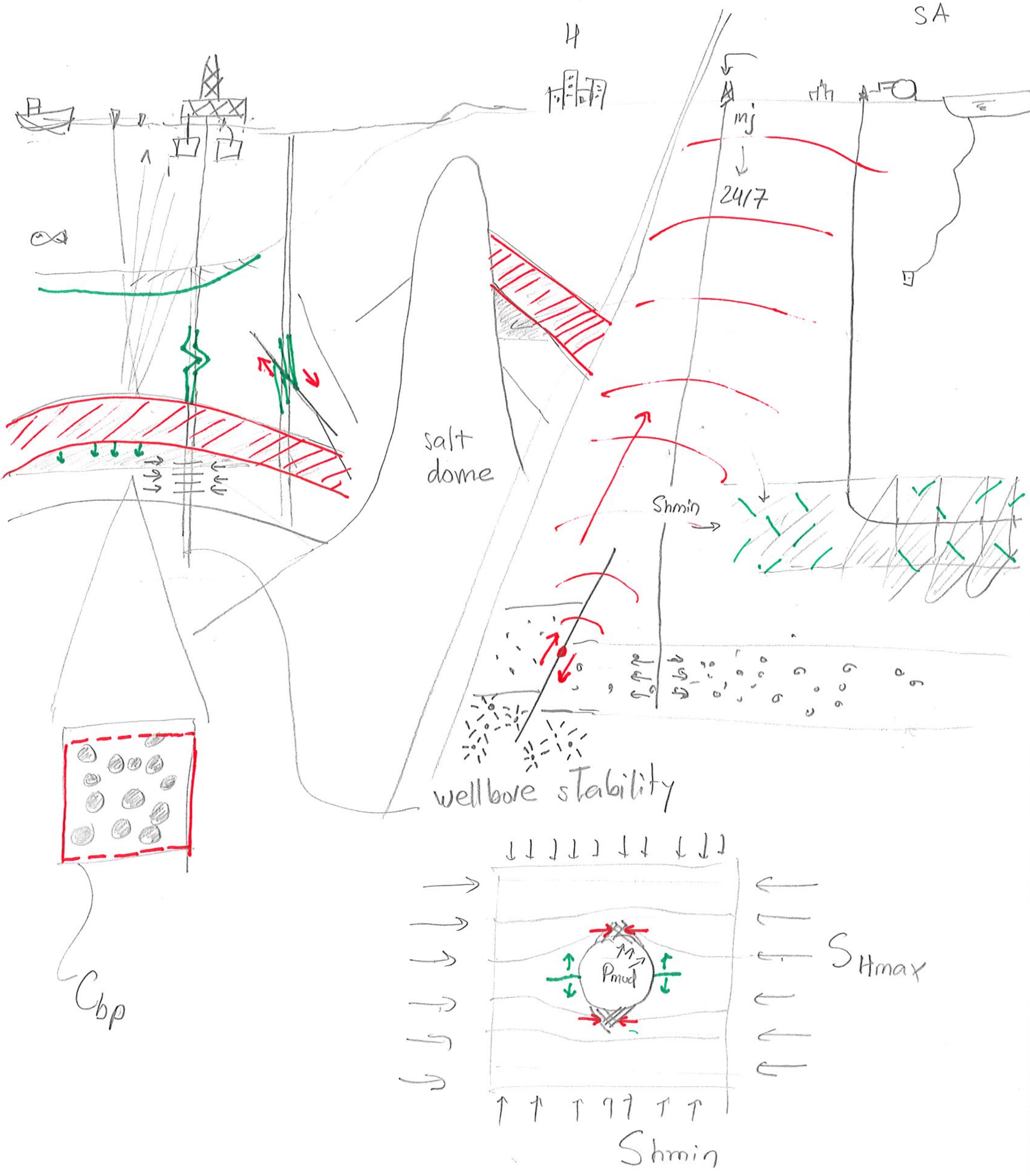
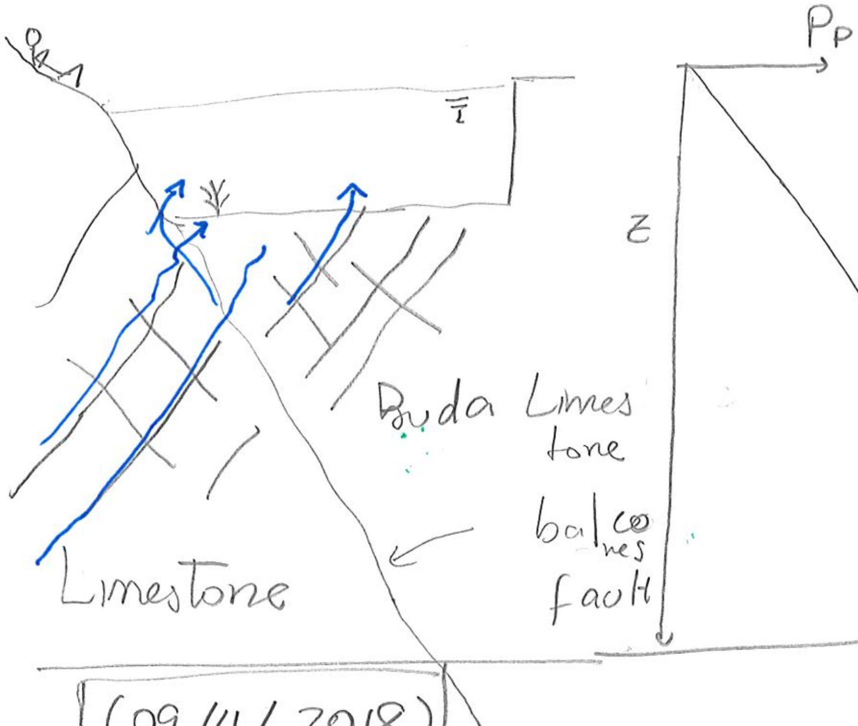
INTRODUCTION (30/8/2018)



Pore pressure



(09/4/2018)

$$P_p = z \underbrace{\rho_w \cdot g}$$

$$\rho_w \cdot g = 1000 \frac{kg}{m^3} \cdot 10 \frac{m}{s^2}$$

$$= 10000 \frac{N}{m^2} \cdot \frac{1}{m}$$

$$= 10^4 \; Pa \frac{1}{10^{-3} \, km}$$

$$= 10^7 \frac{Pa}{km}$$

SI units

$$\boxed{\rho_w g = 10 \frac{MPa}{km}}$$

10 MPa ~ 1500 psi

Hydrostatic gradient

$$P_p = \underbrace{z}_{\text{depth}} \cdot \underbrace{\rho_w \cdot g}_{\text{gradient}}$$

$\sim 0.44 \frac{psi}{ft}$

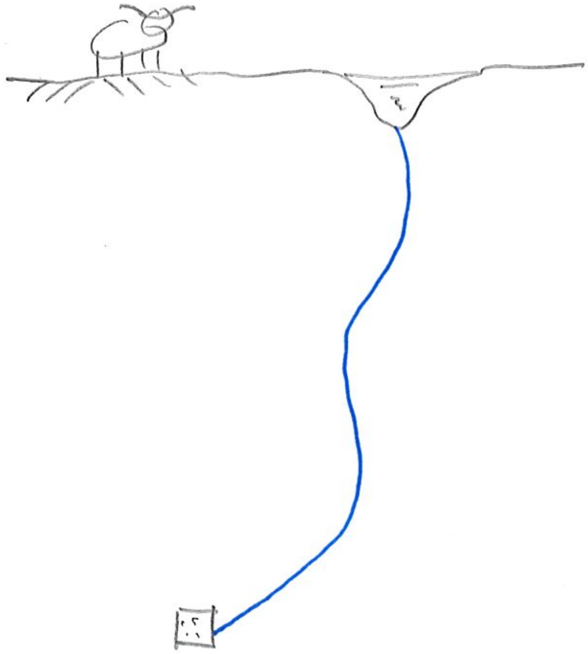
$$\rho_w = 62.4 \frac{lb}{ft^3}$$

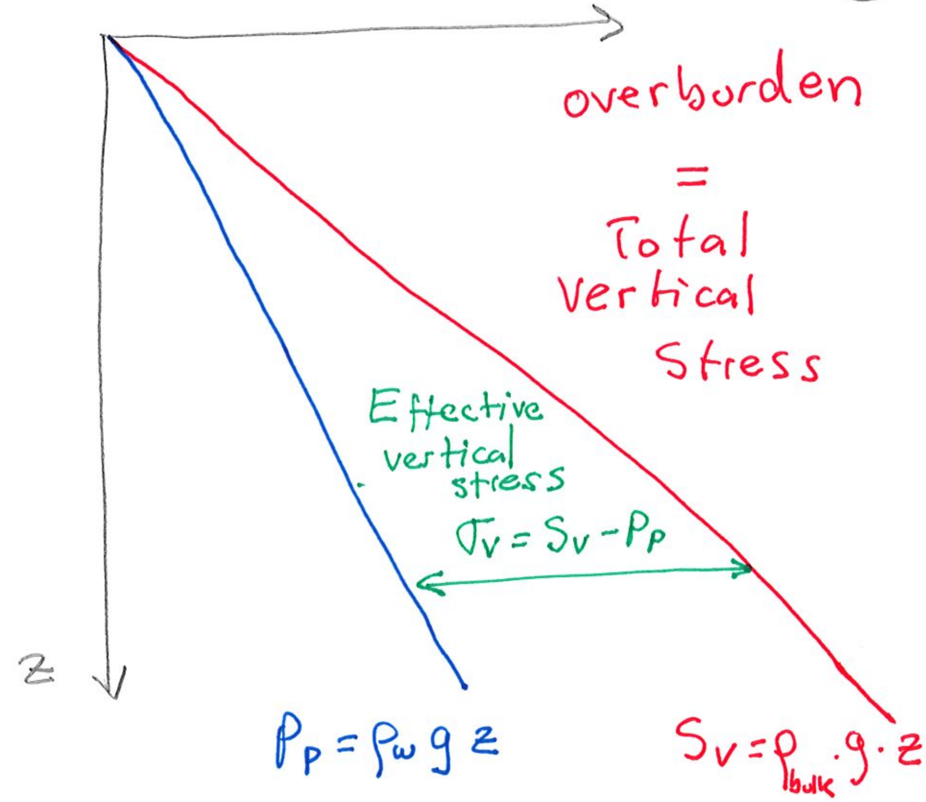
$$\rho_w g = 62.4 \frac{\vec{lb}}{ft^3}$$

$$= 62.4 \frac{\vec{lb}}{(12 in)^2} \frac{1}{ft}$$

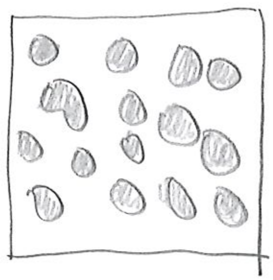
$$\boxed{\rho_w g = 0.433 \frac{psi}{ft}}$$

field units





$$\underbrace{\rho_{m1}\cdot C_{m1} + \rho_{m2}\cdot C_{m2}} + \dots + \rho_{mi}\cdot C_{mi}$$



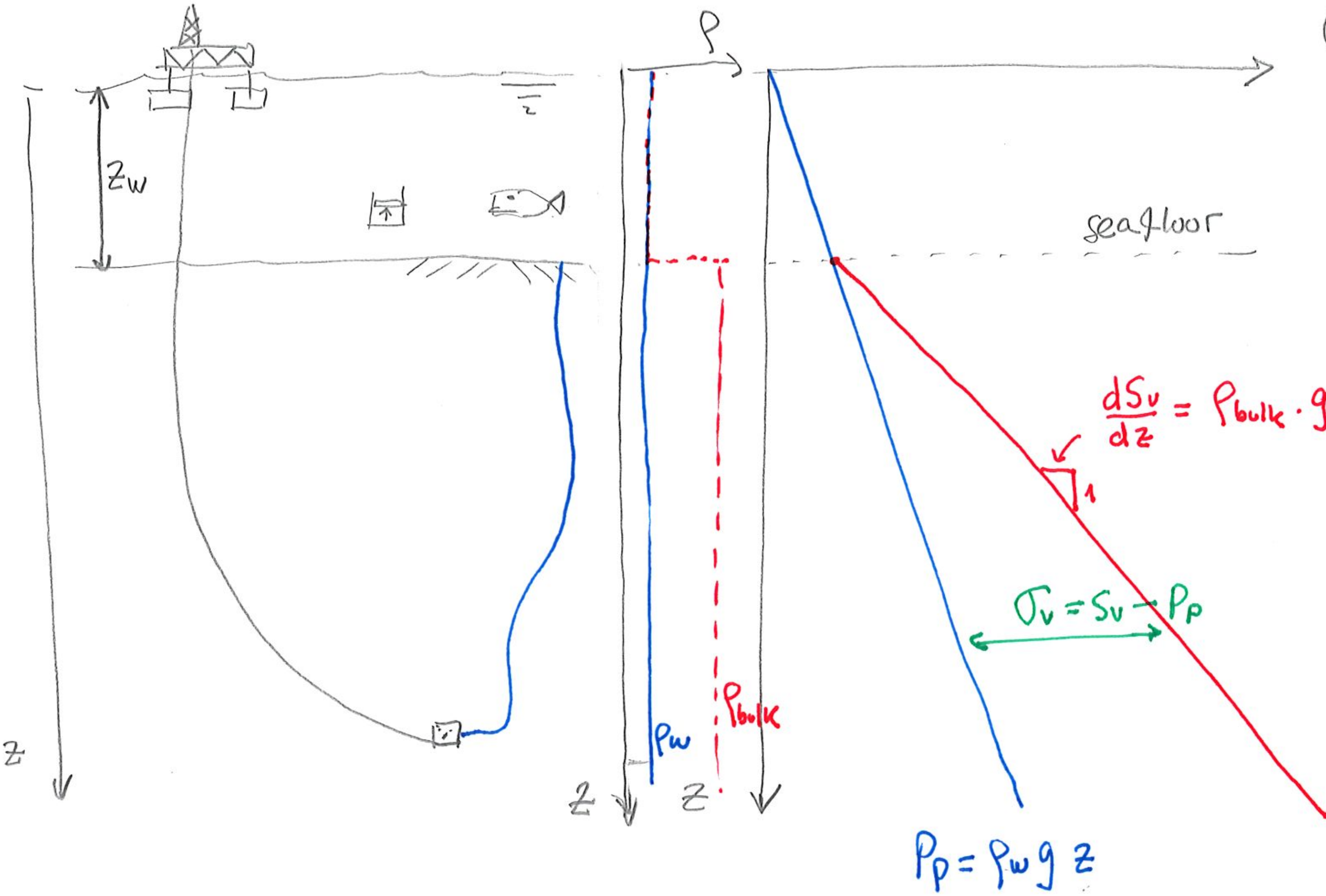
$$\rho_{bulk} = \rho_m (1-\phi) + \rho_w (\phi)$$

sandstone
$$\begin{cases} \phi = 0.20 \\ \rho_m = \rho_{quartz} = 2.65 \text{ g/cc} = 2650 \text{ kg/m}^3 \\ \rho_w = 1000 \text{ kg/m}^3 \end{cases}$$

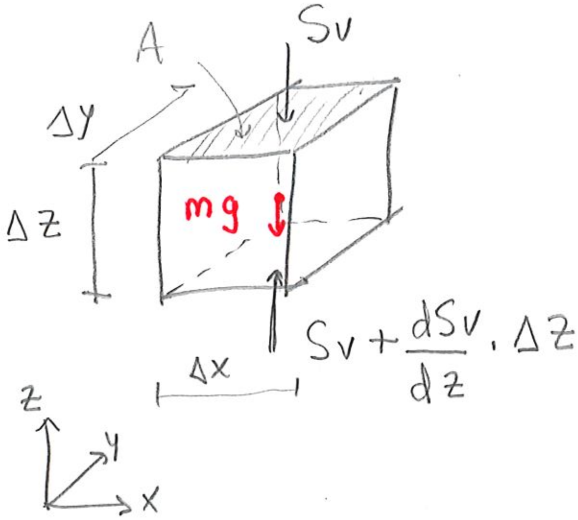
$$\rho_{bulk} = 2320 \text{ K/m}^3$$

total vertical stress gradient $\boxed{\rho_{bulk}\, g \cong 23 \text{ MPa/Km}}$ — 1 psi/ft

hydrostatic Pore pressure gradient $\boxed{\rho_w \cdot g = 10 \text{ MPa/Km}}$ — 0.433 $\frac{psi}{ft}$



$$\boxed{S_v = \rho_w g \cdot z_w + \rho_{bulk} \cdot g \, (z - z_w)}$$



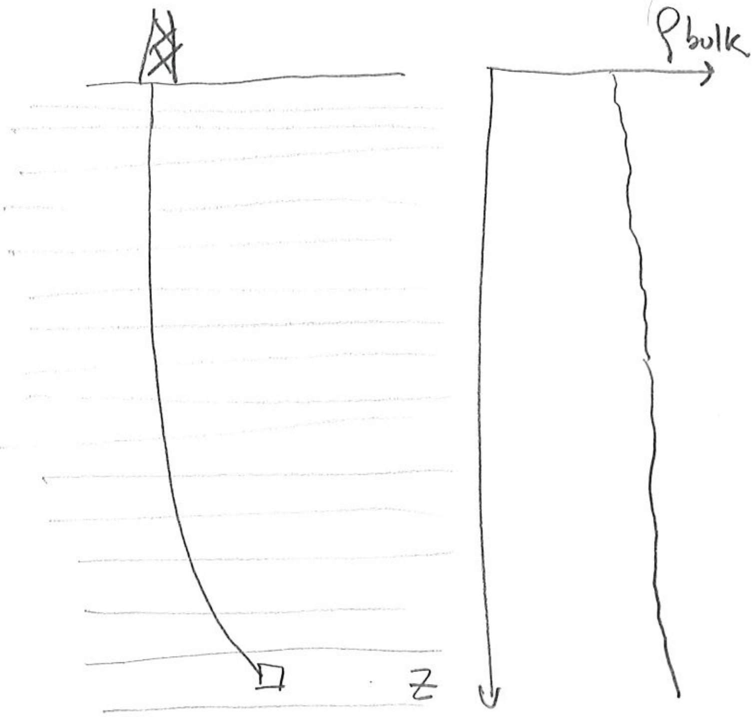
$$\sum F_z = 0$$

$$= -S_v \cdot A - mg + \left(S_v + \frac{dS_v}{dz} \cdot \Delta z\right) \cdot A$$

$$= \cancel{-S_v} \cdot \Delta x \cdot \Delta y - \rho_{bulk} \Delta x \Delta y \Delta z g$$

$$+ \left(\cancel{S_v} + \frac{dS_v}{dz} \Delta z\right) \Delta x \Delta y$$

$$\boxed{\frac{dS_v}{dz} = \rho_{bulk} \cdot \rho}$$



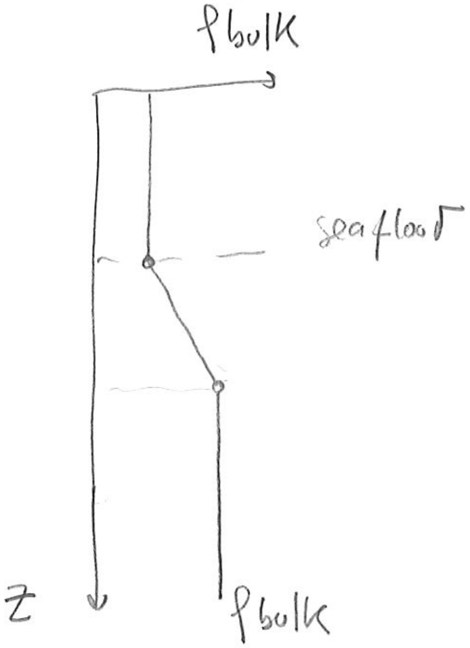
$$\frac{dS_v}{dz} = \rho_{bulk}\, g$$

$$\int_{S_v(z=0)}^{S_v(z)} dS_v = \int_{z=0}^{z} \rho_{bulk}(z)\, g \cdot dz$$

Assume $\{\ S_v(z=0) = 0$

$$S_v(z) - \cancel{S_v(z=0)}^{\,0}$$

$$\boxed{S_v(z) = \int_0^z \rho_{bulk}(z)\, g\, dz}$$

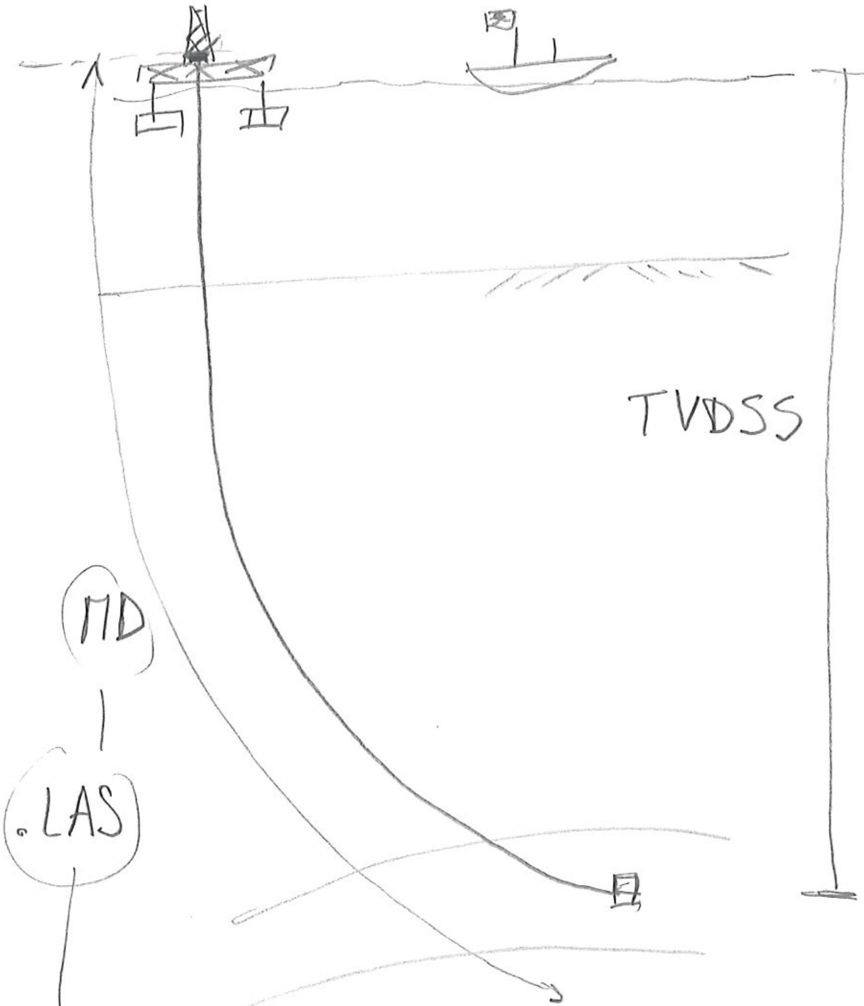


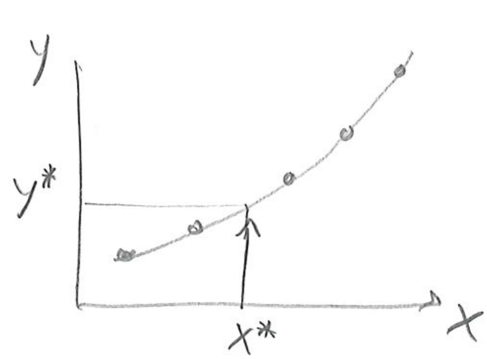
$$S_v(z_i) = \sum_{z=0}^{z_i} \frac{\rho_{bulk}(z_i) + \rho_{bulk}(z_{i-1})}{2} \cdot g \cdot \Delta z$$

matlab, python $\}\rightarrow$ for loop

excel $\{$

z	Sv
	↑
	$= T + \frac{dS_v}{dz} \cdot \Delta z$
	⋮





$$TVDSS = f(MD)$$

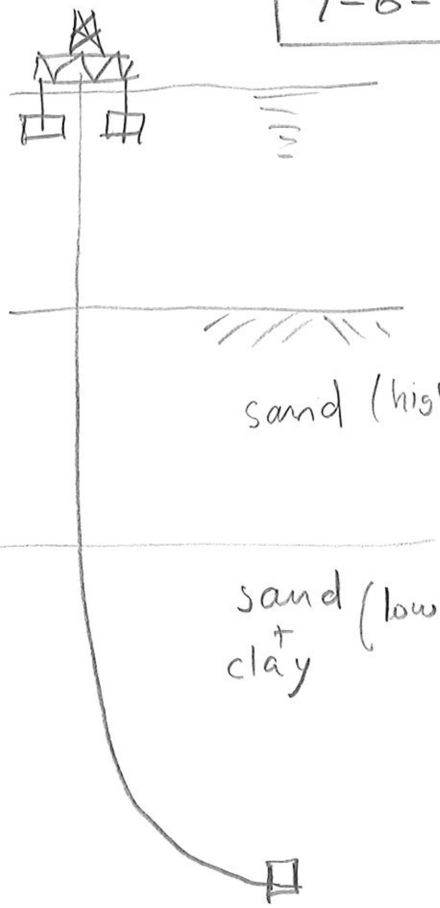
$$y^* = \text{interp}(\underline{x}, \underline{y}, x^*)$$

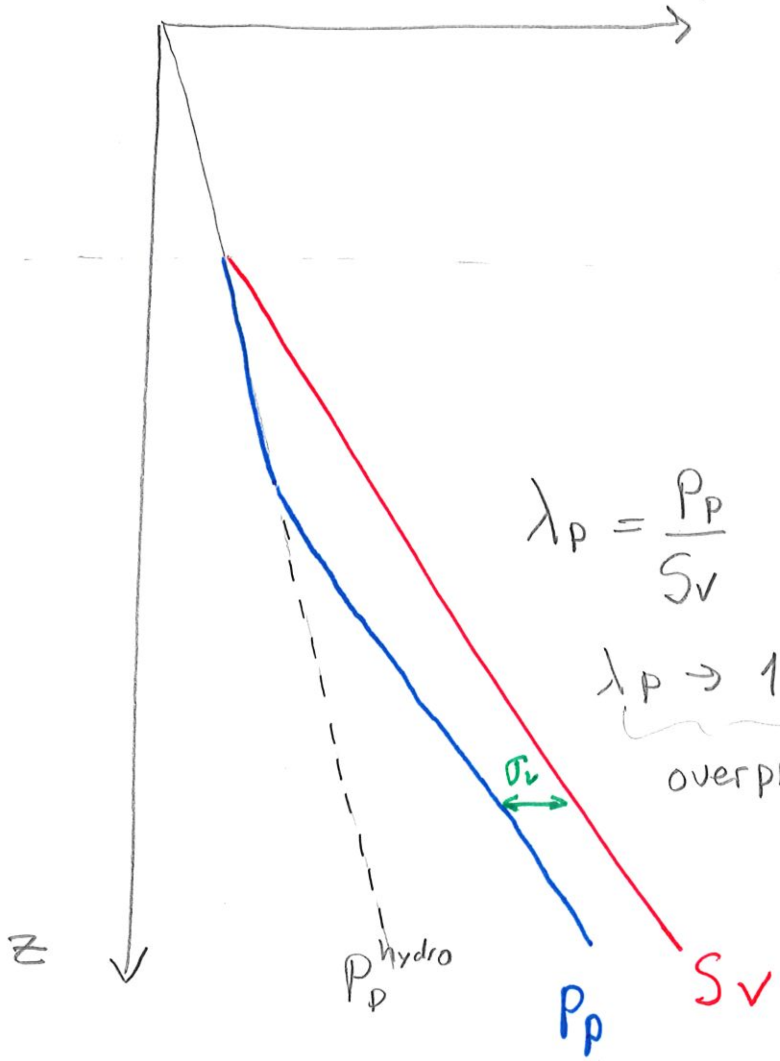
.DEV

MD [ft]	RHOB	GRR	DT

MD	TVDSS	X	Y

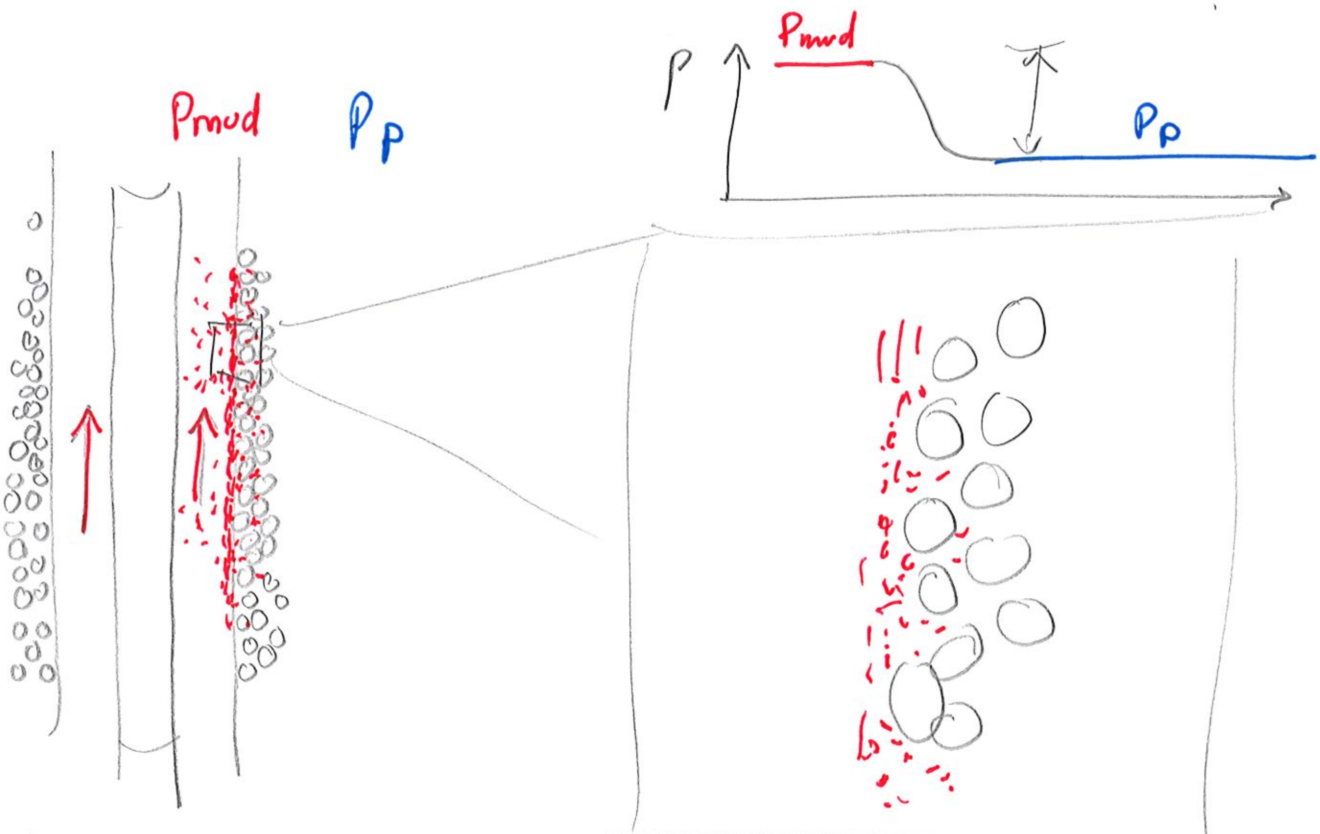
9-6-2018

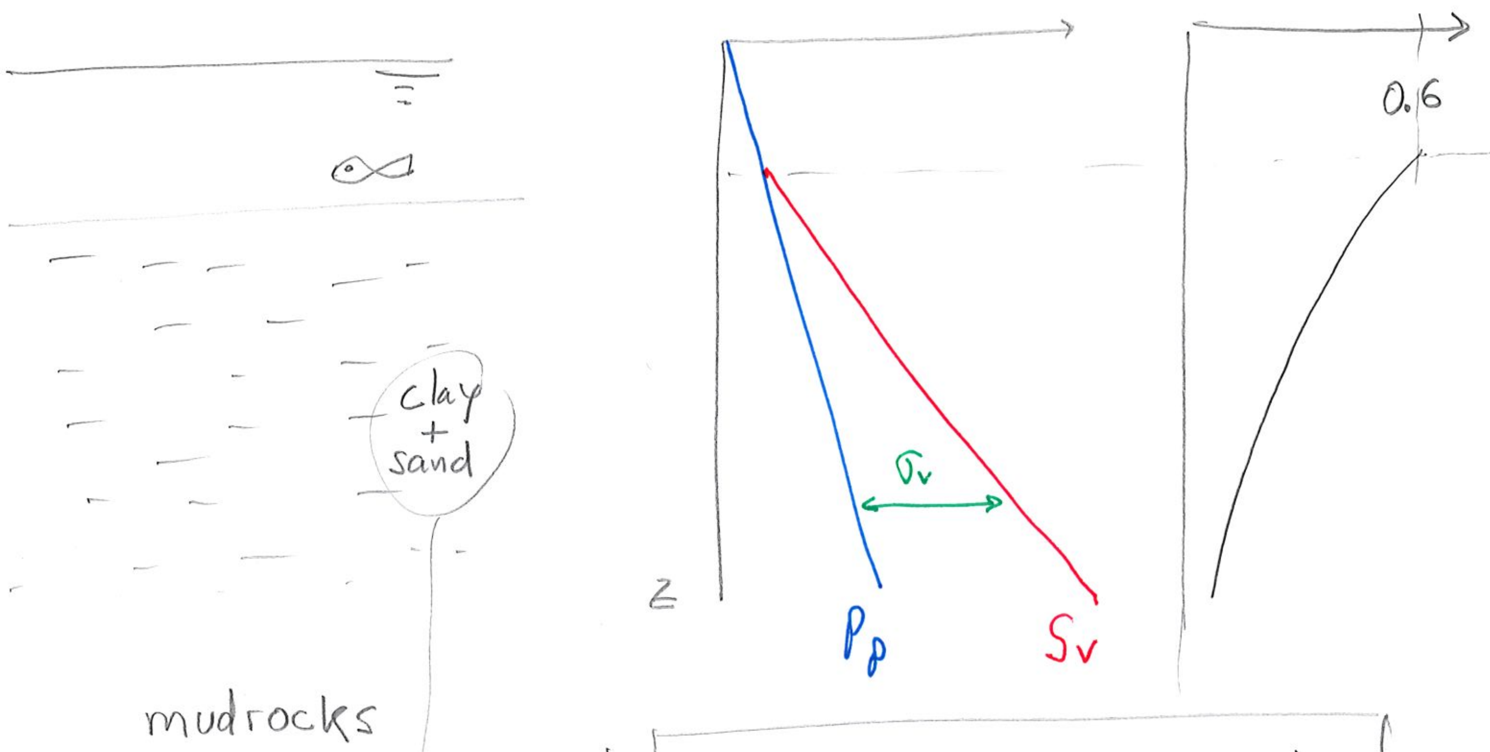




$$\lambda_p = \frac{P_p}{S_v}$$

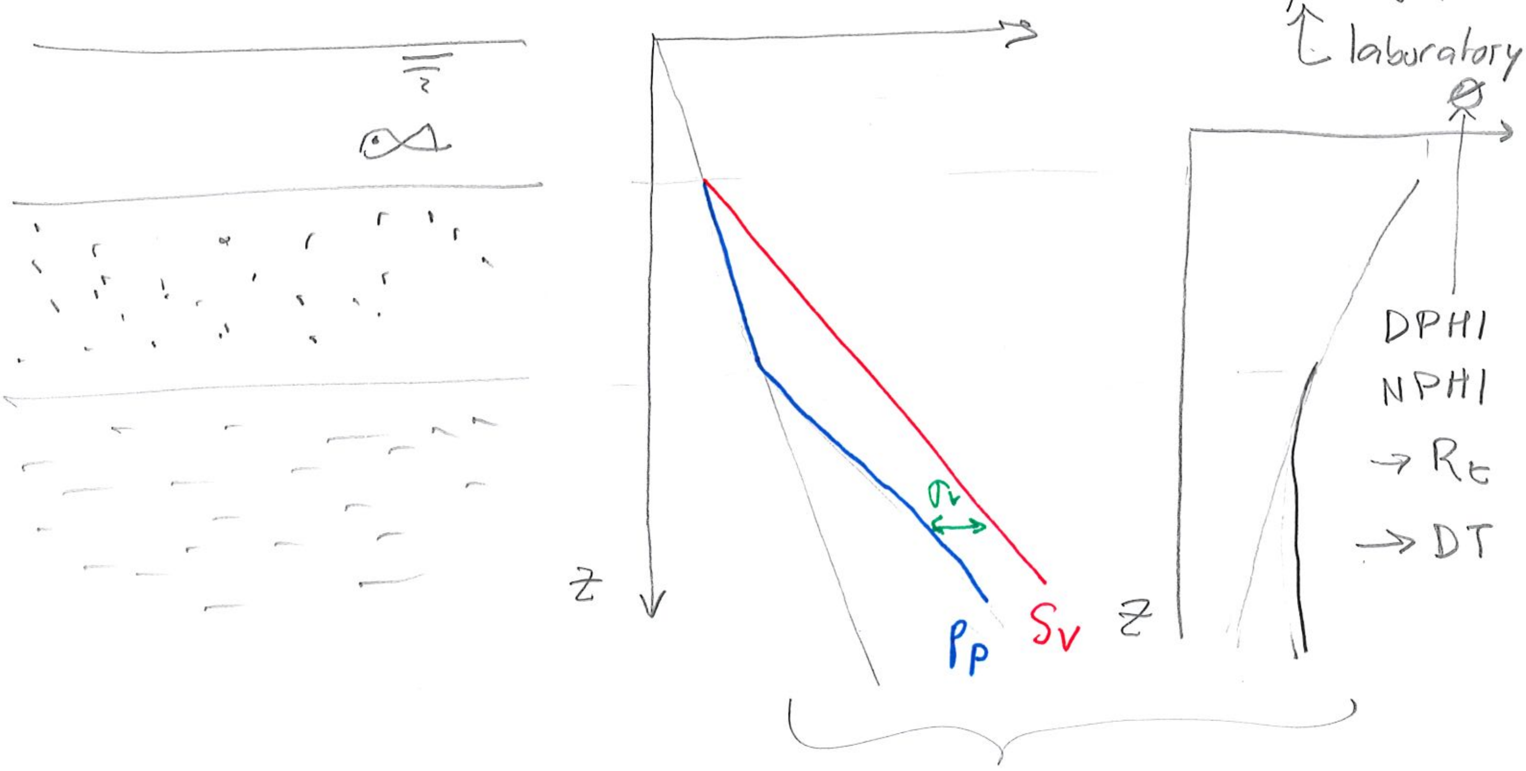
$$\underbrace{\lambda_p \rightarrow 1}_{\text{overpressure}}$$





$$\varnothing = \varnothing_0 \exp(-\beta \sigma_v)$$

fitting param
↑ laboratory



Underconsolidation

Disequilibrium compaction

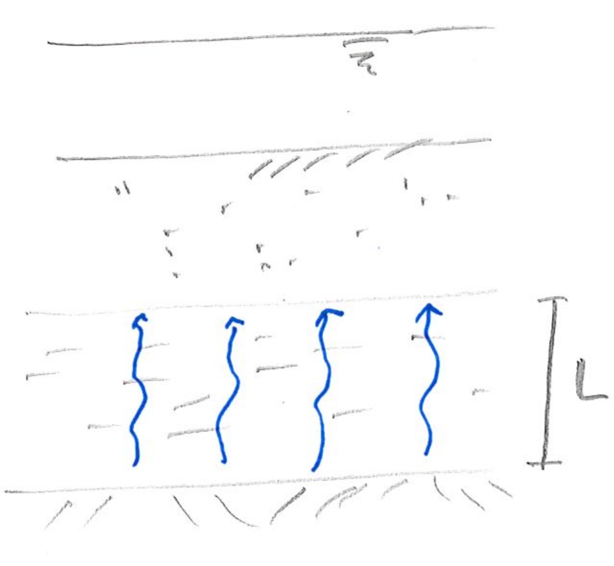
$$\overset{LT}{\phi} = \underset{lab}{\phi_0} \exp\left[-\underset{lab}{\beta}\left(\overset{\int \rho_{bulk}}{S_v} - P_p\right)\right]$$

$$\frac{\phi}{\phi_0} = \exp\left[-\beta\left(S_v - P_p\right)\right]$$

$$\ln\left(\frac{\phi}{\phi_0}\right) = -\beta\left(S_v - P_p\right)$$

$$\beta P_p = \ln\left(\frac{\phi}{\phi_0}\right) + \beta S_v$$

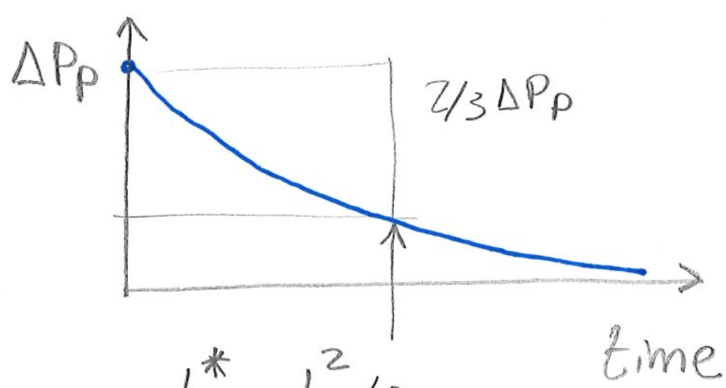
$$\boxed{P_p = S_v + \frac{\ln(\phi/\phi_0)}{\beta}}$$



undrained loading

$$\left.\frac{dP_p}{dt} = \mathbb{D}_h \frac{d^2 P_p}{dz^2}\right\}$$ Diffusivity Eq for pore pressure

$$\mathbb{D}_h = \frac{M_{bulk} \cdot K}{\mu_w}$$



$$t^* = L^2 / \mathbb{D}_h$$

Case 1: sand 100 mD

Case 2: mudrock 100 nD

$1\,D = 10^{-12}\,m^2$

$1\,GPa = 10^9\,Pa$

$\mu_w = 10^{-3}\,Pa\cdot s$, $M_{bulk} = 1\,GPa$, $L = 100\,m$

$$t^* = \begin{cases} \sim 1 \text{ day} & \text{sand} \\ \sim 3000 \text{ years} & \text{shale} \end{cases}$$

Other sources of overpressure

→ ΔT → thermal dilation → ΔP_p

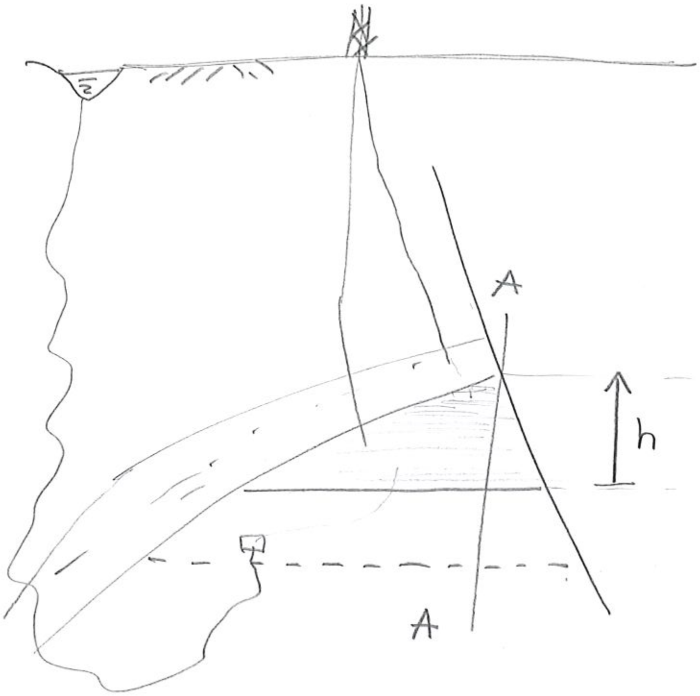
→ Hydrocarbon generation → (P, T) → organics → hydrocarbons

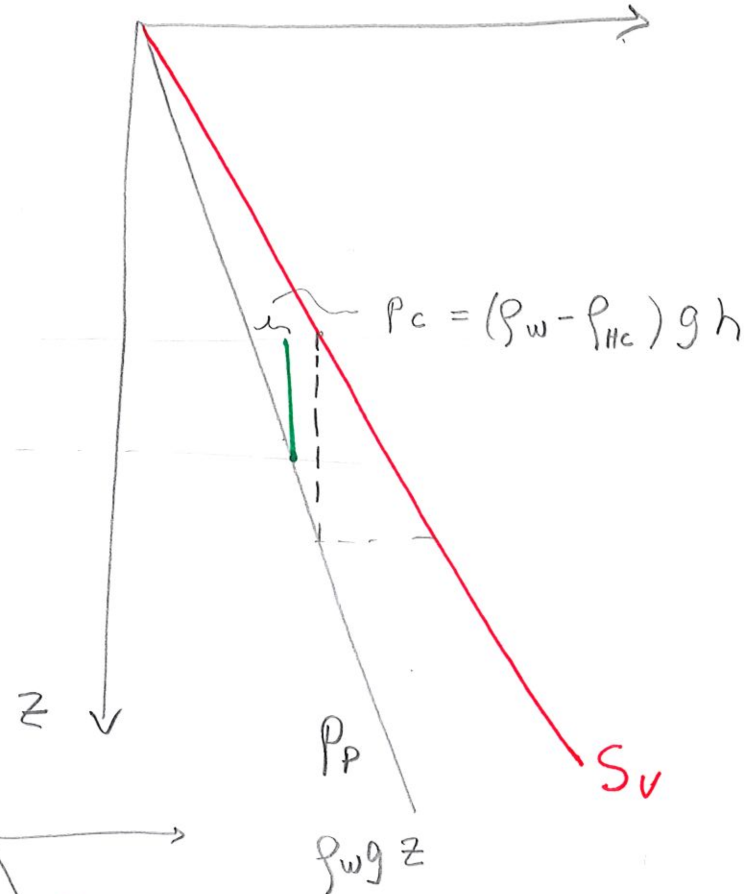
oil and gas window

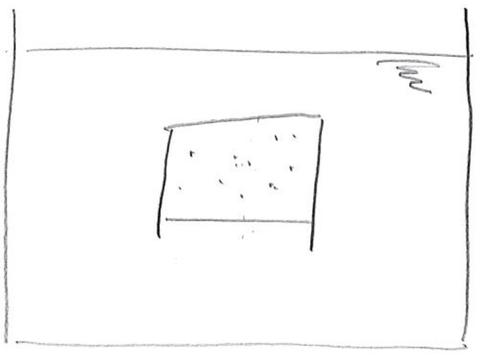
→ ΔP_p → shales → organic-rich mudrocks in the oil and gas window

