Demand-System Asset Pricing: Theoretical Foundations*

William Fuchs† Satoshi Fukuda‡ Daniel Neuhann§

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Abstract

Recent approaches to asset pricing involve the estimation of asset demand systems in which investors may have non-pecuniary tastes (or dogmatic beliefs) over asset characteristics. We investigate theoretical foundations of demand-system asset pricing by incorporating tastes into canonical models of portfolio choice. Our analysis raises several conceptual issues, including the notion of no arbitrage with tastes, the measurement of cross-asset demand spillovers, and the identification of structural parameters for counterfactuals. Imperfectly accounting for cross-asset spillovers can lead to low *measured* demand elasticities even when true elasticities are near infinite. We discuss several methodological approaches to address these concerns.

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†University of Texas at Austin, UC3M, FTG, and CEPR. william.fuchs@mccombs.utexas.edu.

‡Bocconi University and IGIER. satoshi.fukuda@unibocconi.it.

§University of Texas at Austin and FTG. daniel.neuhann@mccombs.utexas.edu.

1 Introduction

Recent approaches to asset pricing following Koijen and Yogo (2019) involve the estimation of *demand systems* for financial assets in which investors are permitted to have *tastes* (or dogmatic beliefs) over both pecuniary and non-pecuniary attributes of financial assets, such as environmental scores or the identity of the issuer. According to this approach, data on portfolio holdings can be used to identify investor tastes and beliefs, and this data encodes information that can inform researchers about the functioning of financial markets and the equilibrium response to a variety of counterfactual shocks, such as evolving tastes or changes in the wealth distribution across investor types.

This focus on investor tastes and individual demand functions represents a sharp break from neoclassical asset pricing, which emphasizes the fungibility of securities up to cash flows and focuses on price data disciplined by no arbitrage (Ross, 2004). To interpret portfolio data, this approach also requires a methodological shift towards demand estimation techniques that were previously predominantly used in industrial organization. While this stark dichotomy may well be an advantage given the imperfect empirical record of neoclassical models, it also means jettisoning much of the well-established theoretical foundations that underlie the neoclassical approach. Hence, it is important to develop theoretical foundations for the demand-system approach.

In this paper, we study the theoretical foundations of demand-based asset pricing by enriching canonical models of portfolio choice with flexible specifications for investor tastes and portfolio constraints. Our main findings are that tastes can lead to violations of the organizing principle of no arbitrage, and that logit asset demand systems of the type currently in use do not adequately account for cross-asset demand complementarities induced by portfolio diversification and/or investment mandates. As such, they may produce low *measured* elasticities even when the true underlying elasticities are very large. Our framework thus provides a simple explanation for the "puzzling" difference between the low asset market elasticities documented by leading demand-system estimates and the high demand elasticities obtained in calibrations of neoclassical models.

Why are "taste differences" important for demand-system asset pricing? The central identification challenge in financial markets is that the outstanding supply of securities is rarely adjusted for exogenous reasons. Hence, the identification of individual demand functions requires exogenous variation in the residual supply faced by a given investor. Since residual supply is gross supply net of other investors' demand, demand identification requires the existence of *demand shocks* to some investors that are suitably exogenous to the (unobserved) demand of other investors. Such orthogonal demand shocks can exist if investors do not update their own return expectations in response to others' demand shocks, or if some investors trade for exogenous reasons. That is, demand system approaches *require* some notion of individual-specific tastes, beliefs, or constraints that do not vary with an appropriate notion of "market-wide" expected returns.

To investigate the asset pricing implications of incorporating tastes, we introduce flexible heterogeneous tastes into a canonical portfolio choice problem. The typical approach in demand-system asset pricing is to model taste and belief heterogeneity directly over asset *returns*. We instead model tastes over *payoffs*, not returns. While taste differences over payoffs naturally induce differences in perceived returns for any given asset price, the distinction is nevertheless important: preferences over payoffs are structural parameters that are invariant to changes in, e.g., the market-wide distribution of tastes and wealth, whereas returns respond to such shocks. Properly accounting for the equilibrium determination of returns allows for a more precise understanding of the identification problem and its implications for equilibrium analysis.

We begin by revisiting some fundamental conceptual issues in theoretical asset pricing, including the appropriate definition of no arbitrage under tastes. When investors have dogmatic tastes over assets, including over non-pecuniary characteristics, two assets with identical cash flows need no longer have the same price. Thus, we must modify the definition of the law of one price to account for taste-based valuation differences. Even with this broader understanding of the value of an asset, we show that with sufficiently dissimilar tastes (given a set of allowable trades) in general there does not exist a pricing function that leaves no arbitrage opportunities. This has direct implications for equilibrium existence, incentives for short sales, and the use of arbitrage-based factor approaches to reducing the dimensionality of the portfolio choice problem in demand estimation.

Our analysis reveals two main ways of restoring no arbitrage in settings with tastes. The first is to impose trading restrictions or to focus on a small number of assets, as is frequently done in models of heterogeneous beliefs. The second is to allow for richer specifications, such as declining marginal tastes for individual assets. While both approaches have merit, we also illustrate potential pitfalls that may arise when using them. In part, this is because integrating tastes with risk and return requires a *cardinal* interpretation of taste parameters. Hence measuring demand requires identifying the intensity of tastes (not just their ordinal ranking), and aggregation into portfolios (such as "high ESG" portfolios) must be weighted by marginal utility.

Having discussed the modeling of tastes, we turn to the appropriate specification of demand systems for financial securities, as well as the interpretation of estimates generated by state-of-the-art demand systems. Because demand estimation is perhaps the central problem in industrial organization, researchers in this tradition have developed a rich set of tools to estimate preferences parameters from observational data (Berry and Haile, 2021). Yet, financial assets present a number of unique challenges that differ from the demand for nonfinancial goods typically studied in industrial organization. Hence, the application of existing techniques to financial securities is far from straightforward.

Perhaps the most important insight of portfolio choice theory is that asset demand is best understood as pertaining to *bundles* of assets, not individual securities. This is because risk-averse investors evaluate each individual asset as a function of its returns and its covariance with the rest of a portfolio. In the language of demand systems, financial markets exhibit investor-level demand complementarities, whereby the marginal value of a given asset depends on the investor's intensive-margin holdings of other assets. These demand complementarities yield rich asset-specific substitution patterns that ultimately determine demand elasticities, the equilibrium price responses to shocks, and endogenous price spillovers to other assets. Demand systems in the logit family, including Koijen and Yogo (2019), do not allow for asset-specific substitution patterns.[1] We argue that failing to account for these complementarities, and the cross-asset price spillovers they induce, can sharply bias the measurement and interpretation of demand elasticities.

To establish how endogenous cross-asset spillovers affect measured demand elasticities, we cast our model in general equilibrium and conduct an ideal experiment inside our model: we exogenously shock the supply of a given asset, and trace out the endogenous portfolio adjustment in response to this shock. Our main observation is that

[1]This issue is grounded in the Independence of Irrelevant Alternatives (IIA) problem in industrial organization. This problem is likely to be particularly severe in asset markets where the *marginal* value of assets depends on the intensive-margin holdings of other securities and the unit of analysis is portfolio shares at the individual level rather than market shares. A common fix in IO is to use nested logit approaches that allow for differential substitution across categories of assets. This requires ex-ante knowledge of substitution patterns, and it does not address the endogenous price spillovers that arise in general equilibrium with demand complementarities.

even such an ideal instrument generates correlated equilibrium price adjustments in substitutable assets. For example, an exogenous supply shock for Apple stock will induce endogenous price adjustments in the prices of both Apple and Microsoft stock. Since price adjustments limit the incentives of an individual investor to adjust her portfolio in response to the shock, even ideal instruments will therefore suggest a low elasticity to exogenous shocks even when the true demand elasticity, *in which all other prices are held fixed*, is very high. Put simply, there is no need for an individual investor to sell Apple and buy Microsoft if both expected returns have already adjusted. Hence, measured elasticities may be low even if the true underlying elasticity is near infinite. This *equilibrium spillover bias* provides a simple explanation of the puzzling observation that demand system estimates indicate low elasticities whereas calibrations of canonical models suggest they should be orders of magnitude higher.

This aspect of our analysis also reveals a deeper conceptual point. In the standard neoclassical view of financial markets, the relevant object of analysis is state-contingent consumption or wealth, not the specific quantities of particular securities. As such, neoclassical approaches suggest high *asset-level* elasticities when multiple assets deliver similar levels of consumption in a given set of states of the world, but are perfectly consistent with low elasticities over state-contingent wealth. Endogenous price spillovers across substitute assets thus determine the extent to which an *asset-level shock* affects the price of *state-contingent wealth*. When there are substitute assets, this pass-through is near perfect and the measured elasticity reveals the low consumption-level elasticity even when the underlying asset-level elasticity is very large. To appropriately compare and contrast leading asset pricing frameworks, it is therefore critical to carefully define the margin of adjustment a given elasticity is intended to measure.

Cross-asset spillovers constrain the set of valid instruments that can identify tastes for specific assets because the *ceteris paribus* condition is unlikely to hold. An implication for empirical work is that researchers must appropriately control for cross-asset spillovers within the estimation procedure. One approach to doing so is jointly estimate a price spillover matrix alongside the set of individual demand functions, checking for consistency between the two using market clearing. Developing the necessary tools to analyze the joint system and applying them to data is a promising avenue for future work.

In the final part of our analysis, we ask whether financial demand elasticities should be interpreted as revealing structural parameters that are likely to be invariant under counterfactuals. Such counterfactuals are of interest to the literature. For example, Koijen, Richmond, and Yogo (2022) aim to estimate asset prices after a shock to the cross-sectional wealth distribution, and Darmouni, Siani, and Xiao (2023) study policy interventions in corporate bond markets. To analyze the structural interpretation of demand elasticities, we study another feature that distinguishes financial markets from many goods markets, which is that financial assets are investment goods whose current value critically depends on their expected resale price.

Using a dynamic variant of our baseline model, we derive demand functions for financial assets that depend both on an individual investor's private tastes and her expectations of market returns. Similar to a Keynesian beauty contest (Keynes, 1936), demand elasticities alone thus cannot distinguish whether an investor chooses her asset holdings based on her own tastes or her expectations of others' tastes. Yet estimating counterfactuals in many cases requires assigning tastes to a particular investor. For example, assessing the asset pricing implications of a (possibly endogenous) change in the cross-sectional wealth distribution requires knowledge of the tastes of the investors who grew richer and those who grew poorer.

We establish a related result for unobservable portfolio constraints. For a range of parameters, tastes for a given asset are observationally equivalent to unobserved mandates that constrain an investor's portfolio choice. Yet an unconstrained investor will respond differently to a price shock than one that is not permitted to change her portfolio. Hence, we illustrate that mistaking tastes for constraints can lead to qualitatively different counterfactual responses to shocks. We also illustrate some additional concerns, such as endogenous capital flows across investment funds with fixed mandates or the misspecification of portfolio constraints.

The paper is structured as follows. The rest of this section discusses related literature. Section 2 discusses the fundamental identification problem and its implications with regards to incorporating non-pecuniary tastes in an asset pricing framework. Here, we also discuss the failure of no arbitrage under tastes, as well as potential methods of restoring no arbitrage. Section 3 presents our general-equilibrium framework. Section 4 discusses measurement and interpretation of asset demand elasticities, and discusses potential methodological adjustments to accounting for cross-asset spillovers. Section

5 studies identification of structural parameters in the context of forward-looking asset demand and discusses other threats to identification from equilibrium play. Section 6 provides concluding remarks. All proofs are in the Appendix.

Related Literature

This paper studies the theoretical foundations of demand-based asset pricing following Koijen and Yogo (2019). This methodological approach has been used to address a number of substantive questions, counterfactual asset prices and price informativeness in response to non-pecuniary tastes shocks or changes in the size distribution of institutional investors (Koijen, Richmond, and Yogo, 2022), global imbalances and currency returns (Jiang, Richmond, and Zhang, 2023), corporate bond returns (Bretscher, Schmid, Sen, and Sharma, 2022), the conduct of monetary policy (Nenova, 2003), the degree of competition in stock markets (Haddad, Huebner, and Loualiche, 2022), and counterfactual policy interventions in corporate bank markets (Darmouni, Siani, and Xiao, 2023). The fact that estimated demand systems appear to reveal that financial institutions exhibit low demand elasticities has also been used to argue that financial markets as a whole are inelastic, with implications for the equity premium (Gabaix and Koijen, 2020).

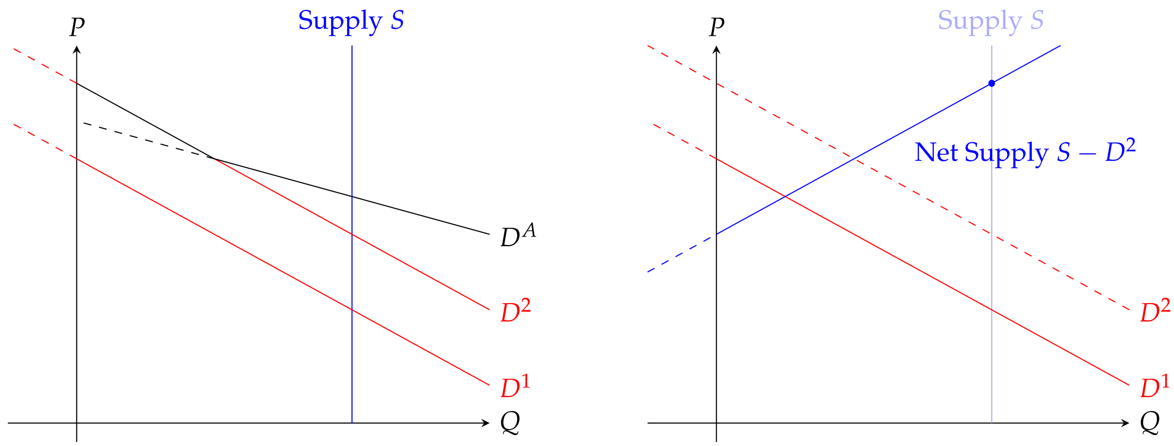
A number of recent papers discuss methodological aspects of demand-system asset pricing. Haddad, Huebner, and Loualiche (2022) generalize the logit framework in Koijen and Yogo (2019) to allow for endogenous demand elasticities due to, e.g., learning from prices, price impact or bounded rationality. van der Beck (2021, 2022) argues that estimating demand elasticities from trades rather than positions circumvents the endogeneity of investment mandates. Darmouni, Siani, and Xiao (2023) study a two-layer system where institutional investors' assets under management are determined by a second logit demand system. Koijen and Yogo (2020) and Chaudary, Fu, and Li (2023) specify nested logit demand systems for international finance and corporate bond markets, respectively. Our analysis highlights the importance of cross-asset demand complementarities, and the endogenous price spillovers they induce, for the estimation of demand elasticities. We also argue that demand elasticities can be interpreted as structural parameters useful for counterfactuals only under stringent assumptions.

