Rules vs. Disclosure: Prudential Regulation and Market Discipline*

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Abstract

Financial regulators can address excessive bank risk-taking through ex-ante rules or by fostering market discipline via disclosure. We study the optimal joint design of these tools and show they are substitutes—targeted state-contingent disclosure reduces the need for stringent ex-ante intervention—yet their combined use lowers the overall regulatory burden. Optimal disclosure is countercyclical, becoming more detailed in adverse states to prevent market freezes. Systemically important institutions require stricter ex-ante oversight yet less ex-post transparency. Methodologically, the paper integrates optimal disclosure with moral hazard, adverse selection, and prudential regulation within a unified framework. (*JEL classification:* G21; G28; D82)

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1 Introduction

One of the central goals of financial regulation is to ensure that banks do not hold excessively risky assets on their balance sheets. This objective can be achieved either by curbing excessive risk-taking at origination, or by enabling banks to transfer risk efficiently to other market participants ex post. In practice, however, adverse selection in asset markets often limits the extent to which banks can offload risk, creating the risk of excessive risk concentration.

To overcome these frictions, regulators employ a broad range of policy instruments to shape banks' incentives and the information environment in financial markets. These include *ex-ante rules and supervision* designed to prevent moral hazard, and *ex-post disclosure*—such as the publication of stress test results—designed to promote market discipline through information. Our goal is to understand how these two regulatory pathways interact in shaping bank incentives and market outcomes, and how they should be jointly designed to achieve prudential objectives. Specifically, we ask: how should a welfare-maximizing regulator optimally combine ex-ante rules and ex-post disclosure? When should regulators rely more on supervision, and when should they rely more on market discipline?

We develop a unified framework that integrates moral hazard and adverse selection with information design and prudential policy. In our model, banks may own low-quality assets either due to moral hazard or because they suffered adverse shocks. These assets can be traded in a secondary market subject to asymmetric information, but markets freeze if adverse selection is severe. There is a role for policy because market participants do not internalize the social benefits from reducing bank risk exposure through asset sales.

We posit that regulators can influence market outcomes through two broad classes of policy instruments. The first class consists of *ex-ante rules and regulation*. In our model, such rules can directly induce banks to work harder in origination, but they have the downside that they cannot respond to shocks. When deciding how to set origination standards, the regulator must therefore balance the costs of upfront effort with the risk of ex-post market shutdowns. When rules are the only available policy tool, the regulator optimally chooses tight *prudential* rules that partially insure against market freezes.

The second class consists of *state-contingent disclosure* of information about realized asset quality. The main benefit of this tool is that it allows the regulator to foster trading by pooling weak and strong banks. Since there is a social benefit to trade, this encourages the regulator to only partially reveal its information. But there is also feedback to incentives. If all banks expect to trade at a common price, there is little benefit to originating good assets. When disclosure is the only tool, the regulator will thus sacrifice some ex-post trade by providing transparency to sustain incentives.

Our main result is that rules and disclosure are *policy substitutes* and, yet, should be used jointly. When designing disclosure, opacity fosters trade but also diminishes incentives. When rules can be used to ensure incentives, regulators can reduce transparency to promote liquidity. Conversely, the ability to release information ex-post reduces the need for tough ex-ante rules. Since effort is costly, a regulatory framework that permits disclosure can thus permit laxer regulation upfront. This offers a clear rationale for both tools within a unified regulatory framework: when regulators must worry about both origination and liquidity, the joint use of disclosure and regulation serves to decouple adverse selection from moral hazard.

Our characterization of optimal policy also shows how the relative emphasis on rules and disclosure should vary with aggregate conditions and the systemic importance of a given bank. In particular, disclosure should be more detailed after adverse shocks, and systemically important institutions should face tighter ex-ante rules but less disclosure. These findings are consistent with the principles of Basel III and the Dodd–Frank Act, which impose heightened supervision on systemically important financial institutions while limiting the extent of public stress test disclosure. They are also consistent with Bird et al. (2023), who document that the Federal Reserve's disclosure of stress test results were more lenient toward more systemically important banks. Importantly, however, these properties need *not* obtain when regulators can only use one tool. In the case of disclosure, for example, regulators who cannot impose rules may provide *more* disclosure in good times to provide incentives. The availability of multiple regulatory tools thus determines the optimal use of each individual instrument.

We deliberately study two dichotomous classes of policy tools in order to capture an essential distinction between *ex-ante mechanisms that deter moral hazard* and *ex-post mechanisms that mitigate adverse selection*. This allows us to highlight common mechanisms underlying diverse policy instruments and to derive general principles for their optimal joint design. It also distinguishes our approach from much of the existing literature, which tends to study individual tools—such as capital requirements or disclosure mandates—in isolation. Importantly, our analysis yields policy and empirical implications that extend beyond the specific instruments we model because other policy tools—such as capital requirements, liquidity ratios, or resolution frameworks—can be interpreted within this broader classification. For example, capital regulation can be viewed as one means of enforcing higher ex-ante effort, while also affecting the social benefit of risk transfer from banks to secondary markets. Hence, our analysis complements, rather than replaces, the study of optimal capital buffers.

Section 5 characterizes optimal regulatory choices from the perspective of self-interested banks. This allows us to distinguish the extent to which the socially-optimal regulation is driven by regulators' ability to enforce commitment versus the need to account for positive externalities from asset sales. We show that banks' preferred regime entails less effort and lower liquidity than what is socially optimal, creating a gap between prudential and self-interested regulation. The difference between bank- and regulator-optimal standards widens as uncertainty rises, reinforcing the need for external supervision in periods of stress.

The rest of the paper is structured as follows. The rest of the Introduction discusses the related literature. Section 2 lays out the model. Section 3 individually studies two policies: information disclosure without regulation (Section 3.1) and regulation without disclosure (Section 3.2). With these in mind, Section 4 studies the optimal joint design of regulation and disclosure. Section 5 discusses banks' optimal regulation. Section 6 provides concluding remarks. The proofs are relegated to Appendix A.

Our paper contributes to the literature applying information design to financial regulation and supervisory disclosure.[1] To our knowledge, we are the first to model the joint design of ex-ante regulation and ex-post disclosure in a framework featuring both ex-ante moral hazard in asset origination and ex-post adverse selection in the secondary asset market. This paper shows that regulatory architecture should be viewed as an integrated system in which rules and market discipline jointly balance ex-ante incentive provision with ex-post transparency.

While prior work shows regulators may optimally obfuscate information to ensure risk-sharing (Goldstein and Leitner, 2018) or prevent bank runs (Bouvard et al., 2015; Faria-e-Castro et al., 2017; Williams, 2017; Moreno and Tuomas, 2025), our model highlights the resulting ex-ante moral hazard created by this pooling. While Bouvard et al. (2015), Williams (2017), and Goldstein and Leitner (2018) find regulators disclose more in bad times, our model highlights a core trade-off: opacity (pooling banks) increases trading opportunities but reduces ex-ante effort.[2] When combined with regulation, the regulator prioritizes the trading effect, and disclosure becomes less informative as the externality rises.

Bouvard et al. (2015) and Parlasca (2024a,b) study disclosure by a privately informed regulator. We differ, as in our model banks' private effort determines systemic soundness, creating an ex-post insurance vs. ex-ante moral hazard trade-off, and we solve for the optimal joint design of regulation and disclosure. Unlike Bouvard et al. (2015), we show ex-ante regulation can alleviate the tension between obfuscation and incentives.[3] Unlike Parlasca (2024a,b), we show test informativeness depends on other available policy instruments.

Williams (2017) finds stress tests and liquidity buffers are policy substitutes as tests reduce run likelihood. Alvarez and Barlevy (2021) argue regulation and mandatory disclosure

[1]See, for instance, Goldstein and Sapra (2013), Hirtle and Lehnert (2015), and Goldstein and Leitner (2022) for surveys on information disclosure through stress testing.

[2]See also Parlatore (2015), Dang et al. (2017), and Monnet and Quintin (2017) for bank opaqueness.

[3]Morrison and White (2013), Shapiro and Skeie (2015), and Shapiro and Zeng (2024) examine a regulator's reputational concerns over forbearance. Rhee and Dogra (2024) find that disclosure commitments can induce a "model monoculture" in bank risk profiles.

can be substitutes. We contribute by providing an optimal joint design of these tools. While we focus on banks' private effort, other work examines banks' private information (Leitner and Yilmaz, 2019; Dai et al., 2024; Quigley and Walther, 2024) or the regulator's optimal learning via stress tests (Parlatore and Philippon, 2023; Ding et al., 2024).

Daley et al. (2020) show that public ratings shift banks from signaling (via asset retention) to an originate-to-distribute model. In their setting, this transparency improves allocative efficiency but weakens screening incentives. We extend this insight to the joint design of prudential rules and disclosure, showing how the information environment impacts both bank incentives and the regulator's optimal balance between supervision and market discipline.

Next, our paper is among the few studying the joint design of prudential policy instruments. While Bhattacharya et al. (2002) study capital regulation with audits and Décamps et al. (2004) examine Basel II (showing market discipline can lower capital regulation), both papers omit the trade-off between ex-post information obfuscation and ex-ante moral hazard.

As Hirtle and Kovner (2022) survey, literature on bank supervision, especially its joint design with regulation, is scarce. While Eisenbach et al. (2016, 2022) and Agarwal and Goel (2024) consider noisy supervision, they focus on resource allocation (Eisenbach et al., 2016, 2022) or misclassification from noisy stress tests (Agarwal and Goel, 2024). Biswas and Koufopoulos (2022) study ex-post moral hazard and ex-ante adverse selection, finding that optimal combinations of disclosure and regulation can improve welfare. Their interaction mechanism differs from ours; in our model, partial disclosure provides ex-post insurance, thereby lowering the required level of regulation. Orlov et al. (2023) analyze sequential stress tests with contingent capital, finding the optimal test involves precautionary recapitalization followed by a more informative test.[4] In contrast to theirs, our model endogenizes the asset quality from a bank's private action, which also interacts with the regulator's policy tools. Faria-e-Castro et al. (2017) link stress testing to a government's fiscal capacity, showing that governments with more capacity conduct more informative tests as they are less worried about bank runs. Their study sheds light on the difference between the stress tests implemented in the United States and in Europe following the Global Financial Crisis.[5]

Finally, we relate to a literature on Bayesian persuasion with moral hazard (Rodina and Farragut, 2018; Boleslavsky and Kim, 2021). The key difference is that our paper focuses on the optimal joint design of an ex-ante regulation and ex-post information disclosure.[6]

[4] Orlov et al. (2023) also allow for correlation in banks' portfolio. See also Gick and Pausch (2012), Huang (2021), Inostroza (2023), Inostroza and Pavan (2023), Leitner and Williams (2023), and Parlatore and Philippon (2023) for various forms of correlation among, for instance, banks or market participants.

[5] Spargoli (2012) studies the trade-off of disclosing negative stress test results: while it prompts banks to replenish capital (reducing default risk), it may also lead to downsizing or require capital injections. Huang et al. (2024) study the policy interaction between deposit insurance and liquidity injections.

[6] Applications beyond banking include schools' grading standards (e.g., Dubey and Geanakoplos, 2010;

2 Model

We consider a prudential regulator who is concerned that banks which hold too many risky assets may suffer financial distress at some future date.[7] To reduce this risk, the regulator would like banks to sell their risky assets to other financial institutions who are less likely to contribute to systemic crises. However, the market for risky assets can break down, creating a role for a regulator to foster liquidity through ex-ante rules and ex-post disclosure.

Specifically, we consider a two-period model in which a bank originates an asset of uncertain quality and may later sell it to a competitive fringe of buyers (the market).[8] Asset quality $q \in \{H, L\}$ (i.e., High or Low) is determined by two components: unobservable bank effort $e \in [0, \frac{1}{2})$ that is exerted in the first period, and an exogenous public shock θ that is realized once effort is sunk. The probability of producing a high-quality asset is

$$\text{Prob}(q = H \mid e) = \theta e.$$

Effort e is costly and cost function $c : [0, \frac{1}{2}) \to [0, \infty)$ is increasing, convex, twice continuously differentiable, and satisfies $c(0) = 0$, $c'(0) = 0$, $\lim_{e \to \frac{1}{2}} c(e) = \infty$, and $\lim_{e \to \frac{1}{2}} c'(e) = \infty$. Two example cost functions which satisfy our assumptions are

$$c(e) = -k(2e + \log(1 - 2e)) \text{ or } c(e) = \frac{ke^2}{1 - 2e}, \text{ where } k > 0.$$

State θ captures economic conditions that influence the future cash flows generated by bank assets. For example, a low realization of θ might reflect a downturn in which bank loans are less likely to be repaid. However, this need not imply that the bank is already in distress. We assume that θ is uniformly drawn from $[1 - \varepsilon, 1 + \varepsilon]$, where $\varepsilon \in (0, 1)$ captures the uncertainty of the environment. Because θ is random, asset quality is uncertain even conditional on effort. The uniform distribution does not play a crucial role and is for ease of analysis. The assumption that effort is bounded by $\frac{1}{2}$ ensures that the probability θe is well-defined (i.e., between 0 and 1).[9]

Boleslavsky and Cotton, 2015; Zubrickas, 2015) and quality certification (e.g., Albano and Lizzeri, 2001; Zapechelnyuk, 2022).

[7] Spatt (2009) this a "financial stability regulator" (as opposed to a "market integrity regulator").

[8] One could interpret the single bank as representing the whole banking system. Our results readily extend to a setting with a continuum of banks. To simplify notation, we consider a single bank.

[9] One could also consider an alternative additive specification where the probability of obtaining a high-quality asset is $e + \theta$ instead of θe. Given some technical modifications so that the conditional probability $e + \theta$ remains well-defined, the only qualitative result affected by this change is the non-monotonicity of optimal disclosure without regulation that is illustrated in the right panel of Figure 3. See footnote 14.

Asset market. Assets can be traded at the beginning of the second period. Assets of quality $q \in \{H, L\}$ have a value v_q for buyers and ρ_q for the seller. There are private gains from trade for high quality assets, but not for low quality assets. This makes it difficult to trade bad assets. Specifically, buyer and seller values satisfy

$$v_H > \rho_H > \rho_L > v_L.$$

There is asymmetric information because the seller is privately informed of the realization of $q \in \{H, L\}$. Given all available information (including potential disclosures by the regulator), buyers form expectations about the quality of the asset and offer a price. Given the highest offer for the asset, which we denote by p, the seller decides to accept or reject.

Externality. We introduce a role for policy by assuming that a regulator would want all assets to ultimately be held by the market rather than the bank. In particular, there is an additional social value g to trading each asset that is not captured by the traders, and it is large enough that the regulator would like to maximize trade:

$$v_L + g > \rho_L.$$

This payoff structure captures the idea that the regulator is worried about the risk of bank failure and cannot commit to not bailing out banks in the event of distress. However, if the risky assets (or non-performing loans) are held outside of the banking system, there is less risk of financial crises and therefore a lower likelihood of bailouts. Since non-bank investors are not expected to be bailed out, they value bad assets less than banks. The value of g then captures the social cost of bailouts not internalized by banks.[10] Section 2.1 provides a more detailed discussion of this payoff structure, including how g might depend on the bank's systemic importance, its capitalization, and the quality of its assets.

Policy instruments. We consider two classes of policy tools: *ex-ante mechanisms that deter moral hazard* and *ex-post mechanisms that mitigate adverse selection*. This sharp dichotomy allows us to highlight common mechanisms underlying diverse policy instruments used in practice, and to derive general principles for their optimal joint design. In practice, bank regulators have access to a wide array of potential policy instruments.

We refer to the first class as *ex-ante rules and regulations*, and take this to mean all reg-

[10] One could also attain similar trade-offs even without an externality in a model with a continuum of types and imperfect disclosure. There are gains from trade for all assets but the adverse selection problem is particularly severe with low quality assets, and thus they would not trade absent an intervention.

ulatory interventions that affect bank incentives. Specifically, we assume that the regulator can set up, before the state of the economy θ is realized, a system that bounds from below the level of effort that the bank must exert (i.e., minimum effort). We refer to this minimum effort level as a *regulation* and denote by e_M. We permit a constraint on how much effort the regulator can induce: $e_M \leq \overline{e}_M$. Parameter $\overline{e}_M$ allows us to capture different levels of regulatory capacity. While the regulator can directly target the minimum effort level, the effort level cannot respond to economic shocks.

