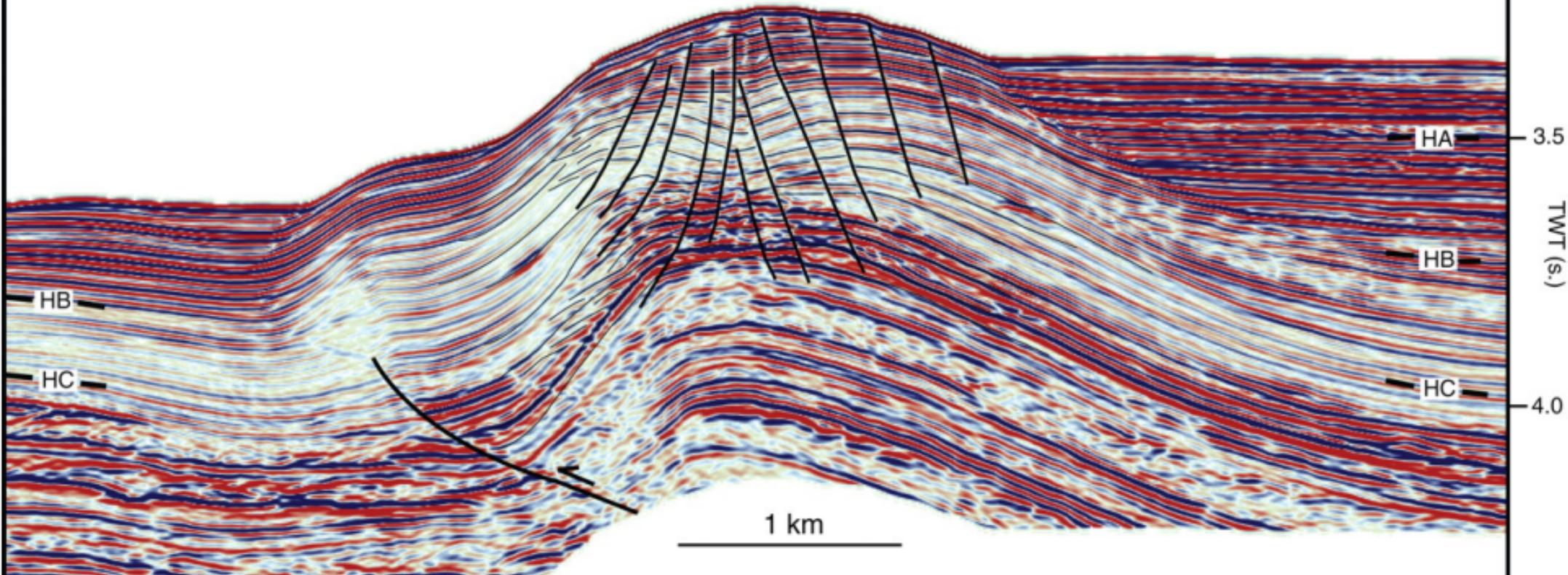


NW

Crestal normal faults

SE

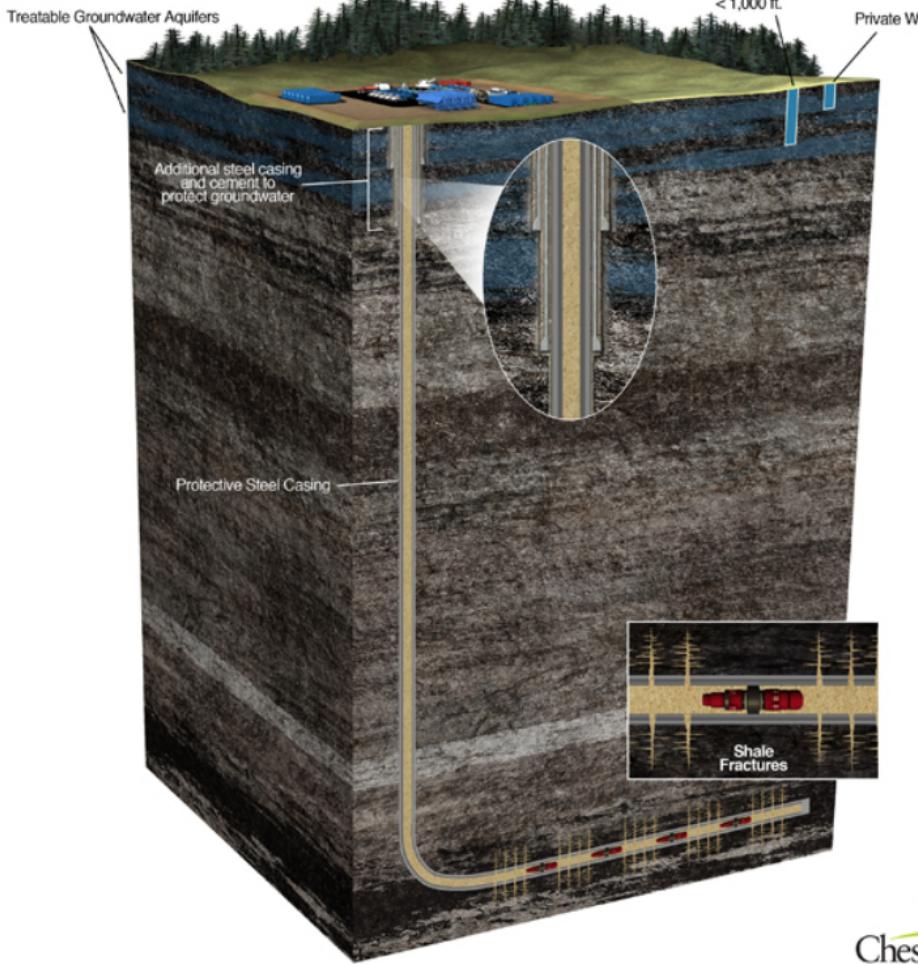


~1850



~2010





Risk-Based
Geomechanical
Screening

Stress Man

MUDLINE SUBSIDENCE



FAULT ACTIVATION

CASING CRUSHING

COMPACTION DRIVE

CASING
SHEAR

SAND PRODUCTION



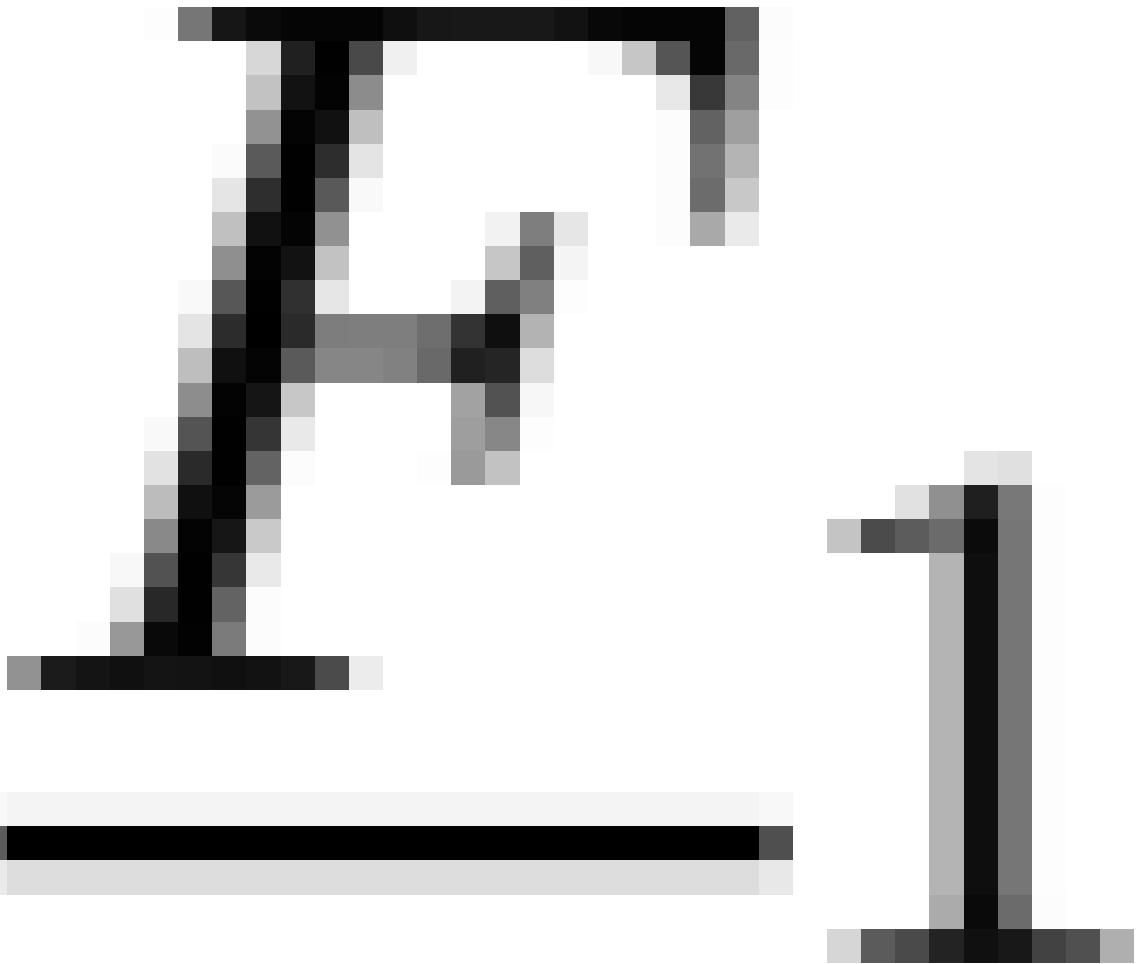
RESERVOIR YIELD

PERMEABILITY LOSS

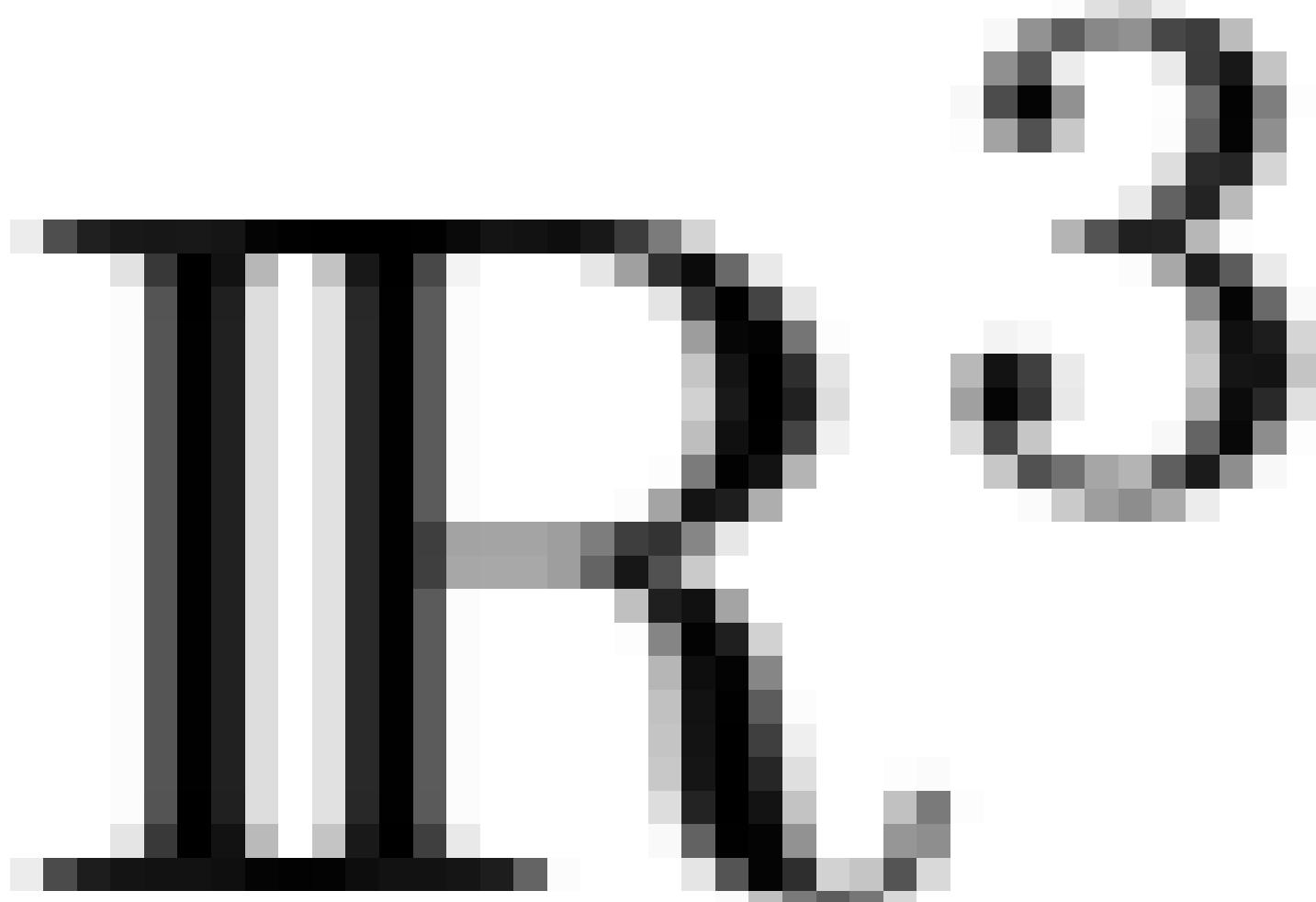
CASING BUCKLING

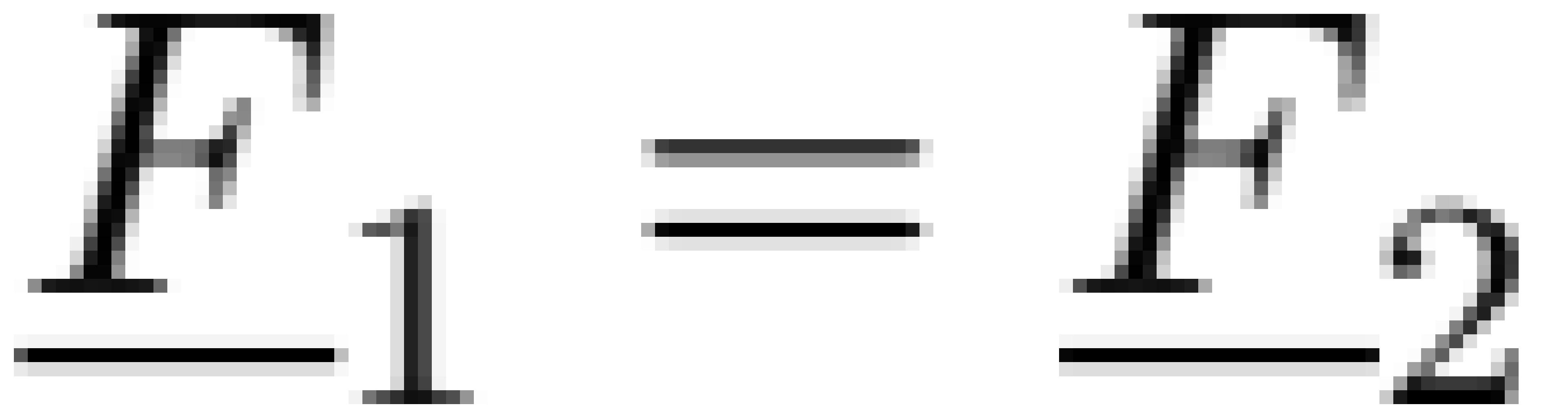
S_V



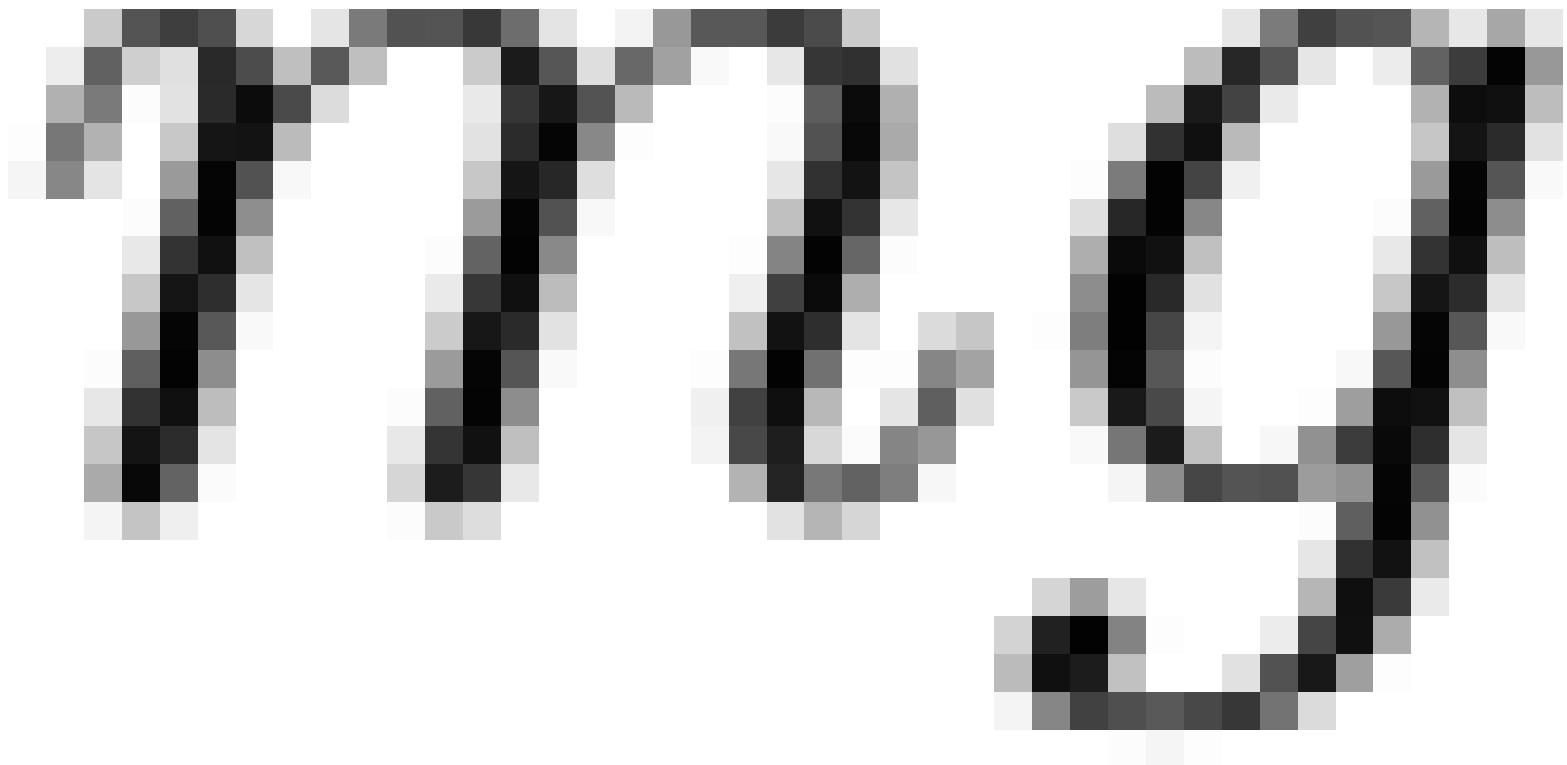


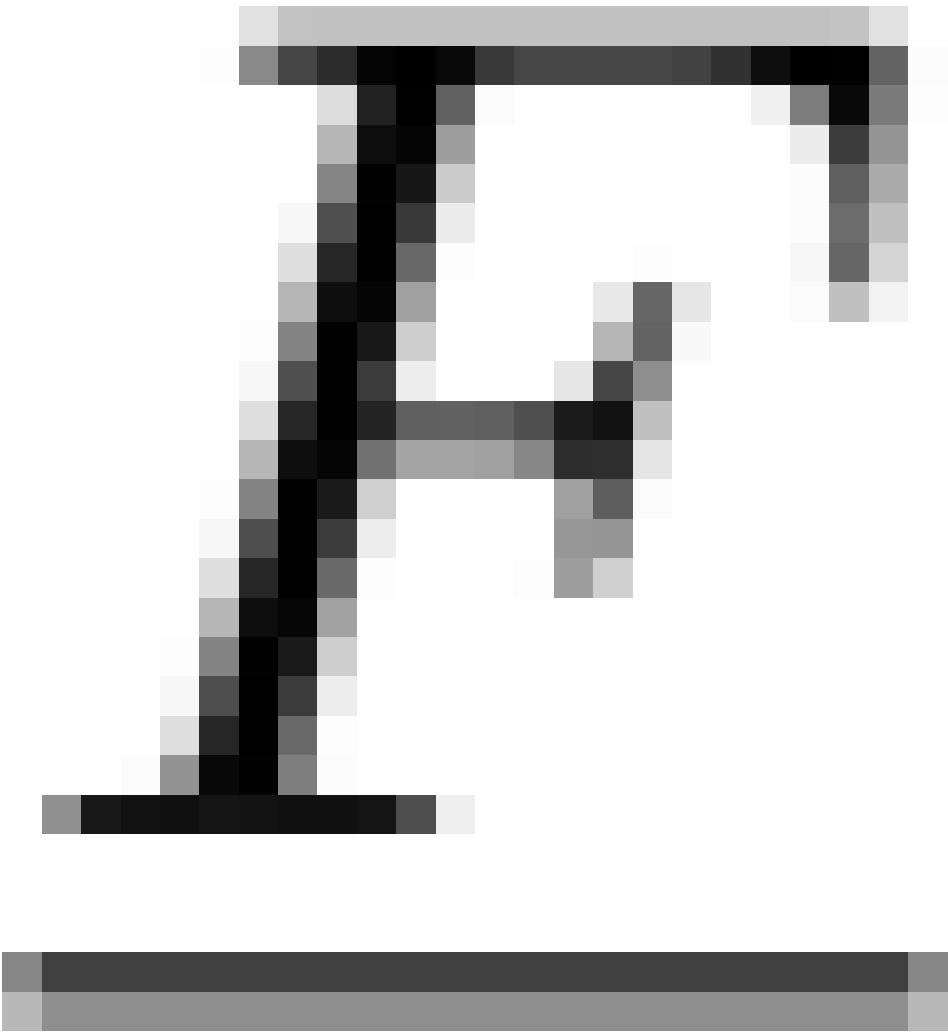


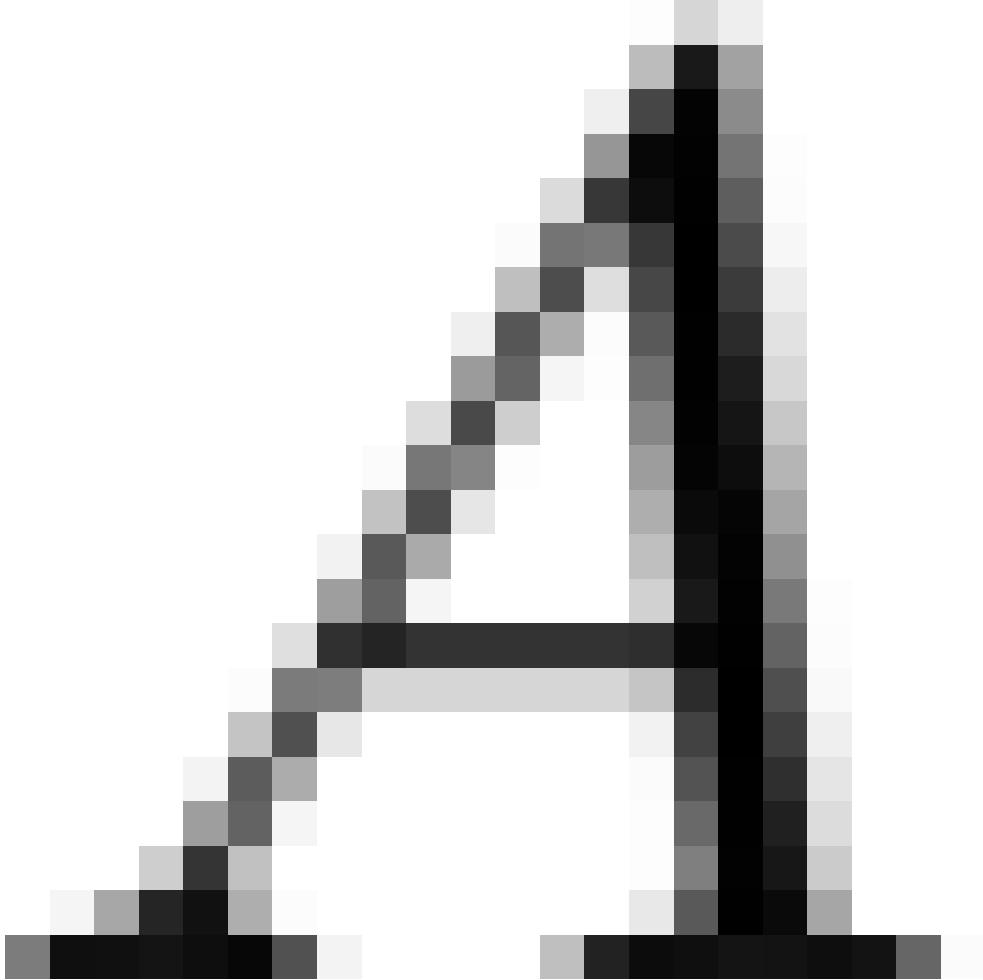












P

S

S

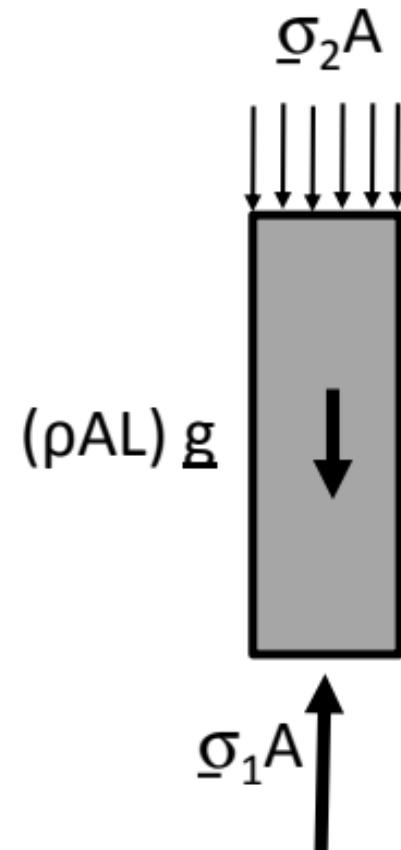
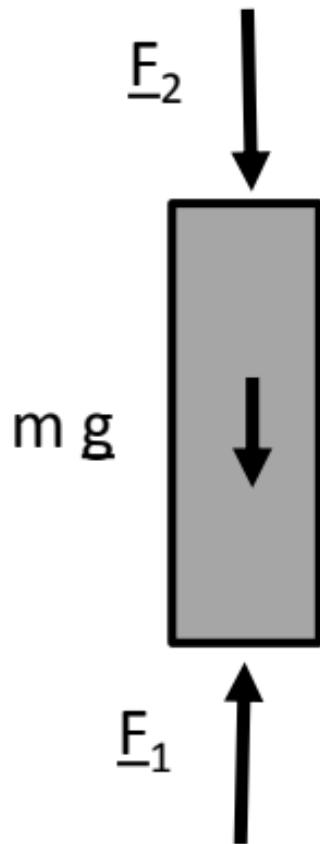
A

S

S

O

S

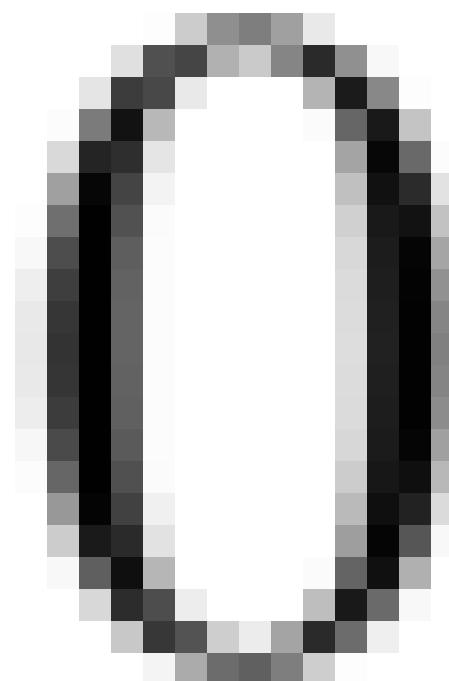
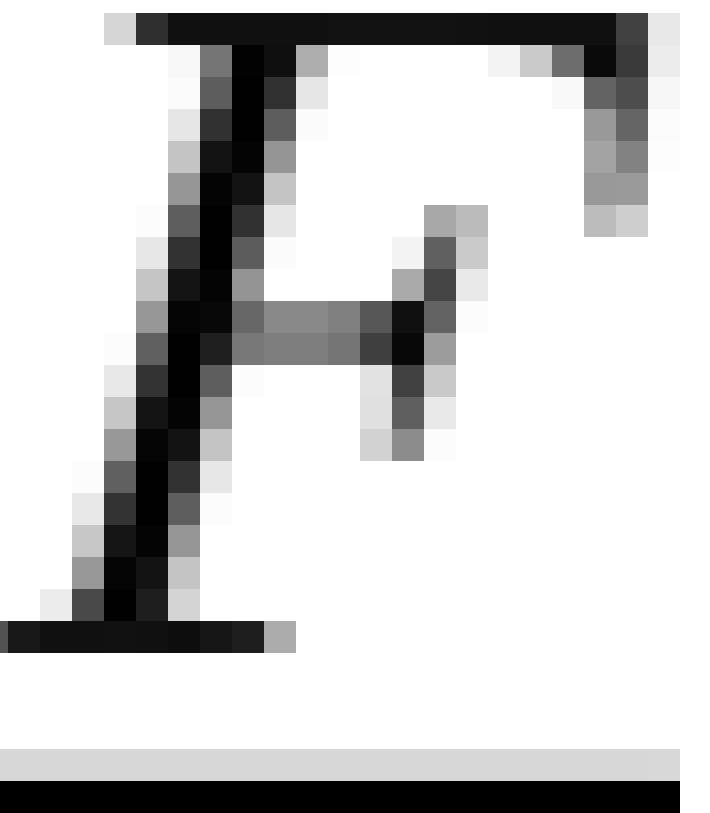
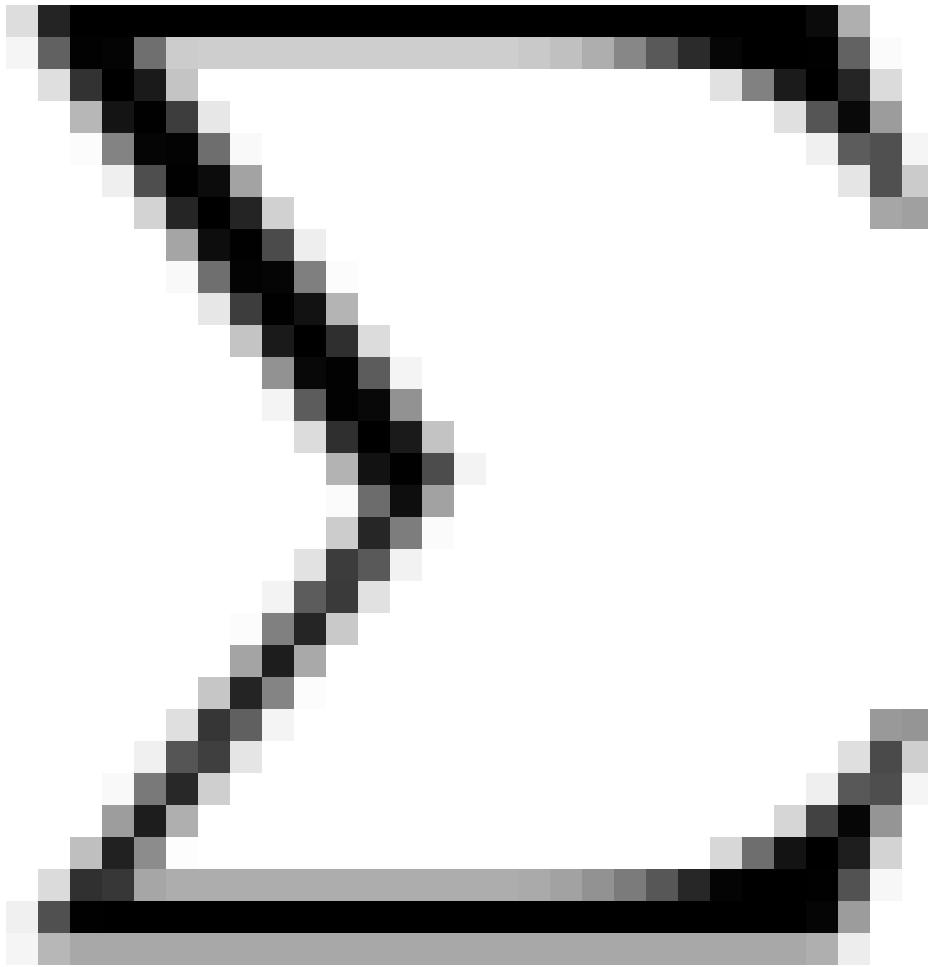


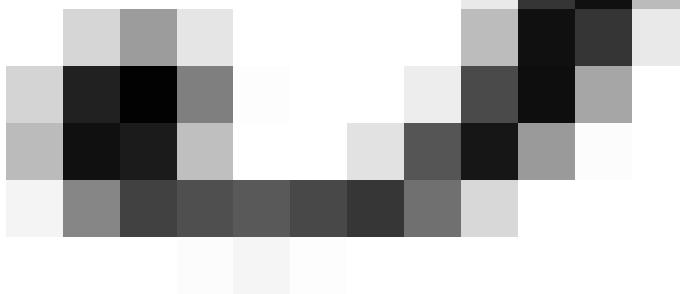
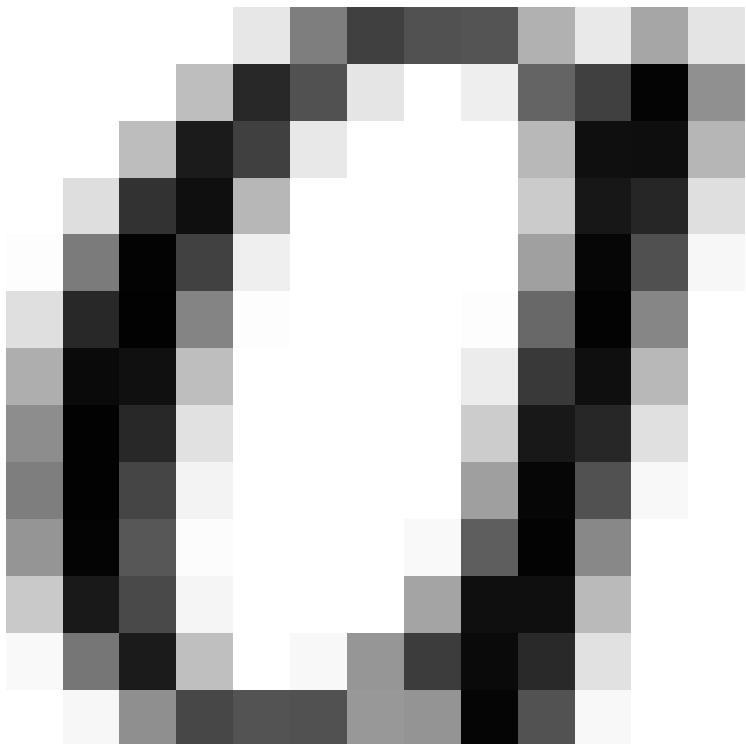
$$\Sigma F_z = +F_1 - F_2 = 0$$

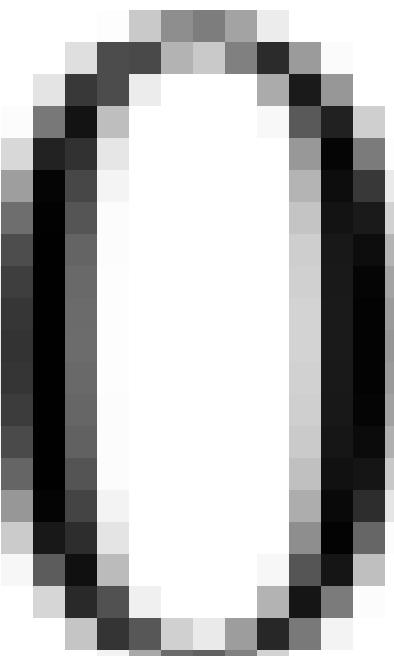
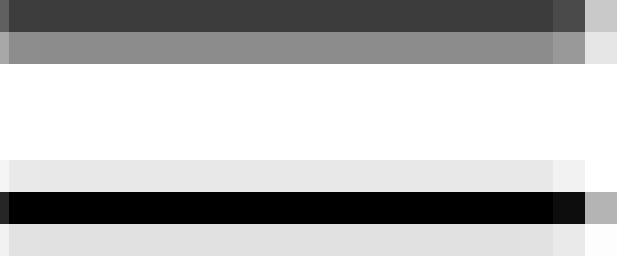
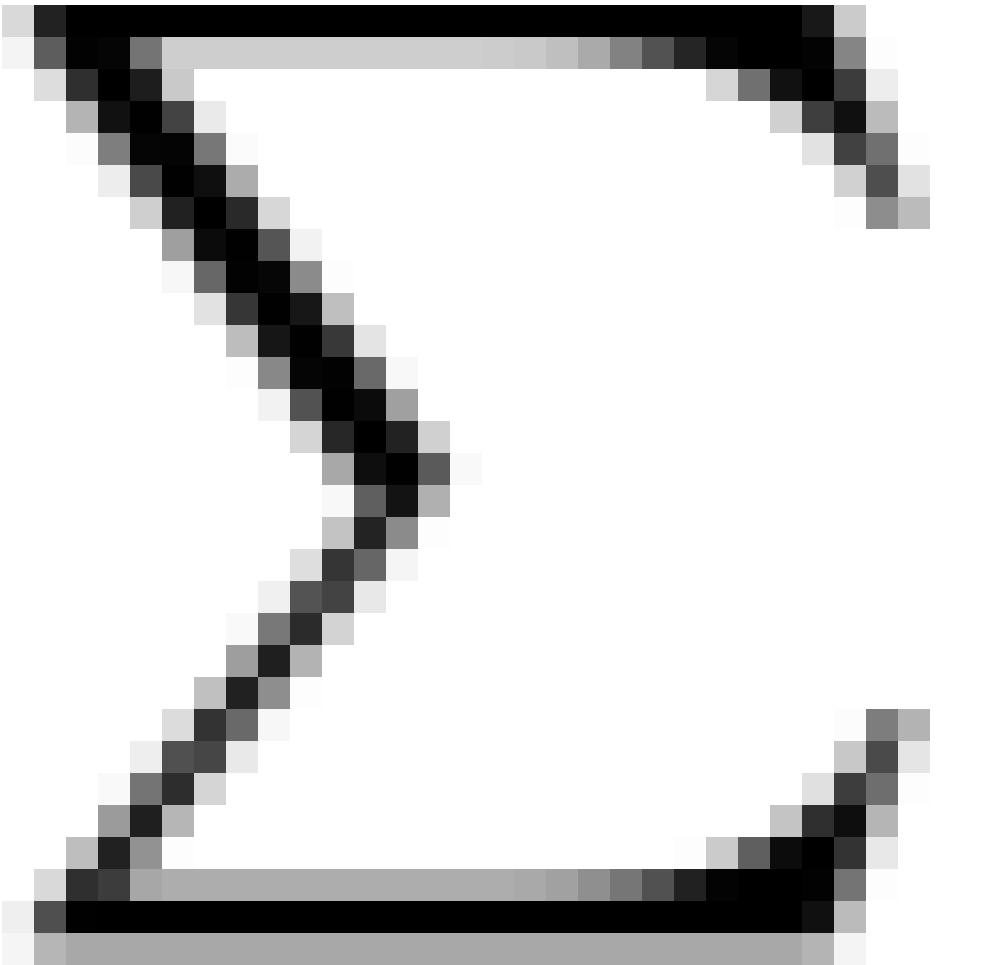
$$\Sigma F_z = +F_1 - m g - F_2$$

$$\Sigma F_z = +\sigma_1 A - (\rho A L)g - \sigma_2 A$$











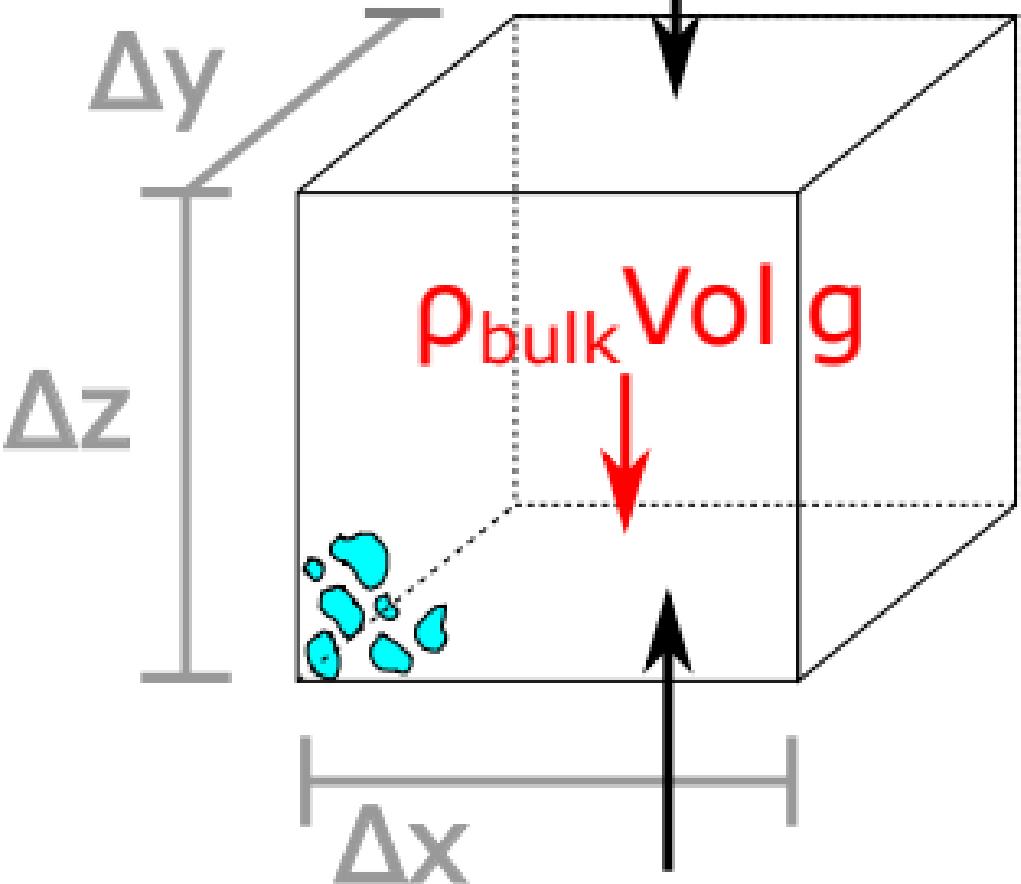


△ S₂

— △ z

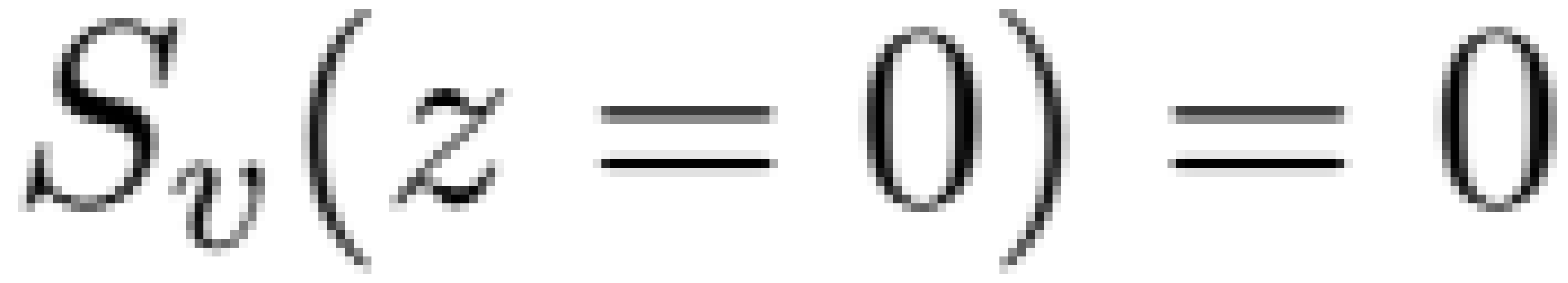
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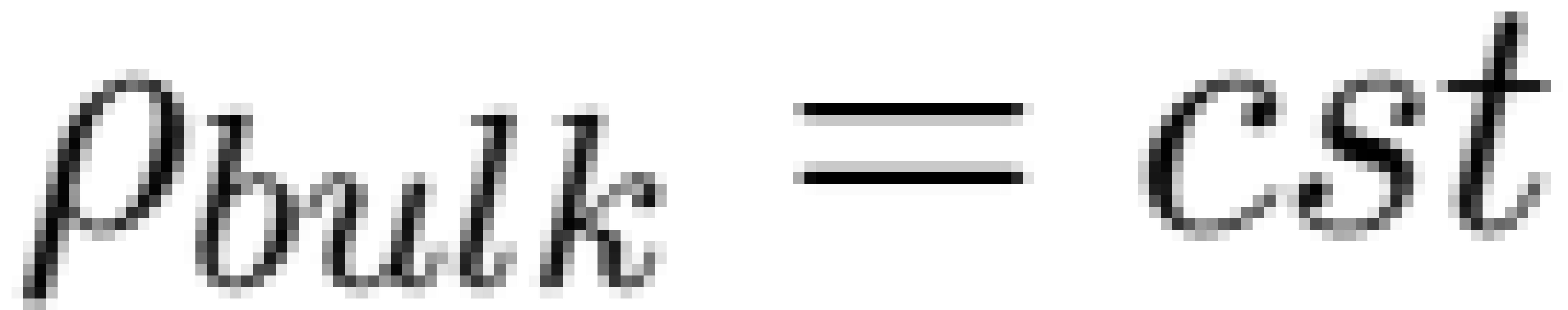
Observe 9

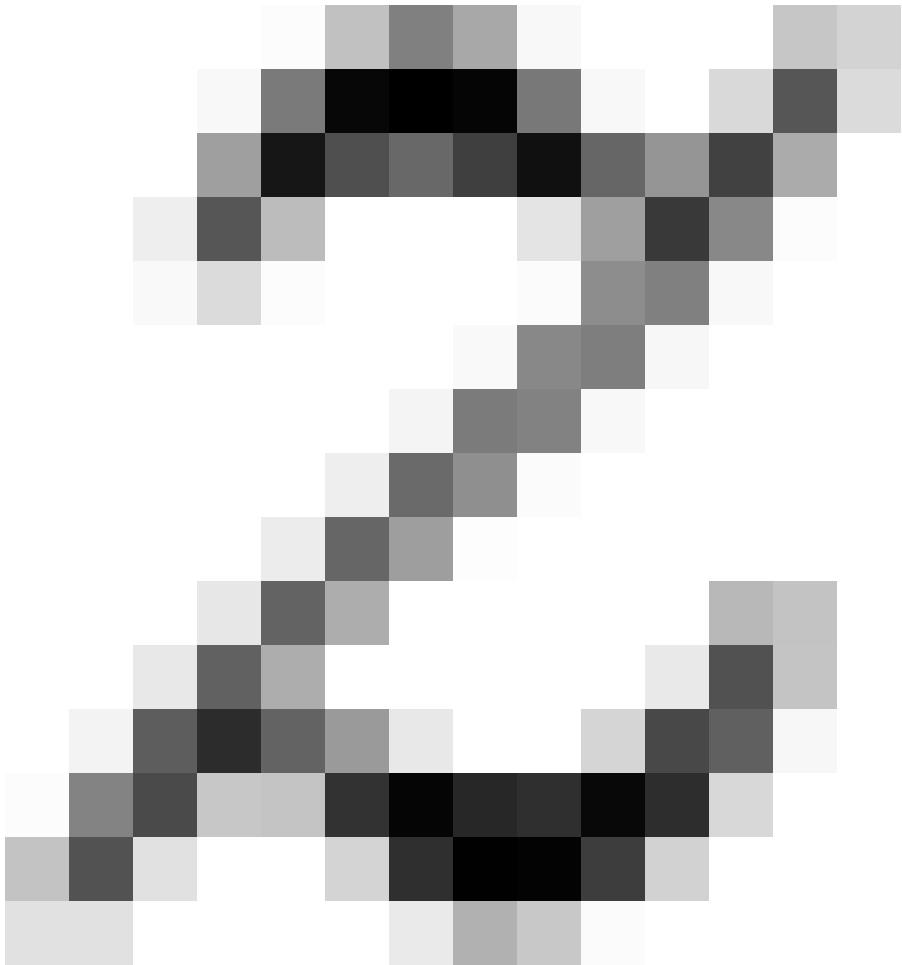
$S_v A$  $S_v A + \Delta S_v A$

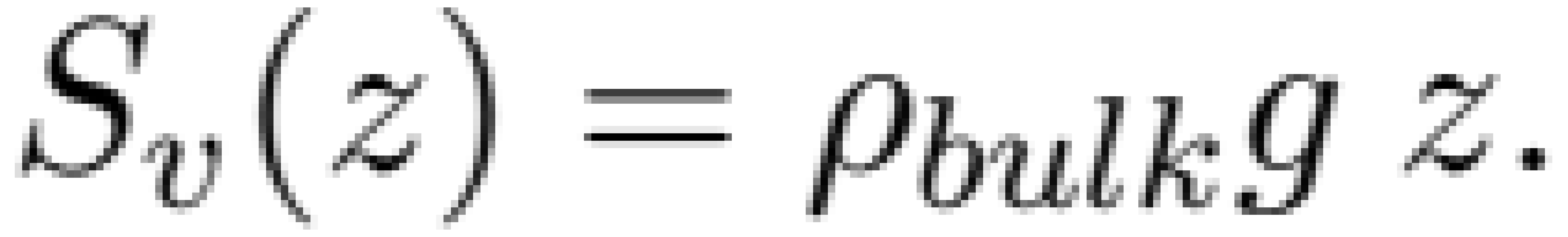
$$\int_0^{S_w(z)} ds_w = \int_0^z \rho_{bulk}(z) g dz$$







