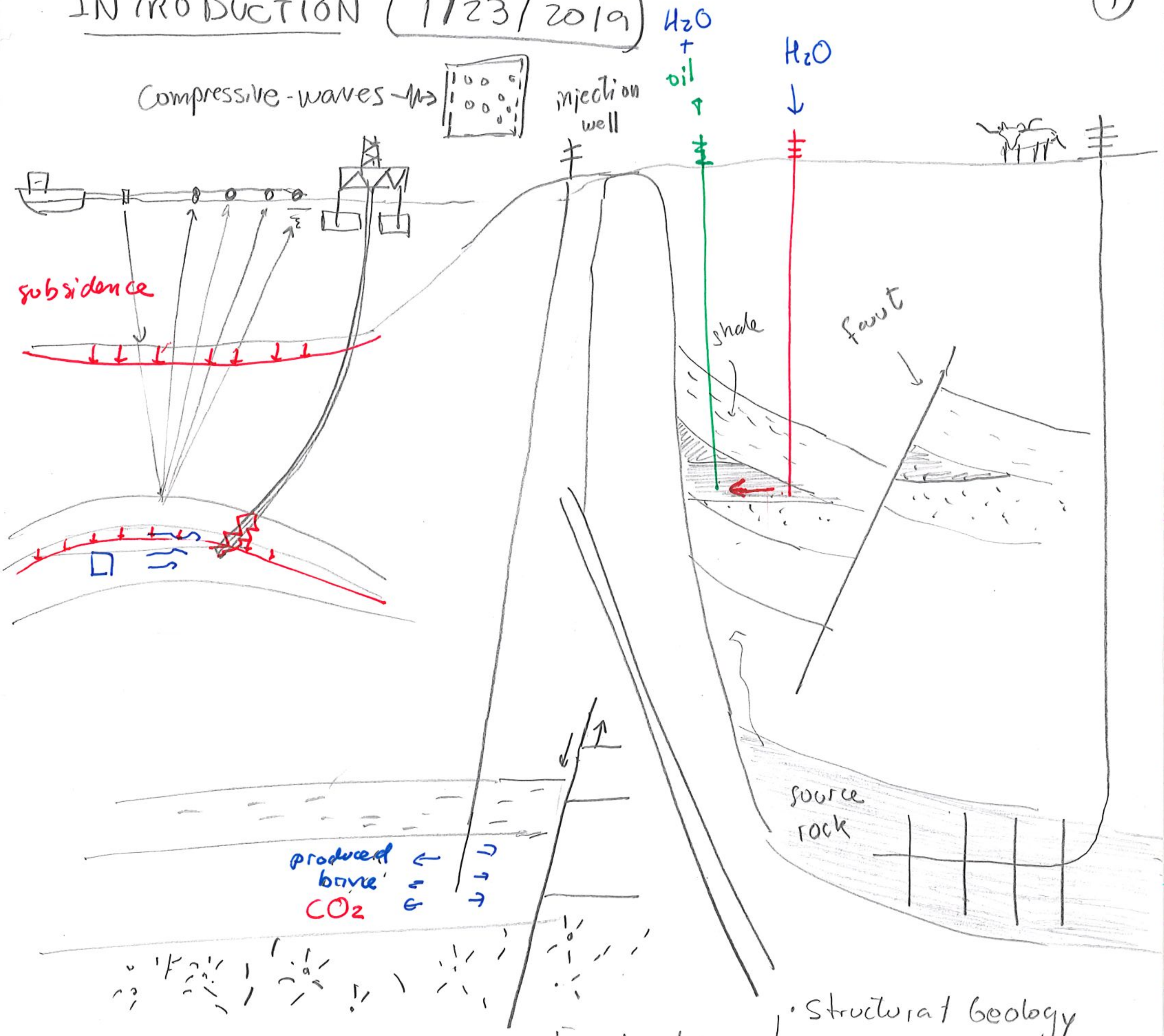
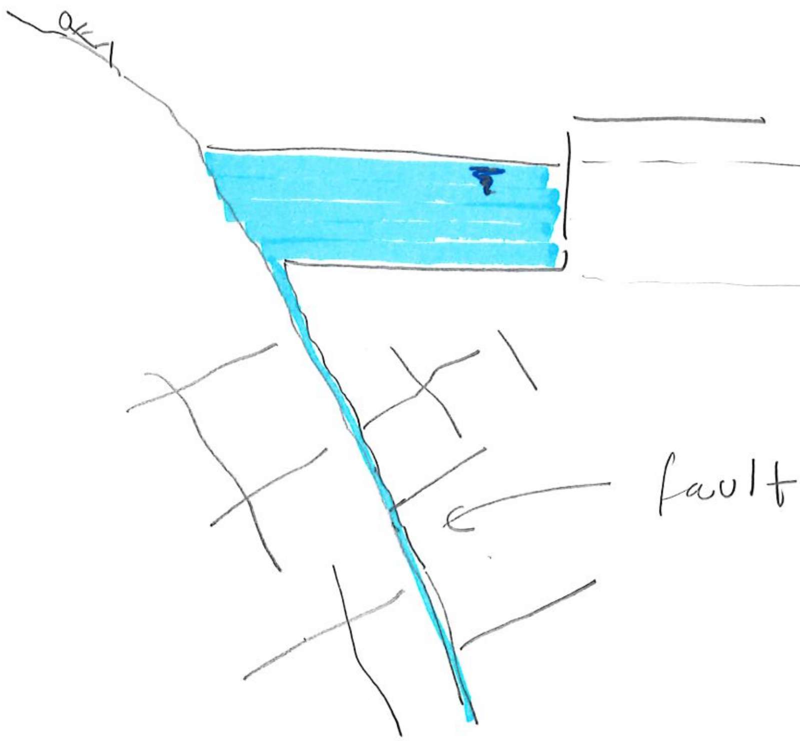
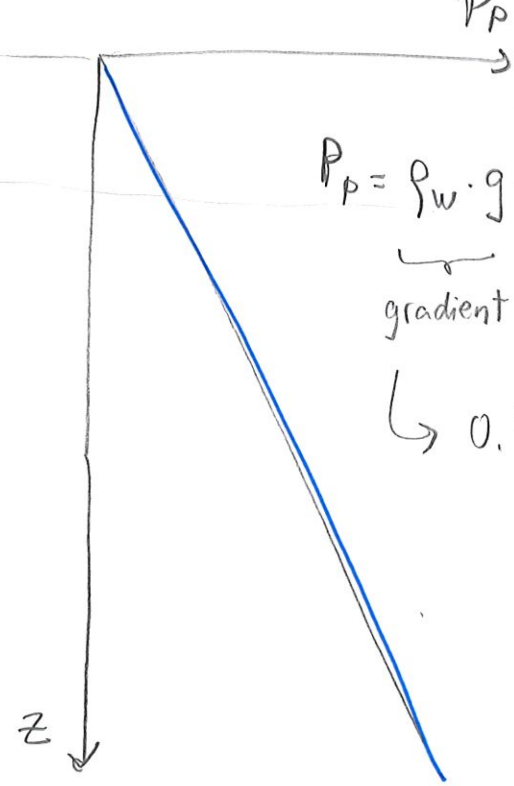
INTRODUCTION (1/23/2019)



- Exploration
  - Structural Geology
  - Seismic surveys
- Drilling & Completions
  - wellbore stability
  - fracturing
- Production
  - compaction
    - ↳ rock compressibility
  - sand production
- Waste disposal
  - brine and CO_2
- well abandonment

Pore pressure (1/25/2019)





$$P_p = \underbrace{\rho_w \cdot g}_{\text{gradient}} \cdot \underbrace{z}_{\text{depth}}$$

↳ $0.44 \ \frac{psi}{ft}$

$$\rho_w \cdot g \cong 1000 \ \frac{kg}{m^3} \cdot 10 \ \frac{m}{s^2}$$

$$= 10^4 \ \frac{N}{m^2} \cdot \frac{1}{m}$$

$$= 10^4 \ Pa \ \frac{1}{10^{-3} \ km}$$

$$= 10 \ 10^6 Pa \ \frac{1}{km}$$

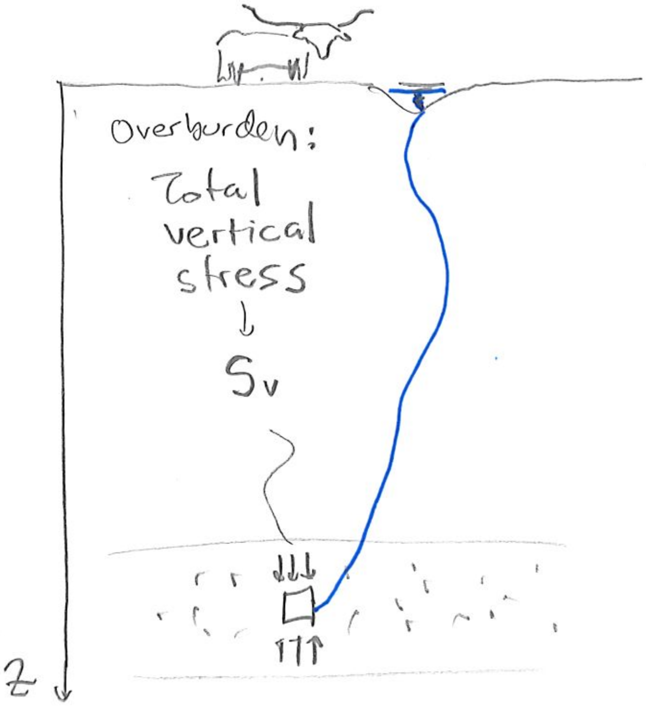
$$\boxed{\rho_w g \cong 10 \ \frac{MPa}{km}}$$

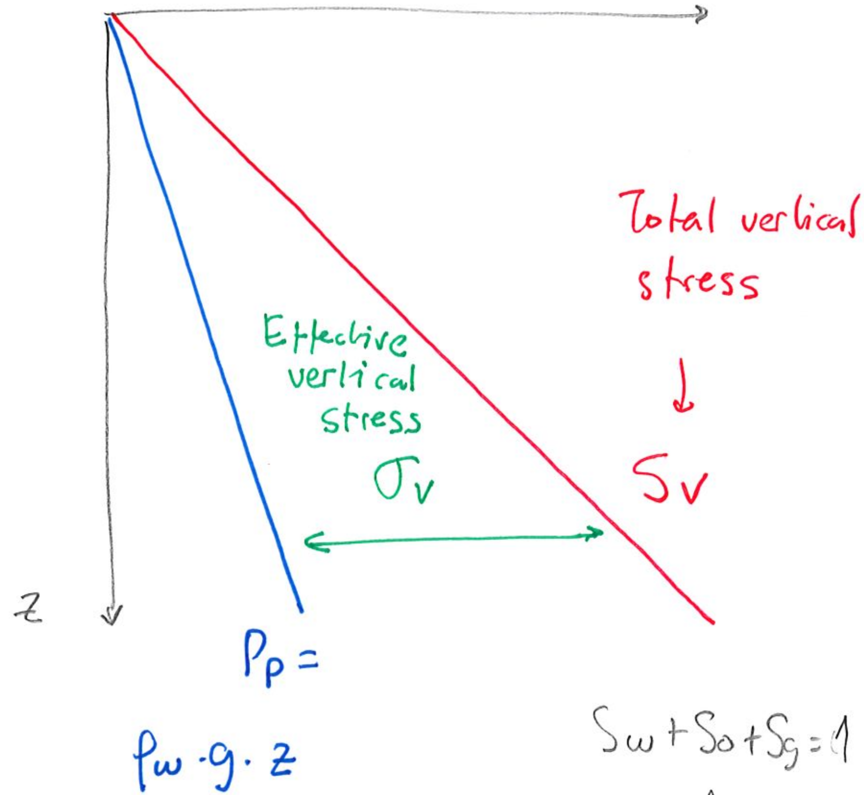
$$\rho_w = 62.4 \ \frac{lb}{ft^3}$$

$$\rho_w \cdot g = 62.4 \ \frac{\vec{lb}}{ft^3}$$

$$\rho_w \cdot g = 62.4 \ \frac{1}{(12in)^2} \ \frac{\vec{lb}}{ft}$$

$$\boxed{\rho_w \cdot g = 0.433 \ \frac{psi}{ft}}$$

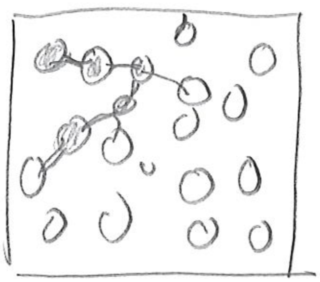




$$S_w + S_o + S_g = 1$$

$$S_v = \rho_{bulk} \cdot g \cdot z$$

$$\rho_F = \rho_w \cdot S_w + \rho_o \cdot S_o + \rho_g \cdot S_g$$



$$\rho_{bulk} = \rho_m (1-\phi) + \rho_F (\phi)$$

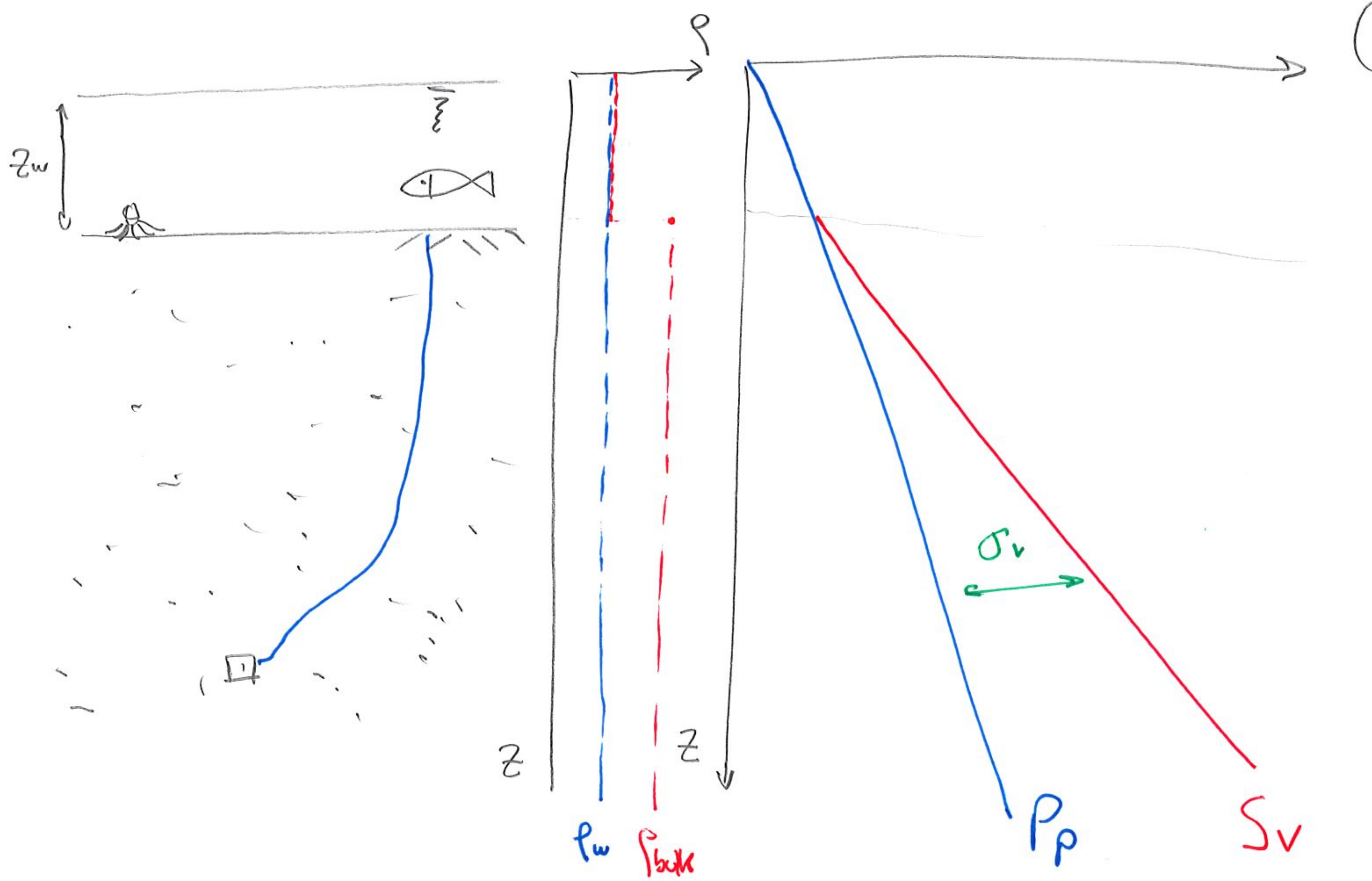
$$\rho_m = C_{m1} \rho_{m1} + C_{m2} \rho_{m2} + \dots + \rho_{mn} C_{mn}$$

$$\sum C_{mn} = 1$$

Sandstone
$\phi = 0.20$
$\rho_{quartz} = 2650 \ kg/m^3$
$\rho_{brine} = 1000 \ kg/m^3$

$\rho_{bulk} = 2320 \ \frac{kg}{m^3}$

$$\boxed{\frac{dS_v}{dz} = \rho_{bulk} \cdot g \cong 23 \ \frac{MPa}{km}}$$



$= \rho_w g z$ $\qquad$ $= \rho_{bulk} g z$

2019/1/28

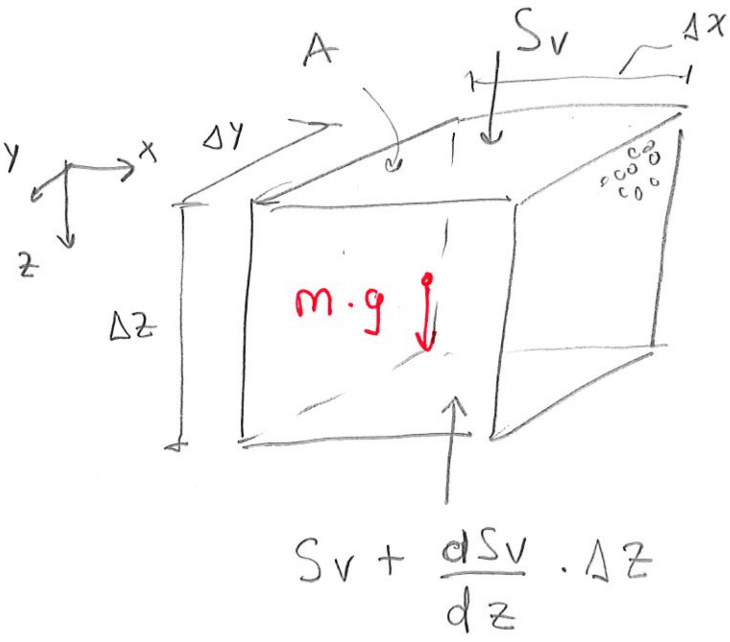
$$S_v = \underbrace{\rho_w g \cdot z_w} + \rho_{bulk} g (z - z_w)$$

