically ration quantities more than buyers (Proposition 2). Moreover, the induced reduction in seller market power can raise ex-ante utility for all investors – since investors are ex-ante identical, each is a buyer and a seller. Example 1 provides a simple illustration of these mechanisms even away from the strategic limit.

Example 1 (Pure risk sharing with two types) *There are two ex-ante identical types $i \in \{1,2\}$ and two equally likely states, $z \in \{1,2\}$. In each state one investor is rich and the other is poor: endowments are $y_1(1) = 2\bar{y}$, $y_1(2) = 0$, $y_2(1) = 0$, and $y_2(2) = 2\bar{y}$. Initial wealth is $w_i = w$. The fringe receives $\frac{\bar{y}}{w}$ in every state. All utility functions are CRRA with risk aversion γ.*

We study the symmetric equilibrium where each strategic investors holds $a_s < 0$ claims on the state with high private income and $a_b > 0$ claims on the state with low private income. The government holds a_g of both claims. Net asset positions are $\hat{a}_s = a_s + \frac{a_g}{2+m_f}$ and $\hat{a}_b = a_b + \frac{a_g}{2+m_f}$. Both prices are $q^* = -\frac{1}{2}u_f'(c_f)$ and price impacts are $q^{*'} = -\frac{1}{2}u_f''(c_f)$, where

$$c_f = \frac{\bar{y}}{w} - \frac{1}{m_f}(\hat{a}_b + \hat{a}_s)$$

and $\hat{a}_b + \hat{a}_s$ is the net demand of strategic investors and the government. The risk sharing gap is

$$\Gamma_{RS} = \frac{\mu}{m_f} q'(z) \frac{(\hat{a}_b - \hat{a}_s)}{2}.$$

Next, consider the direct effect of policy, holding fixed private trading strategies fixed at some $\{a_b^0, a_s^0\}$. The change in fringe consumption induced by a change in policy from 0 to a_g is

$$\Delta c_f = -\frac{1}{m_f} \frac{2a_g}{(2+m_f)}$$

Consistent with Proposition 3, public bond sales thus lower price impact while purchases raise it. Since $\hat{a}_b - \hat{a}_s = a_b - a_s$ for any a_g, moreover, quantity dispersion is invariant in the policy. Hence the direct effect of bond sales is to lower the risk sharing gap, whereas asset purchases raise it.

The total effect of the policy then depends on whether strategic investors' endogenous quantity response can overturn the effects of the change in price impact. For the case of the strategic limit, Proposition 4 shows that this is not the case. For the case of a large fringe ($\mu_f >> 0$), Appendix 7 shows a sufficient condition: if $\frac{\gamma}{m_f}$ is sufficiently large, holding $\frac{\mu}{m_f}$ fixed, then risk sharing gap $\Gamma_{RS}(z)$ to monotonically increase with a_g in both states. In both cases, the key mechanism is that buyers of a given have a higher marginal value of state-contingent consumption than sellers, so that a uniform increase in quantities reduces the misalignment in state prices.

The same mechanism is also apparent when measuring risk-sharing efficiency using the risk sharing wedge Ξ_{RS}. In the strategic limit of the pure risk sharing economy where the policy instrument is risk-free debt, this wedge is equal to

$$\Xi_{RS}(z) = \frac{r_f}{1+r_f} \sum_{i=1}^{N} \frac{\Lambda_i(z)}{\pi(z)} \frac{d\hat{a}_i(z)}{da_g}, \tag{25}$$

where the risk-free rate $r_f = 1/\sum_{z \in \mathcal{Z}} \Lambda_{i'}(z)$ is the same for all types, and the choice of state z is arbitrary by symmetry. The wedge is proportional to the sum of induced changes in strategic agents' net asset positions multiplied by their state prices. Since buyers endogenously have higher state prices than sellers, we obtain the same intuition that raising net supply raises risk sharing efficiency.

4.3 Intertemporal Smoothing

We now analyze how government trade in risk-free bonds affects intertemporal smoothing. By analogy with our approach to risk sharing, we study the strategic limit of an economy in which all gains from trade stem from intertemporal income differences.

Definition 4 (Pure Intertemporal Smoothing Economic) *A pure intertemporal smoothing economy is a deterministic economy in which strategic types differ only with respect to their endowments at dates* 1 *and* 2*.*

The key difference to risk sharing is that government purchases of risk-free bonds now have *direct* distributional effects: since all gains from trade relate to intertemporal trade, government purchases benefit prospective savers at the expense of prospective buyers.

Nevertheless, Proposition 5 shows that, as in the case of risk sharing, public purchases of risk-free debt increase the intertemporal smoothing wedge gap Γ_{IS}, whereas sales reduce it. The reason is that a reduction in net supply disproportionately hurts buyers of risk-free assets, who have steeper state prices than sellers. That is, government purchases amplify the "strategic scarcity" of risk-free debt in intertemporal smoothing.

Proposition 5 (Effects of Government Trading on Intertemporal Smoothing) *In the strategic limit of a pure intertemporal smoothing economy, the intertemporal smoothing gap Γ_{IS}, is increasing in the government bond position, a_g.*

To see the connection between our dispersion-based measures and the intertemporal sharing wedge Ξ_{IS}, we simplify Ξ_{IS} to the pure intertemporal smoothing economy in the strategic limit in which the policy tool is risk-free debt, i.e., $x = a_g$. In this case,

$$\Xi_{IS} = -\frac{1}{N}\sum_{i=1}^{N}\frac{1}{1+\Lambda_i}\left((1+q)\frac{d\hat{a}_i}{da_g} + \frac{dq}{da_g}\hat{a}_i\right), \tag{26}$$

The intertemporal-sharing wedge is minus the sum of (one plus) the inverse of each strategic agent's state prices weighted by the incremental impact of the government increasing its position in risk-free debt on her change in consumption between dates 1 and 2, $\frac{d\left(c_{2,i}-c_{1,i}\right)}{da_g} = (1+q)\frac{d\hat{a}_i}{da_g} + \frac{dq}{da_g}\hat{a}_i$. Because trading less risk-free debt shifts more consumption of sellers to date 2 and of buyers to date 1, $\frac{d\left(c_{2,i}-c_{1,i}\right)}{da_g}$ is positive for sellers and negative for buyers. In both measures, the efficiency consequences of debt sales are linked to the ability of the government to alleviate "strategic scarcity."

The following example illustrates our findings in an analytically tractable pure intertemporal economy with two strategic types and a "large" competitive fringe. This shows that the strategic limit is not necessary for our results.

Example 2 (Pure intertemporal smoothing with two types) *There are two types of strategic agents, $i \in \{1, 2\}$. Endowments are $y_1 = 2y$, and $y_2 = 0$. Type 1 agents have zero initial wealth, Type 2 agents have initial wealth $2y$. Both types have CRRA preferences with relative risk aversion γ. The fringe receives endowment 1 at date 2. There is a risk-free bond with equilibrium price q^*.*

We search for an equilibrium where Type 1 agent sells $a_s < 0$ units of risk-free debt and Type 2 agents buy $a_b > 0$ units of risk-free debt. Define $\hat{a}_s = a_s + \frac{a_g}{2+m_f}$ and $\hat{a}_b = a_b + \frac{a_g}{2+m_f}$. Risk-free debt has a price q^* based on the fringe's marginal utility, $q^* = u'\left(1 - \frac{1}{m_f}(\hat{a}_b + \hat{a}_s)\right)$. and price impact is q'^*. As such, asset positions satisfy

$$\text{Seller optimality:} \qquad \frac{u'(2y + \hat{a}_s)}{u'(-q^*\hat{a}_s)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{a_g}{2+m_f}\right), \tag{27}$$

$$\text{Buyer optimality:} \qquad \frac{u'(\hat{a}_b)}{u'(2y - q^*\hat{a}_b)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{a_g}{2+m_f}\right). \tag{28}$$

With perfect competition (i.e., $\mu = 0$), $a_b = -a_s = y$, $q^* = u'(1) = 1$, and government intervention in financial markets has no real effects. By contrast, with market concentration, Type 1 agents sell fewer claims ($a_s > -y$) while Type 2 agents buy fewer claims ($a_b < y$) because of price impact. As a result, the assets of Type 1 agents have too long a duration, and the assets of Type 2 agents have too short a duration.

It is sufficient, although not necessary, that $\gamma \in (0, 2]$ and $\frac{\gamma}{m_f}$ is sufficiently small, holding $\frac{\mu}{m_f}$ fixed, for the intertemporal smoothing wedge, Γ_{IS}, to monotonically increase with a_g.[6] *If the effective risk-bearing capacity of the fringe, $\frac{\gamma}{m_f}$, is small, then the increase in price impact from increasing a_g dominates the reduction in traded quantities to raise Γ_{IS}.*

4.4 General Gains from Trade

Having considered risk sharing and intertemporal smoothing in isolation, we now consider an economy in which both kinds of gains from trade are present. Similar to before, we find that government debt sales, on the margin, improve risk sharing, while asset

[6] See Appendix C for the formal argument.

purchases worsen risk sharing if the fringe has sufficiently limited risk-bearing capacity.[7] However, the government faces a trade-off between the two sources of gains from trade: debt sales may locally worsen intertemporal smoothing even if risk sharing improves. Since this limits the ability of the government to improve welfare through better risk sharing, the government faces an endogenous "absorption capacity" beyond which financial markets cannot appropriately accommodate government interventions.

Proposition 6 (General Gains from Trade) *Let $\underline{\gamma}$ be the competitive fringe's minimum coefficient of absolute risk aversion across all asset markets z in the absence of government intervention. Government asset purchases cannot achieve the competitive outcome. In addition:*

(i) Large asset purchases drive $\Gamma_{RS}(z)$ for each z and Γ_{IS} to their autarky values while large asset sales drive them to asymptotic, but positive lower bounds.

(ii) For each state z, if $\frac{\underline{\gamma}}{m_f}$ is sufficiently large, then small asset purchases by the government of risk-free bonds raise the risk sharing wedge, $\Gamma_{RS}(z)$, while small asset sales lower it. By contrast, small asset purchases and sales by the government of risk-free bonds can have an ambiguous impact on the intertemporal smoothing measure, Γ_{IS}.

Figure 1 illustrates our results by plotting equilibrium outcomes as a function of the government's position in risk-free debt, a_g. We study the strategic limit of the simple two-state, two-type from Example 1, enriched with intertemporal gains from trade due to differences in initial wealth. The top left panel shows that (the negative) of our risk-sharing measures and that of Davila and Schaab (2024) attenuate as the government sells more risk-free debt. The top right panel shows that this is also true for our intertemporal smoothing measure and the intertemporal-sharing wedge of Davila and Schaab (2024).[8]

The bottom left panel shows that the expected utility of type-2 strategic agents increases with government asset sales, while that of type-1 agents is hump-shaped. This is because government asset sales depress the price of the risk-free asset, which harms type-1 agents who sell it to type-2 agents. As a result, utilitarian welfare (given in Equation (21)) is hump-shaped in government asset sales: increasing for relatively small interventions, but falling for sufficiently large interventions once $a_g = -0.43$. The bottom right

[7] While this condition is sufficient and intuitive, it is not necessary: we find that the same mechanism holds numerically under quite general conditions.

[8] We find that the congruence between our measures and that of Davila and Schaab (2024) holds more generally even when we allow for asymmetries in endowments and varying wealth disparities.

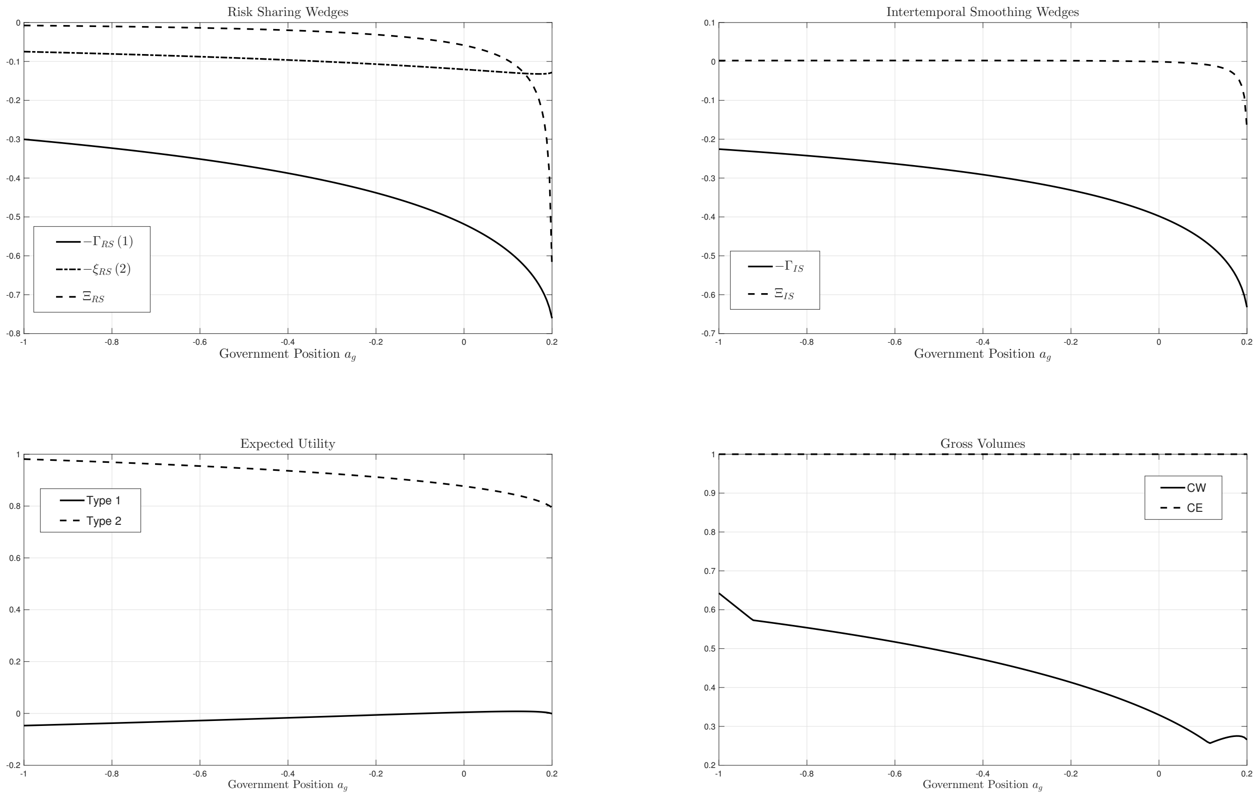

Figure 1: Effects of government trading given intertemporal and risk sharing gains from trade.
Figure notes. We compute equilibrium allocations in the strategic limit of the two-state, two-type economy discussed in Example 1, with the modification that types are also allowed to differ in their initial wealth. The ratio of market concentration to the fringe's mass is $\kappa = 1$. Average income in every state is $\bar{y} = 1$. The within-state income dispersion that determines risk sharing needs is $\Delta = 0.5$. Type 1's initial wealth is $w_1 = 1$. There are intertemporal gains from trade because Type 2's initial income is higher, $w_2 = 2.5$. We plot the negative of our risk-sharing, $\Gamma_{RS}(z)$, and intertemporal smoothing, Γ_{IS}, wedges for comparison to those of Davila and Schaab (2024), Ξ_{RS} and Ξ_{IS}, in the top left and right panels, respectively. The bottom left panel plots the expected utility of both strategic agent types, and the bottom right panel plots gross quantities traded as the sum of all gross Arrow security positions.

panel shows that although government trading raises the gross quantity of assets traded, both with purchases and sales, it need not reflect an improvement in welfare. This shows the importance of an endogenous "absorption capacity". Moreover, risk sharing and intertemporal smoothing statistics are not sufficient for identifying welfare-improving policies because the impact of government trading on asset prices can be distributional.

