A Trilemma for Asset Demand Estimation

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Motivation

Growing interest in using portfolio holdings data to answer questions in finance and macro.

1. What is the effect of quantitative easing?
2. How do "shocks" to large financial institutions affect equilibrium prices.

Current paradigm: adapt demand estimation approaches from IO and measure *asset demand functions*.

We are interested in the conceptual foundations of this approach

1. Is asset-level demand estimation the right paradigm for empirical analysis of portfolio data?

2. When can asset-level elasticities be (nonparametrically) identified from observational data?
   – Can ideal asset-level supply shocks reveal substitution patterns beyond those implied by theory?

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2. When can asset-level elasticities be (nonparametrically) identified from observational data?
   – Can ideal asset-level supply shocks reveal substitution patterns beyond those implied by theory?

Short answer: no and no, except in unrealistic knife-edge cases. (But maybe there are solutions?)

Why might finance be different?

1. Investors (generally) do not care about assets per se – they care about asset *payoffs*.
   - Standard approach: expected utility over state-contingent consumption or wealth.
   - Instrumental demand for assets to achieve desired consumption profile $\{c(z)\}$.

2. Flexible bundling and unbundling of assets into payoffs through *portfolio formation*.
   - Continuous choice over bundles with endogenous complementarity, disciplined by *relative* prices.

3. No sharp distinction between buyers and sellers – anybody can switch depending on prices.

4. Assets are investment goods whose value depends on future prices (i.e., others' demand).

The standard model

- Investor i can choose consumption at date 0 or at date 1.
- At date 1, tate $z \in \mathcal{Z} \equiv \{1, \ldots, Z\}$ is realized with probability $\pi_z \in (0, 1)$.

- Assets $\mathcal{J} \equiv \{1, \ldots, J\}$ with price p_j and state-contingent cash flows $y_j(z)$.
  - Completely generic definition of assets: encompasses bonds, stocks, derivatives, or any bundles.
  - $J \times Z$ payoff matrix Y known to investors but unobserved by econometrician. Typically assume $Z \geq J$ and $rank(Y) = J$.

- Investors choose a portfolio $(a_j^i)_{j \in \mathcal{J}}$ to maximize expected utility over consumption.
  - Continuous choice. In general, a convex combination of feasible bundles is also a feasible bundle.

- Investors receive asset endowments e_j^i and non-marketable endowment w_0^i and $w_1^i(z)$.
- Prices taken as given by each investor, ultimately determined by market clearing.

Standard portfolio choice problem under expected utility

$$\max_{(a^i_1, a^i_2, \ldots, a^i_J)} \quad (1-\delta)u(c^i_0) + \delta \sum_z \pi_z u\Big(\tilde{c}^i(z)\Big)$$

$$\text{s.t.} \quad c^i_1(z) \equiv \sum_j y_j(z) a^i_j + w^i_1(z),$$

$$c^i_0 + \sum p_j (a^i_j - e^i_j) = w^i_0,$$

& potential portfolio restrictions or mandates.

NB: This is separable across states. Recursive utility models as in Epstein-Zin or Kreps-Porteus are not.

To ensure well-behaved portfolio choices, finance relies on "no arbitrage"

- Principle of NA: price system should not allow you to get "something for nothing."

- Example with three assets with prices p_1, p_2 and p_3 and payoff matrix

$$Y = \begin{array}{c c} & \begin{array}{cc} \text{State 1} & \text{State 2} \end{array} \\ \begin{array}{c} \text{Asset 1} \\ \text{Asset 2} \\ \text{Asset 3} \end{array} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \end{array}$$

Prices better satisfy $p_3 = p_1 + p_2$. If not, any investor can generate infinite wealth by shorting one side.

Implications of no arbitrage

- If there is no arbitrage, then there exists a $Z \times 1$ **state price vector** q such that

$$p = Yq.$$

  - Interpretation: q measures the cost of a unit state-contingent consumption.
  - State prices are central because optimal consumption plans depend on the cost of consumption.

- Complete markets: q is unique. Incomplete markets: we may "disagree" outside of the asset span.

Main result: limits to asset demand identification

Assume that payoff matrix Y is unobserved. It is impossible to simultaneously maintain that:

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2. prices satisfy no arbitrage,
3. asset-level demand elasticities are identified from supply shocks to individual securities.

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Reason: cross-asset restrictions from (1) and (2) prevent *ceteris paribus* price variation.

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Exceptions: the asset menu consists of Arrow securities, or one has *many* independent experiments.

Approach

Ideal benchmark: **assume the existence of exogenous asset-level supply shocks.**

Can we (nonparametrically) identify demand elasticities from price variation based on these shocks?

Demand elasticities in “state price language”

In arbitrage-free markets, state prices are what matters for optimal plans.

“How does asset demand change if asset price p_j changes but all other asset prices remain fixed?”

– Formally, a **partial derivative** with respect to a single price.

$\Rightarrow$ *“How do investors respond to the state price changes induced by the shock to asset prices?”*

State price changes given an ideal "ceteris paribus" asset price change

Let Y^+ denote the Moore-Penrose inverse. Given no arbitrage, state prices satisfy

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Lemma. Let v_j denote the unit vector in $\mathbb{R}^J$ with 1 in the j-th position and zeros elsewhere. Then the changes in state prices given the exogenous variation in a single price p_j satisfy

$$\boxed{\Delta \mathbf{q}_j^{\text{ideal}} \equiv \frac{\partial q}{\partial p_j} = Y^+ v_j.}$$

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Ideal experiment requires state price variation proportional to the **inverse payoff matrix**.

