ABSTRACT

A **revolving credit** line allows borrowers to withdraw funds multiple times up to a certain credit limit, paying interest only on the withdrawn amount rather than receiving a lump sum as with traditional loans. This **flexibility** aids businesses and individuals in managing cash flow more efficiently, addressing unpredictable short-term financial needs. To issue a reasonable revolving credit contract, banks need to manage **risks** associated with inflation, interest rates, and customer behavior while ensuring sufficient profitability for operations.

To provide a clear and comprehensive view, this report first addresses the posed questions by examining the value of money over time and the value of withdrawals at specific interest rates. Then, the report builds models to simulate and predict borrower behavior using recursive calculations. Based on customer behavior, we constructed a model to predict withdrawal patterns and repayment times using a **decision tree regressor** algorithm and an improved **random forest regressor** algorithm with higher accuracy based on the divide-and-conquer principle.

Our results produce a mathematical model that can price revolving credit contracts, helping to assess the benefits a company (B) can gain based on the contract's interest rates. This assists banks in setting the most reasonable interest rates and applying them in practice. The model is mathematically optimal and beneficial for both parties. Additionally, its **flexibility** and lack of dependence on specific data allow our model to be applied in various contexts.

Keywords: Revolving credit, flexibility, risk, Decision Tree Regressor, Random Forest Regressor.

TABLE OF CONTENTS

I. PROBLEM STATEMENT

This form of credit is particularly popular in business and personal finance due to its flexibility and convenience, helping companies and individuals manage cash flow more effectively, which is very useful for meeting short-term and unpredictable financial needs. Consequently, companies can focus on business and contribute significantly to GDP and economic development. Revolving credit helps borrowing companies minimize risks in managing capital and delegate this responsibility to banks, which have higher expertise. The challenge is for banks to set an appropriate contract interest rate to ensure sufficient profitability amid market risks and fluctuations, such as the time value of money, market interest rates, and customer behavior, while also being suitable enough for the borrowing company to agree to the contract.

Recognizing this urgent need, our group considered building a model to regulate the timing of withdrawals, repayments, and the amount of each "bullet" to price the contract. This led to the problem and model presented. However, due to limited data and time, we only consider a sufficiently long and general period here. Expanding this on a larger scale could address a much greater need than the model.

II. PROBLEMS MODELING AND ASSUMPTIONS

Given the complex conditions affecting the problem, we simplified the model with some assumptions to facilitate its construction. However, because the overall problem is divided into smaller issues, we will detail the assumptions in each issue.

III. APPROACH

1. Value of Money

Problem: Assume company A and bank B sign a revolving credit contract in May 2024 for 1,000 billion VND, with an interest rate of 3.5% and a term of 10 years. The current market interest rate is 2.9%. One year later, in May 2025, company A withdrew 100 billion VND for a term of 1 year. However, the market interest rate at that time is 3.75%, so bank B must raise capital at 3.75% to lend to company A at 3.5% as per the contract.

1.1. Calculate the value of this withdrawal for bank B (at present time).

1.2. What should be the contract interest rate to make both A and B willing to sign the contract?

1.1. Issue 1.1

Let:

- r: current market interest rate (2.9%)
- T: number of years
- e^{-rT}: the value of 1 unit at time T
- Given the assumptions: 1 unit at time T years will have a value of e^{-rT} at the present time (time value of money).

Replacing T = 1 and r = 0.029 (= 2.9%), we get 1 unit at 05/2025 corresponding to the value at 05/2024:

$$e^{-0.029*1} \approx 0.9714 \text{ (VND)}$$

Thus, the value of this withdrawal for bank B (at the present time) is:

$$e^{-0.029*1} \times 1 \approx 0.9714 \text{ (hundred billion VND)}$$

1.2. Issue 1.2

Assume:

- Company A repays the bank B in full at once, ensuring consistency in the value of money over time.
- Company A will borrow at times when the market interest rate is highest. This means we only analyze the most unfavorable situations for bank B; simpler cases will be handled similarly.

Given these assumptions, bank B needs to set the interest rate at least equal to the highest market rate in 10 years to ensure no loss, but this limits company A's benefits. To accurately balance both parties' interests, we need to consider the probabilistic qualitative factor of market interest rates over 10 years, which will be detailed in Issue 2.2.

2. Interest Rate Risk

Problem: In the above section, we assumed the market interest rate to be 3.75% in one year. However, the market rate at May 2025 is a random future variable unknown to bank B. Through market research, they know that for the next year:

- p_{up} = 25%: Market interest rate will increase by 1%
- p_{down} = 25%: Market interest rate will decrease by 0.5%
- $p = 1 - p_{up} - p_{down}$ = 50%: Market interest rate will remain 2.9%.

2.1. Calculate the value of the withdrawal (at present time) with bank B's market expectations.
2.2. What should the contract interest rate be to make both A and B willing to sign the contract?
2.3. Use past market interest rate data to estimate p_{up} and p_{down}, then price the bullet according to the estimated parameters.

2.1. Issue 2.1

Similar to Issue 1.1, first, we calculate the expected value. The required answer is the product of the expected value with $e^{-0.029*1}$. Given, p_{up} = 25%: market rate increases by 1%, the money value at this rate is:

$$25\% \times 100 \times (1 + 3.9\%) = 25.975 \text{ (billion VND)}.$$

We proceed similarly for other probabilities and sum to get the expected value:

$$25.975 + 50\% \times 100 \times (1 + 2.9\%) + 25\% \times 100 \times (1 + 2.4\%) = 103.025 \text{ (billion VND)}.$$

The value of the withdrawal (at present time) with bank B's market expectations is:

$$e^{-0.029*1} \times 103.025 \approx 100.08 \text{ (billion VND)} .$$

2.2. Issue 2.2

To solve for the appropriate contract interest rate for both company A and bank B, satisfying two conditions, where the first condition is prioritized:

- Bank B ensures no loss.
- Company A receives the lowest interest rate possible.

Define the expected value (EV):

- EV of a random variable is the weighted average of all its specific values' probabilities.
- $EV = \sum_{i=1}^{n} x_i \times p_i \ (i = \overline{1, n})$, given:
  + x_i is the value of each variable.
  + p_i is the probability of x_i (%).