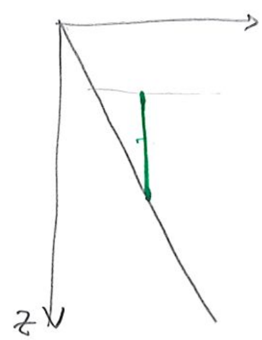
→ Clay Diagenesis

09/11/2018

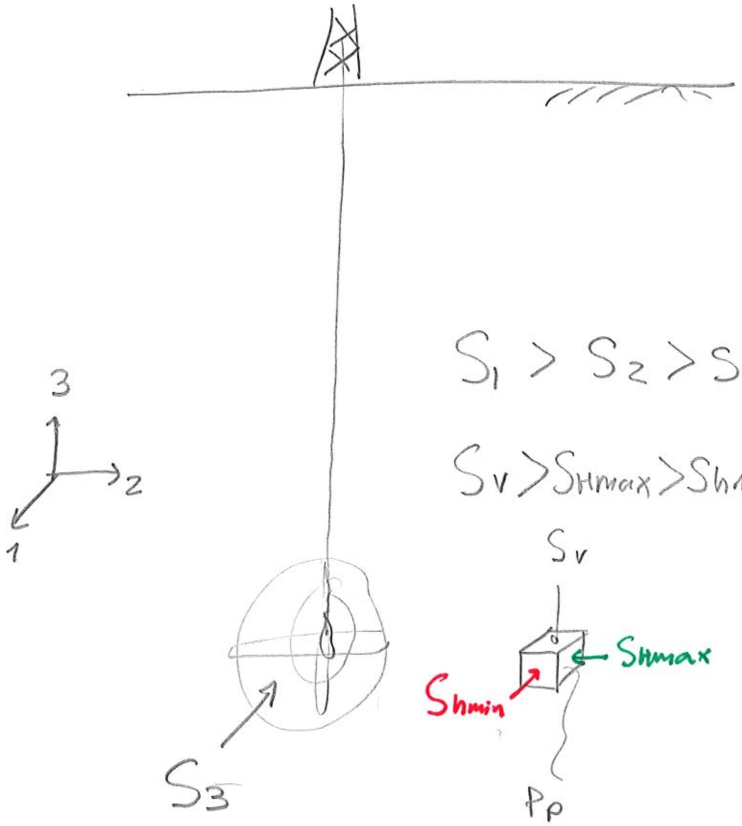






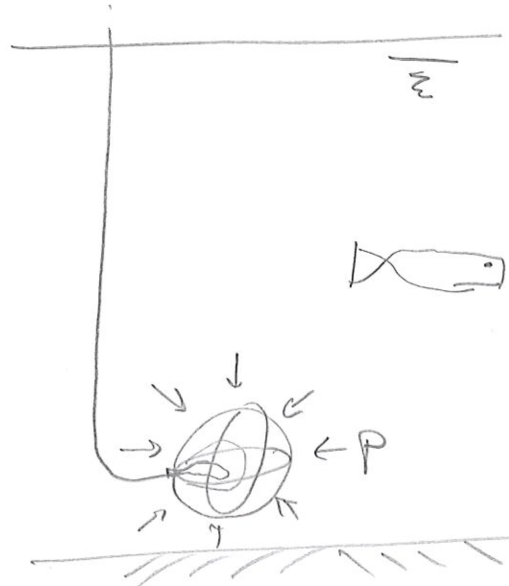


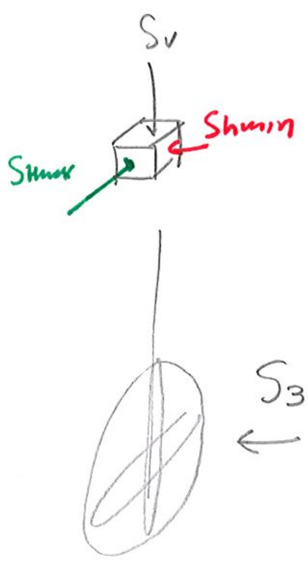
Horizontal stress



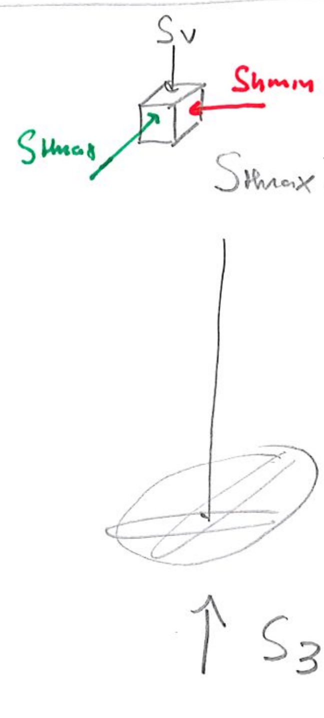
$S_1 > S_2 > S_3$

$S_v > S_{Hmax} > S_{hmin}$



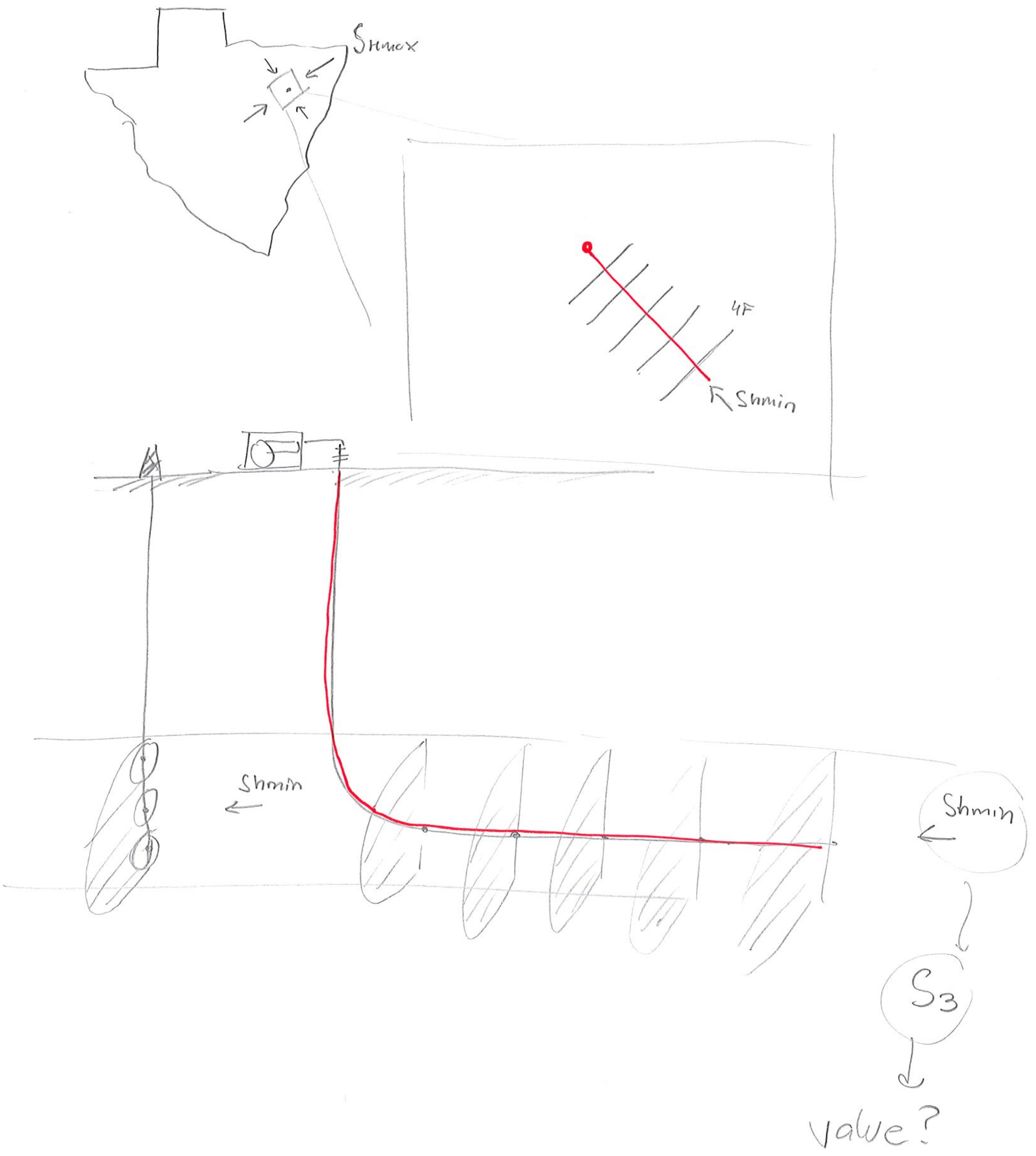


$S_{Hmax} > S_v > S_{hmin}$

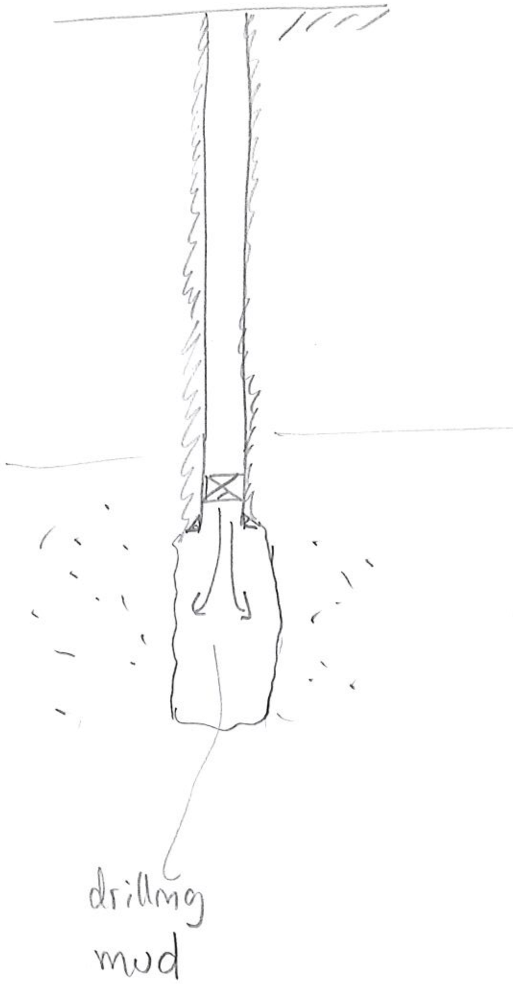


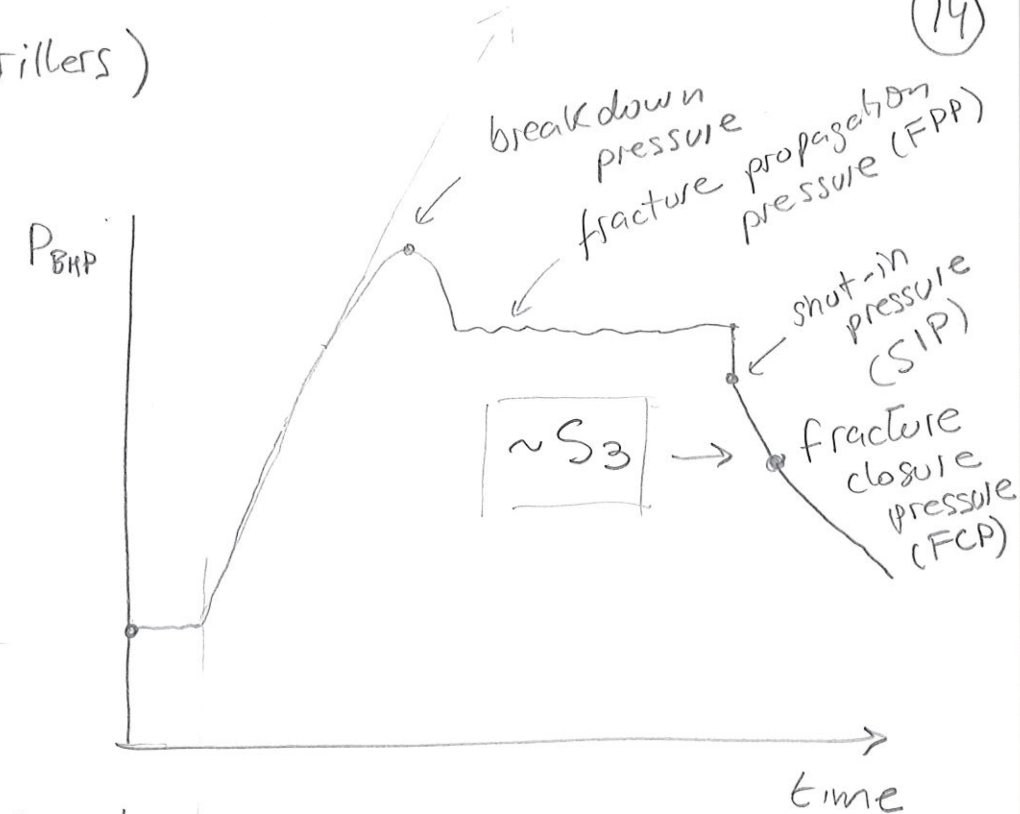
$S_{Hmax} > S_{hmin} > S_v$

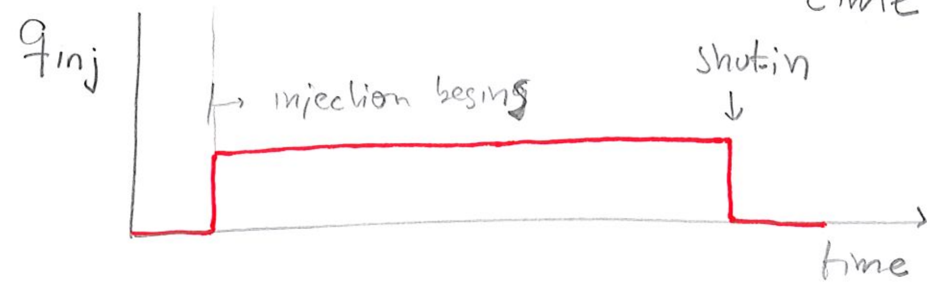
$S_v > S_{Hmax} > S_{hmin}$



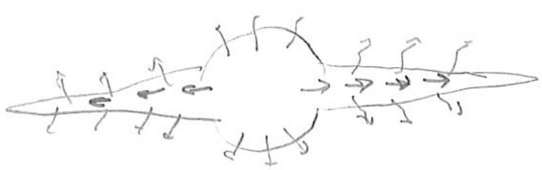
Leak-off test (drillers)



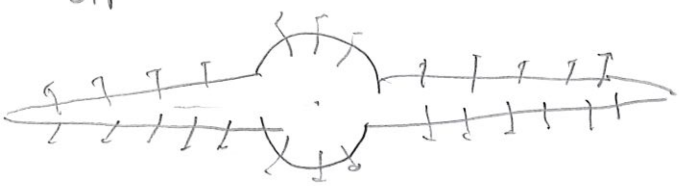




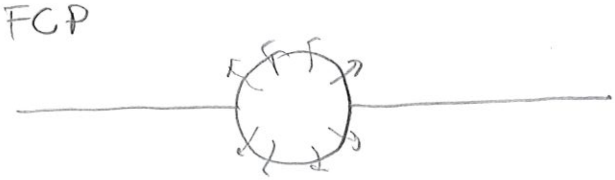
FPP



SIP



FCP



Mini-frac test (Completions)

- ↳ fracturing fluid
- ↳ before a large fracture

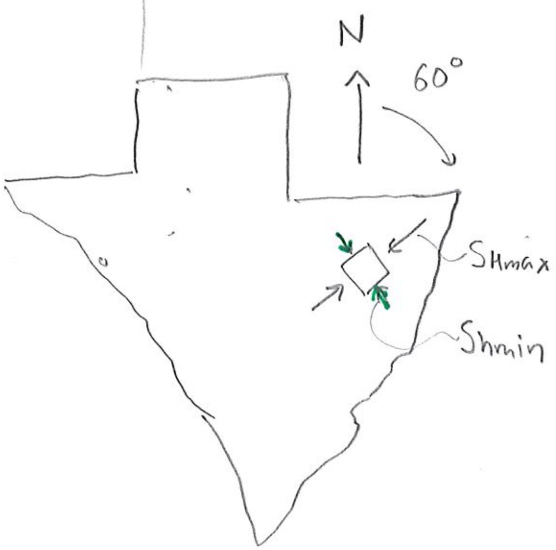
Eaton's Eq

$$\boxed{\sigma_h = \frac{\nu}{1-\nu}\sigma_v}$$

↳ Linear Elasticity

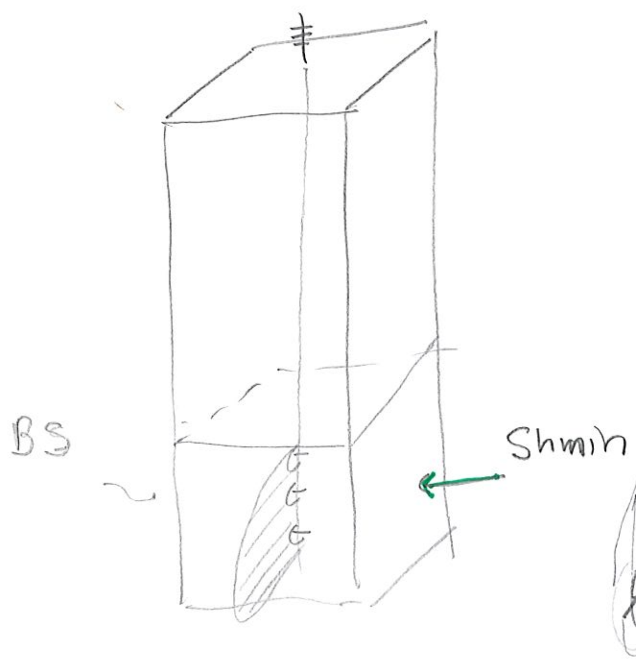
Examples	S_1 ≥	S_2 ≥	S_3	
Permian Bas Eagle Ford Barnett	S_v	S_{Hmax}	S_{hmin}	→ Tectonically Passive → Normal Faulting → Extensional Environment
California San Andreas Fault	S_{Hmax}	S_v	S_{hmin}	→ Strike Slip → Transform boundary
Australia China (W) Argentina (W)	S_{Hmax}	S_{hmin}	S_v	→ Reverse Faulting → Thrust Environment

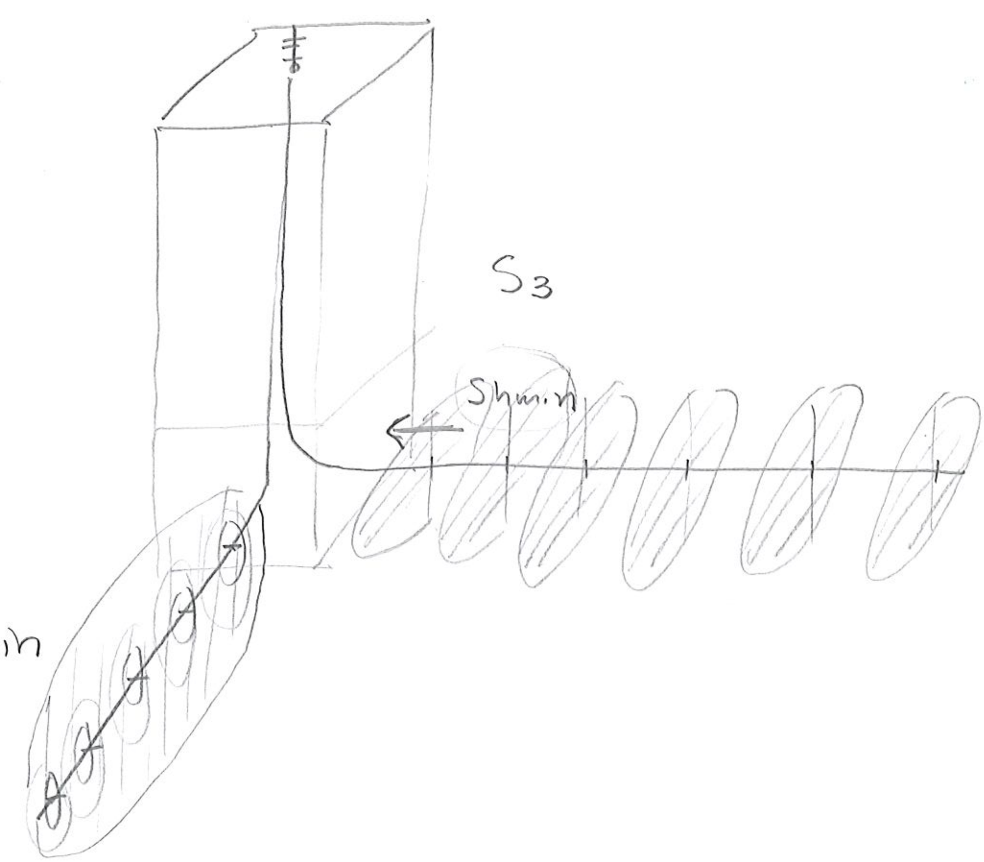
$\boxed{09/13/2018}$

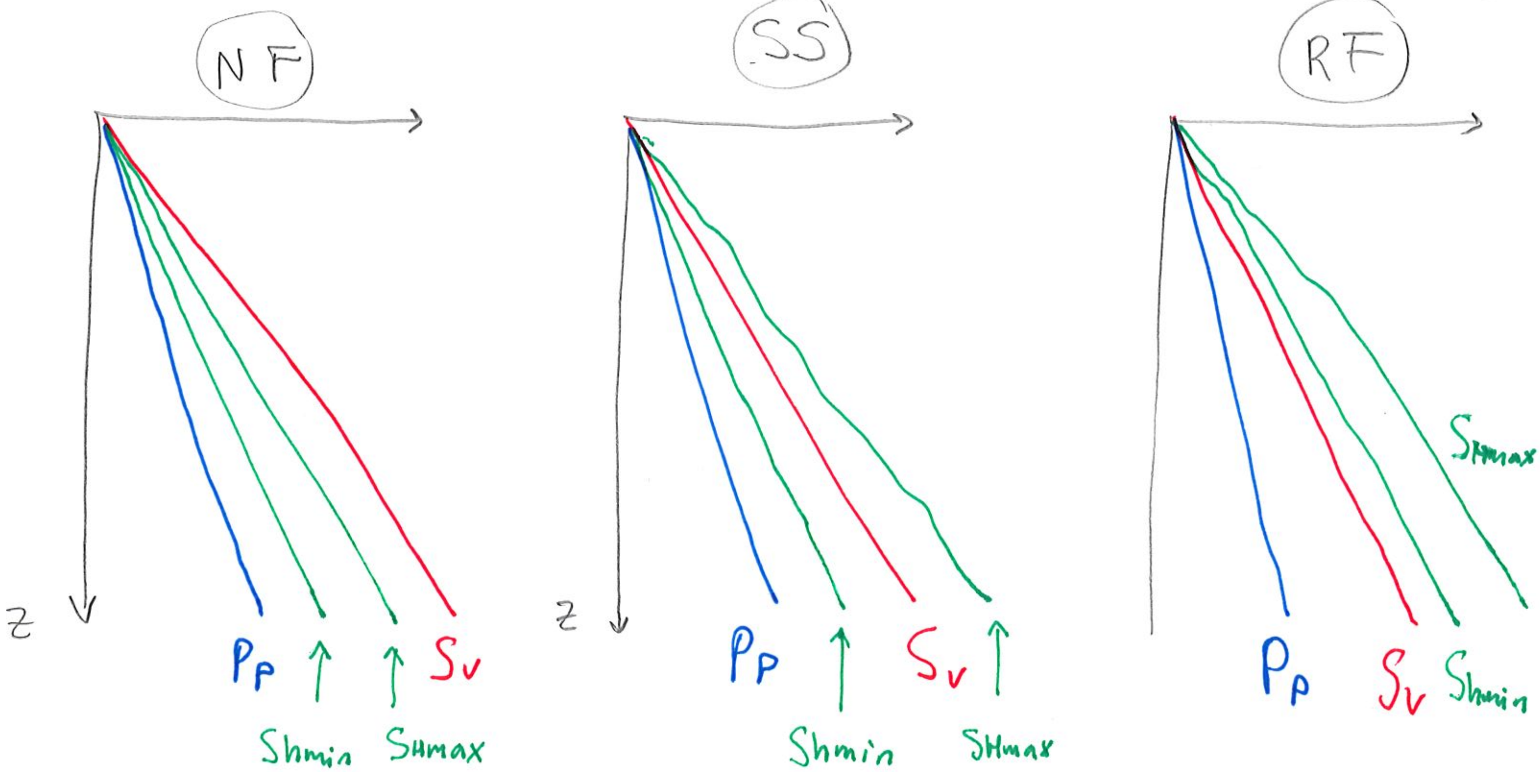


$S_v > S_{Hmax} > S_{hmin}$

S_3



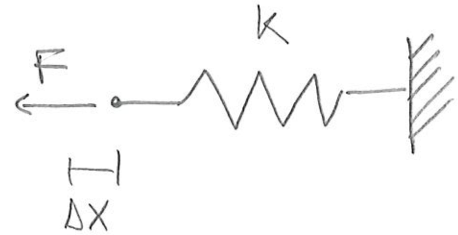



NF
z
Pp
Shmin
SHmax
Sv
SS
z
Pp
Shmin
Sv
SHmax
RF
SHmax
Pp
Sv
Shmin

Elasticity

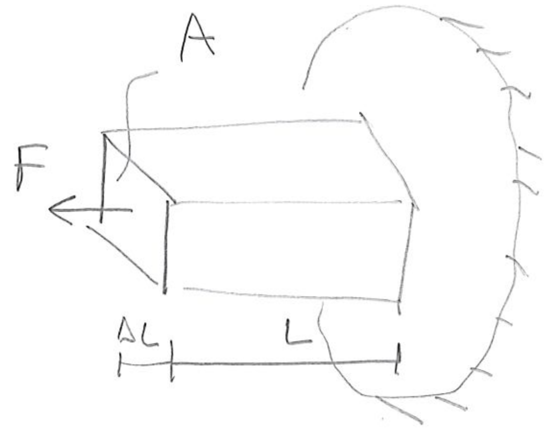
(1D) Hooke's Law

$$F = K\,\Delta X$$



(1D)

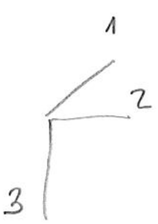
$$\frac{F}{A} = E \cdot \frac{\Delta L}{L}$$



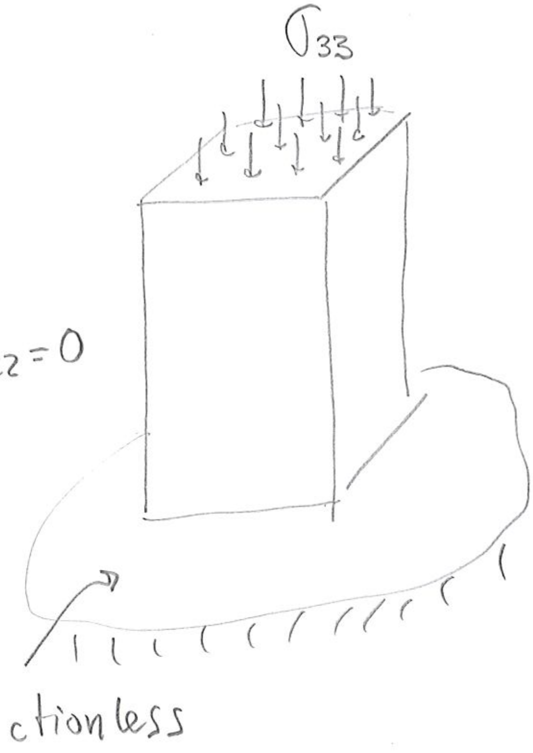
$$\sigma = E \cdot \varepsilon$$

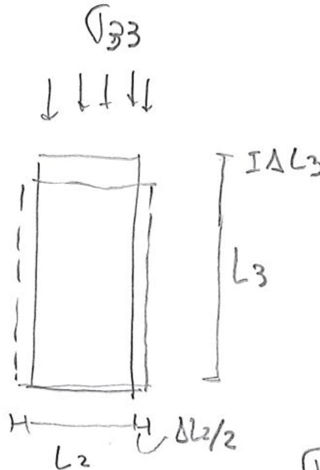
$$[Pa] = [Pa]\,[-]$$

(3D)



$\sigma_{11} = \sigma_{22} = 0$





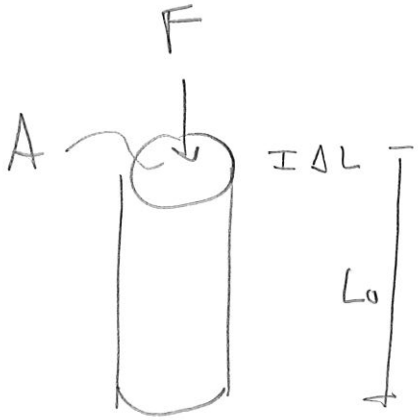
$$\varepsilon_{33} = \frac{\Delta L_3}{L_3}$$

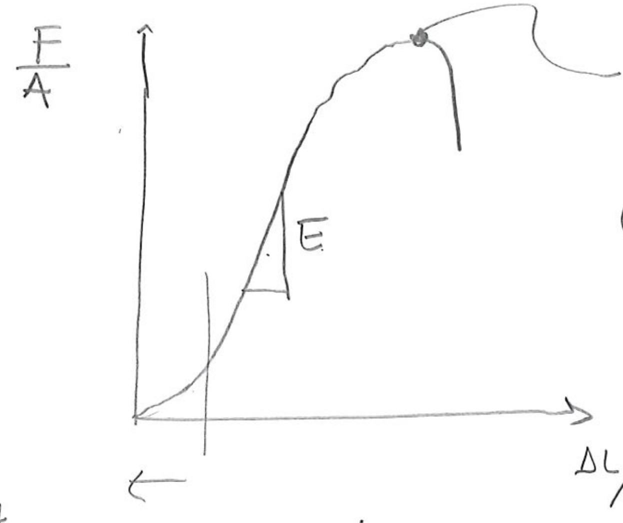
$$\varepsilon_{22} = \frac{\Delta L_2}{L_2}$$

Young's Modulus $\left\{ E = \dfrac{\sigma_{33}}{\varepsilon_{33}} \right.$

Poisson's Ratio $\left\{ \nu = -\dfrac{\varepsilon_{22}}{\varepsilon_{33}} = -\dfrac{\varepsilon_{11}}{\varepsilon_{33}} \right.$

$\nu = [0, 0.5] \rightsquigarrow$ rocks ~0.1–0.4

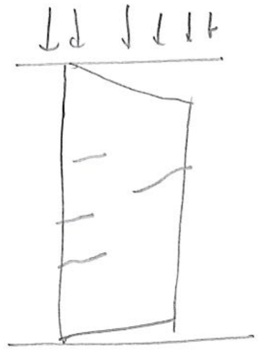




peak stress → unconfined → unconfined compression strength → $\boxed{\text{UCS}}$

ends effects

fractures

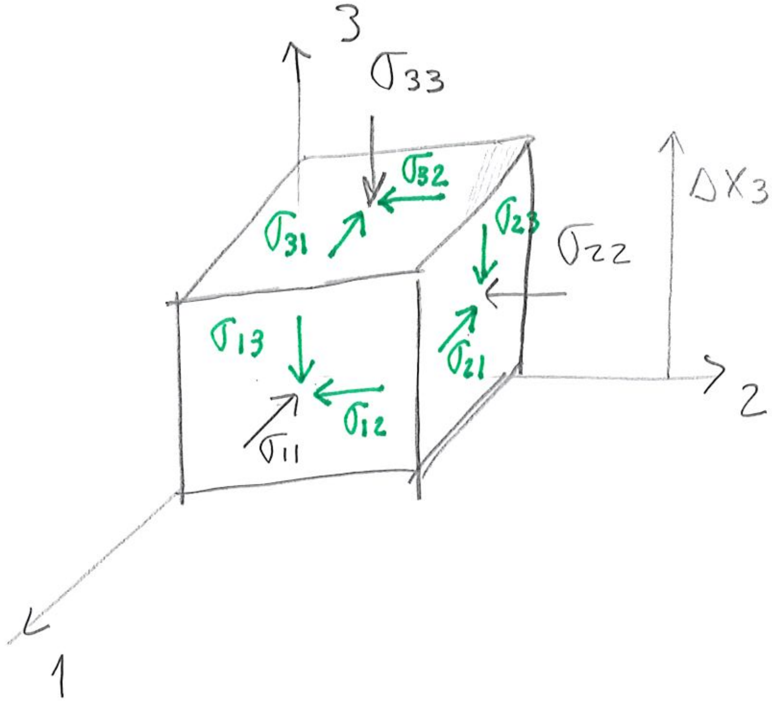


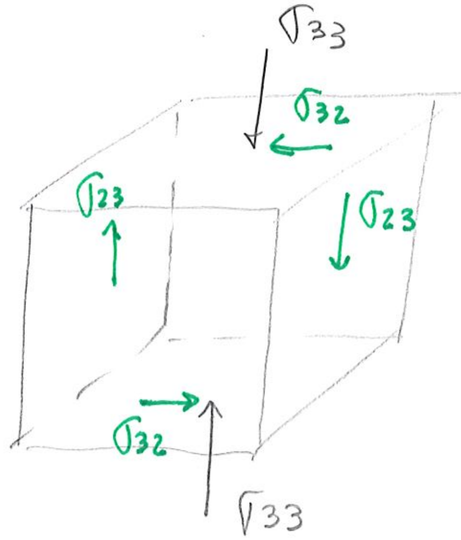
$A = 1\,cm^2$

→ $F = 20\,\vec{lb} \cong 10\,\vec{kg} \cong 100\,N$

$\Delta L / L = 0{,}10$

$$\boxed{E = 10\,MPa}$$

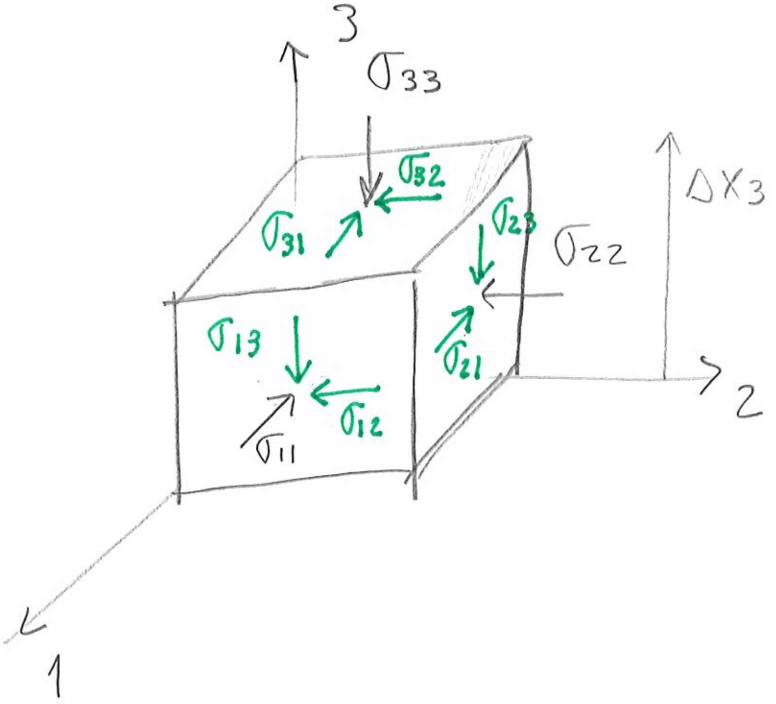


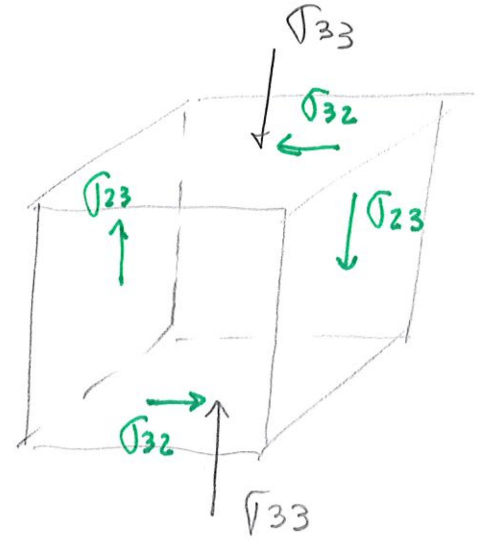


$$\sigma_{32} = \sigma_{23}$$

σ_{ij} → direction (j)

i → face

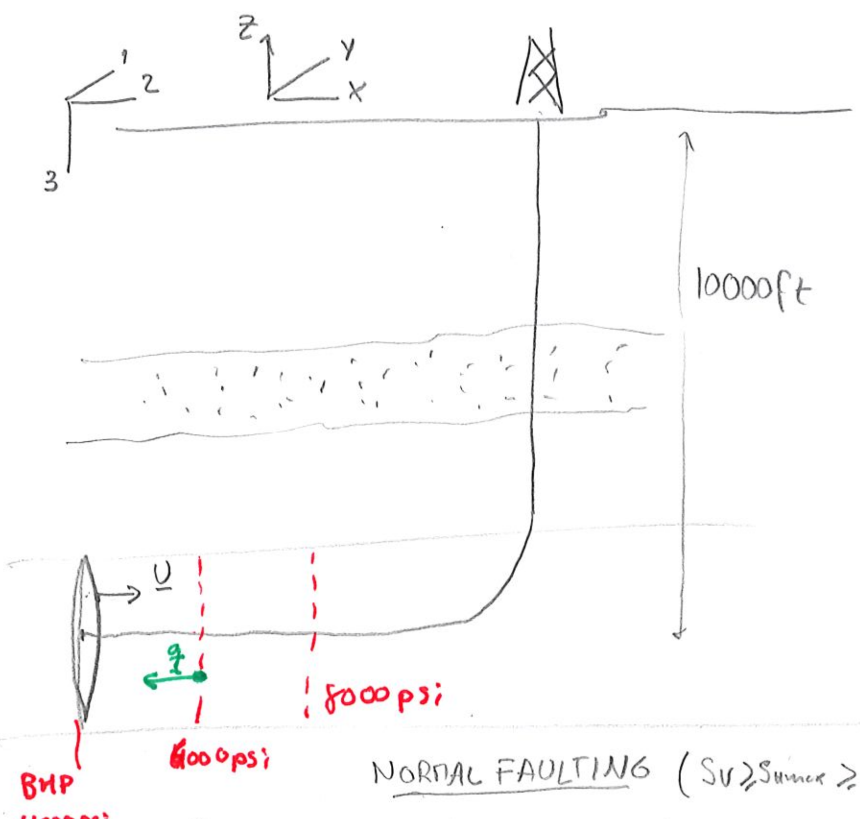




$\sigma_{32} = \sigma_{23}$

σ_{ij} → direction (j)
face (i)

$\boxed{9/18/2018}$



scalar
- $P_p = 8000$ psi
- $T = 165\,°F$
- $S_o = 0.6$

vectors (1,2,3) CS
- $\underline{U} = (0, 0.5, 0)$ in
- $\underline{q} = (0, -1, 0)$ $\frac{in}{hr}$

tensor

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

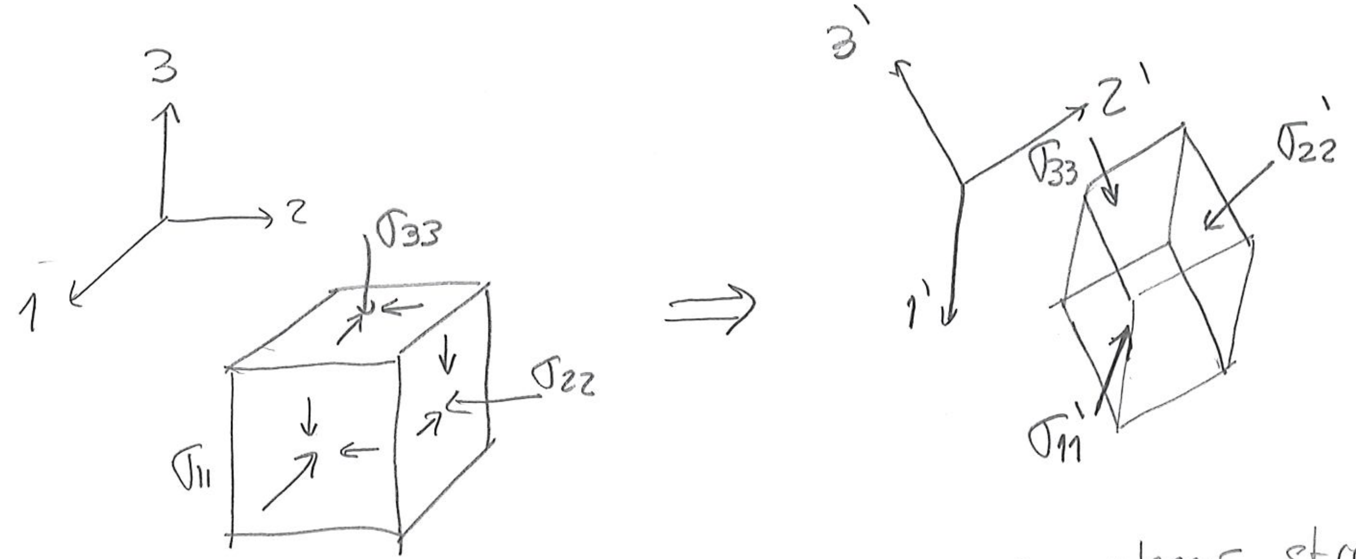
NORMAL FAULTING ($S_v \geq S_{Hmax} \geq S_{hmin}$)

$S_v = 10\,000$ psi → Principal stress

$S_{Hmax} = 9200$ psi

$S_{hmin} = 9000$ psi

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 9200 & 0 & 0 \\ 0 & 9000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}$$



no shear stress

$$\begin{bmatrix}\sigma_{11} & \sigma_{12} & \sigma_{13}\\ \sigma_{21} & \sigma_{22} & \sigma_{23}\\ \sigma_{31} & \sigma_{32} & \sigma_{33}\end{bmatrix}_{123} \xRightarrow[\text{eigen vectors}]{\text{eigen values}} \begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix}_{123'}$$

$\sigma_{12} \neq \sigma_{13} \neq \sigma_{23} \neq 0$

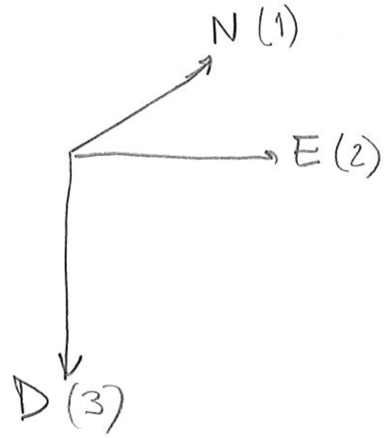
$\sigma_1 > \sigma_2 > \sigma_3 \Rightarrow$ principal stresses

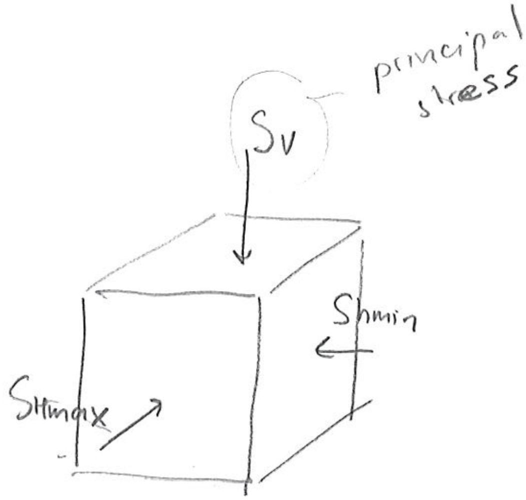
$$\sigma_v = S_v - P_p$$

$$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \underline{\underline{I}} = \begin{bmatrix}S_{11} & S_{12} & S_{13}\\ & S_{22} & S_{23}\\ & & S_{33}\end{bmatrix} - P_p \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$S_{ij} = S_{ji}$ for $i \neq j$

$$= \begin{bmatrix}S_{11} - P_p & S_{12} & S_{13}\\ & S_{22} - P_p & S_{23}\\ & & S_{33} - P_p\end{bmatrix}$$

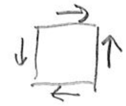


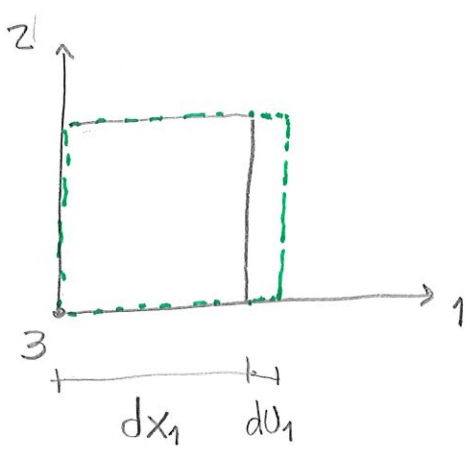


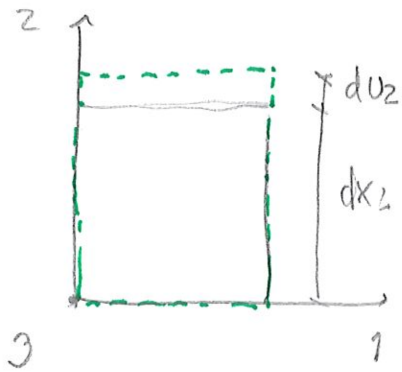
$$\underline{\underline{S}} = \begin{bmatrix} S_{Hmax} & 0 & 0 \\ 0 & S_{hmin} & 0 \\ 0 & 0 & S_V \end{bmatrix}_{NED}$$

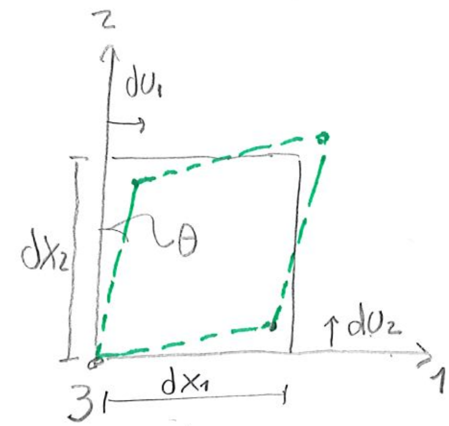
$$\underline{\underline{S}} = \begin{bmatrix} S_{NN} & S_{NE} & S_{ND} \\ S_{EN} & S_{EE} & S_{ED} \\ S_{DN} & S_{DE} & S_{DD} \end{bmatrix}$$