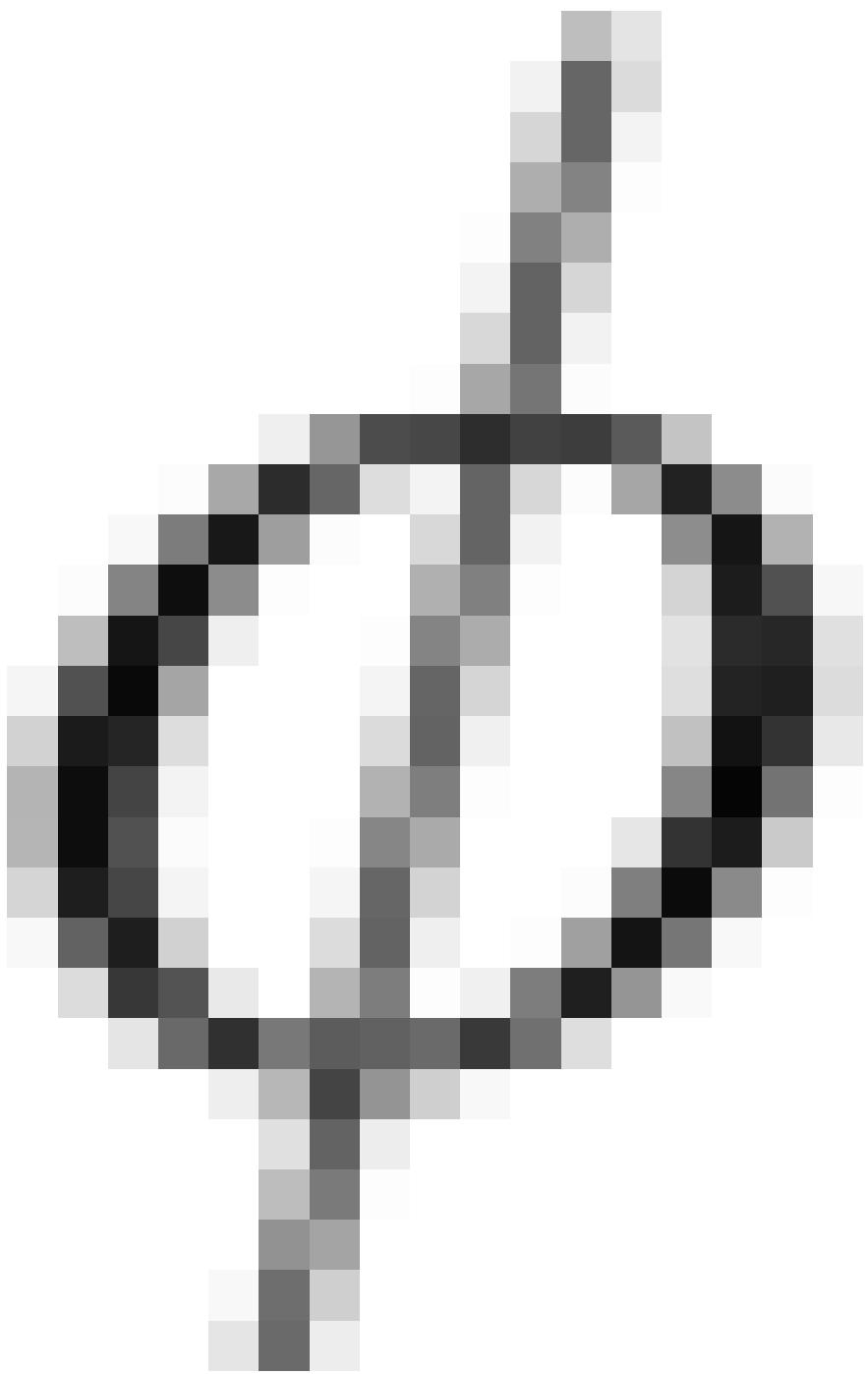


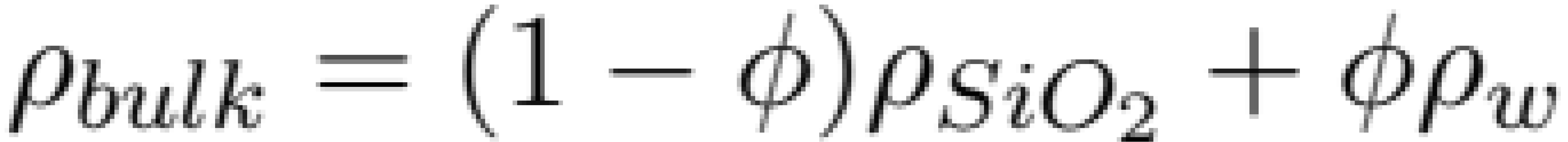




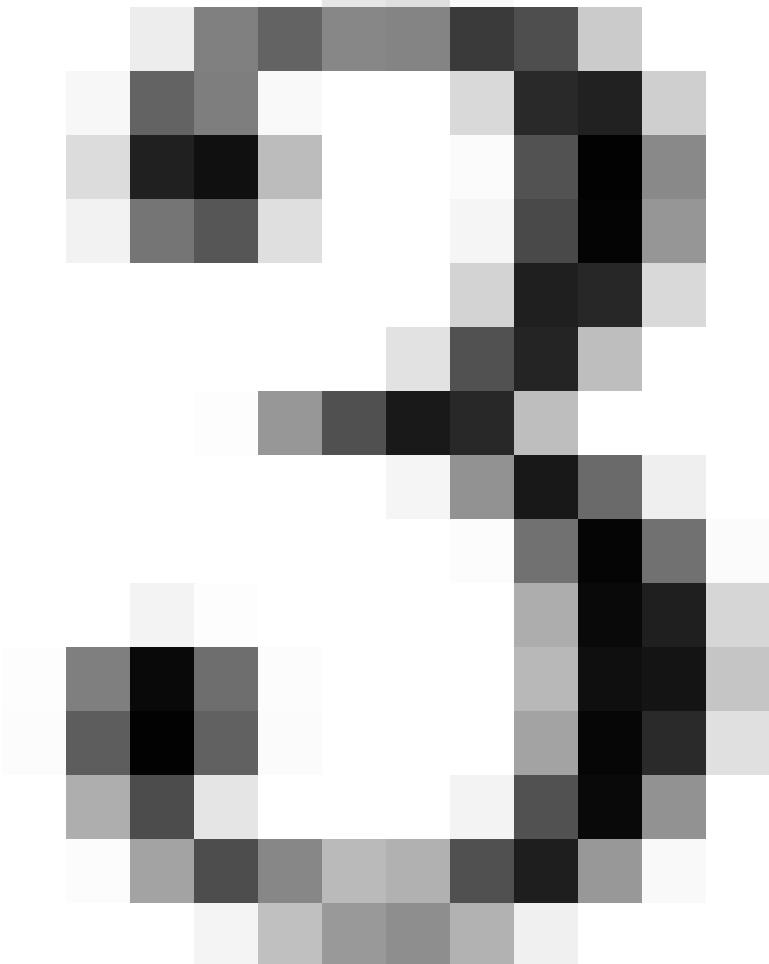


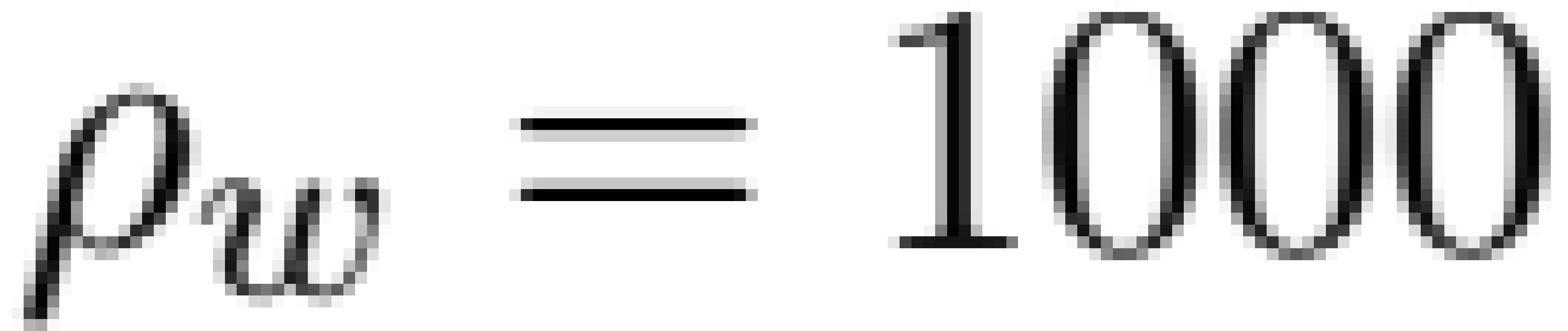


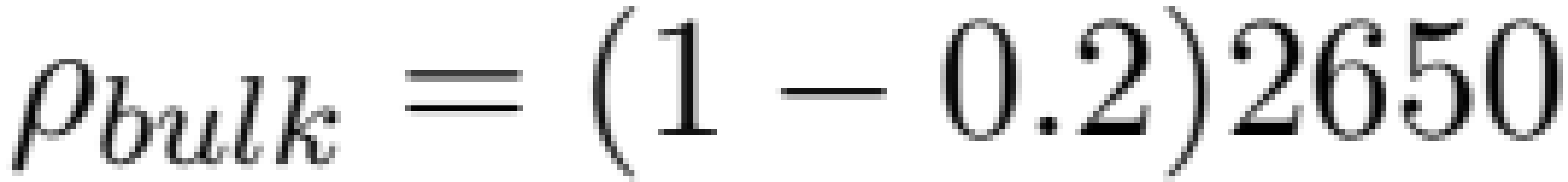


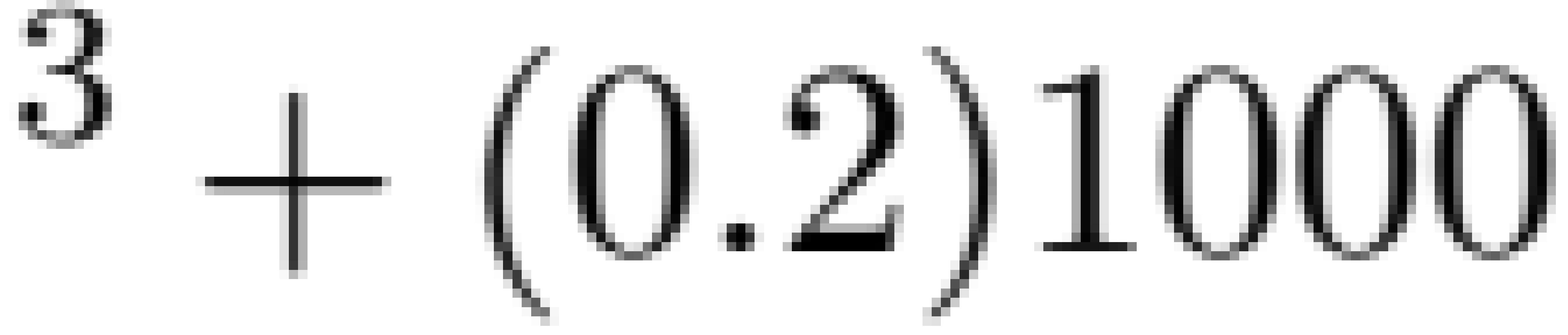




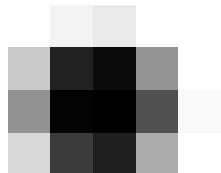
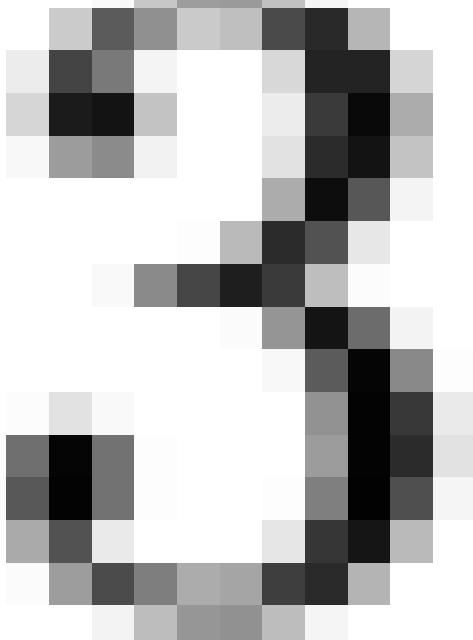


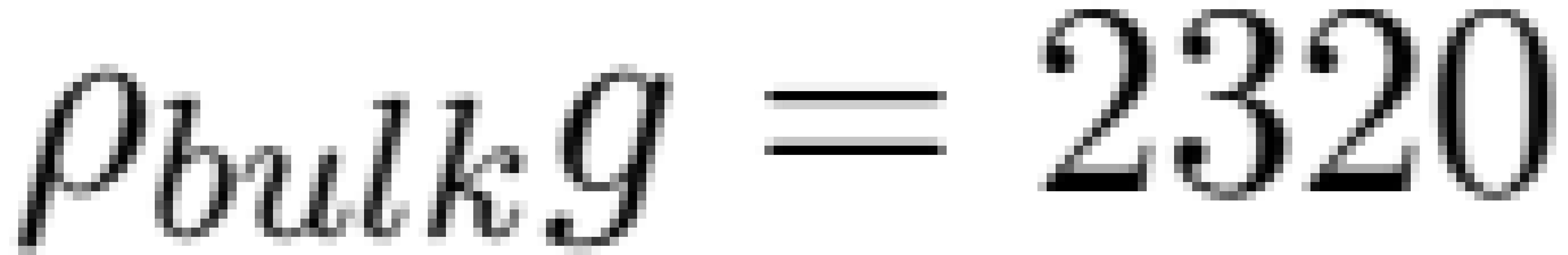






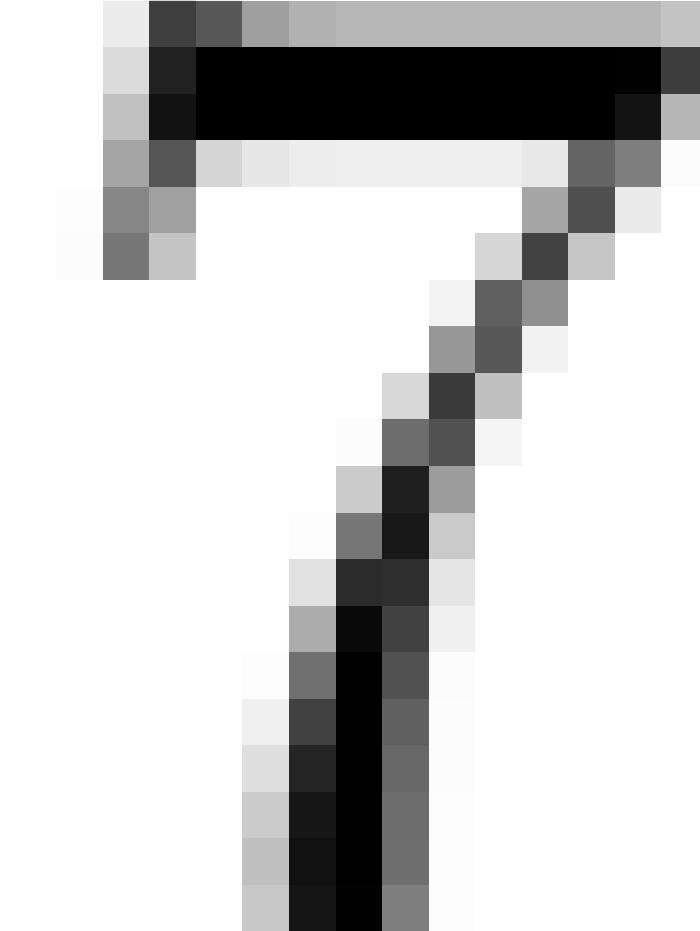
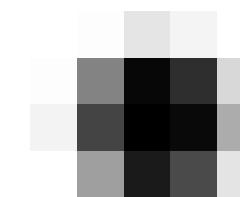
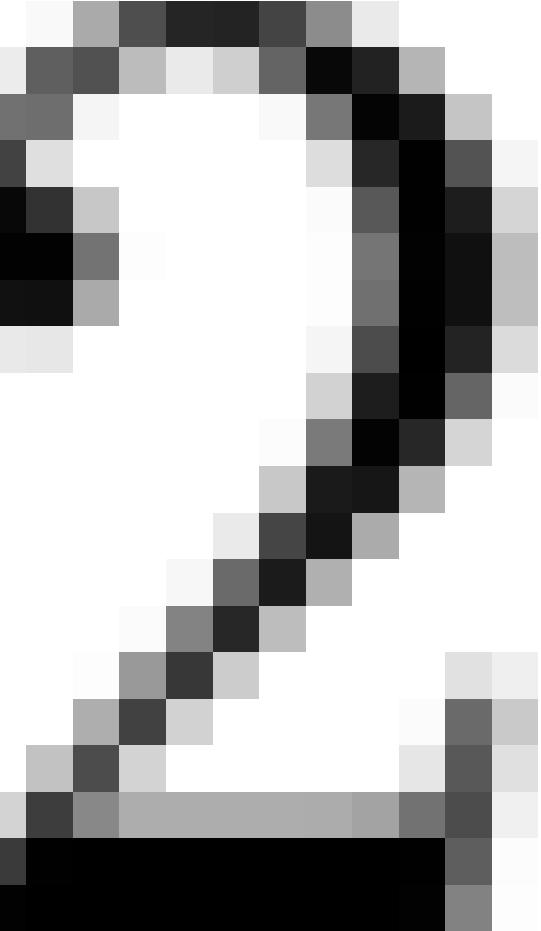
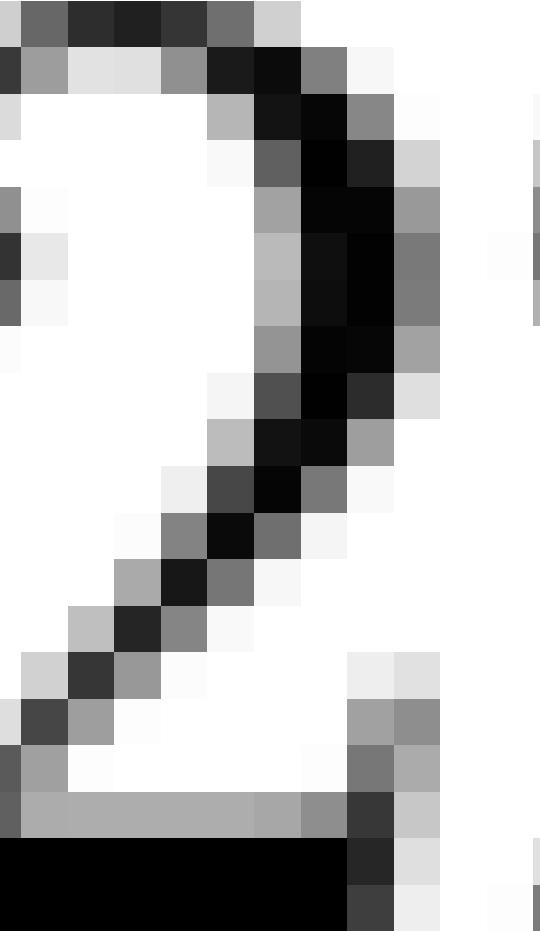


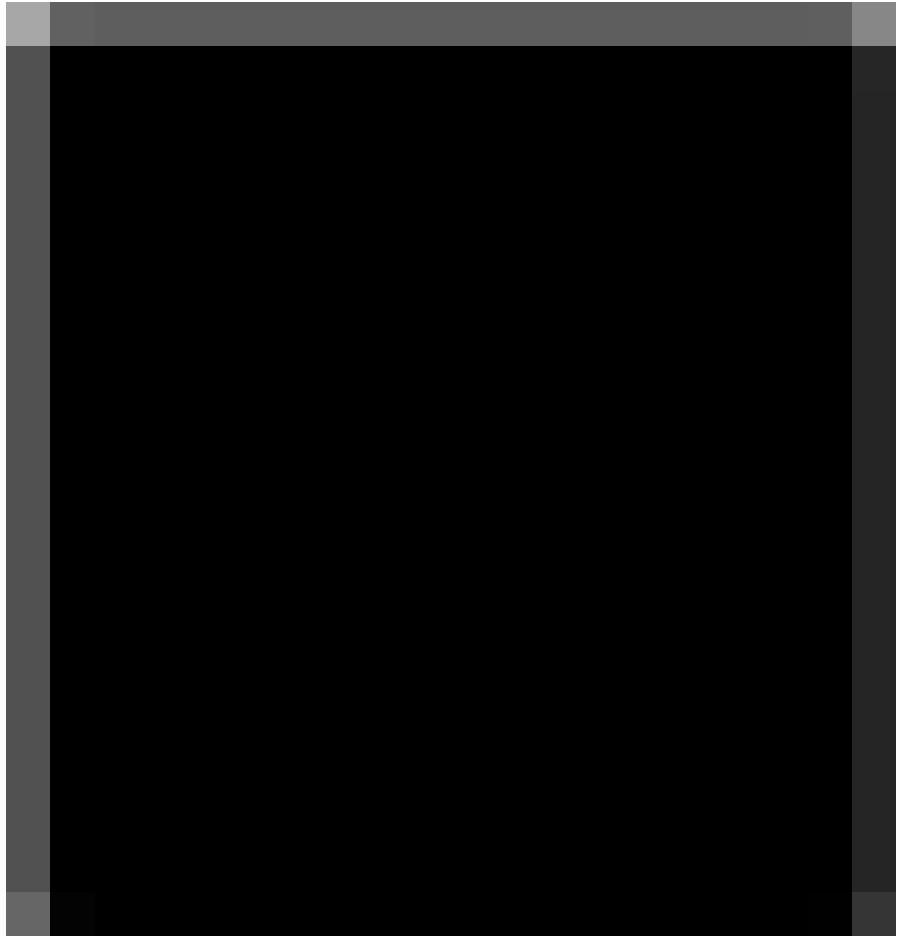
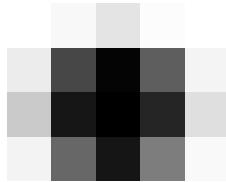


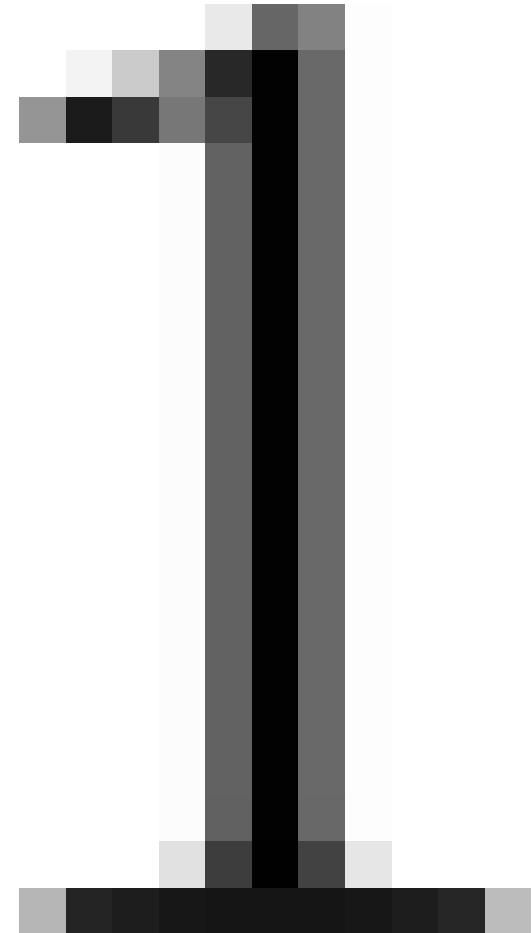
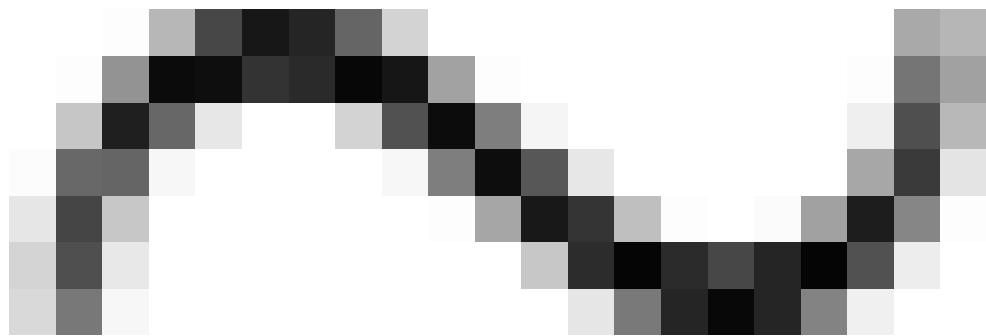


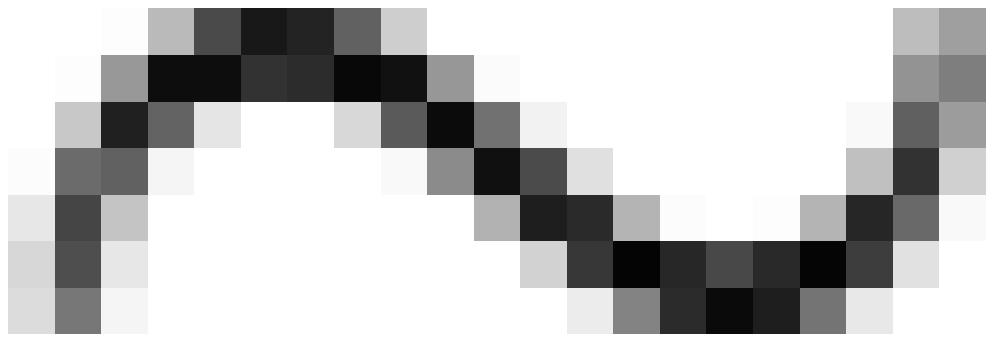


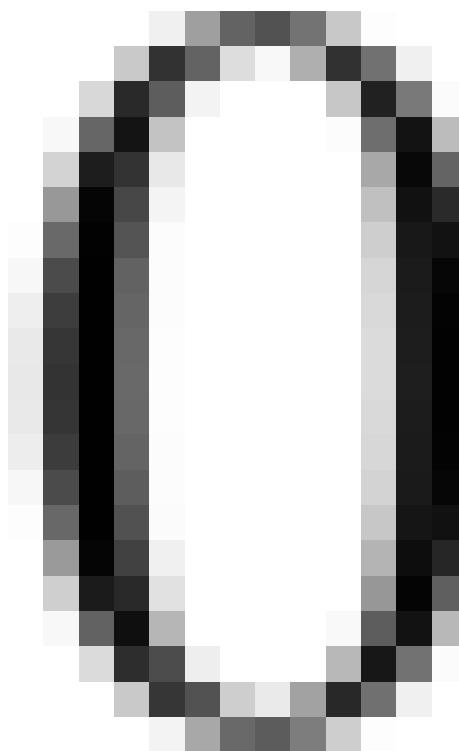
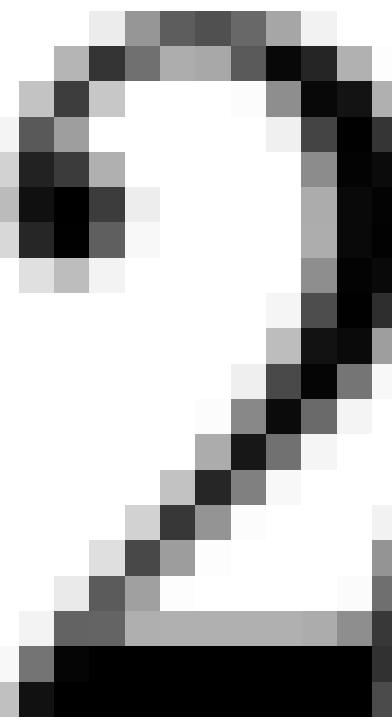
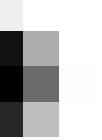
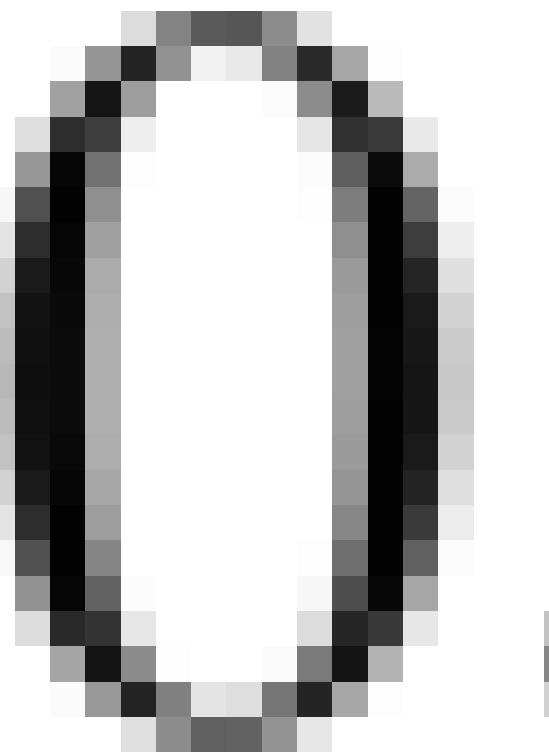
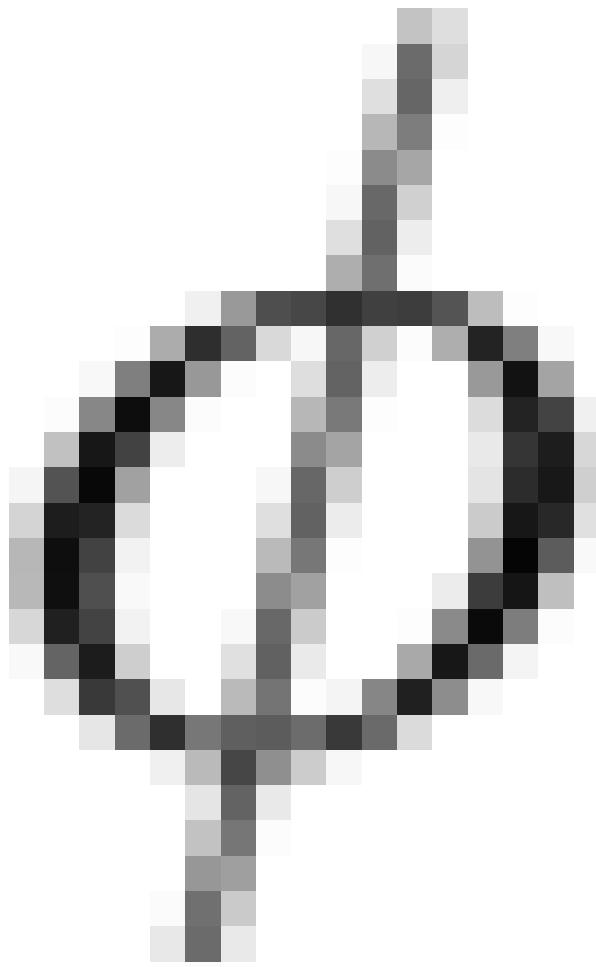


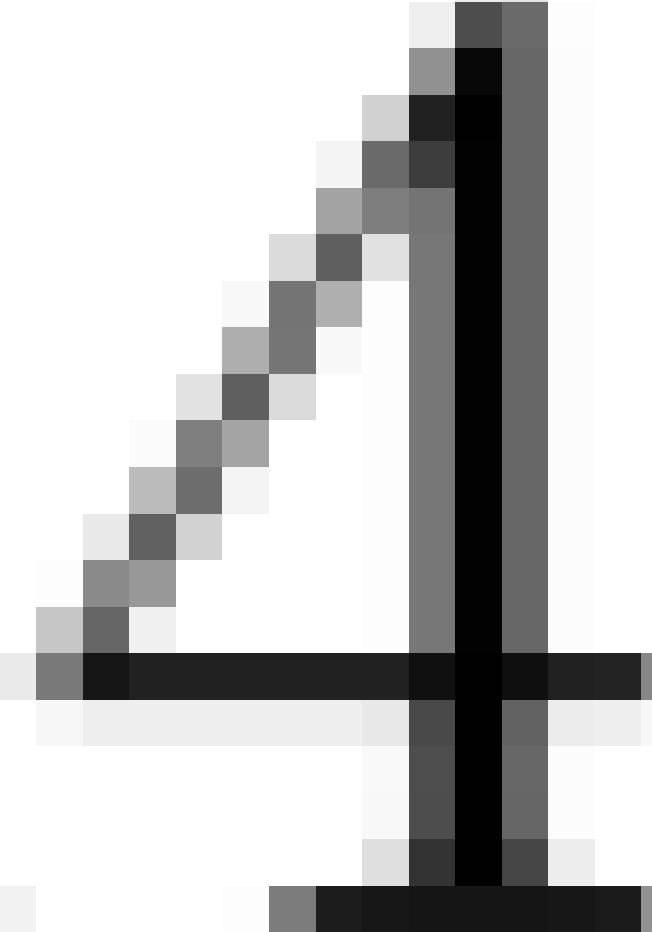
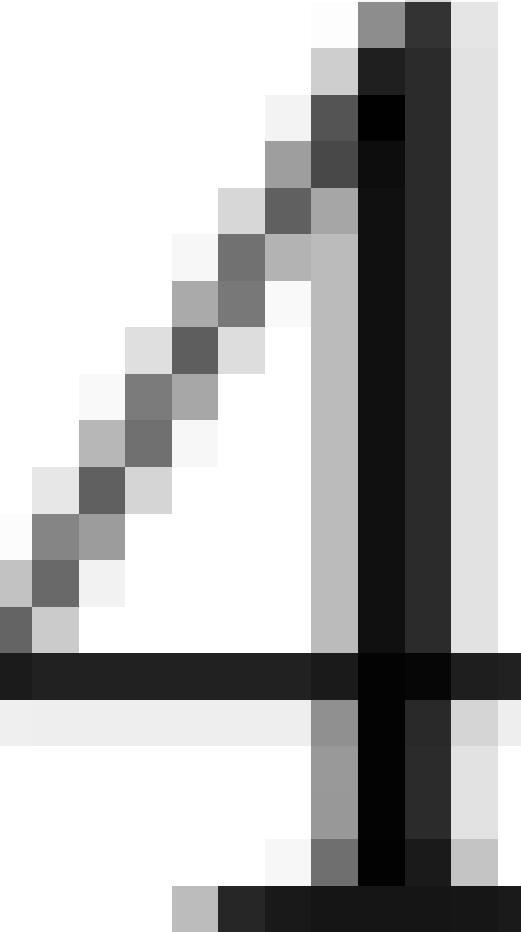
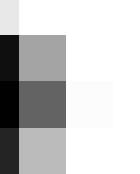
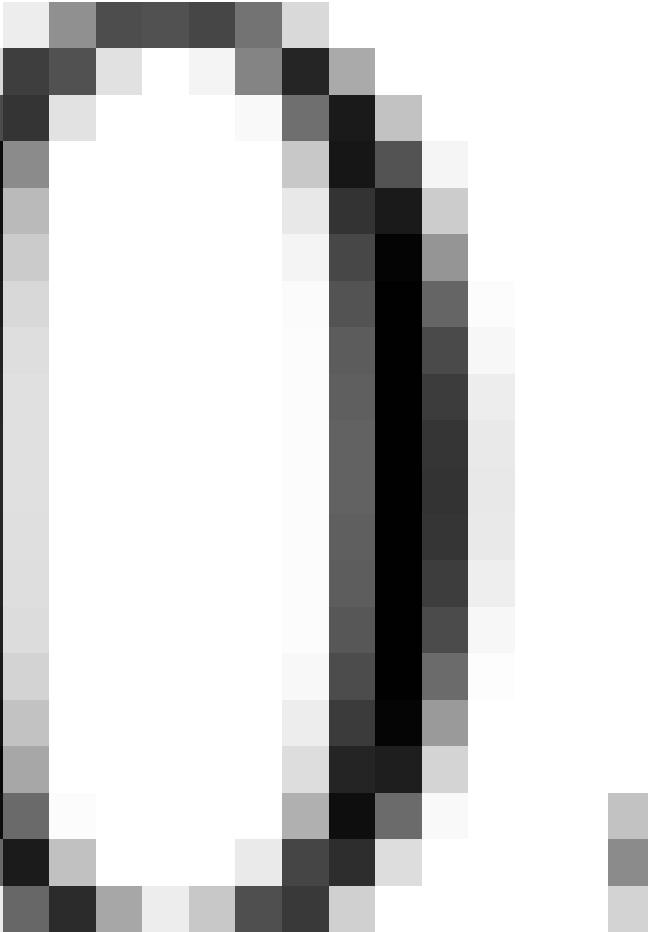
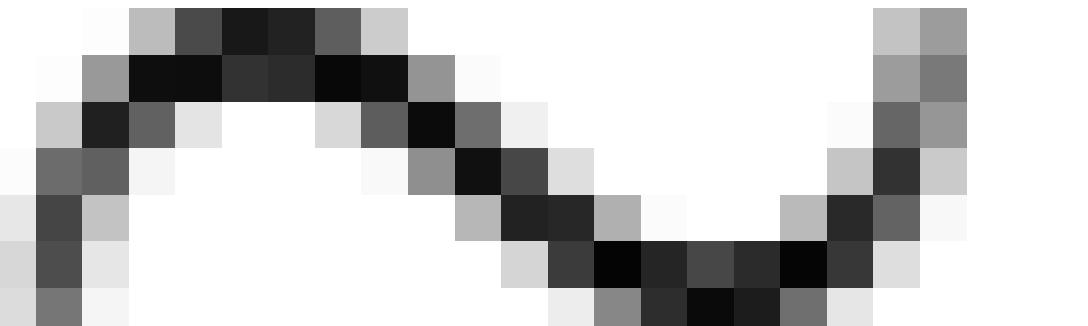


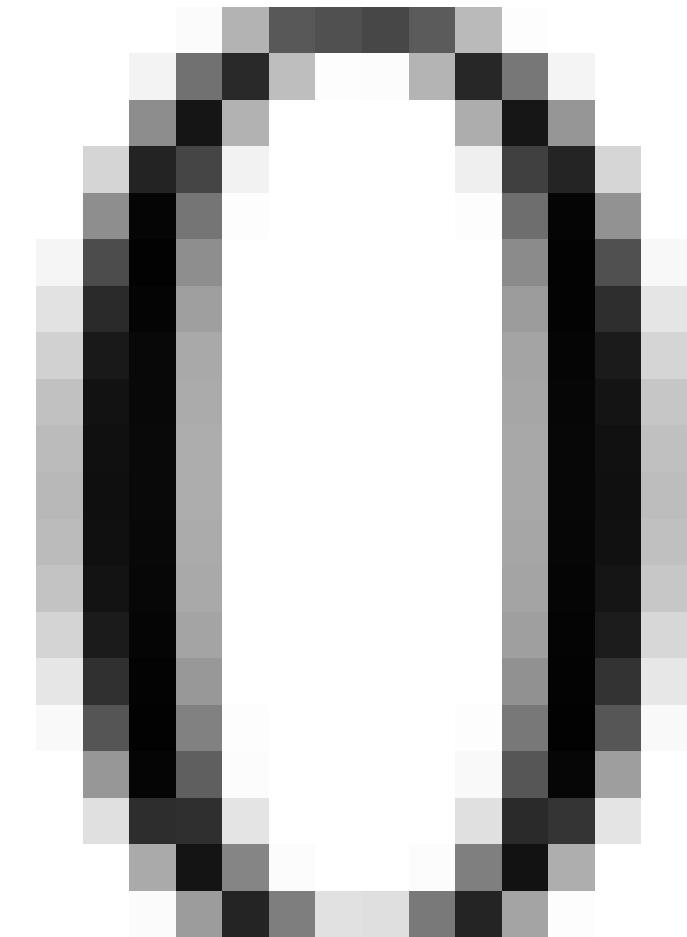
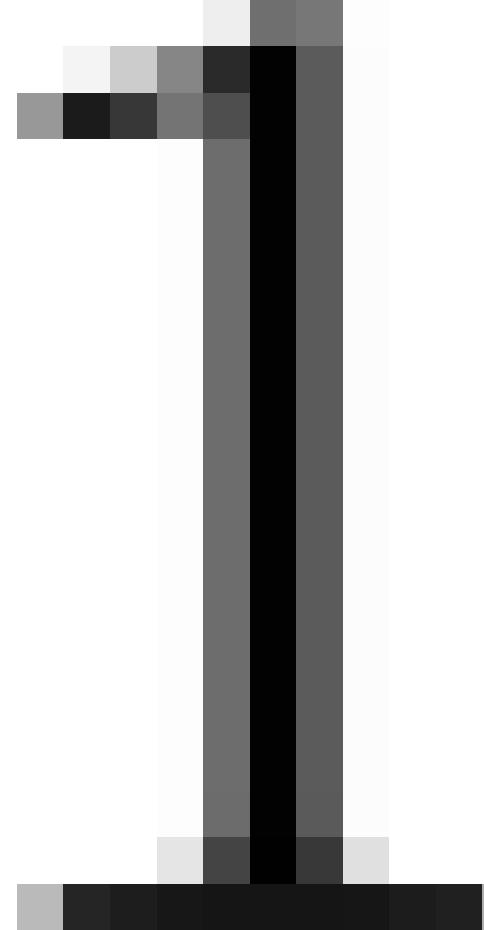
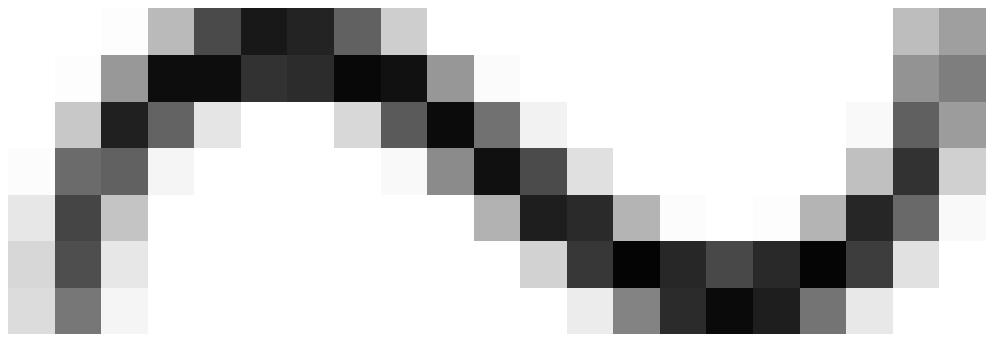




