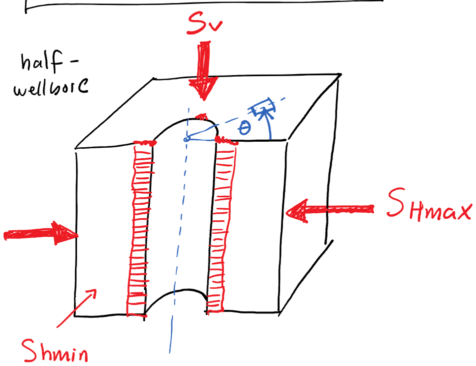
Break down Pressure

Tuesday, November 10, 2020 4:15 PM



- Normal faulting ✓

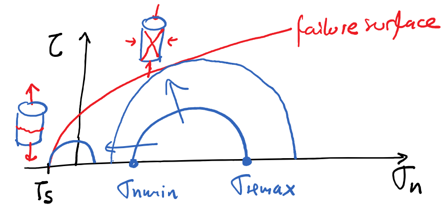
$S_v > S_{Hmax} > S_{hmin}$

- Kirsch solution ✓

$$\sigma_{\theta\theta}(r=a,\theta) = -(P_w - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) \ldots$$
$$- 2(\sigma_{Hmax} - \sigma_{hmin}) \cos(2\theta)$$

Solve for $\begin{cases} \theta = 0 \text{ or } \pi \\ \sigma_{\theta\theta} = -T_s \end{cases}$

$\Downarrow$



Break down pressure

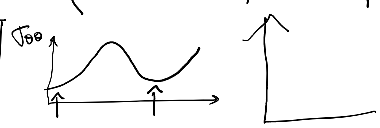
$$\boxed{P_w = P_b = \underbrace{P_p}_{\text{Pore pressure}} \underbrace{- \sigma_{Hmax} + 3\sigma_{hmin}}_{\text{Stress Anisotropy}} \underbrace{+ T_s}_{\text{Tensile strength}}}$$

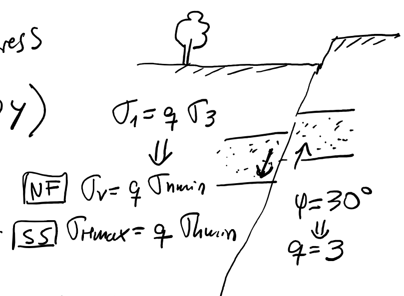
(Case 1) $\sigma_{Hmax} \approx \sigma_{hmin} = \sigma_h$ (isotropic, horizontal stress anisotropy)

$$\boxed{P_b = P_p + 2\sigma_h + T_s}$$

(Case 2) $\sigma_{Hmax} \approx 3\sigma_{hmin}$ (maximum horizontal stress anisotropy)

$$\boxed{P_b = P_p + T_s}$$

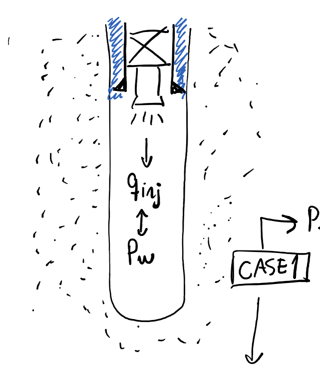




Leak-off test ↔ Diagnostic Fracture Initiation Test (DFIT)

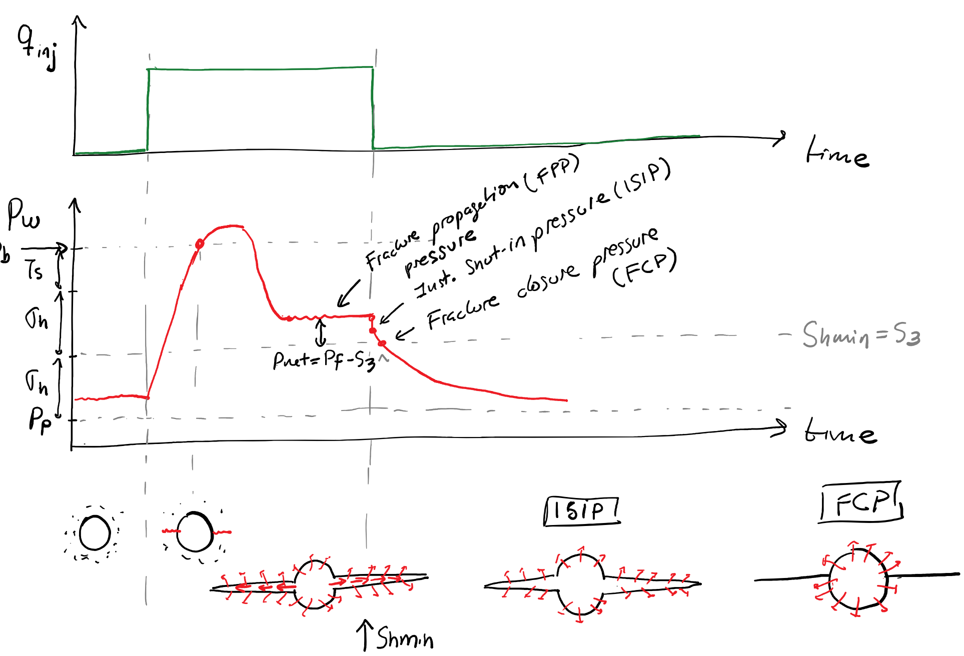
DRILLING

COMPLETIONS AND HF



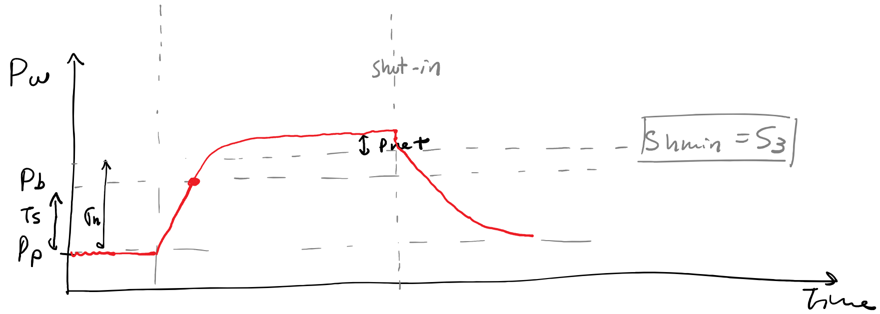
$\sigma_{Hmax} = \sigma_{hmin} = \sigma_h$

