Demand-System Asset Pricing: Theoretical Foundations

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A Trilemma for Asset Demand Estimation

William Fuchs, Satoshi Fukuda and Daniel Neuhann

UCL

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2. Object of interest: the asset-level demand elasticity – a **partial derivative.**
3. Allow investor-specific "tastes" for financial assets to account for portfolio heterogeneity.

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We study the foundations and interpretation of this methodology.

A brief history of thought

1. "Neoclassical asset pricing:" focus on relative prices, quantities mostly irrelevant.
2. Demand effects a la index inclusion: a notion of an *aggregate* demand curve.
3. High-frequency identification of aggregate effects of QE.
4. Intermediary asset pricing.
5. Asset demand systems: **structurally estimate** individual- and asset-level demand on portfolio data.

Existing methods hew closely to IO, but financial assets present unique challenges

1. Central role of **portfolios** and **relative prices** rather than good-specific demand functions.
    - Instrumental preferences over assets; flexible bundling and unbundling.

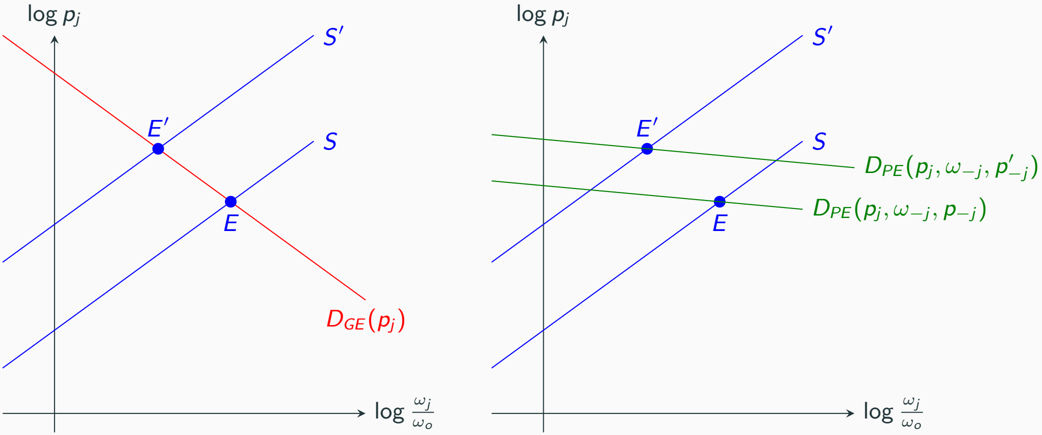
2. Cross-asset price spillovers through **general equilibrium price determination** and **no arbitrage**.

3. **Resale** considerations: current demand depends on future prices.

Main Insights

1. Cross-asset portfolio restrictions and equilibrium spillovers can heavily bias measured elasticities.
    - In current frameworks, measured elasticities may be near one even if true elasticities are near infinite.
    - Control variables do not address this issue – in fact, they change the object of analysis.
2. A general tension between demand estimation and no arbitrage.
    - Except in narrow edge cases, asset-level demand elasticities are not (non-parametrically) identified.
    - In most instances, demand elasticities primarily reflect ex-ante theoretical restrictions.
3. Elasticities are to be interpreted as structural objects only under strong assumptions.
    - In addition to spillovers, forward-looking demand leads depends on others' tastes.

Summary graph



(a) Logit demand **(b)** Demand with cross-asset complementarities

Framework

Basic considerations

We want a simple theoretical laboratory to model asset demand estimation.

To align with current the dominant approach, we allow **heterogeneous tastes** for financial assets.

Paper analyzes this in more detail.

1. Taste disagreement is critical for identification – need mutually orthogonal demand shocks.
2. This disagreement can lead to violations of no arbitrage – precisely because valuations differ.

   Practical concern because most applications rely on assumptions like factor structures.

For today, will simply assume that I have access to a clean asset-level supply shock.

Framework

- Investor i can choose consumption at date 0 or at date 1. Assume log utility.
- State $z \in \mathcal{Z} \equiv \{1, \ldots, Z\}$ with probability $\pi_z \in (0, 1)$.
- Assets $\mathcal{J} \equiv \{1, \ldots, J\}$ with price p_j and state-contingent cash flows $y_j(z)$.
- A portfolio of assets $(a^i_j)_{j \in \mathcal{J}}$.
- Investors receive asset endowments e^i_j and non-marketable endowment w^i_0 and $w^i_1(z)$.
- Prices taken as given by each investors, ultimately determined by market clearing.

Incorporating tastes

Given asset-specific taste parameter θ_j^i, investor i evaluates asset j's payoff $y_j(z)$ as $\theta_j^i y_j(z)$.