Pore pressure gradient (hydrostatic) : $0.44 \frac{psi}{ft} = 10 \frac{MPa}{km}$

Total vertical stress gradient : $\underbrace{1 \frac{psi}{ft}}_{1000 \frac{kg}{m^3}} = \underbrace{23 \frac{MPa}{Km}}_{2320 \frac{Kg}{m^3}}$

lithostatic

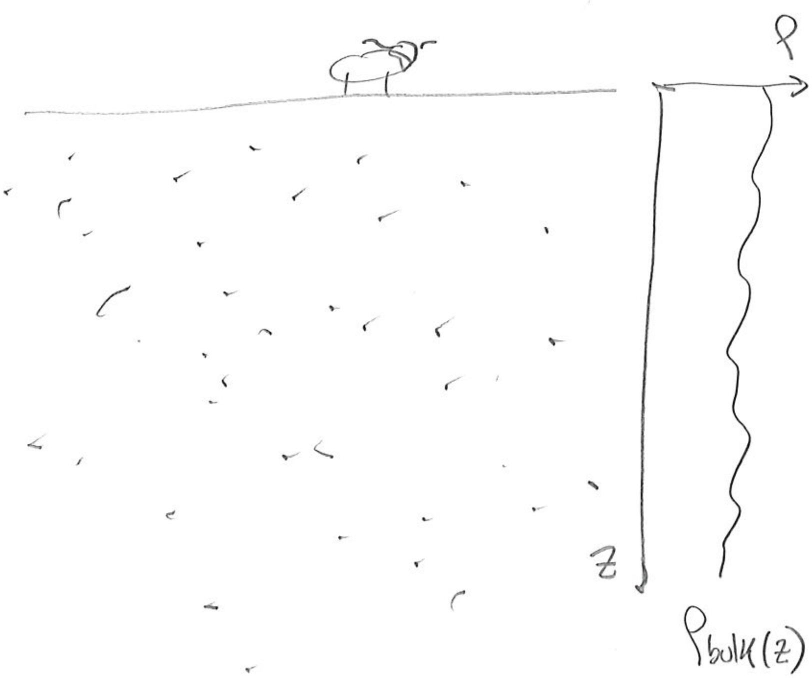
General solution for vertical stress



$$\Sigma F_z = S_v \cdot A + m \cdot g - \left(S_v + \frac{dS_v}{dz} \cdot \Delta z\right) A$$

$$= \underbrace{\cancel{S_v\,(\Delta x\,\Delta y)}} + \rho_{bulk}\,(\Delta x\,\Delta y\,\Delta z)\,g - \left(\cancel{S_v} + \frac{dS_v}{dz}\,\Delta z\right) \Delta x\,\Delta y = 0$$

$$\frac{dS_v}{dz}\,\cancel{\Delta z\,\Delta x\,\Delta y} = \rho_{bulk} \cdot g\,\cancel{\Delta x\,\Delta y\,\Delta z}$$



$$\boxed{\frac{dS_v}{dz} = \rho_{bulk}(z) \cdot g}$$

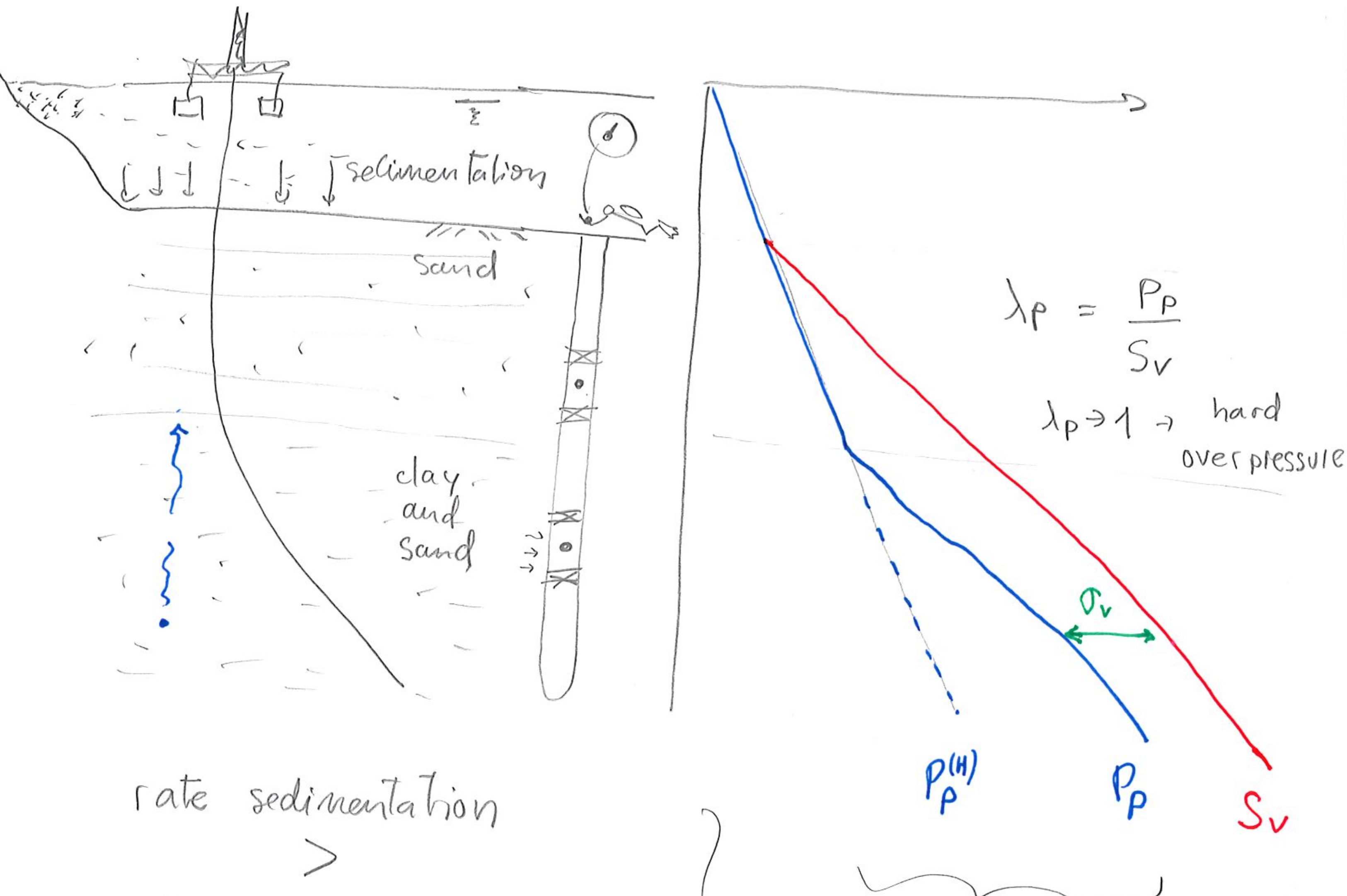
$$\int_{S_v(z=0)}^{S_v(z)} dS_v = \int_0^z \rho_{bulk}(z)\,g\,dz$$

$$S_v(z=0) = 0$$

$$\boxed{S_v(z) = \int_0^z \rho_{bulk}(z) \cdot g \cdot dz}$$

density log

Non-hydrostatic pore pressure



rate sedimentation

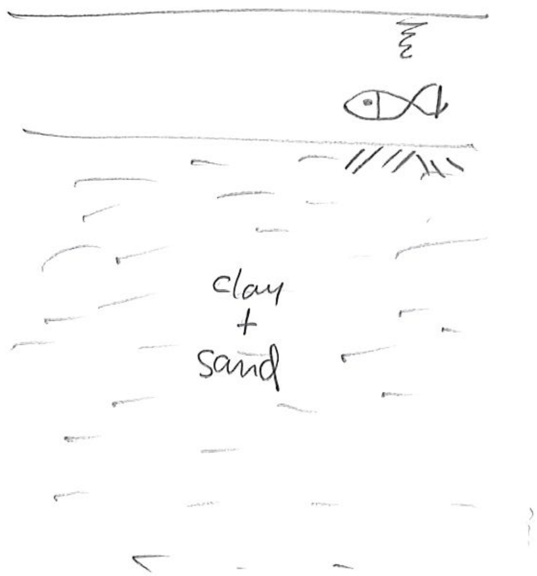
$>$

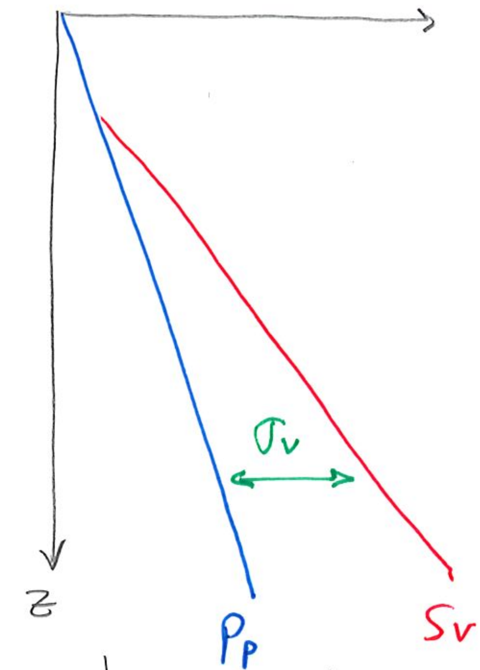
rate of pore pressure diffusion $\Rightarrow$ disequilibrium compaction

$\hookrightarrow$ overpressure

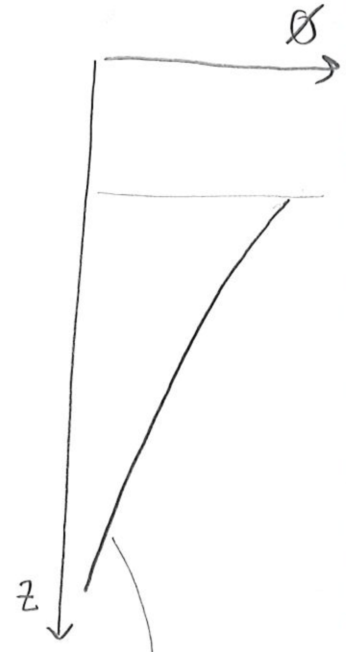
$\to$ underconsolidation

2019/1/30





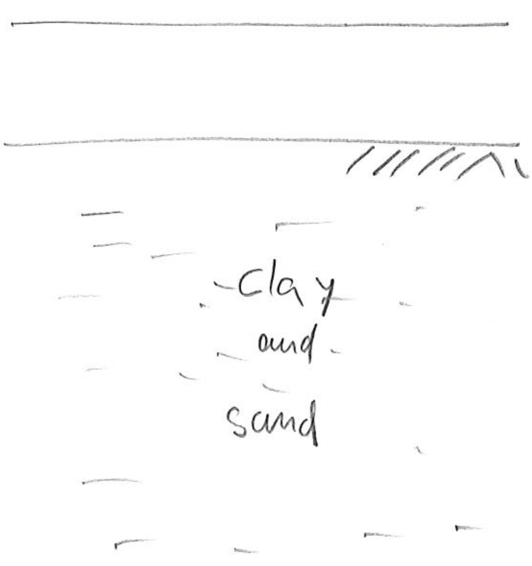
normal compaction

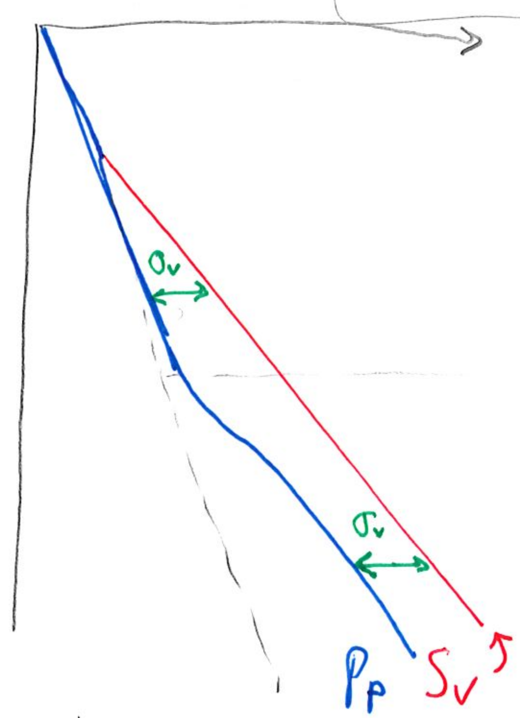


$$\boxed{\emptyset = \underline{\emptyset_0} \exp(-\underline{\beta} \cdot \sigma_v)}$$

fitting param

↳ laboratory





disequilibrium compaction

actual porosity

DPHI

NPHI

RHO

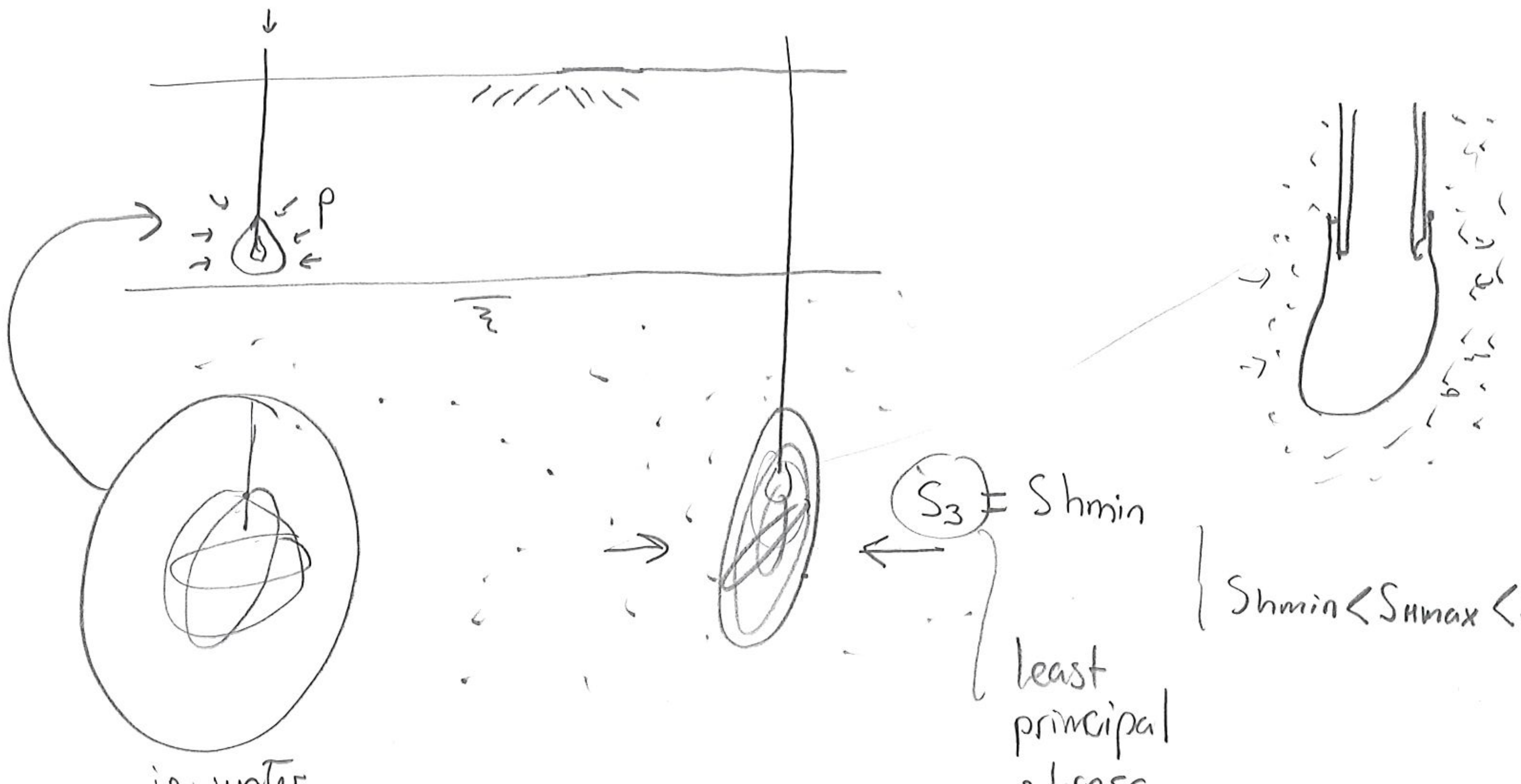
DT

algebra ($\sigma_v = S_v - P_p$)

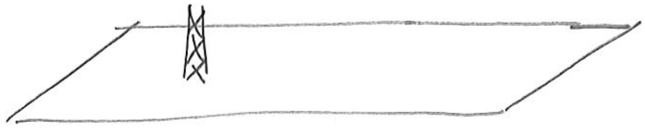
$$P_p = S_v + \underbrace{\frac{\ln(\phi/\phi_0)}{\beta}}_{<0}$$

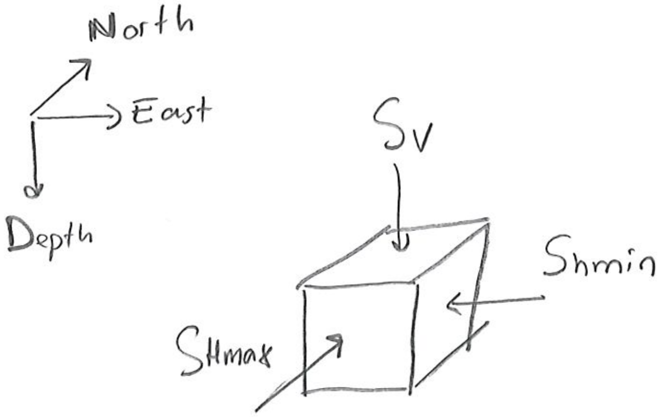
WORKFLOW:
1) Calculate S_v
2) Determine σ_v from ϕ, ϕ_0, β
3) $P_p = S_v - \sigma_v$

Horizontal stresses



$S_{hmin} < S_{Hmax} < S_v$





- 3 values
- 3 directions
- Full knowledge of stress state

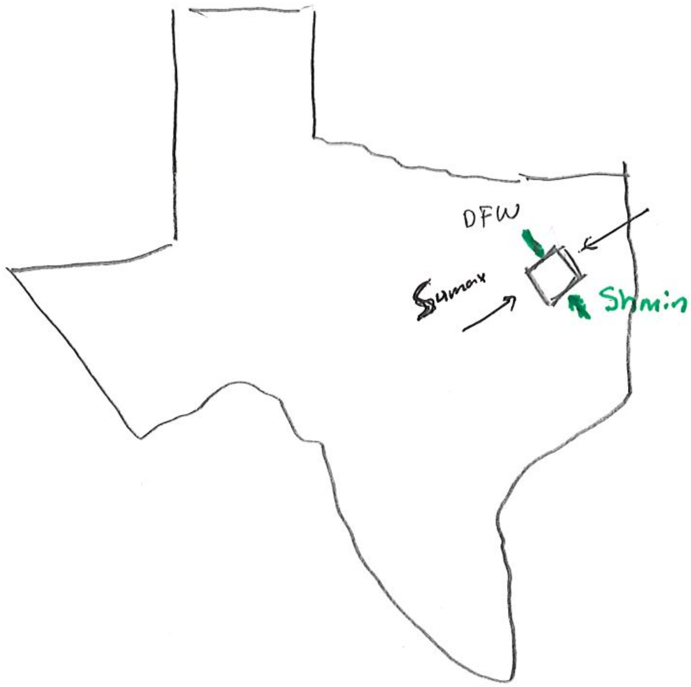
Stress regime	S_1 >	S_2 >	S_3
Normal Faulting (→ Extensional) • Permian, EF	S_v	S_{Hmax}	S_{hmin}
Strike Slip • California	S_{Hmax}	S_v	S_{hmin}
Reverse/Thrust Faulting • Argentina (Vaca Muerta) • Australia	S_{Hmax}	S_{hmin}	S_v