The second class is *ex-post information disclosure*. In particular, the regulator can commit to a disclosure rule which publicly reveals some information about the bank's asset quality conditional on state θ. Formally, the disclosure rule is a pair (π_H, π_L), where

$$\pi_H, \pi_L : [1-\varepsilon, 1+\varepsilon] \to \Delta(\{h, \ell\}).$$

Thus, when the quality of the asset is H (resp. L), given a state $\theta \in [1-\varepsilon, 1+\varepsilon]$, the regulator stochastically announces either h (a good report) or ℓ (a bad report) according to the policy π_H (resp. π_L). In practice, such disclosure might occur in the context of stress testing, but our model need not necessarily align with existing practice.

Figure 1 shows the timeline. While our model is stylized, it captures central elements of current regulatory frameworks. For example, the Fed currently conducts the stress testing programs called the CCAR (Comprehensive Capital Analysis and Review) and the DFAST (Dodd-Frank Act Stress Tests), which allow for bank-specific disclosure of stress test results (e.g., Hirtle and Lehnert, 2015). In addition, the Fed is responsible for ex-ante regulations (such as the Dodd-Frank Act) pertaining to the oversight of the financial system.

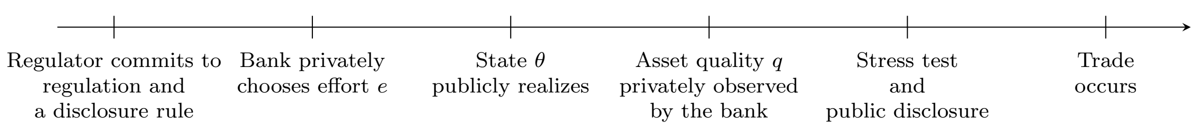

Figure 1: Model timeline

2.1 Model Discussion

We now discuss how key model parameters and assumptions should be interpreted, and how our model maps to prominent policy instruments currently in use.

Interpretation of the payoff structure. Recall that buyer and seller values satisfy:

$$v_H > \rho_H > \rho_L > v_L \text{ and } v_L + g > \rho_L.$$

As an alternative interpretation of the assumption $\rho_L > v_L$, suppose that there are private gains from trade from both asset types: $v_L > \tilde{\rho}_L$. However, given the implicit bailout guarantee g_B if the asset is held by the bank, the relevant asset value to the bank is:

$$\rho_L = \tilde{\rho}_L + g_B > v_L.$$

In contrast, the expected cost of financial distress and this bailout guarantee from the regulator perspective is g such that

$$\rho_L = \tilde{\rho}_L + g_B < v_L + g.$$

With this, and assuming for simplicity that $g_B = g$, we recover our original specification.

Finally, note that if there were no disagreement between regulator and market participants about gains from trade, the optimal disclosure policy is trivially to perfectly reveal asset quality to the market. Hence, the case we study is of more interest.

Capital Requirements and Systemic Importance. To isolate the key forces underlying optimal prudential policy, our model assumes a stark dichotomy between ex-ante regulation and ex-post disclosure. While this approach allows us to derive several insights into the overarching design of prudential frameworks, it does not directly address the interpretation of some prominent regulatory tools, such as capital requirements. In our framework, capital requirements can be viewed primarily as an ex-ante regulatory tool: higher equity capital corresponds to higher effective effort e_M. In addition, better-capitalized banks are less likely to face financial distress and impose lower expected bailout costs, so the externality parameter g decreases with capitalization. Conversely, systemically important institutions can be represented by a higher g, reflecting their larger potential spillovers. Comparative statics with respect to g therefore describe how optimal regulation and disclosure vary with bank size and capitalization without explicitly modeling capital buffers.

Indivisible Asset. We model the asset as indivisible. Allowing banks to retain fractions of a divisible asset could, in principle, introduce signaling motives. We do not consider this extension for several reasons. First, since we combine moral hazard, adverse selection, and information design—dimensions that are rarely analyzed jointly—the current setup is a natural starting point for understanding their interaction. Second, Fuchs et al. (2024) show that, without commitment, time rather than retained quantities is the more effective signaling device, making a multi-period extension the more natural direction. Finally, in the policy context we study, retention would not improve outcomes: if banks expect bailouts when holding bad assets, the concern is not signaling quality but excessive risk retention. If we interpret asset quality as pertaining to the bank's overall balance sheet, the indivisible-asset assumption captures this concern without changing the key trade-offs of the model.

2.2 Central tradeoffs

We now briefly describe the main tradeoffs underlying the costs and benefits of both classes of policy tools. Both rules and disclosure can be used to affect asset quality and market liquidity. Rules directly target effort, and thus reduce the scope for moral hazard. Rules also promote liquidity: since effort increases asset quality on average, trade can take place even for relatively low values of θ. However, since rules are not state-contingent, using rules to ensure trade for all θ requires imposing high effort up front. Since effort is costly, the lack of state-contingency constrains the use of ex-ante mechanisms in liquidity management. Disclosure can be used to foster liquidity by pooling good and bad banks—namely, to announce an asset is of good quality even when it is not. However, the risk of a market freeze limits the ability of the regulator to obfuscate. This results in more disclosure in bad states. There is also another downside to obfuscation, which is that pooling in the asset market weakens incentives to produce high-quality assets. As we will show, these limitations create a rationale for the joint use of both policy tools in prudential regulation. To derive these results in transparent fashion, we begin by discussing each policy tool in isolation.

3 Optimal Design of Individual Policies

We begin by studying the strengths and weaknesses of each policy tool in isolation. Of particular interest in this section are our results on the trade-off between ex-ante incentives and ex-post insurance induced by disclosure in an environment with risky asset quality. Section 4 considers the joint design problem.

3.1 Information Disclosure without Regulation

We begin by studying the optimal information disclosure policy in the absence of regulation. The regulator can commit to an information disclosure policy ex-ante, and trades off the liquidity benefits of opacity with its costs for ex-ante incentives. To establish this trade-off, we first study two benchmarks in which the regulator opts for no information and full information. In the no-information case, the bank has little incentive to exert effort. In the full-information case, effort is high but there is too little trade from the regulator's perspective because low types would not trade. We then show that the optimal information disclosure policy may call for some obfuscation of information.

3.1.1 No Information Benchmark

Suppose that buyers are uninformed about asset quality while the bank knows the realized quality of the asset. Under these assumptions, there is no equilibrium in which trade always occurs for all θ. This is because, in such an equilibrium, prices must be independent of asset quality and the bank would have no incentive to exert any effort. Hence, buyers would be willing to offer at most v_L, which is less than bank's reservation value ρ_H. Conversely, depending on the degree of uncertainty ε and the relative valuations of buyers and sellers, there may exist an equilibrium where trade never occurs.

Going forward, we focus on the more interesting case in which trade occurs for some realizations of θ but not for others. In Appendix A.1, we show that the necessary and sufficient condition for this to be the case is:

$$(c')^{-1}(\rho_H - \rho_L) > \frac{\rho_H - v_L}{v_H - v_L} \frac{1}{1+\varepsilon}. \tag{1}$$

This condition requires that the volatility of θ, which is determined by ε, is large relative to the difference in buyer and seller valuations.

The bank's optimal decision is then determined by how effort e and state θ jointly determine the probability of trade. Fixing some effort level e, trade occurs if and only if the state θ is sufficiently favorable. Let $\theta^*(e)$ be the state at which the quality of the asset, $\theta^*(e)ev_H + (1 - \theta^*(e)e)v_L$, is equal to ρ_H. We can pin down this threshold value as

$$\theta^*(e) := \frac{e^*}{e} \text{ with } e^* := \frac{\rho_H - v_L}{v_H - v_L}, \tag{2}$$

where e^* is an effort level for which the *average* asset quality is equal to ρ_H. Note that for all $\theta < \theta^*(e)$ there is no trade. The probability of $\theta \in [1 - \varepsilon, \theta^*(e))$ is important since in these cases the bank will retain the asset and thus has incentives to exert effort.

The equilibrium effort level e^{NI} with no information is an effort level that maximizes the bank's expected payoff given that trade occurs if and only if $\theta \geq \theta^*(e^{\mathrm{NI}})$:

$$\max_{e\in[0,\frac{1}{2})} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{\theta^*(e^{\mathrm{NI}})} (\theta e \rho_H + (1-\theta e)\rho_L)\, d\theta + \frac{1}{2\varepsilon}\int_{\theta^*(e^{\mathrm{NI}})}^{1+\varepsilon} \left(\theta e^{\mathrm{NI}} v_H + (1-\theta e^{\mathrm{NI}}) v_L\right) d\theta - c(e). \quad (3)$$

The first term captures the bank's payoff under no trade. The second term captures the bank's payoff when trade occurs at price $\theta e^{\mathrm{NI}} v_H + (1-\theta e^{\mathrm{NI}}) v_L$ when $\theta \geq \theta^*(e^{\mathrm{NI}})$. The no-information equilibrium effort level e^{NI} solves Problem (3), which is characterized as follows:

Lemma 1. *A unique solution $e^{\mathrm{NI}} \in \left(\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right)$ exists and is characterized by:*

$$\frac{\rho_H - \rho_L}{4\varepsilon}\left(\left(\frac{e^*}{e^{\mathrm{NI}}}\right)^2 - (1-\varepsilon)^2\right) = c'(e^{\mathrm{NI}}). \quad (4)$$

The left-hand side of Expression (4) is the marginal benefit of exerting effort. It is given by the expected conditional marginal increase in asset quality times the probability of the asset not being traded. Note that given buyer's beliefs, neither prices nor the probability of trade are affected by effort. The right-hand side is the marginal cost of effort.

3.1.2 Full Information Benchmark

Next, we consider the bank's problem in the full-information benchmark where buyers are perfectly informed about asset quality. When asset quality is high, trade occurs at the price v_H. When asset quality is low, no trade occurs. Thus, the bank would choose the full-information effort level e^{FI} that maximizes its payoff:

$$\mathbb{E}\left[\theta e v_H + (1-\theta e)\rho_L\right] - c(e) = e v_H + (1-e)\rho_L - c(e).$$

The first-order condition then uniquely pins down the full-information effort level,

$$e^{\mathrm{FI}} = (c')^{-1}(v_H - \rho_L).$$

Lemma 1 implies that banks exert more effort under full information than under no information: $e^{\mathrm{NI}} < (c')^{-1}(\rho_H - \rho_L) < e^{\mathrm{FI}}$. Despite this positive effect on expected asset quality, providing full information may not be optimal because it precludes trade of bad assets. In this case, the regulator must balance the benefits of fostering trade through opacity with the costs of reducing asset quality on average.

3.1.3 Optimal Disclosure without Regulation

We now characterize the optimal disclosure policy absent regulation. Appendix A.2 shows that it is without loss to consider disclosure policies of the following form. For any θ,

1. if asset quality is H, report h with probability 1
2. if asset quality is L, report h with probability $\beta(\theta)$ and ℓ with probability $1-\beta(\theta)$.

Intuitively, since full information prices satisfy $p^{\mathrm{FI}} = v_H > \rho_H$, the regulator can induce trade in some bad assets by reporting that they are of high quality. Ex-post this increases trade, but it also lowers the bank's ex-ante incentive to exert effort since the price decreases.

Given this restriction, we denote by $p(\theta \mid e, \beta)$ the price in state θ after the regulator has announced the good signal. By Bayes rule, this price is

$$p(\theta \mid e, \beta) := \frac{\theta e v_H + (1-\theta e)\beta(\theta) v_L}{\theta e + (1-\theta e)\beta(\theta)}.$$

To allow for trade, price $p(\theta \mid e, \beta)$ must be at least as high as ρ_H. With this in mind, the regulator's problem is:

$$\max_{e,\beta} \frac{1}{2\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e(v_H + g) + (1-\theta e)\left(\beta(\theta)(v_L + g) + (1-\beta(\theta))\rho_L\right)\right) d\theta - c(e)$$

$$\text{subject to } p(\theta \mid e, \beta) \geq \rho_H \text{ for each } \theta \in [1-\varepsilon, 1+\varepsilon] \text{ and} \tag{5}$$

$$e \in \underset{\hat{e}\in[0,\frac{1}{2})}{\operatorname{argmax}} \frac{1}{2\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} \left((\theta\hat{e} + (1-\theta\hat{e})\beta(\theta))p(\theta \mid e, \beta) + (1-\theta\hat{e})\left(1-\beta(\theta)\right)\rho_L\right) d\theta - c(\hat{e}). \tag{6}$$

Condition (5), which requires the price after the good report to be at least as high as ρ_H, ensures that trade takes place after the good report. Condition (6) is the bank's IC (or Obedience) constraint, which ensures that the bank has an incentive to exert the effort level that the regulator has specified. Denote by $(e^{\mathrm{D}}, \beta^{\mathrm{D}})$ the optimal Disclosure policy.

Proposition 1 shows that, for any cost function, full information is optimal when the externality g is sufficiently small. It also shows that, when g is sufficiently large, there exists a cost function such that information obfuscation (i.e., $\beta^{\mathrm{D}}(\theta) > 0$ for a set of θ with positive measure) is optimal.

Proposition 1 (Optimal Disclosure without Regulation)**.** *The optimal disclosure policy absent regulation satisfies:*

1. *For any cost function c, there exists $\underline{g} > \rho_L - v_L$ such that if $g \in (\rho_L - v_L, \underline{g})$, then full information is optimal: $(e^{\mathrm{D}}, \beta^{\mathrm{D}}) = (e^{\mathrm{FI}}, 0)$.*

2. *There exist a cost function* c *and* $\overline{g} > \rho_L - v_L$ *such that if* $g > \overline{g}$*, then information obfuscation is optimal:* $e^{\mathrm{D}} < e^{\mathrm{FI}}$ *and* $\beta^{\mathrm{D}}(\theta) > 0$ *for some set of* θ *with positive measure.*

Figure 2 illustrates Proposition 1. The solid curve depicts the optimal effort level under the information disclosure problem relative to the full-information effort level e^{FI}. As long as $g > \rho_L - v_L$ is sufficiently low, as shown in Proposition 1 and depicted in Figure 2, full disclosure is optimal: $(e^{\mathrm{D}}, \beta^{\mathrm{D}}) = (e^{\mathrm{FI}}, 0)$. Note first that, in the limit $g = \rho_L - v_L$, full disclosure is optimal, since there is no welfare gain from trading low quality assets. When g is close to this limit, the regulator could reduce disclosure and create a second-order gain. Yet the associated reduction in effort would imply a first-order cost in terms of average asset quality. Thus, there is an interval of values of g under which full disclosure is optimal. However, when g is large, the regulator may want to cross-subsidize some of low realizations to capture g. The trade-off is that, for incentive compatibility, the proposed effort level must be lowered. Thus, there exists $\overline{g}$ (under certain cost functions) such that if $g > \overline{g}$ then $e^{\mathrm{D}} < e^{\mathrm{FI}}$ and $\beta^{\mathrm{D}}(\theta) > 0$ for some set of θ with positive measure.

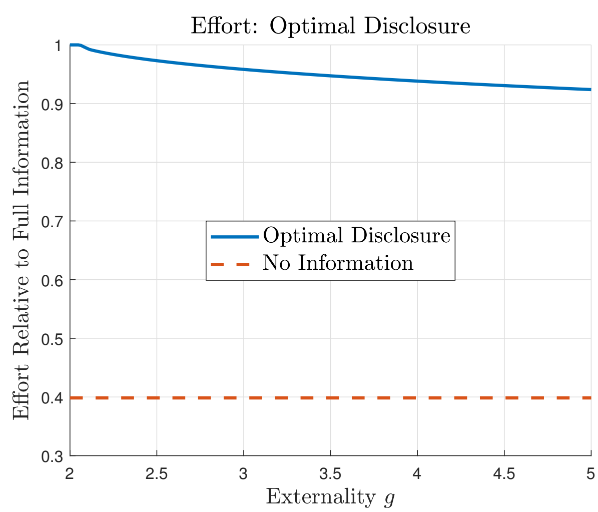

Figure 2: Full versus Partial Information Disclosure (Proposition 1)

As g increases, the regulator faces a more intense trade-off between an ex-ante effort provision and ex-post insurance. Thus, the regulator would be willing to decrease the effort level from the full-information benchmark to increase β more (in an incentive-compatible fashion). If ρ_H is high enough so that Condition (5) starts to bind, then the solution $(e^{\mathrm{D}}, \beta^{\mathrm{D}})$ no longer depends on g. Otherwise, e^{D} is decreasing in g. If g is interpreted as capturing the systemic importance of an institution, this suggests that the regulator is more opaque about large institutions. As a result, these institutions would produce worse assets on average. As we will show later, the regulator will then have an incentive to use other policy tools to induce effort by more systemically important institutions. For instance, the “final tailoring rules” to tailor Federal Reserve’s Enhanced Prudential Standards and Basel III, which became effective in December 2019, apply prudential regulations to banking organizations, with increasingly stringent requirements for larger and more complex (i.e., systemically important) ones.[11]

To get a better sense of the optimal disclosure policy for a given value of g, we consider the optimal disclosure policy when externality g is high so that (partial) information obfuscation is optimal.[12] Figure 3 depicts the optimal disclosure policies for various uncertainty levels ε. The left panel illustrates the effort level given by the optimal information disclosure and the no-information effort levels, normalized by the full-information effort level. The right panel depicts the information obfuscation probability β for various ε.

