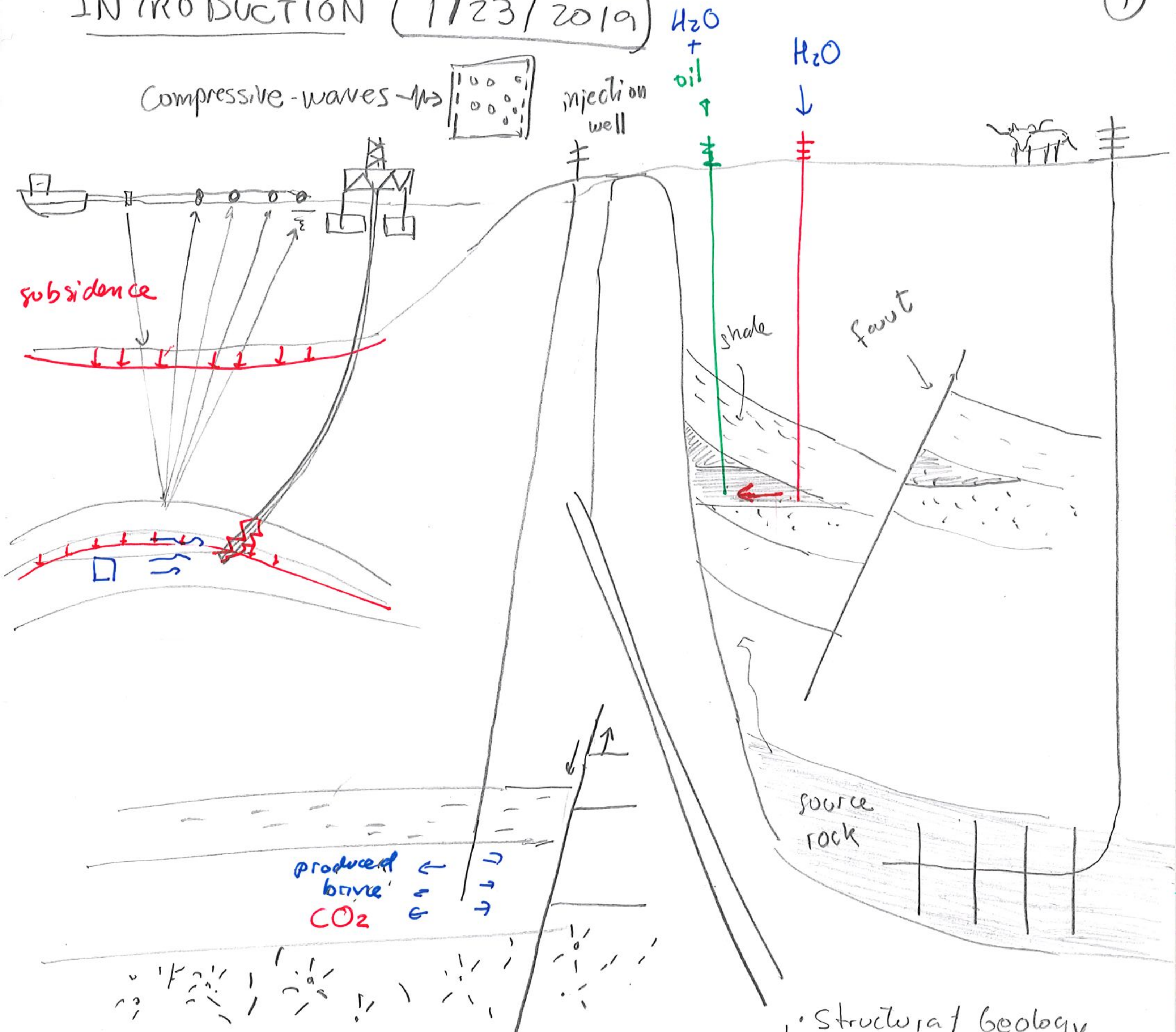
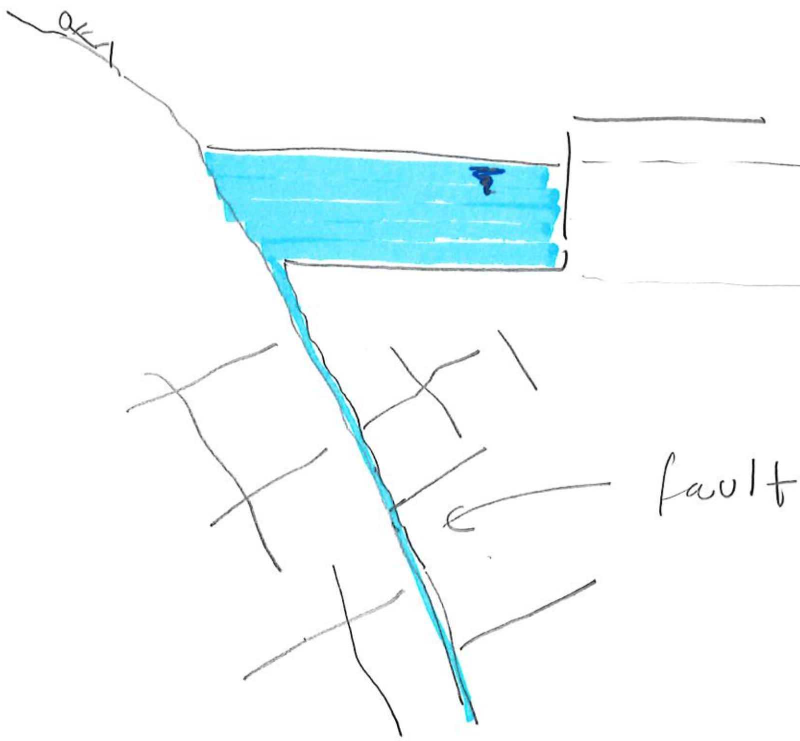
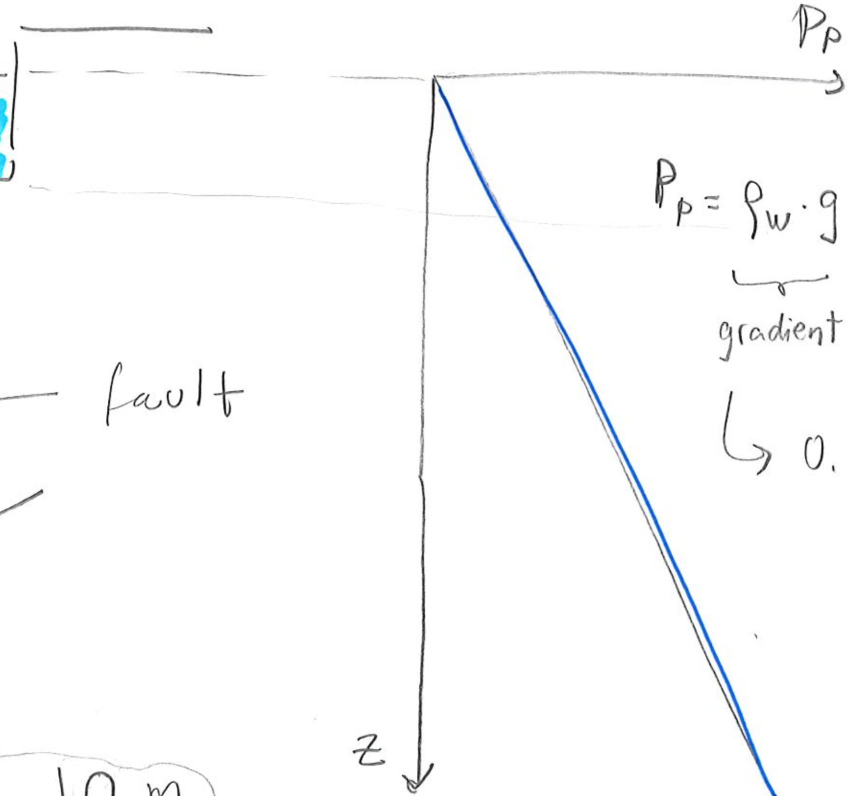
INTRODUCTION (1/23/2019)



- Exploration
  - Structural Geology
  - Seismic surveys
- Drilling & Completions
  - wellbore stability
  - fracturing
- Production
  - compaction
    - ↳ rock compressibility
  - sand production
- Waste disposal
  - brine and CO_2
- well abandonment

Pore pressure (1/25/2019)





$$P_p = \underbrace{\rho_w \cdot g}_{\text{gradient}} \cdot \underbrace{z}_{\text{depth}}$$

$\hookrightarrow 0.44 \, \frac{psi}{ft}$

$$\rho_w \cdot g \cong 1000 \, \frac{kg}{m^3} \cdot 10 \, \frac{m}{s^2}$$

$$= 10^4 \, \frac{N}{m^2} \cdot \frac{1}{m}$$

$$= 10^4 \, Pa \, \frac{1}{10^{-3} \, km}$$

$$= 10 \; 10^6 Pa \, \frac{1}{km}$$

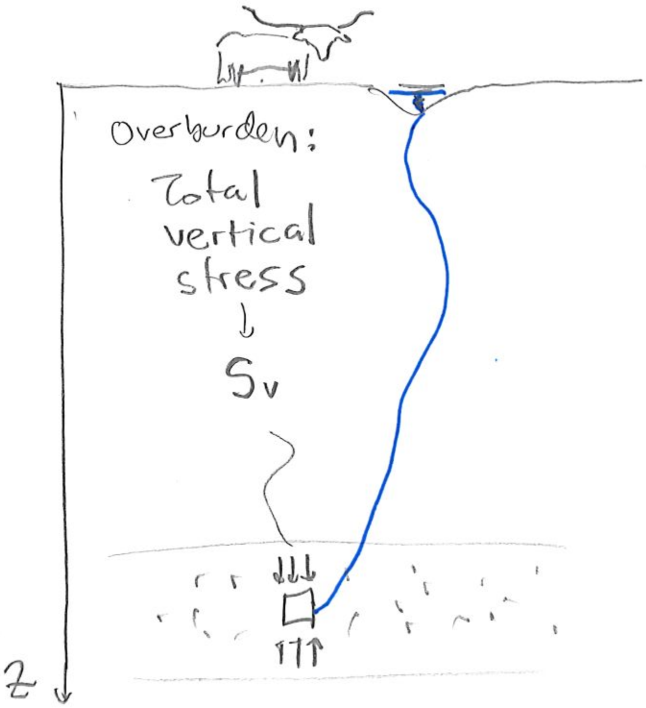
$$\boxed{\rho_w \, g \cong 10 \, \frac{MPa}{km}}$$

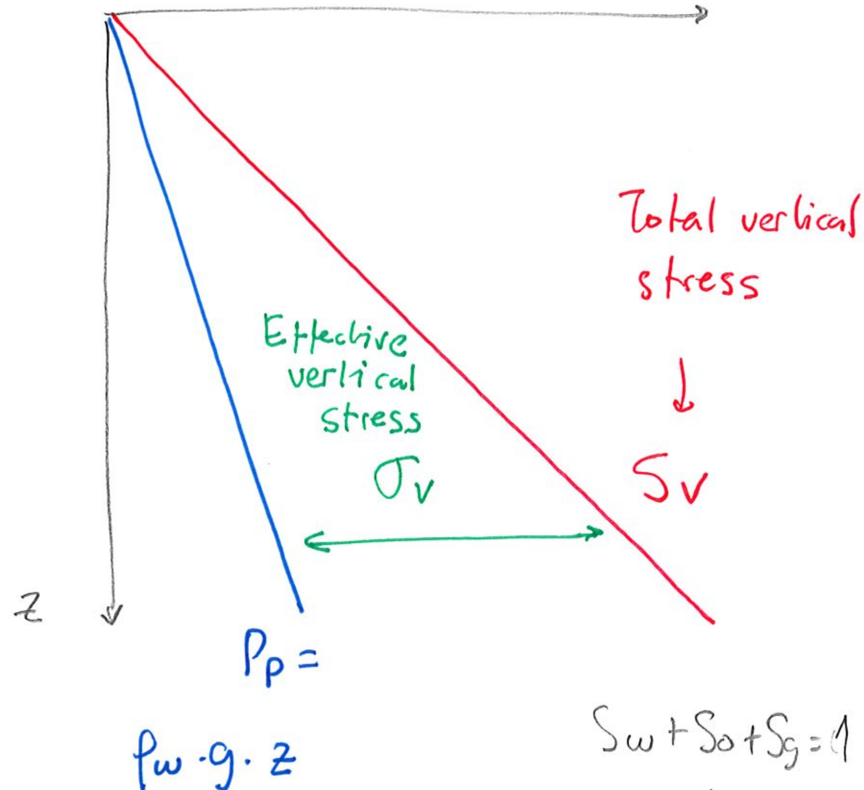
$$\rho_w = 62.4 \, \frac{lb}{ft^3}$$

$$\rho_w \cdot g = 62.4 \, \frac{\vec{lb}}{ft^3}$$

$$\rho_w \cdot g = 62.4 \, \frac{1}{(12in)^2} \, \frac{\vec{lb}}{ft}$$

$$\boxed{\rho_w \cdot g = 0.433 \, \frac{psi}{ft}}$$

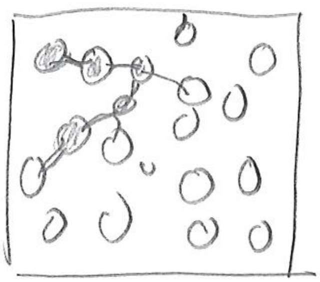




$$S_w + S_o + S_g = 1$$

$$\rho_F = \rho_w \cdot S_w + \rho_o \cdot S_o + \rho_g \cdot S_g$$

$$S_v = \rho_{bulk} \cdot g \cdot z$$



$$\rho_{bulk} = \rho_m (1-\phi) + \rho_F (\phi)$$

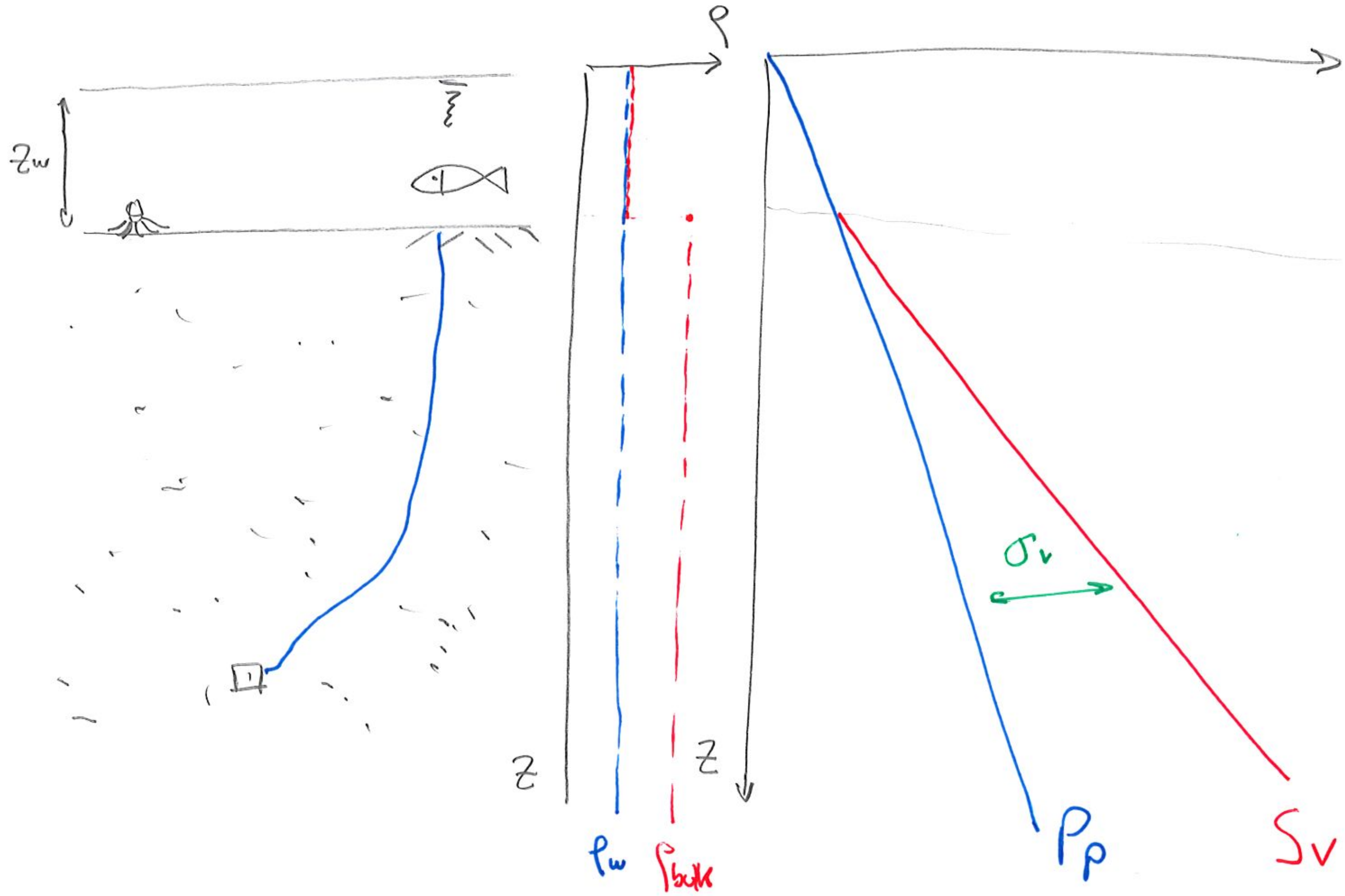
$$\rho_m = C_{m1} \rho_{m1} + C_{m2} \rho_{m2} + \dots + \rho_{mn} C_{mn}$$

$$\sum C_{mn} = 1$$

Sandstone
$\phi = 0.20$
$\rho_{quartz} = 2650 \ kg/m^3$
$\rho_{brine} = 1000 \ kg/m^3$

$$\rho_{bulk} = 2320 \ \frac{kg}{m^3}$$

$$\boxed{\frac{dS_v}{dz} = \rho_{bulk} \cdot g \cong 23 \ \frac{MPa}{km}}$$



$= \rho_w g z$ $\qquad$ $= \rho_{bulk} g z$

2019/1/28

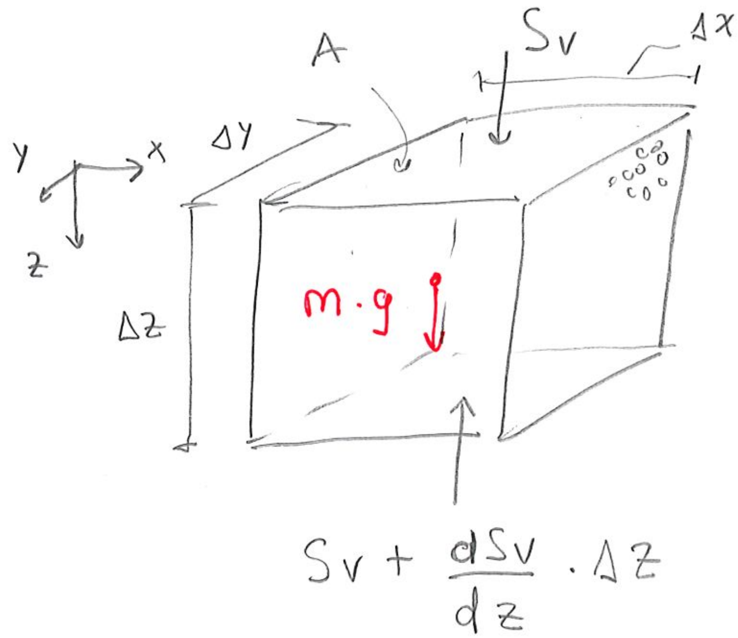
$$S_v = \underbrace{\rho_w g \cdot z_w} + \rho_{bulk} g (z - z_w)$$

Pore pressure gradient (hydrostatic) : $0.44 \frac{psi}{ft} = 10 \frac{MPa}{km}$

Total vertical stress gradient : $\underbrace{1 \frac{psi}{ft}}_{1000 \frac{kg}{m^3}} = \underbrace{23 \frac{MPa}{km}}_{2320 \frac{kg}{m^3}}$

lithostatic

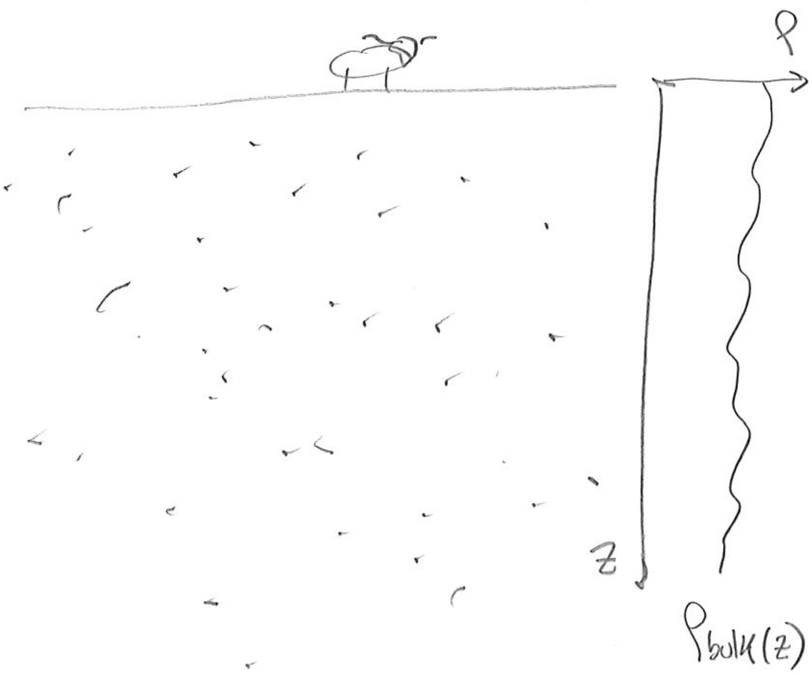
General solution for vertical stress



$$\Sigma F_z = S_v \cdot A + m \cdot g - \left(S_v + \frac{dS_v}{dz} \cdot \Delta z\right) A$$

$$= \underbrace{\cancel{S_v (\Delta x \Delta y)}} + \rho_{bulk} (\Delta x \Delta y \Delta z) g - \left(\cancel{S_v} + \frac{dS_v}{dz} \Delta z\right) \Delta x \Delta y = 0$$

$$\frac{dS_v}{dz} \cancel{\Delta z \Delta x \Delta y} = \rho_{bulk} \cdot g \cancel{\Delta x \Delta y \Delta z}$$



$$\boxed{\frac{dS_v}{dz} = \rho_{bulk}(z) \cdot g}$$

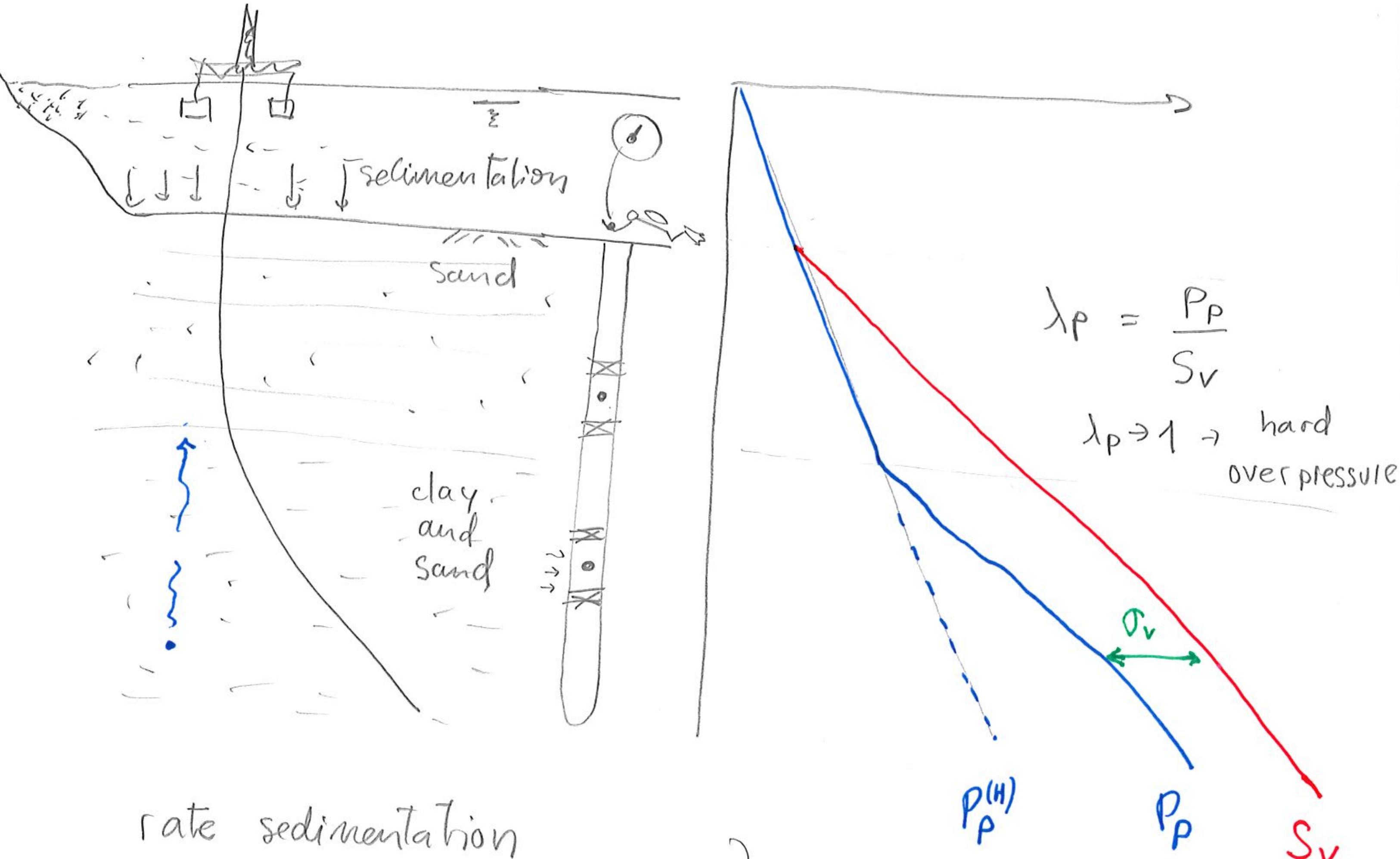
$$\int_{S_v(z=0)}^{S_v(z)} dS_v = \int_0^z \rho_{bulk}(z)\, g\, dz$$

$$S_v(z=0) = 0$$

$$\boxed{S_v(z) = \int_0^z \rho_{bulk}(z) \cdot g \cdot dz}$$

density log

Non-hydrostatic pore pressure



$$\lambda_p = \frac{P_p}{S_v}$$

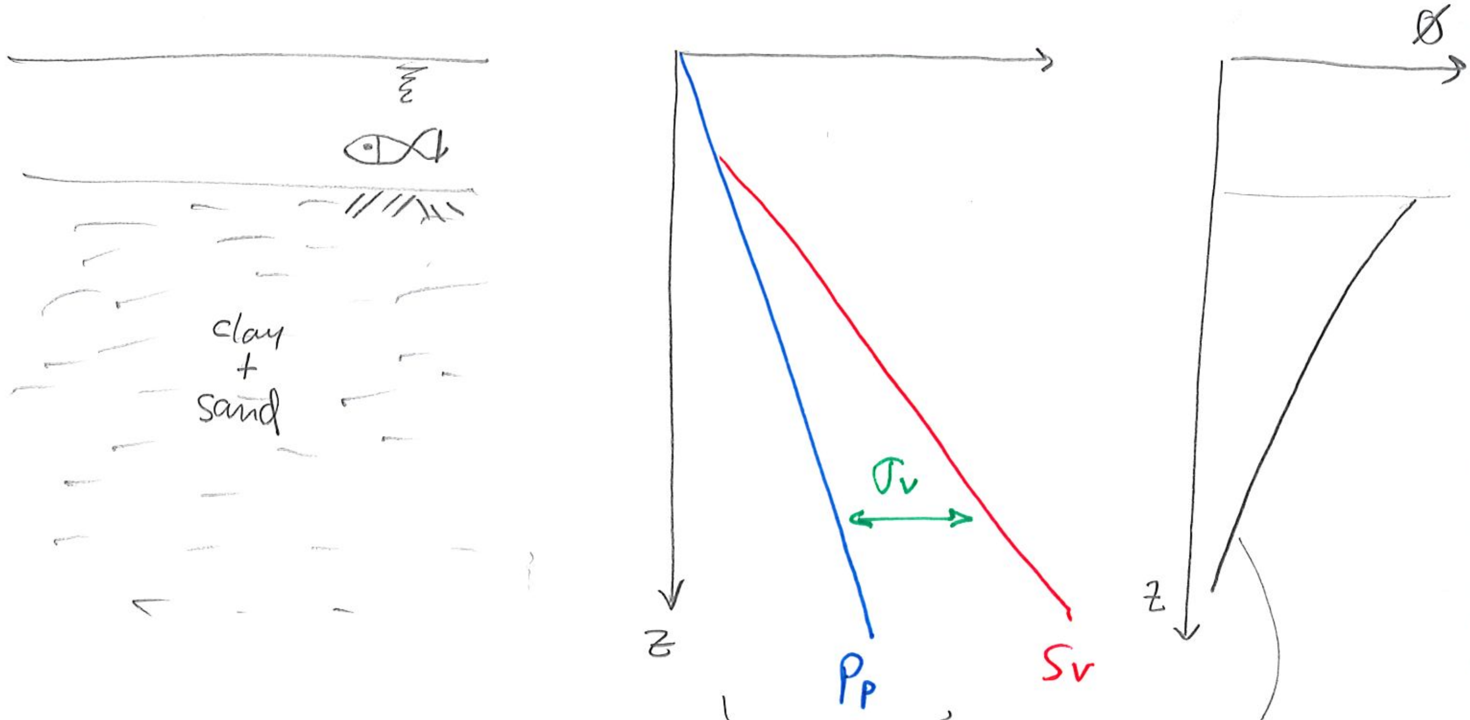
$\lambda_p \to 1 \to$ hard overpressure

rate sedimentation
$>$
rate of pore pressure diffusion $\Big\}$ $\Rightarrow$ disequilibrium compaction

→ overpressure

→ underconsolidation

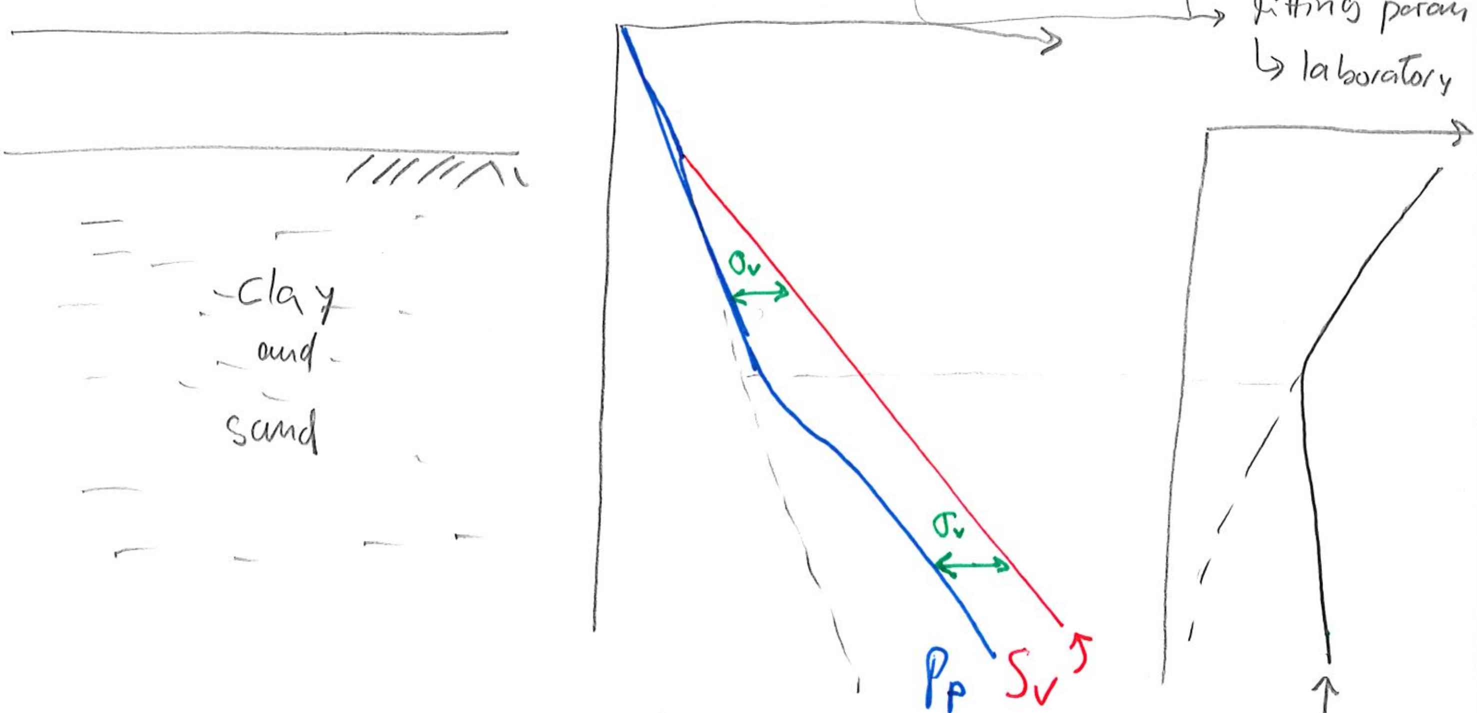
2019/1/30



$\underbrace{\qquad\qquad}_{\text{normal compaction}}$

$$\boxed{\varnothing = \varnothing_0 \exp(-\beta \cdot \sigma_v)}$$

fitting param
↳ laboratory



$\underbrace{\qquad\qquad}_{\text{disequilibrium compaction}}$

- DPHI
- NPHI
- RHO
- DT

← actual porosity

algebra ($\sigma_v = S_v - P_p$)

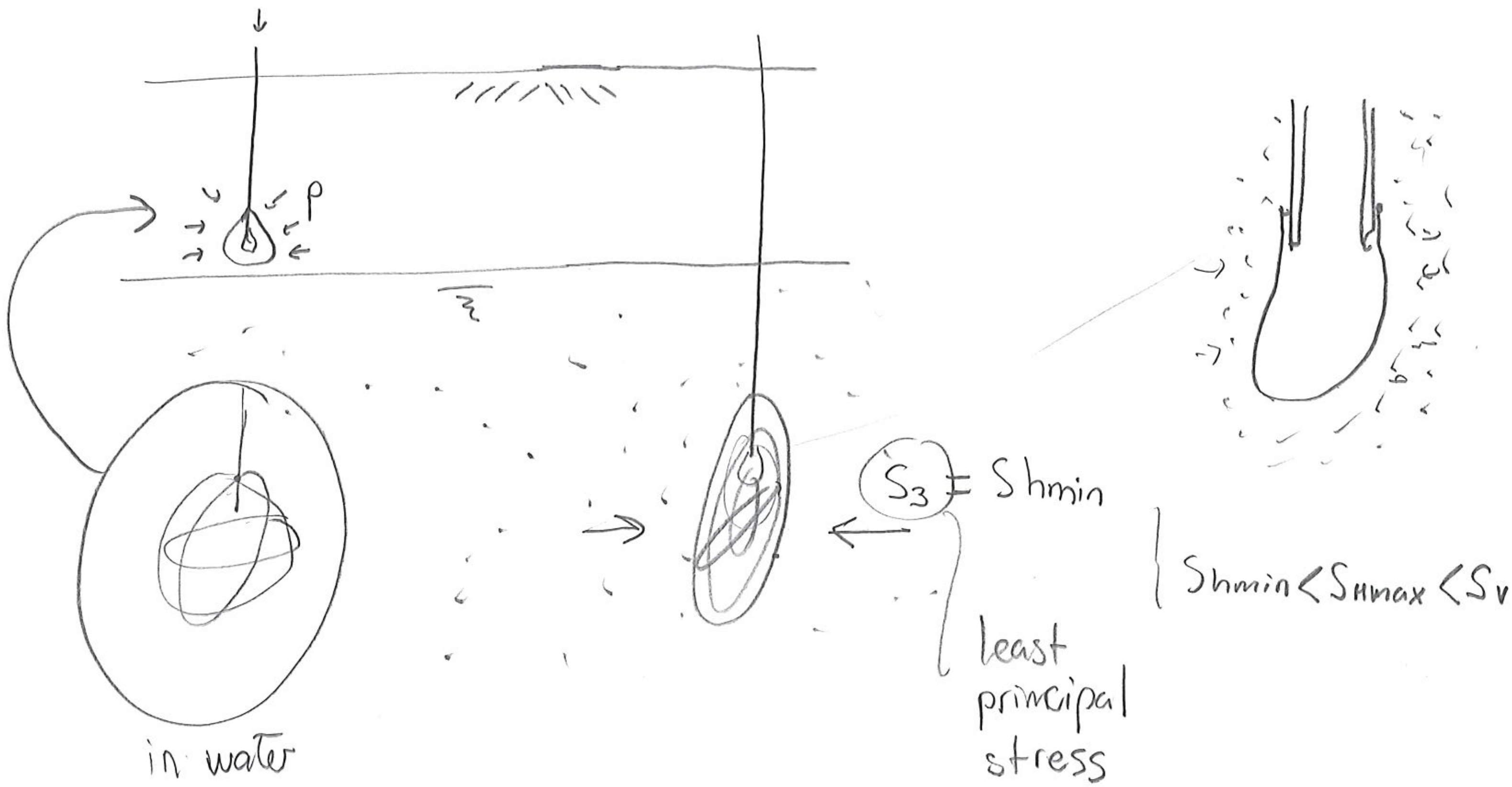
$$P_p = S_v + \underbrace{\frac{\ln(\phi/\phi_0)}{\beta}}_{<0}$$

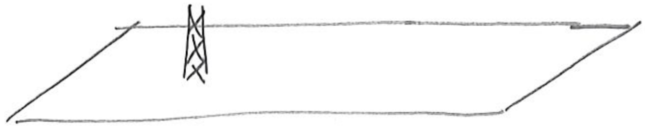
WORKFLOW: 1) Calculate S_v

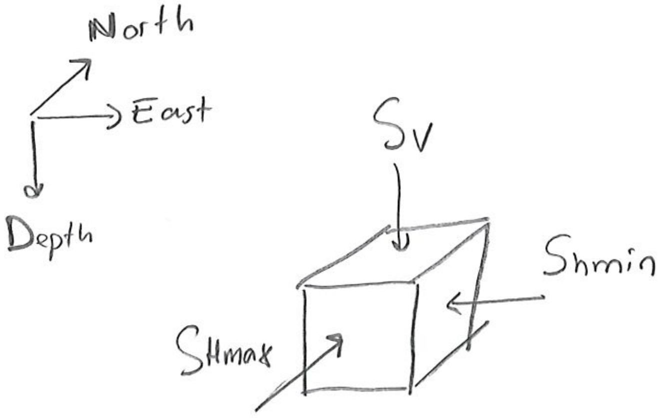
2) Determine σ_v from ϕ, ϕ_0, β

3) $P_p = S_v - \sigma_v$

Horizontal stresses







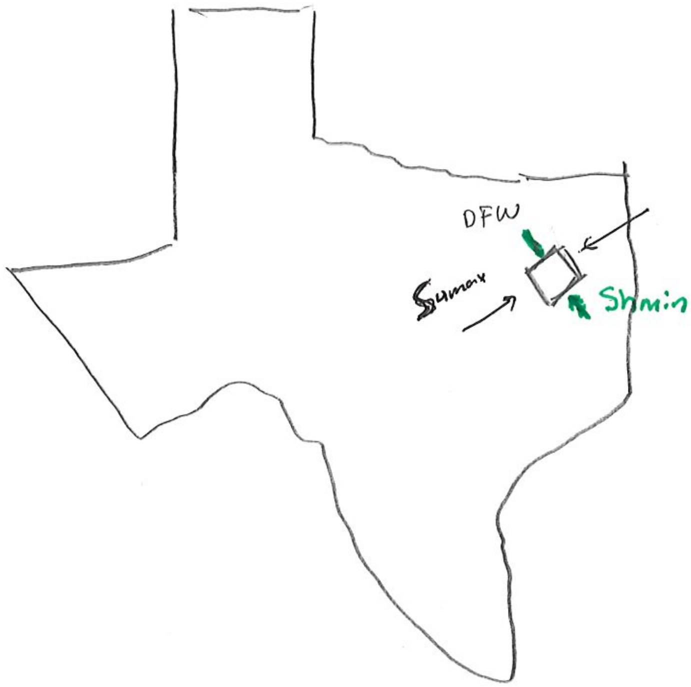
- 3 values
- 3 directions
- Full knowledge of stress state

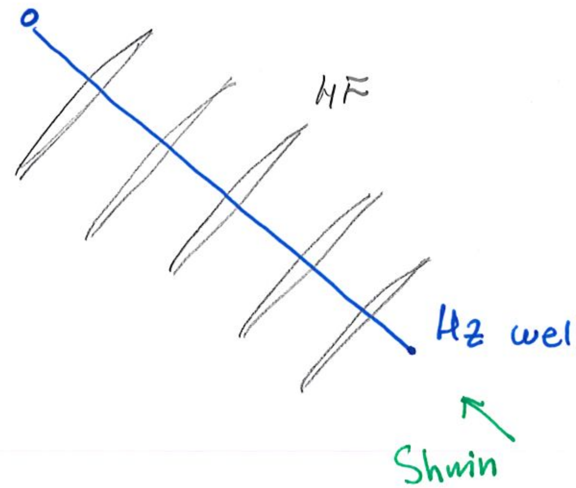
Stress regime	S_1 >	S_2 >	S_3
Normal Faulting (Extensional) • Permian, EF	S_V	S_{Hmax}	S_{hmin}
Strike Slip • California	S_{Hmax}	S_V	S_{hmin}
Reverse/Thrust Faulting • Argentina (Vaca Muerta) • Austalia some depths	S_{Hmax}	S_{hmin}	S_V

Barnett shale

Normal Faulting

$S_v > \underbrace{S_{Hmax} > S_{hmin}}_{?}$





wellbore failure

2/1/2019

S_v | S_{Hmax} | S_{hmin}

$\int_0^z \rho_{bulk} \cdot g \cdot dz$

S_{hmin} ↓ if S_3 ↓ conduct hydraulic fracture test

HW2

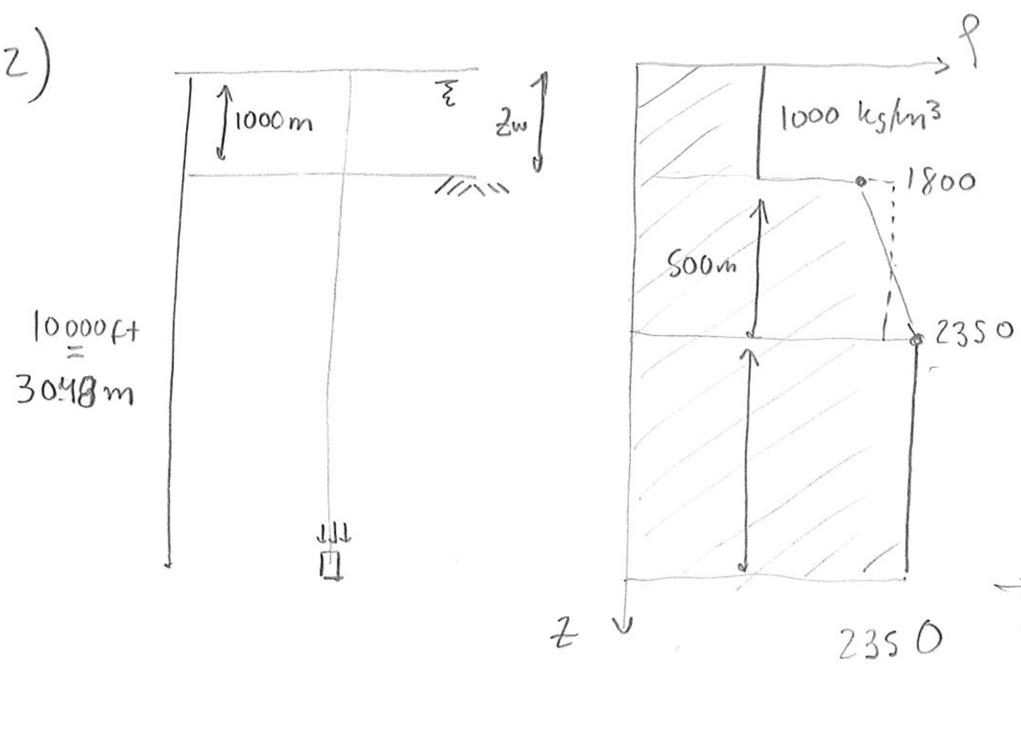
1) $1\ g/cm^3 = 1\ g/cc = 1000\ Kg/m^3$

$\rho_{bulk} = $ ________

$\rho_{bulk} \cdot g = \left|\begin{array}{l} \sim 26\ MPa/Km \\ \sim 1.15\ psi/ft \\ \sim 22\ ppg \end{array}\right.$

