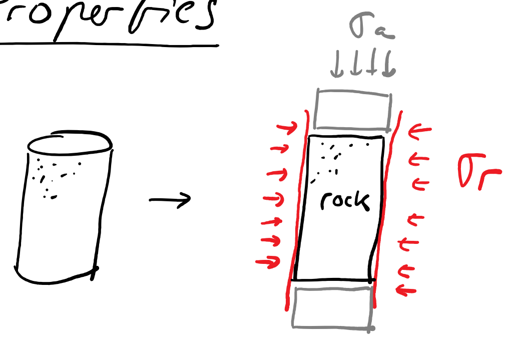
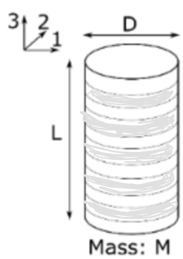
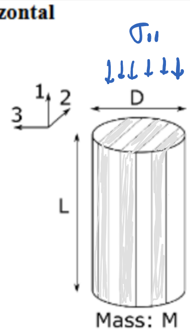
Transverse Vertical Isotropic Rocks

Monday, September 21, 2020 10:58 AM

TVI Static Elastic Properties

- Conventional Method
  - ↳ Axisymmetric Triaxial Cell
  - ↳ Deviatoric Loading Stress Path
    - ↳ $\sigma_r = cst$
    - ↳ $\Delta(\sigma_a - \sigma_r) = \Delta\sigma_a \quad , \quad \sigma_a \geq \sigma_r$

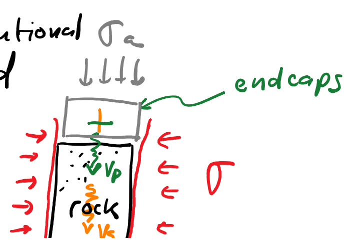


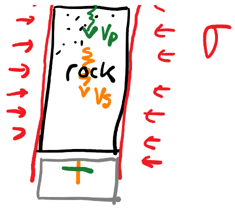
Sample	Quasi-static
Vertical 	Vertical Young modulus $E_v = \frac{\Delta\sigma_{33}}{\Delta\varepsilon_{33}}\Big\vert_{\sigma_{11},\sigma_{22}}$ Vertical Poisson ratio $\nu_v = -\frac{1}{2}\left(\frac{\Delta\varepsilon_{11}}{\Delta\varepsilon_{33}} + \frac{\Delta\varepsilon_{22}}{\Delta\varepsilon_{33}}\right) = \overbrace{-\frac{\Delta\varepsilon_{11}}{\Delta\varepsilon_{33}}}^{\nu_{13}} = \overbrace{-\frac{\Delta\varepsilon_{22}}{\Delta\varepsilon_{33}}}^{\nu_{23}}$
Horizontal 	Horizontal Young modulus $E_h = \frac{\Delta\sigma_{11}}{\Delta\varepsilon_{11}}$ Vertical Poisson ratio $\nu_v = -\frac{\Delta\varepsilon_{33}}{\Delta\varepsilon_{11}} = \nu_{31} \leftarrow = \nu_{13}$ Horizontal Poisson ratio $\nu_h = -\frac{\Delta\varepsilon_{22}}{\Delta\varepsilon_{11}} = \nu_{21} \Rightarrow = \nu_{12}$

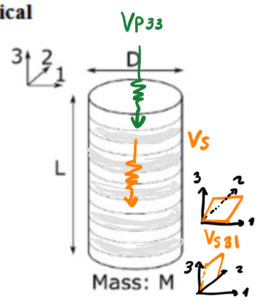
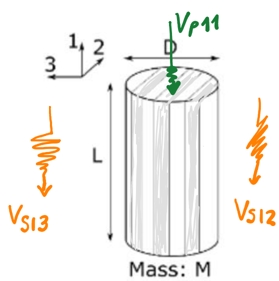
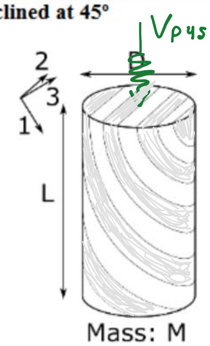
4 parameters