Applying this to calculate the expected value based on interest rates:

$$EV = \left(0.25 \times \frac{100(x - 3.9)}{1.029}\right) + \left(0.5 \times \frac{100(x - 2.9)}{1.029}\right) + \left(0.25 \times \frac{100(x - 2.4)}{1.029}\right)$$

To ensure both conditions, EV must equal zero:

$$\Leftrightarrow (0.25 \times \frac{100(x-3.9)}{1.029}) + (0.5 \times \frac{100(x-2.9)}{1.029}) + (0.25 \times \frac{100(x-2.4)}{1.029}) = 0$$
$$\Leftrightarrow x = 3.025\%$$

To conclude the ideal contract interest rate, add a rate differential to account for risk:

$$r_{RCF} = r_m + s$$

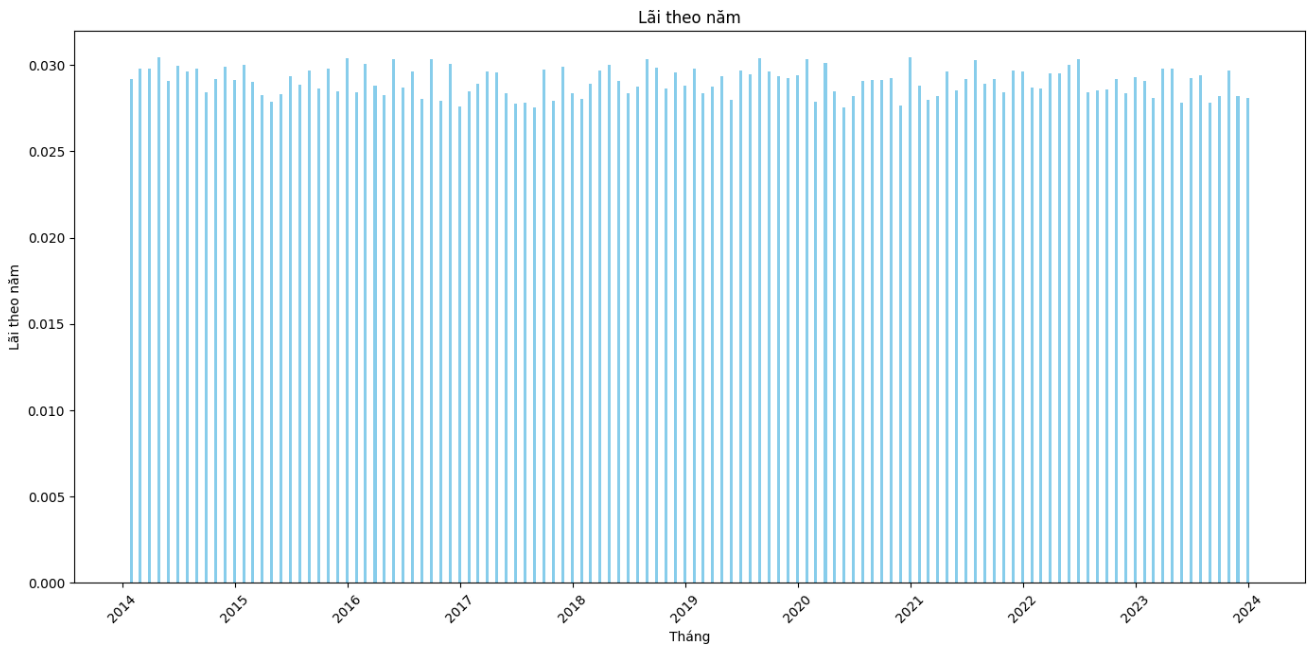
Replacing $r_m = 3.025\%$ và $s = (3.5\% - 2.9\%) = 0.6\%$, we get:

$$y = 3.025\% + 0.6\% = 3.625\%$$

This is the ideal interest rate. This section assumes a known interest rate but Issue 2.3 solves this issue to estimate interest rate based on past data.

2.3. Issue 2.3

To address the problem, we first collected comprehensive data on market interest rates as referenced in the appendix. Here, we present the data in the form of a bar chart:



Hình 1. Biểu đồ lãi theo năm từ năm 2014 đến 2024

We observed that the market interest rates from 2016 to 2023 fluctuated around 2.9%. Specifically:

- If the rates are below 2.9%, the difference between 2.9% and these rates is approximately 0.5%.
- If the rates are above 2.9%, the difference between these rates and 2.9% is approximately 1%.

Therefore, to simplify calculations, we will round the rates as follows:

- Rates lower than 2.9% are rounded to 2.4%.
- Rates higher than 2.9% are rounded to 3.9%.

Next, we will count the number of rates greater than 3.9% and those less than 2.4% to determine p_{up}, p_{down}. From this, we find $p_{up} = 25\%$, $p_{down} = 25\%$ and $p = 50\%$. From there, we need to calculate the present value of 100 billion VND that Company A will withdraw in May 2025 based on future interest rate probabilities. According to the established formula for expected value:

$$EV = 0.25 \times \frac{100}{1+0.039} + 0.25 \times \frac{100}{1+0.024} + 0.50 \times \frac{100}{1+0.029} \simeq 97.05 \text{ (billion VND)}$$

Assuming the contract interest rate is *r*, the expected value of the 100 billion VND is:

$$EV = \frac{100}{1+r}$$

$$\Leftrightarrow 1 + r = \frac{100}{97.05}$$

$$\Leftrightarrow r = \frac{100}{97.05} - 1 \simeq 0.0304$$

Thus, the ideal contract interest rate is 3.04%, at which both parties would agree to sign the contract. From the ideal contract interest rate, we can easily determine the value of the bullets in the revolving credit loan contract.

3. Modeling the Withdrawal Patterns

Assume that Company A only withdraws and repays money at the end of the month. Additionally, assume that the market interest rate will remain constant at 2.9% indefinitely. The value of the revolving credit contract now only depends on the withdrawal pattern of Company A (or Company A's behavior).

Model the withdrawal rule of Company A, including:

- Withdrawal timing.
- Maturity time for each withdrawal.
- Withdrawal amount each time.
- Or all of the above variables for each bullet to price the revolving credit contract.

To simplify the model given the complex conditions affecting the problem, this report has made the following assumptions:

[1] Company A always has the ability to repay the loan when due.

[2] At any time during the contract's validity, Company A can always withdraw a loan with a maturity cycle of at least 1 month and a maximum of 14 months.