$0.44\ \frac{psi}{ft} = 8.3\ ppg$

2)



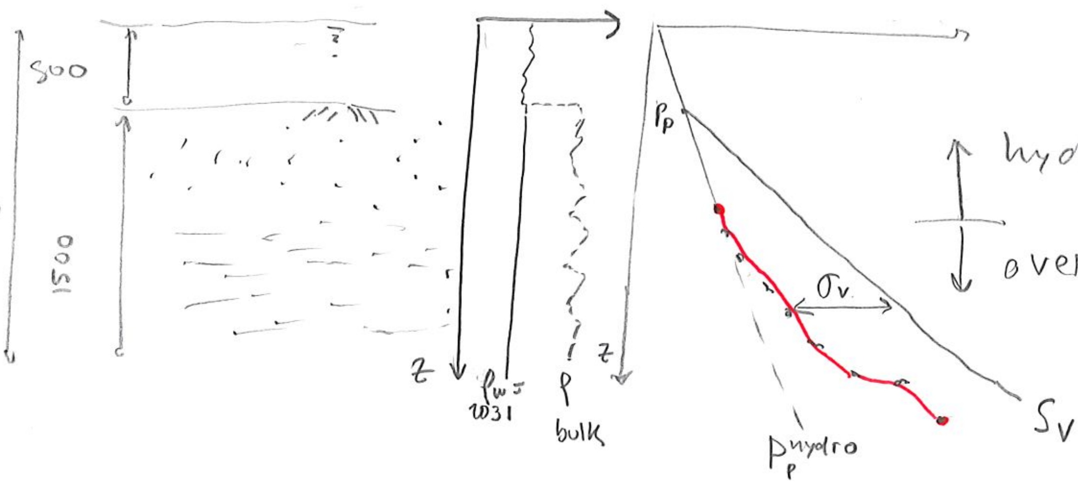
$3000\ psi = 20.6\ MPa$

$1\ MPa = 145\ psi$

$\rightarrow S_v = \int_0^z \rho_{bulk} \cdot g \cdot dz$

$S_v \cong 56\ MPa$

3)



↑ hydrostatic

↓ overpressure

$$\lambda_p = \frac{P_p}{S_v} \ll 1$$

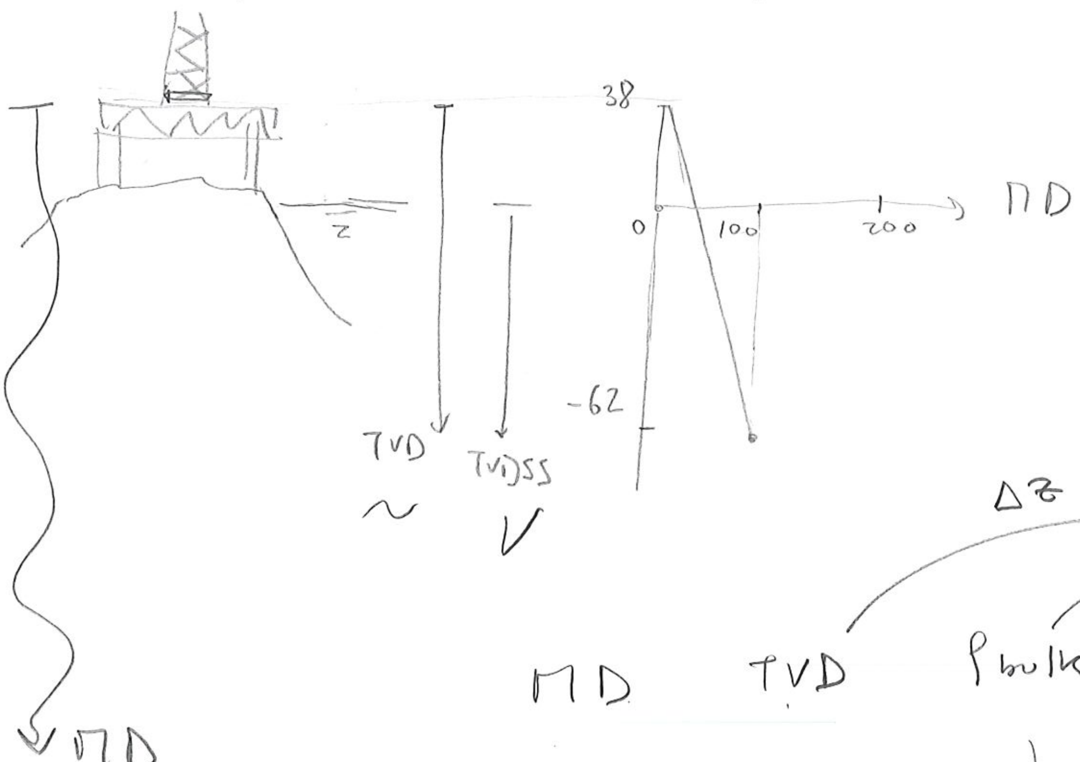
1) Hypothesis: hydrostatic

2) Shale porosity → actual pore pressure

4) Plaquemines Parish, LA

.LAS | EKB^bushing 36 ft

data 1864 - 7551 ft



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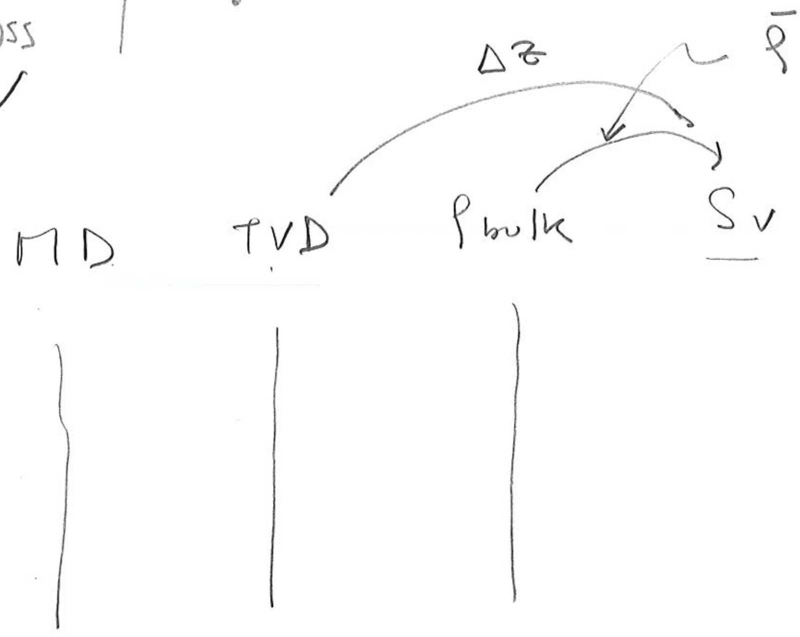
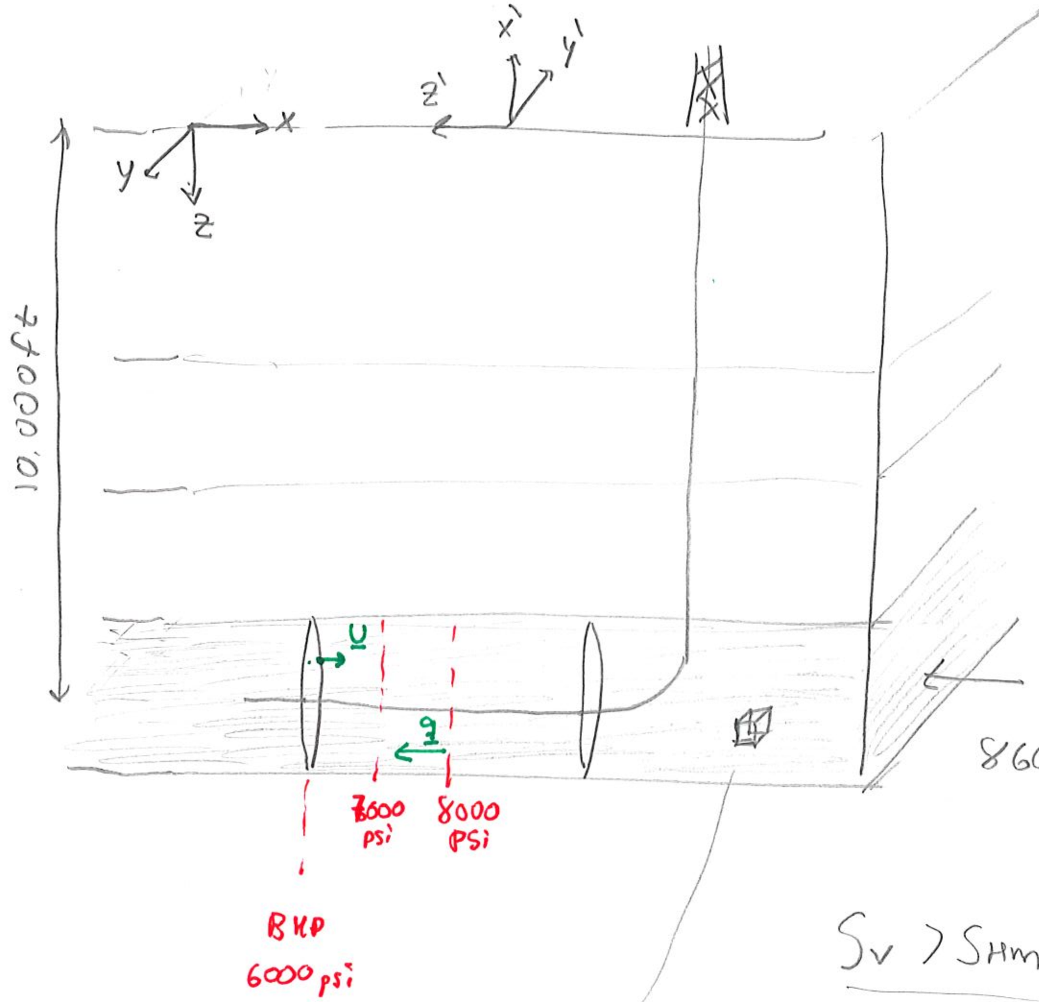


$\boxed{2/4/2019}$



scalar
$P = 8000\ psi$
$T = 200°F$
$S_o = 0.7$

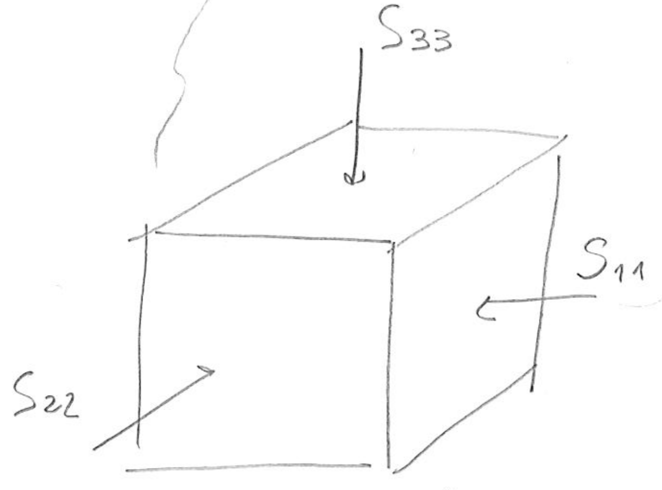
vector
$\underline{U} = (2mm, 0mm, 0mm)$
$\underline{q} = \left(1\frac{ft}{day}, 0\frac{ft}{day}, 0\frac{ft}{day}\right)$

$\underline{\underline{S}}hmin$
8600psi

tensor

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

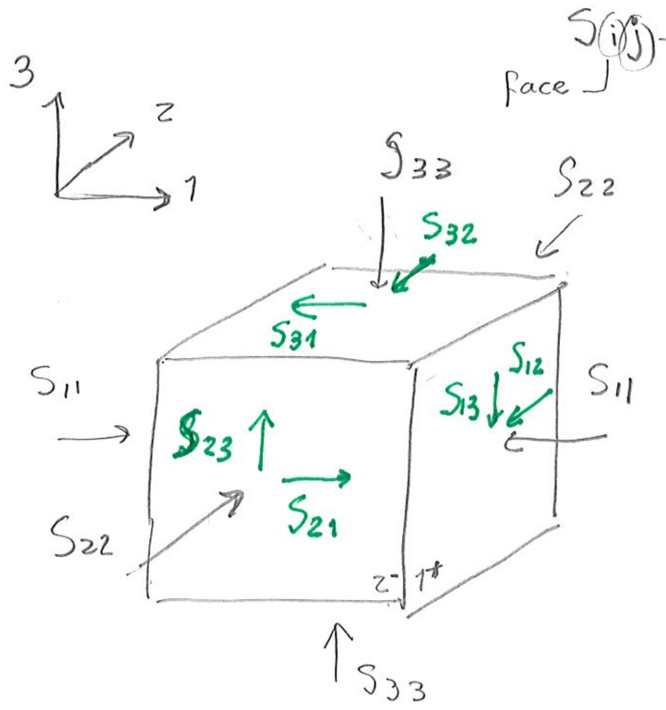
$S_v > S_{Hmax} > S_{hmin}$



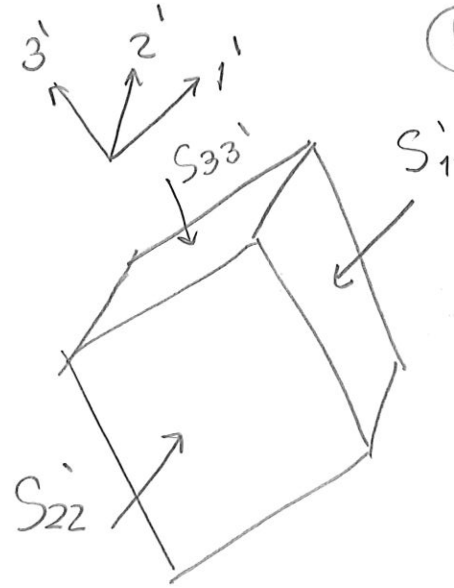
$$\underline{\underline{S}} = \begin{bmatrix} S_{hmin} & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_v \end{bmatrix}$$

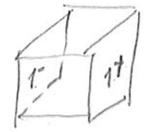
$$\underline{\underline{S}} = \underbrace{\begin{bmatrix} 8600 & 0 & 0 \\ 0 & 9000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}}_{\text{principal stresses} \equiv \text{eigen values}} psi$$

$1 \to x$
$2 \to y$
$3 \to z$



principal stresses & directions $\Longrightarrow$





normal stress positive → compression

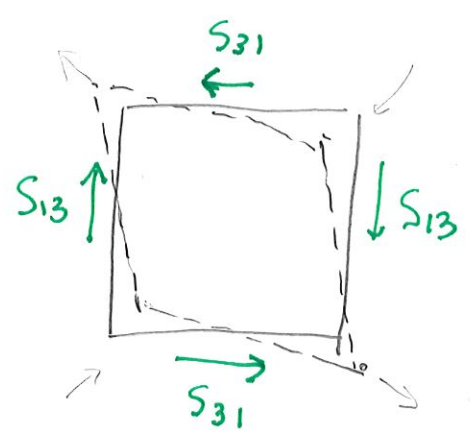
all shear stresses are zero

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \xrightarrow{\text{eig}} \underline{\underline{S}}' = \begin{bmatrix} S_{11}' & 0 & 0 \\ 0 & S_{22}' & 0 \\ 0 & 0 & S_{33}' \end{bmatrix}$$

symmetric

$$\underline{\underline{S}}' = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$S_1 \geq S_2 \geq S_3$$



Mohr circle

Eigenvalues → eig(_)

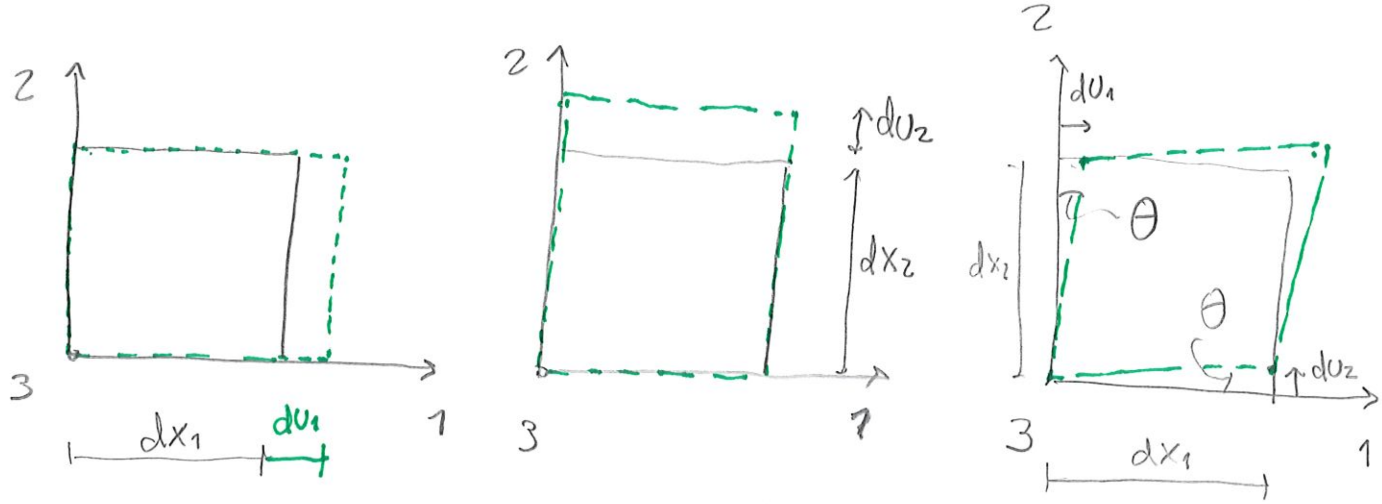
$S_{31} = S_{13}$ → angular momentum Equilibrium

$$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \underline{\underline{I}}$$

$$\underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{bmatrix}}_{\text{Effective stress}} = \underbrace{\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ & S_{22} & S_{23} \\ & & S_{33} \end{bmatrix}}_{\text{Total stress}} - \underbrace{\begin{bmatrix} P_p & 0 & 0 \\ & P_p & 0 \\ & & P_p \end{bmatrix}}_{\text{Pore pressure}}$$

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11}-P_p & S_{12} & S_{13} \\ S_{21} & S_{22}-P_p & S_{23} \\ S_{31} & S_{32} & S_{33}-P_p \end{bmatrix}}_{\text{Effective stress}}$$

Strains (deformation)



$$\varepsilon_{11} = \frac{du_1}{dx_1} \; ; \quad \varepsilon_{22} = \frac{du_2}{dx_2} \; ; \quad \varepsilon_{12} = \frac{1}{2}\left(\frac{du_1}{dx_2} + \frac{du_2}{dx_1}\right)$$

$$\underbrace{\varepsilon_{33} = \frac{du_3}{dx_3}}_{\text{linear strains}}$$

$\hookrightarrow \Delta Vol$

$$\underbrace{\tan\theta = \frac{du_1}{dx_2} = \frac{du_2}{dx_1}}_{\text{shear strain}}$$

$\downarrow$

no volume change

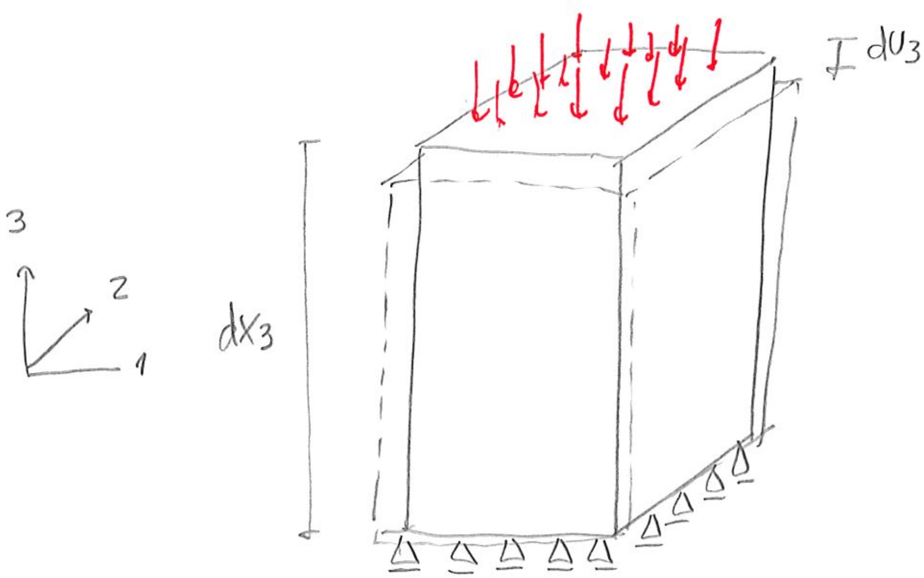
strain tensor

$\downarrow$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \frac{du_1}{dx_1} & \frac{1}{2}\left(\frac{du_1}{dx_2} + \frac{du_2}{dx_1}\right) & \frac{1}{2}\left(\frac{du_1}{dx_3} + \frac{du_3}{dx_1}\right) \\ & \frac{du_2}{dx_2} & \frac{1}{2}\left(\frac{du_2}{dx_3} + \frac{du_3}{dx_2}\right) \\ & & \frac{du_3}{dx_3} \end{bmatrix}$$

$$\left.\begin{aligned}\underline{\underline{\sigma}} &= f(\underline{\underline{\varepsilon}})\\ \underline{\underline{\varepsilon}} &= f(\underline{\underline{\sigma}})\end{aligned}\right\} \ ?$$

Linear Elasticity



$$\varepsilon_{33} = \frac{dU_3}{dX_3}$$

$$\boxed{E = \frac{\sigma_{33}}{\varepsilon_{33}}}$$ Young's Modulus

$\hookrightarrow \sigma_{33} = E \cdot \varepsilon_{33}$

$\hookrightarrow \varepsilon_{33} = \dfrac{\sigma_{33}}{E}$

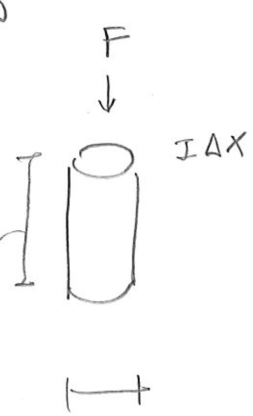
$$\boxed{\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}} = -\frac{\varepsilon_{22}}{\varepsilon_{33}}}$$

Poisson's ratio

Unconfined Loading $\rightarrow$ in one direction

Class Exp

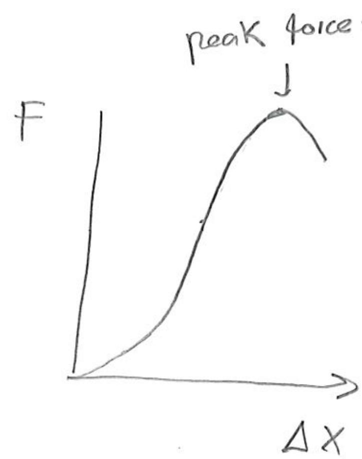
Berea Sandstone



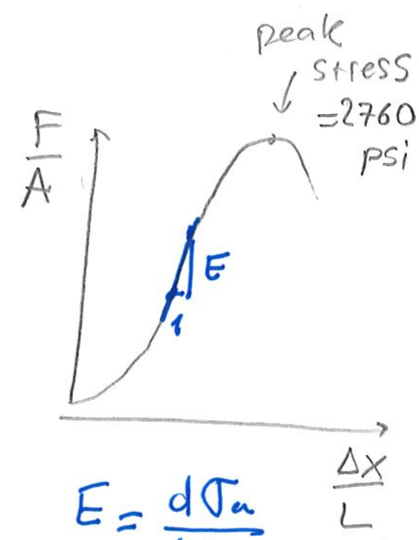
L = 0.8 in

D = 0.38 in

A = 0.11 in^2



$\Rightarrow$



1 turn = 0.15 mm
= 0.006 in

$$E = \frac{d\sigma_a}{d\varepsilon_a}$$

$E = 94 \cdot 10^3$ psi → H

$282 \cdot 10^3$ psi → K, M ✓

$8.9 \cdot 10^4$ MPa → B

ΔF = 110 lb

↓ /A

Δσ = 1000 psi

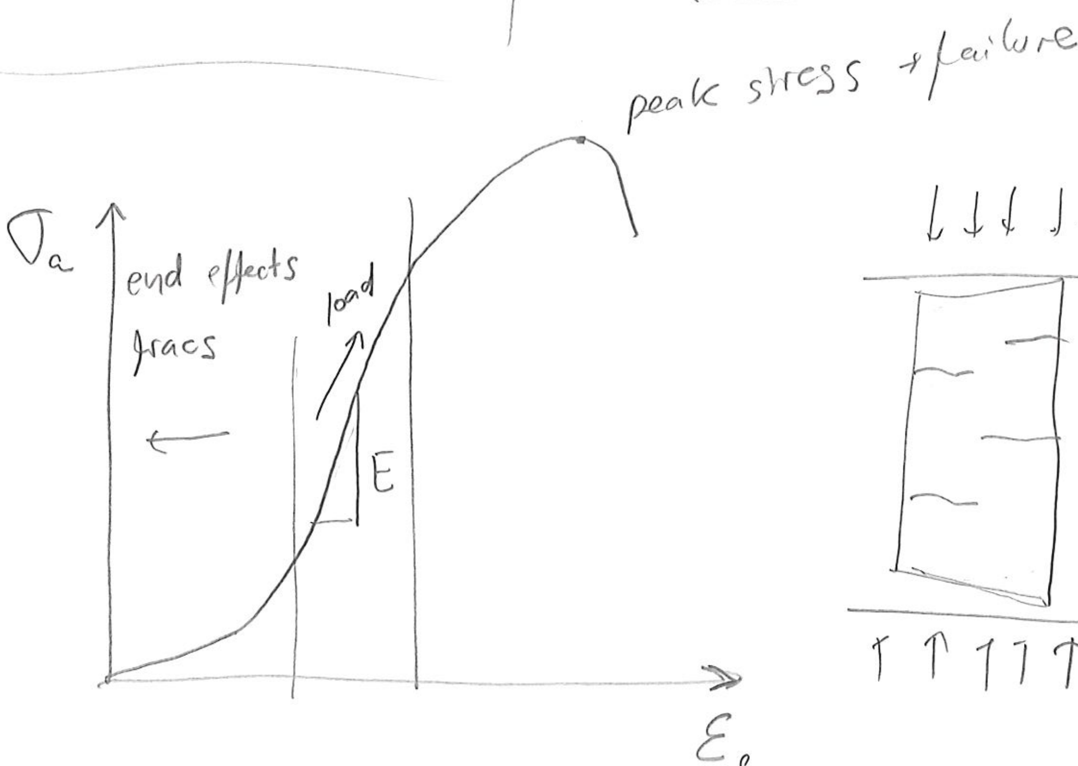
10 kg	0 turn
50 kg	0.75
100 kg	1.25
138 kg	broke

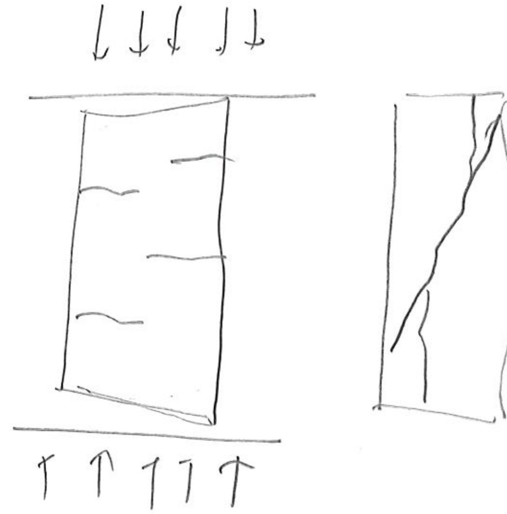
0.003/0.8

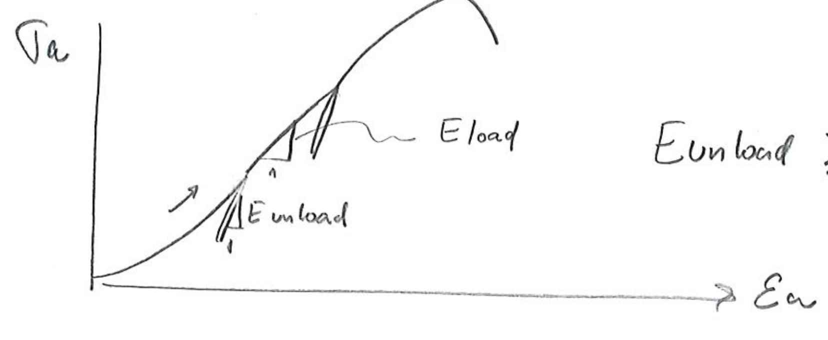
$\Delta X = 0.003 \text{ in} \Rightarrow \varepsilon_a = 3.75 \cdot 10^{-3}$

$\boxed{E = 266 \cdot 10^3 \text{ psi}}$

$\boxed{E = 1.84 \cdot 10^9 \text{ Pa}}$

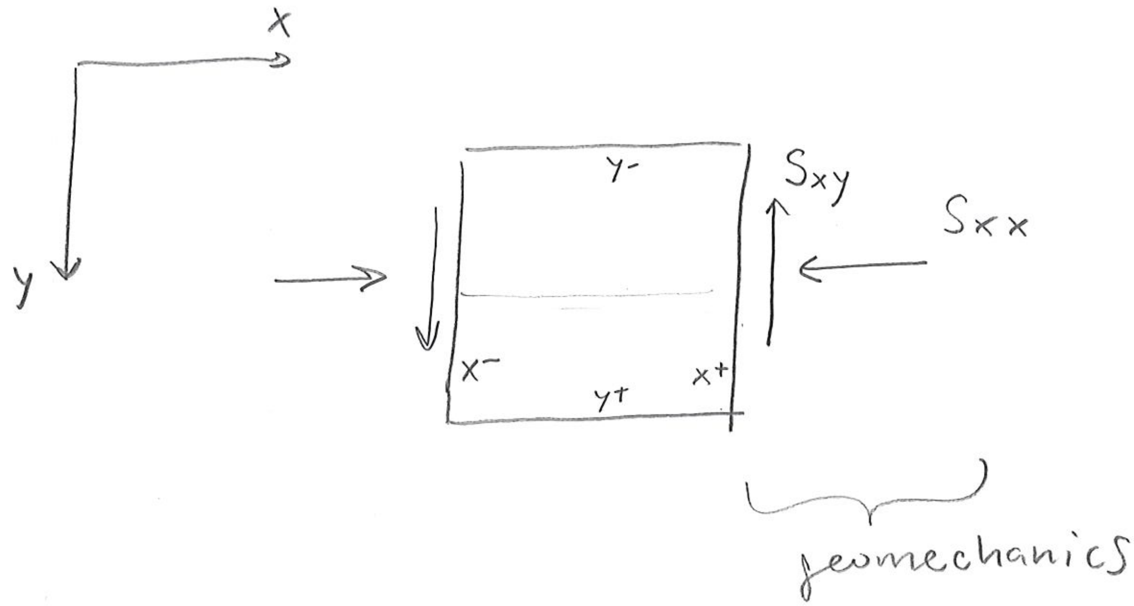






$E_{unload} \geq E_{load}$

HW 3



(2019 / 2 / 11)

$$\begin{cases} \varepsilon_{11} = \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ \varepsilon_{22} = -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ \varepsilon_{33} = -\frac{\nu}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{cases}$$

$\leftarrow \varepsilon_{11} = -\nu\, \varepsilon_{33}$

$$\begin{cases} 2\cdot\varepsilon_{12} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{12} \\ 2\cdot\varepsilon_{13} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{13} \\ 2\cdot\varepsilon_{23} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{23} \end{cases}$$

G: shear modulus, $G = \frac{E}{2(1+\nu)}$

$$\underbrace{\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}_{6\times1}} = \underbrace{\left[\begin{array}{ccc|ccc} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{array}\right]_{6\times6}}_{\text{compliance matrix}} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}_{6\times1}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{D}}} \cdot \underline{\underline{\sigma}} \implies \underset{\text{vector}}{\underline{\varepsilon}} = \underset{\text{matrix}}{\underline{D}} \cdot \underset{\text{vector}}{\underline{\sigma}}$$

VOIGT NOTATION

$$\underline{\underline{\sigma}} = \underbrace{\underline{\underline{C}}}_{\text{stiffness matrix}} \underline{\underline{\varepsilon}}$$

$$\underline{\underline{\sigma}} = \underline{\underline{D}}^{-1} \underline{\underline{\varepsilon}}$$

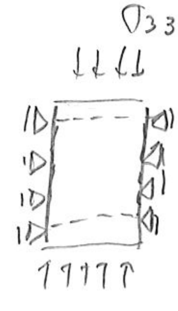
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \left[\begin{array}{ccc|ccc} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{array}\right] \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$

Uniaxial-strain stress path

no tectonic strains $\left\{ \begin{array}{l} \varepsilon_{11} = \varepsilon_{22} = 0 \qquad \varepsilon_{ij} = 0 \quad i \neq j \\ \varepsilon_{33} \neq 0 \end{array} \right.$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \varepsilon_{33} \end{bmatrix}$$

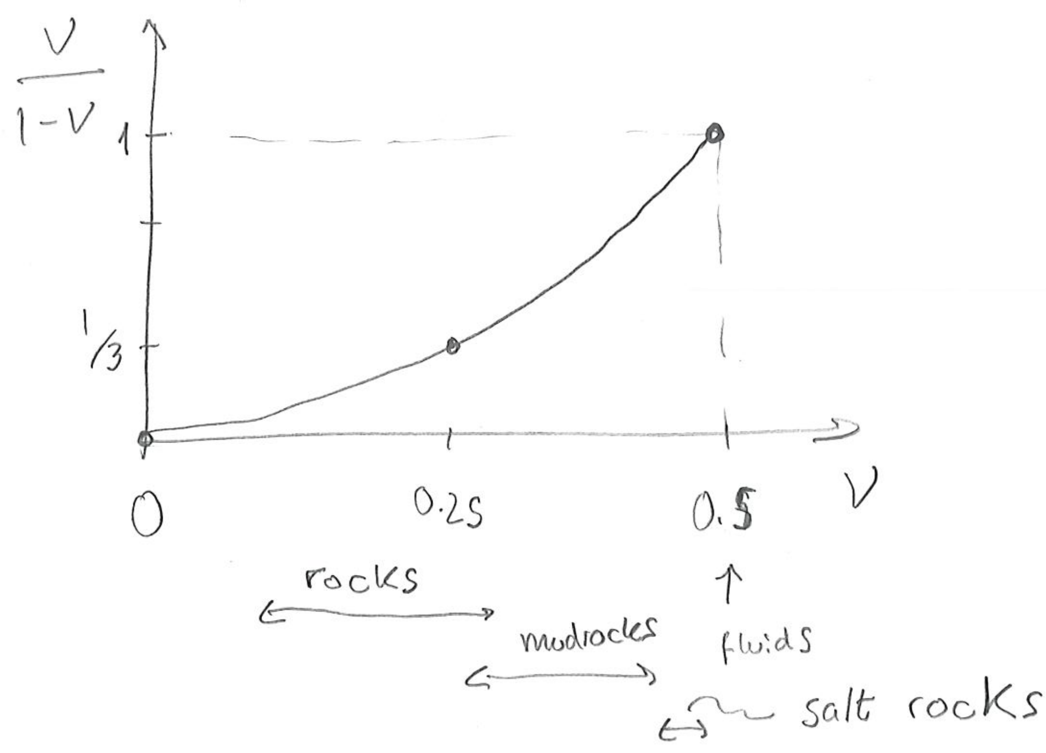
$$\begin{cases} \sigma_{33} = \overbrace{\frac{(1-\nu)\,E}{(1+\nu)(1-2\nu)}}^{M:\ \text{constrained modulus}} \cdot \varepsilon_{33} \\[2ex] \sigma_{11} = \sigma_{22} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\, \varepsilon_{33} = \dfrac{\nu \cancel{E}}{\cancel{(1+\nu)(1-2\nu)}} \dfrac{\cancel{(1+\nu)(1-2\nu)}}{(1-\nu)\cancel{E}}\, \sigma_{33} \end{cases}$$



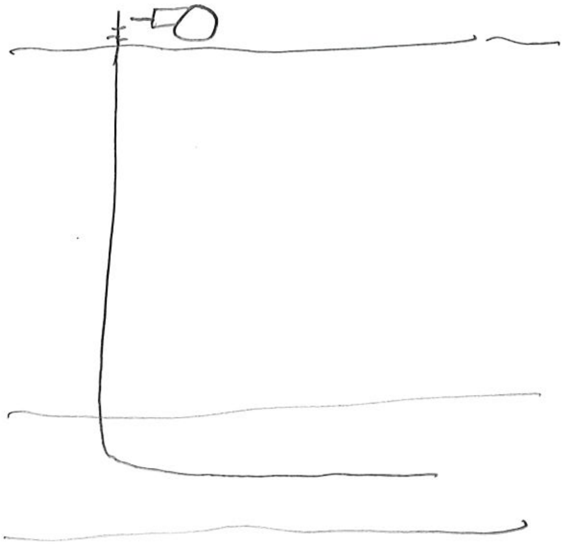
$M > E$

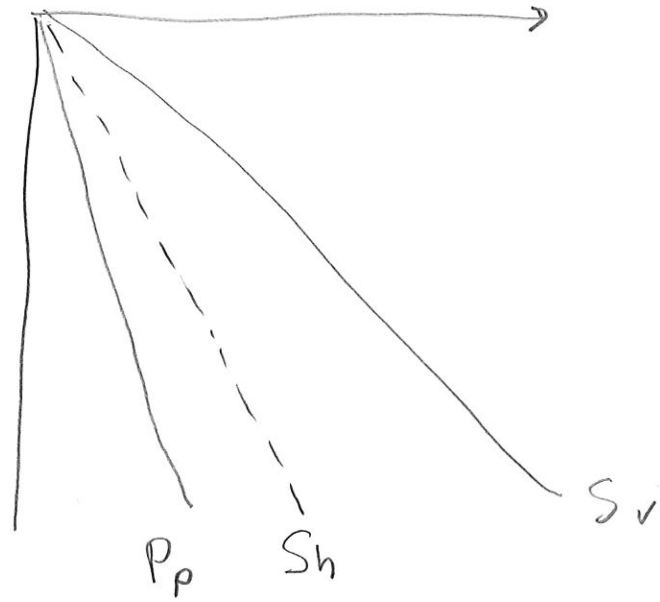
$$\boxed{\sigma_{22} = \underbrace{\frac{\nu}{1-\nu}}_{\text{lateral effective stress coefficient}} \sigma_{33}}$$

→ valid for effective stresses



$S_v \geq S_{Hmax} > S_{hmin}$





$$S_{hmin} = S_{Hmax} = S_h$$

$$\underbrace{S_h}_{\text{Total}} = \underbrace{\sigma_h}_{\text{effective}} + P_p$$

$$S_h = \frac{\nu}{1-\nu}\sigma_v + P_p$$

$$S_h = \frac{\nu}{1-\nu}\left(S_v - P_p\right) + P_p$$

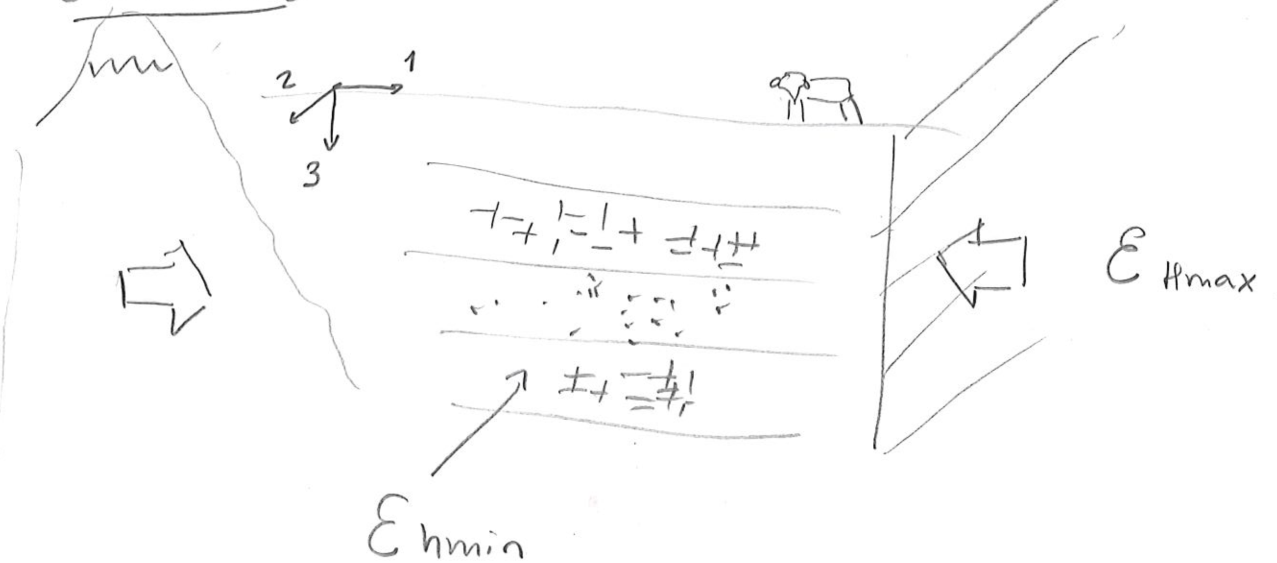
absolute values

$$S_h = \frac{\nu}{1-\nu} S_v + \frac{1-2\nu}{1-\nu} P_p$$

gradient

$$\underbrace{\frac{\Delta S_h}{\Delta z}}_{\text{frac gradient}} = \frac{\nu}{1-\nu} \underbrace{\frac{\Delta S_v}{\Delta z}}_{\text{lithostatic gradient}} + \frac{1-2\nu}{1-\nu} \underbrace{\frac{\Delta P_p}{\Delta z}}_{\text{pore pressure gradient}}$$

TECTONIC STRESSES (2/13/2019)



E': plane strain modulus

$$
\begin{cases}
\sigma_{11} = \dfrac{E}{1-\nu^2}\,\varepsilon_{11} + \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{22} + \dfrac{\nu}{1-\nu}\,\sigma_{33} \\[2ex]
\sigma_{22} = \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{11} + \dfrac{E}{1-\nu^2}\,\varepsilon_{22} + \dfrac{\nu}{1-\nu}\,\sigma_{33} \\[2ex]
\sigma_{33} = \displaystyle\int_0^z \rho_{bulk}\, g\, dz - P_p
\end{cases}
$$

$$\frac{0.25}{1-0.25} = \underline{\underline{0.333}}$$

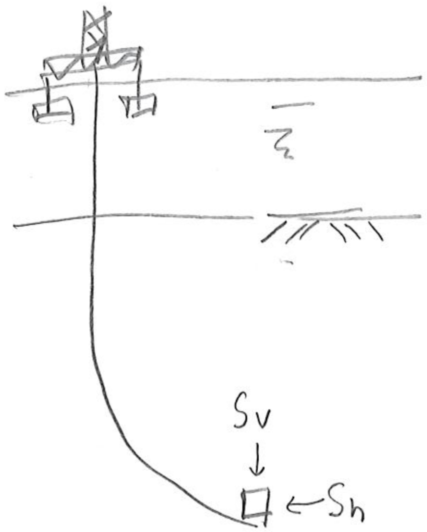
$$
\begin{cases}
\sigma_{Hmax} = \dfrac{E}{1-\nu^2}\,\varepsilon_{Hmax} + \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{hmin} + \dfrac{\nu}{1-\nu}\,\sigma_{33} \\[2ex]
\sigma_{hmin} = \dfrac{\nu E}{1-\nu}\,\varepsilon_{Hmax} + \dfrac{E}{1-\nu^2}\,\varepsilon_{hmin} + \dfrac{\nu}{1-\nu}\,\sigma_{33}
\end{cases}
$$

Measure
↓
- wellbore failure
- hyd frac test

Well logging surveys (Δt_p, Δt_s)
↕
laboratory test

Tectonic strains → calibrate

General procedure to calculate <u>total</u> Hz stress with linear elasticity



1. S_v (Total vertical stress)

2. P_p <
   - hydrostatic
   - non hydrostatic <
     - $\lambda_p = \frac{P_p}{S_v}$
     - $\emptyset_{shale}$

3. $\sigma_v = S_v - P_p$

4. Elasticity <
   - $\varepsilon_h = 0 \rightarrow \sigma_h = \frac{\nu}{1-\nu}\sigma_v$
   - $\varepsilon_{hmin} \neq$, $\varepsilon_{Hmax} \neq 0 \rightarrow$ $\sigma_{Hmax} = \ldots$, $\sigma_{hmin} = \ldots$

5. $$\underbrace{S_{hmin}}_{\text{total}} = \underbrace{\sigma_{hmin}}_{\text{effective}} + P_p$$
   $$S_{Hmax} = \sigma_{Hmax} + P_p$$