More broadly, our paper highlights the importance of accounting for equilibrium spillovers in settings with investor-level demand complementarities. So far, there exist limited methods for dealing with these complementarities. In the context of industrial organization, existing applications mainly focus on small menus. Gentzkow (2007) studies newspaper demand allowing for a discrete choice between print, online, and print plus online. More recently, Allen, Kastl, and Wittwer (2020) make progress towards estimating models of demand complementarities in financial markets in the context of simultaneous auctions for multiple maturities of Canadian treasury bonds. In contrast to equity markets, they have the advantage of a small number of assets, observations on entire demand schedules, and market participants (dealers) that expect to resell their assets without holding long-term risk exposure. Hence, the portfolio choice problem in their model is comparatively simple.

By allowing for general tastes over assets, our paper also contributes to a growing literature in which investors may hold financial securities because of non-pecuniary values associated with them (Starks, 2023). Valued-based approaches have been used to study investment in so-called "green assets," such as stocks or bonds associated with sustainable, environmentally-friendly firms or government expenditures. Pastor, Stambaugh, and Taylor (2021) provide an equilibrium model of such sustainable investment, whereby firms differ in their "green scores," but the set of marketable securities consists only of firm shares and a risk-free asset. We address implications and micro-foundations of such sustainable investing for financial market equilibrium with redundant securities under various forms of tastes, and ask how such tastes might be identified in equilibrium. Pastor, Stambaugh, and Taylor (2022) provide first evidence that green tastes are reflected in equilibrium asset returns. However, in line with Section 5 of our paper, D'Amico, Klausmann, and Pancost (2023) show that accounting for expectations over the path of green returns affects the level and variance of the premium on green assets.

2 The Identification Problem and its Implications

We begin by reviewing the fundamental identification problem as it relates to estimating demand systems for financial assets from observational data. Since demand estimation is a classic issue, particularly in industrial organization, we focus mainly on the novel features introduced by modeling demand for financial assets. We then examine the theoretical implications of models that permit features thought to be useful for identification.



(a) Aggregate Supply and Demand.

(b) Net Supply and Demand.

Figure 1: The basic identification issue in an endowment economy.

In general, the goal of demand estimation is to measure market participants' willingness to pay for different assets. The main difficulty is that, since quantities and prices are generally jointly determined, simple regressions of quantities on prices do not identify structural parameters. Figure 1 illustrates this basic problem, as well as a potential solution. The left panel shows the canonical supply and demand diagram in an endowment economy for financial assets where the supply curve S is vertical. In the panel, D^1 and D^2 are demand curves for individual market participants, and D^A is aggregate demand. Quantities may become negative because financial assets can be sold short.

A common empirical strategy is to trace out demand response to exogenous changes in supply. However, this is not feasible in an endowment economy with fixed supply, which is the basic framework used in demand-system asset pricing. Even outside of endowment economies, it is difficult to find changes in the supply of a given financial asset that are not influenced by prevailing prices. To circumvent this issue, the literature on demand-system asset pricing focuses on variation in *net supply*, defined as aggregate supply minus demand of a subset of market participants. The basic idea is illustrated in the right panel: rather than finding exogenous shocks to aggregate supply S, researchers aim to estimate the structural parameters of demand function D^1 by finding exogenous variation in residual supply $S - D^2$. Put differently, the empirical strategy is to construct exogenous shocks to the residual supply curve faced by a particular investor by finding exogenous shocks to the demand functions of other investors.

This approach places stringent constraints on the type of variation that can be used to identify demand systems. In particular, researchers must find settings in which there are changes in residual supply (which is itself a type of demand shock) that are uncorrelated with the demand of remaining investors. In the context of financial markets, this implies that one must find changes to market prices that are not driven by correlated shocks to discount rates and/or expected payoffs. Since financial assets are investment goods whose current value generically depends on their resale value, these requirements extend not only to preferences over current cash flows, but also expected future prices.

Two broad microfoundations for such shocks have been proposed. The first relies on cross-investor heterogeneity in *tastes* for particular assets, holding fixed a certain notion of expected cash flows. These could arise from differences in investor preferences over the provenance of cash flows, such as when some investors prefer to invest in environmentally-friendly firms. Or, investors may have dogmatic heterogeneous beliefs about returns that are orthogonal to the beliefs of other investors. The second relies on constraints or investment mandates that prevent some market participants from investing in a particular asset for *exogenous* reasons. This approach is implemented empirically by trying to infer (unobservable) constraints that affect investor's investment opportunities, such as differences in investment mandates across investment funds.

Both approaches are potentially relevant in practice: a growing literature documents that investors may care about non-pecuniary asset characteristics such as ESG scores (Starks, 2023), and models with belief differences have a long history in asset pricing. Finally, a wide array of funds are subject to a variety of investment mandates that constrain their portfolio choices. At the same time, there are a number of open questions regarding the theoretical foundations of asset pricing with heterogeneous tastes, and about the feasibility of identifying demand systems in the presence of tastes and (unobserved) constraints. Moreover, well-established tools from industrial organization may have only limited applicability to financial markets because financial assets differ from consumer goods in at least three important ways: (i) assets can be flexibly bundled and unbundled using portfolios, and there may be *redundant* assets that offer identical cash flows as other portfolios; (ii) preferences must admit a cardinal (rather than merely ordinal) interpretation; and (iii) portfolio choice features demand complementarities whereby marginal valuations of a given asset depends on the overall portfolio.

The rest of this section analyzes the implications of these issues in detail. Section

2.1 provides two ways of incorporating tastes in a canonical asset-pricing framework that respects standard notions of expected utility theory and clarifies an important distinction between structural and reduced-form approaches (i.e., tastes over payoffs rather than returns). Based on these foundations, Section 2.2 argues that asset pricing with tastes requires cardinal interpretations of taste parameters. Hence identifying these parameters presents greater challenges than in many classical settings from industrial organization that aim to identify ordinal rankings only. This means that portfolio choice will generally be sensitive to assumptions on the security menu, and also that introducing tastes will generally require supplemental assumptions on the strategy space. Section 2.3 shows that dogmatic taste differences may invalidate no arbitrage, a key organizing principle of portfolio choice and asset pricing. Section 2.4 shows that typical (combinations of) utility functions, asset menus, and investment constraints generate cross-asset complementarities in portfolio choice that are critical for identifying and interpreting estimated demand elasticities and assessing instrument validity. Based on these findings, Section 4 shows that failing to account for these complementarities, and the cross-asset price spillovers they induce, can lead to measured demand elasticities of the same magnitude as in Koijen and Yogo (2019) even when the true underlying elasticity is near infinite.

2.1 Introducing Tastes for Financial Assets

Demand-based asset pricing allows investors to have preferences over asset characteristics that need not be directly related to cash flows. In this section, we discuss theoretical approaches, and implications of, incorporating tastes within canonical asset pricing frameworks. We show that this requires making a number of conceptual decisions that can alter some of the key theoretical underpinnings of asset pricing theory. An important feature of our approach is that we explicitly model tastes over *payoffs*, not returns. Koijen and Yogo (2019) and related papers instead specify tastes and dogmatic beliefs directly over asset returns. While differences in tastes over payoffs naturally induce differences in perceived returns for any given asset price, the distinction is nevertheless important. Preferences over payoffs are structural parameters that are invariant to changes in, e.g., the market-wide distribution of tastes and wealth, whereas returns are endogenous to such shocks. Hence, specifying tastes over payoffs allows us to clarify the exact nature of the identification problem, as well as implications for counterfactual analysis.

The basic framework is standard. We consider a one-shot portfolio choice problem in which an investor can choose to consume at date 0 and/or at date 1.[2] A random state of the world $z \in \mathcal{Z} \equiv \{1, \ldots, Z\}$ is realized at date 1, and the probability of state z is $\pi_z \in (0, 1)$. The set of assets is $\mathcal{J} \equiv \{1, \ldots, J\}$. Asset $j \in \mathcal{J}$ offers state-contingent cash flows $y_j(z)$ in state z. There is a set of investors indexed by i. Investor i has a von Neumann-Morgenstern utility function defined over lotteries.

Within this framework, we consider two main taste specifications. The first is payoff-augmenting tastes, defined as additional "consumption-equivalent" value that is generated by an asset of particular *provenance*. The second is additive-separable tastes, by which we mean that the investor obtains some additional value (or disutility) from holding certain assets that is separable from risk-return considerations. We show that both formulations deliver essentially identical conclusions.

Payoff-Augmenting Tastes. Under payoff-augmenting tastes, investor i evaluates her payoffs from holding portfolio $(a^i_j)_{j \in \mathcal{J}}$ by both the cash flows it generates and her *tastes* $(\theta^i_j)_{j \in \mathcal{J}}$ over assets, where $\theta^i_j > 0$. In particular, we assume that preferences are defined over the *effective units of consumption* delivered by portfolio $(a^i_j)_{j \in \mathcal{J}}$ for investor i in state z, and define these as

$$\tilde{c}^i_1(z) \equiv \sum_{j \in \mathcal{J}} \theta^i_j y_j(z) a^i_j + w^i_1(z),$$

where $w^i_1(z) \geq 0$ is a non-marketable endowment in state z.[3]

Investor i's portfolio choice problem is to maximize expected utility over effective consumption subject to budget balance:

$$\begin{aligned} \max_{(a^i_j)_{j \in \mathcal{J}}} \quad & (1-\delta) u^i(c^i_0) + \delta \sum_{z \in \mathcal{Z}} \pi_z u^i(\tilde{c}^i_1(z)) \qquad \text{(P-PA)} \\ \text{s.t.} \quad & c^i_0 = w^i_0 - \sum_{j \in \mathcal{J}} p_j (a^i_j - e^i_j) \\ & \tilde{c}^i_1(z) = \sum_{j \in \mathcal{J}} \theta^i_j y_j(z) a^i_j + w^i_1(z), \end{aligned}$$

where δ is the discount factor, u^i is the utility function, p_j is the price of asset j, w^i_0 is initial

[2]We consider a dynamic portfolio choice problem in Section 5.1

[3]One can also define payoff-augmenting tastes in an additive manner: $\tilde{c}^i(z) = \sum_j (\theta^i_j + y_j(z)) a^i_j + w^i_1(z)$. The main difference is that tastes operate like a "risk-free" component of returns for every asset, with obvious implications for portfolio choice. Overall, however, the main conclusions are unchanged.

wealth, e_j^i is investor i's endowment of asset j, and c_0^i is consumption in period 0.

Effective consumption is useful for capturing the notion that an investor may, for example, value cash flows produced by environmentally-friendly firms more than an identical cash-flow stream produced by other firms. As such, tastes differentiate effective consumption from pure consumption $c_1^i(z) = \sum_{j \in \mathcal{J}} y_j(z) a_j^i + w_1^i(z)$.

Remark 1 (Heterogeneous Beliefs) *There is a close correspondence between payoff-augmenting tastes and heterogeneous beliefs. In particular, one can generally enrich the state space over which payoffs are defined to include "taste-based payoffs." Heterogeneous tastes can then be mapped into heterogeneous beliefs if investors differ in their probability assessments over this augmented state space. An important consideration in this regard is that such taste-related beliefs are dogmatic: investors must agree to disagree, and in particular they may disagree on whether a particular state of the world can be realized. Interestingly, such strong disagreement is desirable when trying to construct instruments for residual demand because it allows for the possibility of orthogonal demand shocks. However, we show below that it also comes with more undesirable consequences.*

Additive Separable Tastes. Next we consider additive-separable tastes. Fix a function G^i that maps portfolio $(a_j^i)_{j \in \mathcal{J}}$ into utils, and define the investor's decision problem as:

$$\begin{aligned} \max_{(a_j^i)_{j \in \mathcal{J}}} \quad & (1-\delta) u^i(c_0^i) + \delta \sum_{z \in \mathcal{Z}} \pi_z u^i(c_1^i(z)) + G^i\left((a_j^i)_{j \in \mathcal{J}}\right) \qquad \text{(P-AS)} \\ \text{s.t.} \quad & c_0^i = w_0^i - \sum_{j \in \mathcal{J}} p_j (a_j^i - e_j^i) \\ & c_1^i(z) = \sum_{j \in \mathcal{J}} y_j(z) a_j^i + w_1^i(z). \end{aligned}$$

In this problem, the utility index u^i is defined in the standard way over pure consumption $c_1^i(z) = \sum_{j \in \mathcal{J}} y_j(z) a_j^i + w_1^i(z)$, but the overall objective is augmented by an additive value to holding a portfolio. This specification captures the idea that an investor may earn a "warm glow" from holding some stocks, or a disutility from holding others. As such, additive tastes are closely related to the way in which some people model "convenience yields," an idea that goes all the way back to Sidrauski (1967)'s model of money demand.