Then define utility over **taste-adjusted consumption**

$$\tilde{c}_1^i(z) \equiv \sum_j \theta_j^i y_j(z) a_j^i + w_1^i(z).$$

This allows us to use the standard machinery of expected utility. Moreover:

- If we let $\theta_j^i = 1$ for all j we recover the standard model.
- Close connection to dogmatic belief over scale of the payoff.

Taste-augmented portfolio choice problem

Our approach leads to a simple generalization of the standard problem:

$$
\begin{aligned}
\max_{(a^i_1, a^i_2, \ldots, a^i_J)} \quad & (1-\delta)u(c^i_0) + \delta \sum_z \pi_z u\Big(\tilde{c}^i(z)\Big) + \text{continuation value} \\
\text{s.t.} \quad & \tilde{c}^i_1(z) \equiv \sum_j \theta^i_j y_j(z) a^i_j + w^i_1(z), \\
& c^i_0 + \sum p_j (a^i_j - e^i_j) = w^i_0, \\
& \text{ad-hoc portfolio restrictions or mandates.}
\end{aligned}
$$

We model tastes over **payoffs**, not returns. **This allows for endogenous return spillovers.**

What is an asset-level demand function?

Portfolio choice models generate predictions for quantities to be bought of any given asset, say

$a^i_j(\vec{p})$: a function of the vector of all asset prices

Asset-level demand functions can be described using the notion of a **demand elasticity**.

This reflects **thought experiment** in which we vary a single asset price. This is a **partial derivative**:

$$\mathcal{E}^i_{js} \equiv -\frac{\partial a^i_j(\vec{p})}{\partial p_s} \frac{p_s}{a^i_j(\vec{p})}.$$

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Identification problem: given shock χ_s to asset s, observational data shows only the **total derivative**:

$$\hat{\mathcal{E}}_{ss}^i \equiv -\frac{\frac{da_j^i}{d\chi_s}}{\frac{dp_s}{d\chi_s}} \cdot \frac{p_s}{a_j^i}.$$

Deriving demand functions

The first-order necessary condition for the optimal choice of with respect to a^i_j is:

$$\underbrace{\theta^i_j \sum_{z \in \mathcal{Z}} y_j(z) \frac{u^{i'}(\tilde{c}^i(z))}{u^{i'}(c^i_0)} \quad + \quad \texttt{Lagrange multipliers} \quad - \quad p_j}_{\equiv \text{ Net marginal value } F^i_j(a^i, p)} = 0.$$

Deriving demand functions

Asset-level demand functions are jointly determined by a **system of equations** (+ constraints):

$$\begin{bmatrix} F_1^i(a^i, p) \\ F_2^i(a^i, p) \\ \vdots \\ F_J^i(a^i, p) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

This system typically exhibits **demand complementarities**: marginal value of asset j depends on a^i_{-j}.

1. Diversification: marginal value depends on covariance with portfolio.
2. Constraints: investment mandates which allow substitution between some assets.

Response to an exogenous supply shock χ_s to asset s.

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Total derivative of i's portfolio in response to the shock is:

$$
\begin{bmatrix} \frac{da_1^i}{d\chi_s} \\ \vdots \\ \frac{da_s^i}{d\chi_s} \\ \vdots \\ \frac{da_J^i}{d\chi_s} \end{bmatrix}
=
\underbrace{\begin{bmatrix} \frac{\partial a_1^i}{\partial p_1} & \cdots & \frac{\partial a_1^i}{\partial p_s} & \cdots & \frac{\partial a_1^i}{\partial p_J} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial a_s^i}{\partial p_1} & \cdots & \frac{\partial a_s^i}{\partial p_s} & \cdots & \frac{\partial a_s^i}{\partial p_J} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial a_J^i}{\partial p_1} & \cdots & \frac{\partial a_J^i}{\partial p_s} & \cdots & \frac{\partial a_J^i}{\partial p_J} \end{bmatrix}}_{\text{Quantity responses}}
\underbrace{\begin{bmatrix} \frac{dp_1}{d\chi_s} \\ \vdots \\ \frac{dp_s}{d\chi_s} \\ \vdots \\ \frac{dp_J}{d\chi_s} \end{bmatrix}}_{\text{Price spillovers}}
+
\underbrace{\begin{bmatrix} \frac{\partial a_1^i}{\partial \chi_s} \\ \vdots \\ \frac{\partial a_s^i}{\partial \chi_s} \\ \vdots \\ \frac{\partial a_J^i}{\partial \chi_s} \end{bmatrix}}_{\text{Income effects}} .
$$

Structural elasticities obscured by cross-asset interactions and equilibrium price spillovers $\mathcal{S}_{js} \equiv \frac{dp_j}{d\chi_s}$.