Reservoir Engineering

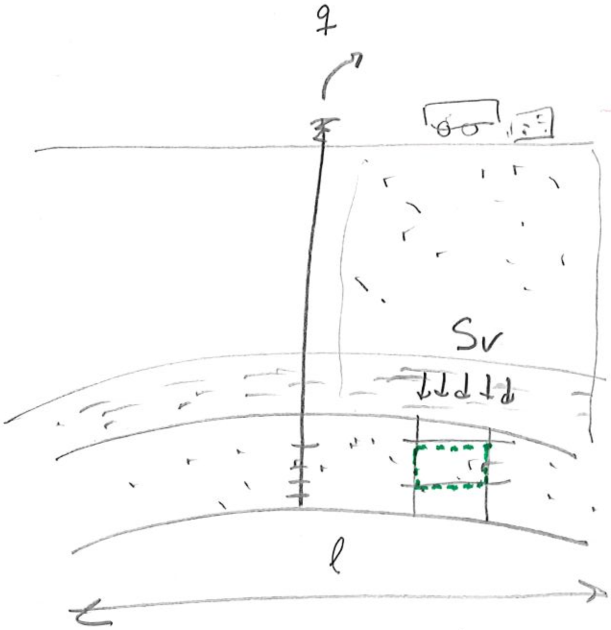
$$\frac{\partial P}{\partial t} = \frac{k}{\mu\, C_t}\,\frac{\partial^2 P}{\partial x^2}$$

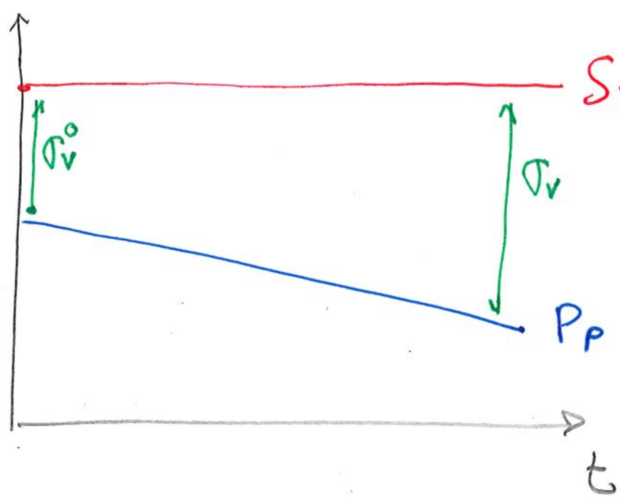
$\mu : 10^{-6}$
M (Mega) : 10^{6}
k (kilo) : 10^{3}

C_t — Total compressibility

$$C_t = C_g S_g + C_w S_w + C_o S_o + C_f$$

C_f → rock compressibility





$h \ll l$

Typical values of C_{bp}
[1 - 20 μsips]
↳ micro compress

Pore (rock) compressibility $\quad \boxed{C_f = C_{pp}} = \dfrac{1}{V_p}\dfrac{dV_p}{dP_p}$

V_p : pore volume
V_b : bulk volume

Bulk rock compressibility $\quad C_{bp} = \dfrac{1}{V_b}\dfrac{dV_b}{dP_p}$

$V_p = V_b$ (assumption)

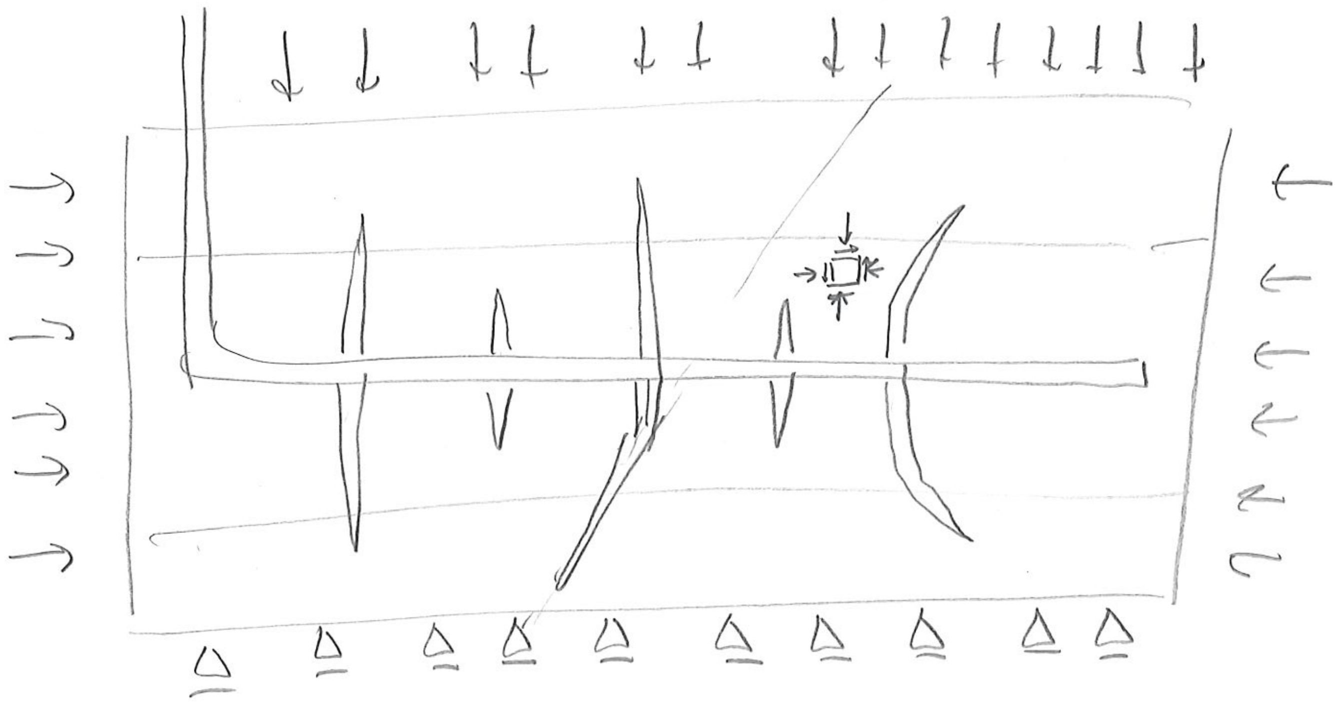
$C_{bp} = \dfrac{1}{M}$

$M = [\text{psi}, \text{MPa}]$

$$C_{pp} = \frac{1}{\frac{V_p}{V_b}}\;\frac{1}{V_b}\frac{dV_b}{dP_p} = \frac{C_{bp}}{\emptyset} = \underbrace{\frac{1}{\emptyset\, M}}$$

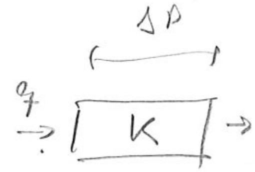
C_{bp} = [1/MPa]
[$10^{-6}\,\text{psi}^{-1}$]
↓
μsip

General solution for a continuum mechanics problem



→ Fluid flow problem

- Darcy : $\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = -\dfrac{K}{\mu} \begin{bmatrix} \partial P/\partial x_1 \\ \partial P/\partial x_2 \\ \partial P/\partial x_3 \end{bmatrix}$



- Mass conservation : $\dfrac{\partial q_1}{\partial x_1} + \dfrac{\partial q_2}{\partial x_2} + \dfrac{\partial q_3}{\partial x_3} = 0$

$$-\frac{K}{\mu}\left(\frac{\partial^2 P}{\partial x_1^2} + \frac{\partial^2 P}{\partial x_2^2} + \frac{\partial^2 P}{\partial x_3}\right) = 0$$

→ Mechanics

- Elasticity (Linear) : $\sigma \leftrightarrow \varepsilon$
- Momentum conservation

$$\underbrace{(\lambda+\mu)}_{\text{Lamé parameters}} \overbrace{\nabla}^{\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)} \cdot (\nabla \cdot \underline{u}) + \mu \overbrace{\nabla^2}^{\left(\frac{\partial^2}{\partial x_1^2}, \frac{\partial^2}{\partial x_2^2}, \frac{\partial^2}{\partial x_3^2}\right)} (\underline{u}) + \underline{b} = 0$$

$\underline{u}$: displacement vector → strain

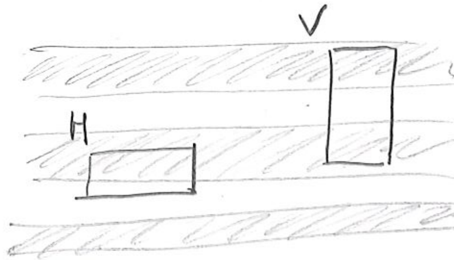
$\underline{b}$: acceleration (gravity)

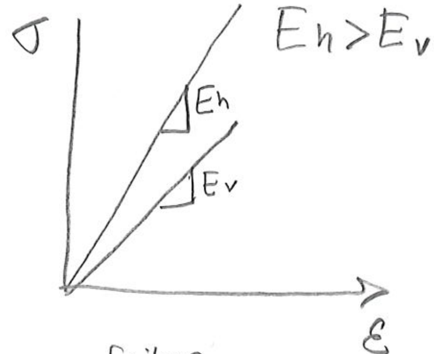
Lamé parameters $\begin{cases} \mu = G \\ \lambda = \dfrac{\nu E}{(1-2\nu)(1+\nu)} \end{cases}$

Solutions
- Analytical
  - Kirsch (wellbore)
  - Griffith (fracture)
- Numerical → Hydraulic Fracturing simulators
  - Finite Differences → CMG
  - Finite Element Method

Real rocks

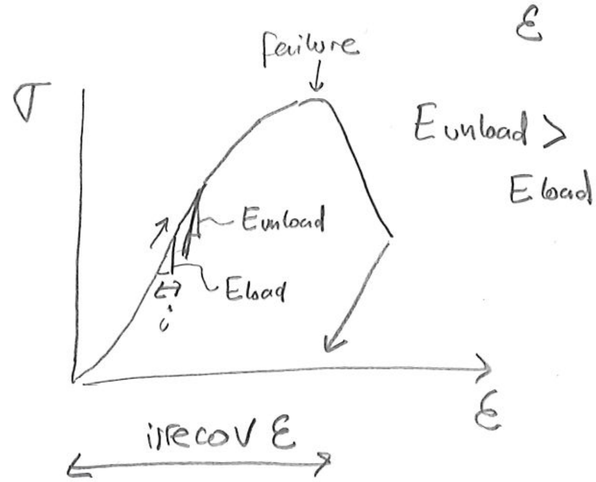
- anisotropy



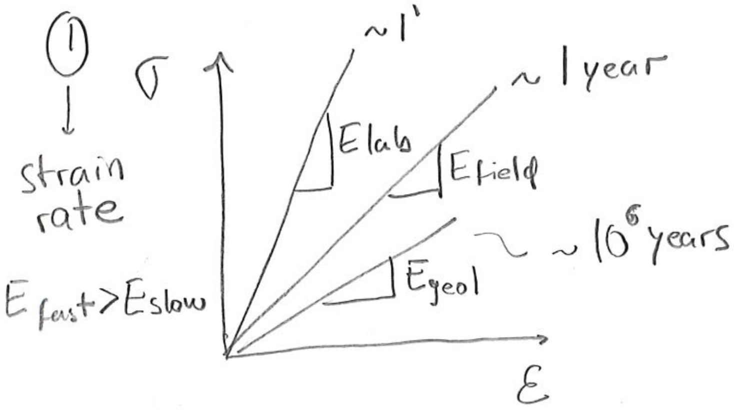


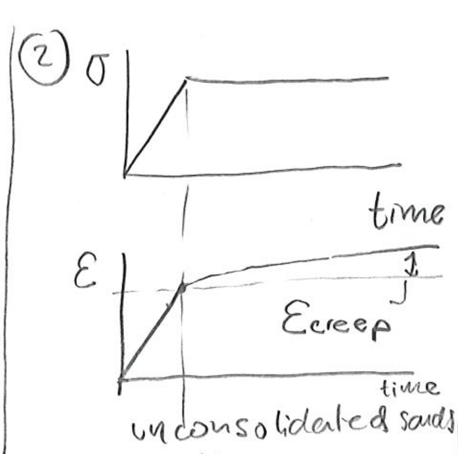
- elasto-plasticity

$\underbrace{\text{elasto}}_{\text{recov}}$-$\underbrace{\text{plasticity}}_{\text{irrecoverable}}$

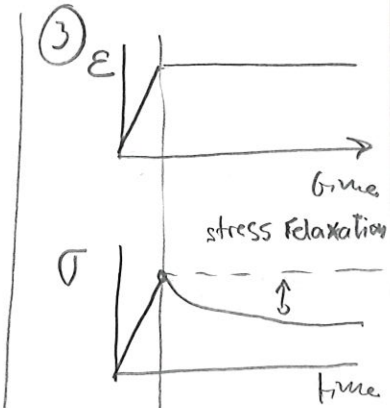


- visco-elasticity





unconsolidated sands
→ salt rocks

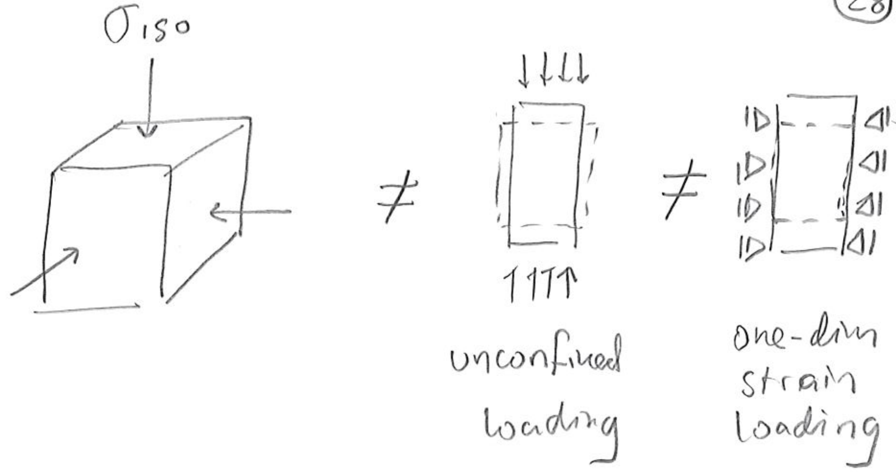


HW 4

3) Isotropic loading

$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{iso}$

$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$



$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{D}}}\ \underline{\underline{\sigma}}$$

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} 1/E & -\nu/E & -\nu/E \\ \text{---} & 1/E & -\nu/E \\ \text{---} & \text{---} & 1/E \end{bmatrix}}\begin{bmatrix} \sigma_{iso} \\ \sigma_{iso} \\ \sigma_{iso} \end{bmatrix}$$

$$\varepsilon_{vol} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

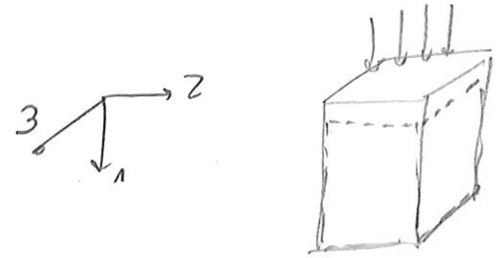
$$\hookrightarrow \varepsilon_{11} = \frac{(1-2\nu)}{E}\sigma_{iso}$$

$$\sigma_{iso} = \underbrace{\boxed{}}\ \varepsilon_{vol}$$

K : Bulk modulus

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{iso} & 0 & 0 \\ 0 & \sigma_{iso} & 0 \\ 0 & 0 & \sigma_{iso} \end{bmatrix} = \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \text{psi}$$

4) $$M = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

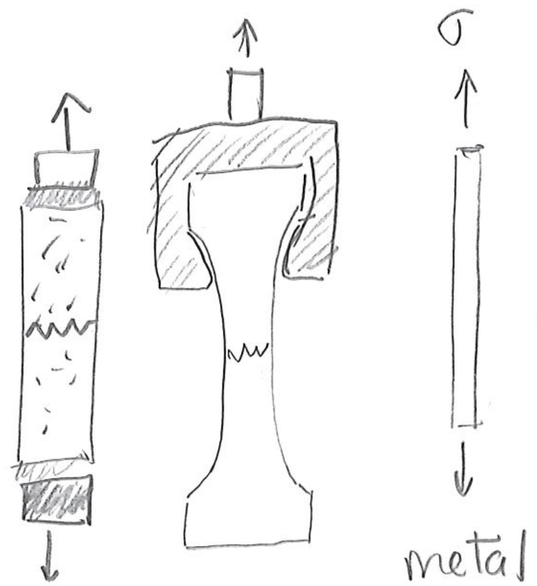


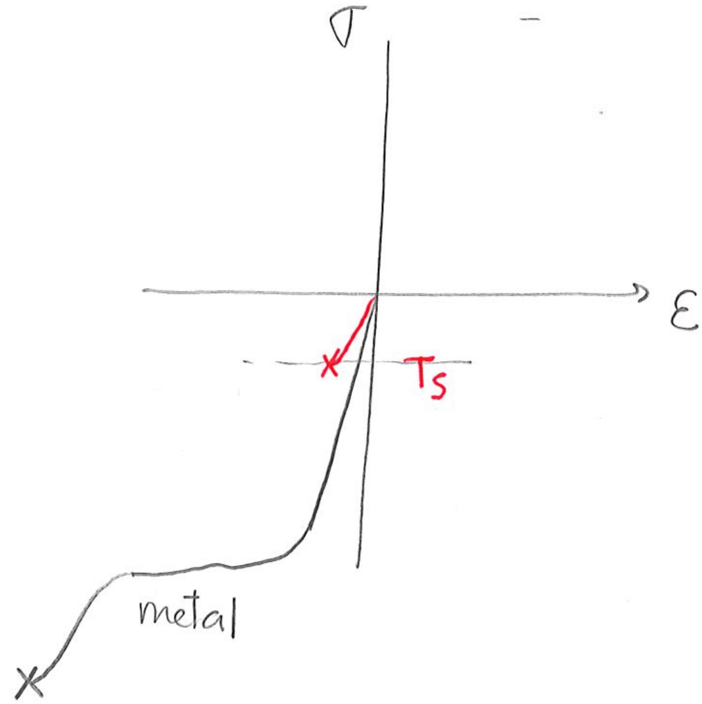
$$\underset{\approx}{\varepsilon} = \begin{bmatrix} \underline{\quad} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; ; \; \underset{\approx}{\sigma} = \begin{bmatrix} \underline{\quad} & 0 & 0 \\ 0 & \underline{\quad} & 0 \\ 0 & 0 & \underline{\quad} \end{bmatrix}$$

5)

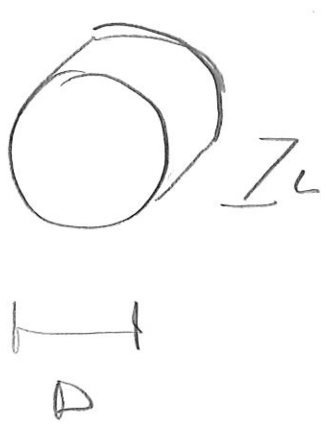
Failure of rocks

- Tensile failure ↔ tensile strength

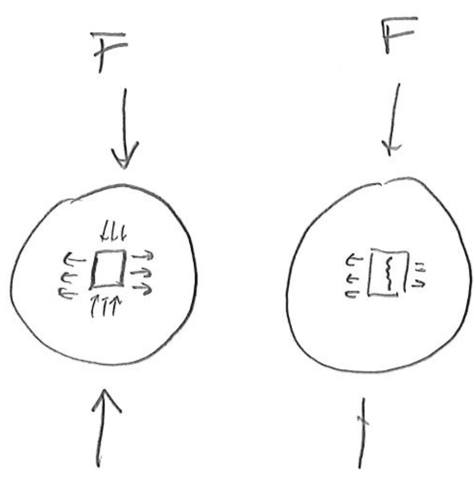


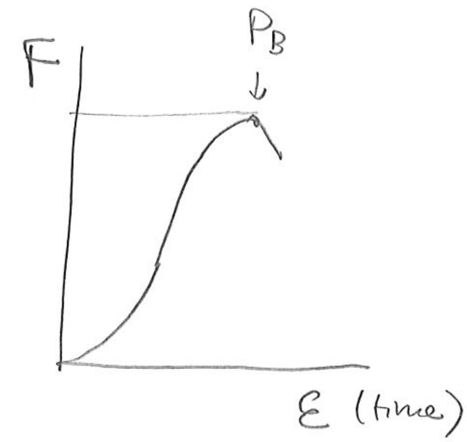


- Brazilian Test



$D > L$

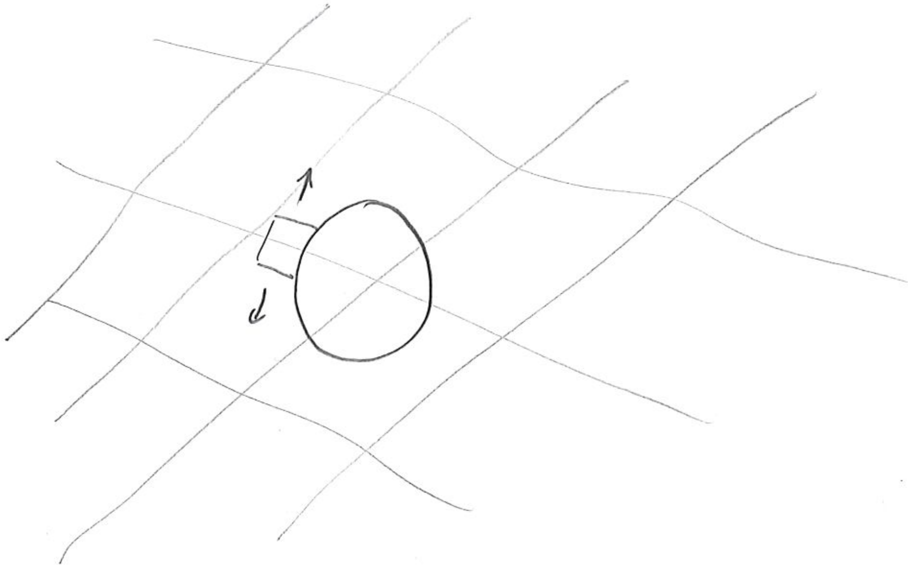




$$T_S = \frac{P_B}{\pi \cdot L \cdot R}$$

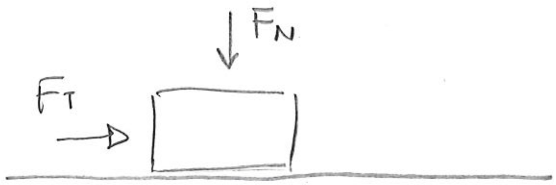
max $\boxed{550\ lb}$

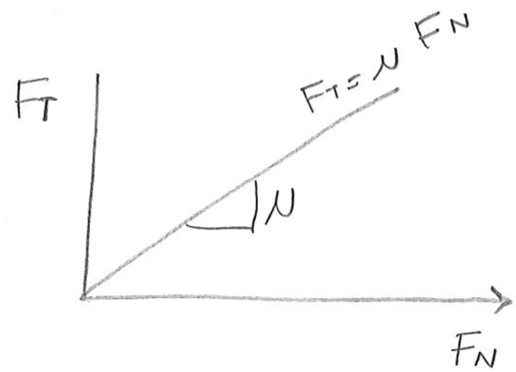
Class Example $T_S = \dfrac{500\,lb}{\pi \cdot 1in \cdot 0.5in} = 318\ psi$

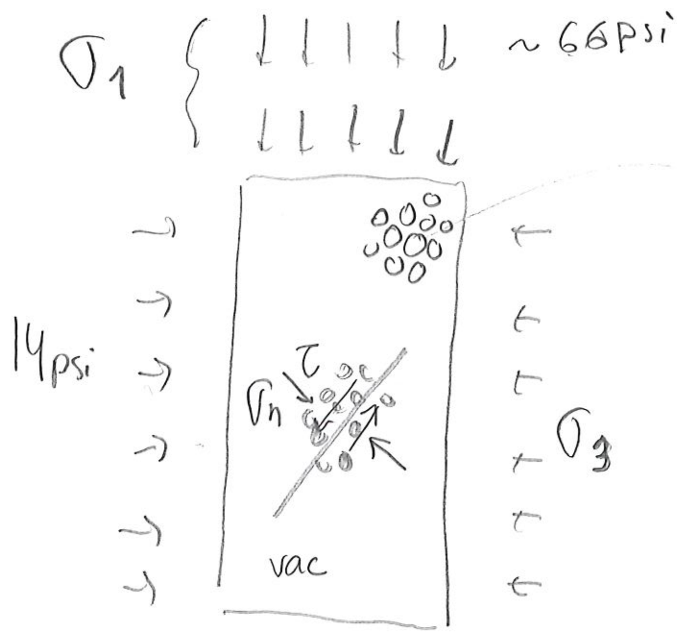


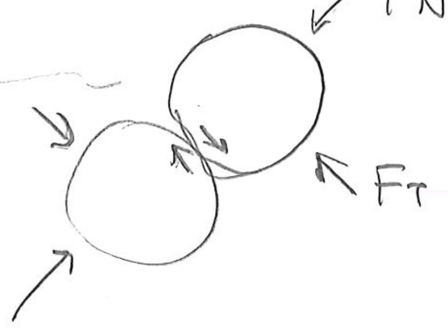
T_s (rocks) < 1500 psi

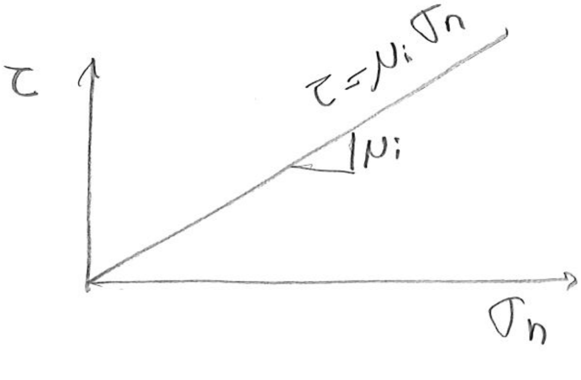
Shear strength





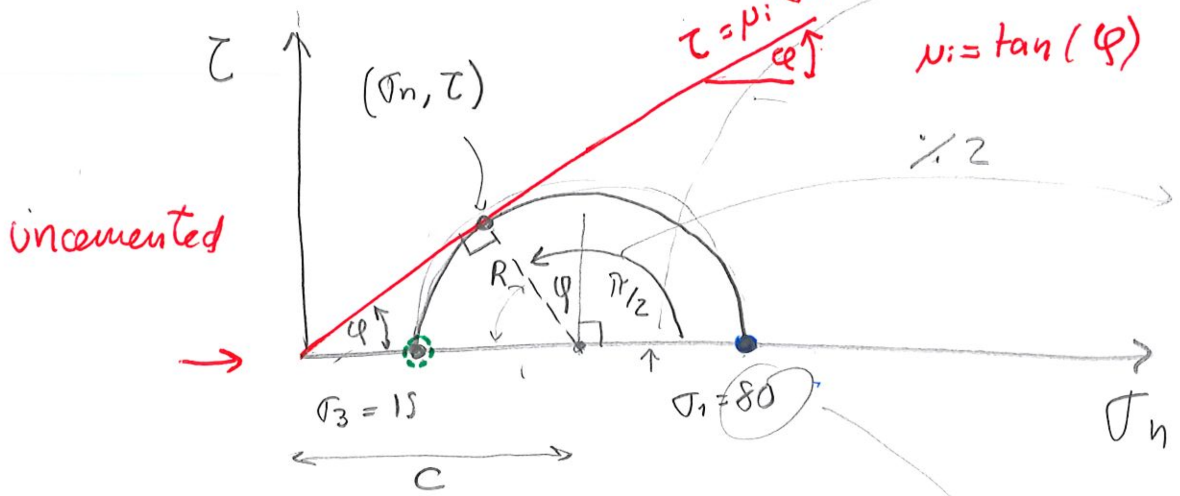


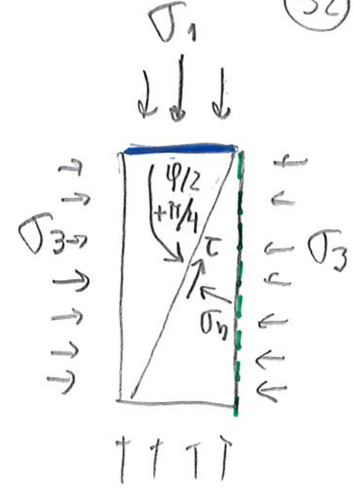




Mohr-Coulomb Failure Criterion

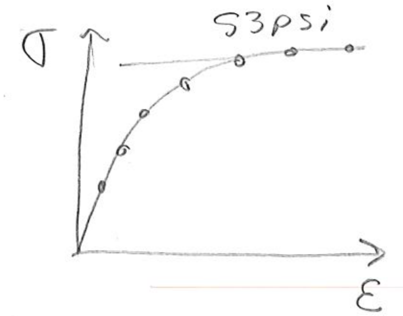
$\mu_i = \tan(\varphi)$





Mohr circle: graphical representation of the stress tensor

→ actually 53 psi



$$\underline{\underline{\sigma}} = \begin{bmatrix} 80 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix} \text{psi} \rightarrow \text{principal}$$

$\varphi \sim 30^\circ$

$\frac{\varphi}{2} + \frac{\pi}{4} = \underline{60^\circ}$

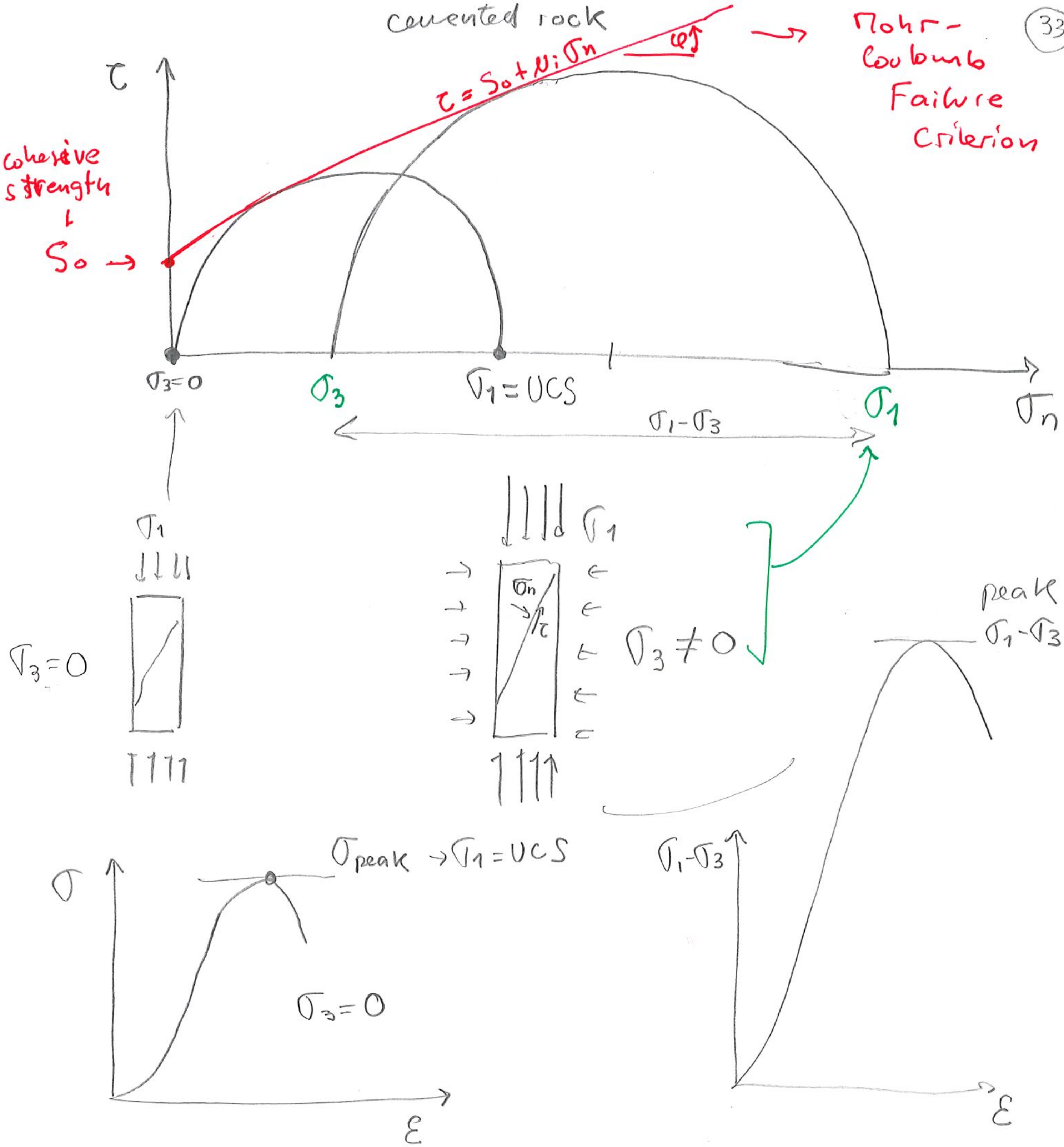
stress anisotropy

$$\frac{\sigma_1}{\sigma_3} = \frac{C+R}{C-R} = \frac{\cancel{C} + \cancel{C}\sin\varphi}{\cancel{C} - \cancel{C}\sin\varphi}$$

$$\frac{\sigma_1}{\sigma_3} = \frac{1+\sin\varphi}{1-\sin\varphi}$$

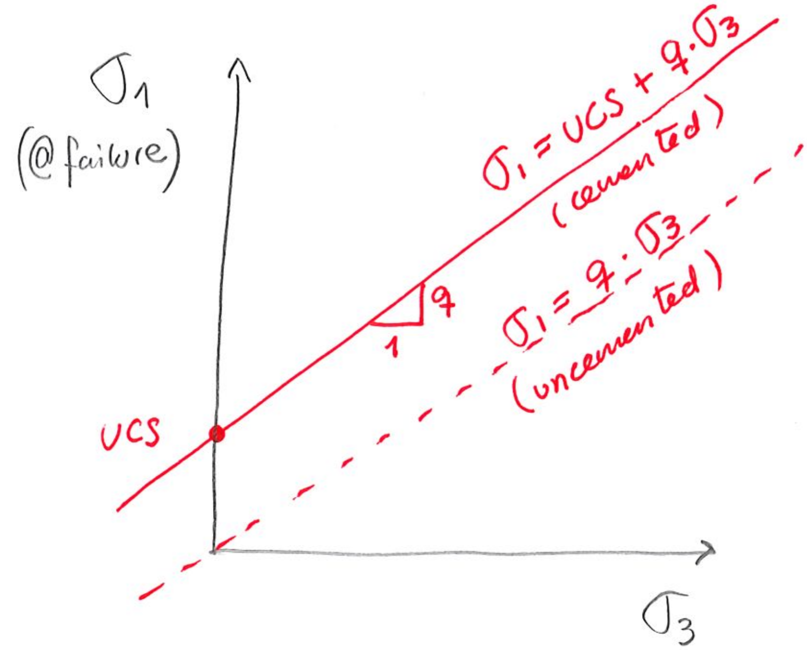
$\varphi = 30^\circ$

$$\boxed{\frac{1+\sin\varphi}{1-\sin\varphi} = 3}$$



UCS
- Berea ~ 3000 psi
- Boise ~ 3500 psi
- Texas cream ~ 2000 psi

Shales ~ 10,000 psi
15,000 psi



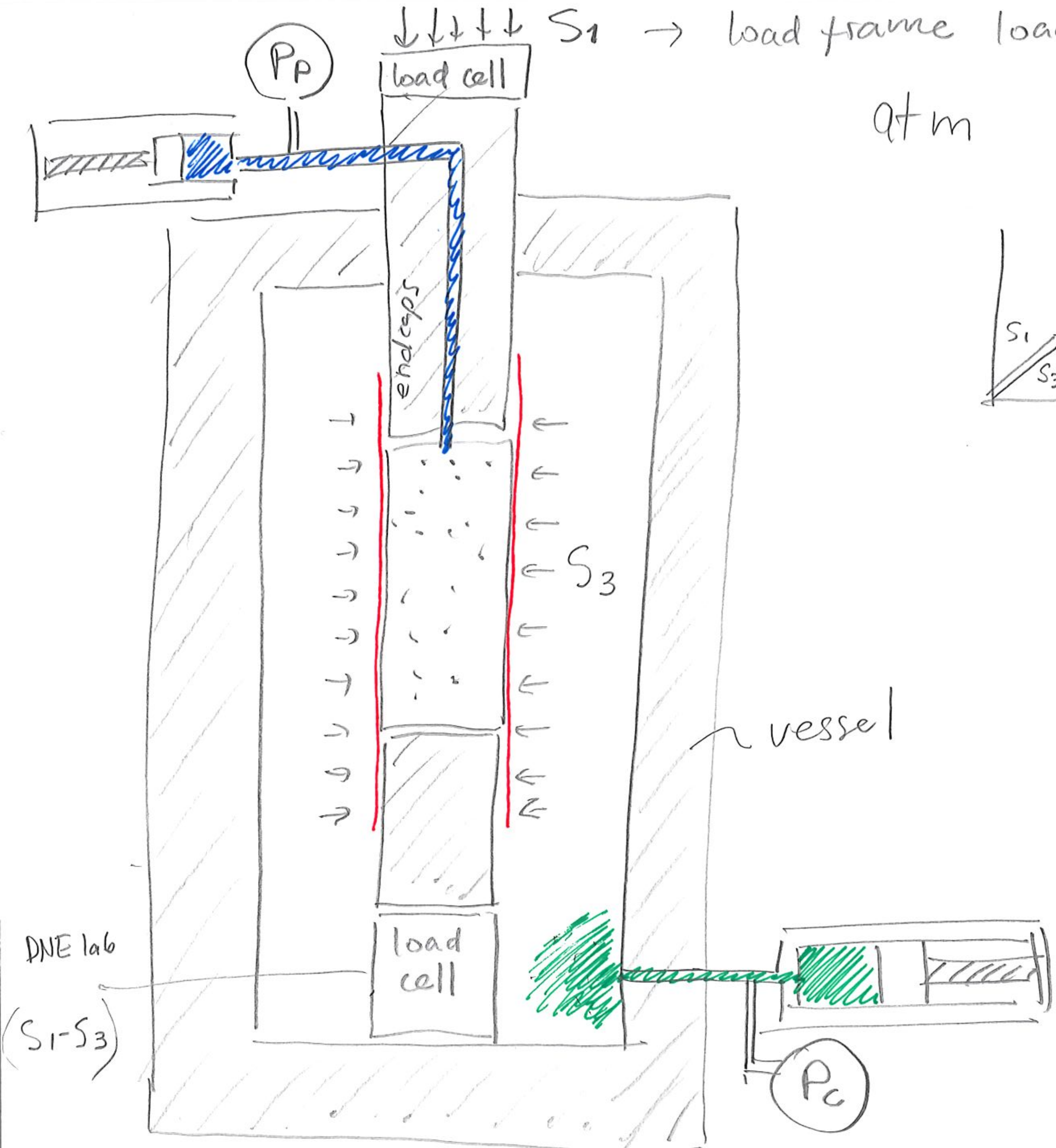
$$q = \frac{1 + \sin\varphi}{1 - \sin\varphi}$$

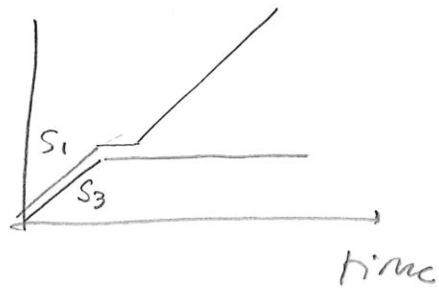
$$UCS = 2 \cdot S_0 \cdot \sqrt{q}$$

$$\varphi = \arctan\left(\frac{q-1}{2\sqrt{q}}\right)$$

$$\sigma_3 = P_c - P_p$$

$$\sigma_1 = \sigma_3 + (\sigma_1 - \sigma_3) \quad ; \quad \overbrace{S_1 - S_3}^{\text{total}} = \overbrace{\sigma_1 - \sigma_3}^{\text{effective}}$$



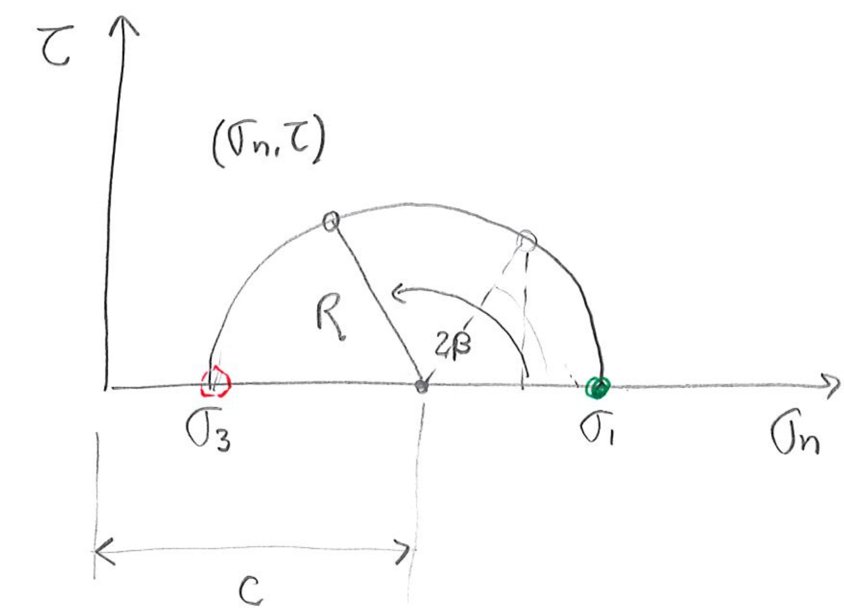


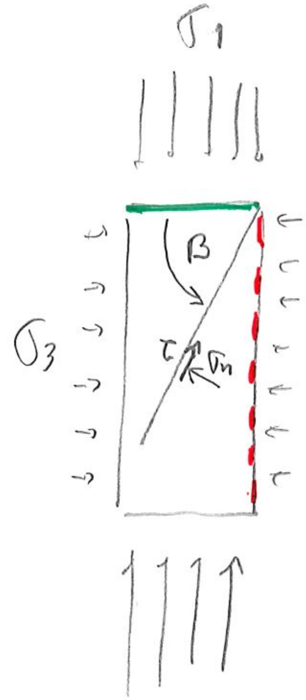
No pore pressure $\Rightarrow$ $S_1 = \sigma_1$; $S_3 = \sigma_3$ (dry rocks)

With pore pressure $\Rightarrow$ $\sigma_3 = S_3 - P_p = P_c - P_p$

$$\sigma_1 = S_1 - P_p$$

Mohr circle

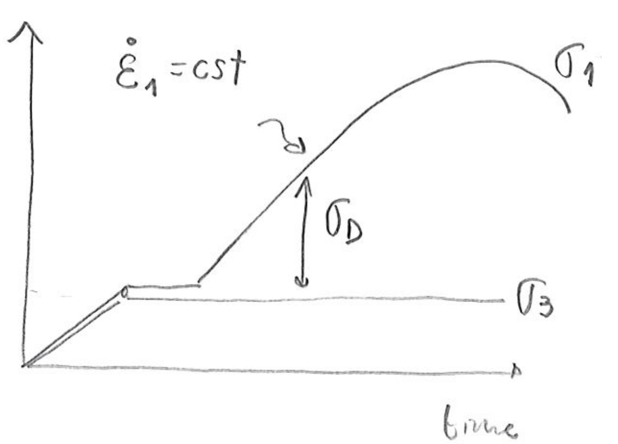


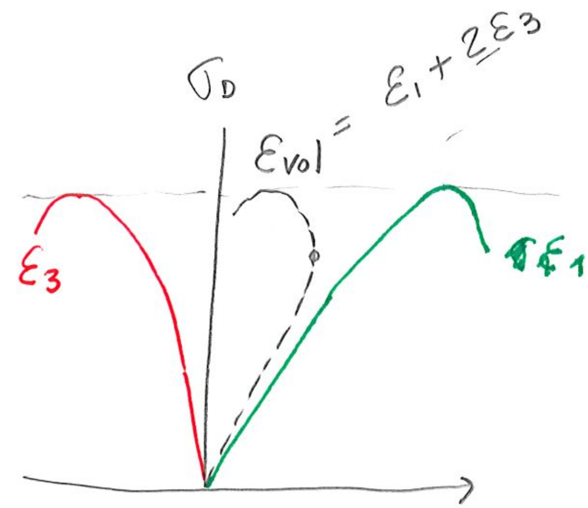


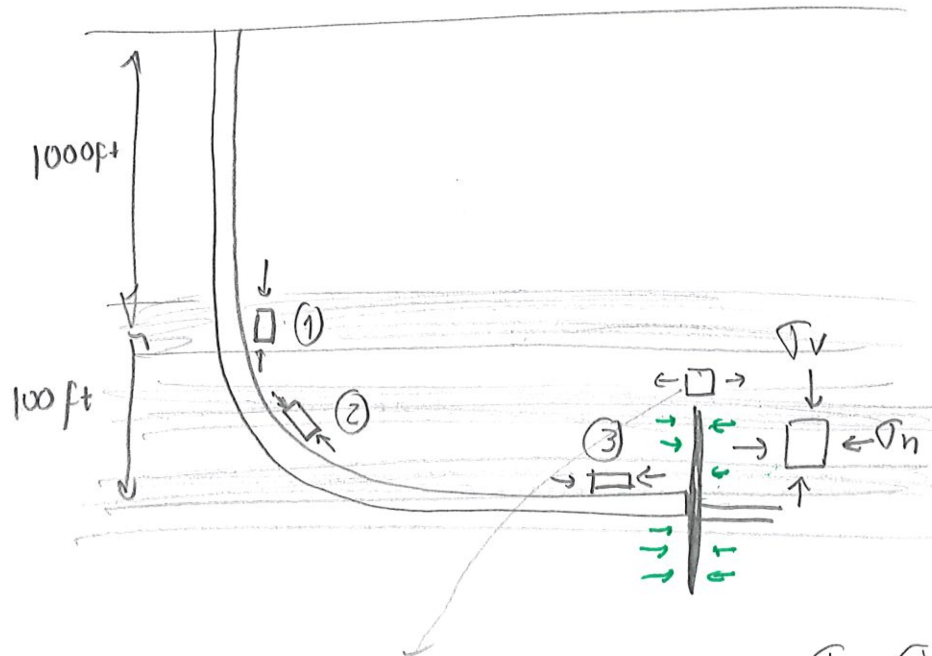
$$\begin{cases} \sigma_n = c + R \cdot \cos 2\beta \\ \tau = R \cdot \sin 2\beta \end{cases}$$