Additive separable tastes differ from payoff-augmenting tastes in that non-pecuniary benefits of holding certain assets do not directly depend on the properties of the utility function. For example, tastes do not necessarily induce wealth or substitution effects in portfolio choice. While this may be an advantage for particular applications, it also has the drawback that it is generally difficult to discipline the particular functional form of G^i, even though the functional form *will* generally determine the trade-off between pecuniary and non-pecuniary aspects of portfolio choice.[4] To ensure a closer correspondence with standard utility theory and existing demand-system approaches, we focus on payoff-augmenting tastes for most of our analysis.

2.2 Cardinal Interpretation of Tastes

Given the nature of the questions studied, standard methods in industrial organization are mainly developed for settings where it is sufficient to identify *ordinal* preferences over alternatives. This is because standard consumer preference rankings can be fully represented by ordinal utility functions. This simplification does not apply in the context of portfolio choice, where expected utility theory requires a cardinal interpretation of utility functions. Hence even linear transformations of tastes may affect portfolio choices.

We start with formulating the optimality conditions under heterogeneous taste specifications. Under payoff-augmenting tastes, investor i's marginal rate of substitution over effective consumption in Program (P-PA) is $\tilde{\Lambda}^i(z) \equiv \frac{\delta \pi_z u^{i'}(\tilde{c}_1^i(z))}{(1-\delta) u^{i'}(c_0^i)}$, where $u^{i'}$ is marginal utility. Under additive-separable tastes, investor i's marginal rate of substitution over pure consumption in Program (P-AS) is analogous: $\Lambda^i(z) \equiv \frac{\delta \pi_z u^{i'}(c_1^i(z))}{(1-\delta) u^{i'}(c_0^i)}$. The optimality condition for a_j^i under payoff-augmenting tastes is $\theta_j^i \sum_{z \in \mathcal{Z}} y_j(z) \tilde{\Lambda}^i(z) = p_j$, while under additive separable tastes it is $\sum_{z \in \mathcal{Z}} y_j(z) \Lambda^i(z) + g_j^i \left((a_j^i)_{j \in \mathcal{J}} \right) = p_j$, where g_j^i is the partial derivative of G^i with respect to a_j^i. These conditions relate the risk-return trade-off, measured using marginal utility, to investor tastes. The solutions to these programs depend on the functional form of tastes and are not invariant to simple monotone transformations. Identifying demand systems thus requires measuring the intensity of tastes, not just their ordinal ranking. Moreover, the *aggregation* of assets into portfolios requires appropriately weighting individual taste parameters by marginal utility.

Proposition 1 (Sensitivity to Rank-preserving Transformations of Tastes) *Portfolio choices are sensitive to rank-preserving transformations of taste parameters. In particular:*

[4] A further consideration is that if we have taste shocks (both aggregate and idiosyncratic) this would be another risk factor to contemplate when forming optimal portfolios. We abstract in the current paper from the additional considerations for identification this would entail.

1. *The solution to* (P-PA) *is sensitive to rank-preserving transformations of* $\theta^i = (\theta^i_j)_{j\in\mathcal{J}}$*, including positive linear transformations and mean-preserving spreads of taste parameters.*

2. *The solution to* (P-AS) *is sensitive to monotone transformations of* G^i*, including linear positive transformations such as the re-scaling of the units in which tastes are measured.*

The intuition for this result is straightforward. Under payoff-augmenting tastes, increasing the nominal value of θ^i_j increases the consumption-equivalent value of holding asset j, leading the investor to allocate more funds to this asset and distorting the overall portfolio. Increasing tastes for all assets simultaneously raises the value of holding all assets, which leaves portfolio weights unchanged but alters the consumption-saving decision between dates 0 and 1. In the case of additive-separable tastes, portfolio choices trade off the marginal increase in non-pecuniary values against the risk-return trade-off as measured by marginal utility over consumption. Since expected utility is cardinal, any rank-preserving transformation (such as a change in units) will alter the optimal portfolio.

2.3 No Arbitrage with Tastes

So far, we have discussed two methods for integrating tastes with portfolio choices and clarified their link to models of heterogeneous beliefs. We also argued that tastes may be useful for identification because they create the possibility of orthogonal demand shocks. At the root of this identification argument is that investors may have dogmatic differences in asset valuations. We now argue that this same feature may invalidate a key organizing principle of asset pricing and portfolio choice, namely no arbitrage. The challenge can be transparently discussed in our setting because we model tastes over *payoffs*, which can be interpreted as structural parameters, rather than *returns*, which are endogenous objects that depend on market participants' preferences, beliefs, and constraints.

No arbitrage is the notion that an equilibrium price system should not admit trades in which an investor receives something for nothing. While this organizing principle is appealing in its own right, it also has important practical implications for demand system estimation. Due to the large number of financial assets, it is infeasible to estimate demand functions for all assets, in particular when some of these assets may have partially redundant cash flows. No arbitrage greatly reduces the dimensionality of the portfolio choice problem by allowing researchers to focus on the pricing state-contingent payoffs (or factor portfolios).[5] Hence it is critical to assess whether no arbitrage applies even when investors have dogmatic taste differences.

Operationalizing no arbitrage requires a theory of value. In the neoclassical approach, the appropriate notion of value is cash flows, and an arbitrage is "something for nothing," or, more formally, "an investment strategy that guarantees a positive payoff in some contingency with no possibility of a negative payoff and no initial net investment" (Ross, 2004). Hence a payoff space X (the set of attainable payoffs, a subset of $\mathbf{R}^Z$) and a pricing function $p : X \to \mathbf{R}$ are sufficient to define no arbitrage.

Definition 1 (No Arbitrage (Cochrane, 2005)) *A payoff space X and pricing function p leave no arbitrage opportunities if, every payoff $x \in X$ that is weakly positive almost surely (i.e., $x \geq 0$) and strictly positive with some positive probability (i.e., $x > 0$) has positive price: $p(x) > 0$.*

This definition is critical for deriving basic properties of price systems, including the existence of positive stochastic discount factors. It also underpins common factor approaches to modeling asset returns using low-dimensional approximations, which is used by Koijen and Yogo (2019) to reduce the dimensionality of the asset space.

When investors differ in their tastes, Definition 1 is not sufficient to define no arbitrage because investors do not evaluate investment opportunities on the basis of cash flows alone. In particular, they may have subjective views on what constitutes "something for nothing" and thus a payoff space alone is not sufficient for defining valuations.

To define no arbitrage with tastes, we provide two preliminary definitions. First, letting a vector subspace $\mathcal{A}$ of $\mathbf{R}^J$ denote the set of feasible portfolios and letting $(p_j)_{j\in\mathcal{J}}$ be a price vector of individual assets, the pricing function $P : \mathcal{A} \to \mathbf{R}$ maps a portfolio $a = (a_j)_{j\in\mathcal{J}}$ into its price according to $P(a) \equiv \sum_{j\in\mathcal{J}} p_j a_j$. Second, investor i has a linear taste function $v^i : \mathcal{A} \to \mathbf{R}^Z$ that maps a portfolio a into a vector $v^i(a)$ of state-contingent taste-augmented payoffs for investor i. Specifically, letting $Y^i \equiv (\theta^i_j y_j(z))_{z,j}$ be the $Z \times J$ matrix of investor i's payoff-augmented cash flows, $v^i(a) \equiv Y^i a$. This generalizes the neoclassical approach in which $\theta^i_j = 1$ for all j.[6] We then have the following definition.

Definition 2 (No Arbitrage with Tastes) *Let taste functions v^i be given for all investors i. The pricing function P leaves no arbitrage opportunities if, for any investor i and any portfolio $a \in \mathcal{A}$*

[5]Such concerns do not arise in, say, consumer good settings, where a consumer is unable to combine parts of multiple cars to arrive at a more desirable bundle of characteristics.

[6]While we focus on payoff-augmenting tastes, analogous results hold for additive-separable tastes.

such that the effective payoff is weakly positive almost surely (i.e., $v^i(a) \geq 0$) and strictly positive with strictly positive probability (i.e., $v^i(a) > 0$), the associated price is positive: $P(a) > 0$.

The definition states that the pricing function P leaves no arbitrage opportunities given taste functions $(v^i)_i$ if and only if, for every i, the pricing function P leaves no arbitrage opportunities in the neoclassical sense when the cash-flow matrix is given as if it were Y^i. The main substantive restriction is that taste functions are linear, in line with Koijen and Yogo (2019). We will discuss how non-linear tastes may restore no arbitrage.

We then have the following result that invalidates no arbitrage if investors have sufficiently heterogeneous tastes.

Theorem 1 (Generic Arbitrage Opportunities with Tastes) *Fix taste functions v^i for all i. There does not exist a pricing function P that leaves no arbitrage opportunities if:*

$$\text{there exist } a, i, \text{ and } i' \text{ such that } v^i(a) > 0 \text{ and } 0 \geq v^{i'}(a). \tag{C}$$

A sufficient condition for (C) is that there exist assets j and j' such that

(i) both assets have identical cash flows:

$$y_j(z) = y_{j'}(z) \text{ for all } z \in \mathcal{Z};$$

(ii) there exist investors i and i' with sufficiently heterogeneous tastes with respect to these assets:

$$\theta^i_j > \theta^i_{j'} \text{ and } \theta^{i'}_j \leq \theta^{i'}_{j'}.$$

Hence, no arbitrage fails as long as tastes are sufficiently heterogeneous and the asset menu is sufficiently rich, such as when the payoffs of a stock over which investors have tastes can be replicated using options, without reference to whether no arbitrage holds with respect to each investor i's taste-augmented cash-flow matrix. The following example illustrates the theorem.

Example 1 (Green and Red Assets) *There are a green asset and a red asset with prices denoted by p_g and p_r, respectively. Both assets deliver a unit payoff with certainty. There are two investor types, denoted by α and β, that differ in their relative taste for the two assets. For each investor* *type i, the taste function is given by $v^i(a_g, a_r) = \theta^i_g a_g + \theta^i_r a_r$ with the following properties: while type α's taste-augmented payoffs for green and red assets satisfy $\theta^\alpha_g > \theta^\alpha_r$, type β has $\theta^\beta_g < \theta^\beta_r$.*

We consider a long-short portfolio consisting of selling one unit of the green asset and buying one unit of the red asset: $a = (-1, 1)$. The price of this portfolio is $P(a) = p_r - p_g$. Investor i's taste-augmented payoff is $v^i(a) = \theta^i_r - \theta^i_g$. If there exists a pricing function P^* that leaves no arbitrage opportunities, then the absence of arbitrage opportunities for type β requires that $P^*(a) > 0$. Since type α can conduct the trade in reverse, no arbitrage for that type requires $P^*(a) < 0$, which is a contradiction. Hence, there does not exist a pricing function that leaves no arbitrage opportunities.

The theorem hinges on two main assumptions. First, taste functions are linear so that taste differences are invariant to quantities. Second, investors can freely short assets and form portfolios. Given these two conditions, price changes alone are not sufficient to equilibriate asset markets. This suggests two possible ways of restoring no arbitrage in models with heterogeneous tastes: trading restrictions or non-linear tastes.

Trading restrictions have a long history in the literature on heterogeneous beliefs.[7] Precisely because dogmatic beliefs can generate problems of arbitrage and mispricing, theoretical models with heterogeneous beliefs generally impose strong restrictions on feasible strategies that preclude the existence of *risk-free* arbitrages. Prominent examples include short-sale constraints or sparse asset menus, such as restricting attention to one risk-free asset and one risky asset, with disagreement only about the dividends of the risky asset. This suggests that researchers can rely on similar restrictions to ensure the existence of equilibria with desirable pricing properties in the presence of tastes differences. While theoretically sound, in empirical applications this approach has the downside that researchers must impose investment constraints on a large number of assets, and that equilibrium outcomes are highly sensitive to the precise form of these restrictions (Detemple and Murthy, 1997). In our framework, Section 5.3.1 shows that misspecified short-sale constraints on one asset can contaminate demand elasticities on complementary assets.

The second approach is to use taste functions with decreasing marginal tastes, whereby the marginal contribution of tastes to investor valuations converge to zero for sufficiently large asset holdings. In principle, this allows investors to agree on marginal valuations even if they differ in their infra-marginal tastes. While appealing, this ap-

[7]See Hong and Stein (2007) for a survey on implications of heterogeneous beliefs on asset pricing.

proach has the downsides that payoffs and co-variances are no longer sufficient for optimal portfolio choice, and that one must identify an entire taste function to estimate demand. Moreover, generically establishing no arbitrage in this case is more complex since no arbitrage only holds at the equilibrium asset holdings. Nevertheless, we view this as a promising avenue for future research.

We conclude this section by discussing the law of one price under tastes.

Remark 2 (Implications for Law of One Price) *Violations of no arbitrage may exist even if the law of one price (LOOP) holds conditional on the asset "color." Given tastes, LOOP can be defined as requiring that two assets which deliver identical taste-augmented payoffs must have the same price. Our example shows that, even if LOOP holds for individual assets, there still exist portfolios over which investors have strict disagreements. This suggests a deeper question, which is to which extent assets that are redundant in terms of their cash flows inherit the non-pecuniary benefits of underlying assets. For example, does a portfolio of an option and a bond generate similar tastes as a stock? Answers to questions such as this appear critical for developing a full-fledged theory of taste-based asset pricing and a necessary first step for proper measurement.*

2.4 Demand Complementarities

Having discussed ways of integrating tastes with portfolio choice, we return to a more formal analysis of the identification challenge for demand systems in financial markets. We emphasize two critical factors that differentiate financial markets from settings typically studied in industrial organization: demand system naturally exhibit *complementarities* in the marginal valuation of various assets, and investors must contend with general equilibrium price determination. We will argue that these considerations render asset demand systems prone to misspecification and, as a result, biased demand elasticities.