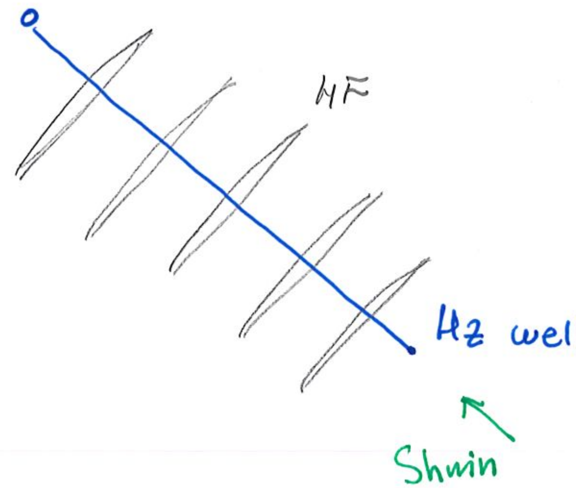
some depths

Barnett shale

Normal Faulting

$$\underbrace{S_v > S_{Hmax} > S_{hmin}}_{?}$$





2/1/2019

S_v

$$\int_0^z \rho_{bulk} \cdot g \cdot dz$$

$$\overbrace{S_{Hmax} \qquad S_{hmin}}^{\text{wellbore failure}}$$

S_{hmin}

↓

if S_3

↓

conduct hydraulic fracture test

HW2

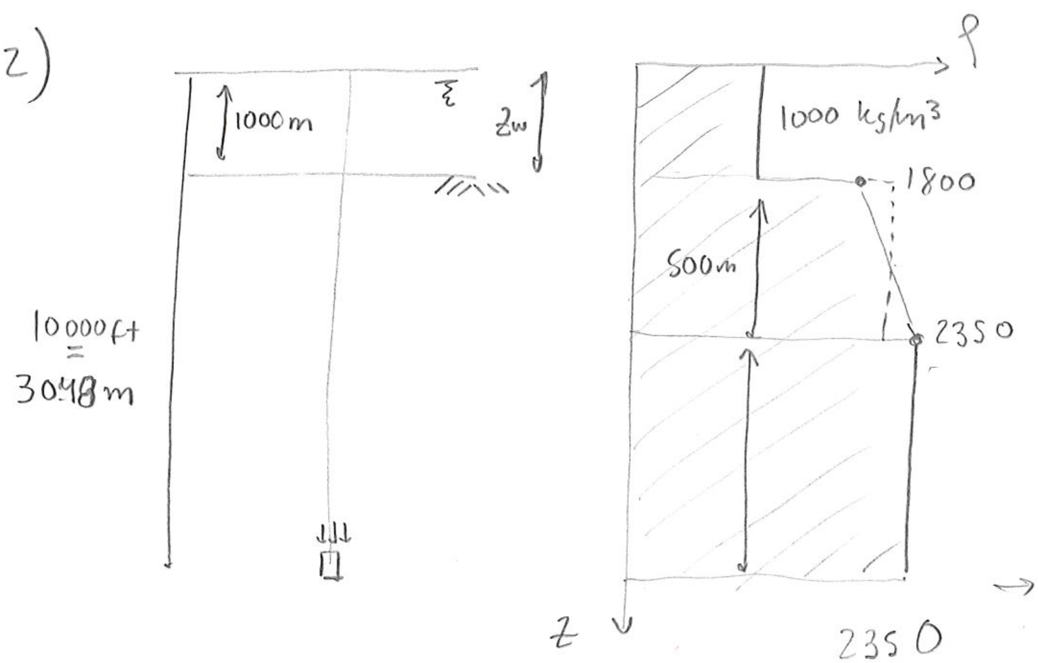
1) $1\ g/cm^3 = 1\ g/cc = 1000\ Kg/m^3$

$\rho_{bulk} = \_\_\_\_\_\_\_\_$

$\rho_{bulk} \cdot g =$ | $\sim 26\ MPa/Km$
$\sim 1.15\ psi/ft$
$\sim 22\ ppg$

$0.44\ \frac{psi}{ft} = 8.3\ ppg$

2)



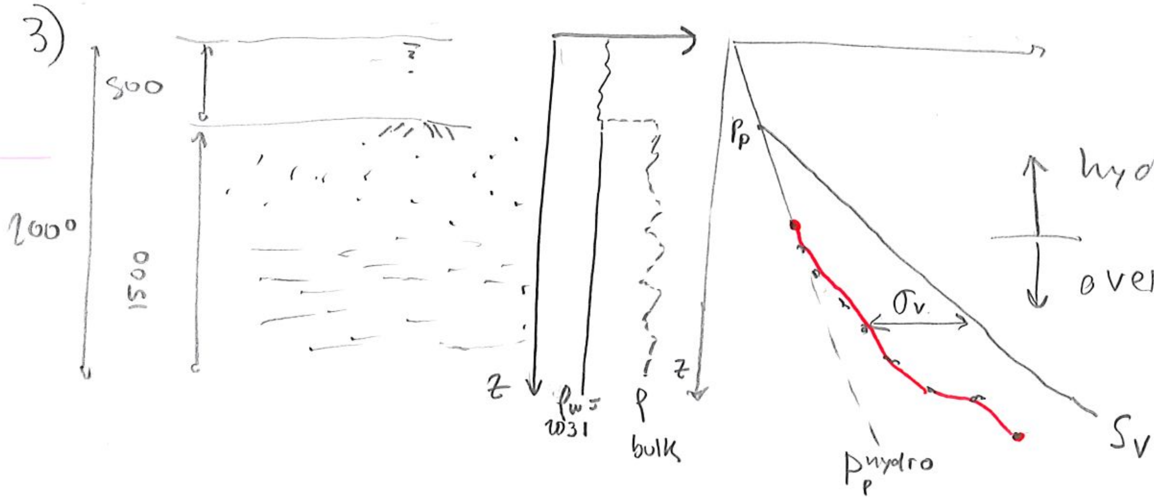
$3000\ psi = 20.6\ MPa$

$1\ MPa = 145\ psi$

$\rightarrow S_v = \int_0^z \rho_{bulk} \cdot g \cdot dz$

$S_v \cong 56\ MPa$

3)



↑ hydrostatic

↓ overpressure

$$\lambda_p = \frac{P_p}{S_v} \ll 1$$

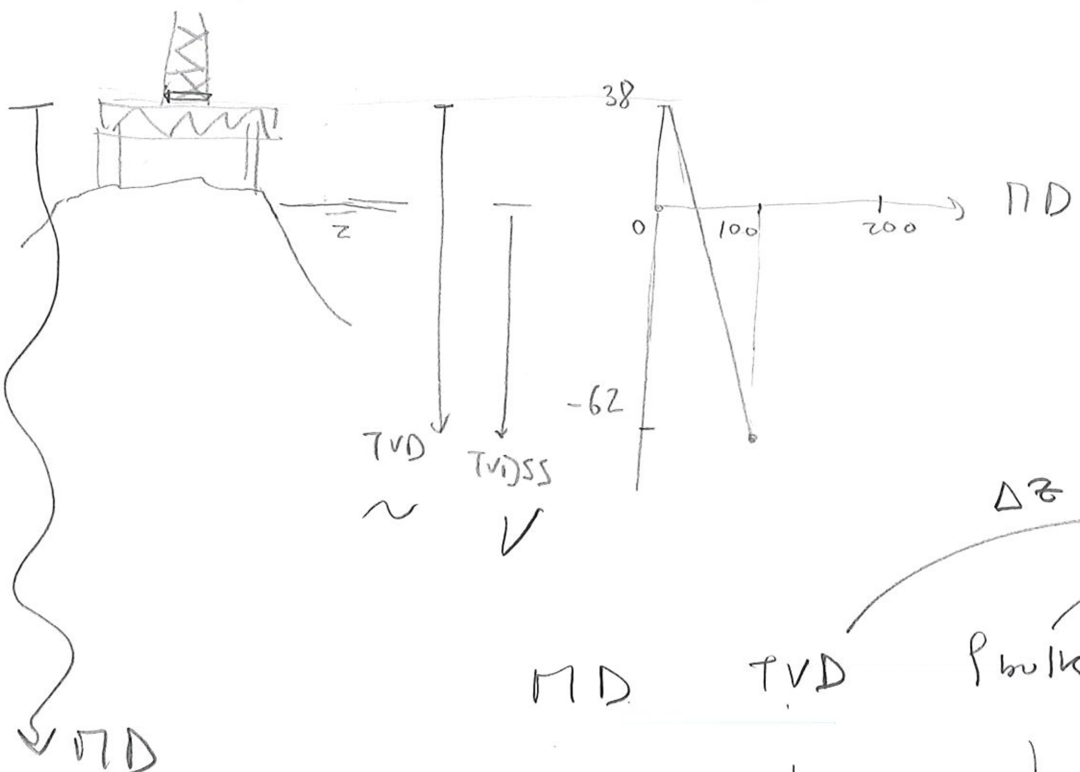
1) Hypothesis: hydrostatic

2) Shale porosity → actual pore pressure

4) Plaquemines Parish, LA

.LAS | EKB$^{\text{bushing}}$ 36 ft

data 1864 - 7551 ft

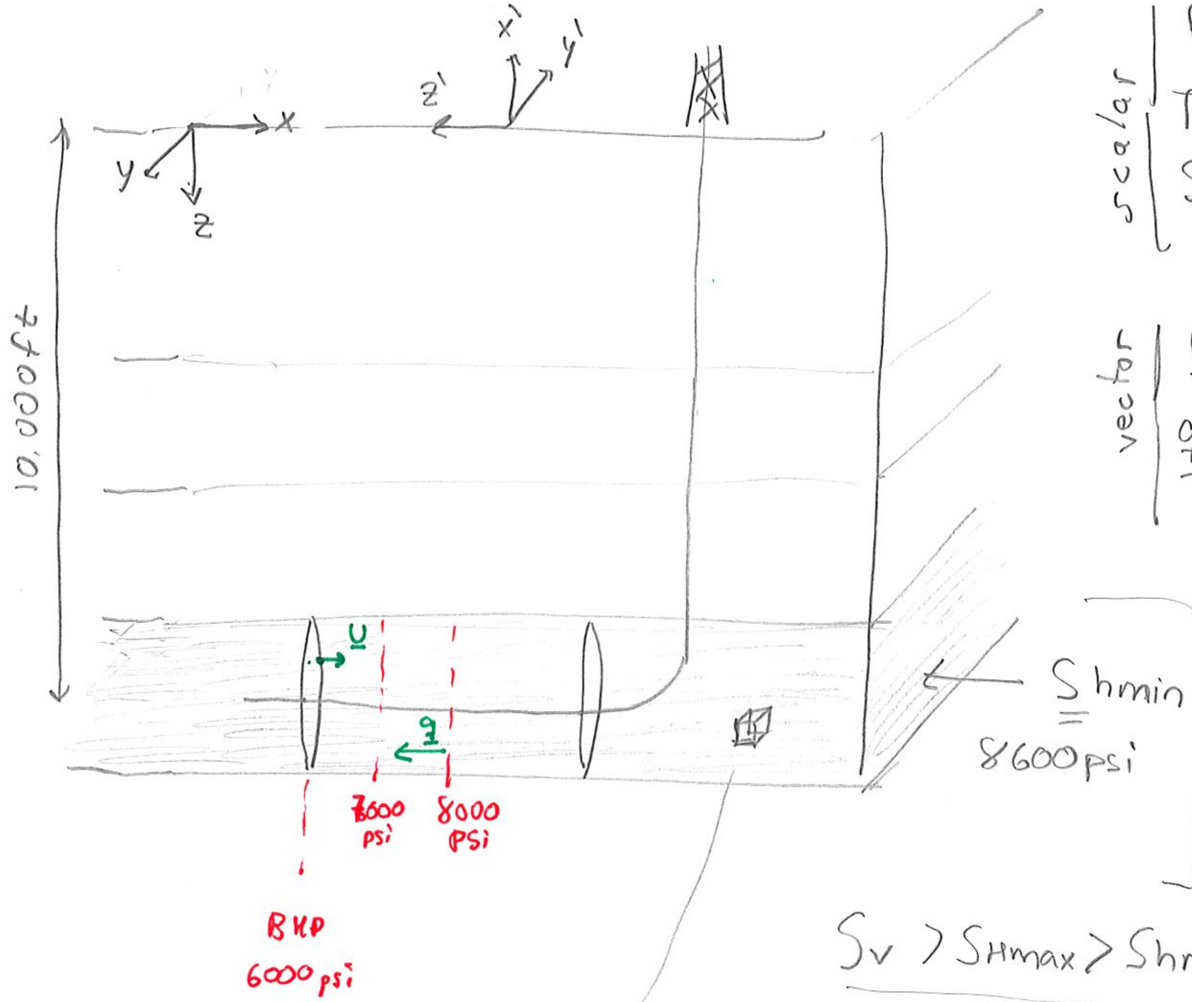


DEV

Δz $\bar{\rho}$

MD TVD ρ_{bulk} $\underline{S_v}$

2/4/2019



scalar

P = 8000 psi

T = 200°F

S_o = 0.7

vector

$\underline{U}$ = (2 mm, 0 mm, 0 mm)

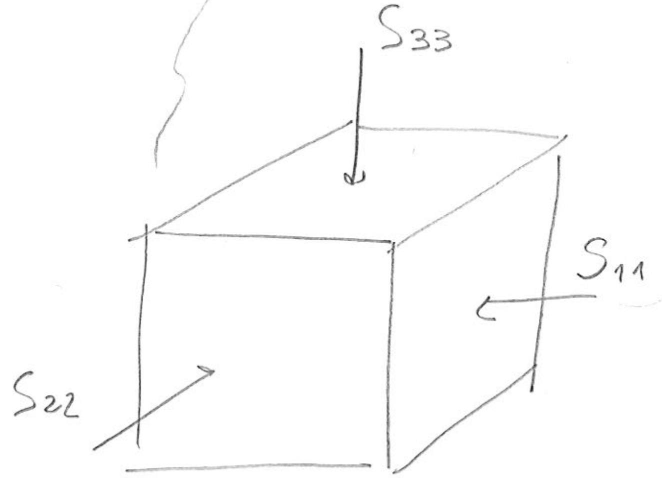
$\underline{q}$ = $\left(1 \frac{ft}{day}, 0 \frac{ft}{day}, 0 \frac{ft}{day}\right)$

tensor

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$S_v > S_{Hmax} > S_{hmin}$

$$\underline{\underline{S}} = \begin{bmatrix} S_{hmin} & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_v \end{bmatrix}$$

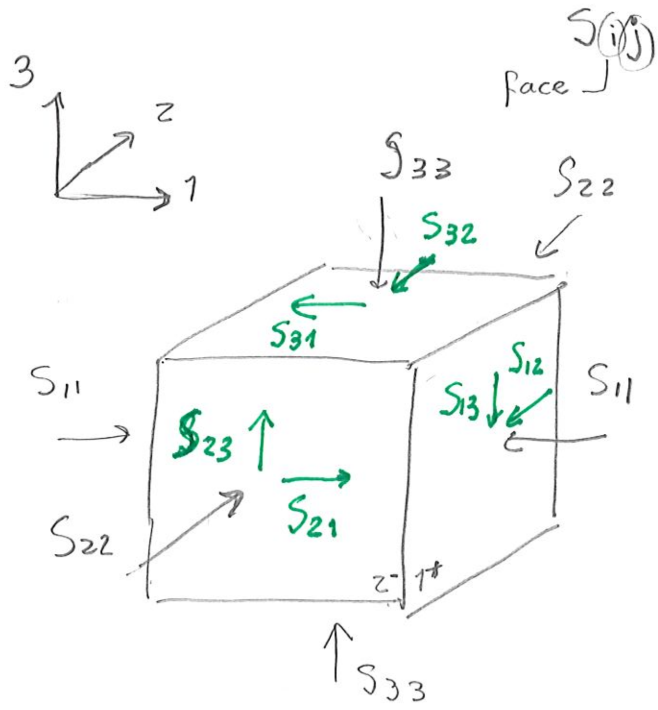


$$\underline{\underline{S}} = \underbrace{\begin{bmatrix} 8600 & 0 & 0 \\ 0 & 9000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}}_{\text{principal stresses} \equiv \text{eigen values}} \text{psi}$$

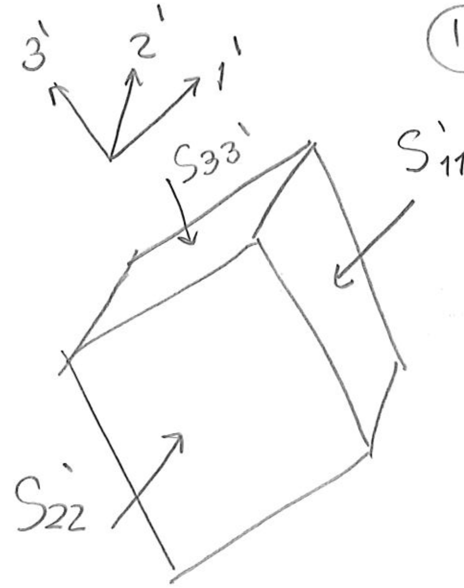
1 → x

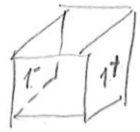
2 → y

3 → z



principal stresses & directions ⇒





normal stress positive → compression

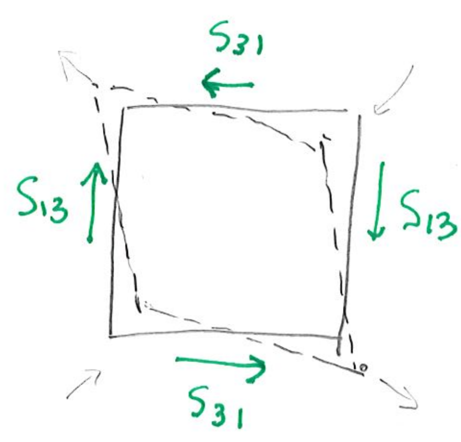
all shear stresses are zero

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \xrightarrow{\text{eig}} \underline{\underline{S}}' = \begin{bmatrix} S'_{11} & 0 & 0 \\ 0 & S'_{22} & 0 \\ 0 & 0 & S'_{33} \end{bmatrix}$$

symmetric

$$\underline{\underline{S}}' = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$S_1 \geq S_2 \geq S_3$$



Mohr circle

Eigenvalues → eig(_)

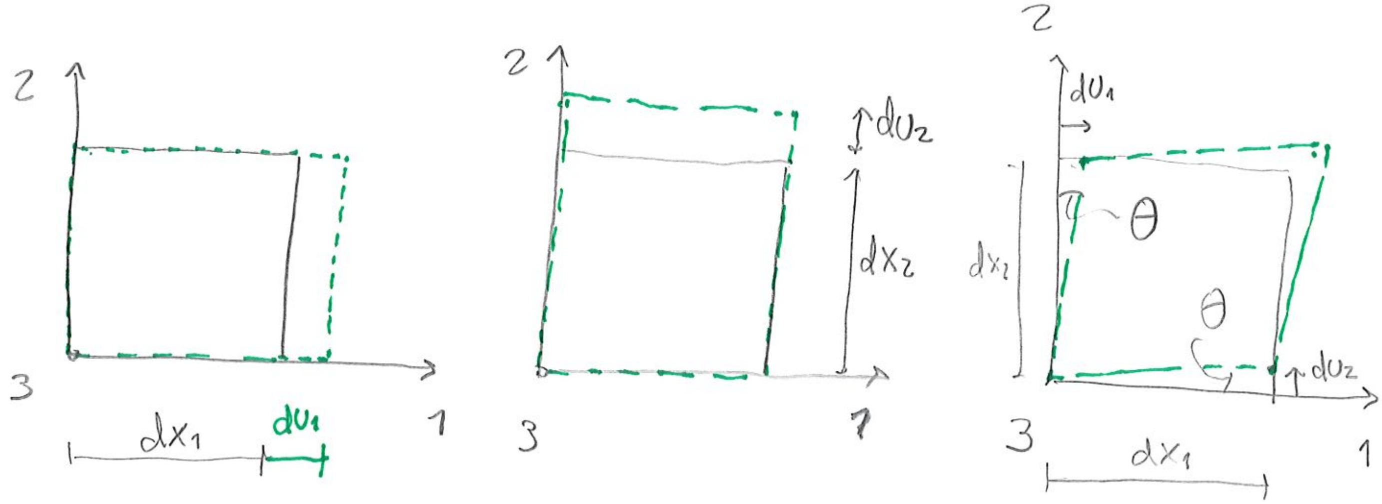
$S_{31} = S_{13}$ → angular momentum Equilibrium

$$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \, \underline{\underline{I}}$$

$$\overbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{bmatrix}}^{\text{Effective stress}} = \overbrace{\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ & S_{22} & S_{23} \\ & & S_{33} \end{bmatrix}}^{\text{Total stress}} - \overbrace{\begin{bmatrix} P_p & 0 & 0 \\ & P_p & 0 \\ & & P_p \end{bmatrix}}^{\text{Pore pressure}}$$