Observed versus structural elasticity

Proposition: The observed own-price elasticity $\hat{\mathcal{E}}^i_{ss}$ can be decomposed as follows:

$$\hat{\mathcal{E}}^i_{ss} = \mathcal{E}^i_{ss} - \underbrace{\sum_{j \neq s} \frac{\mathcal{S}_{js} \frac{1}{p_j}}{\frac{dp_s}{d\chi_s} \frac{1}{p_s}} \mathcal{E}^i_{sj}}_{\text{Complementarities}} - \underbrace{\frac{\frac{\partial a^i_s}{\partial \chi_s} \frac{1}{a^i_s}}{\frac{dp_s}{d\chi_s} \frac{1}{p_s}}}_{\text{Income effects}} . \tag{1}$$

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Intuition in a simple example. Investor chooses between bond and two similar stocks.

1. Holding other price fixed, a price change triggers rapid reallocation to other stock ($\mathcal{E}_{sj}^i$ is large).
2. If many investors attempt to do this, other price must adjust.
3. Once prices adjust, there is no need to adjust your portfolio.

How to isolate the structural elasticity from the observed one?

Generic answer: one must place restrictions on the substitution matrix.

Approach from IO: discrete choice over goods with homogeneous substitution to an "outside good."

KY use a similar system: "logit" demand for financial assets conditional on characteristics.

1. Impose assumptions on *returns* to sharply reduce scope for cross-asset spillovers.
2. Yields homogeneous substitution in units of portfolio weights relative to an outside asset.

The KY logit demand framework

Assume equilibrium **returns** follow a factor structure with **diagonal** conditional covariance matrix.

This means there are no complementarities left to worry about conditional on the factors.

Demand in units of relative portfolio weights assumed to depend only on own prices and characteristics:

$$\frac{\omega_j(p)}{\omega_{out}(p)} = \frac{\omega_j}{\omega_{out}}(p_j) = \exp\left\{\beta_0 \log p_j + \sum_{k=1}^{K-1} \beta_k x_k(j) + \beta_K\right\}\zeta(j),$$

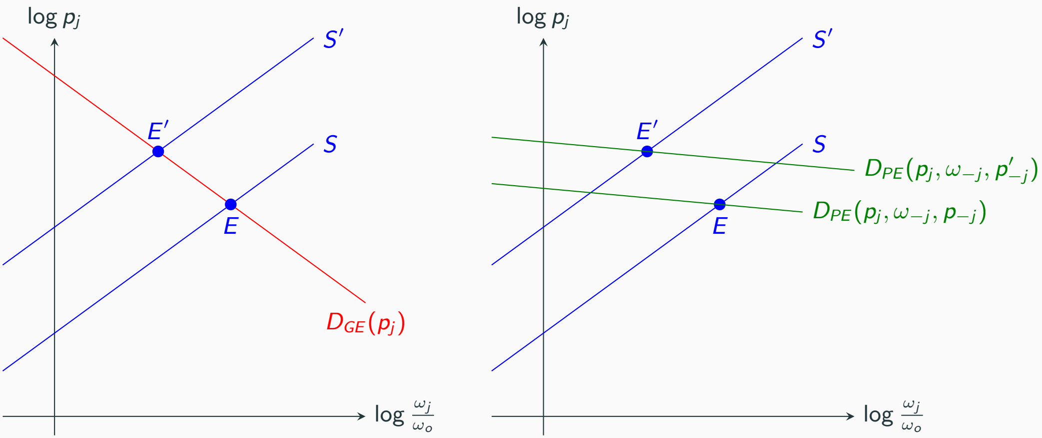
Identification: if $x_k(j)$'s are invariant to supply shocks, observed and structural elasticity are identical.

$\Rightarrow$ can identify β_0 from *observed* portfolio changes given exogenous price changes.

$\Rightarrow$ because demand is separable across assets, need only one price instrument per asset.

Problem: Returns and substitution are **endogenous**. Does logit demand generate a factor structure?

Back to our graph



(a) Logit demand

(b) Demand with cross-asset complementarities

Do equilibrium returns and substitution patterns satisfy the logit structure?

Our approach

We derive equilibrium portfolio choices alongside returns in a canonical framework.

- In particular, enrich Lucas '78 with payoff-augmenting tastes and mandates.
- As in KY, use log utility for simplicity (but this isn't necessary).

Use this model to compute true and measured elasticities based on the logit structure.

A minimal asset menu

- Two aggregate states, $z = 1, 2$ with prob. $\pi_z = \frac{1}{2}$. For each z, one tree that pays 1 in z only.

