Portfolio Regulation of Large Financial Institutions*

Daniel Neuhann† Mahyar Sefidgaran‡ Michael Sockin§

December 2024

Abstract

We examine how portfolio regulations affect risk sharing between financial institutions with market power. Unconstrained access to complete markets permits flexible exploitation of market power and induces inefficient risk sharing. Appropriate portfolio restrictions counteract this, improving liquidity and risk sharing by bundling securities with offsetting strategic incentives. However, excessive regulation can counterproductively destroy gains from trade. An application of our theory shows that cross-asset spillovers are critical for policy evaluation: in general equilibrium, risk sharing can improve even if certain asset-specific liquidity measures deteriorate. We also discuss the effects of asymmetric regulation for different institutions.

*This paper was previously titled "Portfolio Regulation of Large Non-Bank Financial Institutions." We thank Gregor Matvos, two anonymous referees, Ana Babus, Bradyn Breon-Drish, Eduardo Dávila, William Fuchs, Kinda Hachem, Ben Hébert, Giorgia Piacentino and a number of seminar audiences for helpful comments.

†University of Texas at Austin. daniel.neuhann@mccombs.utexas.edu. (Corresponding author).

‡University of Texas at Austin. mahyar.sefidgaran@utexas.edu.

§University of Texas at Austin. michael.sockin@mccombs.utexas.edu.

1 Introduction

An important objective of financial regulation is to foster sound risk management among systemically important financial institutions, such as large banks, pension funds, insurance companies and dealers. Part of the regulatory toolkit consists of enforcing limits on the types of assets and portfolios that financial institutions can hold. For example, some jurisdictions allow a wide variety of financial institutions to trade in complex securities, such as interest rate and foreign exchange derivatives, while others restrict some of them to trade only in common securities such as stocks and bonds.[1] Financial institutions may also face constraints on particular trading strategies, such as bans on "naked" trades of credit default swaps if buyers do not also hold the underlying bond.

There are natural benefits to allowing financial institutions to make portfolio decisions without restrictions. Absent others frictions, access to a richer trading opportunities allows them to realize more gains from trade. However, recent evidence suggests that many markets in which these investors trade, such as those for interest rate swaps, corporate bonds, credit default swaps, and asset-backed securities, have limited competition and liquidity, and that these factors can induce institutions to distort their portfolios and take on excessive risk.[2] It is thus unclear whether unfettered access to these markets indeed promotes better risk management by these institutions.

In this paper, we investigate how imposing portfolio constraints on financial institutions impacts how they share risks in illiquid financial markets. Our key insight is that, if there are large investors who trade strategically with price impact, suitably chosen portfolio constraints which render markets incomplete can improve risk sharing and generate Pareto improvements relative to complete markets. The reason is that market incompleteness deters the strategic exploitation of market power by large investors. As an application of our theory, we show that portfolio restrictions on credit default swaps (CDS), such as bans on "naked" CDS positions of the

[1] Portfolio regulation of financial institutions has historically received varying treatment. For instance, although the European Union and South Africa do not allow pension funds to trade derivatives, the United States and United Kingdom have been more permissive. New York recently passed a bill raising the investment cap for pension funds on alternative assets, while the United Kingdom relaxed its stringent default fund charge cap to ease investment in alternative assets.

[2] For evidence of market concentration and limited liquidity in interest rates swap, credit default swap, corporate bond, and asset-backed security markets, see Khetan, Li, Neamtu, and Sen (2023), D'Errico, Battiston, Peltonen, and Scheicher (2018) and Peltonen, Scheicher, and Vuillemey (2014), Goldstein and Hotchkiss (2020), and Harkrader and Puglia (2020), respectively. For evidence of strategic trading by financial institutions in these markets that led to poor risk sharing, in particular in the context of the 2022 UK Gilt crisis, see Pinter and Walker (2023), Becker, Opp, and Saidi (2022), Pinter, Siriwardane, and Walker (2024).

type previously imposed in Europe, can increase overall risk-sharing efficiency even if they raise asset-specific measures of illiquidity, such as bond bid-ask spreads.

Understanding the costs and benefits of portfolio regulation is not straightforward. The key issue is cross-asset spillovers: imposing restrictions on one market may change liquidity in other markets, and it is the aggregate effect which is relevant for overall efficiency. To address this issue, we propose a general equilibrium model of strategic trading in imperfectly competitive financial markets subject to arbitrary constraints on portfolio formation. Our model accounts for spillovers across assets, satisfies no arbitrage despite strategic trading, features non-linear price impact, and permits flexible heterogeneity in gains from trade. As such, it is a useful laboratory to assess the general equilibrium effects of portfolio regulation on liquidity and welfare.

We study an endowment economy in which risk averse investors with convex marginal utility (i.e., CRRA preferences) trade to share risks. There are competitive investors as well as a finite number of large investors with price impact. The fundamental friction is that investors with price impact distort their portfolios to capture price concessions. These distortions take the form of asset-level wedges between private valuations and market prices, with sellers rationing supply to raise prices and buyers rationing demand to lower prices. In equilibrium, this leads to inefficient risk sharing relative to the first best because there are lost gains from trade. These distortions are severe when price impact is high (so that small changes in volume have large effects), and when investors have relatively flat marginal utility (so that changes in volumes have low private costs). They are also amplified by an illiquidity externality that arises from non-linear price impact.

Given that investors trade across many asset markets, knowledge of asset-level wedges is not sufficient to characterize the aggregate consequences of strategic rationing. We therefore develop a method which allows us to characterize the aggregate distortions from price impact directly in terms of risk exposures and gains from trade rather than asset-level wedges. In particular, given investors' state prices we can determine which risk exposures are efficiently allocated and which gains from trade are left unrealized. Because distortions vary with the set of portfolios that can be traded, i.e., the asset span, they can be affected by regulation.

We then assess the scope for regulatory interventions by characterizing *constrained efficient asset spans*. In line with the classic compensation principle (Boadway and Bruce (1984)), we say that an asset span is constrained efficient if the regulator cannot engineer a Pareto improvement by altering the asset span and imposing budget-balanced transfers across agents at date one. Our main insight is that, under a relatively weak technical condition, unrestricted access to complete markets (i.e., unfettered access to the full set of Arrow securities) is constrained inefficient, whereas judiciously chosen forms of market incompleteness are constrained efficient.

Under imperfect competition, the drawback of complete markets is that they allow investors to flexibly exploit their price impact in a fully state-contingent manner. Putting constraints on portfolio formation can mitigate this behavior by linking distortions across states. In particular, if cross-state linkages create offsetting incentives, regulation can achieve lower average distortions than complete markets. This benefit must be traded off against the direct cost of market incompleteness, which is that incomplete markets do not permit certain risks to be traded.

This trade-off can be addressed by bundling securities into *composite assets* that still permit gains from trade to be realized. These composite assets can be constructed by forming an asset (or, equivalently, a portfolio strategy) that pays out the net cash flows investors would exchange. Restricting trade to the composite assets leads to less exploitation of market power than when investors can trade its constituent securities. This is because distorting trade in the composite asset simultaneously distorts multiple margins of adjustment, whereas trade in individual securities allows investors to optimally distort every margin.

To provide a concrete example, imagine two groups of investors facing idiosyncratic risk: half are rich in state 1 and poor in state 2, and the other half is rich in state 2 and poor in state 1. Efficiency requires investors to trade offsetting claims on consumption to eliminate all idiosyncratic risk. Our results show that restricting investors to trade only the swap with payoffs $[1, -1]$ rather than being allowed to trade the full of set of Arrow securities leads to a Pareto improvement. With access to Arrow securities, investors disproportionately distort risk sharing in the state where they are rich because being rich means that marginal utility is relatively flat. Restricting markets to the swap mitigates market power by creating an offsetting effect: rationing trade in the swap also affects consumption when the investor is poor. As a result, overall risk sharing improves.

This insight can be applied to prominent regulatory debates. In the aftermath of the Eurozone crisis of 2010, European regulators proposed a ban on "naked" position in credit default swaps (CDS) on sovereign bonds. Naked trades are positions in CDS that are not associated with a position in the underlying bond. Instead, regulators forced investors to hold "covered" positions, which are trading strategies in which an investor holds CDS and the underlying sovereign bond in some proportion. Our model is well-suited to analyze the efficacy of these regulations because they are a form of asset bundling, and because CDS markets are highly concentrated.

We calibrate our model to data from CDS and bond markets using bid-ask spreads to assess the size of price impact frictions in these markets. We show that the banning of "naked" CDS positions can be constrained efficient when the required ratio of CDS to the underlying bond aligns sufficiently with gains from trade. However, absent direct transfers from clients to dealers, we estimate that the policy is distributional, with clients benefiting at the expense of dealers. This aligns closely with dealer complaints about the regulation during this time. We also show that considering only asset-level indicators of liquidity can be misleading. In our calibration, overall risk sharing improves even though bid-ask spreads on bonds increase. The reason is that bid-ask spreads on CDS decrease, and this is enough to offset the decline in bond liquidity. Our application consequently emphasizes the importance of taking a general equilibrium perspective regarding portfolio regulation, and to assess liquidity at the level of risk exposures, not securities.

We conclude by offering a broader discussion our findings. First, we consider regulation that is imposed on a subset of investors rather than market-wide. The main mechanisms remain robust, but asymmetric regulation also redistributes market power from regulated institutions, such as banks, pension funds, and insurance companies to unregulated investors, such as asset managers and hedge funds (e.g., Khetan, Li, Neamtu, and Sen (2023), Pinter and Walker (2023)). Thus, portfolio regulations are likely to be most effective when they are applied market-wide. Moreover, if restrictions are improperly chosen, this can lead to a buildup of diversifiable risk among large financial institutions. Second, we discuss two additional settings to which our theory might be fruitfully applied: capital guarantee products and variable annuities. Both are structured financial products which allow investors to obtain exposure to aggregate risk *bundled* with insurance against downside risk. Calvet, Celerier, Sodini, and Vallee (2023) and Koijen and Yogo (2022) show that the pricing and supply of these products is hampered by market power and illiquidity, suggesting that appropriately designed restrictions on the precise structure of these bundles may raise trading efficiency in these settings.

1.1 Related literature

Our paper contributes to a growing literature examines how regulation affects the industrial organization of financial markets. Egan, Hortacsu, and Matvos (2017) investigates the role of regulation in promoting financial stability among large banks that hold uninsured deposits, while Buchak, Matvos, Piskorski, and Seru (2024) studies how changes to capital requirements and quantitative easing impact competition among traditional and shadow banks. Basak and Pavlova (2013) shows that financial institutions distort portfolios and asset prices when they take into account the performance of a benchmark. Hachem and Song (2021) examines liquidity booms in imperfectly competitive interbank markets. We instead study how the risk management of large financial institutions with price impact interacts with the set of risk exposures they can trade.

Our equilibrium concept is Cournot-Walras in the tradition of Gabszewicz and Vial (1972). The main benefit of this concept is that we can incorporate rich heterogeneity in preferences and income risk, arbitrary asset spans, asymmetric strategies, and nonlinear price impact, all of which are important for our analysis. In similar frameworks, Basak (1997) studies a monopolistic investor who shares risks with price-taking agent in an Arrow-Debreu economy, Rahi and Zigrand (2009) models the incentives of large traders to arbitrage across segmented markets, Eisenbach and Phelan (2022) studies fire sales externalities, and Kacperczyk, Nosal, and Sundaresan (2024) considers asset price informativeness. Different from these studies, we examine how risk sharing among oligopolistic investors interacts with regulatory constraints on portfolio formation.

A related approach based on Kyle (1989) studies equilibrium-in-demand-schedules. This allows for richer strategic interactions at the cost of stronger assumptions on preferences and payoffs (i.e., symmetry, CARA-normal settings, restricted asset spans) for tractability. In this tradition, Malamud and Rostek (2017) and Rostek and Yoon (2021) show that introducing redundant assets or restricting trading partners through decentralization in over-the-counter markets can improve risk sharing by redistributing price impact across traders. We highlight the complementary result that restricting the asset span by bundling securities can mitigate incentives to *exploit* price impact.

Our findings are also related to the literature on financial innovation and market design (e.g., Demange and Laroque (1995), Pesendorfer (1995)). Athanasoulis and Shiller (2000) shows a social planner in a CARA-normal setting will first open asset markets most aligned with competitive agents' endowments. Previous papers establish that closing markets may be optimal when financial markets are constrained inefficient, such as with multiple goods (e.g., Cass and Citanna (1998), Elul (1995)), asymmetric information (e.g., Marin and Rahi (2000)), or heterogeneous beliefs (e.g., Blume, Cogley, Easley, Sargent, and Tsyrennikov (2018)). Carvajal, Rostek, and Weretka (2012) shows how profit maximizing security design by competitive agents may involve leaving markets incomplete. Babus and Hachem (2023) and Babus and Hachem (2021) considers how private incentives to design risky securities depend on the competitiveness of the demand side, while Babus and Parlatore (2021) explores market fragmentation with endogenous liquidity. In contrast to this literature, we find that unrestricted trading in complete markets *increases* the scope for privately optimal (but socially inefficient) rent extraction.

More broadly, we relate to the literature on endogenous market incompleteness in which risk sharing is limited by constraints such as limited commitment or borrowing constraints (e.g., Alvarez and Jermann (2000), Dávila and Korinek (2018)). In these models, complete markets still allow the maximum feasible gains from trade to be realized and state prices are always fully aligned. This is not the case in our model, in which risk sharing distortions are purely strategic. We show that this allows regulators to foster liquidity through constraints on portfolio formation.

2 Model

We study an endowment economy with a finite number of large strategic agents with price impact, a continuum of price-taking agents (the competitive fringe) who enforce no arbitrage even in the presence of strategic interaction, and arbitrary market structures (complete and incomplete). In what follows, bold symbols indicate vectors and $\{i, j\}$ subscripts on capital letters indicate the i^{th} row and j^{th} column element of a matrix.

2.1 Agents, endowments, and preferences.

There are two dates, $t \in \{1, 2\}$ and a single numeraire good. Uncertainty is represented by a discrete set of states of the world $\mathcal{Z}$ with cardinality $Z = |\mathcal{Z}|$. State $z \in \mathcal{Z}$ is realized at date 2 with probability $\pi(z) \in (0, 1)$.

There are two broad classes of agents: a competitive fringe of mass m_f which takes prices as given, and a finite number of strategic agents who internalize that their actions affect equilibrium prices. There are $N > 1$ types of strategic agents, indexed by $i \in \{1, 2 \ldots, N\}$ and $1/\mu_i$ agents of type i, each with mass μ_i, so that the total mass of all strategic agents is N. We assume $N \leq Z$ so that there are weakly fewer agent types than states of the world, although this assumption is only required for Propositions 4 and 5.

Strategic agents may differ in endowments and preferences. An agent of type i receives state-contingent income $\mu_i y_i(z)$ in state z, and an initial endowment of $\mu_i w_i$ at date 1. These agents represent large financial institutions, such as banks, insurance companies, pension funds, broker dealers or large asset managers (i.e., hedge funds). In the former case, their incomes are the net state-contingent payoffs from loan portfolios, premiums less payouts to insurees, or defined benefit pensioners, respectively, while in the latter they are state-contingent net fund flows.[3] The fringe, which represents smaller investors without price impact, receives state-contingent income $y_f(z)$ and initial endowment w_f. All endowments are bounded.

Remark 1 *What is important for our results is that there are gains from trade between investors. While we model gains from trade as coming from differences in endowments and/or preferences, in practice they may also reflect induced preferences from other regulations, such as risk-weighted capital requirements, which create heterogenous marginal valuations for financial assets.*

For an individual agent of type i, its size, μ_i, parameterizes its market power because it represents the fraction that it has of the total income and wealth of type i, $y_i(z)$ and w_i. Controlling a larger share of income and wealth translates into higher price impact. Parameter μ_i thus represents what we refer to as market concentration, and higher μ_i reflects a higher degree of concentration of type i agents. The competitive equilibrium corresponds to the special case in which $\mu_i = 0$ for all i. In what follows, we focus on the case in which all agents within a type follow symmetric strategies.

All agents have homothetic and concave utility functions $u_{i,t}(\cdot)$ over consumption $c_{i,t}$ at date t. Risk aversion captures the notion that financial institutions can exhibit risk aversion under a variety of frictions, generating a motive for risk sharing. Beyond this, preferences can be heterogenous. The competitive fringe has linear utility over date 1 consumption, $u_{f,1}(c_{f,1}) = c_{f,1}$,

[3] Since we work in a complete-markets setting with essentially unrestricted income processes, our model can also be used to capture interest risk through a suitable redefinition of the states of the world.

and concave utility $u_f(\cdot)$ over date 2 consumption (i.e., quasi-linear preferences). Both $u_{i,t}(\cdot)$ and $u_f(\cdot)$ are $\mathcal{C}^2$, strictly increasing, and with convex marginal utility. A quasi-linear fringe is analytically convenient because the pricing functional depends only on the fringe's marginal utility at $t = 2$, but this is not essential.

2.2 Financial markets

Markets are, in principle, complete in that all risks are tradeable in the absence of mandates or regulation. However, regulatory constraints may effectively restricts the asset span that they can trade. To capture this, we allow for arbitrary asset spans, and we refer to the asset span generated by the set of market-wide regulations on portfolio formation as the prevailing *market structure*. Consequently, financial markets consist of $J \leq Z$ securities with bounded payoffs $\{x_j(z)\}_{j=1}^{J}$. A market structure is indexed by payoff matrix $X \in \mathbb{R}^{J\times Z}$, such that $\boldsymbol{x}(z) = X\boldsymbol{\delta}(z)$, where $\boldsymbol{\delta}(z)$ is the $Z \times 1$ vector whose z^{th} entry is 1 and 0 otherwise. Trading takes place at date 1 and assets pay out at date 2. We distinguish between complete and restricted market structures.

Definition 1 *A market structure X is complete if $rank(X) = Z$ and restricted if $rank(X) < Z$.*

Type i's asset position in security j is $a_{i,j} \in \mathbb{R}$, where $a_{i,j} < 0$ denotes a sale. Given that all agents within a type follow symmetric strategies, an individual agent of type i holds $\mu_i a_{i,j}$ units of the asset, and the total holdings of all agents of type i is $a_{i,j}$. The competitive fringe's asset position in security j by $a_{f,j}$. The market clearing conditions are

$$\sum_{i=1}^{N} a_{i,j} + m_f a_{f,j} = 0 \qquad \text{for all } j. \tag{1}$$

Because equilibrium prices depend on asset positions, we denote the market-clearing pricing functional by $P_j(A)$, where $A = [\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots \boldsymbol{a}_N]$ is the matrix of all strategic agents' asset holdings and $\boldsymbol{a}_i$ the $J \times 1$ vector of asset holdings of type i agents. The *price impact* of an agent of type i for asset j is the marginal change in the equilibrium price of asset j given a marginal change in the agent's asset position, holding fixed other large agents' positions. We denote the price impact of the representative large agent of type i by the $J \times J$ matrix $\Lambda_i(A)$. As the change in aggregate quantities induced by a change in the portfolio of an *individual* agent of type i scales with its mass

μ_i, so will its price impact. We characterize the properties of equilibrium prices in the next section.

2.3 Equilibrium concept

Our equilibrium concept is Cournot-Walras Equilibrium. In this approach, strategic agents submit demand schedules taking as given other strategic investors' demand and internalizing their effect on equilibrium prices through their influence on the competitive fringe's portfolio choices.[4]

At date 1, strategic type i allocates endowment $\mu_i w_i$ between immediate consumption and asset purchases. At date 2, the agent consumes its income and asset payoffs. The decision problem of the representative strategic investor of type i is:

$$\begin{aligned} U_i = \max_{\left\{c_{1,i}, \{a_i^j\}_{j=1}^J\right\}} \quad & u_{i,1}\left(c_{1,i}\right) + \sum_{z \in \mathcal{Z}} \pi\left(z\right) u_{i,2}\left(c_{2,i}\left(z\right)\right) \\ \text{s.t.} \quad & \mu_i c_{1,i} = \mu_i w_i - \mu_i \sum_j P_j(A) a_{i,j}, \\ & \mu_i c_{2,i}\left(z\right) = \mu_i y_i\left(z\right) + \mu_i \sum_j x_j(z) a_{i,j}. \end{aligned} \tag{2}$$

We define the controls of strategic agents in this manner while recognizing that the consumption of the representative strategic agent of type i is actually $\mu_i c_{1,i}$ and $\mu_i c_{2,i}\left(z\right)$ at dates 1 and 2, respectively, and similarly with optimal asset holdings, $\mu_i a_{i,j}$. This is because under homothetic preferences, optimal policies are invariant to μ_i.

