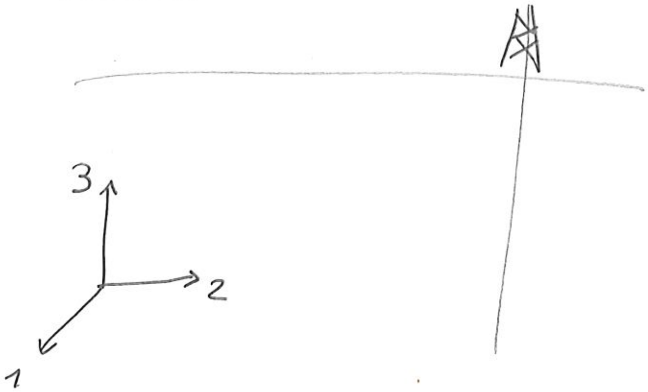
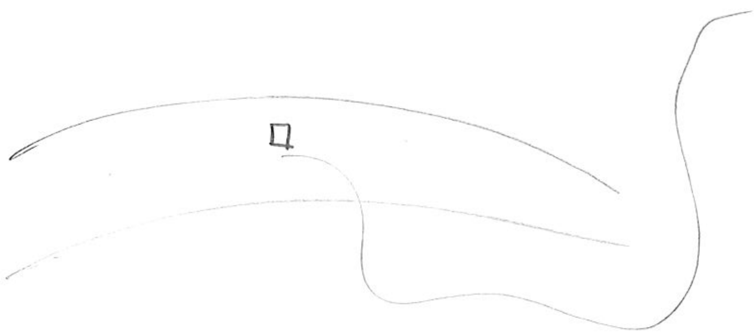
Intro

(9-4-2018)



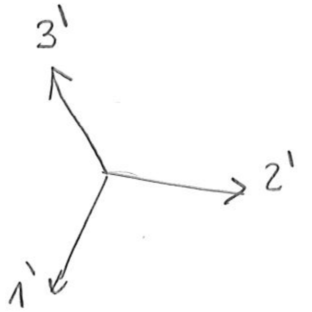
$$\underline{\underline{\sigma}} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$



$\sigma_{ij} = \sigma_{ji}$, symmetry

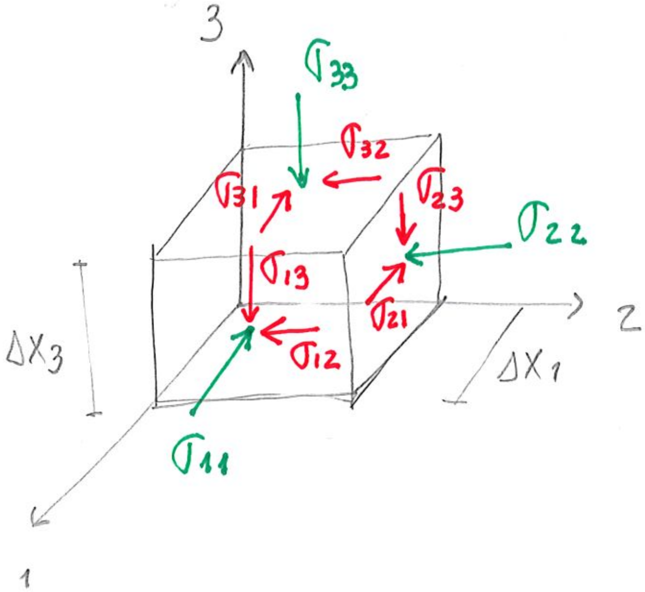
$$\underbrace{\sigma_{ij} \in \mathbb{R}}$$

eigen values are real

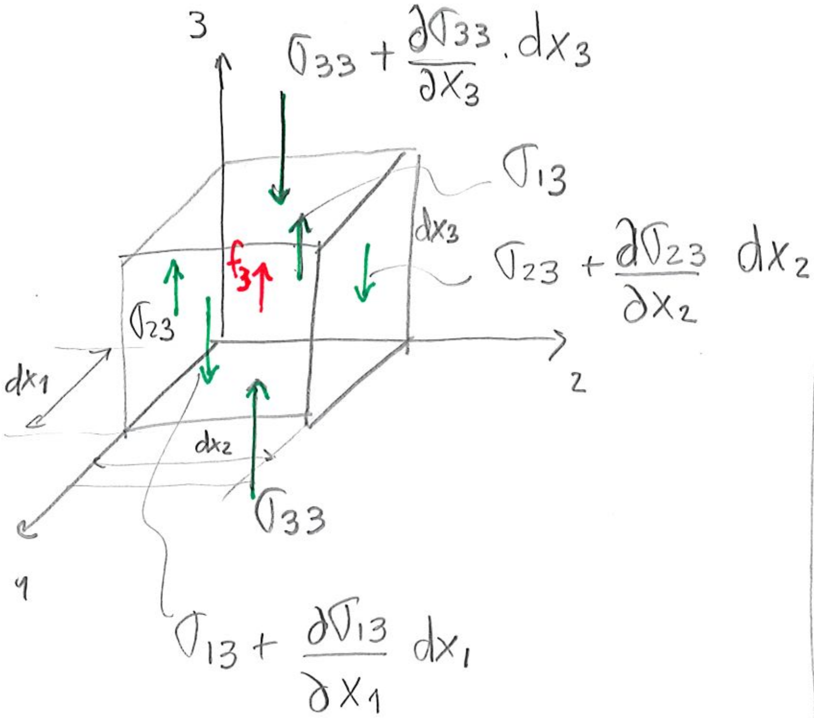


$$\underline{\underline{\sigma}} = \begin{vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{vmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$



$$\underline{\underline{\sigma}} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$



$$dx_1\, dx_2\, dx_3 = Vol$$

$$\Sigma F_3 = 0 = m \cancel{a} = \rho \cdot Vol \cdot \cancel{a}$$

$$\cancel{\sigma_{33}\, dx_1 dx_2} - \left(\cancel{\sigma_{33}} + \frac{\partial \sigma_{33}}{\partial x_3} dx_3\right) dx_1 dx_2 +$$

$$\cancel{\sigma_{13}\, dx_2 dx_3} - \left(\cancel{\sigma_{13}} + \frac{\partial \sigma_{13}}{\partial x_1} dx_1\right) dx_2 dx_3 +$$

$$\cancel{\sigma_{23}\, dx_1 dx_3} - \left(\cancel{\sigma_{23}} + \frac{\partial \sigma_{23}}{\partial x_2} dx_2\right) dx_1 dx_3$$

$$+ f_3 = 0$$

$$\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} - \frac{f_3}{Vol} = 0$$

$$\rho = m / Vol \quad , \quad b_3 \cdot \rho$$

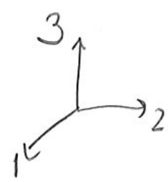
Cauchy's equilibrium equations

$$\frac{\partial \sigma_{ij}}{\partial x_j} - b_i \rho = 0$$

$$\left\{\begin{aligned}
\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} - b_1 \rho = 0 \\
\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} - b_2 \rho = 0 \\
\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} - b_3 \rho = 0
\end{aligned}\right.$$

$$\nabla \cdot \underline{\underline{\sigma}} - \underline{b} = 0$$

$$\sigma_{ij} = \sigma_{ji}$$



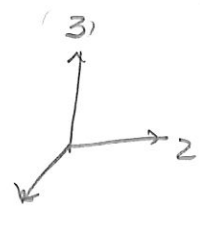
If σ_v is a principal stress $\Rightarrow$ the other two principal stresses are horizontal

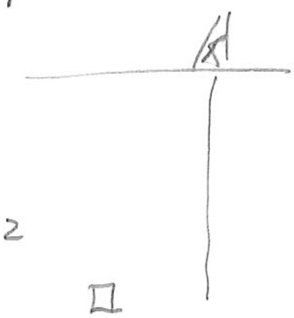
$$\sigma_1 \perp \sigma_2 \perp \sigma_3$$

$$\sigma_v \perp \sigma_{Hmax} \perp \sigma_{hmin}$$

Effective stress tensor $\underline{\underline{\sigma}} = \begin{vmatrix} \sigma_{11} & 0 & 0 \\ & \sigma_{22} & 0 \\ & & \sigma_{33} \end{vmatrix} = \begin{vmatrix} \sigma_{hmin} & 0 & 0 \\ 0 & \sigma_{Hmax} & 0 \\ 0 & 0 & \sigma_v \end{vmatrix}$

Total stress tensor $\underline{\underline{S}} = \begin{vmatrix} S_{hmin} & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_v \end{vmatrix}$



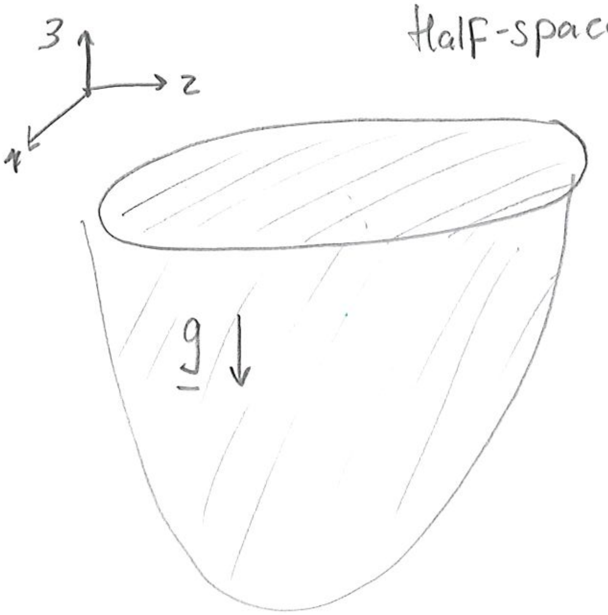


$$\boxed{S_v = \int_0^z \rho_{bulk}(z) \cdot g \cdot dz}$$

ρ_{bulk} measured from → gamma ray density Tool; → sampling

$\rho_{bulk} \cdot g \sim 23 \frac{MPa}{Km}$

$\sim 1 \frac{psi}{ft}$



Half-space $\Rightarrow \dfrac{\partial}{\partial x_1} = \dfrac{\partial}{\partial x_2} = 0$

$$\frac{d S_{33}}{d x_3} = \rho g$$

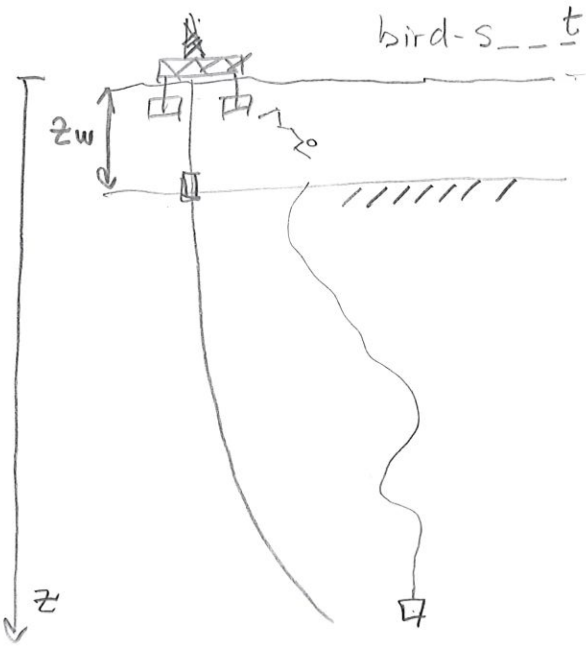
$$\int_{S_{33}(x_3=0)}^{S_{33}(x_3)} d S_{33} = \int_0^{x_3} \rho g \, dx_3$$

$$\Big\downarrow \; S_{33}(x_3 = 0) = 0$$

$$\boxed{S_{33}(x_3) = \int_0^{x_3} \rho(x_3)\, g \, dx_3}$$

$$\Big\downarrow \; \rho(x_3) = \rho \text{ (constant)}$$

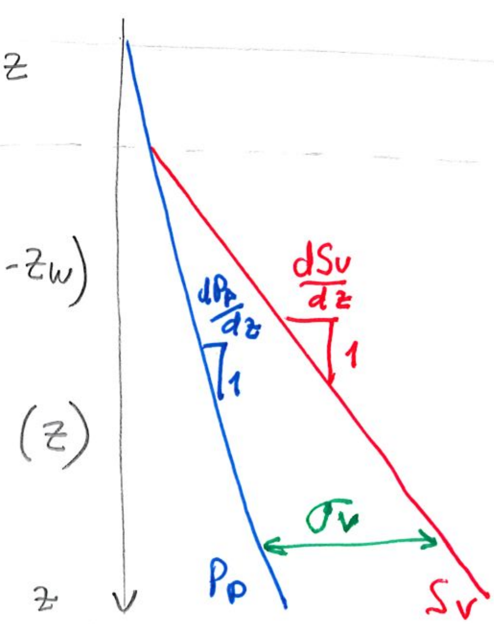
$$S_{33}(x_3) = \rho g x_3$$

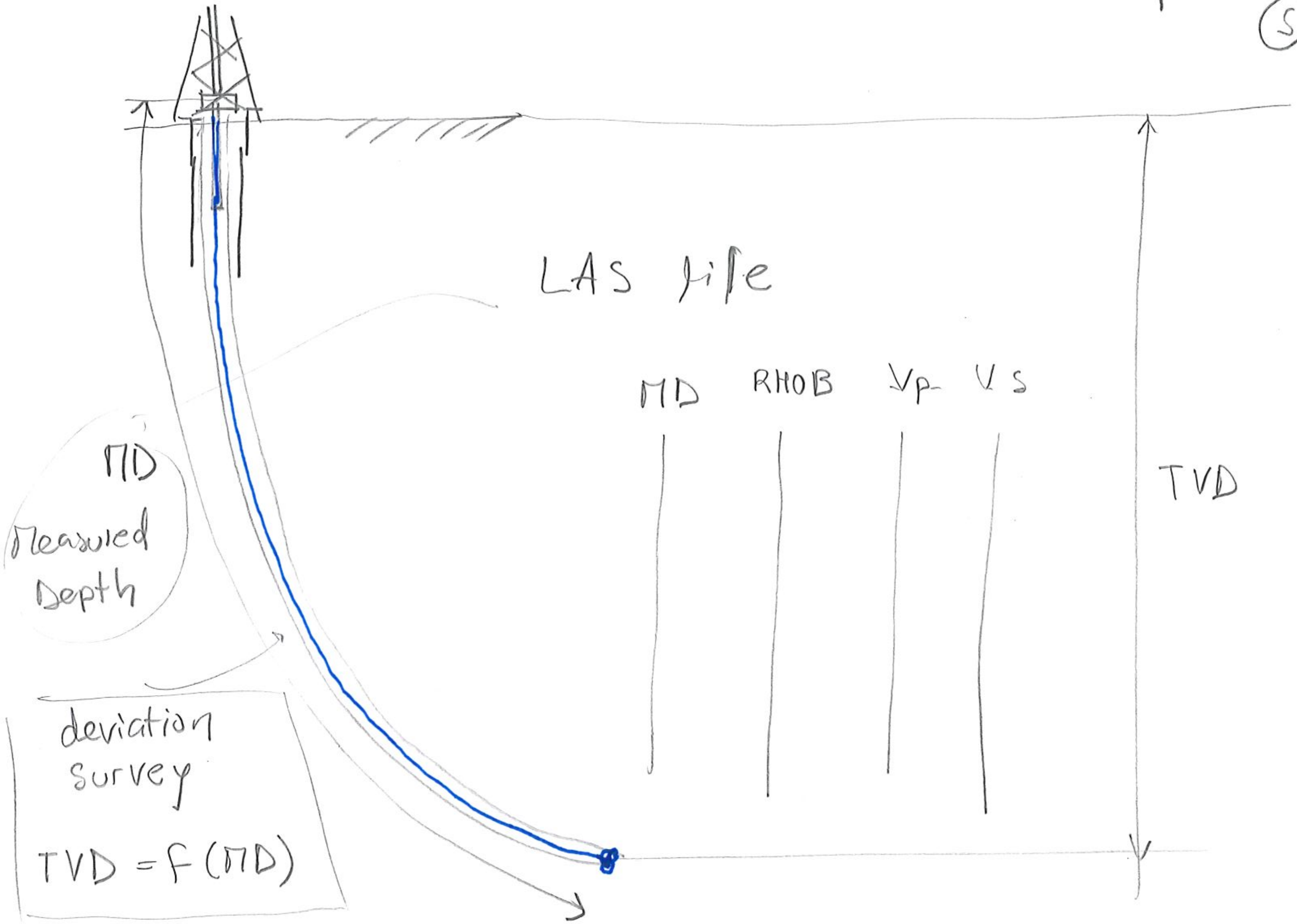


$$P_p(z) = \rho_w g z$$

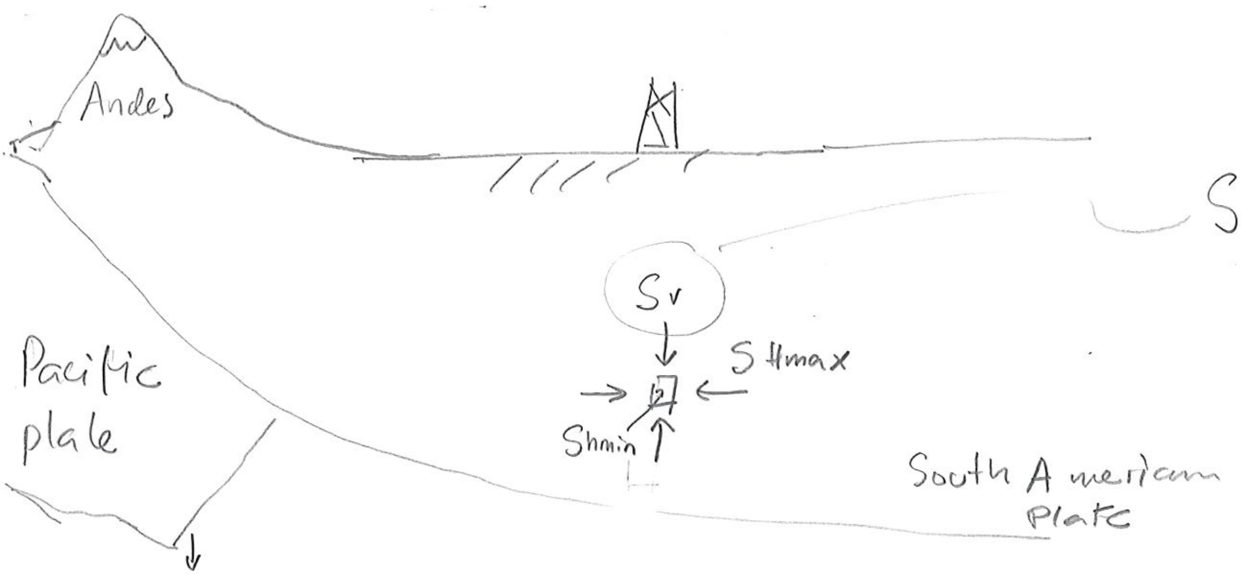
$$S_v(z) = \rho_w g z_w + \rho_{bulk} g (z - z_w)$$

$$\sigma_v(z) = S_v(z) - P_p(z)$$

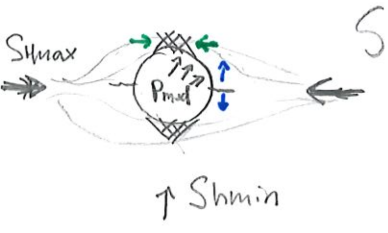




9/6/2018



$$S_v = \int_0^z \rho_{bulk}(z)\, g\, dz$$



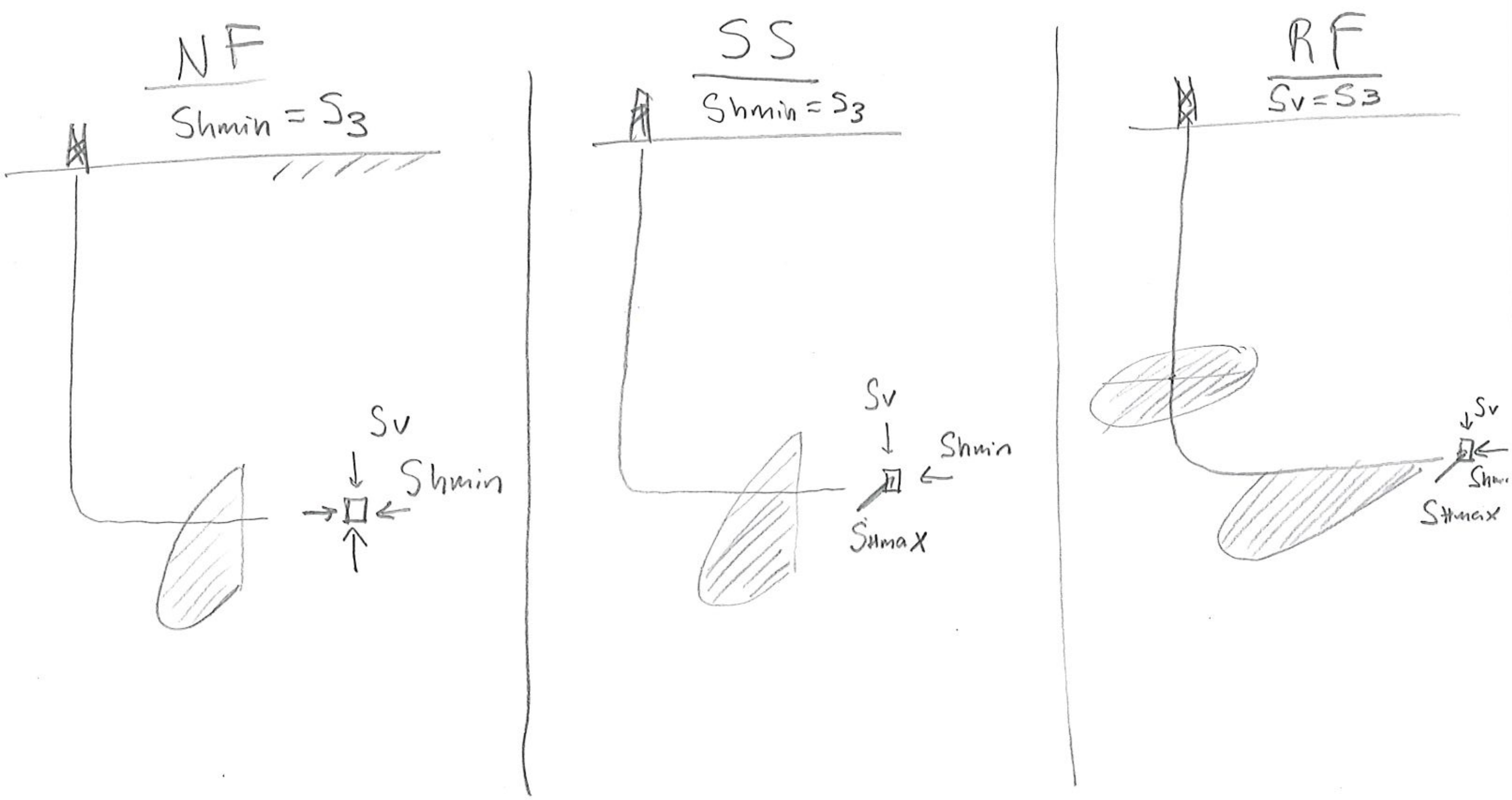
Shmin ⟷ HF

Faults

Wellbore failure

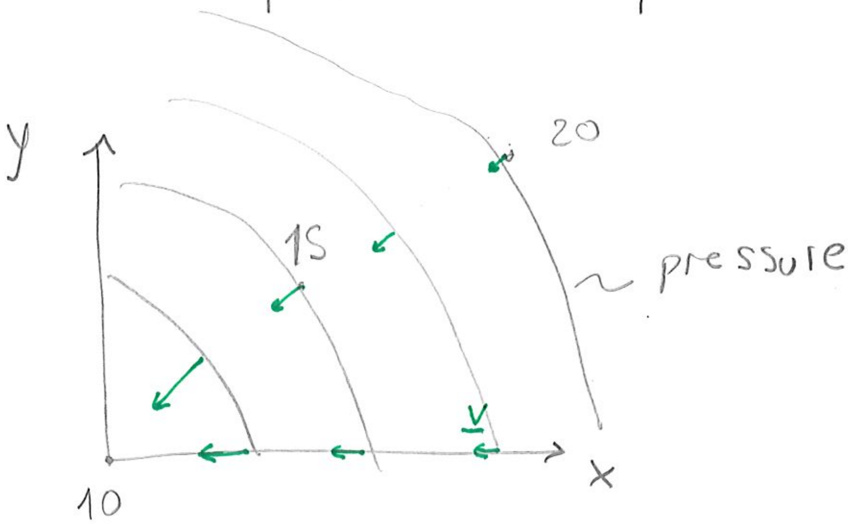
} SHmax

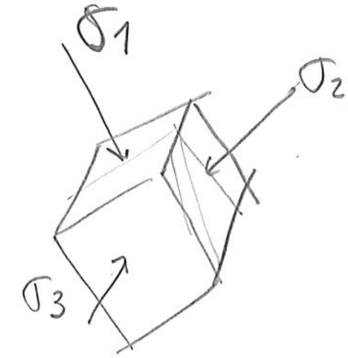
$S_1 >$	$S_2 >$	S_3	
S_v	S_{Hmax}	S_{hmin}	→ Normal Faulting
S_{Hmax}	S_v	S_{hmin}	→ Strike Slip F
S_{Hmax}	S_{Hmin}	S_v	→ Reverse Faulting

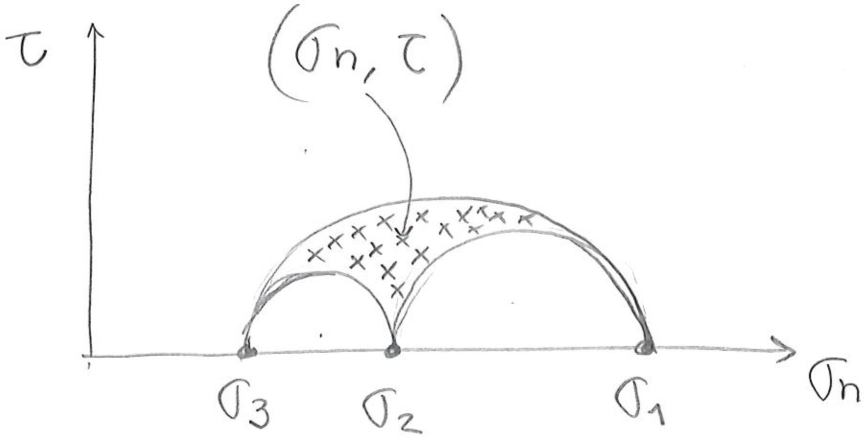


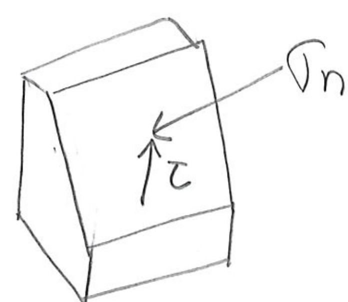
3D Mohr Circle

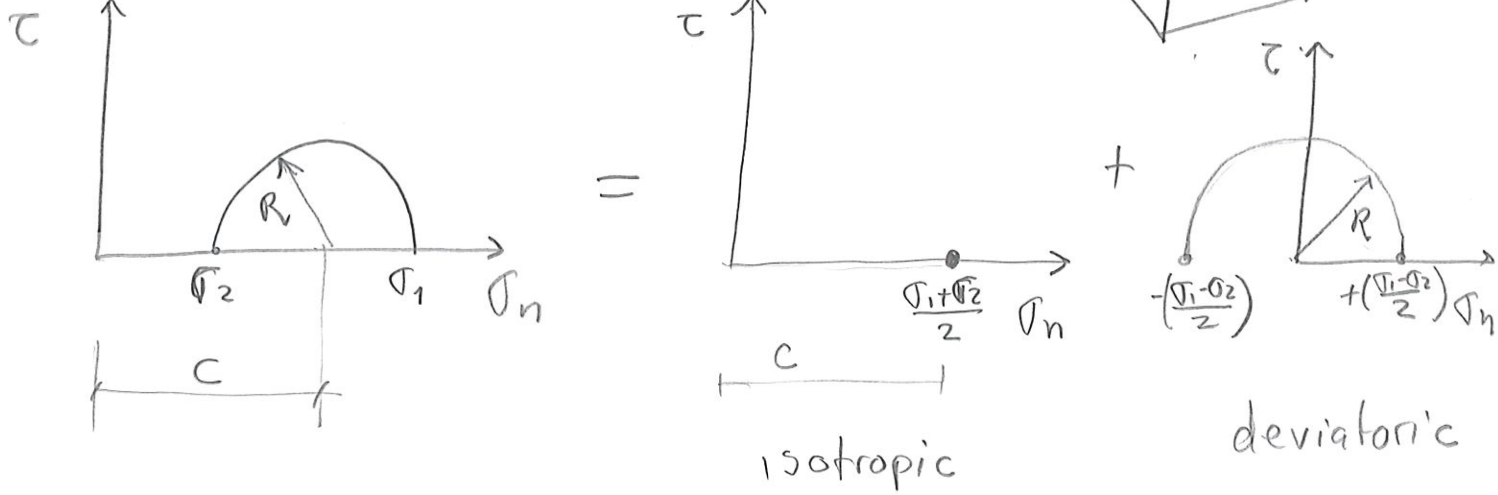
$$\underline{\underline{S}} = \begin{vmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{vmatrix} \rightarrow \underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \underline{\underline{I}} = \begin{vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{vmatrix}$$











$$
\begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix} = \overbrace{\begin{vmatrix} \sigma_m & 0 & 0 \\ & \sigma_m & 0 \\ & & \sigma_m \end{vmatrix}}^{\text{isotropic}} + \overbrace{\begin{vmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ & \sigma_{22}-\sigma_m & \sigma_{23} \\ & & \sigma_{33}-\sigma_m \end{vmatrix}}^{\text{deviatoric comp}}
$$

$$\sigma_m \triangleq \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3}$$

$$\underline{\underline{\sigma}} = \sigma_m \underline{\underline{I}} + \underline{\underline{S}}_d$$

Invariants

$$I_1(\underline{\underline{\sigma}}) = \sigma_{11}+\sigma_{22}+\sigma_{33} = tr(\underline{\underline{\sigma}}) \rightarrow \sigma_m = I_1/3$$

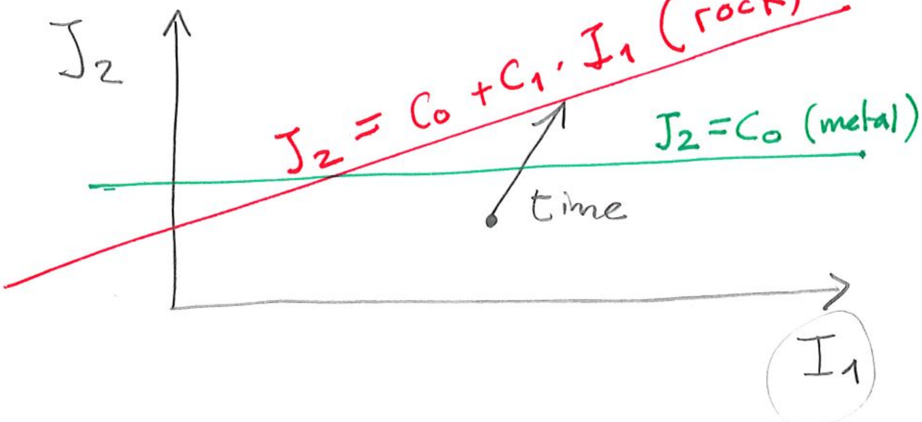
$$I_2(\underline{\underline{\sigma}}) = \sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33}+\sigma_{22}\sigma_{33}-\sigma_{12}^2-\sigma_{13}^2-\sigma_{23}^2$$

$$I_3(\underline{\underline{\sigma}}) = \det(\underline{\underline{\sigma}})$$

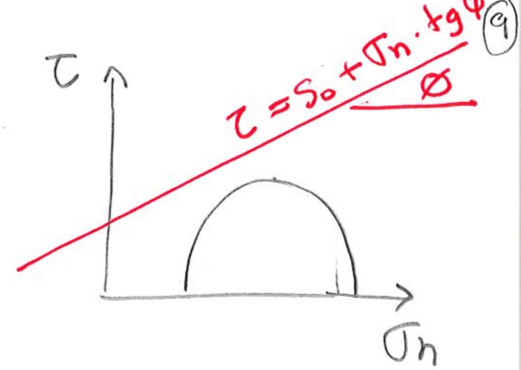
$$J_1(\underline{\underline{S}}_d) = 0$$

$$J_2(\underline{\underline{S}}_d) = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]$$

$$J_3(\underline{\underline{S}}_d) = \ldots$$



$\approx$



$$\left\{\begin{array}{l} p' = \sigma_m = \dfrac{I_1(\underline{\sigma})}{3} \\ q = \sqrt{3 \cdot J_2} \end{array}\right\}$$

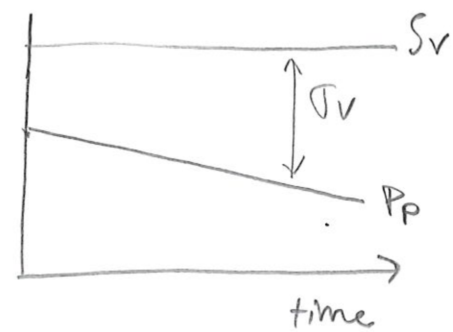
critical state soil mechanics

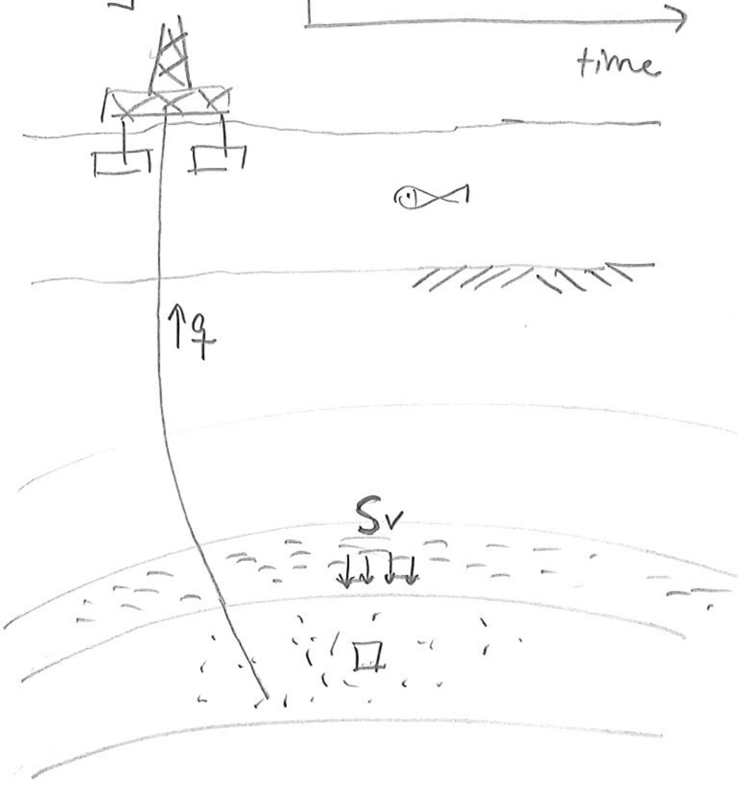
$\sigma_1, \quad \sigma_2 = \sigma_3$

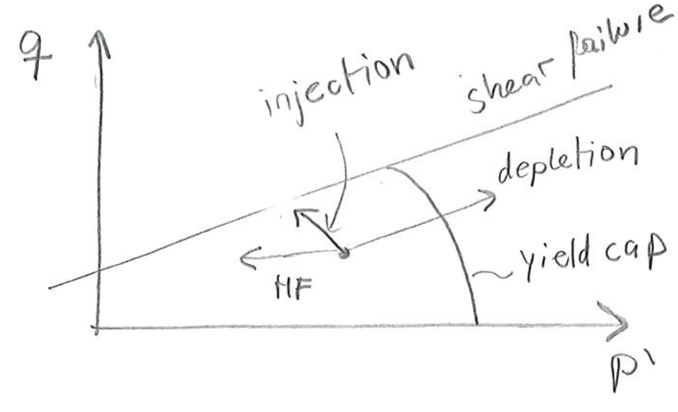
$$q = \sqrt{3 \, \frac{1}{6}\left[(\sigma_1-\sigma_3)^2 + (\sigma_1-\sigma_3)^2\right]}$$

$$\boxed{q = \sigma_1 - \sigma_3}$$

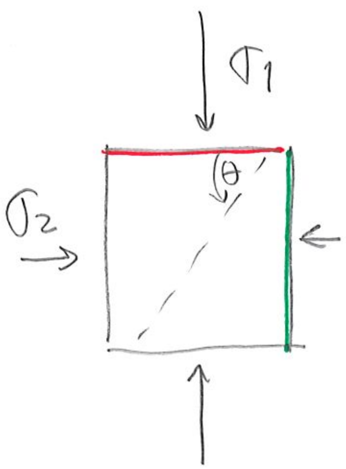
$$\boxed{p = \frac{\sigma_1 + 2\sigma_3}{3}}$$

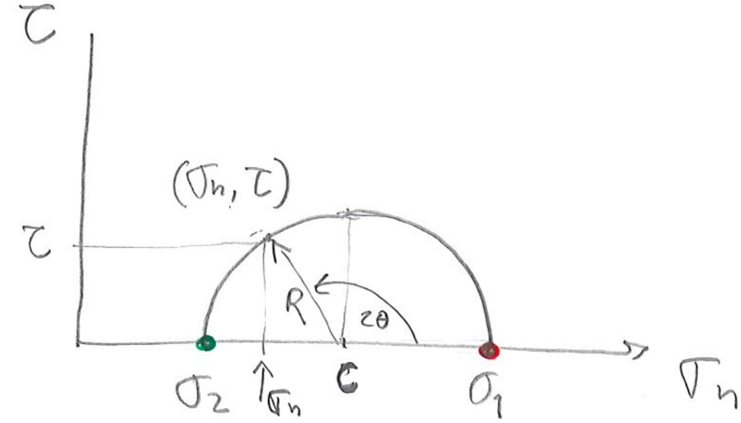






Stress projection on a plane



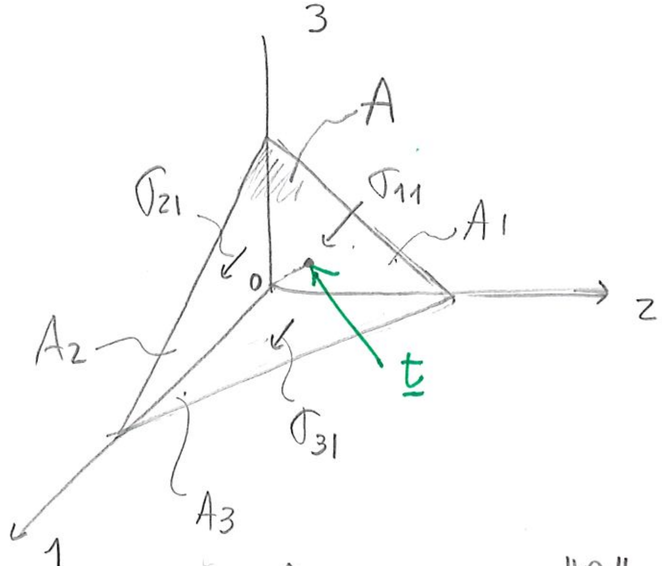


$$C = \frac{\sigma_1 + \sigma_2}{2} \quad ; \quad R = \frac{\sigma_1 - \sigma_2}{2}$$

$$\begin{cases} \sigma_n = C + R \cdot \underline{\cos(2\theta)} \\ \tau = R \sin(2\theta) \end{cases}$$

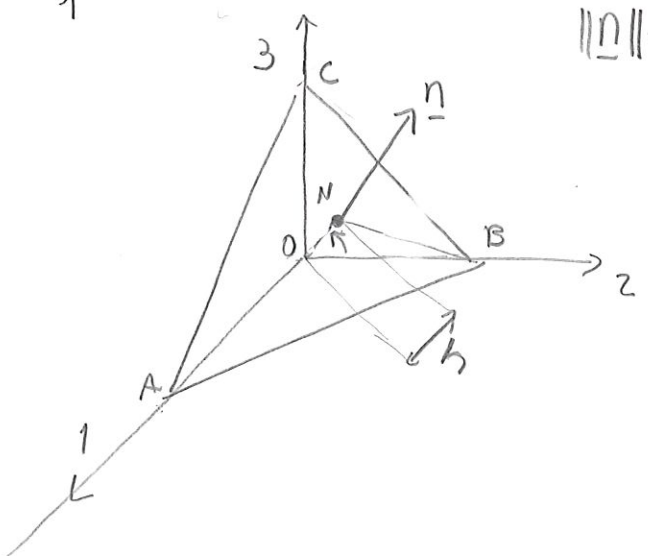
Stress projection (3D)

$\Sigma F_1 = 0$



① $\sigma_{11} A_1 + \sigma_{21} A_2 + \sigma_{31} A_3 = t_1 A$

② $$\begin{cases} \text{Vol} = \frac{1}{3} A \cdot h \\ \quad = \frac{1}{3} A_1 \overline{OA} \\ \cancel{\frac{1}{3}} A h = \cancel{\frac{1}{3}} A_1 \overline{OA} \end{cases}$$



$\|\underline{n}\| = 1$

$$A_1 = \frac{h}{\overline{OA}} A$$

$$\begin{cases} A_1 = \cos \widehat{AON} \cdot A \\ A_2 = \cos \widehat{BON}\ A \\ A_3 = \cos \widehat{CON}\ A \end{cases}$$

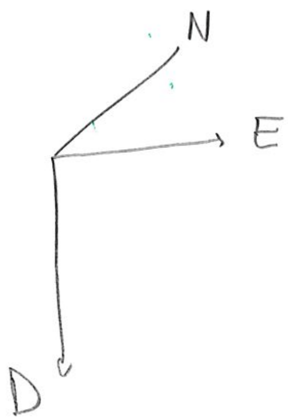
$$\underline{A_i = n_i A}$$

①-② $\sigma_{11} n_1 \cancel{A} + \sigma_{21} n_2 \cancel{A} + \sigma_{31} n_3 \cancel{A} = t_1 \cancel{A}$

$\underline{t} = \underline{\sigma} \cdot \underline{n} \leftarrow$ ③ $$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

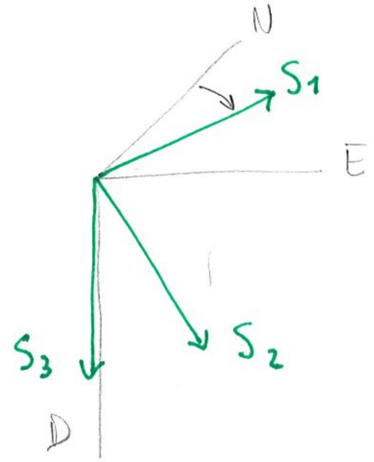
$$\begin{cases} \sigma_n = \underline{t} \cdot \underline{n} \\ \tau^2 = t^2 - \sigma_n^2 \end{cases}$$

N - E - D



Geographical CS

$S_1 - S_2 - S_3$



Principal Stress CS

$$\underline{\underline{S}}_G = \begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix} \leftarrow \underline{\underline{S}}_P = \begin{vmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{vmatrix}$$

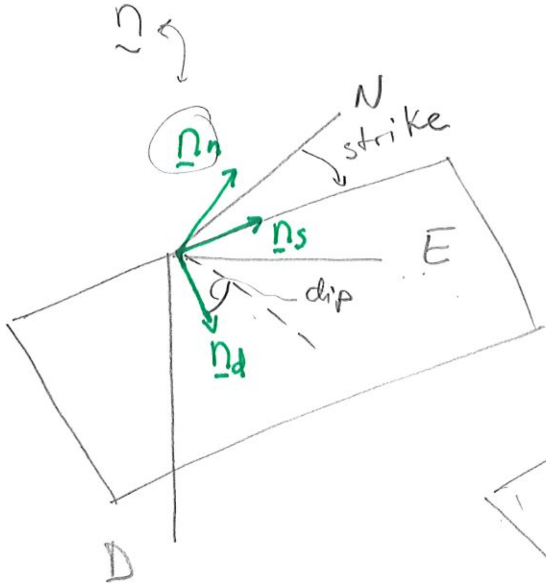
S_1 at $\overbrace{30^\circ N}^{\text{azimuth}}$

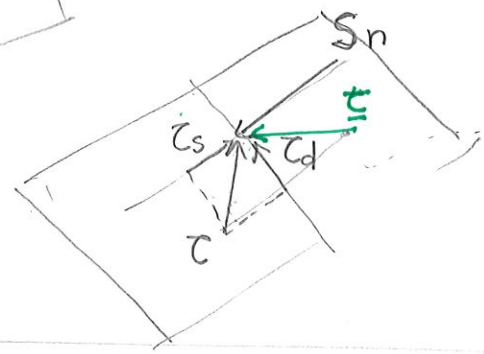
S_3 is vertical

$$\underline{\underline{S}}_G = R_{PG}^T \, \underline{\underline{S}}_P \, R_{PG}$$

$$R_{PG} = \underbrace{\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix}}_{f = (\alpha, \beta, \gamma)}$$

d-s-n





$$\underline{n}_n = \begin{bmatrix} - \\ - \\ - \end{bmatrix}; \quad \underline{n}_d = \begin{bmatrix} - \\ - \\ - \end{bmatrix}; \quad \underline{n}_s = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$$

f (dip, strike)

$$\underline{t}' = \underline{\underline{\sigma}}_G \cdot \underline{n}_n$$

$$\underline{t} = \underline{\underline{S}}_G \cdot \underline{n}_n$$

$S_n = \underline{t} \cdot \underline{n}_n$ } Total normal stress

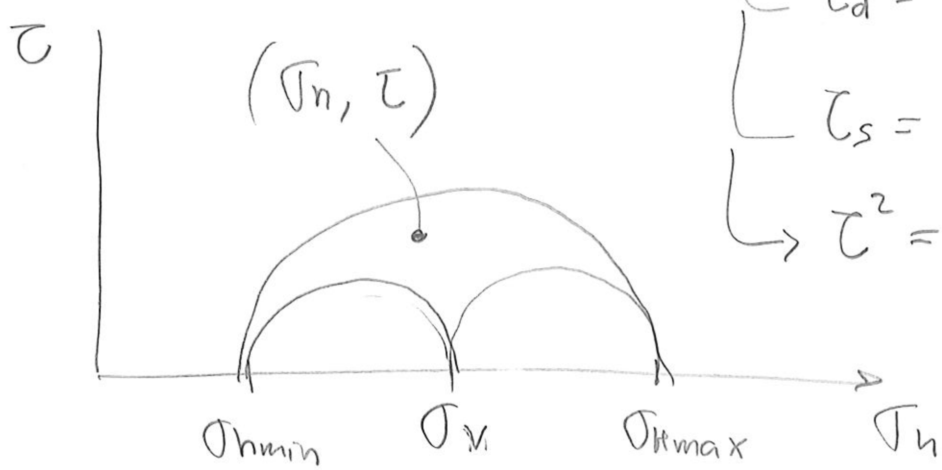
$\sigma_n = S_n - P_p$ } Eff normal stress

$\tau = \sqrt{\|\underline{t}\|^2 - S_n^2}$ } Shear stress

$\tau_d = \underline{t} \cdot \underline{n}_d$

$\tau_s = \underline{t} \cdot \underline{n}_s$

$\tau^2 = \tau_d^2 + \tau_s^2$



General solution to an elasticity problem

Equilib

$$\nabla \cdot \underline{\underline{\sigma}} + \underline{f} = (\rho \, Vol) \, \underline{a} = 0 \quad \Rightarrow \underline{F} = m \cdot \underline{a}$$

$$\underline{\underline{\varepsilon}} = f_1(\underline{u}) \leftarrow \text{Kinematic Equations} \begin{cases} \text{large strain} \\ \text{small strain} \end{cases}$$