To discuss the general identification problem, we enrich decision problem (P-PA) by assuming that an investor faces (unobserved) constraints on portfolio choices. This is relevant in practice and important for current methodologies: the instrumental variable strategy in Koijen and Yogo (2019) relies on "investment mandates" that create exogenous variation in the net supply of certain securities. Formally, we assume that investor i faces (unobserved) $K \geq 0$ investment constraints on portfolio choices. The k-th investment constraint is defined as

$$M_k^i(a^i, p) \leq 0,$$

where $a^i = (a^i_j)_{j\in\mathcal{J}}$ is the vector of investor i's portfolio holdings, $p = (p_j)_{j\in\mathcal{J}}$ is the price vector, and the function $M^i_k(\cdot)$ is twice continuously differentiable in a^i_j for all j. We assume that the set of feasible portfolios induced by these constraints is convex. The types of portfolio mandates observed in practice would naturally satisfy this assumption.

Remark 3 (Investment Constraints) *A variety of constraints fit our approach. First, a short-sale constraint on asset j is $a^i_j \geq -\underline{a}_j$ for some $\underline{a}_j$. Second, asset j is outside investor i's investment universe if $a^i_j \leq 0$ and $a^i_j \geq 0$. Third, an investment mandate which constrains portfolio weights can be modeled as follows. Let χ denote a vector of asset characteristics, and let $\mathcal{C}(\chi)$ be the set of assets with these characteristics. The portfolio weight of assets with characteristic χ in investor i's portfolio is $\omega^i(\chi) = \frac{\sum_{j\in\mathcal{C}(\chi)} p_j a^i_j}{\sum_{j\in\mathcal{J}} p_j a^i_j}$. An investment mandate requires that $\omega^i(\chi) \geq \underline{\omega}(\chi)$ and $\omega^i(\chi) \leq \overline{\omega}(\chi)$ for some parameters $\underline{\omega}(\chi)$ and $\overline{\omega}(\chi)$.*

Given these assumptions, investor i's portfolio choice problem is:

$$
\begin{aligned}
\max_{a^i} \quad & (1-\delta)u^i(c^i_0) + \delta \sum_{z\in\mathcal{Z}} \pi_z u^i(\tilde{c}^i_1(z)) \\
\text{s.t.} \quad & c^i_0 = w^i_0 - \sum_{j\in\mathcal{J}} p_j(a^i_j - e^i_j) \\
& \tilde{c}^i_1(z) = \sum_{j\in\mathcal{J}} \theta^i_j y_j(z) a^i_j + w^i_1(z) \text{ for all } z \\
& M^i_k(a^i, p) \leq 0 \text{ for all } k,
\end{aligned}
$$

where the first two constraints are the budget constraints at time 0 and time 1 in state z. Denote by λ^i_k the Lagrange multiplier associated with the k-th investment constraint, and $m^i_{k,j}(a^i, p)$ the partial derivative of $M^i_k(a^i, p)$ with respect to a^i_j.

The first-order necessary condition for investor i's choice of holdings of asset j is:

$$
F^i_j(a^i, p) \equiv \theta^i_j \sum_{z\in\mathcal{Z}} y_j(z)\tilde{\Lambda}^i(z) + \sum_k \lambda^i_k \frac{m^i_{k,j}(a^i, p)}{(1-\delta)u^{i\prime}(c^i_0)} - p_j = 0. \tag{1}
$$

We will refer to $a^i_j(a^i_{-j}, p)$ as investor i's demand function for asset j, which highlights that the demand for a particular asset may depend on the holdings and prices of all assets. The system of J equations in J unknowns that determines i's optimal portfolio is:

$$\begin{bmatrix} F_1^i(a^i, p) \\ F_2^i(a^i, p) \\ \vdots \\ F_J^i(a^i, p) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

This system generically features cross-asset *demand complementarities* whereby the optimal choice of a_j^i depends on holdings of other assets. There are two sources of such cross-asset restrictions. The first is the canonical notion of portfolio diversification, whereby an investor's marginal valuation of one particular asset depends on the holdings of all other securities (in particular, their covariance with the overall portfolio).

The second arises from portfolio constraints that permit some degree of substitutability between assets, as in the following three examples. The first example is a bond fund which may face a requirement to invest a certain proportion of its wealth in high-yield bonds but has flexibility over which particular bonds to invest in. The second example is a fund that is designed to track a particular index but is permitted to have some degree of tracking error. The third example is a constraint on market-weighted portfolio shares held in different asset classes. For example, an investment fund may be required to hold x% of its assets under management in stocks with a high ESG score, or must hold a portfolio whose value-weighted ESG score exceeds some threshold.

Demand complementarities create difficulties for identifying structural parameters in demand system estimation. In particular, given demand complementarities, even exogenous variation in a single price is generally not sufficient to identify specific demand parameters such as own-price elasticities (Berry and Haile, 2021). This is because complementarities induce endogenous movements in the prices of related goods, thereby contaminating the demand response to price changes.

To see how these demand complementarities affect the identification problem in our particular context, assume that we have an ideal instrument in hand, namely a purely exogenous supply shock χ_s that directly affects only the supply of asset s. This could be an agent outside the model taking some fraction of the supply of asset s. By the implicit function theorem, the optimal change of i's portfolio in response to this shock satisfies:

$$\begin{bmatrix} \sum_{j=1}^{J} \left\{ \frac{\partial F_1^i(a^i,p)}{\partial p_j} \frac{\partial p_j}{\partial \chi_s} + \frac{\partial F_1^i(a^i,p)}{\partial a_j^i} \left[\sum_{j' \neq j} \frac{\partial a_j^i(a_{-j}^i,p)}{\partial a_{j'}^i} \frac{\partial a_{j'}^i(a_{-j'}^i,p)}{\partial p_{j'}} \frac{\partial p_{j'}}{\partial \chi_s} \right] \right\} \\ \sum_{j=1}^{J} \left\{ \frac{\partial F_2^i(a^i,p)}{\partial p_j} \frac{\partial p_j}{\partial \chi_s} + \frac{\partial F_2^i(a^i,p)}{\partial a_j^i} \left[\sum_{j' \neq j} \frac{\partial a_j^i(a_{-j}^i,p)}{\partial a_{j'}^i} \frac{\partial a_{j'}^i(a_{-j'}^i,p)}{\partial p_{j'}} \frac{\partial p_{j'}}{\partial \chi_s} \right] \right\} \\ \vdots \\ \sum_{j=1}^{J} \left\{ \frac{\partial F_J^i(a^i,p)}{\partial p_j} \frac{\partial p_j}{\partial \chi_s} + \frac{\partial F_J^i(a^i,p)}{\partial a_j^i} \left[\sum_{j' \neq j} \frac{\partial a_j^i(a_{-j}^i,p)}{\partial a_{j'}^i} \frac{\partial a_{j'}^i(a_{-j'}^i,p)}{\partial p_{j'}} \frac{\partial p_{j'}}{\partial \chi_s} \right] \right\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{2}$$

The goal of demand system estimation is to identify structural parameters that determine asset demand. For example, researchers might be interested in estimating asset j's own-price elasticity $-\frac{\partial a_j^i(a_{-j}^i,p)}{\partial p_j} \frac{p_j}{a_j^i(a_{-j}^i,p)}$, which is the *ceteris paribus* percentage change in investor i's demand for asset j given a percentage change in the asset's price. The system of equations (2) shows why estimating this elasticity is difficult in the presence of demand complementarities. If investor i expects other investors' demand systems to exhibit complementarities, she will expect the prices of related assets to change as well. This is formally captured by the price spillovers $\frac{\partial p_j}{\partial \chi_s}$ (shown in red). If her demand exhibits complementarities, then her own response will be contaminated by induced changes in her holdings of other assets, as determined by terms $\frac{\partial F_j^i}{\partial p_{j'}}$ and $\frac{\partial F_j^i}{\partial a_{j'}^i}$ (shown in dark green) and their summation across all other assets.

Price spillovers and demand complementarities are naturally linked to each other: price spillovers occur in equilibrium if and only if at least *some* investors exhibit demand complementarities with respect to the shocked asset. Hence, specifying the demand system requires some *theory* for what these complementarities are, and how investors reason about equilibrium price spillovers. The neoclassical approach is to assume investors care about cash flows only, and use no arbitrage and rational expectations to discipline cross-asset spillovers. However, this is difficult to maintain in a world with non-pecuniary tastes in which investors may have different views about the degree of complementarity between assets, all of which must be jointly inferred from the data.

To circumvent these issues, Koijen and Yogo (2019) impose a-priori restrictions on preferences, payoffs and mandates under which demand complementarities are de facto ruled out (i.e, $\frac{\partial F_j}{\partial a^i_{j'}} = 0$ for $j' \neq j$).[8] With respect to mandates, they rule out constraint-based complementarities by considering only *extensive margin quantity* restrictions on individual assets. Such purely asset-specific mandates rule out the empirically-relevant cases in which mandates permit some substitution across assets. With respect to payoffs and preferences, they consider log preferences with log-normal returns, and their Assumption 1 mutes diversification considerations by imposing a one-factor structure on returns in which the conditional covariance matrix is diagonal. Rationales for this characteristics-based approach must be statistical, since no arbitrage may not hold under tastes. As such, their Proposition 1 then shows that each asset's *own* characteristics are determining for deriving own portfolio weights relative to the outside asset. As we show in Section 4, imposing these restrictions on returns rather than cash flows can bias elasticity estimates. This allows us to reconcile low estimates of financial demand elasticities obtained using the Koijen and Yogo (2019) methodology with the significantly higher estimates of the neoclassical macro-finance literature.

More broadly, portfolio-level demand complementarities may undermine the validity of asset-level instrumental variables: the exclusion restriction fails if shocks to one asset are passed through to other assets via investors' endogenous demand responses. Determining whether this problem is relevant in a particular setting requires knowledge of (or structural assumptions on) the degree of substitutability across assets, which in turn depends on investors unobserved preferences and tastes.

3 Equilibrium Framework

The previous section offered a theoretical analysis of central modeling and identification issues in demand-system asset pricing. We now construct a fully-specified general equilibrium economy based on Lucas (1978) that allows us to assess potential biases introduced by the aforementioned theoretical considerations. In particular, because the model allows us to flexibly model patterns of substitution and complementarity among assets,

[8] In particular, they work in the logit-demand framework, where substitution patterns are constrained to be symmetric for all alternative assets. In industrial organization, this leads to the well-known Independence of Irrelevant Alternatives (IIA) problem. However, this problem is more severe in financial markets where there are direct demand complementarities between assets. One way of addressing heterogeneous substitution patterns is to use *nested logit* frameworks. This requires taking an ex-ante stance on substitutability, which is difficult when estimating demand systems within an asset class (i.e., stocks) without ex-ante restrictions on taste parameters.

we can use it to assess whether existing methods are able to accurately infer structural demand elasticities. We are also able to directly compute "true" counterfactuals and ask how accurately they can be estimated using parameters inferred from equilibrium play. We use the payoff-augmenting approach to modeling tastes because it aligns more closely with standard expected utility theory and existing demand-system approaches.

Environment. There is a single period of trading and all information is public. Asset payoffs depend on the realization of an aggregate state $z \in \{1, 2\}$ that is realized at date 1. The probability of state z is given by $\pi_z \in (0, 1)$. Associated with each state z is a Lucas tree that pays off $y(z)$ if the state is z. Trees are perfectly divisible, and the aggregate supply of each tree is equal to one. To nest "demand effects" and investor tastes, we assume that the tree referencing aggregate state 1 consists of two equally-sized sub-trees: the *green tree* and the *red tree*. In aggregate state 1, the green tree pays $y_g(\iota)$ and the red tree pays $y_r(\iota)$, where $\iota \in \{g, r\}$ is a distributional shock that determines which of the two trees offers more cash flows, holding total cash flows in state 1 fixed. In particular,

$$y_g(\iota) = \begin{cases} y(1) + \epsilon & \text{if } \iota = g \\ y(1) - \epsilon & \text{if } \iota = r \end{cases} \quad \text{and} \quad y_r(\iota) = \begin{cases} y(1) - \epsilon & \text{if } \iota = g \\ y(1) + \epsilon & \text{if } \iota = r \end{cases},$$

which implies that the distributional shock is fully diversifiable:

$$\frac{1}{2} y_g(\iota) + \frac{1}{2} y_r(\iota) = y(1) \text{ for all } \iota \in \{g, r\}.$$

The probability of the distributional shock favoring red is $\Pr(\iota = r) = \rho$. We use parameter $\epsilon \in [0, y(1))$ to vary the substitutability between green and red trees. If $\epsilon = 0$, then green and red trees are perfect substitutes with respect to their cash flows (but perhaps not with respect to their tastes). If $\epsilon > 0$, they are complements because holding both serves to diversify distributional risk. Table 1 summarizes the payoff structure.

Given this structure, one can think of assets as having two "characteristics:" the aggregate state of the world in which their payoffs accrue (i.e., 1 or 2), and their color. These characteristics determine in which states cash flows accrue (and thus serve as useful statistical summaries of the overall cash flow distribution), and they can also be used to define non-pecuniary tastes. When state 1 trees are perfect substitutes ($\epsilon = 0$), the color characteristic is irrelevant for cash flows and *only* matters through its link with tastes.

		State 1 (π_1)		State 2 (π_2)
		$\iota = g\ (1-\rho)$	$\iota = r\ (\rho)$	
Tree 1	*green*	$y(1) + \epsilon$	$y(1) - \epsilon$	0
	red	$y(1) - \epsilon$	$y(1) + \epsilon$	0
Tree 2		0		$y(2)$

Table 1: The Description of the Payoff Structure.

When state 1 trees are imperfect substitutes ($\epsilon > 0$), even investors without tastes ($\theta^i_j = 1$) care about the color characteristic because it summarizes cash flow risk. Finally, the limit $\epsilon \to 1$ leads to three distinct states of the world, each associated with a single tree that cannot be substituted for each other. Hence, ϵ also modulates how green and red trees should be interpreted. When ϵ is small, they might represent two stocks from the same industry with similar cash flow processes. When ϵ is high, they might represent different asset *classes* that pay off in different states of the world.