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11}-P_p & S_{12} & S_{13} \\ S_{21} & S_{22}-P_p & S_{23} \\ S_{31} & S_{32} & S_{33}-P_p \end{bmatrix}}_{\text{Effective stress}}$$

Strains (deformation)



$$\varepsilon_{11} = \frac{dU_1}{dX_1} \quad ; \quad \varepsilon_{22} = \frac{dU_2}{dX_2} \quad ; \quad \varepsilon_{12} = \frac{1}{2}\left(\frac{dU_1}{dX_2} + \frac{dU_2}{dX_1}\right)$$

$$\underbrace{\varepsilon_{33} = \frac{dU_3}{dX_3}}_{\text{linear strains}}$$

$\hookrightarrow \Delta Vol$

$$\underbrace{\tan\theta = \frac{dU_1}{dX_2} = \frac{dU_2}{dX_1}}_{\text{shear strain}}$$

↓

no volume change

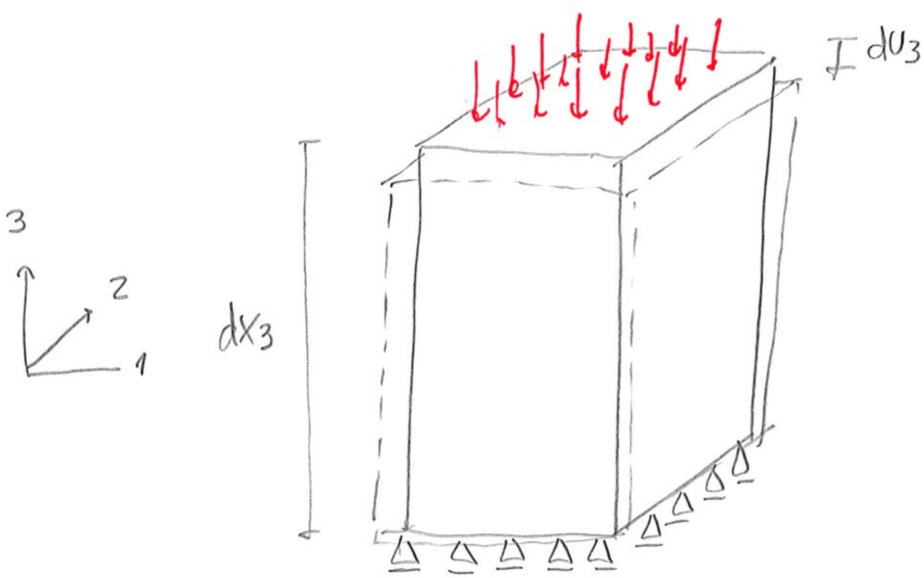
strain tensor

↓

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \frac{dU_1}{dX_1} & \frac{1}{2}\left(\frac{dU_1}{dX_2} + \frac{dU_2}{dX_1}\right) & \frac{1}{2}\left(\frac{dU_1}{dX_3} + \frac{dU_3}{dX_1}\right) \\ & \frac{dU_2}{dX_2} & \frac{1}{2}\left(\frac{dU_2}{dX_3} + \frac{dU_3}{dX_2}\right) \\ & & \frac{dU_3}{dX_3} \end{bmatrix}$$

$$\left.\begin{array}{l} \underline{\underline{\sigma}} = f(\underline{\underline{\varepsilon}}) \\ \\ \underline{\underline{\varepsilon}} = f(\underline{\underline{\sigma}}) \end{array}\right\} \; ?$$

Linear Elasticity



$$\varepsilon_{33} = \frac{dU_3}{dX_3}$$

$$\boxed{E = \frac{\sigma_{33}}{\varepsilon_{33}}}$$ Young's Modulus

$\hookrightarrow \sigma_{33} = E \cdot \varepsilon_{33}$

$\hookrightarrow \varepsilon_{33} = \dfrac{\sigma_{33}}{E}$

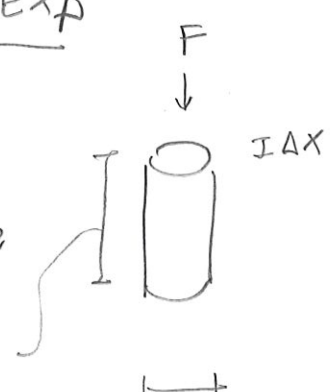
$$\boxed{\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}} = -\frac{\varepsilon_{22}}{\varepsilon_{33}}}$$

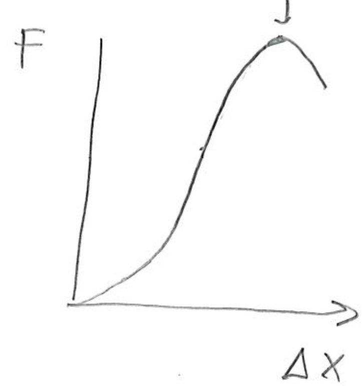
Poisson's ratio

Unconfined Loading → in one direction

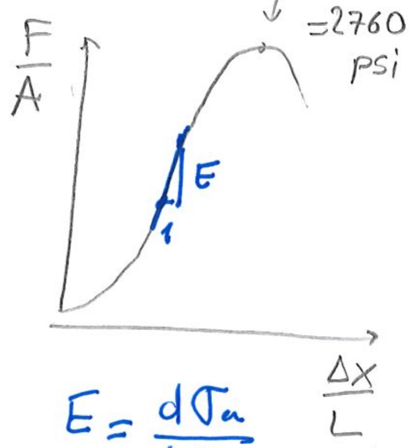
Class Exp

Berea Sandstone





⇒



$L = 0.8\text{ in}$

$D = 0.38\text{ in}$

$A = 0.11\text{ in}^2$

1 turn = 0.15 mm = 0.006 in

$$E = \frac{d\sigma_a}{d\varepsilon_a}$$

$E = 94 \cdot 10^3$ psi $\rightarrow$ H

$262 \cdot 10^3$ psi $\rightarrow$ K, M ✓

$8.9 \cdot 10^4$ MPa $\rightarrow$ B

$\Delta F = 110$ lb

$\downarrow$ /A

$\Delta\sigma = 1000$ psi

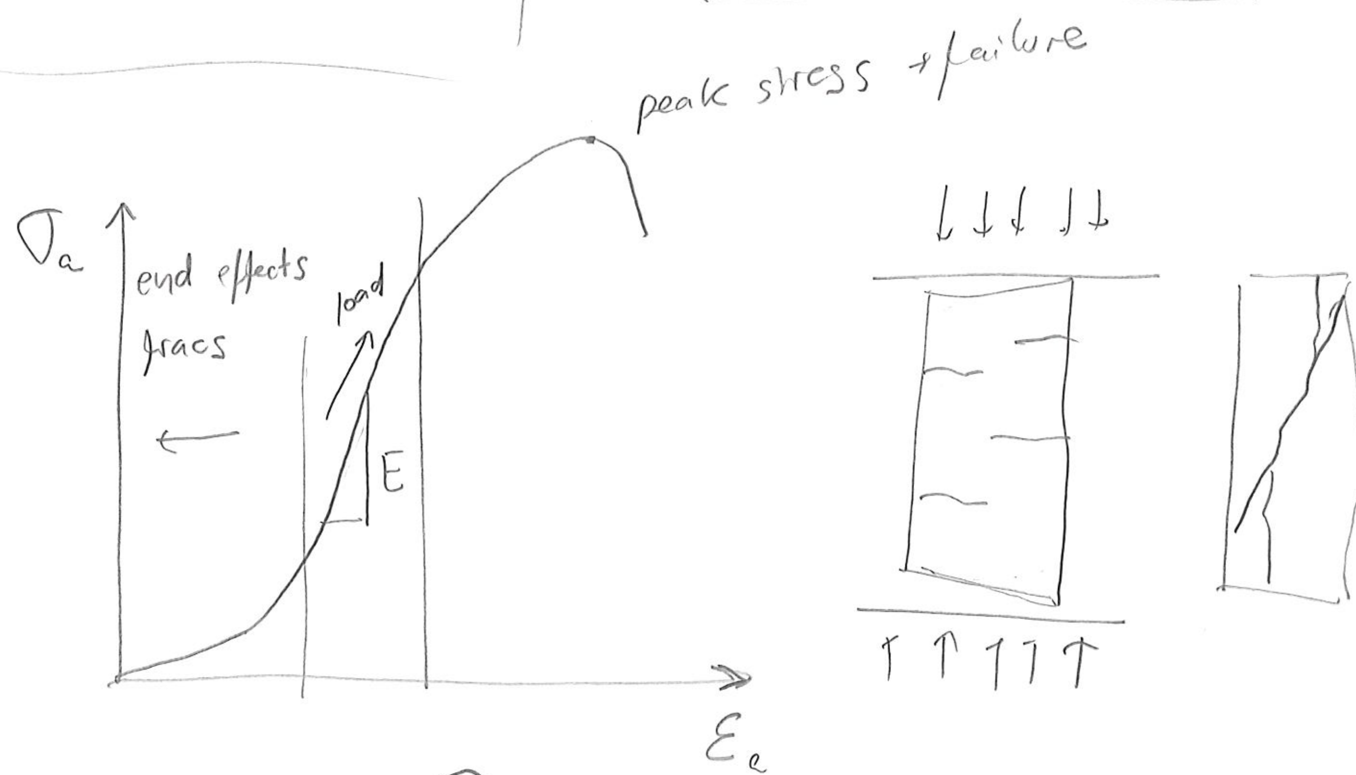
10 kg	0 turn
50 kg	0.75
100 kg	1.25
138 kg	broke

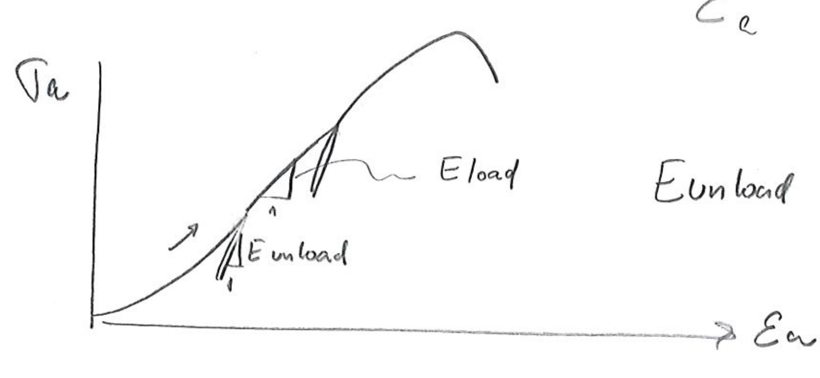
0.003/0.8

$\Delta X = 0.003$ in $\Rightarrow \varepsilon_a = 3.75 \cdot 10^{-3}$

$\boxed{E = 266 \cdot 10^3 \text{ psi}}$

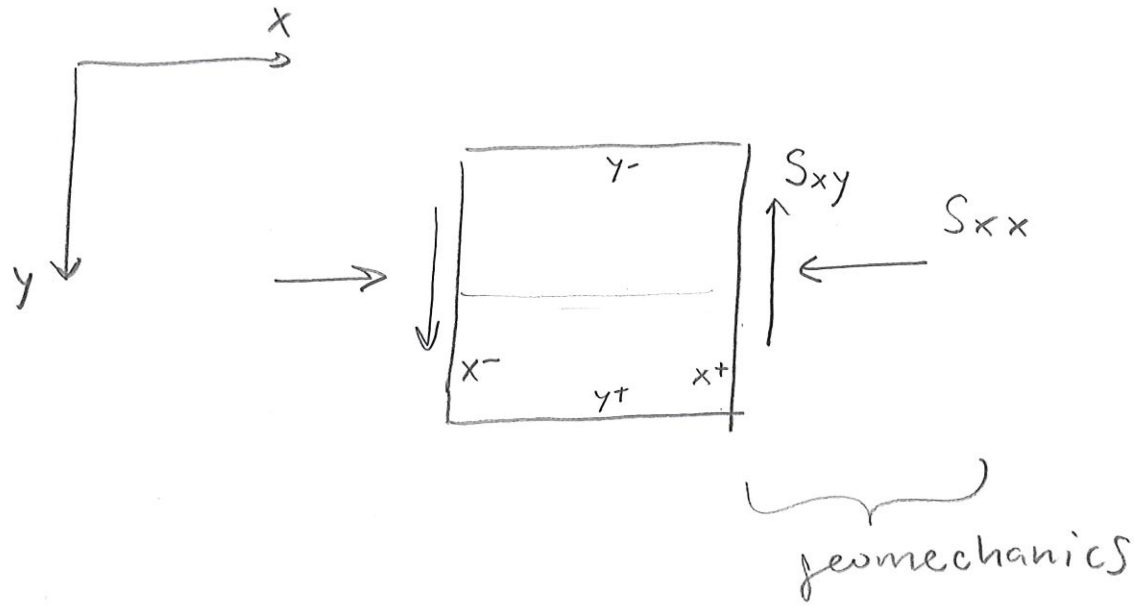
$\boxed{E = 1.84 \cdot 10^9 \text{ Pa}}$





Eunload $\geq$ Eload

HW3



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$$\begin{cases} \varepsilon_{11} = \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} & \leftarrow \varepsilon_{11} = -\nu\,\varepsilon_{33} \\ \varepsilon_{22} = -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ \varepsilon_{33} = -\frac{\nu}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{cases}$$

$$\begin{cases} 2\cdot\varepsilon_{12} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{12} \\ 2\cdot\varepsilon_{13} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{13} \\ 2\cdot\varepsilon_{23} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{23} \end{cases}$$

G: shear modulus, $G = \frac{E}{2(1+\nu)}$

$$\underbrace{\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}}_{6\times1} = \underbrace{\left[\begin{array}{ccc|ccc} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{array}\right]}_{6\times6} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}_{6\times1}$$

compliance matrix

$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{D}}}} \cdot \underline{\underline{\sigma}} \Longrightarrow \underset{\text{vector}}{\underline{\varepsilon}} = \underset{\text{matrix}}{\underline{D}} \cdot \underset{\text{vector}}{\underline{\sigma}}$$

VOIGT NOTATION

$$\underline{\underline{\sigma}} = \underbrace{\underline{\underline{C}}}_{\text{stiffness matrix}} \underline{\underline{\varepsilon}}$$

$$\underline{\underline{\sigma}} = \underline{\underline{D}}^{-1} \underline{\underline{\varepsilon}}$$

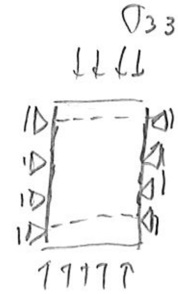
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \left[\begin{array}{ccc|ccc} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{array}\right] \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \hline 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$

Uniaxial-strain stress path

no tectonic strains $\left\{ \begin{array}{l} \varepsilon_{11} = \varepsilon_{22} = \emptyset \quad\quad \varepsilon_{ij} = 0 \quad i \neq j \\ \varepsilon_{33} \neq 0 \end{array} \right.$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \varepsilon_{33} \end{bmatrix}$$

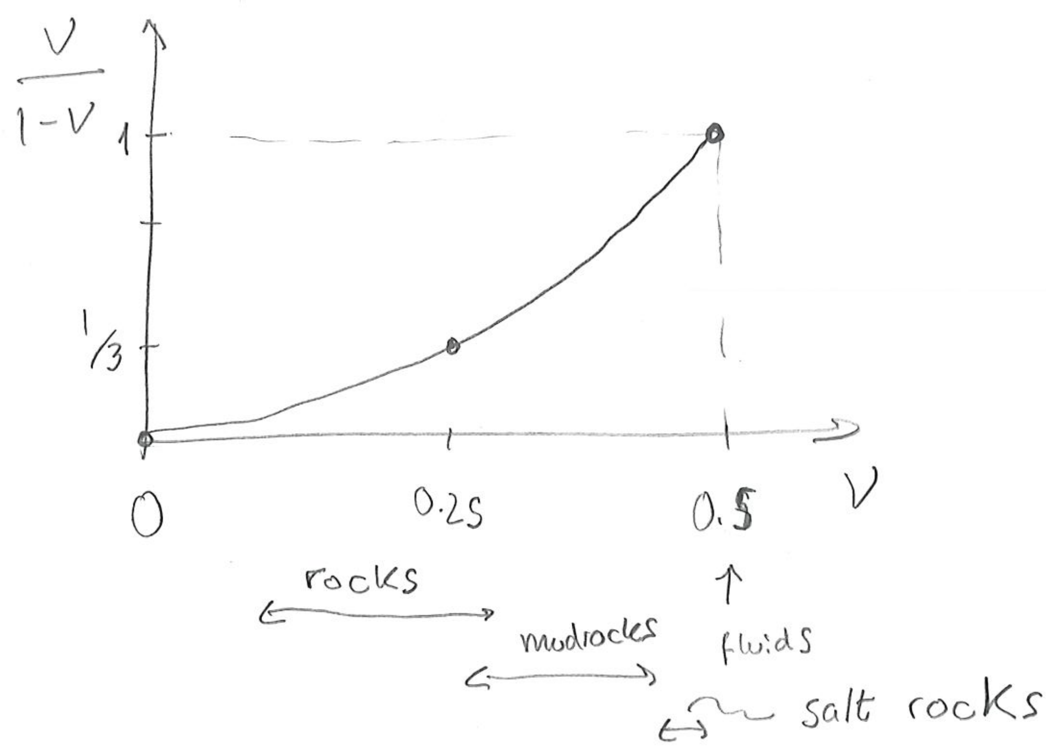
$$\begin{cases} \sigma_{33} = \overbrace{\boxed{\dfrac{(1-\nu)\,E}{(1+\nu)(1-2\nu)}}}^{M:\ \text{constrained modulus}} \cdot \varepsilon_{33} \\[2ex] \sigma_{11} = \sigma_{22} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\,\varepsilon_{33} = \dfrac{\nu \cancel{E}}{\cancel{(1+\nu)(1-2\nu)}}\,\dfrac{\cancel{(1+\nu)(1-2\nu)}}{(1-\nu)\cancel{E}}\,\sigma_{33} \end{cases}$$



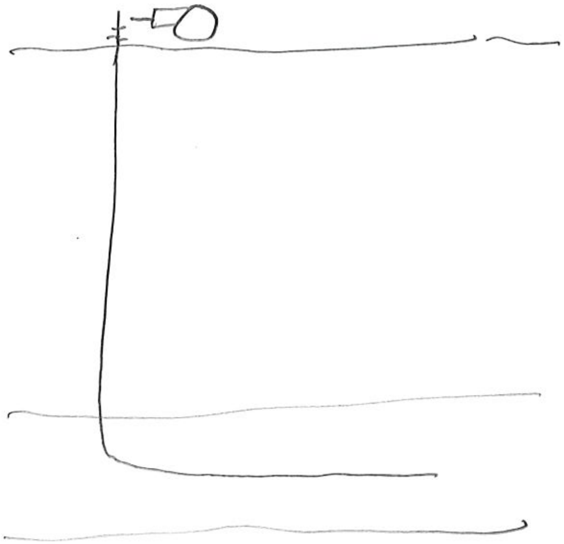
$M > E$

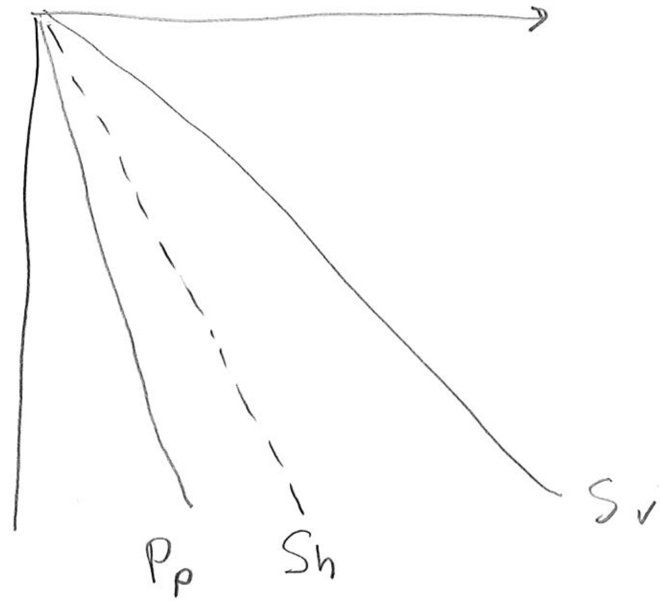
$$\underbrace{\boxed{\sigma_{22} = \frac{\nu}{1-\nu}\,\sigma_{33}}}_{\text{lateral effective stress coefficient}}$$

