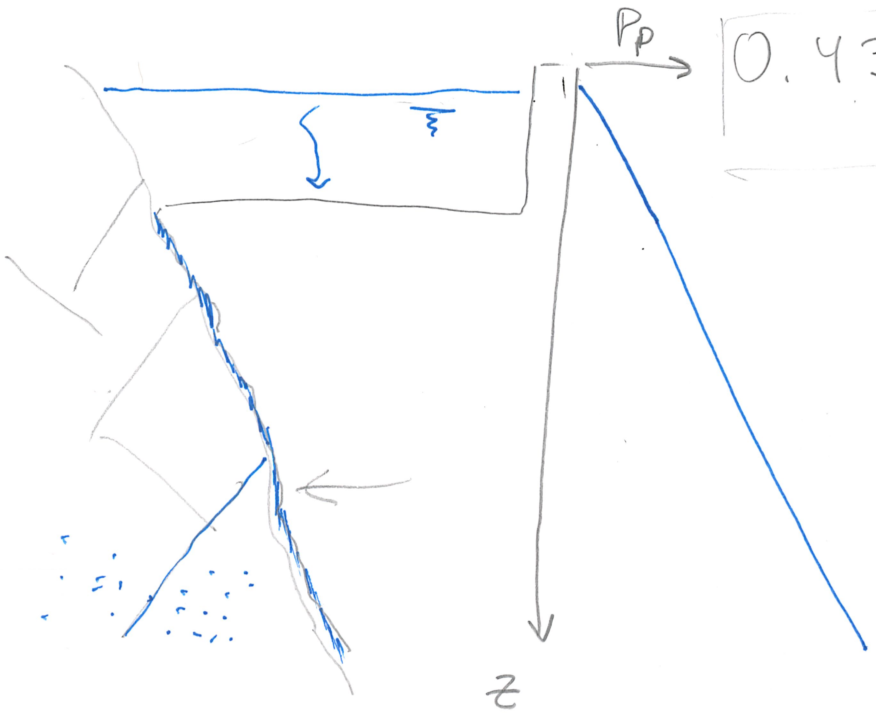
Pore pressure

9/4/2019

hydrostatic - water



$\boxed{0.43 \ \frac{psi}{ft}}$

$P_p = \rho_w g \cdot z$

$$\rho_w g = 1000 \frac{kg}{m^3} \cdot 9.8 \frac{m}{s^2}$$

$$\sim 10{,}000 \frac{N}{m^2} \cdot \frac{1}{m}$$

Pa

$$\boxed{\rho_w g \sim 10 \ \frac{MPa}{Km}}$$

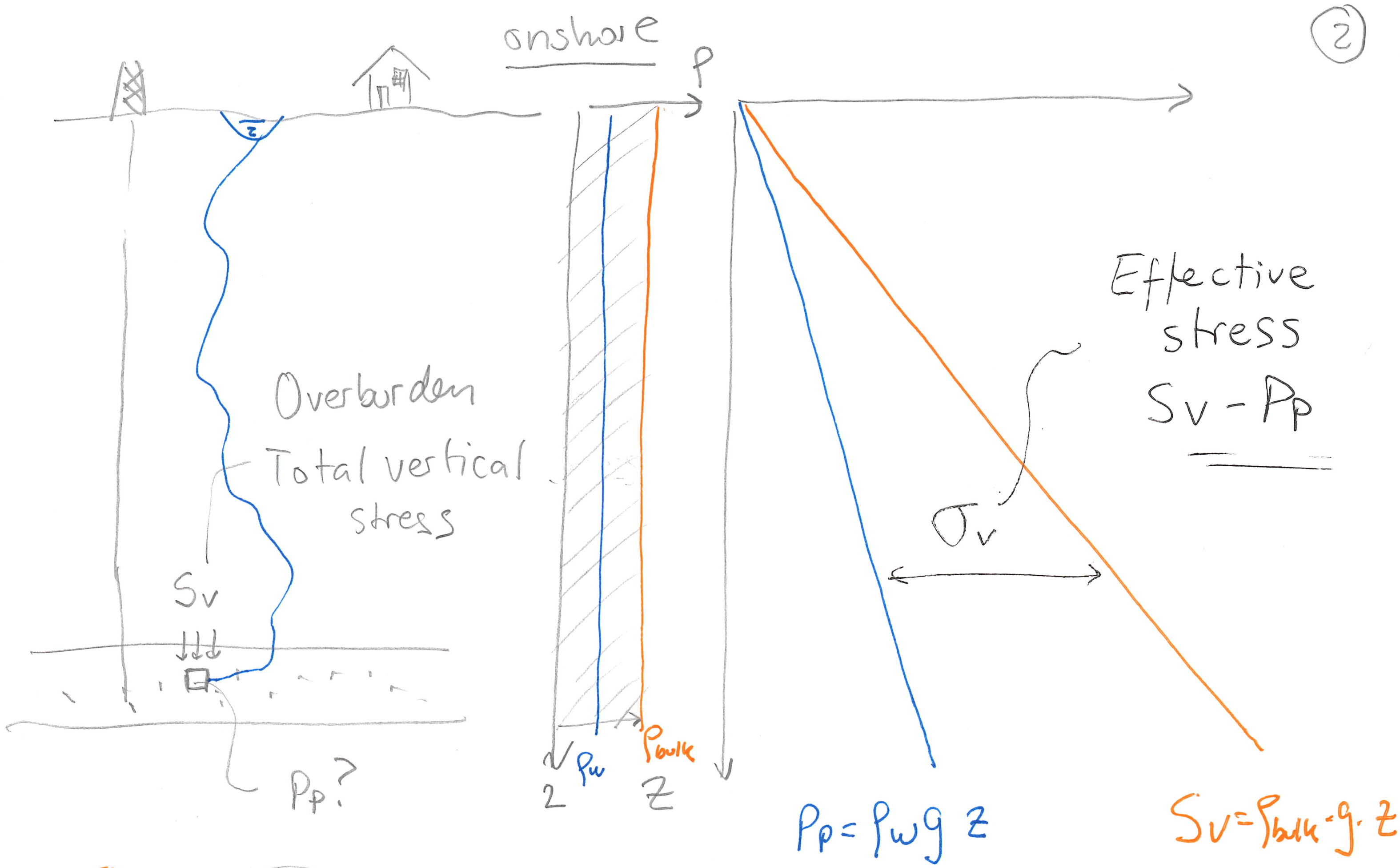
$$\rho_w = 62.4 \frac{lb}{ft^3}$$

$$\rho_w g = 62.4 \frac{\vec{lb}}{ft^3}$$

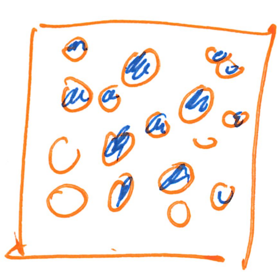
$$= 62.4 \frac{\vec{lb}}{(12in)^2} \frac{1}{ft}$$

$$= 0.433 \frac{lb}{in^2} \frac{1}{ft}$$

$$\boxed{\rho_w g = 0.433 \ psi/ft}$$



$$S_v = \rho_{bulk}\, g\, z$$



$$\rho_{bulk} = \underbrace{\rho_{min}(1-\phi)} + \underbrace{\rho_w\, \phi}$$

V_{quartz}	S_w
$V_{dolomite}$	S_o
$V_{calcite}$	S_g
V_{OM}	S_{CO_2}

vertical stress gradient

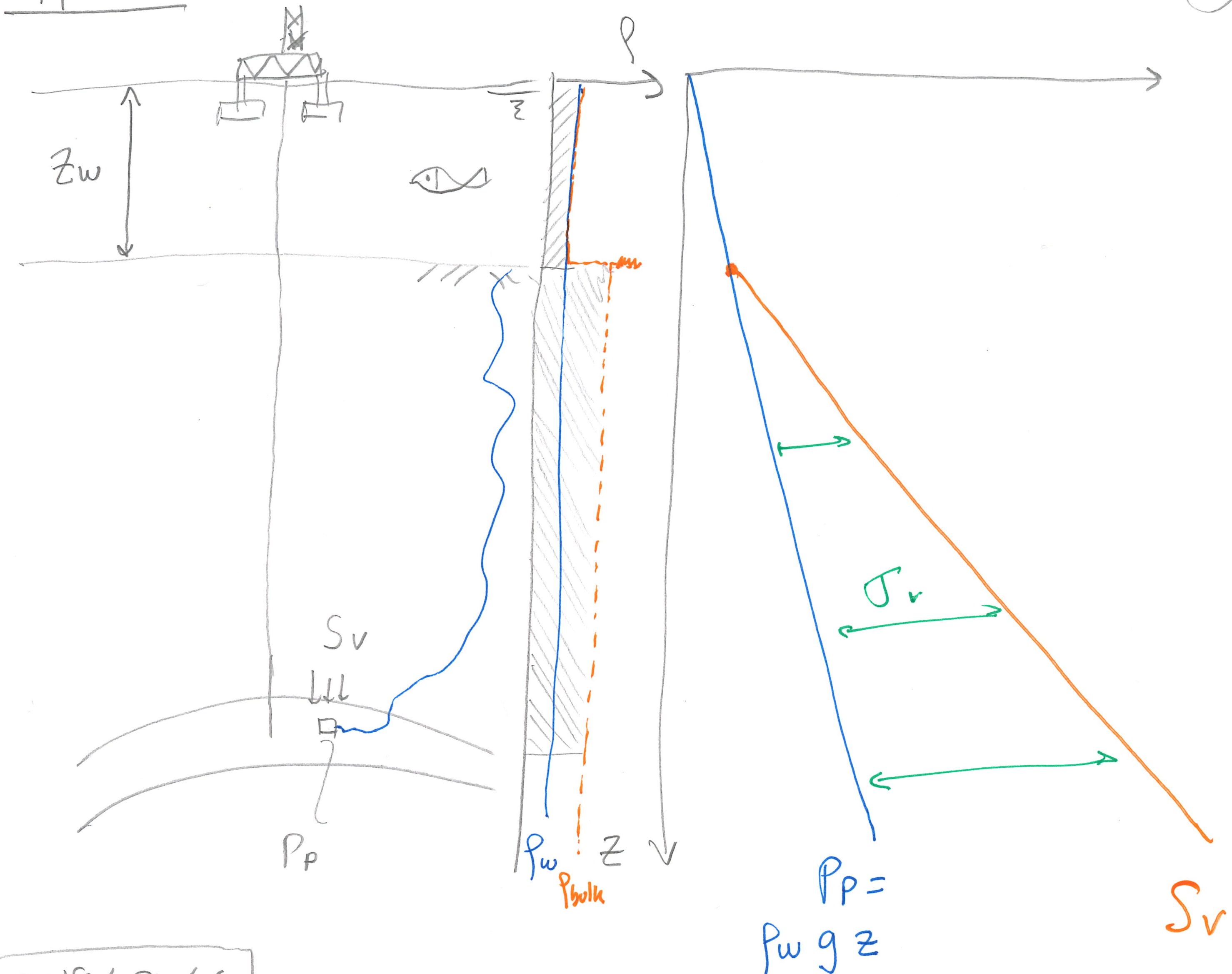
$\phi = 0.20$

$\rho_{min} = \rho_{SiO_2} = 2650 \dfrac{kg}{m^3}$

$\rho_{fluid} = \rho_w = 1000\ kg/m^3$

$$\rho_{bulk}\, g = 2320 \frac{kg}{m^3} \cdot 9.8 \frac{m}{s^2} \simeq \boxed{23 \frac{MPa}{km} \simeq 1 \frac{psi}{ft}}$$

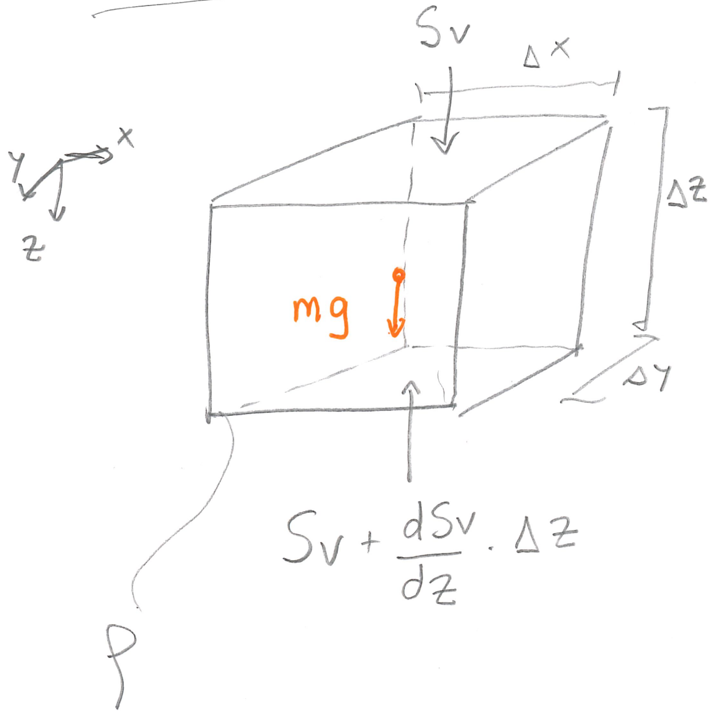
offshore



2019/9/6

$$S_v = \underbrace{\rho_w g z_w}_{\text{weight of water}} + \underbrace{\rho_{bulk} g (z - z_w)}_{\text{weigth of rock with water/fluids}}$$

General solution for vertical stress



$$\Sigma F_z = 0$$

$$\left(\cancel{S_v} + \frac{dS_v}{dz}\Delta z\right)\Delta X \Delta Y - \cancel{S_v \Delta x \Delta y} - mg = 0$$

$$\frac{dS_v}{dz}\Delta z \cancel{\Delta x \Delta y} = \rho \cancel{\Delta x \Delta y}\Delta z \cdot g$$

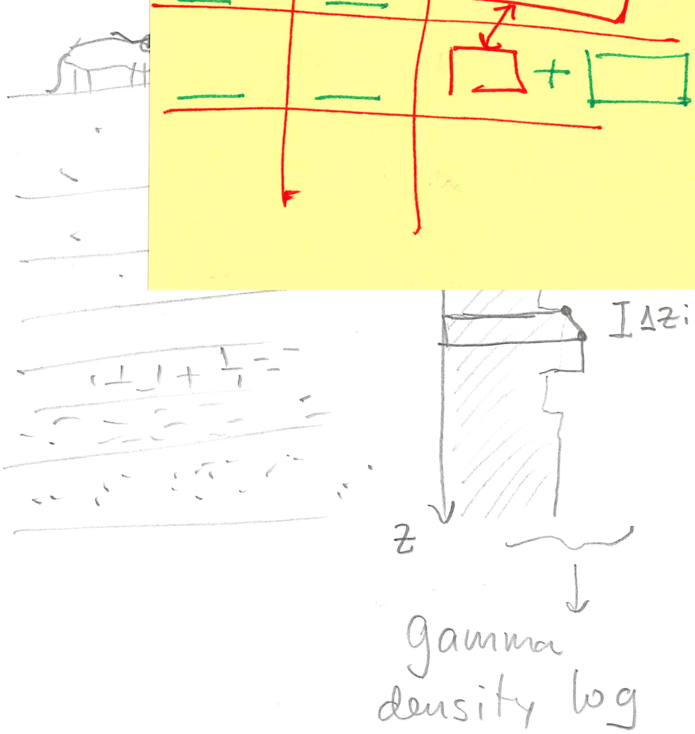
Total vertical stress gradient $\Big\}$ $$\frac{dS_v}{dz} = \rho g$$

z	ρ	Sv
0	—	0
100	—	
—	—	+

$$\frac{dS_v}{dz} = \rho_{bulk}(z)\, g$$

$$\int_0^{S_v(z)} dS_v = \int_0^z \rho_{bulk}(z)\, g\, dz$$

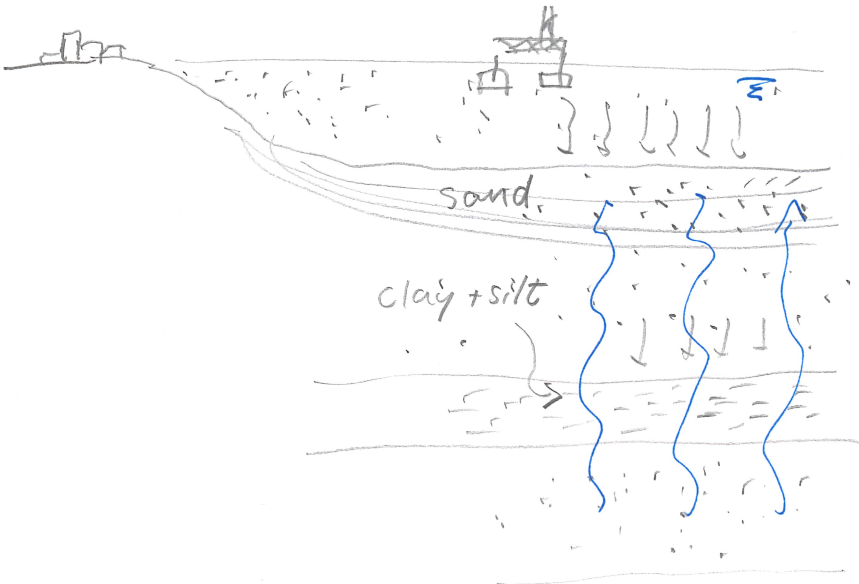
$$\boxed{S_v(z) = \int_0^z \rho_{bulk}(z)\, g\, dz}$$