The competitive fringe differs from strategic agents in that it takes prices as given when forming its portfolio. Hence its decision problem is:

$$\begin{aligned} U_f = \max_{\left\{c_{f1}, \{a_f^j\}_{j=1}^J\right\}} \quad & c_{f1} + \sum_{z \in \mathcal{Z}} \pi\left(z\right) u_{f,2}\left(c_{2,f}\left(z\right)\right) \\ \text{s.t.} \quad & c_{f1} = w_f - \sum_j P_j a_{f,j}, \\ & c_{2,f}\left(z\right) = y_f\left(z\right) + \sum_j x_j(z) a_{f,j}. \end{aligned} \tag{3}$$

We call the equilibrium with price impact a *market equilibrium.*

[4] Neuhann and Sockin (2024) provide a detailed exposition of this approach.

Definition 2 (Market equilibrium) *A market equilibrium is a Cournot-Walras equilibrium consisting of strategy profiles* $\sigma_i = \left(c_{1,i}, \{a_{i,j}(z)\}_{j=1}^{J}\right)$ *for each representative agent of type i and* $\sigma_f = \left(c_{f1}, \{a_{f,j}(z)\}_{j=1}^{J}\right)$ *for the competitive fringe, pricing functions* $P_j(A)$*, and associated price impact function* $\Lambda_i(A)$*, such that:*

1. *Policy* σ_i *solves decision problem* (2) *for each i given* σ_{-i} *and the set of pricing functions.*

2. *Each market clears with zero excess demand according to* (1).

3. *Price impact functions are consistent with pricing functions for all assets.*

4. *All agents have rational expectations with respect to their equilibrium price impact.*

For clarity of exposition we focus on the case in which all strategic agents have the same size, $\mu_i = \mu$ for all $i \in \{1, ..., N\}$. However, our results remain valid with the appropriate generalization to heterogeneous sizes.

2.4 Equilibrium

We now characterize equilibrium in our economy. To characterize the optimal portfolio of strategic agents, it is useful to define the stochastic discount factor (SDF) for agent i as the marginal rate of substitution between time 1 and state z:

$$m_i(z) \equiv \frac{u'_{i,2}(c_{2,i}(z))}{u'_{i,1}(c_{1,i})},$$

where $f'(\cdot)$ is the derivative of $f(\cdot)$. To compactly describe key properties of equilibrium, further let $\boldsymbol{M}_i$ be the vector of agent i's SDFs, $\boldsymbol{a}_i$ the $J \times 1$ vector of asset demands, and Π the $Z \times Z$ diagonal matrix of objective probabilities with diagonal entries $\Pi_{zz} = \pi(z)$.

We begin by establishing fundamental properties of the market equilibrium.

Proposition 1 *In a market equilibrium:*

1. *Securities prices satisfy the law of one price and are given by* $P_j(A) = p_j(A)$ *where:*

$$p_j(A) = \sum_{z \in \mathcal{Z}} \pi(z)\, x_j(z) m_f(z) \quad \textit{and} \quad m_f(z) = u'_{f,2}\left(c_{2,f}(z)\right).$$

The fringe's SDF, $m_f(z)$, is decreasing in consumption $c_{2,f}(z)$ and, by market-clearing,

$$c_{2,f}(z) = y_f(z) - \sum_j x_j(z) \frac{1}{m_f} \sum_{i=1}^{N} a_{i,j}.$$

2. *The price impact matrix of the representative agent of a given strategic agent type is symmetric across $i \in \{1, ..., N\}$,*

$$\Lambda(A) = \frac{\mu}{m_f} \mathrm{X}\Pi\Gamma\mathrm{X}' \quad \textit{where} \quad \Gamma_{j,z} = -m_f'(z)\,\mathbf{1}_{\{j=z\}} \geq 0.$$

3. *The optimal asset portfolio of type $i \in \{1., ..., N\}$ satisfies the first-order necessary condition:*

$$\mathrm{X}\Pi\boldsymbol{M}_i = \boldsymbol{p}(A) + \frac{\mu}{m_f} \mathrm{X}\Pi\Gamma\mathrm{X}'\boldsymbol{a}_i.$$

   If, in addition, the condition in Eq. (A.16) is satisfied, then the agent's optimal asset portfolio is also unique. It is sufficient, although not necessary, that the competitive fringe has both constant relative risk aversion with relative risk aversion coefficient ≤ 1 and an endowment that is sufficiently large in every state for this to be satisfied.

4. *Consumption allocations are invariant to trading alternative sets of securities with the same span.*

Proposition 1 shows the first-order condition for portfolio optimality of the competitive fringe immediately generates a closed-form demand system for all assets. Specifically, prices are equal to the marginal utility of the fringe evaluated at their equilibrium consumption level. Price impact is then the marginal change in the fringe's marginal utility induced by a change in strategic agents' quantities. By market-clearing, this can be inferred by simply writing the fringe's consumption as a function of the strategic agents' portfolios. This leads to a closed-form expression for price impact that is linear in the fringe's marginal utility across states because of the quasi-linearity of its utility. Because the fringe enforces no arbitrage, consumption allocations are also invariant to the set of traded securities provided that they have equivalent asset spans.[5]

Given the pricing system, the optimal portfolio choice of strategic agents equates asset

[5]Strategic interaction in our model is intermediated by the competitive fringe. Although a strategic agent takes the asset positions of other strategic agents as given, it does internalize how its own asset demand impacts equilibrium asset prices by altering the marginal utility of the fringe. Through this channel, how one strategic agent type trades indirectly affects how another strategic agent type trades by altering the price (and price impact) that agent type faces.

prices to expected state prices (i.e., probabilities $\pi(z)$ multiplied by a type i agent's SDF $m_i(z)$) across all states in which the asset pays a dividend, plus an endogenous distortion from its price impact that scales with position size. This wedge distorts down the demand of buyers to lower prices, and the supply of sellers to raise prices. This leads to unrealized gains from trade in the precise sense that expected state-contingent valuations of marginal changes in consumption are dispersed across buyers and sellers. Hence, risk sharing is inefficient relative to the first best that obtains under perfect competition. These wedges exist although there are no exogenous barriers that would prevent strategic agents from realizing all feasible gains from trade. In the sequel, we examine these wedges more carefully by putting more structure on the set of tradeable assets.

Notably, there exists a unique market equilibrium for any asset span, which ensures that regulation has a unique and well-defined implementation in our economy. Existence follows from standard arguments, while we establish uniqueness by demonstrating the global concavity of a potential function that aggregates all agents' utility functions, accounting for Cournot-Walras equilibrium when taking first-order conditions.[6]

Proposition 2 *There exists an unique market equilibrium.*

3 Benchmark Without Portfolio Constraints

In the previous section, we characterized basic properties of equilibria that obtain under any market structure. We now provide a characterization of the distortions from market concentration when there are no exogenous impediments to trade. This is the case when markets are complete and investors face no portfolio constraints. This setting provides a benchmark to understand how portfolio regulation may improve trading efficiency in imperfectly competitive financial markets.

3.1 Privately-optimal distortions and externalities

To understand how price impact distorts portfolios in the absence of regulation, we adapt Proposition 1 to the complete market structure with the full set of Arrow securities. (Since equilibrium

[6]The test of global concavity reduces to examining how a change in a strategic agent's asset demand impacts its own utility to second-order, directly by altering its portfolio holdings and indirectly by changing price impact in all asset markets. The direct effect is globally concave, the indirect effect cancels out because of symmetry in how the price impact in asset j spills over to j' and from j' to j.

allocations are invariant to the particular set of tradeable assets, our results apply to *any* complete market structure.) We then have the following characterization.

Proposition 3 (Optimal portfolios in complete markets) *Let X equal the identity matrix, and let $p(z)$ and $a_i(z)$ denote the price and quantities of the Arrow security referencing state z, respectively. Then prices and price impact for the Arrow security z are given by $p\left(z\right) = \pi\left(z\right) m_f\left(z\right)$ and $\frac{\mu}{m_f} p'_z(A) = \Lambda_{zz}\left(A\right) = -\frac{\mu}{m_f}\pi\left(z\right) m'_f\left(z\right) = \frac{\mu}{m_f} p'\left(z\right) > 0$, respectively. The SDF for a strategic agent of type i is:*

$$m_i(z) \equiv \frac{u'_{i,2}(y_i(z) + a_i(z))}{u'_{i,1}(c_{1,i})},$$

The optimal holdings of Arrow security z satisfies the following first-order condition:

$$\pi\left(z\right) m_i(z) = p(z) + \frac{\mu}{m_f} p'(z) a_i(z) \qquad \textit{for all } z.$$

Sellers of assets (agents with $a_i(z) < 0$) increase their position relative to the optimal portfolio under perfect competition, while buyers (agents with $a_i(z) > 0$) reduce their position. Hence, marginal valuations are misaligned between buyers and sellers in every state of the world.

The propositions show that imperfect competition induces agents to implement rent-seeking distortions at the cost of lost gains from trade. Rather than aligning marginal valuations $m_i(z)$ with the associated Arrow security price $p(z)$ (and thus with all other investors' marginal valuations), investors with price impact opt to ration trades in order to extract infra-marginal price concessions. This leads to excess consumption volatility relative to the perfect competition benchmark. When markets are complete, privately optimal distortions are captured by a state-specific wedge between private valuations and market prices, namely $|\pi(z)m_i(z) - p(z)|$.

Given complete markets, these distortions can be chosen in a state-contingent manner. Investors optimally choose to distort more in states in which price impact is high (leading to high marginal benefits to quantity distortions) and where changes in traded quantities have relatively small private costs. Private costs are determined by marginal utility, and thus are low in states in which marginal utility is relatively flat. This observation implies that one might temper overall distortions by bundling multiple Arrow securities into composite securities. The reason is that such bundling reduces the scope for investors to engage in state-specific distortions. Since forced bundling of Arrow securities renders markets incomplete, incomplete markets may thus lead to better risk sharing than complete markets when portfolio choice is distorted by price impact.

There is a further motive for regulatory interventions, which is that privately optimal distortions impose an *illiquidity externality* on other investors. This effect is most transparent with a second-order approximation to the pricing function in Proposition 3,

$$\Delta p(z) \approx p'(z) \sum_i \Delta a_i(z) + \frac{1}{2} p''(z) \sum_i \sum_k \Delta a_i(z) \cdot \Delta a_k(z). \tag{4}$$

The first linear term on the right-hand side of Eq. (4) is the direct price change resulting from a change in agent demands evaluated at the slope of the pricing function, whereby more demand raises prices.[7] The second-order term for Eq. (4) represents the illiquidity externality that arises only because our model allows for non-linear price impact, and it reflects strategic interactions among large investors. If the fringe has convex marginal utility, $p''(z) > 0$, a supply reduction by one seller increases price impact, raising the marginal benefit for another seller to also ration supply. Conversely, demand reductions lower price impact and induce other buyers to buy more.

We now illustrate these mechanisms using a canonical setting with pure diversifiable risk. The setting, which we return to throughout the paper, is as follows.

Setting 1 (Purely Diversifiable Risk) *There are two types of strategic agents, $i \in \{1, 2\}$, that have the same concave utility function, $u(\cdot)$, and the fringe's date-2 utility function is also $u(\cdot)$. All strategic agents and the fringe have an initial endowment of $\bar{y}$. There are equally-likely states, $z \in \{1, 2\}$. Strategic agents are ex-ante symmetric and face pure idiosyncratic risk: $y_i(i) = \bar{y} + \Delta$ and $y_i(3-i) = \bar{y} - \Delta$, i.e., in every state, one type is income rich and the other type is poor. The fringe receives $\bar{y}$ in both states.*

The distortions to risk sharing which arise in this setting are as follows.

Example 1 (Market power rations trade of diversifiable risk) *Consider the environment from Setting 1. By the ex-ante symmetry of strategic agents, in equilibrium each strategic agent sells $a_S < 0$ claims on the state with high private income, and buys a_B claims on the state with low private income. Since the fringe has constant endowment, both securities have the same price p and price impact p'. Hence strategic agents choose the same consumption at date 1.*

[7] This direct effect is also present in models of strategic trading that feature affine prices, such as those analyzed in the CARA-normal tradition of Kyle (1989). Because the competitive fringe's utility function over date-2 consumption is concave, $p'(z) > 0$, and an increase in demand / reduction of supply by any strategic agent increases the asset price.

Perfect risk sharing requires $a_S = -\Delta$ and $a_B = \Delta$. Since price impact induces imperfect risk sharing, $a_S = -\Delta + \delta_S$ and $a_B = \Delta - \delta_B$ for some privately optimal distortions δ_S and δ_B for sales and buys, respectively. The first-order conditions determining these distortions are

$$\text{Sale distortion:} \qquad \left| \frac{\frac{1}{2}u'(\bar{y}+\delta_S)}{u'(\bar{y}+p\cdot(\delta_S-\delta_B))} - p \right| = \frac{\mu}{m_f} p'\Big(\Delta - \delta_S\Big), \tag{5}$$

$$\text{Buy distortion:} \qquad \left| \frac{\frac{1}{2}u'(\bar{y}-\delta_B)}{u'(\bar{y})-p\cdot(\delta_S-\delta_B))} - p \right| = \frac{\mu}{m_f} p'\Big(\Delta - \delta_B\Big), \tag{6}$$

where the left-hand side measures the distortion between private marginal valuations of future consumption and the asset price. Eqs. 5 *and* 6 *show that these wedges are risk-sharing distortions that stem from price impact. It is easy to verify that sales are distorted more than buys, $\delta_S > \delta_B > 0$.*[8] *This is because every agent sells claims on the state with high income, which means that distortions are less costly because marginal utility is flatter. As a result, prices and price impact rise, thereby generating the illiquidity externality.*

3.2 Aggregate consequences of strategic distortions

In the previous subsection, we show that privately-optimal portfolio distortions because of price impact can be cast as a set of state-contingent wedges that capture the private marginal benefit of rationing trades. These make clear that optimal distortions are sensitive to an agent's state-contingent income, its trading needs, and its price impact. However, because these wedges are defined at the individual level over state-contingent income exposures, they provide limited insight into the general equilibrium distortions that they entail. Since policy must reckon with equilibrium effects, we now develop a method to map state-contingent wedges into market-wide risk-sharing arrangements to characterize the aggregate consequences of strategic trading.

Specifically, we construct a map between the equilibrium of our model with complete markets (Proposition 3) and an equivalent *counterfactual economy with competitive trading but incomplete markets*. Such a mapping exists because market power leads to misaligned marginal valuations (i.e., state prices), similar to what occurs in competitive models with incomplete markets. The particular form of the counterfactual asset span (i.e., the markets that appear to be missing) then provides an interpretable measure of the directions of trade that are distorted by strategic trading.

[8] If instead $\delta_S = \delta_B$, then the right-hand side is the same for buyers and sellers. However, the left-hand side is strictly smaller for sellers than buyers for any given distortion; as such, $\delta_S > \delta_B > 0$.

Our approach is similar to Constantinides and Duffie (1996). We first endow all agents in the counterfactual economy with the consumption allocation they obtain in the market equilibrium (i.e., $c_{1,i}$ at date 1 and $c_{2,i}(z)$ at date 2). Next, we consider the set of market structures that generate no further trade away from this allocation if agents behave competitively. Since such market structures always exist (autarky being one example), we consider the *maximum rank* restricted asset span and show it is less than full rank. The formal definition of such a counterfactual economy is as follows.

Definition 3 (Counterfactual Economy with Competitive Trading) *A counterfactual economy with competitive trading of rank K consists of a state-contingent endowments process* $\mathbf{E} = \{c_{1,i}, \{c_{2,i}(z)\}_{i=1}^{N}\}$ *for all agents at all dates and an* $K \times Z$ *asset return matrix* $\tilde{X}$*, such that:*

1. *The endowment process is the consumption process from the equilibrium in Proposition 3.*
2. *Taking the payoff matrix and all prices as given, all agents solve their decision problem (2).*
3. *No trade is a solution to all agents' decision problems.*

Let $\mathcal{M}$ be the $N \times Z$ matrix of strategic agents' stochastic discount factors (i.e., $\mathcal{M}_{iz} = m_i(z)$). In addition, let $\boldsymbol{\iota}_N$ be the $N \times 1$ vector of ones and superscript T denote the transpose. We then have Proposition 4.

Proposition 4 *There exists a counterfactual economy with competitive trading in which the return matrix* $\tilde{X}$ *has maximal rank K less than the number of states Z (i.e.,* $\tilde{X}$ *is rank deficient), where* $\tilde{X}$ *is the largest rank matrix whose columns satisfy:*

$$\mathcal{M}\Pi\tilde{x}_k = \boldsymbol{\iota}_N \qquad \textit{for all } k,$$

It is sufficient (although not necessary) that second moment matrix of agents' state prices ($\mathcal{M}\mathcal{M}^T$) *has full rank for* $\tilde{X}$ *to have a nontrivial solution. The extent of implied market incompleteness, as measured by unrealized gains from trade, satisfies:*

$$Cov\left(m^*\left(z\right), m_i\left(z\right) - m_{i'}\left(z\right)\right) = 0,$$

where $m^*(z)$ *(given in Eq. (A.55)) is the projection of the SDF onto the incomplete return space.*

To construct the restricted asset span, we collect all no-arbitrage conditions based on the SDFs of the N strategic types from the market equilibrium, but now assume they price assets competitively. Given a market structure indexed by a matrix $\tilde{X}$ of K asset returns by security and state, we use agents' Euler equations to solve for each asset payoff vector, $\tilde{x}_k \in \tilde{X}$. This $\tilde{X}$ is the fictitious restricted asset span, and we look for the largest K for which a solution with linearly independent asset returns exists.[9]

The second part of Proposition 4 states that the restricted asset span is such that any potential gains from trade are unpriced by the unique SDF implied by market returns. When markets are incomplete, one can always recover such a market-implied SDF (e.g., Hansen and Jagannathan (1991)). Any unrealized gains from trade are consequently interpreted as untradeable under the hypothesis of perfect competition. Our analysis inverts the insights of Hansen and Jagannathan (1991) to show one can recover the implied asset span from the cross-section of state prices.[10]

The restricted asset span provides a taxonomy of risks that can be fully shared under market power (i.e., lie within the span) and those not shared enough (i.e., are orthogonal to it). The former are appropriately tradable, while the latter are strategically rationed to extract price concessions. This allows us to characterize distortions from market power directly in terms of the *risk exposures* they generate, which is the ultimate object of preferences, rather than quantities traded of particular assets, which can be arbitrary distorted by the presence of redundant securities. The main practical upshot of the construction is that it provides guidance into the optimal design of portfolio constraints. In particular, we will show that bundling securities into composite asset can mute market power, but that doing so efficiently requires the composite asset still facilitate gains from trade. The counterfactual asset span summarizes these gains from trade.

Before turning to the optimal design of regulation, we use two canonical special cases of our model to illustrate the counterfactual asset span. We first return to Setting 1 with homogeneous preferences and pure diversifiable risk. We then consider a setting with aggregate risk and heterogeneous preferences. Appendix B provides the derivations and an interpretation of the

[9] We do not need to include the competitive fringe in this calculation because we can use the fringe's state prices to pin down asset prices once we have recovered the return matrix, $\tilde{X}$. This is because $\tilde{X}$ is a matrix of dividend yields (dividend x divided by price p), and we are free to specify the two separately such that the fringe's no arbitrage conditions hold.

[10] With more than two states, the dividend-yield matrix $\tilde{X}$ is not unique. Any multiplication of $\tilde{X}$ by an invertible $J \times J$ matrix, O, whose columns sum to 1 would also yield a return matrix that satisfies Proposition 4. This has the interpretation of rearranging the J assets into J portfolios that represent new assets with an equivalent asset span.

counterfactual span in terms of the unique market-implied SDF.

Example 2 (The counterfactual asset span with pure diversifiable risk is a risk-free bond) *In Setting 1, price impact deters investors from perfectly insuring purely diversifiable risk (see Example 1). As a result, the counterfactual asset span is a risk-free bond, and the rationed asset that would allow the gains from trade to be realized is an idiosyncratic risk swap with payoffs* $[-1, 1]$.[11]

Under the maintained hypothesis of perfect competition, the only deterrent to sharing diversifiable risk is that these risks are not tradeable in the market. Therefore, the counterfactual asset span is a risk-free bond, which does not allow for risk sharing, and the missing (or rationed) asset is risk swap that allows risk transfer.

Next we consider a setting with aggregate risk and heterogeneous risk attitudes.

Setting 2 (Heterogeneous Preferences) *There are two types of strategic agents,* $i \in \{1, 2\}$, *and two equally-likely states of the world,* $z \in \{l, h\}$. *All strategic agents and the competitive fringe have an initial endowment of* $\bar{y}$. *There is only aggregate risk,* $y_i(h) = y_h$ *and* $y_i(l) = y_l$ *for all* i. *Risk attitudes are heterogeneous: Type 1 is strictly risk-averse and Type 2 is risk-neutral. As a result, it is efficient for the risk-neutral investor to carry all exposure to aggregate risk. The fringe has the same utility over date 2 consumption as the risk-averse agent.*

The counterfactual asset span and the missing asset in this setting are as follows.