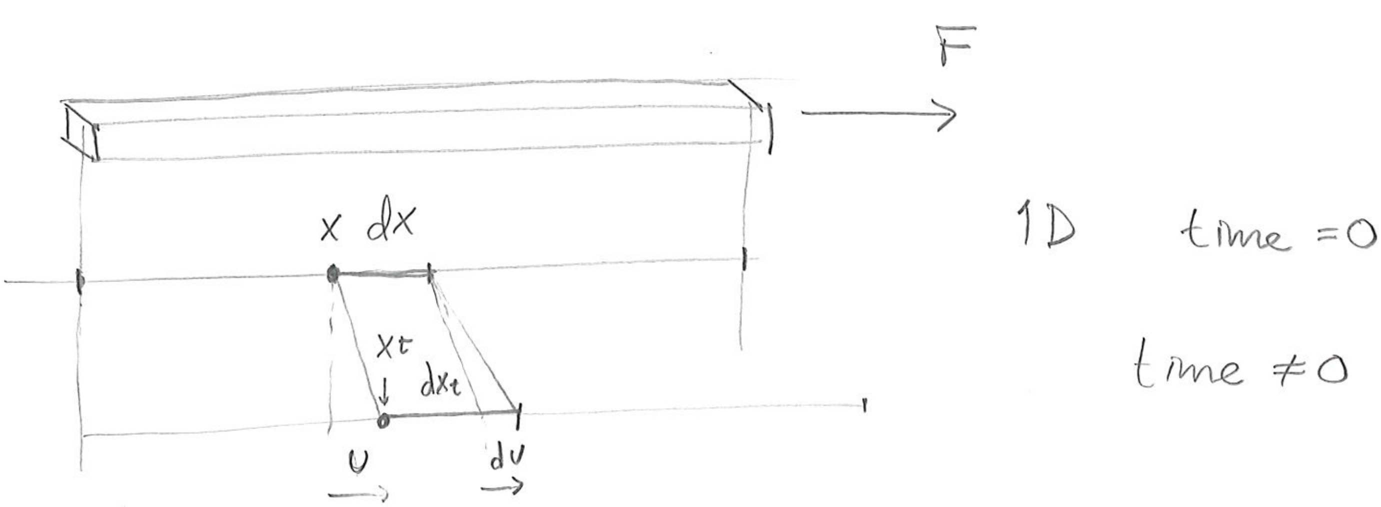
$$\underline{\underline{\sigma}} = f_2(\underline{\underline{\varepsilon}}) \leftarrow \text{Constitutive Equations}$$

properties of the material

$$\nabla \cdot \left(f_2\left(f_1\left(\underline{u}\right)\right)\right) + \underline{f} = 0$$

unknown

Kinematic equations; small strains



$$\varepsilon = \frac{dx_t - dx}{dx} = \frac{\left[\cancel{x+u}+\cancel{dx} + du - \cancel{(x+u)}\right] - \cancel{dx}}{dx}$$

$$\boxed{\varepsilon = \frac{du}{dx}}$$ → in the same direction

$$\underline{\underline{\varepsilon}} = \begin{vmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ & \varepsilon_{22} & \varepsilon_{23} \\ & & \varepsilon_{33} \end{vmatrix} = \underbrace{\begin{vmatrix} \frac{\partial u_1}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_3}\right) \\ & & \frac{\partial u_3}{\partial x_3} \end{vmatrix}}_{\text{small strains}}$$

$$\underline{\underline{\varepsilon}} = f_1(\underline{u})$$

Constitutive Equations

$$\underline{\underline{\sigma}} = f_1(\underline{\underline{\varepsilon}})$$

$$\downarrow$$

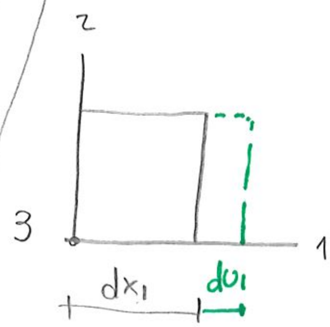
$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}}$$

9 var.	81 param	9 var.	} no symm
6	36 param	6	} symm
	6x6		

$$\underline{\underline{\sigma}} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{vmatrix} \xrightarrow{\text{Voigt Notation}} \begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & \cdots & C_{16} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \cdots & & & & C_{66} \end{vmatrix} \begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{vmatrix}$$

Jacobian

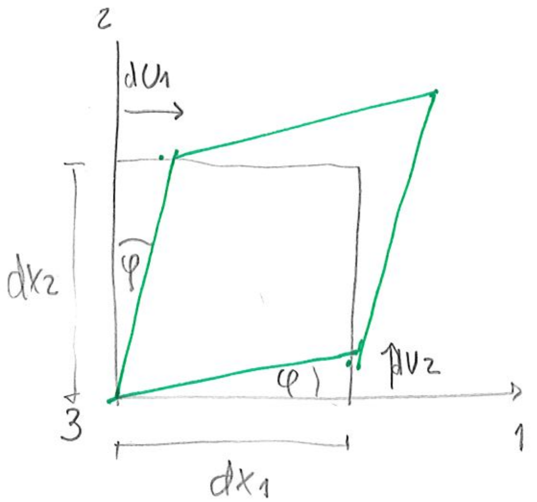
$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial u_2}{\partial x_2} & 0 \\ 0 & 0 & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$



$$\underline{\varepsilon_{11} = \frac{du_1}{dx_1}}$$

$$+ \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right) \\ - & - & 0 \end{bmatrix} \rightarrow \text{Symm}$$

$$\Big\} \ \underline{\underline{\varepsilon}}$$



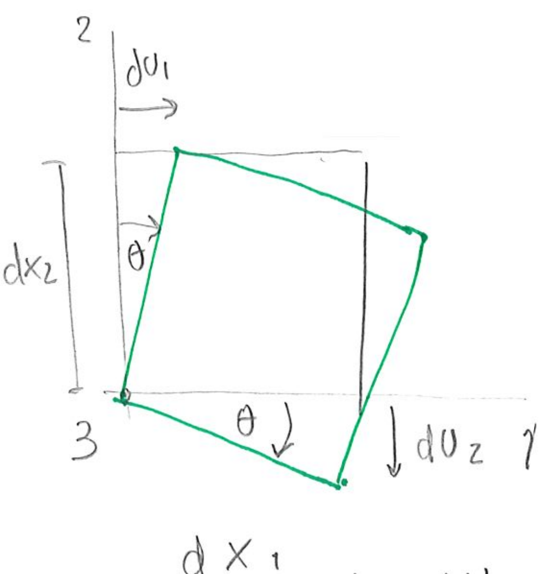
$$\varepsilon_{12} \cong \frac{1}{2}(2 \, tg\, \varphi) = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)$$

$$+ \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}-\frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}-\frac{\partial u_3}{\partial x_2}\right) \\ & & 0 \end{bmatrix} \Rightarrow \underline{\underline{\omega}}$$

rotation matrix

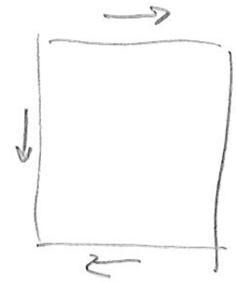
$\rightarrow \neq$ Symm

Skew Symm



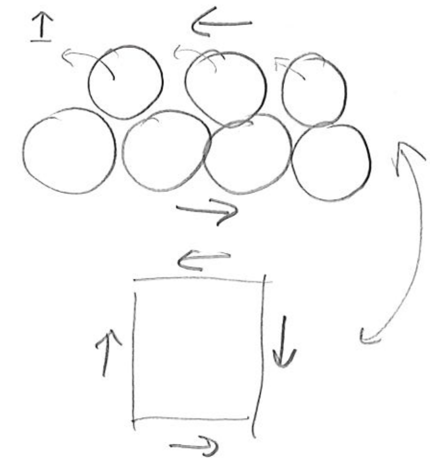
$$\omega_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}-\frac{\partial u_2}{\partial x_1}\right)$$

$$
\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}
=
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
\underbrace{0 \quad 0 \quad 0}_{} & & & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}
$$



shear decoupling

sand dilation is an exception

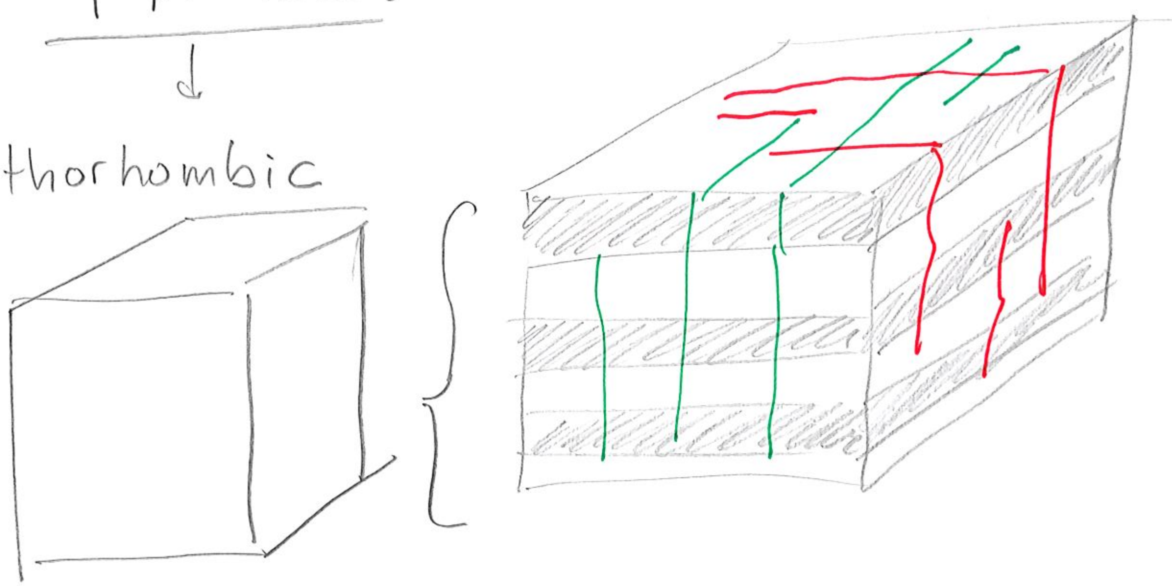


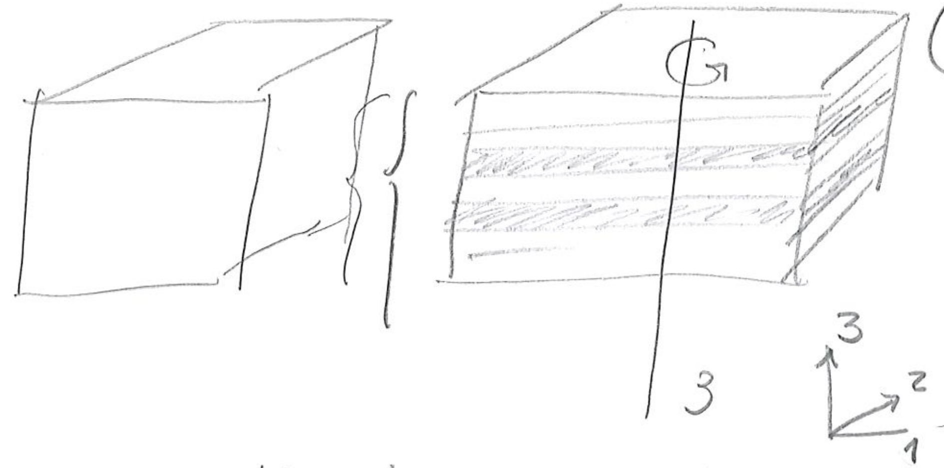
↓

9 parameters

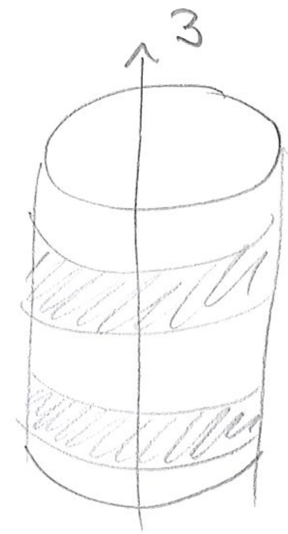
↓

orthorhombic



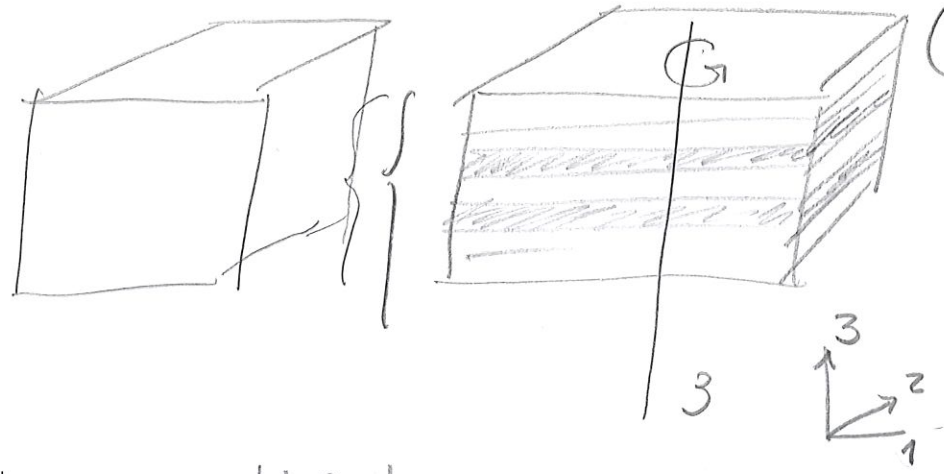


$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix}
=
\left[\begin{array}{ccc|ccc}
C_{11} & C_{12} & C_{13} & & & \\
 & C_{11} & C_{13} & & & \\
 & & C_{33} & & & \\
\hline
 & & & C_{44} & & \\
 & & & & C_{44} & \\
 & & & & & C_{66}
\end{array}\right]
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \hline 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{Bmatrix}
$$

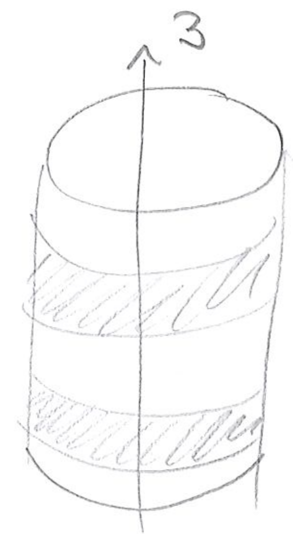


$$C_{66} = \frac{C_{11} - C_{12}}{2}$$

5 ind parameters $\Rightarrow$ Transverse Isotropy



$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & & & \\ & C_{11} & C_{13} & & & \\ & & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{44} & \\ & & & & & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{pmatrix}$$



$$\underbrace{C_{66} = \frac{C_{11} - C_{12}}{2}}$$

5 ind parameters $\Longrightarrow$ Vertical Transverse Isotropy $\rightarrow$ VTI

$$\underline{\underline{\sigma}} = f_2 (\underline{\underline{\varepsilon}})$$

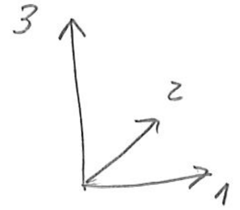
$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}}$$

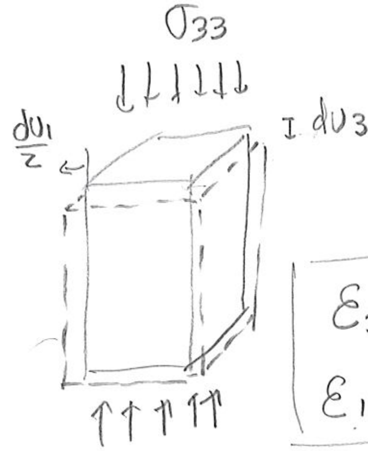
$$\underline{\underline{\sigma}} (\underline{\underline{\varepsilon}}_A + \underline{\underline{\varepsilon}}_B) = \underline{\underline{\sigma}} (\underline{\underline{\varepsilon}}_A) + \underline{\underline{\sigma}} (\underline{\underline{\varepsilon}}_B)$$

$$\underbrace{\underline{\underline{\sigma}} (a \underline{\underline{\varepsilon}}) = a \cdot \underline{\underline{\sigma}} (\underline{\underline{\varepsilon}})}$$

- suporposition principle
- convolution (time)

Homogeneous linear elastic isotropic solid





$$\boxed{\begin{aligned}\varepsilon_{33} &= \frac{\sigma_{33}}{E}\\ \varepsilon_{11} &= -\nu\cdot\varepsilon_{33}\end{aligned}}$$

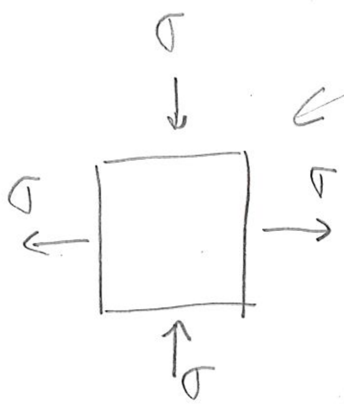
$$\underline{\underline{\sigma}} = \begin{vmatrix} 0 \\ 0 \\ \sigma_{33} \\ \hline 0 \\ 0 \\ 0 \end{vmatrix} \; ; \quad \underline{\underline{\varepsilon}} = \begin{vmatrix} -\nu\varepsilon_{33} \\ -\nu\,\varepsilon_{33} \\ \sigma_{33}/E \\ \hline 0 \\ 0 \\ 0 \end{vmatrix}$$

$\sigma_{11} = \sigma_{22} = 0$
↳ unconfined

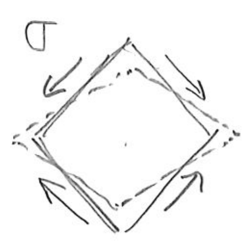
$$\varepsilon_{33} = \frac{dU_3}{dX_3} \; ; \quad \varepsilon_{11} = \frac{dU_1}{dX_1} = \frac{dU_2}{dX_2}$$

$$\begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \hline 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{vmatrix} = \begin{vmatrix} 1/E & -\nu/E & -\nu/E & & 0 & \\ -\nu/E & 1/E & -\nu/E & & & \\ -\nu/E & -\nu/E & 1/E & & & \\ \hline & & & \frac{2(1+\nu)}{E} & 0 & 0 \\ & 0 & & 0 & \frac{2(1+\nu)}{E} & 0 \\ & & & 0 & 0 & \frac{2(1+\nu)}{E} \end{vmatrix} \begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{vmatrix}$$

G^{-1}

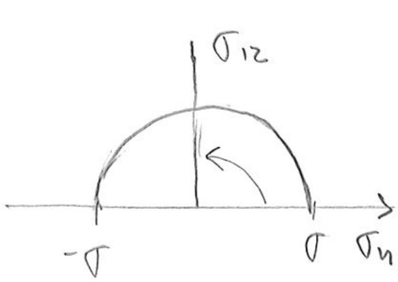


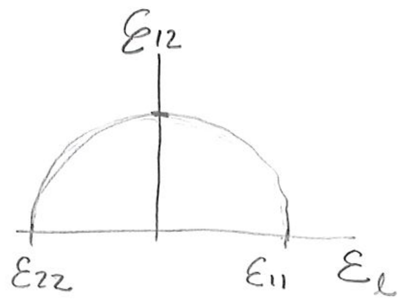
$\Rightarrow$



$$\begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \hline 2\varepsilon_{12} \end{vmatrix} = \begin{vmatrix} 1/E & -\nu/E & 0 \\ -\nu/E & 1/E & 0 \\ \hline 0 & 0 & \_ \end{vmatrix} \begin{vmatrix} \sigma \\ -\sigma \\ \hline 0 \end{vmatrix}$$

$$\begin{cases} \varepsilon_{11} = \dfrac{1+\nu}{E}\cdot\sigma \\ \varepsilon_{22} = -\dfrac{1+\nu}{E}\,\sigma \end{cases}$$





$$\varepsilon_{12} = [(1+\nu)/E]\,\sigma$$

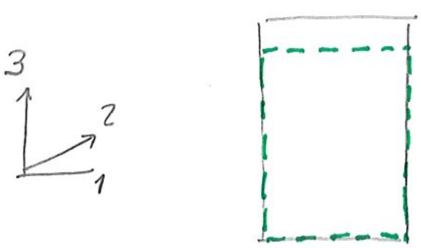
$$2\,\varepsilon_{12} = \frac{2(1+\nu)}{E}\,\sigma_{12}$$

$$\underline{\underline{\varepsilon}} = \underbrace{\underline{\underline{\underline{\underline{D}}}}}_{\text{compliance}} \cdot \underline{\underline{\sigma}} \rightarrow \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{D}}}}^{-1} \underline{\underline{\varepsilon}}$$

$$\underline{\underline{\sigma}} = \underbrace{\underline{\underline{\underline{\underline{C}}}}}_{\text{stiffness}} \underline{\underline{\varepsilon}}$$

$$\underbrace{\begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{vmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{vmatrix} 1-\nu & \nu & \nu & & & \\ & 1-\nu & \nu & & 0 & \\ & & 1-\nu & & & \\ \hline & & & \frac{1-2\nu}{2} & & \\ & 0 & & & \frac{1-2\nu}{2} & \\ & & & & & \frac{1-2\nu}{2} \end{vmatrix}}_{\underline{\underline{\underline{\underline{C}}}}} \begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \hline 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{vmatrix}$$

longitudinal modulus
constrained "
oedometric "



one-dimensional strain

$\varepsilon_{33} \neq 0$

$\varepsilon_{11} = \varepsilon_{22} = 0$

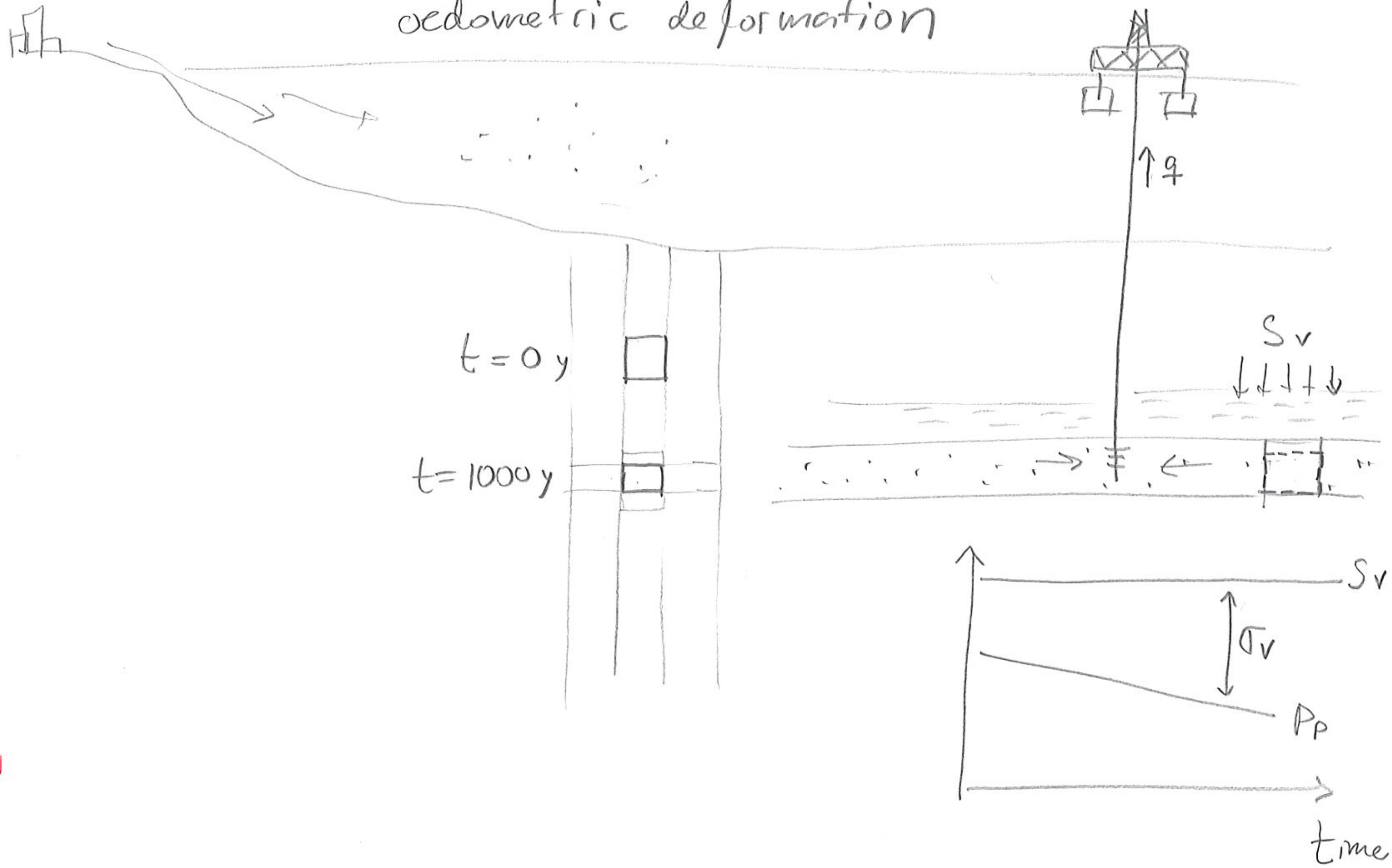
$$\sigma_{33} = M \varepsilon_{33} = \left[\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\right] \varepsilon_{33}$$

$$\sigma_{11} = \sigma_{22} = \boxed{\frac{\nu}{1-\nu}} \sigma_{33}$$

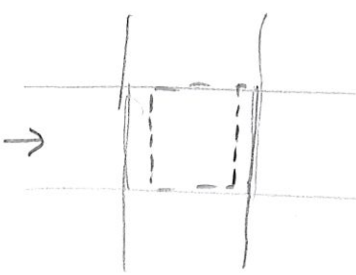
lateral stress coeff $= K$

$$S_{11} = \sigma_{11} + P_p$$

uniaxial deformation
oedometric deformation

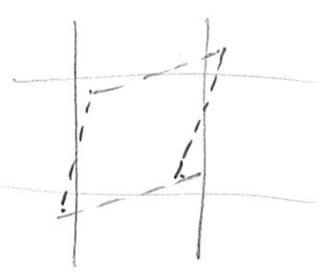


$$V_p = \sqrt{\frac{M}{\rho}} \rightarrow$$



$$M = \frac{E\,(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$V_s = \sqrt{\frac{G}{\rho}} \rightarrow$$



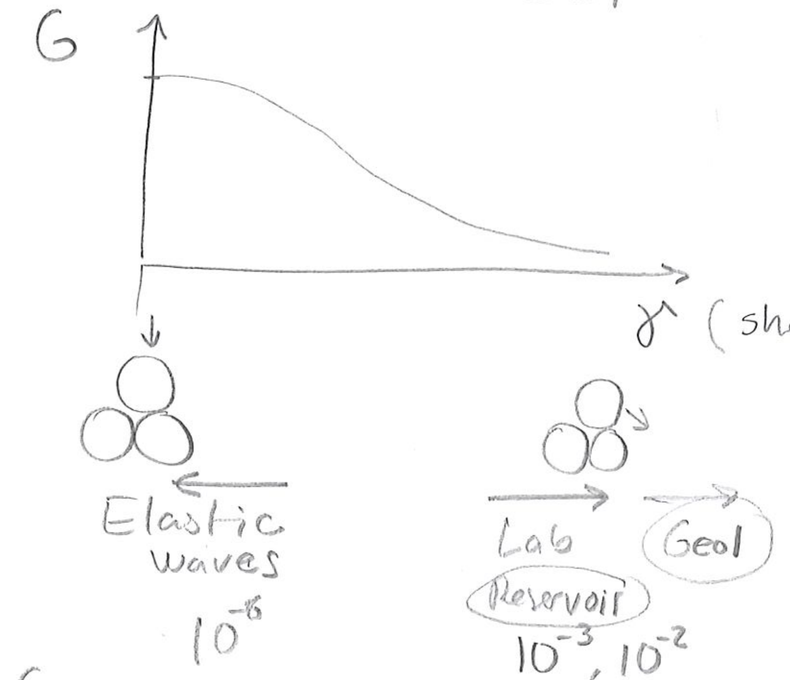
$$G = \frac{E}{2(1+\nu)}$$

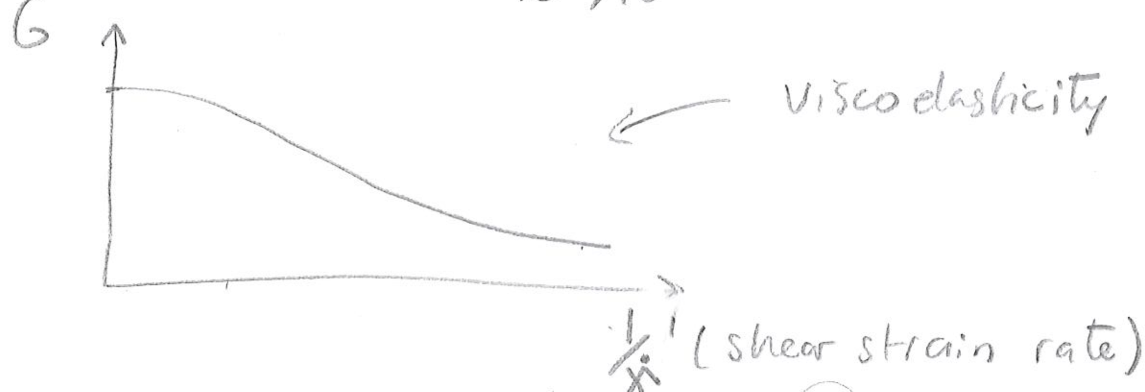
$$\begin{cases} E_{dyn} = \rho\, V_s^2 \left(\dfrac{3 V_p^2 - 4 V_s^2}{V_p^2 - V_s^2} \right) \\[2ex] \nu_{dyn} = \dfrac{V_p^2 - 2 V_s^2}{2\,(V_p^2 - V_s^2)} \end{cases}$$

$$E_{dyn} \neq E_{st}$$
$$\nu_{dyn} \neq \nu_{st}$$

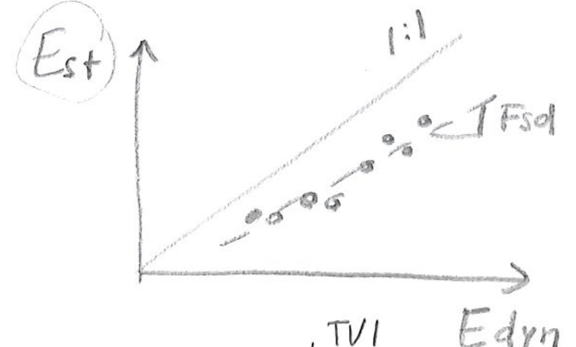
$$G_{dyn} \neq G_{st}$$
$$G_{dyn} > G_{st}$$

$$\tau = G \cdot \underbrace{\gamma}_{\text{shear strain}}$$

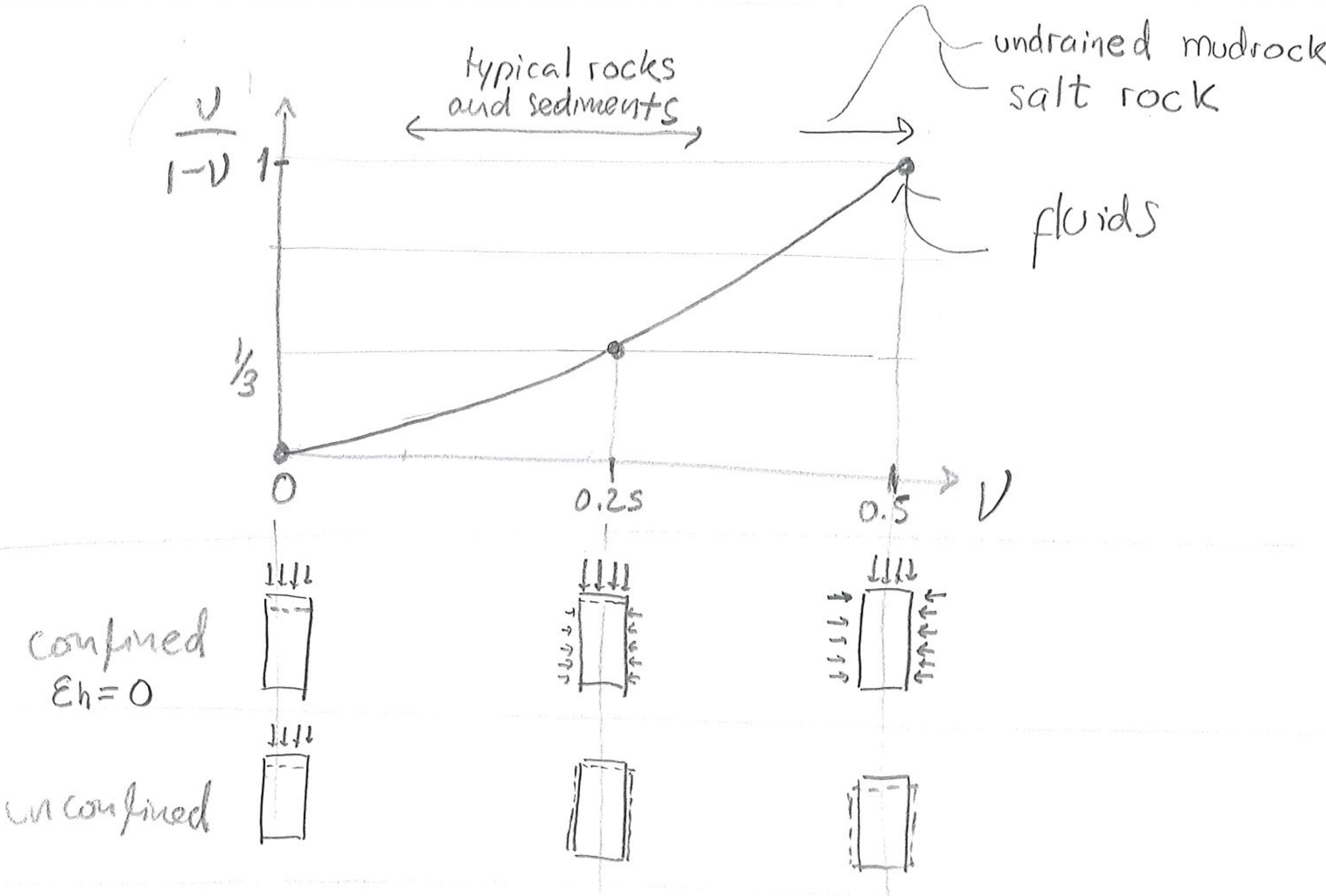




$$\boxed{\begin{aligned} \nu_{st} &= \nu_{dyn} \\ E_{st} &= F_{sd} \cdot E_{dyn} \end{aligned}}$$



$$C_{ij}^{st} = F_{ij} \, C_{ij}^{dyn} - \left| \begin{array}{l} \text{TVI} \\ C_{11}, C_{33} \\ C_{13}\ C_{12}, C_{44} \end{array} \right.$$



$$\varepsilon_{11} \neq 0; \quad \varepsilon_{22} \neq 0$$

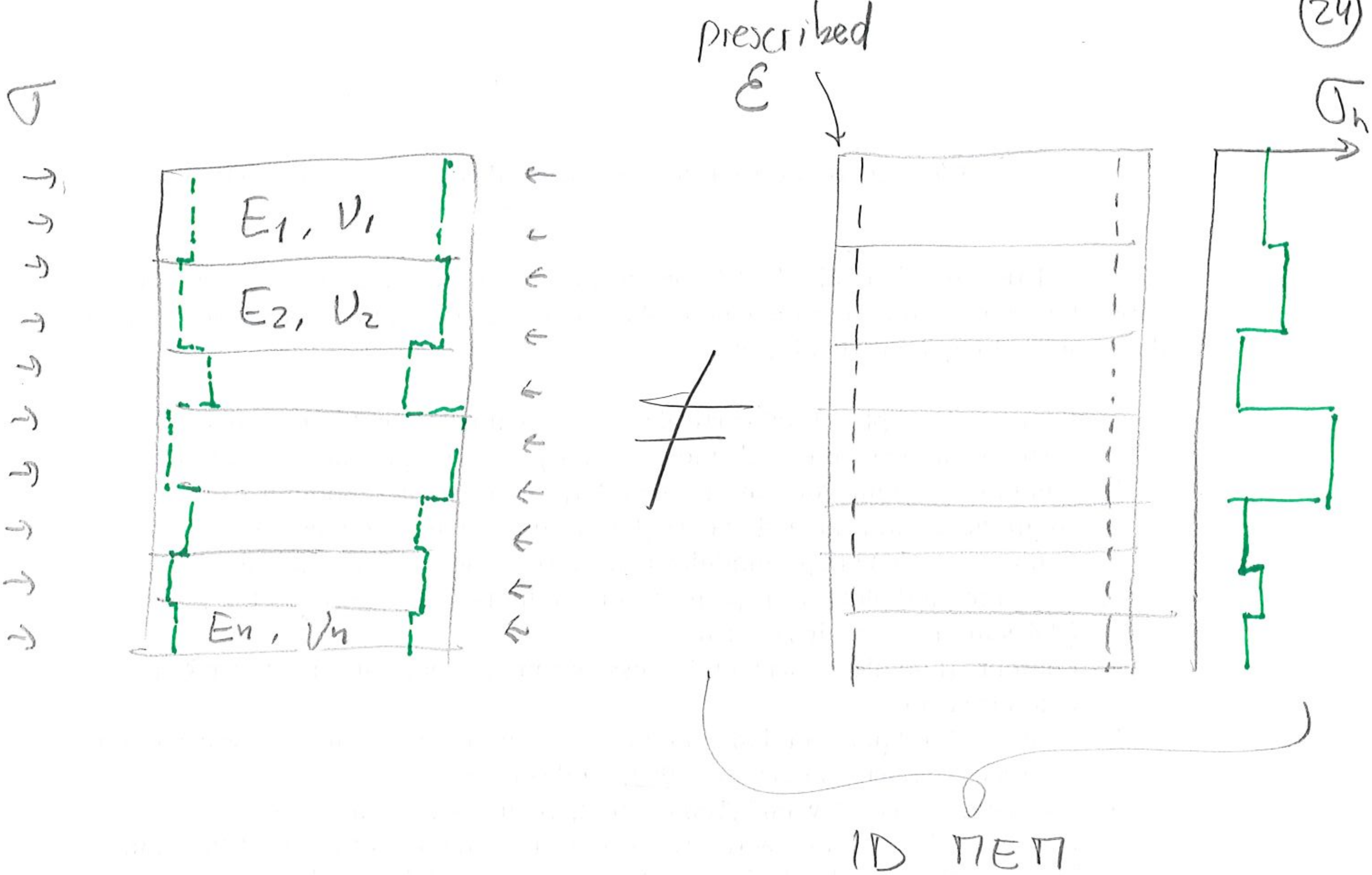
$$\begin{cases} \sigma_{11} = \dfrac{E}{1-\nu^2}\varepsilon_{11} + \dfrac{\nu E}{1-\nu^2}\varepsilon_{22} + \dfrac{\nu}{1-\nu}\sigma_{33} \\ \sigma_{22} = \underbrace{\dfrac{\nu \cdot E}{1-\nu^2}\varepsilon_{11} + \dfrac{E}{1-\nu^2}\varepsilon_{22}}_{\text{tectonic stresses}} + \underbrace{\dfrac{\nu}{1-\nu}\sigma_{33}}_{\text{overburden component}} \end{cases}$$

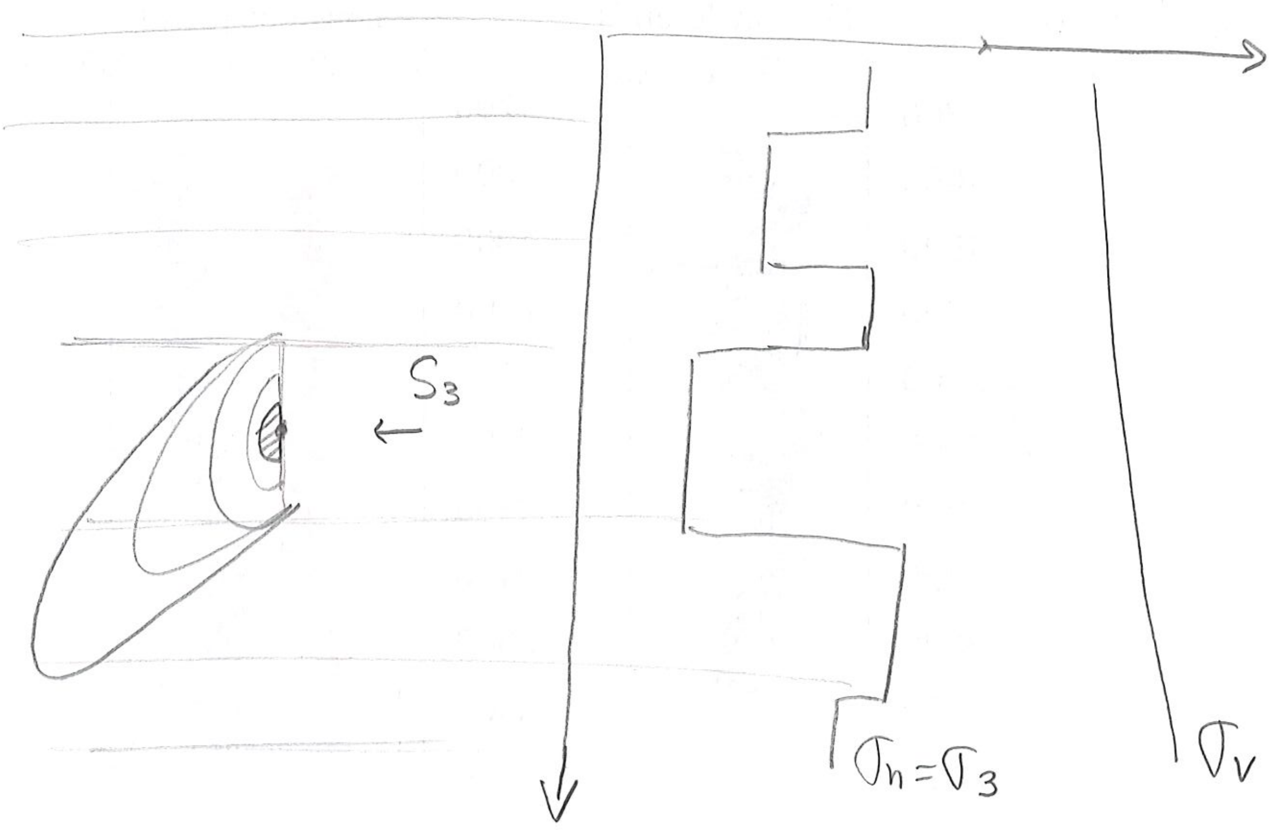
$\varepsilon_{11}, \varepsilon_{22} \rightarrow$ tectonic strains

$$E' = \frac{E}{1-\nu^2} \quad \text{(plane strain modulus)}$$

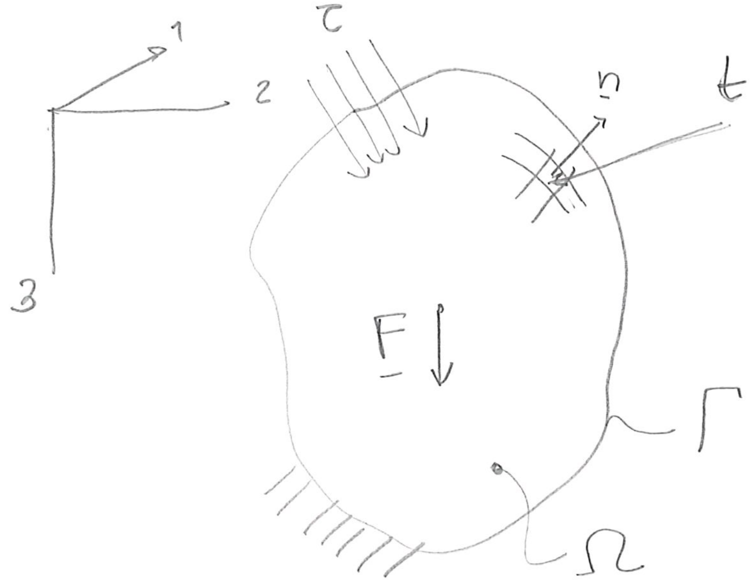
$\rightarrow \sigma_{11}, \sigma_{22} \rightarrow S_{11}, S_{22}$

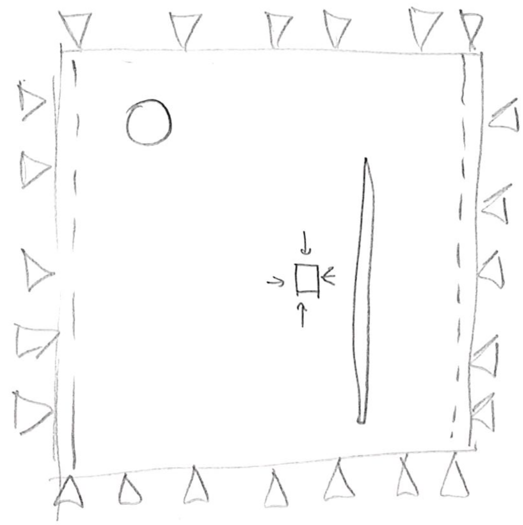
$\rightarrow$ 1D Mechanical Earth Model ($\underline{E_{st}, \nu_{st}}$)


σ
E1, ν1
E2, ν2
En, νn
≠
prescribed ε
σh
1D MEM


S3
σh = σ3
σv

General problem





$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} \overbrace{1-\nu}^{(1-2\nu+\nu)} & \nu & \nu & & & \\ & 1-\nu & \nu & & 0 & \\ & & 1-\nu & & & \\ & & & \frac{1-2\nu}{2} & & \\ & 0 & & & \frac{1-2\nu}{2} & \\ & & & & & \frac{1-2\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\\ 2\varepsilon_{12}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ & \lambda+2\mu & \lambda & & & \\ & & \lambda+2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\\ 2\varepsilon_{12}\end{bmatrix}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad G = \mu = \frac{E}{2(1+\nu)}$$

$$\nabla \cdot \underline{\underline{\sigma}} + \underline{f} = 0 \qquad (1)$$

$$\underline{\underline{\varepsilon}} = F(\underline{u}) \qquad (2)$$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} \qquad (3)$$

$$\underline{q} = -\frac{K}{\mu} \nabla \underset{\sim}{P}$$

coord 1 $\rightarrow \dfrac{\partial \sigma_{11}}{\partial x_1} + \dfrac{\partial \sigma_{12}}{\partial x_2} + \dfrac{\partial \sigma_{13}}{\partial x_3} + f_1 = 0$

(3) $\rightarrow$ (1) : (4)

$$\frac{\partial}{\partial x_1}\Big(\lambda(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}) + 2\mu\varepsilon_{11}\Big) + \frac{\partial}{\partial x_2}\Big(2\mu\varepsilon_{12}\Big) + \frac{\partial}{\partial x_3}\Big(2\mu\varepsilon_{13}\Big) + f_1 = 0$$

(2) $\rightarrow$ (4)

$$\frac{\partial}{\partial x_1}\left(\lambda\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)+2\mu\frac{\partial u_1}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\cancel{2}\mu\frac{1}{\cancel{2}}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)\right)$$

$$+\frac{\partial}{\partial x_3}\left(\cancel{2}\mu\frac{1}{\cancel{2}}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)\right) + f_1 = 0$$

$$\lambda\left[\frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)\right] + 2\mu\frac{\partial^2 u_1}{\partial x_1^2} + \mu\frac{\partial^2 u_1}{\partial x_2^2} + \mu\frac{\partial}{\partial x_2}\left(\frac{\partial u_2}{\partial x_1}\right) + \mu\frac{\partial^2 u_1}{\partial x_3^2} + \mu\frac{\partial}{\partial x_3}\left(\frac{\partial u_3}{\partial x_1}\right) + f_1 = 0$$

$$\frac{\partial^2(f(x,y))}{\partial x\,\partial y} = \frac{\partial^2(f(x,y))}{\partial y\,\partial x}$$

$$\lambda\left[\frac{\partial}{\partial x_1}\underbrace{\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)}_{\nabla\cdot\underline{u}}\right] + \mu\nabla^2 u_1 + \mu\left[\frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}\right)\right] + f_1 = 0$$

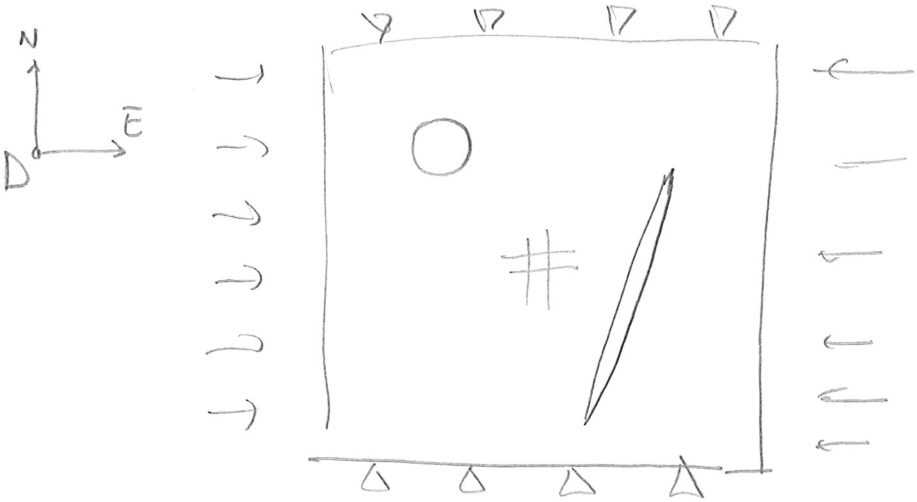
$$\nabla^2 u_1 = \frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} + \frac{\partial^2 u_1}{\partial x_3^2}$$

$$\boxed{(\lambda+\mu)\nabla(\nabla\cdot\underline{u}) + \mu\nabla^2\underline{u} + \underline{F} = 0}$$

Navier's Equation
linear elasticity

Analytical solution

- Belttrami-Mitchell Eq
  - strain compatibility
    - $\underline{u} \rightarrow \underline{\underline{\varepsilon}}$ ✓
    - $\underline{\underline{\varepsilon}} \rightarrow \underline{u}$ not straightforward
- Airy's equation
  - $\nabla^4 \varphi = 0$
    - $\frac{\partial^2 \varphi}{\partial x_1^2} = \sigma_{11}$; $\frac{\partial^2 \varphi}{\partial x_2} = \sigma_{22}$; $\frac{\partial^2 \varphi}{\partial x_1 \partial x_2} = \sigma_{12}$
- Kirsch → Jgor, Zoback, Jaeger
- Griffith → Jaeger,
- Sneddon