Investors. There are different investor types indexed by i, where types determine investors' endowment, tastes, and investment constraints. Investor i takes positions a^i_j in asset $j \in \mathcal{J} \equiv \{g, r, 2\}$, subject to feasibility with respect to any investment constraints or mandates. To remove taste-based distortions of the consumption-savings margin, we assume that investors care only about consumption at date 1, and evaluate the payoffs of their portfolios using *effective units of consumption*. Based on Section 2.1, we define this as

$$\tilde{c}^i(\iota) \equiv \sum_{j \in \mathcal{J}} \theta^i_j y_j(\iota) a^i_j \text{ for each } \iota \in \{g, r, 2\},$$

where $\tilde{c}^i(r)$ and $\tilde{c}^i(g)$ represent effective consumption as a function of the distributional shock and $\tilde{c}^i(2)$ is effective consumption in state 2. For simplicity, we normalize tastes for tree 2 to be equal to one: $\theta^i_2 = 1$.

Investor i is endowed with e^i_g, e^i_r and e^i_2 units of green, red, and state 2 trees. The aggregate endowment of tree j is E_j, with $E_j = \frac{1}{2}$ for each $j \in \{g, r\}$, $E_2 = 1$, and

$$\sum_i e^i_j = E_j \text{ for each } j \in \{g, r, 2\}.$$

Investor preferences over state-contingent effective units of consumption are given by CRRA utility function u. Normalizing the price of tree 2 to $p_2 = 1$ and denoting the prices of green and red trees by p_g and p_r, we define the market value of type i's endowment as $W^i \equiv e_2^i + p_g e_g^i + p_r e_r^i$. Then the budget constraint is

$$a_2^i + p_g a_g^i + p_r a_r^i = W^i. \tag{3}$$

Using this equation to substitute out holdings of tree 2 yields the decision problem

$$\begin{aligned} \max_{a_g^i, a_r^i} \quad & \pi_1 \left[\rho u \left(\theta_g^i y_g(r) a_g^i + \theta_r^i y_r(r) a_r^i \right) + (1-\rho) u \left(\theta_g^i y_g(g) a_g^i + \theta_r^i y_r(g) a_r^i \right) \right] \\ & + \pi_2 u \left(y(2)(W^i - p_g a_g^i - p_r a_r^i) \right) \\ \text{s.t.} \quad & M_k^i(a^i, p) \leq 0 \text{ for all } k, \end{aligned} \tag{4}$$

where $M_k^i(a, p)$ again denotes the k-th investment constraint for investor i.

Equilibrium concept. Our equilibrium concept is competitive equilibrium.

Definition 3 (Competitive Equilibrium) *A competitive equilibrium consists of asset prices* (p_g, p_r) *and portfolios* (a_g^i, a_r^i, a_2^i) *for each i such that:*

1. *Given asset prices, portfolios solve decision problem* (4) *for each i.*

2. *The goods market clears:* (3) *holds for each i.*

3. *Markets clear for every asset:* $\sum_i a_j^i = E_j$ *for each* $j \in \{g, r, 2\}$.

Simplifying Assumption. As in Koijen and Yogo (2019), we assume log utility. While this is not essential, in our setting it has the added benefit that the income and substitution effects from a shock to taste intensity θ^i exactly cancel out. Hence, inference is likely to be even more difficult with general utility functions.

4 Measuring and Interpreting Demand Elasticities

One of the central ideas of neoclassical finance is that financial markets are quick to reallocate capital in response to changes in expected returns, leading to high demand elasticities for financial assets. A central claim of demand-system asset pricing is that this view may be incorrect, with far-reaching implications. For example, Koijen and Yogo (2021) state:

> Our estimates, which agree with demand elasticities estimated by others [that follow the same methodology], are three orders of magnitude smaller than those implied by calibrations of standard asset pricing models. For example, a calibration of the capital asset pricing model (CAPM) implies a demand elasticity for individual stocks that exceeds 5,000. Investors should easily arbitrage any deviation from the CAPM because with limited idiosyncratic risk at the individual stock level, there is a high elasticity of substitution across stocks.

We now assess whether this discrepancy is indeed due to the way financial markets function, or may at least in part stem from measurement errors induced by misspecified demand systems that ignore complementarities of the form discussed in Section 2.4. In particular, we argue that estimated elasticities are highly sensitive to the manner in which demand systems specify substitutability and price spillovers across assets, and existing approaches based on logit-type demand systems as in Koijen and Yogo (2019) *assume* that these substitution patterns cannot be asset-specific. This can result in low *measured* elasticities even when the "true" underlying elasticities are large.

The most straightforward version of this argument can be obtained in the most basic version of our model in which investors are ex-ante identical, have no non-pecuniary tastes ($\theta^i_j = 1$) and face no portfolio constraints or mandates. Since the model then collapses to the standard representative-agent model, the first-order conditions for optimal holdings of green and red trees hold with equality, and are given by:

$$\begin{aligned}
a^i_g: \qquad (1-\pi_1)p_g &= \pi_1\rho\frac{y(1)-\epsilon}{y(2)}u'\left(\frac{\tilde{c}^i(r)}{\tilde{c}^i(2)}\right) + \pi_1(1-\rho)\frac{y(1)+\epsilon}{y(2)}u'\left(\frac{\tilde{c}^i(g)}{\tilde{c}^i(2)}\right); \\
a^i_r: \qquad (1-\pi_1)p_r &= \pi_1\rho\frac{y(1)+\epsilon}{y(2)}u'\left(\frac{\tilde{c}^i(r)}{\tilde{c}^i(2)}\right) + \pi_1(1-\rho)\frac{y(1)-\epsilon}{y(2)}u'\left(\frac{\tilde{c}^i(g)}{\tilde{c}^i(2)}\right).
\end{aligned}$$

The economic content of these relation is well understood. Asset prices are related to the marginal rate of substitution between aggregate states 1 and state 2, with the former accounting for variation in rates of returns and consumption levels generated by the distributional shock $\iota \in \{g, r\}$. Important for our purposes is that this simple system naturally generates heterogeneous substitution patterns: because green and red trees pay off in the same aggregate state, their marginal valuations depend more closely on consumption in aggregate state 1 than does the marginal valuation of tree 2. Hence, as long as ϵ is not too large, green and red trees are closer substitutes for each other than for tree 2, and their prices will be more closely correlated. Such heterogeneous substitution patterns are not present in logit-demand systems of the type used in Koijen and Yogo (2019).

The appropriate theoretical notion of a demand elasticity is the *ceteris paribus* percentage change in quantity relative to a percentage change in the asset price,

$$\eta_j^i \equiv -\frac{\partial a_j^i}{\partial p_j}\frac{p_j}{a_j},$$

with *all prices other than p_j held fixed.* Since investors are endowed with financial assets, we do not assume that *wealth* is held fixed. Instead, endowments are repriced alongside traded assets, albeit not necessarily in a manner that holds expected utility fixed. The underlying feature that investors are both "suppliers" and "consumers" of financial assets thus differentiates financial demand from standard Marshallian or Hicksian demand.

To simplify notation, henceforth we assume that aggregate states 1 and 2 have the same total output, $y(1) = y(2) = 1$, and that the probability of each aggregate state is equal to one half, $\pi_1 = \frac{1}{2}$. This allows us to focus on the relative pricing of green and red trees without worrying about aggregate risk. In the representative-agent benchmark, we then recover the standard result that demand elasticities for green and/or red trees are infinite when these assets are perfect substitutes for each other.

Proposition 2 (Representative-Agent Benchmark) *Let $y(1) = y(2) = 1$ and $\pi_1 = \frac{1}{2}$. In the representative-agent benchmark without tastes or mandates, prices are*

$$p_g = 1 - (2\rho - 1)\epsilon \qquad \text{and} \qquad p_r = 1 + (2\rho - 1)\epsilon.$$

If green and red trees are perfect substitutes ($\epsilon = 0$), then the green elasticity is $\eta_g^i = \infty$.

Can one recover the "true" elasticity using a demand system that does not account for endogenous cross-asset spillovers and demand complementarities at the investor level? To answer this question, we conduct the following "ideal" thought experiment: we exogenously adjust the supply of green trees by ψ, and distribute these trees to existing investors pro rata (the proof shows that similar results obtain if only a subset of investors receive additional trees). Such a supply shock is what commonly-used instrumental variable strategies aim to approximate. For example, the Koijen and Yogo (2019) instrument relies on cross-sectional variation in investment universes (i.e., variation in the ability of investors to hold a particular asset). Our shock can be interpreted as a change in investment mandates that forces an outside investor to relinquish their holdings of green trees. Since a clean shock to an asset price is available, under the assumption that all other prices remain equal one can then estimate the price coefficient in the demand function for green trees using the observed change in green holdings.

While initially there was no uncertainty about total output in aggregate state 1, the supply shock creates some. Let $\epsilon = \tilde{\epsilon} y(1)$, where $\tilde{\epsilon}$ is the distributional shock in percentage terms. The output available for consumption in aggregate state 1 depending on the distributional shock $\iota \in \{g, r\}$, denoted by $\hat{y}(1, \iota)$, is then given by

$$\hat{y}(1, g) = (1 + \psi + \psi\tilde{\epsilon})y(1) \qquad \text{and} \qquad \hat{y}(1, r) = (1 + \psi - \psi\tilde{\epsilon})y(1).$$

The supply shock to green trees leads to both an aggregate consumption shock of size $1 + \psi$ and additional sensitivity to the distributional shock $\pm\psi\tilde{\epsilon}$. To get simple expressions, assume for now that the probability of the distributional shock is $\rho = \frac{1}{2}$. Then, conditional on the supply shock, asset prices are

$$\begin{aligned} p_g &= \frac{1}{2}\left[\frac{1-\tilde{\epsilon}}{1+\psi-\psi\tilde{\epsilon}} + \frac{1+\tilde{\epsilon}}{1+\psi+\psi\tilde{\epsilon}}\right] \\ p_r &= \frac{1}{2}\left[\frac{1+\tilde{\epsilon}}{1+\psi-\psi\tilde{\epsilon}} + \frac{1-\tilde{\epsilon}}{1+\psi+\psi\tilde{\epsilon}}\right] \end{aligned},$$

which is state 1 consumption growth average across distributional shocks and adjusted by the distributional shock loadings of green and red trees. The central upshot is that, because green and red trees are substitutes, equilibrium considerations will lead to a *repricing of red trees* even though there was no direct shock to their supply. In particular, when assets are perfect substitutes (i.e., $\epsilon = 0$), green and red trees have the same expected returns after the shock. As the next result shows, this endogenous *return response* implies that investors do not adjust their portfolios much at all after the supply shock even when the true underlying elasticity (in which the price of red trees is held fixed) is infinite.

Proposition 3 (Measured vs True Elasticities) *Let $\epsilon = 0$. Consider a purely exogenous shock*

ψ to the supply of green trees. Since $p_g = \frac{1}{1+\psi}$, the change in the green price is $\frac{1}{1+\psi} - 1$. Quantities of the green asset are $a_g = \frac{1}{2} + \psi$, and so the change in quantity is ψ. Hence, the observed own-price elasticity of green trees is

$$-\frac{\Delta a_g}{\Delta p_g}\frac{p_g^0}{a_g^0} = 2(1+\psi),$$

even though the true structural elasticity is infinity.

The example is stark but it reveals a robust mechanism: when *infinitely* elastic markets rapidly reprice substitute assets in response to shocks, *equilibrium* portfolio responses may suggest very low elasticities even when the true structural elasticity is high. The reason is that individual demand responses are *strategic substitutes*: when other investors adjust their portfolios, the resulting price response implies that a given individual investor will rebalance her portfolio less than she otherwise would.

Figure 2 shows that this is true even when green and red trees are partial substitutes (see the Appendix for the closed-form expressions). In this figure, true and measured elasticities are plotted on a log scale. Measured elasticities are small throughout and of the same order of magnitude as leading estimates in the demand-system literature, namely around 2-3. The true elasticity instead approaches infinity as ϵ goes to zero (such as when there are two closely substitutable stocks), and falls below the measured elasticity when $\epsilon \to 1$. In this latter limit, there is no substitutability: there are three states, all of which are associated with a single tree. Hence, a shock to the supply of green trees affects consumption only in a single state, leading to low underlying elasticities.[9]

These observations reveal a deeper conceptual point. In neoclassical finance, the relevant object of analysis is state-contingent consumption, not asset holdings.[10] As such, neoclassical approaches are perfectly consistent with low elasticities over state-contingent consumption, but suggest high *asset-level* elasticities when multiple assets deliver similar

[9]Chaudary, Fu, and Li (2023) use a nested-logit demand system for the corporate bond market to document empirically that substitution patterns do indeed depend on "similarity" between assets, and that there is less substitution at higher levels of aggregation. However, because the nested logic specification requires an ex-ante specification of substitution patterns, such approaches may be less suited to the cross-section of stocks (or general financial assets). An and Huber (2024) study intermediary effects in foreign exchange by specifying elasticities over risk factors, thereby circumventing the problem of asset substitution. However, this methodology requires ex-ante knowledge of the risk factors, and these can be identified separately from intermediary preferences only if long-run risk prices are invariant in intermediary behavior.

[10]The introduction of tastes over specific holdings will imply a departure from this fundamental feature.

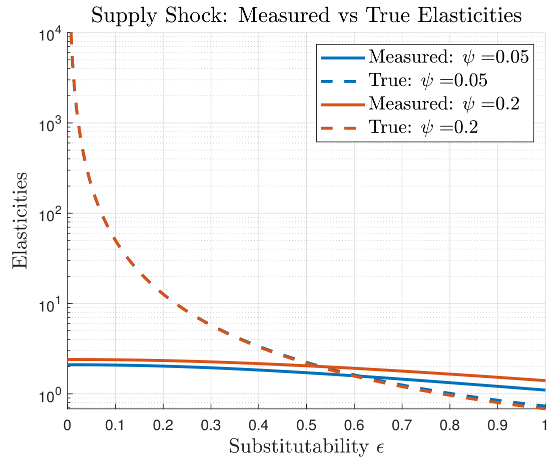


Figure 2: Supply Shock: Measured vs True Elasticities.

levels of consumption in a given set of states of the world. In our example economy, endogenous price spillovers therefore capture the extent to which an *asset-level shock* affects the price of state-contingent consumption. When there are substitute assets, this pass-through is near perfect and the measured elasticity reveals the low consumption-level elasticity even when the underlying asset-level elasticity is very large.