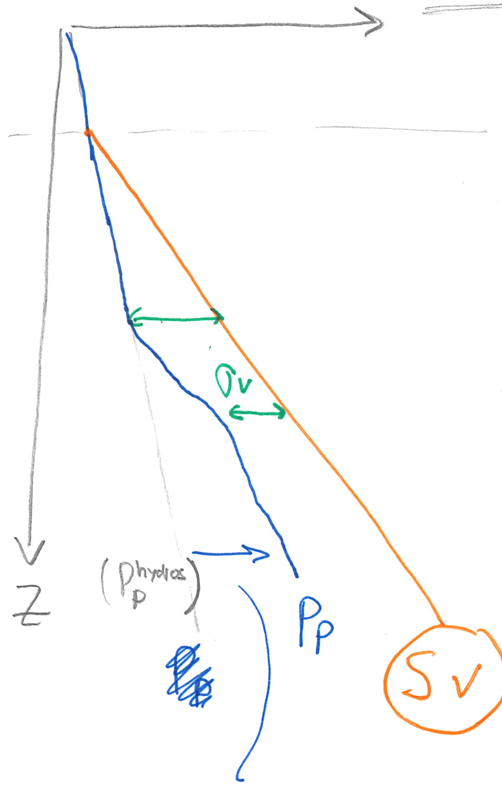


$$S_v(z) = \sum_{i=0}^{n} \left[\frac{\rho_{bulk}(z_i) + \rho_{bulk}(z_{i+1})}{2}\right] g \Delta z_i$$

Non-hydrostatic pore pressure

- HC generation
- Difference mass density
- Disequilibrium compaction → GOM



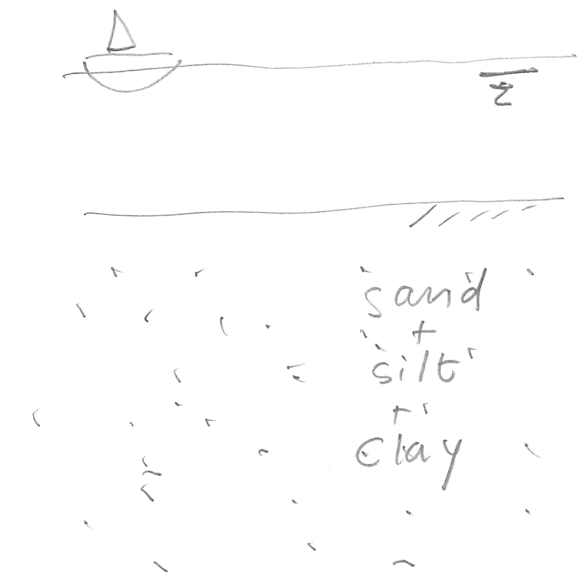


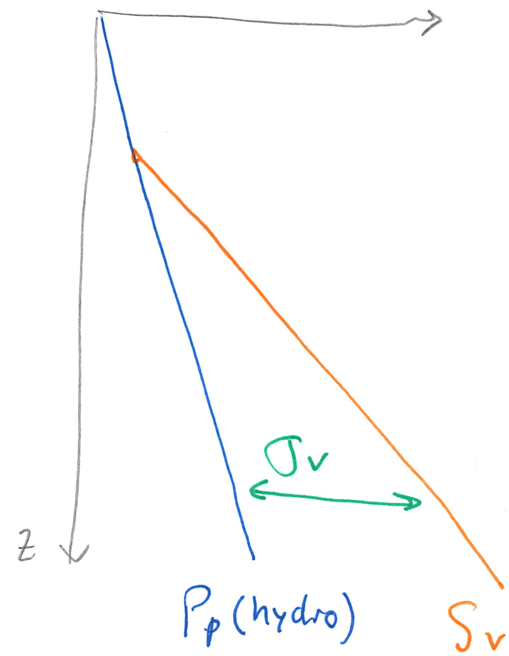
over pressure

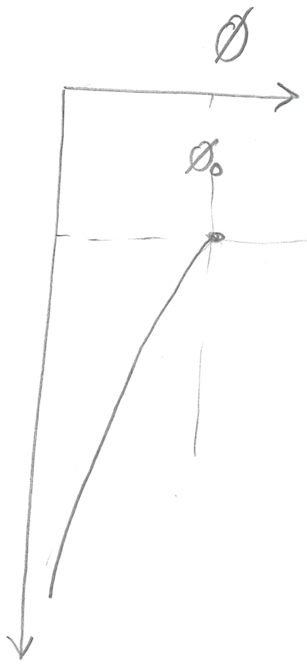
$$\underbrace{\text{rate of sedimentation} > \text{rate of pore pressure diffusion}}_{\text{Diseq. Comp.}}$$

$$\lambda_p = \frac{P_p}{S_v}$$

$\boxed{2019/9/9}$



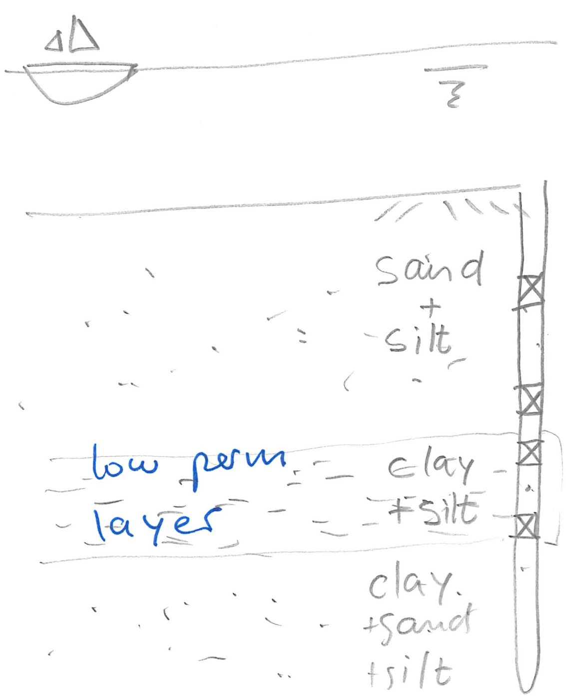


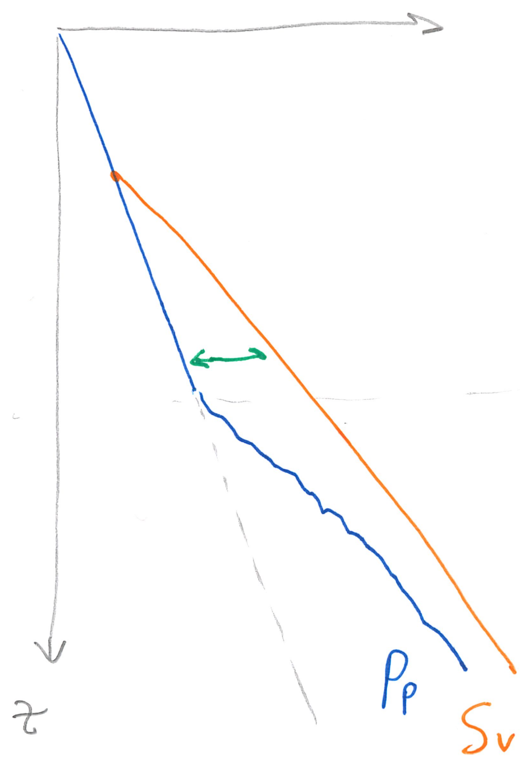


normal compaction

logs ←
labs ←

$$\boxed{\varnothing = \underbrace{\varnothing_0}_{\text{parameters}} \exp(-\underbrace{\beta}_{\text{parameters}} \sigma_v)}$$





gamma ↔ NPHI, DPHI, DT, cores → actual porosity

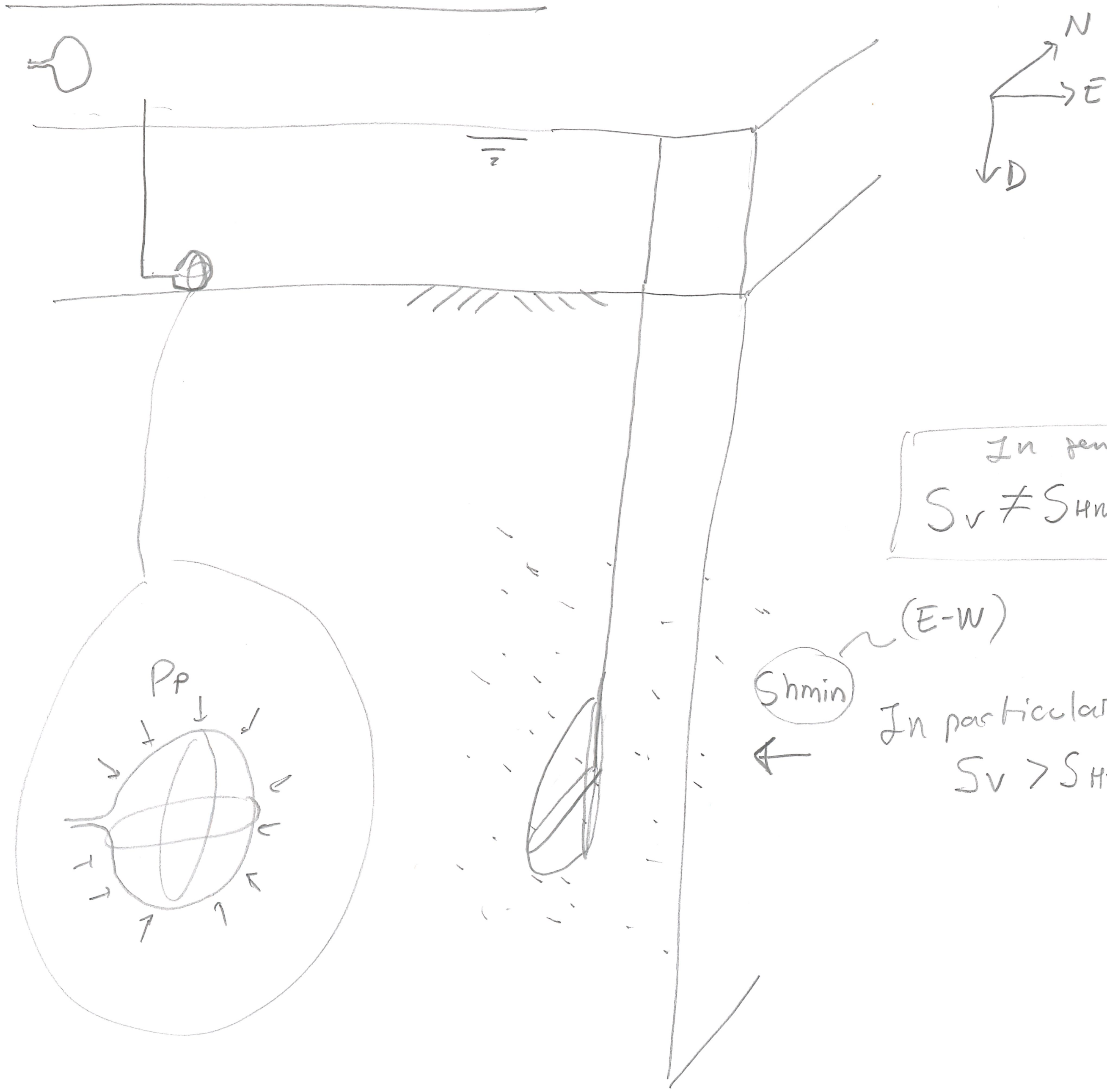
WORKFLOW

1) Calculate S_v

2) Read ϕ, calculate $\boxed{\sigma_v = -\frac{\ln(\phi/\phi_0)}{\beta}}$

3) Determine $P_p = \underline{S_v} - \underline{\sigma_v}$

Horizontal Stresses



In general

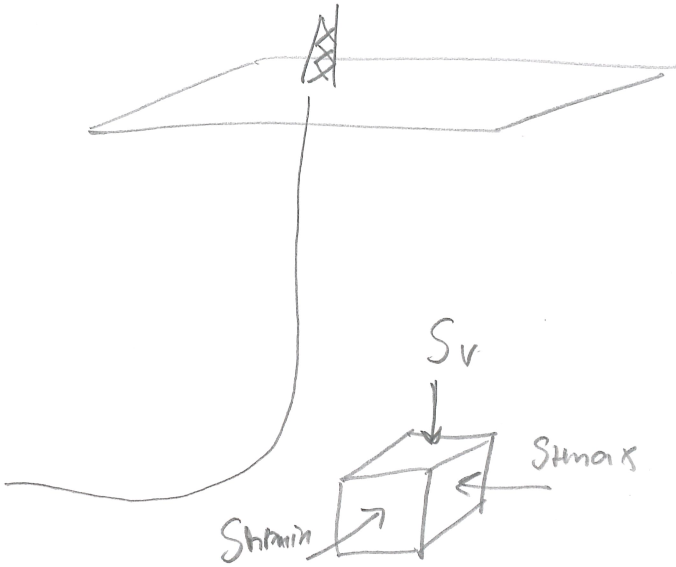
$\boxed{S_v \neq S_{Hmax} \neq S_{hmin}}$