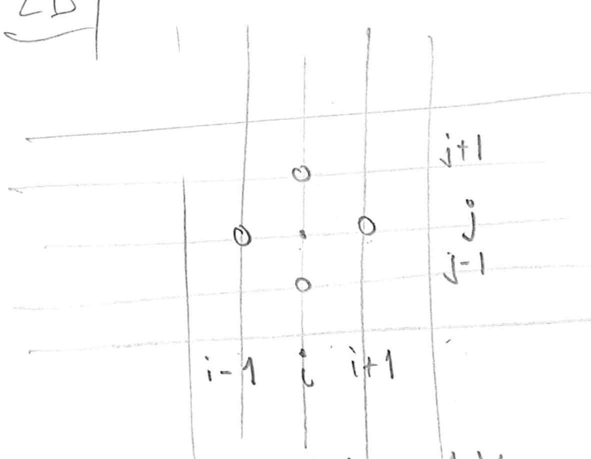
- Finite differences (FLAC)

$$\begin{cases} \underline{q} = -\dfrac{K}{\mu} \nabla P & \text{(constitutive)} \\ \nabla \cdot \underline{q} = 0 & \text{(mass conserv)} \end{cases}$$

$$-\frac{K}{\mu}\left(\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial P}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial P}{\partial z}\right)\right) = 0$$

$$\boxed{\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = 0}$$

$\boxed{2D}$



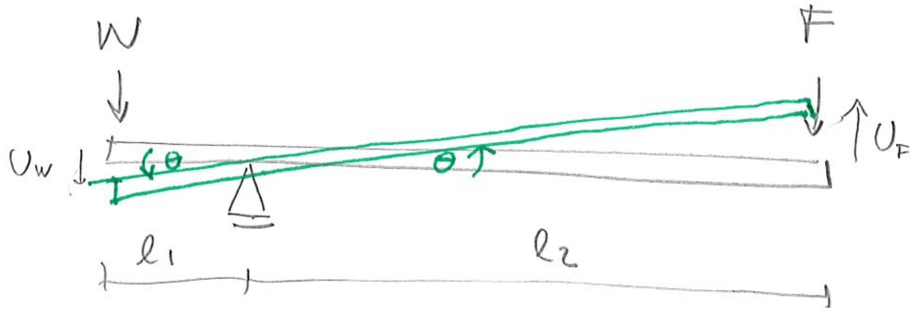
$\Delta x = \Delta y$

$$0 = \frac{\dfrac{P_{i+1}^{j} - P_i^j}{\Delta x} - \dfrac{P_i^j - P_{i-1}^j}{\Delta x}}{\Delta x} + \frac{\dfrac{P_i^{j+1} - P_i^j}{\Delta y} - \dfrac{P_i^j - P_i^{j-1}}{\Delta y}}{\Delta y}$$

$$\boxed{P_i^j = \frac{P_{i+1}^j + P_{i-1}^j + P_i^{j+1} + P_i^{j-1}}{4}}$$

Finite Element Method

Eq Ang Momentum



$$W l_1 - F l_2 = 0$$

$$\boxed{F = \frac{l_1}{l_2} W}$$

Energy conservation → Principle of virtual work

$$W \cdot \cancel{\tan} \theta \, l_1 = F \, \cancel{\tan} \theta \, l_2 \rightarrow \boxed{F = \frac{l_1}{l_2} W}$$

$$\nabla \cdot \underline{\underline{\sigma}} + \underline{F} = 0$$

$$-\nabla \cdot \sigma = \underline{F}$$

virtual displacement

$$\int_\Omega \delta \underline{u} \cdot (-\nabla \cdot \underline{\underline{\sigma}}) = \int_\Omega \delta \underline{u} \cdot \underline{F}$$

↓ Green's Theorem

$$\underbrace{\int_\Omega \nabla \delta \underset{\sim}{u} \cdot \underline{\underline{\sigma}}} - \int_\Gamma \delta \underset{\sim}{u} \cdot (\underline{\underline{\sigma}} \cdot \underline{n}) = \int_\Omega \delta \underline{u} \cdot \underline{F}$$

variational form ↓ weak form

Tensorial product

$$\int_\Omega \underbrace{\varepsilon(\nabla \underset{\sim}{u}) : \underline{\underline{\sigma}}(\underline{u})}_{\text{strain energy}} = \underbrace{\int_\Gamma \delta \underset{\sim}{u} \cdot (\underline{\underline{\sigma}} \cdot \underline{n})}_{\text{boundary conditions}} + \underbrace{\int_\Omega \delta \underset{\sim}{u} \cdot \underline{F}}_{\text{body forces}}$$

$$\varepsilon(\nabla u) : \underline{\underline{\sigma}}(u) = \varepsilon_{11} \cdot \sigma_{11} + \varepsilon_{22} \cdot \sigma_{22} + \varepsilon_{33}\, \sigma_{33} + \varepsilon_{12} \cdot \sigma_{12} + \ldots$$

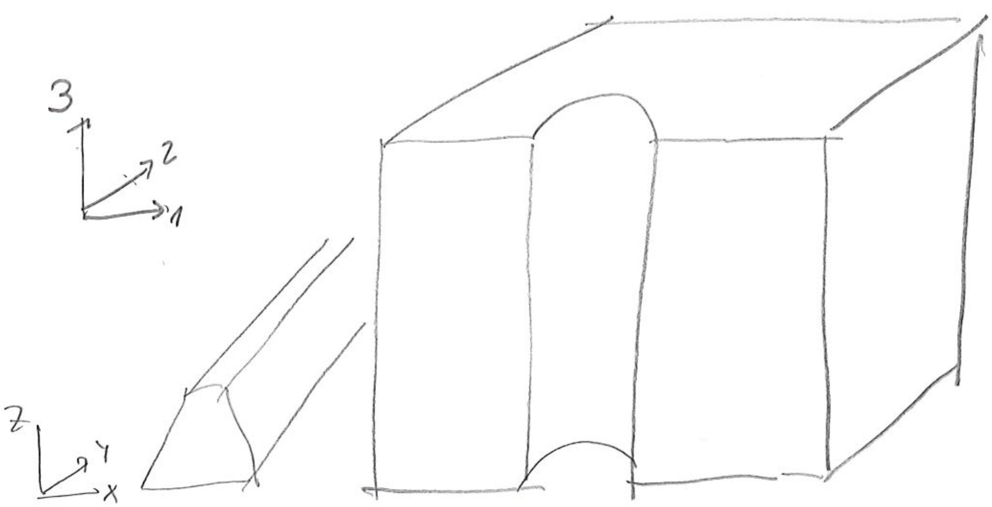
$$E = P \cdot V$$

$$\frac{E}{V} = \sigma \cdot \frac{V}{V}$$

→ unknowns → $\delta \underline{u}$, $\underline{u}$ actual displacements

Plane strain solution

$$\frac{\partial}{\partial x_3} = 0$$



$$\frac{\partial u_3}{\partial x_3} = 0 \rightarrow \varepsilon_{33} = 0$$

$$\frac{\partial u_1}{\partial x_3} = 0 \rightarrow \varepsilon_{13} = 0 \Rightarrow \sigma_{13} = 0$$

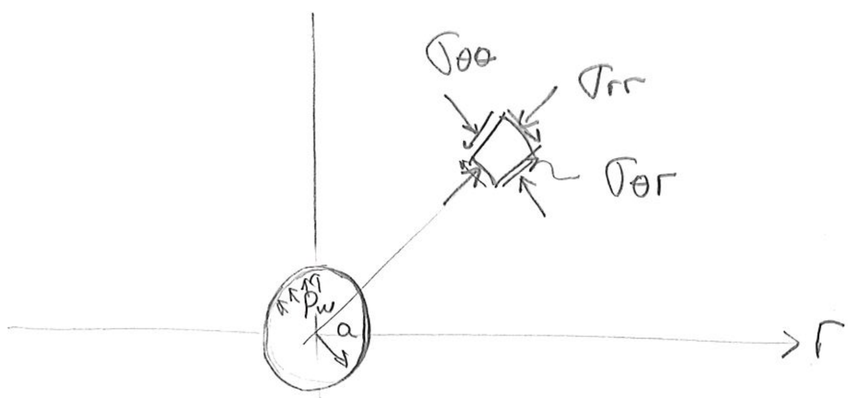
$$\frac{\partial u_2}{\partial x_3} = 0 \rightarrow \varepsilon_{23} = 0 \Rightarrow \sigma_{23} = 0$$

$$\begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \cancel{\sigma_{23}} \\ \cancel{\sigma_{13}} \\ \sigma_{12} \end{vmatrix} = \begin{vmatrix} & \\ & \end{vmatrix} \begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \cancel{\varepsilon_{33}} \\ \hline 2\cancel{\varepsilon_{23}} \\ 2\cancel{\varepsilon_{13}} \\ 2\varepsilon_{12} \end{vmatrix} 0 \Rightarrow \begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{vmatrix} = \begin{vmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ \hline 0 & 0 & \mu \end{vmatrix} \begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{vmatrix}$$

$$\sigma_{33} = \nu\left(\sigma_{11} + \sigma_{22}\right)$$

Plane stress solution $\rightarrow$ $\sigma_{33} = 0$

$\downarrow$ <u>Kirsch equations</u>



$$r = a \begin{cases} \sigma_{\theta r} = 0 \\ \sigma_{rr} = (P_w - P_p) \end{cases}$$

$P_w = 0$

$\sigma_{\theta\theta} < -T_s$ tensile strength

$\llcorner$ Tensile frac

$\sigma_{\theta\theta} > UCS$

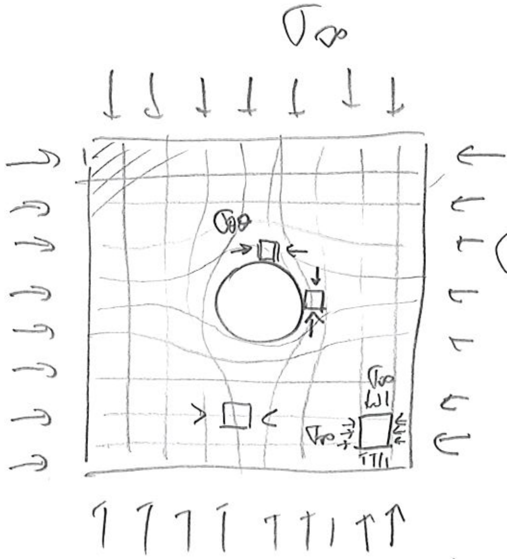
$\llcorner$ shear failure

$\hookrightarrow$ breakout

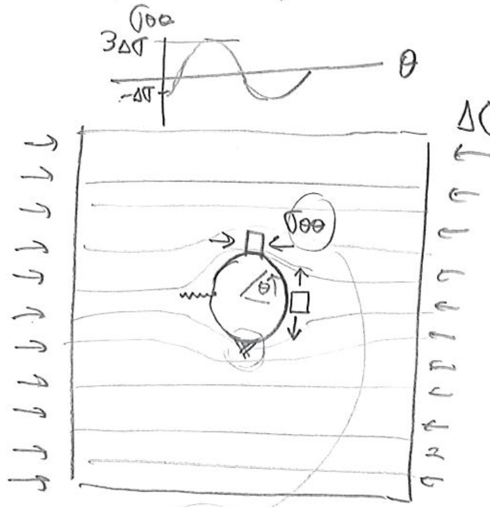
$$\sigma_1 = S_1 - P_p$$
$$\sigma_2 = S_2 - P_p$$

Kirsch

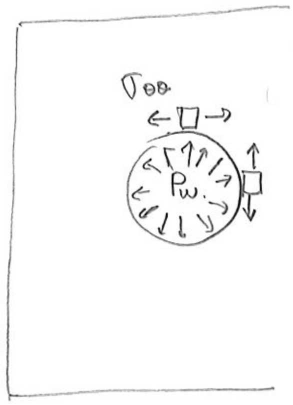
=



+



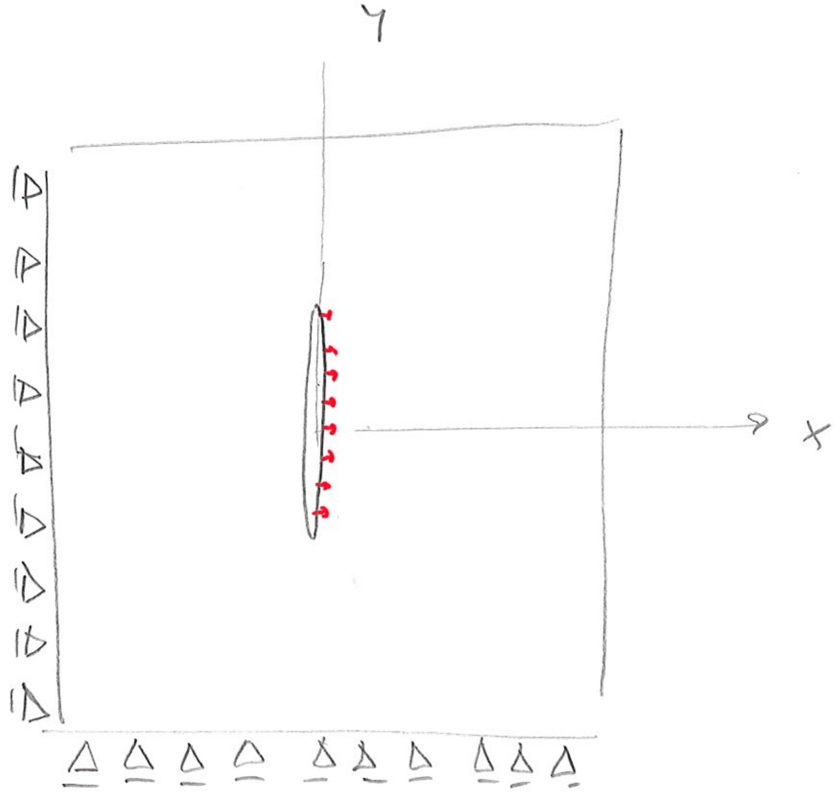
+



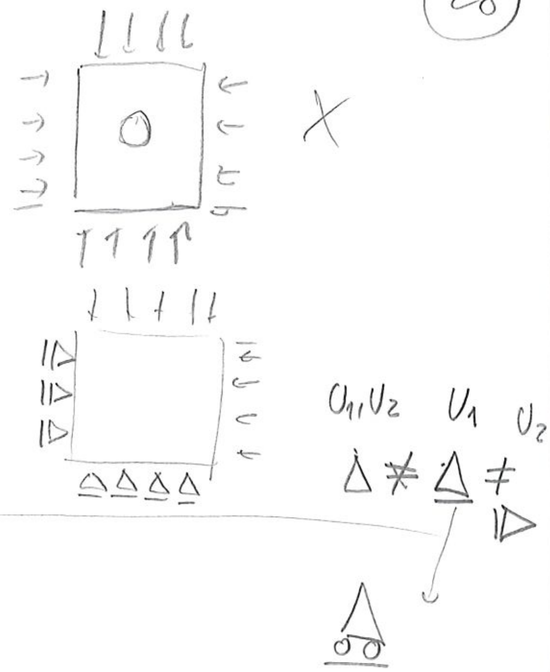
$\frac{\sigma_{\theta\theta}}{\sigma_\infty} = 2 \leftarrow$ stress intensity factor

$\frac{\sigma_{\theta\theta}}{\Delta\sigma} = 3 \rightarrow$

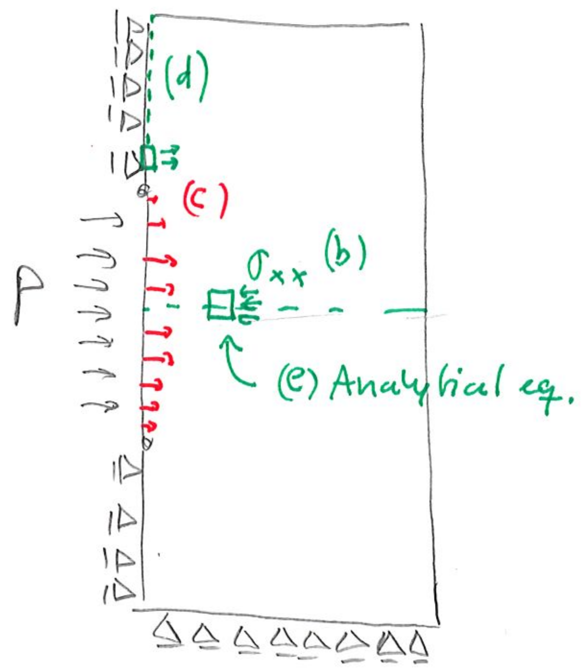
$\frac{\sigma_{\theta\theta}}{P_w} = -2$



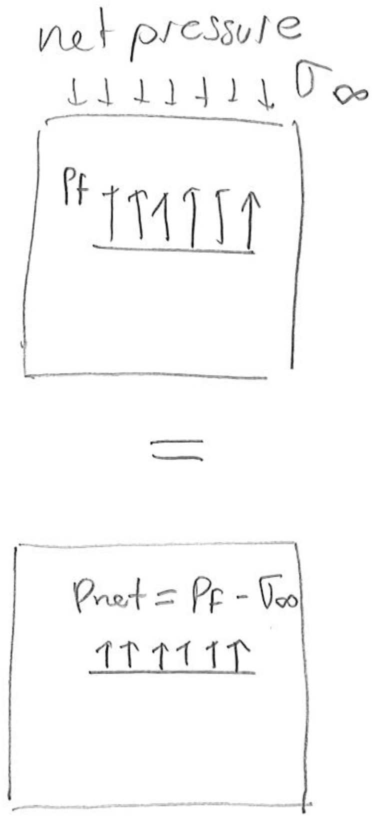
Y
x



X
U_1, U_2
U_1
U_2

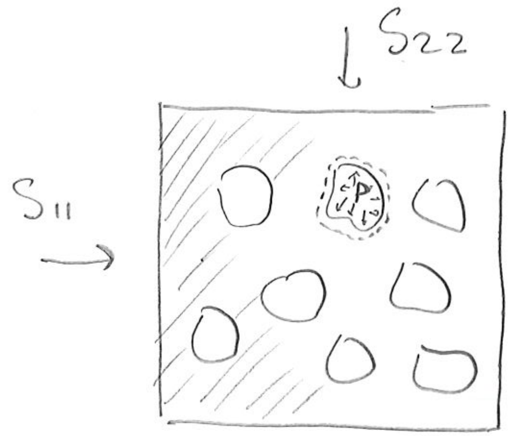


(d)
(c)
σ_{xx} (b)
P
(e) Analytical eq.



net pressure
σ_∞
P_f
=
$P_{net} = P_f - \sigma_\infty$

The porous solid (Coussy, 2010) Ch3 - Ch4



- pores → fluids, pressure, no shear
- deformable pore walls
- bulk volume $d\Omega$
  ↳ original $d\Omega_0$

@ current state $n = \dfrac{\square}{▨ + \square}$

Eulerian porosity

At any moment $n \cdot d\Omega = \underbrace{\phi}_{\text{Lagrangian porosity}} d\Omega_0$

$$\phi = \frac{n\,d\Omega}{d\Omega_0}$$ } Pore volume at time t / } Original bulk volume

$\varphi = \phi - \phi_0$ } Change of porosity

Change of $d\Omega \leftarrow \left(\overbrace{\epsilon_s}^{\text{solid strain}}, \overbrace{\varphi/\phi_0}^{\text{porosity strain}} \right)$

Solid

$\epsilon = \varepsilon_{vol}$

$$d\Omega^s = (1+\underset{\uparrow\ \text{def}}{\epsilon_s})\, d\Omega_0^s$$

$$(1-n)\, d\Omega = (1+\epsilon_s)(1-\phi_0)\, d\Omega_0$$

$$\left(1-\frac{\phi\, d\Omega_0}{d\Omega}\right)\boxed{d\Omega} = (+\epsilon_s)(1-\phi_0)\boxed{d\Omega_0}$$

$$\downarrow\ \epsilon = \frac{d\Omega}{d\Omega_0}-1$$

$$\left(1-\frac{\phi}{1+\epsilon}\right) = \frac{1+\epsilon_s}{1+\epsilon}(1-\phi_0)$$

$$1+\epsilon-\phi = (1+\epsilon_s)(1-\phi_0)$$

$$\boxed{\epsilon = (1-\phi_0)\,\epsilon_s + \overbrace{\underbrace{\phi-\phi_0}_{\frac{\varphi}{\phi_0}\cdot\phi_0}}^{\varphi}}$$

bulk vol strain

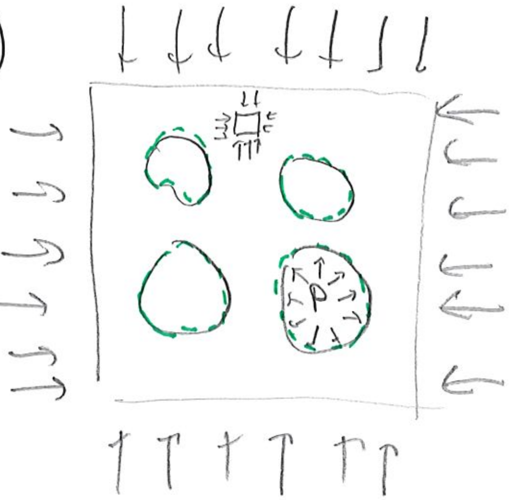
---

(1)

$$\cancelto{0}{\epsilon} = (1-\phi_0)\,\epsilon_s + \varphi$$

$$\boxed{\epsilon_s = -\frac{\varphi}{(1-\phi_0)}}$$

2

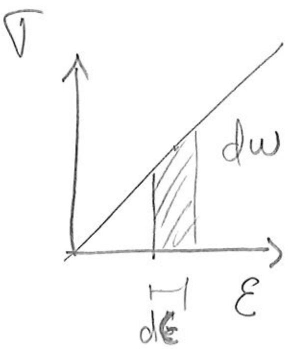


$\varphi = 0$

$$\boxed{\epsilon_s = \frac{\epsilon}{1-\phi_0}}$$

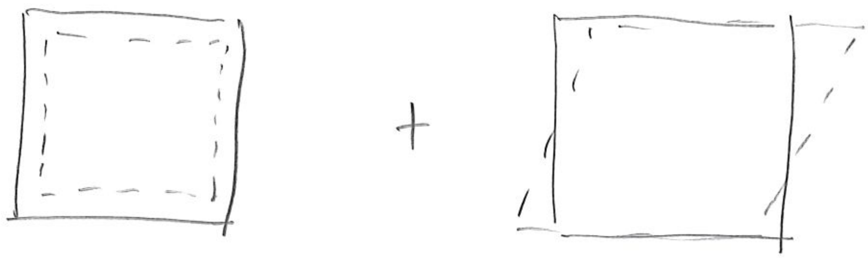
Free energy of the porous solid

→ strain energy



$\frac{1}{2}\sigma\, d\epsilon$

$\frac{1}{2} k (d\epsilon)^2$



$$dW = \underbrace{S_m}_{\substack{\text{mean stress} \\ \frac{S_{11}+S_{22}+S_{33}}{3}}} \cdot d\epsilon + \underbrace{S_{ij}}_{\substack{\text{deviatoric} \\ \text{stress tensor}}} \cdot \underbrace{de_{ij}}_{\substack{\text{deviatoric} \\ \text{strain tensor}}}$$

$$dW = S_m\, d\epsilon + S_{ij}\, de_{ij} + p \cdot \underset{d\phi}{d\varphi}$$

$=$

linear elasticity dW ↓ isotropic

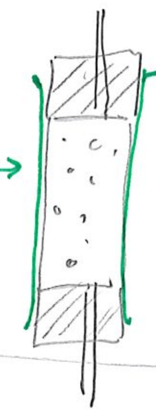
$$\eta_s = \frac{1}{2}\overset{\text{bulk mod}}{K}\epsilon^2 + \overset{\text{shear mod}}{G}\, e_{ij}\cdot e_{ij} - \underbrace{\overset{\text{cst}}{(\alpha)}}_{\text{unk}}\epsilon p - \underbrace{\overset{\text{cst}}{\left(\frac{1}{2}\frac{1}{N}\right)}}_{\text{unk}}p^2$$

$$\frac{\partial \eta_s}{\partial \epsilon} = S_m = K\epsilon - \alpha p \quad (1)$$

$$\frac{\partial \eta_s}{\partial e_{ij}} = s_{ij} = 2G e_{ij} \quad (2) \Longrightarrow \quad \alpha = ? \quad N = ?$$

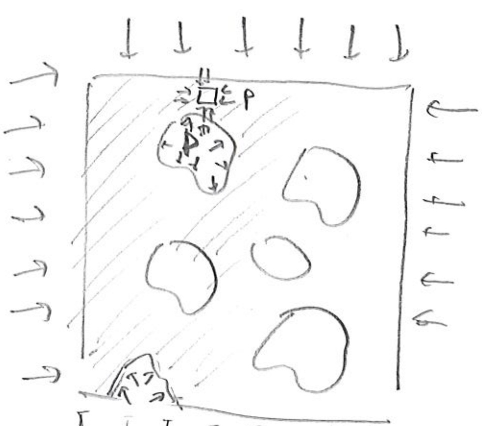
$$-\frac{\partial \eta_s}{\partial p} = \varphi = \alpha\epsilon + \frac{p}{N} \quad (3)$$

jacket →



→ ∅ all connected

Unjacketed loading



P

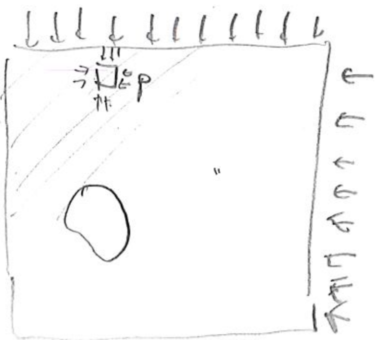
(1) $-P = K\epsilon - \alpha P$

↳ $\epsilon = -(1-\alpha)\dfrac{P}{K}$ → unj

$$\epsilon_s = -\frac{P}{K_s}$$

→ $\epsilon = \epsilon_s$ | porosity connected, isotropic homogeneous solid

no pores



$$(1-\alpha)\frac{\cancel{P}}{K} = \frac{\cancel{P}}{K_s}$$

Biot's coefficient $\boxed{\alpha = 1 - \dfrac{K}{K_s}}$

K: bulk drained "dry" modulus

K_s: solid matrix bulk modulus

$\alpha \leq 1$

3 $\varphi = \alpha \, \epsilon + P/N$

$$\varphi = \alpha\left(-\frac{P}{K_s}\right) + \frac{P}{N}$$

$$\varphi/\phi_0 = -\frac{P}{K_s}$$

$$\phi_0\left(-\frac{\cancel{P}}{K_s}\right) = \alpha\left(-\frac{\cancel{P}}{K_s}\right) + \frac{\cancel{P}}{N}$$

$$\boxed{\frac{1}{N} = \frac{\alpha - \phi_0}{K_s}}$$

second poromechanical modulus

→ pore modulus

→ $[N] = MPa$, $N > 0$

→ $\phi_0 \leq \alpha \leq 1$

Measurement of Biot's coefficient

→ Maurice (Belgium)

(1) $\alpha = 1 - \dfrac{K}{K_s}$

drained bulk modulus

→ Estimate $K_s \leftarrow K_s$ (quartz)

- if occluded ∅ X

(2) $\alpha = 1 - \dfrac{K_{jack}}{K_{unj}}$

drained bulk modulus ✓

↳ jacketed

water → salinity → sensitive

→ short circuit transducers

gas → 

oil → viscosity too high

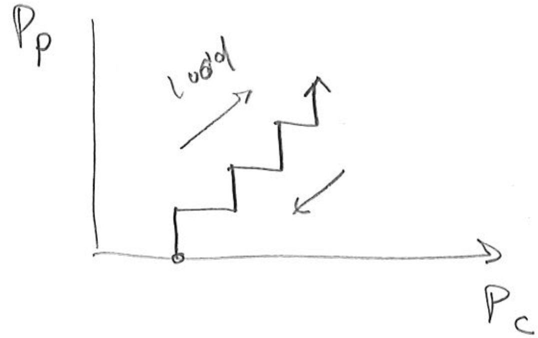
long time

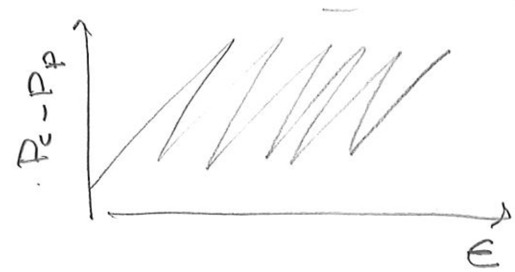
(3) $S_m = K\epsilon - \alpha p$

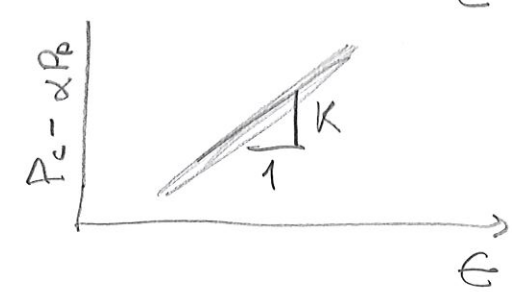
$$K\epsilon = \underbrace{S_m + \alpha P}_{\text{Biot's Eff stress}}$$

$$\epsilon = \frac{1}{K}\left(S_m + \alpha p\right)$$

$\alpha = \ldots$







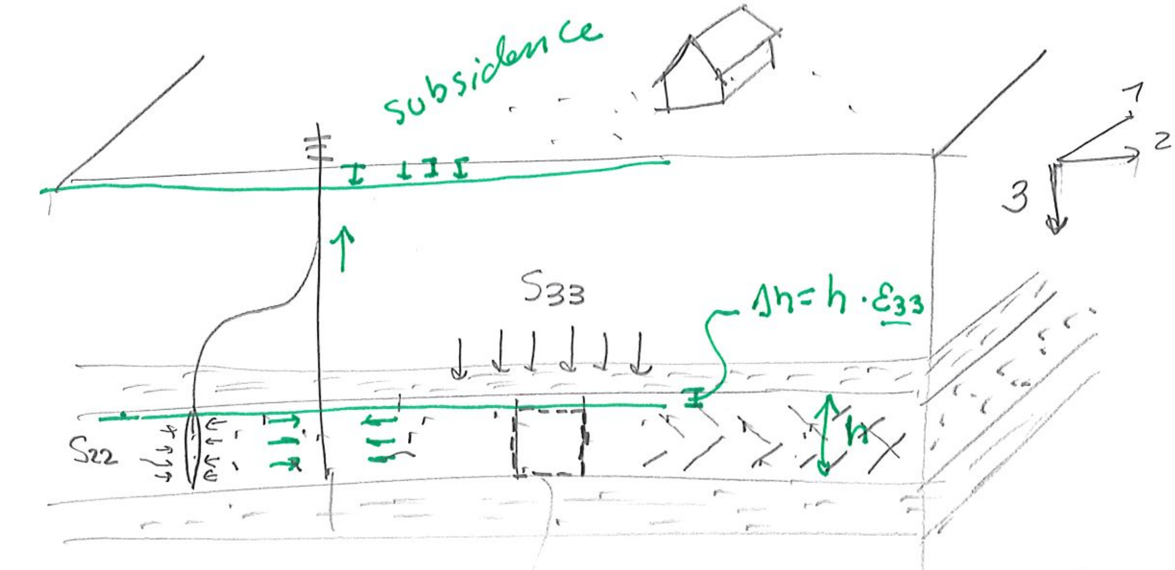
$$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \underline{\underline{I}}$$ } Terzaghi's effective stress

→ $$\underline{\underline{\sigma}} = \underline{\underline{S}} - \alpha P_p \underline{\underline{I}}$$ } Biot's effective stress

- Poroelaslicity
- φ → changes of porosity
- K_u → undrained bulk modulus
- undrained loading → pore pressure change
- "squirt flow" → fluid flow induced by changes in bulk strain

end-members of actual solution: drained / undrained } combination

Drained problem of reservoir depletion



1D strain condition $\varepsilon_{33} \neq 0$; $\varepsilon_{11} = \varepsilon_{22} = 0$

$$\Downarrow$$

$$S_{11} = S_{22}$$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} \rightarrow \underline{\underline{S}} - \alpha P_p \underline{\underline{I}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{D}}}} \cdot \underline{\underline{\sigma}}$$

linear isotropic elasticity

ΔP_p

$$\begin{bmatrix} \cancel{\varepsilon_{11}} \\ \cancel{\varepsilon_{22}} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & & 0 & \\ -\nu/E & 1/E & -\nu/E & & & \\ -\nu/E & -\nu/E & 1/E & & & \\ & & & 1/G & & \\ & 0 & & & 1/G & \\ & & & & & 1/G \end{bmatrix} \begin{bmatrix} S_{11} - \alpha P_p \\ S_{22} - \alpha P_p \\ S_{33} - \alpha P_p \\ S_{23} \\ S_{13} \\ S_{12} \end{bmatrix}$$

(37)

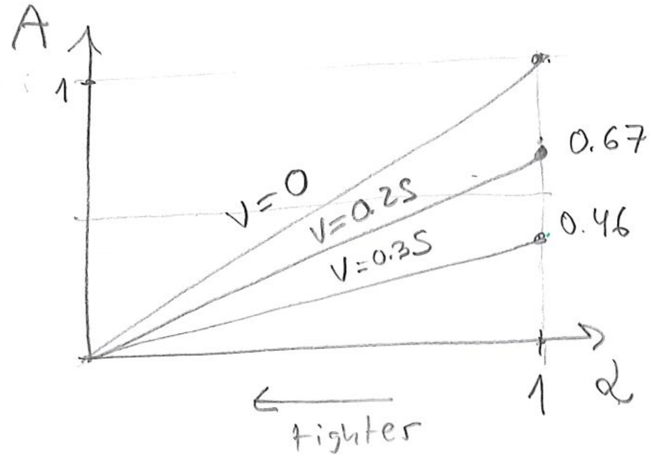
$$\begin{cases} \varepsilon_{11} = 0 = \left(\frac{1-\nu}{E}\right)(S_{22} - \alpha P_p) - \frac{\nu}{E}(S_{33} - \alpha P_p) & (1) \\ \varepsilon_{33} = -\frac{2\nu}{E}(S_{22} - \alpha P_p) + \frac{1}{E}(S_{33} - \alpha P_p) & (2) \end{cases}$$

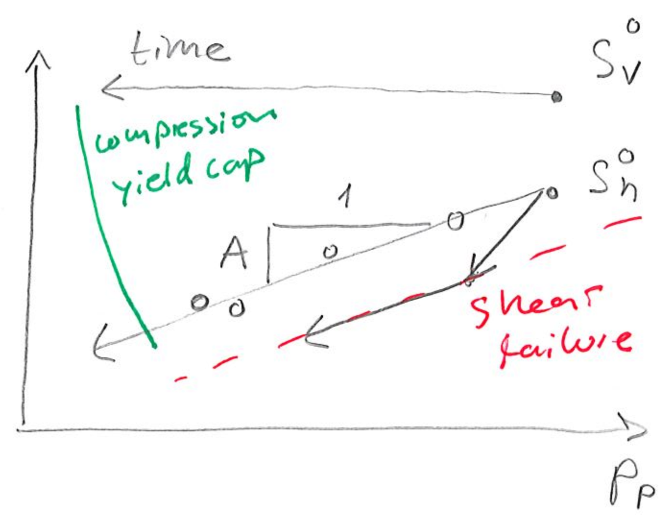
$$(1) \quad \underbrace{(S_{22} - \alpha P_p)}_{\sigma_{22}} = \frac{\nu}{1-\nu} \underbrace{(S_{33} - \alpha P_p)}_{\sigma_{33}} \quad \Bigg\} \text{ already known}$$

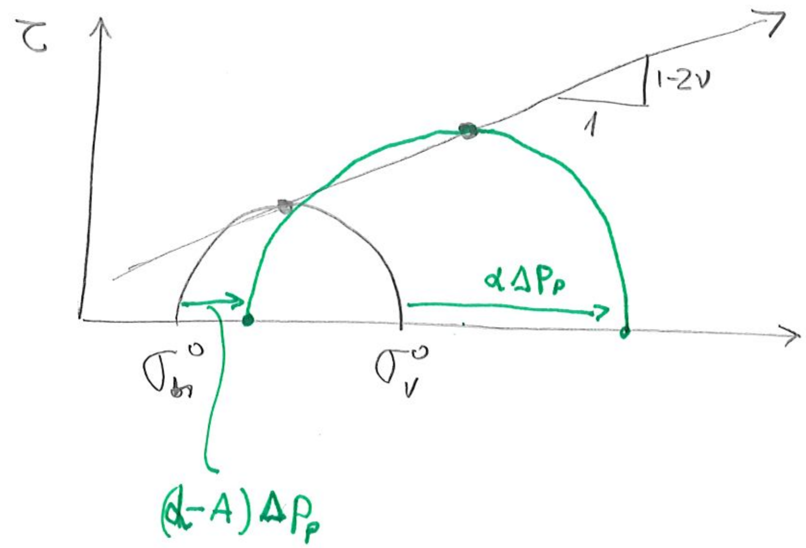
$$S_{22} = \frac{\nu}{1-\nu} S_{33} + \left[1 - \frac{\nu}{1-\nu}\right] \alpha P_p$$

$$\boxed{S_{22} = \frac{\nu}{1-\nu} S_{33} + \left(\frac{1-2\nu}{1-\nu}\right) \alpha P_p} \quad (3)$$

$$\left.\frac{\partial S_{22}}{\partial P_p}\right|_{S_{33}} = \alpha \frac{1-2\nu}{1-\nu} = A$$

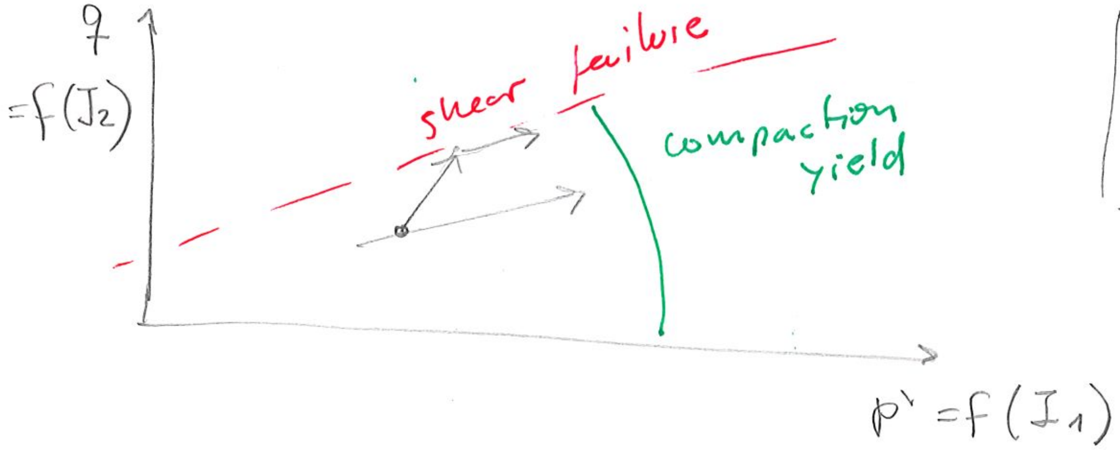


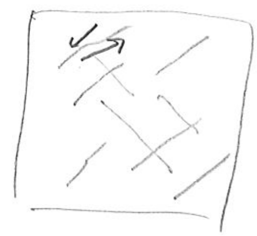




} for $\Delta P < 0$

ν	$1-2\nu$
0.1	0.8
0.2	0.6
0.3	0.4





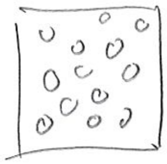
(2) + (3)

$$\boxed{\varepsilon_{33} = \underbrace{\frac{1}{E}\frac{(1-2\nu)(1+\nu)}{(1-\nu)}}_{1/M \,\to\, \text{constrained modulus}} \left(S_{33} - \alpha P_p \right)}$$

$$\left.\frac{\partial \varepsilon_{33}}{\partial P_p}\right|_{S_{33}} = -\frac{\alpha}{M} \quad \longrightarrow \quad \boxed{\Delta \varepsilon_{33} = -\frac{\alpha}{M} \Delta P_p}$$

subsidence $\rightarrow$ $\Delta h = h \left(-\frac{\alpha}{M} \Delta P_p \right)$

Undrained Loading (no fluid comes in/out from the pore space)



$$d(\phi \rho_F) = \rho_F \, d\phi + \phi \, d\rho_F$$

$$C_F = K_F^{-1}$$

$$\frac{d(\phi \rho_F)}{\rho_F} = d\phi + \phi \frac{d\rho_F}{\rho_F}$$

$$\varphi = \alpha \epsilon + \frac{P}{N} \qquad d\rho_F = \frac{\rho_F}{K_F} dP$$

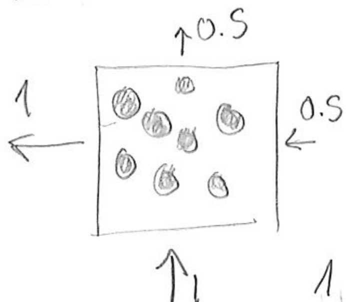
$$= \left(\alpha \, d\epsilon + \frac{1}{N} dP\right) + \phi \left(\frac{1}{K_F} dP\right)$$

Small strains $d\phi \ll \phi_0$

$$\frac{d(\phi \rho_F)}{\rho_F} = \alpha \, d\epsilon + \underbrace{\left(\frac{1}{N} + \frac{\phi_0}{K_F}\right)}_{\frac{1}{M^*}} dP \qquad \frac{1}{N} = \frac{\alpha - \phi_0}{K_S}$$

M^* → Biot Modulus (constant)

Continuity equation



$$\frac{d(\rho_F \phi)}{dt} + \nabla \cdot (\rho_F \, q) = 0$$

Darcy's eq (isotropic)

$$\frac{1}{dt}\left(\cancel{\rho_F}\left[\alpha \, d\epsilon + \frac{1}{M^*} dP\right]\right) + \nabla \cdot \left(\cancel{\rho_F}\left(-\frac{K}{\nu}\right) \nabla P\right) = 0$$

homogeneous

$$\alpha \frac{d\epsilon}{dt} + \frac{1}{M^*} \frac{dP}{dt} + \left(-\frac{K}{\nu} \nabla^2 P\right) = 0$$

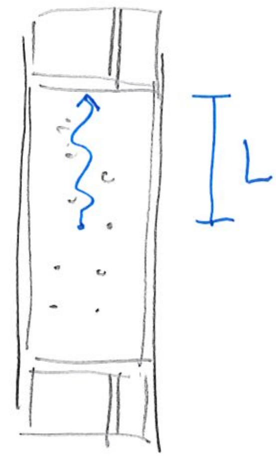
$$\boxed{\frac{dP}{dt} = \frac{K M^*}{\nu} \nabla^2 P - \alpha M^* \frac{d\epsilon}{dt}}$$

Diffusivity eq coupled with poroelasticity

$$\frac{dP}{dt} \sim \mathbb{D}\, \nabla^2 P$$

$$\boxed{t_{ch} = \frac{L^2}{\mathbb{D}}}$$

L : drainage length

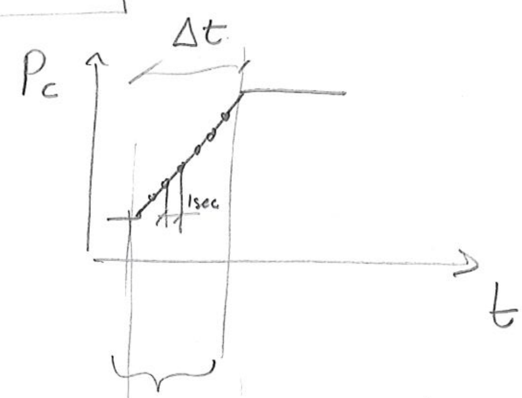


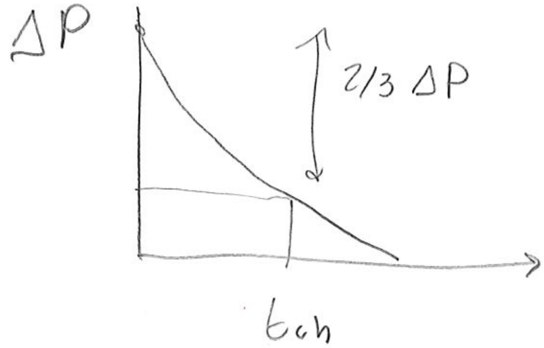
$$\mathbb{D} = \frac{K_h\, \Pi^*}{\mu}$$

$$= \frac{K_h}{\mu} \left(\frac{\alpha - \phi_0}{K_s} + \frac{\phi_0}{K_F} \right)^{-1}$$

$$\mathbb{D} = \frac{K_h}{\mu} \left(\frac{(1-\alpha)(\alpha-\phi_0)}{K} + \frac{\phi_0}{K_F} \right)^{-1}$$