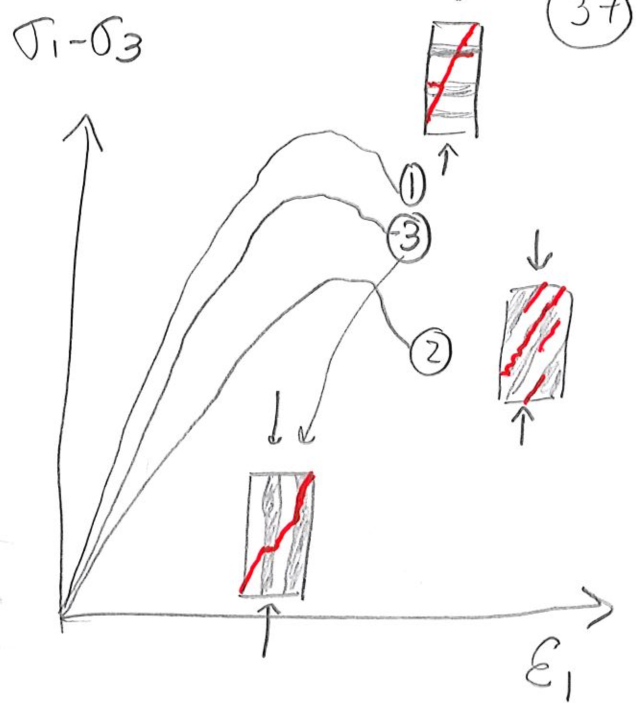
$$2\beta = (0, 180°)$$

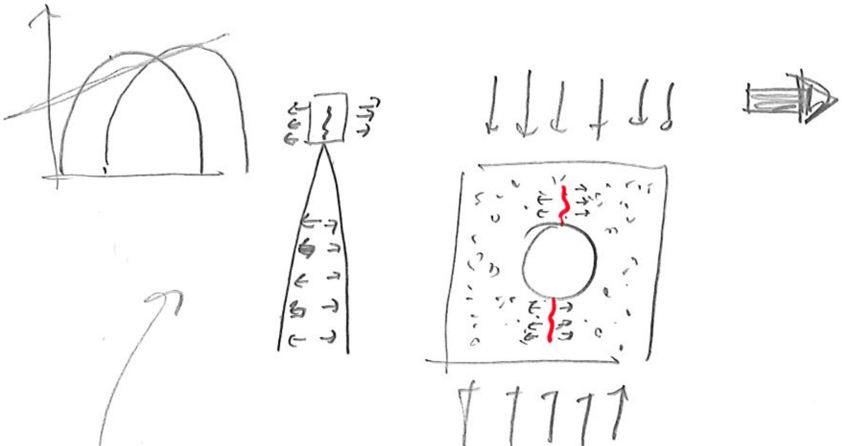
$$\dot{\varepsilon}_1 = \frac{d\varepsilon}{dt}$$

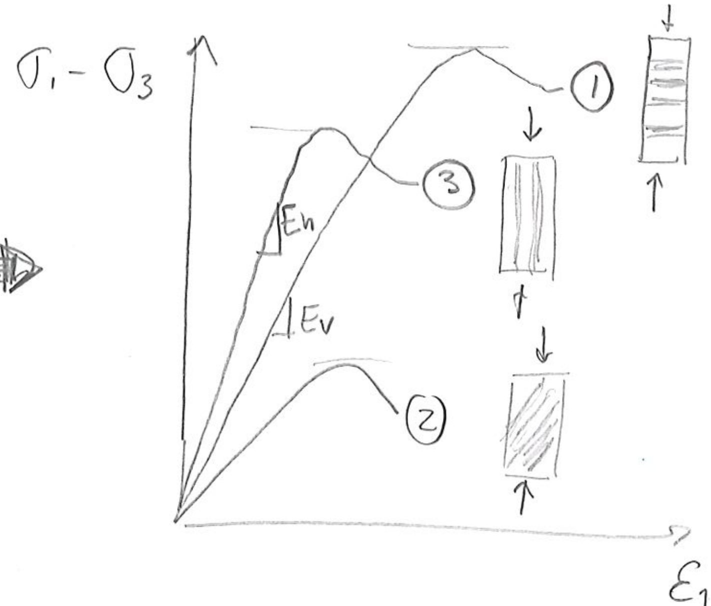


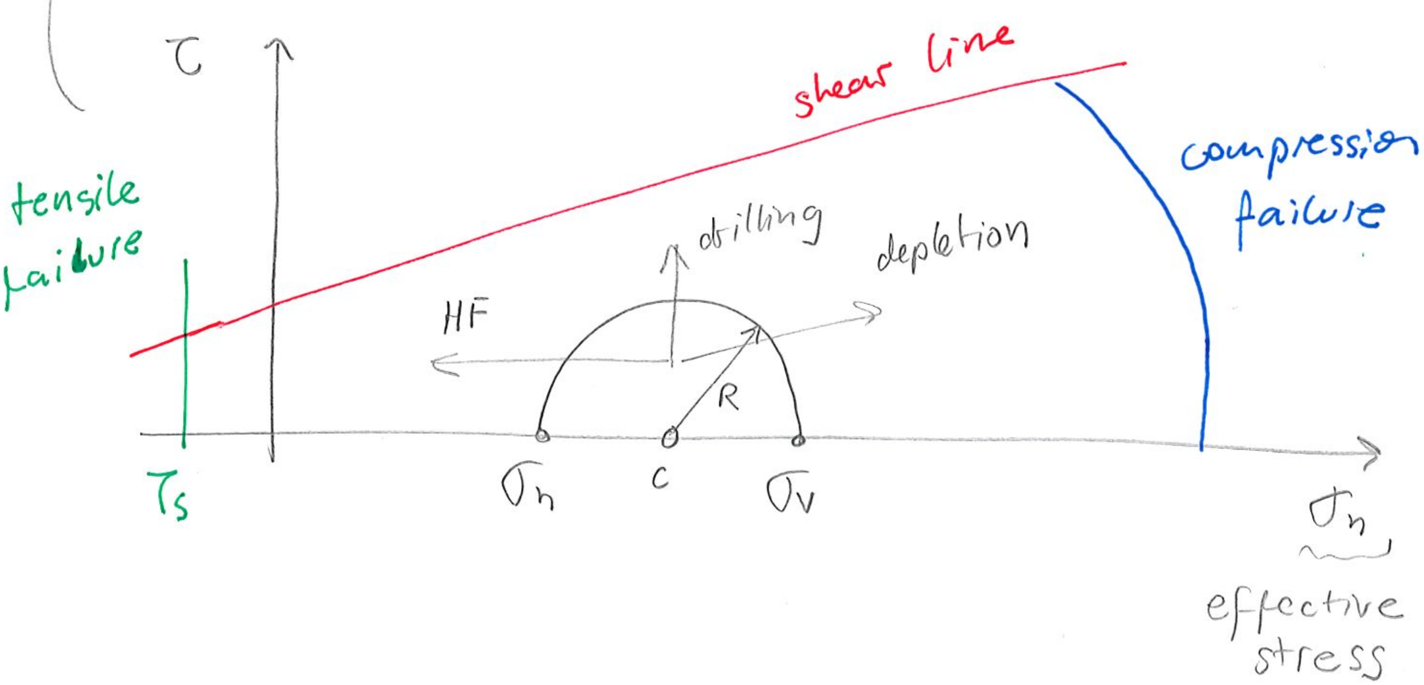



1000ft
100 ft
1
2
3
σ_v
σ_h

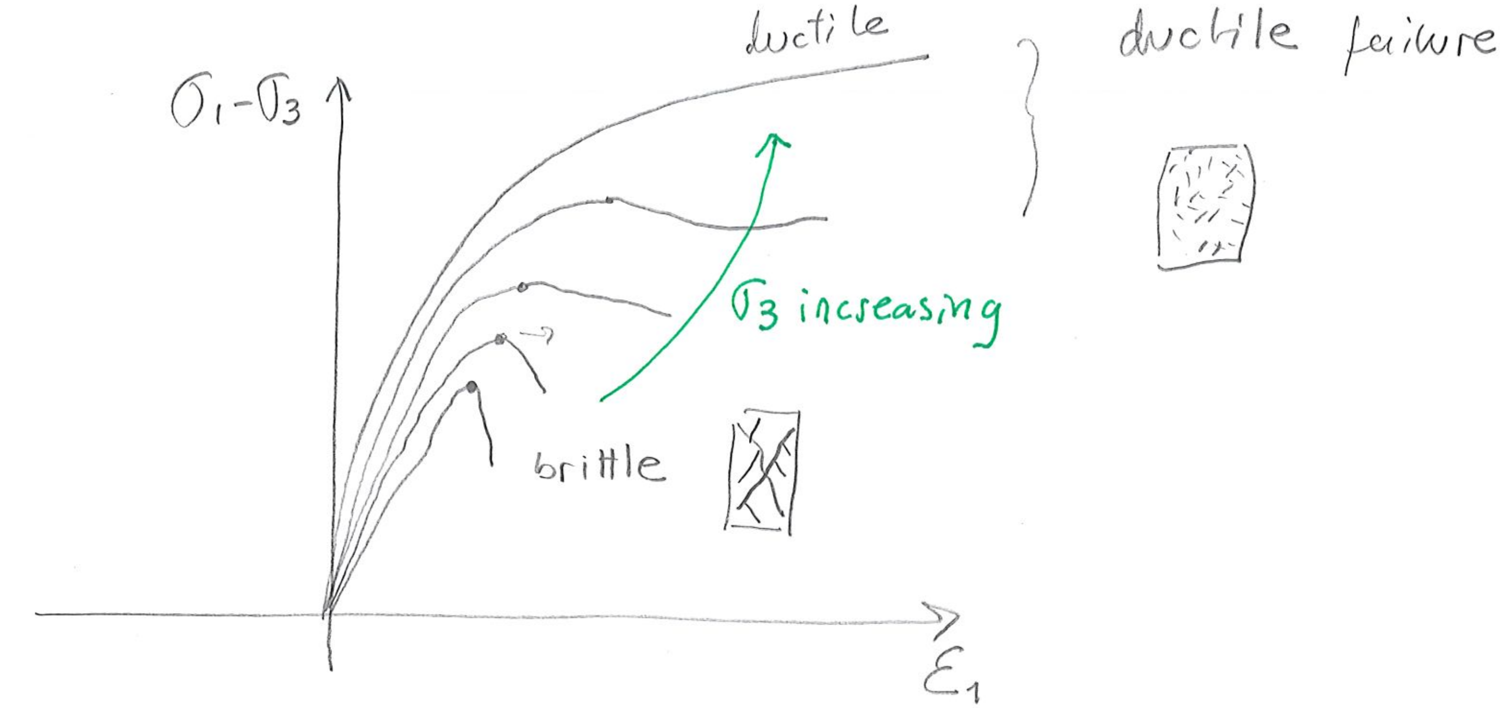

$\sigma_1 - \sigma_3$
1
3
2
ε_1



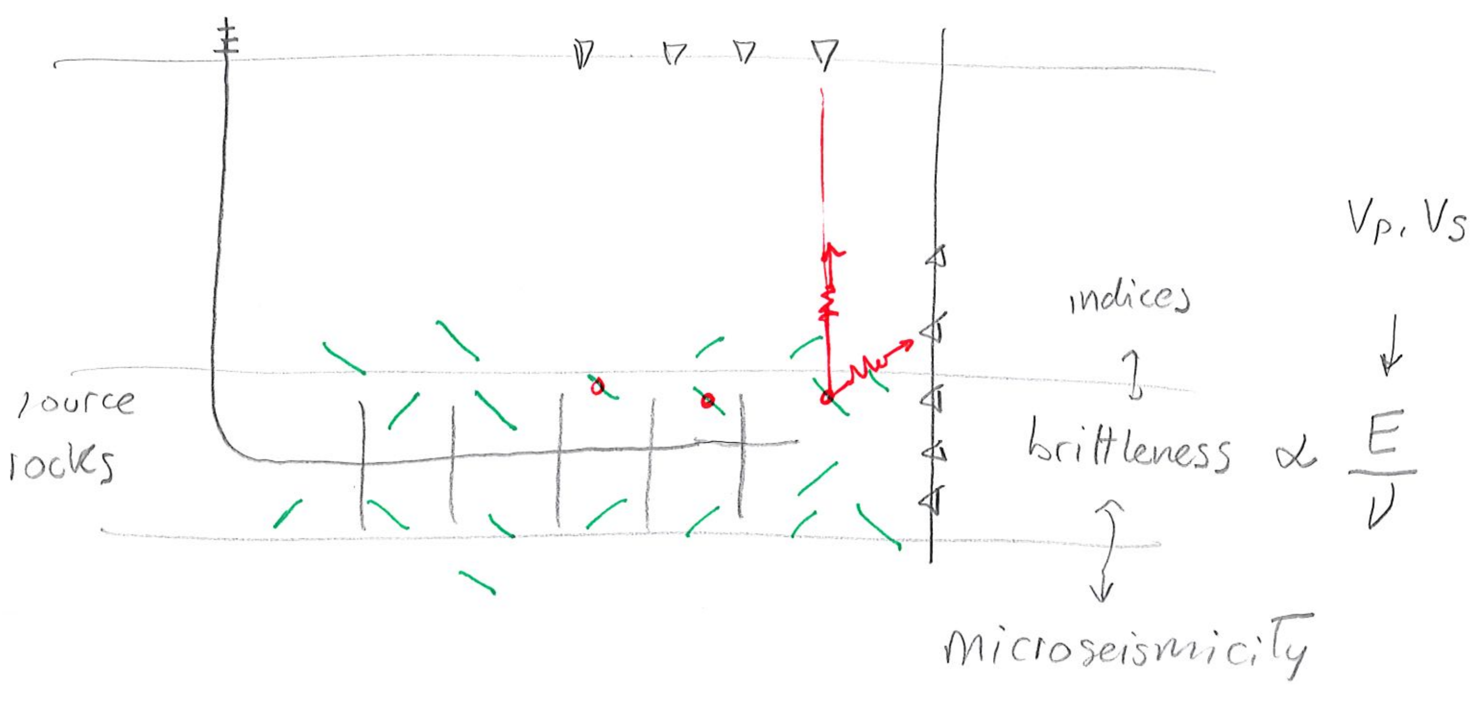

$\sigma_1 - \sigma_3$
1
3
E_h
E_v
2
ε_1


τ
shear line
tensile failure
compression failure
drilling
depletion
HF
R
T_s
σ_h
c
σ_v
σ_n
effective stress

Brittle to ductile transition

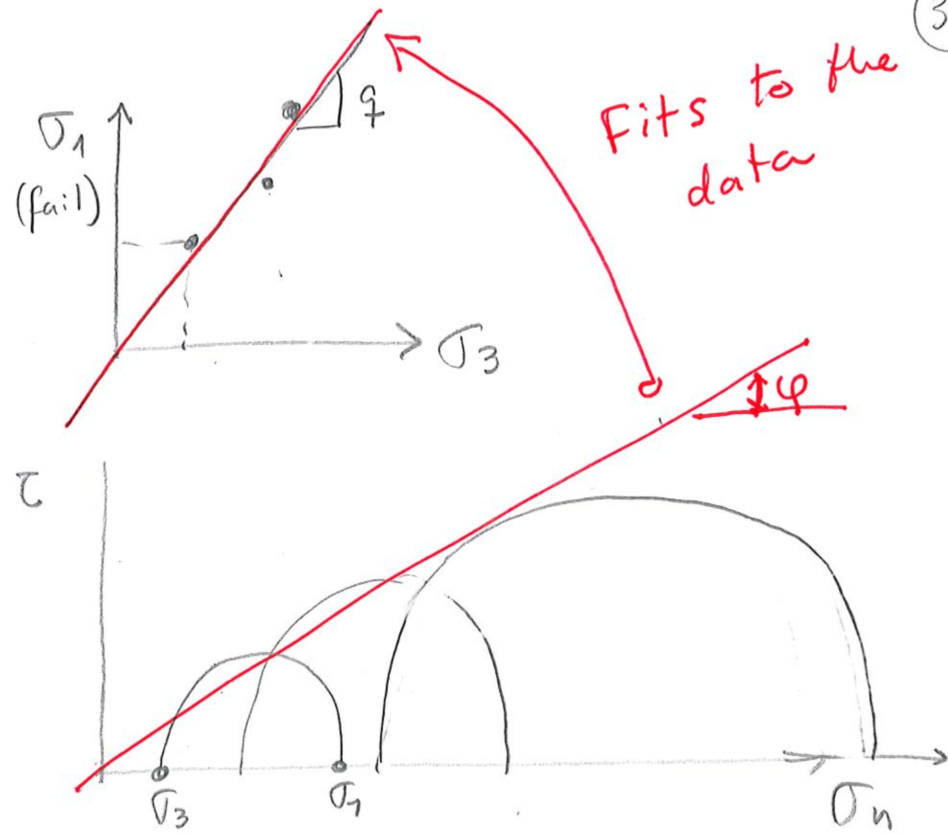


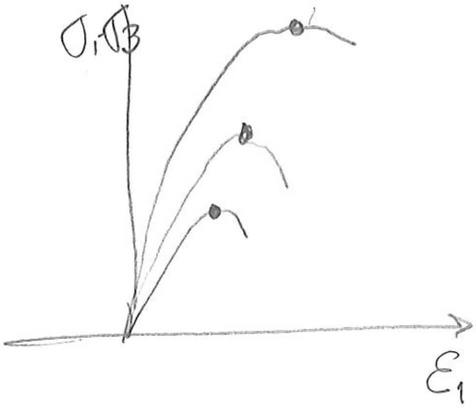
brittle → ductile $\left\{ \begin{array}{l} \uparrow \text{temperature} \\ \uparrow \text{time frame} \end{array} \right.$



HW 5

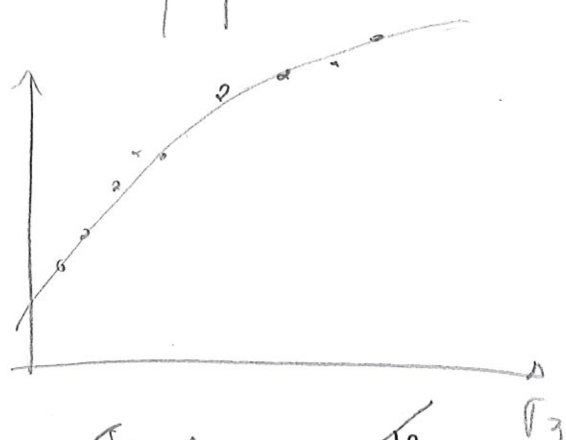
1) uncemented





σ_n	τ
0	—
10	—

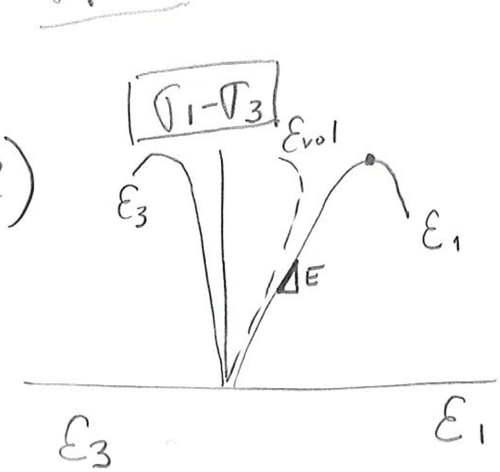
$\tau = \sigma_n \cdot \tan\varphi$

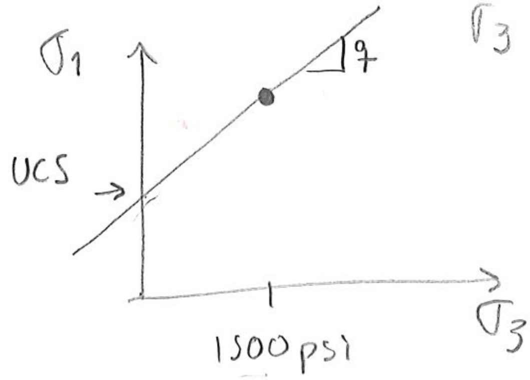


$\sigma_1 = \sigma_D + \sigma_3$

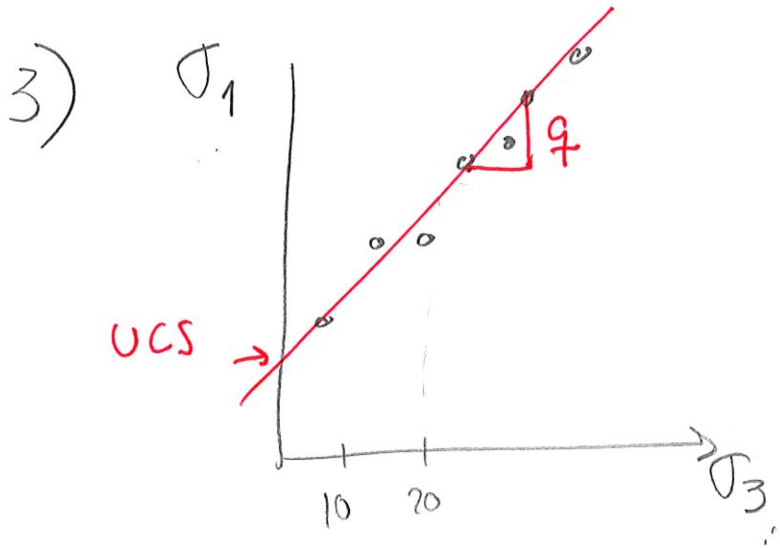
$\sigma_1 = \sigma_1 - \cancel{\sigma_3} + \cancel{\sigma_3}$

2)





3)



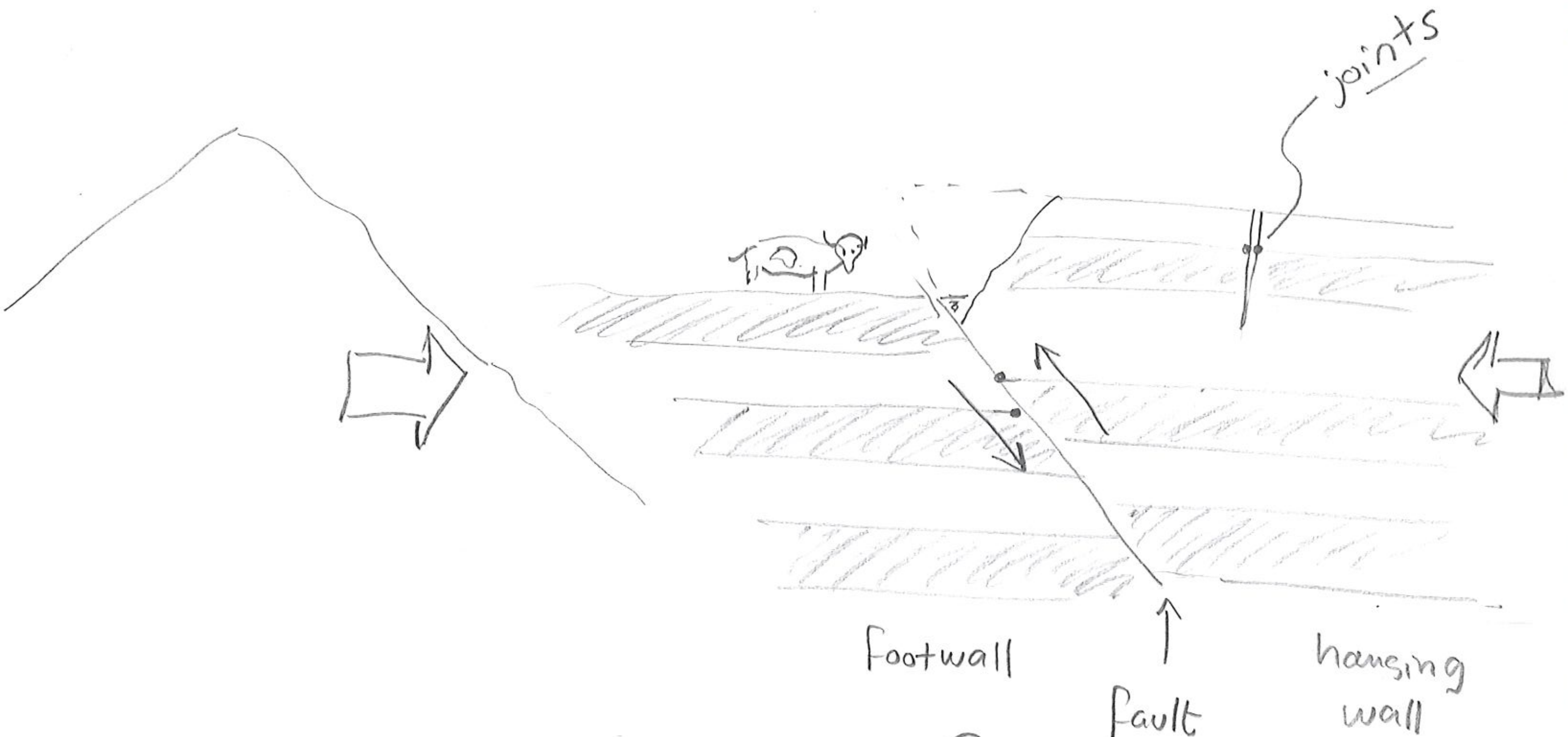
Example $S_3 = 20$ MPa (both cases)

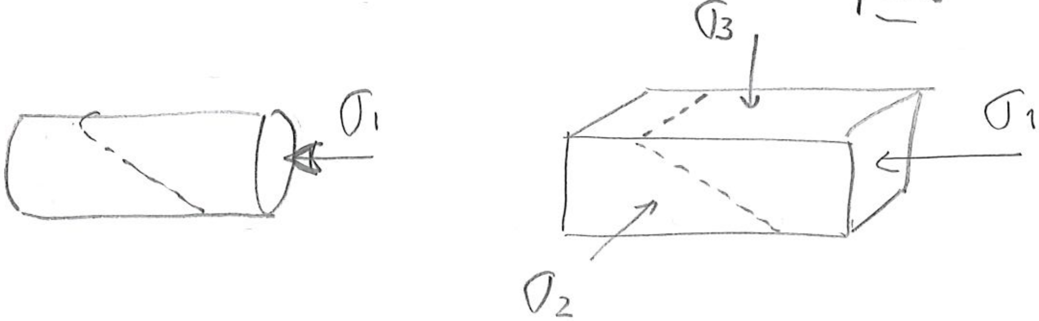
$P_p = 10$ MPa $\rightarrow$ $\sigma_3 = 10$ MPa

$P_p = 0$ MPa $\rightarrow$ $\sigma_3 = 20$ MPa

Faults and fractures (shear)

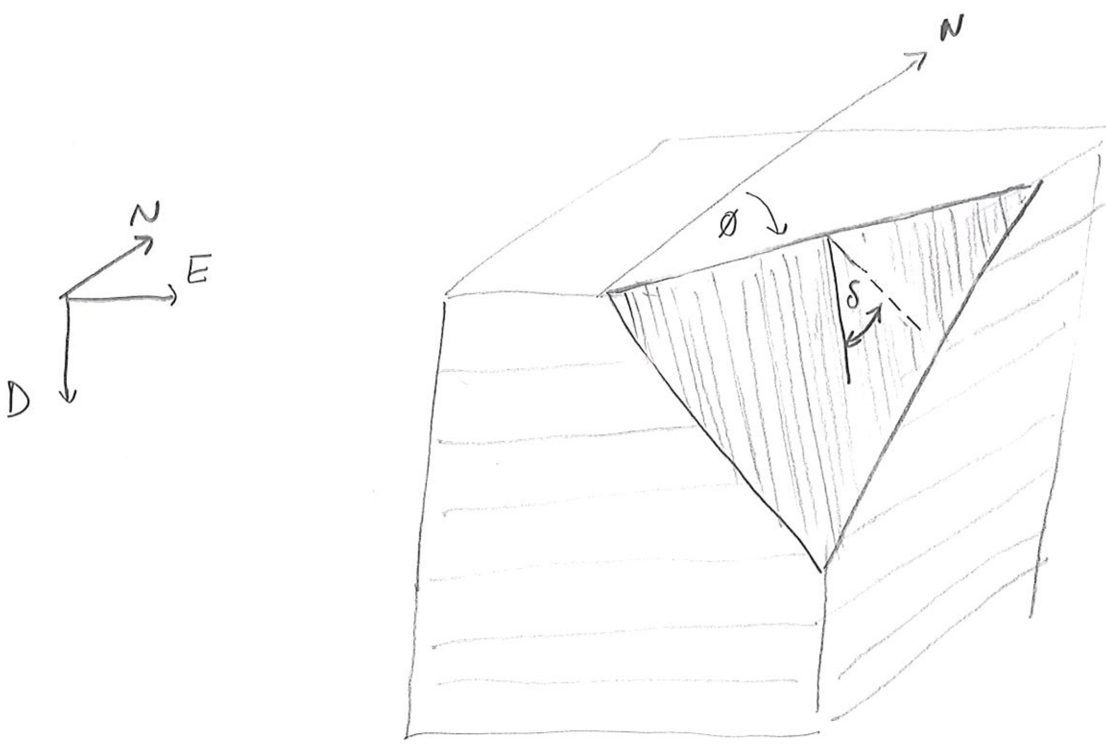
- crème brûlée
- frade oil field





Methods to map faults and fractures

- seismic | big faults
- well bore imaging | big and small faults
- ...



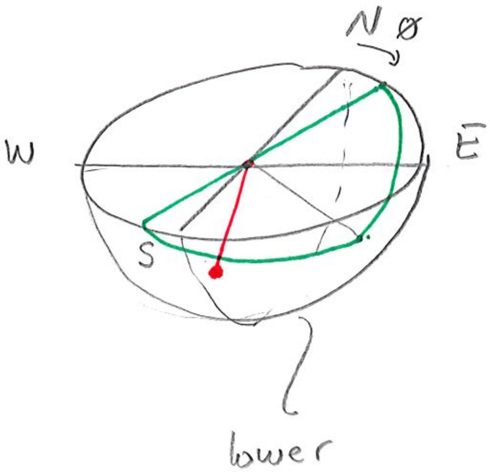
ø: strike

δ: dip

δ: | 0 - 90°
| = 45°

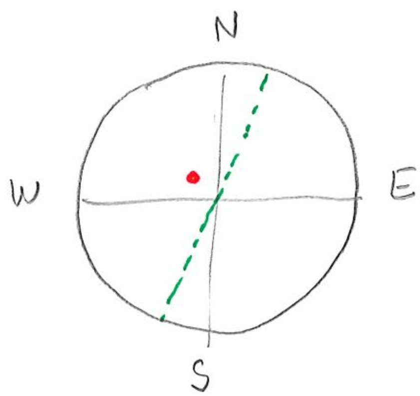
ø: | Quadrant convention
| = N 40° E
| = E 50° N } field
| Azimuth convention
| = 040° (clockwise) } computations

stereonets : 3D → 2D



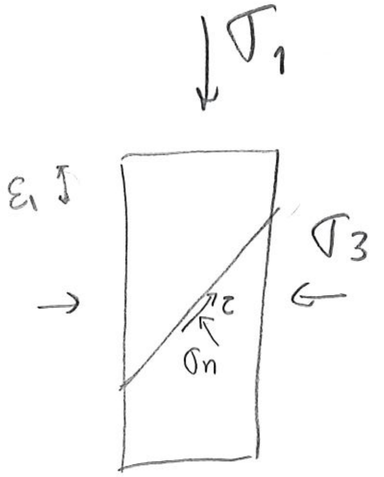
lower hemisphere

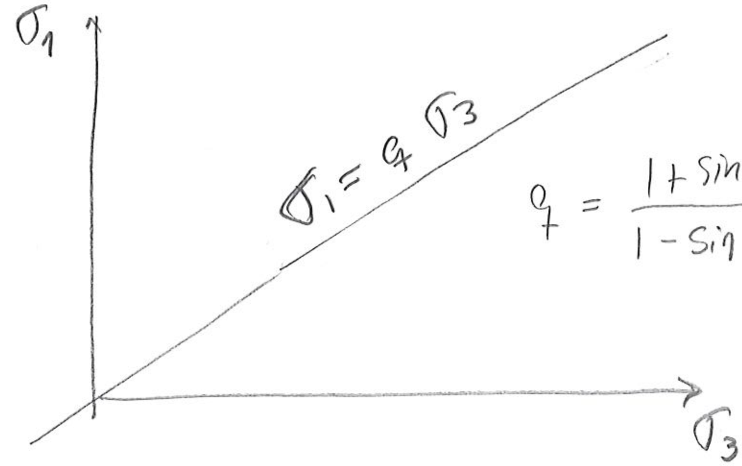
3D



2D

Strength of faults



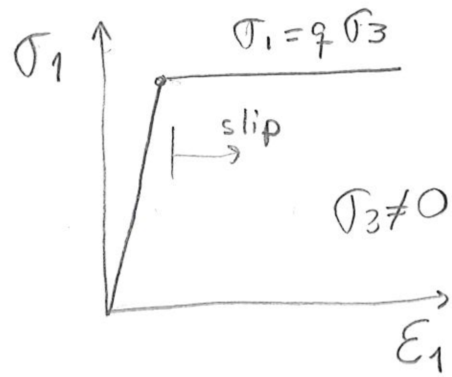


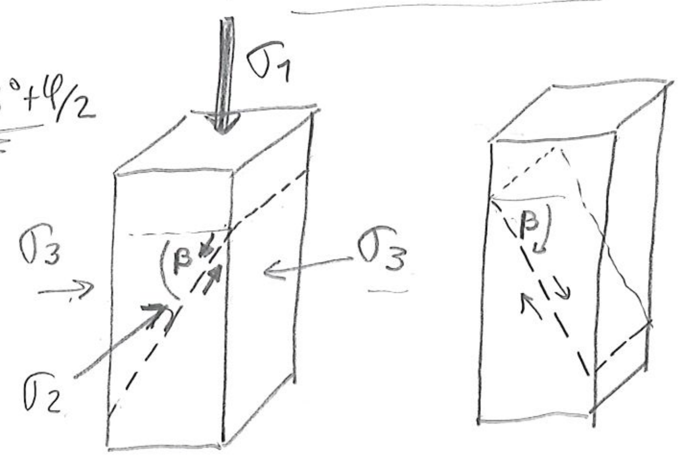
$$q = \frac{1+\sin\varphi}{1-\sin\varphi}$$

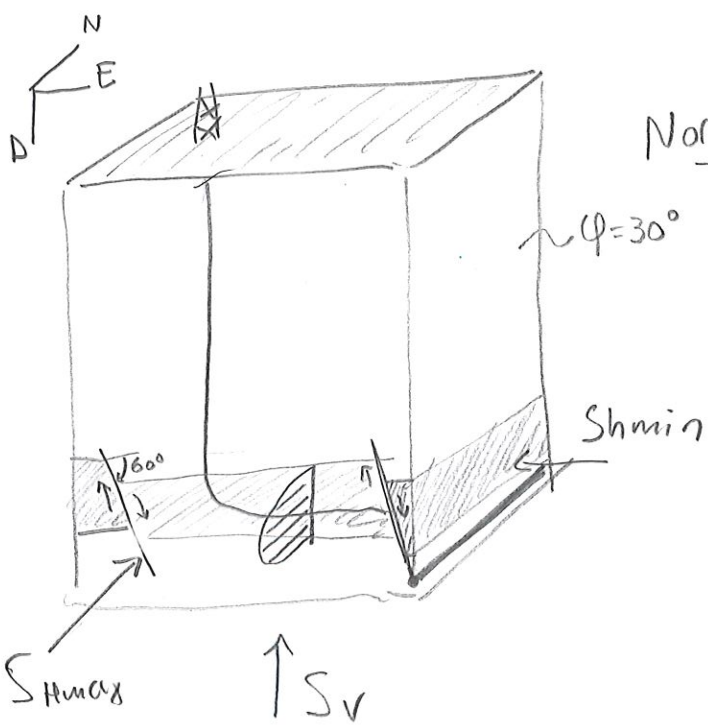
interface | • no cohesion
| • frictional strength

$\varphi = 30° \Rightarrow q = 3$

$\beta = 45° + \varphi/2$







Normal faulting ($S_v > S_{Hmax} > S_{hmin}$)

Faults

strike ϕ: 000°

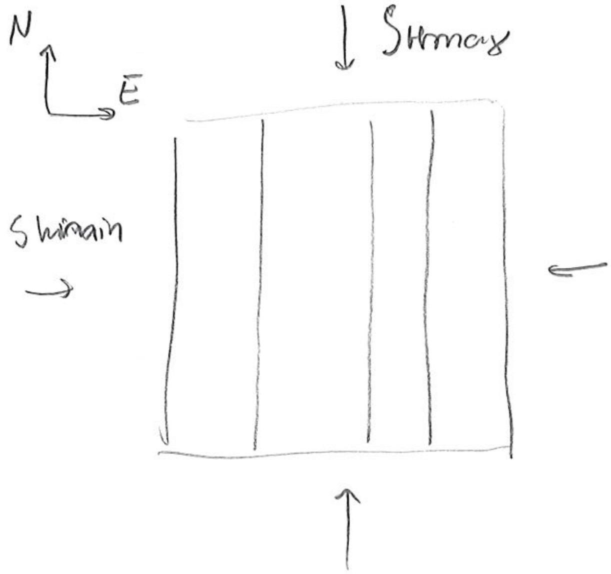
dip δ: 60°E

HF

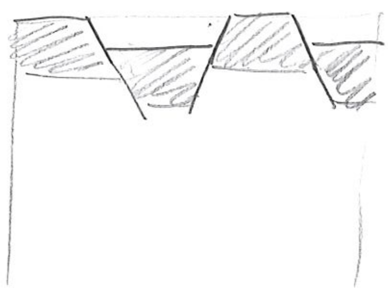
strike: 000°

dip: 90°

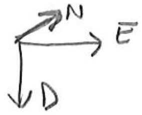
Top view (Normal faulting



side view



Ideal orientation of faults

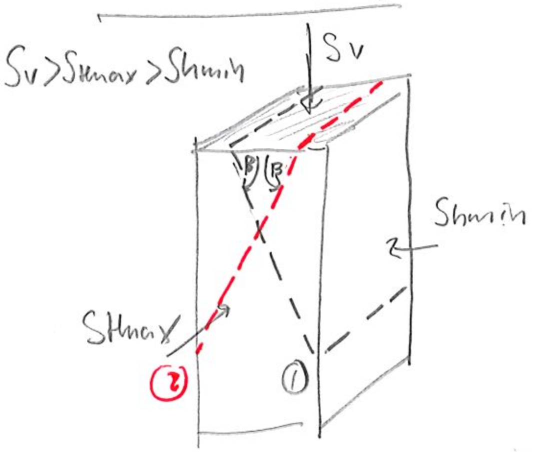


$\varphi = 30°$ all cases

$\beta = 60° = 45° + \varphi/2$

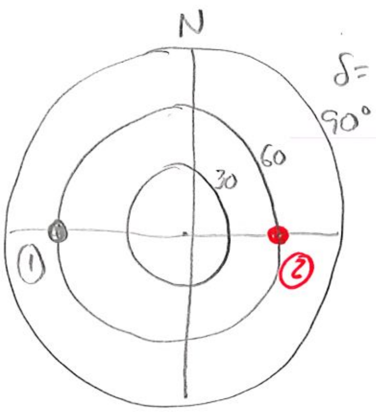
Normal Faulting

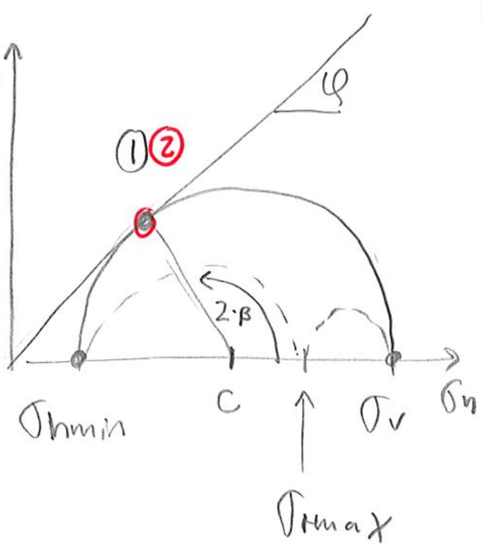
$S_v > S_{Hmax} > S_{hmin}$



① $\phi = 000°$, $\delta = 60°E$

② $\phi = N\text{-}S$, $\delta = 60°W$



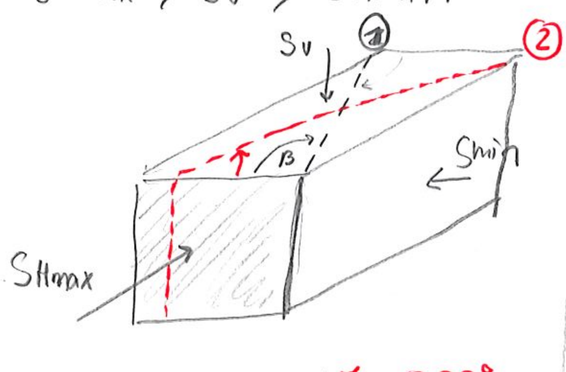


where $B = 45° + \varphi/2$

all cases

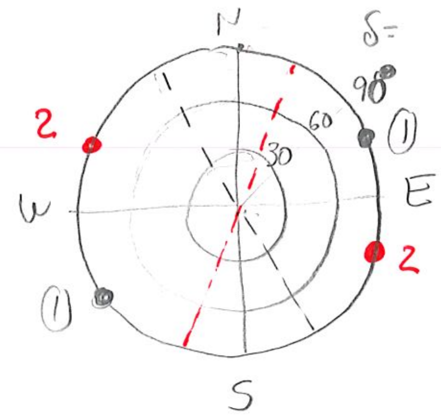
Strike Slip

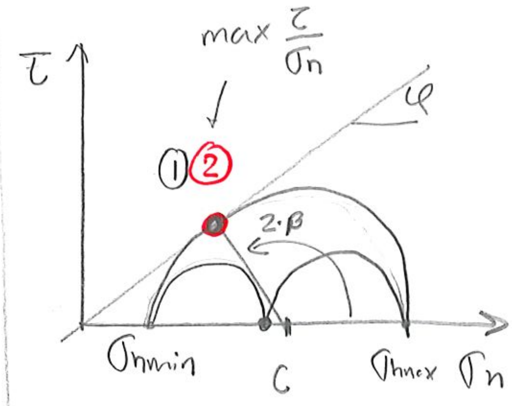
$S_{Hmax} > S_v > S_{hmin}$



① $\phi = N30°W$, $\delta = 90°$

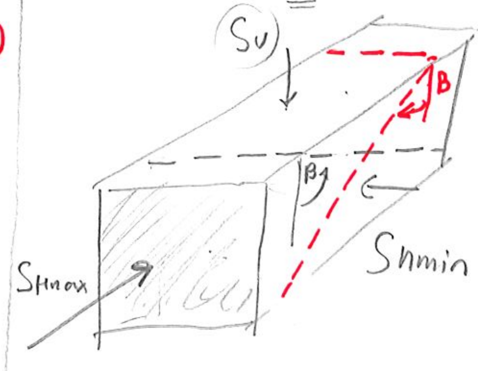
② $\phi = 030°$, $\delta = 90°$





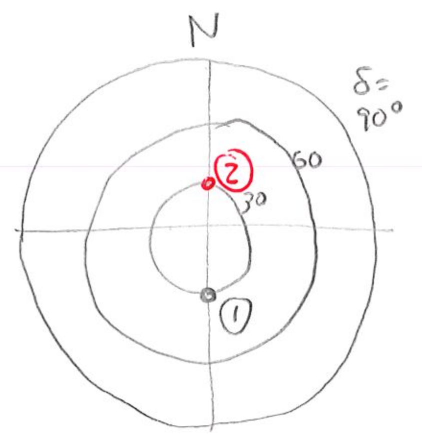
Reverse Faulting

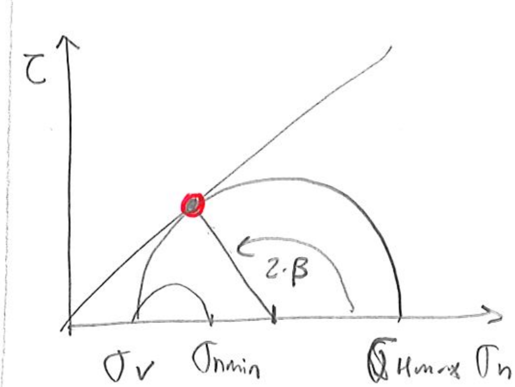
$S_{Hmax} > S_{hmin} > S_v$



① $\phi = 090°$, $\delta = 30°N$

② $\phi = E\text{-}W$, $\delta = 30°S$



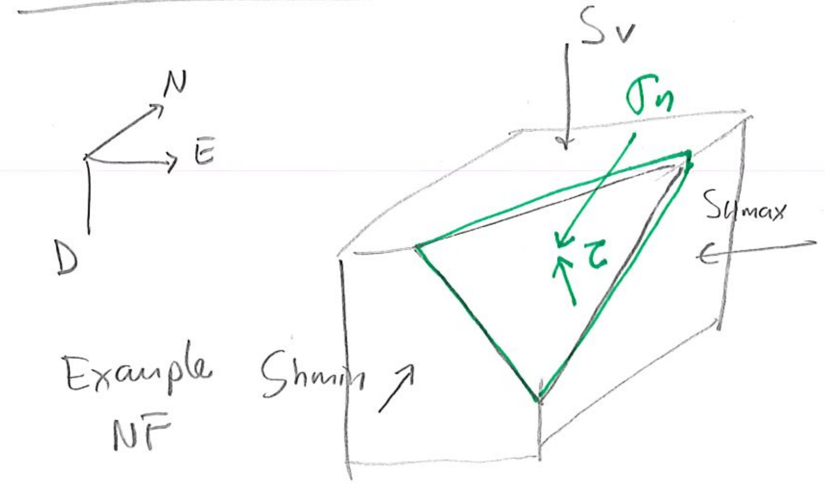


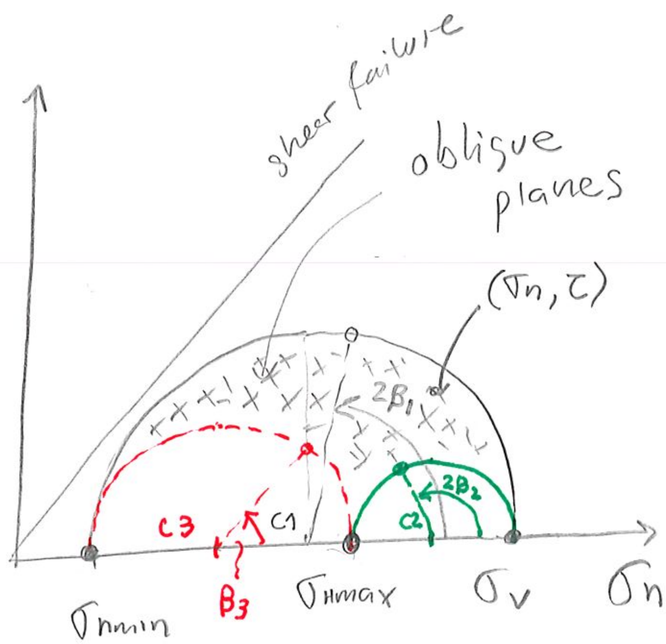
Applications:

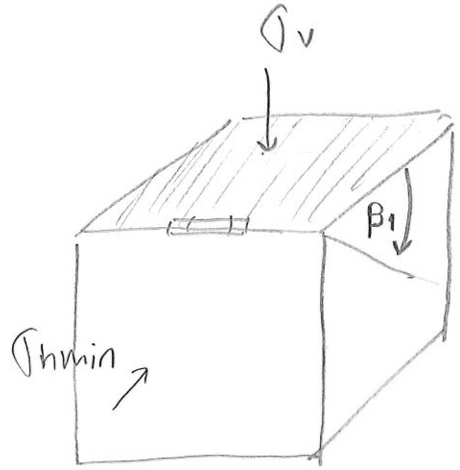
① Ideal orientation of faults
$\Updownarrow$
orientation of principal stresses

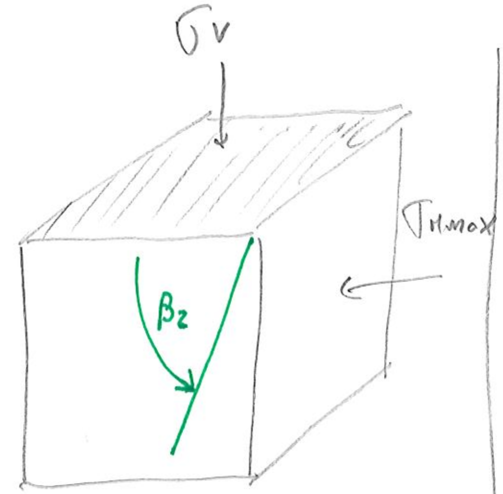
② Fracture reactivation (shear) or fault reactivation due to large τ/σ_n
$\hookrightarrow$ calculate τ and σ_n

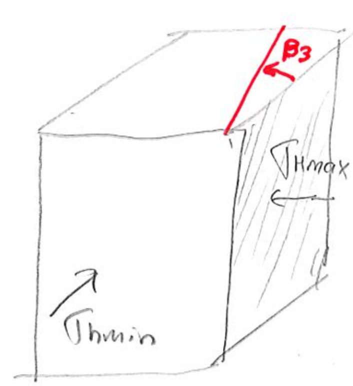
3D Mohr-circle











Problem

Stress

- $S_v = 23$ MPa
- $P_p = 10$ MPa
- $S_{Hmax} = 20$ MPa
- $S_{hmin} = 13.8$ MPa (Azimuth = 090°)

$\sigma_v = 13$ MPa

$\sigma_{Hmax} = 10$ MPa

$\sigma_{hmin} = 3.8$ MPa

Fault

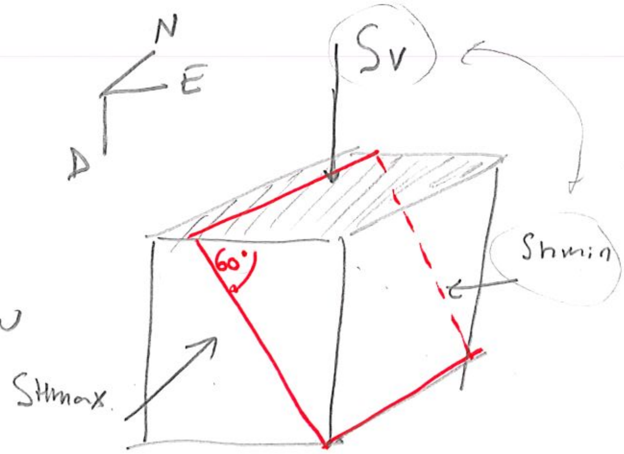
- Strike = 000°
- dip = 60°E

Calculate

- σ_n, τ
- τ / σ_n

---

1) Identify stress regime → NF



2) Draw block diagram or top view

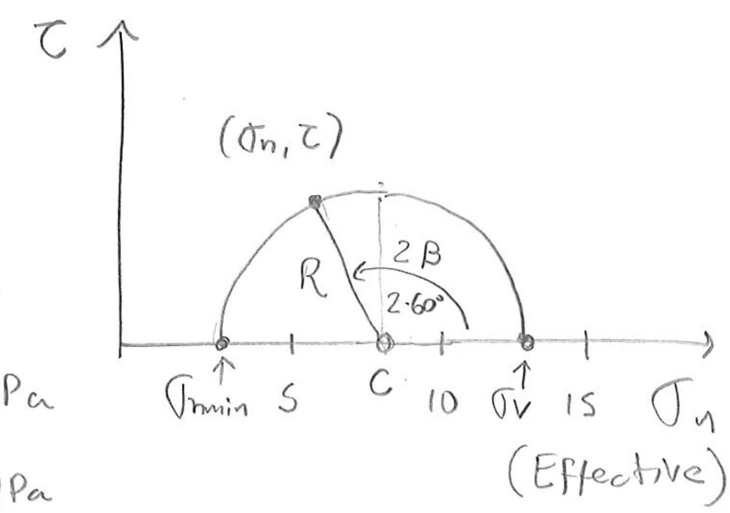
3) Identify fault and corresponding (circle, angle)

$$C = \frac{13\text{ MPa} + 3.8\text{ MPa}}{2} = 8.4\text{ MPa}$$

$$R = \frac{13\text{ MPa} - 3.8\text{ MPa}}{2} = 4.6\text{ MPa}$$

$$\sigma_n = C + \cos(2\beta) \cdot R = 6.1\text{ MPa}$$

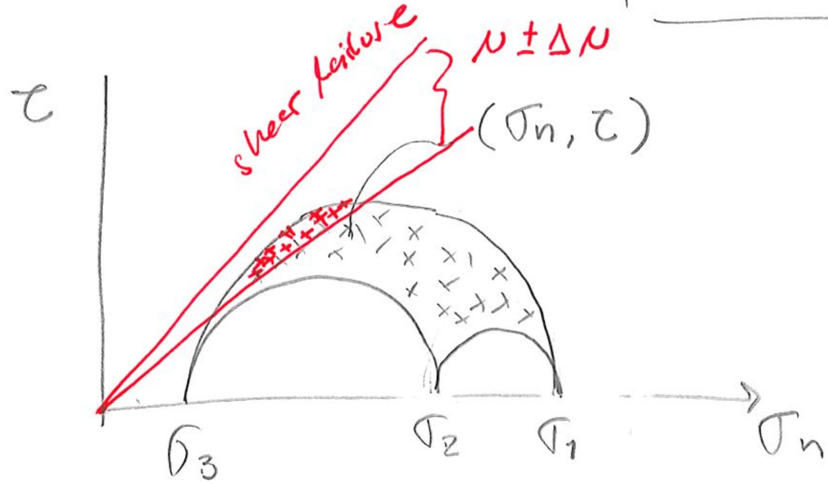
$$\tau = R \cdot \sin(2\beta) = 4.0\text{ MPa}$$



Applications of stress projection on a plane

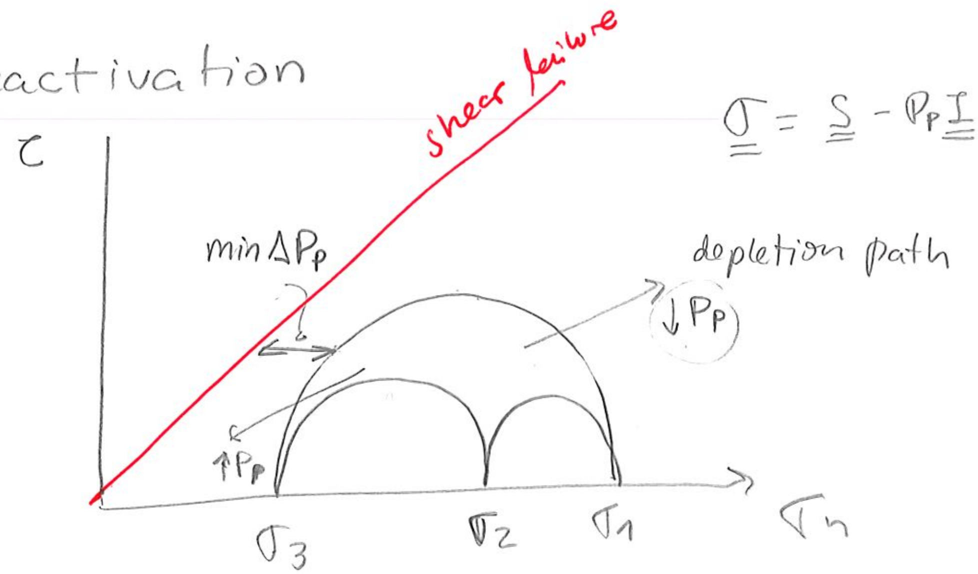
brittle rocks

1. Critically stressed fractures $\Longleftrightarrow$ $K_{frac} \propto \dfrac{\tau}{\sigma_n}$



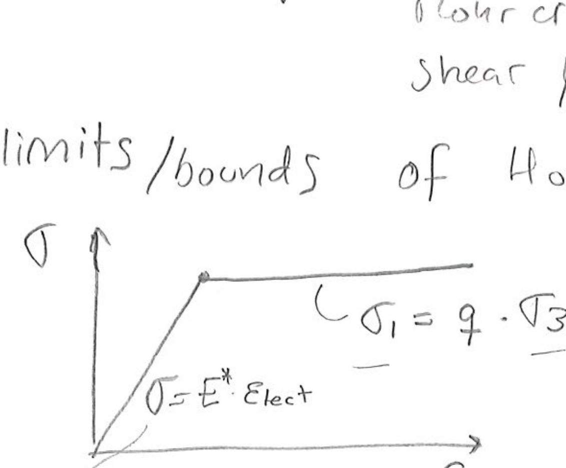
2. Fault reactivation

$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \underline{\underline{I}}$



min ΔP_p = distance between Mohr circle and shear failure line

3. Determination of limits/bounds of Horizontal stress



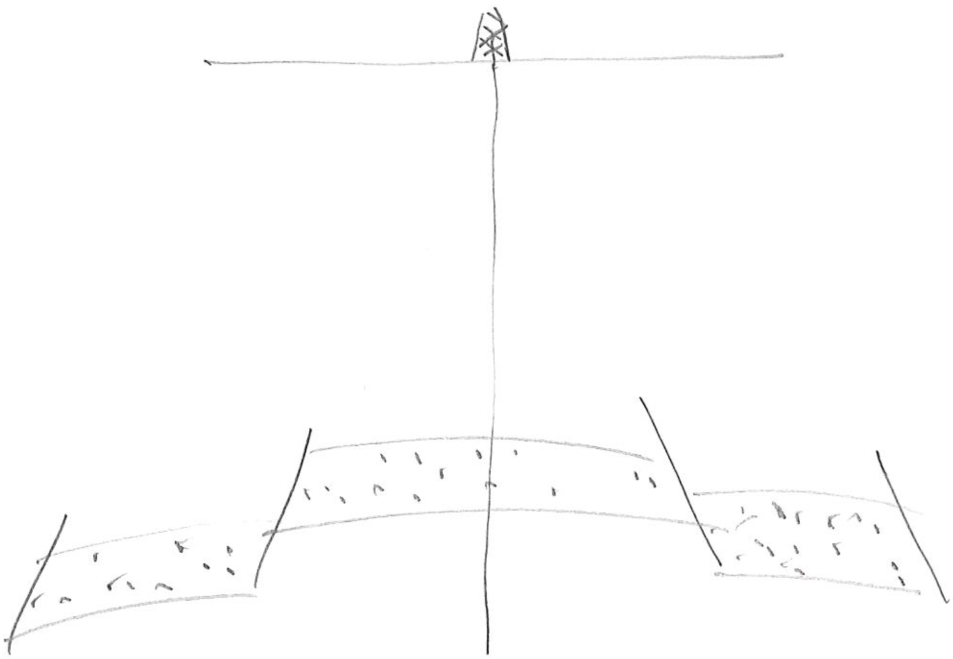
$\sigma_n = \frac{\nu}{1-\nu}\sigma_v + \frac{E}{1-\nu^2}\varepsilon_{hmax} \ldots$

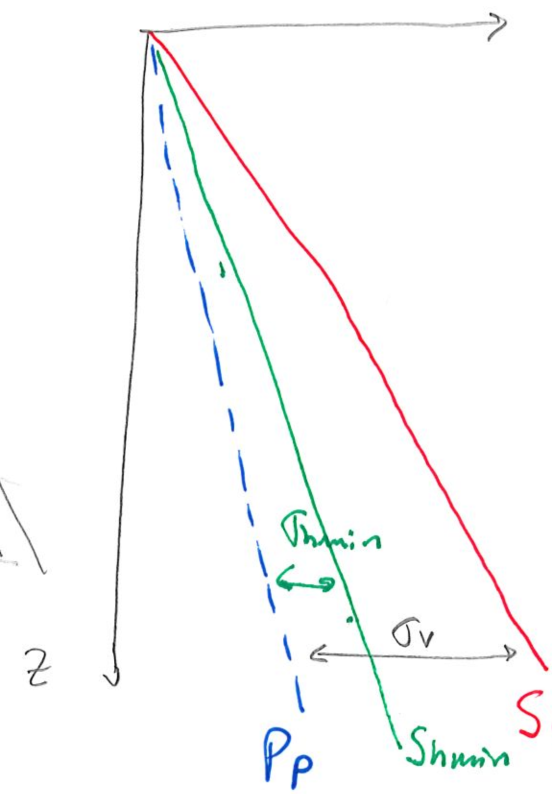
Normal faulting : $\sigma_1 = q\,\sigma_3$

$$\boxed{\sigma_v = q\,\sigma_{hmin}}$$

usually known

$\sigma_{hmin} = \dfrac{\sigma_v}{q}$





assume

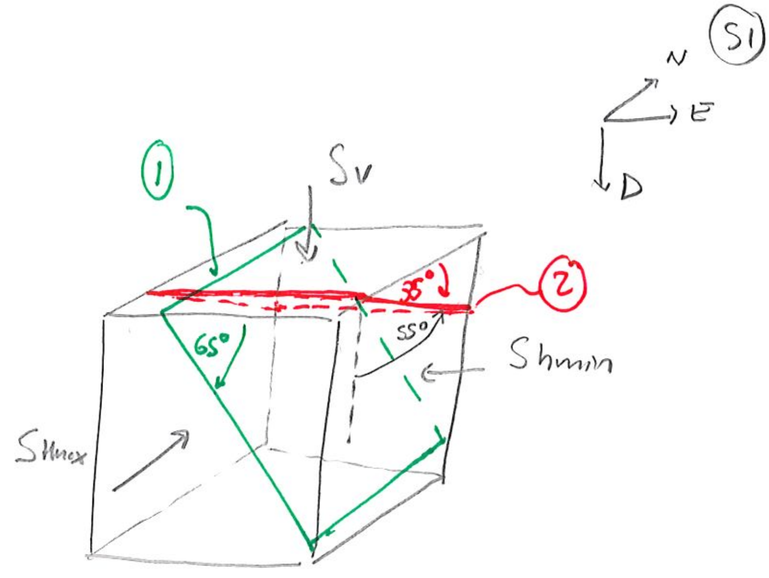
$$\boxed{q = 3}$$

HW 7

5) • Draw Block diagram

$S_{Hmax} > S_v > S_{hmin}$

$P_p = 20\ MPa$

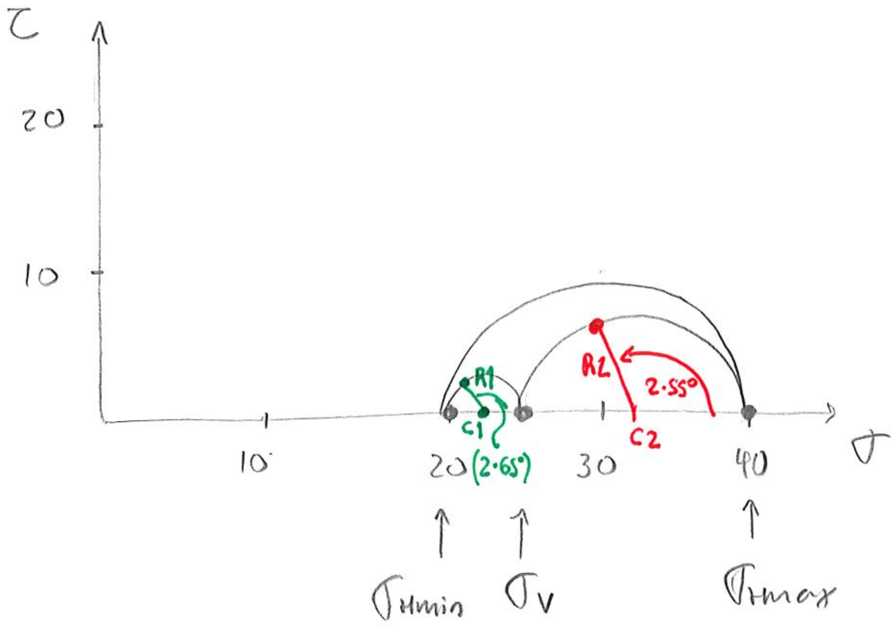


• Calculate effective principal stresses

$\sigma_{Hmax} = 40\ MPa \quad ; \quad \sigma_v = 25\ MPa$ Principal stresses

$\sigma_{hmin} = 20\ MPa$

• Draw Mohr circles



① $\tau =$

$\sigma_n =$

(S2)

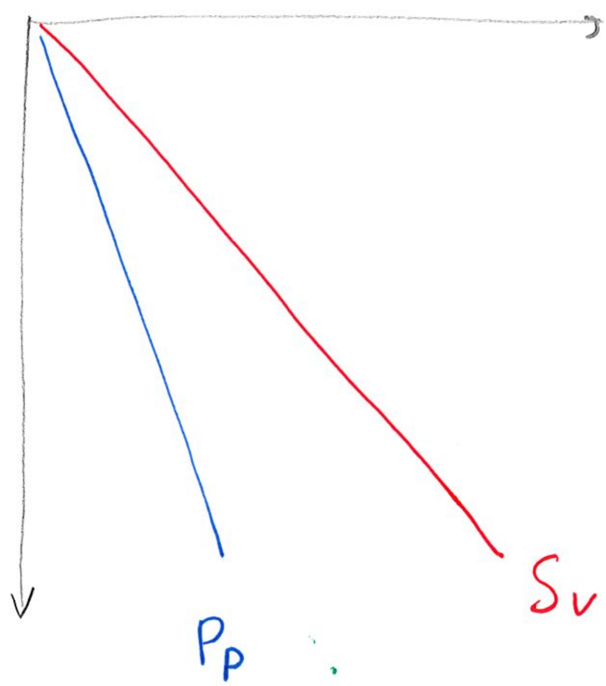
7) limits of S_1 and S_3 for NF and RF

$\rightarrow$ S_v —

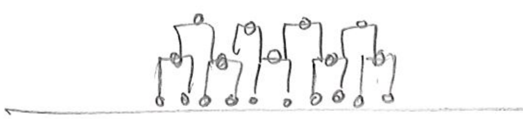
NF: $\sigma_{hmin} = \dfrac{\sigma_v}{q}$

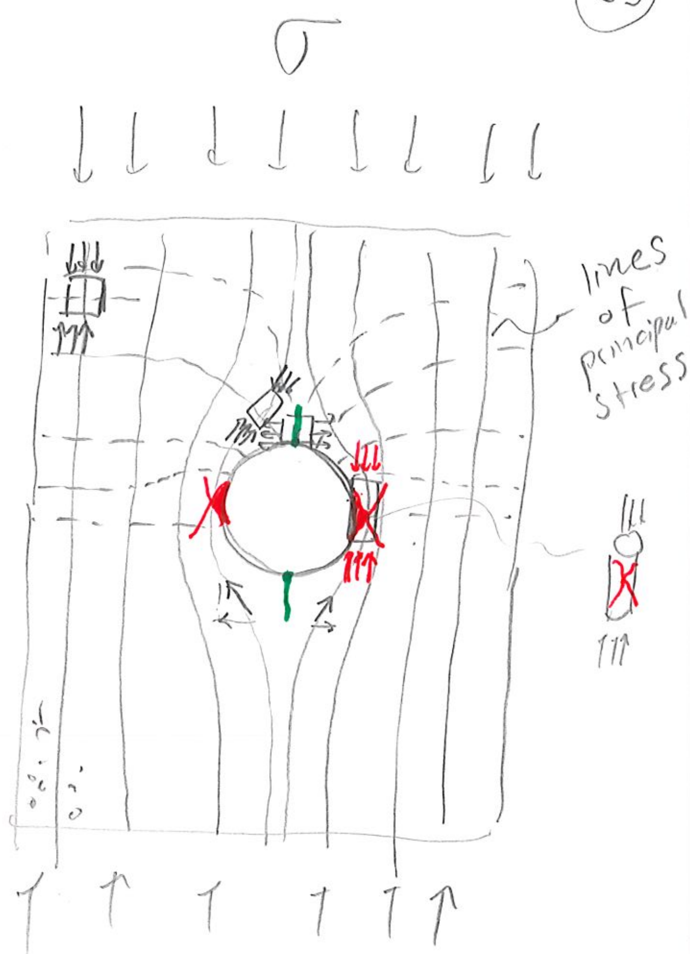
RF: $\sigma_{Hmax} = q \cdot \sigma_v$

$$\sigma_1 = q\,\sigma_3$$



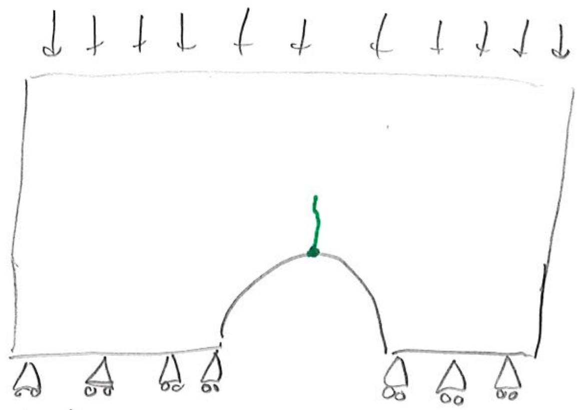
Wellbore Stability





$\equiv$

- Generalized solution of continuum mechanics problem



$$(\lambda + G)\, \nabla \cdot (\nabla \underline{U}) + G \nabla^2 \underline{U} = 0$$

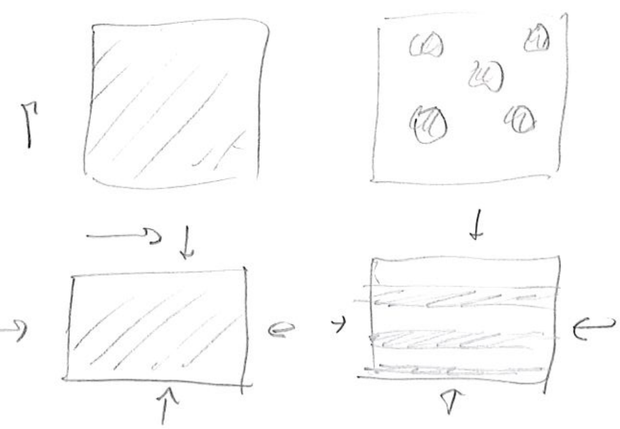
↓

analytical

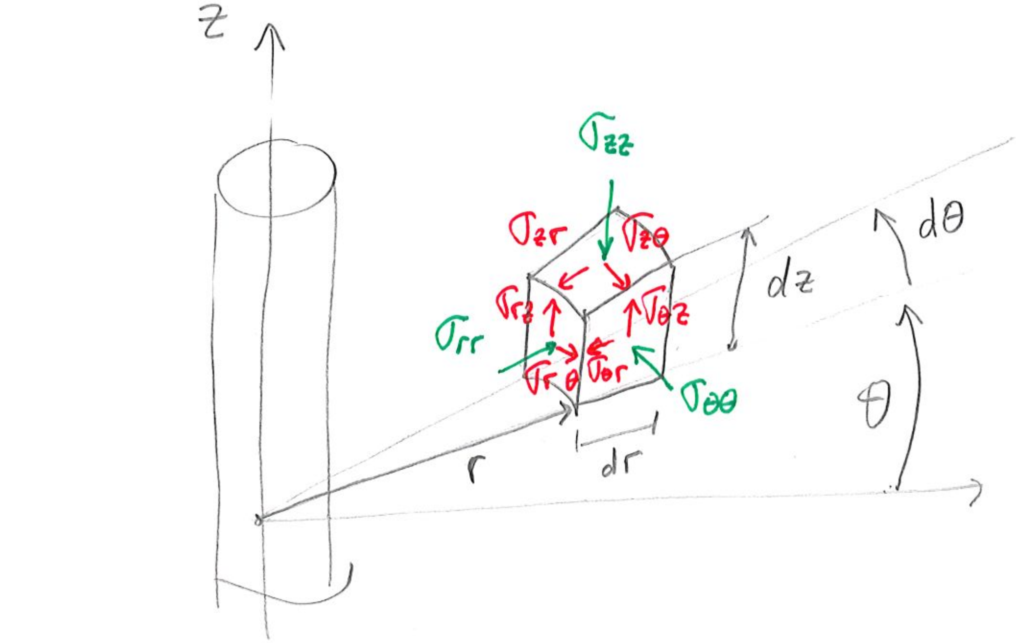
↓

Kirsch solution

- linear elastic solid
- homogeneous
- isotropic

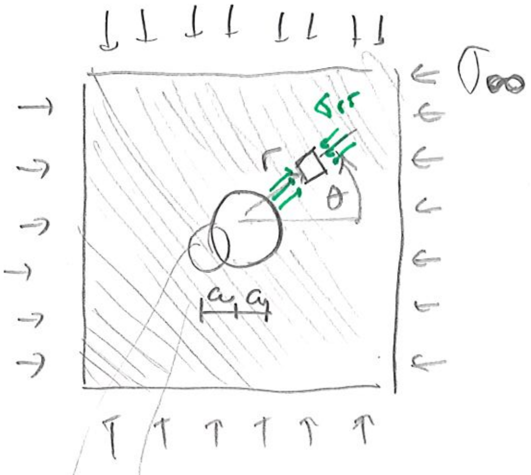


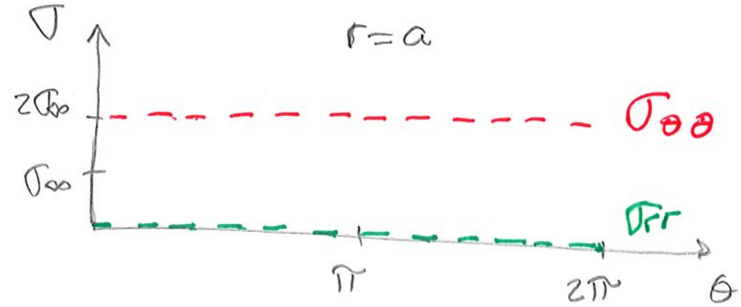
cylindrical coordinates

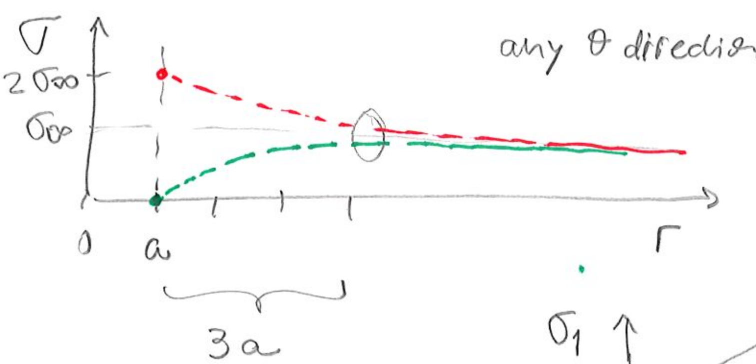


normal $\left\{ \begin{array}{l} \sigma_{zz}: \text{axial stress} \\ \sigma_{rr}: \text{radial stress} \\ \sigma_{\theta\theta}: \text{tangential} \\ \quad \text{circumfe...} \\ \quad \text{hoop stress} \end{array} \right.$

$\left. \begin{array}{l} \sigma_{zr} \\ \sigma_{\theta r} \\ \vdots \end{array} \right\}$ shear stresses

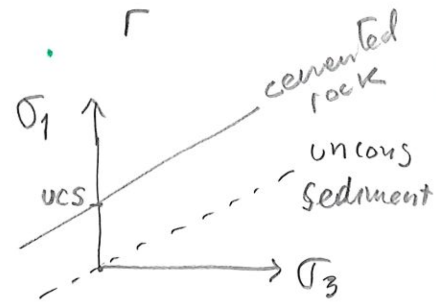




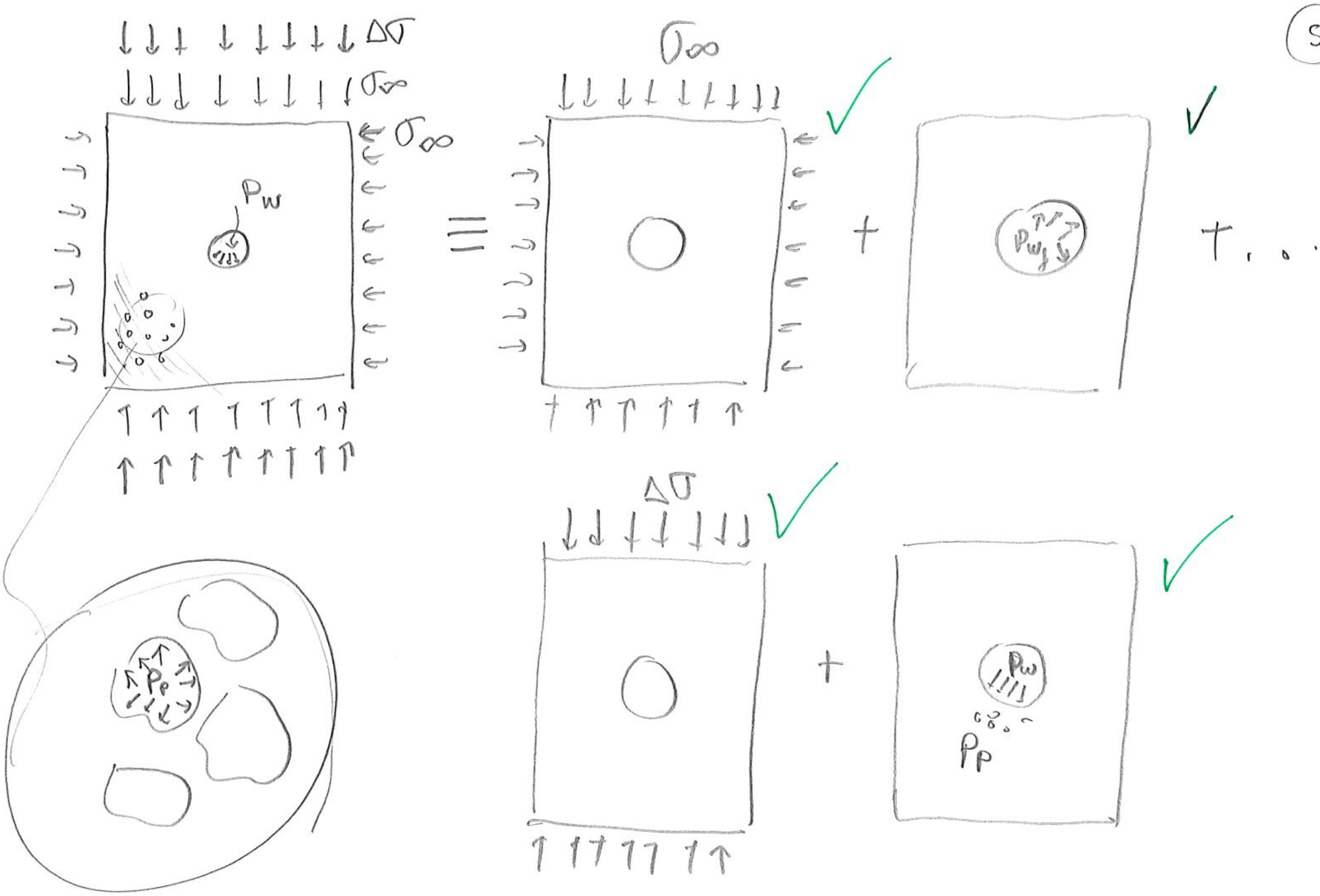


$$\sigma_{rr} = \left(1 - \frac{a^2}{r^2}\right)\sigma_\infty$$

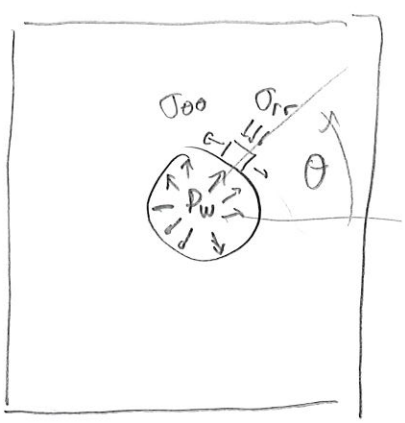
$$\sigma_{\theta\theta} = \left(1 + \frac{a^2}{r^2}\right)\sigma_\infty$$



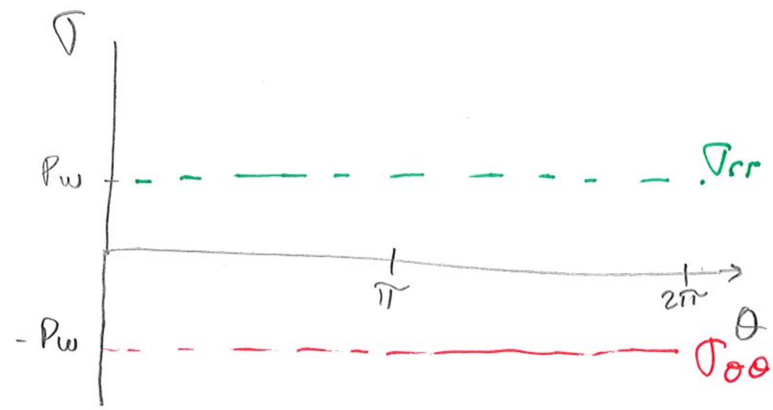
$$\underbrace{\frac{\sigma_{\theta\theta}(r=a)}{\sigma_\infty}}= 2 \quad ; \quad \left.\frac{\sigma_{\theta\theta}}{\sigma_{rr}}\right|_{r=a} = \infty$$



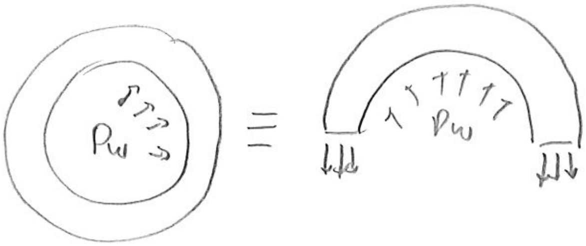
Δσ
σ∞
σ∞
Pw
≡
σ∞
+
Pw
+ . . .
Pp
Δσ
+
Pw
Pp



σθθ
σrr
Pw
θ



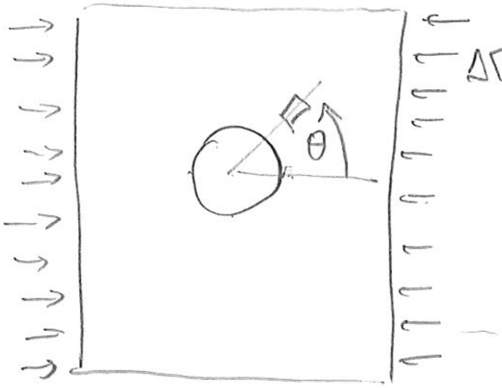
σ
Pw
σrr
π
2π
θ
-Pw
σθθ

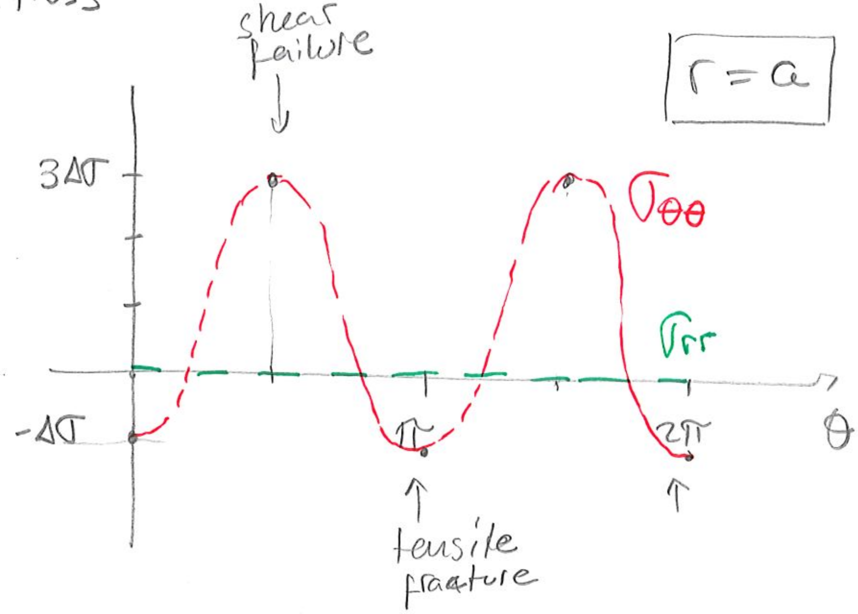


Pw
=
Pw

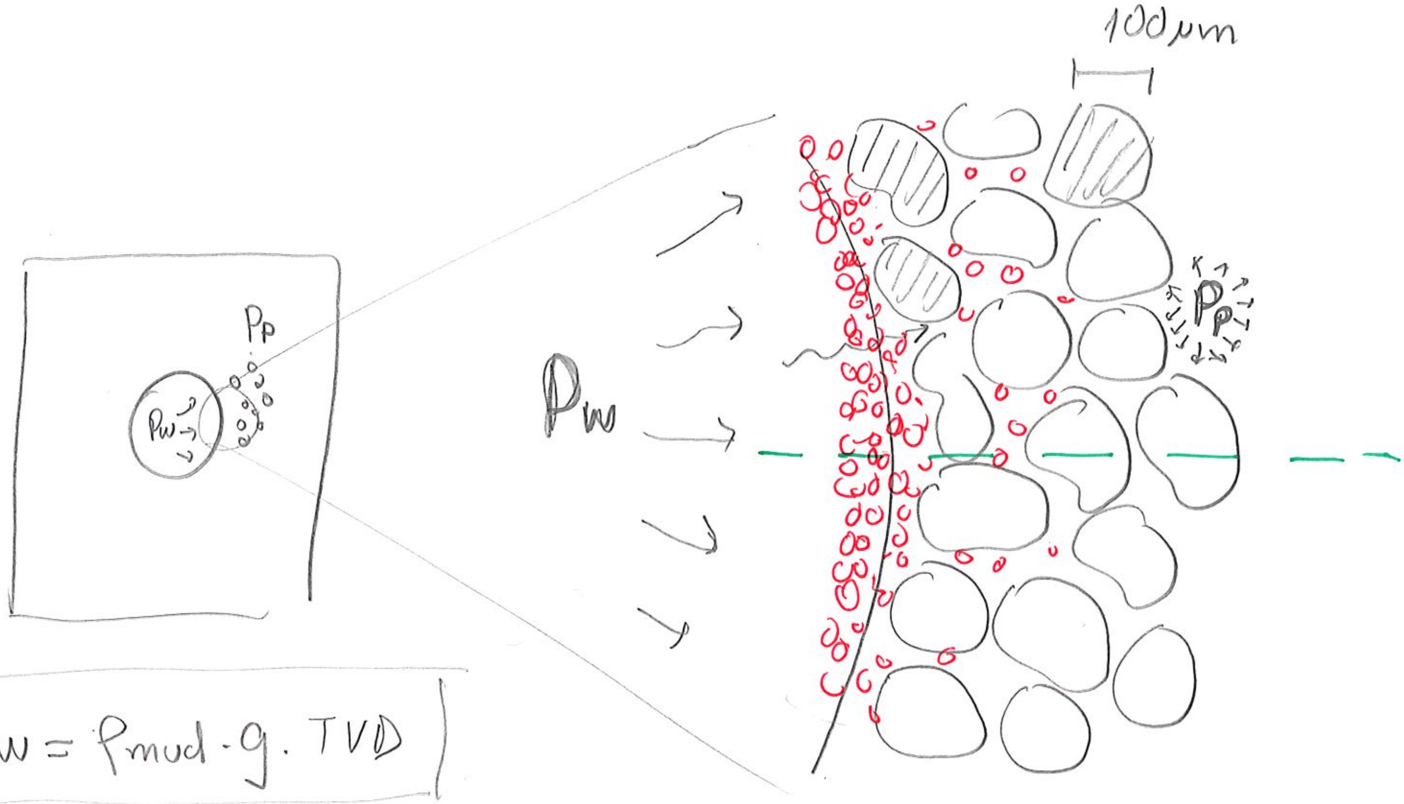
$\Delta\sigma$: differential stress

$\boxed{r = a}$



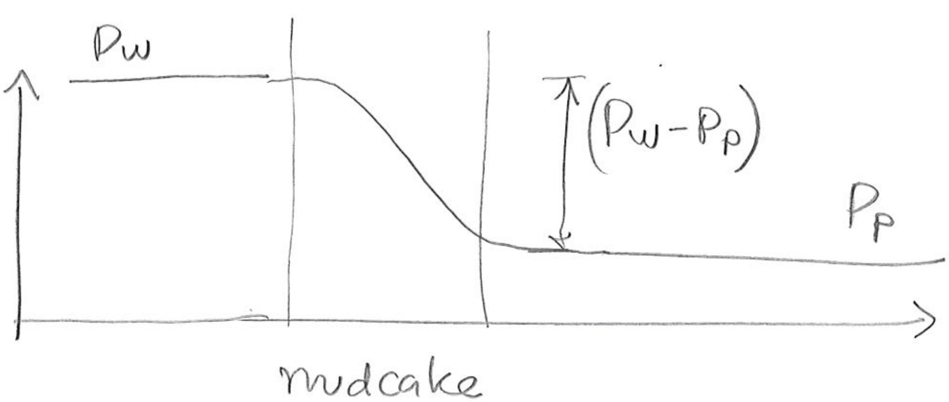


$$\left.\frac{\sigma_{\theta\theta}}{\Delta\sigma}\right|_{r=0,\ \theta=\pi/2} = 3$$



$$\boxed{P_w = \rho_{mud} \cdot g \cdot TVD}$$

↳ hydrostatic



Stresses at the well bore wall

$$
r = a \quad \left|\;
\begin{aligned}
\sigma_{rr} &= (P_w - P_p) \\
\sigma_{\theta\theta} &= -(P_w - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos 2\theta \\
\sigma_{\theta r} &= 0
\end{aligned}
\right.
$$