- Split Tree 1 into **green** and **red** halves with diversifiable risk. Green half pays better in green state.

		State 1		State 2
		Green shock $(1-\rho)$	Red shock (ρ)	
Tree 1	*green*	$1+\epsilon$	$1-\epsilon$	0
	red	$1-\epsilon$	$1+\epsilon$	
Tree 2		0		1

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- Parameter ϵ measures complementarity between assets. Tastes can be defined over colors: (θ^i_g, θ^i_r).
- Contrast with logit: heterogeneous substitution across assets if and only if $\epsilon < 1$.

Endowments and demand system implementation

- Tree 2 is the *outside asset* with normalized price $p_2 \equiv 1$. Relative prices p_r and p_g.
- Endowments $E_2 = 1$, $E_r = \frac{1}{2}$ and $E_g = \frac{1}{2} + \psi$. Use ψ as an **exogenous supply shock.**

- As in KY, define demand in units of portfolio shares relative to the outside good, $\frac{\omega_j^i(p)}{\omega_2^i(p)}$, so that

$$\mathcal{E}_{jj}^i \equiv -\frac{\partial(\omega_j^i(p)/\omega_2^i(p))}{\partial p_j}\frac{p_j}{\omega_j^i(p)/\omega_2^i(p)} \qquad \text{and} \qquad \hat{\mathcal{E}}_{jj}^i \equiv -\frac{d(\omega_j^i(p)/\omega_2^i(p))}{d p_j}\frac{p_j}{\omega_j^i(p)/\omega_2^i(p)}.$$

- NB: For the basic point, sufficient to assume no constraints or tastes over assets, $\theta_j^i = 1$.

The logit bias in portfolio shares

Supply variation ψ is a clean instrument for p_g: fully exogenous to all investors in the model.

Under the hypothesis of logit demand, **structural elasticity = observed elasticity**, $\mathcal{E}^i_{jj} = \hat{\mathcal{E}}^i_{jj}$.

Proposition: Let $\mathcal{B}^i_{jj} \equiv \mathcal{E}^i_{jj} - \hat{\mathcal{E}}^i_{jj}$ denote the "logit bias." This bias is given by

$$\mathcal{B}^i_{jj} = -\underbrace{\frac{\partial\left(\omega^i_j(p)/\omega^i_2(p)\right)}{\partial p_{-j}}}_{\text{Complementarity}} \underbrace{\frac{p_{-j}}{\left(\omega^i_j(p)/\omega^i_2(p)\right)} \frac{p_j}{p_{-j}}}_{\text{Scaling terms}} \underbrace{\frac{dp_{-j}}{dp_j}}_{\text{Price Spillover}} .$$

Equilibrium demand functions

Contra logit, equilibrium demand functions **depend on both prices** for all $\epsilon < 1$.

$$\frac{\omega_g^i(p_g, p_r)}{\omega_2^i(p_g, p_r)} = \theta_r^i \frac{\pi_1}{\pi_2} p_g \cdot \frac{(\theta_r^i p_g + \theta_g^i p_r)\epsilon^2 - (\theta_r^i p_g - \theta_g^i p_r) + 2\theta_g^i p_r \epsilon(1-2\rho)}{(\theta_r^i p_g + \theta_g^i p_r)^2 \epsilon^2 - (\theta_r^i p_g - \theta_g p_r)^2}; \tag{2}$$

$$\frac{\omega_r^i(p_g, p_r)}{\omega_2^i(p_g, p_r)} = \theta_g^i \frac{\pi_1}{\pi_2} p_r \cdot \frac{(\theta_r^i p_g + \theta_g^i p_r)\epsilon^2 + (\theta_r^i p_g - \theta_g^i p_r) - 2\theta_r^i p_g \epsilon(1-2\rho)}{(\theta_r^i p_g + \theta_g^i p_r)^2 \epsilon^2 - (\theta_r^i p_g - \theta_g^i p_r)^2}. \tag{3}$$