In particular

$S_v > S_{Hmax} > \underbrace{S_{hmin}}_{S_3}$

$\perp$

HF



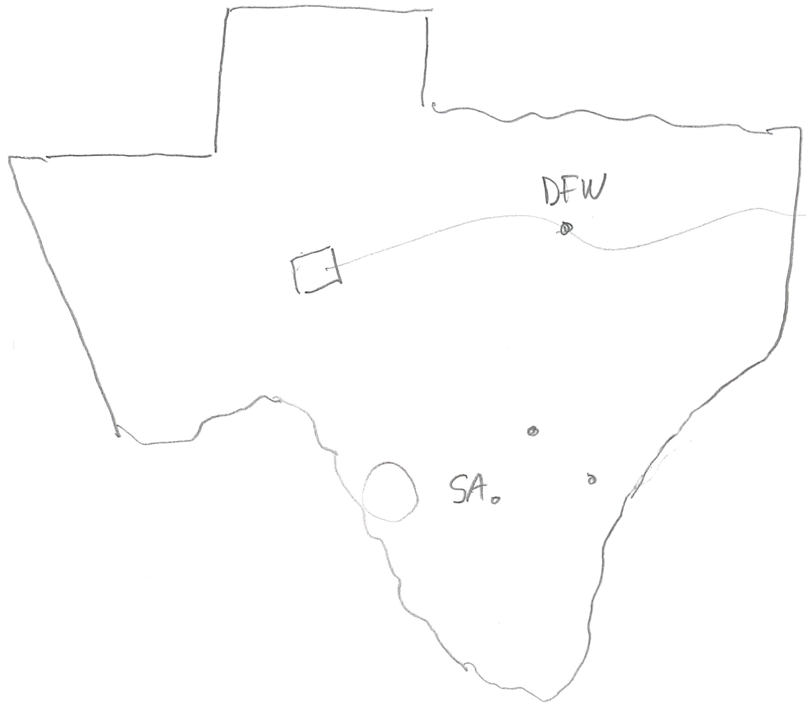
3 values of stresses

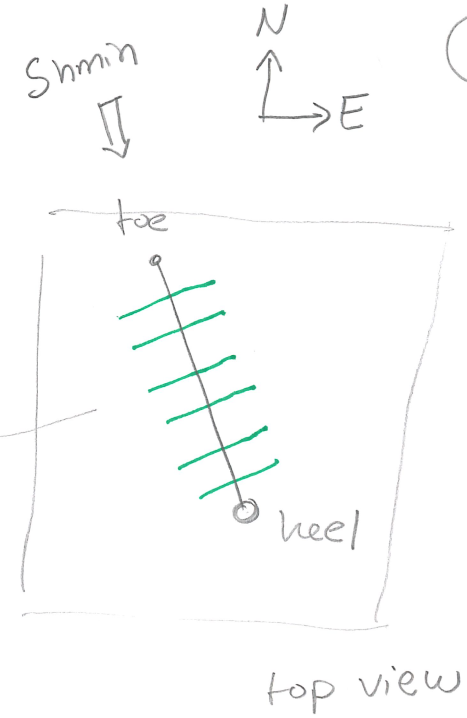
3 directions

$\underbrace{\qquad\qquad\qquad}_{\text{Fully define the state of stress}}$

	Max	Interm	Least	
Stress regime	S_1 >	S_2 >	S_3	
Normal Faulting	Sv	SHmax	Shmin	most sedim US
Strike Slip	SHmax	Sv	Shmin	California San Andreas
Reverse Faulting (Thrust)	SHmax	SHmin	Sv	tectonic activity

9/11/2019





$$S_v > S_{Hmax} > \underbrace{S_{hmin}}_{S_3}$$

$\Rightarrow$ S_v $S_{Hmax} \longleftrightarrow S_{hmin}$ $\Leftarrow$

S_v

$$\int_0^z \rho_{bulk}\, g\, dz$$

$\downarrow$

Δz

$\downarrow$

TVD

S_{Hmax}

if S_2

- wellbore stability
- fault equilibrium
- Mechanical model

S_{hmin}

if S_3

- Hydraulic Test) Direct
- Wellbore stability
- Fault equilibrium } Indirect
- Mechanical model (elasticity)

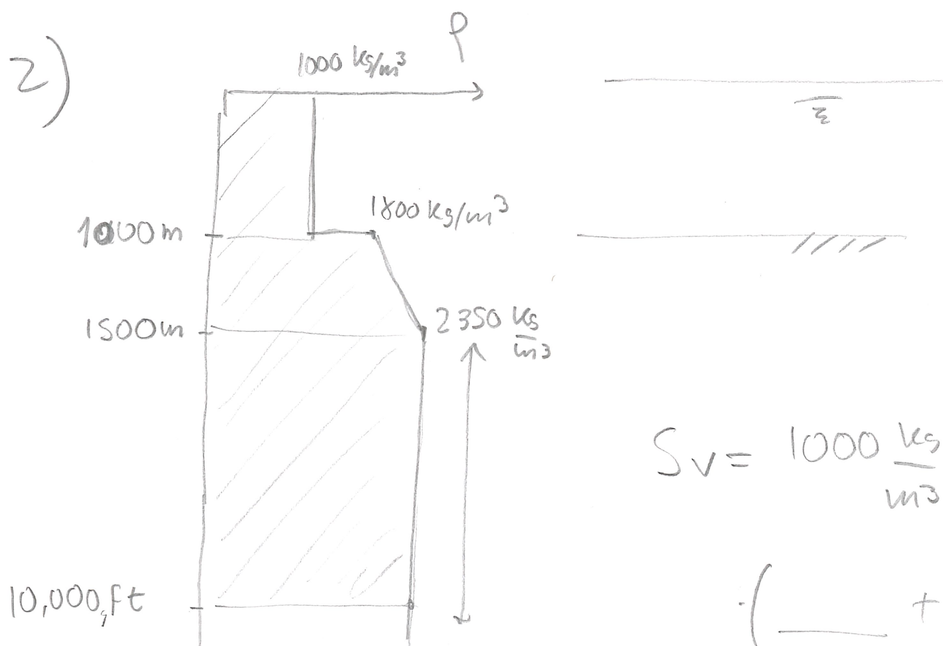
HW2

1) $\rho_{bulk} = \rho_{min} (1-\varnothing) + \rho_{fluid} (\varnothing)$

ρ_{min}: V_{SiO_2}, V_{CaCO_3}, V_{clay}, $\vdots$

ρ_{fluid}: S_w, S_o, S_g

2)



9.8 $\frac{m}{s^2}$

$$S_v = 1000 \frac{kg}{m^3} \cdot g \cdot 1000\,m +$$

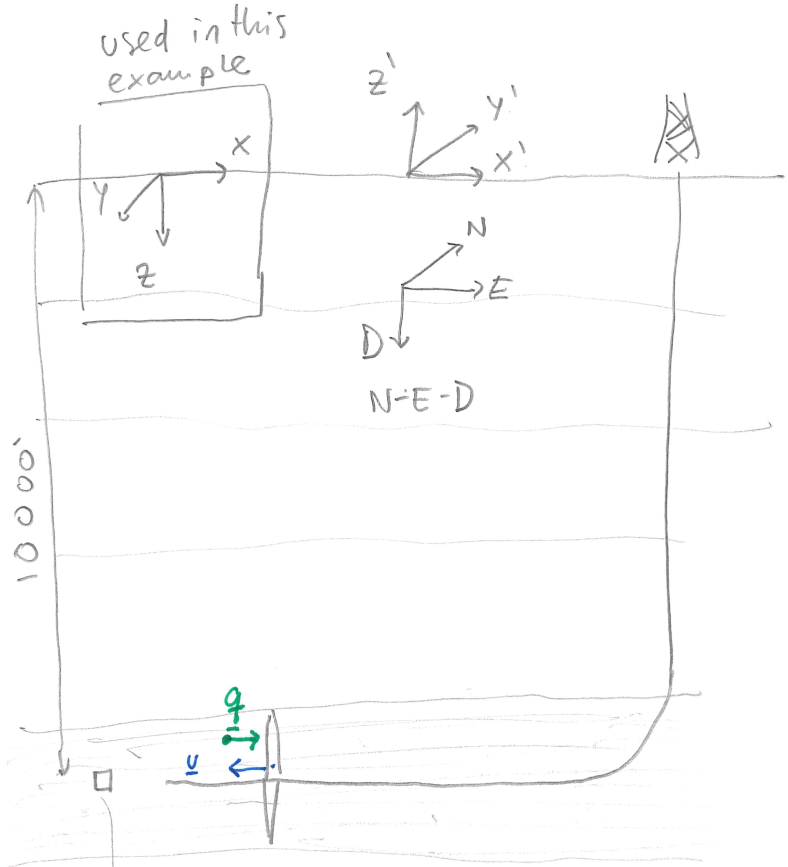
$$\frac{(\_\_\_ + \_\_\_)}{2} g \cdot 500\,m +$$

$$2350 \ldots$$

$$\boxed{\begin{array}{l} S_v < 25{,}000 \text{ psi} \\ S_v < 150 \text{ MPa} \end{array}}$$

SI → MKS (meter, kilogram, second)

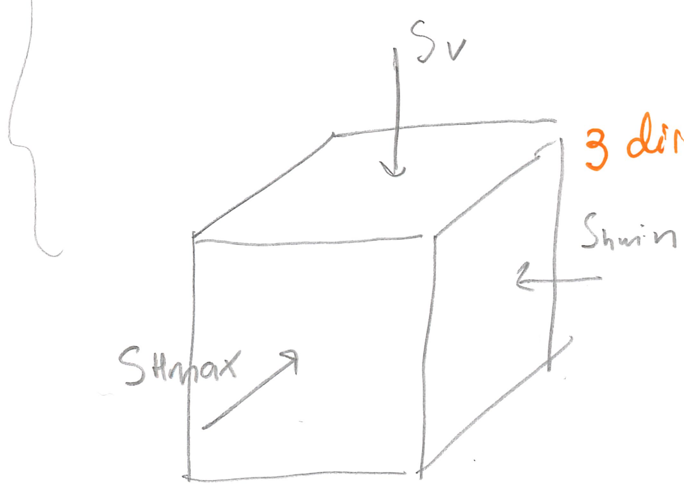
↳ $Pa = \left[\frac{N}{m^2}\right]$



scalar
$$\begin{cases} P_p = 4300 \text{ psi (hydro)} \\ T = 180^\circ F \\ S_o = 0.80 \end{cases}$$

vector
$$\begin{cases} \underline{q} = [0.2, 0, 0] \text{ ft/sec} \\ \underline{u} = [-0.5, 0, 0] \text{ in} \end{cases}$$

tensor
$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$



3 directions + 3 values

$$\underline{\underline{S}} = \begin{bmatrix} S_{hmin} & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_v \end{bmatrix}$$

off diagonal: shear stresses

diagonal: normal stresses

Principal stresses

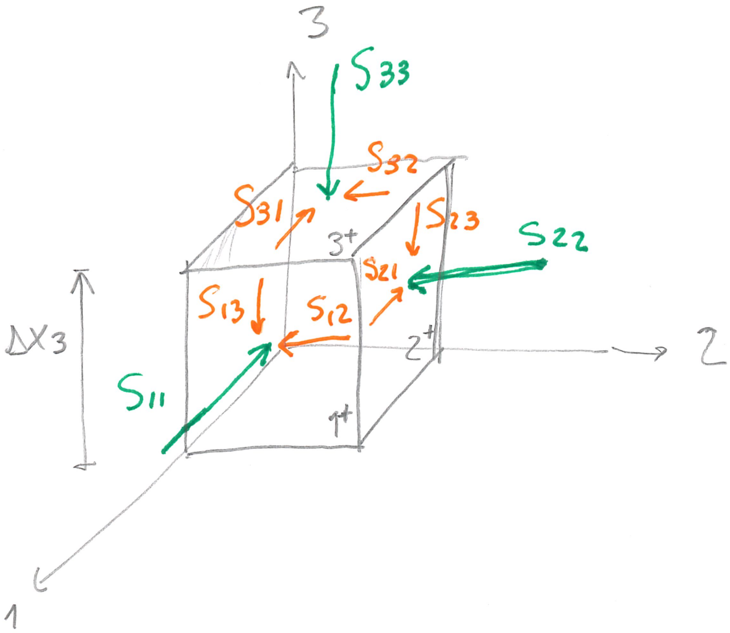
$$\underbrace{S_{hmin}}_{S_3} < S_{Hmax} < \underbrace{S_v}_{S_1}$$

$\Downarrow$

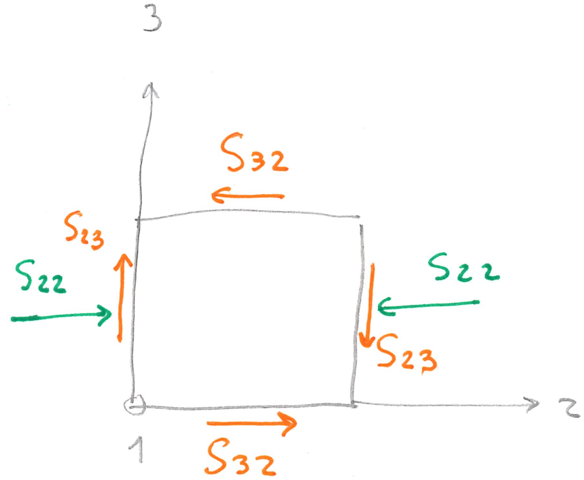
$$\underbrace{\sim (60-70\%)\, S_v}_{\text{frac gradient}}$$

$$\underline{\underline{S}} = \begin{bmatrix} 6500 & 0 & 0 \\ 0 & 7000 & 0 \\ 0 & 0 & 10000 \end{bmatrix} \text{ psi}$$

x - y - z
i - j - k



$S > 0 \Rightarrow$ compression



$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Equil. angular momentum

$S_{32} = S_{23}$

$$\downarrow$$

$$\begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 7.28 & 0 & 0 \\ 0 & 0.41 & 0 \\ 0 & 0 & -1.69 \end{bmatrix}$$

Principal stresses

$$\underbrace{S_1}_{\text{Max}} \geq \underbrace{S_2}_{\text{Interm}} \geq \underbrace{S_3}_{\text{Least total principal stress}}$$

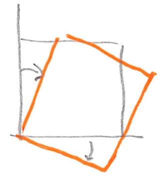
Effective stress = Total stress − Pore pressure

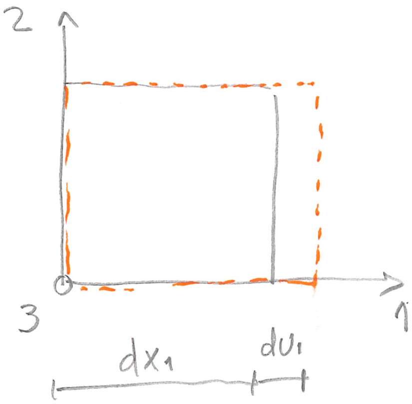
$$\sigma_V = S_V - P_p$$

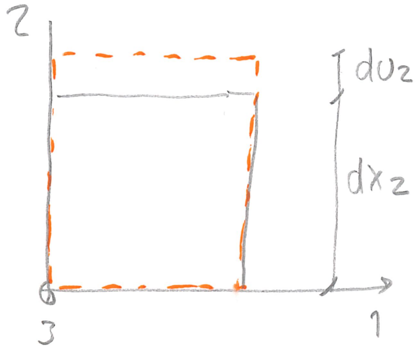
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} - \begin{bmatrix} P_p & 0 & 0 \\ 0 & P_p & 0 \\ 0 & 0 & P_p \end{bmatrix}$$

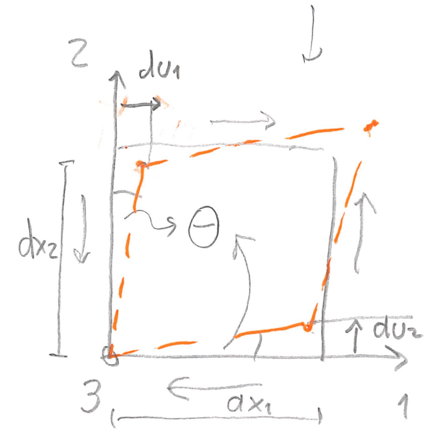
$$= \underbrace{\begin{bmatrix} S_{11}-P_p & S_{12} & S_{13} \\ S_{21} & S_{22}-P_p & S_{23} \\ S_{31} & S_{32} & S_{33}-P_p \end{bmatrix}}_{\text{effective stress}}$$

Strain (deformation)









$$\tan\theta = \frac{du_1}{dx_2} = \frac{du_2}{dx_1}$$

$$\underbrace{\varepsilon_{11} = \frac{du_1}{dx_1} \qquad \varepsilon_{22} = \frac{du_2}{dx_2}}_{+\varepsilon_{33}}$$