Strains









$$\varepsilon_{11} = \frac{dU_1}{dX_1}$$

$$\varepsilon_{22} = \frac{dU_2}{dX_2}$$

$$\tan\theta = \frac{dU_1}{dX_2}$$

$$\theta \cong \frac{dU_1}{dX_2}$$

└ linear strain

└ volumetric strain

$$\underbrace{\varepsilon_{12} = \frac{1}{2}\left(\frac{dU_1}{dX_2} + \frac{dU_2}{dX_1}\right)}_{\text{shear strain}}$$

no volume change

$$\underline{\underline{\varepsilon}} = \begin{vmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ & \varepsilon_{22} & \varepsilon_{23} \\ & & \varepsilon_{33} \end{vmatrix} = \begin{vmatrix} \frac{\partial U_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial U_1}{\partial x_2} + \frac{\partial U_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial U_1}{\partial x_3} + \frac{\partial U_3}{\partial x_1}\right) \\ & \frac{\partial U_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial U_2}{\partial x_3} + \frac{\partial U_3}{\partial x_2}\right) \\ & & \frac{\partial U_3}{\partial x_3} \end{vmatrix}$$

Symmetric

$$\underline{\varepsilon} = \overbrace{\underline{\underline{\underline{D}}}}^{\text{compliance matrix}} \cdot \underline{\underline{\sigma}} \quad \leftarrow \quad \boxed{\varepsilon = \frac{\sigma}{E}} \quad \boxed{\sigma = E\,\varepsilon}$$

Hooke's law
1D - Stress

↓ Voigt Notation

$$\begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\cdot\varepsilon_{12} \\ 2\cdot\varepsilon_{13} \\ 2\cdot\varepsilon_{23} \end{vmatrix}_{6\times1} = \underbrace{\begin{vmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{vmatrix}}_{6\times6} \begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{vmatrix}_{6\times1}$$

$$\underline{\underline{\underline{D}}}^{-1} \cdot \underline{\underline{\varepsilon}} = \overbrace{\underline{\underline{\underline{D}}}^{-1} \cdot \underline{\underline{\underline{D}}}}^{I}\ \underline{\underline{\sigma}}$$

$$\hookrightarrow \underline{\underline{\sigma}} = \underline{\underline{\underline{C}}}\ \underline{\underline{\varepsilon}}$$

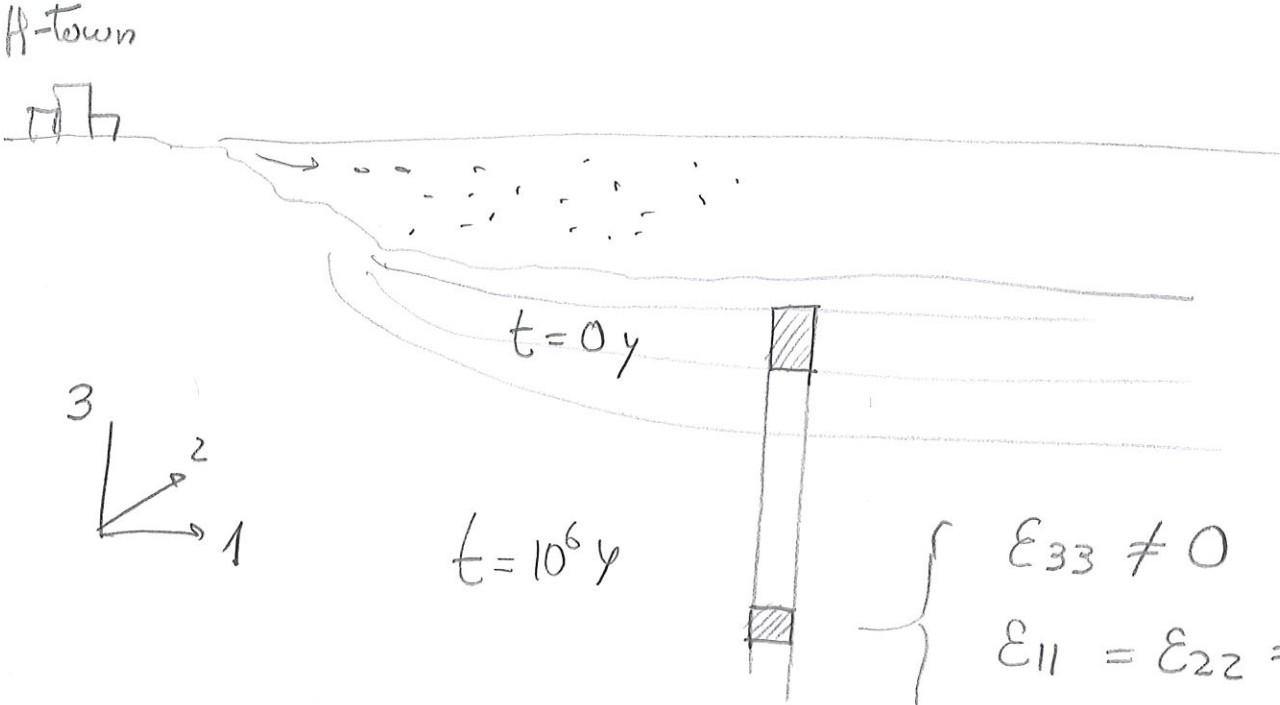
stiffness matrix
$\underline{\underline{\underline{D}}}^{-1}$

Generalized Hooke's law in 3D (isotropic linear elastic solid)
↳ 2 independent coeff.

$$
\left\{
\begin{aligned}
\varepsilon_{11} &= \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\
\varepsilon_{22} &= -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\
\varepsilon_{33} &= -\frac{\nu}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \\
2\varepsilon_{12} &= \frac{1}{G}\sigma_{12} + 0\,\sigma_{33} \\
2\varepsilon_{13} &= \frac{1}{G}\sigma_{13} + 0\,\sigma_{33} \\
2\varepsilon_{23} &= \frac{1}{G}\sigma_{23} + 0\,\sigma_{33}
\end{aligned}
\right.
$$

$$\underbrace{G = \frac{E}{2(1+\nu)}}_{\text{Shear modulus}}$$

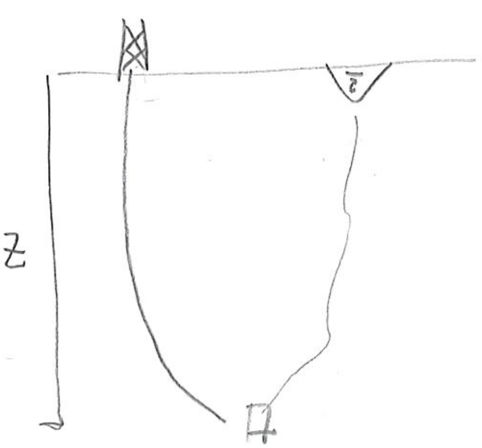
$$
\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}
= \frac{E}{(1+\nu)(1+2\nu)}
\begin{bmatrix}
1-\nu & \nu & \nu & & & \\
\nu & 1-\nu & \nu & & 0 & \\
\nu & \nu & 1-\nu & & & \\
 & & & \frac{1-2\nu}{2} & 0 & 0 \\
 & 0 & & 0 & \frac{1-2\nu}{2} & 0 \\
 & & & 0 & 0 & \frac{1-2\nu}{2}
\end{bmatrix}
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{Bmatrix}
$$



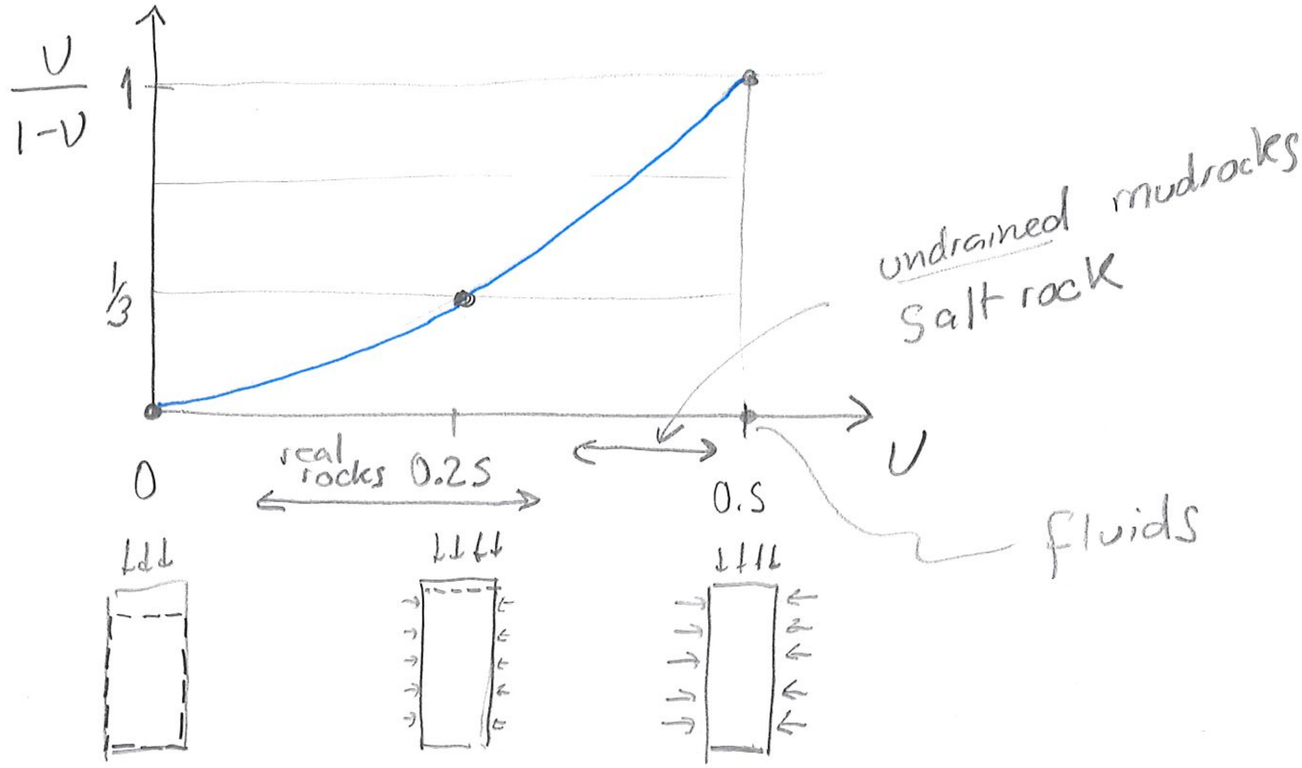
$$\left\{ \begin{array}{l} \varepsilon_{33} \neq 0 \\ \varepsilon_{11} = \varepsilon_{22} = 0 \\ \varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} \sigma_{33} = \dfrac{(1-\nu)\,E}{(1+\nu)\,(1+2\nu)}\,\varepsilon_{33} = M\,\varepsilon_{33} \\[2ex] \underbrace{\sigma_{11}}_{Hz} = \dfrac{\nu}{1-\nu}\,\underbrace{\sigma_{33}}_{Vert} \end{array} \right.$$

Procedure to determine total Hz stress with linear elasticity



1. S_v
2. Determine P_p $<$ hydrostatic ✓ / non-hydrostatic $<$ λ_p (σ_v) / $\phi = \phi_0 e^{(-\beta\sigma_v)}$
3. $\sigma_v = S_v - P_p$
4. $\sigma_h = \dfrac{\nu}{1-\nu}\,\sigma_v^{(*)}$ (simplification)
5. $S_h = \sigma_h + P_p$

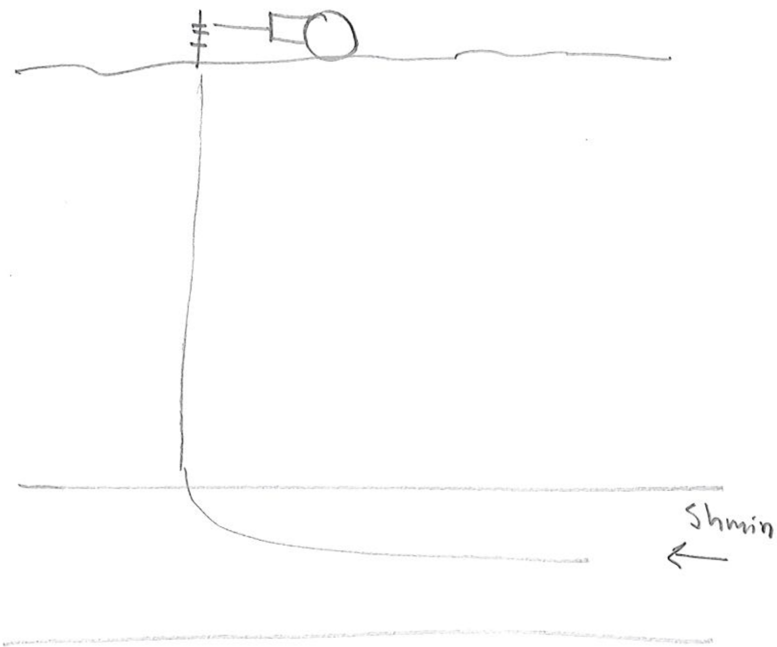


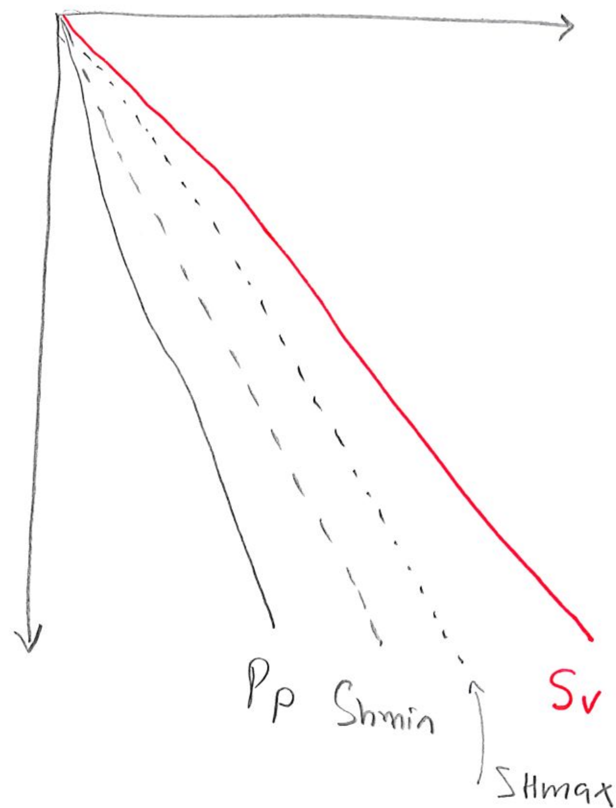
$$\varepsilon_{11} \neq 0 \; ; \; \varepsilon_{22} \neq 0 \; ; \; \varepsilon_{33} \neq 0 \; ; \; \varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0$$

$$\underbrace{\varepsilon_{11} \neq 0 \; ; \; \varepsilon_{22} \neq 0}_{\text{Horizontal}} \; ; \; \underbrace{\varepsilon_{33} \neq 0}_{\text{vertical}}$$

$$\begin{cases} \sigma_{11} = \dfrac{E}{1-\nu^2}\varepsilon_{11} + \dfrac{\nu E}{1-\nu^2}\varepsilon_{22} + \dfrac{\nu}{1-\nu}\sigma_{33} \\[2ex] \sigma_{22} = \underbrace{\dfrac{\nu E}{1-\nu^2}\varepsilon_{11} + \dfrac{E}{1-\nu^2}\varepsilon_{22}}_{\text{Tectonic stress}} + \underbrace{\dfrac{\nu}{1-\nu}\sigma_{33}}_{\text{Hz stress due to overburden}} \end{cases}$$

$\varepsilon_{11}, \varepsilon_{22}$, tectonic strains





$$S_{hmin} = S_{Hmax} = S_h$$

$$S_h = \sigma_h + P_p$$

$$S_h = \frac{\nu}{1-\nu}\sigma_v + P_p$$

$$S_h = \frac{\nu}{1-\nu}\left(S_v - P_p\right) + P_p$$

$$S_h = \frac{\nu}{1-\nu} S_v + \frac{1-2\nu}{1-\nu} P_p$$

$$\underbrace{\frac{\Delta S_h}{\Delta z}}_{\text{Fracture gradient}} = \frac{\nu}{1-\nu}\underbrace{\frac{\Delta S_v}{\Delta z}}_{\substack{\text{- Total vert stress grad} \\ \text{- Lithostic grad}}} + \frac{1-2\nu}{1-\nu}\underbrace{\frac{\Delta P_p}{\Delta z}}_{\text{Pore pressure gradient}}$$

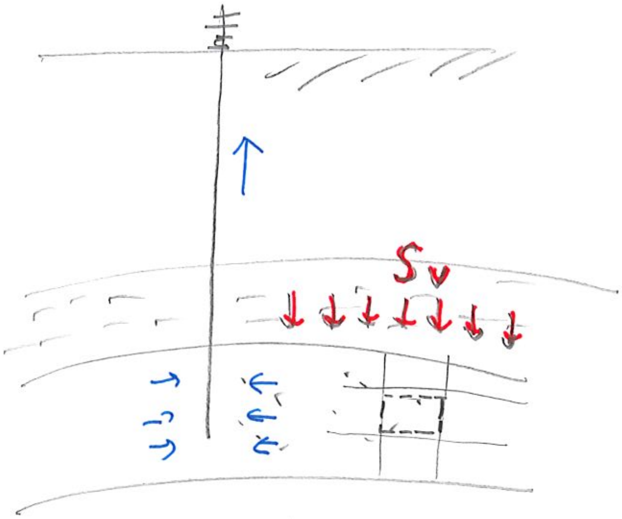
Reservoir Engineering

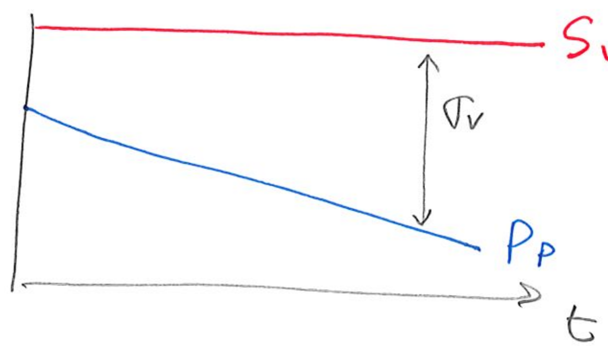
$$\frac{\partial P}{\partial t} = \frac{K}{\mu C_t} \frac{\partial^2 P}{\partial x^2}$$

C_t total compressibility

$$C_t = S_g C_g + S_o C_o + S_w C_w + C_f$$

C_f Formation compress / rock compressibility



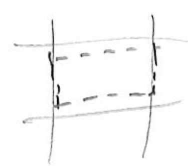


Pore Compressibility $\quad C_f = C_{pp} = \dfrac{1}{V_p} \dfrac{\partial V_p}{\partial P_p} \Big|_{S_v, \varepsilon_h}$

for $\nu > 0$
$M > E$

Bulk Compress $\quad C_{bp} = \dfrac{1}{V_b} \dfrac{dV_b}{dP_p} \Big|_{S_v, \varepsilon_h}$

M: constrained modulus

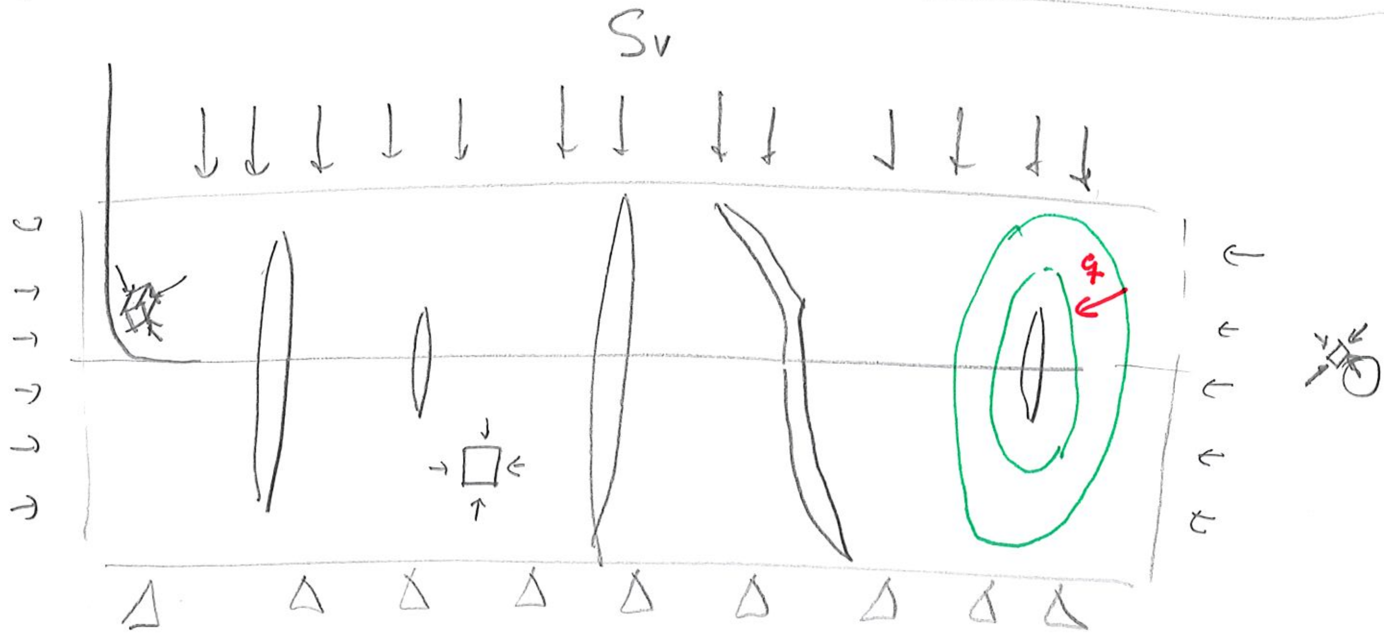


$$C_{bp} = \frac{1}{M}$$

If $dV_p = dV_b$

$$C_{pp} = \frac{1}{\frac{V_p}{V_b}} \frac{1}{V_b} \frac{dV_b}{dP_p} \Big|_{S_v, \varepsilon_h} = \frac{C_{bp}}{\phi} = \frac{1}{\phi M}$$

General solution for a continuum mechanics problem



Fluid flow problem

Mass conservation

$$\underbrace{\frac{\partial q_1}{\partial x_1} + \frac{\partial q_2}{\partial x_2} + \frac{\partial q_3}{\partial x_3}}_{\text{divergence}} = 0$$

Darcy eq

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = -\frac{K}{\mu} \cdot \underbrace{\begin{bmatrix} \frac{\partial P}{\partial x_1} \\ \frac{\partial P}{\partial x_2} \\ \frac{\partial P}{\partial x_3} \end{bmatrix}}_{\text{gradient}}$$

$$-\frac{K}{\mu}\left(\frac{\partial^2 P}{\partial x_1^2} + \frac{\partial^2 P}{\partial x_2^2} + \frac{\partial^2 P}{\partial x_3^2}\right) = 0$$

$\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$

Laplacian $\left(\frac{\partial^2}{\partial x_1^2}, \frac{\partial^2}{\partial x_2^2}, \frac{\partial^2}{\partial x_3}\right)$

$$\boxed{(\lambda + \mu)\, \nabla \cdot (\nabla \cdot \underline{U}) + \mu \nabla^2 \underline{U} + \underline{F} = 0}$$

displacements $= \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

body forces
↳ gravity

Lamé Parameters

$\mu = G$

$\lambda = \frac{\nu E}{\ldots}$

Solutions
- Analytical
  - Kirsch ↔ wellbores
  - Sneddon ↔ fractures
  - ....
- Numerical
  - finite differences ↔ CMG
  - finite element method

Real rocks

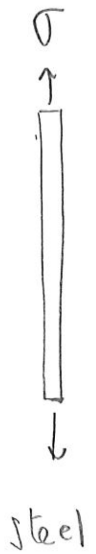
. anisotropy

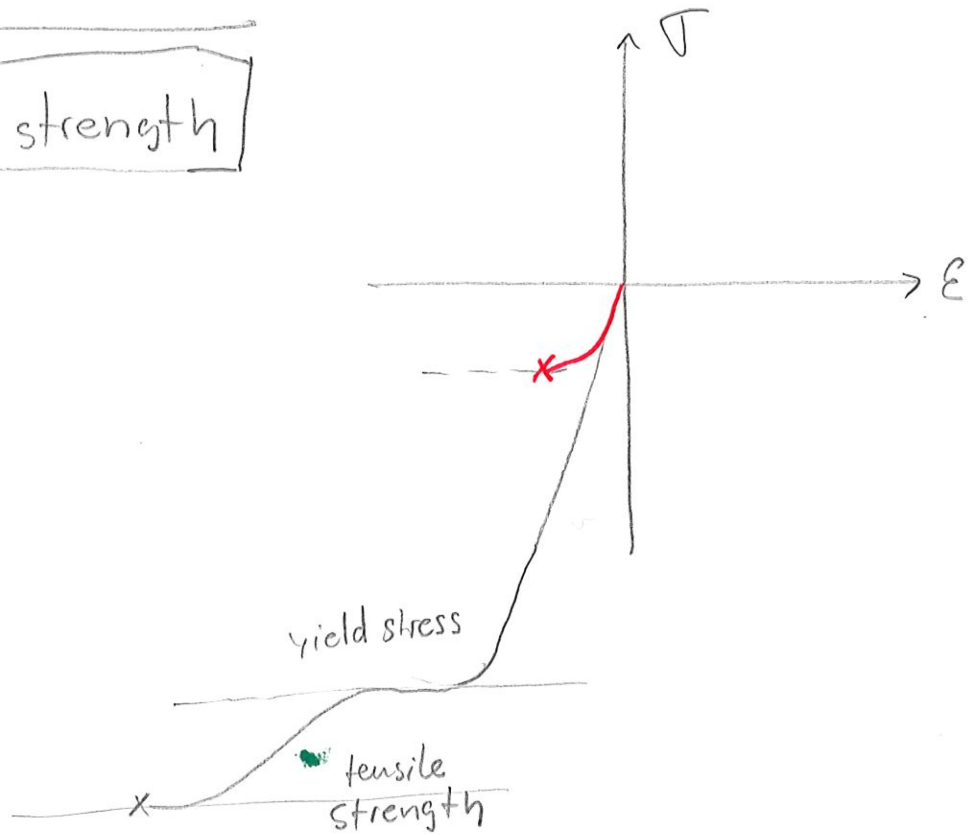
. elastoplasticity

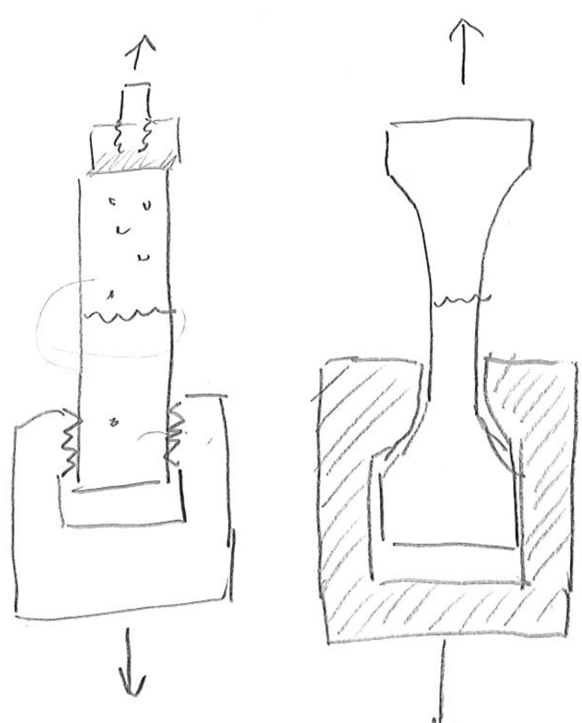
. visco-elasticity

Failure of Rocks

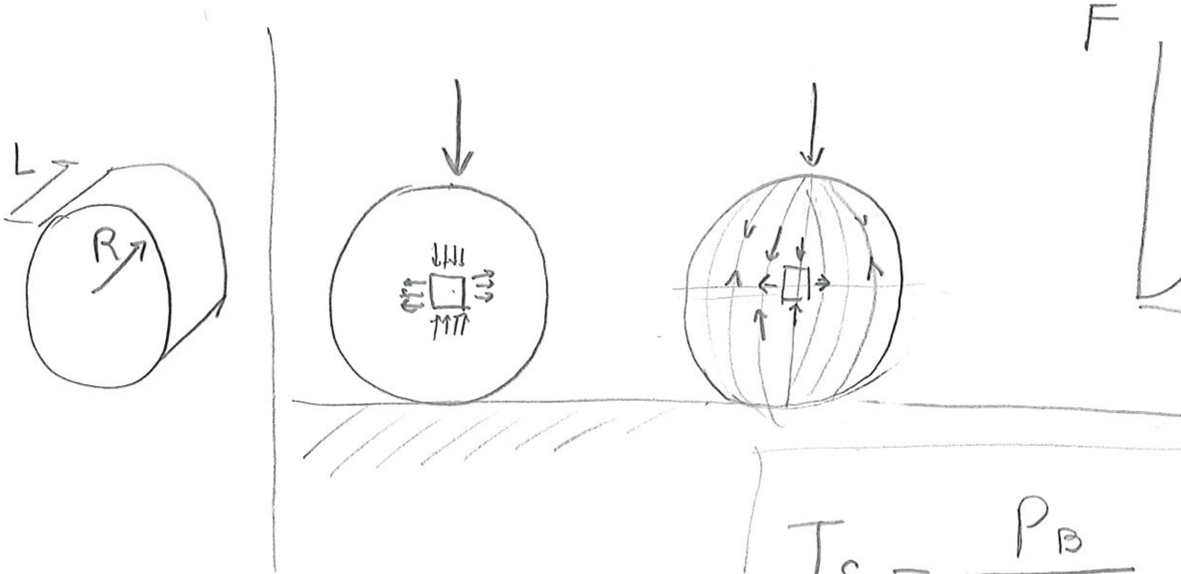
Tensile strength

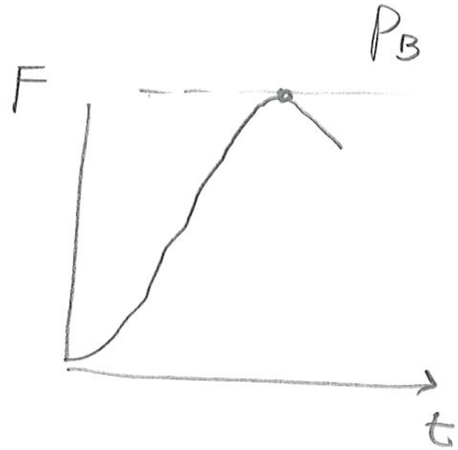






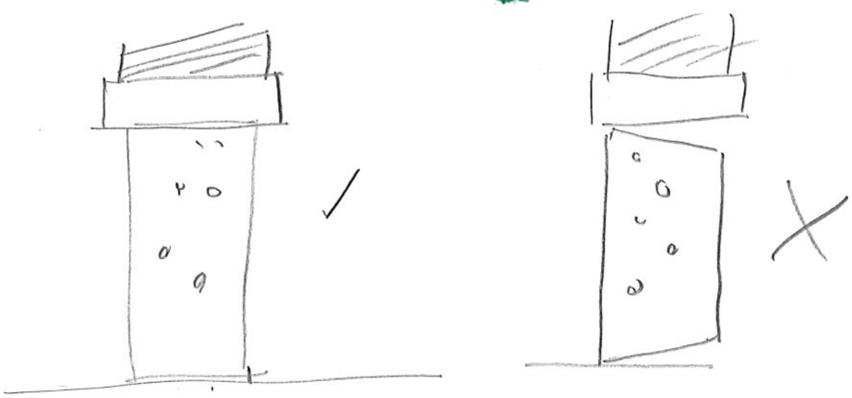
Brazilian Test



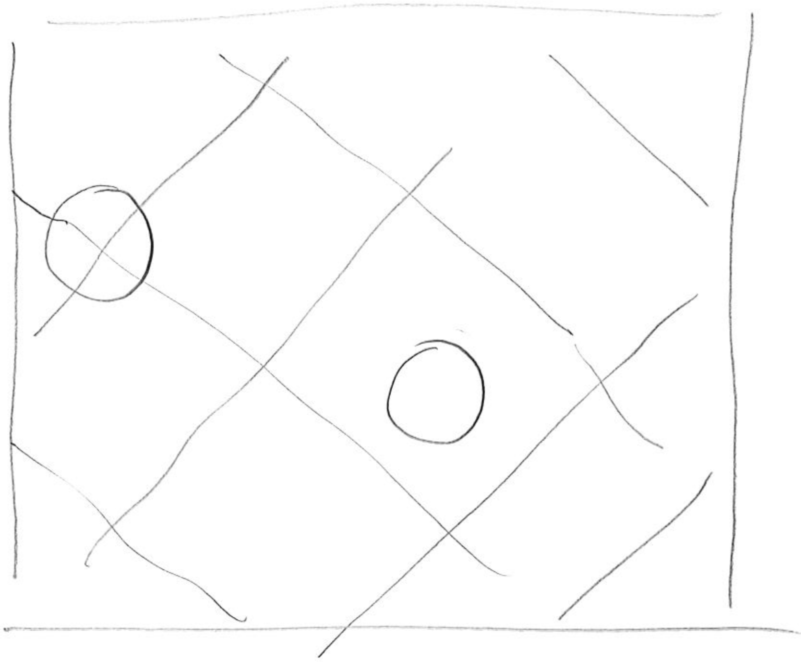


$$\boxed{T_S = \frac{P_B}{\pi L R}}$$

$L \leq 2R$

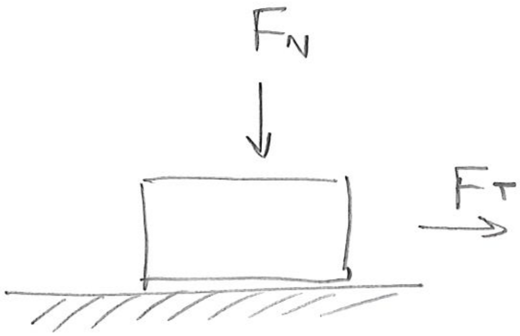


15 cars

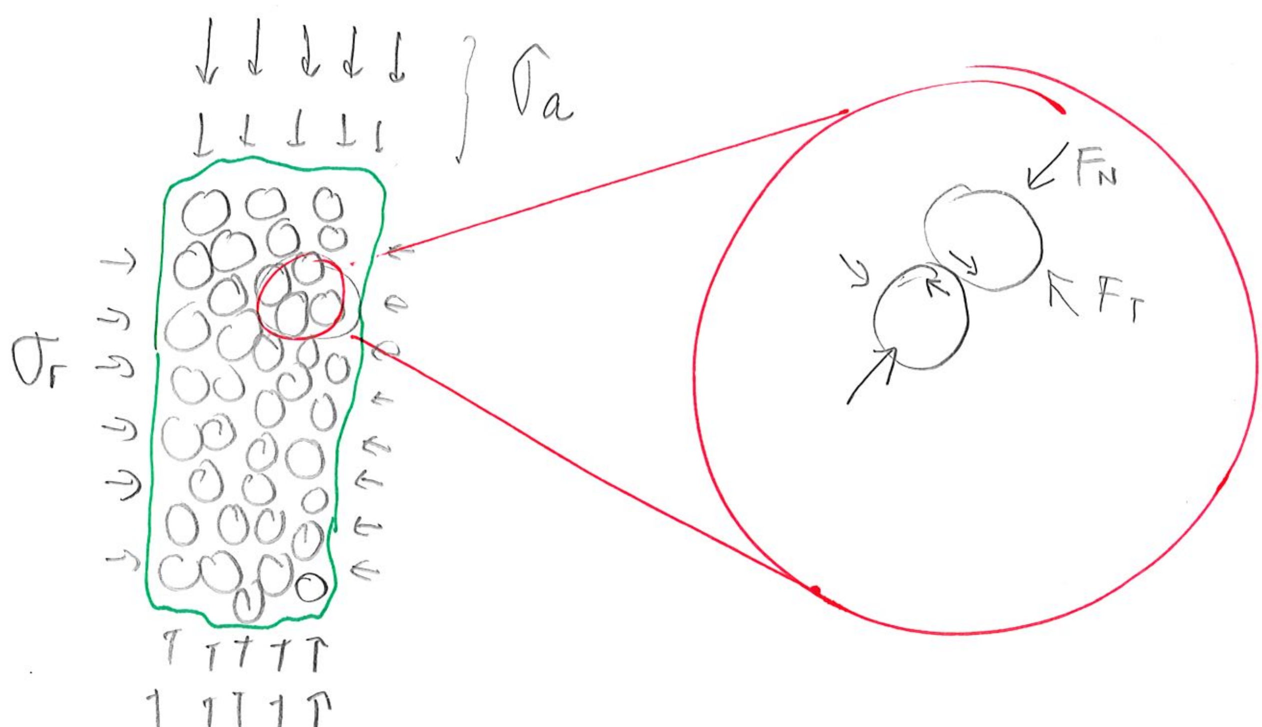


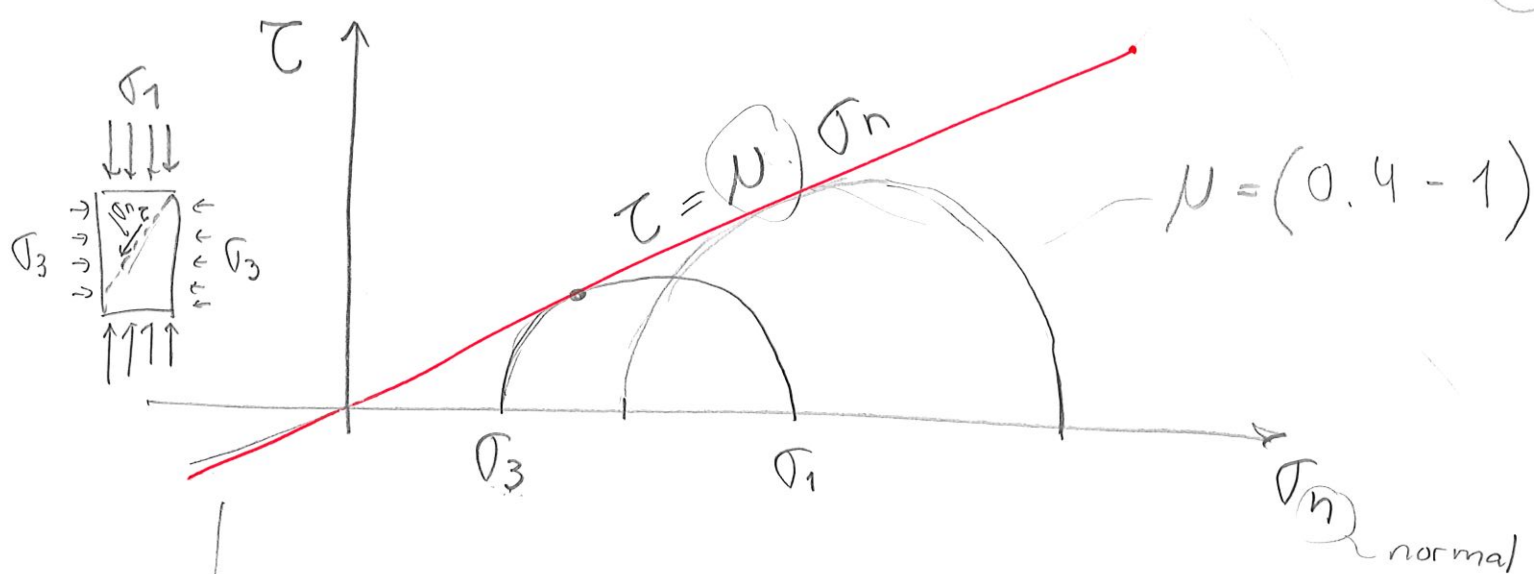
Strength is scale dependent

Shear strength

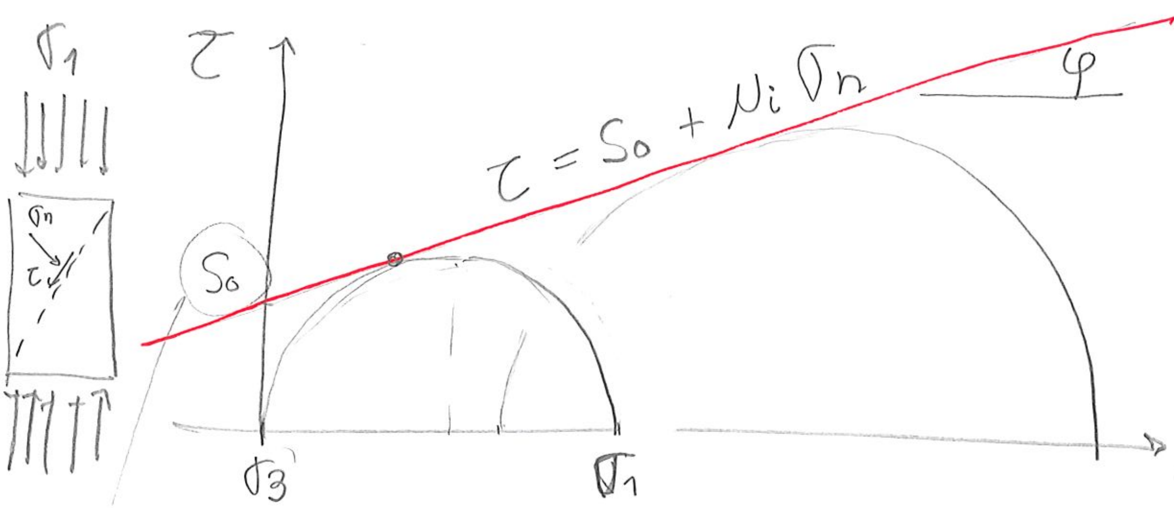


$F_T = \mu \cdot F_N$



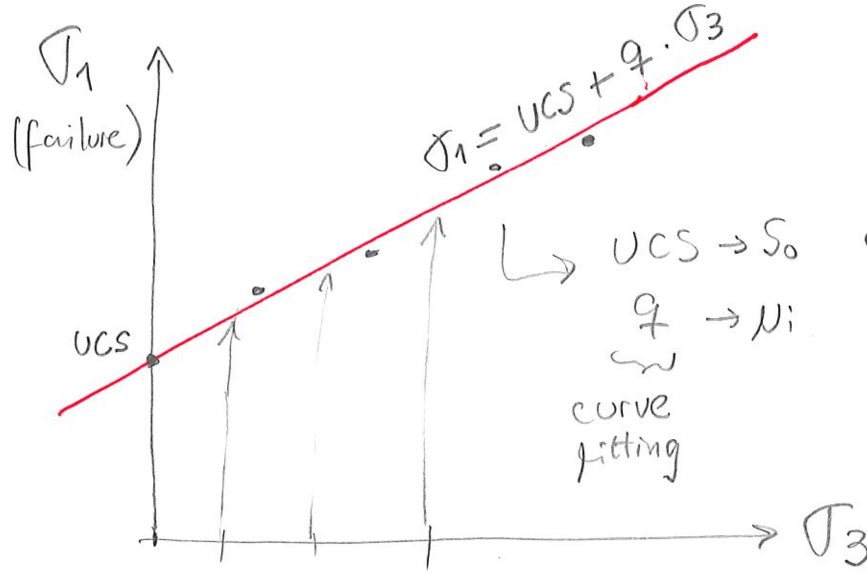


shear strength of uncemented sediments



If $\sigma_3 = 0 \Rightarrow \sigma_1(\text{failure}) = UCS$ – Unconfined compressive strength

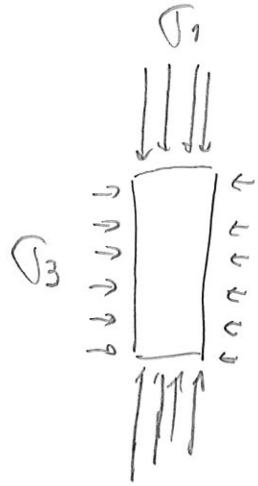
Cohesive strength



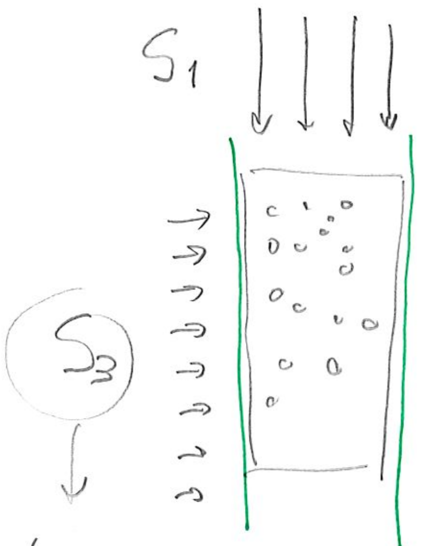
↳ $UCS \rightarrow S_0$

$q \rightarrow \mu_i$

curve fitting



$$q = \frac{1 + \sin\varphi}{1 - \sin\varphi} \quad ; \quad \mu_i = \tan\varphi$$



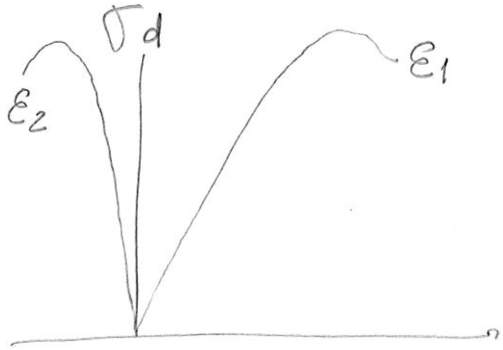
Lab

P_c

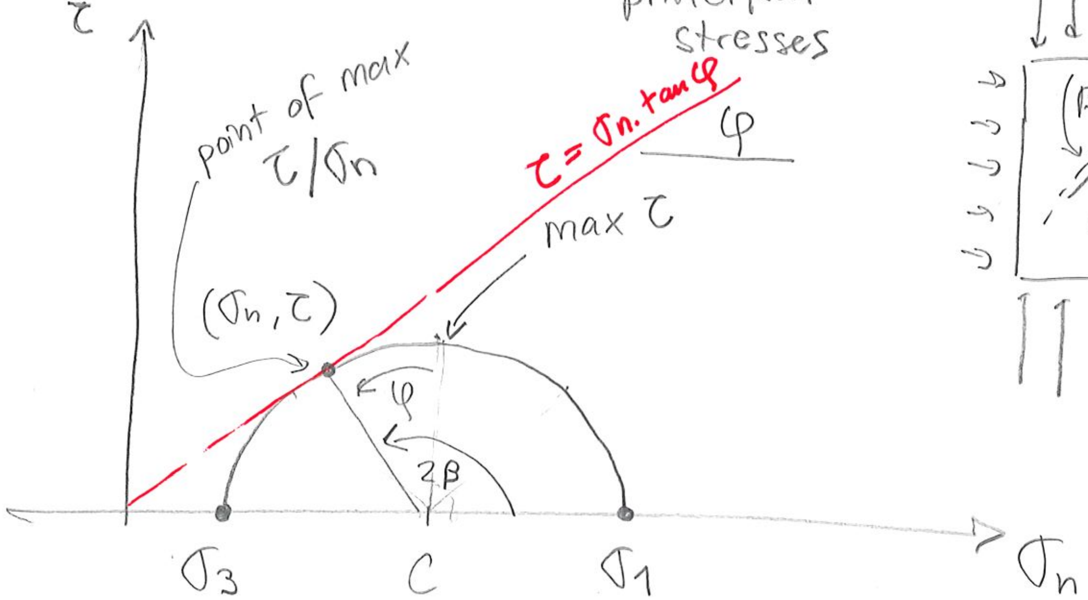
Confining pressure

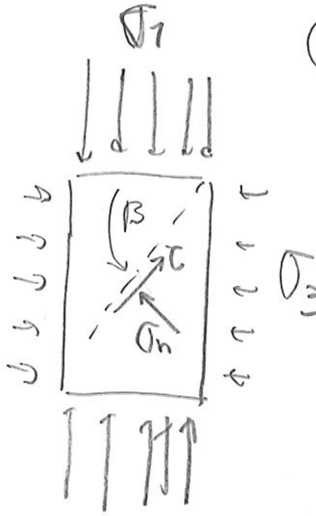
$\longrightarrow$

$$\sigma_1 = S_1 - P_p$$
$$\sigma_3 = S_3 - P_p$$
$$S_1 - S_3 = S_d$$
$$\sigma_1 - \sigma_3 = S_d = \sigma_d$$



