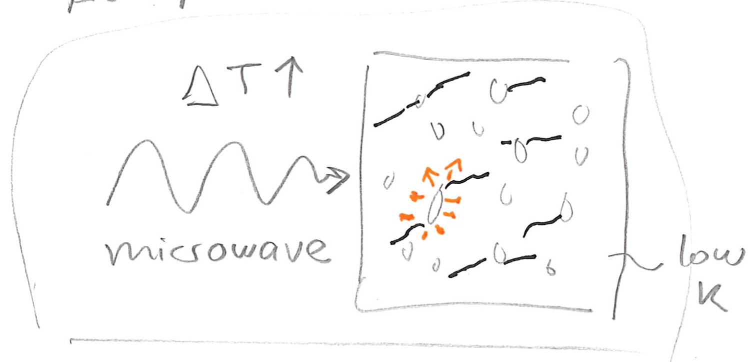
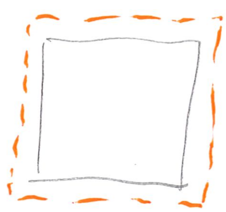
Thermo-elasticity

- applications
  - geothermal energy
    - shallow
    - deep
      - • HF
      - • ↑ τ/σ_n
  - drilling — tensile $\sigma_{\theta\theta}$
  - fluid injection
    - EOR
      - water
        - liquid
        - steam vapor
      - CO_2
      - chemical
    - injection disposal
      - CO_2
      - water
  - hydraulic fracturing (unconv) tight form
    - cold → cryogenic fracturing
    - hot → microwave

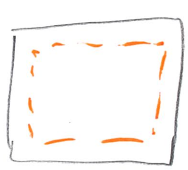


$\Delta T > 0$



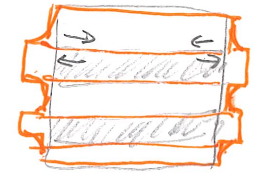
$\varepsilon_{vol} = \epsilon < 0$ (Dilation)

$\Delta T < 0$



$\varepsilon_{vol} > 0$ (contraction)

$\Delta T \lessgtr 0$ → no shear strains (isotropic) homogeneous



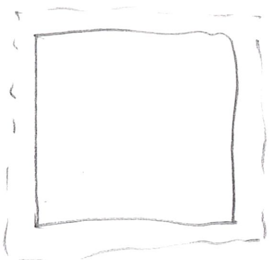
heterogeneous, $\neq \beta$ dilation coeff

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} + 3\underbrace{\beta}_{\text{linear thermal dilation coefficient}} \overbrace{K}^{\text{Bulk modulus}} \underbrace{(\Delta T)}_{\text{change of temp}} \underbrace{\underline{\underline{I}}}_{\text{identity matrix}}$$

identity matrix:
- isotropic
- no shear comp

Dilation

Free dilation (unconstrained)



$$\cancelto{0}{\underline{\underline{\sigma}}} = \underline{\underline{\underline{\underline{C}}}}\ \underline{\underline{\varepsilon}} + 3\beta K(\Delta T)\underline{\underline{I}}$$

$$\underline{\underline{\varepsilon}} = -\underline{\underline{\underline{\underline{D}}}}\left(3\beta K(\Delta T)\underline{\underline{I}}\right)$$

$$\begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{vmatrix} = \begin{vmatrix} 1/E & -\nu/E & -\nu/E \\ & 1/E & -\nu/E \\ & & 1/E \end{vmatrix} \begin{vmatrix} -3\beta K(\Delta T) \\ -3\beta K(\Delta T) \\ -3\beta K(\Delta T) \end{vmatrix}$$

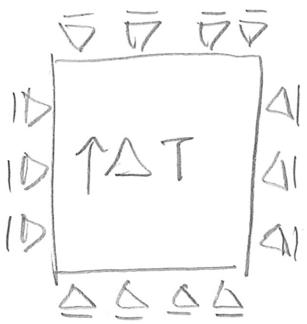
$$\varepsilon_{11} = \underbrace{\frac{(1-2\nu)}{E}\left(-3\right.}_{K}\left.\beta K(\Delta T)\right)$$

$$\boxed{\varepsilon_{11} = -\beta(\Delta T)}$$

- $\Delta T > 0 \rightarrow \Delta\varepsilon < 0$ dilation
- $\Delta T < 0 \rightarrow \Delta\varepsilon > 0$ contraction

$$\left.\begin{array}{l} \beta = 10^{-6} - 10^{-5}\ \frac{1}{°K} \\ \Delta T > 100°K \end{array}\right\} \varepsilon = 10^{-4} - \underbrace{10^{-3}}_{\text{thousandths}}$$

Constrained dilation



$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\;\cancelto{0}{\underline{\underline{\varepsilon}}} + 3\beta K \Delta T\, \underline{\underline{I}}$$

$$\begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{vmatrix} = \begin{vmatrix} +3\beta K \Delta T \\ +3\beta K \Delta T \\ +3\beta K \Delta T \end{vmatrix}$$

$$\sigma_{11} = 3\,\beta\,K\,\Delta T$$

$$\left.\begin{aligned} K &= 10 \text{ GPa} \\ \Delta T &= 100^\circ K \end{aligned}\right\} \quad \sigma_{11} = 3 \cdot 10^{-5} \frac{1}{\cancel{K}} \cdot 10^{10}\,\text{Pa} \cdot 100\,\cancel{K}$$

$$\beta = 10^{-5}\ 1/K \qquad \sigma_{11} = 3 \cdot 10^{7}\ \text{Pa}$$

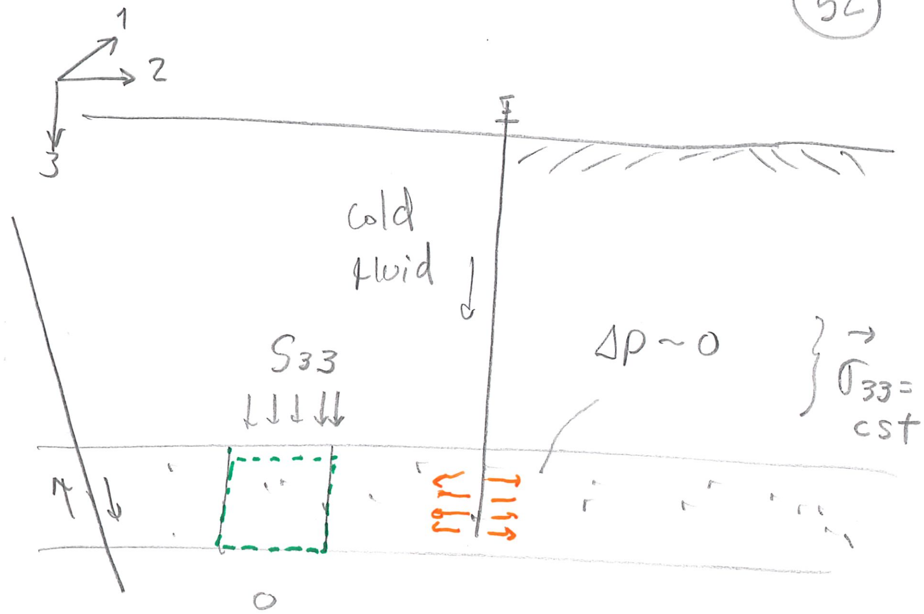
$$= 30\ \text{MPa}$$

$$\underbrace{\cong 4400\ \text{psi}}_{\text{compression}}$$

1D strain case

$\Delta T = T - T_0$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\;\underline{\underline{\varepsilon}} + 3\beta K \tilde{\theta}\,\underline{\underline{I}}$$



$$\begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{vmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{vmatrix} 1-\nu & \nu & \nu \\ & 1-\nu & \nu \\ & & 1-\nu \end{vmatrix} \begin{vmatrix} \cancel{\varepsilon_{11}} \xrightarrow{} 0 \\ \cancel{\varepsilon_{22}} \xrightarrow{} 0 \\ \varepsilon_{33} \end{vmatrix} + \begin{vmatrix} 3\beta K\theta \\ 3\beta K\theta \\ 3\beta K\theta \end{vmatrix}$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)}\,\varepsilon_{33} + 3\beta K\theta$$

$$\sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\,\varepsilon_{33} + 3\beta K\theta$$

$$\sigma_{11} = \frac{\nu \cancel{E}}{\cancel{(1+\nu)}\cancel{(1-2\nu)}}\left(\sigma_{33} - 3\beta K\theta\right)\frac{\cancel{(1+\nu)}\cancel{(1-2\nu)}}{\cancel{E}(1-\nu)} + 3\beta K\theta$$

$$\sigma_{11} = \frac{\nu}{1-\nu}\,\sigma_{33} + 3\beta K\theta\left(1 - \frac{\nu}{1-\nu}\right)$$

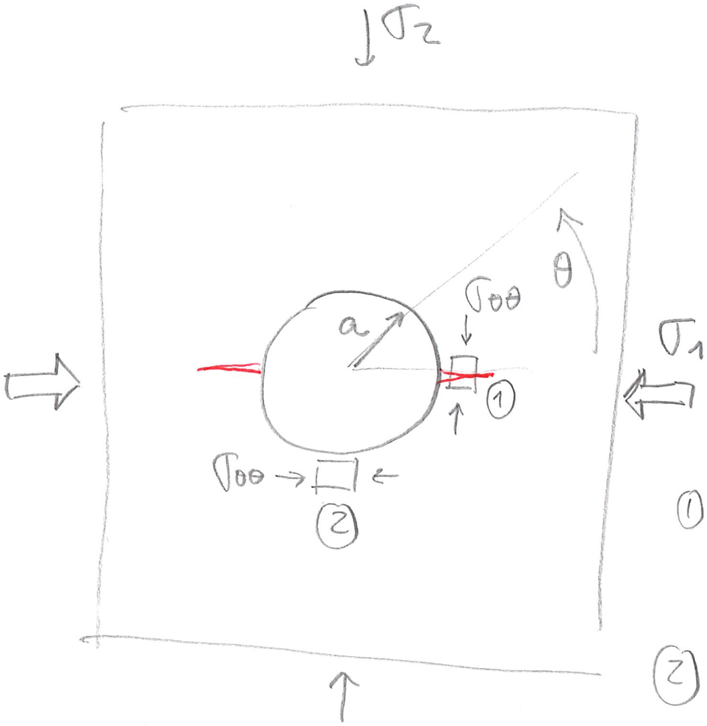
$\beta = 10^{-5}\ 1/K$
$E = 10\ GPa$
$\nu = 0.2$

$$\frac{\partial\sigma_{11}}{\partial\theta} = 0.1\ \frac{MPa}{K}$$

$$10^{5}\ \frac{Pa}{K} = 0.1 \cdot 10^{6}\ \frac{Pa}{K}$$

$$\sim \frac{10^{-5}\ \frac{1}{K}\ 10^{10}\ Pa}{1}$$

$$\left.\frac{\partial\sigma_{11}}{\partial\theta}\right|_{\sigma_{33}} = 3\beta K\left(\frac{1-2\nu}{1-\nu}\right) = \cancel{3}\beta\,\frac{E}{\cancel{3}\cancel{(1-2\nu)}}\,\frac{\cancel{(1-2\nu)}}{(1-\nu)} = +\frac{\beta E}{1-\nu}$$



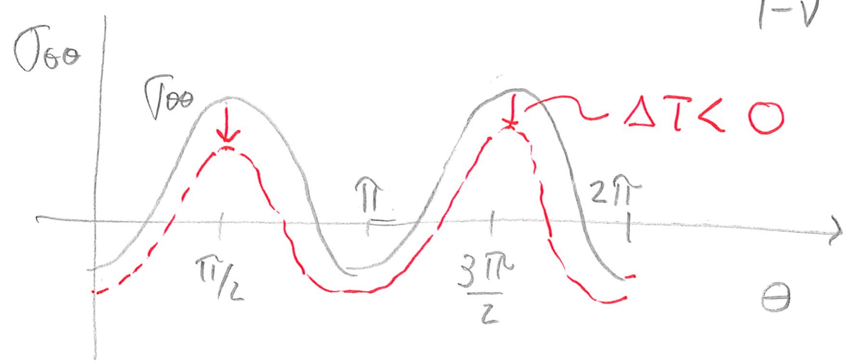
$$\boxed{\frac{\partial \sigma_{\theta\theta}^T}{\partial \theta} = \frac{\beta E}{1-\nu}}$$

↳ steady state conditions

$$① \; \sigma_{\theta\theta}(r=a) = -(P_w - P_p) - \sigma_1 + 3\sigma_2 + \Delta\sigma_{\theta\theta}^T$$

$$② \; \sigma_{\theta\theta}(r=a) = \underbrace{-(P_w - P_p) + 3\sigma_1 - \sigma_2}_{\text{Kirsch}} + \underbrace{\Delta\sigma_{\theta\theta}^T}_{\text{Thermal}}$$

$$\Delta\sigma_{\theta\theta}^T = \frac{\beta E}{1-\nu} \cdot \Delta T$$



General problem of thermo-elasticity (f(time))

$$\nabla \underline{\sigma} + \underline{F} = 0 \; \text{— Equil}$$

$$\underline{\underline{\varepsilon}} = \tfrac{1}{2}(\nabla \underline{u} + \nabla \underline{u}^T) \; \text{— Kinematic}$$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{C}}}\, \underline{\underline{\varepsilon}} + 3\beta K \theta \underline{\underline{I}} \; \text{— Constitutive}$$

$$\frac{d\theta}{dt} = \underbrace{\frac{K_T}{\rho C_v}}_{\text{thermal diffusivity}} \underbrace{\nabla^2}_{\text{Laplacian operator}} \theta + \frac{3K\beta T_0}{\rho C_v} \frac{d\varepsilon_{vol}}{dt}$$

typical param: $\sim 50K \cdot 0.01 = 0.5K$

Thermo-poro-elasticity (Coussy 4.3)

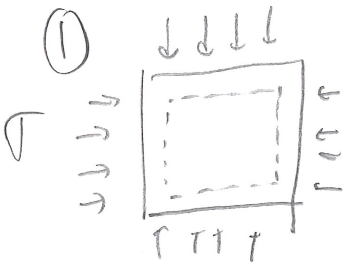
↳ new energy term → entropy heat ↔ $\Delta T = T - T_0$

$$
\left.\begin{array}{ll}
\text{mean stress} & S_m = K\epsilon - \alpha p - 3\beta K (T - T_0) \\
\text{sheer stress} & S_{ij} = 2 G e_{ij} \\
\Delta \text{ porosity} & \varphi = \alpha\epsilon + P/N - 3\beta_\varphi (T - T_0) \quad ; \ \beta_\varphi = \beta_{solid}(\alpha - \phi_0) \\
\text{entropy} & S_s = S_{s_0} + 3\beta K\epsilon - 3\beta_\varphi p + \dfrac{C_v (T - T_0)}{T_0}
\end{array}\right.
$$

Visco-elasticity

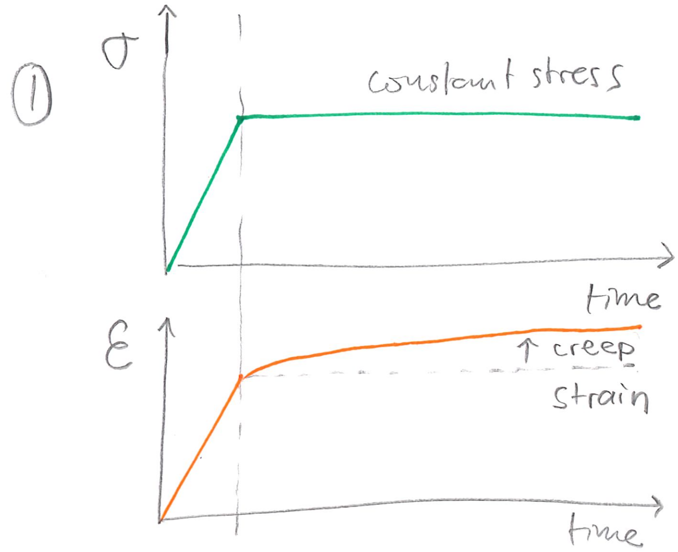
Example

(1)

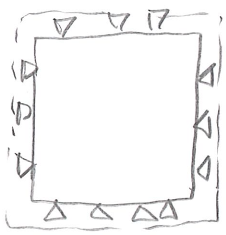


creep strain

(1)

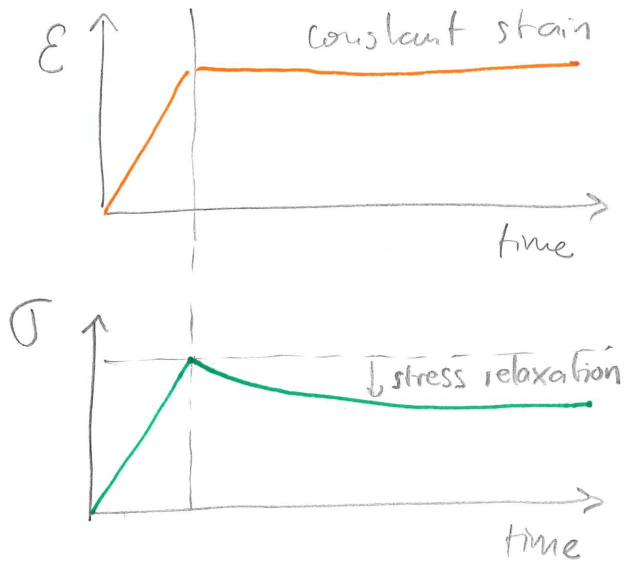


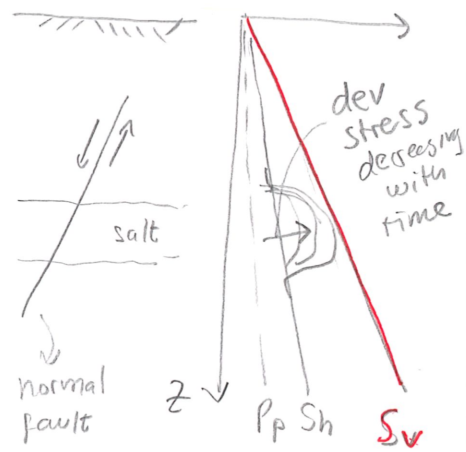
(2)



or

stress relaxation

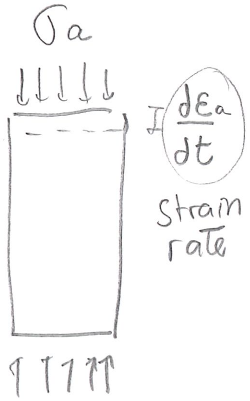




$$\sigma_v / \sigma_h > \frac{1+\sin\varphi}{1-\sin\varphi} \sim 3$$

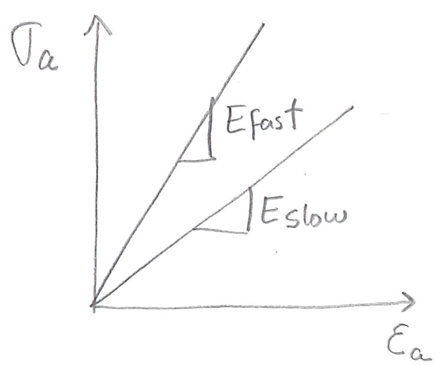
$\varphi = 30°$

(3)



strain rate

strain-rate dependent stiffness



fold

sandstone: deformation over millions of years

particle rearrangement + crushing

- subcritical fracture propagation

Elasticity

→ Poro-elasticity ✓

→ Thermo-elasticity ✓

→ Visco-elasticity ✓

→ Chemo-elasticity ✓

→ Elasto-<u>plasticity</u>

<u>Chemo-elasticity</u> ⟷ <u>chemo-plasticity</u>

$$\underline{\underline{S}} = \underline{\underline{\underline{\underline{C}}}}\ \underline{\underline{\varepsilon}} + \underbrace{\alpha}\ p\ \underline{\underline{I}} + \underbrace{3\beta}_{\text{coeff}} K \theta\, \underline{\underline{I}} + \underbrace{\gamma}_{\text{coeff}}\ \overset{\text{chemical potential}}{\mu}\ K\, \underline{\underline{I}}$$

Mech — Poro Hydro — thermal — chemical

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}$$

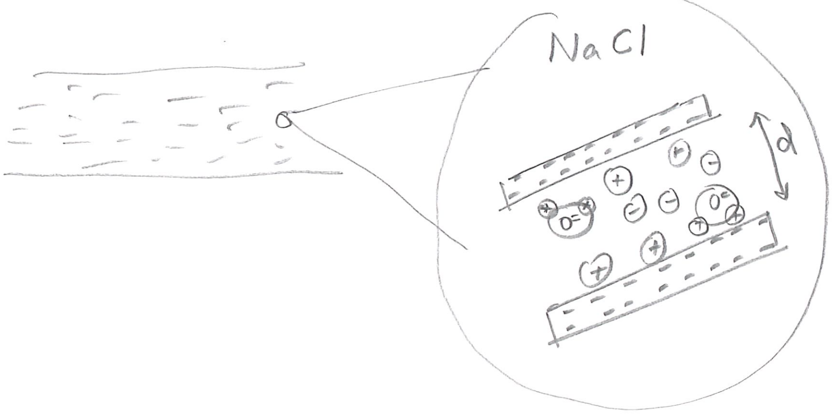
T H C M processes

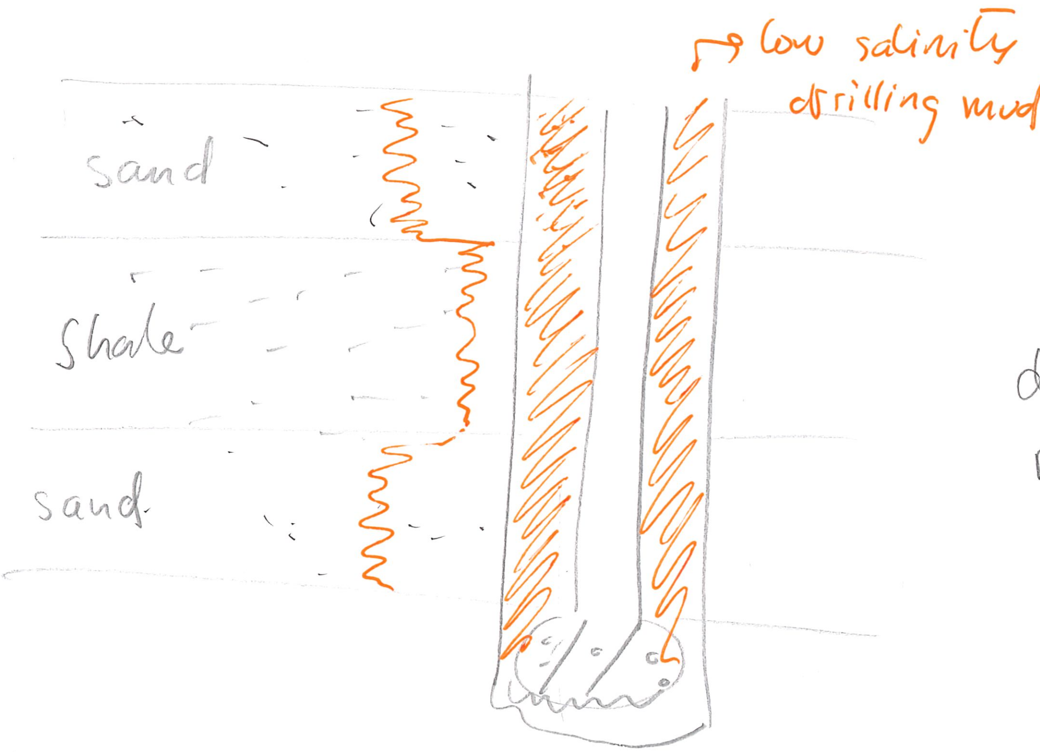
Multiphysics problems

"emergent phenomena"