5 Optimal Trading Rules

Thus far we have characterized only the positive consequences of public asset market interventions. We now examine how a government should *optimally* buy or sell financial assets to maximize utilitarian welfare, defined as in Equation (21) by

$$W = \sum_{i=1}^{N} u\left(c_{1,j,i}\right) + \sum_{z\in\mathcal{Z}} \pi\left(z\right) u\left(c_{2,j,i}\right) + m_f \left(\sum_{z\in\mathcal{Z}} \pi\left(z\right) u\left(c_{2,f}\right) - c_{1,f}\right). \tag{29}$$

We assume the government is a Stackelberg leader who declares a budget-balanced tax and trading policy and internalizes all investors' reactions to this policy. Accordingly, we define *public price impact* as the *total* derivative of an Arrow security's price with respect to a generic change in government policy dx, $\frac{dq(z)}{dx}$. Note that public price impact differs from private price impact in that it incorporates all subsequent equilibrium responses to the policy, including those of strategic investors. We do not restrict the sources of gains from trade, but consider different restrictions on the set of assets traded by the government.

5.1 Trading Rules for Risk-free Debt

We begin with the benchmark in which the government is restricted to trading risk-free debt. The policy instrument thus is the quantity a_g traded of every Arrow security.

Proposition 7 characterizes the optimal intervention under this assumption. We find that a_g should be chosen to align particular weighted averages of public and private price impact across states of the world.

Proposition 7 (Optimal Government Position in Risk-free Debt) *The government's optimal holding of risk-free debt, a_g, satisfies the necessary condition:*

$$\sum_{z\in\mathcal{Z}} \left(w_{pri}\left(z\right) \frac{\mu}{m_f} q'\left(z\right) - w_{pub}\left(z\right) \frac{dq\left(z\right)}{da_g} \right) \begin{cases} = 0 & \text{if } a_g \in (-\infty, \infty) \\ \leq 0 & \text{if } a_g = -\infty, \\ \geq 0 & \text{if } a_g = +\infty. \end{cases} \tag{30}$$

where $w_{pri}\left(z\right) = \frac{1}{2}\sum_{i=1}^{N} u'\left(c_{1,i}\right) \frac{d\hat{a}_i(z)^2}{da_g}$ and $w_{pub}\left(z\right) = \sum_{i=1}^{N} \left(u'\left(c_{1,i}\right) - m_f\right) \hat{a}_i\left(z\right)$ are the weights on private and public price impact, respectively. In the benchmark with perfect competition, this condition is satisfied even without government intervention.

The weight on private price impact captures the marginal effect of government trading on the gains from trade realized by strategic agents, as measured by the cross-sectional average of marginal utility times the change in net trading quantities squared. The weight on public price impact captures the marginal impact of government trading on the cost of trade for all agents; imposing market-clearing, it is the difference between the increase in portfolio costs for strategic agents and the decline for the competitive fringe.

When the difference between weighted public and private price impact is negative, the government sells debt to lower portfolio costs and to promote trade by reducing private price impact. When it is positive, the government buys debt to raise portfolio costs, which is redistributive because it benefits asset sellers at the expense of buyers. The equalization of private and public price impact consequently balances the improvement in risk sharing and intertemporal smoothing against the redistributive costs of distorting portfolio prices, and can lead to an interior optimal choice of risk-free debt.

5.2 Trading Rules for Other Securities

We can adapt our optimal trading rule to allow the government to trade a richer set of linearly-independent securities. This is of interest because many governments now routinely trade in securities other than government debt. For example, the US and EU intervened in corporate bond markets during the 2020 Covid crisis, the Japanese government is now the largest holder of Japanese equities, and countries often intervene in foreign exchange while managing foreign reserves (e.g., Amador, Bianchi, Bocola, and Perri, 2020). To this end, Proposition 8 provides a characterization of the optimal choice of the government's position, a_{gx}, for a given security with state-contingent payoff vector $X(z)$.

Proposition 8 (Optimal Targeted Interventions) *The government's optimal holding of a security with state-contingent payoff vector $X(z)$, a_{gX}, satisfies the necessary condition:*

$$\sum_{z\in\mathcal{Z}}\left(w_{pri,X}\left(z\right)\frac{\mu}{m_f}q'\left(z\right)-w_{pub}\left(z\right)\frac{dq\left(z\right)}{da_{gX}}\right)=\begin{cases}=0 & \text{if } a_{gX}\in(-\infty,\infty),\\ \leq 0 & \text{if } a_{gX}=-\infty,\\ \geq 0 & \text{if } a_{gX}=+\infty.\end{cases} \tag{31}$$

where $w_{pri,X}\left(z\right)=\frac{1}{2}\sum_{i=1}^{N}u'\left(c_{1,i}\right)\frac{d\hat{a}_i(z)^2}{da_{gX}}$ and $w_{pub}\left(z\right)$ is given in Proposition 7.

As in the case of risk-free debt, the government aims to balance weighted averages of public and private price impact to maximize utilitarian welfare. The key difference is that the security may allow the government to differentially affect different states of the world, thereby allowing for more targeted interventions. For example, the government may opt to trade relatively more of a security which pays off disproportionately in a state of the world with low liquidity.

This suggests a natural pecking-order to asset interventions. If the government sells assets to improve risk sharing and the risk management practices of market participants, it should prioritize trading in selling illiquid assets that have a larger impact on market liquidity. By contrast, if it buys assets to accomplish other policy objectives, such as to lower interest rates, then it should prioritize buying more liquid securities to minimize its adverse impact on market liquidity.

To see this insight, it is sufficient to restrict our attention to the government trading Arrow securities with positions $\{a_g(z)\}_{z\in\mathcal{Z}}$. Suppose that the government aims to maximize welfare according to the objective (29) but must now also satisfy a policy constraint:

$$\sum_{z\in\mathcal{Z}} q(z)\, a_g(z) = G, \tag{32}$$

such that total purchases and sales in the numeraire must equal some target, G, that can be positive (net asset purchases) or negative (net asset sales). Let θ be the shadow cost of this policy constraint, which is positive if the government's marginal trade to target G raises welfare, and negative otherwise. Since government asset purchases raise price impact and portfolio costs while sales lower them, our analysis suggests that $\theta G \leq 0$. Applying Equation (33) from Proposition 8 to Arrow securities, the optimal position $a_g(z)$ in security z at an interior optimum satisfies:

$$\frac{\sum_{z'\in\mathcal{Z}}\left(w_{pri,\delta(z)}(z')\frac{\mu}{m_f}q'(z') - w_{pub}(z')\frac{dq(z')}{da_g(z)}\right)}{q(z)+\sum_{z'\in\mathcal{Z}}\frac{dq(z')}{da_g(z)}a_g(z')} = \theta. \tag{33}$$

Similar to a standard portfolio choice problem with a budget constraint, Equation (33) equates the ratio of the marginal benefit to the marginal cost of buying another unit of asset z (the left-hand side) to the shadow cost of the budget constraint (the right-hand side). The marginal benefit is the increase in utilitarian welfare, which can be negative, while the marginal cost is the increase in the cost of the government's portfolio. Different from a standard portfolio choice problem, however, the government internalizes the impact of its trading on all asset prices, which is why equation (33) includes cross-asset spillovers. Notably, the right hand side is independent of all assets. Starting at any given initial government allocation, assets can thus be ranked by the left-hand side with respect to their marginal net effect on welfare. We interpret this as a pecking order across assets.

5.3 Implications for Public Asset Purchases and Sales

Our findings suggest a role for a government to intervene in financial markets to manage liquidity when price impact distorts trade among large market participants. The trade-offs we highlight between improving risk sharing and intertemporal smoothing and re-distributing trading profits by altering portfolio costs lead to an optimal balancing of public versus private price impact, where the weights on each reflect the underlying gains from trade. More generally, this prescription needs to be weighed against the other motives for asset market interventions, such as altering the risk-bearing capacity of financial markets and the cost of capital for households and corporations.

In terms of when and how a government should intervene, our findings suggest that mitigating financial market concentration should make government asset sales more countercyclical. A government managing financial market liquidity will sell more assets when financial markets are relatively more illiquid, such as during business cycle troughs, compared to when they are relatively more liquid, such as during business cycle booms. Because price impact is proportional to prices, our model implies that in the cross-section of assets, those that are more illiquid have higher state prices, i.e., lower returns, and can be targeted appropriately. Our analysis further suggests that the government should assess the incremental, rather than cumulative, impact of its asset purchases and sales because it is changes in its trading positions that determine asset market liquidity.

6 An Example with the Eurozone

We conclude by illustrating our findings using a numerical example calibrated to the European Central Bank's 2014-2017 large-scale asset purchase program. Our goal is not to provide an exhaustive assessment of quantitative easing policies on the macroeconomy (for example, by accounting for its spillovers to household and firms), but rather to provide a theory-guided analysis of implications for market functioning and trading among financial institutions. The 2014 intervention is a useful case study because the ECB conducted large-scale asset purchases outside of a crisis period, which allows us to examine how such purchases impact market liquidity absent forced asset sales.

In what follows, we consider date 1 to represent one year and date 2 to represent ten years; as such, risk-free debt in our model is akin to a ten-year government bond and the risk-free rate the ten-year rate on government bonds. We assume that there are three groups of strategic agents: 1) insurance companies and pension funds (ICPFs) that have long duration portfolios; 2) banks and corporations that have short duration portfolios; and 3) mutual funds and hedge funds that have portfolios of intermediate duration. All strategic agents have constant relative risk aversion (CRRA) preferences with risk aversion γ, as does the competitive fringe at date 2. To focus on trading among these institutional investors, we examine the strategic limit from Definition 2.

At date 2, there are two possible states $z \in \{1,2\}$ with $\pi(z) = \frac{1}{2}$. Strategic agents of type $i \in \{1,2\}$ that represent pension funds and insurance companies receive initial wealth $(1-k_1)\,\bar{y}$ and an endowment at date 2, $y_i(i) = k_1\bar{y}\,(1+\Delta)$ and $y_i(-i) = k_1\bar{y}\,(1-\Delta)$. That is, in every state one of the two types has high income and the other has low income. Similarly, strategic agents of type $i \in \{3,4\}$ that represent banks and corporations receive initial wealth $(1-k_2)\,\bar{y}$ and an endowment at date 2, $y_i(i-2) = k_2\bar{y}\,(1+\Delta)$ and $y_i(5-i) = k_2\bar{y}\,(1-\Delta)$. Strategic agents of type $i = 5$ that represent hedge funds and mutual funds receive initial wealth $(1-k_3)\,\bar{y}$ and a certain endowment at date 2 of $k_3\bar{y}$. The competitive fringe receives $\bar{y}$ at both dates. We set $\bar{y} = 10$, $\kappa = 1$, and calibrate the remaining parameters $\{\gamma, k_1, k_2, k_3, \Delta, a_g\}$ following a procedure described in Appendix D. The relevant data moments are from Koijen, Koulischer, Nguyen, and Yogo (2021). Table A.1 in the appendix reports the parameters.

Figure 2 shows our main findings. We simulate the equilibrium for different values of the government's position, a_g. The vertical line plots the calibrated "initial position" of the government. Starting from this point, asset purchases thus represent a rightward move along the x-axis. Given our no-arbitrage framework with complete markets, we can rewrite Arrow security positions in terms of interpretable assets. In the left panel, we plot the net positions of ICPFs in terms of a risk-free bond $\hat{a}_1(rf)$ and a swap that pays 1 in state 2 and -1 in state 1, $\hat{a}_1(swap)$. In the right panel, we plot the risk-free rate.

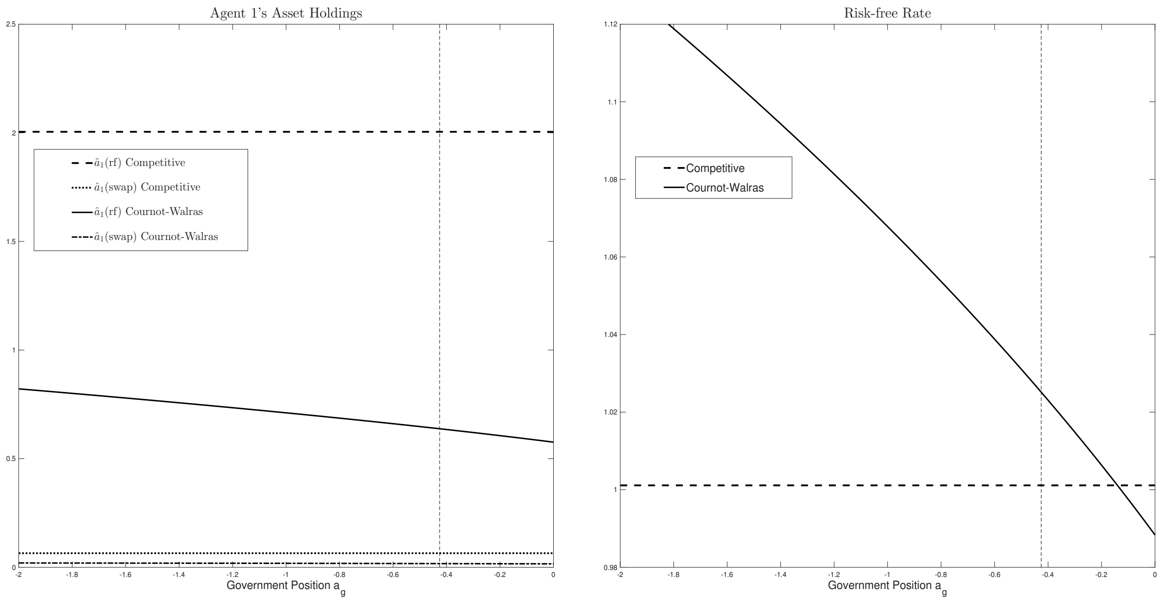

Figure 2: Agent 1's Asset Holdings (Left Panel) and Risk-free Rate (Right Panel) across Government Positions for the parameters in Table A.1

Price impact induces ICPFs to ration trades. In the competitive equilibrium, they would trade 2.005 shares of risk-free debt and 0.0648 shares of the swap. Under imperfect competition, they trade only 0.6232 units of risk-free debt and 0.0169 units of the swap. As the government sells more assets and interest rates rise, their positions in risk-free debt and the swap increase to 0.7210 and 0.0191 units, respectively when $a_g = -1.25$. However, because ICPFs portfolio quantities change very little in response to price changes, public trading has only limited impact on risk sharing and intertemporal smoothing.

In Figure 3, we plot utilitarian welfare according to the objective (21) and the risk sharing and intertemporal smoothing wedges characterized in Section 3. (Our risk sharing measure, $\Gamma_{RS}(z)$, is identical for both states in this exercise.) The left panel shows that risk sharing improves as the government sells more assets and reduces price impact, although it cannot achieve the level of risk sharing in the competitive equilibrium. The middle panel similarly demonstrates that intertemporal smoothing also improves as the government sells more assets. Interestingly, the measures of Davila and Schaab (2024) reveal that the government marginally improves utilitarian welfare more along the intertemporal smoothing compared to the risk sharing margin, which is consistent with duration mismatch being the primary source of gains from trade based on our calibration. In this sense, our modeling framework can be used to interpret quantity and price data to reveal the underlying sources of illiquidity and gains from trade.

Consistent with our theoretical analysis, welfare is not only strictly below the competitive benchmark (-0.84% in consumption equivalent terms), it is decreasing in public debt purchases but increasing in debt sales. This welfare loss stems from both worsening risk management and duration mismatch among the three types of financial institutions. However, because all types have very inelastic demands, the 26% large-scale asset purchase of government bonds by the ECB over 2014-2017 mostly affected prices rather than allocations. As a result, it modestly reduced consumption-equivalent welfare by -0.012%.