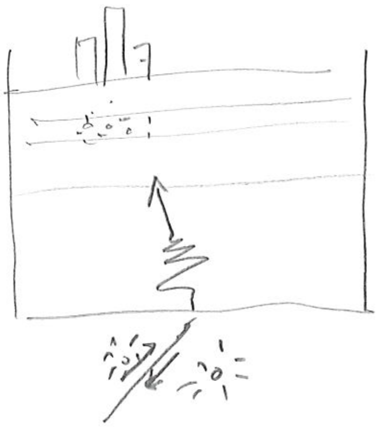
↳ t_{ch} = _ _ _ sec

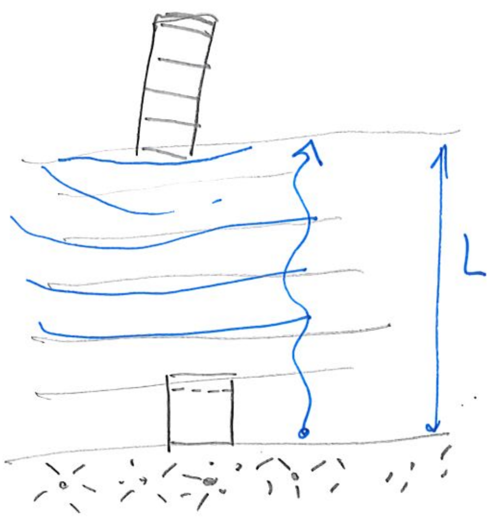




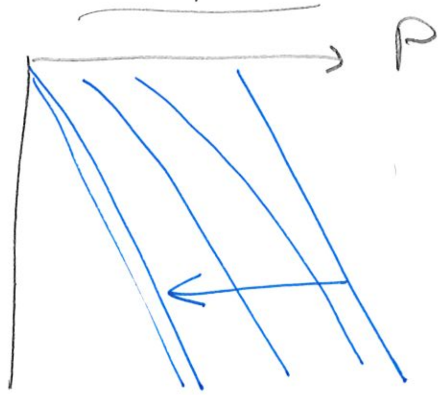
Undrained loading

Example 1: liquefaction
$\updownarrow$
quick sands





Example 2



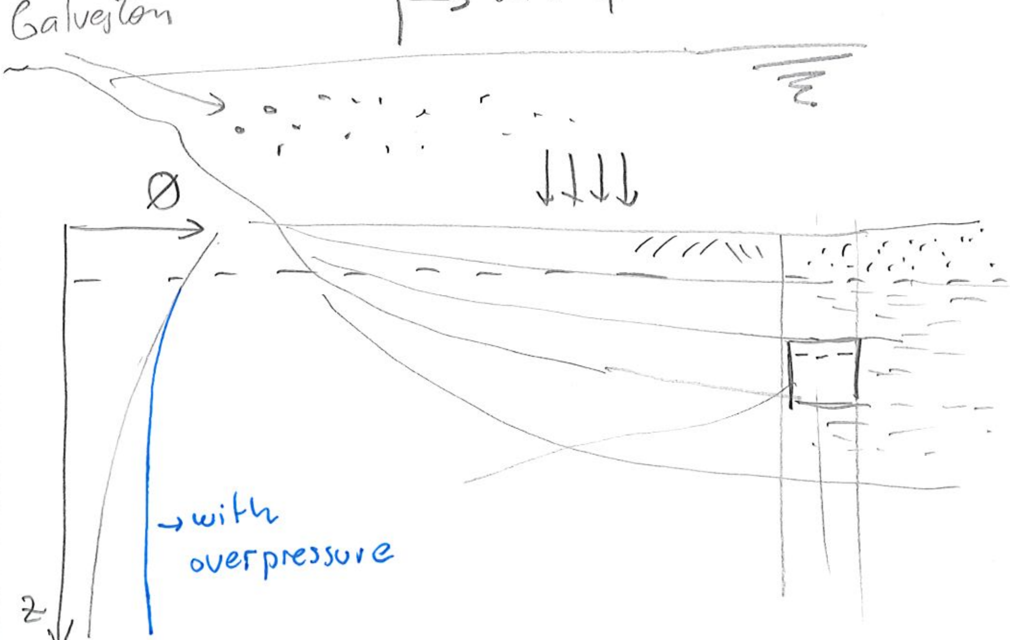
$$\frac{E(\quad \nu)}{(1+\nu)(\quad)}$$

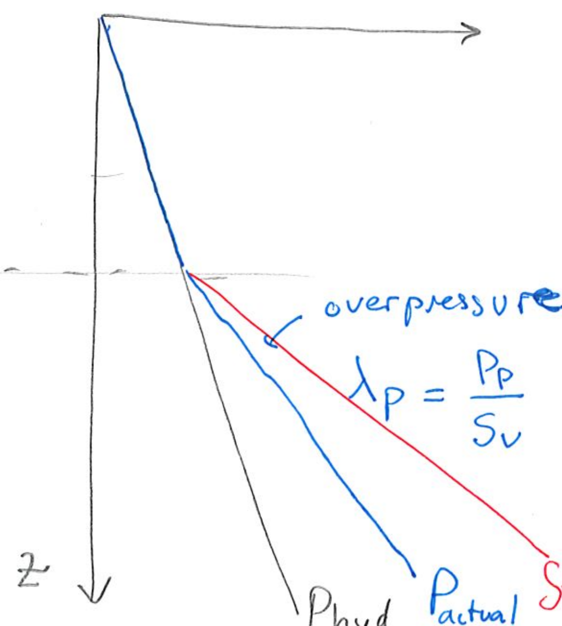
$$\mathbb{D}_h \cong \frac{k M}{\nu}$$

$$t_{ch} = L^2 / \mathbb{D}_h$$

↳ consolidation problem

↳ disequilibrium compaction : $\dot{sed} \gg \dot{P}_p$

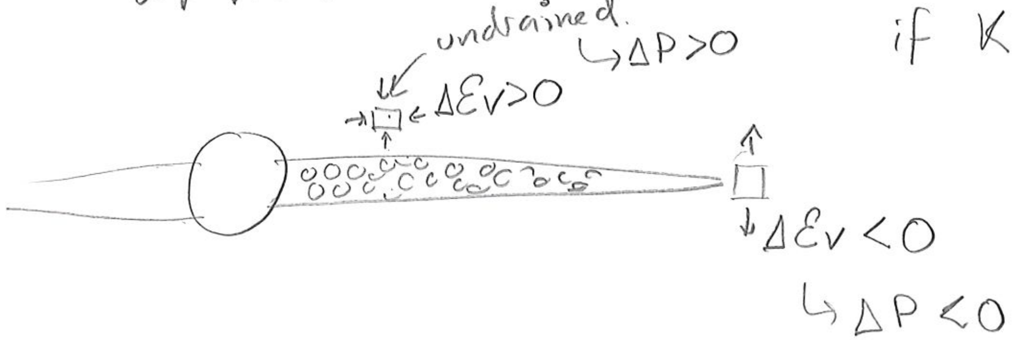


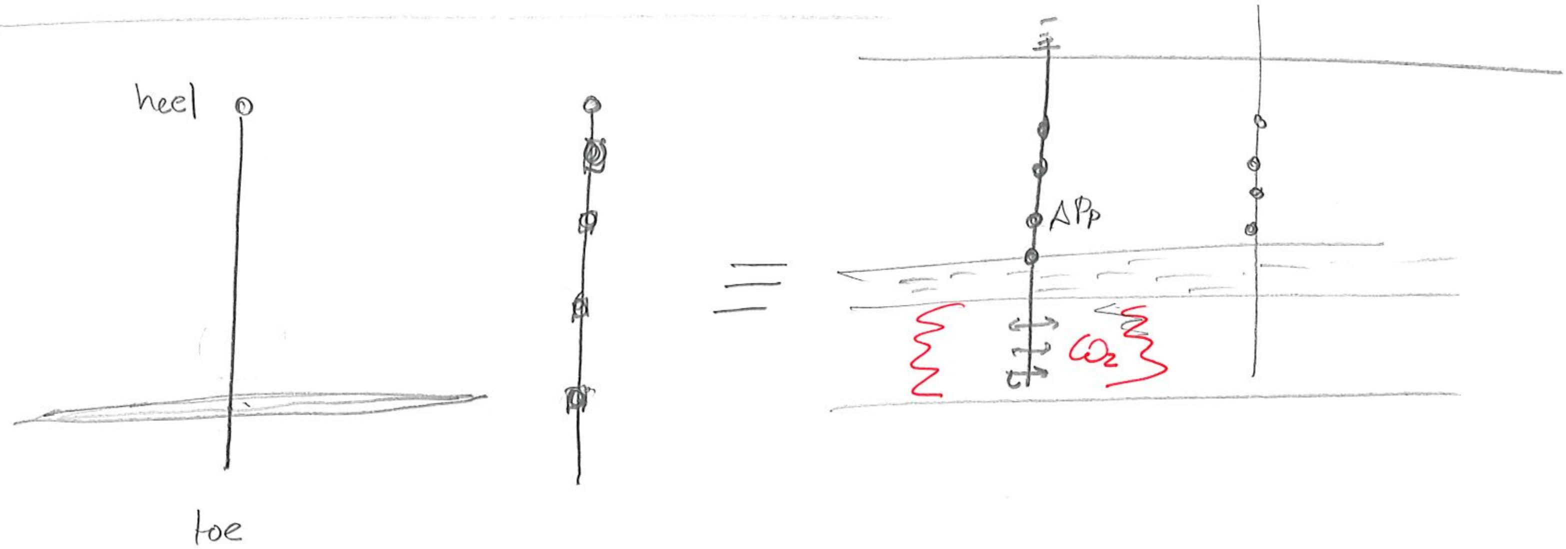


Undrained loading

Top view

if $K \lesssim 1\mu D$

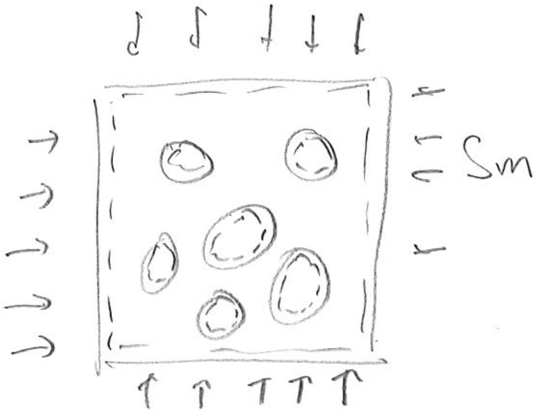




$\uparrow S_3 = S_{hmin}$

1. Pore pressure change due to volumetric strain

$$\left.\frac{\partial P}{\partial \epsilon}\right|_{d(\rho_F \phi)} = -\alpha M^* = -\alpha \left(\frac{\alpha - \phi_0}{K_S} + \frac{\phi_0}{K_F} \right)^{-1}$$



2. Undrained Bulk Modulus

$$\left.\frac{\partial S_m}{\partial \epsilon}\right|_{\text{drained, dry}} = K$$

Bulk modulus porous solid

$$\left.\frac{\partial S_m}{\partial \epsilon}\right|_{\substack{d(\rho_F \phi) \\ \text{undrained}}} = K_U$$

$$S_m = K\epsilon - \alpha P$$

$$\frac{\partial S_m}{\partial \epsilon} = K - \alpha \frac{\partial P}{\partial \epsilon}$$

$$= K - \alpha(-\alpha M^*)$$

$$\frac{\partial S_m}{\partial \epsilon} = \boxed{K + \alpha^2 M^* = K_U}$$

$$K + \alpha^2 \left(\frac{K \cdot K_F}{K_F(\alpha - \phi_0)(1-\alpha) + K \phi_0} \right) = K_U$$

if

$K_F(\text{water}) \gg K_F(\text{gas})$

$C_{gas} \gg C_{water}$

(3) Skempton's parameter

$$\frac{\partial P}{\partial S_m} = -B$$

$$S_m = K\epsilon - \alpha P$$

$$\frac{\partial S_m}{\partial p} = K\frac{\partial \epsilon}{\partial p} - \alpha$$

$$\frac{\partial S_m}{\partial p} = \underbrace{K\left(-\frac{1}{\alpha M^*}\right) - \alpha}$$

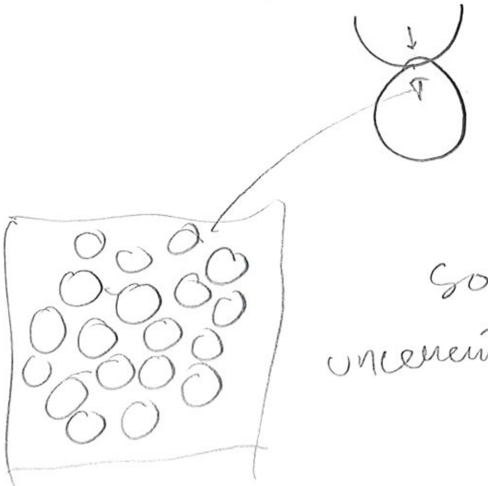
$$B = \left(\alpha + \frac{K}{\alpha M^*}\right)^{-1}$$

$$\boxed{B^{-1} = \frac{1}{\alpha}\left(\frac{K}{M^*} + \alpha^2\right)}$$

$B \rightarrow \sim 1 \rightarrow$ water saturated

$B \ll 1 \rightarrow$ gas in pore space

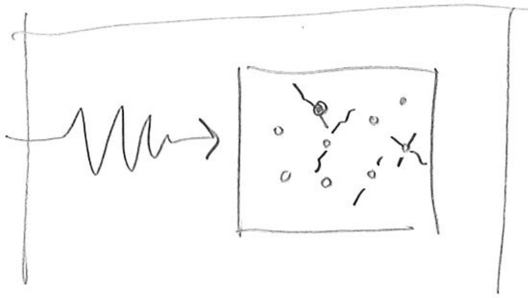
$$\frac{K}{M^*} = \frac{K(\alpha - \phi_0)}{K_s} + \frac{K\phi_0}{K_F}$$



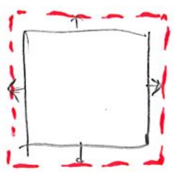
soils
uncemented sed $\rightsquigarrow$ $\underbrace{K \ll K_s \; ; \; K \ll K_F}_{B \rightarrow 1}$

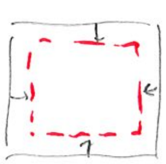
Thermo-elasticity

applications
- geothermal energy
  - shallow
  - deep } ↑ τ/σ_n
- drilling - wellbore } tensile $\sigma_{\theta\theta}$
- fluid injection
  - water
    - liquid
    - vapore ← heavy oil
  - CO_2
  - chem (foam, polymers)
- hydraulic fracturing
  - → freeze fracking
    - ↳ liquid N_2
  - → heating → microwave
    - ↙ heat up

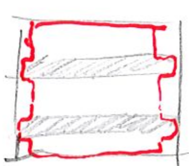


high k	low
↓	↓
steam	steam
drained	undrained
	↓
	cracking

$\Delta T > 0$  $\Delta \epsilon < 0$ dilation

$\Delta T < 0$  $\Delta \epsilon > 0$ contraction

heterogeneous



$\neq$

$\Delta T \lessgtr 0$ → no shear strains ← isotropic homogeneous solid

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\ \underline{\underline{\varepsilon}} + 3\underbrace{\beta}_{\text{linear expansion coefficient}}\ \underbrace{K}_{\text{Bulk modulus}}\ \underbrace{\Delta T}\ \underline{\underline{I}}$$

$\underline{\underline{I}}$ ← isotropic
← no sheer strains

Dilation

$$\cancel{\underline{\underline{\sigma}}}^{\,0} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} + 3\beta K \Delta T\ \underline{\underline{I}}$$

$$\hookrightarrow\ \underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{D}}}} \cdot \left(-3\beta K \Delta T\ \underline{\underline{I}}\right)$$

$C^{-1} \rightarrow D$

$$\begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{vmatrix} = \begin{vmatrix} 1/E & -\nu/E & -\nu/E \\ & 1/E & -\nu/E \\ & & 1/E \end{vmatrix} \begin{vmatrix} -3\beta K \Delta T \\ '' \\ '' \end{vmatrix}$$

$$\varepsilon_{11} = \underbrace{\frac{(1-2\nu)}{E}}_{K^{-1}} \left(-3\beta K \Delta T\right)$$

thermal dilation

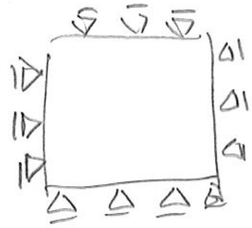
$$\boxed{\varepsilon_{11} = -\beta \Delta T}$$

heating: $\Delta T > 0 \rightarrow \varepsilon_{11} < 0$ dilation

cooling: $\Delta T < 0 \rightarrow \varepsilon_{11} > 0$ contraction

$$\left.\begin{aligned} \beta &= 10^{-6} - 10^{-5}\ \frac{1}{{}^{\circ}K} \\ \Delta T &= 100^{\circ} K \end{aligned}\right\}\ \varepsilon \sim 10^{-4} - 10^{-3}$$

Constrained dilation



$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\ \cancel{\underline{\underline{\varepsilon}}}^{\,0} + 3\beta K \Delta T\ \underline{\underline{I}}$$

thermal stress

$$\begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{vmatrix} = \begin{vmatrix} +3\beta K \Delta T \\ '' \\ '' \end{vmatrix}$$

$$\sigma_{11} = 3\beta K \Delta T$$

$K = 10\ GPa$

$\Delta T = 100\,°C$

$\beta = 10^{-5}\ 1/°C$

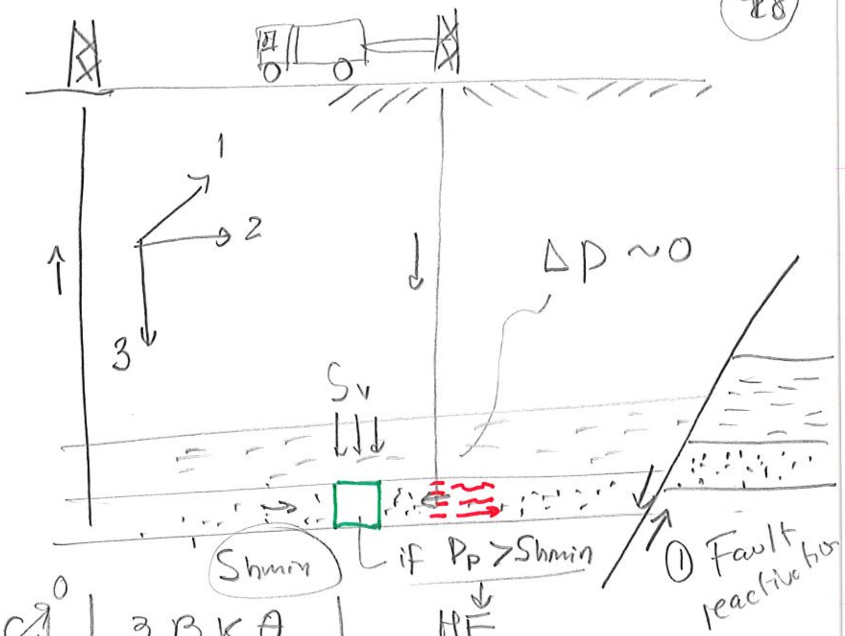
$$\sigma_{11} = 3 \cdot 10^{-5} \frac{1}{\cancel{°C}} \cdot 10^{10}\,Pa \cdot 10^{2}\,\cancel{°C}$$

$$\sigma_{11} = +30\ MPa$$

compression

1-D strain case

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\,\underline{\underline{\varepsilon}} + 3\beta K \underbrace{\theta}_{T-T_0}\,\underline{\underline{I}}$$



$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu\\ & 1-\nu & \nu\\ & & 1-\nu\end{bmatrix}\begin{bmatrix}\cancelto{0}{\varepsilon_{11}}\\ \cancelto{0}{\varepsilon_{22}}\\ \varepsilon_{33}\end{bmatrix} + \begin{bmatrix}3\beta K\theta\\ 3\beta K\theta\\ 3\beta K\theta\end{bmatrix}$$

$\varepsilon_{11} = \varepsilon_{22} = 0$

$\sigma_{33} = cst$

$$\sigma_{11} = \frac{E\,\nu}{(1+\nu)(1-2\nu)}\,\varepsilon_{33} + 3\beta K\theta$$

$$\sigma_{33} = \frac{E\,(1-\nu)}{(1+\nu)(1-2\nu)}\cdot\varepsilon_{33} + 3\beta K\theta$$

$$\sigma_{11} = \frac{\nu}{1-\nu}\left(\sigma_{33} - 3\beta K\theta\right) + 3\beta K\theta$$

just mechanics
$\sigma_{11} = \frac{\nu}{1-\nu}\sigma_{33}$
Δ pore pressure
$\left.\frac{\partial\sigma_{11}}{\partial P}\right\vert = \alpha\left(\frac{1-2\nu}{1-\nu}\right)$
Δ temperature
$\left.\frac{\partial\sigma_{11}}{\partial\theta}\right\vert = \frac{\beta E}{1-\nu}$

$$\left.\frac{\partial\sigma_{11}}{\partial\theta}\right|_{\sigma_{33}} = \frac{\nu}{1-\nu}\left(-3\beta K\right) + 3\beta K = 3\beta\underbrace{\frac{E}{3(1-2\nu)}}_{K}\left(1-\frac{\nu}{1-\nu}\right)$$

$$\boxed{\left.\frac{\partial\sigma_{11}}{\partial\theta}\right|_{\sigma_{33}} = +\frac{\beta E}{1-\nu}}$$

$$\left.\begin{aligned}\beta &= 10^{-5}\ 1/°C\\ E &= 10\ GPa\\ \nu &= 0.2\end{aligned}\right\} \sim 0.1\ \frac{MPa}{°C}$$

General problem of thermo-elasticity

$$\begin{cases} \nabla \cdot \underline{\underline{\sigma}} + \underline{f} = 0 & \text{(Equil)} \\ \underline{\underline{\varepsilon}} = \frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right) & \text{(Kinem)} \\ \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\,\underline{\underline{\varepsilon}} + 3\beta K \theta \underline{\underline{I}} & \text{(Constitutive)} \\ \dfrac{\partial \theta}{\partial t} = \dfrac{K_T}{\rho C_v} \nabla^2 \theta + \underbrace{3\dfrac{K B T_0}{\rho C_v}}\dfrac{\partial \varepsilon_{vol}}{\partial t} \end{cases}$$

K_T: thermal conductivity

$$\frac{3\ 10^{10}\,\text{Pa}\ 10^{-5}\,\text{K}^{-1}\ 300\,\text{K}}{1.76 \cdot 10^{6}\ \frac{\text{J}}{\text{m}^3\text{K}}} \sim 50\,\text{K}$$

$\varepsilon_{vol} = 0.01$

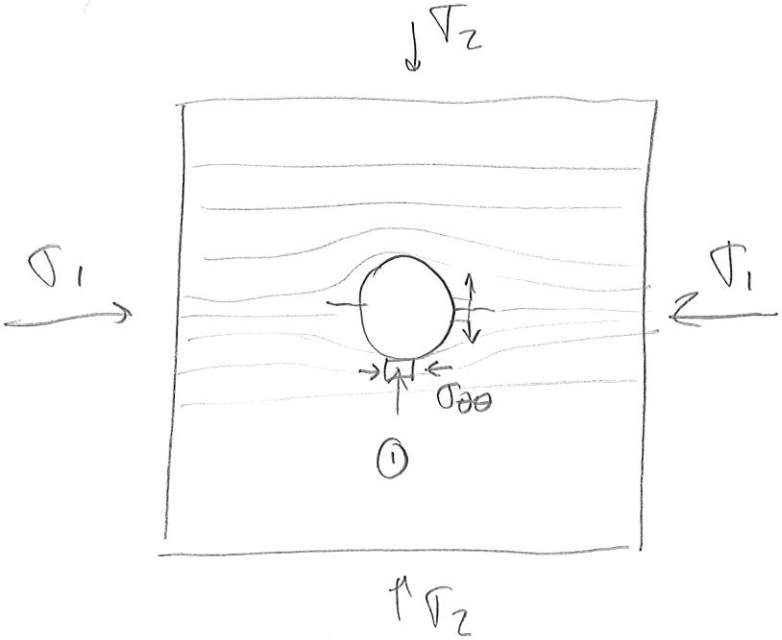
$\rightarrow 0.5\,\text{K}$

Thermo-poro-elasticity (Coussy 4.3)

└ Add new energy term → heat → $\Delta T = T - T_0$

mean stress	$S_m = K\epsilon - \alpha P - 3\beta K (T - T_0)$
shear stress	$S_{ij} = 2 G e_{ij}$
Δ porosity	$\varphi = \alpha \epsilon + P/N - 3\beta_\varphi (T - T_0)$
entropy	$S_s = S_{s0} + 3\beta K \epsilon - 3\beta_\varphi P + \dfrac{C_v (T - T_0)}{T_0}$

$$\beta_\varphi = \beta_{solid}(\alpha - \phi_0)$$



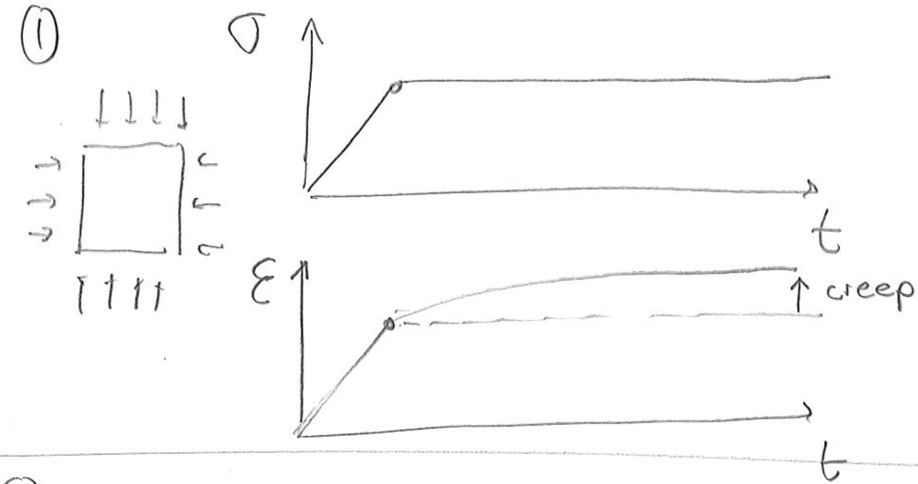
① $\sigma_{\theta\theta} = -(P_w - P_p) + 3\sigma_1 - \sigma_2 + \Delta\sigma_{\theta\theta}^T$

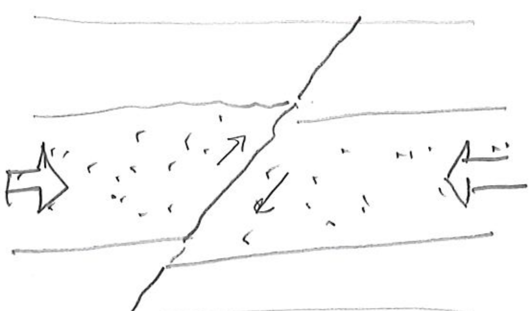
② $\sigma_{\theta\theta} = -(P_w - P_p) - \sigma_1 + 3\sigma_2 + \Delta\sigma_{\theta\theta}^T$

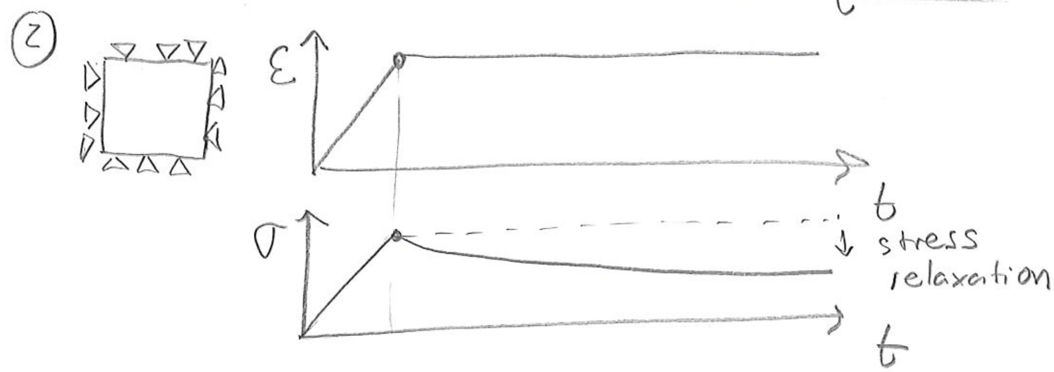
$$\boxed{\Delta\sigma_{\theta\theta}^T = \frac{\beta E \Delta T}{1-\nu}}$$

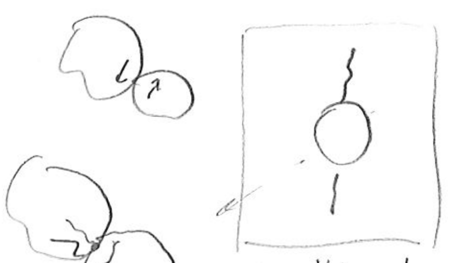
↳ steady state

Visco-elasticity

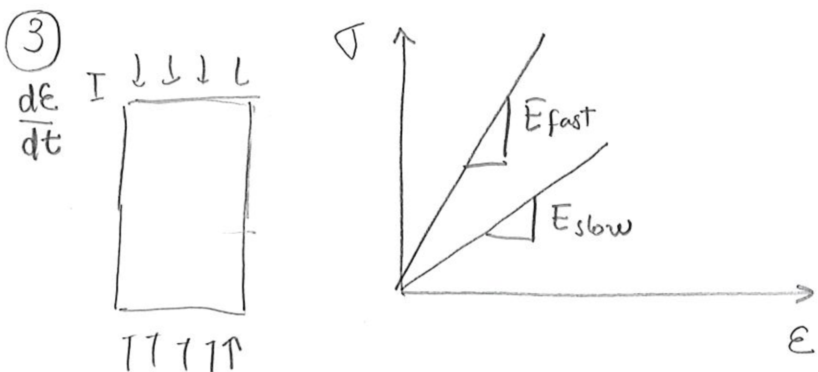






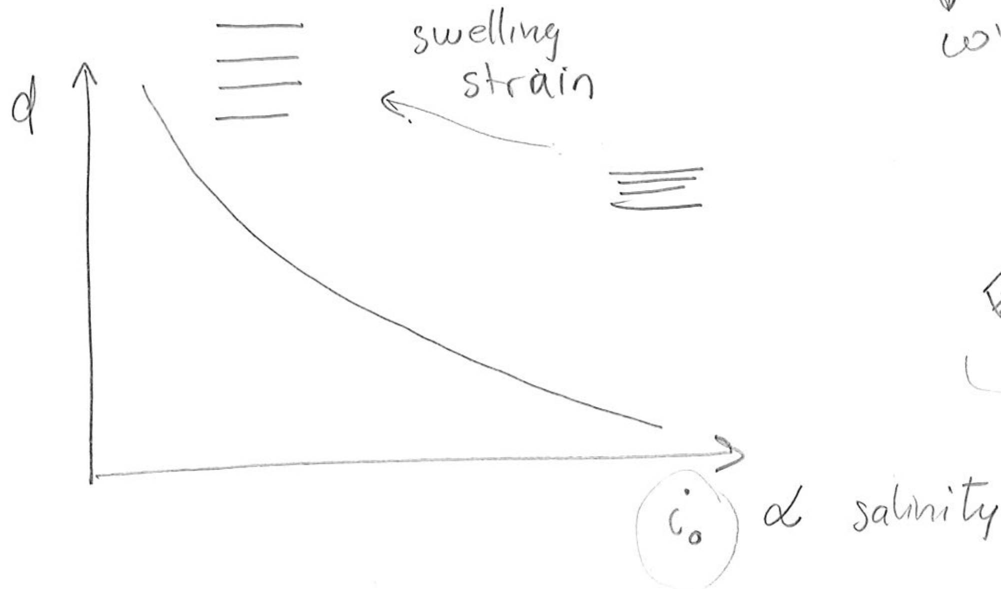


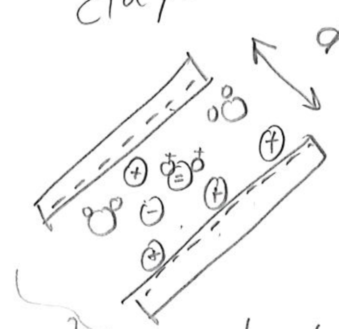
subcritical fracture propagation



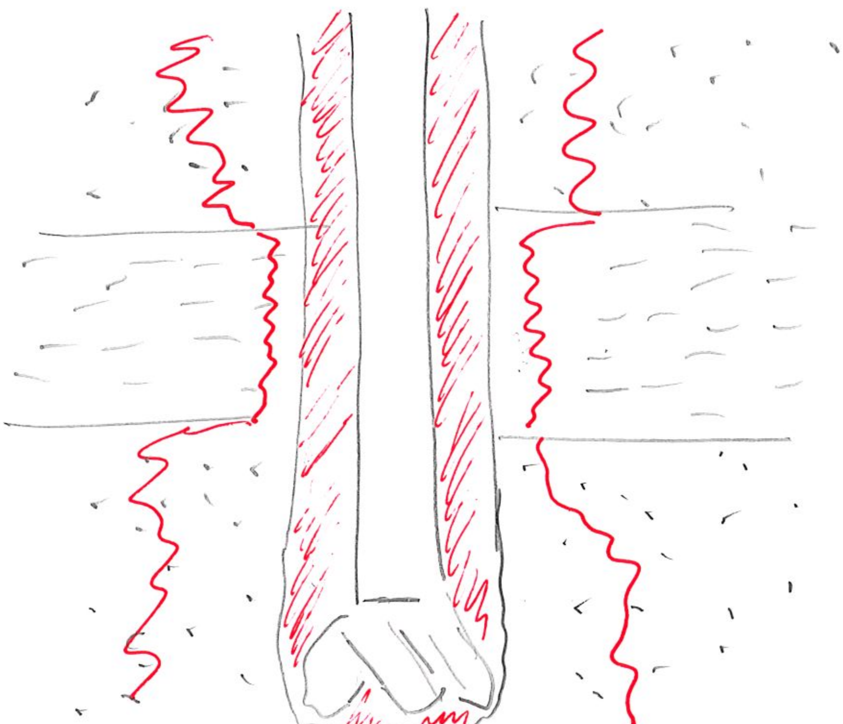
Example 1 : swelling in shales

composed of clays

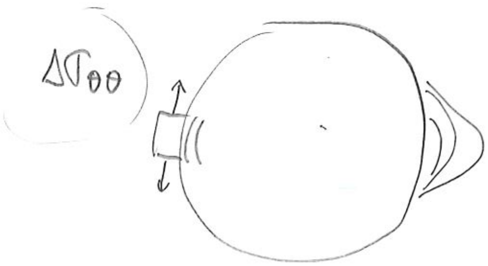




electrical repulsion
↕
Double Layer



drilling mud
- water-based
  - bentonite ✓
  - water ✓
  - barite ↔ heavy
  - salt
- oil-based
- underbalanced drilling



Elasticity

- → Poro elasticity
- → Thermo elasticity
- → Visco-elasticity
- → Chemo - elasticity ⟷ Chemo - plasticity
  - → $\Delta \mu$ ⇝ chemical potential
    - → fluid
    - ↕
    - → matrix solid

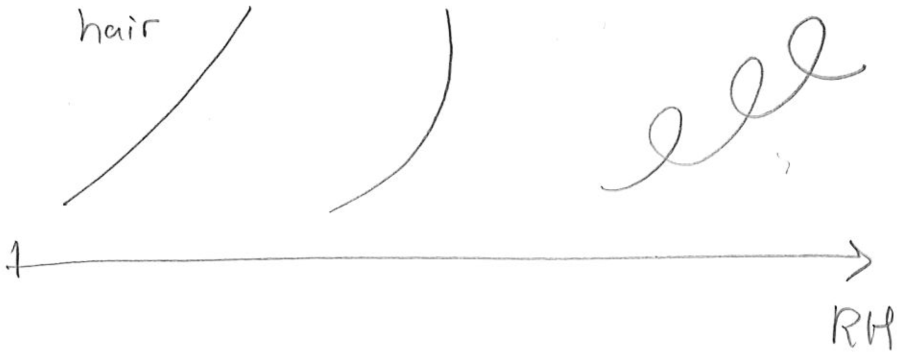
$$\underline{\underline{\sigma}} = \underbrace{\underline{\underline{\underline{C}}}\,\underline{\underline{\varepsilon}}}_{\text{eff stress}} + \underbrace{\alpha P \underline{\underline{I}}}_{\text{pore pressure}} + \underbrace{\gamma \mu K}_{\text{chemical term}} \underline{\underline{I}} + \theta + \ldots$$

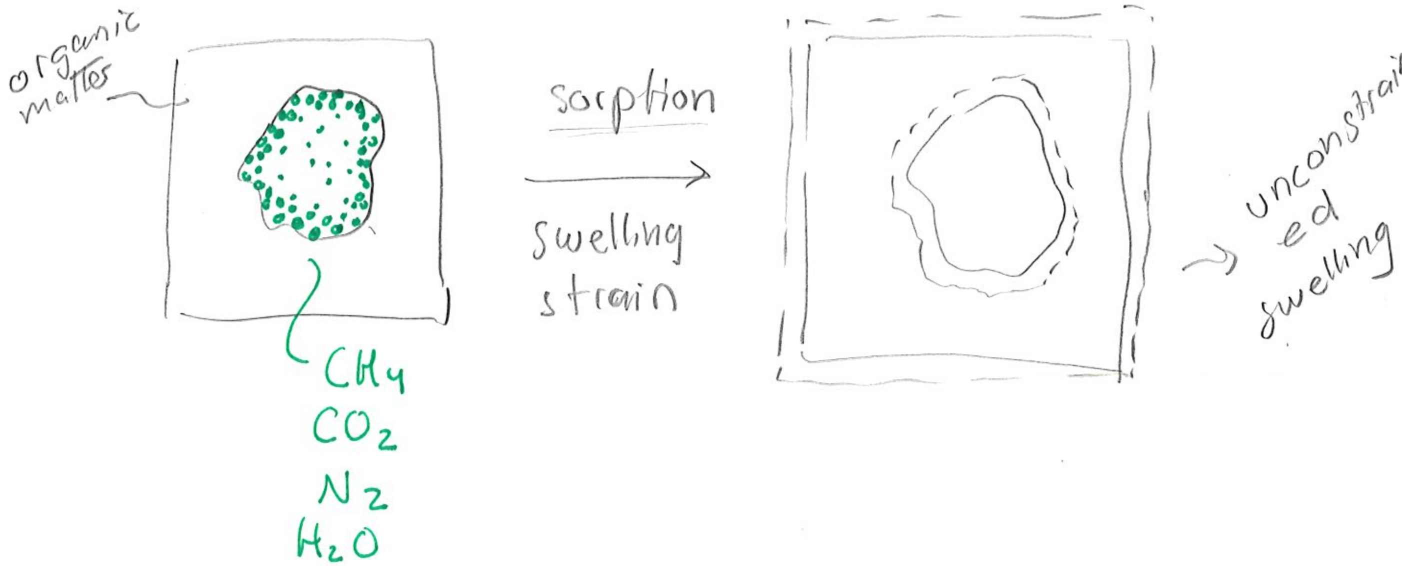
THCM problems

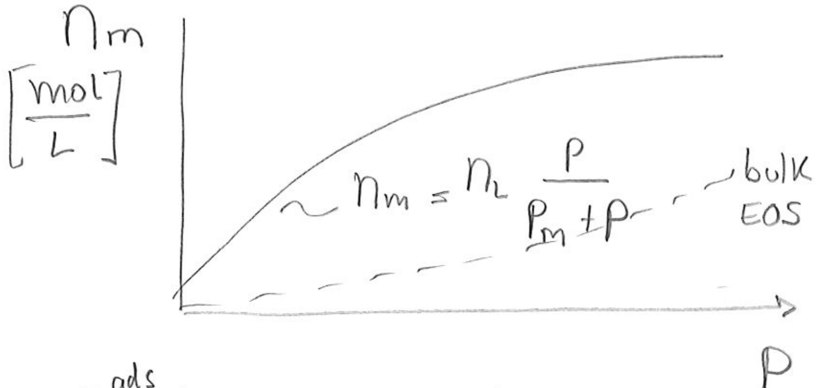
Thermo
Hydro
Chemo
Mechanical

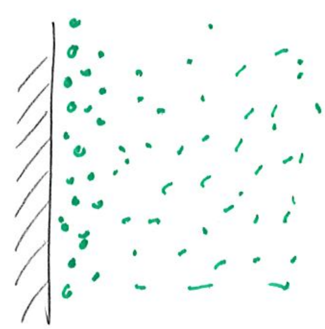
} Multiphysics problems
↓
"emergent phenomena"

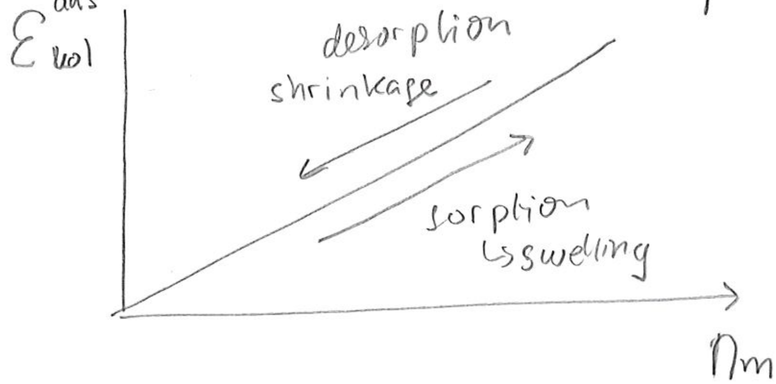
Example 2: Adsorption-induced deformation









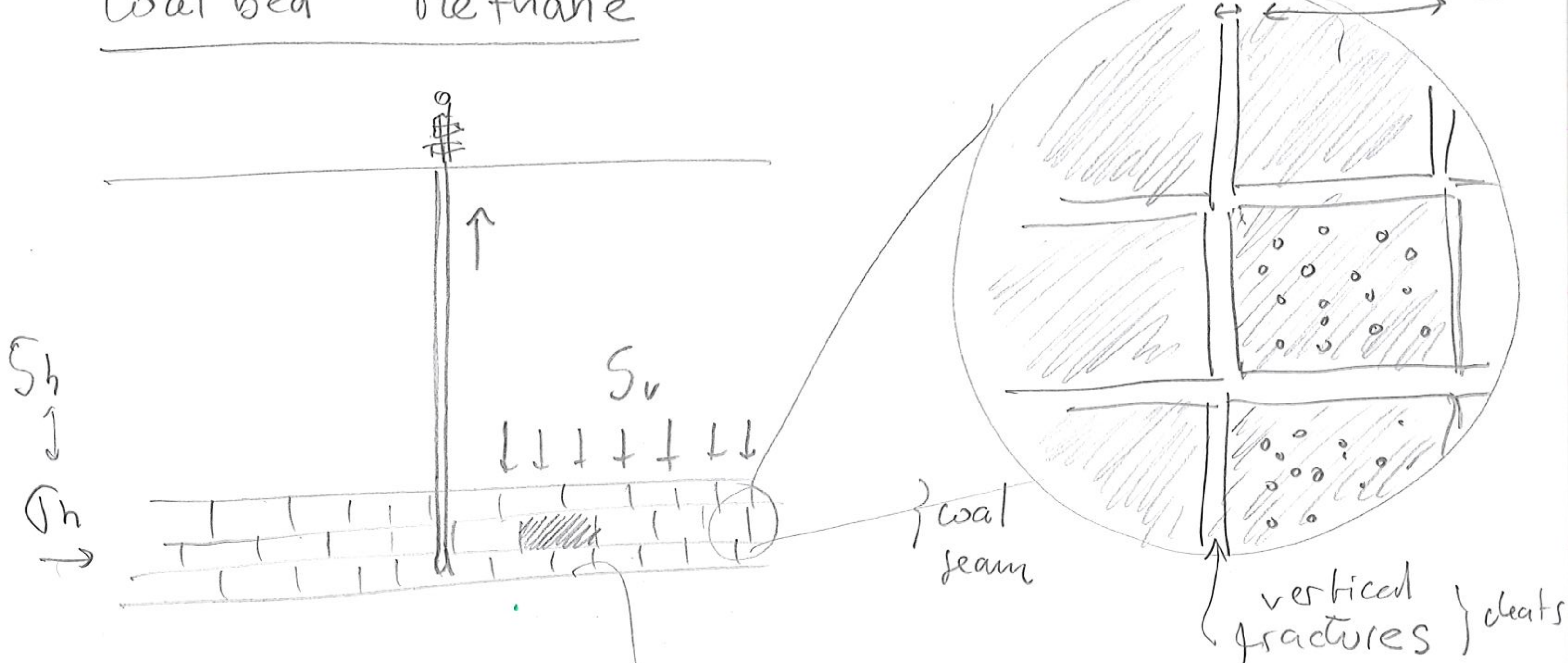


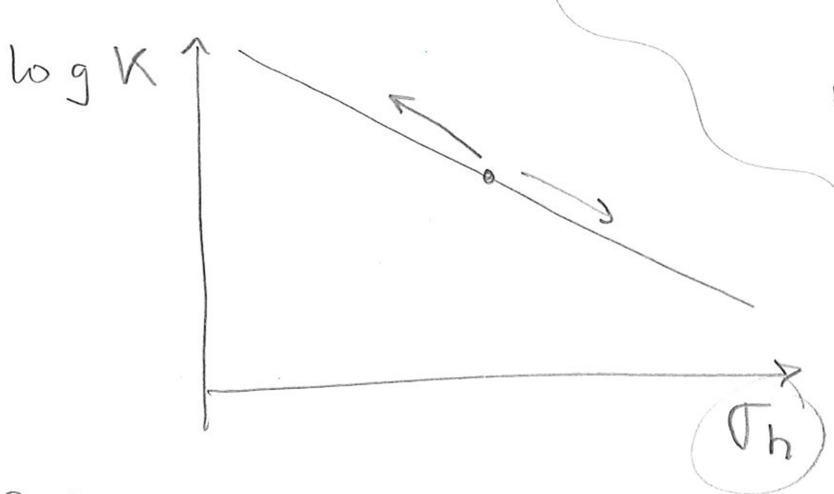
adsorption stress

$$\underline{\underline{S}} = \underline{\underline{\underline{\underline{C}}}}\,\underline{\underline{\varepsilon}} + \alpha p \underline{\underline{I}} + (1-\alpha)\, \underline{\underline{S_a(p)}}\,\underline{\underline{I}}$$

(non linear with pressure)

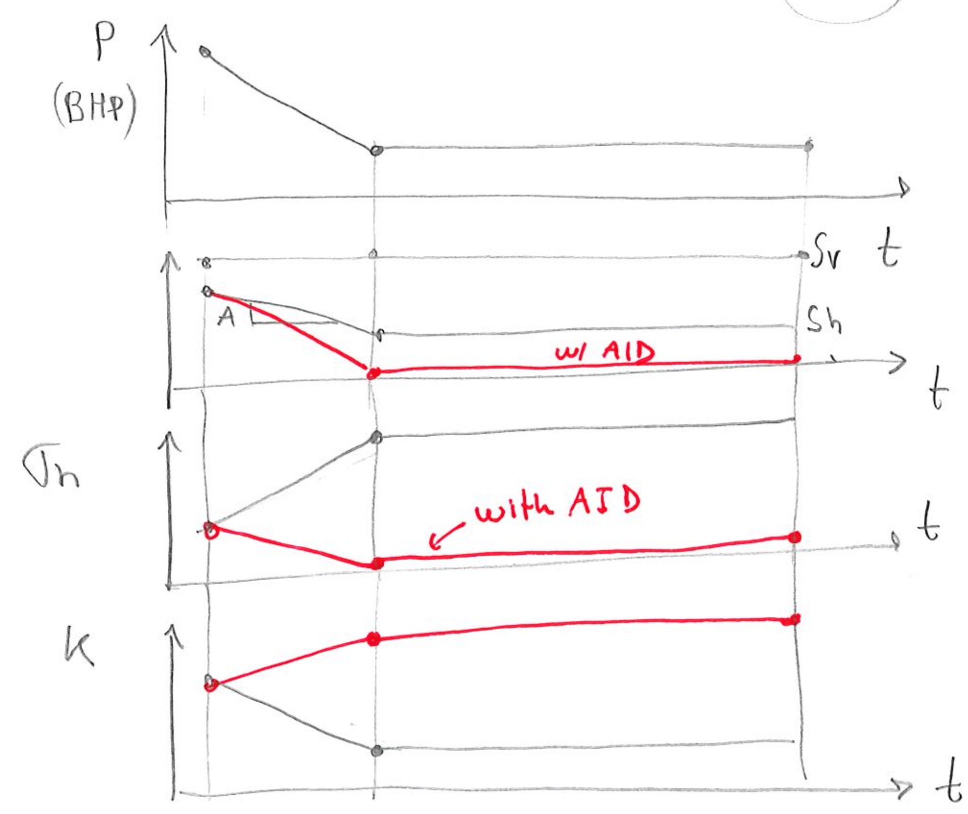
Coal bed Methane

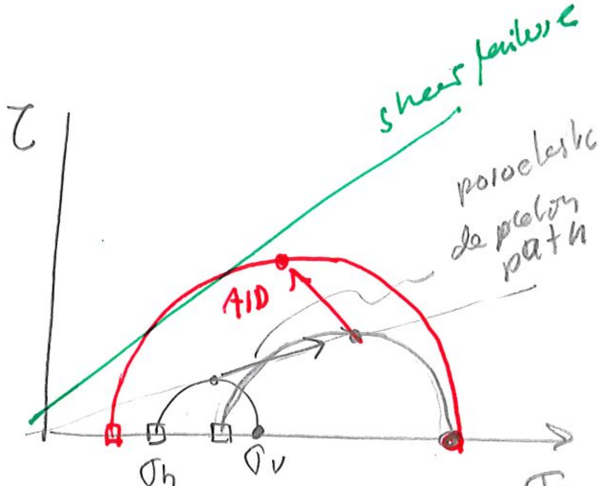




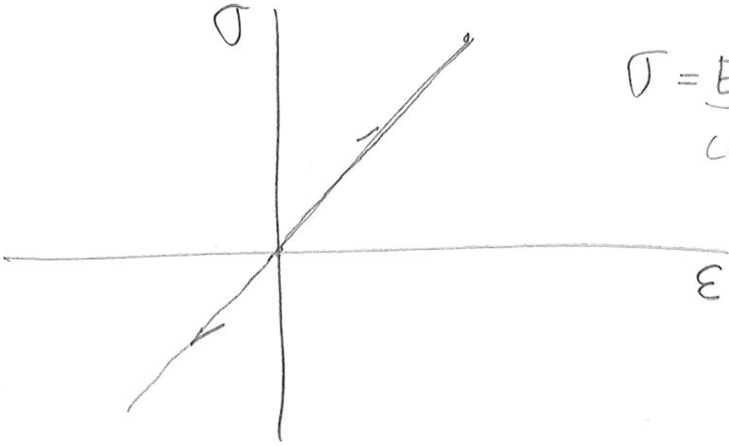
$$K = \frac{h^3}{12 \cdot S} \Big\} \; \Delta h \leftrightarrow C_f$$

adsorption - induced deformation



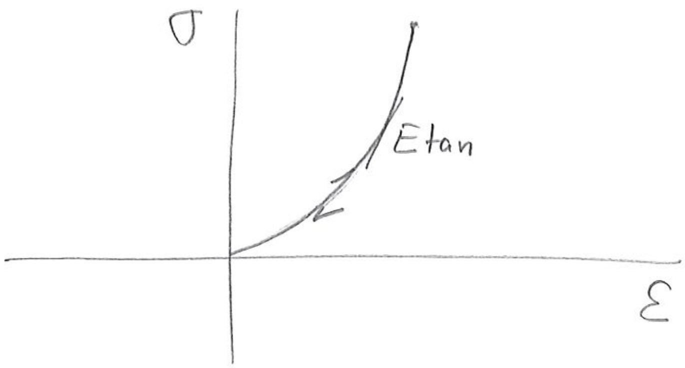


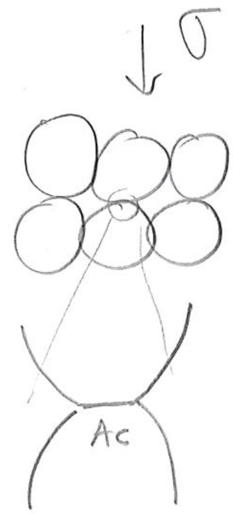
Plasticity and Inelasticity

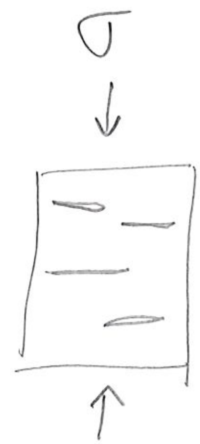


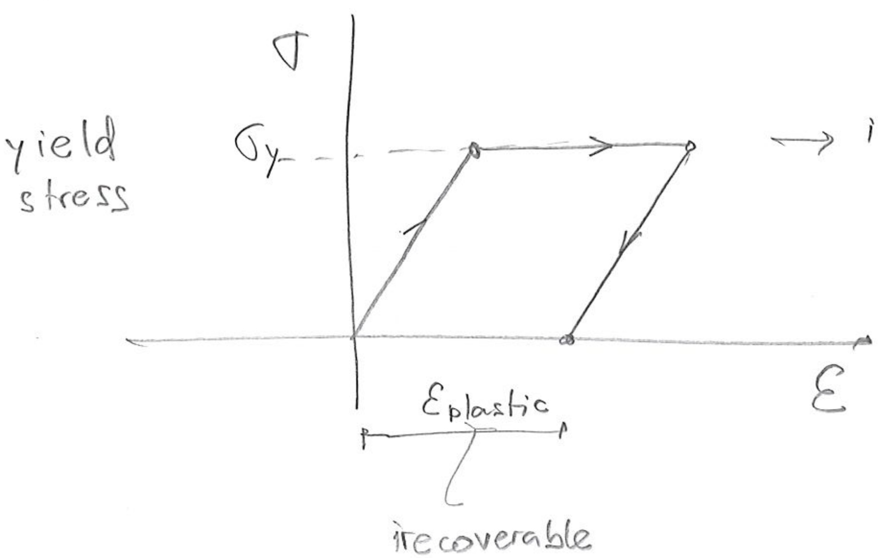
$$\sigma = \underset{\smile}{E} \cdot \varepsilon$$

constant → linear



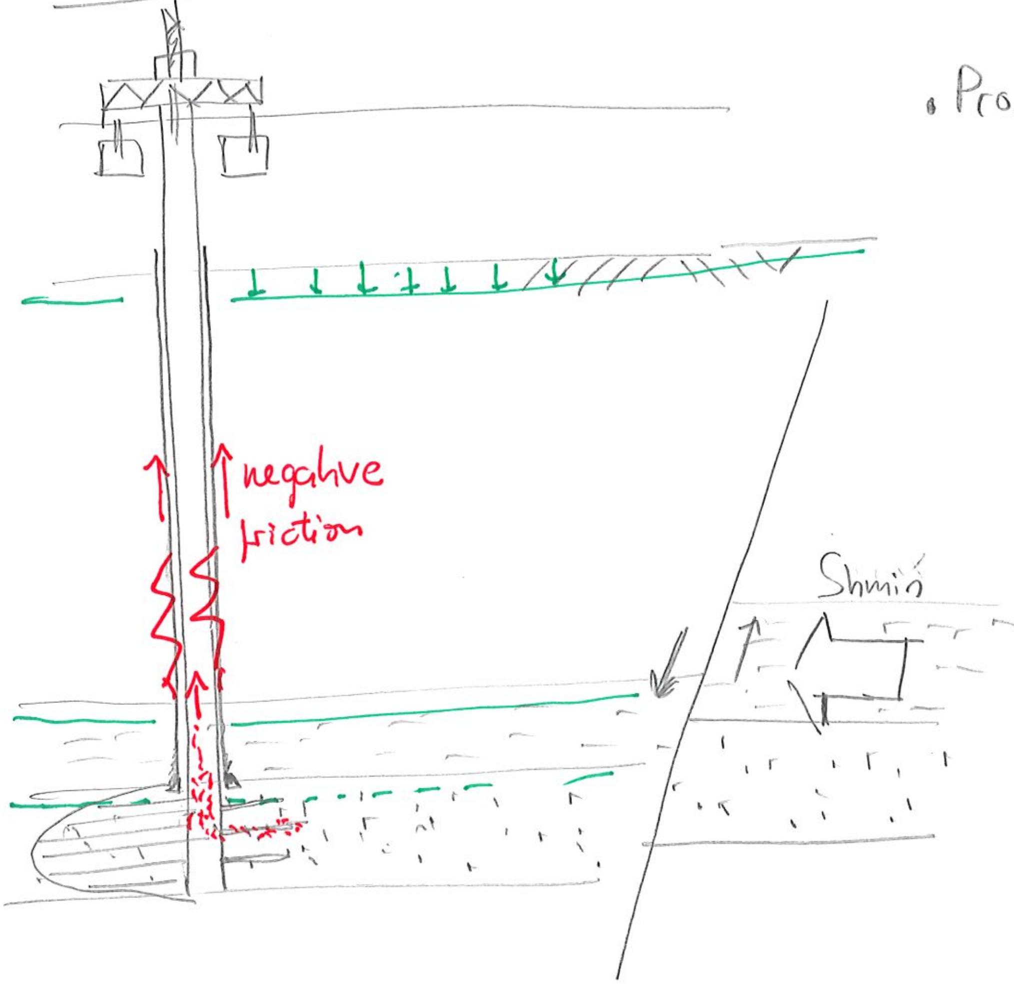






→ ideal plastic deformation

Examples of inelastic strains



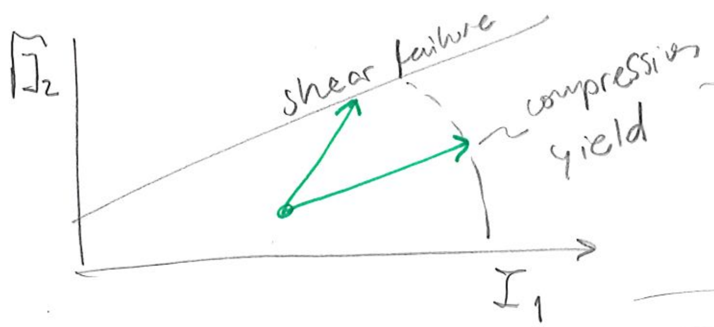
- Prospecting | Determine in-situ stress
  $\sigma_1 = \frac{1+\sin\varphi}{1-\sin\varphi}\sigma_3$