Tensile failure ; Breakdown Pressure P_b

$$\theta = 0^\circ \text{ or } 180^\circ$$

$$\sigma_{\theta\theta} = -T_s$$

$$-T_s = -(\underline{P_w} - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})$$

$$\boxed{P_w = P_b = \underbrace{P_p}_{\text{pore press \} field}} + \underbrace{3\sigma_{hmin} - \sigma_{Hmax}}_{\text{stress anisotropy \} field}} + \underbrace{T_s}_{\text{tensile strength \} lab}} \;\Big|\; \overbrace{-\sigma^{\Delta T}}^{\text{thermal stress}}}$$

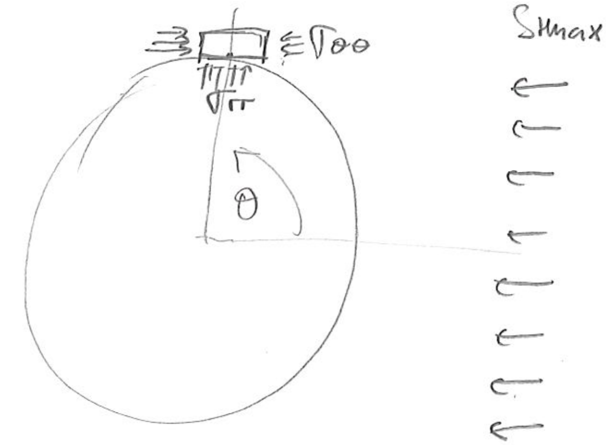
$$P_w > P_b \Rightarrow \left|\; \begin{array}{l} \text{tensile fracture} \\ \text{wellbore breakdown} \end{array}\right.$$

Sheer failure: Wellbore breakouts

$\theta = 90^\circ$ or 270°

$\sigma_{\theta\theta} = -(P_w - P_p) + 3\sigma_{Hmax} - \sigma_{hmin}$

$\sigma_{rr} = (P_w - P_f)$



sheer failure: $\boxed{\sigma_1 = UCS + \sigma_3 \cdot q}$

$\sigma_1 \rightarrow \sigma_{\theta\theta}$ $\qquad$ $\sigma_3 \rightarrow \sigma_{rr}$

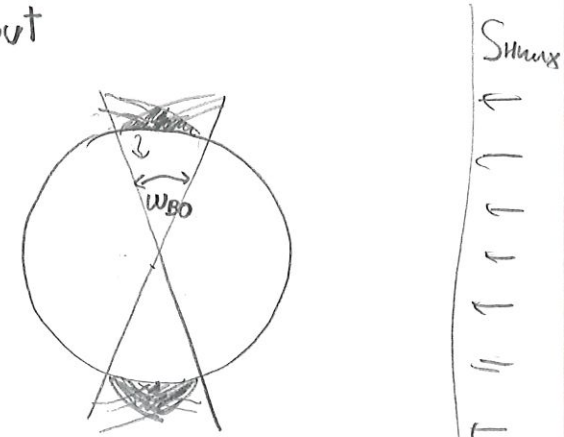
$$\boxed{P_{w\,shear} = P_p + \frac{3\sigma_{Hmax} - \sigma_{hmin} - UCS}{1+q}}$$

$$\boxed{P_{wBO} = P_p + \dots \frac{(\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(\pi - W_{BO}) - UCS}{1+q}}$$

valid for

$W_{BO} \lesssim 70^\circ$

Breakout

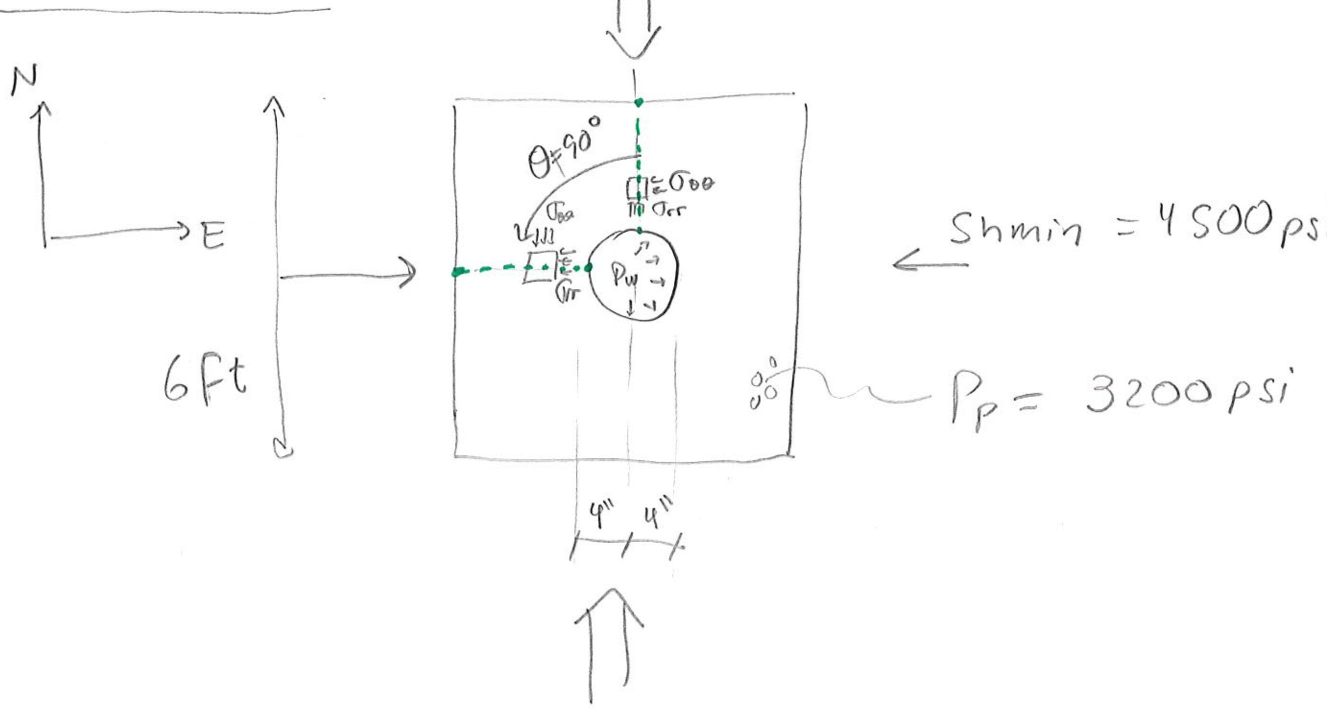


↳ back-calculate $\boxed{S_{Hmax}}$ for a vertical wellbore

$\boxed{\text{HW \#8}}$

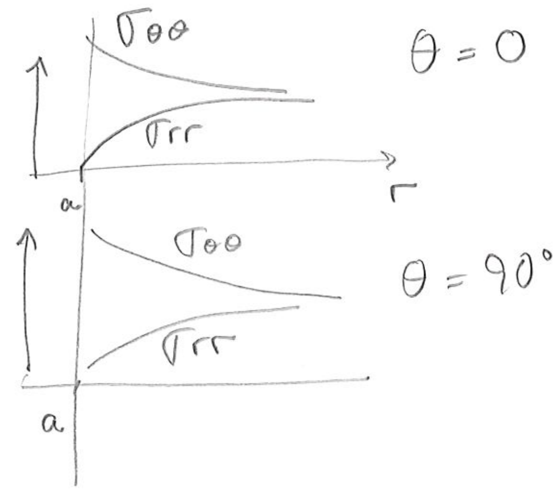
Problem 6.9.1

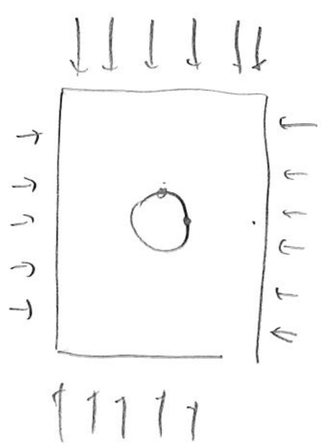
$S_{Hmax} = 6000\,psi$

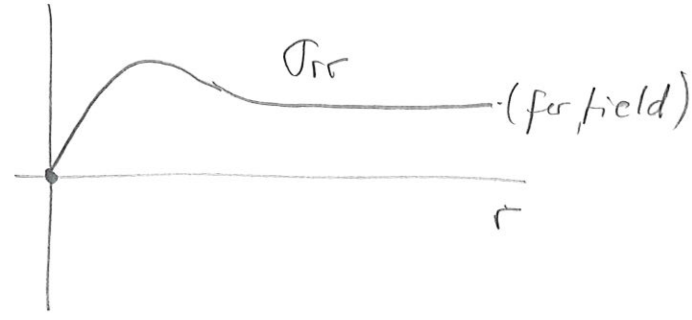


1) $P_w = 3200\,psi$

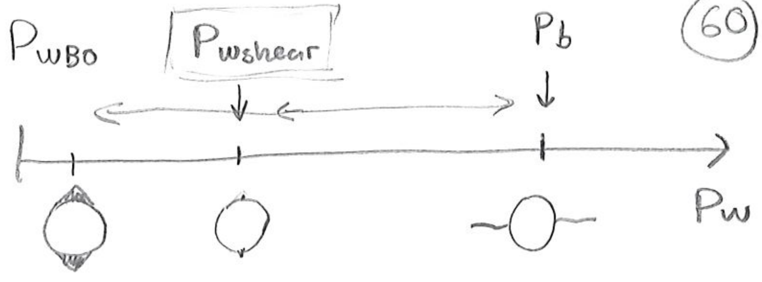
2) $P_w = 4000\,psi$

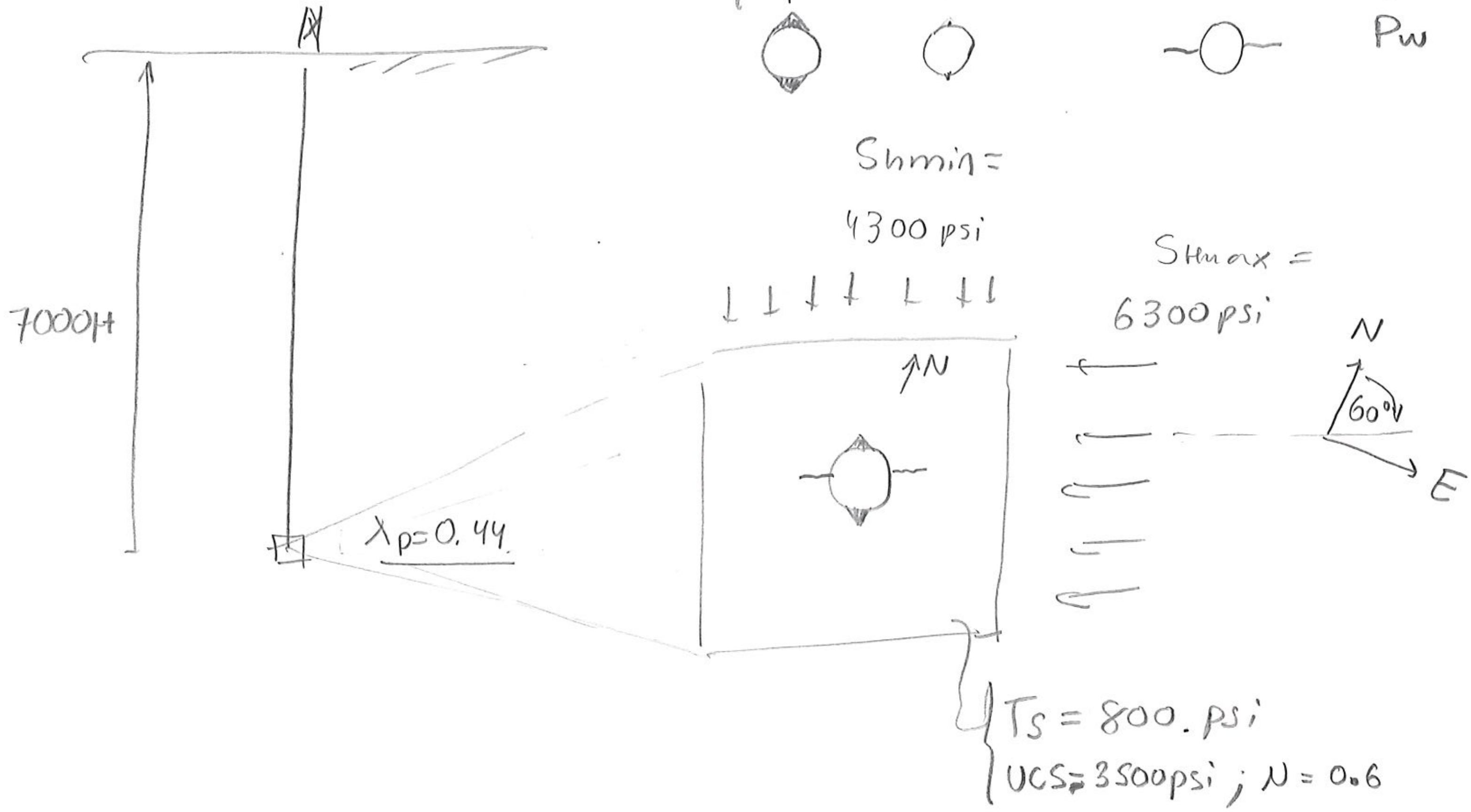






Problem 6.9.2





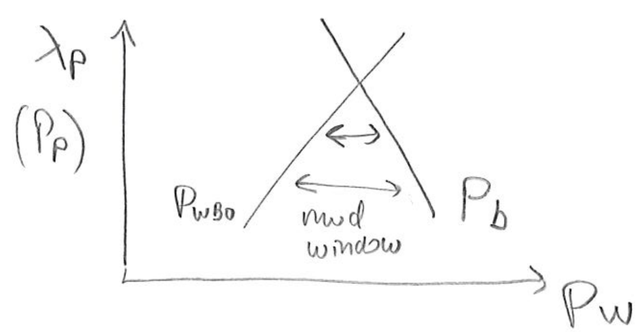
$$P_b = 3080\,psi + 3 \cdot 1220\,psi - 3220\,psi + 800\,psi = \boxed{4320\ psi}$$

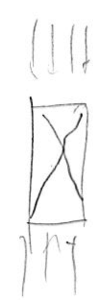
$= 11.60\ ppg$

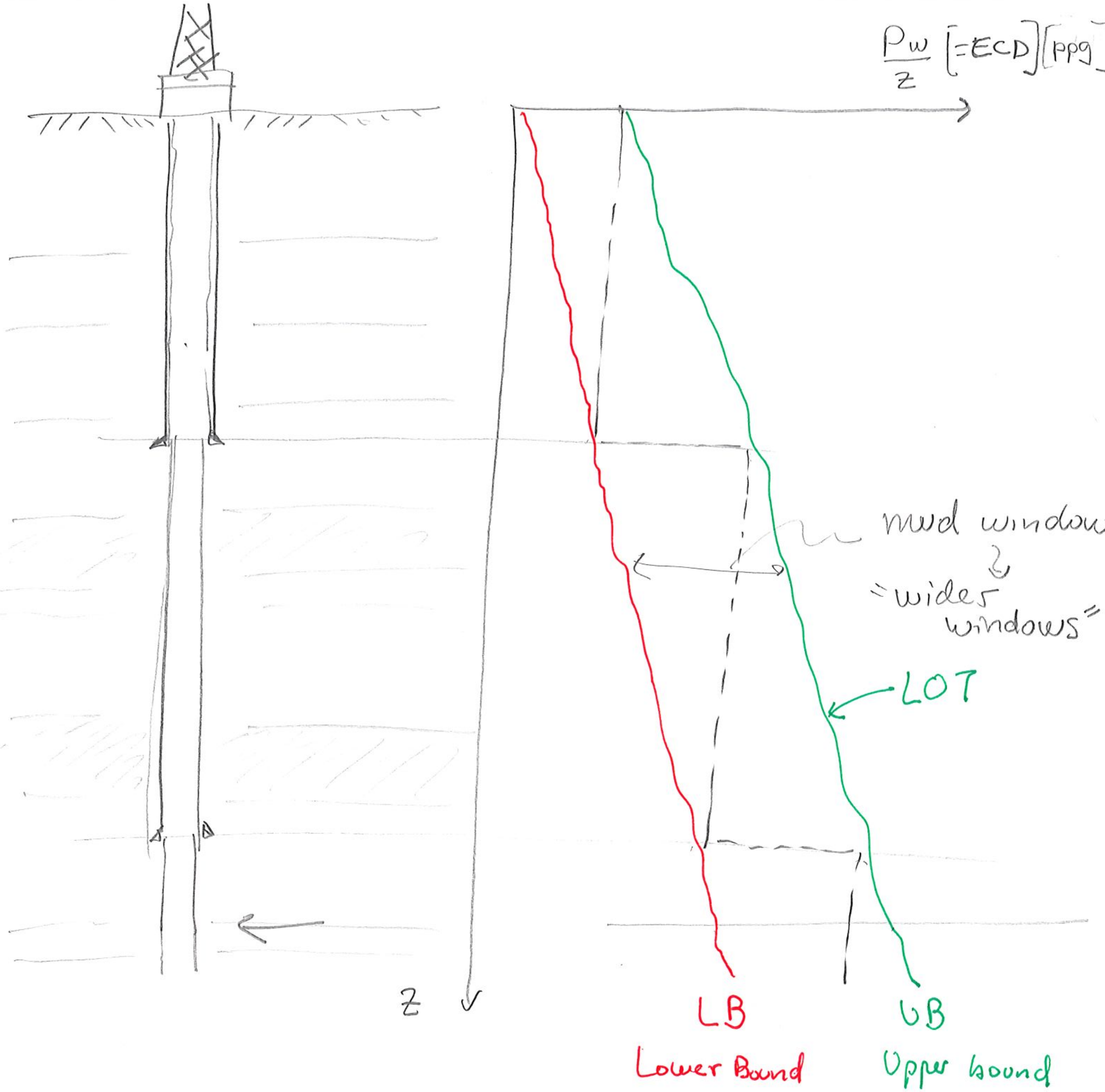
$$P_{w\,shear} = 3080\ psi + \frac{3 \cdot 3220\,psi - 1220\,psi - 3500\,psi}{1 + 3.12} = \boxed{4280\ psi}$$

$= 11.53\ ppg$

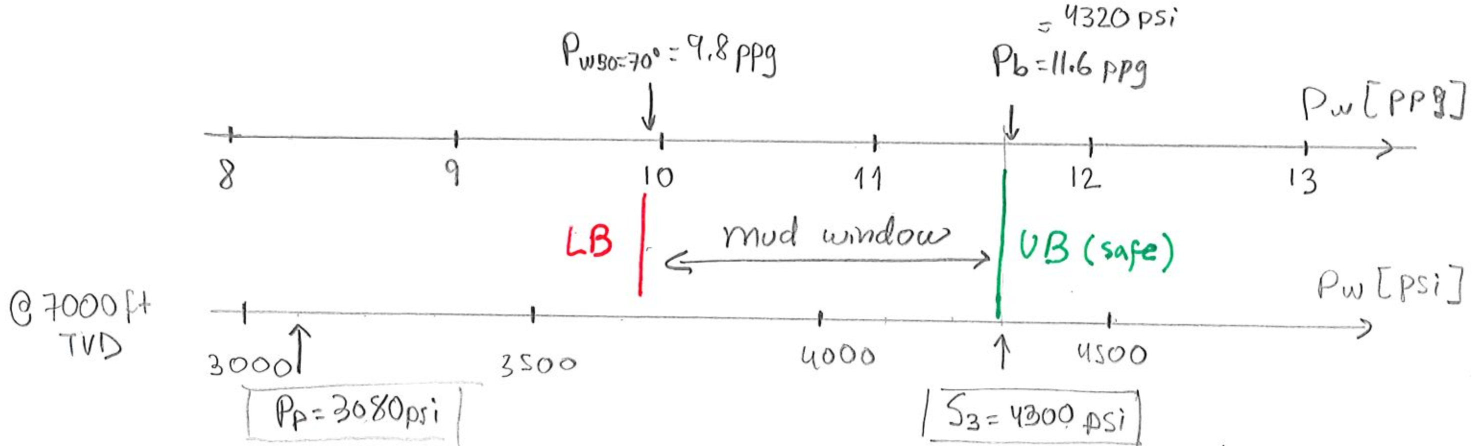
$$P(w_{BO} = 70°) = \ldots\ldots = \boxed{3640\ psi} = 9.8\ ppg$$





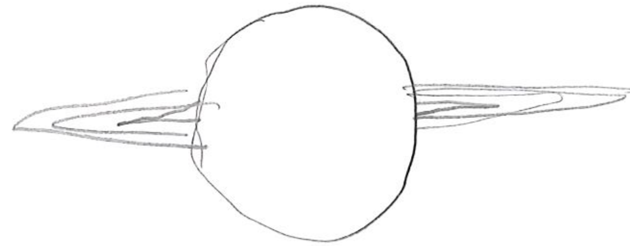


$\frac{P_w}{z}$ [=ECD] [ppg]
mud window
"wider windows"
LOT
z
LB
Lower Bound
UB
Upper bound



= 4320 psi
$P_{w \beta 0 = 70°}$ = 9.8 ppg
P_b = 11.6 ppg
P_w [ppg]
8
9
10
11
12
13
LB
mud window
UB (safe)
P_w [psi]
@ 7000 ft TVD
3000
3500
4000
4500
P_p = 3080 psi
S_3 = 4300 psi

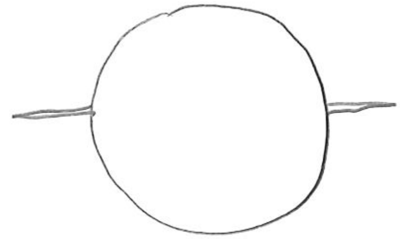
$\boxed{P_b > S_3}$; $P_w > P_b$



↳ large hydraulic fracture

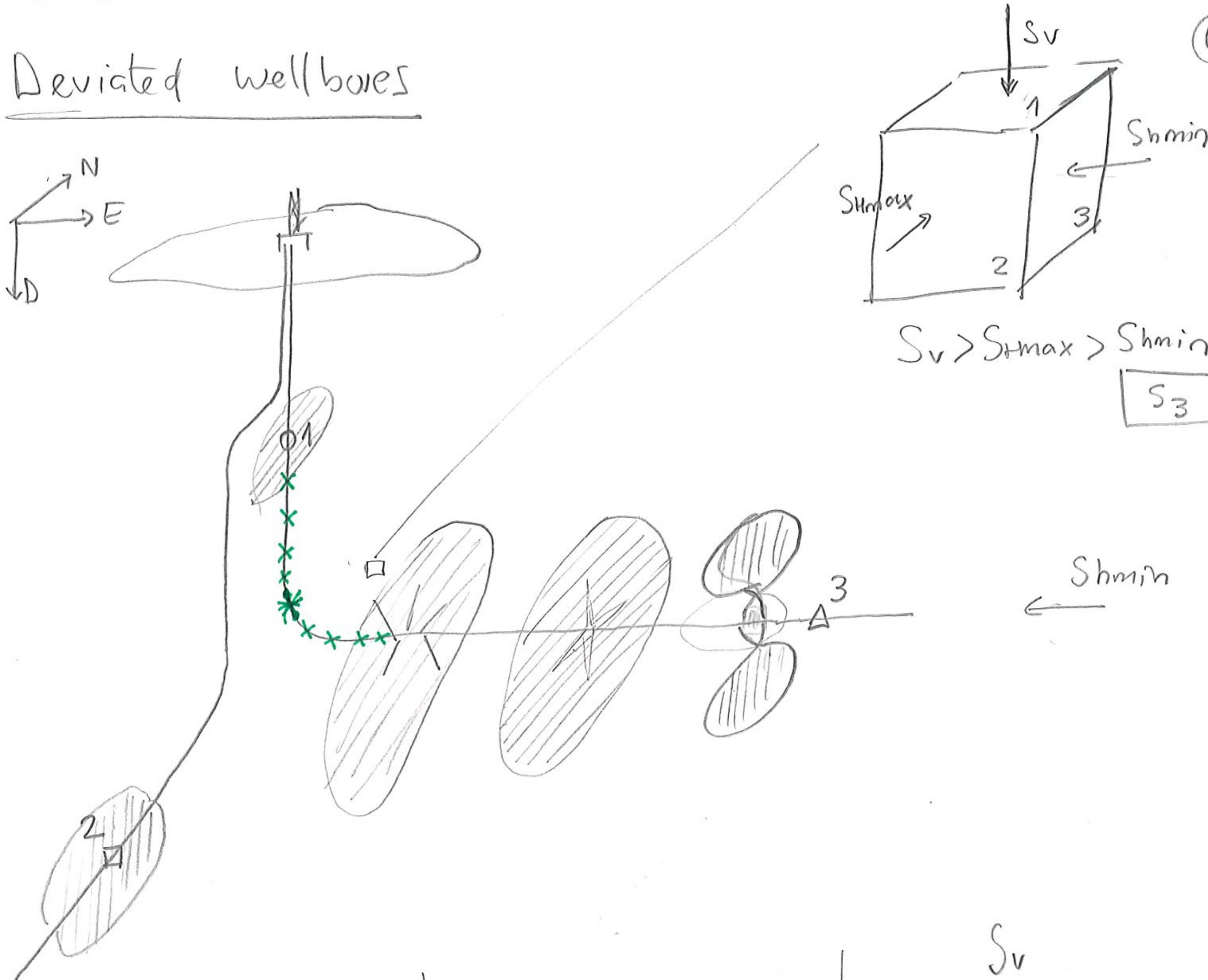
$\boxed{S_3 > P_b}$; $P_w > P_b$

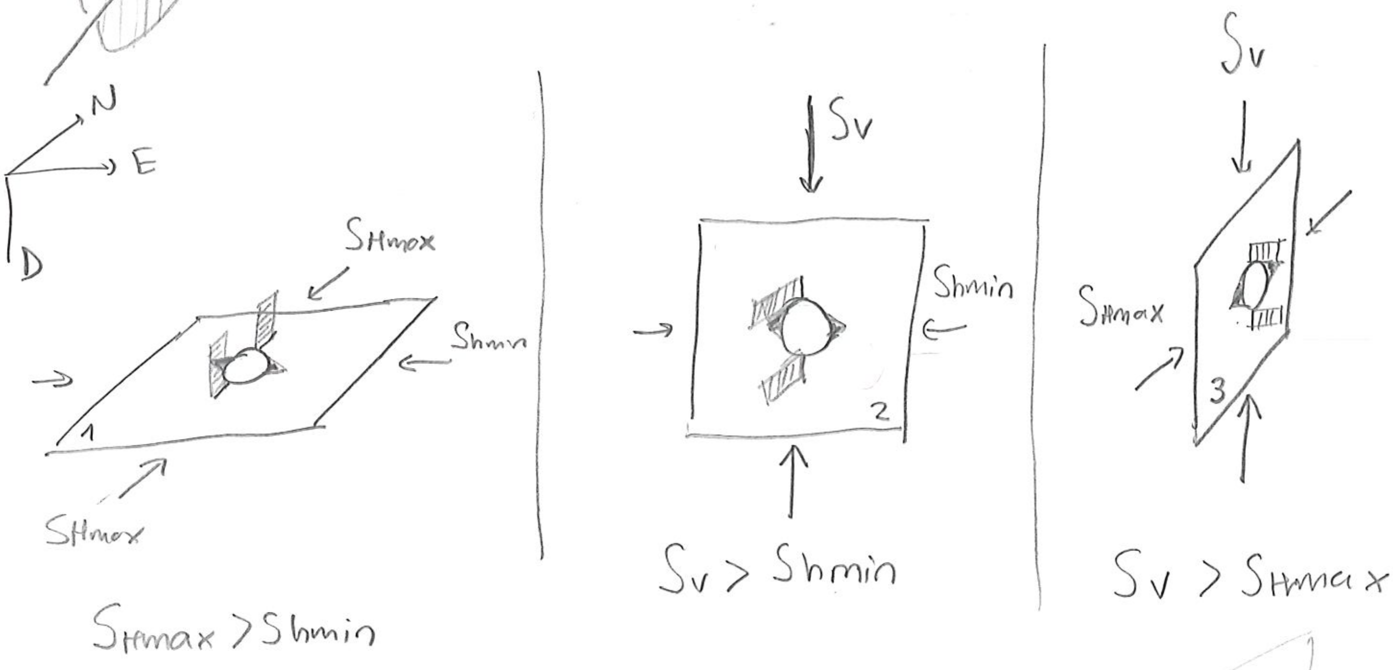
$P_w < S_3$

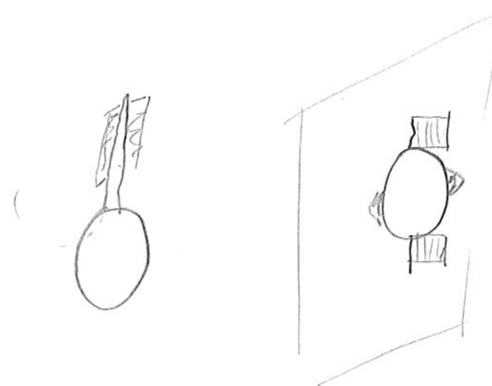


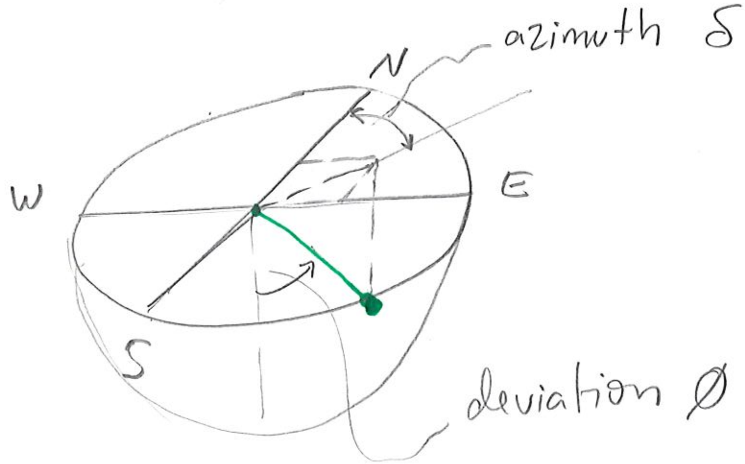
↳ small tensile fracture

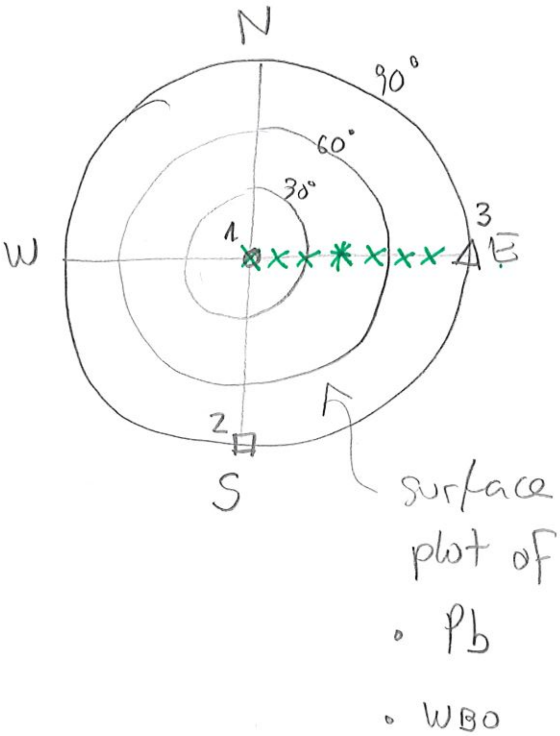
Deviated wellbores









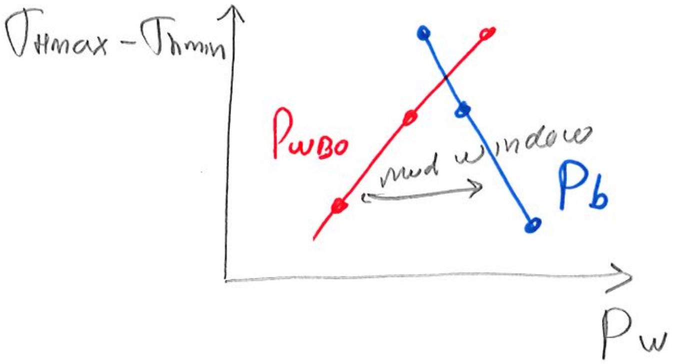


Calculation of wellbore stability for an arbitrary deviation

↳ see book notes

HW #9

6.9.3 → P_{wBO} , $P_{w shear}$, P_b



6.9.4 → Offshore

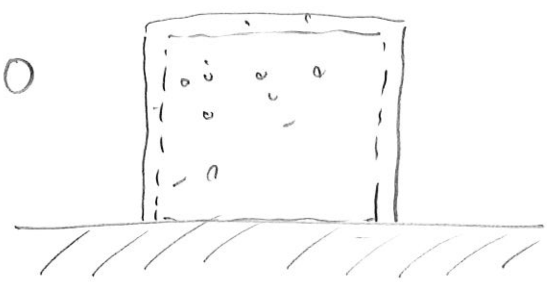
6.9.5 → Laterals : Hz wellbores

↳ $w_{BO} = 45°$

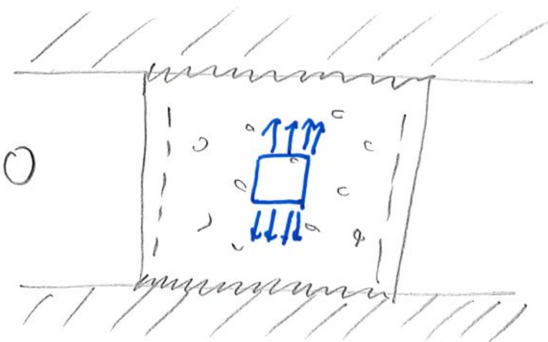
Others factors that affect wellbore stability

(1) Temperature

$\Delta T < 0$



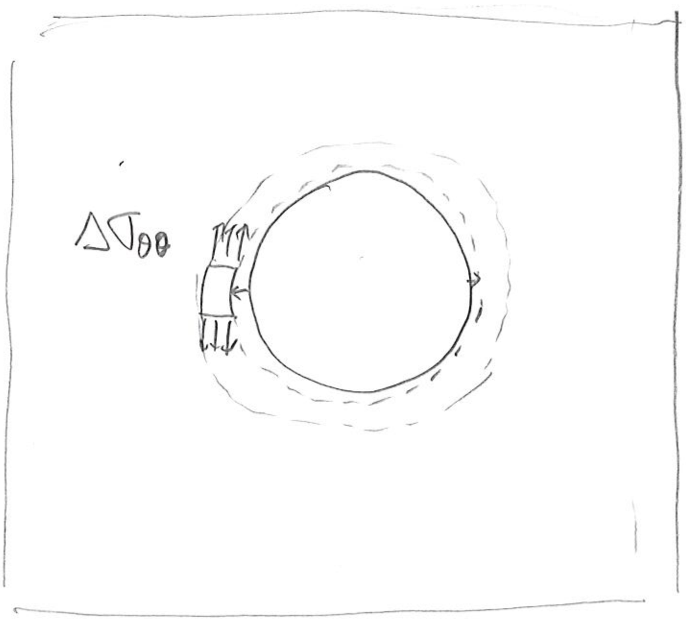
$\Delta T < 0$



Thermal strain

Thermal stress

$$\frac{\Delta L}{L} = \alpha_T \, \Delta T \quad \text{(1D experiment)}$$



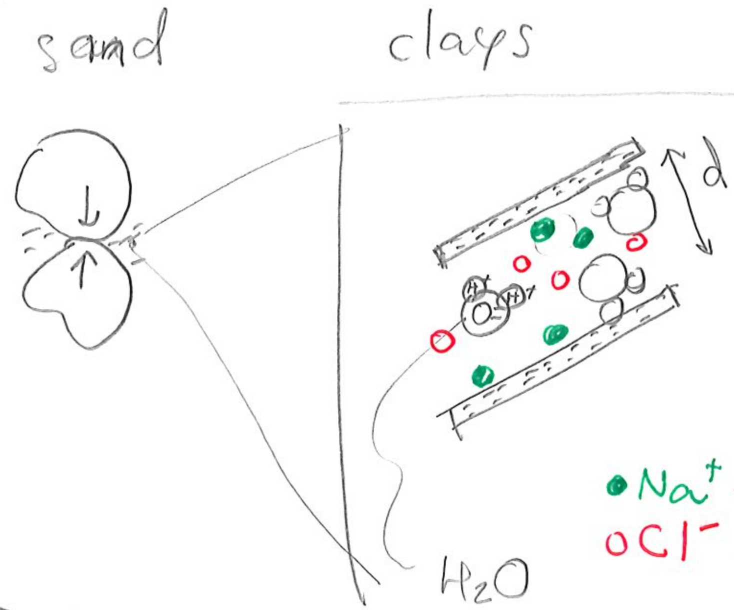
thermoelasticity

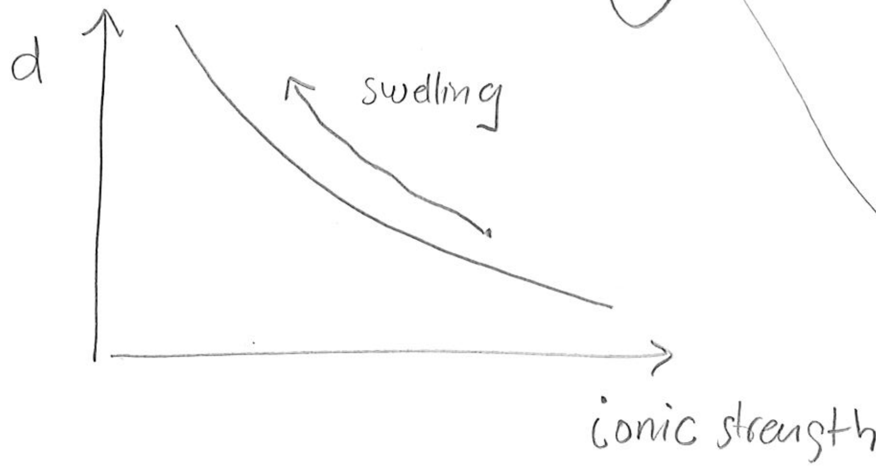
$$\Delta \sigma_{\theta\theta} = \frac{E}{1-\nu} \alpha_T \, \Delta T$$

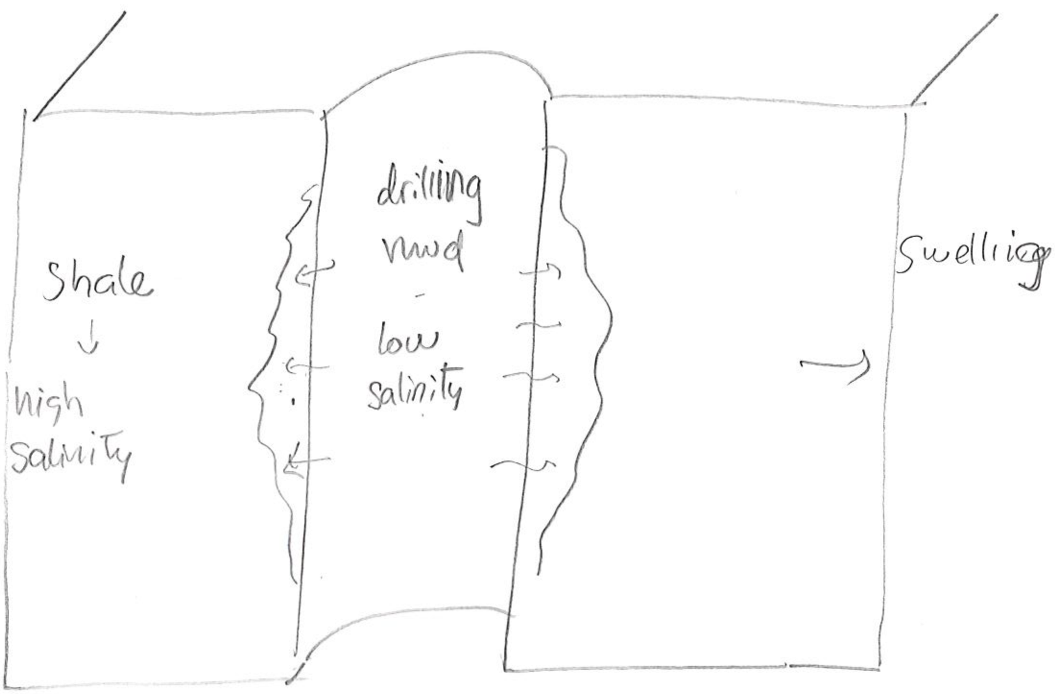
$\hookrightarrow \sigma^{\Delta T} \rightarrow$ use in $\underline{P_b}$

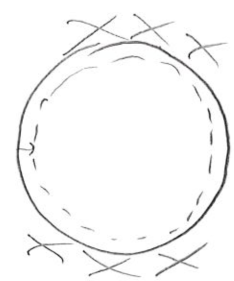
(2) Shale instability

shale ↔ clays







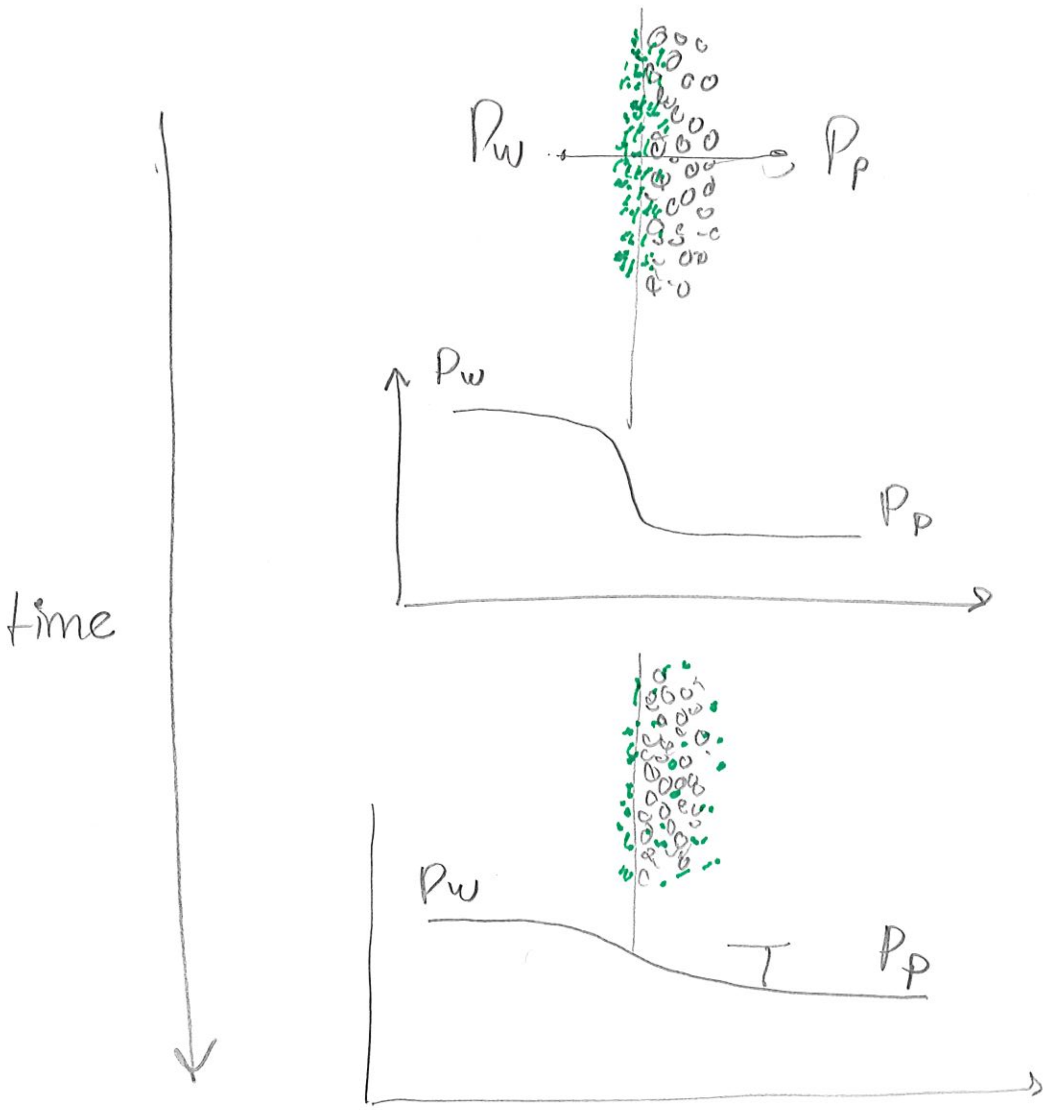


Solutions

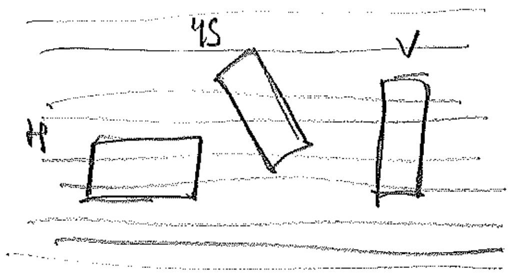
- oil-based mud
- high-salinity – K I
- underbalanced drilling (no filtrate water)