As illustrated in the figure, the function β is not monotone. For low θ (i.e., in “bad times”), Condition (5) is binding: in order for the price $p(\theta \mid e, \beta)$ to be at least as high as ρ_H, information has to be disclosed to the extent that the condition is satisfied.[13] In contrast, for high θ (i.e., in “good times”), Condition (5) is not binding. Then, decreasing $\beta(\theta)$ (more information disclosure) relaxes the IC constraint (6) or incentivizes the bank to exert a higher level of effort. In our model in which the probability of a high quality asset is multiplicatively given by θe, lowering $\beta(\theta)$ for high θ relaxes incentives and the loss the regulator incurs from foregoing trades for low quality assets is least.[14] Thus, as shown in Proposition 4 in Appendix A.2, there exists $\overline{\theta}$ such that (i) $\beta(\theta) = 0$ when $\theta > \overline{\theta}$ and (ii) $\beta(\theta)$ is the maximum obfuscation probability constrained by Condition (5) when $\theta < \overline{\theta}$. That is, the optimal disclosure policy takes a “bang-bang” disclosure policy.

3.2 Regulation without Disclosure

We have shown that the regulator might want to obfuscate information to promote trade, yet worries that doing so creates a moral hazard problem. Thus, a separate policy that can induce effort, namely regulation, may be useful. We now consider how the regulator would

[11] The opening statement by Federal Reserve Chair Jerome H. Powell emphasizes that “all of our rules keep the toughest requirements on the largest and most complex firms, because they pose the greatest risks to the financial system and our economy” (https://www.federalreserve.gov/newsevents/pressreleases/powell-opening-statement-20191010.htm; Date of Access: April 16, 2024).

[12] For the purpose of illustration, we take $g = 10$ in Figure 3 so that the optimal effort level e^{D} is lower than the one depicted in Figure 2.

[13] The kink on the curve e^{D} in the left panel of the figure at around $\varepsilon = 0.42$ corresponds to the fact that Condition (5) starts to bind around that point.

[14] If effort and the state entered additively as discussed in footnote 9, then this effect would be absent. We note that this non-monotonicity also disappears under the multiplicative specification once we allow for regulation in Section 4. That is, if effort and the state were additive, then the right panel of Figure 3 would then look similar to the right panel of Figure 5.

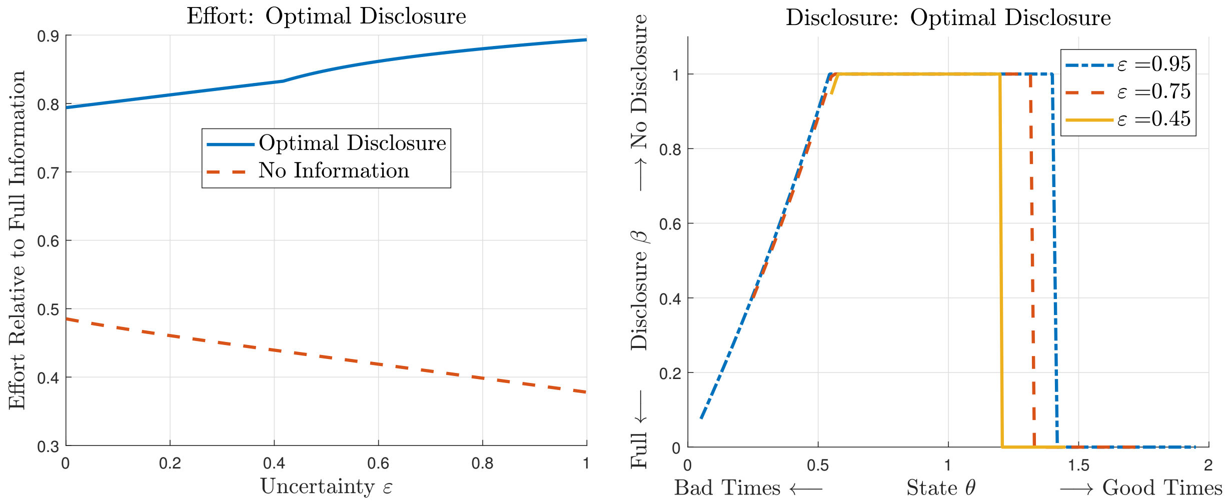

Figure 3: Optimal Information Disclosure Policy $(e^{\mathrm{D}}, \beta^{\mathrm{D}})$

optimally set such rules on effort in the absence of disclosure.

For simplicity, we directly assume that the regulator can choose the minimum effort level $e_M \in [0, \frac{1}{2})$ under the regulatory capacity constraint $e_M \leq \overline{e}_M$. Requiring e_M would correspond to a certain amount of paperwork or conditions to be checked in order to guarantee a minimal loan quality, while the regulatory capacity constraint $\overline{e}_M$ captures a regulatory limitation. If the regulation binds, the bank exerts effort level e_M. For ease of notation, therefore, we suppose that the regulator directly chooses the effort level $e \in [0, \frac{1}{2})$ under the regulatory capacity constraint $e \leq \overline{e}_M$. For exposition, we first assume that the regulatory capacity constraint is not binding, and then consider the case in which it is binding.

Suppose first that there is no uncertainty: $\varepsilon = 0$. The regulator then chooses optimal effort under the presumption that all assets trade. Since the corresponding social welfare is

$$\mathbb{E}\left[\theta e(v_H + g) + (1 - \theta e)(v_L + g)\right] - c(e) = ev_H + (1 - e)v_L + g - c(e),$$

the efficient effort level $e^\diamond$ is given by the first-order condition: $e^\diamond := (c')^{-1}(v_H - v_L)$.

This efficient effort level is higher than the full-information effort level $e^{\mathrm{FI}} = (c')^{-1}(v_H - \rho_L)$. Average asset quality satisfies $ev_H + (1 - e)v_L \geq \rho_H$ if and only if $e \geq e^*$, where $e^* = \frac{\rho_H - v_L}{v_H - v_L}$. Since $(c')^{-1}(v_H - v_L) > e^*$ by Condition (1) so that all assets trade, the optimal regulation e^{R} in the absence of a binding regulatory capacity constraint is:

$$e^{\mathrm{R}} = (c')^{-1}(v_H - v_L).$$

If the capacity constraint is binding, then $e^{\mathrm{R}} = \overline{e}_M$. In the absence of uncertainty, regulation can thus attain the efficient level and all assets always trade. However, since regulation is set ex-ante and cannot respond to shocks, it may be not be sufficient to achieve efficiency in a stochastic environment (i.e., $\varepsilon > 0$). Hence we now consider the case in which $\varepsilon > 0$.

Given $\varepsilon > 0$, the regulator must decide how much to insure against market freezes in bad states by requiring more cumbersome regulation. In particular, when the regulator chooses an effort level e, there exists a unique cutoff $\theta^*(e) = \frac{e^*}{e}$ such that $\theta e v_H + (1 - \theta e)v_L \geq \rho_H$ if and only if $\theta \geq \theta^*(e)$. Using the median $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon)$, this means that trade occurs if and only if $\theta \in [\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon), 1+\varepsilon]$.[15] By choosing the effort level, the regulator thus chooses the set of states for which trade occurs. Formally, the regulator's problem is:

$$\begin{aligned}\max_{e \in [0, \frac{1}{2})} \frac{1}{2\varepsilon} \int_{1-\varepsilon}^{\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon)} & (\theta e \rho_H + (1-\theta e)\,\rho_L)\, d\theta \\ + \frac{1}{2\varepsilon} \int_{\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon)}^{1+\varepsilon} & (\theta e v_H + (1-\theta e)\, v_L + g)\, d\theta - c(e). \end{aligned} \tag{7}$$

The next proposition formally characterizes e^{R} in the relaxed problem without the regulatory capacity constraint. The proof shows that the optimal regulation is given by $\overline{e}_M$ when the regulatory capacity constraint binds, as social welfare is increasing on $e \in [0, e^{\text{R}}]$.

Proposition 2 (Optimal Regulation Without Disclosure)**.** *The optimal regulation* e^{R} *is uniquely given by:*

$$e^{\text{R}} = \begin{cases} e^{\diamond} = (c')^{-1}(v_H - v_L) & \text{if } \varepsilon \leq 1 - \frac{e^*}{e^{\diamond}} \\ \min\left(e^{\dagger}, \frac{e^*}{1-\varepsilon}\right) & \text{if } \varepsilon > 1 - \frac{e^*}{e^{\diamond}} \end{cases}, \tag{8}$$

where $e^{\dagger} \in (e^{\diamond}, \frac{1}{2})$ *is a unique solution satisfying*

$$\frac{v_H - v_L}{4\varepsilon}(1+\varepsilon)^2 - \frac{\rho_H - \rho_L}{4\varepsilon}(1-\varepsilon)^2 + \frac{(v_H - \rho_H + \rho_L - v_L)e^* + 2(v_L + g - \rho_L)}{4\varepsilon} \frac{e^*}{(e^{\dagger})^2} = c'(e^{\dagger}). \tag{9}$$

The main trade-off is that ensuring trade in all states requires high upfront effort. Since effort is costly, the regulator may thus choose to accept the risk of market freezes after negative shocks. The next result shows that this is the case under the optimal policy as long as there is sufficient fundamental uncertainty.

Corollary 1. *There exists* $\overline{\varepsilon} \in (0, 1)$ *with the following two properties:*

1. *If* $\varepsilon \leq \overline{\varepsilon}$*, then* $\theta^*(e^{\text{R}}) \leq 1 - \varepsilon$*, i.e., trade always occurs.*

[15] Technically, we would need a slight modification at the end points $\theta \in \{1-\varepsilon, 1+\varepsilon\}$ when $\theta^*(e)$ is outside of $[1-\varepsilon, 1+\varepsilon]$. Since the regulator's value at the end points would not change her objective function, we use this convenient notation.

2. *If $\varepsilon > \overline{\varepsilon}$, then $\theta^*(e^{\mathrm{R}}) \in (1-\varepsilon, 1+\varepsilon)$, i.e., trade takes place for some and only some θ.*

Figure 4 shows that the optimal regulation may be non-monotone in fundamental uncertainty. We plot the ratio of optimal regulation to full-information effort $\frac{e^{\mathrm{R}}}{e^{\mathrm{FI}}}$ as a function of uncertainty ε. For even low levels of ε (when trade takes place in all states) the regulator still sets regulation above the effort induced by full information. Thus, market discipline by itself is not sufficient to induce the optimal level of effort. Next, when uncertainty increases, the regulator increases the level of effort even further to guarantee trade after all realizations of θ. We refer to setting regulation above the productively efficient effort as *prudential regulation*. At first, optimal regulation increases in ϵ such that trade continues to occur for all θ. As ε further increases, full insurance is too costly, and "luck" plays a larger role. Thus, optimal regulation is decreasing in ε. Yet, we show that regulation is still above the efficient effort level (and consequently the full-information effort level) to increase the probability of trade even in bad states.

Figure 4: Optimal Regulation (Proposition 2)

It can also be shown that the optimal regulation e^{R} is non-decreasing in g. This implies that the regulator would impose higher standards for more systemically important banks (high g). Corollary 3 in Appendix A.3 formally presents this result. The next section shows that this comparative static remains robust when the regulator also has access to disclosure.

4 Joint Design of Regulation and Disclosure

We now characterize the optimal joint design of ex-ante regulation and ex-post disclosure, and show that there is a rationale for including both in an overarching framework. In particular, the ability to provide state-contingent disclosure allows the regulator to maintain asset market liquidity even with lower ex-ante effort.

As in the case of disclosure without regulation, the regulator faces the constraint that disclosure must be sufficiently informative such that the price conditional on a good signal is weakly larger than the seller's value of a good asset: $p(\theta \mid e, \beta) \geq \rho_H$. When the regulator cannot impose ex-ante rules, this constraint is not necessarily binding because increased transparency improves incentives. The next result establishes that this role for disclosure is *not* needed when regulation is available: the ability to impose rules allows the regulator to use disclosure only to manage liquidity.

Lemma 2 (Minimal disclosure)**.** *Given effort level e, let $\theta^*(e) = \frac{e^*}{e}$ be the cutoff state under which the average quality of the asset is ρ_H. Then the regulator chooses the disclosure rule*

$$\beta(\theta) = \begin{cases} \frac{v_H - \rho_H}{\rho_H - v_L} \frac{\theta e}{1 - \theta e} & \textit{if } \theta \in [1-\varepsilon, \mathrm{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon)) \\ 1 & \textit{if } \theta \in [\mathrm{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon), 1+\varepsilon] \end{cases} .^{16} \tag{10}$$

Lemma 2 implies that information disclosure is *countercyclical*: β is non-decreasing in θ. This is because the regulator discloses information such that the average quality is just equal to ρ_H, and this requires more disclosure when expectations of asset quality would otherwise be low. This monotonicity does not necessarily hold in the absence of regulation, as the regulator may then have an incentive to reveal bad assets in good times to promote incentives.[17] Moreover, asset quality is never fully disclosed when the regulator can enforce effort. This is in contrast to Proposition 1, where full disclosure is sometimes used to ensure incentives in the absence of regulation.

More generally, the next result shows that regulation can *substitute* for disclosure in incentive provision, in the precise sense that the ability to set rules results in more opacity.

Lemma 3 (Regulation substitutes for disclosure)**.** *If regulators have access to regulation:*

1. *Information will never be fully disclosed for any state θ, $\beta(\theta) > 0$ for all $\theta \in [1-\varepsilon, 1+\varepsilon]$ and for any cost function.*

[16] Technically, we would need a slight modification at the end points when $\theta^*(e)$ is outside of $[1-\varepsilon, 1+\varepsilon]$. Since the value of β at the end points would not change the integral, we define β in this way.

[17] One could apply this result to detailed disclosure of stress tests during the crisis times. For example, the 2011 Irish and 2011 Europe-wide EBA (European Banking Authority) stress tests are considered to be detailed, including comparisons of bank and third-party estimates of losses in the Irish case and data in electronic and downloadable form in the EBA case. Schuermann (2014) states that the benefit of detailed bank-specific stress test disclosure is significant during the crisis times and that this is precisely what was done in the 2011 Irish stress test.

2. *The more demanding the regulation, the less disclosure occurs. Formally, for any* (e, β) *and* $(\tilde{e}, \tilde{\beta})$ *satisfying Expression (10) and* $\tilde{e} \geq e$, $\tilde{\beta} \geq \beta$.

Based on these results, we can now fully characterize the optimal joint design. To do so, we start with the relaxed problem that ignores the regulatory capacity constraint $e \leq \overline{e}_M$. Since e determines β through Expression (10), the regulator chooses effort e to solve:

$$\max_{e\in[0,\frac{1}{2})} \frac{1}{2\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e(v_H + g) + (1-\theta e)\left(\beta(\theta)(v_L + g) + (1-\beta(\theta))\rho_L\right)\right) d\theta - c(e).$$

Substituting β from Expression (10), we can write the regulator's problem as:

$$\begin{aligned}\max_{e\in[0,\frac{1}{2})} \frac{1}{2\varepsilon} \int_{1-\varepsilon}^{\operatorname{med}(1-\varepsilon,\theta^*(e),1+\varepsilon)} & \left(\frac{e\theta}{e^*}(\rho_H + g) + \left(1 - \frac{e\theta}{e^*}\right)\rho_L\right) d\theta \\ +\frac{1}{2\varepsilon} \int_{\operatorname{med}(1-\varepsilon,\theta^*(e),1+\varepsilon)}^{1+\varepsilon} & \left(\theta e v_H + (1-\theta e)\, v_L + g\right) d\theta - c(e). \end{aligned} \tag{11}$$

The first term is social welfare when $\theta \leq \theta^*(e)$. With probability $e\theta + (1-e\theta)\beta(\theta) = \frac{e\theta}{e^*}$, the regulator announces h, and trade occurs with price ρ_H which also yields the externality g. With the complementary probability, trade does not occur, and the bank's valuation is ρ_L. The second term is social welfare when $\theta \geq \theta^*(e)$, in which trade occurs with no disclosure.