σ_1, σ_3 are principal stresses



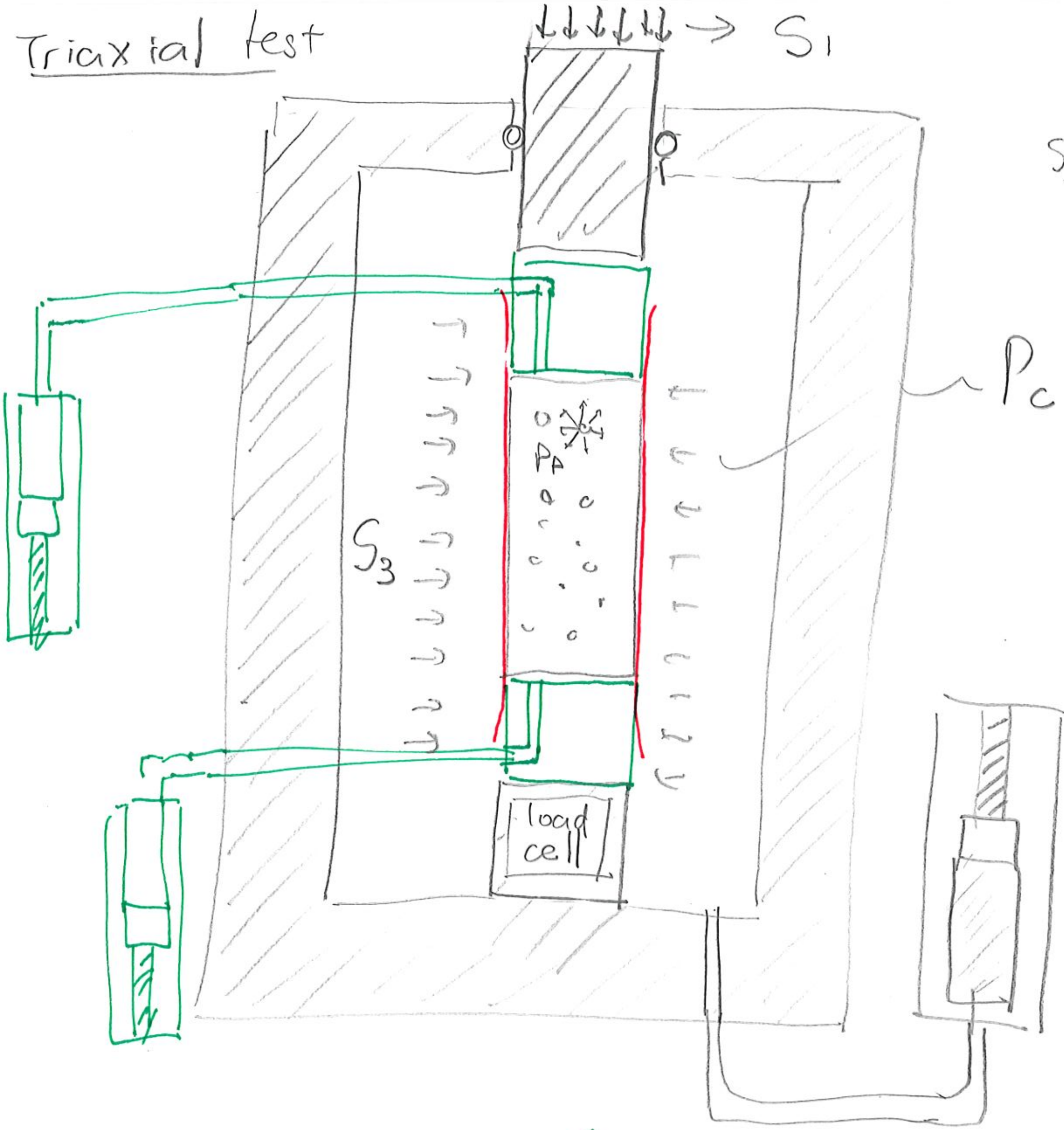


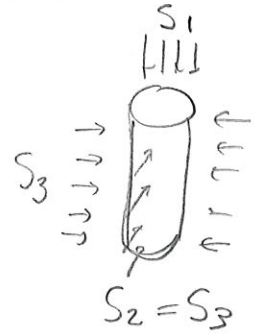
$$\begin{cases} C = \dfrac{\sigma_1 + \sigma_3}{2} \\ R = \dfrac{\sigma_1 - \sigma_3}{2} \end{cases} \Rightarrow \begin{cases} \sigma_n = C + R \cdot \cos 2\beta \\ \tau = R \cdot \sin 2\beta \end{cases}$$

Failure angle

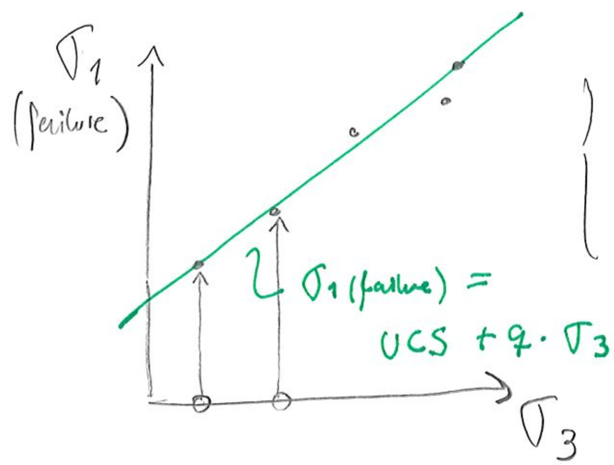
$$2\beta = \pi/2 + \varphi \Rightarrow \underset{\text{max } \tau/\sigma_n}{\beta} = 45° + \varphi/2$$

Triaxial test





P_c: confining pressure remains constant during the test

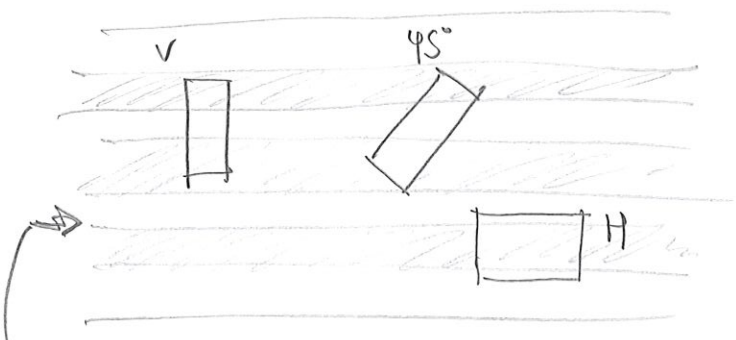


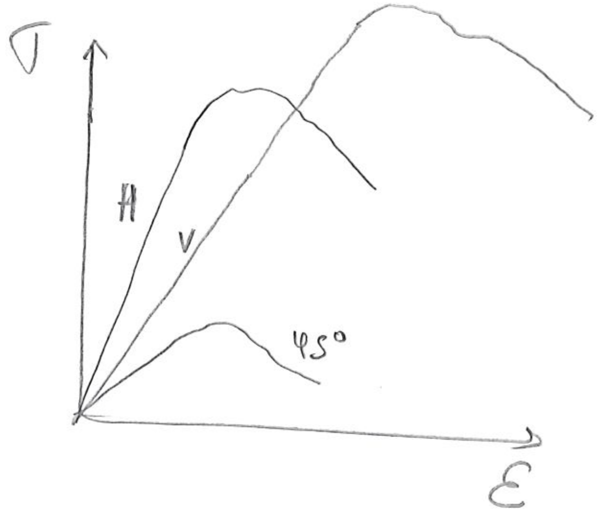
$$\begin{cases} \sigma_1 = S_1 - P_p \\ \sigma_3 = S_3 - P_p \end{cases}$$

$S_3 = P_c$

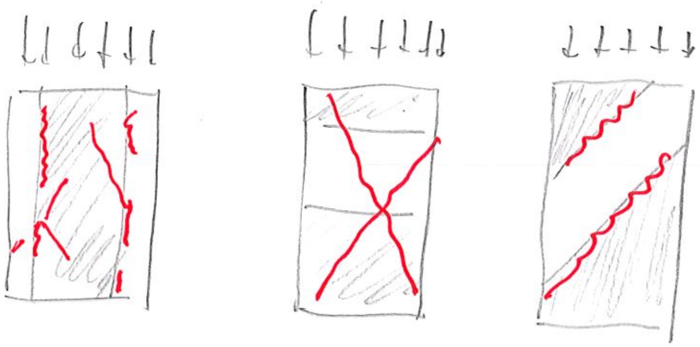
S_1
- → UG lab → measured directly
- → load cell → $S_d = S_1 - S_3 = \sigma_1 - \sigma_3$

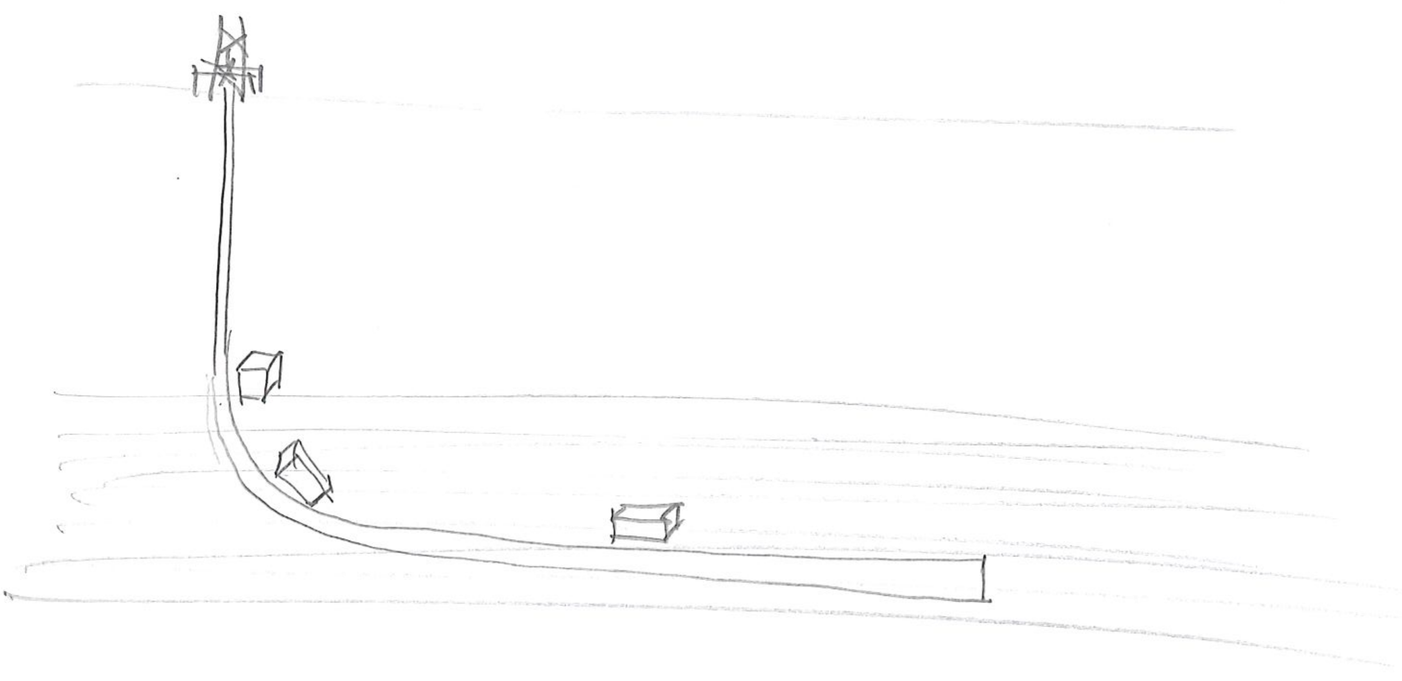
Strength anisotropy



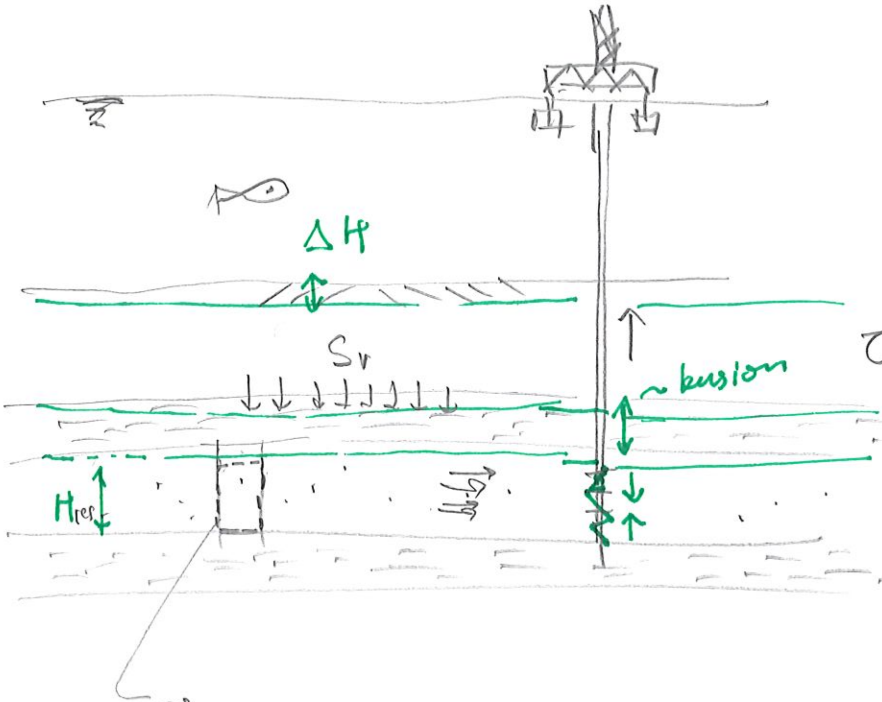


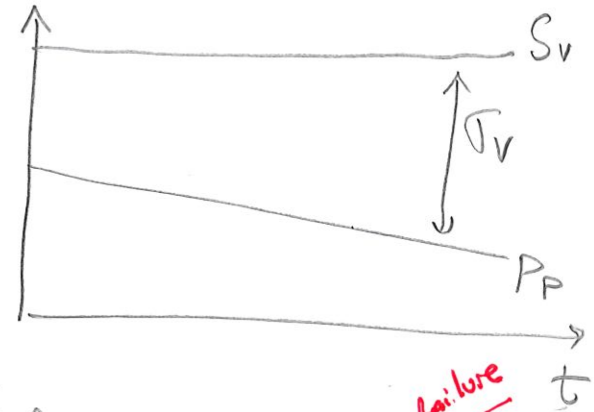
weak interfaces





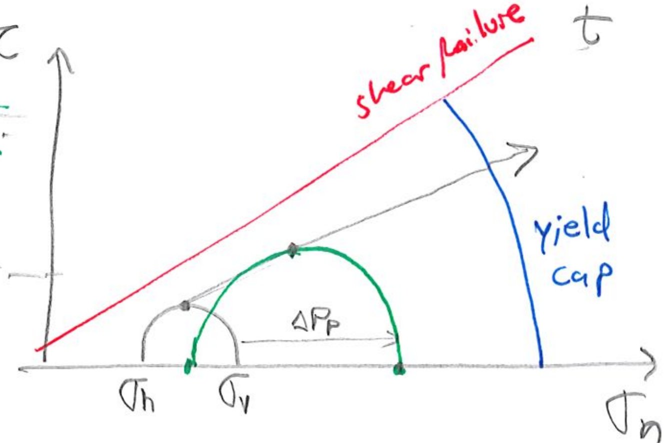
Compression failure

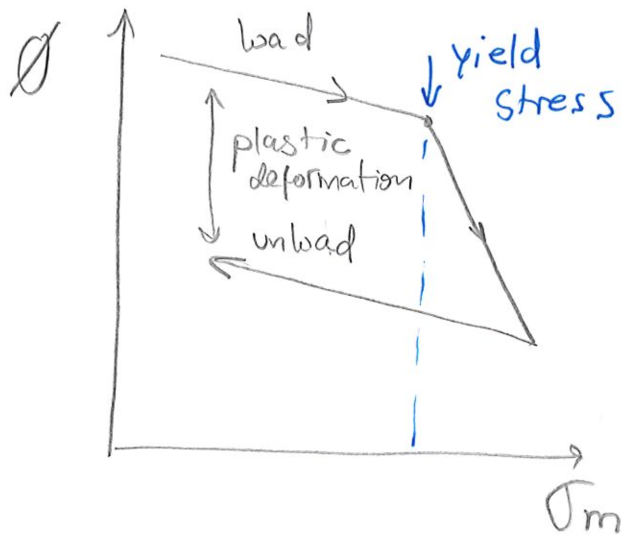


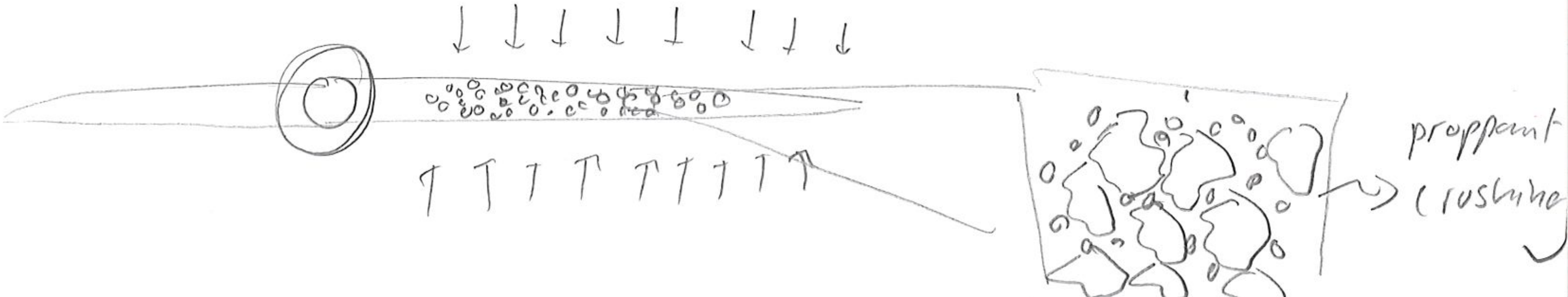


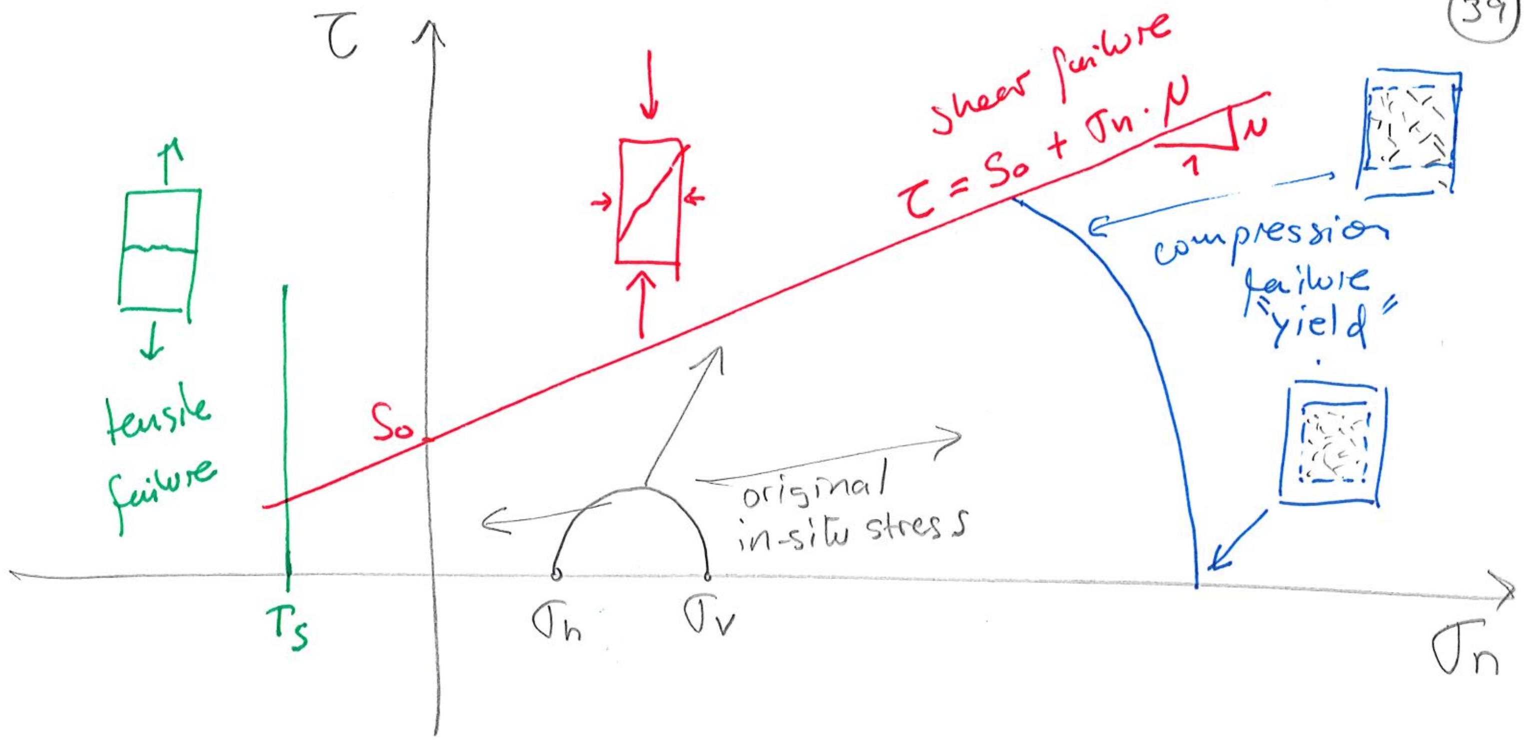
$\varepsilon_v^{res} = 20\%$

$\Delta H = \varepsilon_v^{res} \cdot H_{res}$

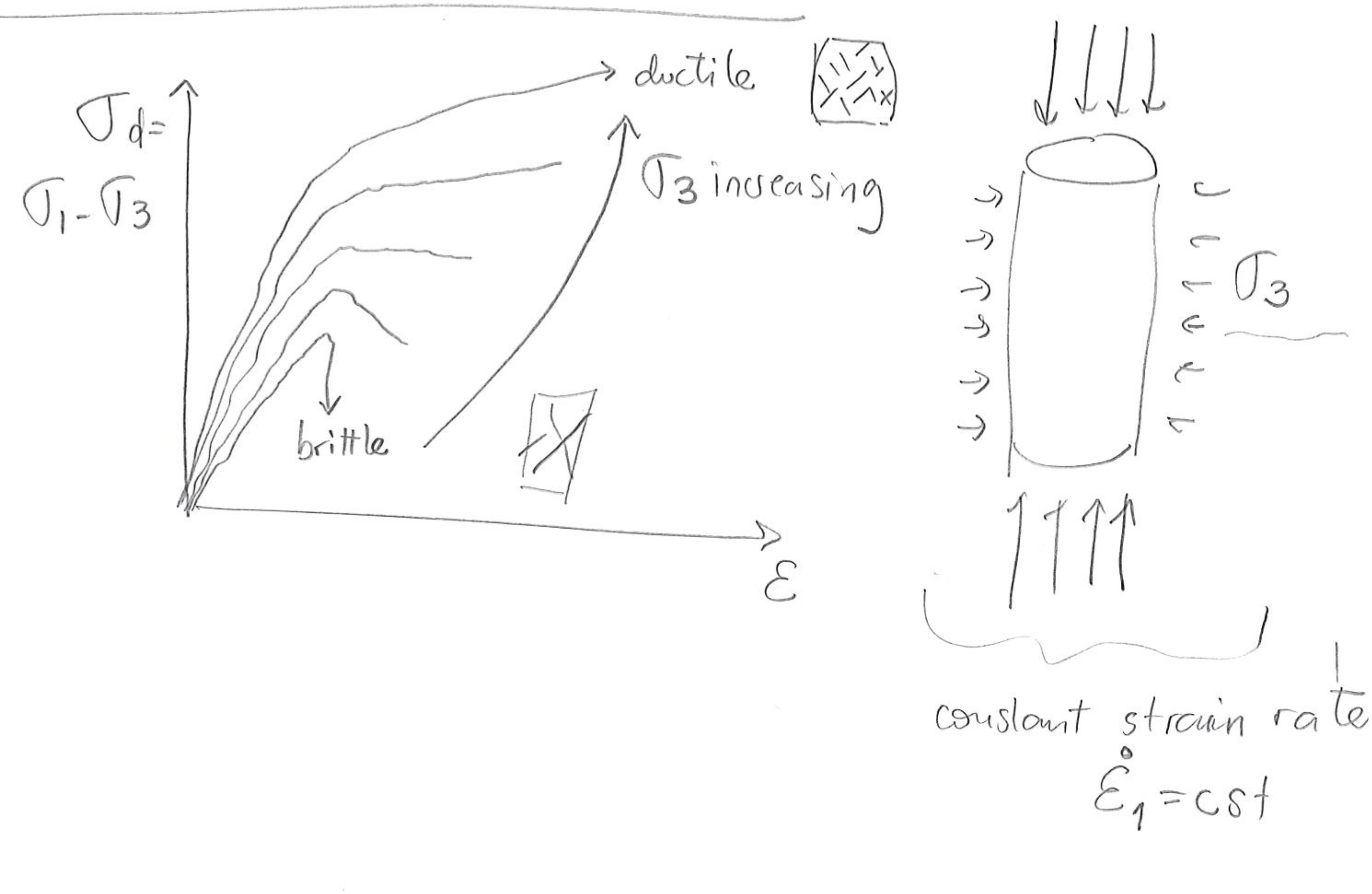


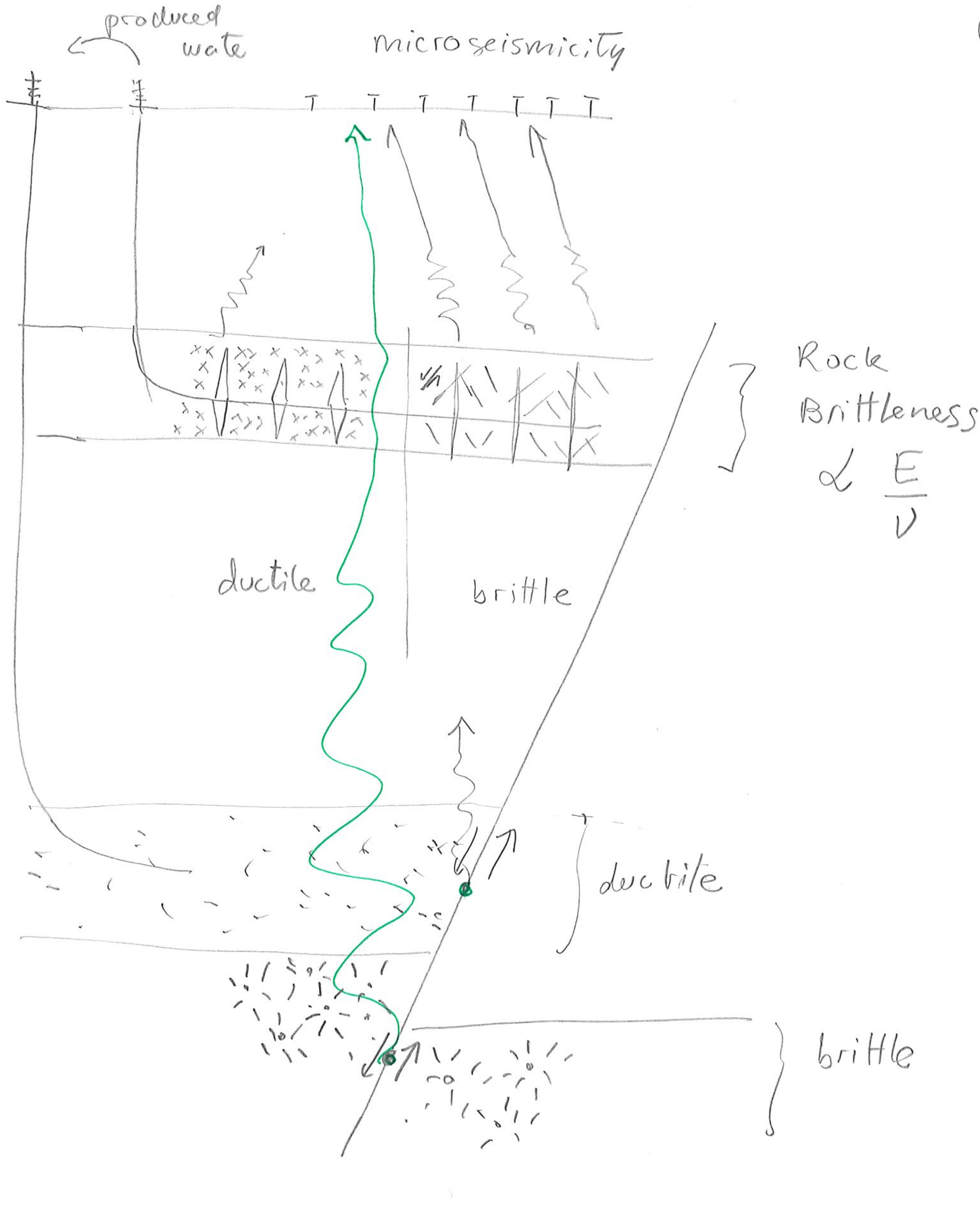




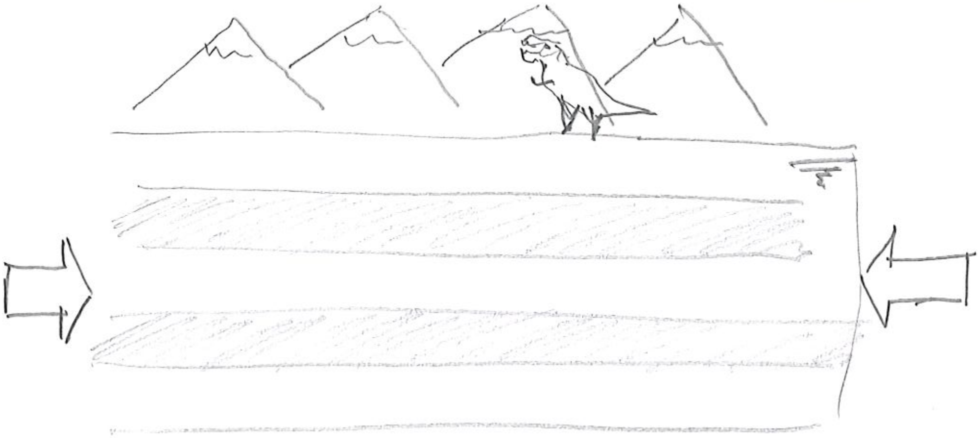


Brittle to ductile transition

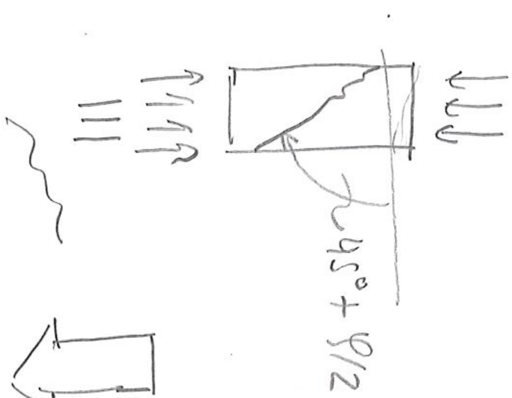


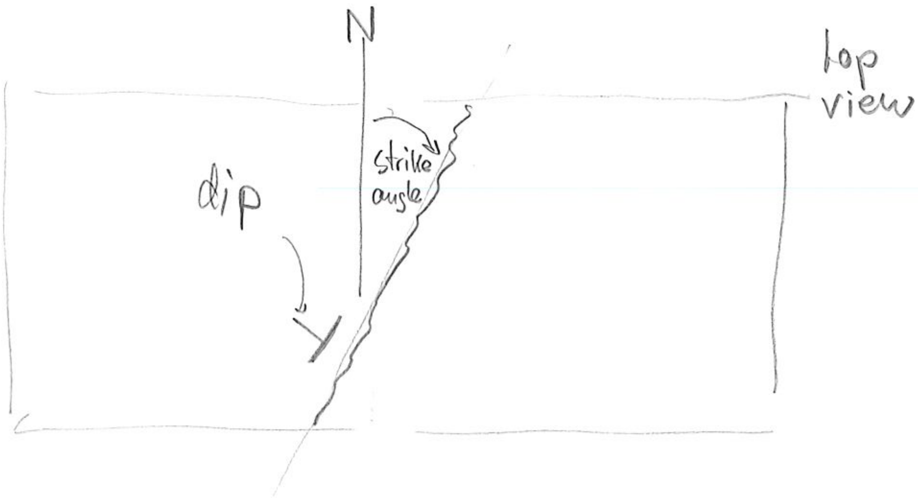

produced
wate
microseismicity
Rock
Brittleness
$\propto \frac{E}{\nu}$
ductile
brittle
ductile
brittle

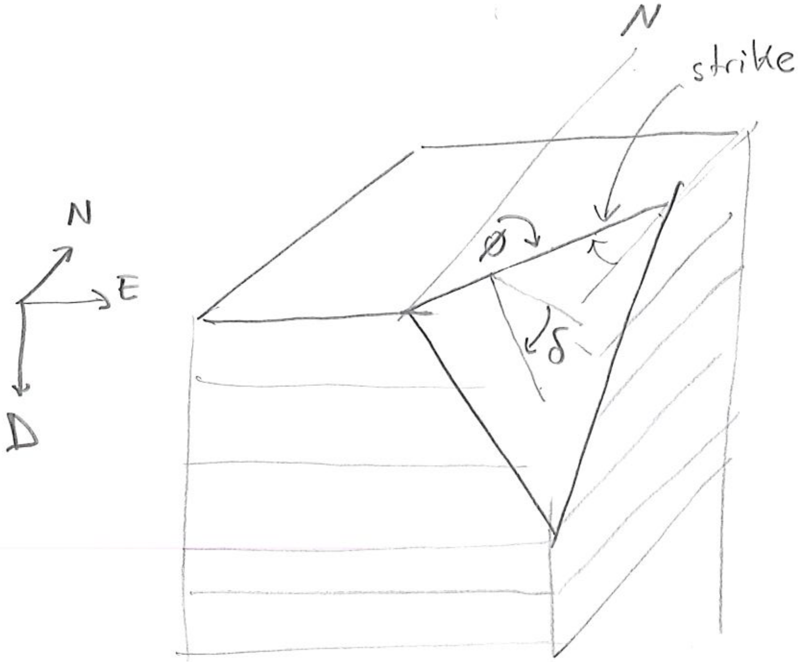
Faults and fractures (shear) → Stresses on a plane



long time







ø: strike angle

δ: dip

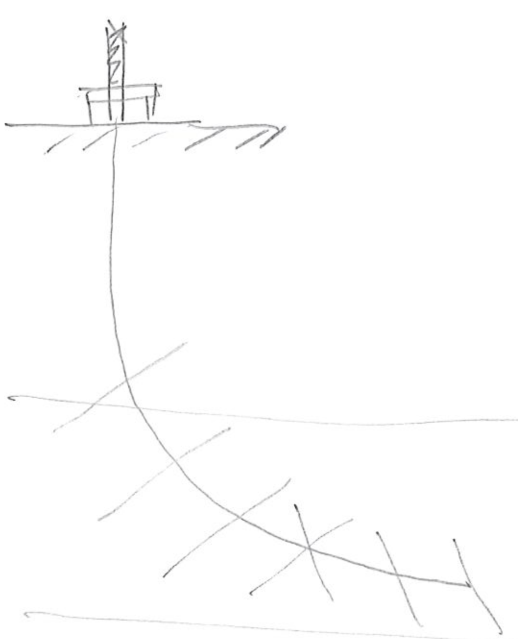
ø | Quadrant convention
= N 30° E
= S 30° W
Azimuth convention
= 030° ↳ clockwise from N
↳ 3 digits

Mapping faults

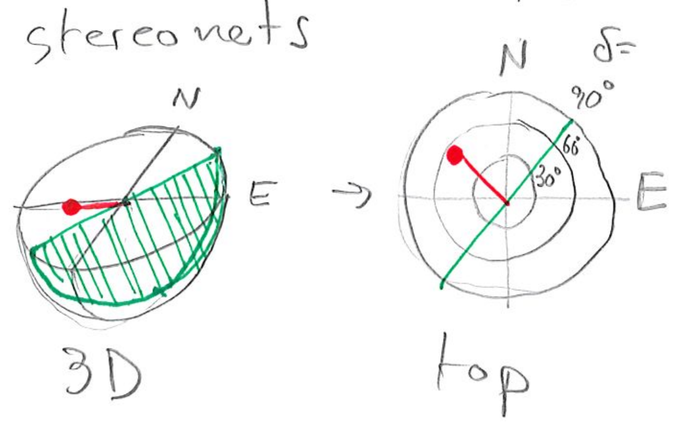
↳ seismic } big faults

↳ wellbore imaging } big faults, small fractures

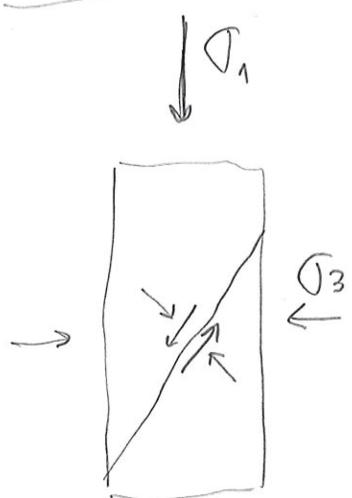
δ | 0-90°
= 50° E-S

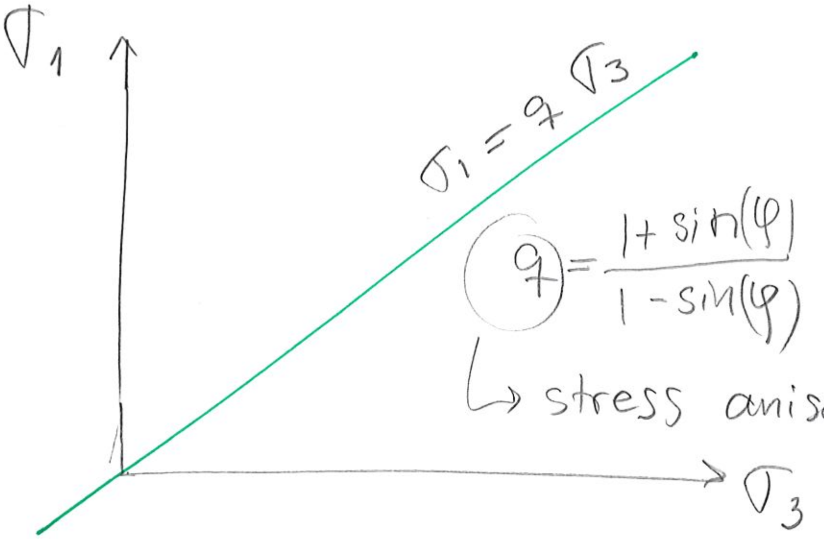


stereonets



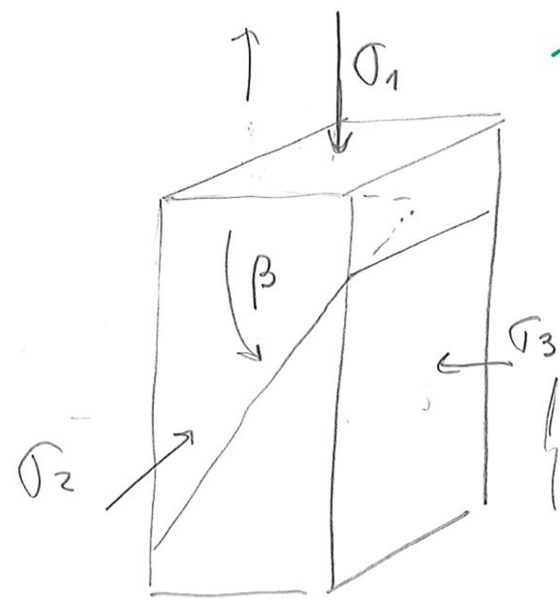
Strength of faults





$$q = \frac{1+\sin(\varphi)}{1-\sin(\varphi)} \quad \begin{cases} \varphi = 30^\circ \\ q = 3 \end{cases}$$

↳ stress anisotropy

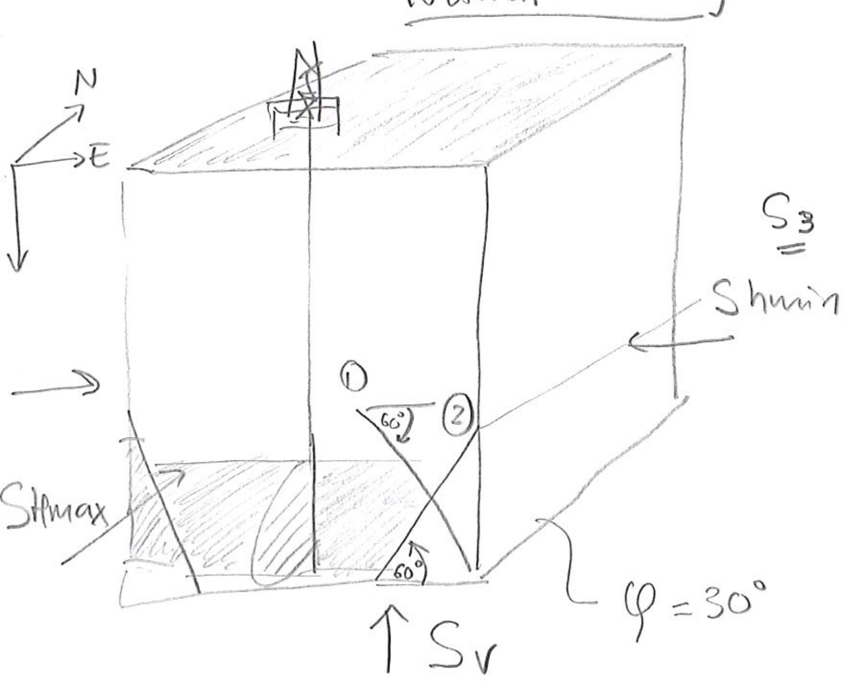


$$\beta = \frac{\pi}{4} + \frac{\varphi}{2} \; ; \quad \varphi = \arctan(\mu)$$

$$\mu = f(q)$$

$\varphi = 30^\circ \rightarrow \beta = 60^\circ$

Normal Faulting

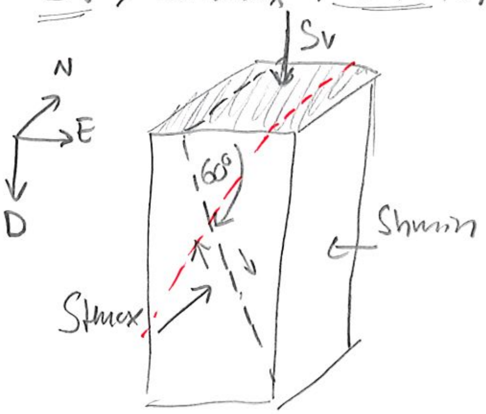


Hydraulic Frac $\begin{cases} \varnothing = 000^\circ \\ \delta = 90^\circ \end{cases}$

Fault $\begin{cases} \varnothing = 000^\circ \\ \delta = 60^\circ E \;\; ① \\ \;\;\; = 60^\circ W \;\; ② \end{cases}$

Normal Faulting

$S_v > S_{Hmax} > S_{hmin}$

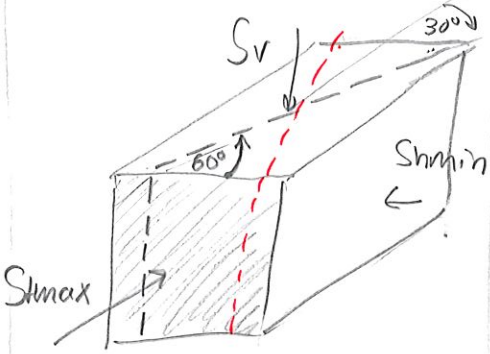


$\phi = 000°$

$\delta = 60°E$

Strike Slip

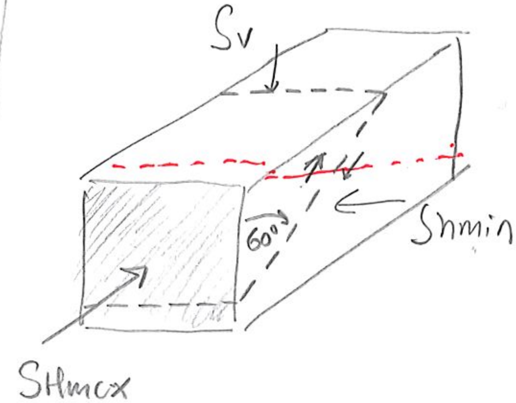
$S_{Hmax} > S_v > S_{hmin}$



$\phi = 030°$

$\delta = 90°$

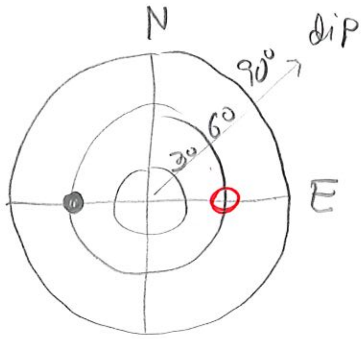
Reverse Faulting

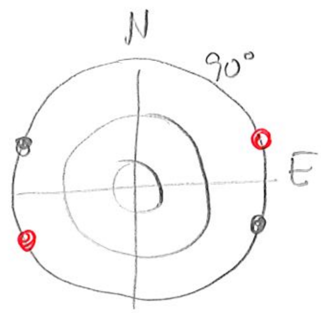


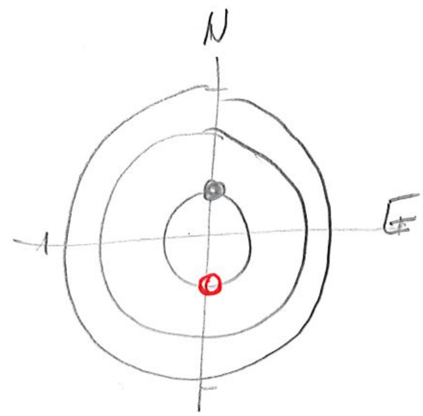
$\phi = 090°$

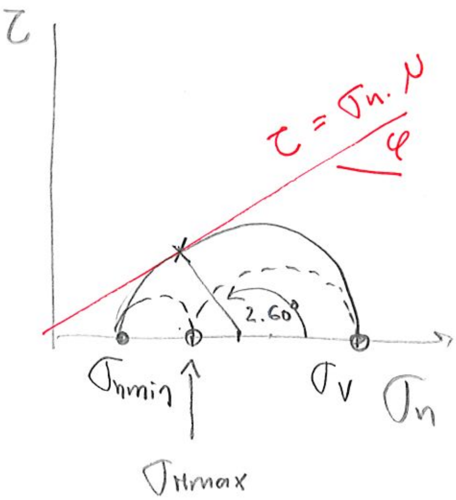
$\delta = 30°S$

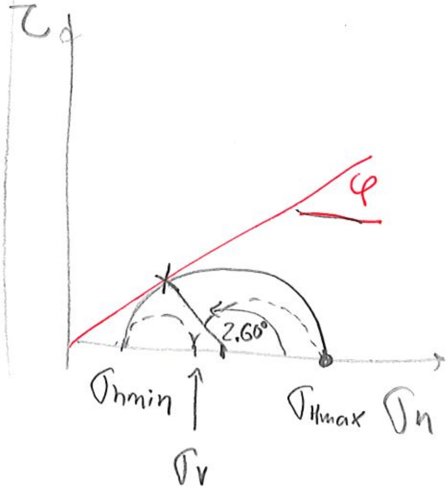
Red: conjugate fault ; Assumption $\varphi = 30°$

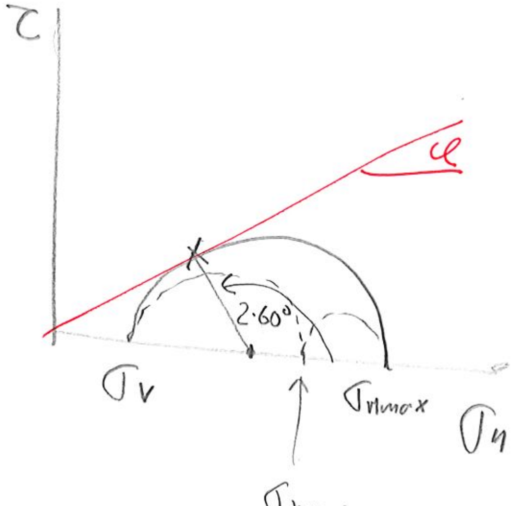










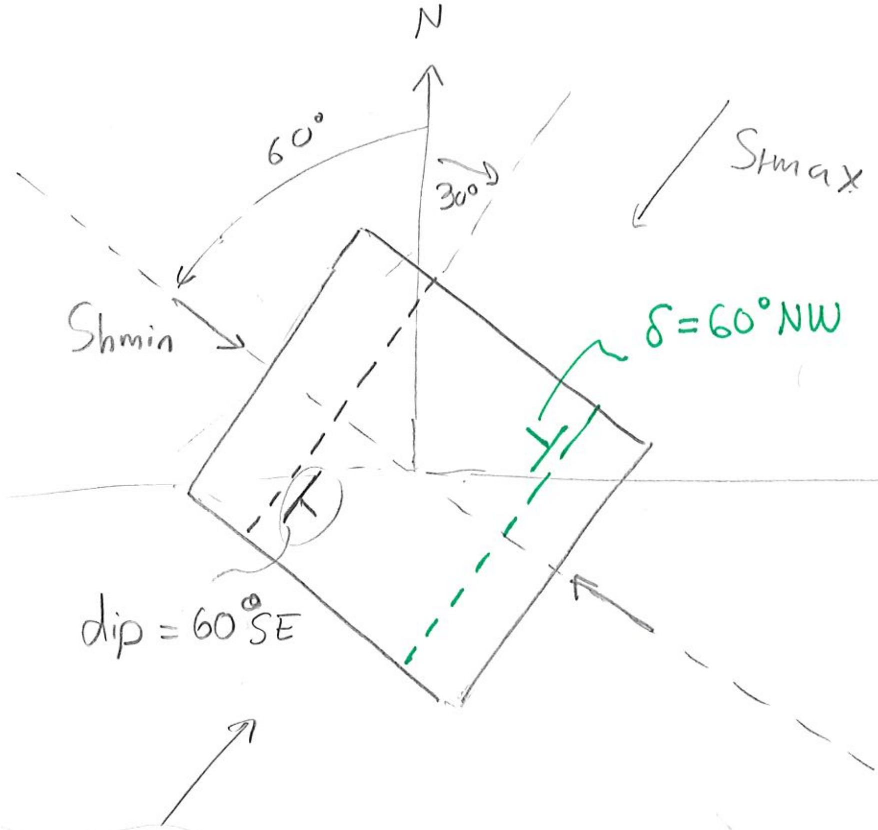


Problem 1

- NF, Sv is a principal stress
- Shmin at Azimuth = N 60° W
- $\varphi = 30°$

ideal orientation?

