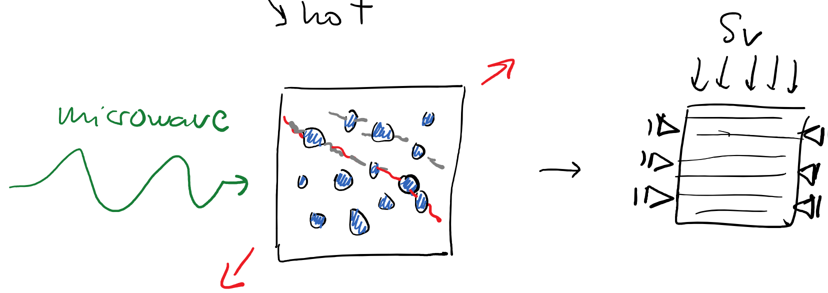
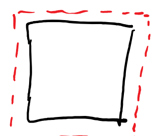
Thermo-elasticity

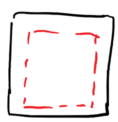
Friday, October 16, 2020 3:55 PM

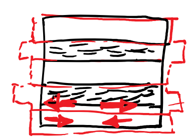
Applications:

- geothermal energy
  - shallow
  - deep {HF, $\tau/\sigma_n \downarrow\uparrow$, $\Delta\sigma$
- drilling — $T_{mud} < T_{form} \Rightarrow \downarrow \sigma_{\theta\theta} \rightarrow$ tensile fractures
- subsurface fluid injection / disposal
  - EOR
    - water
      - liquid
      - steam
    - CO_2
    - chemicals
  - disposal
    - produced water, HF water
    - CO_2
- hydraulic fracturing
  - cold $\rightarrow$ cryogenic fracturing
  - hot



$\Delta T > 0 \Rightarrow$  $\varepsilon_{vol} < 0$ (Dilation)

$\Delta T < 0 \Rightarrow$  $\varepsilon_{vol} > 0$ (Contraction)

$\Delta T \lessgtr 0 \Rightarrow$ no shear strains (isotropic homogeneous) $\neq$  layers $\neq \beta$

Identity matrix

$$\boxed{\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} + 3\beta K (\Delta T) \underline{\underline{I}}}$$

β: Linear thermal dilation coefficient

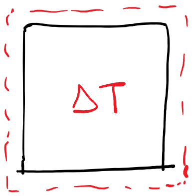
K: Bulk Modulus

ΔT: Change of temperature

$\Delta T = T - T_0$

$\Delta T = \theta$

Dilation



~ • unconstrained
• free dilation

$$\overset{0}{\cancel{\underline{\sigma}}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} + 3\beta K (\Delta T)\, \underline{\underline{I}}$$

$$\underline{\underline{\varepsilon}} = - \underline{\underline{\underline{\underline{D}}}} \left(3\beta K \theta\, \underline{\underline{I}}\right)$$

$$\begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{vmatrix} = \begin{vmatrix} 1/E & -\nu/E & -\nu/E \\ -\nu/E & 1/E & -\nu/E \\ -\nu/E & -\nu/E & 1/E \end{vmatrix} \begin{vmatrix} -3\beta K\theta \\ -3\beta K\theta \\ -3\beta K\theta \end{vmatrix}$$

$$\varepsilon_{11} = -\frac{(1-2\nu)}{\cancel{E}}\left[+3\beta \cancel{K}\theta\right]$$

$1/K$

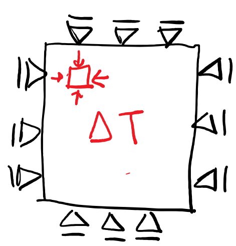
$$\boxed{-\frac{\partial \varepsilon_{11}}{\partial \theta} \overset{def}{=} \beta} \longleftrightarrow \boxed{\varepsilon_{11} = -\beta\,\theta}$$

$\theta = \Delta T > 0 \Rightarrow \Delta\varepsilon < 0$ (Dilation)