The following theorem characterizes the optimal policy, provided that the regulatory capacity constraint is not binding. We discuss the case of a binding regulatory capacity constraint later.

Theorem 1 (Optimal Joint Design)**.** *In the optimal joint design of regulation and disclosure,*

1. *The optimal regulation* e^{RD} *is uniquely given by:*

$$e^{\mathrm{RD}} = \begin{cases} e^{\diamond} = (c')^{-1}(v_H - v_L) & \text{if } \varepsilon \leq 1 - \frac{e^*}{e^{\diamond}} \\ e^{\ddagger} & \text{if } \varepsilon > 1 - \frac{e^*}{e^{\diamond}} \end{cases}, \tag{12}$$

where $e^{\ddagger} \in \left(e^{\diamond}, \frac{e^*}{1-\varepsilon}\right)$ *is a unique solution satisfying*

$$\frac{v_H - v_L}{4\varepsilon}(1+\varepsilon)^2 - \frac{\rho_H - \rho_L + g}{4\varepsilon e^*}(1-\varepsilon)^2 + \frac{v_L + g - \rho_L}{4\varepsilon}\frac{e^*}{(e^{\ddagger})^2} = c'(e^{\ddagger}). \tag{13}$$

2. *The optimal disclosure policy* β^{RD} *is given through Expression (10).*

   (a) If $\varepsilon \leq 1 - \frac{e^*}{e^{\diamond}}$*, then:* $\beta^{\mathrm{RD}}(\theta) = 1$ *for all* $\theta \in [1-\varepsilon, 1+\varepsilon]$.

(b) If $\varepsilon > 1 - \frac{e^*}{e^\diamond}$, then: (i) $\beta^{\mathrm{RD}}(\theta) \in (0,1)$ for all $\theta \in [1-\varepsilon, \theta^*(e))$; and (ii) $\beta^{\mathrm{RD}}(\theta) = 1$ for all $\theta \in [\theta^*(e), 1+\varepsilon]$.

This characterization allows us to show that disclosure also substitutes for regulation: the regulator sets laxer effort requirements when she also sets the optimal disclosure policy. We can also show that systemically-important institutions (high g) should face higher rules but weaker disclosure.

Corollary 2 (Properties of the optimal design)**.** *In the optimal joint design:*

1. *Optimal regulation is laxer when disclosure is available. Denote by $e^{\mathrm{R}}(\varepsilon)$ and $e^{\mathrm{RD}}(\varepsilon)$ the optimal regulation without disclosure and with disclosure for every $\varepsilon \in (0,1)$, respectively. Then, $e^{\mathrm{RD}}(\varepsilon) \leq e^{\mathrm{R}}(\varepsilon)$ for all $\varepsilon \in (0,1)$.*

2. *e^{RD} is non-decreasing in g. Consequently, β^{RD} is non-decreasing in g.*

The notion that systemically-important institutions should face tigther regulation is consistent with the Basel III framework and the Dodd-Frank Act. In the Basel III framework,, the Financial Stability Board, in consultation with Basel Committee on Banking Supervision and national authorities, requires more scrutiny on global systemically important banks (G-SIBs). The provisions of the Dodd-Frank Act state: "Designated [systemically important] FMUs will become subject to the heightened prudential and supervisory provisions of Title VIII, which promote robust risk management and safety and soundness, including conducting their operations in compliance with applicable risk-management standards; providing advance notice and review of changes to their rules, procedures, and operations that could materially affect the nature or level of their risks; and being subject to relevant examination and enforcement provisions."[18] We could interpret higher standards for systemically important institutions not only as higher risk-management standards and capital requirements but also the creation of resolution plans known as "living wills" in order to alleviate a moral hazard problem that information obfuscation creates. This is also consistent with Schneider et al. (2023), who show that large banks face more scrutiny on risk management practices and governance in the qualitative parts of the Fed's recent stress tests. In our model, this can be interpreted precisely as demanding higher effort. Finally, Bird et al. (2023) empirically demonstrate that the Federal Reserve's disclosure of CCAR stress test results was softer for systemically important banks.

[18]https://home.treasury.gov/policy-issues/financial-markets-financial-institutions-and-fiscal-service/fsoc/designations (Date of Access: February 19, 2024).

Illustration. Figure 5 illustrates our results. The left panel depicts optimal regulation e^{RD} with disclosure. The right panel depicts the optimal information disclosure policy β^{RD} for various ε. When uncertainty ε is small, trade always occurs under regulation alone. In this case, information disclosure is not needed (i.e., $e^{\mathrm{RD}} = e^{\mathrm{R}}$), and regulation coincides with the efficient effort. When $\varepsilon = 0.5$, the information disclosure policy β satisfies $\beta(\cdot) = 1$.

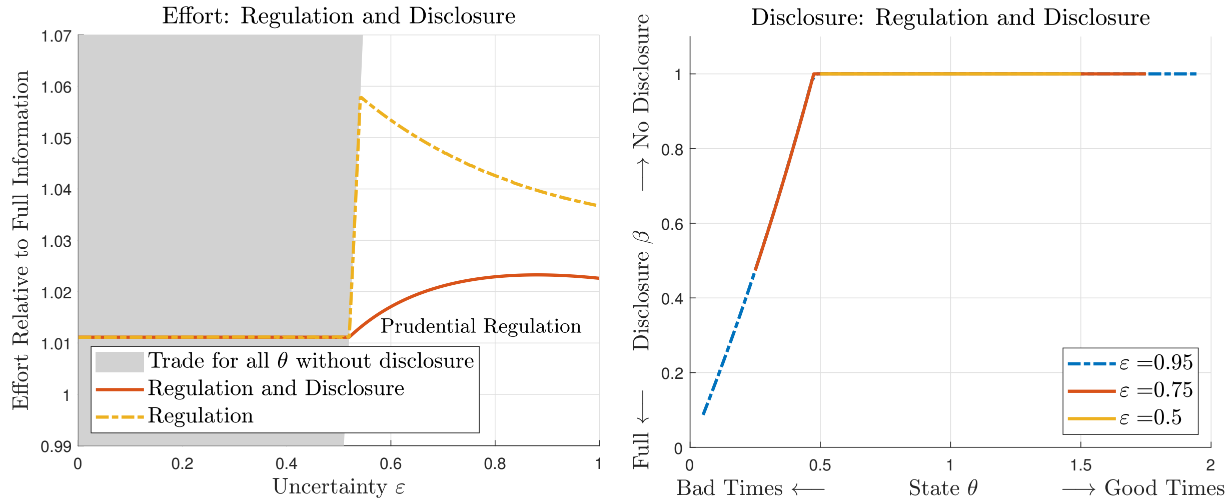

Figure 5: Optimal Regulation and Disclosure (Theorem 1 and Corollary 2)

Once ε is sufficiently large (precisely $\varepsilon > 1 - \frac{e^*}{e^\diamond}$), trade occurs only for some θ under this effort level. On the one hand, when regulation is the only tool, since losing trade opportunities for some realizations of θ is a first-order loss as compared to a second-order cost of increasing effort, the regulator requires prudential regulation so that trade always occurs. As was seen, this corresponds to the increased dashed curve in the left panel of Figure 5. On the other hand, when the regulator can utilize both regulation and disclosure, the regulator can increase the average asset quality and thus enhance trade either by requiring a higher level of effort or by more detailed information disclosure. Thus, the regulator can ensure trade from information disclosure. In other words, information disclosure can reduce the burden of regulation. As we formally show below, the left panel of Figure 5 illustrates that the optimal effort level e^{RD} is lower than e^{R}.

The left panel Figure 5 shows that the optimal regulation e^{RD} with disclosure is still at least as high as the efficient effort level and in fact it is strictly higher whenever uncertainty ε is sufficiently high. Thus, the regulator uses both prudential regulation and disclosure together to increase the likelihood of trade. Note that e^{RD} need not be monotone in uncertainty ε. As we discussed before, when uncertainty is too high, luck plays a larger role than effort, and this might lead to a decrease in the level of regulation for sufficiently high uncertainty. The regulator can then use higher levels of disclosure (as an ex-post partial substitute) when the realized state is indeed low. Since e^{RD} is similar for $\varepsilon \in \{0.75, 0.95\}$ (right panel of Figure 5), the optimal disclosure policies β are also similar.

Binding regulatory capacity constraint. When the regulatory constraint $e \leq \overline{e}_M$ is binding, we must distinguish two cases. When the capacity constraint $\overline{e}_M$ is not too low, the regulator continues to solely use regulation to provide incentives. In this case, $e^{\text{RD}} = \overline{e}_M$ and the disclosure policy follows from Expression (10). Figure 6 illustrates the optimal disclosure policy β in this case. With lower $\overline{e}_M$, more disclosure is needed in bad states in order to prevent market freezes. If the constraint $\overline{e}_M$ is sufficiently tight, rules provides no value to the regulator and the solution jumps to the pure-disclosure solution from Section 3.1.3.

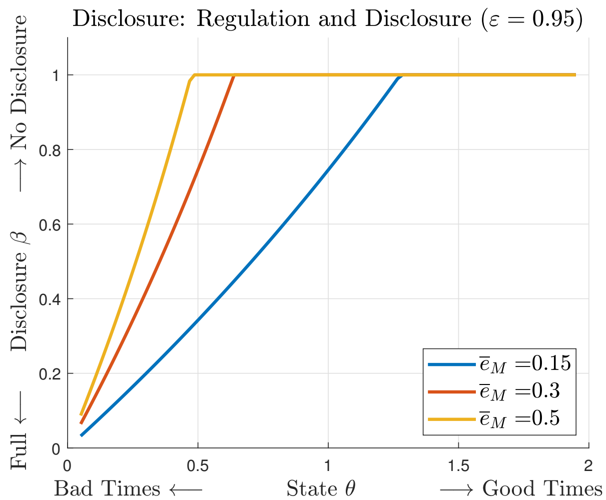

Figure 6: Optimal Disclosure Policy β for Various $\overline{e}_M$

5 Bank-Optimal Design

We conclude our analysis by studying optimal regulation and disclosure if banks were to "self-regulate" by choosing a regulatory framework that maximizes their own expected payoff. This allows us to distinguish the extent to which the socially-optimal regulation is driven by regulators' ability to enforce commitment versus the need to account for positive externalities from asset sales. It is also useful for understanding financial oversight in practice because international framework such as Basel III may lead to a regulatory "race to the bottom" in which national authorities implement a framework to benefit domestic banks. As before, we first study each tool in isolation and then their optimal joint design.

5.1 Bank-Optimal Disclosure

Suppose that banks can credibly commit to a fixed disclosure rule. Given linear payoffs in perceived asset quality and a competitive secondary market, banks would then prefer to eliminate the ex-post adverse selection problem by committing to full disclosure.

In reality, banks are unlikely to be able to commit to a disclosure policy, and those holding low-quality assets would prefer to obfuscate ex-post.[19] The desirability of full disclosure thus illustrates the mechanics of our model, but is not intended as an empirical prediction.

5.2 Bank-Optimal Regulation without Disclosure

Next, we consider the bank's preferred regulation in the absence of disclosure. The benefit of regulation is that it allows the bank to commit to a minimum effort level, helping it alleviate the ex-post adverse selection problem. However, given that the bank does not internalize the externality g from asset sales, it would prefer a lower effort level than the regulator.

To characterize the bank-optimal regulation, we make two observations. First, if trade occurs for all θ under the bank-optimal regulation, then the bank's effort level would be the minimum effort level under which the average quality of the asset is at least as high as ρ_H for all state θ. Second, suppose that trade never occurs under the bank-optimal regulation. Since the bank's payoff is $e\rho_H + (1-e)\rho_L - c(e)$, its effort level is $e = (c')^{-1}(\rho_H - \rho_L)$. However, similarly to Section 3.1.1, Condition (1) rules out this no-trade case.

Denoting by e^{BR} the Bank-optimal Regulation, it follows from these two observations that trade occurs if and only if $\theta \geq \theta^*(e^{\mathrm{BR}})$. The bank's problem is:

$$\max_{e\in[0,\frac{1}{2})} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{\mathrm{med}(1-\varepsilon,\theta^*(e),1+\varepsilon)} (\theta e\rho_H + (1-\theta e)\rho_L)\, d\theta + \frac{1}{2\varepsilon}\int_{\mathrm{med}(1-\varepsilon,\theta^*(e),1+\varepsilon)}^{1+\varepsilon} (\theta e v_H + (1-\theta e)v_L) d\theta - c(e).$$

The first term corresponds to the payoff from no trade, while the second the payoff from trade. Note that the difference from Expression (7) is that the externality g is absent.

When $\theta \leq \theta^*(e^{\mathrm{BR}})$, no trade takes place as the expected quality of the asset is below ρ_H. Since the second observation implies that some trade occurs, we have to have $\theta^*(e^{\mathrm{BR}}) < 1+\varepsilon$, i.e., $\frac{e^*}{1+\varepsilon} < e^{\mathrm{BR}}$. In contrast, when $\theta \geq \theta^*(e^{\mathrm{BR}})$, trades take place at the price $\theta e^{\mathrm{BR}} v_H + (1-\theta e^{\mathrm{BR}})v_L$. The second observation implies that the bank does not have an

[19] Although full disclosure could in principle emerge through an unraveling argument, this requires credible signaling that may be limited in practice. While banks may rely on credit rating agencies to convey information, conflicts of interest and rating shopping constrain the credibility of such signals, often more so than for regulatory disclosure. In addition, maintaining good relationships with supervisors may give banks further incentives to withhold information even when disclosure is feasible. Finally, factors such as franchise value or concave payoffs in asset quality could also discourage voluntary disclosure.

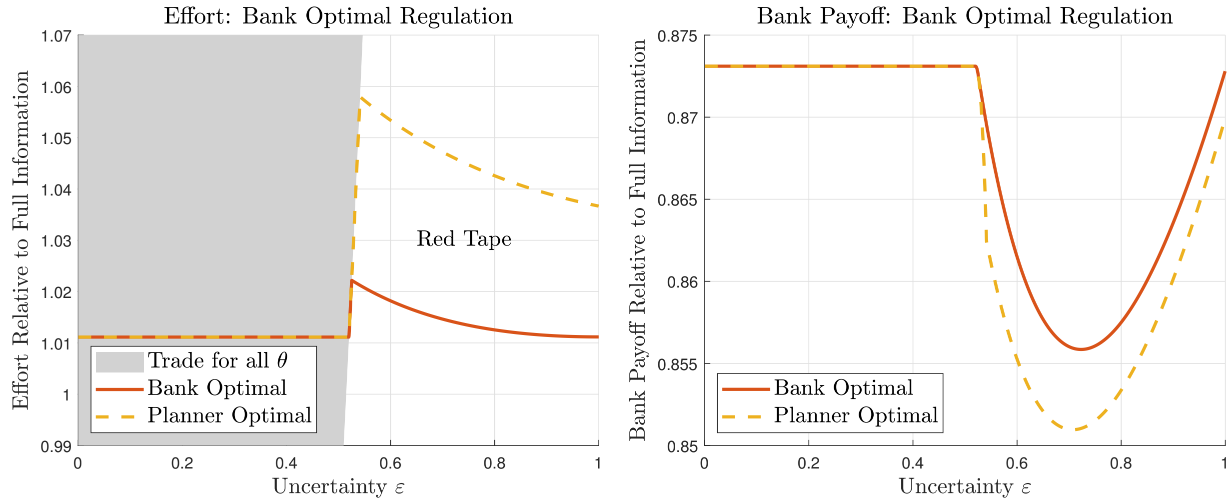

Figure 7: Bank Optimal Regulation (Proposition 3) and Bank's Payoff

incentive to exert e^{BR} with $\theta^*(e^{\mathrm{BR}}) < 1-\varepsilon$, as trade occurs for all θ once $\theta^*(e^{\mathrm{BR}}) \leq 1-\varepsilon$. Thus, we have $\theta^*(e^{\mathrm{BR}}) \geq 1-\varepsilon$, i.e., $\frac{e^*}{1-\varepsilon} \geq e^{\mathrm{BR}}$. Below we formally characterize the bank-optimal effort level e^{BR}.