TVI Dynamic Elastic Properties

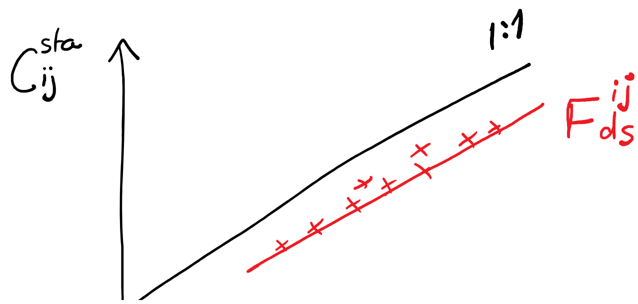
- Conventional Method

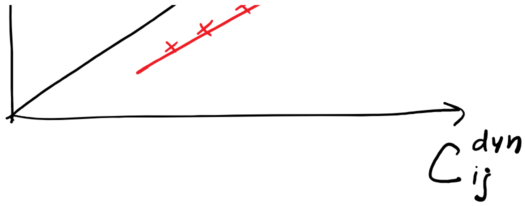




Sample	Dynamic
Vertical 	P-wave stiffness perpendicular to bedding $C_{33} = \rho(V_{p33})^2$ S-wave stiffness perpendicular to bedding $C_{44} = \frac{1}{2}[\rho(V_{s31})^2 + \rho(V_{s32})^2]$ $C_{44} = \rho(V_{s31})^2 = \rho(V_{s32})^2$
Horizontal 	P-wave stiffness parallel to bedding $C_{11} = \rho(V_{p11})^2$ S-wave stiffness perpendicular to bedding $C_{44} = \rho(V_{s13})^2$ ✓ S-wave stiffness in the plane of bedding $C_{66} = \rho(V_{s12})^2$
Inclined at 45° 	Off-diagonal stiffness $C_{13} = -C_{44} + [4\rho^2 V_{p45}^4$ $-2\rho V_{p45}^2(C_{11} + C_{33} + 2C_{44})$ $+(C_{11} + C_{44})(C_{33} + C_{44})]^{1/2}$

Dynamic to Static conversion





Quantification of anisotropy

Static

Young modulus anisotropy

$$\frac{E_h}{E_v} \; ; \; E_h > E_v$$

Poisson's ratio anisotropy

$$\frac{\nu_v}{1-\nu_h}$$ } effective lateral coefficient

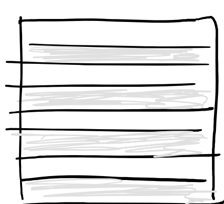
Dynamic

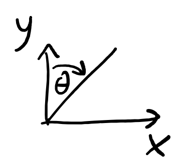
Thomsen Parameters :

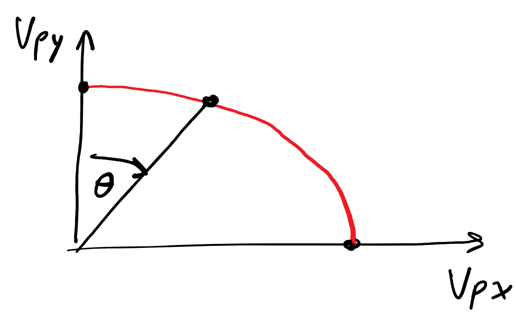
$$\varepsilon = \frac{V_{p11}^2 - V_{p33}^2}{2\, V_{p33}^2} = \frac{C_{11} - C_{33}}{2\, C_{33}}$$

$$\gamma = \frac{C_{66} - C_{44}}{2\, C_{44}}$$

$$\delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2\, C_{33}\,(C_{33} - C_{44})}$$



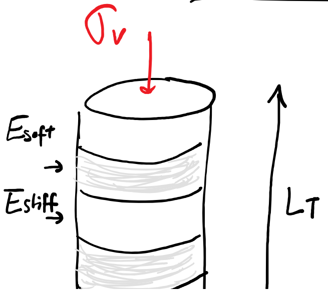




Weak anisotropy

$$V_p(\theta) = V_{p33}\left[1 + \delta \sin^2\theta \cos^2\theta + \varepsilon \sin^4\theta\right]$$

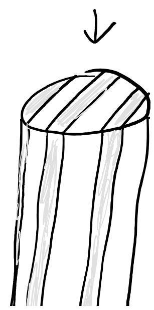
Iso-stress (Reuss Average)