$\theta = \Delta T < 0 \Rightarrow \Delta\varepsilon > 0$ (Contraction)

$$\left.\begin{array}{l} \beta = 10^{-6} \text{ to } 10^{-5}\ \frac{1}{{}^\circ K} \\ \Delta T \sim 100\,{}^\circ C \end{array}\right\} \varepsilon \sim 10^{-4} \text{ to } 10^{-3}$$

Constrained dilation



$\theta = \Delta T > 0$

$\Downarrow$

$\Delta\sigma > 0$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\ \overset{0}{\cancel{\underline{\underline{\varepsilon}}}} + 3\beta K \Delta T\, \underline{\underline{I}}$$

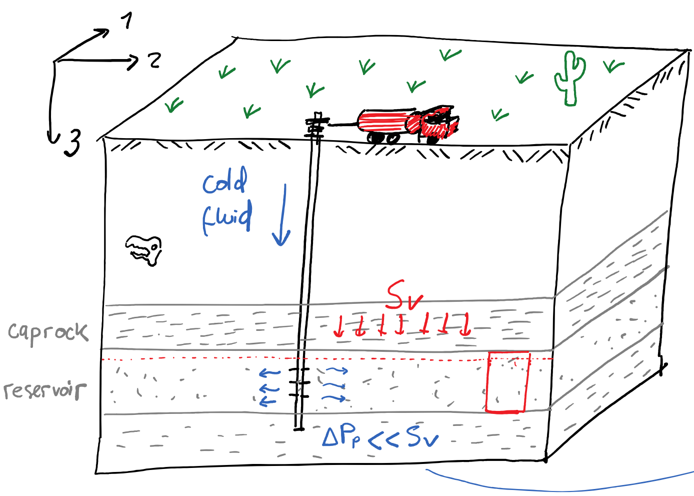
$$\begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{vmatrix} = \begin{vmatrix} +3\beta K\theta \\ +3\beta K\theta \\ +3\beta K\theta \end{vmatrix}$$

$$\sigma_{11} = +3\beta K \theta$$

$\beta = 10^{-5}\ 1/{}^\circ K$ ... 10^{-5} ... 10^{10} ... $10^{2}\ {}^\circ K$

$$\left.\begin{aligned} \beta &= 10^{-5}\ 1/{}^\circ K \\ \theta &= 100\,{}^\circ K \\ K &= 10\ \text{GPa} \end{aligned}\right\} \quad \begin{aligned} \sigma_{11} &= 3 \cdot 10^{-5} \frac{1}{\cancel{{}^\circ K}} \cdot 10^{10}\,\text{Pa} \cdot 10^{2}\,\cancel{{}^\circ K} \\ &= 3 \cdot 10^{7}\,\text{Pa} = \underbrace{30\,\text{MPa} \cong 4400\,\text{psi}}_{\text{compression}} \end{aligned}$$

Thermal stress under uni-axial strain condition



$$\boxed{\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\ \underline{\underline{\varepsilon}} + 3 \beta K \underbrace{\theta}_{= \Delta T = T - T_o} \underline{\underline{I}}}$$

No horizontal strains

$\varepsilon_{11} = \varepsilon_{22} = 0$

$\sigma_{33} \sim$ constant

$\Downarrow$

Uniform cooling in reservoir

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \begin{bmatrix} \cancel{\varepsilon_{11}} \ 0 \\ \cancel{\varepsilon_{22}} \ 0 \\ \varepsilon_{33} \end{bmatrix} + \begin{bmatrix} 3\beta K \theta \\ 3\beta K \theta \\ 3\beta K \theta \end{bmatrix}$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)} \varepsilon_{33} + 3 \beta K \theta$$

$$\uparrow$$

$$\sigma_{33} = \frac{(1-\nu) E}{(1+\nu)(1-2\nu)} \varepsilon_{33} + 3 \beta K \theta$$

$$\sigma_{11} = \frac{\nu \cancel{E}}{\cancel{(1+\nu)(1-2\nu)}} \left[\frac{\cancel{(1+\nu)(1-2\nu)}}{(1-\nu) \cancel{E}} \left(\sigma_{33} - 3\beta K \theta \right) \right] + 3 \beta K \theta$$