Proposition 3. *A unique solution* e^{BR} *exists and satisfies the following:*

$$e^{\mathrm{BR}} = \begin{cases} e^{\diamond} = (c')^{-1}(v_H - v_L) & \text{if } \varepsilon \leq 1 - \frac{e^*}{e^{\diamond}} \\ \min\left(e^{\circ}, \frac{e^*}{1-\varepsilon}\right) & \text{if } \varepsilon > 1 - \frac{e^*}{e^{\diamond}} \end{cases}, \tag{14}$$

where $e^{\circ} \in \left(\frac{e^*}{1+\varepsilon}, e^{\mathrm{R}}\right)$ *is a unique solution satisfying*

$$\frac{v_H - v_L}{4\varepsilon}(1+\varepsilon)^2 - \frac{\rho_H - \rho_L}{4\varepsilon}(1-\varepsilon)^2 + \frac{(v_H - \rho_H + \rho_L - v_L)e^* + 2(v_L - \rho_L)}{4\varepsilon}\frac{e^*}{(e^{\circ})^2} = c'(e^{\circ}). \tag{15}$$

The left panel of Figure 7 illustrates Proposition 3: the solid curve depicts bank-optimal regulation, compared with the dashed curve which depicts optimal regulation e^{R} (provided $e^{\mathrm{R}} \leq \bar{e}_M$). When the degree of uncertainty is low, the bank could self-regulate since it would pick the same effort level to guarantee high quality as the regulator. Instead, when the degree of uncertainty is sufficiently large, supervision by the regulator is necessary since the level chosen by the regulator is significantly higher than the one chosen by the bank. This is because, due to the externality, the regulator cares more about the probability of trade than the bank. Thus, although both increase the effort once trade is not always guaranteed, the regulator would demand more prudential regulation than the bank would like to commit to. We refer to the difference between the optimal regulation and the bank-optimal regulation as "red tape" from the perspective of the bank. As illustrated in the left panel of Figure 7, there exists $\overline{\varepsilon} \in (0,1)$ such that if $\varepsilon \leq \overline{\varepsilon}$, then trade occurs for all θ; and if $\varepsilon > \overline{\varepsilon}$, then trade occurs for only some θ.

The right panel of Figure 7 depicts the bank's payoff from bank-optimal regulation (solid curve) and from optimal regulation (dashed curve). The bank's payoff is non-monotonic in uncertainty. At first uncertainty is not relevant since trade happens with probability 1. Eventually, as uncertainty increases, there are sufficiently bad states that, absent an increase in effort, there would be no trade after those realizations. At first additional effort is used to reduce said possibility. Of course, this is costly, so payoffs decrease. Payoffs continue to decrease as the probability of trade is reduced. Eventually though, there is an effect in the opposite direction coming from the fact that the price conditional on trade is more sensitive than the value conditional on not trading. This option-like feature leads bank's payoff to increase for high levels of uncertainty.

Finally, we make a technical remark on Proposition 3. While Expression (15) corresponds to Expression (9) with $g = 0$, since the bank's objective function in the bank-optimal regulation problem may not be concave, we show that the unique solution to the bank's optimal regulation problem is indeed characterized by Expression (15).

5.3 Bank-Optimal Regulation and Disclosure

We conclude by characterizing bank-optimal regulation with disclosure. By Section 5.1, the bank would like to commit to full disclosure to maximize the bank's private gains from trade. Given full information, the full information effort level e^{FI} is optimal for the bank, and there is no additional benefit from committing to effort through regulation.

In practice, banks are limited in their ability to commit to the full disclosure policy. With less than full disclosure, commitment to a given effort level is strictly valuable for the bank.

To illustrate, we focus on the case where the disclosure policy is the regulator's optimal disclosure policy β^{RD} characterized in Section 4. First, we can show that there exists a threshold level of uncertainty $\overline{\varepsilon} \in \left(1 - \frac{e^*}{e^\diamond}, 1\right]$ such that for low levels of uncertainty $\varepsilon < \overline{\varepsilon}$ the bank-optimal regulation coincides with the regulator's optimal regulation e^{RD} with disclosure. This implies that the bank can self-regulate in environments of low uncertainty. Instead, for high levels of uncertainty $\varepsilon > \overline{\varepsilon}$, the bank would like to commit to lower standards than those imposed by the regulator. In fact, the bank would commit to the bank-optimal regulation e^{BR} without disclosure.[20] In this case, what the regulator views as prudential regulation is mostly considered "red tape" by the bank.

[20] This is because, if the bank's effort level is below the regulator's optimal regulation e^{RD}, then trade occurs when the state θ is high enough so that disclosure does not occur under β^{RD}.

6 Conclusion

We investigate the optimal joint design of two classes of regulatory tools available in bank oversight: ex-ante rules and market discipline fostered by ex-post information disclosure. Using a framework that combines moral hazard, adverse selection, and information design, we determine how regulators should optimally deploy these instruments to manage bank risk. We establish that prudential rules and mandatory disclosure are *substitutes* in the regulatory toolkit—meaning that targeted state-contingent disclosure reduces the need for strict ex-ante rules, and vice-versa. Yet, it is optimal to use both tools *jointly*, as this significantly lowers banks' regulatory burden and improves the liquidity of bank assets.

Our model further demonstrates that optimal disclosure—and the associated market discipline—is inherently *countercyclical*: information should be more detailed and transparent in adverse conditions to prevent market freezes, but more opaque in good times to permit even relatively risky banks to trade in liquid markets. We also derive distinct policy prescriptions based on a bank's systemic importance. Systemically-important financial institutions require stricter ex-ante oversight, but should be subject to less market discipline ex post.

Our findings offer guidance for financial regulatory bodies. They support a shift toward *state-contingent regulation* in which disclosure requirements can be adjusted based on the aggregate state of the financial system. They also provide theoretical justification for a degree of "constructive ambiguity" in the market's view of financial institutions.

While we focus on the optimal design of two broad classes of regulatory tools, a promising direction for future research lies in a more granular examination of these categories. For example, given the central role of capital regulation in global policy, an important extension is to model the optimal design of bank capital requirements alongside other rules and disclosure.

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A Proofs

A.1 Section 3.1.1

A.1.1 Derivation of Condition (1)

We first derive Condition (1) under which there is no equilibrium in which trade never occurs. If there is an equilibrium in which trade never occurs, then the bank would maximize the expected private value of the asset net of costs:

$$\mathbb{E}\left[\theta e \rho_H + (1-\theta e)\rho_L\right] - c(e) = e\rho_H + (1-e)\rho_L - c(e).$$

Given our assumptions on the cost function, the first-order condition uniquely pins down the bank's effort level as

$$e = (c')^{-1}(\rho_H - \rho_L).$$

This condition relates the marginal cost of increasing the probability of obtaining a good asset to the difference in bank valuations between high and low quality assets.

For no trade not to constitute an equilibrium, it must be that at this effort level, the equilibrium price (the average quality of the asset) is as high as ρ_H for some set of θ with positive measure. This condition is exactly Condition (1).

A.1.2 Proof of Lemma 1

Proof of Lemma 1. In the no-information equilibrium, the bank's effort level e^{NI} has to solve Problem (3), which reduces to:

$$\max_{e\in[0,\frac{1}{2})} \frac{\rho_H-\rho_L}{2\varepsilon}\int_{1-\varepsilon}^{\theta^*(e^{\mathrm{NI}})} \theta d\theta \cdot e - c(e).$$

Given our assumptions on the cost function, the problem has a unique solution, which has to coincide with e^{NI}. The first-order condition at $e = e^{\mathrm{NI}}$ is:

$$c'(e^{\mathrm{NI}}) = \frac{\rho_H-\rho_L}{2\varepsilon}\int_{1-\varepsilon}^{\theta^*(e^{\mathrm{NI}})} \theta d\theta = \frac{\rho_H-\rho_L}{4\varepsilon}\left(\left(\frac{e^*}{e^{\mathrm{NI}}}\right)^2 - (1-\varepsilon)^2\right),$$

which coincides with Expression (4). By the arguments in the main text, $1-\varepsilon < \theta^*(e^{\mathrm{NI}}) < 1+\varepsilon$, which implies that $e^{\mathrm{NI}} \in \left(\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right)$. □

A.2 Section 3.1.3

Appendix A.2.1 formulates the regulator's optimal disclosure problem without regulation as discussed in the main text. Appendix A.2.2 provides the proof of Proposition 1. Appendix A.2.3 provides the formal characterization of the optimal disclosure policy discussed in the main text.

A.2.1 Regulator's Problem

We first show that one can restrict attention to the disclosure policies of the following form.

Lemma 4. *It is sufficient to consider a policy (π_H, π_L) with the following properties: (i) π_H reports h with probability $\alpha(\theta)$ and ℓ with probability $1-\alpha(\theta)$ for each θ; and (ii) π_L reports h with probability $\beta(\theta)$ and ℓ with probability $1-\beta(\theta)$ for each θ.*

Proof of Lemma 4. Let $(S, (\pi_H, \pi_L))$ be a signal, i.e., S is a set of signal realizations and $\pi_H, \pi_L : \Theta \to \Delta(S)$ is a (measurable) policy. We show that, without loss, we can let $S = \{h, \ell\}$. To see this, let T_θ be a set of signal realizations under which trade takes place for a given θ. Then, we can define $(\tilde{S}, (\tilde{\pi}_H, \tilde{\pi}_L))$ by $\tilde{S} = \{h, \ell\}$, $\tilde{\pi}_H(\theta)(h) = \pi_H(\theta)(T_\theta)$, and $\tilde{\pi}_L(\theta)(h) = \pi_L(\theta)(T_\theta)$. Thus, we can consider $(S, (\pi_H, \pi_L))$ such that $S = \{h, \ell\}$, $\pi_H(\theta)(h) = \alpha(\theta)$, and $\pi_L(\theta)(h) = \beta(\theta)$. □

By Lemma 4, the regulator's problem is:

$$\max_{e,\alpha,\beta} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e\left(\alpha(\theta)(v_H+g)+(1-\alpha(\theta))\rho_H\right)+(1-\theta e)\left(\beta(\theta)(v_L+g)+(1-\beta(\theta))\rho_L\right)\right)d\theta - c(e)$$

subject to $p(\theta \mid e,\alpha,\beta) := \dfrac{\theta e\alpha(\theta)v_H+(1-\theta e)\beta(\theta)v_L}{\theta e\alpha(\theta)+(1-\theta e)\beta(\theta)} \geq \rho_H$ for each $\theta \in [1-\varepsilon, 1+\varepsilon]$ and

$$e \in \operatorname*{argmax}_{\hat{e}\in[0,\frac{1}{2})} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left((\theta\hat{e}\alpha(\theta)+(1-\theta\hat{e})\beta(\theta))p(\theta \mid e,\alpha,\beta)+\theta\hat{e}(1-\alpha(\theta))\rho_H+(1-\theta\hat{e})\left(1-\beta(\theta)\right)\rho_L\right)d\theta - c(\hat{e}).$$

The policy (e,α,β) is *incentive-feasible* if it satisfies the above two constraints. We show that one can restrict attention to the disclosure policies with $\alpha = 1$.

Lemma 5. *It is without loss to consider the policies with $\alpha = 1$.*

Proof of Lemma 5. If $\alpha = 1$ almost surely, then it is without loss to redefine $\alpha = 1$. Thus, consider an incentive-feasible policy (e,α,β) with $\alpha(\theta) < 1$ on some set with positive measure. We show in two steps that there exists an incentive-feasible policy $(\tilde{e},1,\beta)$ which improves the welfare.

First, since (e,α,β) is incentive-feasible, we derive the first-order condition of the bank's IC constraint. The bank's payoff from taking $\hat{e}$ is:

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left((\theta\hat{e}\alpha(\theta)+(1-\theta\hat{e})\beta(\theta))p(\theta \mid e,\alpha,\beta)+\theta\hat{e}(1-\alpha(\theta))\rho_H+(1-\theta\hat{e})\left(1-\beta(\theta)\right)\rho_L\right)d\theta - c(\hat{e}),$$

and the derivative with respect to $\hat{e}$ is:

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \theta\left\{(\alpha(\theta)p(\theta \mid e,\alpha,\beta)+(1-\alpha(\theta))\rho_H)-(\beta(\theta)p(\theta \mid e,\alpha,\beta)+(1-\beta(\theta))\rho_L)\right\}d\theta - c'(\hat{e}).$$

Hence, e satisfies:

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \theta\left\{(\alpha(\theta)p(\theta \mid e,\alpha,\beta)+(1-\alpha(\theta))\rho_H)-(\beta(\theta)p(\theta \mid e,\alpha,\beta)+(1-\beta(\theta))\rho_L)\right\}d\theta = c'(e).$$

Second, since $p(\theta \mid e,\alpha,\beta)$ is increasing in α, if $\alpha = 1$ then we have:

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \theta(1-\beta(\theta))(p(\theta \mid e,\beta)-\rho_L)d\theta > c'(e).$$

Then, there exists $\tilde{e} > e$ such that

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \theta(1-\beta(\theta))(p(\theta \mid \tilde{e},\beta)-\rho_L)d\theta = c'(\tilde{e}).$$

This is because the right-hand side is increasing in $\tilde{e}$ and diverges to infinity as $\tilde{e} \uparrow \frac{1}{2}$, while the left-hand side is bounded. Moreover, it can be seen that $(\tilde{e}, 1, \beta)$ satisfies the bank's IC constraint (more formally, Lemma 6 shows that the first-order approach is valid). As $(\tilde{e}, 1, \beta)$ improves the welfare, the proof is complete. □

Thus, the regulator's problem reduces to the one in Section 3.1.3. We call a policy (e, β) to be *incentive-feasible* if it satisfies Conditions (5) and (6). The rest of Appendix A.2.1 provides some preliminary observations regarding the regulator's problem.

We show that the first-order approach is valid.

Lemma 6. *The bank's IC constraint (6) can be replaced with its first-order condition:*

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\theta(1-\beta(\theta))(p(\theta \mid e,\beta)-\rho_L)d\theta = c'(e). \tag{16}$$

Proof of Lemma 6. First, we show that the first-order condition of the bank's IC constraint yields Expression (16). Given (e, β), the bank's payoff is $F(\hat{e} \mid e, \beta) - c(\hat{e})$, where

$$F(\hat{e} \mid e,\beta) := \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\left((\theta\hat{e} + (1-\theta\hat{e})\beta(\theta))p(\theta \mid e,\beta) + (1-\theta\hat{e})\left(1-\beta(\theta)\right)\rho_L\right)d\theta.$$

Denoting $F_1(\hat{e} \mid e, \beta) = \frac{\partial F}{\partial \hat{e}}(\hat{e} \mid e, \beta)$, the derivative of the bank's payoff with respect to $\hat{e}$ is:

$$F_1(\hat{e} \mid e,\beta) - c'(\hat{e}) = \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\theta(1-\beta(\theta))\left(p(\theta \mid e,\beta)-\rho_L\right)d\theta - c'(\hat{e}).$$

The first-order condition with respect to $\hat{e}$ at $\hat{e} = e$ is given by Expression (16).

Conversely, assume Expression (16). Hence,

$$F_1(x \mid x, \beta) = c'(x).$$

First, let $e > \hat{e}$. Then,

$$\begin{aligned} c(e) - c(\hat{e}) = \int_{\hat{e}}^{e} c'(x)dx &= \int_{\hat{e}}^{e} F_1(x \mid x,\beta)dx \\ &\leq \int_{\hat{e}}^{e} F_1(x \mid e,\beta)dx = F(e \mid e,\beta) - F(\hat{e} \mid e,\beta), \end{aligned}$$

where the inequality follows because $F_1(x \mid e, \beta)$ is non-decreasing in e. This shows that

$$F(e \mid e,\beta) - c(x) \geq F(\hat{e} \mid e,\beta) - c(\hat{e}).$$

Similarly, let $e < \hat{e}$. Then,

$$\begin{aligned} c(\hat{e}) - c(e) = \int_e^{\hat{e}} c'(x)dx &= \int_e^{\hat{e}} F_1(x \mid x, \beta)dx \\ &\geq \int_e^{\hat{e}} F_1(x \mid e, \beta)dx = F(\hat{e} \mid e, \beta) - F(e \mid e, \beta), \end{aligned}$$

where the inequality follows because $F_1(x \mid e, \beta)$ is non-decreasing in e. This shows that

$$F(e \mid e, \beta) - c(x) \geq F(\hat{e} \mid e, \beta) - c(\hat{e}).$$

□

Next, we prove some preliminary observations.