The demand system in Koijen and Yogo (2019) does not capture this adjustment mechanism to the shock for two reasons. First, logit-style demand systems do not permit asset-specific substitution patterns (i.e., they restrict substitution between green and red trees to be the same as between green tree and tree 2).[11] Second, investors are assumed to have heterogeneous beliefs (or tastes) over *returns*, not payoffs, and these do not vary with the instruments. Hence, there is no endogenous mechanism by which the shock to asset j leads to an adjustment to the expected returns to asset $j' \neq j$.

In our model, the true elasticity can be backed out from equilibrium trading behavior because the model fully specifies the degree of substitutability between assets (i.e., the payoff structure and investor taste parameters), and the equilibrium price adjustment mechanism with respect to shocks (i.e., full information general equilibrium). More broadly, estimation procedures could control for price changes in substitute assets

[11]The foundations of this problem relate to well-known Independence of Irrelevant Alternatives (IIA) property of logit demand. In industrial organization, a common solution to this problem is to introduce random coefficients in the cross-section of consumers, which weakens the IIA property at the level of *market shares* even when it persists at the consumer level. In contrast, demand-system asset pricing studies individual-level portfolio shares rather than market shares. Random coefficients also do not address endogenous cross-asset spillovers driven by portfolio choices.

by estimating a fixed point between individual demands and a matrix of price spillovers.

While theoretically appropriate, the empirical implementation of such a fixed point approach may be challenging. Price spillovers depend on the market-wide degree of substitutability between assets, which in turn depends on the unobserved cross-sectional distribution of taste parameters. Hence researchers would have to jointly estimate the spillover matrix alongside individual demand functions, checking for consistency between the two using market clearing. This difficulty suggests that demand systems may be simpler to estimate for more aggregated categories of assets, such as stocks versus bonds, where cross-asset spillovers are likely to be relatively small. At the same time, it may be harder to find appropriate instruments for such settings.

Remark 4 (Supply side) *One challenge in measuring investor-level demand elasticities in general equilibrium endowment economies is that investors face residual supply curves that are purely driven by other investors' demand. Since aggregate quantities are fixed, market clearing in response to aggregate shocks must therefore occur through prices, rendering elasticities relatively uninformative because they do not reflect an individual's ideal quantity response to a price change. This differentiates financial markets from, e.g., the approach in Berry, Levinsohn, and Pakes (1995), who study a nationwide market but allow for quantity adjustments on the supply side. One way of incorporating a richer supply side in financial markets is to study investors with price impact, in which case trade quantities are informative about marginal valuations. Neuhann and Sockin (2024) provide a framework with price impact that allows for rich investor heterogeneity.*

5 Identifying Structural Parameters

The previous section discussed the measurement and appropriate interpretation of demand elasticities but did not ask whether elasticities are themselves sufficient to identify structural parameters (i.e., parameters that are invariant in counterfactuals). We now turn to the question of whether estimated demand elasticities, if measured correctly, are sufficient to assess counterfactuals. For example, Koijen, Richmond, and Yogo (2022) want to estimate asset prices after a shock to the cross-sectional wealth distribution. This requires identifying preference and constraint parameters that are invariant to a change in the wealth distribution.

We discuss two main challenges to the identification of structural parameters. The first is that investors in financial markets care about the resale value of their assets. This means that demand functions will generally reflect not only the investor's individual tastes for an asset, but also her expectations about *other* investors' future valuations. Counterfactuals require knowledge of the individual-specific taste distribution. Second, it is difficult to separately identify tastes and constraints, but constrained investors will respond differently to counterfactual shocks than unconstrained investors.

5.1 Dynamic trading: whose preferences are being measured?

We begin by discussing the role of dynamic trading and resale considerations, by which we mean the fact that asset valuations are at least partially determined by expected future resale prices. Our main result is that forward-looking demand complicates the identification of individual and "market-wide" tastes. In particular, investors may be willing to buy an asset that they personally dislike if they believe that other investors will pay a high price for it in the future. The basic mechanism underlying this identification goes back all the way to Keynesian beauty contests (Keynes, 1936) and models of speculative demand under heterogeneous tastes (Harrison and Kreps, 1978).

To analyze how resale considerations alter the identification of structural taste parameters, we use a simple two-period variant of our baseline framework. Trees are durable assets which pay dividends in two periods. Investors can trade trees at the beginning of each period, and consume the per-period payoffs generated by their trees at the end of each period. The payoff structure is the same as in our baseline framework, and the aggregate state processes $z \in \{1, 2\}$ and $\iota \in \{g, r\}$, which determine payoffs, are *i.i.d.* across periods.

We denote investor i's purchases of trees at the beginning of period $t \in \{1, 2\}$ by $\mathbf{a}^i_t = (a^i_{g,t}, a^i_{r,t}, a^i_{2,t})$, and we denote by $\mathbf{a}^i_0$ investor i's exogenous endowment. The investor i's state variable at the beginning of period $t \in \{1, 2\}$ consists of the portfolio of asset positions purchased in the previous period, $\mathbf{a}^i_{t-1}$, and the current-period taste parameters $\Theta^i_t = (\theta^i_{j,t})_{j \in \{g,r,2\}}$, which are permitted to evolve stochastically over time. The individual state at the beginning of period t is therefore $\mathbf{s}^i_t = (\mathbf{a}^i_{t-1}, \Theta^i_t)$.

We denote by $\mathbf{S}_t$ the aggregate state variable sufficient for determining prices, which naturally includes the aggregate distribution over investor wealth and tastes. We use this structure to introduce potential variations in market prices over time. In particular, investors know the realization of $\mathbf{S}_1$ when forming portfolio allocations at date 1, and their choices are also influenced by their expectations of $\mathbf{S}_2$.

We write the price of asset $j \in \mathcal{J} \equiv \{g, r, 2\}$ as a function of the aggregate state: $p_{j,t} = P_j(\mathbf{S}_t)$ for some endogenous function P_j. The state-contingent gross return on asset $j \in \mathcal{J}$ is $R_j(\mathbf{S}_2) = \frac{P_j(\mathbf{S}_2)}{P_j(\mathbf{S}_1)}$. Given heterogeneity in tastes, a single investor will thus be concerned with the fact that changes in the preferences or wealth of other investors can induce changes in prices and, therefore, her perceptions of expected returns.

It is sufficient for our purposes to consider the decision problem of a single investor who takes as given the stochastic process over the aggregate state. The investor's wealth at the beginning of period t is determined by the realized state and previous asset positions: $W_t^i(\mathbf{a}_{t-1}^i, \mathbf{S}_t) = \sum_{j \in \mathcal{J}} P_j(\mathbf{S}_t) a_{j,t-1}^i$. We solve the problem by backwards induction, assuming that investors face short-sale constraints. For ease of exposition, assume that green and red trees are perfect substitutes: $\epsilon = 0$. In this case, the second-period choice between green and red trees is bang-bang: the investor buys only green trees if $\theta_{g,2}^i / P_g(\mathbf{S}_2) > \theta_{r,2}^i / P_r(\mathbf{S}_2)$, and only red trees if the inequality is reversed. We then have the following characterization of the second-period value function.

Lemma 1 (Value function) *Let $\epsilon = 0$ and $u = \log$. The second-period value function satisfies:*

$$V_2^i(\mathbf{s}_2^i, \mathbf{S}_2) = H(\Theta_2^i, \mathbf{S}_2) + \log\left(W_2^i(\mathbf{a}_1^i, \mathbf{S}_2)\right),$$

where $H(\Theta_2^i, \mathbf{S}_2)$ depends on investor i's tastes and market prices in period 2, but is independent of any investor choices at date 1.

The separability of the value function into a taste component and a wealth component is a result of log utility. However, this feature is not essential for our results below. What is important is that investors take into account future market prices when forming portfolios. Indeed, weakening separability would further complicate identification.

Now turn to the first-period decision problem. Maintaining the assumptions of log utility and normalizing $P_2(\mathbf{S}_1) = 1$, we can then write the period 1 decision problem under discount factor δ as:

$$V_1^i(\mathbf{s}_1^i, \mathbf{S}_1) = \max_{\mathbf{a}_1^i \geq 0} (1-\delta) \Big[\pi_1 \log \Big(y(1) \left(\theta_{g,1}^i a_{g,1}^i + \theta_{r,1}^i a_{r,1}^i \right) \Big)$$
$$+ \pi_2 \log \left(y(2) \Big(W_1^i(\mathbf{a}_0^i, \mathbf{S}_1) - P_g(\mathbf{S}_1) a_{g,1}^i - P_r(\mathbf{S}_1) a_{r,1}^i \Big) \right) \Big]$$
$$+ \delta \mathbb{E}^i \Big[H(\Theta_2^i, \mathbf{S}_2) + \log \Big(\sum_{j \in \mathcal{J}} P_j(\mathbf{S}_2) a_{j,1}^i \Big) \Big],$$

where the expectation $\mathbb{E}^i$ is taken with respect to private tastes and the aggregate state variable at date 2. The first two lines exactly correspond to static decision problem (4), whereby the investor chooses the quantity of green and red trees to maximize expected utility in states 1 and 2. The third line captures dynamic considerations, whereby the investor takes into account how current asset positions affect future wealth.

Since green and red trees are perfect substitutes, let $\tilde{c}_1^i(1) = y(1) \left(\theta_{g,1}^i a_{g,1}^i + \theta_{r,1}^i a_{r,1}^i \right)$ denote taste-augmented consumption in state 1 at date 1, and $c_1^i(2)$ consumption in state 2 at date 1. The first-order condition for interior holdings of asset $j \in \{r, g\}$ is given by

$$\frac{1-\delta}{P_j(\mathbf{S}_1)} \frac{\pi_1 y(1) \theta_{j,1}^i}{\tilde{c}_1^i(1)} = \frac{(1-\delta)\pi_2 y(2)}{c_1^i(2)} + \delta \mathbb{E}^i \left[\frac{R_2(\mathbf{S}_2) - R_j(\mathbf{S}_2)}{W_2^i(\mathbf{a}_1^i, \mathbf{S}_2)} \right], \tag{5}$$

where it is clear that at least one of the green and red trees must be purchased in positive quantities. The left-hand side is the period-1 marginal utility benefit of buying more of tree j at current price $P_j(\mathbf{S}_1)$, taking into account current tastes for the specific asset. The right-hand side is the cost of buying more of the tree, which is composed of two elements: the marginal utility loss from consuming less in state 2 at date 1, and the expected return reduction from carrying wealth forward in the form of tree j rather than tree 2, weighted by the marginal value of wealth at date 2.

Equation (5) reveals the identification challenge introduced by dynamic trading. Demand is affected positively by both private tastes and expectations of market returns. Since market returns are determined by the tastes of tomorrow's marginal investor, observing demand alone is not sufficient to assess whether an investor is buying an asset based on her own preferences or her beliefs over future market-wide preferences. This distinction is absent in demand systems which specify tastes or heterogeneous beliefs directly over returns, and where preference parameters $\theta_{j,t}^i$ and return expectations

$\mathbb{E}^i[R_j(\mathbf{S}_2)]$ are both jointly interpreted as "tastes." Yet distinguishing between structural taste parameters and endogenous return expectations is critical for counterfactuals. For example, a wealth shock to investors with a taste for green trees may have different pricing consequences than a similar shock to investors with a taste for red trees.

This result also has implications for interpreting demand systems estimated from "event studies" such as index deletions or additions. To the extent that deletions can be forecast, forward-looking trading by unconstrained investors may muddle the structural interpretation of estimated elasticities. Nevertheless, equation (5) does not suggest an impossibility result. At least in the special case of log preferences, tastes $\theta^i_{j,t}$ and return expectations $\mathbb{E}^i(R_j(\mathbf{S}_2))$ do appear separately in the demand functions. A suitably-specified demand system may therefore separate both elements with appropriate instruments.

5.2 Tastes versus constraints

We now use our benchmark static model to investigate the identification of structural parameters when investors are allowed to differ in their tastes and unobservable investment mandates. The basic insight is apparent directly from the general first-order condition (1) for asset holdings a^i_j, which states that the optimal portfolio satisfies:

$$\theta^i_j \sum_{z\in\mathcal{Z}} y_j(z)\tilde{\Lambda}^i(z) + \sum_k \lambda^i_k \frac{m^i_{k,j}(a^i, p)}{(1-\delta)u^{i\prime}(c^i_0)} = p_j.$$

The first-term on the left-hand side depends on preferences, the second on constraints. A given portfolio choice can therefore be driven by either tastes or portfolio constraints. We now construct a simple example economy to show that (unobserved) constraints may be observationally equivalent to tastes in a given equilibrium but yield different counterfactuals. We make the following simplifying assumptions in this section:

(i) There are two types of investors, $i \in \{\alpha, \beta\}$, each of which face short-sale constraints. Tastes are of equal magnitude for each type: $\theta^\alpha_g = 1+t$ and $\theta^\alpha_r = 1-t$, while $\theta^\beta_g = 1-t$ and $\theta^\beta_r = 1+t$. Hence, type α prefers green while type β prefers red.

(ii) Type α owns share $\omega \geq \frac{1}{2}$ of the aggregate endowment of each tree.

(iii) There is no aggregate risk, $y(1) = y(2) = 1$ and $\pi_1 = \frac{1}{2}$.

We begin by considering the baseline case where investors face no investment mandates other than short-sale constraints. Taste differences can then lead to endogenous sorting in equilibrium: type α investors hold the green tree and tree 2, while type β investors hold the red tree and tree 2. However, an increase in the wealth of type α raises the price of green trees and induces type α investors to buy both green and red trees. We obtain closed-form solutions if green and red trees are perfect substitutes ($\epsilon = 0$).