$$\sigma_{11} = \frac{\nu}{1-\nu} \sigma_{33} + \left(1 - \frac{\nu}{1-\nu} \right) 3 \beta K \theta$$

$$\sigma_{11} = \frac{\nu}{\ldots} \sigma_{\ldots} \ldots \left(\frac{1-2\nu}{\ldots} \right) 3 \beta K \theta$$

$$\boxed{\sigma_{11} = \frac{\nu}{1-\nu}\,\sigma_{33} + \left(\frac{1-2\nu}{1-\nu}\right) 3\beta K \theta}$$

$$\left.\frac{\partial \sigma_{11}}{\partial \theta}\right|_{\sigma_{33}} = \left(\frac{1-2\nu}{1-\nu}\right) 3\beta K = \frac{\cancel{1-2\nu}}{1-\nu}\cdot\frac{E}{\cancel{3}\,\cancel{(1-2\nu)}}\cdot \cancel{3}\,\beta$$

$$\boxed{\left.\frac{\partial \sigma_{11}}{\partial \theta}\right|_{\sigma_{33}} = +\frac{\beta E}{1-\nu}}$$

$$\begin{cases} \downarrow \theta \Rightarrow \downarrow \sigma_{11} \\ \uparrow \theta \Rightarrow \uparrow \sigma_{11} \end{cases}$$

just mechanics

$$\sigma_{11} = \frac{\nu}{1-\nu}\,\sigma_{33}$$

ΔP_p : pore pressure

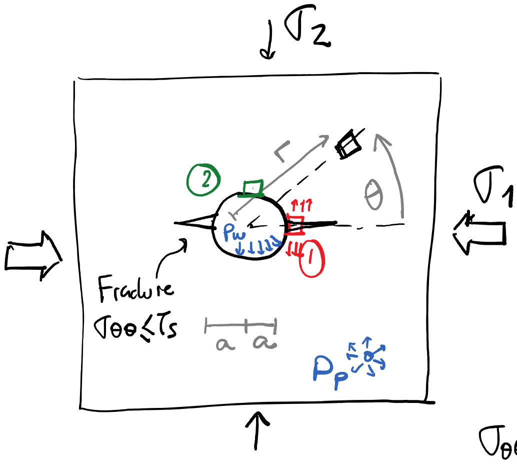
$$\frac{\partial \sigma_{11}}{\partial P_p} = \alpha\left(\frac{1-2\nu}{1-\nu}\right)$$

Δ Temp

$$\frac{\partial \sigma_{11}}{\partial \theta} = \frac{\beta E}{1-\nu}$$

$$\begin{cases} \beta = 10^{-5}\ 1/{}^\circ K \\ E = 10\ GPa \\ \nu = 0.2 \end{cases} \Rightarrow \frac{\partial \sigma_{11}}{\partial \theta} = 0.1\ \frac{MPa}{{}^\circ K}$$

Changes of stresses due to temperature around a wellbore

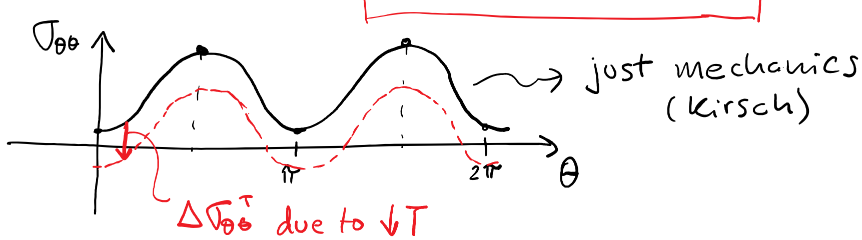


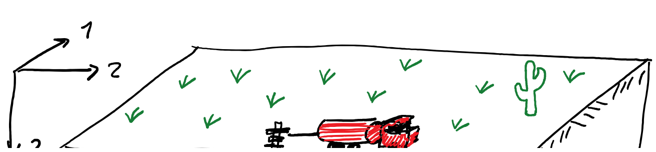
$$① \ \sigma_{\theta\theta}(r=a, \theta=0^\circ) = \overbrace{-(P_w - P_p) - \sigma_1 + 3\sigma_2}^{\text{Kirsch}} + \overbrace{\Delta \sigma_{\theta\theta}^T}^{\text{Thermal stress}}$$