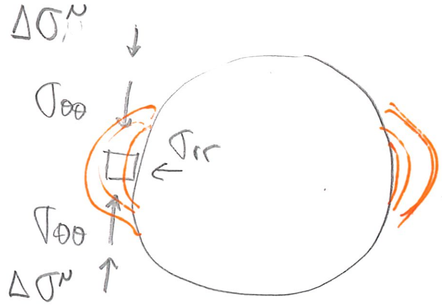
1 Chemical sensitivity of shales

electrical repulsion
↳ double layer

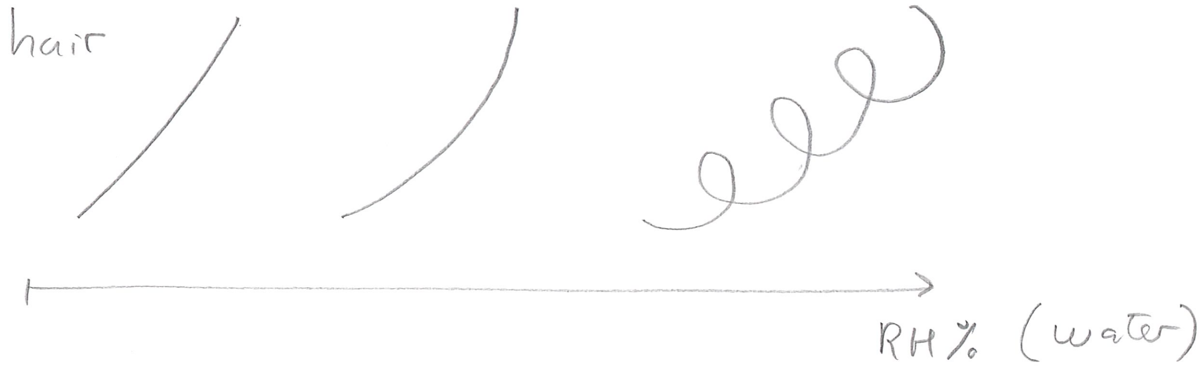


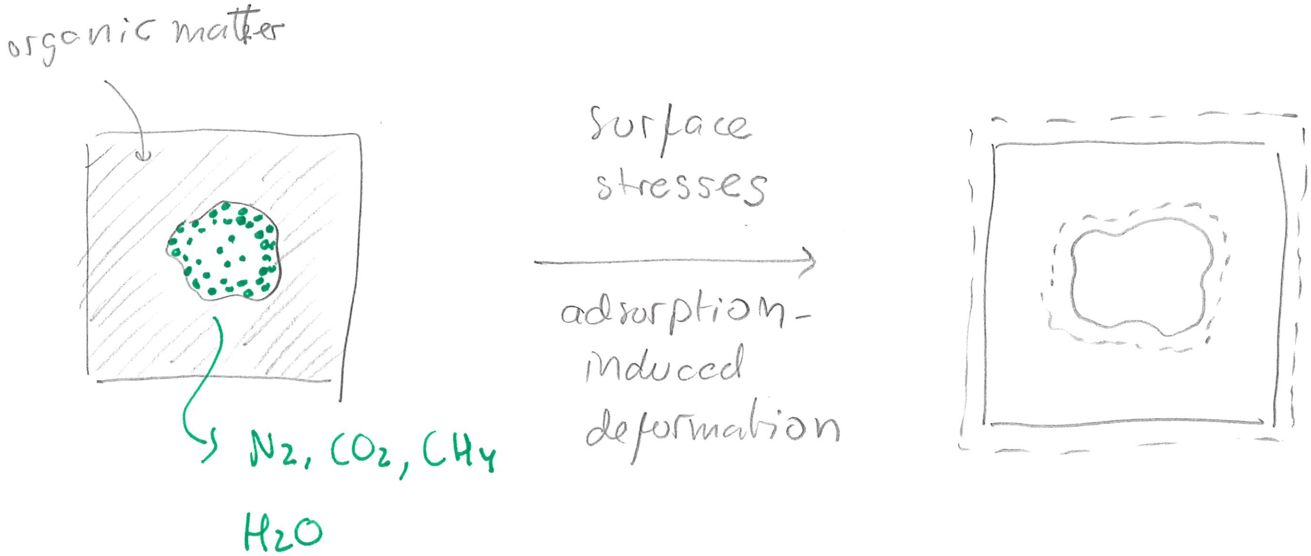


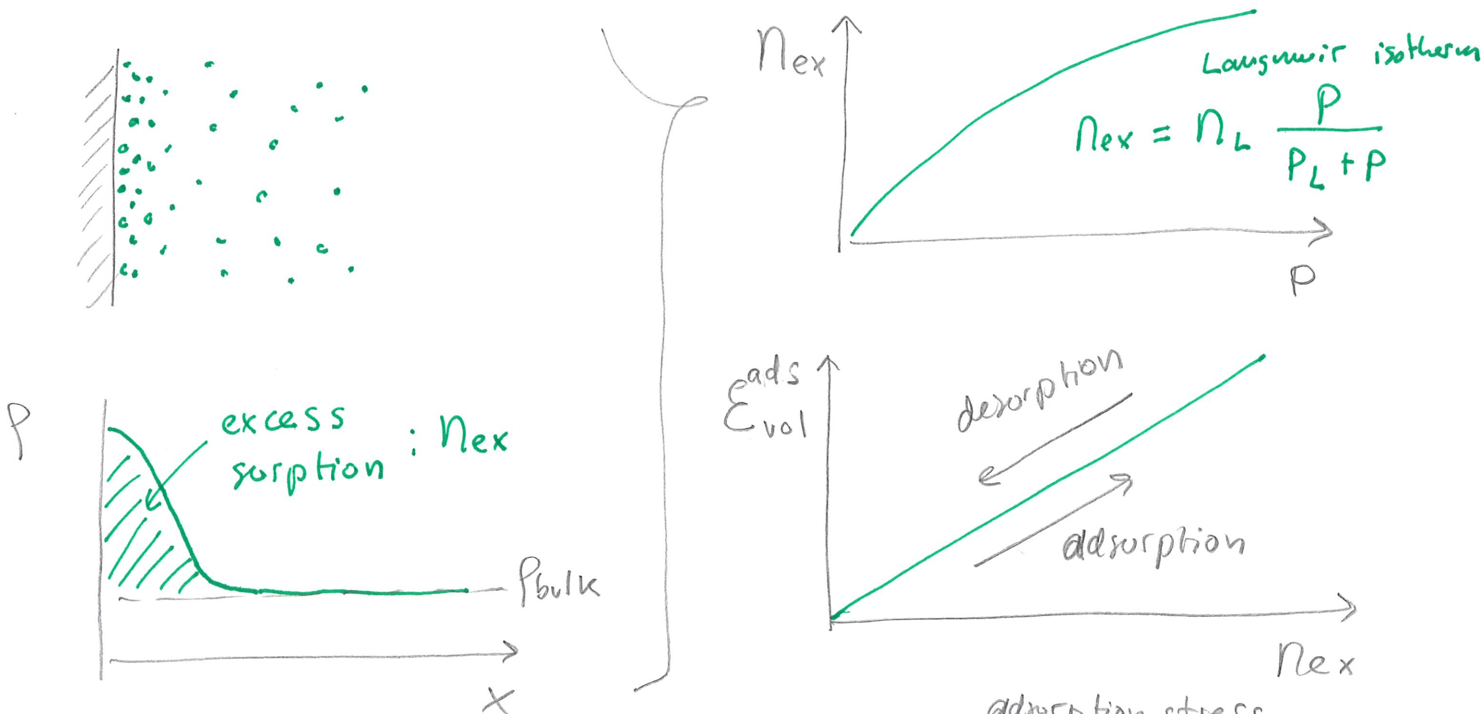
drilling mud
- water based
  - ↑ salinity
- oil-based mud
  ↳ doesn't hydrate clay
- underbalanced drilling
  $P_w < P_p$



Example 2: Adsorption-induced deformation

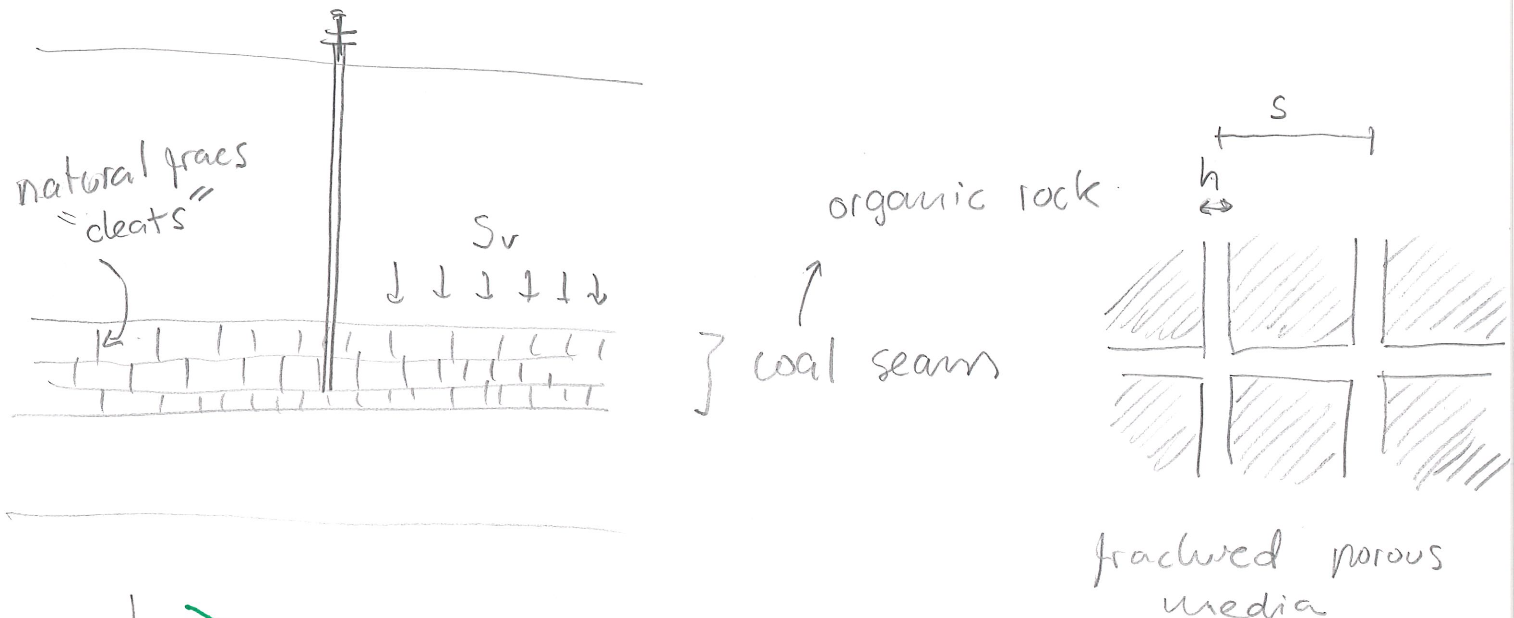


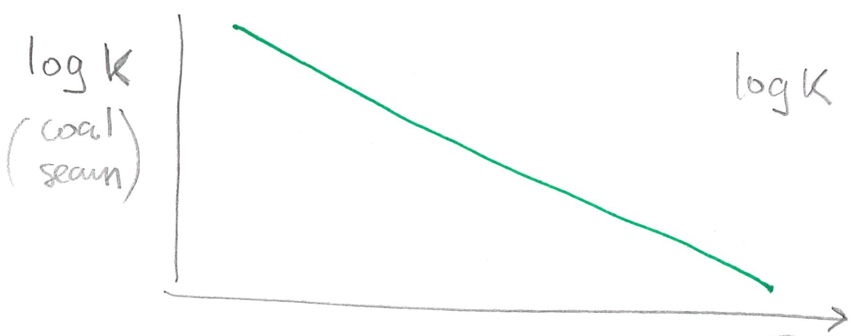




$$\underline{\underline{S}} = \underline{\underline{\underline{\underline{C}}}}\;\underline{\underline{\varepsilon}} + \alpha p \underline{\underline{I}} + (1-\alpha) \overbrace{S_a(p)}^{\text{adsorption stress}} \underline{\underline{I}}$$