[3] Company A only withdraws and repays money at the end of the month.

[4] The market interest rate will remain constant at 2.9% indefinitely

[5] The average maturity time is 3 months.

[6] The simulation period is 10 years.

[7] The credit limit is 1000 billion VND.

[8] The average withdrawal amount each time is 100 billion VND.

To model the behavior of withdrawing money from the revolving credit contract, we build an algorithm to graph the data of withdrawal amounts, maturity times, and remaining balances at each time. First, we will use the data set of withdrawal amounts and maturity times (1) in the appendix, where the average withdrawal amount is 100 billion VND, the average maturity time is 3 months, the average market interest rate is 2.9%, and the data includes a total of 120 months from January 2014 to December 2023, with withdrawals made at the end of each month. Moreover, this model considers that Company A can borrow multiple times with various maturity cycles as long as the balance in the revolving credit account does not become negative (initially, this account has 1000 billion VND). We then construct the balance calculation function as follows:

$$B_t = B_{t-1} - W_t$$

Where B_t is the balance at month t; W_t is the withdrawal amount at month t.

Next, we define the recursive function $H(t)$ to update the balance as follows:

$$H(t) = \{(A_{t-i}, D_{t-i} - 1) | (A_{t-i}, D_{t-i}) \in H(t-1), D_{t-i} > 1\}$$

Where for each month, A_{t-i} is the available amount from the previous month; D_{t-i} is the remaining time until the amount is repaid to the balance.

When $D_{t-i} = 1$, we add the amounts together: $B_t = B_t + A_{t-i}$.

Additionally, we calculate the monthly interest rate from the annual interest rate using the formula: $I_t = (1 + I_n)^{\frac{1}{12}} - 1$ where I_t is the monthly interest rate and I_n is the annual interest rate.

To complete this model, we first implemented the algorithm into Python code (available in the appendix). We then used the matplotlib library with Python 3.11 to graph the data, resulting in the following graphs:

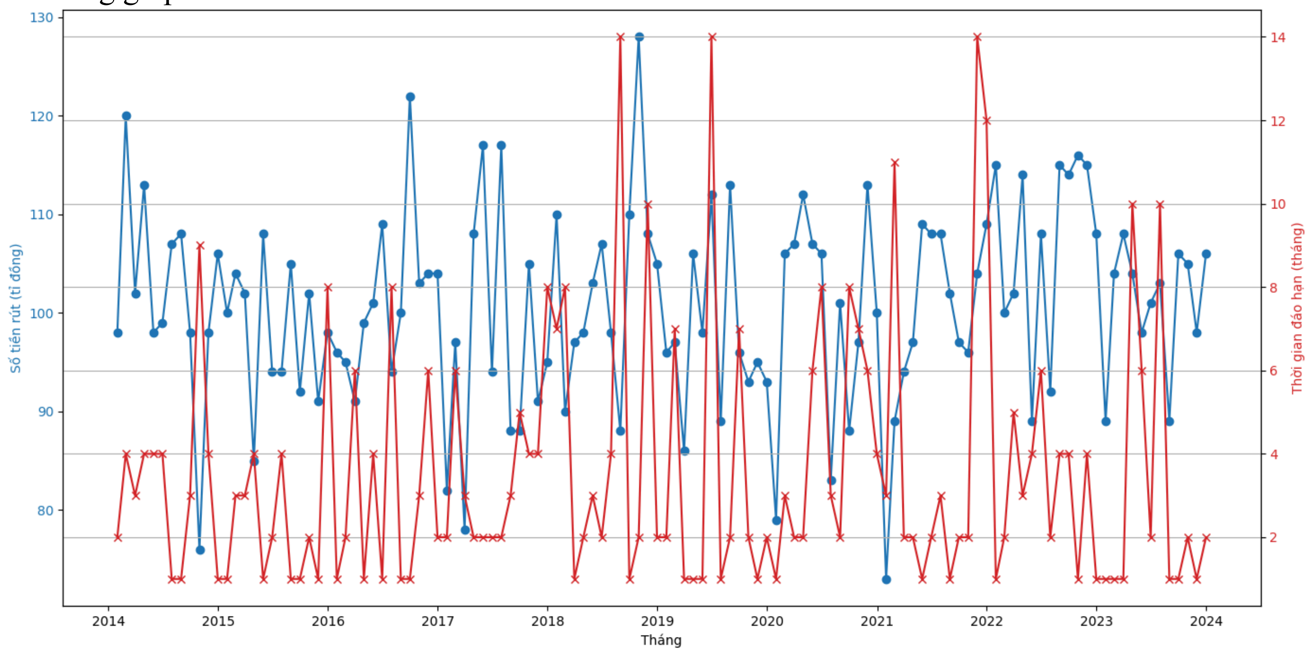


Figure 2. Correlation between Withdrawal Amounts and Maturity Times over 120 Months

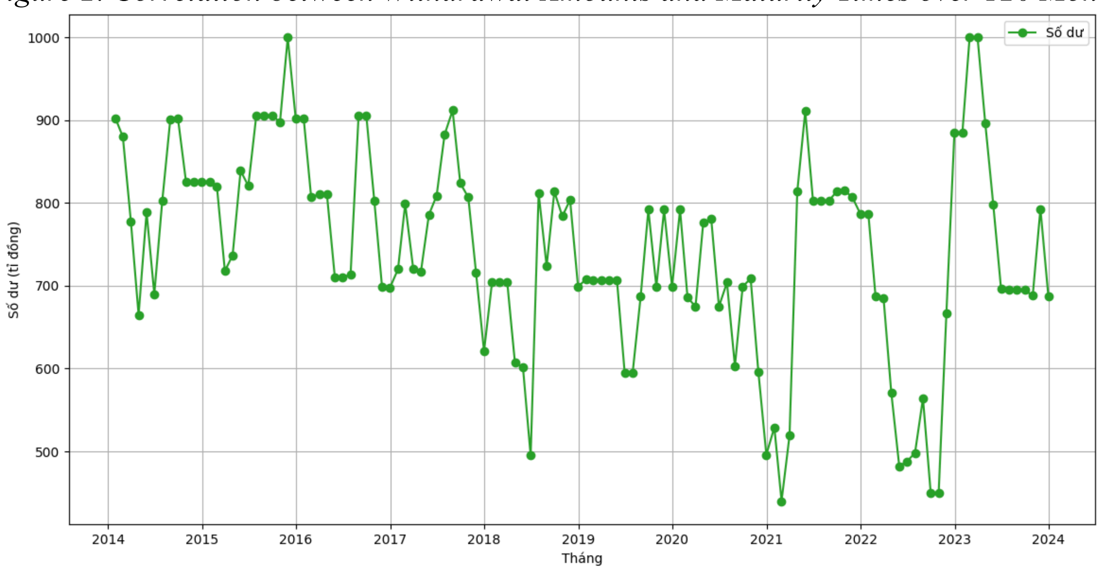


Figure 3. Remaining Balance over Time

This model illustrates the balance in the revolving credit loan, helping Company A manage the loan easily and strategize the most reasonable borrowing.