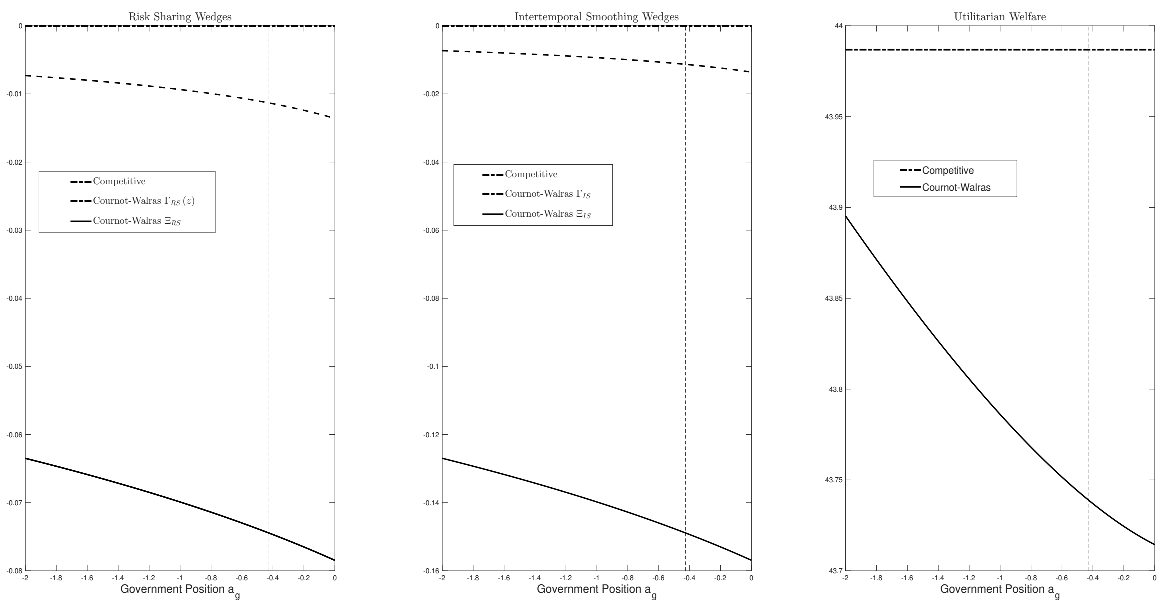

Figure 3: We plot the negative of our risk sharing, $\Gamma_{RS}(z)$, and intertemporal smoothing, Γ_{IS}, wedges for comparison to those of Davila and Schaab (2024), Ξ_{RS} and Ξ_{IS}, in the left and middle panels, respectively, and utilitarian welfare in the right panel across government positions for the parameters in Table A.1

7 Conclusion

We provide a novel perspective on how the public provision of liquidity impacts risk sharing and intertemporal trading among investors with price impact. Our key finding is that government asset sales can improve liquidity by alleviating strategic distortions stemming from market power. This holds true even though all interventions are fully funded, budget neutral and occur in complete markets. By contrast, government purchases which lower interest rates induce investors to intensify rent-seeking at the expense of risk management. This suggests that standard "expansionary" interventions can have the unintended consequence of harming the efficiency of trade.

More generally, we show that a government may face a trade-off between improving risk sharing and distorting intertemporal trade, which leads to endogenous limits on the optimal scale of asset market interventions. Our results on optimal policy deliver insights into the optimal management of public portfolios and central bank balance sheets, and highlight that market liquidity introduces a pecking order to how a government should intervene in asset markets. Finally, cross-asset spillovers imply that the (even large) changes in asset-specific yields or liquidity measures are insufficient for evaluating the transmission of large-scale asset purchases to large investor portfolios.

References

AMADOR, M., J. BIANCHI, L. BOCOLA, AND F. PERRI (2020): "Exchange Rate Policies at the Zero Lower Bound," *The Review of Economic Studies*, 87(4), 1605–1645. [Cited on pages 3 and 23.]

ANGELETOS, G.-M. (2002): "Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure," *Quarterly Journal of Economics*, 117, 1105–1131. [Cited on page 4.]

ANGELETOS, G.-M., F. COLLARD, AND H. DELLAS (2023): "Public Debt as Private Liquidity: Optimal Policy," *Journal of Political Economy*, 131, 3233–3264. [Cited on page 4.]

BASAK, S. (1997): "Consumption Choice and Asset Pricing with a Non-price-taking Agent," *Economic Theory*, 10(3), 437–462. [Cited on page 4.]

BASAK, S., AND A. PAVLOVA (2004): "Monopoly Power and the Firm's Valuation: A Dynamic Analysis of Short versus Long-Term Policies," *Economic Theory*, 24, 503–530. [Cited on page 4.]

BERNANKE, B. (2012): "Monetary Policy Since the Onset of the Crisis," Speech at the Federal Reserve Bank of Kansas City Economic Symposium at Jackson Hole, Wyoming. [Cited on page 2.]

BEWLEY, T. (1986): "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers," in *Contributions to Mathematical Economics in Honor of Gerard Debreu.* Amsterdam:North-Holland. [Cited on page 4.]

BRECKENFELDER, J., P. COLLIN-DUFRESNE, AND S. CORRADIN (2024): "Is the Bond Market Competitive? Evidence from the ECB's Asset Purchase Program," *Swiss Finance Institute Research Paper 23-104*. [Cited on page 1.]

CARAMP, N., AND S. J. SINGH (2023): "Bond Premium Cyclicality and Liquidity Traps," *Review of Economic Studies*, 90(6), 2822–2879. [Cited on page 4.]

CARVAJAL, A. (2018): "Arbitrage pricing in non-Walrasian financial markets," *Economic Theory*. [Cited on page 35.]

CHOI, J., R. KIRPALANI, AND D. J. PEREZ (2022): "The Macroeconomic Implications of US Market Power in Safe Assets," *Working Paper*. [Cited on page 4.]

COEURÈ, B. (2015): "Embarking on Public Sector Asset Purchases," Speech at the Second International Conference on Sovereign Bond Markets. [Cited on page 2.]

DAVILA, E., AND A. KORINEK (2018): "Pecuniary Externalities in Economies with Financial Frictions," *Review of Economic Studies*, 85, 352–395. [Cited on page 3.]

DAVILA, E., AND A. SCHAAB (2024): "Welfare Assessments with Heterogeneous Individuals," *NBER Working Paper*. [Cited on pages 1, 13, 20, 21, 27, 28, 37, and 38.]

EISENBACH, T., AND G. PHELAN (2022): "Cournot Fire Sales," *American Economic Journal: Macroeconomics*, 14, 508–42. [Cited on page 4.]

EISENSCHMIDT, J., Y. MA, AND A. L. ZHANG (2022): "Monetary Policy Transmission in Segmented Markets," *Working Paper*. [Cited on page 4.]

GERTLER, M., AND P. KARADI (2013): "QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool," *International Journal of Central Banking*, 9, 1–53. [Cited on page 4.]

HE, Z., A. KRISHNAMURTHY, AND K. MILBRADT (2019): "A Model of Safe Asset Determination," *American Economic Review*, 109, 1230–1262. [Cited on page 4.]

HOLMSTROM, B., AND J. TIROLE (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106, 1–40. [Cited on page 4.]

HUBER, A. W. (2023): "Market power in wholesale funding: A structural perspective from the triparty repo market," *Journal of Financial Economics*, 149(2), 235–259. [Cited on page 4.]

JOYCE, M. A., Z. LIU, AND I. TONKS (2017): "Institutional Investors and the QE Portfolio Balance Channel," *Journal of Money, Credit and Banking*, 49(11), 1225–1246. [Cited on page 52.]

KACPERCZYK, M., J. NOSAL, AND S. SUNDARESAN (2021): "Market Power and Price Informativeness," *Working Paper*. [Cited on page 4.]

KOIJEN, R. S., F. KOULISCHER, B. NGUYEN, AND M. YOGO (2021): "Inspecting the Mechanism of Quantitative Easing in the Euro Area," *Journal of Financial Economics*, 140, 1–20. [Cited on pages 26, 62, and 63.]

KYLE, A. S. (1989): "Informed Speculation with Imperfect Competition," *The Review of Economic Studies*, 56(3), 317–355. [Cited on page 7.]

LOGAN, L., AND U. BINDSEIL (2019): "Large Central Bank Balance Sheets and Market Functioning," Discussion paper, Bank for International Settlements. [Cited on page 2.]

MARTINEZ-MIERA, D., AND R. REPULLO (2017): "Search for Yield," *Econometrica*, 85, 351–378. [Cited on page 4.]

NEUHANN, D., M. SEFIDGARAN, AND M. SOCKIN (2025): "Portfolio Regulation of Large Non-Bank Financial Institutions," *Review of Financial Studies*, accepted. [Cited on pages 4 and 35.]

NEUHANN, D., AND M. SOCKIN (2022): "Strategic Savings and Capital Flows," *Working Paper*. [Cited on page 35.]

——— (2024): "Financial market concentration and misallocation," *Journal of Financial Economics*, 159, 103875. [Cited on pages 4, 7, and 14.]

PELIZZON, L., M. G. SUBRAHMANYAM, AND D. TOMIO (2022): "Central Bank-Driven Mispricing," *Working Paper*. [Cited on page 1.]

PINTER, G., E. SIRIWARDANE, AND D. WALKER (2024): "Fire Sales of Safe Assets," *Working Paper*. [Cited on page 1.]

PINTER, G., AND D. WALKER (2023): "Hedging, Market Concentration and Monetary Policy: a Joint Analysis of Gilt and Derivatives Exposures," *Bank of England Staff Working Paper No. 1,032*. [Cited on page 2.]

VAYANOS, D., AND J. VILA (2021): "A Preferred-Habitat Model of the Term Structure of Interest Rates," *Econometrica*, pp. 77–112. [Cited on page 4.]

WALLEN, J. (2020): "Markups to Financial Intermediation in Foreign Exchange Markets," *Working Paper*. [Cited on page 4.]

WALLEN, J., AND J. C. STEIN (2023): "The Imperfect Intermediation of Money-Like Assets," *Working Paper*. [Cited on page 1.]

WANG, J. (2023): "Government Intervention in the Financial Market," *Working Paper*. [Cited on page 4.]

WANG, O. (2018): "Banks, Low Interest Rates, and Monetary Policy Transmission," *Working Paper*. [Cited on page 4.]

Appendix A: Proofs of Propositions

Proof of Lemma 1:

As a preliminary, suppose we have some arbitrary asset span indexed by the $|\mathcal{Z}| \times |\mathcal{Z}|$ matrix X that is of full rank. In the special case of Arrow-Debreu assets, $X = I_{|\mathcal{Z}|}$, i.e., the identity matrix of rank $|\mathcal{Z}|$. Let x_k index the k^{th} row vector of X, and $x_k(z)$ be the dividend asset k pays in state z.

If the competitive fringe trades assets with span X, it is immediate from the first-order conditions of the fringe's problem that the vector of asset prices $\vec{q}_X$ satisfies:

$$\vec{q}_X = X\vec{\Lambda}_f, \tag{A.1}$$

where $\vec{\Lambda}_f$ is vector of the fringe's state prices. Since the quasi-linear competitive fringe now maximizes $u_f\left(y_f(z) - \sum_{k=1}^{|\mathcal{Z}|} x(z)\, x_k(z)\, A_{x_k}(z)\right) + \sum_{k=1}^{|\mathcal{Z}|} x(z)\, q_{x_k} A_{x_k}(z)$, where $A_{x_k}(z)$ is the total demand for asset k of the strategic agents, the price impact function can be summarized by the matrix Γ:

$$\Gamma = XUX', \tag{A.2}$$

where U is the diagonal matrix with diagonal entries $-\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2,f}(z)\right)$.

Step 1: The Problem of the Fringe:

From the first-order condition for $a_f(z)$ from the competitive fringe's problem (8), we can recover the pricing Equation of the Arrow-Debreu claim to security z:

$$\tilde{q}(z) = \pi(z)u_f'\left(c_{2,f}(z)\right) = \Lambda_f(z), \tag{A.3}$$

where $\Lambda_f(z)$ is the competitive fringe's state price. Since $c_{2,f}(z) = y_f(z) - \tau_2(z) + a_f(z)$, imposing the market-clearing condition, (4), reveals:

$$\tilde{q}(z) = \pi(z)u_f'\left(y_f(z) - \tau_2(z) - \frac{1}{m_f}\left(A(z) + a_G(z)\right)\right). \tag{A.4}$$

In equilibrium, this must be the realized price of the claim, $Q(\mathbf{A}, z)$. As this price is a function of state variables from the perspective of the fringe, we designate the realized price more concisely as:

$$q\left(z\right)=Q(\mathbf{A},z). \tag{A.5}$$

Step 2: Equilibrium Price Impact:

Because agents of type i take the demands of other agents (even within their type) as given, $u_f\left(z\right)$ is twice continuously differentiable, and each agent's position size scales by its mass μ, we can derive each agent's perceived price impact:

$$\frac{\partial\tilde{Q}_{j,i}(\mathbf{A},z)}{\partial a_i\left(z\right)}=-\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2,f}(z)\right)=-\frac{\mu}{m_f}\frac{\partial q(z)}{\partial A\left(z\right)}, \tag{A.6}$$

which also implies that price impact is symmetric across all strategic agents. Defining $q'\left(z\right)=\frac{\partial q(z)}{\partial A(z)}$ yields the expression in the statement of the proposition.

Finally, we recognize price impact $q'\left(z\right)$ is convex because of the convex marginal utility of the fringe. It is straightforward to see:

$$q''\left(z\right)=\frac{\mu}{m_f}\pi(z)u_f'''\left(c_{2,f}(z)\right)>0, \tag{A.7}$$

$$q'''\left(z\right)=-\left(\frac{\mu}{m_f}\right)^2\pi(z)u_f''''\left(c_{2,f}(z)\right)>0. \tag{A.8}$$

Price impact is consequently convex in the net demand of strategic agents. In addition, because $-u_f''\left(c_{2,f}(z)\right)$ is (weakly) decreasing in $c_{2,f}(z)$ with increasing, concave utility, it follows prices and price impact are both increasing in the net demand of strategic agents and the government $A\left(z\right)$. Because $c_{2,f}(z)$ is a sufficient statistic for both price impact and the price level, it follows price impact is increasing in the price level.

Step 3: Strategic Agent Demand:

Consider the optimization problem of strategic agent j of type i, (7). We attach the Lagrange multiplier φ_i to the budget constraint. The first-order necessary conditions for

$c_{i,j,1}$ and $\{a_{i,j}(z)\}_{z\in\mathcal{Z}}$ are given by:

$$c_{1,j,i} \;:\; u'(c_{1,j,i}) - \varphi_{j,i} \le 0 \;\; (= \; if \; c_{1,j,i} > 0), \tag{A.9}$$

$$a_{j,i}(z) \;:\; -\pi(z)\, u_2^{i\prime}(c_{2,j,i}(z)) + \varphi_{j,i}\left(\tilde{Q}_{j,i}(\mathbf{A},z) + \frac{\partial \tilde{Q}_{j,i}(\mathbf{A},z)}{\partial a_{j,i}(z)} a_{i,j}(z)\right) = 0. \tag{A.10}$$

The above represents the first-order necessary conditions for agent $i's$ problem. Because $u(\cdot)$ satisfies the Inada condition, $c_{1,j,i} > 0$ and (A.9) binds with equality.

Because strategic agent i has rational expectations, her perceived price impact must coincide with actual price impact from (10). Consequently, Equation (A.10) reduces to:

$$a_{j,i}(z) : \Lambda_{j,i}(z) = q(z) + \frac{\mu}{m_f} q'(z) a_{j,i}(z) \,\forall\, z \in \mathcal{Z}. \tag{A.11}$$

Step 4: The Law of One Price:

Let $\vec{q}$ be the vector of Arrow asset prices. It is immediate from Equation (A.1) because $\vec{q} = \vec{\Lambda}_f$:

$$\vec{q}_X = X\vec{q}. \tag{A.12}$$

The Law of One Price thus holds for redundant assets in the complete markets economy.