$$② \ \sigma_{\theta\theta}(r=a, \theta=90^\circ) = -(P_w - P_p) + 3\sigma_1 - \sigma_2 + \Delta \sigma_{\theta\theta}^T$$

steady state solution

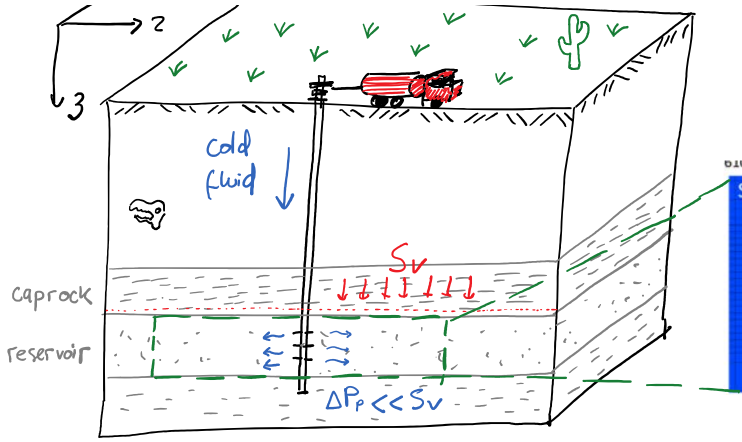
$$\boxed{\Delta \sigma_{\theta\theta}^T = \frac{\beta E}{1-\nu}\cdot \Delta T}$$



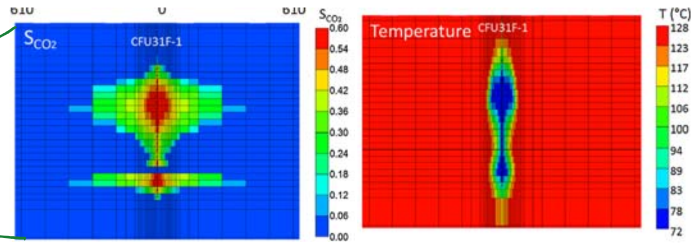


$= \Delta T = T - T_o$

$$\boxed{\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\ \underline{\underline{\varepsilon}} + 3\beta K \theta\, \underline{\underline{I}}}$$



$$\boxed{\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\;\underline{\underline{\varepsilon}} + 3\beta K \theta \underline{\underline{I}}}$$



Coupled problem of thermo-elasticity (time variation)

$$\begin{cases} \nabla \underline{\underline{\sigma}} + \underline{F} = \underline{0} & \longrightarrow \text{Equilibrium} \\ \underline{\underline{\varepsilon}} = \frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right) & \longrightarrow \text{Kinematic} \\ \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} + 3\beta K \theta \underline{\underline{I}} & \rightarrow \text{Constitutive} \quad (\theta = \Delta T = T - T_0) \\ \dfrac{\partial \theta}{\partial t} = \boxed{\dfrac{K_T}{\rho C_v}} \nabla^2 \theta + \underbrace{\dfrac{3\beta K T_0}{\rho C_v}} \cdot \dfrac{\partial \varepsilon_v}{\partial t} & \rightarrow \text{Diffusivity equation} \end{cases}$$

Thermal Diffusivity parameter
K_T : thermal conductivity
C_v : heat capacity

$$\frac{3 \cdot 10^{-5}\, K^{-1} \cdot 10^{10}\, Pa \cdot 300°K}{1.76 \cdot 10^{6}\, \frac{J}{m^3 K}} \sim 50°K \quad \Big\} \quad \varepsilon_{vol} = 0.01 \Downarrow \underline{0.5°K}$$

