# Demand-System Asset Pricing: Theoretical Foundations

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2. Investors may have "tastes" over assets (rather than cash flows). ESG, dogmatic beliefs, latent constraints.

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**We study the foundations and interpretation of this methodology.**

## A brief history of thought

1. "Neo-classical asset pricing:" focus on relative prices, quantities mostly irrelevant.

2. Demand effects a la index inclusion: a notion of an *aggregate* demand curve.
3. High-frequency identification of monetary policy or QE shocks.
4. Intermediary asset pricing: some specific quantities and portfolios matter.

5. Asset demand systems: **structurally estimate** individual- and asset-level demand on portfolio data.

# Existing methods hew closely to IO, but financial assets present unique challenges

1. Central role of *portfolios* and relative prices rather than asset-specific demand functions.
2. Cross-asset price spillovers through *general equilibrium price determination*.
3. *Resale* considerations: current demand depends on future prices.

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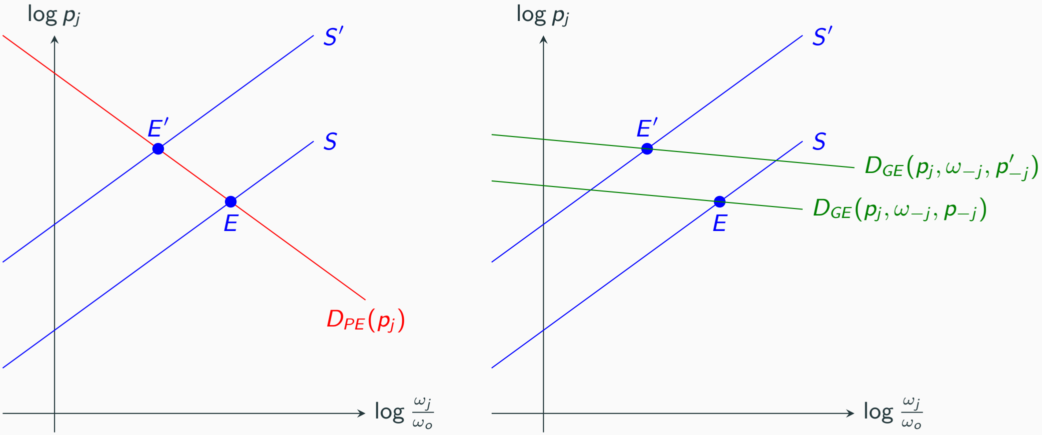
2. Cross-asset portfolio linkages and price spillovers can heavily bias measured elasticities.
    - In current frameworks, measured elasticities may be near zero even if true elasticities are near infinite.
    - Control variables do not address this issue.

3. Elasticities should be interpreted as structural objects only under strong additional assumptions.
    - Identification challenges: preferences versus latent constraints; Keynesian beauty contests.