coal bed methane

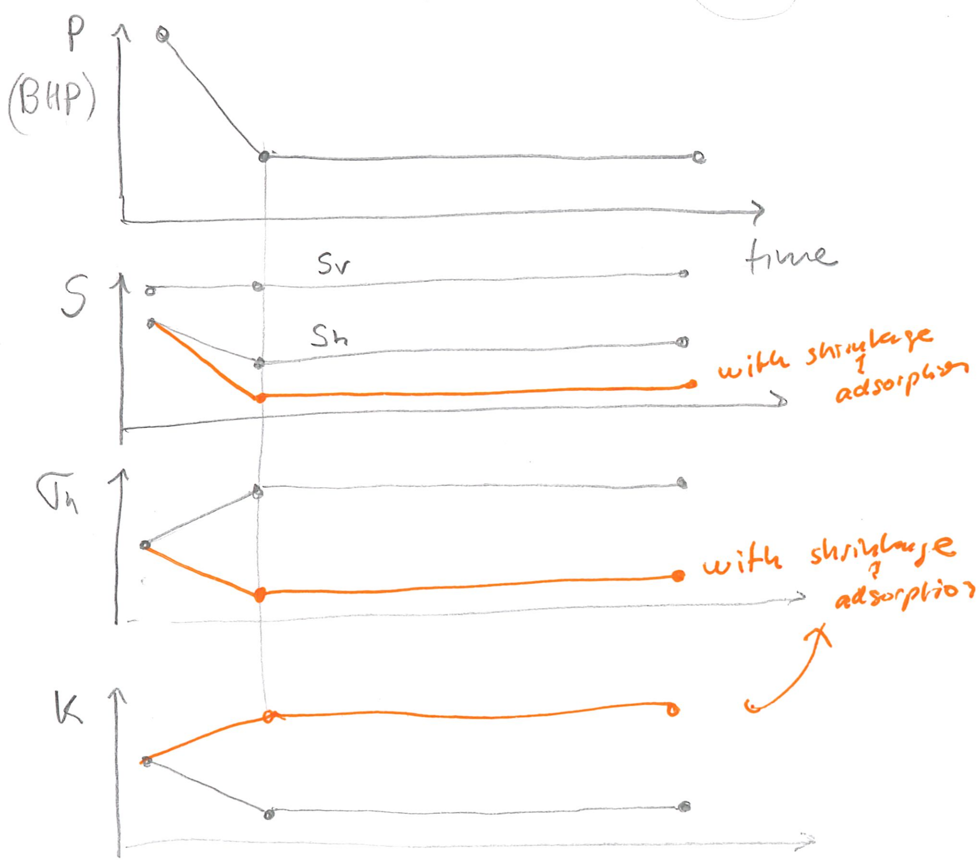


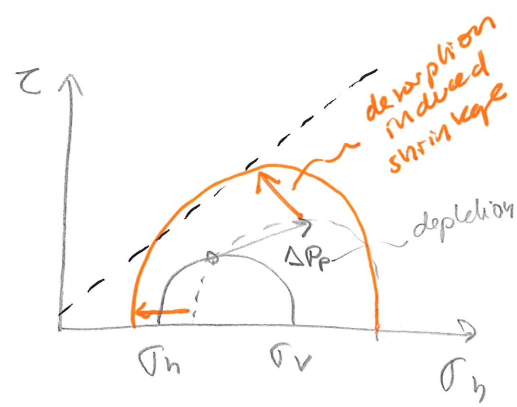


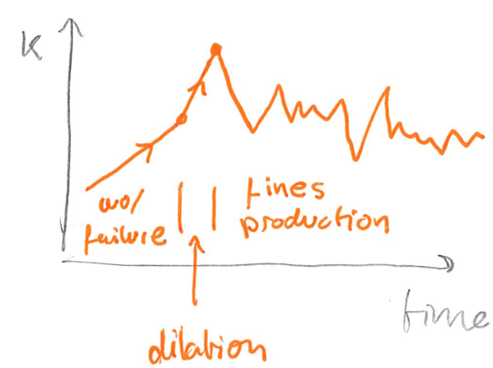
$$\log k \propto \frac{1}{\sigma_h}$$

$$\boxed{k = \frac{h^3}{12 \cdot s}}$$

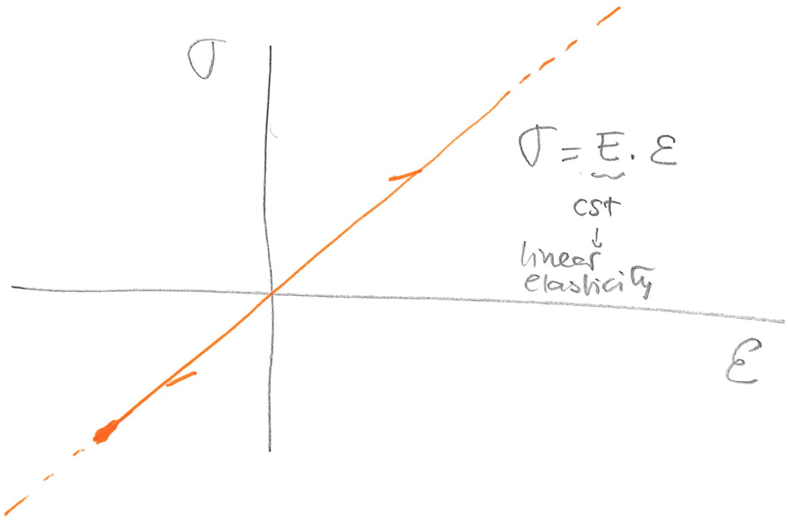
σ_h (effective stress) ← horizontal

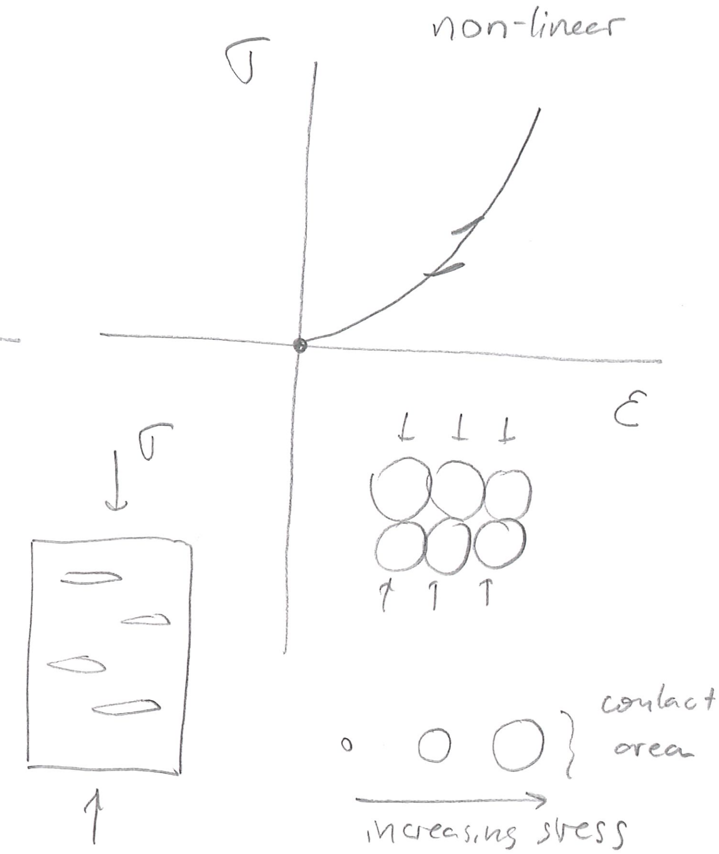


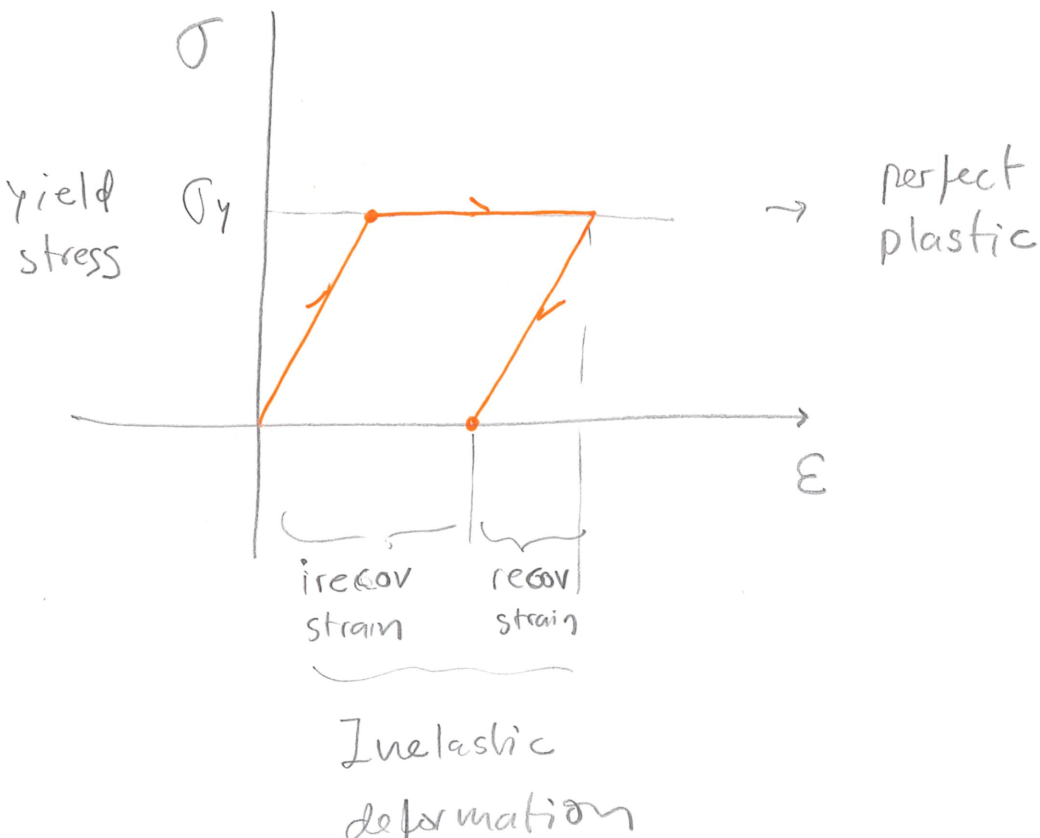




Plasticity and Inelasticity

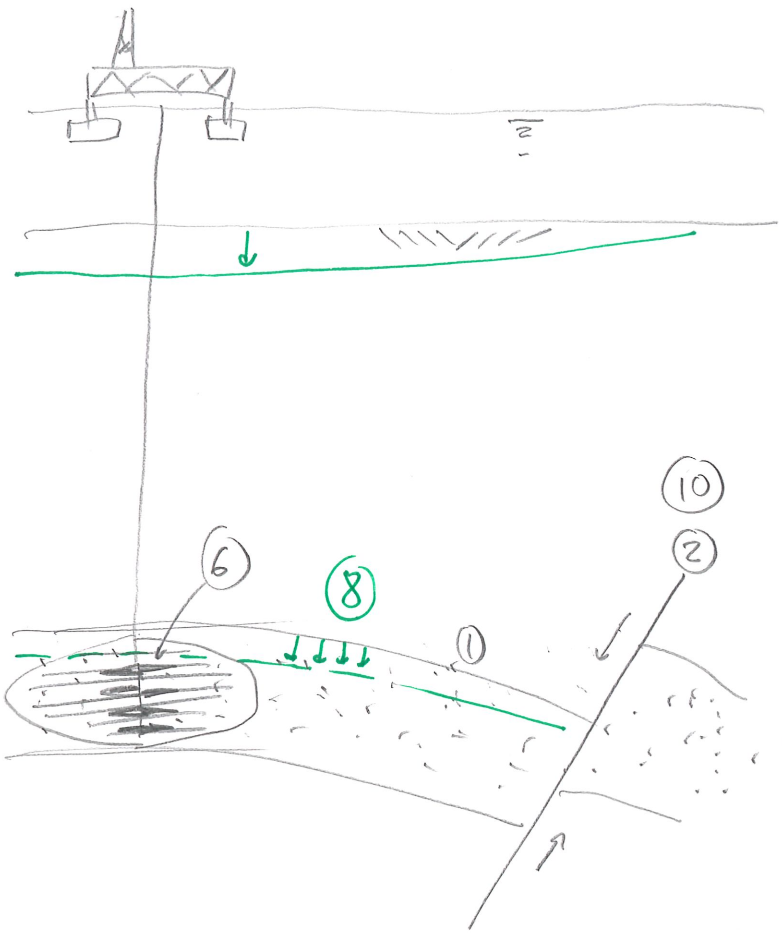


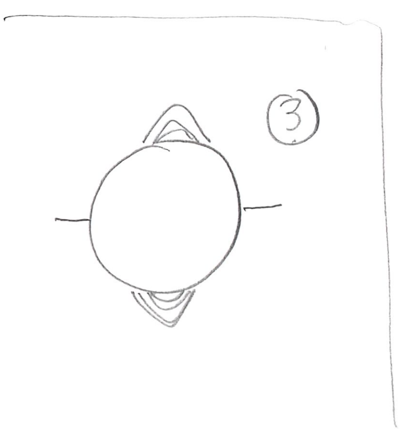


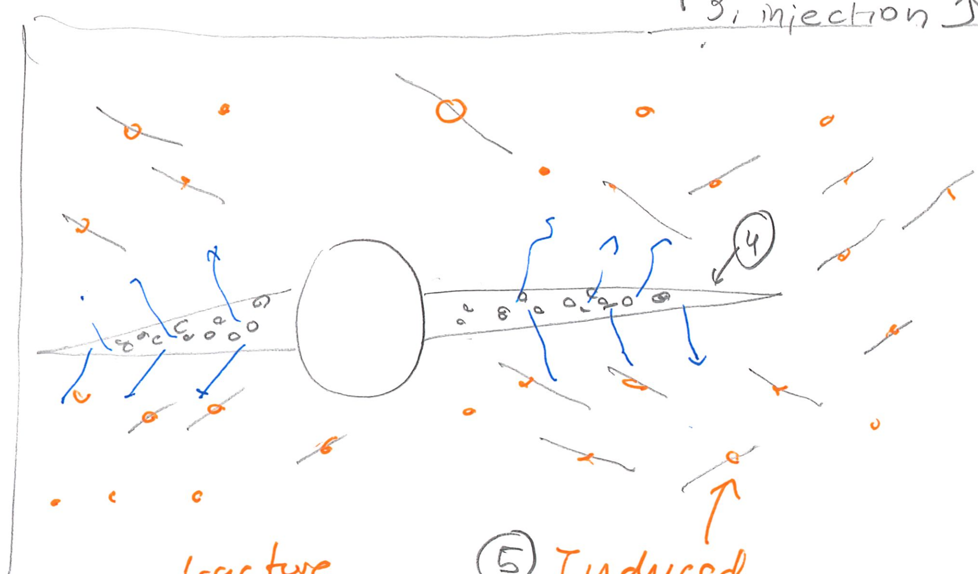


Examples

- Exploration
  - folding (1)
  - faulting
  - $\sigma_1 = \frac{1+\sin\varphi}{1-\sin\varphi}\sigma_3$ (2)
- Drilling
  - breakouts and tensile fracs (3)
  - wellbore cuttings
- Completion
  - fracturing (4)
  - induced micro-seismicity (5)
  - perforations (6)
  - reduced stress-shadow (7)
- Production
  1. compaction (8)
  2. fines (sand) production (9)
     ↳ fault reactivation (10)
  3. injection ↑





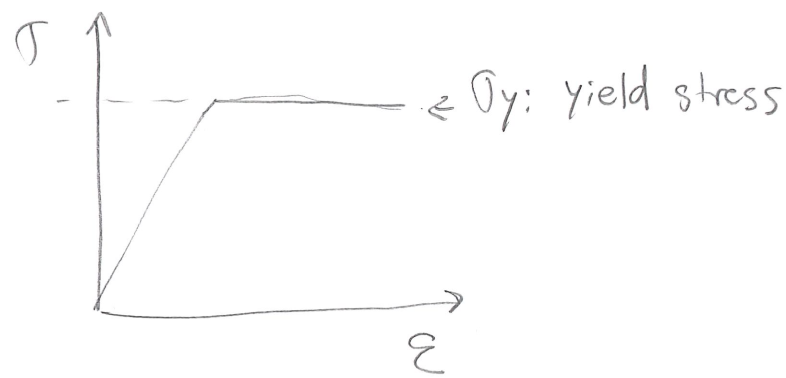


fracture reactivation

$\sigma_1 = q \cdot \sigma_3$

$q = \frac{1+\sin\varphi}{1-\sin\varphi}$

(5) Induced microseismicity



$\sigma_1 = \sigma_2 = 0$

$\sigma_3 (@\text{yield}) = \sigma_y$

$f(\underline{\underline{\sigma}}) = K \rightarrow$ failure criterion
yield

$f(\sigma_1, \sigma_2, \sigma_3, \text{directions } \sigma_i) = K$

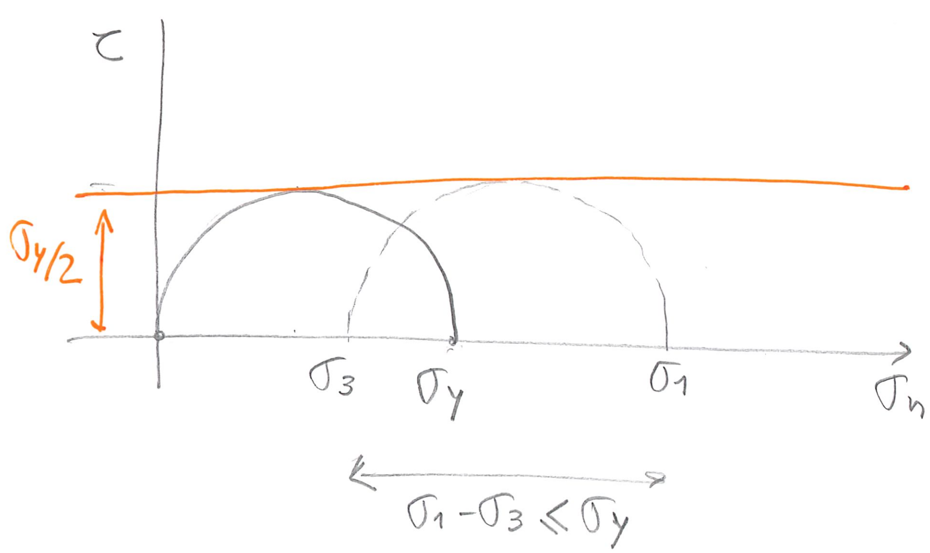
↓ assuming isotropy

$f(\sigma_1, \sigma_2, \sigma_3) = K$

$f^*(I_1, I_2, I_3) = K^*$

Tresca

$\sigma_1 - \sigma_3 \leqslant \sigma_y$



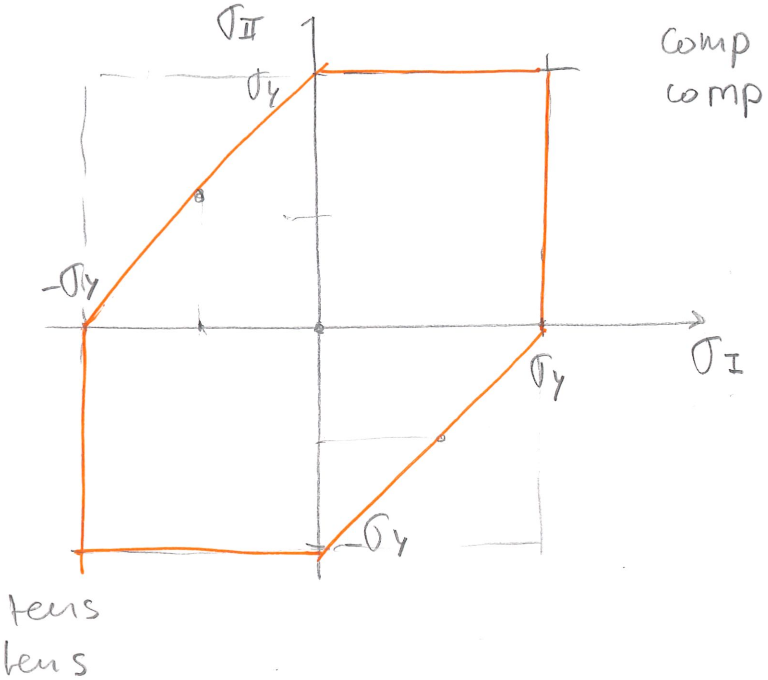
$|\sigma_I - \sigma_{III}| \leqslant \sigma_y$

$|\sigma_I - \sigma_{II}| \leqslant \sigma_y$

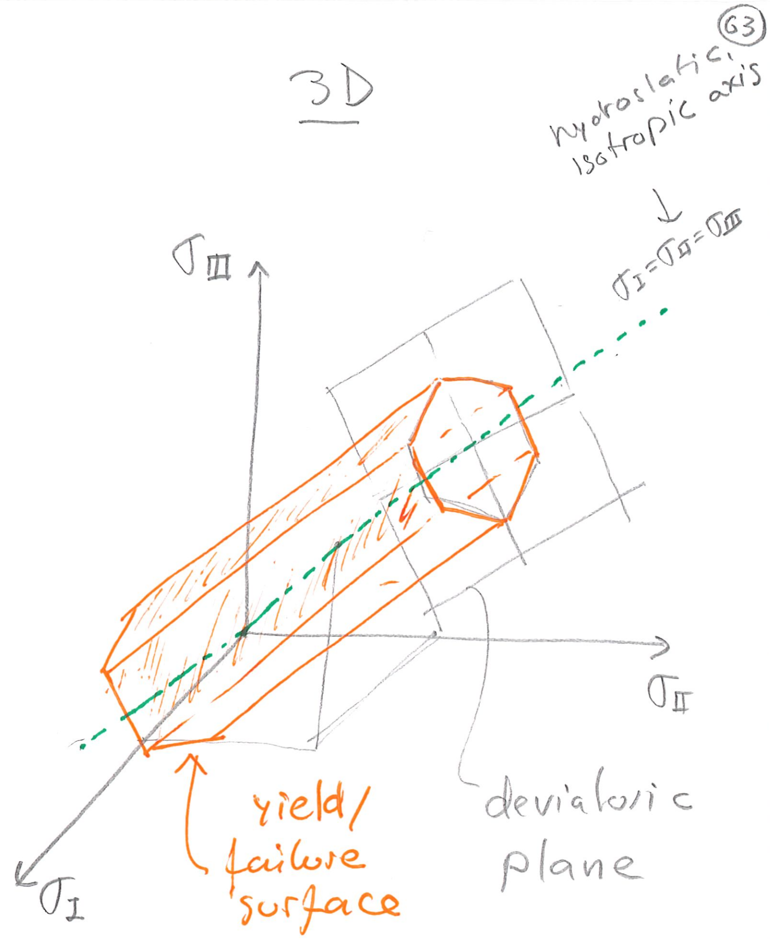
$$\underbrace{|\sigma_{II} - \sigma_{III}|}_{\text{principal stresses not ordered}} \leqslant \sigma_y$$

Principal stresses space

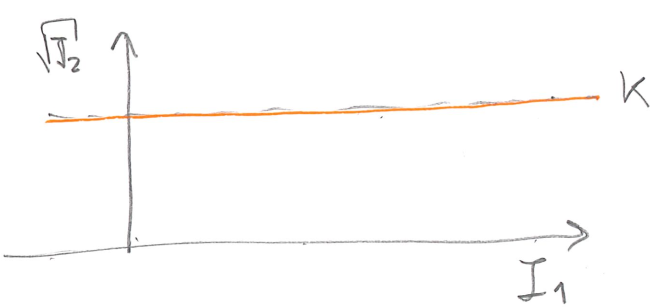
2D ($\sigma_{III} = 0$)



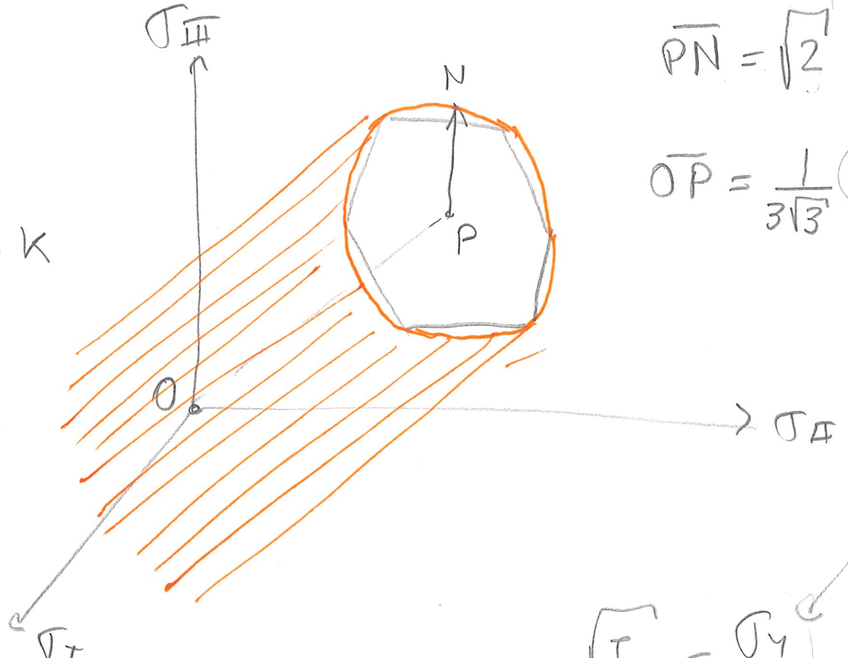
3D



von Mises



$\sqrt{J_2} = K$

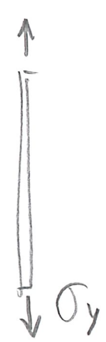


$\overline{PN} = \sqrt{\frac{2}{3}}\,\sigma_y$

$\overline{PN} = \sqrt{2}\,\sqrt{J_2}$

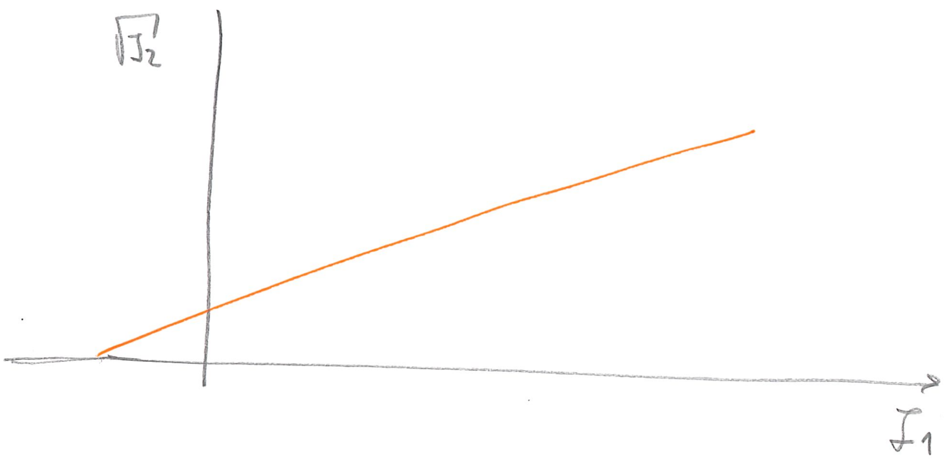
$\overline{OP} = \frac{1}{3\sqrt{3}}\, I_1$

$\sqrt{J_2} = \frac{\sigma_y}{\sqrt{3}}$



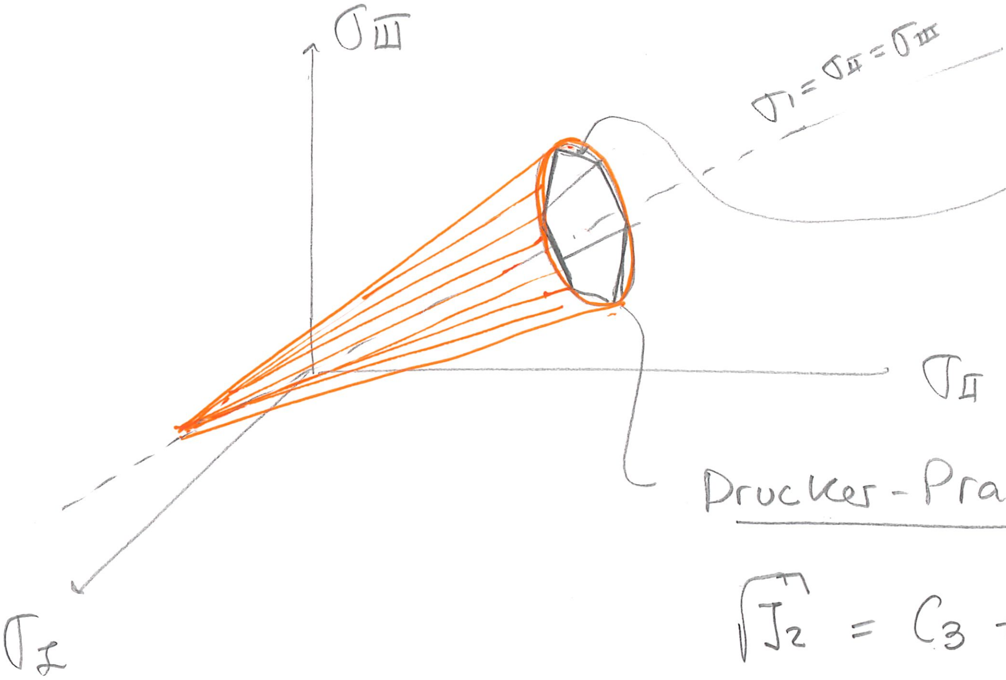
σ_y measured from a uniaxial stress test