Coupled problem of thermo-poro-elasticity

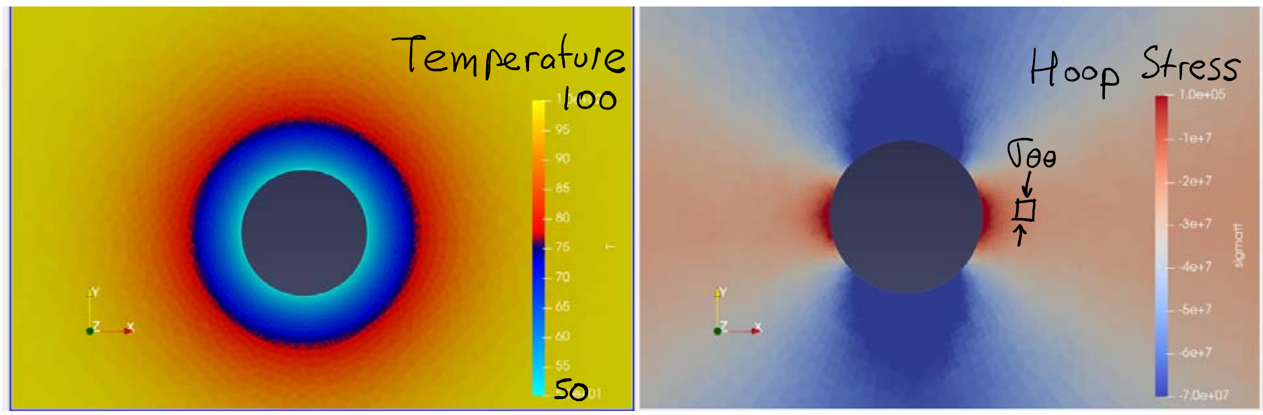
↳ O. Coussy - Ch. 4.3

$$\begin{cases} S_m = K\epsilon - \alpha P - 3\beta K (T - T_0) \\ s_{ij} = 2 G e_{ij} \\ \varphi = \alpha \epsilon + P/N - 3\beta_\varphi (T - T_0) \quad \rightarrow \beta_\varphi = \beta_{solid}(\alpha - \phi_0) \\ S_s = S_{s_0} + 3\beta K \epsilon - 3\beta_\varphi P + C_v \dfrac{(T - T_0)}{T_0} \end{cases}$$

Entropy

+ Diffusivity equations for heat and pore pressure

$$\begin{cases} \nabla \underline{\underline{\sigma}} + \underline{F} = \underline{0} & \longrightarrow \text{Equilibrium} \\ \underline{\underline{\varepsilon}} = \tfrac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^{T} \right) & \longrightarrow \text{Kinematic} \\ \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} + 3 \beta K \theta \, \underline{\underline{I}} & \rightarrow \text{Constitutive} \quad (\theta = \Delta T = T - T_0) \\ \dfrac{\partial \theta}{\partial t} = \dfrac{k_T}{\rho C_v} \nabla^2 \theta + \dfrac{3 \beta K T_0}{\rho C_v} \cdot \dfrac{\partial \varepsilon_v}{\partial t} & \rightarrow \text{Diffusivity} \end{cases}$$



Chemo-mechanical coupled processes

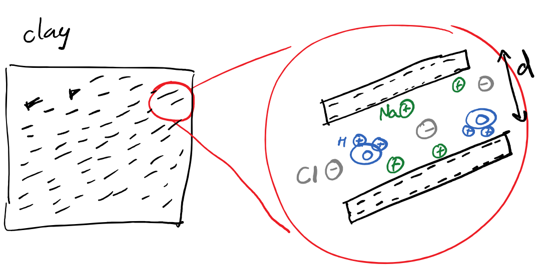
- chemo-plasticity : inelastic deform assisted by chem processes
- chemo-elasticity