$S_3 = S_{hmin} = P_p + \sigma_h$



Case 2

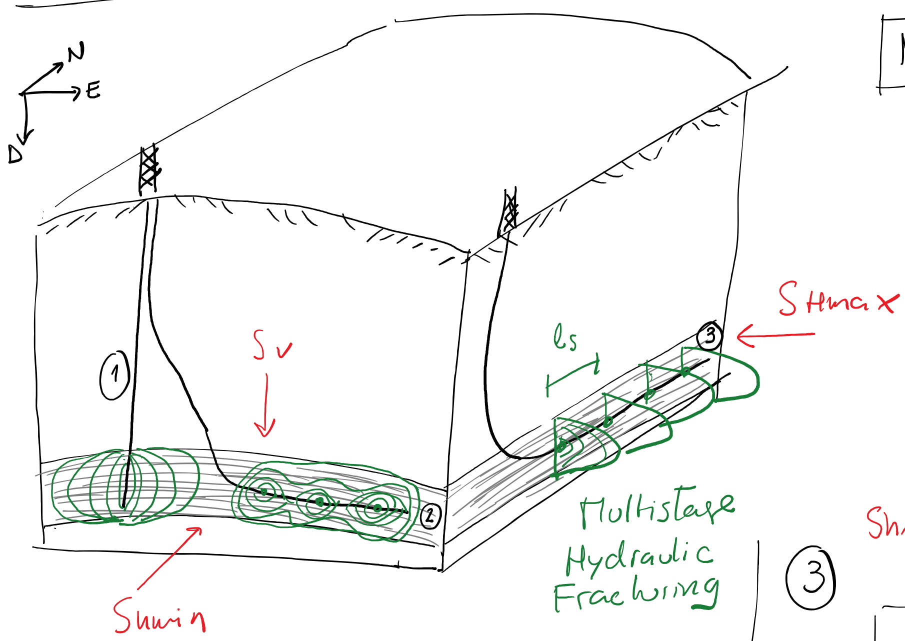
$P_b = P_p + T_s$

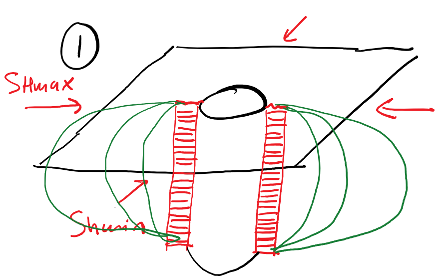


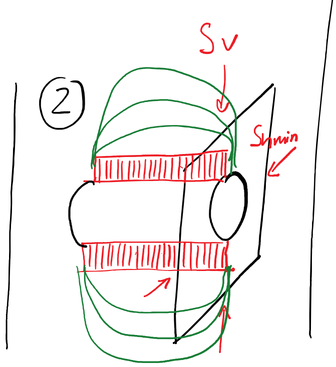
Ideal orientation of HF growing from wellbores and perforations

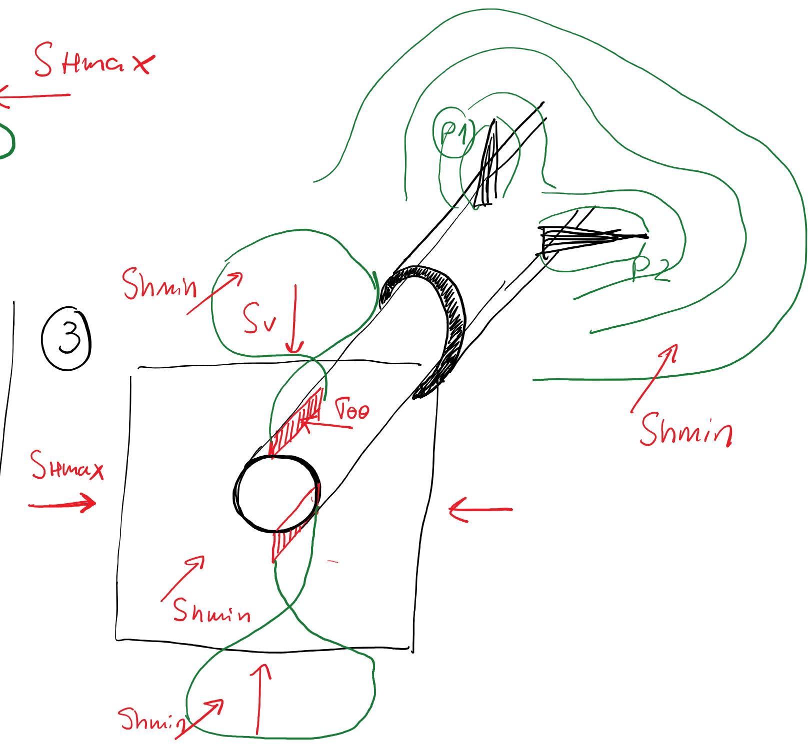
NF: $S_v > S_{Hmax} > S_{hmin}$

S_3





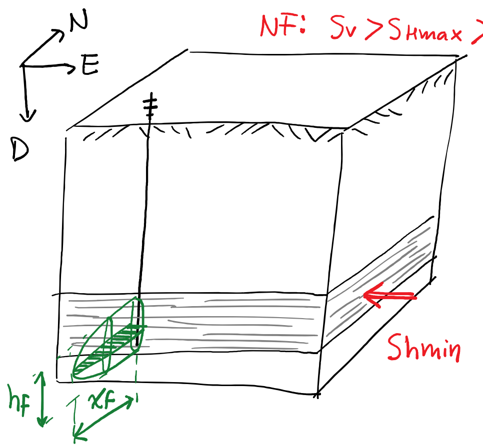




Coupled fluid-driven fracture problem ← Valko and Economides *Hydraulic Fracture Mechanics*

Thursday, November 12, 2020 2:30 PM

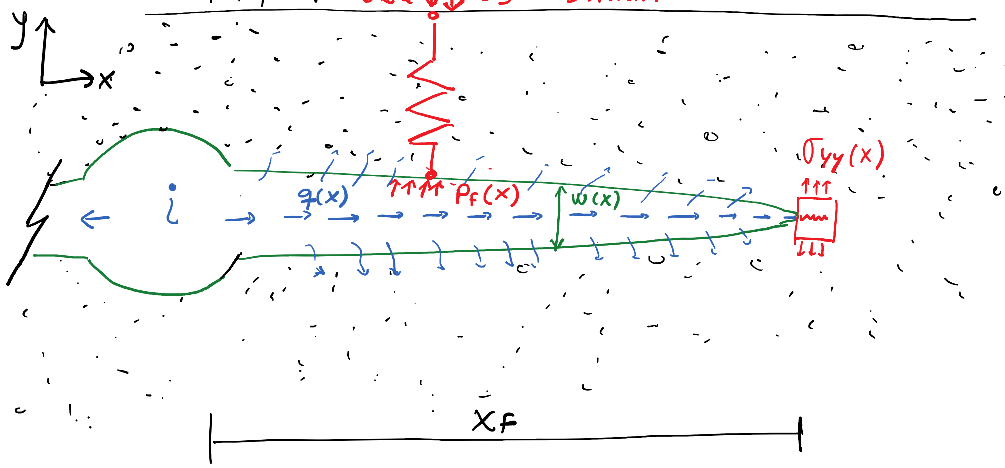
NF: $S_v > S_{Hmax} > S_{hmin}$



⇒

Horizontal cross section (plane N-E)

Far field $S_3 = S_{hmin}$

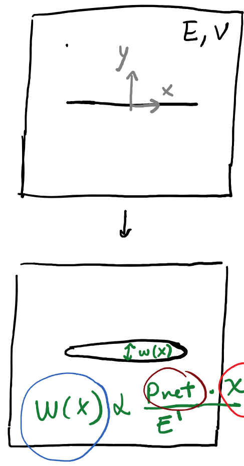


$$P_{net} = P_f - S_3 = P\left(\begin{matrix}\text{elastic}\\ \text{deformation}\end{matrix}\right) + P\left(\begin{matrix}\text{viscous}\\ \text{losses}\end{matrix}\right) + P\left(\begin{matrix}\text{new rock}\\ \text{surface}\end{matrix}\right)$$

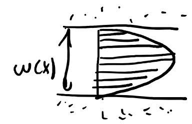
$$\nabla\cdot\underline{\underline{\sigma}} = 0$$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}} \rightarrow$$

$$\underline{\underline{\varepsilon}} = \frac{1}{2}(\nabla u + \nabla u^T)$$



$$w(x) \propto \frac{P_{net}}{E'} \cdot x_f$$



- $q(x) \propto \frac{[w(x)]^3}{\mu} h_f \cdot \frac{dP(x)}{dx}$

- leak-off

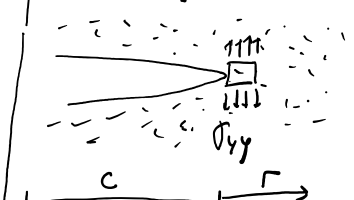
$$q_m = -\frac{k}{\mu}\nabla P$$

K_{IC} : Fracture toughness

- $K_I > K_{IC} \Rightarrow$ Frac propag
- $K_I < K_{IC} \Rightarrow$ Frac does not propagate