Stress sensitive yield criteria



Modified - Tresca

$$|\sigma_1 - \sigma_3| = C_1 + C_2 \cdot \sigma_m$$



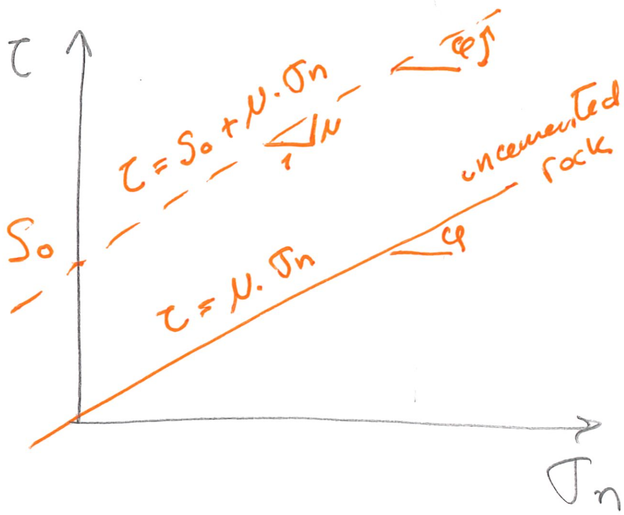
Drucker - Prager

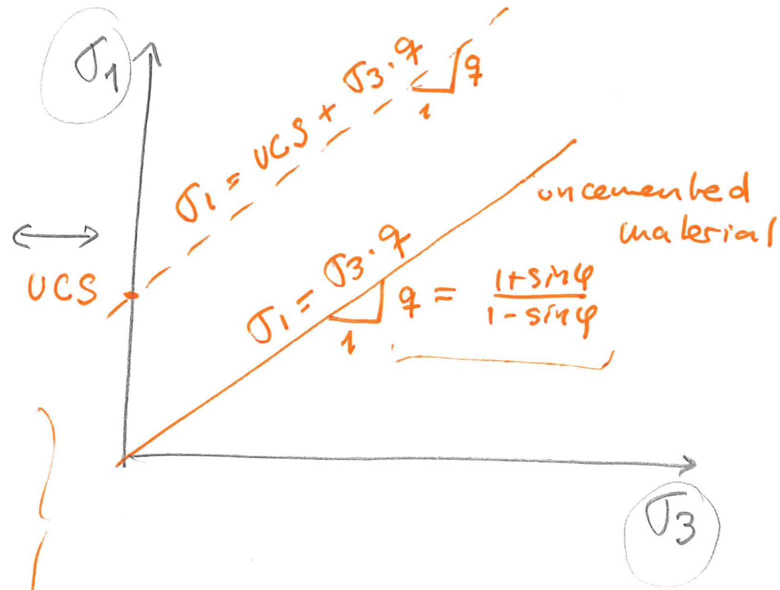
$$\sqrt{J_2} = C_3 + C_4 \cdot I_1$$

cohesive strength

friction angle

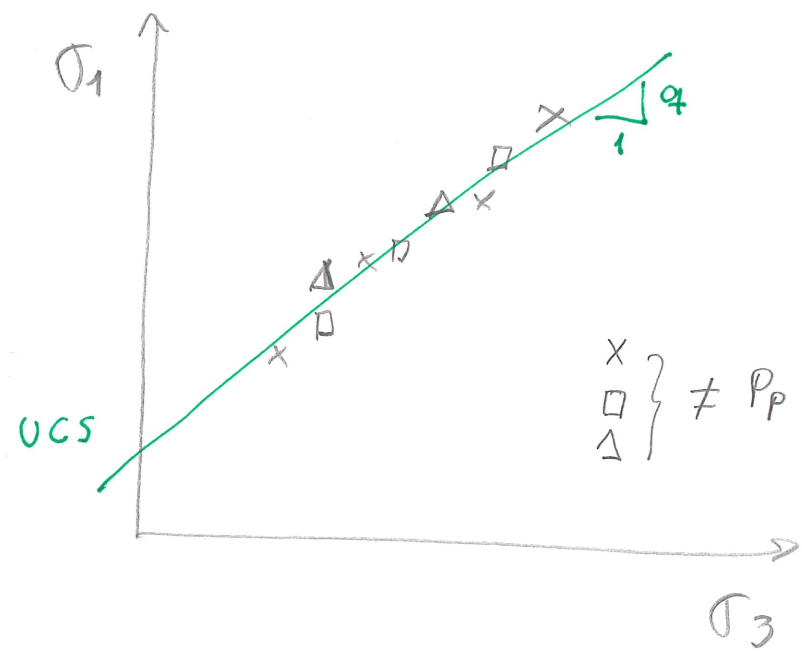
$$\begin{cases} C_3 = \dfrac{6 \cdot S_0 \cos \varphi}{\sqrt{3}\,(3 - \sin\varphi)} \\[2ex] C_4 = \dfrac{2}{\sqrt{3}} \dfrac{\sin\varphi}{(3 - \sin\varphi)} \end{cases}$$





$$UCS = 2 \cdot S_0 \sqrt{q}$$

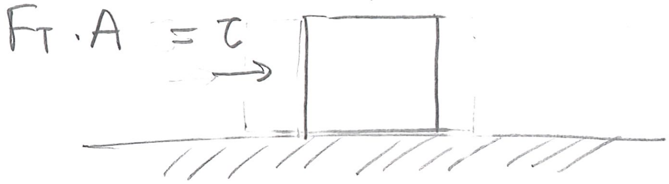
Triaxial (axisymmetric) test with pore pressure



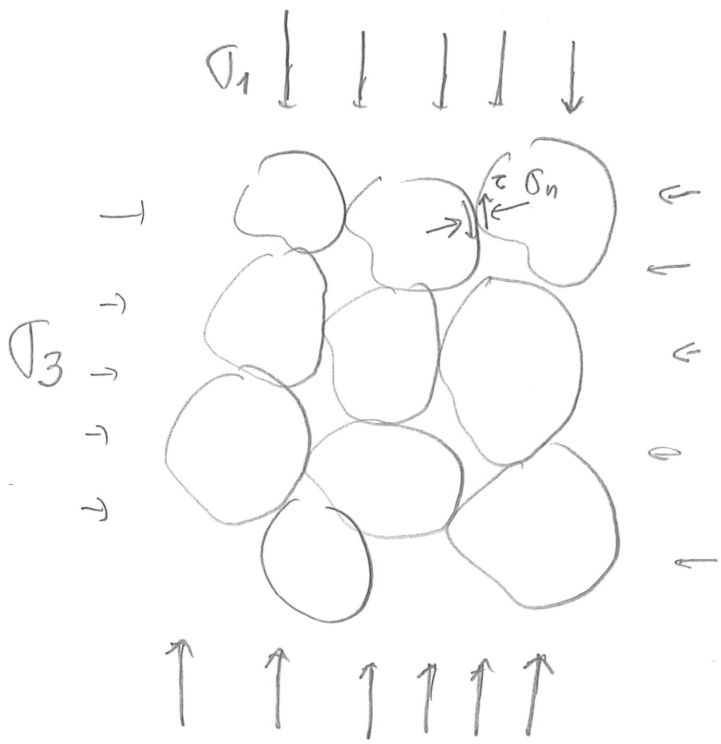
Stress sensitive geomaterials

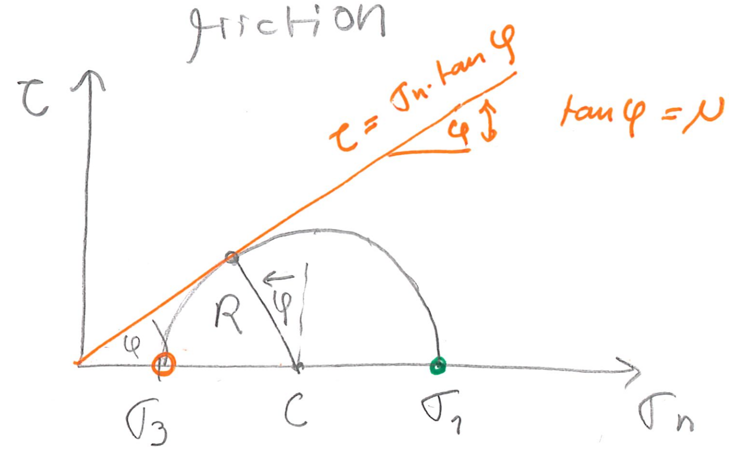
$$\sigma_n = F_N \cdot A$$

$$F_T \cdot A = \tau$$



$$\tau = \underbrace{\mu}_{\text{coefficient of friction}} \sigma_n$$

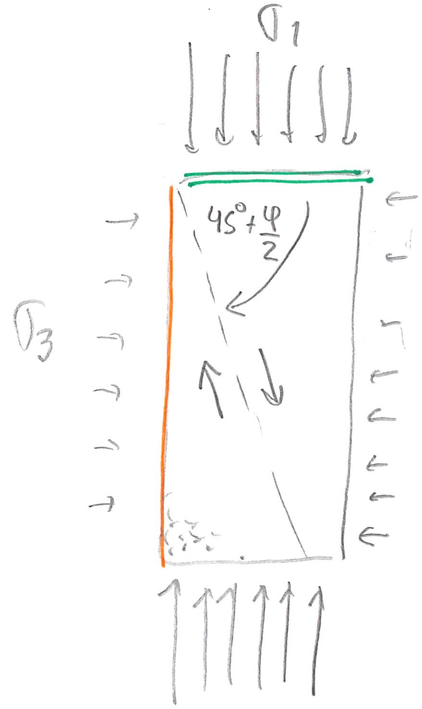




$$\frac{\sigma_1}{\sigma_3} = \frac{C + R}{C - R}$$

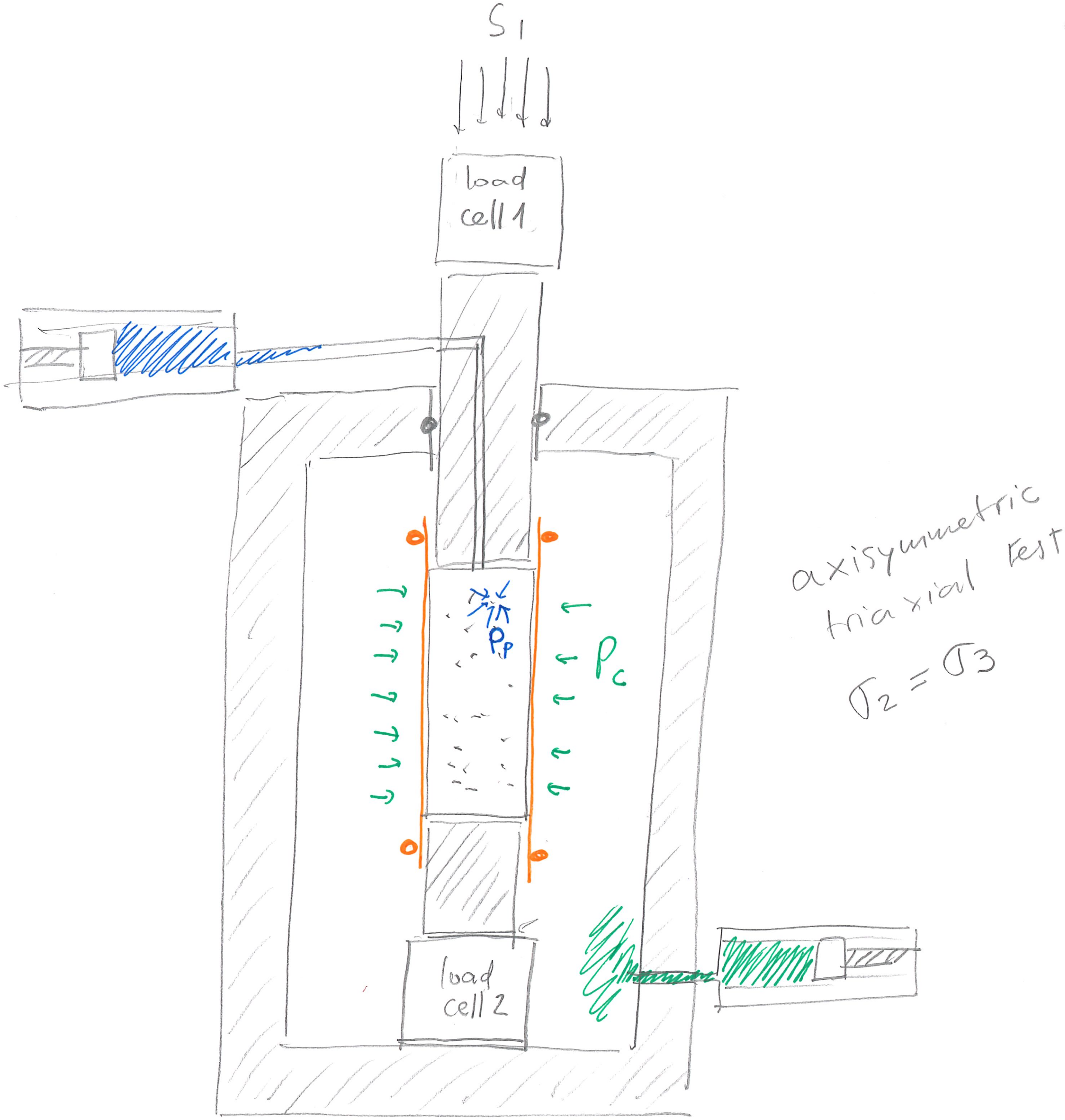
$$\frac{\sigma_1}{\sigma_3} = \frac{\cancel{C} + \cancel{C}\cdot \sin\varphi}{\cancel{C} - \cancel{C}\sin\varphi}$$

$$\left\{ \frac{\sigma_1}{\sigma_3} = \frac{1+\sin\varphi}{1-\sin\varphi} \right\}$$ stress anisotropy ratio



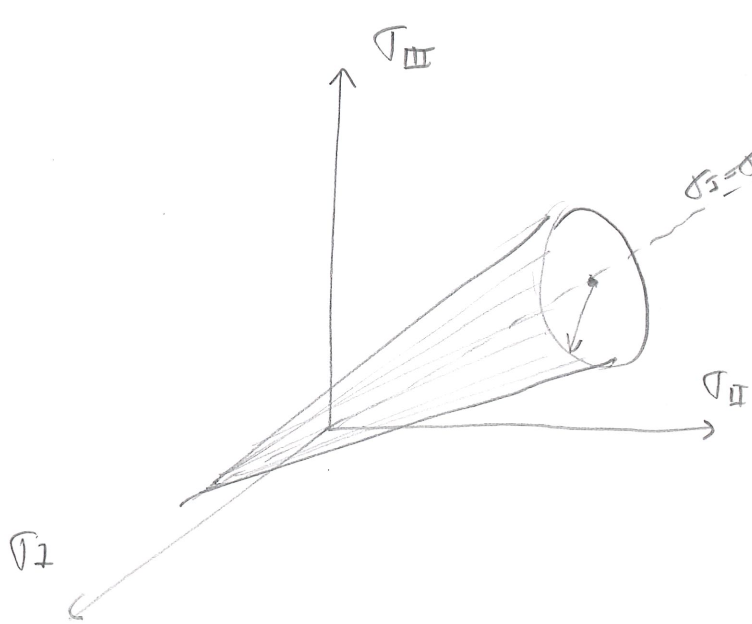
$$\sigma_1 = \underbrace{\left(\frac{1+\sin\varphi}{1-\sin\varphi}\right)}_{q} \cdot \sigma_3$$

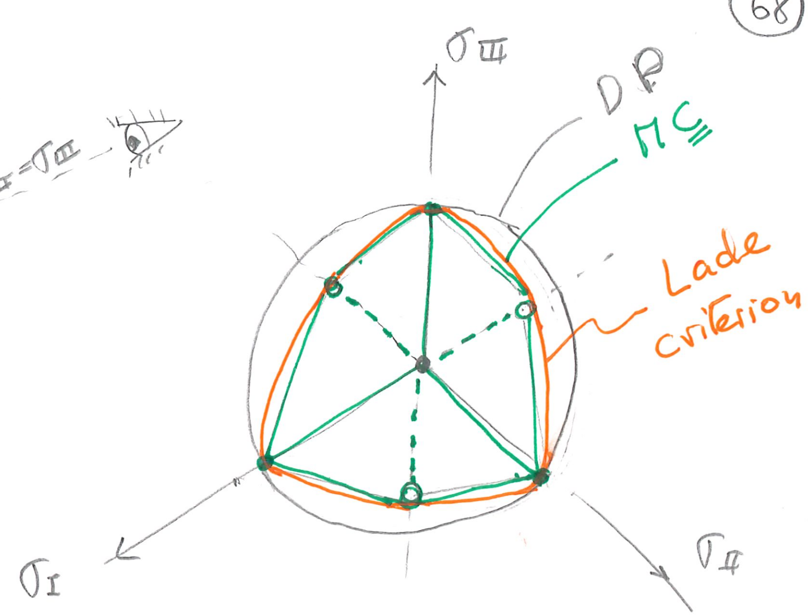
q : { if $\varphi = 30° \Rightarrow q = 3$

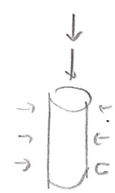


$$\left| \begin{array}{l} S_3 = P_c \\ S_1 \rightarrow \text{load cell 1} \end{array} \right. \Rightarrow \left| \begin{array}{l} \sigma_3 = P_c - P_p \\ \sigma_1 = S_1 - P_p \end{array} \right.$$

$$S_D = S_1 - S_3 \rightarrow \text{load cell 2} \rightarrow = \underline{\sigma_1 - \sigma_3 = \sigma_d}$$

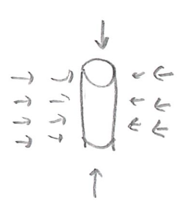






Triaxial compression

$\boxed{p_c =} \sigma_2 = \sigma_3 < \sigma_1$



Triaxial extension

$\sigma_3 < \sigma_1 = \sigma_2$

Modified Lade criterion

$f(I_1, I_3) = K$ } + ? check book

$$\boxed{\frac{(I_1^*)^3}{I_3^*} = 27\eta}$$

$I_1^* = \sigma_1^* + \sigma_2^* + \sigma_3^*$

$I_3^* = \sigma_1^* \cdot \sigma_2^* \cdot \sigma_3^*$

eff stress

where $\sigma_i^* = \sigma_i + S$

$S = S_0 / \tan\varphi$

$$\eta = \frac{4(\tan\varphi)^2(9 - 7\sin\varphi)}{1 - \sin\varphi}$$

Project # 6

Step 1) → Get equations for failure criteria

- Coulomb
- Lade (modified)
- Drucker-Prager

→ T_s : tensile strength

Step 2) Calculate stresses around the wellbore wall

vertical well

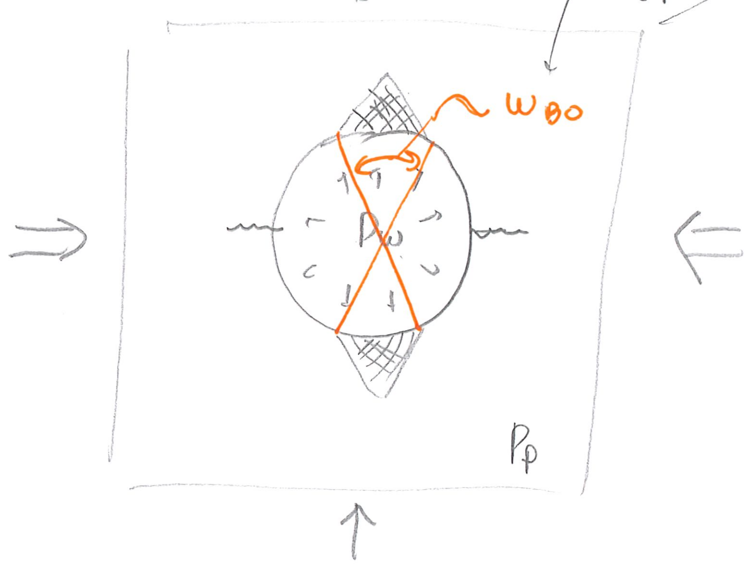
↓

Kirsch

↓

region in the wellbore

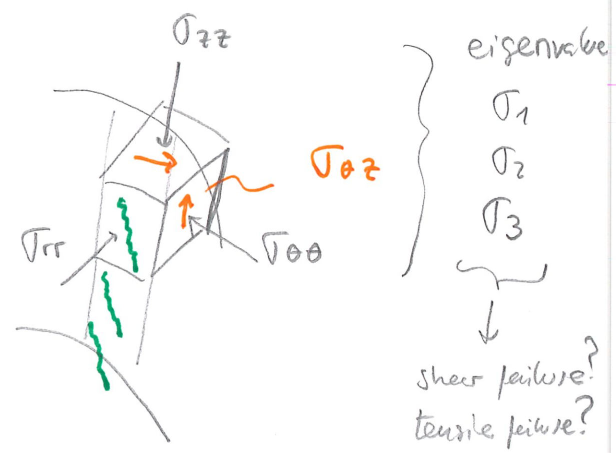
$\sigma_1 > UCS + \sigma_3 \cdot q$

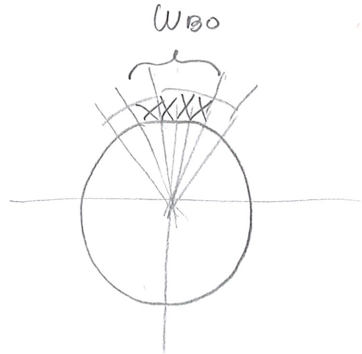


deviated well

↓

Kirsch + shear





for $\delta = 0$ to $360°$

for $\phi = 0$ to $90°$

for $\alpha = 0°$ to $360°$ ($1°$++)

$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{\theta z}, \sigma_{zz} \rightarrow \sigma_1, \sigma_2, \sigma_3$

shear failure?

