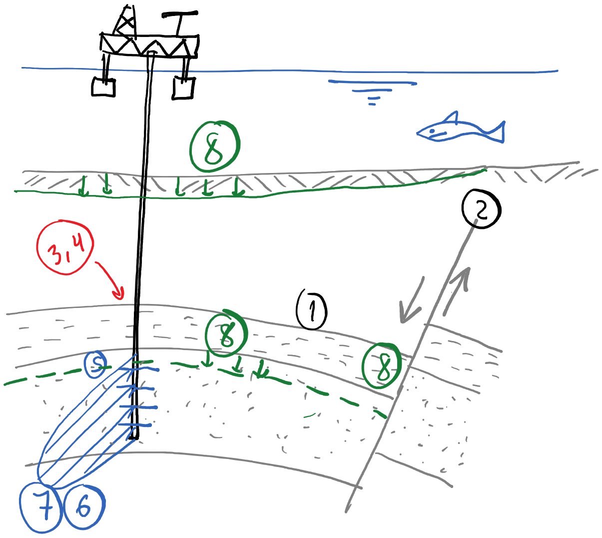
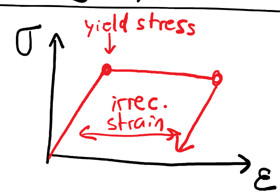
Inelasticity in subsurface engineering applications

Monday, October 26, 2020 5:17 PM



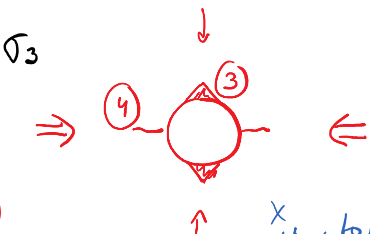


Inelasticity
- limits max stresses
- strength passed yield stress
- permanent strain

- **Exploration**
  - folding (1)
  - faulting (2)

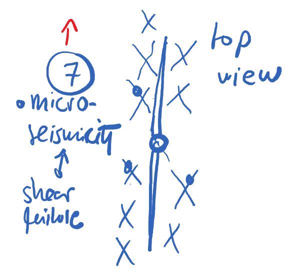
    $\sigma_1 = \frac{1+\sin\varphi}{1-\sin\varphi}\,\sigma_3$

- **Drilling**
  - well bore cuttings
  - breakouts (3) and tensile fractures (4)



- **Production**
  - reservoir compaction (8)
    - → fault reactivation
  - injection
  - sand production

- **Completion**
  - perforation (5)
  - hydraulic fracture (6)
  - fracture react → microseism. (7)



Stress-insensitive yield criteria

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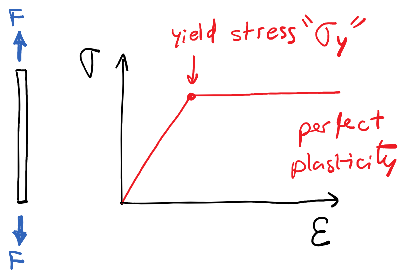
Yield criteria insensitive to mean (effective) stress



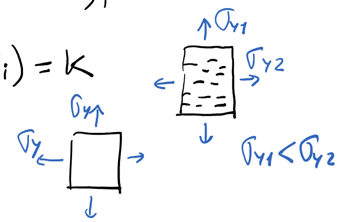
$f(\underline{\sigma}) = K \rightarrow$ yield criterion, failure criterion

$f(\sigma_1, \sigma_2, \sigma_3, \text{direction } \sigma_i) = K$

$\Downarrow$ isotropy

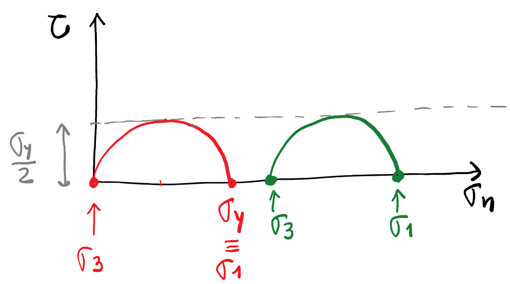
$f(\sigma_1, \sigma_2, \sigma_3) = K$

$f(I_1, I_2, I_3) = K^*$



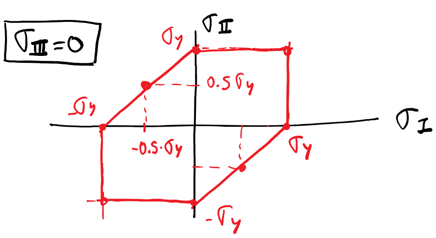
Tresca

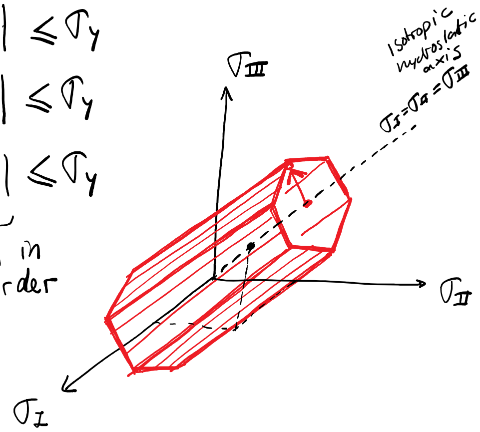
$$\boxed{\sigma_1 - \sigma_3 \leq \sigma_y}$$



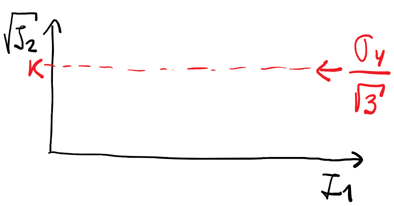
$$\left.\begin{array}{l} |\sigma_I - \sigma_{II}| \leq \sigma_y \\ |\sigma_I - \sigma_{III}| \leq \sigma_y \\ |\sigma_{II} - \sigma_{III}| \leq \sigma_y \end{array}\right.$$

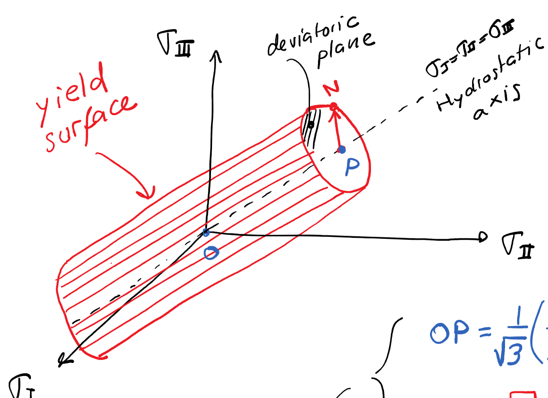
not ordered in particular order





Von Mises





$$J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right]$$

$\sigma_1 = \sigma_y \; ; \quad \sigma_2 = \sigma_3 = 0$

$$J_2 = \frac{1}{6}\left[2\sigma_y^2\right]$$

$$\boxed{\sqrt{J_2} = \frac{\sigma_y}{\sqrt{3}}} \longrightarrow PN = \underbrace{\sqrt{\frac{2}{3}} \cdot \sigma_y}_{\text{Von Mises stress}}$$

$$\left\{\begin{array}{l} OP = \frac{1}{\sqrt{3}}\left(\frac{I_1}{3}\right) \\ PN = \sqrt{2}\,\sqrt{J_2} \end{array}\right.$$

DEMO

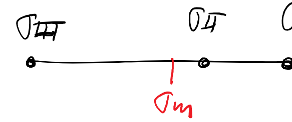
For $\underline{\sigma} = \frac{I_1}{3} \cdot \underline{\underline{I}} + \underline{\underline{S_d}}$ where $\underline{\sigma} = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \end{bmatrix}$; $\underline{\underline{S_d}} = \begin{bmatrix} \sigma_I - \sigma_m & 0 & 0 \\ 0 & \sigma_{II} - \sigma_m & 0 \end{bmatrix}$