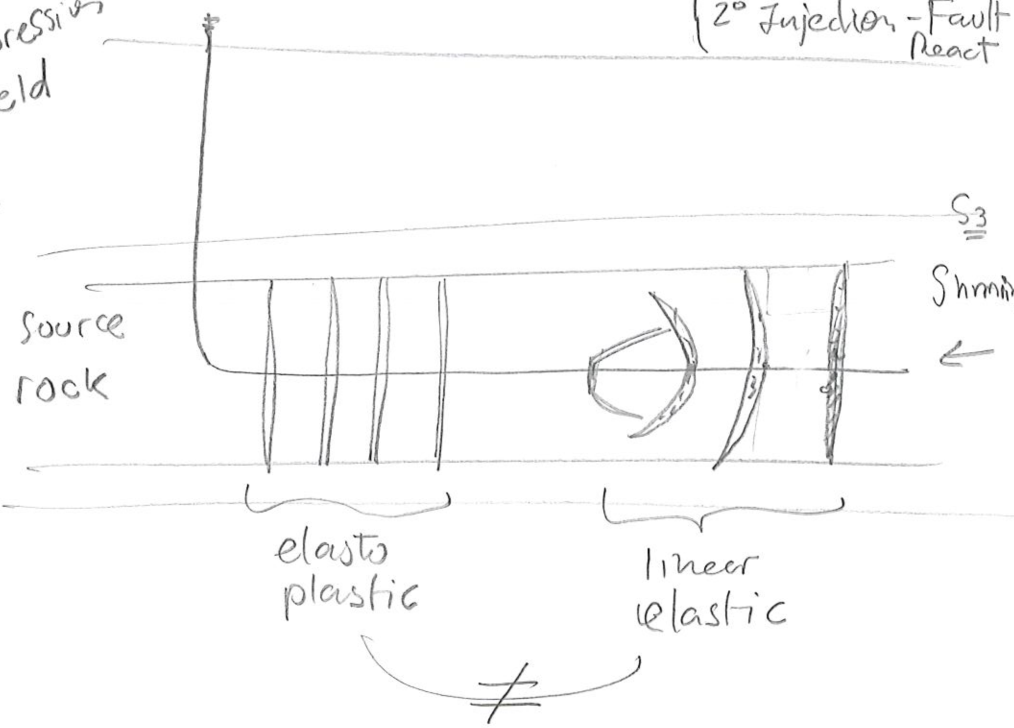
  $\boxed{\sigma_v = \frac{1+\sin\varphi}{1-\sin\varphi} S_{hmin}}$

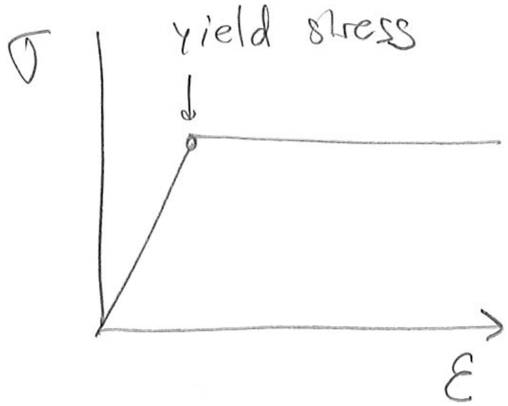
- Drilling | perforation ↳ cutting
  $\boxed{\text{wellbore stability}}$

- Completion | perforations
  HF, stress shadow

- Production | 1° Compaction, fault reactivation, sand production
  2° Injection - Fault React

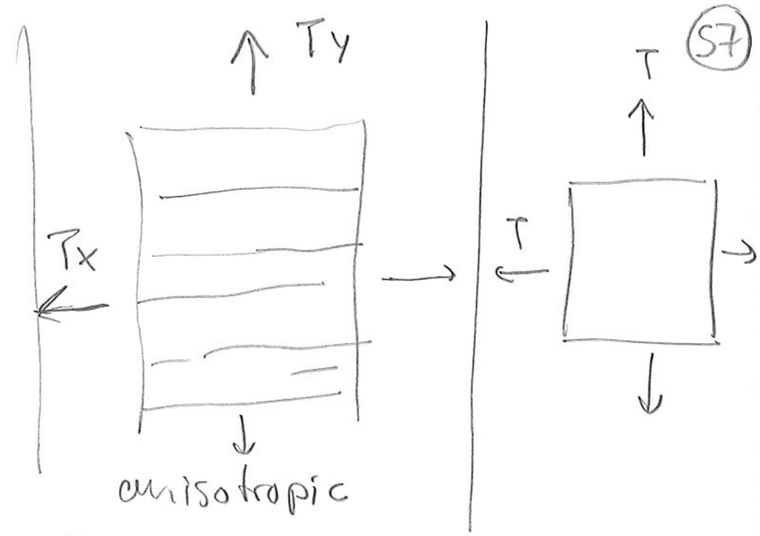






perfect - plasticity

$\sigma = \sigma_y$



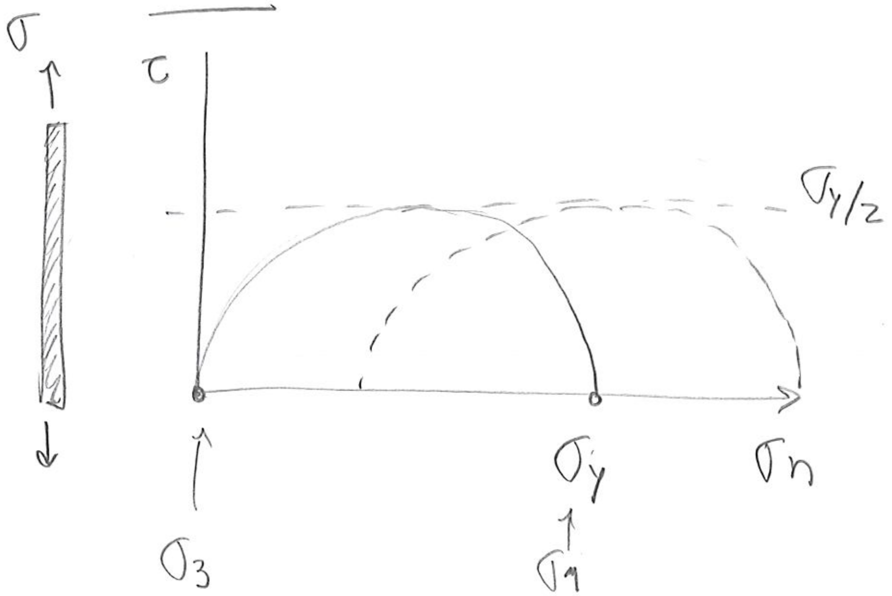
$$f(\underline{\underline{\sigma}}) = K \rightarrow \text{failure criterion}$$

$$f(\sigma_1, \sigma_2, \sigma_3, \text{direction } \sigma_i) = K$$

isotropy

$$f(\sigma_1, \sigma_2, \sigma_3) = K \longleftrightarrow f^*(I_1, I_2, I_3) = k^*$$

Tresca



$$\boxed{\sigma_1 - \sigma_3 \leq \sigma_y}$$

$$|\sigma_I - \sigma_{II}| \leq \sigma_y$$

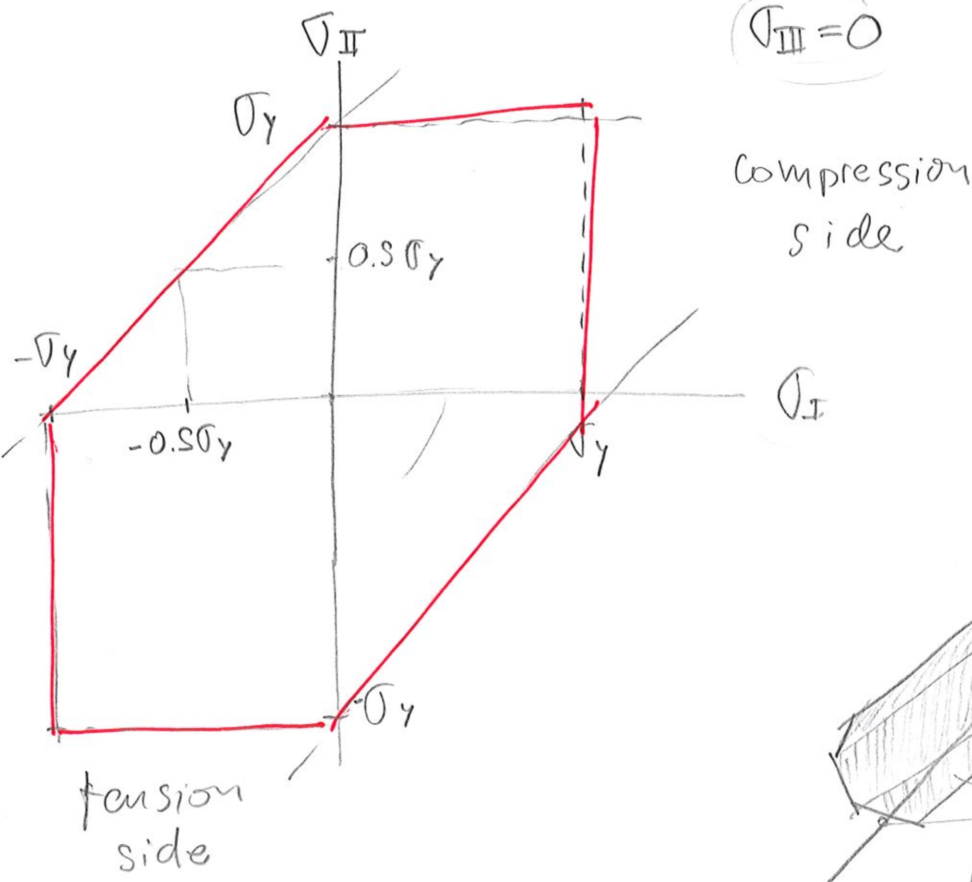
$$|\sigma_I - \sigma_{III}| \leq \sigma_y$$

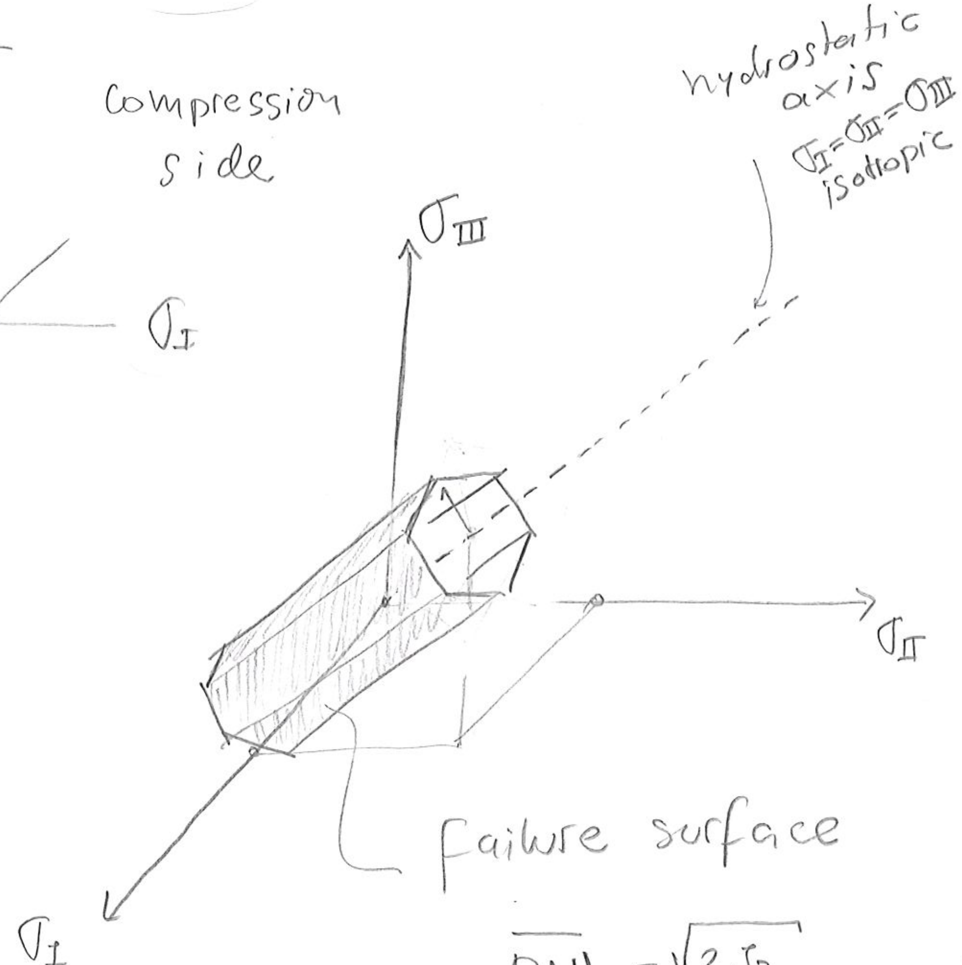
$$|\sigma_{II} - \sigma_{III}| \leq \sigma_y$$

$\sigma_I, \sigma_{II}, \sigma_{III}$

not sorted by magnitude

$\sigma_{III} = 0$



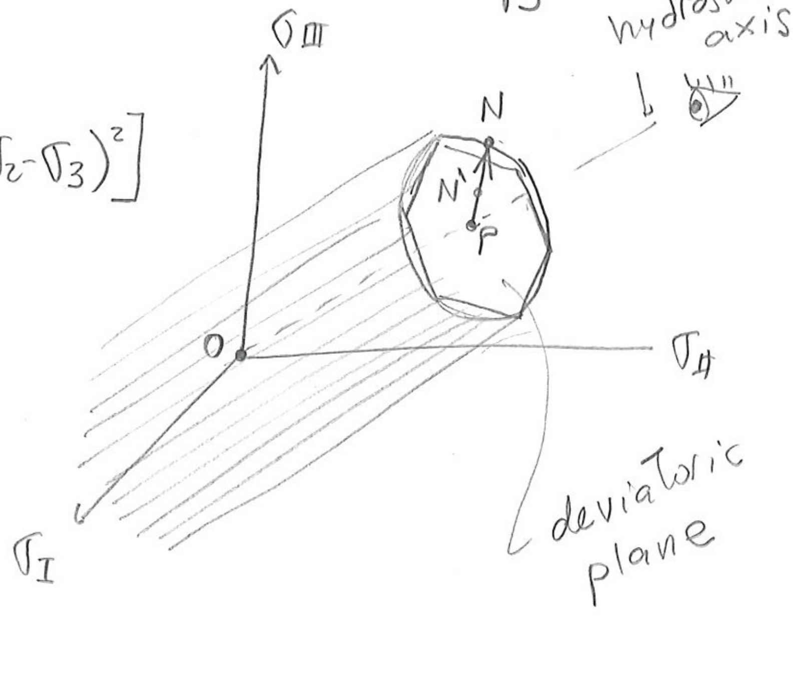


$$\overline{PN'} = \sqrt{2 J_2}$$

$$\overline{PN} = \sqrt{\frac{2}{3}} \cdot \sigma_y$$

$$\overline{OP} = \frac{\sigma_m}{\sqrt{3}}$$

von Mises

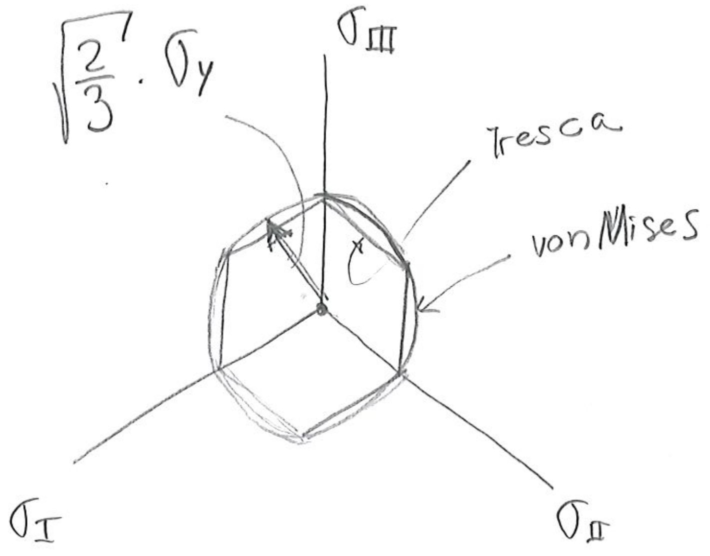


$$J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]$$

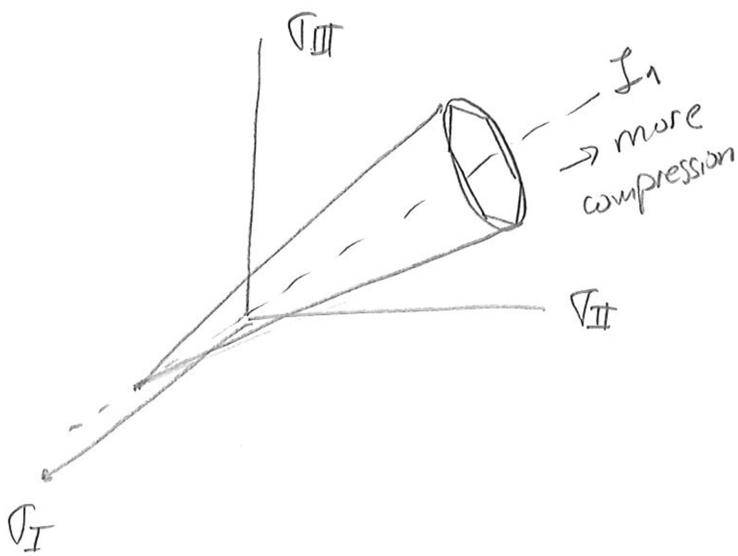
$$\sigma_1 = \sigma_y, \quad \sigma_2 = \sigma_3 = 0$$

$$J_2 = \frac{1}{\cancel{6}\,3} \cancel{2}\sigma_y^2$$

$$\boxed{\sqrt{J_2} = \frac{\sigma_y}{\sqrt{3}} = K}$$



(effective mean) stress sensitive

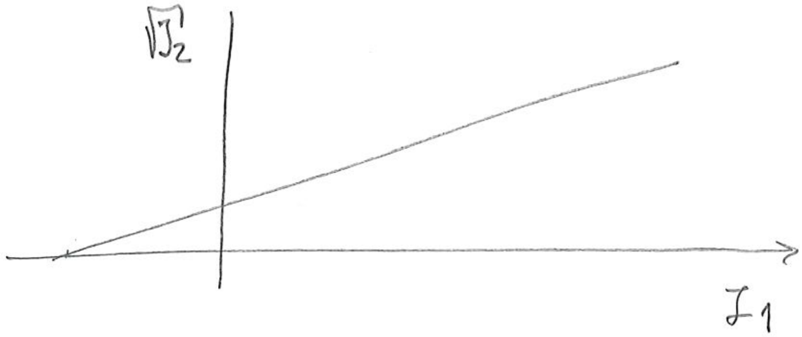


Modified Tresca

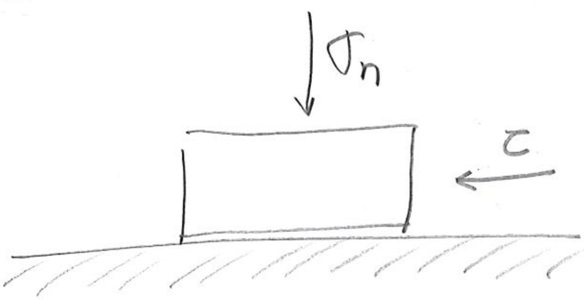
$$|\sigma_1 - \sigma_3| = C_1 + C_2 \cdot \sigma_m$$

Drucker - Prager

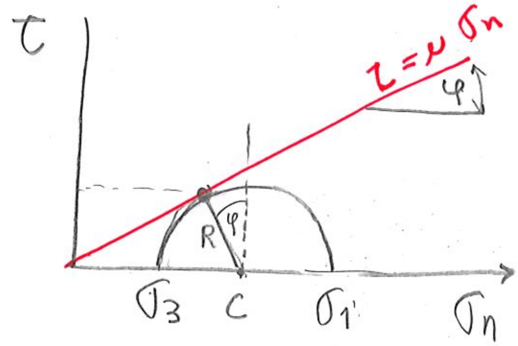
$$\sqrt{J_2} = C_3 + C_4 \cdot I_1$$

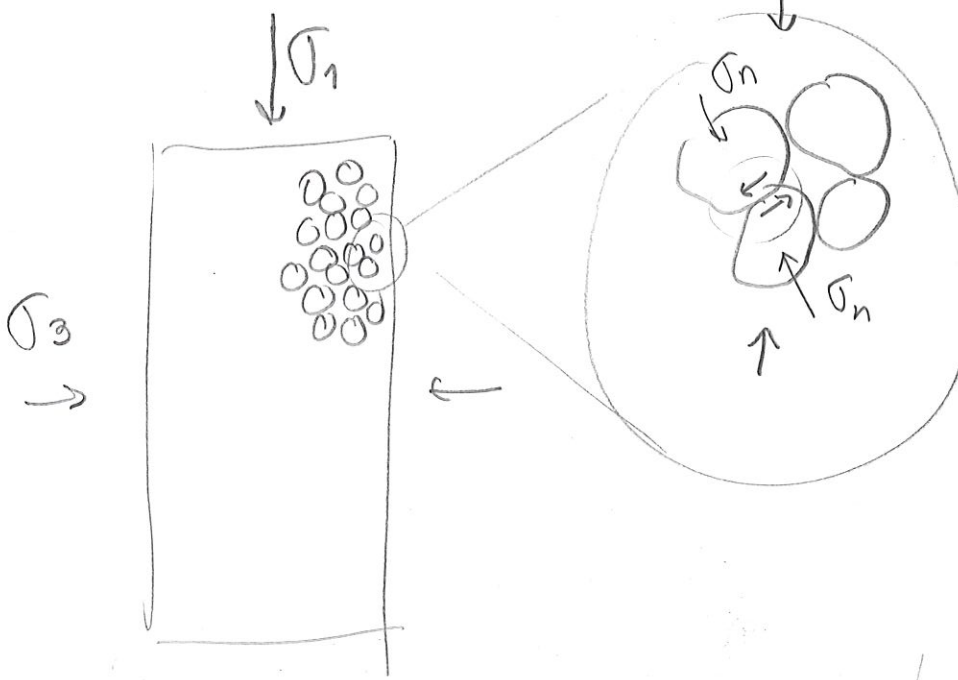


Stress sensitive geomaterials



$$\tau = \mu \cdot \sigma_n$$



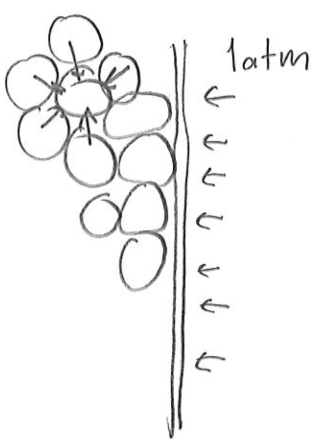


$$\tau = \mu \sigma_n$$

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) \cos\varphi = \tan\varphi \cdot \left(\quad \right)$$

$$\frac{\sigma_1 + \sigma_3}{2} - \sin\varphi \cdot \left(\frac{\sigma_1 - \sigma_3}{2}\right)$$

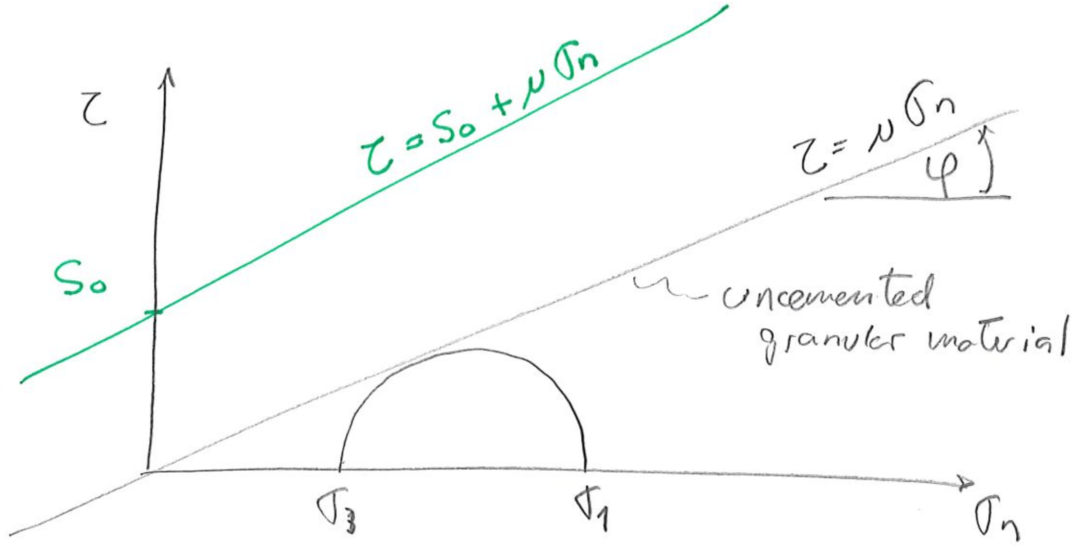
$$\boxed{\sigma_1 = \frac{1+\sin\varphi}{1-\sin\varphi} \cdot \sigma_3}$$



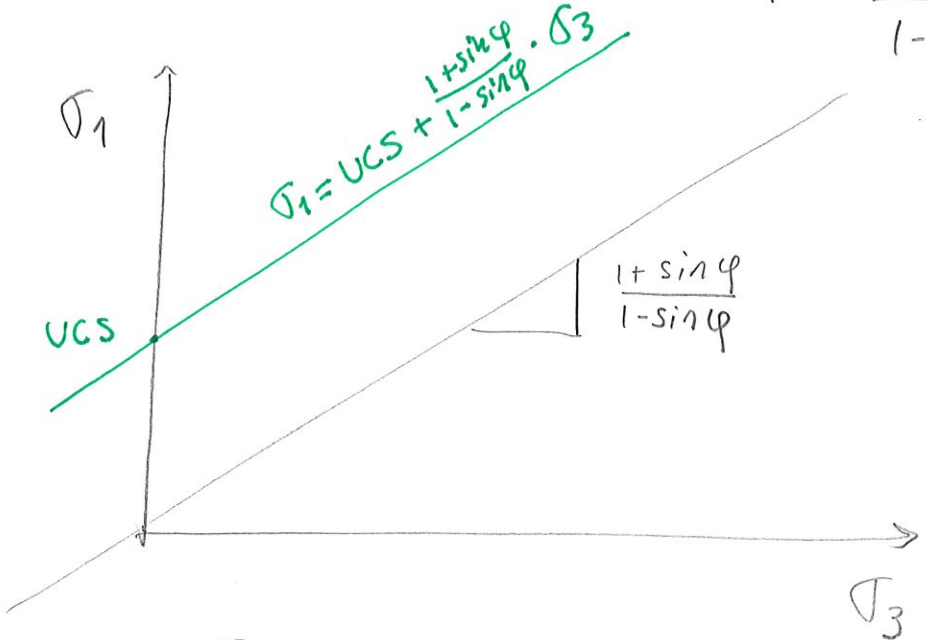
$$\varphi = 30^\circ$$

$$\mu \cong 0.58$$

$$q = \frac{1+\sin\varphi}{1-\sin\varphi} = 3$$

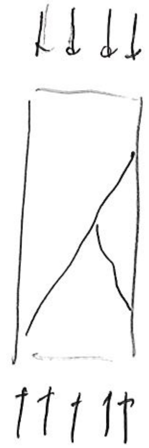


$$\sigma_1 = \frac{1+\sin\varphi}{1-\sin\varphi} \cdot \sigma_3$$

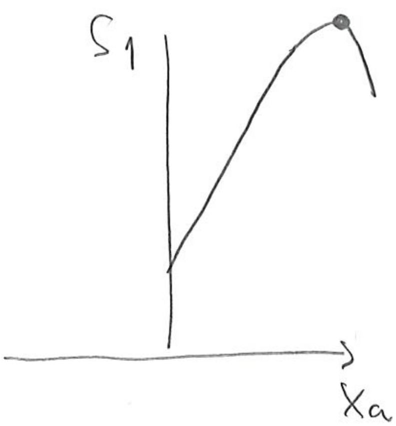


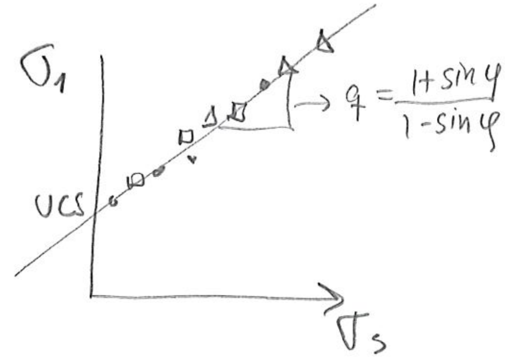
$\sigma_1 = UCS$

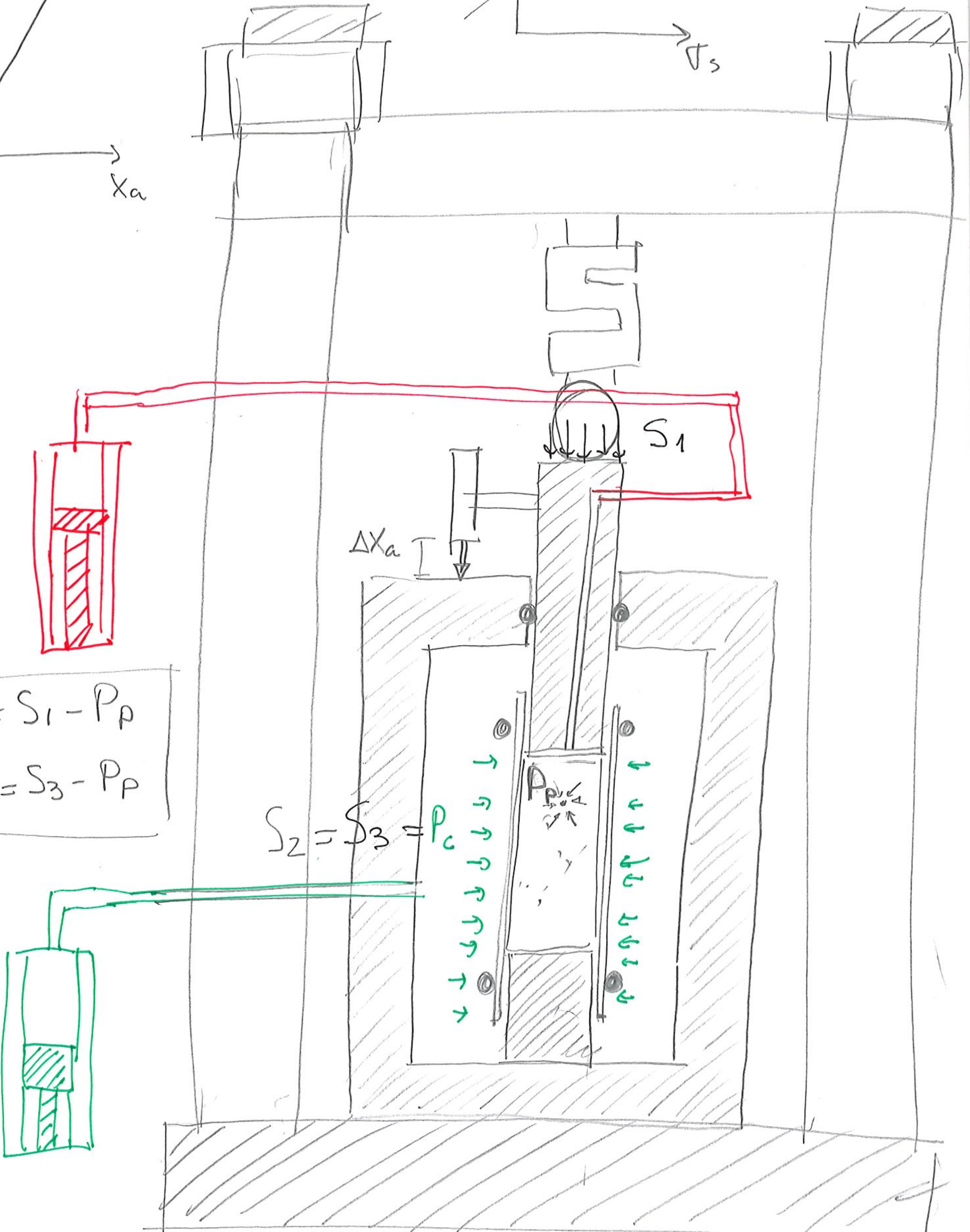
$\sigma_3 = 0$



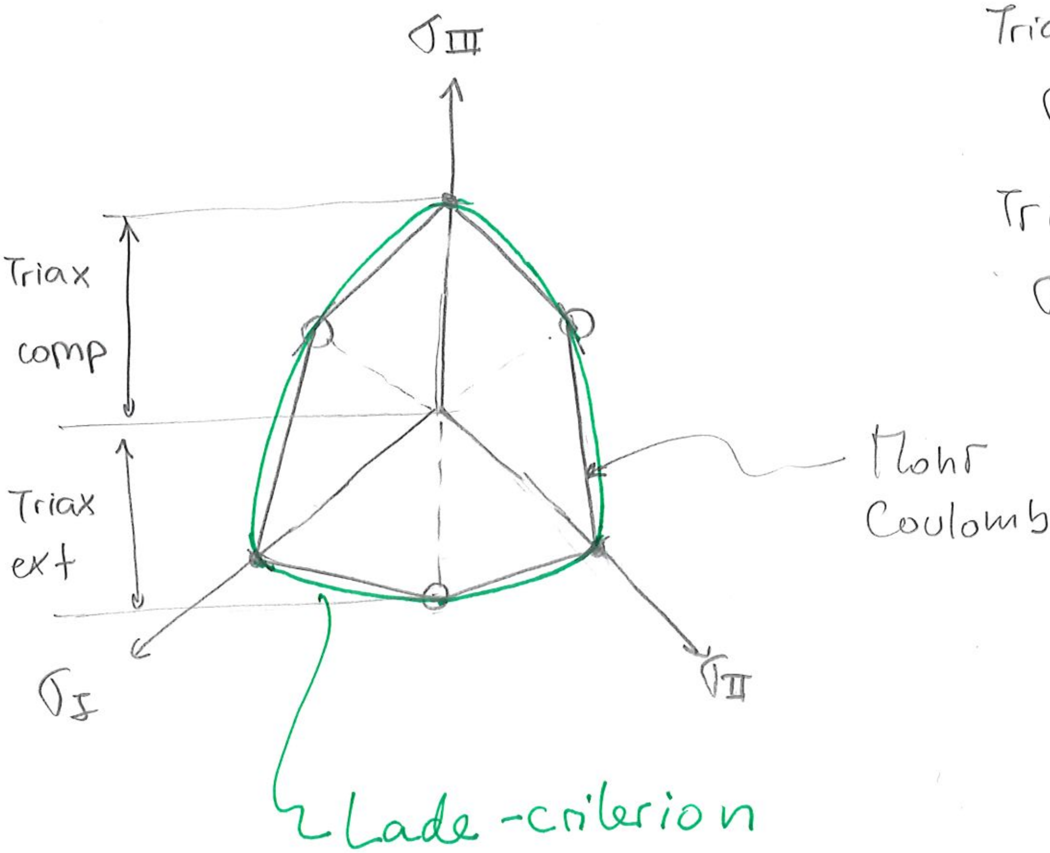
$$UCS = 2 \cdot S_0 \sqrt{\frac{1+\sin\varphi}{1-\sin\varphi}}$$







$$\boxed{\begin{aligned}\sigma_1 &= S_1 - P_p\\ \sigma_3 &= S_3 - P_p\end{aligned}}$$



Triaxial compresion ———

$\sigma_1 > \sigma_2 = \sigma_3$

Triaxial extension - - - -

$\sigma_1 = \sigma_2 > \sigma_3$

Modified Lade criterion

$$f(I_1, I_3) = K$$

$$\boxed{\frac{(I_1^*)^3}{I_3^*} = 27\,\eta}$$

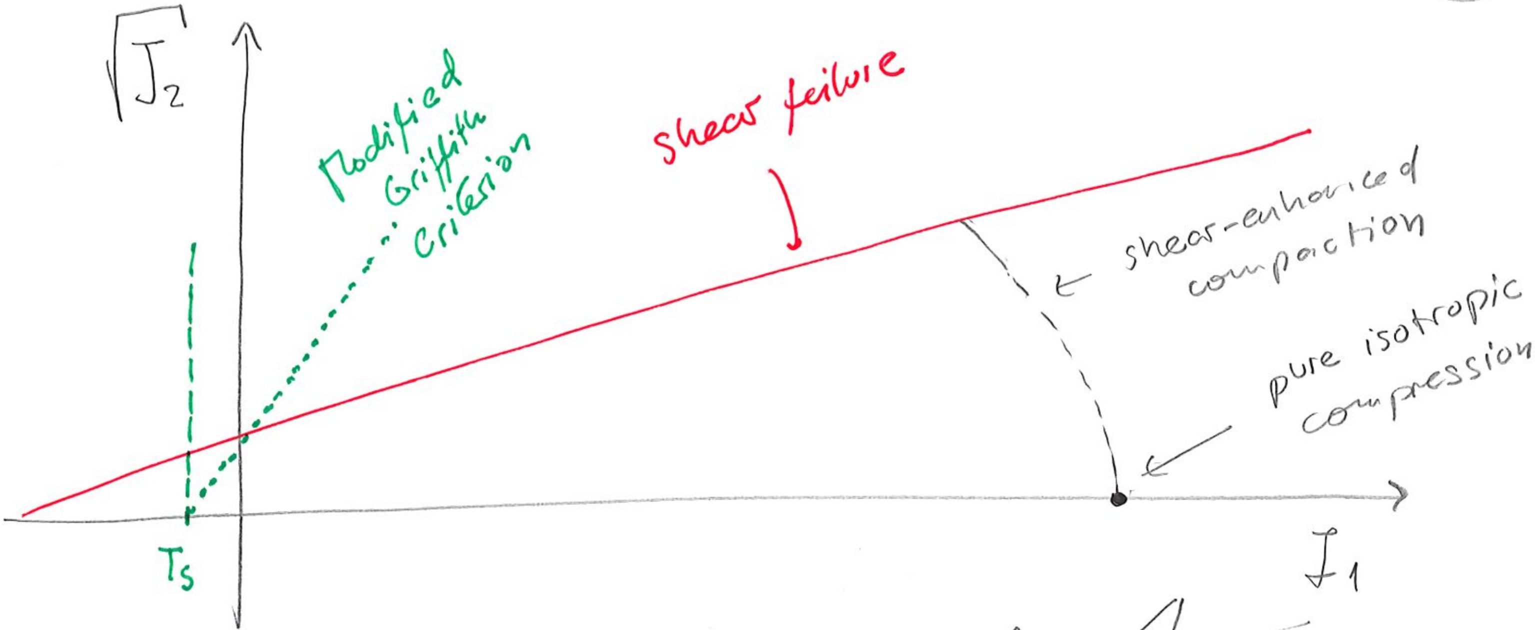
where $\sigma_i^* = \sigma_i + S$

$S = S_0/\tan\varphi$

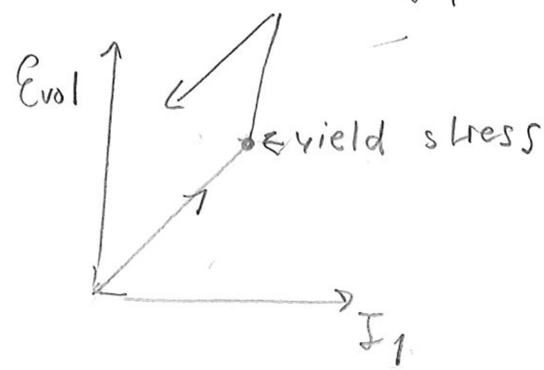
$$\eta = \frac{4(\tan\varphi)^2(9-7\sin\varphi)}{1-\sin\varphi}$$

$I_1^* = \sigma_1^* + \sigma_2^* + \sigma_3^*$

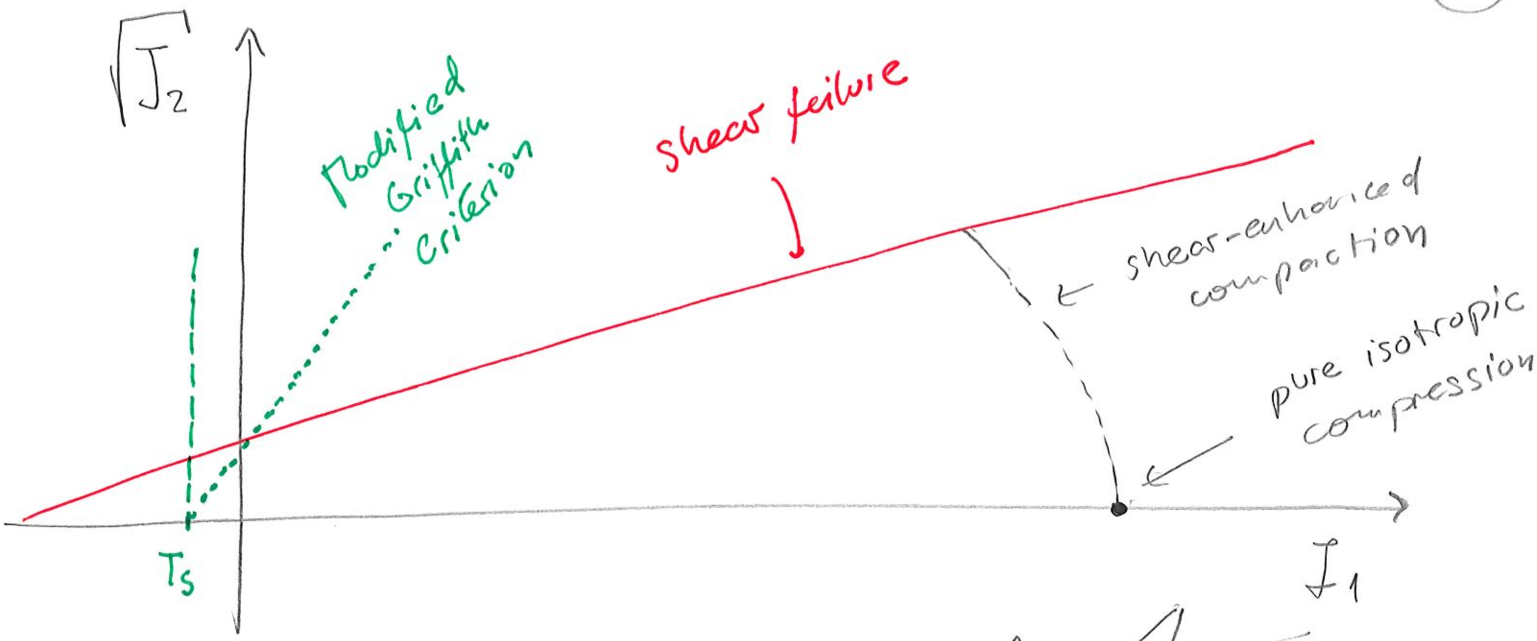
$I_3^* = \sigma_1^* \cdot \sigma_2^* \cdot \sigma_3^*$

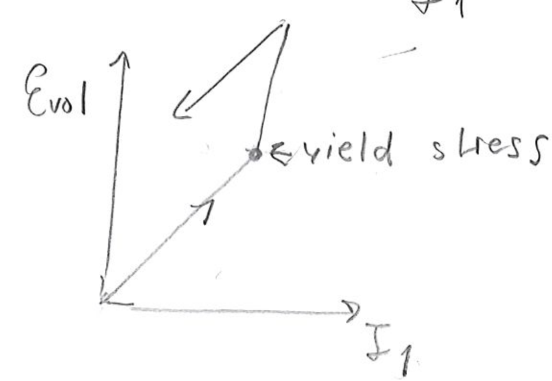


√J2
Modified Griffith criterion
shear failure
shear-enhanced compaction
pure isotropic compression
Ts
I1



εvol
yield stress
I1





Beyond the yield point

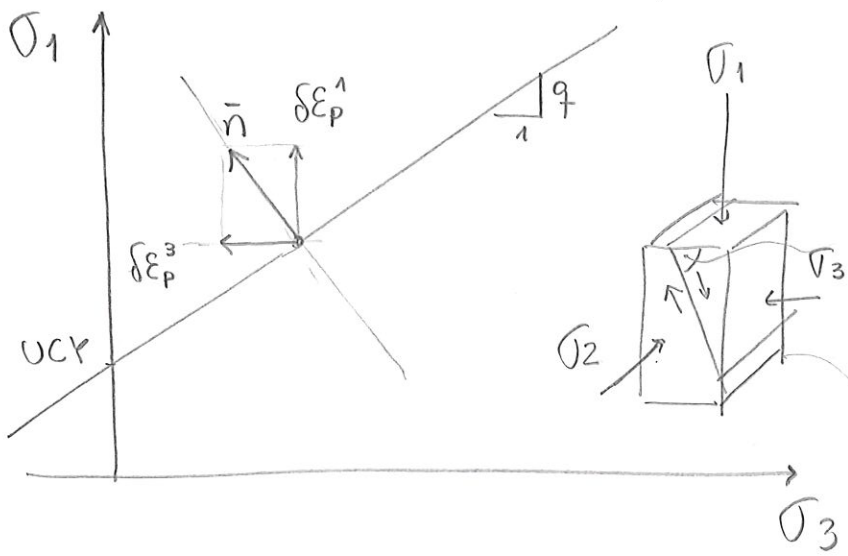
↳ predict plastic strains $d\varepsilon^p \leftrightarrow d\sigma_{ij}$