→ valid for effective stresses



$S_v \geq S_{Hmax} > S_{hmin}$





$$S_{hmin} = S_{Hmax} = S_h$$

$$\underbrace{S_h}_{\text{Total}} = \underbrace{\sigma_h}_{\text{effective}} + P_p$$

$$S_h = \frac{\nu}{1-\nu}\sigma_v + P_p$$

$$S_h = \frac{\nu}{1-\nu}\left(S_v - P_p\right) + P_p$$

absolute values

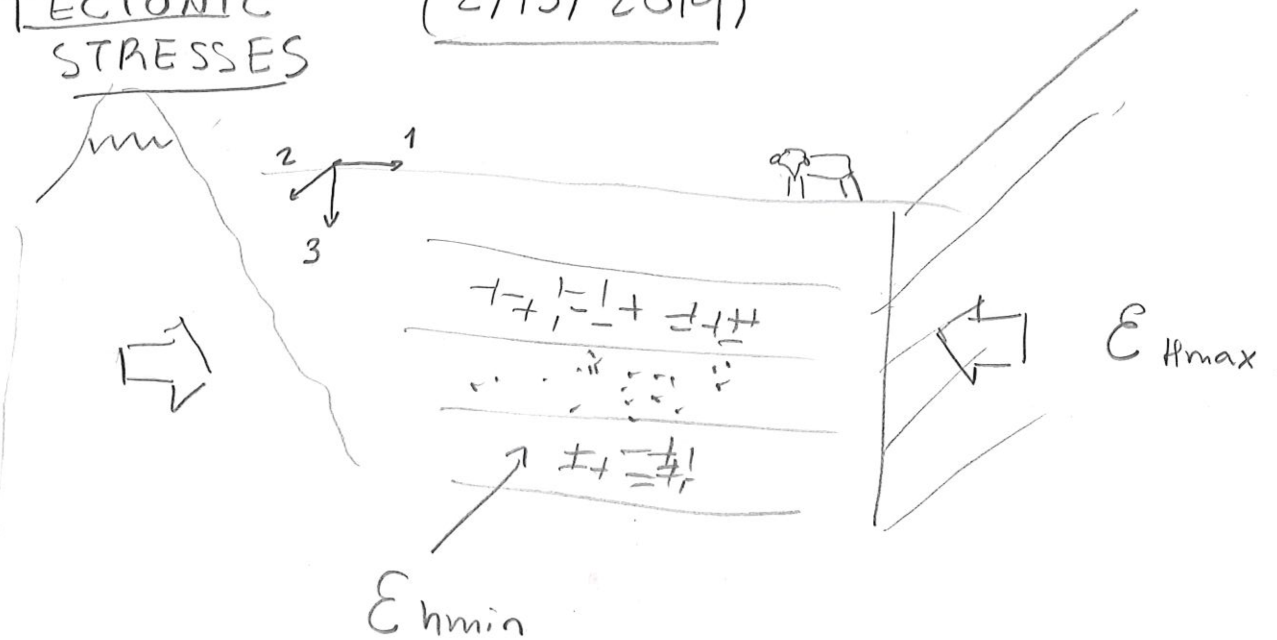
$$S_h = \frac{\nu}{1-\nu} S_v + \frac{1-2\nu}{1-\nu} P_p$$

gradient

$$\underbrace{\frac{\Delta S_h}{\Delta z}}_{\text{frac gradient}} = \frac{\nu}{1-\nu}\underbrace{\frac{\Delta S_v}{\Delta z}}_{\text{lithostatic gradient}} + \frac{1-2\nu}{1-\nu}\underbrace{\frac{\Delta P_p}{\Delta z}}_{\text{pore pressure gradient}}$$

TECTONIC STRESSES

(2/13/2019)



E': plane strain modulus

$$
\begin{cases}
\sigma_{11} = \dfrac{E}{1-\nu^2}\,\varepsilon_{11} + \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{22} + \dfrac{\nu}{1-\nu}\,\sigma_{33} \\[2ex]
\sigma_{22} = \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{11} + \dfrac{E}{1-\nu^2}\,\varepsilon_{22} + \dfrac{\nu}{1-\nu}\,\sigma_{33} \\[2ex]
\sigma_{33} = \displaystyle\int_0^z \rho_{bulk}\, g\, dz - P_p
\end{cases}
$$

$$\frac{0.25}{1-0.25} = \underline{\underline{0.333}}$$

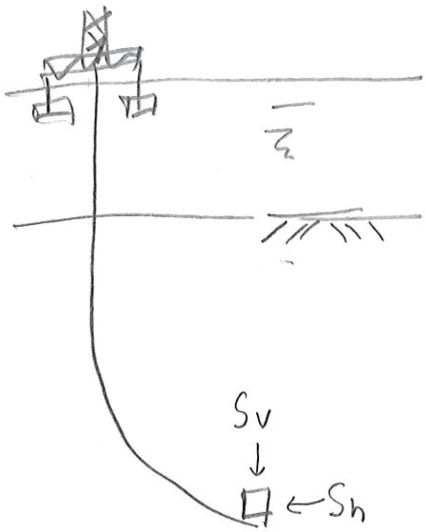
$$
\begin{cases}
\sigma_{Hmax} = \dfrac{E}{1-\nu^2}\,\varepsilon_{Hmax} + \dfrac{\nu E}{1-\nu^2}\,\varepsilon_{hmin} + \dfrac{\nu}{1-\nu}\,\sigma_{33} \\[2ex]
\sigma_{hmin} = \dfrac{\nu E}{1-\nu}\,\varepsilon_{Hmax} + \dfrac{E}{1-\nu^2}\,\varepsilon_{hmin} + \dfrac{\nu}{1-\nu}\,\sigma_{33}
\end{cases}
$$

Measure
↓
- wellbore failure
- hyd frac test

Well logging surveys (Δt_p, Δt_s)
↕
laboratory test

Tectonic strains → calibrate

General procedure to calculate total Hz stress with linear elasticity



① S_v (Total vertical stress)

② P_p — hydrostatic / non hydrostatic — $\lambda_p = \frac{P_p}{S_v}$ / ϕ_{shale}

③ $\sigma_v = S_v - P_p$

④ Elasticity

- $\varepsilon_h = 0 \rightarrow \sigma_h = \frac{\nu}{1-\nu}\sigma_v$
- $\varepsilon_{hmin} \neq$, $\varepsilon_{Hmax} \neq 0 \rightarrow$ $\sigma_{Hmax} = \ldots$, $\sigma_{hmin} = \ldots$

⑤

$$\underbrace{S_{hmin}}_{\text{total}} = \underbrace{\sigma_{hmin}}_{\text{effective}} + P_p$$

$$S_{Hmax} = \sigma_{Hmax} + P_p$$

total effective

Reservoir Engineering

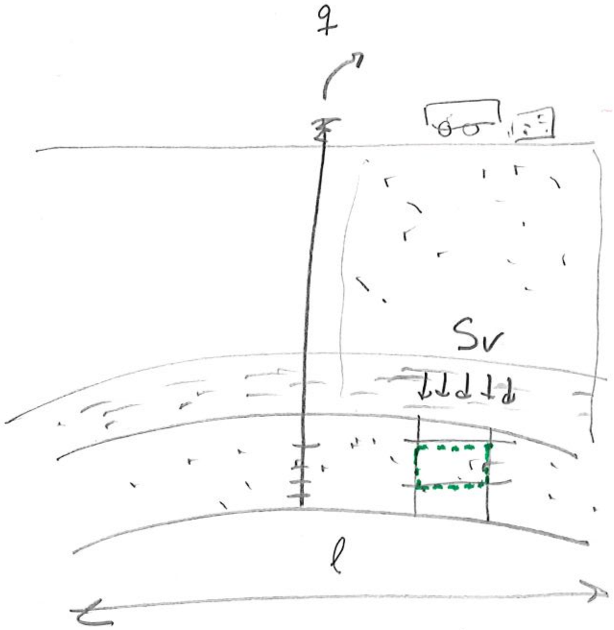
$$\frac{\partial P}{\partial t} = \frac{k}{\mu C_t}\frac{\partial^2 P}{\partial x^2}$$

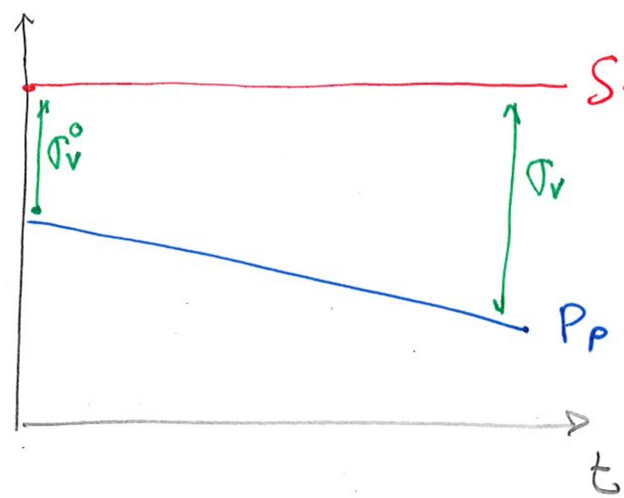
$\mu : 10^{-6}$
M (Mega) $: 10^{6}$
k (kilo) $: 10^{3}$

Total compressibility

$$C_t = C_g S_g + C_w S_w + C_o S_o + C_f$$

rock compressibility





$h << l$

Typical values of C_{bp}

$[1 - 20\ \mu sips]$
↳ micro compress

Pore (rock) compressibility $\boxed{C_f = C_{pp}} = \frac{1}{V_p}\frac{dV_p}{dP_p}$

V_p : pore volume
V_b : bulk volume

Bulk rock compressibility $C_{bp} = \frac{1}{V_b}\frac{dV_b}{dP_p}$

$V_p = V_b$ (assumption)

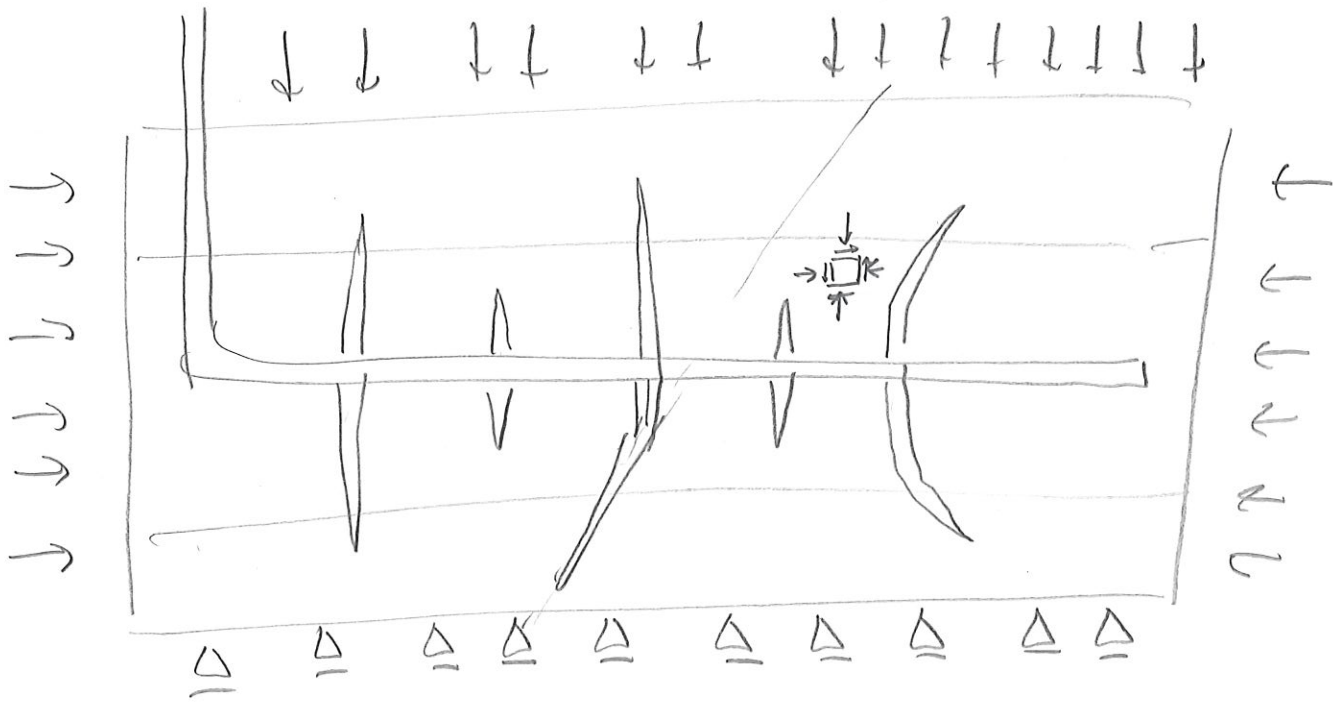
$C_{bp} = \frac{1}{M}$

$M = [psi, MPa]$

$$C_{pp} = \frac{1}{\frac{V_p}{V_b}}\left(\frac{1}{V_b}\frac{dV_b}{dP_p}\right) = \frac{C_{bp}}{\emptyset} = \underbrace{\frac{1}{\emptyset M}}$$

$C_{bp} =$
$[1/MPa]$
$[10^{-6} psi^{-1}]$
↓
μsip

General solution for a continuum mechanics problem



→ **Fluid flow problem**

- Darcy :

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = -\frac{K}{\mu}\begin{bmatrix} \partial P/\partial x_1 \\ \partial P/\partial x_2 \\ \partial P/\partial x_3 \end{bmatrix}$$

- Mass conservation :

$$\frac{\partial q_1}{\partial x_1} + \frac{\partial q_2}{\partial x_2} + \frac{\partial q_3}{\partial x_3} = 0$$

$$-\frac{K}{\mu}\left(\frac{\partial^2 P}{\partial x_1^2} + \frac{\partial^2 P}{\partial x_2^2} + \frac{\partial^2 P}{\partial x_3}\right) = 0$$

→ **Mechanics**

- Elasticity (Linear) : $\sigma \leftrightarrow \varepsilon$
- Momentum conservation

$$(\lambda+\mu)\,\nabla\cdot(\nabla\cdot \underline{U}) + \mu\,\nabla^2(\underline{U}) + \underline{b} = 0$$

∇ : $\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$

∇^2 : $\left(\frac{\partial^2}{\partial x_1^2}, \frac{\partial^2}{\partial x_2^2}, \frac{\partial^2}{\partial x_3^2}\right)$

$\underline{b}$: acceleration (gravity)

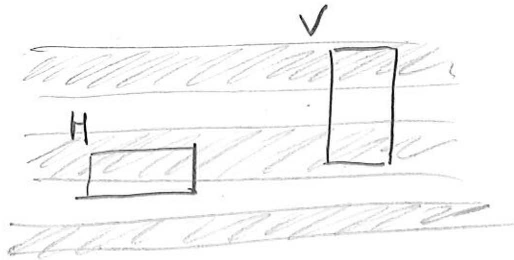
$\underline{U}$: displacement vector → strain

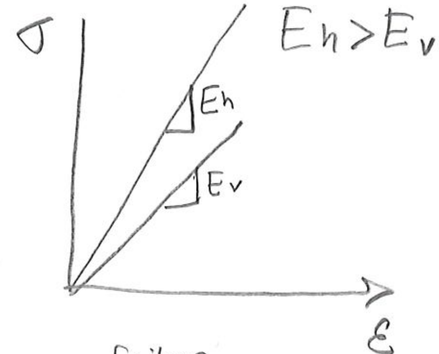
Lamé parameters $\begin{cases} \mu = G \\ \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \end{cases}$

Solutions
- Analytical
  - Kirsch (wellbore)
  - Griffith (fracture)
- Numerical → Hydraulic Fracturing simulators
  - Finite Differences → CMG
  - Finite Element Method

Real rocks

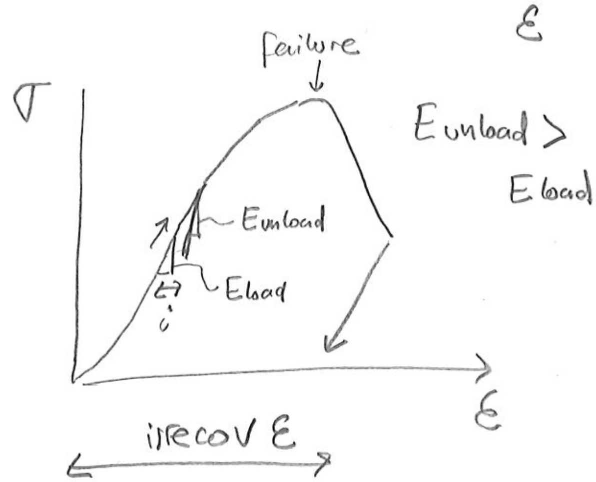
- anisotropy





- elasto-plasticity

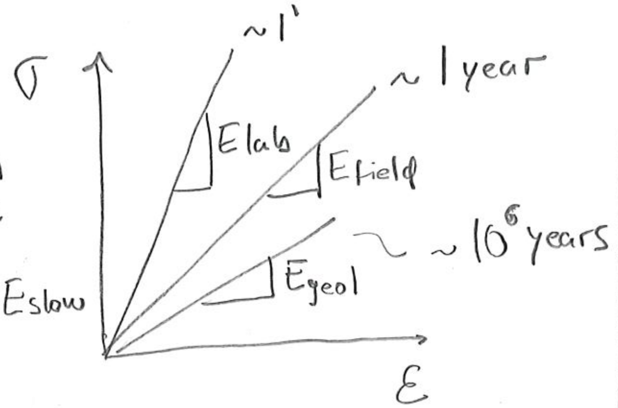
$\underbrace{\text{elasto}}_{\text{recov}}$ - $\underbrace{\text{plasticity}}_{\text{irrecoverable}}$



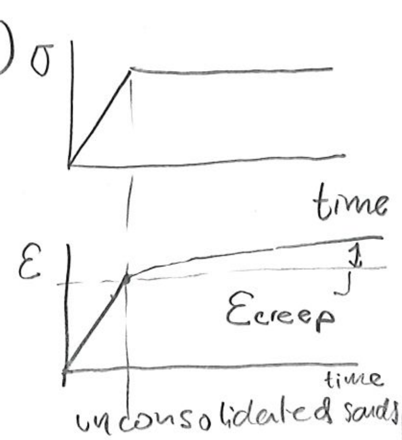
- visco-elasticity

(1) → strain rate

$E_{fast} > E_{slow}$

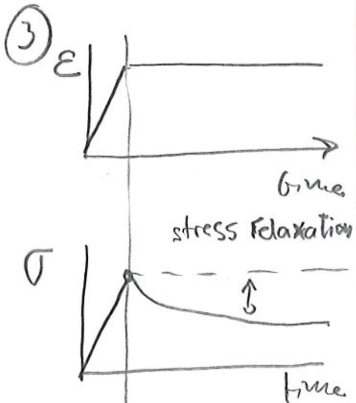


(2)



unconsolidated sands
→ salt rocks

(3)