Example 3 (The counterfactual asset span with heterogeneous preferences is a market index) *While it is efficient for the risk-neutral agent to hold all risk exposure, under price impact the risk-averse agent remains exposed to some risk. Hence, the risk-neutral agent has a constant SDF for all states,* $m_2(z) = m_{rn} > 0$, *while the risk-averse agent has two distinct state prices satisfying* $m^h < m_{rn}$ *and* $m^l > m_{rn}$. *The implied restricted asset span from Proposition 4 has one asset with dividend-yield vector* $[\tilde{x}_l, \tilde{x}_h]$ *satisfying:*

$$\frac{1}{2} \begin{bmatrix} m_{rn} & m_{rn} \\ m^h & m^l \end{bmatrix} \begin{bmatrix} \tilde{x}_h \\ \tilde{x}_l \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

[11] The equivalent restricted asset span can also be characterized using the market-implied SDF m^* from Proposition 4. Because only a risk-free asset is traded, m^* is $\frac{1}{r_f^*}$ in both states. The representative investor consequently owns a risk-less portfolio. We thank an anonymous referee for this helpful suggestion.

Solving this equation gives

$$\tilde{x}_h = \frac{m^l - m_{rn}}{\frac{1}{2} m_{rn} \left(m^l - m^h\right)} \qquad \text{and} \qquad \tilde{x}_l = \frac{m_{rn} - m^h}{\frac{1}{2} m_{rn} \left(m^l - m^h\right)}.$$

The asset is exposed to aggregate risk since $\tilde{x}_h > \tilde{x}_l$ by Jensen's inequality. Therefore, it is a levered market index. This has a natural interpretation: the risk-averse agent can only trade a market index with a degree of risk exposure that provides no additional insurance. The missing asset is one that would allow for insurance against aggregate risk. This is an aggregate risk swap with payoffs $[m_{rn} - m^h, m_{rn} - m^l]$, i.e. it pays in the high state and loses money in the bad state.

In both settings, the counterfactual asset span clarifies which risks are poorly shared because of distortions from price impact. We later return to these settings and show that the missing assets we derive here are in fact constrained efficient asset spans. As such, they provide direct guidance into how a regulator should bundle securities to attenuate market power while still facilitating the realization of gains from trade.

4 Equilibrium consequences of portfolio constraints

Section 3 established that price impact distorts risk sharing, and that complete markets allow investors to choose state-contingent wedges that maximize their private benefit from price impact. We now show that imposing portfolio constraints in the form of regulatory restrictions on what investors can trade hampers their ability to exert market power. Proposition 5 provides a general characterization of constrained efficient asset spans, and a sufficient condition such that complete markets are *not* constrained efficient. Proposition 6 derives the constrained efficient market structure in our two canonical settings.

For now, we assume that regulator can impose *market-wide* constraints on the economy. this we mean that all investors can trade the same set of portfolios. We discuss incomplete regulatory coverage in Section 6.1. We capture market-wide restrictions by assuming that the regulator can choose the asset span X for the economy, and search for constrained efficient asset spans. Based on the classic compensation principle (Boadway and Bruce (1984)), and in line with standard definitions of constrained efficiency, such as Geanakoplos, Magill, Quinzii, and Dreze (1990), the regulator can also impose a set of budget-balanced transfers at time one.

Assumption 1 (Regulatory tools) *The regulator chooses the market structure, as defined by the $J \times Z$ matrix of payoffs X, and a set of budget-balanced date-1 transfers between investors.*

Our definition of constrained efficiency is as follows.

Definition 4 *An asset span is constrained efficient if a planner cannot implement a Pareto improvement by altering the asset span and introducing budget-balanced date-1 transfers.*

Proposition 5 uses a local perturbation approach to characterize a necessary condition that a constrained efficient asset span must satisfy.[12] In a second step, it then establishes a sufficient condition for complete markets (i.e., the unrestricted asset span) to be constrained *inefficient*. In what follows, $\otimes$ indicates the Kronecker product and $\nabla_{\vec{X}_J}$ is the Gateaux derivative with respect to the $ZJ \times 1$ asset span payoff vector $\vec{\boldsymbol{X}}_J$, and $\mathbf{0}_{ZJ}$ is the $ZJ \times 1$ of zeroes. Recall that $\boldsymbol{M}_i$ represents the stochastic discount factor for agent i, that the stochastic discount factor for the competitive fringe, $\boldsymbol{M}_f$, is aligned with market prices, and that Π is the vector of physical probabilities.

Proposition 5 *A constrained efficient asset span satisfies the necessary condition:*

$$\sum_{i=1}^{N} \underbrace{\Pi\left(\boldsymbol{M}_i - \boldsymbol{M}_f\right) \otimes \boldsymbol{a}_i}_{\textit{Change in Risk Sharing Efficiency}} + \underbrace{\frac{\mu}{m_f} \nabla_{\vec{X}_J} \boldsymbol{a}_i X \Gamma X^T \boldsymbol{a}_i}_{\textit{Change in Price Impact Rents}} = \mathbf{0}_{ZJ}, \tag{7}$$

If $N < Z$, then complete markets (i.e., no restrictions) is constrained inefficient.

Eq. (7) decomposes the effects of changes in the asset span into two channels. The first channel operates by directly altering which risks are traded: changes in the asset span affect trading efficiency by determining which consumption profiles cannot be traded. The decomposition shows that unrealized gains from trade can be measured by expected quantity-weighted differences in stochastic discount factors. The second channel operates by modulating the distortion from price impact. Since investors have state-contingent motives for exploiting market power,

[12] As is well- appreciated (e.g., Cass and Citanna (1998)), local arguments are invalid when the number of assets changes. We therefore consider perturbations of the asset span for a fixed number of assets, J, with candidate asset spans evaluated for each choice of $J \in \{1, ...Z\}$. If $N < Z$, we can use this condition to compare complete markets to restricted asset spans because we can replicate the complete markets allocation with a restricted asset span of N bespoke assets. The proof provides details.

varying the asset span mitigates or amplifies these distortions. This is reflected in price impact terms. Constrained efficient asset spans optimally trade off these two channels.

The second part of Proposition 5 establishes a sufficient condition for the constrained inefficiency of complete markets. Whereas Section 3.1 showed that complete markets allow for privately optimal state-contingent exploitation of market power, our results here show that appropriately chosen forms of market incompleteness can dampen the resulting distortions. This finding contrasts that of the canonical theory of the second-best with competitive rational agents (e.g., Geanakoplos and Polemarchakis (1986)) in which moving to complete markets achieves the first-best outcome. Here, complete markets amplify distortions from market power relative to appropriately incomplete markets.

To characterize constrained efficient market structures in more detail, it is necessary to place further restrictions on endowments and preferences. Hence we return to our two canonical settings.[13] For these settings, Proposition 6 shows that a swap, which is the "rationed asset" from Section 3, generates *Pareto* improvements relative to complete markets by improving risk sharing, and this does not require initial transfers.

Proposition 6 (Pareto-improving market incompleteness in canonical settings) *The optimal asset span in our two canonical settings can be described as follows:*

(i) ***[Setting 1]*** *If $u'(\bar{y}) \geq 1$, then restricting trade to a swap that pays $[1, -1]$ achieves a Pareto improvement over no restrictions.*

(ii) ***[Setting 2]*** *Consider the limit of the economy in which $m_f \to 0$, holding μ / m_f fixed, and one type of strategic agent is strictly less risk averse than the other. Then, there exists a swap with payoffs $[-1, x_l]$ and $x_l > 0$, such that restricting agents to trade it achieves a Pareto improvement over no restrictions.*

The intuition for the first setting is as follows. With ex-ante symmetric agents, trading a swap is sufficient to realize all gains from trade. With Arrow securities, because agents are ex-post heterogeneous, they ration sales more than purchases since sales occur when the agent is rich and marginal utility is relatively flat. By bundling states, the swap lowers the incentives to distort sales,

[13] To isolate the asymmetric incentives of strategic agents to distort in the aggregate risk example, we assume that one strategic agent type is less-averse than the other (rather than risk-neutral), in the sense that the other's utility function is a concave transformation of its own at both dates. In addition, we focus on the limit in which the competitive fringe become arbitrarily small, but price impact remains well-defined in the limit. See the proof for further details.

which improves risk sharing and liquidity. As a result, all agents have strictly higher utility when the swap is the only available asset. Setting 2 with aggregate risk follows similar logic. Although the less risk-averse agent can exert more market power in all states, it does so particularly when the risk-averse agent is desperate for additional consumption (i.e., the low state). By fixing the terms of trade across high and low states, the restricted asset span leads to weaker distortions to asset positions. As a result, the more risk-averse agent is strictly better off when trading the swap, while the less risk-averse agent is indifferent (i.e., no worse off) because the loss in inframarginal trading rents is offset by the increase in volume.

5 Application: regulation of credit default swaps

In this section, we use our framework to shed light on a prominent regulatory debate regarding credit default swaps for sovereign bonds. The basic question is whether market participants should be allowed to buy CDS contracts only if they have sufficient exposure to the underlying asset (a so-called "covered" position), or if they should be allowed to buy CDS contracts without any underlying exposure (a "naked" position). Germany temporarily banned "naked" CDS positions on Eurozone sovereign debt in May 2010, while the European Union permanently banned them in November 2011. Advocates of the ban argue that naked positions encourage speculation, while critics counter that bans harm liquidity in underlying bond markets. For instance, Oehmke and Zawadowski (2015) shows that the introduction of CDS contracts lowers yields but can raise borrowing costs for traders. Our model can inform this debate because it can account for cross-asset spillovers under flexible portfolio constraints. Moreover, consistent with our model, CDS markets are known to be concentrated: a small number of dealers account for a large majority of CDS volume, and high observed bid-ask spreads are indicative of limited liquidity (see the evidence in D'Errico, Battiston, Peltonen, and Scheicher (2018) and Chaumont, Gordon, Sultanum, and Tobin (2023)). Our approach is to measure price impact from bid-ask spreads and use our model to assess different regulatory treatments of CDS contracts.

We consider the following special case of our model. There are two states, $z \in \{1, 2\}$, two types of strategic agents and a competitive fringe, all with log utility. The aggregate endowment is Y at date 0 and in state 1, but it is $\delta Y \leq Y$ in state 2. Clients receive a share α_0 of the aggregate endowment at date 0, and a share $\alpha_1(z)$ in state z at date 1. We call the first strategic type *clients*, and the second type *dealers*. This is because dealers end up selling bonds and CDS to clients.[14]

We then study two market structures. In the first (*complete markets*), there is a risky bond and a CDS contract. The bond has face value B but pays zero upon a default. Default occurs in state 2, the state with the low aggregate endowment. The CDS contract pays 0 in state 1 and B in state 2. Hence, the CDS can be used to insure against default, and the combination of both assets leads to complete markets. In the second structure (*regulated markets*), agents can trade only a single security: a covered bond which pays B in state 1 and ρB in state 2. This structure can be achieved by requiring market participants to hold the bond and CDS in fixed proportion, as in a requirement to trade only covered positions. Because B is a normalization, the key parameter determining this market structure is ρ.

An important obstacle to taking our model to the data is that the cross-section of endowments and trading positions is not readily observable to the econometrician, even if it may be (partially) observable to traders and regulators. To overcome this issue, we set endowments by assuming the regulated market structure is well-calibrated to the prevailing gains from trade. In particular, we assume that under perfect competition, the covered bond is sufficient to realize all gains from trade. This obtains if clients would like to buy 1 unit of the risky bond and ρ units of the CDS contract from the dealers. Given this assumption, ρB then determines the cross-state dispersion of endowments. Appendix C considers the case where the covered bond is not sufficient to realize all gains from trade, so that the policy is poorly calibrated to the underlying fundamentals.

We calibrate the key parameters of our model by matching model moments generated under complete markets to data from global bond and CDS markets. We target the mean sovereign bond mid-price yield, mean sovereign bond bid-ask spread, and the mean sovereign CDS bid-ask spread across 65 sovereigns from 2004 to 2012 reported in Table 1 of Sambalaibat (2023). We also target a CDS-bond basis of -1% for speculative-grade countries from Figure 7 of Gilchrist, Wei, Yue, and Zakrajšek (2022).[15] Our calibration should be understood as capturing broad empirical

[14] To avoid having to calibrate an endowment process for the competitive fringe, we study the limit of our economy as the fringe becomes arbitrarily small, $m_f \to 0$, but the relative size of strategic agents remains constant in the limit, $\frac{\mu}{m_f} = \kappa$. Neuhann and Sockin (2024) formally study this limit.

[15] That is, we calibrate the model assuming the data were generated in a time period when "naked" CDS were available to trade. Since the EU ban applied only in some countries and only in the last year of the sample, this assumption appears reasonable. Appendix C describes how we map our model into these empirical moments in more detail.

patterns across a range of countries.

Data Moment	Value	Parameter	Calibrated Value
Aggregate steady state endowment	–	Y	2
Face value of bond	–	B	0.05
Bond yield (%)	5.08%	Default prob. π	0.048
Bid-ask spread for bonds (% of par)	1.00%	Relative size $\frac{\mu}{m_f}$	0.133
Bid-ask spread for CDS (% of mid-price)	13.47%	Agg. shock δ	0.994
CDS-bond basis (%)	−1.00%	Dispersion ρB	0.6917

Table 1: Calibration targets and associated parameters. Y and B fix the scale of the economy and thus have no direct counterparts in the data. All market participants have log utility.

Table 1 provides the target moments and associated parameters. The key parameters are the shock to the aggregate endowment δ, the probability of default π, the relative size of strategic agents μ/m_f, and income dispersion ρ. While all parameters interact non-linearly, identification broadly works as follows. The bond yield is directly influenced by the default probability π. Relative size μ/m_f directly affects the price impact friction in investors' optimality conditions, and we can therefore discipline its level using the bid-ask spreads. The bid-ask spread for bonds depends on marginal valuations in both states of the world, whereas the CDS bid-ask spread depends on marginal valuations in state 2 only. Hence aggregate shock δ separately modulates the CDS bid-ask spread. The CDS-bond basis measures differential expected returns across bonds and CDS. Because these reflect marginal utilities, the basis pins down the dispersion in endowments.

Table 2 reports model outcomes for our calibrated parameters from the complete-markets benchmark. Given that we set endowments in our calibration, we can fit the targeted moments exactly. The main object of interest is therefore the counterfactual equilibrium in the regulated market structure, which is reported in the third column. Banning naked "CDS" positions lowers sovereign bond yields by 67 basis points because investors buy more bonds to maintain a covered CDS position. This higher demand for bonds increases the bid-ask spread (as % of par) by 32.0%. In contrast, the lower demand for CDS positions reduces the CDS bid-ask spread (as % of mid-price) by 38.5% and the CDS-bond basis (in absolute value) by 44%. Consequently, sovereign borrowing costs fall and CDS markets become more liquid. Although utilitarian welfare increases by 2.8%, expected utility rises for clients by 7.9% but falls for dealers by 12.5%.

Absent transfers, the regulation is distributional because bundling sovereign bond and CDS positions attenuates sellers' market power in each market, and clients are buyers in both.

That it harms dealers is consistent with their protest of the E.U. regulation that led to their exemption.[16] However, we also find that date-1 transfers of 0.3-0.46 basis points of type-1 agent's initial wealth would render the regulation a Pareto improvement.[17] In practice, such transfers would have the interpretation of clients paying bond and CDS dealers a fixed fee to trade with them.

Data Moment	**Complete markets**	**Covered Bond only**
Bond yield (%)	5.08	5.05
Bid-ask spread for bonds (% of par)	1.00	1.32
Bid-ask spread for CDS (% of mid-price)	13.47	8.29
CDS-bond basis (%)	-1.00	-0.56
Change in client expected utility (%)	–	7.9
Change in dealer expected utility (%)	–	-12.5
Transfers for Pareto improvement (bps)	–	0.3–0.46

Table 2: Outcomes for complete markets benchmark and regulated markets with covered bond only. Percentage changes in welfare and utility are defined relative to complete markets, and computed assuming no transfers between agents. Transfers required for a Pareto improvement are date-1 transfers from clients to dealers, and are measured relative to initial date-1 client wealth.

The increase in overall welfare indicates that the portfolio regulation fosters higher trading efficiency. Interestingly, this is the case even though important asset-level measures of liquidity, such as the bond bid-ask spread, deteriorate. The reason is that the decline in bond liquidity is more than offset by an increase in CDS liquidity, leading to higher overall trading efficiency. This highlights the importance of cross-asset spillovers for policy evaluation, and suggests that changes in bid-ask spreads are insufficient for gauging the impact of banning "naked" CDS positions.

6 Broader implications

In this section, we briefly discuss some broader implications of our analysis. First, we study asymmetric regulatory coverage in which regulation may apply only to some investors. Second, we discuss two additional markets where our findings are relevant.

6.1 Portfolio constraints on specific institutions

Until now, we assumed regulatory portfolio constraints were imposed on all market participants. However, in practice certain regulations apply only to a subset of financial institutions, either by

[16]See, for instance, "Dealers and Issuers Protest European Ban on Naked CDS".

[17]While the classic compensation principle considers hypothetical transfers, here we actually change the allocation of wealth and recompute equilibrium given the transfers.

design or because of limited enforcement. We now consider how imperfect regulatory coverage affects our results. We model asymmetric portfolio constraints by assuming markets are composed of Arrow securities and some agents face constraints on the combinations of securities they can trade. For example, a portfolio constraint might require agent i to take position $a_i(z) = \psi(z)a_i(z^*)$ in Arrow security z if she holds a position $a_i(z^*)$ in Arrow security z^*, where $\psi(z)$ is a parameter.

Since our aim is to understand how partial coverage alters our insights on the effectiveness of portfolio regulation, we return to our two canonical settings. In Setting 1, mandates still improve equilibrium outcomes even if coverage is imperfect.

Example 4 (Mandates with imperfect coverage also reduce price impact) *Consider Setting 1 in which two types ex-ante symmetric agents share diversifiable risk. We impose portfolio constraints on a subset of large agents by assuming a fraction χ of each type is restricted to take a position in asset 2 that is the negative of its position in asset 1, i.e. $a(2) = -a(1)$. This is equivalent to forcing these agents to trade only the swap with payoffs $[1, -1]$, which we identified as the optimal asset span in this setting.*

Perfect risk sharing using the swap requires $a(1) = \Delta$ for a portfolio-constrained agent of type 2 and $a(1) = -\Delta$ for a constrained agents of type 1. Let δ^{PR} denote the absolute deviation from perfect risk sharing for an agent with trading restrictions (that is, $a(1) = \Delta - \delta^{PR}$ if the agent is type 2 and $a(1) = -\Delta + \delta^{TR}$ if the agent is of type 1). Because prices of the underlying securities remain symmetric, the net cost of the swap is zero and constrained investor have zero net expenditures on financial assets. The optimality condition determining distortion δ^{PR} is

$$\frac{1}{2}\left|\frac{\frac{1}{2}u'(\bar{y}+\delta^{PR})}{u'(\bar{y})} - p\right| + \frac{1}{2}\left|\frac{\frac{1}{2}u'(\bar{y}-\delta^{PR})}{u'(\bar{y})} - p\right| = \frac{\mu}{m_f}p'\left(\Delta - \delta^{PR}\right).$$

Comparing with the analogous conditions for unconstrained investors (see Example 1), constrained investor have initial consumption $\bar{y}$ rather than $\bar{y} - p^*(\delta_S - \delta_B)$. Since $\delta^S > \delta^B$, constrained investors face a higher cost of distorting portfolios at date 2, and therefore distort less. Given that constrained agents have exactly offsetting demand for both Arrow securities, fringe consumption is $c_{2,f} = \bar{y} - \frac{1-\chi}{m_f}(\delta_S - \delta_B)$. Since c_f is strictly increasing in the constrained share χ, price impact is strictly decreasing. As such, portfolio constraints with partial coverage improve liquidity.