Bias between observed and structural elasticity

Proposition. For a small shock to green supply ψ, the logit bias satisfies

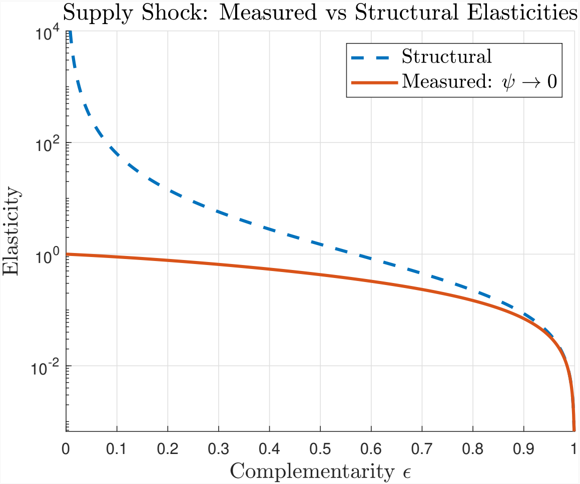
$$\mathcal{B}_{gg} = \frac{(1-\epsilon^2)^2(1+(1-2\rho)\epsilon)}{8\epsilon^2\rho(1-\rho)((1+\epsilon)^2-4\epsilon\rho)}.$$

The bias is positive, is strictly decreasing in ϵ, goes to infinity as $\epsilon \to 0$, and is zero iff $\epsilon = 1$.

In particular, in the limit as red and green assets become perfect substitutes, we have

$$\lim_{\epsilon\to 0} \mathcal{E}_{gg} = \infty \qquad \text{and} \qquad \lim_{\epsilon\to 0} \hat{\mathcal{E}}_{gg} = 1$$

Measured versus structural elasticities (log scale)



Measured is *always* low; true is low when $\epsilon \to 1$ $\quad (\rho = \frac{1}{4})$.

Interpretation

- When $\epsilon < 1$, controlling for outside demand not sufficient to capture actual substitution patterns.

- Given spillovers, observed elasticity is determined by substitution towards **state-1 consumption**.
- Consumption elasticities can be low even if the asset-level elasticity is extremely high.

- The response in KY25 does not address this problem – their estimator performs even worse.

Details

Control variables change the the estimand

One proposed solution to spillover biases is the use of controls, such as factors or characteristics.

– "If spillovers occur mainly among similar assets, we should control for asset similarity."

This changes the unit of analysis to *residual cash flows* (rather than the asset).

– Assets might be substitutable precisely because they have common factor exposures.

Problem: **residual cash flow elasticities may be uninformative about asset-level elasticities.**

Details

Trilemma for Asset Demand Estimation

Two questions

1. When can asset-level elasticities be (nonparametrically) identified from observational data?

   Can ideal asset-level supply shocks reveal substitution patterns beyond those implied by theory?

2. Is asset-level demand estimation the right paradigm for empirical analysis of portfolio data?

General limits to asset demand identification

It is impossible to simultaneously maintain that:

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Reason: cross-asset restrictions from (1) and (2) prevent *ceteris paribus* price variation.

Exceptions: the asset menu consists of Arrow securities, or one has *many* independent experiments.

Environment is the same as before

- Disregard "tastes" for now ($\theta^i_j = 1$).
- **Markets can be complete or incomplete.**
- p is the vector of asset prices, q is the vector of state prices (need not be unique).

Benefits of No Arbitrage in Demand Estimation

1. Ensures well-defined demand functions.
2. It is *practically important* to reduce dimensionality of choice sets (e.g., using characteristics).
3. It is a weak requirement on equilibrium play that ensures consistency in counterfactuals.

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Assumption 1. There is no arbitrage,

$$p = Yq.$$

We use a strict definition, but similar issues are at play with "approximate arbitrage."

Translating demand elasticities

"How does asset demand change if asset price p_j changes but all other asset prices remain fixed?"

$\Rightarrow$ *"How do investors respond to the state price changes induced by an shock to asset prices?"*

State price changes in an ideal experiment

Let Y^+ denote the Moore-Penrose inverse. Given no arbitrage, $q = Y^+ p$.

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Lemma. Let v_j denote the unit vector in $\mathbb{R}^J$ with 1 in the j-th position and zeros elsewhere. Then the changes in state prices given the exogenous variation in a single price p_j satisfy

$$\boxed{\boldsymbol{\Delta}\mathbf{q}^{\mathrm{ideal}} \equiv \frac{\partial q}{\partial p_j} = Y^+ v_j.}$$

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Elasticity measurement requires state price variation proportional to the **inverse payoff matrix**.

Identification from (ideal) supply shocks

Assume that we have an "ideal" laboratory with exogenous shocks to the supply of some asset j.

- For example, an outside investor helicopter drops an asset for purely exogenous reasons.

Does the resulting price variation identify a demand elasticity with respect to price p_j?

To understand equilibrium effects, we impose only a weak (and favorable) condition

Definition (Separable downward-sloping consumption demand)

Consumption demand is separable downward-sloping if $\exists$ a strictly positive $Z \times Z$ diagonal matrix V s.t.

$$\boxed{\Delta \mathbf{q}_j^{\text{supply}} \equiv \frac{\partial q}{\partial E_j} = -V y_j^{\mathrm{T}} \qquad \text{for all assets } j,}$$

– Standard interpretation: V captures the marginal utility of the marginal investor.