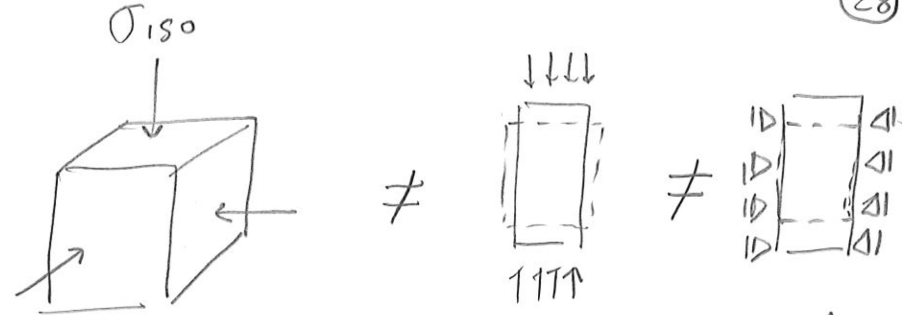


HW 4

3) Isotropic loading

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{iso}$$

$$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$$



unconfined loading

one-dim strain loading

$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{D}}}\ \underline{\underline{\sigma}}$$

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} \\ \text{—} & \frac{1}{E} & \frac{-\nu}{E} \\ \text{—} & \text{—} & \frac{1}{E} \end{bmatrix}} \begin{bmatrix} \sigma_{iso} \\ \sigma_{iso} \\ \sigma_{iso} \end{bmatrix}$$

$$\varepsilon_{vol} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

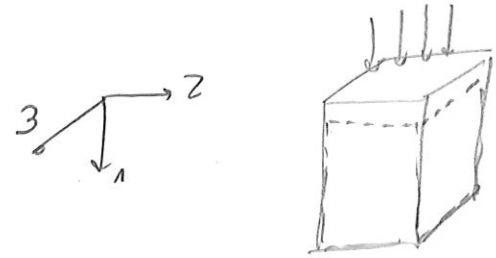
$$\hookrightarrow \varepsilon_{11} = \frac{(1-2\nu)}{E}\sigma_{iso}$$

$$\sigma_{iso} = \underbrace{\boxed{}}\ \varepsilon_{vol}$$

K : Bulk modulus

$$\underline{\sigma} = \begin{bmatrix} \sigma_{iso} & 0 & 0 \\ 0 & \sigma_{iso} & 0 \\ 0 & 0 & \sigma_{iso} \end{bmatrix} = \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \text{psi}$$

4) $$M = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

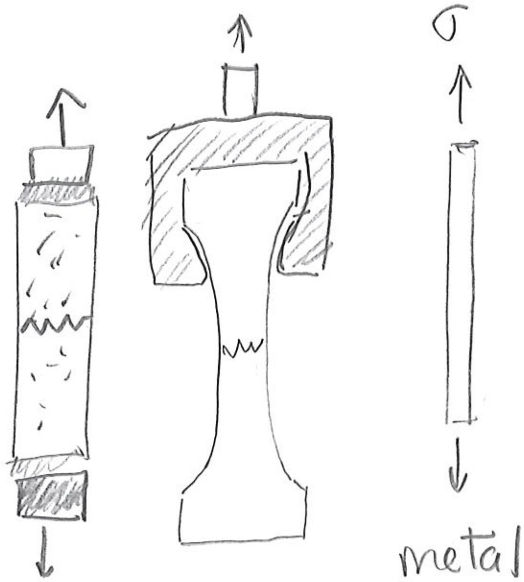


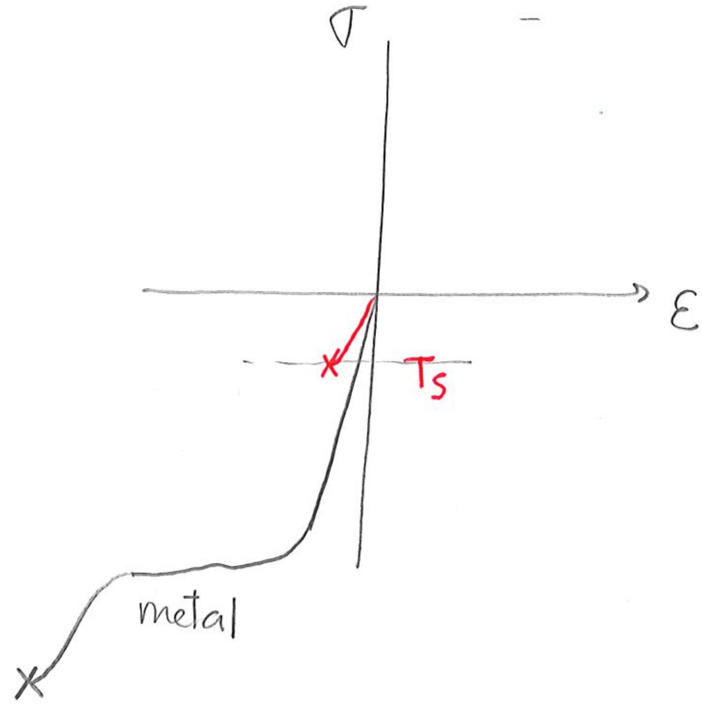
$$\underset{\approx}{\varepsilon} = \begin{bmatrix} \text{—} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; ; \; \underset{\approx}{\sigma} = \begin{bmatrix} \text{—} & 0 & 0 \\ 0 & \text{—} & 0 \\ 0 & 0 & \text{—} \end{bmatrix}$$

5)

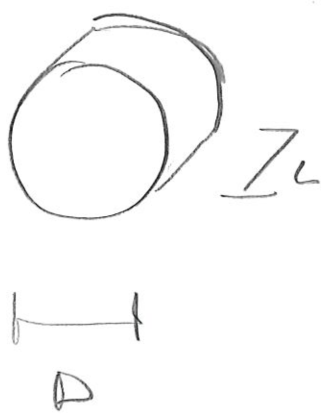
Failure of rocks

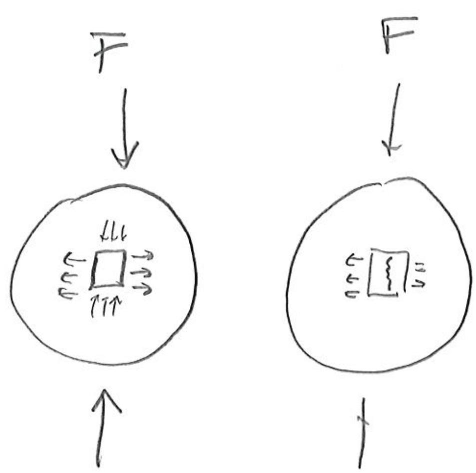
- Tensile failure ↔ tensile strength

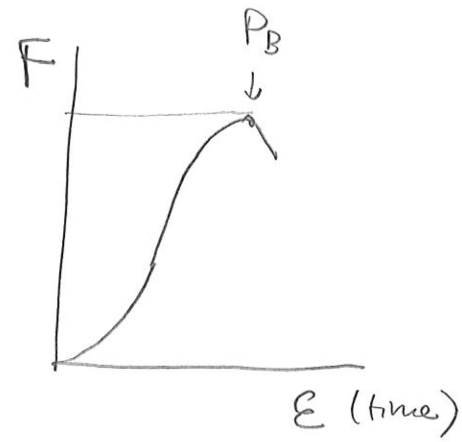




- Brazilian Test





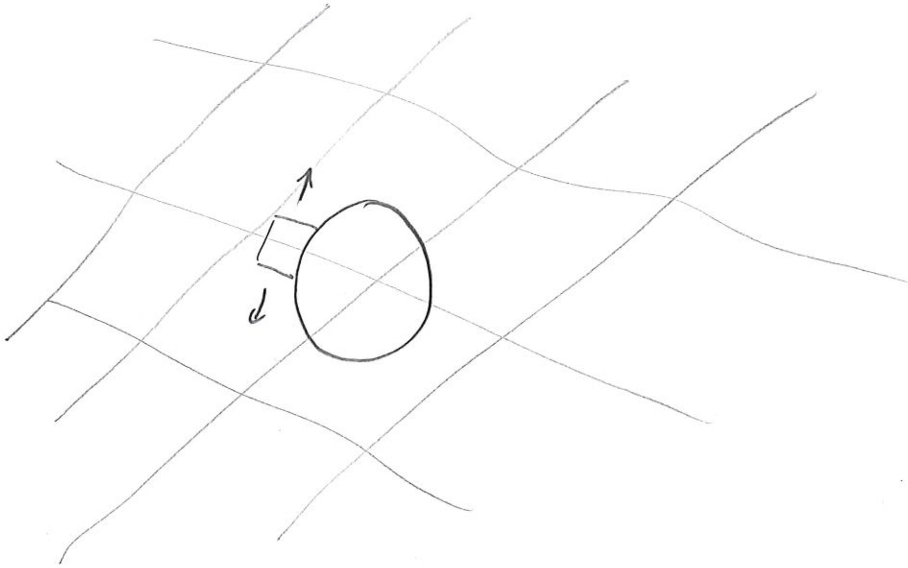


$D > L$

$$T_S = \frac{P_B}{\pi \cdot L \cdot R}$$

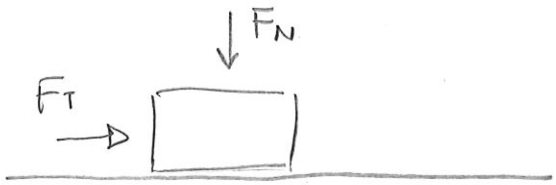
max $\boxed{550\ \text{lb}}$

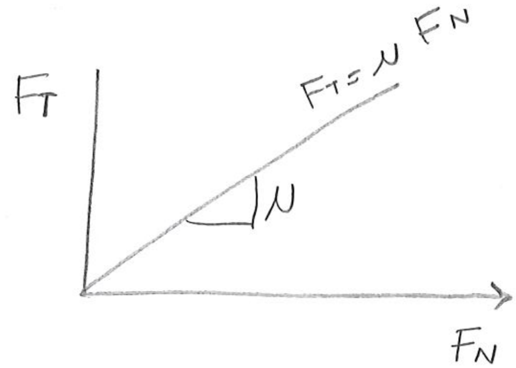
Class Example $T_S = \dfrac{500\ \text{lb}}{\pi \cdot 1\text{in} \cdot 0.5\text{in}} = 318\ \text{psi}$

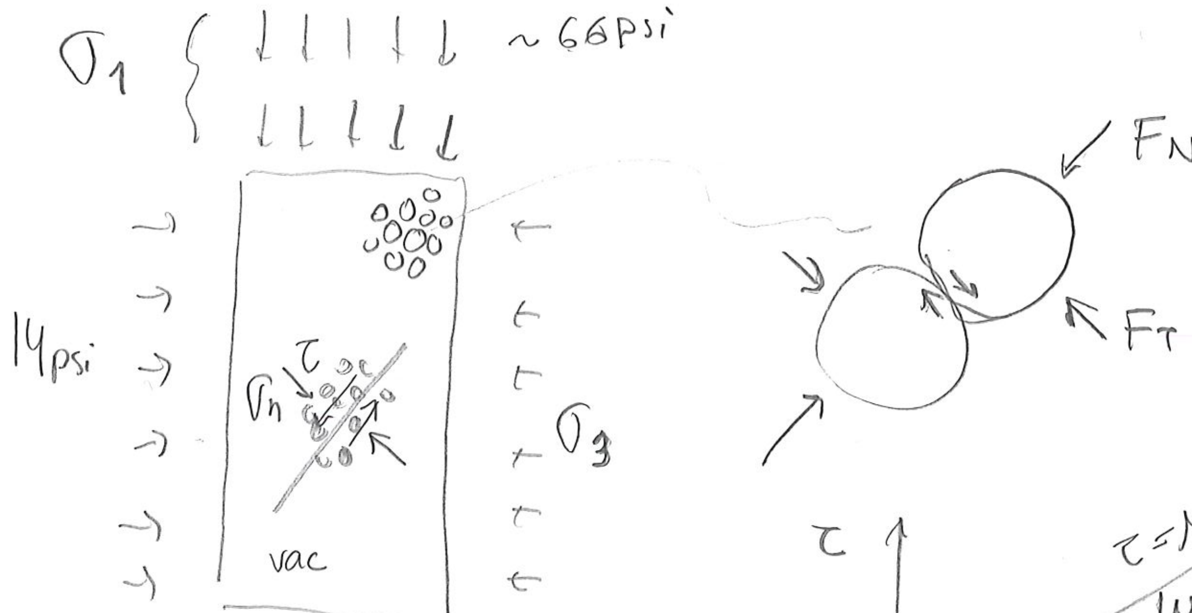


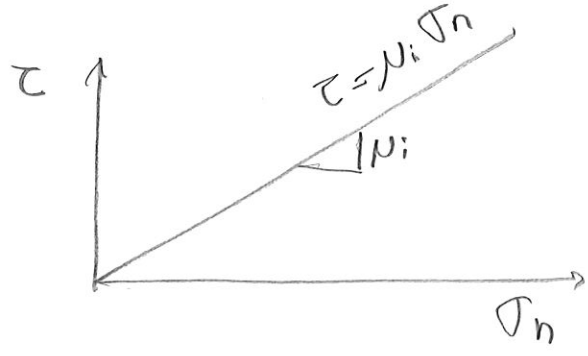
T_S (rocks) < 1500 psi

Shear strength

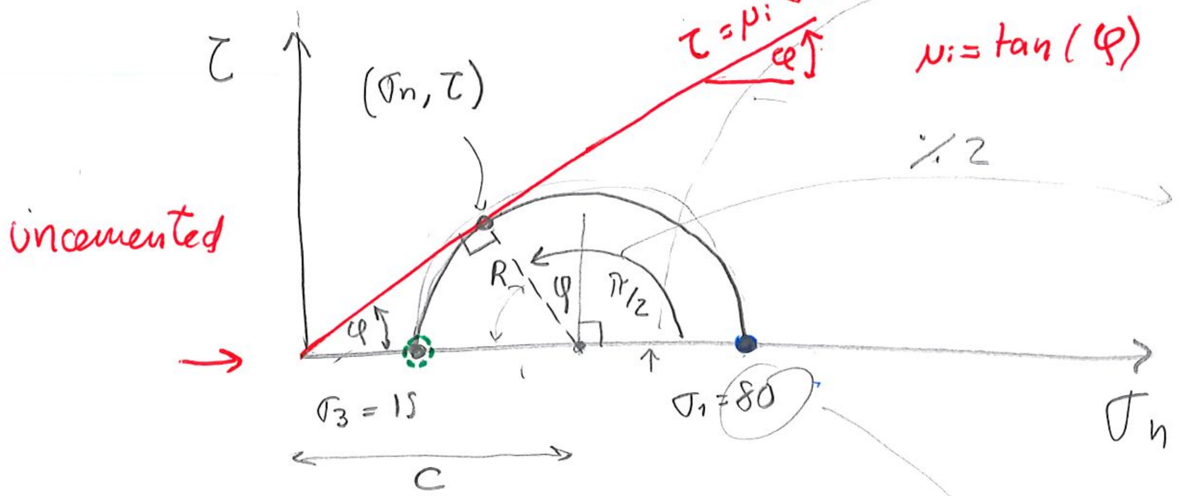


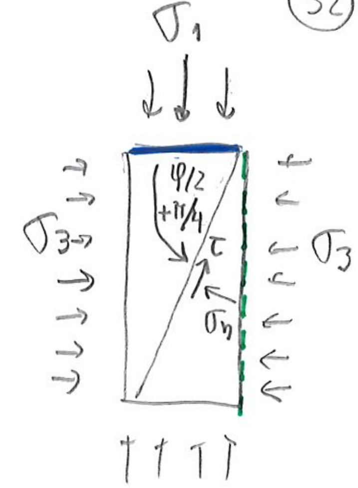






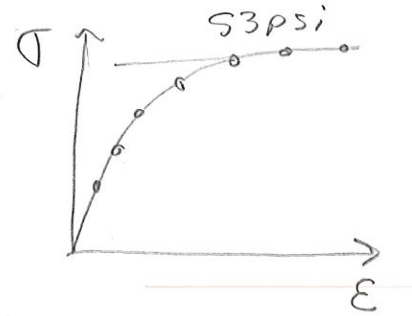
Mohr-Coulomb Failure Criterion





Mohr circle: graphical representation of the stress tensor

→ actually $\boxed{53\ psi}$



$$\underline{\underline{\sigma}} = \begin{bmatrix} 80 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix} \text{psi} \rightarrow \text{principal}$$