- HF = δ, $\emptyset$
- Faults = δ, $\emptyset$



$$\text{HF} = \begin{cases} \emptyset = 030° \\ \delta = 90° \end{cases}$$

$$60° = 45° + \varphi/2$$

$$\text{Faults} = \begin{cases} \emptyset = 030° \\ \delta_1 = 60° \text{SE} \\ \delta_2 = 60° \text{NW} \end{cases}$$

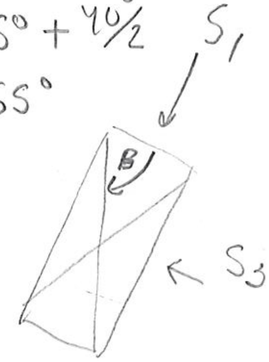
Problem 2

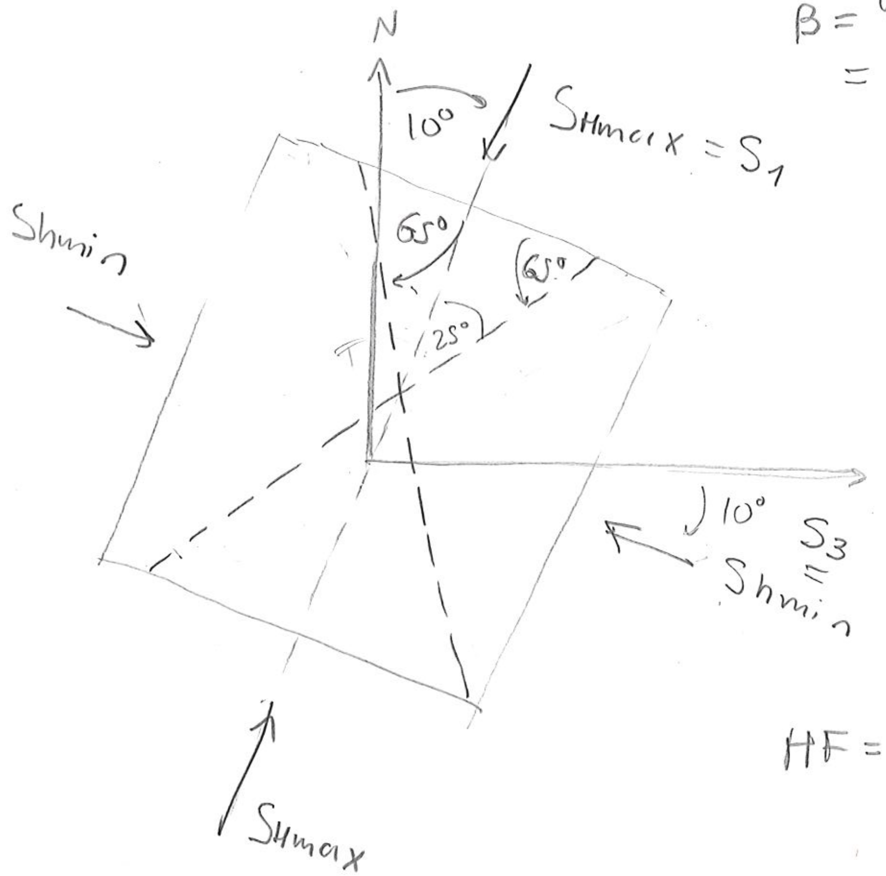
ideal orientation?

- SS, S_v = principal stress
- S_{Hmax} @ 010°
- $\varphi = 40°$

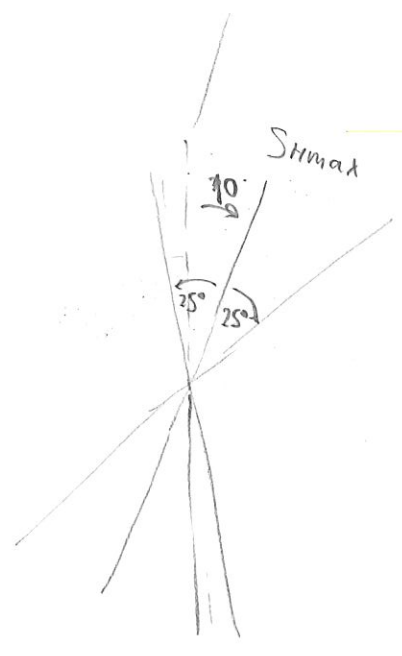
- HF
- Faults

$$\beta = 45° + 40°/2 = 65°$$





$$HF = \left| \begin{array}{l} \varnothing = 010° \\ \delta = 90° \end{array} \right.$$



$$\text{Faults} \left| \begin{array}{l} \varnothing = N15°W \\ \varnothing = N\overset{3}{\cancel{4}}5°E \\ \delta = 90° \end{array} \right.$$

Applications

(1) Ideal orientation of faults

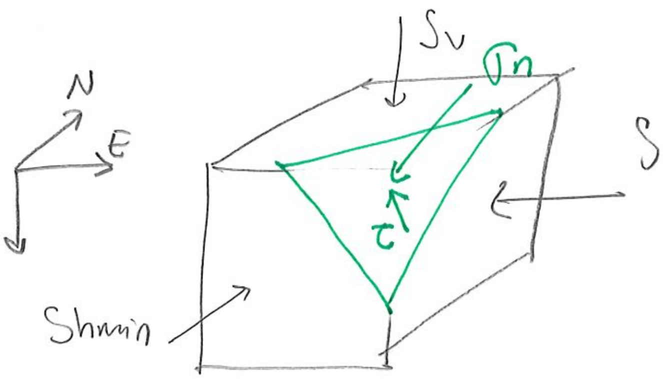
$\updownarrow$

Orientation of principal stresses

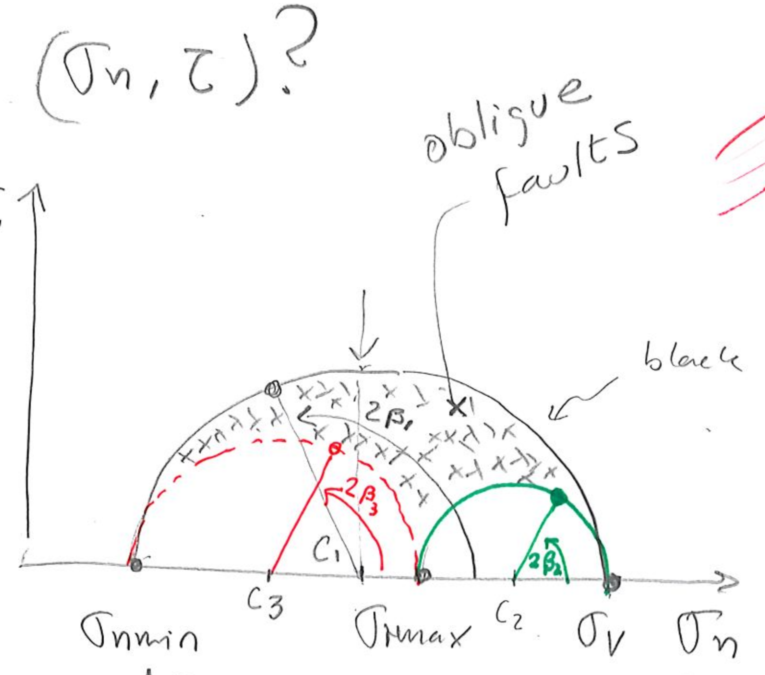
(2) Fracture reactivation (shear) or Fault reactivation due to $\uparrow \tau / \sigma_n$

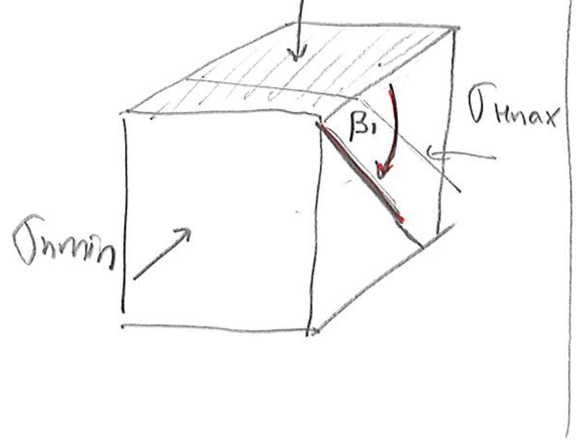
$\hookrightarrow$ calculate τ and σ_n

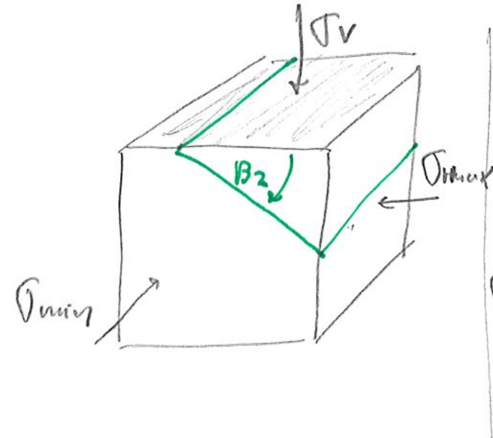
3D Mohr-Circle

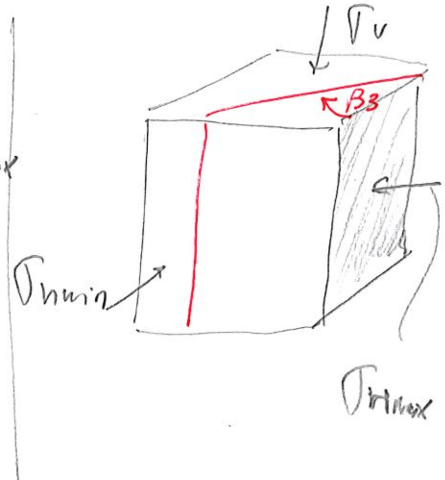


Normal Faulting









Problem 3

Azimuth = 090°

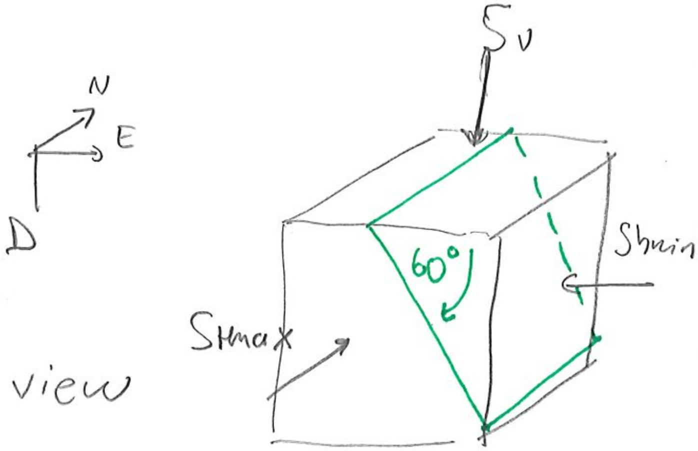
Stress | $S_v = 23\,MPa$; $S_{Hmax} = 20\,MPa$; $S_{hmin} = 13.8\,MPa$
| $P_p = 10\,MPa$

Fault { strike = 000°
dip = δ = 60°E

$\boxed{\sigma_n, \tau ? \quad \tau/\sigma_n}$

1) Identify stress regime

2) Draw block diagram or top view



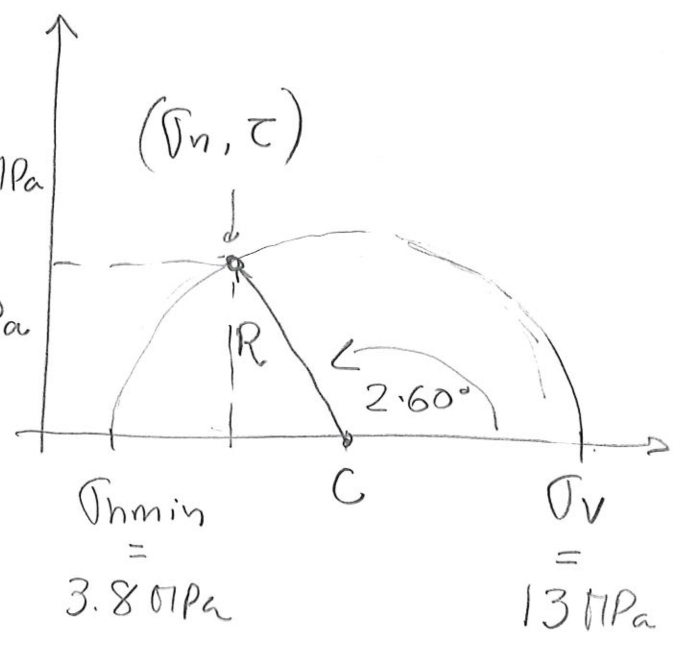
3) Identify fault and corresponding angle in the Mohr Circle

$\sigma_n = C + R \cdot \cos(2\ 60°) = 6.1\,MPa$

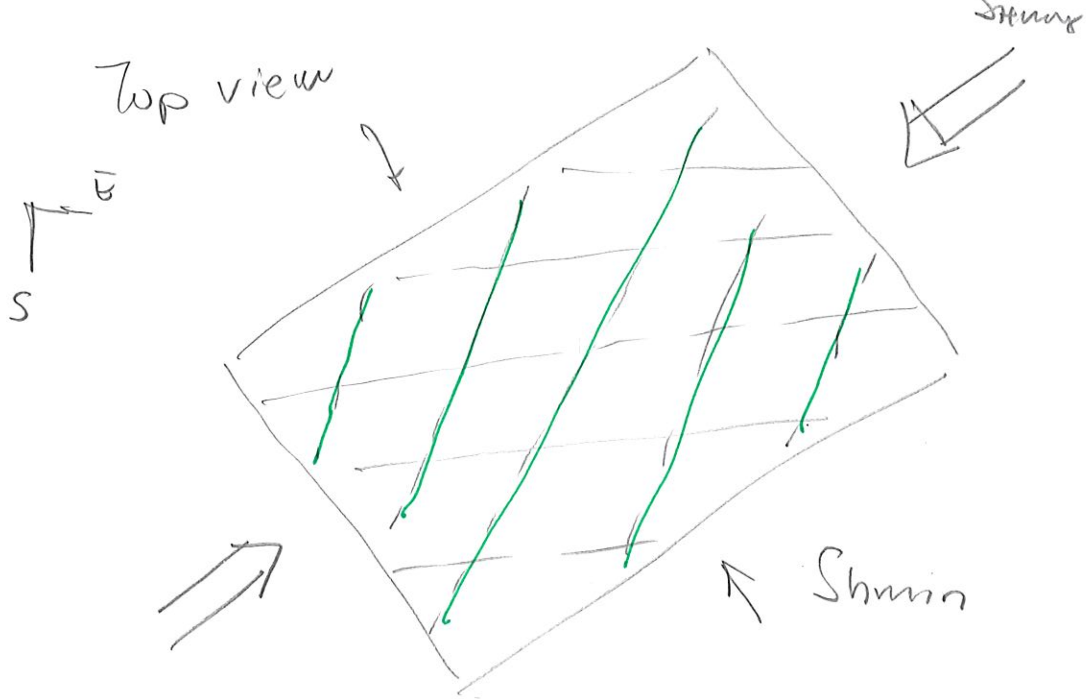
$\tau = R \cdot \sin(2 \cdot 60°) \cong 4\,MPa$

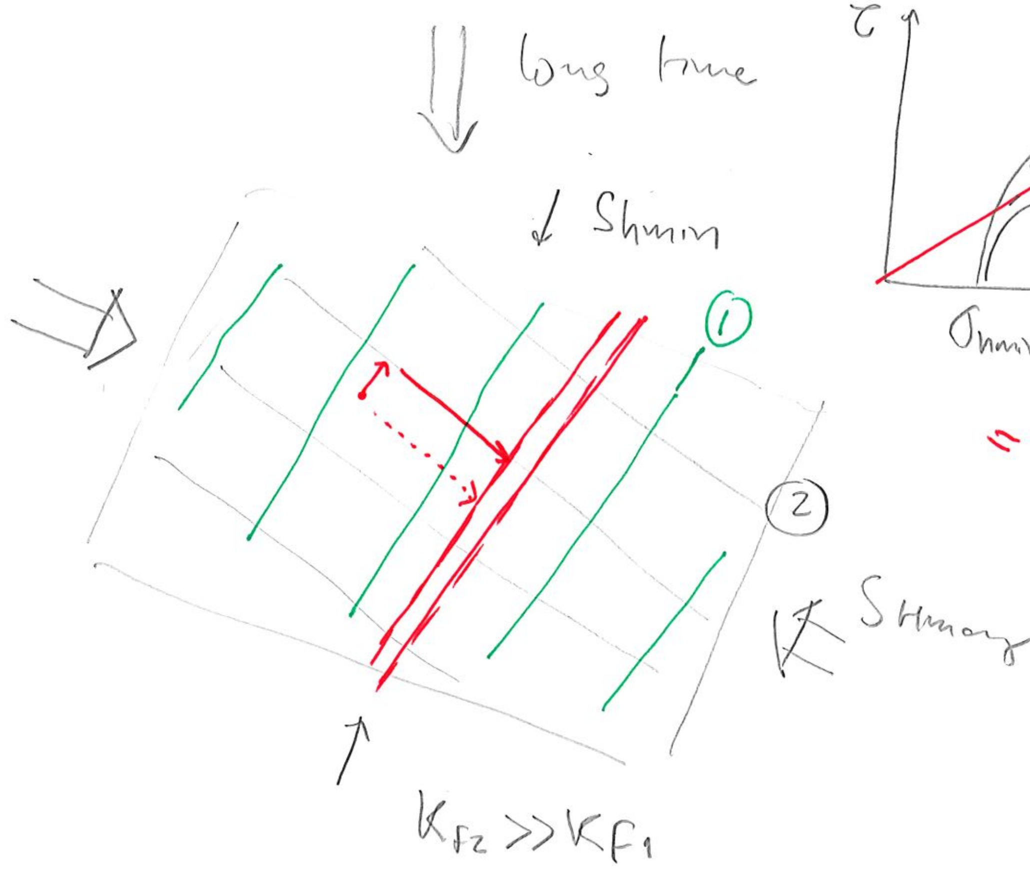
$C = (\sigma_v + \sigma_{hmin})/2$

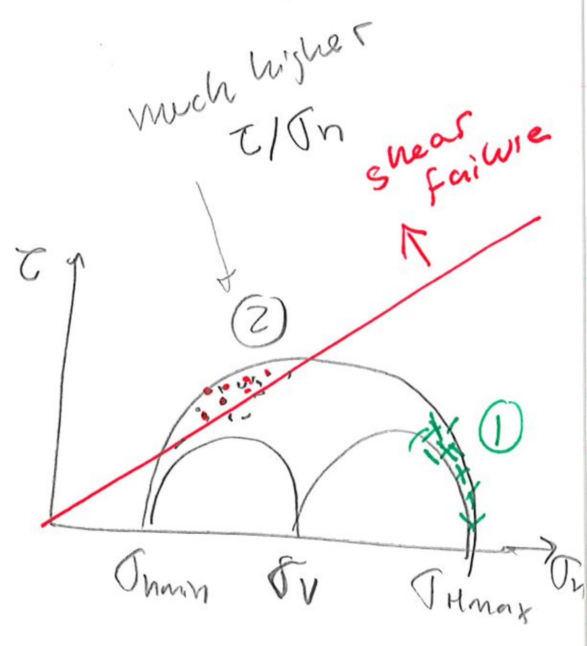
$R = (\sigma_v - \sigma_{hmin})/2$



Strike-Slip







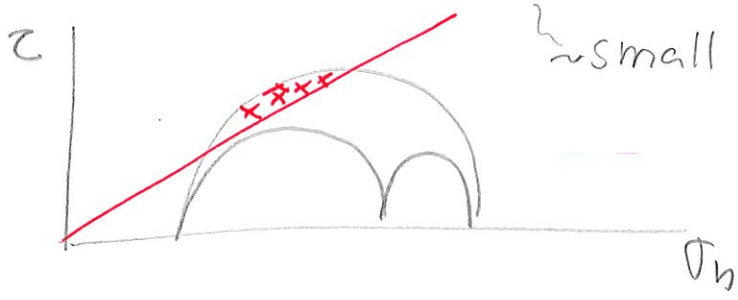
"critically stressed fractures"

↓

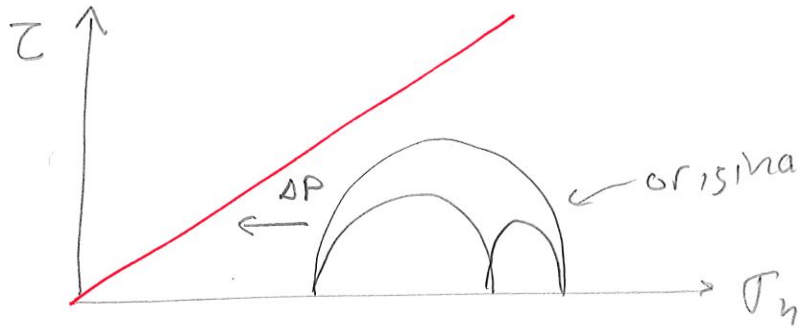
high perm

Applications for Reservoir Geomechanics

1 Critically stressed fractures



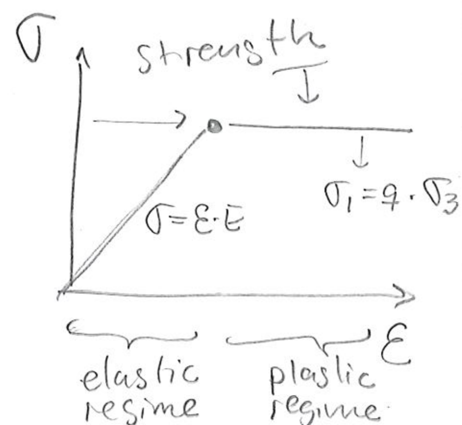
2 Fault reactivation



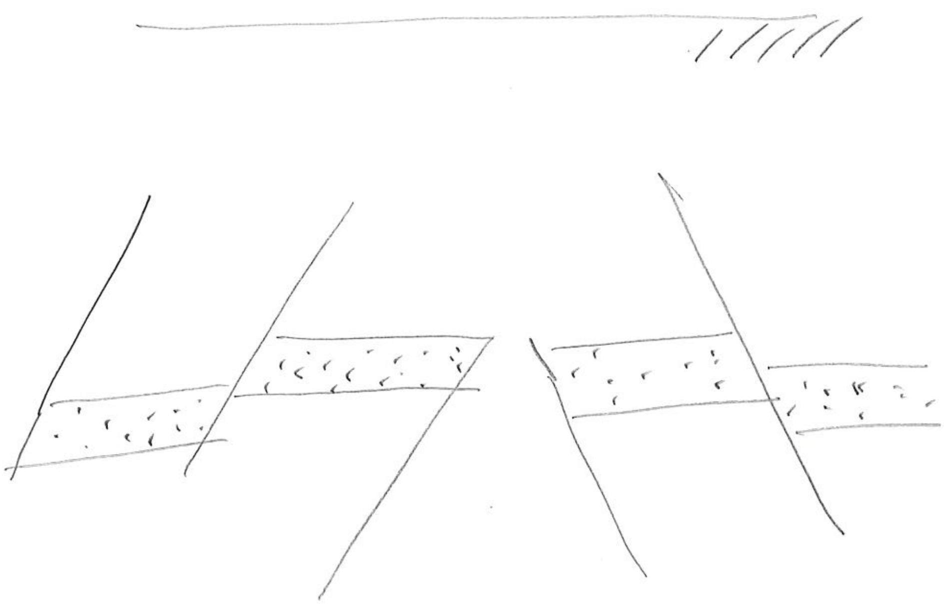
$$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \underline{\underline{I}}$$

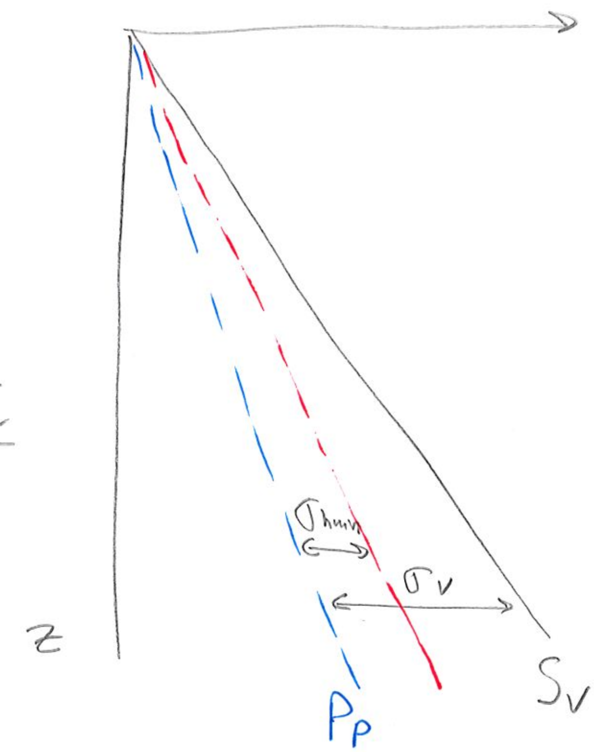
3 Determination of stresses based on fault equilibrium

$$\sigma_h = \frac{\nu}{1-\nu}\sigma_v + \frac{E}{1-\nu^2}\varepsilon_{Hmax} \ldots$$



Normal Faulting



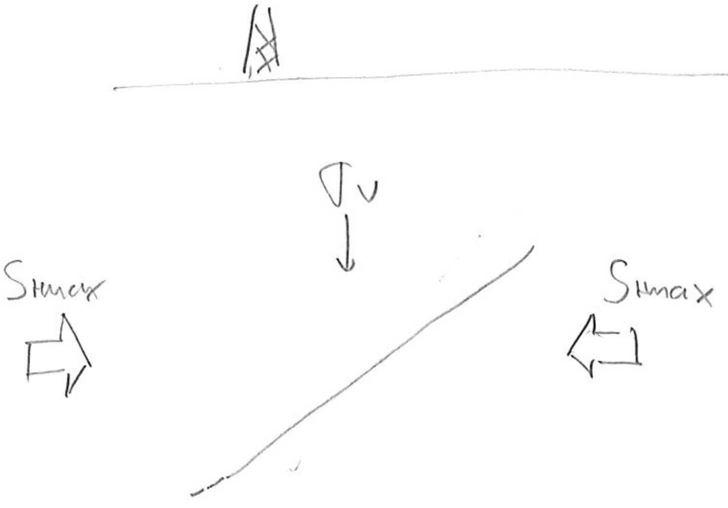


$$\sigma_{hmin} = \sigma_v / q$$

$$\sigma_3 = \sigma_1 / q$$

$$\varphi = 30^\circ \Rightarrow q = 3$$

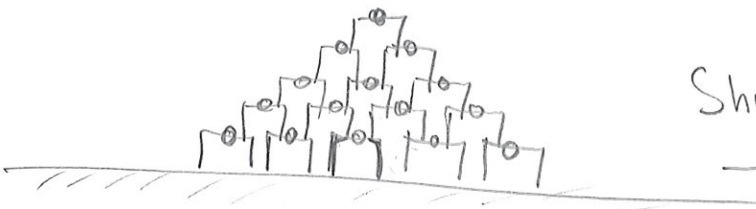
Reverse Faulting

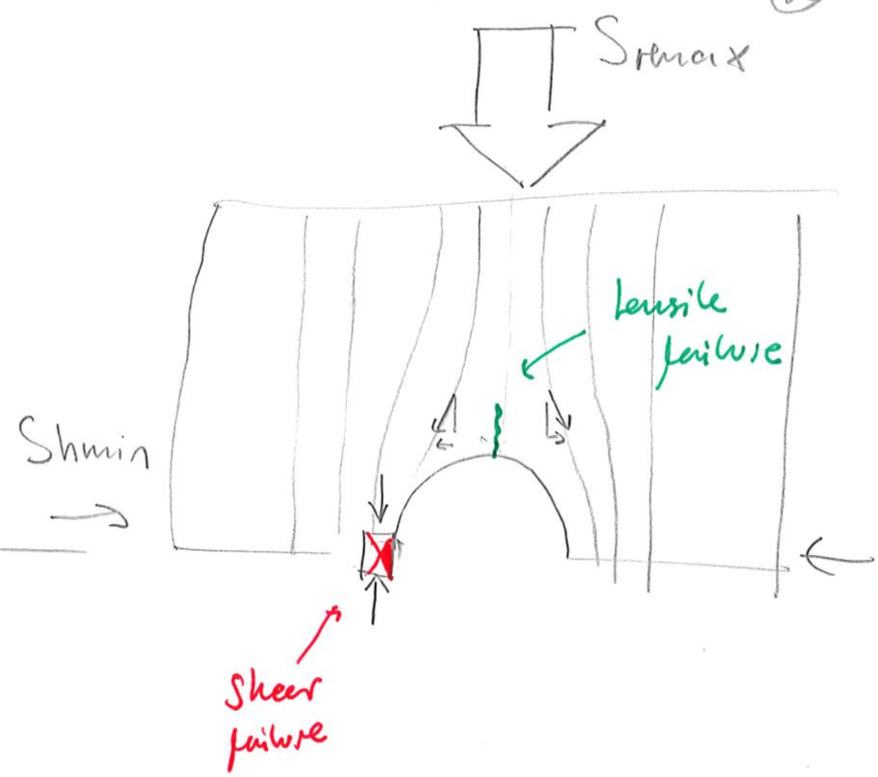


$$\sigma_{Hmax} = q \cdot \sigma_v$$

$$\sigma_1 = q \cdot \sigma_3$$

Wellbore stability





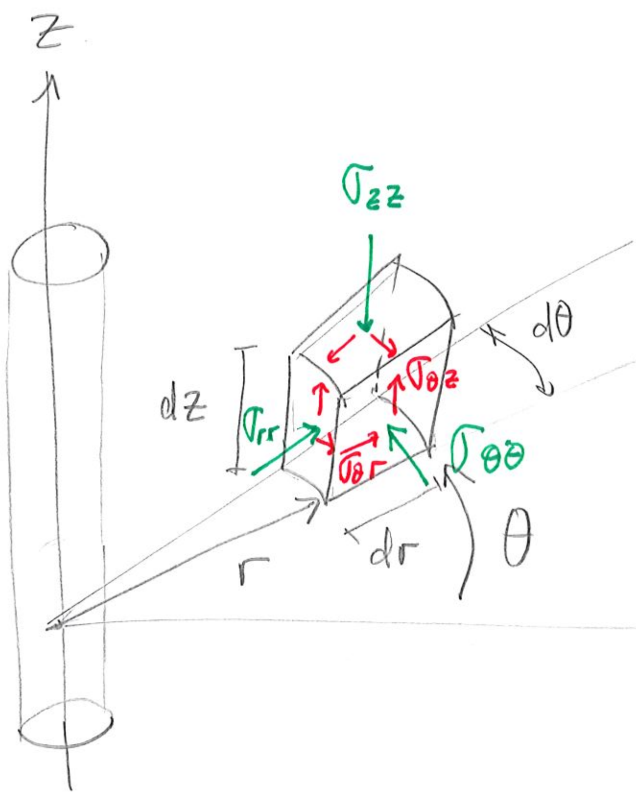
1st Lamé parameter — shear modulus — displacement ⇩ strain

$$(\lambda + G)\,\nabla \cdot (\nabla \underline{u}) + G\,\nabla^2 \underline{u} = 0$$

easy geometry
easy boundary conditions
⇩
analytical equation

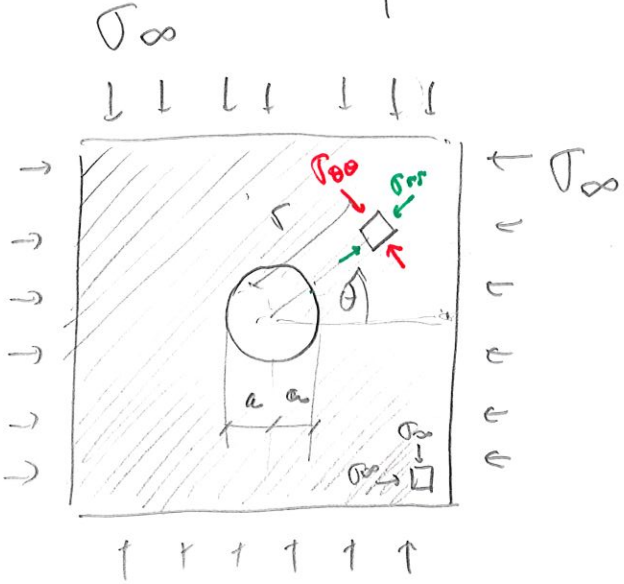
Kirsch equation (stresses around a cylindrical cavity with linear elastic isotropic solids)

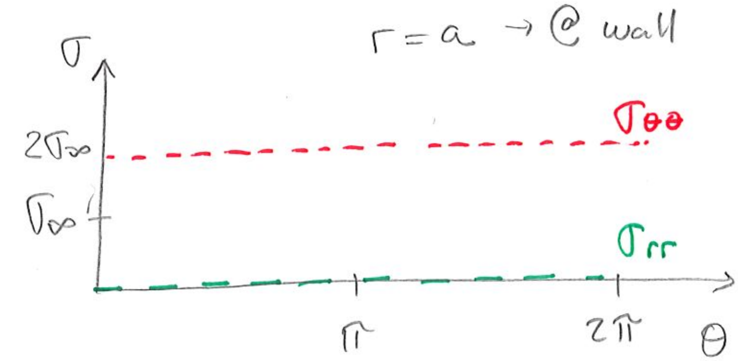
cylindrical coordinates

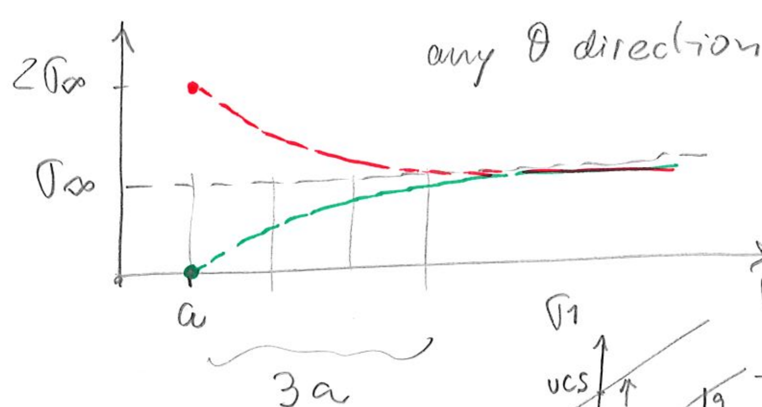


σ_{rr} = radial stress

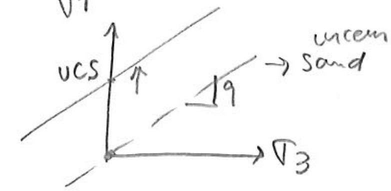
$\sigma_{\theta\theta}$ = tangential
circumferential
hoop stress



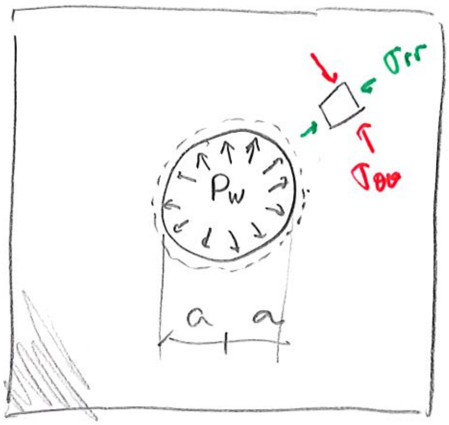


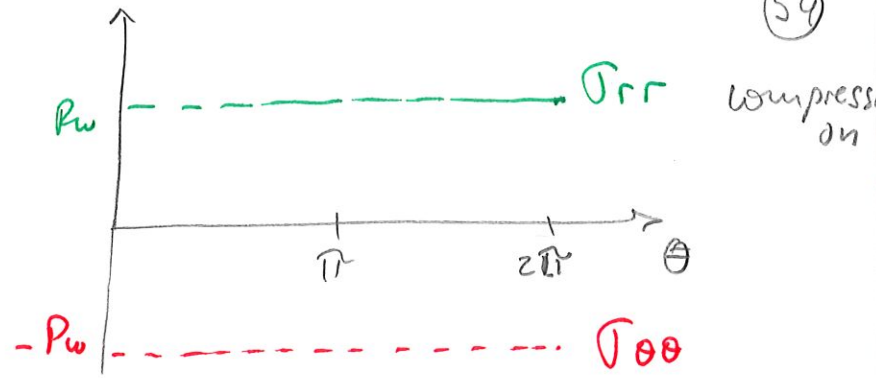


$$\begin{cases} \sigma_{rr} = \left(1 - \dfrac{a^2}{r^2}\right)\sigma_\infty \\ \sigma_{\theta\theta} = \left(1 + \dfrac{a^2}{r^2}\right)\sigma_\infty \end{cases}$$

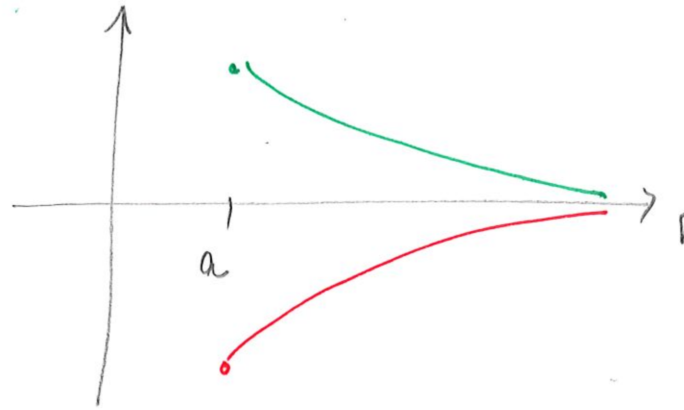


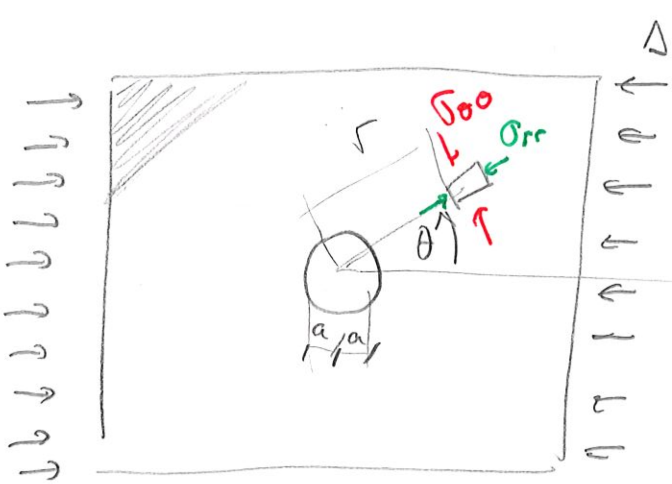
stress intensification $\dfrac{\sigma_{\theta\theta}(r=a)}{\sigma_\infty} = 2$; stress anisotropy ratio $= \dfrac{2\sigma_\infty}{0} \to \infty$

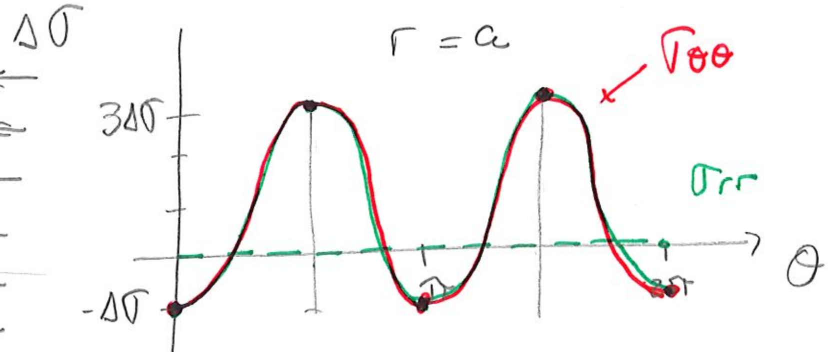




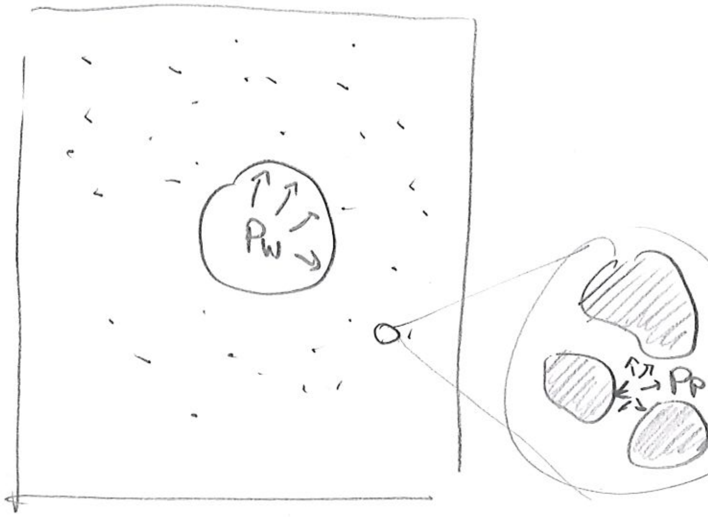
$$\left\{ \begin{aligned} \sigma_{rr} &= +P_w \frac{a^2}{r^2} \\ \sigma_{\theta\theta} &= -P_w \frac{a^2}{r^2} \end{aligned} \right.$$

