Roman, *Advanced Linear Algebra*

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1 • Vector Spaces

EXERCISE 1.11

Show that if S is a subspace of a vector space V, then $\dim S \leq \dim V$. Furthermore, if $\dim S = \dim V < \infty$ then $S = V$.

SOLUTION. Let $\mathcal{B}$ be a basis for S. Then this is linearly independent as a subset of V, hence is contained in a basis $\mathcal{B}'$ for V by Theorem 1.9. Then $\mathcal{B} \subseteq \mathcal{B}'$, so it follows that $\dim S \leq \dim V$.

Now assume that $\dim S = \dim V < \infty$. Then $|\mathcal{B}| = |\mathcal{B}'|$, but since each basis is finite and one is contained in the other, we must have $\mathcal{B} = \mathcal{B}'$. Hence $S = V$. □