Volometric strains

Linear strains

$$\underbrace{\varepsilon_{12} = \frac{1}{2}\left(\frac{du_1}{dx_2} + \frac{du_2}{dx_1}\right)}$$

shear strains

Strain tensor (small strain)

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \frac{du_1}{dx_1} & \frac{1}{2}\left(\frac{du_1}{dx_2} + \frac{du_2}{dx_1}\right) & \frac{1}{2}\left(\frac{du_1}{dx_3} + \frac{du_3}{dx_1}\right) \\ - & \frac{du_2}{dx_2} & \frac{1}{2}\left(\frac{du_2}{dx_3} + \frac{du_3}{dx_2}\right) \\ - & - & \frac{du_3}{dx_3} \end{bmatrix}$$

$\underline{\underline{\sigma}} \longleftrightarrow \underline{\underline{\varepsilon}} \longrightarrow$ Constitutive equations – E.g. linear elasticity

$$\underline{\underline{\sigma}} \longleftrightarrow \underline{\underline{\varepsilon}}$$ → Constitutive relationships

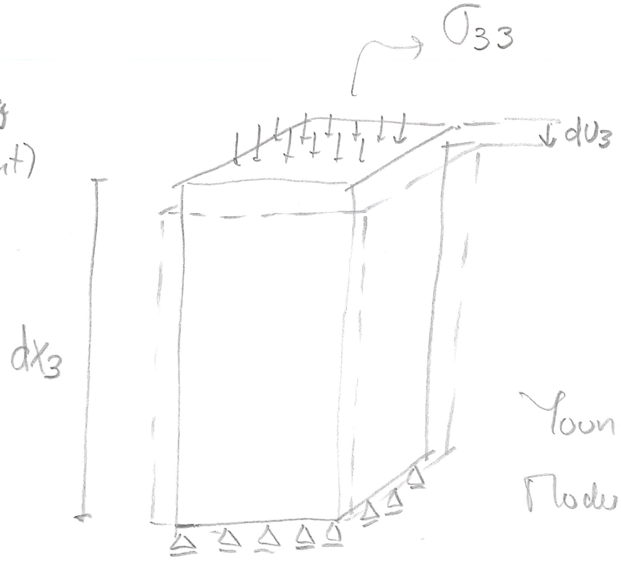
$$\underline{\underline{\varepsilon}} = f(\underline{\underline{\sigma}})$$

$$\underline{\underline{\sigma}} = f(\underline{\underline{\varepsilon}})$$

Linear elasticity (Hooke's law 1D $\sigma = E\,\varepsilon$)

isotropic

Test: axial loading with no (constant) confinement



$$\varepsilon_{33} = \frac{dU_3}{dx_3}$$

$$\varepsilon_{33} = \frac{\sigma_{33}}{E}$$

Young's Modulus $$\boxed{E = \frac{\Delta\sigma_{33}}{\Delta\varepsilon_{33}}} \quad \left[\frac{F}{L^2}\right]$$

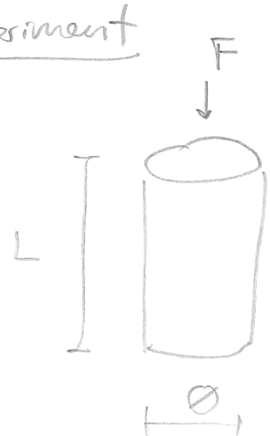
rocks → $[1\,\text{GPa}, 50\,\text{GPa}]$

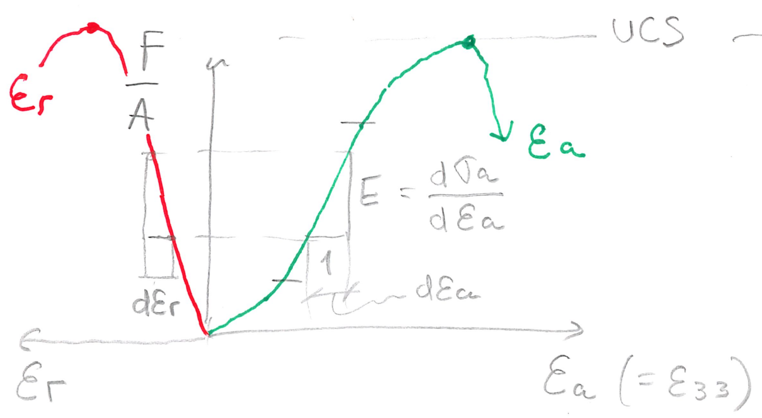
$[<10^6\,\text{psi}, \sim 7 \cdot 10^6\,\text{psi}]$

Poisson ratio $$\boxed{\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}} = -\frac{\Delta\varepsilon_{11}}{\Delta\varepsilon_{33}}}$$

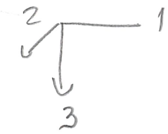
rocks → $[0.1 - 0.4]$

Experiment





$$
\begin{cases}
\varepsilon_{11} = \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} + 0 \\
\varepsilon_{22} = -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} + 0 \\
\varepsilon_{33} = -\frac{\nu}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} + 0 \\
2\cdot\varepsilon_{12} = 0 + 0 + 0 + \frac{\sigma_{12}}{G} + 0 + 0 \\
2\cdot\varepsilon_{13} = 0 + 0 + 0 + 0 + \frac{\sigma_{13}}{G} + 0 \\
2\cdot\varepsilon_{23} = 0 + 0 + 0 + 0 + 0 + \frac{\sigma_{23}}{G}
\end{cases}
$$



→ 2 independent parameters

$$G = \frac{E}{2(1+\nu)}$$

G → Shear Modulus

$$
\underbrace{\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}}_{6\times 1}^{\text{OUTPUT}}
=
\underbrace{\begin{bmatrix}
1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\
-\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\
-\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G
\end{bmatrix}}_{6\times 6}^{\text{compliance matrix}}
\underbrace{\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}}_{6\times 1}^{\text{INPUT}}
$$

$$\underline{\varepsilon} = \underline{\underline{D}} \cdot \underline{\sigma}$$

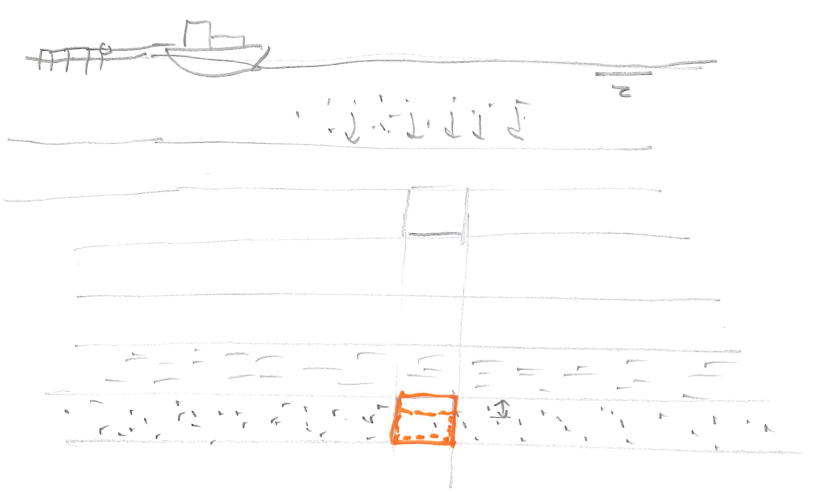
$$\underline{\underline{D}}^{-1} \underline{\varepsilon} = \underline{\underline{D}}^{-1} \cdot \underline{\underline{D}} \, \underline{\sigma}$$

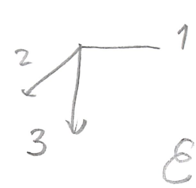
$$\underline{\sigma} = \underline{\underline{D}}^{-1} \, \underline{\varepsilon}$$

$$\underline{\sigma} = \underbrace{\underline{\underline{C}}}_{\text{stiffness matrix}} \cdot \underline{\varepsilon}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \left[\begin{array}{ccc|ccc} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{array}\right] \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \hline 2\cdot\varepsilon_{12} \\ 2\cdot\varepsilon_{13} \\ 2\cdot\varepsilon_{23} \end{bmatrix}$$

Uniaxial-strain stress path



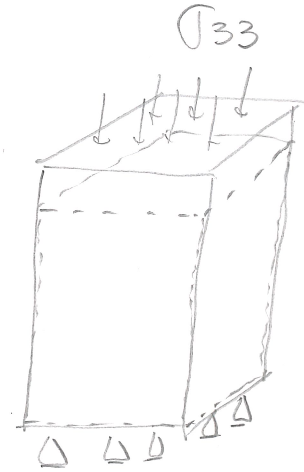


$$\left.\varepsilon_{11} = \varepsilon_{22} = 0 \right\} \text{tectonic strains}$$

$$\varepsilon_{33} \neq 0$$

$$\underbrace{\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0}_{\text{tectonically passive environment}}$$

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu\\ \nu & 1-\nu & \nu\\ \nu & \nu & 1-\nu\end{bmatrix}\begin{bmatrix}\emptyset\\ \emptyset\\ \varepsilon_{33}\end{bmatrix}$$



$$\boxed{\sigma_{33} = \underbrace{\frac{(1-\nu)\,E}{(1+\nu)(1-2\nu)}}_{M:\ \text{constrained modulus}}\ \varepsilon_{33}}$$

M: constrained modulus

$$E \leqslant M$$

bulk compressibility^{-1}

$\updownarrow$

pore compressibility

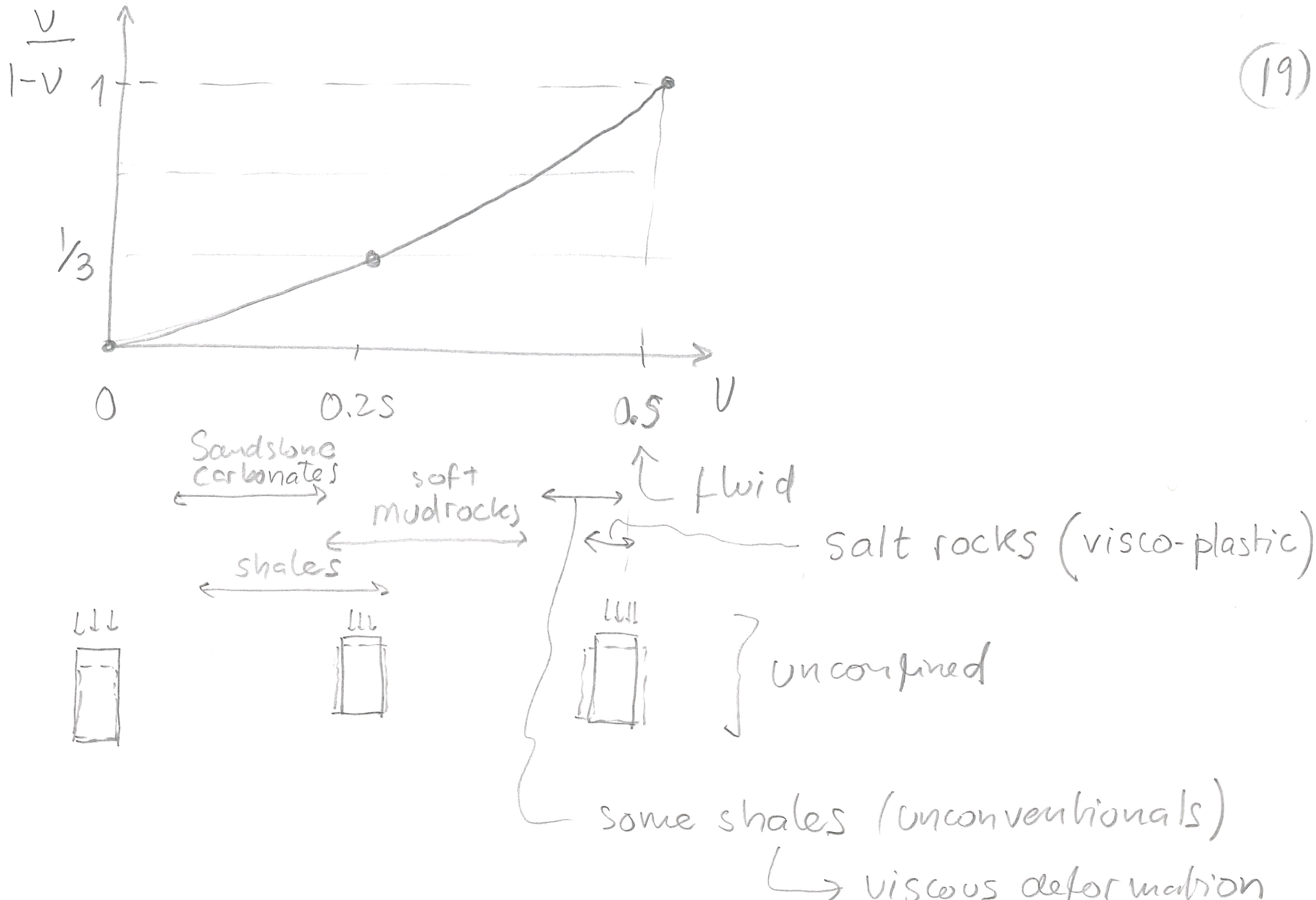
$$\sigma_{11} = \nu\,\varepsilon_{33}\,\frac{E}{(1+\nu)(1-2\nu)} = \nu\left[\frac{\cancel{(1+\nu)}\,\cancel{(1-2\nu)}}{(1-\nu)\,\cancel{E}}\,\sigma_{33}\right]\frac{\cancel{E}}{\cancel{(1+\nu)}\,\cancel{(1-2\nu)}}$$

$$\boxed{\begin{aligned}\sigma_{11} &= \frac{\nu}{1-\nu}\cdot\sigma_{33}\\ \sigma_{22} &= \frac{\nu}{1-\nu}\cdot\sigma_{33}\end{aligned}} \longrightarrow \boxed{\begin{aligned}S_{11} &= \sigma_{11} + P_p\\ S_{22} &= \sigma_{22} + P_p\end{aligned}}$$