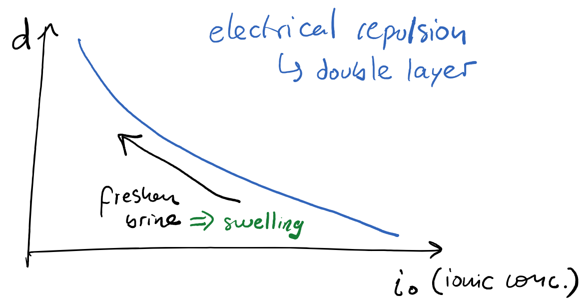
$$\hookrightarrow \underline{\underline{S}} = \underline{\underline{\underline{\underline{C}}}}\;\underline{\underline{\varepsilon}} + \alpha P \underline{\underline{I}} + 3\beta K \theta \underline{\underline{I}} + \underbrace{\gamma}_{\text{coeff}} \overset{\text{chem potential}}{\mu} K \underline{\underline{I}}$$

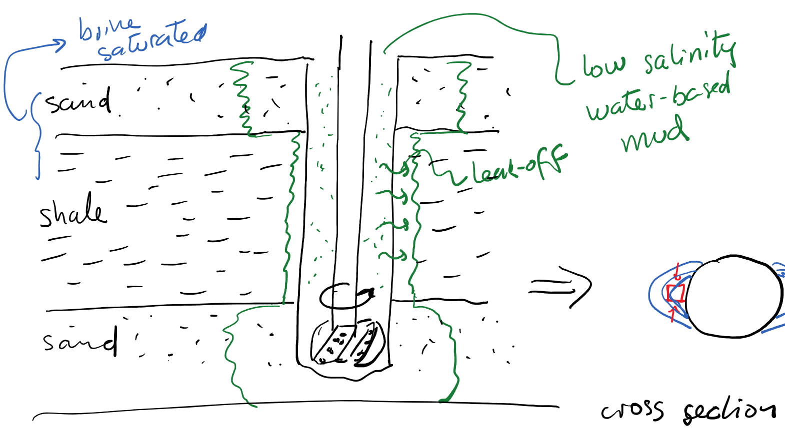
$$\underbrace{\text{Mech} \quad \begin{matrix}\text{poro}\\ \text{hydro}\end{matrix} \quad \text{thermal} \quad \text{chemical}}$$

$\underbrace{\text{THCM}}$ coupled processes $\Longrightarrow$ emergent phenomena