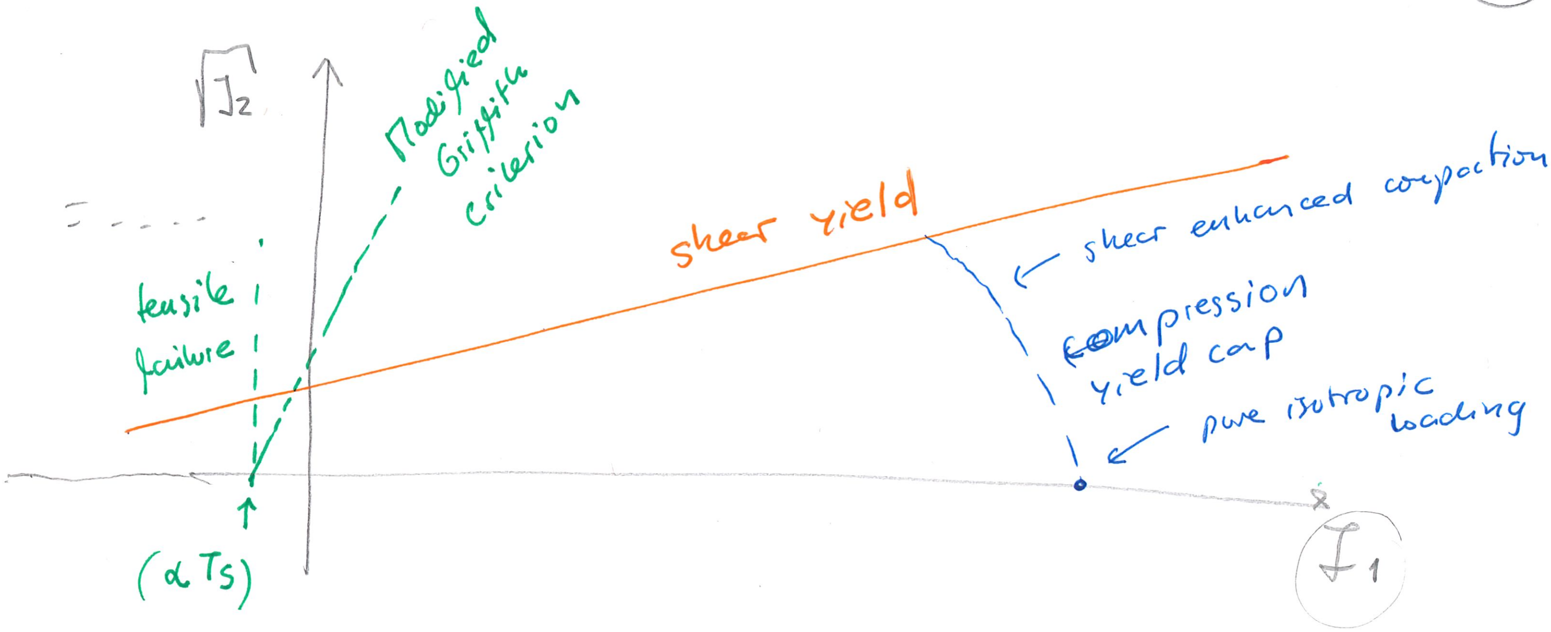
tensile fracture?

end

end

end

end



mean stress

$= \sigma_1 + \sigma_2 + \sigma_3$

Beyond the yield point (Fjaer et al)

$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{D}}} \cdot \underline{\underline{\sigma}}$$

- predict strains
  - elastic – $\varepsilon_{ij} = D_{ijmn}\,\sigma_{mn} \rightarrow (d\varepsilon_{ij}) = D_{ijmn}(d\,\sigma_{mn})$
  - plastic – $d\varepsilon^p_{ij} \leftrightarrow d\,\sigma_{ij}$
- • small-strain
  • continuous strain field
  • rate-independent

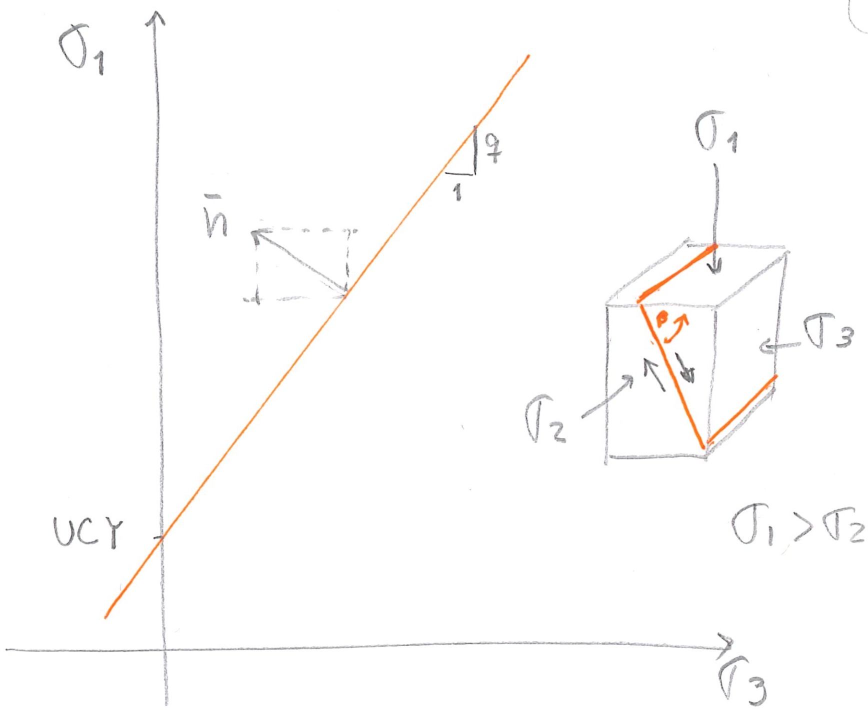
1. yield criterion
   $f(\sigma_{ij}) = Y$
2. strain-hardening rule
   $Y = f^2(\delta\varepsilon^p)$
3. strain de composition
   $\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^p_{ij}$
4. plastic flow rule
   $d\varepsilon^p_{ij} \longleftrightarrow d\,\sigma_{ij}$
5. elastic unloading criterion

Example: Mohr-Coulomb

$$\tau = S_0 + \sigma_n \cdot \tan\varphi$$

$$\underbrace{\sigma_1 = UCY + q\,\sigma_3}$$

$$f = \sigma_1 - UCY - q\,\sigma_3$$



$$\beta = 45° + \varphi/2$$

$$\sigma_1 > \sigma_2 > \sigma_3$$

$$\bar{n} = \left(\frac{\partial f}{\partial \sigma_1}, \frac{\partial f}{\partial \sigma_2}, \frac{\partial f}{\partial \sigma_3} \right)$$

Flow rule
(associated)

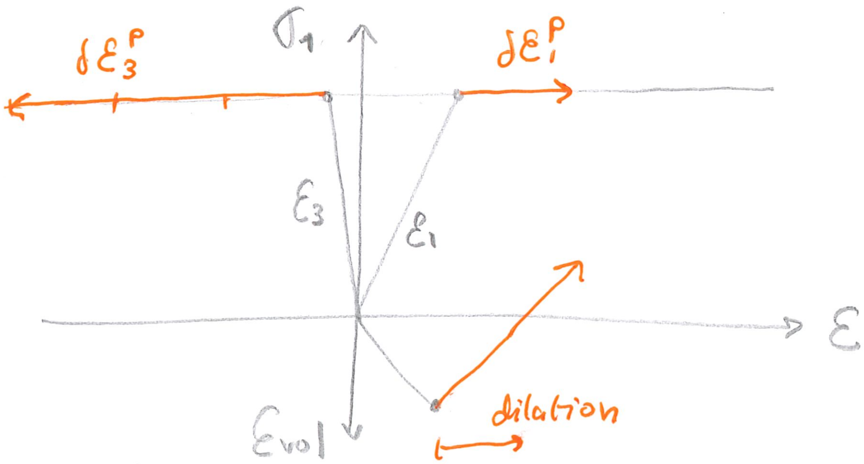
$$\boxed{d\varepsilon_{ij}^p = \underbrace{d\lambda}_{cst} \cdot \frac{\partial f}{\partial \sigma_{ij}}}$$

$$\bar{n} = (1, 0, -q)$$

$$\varphi = 30° \Rightarrow q = 3$$

$$q = \frac{1+\sin\varphi}{1-\sin\varphi}$$

$$\left.\begin{aligned} d\varepsilon_1^p &= d\lambda \cdot 1 \\ d\varepsilon_2^p &= d\lambda \cdot 0 \\ d\varepsilon_3^p &= d\lambda\,(-q) \end{aligned}\right\} \rightarrow d\varepsilon_{vol} = \underbrace{d\lambda\,(1-q)}_{\text{dilation}}$$



— elastic

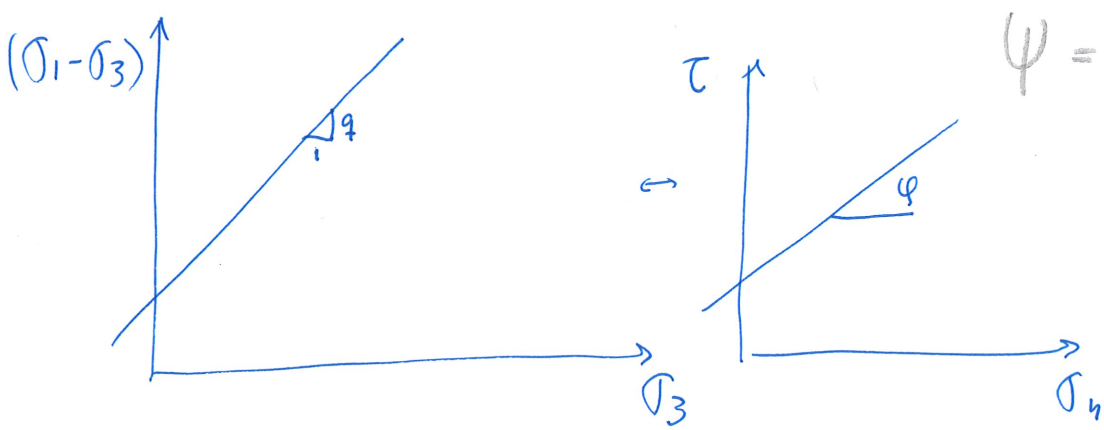
— plastic
- perfect plasticity
- no hardening

$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial F}{\partial \sigma_{ij}}$$ ~ yield surface $\Rightarrow$ Associated flow rule

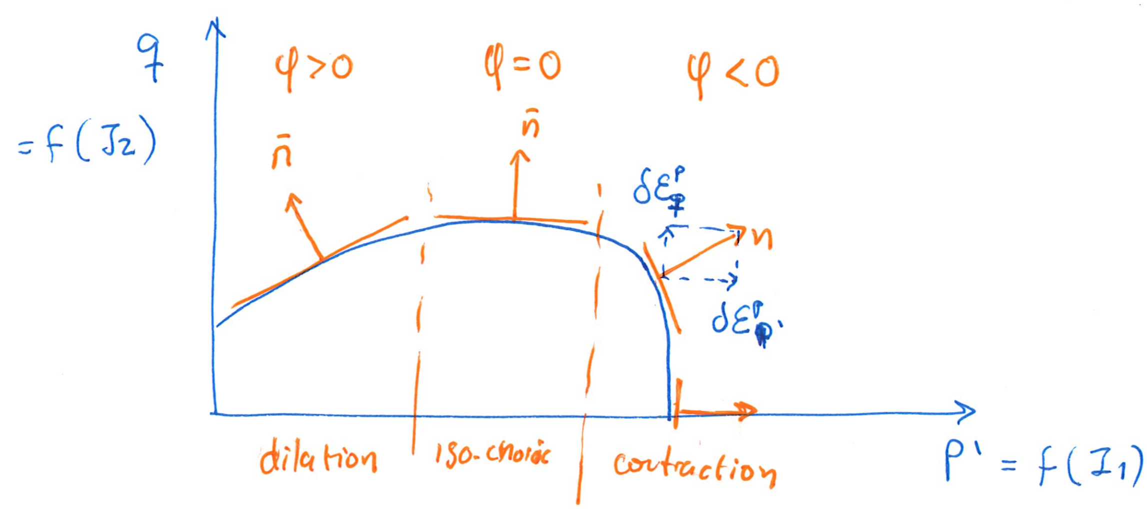
$$d\varepsilon_{ij}^{p} = d\lambda \frac{\partial g}{\partial \sigma_{ij}}$$ ~ plastic potential function $\Rightarrow$ $g \neq F$ Non-associated flow rule

$$g = \sigma_1 - UCY - \frac{1+\sin\psi}{1-\sin\psi}\,\sigma_3 \qquad \psi < \varphi$$

dilation angle

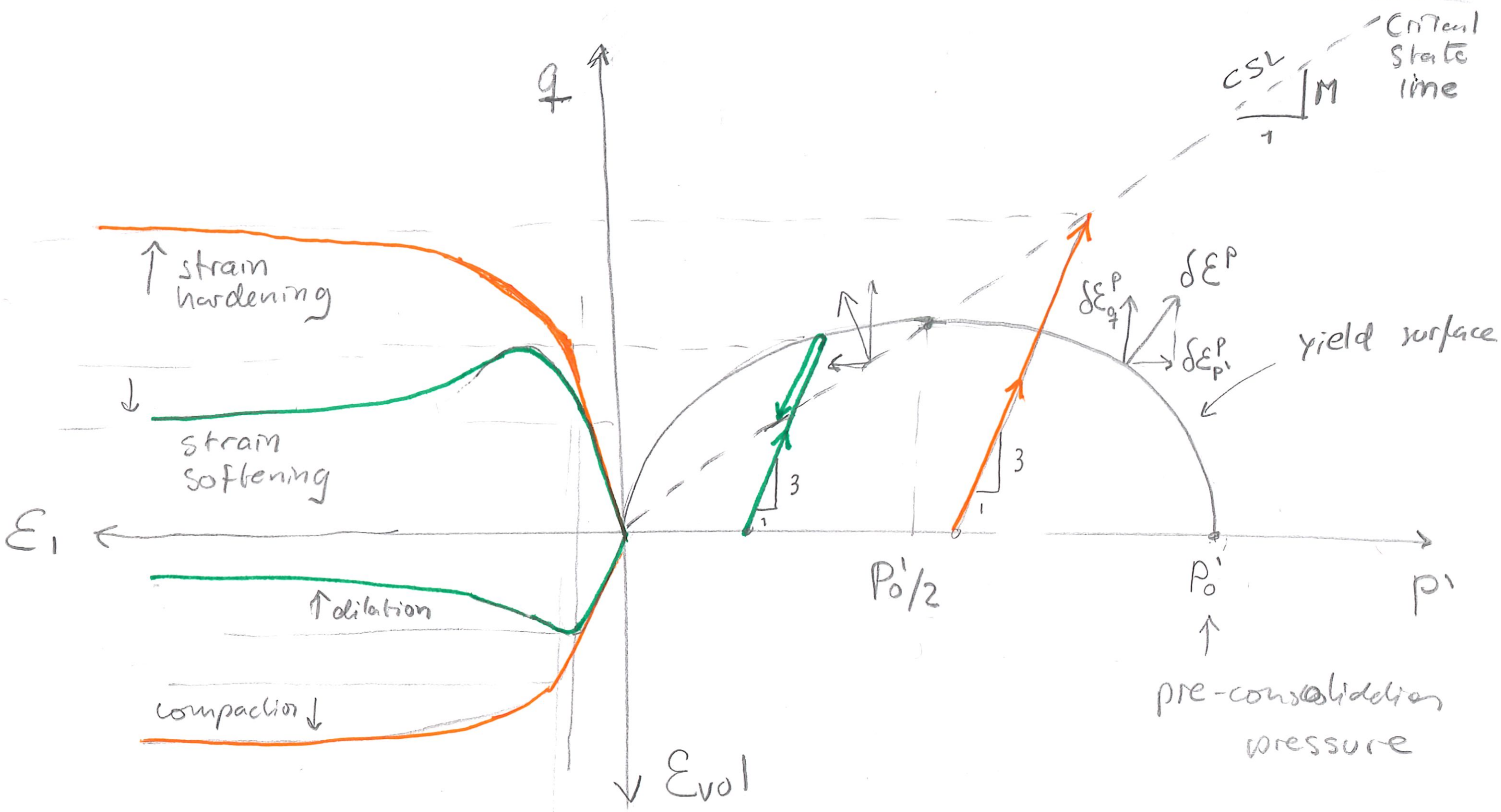


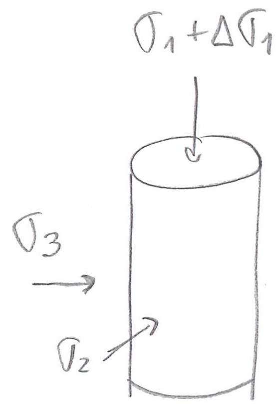
$$\psi = \begin{cases} \psi > 0 \Rightarrow \text{dilation} \\ \psi = 0 \Rightarrow \text{iso-choric} \\ \psi < 0 \Rightarrow \text{contraction} \end{cases}$$



Critical State Soil Mechanics

↳ Cam-Clay Model (uncemented sediments)





Triaxial Test with deviatoric compression –

$$p' = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

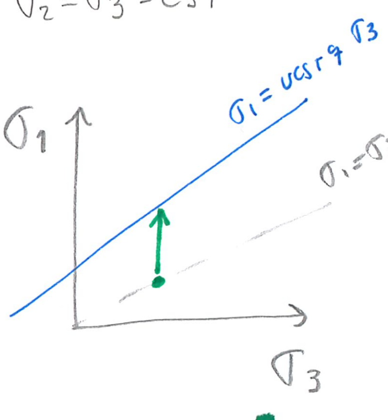
$\Delta\sigma_1$

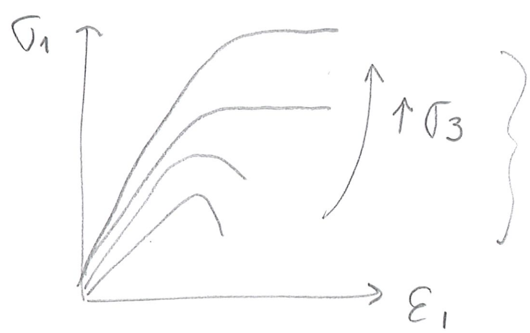
$$\Delta p' = \frac{\Delta\sigma_1}{3}$$

$$\Delta q = \Delta\sigma_1$$

$$\frac{\Delta q}{\Delta p'} = \frac{\Delta\sigma_1}{\Delta\sigma_1/3} = 3$$

$\sigma_2 = \sigma_3 = cst$





brittle to ductile transition

critical state line

$$q = M \cdot p' \quad (@\ CSL)$$

$\varphi_{cs} = 30^\circ \longrightarrow \sigma_1 = 3\sigma_3$

$$\hookrightarrow M = \frac{q}{p'} = \frac{\sigma_1 - \sigma_3}{\frac{\sigma_1 + 2\sigma_3}{3}} \Longrightarrow M = \frac{2\sigma_3}{\frac{5\sigma_3}{3}} = 1{,}2$$

In general

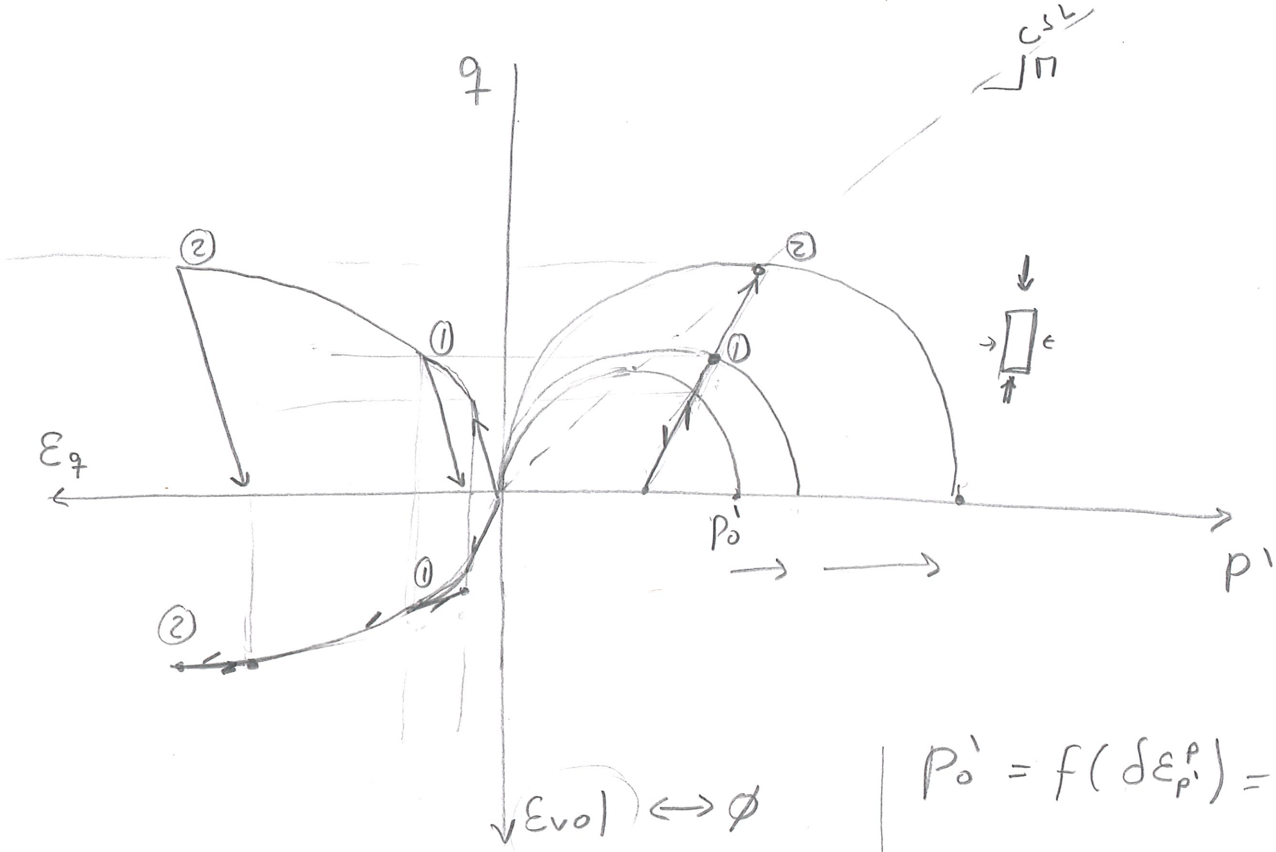
$$\boxed{M = \frac{3\left(\frac{1+\sin\varphi_{cs}}{1-\sin\varphi_{cs}} - 1\right)}{2 + \frac{1+\sin\varphi_{cs}}{1-\sin\varphi_{cs}}}}$$

$$f(q, p', p_0') = q^2 - M^2 p'(p_0' - p') = 0 \leftarrow \text{yield surface}$$

$\hookrightarrow q = 0 \Rightarrow p' = p_0'$

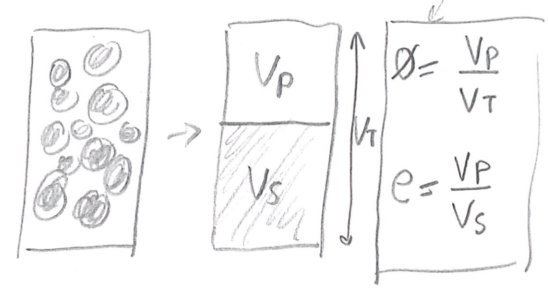
$\hookrightarrow q = M p' \Rightarrow p' = \frac{p_0'}{2}$