K_I : Fracture Intensity

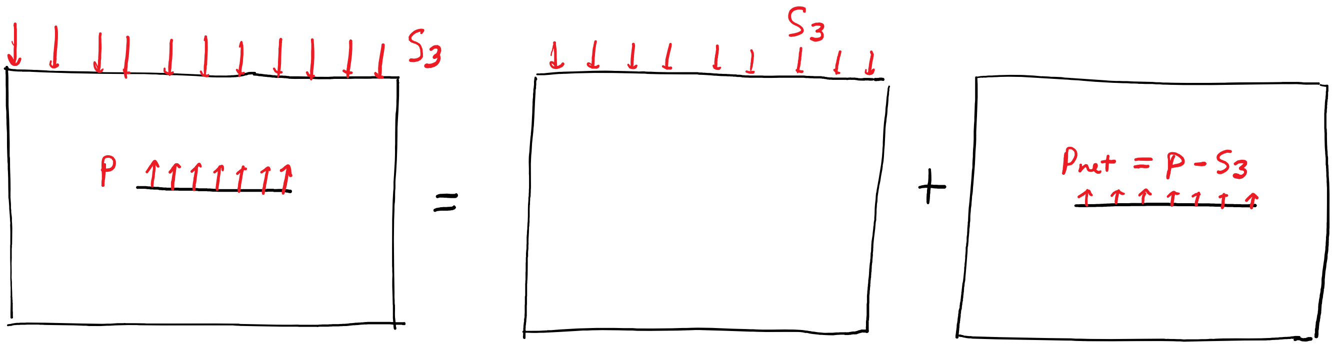
$$K_I = \lim_{r\to 0^+}\left[\sqrt{2\pi r}\cdot\sigma_{yy}(c+r,0)\right]$$



$$K_I = f(P_{net}, x_f)$$

Linear Elastic Fracture Mechanics (LEFM)

Monday, November 16, 2020 11:21 AM

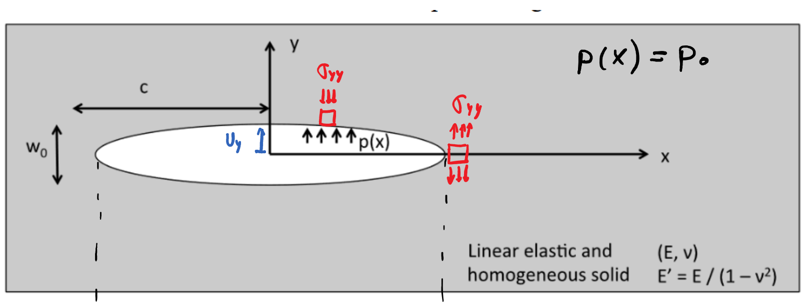


S_3: far-field least principal stress

P_{net}: net pressure

Displacements and stresses around a linear fracture (plane-strain)

Griffith solution for Navier's Eq. with a Linear Elastic Isotropic Solid



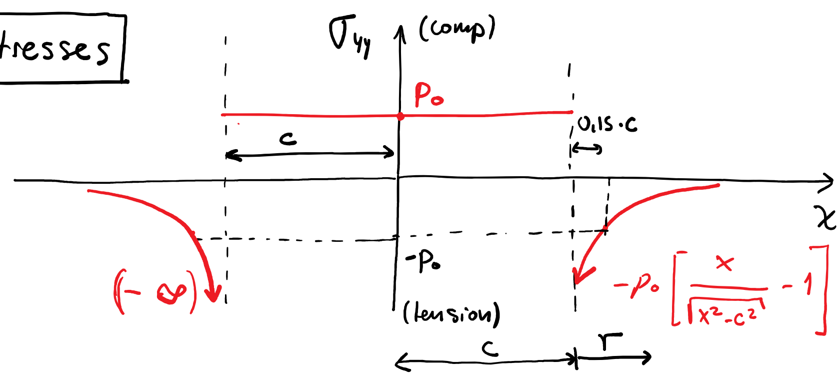
Displacements

$$U_y(x,0) = \begin{cases} \dfrac{2P_o}{E'}\sqrt{c^2-x^2}\ ; & 0 \le x \le c \\ 0\ ; & x > c \end{cases}$$

$$\Downarrow$$

$$W_o = 2\,U_y(0,0) = \frac{4P_oc}{E'}$$

$\boxed{\text{Stresses}}$

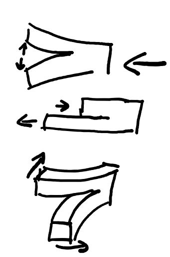


$$\sigma_{yy} = \begin{cases} P_o\ ; & 0 \le x < c \\ -P_o\left[\dfrac{x}{\sqrt{x^2-c^2}} - 1\right]; & x \ge c \end{cases}$$

Stress intensity factor

$$\rightarrow K_I = \lim_{r\to 0^+}\left[\sqrt{2\pi r}\cdot\left(-\sigma_{yy}(c+r,0)\right)\right] \leftarrow$$

Fracture Modes
- I : open-mode
- II : shear in-plane
- III : shear out-of-plane



$$= \lim_{r\to 0^+}\left[\sqrt{2\pi r}\cdot\left(+P_o\left[\frac{c+r}{\sqrt{(c+r)^2-c^2}} - 1\right]\right)\right] \quad ; x = c + r$$

$$\lim_{r\to 0^+}\left[\cdots \left(\cdots \sqrt{(c+r)^2-c^2} \cdots \right)\right]$$

$$= p_0\sqrt{2\pi}\cdot\lim_{r\to 0^+}\left[r^{1/2}\cdot\left(\frac{c+r}{\sqrt{2cr+r^2}}-1\right)\right]$$

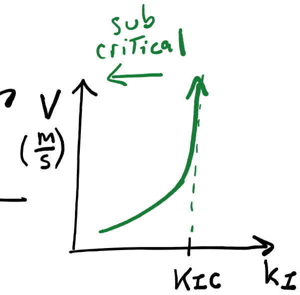
$$\boxed{K_I = p_0\cancel{\sqrt{2}}\sqrt{\pi}\left(\frac{c}{\cancel{\sqrt{2}}\sqrt{c}}\right) = p_0\sqrt{\pi c}}$$

p_0: constant pressure

LEFM criterion for fracture propag.

$$\begin{cases} K_I \geq K_{IC} \Rightarrow \text{fracture propagation} \\ K_I < \underbrace{K_{IC}} \Rightarrow \text{no fracture propagation} \end{cases}$$

Fracture toughness (c: critical) ←



Typical values of

$K_{IC} \sim 0.2 - 2$ $\underline{MPa\cdot m^{1/2}}$ (geological materials)

↑ Mode 1

└ measured through "notch experiments"