1) Chemical sensitivity of shales

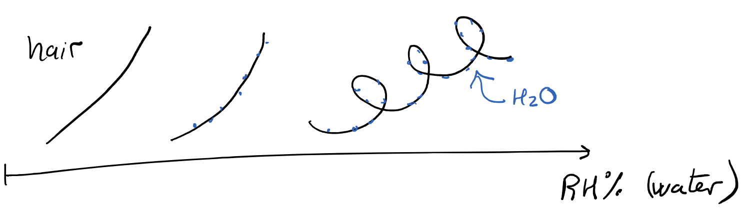


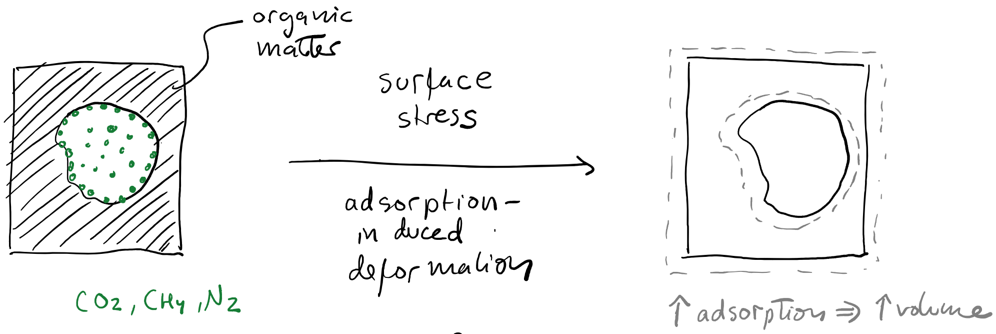


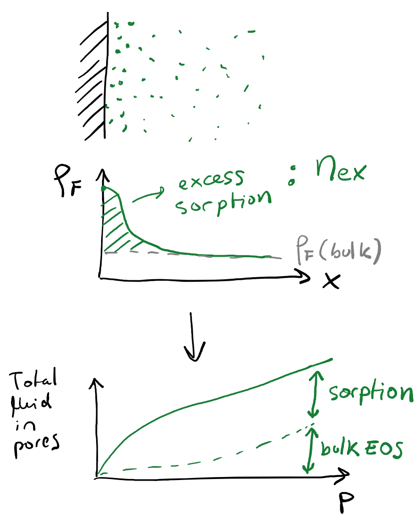


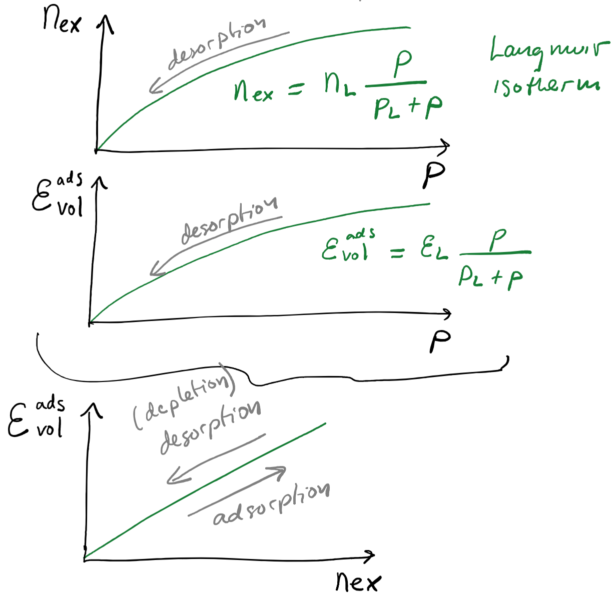
- drilling mud
  - water-based mud
    ↳ ↑ salinity
  - oil-based mud
    ↳ non-polar fluid
  - under balanced drilling
    $P_{mud} < P_p$

2) Adsorption-induced deformation





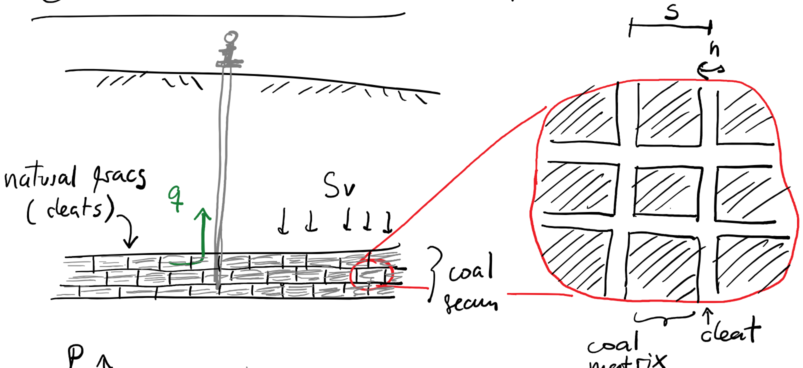




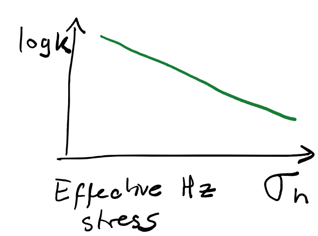
$$\underline{\underline{S}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} + \alpha P \underline{\underline{I}} + (1-\alpha)\, S_a(P)\, \underline{\underline{I}}$$

