Characteristic functions

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Define a function $s_n \colon \mathbb{R}^{nd} \to \mathbb{R}^d$ by

$$s_n(x_1, \dots, x_n) = x_1 + \cdots + x_n.$$

This is Borel-measurable, so the following definition makes sense:

DEFINITION 1.1: *Convolution of measures*

Let $\mu_1, \dots, \mu_n$ be finite measures on $(\mathbb{R}^d, \mathcal{B}(\mathbb{R}^d))$. The image measure

$$\mu_1 * \cdots * \mu_n = (\mu_1 \otimes \cdots \otimes \mu_n) \circ s_n^{-1}$$

on $\mathcal{B}(\mathbb{R}^d)$ is called the *convolution* of the measures $\mu_1, \dots, \mu_n$.

It is easy to show that the convolution product is associative, so it is enough to study the convolution of two measures.

If μ has density $f \in \mathcal{L}(\lambda)^+$