$\varphi \sim 30^\circ$

$\frac{\varphi}{2} + \frac{\pi}{4} = \underline{60^\circ}$

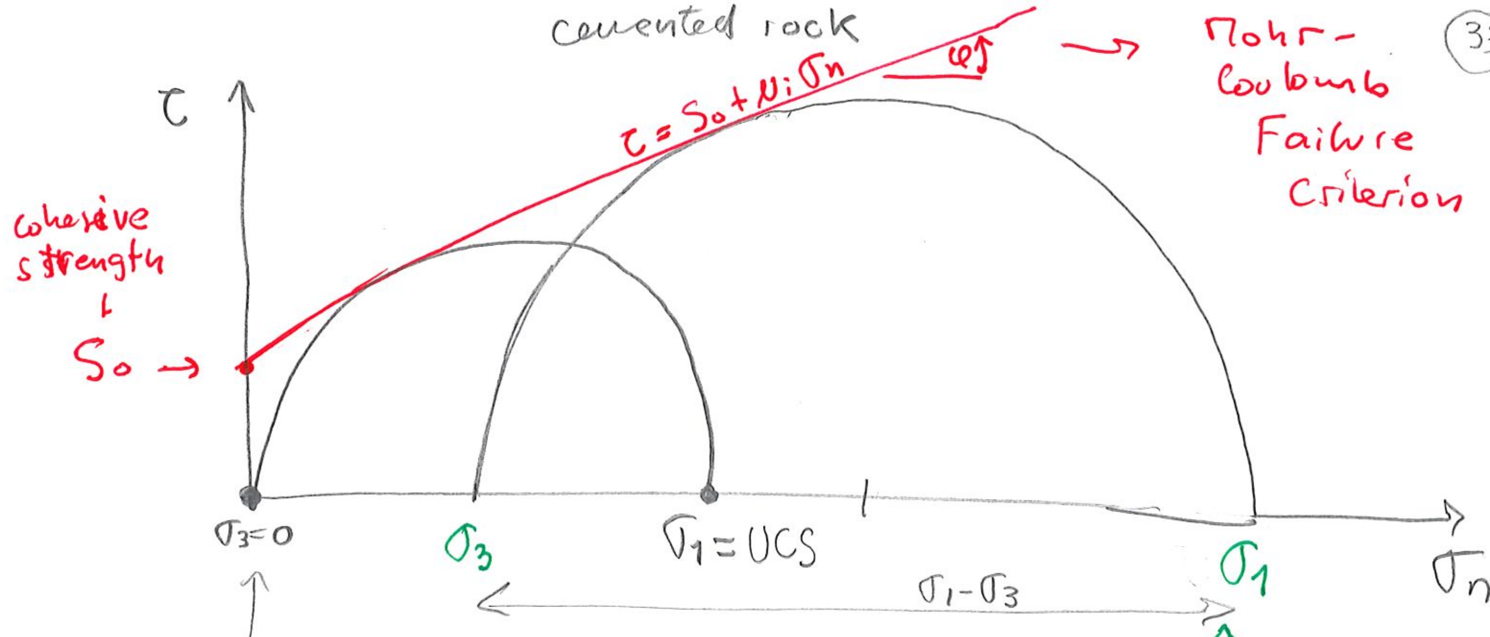
stress anisotropy

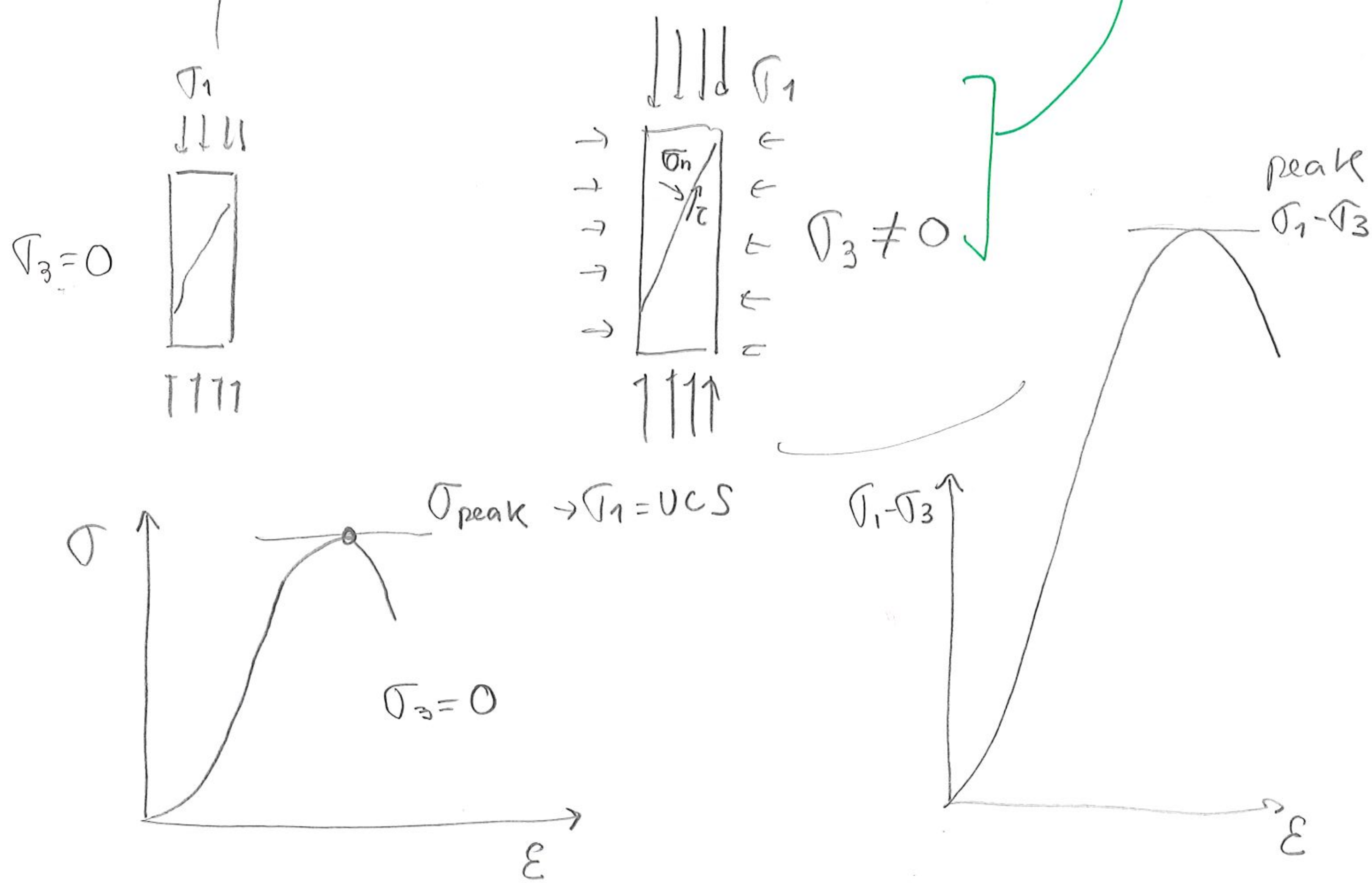
$$\frac{\sigma_1}{\sigma_3} = \frac{C + R}{C - R} = \frac{\cancel{C} + \cancel{C}\sin\varphi}{\cancel{C} - \cancel{C}\sin\varphi}$$

$\varphi = 30^\circ$

$$\frac{\sigma_1}{\sigma_3} = \frac{1+\sin\varphi}{1-\sin\varphi}$$

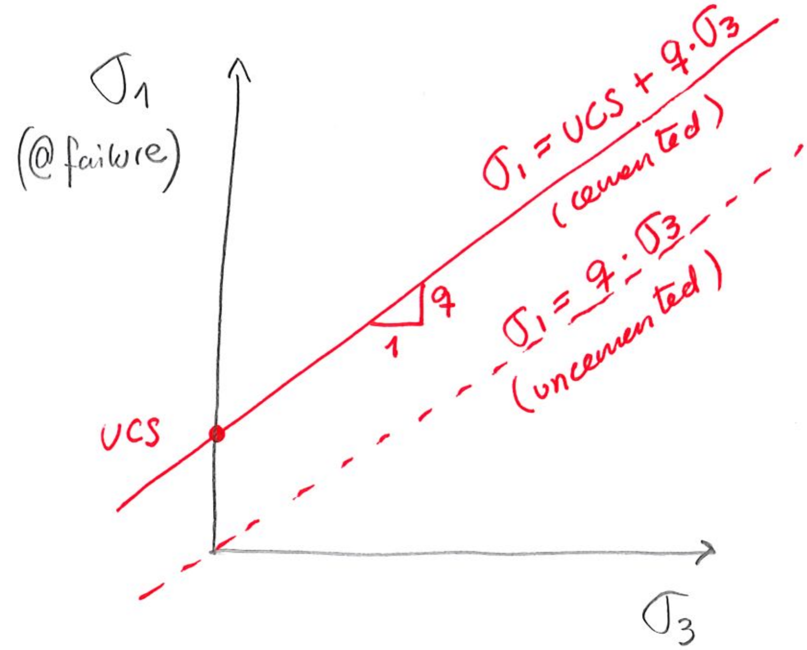
$$\boxed{\frac{1+\sin\varphi}{1-\sin\varphi} = 3}$$





UCS
- Berea ~ 3000 psi
- Boise ~ 3500 psi
- Texas cream ~ 2000 psi

Shales ~ 10,000 psi
15,000 psi



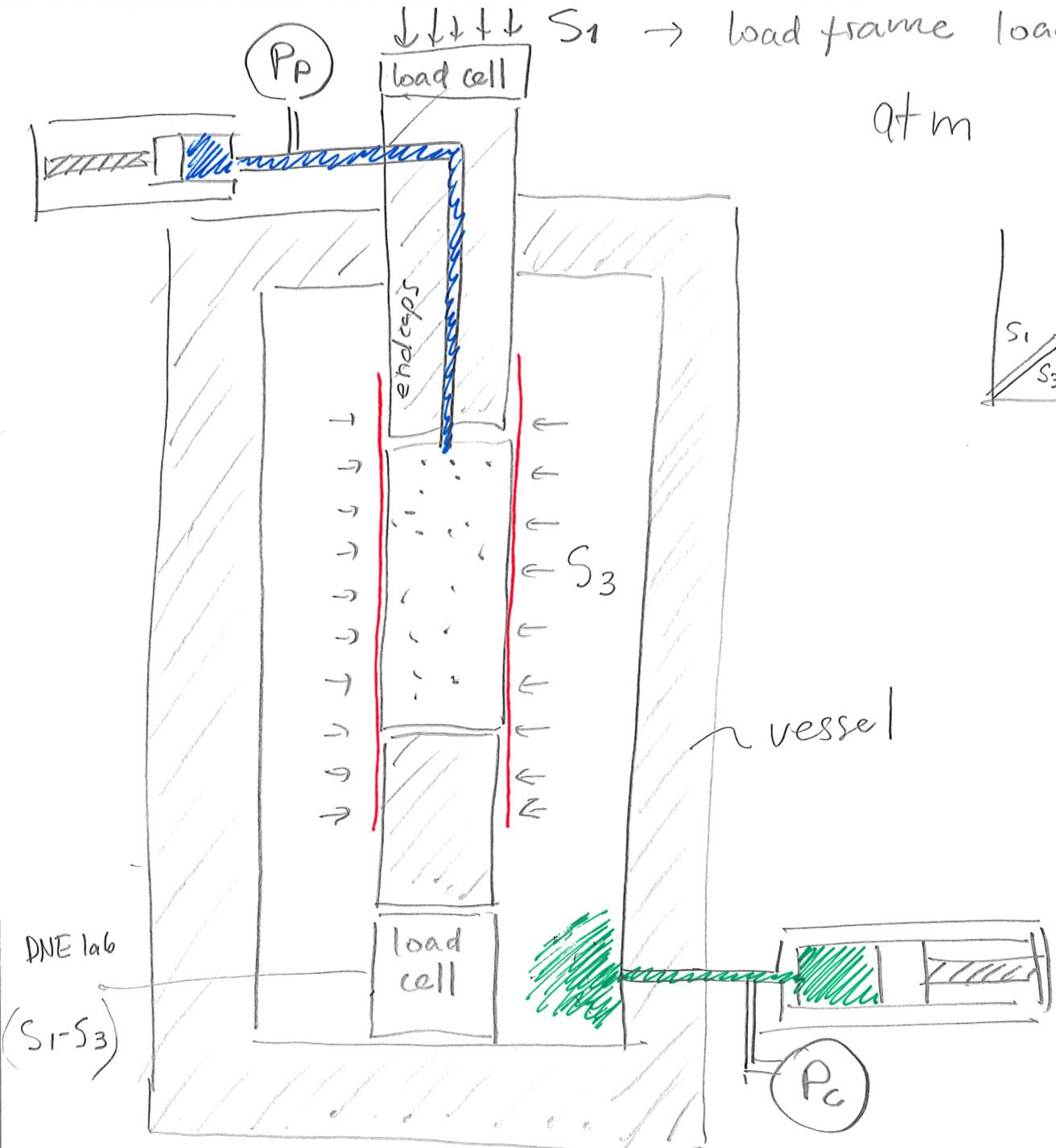
$$q = \frac{1 + \sin\varphi}{1 - \sin\varphi}$$

$$UCS = 2 \cdot S_0 \cdot \sqrt{q}$$

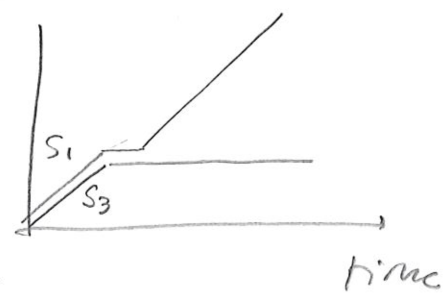
$$\varphi = \arctan\left(\frac{q - 1}{2\sqrt{q}}\right)$$

$$\sigma_3 = P_c - P_p$$

$$\sigma_1 = \sigma_3 + (\sigma_1 - \sigma_3) \quad ; \quad \overbrace{S_1 - S_3}^{\text{total}} = \overbrace{\sigma_1 - \sigma_3}^{\text{effective}}$$



atm

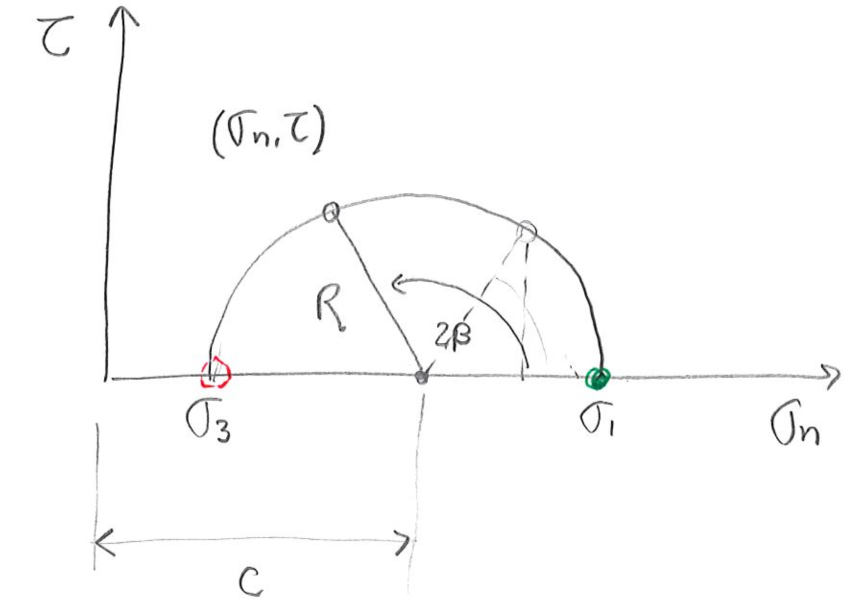


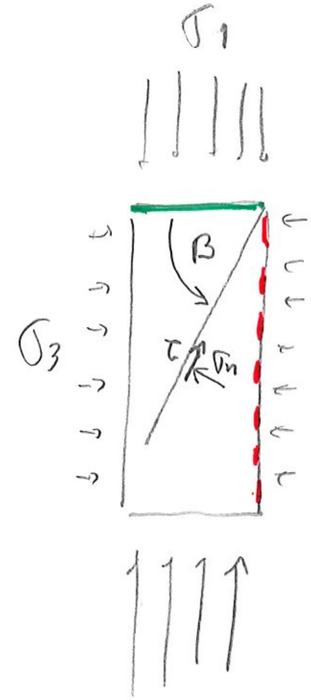
No pore pressure $\Rightarrow S_1 = \sigma_1 \; ; \; S_3 = \sigma_3$ (dry rocks)

With pore pressure $\Rightarrow \sigma_3 = S_3 - P_p = P_c - P_p$

$$\sigma_1 = S_1 - P_p$$

Mohr circle

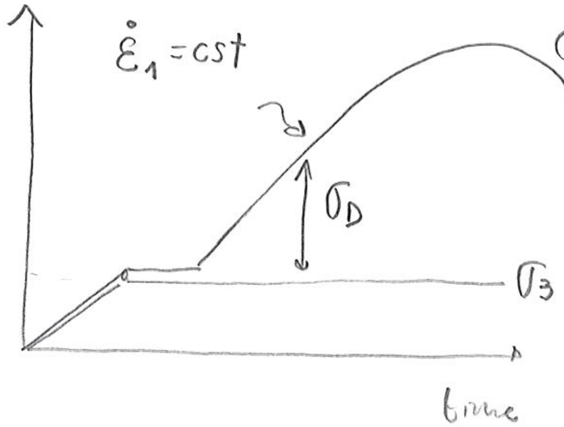


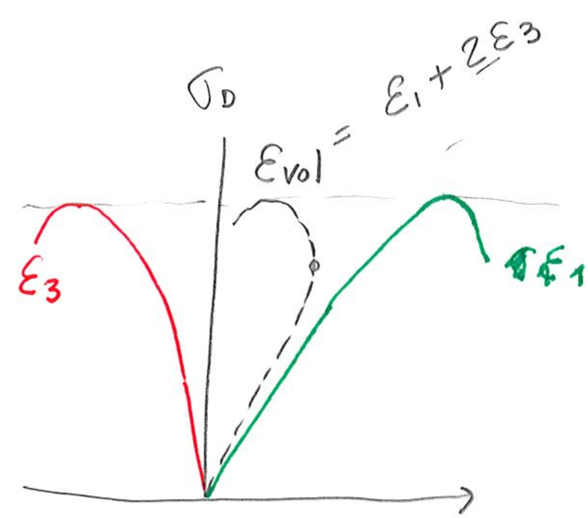


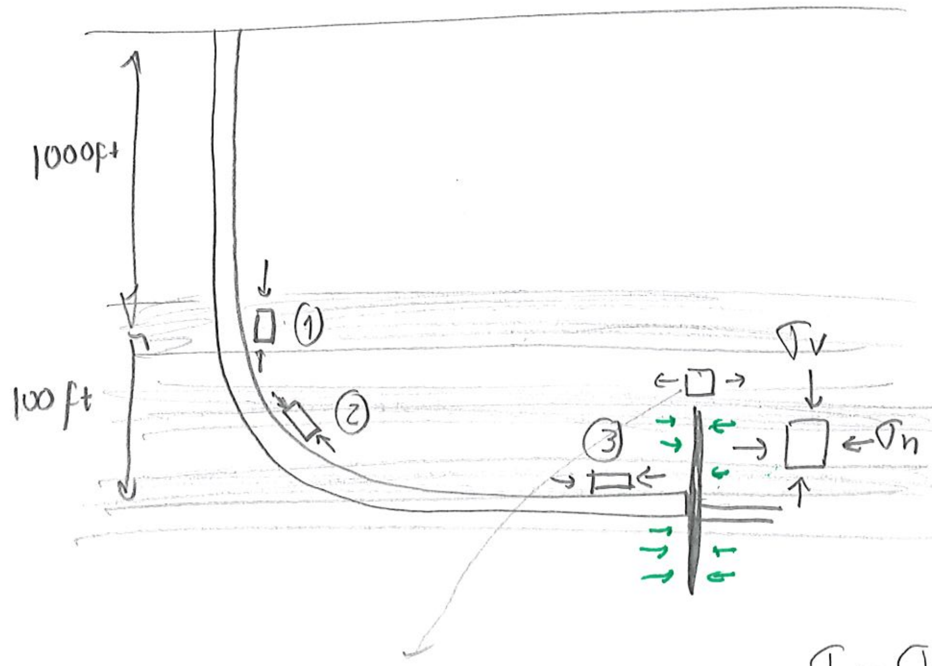
$$\begin{cases} \sigma_n = c + R \cdot \cos 2\beta \\ \tau = R \cdot \sin 2\beta \end{cases}$$

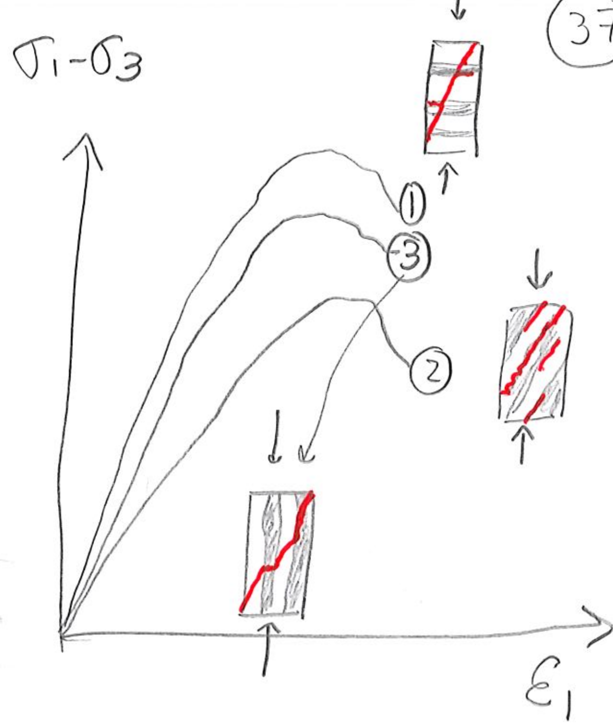
$2\beta = (0, 180°)$

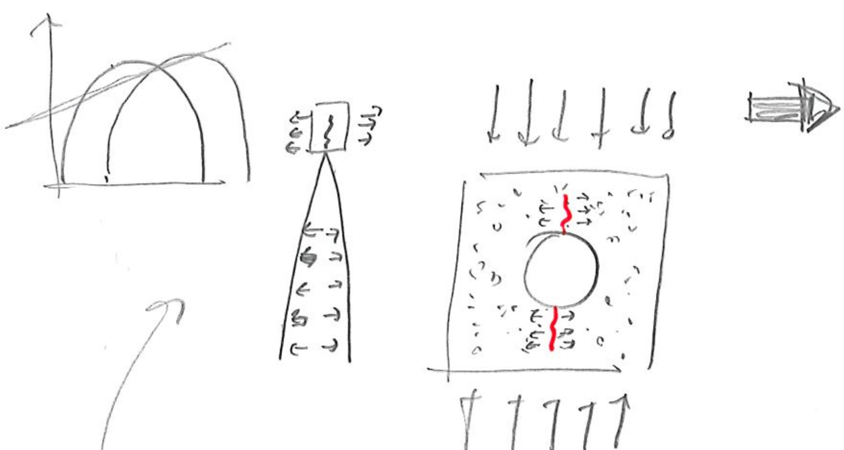
$\dot{\varepsilon}_1 = \dfrac{d\varepsilon}{dt}$