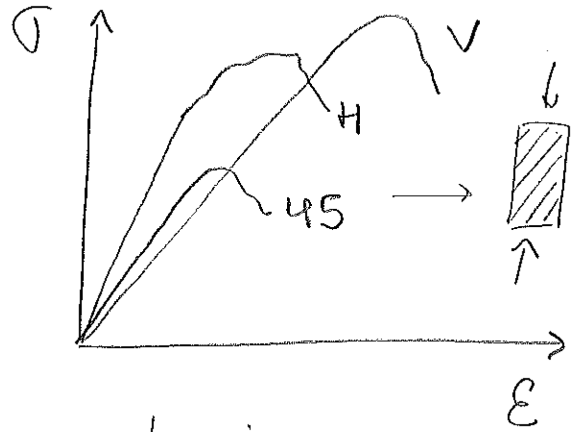
3 Loss of mud support

$$\sigma_{rr} = (P_w - P_p) \Leftarrow \text{filter cake, mud cake} \checkmark$$

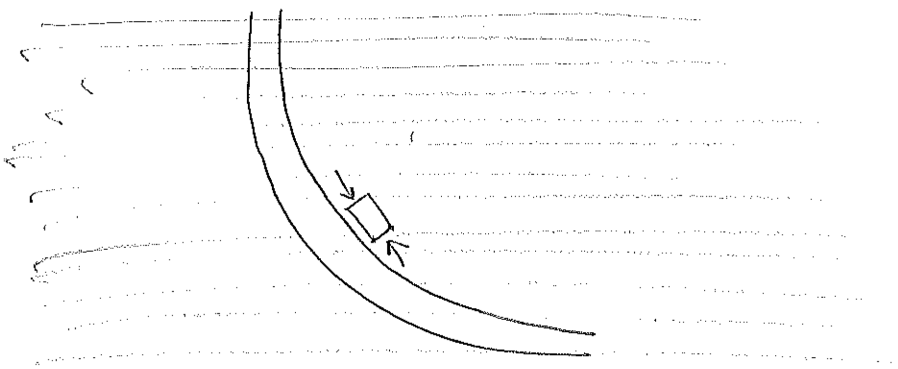


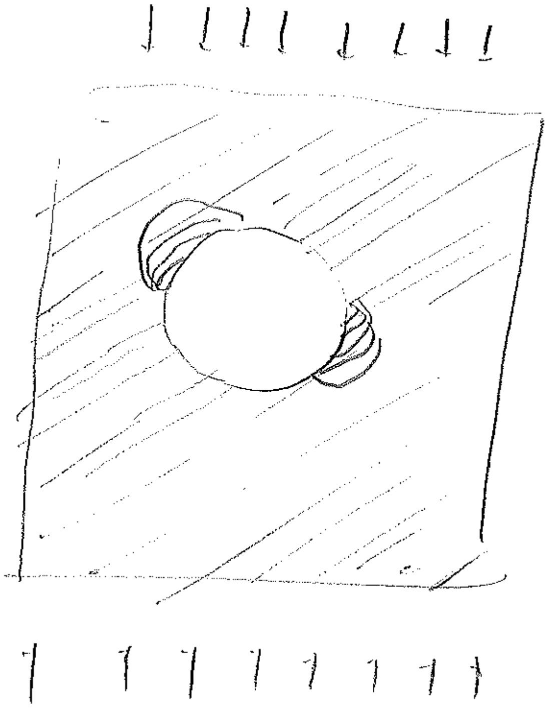
4) Strength anisotropy ⟷ deviated wellbores





→ tension
↳ tensile strength



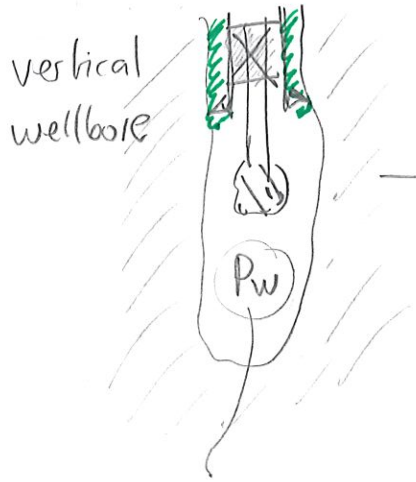


→ breakouts in directions/locations different from the ones in isotropic rock

Hydraulic Fracturing

- Fluid-driven fractures in nature (→ book)
- Hydraulic fractures in well testing

1) Leak-off test (Drilling) → drilling mud



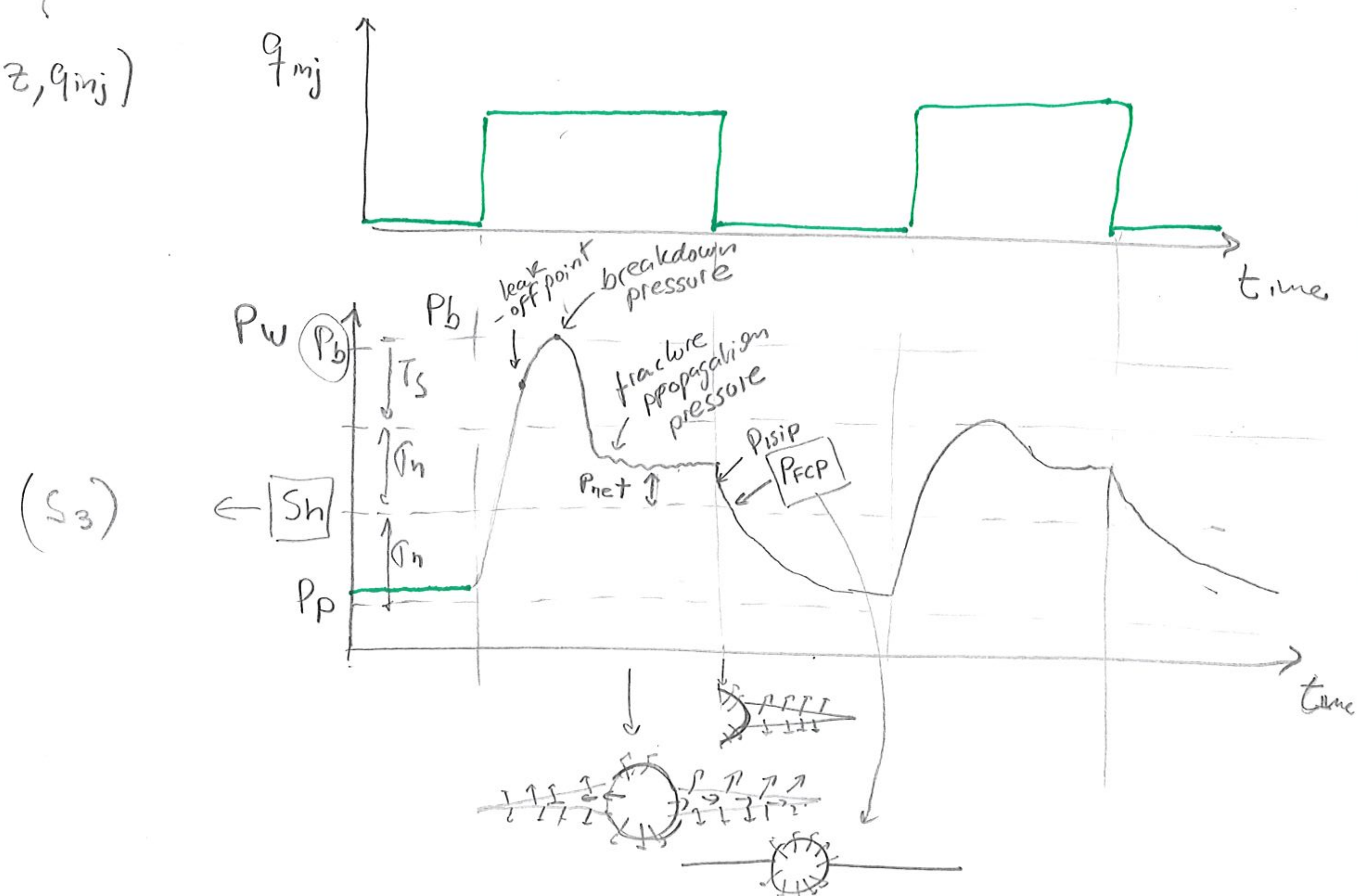
$F(\rho_{mud}, z, q_{inj})$

Normal Faulting ($S_v > S_{Hmax} > S_{hmin}$)

$$S_{Hmax} = S_{hmin} = S_h \rightarrow \sigma_h$$

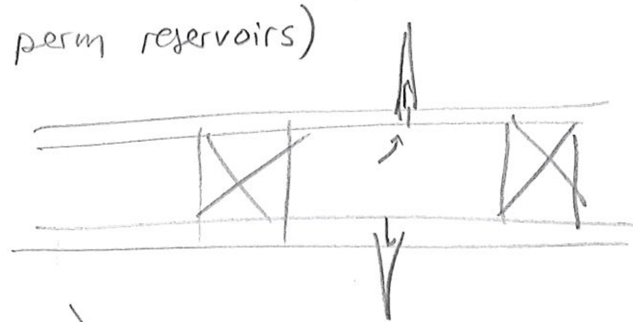
$$P_b = P_p + 3\sigma_{hmin} - \sigma_{Hmax} + T_s$$

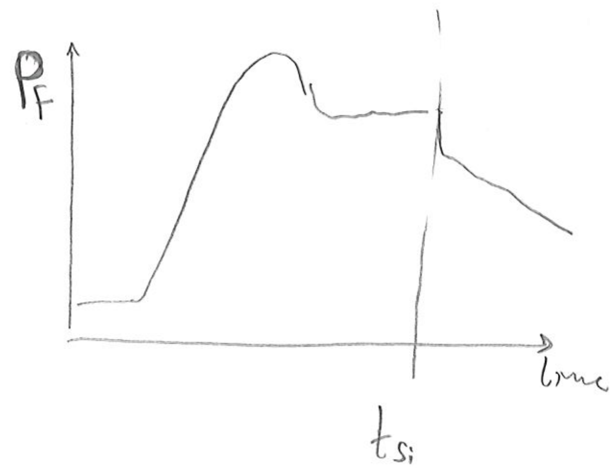
$$\boxed{P_b = P_p + 2\sigma_h + T_s}$$



2) DFIT (Diagnostics fracture initiation Test)

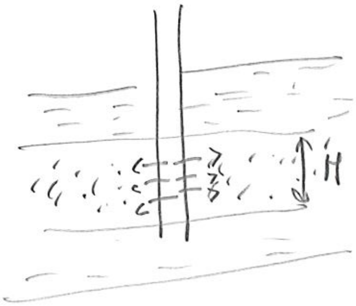
- Mini-frac test → (S_3, $K_{formation}$)
- Completion (low perm reservoirs)
- → Fracturing Fluid
- → small volumes (≪10 bbl)
- → small rates (≤ 3 bbl/min)

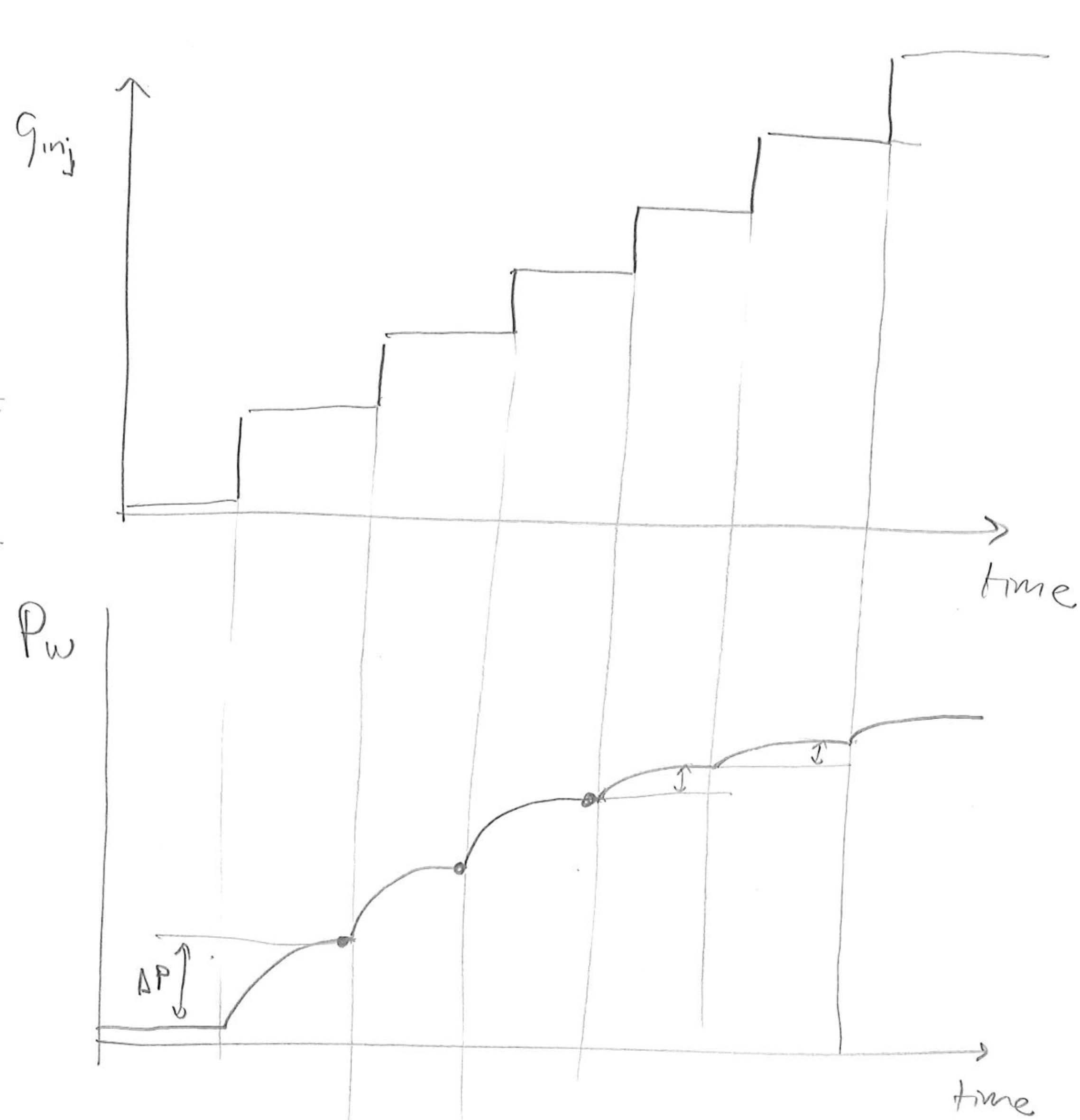




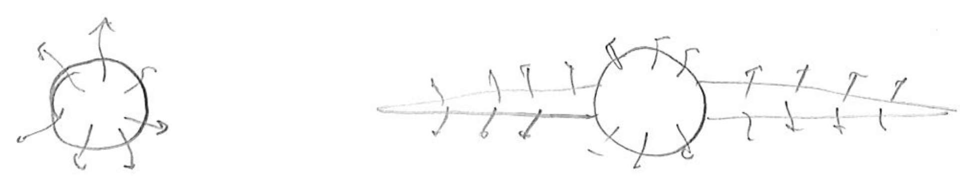
3) Step Rate test

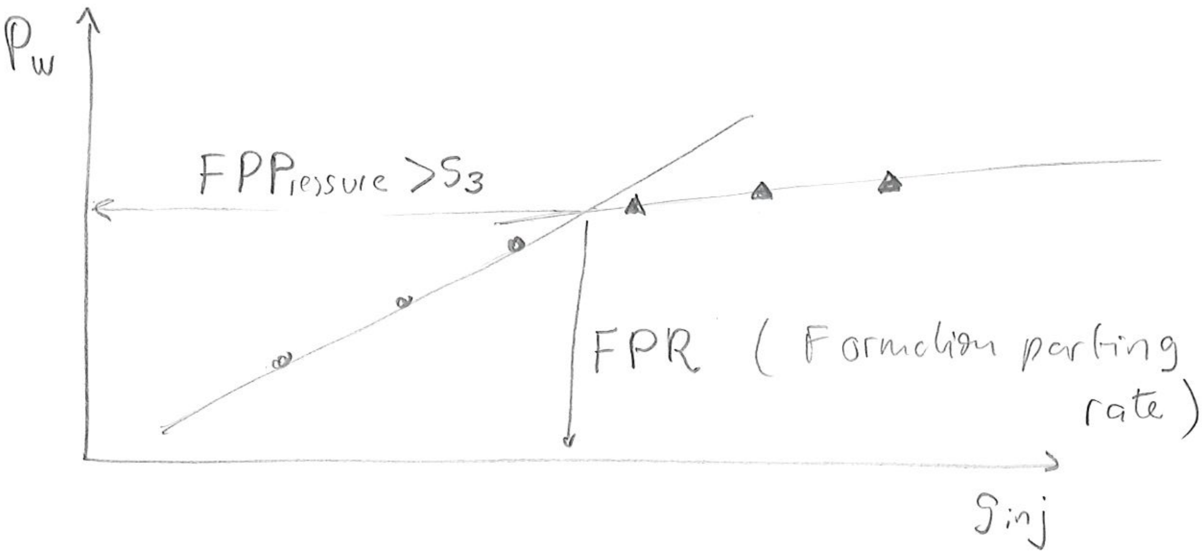
- → Max injection pressure ↔ Max injection rate
- → continuos injection
  - disposal water (production, EOR fracturing)
  - water for EOR
  - CO_2, N_2, polymer
  - steam
- → permeable formation
- → regulatory agencies



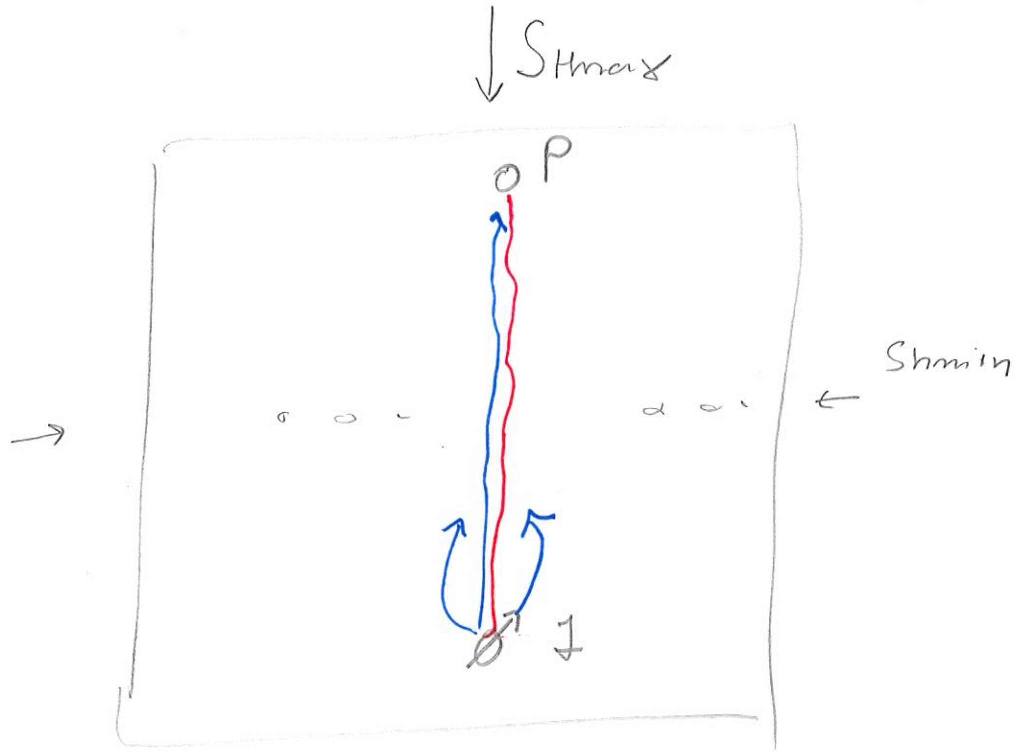


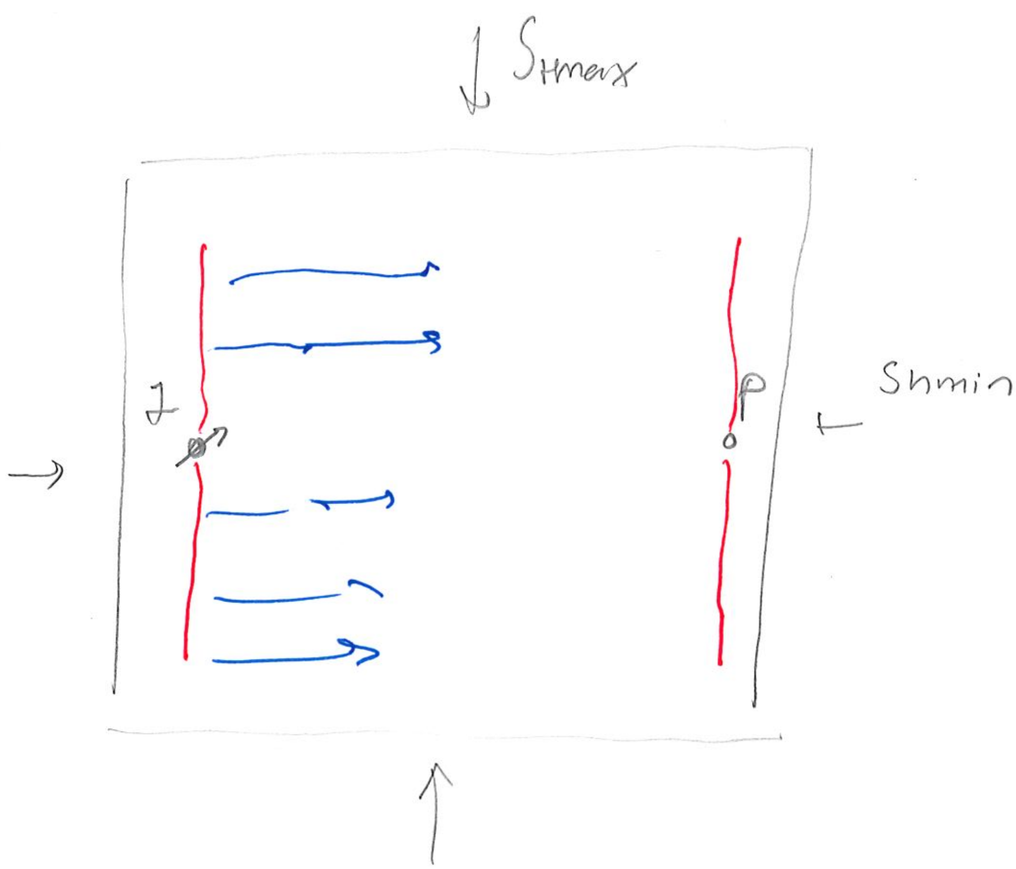
assumes
infinitely
large
reservoir





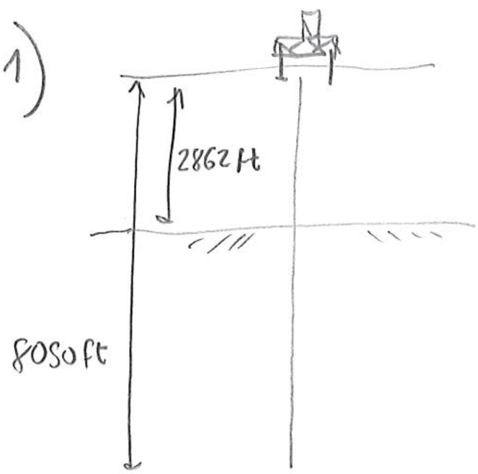
Normal faulting





HW #10

1)



$S_v = 6447$ psi

$S_3 = 5240$ psi

$S_3 < S_v \Rightarrow$ either NF or SS

$\sigma_{hmin} =$

$\sigma_v =$

$$\boxed{\frac{\sigma_v}{\sigma_{hmin}} = 3.24} \xrightarrow{\text{likely}} NF$$

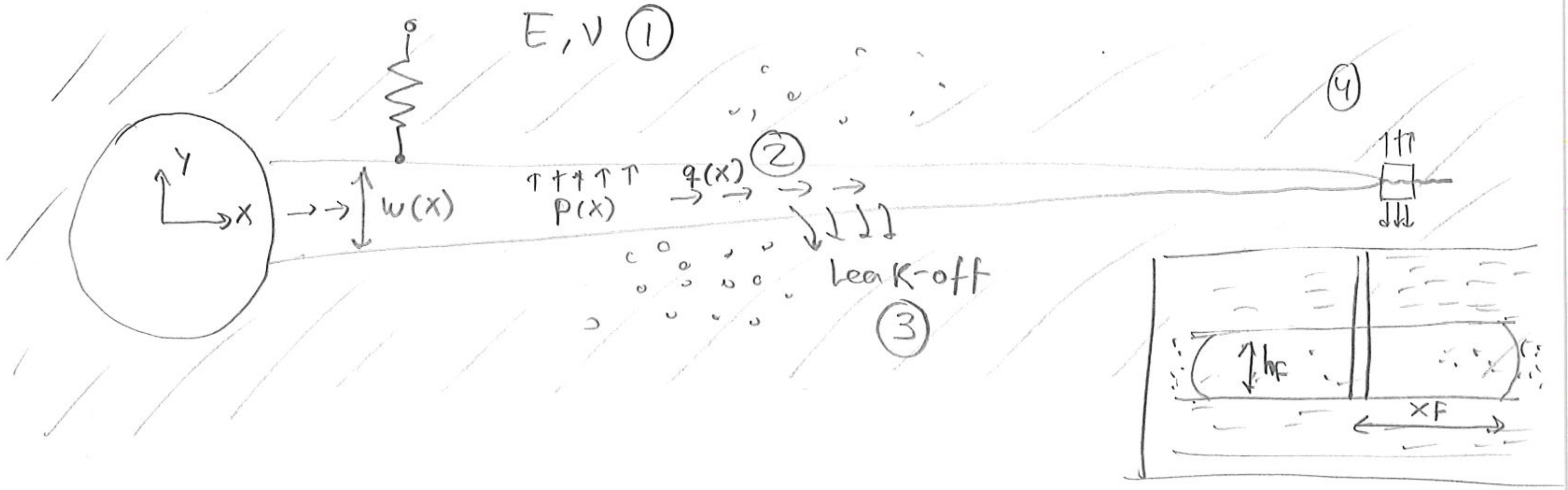
$$\xrightarrow{\text{likely}} \frac{\sigma_{Hmax}}{\sigma_{hmin}} < 3.24$$

$\rho_{mud} =$

2) Plot $p(t)$ and $q(t)$

$$S_3 = \underbrace{7503 \text{ ft } 0.44 \frac{psi}{ft}}_{\substack{\text{hydrostatic}\\\text{pressure}\\\text{within wellbore}}} + \underbrace{FCP}_{\substack{\text{surface}\\\text{reading}}}$$

Coupled hydraulic fracture propagation problem



(1) Solid Mechanics (Elasticity)

$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\sigma}} \xrightarrow[\text{Eq}]{\text{kin eq.}} \text{Navier's eq}$$

$$w(x) \propto \frac{P(x)}{E}$$

(2) Fluid Mechanics

$$\hookrightarrow \quad q(x) = \frac{(W(x))^3 \cdot hF}{12\,\mu} \frac{\Delta P}{\Delta X}$$

laminar flow

Newtonian fluid

(3) Leak-off

$$\underbrace{V_i}_{\text{total}} = \overbrace{V}^{\text{frac}} + \overbrace{V_L}^{\text{Leaks}}$$

$$V_i = i \cdot t \quad \} \; i: \text{one wing}$$

$$V_L = A_L \left(2 C_L \sqrt{t} + S_p\right)$$

S_p: spurt loss

C_L: Carter Leak-off coeff

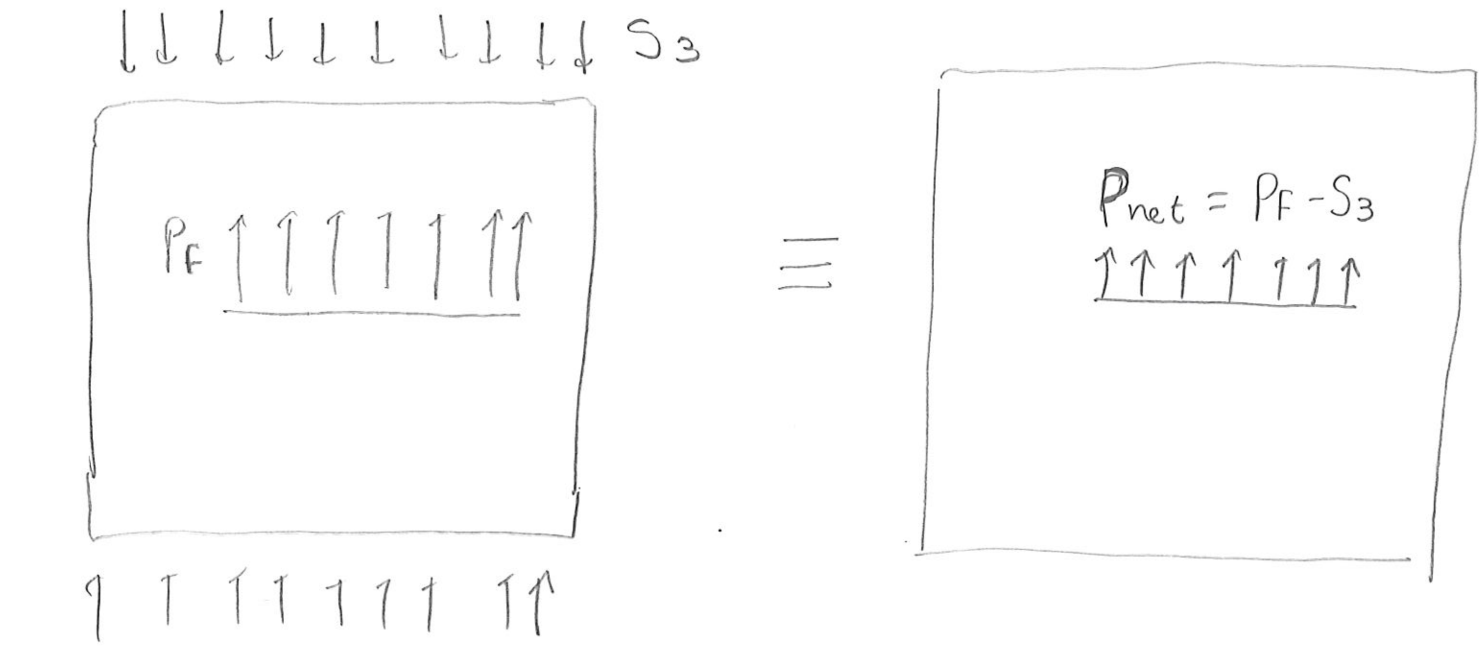
$$\eta = \frac{V}{V_i}$$

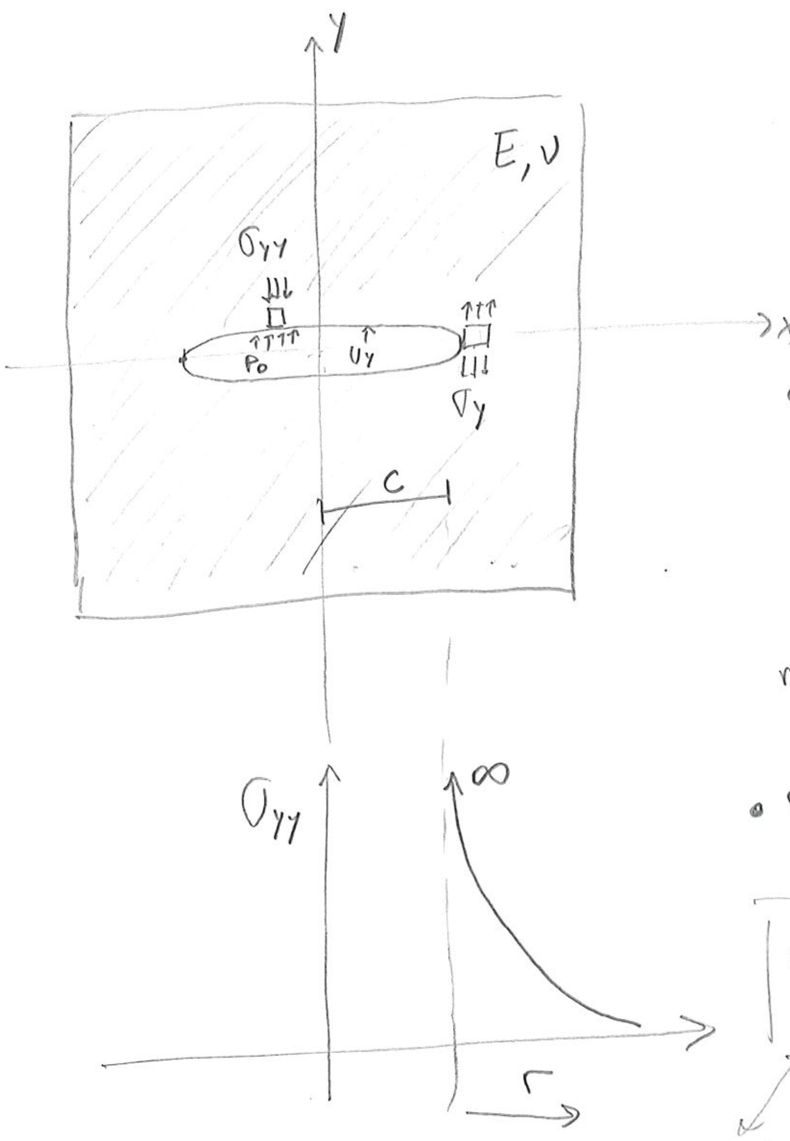
(4) Creation of new surface

↳ fracture mechanics

Fracture propagates $\underbrace{K_I}_{\text{stress intensity}} > \underbrace{K_{IC}}_{\text{fracture Toughness}}$

Griffith line crack problem





$$p(x) = P_o$$

displacements

$$(\lambda + G)\nabla(\nabla \cdot \underline{U}) + G\nabla^2\underline{U} + \rho\,\underline{b} = 0$$

↓ Griffith

- $$U_y(x,0) = \frac{2P_o}{E'}\sqrt{c^2 - x^2}\;\Big|^{0<x\le c}$$ ↝ ellipse

plane-strain modulus $E' = E/(1-\nu^2)$

max width $\rightarrow W(x=0) = \dfrac{4P_o c}{E'}$

- $$\sigma_{yy}(x>c,0) = -P_o\left[\frac{x}{\sqrt{x^2-c^2}} - 1\right]$$

$$\boxed{K_I = \lim_{r\to 0^+}\left[(2\pi r)^{1/2}(-\sigma_{yy})\right]}$$

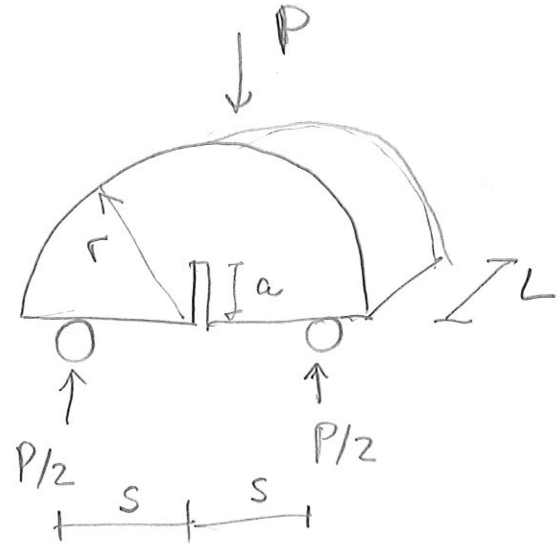
Fracture intensity

Fracture Toughness

$$K_I(p(x)=P_o) = P_o(\pi c)^{1/2} \leftrightarrow K_{Ic}$$

opening mode

Semicircular bending test



$$K_{IC} = \frac{P_{max}\,(\pi a)^{1/2}}{2 r L} Y_I$$

B 84 Indiana Limestone

$r = 0.019$ m

$a = 0.004$ m

$L = 0.013$ m

$Y_I = 4.5$

$P_{max} = 93$ lb $= 413$ N

$K_{IC} = 420\,000 \ \frac{N\,m^{1/2}}{m^2}$

$K_{IC} = 0.42$ MPa·m$^{1/2}$

$=$ psi · in$^{1/2}$

A 83

$r = 0.019$ m

$a = 0.0044$ m

$L = 0.013$ m

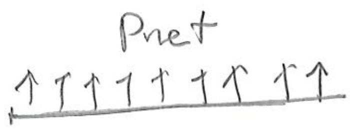
$Y_I = 4.5$

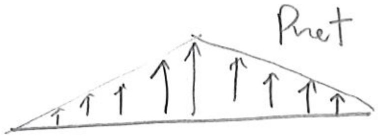
$P_{max} = 11.4$ lb

$K_{IC} =$ MPa·m$^{1/2}$

Typical values K_{IC} > 0.05 MPa·m$^{1/2}$

< 2.0 MPa m$^{1/2}$

a) 

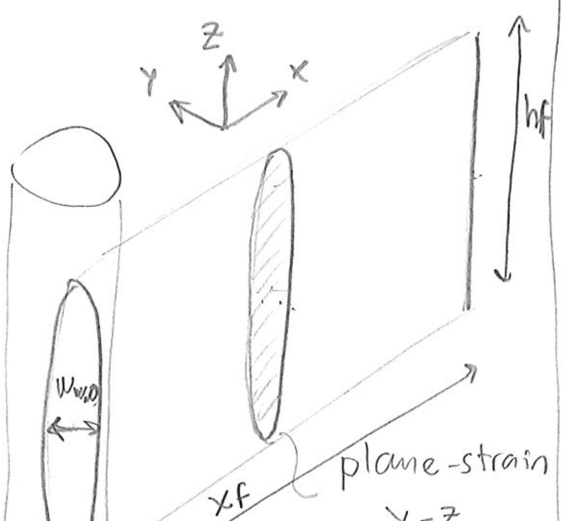
b) 

→ more realistic

$$\xrightarrow{q}$$
dP/dx

Models of fracture propagation (planar) 70's

PKN



- constant $i(t) = cst$
- no leak-off
- $K_{Ic} = 0$
- $p_{net}(tip) = 0$

$$x_f = \left(\frac{625}{512\pi^3}\right)^{1/5}\left(\frac{i^3 E'}{\mu\, h_f^4}\right)^{1/5} t^{4/5}$$

$P_n(x,0) = \ldots \quad t^{1/5}$

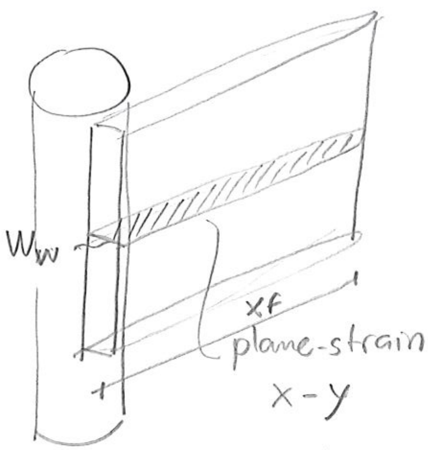
$W_{w,0} = \ldots$

$$\boxed{V_{frac} = \bar{w}\cdot x_f \cdot h_f = \frac{\pi}{5} W_{w,0}\cdot x_f \cdot h_f}$$