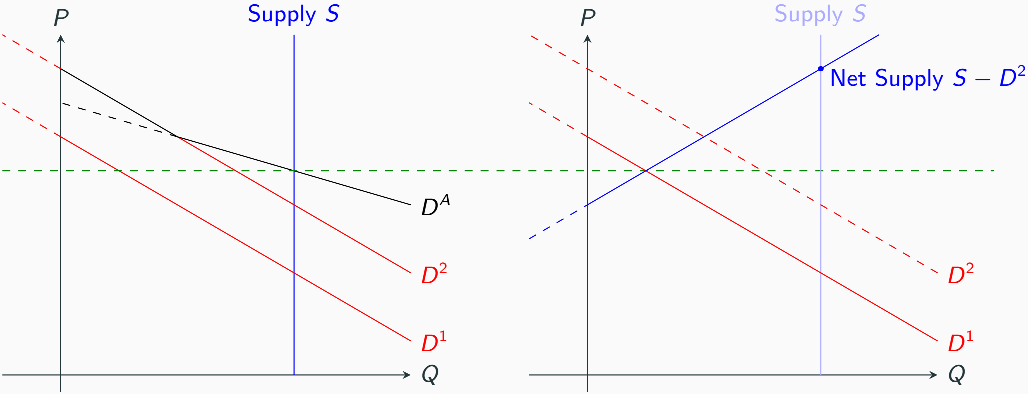
# Summary graph




**(a)** Logit demand **(b)** Demand with cross-asset complementarities

# Framework: Portfolio choice with tastes

# Residual supply shocks



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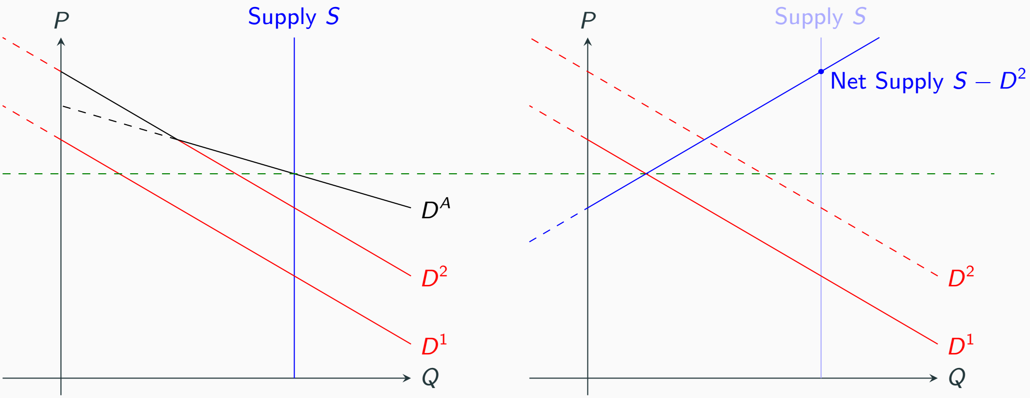


- Identification requires heterogeneous **tastes** which do not affect others' valuations.

  (i) non-pecuniary preferences over characteristics, (ii) dogmatic beliefs, (iii) latent constraints.

# Incorporating demand effects in a canonical framework

A one-shot portfolio choice problem:

- Investor $i$ can choose consumption at date 0 or at date 1.
- State $z \in \mathcal{Z} \equiv \{1, \ldots, Z\}$ with probability $\pi_z \in (0, 1)$.
- Assets $\mathcal{J} \equiv \{1, \ldots, J\}$ with price $p_j$ and state-contingent cash flows $y_j(z)$.
- A portfolio of assets $(a_j^i)_{j \in \mathcal{J}}$.
- Investors receive asset endowments $e_j^i$ and non-marketable endowment $w_0^i$ and $w_1^i(z)$.

# Two specifications for tastes

1. **Payoff-augmenting:** Investor $i$ evaluates asset $j$'s payoff $y_j(z)$ as $\theta^i_j y_j(z)$.

$$\tilde{c}^i_1(z) \equiv \sum_j \theta^i_j y_j(z) a^i_j + w^i_1(z).$$

   - If we let $\theta^i_j = 1$ for all $j$ we recover the standard model.
   - Close connection to dogmatic belief over the scale of the payoff.

2. **Utility-augmenting:** Define a taste function $G(\cdot)$. Utility from portfolio $a^i$ is

$$U^i(c(a^i)) + G^i(a^i).$$

   - If we let $G^i(a^i) = 0$ we recover the standard model.

# Embedding payoff-augmenting tastes into portfolio choice

Our approach leads to a simple generalization of the standard problem:

$$\max_{(a_1^i, a_2^i, \dots, a_J^i)} \quad (1-\delta)u(c_0^i) + \delta \sum_z \pi_z u\Big(\tilde{c}^i(z)\Big) + \text{continuation value}$$

$$\text{s.t.} \quad \tilde{c}_1^i(z) \equiv \sum_j \theta_j^i y_j(z) a_j^i + w_1^i(z),$$

$$c_0^i + \sum p_j (a_j^i - e_j^i) = w_0^i,$$

ad-hoc portfolio restrictions or mandates.