Step 5: Invariance:

Let us conjecture that consumption allocations are unchanged if strategic agents and the fringe instead trade with asset span X. Because the fringe's consumption is unchanged, its state prices $\vec{\Lambda}_f$ and consequently Arrow prices $\vec{q}$ are unchanged.

Notice next we can stack the first-order conditions for strategic agent i with asset span $I_{|\mathcal{Z}|}$ from Equation (A.11) as:

$$\vec{\Lambda}_i = \vec{\Lambda}_f + U\vec{a}_i, \tag{A.13}$$

where $\vec{\Lambda}_i$ are the stacked state prices of agent i, $\vec{a}_i$ is the vector of her asset positions, and we have substituted for Arrow-Debreu prices $\vec{q}$ with $\vec{\Lambda}_f$.

Let $\vec{a_{i,x}}$ be the vector of asset positions of agent i when she instead trades with asset span X. Imposing invariance of the consumption allocations of strategic agent i requires:

$$\vec{a_i} = X' \vec{a_{i,x}}. \tag{A.14}$$

Substituting with Equation (A.14), we can manipulate Equation (A.13) to arrive at:

$$X\vec{\Lambda}_i = X\vec{\Lambda}_f + XUX'\vec{a_{i,x}} = X\vec{\Lambda}_f + \Gamma\vec{a_{i,x}}, \tag{A.15}$$

where we have also substituted with Equation (A.2). This is the identical stacked first-order conditions if strategic agent instead traded asset span X.

Both strategic agents and the competitive fringe therefore choose the same state-specific asset exposures under both asset spans. Finally, because the Law of One Price holds, the cost of each agent's portfolio is the same under both asset spans. We conclude that consumption allocations are invariant to the complete markets asset span.[9]

Step 6: Existence and Uniqueness:

Existence and uniqueness in the absence of a government are established in Proposition 2 of Neuhann, Sefidgaran, and Sockin (2025) for arbitrary asset spans. We can consequently apply their arguments to the case of complete markets to conclude that an equilibrium exists and is unique. Government trading does not impact these results because its trading is budget-balanced at both dates through taxes and transfers, τ_1 and $\tau_2(z)$ for $z \in \mathcal{Z}$, and consequently does not alter aggregate resources at both dates.

Proof of Proposition 1:

When all agents are competitive, and $\mu = 0$, Equation (16) reduces to:

$$q(z) = \frac{\pi(z)\, u'(c_{2,i}(z))}{u'(c_{1,i})} = \pi(z)\, u'_f\left(c_f(z)\right) = \Lambda^{CE}(z), \tag{A.16}$$

[9] This result is true for any full-rank asset span X. See, for instance, Carvajal (2018) and Neuhann and Sockin (2022). Neuhann and Sockin (2022) show that these results hold more generally with incomplete markets among equivalent asset spans.

Hence there is perfect risk sharing in the competitive equilibrium. This implies for the N types of agents with homothetic preferences:

$$\frac{c_{2,i}\left(z\right)}{c_{1,i}}=\frac{c_{2,j}\left(z\right)}{c_{1,j}}=\eta\left(z\right), \tag{A.17}$$

and for the competitive fringe:

$$c_{f}\left(z\right)=\eta_{f}\left(z\right)=u_{f}^{-1}\left(u'\left(\eta\left(z\right)\right)\right). \tag{A.18}$$

Substituting for date 2 consumption into the budget constraint at date 1, the intertemporal budget constraint for agents of type i is:

$$c_{1,i}+\sum_{z\in\mathcal{Z}}q\left(z\right)c_{2i}\left(z\right)=w_{i}-\tau_{1}+\sum_{z\in\mathcal{Z}}q\left(z\right)\left(y_{i}\left(z\right)-\tau_{2}\left(z\right)\right). \tag{A.19}$$

Recognizing that $\tau_1=-\sum_{z\in\mathcal{Z}}q\left(z\right)\tau_2\left(z\right)$, it follows that Equation (A.19) reduces to:

$$c_{1,i}+\sum_{z\in Z}q\left(z\right)c_{2i}\left(z\right)=w+\sum_{z\in\mathcal{Z}}q\left(z\right)y_{i}\left(z\right). \tag{A.20}$$

Finally, substituting with Equation (A.79), we arrive at:

$$c_{1,i}=\frac{w+\sum_{z\in\mathcal{Z}}q\left(z\right)y_{i}\left(z\right)}{1+\sum_{z\in\mathcal{Z}}q\left(z\right)\eta\left(z\right)}. \tag{A.21}$$

Suppose the consumption of the fringe is invariant to the government's policies, then so is $q\left(z\right)$ because it is equal to the state prices of the fringe. If prices are unchanged, then from Equation (A.21) $c_{1,i}$ is also unchanged, and so are $c_{2,i}\left(z\right)=\eta\left(z\right)c_{1,i}$. By market-clearing then, so is the consumption of the fringe, confirming the conjecture.

Proof of Lemma 2:

The first-order necessary condition Equation (A.11) is derived in Step 3 of Lemma 1. Substituting for $a_{j,i}\left(z\right)$ with Equation (12), we arrive at the stated result.

Proof of Lemma 3:

Step 1: Risk-sharing wedge of Davila and Schaab (2024):

Following the notation of Davila and Schaab (2024), in our two-date setting, we can define the date-2 stochastic weights for type-i agents

$$\omega_{2,i}(z) = \frac{\pi(z)\, u'(c_{2,i}(z))}{\sum_{z' \in \mathcal{Z}} \pi(z')\, u'(c_{2,i}(z'))} = \frac{\Lambda_i(z)}{\sum_{z' \in \mathcal{Z}} \Lambda_i(z')}, \tag{A.22}$$

while the date-t dynamic weights for type-i agents are

$$\omega_{1,i} = \frac{1}{1 + \sum_{z' \in \mathcal{Z}} \Lambda_i(z')}, \tag{A.23}$$

$$\omega_{2,i} = \frac{\sum_{z' \in \mathcal{Z}} \Lambda_i(z')}{1 + \sum_{z' \in \mathcal{Z}} \Lambda_i(z')}, \tag{A.24}$$

for $i \in \{1, ..., N, f\}$, and date-t weights are:

$$\omega_t = \frac{1}{N + m_f} \sum_{i \in \{1,2,...,N,f\}} m_i \omega_{t,i}, \tag{A.25}$$

for population mass $m_i = 1$ if $i \in \{1, 2, ..., N\}$ and $m_i = m_f$ for $i = f$.

Because we study an endowment economy, any change in date 1 or 2 consumption among agents because of a change in the government's position, a_g, must be purely redistributive, i.e., at date 1, $\mathbb{E}^*\left[\frac{dc_{1,i}}{dx}\right] = 0$, while in state z at date 2, $\mathbb{E}^*\left[\frac{dc_{2,i}(z)}{dx}\right] = 0$.

We can then express their risk-sharing wedge from Equation (11) in Davila and Schaab (2024) as:

$$\begin{aligned}
\Xi_{RS} &= \omega_1 Cov^*\left[\omega_{1,i}, \frac{dc_{1,i}}{dx}\right] + \omega_2 \sum_{z \in \mathcal{Z}} Cov^*\left[\omega_{2,i}(z), \frac{dc_{2,i}(z)}{dx}\right] \\
&= \omega_1 E^*\left[\omega_{1,i} \frac{dc_{1,i}}{dx}\right] + \omega_2 \sum_{z \in \mathcal{Z}} E^*\left[\omega_{2,i}(z) \frac{dc_{2,i}(z)}{dx}\right], \\
&= \sum_{z \in \mathcal{Z}} \omega_2 E^*\left[\omega_{2,i}(z) \frac{d\hat{a}_i(z)}{dx}\right] - \omega_1 E^*\left[\omega_{1,i} \frac{d(q(z)\,\hat{a}_i(z))}{dx}\right],
\end{aligned} \tag{A.26}$$

where we can recover the derivatives $\frac{da_i(z)}{dx}$ for strategic agents by applying the Implicit Function Theorem to the first-order conditions for strategic agents' optimal asset positions based on Equation (16)

$$\nabla_x \hat{a} = -\left[\nabla_{\hat{a}} vec\left(G\right)\right]^{-1} \nabla_x vec\left(G\right), \tag{A.27}$$

where $\hat{a}$ is $NZ \times 1$ vector of all strategic agents' asset positions, $\iota_{N\times Z}$ is the $N \times Z$ vector of ones, and $G\left(i,z\right)$ is the $N \times Z$ matrix

$$G\left(i,z\right) = \frac{a_g}{N+m_f} + \frac{m_f}{\mu} q'\left(z\right)^{-1}\left(\Lambda_i\left(z\right) - q\left(z\right)\right) - \hat{a}_i\left(z\right), \tag{A.28}$$

and we can recover from the market-clearing conditions (4) that $\frac{d\hat{a}_f(z)}{dx} = -\sum_{i=1}^{N} \frac{d\hat{a}_i(z)}{dx}$, where $\frac{d\hat{a}_i(z)}{dx}$ are the appropriate elements of $\nabla_x \hat{a}$.

In the special case of the government's risk-free debt position, $\nabla_x vec\left(G\right) = \frac{1}{N+m_f}\iota_{NZ\times 1}$, where $\iota_{NZ\times 1}$ is the $NZ \times 1$ vector of ones, and

$$\nabla_x \hat{a} = -\frac{1}{N+m_f}\left[\nabla_{\hat{a}} vec\left(G\right)\right]^{-1} \iota_{NZ\times 1}. \tag{A.29}$$

Step 2: Intertemporal-sharing wedge of Davila and Schaab (2024):

Similarly, we can compute the intertemporal-sharing wedge from Equation (11) in Davila and Schaab (2024) as

$$\begin{aligned} \Xi_{IS} &= Cov^*\left[\omega_{1,i}, \frac{dc_{1,i}}{dx}\right] + \sum_{z\in\mathcal{Z}} Cov^*\left[\omega_{2,i}, \omega_{2,i}\left(z\right)\frac{dc_{2,i}\left(z\right)}{dx}\right] \\ &= \sum_{z\in\mathcal{Z}} Cov^*\left[\omega_{2,i}, \omega_{2,i}\left(z\right)\frac{d\hat{a}_i\left(z\right)}{dx}\right] - E^*\left[\omega_{1,i}\frac{d\left(q\left(z\right)\hat{a}_i\left(z\right)\right)}{dx}\right]. \end{aligned} \tag{A.30}$$

Step 3: Aggregate efficiency wedge of Davila and Schaab (2024):

Because our economy is an endowment economy, $\sum_{i=1}^{N} m_i \frac{dc_i(z)}{dx} = 0$. As such, from Equations (6) and (11) of Davila and Schaab (2024), the Aggregate Efficiency wedge is zero, i.e., $\Xi_{AE} = 0$. Because the government cannot alter the total endowment of the economy in any state, it cannot alter aggregate efficiency in the economy by changing its demand for risk-free debt

Proof of Proposition 2:

In the absence of government trading, the optimal asset position of type-j strategic agents from Equation (16) reduces to

$$\Lambda_{j,i}(z) - q(z) = \frac{\mu}{m_f} q'(z) a_i(z). \tag{A.31}$$

Summing over condition (A.31) and imposing market-clearing in the Strategic Equilibrium (in which $m_f \approx 0$) yields:

$$q(z) = E^*[\Lambda_{j,i}(z)]. \tag{A.32}$$

To compare Arrow asset prices in the Cournot-Walras to those in the competitive equilibrium, we first characterize state prices in the latter in the following lemma.

Lemma 4 (State Prices in the Competitive Equilibrium) *State prices in the competitive equilibrium in which $m_f \approx 0$, $\Lambda^{CE}(z)$, satisfy:*

$$\Lambda^{CE}(z) = \pi(z) u' \left(\frac{\frac{1}{N} Y(z)}{w} \right). \tag{A.33}$$

Proof. In the competitive equilibrium in which $\mu = 0$, Equation (A.31) implies $q(z) = \Lambda_{j,i}(z) = \Lambda_f(z) = \Lambda^{CE}(z)$, which imposes:

$$\Lambda^{CE}(z) = \frac{\pi(z) u'(c_{2,i}(z))}{u'(c_{1,i})} = \pi(z) u'_f(c_f(z)) = q(z). \tag{A.34}$$

Equation (A.34) implies for the N types of agents with homothetic preferences:

$$\frac{c_{2,i}(z)}{c_{1,i}} = \frac{c_{2,j}(z)}{c_{1,j}} = \eta(z), \tag{A.35}$$

and for the competitive fringe:

$$c_f(z) = \eta_f(z) = u_f^{-1}(u'(\eta(z))). \tag{A.36}$$

From Equations (A.34) and (A.35), we recognize:

$$\Lambda^{CE}(z) = \pi(z)\, u'(\eta(z)), \tag{A.37}$$

and that summing over the set of strategic agents in Equation (A.35), we can recover $\eta(z)$:

$$\eta(z) = \frac{\sum_{i=1}^{N} c_{2,i}(z)}{\sum_{i=1}^{N} c_{1,i}}. \tag{A.38}$$

Substituting the market-clearing conditions for consumption, Equations (1) and (2), into (A.38), and substituting this expression for $\eta(z)$ into Equation (A.37), we have:

$$\Lambda^{CE}(z) = \pi(z)\, u'\left(\frac{Y(z) + A^{CE}(z)}{\sum_{i=1}^{N} w_i + m_f\left(w_f - c_{f1}^{CE}\right)}\right). \tag{A.39}$$

In the Strategic Equilibrium in which $m_f \approx 0$, and consequently $A^{CE}(z) \approx 0$, (A.39) reduces to:

$$\Lambda^{CE}(z) = \pi(z)\, u'\left(\frac{\frac{1}{N} Y(z)}{w}\right). \tag{A.40}$$

■

If all agents are type-symmetric, then $\sum_{z\in\mathcal{Z}} q(z)\, a_i(z) = 0$ and $c_{1,i} = w$ for all i in the Cournot-Walras Equilibrium. We can then apply Jensen's Inequality to (A.32) and invoke Lemma 4 to conclude that:

$$\begin{aligned} E^*[\Lambda_i(z)] = \frac{1}{N}\sum_{n=1}^{N} \pi(z)\, u'\left(\frac{c_{2,i}(z)}{w}\right) &\geq \pi(z)\, u'\left(\frac{\frac{1}{N}\sum_{n=1}^{N} c_{2,i}(z)}{w}\right) = \pi(z)\, u'\left(\frac{\frac{1}{N}Y(z)}{w}\right) \\ &= \Lambda^{CE}(z), \end{aligned} \tag{A.41}$$

where the third step holds by market-clearing (4). Notice that this is true for all $\mu > 0$. It follows that $r_f = [\sum_{z\in\mathcal{Z}} q(z)]$ is lower than in the competitive equilibrium.

Proof of Proposition 3:

Consider the Arrow price in state z, $q(z)$. From Equations (9) and (15), the change in $q(z)$ with respect to a change in the government's risk-free debt position, a_g, is:

$$\frac{dq(z)}{da_g} = -\pi(z)u_f''(c_{2,f}(z))\frac{dc_{2,f}(z)}{da_g} = \frac{1}{m_f}q'(z)\sum_{i=1}^{N}\frac{d\hat{a}_i(z)}{da_g}. \tag{A.42}$$

Recognizing that $q'(z) \geq 0$, we focus on the sign of $\frac{d\hat{a}_i(z)}{da_g}$.