We next consider Setting 2 with heterogeneous preferences and pure aggregate risk. Here, asymmetric mandates can have differential effects on the liquidity of different securities because asymmetric mandates reallocate market power across financial institutions. Specifically, liquidity improves in markets where unconstrained institutions are buyers, and deteriorates in markets where they are sellers, and this induces reallocation in surplus across institutions as well.[18]

Example 5 (Asymmetric mandates can redistribute market power and liquidity) *Consider Setting 2. Further assume that the fringe has wealth 1 in every state, and strategic agents' initial wealth is $\bar{y} = \frac{y_h + y_l}{2}$. Under perfect competition, the risk-averse agents obtains full insurance by buying $\frac{1}{2}(y_l - y_h)$ units of a swap with payoffs $[1, -1]$ (or by buying the underlying Arrow positions directly). By market clearing, the risk-neutral agent takes the offsetting position.*

Now suppose that the risk-neutral type is restricted to trade only the swap. Let δ^{rn} denote the absolute deviation from perfect risk sharing for the risk-neutral agent with portfolio restrictions, i.e., $a_{rn} = \frac{1}{2}(y_l - y_h) + \delta^{rn}$. The first-order condition pinning down this distortion is

$$p(h) - p(l) = \frac{\mu}{m_f}\left(p'(l) + p'(h)\right)\frac{1}{2}\left(y_l - y_h\right) + \frac{\mu}{m_f}\left(p'(l) + p'(h)\right)\delta^{rn}.$$

where by no arbitrage the swap price is $p(l) - p(h)$ and price impact is $\frac{\mu}{m_f}(p'(h) + p'(l))$. Next, consider how much the unconstrained investor distorts her portfolio, and let δ^{ra}_z denote the state-z distortion from perfect risk sharing. Taking differences of the optimality conditions for the individual securities yields the implied distortion to the swap, which usefully summarizes net distortions:

$$\frac{\frac{1}{2}u'\left(\bar{y} - \delta^{ra}_l\right) - \frac{1}{2}u'\left(\bar{y} + \delta^{ra}_h\right)}{u'\left(\bar{y} + \frac{1}{2}\left(p(l) - p(h)\right)\left(y_h - y_l\right) + p(l)\delta^{ra}_l - p(h)\delta^{ra}_h\right)} = \frac{\mu}{m_f}\sum p'(z)(\delta^{ra}_z - \delta^{rn})$$

Portfolio constraints prevent the risk-neutral agent from fully exerting its market power. This allows the risk-averse agent to more successfully raise the price of the high state claim, which raises price impact in this market, and to acquire low-state consumption relatively cheaply and with comparatively low price impact. As such, asymmetric portfolio distortions redistribute liquidity from the high state to the low state, and utility from constrained to unconstrained investors. In practice, this suggests that mandates on regulated financial institutions, such as banks or portfolio insurance, may reallocate pricing power to those with fewer

[18] This insight extends beyond asymmetric mandates to other restrictions on strategic agents' portfolios, such as position limits and risk management constraints. Because position limits and risk management constraints mainly affect buyers rather than sellers of assets, they are less likely to be as effective as market-wide regulation on what agents can trade.

constraints, such as hedge funds.

6.2 Implications for other markets and products

In the previous sections, we provided a general theoretical analysis of imposing portfolio restrictions in financial markets, and applied our theory to a prominent regulatory debate surrounding credit default swaps. In this section, we briefly discuss other financial products and settings where our analysis can provide new insights for regulation.

6.2.1 Capital guarantee products

Calvet, Celerier, Sodini, and Vallee (2023) document the comparatively fast adoption of so-called "capital guarantee" products among Swedish households. These are structured products that offer substantial exposure to aggregate risk, capturing about half of the equity premium, but also provide protection against downside risk relative to buying a standard market index. They therefore can be understood as a *bundle* of a market index and an aggregate risk swap of the type that we characterize in Example 3. Calvet, Celerier, Sodini, and Vallee (2023) document substantial markups in this market, on the order of 1.5% of invested capital per year. This suggests that there are gains from trade in aggregate risk, but also that the bundle currently offered by insurance companies allows them to extract significant rents. Through the lens of our model, this suggests that appropriate constraints on the design of capital guarantee products can attenuate market power. In particular, our characterization of the rationed asset span in Example 2 indicates that the optimal security design should be sensitive to marginal valuations, which depend on household preferences, among other factors.

6.2.2 Variable annuities

Koijen and Yogo (2022) discuss the supply of variable annuity products in the U.S. life insurance industry. These are structured products that bundle mutual funds with minimum return guarantees over long horizons. They proved popular, accounting for 35% of U.S. life insurer liabilities in 2015, but also fragile, with insurers reducing supply of variable annuities during periods of financial stress. Consistent with the key friction underpinning our theory, Koijen and Yogo (2022) emphasizes that the supply and demand for variable annuities is driven by both the low liquidity of long-term options markets that would otherwise allow insurance companies to hedge aggregate risk, and by market power in the provision of these products. Changing financial product design appears to be a partial solution to this problem, with insurers now selling "short put" products to clients willing to bear aggregate risk (Barbu and Sen (2024)). Our framework provides tools to assess how regulating products for aggregate risk transfer impacts liquidity and trading efficiency.

7 Conclusion

We explore how portfolio restrictions on large financial institutions affect their risk management. Because such investors often trade in relatively illiquid markets, we examine this issue through the lens of imperfect competition in financial markets. Our analysis delivers two key insights. First, the portfolio distortions from market power are highly state-contingent, and that risk sharing is most impaired when gains from trade are large and strategic investors face few restrictions on portfolio formation. Second, optimally-chosen broad regulation that restricts the trading opportunities of all large financial institutions can improve risk sharing by attenuating rent-seeking behavior. Applying our model to the banning of "naked" sovereign CDS positions, we find that the policy improves risk sharing even though it increases bid-ask spreads in bond markets. This highlights the importance of accounting for cross-asset spillovers when evaluating policy. Overall, our findings suggest that relaxing portfolio constraints on large financial institutions can contribute to a build-up of diversifiable risk in the financial sector if markets are not perfectly competitive.

References

ALVAREZ, F., AND U. JERMANN (2000): "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, 68(4), 775–797.

ATHANASOULIS, S., AND R. SHILLER (2000): "The Significance of the Market Portfolio," *Review of Financial Studies*, 13, 301–329.

BABUS, A., AND K. HACHEM (2021): "Regulation and Security Design in Concentrated Markets," *Journal of Monetary Economics*, 121(1), 139–151.

——— (2023): "Markets for financial innovation," *Journal of Economic Theory*, 208, 105615.

BABUS, A., AND C. PARLATORE (2021): "Strategic Fragmented Markets," *Journal of Financial Economics*.

BARBU, A., AND I. SEN (2024): "Hedging through Product Design," *Unpublished working paper, INSEAD and Harvard*.

BASAK, S. (1997): "Consumption Choice and Asset Pricing with a Non-price-taking Agent," *Economic Theory*, 10(3), 437–462.

BASAK, S., AND A. PAVLOVA (2013): "Asset Prices and Institutional Investors," *American Economic Review*, pp. 1728–58.

BECKER, B., M. M. OPP, AND F. SAIDI (2022): "Regulatory Forbearance in the U.S. Insurance Industry: The Effects of Removing Capital Requirements for an Asset Class," *The Review of Financial Studies*, 35, 5438–5482.

BLUME, L. E., T. COGLEY, D. A. EASLEY, T. J. SARGENT, AND V. TSYRENNIKOV (2018): "A case for incomplete markets," *Journal of Economic Theory*, 178, 191–221.

BOADWAY, R., AND N. BRUCE (1984): *Welfare Economics*. Basil Blackwell, Oxford.

BUCHAK, G., G. MATVOS, T. PISKORSKI, AND A. SERU (2024): "Beyond the balance sheet model of banking: Implications for bank regulation and monetary policy," *Journal of Political Economy*, 132(2), 616–693.

CALVET, L., C. CELERIER, P. SODINI, AND B. VALLEE (2023): "Can Security Design Foster Household Risk Taking?," *Journal of Finance*, 78(4), 1917–1966.

CARVAJAL, A., M. ROSTEK, AND M. WERETKA (2012): "Competition in Financial Innovation," *Econometrica*, 80, 1895–1936.

CASS, D., AND A. CITANNA (1998): "Pareto Improving Financial Innovation in Incomplete Markets," *Economic Theory*, 11, 467–494.

CHAUMONT, G., G. GORDON, B. SULTANUM, AND E. TOBIN (2023): "Sovereign Debt and Credit Default Swaps," *Working Paper*.

CONSTANTINIDES, G. M., AND D. DUFFIE (1996): “Asset Pricing with Heterogeneous Consumers,” *Journal of Political Economy*, 104(2), 219–240.

DÁVILA, E., AND A. KORINEK (2018): “Pecuniary Externalities in Economies with Financial Frictions,” *Review of Economic Studies*, 85.

DEMANGE, G., AND G. LAROQUE (1995): “Optimality of Incomplete Markets,” *Journal of Economic Theory*, 65, 218–232.

D’ERRICO, M., S. BATTISTON, T. PELTONEN, AND M. SCHEICHER (2018): “How does Risk Flow in the Credit Default Swap Market?,” *Journal of Financial Stability*, 35, 53–74.

EGAN, M., A. HORTACSU, AND G. MATVOS (2017): “Deposit Competition and Financial Fragility: Evidence from the US Banking Sector,” *American Economic Review*, 107, 169–216.

EISENBACH, T., AND G. PHELAN (2022): “Cournot Fire Sales,” *American Economic Journal: Macroeconomics*, 14, 508–42.

ELUL, R. (1995): “Welfare Effects of Financial Innovation in Incomplete Markets Economies with Several Consumption Goods,” *Journal of Economic Theory*, (65), 43–78.

GABSZEWICZ, J. J., AND J.-P. VIAL (1972): “Oligopoly ”a la Cournot” in a General Equilibrium Analysis,” *Journal of Economic Theory*.

GEANAKOPLOS, J., M. MAGILL, M. QUINZII, AND J. DREZE (1990): “Generic Inefficiency of Stock Market Equilibrium when Markets are Incomplete,” *Journal of Mathematical Economics*, 19, 113–151.

GEANAKOPLOS, J., AND H. M. POLEMARCHAKIS (1986): “Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete,” *Essays in Honor of Kenneth J. Arrow: Uncertainty, Information, and Communication*, 3, 65–95.

GILCHRIST, S., B. WEI, V. Z. YUE, AND E. ZAKRAJŠEK (2022): “Sovereign Risk and Financial Risk,” *Journal of International Economics*, 136, 103593.

GOLDSTEIN, M. A., AND E. S. HOTCHKISS (2020): “Providing Liquidity in an Illiquid Market: Dealer Behavior in US Corporate Bonds,” *Journal of Financial Economics*, 135, 16–40.

Hachem, K., and Z. M. Song (2021): "Liquidity Rules and Credit Booms," *Journal of Political Economy*, 129(10), 2721–2765.

Hansen, L. P., and R. Jagannathan (1991): "Implications for Security Market Data for Models of Dynamic Economies," *Journal of Political Economy*, 99, 225–262.

Harkrader, J. C., and M. Puglia (2020): "Fixed Income Market Structure: Treasuries vs. Agency MBS," *FEDS Notes*.

Kacperczyk, M., J. Nosal, and S. Sundaresan (2024): "Market power and price informativeness," *Review of Economic Studies*, p. rdae077.

Khetan, U., J. Li, I. Neamtu, and I. Sen (2023): "The Market for Sharing Interest Rate Risk: Quantities and Asset Prices," *Available at SSRN 4517795*.

Koijen, R. S., and M. Yogo (2022): "The Fragility of Market Risk Insurance," *Journal of Finance*, 77(2), 809–1431.

Kyle, A. S. (1989): "Informed Speculation with Imperfect Competition," *The Review of Economic Studies*, 56(3), 317–355.

Malamud, S., and M. Rostek (2017): "Decentralized Exchange," *American Economic Review*, 107, 3320–3362.

Marin, J. M., and R. Rahi (2000): "Information Revelation and Market Incompleteness," *Review of Economic Studies*, 67, 455–481.

Neuhann, D., and M. Sockin (2024): "Financial Market Concentration and Misallocation," *Journal of Financial Economics*.

Oehmke, M., and A. Zawadowski (2015): "Synthetic or Real? The Equilibrium Effects of Credit Default Swaps on Bond Markets," *The Review of Financial Studies*, 28, 3303–3337.

Peltonen, T. A., M. Scheicher, and G. Vuillemey (2014): "The Network Structure of the CDS Market and Its Determinants," *Journal of Financial Stability*, 13, 118–133.

Pesendorfer, W. (1995): "Financial Innovation in a General Equilibrium Model," *Journal of Economic Theory*, 65, 79–116.

PINTER, G., E. SIRIWARDANE, AND D. WALKER (2024): "Fire Sales of Safe Assets," *Working Paper*.

PINTER, G., AND D. WALKER (2023): "Hedging, market concentration and monetary policy: a joint analysis of gilt and derivatives exposures," *Available at SSRN 4521537*.

RAHI, R., AND J.-P. ZIGRAND (2009): "Strategic Financial Innovation in Segmented Markets," *Review of Financial Studies*, 22, 2941–2971.

ROSEN, J. B. (1965): "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games," *Econometrica*, 33, 520–534.

ROSTEK, M., AND J. H. YOON (2021): "Exchange Design and Efficiency," *Econometrica*, 89, 2887–2928.

SAMBALAIBAT, B. (2023): "The Challenges of Policy Extrapolation: A Case Study of European Naked CDS Bans," *Working Paper*.

Appendix A: Proofs of Propositions

In what follows, we define the following objects. Let I_k be the $k \times k$ identity matrix, $\boldsymbol{\iota}_k$ be the $k \times 1$ vector of ones, $\boldsymbol{e}_j$ the $j \times 1$ Euclidian vector whose j^{th} entry is 1, and all other entries are zero. In addition, let $\boldsymbol{x}_j$ be the j^{th} $1 \times Z$ row vector of the payoff matrix, X.

Proof of Proposition 1

Step 1: The Problem of the Competitive Fringe:

From the first-order condition for the optimal holdings of asset j, $a_{f,j}$, from the competitive fringe's problem (3), we can recover the pricing equation:

$$p_j = \sum_{z \in \mathcal{Z}} x_j(z)\, \pi(z)\, u'_{f,2}\left(c_{2,f}(z)\right) = \sum_{z \in \mathcal{Z}} \pi(z)\, x_j(z)\, m_f(z), \tag{A.1}$$

where $M_f(z) = u'_{f,2}\left(c_f(z)\right)$ is the SDF of the competitive fringe in state z. Because the fringe's consumption in state z satisfies Eq. (3) and $a_{f,j}$ satisfies market clearing in Eq. (1), it is immediate from (A.1) that price impact satisfies the matrix:

$$\Lambda_i(A) = \frac{\mu}{m_f} X\Pi\Gamma X' \text{ where } \Gamma_{j,z} = -m_f'(z)\,\mathbf{1}_{\{j=z\}} \geq 0, \tag{A.2}$$

which is the same for all strategic agents (i.e., price impact is anonymous). We define the equilibrium price function to be $p_j(A) = P_j(A)$.

Substituting the market-clearing condition in Eq. (1) into the fringe's consumption at date 2 from (3), $c_f(z)$ can be expressed as:

$$c_{2,f}(z) = y_f(z) + \sum_j x_j(z)\left(-\frac{1}{m_f}\sum_{i=1}^{N} a_{i,j}\right).$$

Step 2: The Law of One Price:

The law of one price holds because the competitive fringe prices all assets. To see this, suppose there are two assets, j and k, with payoffs $x_j(z)$ and $x_k(z)$. Then:

$$p_j(A) = \sum_{z\in\mathcal{Z}} \pi(z)\, x_j(z)\, m_f(z) = \sum_{z\in\mathcal{Z}} \pi(z)\, x_k(z)\, m_f(z) = p_k(A).$$

Since the competitive fringe participates in all asset markets, no arbitrage is satisfied in our setting.

Step 3: The Problem of Strategic Agents:

To economize on notation, we consider a representative agent of type i. Let φ_i be the Lagrange multiplier on strategic agent i's budget constraint. The first-order condition for optimal initial consumption $c_{1,i}$ from strategic agent i's problem (2) is:

$$u_{i,1}'(c_{1,i}) - \varphi_i = 0. \tag{A.3}$$

The first-order condition for optimal asset holdings of asset j, $a_{i,j}$, is:

$$\sum_{z\in\mathcal{Z}} \pi(z)\, x(z)\, u_{i,2}'(c_{2,i}(z)) - \varphi_i\left(P_j(A) + \Lambda_{i,j}(A)\,\boldsymbol{a}_i\right) = 0, \tag{A.4}$$

where $\Lambda_{i,j}(A)$ is the j^{th} row of $\Lambda_i(A)$ and $\boldsymbol{a}_i$ is the vector of agent i's asset demands. We can rewrite (A.4) with strategic agent i's state price $M_i(z)$, substituting for φ_i with (A.3):

$$\sum_{z\in\mathcal{Z}} \pi(z)\, m_i(z)\, x(z) - P_j(A) - \Lambda_{i,j}(A)\, \boldsymbol{a}_i = 0. \tag{A.5}$$

We next recognize that strategic agent i has rational expectations, and its perceived price impact must be its actual price impact. It then follows that $\Lambda_i(A) = X\Gamma X'$.

Let $\boldsymbol{M}_i$ be the vector agent i's SDFs and Γ the diagonal matrix with entries $\Gamma_{j,z} = -m_f'(z)\,\mathbf{1}_{\{j=z\}} \geq 0$. Since this price impact function is pinned down by the fringe from (A.2), and the equilibrium price function is $p_j(A)$, (A.5) becomes:

$$\boldsymbol{p}(A) = X\Pi\boldsymbol{M}_i - \frac{\mu}{m_f} X\Pi\Gamma X'\boldsymbol{a}_i, \tag{A.6}$$

because a strategic agent of type i has mass μ.

Since the budget constraint of strategic agent i will hold with equality in equilibrium by efficiency, the optimal asset holdings $a_{i,j}$ determine both date 1 and date 2 consumption, $c_{1,i}$ and $c_{2,i}(z)$, respectively.

We now recognize that no strategic agent would ever take an infinite position in any asset. To see this, notice that optimal asset holdings satisfy the FONC (A.6), which we rewrite as:

$$X\Pi\boldsymbol{M}_i = \boldsymbol{p}(A) + \frac{\mu}{m_f} X\Pi\Gamma X'\boldsymbol{a}_i. \tag{A.7}$$

Because all endowments at all dates are bounded, the asset positions, and consequently consumption at each date, of all agents are also bounded. Consequently, we can bound all controls of strategic agent i's problem, $\{c_{1,i}, \{a_{i,j}\}_{j=1}^{J}\}$, in a closed and bounded set. By the Heine-Borel Theorem, this set is compact.

Because the state prices of the strategic agents and the price impact functional are continuous, because all utility functions are $\mathcal{C}^2$, strategic agent i's choice correspondence set is also continuous in the optimization problem's primitives (i.e., endowment processes, asset positions of other agents, and initial wealth). Consequently, the choice correspondence of strategic agent i's problem is continuous and compact-valued.

It then follows because the objective function of strategic agent i is continuous (in fact, differentiable), and the choice correspondence is continuous and compact-valued, that by Berges' Theory of the Maximum a solution to the decision problem of strategic agent i exists and the optimal policy correspondence is upper-hemicontinuous and compact-valued. As the choice of i was arbitrary, this holds for all agents of type i and all types $i \in \{1, ..., N\}$.

Step 4: Uniqueness of Strategic Agent's Optimal Portfolios:

Case 1: Complete Markets: We begin with the complete markets case. In this case, the price of Arrow security z is $p(z) = -\Lambda_f(z)$. For notational convenience, we designate the price impact in asset z as $p'(z) = -\Lambda_f'(z)$ and the derivative of price impact as $p''(z) = -\Lambda_f''(z)$.

Taking the second-order condition for the optimal asset position $a_i(z)$, and dividing by $u'(c_{1,i}) > 0$ and substituting with Eq. A.6, we have that:

$$\frac{\partial^2 U_i}{\partial a_i(z)^2} \propto \frac{u''(c_{2,i}(z))}{u'(c_{1,i})} - \frac{u''(c_{1,i})}{u'(c_{1,i})} p(z) M_{j,i}(z) - \frac{\mu}{m_f} p'(z) \left(2 + \frac{p''(z)}{p'(z)} a_i(z)\right), \tag{A.8}$$

which we can rewrite as:

$$\frac{\partial^2 U_{j,i}}{\partial a_i(z)^2} \propto \pi(z) \frac{\partial m_i(z)}{\partial a_i(z)} - \frac{\mu}{m_f} p'(z) \left(2 + \theta(z) \frac{a_i(z)}{c_{2,f}(z)}\right), \tag{A.9}$$

where $\frac{p''(z) c_{2,f}(z)}{p'(z)} = -\frac{u'''(c_{2,f}(z)) c_{2,f}(z)}{u''(c_{2,f}(z))} = \theta(z)$ is the fringe's coefficient of relative prudence. Notice that $\frac{\partial m_{j,i}(z)}{\partial a_i(z)} \leq 0$ because buying more of the asset referencing state z lowers strategic agent j of type i's marginal valuation of consumption in that state. Because the competitive fringe has convex marginal utility, $\theta(z) \geq 0$.