Lemma 7. *1. The full-information policy $(e^{\mathrm{FI}}, 0)$ is incentive-feasible.*

2. If a policy (e, β) is incentive-feasible, then $e \leq e^{\mathrm{FI}}$.

3. There is no incentive-feasible policy (e, β) with $\beta(\cdot) = 1$. No disclosure for all state realizations, i.e., $\beta(\cdot) = 1$, is never optimal.

Proof of Lemma 7. 1. We consider the full-information benchmark. To that end, consider a policy (e, β) with $e > 0$ and $\beta(\cdot) = 0$. Since $p(\theta \mid e, \beta) = v_H$, the bank's IC constraint is:

$$e \in \underset{\hat{e} \in [0, \frac{1}{2})}{\operatorname{argmax}} \frac{1}{2\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} \left(\theta \hat{e} v_H + (1 - \theta \hat{e})\, \rho_L\right) d\theta - c(\hat{e}).$$

This is the same problem as the full-information benchmark. Hence, $(e^{\mathrm{FI}}, 0)$ is incentive-feasible.

2. We show that any effort level $e > e^{\mathrm{FI}}$ cannot be implemented. To see this, we consider the bank's first-order condition. The bank's payoff is given as:

$$\frac{1}{2\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} \left((\theta \hat{e} + (1 - \theta \hat{e})\beta(\theta))p(\theta \mid e, \beta) + (1 - \theta \hat{e})\,(1 - \beta(\theta))\rho_L\right) d\theta - c(\hat{e}).$$

The derivative of the bank's payoff with respect to $\hat{e}$ is given as:

$$\frac{1}{2\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} \theta(1 - \beta(\theta))(p(\theta \mid e, \beta) - \rho_L) d\theta - c'(\hat{e}) \leq v_H - \rho_L - c'(\hat{e}).$$

If $e > e^{\mathrm{FI}}$ is implemented, then at $\hat{e} = e > e^{\mathrm{FI}}$, the bank's marginal payoff is negative.

3. Consider $\beta(\cdot) = 1$. Then the bank's problem is:

$$\max_{\hat{e}\in[0,\frac{1}{2})} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} p(\theta \mid e, \beta)d\theta - c(\hat{e}).$$

Thus, the bank would take $\hat{e} = 0$. Hence, if (e, β) is incentive-feasible, then $e = 0$. However, under such (e, β), the price is $p(\theta \mid e, \beta) = v_L < \rho_H$ for all θ. Thus, there is no incentive-feasible policy (e, β) such that $\beta(\cdot) = 1$.

□

The last part of the lemma states that the stress test that always gives the "passing grade" is not incentive feasible. As discussed in the main text, we will show below that if externality g is low enough then the full information will be optimal.

A.2.2 Proof of Proposition 1

We prove Proposition 1. To prove the second part, we prove the following two lemmas. The first lemma provides a sufficient condition under which full information is not optimal. The second lemma establishes conditions under which the above sufficient condition holds.

Lemma 8. *If*

$$g\frac{v_H - ((1-e^{\mathrm{FI}})v_L + e^{\mathrm{FI}}\rho_L)}{c''(e^{\mathrm{FI}})e^{\mathrm{FI}}} < (1-e^{\mathrm{FI}})(v_L + g - \rho_L), \tag{17}$$

then $(e^{\mathrm{FI}}, 0)$ *is not optimal.*

Proof of Lemma 8. To derive the sufficient condition (17), we consider a policy (e, β) such that $\beta(\cdot)$ is constant. Since the constraint on the price is slack when $\beta(\cdot) = 0$, we consider a relaxed problem in which the relevant constraint is the IC constraint. By Lemma 6, the problem is:

$$\max_{e,\beta} e(v_H + g) + (1-e)\left(\beta(v_L + g) + (1-\beta)\rho_L\right) - c(e)$$
$$\text{subject to } \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\theta(1-\beta)(p(\theta \mid e, \beta) - \rho_L)d\theta = c'(e).$$

Then, the Lagrangian is:

$$\begin{aligned}\mathcal{L} = {} & ev_H + (1-e)(\beta v_L + (1-\beta)\rho_L) - c(e) + (e + (1-e)\beta)g \\ & + \lambda\left(\frac{1-\beta}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\theta\left(\frac{\theta e v_H + (1-\theta e)\beta v_L}{\theta e + (1-\theta e)\beta} - \rho_L\right)d\theta - c'(e)\right) + \mu\beta,\end{aligned}$$

where μ is the Lagrange multiplier associated with $\beta \geq 0$.

The first-order condition with respect to e is:

$$v_H - (\beta v_L + (1-\beta)\rho_L) - c'(e) + (1-\beta)g + \lambda\left(\frac{1-\beta}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\frac{\theta^2\beta(v_H - v_L)}{(\theta e + (1-\theta e)\beta)^2}d\theta - c''(e)\right) = 0.$$

The first-order condition with respect to β is:

$$\begin{aligned}0 &= (1-e)(v_L + g - \rho_L) + \mu \\ &- \lambda\left(\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\theta\left(\frac{\theta e v_H + (1-\theta e)\beta v_L}{\theta e + (1-\theta e)\beta} - \rho_L\right)d\theta + \frac{1-\beta}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\theta\frac{(1-\theta e)\theta e(v_H - v_L)}{(\theta e + (1-\theta e)\beta)^2}d\theta\right).\end{aligned}$$

Suppose that $(e,\beta) = (e^{\mathrm{FI}}, 0)$ is optimal. We derive the first-order necessary conditions. At $(e,\beta) = (e^{\mathrm{FI}}, 0)$, the first-order condition with respect to e reduces to:

$$v_H - \rho_L - c'(e^{\mathrm{FI}}) + g = \lambda c''(e^{\mathrm{FI}}), \text{ that is, } g = \lambda c''(e^{\mathrm{FI}}).$$

At $(e,\beta) = (e^{\mathrm{FI}}, 0)$, the first-order condition with respect to β reduces to:

$$0 = (1-e^{\mathrm{FI}})(v_L + g - \rho_L) - \lambda\frac{v_H - ((1-e^{\mathrm{FI}})v_L + e^{\mathrm{FI}}\rho_L)}{e^{\mathrm{FI}}} + \mu.$$

In order for $(e,\beta) = (e^{\mathrm{FI}}, 0)$ to be an optimum, it is necessary that $\mu \geq 0$. Intuitively, the benefit from infinitesimally increasing e from e^{FI} exceeds the benefit from infinitesimally increasing β from 0. Hence, it is necessary that:

$$0 \leq \mu = g\frac{v_H - ((1-e^{\mathrm{FI}})v_L + e^{\mathrm{FI}}\rho_L)}{c''(e^{\mathrm{FI}})e^{\mathrm{FI}}} - (1-e^{\mathrm{FI}})(v_L + g - \rho_L).$$

Thus, under Expression (17), $(e,\beta) = (e^{\mathrm{FI}}, 0)$ cannot be optimal. □

Moving on to the second lemma:

Lemma 9. *If*

$$v_H - ((1-e^{\mathrm{FI}})v_L + e^{\mathrm{FI}}\rho_L) < (1-e^{\mathrm{FI}})c''(e^{\mathrm{FI}})e^{\mathrm{FI}}, \tag{18}$$

then there exists $\overline{g} > \rho_L - v_L$ such that Condition (17) holds if and only if $g > \overline{g}$. For instance, Expression (18) holds when $c(e) = \frac{ke^2}{1-2e}$ with $k > 0$.

Proof of Lemma 9. First, if Expression (18) holds, then there exists

$$\overline{g} = \frac{v_H - ((1-e^{\mathrm{FI}})v_L + e^{\mathrm{FI}}\rho_L)}{(1-e^{\mathrm{FI}})c''(e^{\mathrm{FI}})e^{\mathrm{FI}} - (v_H - ((1-e^{\mathrm{FI}})v_L + e^{\mathrm{FI}}\rho_L))}(\rho_L - v_L) > \rho_L - v_L$$

such that Condition (17) holds if and only if $g > \overline{g}$.

Second, assume $c(e) = \frac{ke^2}{1-2e}$. Then, $c'(e) = \frac{k}{2}\left(\frac{1}{(1-2e)^2} - 1\right)$, $(c')^{-1}(y) = \frac{1}{2}\left(1 - \sqrt{\frac{k}{k+2y}}\right)$, and $c''(e) = \frac{2k}{(1-2e)^3}$. Since

$$e^{\mathrm{FI}} = \frac{1}{2}\left(1 - \sqrt{\frac{k}{k+2(v_H - \rho_L)}}\right) \text{ and } 1 - e^{\mathrm{FI}} = \frac{1}{2}\left(1 + \sqrt{\frac{k}{k+2(v_H - \rho_L)}}\right),$$

Expression (18) reduces to:

$$\begin{aligned}&\frac{1}{2}\left(1 + \sqrt{\frac{k}{k+2(v_H - \rho_L)}}\right)(v_H - v_L) + \frac{1}{2}\left(1 - \sqrt{\frac{k}{k+2(v_H - \rho_L)}}\right)(v_H - \rho_L)\\ <&(v_H - v_L)\left(\frac{k}{k+2(v_H - \rho_L)}\right)^{-\frac{1}{2}}.\end{aligned}$$

Since $\rho_L > v_L$, this inequality holds because

$$\frac{k}{k+2(v_H - \rho_L)} < 1.$$

□

Now, we provide the proof of Proposition 1.

Proof of Proposition 1. 1. We denote by W the social welfare under a policy $(e, \beta(\cdot \mid e))$, where, since we will vary e, we denote by $\beta(\cdot \mid e)$. Then, we have:

$$W = e(v_H + g) + (1-e)\rho_L + \mathbb{E}\left[(1-\theta e)\beta(\theta \mid e)\right](v_L + g - \rho_L) - c(e).$$

Also, we denote by W^{FI} the social welfare under the full-information benchmark. Since trade yields externality g per unit of trading,

$$W^{\mathrm{FI}} = e^{\mathrm{FI}}(v_H + g) + (1 - e^{\mathrm{FI}})\rho_L - c(e^{\mathrm{FI}}).$$

Then, we have:

$$\frac{W^{\mathrm{FI}} - W}{e^{\mathrm{FI}} - e} = (v_H - \rho_L + g) - \frac{c(e^{\mathrm{FI}}) - c(e)}{e^{\mathrm{FI}} - e} + \frac{\mathbb{E}\left[(1-\theta e)(0 - \beta(\theta \mid e))\right]}{e^{\mathrm{FI}} - e}(v_L + g - \rho_L).$$

Since c is convex, we have

$$(v_H - \rho_L + g) - \frac{c(e^{\mathrm{FI}}) - c(e)}{e^{\mathrm{FI}} - e} \geq (v_H - \rho_L) - c'(e^{\mathrm{FI}}) + g = g.$$

Thus,

$$\frac{W^{\mathrm{FI}} - W}{e^{\mathrm{FI}} - e} \geq g + \frac{\mathbb{E}\left[(1-\theta e)(0 - \beta(\theta \mid e)\right]}{e^{\mathrm{FI}} - e}(v_L + g - \rho_L).$$

Hence, letting

$$M = \sup_{e \in [0, e^{\mathrm{FI}}]} -\frac{\mathbb{E}\left[(1-\theta e)(0 - \beta(\theta \mid e)\right]}{e^{\mathrm{FI}} - e},$$

if $M \leq 1$, then $W^{\mathrm{FI}} \geq W$; if $M > 1$, then $W^{\mathrm{FI}} \geq W$ if

$$g \leq \frac{M}{M-1}(\rho_L - v_L).$$

2. Lemma 8 establishes a sufficient condition under which full information is not optimal. Lemma 9 establishes conditions under which Lemma 8 holds, for instance, when the cost function is given by $c(e) = \frac{ke^2}{1-2e}$.

□

A.2.3 Characterization of the Optimal Disclosure Policy

As discussed in the main text, we characterize the form of the optimal disclosure policy β^{D} when partial disclosure is optimal (i.e., when full information disclosure is not optimal).

Proposition 4. *Suppose that an optimal policy $(e^{\mathrm{D}}, \beta^{\mathrm{D}})$ entails partial disclosure. Then, there exists $\overline{\theta}$ such that*

$$\beta^{\mathrm{D}}(\theta) = \begin{cases} \min\left(1, \frac{v_H - \rho_H}{\rho_H - v_L}\frac{\theta e^{\mathrm{D}}}{1-\theta e^{\mathrm{D}}}\right) & \text{if } \theta < \overline{\theta} \\ 0 & \text{if } \theta \geq \overline{\theta} \end{cases}.$$

Proof of Proposition 4. We consider the best disclosure policy β given e. Given e, we consider disclosure policies β that satisfy the constraint on the price:

$$\beta(\theta) \leq \frac{v_H - \rho_H}{\rho_H - v_L}\frac{\theta e}{1-\theta e} \text{ for all } \theta \leq \theta^*(e).$$

The regulator's objective then amounts to maximizing

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}(1-\theta e)\left(\beta(\theta)(v_L+g)+(1-\beta(\theta))\rho_L\right)d\theta$$

among e and β of the above form subject to the IC constraint

$$c'(e)=\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\theta(1-\beta(\theta))(p(\theta\mid e,\beta)-\rho_L)d\theta.$$

For the regulator, decreasing $\beta(\theta)$ leads to an instantaneous loss of $(1-\theta e)(v_L+g-\rho_L)$, which is decreasing in θ and e. Decreasing $\beta(\theta)$ relaxes the constraints (for the IC constraint, the right-hand side is decreasing in $\beta(\theta)$, as $p(\theta\mid e,\beta)$ is higher when θ is higher and $\beta(\theta)$ is lower). Thus, the optimal disclosure policy takes a bang-bang solution of the form

$$\beta(\theta)=\begin{cases}\min\left(1,\frac{v_H-\rho_H}{\rho_H-v_L}\frac{\theta e}{1-\theta e}\right) & \text{if } \theta<\overline{\theta}(e)\\ 0 & \text{if } \theta\geq\overline{\theta}(e)\end{cases}.$$

To see this, for any given incentive-feasible disclosure policy $(\tilde{e},\tilde{\beta})$, there exists an incentive-feasible disclosure policy (e,β) such that $e\geq\tilde{e}$ and β is of the above form. Moreover, if $(\tilde{e},\tilde{\beta})$ is not of this form for a set of θ with strictly positive measure, then the new disclosure policy (e,β) leads to a strict improvement. Note that the value of β at $\theta=\overline{\theta}(e)$ does not affect the regulator's objective and the constraints. The proof is complete by letting $\overline{\theta}=\overline{\theta}(e^{\mathrm{D}})$. □

We remark that, in principle, Proposition 4 implies that finding an optimal disclosure policy reduces to a finite-dimensional problem of finding $(e,\overline{\theta})$ that maximizes the regulator's objective function subject to the bank's IC constraint. Yet, since the first-order conditions of the regulator's problem are convoluted and add little economic insights, we characterize the form of the optimal disclosure policy in terms of the threshold $\overline{\theta}$.

A.3 Section 3.2

Proof of Proposition 2. The proof consists of five steps. In the first step, since the regulator's objective function depends on $\mathrm{med}(1-\varepsilon,\theta^*(e),1+\varepsilon)$, we categorize the following two cases. In the first case, the objective function is a piece-wise continuous function over three intervals. In the second case, the objective function is a piece-wise continuous function over two intervals.

1. The first case is when $\varepsilon \in (0,1)$ satisfies:

$$\frac{e^*}{1-\varepsilon} < \frac{1}{2}, \text{ that is, } \varepsilon < 1 - 2e^*. \tag{19}$$

   In this case, we need to consider the following three sub-cases on $e \in \left[0, \frac{1}{2}\right)$:

   (a) $e \in \left[0, \frac{e^*}{1+\varepsilon}\right]$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = 1+\varepsilon$;

   (b) $e \in \left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = \theta^*(e)$; and

   (c) $e \in \left[\frac{e^*}{1-\varepsilon}, \frac{1}{2}\right)$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = 1-\varepsilon$.

2. The second case is when $\varepsilon \in (0,1)$ satisfies:

$$\frac{1}{2} \leq \frac{e^*}{1-\varepsilon}, \text{ that is, } \varepsilon \geq 1 - 2e^*. \tag{20}$$

   In this case, we need to consider the following four sub-cases on $e \in [0, 1]$:

   (a) $e \in \left[0, \frac{e^*}{1+\varepsilon}\right]$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = 1+\varepsilon$; and

   (b) $e \in \left[\frac{e^*}{1+\varepsilon}, \frac{1}{2}\right)$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = \theta^*(e)$.