– V diagonal is perhaps the best case – no direct cross-state spillovers from supply shocks.

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State price changes induced by supply shocks are proportional to the **payoff matrix itself**.

Do supply shocks generate the “ideal” state price variation? (Strict version)

Condition 1 (Identical Variation)

A supply shock to asset j **generates the ideal state price variation** if there exists a scalar k_j such that

$$\Delta \mathbf{q}_j^{\text{ideal}} = k_j \Delta \mathbf{q}_j^{\text{supply}}.$$

Do supply shocks generate the "ideal" state price variation? (Weak version)

Condition 2 (Variation of the same sign)

The supply shock generates **state price variation of the same sign** if, element by element,

$$\mathrm{sign}(\Delta \mathbf{q}_j^{\mathrm{ideal}}) = \mathrm{sign}(\Delta \mathbf{q}_j^{\mathrm{supply}})$$

Since Y has weakly positive entries, this condition holds for all j if Y^+ has only weakly positive entries.

Trilemma

Definition. Assets j and j' **have overlapping payoffs** if there exists at least one state z such that $y_j(z) > 0$ and $y_{j'}(z) > 0$.

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Theorem (Trilemma). If Conditions 1 or 2 are satisfied, then YY^{T} is diagonal, and:

(i) If YY^{T} is diagonal, then **there are no assets with overlapping payoffs.**

(ii) If markets are complete, then YY^{T} is diagonal if and only if Y **is diagonal up to permutations.**

"Proof." Plemmons and Cline (PAMS, 1972).

Interpretation

1. Asset supply shocks affect behavior because change the cost of consumption (i.e., state prices).
2. Under no arbitrage, supply shocks affects the price of other assets with overlapping payoffs.
3. Only exception: Arrow securities (or a suitable generalization to incomplete markets).
4. **No overlapping payoffs is much stronger than orthogonal payoff distributions.**

Arrow securities eliminate the distinction between asset demand and consumption demand.

Overcoming the Trilemma?

1. Multiple independent experiments

Suppose you were willing to work with linear demand. Write investor i's asset demand function as

$$a_i \;=\; \bar{a}_i \;+\; S_i\,(p - \bar{p}) \;+\; \varepsilon_i, \tag{4}$$

Suppose we have N **distinct experiments** generating data on prices and quantities for investor i,

$$G \equiv \left[\, \Delta p^{(1)}, \ldots, \Delta p^{(N)} \,\right] \in \mathbb{R}^{J \times N}, \tag{5}$$

$$\Delta A_i \equiv \left[\, \Delta a_i^{(1)}, \ldots, \Delta a_i^{(N)} \,\right] \in \mathbb{R}^{J \times N}. \tag{6}$$

Then we can write this as the linear system:

$$\Delta A_i \;=\; S_i\,G \;+\; U_i, \tag{7}$$

Complete Identification given $N = J$

Let $N = J$. The unique ordinary least-squares estimator of S_i is

$$\hat{S}_i \;=\; \Delta A_i \, G^{+}, \tag{8}$$

where $\hat{S}_i$ is an **unbiased and consistent estimator** of S_i. When $U_i = 0$, $\hat{S}_i = S_i$.

Incomplete Identification with $N < J$

Let $P_G \equiv GG^+$ be the orthogonal projector onto $\mathrm{col}(G)$, the column space of the matrix of observed price changes G. Then the general solution to the least-squares problem is

$$S_i = \Delta A_i G^+ + B_i(I - P_G), \quad B_i \in \mathbb{R}^{J \times J} \tag{9}$$

where B_i is an **arbitrary matrix that is entirely unrestricted by the data.**

Only the projection onto observed shocks is identified; elasticities in the null space are unbounded.

2. Theoretical restrictions

One can always achieve parametric identification using structural restrictions (*aka* a model).

These restrictions must be evaluated on first principles: the data is silent.

Specifically, structural models must capture the cross-asset spillovers which are endemic to asset pricing.

Relative elasticities

An alternative: measuring relative elasticities

- Haddad et al. (2025): under slightly weaker symmetry, one can identify the "relative elasticity."
  i.e., the own-price minus the cross-price elasticity.

- Clarifies the identification challenge and partially circumvents it using a **different estimand.**
- Substitution matrix must be decomposed into unobservables and observable "controls."

- Controls strongly change the interpretation, and small misspecification can lead to large biases.