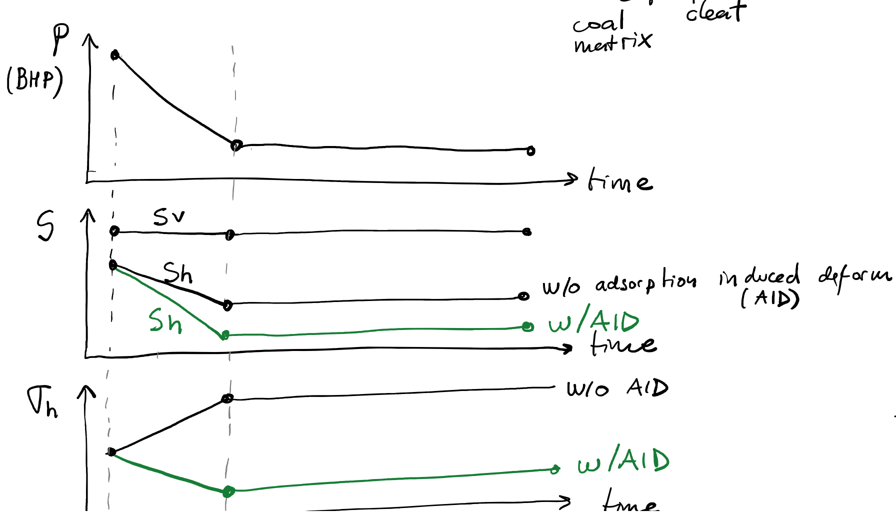
→ Adsorption stress : $S_a(P)$

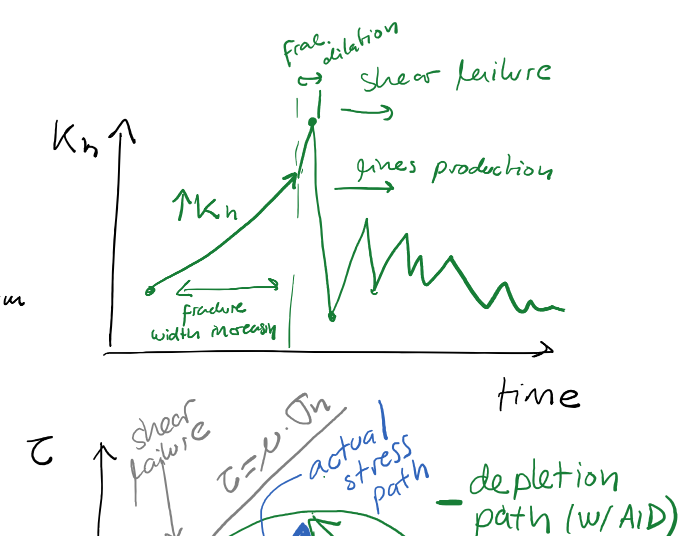
Coal bed methane (desorption-induced shrinkage and stress relaxation)

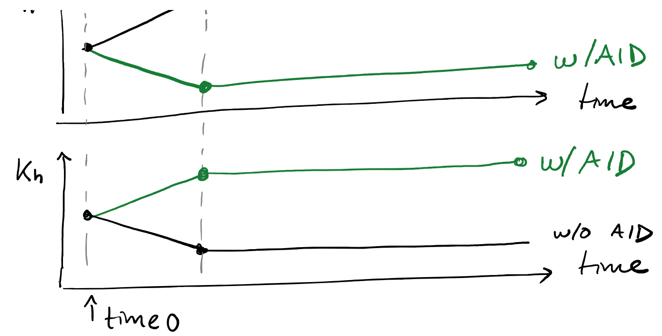


$$K = \frac{h^3}{12 \cdot s}$$

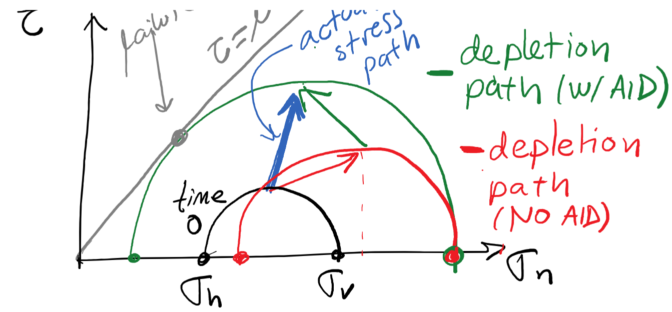








w/AID
time
K_h
w/AID
w/o AID
time
time 0



τ
$\tau=\mu$
actual stress path
depletion path (w/ AID)
depletion path (NO AID)
time 0
σ_h
σ_v
σ_n