→ long fractures

$x_f > h_f$, → completions

KGD



- constant $i(t)$
- no leak-off
- $\boxed{K_{Ic} = 0}$
- $P_{net}(tip) = 0$

$x_f =$

$P_n = \ldots\ldots \quad t^{-1/3}$

$W_w =$

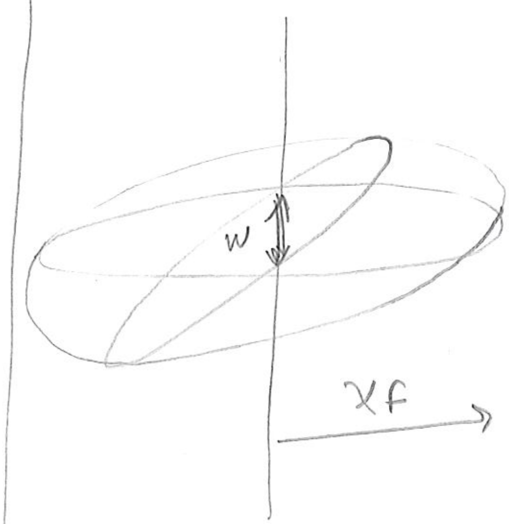
→ short fractures

$x_f < h_f$

→ leak-off tests

→ frac-pack completions

Radial

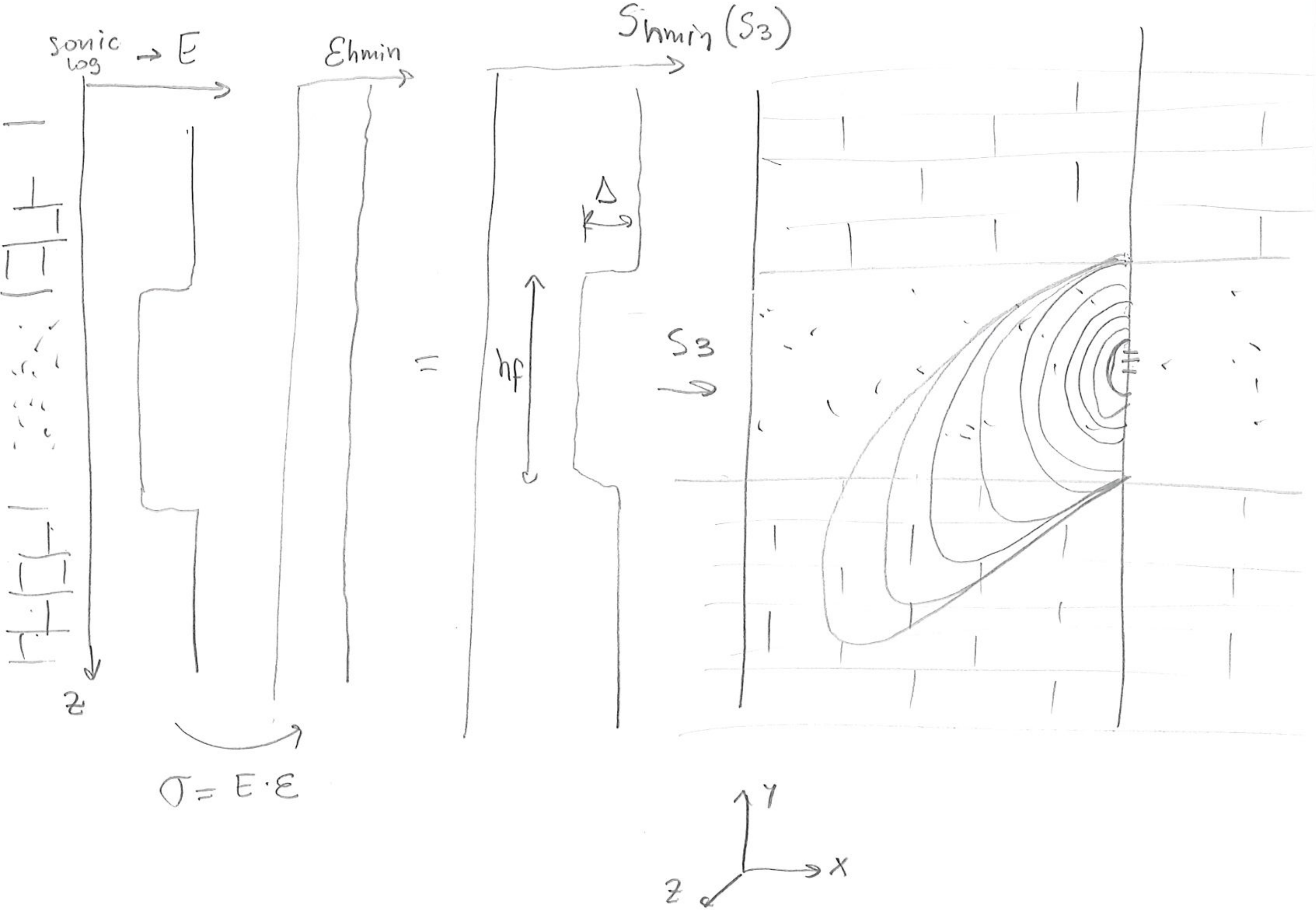


$x_f =$

$P_n =$

$W_w =$

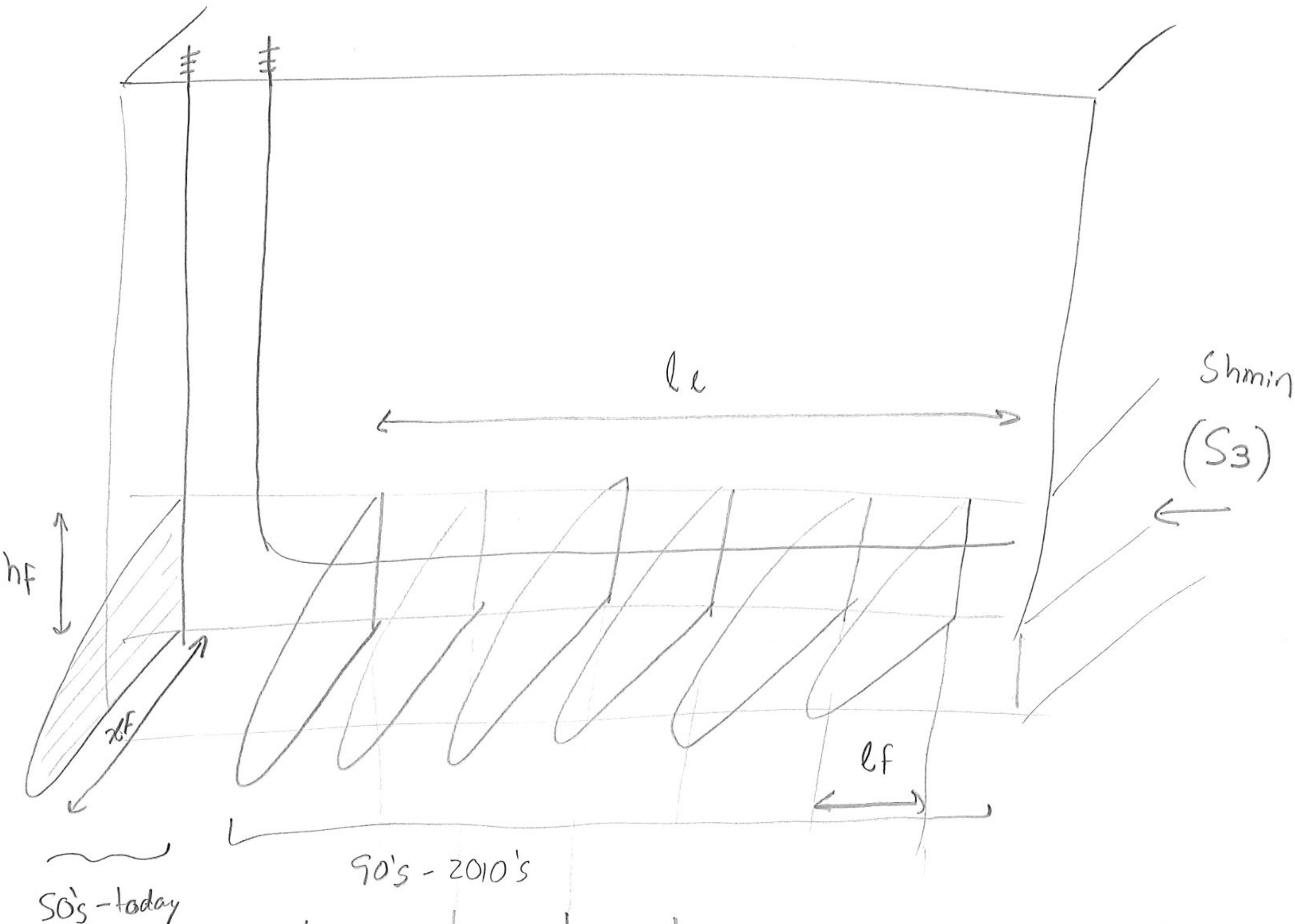
Determination of fracture height ⟷ Stress log

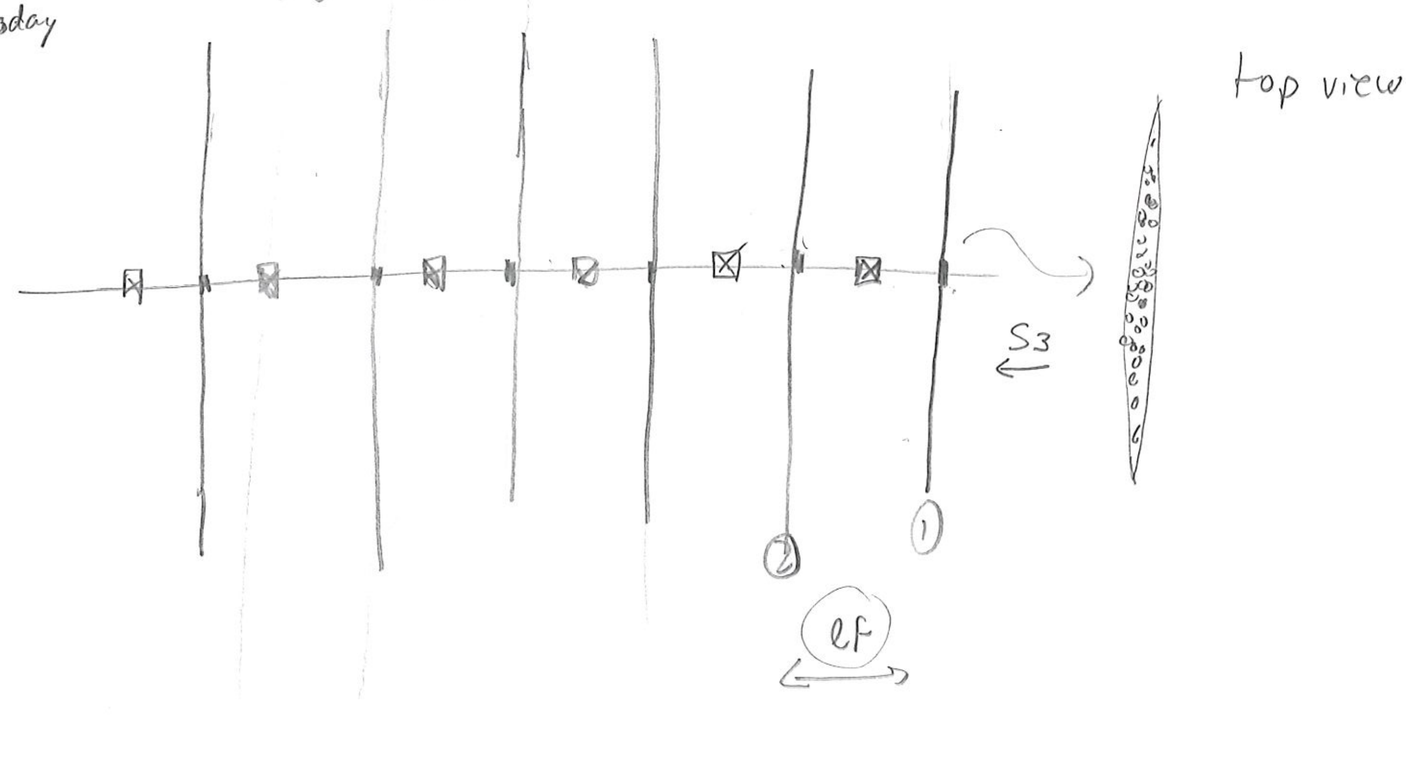


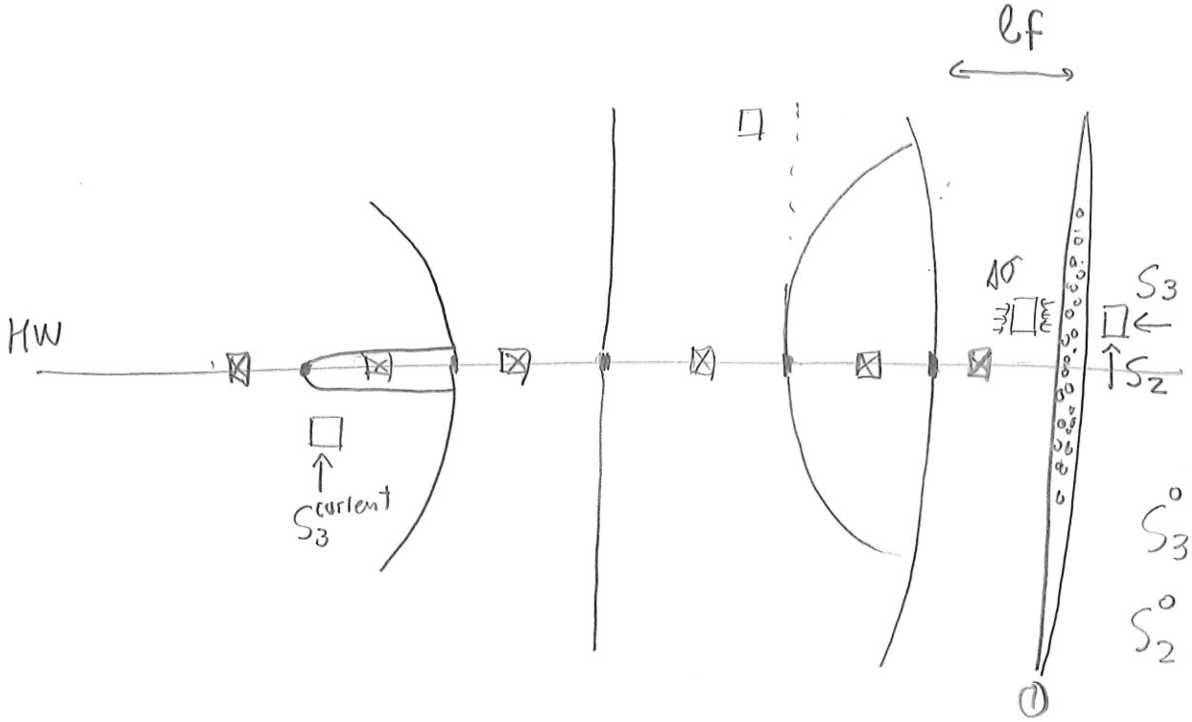
→ Frac Pro
→ Gohfer

} MEM + fluid injection
leak off
fracture geometry } → fracture

Multistage hydraulic Fracturing







$\sigma = E \cdot \frac{\varepsilon}{l}$

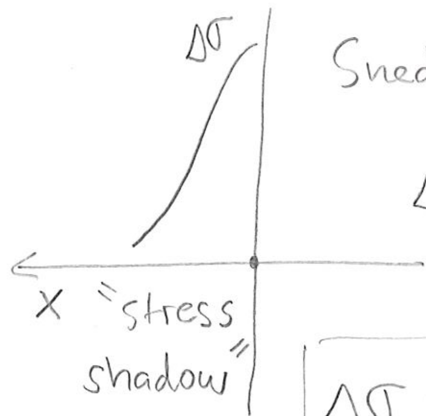
Young's width

$S_3^0 = S_{hmin}$

$S_2^0 = S_{Hmax}$

$$\boxed{l_f \propto \frac{P_{net}\, h_f}{(S_2 - S_3)}}$$

$P_{net} \propto W \cdot E$



Sneddon's

$$\Delta\sigma \propto \frac{P_{net} \cdot h_f}{x}$$

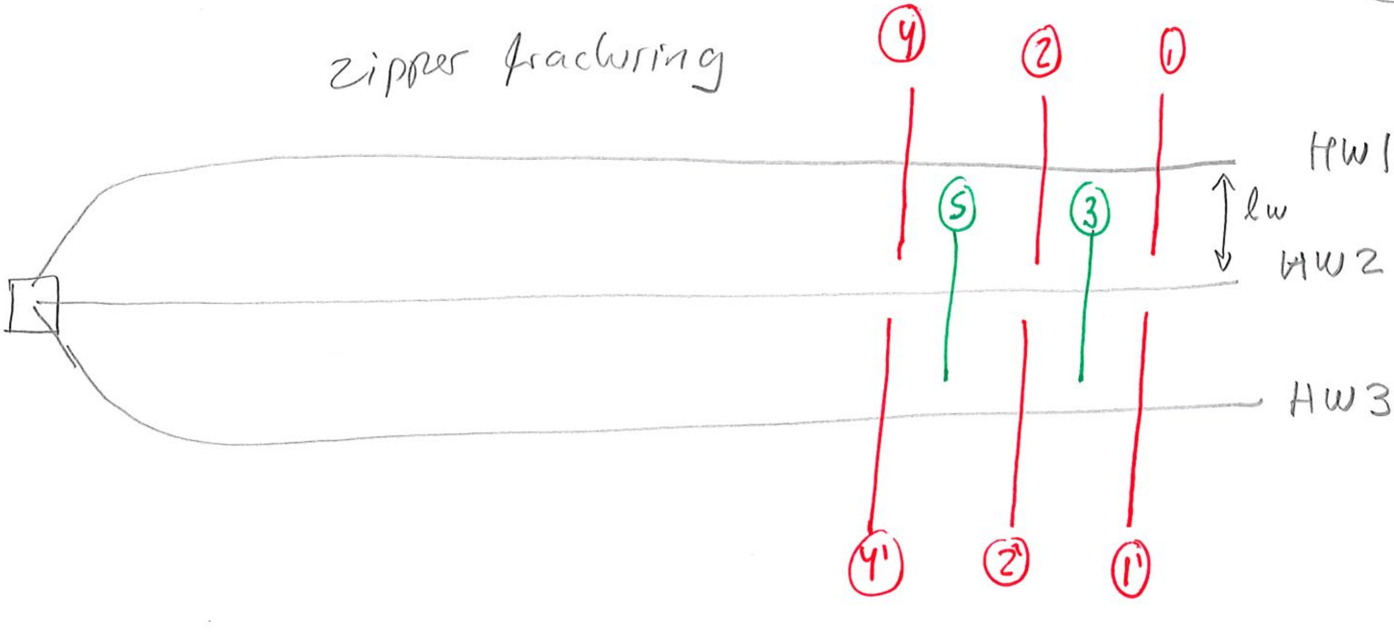
$$\boxed{\Delta\sigma \leftrightarrow \underbrace{S_2^0 - S_3^0}_{\text{differential stress}}}$$

$P_{net} < (S_2 - S_3) \rightarrow$ no frac interference

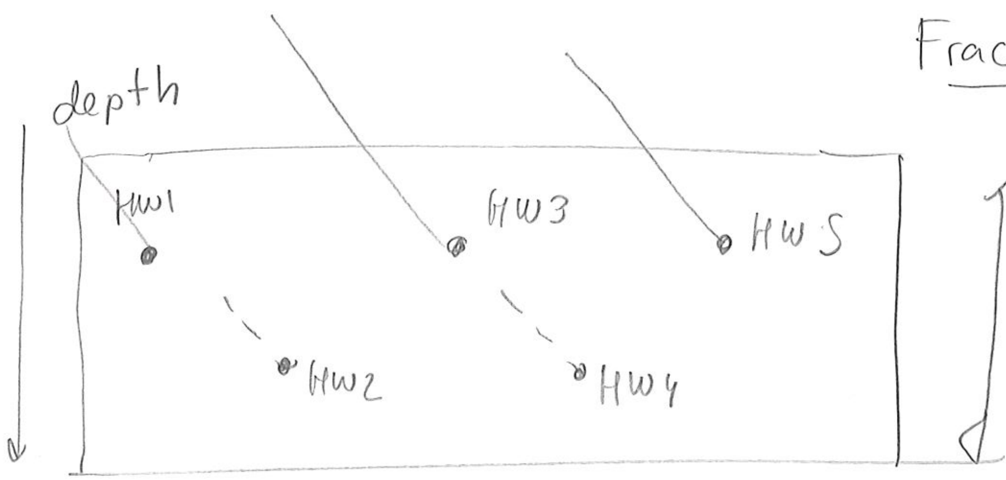
$P_{net} > (S_2 - S_3) \rightarrow$ frac interference

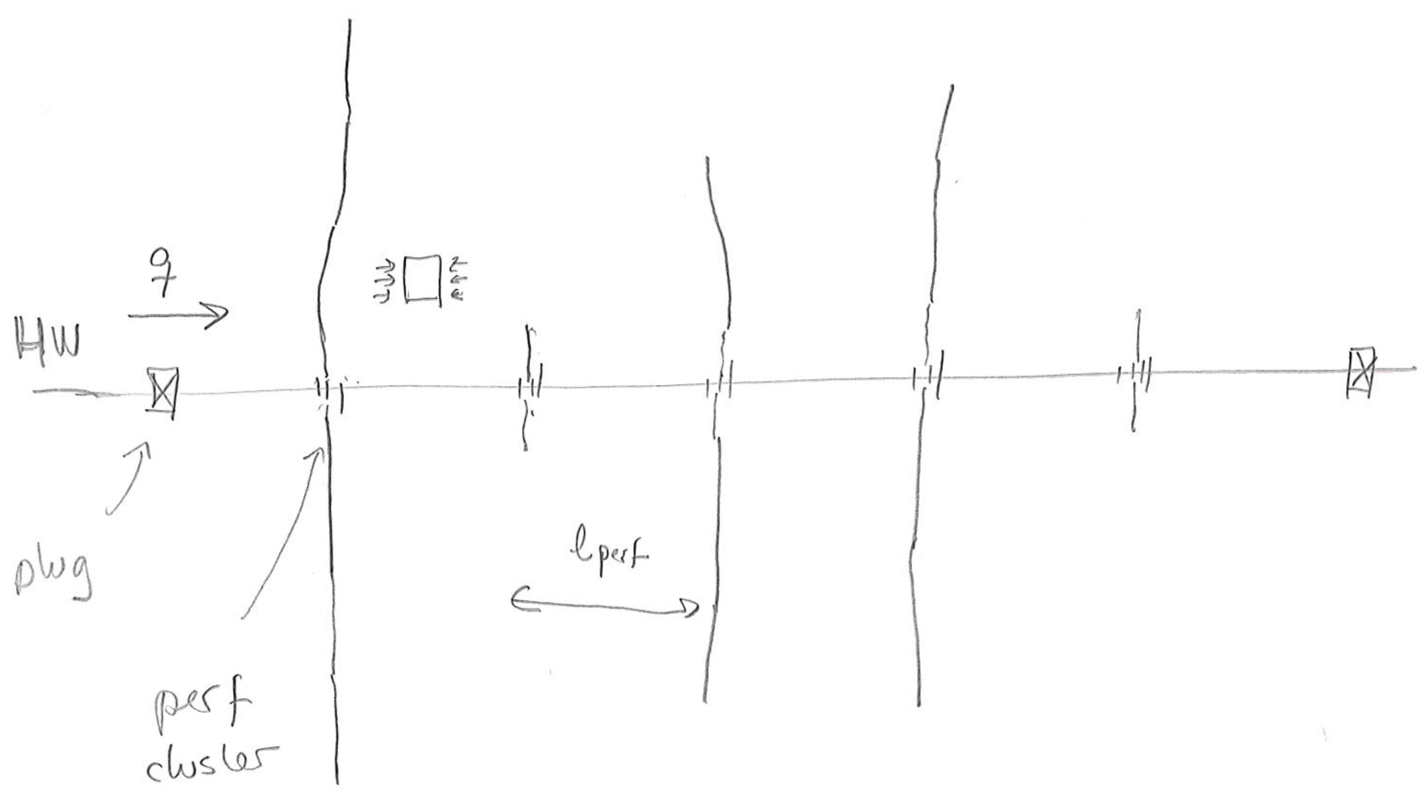
TOP VIEW

zipper fracturing



Frac hit:



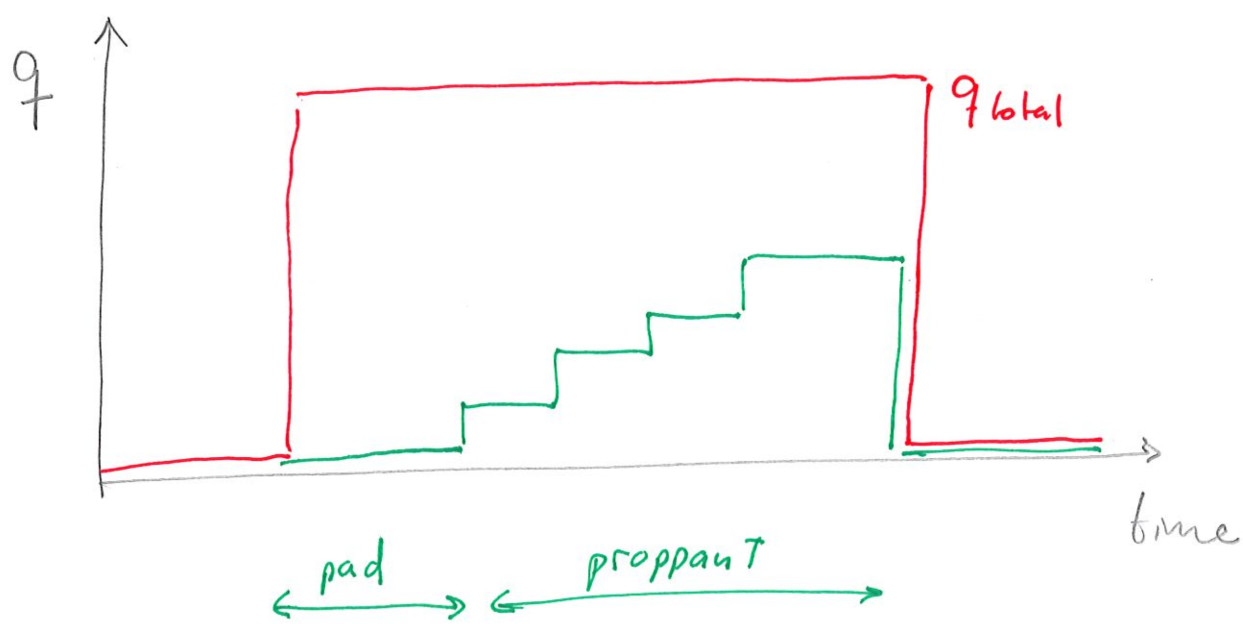


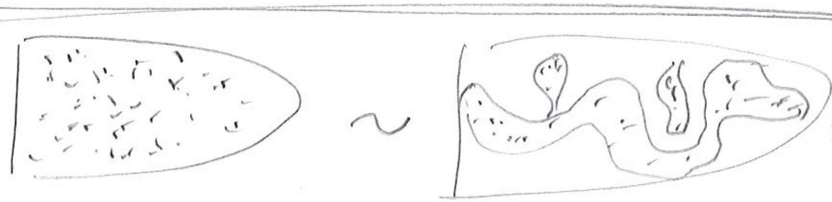
Typical fracturing completion inthe Permian today

- lenght of lateral ~ 10,000 ft
- 40 stages
- 4 - 15 clusters/stage ⟷ 60% clusters open
- each cluster 6 perf/foot → 60° each other

- 2 bbl/min/perf
  - → 100 bbl/min
  - → 2500 gal/LF
- 2000 $\frac{\text{lb (proppant)}}{\text{LF}}$

$$0.8 = \frac{\text{lb (proppant)}}{\text{gal (frac fluid)}}$$

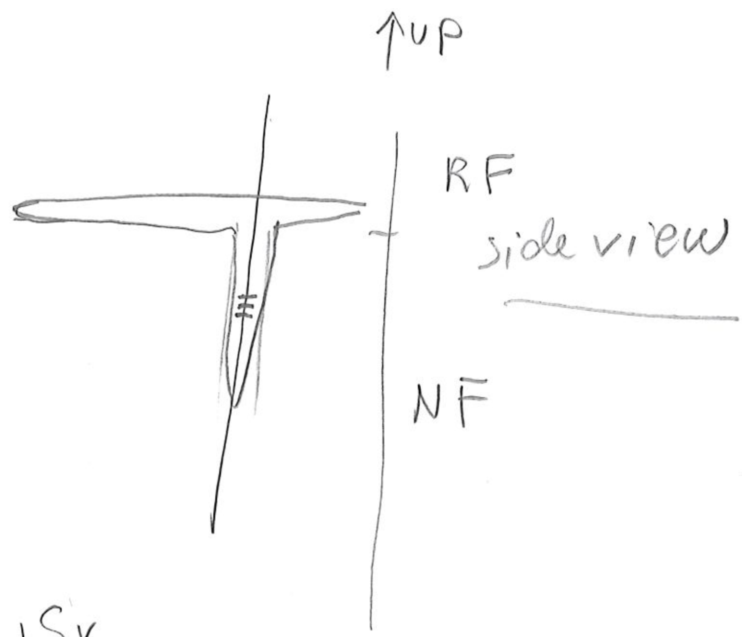


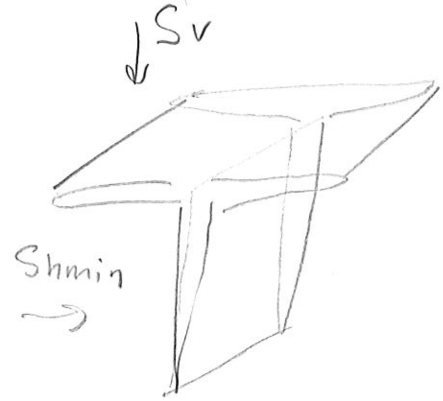


~ slug injection of proppant

HW #11

→ T-shaped fracture
↓
reversal of stress regime





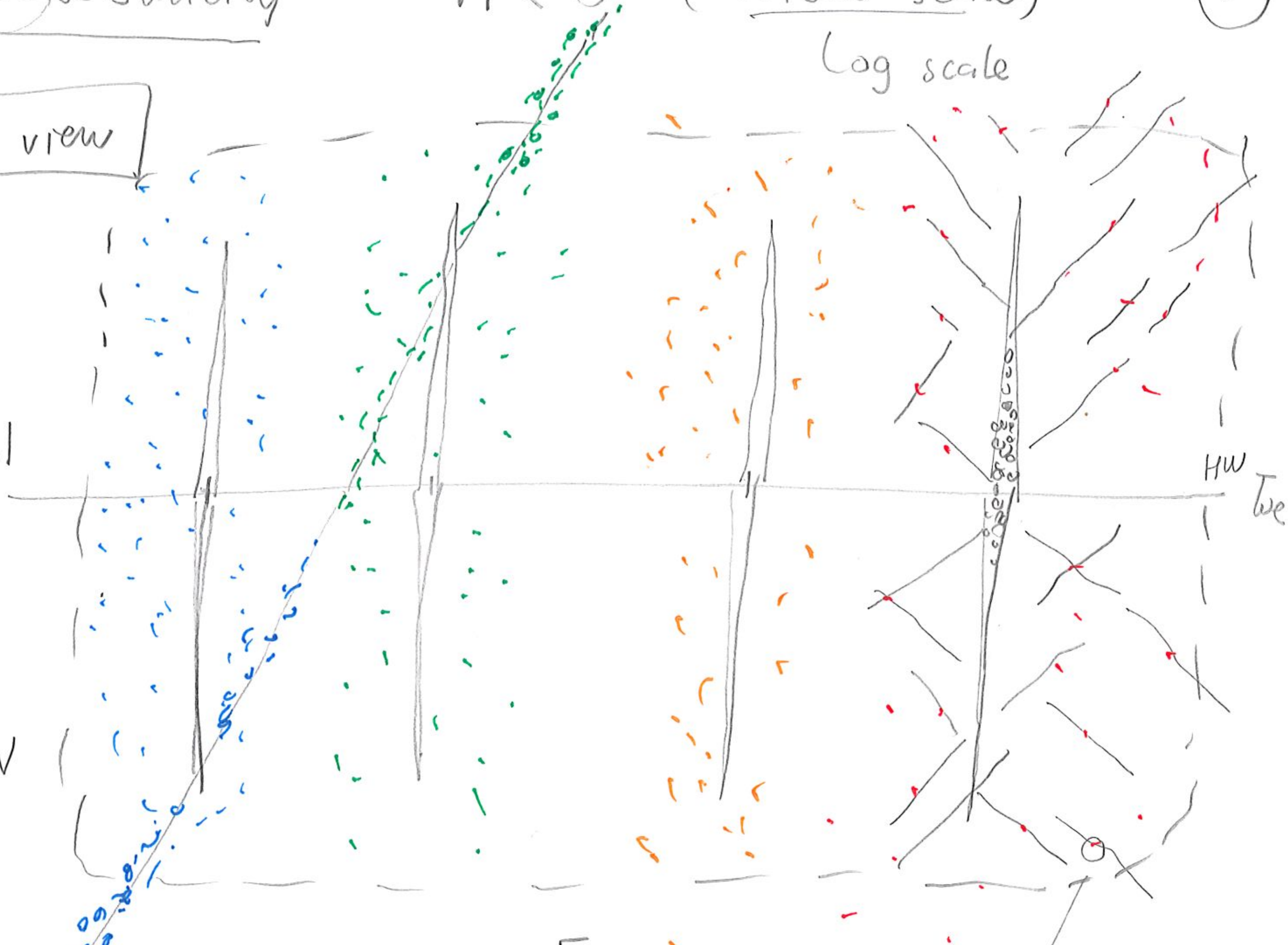
Microseismicity M < 0 (Ritcher scale)

Log scale

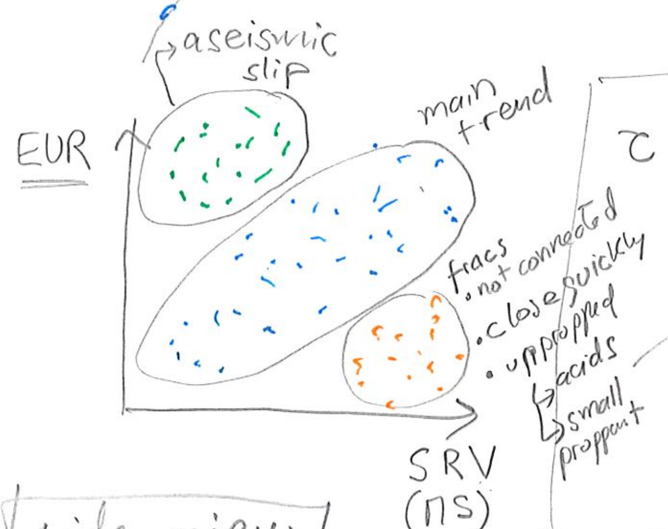
Top view

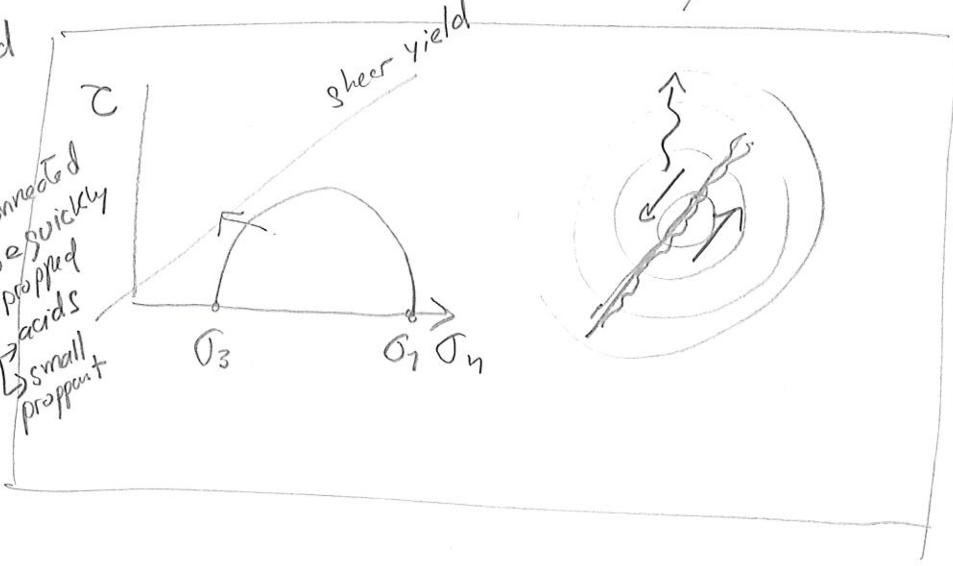
heel

SRV



Brittleness $\sim \frac{E}{\nu}$





side view

$$EUR = RF \cdot OOIP$$

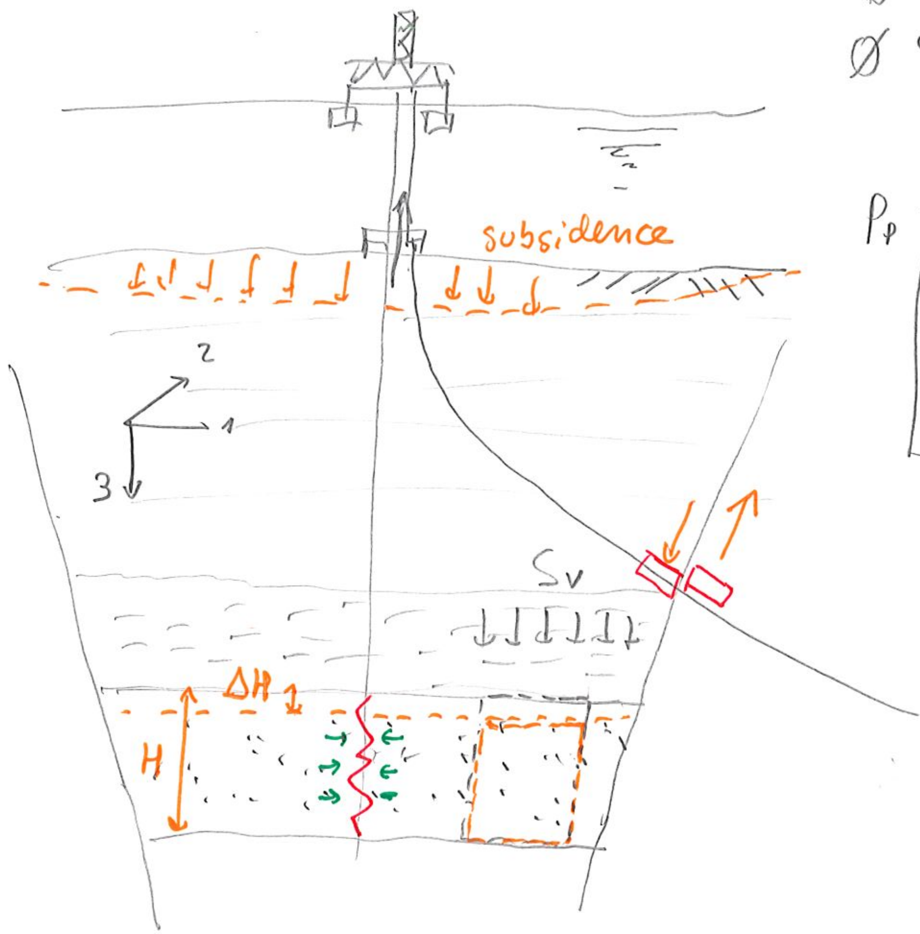
$$= RF \cdot \frac{\phi (1-S_w) \, Vol}{B_{oi}}$$

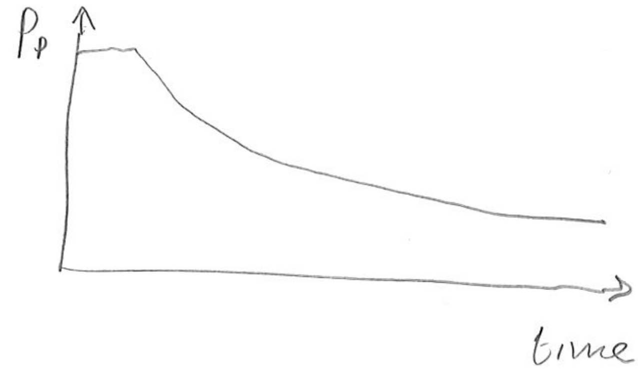
$$EUR = RF \cdot \frac{\phi (1-S_w) \, SRV}{B_{oi}}$$

→ depends on both stimulation and net pay

Reservoir depletion → predict $K = f(\text{time} \leftrightarrow \text{depletion})$

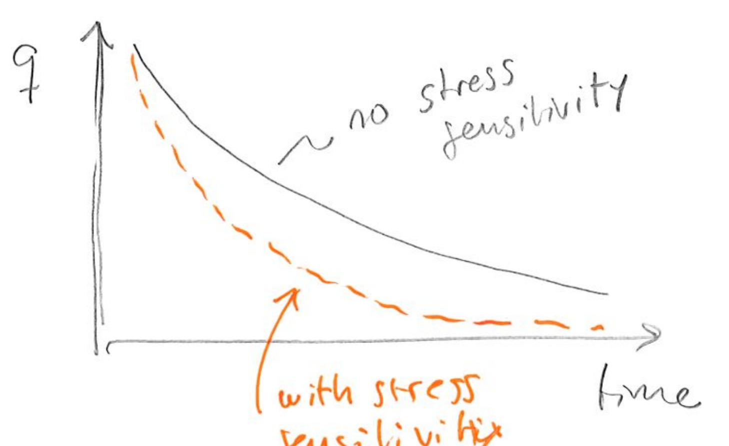
$$\begin{array}{c} K \\ \updownarrow \\ \varnothing \iff \underline{\underline{\sigma}} \text{ (eff. stresses)} \end{array}$$





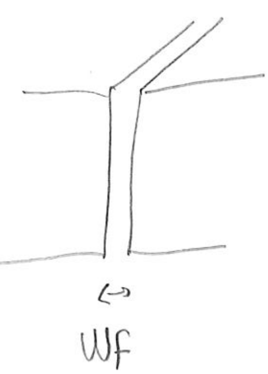
$$\left.\begin{array}{l} \varepsilon_{33} = \dfrac{\Delta H}{H} \\ \varepsilon_{11} = \varepsilon_{22} = \varnothing \end{array}\right\} \text{one-D strain condition}$$

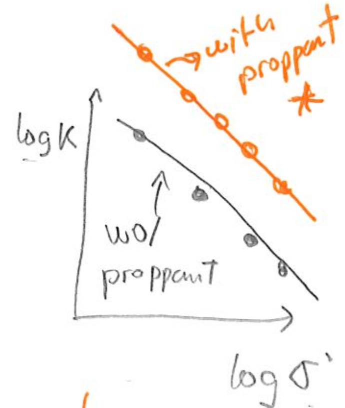
$$\downarrow P_p \rightarrow \text{compaction} \left[\begin{array}{l} \downarrow \varnothing \rightarrow \downarrow K \leftrightarrow K \propto \varnothing \cdot d_p^2 \\ \uparrow \sigma \rightarrow \text{fractured rocks} \end{array}\right.$$



$K = f(P_p) = f(\underline{\underline{\sigma}})$

$K \propto w_f^3$





* • proppant crushing
• proppant embedment

$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \underline{\underline{I}}$ → Terzaghi's effective stress

$\underline{\underline{\sigma}} = \underline{\underline{S}} - \alpha P_p \underline{\underline{I}}$ → Biot's effective stress

→ $\alpha \cong (1, 0.5)$ Biot's coefficient

1 ↑ soft rocks – uncons sand

0.5 ↑ stiff rocks – stiff shales

$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}}$

1-D strain ↓

$$\begin{bmatrix} S_{11} - \alpha P_p \\ S_{22} - \alpha P_p \\ S_{33} - \alpha P_p \\ \hline S_{12} \\ S_{13} \\ S_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \left[\begin{array}{ccc|ccc} 1-\nu & \nu & \nu & & & \\ & 1-\nu & \nu & & 0 & \\ & & 1-\nu & & & \\ \hline & & & - & & \\ & 0 & & & - & \\ & & & & & - \end{array}\right] \begin{bmatrix} \cancel{\varepsilon_{11}} \\ \cancel{\varepsilon_{22}} \\ \varepsilon_{33} \\ \hline 2\cdot\varepsilon_{12} \\ 2\cdot\varepsilon_{13} \\ 2\cdot\varepsilon_{23} \end{bmatrix}$$

3^{rd} row $\Rightarrow$ $\varepsilon_{33} = \dfrac{\overset{cst}{S_{33}} - \alpha P_p}{\underbrace{\left[\dfrac{E(1-\nu)}{(1+\nu)(1-2\nu)}\right]}_{M}}$ $\Rightarrow$ $\boxed{\dfrac{\Delta \varepsilon_{33}}{\Delta P_p} = -\dfrac{\alpha}{M}}$

1^{st} row + 3^{rd} row $\Rightarrow$ $\sigma_{11} = \dfrac{\nu}{1-\nu}\sigma_{33}$ $\Rightarrow$ $S_{11} = \dfrac{\nu}{1-\nu}(S_{33} - \alpha P_p) + \alpha P_p$

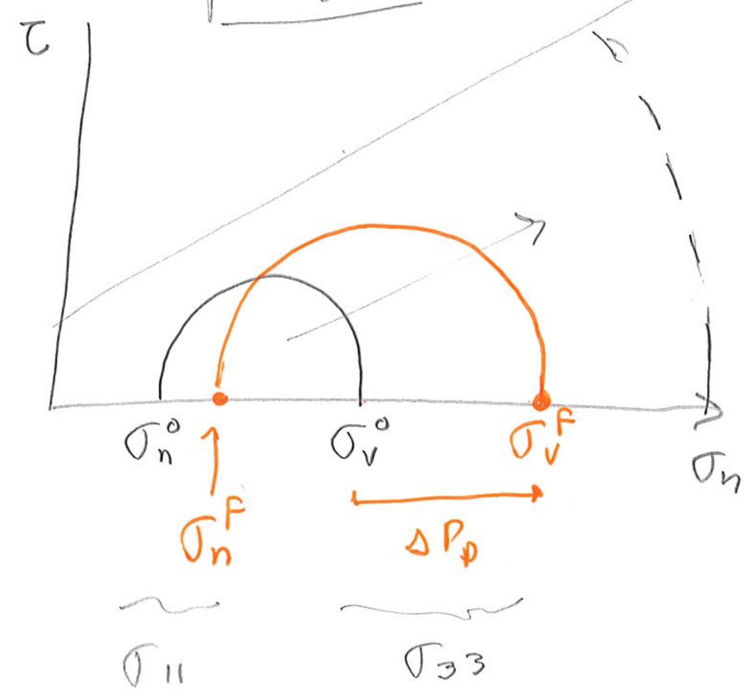
$\boxed{\dfrac{\Delta S_{11}}{\Delta P_p} = \alpha \dfrac{1-2\nu}{1-\nu}} = A$

Effective stresses

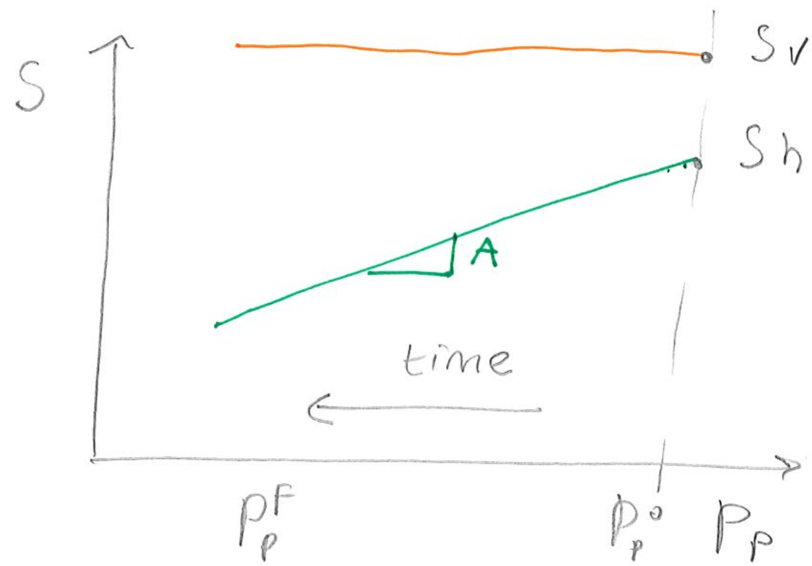
- $$\frac{\Delta\sigma_{11}}{\Delta P_p} = -\alpha \frac{\nu}{1-\nu}$$

- $$\frac{\Delta\sigma_{33}}{\Delta P_p} = -\alpha$$

NF, SS



Total stress



→ See example problem in book notes