From Equation (16) when the government trades risk-free debt, the first-order condition for the optimal net positions of type-i strategic agents, $\hat{a}_i(z)$, reduces to:

$$\Lambda_i(z) - q(z) = \frac{\mu}{m_f}q'(z)\left(\hat{a}_i(z) - \frac{a_g}{N+m_f}\right) \tag{A.43}$$

Examining Equations (14) and (15), we recognize that state prices, $\Lambda_i(z)$, and asset prices, $q(z)$, depend only on net asset positions, $\hat{a}_i(z)$. Therefore, the government's position in the risk-free asset, a_g, only directly enters the first-order condition through the right-hand side.

Suppose that $m_f > 0$ and the government increases its risk-free debt position, a_g, by $\epsilon > 0$. Then, let $\tilde{\Lambda}_i(z)$ be agent i's state price after she adjusts her portfolio to her new position $\tilde{\hat{a}}_i(z)$, $\Delta\hat{a}_i(z) = \tilde{\hat{a}}_i(z) - \hat{a}_i(z)$ the change in her holdings, and $\Delta q(z) = \tilde{q}(z) - q(z)$ the change in the asset price. Then Equation (A.43) implies:

$$\tilde{\Lambda}_i(z) - \Lambda_i(z) - \Delta q(z) - \frac{\mu}{m_f}\Delta q'(z)\hat{a}_i(z) = \frac{\mu}{m_f}q'(z)\left(\Delta\hat{a}_i(z) - \frac{\epsilon}{N+m_f}\right). \tag{A.44}$$

Suppose, by way of contradiction, that $\Delta\hat{a}_i(z) < 0$ if i is a buyer and $\hat{a}_i(z) > 0$. Then, the right-hand side of Equation (A.44) is negative. However, if $\Delta\hat{a}_i(z) < 0$, then agent i buys less / sells more of asset z. This raises her state price in state z, i.e., $\tilde{\Lambda}_i(z) > \Lambda_i(z)$. By reducing demand, it also lowers the asset price, i.e., $\Delta q(z) < 0$ and price impact term, i.e., $\Delta q'(z)\hat{a}_i(z) < 0$. As a consequence, the left-hand side of Equation (A.44) is positive, which is a contradiction. As such, $\Delta\hat{a}_i(z) > 0$ and $\frac{d\hat{a}_i(z)}{da_g} > 0$.

Suppose i is a seller and $\hat{a}_i < 0$. We recognize that because $\frac{d\hat{a}_i(z)}{da_g} > 0$ when $\hat{a}_i \geq 0$, by continuity, $\frac{d\hat{a}_i(z)}{da_g} > 0$ for $\hat{a}_i < 0$ sufficiently small. For more negative $\hat{a}_i < 0$, notice from Equation (16) that holding prices fixed, the market power wedge, $\Lambda_i(z) - q(z) < 0$, is increasing in a_g when $\hat{a}_i < 0$. Combining this observation with $\frac{d\hat{a}_i(z)}{da_g} > 0$ as $\hat{a}_i < 0$ goes to zero, we conclude $\frac{d\hat{a}_i(z)}{da_g} > 0$ more generally when i is a seller in market z.

Because $\frac{d\hat{a}_i(z)}{da_g} > 0$ for all strategic agents, we have $\sum_{i=1}^{N} \frac{d\hat{a}_i(z)}{da_g} > 0$. Given that $q'(z) \geq 0$ and $\sum_{i=1}^{N} \frac{d\hat{a}_i(z)}{da_g} > 0$, we conclude from Equation (A.42) that $\frac{dq(z)}{da_g} > 0$.

Next, we recognize that the change in price impact, $q'(z)$, with respect to a change in the government's risk-free debt position, a_g, is:

$$\frac{dq'(z)}{da_g} = \frac{1}{m_f} \frac{u'''\left(c_{2,f}(z)\right)}{-u''\left(c_{2,f}(z)\right)} q'(z) \sum_{i=1}^{N} \frac{d\hat{a}_i(z)}{da_g}, \tag{A.45}$$

where $\frac{u'''\left(c_{2,f}(z)\right)}{-u''\left(c_{2,f}(z)\right)} > 0$ because the fringe has convex marginal utility. Since $q'(z) > 0$ and $\sum_{i=1}^{N} \frac{d\hat{a}_i(z)}{da_g} > 0$, $\frac{dq'(z)}{da_g} > 0$. As our choice of z was arbitrary, $\frac{dq(z)}{da_g}, \frac{dq'(z)}{da_g} > 0$ for all z.

Finally, if $\frac{dq(z)}{da_g} > 0$ for all z, then $\sum_{z\in\mathcal{Z}} \frac{dq(z)}{da_g} > 0$. Because the risk-free rate, r_f, is the inverse of the sum of Arrow asset prices, $\sum_{z\in\mathcal{Z}} q(z)$, it also lowers r_f, i.e., $\frac{dr_f}{da_g} < 0$.

Suppose now that $m_f \approx 0$, which we refer to as the strategic limit in the main text, then $\sum_{i=1}^{N} \hat{a}_i(z) \approx 0$ by market clearing. In this case, $\sum_{i=1}^{N} \frac{d\hat{a}_i(z)}{da_g} \approx 0$, and demand must internally clear among strategic agents. However, by continuity as $m_f \to 0$ and $\frac{\mu}{m_f} \to \kappa$, we continue to have the same comparative statics with respect to a_g.

Proof of Proposition 4:

Since agents are type-symmetric, it must be that $\sum_{z\in\mathcal{Z}} q(z) a_i(z) = 0$ for all i. To see this, notice by way of contradiction that if $\sum_{z\in\mathcal{Z}} q(z) a_i(z) > 0$ for some type i, then it must be negative for some other type i' by market-clearing, which contradicts that the agents are type-symmetric. Consequently, $\sum_{z\in\mathcal{Z}} q(z) a_i(z) \equiv 0$ for all i, and therefore all agents consume the same initial consumption $c_{1,i} = w$ at date 1. The state price of a strategic agent of type i for state z is then

$$\Lambda_i(z) = \frac{u'\left(y_i(z) + \hat{a}_i(z)\right)}{u'(w)}, \tag{A.46}$$

and moves inversely with her asset position in that state, $\hat{a}_i(z)$. Given that agents are type-symmetric, it is sufficient for us to characterize one asset market z because they each behave identically.

Because $\Gamma_{RS}(z)^2 = Var^i[\Lambda_i(z)]$, we recognize:

$$\begin{aligned}
\frac{1}{2}\frac{d\Gamma_{RS}(z)^2}{da_g} &= \frac{1}{2u'(w)^2}\frac{dVar[u'(y_i(z)+\hat{a}_i(z))]}{da_g} \\
&= \sum_{i=1}^{N}\frac{u'(y_i(z)+\hat{a}_i(z)) - \frac{1}{N}\sum_{i'=1}^{N}u'(y_{i'}(z)+\hat{a}_{i'}(z))}{Nu'(w)^2}u''(y_i(z)+\hat{a}_i(z))\frac{d\hat{a}_i}{da_g} \\
&= Cov^i\left[\Lambda_i(z) - q(z), \frac{u''(y_i(z)+\hat{a}_i(z))}{u'(w)}\frac{d\hat{a}_i}{da_g}\right] \\
&= -\frac{\kappa q'(z)}{u'(w)}Cov^i\left[a_i(z), |u''(y_i(z)+\hat{a}_i(z))|\frac{d\hat{a}_i}{da_g}\right], \qquad \text{(A.47)}
\end{aligned}$$

where the last step follows from Equation (16). Substituting $a_i(z) = \hat{a}_i(z) + \frac{a_g}{N}$ and recognizing that $\sum_{i=1}^{N} a_i(z) = 0$ and $\sum_{i=1}^{N}\frac{d\hat{a}_i}{da_g} = 0$, we have that:

$$\frac{1}{2}\frac{d\Gamma_{RS}(z)^2}{da_g} = -\frac{\kappa q'(z)}{u'(w)}Cov^i\left[\hat{a}_i(z)|u''(y_i(z)+\hat{a}_i(z))|, \frac{d\hat{a}_i}{da_g}\right]. \qquad \text{(A.48)}$$

Note that $|u''(y_i(z)+\hat{a}_i(z))|\hat{a}_i(z)$ is increasing in $\hat{a}_i(z)$ because strategic agents' have convex marginal utility

In addition, from Equation (16), we can calculate that in the case of type-symmetric agents in which there are no cross-asset spillovers that

$$\frac{d\hat{a}_i}{da_g} = \frac{1}{N}\frac{1}{\frac{|u''(y_i(z)+\hat{a}_i(z))|}{u'(w)} + 2\kappa q'(z) + \kappa q''(z)a_i(z)}. \qquad \text{(A.49)}$$

Across agents, i.e., holding prices and price impact fixed, the denominator is larger for buyers than sellers since $|u''(y_i(z)+\hat{a}_i(z))|\hat{a}_i(z)$ is increasing in $\hat{a}_i(z)$ because of convex marginal utility. Consequently, $\frac{\partial}{\partial\hat{a}_i(z)}\frac{d\hat{a}_i(z)}{da_g} < 0$. This conveys the key intuition for our result: sellers' supply is more elastic than buyers' demand, and consequently their reduction in supply dominates the change in buyers' demand, increasing price impact and worsening risk sharing. As such:

$$Cov^{i}\left[\hat{a}_{i}\left(z\right)\left|u''\left(y_{i}\left(z\right)+\hat{a}_{i}\left(z\right)\right)\right|,\frac{d\hat{a}_{i}}{da_{g}}\right]<0, \tag{A.50}$$

and therefore from Equation (A.48).

Since $\Gamma_{RS}\left(z\right)>0$, we conclude $\frac{d\Gamma_{RS}(z)}{da_{g}}>0$. By symmetry, this holds for all z.

Proof of Proposition 5:

Because the economy is deterministic and all agents risk-averse, only a risk-free bond is traded. Consequently, strategic agents of type i only have one state price:

$$\Lambda_{i}=\frac{u'\left(\bar{y}_{i}+\hat{a}_{i}\right)}{u'\left(w_{i}-q\hat{a}_{i}\right)}, \tag{A.51}$$

that maps the consumption numeraire at date 2 into the consumption numeraire at date 1, which is decreasing in $\hat{a}_{i}$. By market clearing, the price of the risk-free bond in the strategic limit is given by $q=\sum_{i=1}^{N}\Lambda_{i}$.

Recognizing the case of one risk-free asset is similar to the type-symmetric case in which all asset markets z are identical, we can apply analogous arguments to those in the proof of Proposition 4 to conclude that in the strategic limit the intertemporal-sharing wedge, Γ_{IS}, is increasing in the government's demand for the risk-free bond, a_{g}.

Proof of Proposition 6:

We begin with the Euler Equations from Equation (16) expressed in terms of market power wedges $\Lambda_{i}\left(z\right)-q\left(z\right)$ and the net asset demands $\hat{a}_{i}$, which we write as

$$\Lambda_{i}\left(z\right)-q\left(z\right)=\frac{\mu}{m_{f}}q'(z)\hat{a}_{i}\left(z\right)-\frac{\mu}{m_{f}}q'(z)\frac{a_{g}}{N+m_{f}}, \tag{A.52}$$

and depends on $\frac{a_{g}}{N+m_{f}}$ only through the last term on the right-hand side. If $a_{g}>0$, then the last term is negative, while if $a_{g}<0$, then the last term is positive.

Notice government asset purchases cannot generically achieve the competitive equilibrium. This is because a_g (which is one degree of freedom) cannot be chosen such that the wedges $\frac{\mu}{m_f}q'(z)\left(\hat{a}_i(z)-\frac{a_g}{N+m_f}\right)$ ($N\times|\mathcal{Z}|$ Equations) are all zero.[10]

In what follows, we measure the aggregate efficiency of risk sharing in an asset market z using $Var^i[\Lambda_i(z)]$ defined in Equation (17), which is the square of $\Gamma_{RS}(z)$. The change in efficiency in risk sharing in state z for a change in government policy a_g is:

$$\Delta\log Var^i[\Lambda_i(z)] = 2\Delta\log q'(z) + \Delta\log Var^i[\hat{a}_i(z)]. \tag{A.53}$$

From Proposition 3, $\frac{\partial q'(z)}{\partial a_g}>0$ and the first-term in Equation (A.53) is positive if $a_g>0$ and negative if $a_g<0$ for all z. We can further approximate Equation (A.53) to first-order for a change in government policy $\Delta a_g\to 0$ as

$$\frac{1}{2}\frac{d\log Var_i(\Lambda_i(z))}{da_g}\approx\frac{\gamma_f(z)}{m_f}\sum_{i=1}^{N}\frac{d\hat{a}_i(z)}{da_g}+\frac{N-1}{N}\sum_{i=1}^{N}w_i\frac{d\hat{a}_i(z)}{da_g}, \tag{A.54}$$

where $\gamma_f(z)$ is the competitive fringe's coefficient of absolute risk aversion in state z, and $w_i=\frac{\hat{a}_i(z)-\frac{1}{N}\sum_{j=1}^{N}\hat{a}_j(z)}{Var^i[\hat{a}_i(z)]}$ and sum to zero.

To first-order, the change in the efficiency of risk sharing is driven by how the change in each agent's net asset position $\hat{a}_i(z)$ impacts not only market liquidity but also their position relative to the mean exposure $\frac{1}{N}\sum_{j=1}^{N}\hat{a}_j(z)$. The first term is increasing in $\frac{\gamma(z)}{m_f}$, which is the inverse of the effective risk-bearing capacity of the fringe. Let $\underline{\gamma}=\min_{z\in\mathcal{Z}}\gamma(z)$, i.e., the minimum coefficient of risk aversion across all asset markets when $a_g=0$.

Step 1: Asymptotic Absorption Capacity:

Consider first large government asset purchases $a_g>>0$. Recall from Proposition 3 that price and price impact are increasing in a_g. From strategic agents' first-order conditions, Equation (16), this is because sellers reduce their supply (even becoming buyers if prices rise sufficiently). As a consequence, the demand from buyers is increasingly being absorbed by the competitive fringe from the market clearing conditions (4) until $\sum_{i=1}^{N}\hat{a}_i(z)=m_f y_f(z)$ for arbitrarily small asset purchases by strategic agents, in which

[10] This is also true even if the government can trade each Arrow asset separately because there are still more Euler Equations than asset markets. Because prices (and consequently price impact) cannot be zero by no arbitrage, these wedge will not vanish if government asset sales are arbitrarily large.

case the asset price in market z, $q(z)$ becomes infinite, as does price impact. This immiseration pushes $\Gamma_{RS}(z) = Var^i[\Lambda_i(z)]$ and $\Gamma_{IS} = Var^i[\sum_{z\in\mathcal{Z}} \Lambda_i(z)]$ to their autarky values. Sufficiently large asset purchases severely worsen our risk sharing and intertemporal smoothing measures.

Consider next large government asset sales $a_g << 0$. In this case, the market is saturated with supply of each Arrow asset and prices and price impact are low from Proposition 3. Because the risk sharing and intertemporal smoothing wedges, $\Gamma_{RS}(z)$ and Γ_{IS}, are bounded from below by 0, and from above because state prices cannot become infinite since strategic agent utility satisfies the Inada condition, $\Gamma_{RS}(z)$ and Γ_{IS} eventually asymptote to their lower bounds.[11] Because from our analysis above, government trading cannot achieve the competitive equilibrium, these limits are all bounded away from 0.

Further, because $\Gamma_{RS}(z)$ is continuous in a_g, it has an even number of turning points in a_g. Given $\Gamma_{RS}(z)$ is driven by two monotonic forces that move in opposite directions, price impact, $q'(z)$, and asset position variance, $Var^i[\hat{a}_i(z)]$, there are either zero or two turning points.