↳ small-strain (continuous) rate-independent plasticity

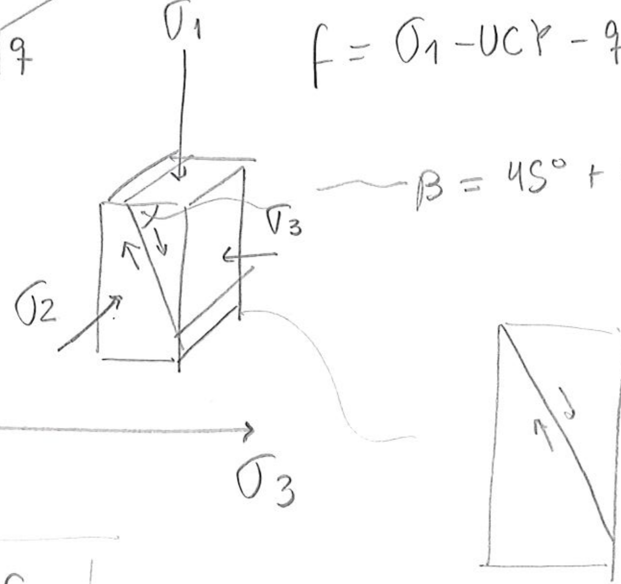
1. yield criterion
$$f(\sigma_{ij}) = Y$$
2. strain hardening rule
$$Y = F'(\varepsilon^p)$$
3. strain decomposition
$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p$$
4. plastic flow rule
$$d\varepsilon_{ij}^p \leftrightarrow d\sigma_{ij}$$
5. elastic unloading criterion

Example: Mohr-Coulomb $\quad \tau = S_0 + \mu \sigma_n$

$$\sigma_1 = UCY + q\,\sigma_3$$



$$f = \sigma_1 - UCY - q\,\sigma_3$$



$\beta = 45° + \varphi/2$

flow rule:
(associated)

$$\boxed{d\varepsilon_{ij}^p = \underbrace{d\lambda}_{\text{param}} \cdot \frac{\partial f}{\partial \sigma_{ij}}}$$

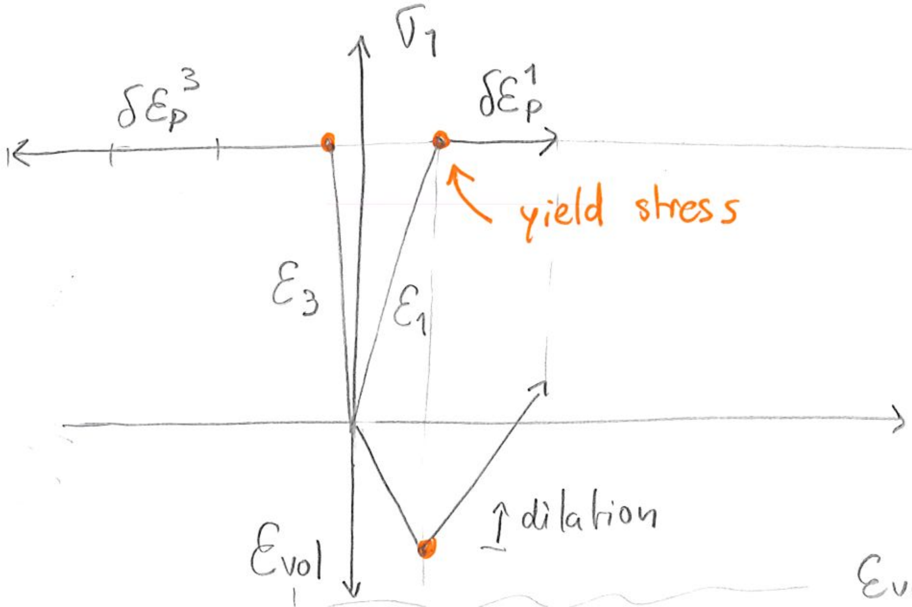
$$\bar{n} = \left(\frac{\partial f}{\partial \sigma_1}, \frac{\partial f}{\partial \sigma_2}, \frac{\partial f}{\partial \sigma_3} \right)$$

$$\bar{n} = (1, 0, -q)$$

$$\begin{cases} \delta\varepsilon_p^1 = d\lambda \cdot 1 \\ \delta\varepsilon_p^2 = d\lambda \cdot 0 \\ \delta\varepsilon_p^3 = d\lambda\,(-q) \end{cases}$$

$$q = \frac{1+\sin\varphi}{1-\sin\varphi} \; ; \text{ if } \varphi = 30° \Rightarrow q = 3$$

$$\delta\varepsilon_p^{vol} = d\lambda\,(1-q) \;\} \text{ Dilation}$$



elasto-plastic
perfect-plasticity
no hardening

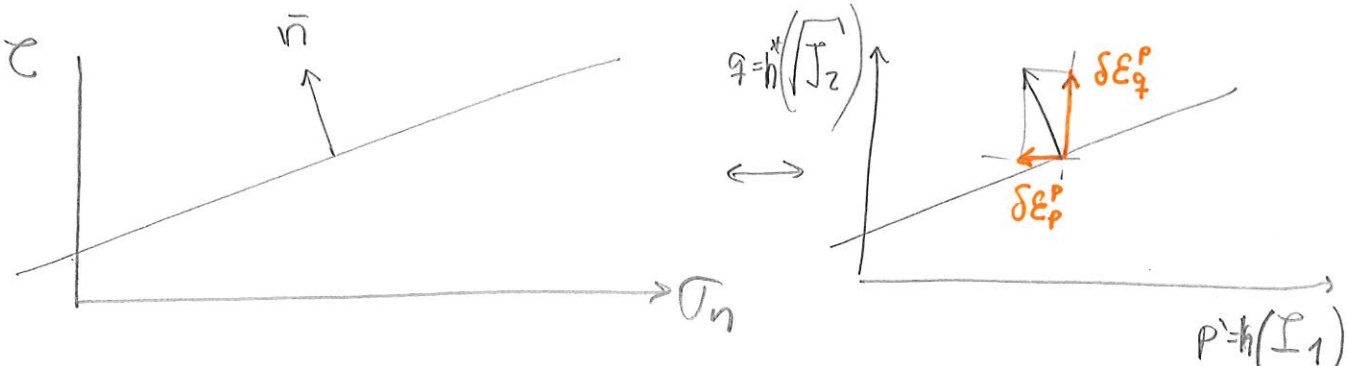
ε_{vol} vs ε_1

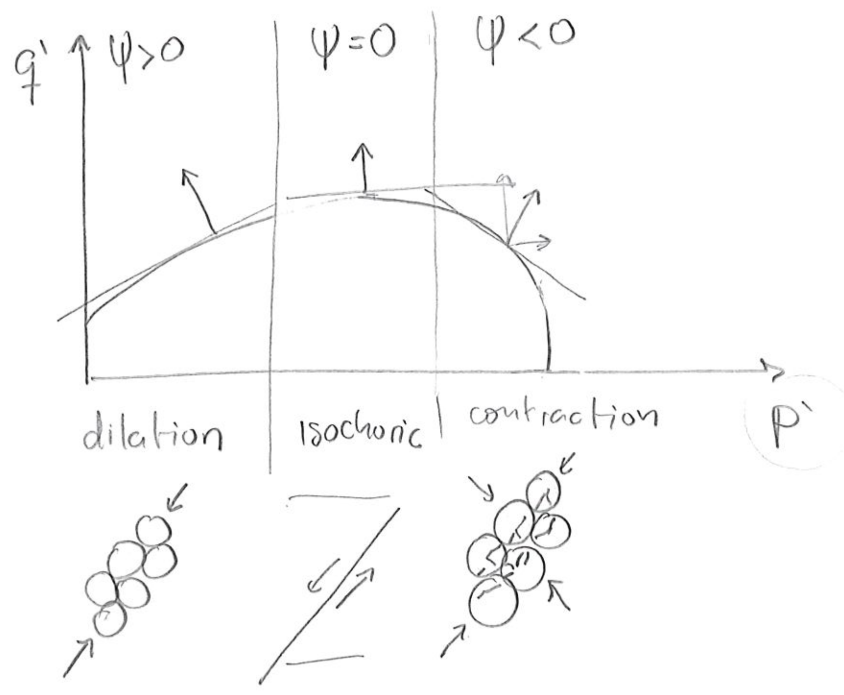
$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial F}{\partial \sigma_{ij}}$$ yield surface $\Rightarrow$ Associated flow rule

$$\partial\varepsilon_{ij}^{p} = d\lambda \cdot \frac{\partial g}{\partial \sigma_{ij}}$$ plastic potential function $\Rightarrow$ $g \neq f$ Non-associated flow rule

$$g = \sigma_1 - UCP - \frac{1+\sin\psi}{1-\sin\psi}\sigma_3 \qquad \psi < \varphi$$

ψ: Dilation angle
- $>0 \rightarrow$ dilation
- $0 \rightarrow$ isochoric
- $<0 \rightarrow$ contraction

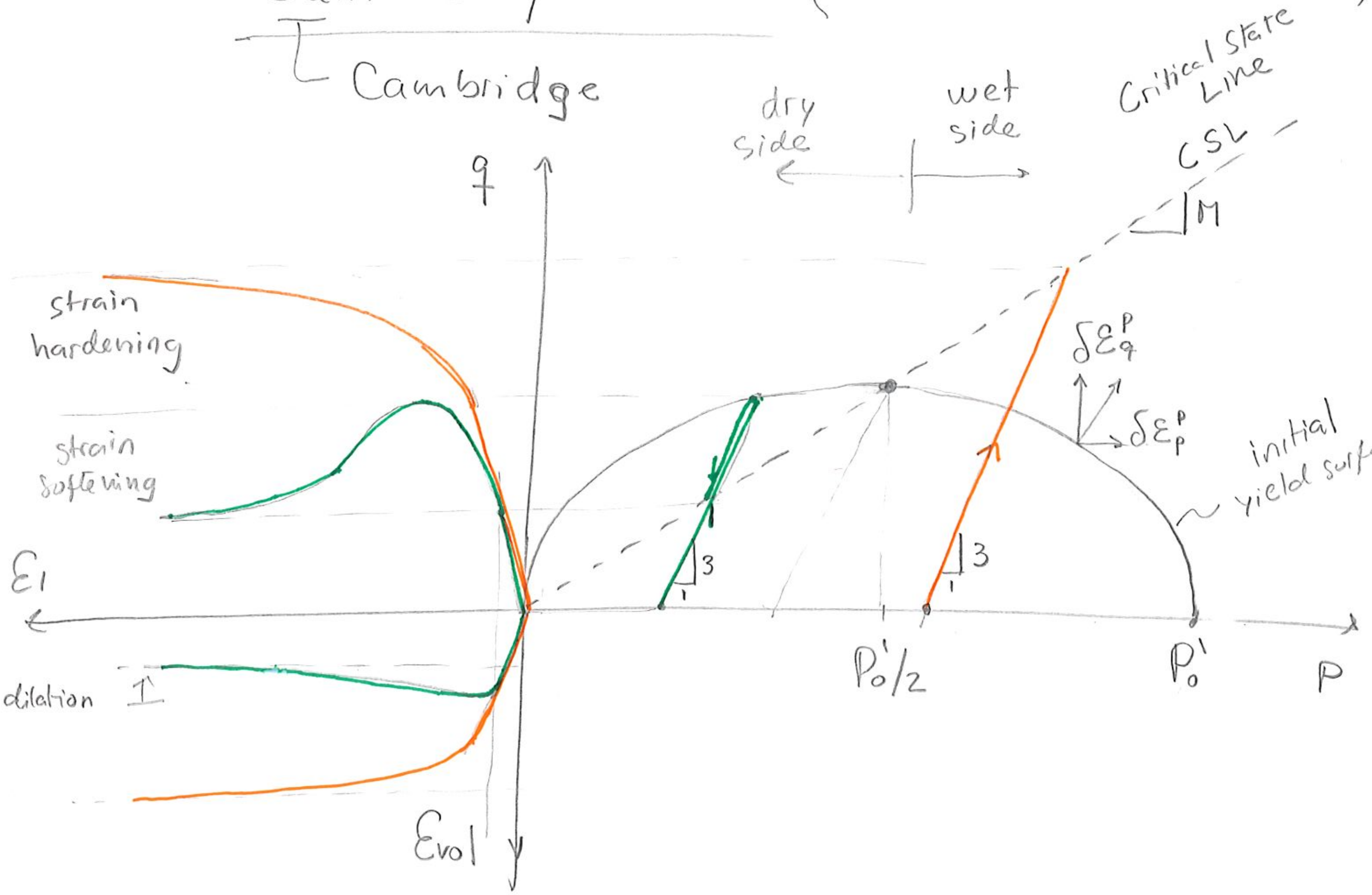


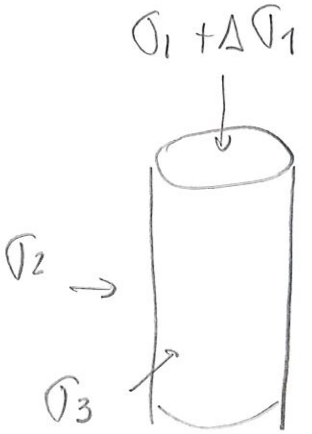


Critical State Soil Mechanics

↳ Cam-Clay Model (uncemented sediments)

Cambridge





$$p' = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

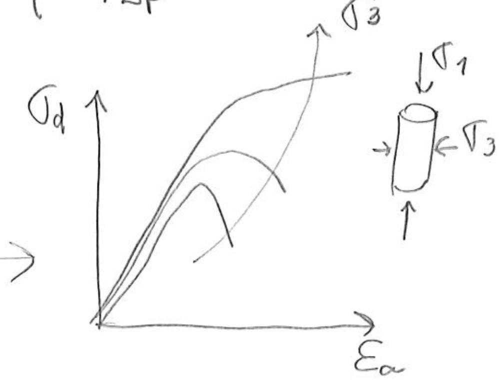
$$q = \sigma_1 - \sigma_3$$

$$\Longrightarrow \Delta\sigma_1 \quad \Delta p' = \frac{1}{3}\Delta\sigma_1$$

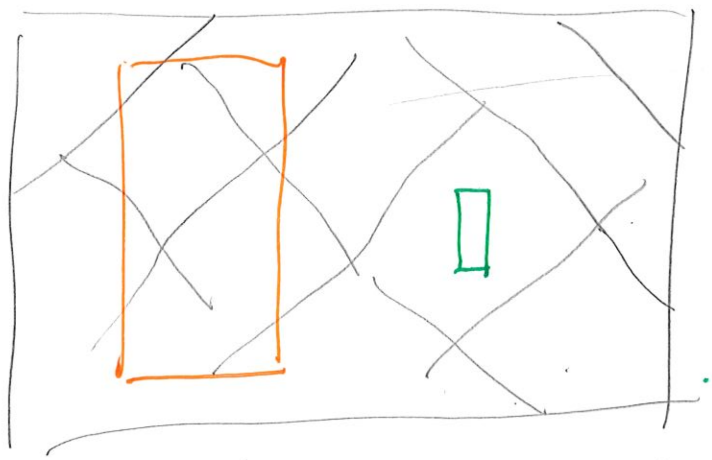
$$\Delta q = \Delta\sigma_1$$

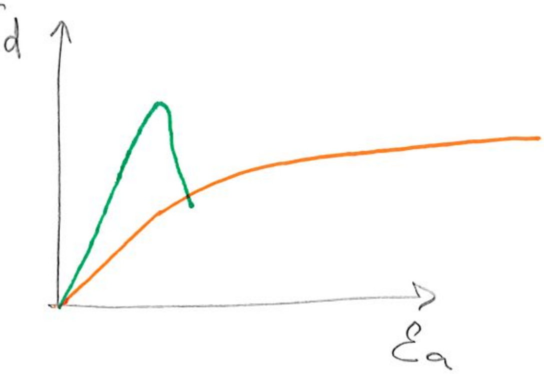
$$\Delta q / \Delta p' = 3$$

"brittle to ductile transition"



other factors brittle to ductile → temperature
→ loading rate (viscoelasticity)
→ length scale





critical state line slope

$$M = \frac{q}{p'}\Big|_{\substack{failure\\ CSL}} = \frac{\sigma_1 - \sigma_3}{\frac{\sigma_1 + 2\sigma_3}{3}} = 3\,\frac{2\cancel{\sigma_3}}{5\cancel{\sigma_3}} = 1.2$$

$$\frac{\sigma_1}{\sigma_3} = \underbrace{\frac{1+\sin\varphi_{cs}}{1-\sin\varphi_{cs}}}_{3} \rightarrow \varphi_{cs} = 30° \Rightarrow \sigma_1 = 3\sigma_3$$

f : yield surface

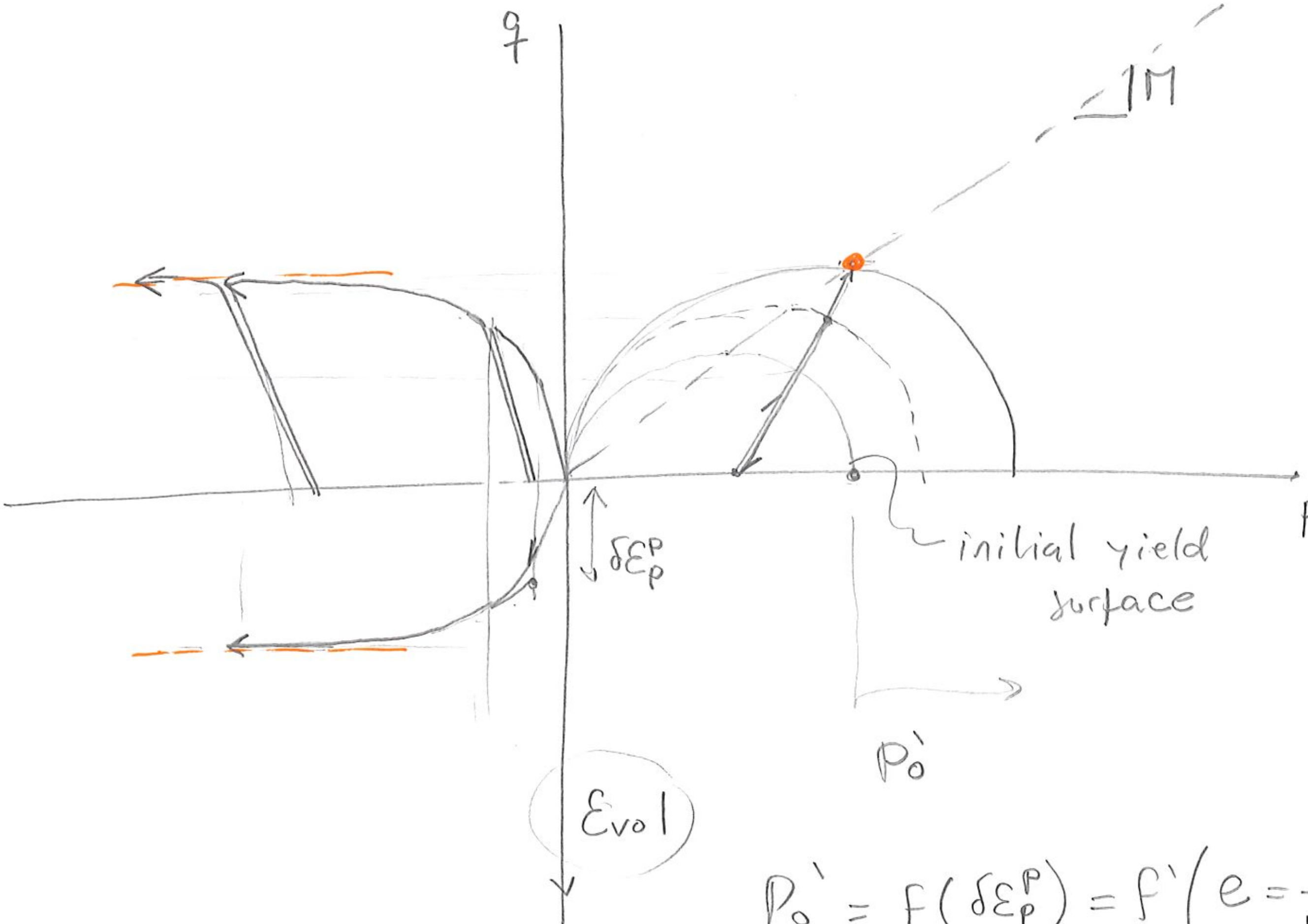
$$f(q, p', p_0') = q^2 - M^2 p'(p_0' - p') = 0 \Rightarrow \text{yield}$$

↳ $q = 0 \Rightarrow p' = p_0'$

↳ $q = Mp' \Rightarrow p' = \frac{p_0'}{2}$

p_0' is a variable ⟷ hardening parameter → $f(\emptyset)$

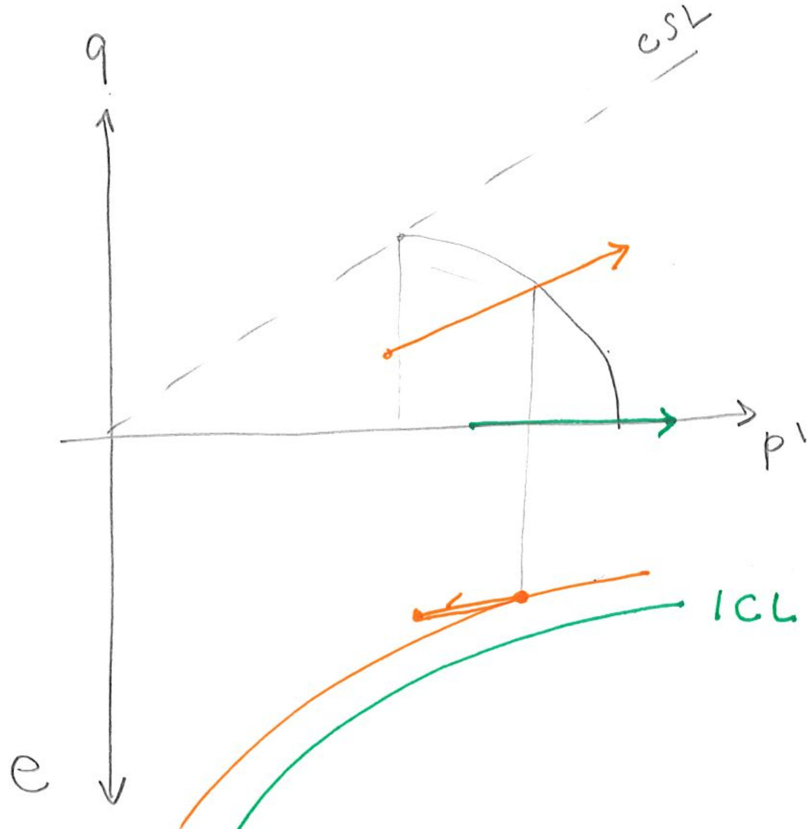
$f(\delta\varepsilon_p^p)$



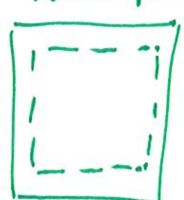
$$p_0' = f(\delta \varepsilon_p^p) = f'\left(e = \frac{\emptyset}{1-\emptyset}\right)$$

$\Delta \delta \varepsilon_p^p > 0 \Rightarrow \downarrow \emptyset \rightarrow$ stronger

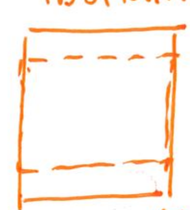
$\Delta \delta \varepsilon_p^p < 0 \Rightarrow \uparrow \emptyset \rightarrow$ weaker



isotropic

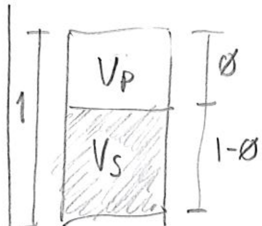


1D strain

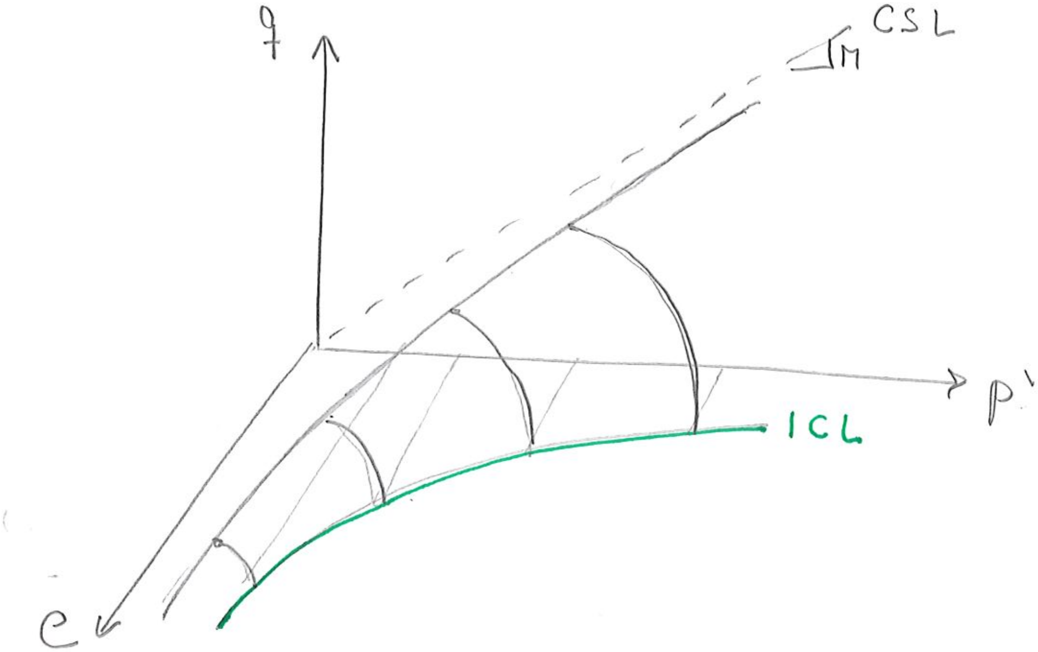


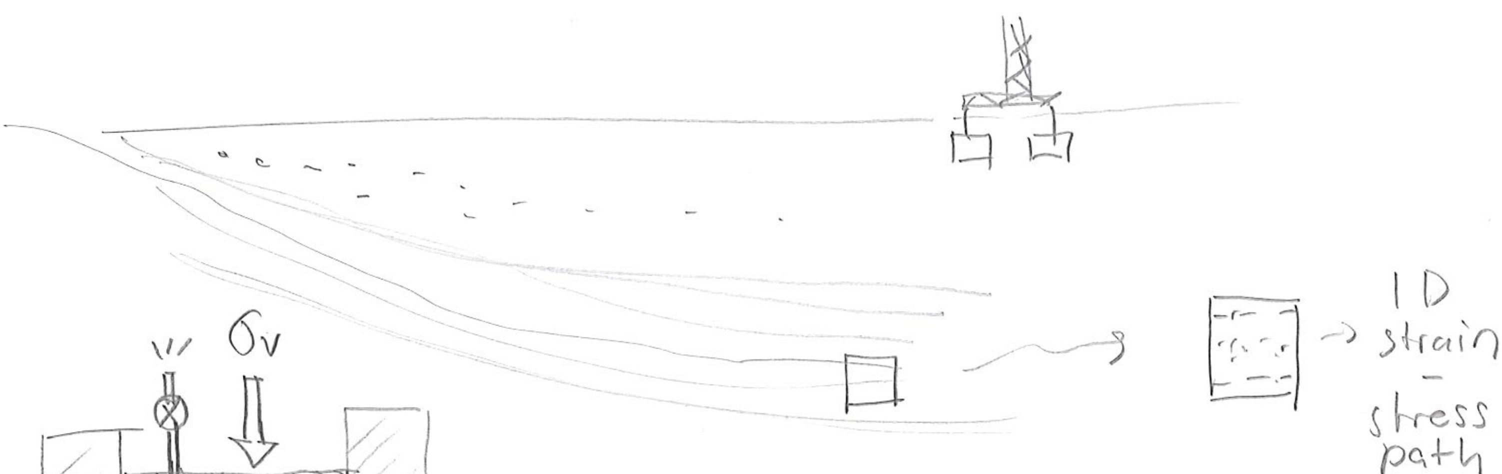
oedometer

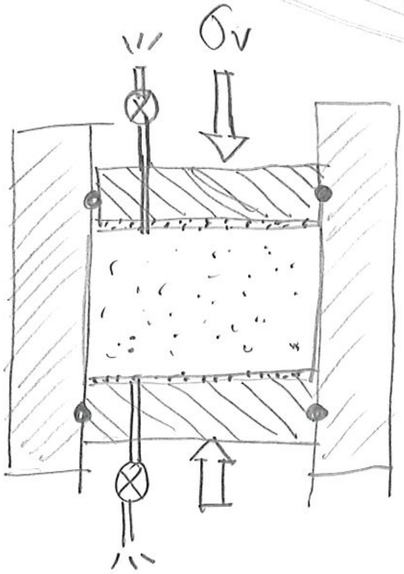
- sedimentation and burial
- depletion

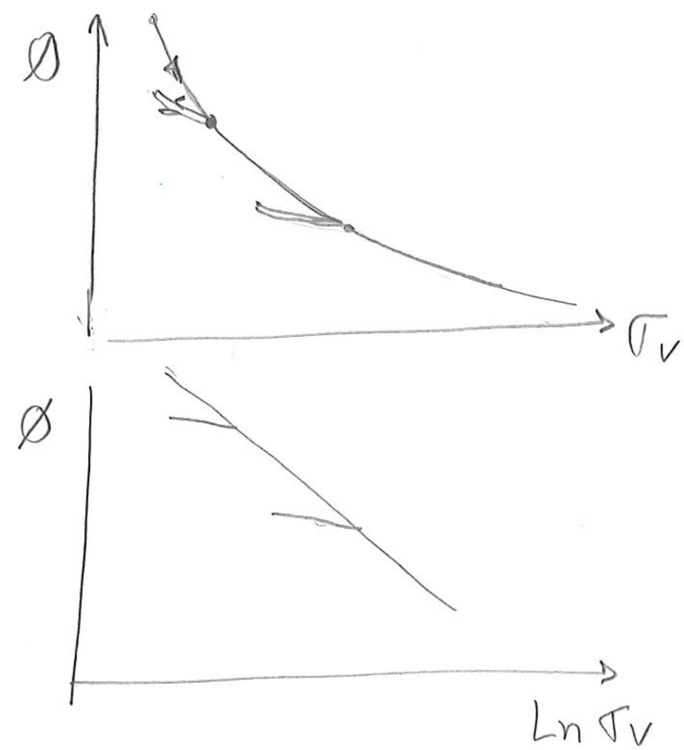


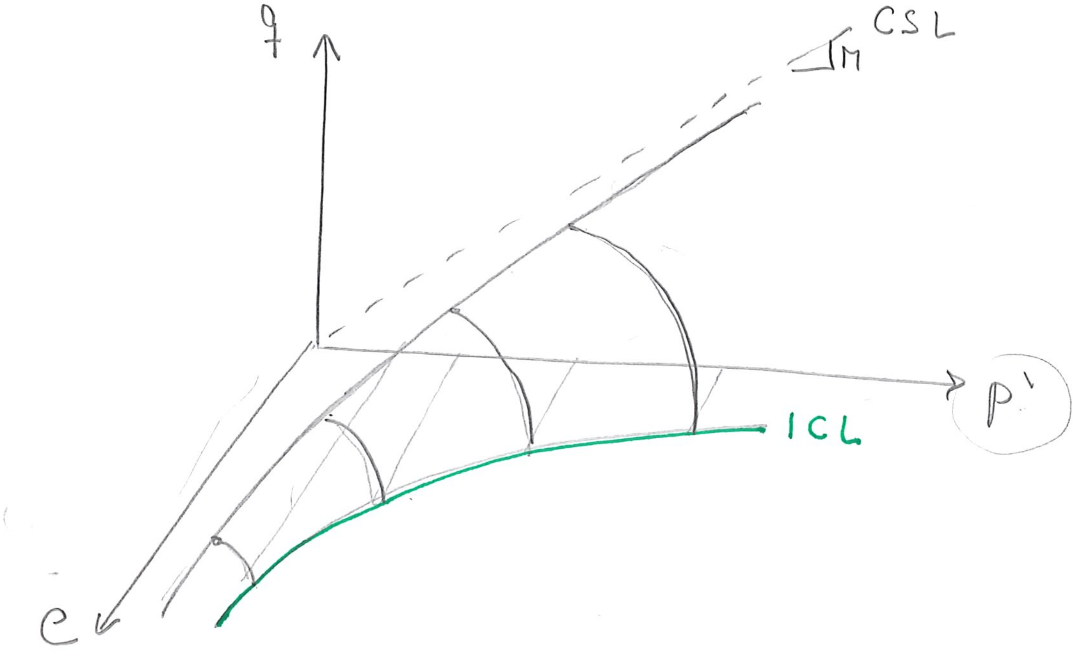
$$e = \frac{V_P}{V_S} = \frac{\emptyset}{1-\emptyset}$$


q
CSL
M
p'
ICL
e

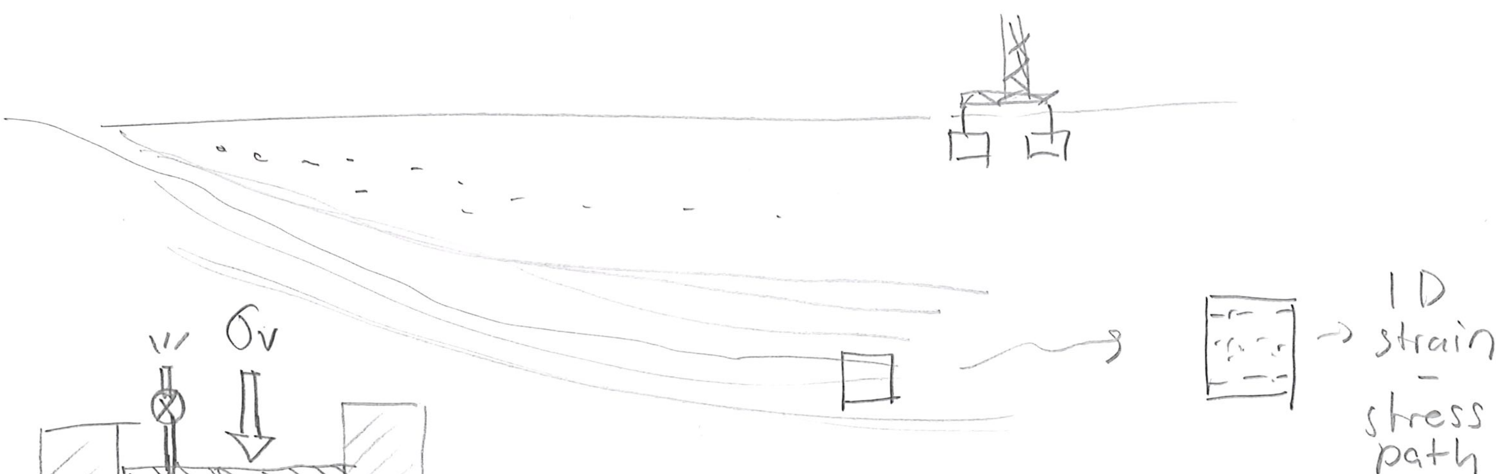

1D
strain
-
stress
path

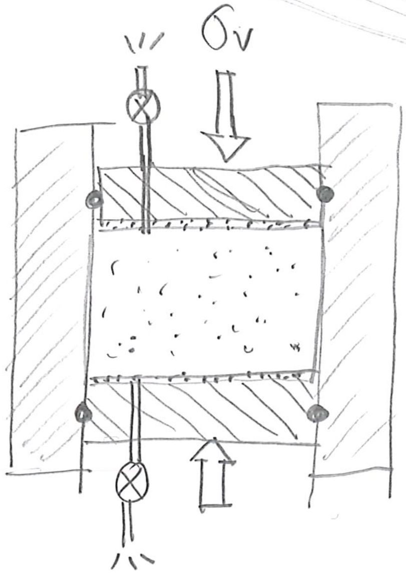

σv

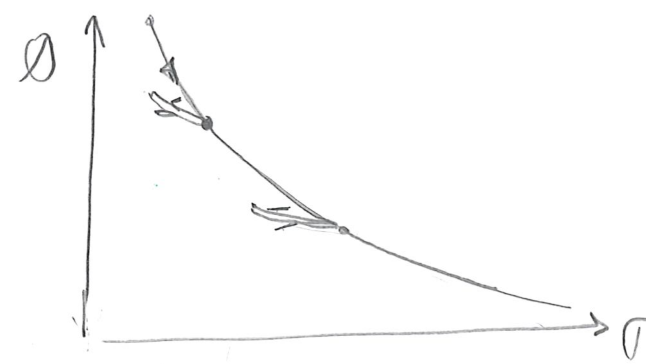

Ø
σv
Ø
Ln σv



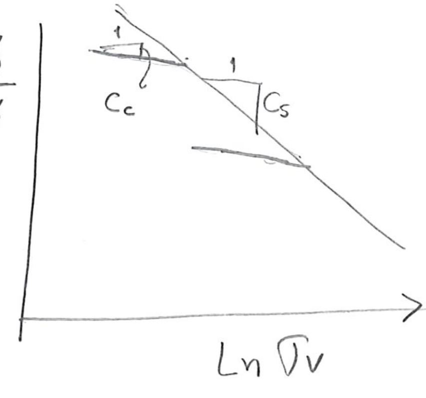
$$= \frac{\varnothing}{1-\varnothing}$$







$$e = \frac{\varnothing}{1-\varnothing}$$

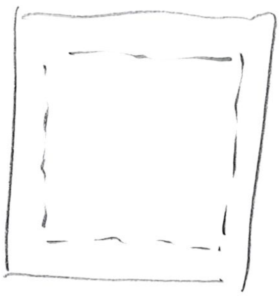


$$\rightarrow \underbrace{e - e_0 = -C_c \ln \frac{\sigma_v}{1 MPa}}$$

elastic | reloading
| $\sigma_v <$ precons stress

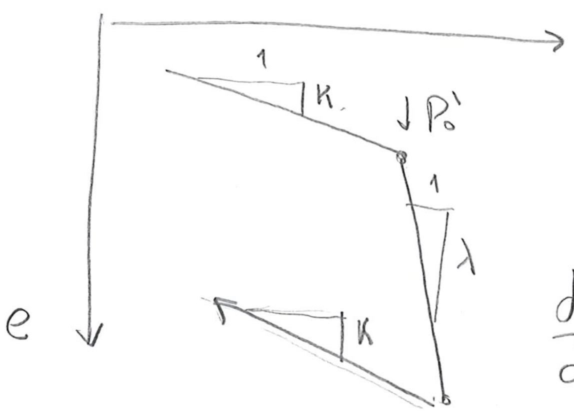
$$\underbrace{e - e_0}_{\text{elasto-plastic}} = -C_s \ln \frac{\sigma_v}{1 MPa}$$

Isotropic stress path



elastic $\Delta e = \kappa \ln \frac{p_0'}{p'}$

plastic $\Delta e = \lambda \ln \frac{p'}{p_0'}$



$e = C - \kappa \ln p'$

$\frac{de}{dp} = -\frac{\kappa}{p'}$

① Elastic strains

$$\delta \varepsilon_p^e = \frac{\kappa}{v} \frac{dp'}{p'}$$

$$\delta \varepsilon_q^e = \frac{dq}{3G}$$

v: specific volume

$v = 1 + e$

$$e = \frac{V_v}{V_b - V_v} \rightarrow de = \quad dV_v + \quad dV_b \quad , \quad \boxed{\delta \varepsilon_p = \frac{dV_b}{V_b}}$$

$\boxed{dV_v = dV_b}$

$(2/3)(\varepsilon_1 - \varepsilon_3)$ $\quad de = -(1+e)\,\delta\varepsilon_p$

$\boxed{\varepsilon_q = \varepsilon_1 - \varepsilon_3}$

$$\begin{bmatrix} \delta \varepsilon_p^e \\ \delta \varepsilon_q^e \end{bmatrix} = \begin{bmatrix} (\kappa/v)(p')^{-1} & 0 \\ 0 & 1/3G \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix}$$

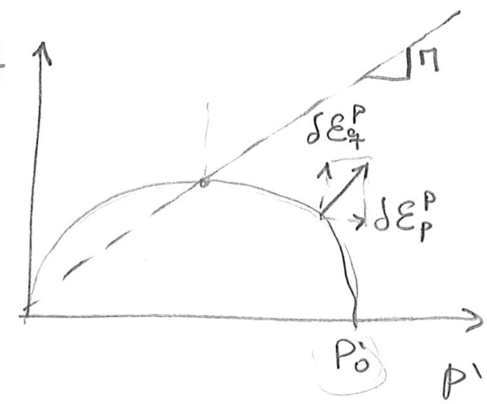
(2) Plastic strains

$$f = q^2 - M^2 p'(p_0' - p')$$

$$\boxed{F^* = \frac{p'}{p_0'} - \frac{M^2}{\eta^2 + M^2} \quad ; \quad \eta = \frac{q}{p'}}$$

derivatives

$$\begin{cases} \dfrac{\partial F^*}{dp'} = p_0' M^2 \left(\dfrac{M^2 - \eta^2}{M^2 + \eta^2} \right) \\[2ex] \dfrac{\partial f^*}{\partial q} = p_0' M^2 \left(\dfrac{2\eta}{M^2 + \eta^2} \right) \\[2ex] \dfrac{\partial F^*}{\partial p_0'} = - \dfrac{p'}{(p_0')^2} \end{cases}$$



Associated flow rule

$$\delta \varepsilon_{ij}^p = \partial \chi \frac{\partial f}{\partial \sigma_{ij}} \Rightarrow \begin{cases} \partial \varepsilon_p^p = \partial \chi \dfrac{\partial f^*}{\partial p'} \\[2ex] \partial \varepsilon_q^p = \partial \chi \dfrac{\partial f^*}{\partial q} \end{cases}$$

$$\begin{cases} \delta \varepsilon_p^p = \partial \chi \left[p_0' M^2 \left(\dfrac{M^2 - \eta^2}{M^2 + \eta^2} \right) \right] \\[2ex] \delta \varepsilon_p^p = \dfrac{\lambda - \kappa}{v} \dfrac{d p_0'}{p_0'} \end{cases}$$

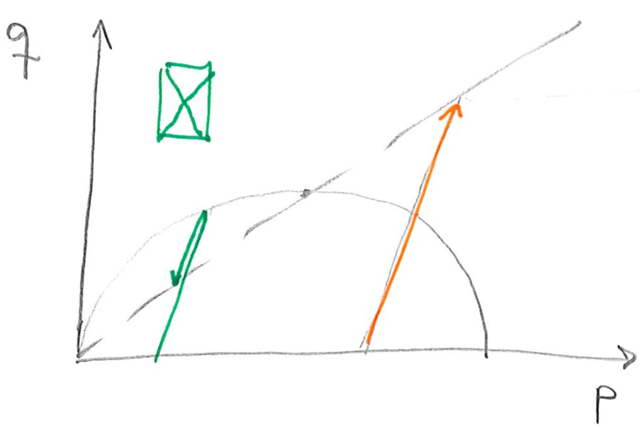
← isotropic elasto-plastic compression

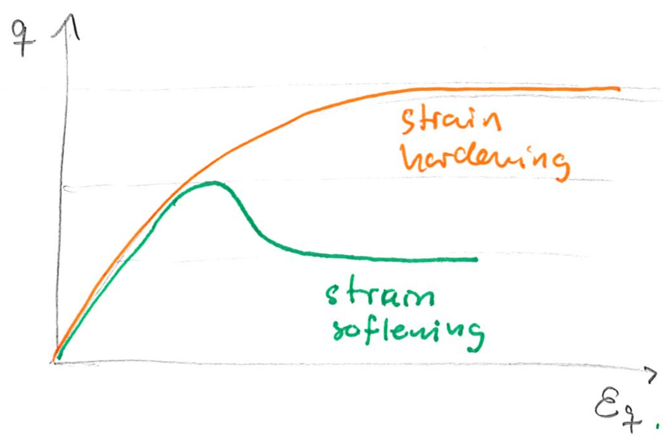
$$\boxed{d p_0' = \frac{v}{\lambda - \kappa} p_0' \, d\varepsilon_p^p} \quad \rightarrow \quad \delta \chi = \frac{\lambda - \kappa}{v p' (M^2 + \eta^2)}$$

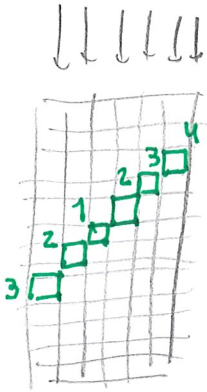
$$\begin{bmatrix}\delta\varepsilon_p^p\\ \delta\varepsilon_q^p\end{bmatrix} = \frac{\lambda-\kappa}{v p'(M^2+\eta^2)}\begin{bmatrix} M^2-\eta^2 & 2\eta \\ 2\eta & \dfrac{4\eta^2}{M^2-\eta^2}\end{bmatrix}\begin{bmatrix}dp'\\ dq\end{bmatrix}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^p$$

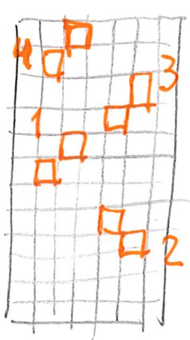
$$d\underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{C}}}}^e d\underline{\underline{\sigma}} + \underline{\underline{\underline{\underline{C}}}}^p d\underline{\underline{\sigma}}$$