Effective horizontal stress

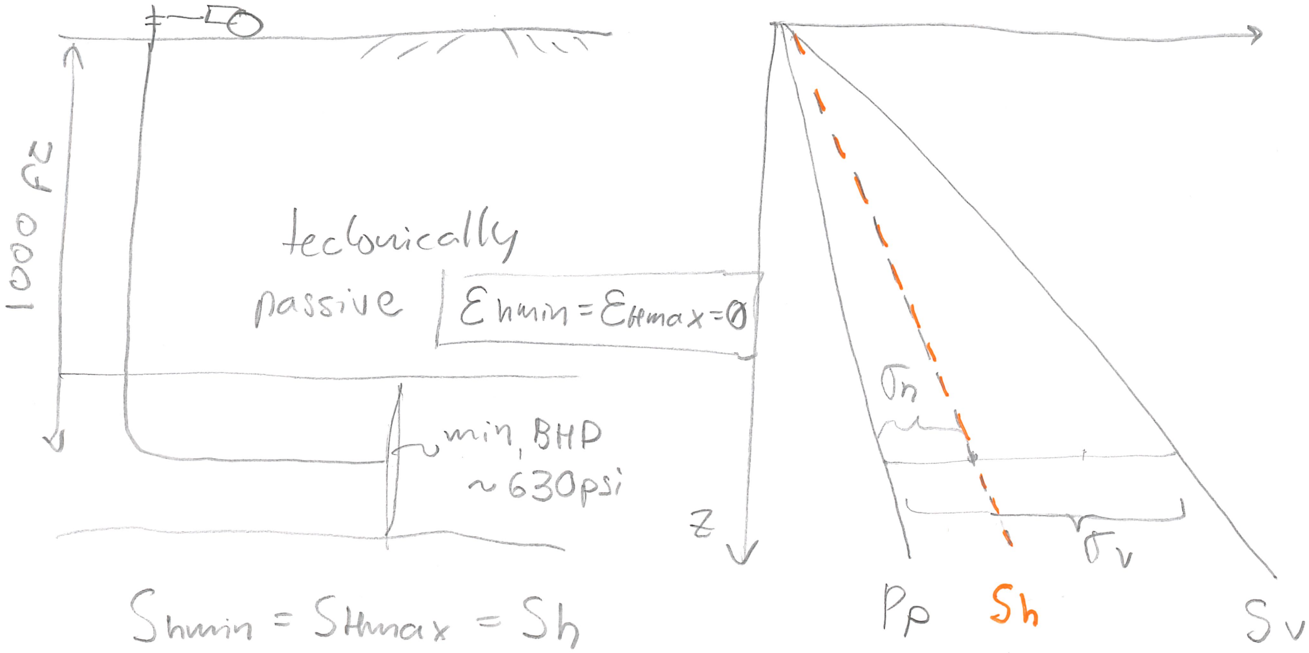
Effective vertical stress

total stress



σ_{33} : effective vertical stress

σ_{11}, σ_{22} : eff horizontal stress → Total Hz stress



$S_{hmin} = S_{Hmax} = S_h$

$S_h < S_v$

$$\underbrace{S_h}_{\text{total}} = \underbrace{\sigma_h}_{\text{effective}} + P_p$$

$$S_h = \frac{\nu}{1-\nu}\sigma_v + P_p$$

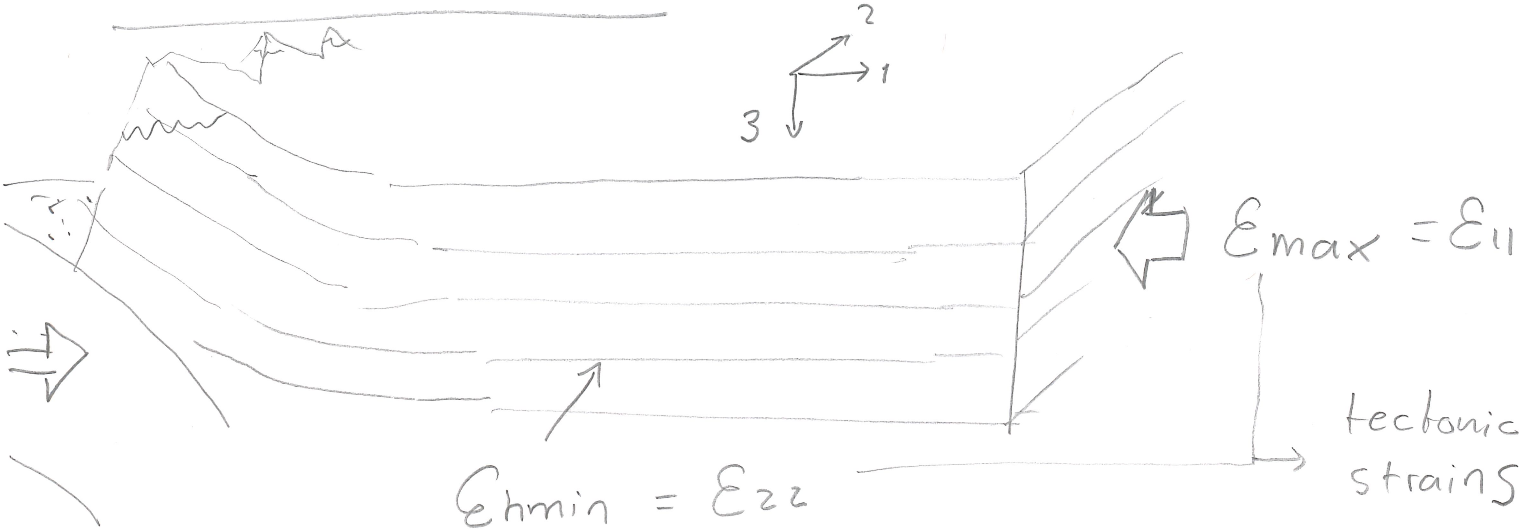
$$S_h = \frac{\nu}{1-\nu}(S_v - P_p) + P_p$$

absolute value → $$S_h = \frac{\nu}{1-\nu} S_v + \frac{1-2\nu}{1-\nu} P_p$$

$$\underbrace{\frac{\Delta S_h}{\Delta z}}_{\text{fracture gradient}} = \frac{\nu}{1-\nu}\underbrace{\frac{\Delta S_v}{\Delta z}}_{\substack{\sim 1\ \text{psi/ft} \\ \text{Total vertical stress gradient}}} + \frac{1-2\nu}{1-\nu}\underbrace{\frac{\Delta P_p}{\Delta z}}_{\substack{\sim 0.44\ \text{psi ft} \\ \text{Pore pressure gradient}}}$$

$\nu = 0.25$ → $\frac{\Delta S_h}{\Delta z} \cong 0.63$

Tectonic stresses



$$
\left|\begin{array}{l}
\sigma_{11} = \boxed{\dfrac{E}{1-\nu^2}}\,\varepsilon_{11} + \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{22} + \dfrac{\nu}{1-\nu}\,\sigma_{33} \\[2ex]
\sigma_{22} = \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{11} + \dfrac{E}{1-\nu^2}\,\varepsilon_{22} + \underbrace{\dfrac{\nu}{1-\nu}\,\sigma_{33}}_{\text{overburden component}} \\[2ex]
\sigma_{33} = \displaystyle\int_0^z \rho_{bulk} \cdot g \cdot dz - P_p
\end{array}\right.
$$

$$E' = \frac{E}{1-\nu^2} \rightarrow \text{plane strain modulus}$$

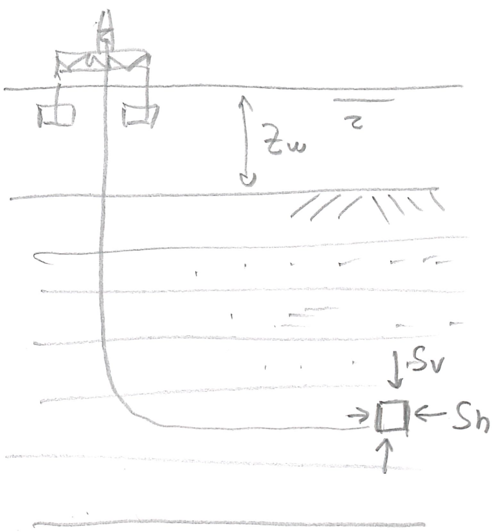
E' → sonic logs (Δt_p, Δt_s) ← $\frac{\nu}{1-\nu}$
laboratory tests

$$
\left|\begin{array}{l}
\sigma_{Hmax} = E'\,\varepsilon_{Hmax} + \nu E'\,\varepsilon_{hmin} + \dfrac{\nu}{1-\nu}\,\sigma_v \\[2ex]
\sigma_{hmin} = \nu E'\,\varepsilon_{Hmax} + E'\,\varepsilon_{hmin} + \dfrac{\nu}{1-\nu}\,\sigma_v \\[2ex]
\sigma_v = \underbrace{\displaystyle\int_0^z \rho_{bulk}\, g\, dz}_{\text{Density log}} - \underbrace{P_p}_{}
\end{array}\right.
$$

ε_{Hmax}, ε_{hmin}: calibrated with field data
- HF
- wellbore failure

P_p:
- hydrostatic
- direct measurement
- porosity

General procedure to calculate total Hz stress



① S_v (Total vertical stress)

② P_p — hydrostatic / non-hydro — Ø shale / $P_p = \boxed{\lambda_p} \cdot S_v$ (λ_p given)

③ $\sigma_v = S_v - P_p$

④ σ_{Hmax}, σ_{hmin} (Effective)

a. Isotropic elasticity
- $\varepsilon_h = 0 \Rightarrow \sigma_h = \frac{\nu}{1-\nu} \sigma_v$
- $\varepsilon_{hmin} \neq \varepsilon_{Hmax} \neq 0$, E, ν
  - $\hookrightarrow \sigma_{hmin} = \dots$
  - $\sigma_{Hmax} = \dots$

b. Plasticity $\Leftrightarrow$ fault equilibrium

c. Visco-elasticity

⑤
$$\begin{cases} S_{hmin} = \sigma_{hmin} + P_p \\ \underbrace{S_{Hmax}}_{\text{total}} = \underbrace{\sigma_{Hmax}}_{\text{effective}} + P_p \end{cases}$$

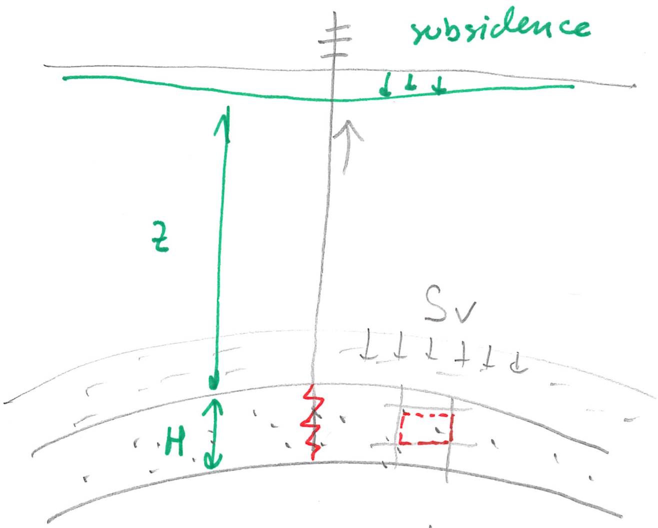
Rock compressibility

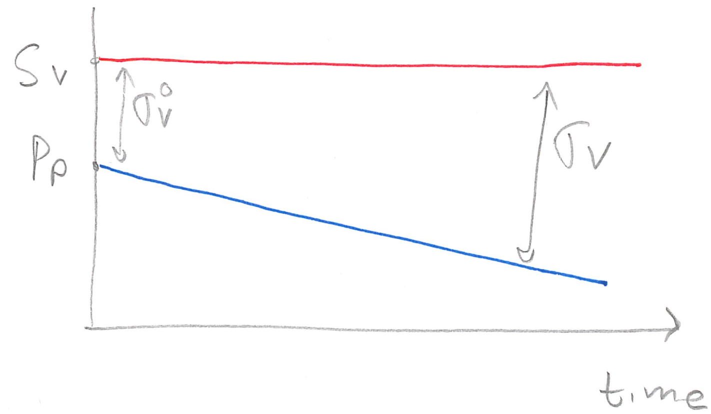
$$\frac{\partial P}{\partial t} = \frac{K}{\mu C_t} \frac{\partial^2 P}{\partial x^2}$$

total compressibility

$$C_t = C_g S_g + C_w S_w + C_o S_o + \boxed{C_F}$$

rock compressibility





$$C_{bp} = \frac{1}{M}$$

$$\boxed{C_{bp} = \frac{1}{V_b} \frac{dV_b}{dP_p}} \rightarrow \frac{dV_b}{V_b} = \varepsilon_{vol}$$

$dV_b = dV_p$ (assumption)

$$\boxed{C_{pp}} = \frac{1}{V_p} \frac{dV_p}{dP_p} = \frac{1}{\frac{V_p}{V_b}} \frac{1}{V_b} \frac{dV_b}{dP_p} = \frac{C_{bp}}{\phi} = \boxed{\frac{1}{M \phi}}$$

[1 − 30 μsips]

$$\mu sip = 10^{-6} \cdot \frac{1}{psi}$$

HW #4

3) Isotropic loading

$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{iso}$

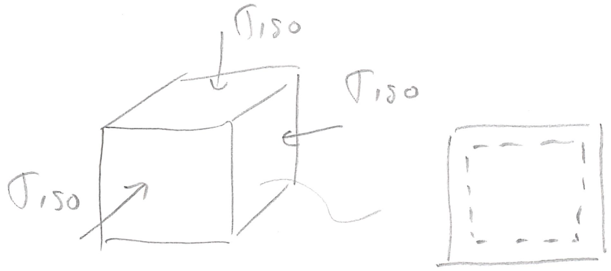
$\sigma_{12} = \sigma_{13} = \sigma_{23} = \emptyset$

$\varepsilon_{11} = \frac{(1-2\nu)}{E} \cdot \sigma_{iso}$

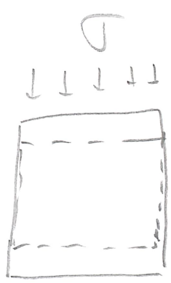
$\varepsilon_{vol} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$

$$\boxed{\sigma_{iso} = \underbrace{\qquad\qquad}_{K\text{ : Bulk modulus }[Pa]} \varepsilon_{vol}}$$

K : Bulk modulus $[Pa] \longrightarrow \frac{1}{K} = C_{bulk} \; [Pa^{-1}]$



4)



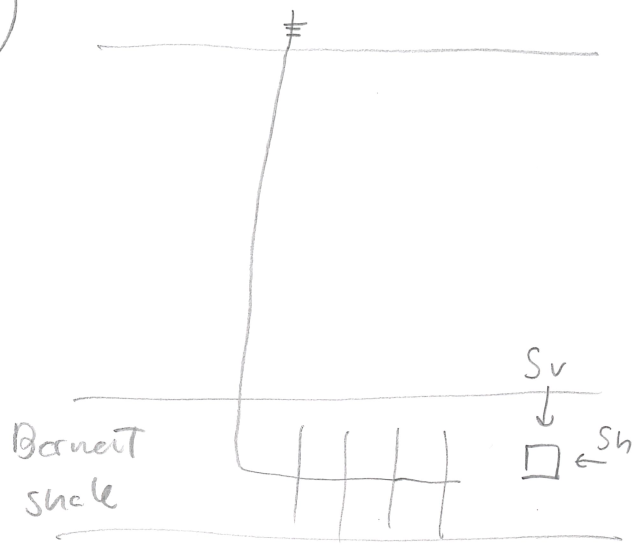
$\longrightarrow \Delta \varepsilon_v = \frac{\Delta \sigma_v}{M}$

M: constrained modulus

$\frac{1}{M}$: bulk compressibility under 1D-strain condition

$\hookrightarrow C_{bp} \rightarrow C_{pp}$

5)