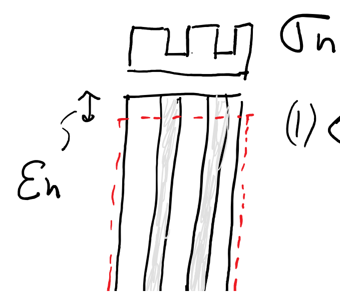


$$\langle E_v \rangle = \frac{\sigma_v}{\langle \varepsilon_v \rangle}$$

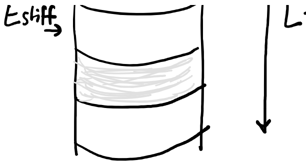
$$\hookrightarrow \langle \varepsilon_v \rangle = \frac{\sigma_v}{\quad} \quad (1)$$

Iso-strain (Voigt Average)





$$(1) \; \langle E_h \rangle = \frac{\langle \sigma_n \rangle}{\varepsilon_h}$$



$$\hookrightarrow \langle \varepsilon_v \rangle = \frac{\sigma_v}{\langle E_v \rangle} \quad (1)$$

L_{soft}, L_{stiff}

$$\frac{\langle \sigma_h \rangle}{\varepsilon_h}$$

$$(2) \quad \langle \varepsilon_v \rangle = \left(\frac{\sigma_v}{E_{stiff}} \cdot L_{stiff} + \frac{\sigma_v}{E_{soft}} \cdot L_{soft} \right) \cdot \frac{1}{L_T}$$

$$= \frac{\sigma_v}{E_{stiff}} \cdot \underbrace{\frac{L_{stiff}}{L_T}}_{f_{stiff}} + \frac{\sigma_v}{E_{soft}} \cdot \underbrace{\frac{L_{soft}}{L_T}}_{f_{soft}}$$

(1,2) →

$$\frac{\cancel{\sigma_v}}{\langle E_v \rangle} = \frac{\cancel{\sigma_v}}{E_{stiff}} \cdot f_{stiff} + \frac{\cancel{\sigma_v}}{E_{soft}} \cdot f_{soft}$$

$$\boxed{\langle E_v \rangle = \left(\frac{f_{stiff}}{E_{stiff}} + \frac{f_{soft}}{E_{soft}} \right)^{-1}}$$

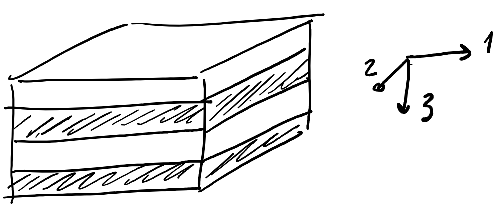
$$(2) \quad \langle \sigma_h \rangle = \sigma_h^{soft} \cdot \underbrace{\frac{L_{soft}}{L_T}} + \sigma_h^{stiff} \cdot \underbrace{\frac{L_{stiff}}{L_T}}$$

(1,2)

$$\cancel{\varepsilon_h} \langle E_h \rangle = \cancel{\varepsilon_h} E_{soft} \cdot f_{soft} + \cancel{\varepsilon_h} \cdot E_{stiff} \cdot f_{stiff}$$

$$\boxed{\langle E_h \rangle = E_{soft} \cdot f_{soft} + E_{stiff} \cdot f_{stiff}}$$

TVI stiffness matrix



$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_h} & -\frac{\nu_h}{E_h} & -\frac{\nu_v}{E_v} \\ -\frac{\nu_h}{E_h} & \frac{1}{E_h} & -\frac{\nu_v}{E_v} \\ -\frac{\nu_v}{E_v} & -\frac{\nu_v}{E_v} & \frac{1}{E_v} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix}$$

For isotropic loading $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_m$

For isotropic loading $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_m$

$$\varepsilon_{11} = \varepsilon_{22} = \left(\frac{1-\nu_h}{E_h} - \frac{\nu_v}{E_v} \right) \sigma_m$$

$$\varepsilon_{33} = \left(-\frac{2\nu_v}{E_v} + \frac{1}{E_v} \right) \sigma_m$$

$$\varepsilon_{vol} = \left[2\left(\frac{1-\nu_h}{E_h} - \frac{\nu_v}{E_v} \right) + \left(\frac{1-2\nu_v}{E_v} \right) \right] \sigma_m$$

$$\boxed{K_{VTI} = \left[\frac{2(1-\nu_h)}{E_h} + \frac{1-4\nu_v}{E_v} \right]^{-1}}$$

if $\nu_h = \nu_v$; $E_h = E_v$

$$K = \left[\frac{2-2\nu+1-4\nu}{E} \right]^{-1} = \frac{E}{3(1-2\nu)} \quad ✓$$