For $\underline{\underline{\sigma}} = \frac{I_1}{3} \cdot \underline{\underline{I}} + \underline{\underline{Sd}}$ where $\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix}$; $\underline{\underline{Sd}} = \begin{bmatrix} \sigma_I - \sigma_m & 0 & 0 \\ 0 & \sigma_{II} - \sigma_m & 0 \\ 0 & 0 & \sigma_{III} - \sigma_m \end{bmatrix}$

$$\overline{OP} \stackrel{def}{=} \sqrt[2]{\sigma_m^2 + \sigma_m^2 + \sigma_m^2}$$ // Euclidean distance

$$\overline{OP} = \sqrt[2]{3\left(\frac{I_1}{3}\right)^2} = \sqrt{3} \cdot \frac{I_1}{3}$$

$$\begin{cases} \overline{PN} \stackrel{def}{=} \sqrt[2]{(\sigma_I - \sigma_m)^2 + (\sigma_{II} - \sigma_m)^2 + (\sigma_{III} - \sigma_m)^2} \\ J_2\,(\underline{\underline{S}}d) \stackrel{def}{=} \frac{1}{6}\left[(\sigma_I - \sigma_{II})^2 + (\sigma_I - \sigma_{III})^2 + (\sigma_{II} - \sigma_{III})^2\right] \end{cases}$$



$$J_2 = \frac{1}{6}\Big[(\sigma_I - \sigma_m)^2 + (\sigma_{II} - \sigma_m)^2 - 2(\sigma_I - \sigma_m)(\sigma_{II} - \sigma_m) +$$
$$(\sigma_I - \sigma_m)^2 + (\sigma_{III} - \sigma_m)^2 - 2(\sigma_I - \sigma_m)(\sigma_{III} - \sigma_m)$$
$$(\sigma_{II} - \sigma_m^2) + (\sigma_{III} - \sigma_m)^2 - 2(\sigma_{II} - \sigma_m)(\sigma_{III} - \sigma_m)\Big]$$

$$(\sigma_I - \sigma_{III})^2 = (\sigma_I - \sigma_m + \sigma_m - \sigma_{III})^2$$
$$= \left[(\sigma_I - \sigma_m) - (\sigma_{III} - \sigma_m)\right]^2$$
$$= (\sigma_I - \sigma_m)^2 - 2(\sigma_I - \sigma_m)(\sigma_{III} - \sigma_m) + (\sigma_{III} - \sigma_m)^2$$

$$J_2 = \frac{2}{6}\Big[(\sigma_I - \sigma_m)^2 + (\sigma_{II} - \sigma_m)^2 + (\sigma_{III} - \sigma_m)^2 - (\sigma_I - \sigma_m)(\sigma_{II} - \sigma_m)$$
$$- (\sigma_I - \sigma_m)(\sigma_{III} - \sigma_m) - (\sigma_{II} - \sigma_m)(\sigma_{III} - \sigma_m)\Big]$$

$$J_2 = \frac{2}{6}\Big\{(\sigma_I - \sigma_m)^2 + (\sigma_{II} - \sigma_m)^2 + (\sigma_{III} - \sigma_m^2) - \left[\sigma_I\sigma_{II} - \sigma_I\sigma_m - \sigma_{II}\sigma_m + \sigma_m^2\right]$$
$$- \left[\sigma_I\sigma_{III} - \sigma_I\sigma_m - \sigma_{III}\sigma_m + \sigma_m^2\right]$$
$$- \left[\sigma_{II}\sigma_{III} - \sigma_{II}\sigma_m - \sigma_{III}\sigma_m + \sigma_m^2\right]\Big\}$$

$$\underset{\substack{\uparrow \\ \text{vector}}}{\underline{OP}} = \left(\frac{\sqrt{3}}{3}I_1, \frac{\sqrt{3}}{3}I_1, \frac{\sqrt{3}}{3}I_1\right) = \left(\sqrt{3}\sigma_m, \sqrt{3}\sigma_m, \sqrt{3}\sigma_m\right)$$

$$\underline{\sigma} = (\sigma_I, \sigma_{II}, \sigma_{III})$$

$$\frac{\sqrt{3}}{3} - 1 = \frac{\sqrt{3} - 3}{3}$$

$$\underline{\sigma} = \underline{OP} + \underline{Sd}$$

$$\underline{Sd} = \underline{\sigma} - \underline{OP} = \left(\sigma_I - \sqrt{3}\sigma_m, \sigma_{II} - \sqrt{3}\sigma_m, \sigma_{III} - \sqrt{3}\sigma_m\right)$$

(1)

$$\|\underline{\sigma}\|^2 = \|\underline{OP}\|^2 + \|\underline{Sd}\|^2$$

$$\|\underline{Sd}\|^2 = \|\underline{\sigma}\|^2 - \|\underline{OP}\|^2$$
$$= (\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2) - (3\sigma_m^2 + 3\sigma_m^2 + 3\sigma_m^2)$$

$$\sigma_m^2 = \left[\frac{(\sigma_I + \sigma_{II} + \sigma_{III})}{3}\right]^2$$

$$\sigma_m^2 = \frac{1}{9}\left(\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 + 2\sigma_I\sigma_{II} + 2\sigma_I\sigma_{III} + 2\sigma_{II}\sigma_{III}\right)$$

$$\|S_d\| = \|\sigma\| - \|\sigma_m\|$$

$$= \left(\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2\right) - \left(3\sigma_m^2 + 3\sigma_m^2 + 3\sigma_m^2\right)$$

$$
\begin{aligned}
= {} & \sigma_I^2 + \sigma_m^2 - 2\sigma_I\sigma_m \\
& \sigma_{II}^2 + \sigma_m^2 - 2\sigma_{II}\sigma_m \\
& \sigma_{III}^2 + \sigma_m^2 - 2\sigma_{III}\sigma_m \\
& - 9\sigma_m^2 - 3\sigma_m^2 + 2\sigma_I\sigma_m + 2\sigma_{II}\sigma_m + 2\sigma_{III}\sigma_m
\end{aligned}
$$

$$= (\sigma_I - \sigma_m)^2 + (\sigma_{II} - \sigma_m)^2 + (\sigma_{III} - \sigma_m)^2 - 12\sigma_m^2 + 2\sigma_I\sigma_m + 2\sigma_{II}\sigma_m + 2\sigma_{III}\sigma_m$$

$$\|S_d\|^2 = \left(\sigma_I - \sqrt{3}\,\sigma_m\right)^2 + \left(\sigma_{II} - \sqrt{3}\,\sigma_m\right)^2 + \left(\sigma_{III} - \sqrt{3}\,\sigma_m\right)^2$$

$$= \sigma_I^2 - 2\sigma_I\sqrt{3}\,\sigma_m + 3\sigma_m^2 + \sigma_{II}^2 - 2\sqrt{3}\,\sigma_{II}\sigma_m + 3\sigma_m^2 + \sigma_{III}^2 - 2\sqrt{3}\,\sigma_{III}\sigma_m + 3\sigma_m^2$$

$$= \ldots$$

$$
\|S_d\| = \sqrt[2]{
\begin{aligned}
& \left[\left(\tfrac{\sqrt{3}-3}{3}\sigma_I\right)^2 + \left(\tfrac{\sqrt{3}}{3}\sigma_{II}\right)^2 + \left(\tfrac{\sqrt{3}}{3}\sigma_{III}\right)^2 + 2\left(\tfrac{\sqrt{3}-3}{3}\right)\sigma_I\left(\tfrac{\sqrt{3}}{3}\right)\sigma_{II} + 2\left(\tfrac{\sqrt{3}}{3}\right)\sigma_I\left(\tfrac{\sqrt{3}-3}{3}\right)\sigma_{II} + 2\left(\tfrac{\sqrt{3}}{3}\right)^2\sigma_{II}\sigma_{III}\right) \\
& + \qquad + \\
& \left(\tfrac{\sqrt{3}}{3}\sigma_I\right)^2 + \left(\tfrac{\sqrt{3}-3}{3}\right)\sigma_{II}^2 + \left(\tfrac{\sqrt{3}}{3}\sigma_{III}\right)^2 + 2\left(\tfrac{\sqrt{3}}{3}\right)\sigma_I\left(\tfrac{\sqrt{3}-3}{3}\right)\sigma_{II} + 2\ldots \\
& \left(\tfrac{\sqrt{3}}{3}\sigma_I\right)^2 + \left(\tfrac{\sqrt{3}}{3}\sigma_{II}\right)^2 + \left(\tfrac{\sqrt{3}-3}{3}\sigma_{III}\right)^2 + \ldots 2\sigma_I\sigma_{II}\left(\tfrac{\sqrt{3}}{3}\right)^2 + \ldots + \ldots
\end{aligned}
}
$$

$$= \frac{\sqrt{3}}{3}\left(3\frac{\sqrt{3}}{3} - 2\right)$$

$$2\left(\frac{\sqrt{3}}{3}\right)^2 - 2\frac{\sqrt{3}}{3} + \left(\frac{\sqrt{3}}{3}\right)^2 = 3\left(\frac{\sqrt{3}}{3}\right)^2 - \frac{2\sqrt{3}}{3}$$

$$= \sqrt[2]{\left(\sqrt{3} - 1\right)\left(\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2\right) + 2\left(2\left(\frac{\sqrt{3}}{3} - 1\right)\frac{\sqrt{3}}{3} + \left(\frac{\sqrt{3}}{3}\right)^2\right)\left(\sigma_I \cdot \sigma_{II} + \sigma_I\sigma_{III} + \sigma_{II}\sigma_{III}\right)}$$

$$= \sqrt[2]{\sqrt{3} - 1\left(\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2\right) + 2\left(\frac{\sqrt{3}}{3}\left(3\frac{\sqrt{3}}{3} - 2\right)\right)\left(\sigma_I\sigma_{II} + \sigma_I\sigma_{III} + \sigma_{II}\sigma_{III}\right)}$$