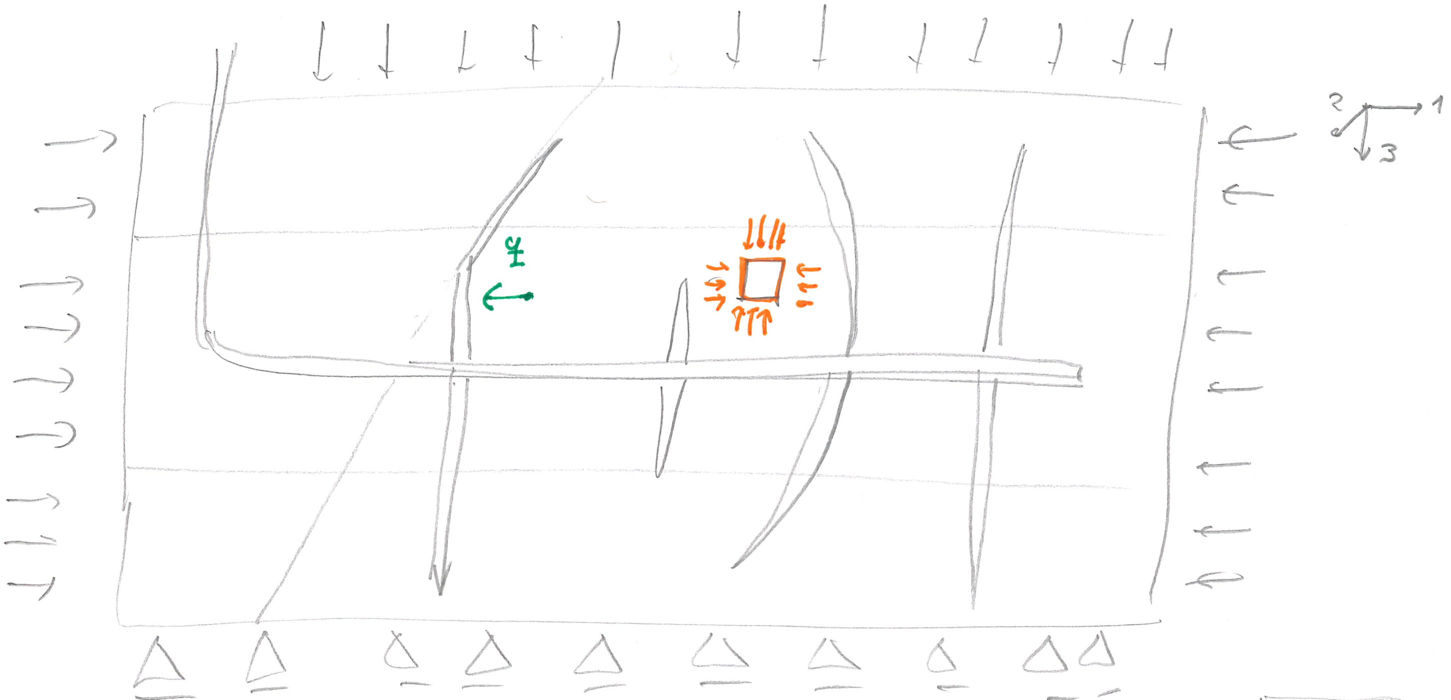


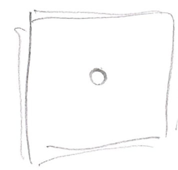
[1-7] → tectonically passsive environment

$$\begin{cases} \sigma_h = \dfrac{\nu}{1-\nu}\,\sigma_v \\ P_p \text{ hydrostatic} \end{cases}$$

[8] → $\begin{cases} \varepsilon_{hmin} = 0 \\ \varepsilon_{Hmax} = 0.0002 \\ P_p = \lambda_p \cdot S_v \end{cases}$ $\Big|$ $E = 5 \cdot 10^6$ psi

General solution for a continuous mechanics problem

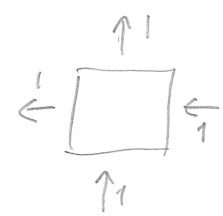




Fluid flow problem

(1) • Darcy →

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = -\frac{K}{\mu} \begin{bmatrix} \partial P/\partial x_1 \\ \partial P/\partial x_2 \\ \partial P/\partial x_3 \end{bmatrix}$$



(2) • Mass conservation

$$\frac{\partial q_1}{\partial x_1} + \frac{\partial q_2}{\partial x_2} + \frac{\partial q_3}{\partial x_3} = 0$$

(1) → (2)

$$\boxed{-\frac{K}{\mu}\left(\frac{\partial^2 P}{\partial x_1^2} + \frac{\partial^2 P}{\partial x_2^2} + \frac{\partial^2 P}{\partial x_3^2}\right) = 0} \rightarrow \text{unknown } P$$

Mechanics

$\underline{u} \rightarrow \underline{\varepsilon} \rightarrow \underline{\sigma}$

- Constitutive: $\underline{\sigma} = \underline{\underline{C}}\ \underline{\varepsilon}$ (Elasticity)
- Conservation: Equilibrium → $\sigma(x, y, z)$
- Kinematic equations $\underline{\varepsilon} \leftrightarrow \underline{u}$

unknown → $\underline{u}$

parameters: G: shear modulus

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

derivatives $\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$

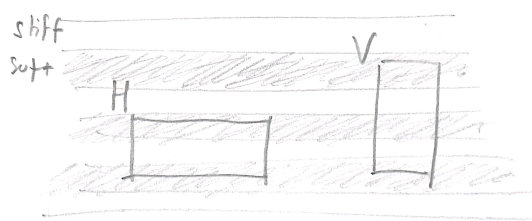
$$\boxed{(\lambda + G)\,\nabla\cdot(\nabla\cdot\underline{u}) + G\,\nabla^2(\underline{u}) + \underline{b} = 0}$$

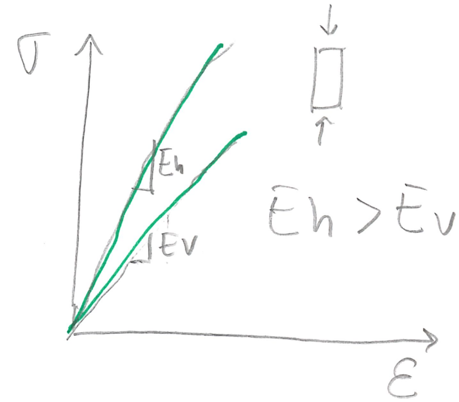
→ body force (e.g. gravity)

Solution mechanics differential equation
- Analytical
  - Kirsch (wellbore)
  - Griffith (fracture)
- Numerical
  - Finite Differences (CMG)
  - Finite Element Method

Real rocks

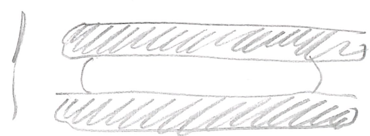
- Anisotropic





$E_h > E_v$

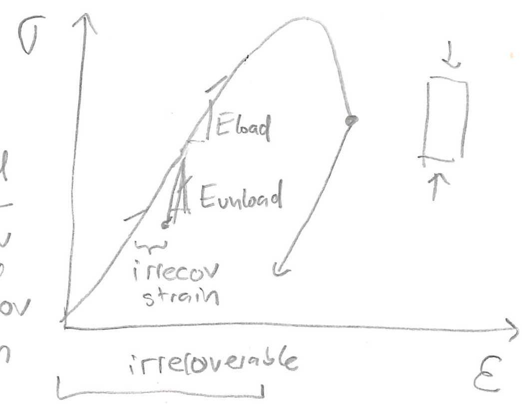
cookie



- Elasto-plastic

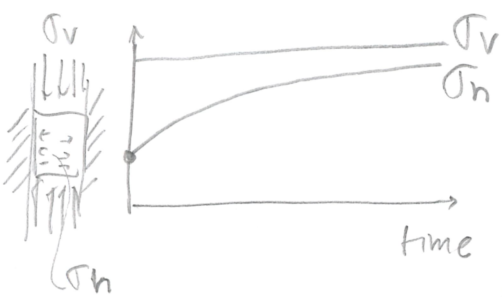
$\underbrace{\text{Elasto}}_{\text{recoverable,}}$ - $\underbrace{\text{plastic}}_{\text{irrecoverable deformation}}$

$\underbrace{E_{unload}}_{\text{recov strain}} > \underbrace{E_{load}}_{\text{recov and irrecov strain}}$

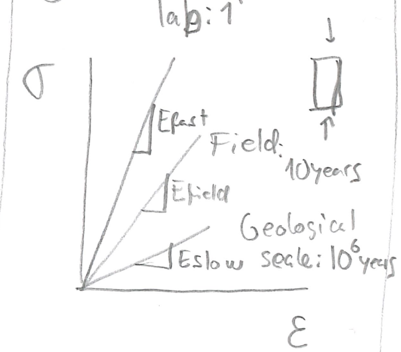


- Visco-elastic

$\underbrace{\text{Visco}}_{\text{time-dependent}}$ - elastic

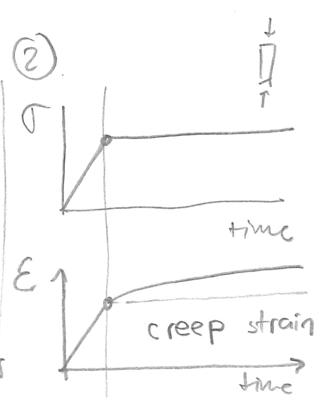


① lab: 1'

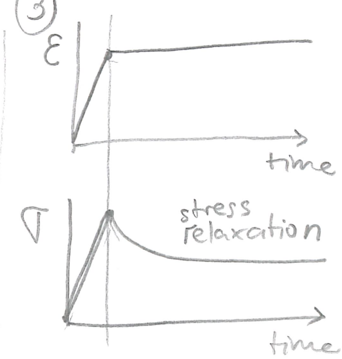


$\dot{\varepsilon}$: strain rate

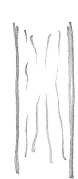
②



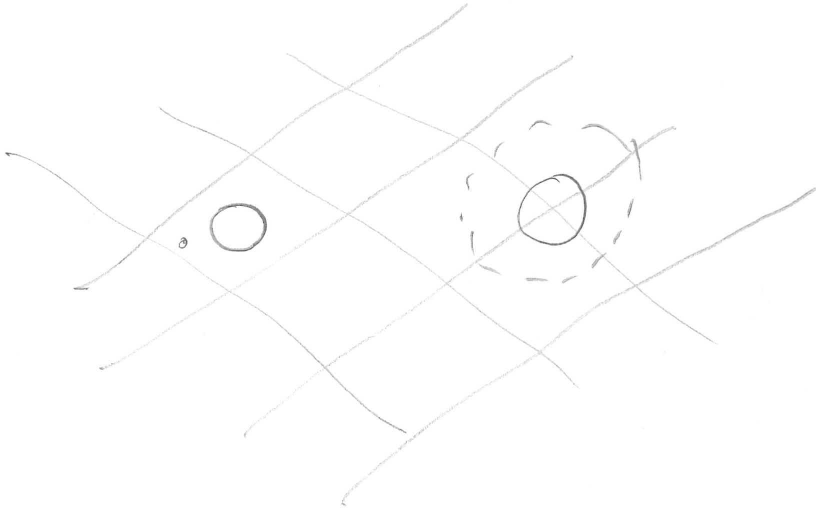
③

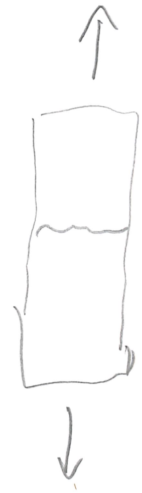


salt rocks
soft mudrocks ⟷ salt rock

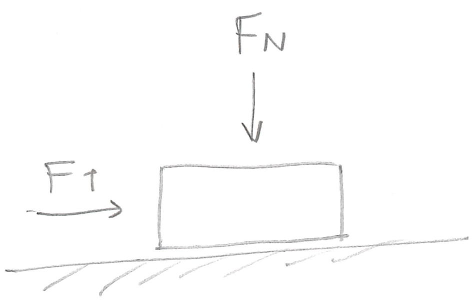


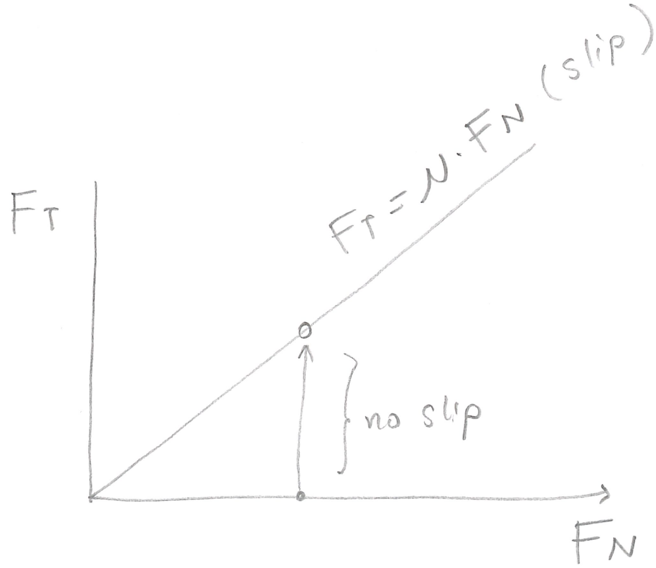
Tensile strength

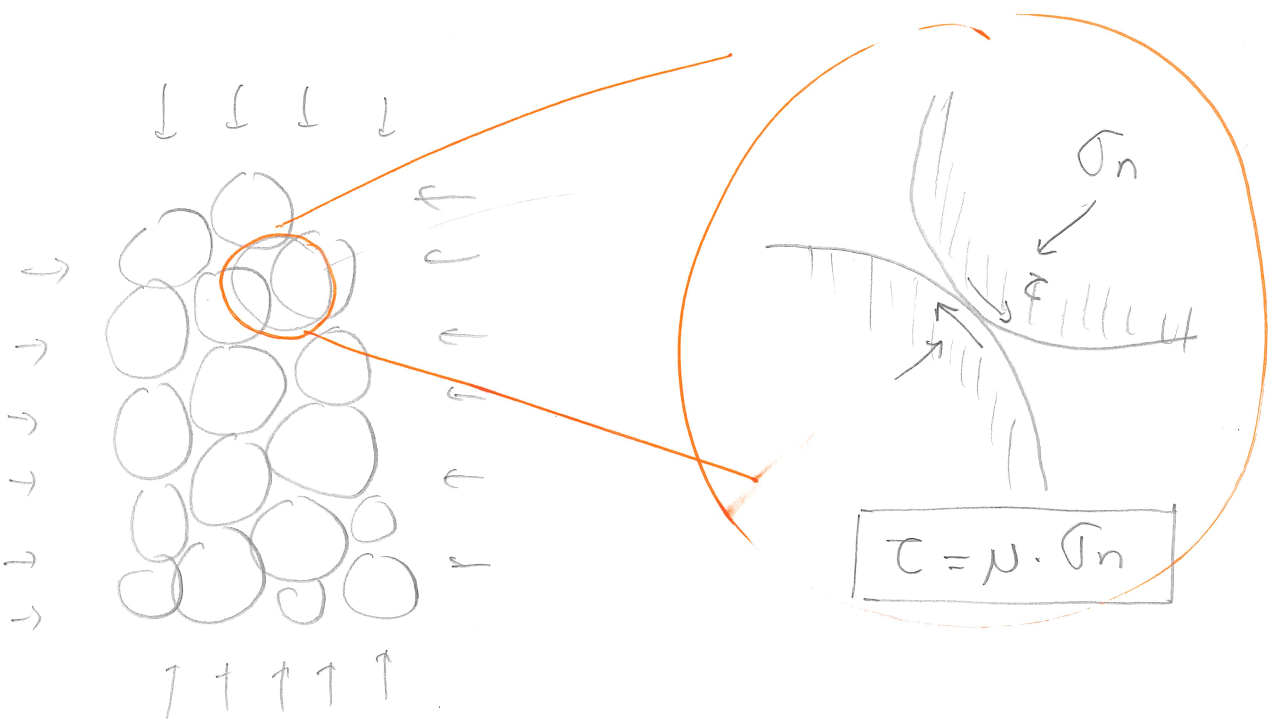




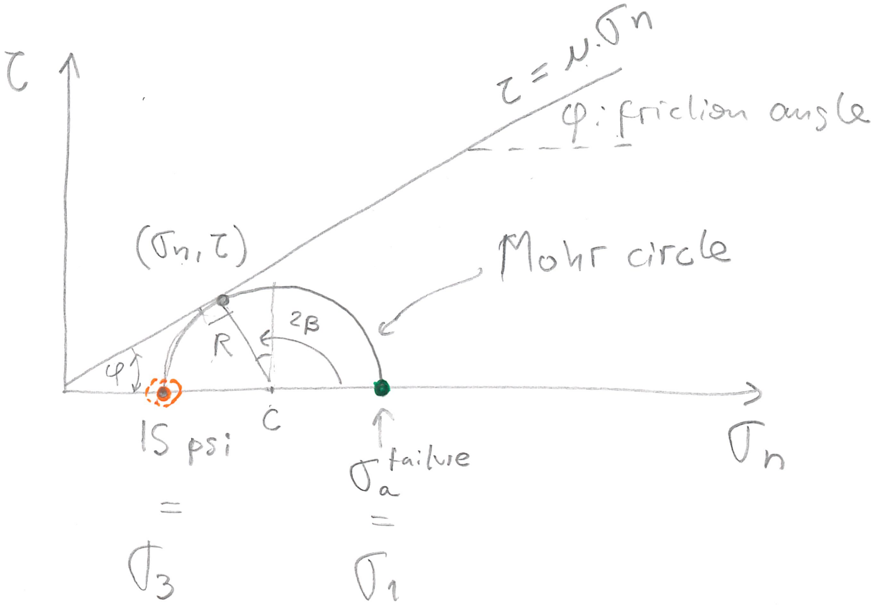
Shear strength

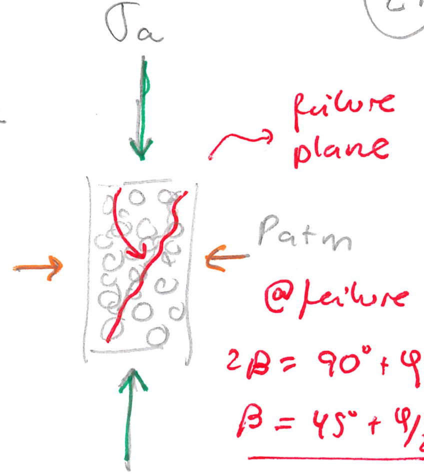






$\sin\varphi = \frac{R}{c}$



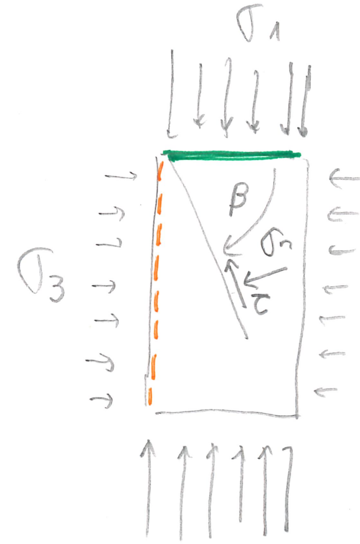


$2\beta = 90° + \varphi$

$\beta = 45° + \varphi/2$

$\varphi = 30°$

$\beta_{fail} = 60°$



$\frac{\sigma_1}{\sigma_3}$ @ failure ?