$\hookrightarrow$ variable $\longleftrightarrow$
- $\rightarrow$ pre-consolidation pressure
- $\rightarrow$ size of the yield surface
- $\hookrightarrow$ hardening parameter
  - $\llcorner\ f(\phi) \sim f(\delta\varepsilon^{p}_{p'})$



$$p_0' = f(\delta \varepsilon_{p'}^{p}) = f'\left(e = \frac{\phi}{1-\phi}\right)$$

void ratio

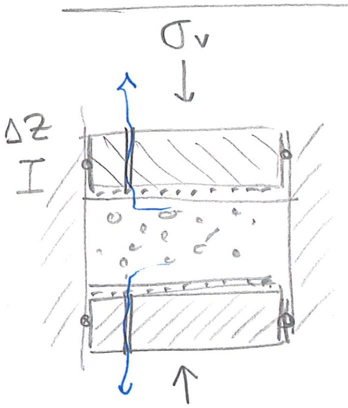


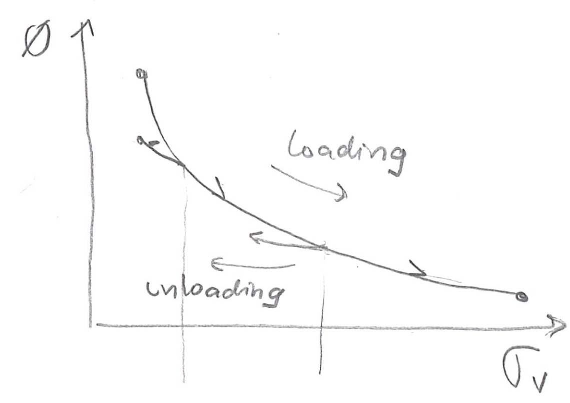
say $\underline{V_T = 1}$ $\quad e = \dfrac{\phi}{1-\phi}$

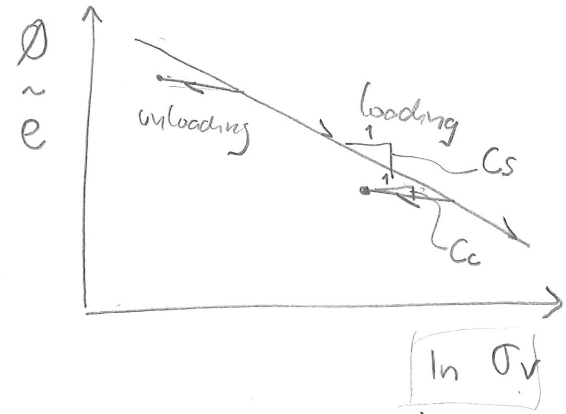
$\Delta \delta \varepsilon_{p'}^{p} > 0 \Rightarrow \downarrow \phi \Rightarrow$ stronger

$\Delta \delta \varepsilon_{p'}^{p} < 0 \Rightarrow \uparrow \phi \Rightarrow$ weaker

Compaction curve

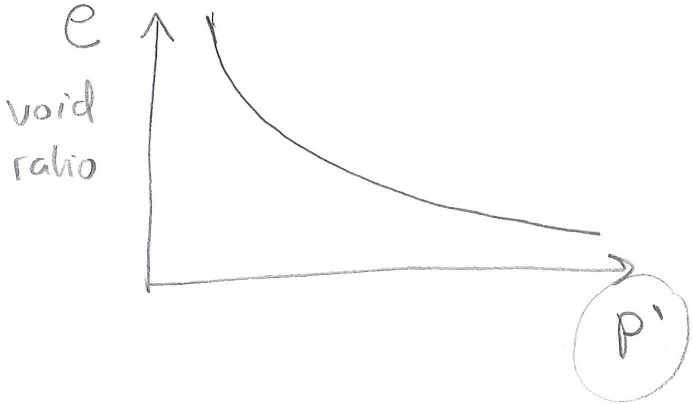


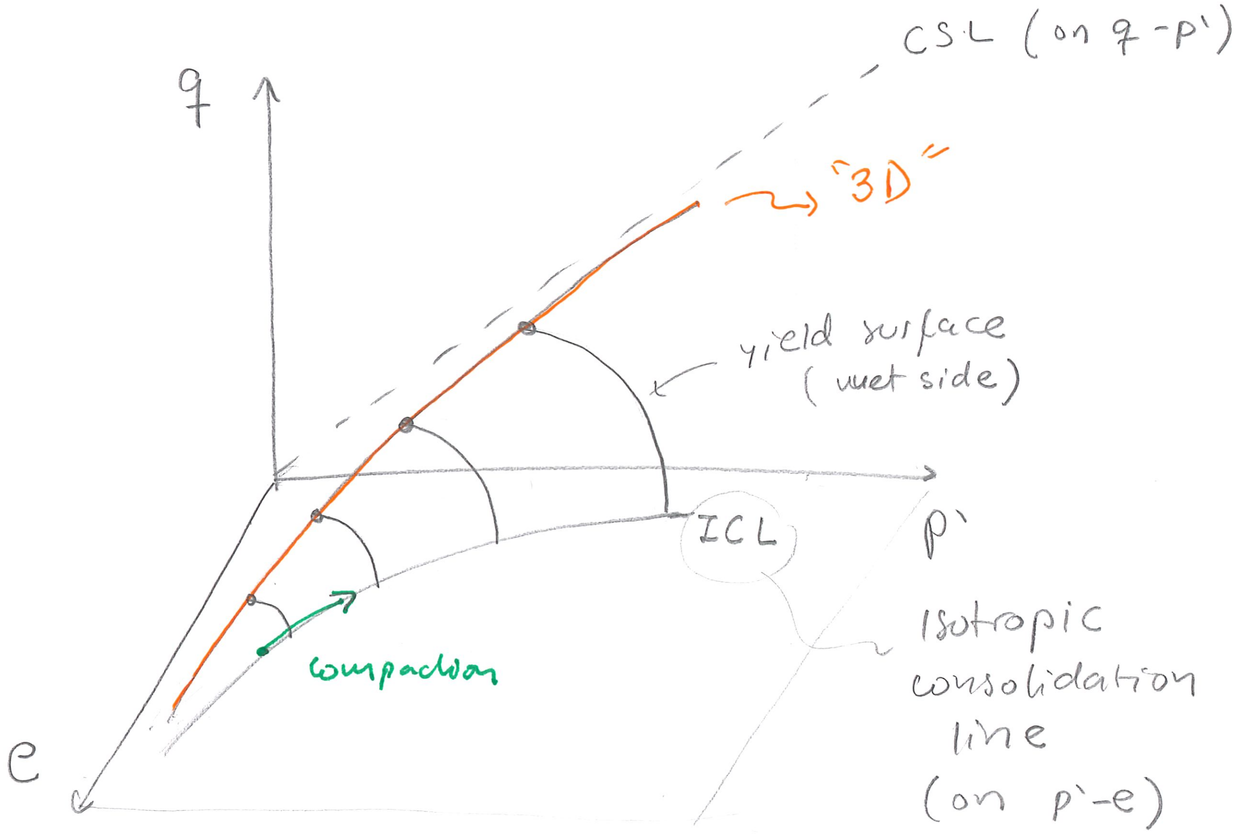


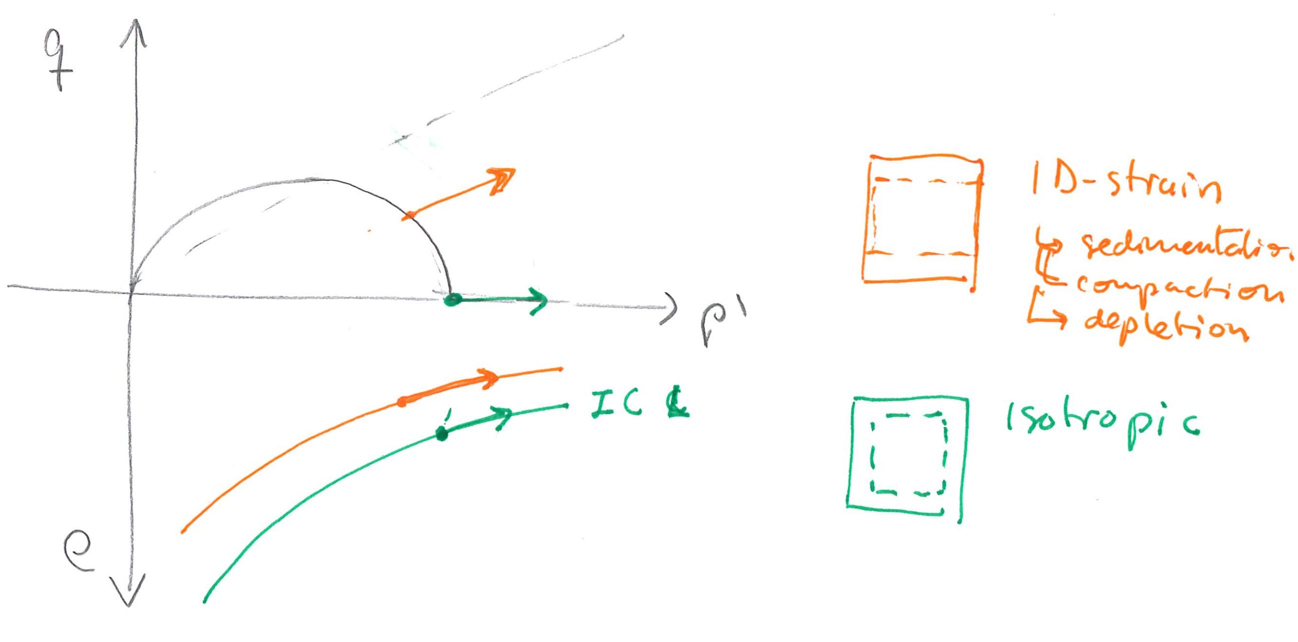


elastic $\left\{ e = e_0 - C_c \ln\left(\dfrac{\sigma_v}{1\,MPa}\right) \right.$

plastic $\left| e = e_0 - C_s \ln\left(\dfrac{\sigma_v}{1\,MPa}\right) \right.$


e
void ratio
p'


CSL (on q-p')
q
"3D"
yield surface (wet side)
p'
ICL
isotropic consolidation line (on p'-e)
compaction
e

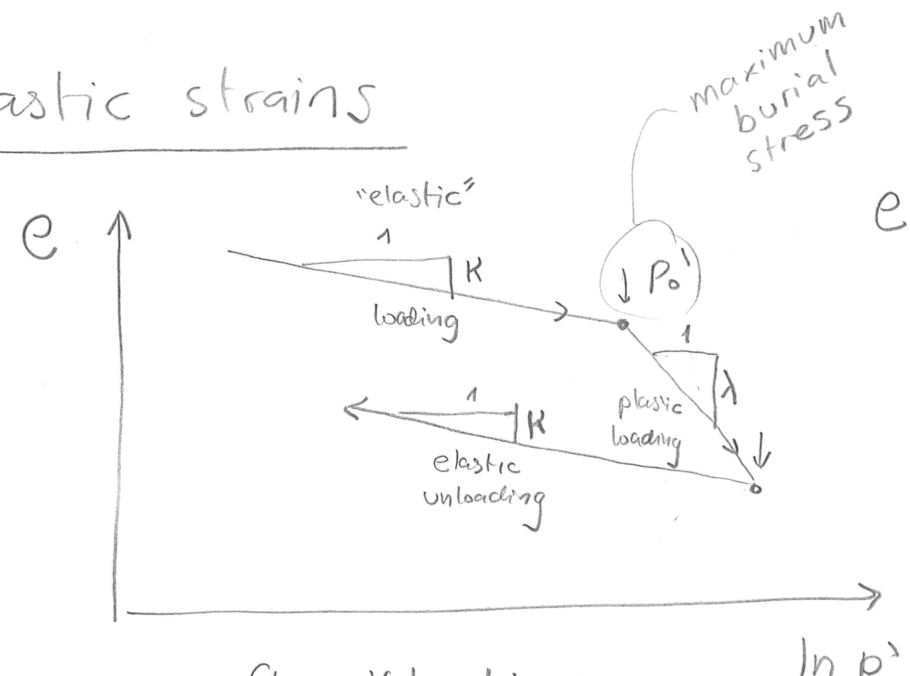

q
p'
ICL
e
1D-strain
sedimentation
compaction
depletion
Isotropic

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^p$$

$$d\underline{\underline{\varepsilon}} = d\underline{\underline{\varepsilon}}^e + d\underline{\underline{\varepsilon}}^p$$

$$d\underline{\underline{\varepsilon}} = \underbrace{\underline{\underline{\underline{\underline{C}}}}^e \cdot d\underline{\underline{\sigma}}}_{\text{elastic strains}} + \underbrace{\underline{\underline{\underline{\underline{C}}}}^p \, d\underline{\underline{\sigma}}}_{\text{plastic strains}}$$

1 Elastic strains



$e = c - K \ln p'$

$$\frac{de}{dp'} = -\frac{K}{p'} \quad (1)$$

$$e = \frac{V_v}{V_b - V_v} = \ldots$$

$de = \ldots dV_v + \ldots dV_b$

assume

$\underline{dV_v = dV_b}$ $(\alpha \simeq 1)$

Change Volumetric strain

$$(2)\ \underline{de} = -(1+e)\, \underline{d\varepsilon_{p'}} \leftarrow \boxed{\delta\varepsilon_{p'} = \frac{dV_b}{V_b} = \frac{dV_v}{V_b}}$$

(1) + (2)

$$\begin{cases} d\varepsilon_{p'}^e = \dfrac{K}{1+e} \cdot \dfrac{dp'}{p'} \\[2ex] d\varepsilon_q^e = \dfrac{1}{3G} \cdot dq \end{cases}$$

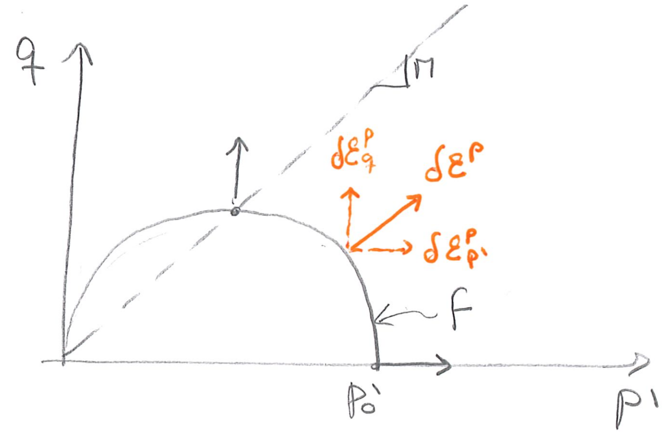
; $1+e = v$: specific volume

; $\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$

$$\underbrace{\begin{bmatrix} d\varepsilon_{p'}^e \\ d\varepsilon_q^e \end{bmatrix}}_{\text{strains}} = \begin{bmatrix} K/[(1+e)p'] & 0 \\ 0 & 1/3G \end{bmatrix} \overbrace{\begin{bmatrix} dp' \\ dq \end{bmatrix}}^{\text{stress}}$$

Elastic (before yield)

(2) Plastic strains



$$f = q^2 - M^2 p'(p_o' - p')$$

$$f^* = \frac{p'}{p_o'} - \frac{M^2}{\eta^2 + M^2}\ ; \quad \eta = \frac{q}{p'}$$

derivatives
$$\begin{cases} \dfrac{df^*}{dp'} = p_o' M^2 \left(\dfrac{M^2 - \eta^2}{M^2 + \eta^2} \right) \\[2ex] \dfrac{df^*}{dq} = p_o' M^2 \left(\dfrac{2\eta}{M^2 + \eta^2} \right) \\[2ex] \dfrac{df^*}{p_o'} = - \dfrac{p'}{(p_o')^2} \end{cases}$$

(3)

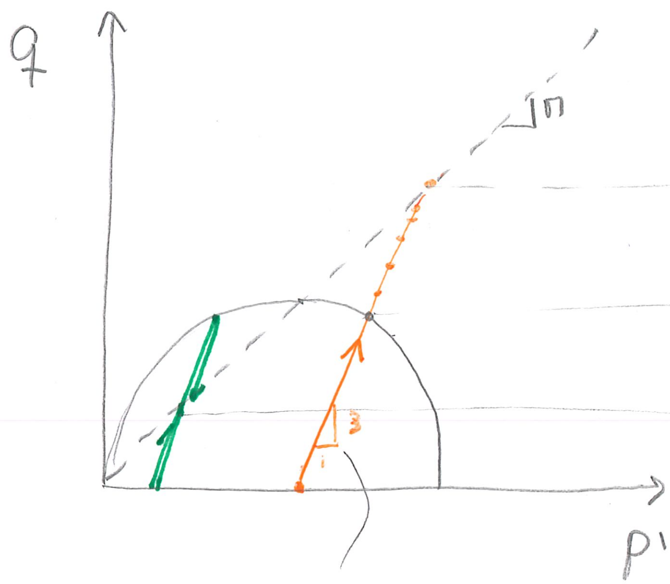
Associated flow rule

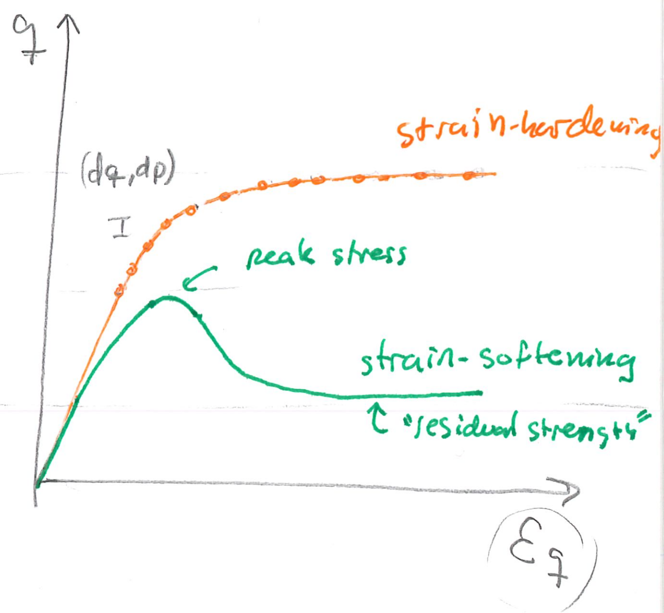
$$\delta\varepsilon^p_{ij} = d\chi \cdot \frac{df}{d\sigma_{ij}} \Rightarrow \begin{cases} d\varepsilon^p_{p'} = d\chi \dfrac{df^*}{dp'} \\[2ex] d\varepsilon^p_q = d\chi \cdot \dfrac{df^*}{dq} \end{cases}$$

from ICL (4) $\quad \delta\varepsilon^p_{p'} = \dfrac{\lambda - \kappa}{1+e} \cdot \dfrac{dp_o'}{p_o'}$

(3)+(4) $\quad d\chi = \dfrac{\lambda - \kappa}{(1+e)\, p' (M^2 + \eta^2)} \quad \leftarrow$ hardening parameter

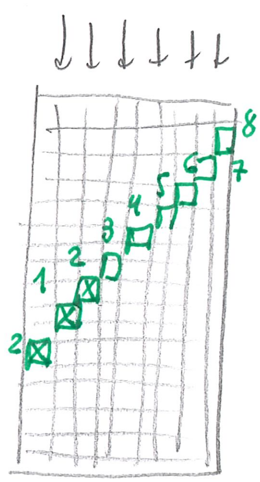
$$\begin{bmatrix} \delta\varepsilon^p_{p'} \\ \delta\varepsilon^p_q \end{bmatrix} = \frac{\lambda - \kappa}{v \cdot p' \cdot (M^2 + \eta^2)} \begin{bmatrix} M^2 - \eta^2 & 2\eta \\ 2\eta & \dfrac{4\eta^2}{M^2 - \eta^2} \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix} \Bigg\} \text{ Plastic strains}$$



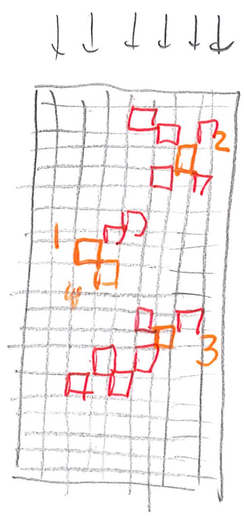


$dq = 3dp'$ → axisymmetric triaxial stress path

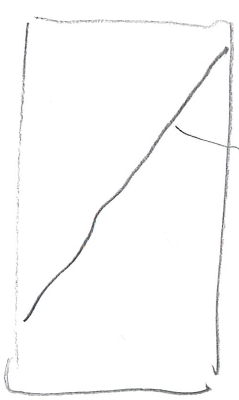
↓ fixed by you



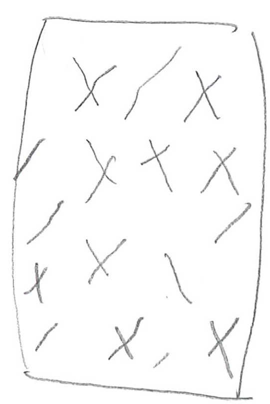
- positive feedback mechanism
- unstable process