Since the objective function is a piece-wise continuous function for each case, optimal regulation e^{R} exists.

The second step shows that $e^{\mathrm{R}} \geq \frac{e^*}{1+\varepsilon}$, that is, Cases (1a) and (2a) are never optimal. In either case, Problem (7) reduces to:

$$\max_{e \in \left[0, \frac{e^*}{1+\varepsilon}\right]} \frac{1}{2\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e \rho_H + (1-\theta e)\,\rho_L\right) d\theta - c(e),$$

that is,

$$\max_{e \in \left[0, \frac{e^*}{1+\varepsilon}\right]} e\rho_H + (1-e)\,\rho_L - c(e).$$

Since the objective function is concave and since Condition (1) implies $\rho_H - \rho_L > c'\left(\frac{e^*}{1+\varepsilon}\right)$, it follows that the objective function is increasing on $\left[0, \frac{e^*}{1+\varepsilon}\right]$, and the proof of the second step is complete.

The third step shows that if $\varepsilon \leq 1 - \frac{e^*}{e^\diamond}$ then a unique optimal regulation is $e^{\mathrm{R}} = e^\diamond$, where $e^\diamond = (c')^{-1}(v_H - v_L)$. Take $\varepsilon \leq 1 - \frac{e^*}{e^\diamond}$. Then, $\frac{e^*}{1-\varepsilon} \leq e^\diamond$ implies that we are in Case 1.

Especially, we consider Case (1c), in which Problem (7) reduces to:

$$\max_{e\in\left[\frac{e^*}{1-\varepsilon},\frac{1}{2}\right)} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e v_H + (1-\theta e)\, v_L + g\right) d\theta - c(e),$$

that is,

$$\max_{e\in\left[\frac{e^*}{1-\varepsilon},\frac{1}{2}\right)} e v_H + (1-e)\, v_L + g - c(e).$$

As the objective function is strictly concave, the unique solution is

$$\max\left(\frac{e^*}{1-\varepsilon}, e^\diamond\right) = e^\diamond.$$

This is a solution of the entire problem, as the efficient value $e^\diamond v_H + (1-e^\diamond)v_L + g - c(e^\diamond)$, which is an upper bound of the entire problem, is attained.

The fourth step analyzes Cases (1b) and (2b). In each case, Problem (7) reduces to:

$$\max_{e\in\left[\frac{e^*}{1+\varepsilon},\frac{e^*}{1-\varepsilon}\right]\cap\left[0,\frac{1}{2}\right)} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{\theta^*(e)} \left(\theta e \rho_H + (1-\theta e)\, \rho_L\right) d\theta + \frac{1}{2\varepsilon}\int_{\theta^*(e)}^{1+\varepsilon} \left(\theta e v_H + (1-\theta e)\, v_L + g\right) d\theta - c(e),$$

that is,

$$\begin{aligned}\max_{e\in\left[\frac{e^*}{1+\varepsilon},\frac{e^*}{1-\varepsilon}\right]\cap\left[0,\frac{1}{2}\right)} &\frac{(v_L+g)(1+\varepsilon)-\rho_L(1-\varepsilon)}{2\varepsilon} + \frac{1}{2\varepsilon}\left(\frac{v_H-v_L}{2}(1+\varepsilon)^2 - \frac{\rho_H-\rho_L}{2}(1-\varepsilon)^2\right) e \\ &- \frac{1}{2\varepsilon}\left(\frac{v_H-\rho_H+\rho_L-v_L}{2}e^* + (v_L+g-\rho_L)\right)\frac{e^*}{e} - c(e).\end{aligned}$$

The objective function is a strictly concave function because the first two terms define an affine function in e and the third and fourth terms are a strictly concave function.

The first-order condition with respect to e is given by Expression (9):

$$\left(\frac{v_H-v_L}{2}(1+\varepsilon)^2 - \frac{\rho_H-\rho_L}{2}(1-\varepsilon)^2\right) + \left(\frac{v_H-\rho_H+\rho_L-v_L}{2}e^* + (v_L+g-\rho_L)\right)\frac{e^*}{e^2} = 2\varepsilon c'(e).$$

For the right-hand side, $c'(0)=0$, c' is increasing, and $\lim_{e\to\frac{1}{2}} c'(e) = \infty$. For the left-hand side, it diverges to infinity as $e \downarrow 0$ and it is decreasing. Hence, there is a unique $e^\dagger \in (0, \frac{1}{2})$ which satisfies the first-order condition. In fact, we show that $e^\dagger \in (e^\diamond, \frac{1}{2})$. To see this, it suffices to show:

$$\left(\frac{v_H - v_L}{2}(1+\varepsilon)^2 - \frac{\rho_H - \rho_L}{2}(1-\varepsilon)^2\right) + \left(\frac{v_H - \rho_H + \rho_L - v_L}{2}e^* + (v_L + g - \rho_L)\right)\frac{e^*}{(e^\diamond)^2}$$
$$> 2\varepsilon c'(e^\diamond).$$

Since $c'(e^\diamond) = v_H - v_L$, the above inequality reduces to:

$$\frac{v_H - \rho_H + \rho_L - v_L}{2}(1-\varepsilon)^2 + \left(\frac{v_H - \rho_H + \rho_L - v_L}{2}e^* + (v_L + g - \rho_L)\right)\frac{e^*}{(e^\diamond)^2} > 0,$$

which follows because $v_H > \rho_H$ and $v_L + g > \rho_L > v_L$.

The fifth step shows that

$$e^{\mathrm{R}} = \min\left(e^\dagger, \frac{e^*}{1-\varepsilon}\right) \text{ when } 1 - \frac{e^*}{e^\diamond} \le \varepsilon, \text{ i.e., } e^\diamond \le \frac{e^*}{1-\varepsilon}.$$

We start with Case 1. We have seen from the analysis of Case (1a) that the objective function is increasing on $\left[0, \frac{e^*}{1+\varepsilon}\right]$. For Case (1c), since $e^\diamond \le \frac{e^*}{1-\varepsilon}$, the objective function is decreasing on $\left[\frac{e^*}{1-\varepsilon}, \frac{1}{2}\right)$. Thus, the objective function is maximized on $\left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$. Since it is a strictly concave function on this interval, it has a unique maximizer $\min\left(e^\dagger, \frac{e^*}{1-\varepsilon}\right)$.

Next, we consider Case 2. We have seen from the analysis of Case (2a) that the objective function is increasing on $\left[0, \frac{e^*}{1+\varepsilon}\right]$. For Cases (2b), as $e^\dagger \in \left[\frac{e^*}{1+\varepsilon}, \frac{1}{2}\right)$, the objective function is uniquely maximized at $e^\dagger = \min\left(e^\dagger, \frac{e^*}{1-\varepsilon}\right)$, as $e^\dagger < \frac{1}{2} \le \frac{e^*}{1-\varepsilon}$.

Thus, $e^{\mathrm{R}} = \min\left(e^\dagger, \frac{e^*}{1-\varepsilon}\right)$ holds for each possibility. □

Proof of Corollary 1. Proposition 2 implies the following. When $\varepsilon \le 1 - \frac{e^*}{e^\diamond}$, trades occur for all $\theta \in [1-\varepsilon, 1+\varepsilon]$. When $1 - \frac{e^*}{e^\diamond} \le \varepsilon$, as long as $\frac{e^*}{1-\varepsilon} \le e^\dagger$, trades occur for all $\theta \in [1-\varepsilon, 1+\varepsilon]$. On the one hand, $\frac{e^*}{1-\varepsilon}$ is increasing in ε and diverges to infinity as $\varepsilon \uparrow 1$. On the other hand, at $\varepsilon = 1 - \frac{e^*}{e^\diamond}$ (i.e., $e^\diamond = \frac{e^*}{1-\varepsilon}$), we have $e^\dagger > \frac{e^*}{1-\varepsilon}$. We show that $e^\dagger$ is decreasing in ε. Since $e^\dagger$ is characterized by Expression (9), differentiating both sides of Expression (9) with respect to ε yields:

$$(v_H - v_L)(1+\varepsilon) + (\rho_H - \rho_L)(1-\varepsilon) - 2c'(e^\dagger)$$
$$= \left(2\left(\frac{v_H - \rho_H + \rho_L - v_L}{2}e^* + (v_L + g - \rho_L)\right)\frac{e^*}{(e^\dagger)^3} + 2\varepsilon c''(e^\dagger)\right)\frac{\partial e^\dagger}{\partial \varepsilon}.$$

Since the left-hand side satisfies

$$\begin{aligned}
&(v_H - v_L)(1+\varepsilon) + (\rho_H - \rho_L)(1-\varepsilon) - 2c'(e^\dagger) \\
<&(v_H - v_L)(1+\varepsilon) + (\rho_H - \rho_L)(1-\varepsilon) - (v_H - v_L)\frac{(1+\varepsilon)^2}{2} + (\rho_H - \rho_L)\frac{(1-\varepsilon)^2}{2} \\
=&-\frac{(1+\varepsilon)(1-\varepsilon)}{2\varepsilon}(v_H - \rho_H + \rho_L - v_L) < 0,
\end{aligned}$$

it follows that $e^\dagger$ is decreasing in ε.

Thus, there exists a unique $\overline{\varepsilon} \in (0,1)$ such that $\frac{e^*}{1-\varepsilon} = e^\dagger$. The statement of the proposition holds with this $\overline{\varepsilon}$. $\square$

Finally, as discussed in the main text, we formulate and prove the following comparative-statics result on the optimal regulation e^{R}.

Corollary 3. *1. e^{R} is not monotone in ε.*

2. e^{R} is non-decreasing in g.

Proof of Corollary 3. 1. First, when $\varepsilon \leq 1 - \frac{e^*}{e^\diamond}$, $e^{\mathrm{R}} = e^\diamond$ does not depend on ε. Second, when $e^\dagger \geq \frac{e^*}{1-\varepsilon}$, $e^{\mathrm{R}} = \frac{e^*}{1-\varepsilon}$ is increasing in ε. Third, when $e^\dagger \leq \frac{e^*}{1-\varepsilon}$, $e^{\mathrm{R}} = e^\dagger$ is decreasing in ε.

2. Since $e^\diamond$ and $\frac{e^*}{1-\varepsilon}$ do not depend on g, it suffices to show that $e^\dagger$ is non-decreasing in g. Take $\tilde{g}$ with $\tilde{g} > g$. Suppose to the contrary that $\tilde{e}^* \leq e^\dagger$. Then, while the left-hand side of Expression (9) is strictly increased, the right-hand side of Expression (9) is weakly decreased. This is a contradiction.

$\square$

A.4 Section 4

Proof of Lemma 2. Since $v_L + g > \rho_L$, the regulator maximizes the social welfare by maximizing $\beta(\theta)$ for each given θ subject to $p(\theta \mid e, \beta) \geq \rho_H$. When $\theta \leq \theta^*(e)$, setting $\beta(\theta)$ as in Expression (10) makes the price at θ equal to ρ_H. For states in which trade would occur irrespective of the disclosure policy, we set $\beta(\theta) = 1$. This leads to Expression (10). $\square$

Proof of Lemma 3. 1. Suppose that $\beta(\theta) = 0$ for some $\theta \in [1-\varepsilon, 1+\varepsilon]$. Then, the price at θ satisfies

$$v_H = \rho_H,$$

which is a contradiction. Hence, $\beta(\theta) > 0$ for all $\theta \in [1-\varepsilon, 1+\varepsilon]$.

2. It is without loss to consider the case in which $\bar{e}_M \geq \tilde{e} \geq e$. It follows from Expression (2) that $\theta^*(\tilde{e}) \leq \theta^*(e)$. For any $\theta \in [1-\varepsilon, 1+\varepsilon]$ with $\theta \geq \theta^*(\tilde{e})$, we have $\tilde{\beta}(\theta) = 1 \geq \beta(\theta)$. For any $\theta \in [1-\varepsilon, 1+\varepsilon]$ with $\theta < \theta^*(\tilde{e})$, it follows from

$$\frac{\partial \beta(\theta)}{\partial e} = \frac{v_H - \rho_H}{\rho_H - v_L} \frac{\theta}{(1-\theta e)^2} > 0$$

that

$$\tilde{\beta}(\theta) = \frac{v_H - \rho_H}{\rho_H - v_L} \frac{\theta \tilde{e}}{1-\theta \tilde{e}} \geq \frac{v_H - \rho_H}{\rho_H - v_L} \frac{\theta e}{1-\theta e} = \beta(\theta).$$

The proof is complete.

□

Proof of Theorem 1. The structure of the proof resembles that of Proposition 2. The proof consists of five steps. In the first step, since the regulator's objective function depends on $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon)$, we categorize the following two cases. In the first case, the objective function is a piece-wise continuous function over three intervals. In the second case, the objective function is a piece-wise continuous function over two intervals.

1. The first case is when $\varepsilon \in (0,1)$ satisfies:

$$\frac{e^*}{1-\varepsilon} < \frac{1}{2}, \text{ that is, } \varepsilon < 1 - 2e^*. \tag{21}$$

   In this case, we need to consider the following three sub-cases on $e \in \left[0, \frac{1}{2}\right)$:

   (a) $e \in \left[0, \frac{e^*}{1+\varepsilon}\right]$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = 1+\varepsilon$;

   (b) $e \in \left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = \theta^*(e)$; and

   (c) $e \in \left[\frac{e^*}{1-\varepsilon}, \frac{1}{2}\right)$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = 1-\varepsilon$.

2. The second case is when $\varepsilon \in (0,1)$ satisfies:

$$\frac{1}{2} \leq \frac{e^*}{1-\varepsilon}, \text{ that is, } 1 - 2e^* \leq \varepsilon. \tag{22}$$

   In this case, we need to consider the following two sub-cases on $e \in \left[0, \frac{1}{2}\right)$:

   (a) $e \in \left[0, \frac{e^*}{1+\varepsilon}\right]$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = 1+\varepsilon$; and

   (b) $e \in \left[\frac{e^*}{1+\varepsilon}, \frac{1}{2}\right)$, in which $\text{med}(1-\varepsilon, \theta^*(e), 1+\varepsilon) = \theta^*(e)$.

Since the objective function is a piece-wise continuous function for each case, optimal regulation e^{RD} exists.

The second step shows that $e^{\mathrm{RD}} \geq \frac{e^*}{1+\varepsilon}$, that is, Cases (1a) and (2a) are never optimal. In either case, Problem (11) reduces to:

$$\max_{e\in\left[0,\frac{e^*}{1+\varepsilon}\right]} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\left(\rho_L+\frac{\rho_H-\rho_L+g}{e^*}\theta e\right)d\theta - c(e),$$

that is,

$$\max_{e\in\left[0,\frac{e^*}{1+\varepsilon}\right]} \rho_L+\frac{\rho_H-\rho_L+g}{e^*}e - c(e).$$

Since the objective function is strictly concave and Condition (1) implies

$$\frac{\rho_H-\rho_L+g}{e^*} > v_H - v_L > c'\left(\frac{e^*}{1+\varepsilon}\right),$$

it follows that the objective function is increasing on $\left[0,\frac{e^*}{1+\varepsilon}\right]$, and the proof of the second step is complete.

The third step shows that if $\varepsilon \leq 1-\frac{e^*}{e^\diamond}$ then a unique optimal regulation is $e^{\mathrm{RD}} = e^\diamond$. Take $\varepsilon \leq 1-\frac{e^*}{e^\diamond}$. Then, $\frac{e^*}{1-\varepsilon} \leq e^\diamond$ and $e^\diamond < \frac{1}{2}$ imply that we are in Case 1. Especially, we consider Case (1c), in which Problem (11) reduces to:

$$\max_{e\in\left[\frac{e^*}{1-\varepsilon},\frac{1}{2}\right)} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon}\left(\theta e v_H+(1-\theta e)v_L+g\right)d\theta - c(e),$$

that is,

$$\max_{e\in\left[\frac{e^*}{1-\varepsilon},\frac{1}{2}\right)} v_L+g+(v_H-v_L)e-c(e).$$

Thus, the analysis reduces to that of Case (1c) in the proof of Proposition 2. As the objective function is strictly concave, the unique solution is $e^\diamond$. This is a solution of the entire problem, as the first-best value $e^\diamond v_H+(1-e^\diamond)v_L+g-c(e^\diamond)$, which is an upper bound of the entire problem, is attained.