Stress-sensitive yield criteria

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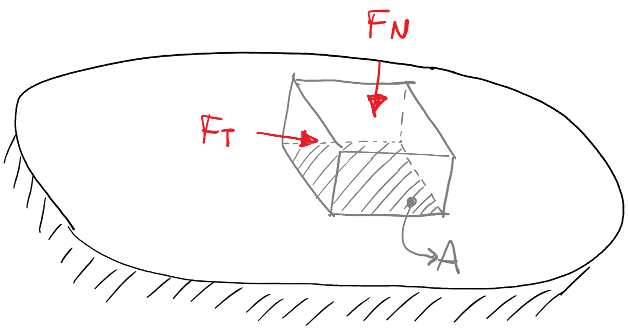
Stress sensitive yield criteria

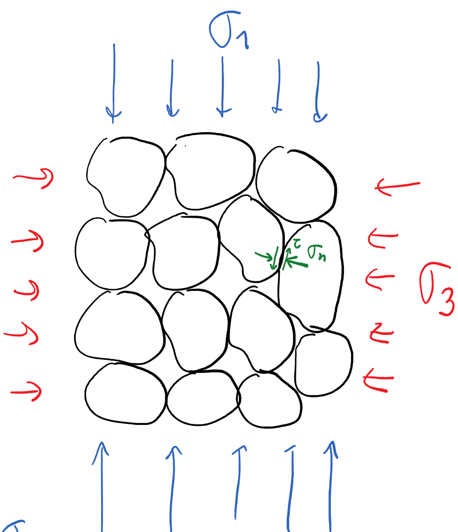
$\hookrightarrow$ Frictional strength

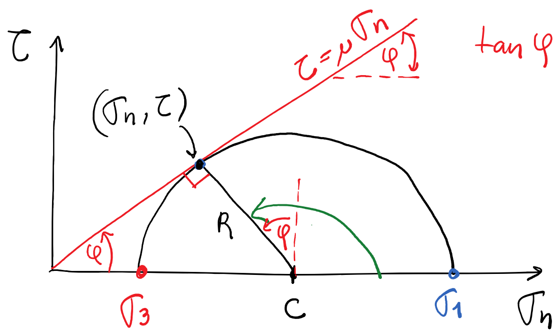


Friction coefficient (~ 0.3 to 1.0)

$$\tau = \mu \cdot \sigma_n$$

$$\frac{F_T}{A} = \mu \frac{F_N}{A}$$



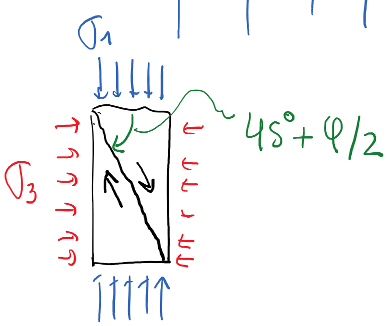


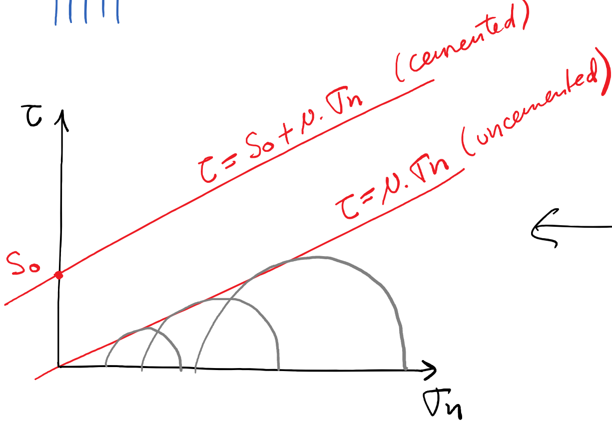
$\tan \varphi = \mu$

$$\frac{\sigma_1}{\sigma_3} = \frac{C + R}{C - R}$$

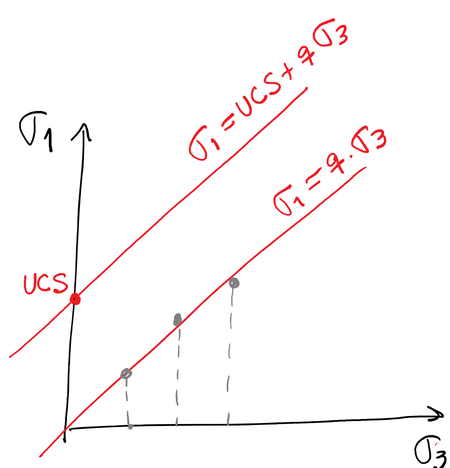
$$\frac{\sigma_1}{\sigma_3} = \frac{\cancel{C} + \sin\varphi \cdot \cancel{C}}{\cancel{C} - \sin\varphi \cdot \cancel{C}} \quad \leftarrow \quad \sin\varphi = \frac{R}{C}$$

$$\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin\varphi}{1 - \sin\varphi} = q; \quad \varphi \sim 30^\circ \Rightarrow q = 3$$





$\longleftrightarrow$



$$\mu = \frac{q - 1}{2 \cdot \sqrt{q}}$$

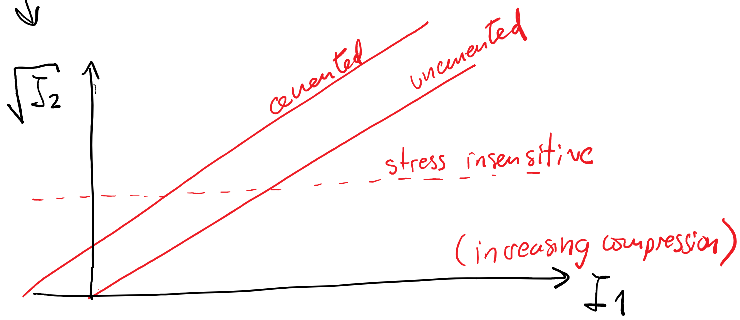
$$UCS = 2 S_0 \sqrt{q}$$

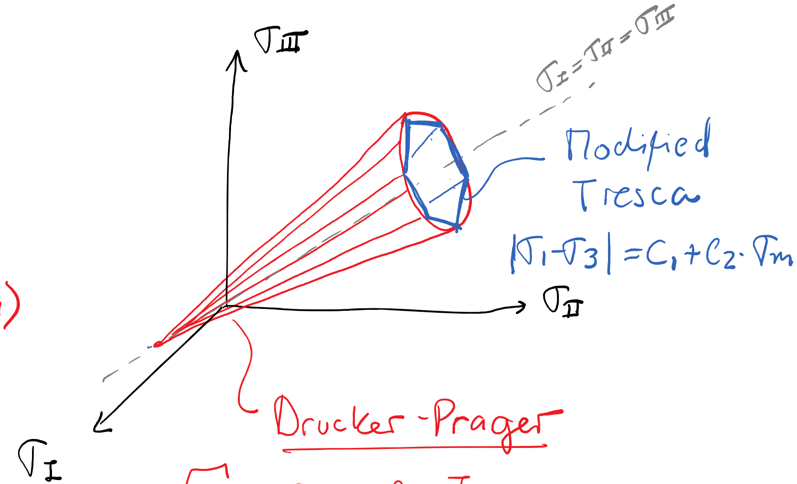
/ Extension to 3D

σ_{III}

$\tau_{II} = \sigma_{III}$

Extension to 3D

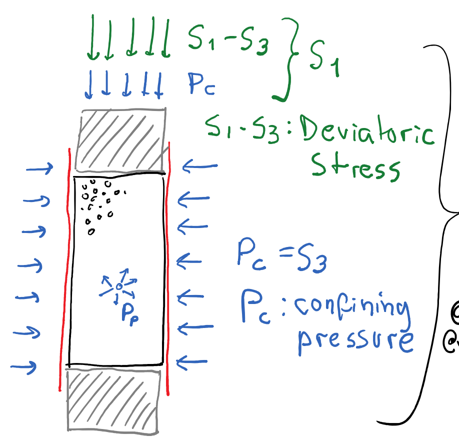




$$\sqrt{J_2} = C_3 + C_4 \cdot I_1$$

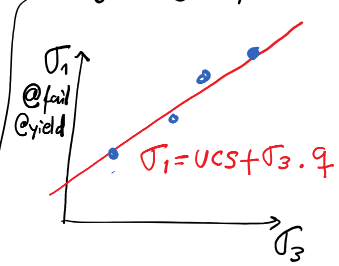
Mohr Coulomb $\Rightarrow$
$$\begin{cases} C_3 = \dfrac{6 \cdot S_0 \cdot \cos\varphi}{\sqrt{3}\,(3-\sin\varphi)} \\[2ex] C_4 = \dfrac{2}{\sqrt{3}} \dfrac{\sin\varphi}{(3-\sin\varphi)} \end{cases}$$