It then follows because $\theta(z) \geq 0$ that for $a_i(z) \geq 0$, $\frac{\partial^2 U_i}{\partial a_i(z)^2} < 0$ based on condition A.14. Consequently, we need only focus on the case in which $a_i(z) < 0$.

Let $\tilde{c}_f(z)$ be the residual consumption of the fringe in state z without agents of type i's supply. When $a_i(z) < 0$, we can rewrite condition (A.14) as::

$$\frac{\partial^2 U_i}{\partial a_i(z)^2} \propto \pi(z) \frac{\partial m_i(z)}{\partial a_i(z)} - \frac{\mu}{m_f} p'(z) \left(2 - \theta(z) \frac{|a_i(z)|}{\tilde{c}_f(z) + |a_i(z)|}\right), \tag{A.10}$$

where we recognize that all agents j of type i behave identically.

Suppose $y_f(z)$ is sufficiently large that $\tilde{c}_f(z) \geq 0$. Then $\frac{|a_i(z)|}{\tilde{c}_f(z)+|a_i(z)|} \leq 1$, and it is sufficient that $\theta(z) \leq 2$ for $\frac{\partial^2 U_i}{\partial a_i(z)^2} < 0$. This is true if the fringe has constant relative risk aversion preferences with CRRA coefficient less than or equal to 1.

Further, notice that with Arrow-Debreu securities:

$$\frac{\partial^2 U_i}{\partial a_i(z)\,\partial a_i(z')} \propto \pi(z)\frac{\partial m_i(z)}{\partial a_i(z')} < 0. \tag{A.11}$$

Recall that $\boldsymbol{M_i}$ is the $Z \times 1$ vector of the SDFs of strategic agent i and $\boldsymbol{a}_i$ the $Z \times 1$ vector of asset positions. We can then express the Hessian as being proportional to $\nabla_{\boldsymbol{a}_i}\boldsymbol{m_i} - D$, where D is a diagonal matrix. That $\nabla_{\boldsymbol{a}_i}\boldsymbol{m_i}$ is negative definite is standard for portfolio choice models with concave utility. It is also sufficient that $y(z)$ is sufficiently large and the fringe has constant relative risk aversion preferences with CRRA coefficient less than or equal to 1 for D to be a diagonal negative definite matrix because all diagonal elements are then negative.

Because the sum of negative definite matrices is also negative definite, it follows that the Hessian is negative definite. As such, in this case the first-order conditions for the optimal asset positions of strategic agent j of type i are also sufficient.

Case 2: Incomplete Markets: The argument for the incomplete markets case is similar to the complete markets case. Let $\boldsymbol{G}$ be the $Z^2 \times 1$ vectorization of the matrix, $\Pi\Gamma$.

For the fourth part of the lemma, taking the second-order condition for the optimal asset position vector $\boldsymbol{a}_i$, and dividing by $u'(c_{1,i}) > 0$ and substituting with Eq. A.6, we have that the Hessian of the agent's unconstrained optimization problem is $\mathcal{H}_i$[1]

$$\mathcal{H}_i = X\Pi\vartheta_i X' - u''(c_{1,i})\,X\Pi\boldsymbol{M}_i\boldsymbol{p}'(A) - u'(c_{1,i})\frac{\mu}{m_f}\left(2X\Pi\Gamma X' + \left(\boldsymbol{a}_i' X \otimes X\right)\partial_{\boldsymbol{a}_i}\boldsymbol{G}\right), \tag{A.12}$$

where $\otimes$ is the Kronecker product and ϑ_i is the $\mathcal{Z} \times \mathcal{Z}$ diagonal matrix with diagonal entries $u''(c_{2,i}(z))$.

[1]Note that:

$$X\Pi\Gamma X'\boldsymbol{a}_i = \left(\boldsymbol{a}_i' X \otimes X\right)\boldsymbol{G}.$$

Let H be the $Z \times Z$ gradient matrix of the vector $\boldsymbol{M_i}$ with respect to $\boldsymbol{a}_i$ and $\hat{\Gamma}$ the diagonal matrix with diagonal entries $-u'''\left(c_{2,f}\left(z\right)\right) > 0$. With some manipulation, and dividing by $u'\left(c_{1,i}\right) > 0$, we can rewrite Eq. A.12 as:

$$\mathcal{H}_i \propto X\Pi H X' - \frac{\mu}{m_f}\left(2X\Pi\Gamma X' + X\Pi\hat{\Gamma}\left(\left(X'\boldsymbol{a}_i\boldsymbol{\iota}_J'\right)\odot X'\right)\right), \tag{A.13}$$

where $\odot$ is the Hadamard product. By the properties of the Hadamard product, Eq. (A.13) reduces to:

$$\mathcal{H}_i \propto X\Pi\left(H - \frac{\mu}{m_f}\left(2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right)\right)\right)X', \tag{A.14}$$

where $Diag$ is the diagonal operator.

Notice that for any vector $J \times 1$ vector $\boldsymbol{x}$ that:

$$\boldsymbol{b}'X\Pi\left(H - \frac{\mu}{m_f}\left(2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right)\right)\right)X'\boldsymbol{b} = \boldsymbol{y}'\Pi\left(H - \frac{\mu}{m_f}\left(2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right)\right)\right)\boldsymbol{y}, \tag{A.15}$$

for $\boldsymbol{y} = X'\boldsymbol{b}$.

Consequently, if:

$$\boldsymbol{y}'\Pi\left(H - \frac{\mu}{m_f}\left(2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right)\right)\right)\boldsymbol{y} < 0\,\forall\,\boldsymbol{a}_i, \tag{A.16}$$

then the second-order condition for strategic agent i's optimization program is satisfied, and its optimal policies are unique.

Because H is negative definite for concave utilities $u_{i,1}\left(\cdot\right)$ and $u_{i,2}\left(\cdot\right)$, $X\Pi H X'$ is negative definite. Because the sum of symmetric negative definite and negative (semi)-definite matrices is negative definite, if $2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right)$ is positive semi-definite, then $X\Pi H X' - \frac{\mu}{m_f}X\Pi\left(2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right)\right)X'$ is negative definite.[2]

We can rewrite $2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right)$ as:

$$2\Gamma + \hat{\Gamma}Diag\left(X'\boldsymbol{a}_i\right) = \Gamma\left(2I_Z + \Theta Diag\left(\boldsymbol{c}_f\right)^{-1}Diag\left(X'\boldsymbol{a}_i\right)\right), \tag{A.17}$$

where I_Z is the $Z \times Z$ identity matrix, Θ is the diagonal $Z \times Z$ matrix of the competitive fringe's

[2]Notice further by Eq. A.15 that Γ is positive definite.

coefficient of relative prudence in each state, $\theta(z)$, and $\boldsymbol{c}_f$ is the vector of the competitive fringe's consumption in each state. Because the competitive fringe has convex marginal utility, $\theta(z) \geq 0$.

Thus, it is sufficient that the diagonal matrix $2I_Z + \Theta Diag\left(\boldsymbol{c}_f\right)^{-1} Diag\left(X'\boldsymbol{a}_i\right)$ has all non-negative entries for it to be positive semi-definite. Suppose the minimal element is achieved in state $\tilde{z}$, and let $x_{:\tilde{z}}$ be the $\tilde{z}^{th}$ column of X.

Notice that $x'_{:\tilde{z}}\boldsymbol{a}_i$ is the total transfer to strategic agents of type i of state $\tilde{z}$ consumption, and let $\tilde{c}_f(z)$ be the residual consumption of the fringe in state z without agents of type i's transfer. We consequently require that:

$$2 + \theta(\tilde{z}) \frac{x'_{:\tilde{z}}\boldsymbol{a}_i}{\tilde{c}_f(z) - x'_{:\tilde{z}}\boldsymbol{a}_i} \geq 0, \tag{A.18}$$

similar to the complete markets case, since $\theta(z)$ is nonnegative because the fringe has convex marginal utility, we need only focus on the case in which $x'_{:\tilde{z}}\boldsymbol{a}_i$ is negative. In this case, Eq. (A.19) becomes:

$$2 - \theta(\tilde{z}) \frac{|x'_{:\tilde{z}}\boldsymbol{a}_i|}{\tilde{c}_f(z) + |x'_{:\tilde{z}}\boldsymbol{a}_i|} \geq 0, \tag{A.19}$$

Suppose $y_f(z)$ is sufficiently large that $\tilde{c}_f(z) \geq 0$. Then $\frac{|x'_{:\tilde{z}}\boldsymbol{a}_i|}{\tilde{c}_f(z)+|x'_{:\tilde{z}}\boldsymbol{a}_i|} \leq 1$, and it is sufficient that $\theta(z) \leq 2$ for $\frac{\partial^2 U_i}{\partial a_i(z)^2} < 0$. This is true if the fringe has constant relative risk aversion preferences with CRRA coefficient less than or equal to 1.

Consequently, we arrive at the same sufficient condition we derived in complete markets.

Step 5: Market Structure Invariance:

Consider a different $Z \times K$ payoff matrix B with an equivalent asset span and asset price vector $\boldsymbol{p}^B\left(A^B\right)$. By this, we mean that there exist vectors $\boldsymbol{a}_i^B$, such that:

$$X'\boldsymbol{a}_i = B'\boldsymbol{a}_i^B. \tag{A.20}$$

Similar exercises can be done for the asset positions $\boldsymbol{a}_f^B$ of the fringe.

It is immediate from Step 1 that the prices $p^B\left(A^B\right)$ satisfy:

$$p^B\left(A^B\right) = B\Pi\boldsymbol{M}_f^B, \tag{A.21}$$

and that

$$\Lambda\left(A^{B}\right) = B\Pi\Gamma^{B}B' \quad \Gamma^{B}_{j,k} = -m^{B\prime}_{f}\left(z\right)\mathbf{1}_{\{j=k\}}.$$

Consider now the analogue of the first-order necessary conditions for optimal asset demand of strategic agent i from Step 2:

$$p^{B}\left(A^{B}\right) = B\Pi\boldsymbol{M}_{i} + B\Pi\Gamma^{B}B'\boldsymbol{a}^{B}_{i}. \tag{A.22}$$

Suppose now that the consumption allocations of strategic agents and the fringe are the same under asset span B as under X. This implies that $\Gamma^{B} = \Gamma$. Notice then that we can manipulate Eq. (A.22), substituting with Eqs. (A.21) and (A.20):

$$\boldsymbol{\Lambda}_{f} = \boldsymbol{\Lambda}_{i} + \Pi\Gamma B'\boldsymbol{a}^{B}_{i} = \boldsymbol{\Lambda}_{i} + \Pi\Gamma X'\boldsymbol{a}_{i}. \tag{A.23}$$

Multiplying by X, we arrive at:

$$p\left(A\right) = X\Pi\boldsymbol{M}_{i} + X\Pi\Gamma X'\boldsymbol{a}_{i},$$

which is the first-order condition for optimal asset demands under asset span X. Consequently, strategic agents choose the same optimal state-contingent exposures as under both asset spans.

Finally, we show that the portfolio of strategic agent i costs the same under asset span X as under B. This is trivial, however, because:

$$p\left(A^{B}\right)'\boldsymbol{a}^{B}_{i} = \boldsymbol{M}'_{f}\Pi B'\boldsymbol{a}^{B}_{i} = \boldsymbol{M}'_{f}\Pi X'\boldsymbol{a}_{i} = \boldsymbol{p}\left(A\right)'\boldsymbol{a}_{i},$$

as required. Consequently, equilibrium allocations are invariant to the market structures that implement the same asset span. ■

Proof of Proposition 2

Step 1: Existence of a Market Equilibrium in Pure Strategies:

As a result of Berge's Theory of the Maximum, the optimal policies of each strategic agent are compact-valued, upper hemi-continuous correspondences. In addition, we recognize that because the budget constraint of each strategic agent is affine in its consumption that the image, $\{c_{1,i} + \sum_j p_j a_{i,j} \leq w_i\}$ at date 1 and $\{c_{2,i}(z) - \sum_j x_j(z) a_{i,j} \leq y_i(z)\}$ at date 2 for $z \in \mathcal{Z}$, are convex sets when optimizing over $\{a_{i,j}\}_{j=1}^{J}$.[3] By Roxin's Condition, then, because the image of the action space to the agent's budget constraint is convex, the agent's constraint set for its optimal policy correspondence is also convex.

We can then construct a mapping from a conjectured set of asset positions for all strategic agents to an optimal set of asset positions using the market-clearing conditions (1) and the optimal policy correspondences as an equilibrium correspondence whose image is a compact, convex set. We can then apply Kakutani's Fixed Point Theorem to conclude that an upper hemicontinuous correspondence from a compact, convex set to itself has at least one fixed point. Consequently, an equilibrium in pure strategies exists.

Step 2: Uniqueness of the Market Equilibrium in Pure Strategies:

To characterize the uniqueness of a market equilibrium in pure strategies, we follow Rosen (1965) and construct the potential function $\phi(\boldsymbol{a}, \boldsymbol{r})$:

$$\phi(\boldsymbol{a}, \boldsymbol{r}) = \sum_{i=1}^{N} r_i \sum_{k=1}^{1/\mu} \phi_{i,k}(\boldsymbol{a}, \boldsymbol{r}) + r_{N+1}\phi_f(\boldsymbol{a}, \boldsymbol{r}), \tag{A.24}$$

where $\boldsymbol{a}$ stacks the vectors $\boldsymbol{a}_i$ $1/\mu$ times from $i = 1$ to $i = N$ and that of the competitive fringe $a_{f,j}$, $\{r_i\}_{i=1}^{N+1}$ form the elements of $\boldsymbol{r}$ and are arbitrary nonnegative weights that sum to 1, and $\phi_i(\boldsymbol{a}, \boldsymbol{r})$ and $\phi_f(\boldsymbol{a}, \boldsymbol{r})$ are given by:

$$\phi_{i,k}(\boldsymbol{a}, \boldsymbol{r}) = u_{i,1}\left(w_i - \sum_{j=1}^{J} p_j a_{i,j}\right) + \sum_{z \in \mathcal{Z}} \pi(z)\, u_{i,2}\left(y_i(z) + \sum_{j=1}^{J} x_j(z)\, a_{i,j}\right), \tag{A.25}$$

and

$$\phi_f(\boldsymbol{a}, \boldsymbol{r}) = w_i - \sum_{j=1}^{J} p_j a_{f,j} + \sum_{z \in \mathcal{Z}} \pi(z)\, u_{f,2}\left(y_i(z) + \sum_{j=1}^{J} x_j(z)\, a_{f,j}\right), \tag{A.26}$$

[3] Although $\sum_{j=1}^{J} p_j a_{i,j}$ is not convex because of market power, consumption can adjust to satisfy the budget constraint.

and where we have substituted for each agent's consumption using their budget constraints from decision problems 2 and 3, respectively. We note the payoff functions are the same for all agents k of a strategic type i.

We can then define the pseudo-gradient of $\phi(\boldsymbol{a})$, $\nabla_{\boldsymbol{a}}\phi(\boldsymbol{a})$ as:

$$\nabla_{\boldsymbol{a}}\phi(\boldsymbol{a},\boldsymbol{r}) = \begin{bmatrix} r_1\nabla_{\boldsymbol{a}_1}\left(u_{1,1}\left(c_{1,1}\right)+\sum_{z\in\mathcal{Z}}\pi(z)\,u_{1,2}\left(c_{2,1}(z)\right)\right) \\ \cdots \\ r_1\nabla_{\boldsymbol{a}_1}\left(u_{1,1}\left(c_{1,1}\right)+\sum_{z\in\mathcal{Z}}\pi(z)\,u_{1,2}\left(c_{2,1}(z)\right)\right) \\ r_2\nabla_{\boldsymbol{a}_2}\left(u_{2,1}\left(c_{1,2}\right)+\sum_{z\in\mathcal{Z}}\pi(z)\,u_{2,2}\left(c_{2,2}(z)\right)\right) \\ \cdots \\ r_2\nabla_{\boldsymbol{a}_2}\left(u_{2,1}\left(c_{1,2}\right)+\sum_{z\in\mathcal{Z}}\pi(z)\,u_{2,2}\left(c_{2,2}(z)\right)\right) \\ \cdots \\ r_N\nabla_{\boldsymbol{a}_N}\left(u_{N,1}\left(c_{1,N}\right)+\sum_{z\in\mathcal{Z}}\pi(z)\,u_{N,2}\left(c_{2,N}(z)\right)\right) \\ \cdots \\ r_N\nabla_{\boldsymbol{a}_N}\left(u_{N,1}\left(c_{1,N}\right)+\sum_{z\in\mathcal{Z}}\pi(z)\,u_{N,2}\left(c_{2,N}(z)\right)\right) \\ r_{N+1}\nabla_{\boldsymbol{a}_f}\left(u_{f,1}\left(c_{f,N}\right)+\sum_{z\in\mathcal{Z}}\pi(z)\,u_{f,2}\left(c_{2,f}(z)\right)\right) \end{bmatrix}, \tag{A.27}$$

which is the $(N/\mu+1)\,J\times 1$ vector of stacked first-order conditions from each agent's optimization program that holds fixed the strategies of the other strategic agents. Our object of interest is the Jacobian of the vector $\nabla_{\boldsymbol{a}}\phi(\boldsymbol{a},\boldsymbol{r})$ with respect to $\boldsymbol{a}$, $\Phi(\boldsymbol{a},\boldsymbol{r})$. The market equilibrium is unique if $\frac{1}{2}\left(\Phi(\boldsymbol{a},\boldsymbol{r})+\Phi'(\boldsymbol{a},\boldsymbol{r})\right)$ is negative definite for all $\boldsymbol{a}$ that satisfy each agent's budget constraint and the market-clearing conditions in Eq. (1).

Notice, however, that because of the anonymity of asset prices p_j and price impact p_j', the Jacobian $\Phi(\boldsymbol{a},\boldsymbol{r})$ has the special partitioned structure:

$$\Phi(\boldsymbol{a},\boldsymbol{r}) = \begin{bmatrix} 1_{\frac{N}{\mu}}\otimes K(\boldsymbol{a},\boldsymbol{r}) \\ 0_{J\times NJ}. \end{bmatrix} \tag{A.28}$$

where $1_{\frac{N}{\mu}}$ is the vector of ones of size $\frac{N}{\mu}$, $\otimes$ is the Kroneker product, $0_{J\times(1+\frac{N}{\mu})J}$ is a matrix of zeroes, and:

$$K(\boldsymbol{a},\boldsymbol{r})=\begin{bmatrix}\Phi_{\{1,1\}} & \mu\Phi_{\{1,-1\}} & \mu\Phi_{\{1,-1\}} & \cdots & \mu\Phi_{\{1,-1\}} & m_f\Phi_{\{1,-1\}}\\ \mu\Phi_{\{2,-2\}} & \Phi_{\{2,2\}} & \mu\Phi_{\{2,-2\}} & \cdots & \mu\Phi_{\{2,-2\}} & m_f\Phi_{\{2,-2\}}\\ \mu\Phi_{\{3,-3\}} & \mu\Phi_{\{3,-3\}} & \Phi_{\{3,3\}} & \cdots & \mu\Phi_{\{3,-3\}} & m_f\Phi_{\{3,-3\}}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \mu\Phi_{\{N,-N\}} & \mu\Phi_{\{N,-N\}} & \mu\Phi_{\{N,-N\}} & \cdots & \Phi_{\{N,N\}} & m_f\Phi_{\{N,-N\}}\end{bmatrix}. \tag{A.29}$$

for $J\times J$ matrices $\Phi_{i,i}$ and $\Phi_{i,-i}$. The diagonal matrices $\Phi_{i,i}$ are the Jacobians with respect to $\boldsymbol{a}_i$ of $\nabla_{\boldsymbol{a}_i}\phi_i(\boldsymbol{a},\boldsymbol{r})$. The off-diagonal matrices $\Phi_{i,-i}$ are the Jacobians with respect to $\boldsymbol{a}_{-i}$ of $\nabla_{\boldsymbol{a}_i}\phi_i(\boldsymbol{a},\boldsymbol{r})$, which are the same for each i' because all agents enter symmetrically into asset prices p_j with a factor μ except for the fringe, which enters with a factor m_f. The second derivatives for the competitive fringe's problem are all identically zero (i.e., the zero matrices) because the asset price is always exactly equal to the fringe's marginal utility from its first-order conditions.