From function T_t, taking t = 120 as the final month in the 10-year contract, we calculate the total interest (contract value) to be approximately 217.48 billion VND. This model can also simulate and evaluate the behavior of other companies depending on the time, withdrawal amounts, and input data for the model.

4. Modeling Interest Rate Fluctuations and Customer Behavior

Data mining is the process of automatically extracting patterns from large datasets, including predictive modeling methods, where regression methods can be used. This report uses the Recursive Partitioning (RPART) method to build a regression model. The splitting criterion is SST − (SSL + SSR), where SST is the total sum of squares for the node, SSR and SSL are the total sum of squares for the right and left child nodes, respectively. We recognize that the Decision Tree Regressor (DTR) is a strong and reliable method in cases like missing values, nonlinear values, or discrepancies between variables. In revolving credit contracts (RCF), we need to identify which factors or variables affect the amount withdrawn in the contract (the average value and variance or standard deviation of the loan). First, we build a predictive model based on the Decision Tree Regressor by setting parameters such as Entropy (the randomness level of the dataset), Information Gain, Gini Index, Gain Ratio, Reduction in Variance, and Chi-Square.

We represent the entropy function by the following mathematical function:

$$E(S) = \sum_{i=1}^{c} - p_i \times log_2(p_i)$$

Where S is the initial value, p_i is the probability of event i (the increase or decrease) of value S. However, in this case, we also need to consider the randomness of the interest rate at each time point, so we use a function to represent the randomness of multiple factors:

$$E(T, X) = \sum_{c \in X} \mathrm{P}(c) \times E(c)$$

Where X is the randomness level of the market interest rate.

Information Gain measures how well the randomness of the dataset is split according to the target. In other words, Information Gain is the reduction in entropy. The mathematical function for Information Gain is expressed as follows:

$$Information\ Gain(T, X) = Entropy(T) - Entropy(T, X)$$

We can simplify this function to:

$$Information\ Gain = Entropy(before) - \sum_{j=1}^{k} \times Entropy(j, after)$$

The Gini Index, otherwise known as a cost function, is used to evaluate small parts of the dataset and is expressed as follows:

$$Gini = 1 - \sum_{i=1}^{C} \times (p_i)^2$$

Gain Ratio is an adjusted function for Information Gain to calculate specific values (more dispersed) in the dataset and is expressed by the function:

$$Gain\ Ratio = \frac{Information\ Gain}{SplitInfo} = \frac{Entropy\ (before) - \sum_{j=1}^{K} Entropy(j, after)}{\sum_{j=1}^{K} w_j \times log_2 w_j}$$

Reduction in Variance is an algorithm used for continuously appearing variables in the dataset, expressed by the following function:

$$Variance = \frac{\sum (X - \overline{X})^2}{n}$$

Finally, the Chi-Square method helps us identify the differences between independent and dependent variables in the data and their randomness. For a function that measures the Chi-Square value, it can be expressed as follows:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

To complete this model, we first implemented the algorithm into Python code (available in the appendix). We then used the matplotlib library with Python 3.11 to graph the data, resulting in the following graphs:

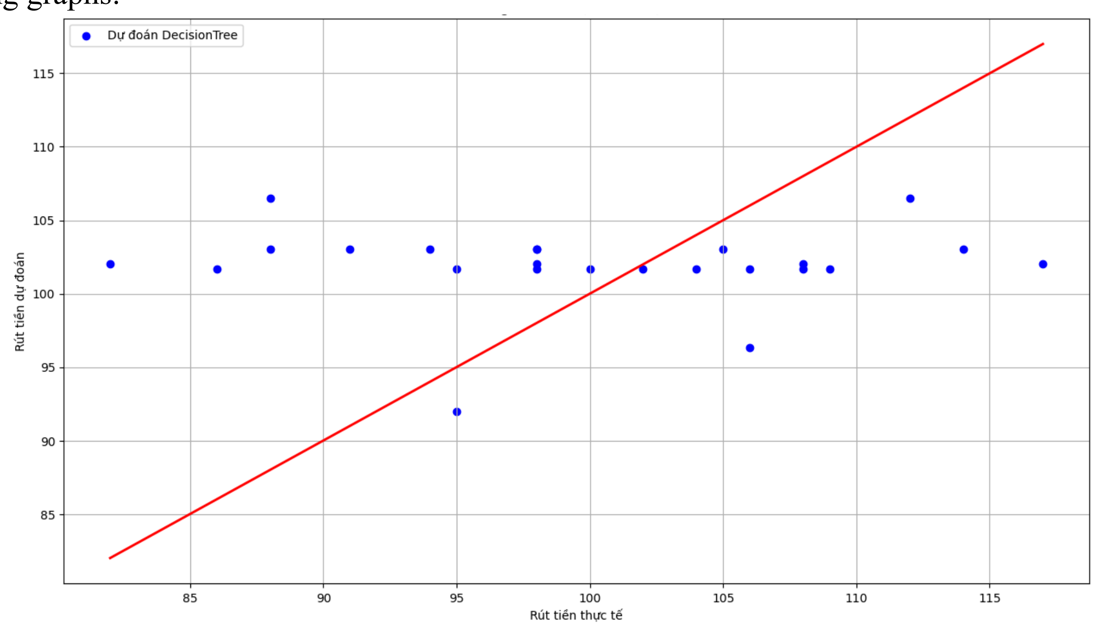


Figure 4. Actual Value and Predicted Value by the Model Based on DecisionTreeRegressor