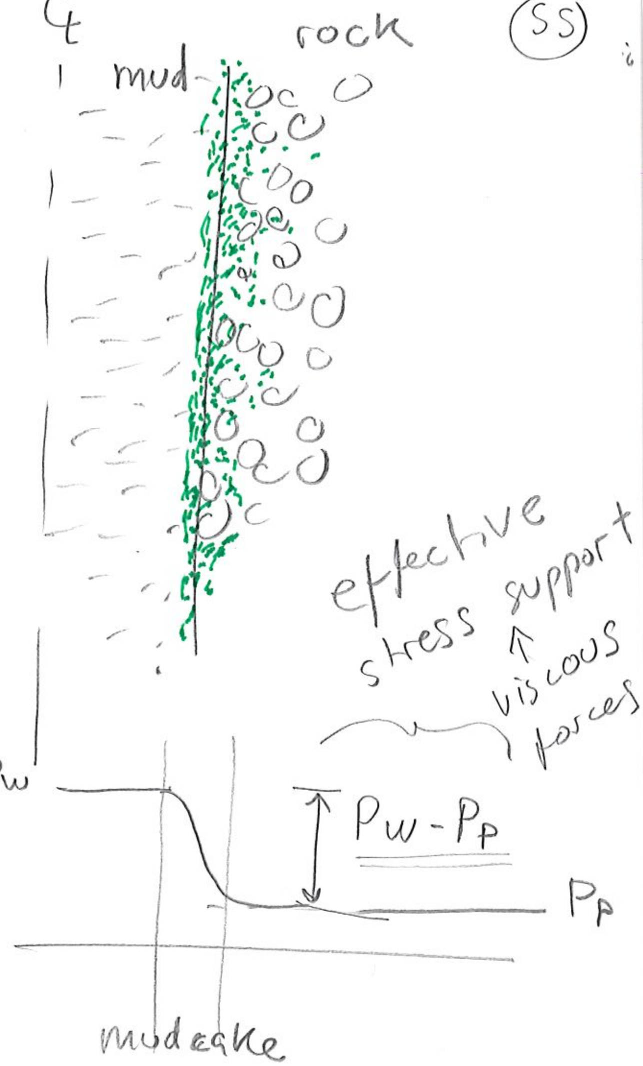






$$\text{stress intensity ratio} = \frac{3\Delta\sigma}{\Delta\sigma} = 3$$





(SS)

$$\underbrace{P_W}_{\text{mud pressure}} = \underbrace{\rho_{mud}\, g}_{\text{ECD}} \ \underline{TVD}$$

$$\sigma_{rr} = \underbrace{+(P_W - P_P)\left(\frac{a^2}{r^2}\right)}_{\text{effective wall support}} + \underbrace{\frac{\sigma_1 + \sigma_3}{2}\left(1 - \frac{a^2}{r^2}\right)}_{\text{far-field isotropic stress}} + \underbrace{\frac{\sigma_1 - \sigma_3}{2}\left(1 - 4\frac{a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos 2\theta}_{\text{differential stress}}$$

$$\sigma_{\theta\theta} = -(P_W - P_P)\left(\frac{a^2}{r^2}\right) + \frac{\sigma_1 + \sigma_3}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_1 - \sigma_3}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\theta$$

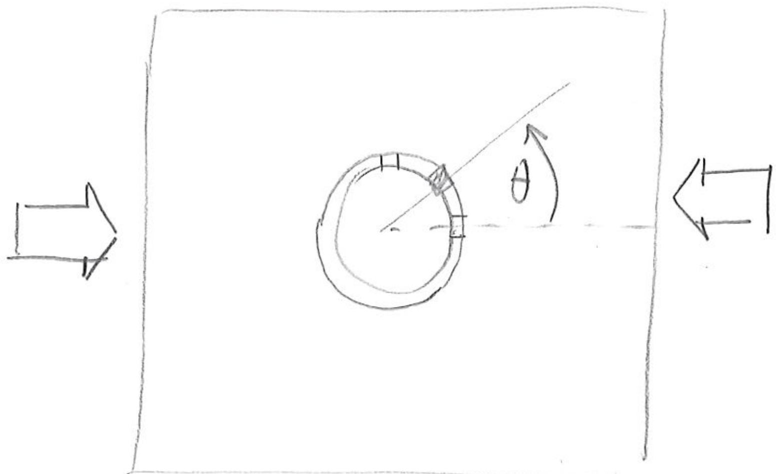
$$\sigma_{\theta r} = -\frac{\sigma_1 - \sigma_3}{2}\left(1 + 2\frac{a^2}{r^2} - \frac{3a^4}{r^4}\right)\sin 2\theta$$

Vertical wellbore

↳ stresses at $r = a$

$$\begin{cases} \sigma_{rr} = (P_w - P_p) \\ \sigma_{\theta\theta} = -(P_w - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos 2\theta \\ \sigma_{\theta r} = 0 \end{cases}$$

- Tensile fractures.
- Shear failure



Tensile fractures → Breakdown pressure

Minimum hoop stress $\theta = 0^\circ$ or π

$$-T_S = -(P_w - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})$$

break down pressure

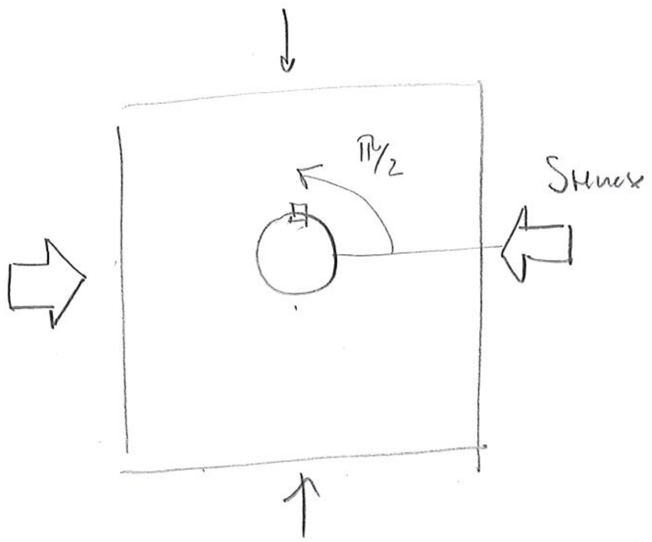
$$\boxed{P_b = \underbrace{P_p}_{\text{Pore pressure}} \underbrace{- \sigma_{Hmax} + 3\sigma_{hmin}}_{\text{Stress anisotropy}} + \underbrace{T_S}_{\text{Tensile stresth}}}$$

↳ If strike slip at limit esuilibrium

If $P_w > P_b$

↳ tensile fractures

Shear fractures → Break outs



$\theta = \pi/2\,;\ 3\pi/2$

$$\left\{\begin{array}{l}\sigma_{\theta\theta} = -(P_w - P_p) + 3\sigma_{Hmax} - \sigma_{hmin}\\ \sigma_{rr} = (P_w - P_p)\end{array}\right.$$

$$\sigma_1 = UCS + q\,\sigma_3$$

$$\sigma_1 \rightarrow \sigma_{\theta\theta} \qquad \sigma_3 \rightarrow \sigma_{rr}$$

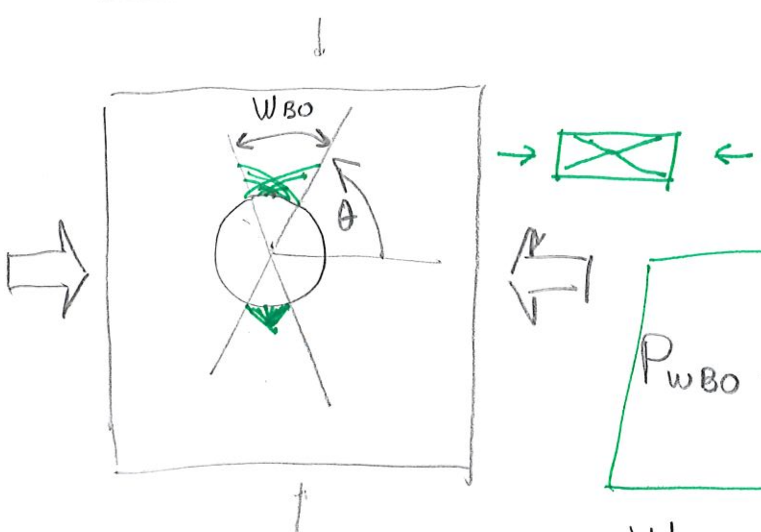
$$-(P_w - P_p) + 3\sigma_{Hmax} - \sigma_{hmin} = UCS + q\,(P_w - P_p)$$

$$\boxed{P_{w\,shear} = \underbrace{P_p}_{\text{pore pressure}} + \frac{\overbrace{3\sigma_{Hmax} - \sigma_{hmin}}^{\text{stress anisotropy}} - UCS}{1+q}}$$

strength of the rock

If $P_w < P_{w\,shear} \Rightarrow$ break outs

Breakout angle



$$\boxed{\theta = \pi/2 - W_{BO}/2}$$

$+$

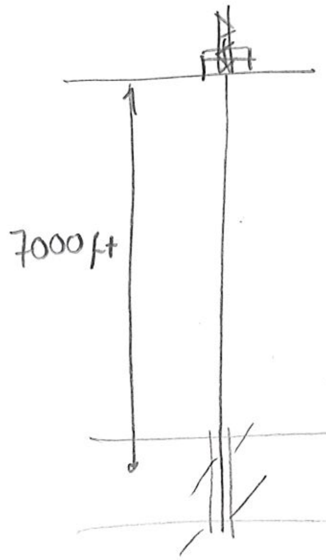
$$\boxed{\sigma_1 = UCS + q\,\sigma_3}$$

$$\boxed{P_{wBO} = P_p + \frac{(\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(\pi - W_{BO}) - UCS}{1+q}}$$

$W_{BO} \leqslant 60°$ (valid for)

(58)

Problem

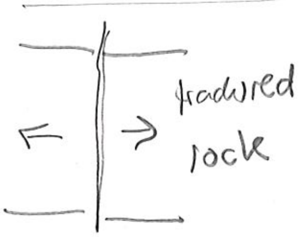


Site: $\frac{\Delta S_v}{dz} = 1$ psi/ft ; $\lambda_p = 0.44$; $S_{hmin} = 4300$ psi ; $S_{Hmax} = 6300$ psi

Rock: $T_s = 800$ psi ; $\mu_i = 0.6 \Rightarrow q = 3.12$; $UCS = 3500$ psi

$$P_b = 3080\,psi - 3220\,psi + 3 \cdot 1220\,psi + 800\,psi = 4320\,psi$$

$$= 11.64\,ppg$$



$T_s = 0$ psi

$\downarrow$

$P_b = 3520$ psi

8.3 ppg — 0.44 psi/ft

11.64 ppg — 0.617 psi/ft

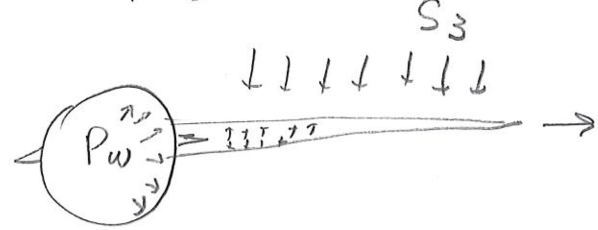
$$P_{w\,shear} = 3080\,psi + \frac{3 \cdot 3220 - 1220 - 3500}{1 + 3.12} = 4279\,psi$$

$$= 11.53\,psi$$

$$P_{w\,BO=70°} = 3640\,psi = 9.8\,ppg$$

WBO = ? for $P_w = 10$ ppg

$P_w > P_b > S_3 \Rightarrow$ uncontrolled fracture propagation



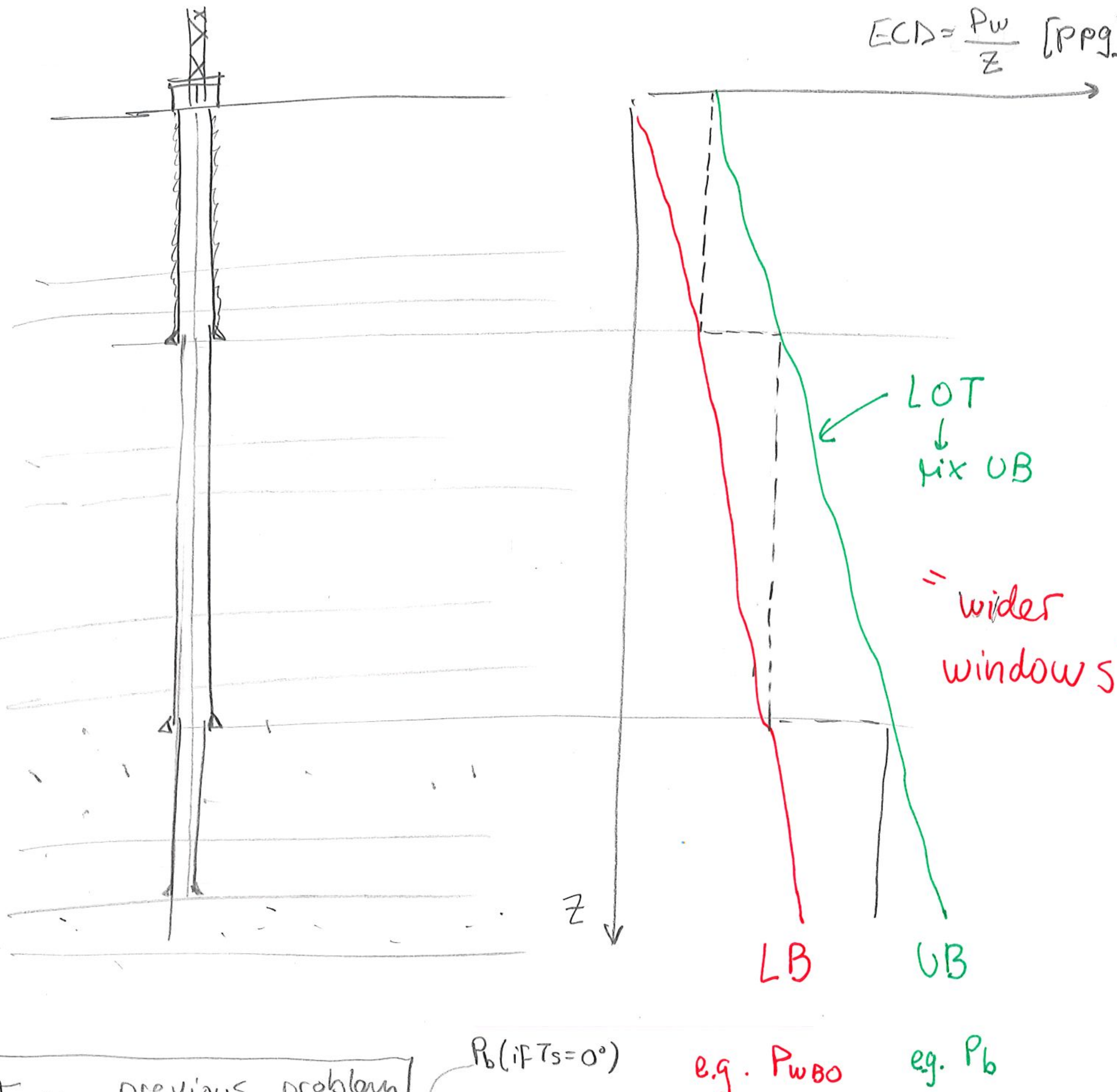
→ lost circulation

$P_b < \underline{P_w} < S_3 \Rightarrow$ short tensile fracture

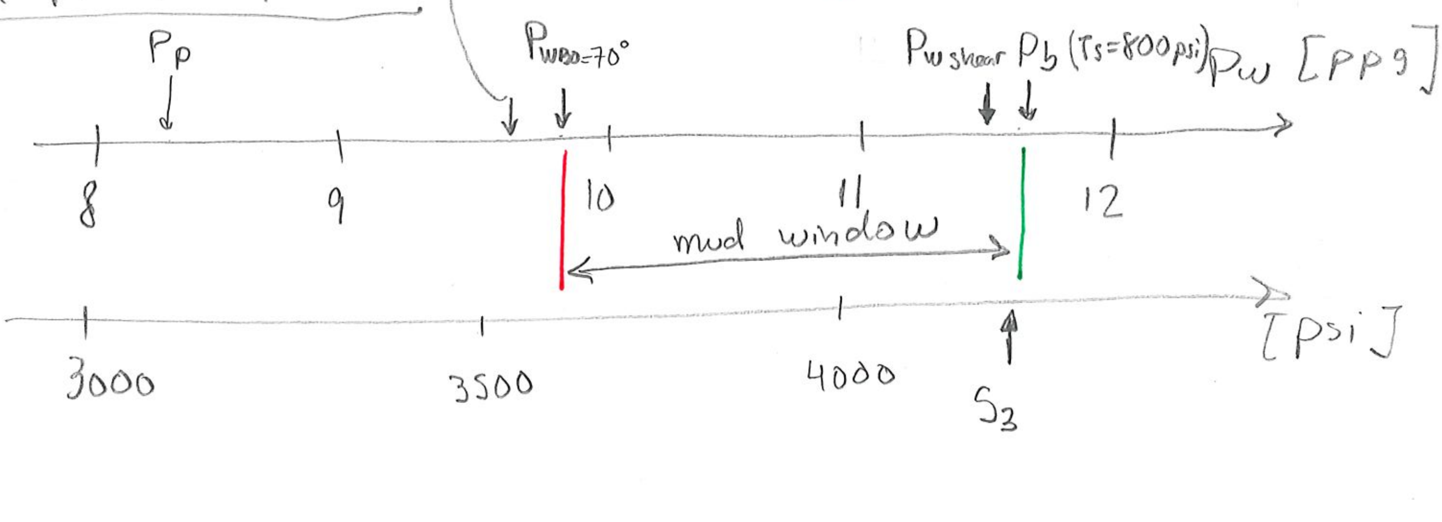
→ does not comprise wellbore stability

$P_w = f(ECD, z)$

$ECD = \frac{P_w}{z}$ [ppg]

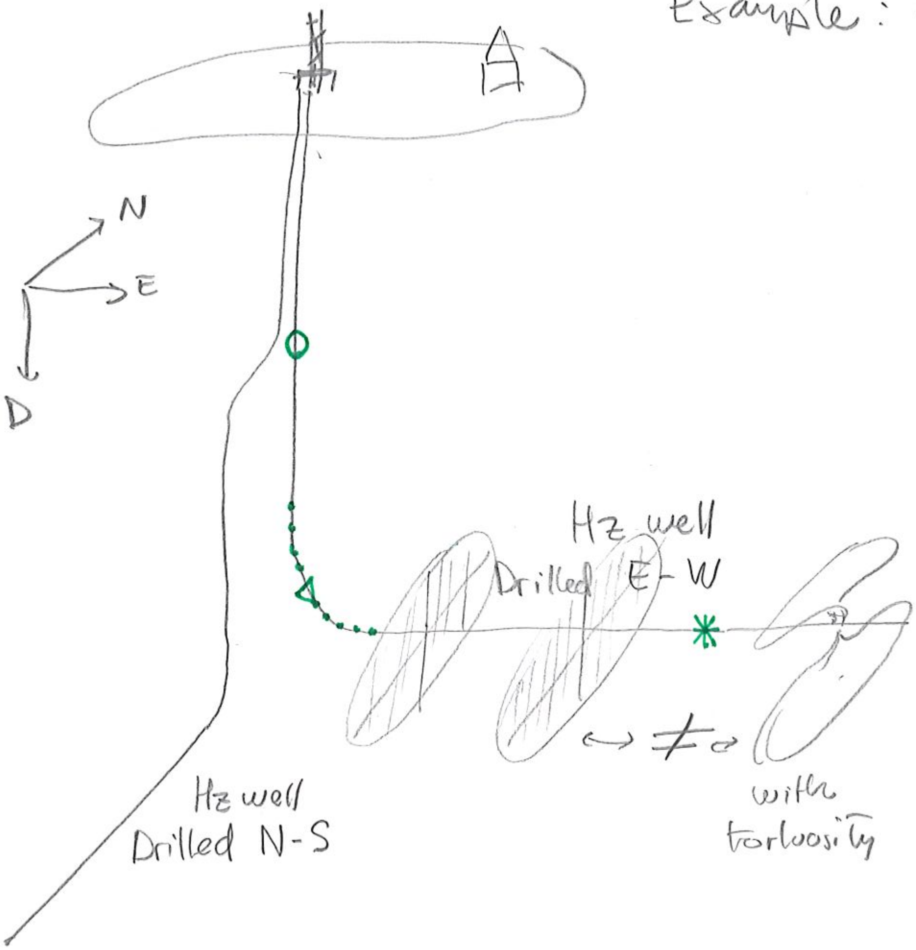


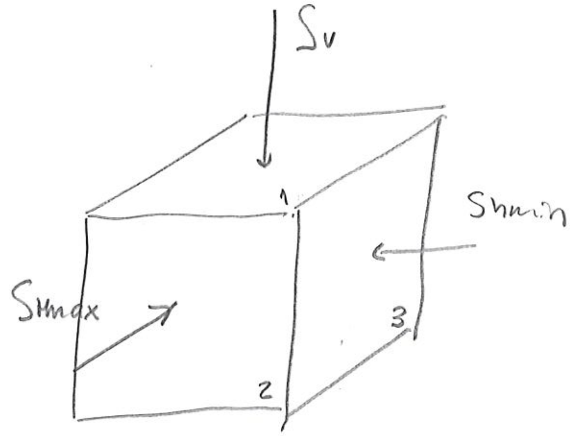
$\boxed{\text{From previous problem}}$

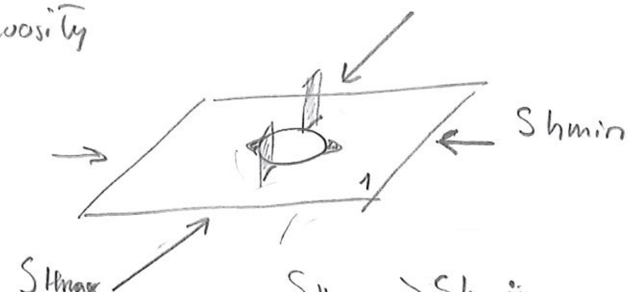


Deviated wellbores

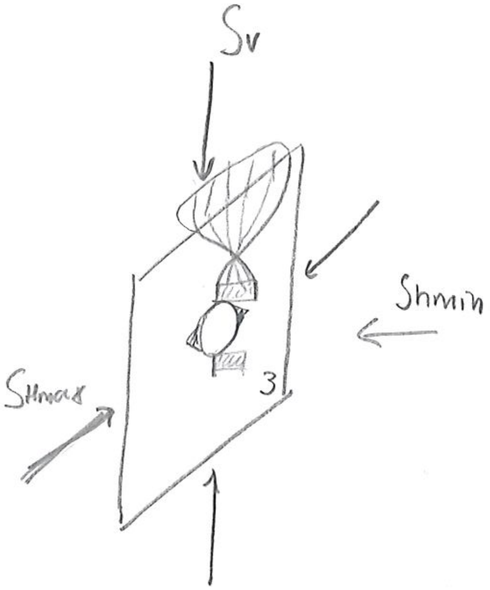
Example: normal faulting $S_v > S_{Hmax} > S_{hmin}$



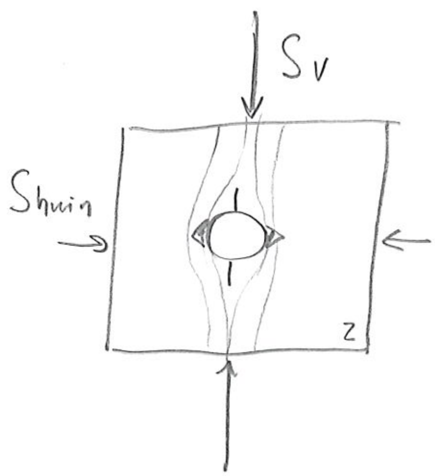




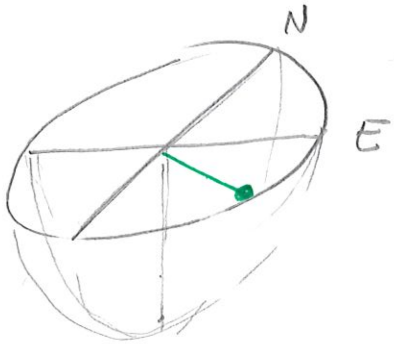
$S_{Hmax} > S_{hmin}$

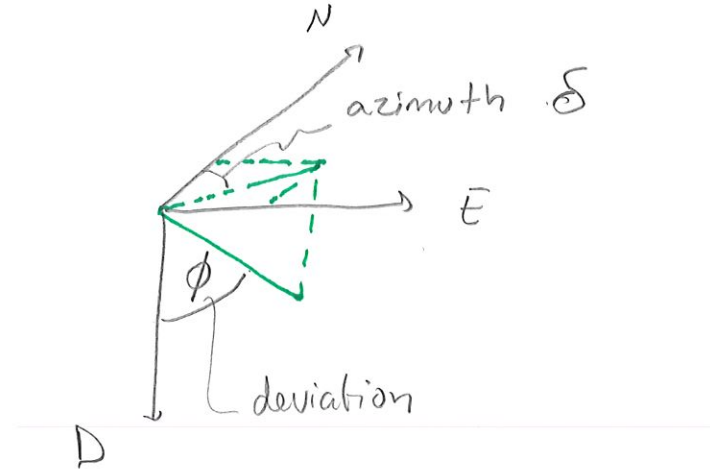


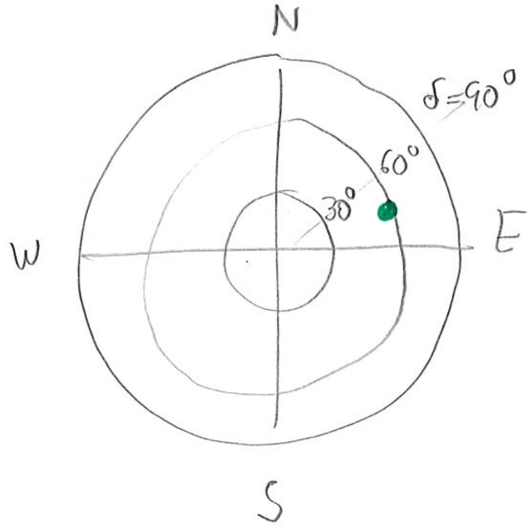
$S_v > S_{Hmax}$ (NF)



$S_v > S_{hmin}$ (NF)

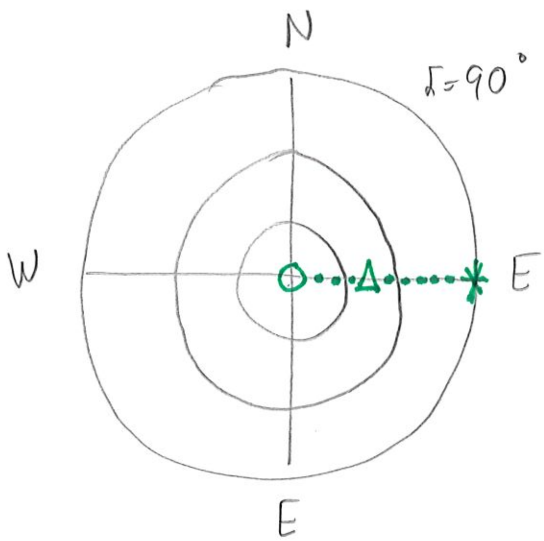


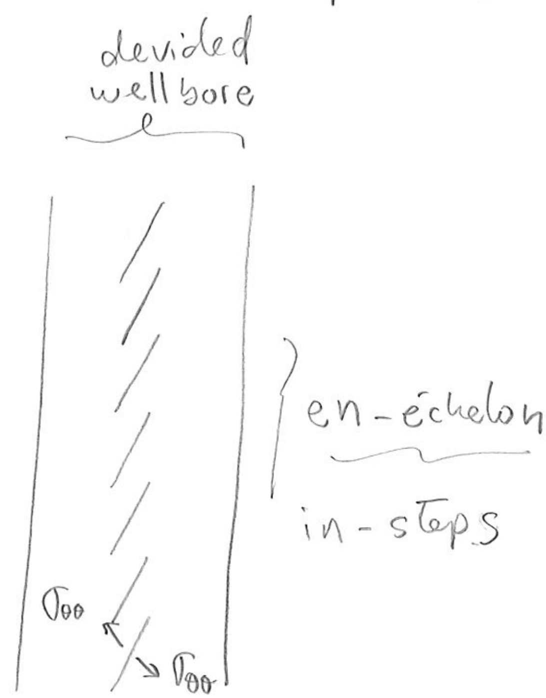




tensile fractures

Example previous page



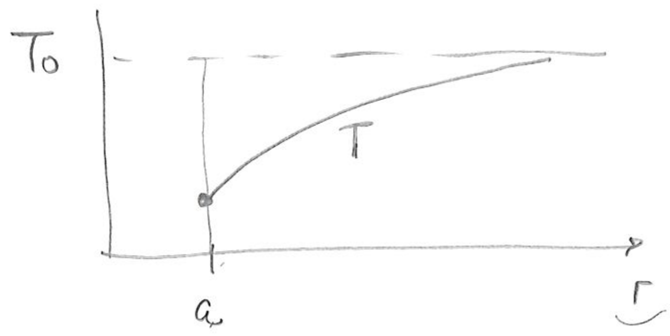


Wellbore stability: other factors

1 Temperature effects

Thermal strain $\longleftrightarrow$ Thermal stress

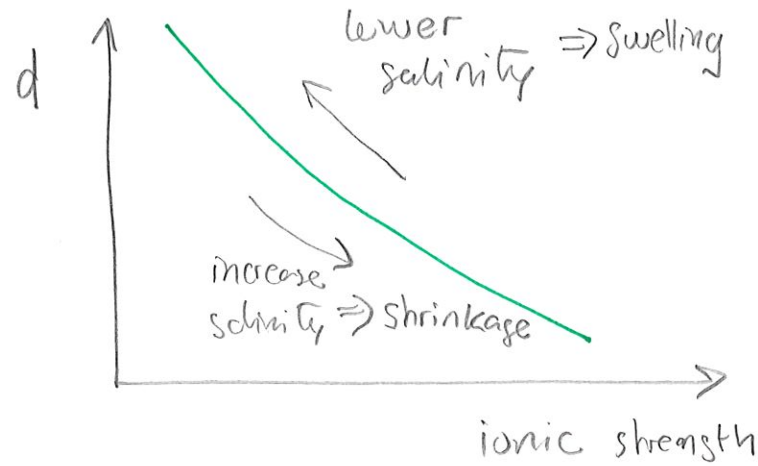
proportional to $\alpha_T \Delta T$

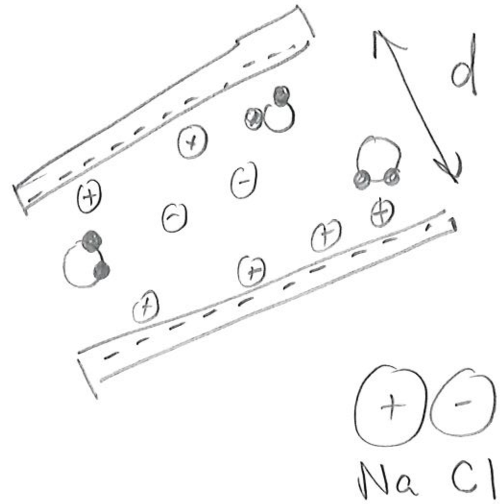


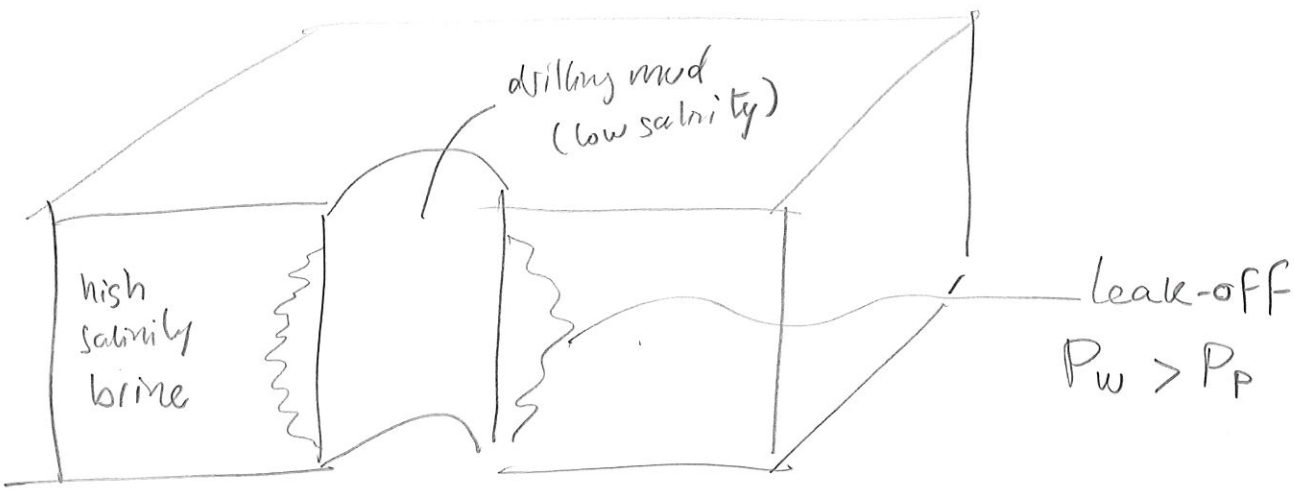
$$\boxed{\Delta\sigma_{\theta\theta} = \frac{E}{1-\nu}\alpha_T \Delta T}$$

↳ use in P_b

② Shale instability







low salinity water invades shale pore space → increased $\sigma_{\theta\theta}$; → weaken shale } break outs

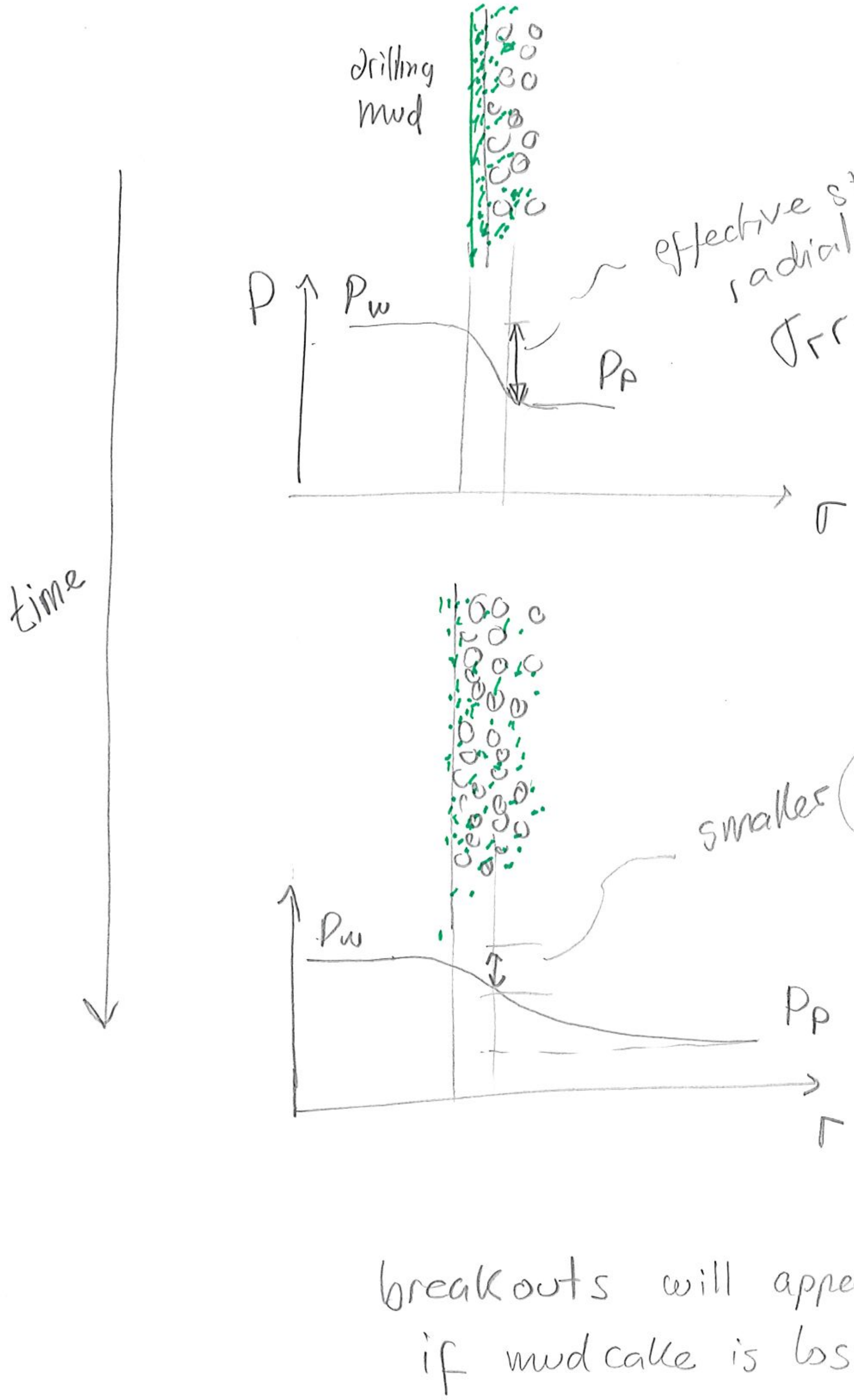
Solutions
- increase mud (water) salinity → KI ← inhibitors
- oil-based mud
- underbalanced drilling (little leak-off)

3 Loss of mud support

perfect mudcake

effective stress radial support

$\sigma_{rr} = P_w - P_p$

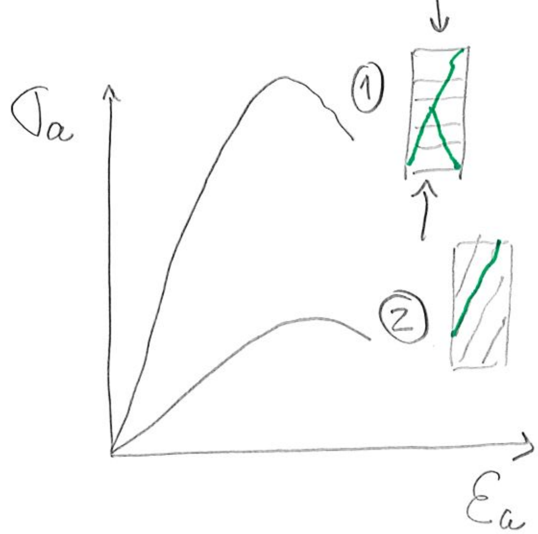


$\sigma_1 = UCS + q\,\sigma_3$

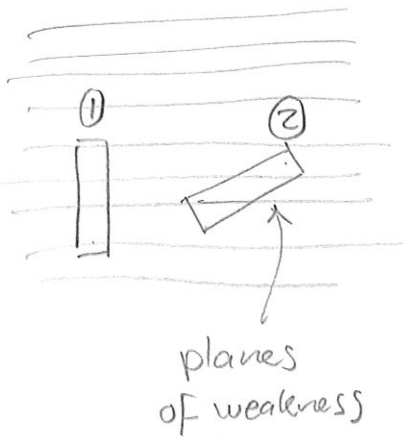
smaller σ_{rr}

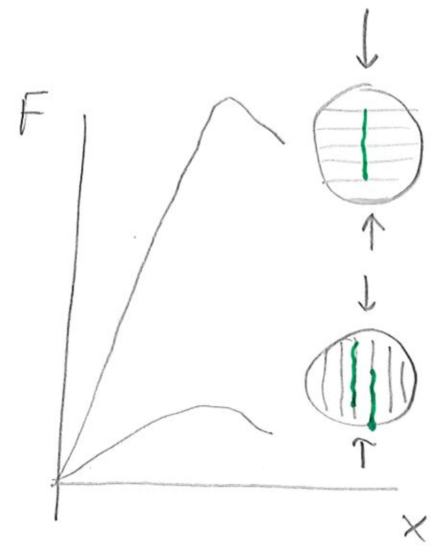
breakouts will appear after some time if mudcake is lost, (w_{BO} increase)

4 Strength anisotropy

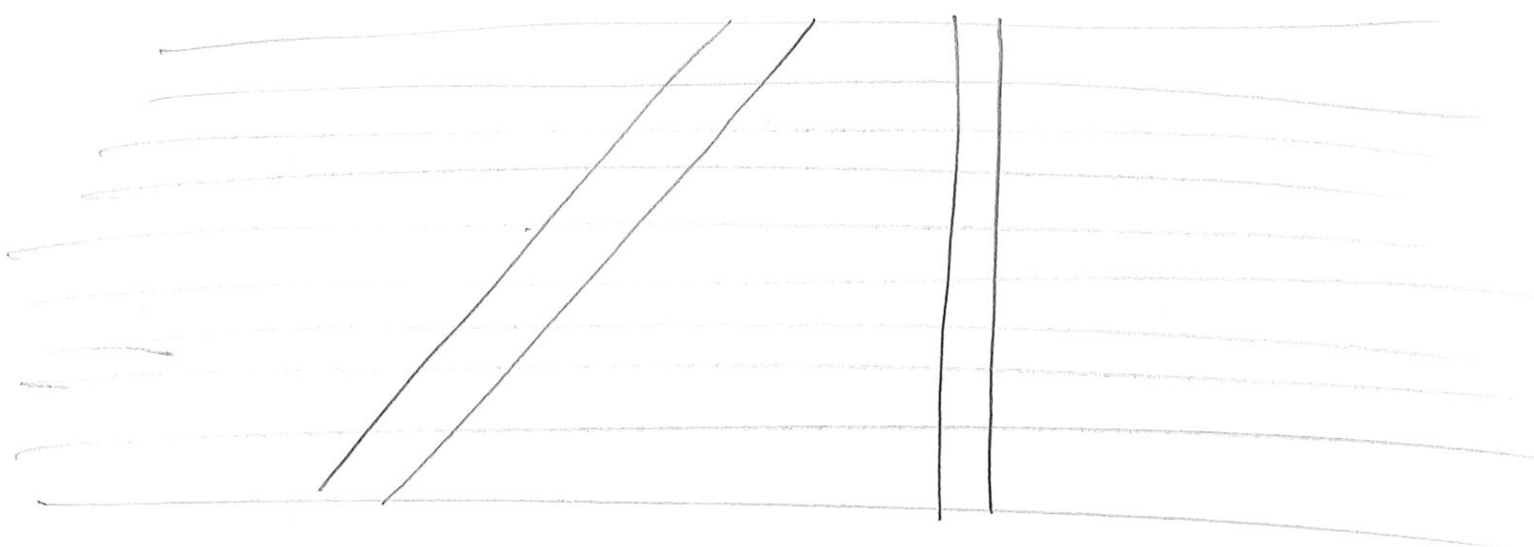


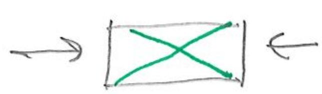
shear strength

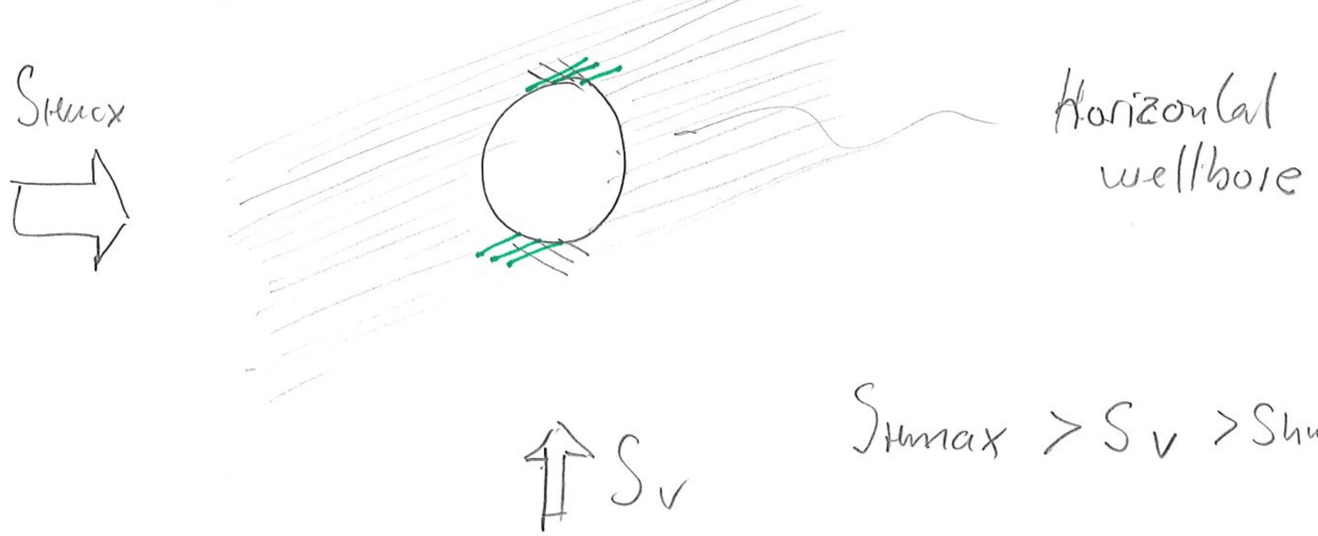




tensile strength

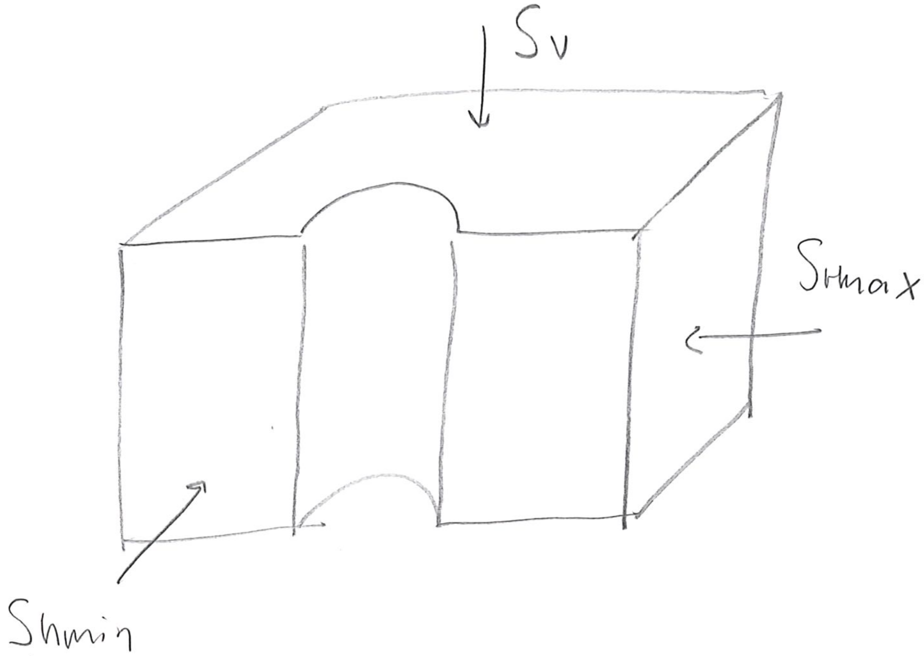






$S_{Hmax} > S_v > S_{hmin}$

Wellbore pressurization (leak-off test)