A perturbed payoff structure and relative elasticities

		State 1		State 2
		Green shock $(1-\rho)$	Red shock (ρ)	
Tree 1	*green*	$1+\epsilon$	$1-\epsilon$	0
	red	$1-\epsilon$	$1+\epsilon$	δ
Tree 2		0		$1-\delta$

Asymmetries and potential bias in relative elasticities

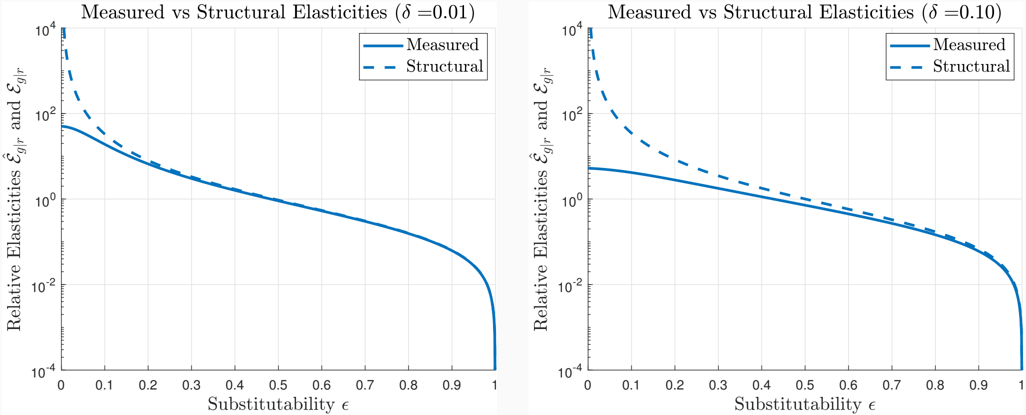

Figure 3: The Measured and True Relative Elasticities: $\delta \in \{0.01, 0.1\}$.

Concern: Assets are more likely going to be closer substitutes when they are more symmetric.

Are demand elasticities structural?

Counterfactuals from demand systems

Growing interest in using demand systems for counterfactuals and policy.

Should demand elasticities be interpreted as “deep” parameters?

- Require invariant parameters to appropriately inform policymakers.

Even beyond spillovers, we find that two problems limit structural interpretation:

1. Dynamic trading: demand elasticities alone cannot identify *whose* tastes affect current demand.
2. One can rationalize a portfolio by constraints **or** preferences – but counterfactuals differ.

Conclusion

We study the methodological foundations of demand-based asset pricing, relying on principles of portfolio choice and equilibrium price determination.

1. Tastes may invalidate the organizing principle of no arbitrage.
2. Price spillovers offer a simple explanation for low *measured* elasticities.
3. Generic tension between no arbitrage and asset-level demand analysis.

. . . Lots of work to be done.

Appendix

Koijen Yogo 2025

Koijen Yogo (2025) argue we used the wrong benchmark

Their abstract states:

> *[...] FFN25 use an incorrect [within-asset] estimator for their central claim that "measured elasticities are near one even if true elasticities are near infinite." The cross-sectional instrumental variables estimator correctly identifies the demand elasticities in KY19 and FFN25.*

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> *[...] FFN25 use an incorrect [within-asset] estimator for their central claim that "measured elasticities are near one even if true elasticities are near infinite." The cross-sectional instrumental variables estimator correctly identifies the demand elasticities in KY19 and FFN25.*

The cross-sectional estimator they propose is:

$$\hat{\beta}_0 = -\frac{\mathrm{Cov}(\log \omega_j(\psi), z_j)}{\mathrm{Cov}(\log p_j(\psi), z_j)}.$$

Importantly, they conduct their analysis with an approximation rather than the exact estimator and their approximation is only valid under knife-edge symmetry assumptions.

Proposition (Bias in the cross-sectional estimator)

If $\rho \neq \frac{1}{2}$, the KY approximation is invalid and the exact value of the KY25 estimator $\hat{\beta}_0$ satisfies

$$\lim_{\psi \to 0} \hat{\beta}_{0,\mathrm{exact}}(\psi) = -1,$$

which is of the opposite sign as, and does not vary with, the structural elasticity $\mathcal{E}_{gg}$.