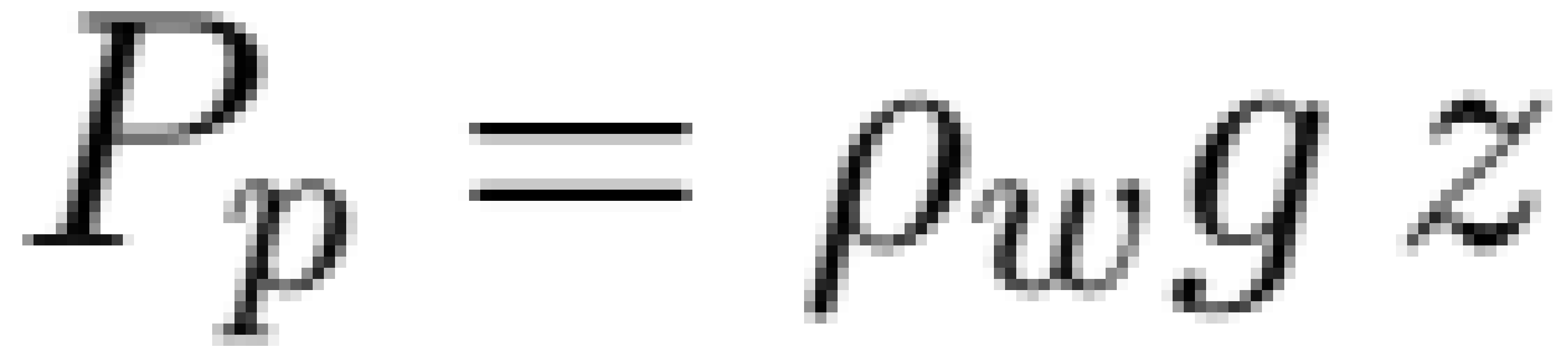


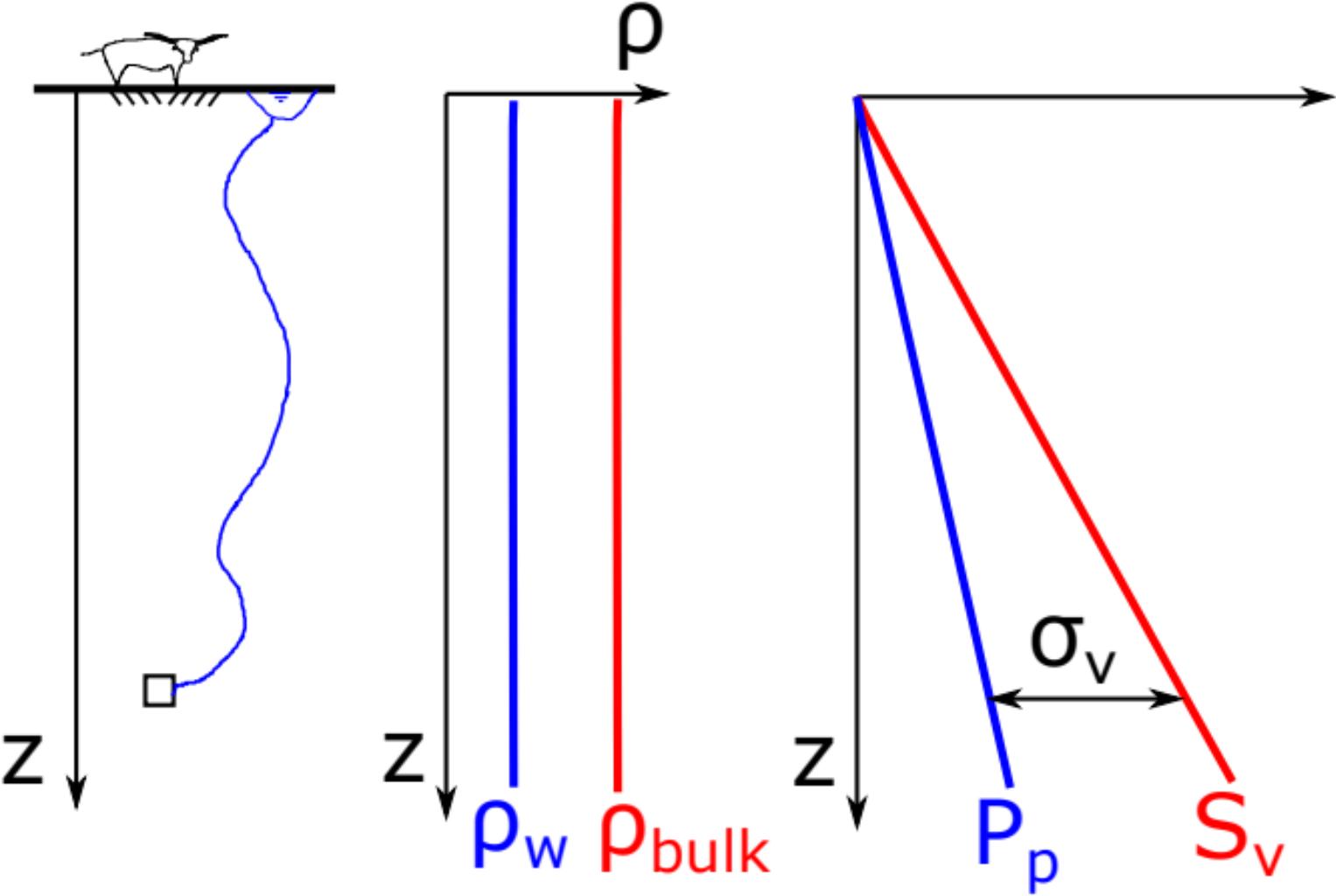




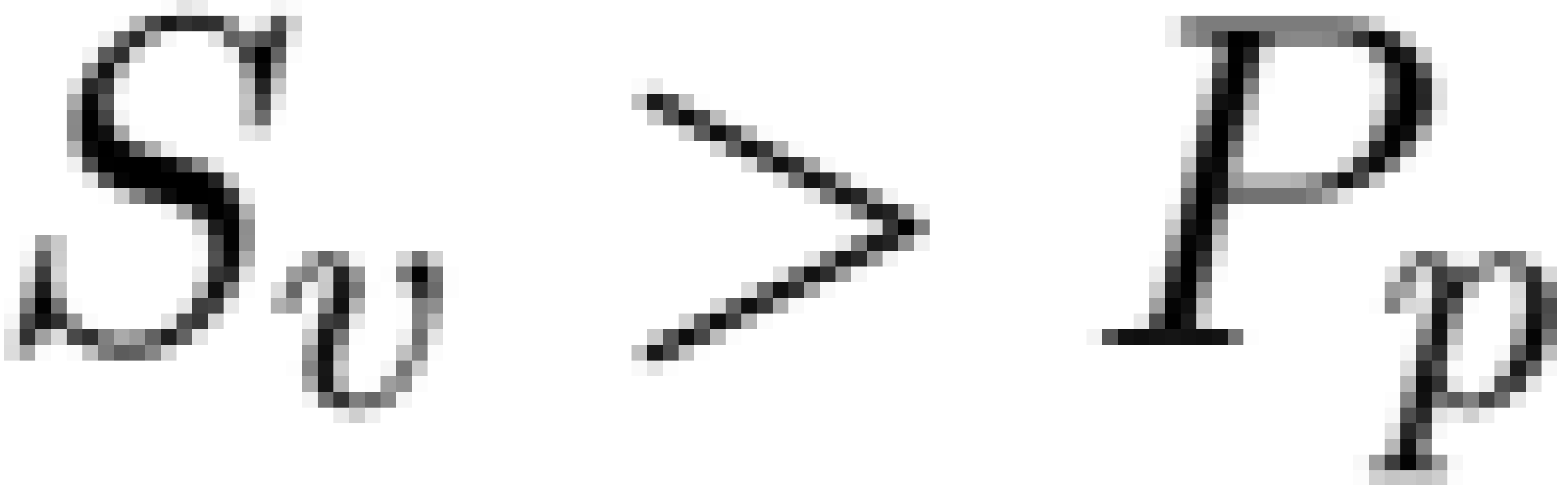








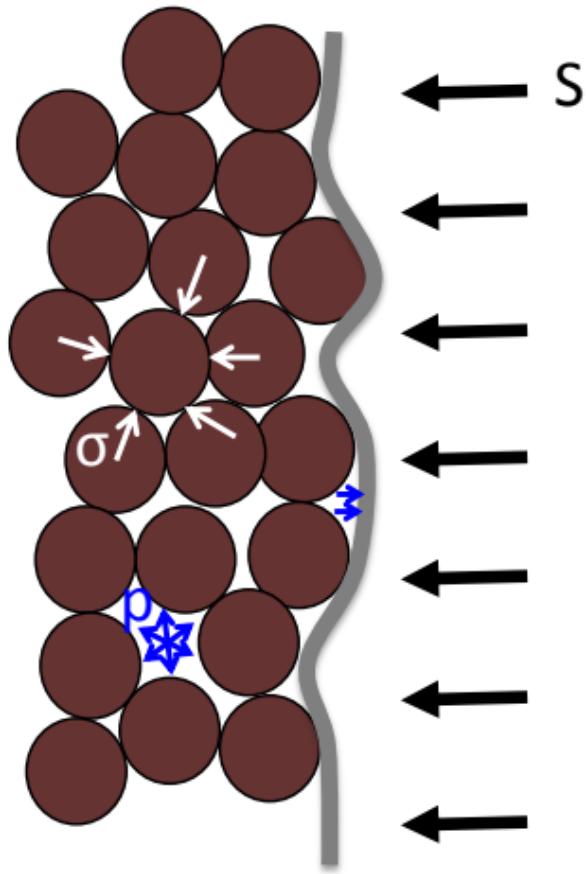




Effective stress =

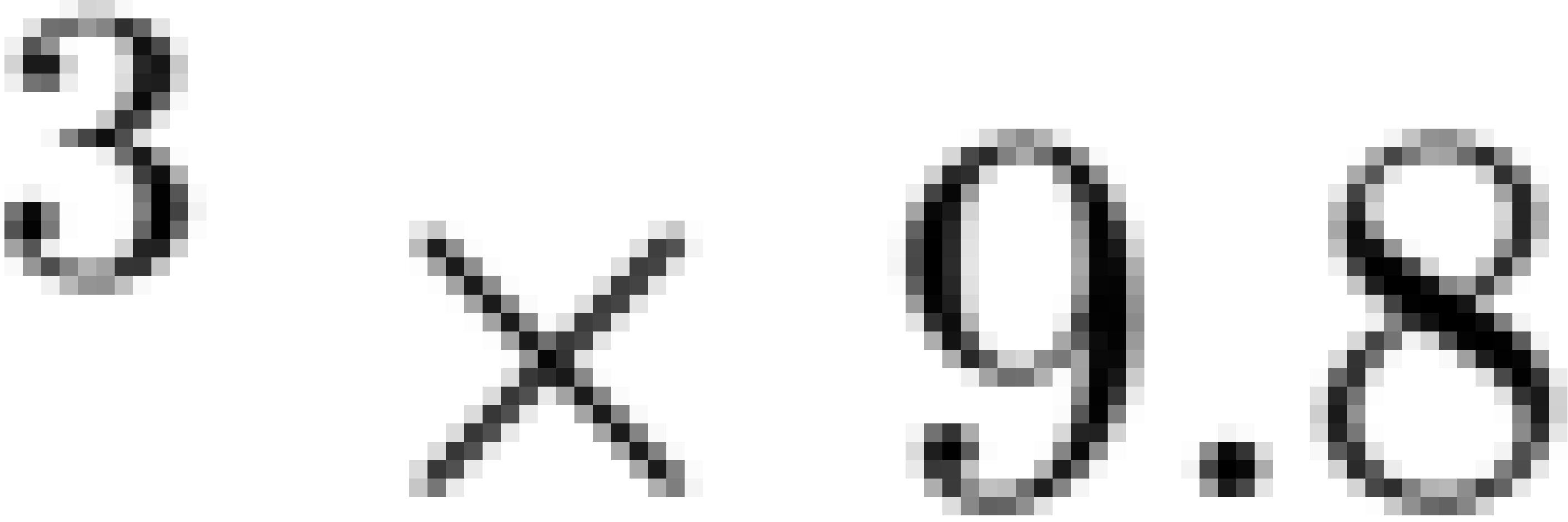
Total stress – Pore pressure

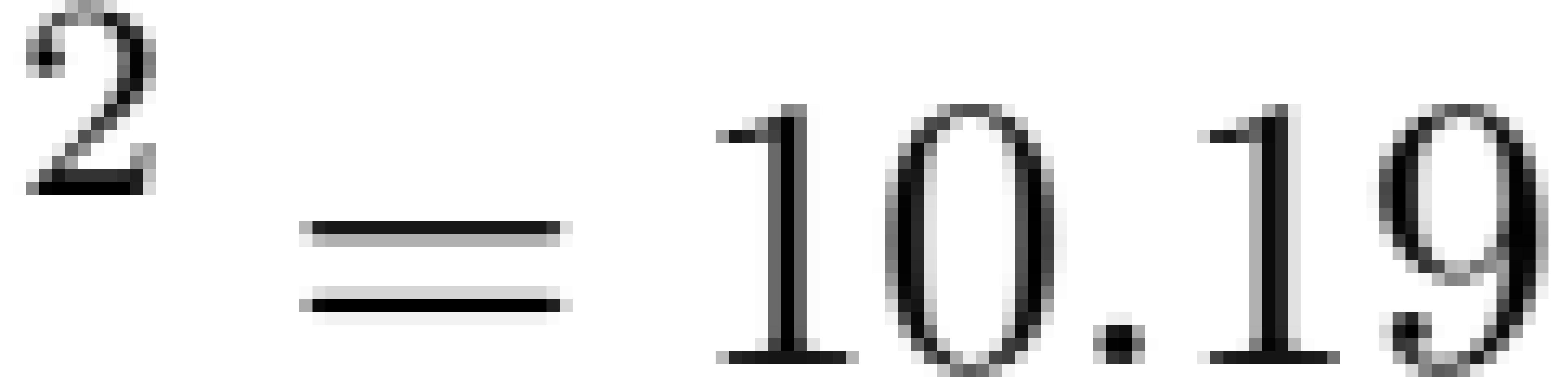
$$\sigma = S - p$$





dP P $\rho_{w,9}$ 1040
 dz





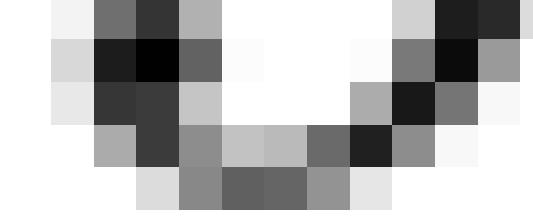
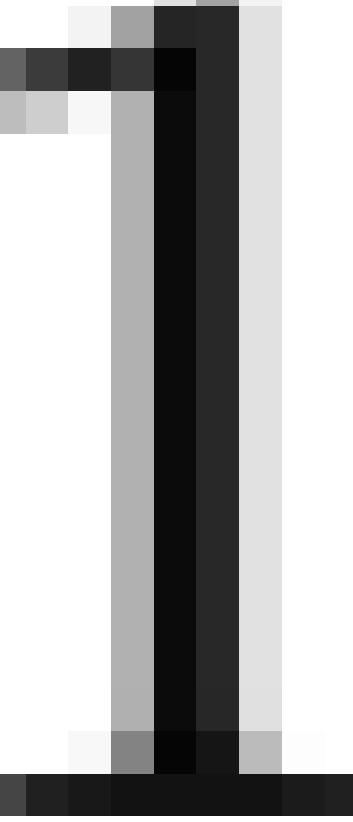
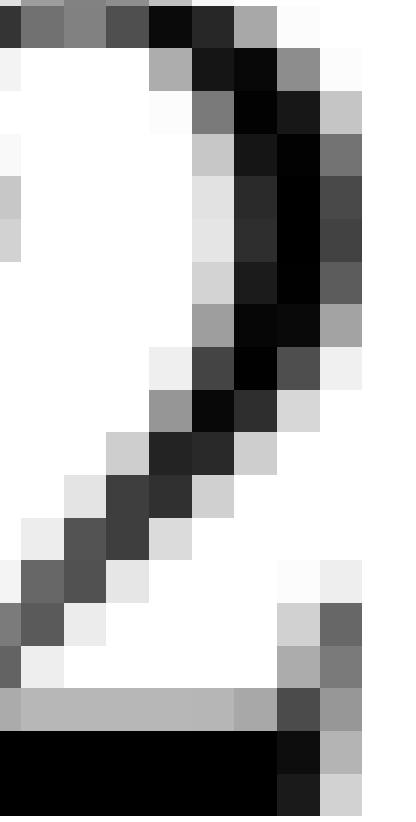
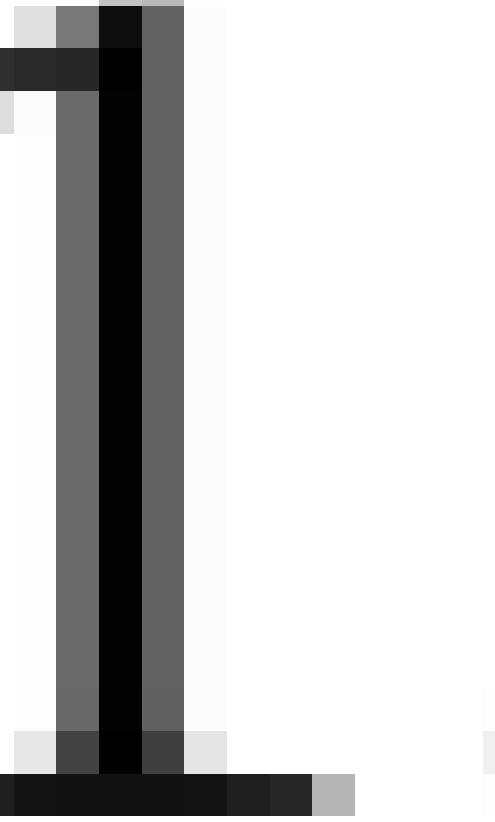
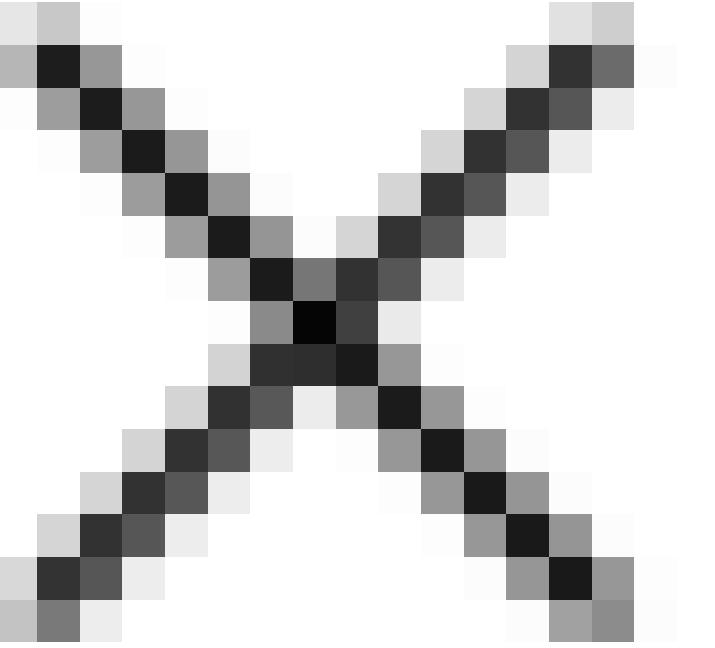
dsu = Paulk 2350

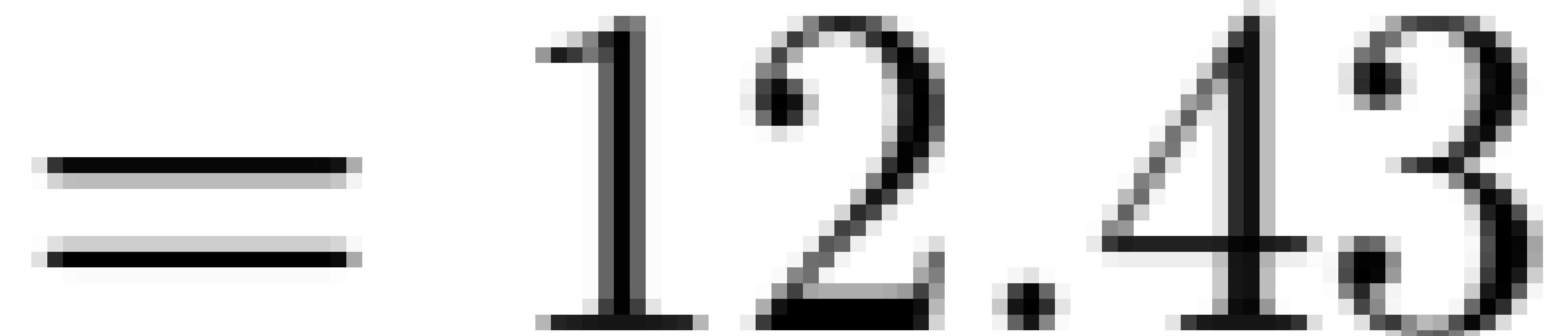


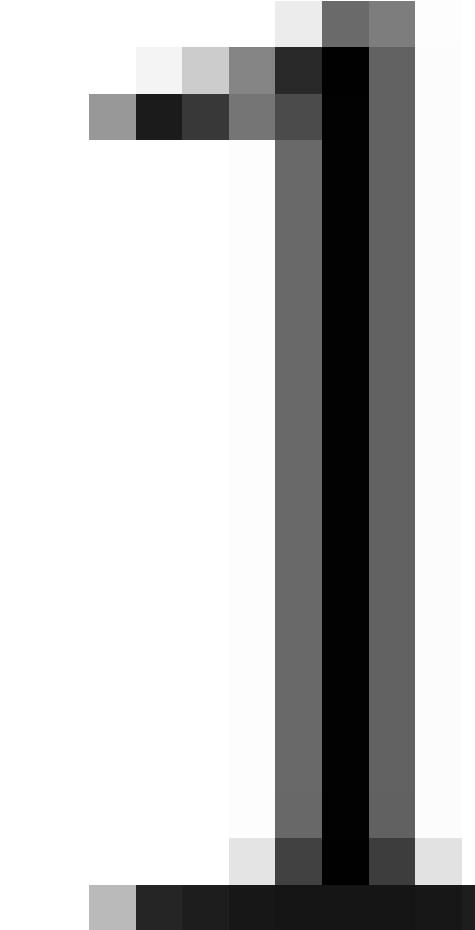
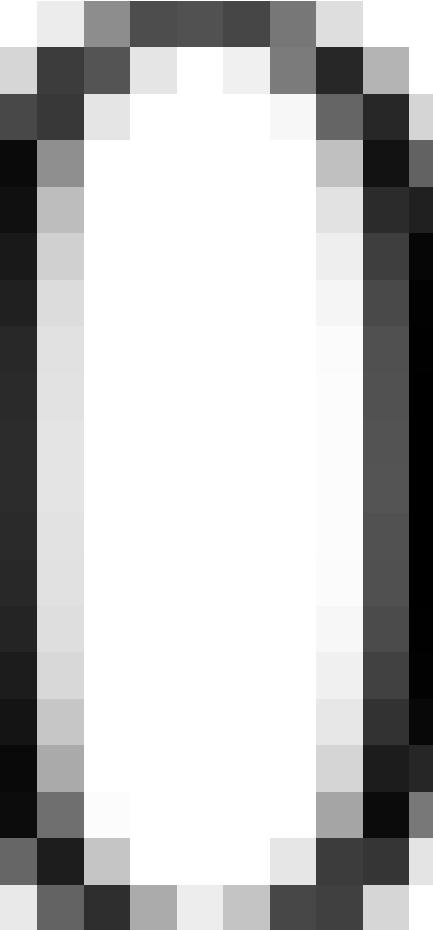
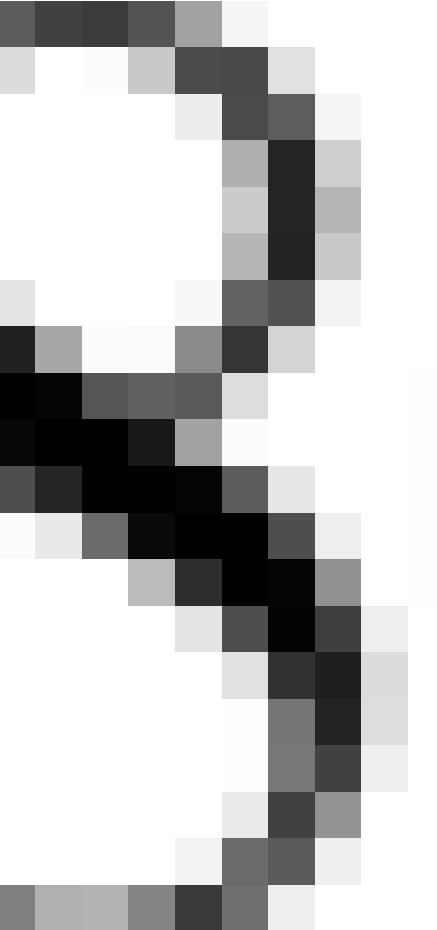
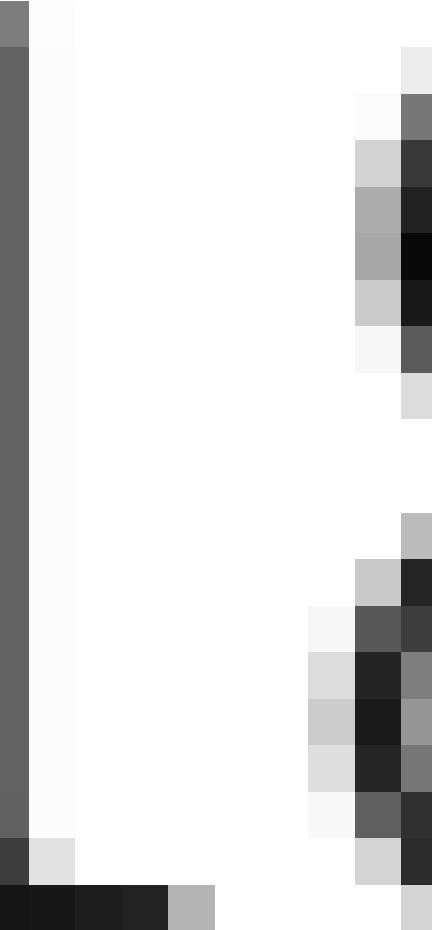
P
P

dP
dP
dZ
dZ

10.19







d_5^s

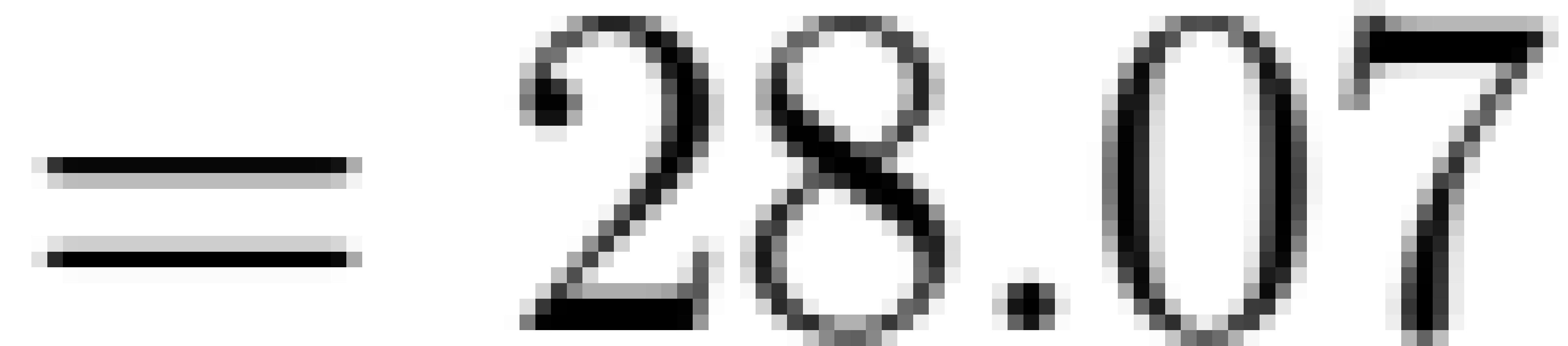


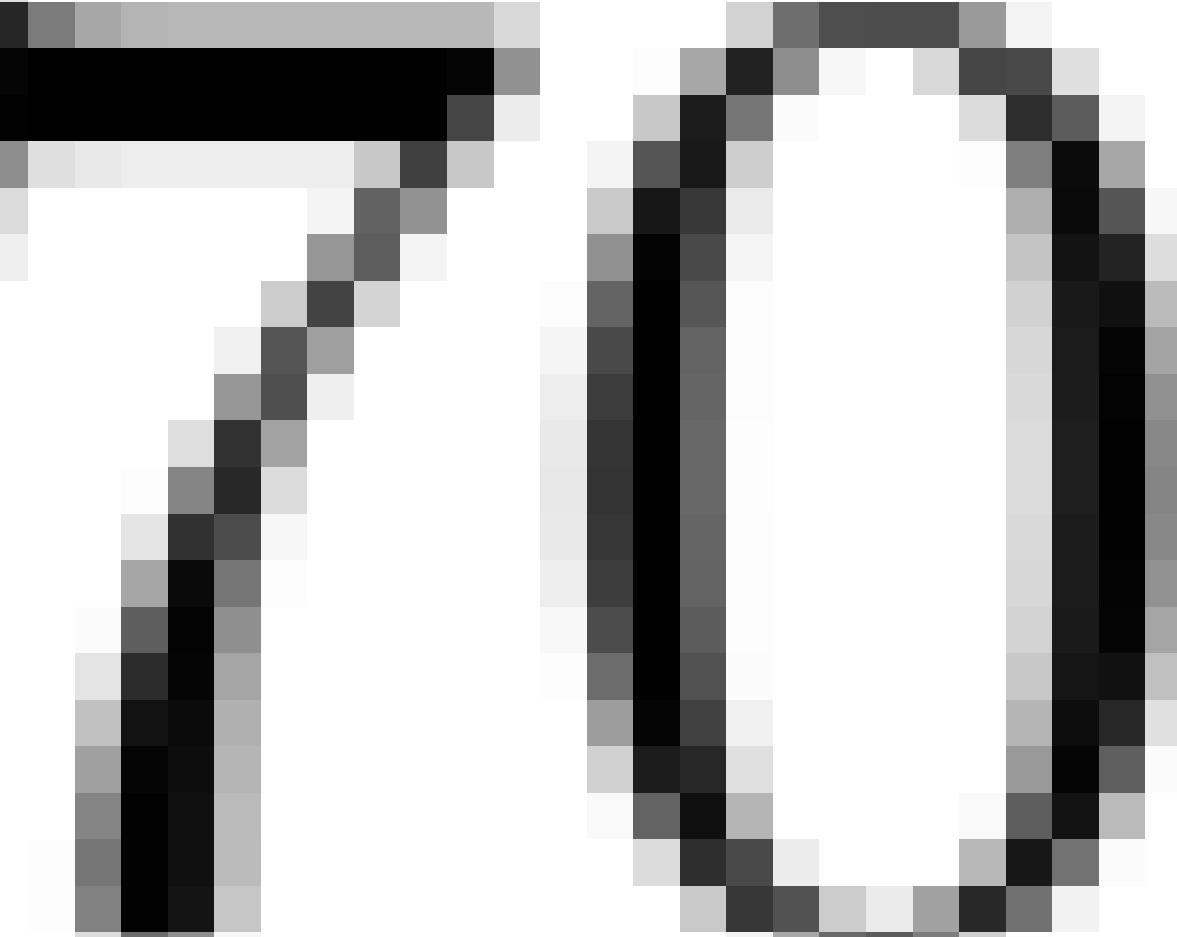
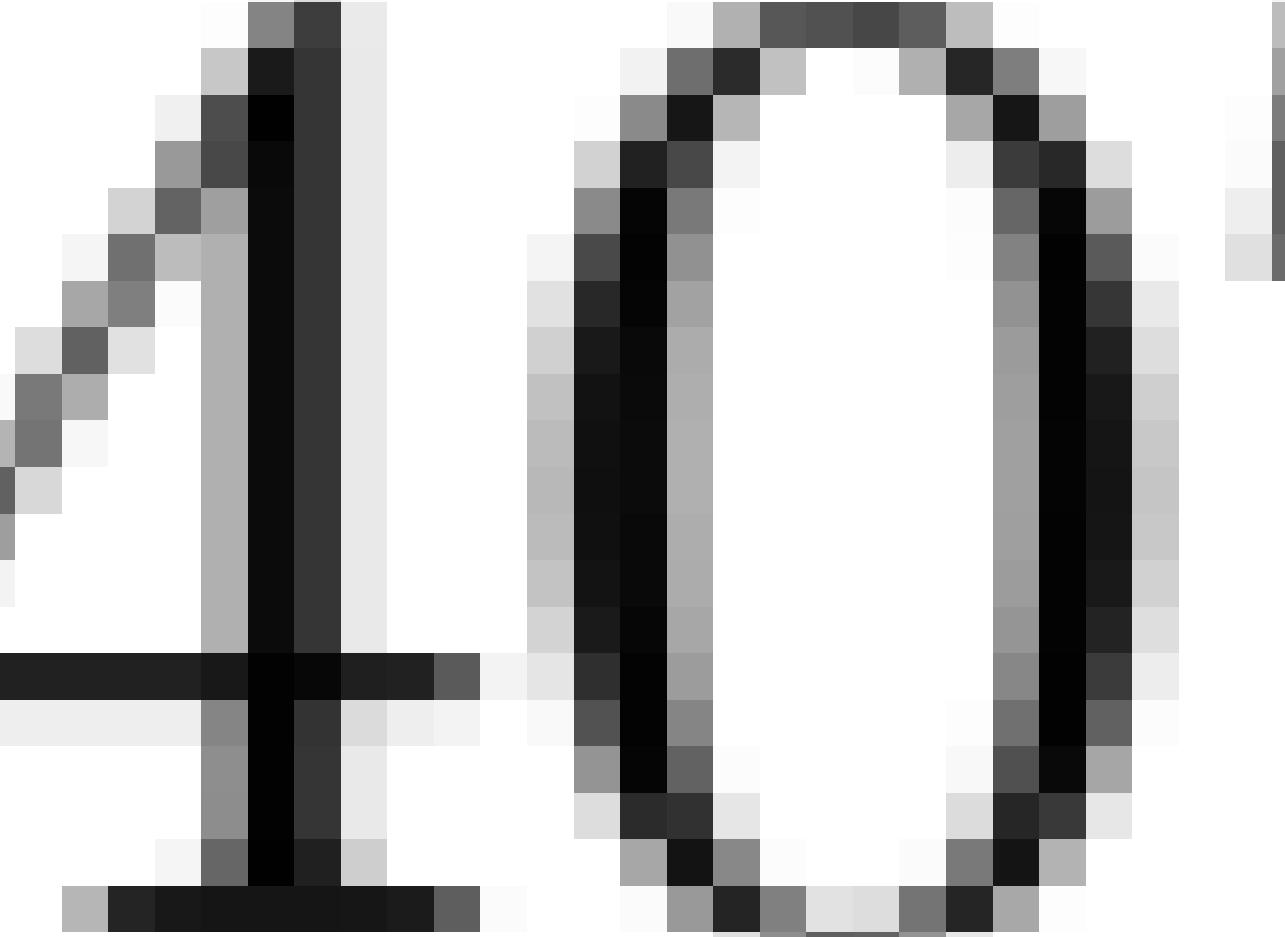
\bar{z}



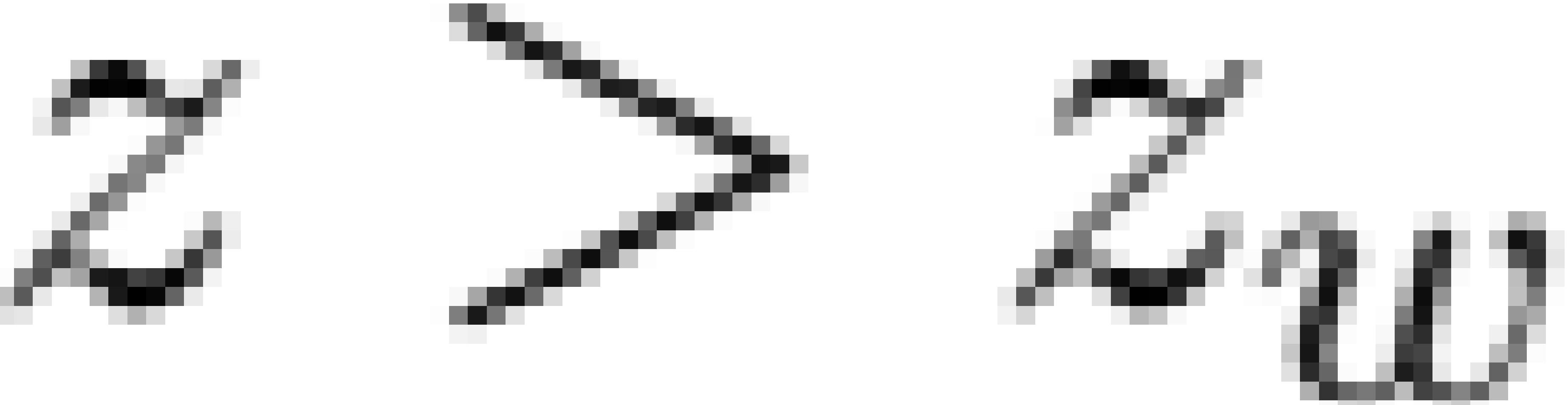
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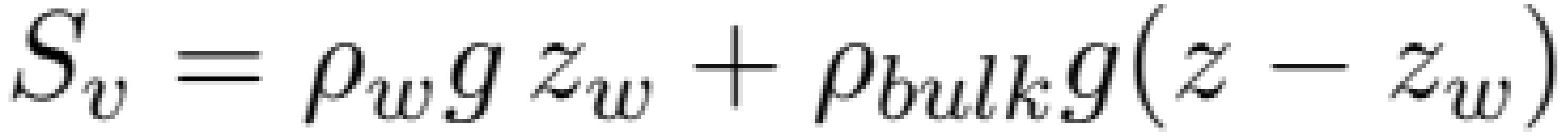
d_2^s

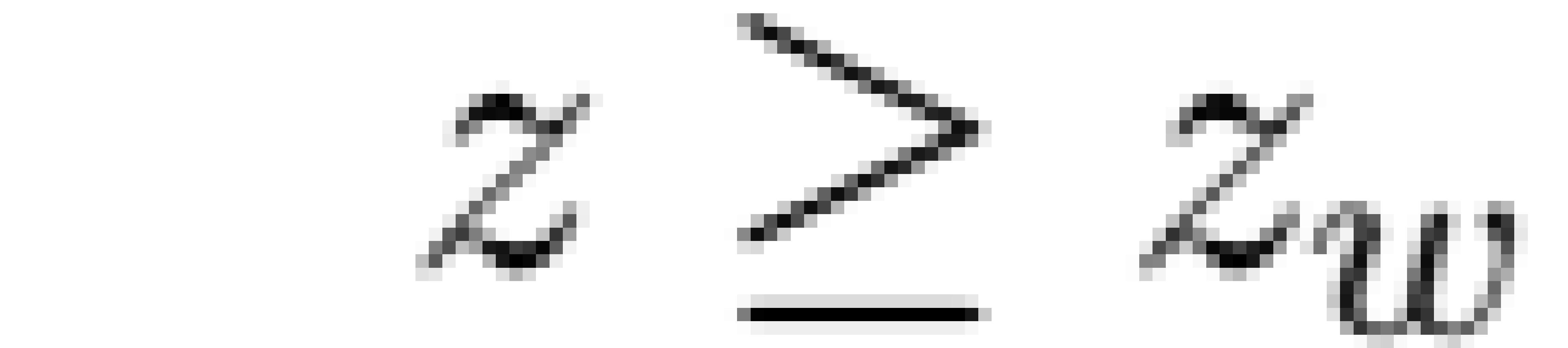


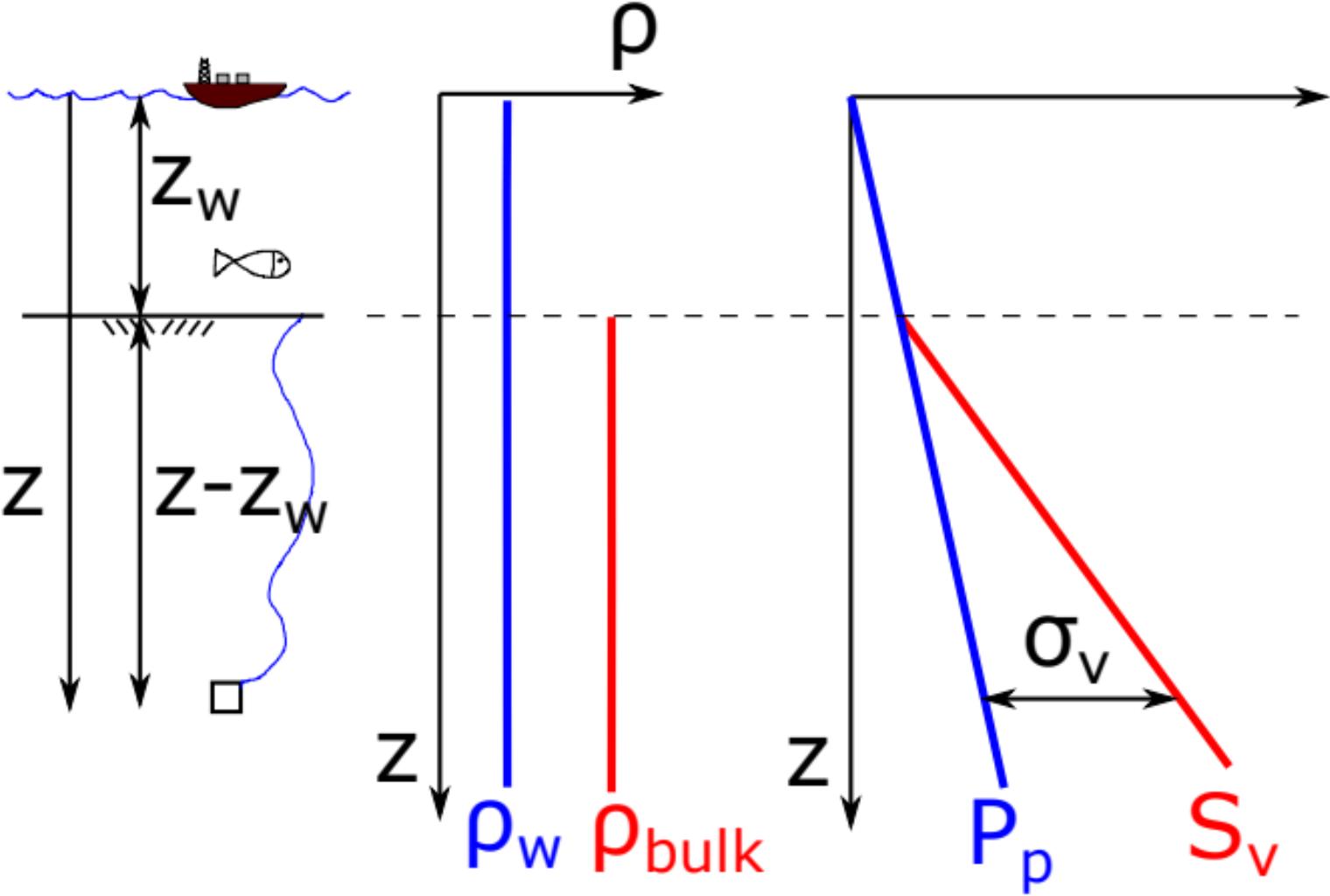




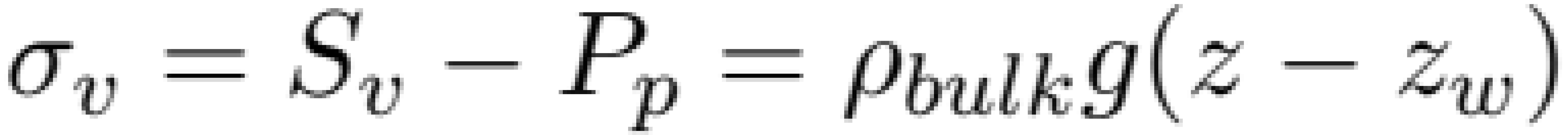




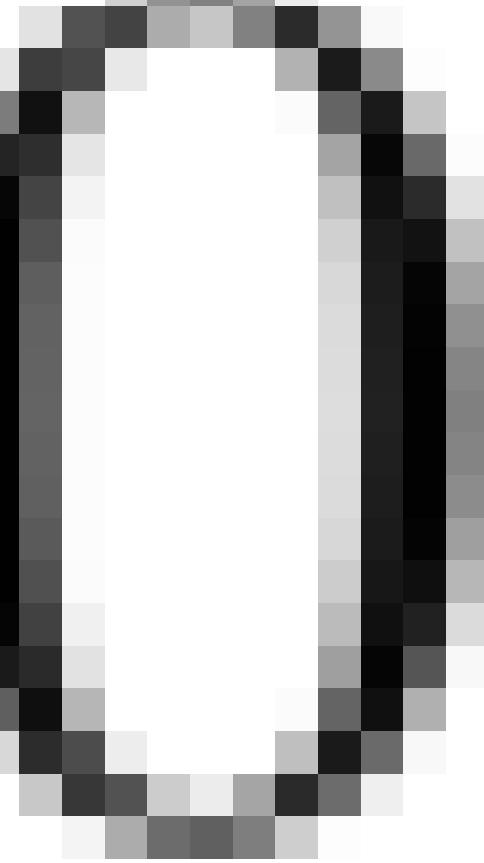
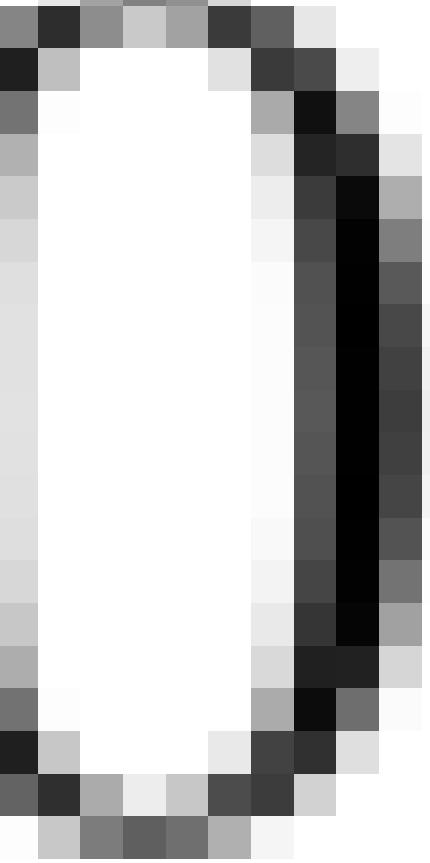
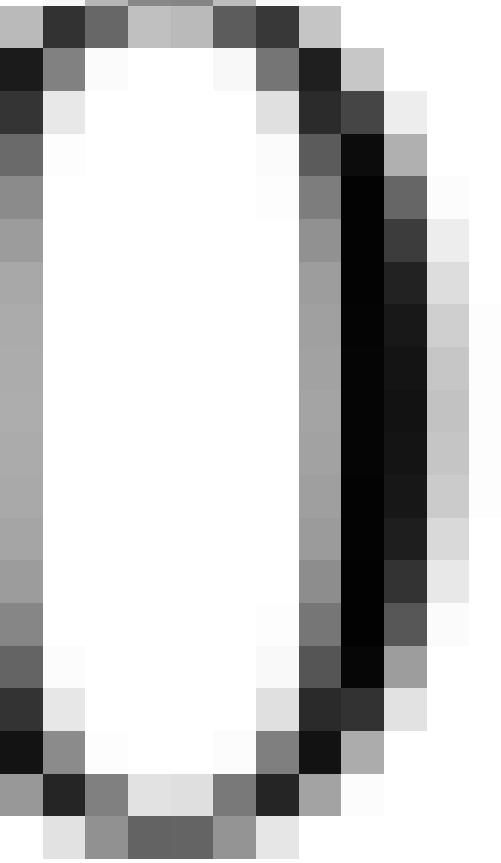
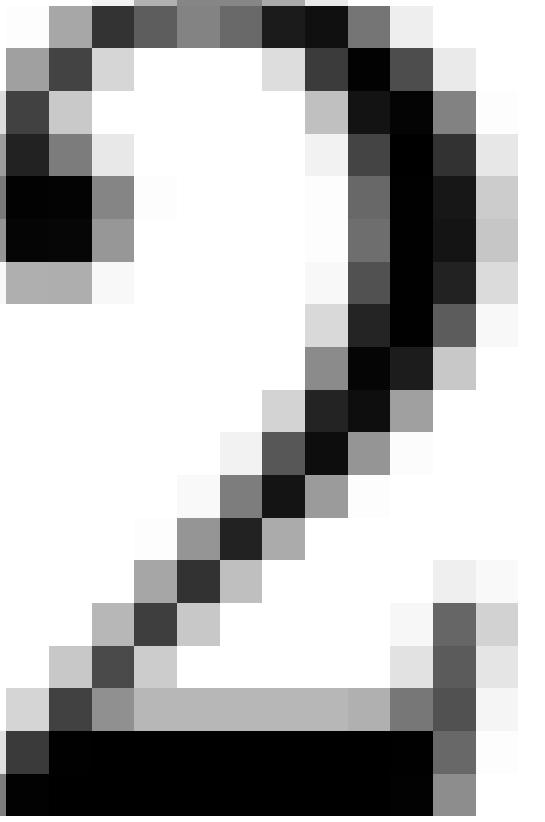
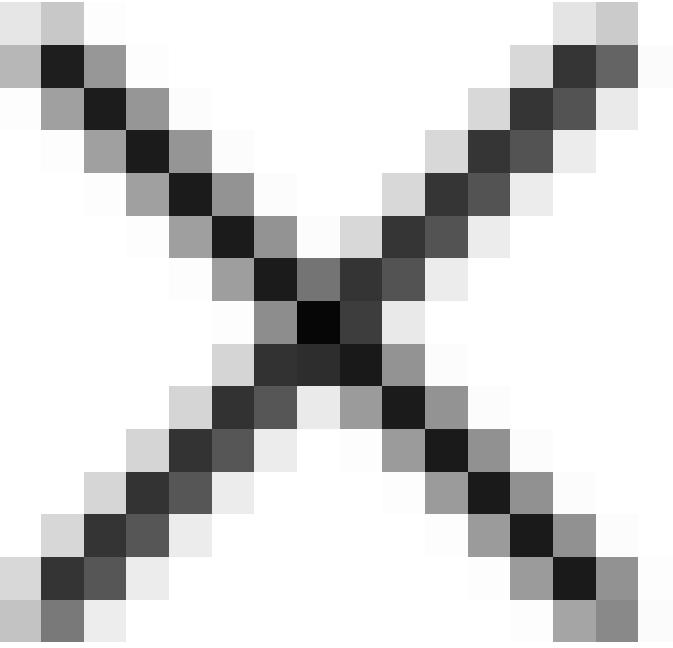