We model tastes over **payoffs**, not returns. **This allows for endogenous return spillovers.**

# Basics: no arbitrage

# No arbitrage in asset demand systems

- It is infeasible to estimate demand systems for the universe of stocks.
- First step based on APT: projection onto a small set of characteristics or risk-factors.
- But: no arbitrage may not hold under heterogeneous tastes.

# No arbitrage with tastes

**"Theorem":** For sufficiently rich security menus and heterogeneous tastes, there do not exist price systems that preclude arbitrage for all investors.

- NA: price should preclude investments that offer "something for nothing."
- The point of tastes is that investors disagree about "something."

# Identifying asset-level demand elasticities

## What is an asset-level demand function?

Portfolio choice models generate predictions for quantities to be bought of any given asset, say

$$a^i_j(p, a^i_{-j}).$$

In standard models, this generically depends on other endogenously chosen asset quantities and prices.

Asset-level demand can be summarized using a **demand elasticity**. This is a **partial derivative**:

$$\mathcal{E}^i_{js} \equiv -\frac{\partial a^i_j(p, a^i_{-j})}{\partial p_s} \frac{p_s}{a^i_j(p, a^i_{-j})}.$$

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**Identification challenge:** observational data show the **total derivative** for all margins of adjustment:

$$\hat{\mathcal{E}}_{ss}^i \equiv -\frac{\frac{da_j^i}{d\chi_s}}{\frac{dp_s}{d\chi_s}} \cdot \frac{p_s}{a_j^i}.$$

# Deriving demand functions

The first-order necessary condition for the optimal choice of with respect to $a_j^i$ is:

$$\underbrace{\theta_j^i \sum_{z \in \mathcal{Z}} y_j(z) \frac{u^{i'}(\tilde{c}^i(z))}{u^{i'}(c_0^i)} \quad + \quad \texttt{Lagrange multipliers} \quad - \quad p_j}_{\equiv \text{ Net marginal value } F_j^i(a^i,\, p)} = 0.$$

# Deriving demand functions

Asset-level demand functions are jointly determined by a **system of equations** (+ constraints):

$$\begin{bmatrix} F_1^i(a^i, p) \\ F_2^i(a^i, p) \\ \vdots \\ F_J^i(a^i, p) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

This system typically exhibits **demand complementarities**: marginal value of asset $j$ depends on $a^i_{-j}$.

1. Diversification: marginal value depends on covariance with portfolio.
2. Constraints: investment mandates which allow substitution between some assets.

# Response to an exogenous supply shock $\chi_s$ to asset $s$.

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Total derivative of $i$'s portfolio in response to the shock is:

$$\begin{bmatrix} \frac{da_1^i}{d\chi_s} \\ \vdots \\ \frac{da_s^i}{d\chi_s} \\ \vdots \\ \frac{da_J^i}{d\chi_s} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial a_1^i}{\partial p_1} & \cdots & \frac{\partial a_1^i}{\partial p_s} & \cdots & \frac{\partial a_1^i}{\partial p_J} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial a_s^i}{\partial p_1} & \cdots & \frac{\partial a_s^i}{\partial p_s} & \cdots & \frac{\partial a_s^i}{\partial p_J} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial a_J^i}{\partial p_1} & \cdots & \frac{\partial a_J^i}{\partial p_s} & \cdots & \frac{\partial a_J^i}{\partial p_J} \end{bmatrix}}_{\text{Quantity responses}} \underbrace{\begin{bmatrix} \frac{dp_1}{d\chi_s} \\ \vdots \\ \frac{dp_s}{d\chi_s} \\ \vdots \\ \frac{dp_J}{d\chi_s} \end{bmatrix}}_{\text{Price spillovers}} + \underbrace{\begin{bmatrix} \frac{\partial a_1^i}{\partial \chi_s} \\ \vdots \\ \frac{\partial a_s^i}{\partial \chi_s} \\ \vdots \\ \frac{\partial a_J^i}{\partial \chi_s} \end{bmatrix}}_{=\text{Income effects}} .$$

Structural elasticities obscured by cross-asset interactions and equilibrium price spillovers $\mathcal{S}_{js} \equiv \frac{dp_j}{d\chi_s}$.