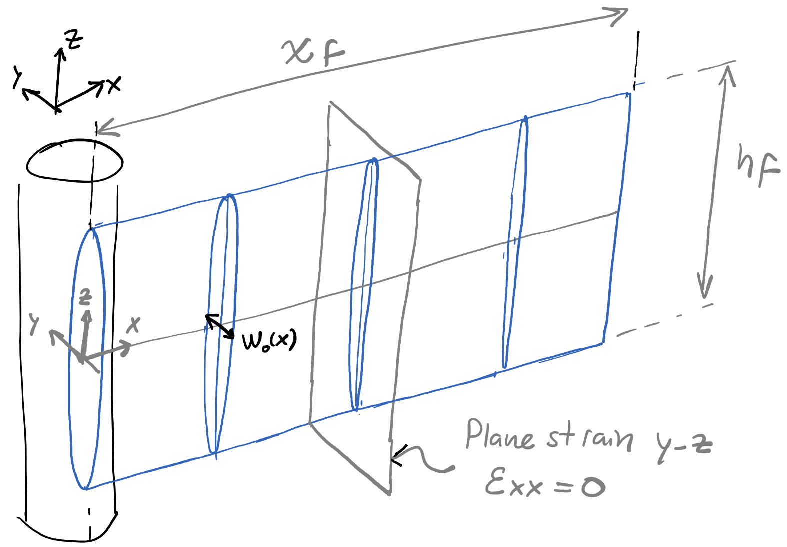
e.g.: semicircular bending experiment

Solution to coupled problem of a fluid-driven fracture PKN

Wednesday, November 18, 2020 5:40 PM

Objective: understand methodology to solve a fluid-driven fracture problem

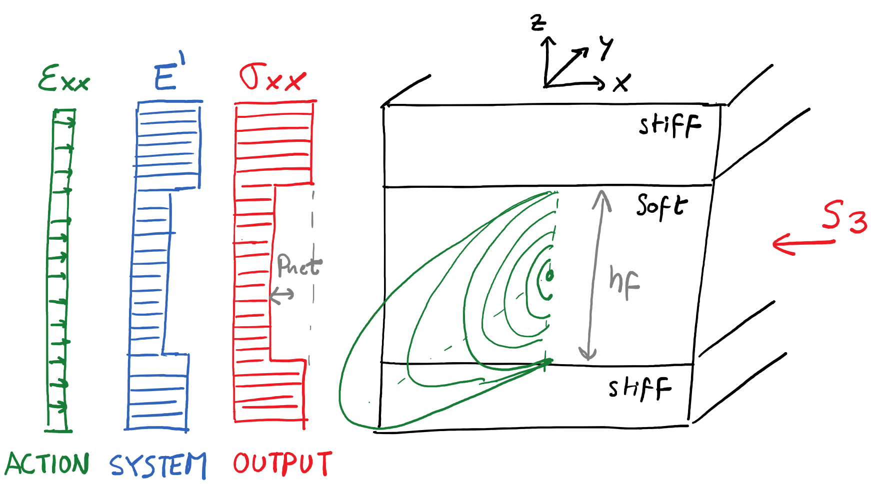
Example: PKN Model (Perkins, Kern, Nordgren)



Main assumptions

- h_F constant ✓
- $x_F > h_F$ ✓
- Plane strain y-z ✓
- $K_{Ic} = 0$ ✓
- Geometry as in figure ✓

① Fracture height ← 1D stress solution



② Fracture width ← Griffith solution

Solution in plane y-z

$$W_o(x) = \frac{4\, P_n(x)\, \overbrace{C}^{h_f/2}}{E'} = \frac{2\, P_n(x)\, h_F}{E'} \qquad \boxed{Eq. 1}$$

③ Fluid flow within fracture ← laminar flow + Newtonian fluid

Flow along x through elliptical channel

$$\frac{dP}{dx} = -\frac{64\,\nu\;q(x)}{\pi\,[W_0(x)]^3\,h_f} \qquad \boxed{Eq.\ 2}$$

④ Mass conservation ← fluid in fracture + leak-off

$$V_i = V + \underbrace{\cancel{V_L}}_{=0} \qquad \boxed{Eq.\ 3}$$

$V_L = 2A_F C_L \sqrt{t}$ or Darcy ✓

$$V_i = i \cdot t \qquad \boxed{Eq.\ 4}$$

i: injection rate (constant) ✓

$$q(x) = \underbrace{i}_{\text{one-wing}} \qquad \boxed{Eq.\ 5}$$

no leak-off

$\boxed{Eq\ 1+2+5}$ → Pressure gradient along frac

$$\frac{dP}{dx} = -\frac{64 \cdot \nu \cdot i}{\pi\,[2\,h_f\,P_n(x)\,E'^{-1}]^3 \cdot h_f}$$

$$\frac{dP}{dx} = -\frac{8\nu\,i\,E'^3}{\pi\,h_f^4\,[P_n(x)]^3}$$

→ Integrate $P_n(x)$

$$\int_{P_n(0)}^{P_n(x_f)} [P_n(x)]^3\,dP = -\int_0^{x_f} \frac{8\nu\,i\,E'^3}{\pi\,h_f^4}\,dx$$

$$\frac{\cancel{[P_n(x_f)]^4}^{=0}}{4} - \frac{[P_n(0)]^4}{4} = -\frac{8\nu\,i\,E'^3}{\pi\,h_f^4}(x_f - \cancel{0})$$

$$P_n(x=0) = \left(\frac{32\,\nu\,i\,E'^3\,x_F}{\pi\,h_f^4}\right)^{1/4} \qquad \boxed{Eq.\ 6}$$

$$W_0(x=0) = \left(\frac{512\,\nu\,i\,x_F}{\pi\,E'}\right)^{1/4} \qquad \boxed{Eq.\ 7}$$

+ Eq. 1

$$W_o(x=0) = \left(\frac{512\,\mu\, i\, x_f}{\pi\, E'}\right)^{1/4} \qquad \boxed{Eq.\ 7}$$

Introducing time into equations

Geometry : $V_{frac} = x_f\, h_f\, \bar{w}$

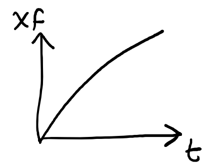
→ where $\bar{w} = \frac{\pi}{5} W_o(x=0)$

Material balance: $V_{frac} = i \cdot t$

$$x_f = \frac{i \cdot t}{h_f \left(\frac{\pi}{5} \cdot W_o(x=0)\right)} \qquad \boxed{Eq.\ 8}$$

PKN Model

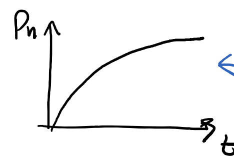
$$x_f = \left(\frac{625}{512 \cdot \pi^3}\right)^{1/5} \left(\frac{i^3 \cdot E'}{h_f^4 \cdot \mu}\right)^{1/5} \cdot t^{4/5}$$



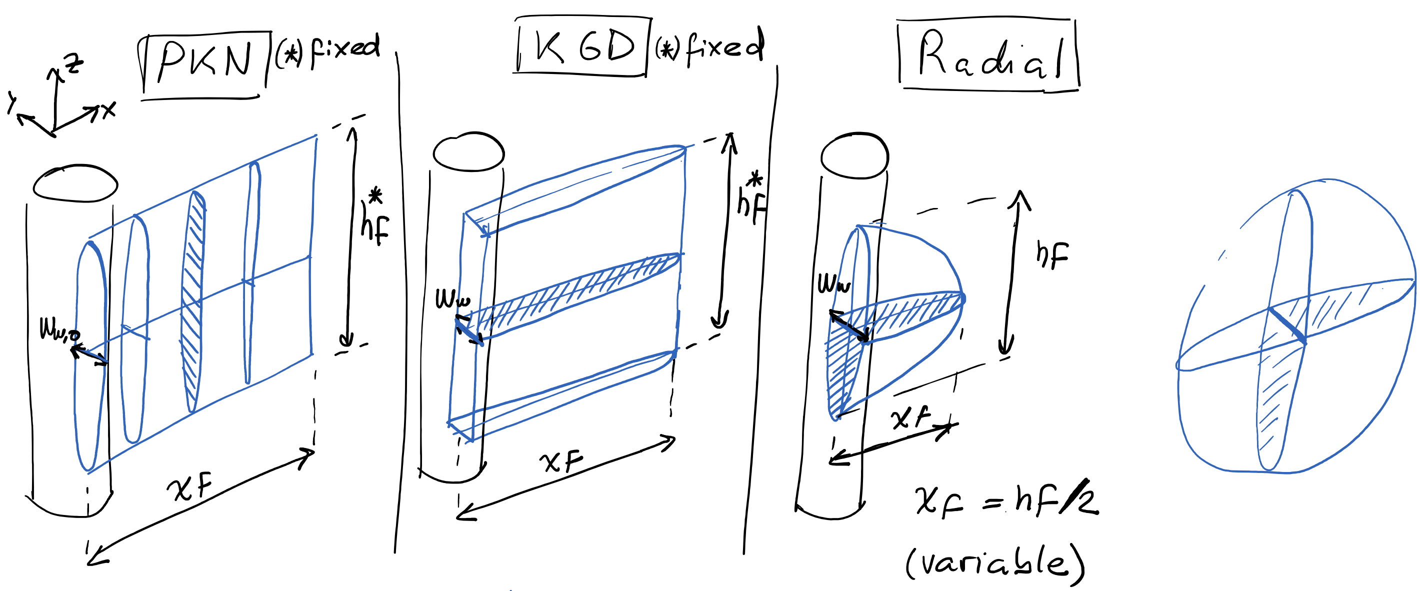
← Eq 7 + 8

$$W_o(x=0) = \left(\frac{2560}{\pi^2}\right)^{1/5} \left(\frac{i^2\, \mu}{E'\, h_f}\right)^{1/5} \cdot t^{1/5}$$

$$P_n(x=0) = \left(\frac{80}{\pi^2}\right)^{1/5} \left(\frac{E'^4\, i^2\, \mu}{h_f^6}\right)^{1/5} \cdot t^{1/5}$$