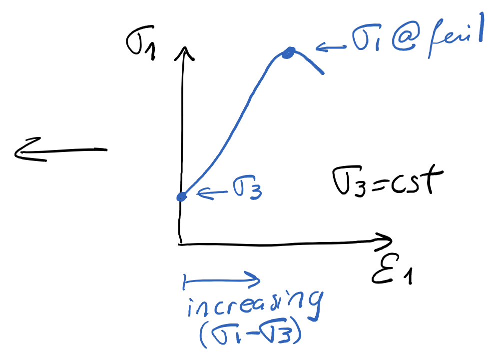
Triaxial tests (axisymmetric)



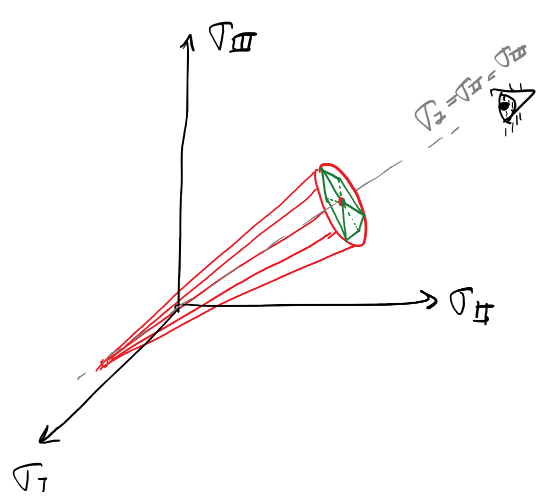
$$S_1 = (S_1 - S_3) + P_c$$

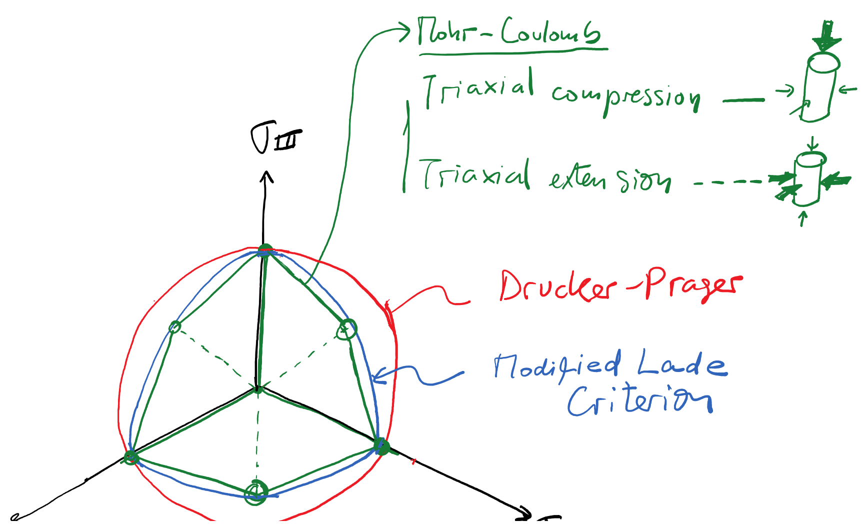
$$\sigma_1 = S_1 - P_p$$
$$\sigma_3 = S_3 - P_p$$

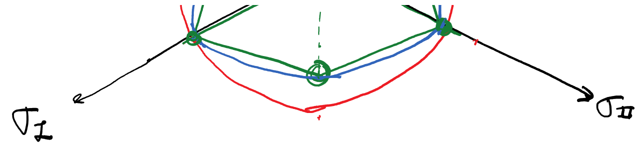




Modified Lade criterion







$$f(I_1, I_3) = K$$

$$\boxed{\frac{(I_1^*)^3}{I_3^*} = 27 + \eta}$$

$$I_1^* = \sigma_1^* + \sigma_2^* + \sigma_3^*$$

$$I_3 = \sigma_1^* \cdot \sigma_2^* \cdot \sigma_3^*$$

$$\sigma_i^* = \sigma_i + S$$

$$S = S_0 / \tan\varphi$$

$$\eta = \frac{4(\tan\varphi)^2(9 - 7\sin\varphi)}{1 - \sin\varphi}$$

Determination of plastic strains: Flow rule

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Beyond the yield point: determination of plastic strains

↳ determination of strains

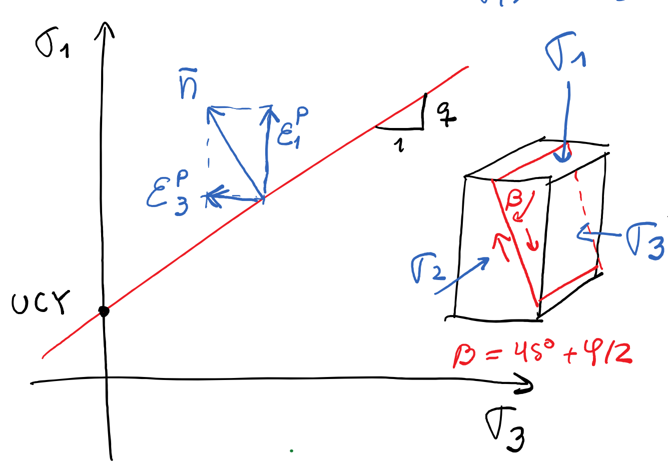
- elastic
$$\begin{cases} \underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{D}}}} \cdot \underline{\underline{\sigma}} \\ \varepsilon_{ij} = D_{ijmn}\,\sigma_{mn} \\ d\varepsilon_{ij} = D_{ijmn}\,d\sigma_{mn} \end{cases}$$
- plastic – $d\varepsilon_{ij}^{p} \leftrightarrow d\sigma_{ij}$

↳
- small strain
- continuous strain field
- strain-rate independent

1. Yield criterion
$$f(\sigma_{ij}) = Y$$
2. Strain-hardening rule
$$Y = F^{*}(\delta\varepsilon^{p})$$
3. Strain decomposition
$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p}$$
4. Plastic flow rule
$$d\varepsilon_{ij}^{p} \leftrightarrow d\sigma_{ij}$$
5. Elastic unloading criterion

Example: Mohr - Coulomb

$\sigma_1 > \sigma_2 > \sigma_3$



$$\tau = S_o + \nu \cdot \sigma_n \;;\quad \nu = \tan\varphi$$
$$\updownarrow$$
$$\boxed{\sigma_1 = UCY + q\,\sigma_3} \;;\quad q = \frac{1+\sin\varphi}{1-\sin\varphi}$$

$\varphi = 30° \Rightarrow q = 3$

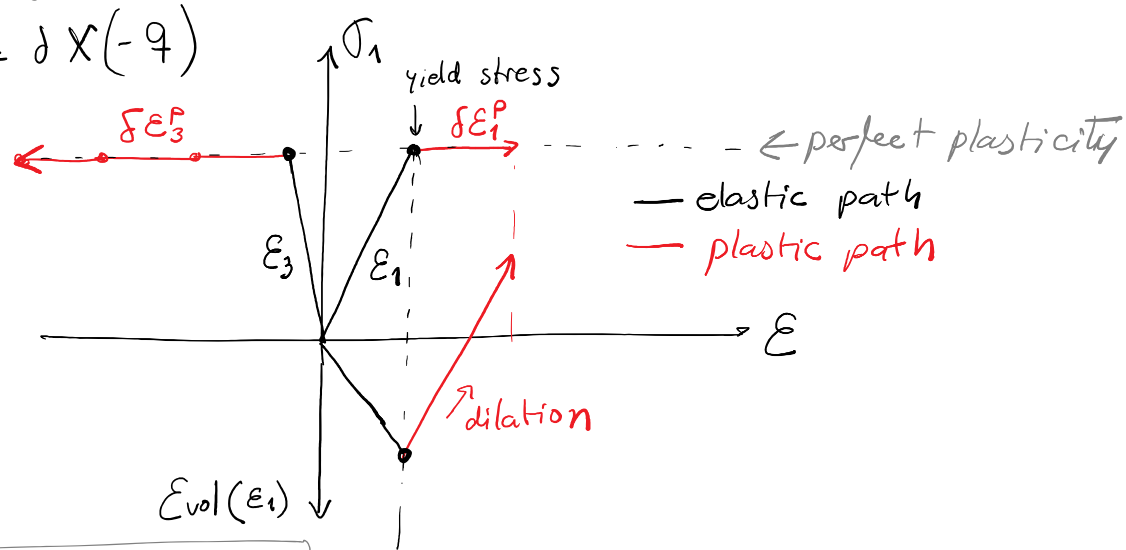
$$\downarrow$$
$$f = \sigma_1 - UCY - q\,\sigma_3$$
$$\downarrow$$
$$\bar{n} = \left(\frac{\partial F}{\partial\sigma_1}, \frac{\partial F}{\partial\sigma_2}, \frac{\partial F}{\partial\sigma_3}\right)$$