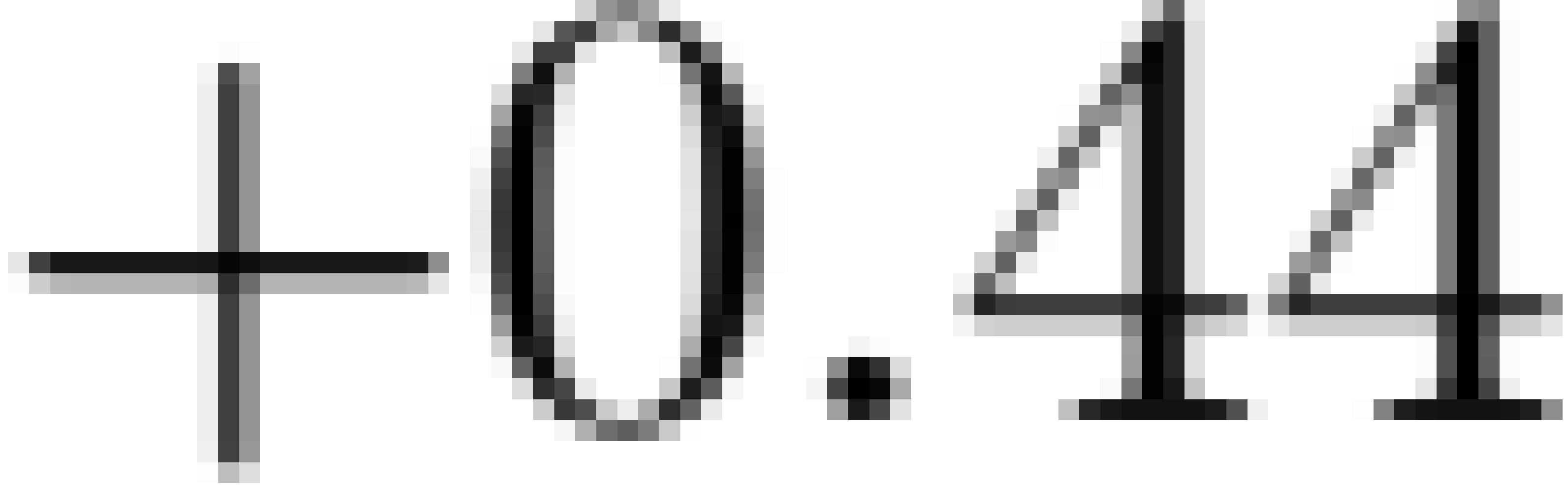


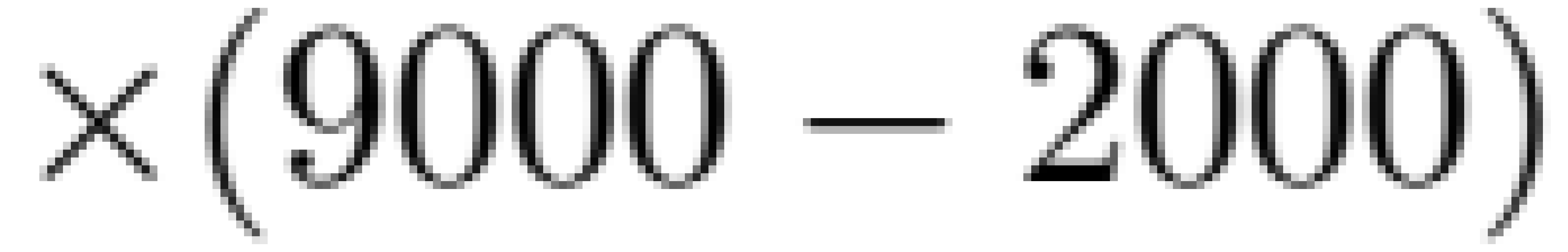


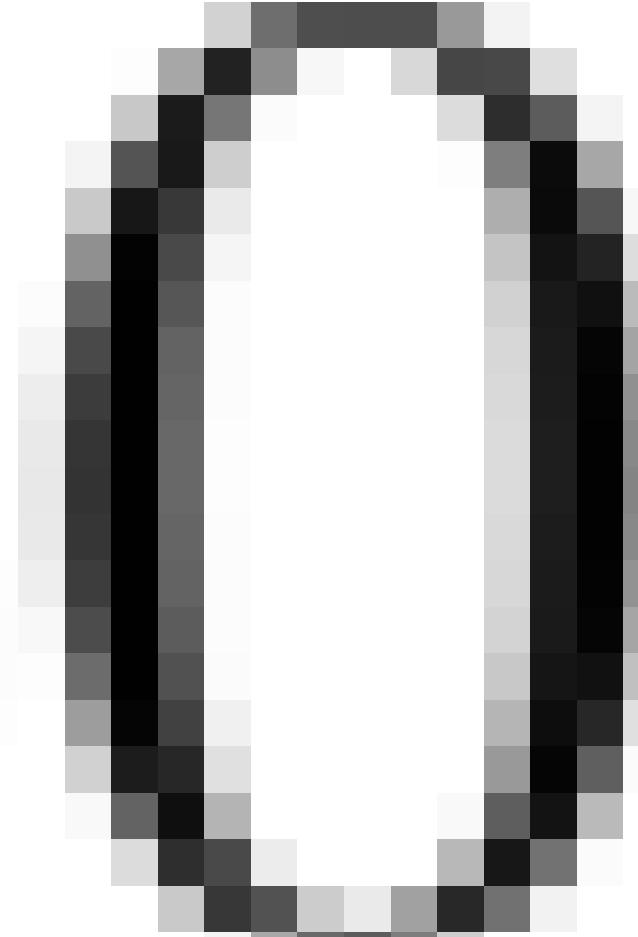
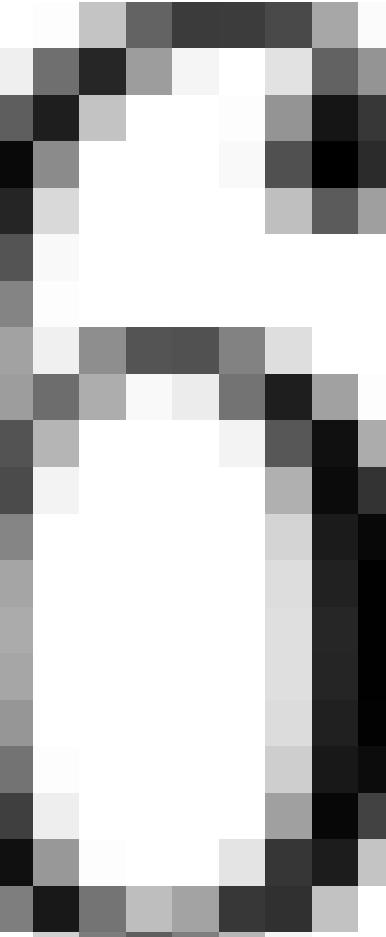
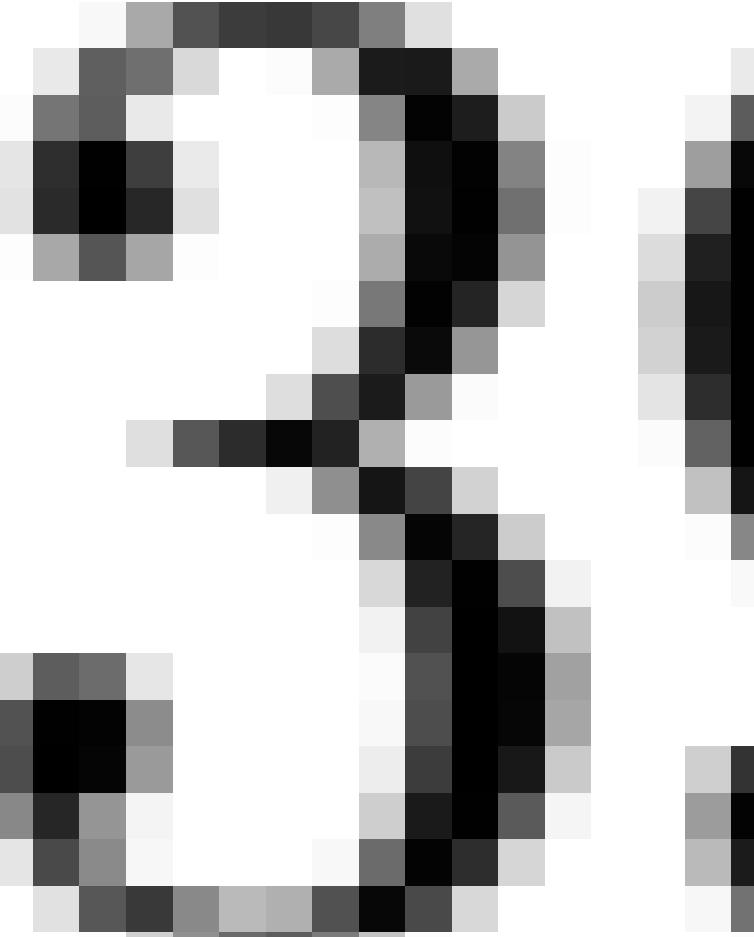


$$P_p = \rho w g z_w + \frac{dP}{dz} (z - z_w) = 0.44$$

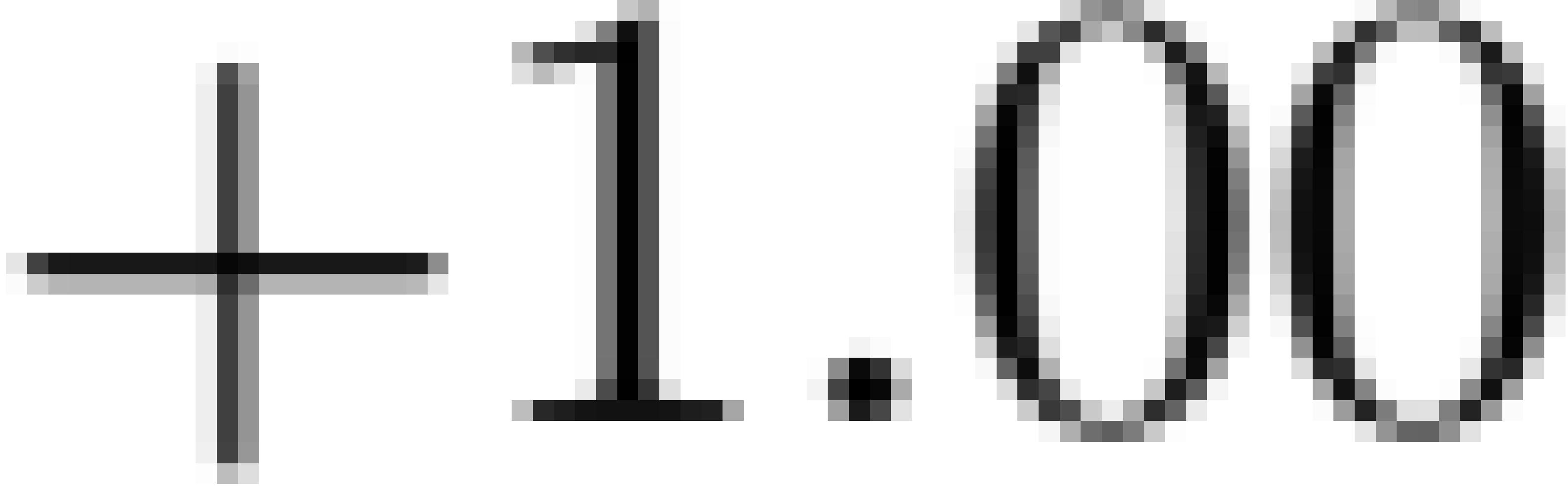


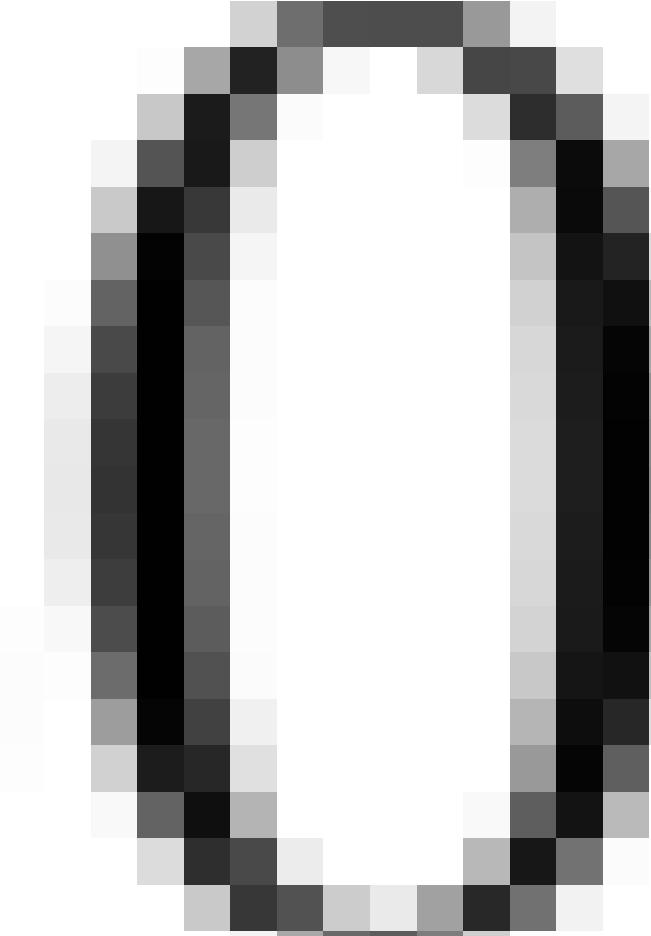
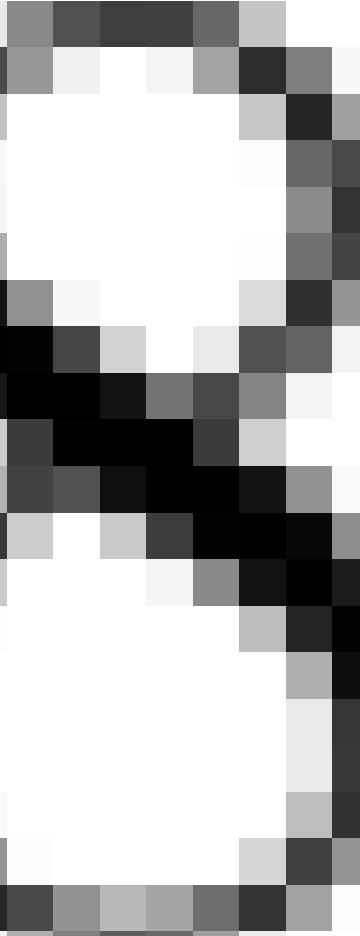
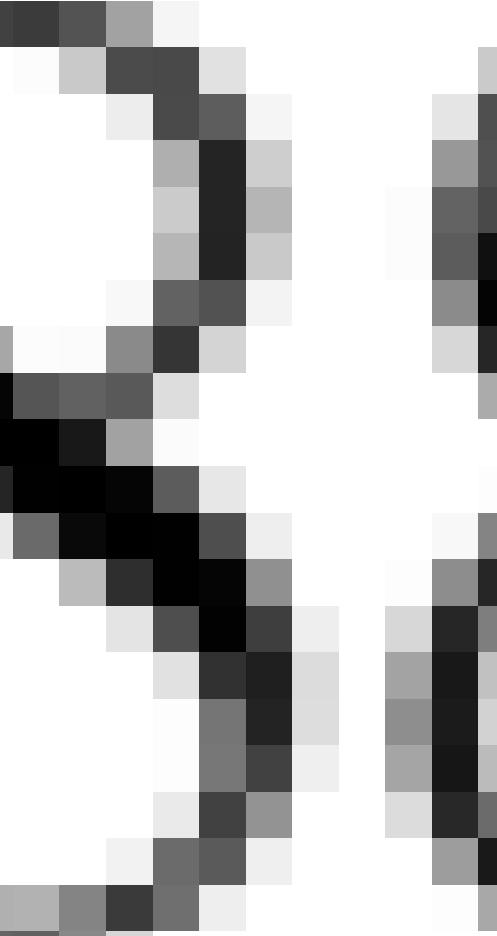


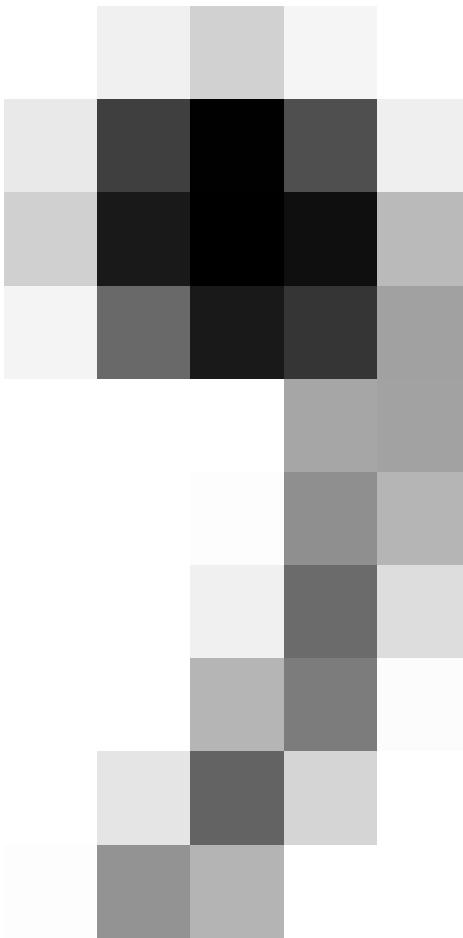


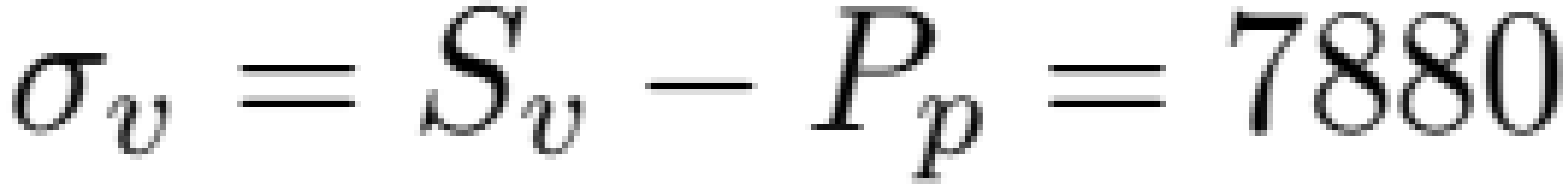


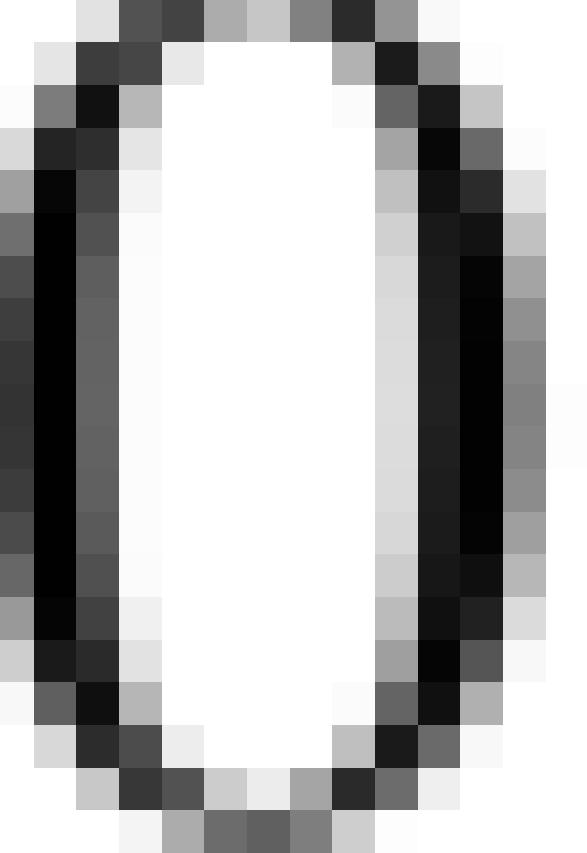
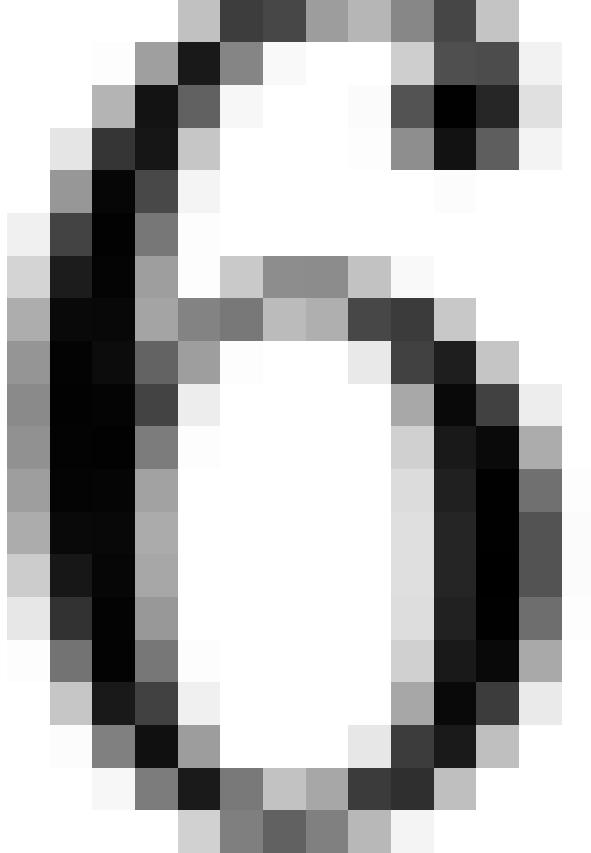
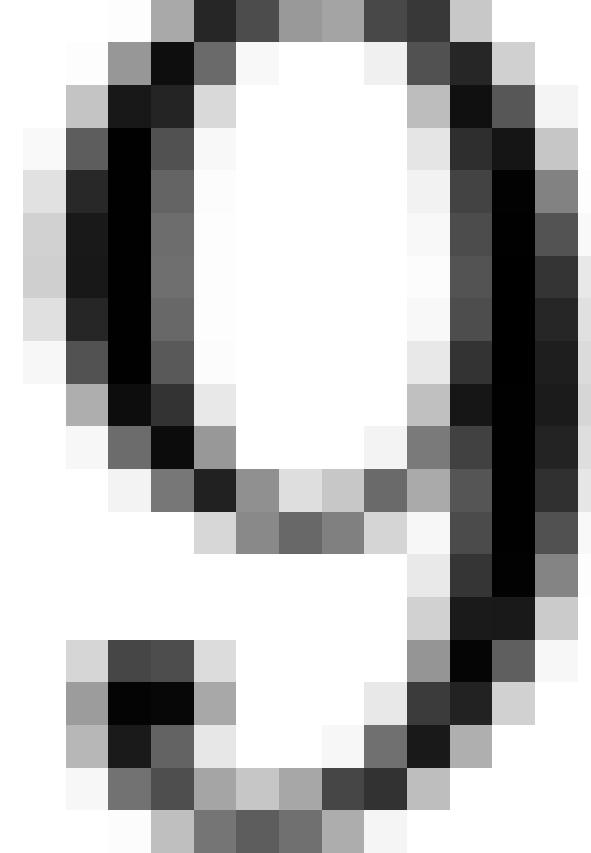
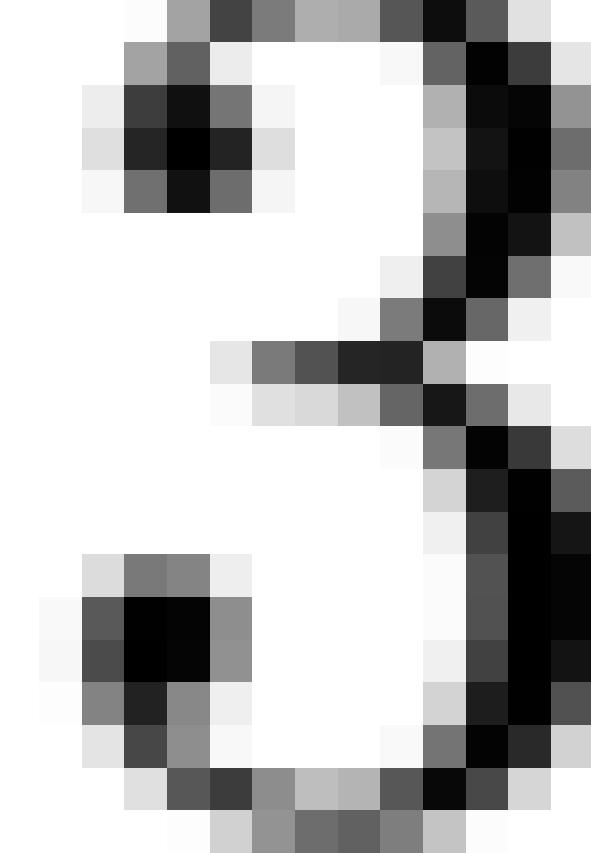
$$S_u = \rho_{u,g} z_u + \frac{dS_u}{dz} (z - z_u) = 0.44$$

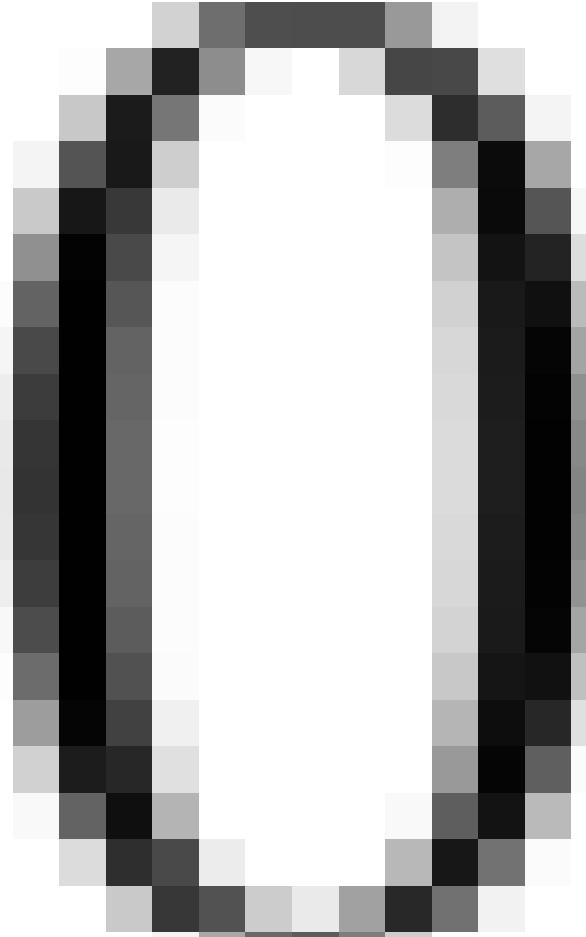
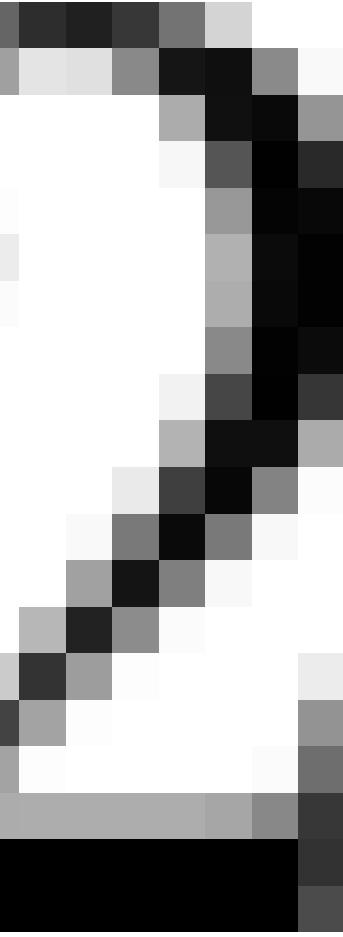
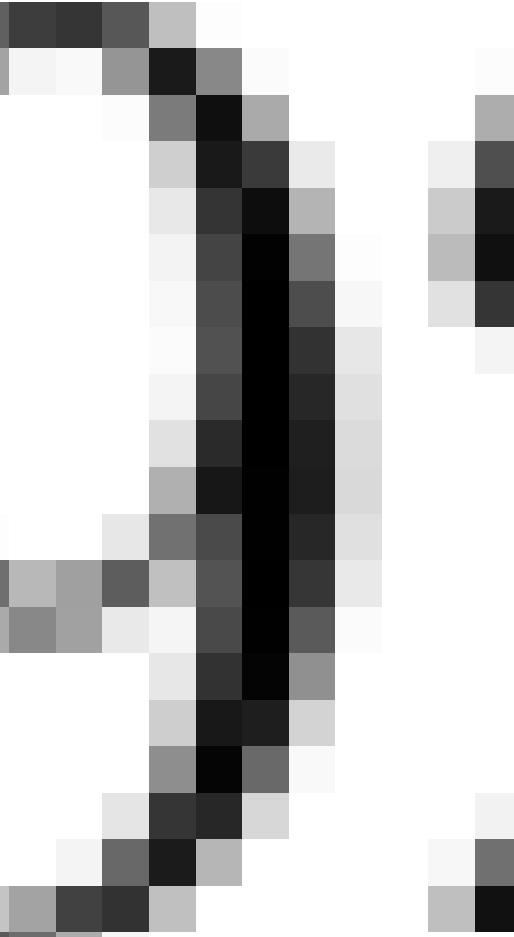
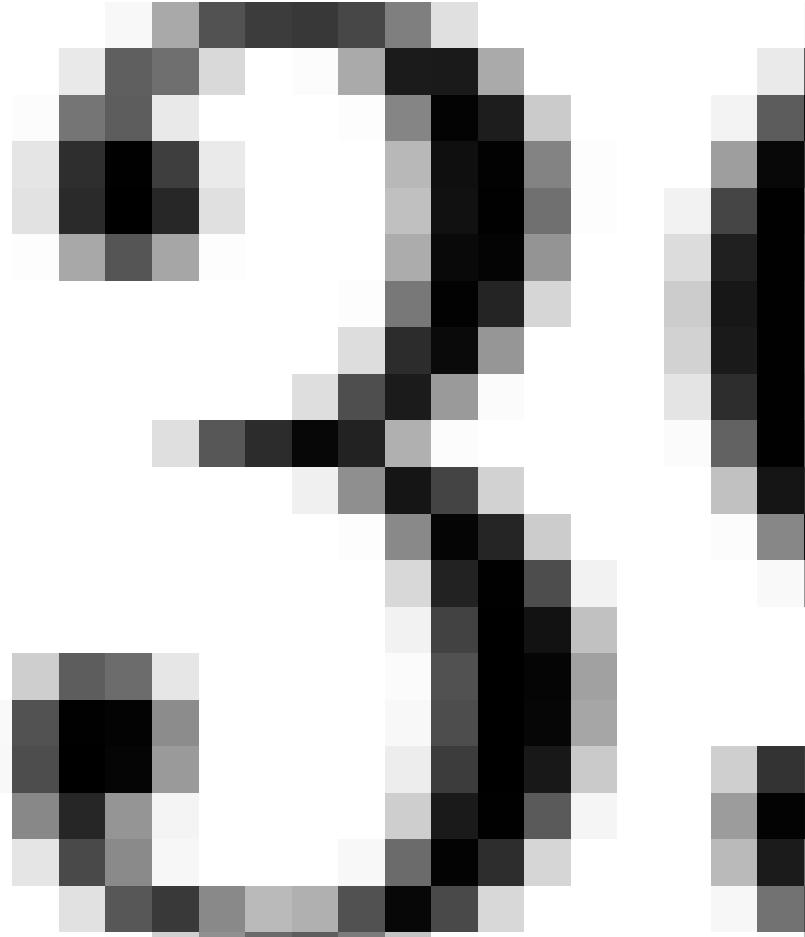


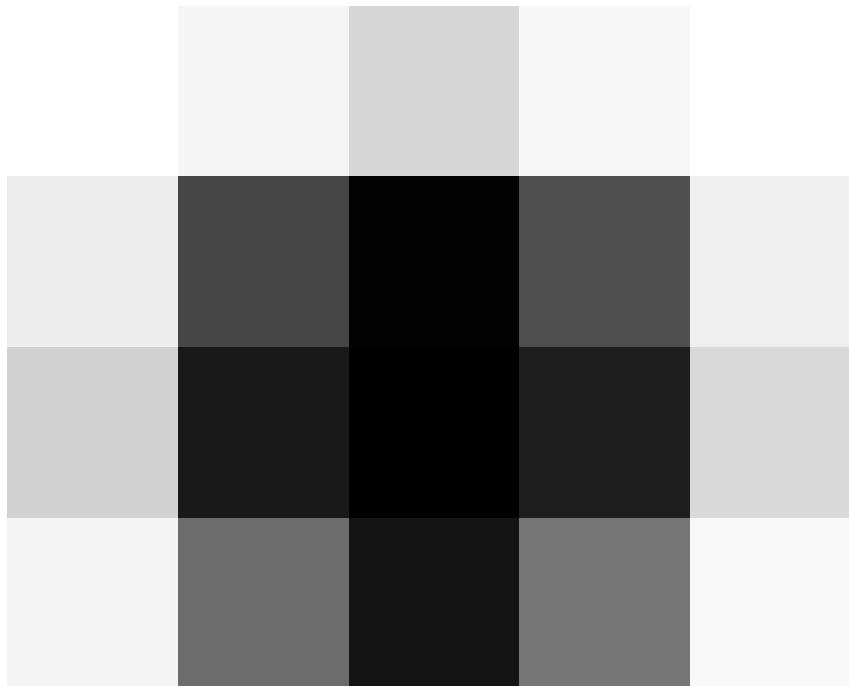


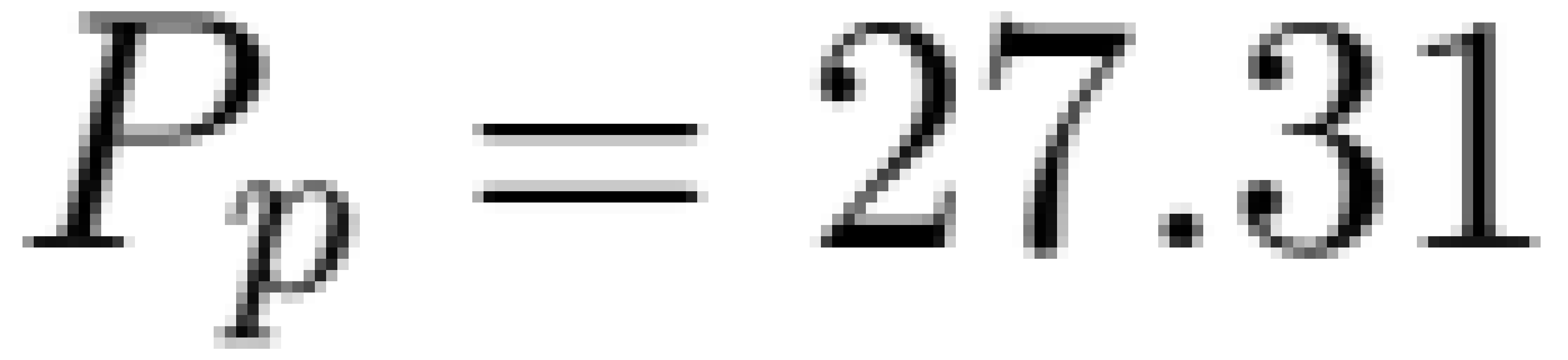






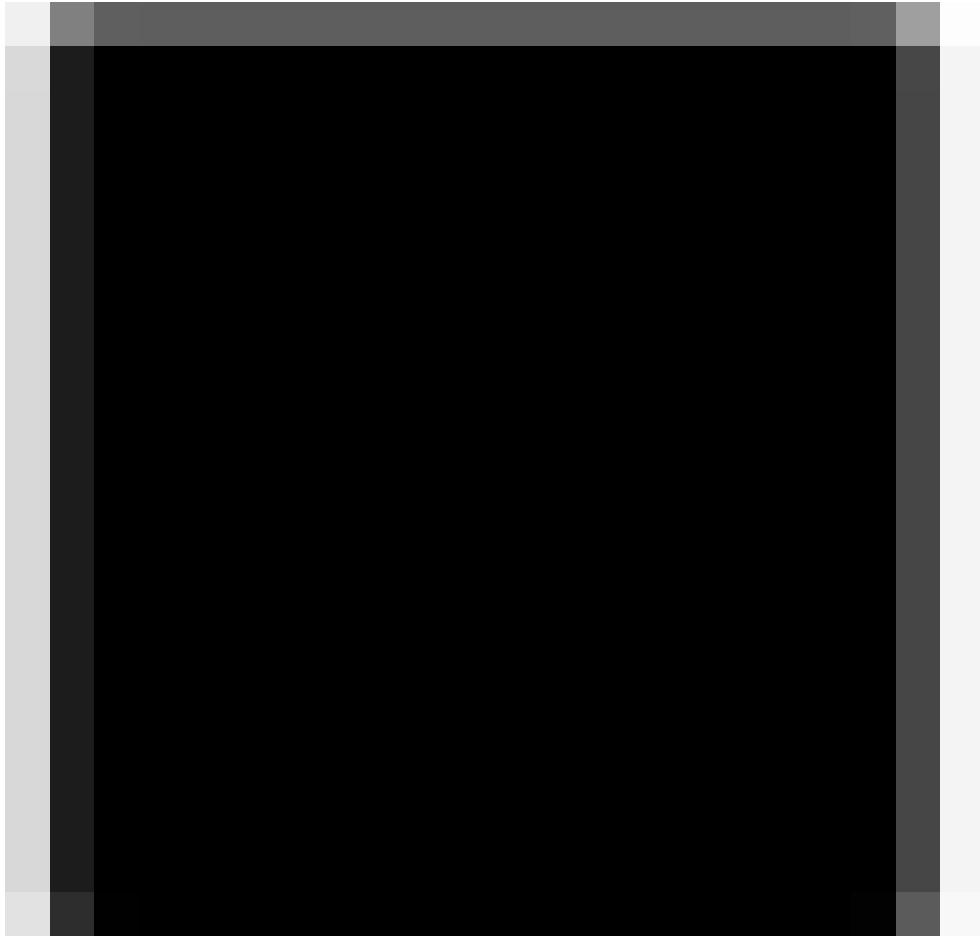








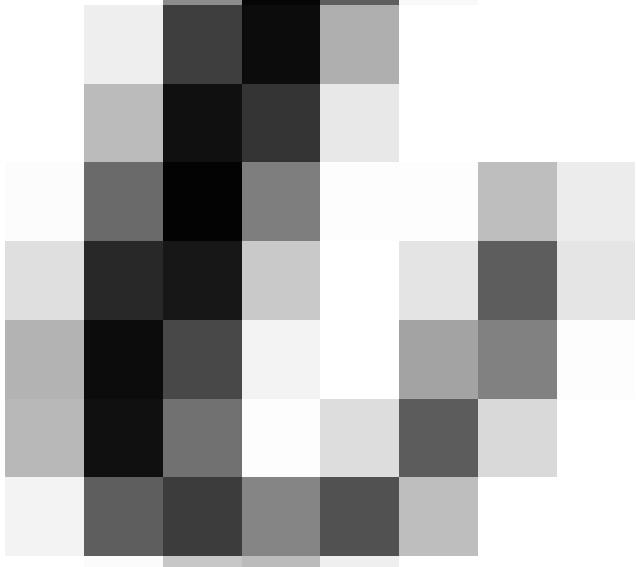
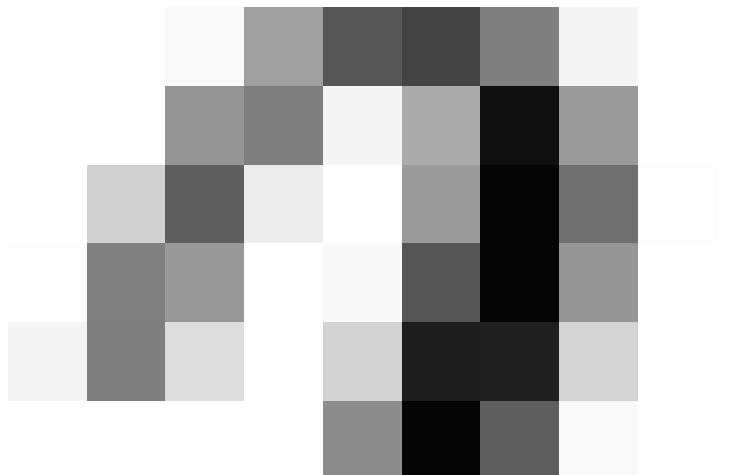
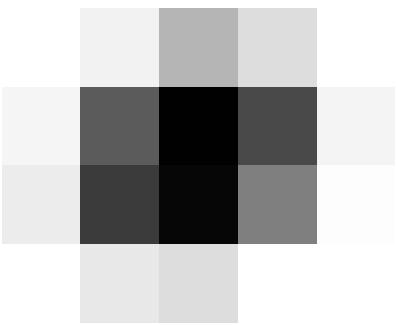


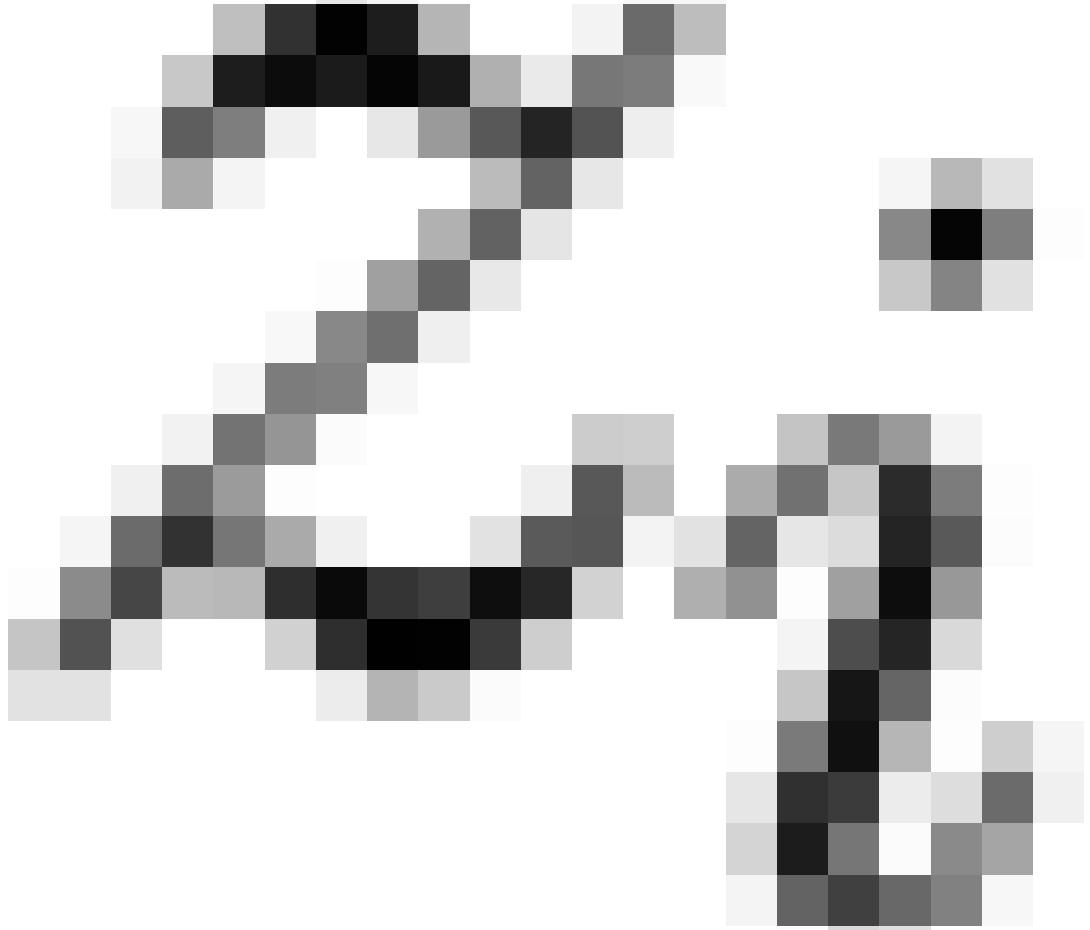


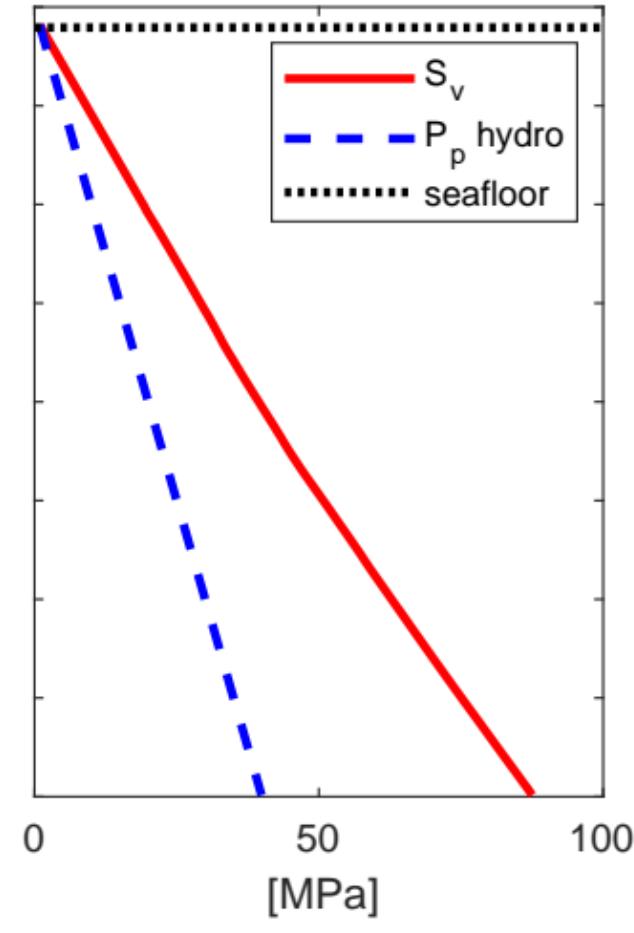
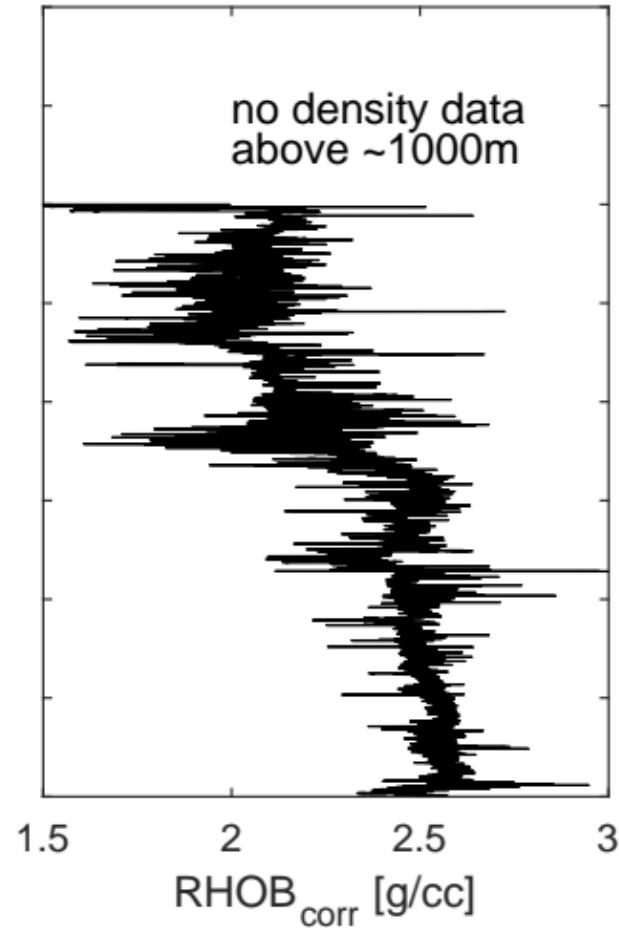
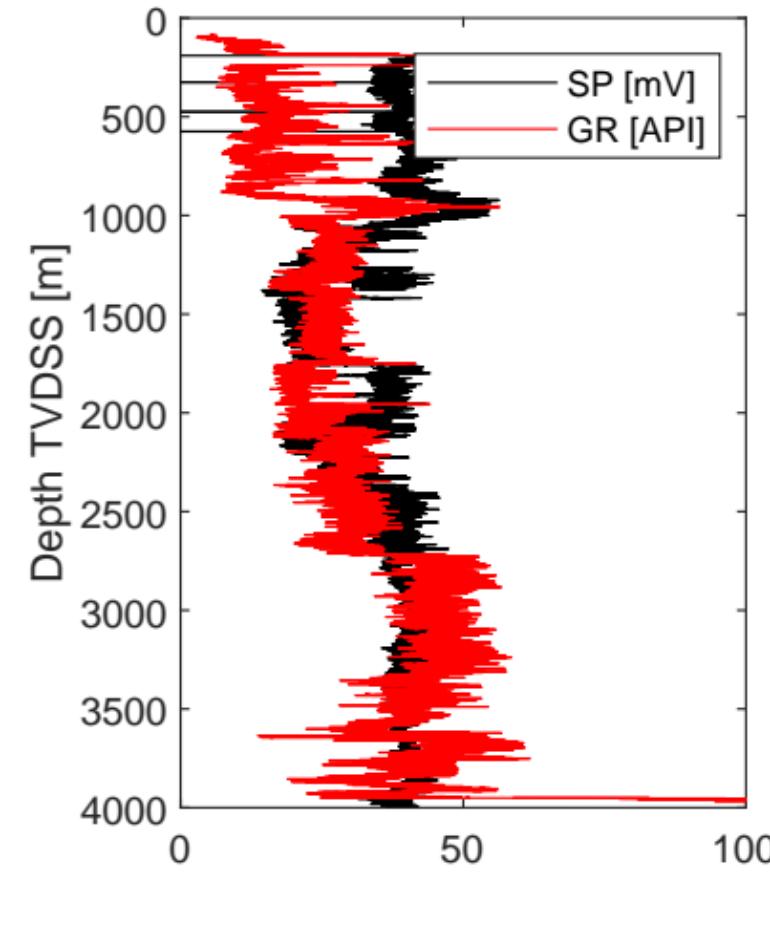
$$S_U(z) = \int_0^z \rho_{\text{bulk}}(z') dz'$$

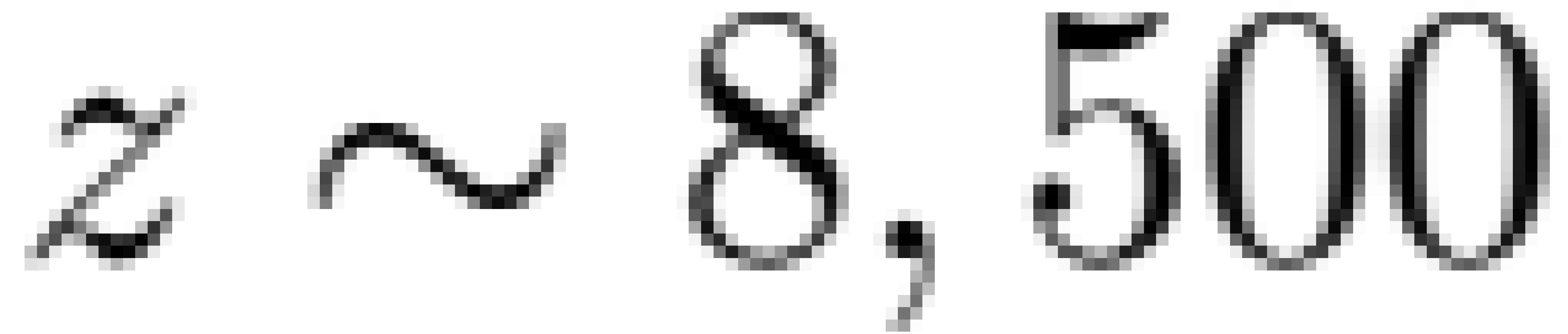


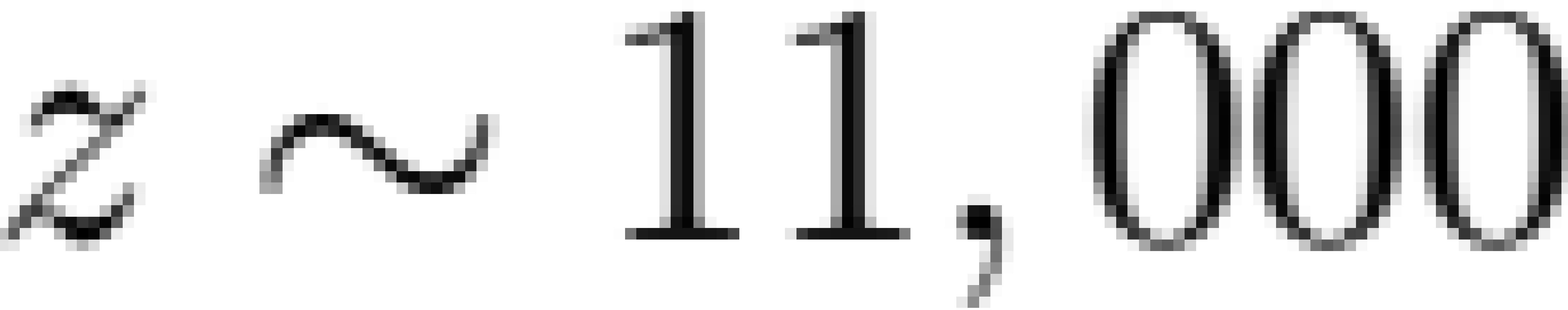
$$S_v(z_i) = \sum_{j=1}^i \rho_{bulk}(z_i) g\Delta z_i$$