Step 2: Risk Sharing Wedge and Small Government Asset Purchases $a_g > 0$:

Suppose the government purchases a small amount of assets, i.e., $a_g > 0$. Consider the first-order (log) change in our risk-sharing wedge, Equation (A.54), of a small increase in a_g from 0 to $\Delta a_g > 0$. From Proposition 3, because all asset prices rise, it must be the case that $\sum_{i=1}^N \frac{\Delta \hat{a}_i(z)}{\Delta a_g} > 0$. We can then express Equation (A.54) as:

$$\frac{d \log Var_i[\Lambda_i(z)]}{da_g} \propto \frac{\gamma_f(z)}{m_f} + \frac{N-1}{N}\sum_{i=1}^N w_i\zeta_i, \tag{A.55}$$

where $\zeta_i = \frac{d\hat{a}_i(z)}{da_g} / \sum_{i=1}^N \frac{d\hat{a}_i(z)}{da_g}$ and sum to 1. From our above discussion, the first piece is positive for all z while the second piece is negative for all z.

[11] From Equation (A.52), for the wedge to asymptote, $\lim_{a_g\to-\infty} q'(z)a_g = 0$ for each z. For this to be the case, $q'(z)$ must fall faster than $|a_g|$ rises (in fact, $q'(z)$ is convex in a_g).

By the Schwartz Inequality, we have that:

$$\left|\frac{1}{N}\sum_{i=1}^{N} w_i \zeta_i\right| \leq \sqrt{\frac{1}{N}\sum_{i=1}^{N} w_i^2 \frac{1}{N}\sum_{i=1}^{N}\zeta_i^2} = \sqrt{\frac{1}{Var^i\left[\hat{a}_i\left(z\right)\right]}\frac{1}{N}\sum_{i=1}^{N}\zeta_i^2}. \tag{A.56}$$

From Equation (A.57), we recognize that $\frac{d\hat{a}_i(z)}{da_g}$ is given by

$$\nabla_{a_g}\hat{a} = \frac{1}{N+m_f}\left[I_{NZ} - \nabla_{\hat{a}} vec\left(H\right)\right]^{-1}\iota_{N\times Z}, \tag{A.57}$$

where $\iota_{N\times Z}$ is the $N \times Z$ vector of ones, I_{NZ} is the $N \times Z$ identity matrix, and $H\left(i,z\right)$ is the $N \times Z$ matrix

$$H\left(i,z\right) = \frac{m_f}{\mu} q'\left(z\right)^{-1}\left(\Lambda_i\left(z\right) - q\left(z\right)\right). \tag{A.58}$$

Given that the derivatives of H are all bounded, and generically, $I - H$, will be invertible, we conclude that the elements of $\nabla_{a_g}\hat{a}$ are bounded. Consequently, all ζ_i are also bounded, and from Equation (A.56), $\left|\frac{1}{N}\sum_{i=1}^{N} w_i\zeta_i\right|$ are also bounded for all z.

It then follows that if $\frac{\gamma}{m_f}$ is sufficiently large, then the change in price impact dominates the change in position variance, and the risk-sharing wedge rises.

Step 3: Risk Sharing Wedge and Small Government Asset Sales $a_g < 0$:

Suppose the government sells a small amount of assets, i.e., $a_g < 0$. From Proposition 3, because all asset prices fall, it must be the case that $\sum_{i=1}^{N}\frac{\Delta\hat{a}_i(z)}{\Delta a_g} < 0$.

By analogous arguments to Step 2, we conclude that if $\frac{\gamma}{m_f}$ is sufficiently large, then the change in price impact again dominates the change in position variance, and the risk-sharing wedge falls.

Step 4: Ambiguous Local Effect on Intertemporal Smoothing:

Notice that our intertemporal smoothing measure from Equation (18) implies:

$$\begin{aligned}\frac{d\Gamma_{IS}^{2}}{da_{g}} &= 2\sum_{z\neq z'\in\mathcal{Z}}\frac{d}{da_{g}}Cov^{i}\left[\frac{\mu}{m_{f}}q'\left(z\right)\left(\hat{a}_{i}\left(z\right)-\mathbb{E}^{i'}\left[\hat{a}_{i'}\left(z\right)\right)\right),\frac{\mu}{m_{f}}q'\left(z'\right)\left(\hat{a}_{i}\left(z'\right)-\mathbb{E}^{i'}\left[\hat{a}_{i'}\left(z'\right)\right]\right)\right] \\ &+\sum_{z\in\mathcal{Z}}\frac{dVar^{i}\left[\frac{\mu}{m_{f}}q'\left(z\right)\hat{a}_{i}\left(z\right)\right]}{da_{g}}.\end{aligned} \tag{A.59}$$

From Steps 2 and 3, if $\frac{\gamma}{m_f}$ is sufficiently large, then the second term in Equation (A.59) is positive for a_g close to zero. However, the first term can be positive or negative depending on the average covariance across strategic agents in asset positions across states, i.e., whether a strategic agent that is a buyer in Arrow asset market z is also a buyer in market z', and similarly with being a seller in asset market z. As such, Γ_{IS} can increase or decrease locally in a_g.

Proof of Proposition 7:

Consider a perturbation to the maximized expected utility of a strategic agent of type i, U_i from decision problem (7), by altering Δa_g:

$$\begin{aligned}\Delta U_{i} &= u'\left(c_{1,i}\right)\left(\frac{dc_{1,i}}{da_{g}}\Delta a_{g}+\sum_{z\in\mathcal{Z}}\pi\left(z\right)\frac{u'\left(c_{2,i}\left(z\right)\right)}{u'\left(c_{1,i}\right)}\frac{dc_{2,i}\left(z\right)}{da_{g}}\Delta a_{g}\right) \\ &= u'\left(c_{1,i}\right)\sum_{z\in\mathcal{Z}}\left(\left(\Lambda_{i}\left(z\right)-q\left(z\right)\right)\frac{d\hat{a}_{i}\left(z\right)}{da_{g}}-\frac{dq\left(z\right)}{da_{g}}\hat{a}_{i}\left(z\right)\right)\Delta a_{g},\end{aligned} \tag{A.60}$$

which, substituting with Equation (16), becomes:

$$\Delta U_{i}=u'\left(c_{1,i}\right)\left(\sum_{z\in\mathcal{Z}}\frac{\mu}{m_{f}}q'\left(z\right)\hat{a}_{i}\left(z\right)\frac{d\hat{a}_{i}\left(z\right)}{da_{g}}-\frac{dq\left(z\right)}{da_{g}}\hat{a}_{i}\left(z\right)\right)\Delta a_{g}, \tag{A.61}$$

where $\frac{dq(z)}{da_g}$ indicates the total change in $q\left(z\right)$ with respect to a_g.

Similarly, for the competitive fringe with maximized expected utility, U_f, from decision problem (8):

$$\Delta U_f = \frac{dc_{1,f}}{da_g}\Delta a_g + \sum_{z\in\mathcal{Z}} \pi(z)\, u'\left(c_{2,f}(z)\right) \frac{dc_{2,f}(z)}{da_g}\Delta a_g$$
$$= \sum_{z\in\mathcal{Z}} \left(\left(\pi(z)\, u'\left(c_{2,f}(z)\right) - q(z)\right) \frac{d\hat{a}_f(z)}{da_g} - \frac{dq(z)}{da_g}\hat{a}_f(z) \right) \Delta a_g, \quad \text{(A.62)}$$

which, substituting with Equations (4) and (9), becomes:

$$\Delta U_f = \sum_{z\in\mathcal{Z}} \frac{dq(z)}{da_g} \sum_{i=1}^{N} \hat{a}_i(z)\, \Delta a_g. \quad \text{(A.63)}$$

Consider now a perturbation to the welfare objective (21):

$$\Delta W = \sum_{i=1}^{N} U_i + m_f U_f, \quad \text{(A.64)}$$

which, substituting with Equations (A.61) and (A.63), becomes:

$$\frac{\Delta W}{\Delta a_g} = \sum_{z\in\mathcal{Z}} \frac{\mu}{m_f} q'(z) \sum_{i=1}^{N} u'(c_{1,i})\, a_i(z) \frac{\Delta \hat{a}_i(z)}{\Delta a_g} - \frac{\Delta q(z)}{\Delta a_g} \sum_{i=1}^{N} \left(u'(c_{1,i}) - m_f\right) \hat{a}_i(z). \quad \text{(A.65)}$$

Taking the limit of equation (A.65) as $\Delta a_g \to da_g$, defining $w_{pri}(z) = \frac{1}{2}\sum_{i=1}^{N} u'(c_{1,i}) \frac{d\hat{a}_i(z)^2}{da_g}$ and $w_{pub}(z) = \sum_{i=1}^{N} \left(u'(c_{1,i}) - m_f\right) \hat{a}_i(z)$, and recognizing that a necessary condition for optimality is $\frac{\partial W}{\partial a_g} = 0$, at the optimal a_g it must be the case from Equation (A.65):

$$\sum_{z\in\mathcal{Z}} w_{pri}(z) \frac{\mu}{m_f} q'(z) - w_{pub}(z) \frac{dq(z)}{da_g} = 0 \; (< if a_g = -\infty, > if a_g = \infty) \quad \text{(A.66)}$$

If all large agents behaved competitively, then $\frac{\mu}{m_f} q'(z) = 0$, and because of Ricardian Equivalence, $\frac{dq(z)}{da_g} = 0$ from Proposition 3. Therefore, the first-order condition reduces to 0 in the competitive equilibrium, and is trivially satisfied.

Thus, increasing the government's position in risk-free debt, a_g, has two effects. First, it marginally shifts all agents' resources from date 1 to date 2. This is because taxes increase at date 1 and are more negative at date 2 to finance the purchase of the debt and lump-sum rebate of its proceeds.

Second, it marginally increases the price of every Arrow asset from the government's increased demand. To first-order, the change in each agent's asset position on her own utility is zero by the Envelope Theorem applied to their respective optimization problems from Lemma (1). However, there is a wealth effect on each agent's utility from having to buy a more expensive asset portfolio, and a strategic indirect effect that a change in large agents' asset positions affects the price impact of other large agents.

Proof of Proposition 8:

We first recognize that by the law of one price, buying a_{gx} units of the asset is equivalent to buying a portfolio of $\{X(z)\, a_{gx}\}_{z \in \mathcal{Z}}$ of Arrow securities. From Equation (A.27), this implies that $\nabla_{a_{gX}} \hat{a}$ is given by:

$$\nabla_{a_{gX}} \hat{a} = -\frac{1}{N + m_f} \left[\nabla_{\hat{a}} vec\left(G\right)\right]^{-1} vec\left(X\left(z\right)\right), \tag{A.67}$$

where now: where $\hat{a}$ is $NZ \times 1$ vector of all strategic agents' asset positions, $\iota_{N\times Z}$ is the $N \times Z$ vector of ones, and $G(i,z)$ is the $N \times Z$ matrix

$$G\left(i,z\right) = \frac{X\left(z\right) a_{gX}}{N + m_f} + \frac{m_f}{\mu} q'\left(z\right)^{-1} \left(\Lambda_i\left(z\right) - q\left(z\right)\right) - \hat{a}_i\left(z\right), \tag{A.68}$$

Consider a perturbation to the maximized expected utility of a strategic agent of type i, U_i, from decision problem (7) by altering Δa_{gx}, and substituting with (16):

$$\Delta U_i = u'\left(c_{1,i}\right) \sum_{z \in \mathcal{Z}} \left(\frac{\mu}{m_f} q'\left(z\right) \hat{a}_i\left(z\right) \frac{d\hat{a}_i\left(z\right)}{da_{gX}} - \frac{dq\left(z\right)}{da_{gX}} \hat{a}_i\left(z\right) \right) \Delta a_{gX}. \tag{A.69}$$

Similarly, for the competitive fringe with maximized expected utility, U_f, from decision problem (8) and substituting with Equations (4) and (9):

$$m_f \Delta U_f = m_f \sum_{z \in \mathcal{Z}} \frac{dq\left(z\right)}{da_{gX}} \sum_{i=1}^{N} \hat{a}_i\left(z\right) \Delta a_{gX}. \tag{A.70}$$

Consider now a perturbation to the welfare objective (21):

$$\Delta W=\sum_{i=1}^{N}U_i+m_fU_f, \tag{A.71}$$

which, substituting with Equations (A.69) and (A.70), becomes:

$$\Delta W=\sum_{z\in\mathcal{Z}}\left(\frac{\mu}{m_f}q'(z)\sum_{i=1}^{N}u'(c_{1,i})\,\hat{a}_i(z)\frac{d\hat{a}_i(z)}{da_{gX}}-\frac{dq(z)}{da_{gX}}\sum_{i=1}^{N}\left(u'(c_{1,i})-m_f\right)\hat{a}_i(z)\right)\Delta a_{gX}. \tag{A.72}$$

Taking the limit of Equation (A.65) as $\Delta a_{gx}\to da_{gX}$, defining $w_{pri,x}(z)=\frac{1}{2}\sum_{i=1}^{N}u'(c_{1,i})\frac{d\hat{a}_i(z)^2}{da_{gX}}$ and $w_{pub}(z)=\sum_{i=1}^{N}\left(u'(c_{1,i})-m_f\right)\hat{a}_i(z)$, and recognizing that a necessary condition for optimality is $\frac{\partial W}{\partial a_{gX}}=0$, at the optimal a_{gX} it must be the case from Equation (A.72) at an interior optimum:

$$\sum_{z\in\mathcal{Z}}\left(\frac{\mu}{m_f}w_{pri,X}(z)\,q'(z)-w_{pub}(z)\frac{dq(z)}{da_{gX}}\right)=0\;(<if a_{gX}=-\infty,>if a_{gX}=\infty). \tag{A.73}$$

Appendix B: Unfunded Bond Market Interventions

As is well-known in the public finance literature, the effects of public policies often depend on how government expenditures are funded. So far, we have assumed that government operations are fully funded by non-distortionary taxes and transfers on market participants. A natural interpretation of these tax-and-trade schemes is that the government exchanges long-term assets for cash or reserves held by large financial institutions. However, in practice these interventions may be "unfunded" from the perspective of market participants. For example, government interventions may be funded by current or future taxation on agents outside of the model. To understand how the funding structure affects the equilibrium outcomes of government interventions, we now weaken the assumption of budget balance in every period. We then show that unfunded asset purchases have the additional effect of shifting aggregate resources across dates.

Suppose the government has an endowment y_{1g} at date 1 and y_{2g} at date 2, and its objective is to trade until it equates its consumption at both dates

$$y_{1g} - \sum_{z \in \mathcal{Z}} q(z)\, a_g = y_{2g} + a_g, \tag{A.74}$$

from which follows, defining $\Delta y_g = y_{1g} - y_{2g}$ that

$$a_g = \frac{\Delta y_g}{1 + \sum_{z \in \mathcal{Z}} q(z)}. \tag{A.75}$$

This has the interpretation of the government wanting to inelastically smooth its expenditures at each date by issuing or purchasing risk-free debt. By varying Δy_g (the endowment mismatch), this fixed rule will provide an exogenous source of demand (or supply) to financial markets at date 1. We focus on the strategic limit from Definition 2 in which $\mu, m_f \to 0$ but $\frac{\mu}{m_f} \to \kappa$ and shut down all taxes and transfers. To examine the role of large-scale asset purchase programs like quantitative easing, we assume $y_{1g} < y_{2g}$ so that $a_g = \bar{a}_g > 0$ and the government buys risk-free debt. The complementary case when $y_{1g} < y_{2g}$, which captures the more quotidian behavior of governments to issue debt to finance expenditures, has analogous results. We then have the following proposition.