- shear and compaction bands
- stable process

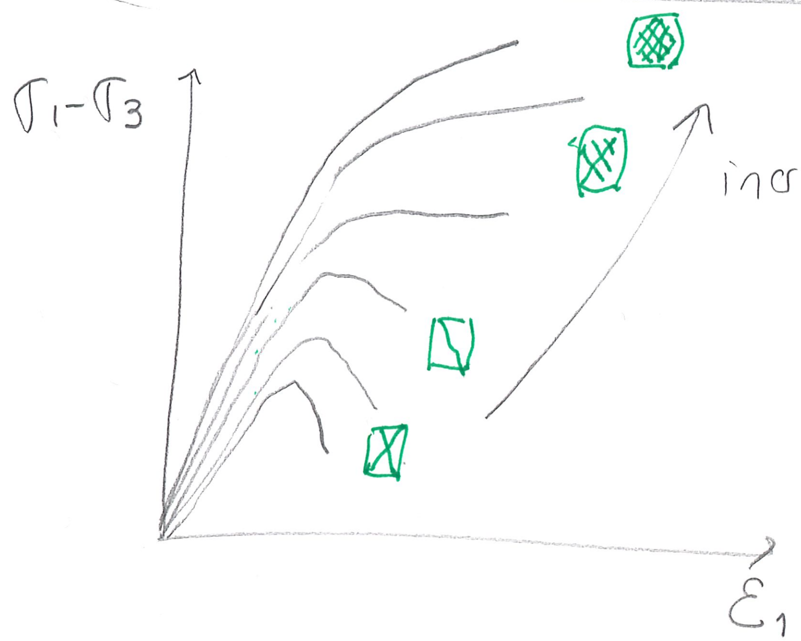


strain localization



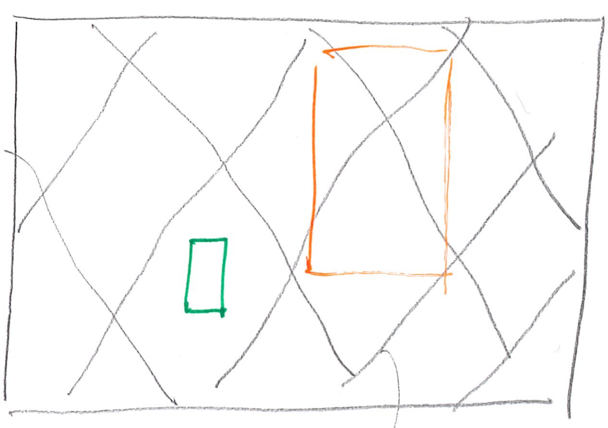
- "diffuse" deformation

Brittle to ductile transition

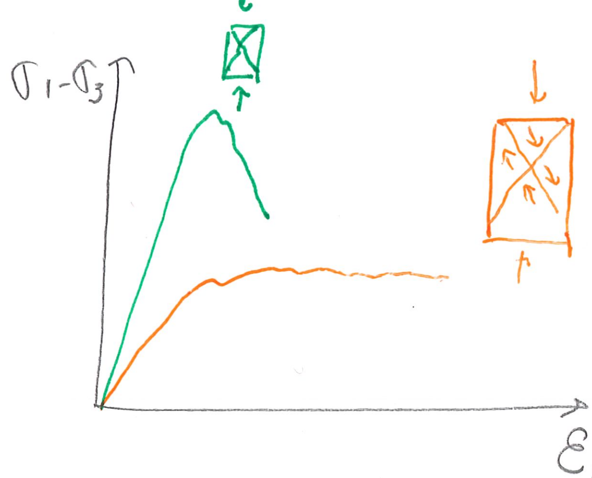


increase

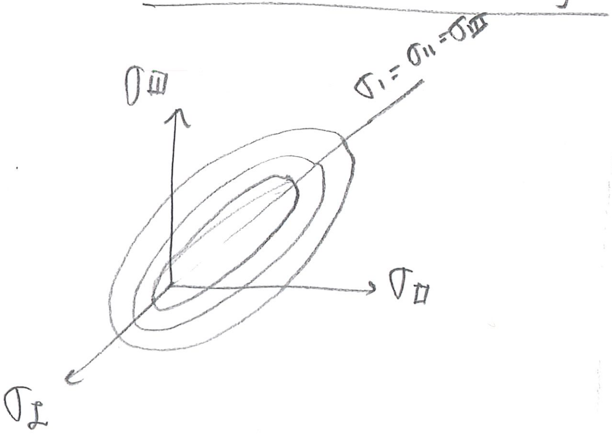
- $\uparrow \sigma_3$ or σ_m or p'
- $\uparrow$ Temperature
- $\downarrow$ loading rate
- shales $\rightarrow$ $\uparrow$ TOC, $\uparrow$ % clay



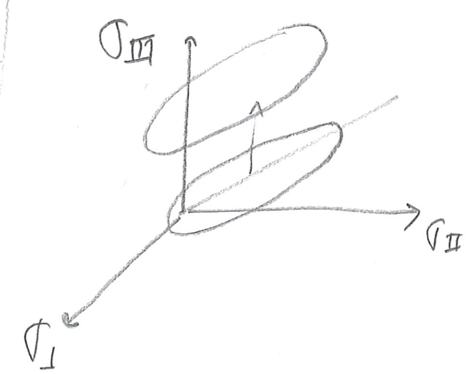
uncemented fractures



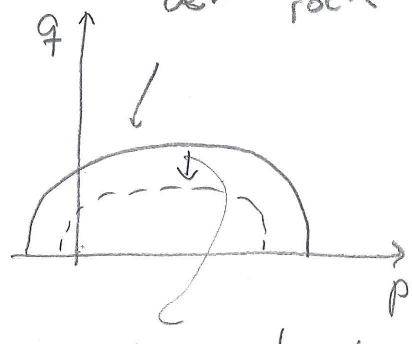
Isotropic hardening ✓



Kinematic hardening

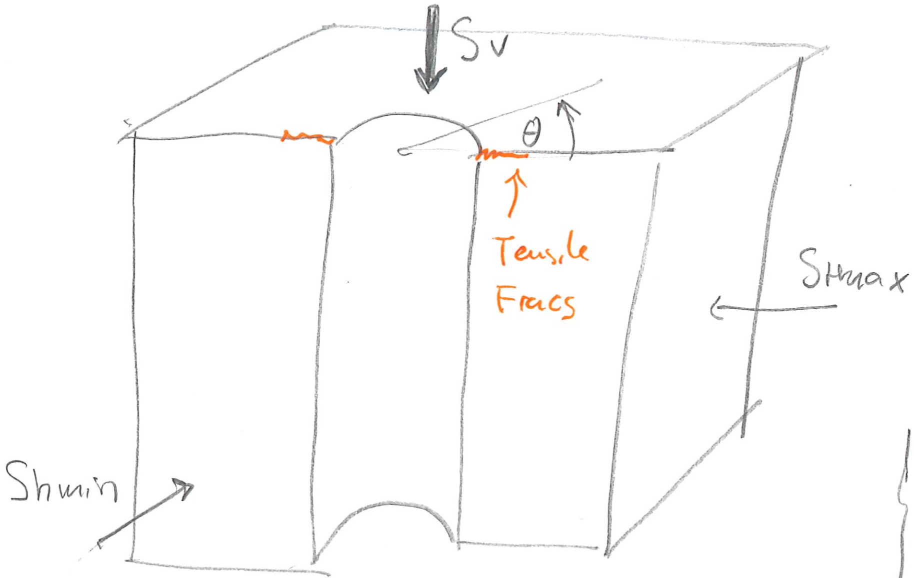


cemented rock



chemical actions
$\downarrow$
"chemo-plasticity"

Hydraulic Fracturing

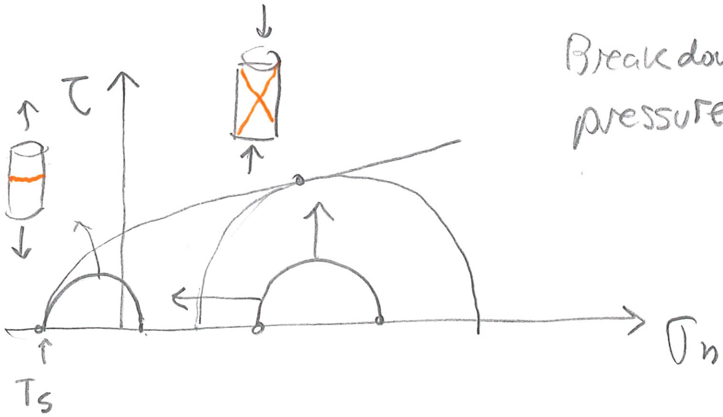


$$S_v > S_{Hmax} > S_{hmin}$$

$$\sigma_{\theta\theta}(r=a,\theta) = -(P_w - P_p) + (\sigma_{Hmax} - \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(2\theta)$$

$$\theta = 0, \pi$$

$$\sigma_{\theta\theta} = -T_s$$



Breakdown pressure

$$\boxed{P_w = P_b = \underbrace{P_p}_{\text{Pore pressure}} \underbrace{-\sigma_{Hmax} + 3\sigma_{hmin}}_{\text{Stress Anisotropy}} + \underbrace{T_s}_{\text{Tensile Strength}}}$$

RF ⇒ $\sigma_{Hmax} = 9 \cdot \sigma_v$

$\sigma_1 = 9 \quad \sigma_3$

1) $\sigma_{Hmax} \approx \sigma_{hmin} = \sigma_h$ (isotropic)

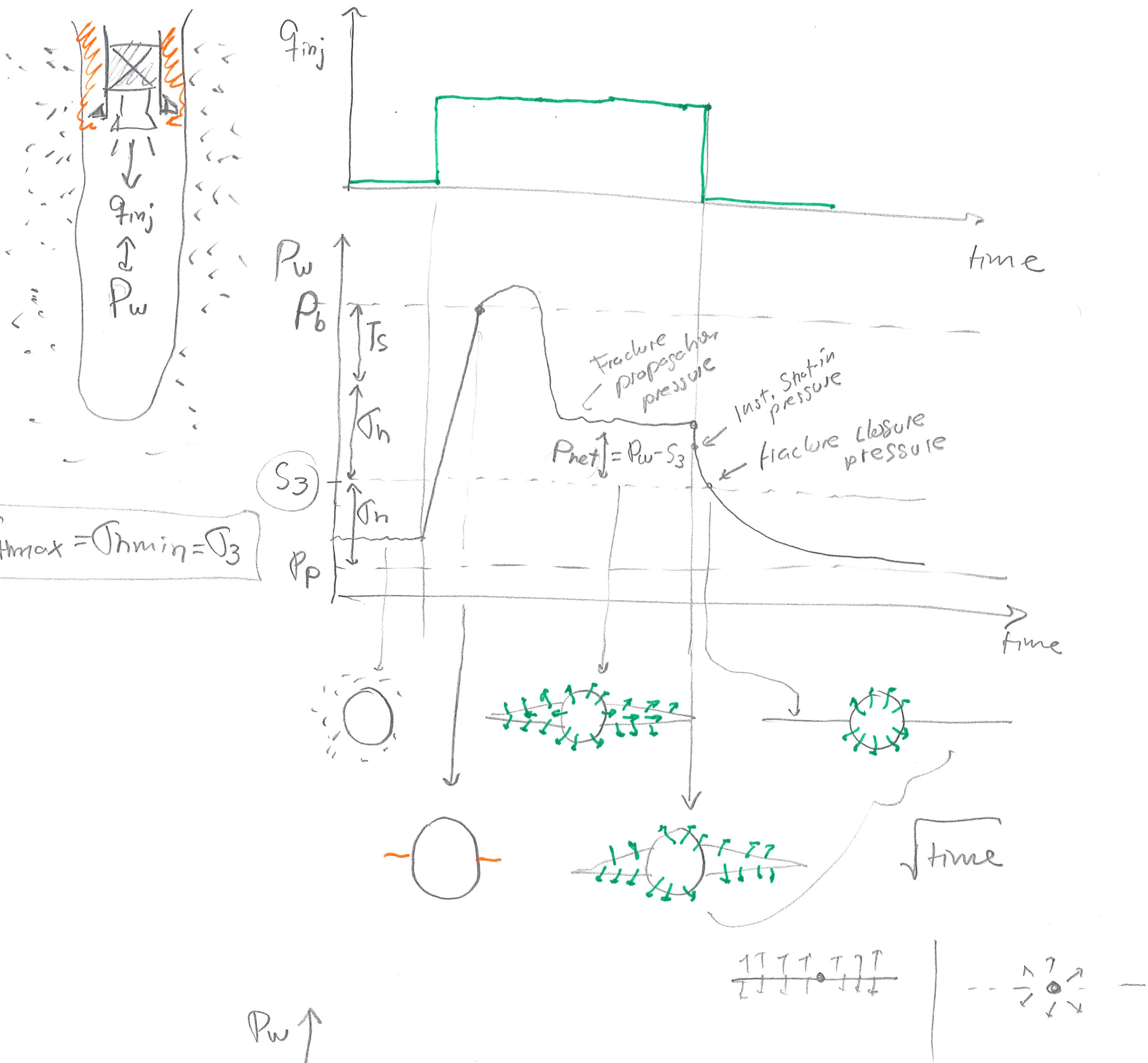
$$\boxed{P_b = P_p + 2\sigma_h + T_s}$$

2) $\sigma_{Hmax} = 3 \cdot \sigma_{hmin}$ (highly anisotropic)

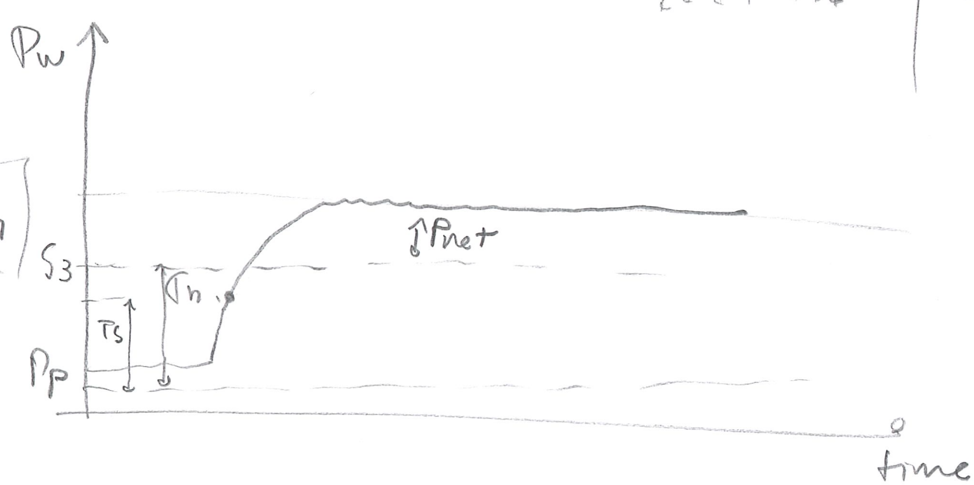
$$P_b = P_p + \cancel{3\sigma_{hmin} - 3\sigma_{hmin}} + T_s$$

DRILLING — COMPLETION - DFIT

Leak-off test (Diagnostic Fracture Initiation Test)



$$\sigma_{Hmax} = \sigma_{hmin} = \sigma_3$$



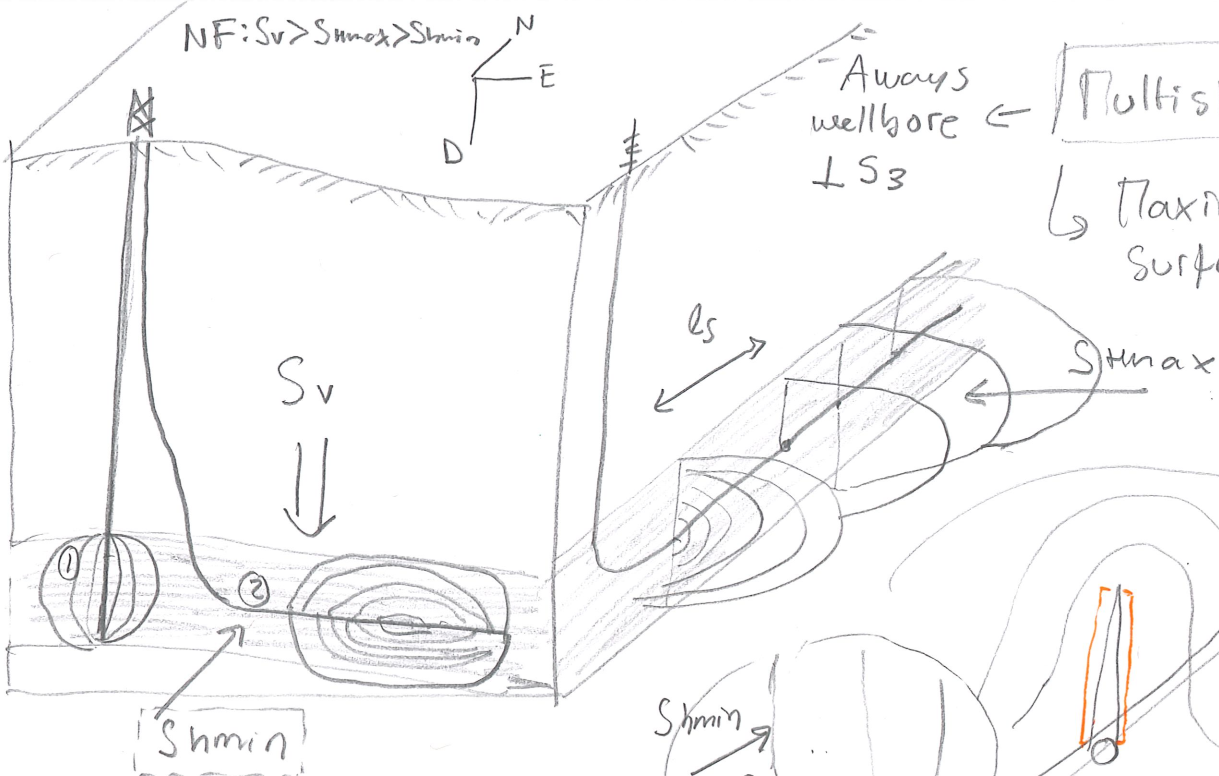
$$\sigma_{Hmax} = 3\,\sigma_{hmin}$$

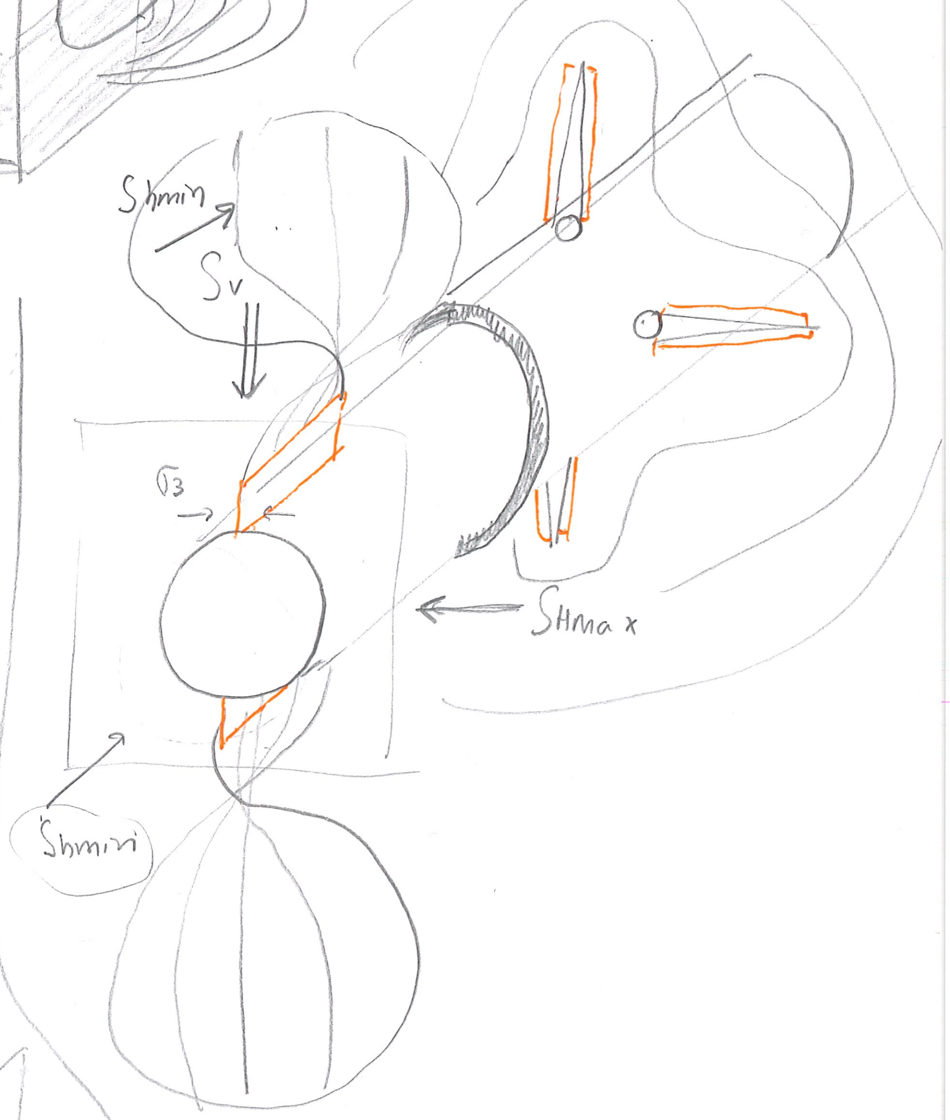
NF: $S_v > S_{Hmax} > S_{hmin}$

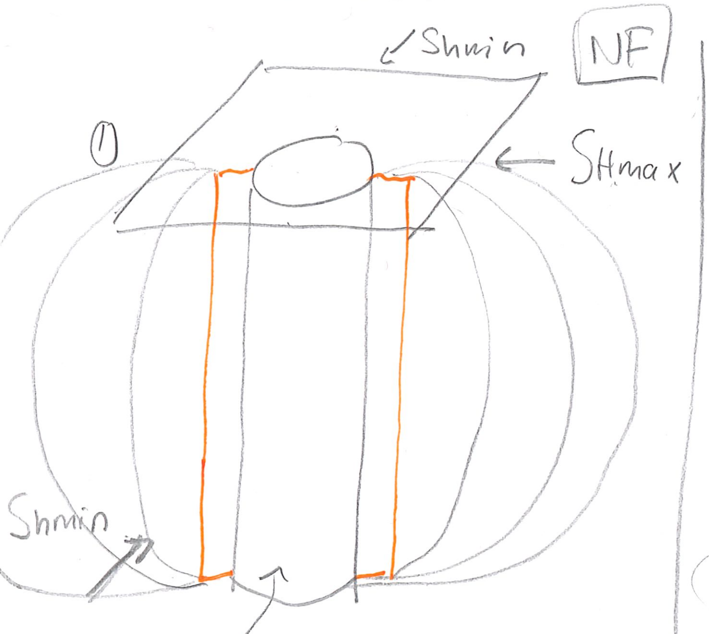
Multistage HF

Always wellbore $\perp S_3$ ← Multistage HF

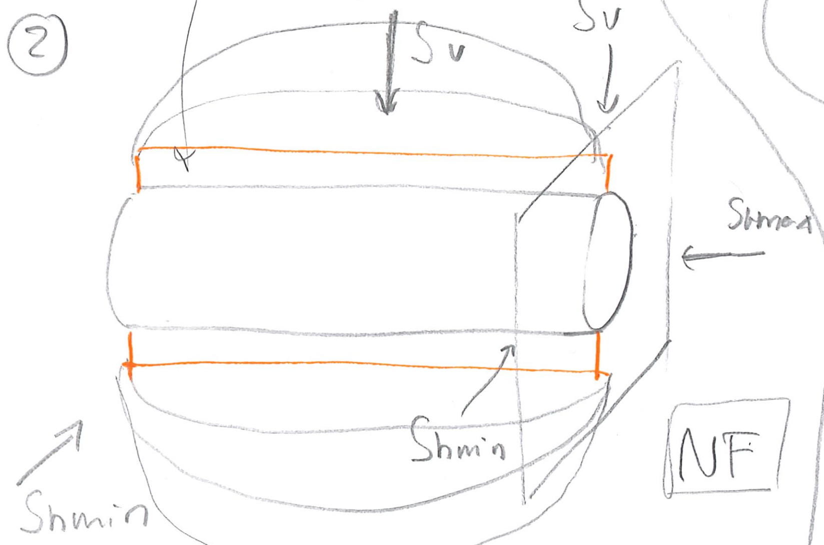
↳ Maximize surface area







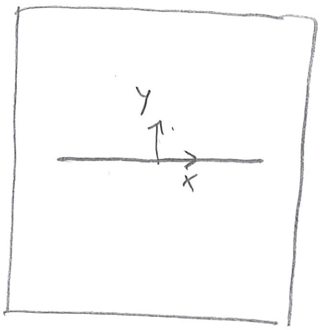
open-hole completion



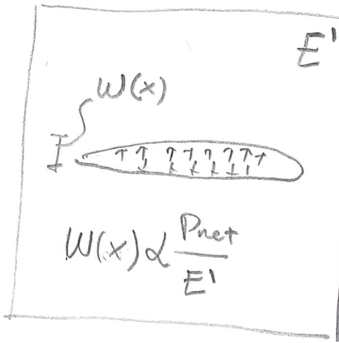
Coupled fluid-driven fracture problem

(Valko and Economides, Hydraulic Fracture Mechanics)

$$P_{net} = P_f - S_3 = \underbrace{P(\text{elasticity})}_{} + P(\text{viscous losses}) + P\left(\begin{matrix}\text{new}\\ \text{rock}\\ \text{surface}\end{matrix}\right)$$