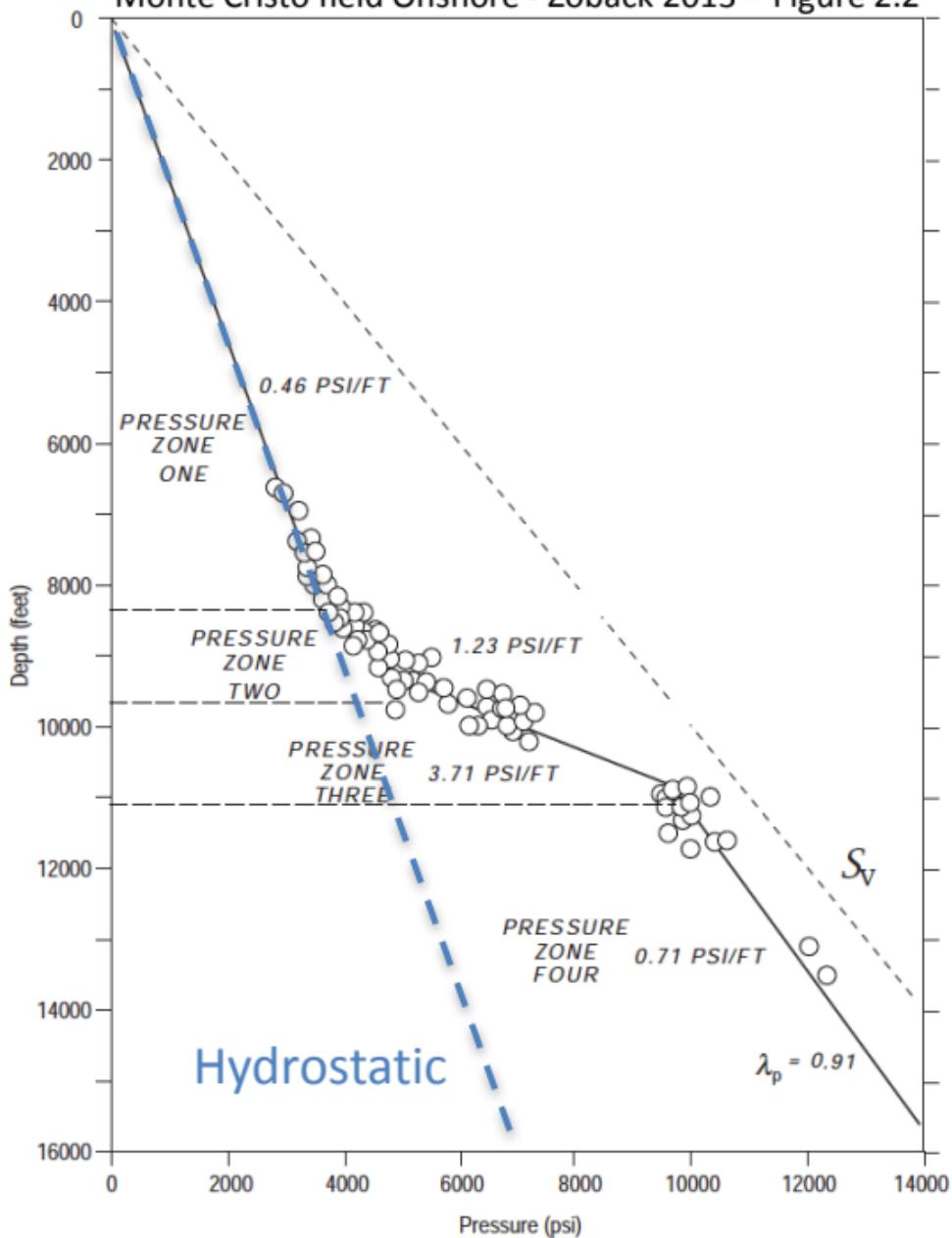








Monte Cristo field Onshore - Zoback 2013 – Figure 2.2

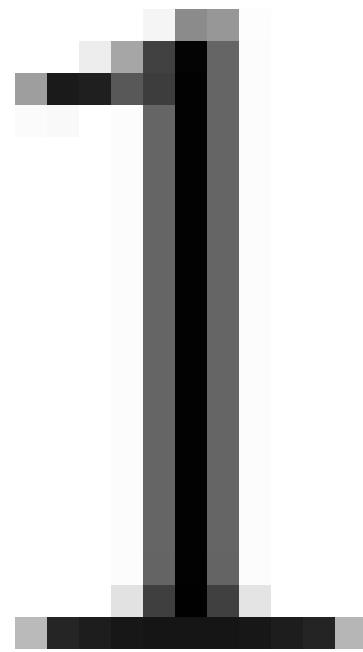
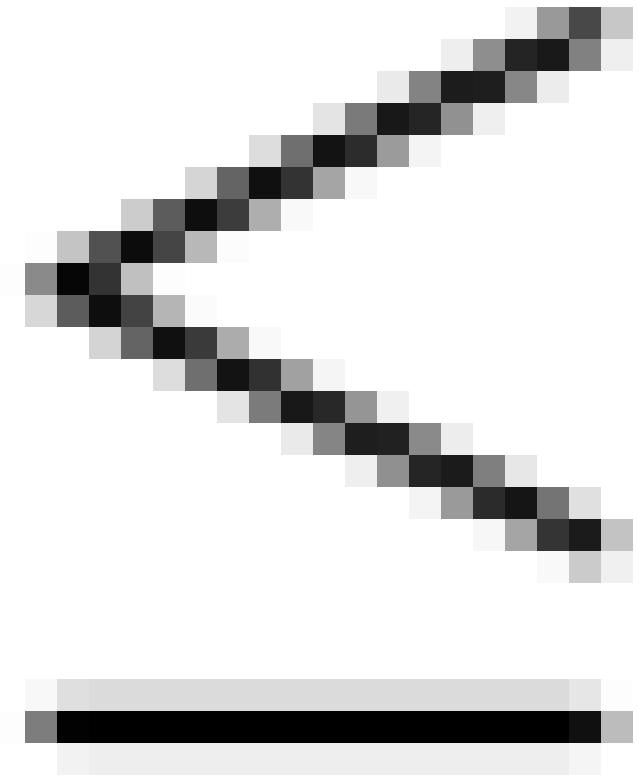


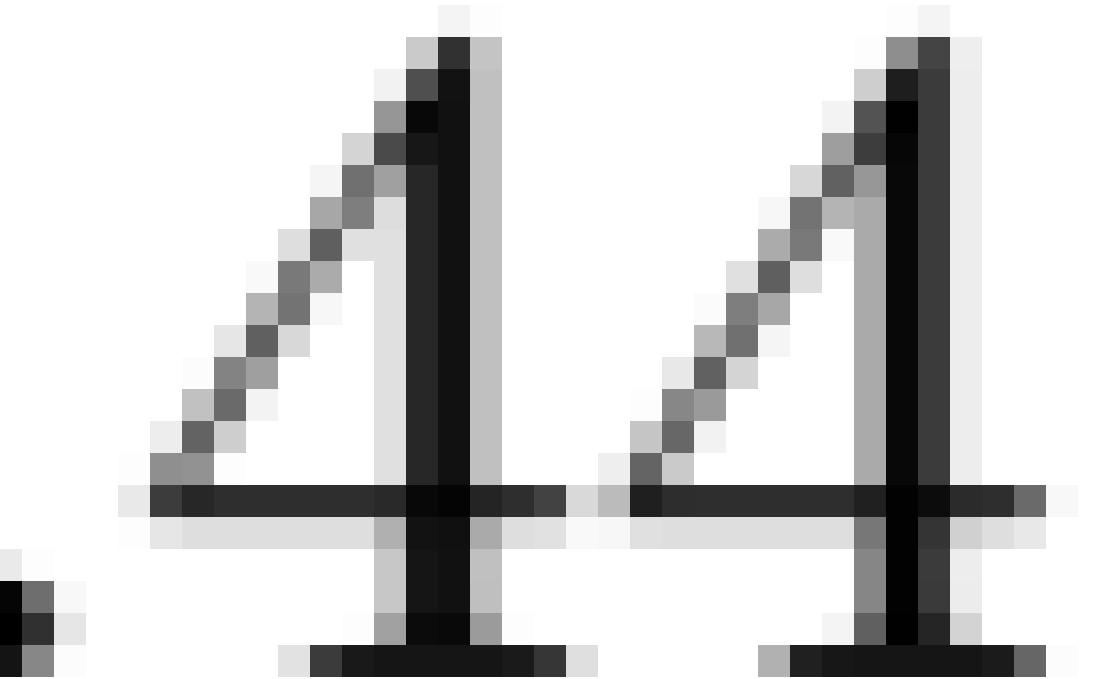
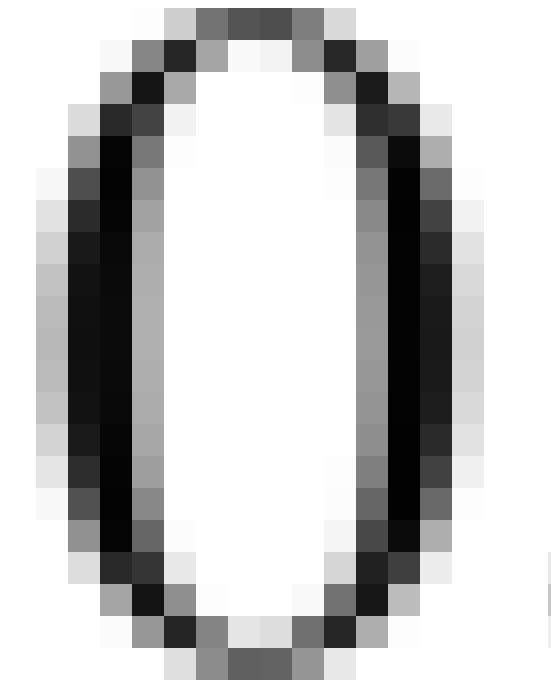
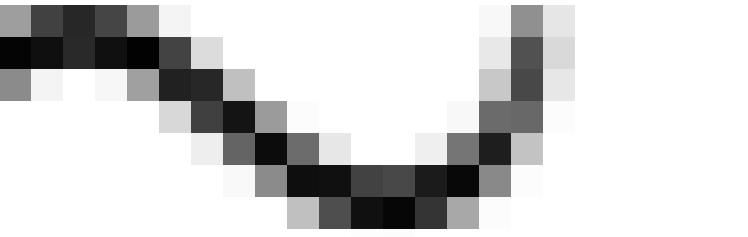


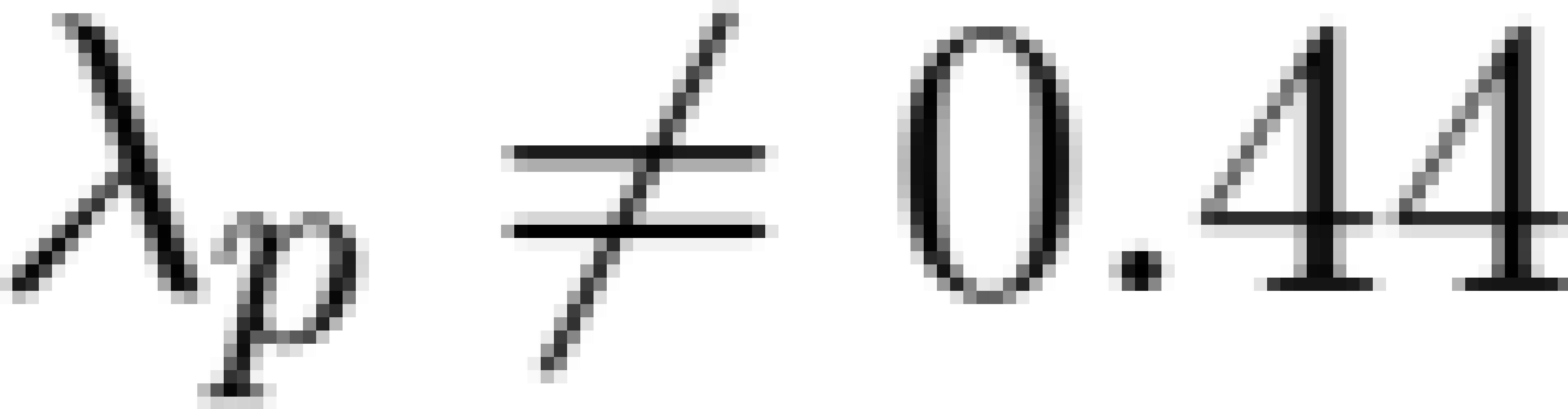
$p(z)$

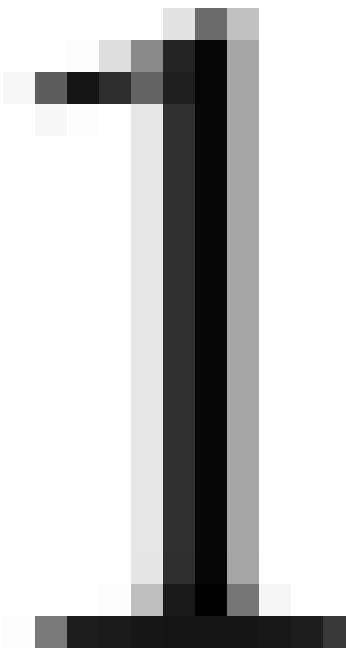
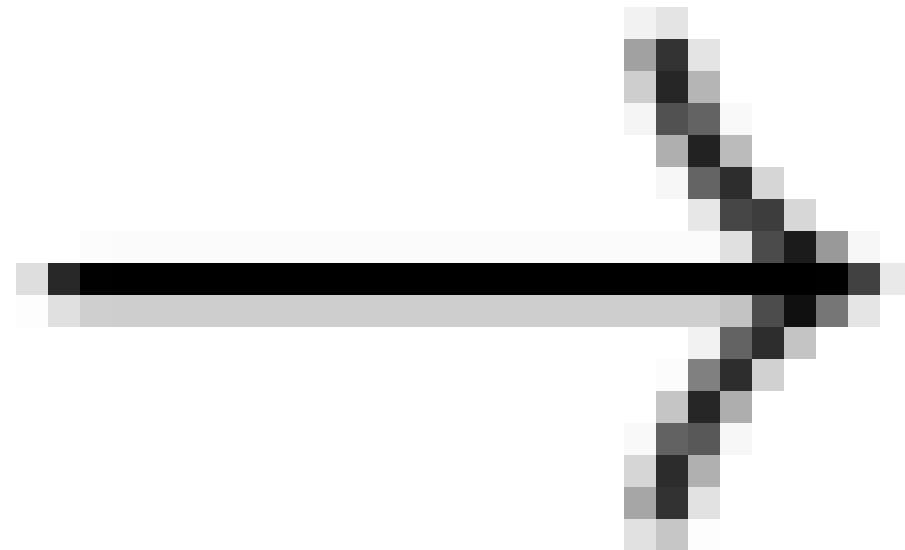


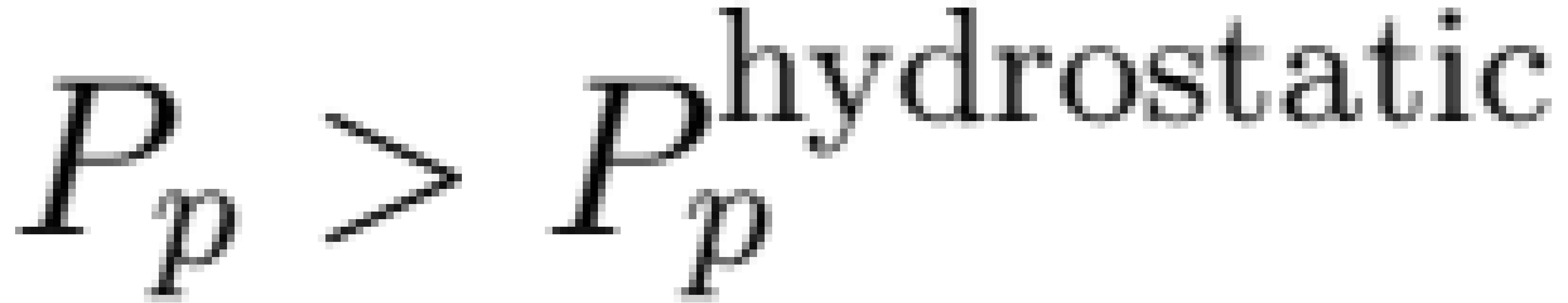
$P_p(z)$
 $S_q(z)$



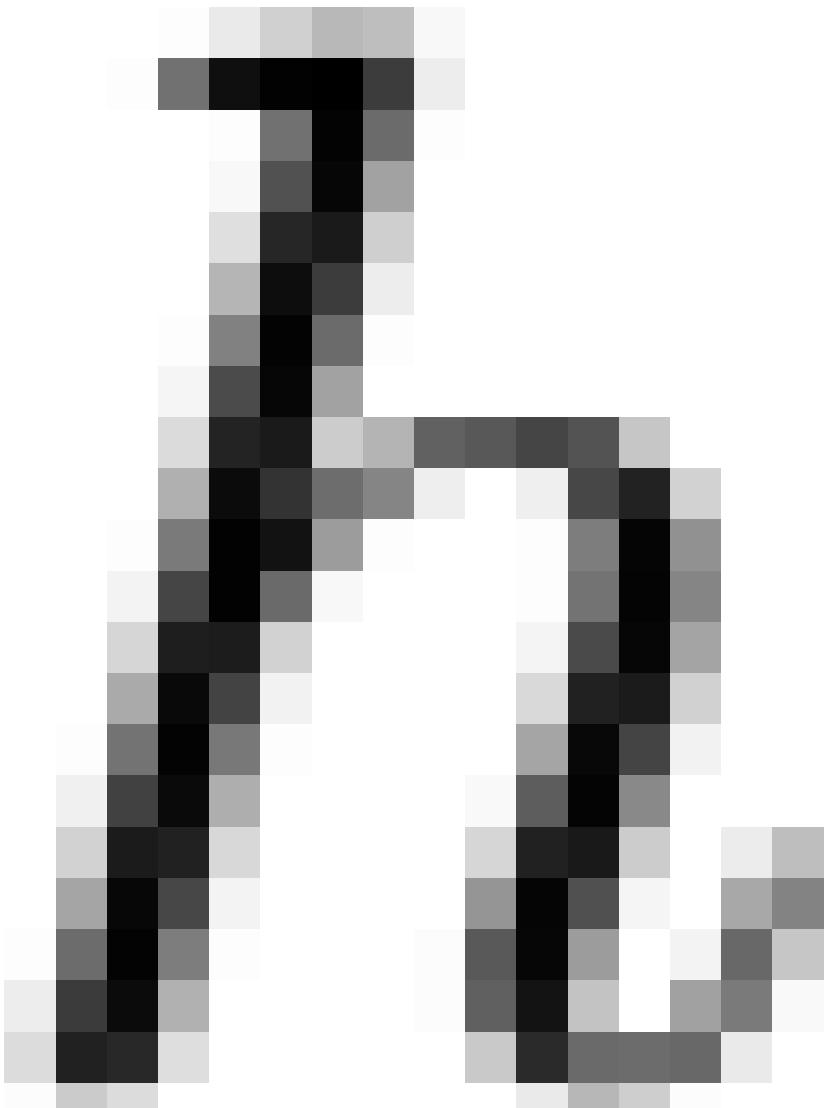














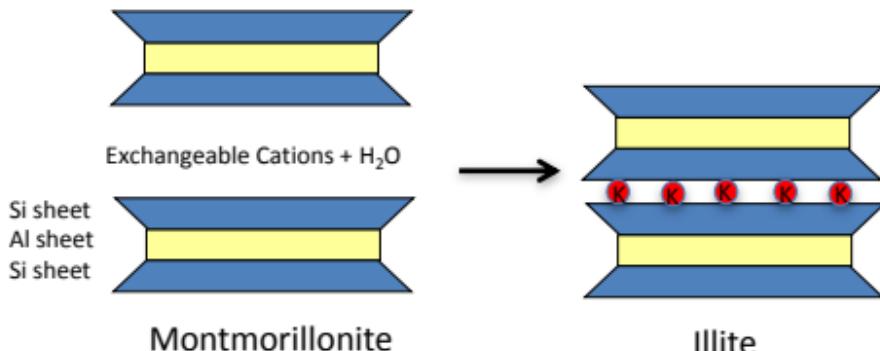


- **Aquathermal pressurization**

- $\Delta T \rightarrow \Delta P$

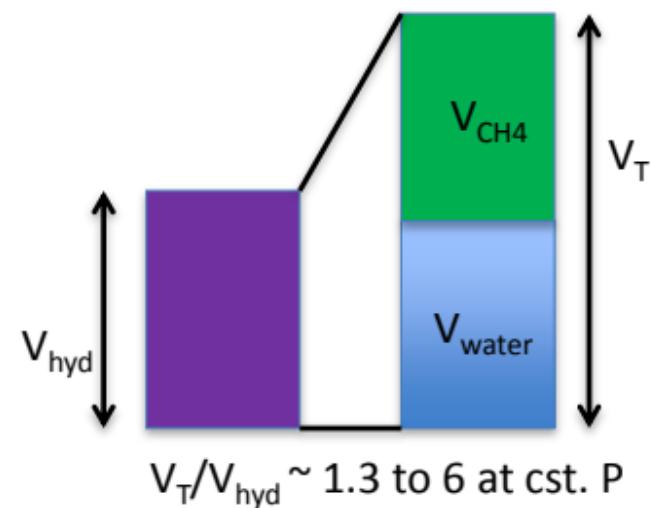
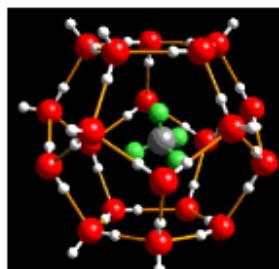
- **Dehydration reactions**

- $\Delta V \rightarrow \Delta P$
 - Montmorillonite to Illite (frees water)

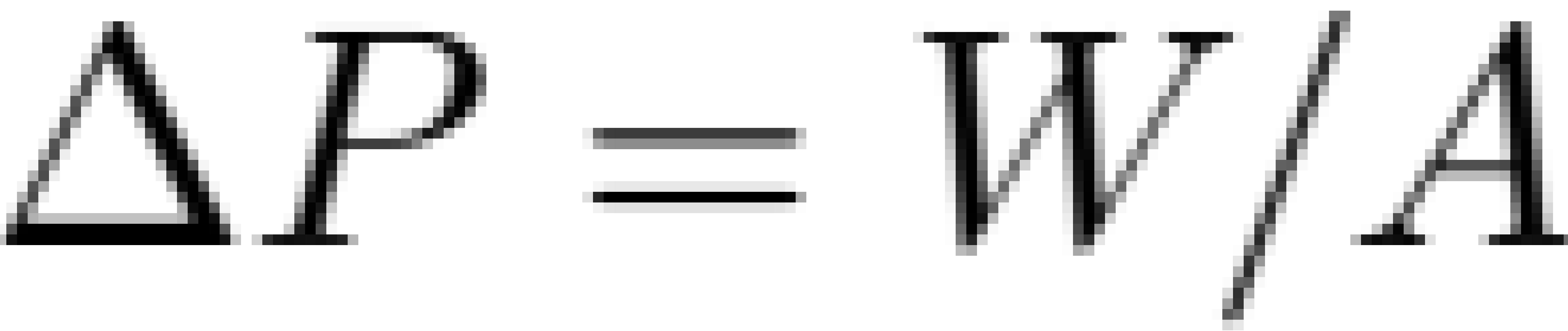


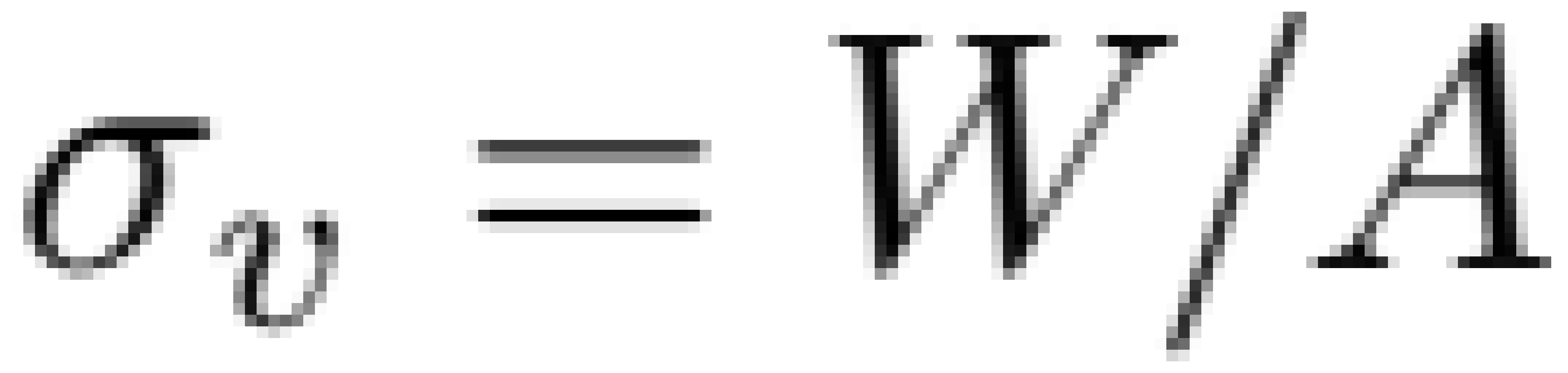
- **Hydrocarbon generation**

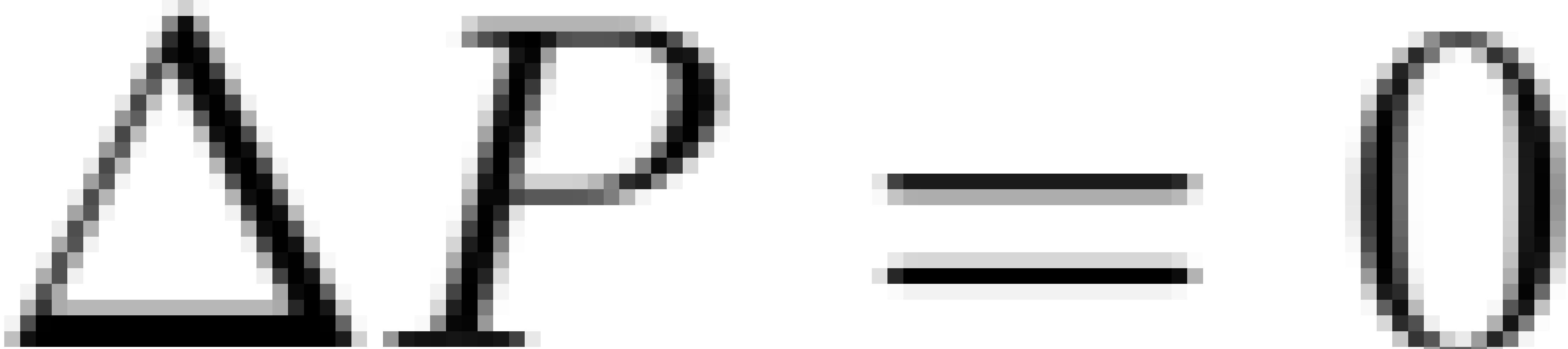
- $\Delta V \rightarrow \Delta P$



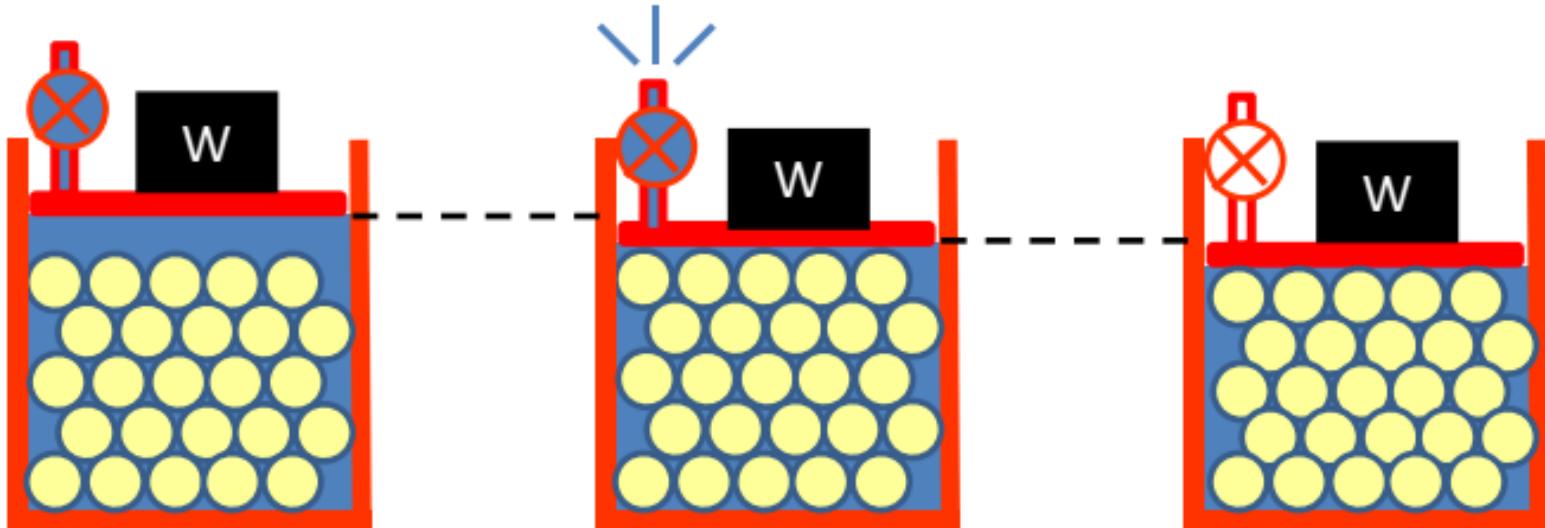




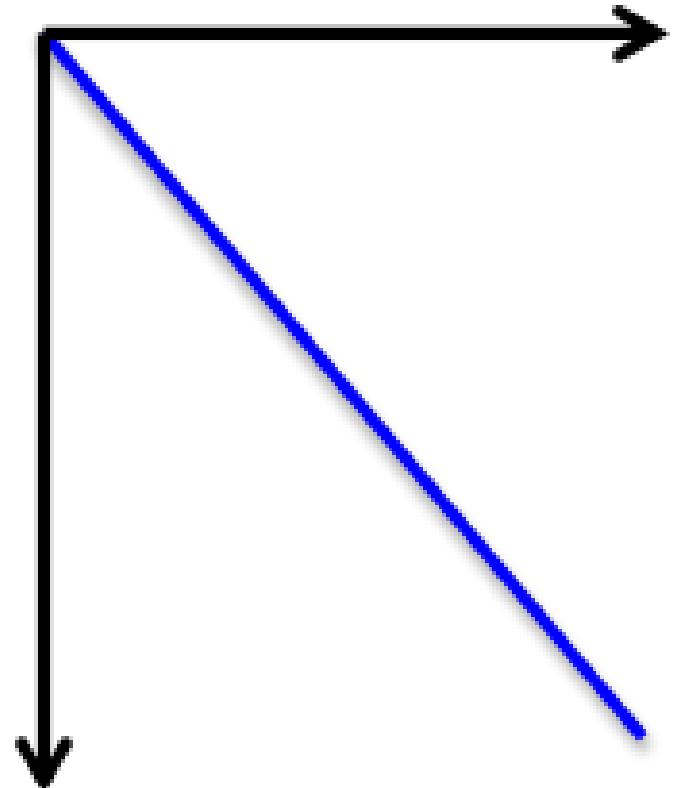
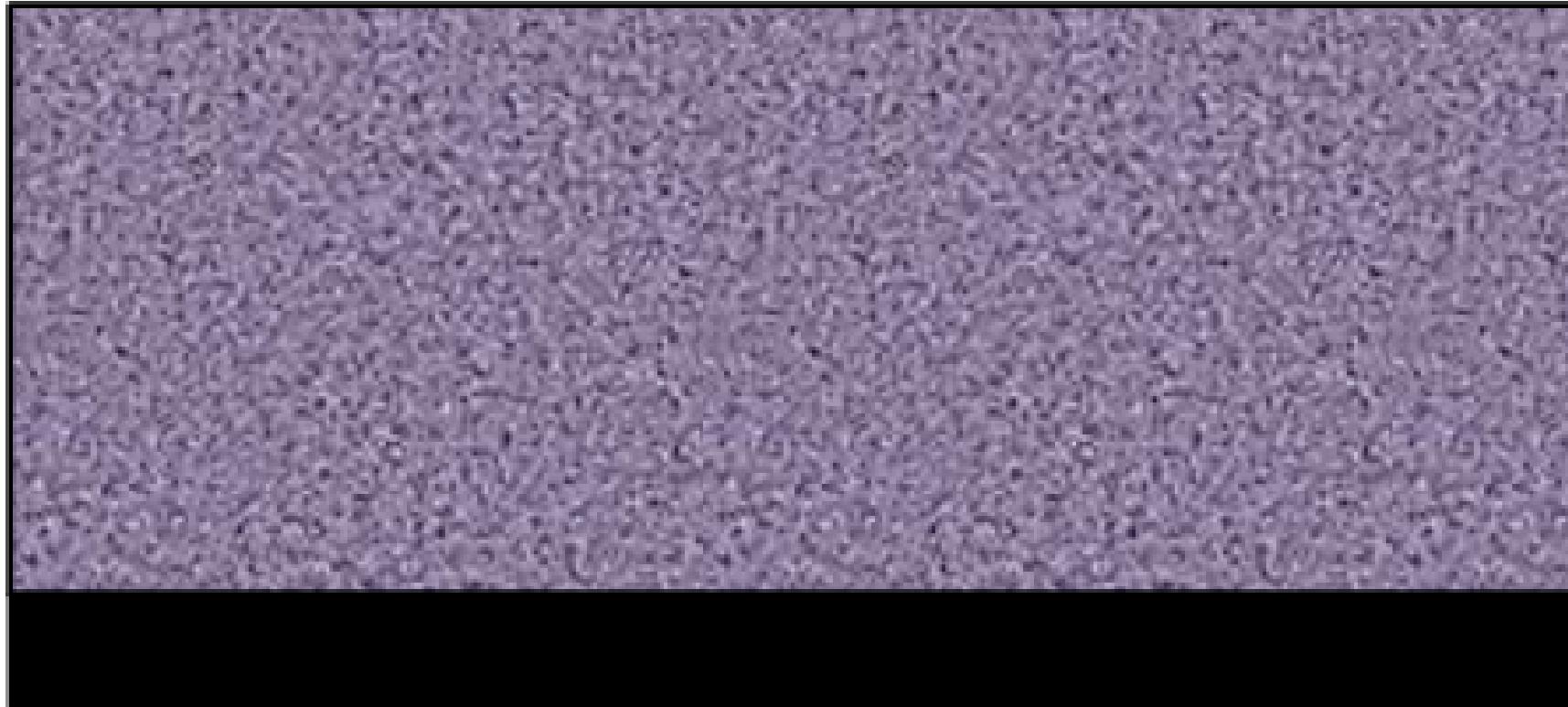


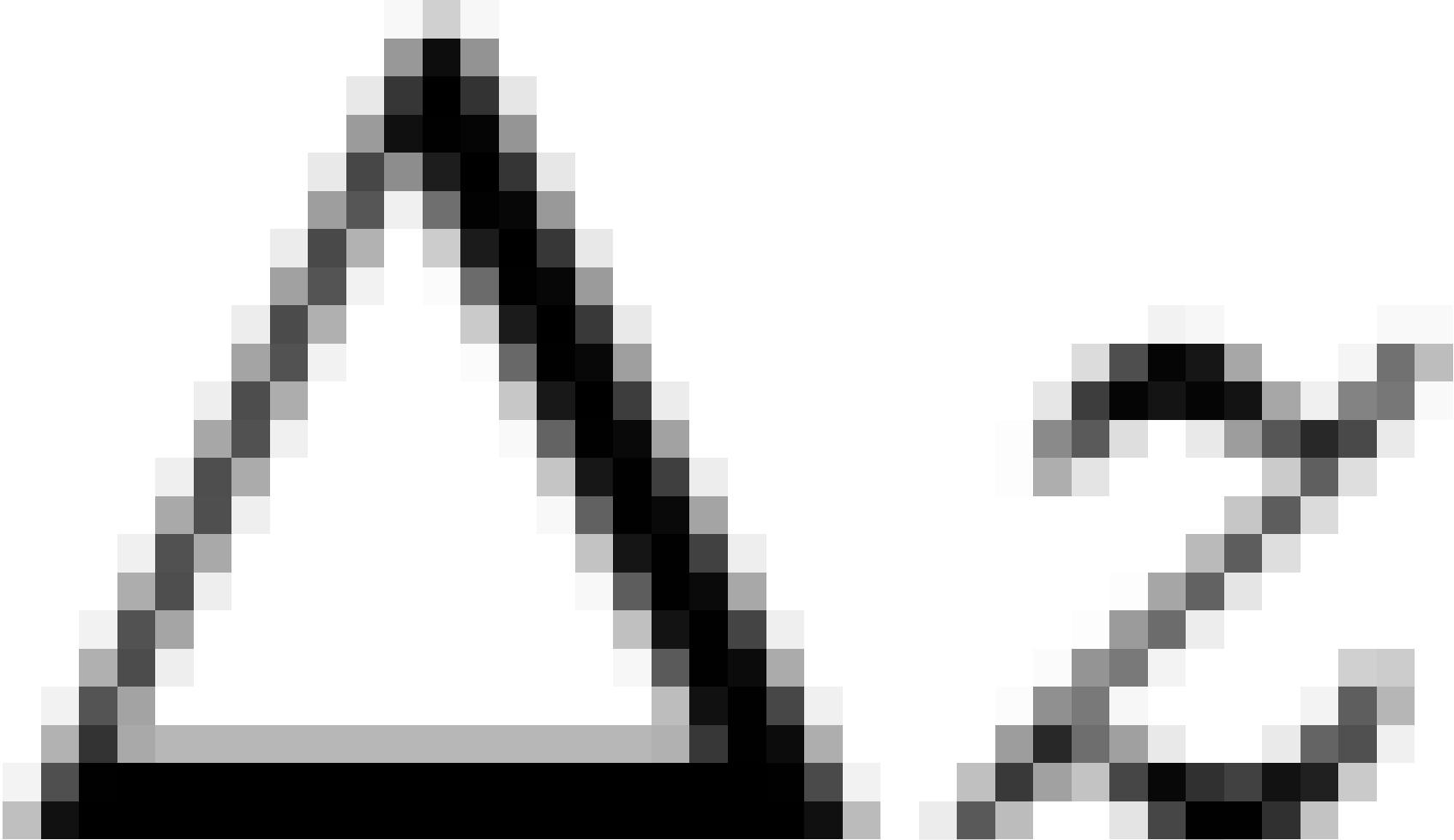


- **Disequilibrium compaction (Underconsolidation)**
 - $\Delta S \rightarrow \Delta P$ (Vertical)
- **Tectonic compression**
 - $\Delta S \rightarrow \Delta P$ (Horizontal)



Pressure water





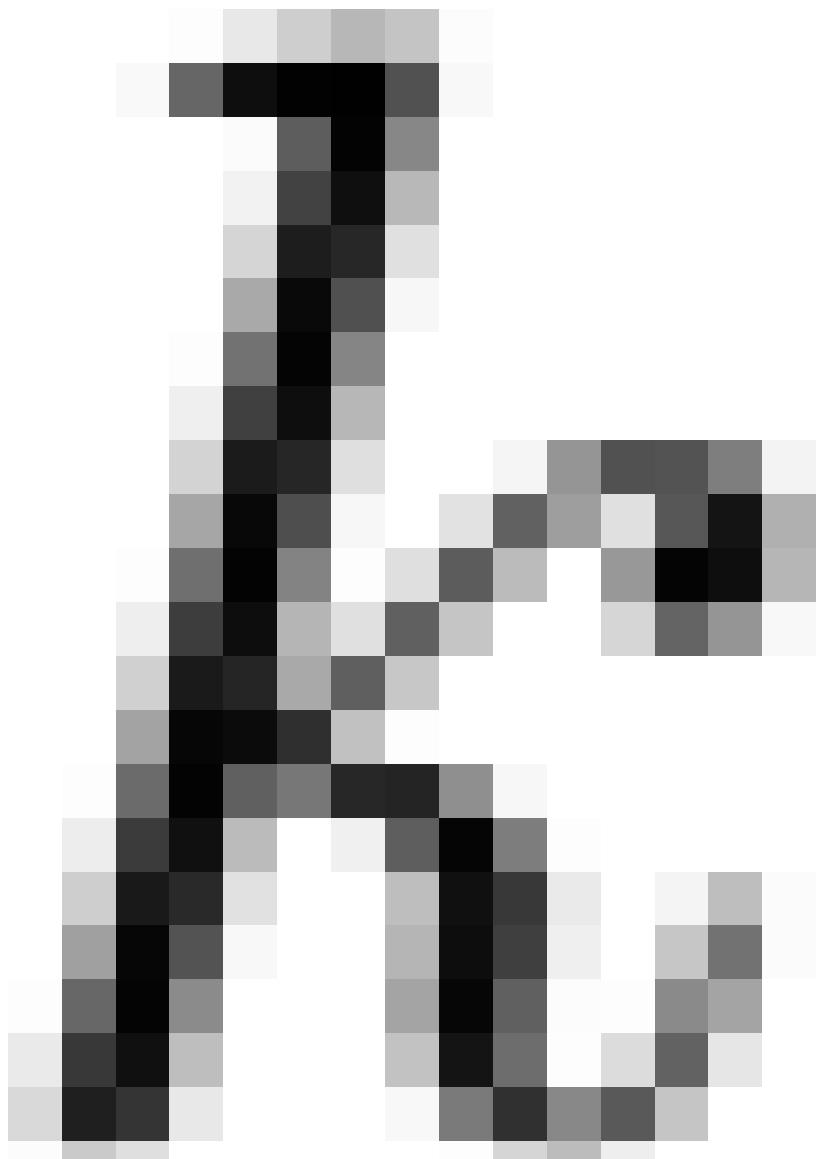
Dh

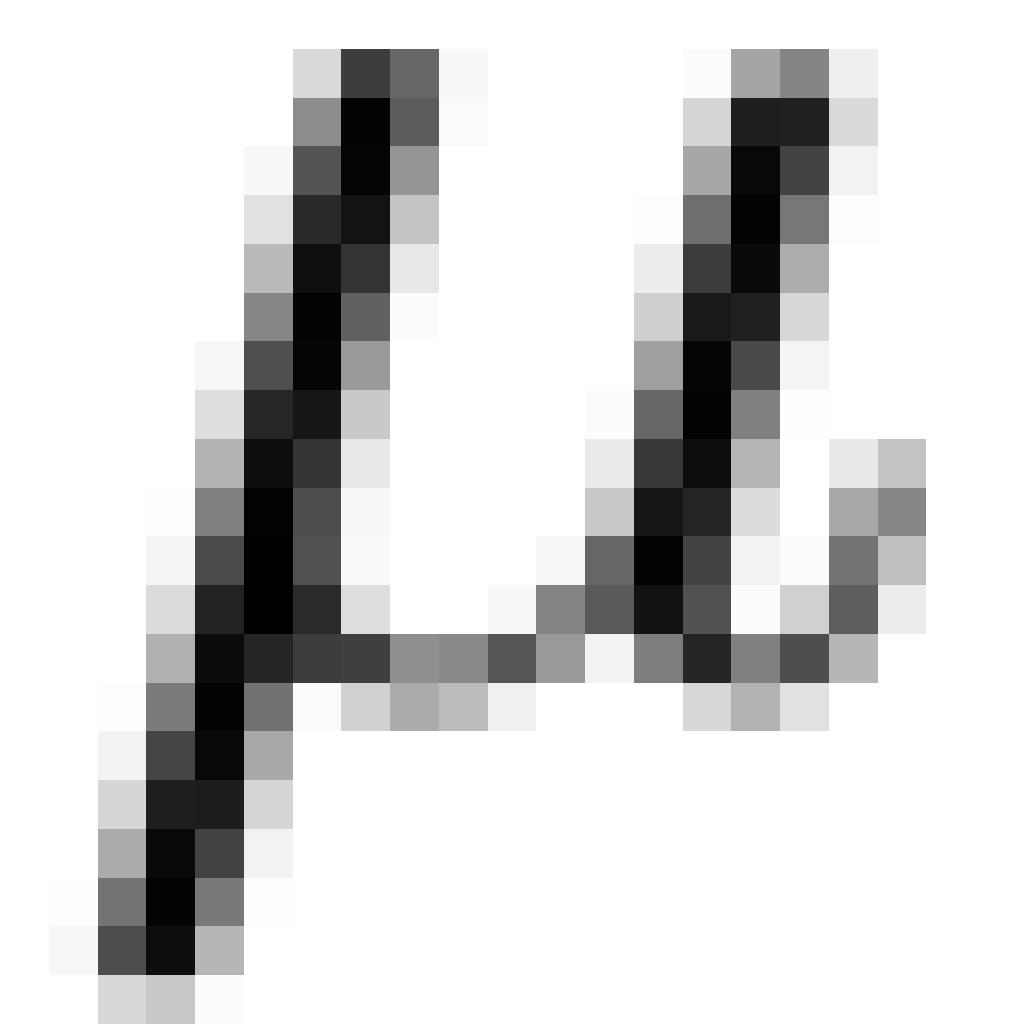
=

MK

ll







αp

βp

αt

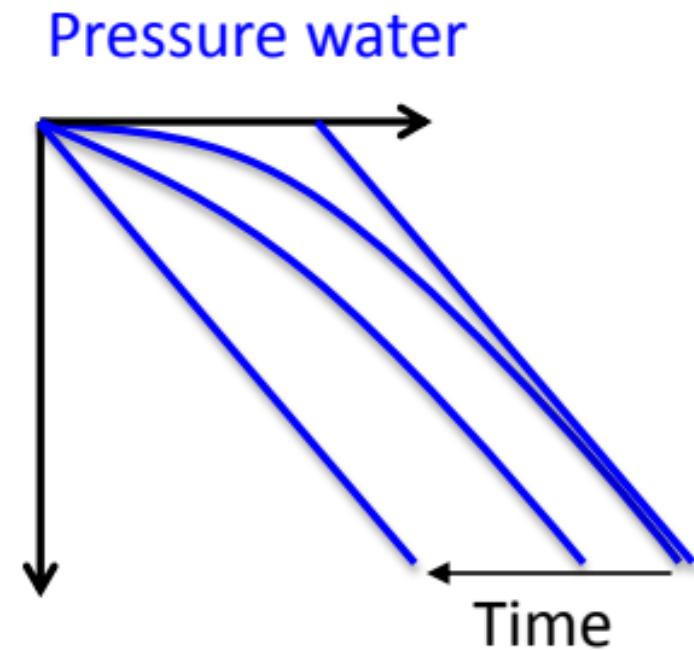
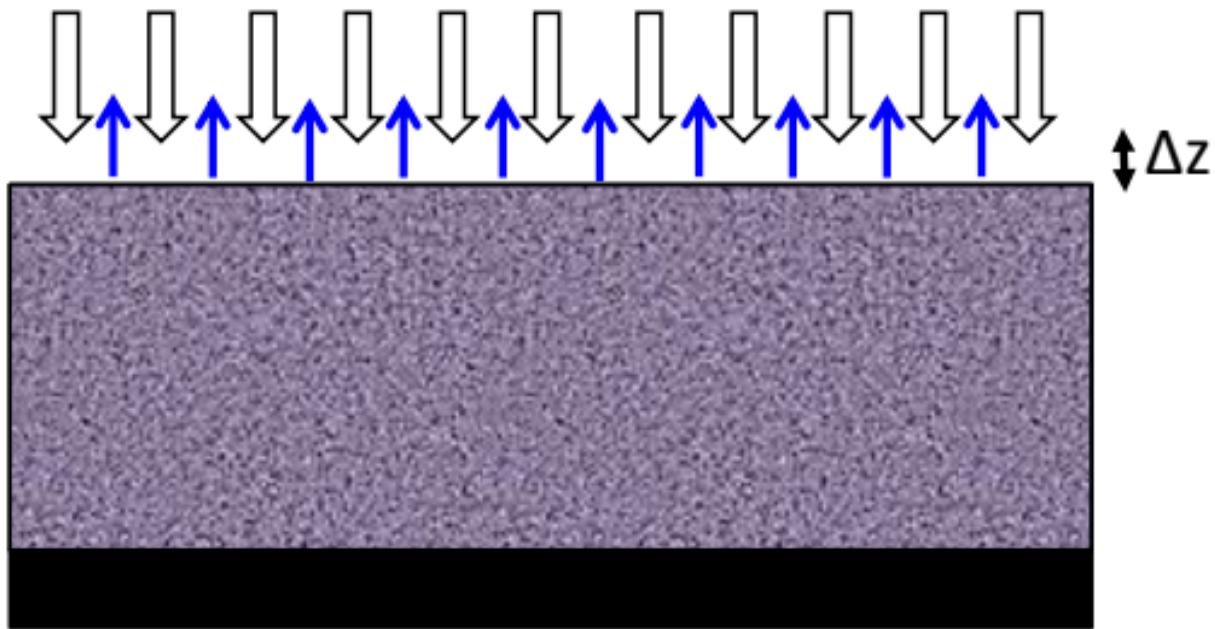
$=$

Dh

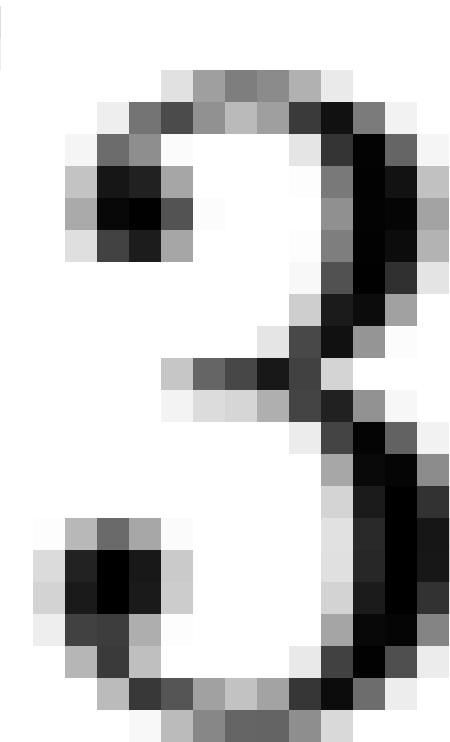
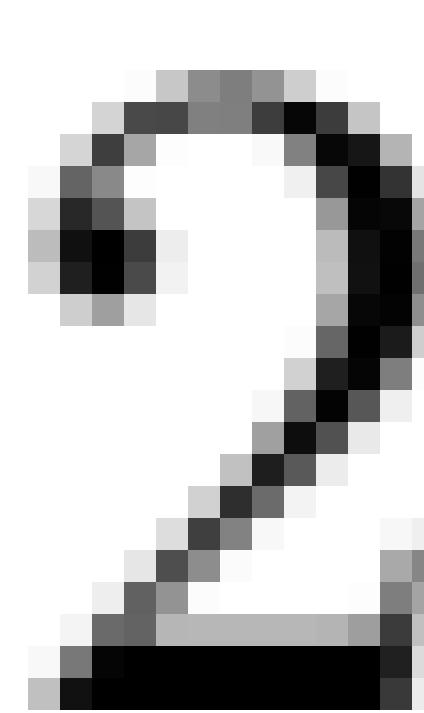
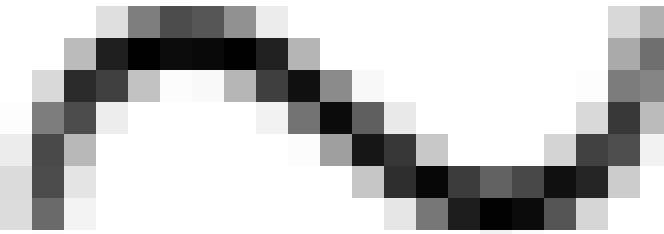
$d^2 p$

dz^2

Rate of sedimentation (loading) and rate of fluid “escape”