It is immediate then that:

$$\begin{aligned}\frac{1}{2}\Delta\boldsymbol{a}'\left(\Phi(\boldsymbol{a},\boldsymbol{r})+\Phi'(\boldsymbol{a},\boldsymbol{r})\right)\Delta\boldsymbol{a} &= \sum_{i=1}^{N}\Delta\boldsymbol{a}_i'\Phi_{\{i,i\}}\Delta\boldsymbol{a}_i+\frac{1}{2}\sum_{i=1}^{N}\Delta\boldsymbol{a}_i'\Phi_{\{i,-i\}}\left(\sum_{i'\neq i}\Delta\boldsymbol{a}_{i'}+m_f\Delta\boldsymbol{a}_f\right)\\ &\quad+\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{i'\neq i}\Delta\boldsymbol{a}_{i'}+m_f\Delta\boldsymbol{a}_f\right)'\Phi_{\{i,-i\}}'\Delta\boldsymbol{a}_i.\end{aligned} \tag{A.30}$$

By market clearing, $\sum_{i=1}^{N}\Delta\boldsymbol{a}_i+m_f\Delta\boldsymbol{a}_f=0$, and consequently Eq. (A.30) becomes:

$$\frac{1}{2}\Delta\boldsymbol{a}'\left(\Phi(\boldsymbol{a},\boldsymbol{r})+\Phi'(\boldsymbol{a},\boldsymbol{r})\right)\Delta\boldsymbol{a} = \sum_{i=1}^{N}\Delta\boldsymbol{a}_i'\left(\Phi_{\{i,i\}}-\frac{1}{2}\mu\left(\Phi_{\{i,-i\}}+\Phi_{\{i,-i\}}'\right)\right)\Delta\boldsymbol{a}_i. \tag{A.31}$$

We define $\boldsymbol{\Lambda}_j(A)=\boldsymbol{x}_j\Gamma X'$ to be the $1\times J$ price impact vector for asset j, and $\boldsymbol{\Lambda}_j'(A)=\boldsymbol{x}_j\Gamma' X'$ its derivative with $\Gamma_{k,l}'=\pi(k)\,u'''\left(c_{2,f}(k)\right)\mathbf{1}_{\{k=l\}}$. Finally, we define:

$$v_{i,j}=\boldsymbol{\Lambda}_j(A)\,\boldsymbol{e}_j+\frac{\mu}{m_f}\boldsymbol{\Lambda}_j'(A)\,\boldsymbol{a}_i. \tag{A.32}$$

Invoking strategic agent i's first-order condition for its optimal asset position, Eq. (A.6), notice that we can then write $\Phi_{\{i,-i\}}$ as:

$$\mu\Phi_{\{i,-i\}} = r_i u''\left(w_i - \sum_{j=1}^{J} p_j a_{i,j}\right)\frac{\mu}{m_f}\begin{bmatrix} (\boldsymbol{x}_1\Pi\boldsymbol{M}_i)\,\boldsymbol{\Lambda}_1(A)\,\boldsymbol{a}_i & \dots & (\boldsymbol{x}_1\Pi\boldsymbol{M}_i)\,\boldsymbol{\Lambda}_J(A)\,\boldsymbol{a}_i \\ (\boldsymbol{x}_2\Pi\boldsymbol{M}_i)\,\boldsymbol{\Lambda}_1(A)\,\boldsymbol{a}_i & \dots & (\boldsymbol{x}_2\Pi\boldsymbol{M}_i)\,\boldsymbol{\Lambda}_J(A)\,\boldsymbol{a}_i \\ \dots & \dots & \dots \\ (\boldsymbol{x}_J\Pi\boldsymbol{M}_i)\,\boldsymbol{\Lambda}_1(A)\,\boldsymbol{a}_i & \dots & (\boldsymbol{x}_J\Pi\boldsymbol{M}_i)\,\boldsymbol{\Lambda}_J(A)\,\boldsymbol{a}_i \end{bmatrix}$$

$$-r_i u'\left(w_i - \sum_{j=1}^{J} p_j a_{i,j}\right)\frac{\mu}{m_f}\begin{bmatrix} v_{i,1} & 0 & \dots & 0 & \\ 0 & v_{i,2} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & v_{i,J} \end{bmatrix}. \tag{A.33}$$

Notice now that because all strategic agents have the same price impact, $\Phi_{\{i,i\}} - \frac{1}{2}\left(\Phi_{\{i,-i\}} + \Phi'_{\{i,-i\}}\right)$ simplifies to the sum of two $J \times J$ matrices, i.e.,

$$\frac{1}{2}\Delta\boldsymbol{a}'\left(\Phi(\boldsymbol{a},\boldsymbol{r}) + \Phi'(\boldsymbol{a},\boldsymbol{r})\right)\Delta\boldsymbol{a} = \sum_{i=1}^{N} r_i \Delta\boldsymbol{a}'_i\left(F_i + \frac{1}{2}u''\left(w_i - \sum_{j=1}^{J} p_j a_{i,j}\right)\frac{\mu}{m_f}B_i\right)\Delta\boldsymbol{a}_i, \tag{A.34}$$

where the first a diagonal matrix A_i contains negative diagonal elements:

$$F_i\{j,j\} = \sum_{z\in\mathcal{Z}} \pi(z)\, x_j(z)^2\, u''\left(y_i(z) + \sum_{j=1}^{J} x_j(z)\, a_{i,j}\right) - u'\left(w_i - \sum_{j=1}^{J} q_j a_{i,j}\right)\frac{\mu}{m_f}\boldsymbol{\Lambda}_j(A)\,\boldsymbol{e}_j, \tag{A.35}$$

and the second is a skew-symmetric matrix B_i with off-diagonal elements:

$$B_i\{j,j'\} = \left(\boldsymbol{x}_j\Pi\boldsymbol{M}_i\right)\boldsymbol{\Lambda}_{j'}(A)\,\boldsymbol{a}_i - \left(\boldsymbol{x}_{j'}\Pi\boldsymbol{M}_i\right)\boldsymbol{\Lambda}_j(A)\,\boldsymbol{a}_i. \tag{A.36}$$

Because F_i is a negative definite matrix, $\sum_{i=1}^{N}\boldsymbol{a}'_i F_i \boldsymbol{a}_i < 0$ for all feasible $\boldsymbol{a}$ that satisfy agents' budget constraints and the market clearing conditions. In addition, because B_i is a skew-symmetric matrix, we have that $\sum_{i=1}^{N}\boldsymbol{a}'_i\left(\frac{1}{2}u''\left(w_i - \sum_{j=1}^{J} p_j a_{i,j}\right)\frac{\mu}{m_f}B_i\right)\boldsymbol{a}_i \equiv 0$.

Therefore,

$$\frac{1}{2}\Delta\boldsymbol{a}'\left(\Phi(\boldsymbol{a},\boldsymbol{r}) + \Phi'(\boldsymbol{a},\boldsymbol{r})\right)\Delta\boldsymbol{a} = \sum_{i=1}^{N} r_i \Delta\boldsymbol{a}'_i A \Delta\boldsymbol{a}_i < 0, \tag{A.37}$$

which establishes the global uniqueness of the equilibrium in pure strategies, conditional on the pricing functional, and this is true for any size of strategic agents, μ, number of assets, J, number of states, $\mathcal{Z}$, and payoff matrix, X. Finally, we recognize that the pricing function is uniquely pinned down by the competitive fringe's marginal utility. This completes the proof.[4] ■

Proof of Proposition 4

Step 1: The Fictitious Asset Span:

We begin with the first part of the claim. Consider the Cournot-Walras equilibrium allocation of strategic agents of type i in the economy with imperfect competition, $\left\{ \left(c_{1,i}, \left\{c_i\left(z\right)\right\}_{z\in\mathcal{Z}}\right) \right\}_{i=1}^{N}$. Define:

$$\pi\left(z\right) m_i\left(z\right) = p\left(z\right) + p'\left(z\right) a_i\left(z\right) \;\forall\; \left(i, z\right), \tag{A.38}$$

to be the implied state price deflator of agent i in state z.

Now consider a fictitious incomplete-markets economy in which all agents behave competitively and take prices as given. In this fictitious economy, all agents have the same state prices that they have in the market equilibrium, except now we counterfactually assume that they traded competitively.

Our goal is to find an equivalent implied market structure, indexed by a set of $K \leq Z$ securities with a $K \times Z$ return matrix $\tilde{X}$ that spans the K (linear combinations of the Z) states, that justifies their ex post dispersion in state prices if these were the assets the agents counterfactually traded. Since our model is static, a security's return is just its dividend yield (i.e., $\tilde{\boldsymbol{X}}_k = \boldsymbol{X}_k / p_k$ for dividend process $\boldsymbol{X}_k$ and price p_k of security k). We later derive the dividends and prices separately using the competitive fringe. Because the synthetic assets can be derivatives, $\tilde{X}$ can also have negative entries.

The no arbitrage condition for a competitive agent i in security k is the standard Euler equation:

$$\sum_{z\in\mathcal{Z}} \pi\left(z\right) m_i\left(z\right) \tilde{x}_k\left(z\right) = 1, \tag{A.39}$$

[4]In fact, we establish the stronger result that this uniqueness holds conditional on any increasing pricing functional that preserves the anonymity of agents' price impact, which is a fairly mild requirement.

which we stack into the matrix equation:

$$\mathcal{M}\Pi\tilde{\boldsymbol{X}}_k = \boldsymbol{\iota}_N. \tag{A.40}$$

For a given security k, there is one condition for each of the N types of strategic agents, giving rise to N conditions for each security and $N \times K$ conditions in total. As shown later, we do not need to include the fringe in this construction. This is because their no arbitrage conditions will be trivially satisfied by the definition of the fictitious incomplete-markets equilibrium because we have the freedom to specify security prices p after recovering $\tilde{X}$.

Our goal is to find as many linearly independent solutions to Eq. (A.39) as is feasible, each corresponding to a different security. The maximum number of securities we can recover is our K, i.e., $K = \max_{k\in\{0,\ldots,Z\}} \left\{rank\left(\tilde{X}\right) = k\right\}$. We want the maximum number of securities because, if there were an additional asset that replicated the asset span, introducing it would not initiate trade because it would already be priced at its no arbitrage value by all strategic agents. Note that the rank of $\mathcal{M}$ satisfies $rank\left(\mathcal{M}\right) \leq min\{N, Z\}$. If $N < Z$, then the system is always under-identified and can have many solutions, while if $N = Z$, then it may have a unique solution if it is identified. Since we assume $N \leq Z$, these are the two cases that we consider in the paper. If instead $N > Z$, it may be over-identified and have no solution (in which case $\tilde{X}$ is the empty set).

To see the content of Eq. (A.40), suppose that all agents were actually competitive instead of strategic. In this case, their state prices would be aligned ex post (i.e., $m_i\left(z\right) = m_{i'}\left(z\right) = m\left(z\right)$ for all i, i'), and $\mathcal{M}$ would reduce to $\boldsymbol{\iota}_N\boldsymbol{M}^T\Pi$, where $\Pi\boldsymbol{M}$ is the vector of unique state prices. In this case, we can stack Eq. (A.40) across K securities to find:

$$\boldsymbol{\iota}_N\boldsymbol{M}^T\Pi\tilde{X}^T = \boldsymbol{\iota}_N\boldsymbol{\iota}_K^T, \tag{A.41}$$

reducing Eq. (A.41) to:

$$\tilde{X}\Pi\boldsymbol{M} = \boldsymbol{\iota}_K. \tag{A.42}$$

It is then clear that a solution to Eq. (A.42) is $\tilde{X}^T = diag\left(\boldsymbol{M}\right)^1$, where $diag$ is the diagonal operator on the vector $\boldsymbol{M}$. As is immediate, this corresponds to the complete market with Arrow-Debreu securities each with payoff 1 and price $\pi\left(z\right)m\left(z\right)$ for state z. Note that this argument does not rely on the number of agents, so state price dispersion from market power is needed for the fictitious asset span not to be complete markets.

With this return matrix $\tilde{X}$, we construct a restricted asset span that measures the degree of market incompleteness by replicating the effective asset span of the Cournot-Walras equilibrium with complete markets. Since at least one state price is misaligned among large agents, $rank(\tilde{X}) < Z$, markets must be incomplete.

We next establish a sufficient condition that ensures a nontrivial solution to Eq. (A.42). By the Fredholm alternative theorem, either Eq. (A.40) has a solution or there exists a nontrivial y, such that:

$$\mathcal{M}\boldsymbol{y} = \mathbf{0}_Z \qquad \text{and} \qquad \boldsymbol{y}^T \boldsymbol{\iota}_N \neq 0. \tag{A.43}$$

Since $N < Z$, we can rewrite the first condition in Eq. (A.44) as:

$$\mathcal{M}^T \mathcal{M}\boldsymbol{y} = \mathbf{0}_N. \tag{A.44}$$

Provided that the $N \times N$ matrix $\mathcal{M}^T\mathcal{M}$ is non-singular, the only solution to Eq. (A.44) is the zero vector. We therefore require that the second moment matrix of state prices is full rank.

Step 2: Equivalence with the Market Equilibrium:

We next verify that the fictitious competitive economy constitutes a competitive equilibrium with no trade.

Note that, by construction, the competitive fringe has the same consumption in both economies. The fringe's state prices are therefore the same in both economies (i.e., $p(z)$). For the fringe to correctly price all assets, we require that:

$$\sum_{z \in \mathcal{Z}} \tilde{x}_k(z)\, p(z) = \boldsymbol{\iota}_K \;\forall\, k \in \{1, ..., K\}. \tag{A.45}$$

Since $\tilde{X}$ is a return matrix (dividend divided by price), we have the freedom to rewrite $\tilde{x}_k(z)$ as $x_k(z)\,/\,p_k$ for security price p_k. Consequently, we can use Eq. (A.45) to find price p_k and dividend processes $x_k(z)$ that satisfy no arbitrage for the fringe. Consequently, asset prices p_k for $k \in$

$\{1, ..., K\}$ satisfy:

$$p_k = \sum_{z \in \mathcal{Z}} x_k(z)\, p(z)\,. \tag{A.46}$$

Note that we construct the fictitious competitive economy to satisfy the Euler equations of strategic agents at no trade when they behave competitively. Since we endow these agents with their consumption allocations in the market equilibrium, the equilibrium consumption allocations for strategic agents are also the same in both economies.

Finally, we check whether the consumption allocation has the same value in the fictitious competitive economy (can be financed with each agent's initial resources). For the allocation to have the same value, the restricted asset span portfolio must have the same cost as the complete markets portfolio for all large agents:

$$\sum_{z \in \mathcal{Z}} p(z)\, a_i(z) = \sum_{k \in \{1,...,K\}} p_k \left(c_{2,i}(z) - y_i(z)\right). \tag{A.47}$$

Substituting Eq. A.46 into Eq. A.48, this condition reduces to:

$$\sum_{z \in \mathcal{Z}} p(z) \left(a_i(z) - \sum_{k \in \{1,...,K\}} x_k(z) \left(c_{2,i}(z) - y_i(z)\right) \right) = 0, \tag{A.48}$$

where we can switch the order of summations because Z is finite and $K \leq Z$. Note, however, that:

$$a_i(z) - \sum_{k \in \{1,...,K\}} x_k(z) \left(c_{2,i}(z) - y_i(z)\right) = 0 \;\forall\, z \in \mathcal{Z}, \tag{A.49}$$

holds trivially by definition for the allocations to be replicated in the fictitious economy. Consequently, the equilibrium allocation has the same value, consistent with no arbitrage for strategic agents' consumption portfolios.

Step 3: Orthogonality to the Market-Implied Stochastic Discount Factor:

Consider the Hansen and Jagannathan (1991) decomposition of an admissible SDFs into:

$$m_i(z) = m(z) + \left(m_i(z) - m(z)\right), \tag{A.50}$$

where a generic $m(z)$ is a SDF. This SDF is implied by market prices if:

$$m(z) = \bar{m} + \left(\tilde{X}(z) - \tilde{X}\Pi\right)\beta(\boldsymbol{M}), \tag{A.51}$$

and $\bar{m}$ is the market-implied mean of the state-price function, i.e., $\bar{m} = \sum_{z\in\mathcal{Z}} p(z)$, or the inverse of the market-implied risk-free rate. As any admissible SDF, $m(z)$, necessarily takes this functional form, so does the unique market-implied SDF, $m^*(z)$, which is also the minimum variance SDF for a given SDF mean.

From Hansen and Jagannathan (1991), $m^*(z)$ is defined by:

$$\beta(\bar{m}) = \Sigma^{-1}\left(\boldsymbol{\iota}_K - \bar{m}\tilde{X}\Pi\boldsymbol{i}_K\right), \tag{A.52}$$

where Σ is the covariance matrix of returns. By construction, one has that:

$$Cov\left(m^*(z), m_i(z) - m^*(z)\right) = 0. \tag{A.53}$$

For any $m_i(z)$ and $m_{i'}(z)$, by the linearity of the covariance operator, we have:

$$Cov\left(m^*(z), m_i(z) - m_{i'}(z)\right) = 0. \tag{A.54}$$

Substituting Eq. (A.52) into Eq. (A.51):

$$m^* = \bar{m}\boldsymbol{i}_K + \left(\tilde{X} - \Pi\tilde{X}\right)\Sigma^{-1}\left(\boldsymbol{\iota}_K - \bar{m}\Pi\tilde{X}\boldsymbol{i}_K\right), \tag{A.55}$$

and it follows that for each security m:

$$Cov\left(\tilde{x}_k, m_i(z) - m_{i'}(z)\right) = 0 \;\forall\; k,\; i,\; i', \tag{A.56}$$

which reveals that the asset span is orthogonal to all residual gains from trade among strategic agents. ■

Proof of Proposition 5

Step 1: Constrained Efficient Asset Span:

Define initial transfers to be τ_i for $i \in \{1, ..., N, f\}$ such that:

$$\sum_{i=1}^{N} \tau_i + m_f \tau_f = 0. \tag{A.57}$$

For Pareto weights, $\omega_i \geq 0$, such that $\sum_{i=1}^{N} \omega_i + \omega_f = 1$, our social welfare criterion is Pareto-weighted welfare:

$$\begin{aligned} \mathcal{W}(\vec{\boldsymbol{X}}_J; \{\omega_i\}_{i=1}^{N}) &= \sum_{i=1}^{N} \omega_i u_{i,1}(w_i + \tau_i - \boldsymbol{M}_f^T \Pi X^T \boldsymbol{a}_i) + \sum_{z \in \mathcal{Z}} \pi(z) \sum_{i=1}^{N} \omega_i u_{i,2}(\boldsymbol{e}_z X^T \boldsymbol{a}_i + y_i(z)) \\ &\quad + m_f \omega_f \left(\sum_{z \in \mathcal{Z}} \pi(z) \, u_{f,2}(\boldsymbol{e}_z X^T \boldsymbol{a}_f + y_f(z)) + w_f + \tau_f - \boldsymbol{M}_f^T \Pi X^T \boldsymbol{a}_f \right) \\ s.t. \quad &: \quad (\text{A.1}), (\text{A.6}), \end{aligned} \tag{A.58}$$

where $\vec{\boldsymbol{X}}_J$ is the vectorization of the $J \times Z$ payoff matrix X^T of size $ZJ \times 1$ for each $J \in \{1, ..., Z\}$. This optimization is subject to the first-order necessary conditions for the asset positions of all strategic agents from Eq. (A.6) and the competitive fringe.[5]

Let $\nabla_{\vec{\boldsymbol{X}}_J}$ be the Gateaux differential operator in the direction $\boldsymbol{\psi}$ for an arbitrary variation $\vec{\boldsymbol{X}}_J + \eta \boldsymbol{\psi}$ as $\eta \to 0$. Then, we have that at the optimal choice of X for J assets. At the optimal choice of $\vec{\boldsymbol{X}}_J$, we have that Invoking the Envelope Condition, the first-order necessary conditions (A.6), and properties of matrix calculus and the Kroneker product, we can characterize the Jacobian of the expected utility of agent i with respect to the payoff matrix, $\nabla_{\vec{\boldsymbol{X}}_J} U_i$, as:

$$\begin{aligned} \nabla_{\vec{\boldsymbol{X}}_J} U_i &= u'_{i,1}(c_{i,1}) \sum_{z \in \mathcal{Z}} \left(\pi(z) \, M_i(z) \, \boldsymbol{e}_z^T - \Pi \boldsymbol{M}_f \right) \otimes \boldsymbol{a}_i \\ &\quad - u'_{i,1}(c_{i,1}) \nabla_{\vec{\boldsymbol{X}}_J} \boldsymbol{M}_f^T \Pi X^T \boldsymbol{a}_i + \frac{\mu}{m_f} u'_{i,1}(c_{i,1}) \nabla_{\vec{\boldsymbol{X}}_J} \boldsymbol{a}_i X \Gamma X^T \boldsymbol{a}_i, \end{aligned} \tag{A.59}$$

[5] The budget constraints of strategic agents and the competitive fringe from Eqs. (2) and (3), respectively, have already been substituted into the welfare objective.

and:

$$\nabla_{\vec{X}_J} U_f \quad = \quad \sum_{z\in\mathcal{Z}} \left(\pi\left(z\right) M_f\left(z\right) \boldsymbol{e}_z^T - \Pi \boldsymbol{M}_f\right) \otimes \boldsymbol{a}_f - \nabla_{\vec{X}_J} \boldsymbol{M}_f^T \Pi X^T \boldsymbol{a}_f, \tag{A.60}$$

where $\nabla_{\vec{X}_J} \boldsymbol{a}_i$ can be recovered from applying the Implicit Function Theorem to equation (A.6).