# Observed versus structural elasticity

**Proposition:** The observed own-price elasticity $\hat{\mathcal{E}}^i_{ss}$ can be decomposed as follows:

$$\hat{\mathcal{E}}^i_{ss} = \mathcal{E}^i_{ss} - \underbrace{\sum_{j \neq s} \frac{\mathcal{S}_{js} \frac{1}{p_j}}{\frac{dp_s}{d\chi_s} \frac{1}{p_s}} \mathcal{E}^i_{sj}}_{\text{Complementarities}} - \underbrace{\frac{\frac{\partial a^i_s}{\partial \chi_s} \frac{1}{a^i_s}}{\frac{dp_s}{d\chi_s} \frac{1}{p_s}}}_{\text{Income effects}} . \tag{1}$$

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**Intuition in a simple example.** Investor chooses between bond and two similar stocks.

1. Holding other price fixed, a price change triggers rapid reallocation to other stock ($\mathcal{E}^i_{sj}$ is large).
2. If many investors attempt to do this, other price must adjust.
3. Once prices adjust, there is no need to adjust your portfolio.

# How to isolate the structural elasticity from the observed one?

Generic answer: one must place restrictions on the substitution matrix.

Canonical IO: discrete choice over goods with homogeneous substitution to an "outside good."

This is the so-called Independence of Irrelevant Alternatives (IIA) property of logit demand.

KY use a similar system: "logit" demand for financial assets conditional on characteristics.

1. Impose assumptions on *returns* to sharply reduce scope for cross-asset spillovers.
2. Yields homogeneous substitution in units of portfolio weights relative to an outside asset.

   Nested logit is more flexible. Does not directly tackle spillovers and requires additional structure.

# The KY logit demand framework

Assume **equilibrium returns** follow a factor structure with diagonal conditional covariance matrix.

Demand in units of relative portfolio weights assumed to depend only on own prices and characteristics:

$$\frac{\omega_j(p)}{\omega_{out}(p)} = \frac{\omega_j}{\omega_{out}}(p_j) = \exp\left\{\beta_0 \log p_j + \sum_{k=1}^{K-1} \beta_k x_k(j) + \beta_K\right\}\theta(j),$$

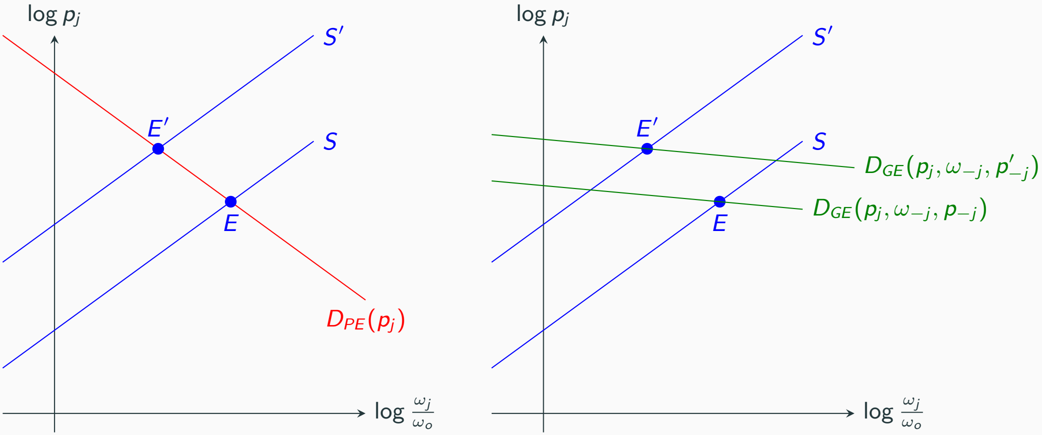
**Identification:** if $x_k(j)$'s are invariant to supply shocks, observed and structural elasticity are **identical**.