Identification from (ideal) supply shocks

Assume that we have an "ideal" laboratory with exogenous shocks to the supply of some asset j.

- For example, an outside investor helicopter drops an asset for purely exogenous reasons.

Does the resulting price variation identify a demand elasticity with respect to price p_j?

To understand equilibrium effects, we impose only a weak (and favorable) condition

Definition (Separable downward-sloping consumption demand)

Consumption demand is separable downward-sloping if $\exists$ a strictly positive $Z \times Z$ diagonal matrix V s.t.

$$\boxed{\Delta\mathbf{q}_j^{\text{supply}} \equiv \frac{\partial q}{\partial E_j} = -V y_j^{\mathrm{T}} \qquad \text{for all assets } j,}$$

– Standard interpretation: V captures the marginal utility of the marginal investor.

– V diagonal is perhaps the best case – no direct cross-state spillovers from supply shocks.

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State price changes induced by supply shocks are proportional to the **payoff matrix itself**.

Do supply shocks generate the "ideal" state price variation? (Strict version)

Condition 1 (Identical Variation)

A supply shock to asset j **generates the ideal state price variation** if there exists a scalar k_j such that

$$\Delta \mathbf{q}_j^{\mathrm{ideal}} = k_j \Delta \mathbf{q}_j^{\mathrm{supply}}.$$

Do supply shocks generate the "ideal" state price variation? (Weak version)

Condition 2 (Variation of the same sign)

The supply shock generates **state price variation of the same sign** if, element by element,

$$\text{sign}(\Delta \mathbf{q}_j^{\text{ideal}}) = \text{sign}(\Delta \mathbf{q}_j^{\text{supply}})$$

Since Y has weakly positive entries, this condition holds for all j if Y^+ has only weakly positive entries.

Trilemma

Definition. Assets j and j' **have overlapping payoffs** if there exists at least one state z such that $y_j(z) > 0$ and $y_{j'}(z) > 0$.

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Theorem (Trilemma). If Conditions 1 or 2 are satisfied, then YY^{T} is diagonal, and:

(i) If YY^{T} is diagonal, then **there are no assets with overlapping payoffs.**

(ii) If markets are complete, then YY^{T} is diagonal if and only if Y **is diagonal up to permutations.**

"Proof." Plemmons and Cline (PAMS, 1972).

$\Rightarrow$ This condition is generically violated by standard primitive assets

Interpretation

1. Asset supply shocks affect behavior because they change the cost of consumption (i.e., state prices).
2. But given no arbitrage, supply shocks affect the price of other assets with overlapping payoffs.
3. Only exception: Arrow securities (or a suitable generalization to incomplete markets).
4. This is typically not the case in reality. (S&P500 payoff matrix barely ever has zeroes.)

Arrow securities eliminate the distinction between asset demand and consumption demand.

Seems natural to study consumption demand, but mapping from assets to consumption is latent.

Overcoming the Trilemma?

1. Multiple independent experiments

Suppose you were willing to work with linear demand. Write investor i's asset demand function as

$$a_i \;=\; \bar{a}_i \;+\; S_i\,(p - \bar{p}) \;+\; \varepsilon_i, \tag{1}$$

Suppose we have N **distinct experiments** generating data on prices and quantities for investor i,

$$G \equiv \left[\,\Delta p^{(1)}, \dots, \Delta p^{(N)}\,\right] \in \mathbb{R}^{J\times N}, \tag{2}$$

$$\Delta A_i \equiv \left[\,\Delta a_i^{(1)}, \dots, \Delta a_i^{(N)}\,\right] \in \mathbb{R}^{J\times N}. \tag{3}$$

Then we can write this as the linear system:

$$\Delta A_i \;=\; S_i\,G \;+\; U_i, \tag{4}$$

Complete Identification given $N = J$

Let $N = J$. The unique ordinary least-squares estimator of S_i is

$$\hat{S}_i \;=\; \Delta A_i \, G^{+}, \tag{5}$$

where $\hat{S}_i$ is an **unbiased and consistent estimator** of S_i. When $U_i = 0$, $\hat{S}_i = S_i$.

Incomplete Identification with $N < J$

Let $P_G \equiv GG^+$ be the orthogonal projector onto $\mathrm{col}(G)$, the column space of the matrix of observed price changes G. Then the general solution to the least-squares problem is

$$S_i = \Delta A_i G^+ + B_i(I - P_G), \quad B_i \in \mathbb{R}^{J \times J} \tag{6}$$

where B_i is an **arbitrary matrix that is entirely unrestricted by the data.**

Only the projection onto observed shocks is identified; elasticities in the null space are unbounded.

2. Theoretical restrictions

One can always achieve parametric identification using structural restrictions.

Challenge: how to accommodate natural complexities in portfolio choice.

– Continuous quantity choice, endogenous demand complementarities, resale prices,...

Conclusion

We study the methodological foundations of demand-based asset pricing, relying on principles of portfolio choice and equilibrium price determination.

1. Generic tension between no arbitrage and asset-level demand analysis.
2. Many questions about how to learn from portfolio data
3. But also many openings for interesting new approaches.

. . . Lots of work to be done.