If $\rho = \frac{1}{2}$, then the KY approximation is accurate and equation (B18) of KY25 holds,

$$\lim_{\psi \to 0} \hat{\beta}_{0,\mathrm{exact}}(\psi) = \frac{1}{\epsilon^2} - 1.$$

Response to the Response

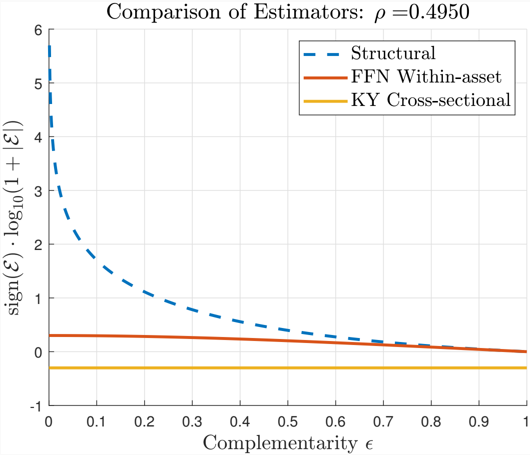

Figure 4: Estimators given a small violation of perfect symmetry ($\rho = 0.495$.) We use an approximate log scale to accommodate negative values.

What's going on?

In the model at hand, the cross-sectional estimator can be computed exactly:

$$\hat{\beta}_{0,\text{exact}} = -\frac{\log(\omega_g(\psi)) - \log(\omega_r(\psi))}{\log(p_g(\psi)) - \log(p_r(\psi))}.$$

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It is exceedingly hard to separate own- and cross-price elasticities from a single equilibrium allocation!

KY25 do not actually use the exact estimator. Instead, approximate demand and prices around $\psi = 0$,

$$\hat{\beta}_0 \approx \hat{\beta}_{0,\text{approx}} \equiv -\frac{Cov\left(\frac{d\log\omega_j}{d\psi}\psi, z_j\right)}{Cov\left(\frac{d\log p_j}{d\psi}\psi, z_j\right)} = -\frac{\frac{d(\log(\omega_g(\psi))-\log(\omega_r(\psi)))}{d\psi}}{\frac{d(\log(p_g(\psi))-\log(p_r(\psi)))}{d\psi}}.$$

Since $\hat{\beta}_0 = F(\psi)/G(\psi)$, this is valid only if $F(0) = G(0) = 0$. This requires **perfect symmetry**, $\rho = \frac{1}{2}$.

Back

Decision Problem with Traded Factors

Decision problem with traded factors

In our setting, the aggregate income in a given state is a factor (there are of course others).

- For example, the green asset has loadings $1+\epsilon$ and $1-\epsilon$ on state g and r income.
- To allow non-factor variation, perturb the model with small idiosyncratic noise, $y_j' = y_j + \eta_j$.

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Given this structure, we can model portfolio choice as a two-step problem:

1. Choose desired factor exposures (i.e. state-contingent consumption c_z) at price q_z.
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Controlling for factor exposures means focusing on the second step: **a conditional decision problem**.

Substitution across assets is now driven only by the idiosyncratic component. **A residual elasticity.**

Factor and residual elasticities

Consider a small perturbation, $Var(\eta_j) \approx 0$. Then optimal factor demand in portfolio shares is

$$\frac{c_z q_z}{W} = \pi_z.$$

The factor-level demand elasticity is zero (and thus very different from the asset elasticity.)

Fix factor exposures at 1 (as in the baseline model). **The elasticity of residual demand is**

$$\frac{\partial \tilde{\omega}_j^*}{\partial \tilde{p}_j} \frac{\tilde{p}_j}{\tilde{\omega}_j} = 1.$$

Highly non-linear: underlying elasticities are low even when the asset-level elasticity is very high.

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Structural Interpretation

Except in special cases, financial assets are **investment goods**.

Investor demand depends on both own prefer and expected market returns (i.e., others' tastes).

$\Rightarrow$ Observed demand elasticities alone cannot identify *whose* tastes affect current demand.

But, for many counterfactuals we do need to be able to attribute tastes to investors.

One can **Problem**: counterfactuals are generally sensitive to the precise micro-foundation.

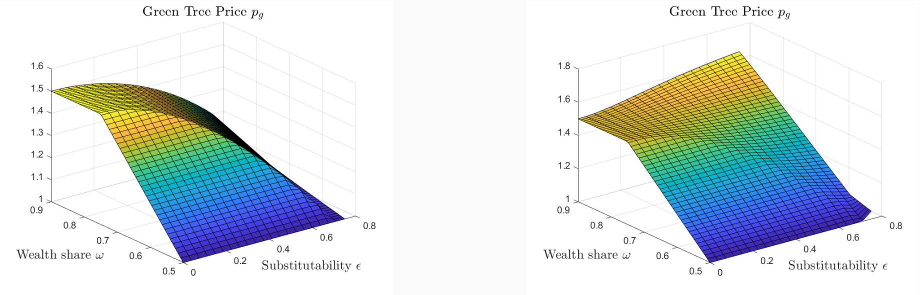

Figure 5: Equilibria in an economy with taste differences and one with portfolio restrictions on green asset share.