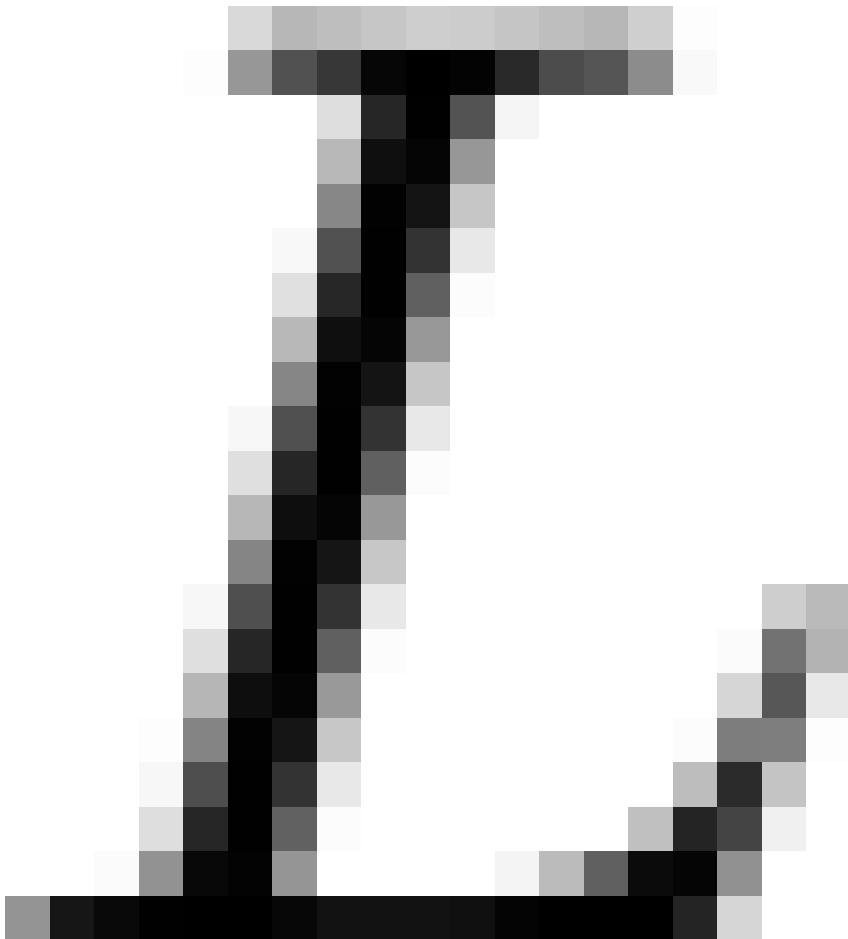
Γ^2

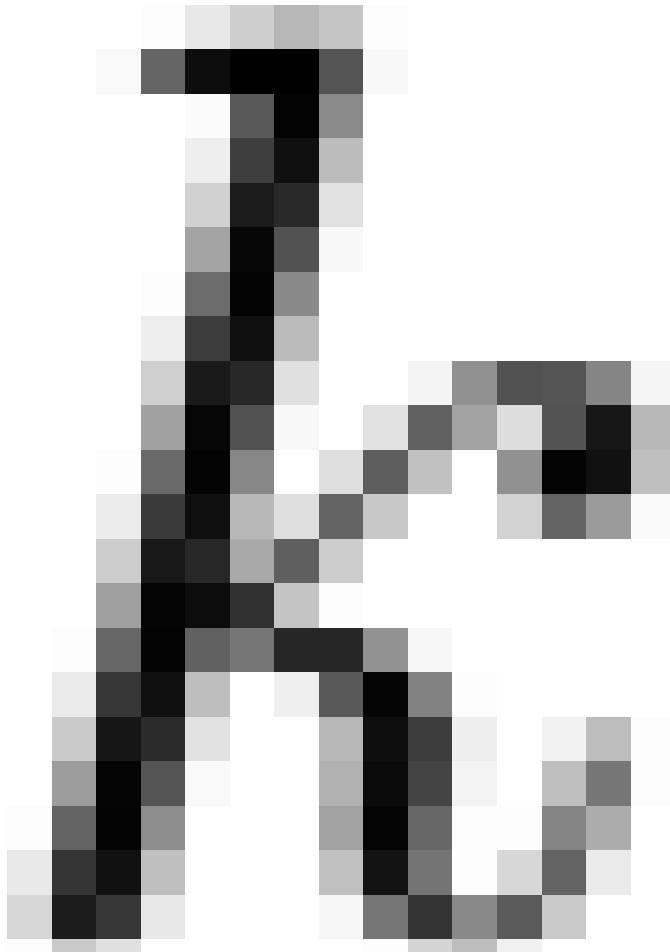
Γ

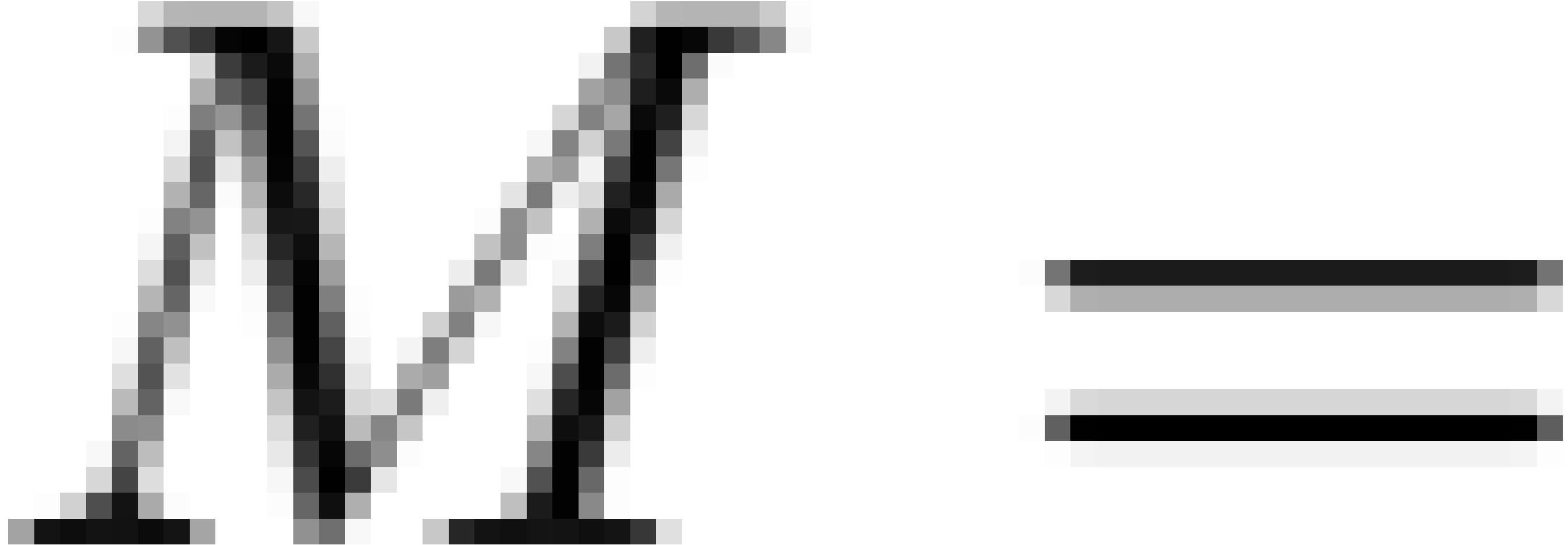
D_h

Γ_{ch}

Γ

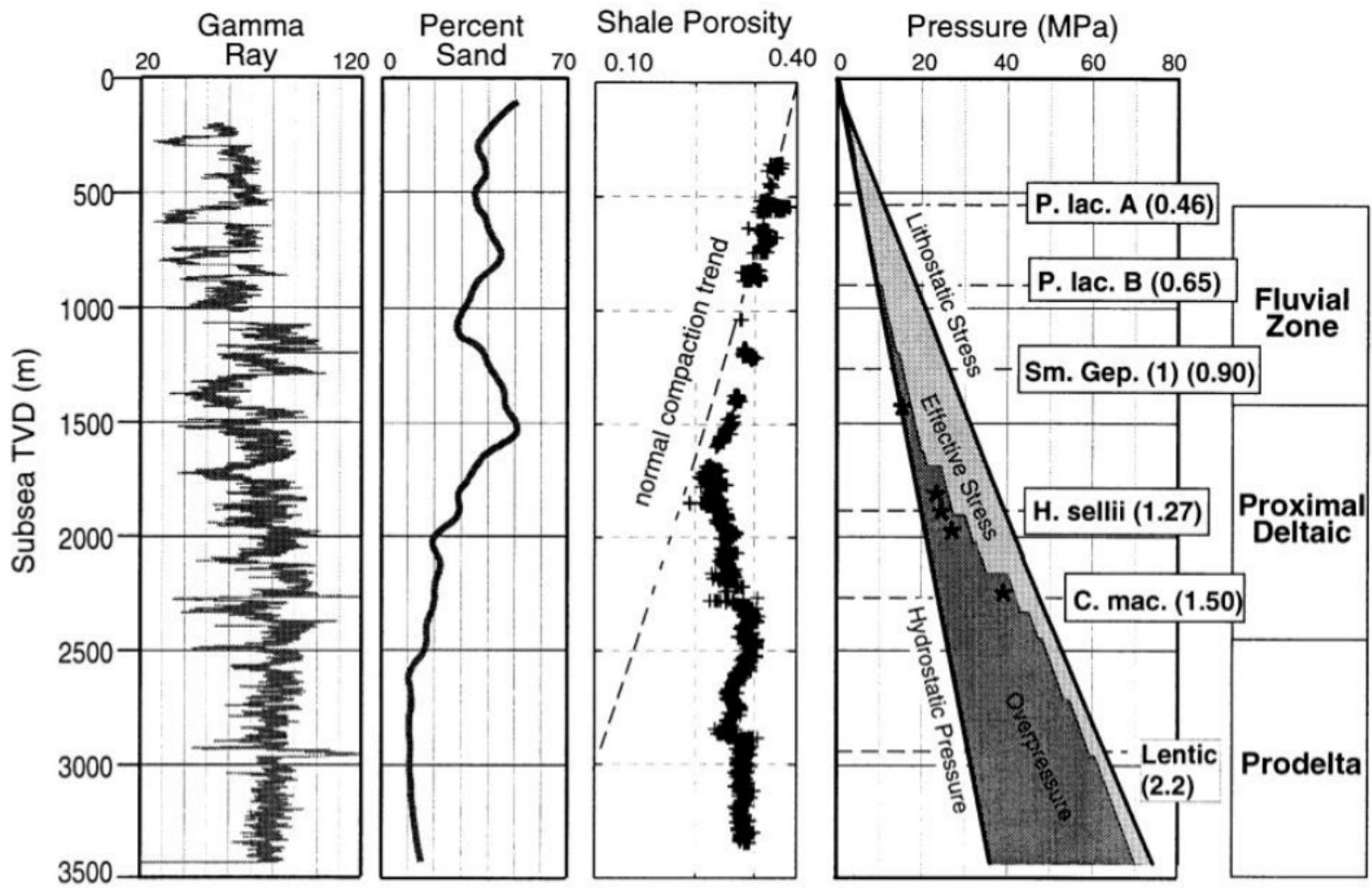




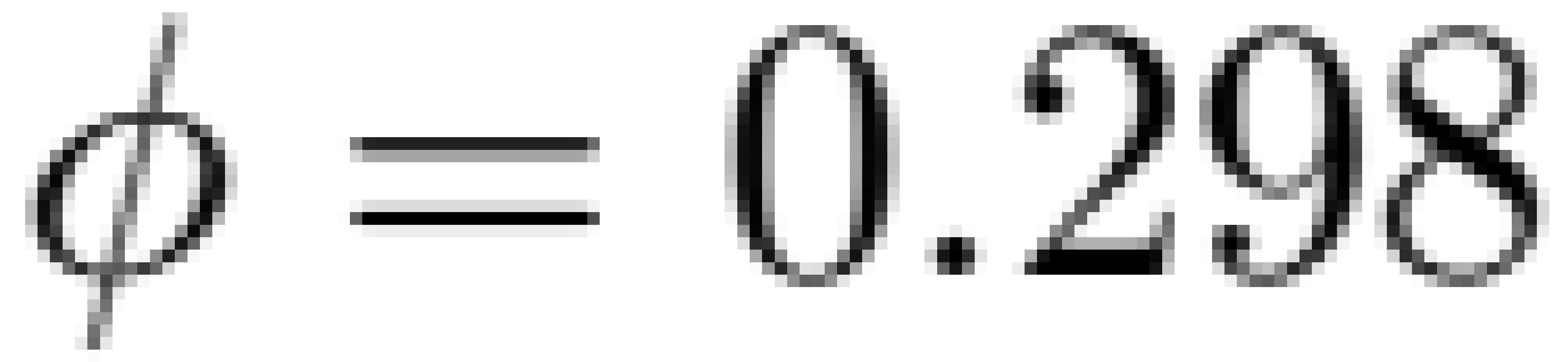


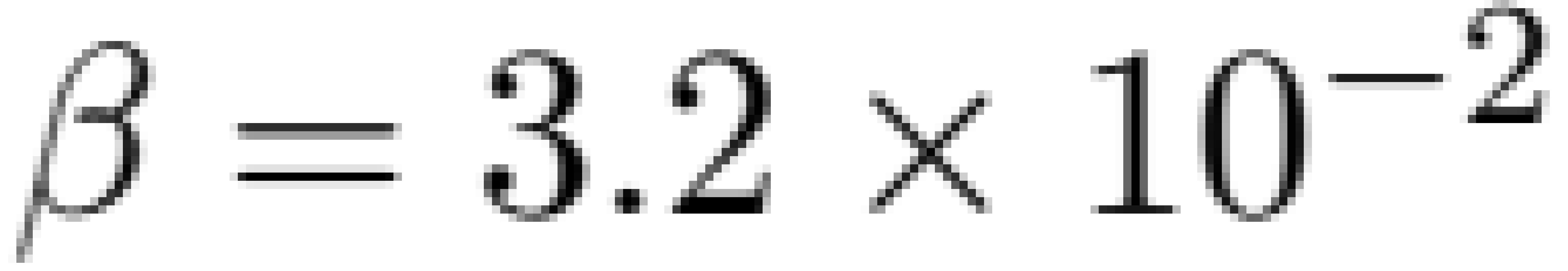


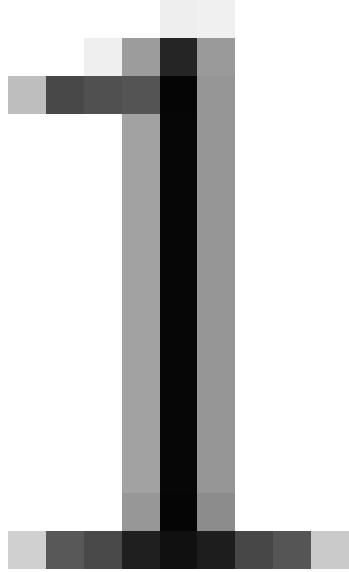


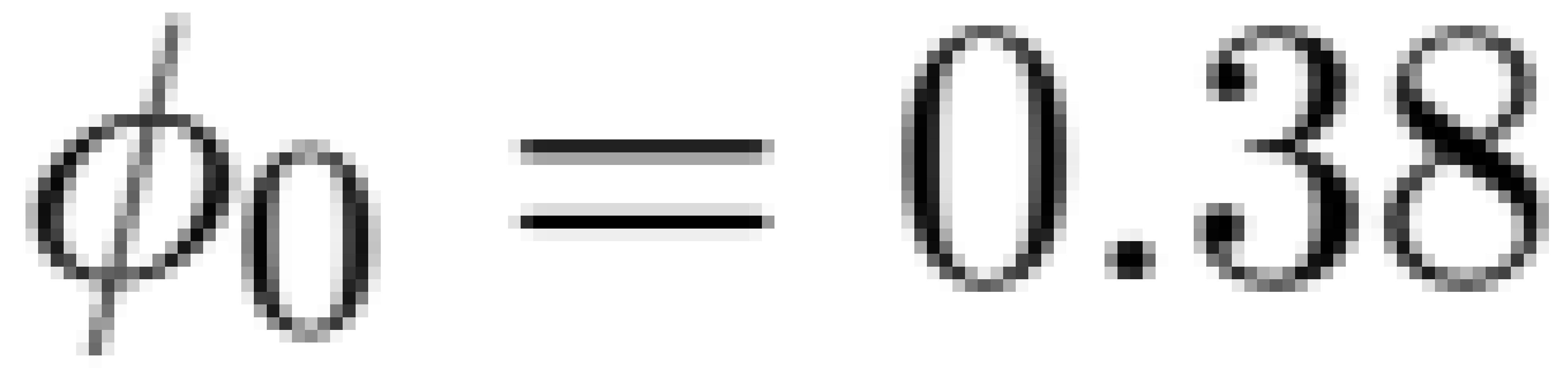


Off-shore Louisiana – Gordon and Flemings (1998) Basin Research

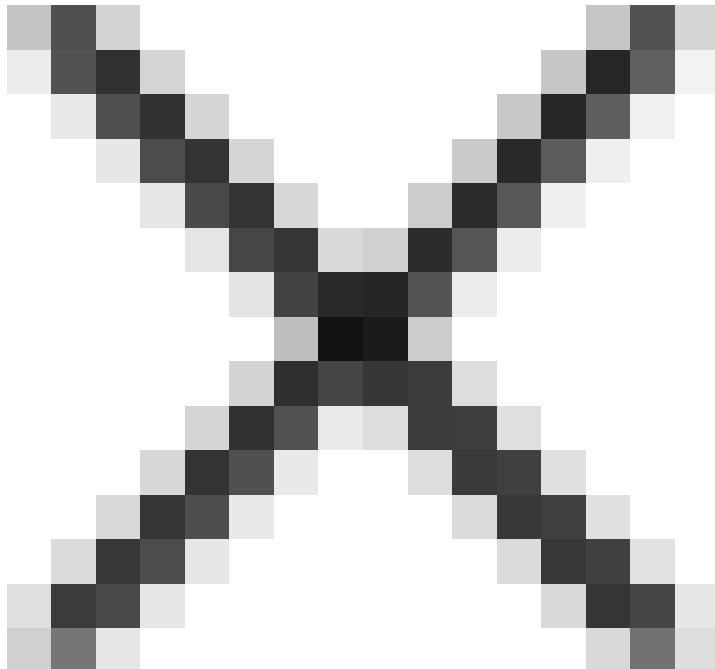




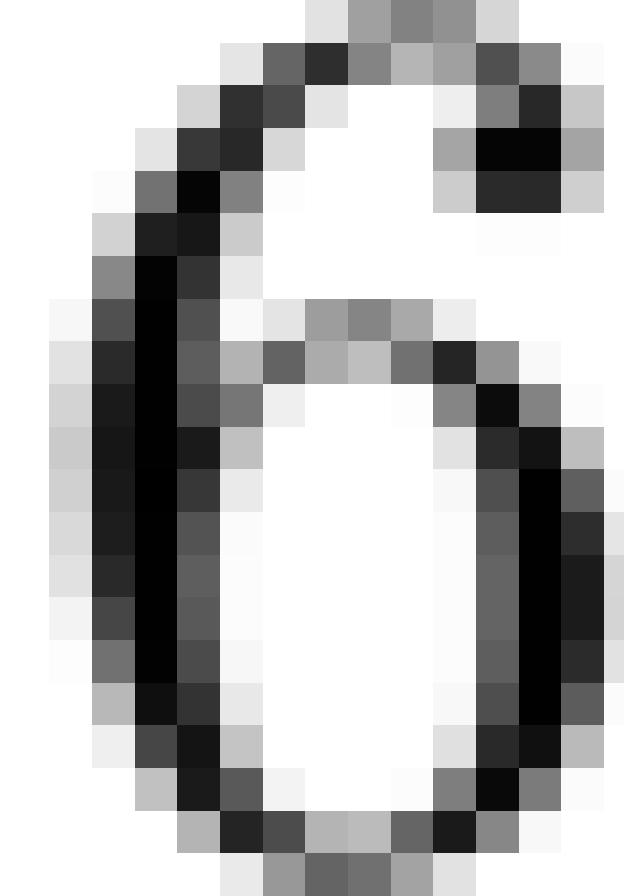
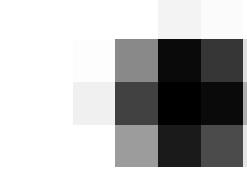
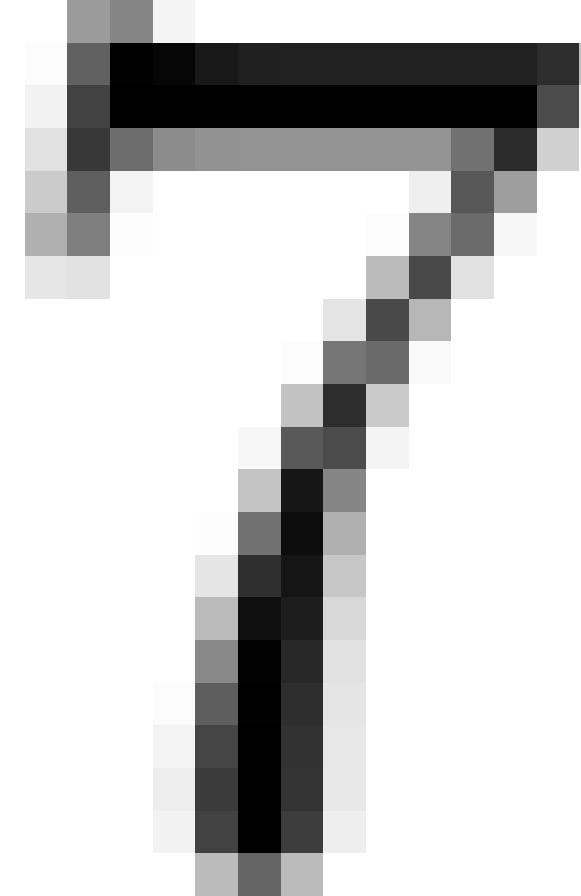


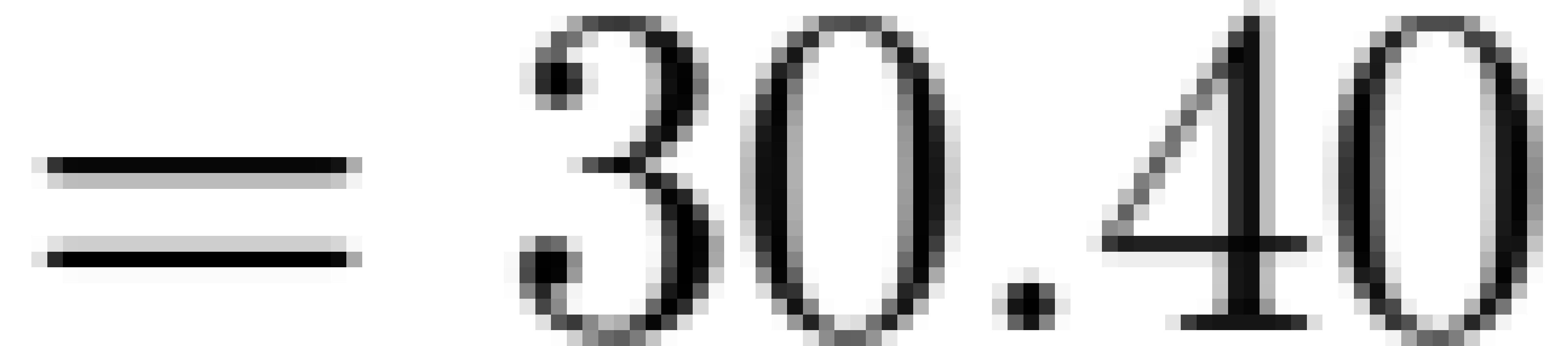




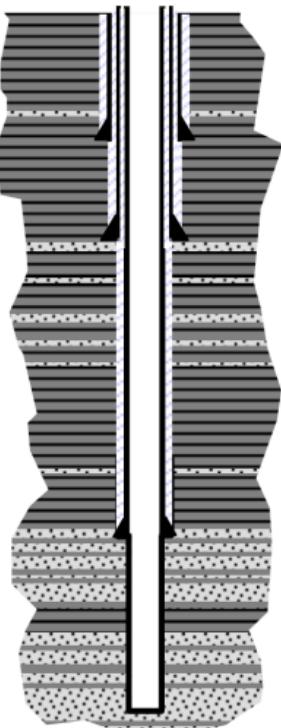
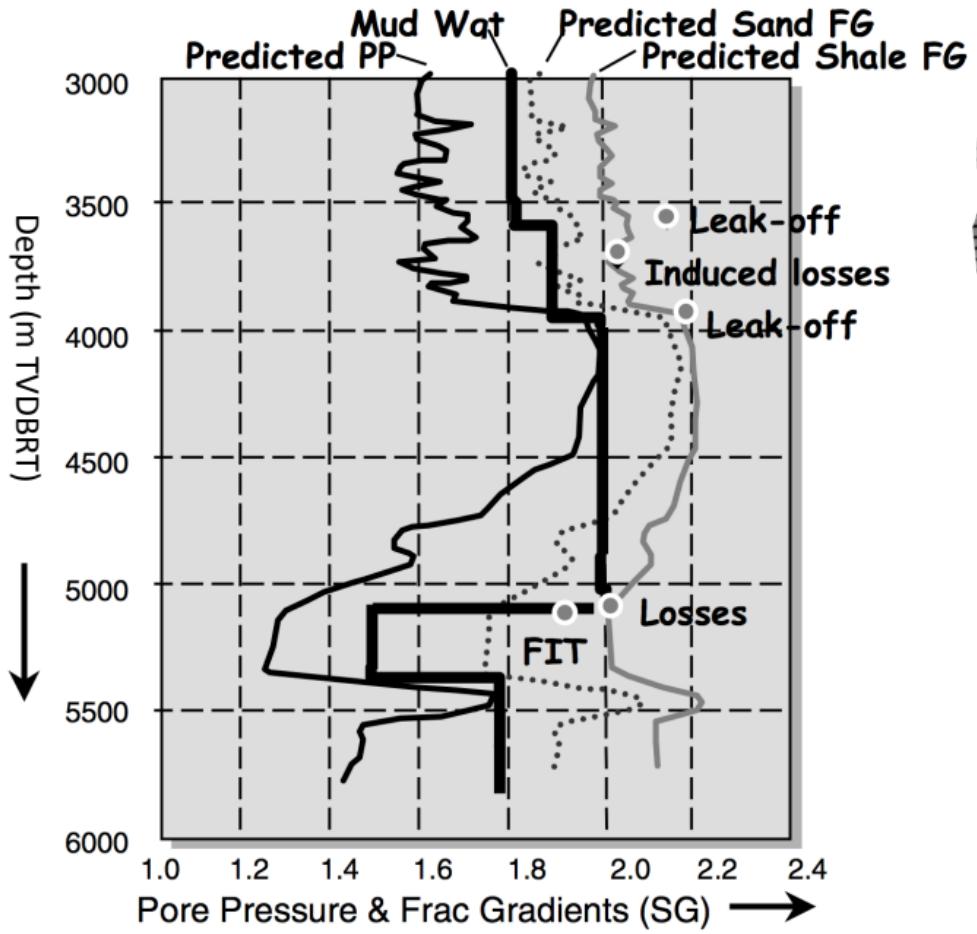


$$P_p = S_p + \ln(\phi/\phi_0) = 38$$





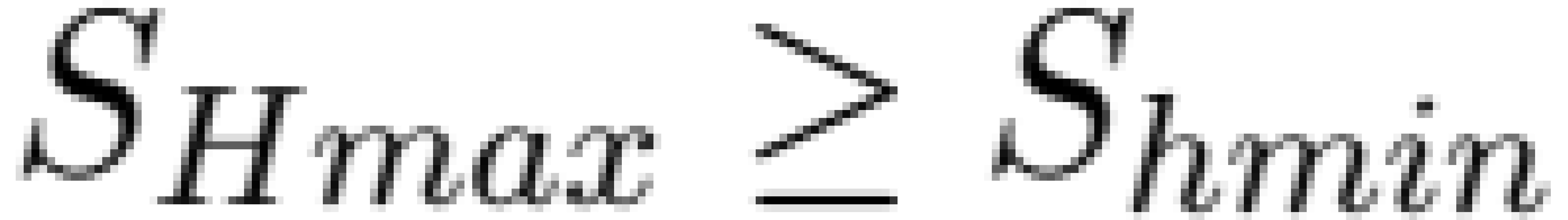
P = 30.40 MPa
 S_2 = 0.8.
38 MPa

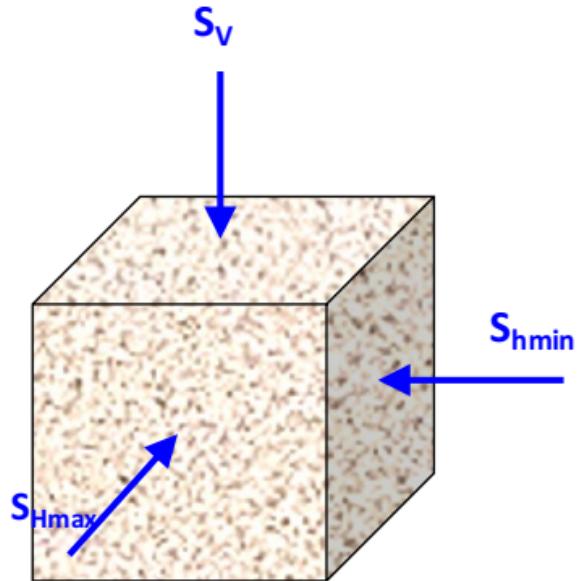
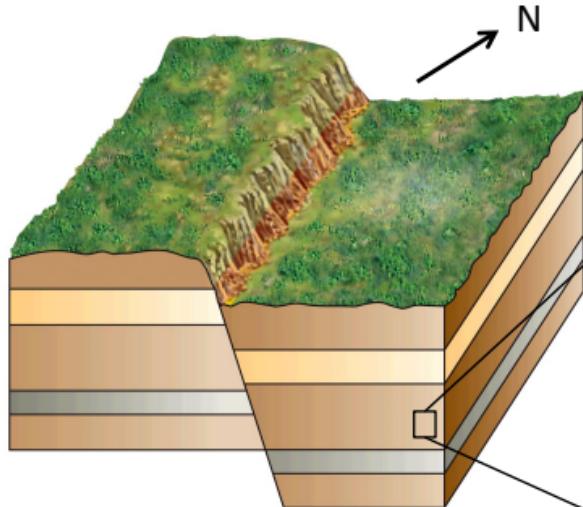


[Caspian Sea, Alberta and McLean – SPE67740]

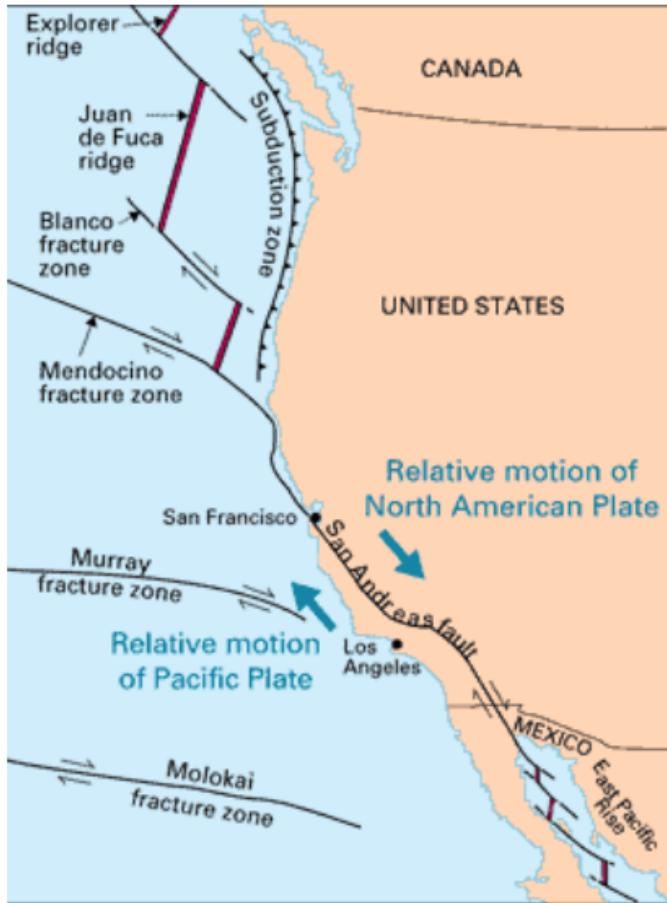






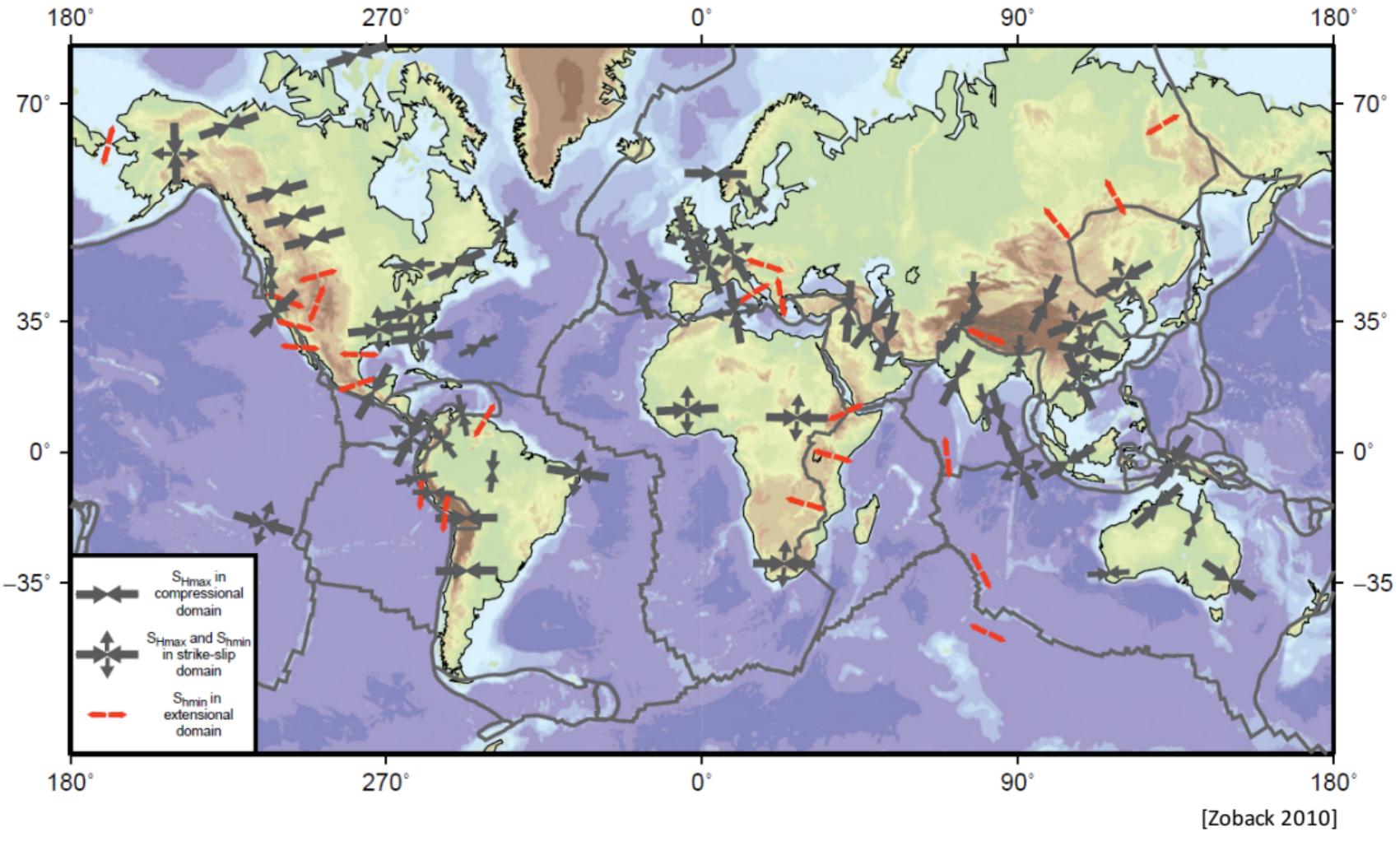


$$\underline{\underline{S}} = \begin{bmatrix} S_V & 0 & 0 \\ 0 & S_{H\max} & 0 \\ 0 & 0 & S_{h\min} \end{bmatrix}$$

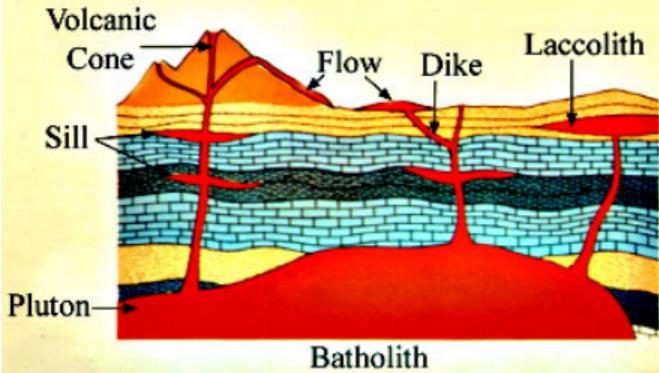


<http://pubs.usgs.gov/gip/dynamic/understanding.html#anchor5798673>

<http://en.wikipedia.org/wiki/File:Aerial-SanAndreas-CarrizoPlain.jpg>



PLUTONS & VOLCANIC LANDFORMS



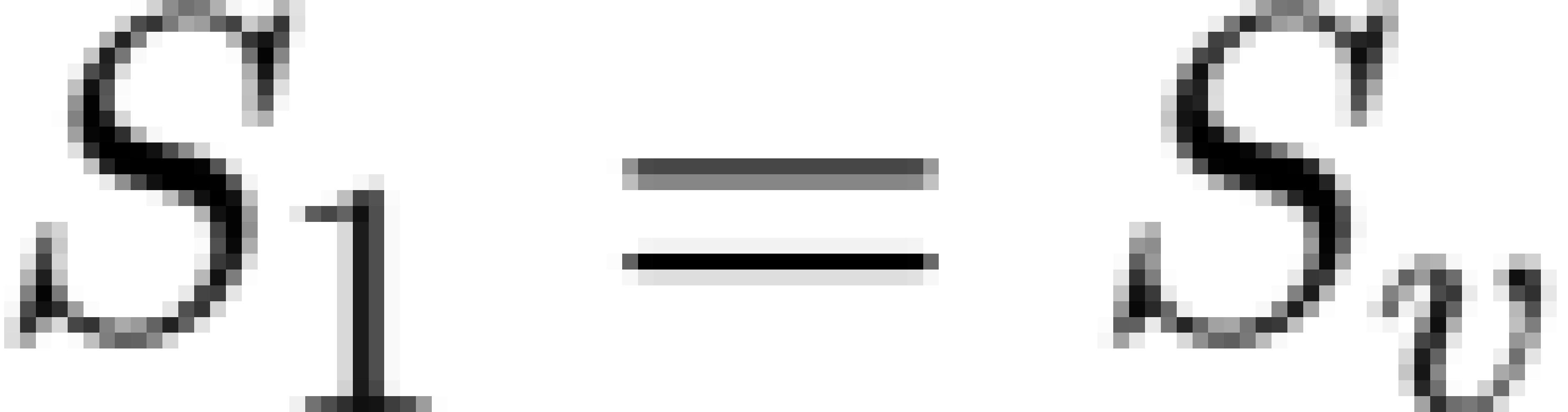
<http://www.indiana.edu/~geol105/1425chap5.htm>
<http://geophysics.ou.edu/geol1114>





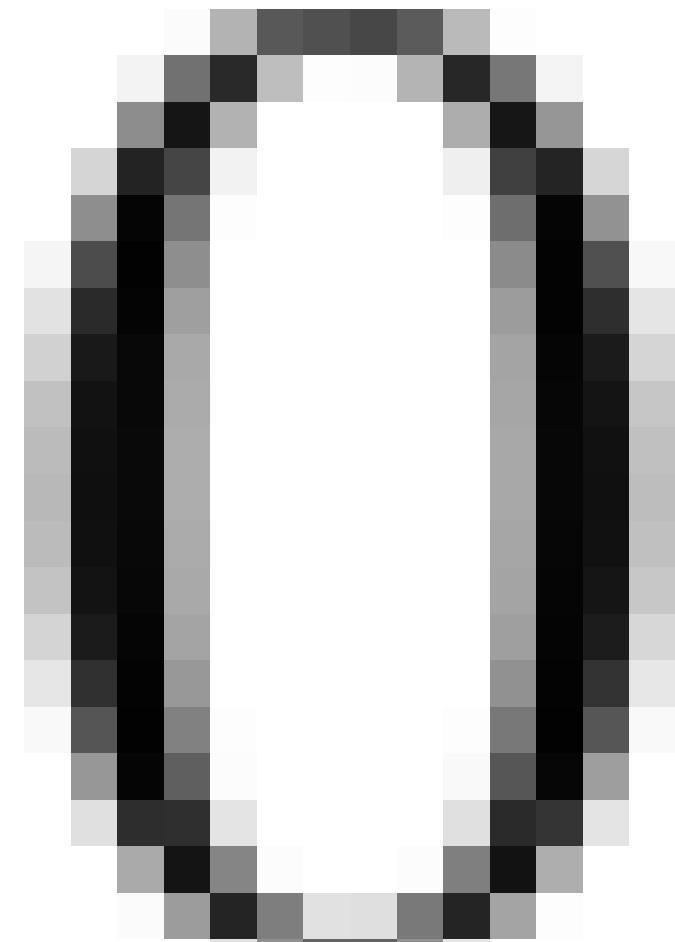
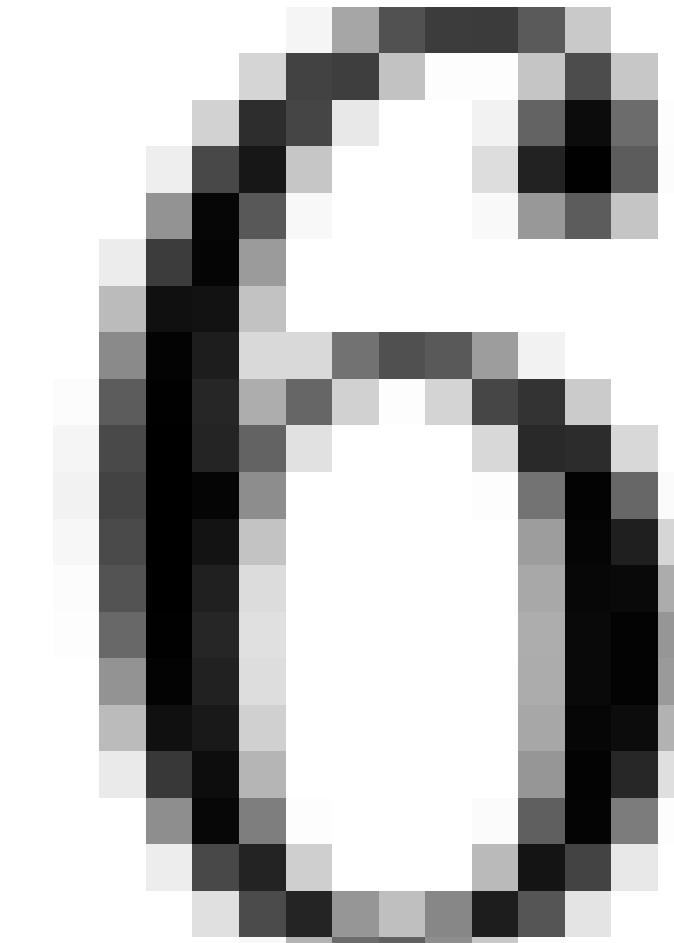
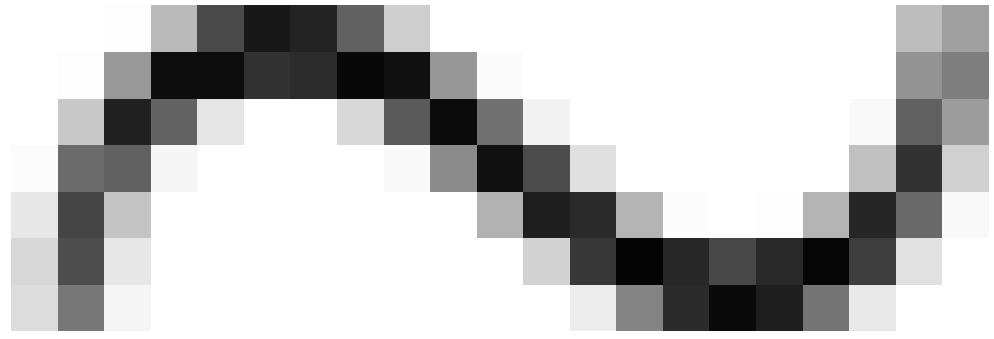








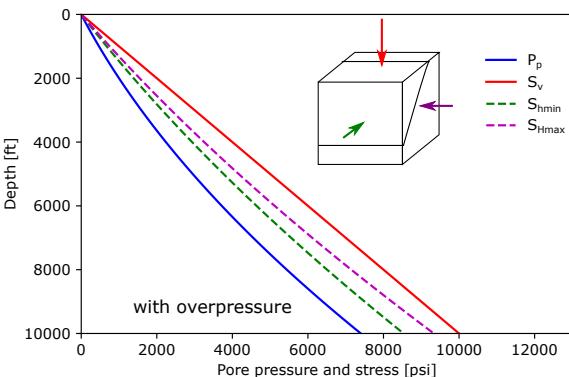
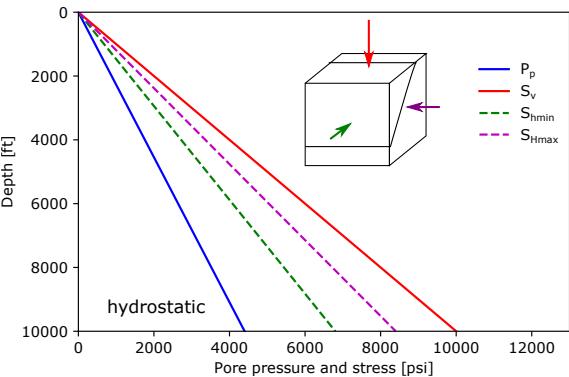




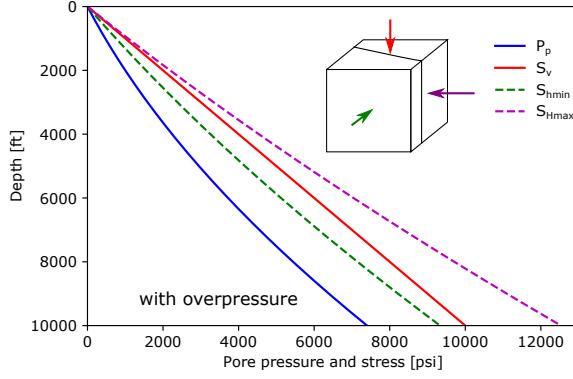
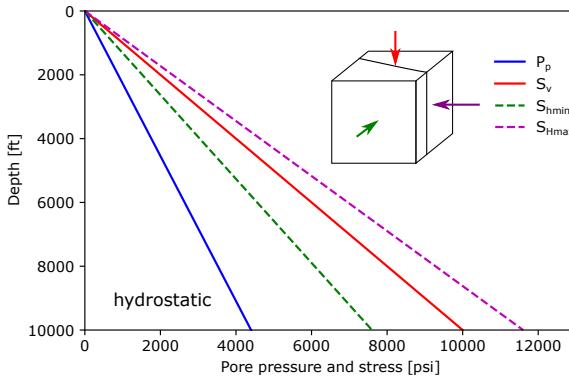




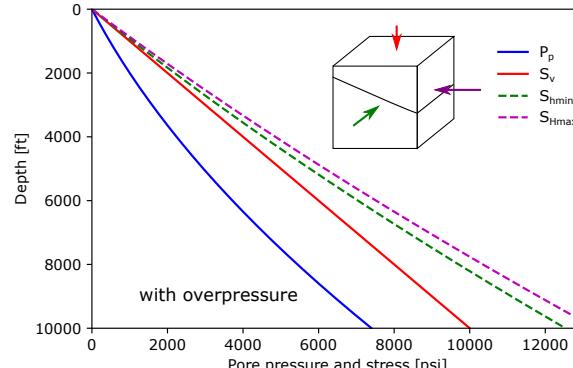
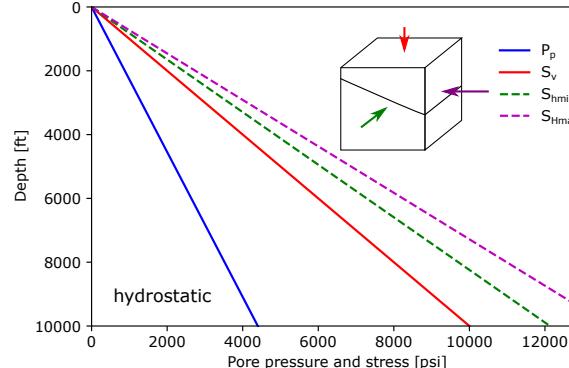
Normal faulting: $S_v > S_{H\max} > S_{h\min}$

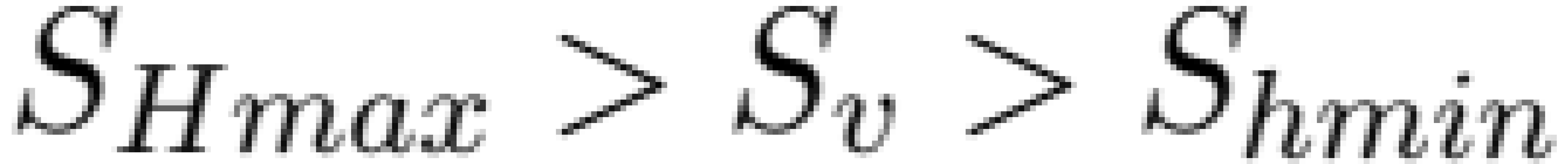


Strike slip faulting: $S_{H\max} > S_v > S_{h\min}$



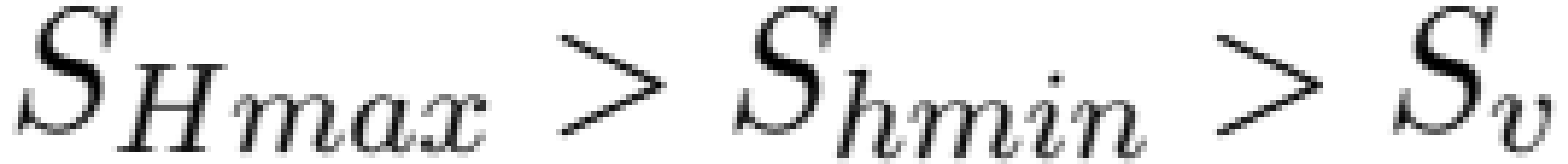
Reverse faulting: $S_{H\max} > S_{h\min} > S_v$





