STAT 886 HW 5

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Text: Problem 5.8. Hint: You may use the fact that $p(\theta|y) = p(\theta)p(y|\theta)/p(y)$. For the example, assume $\lambda_1 = 0.3$ and plot the prior and posterior densities

```
# set priors
priorTau0 <- (0.5^2) # prior variance for 1 and 2
precision0 <- 1/priorTau0
priorMu1 <- 1
priorMu2 <- -1
precision1 <- 1 # known precision
variance <- 1 # known variance
n <- 10 # sample size
dataMean <- -0.25 # observed mean

lambda1 <- 0.3
lambda2 <- 1 - lambda1

# calculate posterior of the mean given data: \theta | y
(w <- (n/variance) / (precision0 + n/variance))
```

```
## [1] 0.7142857
```

```
(postMu1 <- w * dataMean + (1 - w) * priorMu1) # mu_1
```

```
## [1] 0.1071429
```

```
(postMu2 <- w * dataMean + (1 - w) * priorMu2) # mu_2
```

```
## [1] -0.4642857
```

```
(postTau <- (priorTau0 * variance / n) / (priorTau0 + variance/n)) # tau_1^2 = tau_2^2
```

```
## [1] 0.07142857
```

```
# calculate marginal probability of observed data
(marg1 <- dnorm(-0.25, mean = priorMu1, sd = sqrt(priorTau0 + variance/n)))
```

```
## [1] 0.07235503
```

```r
(marg2 <- dnorm(-0.25, mean = priorMu2, sd = sqrt(priorTau0 + variance/n)))
```

```
## [1] 0.3019183
```

```r
(marg <- lambda1 * marg1 + lambda2 * marg2)
```

```
## [1] 0.2330493
```

```r
# updated weights
(lambda1_new <- lambda1 * marg1 / marg)
```

```
## [1] 0.09314127
```

```r
(lambda2_new <- lambda2 * marg2 / marg)
```

```
## [1] 0.9068587
```

```r
# plot posteriro and prior
curve(lambda1_new * dnorm(x, mean = postMu1, sd = postTau) + lambda2_new * dnorm(x, mean = postMu2, sd
      xlim = c(-3,3), col = "red", ylab = "probability density", main = "Prior and posterior densities"
curve(lambda1 * dnorm(x, mean = priorMu1, sd = sqrt(priorTau0)) + lambda2 * dnorm(x, mean = priorMu2, s
      xlim = c(-3,3), add = TRUE)
legend(x = 1.5, y = 4.75, legend = c("posterior", "prior"), col = c("red", "black"), lty = c(1,1))
```