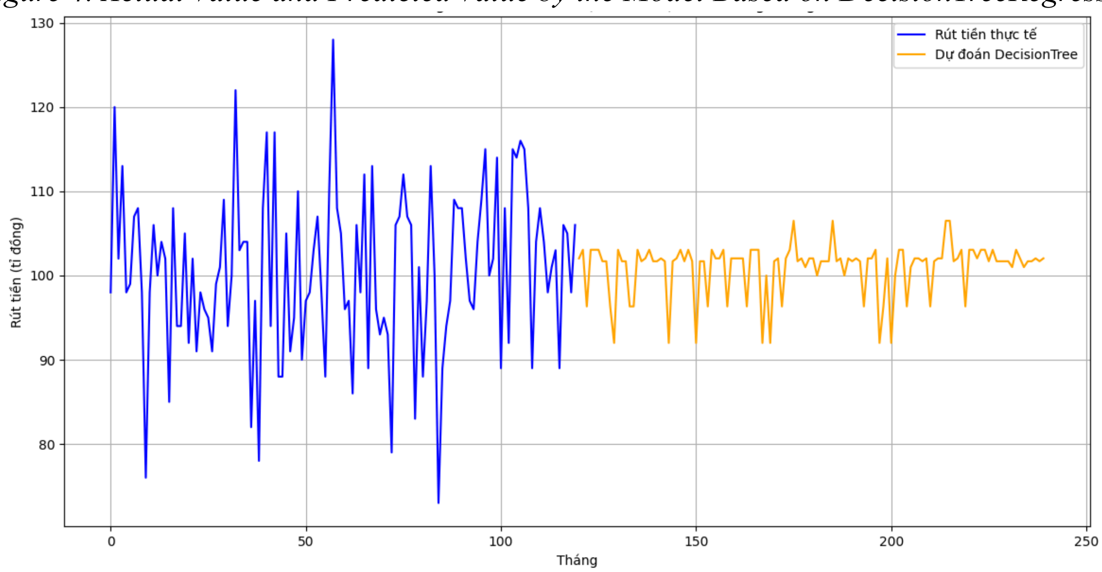


Figure 5. Value Predicted by the Model Based on DecisionTreeRegressor for the Next 10 Years

Based on the data from 2014 to 2023 (120 months), we applied a machine learning model using the constructed algorithm to predict customer behavior based on the learned data.

We noted that the mean squared error (MSE) for this model is relatively high (91.90). The MSE is calculated using the following formula:

$$Mean\ squared\ error = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$$

Where n is the number of elements in the dataset, y_i is the actual value, $\widehat{y}_i$ is the predicted value.

We improved the model by enhancing the algorithm to handle larger datasets. This model is a simple improvement of the model using the Decision Tree Regressor algorithm, but the data is split into smaller portions and processed by multiple Decision Trees.

Although this model requires more computational resources due to the use of multiple Decision Trees, by employing the Random Forest Regressor library, the new model can split the data and process it in parallel, increasing the training and prediction speed. Moreover, this model reduces overfitting by dividing the dataset to use the ensemble method of multiple decision trees, employing random sampling, and limiting the complexity of each decision tree. Finally, this new model also achieves higher accuracy due to multiple decision trees, reducing bias and variance in predictions.

Therefore, we propose an additional prediction model with higher accuracy, particularly when user behavior data is widely distributed, constructed as follows:

First, we create B sub-datasets from the initial dataset $D = \{(x_i, y_i)\}_{i=1}^{N}$:

$$D^b = \{(x_i^b, y_i^b)\}_{i=1}^{N}$$

Where b ∈ {1,2,...,B}

Next, for each sub-dataset D^b , we train a decision tree T^b. At each split node in the tree, we select a random subset of input variables $M \subseteq \{1, 2, ..., p\}$ với $|M| = m$,where p is the number of initial input variables, and m is the number of randomly selected input variables. At each node j in the tree, we select variable k ∈ M and split t to optimize the splitting criterion using the following function:

$$min_{k,t}\left(\sum_{i\in left(j)}(y_i - \overline{y}_{left(j)})^2 + \sum_{i\in right(j)}(y_i - \overline{y}_{right(j)})^2\right)$$

Where $left(j)$ and $right(j)$ are the sets of samples in the left and right branches after splitting at node j, and $\overline{y}_{left(j)}$, $\overline{y}_{right(j)}$ are the means of the values in those sets. After training, each decision tree T^b will predict value $\widehat{y}_b$ for sample x based on the learned data:

$$T_b(x) = \widehat{y}_b$$

Finally, we give the prediction result by averaging all the predictions based on the trees:

$$\widehat{y} = \frac{1}{b}\sum_{b=1}^{B}\widehat{y}_b$$

We used the matplotlib library with Python 3.11 to graph the data and implement the algorithm, resulting in the following graphs comparing the data predicted by the new model with the actual data:

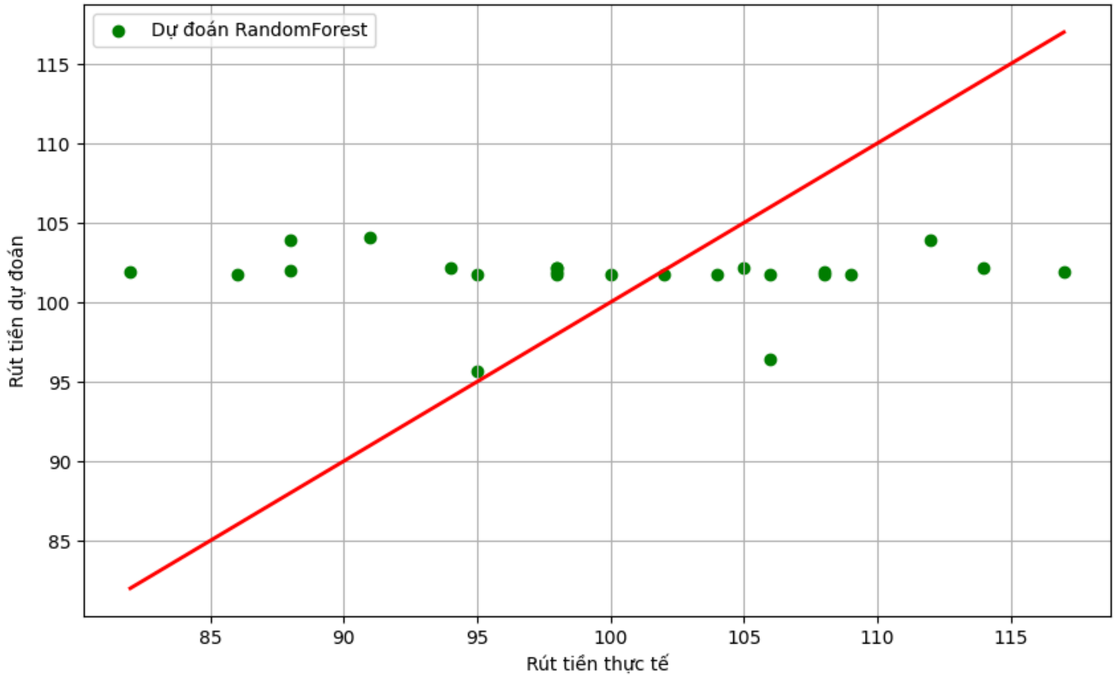


Figure 6. Actual Value and Predicted Value by the Model Based on RandomForestRegressor

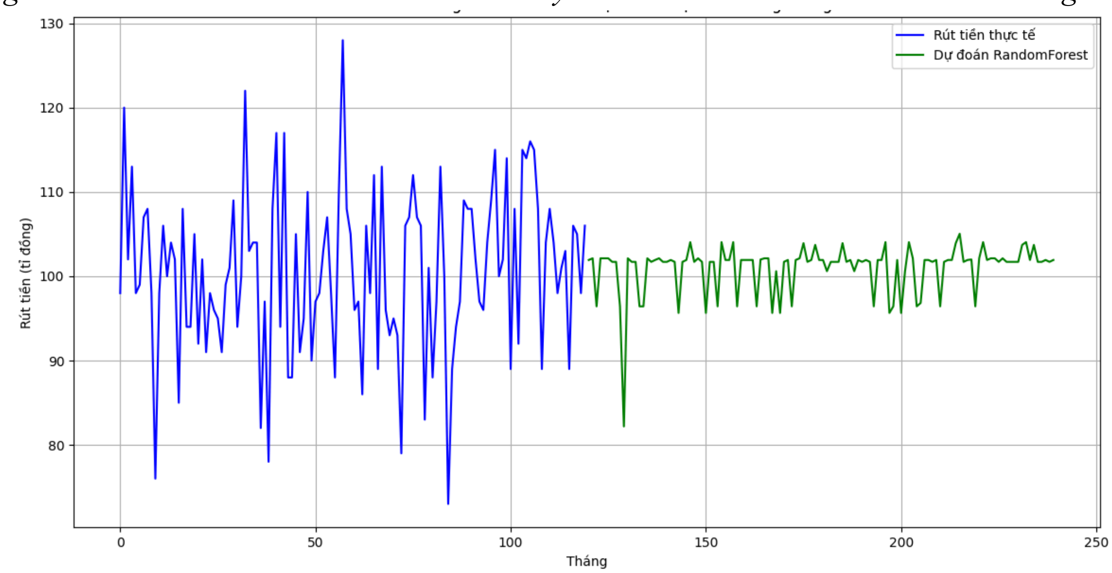


Figure 7. Value Predicted by the Model Based on RandomForestRegressor for the Next 10 Years

We observed that the mean squared error of this model has decreased to 88.66. From this, we can evaluate the improved model's effectiveness, as it provides more accurate results compared to the actual data and can reliably forecast future customer behavior.

In summary, we can apply the improved model using the Random Forest Regressor library combined with the H(t) function to calculate the contract value by defining the total profit for Bank B based on the contract interest rate. This is illustrated in the following graph:

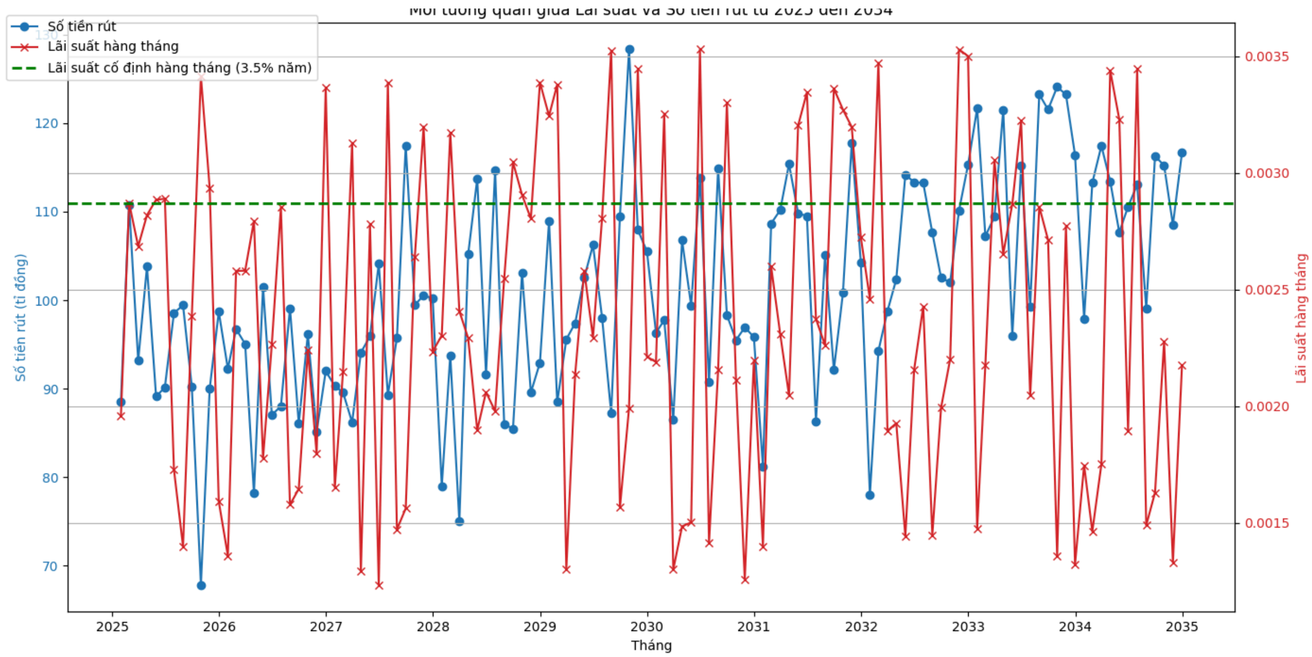


Figure 8. Correlation between Interest Rate and Withdrawn Amount Predicted from 2025 to 2035 Based on Existing Data

In this, we assume the contract is signed with an annual fixed interest rate of 2.9%, and then we calculate the monthly interest using the formula $I_t = (1 + I_n)^{\frac{1}{12}} - 1$ where I_t is the monthly interest rate and I_n is the annual interest rate. Then, we use the H(t) function defined in Section 3 to calculate the total profit, which is approximately 217.48 billion VND.