The change in welfare for agent $i \in \{1, ..., N, f\}$ for an arbitrary market structure and initial transfer policy is then:

$$dU_i = \nabla_{\vec{X}_J}^T U_i d\vec{\boldsymbol{X}}_J + u_{i,1}'\left(c_{i,1}\right) d\tau_i. \tag{A.61}$$

Let $V = \sum_{i=1}^N dU_i / u_{i,1}'\left(c_{i,1}\right) + m_f dU_f$. Substituting $\sum_{z\in\mathcal{Z}} \pi\left(z\right) M_i\left(z\right) \boldsymbol{e}_z^T \otimes \boldsymbol{a}_i = \Pi \boldsymbol{M}_i \otimes \boldsymbol{a}_i$, $\sum_{z\in\mathcal{Z}} \pi\left(z\right) M_f\left(z\right) \boldsymbol{e}_z^T \otimes \boldsymbol{a}_i = \Pi \boldsymbol{M}_f \otimes \boldsymbol{a}_i$, Eq. (A.57), and market clearing conditions in Eq. (1), we can express V using Eqs. (A.59), (A.60), na as:

$$V = \left(\sum_{i=1}^N \Pi\left(\boldsymbol{M}_i - \boldsymbol{M}_f\right) \otimes \boldsymbol{a}_i + \frac{\mu}{m_f} \nabla_{\vec{X}_J} \boldsymbol{a}_i X \Gamma X^T \boldsymbol{a}_i\right)^T d\vec{\boldsymbol{X}}_J, \tag{A.62}$$

where we recall that $X\Gamma X^T$ is the matrix of price impacts and $\nabla_{\vec{X}_J} \boldsymbol{a}_i$ is governed by the first-order conditions in Eq. (A.6).

It is immediate that any constrained-efficient asset span will satisfy the necessary condition $V = 0$, which to hold generically requires from Eq. (A.62) that:

$$\sum_{i=1}^N \Pi\left(\boldsymbol{M}_i - \boldsymbol{M}_f\right) \otimes \boldsymbol{a}_i + \frac{\mu}{m_f} \nabla_{\vec{X}_J} \boldsymbol{a}_i X \Gamma X^T \boldsymbol{a}_i = \boldsymbol{0}_{ZJ}, \tag{A.63}$$

for the $ZJ \times 1$ vector of zeroes, $\boldsymbol{0}_{ZJ}$ and asset allocations that satisfy the first-order conditions (A.6). If a payoff matrix X maximizes each agent's welfare, then Eq. (A.63) is satisfied, and welfare is maximized regardless of the planner's Pareto weights.

The first term in Eq. (A.63) reveals that the planner will raise asset payoffs in states in which, on average, strategic agents are buyers and have higher stochastic discount factors than the fringe, which improves risk sharing. The second term is how changing asset payoffs mitigates or exacerbates the distortions to strategic agents' portfolios because of price impact. If the strategic agent is a buyer in the asset, then welfare is improved by having the strategic agent buy more through changing the asset span (i.e., elements for which $\nabla_{\vec{X}_J} a_{i,j} > 0$).

Step 2: Construction of Equivalent Economy with Restricted Asset Span:

First, consider an arbitrary payoff vector X with K assets and prices p. Suppose we can construct asset positions for each type of agent, a^R_{ik}, for $i \in \{1, ..., N\}$ and $k \in \{1, ..., K\}$, that replicate the consumption allocations in complete markets without any trading restrictions:

$$X^T \boldsymbol{a}^R_i = \boldsymbol{a}^C_i \ \forall\, i \in \{1, ..., N\}, \tag{A.64}$$

where $\boldsymbol{a}^R_i$ and $\boldsymbol{a}^C_i$ are the vectors of asset positions for the restricted and complete markets, respectively.

Second, because all consumption allocations at date 2 are the same, this also applies to the fringe. Stochastic discount factors must therefore be the same under both market structures, $\boldsymbol{M}^R_f = \boldsymbol{M}^C_f$. Since the fringe's state prices are the Arrow prices with complete markets, $\boldsymbol{p}$, and there is no arbitrage, asset prices in the restricted asset span, $\boldsymbol{q}$, satisfy:

$$\boldsymbol{q} = X\boldsymbol{p} = X\boldsymbol{M}^C_f. \tag{A.65}$$

Third, we establish consumption allocations are the same at date 1, i.e., that the complete markets consumption allocations are marketed with the restricted asset span. Given Eqs. (A.64) and (A.65), however, this result is immediate since:

$$\boldsymbol{q}^T \boldsymbol{a}^I_i = \boldsymbol{p}^T X^T \boldsymbol{a}^I_i = \boldsymbol{p}^T \boldsymbol{a}^C_i \ \forall\, i \in \{1, ..., N\}.$$

Therefore, the complete markets equilibrium allocations are marketed.

Fourth, we establish that all agents' decision problems are satisfied. The fringe's Euler equations are trivially satisfied because asset prices are derived from its state prices. For strategic agent i, we can rewrite its first-order necessary condition for its optimal asset demand as:

$$\boldsymbol{q} + \frac{\mu}{m_f} \Gamma^R \boldsymbol{a}^R_i = X\Pi \boldsymbol{M}^R_i, \tag{A.66}$$

where Γ^R is the price impact matrix under the restricted asset span. By no arbitrage from Eq. (A.65), we recognize:

$$\Pi\Gamma^R = X\Pi\Gamma^C X^T, \tag{A.67}$$

where Γ^C is the diagonal complete markets price impact matrix with diagonal entries $\Gamma^C_{z,z} = -u''\left(c_{2,f}(z)\right)$. It is straightforward to see this matrix is symmetric and full rank.

Substituting these results into Eq. (A.66), and recognizing $\boldsymbol{M}_i^R = \boldsymbol{M}_i^C$ because equilibrium allocations are the same, the first-order necessary conditions then reduce to:

$$\boldsymbol{a}_i^R = \frac{m_f}{\mu}\left(X\Pi\Gamma^C X^T\right)^{-1} X\Pi\left(\boldsymbol{M}_i^C - \boldsymbol{M}_f^C\right). \tag{A.68}$$

Note that when $X = Id_Z$, so that markets are complete, then:

$$\boldsymbol{a}_i^C = \frac{m_f}{\mu}\left(\Gamma^C\right)^{-1}\left(\boldsymbol{M}_i^C - \boldsymbol{M}_f^C\right). \tag{A.69}$$

Substituting Eq. (A.69) into Eq. (A.68), we arrive at:

$$\boldsymbol{a}_i^R = \left(X\Pi\Gamma^C X^T\right)^{-1} X\Pi\Gamma^C \boldsymbol{a}_i^C. \tag{A.70}$$

Substituting Eq. (A.64) into Eq. (A.72), we arrive at the identity $\boldsymbol{a}_i^R = \boldsymbol{a}_i^R$.[6] Consequently, the first-order necessary conditions of strategic agents are also satisfied.

To see this set of payoff matrices is non-void, suppose we choose $J = N$ and the payoff matrix X to be such that $\boldsymbol{a}_i^R = \boldsymbol{e}_i$, i.e., each strategic agent type trades only one asset. Stacking all strategic agent's complete-markets asset positions into the $Z \times N$ matrix $A^C = \left[\boldsymbol{a}_1^C\ \boldsymbol{a}_2^C \ldots\ \boldsymbol{a}_N^C\right]$, we have from Eq. (A.72) that:

$$X\Pi\Gamma^C\left(X^T - A^C\right) = 0_{N\times N}, \tag{A.71}$$

where $0_{N\times N}$ is the $N \times N$ matrix of zeroes, from which follows that one solution is

$$X = \left(A^C\right)^T = \tilde{M}\left(\Gamma^C\right)^{-1}, \tag{A.72}$$

[6] We can derive Eq. (A.72) from Eq. (A.64) by using the price impact matrix, $\left(\Pi\Gamma^C\right)^{-1}$, as a weighting matrix to project X onto $\boldsymbol{a}_i^C$.

where $\tilde{M}$ is the $N \times Z$ matrix with column vectors that are the difference between each strategic agent's and the competitive fringe's SDFs, $\tilde{\boldsymbol{M}}_i = \left(\frac{\mu}{m_f}\right)^{-1}\left(\boldsymbol{M}_i - \boldsymbol{M}_f\right)$. X is therefore the payoff matrix whose payoffs are precisely the asset positions the strategic agents would have taken in complete markets with Arrow assets. Because $N < Z$, this is a restricted asset span.

Step 3: Complete Markets:

Finally, we establish that imposing no portfolio restrictions (i.e., complete markets) is constrained inefficient in the presence of market power. By contrast, if there were perfect competition and complete markets, then $\boldsymbol{M}_i^T = \boldsymbol{M}_f^T$ and $\frac{\mu}{m_f} = 0$, and the two terms in Eq. (A.62) are zero. Therefore $V = 0$, and laissez faire complete markets with perfect competition is constrained-efficient.

Consider now the bespoke $N \times Z$ payoff matrix X that replicates the complete markets equilibrium from Step 1 in which $J = N$. From Step 2, we can replicate the market equilibrium without portfolio restrictions with a $N \times Z$ payoff matrix $\tilde{M}\left(\Gamma^C\right)^{-1}$ that has a restricted asset span.

Conditional on this payoff matrix, we have that $\boldsymbol{a}_i = \boldsymbol{e}_i$ and the first-order conditions of all strategic agents are satisfied. If complete markets is optimal, the vector Eq. (A.62) is satisfied with equality at $X = \tilde{M}\left(\Gamma^C\right)^{-1}$:

$$V = \sum_{i=1}^{N} \Pi\left(\boldsymbol{M}_i - \boldsymbol{M}_f\right) \otimes \boldsymbol{e}_i + \frac{\mu}{m_f} \nabla_{\vec{\mathbf{X}}_J} \boldsymbol{a}_i|_{\boldsymbol{a}_i = \boldsymbol{e}_i} \tilde{M}\left(\Gamma^C\right)^{-1} \tilde{M}^T \boldsymbol{e}_i. \tag{A.73}$$

However, the first term in Eq. (A.73) is nonzero from our analysis without portfolio regulation because marginal valuations across agents are misaligned state-by-state, and the sum of the two terms is generically nonzero. Consequently, the market equilibrium without portfolio restrictions is constrained inefficient. ■

Proof of Proposition 6

Step 1: Pure Risk Sharing Case:

First, consider the case of complete markets without any restrictions on what agents can trade.

In this case, the asset price is the same for both the high and low states by symmetry, and we designate it p_{cmp}. Let $a_B > 0$ be the position of an agent when it buys the asset and $a_S < 0$ be the position when it sells. Imposing market clearing, welfare for a strategic agent is:

$$U_i(I_2, \mu) = u\left(\bar{y} - p_{cmp}\left(a_B + a_S\right)\right) + \frac{1}{2}u\left(\bar{y} - \Delta + a_B\right) + \frac{1}{2}u\left(\bar{y} + \Delta + a_S\right),$$

while for the fringe, it is:

$$U_f(I_2, \mu) = 2p_{cmp}\left(a_B + a_S\right) + m_f u\left(\bar{y} - \frac{a_B + a_S}{m_f}\right),$$

where:

$$p_{cmp} = \frac{1}{2}u'\left(\bar{y} - \frac{a_B + a_S}{m_f}\right).$$

Recall the competitive fringe has the same date 2 preferences as the strategic agents. As a result of the convexity of marginal utility with symmetric preferences, sellers restrict their asset positions more than buyers. As such, $a_B + a_S > 0$.

By contrast, with the aggregate swap, both agents take a symmetric position of a when buying and $-a$ when selling, and internally clear the swap market without the fringe. Let the price of an Arrow asset when only a swap is traded be p_{in}. Welfare for a strategic agent is then:

$$U_i([1 -1], \mu) = u\left(\bar{y}\right) + \frac{1}{2}u\left(\bar{y} - \Delta + a\right) + \frac{1}{2}u\left(\bar{y} + \Delta - a\right),$$

while for the fringe is now:

$$U_f([1 -1], \mu) = m_f u\left(\bar{y}\right).$$

Since the competitive fringe has concave utility, prices are increasing in the net demand of strategic agents, and it follows:

$$p_{cmp} = \frac{1}{2}u'\left(\bar{y} - \frac{a_B + a_S}{m_f}\right) > \frac{1}{2}u'\left(\bar{y}\right) = p_{inc}.$$

As such, $|a_S| < a_B < a$ because insurance is more expensive in complete markets.

We first show that the utility of the competitive fringe is higher when only the swap is traded. This is because, to first-order, there is no gain with complete markets over the swap because the competitive fringe prices both Arrow securities based on its marginal utility. However, to second-order, the fringe has lower consumption at date 2 in both states. Formally, approximating $u\left(\bar{y}-\frac{a_B+a_S}{m_f}\right)$ around $\bar{y}$ to second-order:

$$
\begin{aligned}
& U_f([1-1],\mu)-U_f(I_2,\mu) \\
= \; & m_f u(\bar{y}) - m_f u\left(\bar{y}-\frac{a_B+a_S}{m_f}\right) - 2p_{cmp}(a_B+a_S) \\
\approx \; & m_f u(\bar{y}) - m_f u(\bar{y}) + m_f u'(\bar{y})\frac{a_B+a_S}{m_f} - 2p_{cmp}(a_B+a_S) - \frac{1}{2}m_f u''(\bar{y})\left(\frac{a_B+a_S}{m_f}\right)^2 \\
= \; & -\frac{1}{2}m_f u''(\bar{y})\left(\frac{a_B+a_S}{m_f}\right)^2 \\
> \; & 0
\end{aligned}
$$

because $p_{cmp}=\frac{1}{2}u'\left(\bar{y}-\frac{a_B+a_S}{m_f}\right)$ and the fringe has concave utility.

We next show that the utility of both strategic agents is higher with the swap. Because of symmetry, we need only show this for one strategic agent. Comparing welfare under complete markets and the swap, and taking a first-order approximation of $u\left(\bar{y}-p_{cmp}(a_B+a_S)\right)$ around $\bar{y}$, we have that:

$$
\begin{aligned}
& U_i([1-1],\mu)-U_i(I_2,\mu) && \text{(A.74)} \\
= \; & u(\bar{y}) - u\left(\bar{y}-p_{cmp}(a_B+a_S)\right) + \frac{1}{2}u(\bar{y}-\Delta+a) + \frac{1}{2}u(\bar{y}+\Delta-a) \\
& -\frac{1}{2}u(\bar{y}-\Delta+a_B) - \frac{1}{2}u(\bar{y}+\Delta+a_S) \\
\approx \; & \frac{1}{2}\left[u(\bar{y}-\Delta+a) + u(\bar{y}+\Delta-a) - \left(u(\bar{y}-\Delta+a_B) + u(\bar{y}+\Delta+a_S)\right)\right] \\
& +u'(\bar{y})\,p_{cmp}(a_B+a_S). && \text{(A.75)}
\end{aligned}
$$

Note that $p_{cmp}(a_B+a_S)>0$ (i.e., both strategic agents must buy some insurance from the fringe), while no money initially changes hands among strategic agents with the swap.

Let $\Delta_a=\Delta-a\geq 0$, $\delta_B=a_B-a\leq 0$, and $\delta_S=a_S+a\geq 0$. Because $|a_S|<a_B$, we have that

$\delta_B + \delta_S = a_B + a_S > 0$. Since $u(\cdot)$ is strictly concave:

$$\begin{aligned} u(\bar{y} - \Delta_a) + u(\bar{y} + \Delta_a) + m_f u(\bar{y}) \quad \geq \quad & u(\bar{y} - \Delta_a + \delta_B) + u(\bar{y} + \Delta_a + \delta_S) \\ & + m_f u\left(\bar{y} - \frac{\delta_B + \delta_S}{m_f}\right), \end{aligned} \tag{A.76}$$

with equality when $\mu = 0$. This is because the arguments in the left-hand and right-hand sides of Eq. (A.76) are just a reshuffling of the allocations:

$$\bar{y} - \Delta_a + \bar{y} + \Delta_a + m_f\bar{y} = \bar{y} - \Delta_a + \delta_B + \bar{y} + \Delta_a + \delta_S + m_f\bar{y} - \delta_B - \delta_S,$$

in which the fringe consumes less and each strategic agent has higher volatility in their consumption.

It then follows from inequalities in Eqs. (A.75) and (A.76), taking another first-order approximation, that:

$$\begin{aligned} U_i([1-1], \mu) - U_i(I_2, \mu) \quad & > \quad \frac{1}{2}m_f\left(u\left(\bar{y} - \frac{a_B + a_S}{m_f}\right) - u(\bar{y})\right) + u'(\bar{y})\, p_{cmp}(a_B + a_S) \\ & \approx \quad \left(u'(\bar{y})\, p_{cmp} - \frac{1}{2}u'(\bar{y})\right)(a_B + a_S) \\ & = \quad \left(u'(\bar{y})\, p_{cmp} - p_{inc}\right)(a_B + a_S) \\ & > \quad \left(u'(\bar{y}) - 1\right) p_{cmp}(a_B + a_S), \end{aligned} \tag{A.77}$$

because $p_{cmp} > p_{inc}$. Consequently, if $u'(\bar{y}) \geq 1$, it follows from inequality (A.77) that:

$$U_i([1-1], \mu) \geq U_i(I_2, \mu), \tag{A.78}$$

and restricting all agents to trade the swap when $\mu > 0$ represents a Pareto improvement over complete markets.

Step 2: Pure Aggregate Risk Case:

We first show that with complete markets, the less risk-averse agent 2 supplies too little of the

Arrow asset claim to the risk-averse agent 1 in the low aggregate state l compared to the competitive equilibrium. From Eq. (A.6), the first-order necessary condition for agent i with complete markets in the low state l is:

$$\frac{1}{2}\frac{u'_{i,2}\left(y_l + a_{i,l}\right)}{u'_{i,1}\left(\bar{y} - p_l a_{i,l} - p_h a_{i,h}\right)} = p_l + \frac{\mu}{m_f} p'_l a_{i,l}, \tag{A.79}$$

and in the high state is:

$$\frac{1}{2}\frac{u'_{i,2}\left(y_h + a_{i,h}\right)}{u'_{i,1}\left(\bar{y} - p_l a_{i,l} - p_h a_{i,h}\right)} = p_h + \frac{\mu}{m_f} p'_h a_{i,h}, \tag{A.80}$$

where $p_h = \frac{1}{2}u'_{1,2}\left(y_h + a_{f,h}\right)$ and $p'_h = \frac{1}{2}\frac{\mu}{m_f}u''_{1,2}\left(y_h + a_{f,h}\right)$, and similarly with the low state claim price and price impact.

Consider the limit where $m_f \to 0$, so that $a_{1,z} + a_{2,z} \to 0$ for $z \in \{l, h\}$, and $\frac{\mu}{m_f} \to \kappa > 0$. Because agent 1 is less risk-averse than agent 2, agent 1 will insure agent 2 by selling the low-state claim and buying the high state claim (i.e., $a_{2,l} < 0 < a_{2,h}$). By contrast, agent 1 buys the low-state claim and sells the high state claim (i.e., $a_{1,h} < 0 < a_{1,l}$).

Let the asset positions in the high-state claim be $a_{1,h} = -a_{2,h} = -a_h$, and in the low-state claim be $a_{1,l} = -a_{2,l} = a_l$. It then follows from Eqs. (A.79) and (A.80) that:

$$\frac{u'_{2,2}\left(y_l + a_l\right)}{u'_{2,2}\left(y_h - a_h\right)} < \frac{u'_{1,2}\left(y_l - a_l\right)}{u'_{1,2}\left(y_h + a_h\right)}. \tag{A.81}$$

In the competitive equilibrium (in which $\kappa = 0$), condition (A.81) instead holds with equality. Let the perfect competition asset positions be a_l^p and a_h^p for the low- and high-state claims, respectively. It then follows that with market power, agent 2 supplies too little of the low-state claim (i.e. $a_l < a_l^p$), and buys too little of the high-state claim (i.e., $a_h < a_h^p$), and this distorts the marginal rates of transformation across states for both agents. That the strategic agents trade in opposite directions (i.e., strategic agent 1 buys low-state claims and sells high-state claims), motivates us to consider a swap as the asset that is traded in the restricted span.

We next show that restricting trade to a swap reduces strategic agents' market power. Let the payoff vector of the swap be $[x_h\ x_l]$ and the price of this swap be p^*. Without loss, we designate $x_l > 0$ and normalize $x_h = -1 < 0$. Because $m_f \to 0$, the market for the swap clears internally between the two strategic agents. Let the position in the swap of agent 1 be a^* and that of agent 2 be $-a^*$.