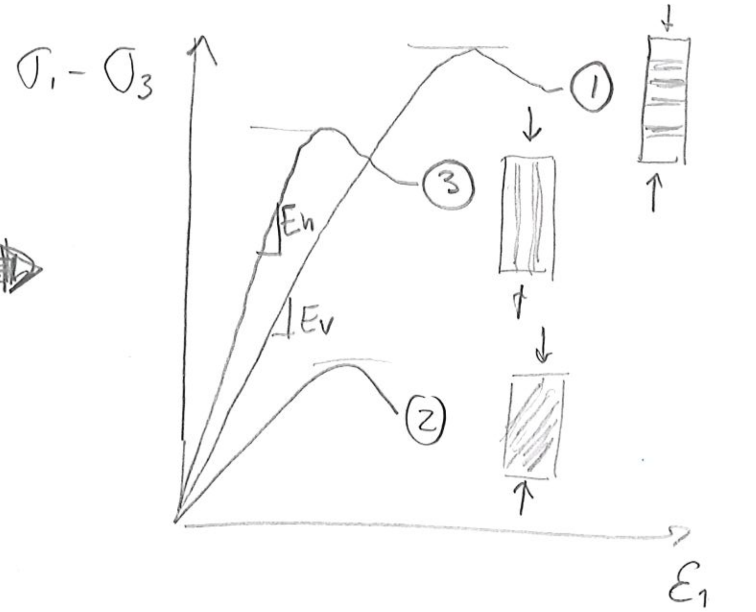


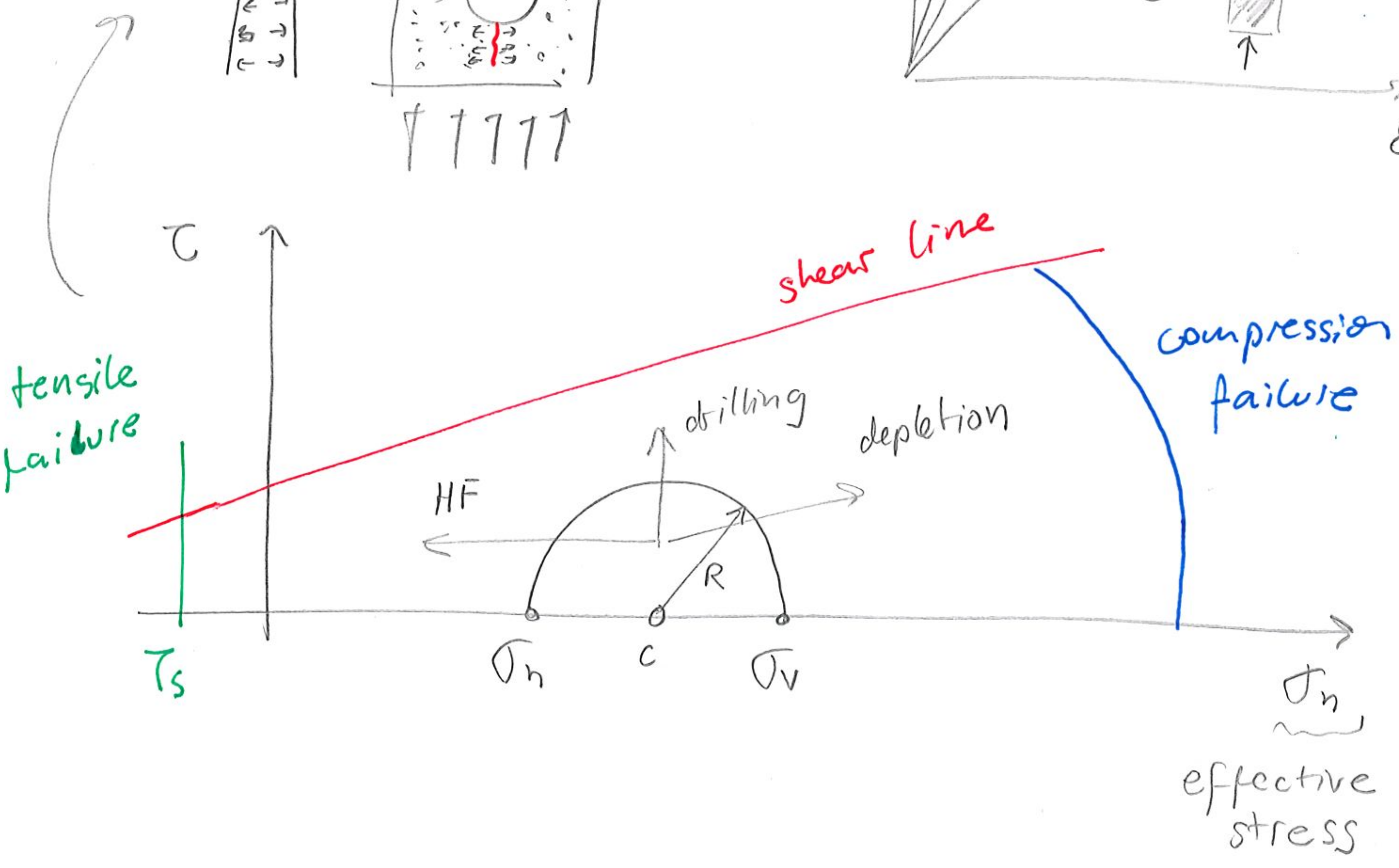



1000ft
100 ft
1
2
3
σ_v
σ_h

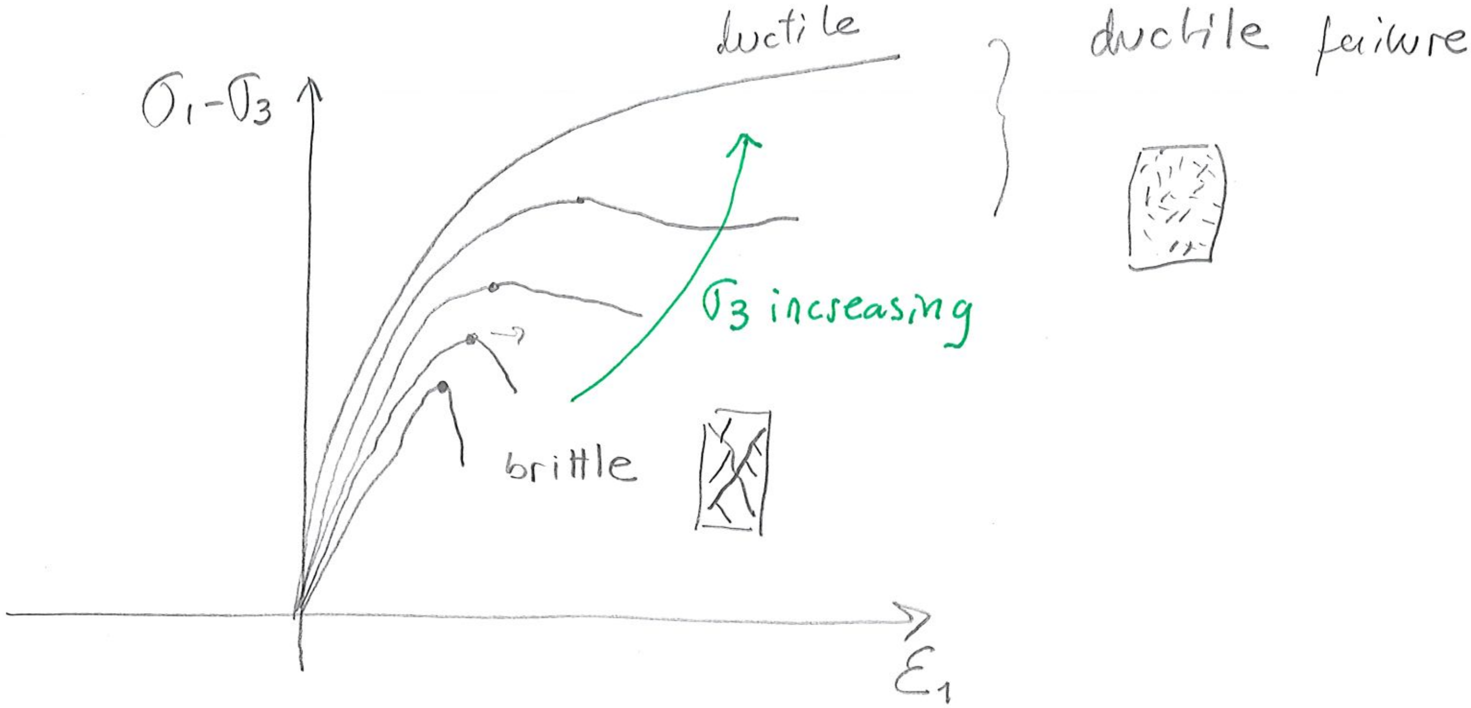

$\sigma_1-\sigma_3$
1
3
2
ε_1




$\sigma_1-\sigma_3$
1
3
E_h
E_v
2
ε_1


τ
shear line
tensile failure
compression failure
drilling
depletion
HF
R
T_s
σ_h
c
σ_v
σ_n
effective stress

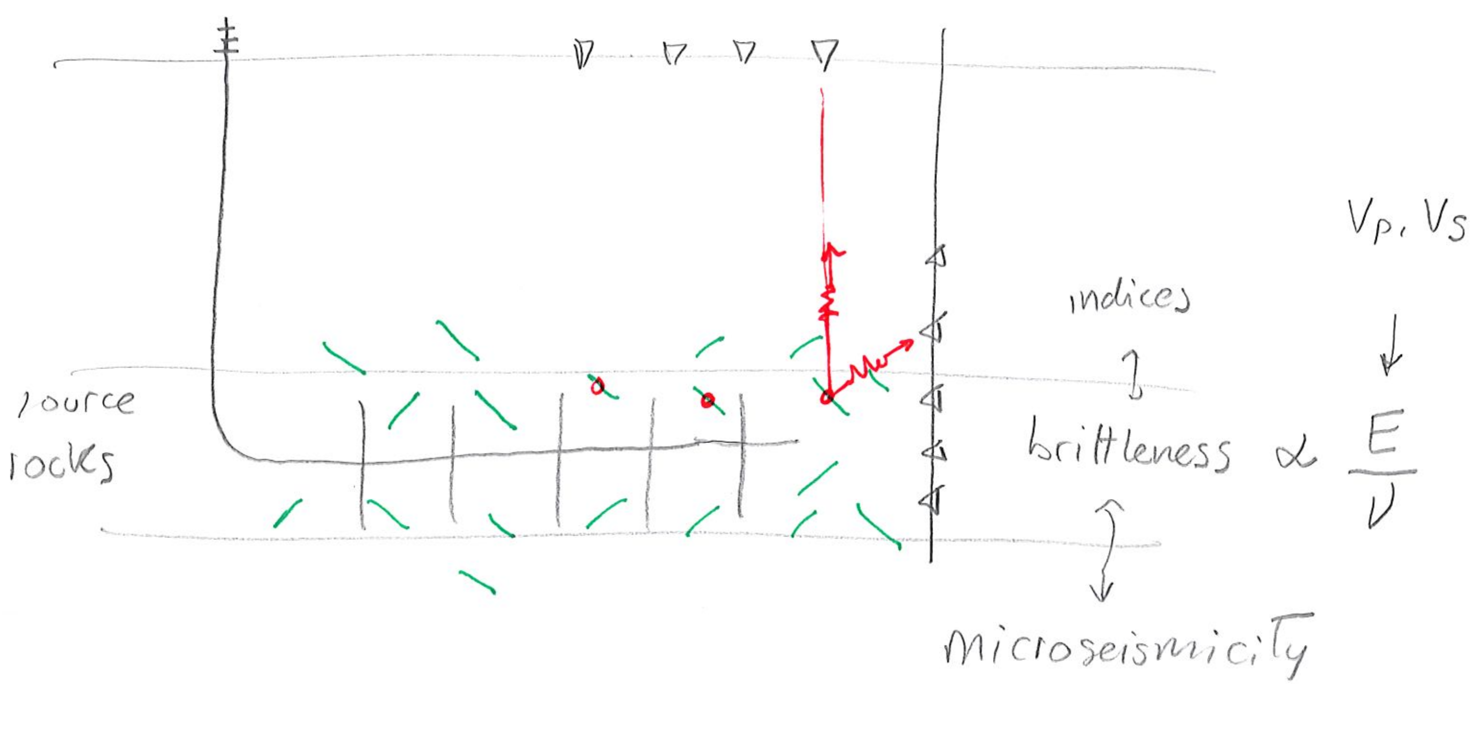
Brittle to ductile transition



brittle → ductile

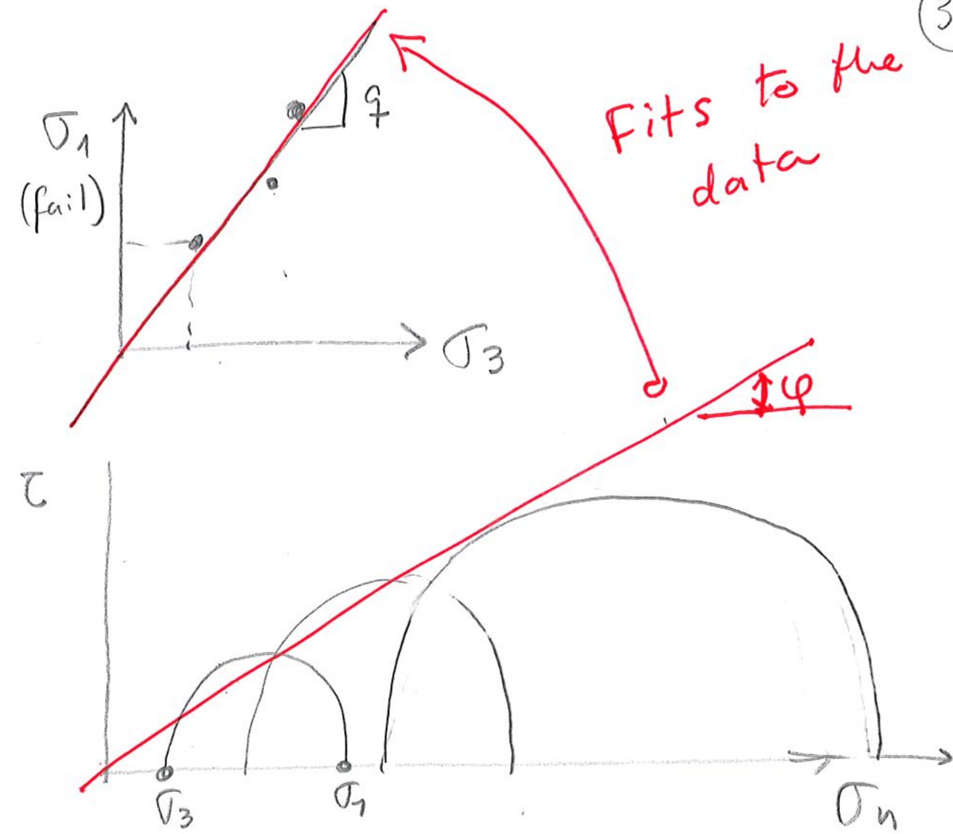
↑ temperature

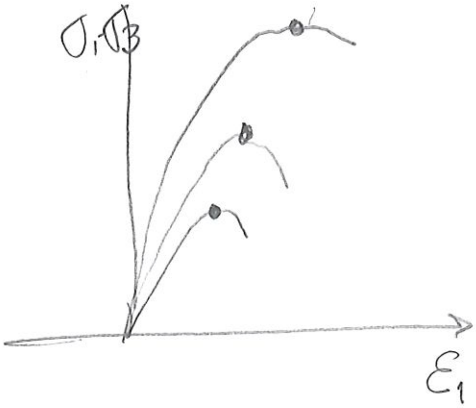
↑ time frame



HW 5

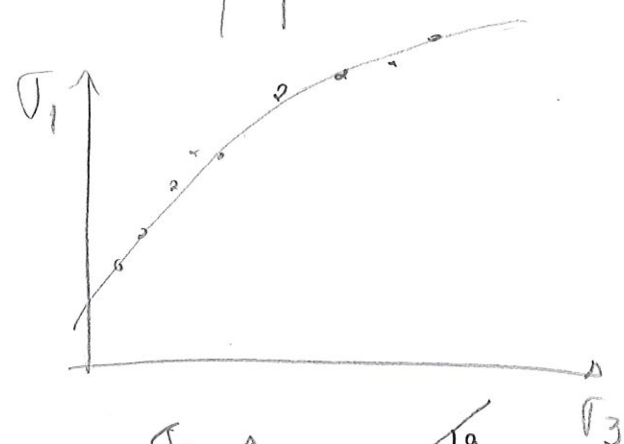
1) uncemented





σ_n	τ
0	—
10	—

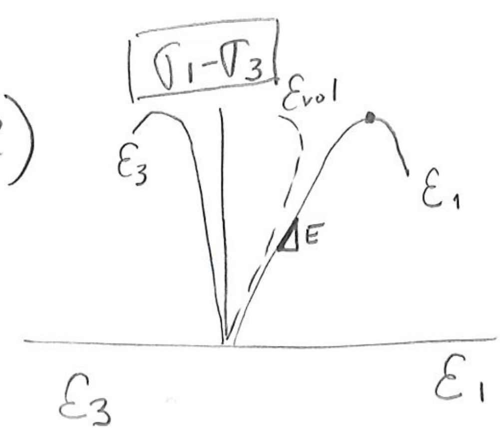
$$\tau = \sigma_n \cdot \tan \varphi$$

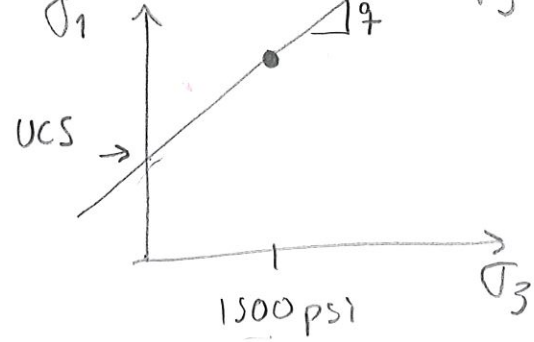


$$\sigma_1 = \sigma_D + \sigma_3$$

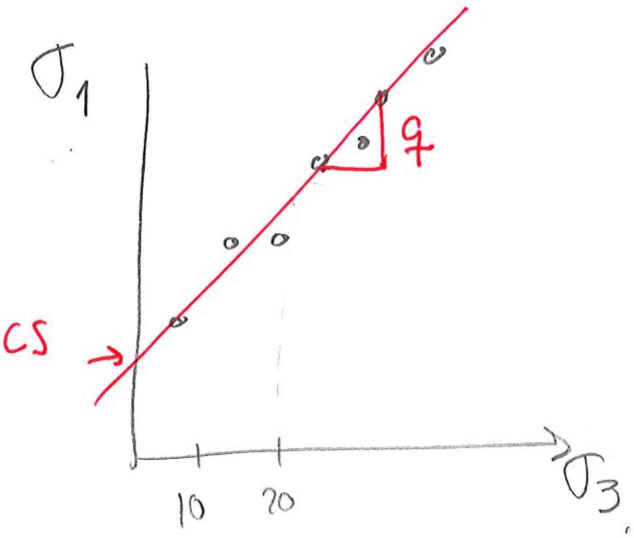
$$\underline{\sigma_1} = \sigma_1 \cancel{- \sigma_3} \cancel{+ \sigma_3}$$

2)





3)



Example $S_3 = 20$ MPa (both cases)

$P_p = 10$ MPa $\rightarrow$ $\sigma_3 = 10$ MPa

$P_p = 0$ MPa $\rightarrow$ $\sigma_3 = 20$ MPa