Looking at the graph, we observe the following: when the annual market interest rate (3.5%) is higher than the contract interest rate, combined with the bank withdrawing large amounts, it will be disadvantageous for the bank in raising capital and causing losses for Bank B. Conversely, if Company A withdraws money when the market interest rate is low, it benefits Bank B and generates higher profits. From the graph, Company A withdrawing money when the market interest rate is below the contract rate is more common than when the market interest rate is higher. However, Company A also withdraws money at times when the market interest rate is higher, making the modeling of Company A's behavior useful for Bank B in setting an appropriate interest rate.

IV. MODEL EVALUATION

1. General Remarks on the Problem

The problem that this model addresses is predicting the withdrawal behavior of Company A from a revolving credit account, aiming to accurately and reasonably price this credit contract. The main goal is to determine the withdrawal patterns of Company A, including factors such as the timing of withdrawals, the maturity period of each withdrawal, and the amount withdrawn each time.

This problem is crucial in practice as it helps Bank B optimize cash flow management and minimize risks in providing credit. Accurate forecasts of customer withdrawal behavior enable the bank to make strategic decisions regarding interest rates and loan terms, ensuring profitability and competitiveness in the financial market. Therefore, this model can be broadly applied in various fields, particularly in credit management and corporate finance.

2. First Model

The first model was constructed using the Decision Tree Regressor (DTR) method to simulate the fluctuations in interest rates and customer behavior. This method can handle missing or nonlinear data effectively. The Decision Tree Regressor helps identify the factors affecting the withdrawal amounts in the revolving credit contract, thereby making more accurate predictions about customer withdrawal behavior. This model can also split data based on parameters like Entropy, Information Gain, and Chi-Square, providing forecasts about Company A’s financial situation.

3. Second Model

The second model was built using the Random Forest Regressor to predict Company A's withdrawal behavior. This model significantly improved accuracy compared to traditional methods. Specifically, the mean squared error of this model decreased to 88.66, indicating its effectiveness in predicting customer behavior and making reliable forecasts about the future. The use of the Random Forest Regressor helps better model nonlinear factors and market fluctuations due to the use of multiple nodes from several Decision Trees, resulting in more accurate financial benefit predictions for the bank.

4. Comparison of the Two Models

Comparing the Random Forest Regressor and Decision Tree Regressor models, each has distinct characteristics and specific advantages and disadvantages. The Random Forest Regressor excels in minimizing prediction errors due to its ability to handle nonlinear factors and data volatility. This model effectively leverages historical data to make reliable forecasts. However, it requires higher computational resources and can become complex to implement. On the other hand, the Decision Tree Regressor is easier to understand and implement and handles missing and nonlinear data well to a limited extent. However, this model is prone to overfitting if not carefully optimized and may not achieve the same high accuracy as the Random Forest Regressor in complex scenarios. Despite the Random Forest Regressor's superiority over the Decision Tree Regressor, it still has not optimized the error, leading to higher biases and inaccurate predictions.

V. PRACTICAL APPLICATIONS OF THE MODEL

The models developed in this report have high applicability in the financial sector, especially in managing revolving credit. Both models help Bank B accurately forecast customer withdrawal behavior, thereby optimizing decisions regarding contract interest rates and effectively managing financial risks. This not only ensures the bank's profitability but also enhances its competitiveness in the market. Additionally, the model can be expanded to apply to various companies and industries, depending on the input data, contributing to improved financial management and forecasting in other financial organizations.

VI. REFERENCES

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VII. APPENDIX

[1] The source code for our team's program can be found at this link: https://github.com/dngvmnh/VM2C_202400000471

[2] Bảng dữ liệu đã dùng trong bài báo cáo:

Thang	Tien rut	Thoi gian	So du	Lai thang	Lai nam
31/01/2025	98	2	902	0.002402	0.029203
28/02/2025	120	4	880	0.002451	0.029808
31/03/2025	102	3	778	0.002447	0.029768
30/04/2025	113	4	665	0.002503	0.030448
31/05/2025	98	4	789	0.002392	0.029084
30/06/2025	99	4	690	0.002463	0.029962
31/07/2025	107	1	803	0.002438	0.029647
31/08/2025	108	1	901	0.002448	0.02978
30/09/2025	98	3	902	0.002336	0.028401
31/10/2025	76	9	826	0.002401	0.029192
30/11/2025	98	4	826	0.002458	0.029894
31/12/2025	106	1	826	0.002396	0.029131
31/01/2026	100	1	826	0.002469	0.03003
28/02/2026	104	3	820	0.002386	0.029012
31/03/2026	102	3	718	0.002327	0.02828
30/04/2026	85	4	737	0.002294	0.027873
31/05/2026	108	1	839	0.002328	0.028297
30/06/2026	94	2	821	0.002413	0.029342
31/07/2026	94	4	906	0.002376	0.028885
31/08/2026	105	1	906	0.002442	0.029698
30/09/2026	92	1	906	0.002356	0.028643

