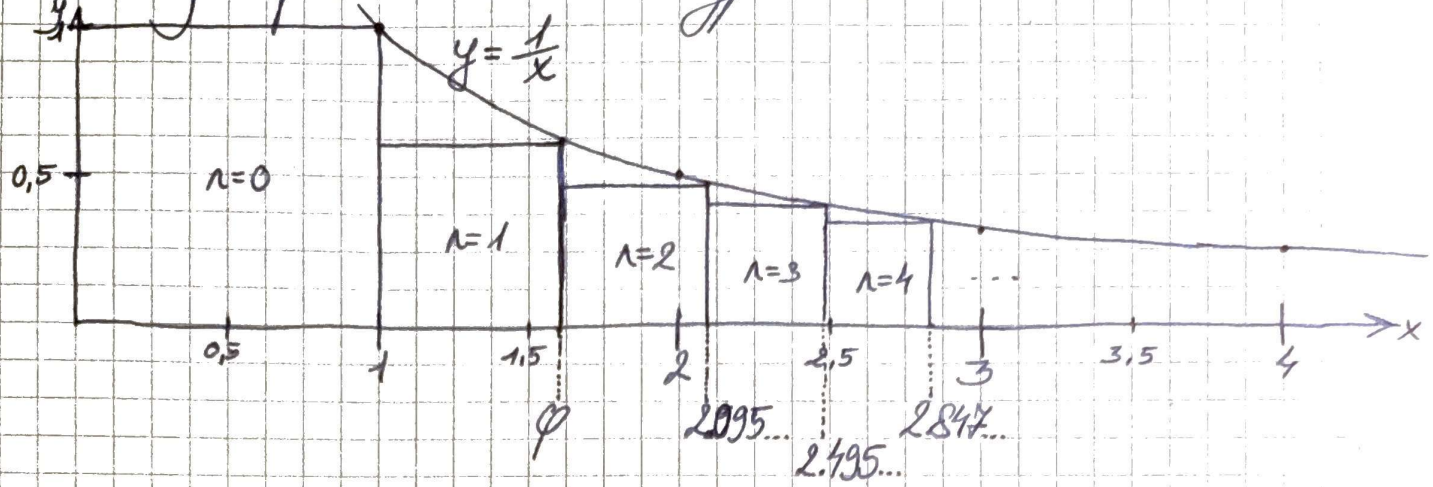
Drawing squares under a hyperbola:



Let n be the index of the square.

If $n=0$ then $y=x$ $\quad x=\frac{1}{x} \Rightarrow x=1$

$n=1$ $\quad y=x-1$ $\quad x-1=\frac{1}{x} \mid \cdot x\neq 0$

$x(x-1)=1$

$x^2-x-1=0$

$\varphi = x = \frac{1+\sqrt{5}}{2}$; $x=\frac{1-\sqrt{5}}{2}<0$

$n=2$ $\quad y=x-\varphi$ $\quad x-\varphi=\frac{1}{x} \mid \cdot x\neq 0$

$x(x-\varphi)=1$

$x^2-\varphi x-1=0$

$x=\frac{\varphi+\sqrt{\varphi^2+4}}{2}$ $\quad x=\frac{\varphi-\sqrt{\varphi^2+4}}{2}<0$

⋮

$n=n$ general case $\quad y=x-x_{n-1}$ $\quad x-x_{n-1}=\frac{1}{x} \mid \cdot x\neq 0$

$x(x-x_{n-1})=1$

$x^2-x_{n-1}x-1=0$

$x=\frac{x_{n-1}+\sqrt{x_{n-1}^2+4}}{2}$

general-case solution