The first-order necessary condition for strategic agent 1's optimal position in the swap is:

$$x_h \frac{1}{2} \frac{u'_{1,2}\left(y_h + x_h a^*\right)}{u'_{1,1}\left(\bar{y} - p^* a^*\right)} + x_l \frac{1}{2} \frac{u'_{1,2}\left(y_l + x_l a^*\right)}{u'_{1,1}\left(\bar{y} - p^* a^*\right)} = p^* + \kappa p^{*\prime} a^*, \tag{A.82}$$

and for agent 2:

$$x_h \frac{1}{2} \frac{u'_{2,2}\left(y_h - x_h a^*\right)}{u'_{2,1}\left(\bar{y} + p^* a^*\right)} + x_l \frac{1}{2} \frac{u'_{2,2}\left(y_l - x_l a^*\right)}{u'_{2,1}\left(\bar{y} + p^* a^*\right)} = p^* - \kappa p^{*\prime} a^*. \tag{A.83}$$

Without loss, we can rewrite the asset positions in the complete markets economy as $a_l = x_l a^c - \delta$ and $a_h = x_h a^c$, and define $p^c = x_l p_l + x_h p_h$ and $p^{c\prime} = x_l^2 p'_l + x_h^2 p'_h$ by no arbitrage based on the fringe's first-order conditions. Multiplying the first-order conditions for the low- and high-state claims by x_l and x_h, Eqs. (A.79) and (A.79) respectively, and adding them together, we find for agent 1:

$$x_l \frac{1}{2} \frac{u'_{1,2}\left(y_l + x_l a^c - \delta\right)}{u'_{1,1}\left(\bar{y} - p^c a^c + p_l \delta\right)} + x_h \frac{1}{2} \frac{u'_{1,2}\left(y_h + x_h a^c\right)}{u'_{1,1}\left(\bar{y} - p^c a^c + p_l \delta\right)} = p^c + \kappa p^{c\prime} a^c - \kappa p'_l x_l \delta. \tag{A.84}$$

and for agent 2:

$$x_l \frac{1}{2} \frac{u'_{2,2}\left(y_l - x_l a^c + \delta\right)}{u'_{2,1}\left(\bar{y} + p^c a^c - p_l \delta\right)} + x_h \frac{1}{2} \frac{u'_{2,2}\left(y_h - x_h a^c\right)}{u'_{2,1}\left(\bar{y} + p^c a^c - p_l \delta\right)} = p^c - \kappa p^{c\prime} a^c + \kappa p'_l x_l \delta. \tag{A.85}$$

If $\delta = 0$, then Eq. (A.84) becomes identical to Eq. (A.82), and similarly with Eqs. (A.85) and (A.83). In this case, complete markets and the restricted asset span achieve the same allocation, and therefore the same welfare. Because of market power, one of the two Arrow assets is undersupplied relative to the other. In what follows, we assume that the low-state claim is undersupplied by the less risk-averse agent, i.e., $\delta > 0$. The complementary case where the high-state claim is undersupplied follows an analogous argument in which we define δ to be the undersupply of the high-state claim.

Consider the special case in which the swap payoff is one-for-one across states (i.e., $x_l = 1$), so that the swap bundles the two Arrow securities into one portfolio. From the first-order conditions (A.79) and (A.80) in complete markets, strategic agents distort in opposite directions across the two markets, lowering the price in the market in which they buy and raising it in the one in which they sell. By bundling the two Arrow securities into a swap with payoff $[-1\ 1]$, a strategic agent's long and short positions are forced to be the same, and that agent can now only distort in one direction (i.e., buy less of the swap or sell less of it). Because this applies to both strategic agents, their ability to distort their asset positions for price concessions (i.e., exert market power) is attenuated.

We define the state prices for strategic agent 1 in complete markets referencing the high and low states when $[x_h\ \ x_l] = [-1\ 1]$ to be:

$$M_{1,h}\left(a^c,\delta\right) = \frac{1}{2}\frac{u'_{1,2}\left(y_h - a^c\right)}{u'_{1,1}\left(\bar{y} - p^c a^c + p_l\delta\right)},\ M_{1,l}\left(a^c,\delta\right) = \frac{1}{2}\frac{u'_{1,2}\left(y_l + a^c - \delta\right)}{u'_{1,1}\left(\bar{y} - p^c a^c + p_l\delta\right)}, \tag{A.86}$$

and for agent 2 to be:

$$M_{2,h}\left(a^c,\delta\right) = \frac{1}{2}\frac{u'_{2,2}\left(y_h + a^c\right)}{u'_{2,1}\left(\bar{y} + p^c a^c - p_l\delta\right)},\ M_{2,l}\left(a^c,\delta\right) = \frac{1}{2}\frac{u'_{2,2}\left(y_l - a^c + \delta\right)}{u'_{2,1}\left(\bar{y} + p^c a^c - p_l\delta\right)}. \tag{A.87}$$

From Eqs. (A.79), (A.84), and (A.85), we have that:

$$p_z = \frac{1}{2}\left(M_{1,z}\left(a^c,\delta\right) + M_{2,z}\left(a^c,\delta\right)\right)\ z \in \{l,h\}, \tag{A.88}$$

$$\kappa p'_z\left(a^c - \delta\right) = \frac{1}{2}\left(M_{1,z}\left(a^c,\delta\right) - M_{2,z}\left(a^c,\delta\right)\right), \tag{A.89}$$

$$p^c = p_l - p_h. \tag{A.90}$$

What remains to be shown is that each strategic agent's welfare is higher when $\delta = 0$ versus $\delta > 0$ in complete markets. Let $\Delta_l\left(a^c,\delta\right) = M_{1,l}\left(a^c,\delta\right) - M_{2,l}\left(a^c,\delta\right) > 0$ because agent 2 sells claims to the low state to agent 1 (who therefore must have a higher state price). Taking the derivative of welfare with respect to δ, substituting with Eqs. (A.88) and (A.89), and recognizing $a^c > 0$ and $\delta < a^c$, we find:[7]

$$\begin{aligned}\frac{\partial U_1\left(I_2,\mu\right)}{\partial\delta} &= u'_{1,1}\left(c_{1,1}\right)\left[p_l - M_{1,l} - \kappa p'_l\left(a^c - \delta\right)\right] \\ &= -u'_{1,1}\left(c_{1,1}\right)\Delta_l\left(a^c,\delta\right) \\ &< 0,\end{aligned} \tag{A.91}$$

[7]Because we are performing a comparative static of how welfare varies with δ, and not solving for the equilibrium choice of δ, we do not have to treat the strategic agent's first-order conditions as constraints.

while

$$\frac{\partial U_2\left(I_2,\mu\right)}{\partial \delta}=u_{2,1}'\left(c_{2,1}\right)\left[-p_l+M_{2,l}+\kappa p_l'\left(a^c-\delta\right)\right]=0. \tag{A.92}$$

It is immediate from Eqs. (A.91) and (A.92) that agent 1 is strictly better off when $\delta = 0$, while agent 2 is indifferent (i.e., no worse off). The strategic agent who is rationed more in complete markets, agent 1, is strictly better off because it shares more risk with the less risk-averse agent 2. Agent 2, in turn, is no worse off because the price improvement per unit traded just offsets the loss in volume traded of the more-rationed Arrow asset when there are only two strategic types and an arbitrarily small competitive fringe.

Since $\delta > 0$ with complete markets and $\delta = 0$ corresponds to the restricted asset span with the swap ($a^c = a^*$), welfare is (weakly) higher for both agents with the swap than with complete markets, and represents a Pareto improvement. Because the planner has the freedom to choose a x_l other than 1, this is a lower bound to how restricting the asset span improves risk sharing. Since the complete markets allocation with perfect competition is feasible if the planner chooses $x_l = \frac{a_l^p}{a_h^p}$, the first-best is feasible with both complete markets and a restricted asset span. ■

Appendix B: Counterfactual Span in Canonical Settings

Example 2

Since strategic agents are ex ante symmetric, there exist two distinct SDF realizations, m^l and $m^h > m^l$, such that $m_i(i) = m^l$ and $m_i(-i) = m^h$. Every agent assigns a low (high) marginal value of consumption to the state with high (low) private returns. Because there are two states, the equivalent restricted asset span has a single asset. The construction from Proposition 4 shows the dividend-yield vector $[\tilde{x}_1\ \tilde{x}_2]$ satisfies:

$$\frac{1}{2}\begin{bmatrix} m^h & m^l \\ m^l & m^h \end{bmatrix}\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Solving this equation gives:

$$\tilde{x}_1=\tilde{x}_2=\frac{1}{\frac{1}{2}m^h+\frac{1}{2}m^l}=r_f^*.$$

This asset is a risk-free bond. With two types of strategic agents and diversifiable risk, consumption allocations are such that an outside observer would infer that only a risk-free bond can be traded. This is because a risk-free bond only allows agents to shift resources across time, whereas there are gains from trade in shifting resources across states. Because these risks are not fully shared, the outside observer interprets this as prima facie evidence this risk cannot be traded.

Alternatively, we can characterize the equivalent restricted asset span using the market-implied SDF m^* from Proposition 4. From Hansen and Jagannathan (1991), this SDF is unique and is the minimum-variance SDF among all admissible ones. Because only a risk-free asset is traded, we find that m^* is $\frac{1}{r_f^*}$ in both states (i.e., the representative investor owns a risk-less portfolio).

We can further identify the "rationed asset" whose return is orthogonal to a risk-free bond that, if it were available under perfect competition, would allow agents to realize the residual gains from trade. In this example, it is a simple swap.

Corollary 1 (Missing Asset with Pure Idiosyncratic Risk) *A security with payoff* $[1\ x]$ *is orthogonal to the risk-free bond if and only if:*

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & x \end{bmatrix}' = 0.$$

This requires $x = -1$. *The "missing asset" that would have allowed for perfect insurance is therefore an idiosyncratic risk swap.*

Example 3

With two aggregate states, the implied restricted asset span from Proposition 4 has one asset with dividend-yield vector $[\tilde{x}_l, \tilde{x}_h]$ satisfying:

$$\frac{1}{2}\begin{bmatrix} m_{rn} & m_{rn} \\ m^h & m^l \end{bmatrix} \begin{bmatrix} \tilde{x}_h \\ \tilde{x}_l \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Solving this equation gives

$$\tilde{x}_h = \frac{m^l - m_{rn}}{\frac{1}{2}m_{rn}\left(m^l - m^h\right)} \qquad \text{and} \qquad \tilde{x}_l = \frac{m_{rn} - m^h}{\frac{1}{2}m_{rn}\left(m^l - m^h\right)}.$$

The asset carries exposure to aggregate risk since $\tilde{x}_h > \tilde{x}_l$ by Jensen's inequality. Therefore, it is a levered market index. Because the risk-neutral agent rations insurance against aggregate shocks, the outside econometrician would conclude there are no assets that offer sufficient protection against aggregate risk. This has a natural interpretation: the risk-averse agent can only trade a market index with a degree of risk exposure that provides no additional insurance opportunities.

Alternatively, we can again characterize the equivalent restricted asset span using the market-implied SDF m^* from Proposition 4. Because only a leveraged market index is traded, if we choose a mean for $m^* = \bar{m}$, we find that $m^*(h) = 2\frac{1-\bar{m}x_l}{\tilde{x}_h - \tilde{x}_l}$ in the high state, and $m^*(l) = 2\frac{\bar{m}\tilde{x}_h - 1}{\tilde{x}_h - \tilde{x}_l}$ in the low state (i.e., the representative investor has a portfolio whose return is positively correlated with the aggregate state). We must specify a mean, $\bar{m}$, for the market-implied SDF because a risk-free asset is not traded, and No arbitrage requires that $\bar{m} \in \left[\frac{1}{x_h}, \frac{1}{x_l}\right]$.

Similar to the case of pure diversifiable risk, we can find the "rationed" asset by searching for the asset whose payoff is orthogonal to the levered market index. Corollary 2 shows that the missing asset is similar to a put on the stock market.

Corollary 2 (Missing Asset with Pure Aggregate Risk) *A security with payoff* $[1\ x]$ *is orthogonal to the levered equity portfolio if and only if:*

$$\begin{bmatrix} 1 & \frac{m_{rn} - m^h}{m^l - m_{rn}} \end{bmatrix} \begin{bmatrix} 1 & x \end{bmatrix}' = 0.$$

This requires $x = -\frac{m^l - m_{rn}}{m_{rn} - m^h} < 0$. *We can rescale this vector as* $\begin{bmatrix} m_{rn} - m^h, & m_{rn} - m^l \end{bmatrix}$*, which is an aggregate risk swap that pays off in the high state and loses money in the low state.*

Appendix C: Regulation of Credit Default Swaps

Mapping the model to data

In this Appendix, we derive the endowment processes of the two strategic agents in our credit default swap application in Section 5. We then provide the model analogues of our empirical asset pricing moments.

Given the aggregate endowment is Y at date 0, and at date 1 is Y in state 1 with probability

$1-\pi$ and δY in state 2 with probability π, we search for income shares for the two strategic agents such that in the competitive equilibrium type 1 agents buy 1 unit of the risky bond and ρ units of the CDS contract. The bond and CDS pay off B and 0 in state 1 and 0 and B in state 2, respectively.

As is standard, optimal risk sharing among the two strategic agents implies from their first-order conditions that the price of the Arrow asset in state z satisfies:

$$p(z) = \Pr(z)\left(\frac{c_{11}(z)}{c_{10}}\right)^{-1} = \Pr(z)\left(\frac{c_{21}(z)}{c_{20}}\right)^{-1}, \tag{A.93}$$

from which we can conjecture that agents 1 and 2 consume fixed fractions, ω_1 and $1-\omega_1$, of the aggregate endowment at each date and state. It is then immediate that:

$$p(1) = 1-\pi, \tag{A.94}$$

$$p(2) = \pi\delta^{-1}. \tag{A.95}$$

Suppose a type 1 agent receives fractions α_0 and α_1 of the initial and date 1 endowments in both states, respectively. The present value of his initial wealth satisfies:

$$\alpha_0 Y + \alpha_1 Y = 2\omega Y. \tag{A.96}$$

Finally, for a type 1 agent to be able to afford buying 1 unit of the risky bond and ρ of the CDS, we require by his budget constraint that:

$$\alpha_0 Y = c_{10} + p(1)B + p(2)\rho B = \omega Y + \left(1-\pi+\pi\delta^{-1}\rho\right)B. \tag{A.97}$$

If we set $\omega = \frac{1}{2}$, then Eqs. (A.96) and (A.97) imply that:

$$\alpha_0 = \frac{1}{2} + \left(1-\pi+\pi\delta^{-1}\rho\right)\frac{B}{Y}, \tag{A.98}$$

$$\alpha_1 = \frac{1}{2} - \frac{1-\pi+\pi\delta^{-1}\rho}{1-\pi+\pi\delta^{-1}\delta}\frac{B}{Y}, \tag{A.99}$$

These income shares implement the competitive equilibrium in which each agent consumes half of the total endowment when trading the risky bond and the CDS contract.

To map our model to the data on sovereign bonds and CDS, we define the bid-ask spread in an asset market to be the difference in marginal valuations between the buyer and seller per trade. To be consistent with Sambalaibat (2023), we define the bid-ask spread of the sovereign bond as a fraction of par and that of the CDS as that of the mid-price, which in our setting is the CDS price because it is equal to half the sum of the marginal valuations of the buyer and seller. For the sovereign bond and CDS contract, these bid-ask spread, b^{bond} and b^{CDS}, are:

$$\begin{aligned} b^{bond} &= (1-\pi)\left(\left(\frac{c_{10}}{c_{11}(1)}\right)-\left(\frac{c_{20}}{c_{21}(1)}\right)\right)\frac{1}{2|a_1|}, && \text{(A.100)} \\ b^{CDS} &= \pi\frac{B}{p_{CDS}}\left(\left(\frac{c_{10}}{c_{11}(2)}\right)-\left(\frac{c_{20}}{c_{21}(2)}\right)\right)\frac{1}{2|a_2|}. && \text{(A.101)} \end{aligned}$$

Let $p_{AD}(z)$ be the price of the Arrow asset referencing state z in the presence of market power. We can define the sovereign bond spread, s^{bond}, as the difference between the bond's yield

$$y^{bond} = \frac{B}{p_{AD}(1)\,B} = \frac{1}{p_{AD}(1)}, \tag{A.102}$$

and the risk-free rate in our model:

$$s^{bond} = \frac{B}{p_{AD}(1)\,B} - \frac{1}{p_{AD}(1)+p_{AD}(2)} = \frac{1}{p_{AD}(1)} - \frac{1}{p_{AD}(1)+p_{AD}(2)}. \tag{A.103}$$

Finally, we define the CDS-bond basis, which is the zero-volatility (z-)spread minus the bond spread. The z-spread, z, is defined as the spread that solves:

$$p_{CDS} = \frac{\pi B}{r+z}, \tag{A.104}$$

where r is the risk-free rate, from which follows:

$$z = \frac{\pi B}{p_{CDS}} - r. \tag{A.105}$$

The CDS-bond basis is then given by:

$$\text{CDS-bond Basis} = z - s^{bond} = \frac{\pi B}{p_{CDS}} - \frac{B}{p_{AD}(1)\,B}. \tag{A.106}$$

We multiply the bid-ask and bond spreads and the CDS-Basis by 100 to convert to percentages.

Mismatch between regulation and gains from trade

Our calibration recovers the unobserved endowments of the two strategic agent types under the assumption that regulators' choice of the low state payoff of the covered bond, ρB, is well-suited to the underlying gains from trade. Under this assumption, it was constrained efficient. However, in practice, a regulator may have limited information about the trading needs of market participants, and may choose a suboptimal value of ρ that can harm rather than improve market outcomes.

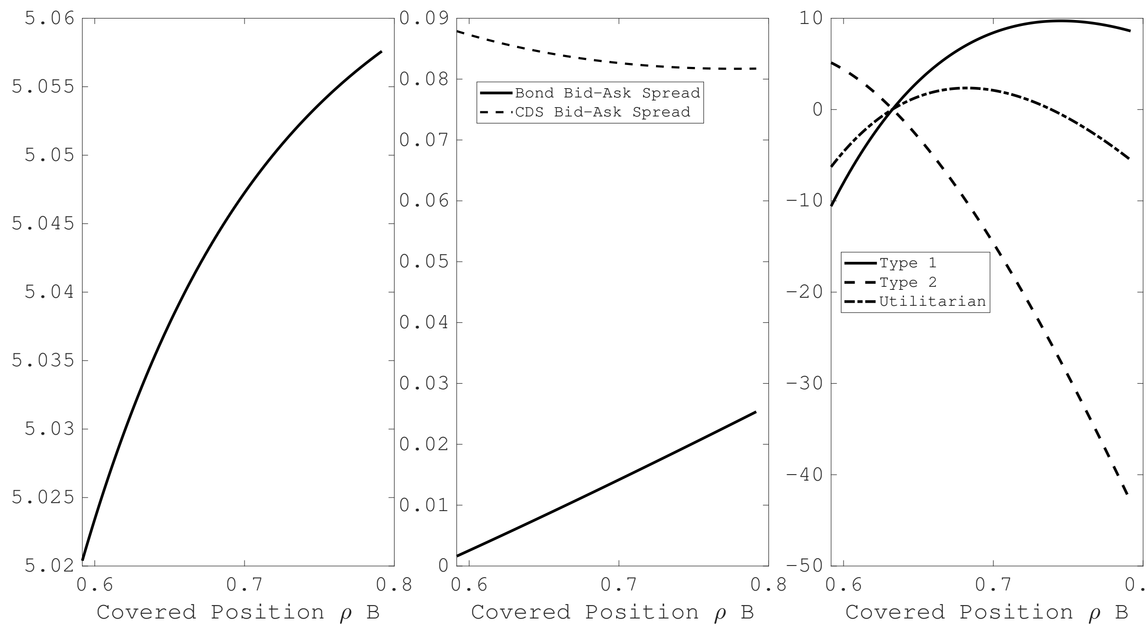

Figure 1: Bond yield (left panel), bond and CDS bid-ask spreads (middle panel), and welfare (right panel) for different values of covered bond positions ρB for the parameters given in Table 1 . The baseline calibration has $\rho B = 0.6917$.

Figure 1 shows equilibrium outcomes for different values of ρ, keeping the underlying endowments fixed. The calibrated economy has $\rho B = 0.6917$, so deviations from this value create a mismatch between regulation and gains from trade. Bond yields and bid-ask spreads are increasing in ρB, while the CDS bid-ask spread is decreasing. While the welfare of type 2 agents is decreasing in ρB, that of type 1 agents is hump-shaped. Hence utilitarian welfare (the sum of both agents' welfares) can be negative if ρB is too high (or too low, if we extend the axes further). Hence for certain choices of ρ the regulation is not constrained efficient.