31/10/2026	102	2	898	0.002449	0.029784
30/11/2026	91	1	1000	0.002343	0.028481
31/12/2026	98	8	902	0.002497	0.030382
31/01/2027	96	1	902	0.002338	0.028416
28/02/2027	95	2	807	0.002471	0.030055
31/03/2027	91	6	811	0.002369	0.028796
30/04/2027	99	1	811	0.002325	0.02826
31/05/2027	101	4	710	0.002493	0.030328
30/06/2027	109	1	710	0.00236	0.028686
31/07/2027	94	8	714	0.002437	0.029637
31/08/2027	100	1	906	0.002306	0.028031
30/09/2027	122	1	906	0.002494	0.030346
31/10/2027	103	3	803	0.0023	0.027952
30/11/2027	104	6	699	0.002473	0.030089
31/12/2027	104	2	698	0.002271	0.027593
31/01/2028	82	2	720	0.002342	0.028469
29/02/2028	97	6	799	0.002379	0.02893
31/03/2028	78	3	721	0.002436	0.029628
30/04/2028	108	2	717	0.002432	0.029574
31/05/2028	117	2	786	0.002333	0.028354
30/06/2028	94	2	809	0.002285	0.027767
31/07/2028	117	2	883	0.00229	0.027832
31/08/2028	88	3	912	0.002267	0.02755
30/09/2028	88	5	824	0.002447	0.029767
31/10/2028	105	4	807	0.002299	0.027943

30/11/2028	91	4	716	0.002459	0.029912
31/12/2028	95	8	621	0.002333	0.028353
31/01/2029	110	7	704	0.002309	0.028059
28/02/2029	90	8	705	0.002377	0.028895
31/03/2029	97	1	705	0.002441	0.02969
30/04/2029	98	2	607	0.002467	0.030006
31/05/2029	103	3	602	0.00239	0.029061
30/06/2029	107	2	495	0.002336	0.028391
31/07/2029	98	4	812	0.002367	0.02878
31/08/2029	88	14	724	0.002492	0.030319
30/09/2029	110	1	814	0.002456	0.029874
31/10/2029	128	2	784	0.002357	0.028659
30/11/2029	108	10	804	0.002431	0.029569
31/12/2029	105	2	699	0.002368	0.028784
31/01/2030	96	2	708	0.002451	0.02981
28/02/2030	97	7	707	0.002335	0.028378
31/03/2030	86	1	707	0.002365	0.028753
30/04/2030	106	1	707	0.002413	0.029347
31/05/2030	98	1	707	0.002304	0.027997
30/06/2030	112	14	595	0.002439	0.029664
31/07/2030	89	1	595	0.002424	0.029483
31/08/2030	113	2	687	0.0025	0.03041
30/09/2030	96	7	792	0.002437	0.029644
31/10/2030	93	2	699	0.002414	0.02936
30/11/2030	95	1	792	0.002406	0.029254

31/12/2030	93	2	699	0.002421	0.029439
31/01/2031	79	1	792	0.002493	0.03033
28/02/2031	106	3	686	0.002293	0.027869
31/03/2031	107	2	675	0.002476	0.030125
30/04/2031	112	2	776	0.002343	0.028483
31/05/2031	107	6	781	0.002268	0.027563
30/06/2031	106	8	675	0.002323	0.028231
31/07/2031	83	3	704	0.002391	0.029074
31/08/2031	101	2	603	0.002398	0.029155
30/09/2031	88	8	699	0.002396	0.029137
31/10/2031	97	7	709	0.002406	0.029263
30/11/2031	113	6	596	0.002275	0.027643
31/12/2031	100	4	496	0.002503	0.030448
31/01/2032	73	3	529	0.002368	0.028787
29/02/2032	89	11	440	0.002305	0.028013
31/03/2032	94	2	519	0.00232	0.028196
30/04/2032	97	2	814	0.002437	0.029643
31/05/2032	109	1	911	0.002346	0.028516
30/06/2032	108	2	803	0.0024	0.029182
31/07/2032	108	3	803	0.002492	0.030314
31/08/2032	102	1	803	0.002377	0.028899
30/09/2032	97	2	814	0.002399	0.029175
31/10/2032	96	2	815	0.00234	0.02845
30/11/2032	104	14	807	0.002439	0.029661
31/12/2032	109	12	787	0.002437	0.02964

31/01/2033	115	1	787	0.00236	0.028692
28/02/2033	100	2	687	0.002356	0.028643
31/03/2033	102	5	685	0.002426	0.029507
30/04/2033	114	3	571	0.002426	0.029506
31/05/2033	89	4	482	0.002467	0.030008
30/06/2033	108	6	488	0.002496	0.030362
31/07/2033	92	2	498	0.002337	0.028412
31/08/2033	115	4	564	0.002348	0.028537
30/09/2033	114	4	450	0.002353	0.028601
31/10/2033	116	1	450	0.002403	0.029214
30/11/2033	115	4	667	0.002335	0.028385
31/12/2033	108	1	885	0.002408	0.029277
31/01/2034	89	1	885	0.002393	0.029091
28/02/2034	104	1	1000	0.002313	0.028111
31/03/2034	108	1	1000	0.00245	0.029803
30/04/2034	104	10	896	0.002448	0.029773
31/05/2034	98	6	798	0.002289	0.027818
30/06/2034	101	2	697	0.002406	0.029261
31/07/2034	103	10	695	0.002418	0.0294
31/08/2034	89	1	695	0.002291	0.027841
30/09/2034	106	1	695	0.002319	0.028183
31/10/2034	105	2	688	0.002439	0.02967
30/11/2034	98	1	793	0.002322	0.028219
31/12/2034	106	2	687	0.002312	0.028104

VIII. WORDS OF GRATITUDE

First of all, we would like to sincerely express our gratitude to the organizers of VM2C for bringing students a meaningful competition. It was a great opportunity for students to learn and experience real-life situations, as well as to have group study sessions and lively discussions. After participating in this competition, we not only gained training with new knowledge and valuable tips through training sessions by the specialized team, but also gained a deeper understanding of the applications of mathematical modeling and the significant career opportunities awaiting us in the future. Moreover, the program has been a stepping stone towards achieving our dreams that we will never forget.

Next, we want to extend our heartfelt thanks to the advisors of the VM2C competition, the professors at VIASM Institute of Advanced Mathematics who supported our team throughout the training process and participation in the program, especially the teachers at our school who introduced us to the competition and guided us from the beginning. They have been our source of motivation to complete this report. Finally, the team would like to thank the University of Natural Sciences along with other sponsoring entities for providing the conditions for the competition to be successfully held.

We deeply appreciate all the contributions and guidance from everyone who accompanied us to create precious memories at the competition. We believe that through this program, students will be equipped with the necessary skills and knowledge to confidently face new challenges in the future. Once again, we sincerely thank the Organizing Committee for their efforts and dedication to the Vietnam Mathematical Modeling Program 2024.