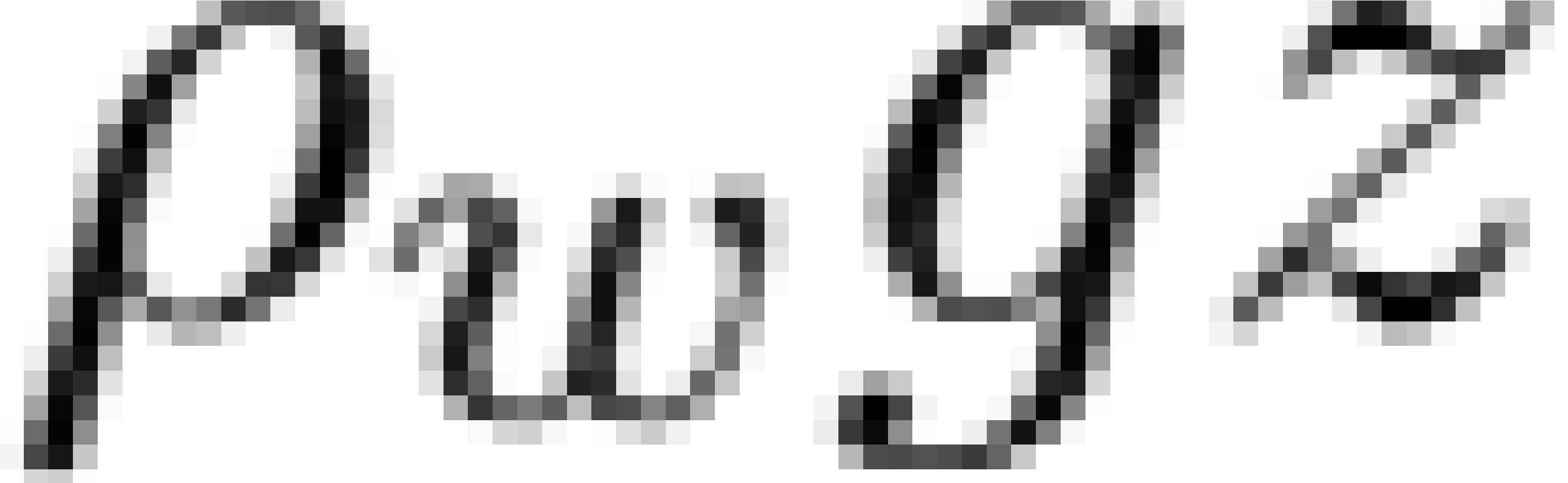










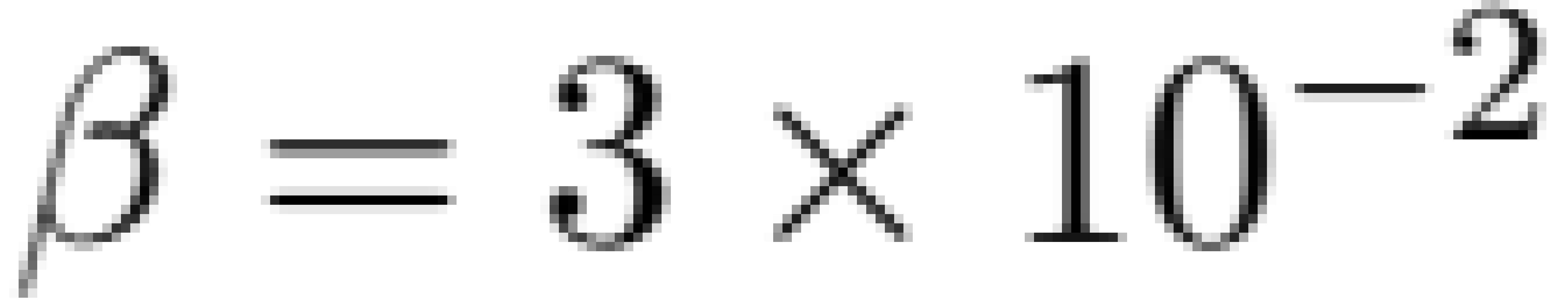


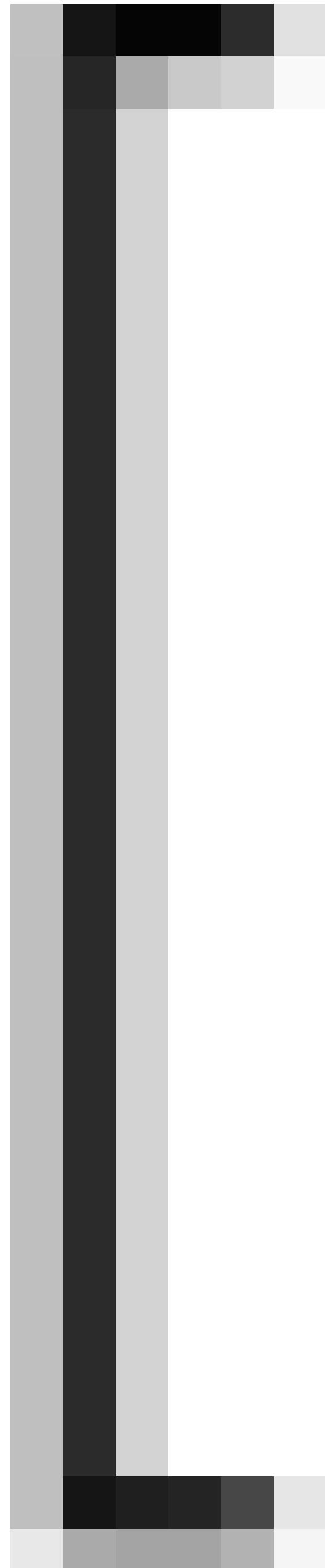


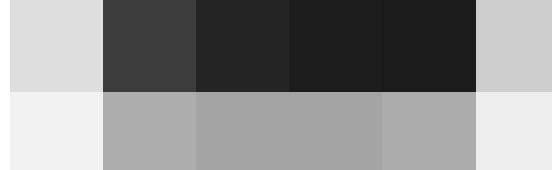


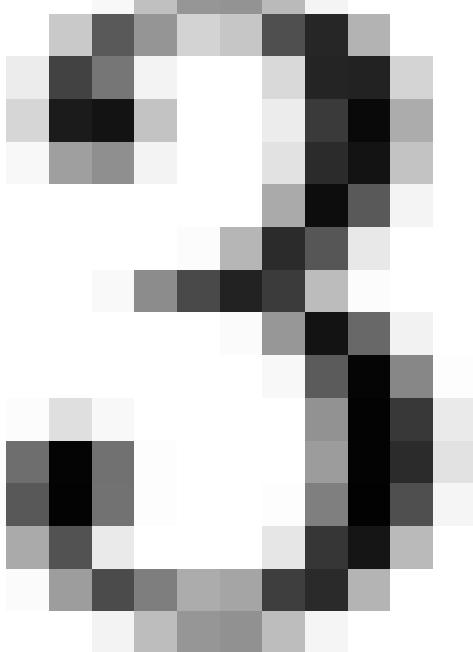


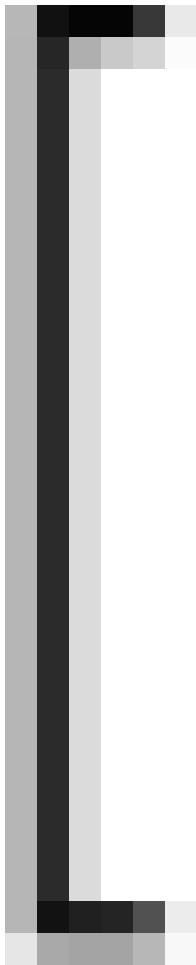


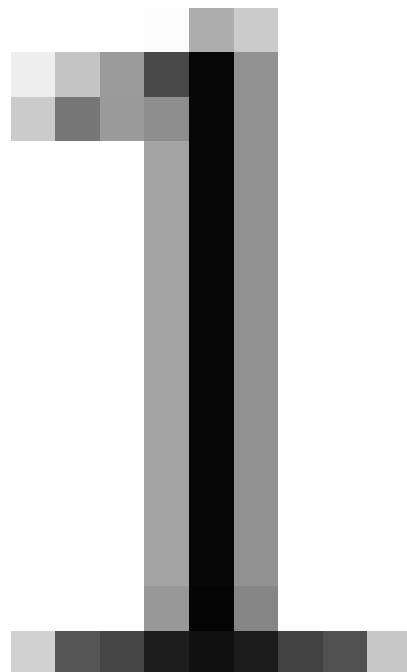
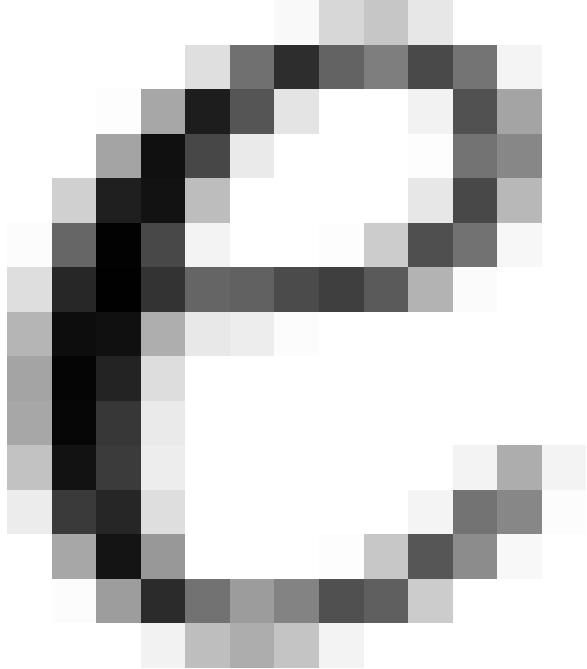


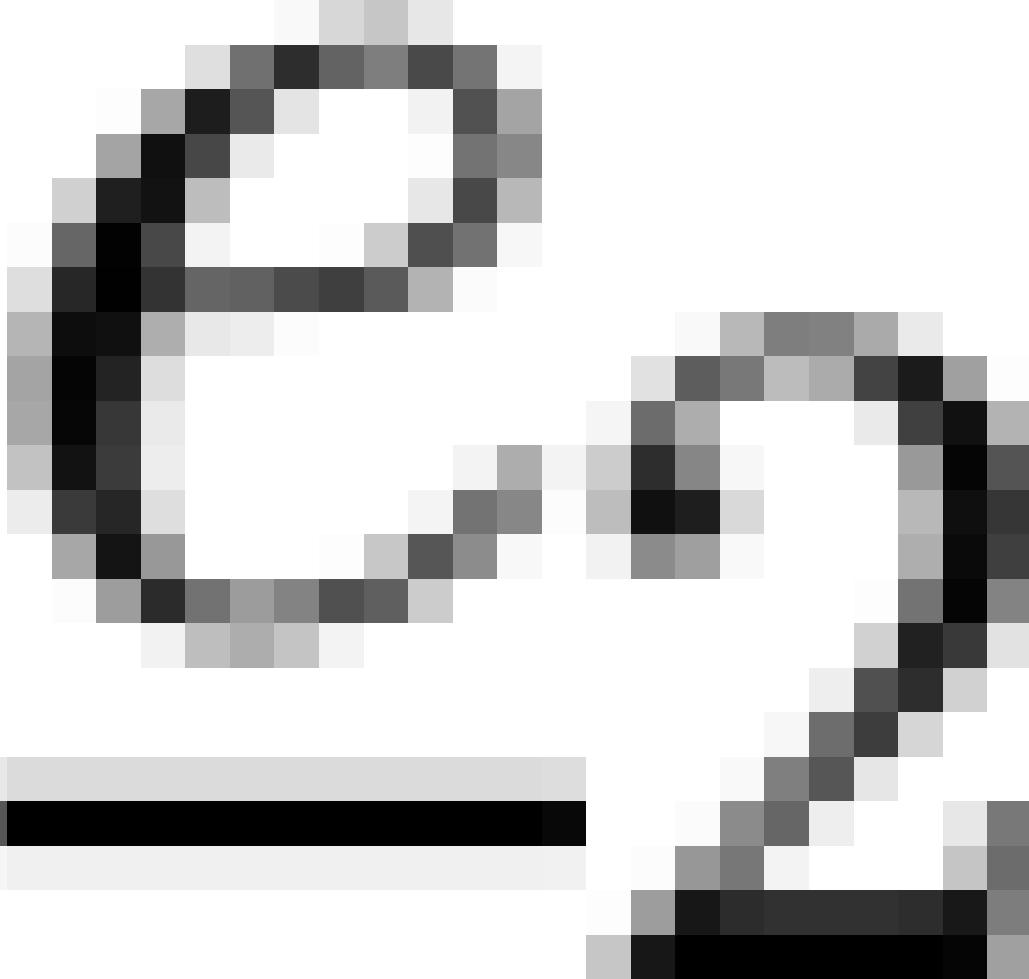


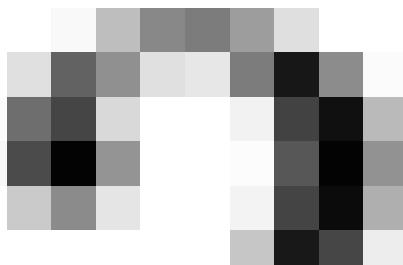
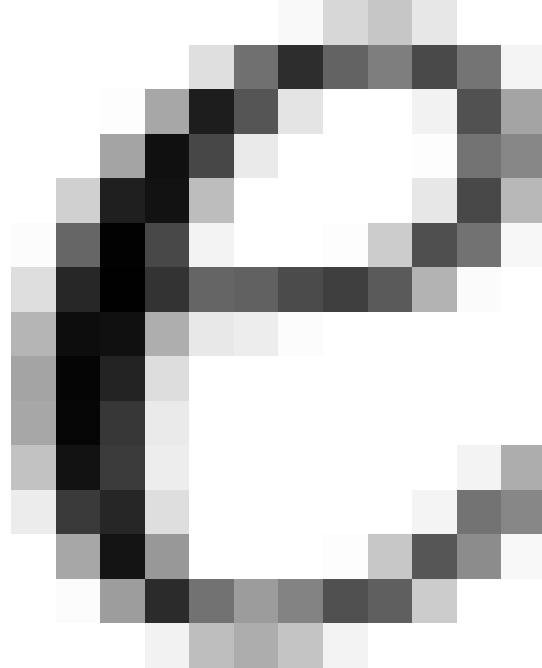


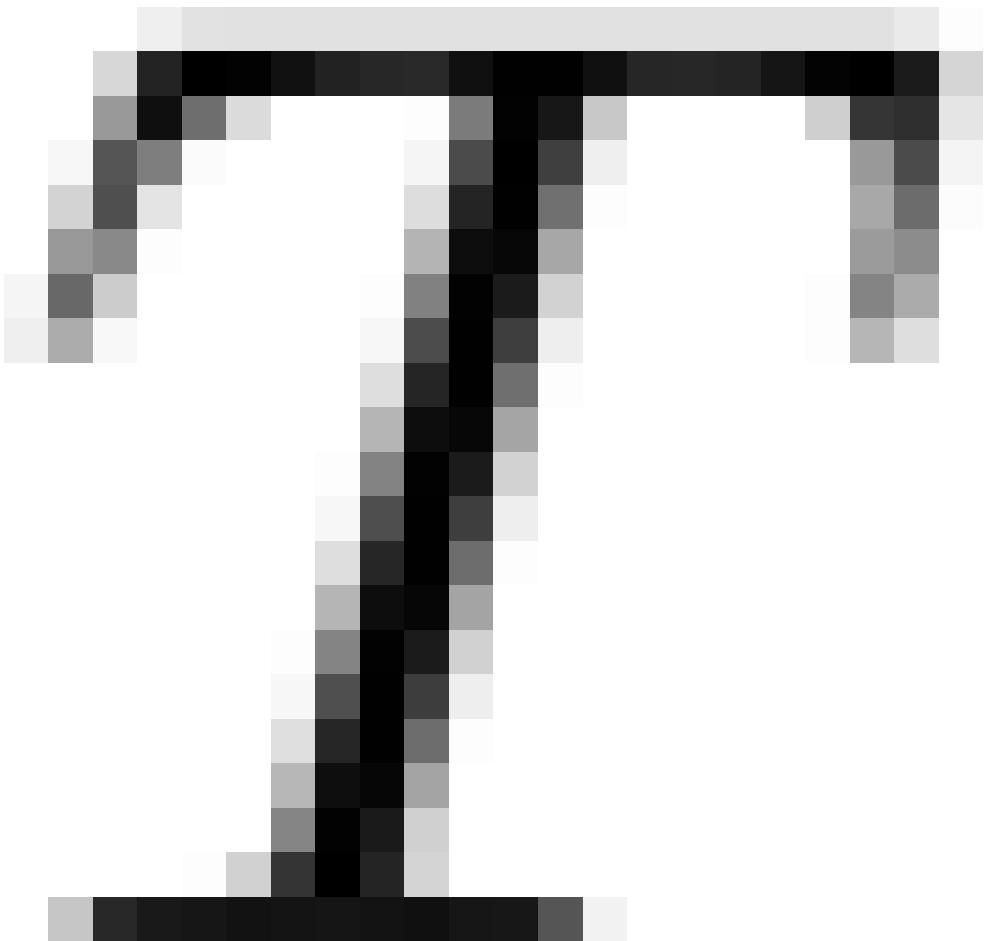




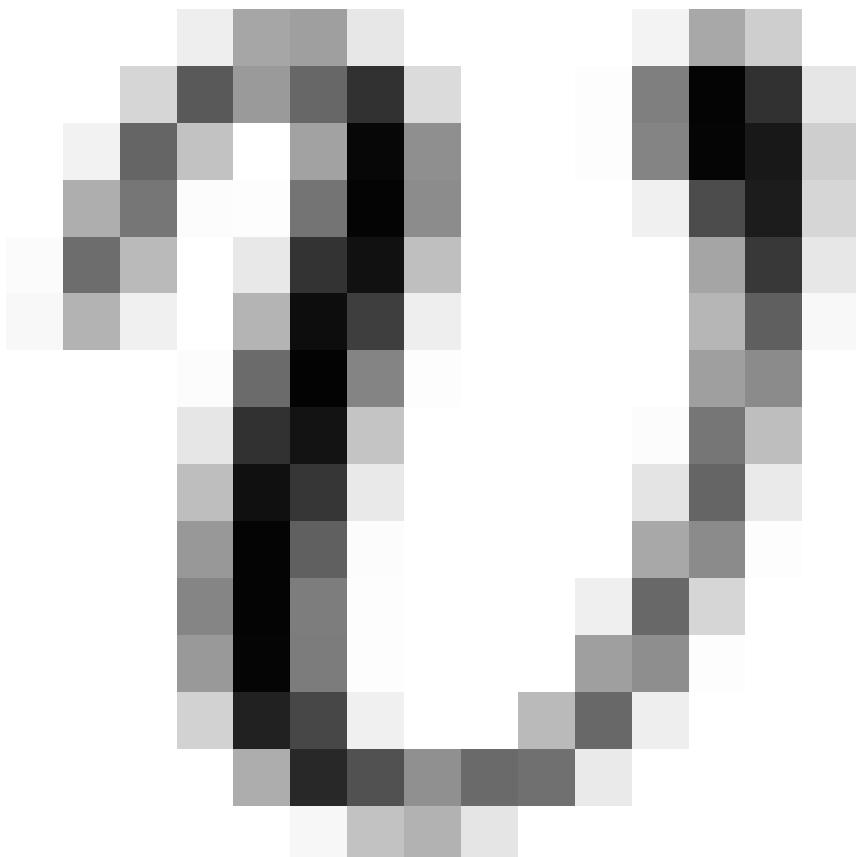


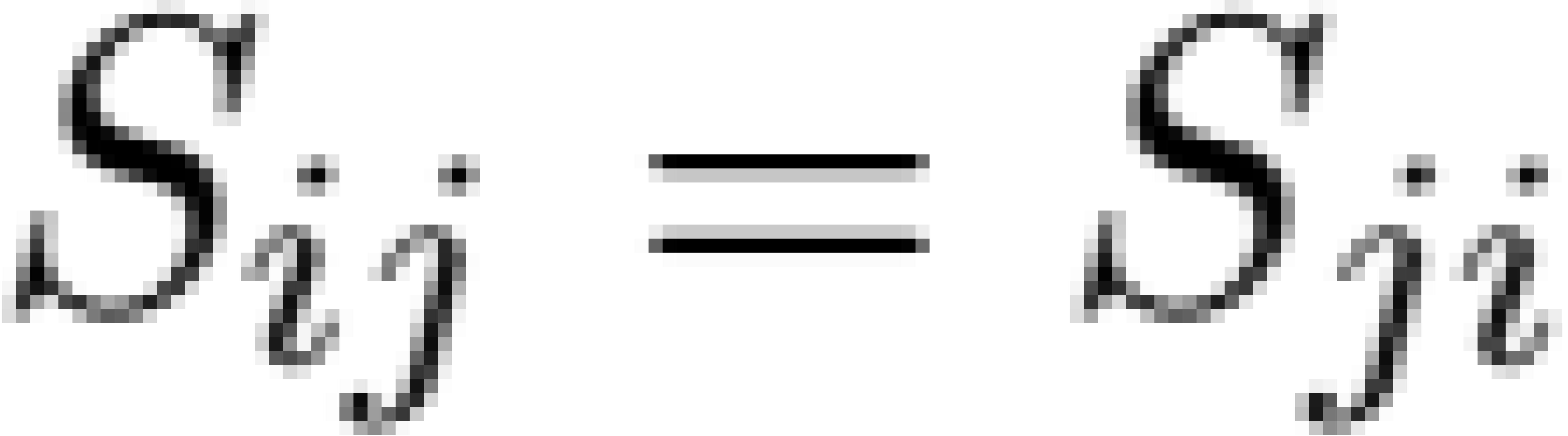


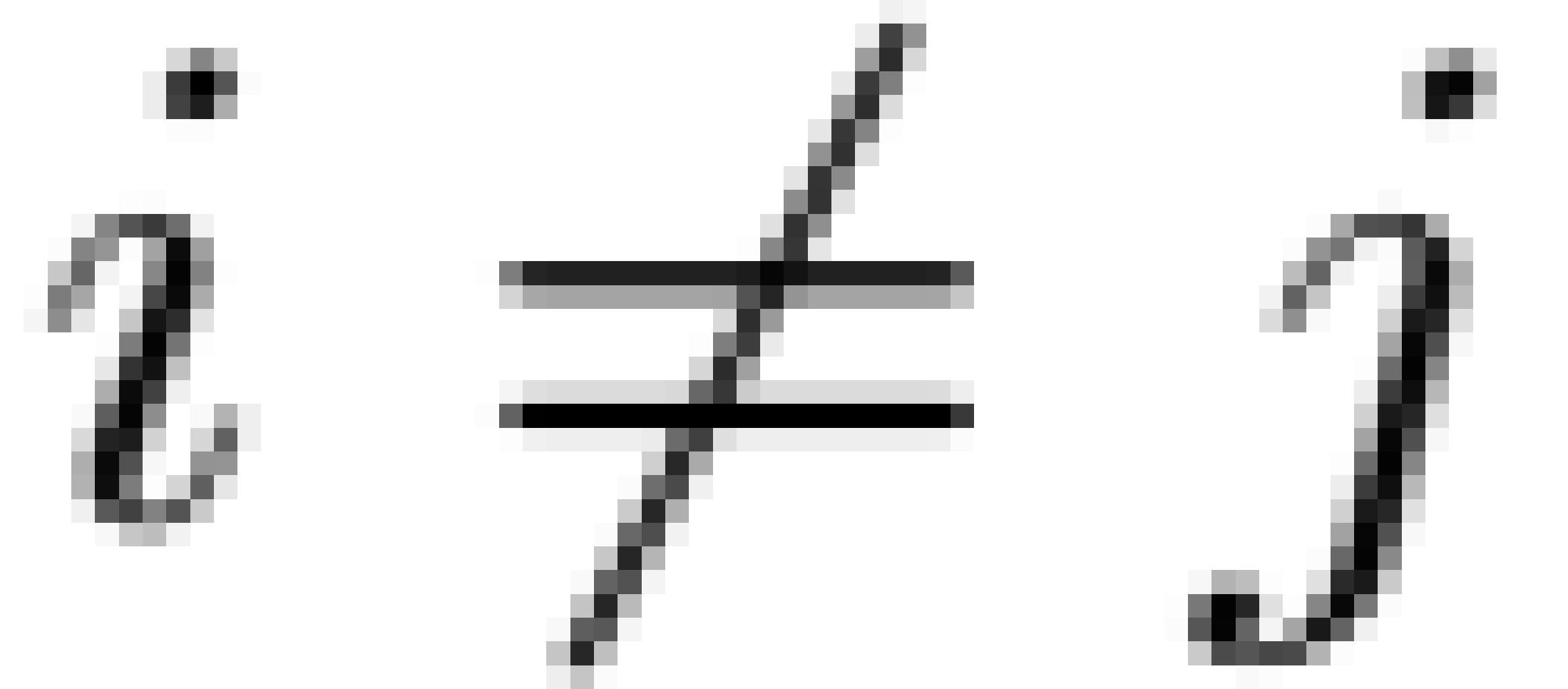


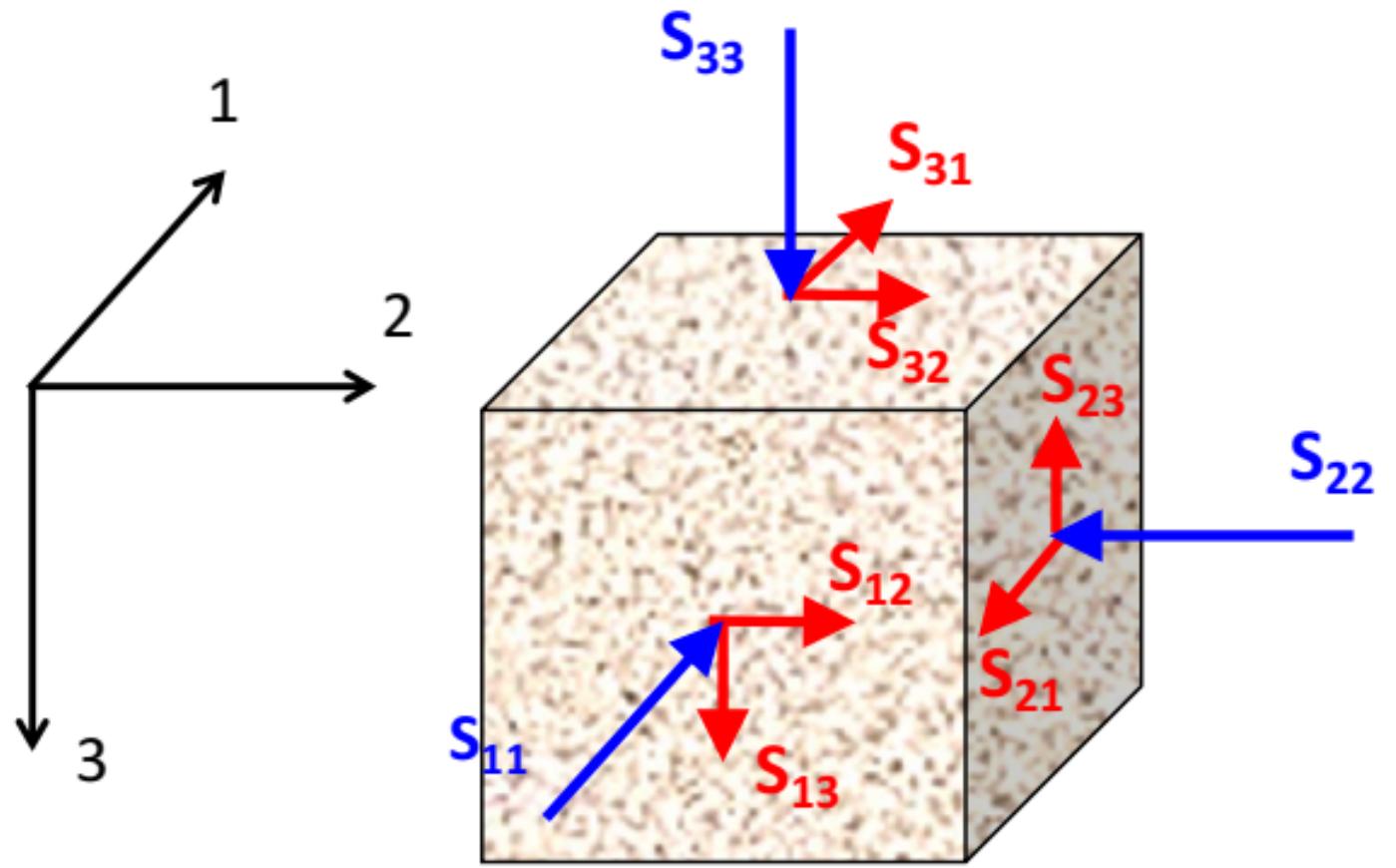






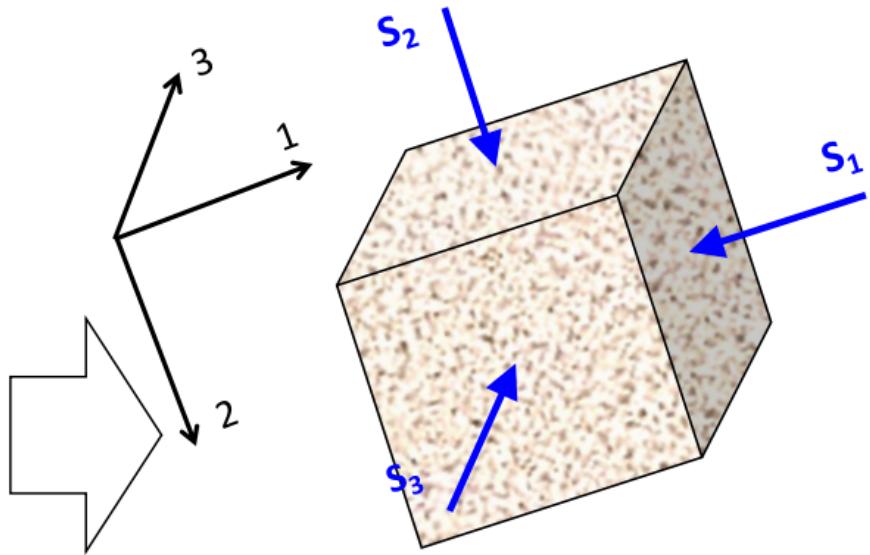
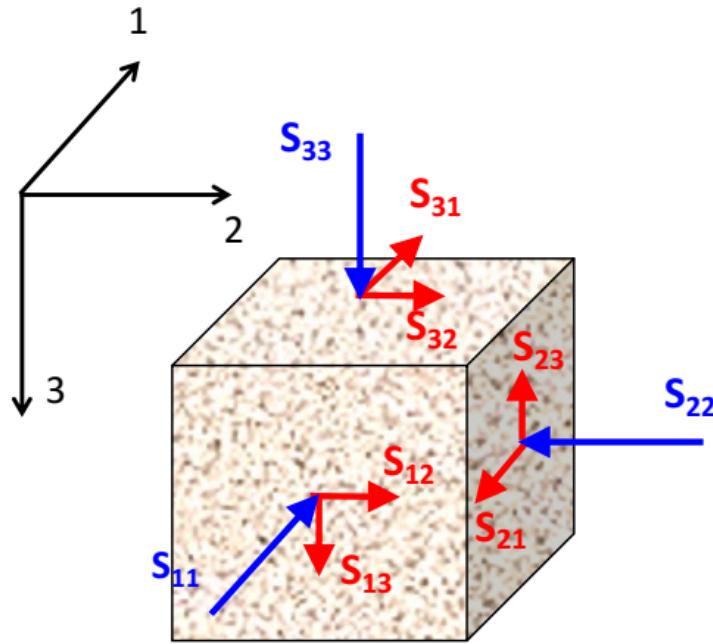






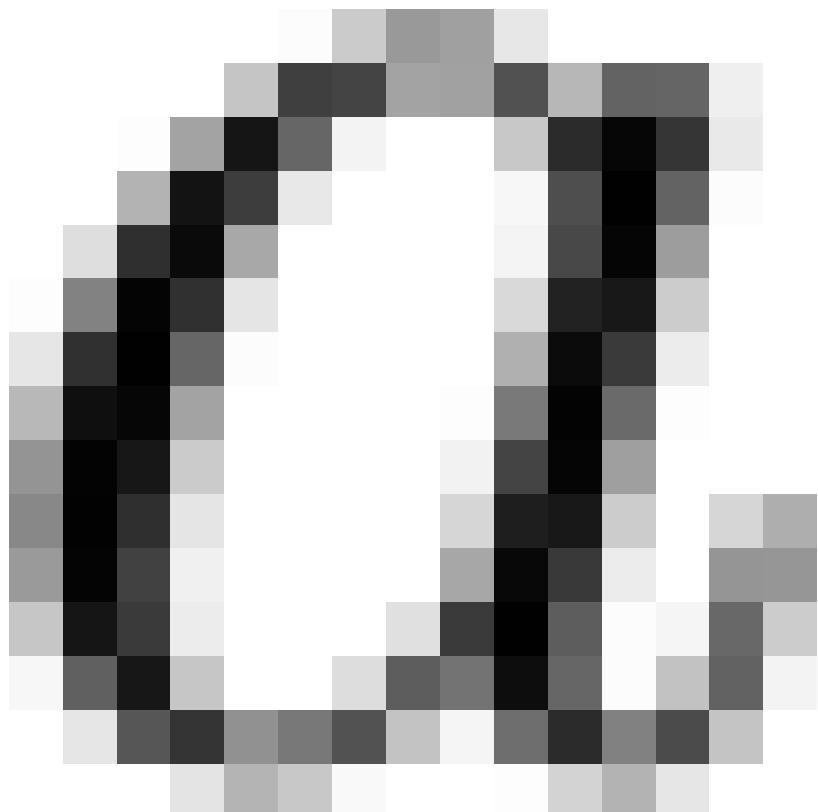
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

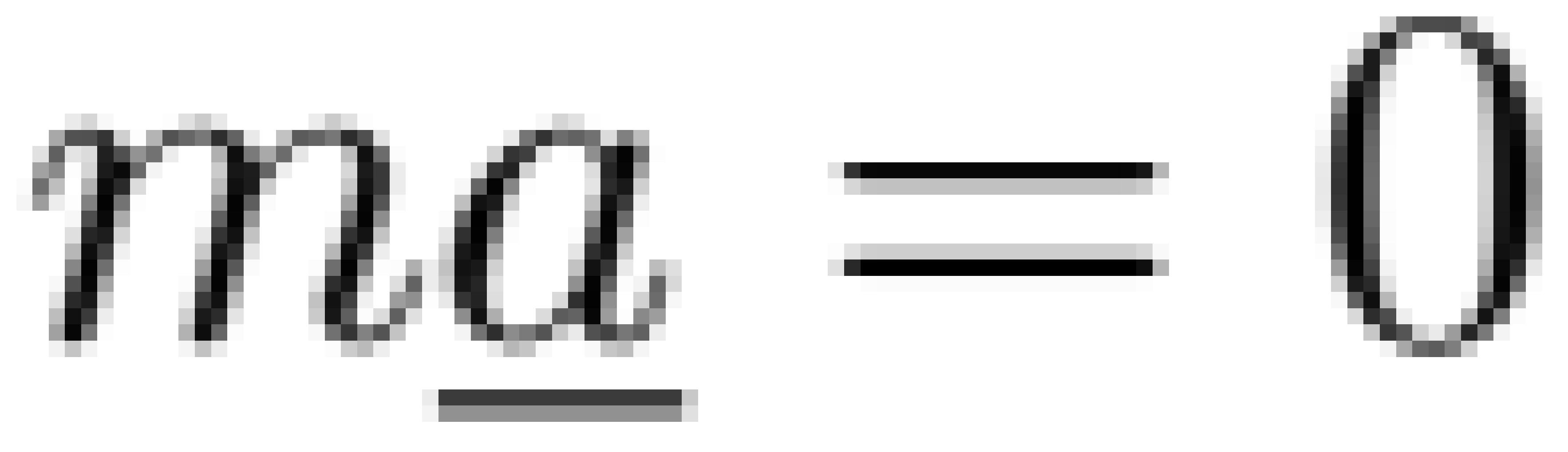


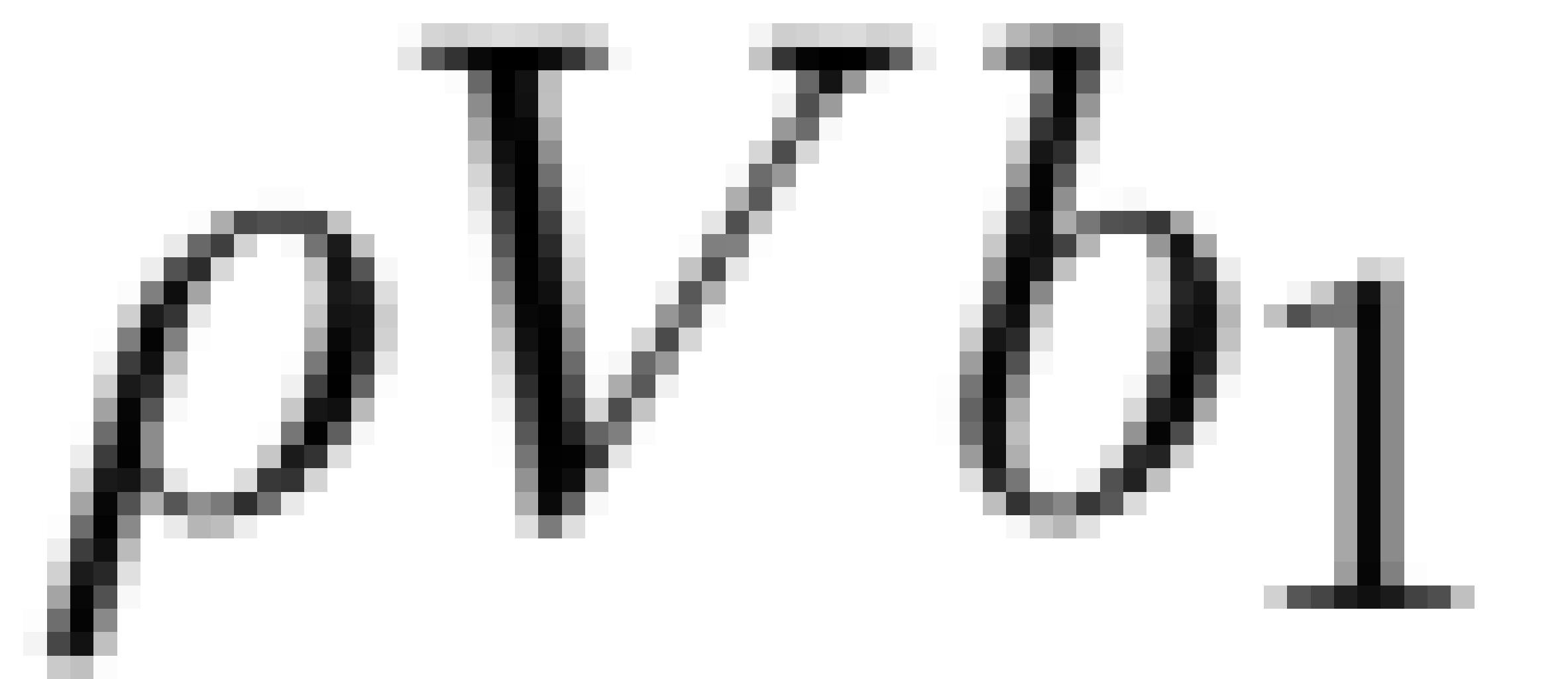


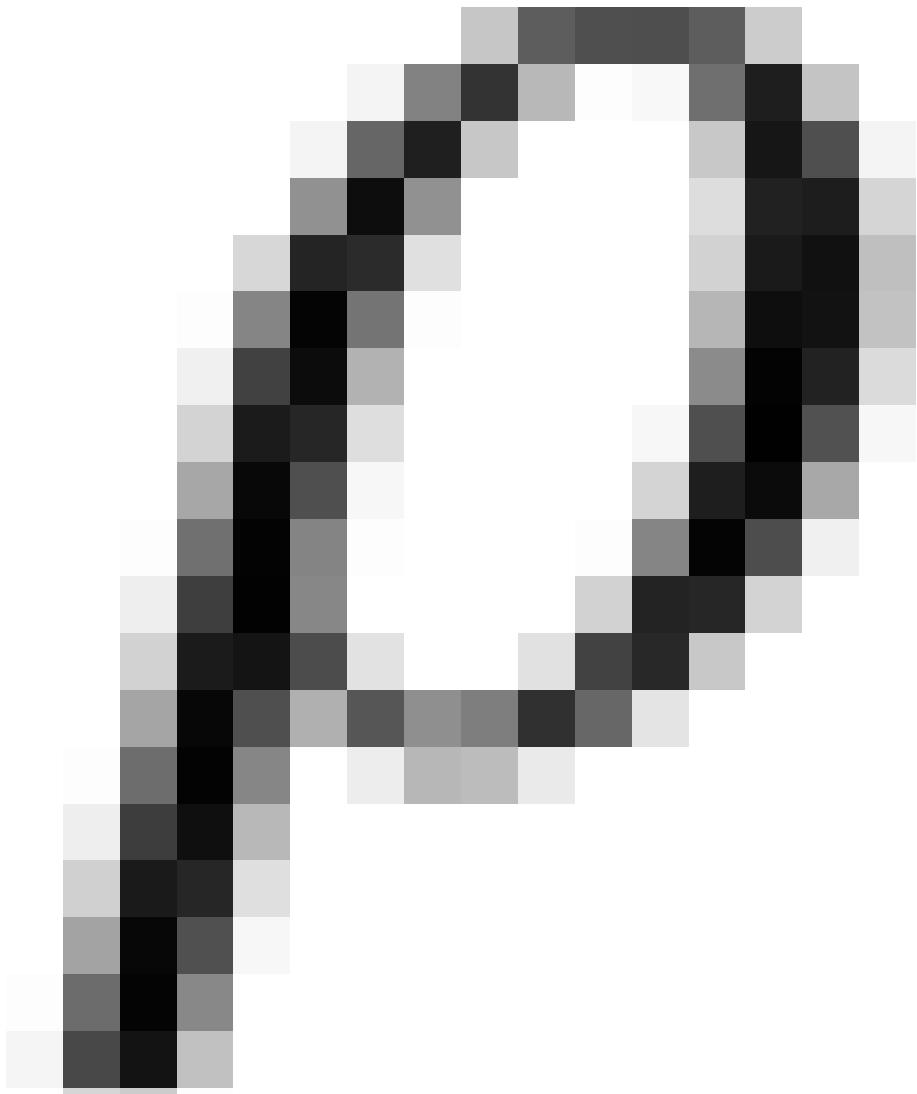
$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

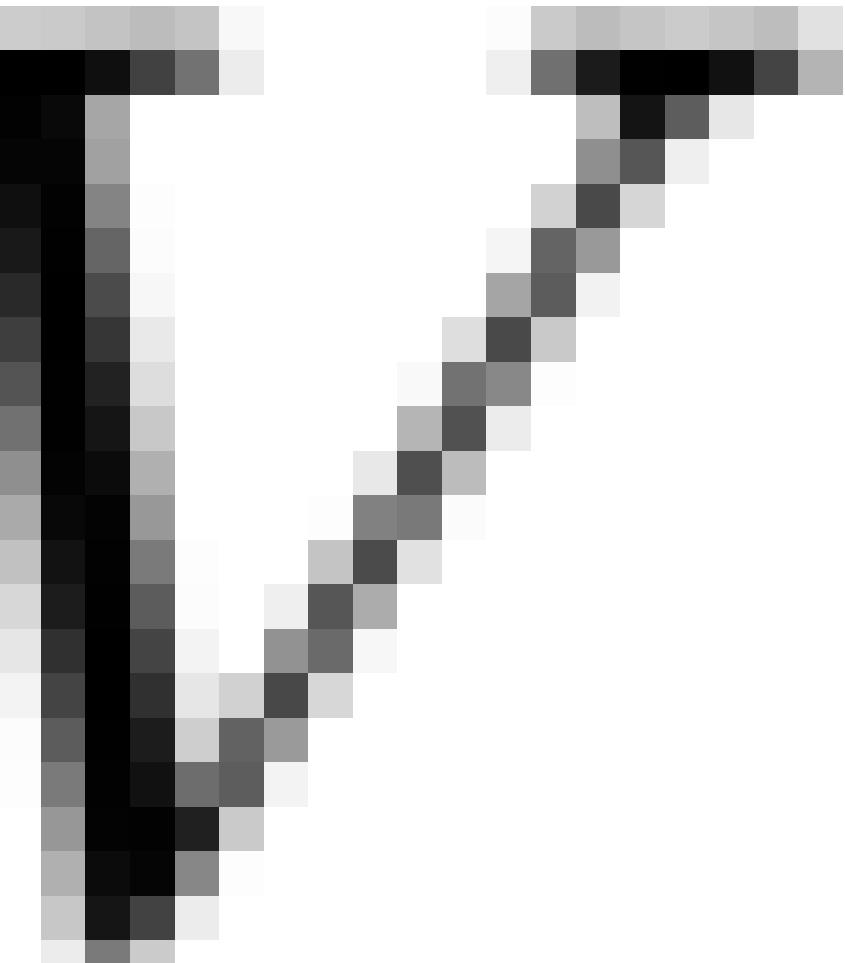
$$\underline{\underline{S}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