Proposition 4 (Equilibrium with tastes) *Let $\epsilon = 0$. There exists a threshold $\overline{\omega}$ for type α's wealth share ω such that type α buys only green trees if $\omega \leq \overline{\omega}$ and buys both green and red trees if $\omega > \overline{\omega}$. When $\omega \leq \overline{\omega}$, prices are fully determined by wealth shares: $\frac{p_r}{p_g} = \frac{1-\omega}{\omega}$. When $\omega > \overline{\omega}$, prices are determined by type α's taste parameters: $p_g = \theta_g^\alpha$ and $p_r = \theta_r^\alpha$.*

We now ask whether tastes can be differentiated from a mandate that forces an investor to, say, only invest in green trees, and, if not, whether these different microfoundations for observed portfolio choices lead to different counterfactuals. To this end, assume that a share m of type α investors is prohibited from holding red trees. Mandates are observed by investors, but not by the econometrician.[12] Hence, mandates and tastes for green assets are *observationally equivalent* if all investors choose to specialize.

Figure 3 illustrates the equilibrium with and without mandates. The left panel shows the price of green trees for entire range of wealth share ω and substitutability ϵ when there are almost no investors subject to the green mandate. The right panel shows equilibrium prices for the same parameters but with a high share of mandate investors.

The equilibrium is identical near the origin where taste-based investors choose to specialize in their preferred color to the same extent as mandate investors. However, the two economies differ when sorting breaks down. When there are almost no investors with mandates, a shock to ϵ creates more demand for diversification. Thus, the price of green trees is decreasing in ϵ if type α investors choose to hold both types of trees. In contrast, mandate investors do not buy red trees at any price. Hence, shocks to ϵ do not reduce their demand for green trees even as type β's demand increases. Thus, the price of green trees *increases* in ϵ when there are sufficiently many mandate investors.

[12] Most mandates in practice are unobserved and must be inferred from equilibrium behavior. Notable exceptions are index deletions as in Chang, Hong, and Liskovich (2015) and Pavlova and Sikorskaya (2023), whereby a certain stock cannot be held by some investors once it is no longer in an index. However, such estimation strategies are naturally confined to short-run elasticities for a limited number of stocks.

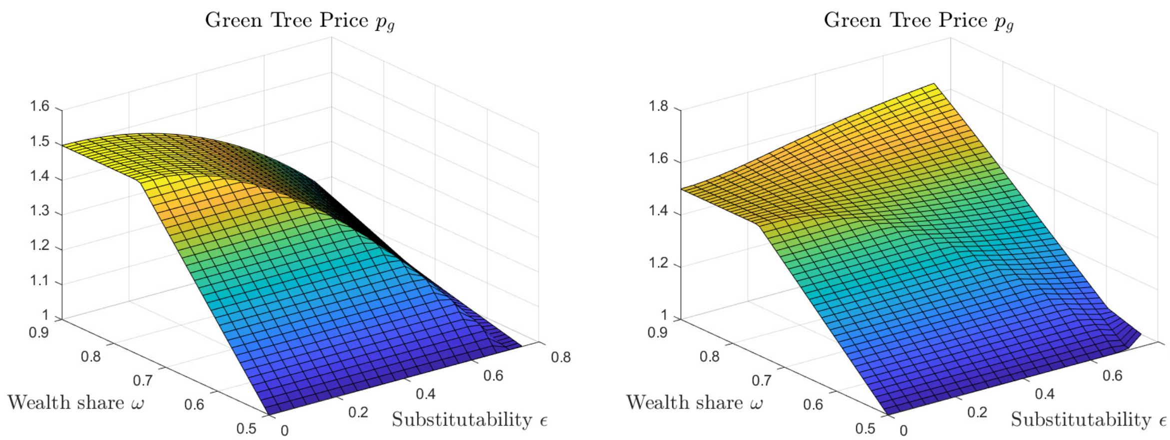


Figure 3: Green Price. Left: Mandate Share $m = 0.01$. Right: Mandate Share $m = 0.85$.

Underlying the result is a general point: observed portfolio choices can always be rationalized by *either* taste parameters *or* constraints, but counterfactuals depend on the precise microfoundation. While we established this result in terms of a mandate that acts on the extensive margin, it is clear that the underlying argument applies also to "intensive margin mandates," such as a requirement to hold a certain fraction of total assets in a particular tree. For example, a certain investor may invest 40% of her state-1 holdings in green trees either because she is forced to or because of a slight preference for red trees.

5.3 Additional Identification Challenges

This section briefly summarizes additional considerations that affect counterfactuals and the structural interpretation of demand elasticities.

5.3.1 Misspecified Constraints

We have shown that it is often necessary to impose constraints on investors to ensure equilibrium existence in the face of heterogeneous tastes. The most common constraint is a prohibition on short sales, which aligns well with data in which *observed* portfolio holdings often show a large number of zero positions. However, because data sources such as Form 13F do not report short sales, it is unclear whether these are "true zeros" or unreported shorts. This leads to the possibility that short-sale constraints are misspecified if they are assumed to be binding whenever a zero position is reported. While data limitations are a concern for any estimation exercise, they are particularly problematic

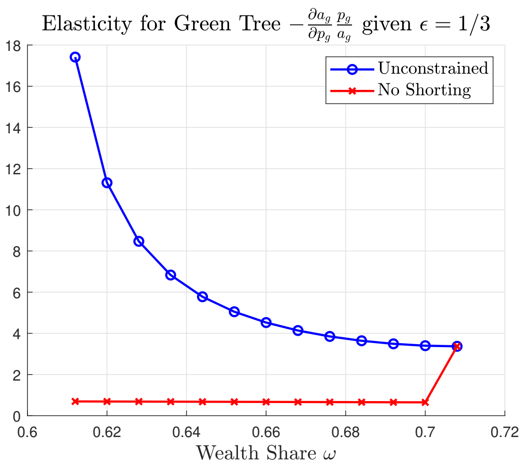


Figure 4: Effects of a misspecified short-sale constraint

when there are demand complementarities and not all relevant dimensions of a decision problem are observed (Samuelson, 1974; Ogaki, 1990).

Figure 4 illustrates this concern in the context of our model. We study equilibrium when green and red trees are partially substitutable ($\epsilon > 0$) and investors have different tastes. Since green and red trees are substitutes, and minus red and green are complements. We compute own-price demand elasticities for an investor with a relative taste for green assets, holding fixed the price of red trees. Without short-sale constraints, such an investor would short red trees to buy more green trees. The blue line plots the elasticity of demand when shorting is allowed, and the red line when shorting is not allowed. If shorting is allowed but not reported, researchers would observe the blue line but interpret it through the lens of a model without short sales. Hence they would estimate a much larger elasticity of substitution between tree 2 and green tree than is warranted by the underlying preferences and behavior. Since this elasticity is sensitive to the price of red trees, the bias varies with the prevailing wealth distribution.

5.3.2 Endogenous capital flows

A common approach in the demand-system literature is based on variations in investment mandates, whereby an asset that cannot be held by many investors may have a lower price than a similar asset that can be purchased by many investors. That is, exogenous cross-sectional variation in investment universes is an instrument for cross-sectional variation in demand. We now argue that identification based on mandates is threatened by the lack of a theory of delegation. Since mandates are in practice imposed on investment funds, not end investors, even very tight mandates are irrelevant as long as end investors flexibly reallocate investments across funds while understanding how funds will allocate their assets under management.[13] This is shown in the following simple proposition, which is reminiscent of the classic two-fund separation theorem (Tobin, 1958).

Proposition 5 *Consider a two-layer structure where households invest through funds, and funds are subject to mandates. Suppose further that there exist at least one green and one red fund. Absent other frictions, equilibrium is invariant in mandates.*

A potential argument in favor of the exogeneity of mandates is that households may be slow to rebalance in response to news, or do so in predictable manners at regular intervals (e.g, at quarter end). This argument is incomplete: if some investors are known to rebalance intermittently, other investors may trade preemptively only to later sell, so that intermittent rebalancers' tastes may be reflected in market demand even when they are not actively trading. This is formally equivalent to our results on dynamic trading in Section 5.1, where demand elasticities are jointly determined by private tastes and expectations over others' tastes. More broadly, Harris, Opp, and Opp (2024) study a general equilibrium framework where specialization, which is observationally equivalent to mandates on the equilibrium path, is endogenously determined by investment opportunities.

6 Conclusion

We present a synthesis between neoclassical asset pricing and recent demand-system approaches to asset pricing, using multiple methods of incorporating tastes into canonical models of portfolio choice. Our analysis highlights several important conceptual concerns, including the definition of no arbitrage under tastes, the measurement of demand elasticities in the presence of cross-asset price spillovers, and the structural interpretation of demand elasticities. Even when purely exogenous supply shocks are available, it is crucial to account for general equilibrium spillover effects when measuring demand elasticities. When the supply of a particular security falls, prices of securities with similar payoff structures will tend to increase as well, and the *equilibrium* portfolio responses may

[13]In the two-layer demand system in Darmouni, Siani, and Xiao (2023), households instead have "tastes" over intermediaries that weaken this reallocation.

be small even when underlying elasticities are near infinite. Using our model, we show that this can lead to severe underestimation of the true elasticities, in particular when assets have close substitutes. As such, our findings can reconcile low demand-system estimates of elasticities with substantially higher elasticities found in canonical models.

Our analysis highlights several potential methodological remedies for the issues we raise. For example, one could restore no arbitrage in taste-based models by specifying non-linear taste functions with decreasing marginal taste intensities, and account for cross-asset substitution by estimating a matrix of price spillovers alongside individual demand functions. Finally, the dynamic version of our model suggests that appropriate instruments might be able to disentangle individual tastes from expectations over market returns even when demand elasticities alone do not provide such identification. We view the empirical implementation of these ideas as fruitful directions for future research.

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A Proofs

Proof of Theorem 1. The proof consists of two parts. The first shows that there does not exist a pricing function P that leaves no arbitrage opportunities if (C) holds. The second establishes the sufficiency condition for (C) as stated in the theorem.

For the first part, suppose that condition (C) holds. Generically, we can assume that $v^i(a) > 0$ and $v^{i'}(a) < 0$. Now, suppose to the contrary that there exists a pricing function P that leaves no arbitrage opportunities. On the one hand, applying Definition 2 to investor i yields $P(a) > 0$. On the other hand, applying Definition 2 to investor i' yields $P(-a) > 0$, i.e., $P(a) < 0$. This is a contradiction.

We move on to the second part. Suppose that the conditions stated in the theorem hold: there exist two assets j and j' and two investors i and i' satisfying the two conditions. Denoting by $v^i_j \equiv (\theta^i_j y_j(z))_z$ investor i's marginal taste with respect to asset j, the two conditions imply:

$$v^i_j > v^i_{j'} \text{ and } v^{i'}_j \leq v^{i'}_{j'}. \tag{C'}$$

We show that condition (C′) implies condition (C). Thus, while the sufficient conditions (i) and (ii) in the statement of the theorem imply that no arbitrage is invalidated if there are two assets with identical cash flows and there are two investors who have sufficiently heterogeneous tastes with respect to these two assets, the fact that condition (C′) turns out to be sufficient implies that the principle of no arbitrage is invalidated as long as there are two investors with sufficiently heterogeneous marginal tastes.

We thus show that condition (C′) implies condition (C). Let $a \in \mathbf{R}^J$ be such that

$$a = \begin{cases} 1 & \text{if } \ell = j \\ -1 & \text{if } \ell = j' \\ 0 & \text{otherwise} \end{cases} .$$

Then, we obtain $v^i(a) = v^i_j - v^i_{j'} > 0$ and $v^{i'}(a) = v^{i'}_j - v^{i'}_{j'} \leq 0$, as desired. ■

Proof of Proposition 2. In the representative-agent benchmark, the agent's portfolio is well-diversified so that the first-order conditions imply:

$$\begin{aligned} p_g &= \frac{\pi_1}{1-\pi_1}\frac{y(1)-(2\rho-1)\epsilon}{y(2)} = 1-(2\rho-1)\epsilon \\ p_r &= \frac{\pi_1}{1-\pi_1}\frac{y(1)+(2\rho-1)\epsilon}{y(2)} = 1+(2\rho-1)\epsilon \end{aligned},$$

where, for each expression, the last equality follows from our simplifying assumptions that $y(1) = y(2) = 1$ and $\pi(1) = \frac{1}{2}$.