$$\bar{n} = \left(\frac{\partial F}{\partial \sigma_1}, \frac{\partial F}{\partial \sigma_2}, \frac{\partial F}{\partial \sigma_3}\right)$$

$$\bar{n} = (1, 0, -q)$$

Flow rule: $d\varepsilon_{ij}^p = \underset{\text{cst}}{\underbrace{d\lambda}} \cdot \dfrac{\partial F}{\partial \sigma_{ij}}$

$$\begin{cases} d\varepsilon_1^p = d\lambda \cdot 1 \\ d\varepsilon_2^p = d\lambda \cdot 0 \\ d\varepsilon_3^p = d\lambda(-q) \end{cases} \Longrightarrow \delta\varepsilon_{vol}^p = d\lambda(1-q)$$



$$\boxed{d\varepsilon_{ij}^p = d\lambda \frac{\partial F}{\partial \sigma_{ij}}}$$ Yield surface

⟹ Associated flow rule
↳ yield surface

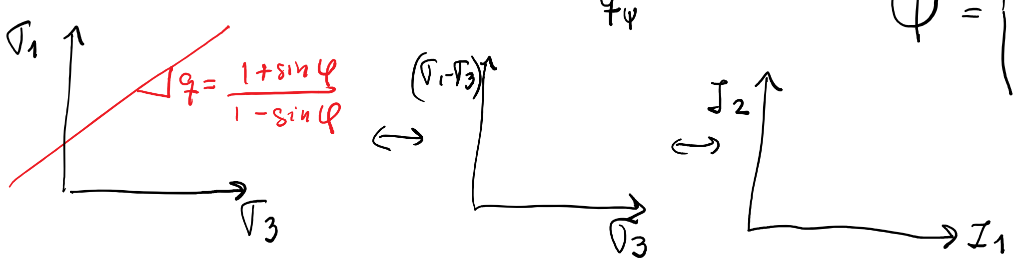
$$\boxed{d\varepsilon_{ij}^p = d\lambda \frac{\partial g}{\partial \sigma_{ij}}}$$ Plastic potential function, $g \neq F$

⟹ Non-associated flow rule

$$g = \sigma_1 - UCY - \underbrace{\frac{1+\sin\psi}{1-\sin\psi}}_{q_\psi}\sigma_3$$

$\psi < \varphi$ ← dilation angle

$$\psi = \begin{cases} \psi > 0 \Rightarrow \text{dilation} \\ \psi = 0 \Rightarrow \text{iso-choric} \\ \psi < 0 \Rightarrow \text{contraction} \end{cases}$$

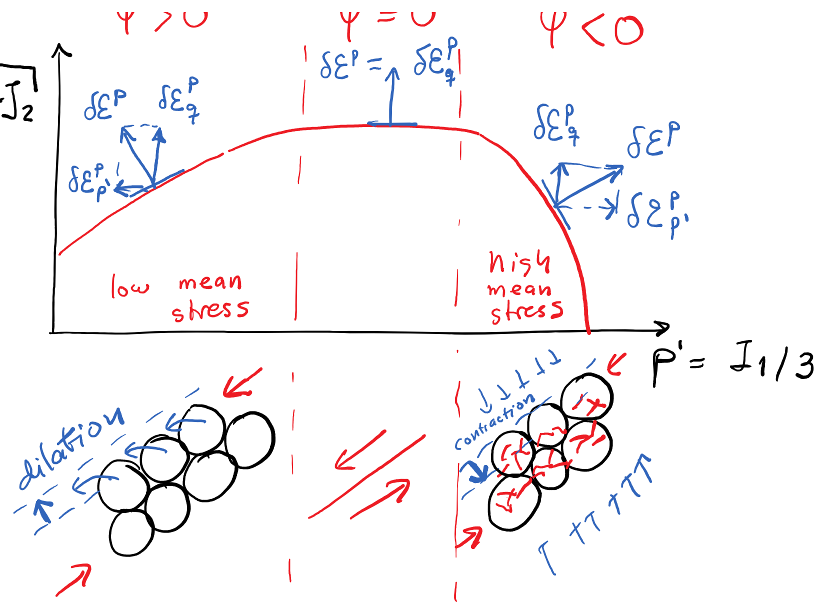


$\psi > 0$ $\psi = 0$ $\psi < 0$

$\boxed{p'-q}$ $q = \dots$ $\delta\varepsilon^p = \delta\varepsilon_q^p$

$\boxed{p'-q \text{ space}}$

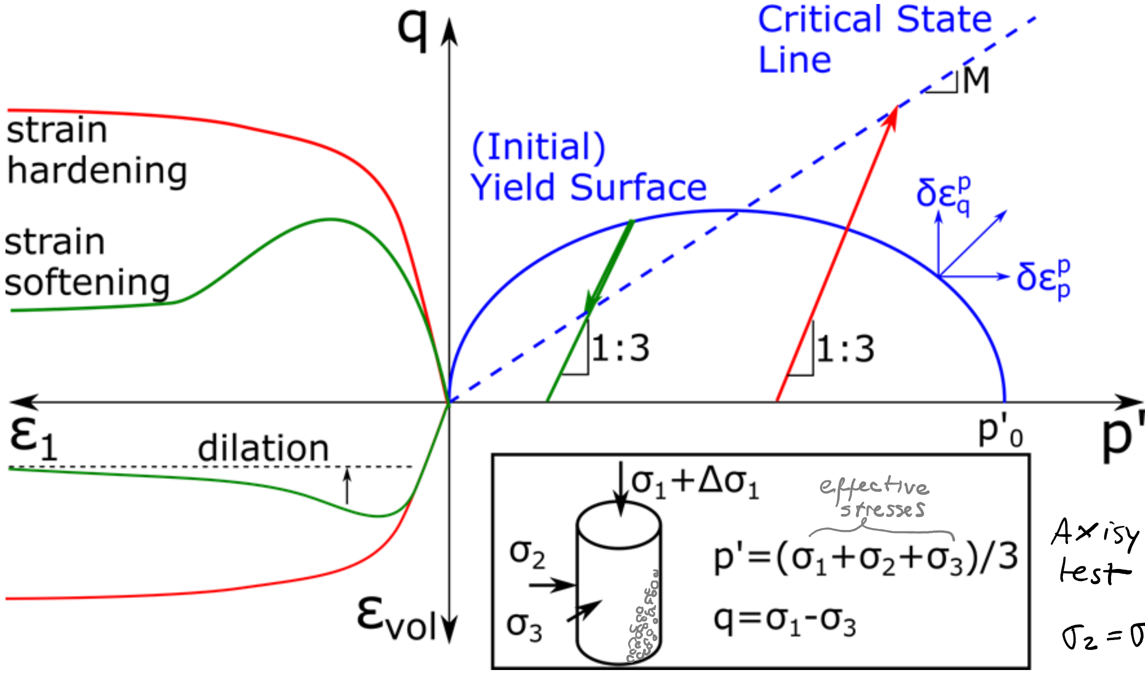
$$q = \sqrt{3 \cdot J_2}$$



$$\boxed{\delta\varepsilon_{ij}^{p} = d\lambda \frac{\partial g}{\partial \sigma_{ij}}}$$

Critical State Soil Mechanics → Cam-clay Model (uncemented sediments)

Monday, November 2, 2020 2:55 PM



Axisymmetric triaxial test (drained)

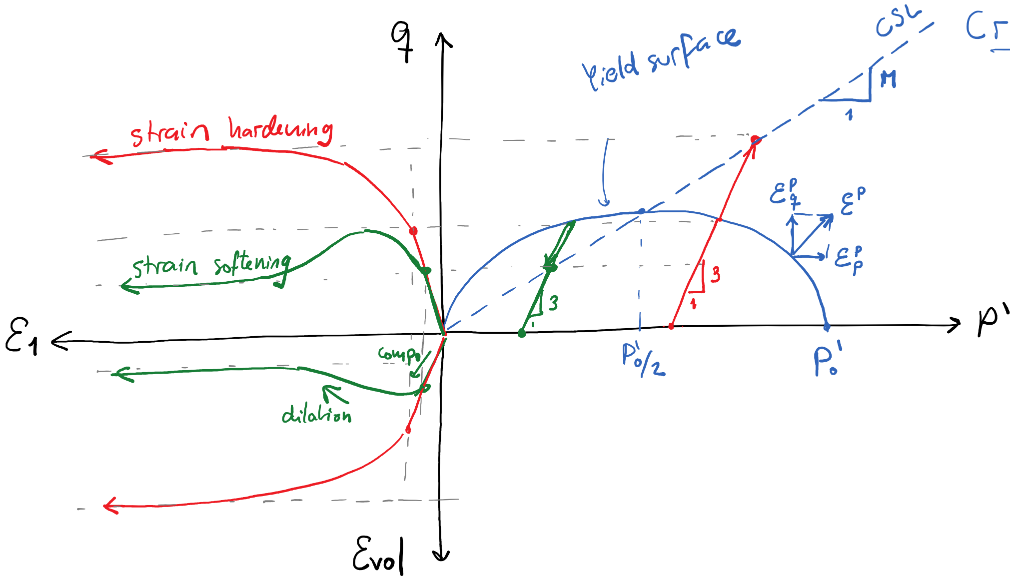
$\sigma_2 = \sigma_3 = cst$ for a given test