← + Eq. 6



plane-strain plane

Demo final PKN equations

$$P_n(x=0) = \left(\frac{32\,\mu\, i\, E'^3\, x_f}{\quad}\right)^{1/4} \qquad \boxed{Eq.\ 6}$$

$$P_n(x=0) = \left(\frac{32\,\nu\, i\, E'^3\, x_F}{\pi\, h_F^4}\right)^{1/4} \qquad \boxed{Eq.6}$$

$$W_0(x=0) = \left(\frac{512\,\nu\, i\, x_F}{\pi\, E'}\right)^{1/4} \qquad \boxed{Eq.7}$$

$$x_F = \frac{i\cdot t}{h_F\left(\frac{\pi}{5}\cdot W_0(x=0)\right)} \qquad \boxed{Eq.8}$$

$\boxed{7+8}$

$$\frac{i\cdot t}{h_F\,\frac{\pi}{5}\cdot x_F} = \left[\frac{512\,\nu\, i}{\pi\, E'}\right]^{1/4} x_f^{1/4}$$

$$x_f^{5/4} = \frac{i\, t}{h_f\,\frac{\pi}{5}}\left(\frac{\pi\, E'}{512\,\nu\, i}\right)^{1/4}$$

$$x_f = \left(\frac{i^{3/4}\cdot 5\, E'^{1/4}}{h_f\,\pi^{3/4}\, 512^{1/4}\,\nu^{1/4}}\right)^{4/5}\cdot t^{4/5}$$

$$\boxed{x_f = \left(\frac{625}{\pi^3\cdot 512}\right)^{1/5}\left(\frac{i^3\cdot E'}{h_f^4\cdot \nu}\right)^{1/5} t^{4/5}} \qquad (9)$$

$$\rightarrow \frac{d x_f}{d t} = \frac{4}{5}\left(\frac{625}{512\cdot\pi^3}\right)^{1/5}\left(\frac{i^3\cdot E'}{h_f^4\cdot\nu}\right)^{1/5}\cdot t^{-1/5} \Rightarrow$$

$\boxed{7+9}$

$$W_0(x=0) = \left(\frac{512\,\nu\, i}{\pi\, E'}\right)^{1/4}\cdot x_f^{1/4}$$

$$= \left(\frac{512\,\nu\, i}{\pi\, E'}\right)^{1/4}\left[\left(\frac{625}{512\cdot\pi^3}\right)^{1/5}\cdot\left(\frac{i^3\cdot E'}{h_f^4\cdot\nu}\right)^{1/5}\cdot t^{4/5}\right]^{1/4}$$

$$= \left(\frac{512\, N\, i}{\pi\, E'}\right) \cdot \left[\left(\frac{625}{512\cdot\pi^3}\right) \cdot \left(\frac{\quad}{h_f^4\cdot N}\right) \cdot t\right]$$

$$= \left[\left(\frac{512}{\pi}\right)^{\frac{5}{4}\cdot\frac{1}{5}} \left(\frac{625}{512\cdot\pi^3}\right)^{\frac{1}{4}\cdot\frac{1}{5}}\right]\left[\left(\frac{N\, i}{E'}\right)^{\frac{5}{4}\cdot\frac{1}{5}} \left(\frac{i^3\cdot E'}{h_f^4\cdot N}\right)^{\frac{1}{4}\cdot\frac{1}{5}}\right] t^{1/5}$$

$$= \left[\frac{512\cdot 5}{\pi^2}\right]^{1/5} \cdot \left[\frac{N\cdot i^2}{E'\cdot h_f}\right]^{1/5} \cdot t^{1/5}$$

$$\boxed{W_0(x=0) = \left(\frac{2560}{\pi^2}\right)^{1/5} \left(\frac{N\cdot i^2}{E'\, h_f}\right)^{1/5} \cdot t^{1/5}} \qquad (10)$$

$\boxed{6+9}$

$$P_n(x=0) = \left(\frac{32\, N\, i\, E'^3}{\pi\, h_f^4}\right)^{1/4} \left[\left(\frac{625}{\pi^3\cdot 512}\right)^{1/5} \left(\frac{i^3\cdot E'}{h_f^4\cdot N}\right)^{1/5} t^{4/5}\right]^{1/4}$$

$$= \left[\left(\frac{32}{\pi}\right)^{\frac{5}{4}\cdot\frac{1}{5}} \left(\frac{625}{\pi^3\, 512}\right)^{\frac{1}{4}\cdot\frac{1}{5}}\right]\left[\left(\frac{N\cdot i\cdot E'^3}{h_f^4}\right)^{\frac{5}{4}\cdot\frac{1}{5}} \cdot \left(\frac{i^3\cdot E'}{h_f^4\cdot N}\right)^{\frac{1}{4}\cdot\frac{1}{5}}\right] t^{1/5}$$

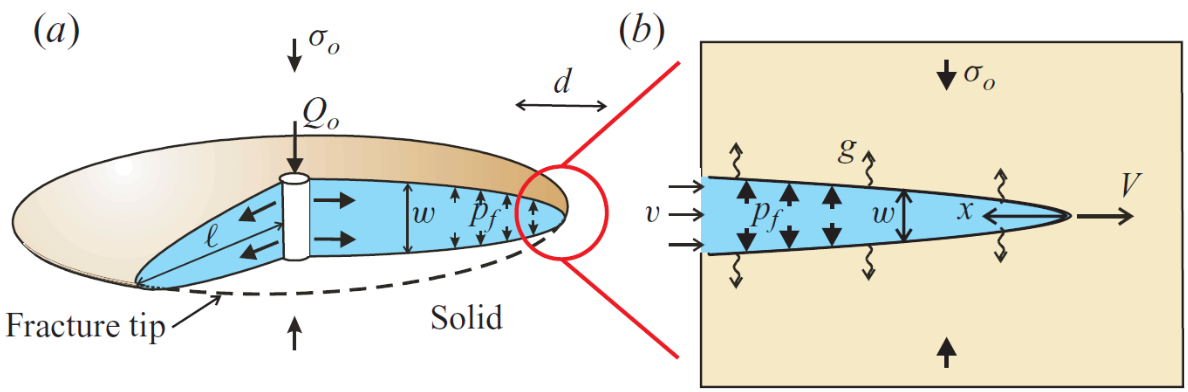
$$= \left[\frac{2^{5\cdot 5/4}\cdot 5^{4/4}}{\pi^{5/4}\,\pi^{\frac{3}{4}}\cdot 2^{\frac{9}{4}}}\right]^{\frac{1}{5}} \cdot \left[\frac{N\cdot i^2\cdot E'^4}{h_f^6}\right]^{1/5} t^{1/5}$$

$$\boxed{P_n(x=0) = \left(\frac{2^4\cdot 5}{\pi^2}\right)^{\frac{1}{5}} \cdot \left(\frac{N\cdot i^2\cdot E'^4}{h_f^6}\right)^{1/5} \cdot t^{1/5}} \qquad (11)$$

Fluid-driven fractures in porous media

Friday, November 20, 2020 4:27 PM

Detournay, Bunger, Lecampion, et al.