$\Rightarrow$



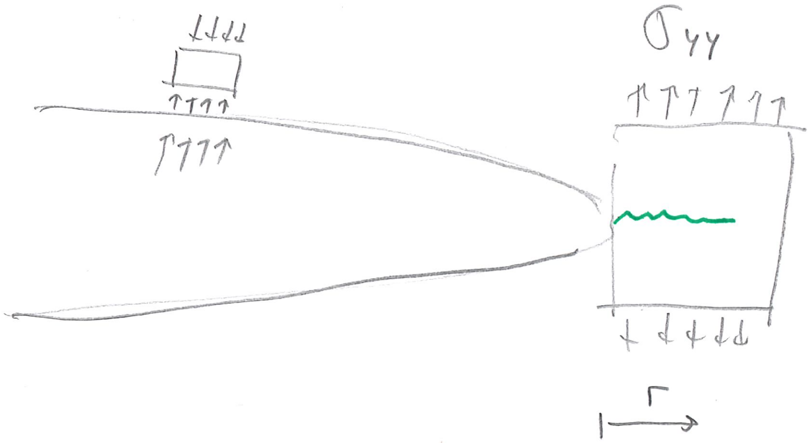
$$W(x) \propto \frac{P_{net}}{E'}$$

$w(x) \longleftrightarrow$ $\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\ \underline{\underline{\varepsilon}}$, $\nabla \cdot \underline{\underline{\sigma}} = 0$, kinem.

leak-off $\propto \dfrac{k_{rock}}{\mu_{fluid}}$

$$q(x) \propto \frac{[w(x)]^3 \cdot h_f}{\mu} \frac{\Delta P(x)}{\Delta x}$$

flow rate in the fracture



$\boxed{\text{LEFM}}$

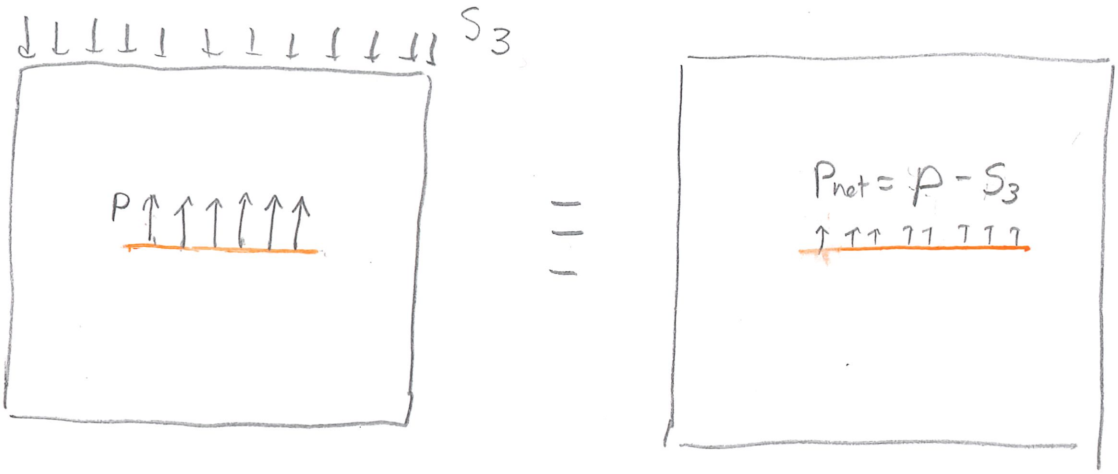
- $K_I > K_{IC}$ → frac propagates
- $K_I < K_{IC}$ → frac does not propagate

K_I: stress intensity, $f(l_f, P_n)$

K_{IC}: toughness ↳ property rock

$$\boxed{K_I = \lim_{r \to 0^+} \left[\sqrt{2\pi r} \cdot \sigma_{yy}(c+r, 0)\right]}$$

Griffith problem

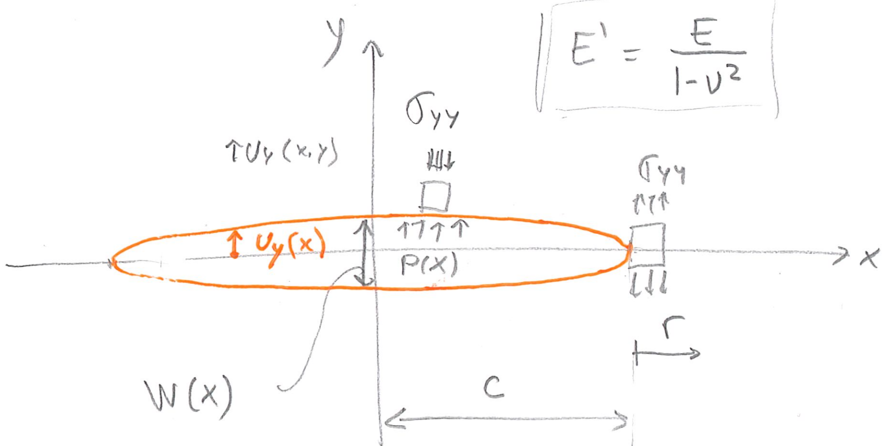


near lastic omogeneous sotropic
LEHI Solid

plane-strain modulus

$$\boxed{E' = \frac{E}{1-\nu^2}}$$

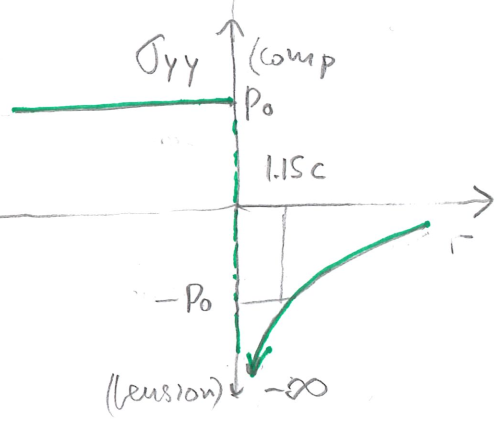
$p(x) = p_0$ (constant)



$$U_y(x,0) = \begin{cases} \frac{2p_0}{E'}\sqrt{c^2 - x^2} & ; 0 \leq x \leq c \\ 0 & ; x > c \end{cases}$$

$$W_0 = \frac{4 p_0 c}{E'}$$

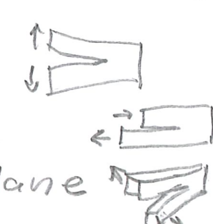
$$\sigma_{yy} = \begin{cases} p_0 & ; 0 \leq x \leq c \\ -p_0\left[\frac{x}{\sqrt{x^2 - c^2}} - 1\right] & ; x \geq c \end{cases}$$



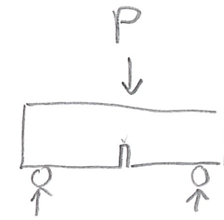
Stress intensity factor

$$K_{I} = \lim_{r \to 0^+} \left[\sqrt{2\pi r} \cdot \sigma_{yy}(c+r, 0)\right] = \underbrace{p_0\sqrt{\pi c}}_{\text{constant pressure solution}}$$

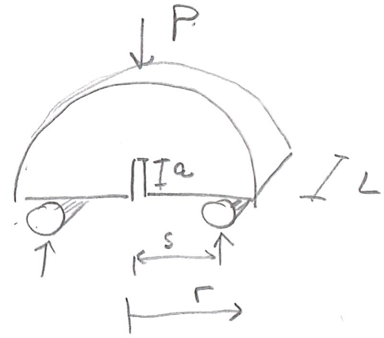
→ Mode
- I - Open mode
- II - shear in-plane
- III - shear out-of-plane



K_{Ic} ← critical stress intensity factor
← property of the rock
← measured in the lab
↳ 3-point loading method



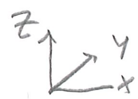
↳ semicircular bending

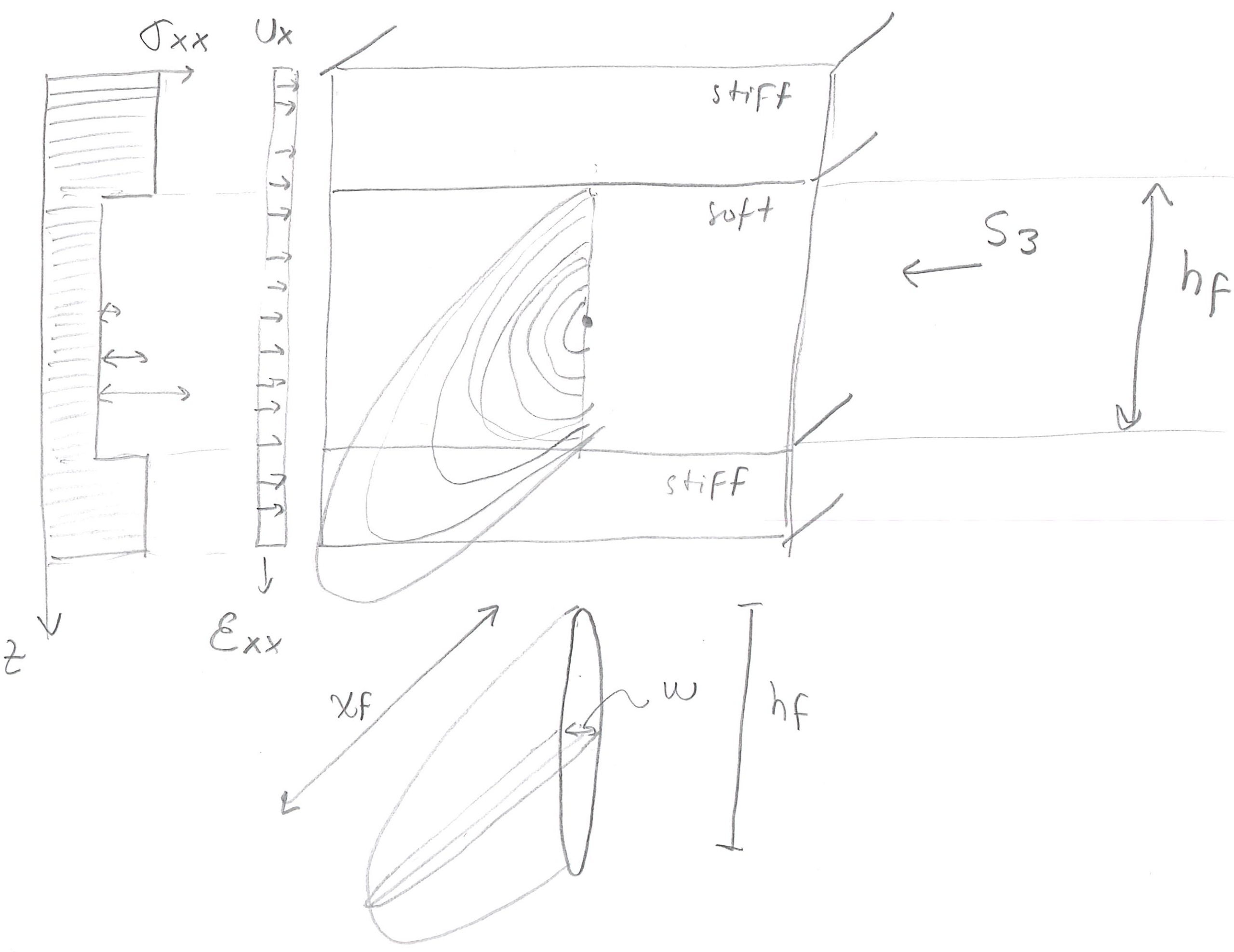


$$K_{Ic} = \frac{P_{max} (\pi a)^{1/2}}{2 r L} Y_I$$

$Y_I(a, r, s) \sim 5$

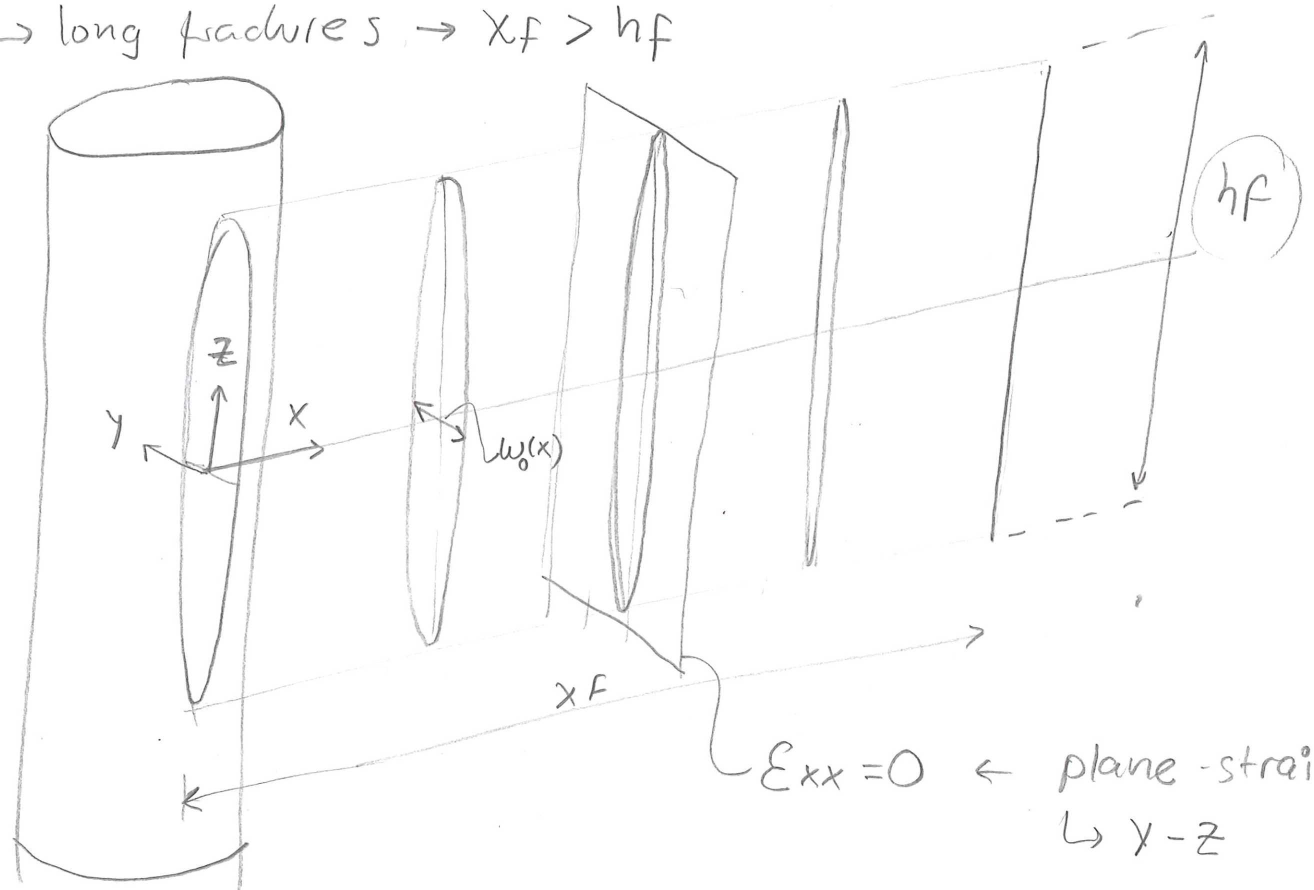
Fracture height





PKN Model (Perkins, Kern, Nordgren)

↳ long fractures → $x_f > h_f$



Linear elasticity

$$w_0(x) = \frac{4\, P_n(x)\,(h_f/2)}{E'} = \frac{2\, P_n(x)\, h_f}{E'} \quad (1)$$

Fluid Mechanics

$$\frac{dP}{dx} = -\frac{64\, N\, q(x)}{\pi\, [w_0(x)]^3\, h_f} \quad (2)$$

elliptical shape
Newtonian fluid
laminar flow

Material Balance

$$V_i = \underbrace{V}_{\text{frac}} + \underbrace{V_L}_{\text{leak-off}} \quad (3)$$

$V_L = 2 \cdot A_f \cdot C_L \cdot \sqrt{t}$

$$\boxed{V_i = i \cdot t} \quad (4)$$

↳ constant injection rate
↳ accounts one-wing

(1),(2), $q(x) = i \Rightarrow \quad \dfrac{dP}{dX} = - \dfrac{64 \mu i}{\pi \left[2 h_f P_n(x) / E' \right]^3 h_f}$

$$\int_{P_n(x=0)}^{P_n(x_f)} [P_n(x)]^3 \, dp = - \int_{x=0}^{x_f} \frac{8 \mu i E'^3}{\pi h_f^4} \, dx$$

$$\cancel{\frac{[P_n(x_f)]^4}{4}} - \frac{[P_n(x=0)]^4}{4} = - \frac{8 \mu i E'}{\pi h_f^4} x_f$$

net pressure at the tip = 0

$$\hookrightarrow P_n(x=0) = \left(\frac{3 \mu i E' x_f}{\pi h_f} \right)^{1/4} \qquad (5)$$

$$\hookrightarrow W_o(x=0) = \left(\frac{512 \mu i x_f}{\pi E'} \right)^{1/4} \qquad (5)+(1)$$

Geometry $\quad V_{frac} = x_f h_f \bar{w} \sim \bar{w} = \frac{\pi}{5} W_o(x=0)$ (6)

Mass balance $\quad V_{frac} = i \cdot t$ (7)

(6)+(7) $\quad x_f = \dfrac{i t}{h_f \left(\frac{\pi}{5} W_o(x=0) \right)}$

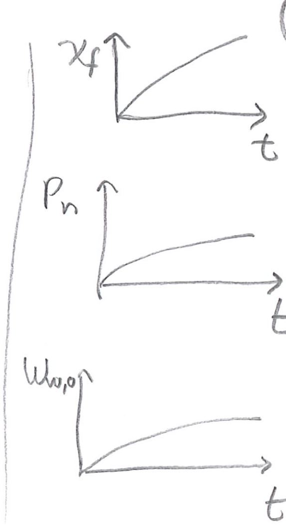
PKN model

- no leak-off
- constant inj. rate

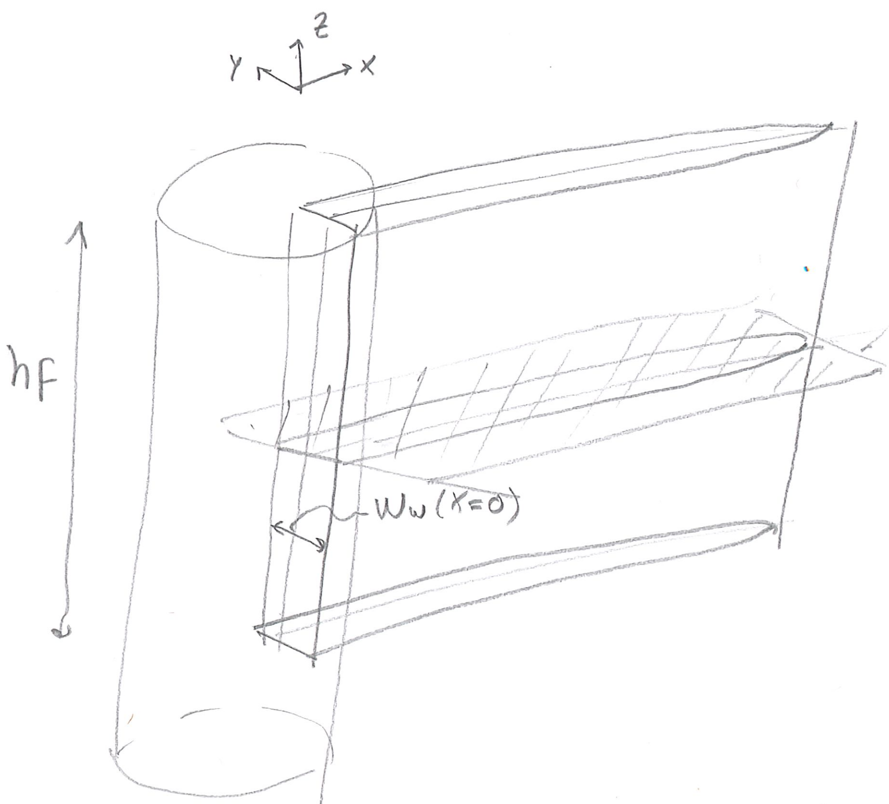
$$X_f = 0.524 \left(\frac{i^3 E'}{\mu h_f^4}\right)^{1/5} t^{4/5}$$

$$P_n(x=0) = 1.520 \left(\frac{E'^4 \mu i^2}{h_f^6}\right) t^{1/5}$$

$$W_0(x=0) = 3.040 \left(\frac{i^2 \mu}{E' h_f}\right) t^{1/5}$$



KGD → short fractures $x_f < h_f$



plane-strain
y-x

$x_f = \qquad t^{2/3}$

$W_w = \qquad t^{1/3}$

$P_{n,w} = \qquad t^{-1/3}$

Radial → penny shape fracture

X_f

w_w

P_n