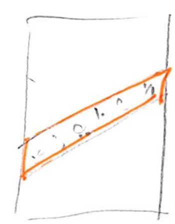


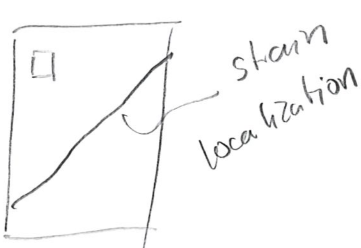


- positive feedback mechanism
- unstable process

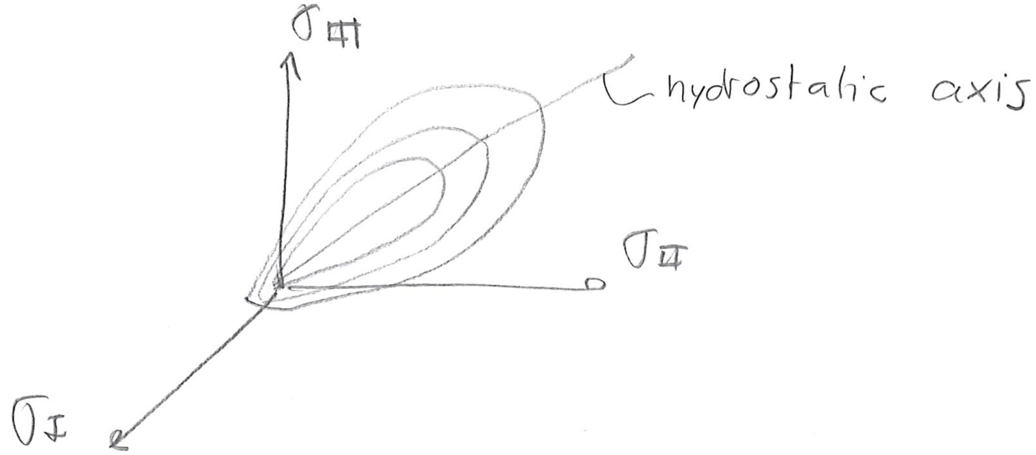


- shear and compaction bands
- stable process

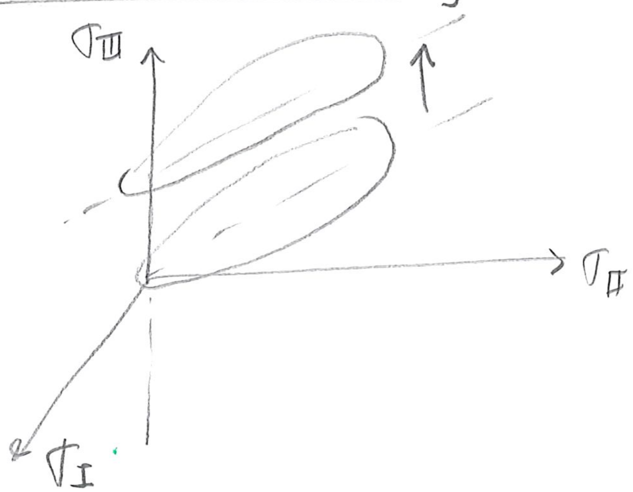




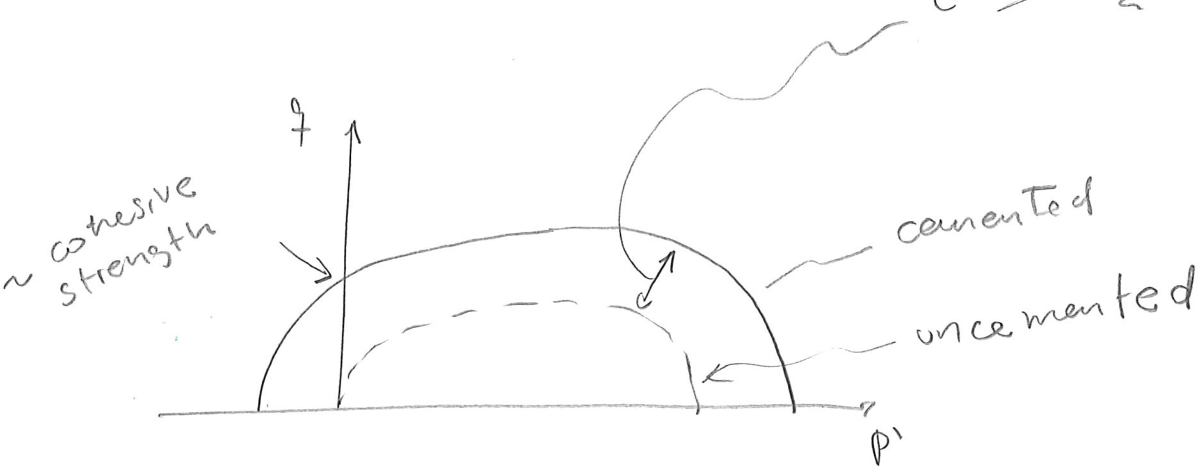
→ Isotropic hardening



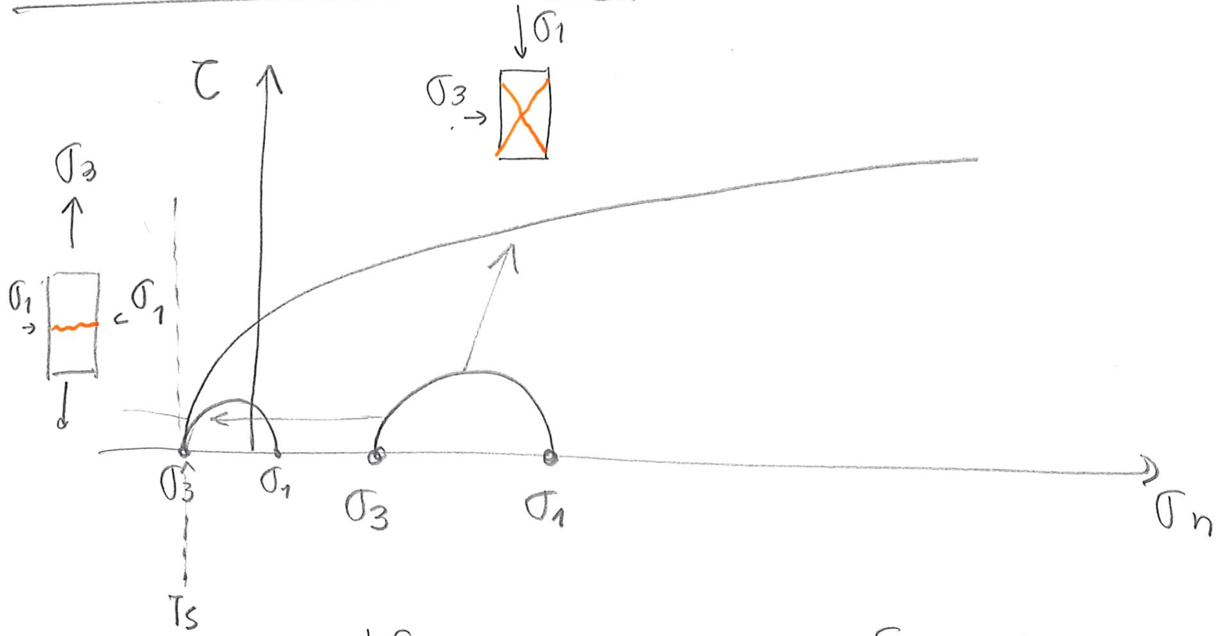
→ kinematic hardening

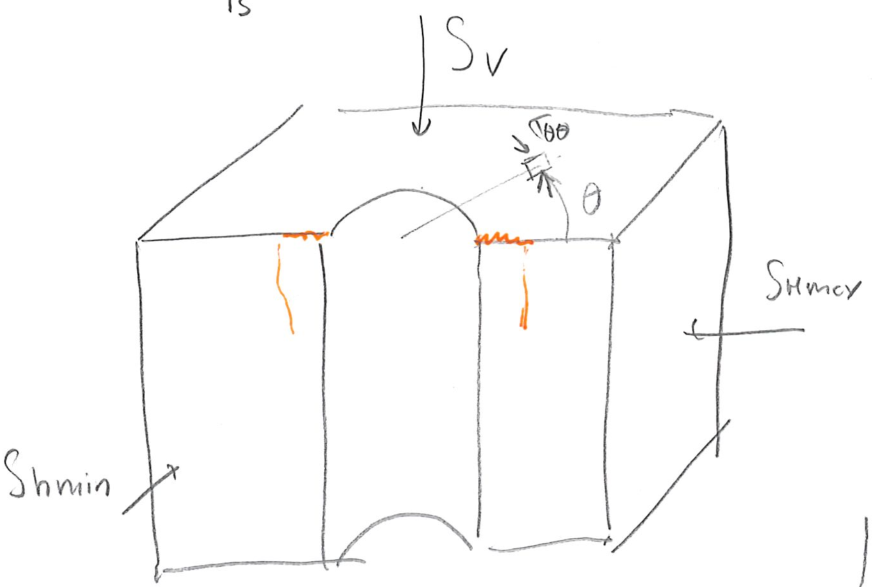


chemo-plasticity



Hydraulic Fracturing





$S_v > S_{Hmax} > S_{hmin}$

Kirsch

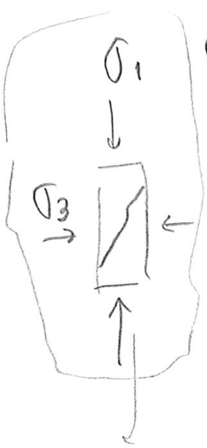
$$\sigma_{\theta\theta}(r=a) = -(P_w-P_p)+(\sigma_{Hmax}+\sigma_{hmin}) - 2(\sigma_{Hmax}-\sigma_{hmin})\cos(2\theta)$$

$$\theta = 0, \pi$$
$$\sigma_{\theta\theta} = -T_s$$

$$-T_s = -(P_w-P_p) - \sigma_{Hmax} + 3\sigma_{hmin}$$

break down pressure

$$\boxed{P_b = P_p \underbrace{- \sigma_{Hmax} + 3\sigma_{hmin}}_{\text{stress anisotropy}} + T_s}$$



$\sigma_1 = q\,\sigma_3$

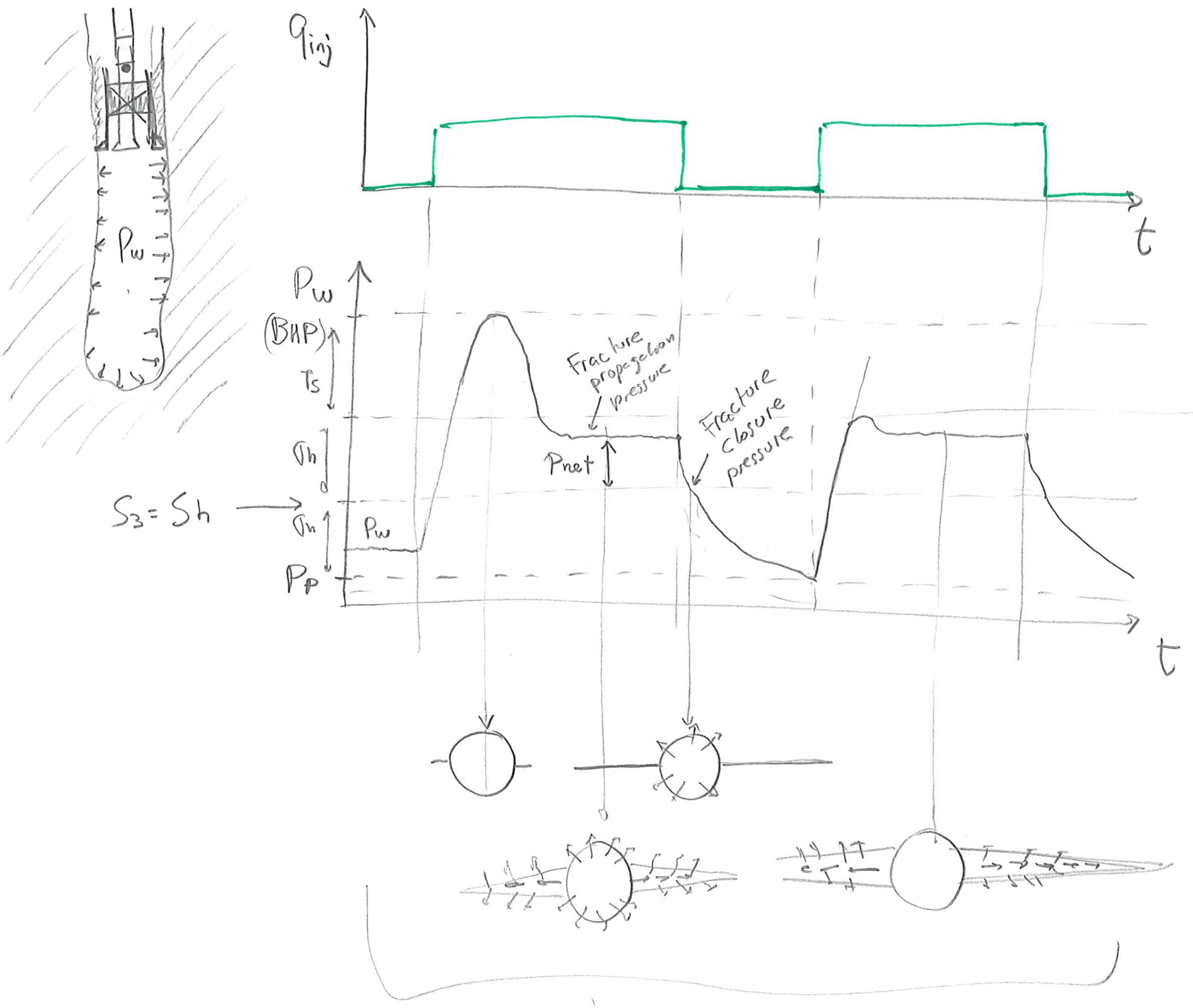
$q = \dfrac{1+\sin\varphi}{1-\sin\varphi}$

$\varphi = 30°$

$\boxed{\sigma_{Hmax} = \sigma_{hmin} = \sigma_h} \Rightarrow \quad P_b = P_p + \sigma_h + \sigma_h + T_s$

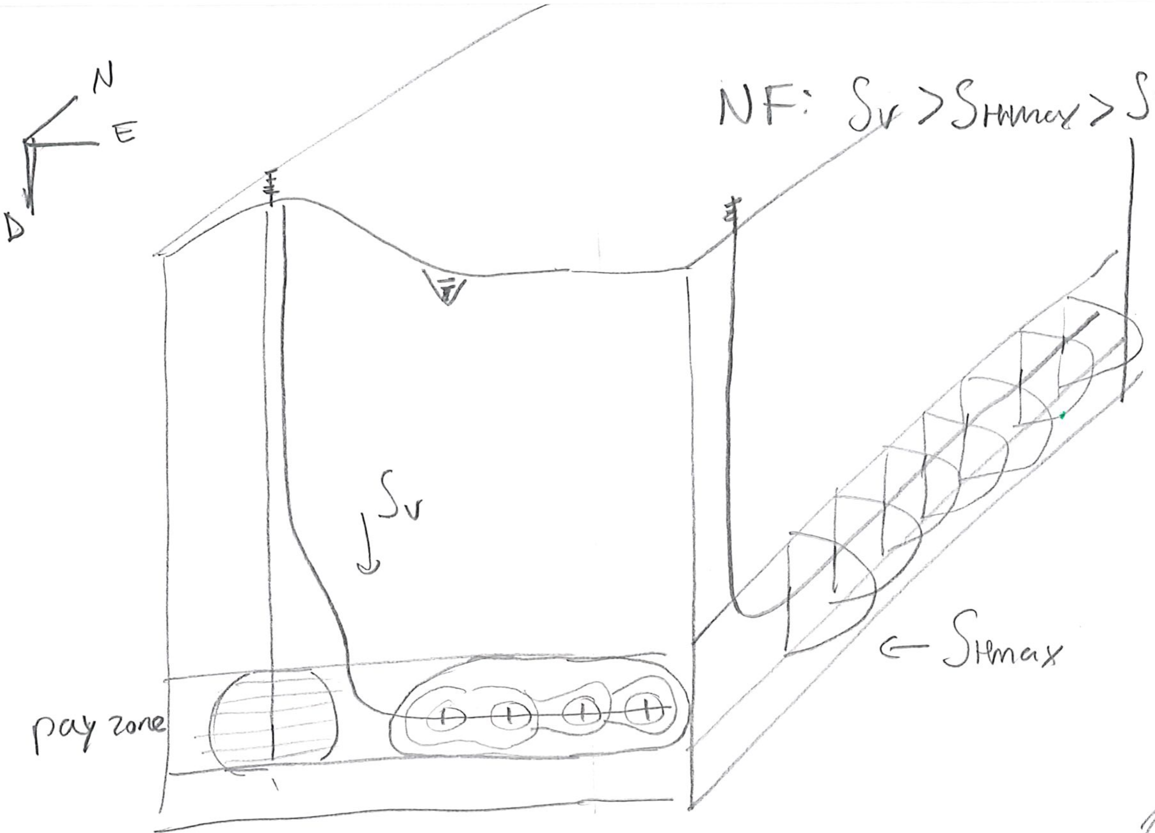
Limit Equil + SS $\Rightarrow \sigma_{Hmax} = 3\sigma_{hmin} \Rightarrow \quad P_b = P_p + \sigma_h^{min} - \sigma_h^{min} + T_s$

$\sigma_{Hmax} = \sigma_{hmin} = \sigma_h$

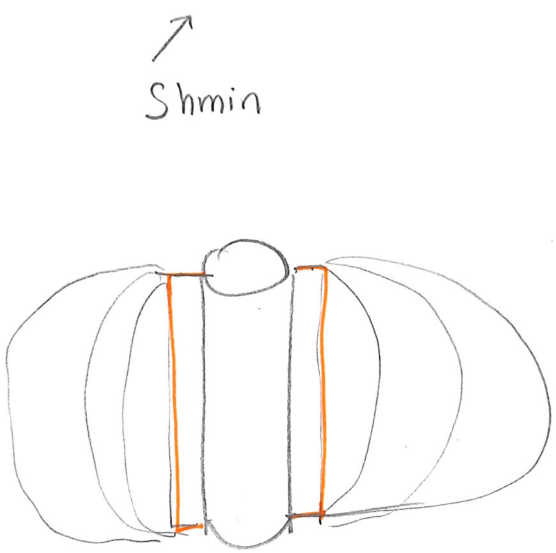


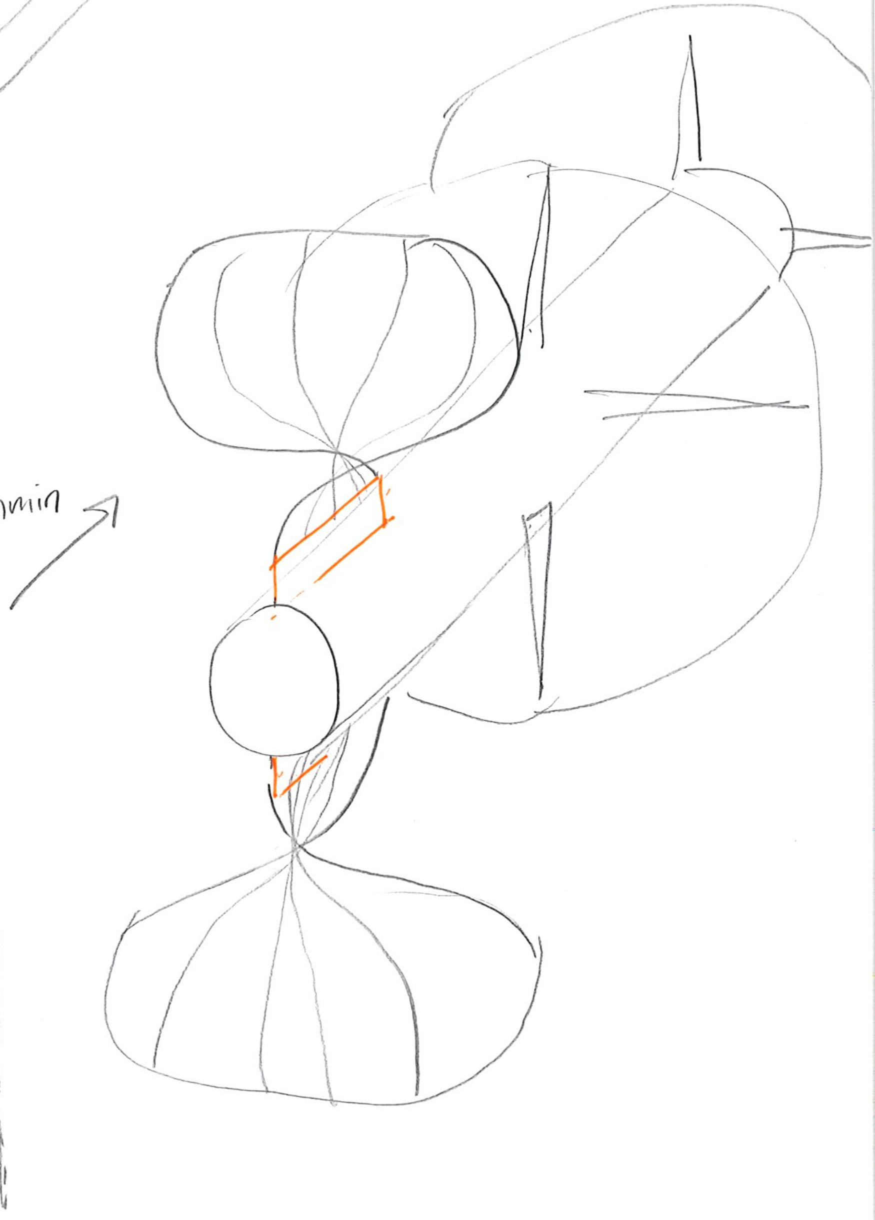
coupled hydraulic fracture propagation problem

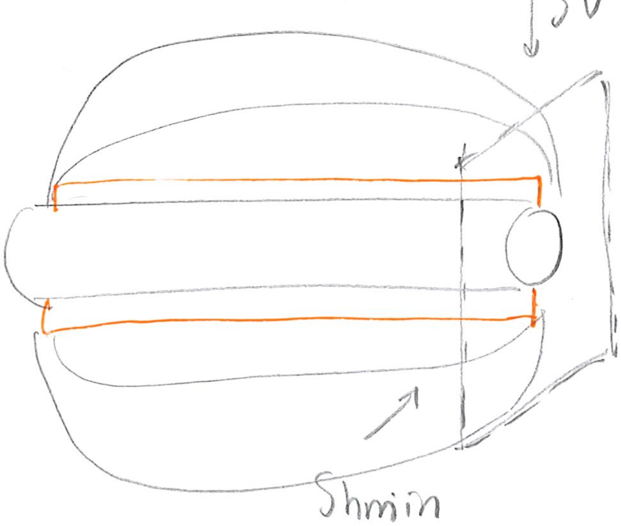
NF: $S_v > S_{Hmax} > S_{hmin}$

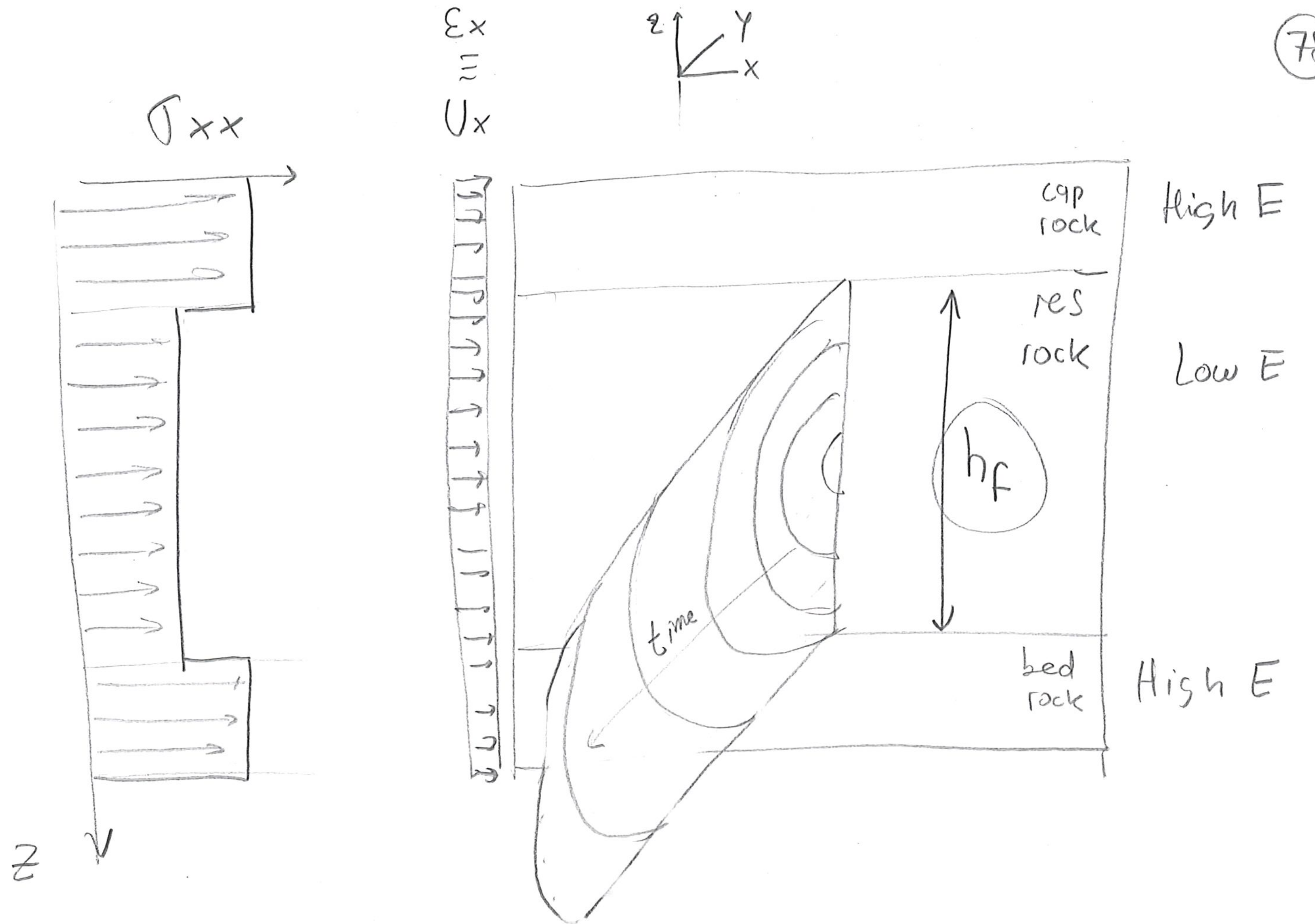


multistage
HF
↓
increase
surface
area

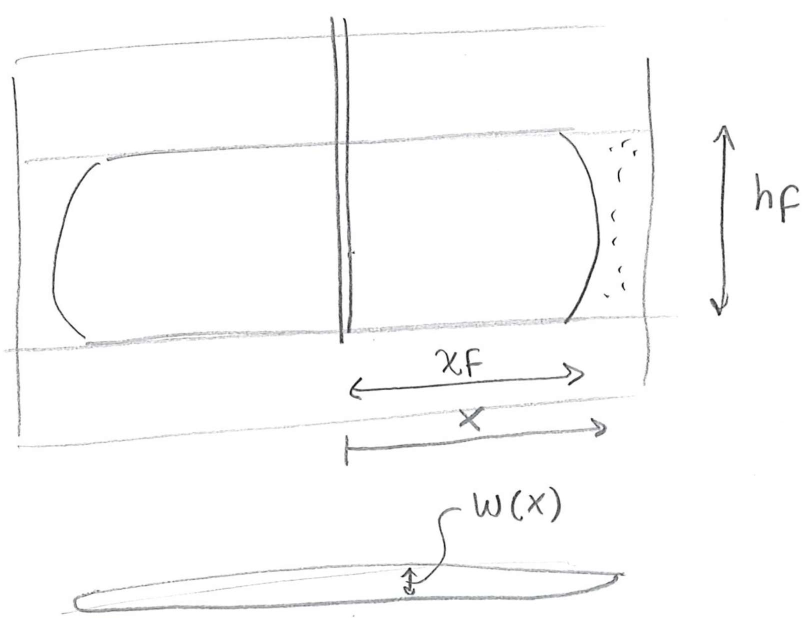






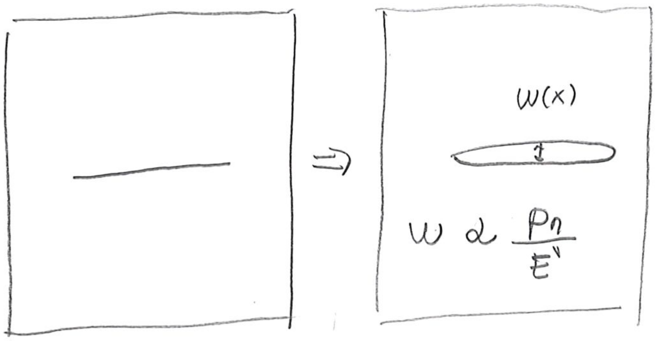


$$\sigma_{hmin} = \frac{\nu}{1-\nu}\sigma_v + E'\varepsilon_{hmin} + E'\nu \cdot \varepsilon_{Hmax}$$



Coupled fracture problem (Valko and Economides Hydraulic Fracture Mechanics)

$$P_n = P(\text{elasticity}) + P(\text{viscous losses}) + P\left(\begin{matrix}\text{new}\\ \text{rock}\\ \text{surface}\end{matrix}\right)$$



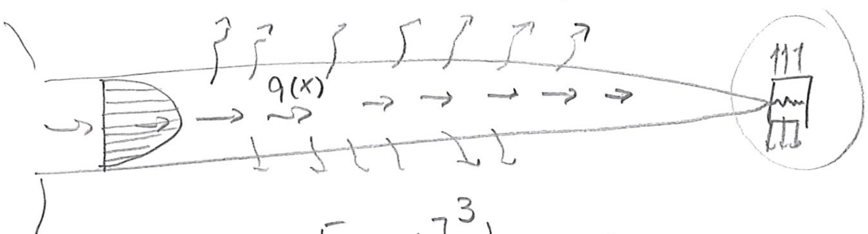
$$w \propto \frac{P_n}{E'}$$

$$\begin{cases} \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} \\ \nabla \cdot \underline{\underline{\sigma}} = 0 \end{cases}$$

If $K_I > K_{Ic}$

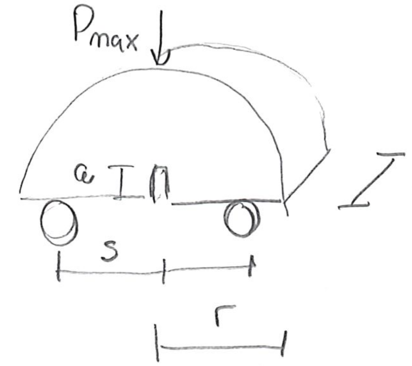
propagation

fracture toughness



$$q(x) \propto \frac{[W(x)]^3 h_F}{\mu} \frac{\Delta P}{\Delta X}$$

$$K_{Ic} = \frac{P_{max} (\pi a)^{1/2}}{2rL} Y_I$$



$Y_I = f(a, s, r) \approx 5$

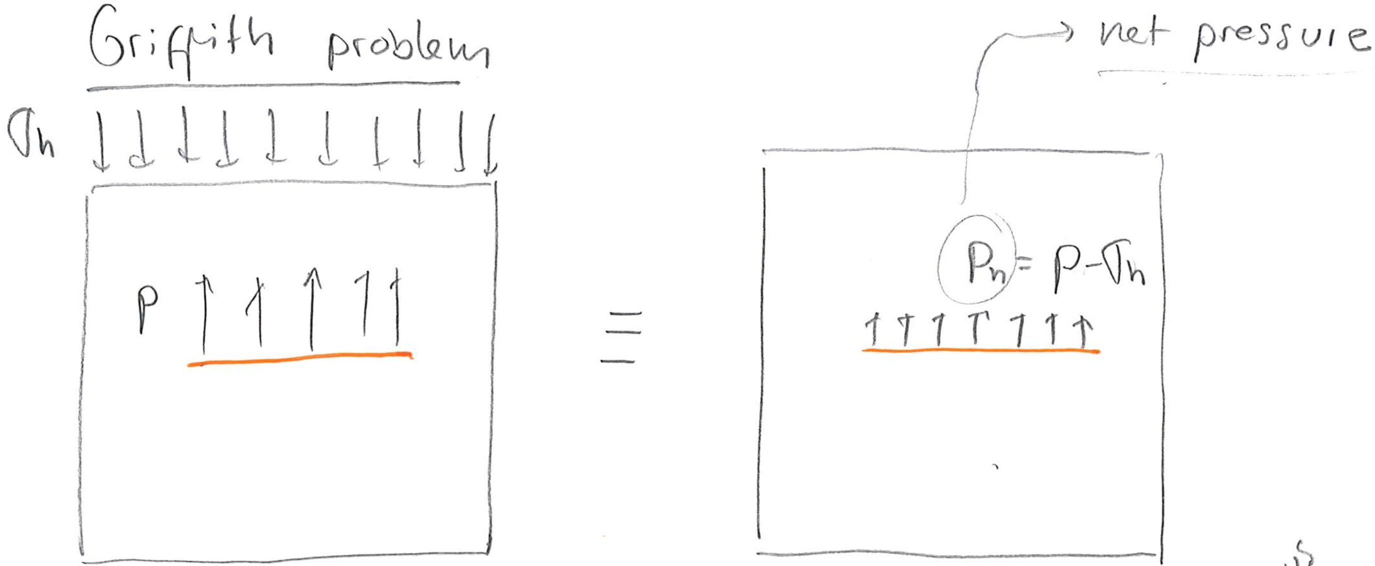
$L = 18.66$ mm

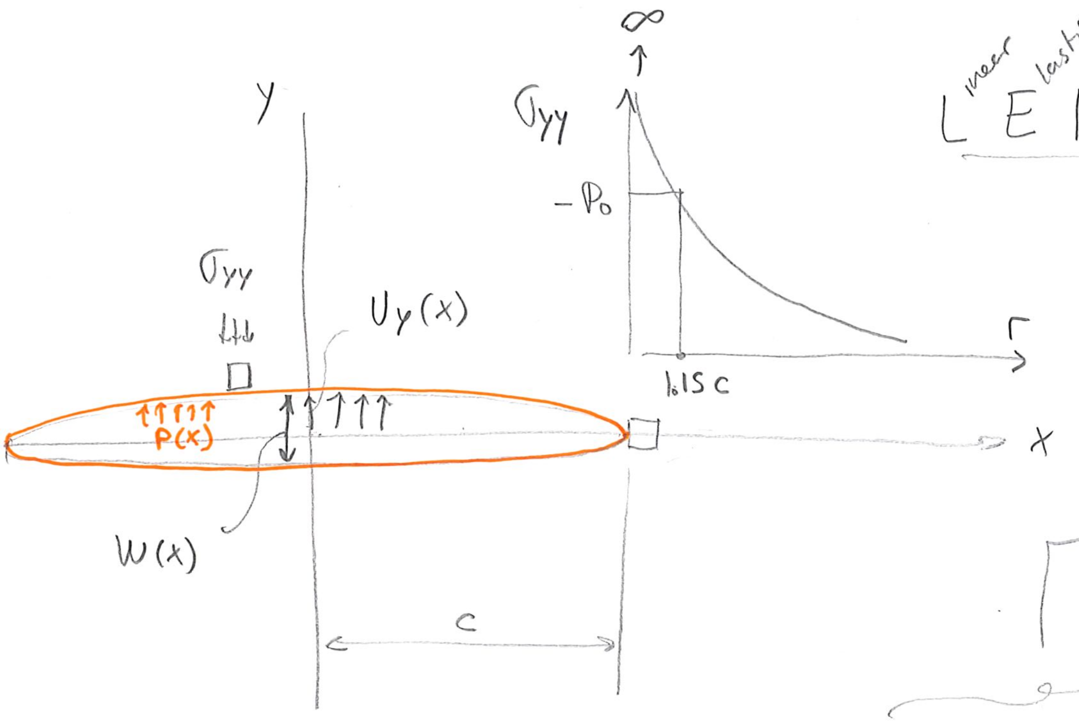
$r = 18.37$ mm

$a = 4.82$ mm

$[K_{Ic}] = \text{MPa}\sqrt{m}$

Griffith problem





Linear Elastic Isotropic Homogeneous Solid

$$E' = \frac{E}{1-\nu^2}$$

- ellipse
- $W_o = \dfrac{4 P_o c}{E'}$

$$p(x) = P_o \text{ (constant)}$$

$$U_y(x,0) = \begin{cases} \dfrac{2P_o}{E'}\sqrt{c^2-x^2} & 0 \leq x \leq c \\ 0 & x \geq c \end{cases}$$

$$\sigma_{yy}(x,0) = \begin{cases} -P_o & 0 \leq x \leq c \\ -P_o\left[\dfrac{x}{\sqrt{x^2-c^2}} - 1\right] & x > c \end{cases}$$

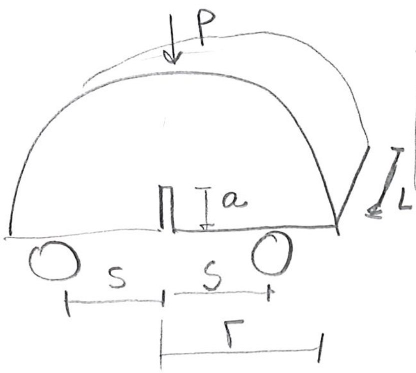
Stress intensity factor

$$K_I = \lim_{r \to 0^+}\left[\sqrt{2\pi \cdot r} \cdot \sigma_{yy}(c+r, 0)\right]$$

constant pressure K_I

$$\boxed{K_I = P_0 \sqrt{\pi c}}$$

→ K_{IC} (constant)

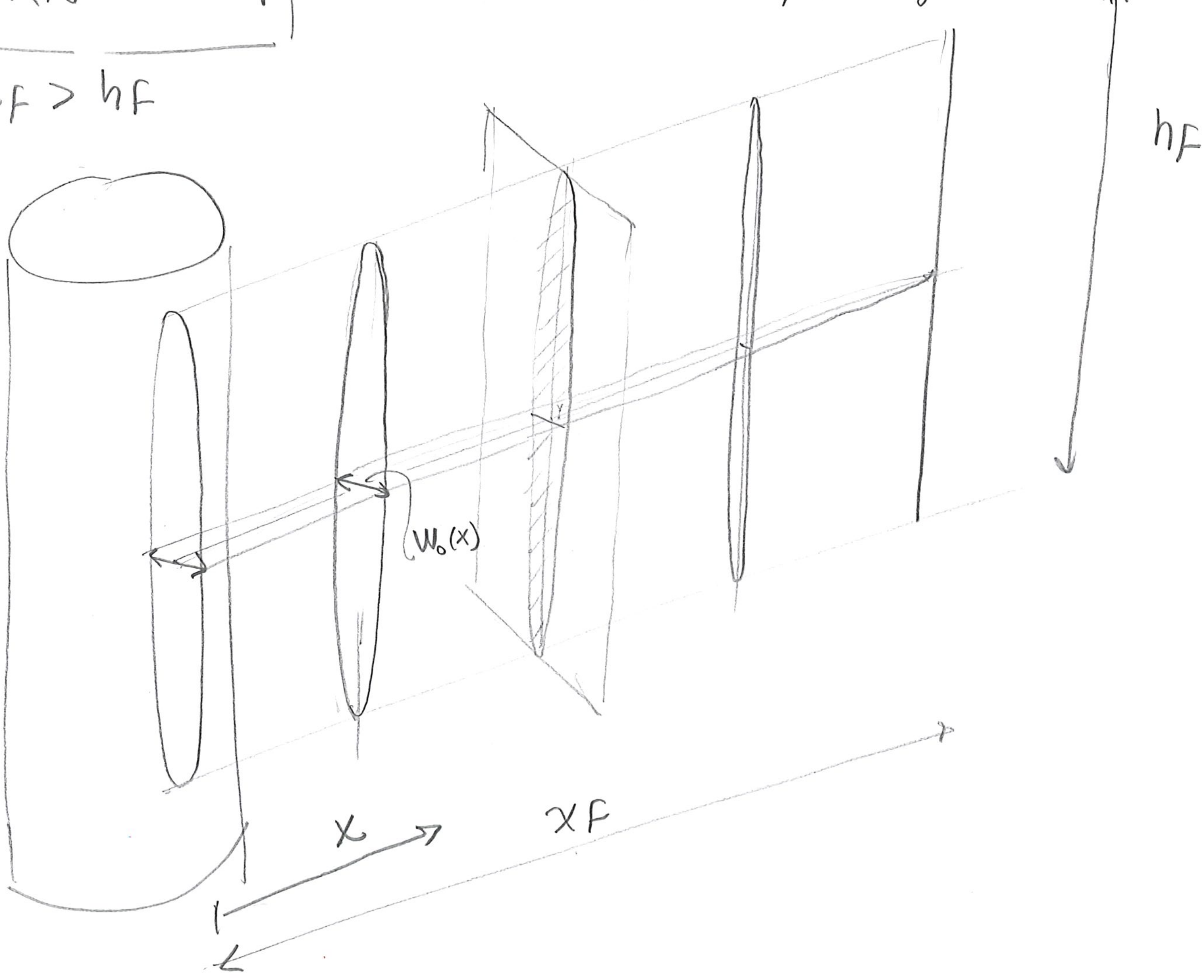


$$\boxed{K_{IC} = \frac{P_{max}\,(\pi a)^{1/2}}{2rL}\,Y_I}$$

$$Y_I(a, r, s) \sim S$$

$\boxed{\text{PKN model}}$ → Perkins, Kern, Nordgren

$x_F > h_F$



$\boxed{\text{Lineer elasticity}}$ $$W_o(x) = \frac{4\,P_n(x)\,(h_f/2)}{E'} = \frac{2\,P_n(x)\,h_f}{E'} \quad (1)$$

$\boxed{\text{Fluid Mechanics}}$ $$\frac{dP}{dX} = -\frac{64\,\mu\,q(x)}{\pi\,[W_o(x)]^3\,h_f} \quad (2)$$

elliptical shape
Newtonian fluid
laminar flow

$\boxed{\text{Material Balance}}$ $$V_i = V + V_L \quad (3)$$

$V_L = 2 A_f C_L \sqrt{t}$

$$V_i = i\,t \quad (4)$$

constant injection rate (one wing)

$\boxed{q(x) = i}$ (no leak-off) (5)

(1) and (2) (5)

$$\frac{dP}{dx} = -\frac{64\,\mu\,i}{\pi\left[2 h_f\,P_n(x)/E'\right]^3 h_f}$$

$$\frac{dp}{dX} = -\frac{8\,\mu\,i\,E'^3}{\pi\,h_f^4\,[P_n(x)]^3}$$

$$\int_{P_n(x=0)}^{P_n(x_f)} [P_n(x)]^3\,dP = -\int_0^{x_f} \frac{8\,\mu\,i\,E'^3}{\pi\,h_f^4}\,dx$$

$$\cancel{\frac{[P_n(x_f)]^4}{4}}^{\,0} - \frac{[P_n(x=0)]^4}{4} = -\frac{8\,\mu\,i\,E'}{\pi\,h_f^4}\,x_f$$

$$\begin{cases} P_n(x=0) = \left(\dfrac{32\,\nu\, i\, E'^3\, x_f}{\pi\, h_f^4}\right)^{1/4} & (6) \\[2em] W_o(x=0) = \left(\dfrac{512\,\nu\, i\, x_f}{\pi\, E'}\right)^{1/4} & (6)+(1) \end{cases}$$

Geometry : $V_{frac} = x_f\, h_f\, \bar{w}$

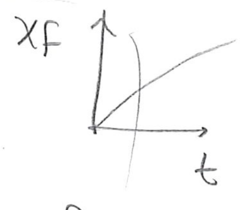
$$\bar{w} = \frac{\pi}{5} W_o(x=0)$$

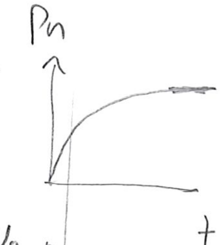
MB : $V_{frac} = i \cdot t$ (4)

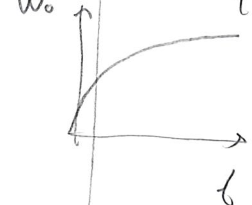
$$x_f = \frac{i \cdot t}{h_f\left(\frac{\pi}{5} W_o(x=0)\right)}$$

PKN model

$$\begin{cases} x_f = \underbrace{0.524}\left(\dfrac{i^3 E'}{\nu\, h_f^4}\right)^{1/5} t^{4/5} \\[2em] P_n(x=0) = \underbrace{1.52}\left(\dfrac{E'^4\, \nu\, i^2}{h_f^6}\right)^{1/5} \cdot t^{1/5} \\[2em] W_o(x=0) = \underbrace{3.04}\left(\dfrac{i^2\, \nu}{E'\, h_f}\right)^{1/5} \cdot t^{1/5} \end{cases}$$







$$P_n(x=0) = \left(\frac{32\,\nu\, i\, E'^3\, x_f}{\pi\, h_f^4}\right)^{1/4} \quad (6)$$

$$W_0(x=0) = \left(\frac{512\,\nu\, i\, x_f}{\pi\, E'}\right)^{1/4} \quad (6)+(1)$$

Geometry : $V_{frac} = x_f\, h_f\, \bar{w}$

$$\bar{w} = \frac{\pi}{5} W_0(x=0)$$

MB : $V_{frac} = i \cdot t$ (4)

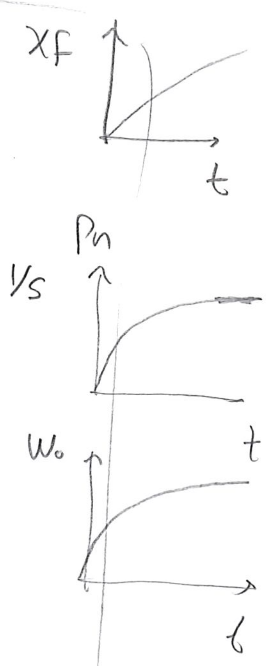
$$x_f = \frac{i \cdot t}{h_f\left(\frac{\pi}{5} W_0(x=0)\right)}$$

PKN model

$$x_f = 0.524 \left(\frac{i^3 E'}{\nu\, h_f^4}\right)^{1/5} t^{4/5}$$

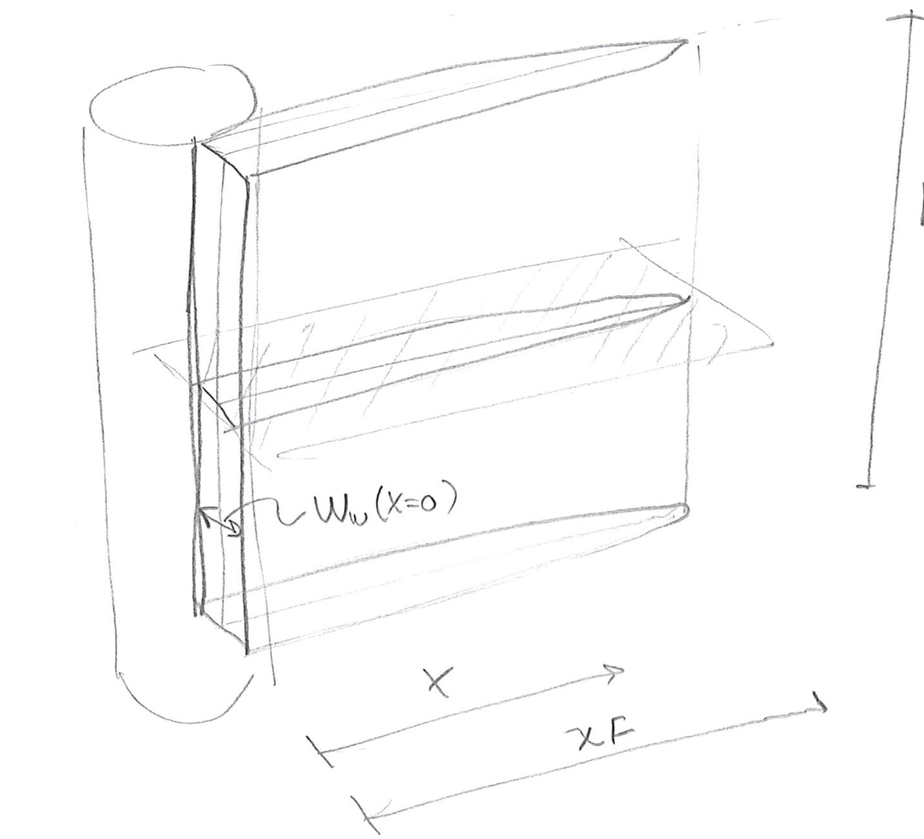
$$P_n(x=0) = 1.52 \left(\frac{E'^4\, \nu\, i^2}{h_f^6}\right)^{1/5} \cdot t^{1/5}$$

$$W_0(x=0) = 3.04 \left(\frac{i^2\, \nu}{E'\, h_f}\right) \cdot t^{1/5}$$



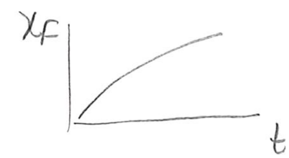
KGD model (Khristianovich, Geertsma, de Klerk)