Garagash et al. 2011

Material parameters

$E' = \frac{E}{1-\nu^2}$; Plane-strain modulus

$K' = \sqrt{\frac{32}{\pi}}\, K_{Ic}$; Toughness

$\mu' = 12\,\mu$; Fluid viscosity

$C' = 2\,C_L$; Leak-off coeff.

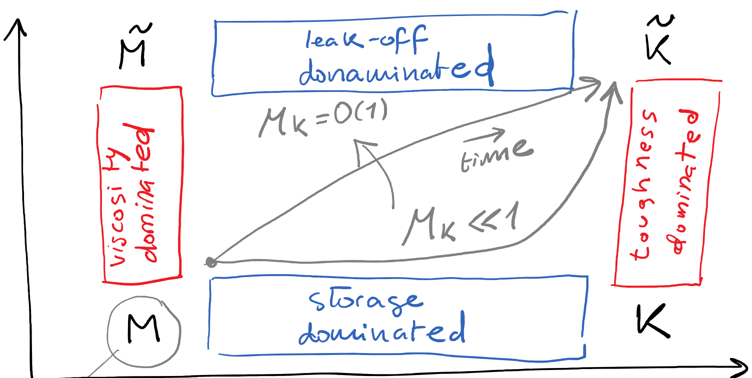
Fracture propagation "regimes" (zero-lag)

Fluid gone into rock / Fluid stored in frac.

Material balance competition

$\updownarrow$

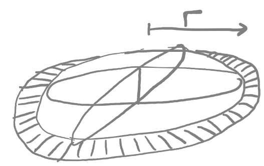
$V_i = V_f + V_{leak\text{-}off}$



low K_{Ic}
high μ

high K_{Ic}
low μ

$$\mathcal{M}_K^{radial} = \mu' \left(\frac{C'^4 E'^{11} Q_o}{K'^{14}} \right)^{1/3}$$



Work creating new rock surface / Work circulating fluid

Competitive dissipative processes

$\boxed{\text{Ex.}}$ M (storage, viscosity) solution for

$K' = 0$;
$C' = 0$;

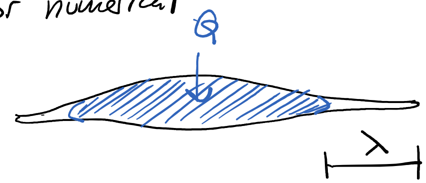
$\sim$ PKN derivation previous lecture

- Energy release rate $G = K_I^2 / E'$
  LEFM

Key points:

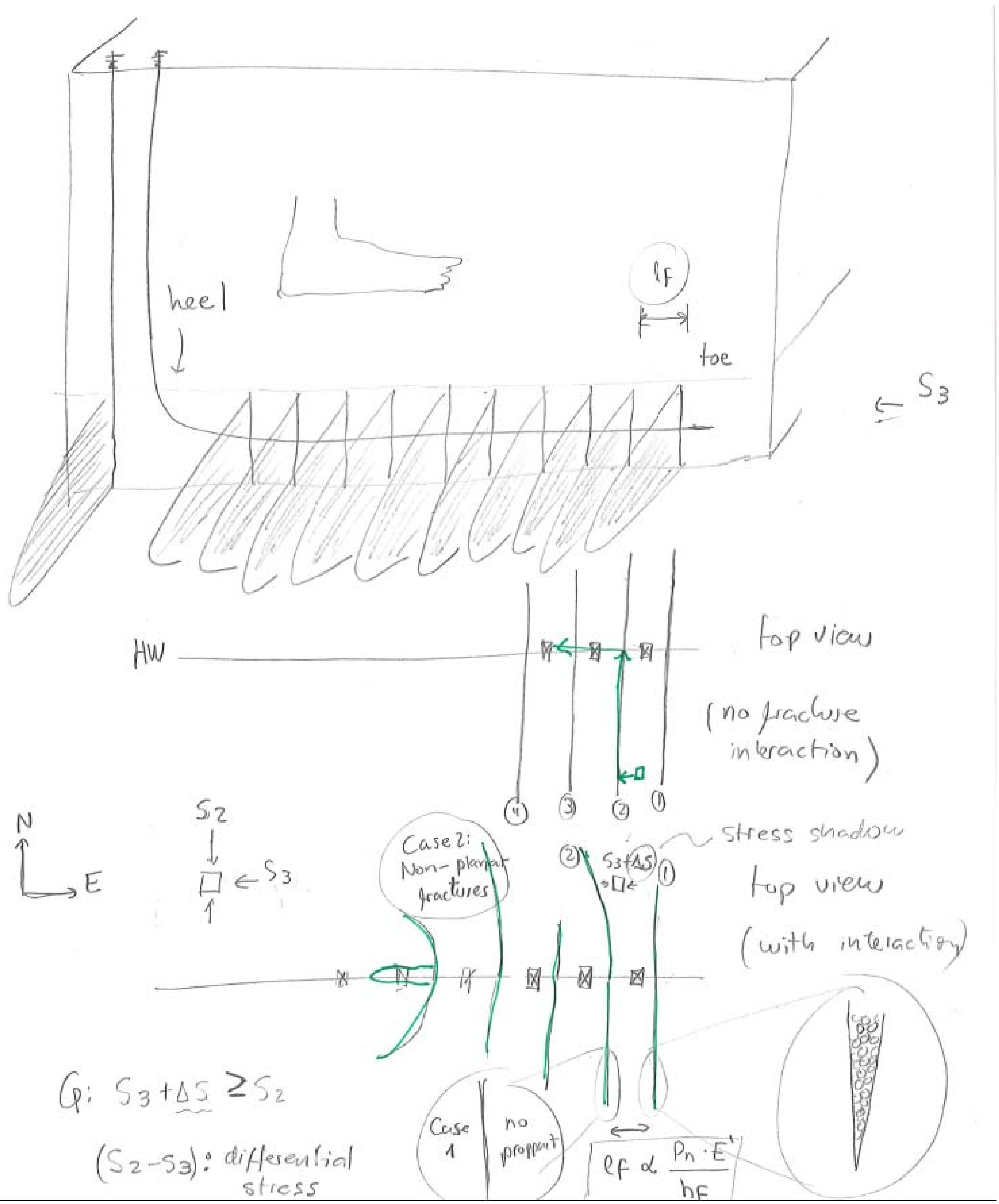
- Subsurface hydraulic fracturing
  $\updownarrow$
  viscosity dominated ✓
- scaling must be taken into account when comparing field and lab HF