$$\frac{\sigma_1}{\sigma_3} = \frac{c+R}{c-R} = \frac{\cancel{c} + \cancel{c}\sin\varphi}{\cancel{c} - \cancel{c}\sin\varphi}$$

stress anisotropy

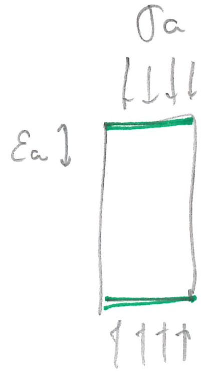
$$\frac{\sigma_1}{\sigma_3} = \frac{1+\sin\varphi}{1-\sin\varphi}$$

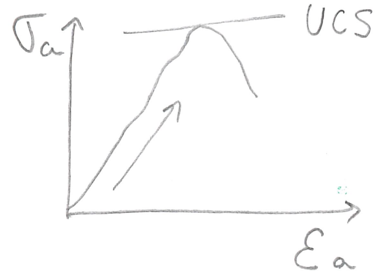
φ : friction angle

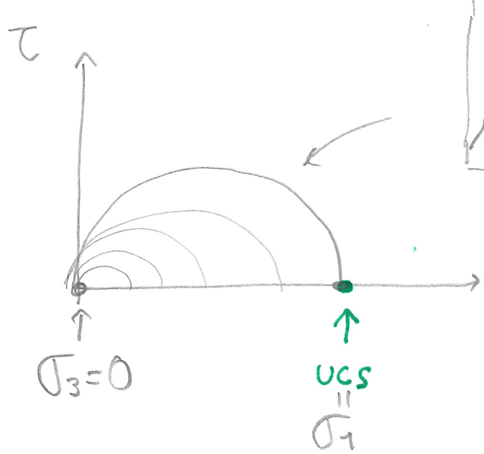
$\sim 30°$ (typical)

$$\varphi = 30° \Rightarrow \frac{\sigma_1}{\sigma_3} = 3 \quad \rightarrow \sigma_3 = \frac{\sigma_1}{3}$$

Unconfined compression strength

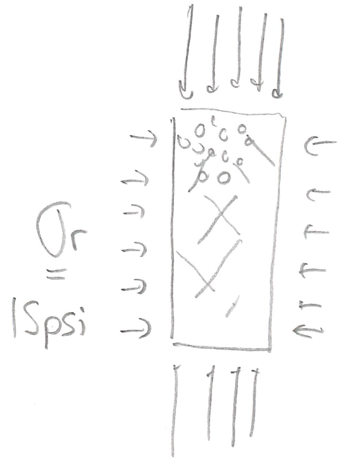


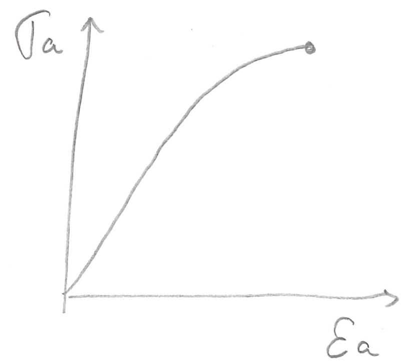


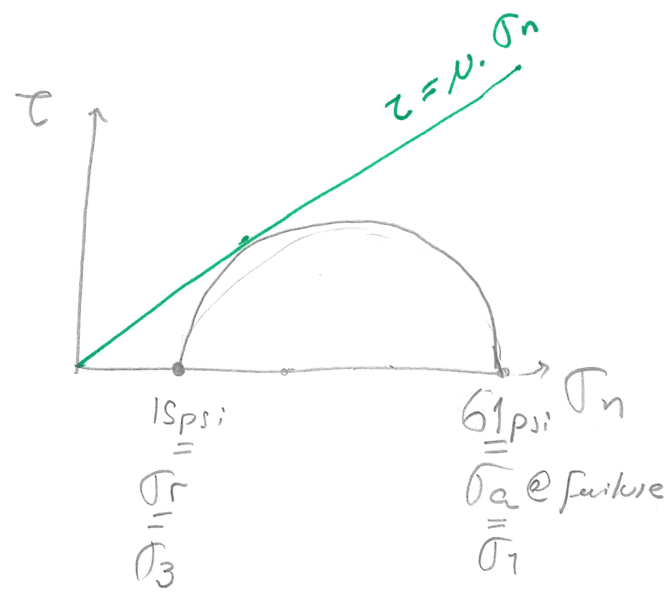


Confined test (Triaxial test)

$\sigma_a = 62$ psi @ failure





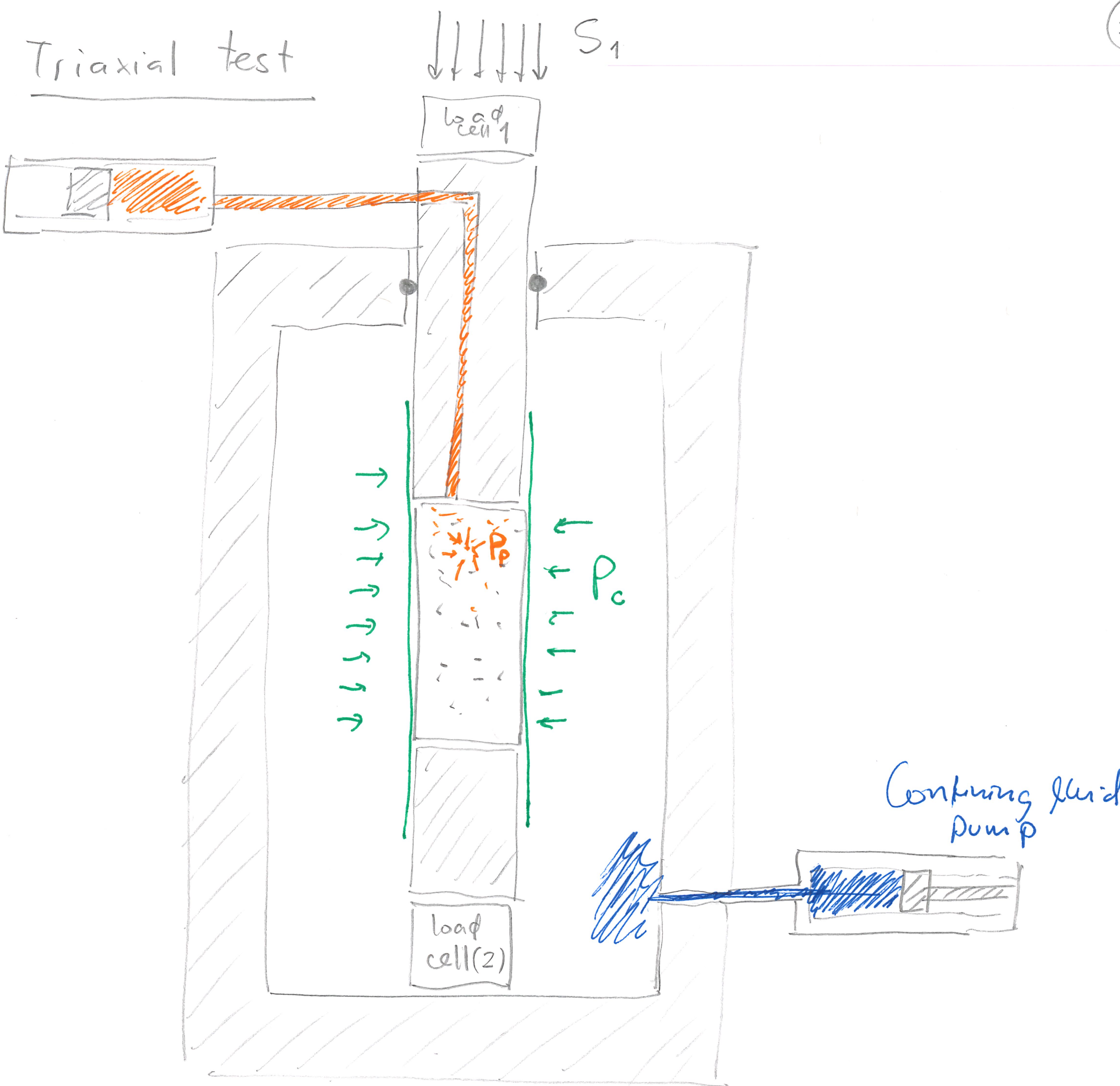


$$\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin\varphi}{1 - \sin\varphi}$$

$$\rightarrow \varphi = \sin^{-1}\left(\frac{\frac{\sigma_1}{\sigma_3} - 1}{\frac{\sigma_1}{\sigma_3} + 1}\right)$$

$$\rightarrow 36^\circ$$

Triaxial test



total stress

Effective stresses
$$\sigma_3 = \widetilde{P_c} - P_p \qquad (S_3 = P_c)$$
$$\sigma_1 = S_1 - P_p \longrightarrow S_1 = F/A \ (\text{Load cell } 1)$$

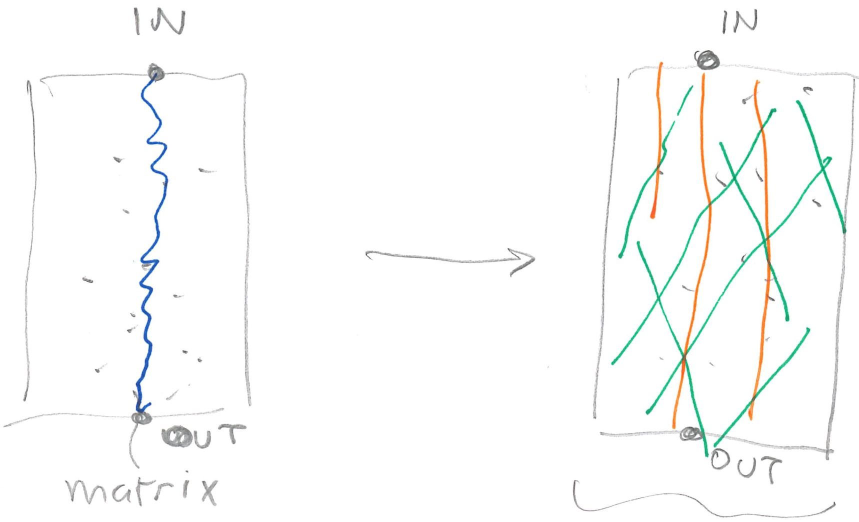
$\underbrace{S_1 - S_3}$ (Load cell 2)

└ Deviatoric stress

↳ $\sigma_1 - \sigma_3 = \sigma_D$

$$\underline{\sigma_1 = \sigma_3 + \sigma_D}$$

Rock failure and permeability



matrix permeability

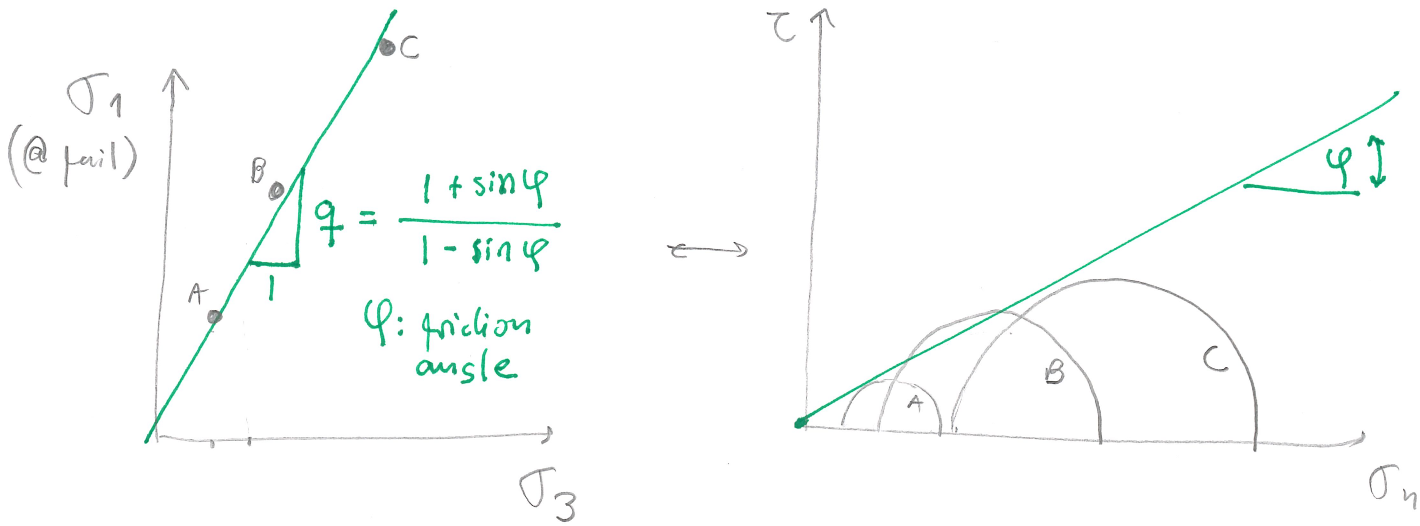
fracture permeability + matrix permeability