Proposition 9 (Unfunded Large-Scale Asset Purchases) *Suppose Δy_g increases and the government demands more risk-free debt. Then, under both perfect and imperfect competition, all asset prices $q(z)$ rise and all agents consume more at date* 1 *and less at date* 2*.*

Proposition 9 shows that unfunded government large-scale asset purchases break Ricardian Equivalence. They effectively reallocate resources among strategic agents from date 2 to date 1 by crowding-out investment in risk-free bonds. This induces agents to consume more at date 1. In a more general setting, such a crowding-out of investment in risk-free bonds can crowd-in investment into alternative investment opportunities, such as riskier equities or corporate debt. For instance, Joyce, Liu, and Tonks (2017) show that the Bank of England's quantitative easing program during the Global Financial Crisis shifted the investments of U.K. insurance companies and pension funds from Gilts toward corporate bonds. Our analysis also clarifies the distinct mechanisms through which government trading influences market liquidity, which operates regardless of the funding scheme, and how it affects the market's risk-bearing capacity, which requires that trading be unfunded.

Proof of Proposition 9:

We first consider how unfunded government purchases $\bar{a}_g > 0$ impact equilibrium allocations in the case of perfect competition. We then consider the complementary case with concentrated financial markets.

Step 1: Perfect Competition:

With perfect competition, it is immediate that the First Welfare Theorem holds and optimal risk-sharing arrangements solve the appropriate social planner's problem. In this case, with symmetric, homothetic preferences, perfect risk sharing calls for $\frac{c_{2i}(z)}{c_{1i}} = \eta(z) = u'^{-1}(q(z))$ for all i. This is Wilson's optimal risk sharing rule. In this case, $\sum_{z\in\mathcal{Z}} q(z) = \sum_{z\in\mathcal{Z}} u'(\eta(z))$

By the intertemporal budget constraint for agent i at date 1

$$c_{1i} + \sum_{z\in\mathcal{Z}} q(z)\, c_{2i}(z) = \left(1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)\right) c_{1i} = w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z), \quad \text{(A.76)}$$

from which follows

$$c_{1i} = \frac{w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z)}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)}. \quad \text{(A.77)}$$

Further, by market clearing in the consumption market at date 2 in the strategic limit in which $m_f \to 0$

$$\sum_{i=1}^{N} c_{2i}(z) + \bar{a}_g = \eta(z)\sum_{i=1}^{N} c_{1i} + \bar{a}_g = \sum_{i=1}^{N} y_i(z), \quad \text{(A.78)}$$

from which follows from Equation (A.77) and $a_g = \frac{y_{1g} - y_{2g}}{1+\sum_{z\in\mathcal{Z}} q(z)}$ that $\eta(z)$ solve

$$\eta(z)\sum_{i=1}^{N} \frac{w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z)}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)} + \frac{\Delta y_g}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))} = \sum_{i=1}^{N} y_i(z). \quad \text{(A.79)}$$

Recovering $\eta(z)$ from Equation (A.79) is consequently sufficient to solve for the competitive equilibrium.

Notice because $\Delta y_g = y_{1g} - y_{2g} > 0$ (i.e., $\bar{a}_g > 0$) and $u'(\eta(z))$ is decreasing in

$\eta(z)$ from applying the Implicit Function Theorem to Equation (A.79) that $\frac{\partial \eta(z)}{\partial \Delta y_g} < 0$. In addition, by the Chain Rule, $\frac{\partial q(z)}{\partial \Delta y_g} = \frac{\partial q(z)}{\partial \eta(z)} \frac{\partial \eta(z)}{\partial \Delta y_g} > 0$ for all $z \in \mathcal{Z}$ because $\frac{\partial q(z)}{\partial \eta(z)} < 0$ with convex marginal utility.

Consequently, as $\Delta y_g > 0$ increases, all agents consume more at date 1 and less at date 2, and asset prices rise state-by-state. Because the government crowds out investment in risk-free bonds, agents are forced to consume more resources at date 1.

Step 2: Imperfect Competition:

Notice in the limit $m_f \to 0$ that asset prices are given by the average of strategic agents' state prices, $q(z) = \frac{1}{N} \sum_{i=1}^{N} \Lambda_i(z)$. Suppose that Δy_g increases to $\Delta y_g + \varepsilon_g$. We conjecture that $\bar{a}_g$ increases and, based on Step 1, all asset prices rise and strategic agents consume more at date 1 and less at date 2, state-by-state. In this case, all strategic agents' state prices (weakly) rise state-by-state, $\Lambda_i(z)$, which raises asset prices and lowers the risk-free rate. Because the risk-free rate falls, the government has to (weakly) buy more assets to achieve its pre-existing date 2 position based on Δy_g, as well as more to cover its incremental position based on ε_g. Therefore, $\bar{a}_g$ increases, confirming the conjecture.

In addition to the direct effect of government purchases, the increase in asset prices raises price impact $q'(z)$ because the competitive fringe's marginal utility is convex. This further attenuates how much risk-free debt is traded among strategic agents. As a result, buyers reduce their purchases of risk-free debt more than with perfect competition, while sellers must sell to fulfill the government's incremental orders, exacerbating the rise in asset prices.

Consequently, as with perfect competition, asset prices rise and strategic agents consume more at date 1 and less at date 2.

Appendix C: Solutions to Examples

In this Appendix, we provide proofs for the additional results in Examples 1 and 2.

Example 1

In what follows, we establish a sufficient condition under which our risk sharing wedge, $\Gamma_{RS}(z)$, is increasing in the government's position in the risk-free asset, a_g, in Example 1.

With CRRA preferences, which are homothetic, let us define the state price of the seller in an asset market as:

$$\Lambda_L = \frac{1}{2}u'\left(\frac{2\bar{y}+\hat{a}_s}{w-q^*\left(\hat{a}_b+\hat{a}_s\right)}\right), \tag{A.80}$$

and that of the buyer:

$$\Lambda_H = \frac{1}{2}u'\left(\frac{\hat{a}_b}{w-q^*\left(\hat{a}_b+\hat{a}_s\right)}\right). \tag{A.81}$$

Since the two strategic agent types are symmetric, we can search for an equilibrium where Type 1 agents sell $a_s < 0$ units of the claim to state 1 and buy $a_b > 0$ units of the claim to state 2. Type 2 agents take the reverse positions. Define $\hat{a}_s = a_s + \frac{a_g}{2+m_f}$ and $\hat{a}_b = a_b + \frac{a_g}{2+m_f}$. The claim in each state has a price q^* based on the fringe's marginal utility, $q^* = u'\left(\frac{\bar{y}}{w} - \frac{1}{m_f}\left(\hat{a}_b+\hat{a}_s\right)\right)$. and price impact is q'^*. As such, asset positions satisfy

$$\text{Seller optimality:} \qquad \Lambda_L = q^* + \frac{\mu}{m_f}q'^*\left(\hat{a}_s - \frac{a_g}{2+m_f}\right), \tag{A.82}$$

$$\text{Buyer optimality:} \qquad \Lambda_H = q^* + \frac{\mu}{m_f}q'^*\left(\hat{a}_b - \frac{a_g}{2+m_f}\right). \tag{A.83}$$

In the two-agent pure risk sharing economy, our risk sharing wedge, $\Gamma_{RS}(z)$, is identical for both states and simplifies to:

$$\Gamma_{RS} = \frac{\Lambda_H - \Lambda_L}{2}. \tag{A.84}$$

Rewriting the first-order necessary conditions for the optimal positions of a strategic agent as:

$$G_1 = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{a_g}{2 + m_f} \right) - \Lambda_H = 0 \tag{A.85}$$

$$G_2 = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{a_g}{2 + m_f} \right) - \Lambda_L = 0, \tag{A.86}$$

we can then apply the Implicit Function Theorem to derive:

$$\begin{bmatrix} \frac{d\hat{a}_b}{da_g} \\ \frac{d\hat{a}_s}{da_g} \end{bmatrix} = \frac{\frac{\mu}{m_f} q'^*}{2 + m_f} \begin{bmatrix} \frac{\mu}{m_f} q^{*\prime} + h_b + \gamma \Lambda_H \frac{1}{\hat{a}_b} & h_b \\ h_s & \frac{\mu}{m_f} q^{*\prime} + h_s + \gamma \Lambda_L \frac{1}{2\bar{y} + \hat{a}_s} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{A.87}$$

where, substituting with Equations (A.82) and (A.83):

$$h_b = \frac{1}{m_f} q^{*\prime} + \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} \left(\hat{a}_b - a_g \right) + \gamma \Lambda_H \frac{\frac{1}{m_f} q^{*\prime} \left(\hat{a}_b + \hat{a}_s \right) + q^*}{w - q^* \left(\hat{a}_b + \hat{a}_s \right)}, \tag{A.88}$$

and

$$h_s = \frac{1}{m_f} q^{*\prime} + \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} \left(\hat{a}_s - a_g \right) + \gamma \Lambda_L \frac{\frac{1}{m_f} q^{*\prime} \left(\hat{a}_b + \hat{a}_s \right) + q^*}{w - q^* \left(\hat{a}_b + \hat{a}_s \right)}. \tag{A.89}$$

Let Δ be the determinant of the matrix in Equation (A.87). It then follows from Equation (A.87) that:

$$\begin{bmatrix} \frac{d\hat{a}_b}{da_g} \\ \frac{d\hat{a}_s}{da_g} \end{bmatrix} = \frac{\frac{\mu}{m_f} q'^*}{2 + m_f} \frac{1}{\Delta} \begin{bmatrix} \frac{\mu}{m_f} q^{*\prime} + h_s - h_b + \gamma \Lambda_L \frac{1}{2\bar{y} + \hat{a}_s} \\ \frac{\mu}{m_f} q^{*\prime} + h_b - h_s + \gamma \Lambda_H \frac{1}{\hat{a}_b} \end{bmatrix}, \tag{A.90}$$

where:

$$h_b - h_s = \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} \left(\hat{a}_b - \hat{a}_s \right) + \gamma \left(\Lambda_H - \Lambda_L \right) \frac{\frac{1}{m_f} q^{*\prime} \left(\hat{a}_b + \hat{a}_s \right) + q^*}{w - q^* \left(\hat{a}_b + \hat{a}_s \right)} > 0, \tag{A.91}$$

because $q^{*\prime\prime} > 0$ since CRRA preferences have convex marginal utility.

Recognizing that $\frac{d\hat{a}_s}{da_g} > 0$ from our analysis of the first-order condition, Equation

(A.82), and that $\frac{\mu}{m_f}q^{*\prime} + h_b - h_s + \gamma\Lambda_H \frac{1}{\hat{a}_b} > 0$, we conclude that $\Delta > 0$.

It is then immediate from Equation (A.106):

$$\frac{d}{da_g}\left(\hat{a}_b + \hat{a}_s\right) = \frac{\frac{\mu}{m_f}q'^{*}}{2+m_f}\frac{1}{\Delta}\left(2\frac{\mu}{m_f}q^{*\prime} + \gamma\Lambda_L\frac{1}{2\bar{y}+\hat{a}_s} + \gamma\Lambda_H\frac{1}{\hat{a}_b}\right) > 0. \tag{A.92}$$

Differentiating $\Lambda_H - \Lambda_L$ with respect to a_g, and recognizing for CRRA preferences, $\frac{-u''(x)x}{u'(x)} = \gamma$, we find:

$$\frac{1}{\gamma}\frac{d\left(\Lambda_H - \Lambda_L\right)}{da_g} = \Lambda_L\frac{d}{da_g}\log\left(\frac{2\bar{y}+\hat{a}_s}{w - q^{*}\left(\hat{a}_b + \hat{a}_s\right)}\right) - \Lambda_H\frac{d}{da_g}\log\left(\frac{\hat{a}_b}{w - q^{*}\left(\hat{a}_b + \hat{a}_s\right)}\right), \tag{A.93}$$

which we can expand with Equations (A.91) and (A.92), and recognizing that $q^{*\prime} = \frac{\gamma q^{*}}{\frac{\bar{y}}{w} - \frac{1}{m_f}\left(\hat{a}_b + \hat{a}_s\right)}$, as:

$$\begin{aligned}\frac{\frac{d(\Lambda_H - \Lambda_L)}{da_g}}{\left(\frac{1}{m_f}q'^{*}\right)^2\frac{1}{2+m_f}\frac{\gamma}{\Delta}} &= \left(\frac{\Lambda_L}{2\bar{y}+\hat{a}_s} + \frac{\Lambda_H}{\hat{a}_b}\right)\frac{1}{m_f}\frac{q^{*\prime\prime}}{q^{*\prime}}\left(\hat{a}_b - \hat{a}_s\right) - \left(\frac{\Lambda_H}{\hat{a}_b} - \frac{\Lambda_L}{2\bar{y}+\hat{a}_s}\right) \\ &\quad -2\left(\Lambda_H - \Lambda_L\right)\frac{\frac{1}{m_f}q^{*\prime}\left(\hat{a}_b + \hat{a}_s\right) + q^{*}}{w - q^{*}\left(\hat{a}_b + \hat{a}_s\right)}.\end{aligned} \tag{A.94}$$

Since the competitive fringe has CRRA preferences, we have that $\frac{q^{*\prime\prime}}{q^{*\prime}}\left(\frac{\bar{y}}{w} - \frac{1}{m_f}\left(\hat{a}_b + \hat{a}_s\right)\right) = 1 + \gamma$. It then follows from Equation (A.94) that:

$$\begin{aligned}\frac{\frac{d(\Lambda_H - \Lambda_L)}{da_g}}{\frac{\left(\frac{1}{m_f}q'^{*}\right)^2\frac{1}{2+m_f}\frac{\gamma}{\Delta}}{\frac{\bar{y}}{w} - \frac{1}{m_f}\left(\hat{a}_b + \hat{a}_s\right)}} &\geq \frac{\Lambda_L}{2\bar{y}+\hat{a}_s}\left(\left(1+\gamma\right)\frac{1}{m_f}\left(\hat{a}_b - \hat{a}_s\right) + \gamma\frac{\bar{y}}{w}\frac{q\left(2\bar{y}+\hat{a}_s\right)}{w - q^{*}\left(\hat{a}_b + \hat{a}_s\right)}\right) \\ &\quad + \frac{\Lambda_H}{\hat{a}_b}\left(-\frac{\gamma}{m_f}\hat{a}_s + \left(\frac{2+\gamma}{m_f} - \frac{\gamma\frac{\bar{y}}{w}q^{*}}{w - q^{*}\left(\hat{a}_b + \hat{a}_s\right)}\right)\hat{a}_b - \frac{\bar{y}}{w}\right).\end{aligned} \tag{A.95}$$

The first term in Equation (A.95) is positive and it follows that if $\frac{\gamma}{m_f}$ is sufficiently large, while $\frac{\mu}{m_f}$ is held fixed, then the second term is nonnegative. If $\frac{\gamma}{m_f}$ is sufficiently large, then from Equation (A.95):

$$\frac{d}{da_g}\Gamma_{RS}(z) = \frac{d}{da_g}\left(\frac{\Lambda_H - \Lambda_L}{2}\right) \;\geq\; 0, \tag{A.96}$$

state-by-state.

Example 2

In what follows, we establish a sufficient condition under which our intertemporal smoothing wedge, Γ_{IS}, is increasing in the government's position in the risk-free asset, a_g, in Example 2.

