

The basic rules of natural deduction:

	<i>introduction</i>	<i>elimination</i>
\wedge	$\frac{\phi \quad \psi}{\phi \wedge \psi} \wedge i$	$\frac{\phi \wedge \psi}{\phi} \wedge e_1 \quad \frac{\phi \wedge \psi}{\psi} \wedge e_2$
\vee	$\frac{\phi}{\phi \vee \psi} \vee i_1 \quad \frac{\psi}{\phi \vee \psi} \vee i_2$	$\frac{\phi \vee \psi \quad \boxed{\begin{smallmatrix} \phi \\ \vdots \\ \chi \end{smallmatrix}} \quad \boxed{\begin{smallmatrix} \psi \\ \vdots \\ \chi \end{smallmatrix}}}{\chi} \vee e$
\rightarrow	$\frac{\boxed{\begin{smallmatrix} \phi \\ \vdots \\ \psi \end{smallmatrix}}}{\phi \rightarrow \psi} \rightarrow i$	$\frac{\phi \quad \phi \rightarrow \psi}{\psi} \rightarrow e$
\neg	$\frac{\boxed{\begin{smallmatrix} \phi \\ \vdots \\ \perp \end{smallmatrix}}}{\neg \phi} \neg i$	$\frac{\phi \quad \neg \phi}{\perp} \neg e$
\perp	(no introduction rule for \perp)	$\frac{\perp}{\phi} \perp e$
$\neg\neg$		$\frac{\neg\neg\phi}{\phi} \neg\neg e$

Some useful derived rules:

$$\frac{\phi \rightarrow \psi \quad \neg\psi}{\neg\phi} \text{MT}$$

$$\frac{\phi}{\neg\neg\phi} \neg\neg i$$

$$\frac{\boxed{\begin{smallmatrix} \neg\phi \\ \vdots \\ \perp \end{smallmatrix}}}{\phi} \text{PBC}$$

$$\frac{}{\phi \vee \neg\phi} \text{LEM}$$

Figure 1.2. Natural deduction rules for propositional logic.