The fourth step analyzes Cases (1b) and (2b). In each case, Problem (11) reduces to:

$$\max_{e\in\left[\frac{e^*}{1+\varepsilon},\frac{e^*}{1-\varepsilon}\right]\cap\left[0,\frac{1}{2}\right)} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{\theta^*(e)}\left(\rho_L+\frac{\rho_H-\rho_L+g}{e^*}e\theta\right)d\theta+\frac{1}{2\varepsilon}\int_{\theta^*(e)}^{1+\varepsilon}\left(\theta e v_H+(1-\theta e)v_L+g\right)d\theta-c(e).$$

Thus, the problem is:

$$\max_{e\in\left[\frac{e^*}{1+\varepsilon},\frac{e^*}{1-\varepsilon}\right]\cap\left[0,\frac{1}{2}\right)} \frac{(v_L+g)(1+\varepsilon)-\rho_L(1-\varepsilon)}{2\varepsilon}+\frac{1}{2\varepsilon}\left(\frac{v_H-v_L}{2}(1+\varepsilon)^2-\frac{\rho_H-\rho_L+g}{2e^*}(1-\varepsilon)^2\right)e$$
$$-\frac{1}{2\varepsilon}\frac{v_L+g-\rho_L}{2}\frac{e^*}{e}-c(e).$$

The objective function is a strictly concave function because the first two terms define an affine function in e and the third and fourth terms are a strictly concave function.

The first-order condition is given by Expression (13), or:

$$\frac{v_H-v_L}{2}(1+\varepsilon)^2-\frac{\rho_H-\rho_L+g}{2e^*}(1-\varepsilon)^2+\frac{v_L+g-\rho_L}{2}\frac{e^*}{e^2}=2\varepsilon c'(e). \tag{23}$$

The right-hand side is increasing in e, approaches 0 as $e\downarrow 0$, and diverges to infinity as $e\uparrow\frac{1}{2}$. The left-hand side goes to infinity as $e\downarrow 0$, and is decreasing in e. Hence, there is a unique solution $e^\ddagger$ of Expression (13) in $(0,\frac{1}{2})$. In fact, we show that $e^\ddagger\geq e^\diamond$ when $\varepsilon\geq 1-\frac{e^*}{e^\diamond}$. To see this, it suffices to show:

$$\frac{v_H-v_L}{2}(1+\varepsilon)^2-\frac{\rho_H-\rho_L+g}{2e^*}(1-\varepsilon)^2+\frac{v_L+g-\rho_L}{2}\frac{e^*}{(e^\diamond)^2}\geq 2\varepsilon c'(e^\diamond).$$

Since $c'(e^\diamond)=v_H-v_L$, the above inequality reduces to:

$$\frac{(1-\varepsilon)^2}{2}\left(v_H-v_L-\frac{\rho_H-\rho_L+g}{e^*}\right)+\frac{v_L+g-\rho_L}{2}\frac{e^*}{(e^\diamond)^2}\geq 0,$$

that is,

$$\frac{v_L+g-\rho_L}{2e^*}\left(\left(\frac{e^*}{e^\diamond}\right)^2-(1-\varepsilon)^2\right)\geq 0,$$

which follows when $\varepsilon\geq 1-\frac{e^*}{e^\diamond}$. In fact, when $\varepsilon=1-\frac{e^*}{e^\diamond}$, $e=e^\diamond$ satisfies the first-order condition.

We also show that $e^\ddagger\geq\frac{e^*}{1-\varepsilon}$ when $\varepsilon\leq 1-\frac{e^*}{e^\diamond}$. To that end, it suffices to show:

$$\frac{v_H-v_L}{2}(1+\varepsilon)^2-\frac{\rho_H-\rho_L+g}{2e^*}(1-\varepsilon)^2+\frac{v_L+g-\rho_L}{2}\frac{(1-\varepsilon)^2}{e^*}\geq 2\varepsilon c'\left(\frac{e^*}{1-\varepsilon}\right),$$

that is,

$$\frac{v_H - v_L}{2}(1+\varepsilon)^2 - \frac{\rho_H - v_L}{2e^*}(1-\varepsilon)^2 \geq 2\varepsilon c'\left(\frac{e^*}{1-\varepsilon}\right).$$

When $\varepsilon \leq 1 - \frac{e^*}{e^\diamond}$, we have $e^\diamond \geq \frac{e^*}{1-\varepsilon}$. Thus, it suffices to show:

$$\frac{v_H - v_L}{2}(1+\varepsilon)^2 - \frac{\rho_H - v_L}{2e^*}(1-\varepsilon)^2 \geq 2\varepsilon c'(e^\diamond) = 2\varepsilon(v_H - v_L),$$

that is,

$$\frac{v_H - v_L}{2\varepsilon}(1-\varepsilon)^2 \geq \frac{\rho_H - v_L}{2\varepsilon e^*}(1-\varepsilon)^2,$$

which holds with equality. Similarly, $e^\ddagger \leq \frac{e^*}{1-\varepsilon}$ when $\varepsilon \geq 1 - \frac{e^*}{e^\diamond}$.

The fifth step shows:

$$e^{\mathrm{RD}} = e^\ddagger \text{ when } 1 - \frac{e^*}{e^\diamond} \leq \varepsilon, \text{ i.e., } e^\diamond \leq \frac{e^*}{1-\varepsilon}.$$

We start with Case 1. We have seen from the analysis of Case (1a) that the objective function is increasing on $\left[0, \frac{e^*}{1+\varepsilon}\right]$. For Case (1c), since $e^\diamond \leq \frac{e^*}{1-\varepsilon}$, the objective function is decreasing on $\left[\frac{e^*}{1-\varepsilon}, \frac{1}{2}\right)$. Thus, the objective function is maximized on $\left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$. Since it is a strictly concave function on this interval, it has a unique maximizer $\min\left(e^\ddagger, \frac{e^*}{1-\varepsilon}\right) = e^\ddagger$.

Next, we consider Case 2. We have seen from the analysis of Case (2a) that the objective function is increasing on $\left[0, \frac{e^*}{1+\varepsilon}\right]$. Thus, the objective function is maximized on $\left[\frac{e^*}{1+\varepsilon}, \frac{1}{2}\right)$. Since it is a strictly concave function on this interval, it has a unique maximizer $e^\ddagger$. □

Proof of Corollary 2. 1. First, when $\varepsilon \leq 1 - \frac{e^*}{e^\diamond}$, we have $e^{\mathrm{R}}(\varepsilon) = e^\diamond = e^{\mathrm{RD}}(\varepsilon)$. Second, when $1 - \frac{e^*}{e^\diamond} \leq \varepsilon$, to show $e^{\mathrm{R}}(\varepsilon) \geq e^{\mathrm{RD}}(\varepsilon)$, it suffices to show that $e^\dagger > e^\ddagger$. To that end, we show:

$$\begin{aligned}
&\left(\frac{v_H - v_L}{2}(1+\varepsilon)^2 - \frac{\rho_H - \rho_L}{2}(1-\varepsilon)^2\right) + \left(\frac{v_H - \rho_H + \rho_L - v_L}{2}e^* + (v_L + g - \rho_L)\right)\frac{e^*}{(e^\ddagger)^2} \\
>&2\varepsilon c'(e^\ddagger) \\
=&\frac{v_H - v_L}{2}(1+\varepsilon)^2 - \frac{\rho_H - \rho_L + g}{2e^*}(1-\varepsilon)^2 + \frac{v_L + g - \rho_L}{2}\frac{e^*}{(e^\ddagger)^2}.
\end{aligned}$$

The above inequality follows because it reduces to:

$$\frac{(\rho_H - \rho_L) + (v_H - v_L)g}{\rho_H - v_L}\frac{(1-\varepsilon)^2}{2} + \left(\frac{v_H - \rho_H + \rho_L - v_L}{2}e^* + \frac{v_L + g - \rho_L}{2}\right)\frac{e^*}{(e^\ddagger)^2} > 0.$$

2. Let g and $\tilde{g}$ be such that $\tilde{g} > g$. Denote by $\Delta := \tilde{g} - g$. Let (e_g, β_g) be a solution to Problem (11) under g, and let $(e_{\tilde{g}}, \beta_{\tilde{g}})$ be a solution to Problem (11) under $\tilde{g}$. Observe that (e_g, β_g) and $(e_{\tilde{g}}, \beta_{\tilde{g}})$ are feasible, i.e., satisfy Expression (10). On the one hand, we have:

$$\begin{aligned}
&\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e_g(v_H+g) + (1-\theta e_g)\left(\beta_g(\theta)(v_L+g) + (1-\beta_g(\theta))\rho_L\right)\right) d\theta - c(e_g)\\
\geq &\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e_{\tilde{g}}(v_H+g) + (1-\theta e_{\tilde{g}})\left(\beta_{\tilde{g}}(\theta)(v_L+g) + (1-\beta_{\tilde{g}}(\theta))\rho_L\right)\right) d\theta - c(e_{\tilde{g}}).
\end{aligned}$$

On the other hand, we have:

$$\begin{aligned}
&\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e_{\tilde{g}}(v_H+\tilde{g}) + (1-\theta e_{\tilde{g}})\left(\beta_{\tilde{g}}(\theta)(v_L+\tilde{g}) + (1-\beta_{\tilde{g}}(\theta))\rho_L\right)\right) d\theta - c(e_{\tilde{g}})\\
\geq &\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e_g(v_H+\tilde{g}) + (1-\theta e_g)\left(\beta_g(\theta)(v_L+\tilde{g}) + (1-\beta_g(\theta))\rho_L\right)\right) d\theta - c(e_g).
\end{aligned}$$

Adding both inequalities yields:

$$\frac{\Delta}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left\{\left(\theta e_{\tilde{g}} - \theta e_g\right)\left(1-\beta_g(\theta)\right) + \left(1-\theta e_{\tilde{g}}\right)\left(\beta_{\tilde{g}}(\theta) - \beta_g(\theta)\right)\right\} d\theta \geq 0. \tag{24}$$

Suppose to the contrary that $e_g > e_{\tilde{g}}$. Then, it follows from Lemma **??** that $\beta_g \geq \beta_{\tilde{g}}$. Then, it follows from Expression (24) that, for all $\theta \in [1-\varepsilon, 1+\varepsilon]$,

$$\left(\theta e_{\tilde{g}} - \theta e_g\right)\left(1-\beta_g(\theta)\right) + \left(1-\theta e_{\tilde{g}}\right)\left(\beta_{\tilde{g}}(\theta) - \beta_g(\theta)\right) = 0.$$

Then, we obtain $\beta_g(\theta) = \beta_{\tilde{g}}(\theta) = 1$ for all $\theta \in [1-\varepsilon, 1+\varepsilon]$. Under $\beta(\cdot) = 1$, consider the problem under generic g:

$$\max_{e\in[0,\frac{1}{2})} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e(v_H+g) + (1-\theta e)\left(v_L+g\right)\right) d\theta - c(e).$$

The solution does not depend on g, as the problem can be written as

$$\max_{e\in[0,\frac{1}{2})} ev_H + (1-e)v_L + g - c(e).$$

Thus, we cannot have $e_g > e_{\tilde{g}}$. Hence, it must be the case that $e_{\tilde{g}} \geq e_g$.

$\square$

A.5 Section 5.2

Proof of Proposition 3. The proof consists of three steps. The first step shows $e^{\mathrm{BR}} \geq \frac{e^*}{1+\varepsilon}$ by showing that the bank's payoff is increasing on $\left[0, \frac{e^*}{1+\varepsilon}\right]$. When $e \leq \frac{e^*}{1+\varepsilon}$, the bank's payoff is:

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \left(\theta e \rho_H + (1-\theta e)\rho_L\right) d\theta - c(e) = e\rho_H + (1-e)\rho_L - c(e).$$

Since it follows from Condition (1) that $(c')^{-1}(\rho_H - \rho_L) > \frac{e^*}{1+\varepsilon}$, the bank's payoff is increasing on $\left[0, \frac{e^*}{1+\varepsilon}\right]$.

The second step shows that $e^{\mathrm{BR}} \leq \max\left(e^\diamond, \frac{e^*}{1-\varepsilon}\right)$. The second step also shows that if $\varepsilon \leq 1 - \frac{e^*}{e^\diamond}$, then $e^{\mathrm{BR}} = e^\diamond$. The proof consists of two sub-steps. In the first sub-step, suppose that $e^\diamond \geq \frac{e^*}{1-\varepsilon}$. Then, if $e = e^\diamond$, the bank's payoff is:

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \theta e v_H + (1-\theta e)v_L d\theta - c(e) = e v_H + (1-e)v_L - c(e),$$

which is an upper bound of the bank's payoff. Thus, if $\varepsilon \leq 1 - \frac{e^*}{e^\diamond}$, then $e^{\mathrm{BR}} = e^\diamond$. In the second sub-step, suppose that $e^\diamond < \frac{e^*}{1-\varepsilon}$. If $e \geq \frac{e^*}{1-\varepsilon}$, then the bank's payoff is:

$$\frac{1}{2\varepsilon}\int_{1-\varepsilon}^{1+\varepsilon} \theta e v_H + (1-\theta e)v_L d\theta - c(e) = e v_H + (1-e)v_L - c(e).$$

Since this is maximized at $e^\diamond < \frac{e^*}{1-\varepsilon}$, it follows that the bank's payoff is decreasing when $e \geq \frac{e^*}{1-\varepsilon}$.

In sum, the second step implies that if $\varepsilon > 1 - \frac{e^*}{e^\diamond}$, then the bank's problem reduces to:

$$\max_{e \in \left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right] \cap \left[0, \frac{1}{2}\right)} \frac{1}{2\varepsilon}\int_{1-\varepsilon}^{\theta^*(e)} \left(\theta e \rho_H + (1-\theta e)\rho_L\right) d\theta + \frac{1}{2\varepsilon}\int_{\theta^*(e)}^{1+\varepsilon} \left(\theta e v_H + (1-\theta e)v_L\right) d\theta - c(e). \tag{25}$$

The third step shows that if $\varepsilon > 1 - \frac{e^*}{e^\diamond}$, then Problem (25) has a unique maximizer in $\left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$, which is specified by the lemma. To that end, differentiating the bank's payoff function in Problem (25) with respect to e yields

$$\begin{aligned}
&\frac{\rho_H - \rho_L}{4\varepsilon}\left(\frac{(e^*)^2}{e^2} - (1-\varepsilon)^2\right) + \frac{v_H - v_L}{4\varepsilon}\left((1+\varepsilon)^2 - \frac{(e^*)^2}{e^2}\right) + \frac{\rho_H - \rho_L}{2\varepsilon}\frac{e^*(1-e^*)}{e^2} - c'(e)\\
=&\frac{v_H - v_L}{4\varepsilon}(1+\varepsilon)^2 - \frac{\rho_H - \rho_L}{4\varepsilon}(1-\varepsilon)^2 + \frac{(v_H - \rho_H + \rho_L - v_L)e^* + 2(v_L - \rho_L)}{4\varepsilon}\frac{e^*}{e^2} - c'(e).
\end{aligned}$$

Thus, we obtain Expression (15) as the first-order condition. If there is no $e \in \left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$

such that Expression (15) holds, then the unique solution of the bank's problem is $e^{\mathrm{BR}} = \frac{e^*}{1-\varepsilon}$. Observe that if this is the case then $\frac{e^*}{1-\varepsilon} < \frac{1}{2}$, as there exists some $e \in \left(\frac{e^*}{1+\varepsilon}, \frac{1}{2}\right)$ which satisfies the first-order condition. This is because, while $c'(e) \uparrow \infty$ as $e \uparrow \frac{1}{2}$, the right-hand side of the first-order condition is bounded on $\left[\frac{e^*}{1+\varepsilon}, \frac{1}{2}\right]$. In fact, such e is unique. Also, comparing Expressions (9) and (15) implies that $e < e^{\mathrm{R}}$.

Thus, suppose that there exists $e \in \left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$ such that Expression (15) holds. If

$$(v_H - \rho_H + \rho_L - v_L)e^* \geq 2(\rho_L - v_L),$$

then the left-hand side of the first-order condition is weakly decreasing while the right-hand side is increasing. Thus, e is the unique solution of the bank's problem. If

$$(v_H - \rho_H + \rho_L - v_L)e^* < 2(\rho_L - v_L),$$

then the left-hand side of the first-order condition is increasing and concave while the right-hand side is increasing and convex. Thus, there exists a unique $e \in \left[\frac{e^*}{1+\varepsilon}, \frac{e^*}{1-\varepsilon}\right]$ that satisfies the first-order condition, and this e is the unique solution of the bank's problem. The proof is complete. □