With CRRA preferences, which are homothetic, let us define the state price of the seller in an asset market as:

$$\Lambda_L = \frac{1}{2}u'\left(\frac{2y + \hat{a}_s}{-q^* \hat{a}_s}\right), \tag{A.97}$$

and that of the buyer:

$$\Lambda_H = \frac{1}{2}u'\left(\frac{\hat{a}_b}{2y - q^* \hat{a}_b}\right). \tag{A.98}$$

In the two-agent pure intertemporal smoothing economy, our intertemporal smoothing wedge, Γ_{IS}, is identical for both states and simplifies to:

$$\Gamma_{IS} = \frac{\Lambda_H - \Lambda_L}{2}. \tag{A.99}$$

From the first-order necessary conditions for the optimal positions of a strategic agent, Equations (28) and (27), we recognize that:

$$\Lambda_H - \Lambda_L = \frac{\mu}{m_f} q^{*\prime} \left(\hat{a}_b - \hat{a}_s\right). \tag{A.100}$$

Rewriting the first-order necessary conditions for the optimal positions of a strategic agent as:

$$G_1 = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{a_g}{2+m_f} \right) - \Lambda_H = 0 \tag{A.101}$$

$$G_2 = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{a_g}{2+m_f} \right) - \Lambda_L = 0, \tag{A.102}$$

we can then apply the Implicit Function Theorem to derive:

$$\begin{bmatrix} \frac{d\hat{a}_b}{da_g} \\ \frac{d\hat{a}_s}{da_g} \end{bmatrix} = \frac{\frac{\mu}{m_f} q'^*}{2+m_f} \begin{bmatrix} \frac{\mu}{m_f} q^{*\prime} + h_b + \gamma \Lambda_H \frac{1}{\hat{a}_b} + \gamma \Lambda_H \frac{q^*}{2y - q^* \hat{a}_b} & h_b \\ h_s & \frac{\mu}{m_f} q^{*\prime} + h_s + \gamma \Lambda_L \frac{1}{2\bar{y} + \hat{a}_s} - \gamma \Lambda_L \frac{1}{\hat{a}_s} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{A.103}$$

where, substituting with Equations (28) and (27):

$$h_b = \frac{\mu}{m_f} q^{*\prime} + \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} \left(\hat{a}_b - a_g \right) + \gamma \Lambda_H \frac{\frac{1}{m_f} q^{*\prime} \hat{a}_b}{2y - q^* \hat{a}_b}, \tag{A.104}$$

and

$$h_s = \frac{\mu}{m_f} q^{*\prime} + \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} \left(\hat{a}_s - a_g \right) - \gamma \Lambda_L \frac{\frac{1}{m_f} q^{*\prime} \hat{a}_s}{q^* \hat{a}_s}. \tag{A.105}$$

Let Δ be the determinant of the matrix in Equation (A.103). It then follows from Equation (A.103) that:

$$\begin{bmatrix} \frac{d\hat{a}_b}{da_g} \\ \frac{d\hat{a}_s}{da_g} \end{bmatrix} = \frac{\frac{\mu}{m_f} q'^*}{2+m_f} \frac{1}{\Delta} \begin{bmatrix} \frac{\mu}{m_f} q^{*\prime} + h_s - h_b + \gamma \Lambda_L \frac{1}{2y + \hat{a}_s} - \gamma \Lambda_L \frac{1}{\hat{a}_s} \\ \frac{\mu}{m_f} q^{*\prime} + h_b - h_s + \gamma \Lambda_H \frac{1}{\hat{a}_b} + \gamma \Lambda_H \frac{q^*}{2y - q^* \hat{a}_b} \end{bmatrix}, \tag{A.106}$$

where:

$$h_b - h_s = \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} \left(\hat{a}_b - \hat{a}_s \right) + \gamma \Lambda_H \frac{\frac{1}{m_f} q^{*\prime} \hat{a}_b}{2y - q^* \hat{a}_b} + \gamma \Lambda_L \frac{1}{m_f} \frac{q^{*\prime}}{q^*}, \tag{A.107}$$

where $q^{*\prime\prime} > 0$ since CRRA preferences have convex marginal utility. It follows:

$$\frac{\mu}{m_f} q^{*\prime} + h_b - h_s + \frac{\gamma \Lambda_H}{\hat{a}_b} + \frac{\gamma \Lambda_H q^*}{2y - q^* \hat{a}_b} = \frac{\mu}{m_f} q^{*\prime} + \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} (\hat{a}_b - \hat{a}_s) + \gamma \Lambda_H \frac{1}{\hat{a}_b} + \gamma \Lambda_H \frac{\frac{1}{m_f} q^{*\prime} \hat{a}_b + q^*}{2y - q^* \hat{a}_b} + \frac{\gamma \Lambda_L}{m_f} \frac{q^{*\prime}}{q^*} > 0. \quad \text{(A.108)}$$

Recognizing that $\frac{d\hat{a}_s}{da_g} > 0$ from our analysis of the first-order condition, Equation (27), and that $\frac{\mu}{m_f} q^{*\prime} + h_b - h_s + \frac{\gamma \Lambda_H}{\hat{a}_b} + \frac{\gamma \Lambda_H q^*}{2y - q^* \hat{a}_b} > 0$, we conclude that $\Delta > 0$.

Further, we have that:

$$\begin{aligned} \frac{d}{da_g} (\hat{a}_b + \hat{a}_s) &= \frac{\frac{\mu}{m_f} q'^*}{2 + m_f} \frac{1}{\Delta} \left(2 \frac{\mu}{m_f} q^{*\prime} - \gamma \Lambda_L \frac{1}{\hat{a}_s} \frac{2y}{2y + \hat{a}_s} + \gamma \Lambda_H \frac{1}{\hat{a}_b} \frac{2y}{2y - q^* \hat{a}_b} \right), \\ \frac{d}{da_g} (\hat{a}_b - \hat{a}_s) &= \frac{\frac{\mu}{m_f} q'^*}{2 + m_f} \frac{1}{\Delta} \left(2h_s - 2h_b - \gamma \Lambda_L \frac{1}{\hat{a}_s} \frac{2y}{2y + \hat{a}_s} - \gamma \Lambda_H \frac{1}{\hat{a}_b} \frac{2y}{2y - q^* \hat{a}_b} \right) \end{aligned} \quad \text{(A.109)}$$

Differentiating $\Lambda_H - \Lambda_L$ from Equation (A.100) with respect to a_g, we find:

$$\frac{d(\Lambda_H - \Lambda_L)}{da_g} = \frac{\mu}{m_f} \frac{1}{m_f} q^{*\prime\prime} (\hat{a}_b - \hat{a}_s) \frac{d(\hat{a}_b + \hat{a}_s)}{da_g} + \frac{\mu}{m_f} q^{*\prime} \frac{d(\hat{a}_b - \hat{a}_s)}{da_g}, \quad \text{(A.110)}$$

which we can expand with Equations (A.109) and (A.109) as:

$$\frac{\frac{d(\Lambda_H - \Lambda_L)}{da_g}}{\left(\frac{\mu}{m_f} q'^*\right)^2 \frac{1}{2 + m_f} \frac{\gamma}{\Delta}} = -\left(1 + \frac{1}{m_f} \frac{q^{*\prime\prime}}{q^{*\prime}} (\hat{a}_b - \hat{a}_s) + \hat{a}_s \frac{2y + \hat{a}_s}{y} \frac{1}{m_f} \frac{q^{*\prime}}{q^*}\right) \frac{\Lambda_L}{\hat{a}_s} \frac{2y}{2y + \hat{a}_s} + \left(\frac{1}{m_f} \frac{q^{*\prime\prime}}{q^{*\prime}} (\hat{a}_b - \hat{a}_s) - 1 - \frac{\hat{a}_b}{y} \frac{1}{m_f} q^{*\prime} \hat{a}_b\right) \frac{\Lambda_H}{\hat{a}_b} \frac{2y}{2y - q^* \hat{a}_b}. \quad \text{(A.111)}$$

Since the competitive fringe has CRRA preferences, we have that $\frac{q^{*\prime\prime}}{q^{*\prime}} \left(1 - \frac{1}{m_f} (\hat{a}_b + \hat{a}_s)\right) =$

$1+\gamma$. Equation (A.111) can be expressed as:

$$\frac{\frac{d(\Lambda_H-\Lambda_L)}{da_g}}{\left(\frac{\mu}{m_f}q'^*\right)^2\frac{1}{2+m_f}\frac{\gamma}{\Delta}} = -\frac{1+\gamma\frac{1}{m_f}\hat{a}_b+\gamma\frac{1}{y}\frac{1}{m_f}\hat{a}_s^2-(2-\gamma)\frac{1}{m_f}\hat{a}_s}{1-\frac{1}{m_f}(\hat{a}_b+\hat{a}_s)}\frac{\Lambda_L}{\hat{a}_s}\frac{2y}{2y+\hat{a}_s}$$
$$+\frac{2\frac{1}{m_f}\hat{a}_b+\gamma\frac{1}{m_f}(\hat{a}_b-\hat{a}_s)-\gamma\frac{1}{y}\frac{1}{m_f}q^*\hat{a}_b^2-1}{1-\frac{1}{m_f}(\hat{a}_b+\hat{a}_s)}\frac{\Lambda_H}{\hat{a}_b}\frac{2y}{2y-q^*\hat{a}_b} \tag{A.112}$$

Notice that in the competitive equilibrium, $q^*\hat{a}_b = y$, and it is less than y otherwise unless government asset sales can achieve the first-best. If $\gamma \in (0,2]$, then $-(2-\gamma)\frac{1}{m_f}\hat{a}_s \geq 0$, and it follows from Equation (A.112) that:

$$\frac{\frac{d(\Lambda_H-\Lambda_L)}{da_g}}{\left(\frac{\mu}{m_f}q'^*\right)^2\frac{1}{2+m_f}\frac{\gamma}{\Delta}} \geq \frac{1}{m_f}\frac{2\hat{a}_b-\gamma\hat{a}_s-m_f}{1-\frac{1}{m_f}(\hat{a}_b+\hat{a}_s)}\frac{\Lambda_H}{\hat{a}_b}\frac{2y}{2y-q^*\hat{a}_b}. \tag{A.113}$$

Finally, if m_f is sufficiently small, holding fixed $\frac{\mu}{m_f}$, then $2\hat{a}_b-\gamma\hat{a}_s$, which is bounded between 0 and $2(2+\gamma)y$), is arbitrarily larger than m_f. Consequently, in this case, we arrive from Equation (A.113) at:

$$\frac{\frac{d(\Lambda_H-\Lambda_L)}{da_g}}{\left(\frac{\mu}{m_f}q'^*\right)^2\frac{1}{2+m_f}\frac{\gamma}{\Delta}} \geq \frac{1}{m_f}\frac{2\hat{a}_b-\gamma\hat{a}_s}{1-\frac{1}{m_f}(\hat{a}_b+\hat{a}_s)}\frac{\Lambda_H}{\hat{a}_b}\frac{2y}{2y-q^*\hat{a}_b} \geq 0. \tag{A.114}$$

It is therefore sufficient, although not necessary, that $\gamma \in (0,2]$ and m_f is sufficiently small (holding fixed $\frac{\mu}{m_f}$) for $\Lambda_H - \Lambda_L$ and consequently Γ_{IS} to be increasing in a_g.

Appendix D: Eurozone Example Calibration

In this Appendix, we describe how we calibrate the Eurozone example in Section 6. There are three groups of strategic agents, insurance companies and pension funds, banks and corporations, and mutual funds and hedge funds, and we examine the strategic limit from Definition 2. The endowments and portfolios of the three types of investors are described in detail in Section 6.

We set $\bar{y} = 10$, $\kappa = 1$, and calibrate the remaining parameters $\{\gamma, k_1, k_2, k_3, \Delta, a_g\}$ as follows. With time-separable utility the coefficient of relative risk aversion γ also determines the elasticity of inter-temporal substitution. Given that inter-temporal smoothing is the key source of gains from trade among the institutions we study, we target γ to match the risk-free rate r_f with the Euro area 10 Years Government Bencnhmark Bond Yield in March 2014 of 2.8%.[12]. We target k_1 , k_2, and k_3 to match the durations of government bond holdings of IPCFs, banks and corporations, and mutual funds from Table 14 of Koijen, Koulischer, Nguyen, and Yogo (2021). We weight the durations of each group across vulnerable and non-vulnerable countries by the size of their holdings to arrive at values of 8.94 years, 4.62 years, and 6.92 years, respectively.[13] We measure duration D in our model using Macaulay's Duration for strategic agent i portfolio, D_i, based on the fraction of present-value consumption derived at each date

$$D_i = \frac{c_{1i}}{w_i + \sum_{z=1}^{2} q\left(z\right) y_i\left(z\right) - \tau_1 - \frac{1}{r_f}\tau_2} + 10 \frac{\sum_{z=1}^{2} q\left(z\right) c_{2i}\left(z\right)}{w_i + \sum_{z=1}^{2} q\left(z\right) y_i\left(z\right) - \tau_1 - \frac{1}{r_f}\tau_2}. \tag{A.115}$$

We target Δ to match the mean demand elasticity for risk-free bonds of ICPFs from Table 13 of Koijen, Koulischer, Nguyen, and Yogo (2021) of -4.04. Defining $\hat{a}_1(rf) = \min_{\{z \in \{1,2\}} a_i\left(z\right)$ to be agent i's holding of risk-free bonds, we can calculate this demand elasticity as $-\frac{d \log|\hat{a}_1(rf)|}{d \log\left(1/r_f\right)}$. Finally, we target the initial size of government trading a_g such that a 26% purchase of outstanding government bonds (the effective size of the Eurozone's asset purchases from 2014-2017) reduces the ten-year yield by 65 bp based on the calculations of Koijen, Koulischer, Nguyen, and Yogo (2021).

Table A.1 in the main paper reports the parameters that we recover from estimating our model using the simulated method-of-moments approach, and Table A.2 compares the calibrated moments that we target in our model with their empirical counterparts. It is worth emphasizing that it is very difficult to match not only asset pricing moments (i.e., the risk-free rate and yield changes), but also portfolio characteristics (i.e., duration and demand elasticities) that are typically ignored in models of strategic trading. Of note is that our calibrated model estimates very realistic demand elasticities for strategic agents

[12]See `https://data.ecb.europa.eu/data/datasets/FM/FM.M.U2.EUR.4F.BB.U2_10Y.YLD`.

[13]Specifically, ICPFs have an average duration of (1284 * 9.8 + 493 * 6.7) / (1284 + 493) = 8.94 years. Banks have an average duration of (1346 * 5.0 + 963 * 4.1)/ (1346 + 963) = 4.62 years. Mutual funds, which include hedge funds, have an average duration of (895 * 7.6 + 333 * 5.1) / (895 + 333) = 6.92 years.

Parameter	Interpretation	Value
$\bar{y}$	Average Endowment	10.000
κ	Relative Size of Strategic Agents	1.000
k_1	Fraction of Bank/Corporate Endowment at Date 2	0.2973
k_2	Fraction of ICPF Endowment at Date 2	0.6198
k_3	Fraction of Mutual/Hedge Fund Endowment at Date 2	0.6703
Δ	Distributional Endowment Risk (% of $\bar{y}$)	0.0109
γ, γ_f	Agent Risk Aversion	0.3117
a_g	Government Initial Asset Position	-0.4258

Table A.1: Parameter choices for the baseline calibration.

compared to the perfect competition benchmark in which elasticities are (locally) infinite. Although we target only the demand elasticity for risk-free debt of ICPFs, those of banks/corporates and mutual/hedge funds in our model are also relatively low at 6.80 and 23.83 (compared to 2.08 and 2.93 in Koijen, Koulischer, Nguyen, and Yogo (2021)), respectively, with mutual and hedge funds sensibly being the most elastic. In addition, price impact in risk-free bonds, i.e., $\kappa\frac{q'}{q}$ for bond price q, is 0.30. While the calibrated coefficient of relative risk aversion is low, we view this as indicating the inter-temporal smoothing is particular important for the institutions we study, so that our preferred interpretation is in the context of an elasticity of inter-temporal substitution.

Moment	Data	Model
Ten-year Risk-free Rate	1.29%	1.43%
Bank / Corporate Duration	4.62	4.21
ICPF Duration	8.94	6.21
Mutual/Hedge Fund Duration	6.92	6.54
ICPF Demand Elasticity	-4.04	-3.86
Asset Purchase Yield Response	65bp	64bp

Table A.2: Model vs empirical moments for the parameters given in Table A.1.