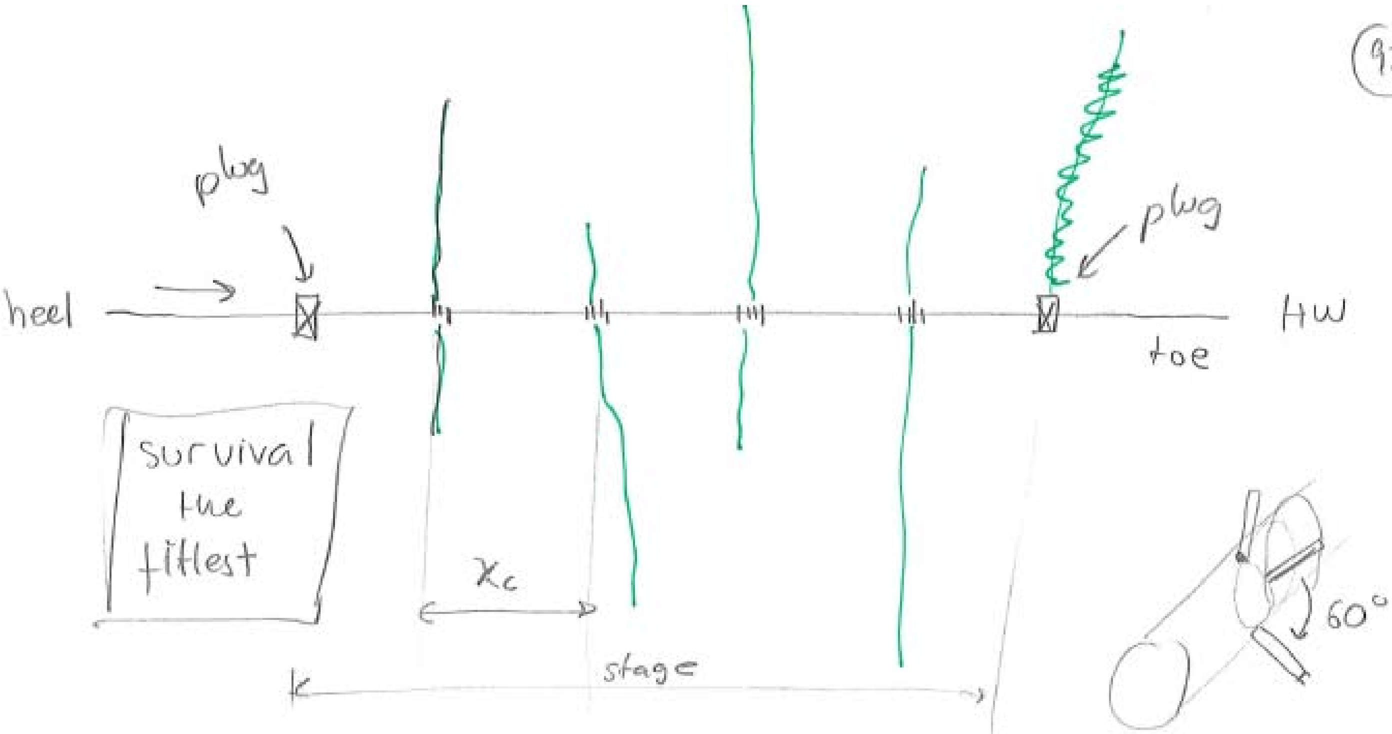
- scaling must be taken into account when comparing field and lab HF
- asymptotic solutions (extreme cases) are useful to conceptualize complex physical processes ✓
- analytical solutions are useful to define properly meshes for numerical solutions ✓
- consider fluid-lag



Multi-stage Hydraulic Fracturing

Monday, November 30, 2020 5:15 PM



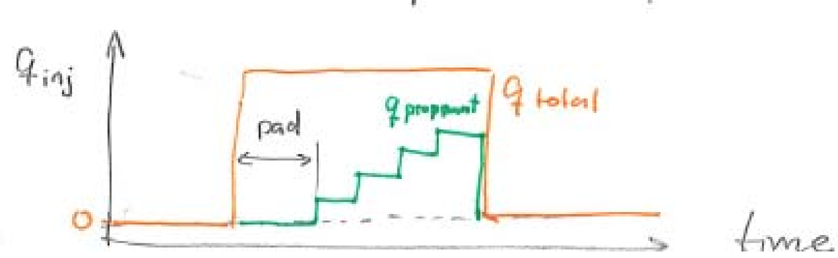


PERMIAN BASIN

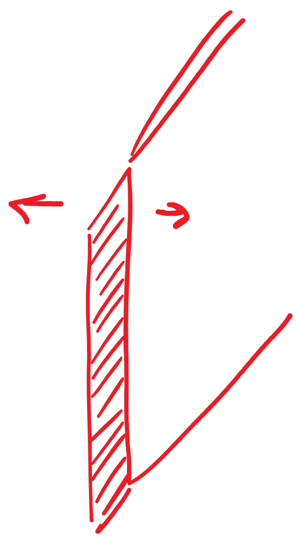
- lateral length ~ 10,000 ft
- 40 stages → stage length ~ 250 ft
- each stage → 4-15 clusters
- 6-20 perforations
- → 2000 $\frac{\text{lb (proppant)}}{\text{LF}}$
- → 2500 $\frac{\text{gallons (frac fluid)}}{\text{LF}}$

0.8 $\frac{\text{lb}}{\text{gall}}$ $\frac{\text{(proppant)}}{\text{(frac fluid)}}$

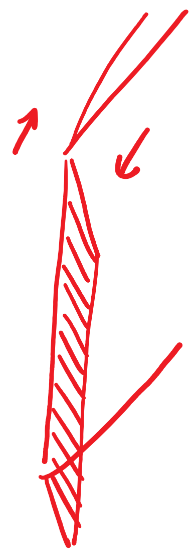
~ 7% $\frac{\text{vol (prop)}}{\text{vol (frac fluid)}}$