$\Rightarrow$ parameter $\beta_0$ is identified from *observed* portfolio changes given price shocks.

$\Rightarrow$ because demand is separable across assets, need only one price instrument per asset.

**Problem:** Returns and substitution are **endogenous**. Standard models naturally violate separability.

# Back to our graph




**(a)** Logit demand

**(b)** Demand with cross-asset complementarities

# Do equilibrium returns and substitution patterns satisfy the logit structure?

# Our approach

We derive equilibrium portfolio choices alongside returns in a canonical framework.

- In particular, enrich Lucas '78 with payoff-augmenting tastes and mandates.
- As in KY, use log utility for simplicity (but this isn't necessary).

Use this model to compute true and measured elasticities based on the logit structure.

**Result**: logit demand elasticities are (strongly) biased under natural conditions.

# A minimal asset menu

- Two aggregate states, $z = 1, 2$ with prob. $\pi_z = \frac{1}{2}$. For each $z$, one tree that pays 1 in $z$ only.
- Split Tree 1 into **green** and **red** halves with diversifiable risk. Green half pays better in green state.

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- Parameter $\epsilon$ measures complementarity between assets. Tastes can be defined over colors: $(\theta^i_g, \theta^i_r)$.
- Contrast with logit: heterogeneous substitution across assets if and only if $\epsilon < 1$.

# Endowments and demand system implementation

- Tree 2 is the *outside asset* with normalized price $p_2 \equiv 1$. Relative prices $p_r$ and $p_g$.
- Endowments $E_2 = 1$, $E_r = \frac{1}{2}$ and $E_g = \frac{1}{2} + \psi$. Use $\psi$ as an **exogenous supply shock.**
- As in KY, define demand in units of portfolio shares relative to the outside good, $\frac{\omega_j^i(p)}{\omega_2^i(p)}$, so that

$$\mathcal{E}_{jj}^i \equiv -\frac{\partial(\omega_j^i(p)/\omega_2^i(p))}{\partial p_j}\frac{p_j}{\omega_j^i(p)/\omega_2^i(p)} \qquad \text{and} \qquad \hat{\mathcal{E}}_{jj}^i \equiv -\frac{d(\omega_j^i(p)/\omega_2^i(p))}{d p_j}\frac{p_j}{\omega_j^i(p)/\omega_2^i(p)}.$$

- NB: For the basic point, sufficient to assume no constraints or tastes over assets, $\theta_j^i = 1$.

# The logit bias in portfolio shares

Supply variation $\psi$ is a clean instrument for $p_g$: fully exogenous to all investors in the model.

Under the hypothesis of logit demand, **structural elasticity = observed elasticity**, $\mathcal{E}^i_{jj} = \hat{\mathcal{E}}^i_{jj}$.

**Proposition:** Let $\mathcal{B}^i_{jj} \equiv \mathcal{E}^i_{jj} - \hat{\mathcal{E}}^i_{jj}$ denote the "logit bias." This bias is given by

$$\mathcal{B}^i_{jj} = -\underbrace{\frac{\partial\left(\omega^i_j(p)/\omega^i_2(p)\right)}{\partial p_{-j}}}_{\text{Complementarity}} \underbrace{\frac{p_{-j}}{\left(\omega^i_j(p)/\omega^i_2(p)\right)} \frac{p_j}{p_{-j}}}_{\text{Scaling terms}} \underbrace{\frac{dp_{-j}}{dp_j}}_{\text{Price Spillover}} .$$

# Equilibrium demand functions

Contra logit, equilibrium demand functions **depend on both prices** for all $\epsilon < 1$.