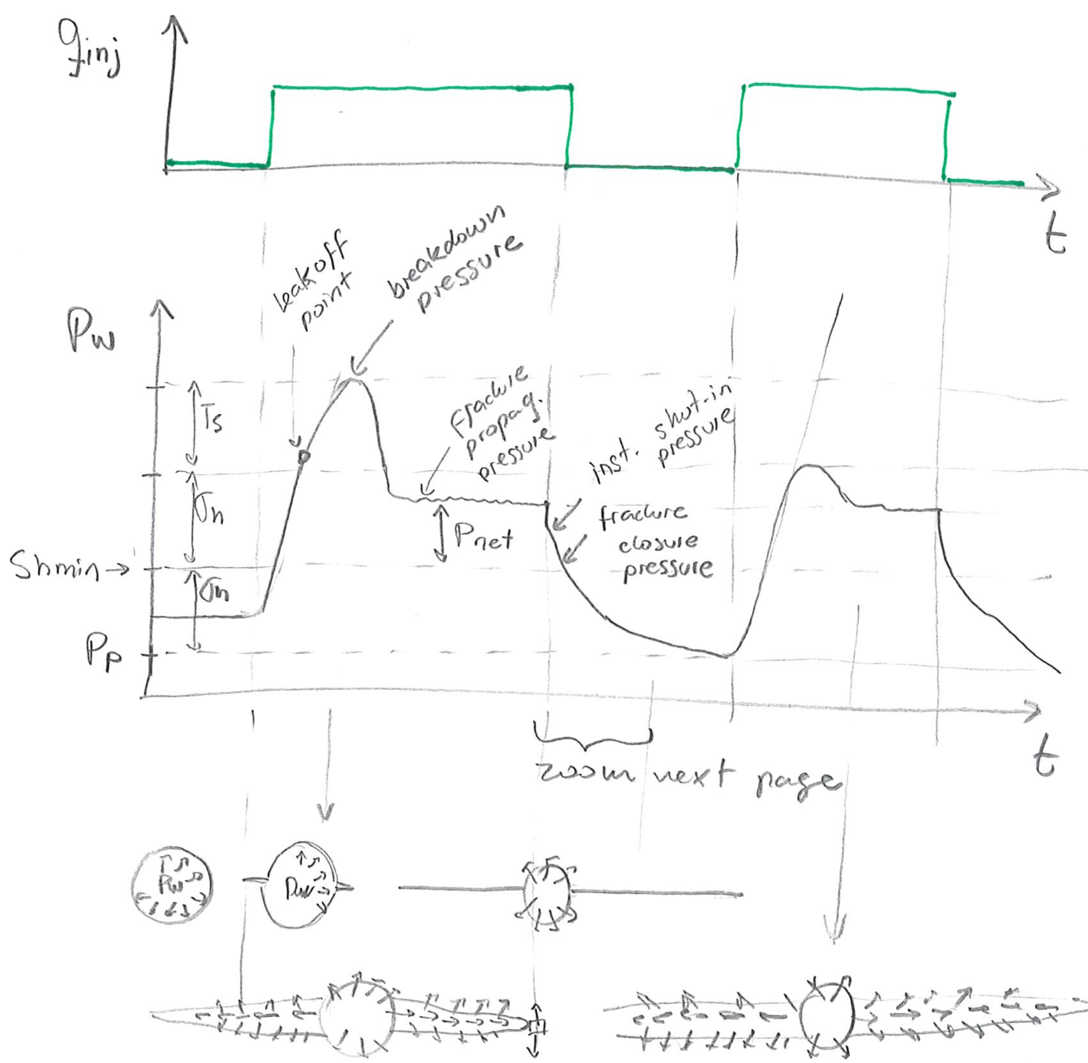
$S_v > S_{Hmax} > S_{hmin}$

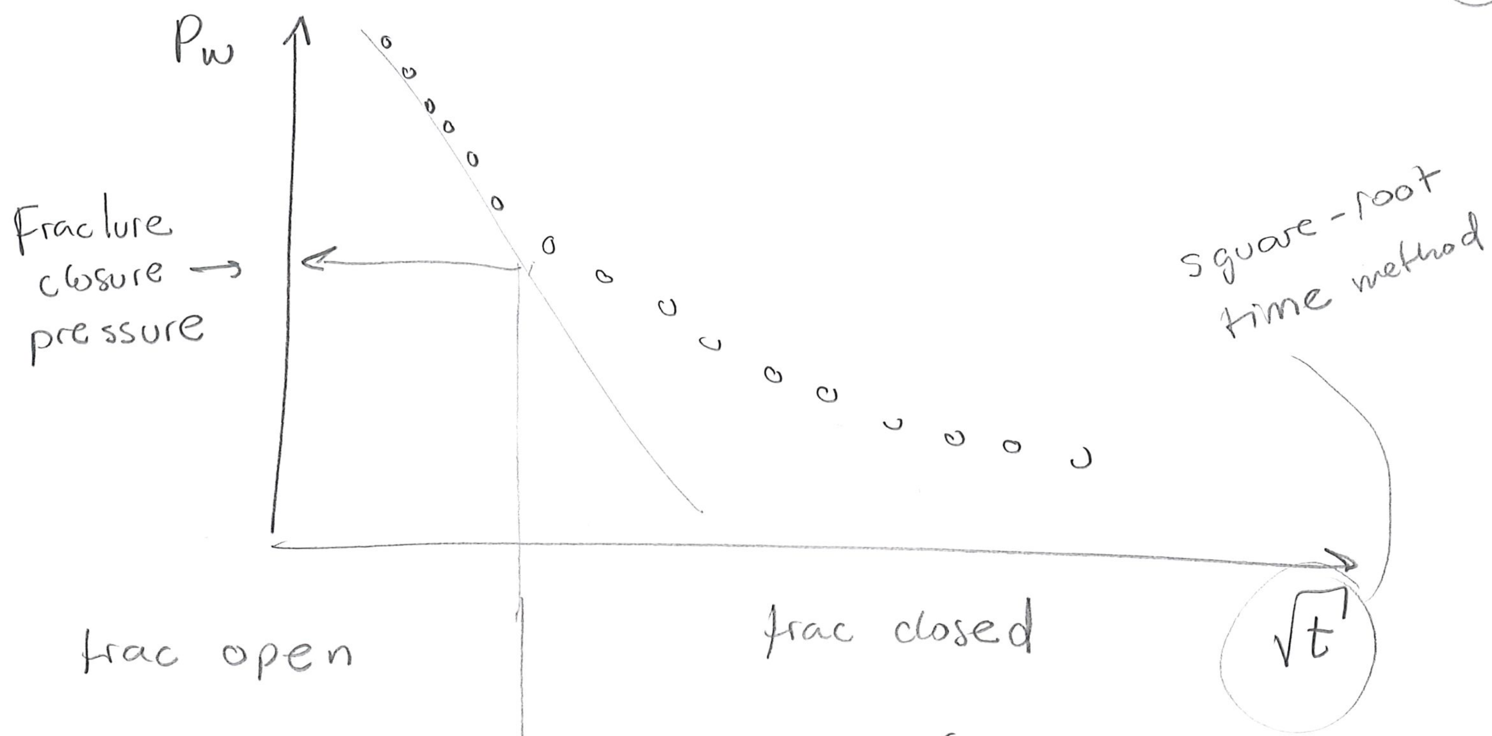
$S_{Hmax} = S_{hmin} = S_h$

$$P_b = P_p + 3\sigma_{hmin} - \sigma_{Hmax} + T_s$$

$$\underbrace{P_b = P_p + 2\sigma_h + T_s}_{\text{isotropic stress field} \perp \text{wellbore}}$$

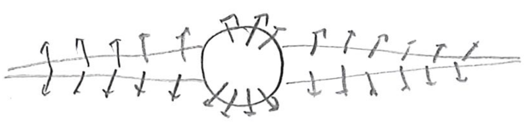
$P_w = f(\rho_{mud}, z, q_{inj})$

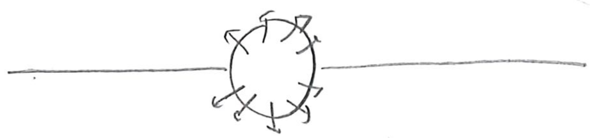




frac open

frac closed



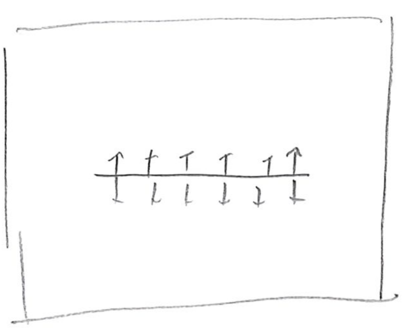


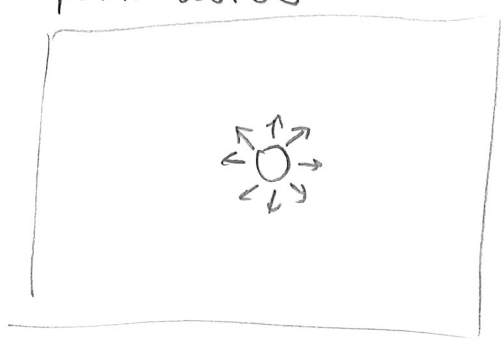
≡

≡

linear source

point source



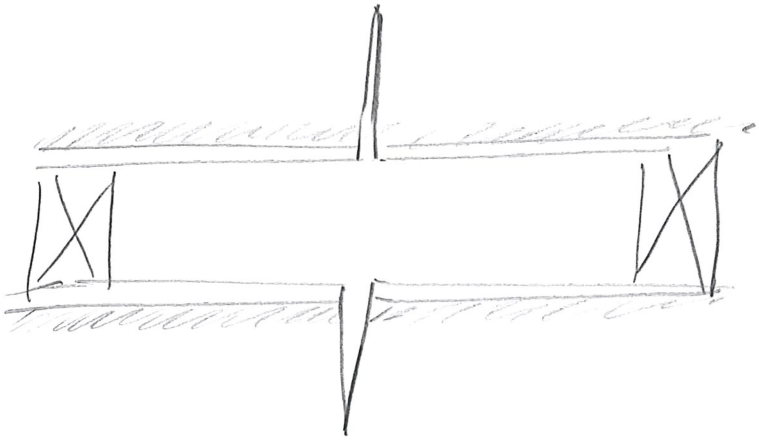


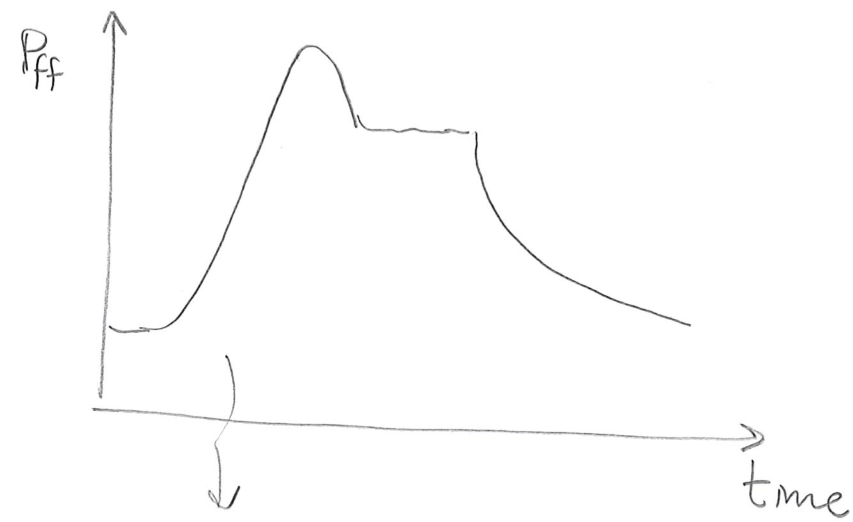
Area of leak-off
↳ wellbore + 2 · area fracture

Area of leak-off
wellbore

DFIT (Diagnostics Fracture initiation test)

- ↳ coumpletion
- ↳ fracturing fluid
- ↳ through perforations
- ↳ before large HF treatment
- ↳ low-perm reservoirs
- ↳ small volumes
- ↳ small rates





G-function

↳ S_3, K_{eff}

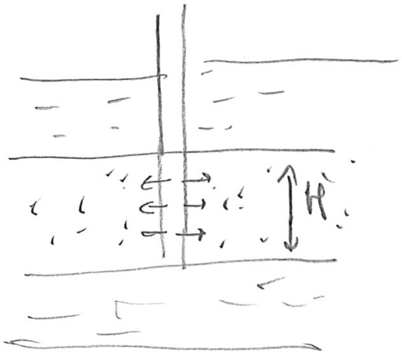
Step-rate test

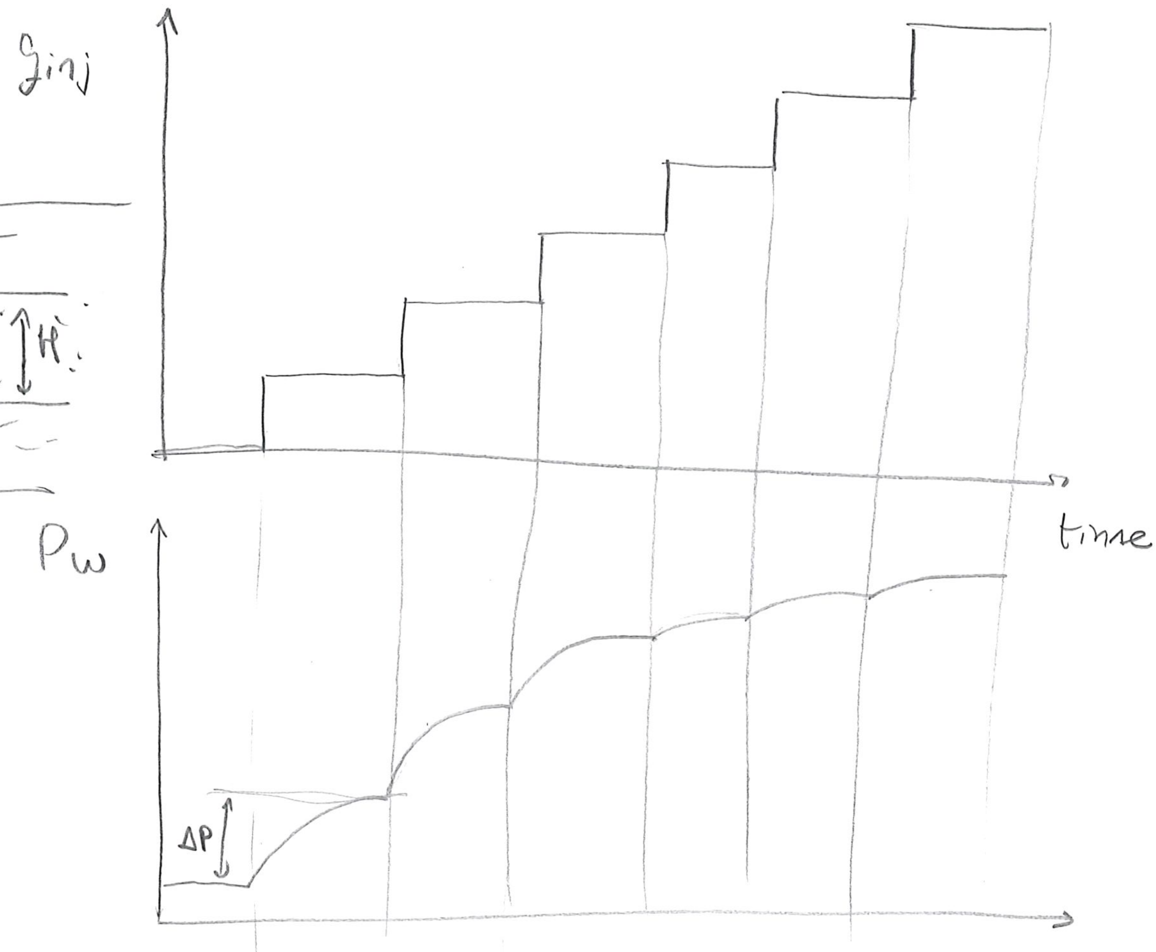
- measure maximum injection pressure
  - inject large amounts of fluid
- EOR ; CO_2, water, polymers, ..., steam
- artificial lift
- waste water disposal.
- permeable formation ✓

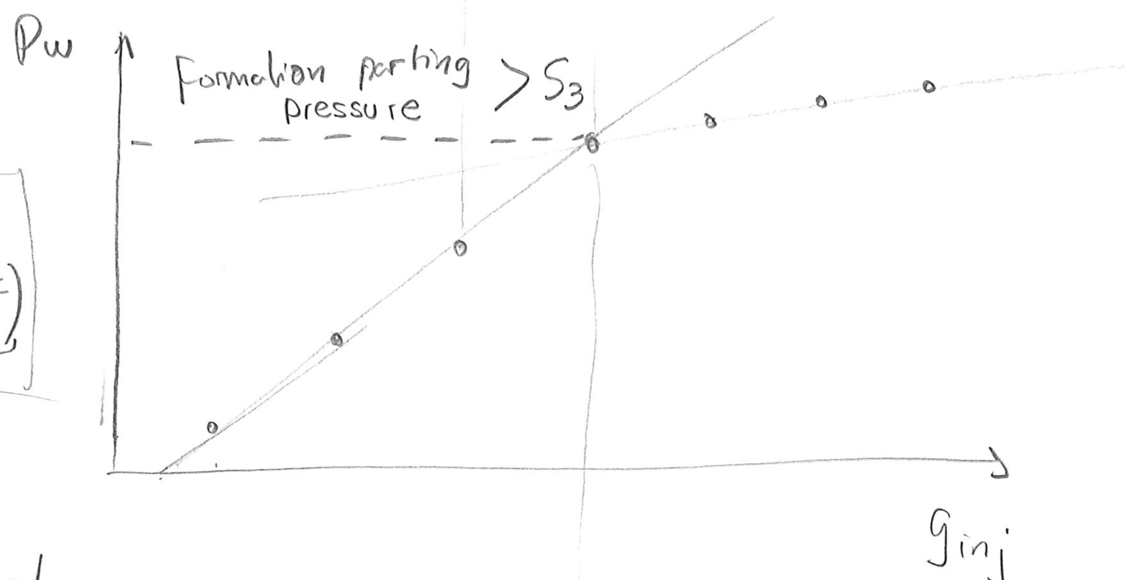
Examples

infinitely large reservoir

- regulatory agencies
  - water disposal
  - regulate injector
    - avoid fracturing

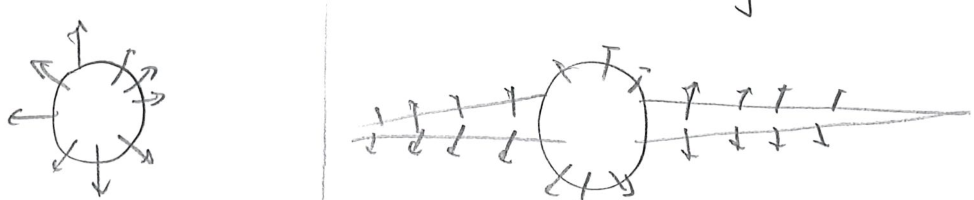




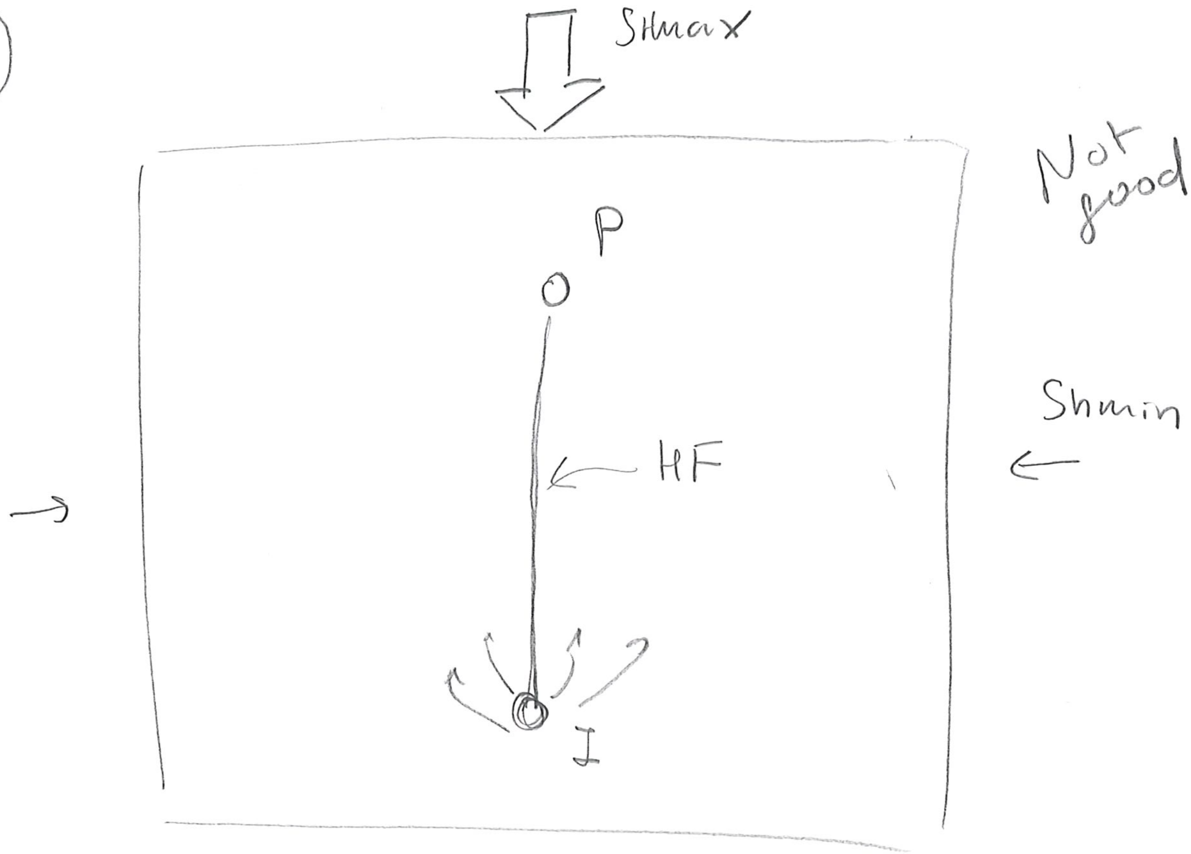


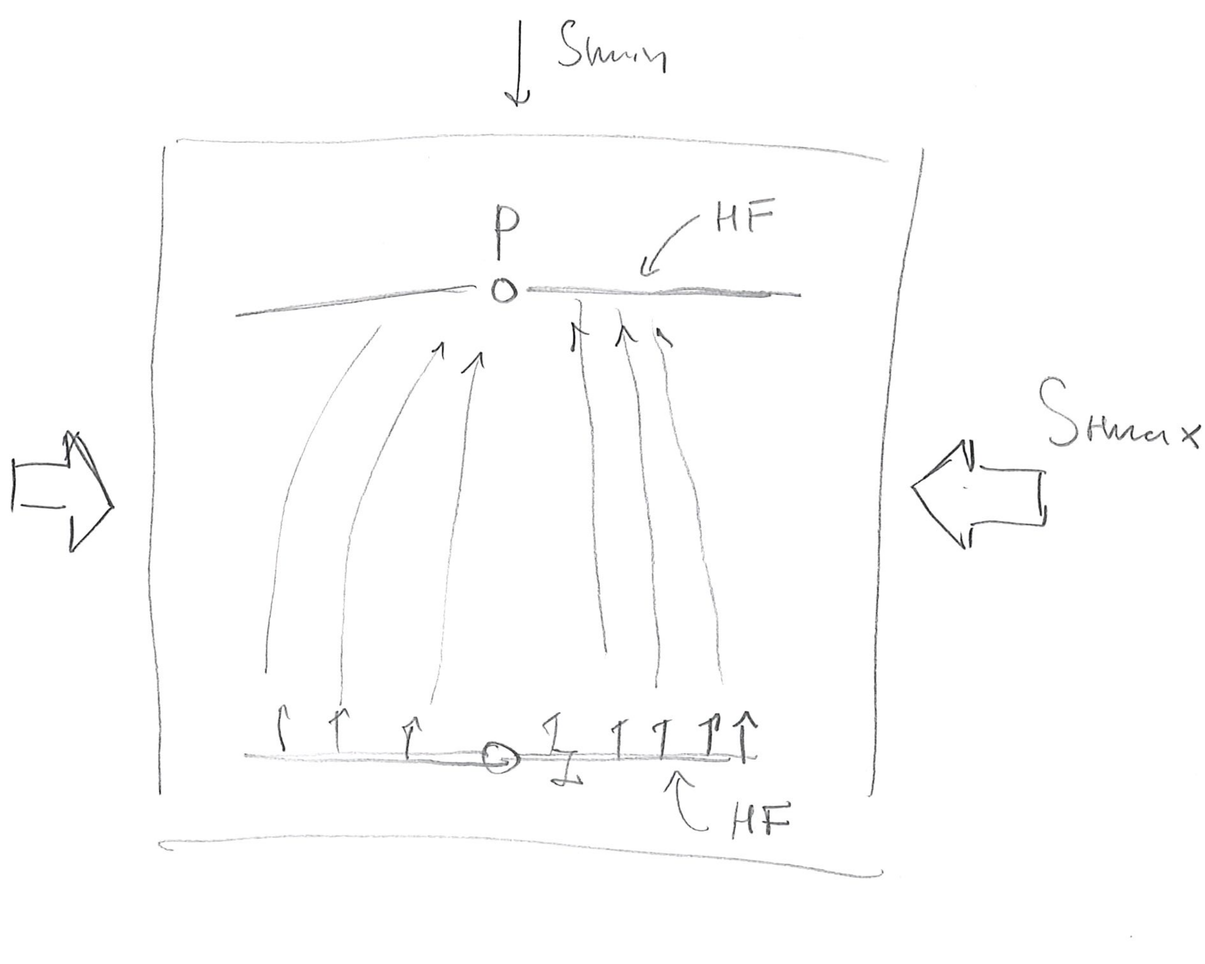
$$\boxed{q = \frac{K}{2\pi H} \cdot \frac{\Delta P}{\underbrace{\ln\left(\frac{r_{ff}}{r_w}\right)}}}$$

when fractured
↓
add skin factor

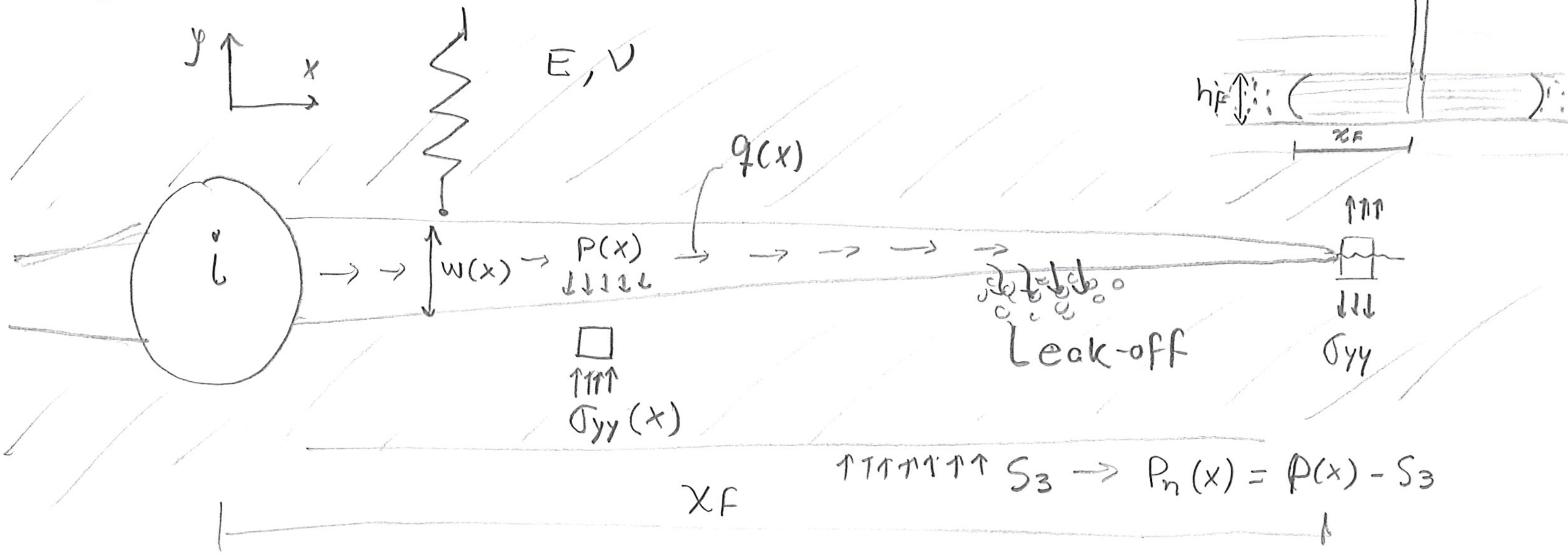


NF





Coupled hydraulic fracture propagation (bi-wing)



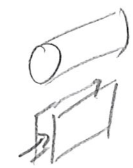
① Solid Mechanics (Elasticity)

$$\boxed{W(x) \propto \frac{P(x)}{E}}$$

$$\underline{\varepsilon} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\sigma}$$

② Fluid Mechanics

↳ parallel plates



$$q(x) \propto \frac{[W(x)]^3 \Delta P \, h_F}{\underbrace{\mu}_{\text{Newtonian fluid}}}$$

③ Leak-off (Material Balance)

$$\underbrace{V_i}_{\text{total injected}} = \underbrace{V}_{\text{frac}} + V_L$$

$V_L = A_L(2C_L\sqrt{t} + S_p)$ — spurt loss (S_p)

$$\eta = \frac{V}{V_i}$$

$\eta \to 1$: high eff HF

$\eta \to 0$: significant leak-off

injection schedule

↳ constant injection rate

$V_i = i \cdot t$ — one-wing

④ Creation of new surface

↳ fracture mechanics

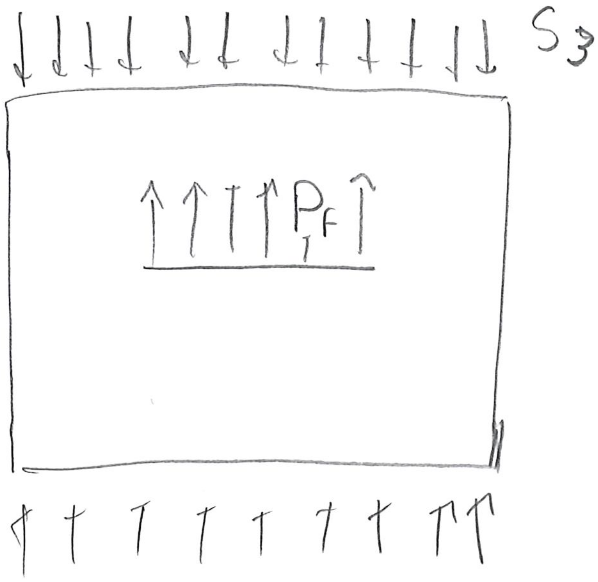
Stress Intensity Factor: $K_I = (P_n, E, \nu, x_f)$

If $K_I > K_{IC}$ — K_{IC}: rock properties, Fracture toughness

↳ fracture propagation

Griffith line crack problem

$$P_n = P_F - S_3$$



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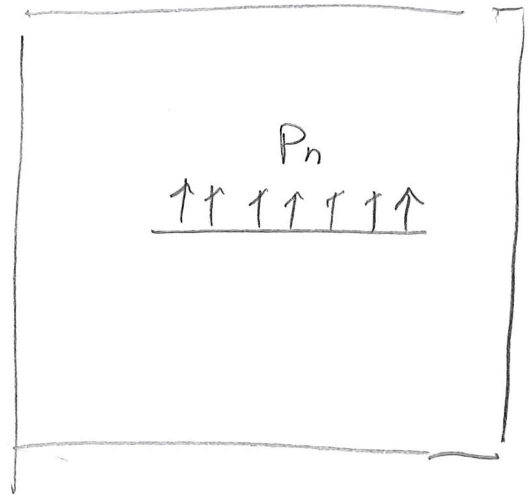
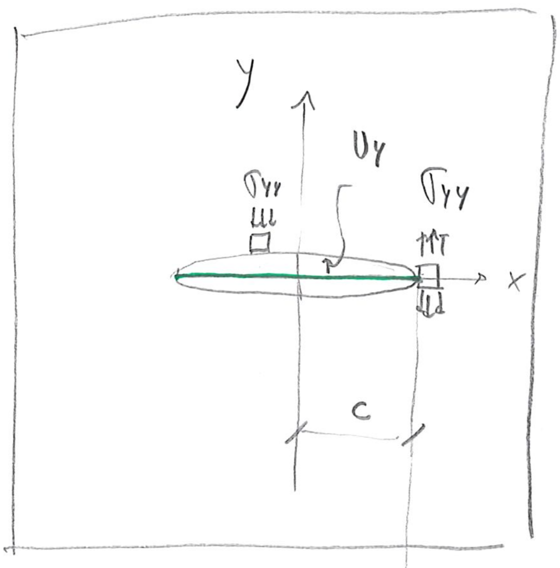


$P_n = P_o \rightarrow$ constant



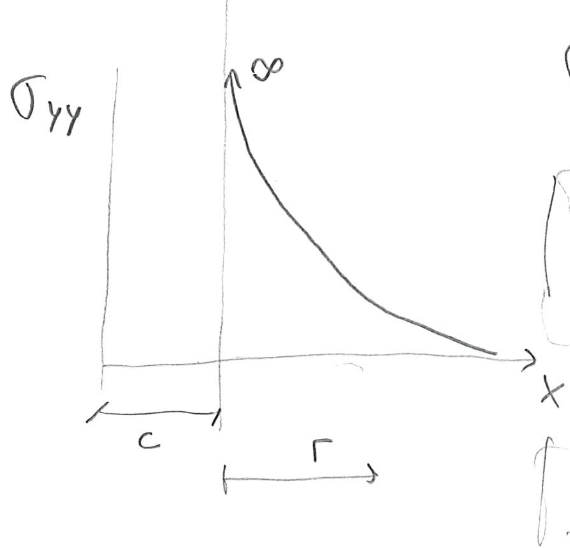
- $U_y(x,0) = \dfrac{2P_o}{E'}\sqrt{c^2 - x^2}$

  $\hookrightarrow E' = \dfrac{E}{1-\nu^2}$

  $\hookrightarrow W(x=0) = \dfrac{4P_o c}{E'}$

- $\sigma_{yy} = P_o\Bigg[\underbrace{\dfrac{x}{\sqrt{x^2-c^2}}}_{\text{tension}} - 1\Bigg]; \quad x \geq c$

  $\sigma_{yy} = P_o \; \}$ comp $; \quad 0 \leq x < c$



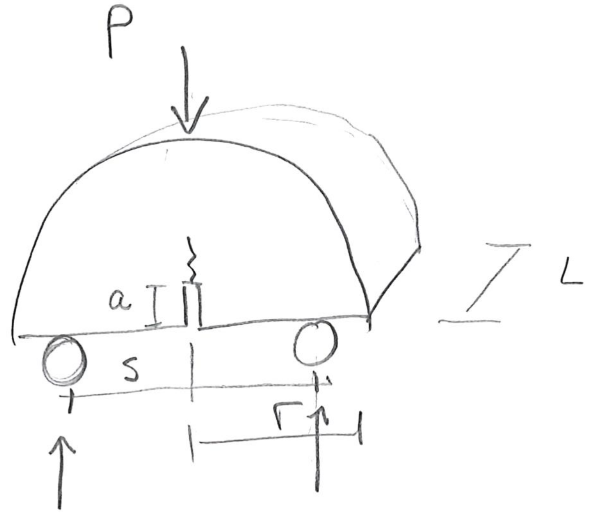
$$\boxed{K_I = \lim_{r \to 0^+}\left[(2\pi r)^{1/2} \cdot \sigma_{yy}\right]}$$

stress intensity

$$\boxed{K_I = P_o(\pi c)^{1/2}}$$ constant pressure

Semicircular bending test (SCB)

~5



$$K_{Ic} \approx \frac{P_{max} (\pi a)^{1/2}}{2 r L} \cdot Y_I$$

K_{Ic} (rocks) ~ 0.05 – 1.5 ($MPa \cdot \sqrt{m}$)

$K_I > K_{Ic} \Rightarrow$ propagation

$K_I < K_{Ic} \Rightarrow$ no propagation

Example

$X_f = 10\,m$ half-length

$S_{hmin} (= S_3) = 20\,MPa$

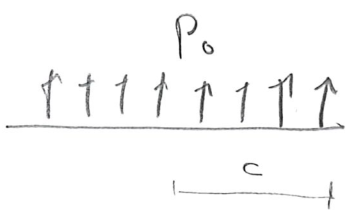
$P_f = 20.5\,MPa$

$E = 1\,GPa$, $\nu = 0.25$

$$E' = \frac{E}{1-\nu^2} = 1070\,MPa = 1.07\,GPa$$

2 cm ~ 1 in

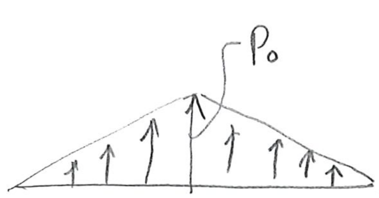
a)



$$W_{w,o} = \frac{4 P_o c}{E'} = \frac{4 \cdot 0.5\,MPa \cdot 10\,m}{1070\,MPa} = 0.019\,m$$

$$K_I (tip) = P_o (\pi c)^{1/2} = 0.5\,MPa\,(\pi \cdot 10\,m)^{1/2} = 2.8\ MPa \cdot m^{1/2}$$

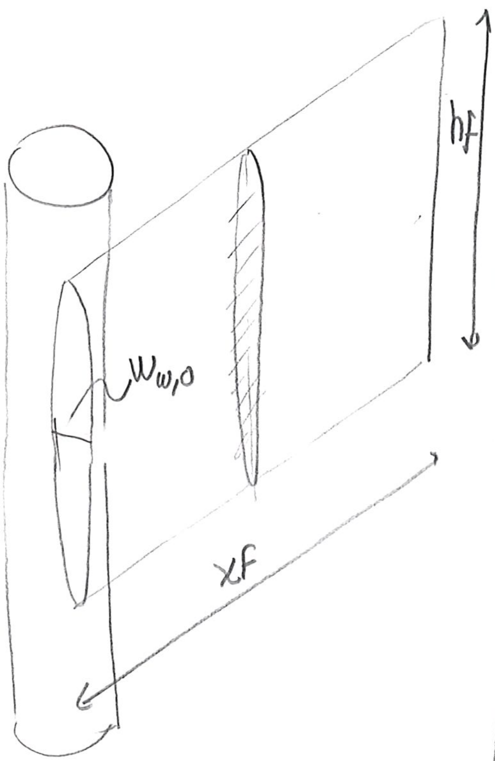
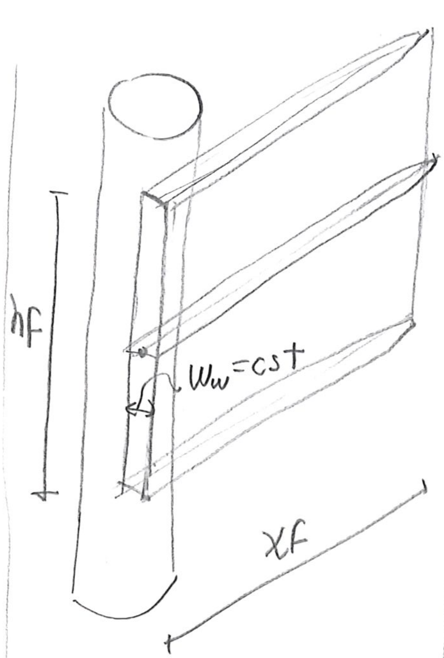
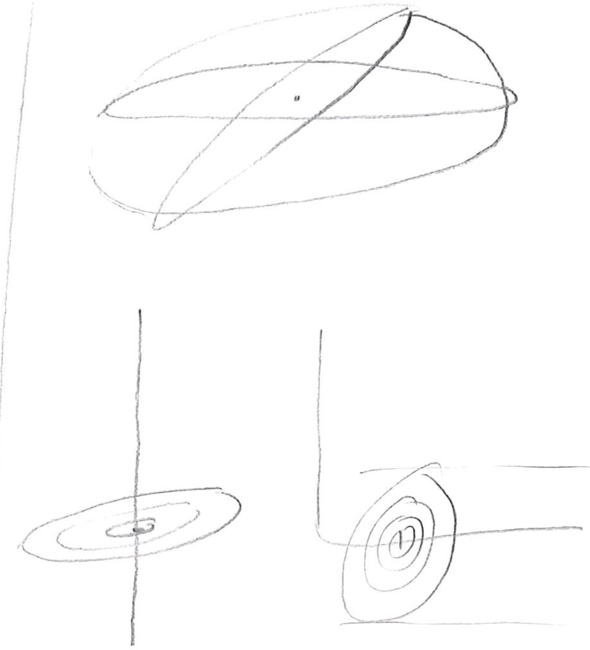
b)



$$W_{w,o} = \frac{2 P_o c}{E'} = 9.5\,mm$$

$$K_I (tip) = \left(1 - \frac{2}{\pi}\right) P_o (\pi c)^{1/2} = 1.02\ MPa \cdot m^{1/2}$$

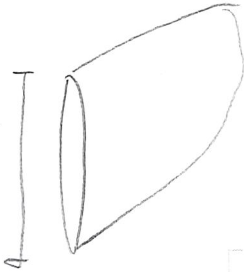
Single planar hydraulic fracture propagation models

PKN	KGD	Penny (Radial)
		
$x_f > h_f$: long fractures - completions	$x_f \leq h_f$ ↳ leak-off FCP ↳ frac-pack	

$$
\begin{cases}
x_f = 0.524 \left(\dfrac{i^3 E'}{\mu\, h_f^4}\right)^{1/5} t^{4/5} \\[2ex]
P_n(x=0) = 1.52 \left(\dfrac{E'^4 \mu\, i^2}{h_f^6}\right)^{1/5} t^{1/5} \\[2ex]
W_{w,0} = 3.04 \left(\dfrac{i^2 \mu}{E' h_f}\right) t^{1/5}
\end{cases}
\rightarrow V_{frac} = \bar{w} \cdot x_f \cdot h_f
$$

- constant injection rate
- $K_I > K_{Ic}$

$$\bar{w}\,(PKN) = \frac{\pi}{5} W_{w,0}$$



PKN Example

hf =	30.48 m	100 ft		
E' =	6.14E+10 Pa	8.9 MMpsi		
mu =	0.001 Pa s	1 cP		
i =	0.0662 m3/s	25.0 bbl/min	(one wing)	
te=	1800 sec	0.5 hour		

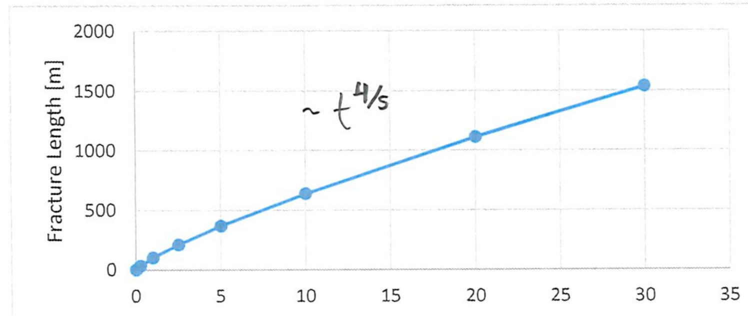
t [min]	t [sec]	x_f [m]	w_w,0 [mm]	p_net [MPa]
0	0	0.0	0.0000	0.0
0.25	15	33.4	1.5561	1.6
1	60	101.1	2.0532	2.1
2.5	150	210.4	2.4662	2.5
5	300	366.4	2.8329	2.9
10	600	637.9	3.2542	3.3
20	1200	1110.7	3.7381	3.8
30	1800	1536.2	4.0538	4.1

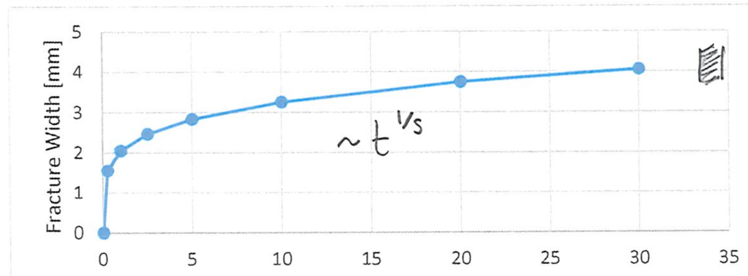
2-wing fracture volume

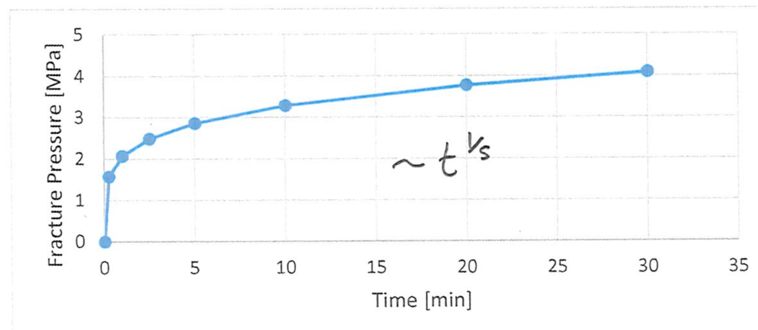
$100\ m^3$

2 Vf =	239	m^3		
water vol = 80 (~~90~~% of 2*Vf)	191	m^3	1.9	swim pools
sand vol = 20 (~~10~~% of 2*Vf)	48	m^3	126.4	metric tons
			6.3	trucks