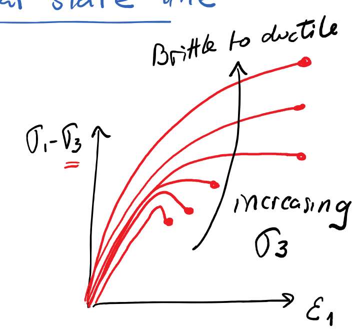
$\Delta\sigma_1 \rightarrow$ until yield

Deviatoric loading

$$\left\{\begin{aligned} \Delta p' &= \frac{\Delta\sigma_1}{3} \\ \Delta q &= \Delta\sigma_1 \end{aligned}\right\} \quad \frac{\Delta q}{\Delta p'} = 3$$

Critical state line





$$q \stackrel{\text{def}}{=} M \cdot p' \quad (@\ CSL)$$

$$M = \frac{q}{p'} = \frac{\sigma_1 - \sigma_3}{\frac{\sigma_1 + 2\sigma_3}{3}} \Longrightarrow M(\varphi_{cs} = 30°) = \frac{2\cancel{\sigma_3}}{\frac{5\cancel{\sigma_3}}{3}} = \frac{6}{5} = 1.2$$

$$\Downarrow$$

$$\sigma_1 = 3\sigma_3$$

$$M \quad 3\left(\frac{1+\sin\varphi_{cs}}{1-\sin\varphi_{cs}} - 1\right)$$

$\sigma_1 = 3\sigma_3$

$$\boxed{M = \frac{3\left(\frac{1+\sin\varphi_{cs}}{1-\sin\varphi_{cs}}-1\right)}{2+\left(\frac{1+\sin\varphi_{cs}}{1-\sin\varphi_{cs}}\right)}}$$

Yield surface

$$f(q, p', \underset{\sim}{p_0'}) = q^2 - M^2 p'(p_0'-p') = 0$$

$q = 0 \Rightarrow \begin{cases} p' = 0 \\ p' = p_0' \end{cases}$

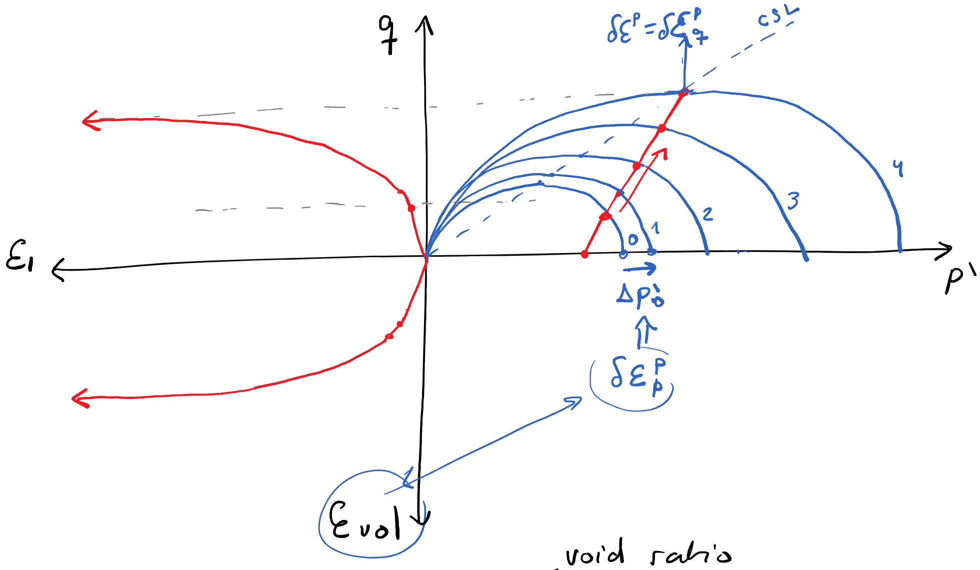
$q = Mp' \Rightarrow p' = \frac{p_0'}{2}$

→ pre-consolidation pressure (stress)

↳ variable

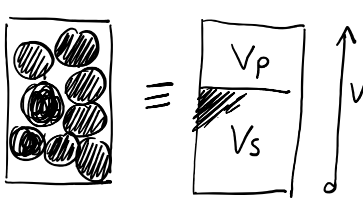
↳ size of the yield surface

↳ hardening parameter $\Leftrightarrow f(\delta\varepsilon_p^p) \sim f(\phi)$



void ratio

$$\underline{p_0'} = f(\delta\varepsilon_p^p) = f^*\left(e = \frac{\phi}{1-\phi}\right)$$

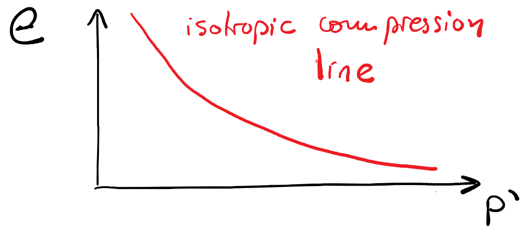


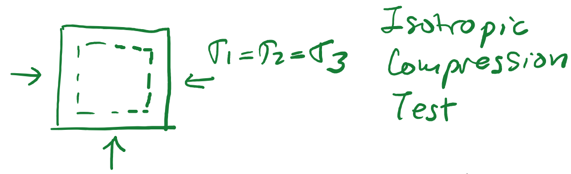
$\phi \overset{def}{=} \frac{V_p}{V_T}$

$e \overset{def}{=} \underline{V_p}$

$V_T = 1$

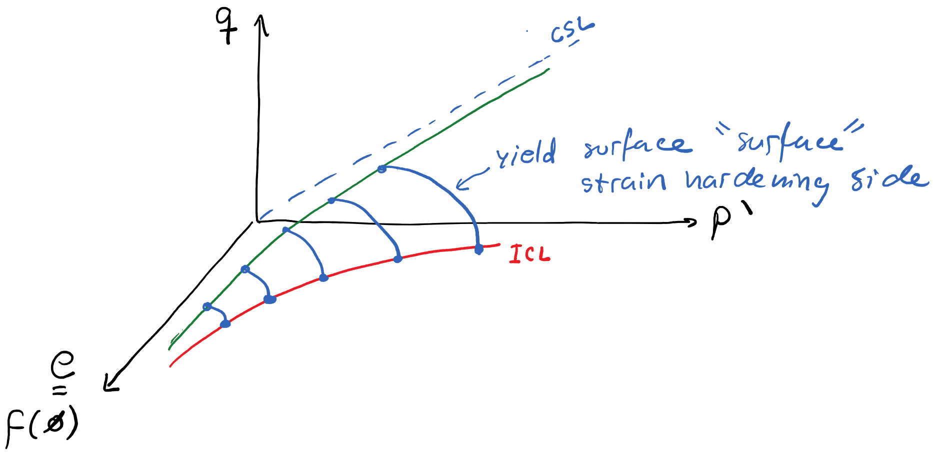
$\Rightarrow e = \frac{V_p}{1-V_p} = \frac{\phi}{1-\phi}$





$$\Rightarrow \ \downarrow \phi \approx \downarrow e \Rightarrow \uparrow p_0'$$

$$\epsilon = \varepsilon_{vol}$$



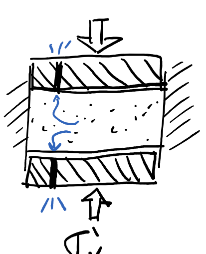
Calculation of elastic and plastic strains

$$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^p \leftarrow \text{strain decomposition}$$