$$\frac{\omega_g^i(p_g, p_r)}{\omega_2^i(p_g, p_r)} = \theta_r^i \frac{\pi_1}{\pi_2} p_g \cdot \frac{(\theta_r^i p_g + \theta_g^i p_r)\epsilon^2 - (\theta_r^i p_g - \theta_g^i p_r) + 2\theta_g^i p_r \epsilon(1 - 2\rho)}{(\theta_r^i p_g + \theta_g^i p_r)^2 \epsilon^2 - (\theta_r^i p_g - \theta_g p_r)^2}; \tag{2}$$

$$\frac{\omega_r^i(p_g, p_r)}{\omega_2^i(p_g, p_r)} = \theta_g^i \frac{\pi_1}{\pi_2} p_r \cdot \frac{(\theta_r^i p_g + \theta_g^i p_r)\epsilon^2 + (\theta_r^i p_g - \theta_g^i p_r) - 2\theta_r^i p_g \epsilon(1 - 2\rho)}{(\theta_r^i p_g + \theta_g^i p_r)^2 \epsilon^2 - (\theta_r^i p_g - \theta_g^i p_r)^2}. \tag{3}$$

# Bias between observed and structural elasticity

**Proposition.** For a small shock to green supply $\psi$, the logit bias satisfies

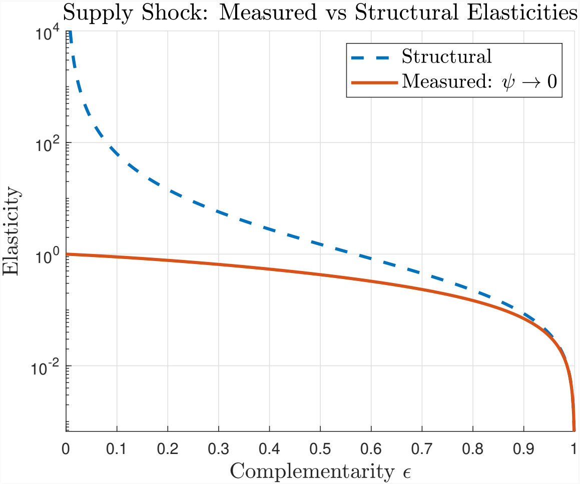
$$\mathcal{B}_{gg} = \frac{(1-\epsilon^2)^2(1+(1-2\rho)\epsilon)}{8\epsilon^2\rho(1-\rho)((1+\epsilon)^2-4\epsilon\rho)}.$$

**The bias is positive, is strictly decreasing in $\epsilon$, goes to infinity as $\epsilon \to 0$, and is zero iff $\epsilon = 1$.**

In particular, in the limit as red and green assets become perfect substitutes, we have

$$\lim_{\epsilon \to 0} \mathcal{E}_{gg} = \infty \qquad \text{and} \qquad \lim_{\epsilon \to 0} \hat{\mathcal{E}}_{gg} = 1$$

# Measured versus structural elasticities (log scale)




Measured is *always* low; true is low when $\epsilon \to 1$ $\quad (\rho = \frac{1}{4})$.

## Interpretation

- When $\epsilon < 1$, substitution between red and green differs from substitution with Tree 2.

  Hence controlling for outside demand not sufficient to capture actual substitution patterns.

- In settings with spillovers, GE elasticities can be very low even if PE elasticities are very large.

  In particular, the bias is large precisely when reallocation would have been very rapid.

- The fact that Arrow securities work is not an accident. (Currently working on this).

# Control variables

# Controlling for cross-asset spillovers?

One proposed solution to spillover biases is the use of controls, such as common factor exposures.

If spillovers occur mainly among similar assets, we should control for asset similarity.

This changes the unit of analysis to *residual cash flows* (rather than the asset).

Assets might be substitutable precisely because they have common factor exposures.

Problem: residual cash flow elasticities may be uninformative about asset-level elasticities.