↳ short fracs $x_f < h_f$

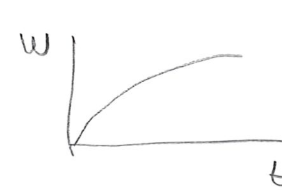


constant rate i, Newtonian fluid, laminar, negligible toughness, no leak-off, $P_n(x=x_f)=0$

$$x_f = 0.539 \left(\frac{i^3 E'}{\mu\, h_f^3}\right) t^{2/3}$$



$$W_w = 2.36 \left(\frac{i^3 \mu}{E'\, h_f^3}\right)^{1/6} t^{1/3}$$

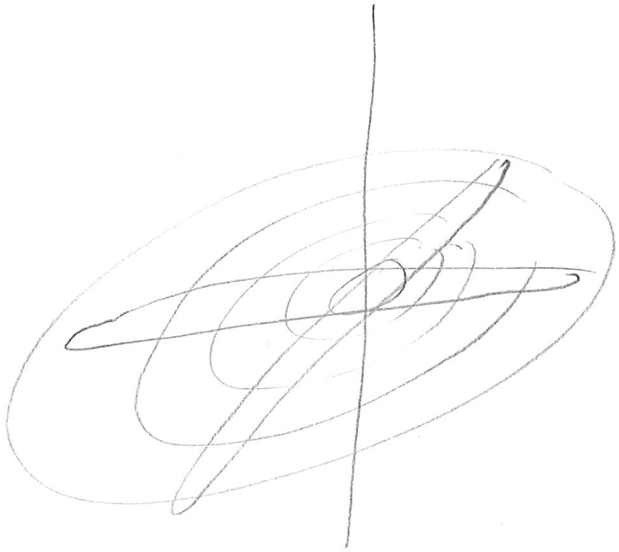


$$P_{n,w} = 1.09 \left(E' \mu\right)^{1/3} \left(t\right)^{-1/3}$$

$\tilde{n}/3$

- short fractures
- independent of i, not true

Penny (radial) fracture

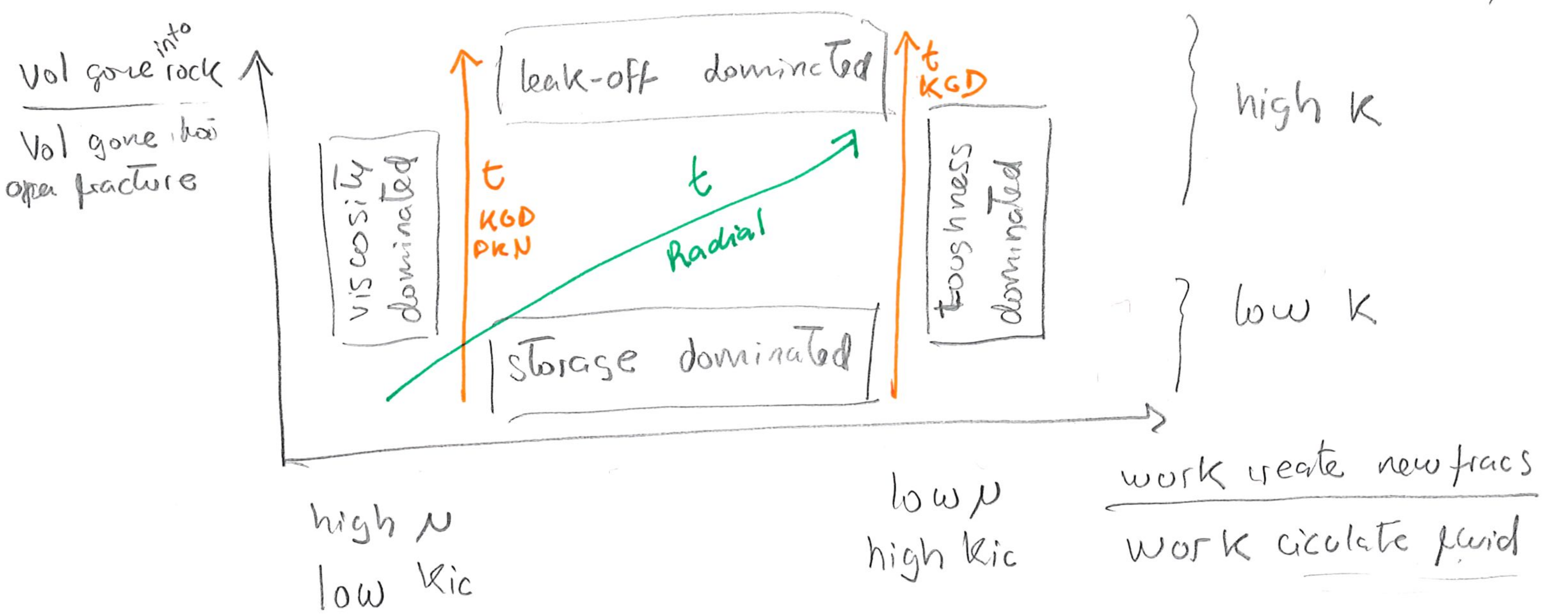


$X_f =$

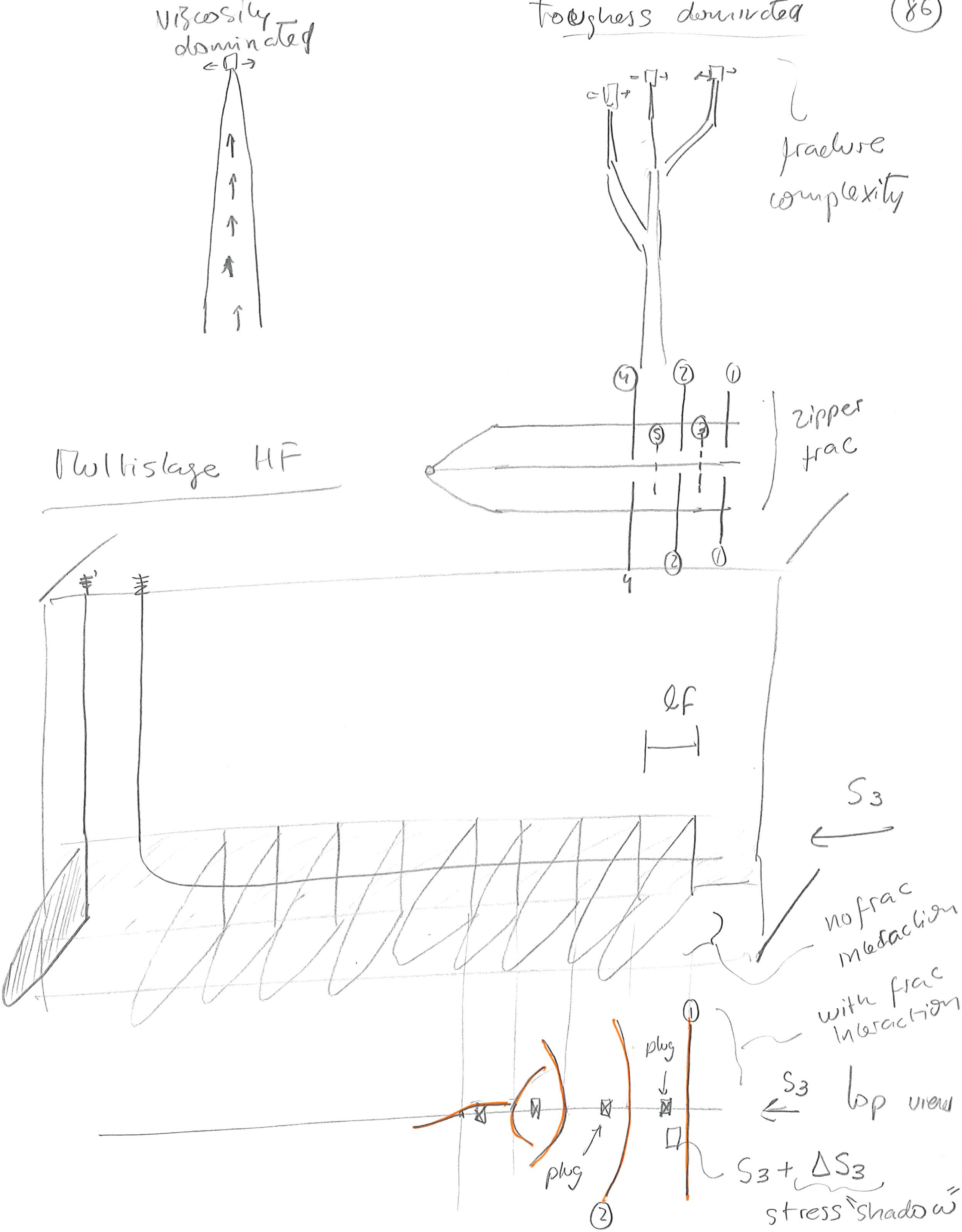
$w =$

$P_n =$

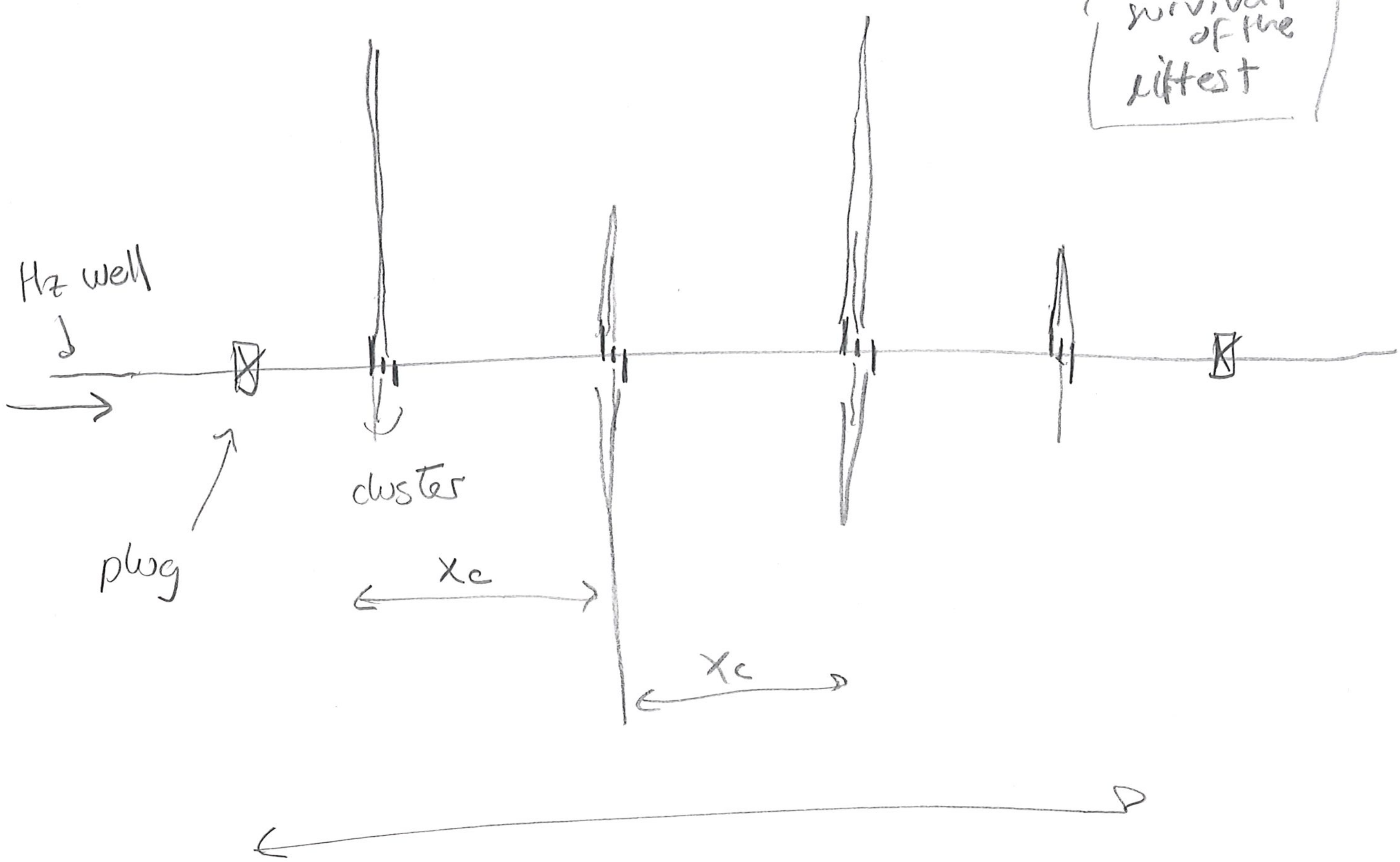
Fluid-driven fractures in porous media (Detournay, U of Minn)



↳ effective stress
↳ leak-off (rigorous)
} → HF in porous media

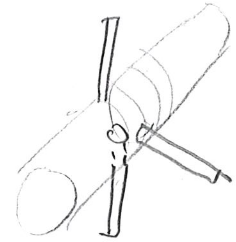

viscosity dominated
toughness dominated
fracture complexity
zipper frac
Multistage HF
lf
S3
no frac interaction
with frac interaction
plug
plug
S3
top view
S3 + ΔS3
stress "shadow"

survival of the fittest



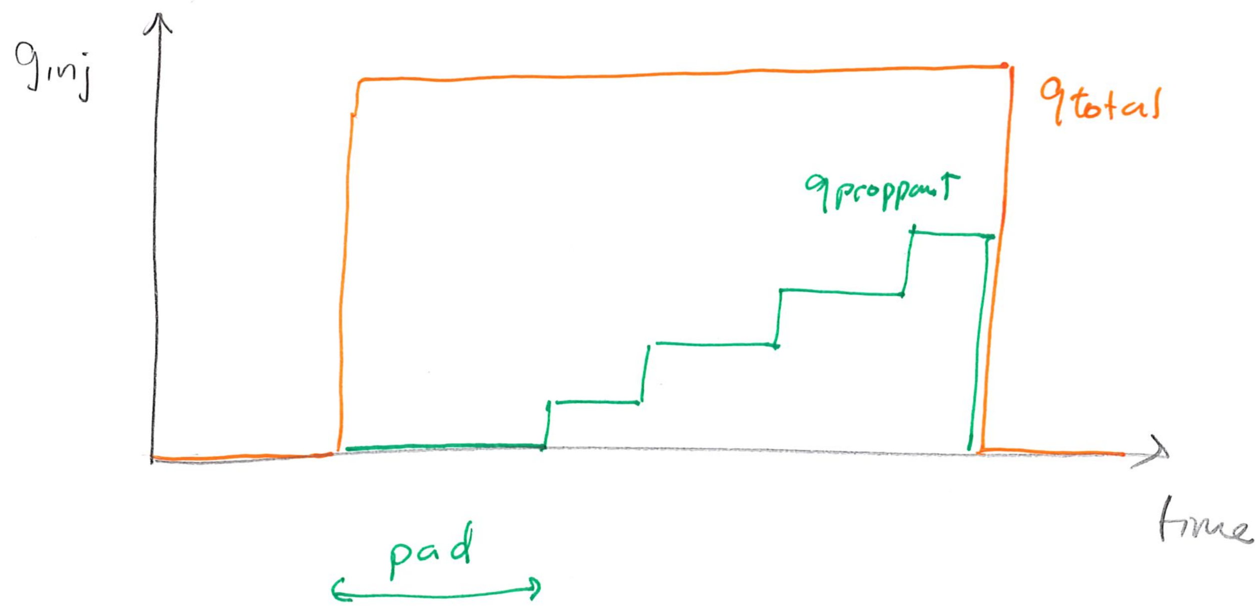
PERMIAN BASIN

- lateral length ~ > 10,000 ft
- → 40 stages
- → each stage ~ 4-15 clusters
- → each cluster - 6-20 perforations
- → 2000 $\frac{\text{lb (proppant)}}{\text{LF}}$ of Hz well
- → 2500 $\frac{\text{gallons (frac fluid)}}{\text{LF}}$



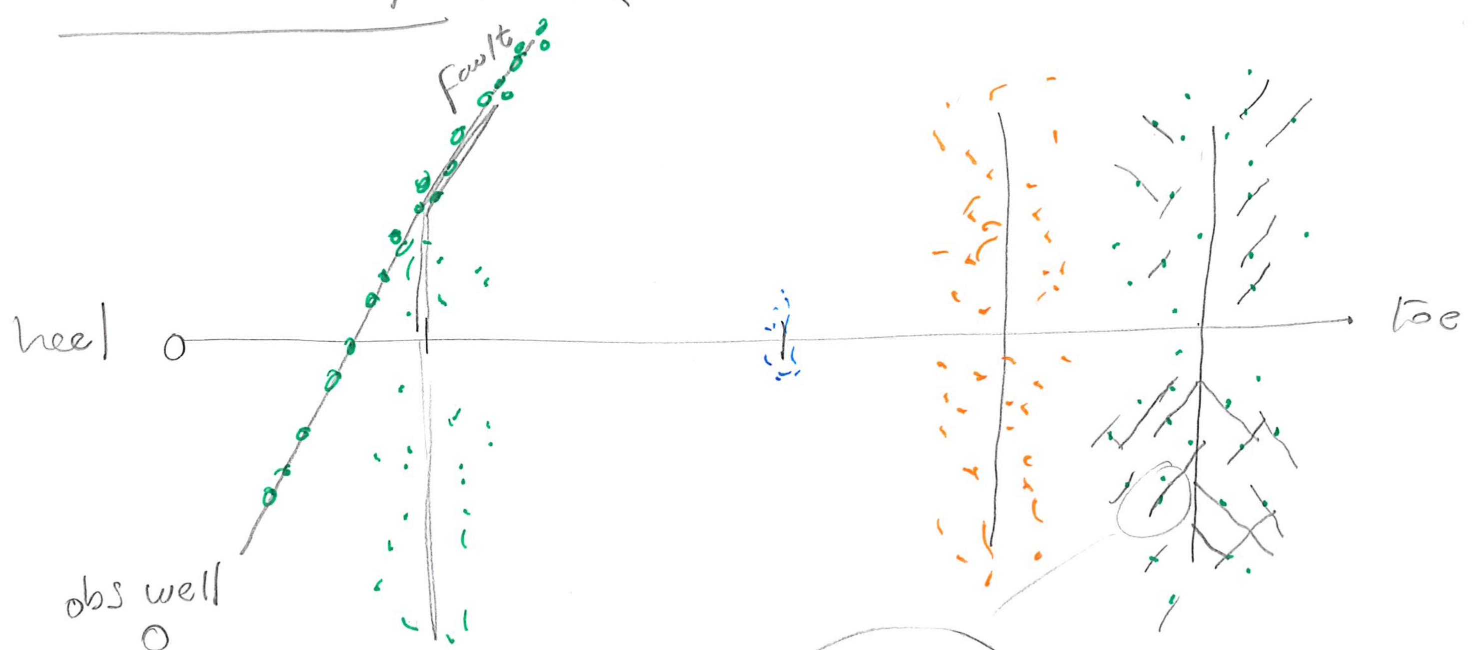
0.8 $\frac{\text{lb (propp)}}{\text{gall (fluid)}}$

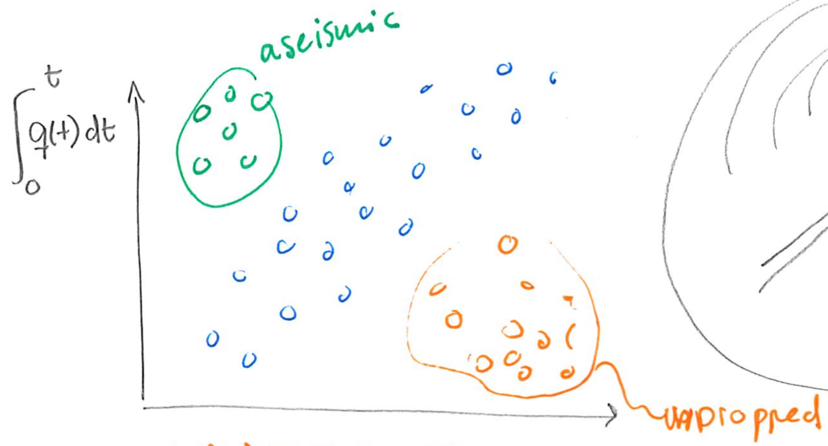
~ 7% $\frac{\text{vol (prop)}}{\text{vol (fluid)}}$

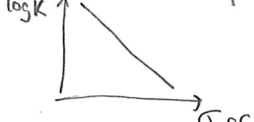


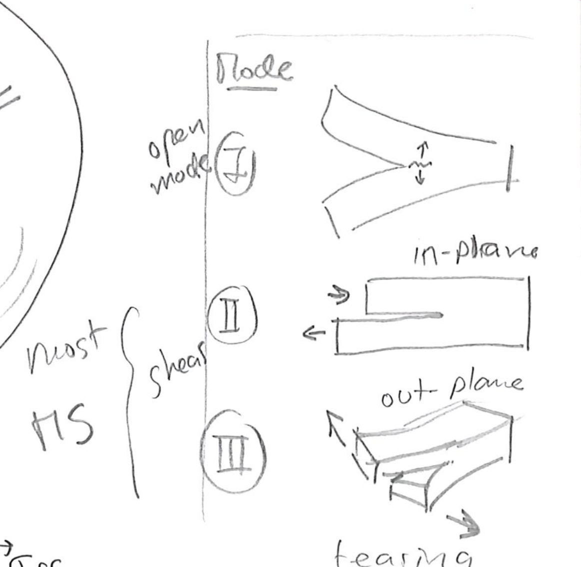
Ritcher Scale (logarithmic scale)

Microseismicity ~ $M \leq 0$



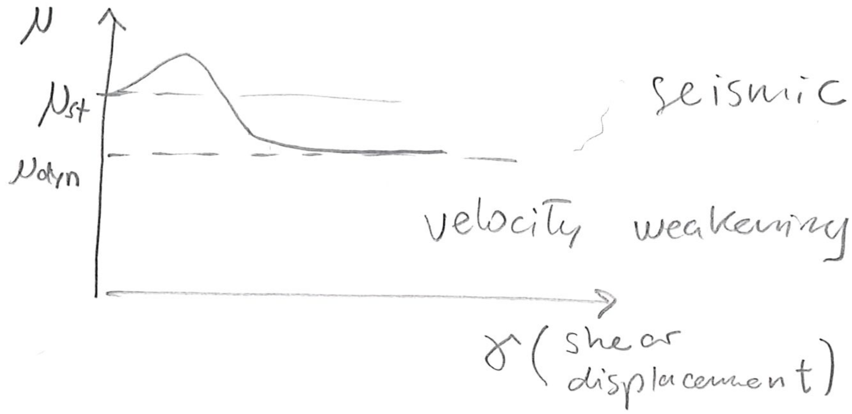




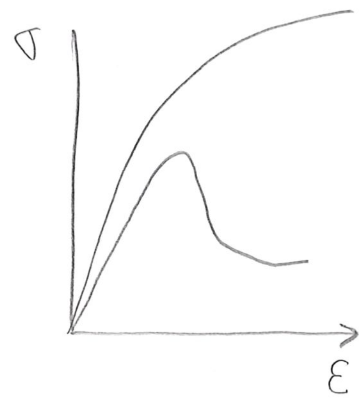


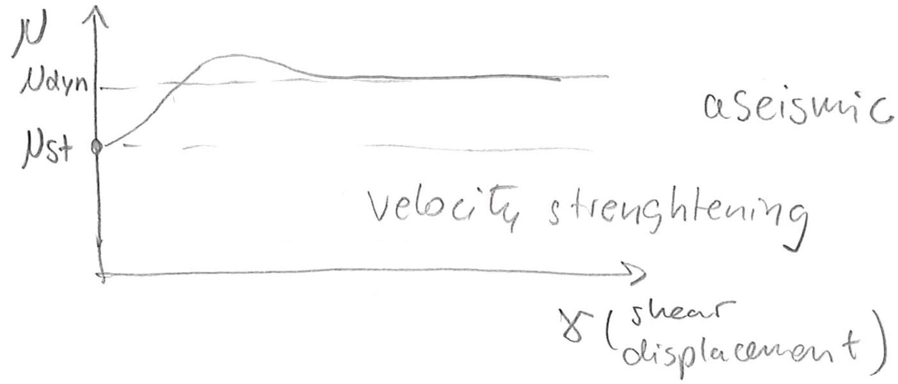
- micro proppants
- acid etching

shear slip < seismic / aseismic



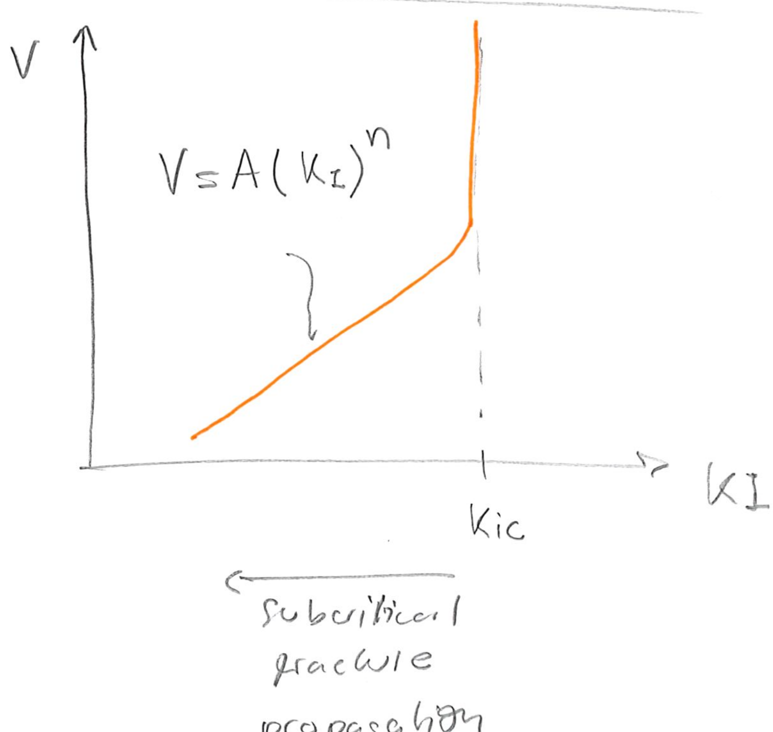
seismic





aseismic

Subcritical fracture propagation



- natural fractures
- hydraulic fracturing

→ modes I, II, III

I: open; II, III: shear

Hydraulic fracturing in unconsolidated sediments

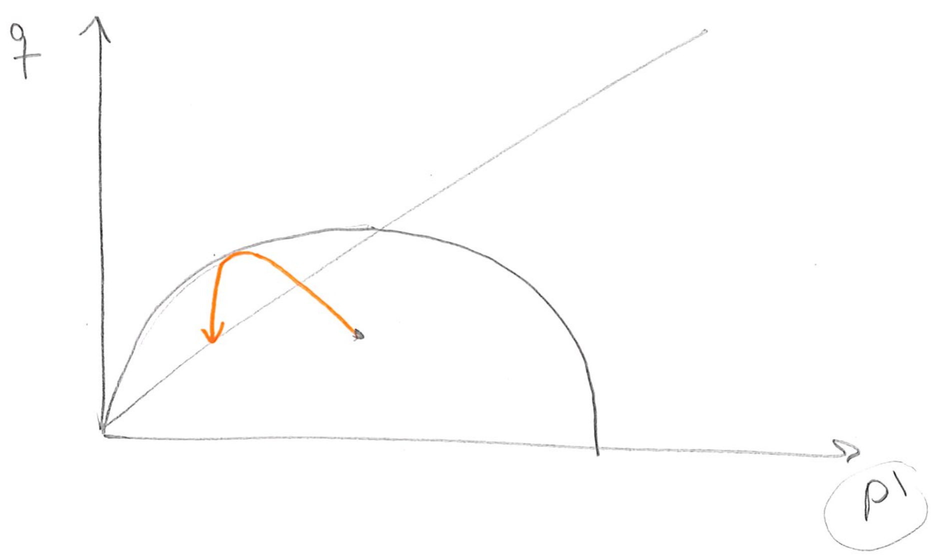
uncemented sediments

↳ tensile strength = ∅

↳ fracture toughness = NA

↳ LEFM does not work

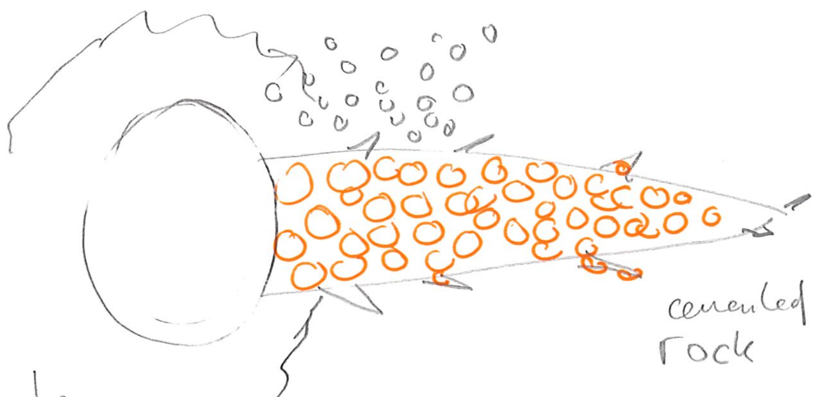
↳ use LEFM, tune (K_{Ic}, E)



$\uparrow P_p \rightarrow \downarrow p'$

anisotropic stress

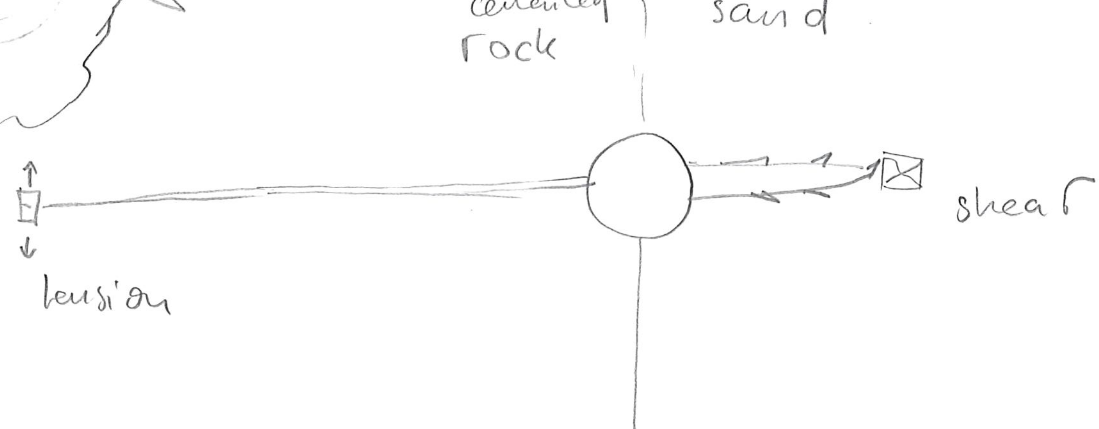
↳ $\uparrow q$



→ avoid sand production

→ by-pass damaged (K) zone

Frac-pack



Lineer Elasticity (Quasi-static)

Quasi-static: inertial forces $\ll$ stresses, body forces

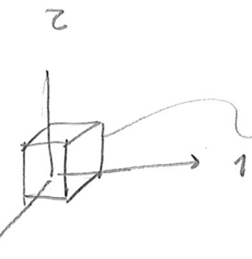
Eq	$\nabla\cdot\underline{\underline{\sigma}} + \underline{f} = \underline{0}$
Kin	$\underline{\underline{\varepsilon}} = \frac{1}{2}\left(\nabla\underline{u} + \nabla\underline{u}^T\right)$
Const	$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\cdot\underline{\underline{\varepsilon}}$

$\rightarrow$ Navier's Eq

$$(\lambda+G)\nabla(\nabla\cdot\underline{u}) + \mu\nabla^2 u + \underline{f} = \underline{0}$$

$\lambda = M - 2G$

$\mu = G$



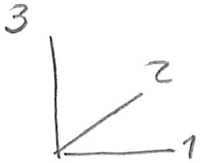
$$\frac{m}{dx_1 dx_2 dx_3} = \rho$$

Dynamic equations

Eq	$\nabla\cdot\underline{\underline{\sigma}} + \overset{0}{\cancel{\underline{f}}} = \rho\frac{d^2\underline{u}}{dt^2}$
Kin	same
Const	same

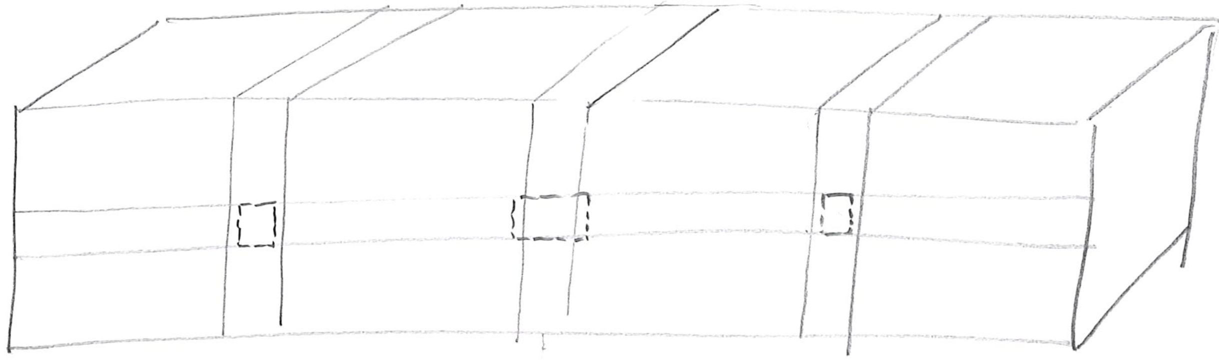
$$\boxed{(M-G)\nabla(\nabla\cdot\underline{u}) + G\nabla^2\underline{u} = \rho\frac{d^2 u}{dt^2}}$$

$\rightarrow$ 3 eq



$U_1 \neq 0 \;;\; U_2 = U_3 = 0$

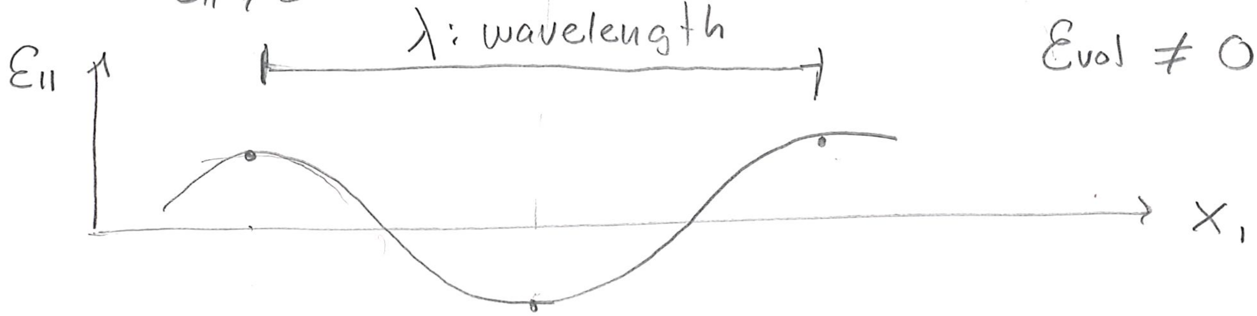
P-wave in direction 1



$\varepsilon_{ij} = 0 \quad i \neq j$

$\varepsilon_{11} \neq 0$

$\varepsilon_{vol} \neq 0$



$$(M-G)\frac{\partial}{\partial x_1}\left(\frac{\partial U_1}{\partial x_1} + \cancel{\frac{\partial U_2}{\partial x_2}} + \cancel{\frac{\partial U_3}{\partial x_3}}\right) + G\left(\frac{\partial^2 U_1}{\partial x_1^2} + \cancel{\frac{\partial^2 U_1}{\partial x_2^2}} + \cancel{\frac{\partial^2 U_1}{\partial x_3^2}}\right) = \rho\frac{\partial^2 U_1}{\partial t^2}$$

$$\frac{\partial^2 U_1}{\partial t^2} = \frac{M}{\rho}\frac{\partial^2 U_1}{\partial x_1^2} \quad \rightarrow$$

$\sqrt{-1}$

$$U_1(x_1,t) = A\, e^{j(\omega t \pm K x_1)}$$

particular case $\quad U_1(x_1,t) = \sin(\omega t + K x_1)$

$$\frac{\partial^2 U_1}{\partial t^2} = -\omega^2 \sin(\omega t + K x_1) \;;\; \frac{\partial^2 U_1}{\partial x_1^2} = -K^2 \sin(\omega t + K x_1)$$

$$\frac{\dfrac{\partial^2 U_1}{\partial t^2}}{\dfrac{\partial^2 U_1}{\partial x_1^2}} = \left(\frac{\omega}{K}\right)^2 = \frac{M}{\rho}$$

ω: angular frequency

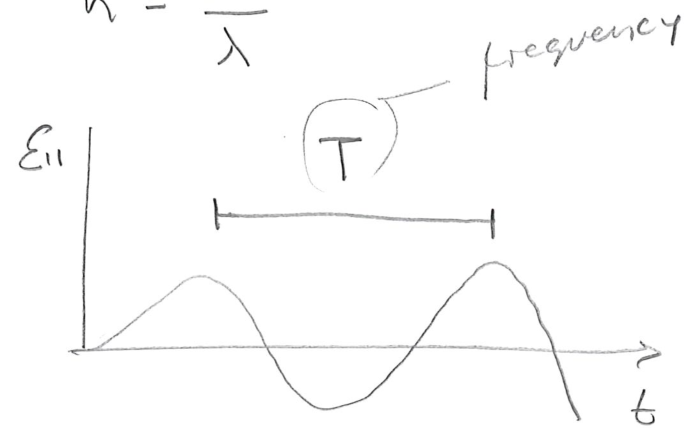
$$\omega = \frac{2\pi}{T}$$

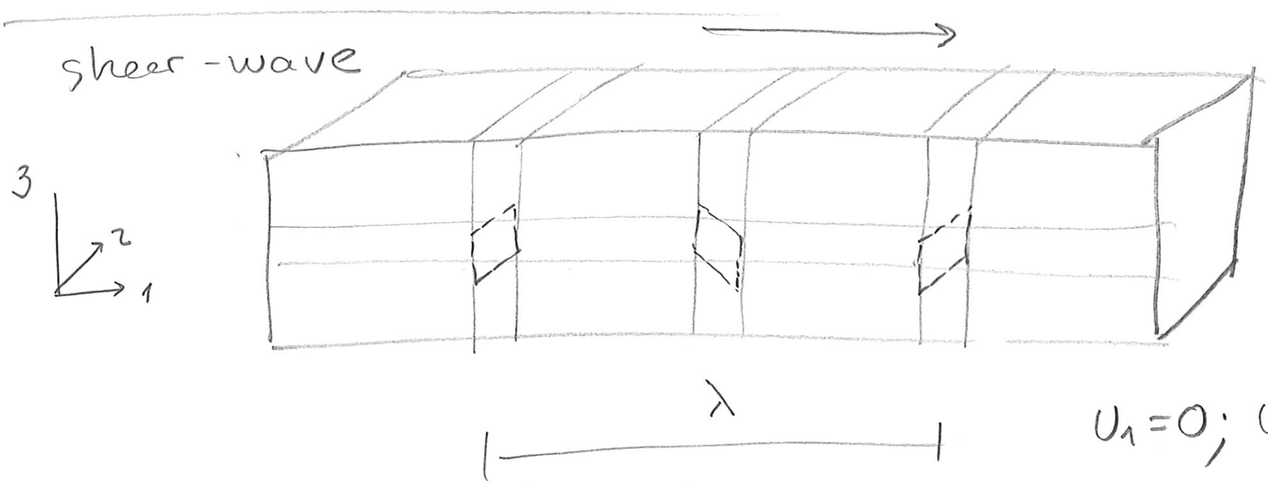
K: wave number

$$K = \frac{2\pi}{\lambda}$$

$$\left(\frac{\dfrac{\cancel{2\pi}}{T}}{\dfrac{\cancel{2\pi}}{\lambda}}\right)^2 = \left(\frac{\lambda}{T}\right)^2 = \frac{M}{\rho}$$

$$\boxed{V_p = \sqrt{\frac{M}{\rho}}}$$





$\varepsilon_{vol} = 0$

$U_1 = 0;\ U_2 = 0;\ U_3 \neq 0$

$$(M-G)\frac{\partial}{\partial x_3}\left(\overset{\varepsilon_{11}}{\cancel{\frac{\partial U_1}{\partial x_1}}} + \overset{\varepsilon_{22}}{\cancel{\frac{\partial U_2}{\partial x_2}}} + \overset{\varepsilon_{33}}{\cancel{\frac{\partial U_3}{\partial x_3}}}\right) + G\left(\frac{\partial^2 U_3}{\partial x_1^2} + \cancel{\frac{\partial^2 U_3}{\partial x_2^2}} + \cancel{\frac{\partial^2 U_3}{\partial x_3^2}}\right) = \rho\frac{\partial^2 U_3}{\partial t^2}$$

$$\boxed{\frac{\partial^2 U_3}{\partial t^2} = \frac{G}{\rho}\frac{\partial^2 U_3}{\partial x_1^2}} \rightarrow V_s = \sqrt{\frac{G}{\rho}}$$

Linear elasticity : E, ν

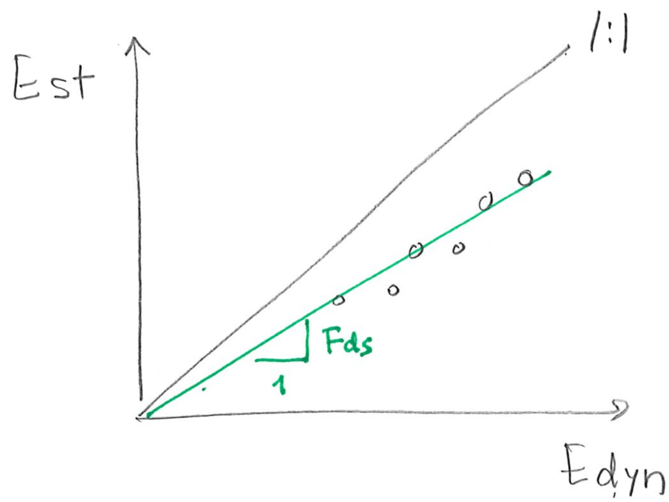
: V_p, V_s

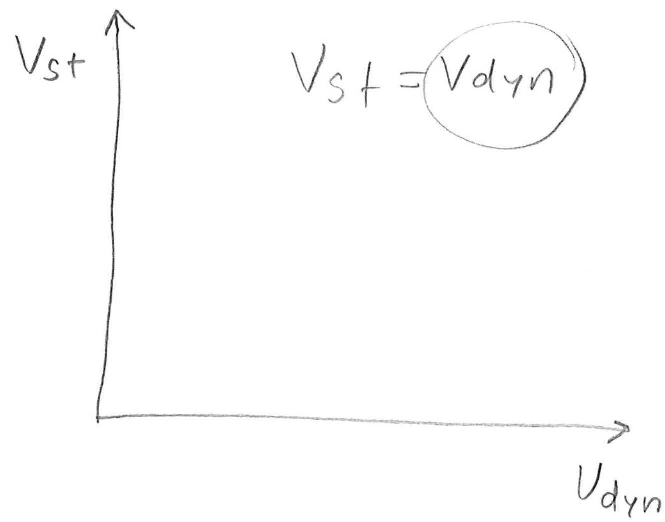
$M = \frac{E}{\cdots}$

$G = \frac{E}{2(1+\nu)}$

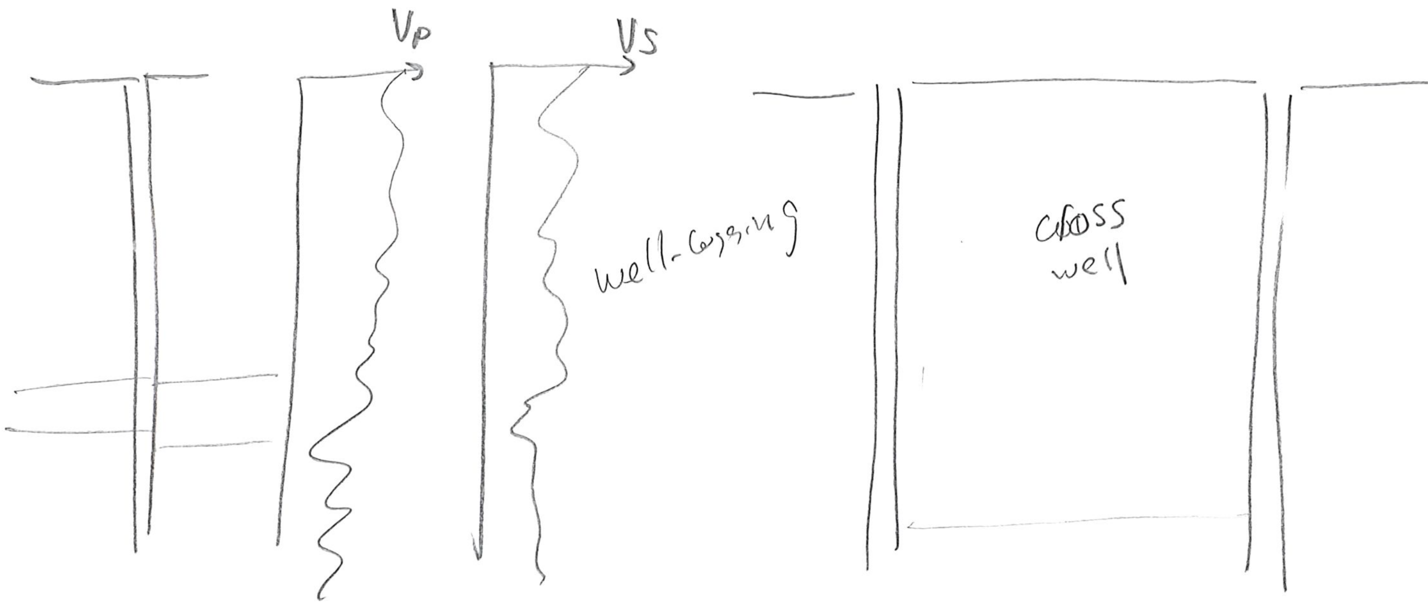
$$E_{dyn} = \rho V_s^2 \frac{(3V_p^2 - 4V_s^2)}{V_p^2 - V_s^2}$$

$$\nu_{dyn} = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)}$$





$\nu_{st} = \nu_{dyn}$



Dynamic to static conversion

$$(V_p\,,\ V_s)_{dyn} \xrightarrow{F_{ds}} E_{st}\,,\ \nu_{st}$$

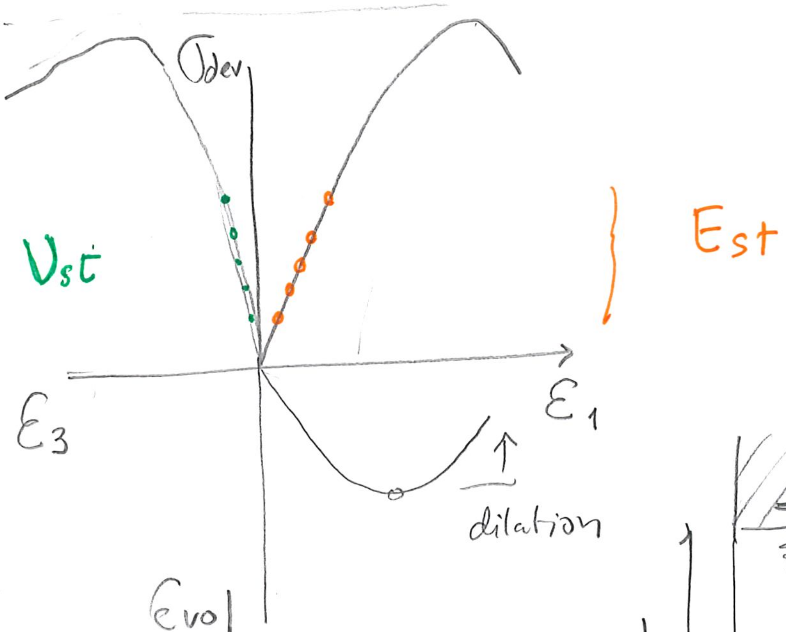
- seismic (active)
- cross well
- passive
- well-logging

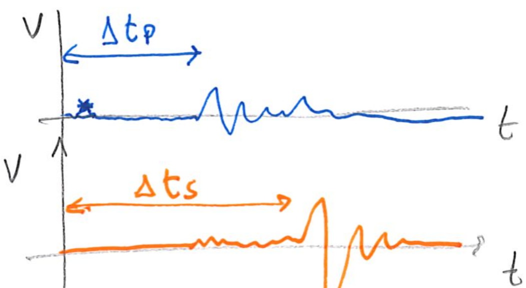
↳ lab

$$\boxed{E_{st} = F_{ds}\, E_{dyn}}$$

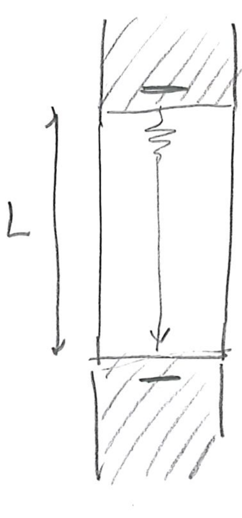
$F_{ds} < 1$

- dispersion
- strain-magnitude
- plastic strains
- visco-elasticity





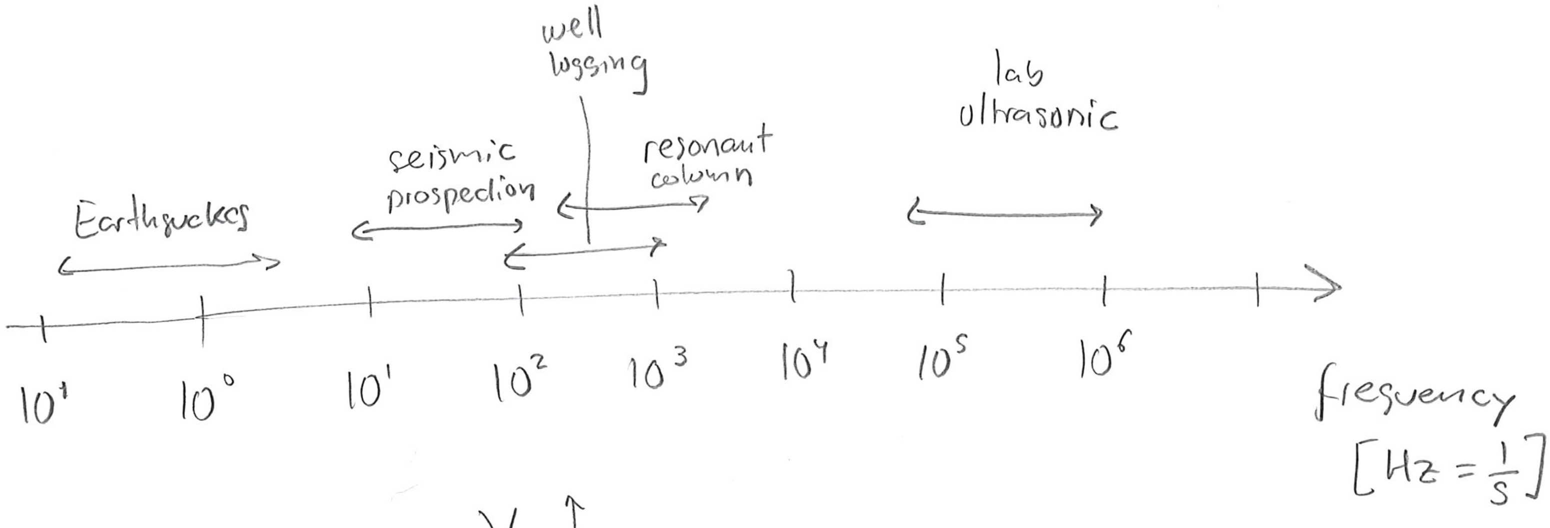
given time

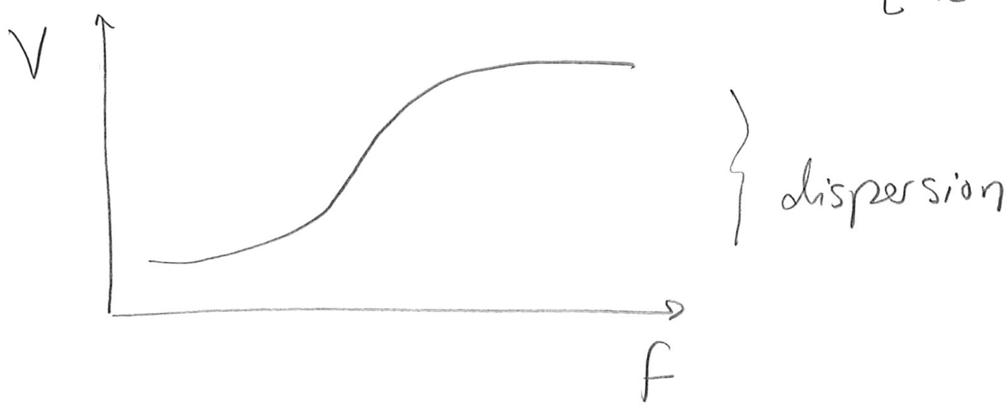


$\Delta t_p^{corr} \simeq 4\,\mu s \rightarrow V_p = \dfrac{L}{\Delta t_p - \Delta t_p^{corr}}$

$\Delta t_s^{corr} \simeq 6\,\mu s$

$V_s =$


well logging
lab ultrasonic
seismic prospection
resonant column
Earthquakes
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
frequency
$[Hz = \frac{1}{s}]$


V
f
dispersion