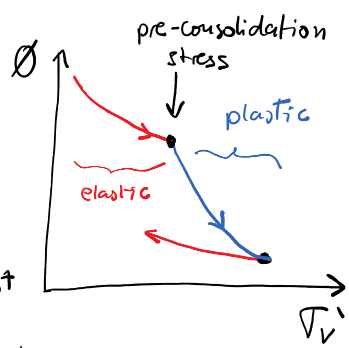
$$d\underline{\underline{\varepsilon}} = d\underline{\underline{\varepsilon}}^e + d\underline{\underline{\varepsilon}}^p \quad \text{(incremental version)}$$

$$d\underline{\underline{\varepsilon}} = \underbrace{\underline{\underline{\underline{\underline{C}}}}^e \, d\underline{\underline{\sigma}}}_{\text{elastic}} + \underbrace{\underline{\underline{\underline{\underline{C}}}}^p \, d\underline{\underline{\sigma}}}_{\text{plastic}}$$

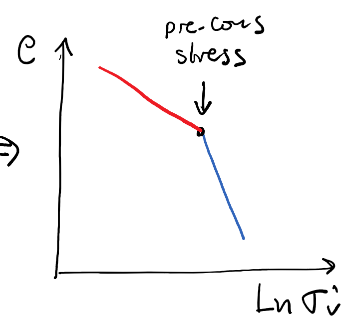
(1) Elastic strains

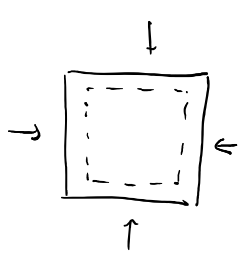


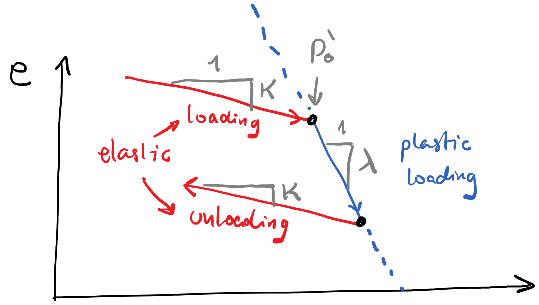
Consolidation Test
(Uniaxial strain)
Oedometric condition



$\Rightarrow$

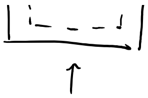


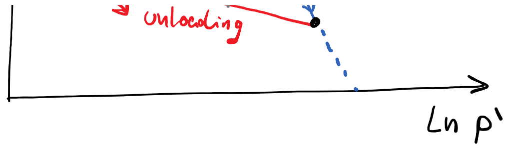


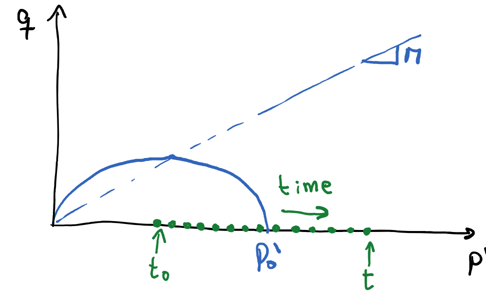


$$e = c - \kappa \cdot Ln\, p'$$

$$\frac{de}{dp'} = -\frac{\kappa}{p'} \quad (1)$$







$$e \overset{def}{=} \frac{V_p}{V_b - V_p} \rightarrow de = \ldots dV_p + \ldots dV_b$$

assumption $dV_b = dV_p \quad (\alpha \approx 1)$

$$\delta \varepsilon_{p'} \equiv \frac{\partial V_b}{V_b} = \frac{\partial V_p}{V_b}$$

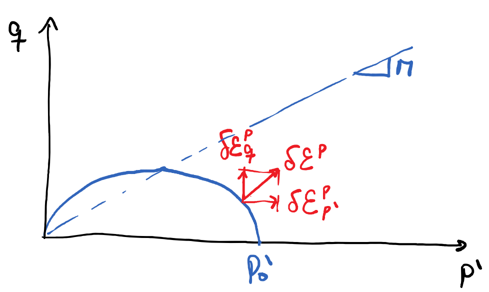
$$de = -(1+e)\,\delta \varepsilon_{p'} \qquad (2)$$

(1)+(2)
$$\begin{cases} \delta \varepsilon_{p'}^{e} = \dfrac{K}{1+e} \cdot \dfrac{dp'}{p'} \\ \delta \varepsilon_{q}^{e} = \dfrac{1}{3G} \cdot dq \end{cases} ; \quad \varepsilon_q \overset{def}{=} \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$$

$$\begin{bmatrix} \delta \varepsilon_{p'}^{e} \\ \delta \varepsilon_{q}^{e} \end{bmatrix} = \begin{bmatrix} \frac{K}{1+e} \cdot \frac{1}{p'} & 0 \\ 0 & \frac{1}{3G} \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix}$$

Elastic strains (before yield)

(2) Plastic strains



$$f = q^2 - M^2 p'(p_0' - p')$$

$$f^* = \frac{p'}{p_0'} - \frac{M^2}{\eta^2 + M^2} \; ; \quad \eta = \frac{q}{p'}$$

derivatives
$$\begin{cases} \dfrac{\partial f^*}{\partial p'} = p_0' M^2 \left(\dfrac{M^2 - \eta^2}{M^2 + \eta^2} \right) \\ \dfrac{\partial f^*}{\partial q} = p_0' M^2 \left(\dfrac{2\eta}{M^2 + \eta^2} \right) \\ \dfrac{\partial f^*}{\partial p_0'} = -\dfrac{p'}{(p_0')^2} \end{cases}$$

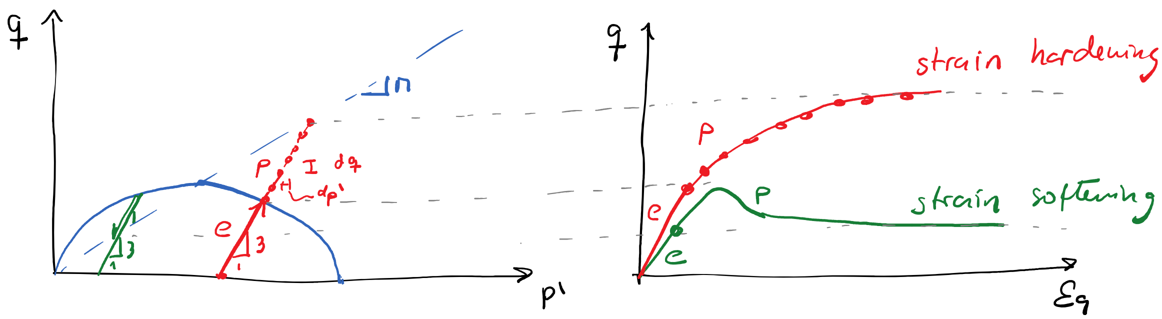
Associated flow rule

$$\delta \varepsilon_{ij}^{p} = dx \frac{\partial f^*}{\partial \sigma_{ij}} \Rightarrow \begin{cases} \delta \varepsilon_{p'}^{p} = dx \dfrac{\partial f^*}{\partial p'} \\ \delta \varepsilon_{q}^{p} = dx \dfrac{\partial f^*}{\partial q} \end{cases} \qquad (3)$$

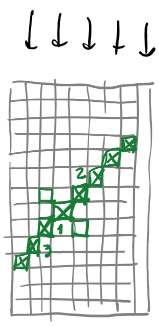
From ICL (4) $\delta\varepsilon^p_{p'} = \dfrac{\lambda-\kappa}{1+e}\,\dfrac{dp_0'}{p_0'}$

(3)+(4) $\rightarrow$ $d\chi = \dfrac{\lambda-\kappa}{(1+e)p'(M^2+\eta^2)}$ ← hardening parameter