NB: we are also ignoring the concern that tastes may violate no arbitrage.

# Decision problem with traded factors

In our setting, the aggregate income in a given state is a factor (there are of course others).

– For example, the green asset has loadings $1 + \epsilon$ and $1 - \epsilon$ on state $g$ and $r$ income.

– To allow non-factor variation, perturb the model with small idiosyncratic noise, $y_j' = y_j + \eta_j$.

Given this, can think of a two-step problem:

1. Choose desired factor exposures (i.e. state-contingent consumption $c_z$) at price $q_z$.
2. Given $c_z$, choose how much idiosyncratic asset exposure $\tilde{w}_j$ to take on at price $\tilde{p}_j$.

Controlling for factor exposures is to focus on the second part: **a conditional decision problem**.

Substitution across assets is driven only by the idiosyncratic component.

# Factor and residual elasticities

Consider a small perturbation, $Var(\eta_j) \approx 0$. Then optimal factor demand in portfolio shares is

$$\frac{c_z q_z}{W} = \pi_z.$$

**The factor-level demand elasticity is zero** (and thus very different from the asset elasticity.)

Fix factor exposures at 1 (as in the baseline model). **The elasticity of residual demand is**

$$\frac{\partial \tilde{\omega}_j^*}{\partial \tilde{p}_j} \frac{\tilde{p}_j}{\tilde{\omega}_j} = 1.$$

Both are small even as the asset-level elasticity can be very high.

# Relative elasticities

## An alternative: measuring relative elasticities

- Haddad et al. (2025): under strong symmetry assumptions, one identify the "relative elasticity."
  i.e., the change in relative quantities over the change in relative prices.

- Clarifies the identification challenge and partially circumvents it using a different estimand.

- Our baseline model satisfies their assumptions! But small perturbations can still lead to biases.
- Our view (maybe wrong) is that the symmetry assumption generically requires controls.

# Are demand elasticities structural?

# Counterfactuals from demand systems

Growing interest in using demand systems for counterfactuals and policy.

- Monetary policy transmission, FX interventions, QE, . . ..

Should demand elasticities be interpreted as "deep" parameters?

- Require invariant parameters to appropriately inform policymakers.

Except in special cases, financial assets are **investment goods**.

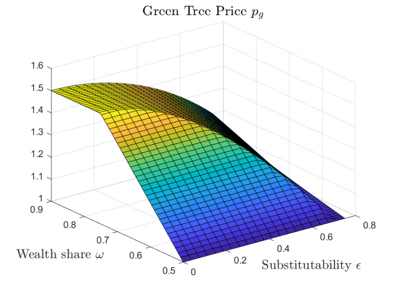
Investor demand depends on both own prefer and expected market returns (i.e., others' tastes).

$\Rightarrow$ Observed demand elasticities alone cannot identify *whose* tastes affect current demand.

But, for many counterfactuals we do need to be able to attribute tastes to investors.

One can generically rationalize a portfolio by constraints **or** preferences.

**Problem**: counterfactuals are generally sensitive to the precise micro-foundation.



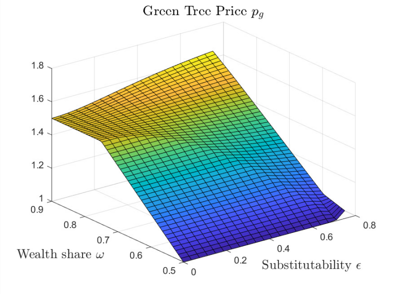




**Figure 3:** Equilibria in an economy with taste differences and one with portfolio restrictions on green asset share.

## Conclusion

We study the methodological foundations of demand-based asset pricing, relying on principles of portfolio choice and equilibrium price determination.

1. Tastes may invalidate the organizing principle of no arbitrage.
2. Price spillovers offer a simple explanation for low *measured* elasticities.
3. Demand elasticities are structural only under stringent restrictions.

. . . Lots of work to be done.