When $\epsilon = 0$, if the price of green trees increases then the agent switches to only holding red trees, yielding $\eta^i_g = \infty$. ■

Proof of Proposition 3. First, under the assumptions that $y(1) = y(2) = 1$ and $\pi(1) = \frac{1}{2}$, no tastes, and $\rho = \frac{1}{2}$, the (interior) first-order conditions are:

$$\begin{aligned} p_g &= \frac{y(1)-\epsilon}{2y(2)}\left(\frac{\tilde{c}^i(2)}{\tilde{c}^i(r)}\right) + \frac{y(1)+\epsilon}{2y(2)}\left(\frac{\tilde{c}^i(2)}{\tilde{c}^i(g)}\right); \\ p_r &= \frac{y(1)+\epsilon}{2y(2)}\left(\frac{\tilde{c}^i(2)}{\tilde{c}^i(r)}\right) + \frac{y(1)-\epsilon}{2y(2)}\left(\frac{\tilde{c}^i(2)}{\tilde{c}^i(g)}\right). \end{aligned}$$

Adding and subtracting these first-order conditions and observing that the representative agent consumes the aggregate endowment, as in the main text, we can show:

$$p_g = \frac{1}{2}\left[\frac{1-\tilde{\epsilon}}{1+\psi-\psi\tilde{\epsilon}} + \frac{1+\tilde{\epsilon}}{1+\psi+\psi\tilde{\epsilon}}\right];$$
$$p_r = \frac{1}{2}\left[\frac{1+\tilde{\epsilon}}{1+\psi-\psi\tilde{\epsilon}} + \frac{1-\tilde{\epsilon}}{1+\psi+\psi\tilde{\epsilon}}\right].$$

Quantities of the green asset are $a_g = \frac{1}{2} + \psi$ as opposed to $a_g^0 = \frac{1}{2}$. We consider computing the elasticity of demand with respect to a change in ψ around 0. Observing that the price of green trees before the shock is:

$$p_g^0 = \frac{\pi_1}{1-\pi_1}\frac{y(1) - \epsilon(2\rho - 1)}{y(2)} = 1,$$

the change in price $\Delta p_g = p_g - p_g^0$ is:

$$\Delta p_g = \frac{1}{2}\left[\frac{-\tilde{\epsilon}(1-\psi) - \psi}{1+\psi-\psi\tilde{\epsilon}} + \frac{(1-\psi)\tilde{\epsilon} - \psi}{1+\psi+\psi\tilde{\epsilon}}\right].$$

The change in quantity is $\Delta a_g = \psi$. Hence, we have

$$-\frac{\Delta a_g}{\Delta p_g}\frac{p_g^0}{a_g^0}\bigg|_{\substack{p_g^0=1 \\ a_g^0=\frac{1}{2}}} = -\frac{\psi}{\frac{1}{2}\left[\frac{-\tilde{\epsilon}(1-\psi)-\psi}{1+\psi-\psi\tilde{\epsilon}} + \frac{(1-\psi)\tilde{\epsilon}-\psi}{1+\psi+\psi\tilde{\epsilon}}\right]}\frac{1}{\frac{1}{2}} = 2\frac{1+\psi(2+\psi-\psi\tilde{\epsilon}^2)}{1+\psi+\tilde{\epsilon}^2-\psi\tilde{\epsilon}^2},$$

which reduces to $2(1+\psi)$ when $\epsilon = 0$, as stated in the proposition. ■

Remark 5 (True Elasticity) *To find the true structural elasticity, observe that the representative agent's first-order conditions given $y(1) = y(2) = 1$ and $\pi(1) = \rho = \frac{1}{2}$ are:*

$$2p_g = (1-\epsilon)\frac{a_2}{(1-\epsilon)a_g^i + (1+\epsilon)a_r} + (1+\epsilon)\frac{a_2}{(1+\epsilon)a_g + (1-\epsilon)a_r};$$
$$2p_r = (1+\epsilon)\frac{a_2}{(1-\epsilon)a_g + (1+\epsilon)a_r} + (1-\epsilon)\frac{a_2}{(1+\epsilon)a_g + (1-\epsilon)a_r}.$$

Substituting the budget constraint

$$a_2^i = 1 + \frac{(1+2\psi)p_g + p_r}{2} - p_g a_g^i - p_r a_r^i$$

and solving for (a_g, a_r), we obtain:

$$a_g = \frac{1}{4} \cdot \frac{(2 + (1+2\psi)p_g + p_r)((1+\epsilon^2)p_r - (1-\epsilon^2)p_g)}{(p_g + p_r)^2\epsilon^2 - (p_g - p_r)^2};$$
$$a_r = \frac{1}{4} \cdot \frac{(2 + (1+2\psi)p_g + p_r)((1+\epsilon^2)p_g - (1-\epsilon^2)p_r)}{(p_g + p_r)^2\epsilon^2 - (p_g - p_r)^2}.$$

After some algebra, the demand elasticity is:

$$\begin{aligned} -\frac{\partial a_g}{\partial p_g}\frac{p_g}{a_g} =& 2p_g \frac{(p_g - p_r)^2(1+p_r) + \left(2p_g p_r^2 - (2+p_r)p_g^2 + 3(2+p_r)p_r^2\right)\epsilon^2 + (p_g+p_r)^2\epsilon^4}{\left((p_g+p_r)^2\epsilon^2 - (p_g-p_r)^2\right)\left(2 + (1+2\psi)p_g + p_r\right)((1+\epsilon^2)p_r - (1-\epsilon^2)p_g)} \\ &+ 2p_g p_r \frac{(p_g-p_r)^2 + 4p_g p_r \epsilon^2 - (p_g+p_r)^2\epsilon^4}{\left((p_g+p_r)^2\epsilon^2 - (p_g-p_r)^2\right)\left(2 + (1+2\psi)p_g + p_r\right)((1+\epsilon^2)p_r - (1-\epsilon^2)p_g)}\psi. \end{aligned}$$

The demand elasticity at $(p_g, p_r) = (1, 1)$ is, as depicted in Figure 2, thus:

$$-\frac{\partial a_g}{\partial p_g}\frac{p_g}{a_g}\bigg|_{\substack{p_g=1\\p_r=1}} = \frac{2 + \psi + (1-\psi)\epsilon^2}{2(2+\psi)\epsilon^2}.$$

where $p_g = p_r = 1$ are the initial equilibrium prices.

Proof of Lemma 1. Following the discussion of the static optimization problem, the second-period value function can be written as

$$\begin{aligned} V_2^i(\mathbf{s}_2^i, \mathbf{S}_2) = \max_{\mathbf{a}_2^i \geq 0}\ & \pi_1 \left[\rho u\left(\theta_{g,2}^i y_g(r) a_{g,2}^i + \theta_{r,2}^i y_r(r) a_{r,2}^i\right) + (1-\rho)u\left(\theta_{g,2}^i y_g(g) a_{g,2}^i + \theta_{r,2}^i y_r(g) a_{r,2}^i\right)\right] \\ &+ \pi_2 u\left(\frac{y(2)}{P_2(\mathbf{S}_2)}\left(W_2^i(\mathbf{a}_1^i, \mathbf{S}_2) - P_g(\mathbf{S}_2)a_{g,2}^i - P_r(\mathbf{S}_2)a_{r,2}^i\right)\right). \end{aligned}$$

Since green and red assets are perfect substitutes (i.e., $\epsilon = 0$), the solution to the second-period decision problem is bang-bang, depending on whether $\theta_g^2/P_g(\mathbf{S}_2) \geq \theta_r^2/P_r(\mathbf{S}_2)$. Suppose that this inequality holds (i.e., green trees are cheap). Then, the value of the second-period problem is:

$$V_{g,2}^i(\mathbf{s}_2^i, \mathbf{S}_2) = \max_{a_{g,2}^i \geq 0} \pi_1 u\left(\theta_{g,2}^i y(1) a_{g,2}^i\right) + u\left(\frac{y(2)}{P_2(\mathbf{S}_2)}\left(W_2^i(\mathbf{a}_1^i, \mathbf{S}_2) - P_g(\mathbf{S}_2)a_{g,2}^i\right)\right).$$

As in the analysis of the static problem, the first-order condition together with log utility yields

$$a^i_{g,2} = \frac{\pi_1 W^i_2(\mathbf{a}^i_1, \mathbf{S}_2)}{P_g(\mathbf{S}_2)}.$$

This means that

$$V^i_{g,2}(\mathbf{s}^i_2, \mathbf{S}_2) = \pi_1 u\Big(\theta^2_g \frac{\pi_1 y(1) W^i_2(\mathbf{a}^i_1, \mathbf{S}_2)}{P_g(\mathbf{S}_2)}\Big) + \pi_2 u\Big(\frac{y(2)}{P_2(\mathbf{S}_2)} \pi_2 W^i_2(\mathbf{a}^i_1, \mathbf{S}_2)\Big).$$

With log utility this can be written as

$$\begin{aligned} V^i_{g,2}(\mathbf{s}^i_2, \mathbf{S}_2) = {} & \pi_1 \log\left(\frac{\theta^i_{g,2}}{P_g(\mathbf{S}_2)}\right) + \pi_2 \log\left(\frac{1}{P_2(\mathbf{S}_2)}\right) + \pi_1 \log\Big(\pi_1 y(1)\Big) + \pi_2 \log\Big(\pi_2 y(2)\Big) \\ & + \log\Big(W^i_2(\mathbf{a}^i_1, \mathbf{S}_2)\Big). \end{aligned}$$

Similarly, if $\theta^2_g / P_g(\mathbf{S}_2) \leq \theta^2_r / P_r(\mathbf{S}_2)$, the value of the second-period problem is:

$$\begin{aligned} V^i_{r,2}(\mathbf{s}^i_2, \mathbf{S}_2) = {} & \pi_1 \log\left(\frac{\theta^i_{r,2}}{P_r(\mathbf{S}_2)}\right) + \pi_2 \log\left(\frac{1}{P_2(\mathbf{S}_2)}\right) + \pi_1 \log\Big(\pi_1 y(1)\Big) + \pi_2 \log\Big(\pi_2 y(2)\Big) \\ & + \log\Big(W^i_2(\mathbf{a}^i_1, \mathbf{S}_2)\Big). \end{aligned}$$

Hence, the second-period value function is written as

$$V^i_2(\mathbf{s}^i_2, \mathbf{S}_2) = \max\Big\{V^i_{g,2}(\mathbf{s}^i_2, \mathbf{S}_2), V^i_{r,2}(\mathbf{s}^i_2, \mathbf{S}_2)\Big\},$$

and it satisfies

$$V^i_2(\mathbf{s}^i_2, \mathbf{S}_2) = H(\Theta^i_2, \mathbf{S}_2) + \log\Big(W^i_2(\mathbf{a}^i_1, \mathbf{S}_2)\Big),$$

where

$$\begin{aligned} H(\Theta^i_2, \mathbf{S}_2) = {} & \pi_1 \max\left\{\log\left(\frac{\theta^i_{r,2}}{P_r(\mathbf{S}_2)}\right), \log\left(\frac{\theta^i_{g,2}}{P_g(\mathbf{S}_2)}\right)\right\} + \pi_2 \log\left(\frac{1}{P_2(\mathbf{S}_2)}\right) \\ & + \pi_1 \log\Big(\pi_1 y(1)\Big) + \pi_2 \log\Big(\pi_2 y(2)\Big). \end{aligned}$$

■

Proof of Proposition 4. First, we consider an equilibrium in which type α investor specializes in green trees while type β investor specializes in red trees. It follows from the first-order conditions with respect to a^α_g and a^β_r in the main text and the aggregate resource constraint on tree 2 that

$$\frac{p_r}{p_g} = \frac{1 - a_2^\alpha}{a_2^\alpha} \qquad \text{and} \qquad p_1 \equiv \frac{p_g + p_r}{2} = \frac{\pi_1}{1 - \pi_1} = 1,$$

where the last equality follows from our simplifying assumptions. Substituting p_1 and the first-order condition with respect to a_g^α into type α investor's budget constraint, together with the aggregate resource constraint, one can show:

$$(a_2^\alpha, a_2^\beta) = (\omega, 1 - \omega).$$

Thus, if sorting occurs in equilibrium, then the prices satisfy

$$(p_g, p_r) = \left(2\omega \frac{\pi_1}{1 - \pi_1}, 2(1 - \omega)\frac{\pi_1}{1 - \pi_1}\right) = (2\omega, 2(1 - \omega))$$

and the investors' portfolio choices are

$$(a_g^\alpha, a_r^\alpha, a_2^\alpha) = \left(\frac{1}{2}, 0, \omega\right) \qquad \text{and} \qquad (a_g^\beta, a_r^\beta, a_2^\beta) = \left(0, \frac{1}{2}, 1 - \omega\right).$$

To show that this constitutes an equilibrium, we need to show that the first-order conditions with respect to a_r^α and a_g^β hold at the zero holding. It can be seen that these first-order conditions are met as long as type α's incentive is satisfied:

$$\omega \leq \overline{\omega} \equiv \frac{\theta_g^\alpha}{\theta_g^\alpha + \frac{(y(1))^2 + \epsilon^2}{(y(1))^2 - \epsilon^2}\theta_r^\alpha} = \frac{\theta_g^\alpha}{\theta_g^\alpha + \theta_r^\alpha}.$$

Next, we consider an equilibrium in which one type of investor, denoted by i, holds both green and red trees while the other type specializes in one tree. For ease of exposition, we take the simplifying assumptions at the outset. Guessing that $\tilde{c}^i(g) = \tilde{c}^i(r) = \tilde{c}^i(2)$, it follows from the first-order conditions with respect to a_g^i and a_r^i that

$$\left(p_g, p_r, p_1\right) = \left(\theta_g^i, \theta_r^i, 1\right).$$

It also follows from the first-order conditions with respect to a_g^i and a_r^i, together with the budget constraint, that

$$a_2^i = (1-\pi_1)e_2^i + \pi_1 \frac{y(1)}{y(2)}(\theta_g^i e_g^i + \theta_r^i e_r^i) = \frac{e_2^i + \theta_g^i e_g^i + \theta_r^i e_r^i}{2}.$$

We also guess and verify that $i = \alpha$. Since type β specializes in red, $a_g^\beta = 0$ and $a_g^\alpha = \frac{1}{2}$. Then, by the aggregate resource constraint,

$$a_r^\alpha = \frac{a_2^\alpha - \theta_g^\alpha E_g}{\theta_r^\alpha}.$$

Since the first-order condition with respect to a_r^β yields $a_r^\beta = \frac{a_2^\beta}{p_r}$, it follows from the budget constraint that

$$a_2^\beta = (1-\pi_1)(e_2^\beta + p_g e_g^\beta + p_r e_r^\beta) = 1-\omega \qquad \text{and} \qquad a_r^\beta = (1-\pi_1)\frac{e_2^\beta + p_g e_g^\beta + p_r e_r^\beta}{p_r} = \frac{1-\omega}{p_r}.$$

Thus, we obtain, as in the statement of the proposition,

$$a_r^\beta = \frac{1-\omega}{2}\frac{E_2 + \theta_g^\alpha E_g + \theta_r^\alpha E_r}{\theta_r^\alpha}.$$

When $\omega > \overline{\omega}$, it can be seen that $a_r^\alpha > 0$ and that the first-order condition with respect to a_g^β at $a_g^\beta = 0$ is also met (i.e., $a_g^\beta = 0$). ■