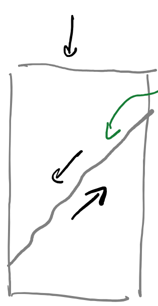
$$\begin{bmatrix}\delta\varepsilon^p_{p'}\\ \delta\varepsilon^p_q\end{bmatrix} = \frac{\lambda-\kappa}{(1+e)p'(M^2+\eta^2)}\begin{bmatrix}M^2-\eta^2 & 2\eta\\ 2\eta & \dfrac{4\eta^2}{M^2-\eta^2}\end{bmatrix}\begin{bmatrix}dp'\\ dq\end{bmatrix} \rightarrow \text{Plastic Strains}$$



strain softening

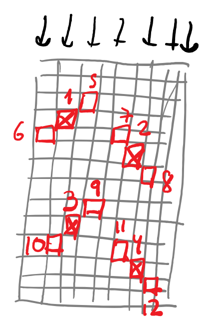


- positive feed back
- unstable process

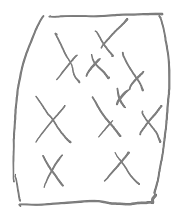


strain localization

strain hardening



- shear and compaction bands
- stable process

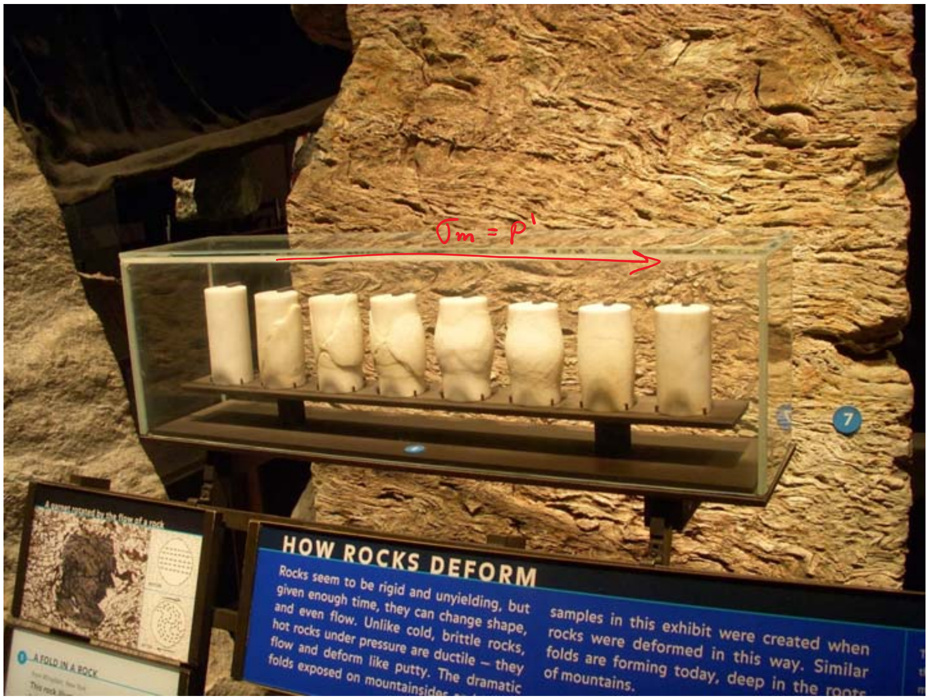


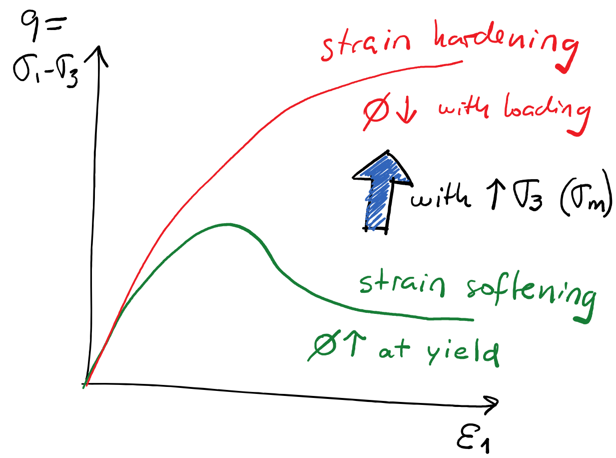
- diffused deformation and plastic strain

Brittle to Ductile Transition, Isotropic and Kinematic hardening

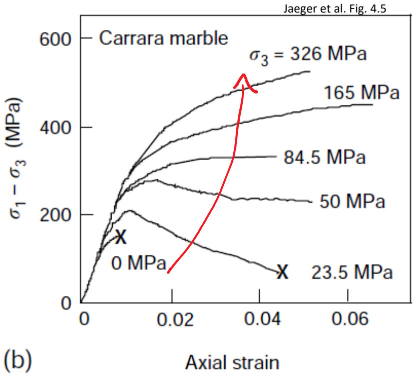
Friday, November 6, 2020 5:42 PM

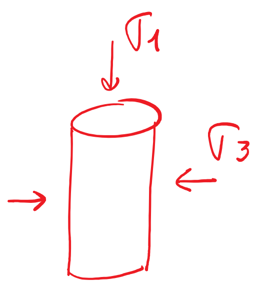
"Brittle to ductile transition"





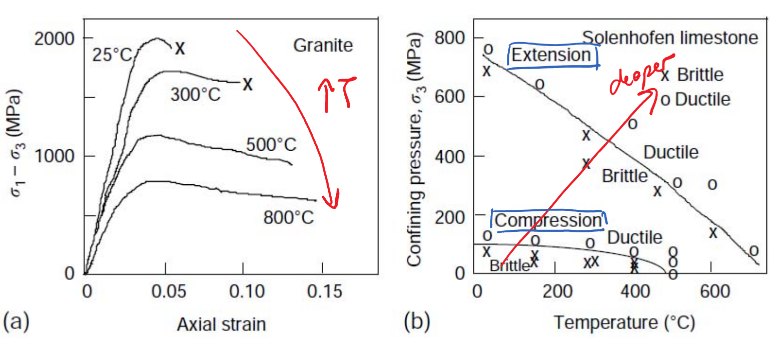
Jaeger et al. Fig. 4.5





$\sigma_2 = \sigma_3$

Jaeger et al., Fig. 4.6

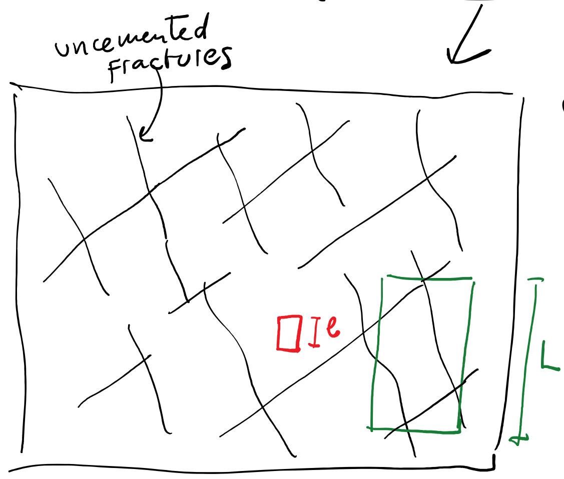


Britte to ductile transition

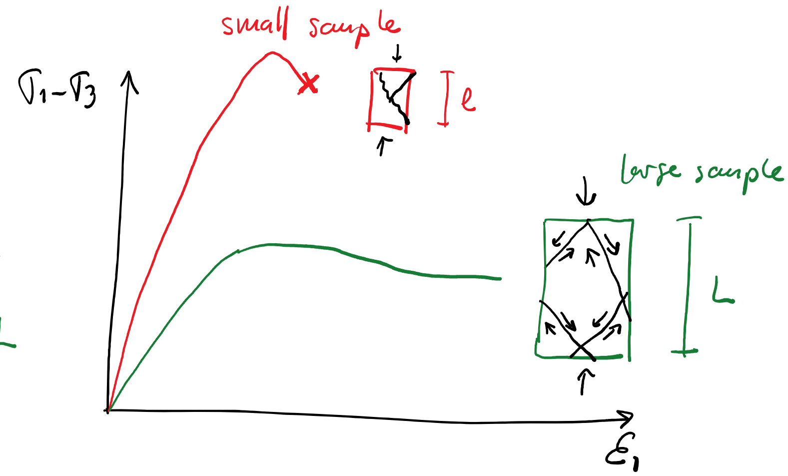
- ↑ σ_m ($= p'$), ↑ σ_3, mean stress
- ↑ T, temperature
- ↓ loading rate

Britte to ductile transition

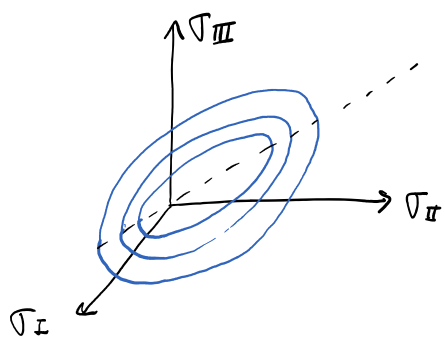
- ↑ T , temperature
- ↓ loading rate
- mineralogy of rocks, organic shales $\left\{ \begin{array}{l} \uparrow \text{TOC} \\ \uparrow \text{clay} \end{array} \right.$
- length scale



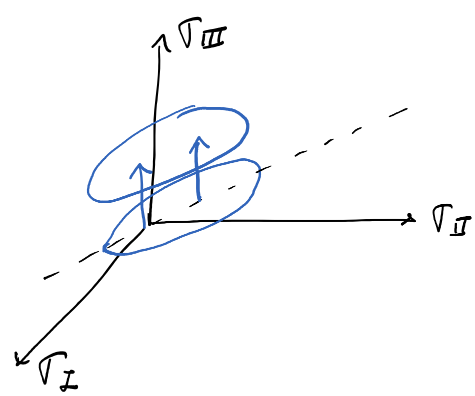
$l \ll L$



Isotropic hardening



Kinematic hardening



Chemo-plasticity