HW #5

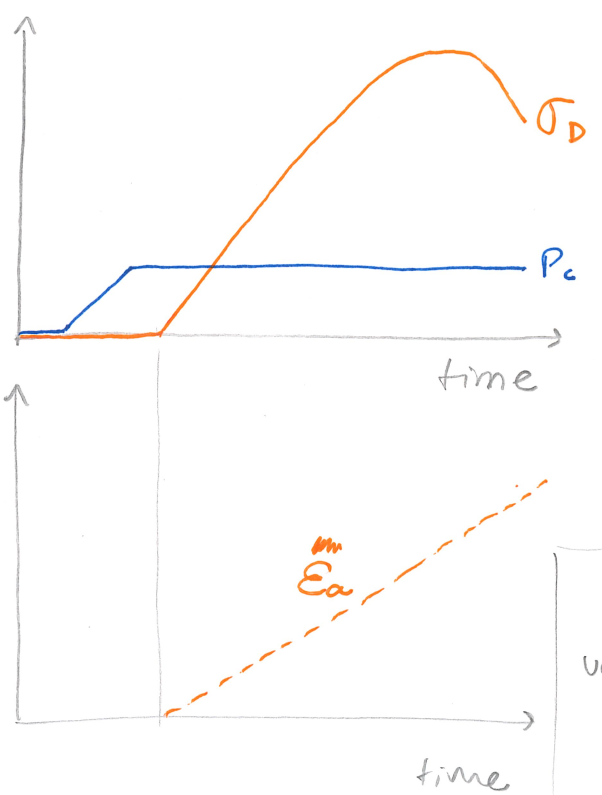
$S_D = \sigma_D$

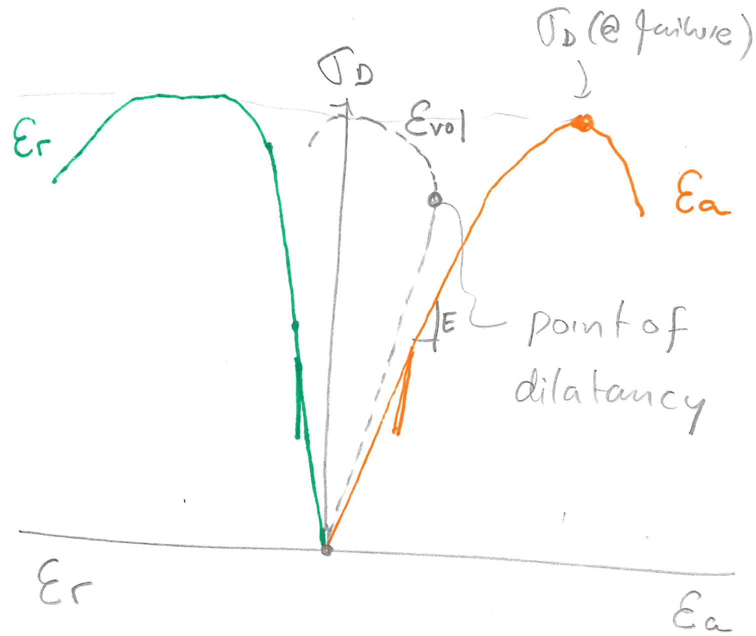
1) $\sigma_3 = P_c - 0$

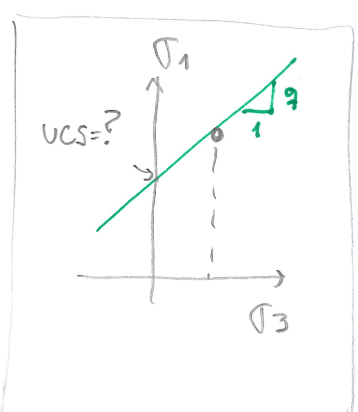
$\sigma_1 = P_c + S_D$

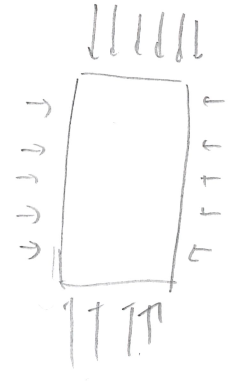


2) Triaxial test









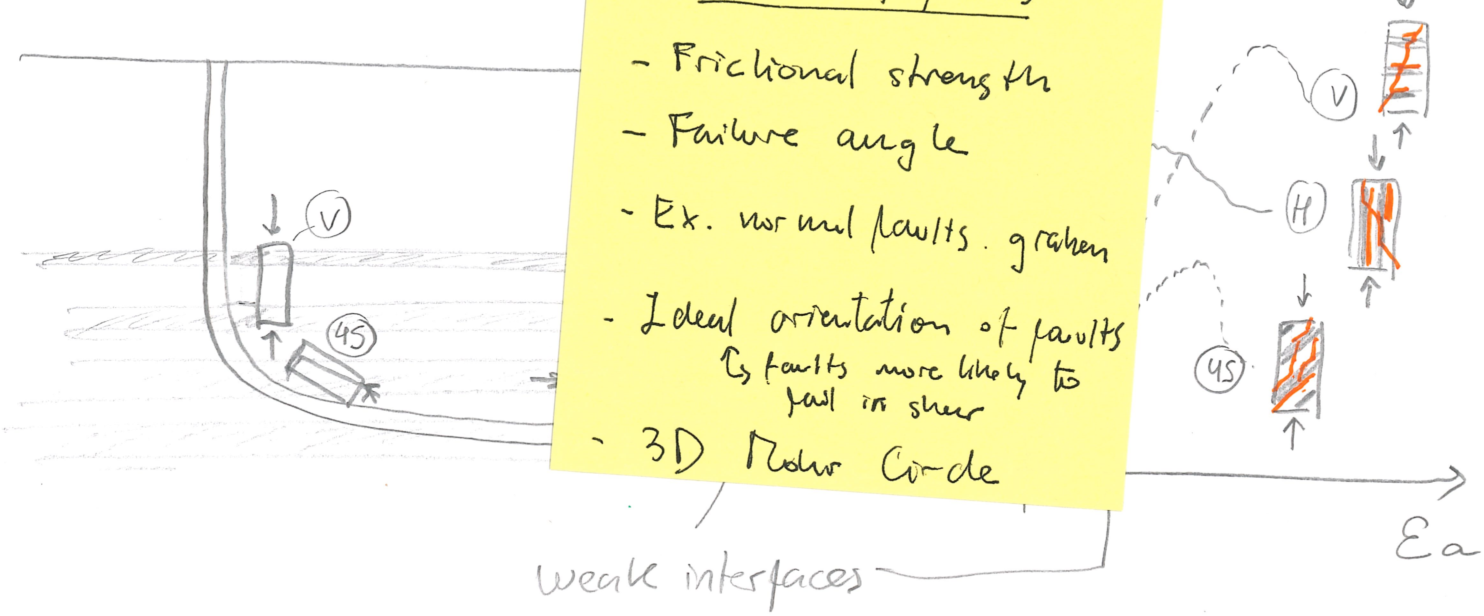
$\varepsilon_{vol} =$

$\varepsilon_1 + \varepsilon_2 + \varepsilon_3$

$= \underbrace{\varepsilon_a + 2\varepsilon_r}_{\text{triaxial test}}$

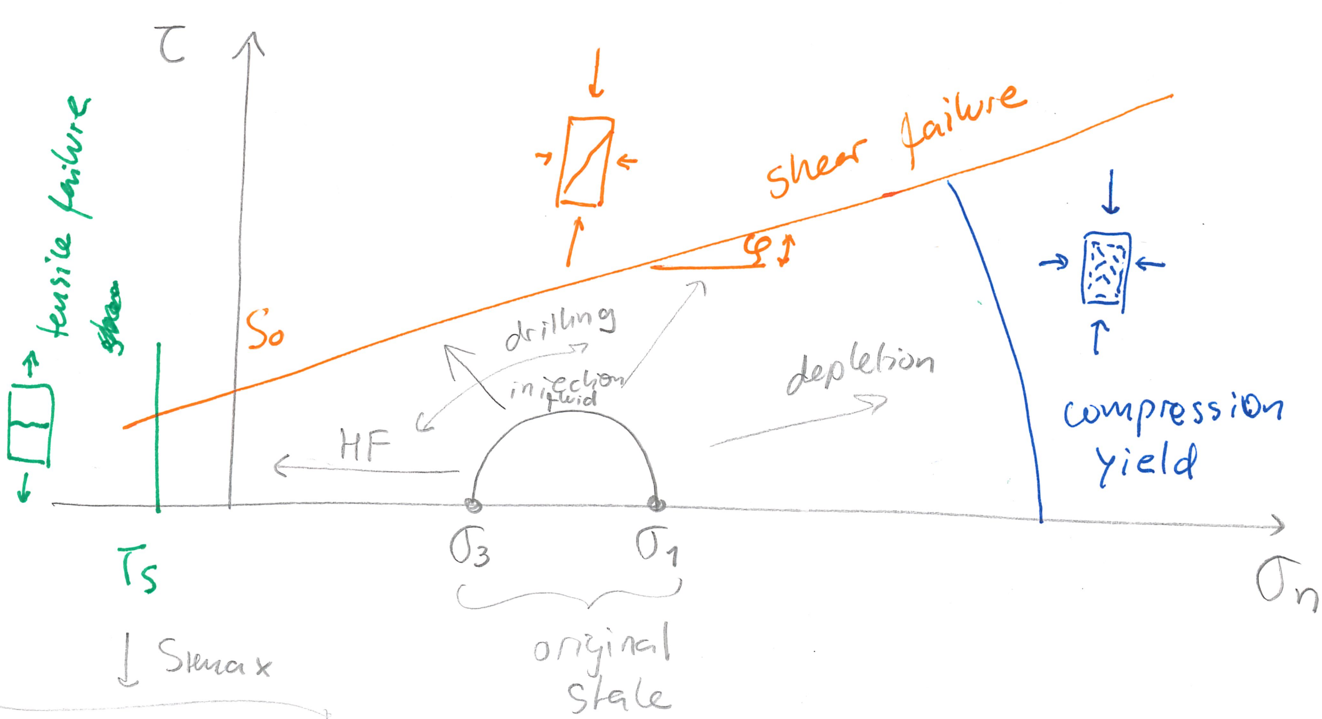
Strength Anisotropy

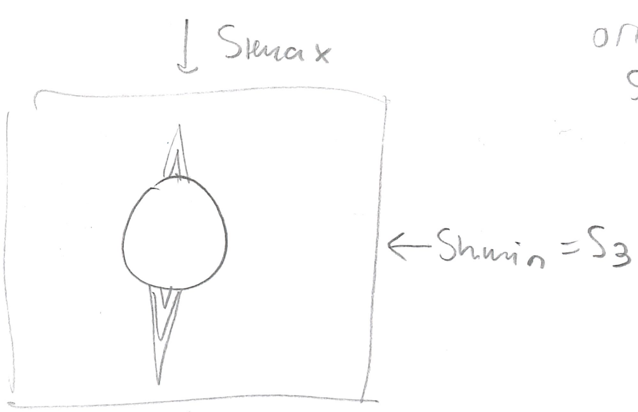
(... elastic ... py)



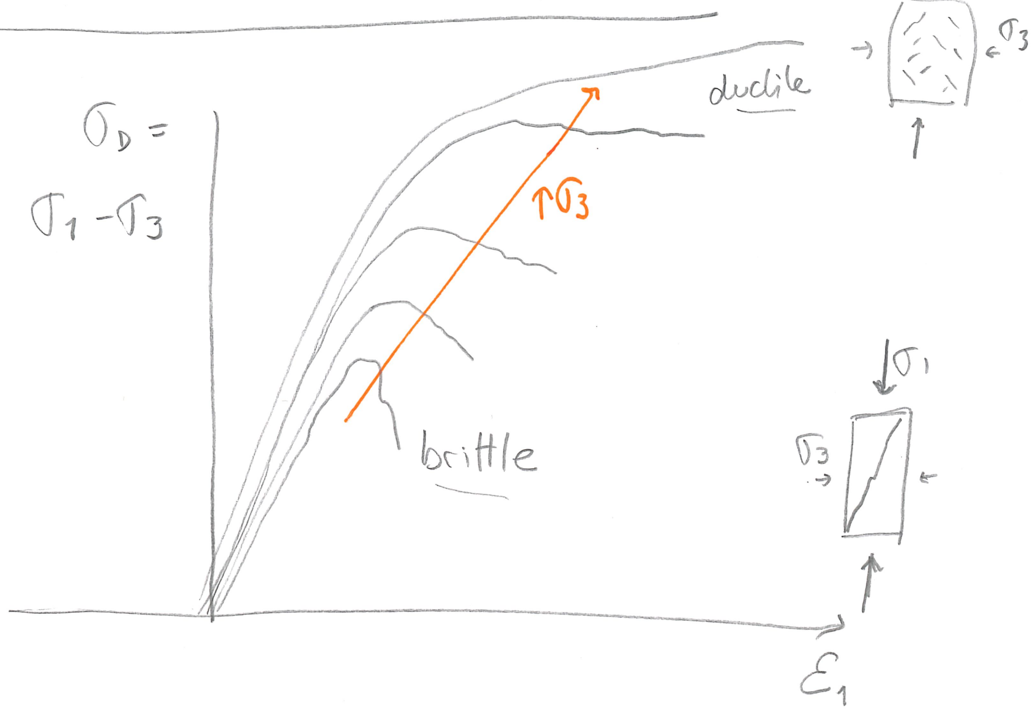
Strength of faults

- Frictional strength
- Failure angle
- Ex. normal faults. graben
- Ideal orientation of faults
  ↳ faults more likely to fail in shear
- 3D Mohr Circle



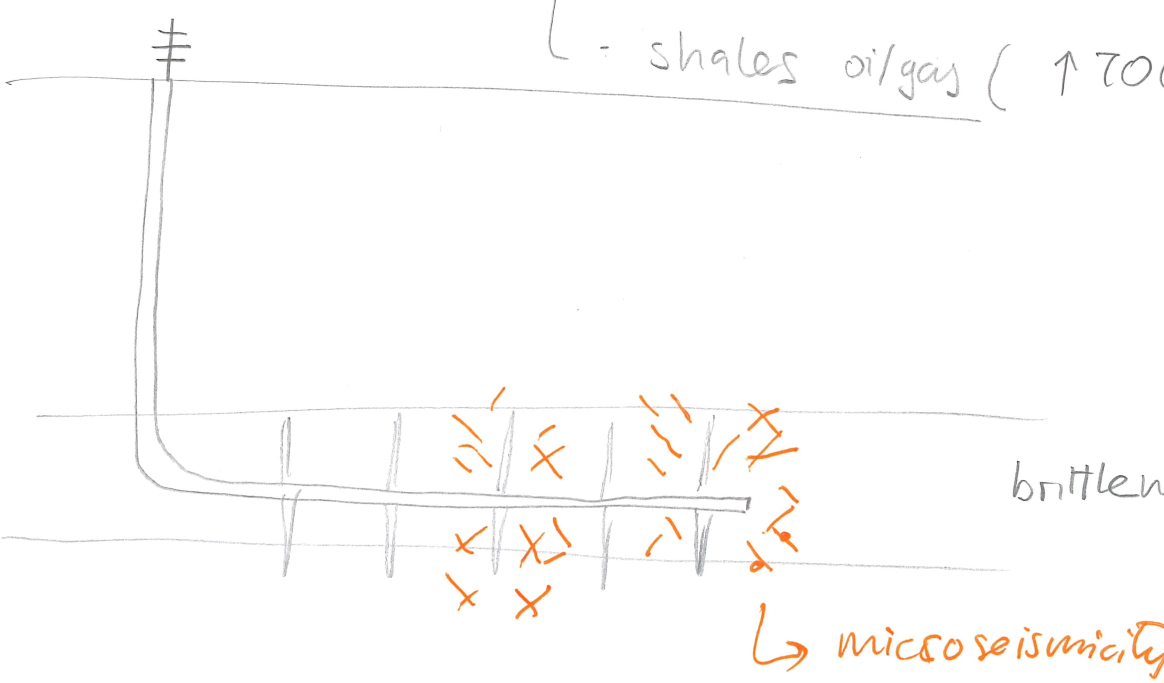


Brittle to ductile failure



factors affecting brittleness

- mean effective stress → $\uparrow \sigma_3 \Rightarrow \uparrow$ ductility
  (depth)
- temperature → $\uparrow T \Rightarrow \uparrow$ ductility
- time → $\uparrow \Delta t = \uparrow$ ductility
- shales oil/gas (↑ TOC, ↑ clay ⇒ ↑ ductility)



brittleness $\sim \dfrac{E}{\nu}$