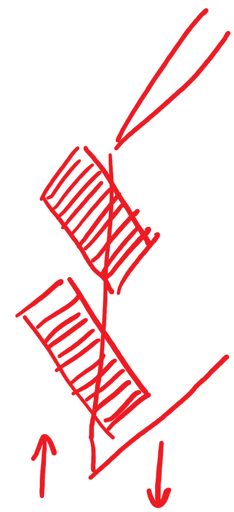
Mode 1



Mode 1+2

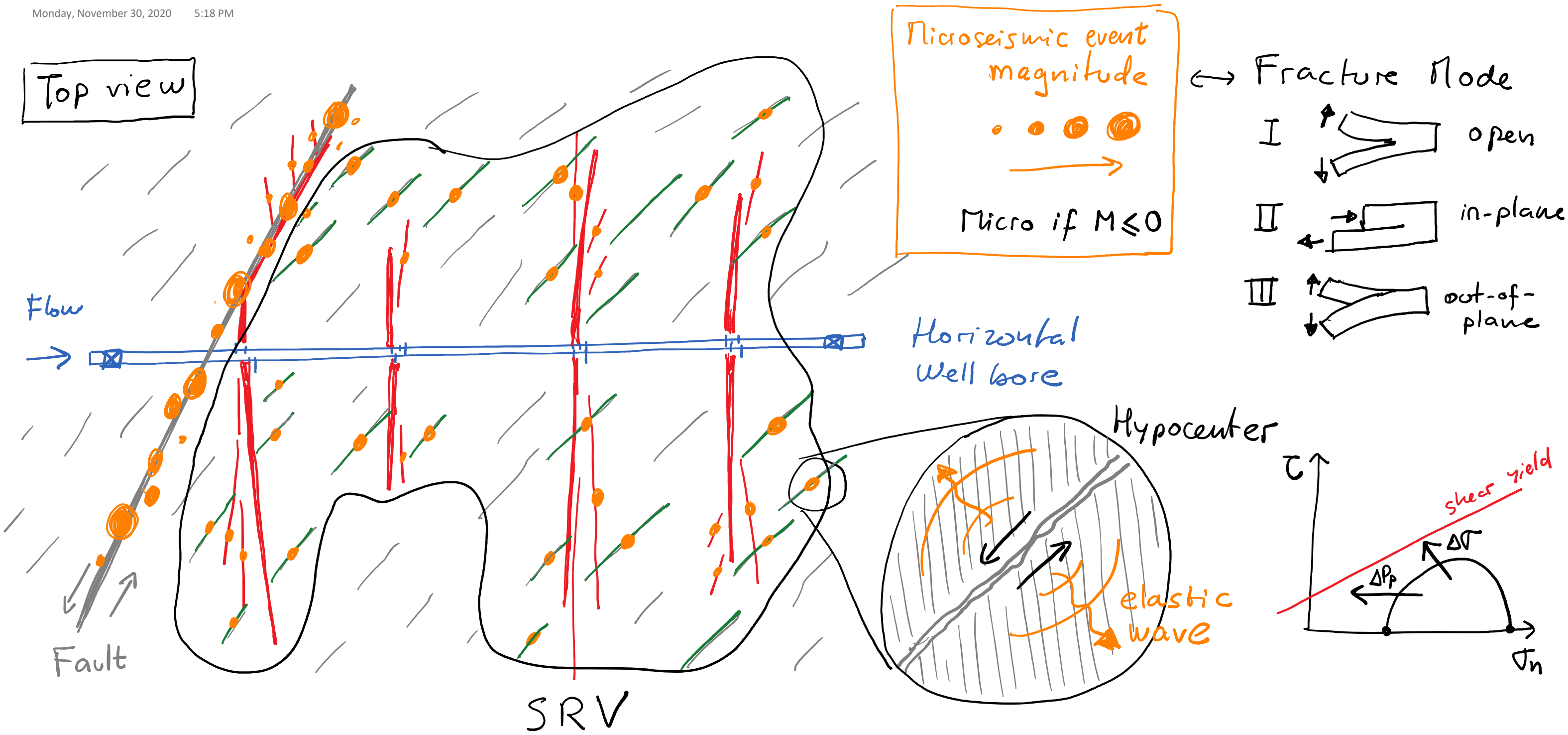


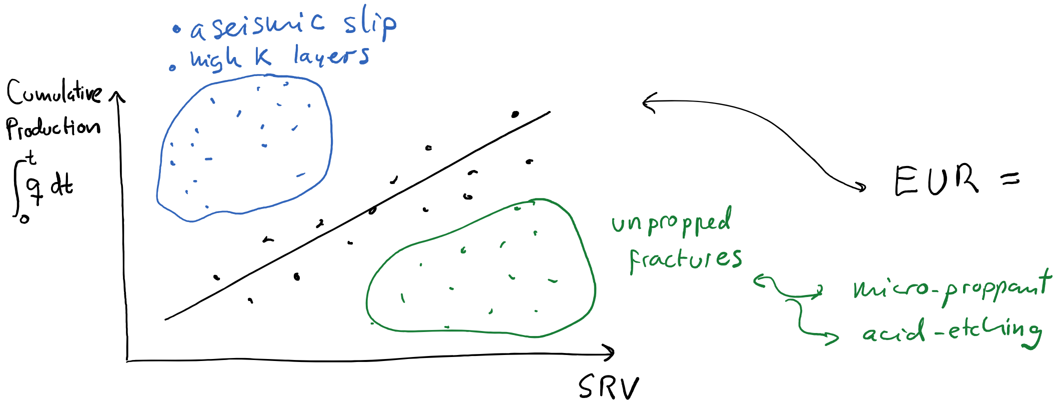
Mode 1+3



Micro-seismicity

Monday, November 30, 2020 5:18 PM

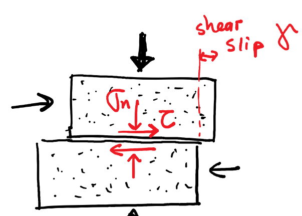




$$EUR = \frac{SRV \cdot \phi \cdot (1-S_w)}{B_o} \cdot RF$$

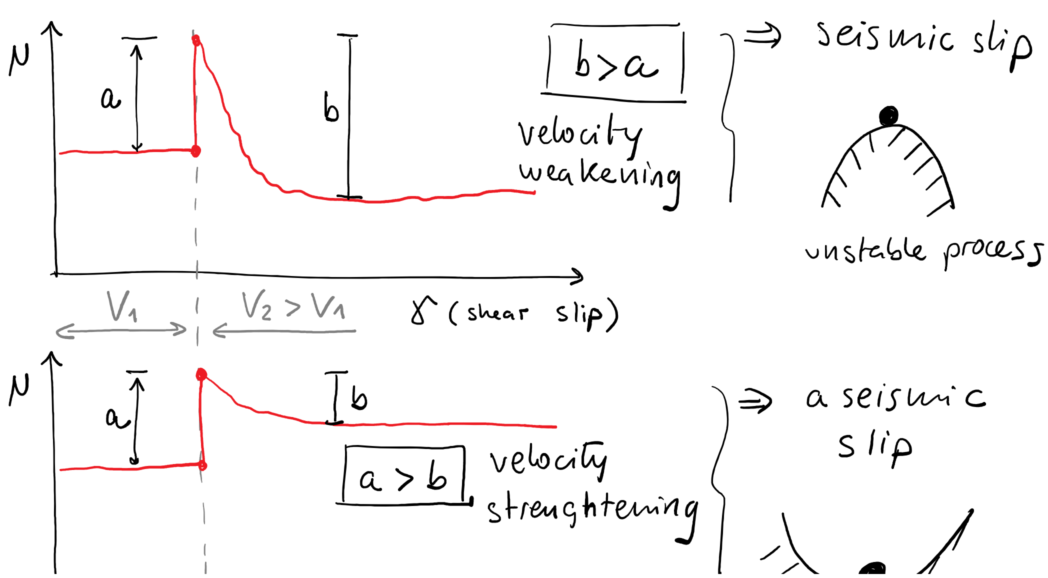
Seismic and aseismic slip

Direct shear experiment



$$\tau = N \cdot \sigma_n$$

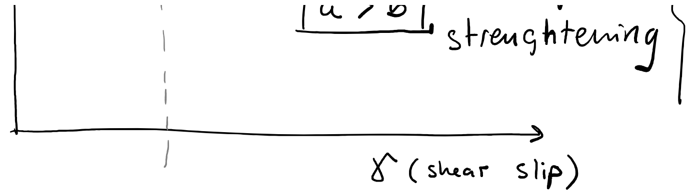
Apparent Friction Coefficient

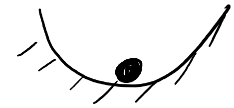


friction
Coefficient

$$\mu = f(\text{rock, slip velocity}, P, T)$$

$$V = \frac{d\gamma}{dt}$$

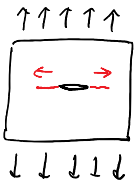




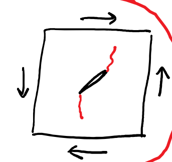
Subcritical fracture propagation

Wednesday, December 2, 2020 11:46 AM

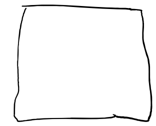
Mode I

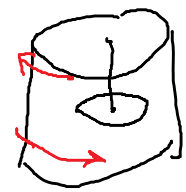


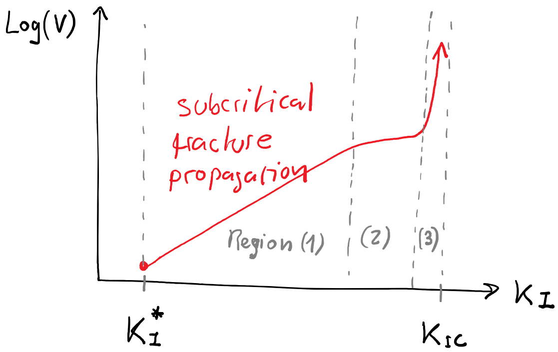
Mode II



Mode III







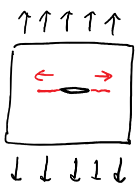
$\log V \propto K_I$

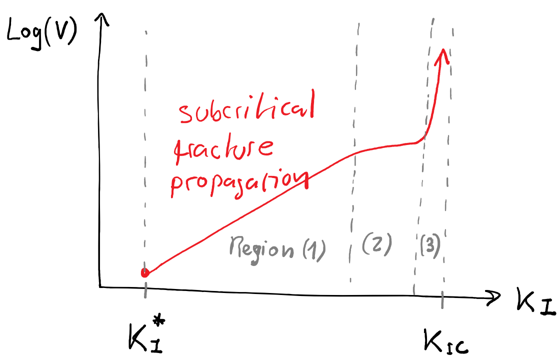
$$\rightarrow \boxed{V = A\left(\frac{K_I}{K_{IC}}\right)^n} \quad \text{Mode I}$$

$$\log V = \log A + n \cdot \log \frac{K_I}{K_{IC}}$$

↳ main parameters
- n: subcritical index (SCI)
- K_{IC}: (critical) fracture toughness

Mode I





$$\boxed{V = A\left(\frac{K_I}{K_{IC}}\right)^n} \quad \text{Mode I}$$