1 truck — 20 tons







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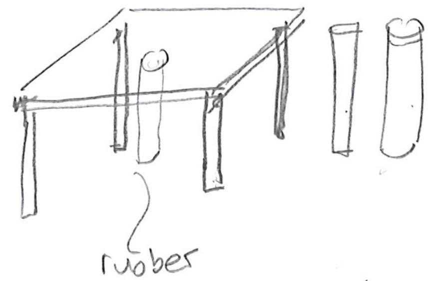
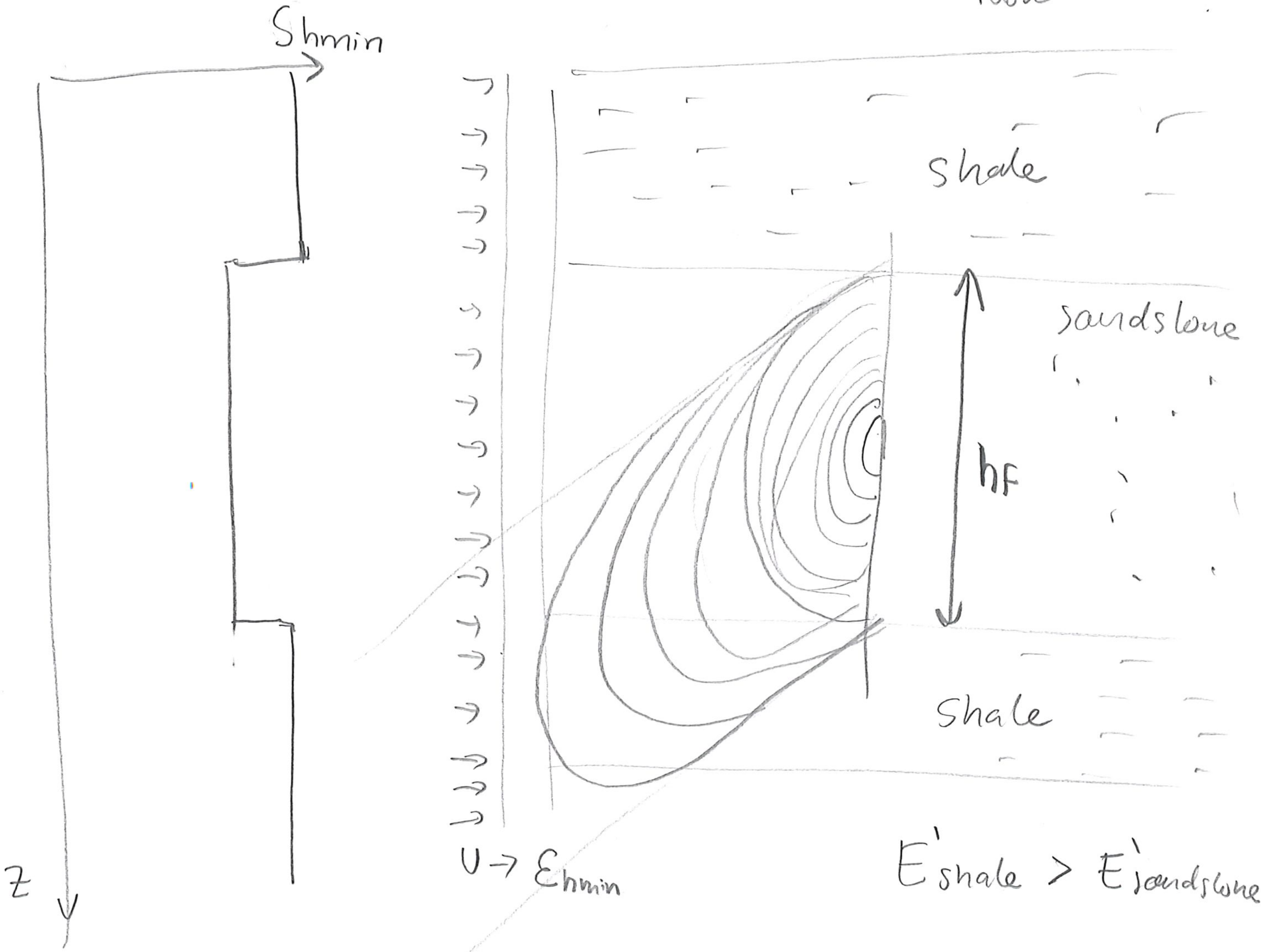
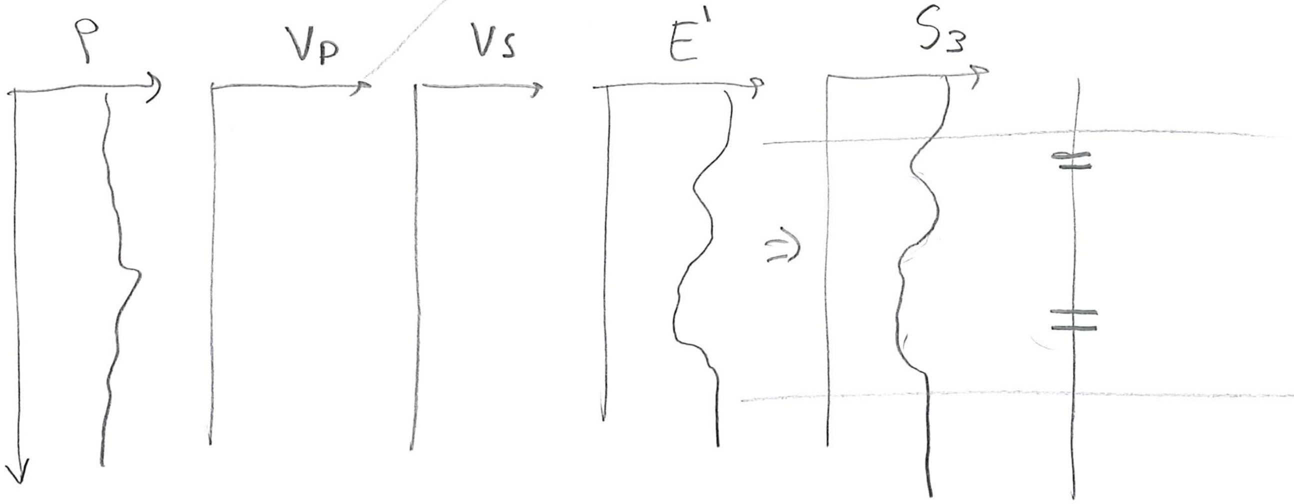
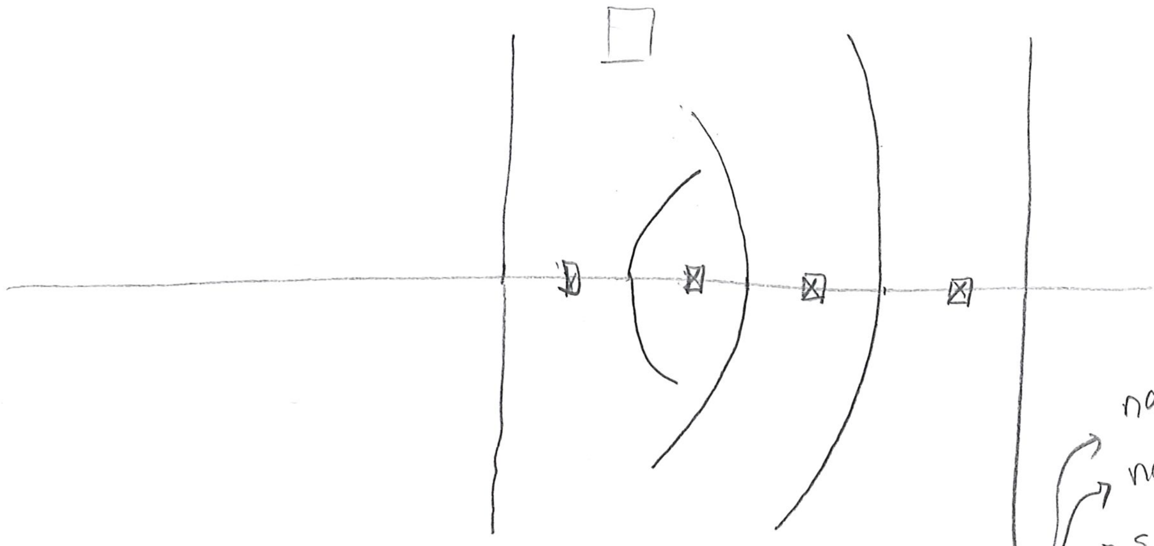
Hydraulic fracture height

- propagates $\perp S_3$
- do work against local S_3

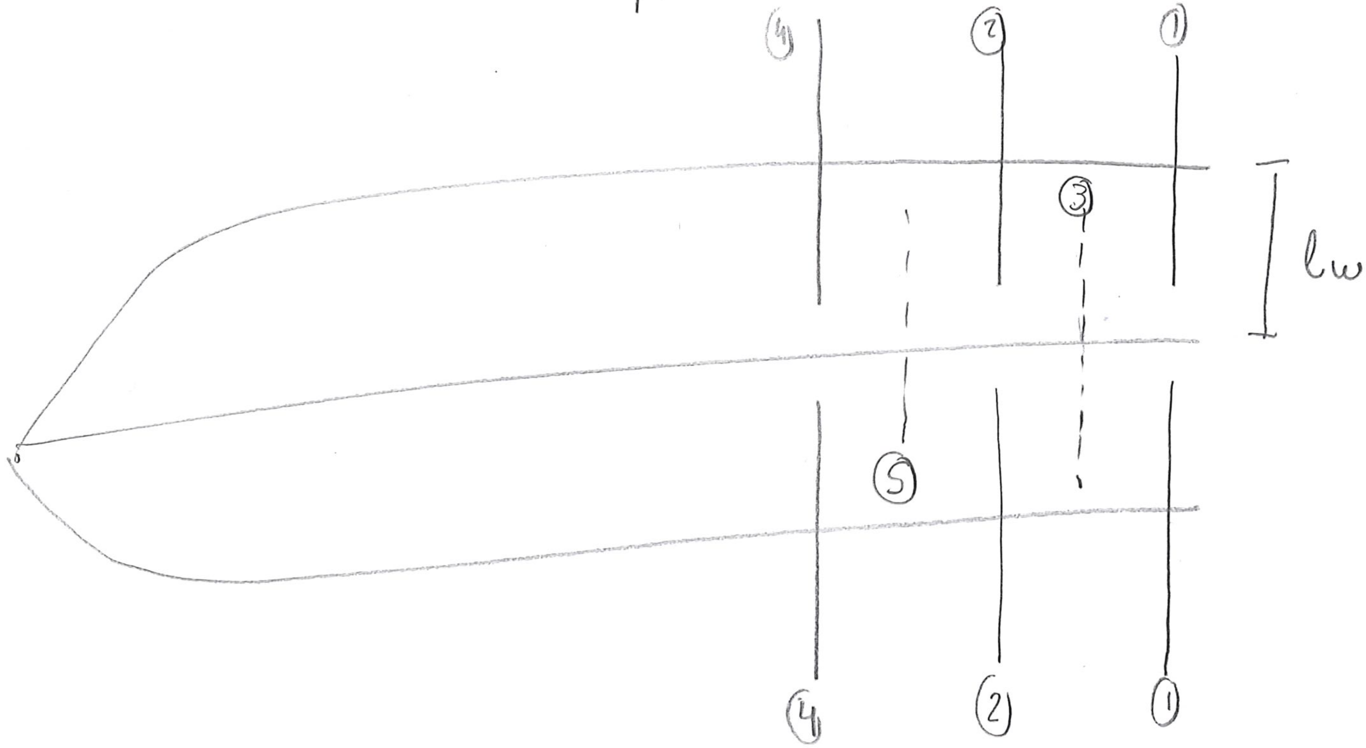


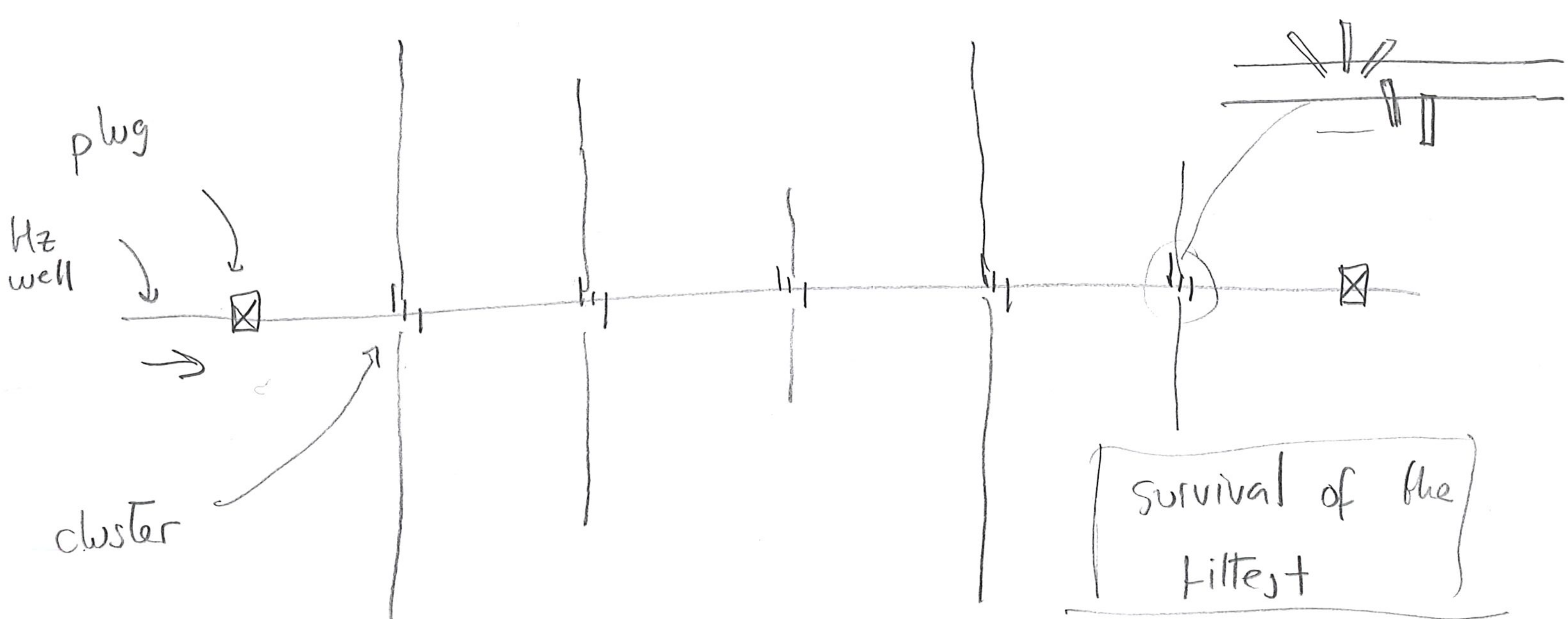




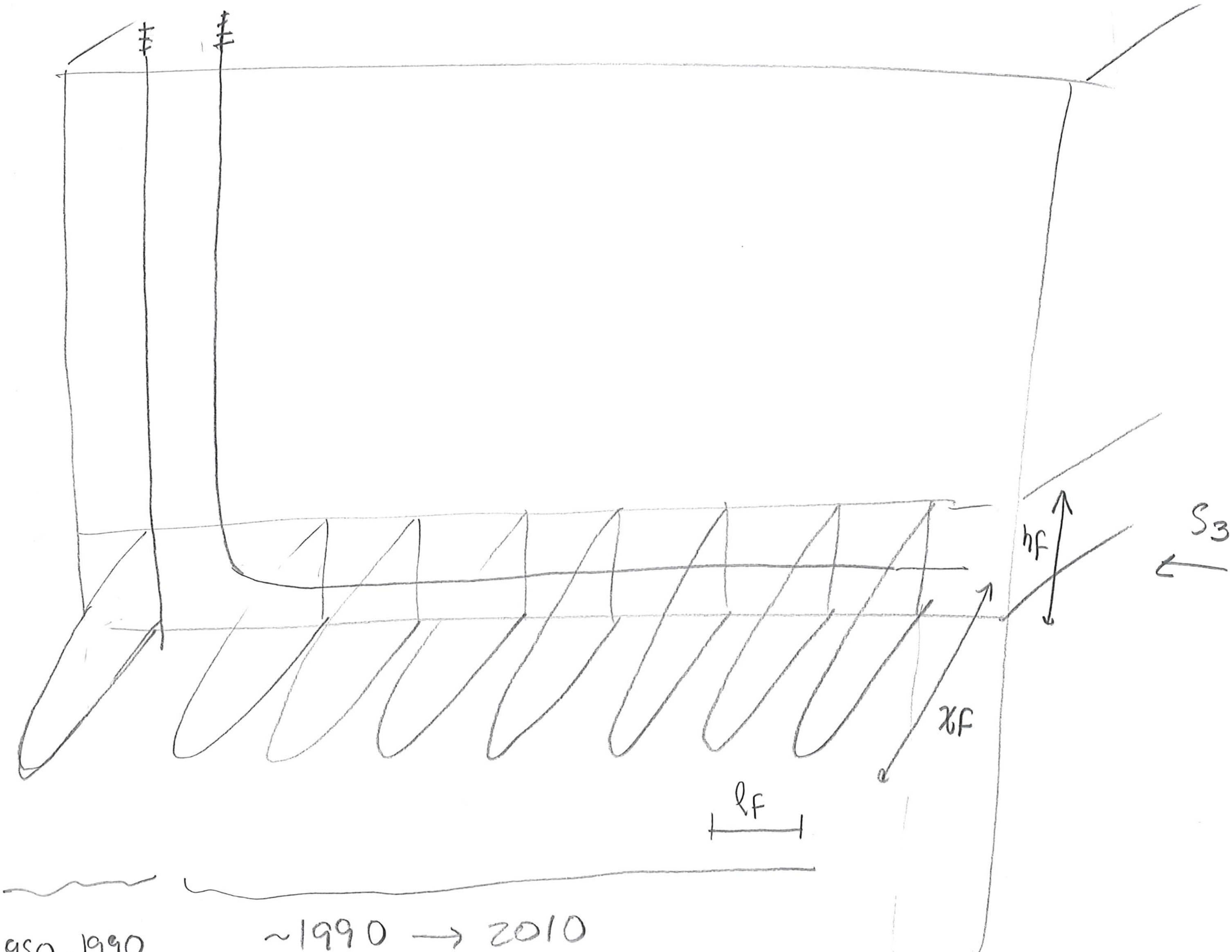


Fracture interaction
- narrower fracs
- non-planer fracs
- shorter fracs

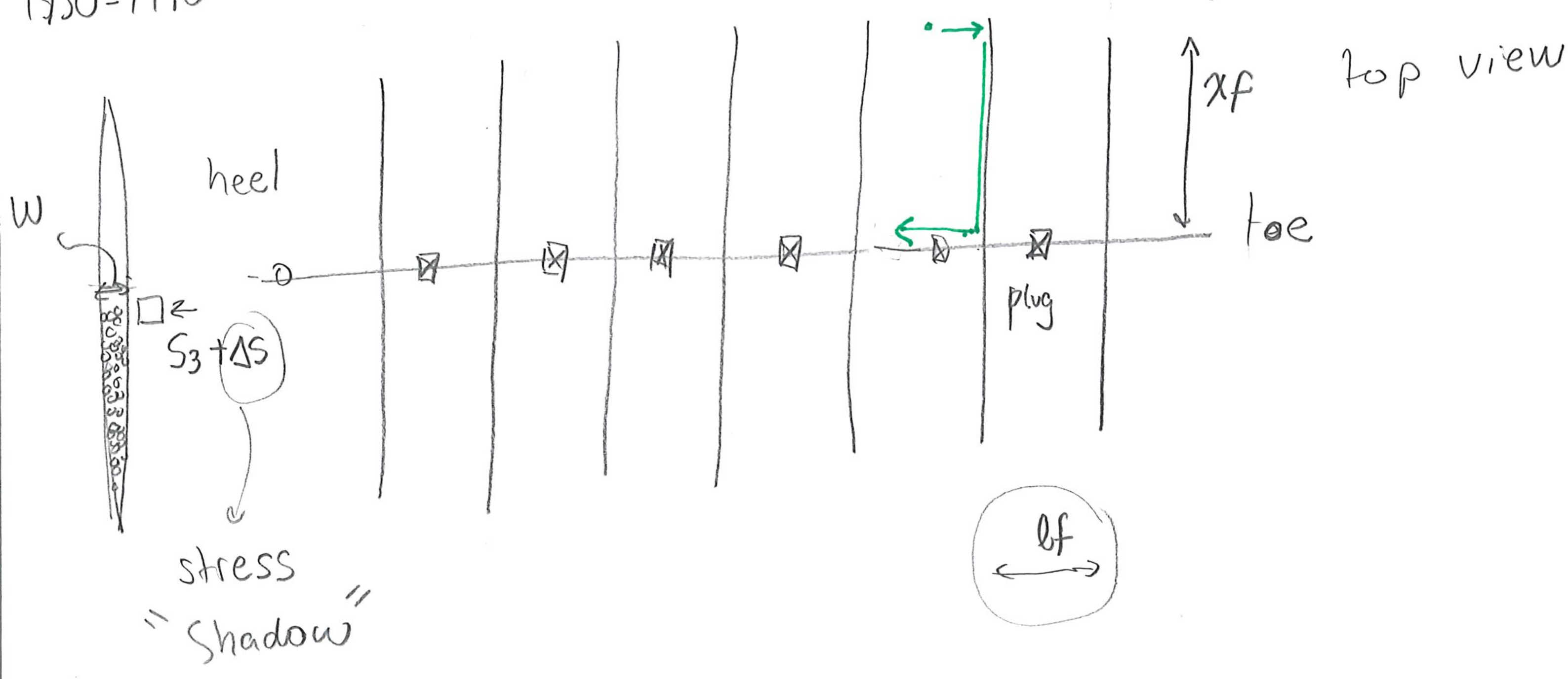




Multistage stage hydraulic fracturing



1950-1990 ~1990 → 2010



today

- length of a lateral : 10'000 ft ~ 2 miles
- 40 stages
- each stage → 4 - 15 clusters → 60% clusters open
- each cluster - 6 perf / foot ~ offset 60°
- 2 bbl/min/perf ~> 100 bbl/min

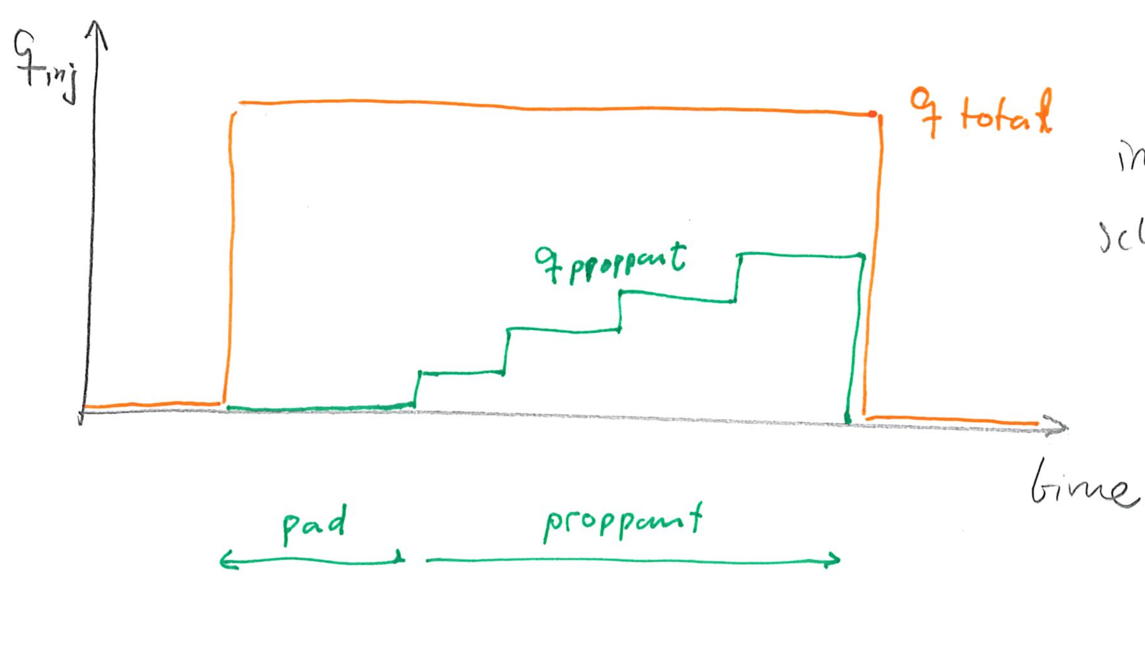
~> 2500 $\frac{\text{gallons}}{\text{LF}}$ of Hz well

↑ 60 bbl/LF

- 2000 $\frac{\text{lb (proppant)}}{\text{LF}}$

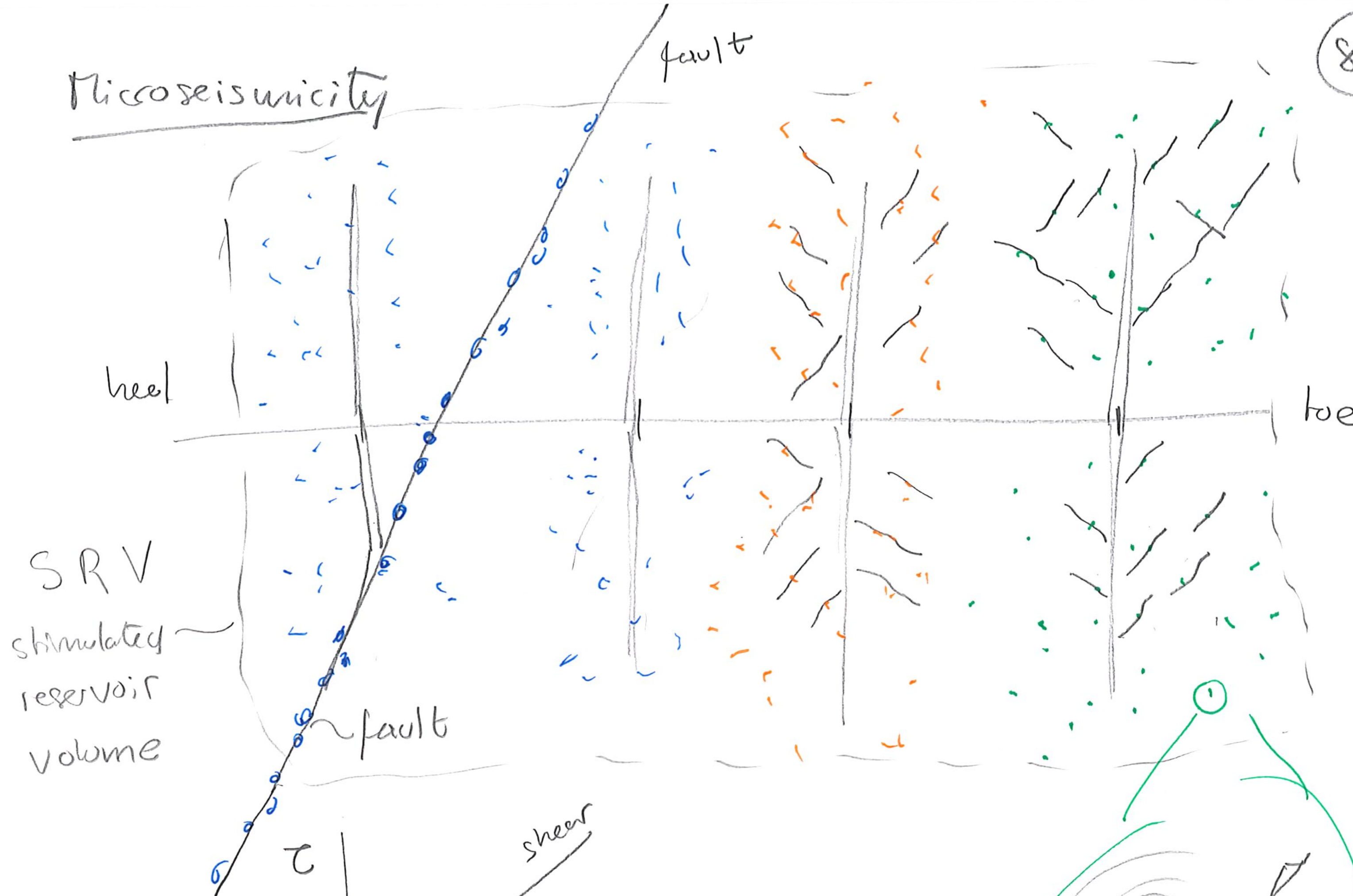
0.8 $\frac{\text{lb}}{\text{gallon}}$

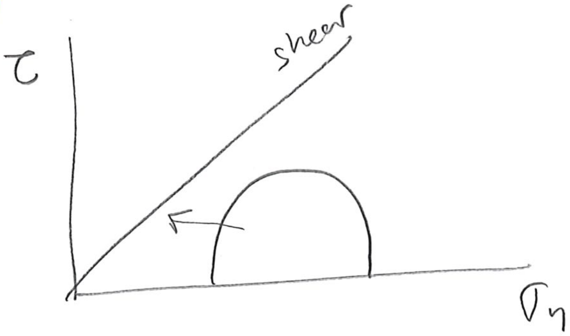
% $\frac{\text{vol (sand)}}{\text{vol (water)}}$

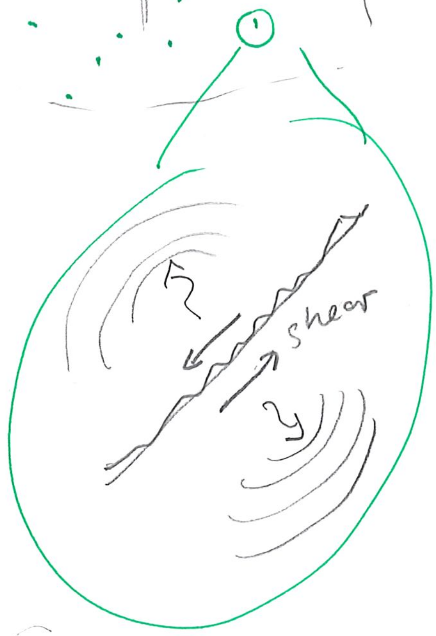


injection schedule

Microseismicity



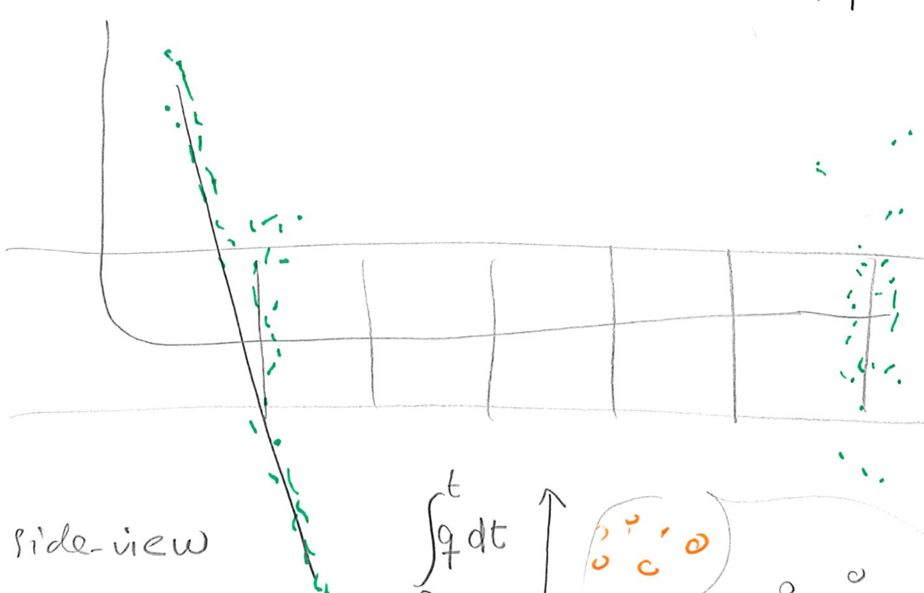




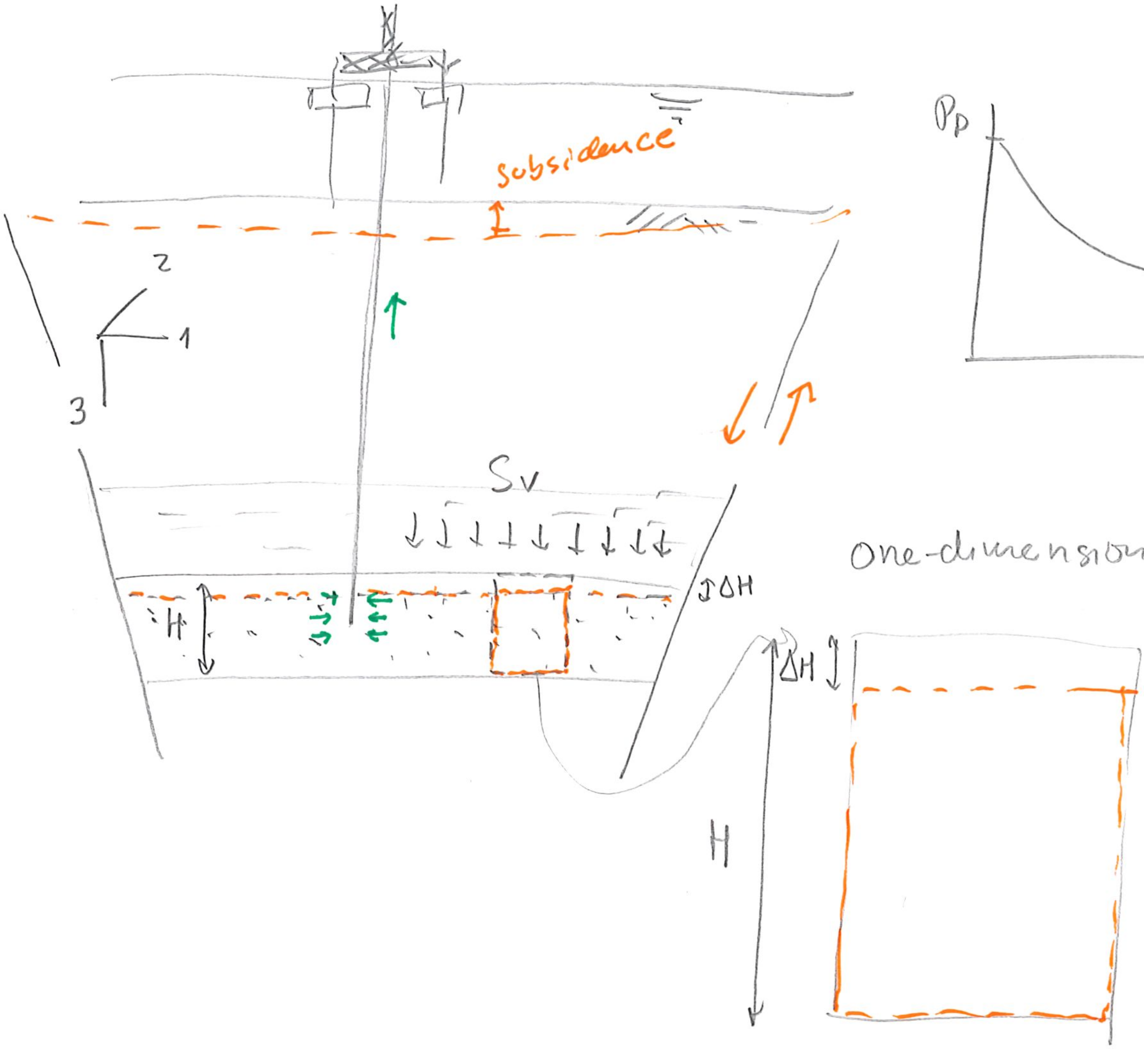
$$M = G \cdot \Delta V_{inj}$$

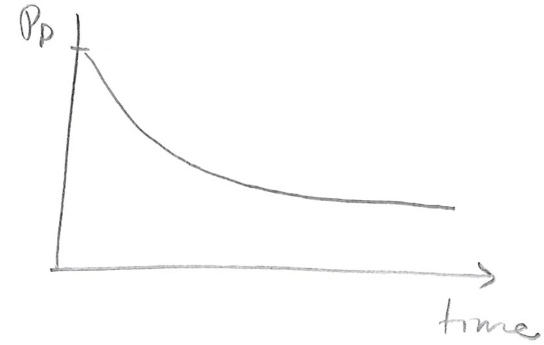
Richter (Log scale)

Microseismicity $M < 0$



Reservoir depletion ↔ evolution of σ and ϕ, and K





one-dimensional strain path

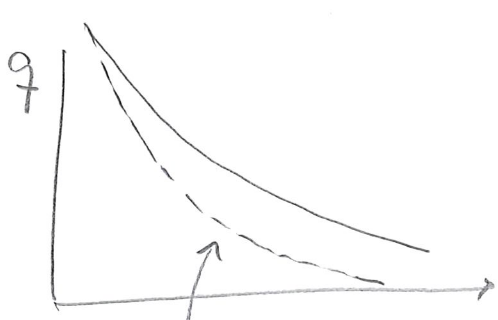
$$\varepsilon_{33} = \frac{\Delta H}{H}$$

$$\varepsilon_{11} = \varepsilon_{22} = 0$$

$\boxed{K \propto d_p^2}$

$\downarrow P_p \rightarrow$ compaction $\rightarrow \downarrow \phi \rightarrow \downarrow K$

$\rightarrow \uparrow \sigma \rightarrow \downarrow\downarrow K$



with reduction of permeability

$\boxed{K \propto w_f^3}$

Porous media

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}} \quad \Big\}\ \text{elasticity}$$

→ Terzaghi's eff stress

$$\sigma = S - P_p$$

$$\underline{\underline{S}} - \alpha \underline{\underline{P_p}} = \underline{\underline{\underline{\underline{C}}}} \cdot \underline{\underline{\varepsilon}}$$

→ Biot's eff stress

$$\sigma = S - \alpha P_p$$

$\alpha = 0.4 - 1$

→ porous sandstones ~0.9

→ tight shales ~ 0.4

$$\rightarrow \begin{bmatrix} S_{11} - \alpha P_{p} \\ S_{22} - \alpha P_{p} \\ S_{33} - \alpha P_p \\ \hline S_{12} \\ S_{13} \\ S_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \left[\begin{array}{ccc|c} 1-\nu & \nu & \nu & \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & \\ \hline & & & - \\ & 0 & & - \\ & & & - \end{array}\right] \begin{bmatrix} \cancel{\varepsilon_{11}} \\ \cancel{\varepsilon_{22}} \\ \varepsilon_{33} \\ \cancel{2\varepsilon_{12}} \\ \cancel{2\varepsilon_{13}} \\ \cancel{2\varepsilon_{23}} \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ \\ \\ \\ \\ \end{array}$$

3rd equation → $\varepsilon_{33} = \dfrac{\overset{cst}{S_{33}} - \alpha P_p}{\underbrace{\dfrac{E(1-\nu)}{(1+\nu)(1-2\nu)}}_{M}} \Rightarrow \boxed{\dfrac{\Delta \varepsilon_{33}}{\Delta P_p} = -\dfrac{\alpha}{M}}$ ∂

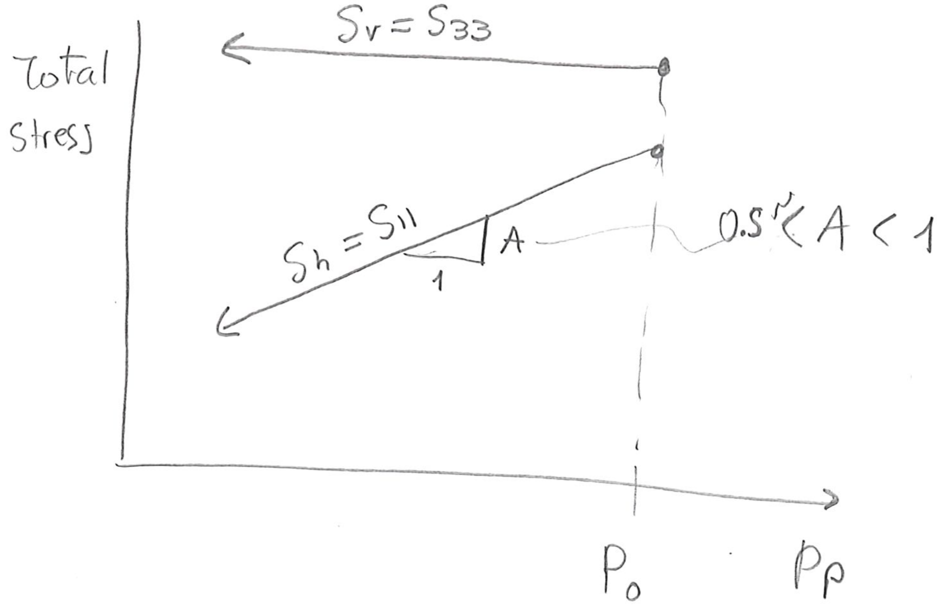
1st, 3rd eqs → $\sigma_{11} = \dfrac{\nu}{1-\nu}\sigma_{33} \Rightarrow S_{11} = \dfrac{\nu}{1-\nu}(\overset{cst}{S_{33}} - \alpha P_p) + \alpha P_p$

$$\boxed{\frac{\Delta S_{11}}{\Delta P_p} = \alpha \frac{1-2\nu}{1-\nu} = A}$$

$$\frac{\Delta \sigma_{11}}{\Delta P_p} = -\alpha \frac{\nu}{1-\nu} \qquad (Hz)$$

$$\sigma_{33} = S_{33} - \alpha P_p \; ; \; \frac{\Delta \sigma_{33}}{\Delta P_p} = -\alpha \qquad (Vt)$$

Total stress path



Reservoir compressibility

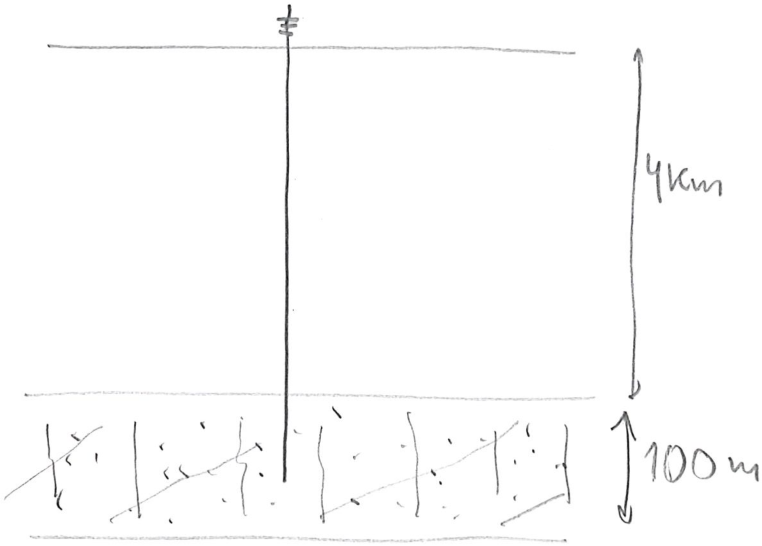
$$\boxed{C_{pp} = \frac{1}{V_p}\frac{dV_p}{dP_p}}$$ ⟶ reservoir engineering
↳ squeezing sponge

$$C_{pp} = \frac{C_{bp}}{\phi}$$

$$C_{bp} = \frac{1}{V_b}\frac{dV_b}{dP_p} = +\frac{\alpha}{M}$$

ε_{vol}

Example



Reservoir rock

Fractured sandstone

$\phi = 0.21$, $\alpha = 0.9$

$E = 2\,MMpsi = 13.8\ GPa$

$\nu = 0.17$

Production

$\Delta P_p = -35\ MPa$

(1) ΔH? displacement at the top of the reservoir

$M = 14.8\ GPa$ $\qquad$ $1\,MPa = 145\,psi$

$\Delta \varepsilon_{33} = 0.21\ \%$

$\Delta H = \frac{0.21}{\cancel{100}}\ \cancel{100}m = 0.21\,m \sim 1\ foot$

$C_{bp} = \frac{0.9}{14.8 \cdot 10^9 Pa} = 0.06 \times 10^{-9} \frac{1}{Pa} = 0.42 \cdot 10^{-6} \frac{1}{psi}$

$C_{pp} = \frac{0.42 \cdot 10^{-6}}{0.21} \frac{1}{psi} \cong 2.0 \cdot 10^{-6} \frac{1}{psi}$

$\boxed{C_{pp} = 2.0\ \mu sip}$

$1\ \mu sip$ —— $20\ \mu sip$

↓ stiff $\qquad$ ↳ soft

(2) What is ΔS_{11} with ΔP_p ?

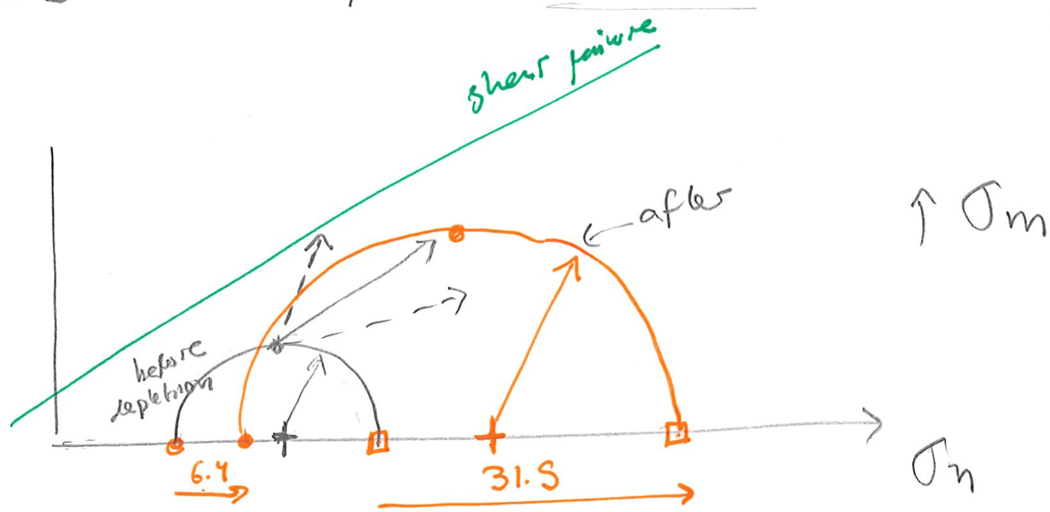
$$A = 0.716$$

$$\Delta S_{11} = 0.716 \cdot (-35\ MPa) \cong -25\ MPa$$

(3) What are $\Delta \sigma_{11}$ and $\Delta \sigma_{33}$?

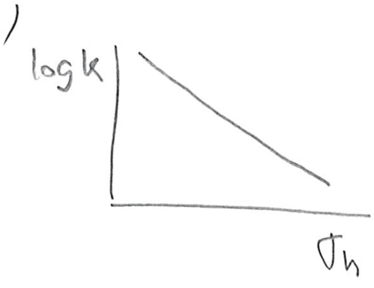
$$\Delta \sigma_{11} = \left(-\alpha \frac{\nu}{1-\nu}\right) \Delta P_p = +6.45\ MPa$$

$$\Delta \sigma_{33} = -\alpha \cdot \Delta P_p = +31.5\ MPa$$



(4) What is the change in permeability

$$\frac{k}{k_0} = \exp\left[-c_f \cdot (\overbrace{\sigma_n - \sigma_{n_0}}^{\Delta\sigma_{11}})\right] \quad ; \quad C_F = 0.25 \frac{1}{MPa}$$



$$\boxed{\frac{k}{k_0} \cong 0.2}$$