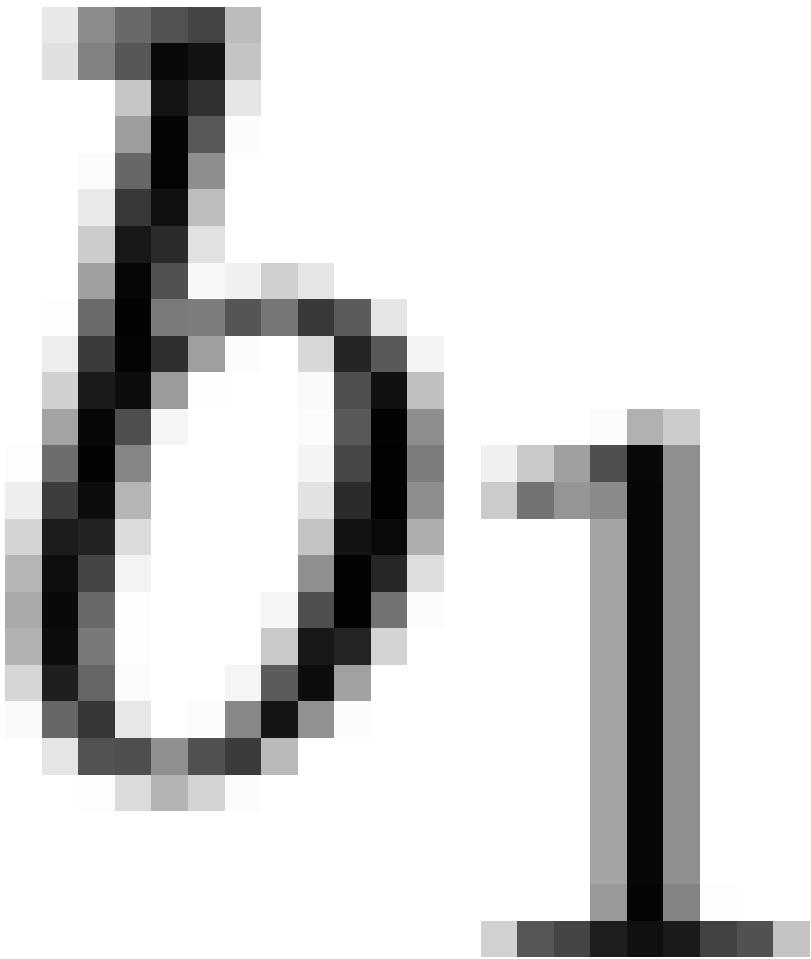








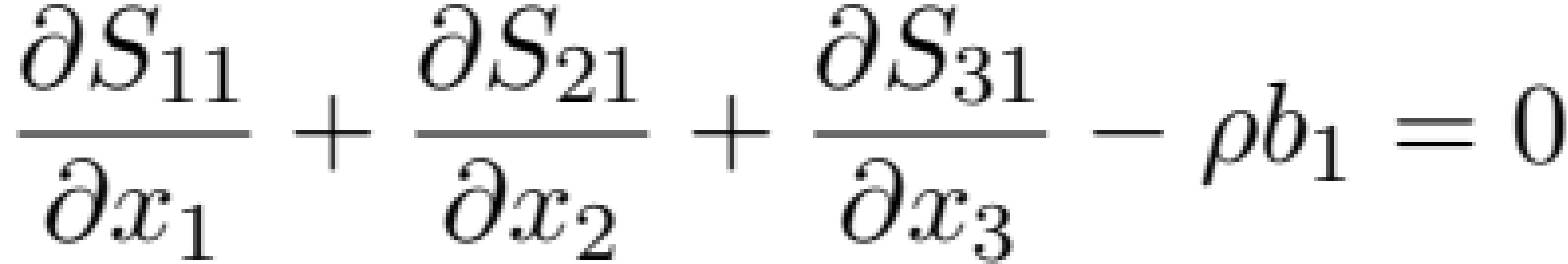


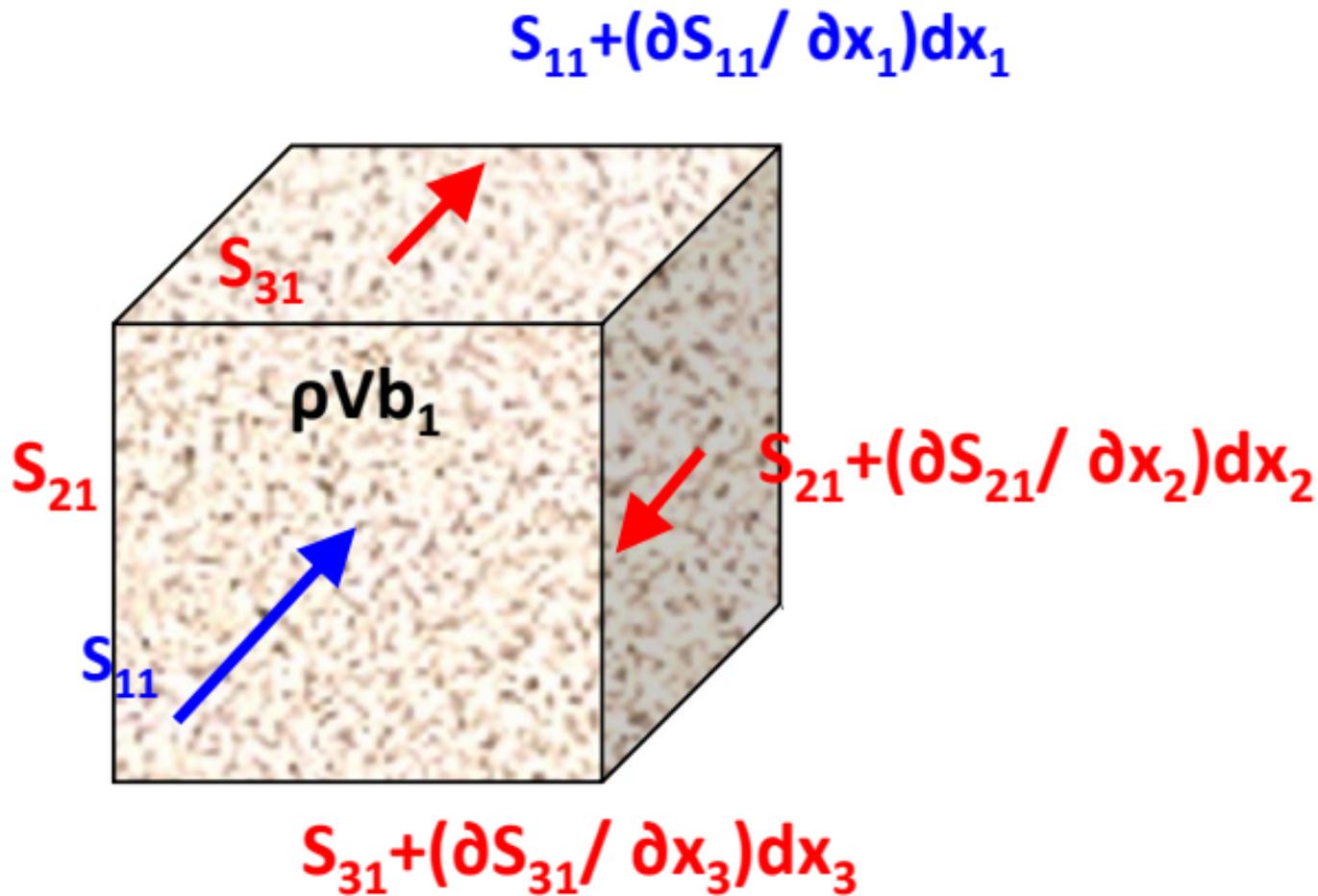
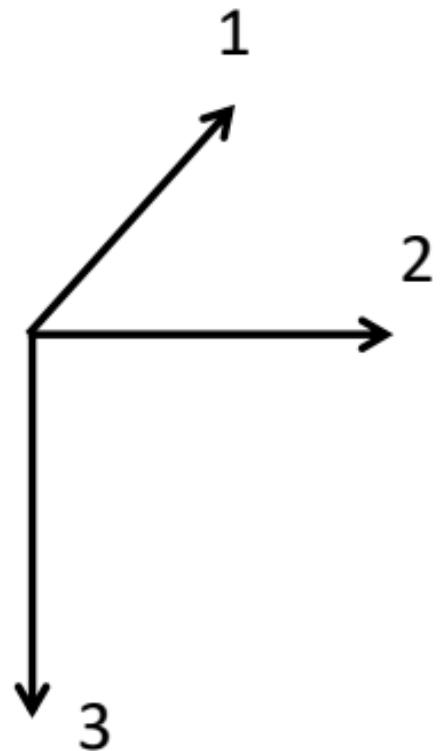


$$\sum F_1 = 0$$

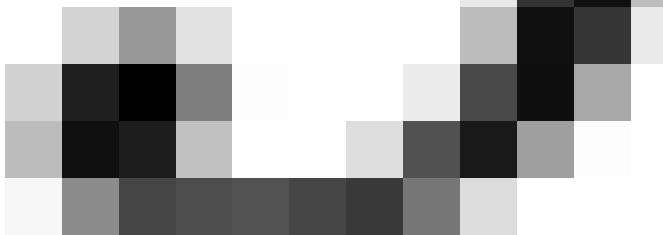
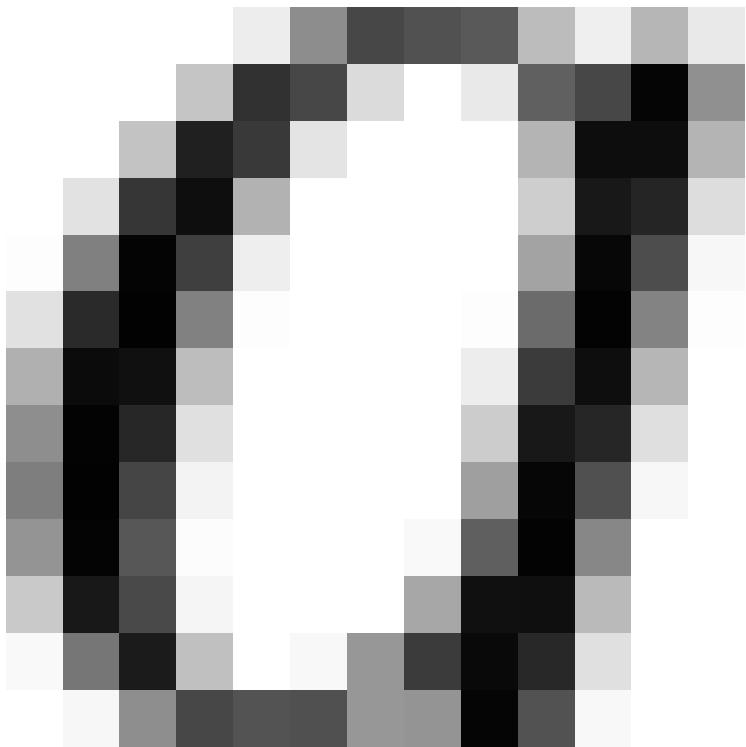
$$\begin{aligned}\sum F_1 &= +S_{11}dx_2dx_3 - \left[S_{11} + \left(\frac{\partial S_{11}}{\partial x_1} \right) dx_1 \right] dx_2dx_3 \\ &\quad + S_{21}dx_1dx_3 - \left[S_{21} + \left(\frac{\partial S_{21}}{\partial x_2} \right) dx_2 \right] dx_1dx_3 \\ &\quad + S_{31}dx_1dx_2 - \left[S_{31} + \left(\frac{\partial S_{31}}{\partial x_3} \right) dx_3 \right] dx_1dx_2 \\ &\quad - \rho(dx_1dx_2dx_3)b_1 = 0\end{aligned}$$

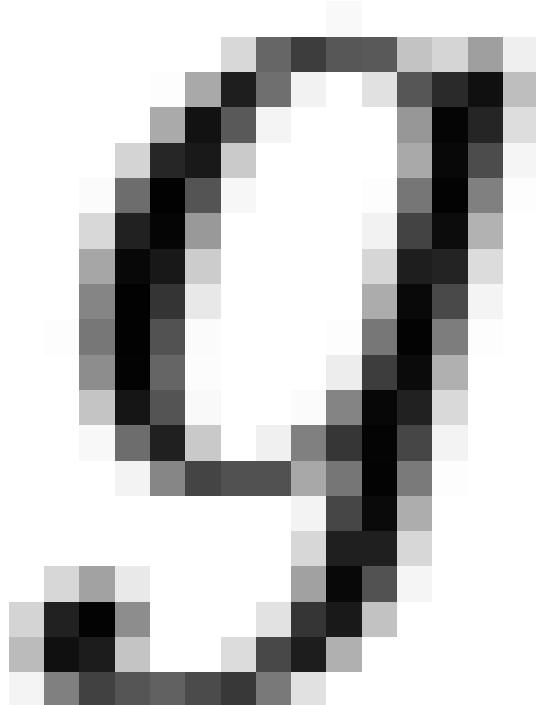






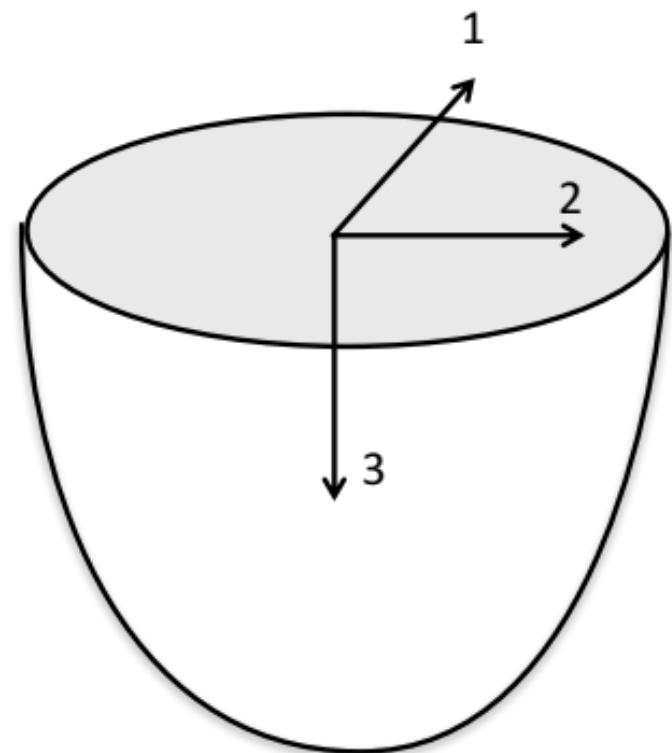
$$\left\{ \begin{array}{l} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} - \rho b_1 = 0 \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} - \rho b_2 = 0 \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} - \rho b_3 = 0 \end{array} \right.$$







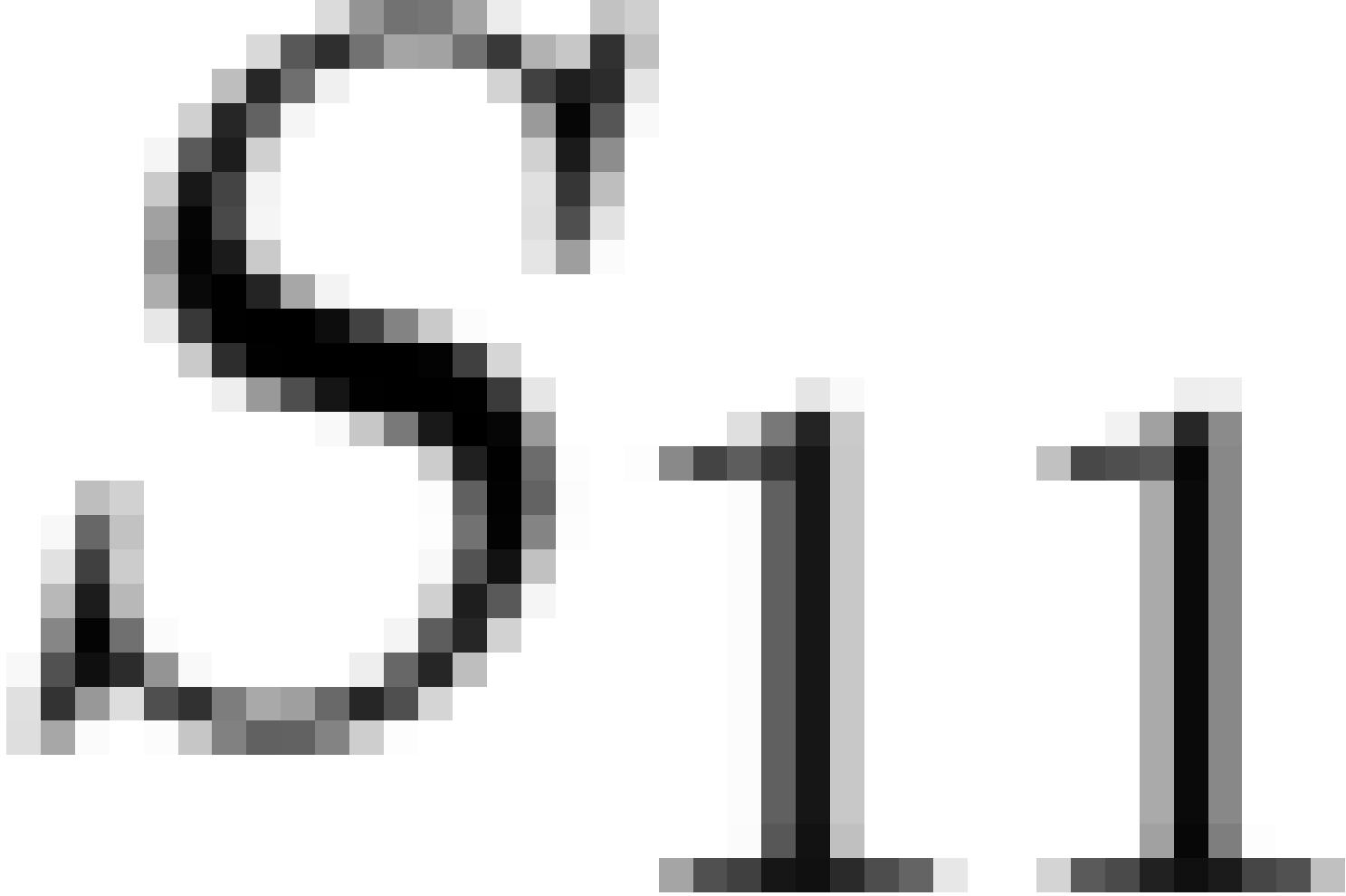
$$s_{33}(c_3) = \rho(c_3) \circ d_{c_3}$$



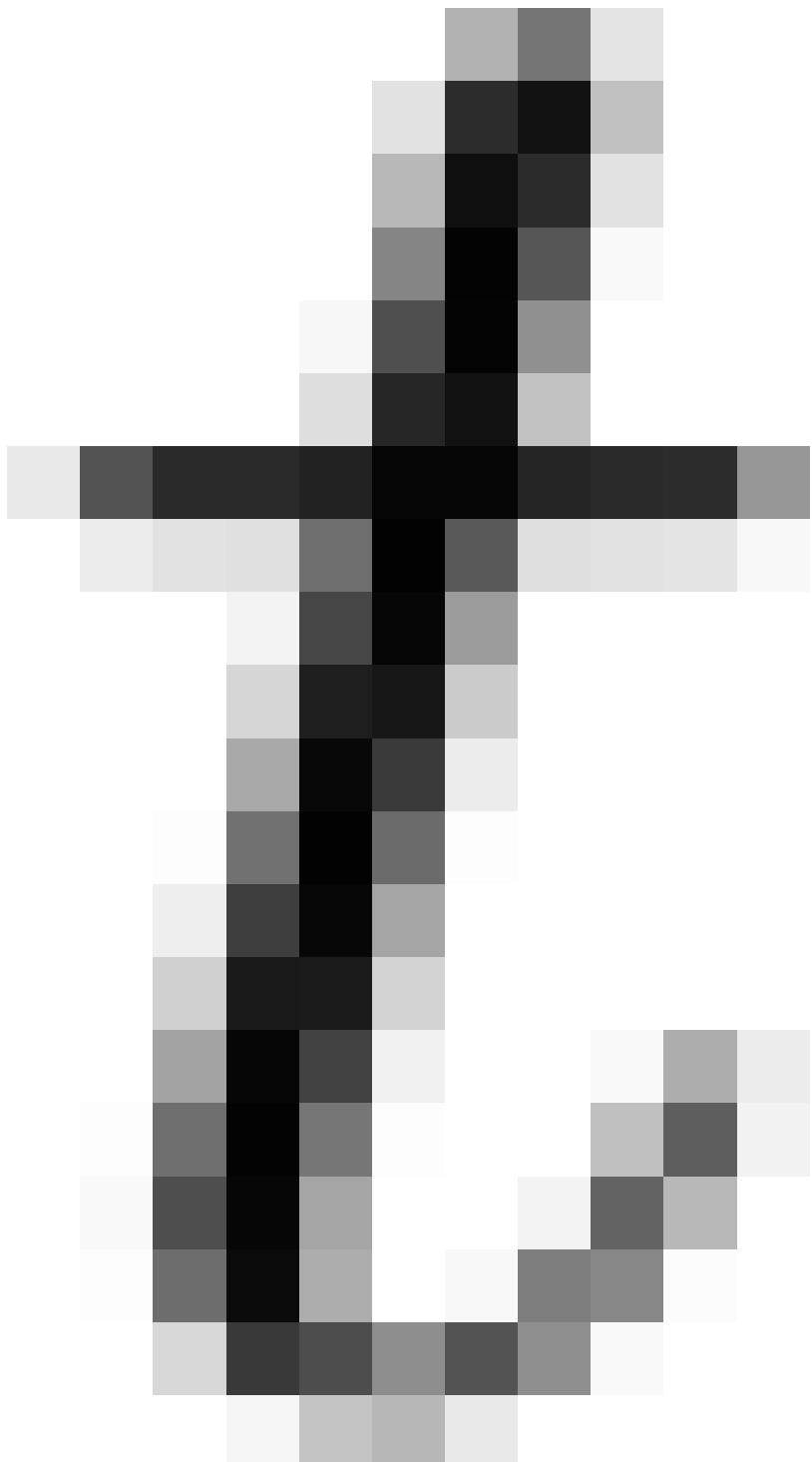
$$\left\{ \begin{array}{l} \cancel{\frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2}} \\ \cancel{\frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2}} \\ \cancel{\frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2}} \end{array} \right.$$

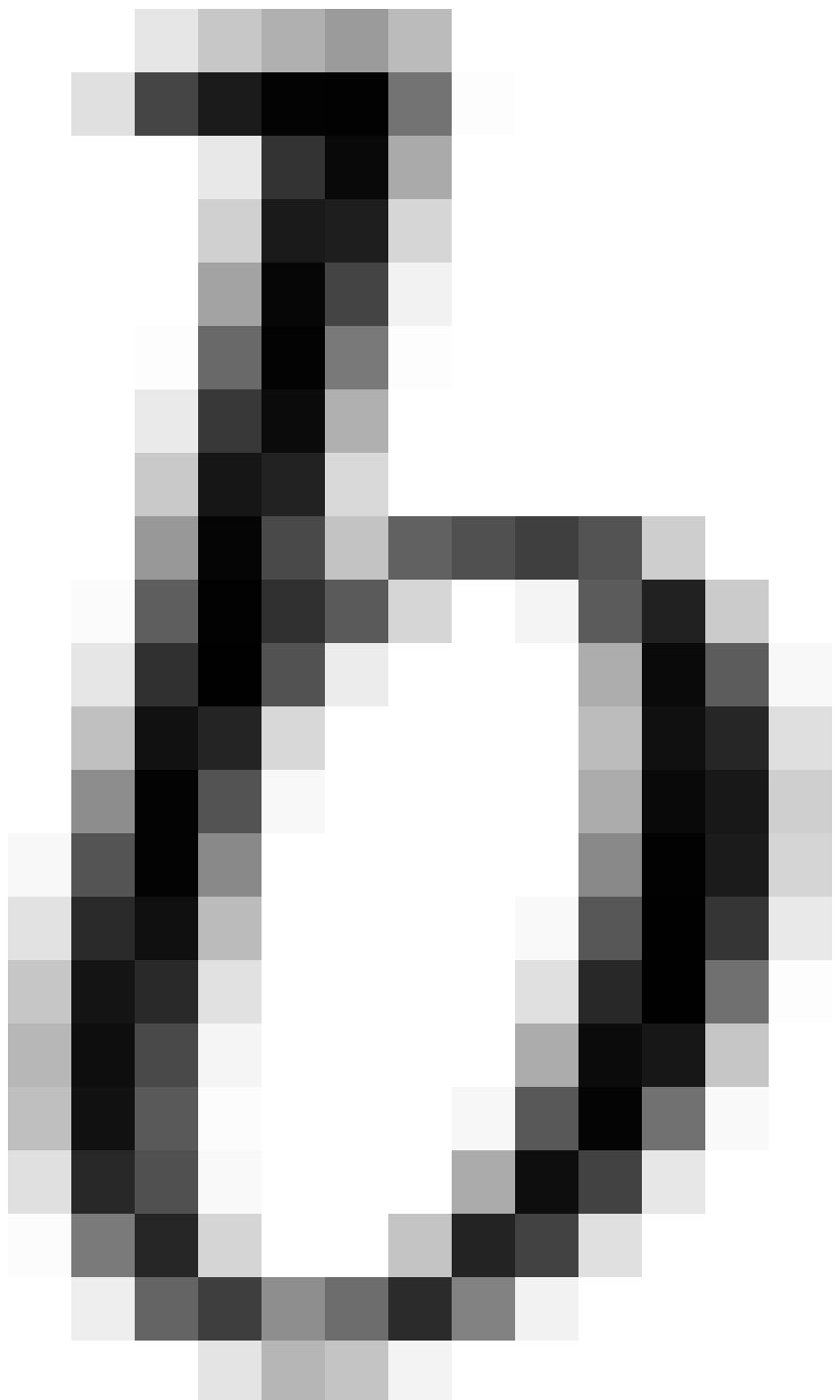
$$\frac{\partial S_{33}}{\partial x_3} - \rho(x_3)g = 0$$

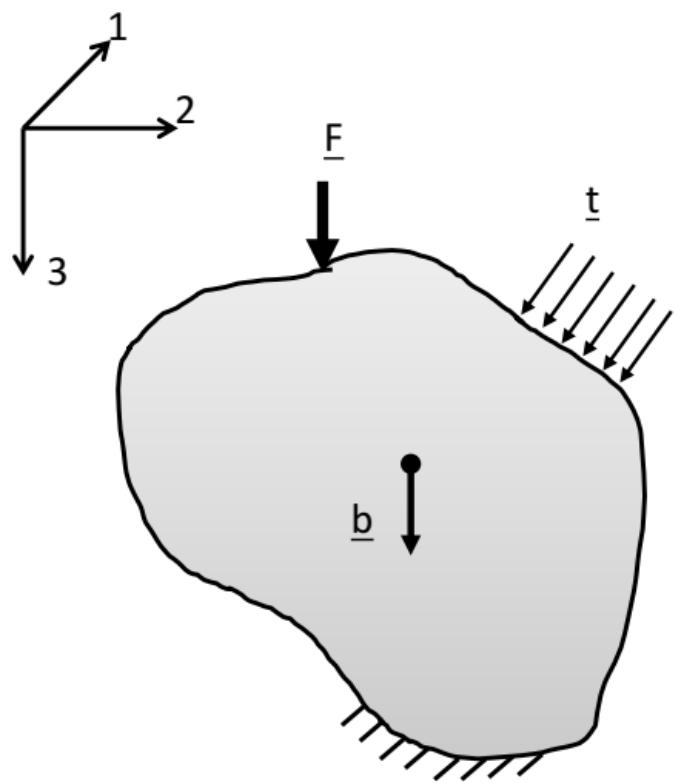
$$S_{33} = \int_0^{x_3} \rho(x_3)g \, dx_3$$











Displacement condition

$$\begin{cases} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2} \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2} \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2} \end{cases}$$

And respect the boundary conditions:

- Displacement
- Boundary stresses
- Boundary Forces
- Body Forces

How do we relate stresses to displacements?

- Displacements → Strains (**Kinematic equations**)
- Strains → Stresses (**Constitutive equations**)



ϵ_{11}



α_1

α_1

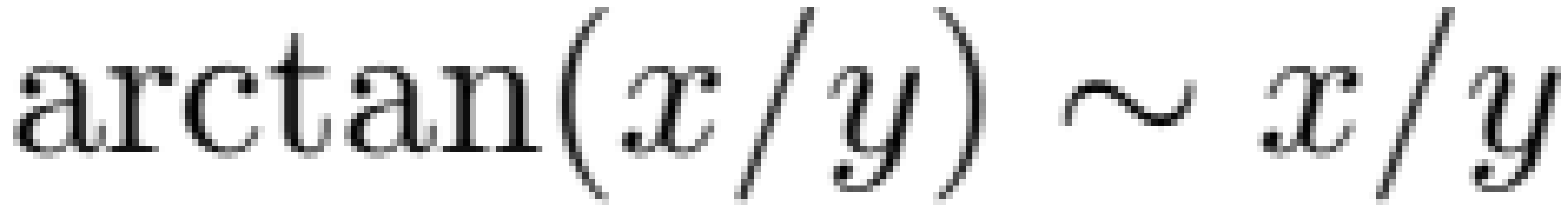
ϵ_{22}

$=$

α_2

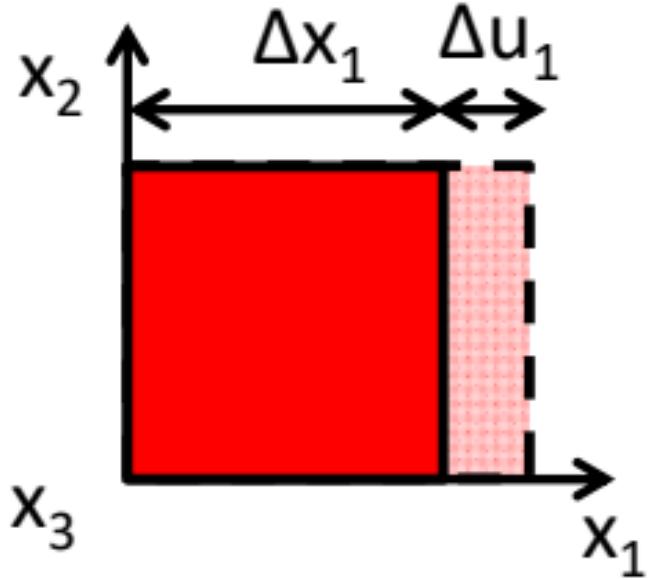
α_2

acc10(ut1) + acc10(ut2) + acc10(ut3)

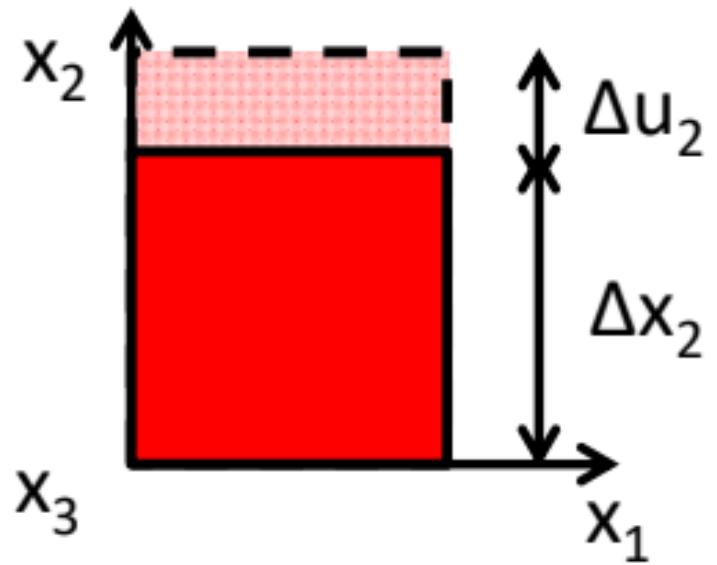


$\epsilon_{12} =$ $\frac{1}{2}$

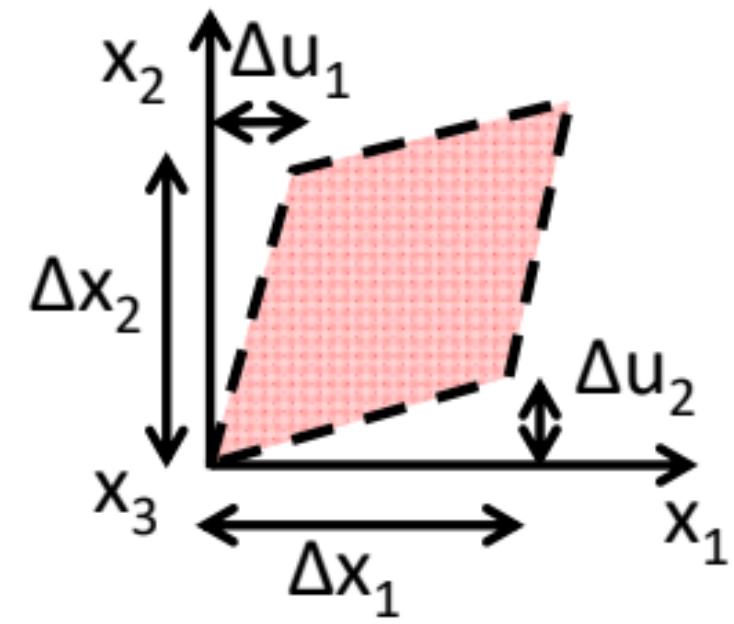
$$\frac{1}{2} \left(u_1 - u_2 + c_1 - c_2 \right)$$



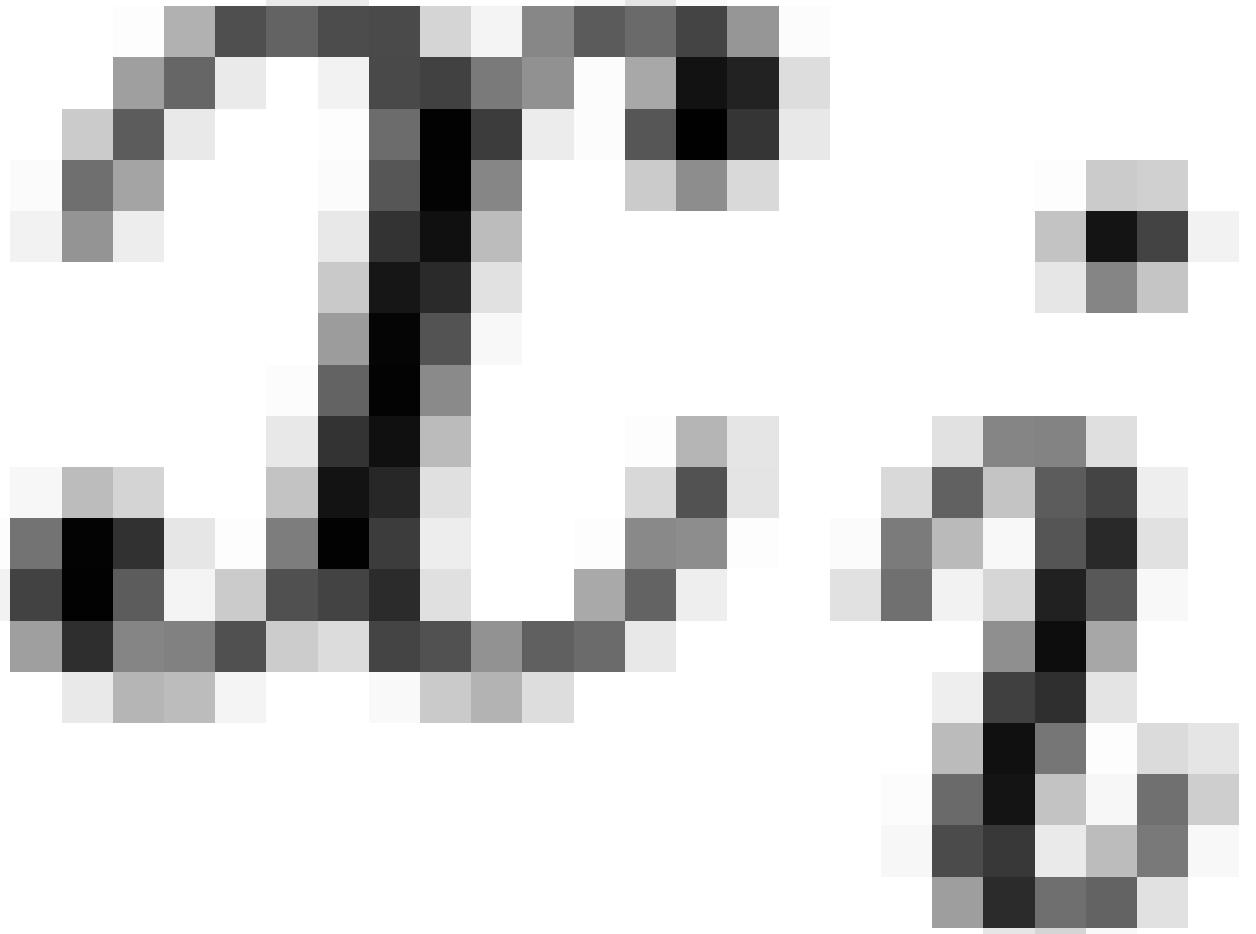
$$\varepsilon_{11} \approx \frac{\Delta u_1}{\Delta x_1}$$

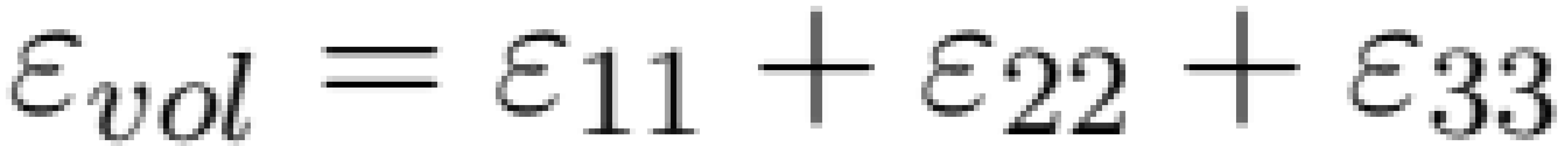


$$\varepsilon_{22} \approx \frac{\Delta u_2}{\Delta x_2}$$

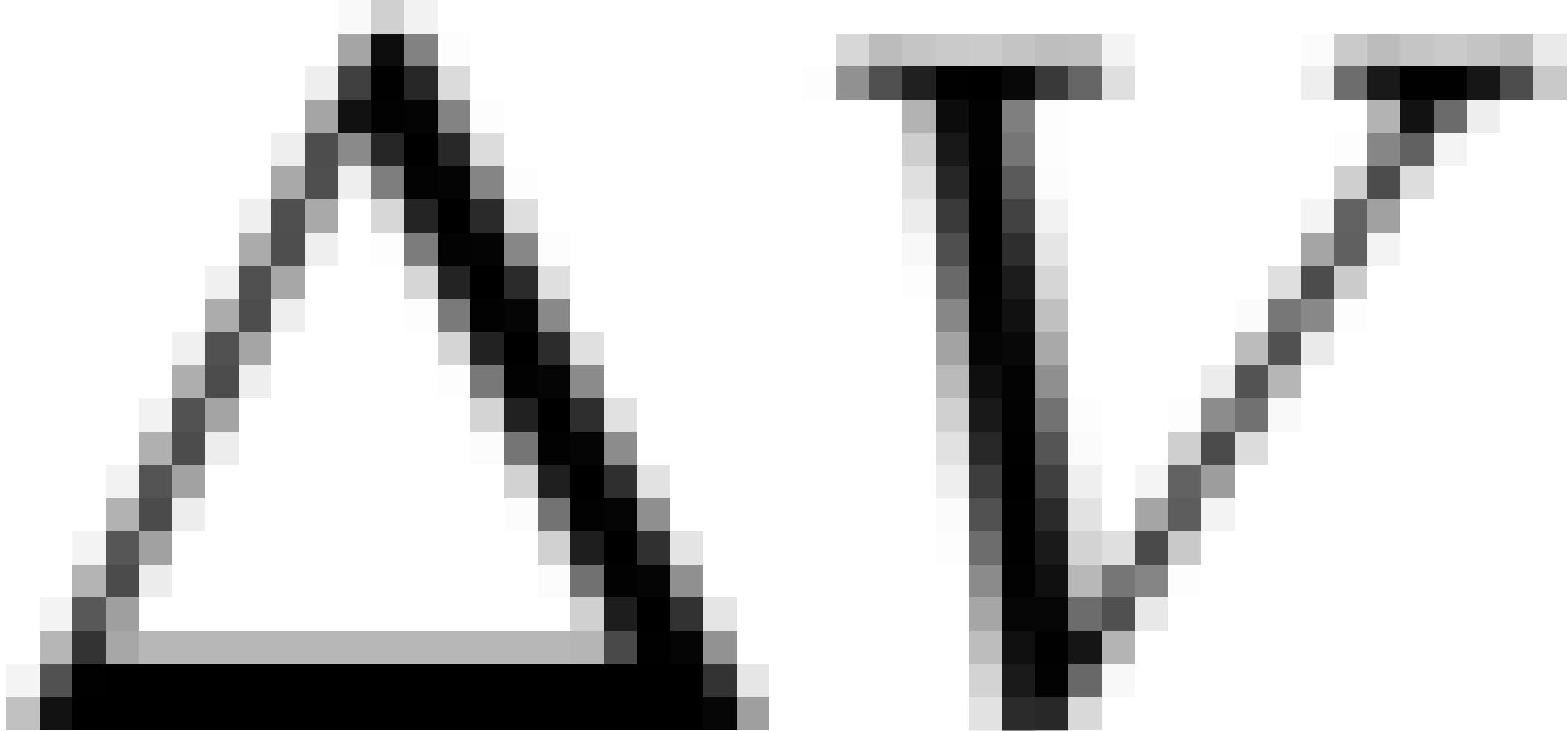


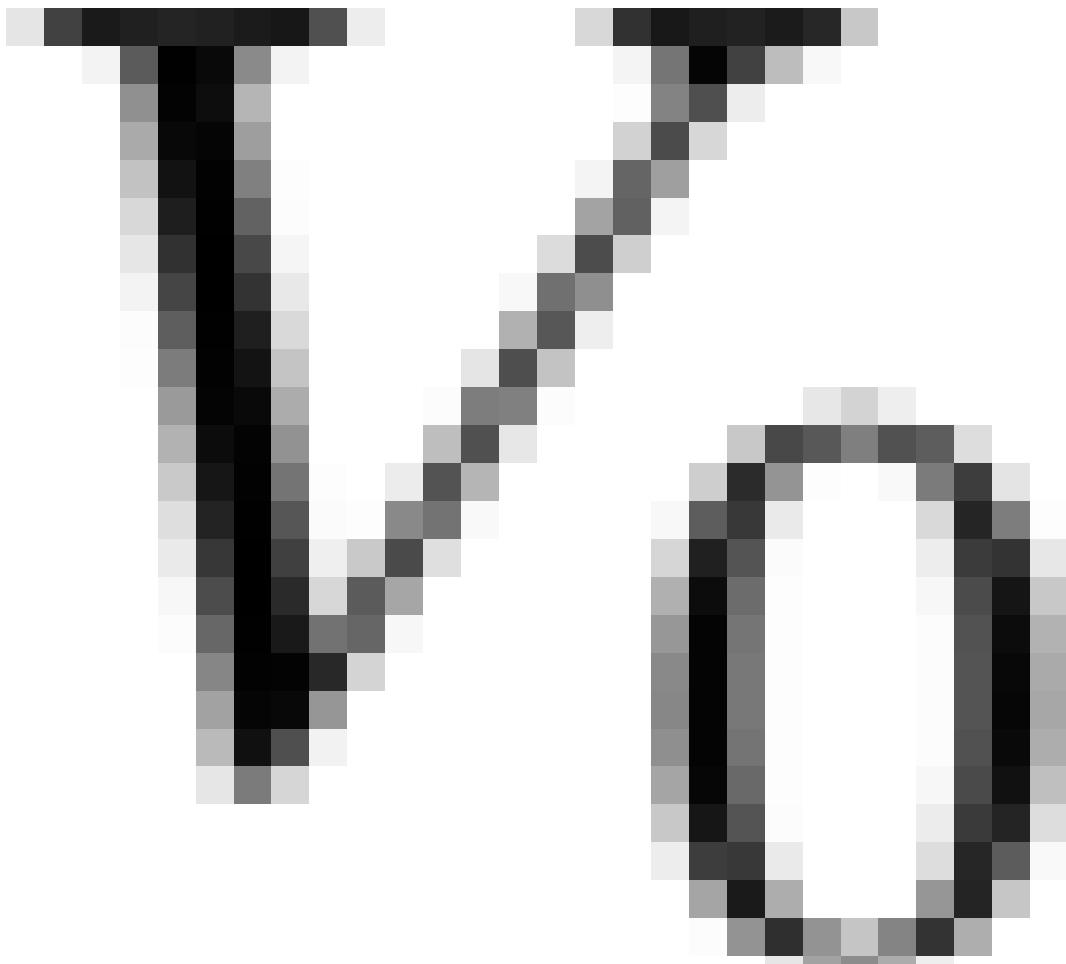
$$\varepsilon_{12} \approx \frac{1}{2} \left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1} \right)$$



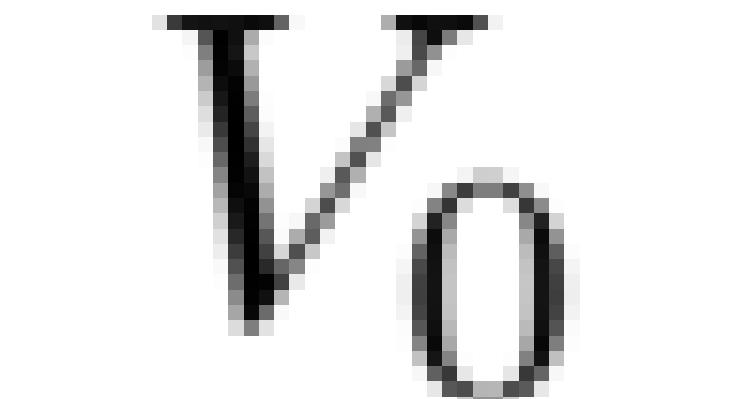
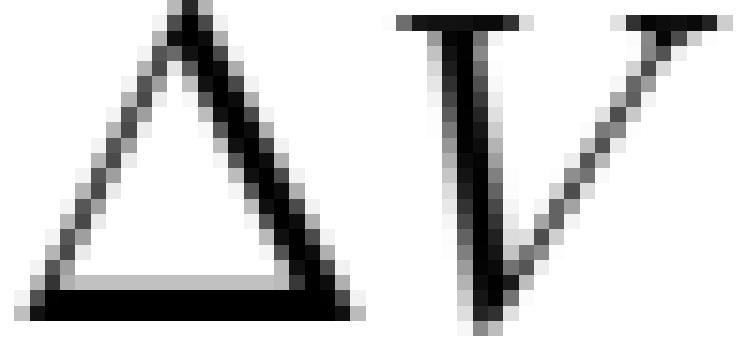




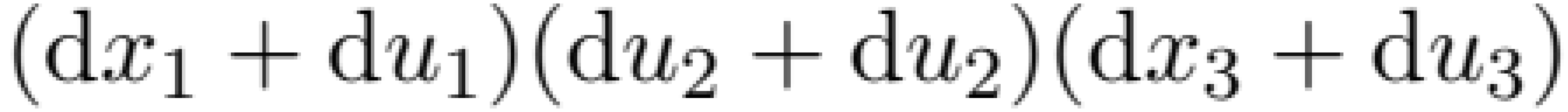




cool





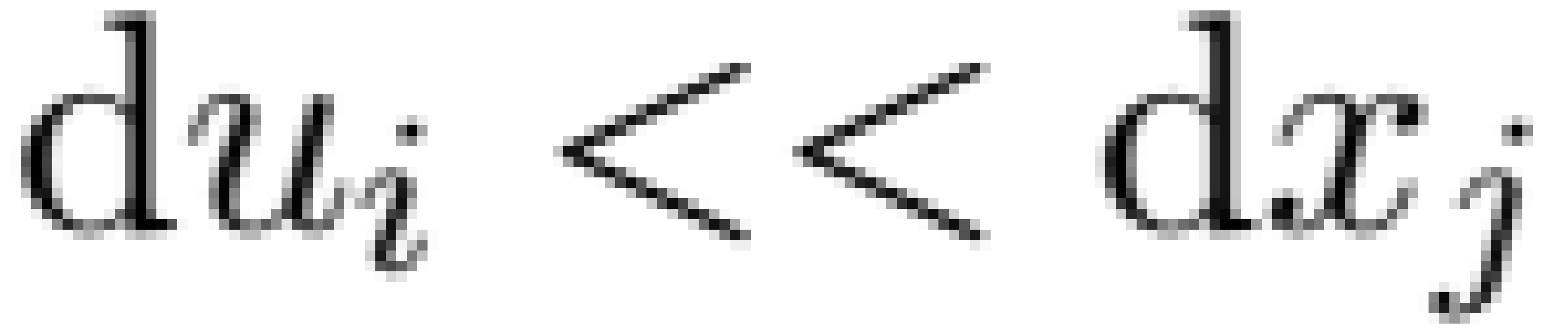


$$\epsilon_{vol} = \frac{[(dx_1 + du_1)(dx_2 + du_3) - (dx_1 dx_2 dx_3)]}{(dx_1 dx_2 dx_3)}$$



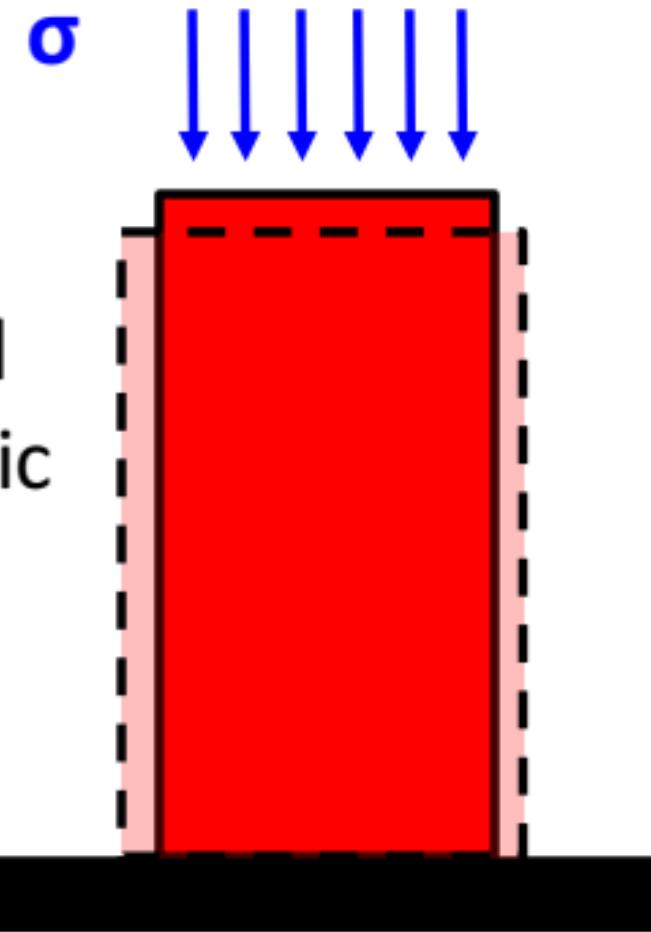




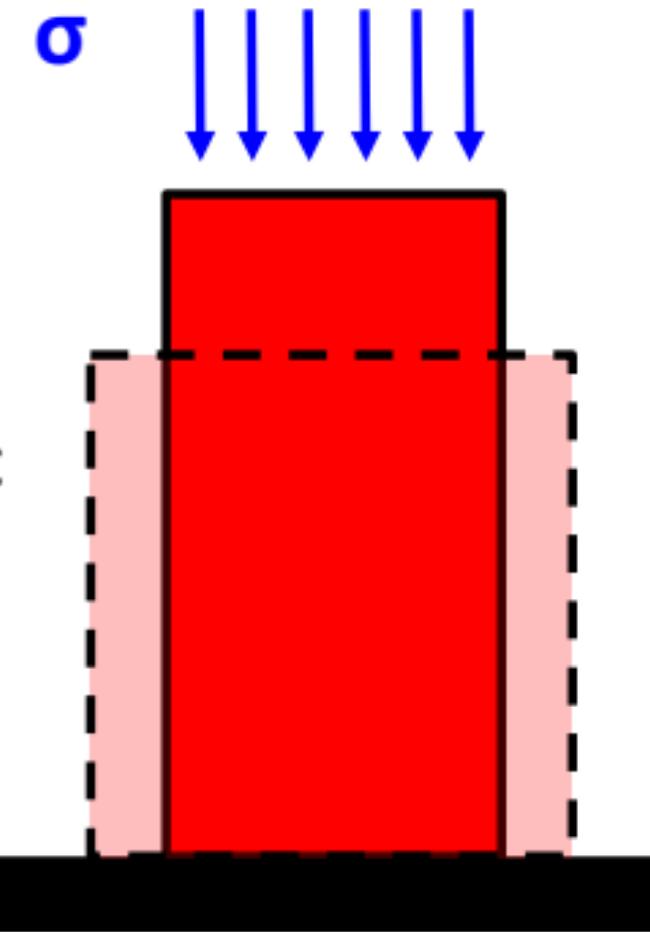


$$\text{vol}^2 \frac{(dx_1 dx_2 du_3 + dx_1 dx_3 du_2 + dx_2 dx_3 du_1)}{(dx_1 dx_2 dx_3)} = \frac{du_1}{\epsilon_{11}} + \frac{du_2}{\epsilon_{22}} + \frac{du_3}{\epsilon_{33}}$$

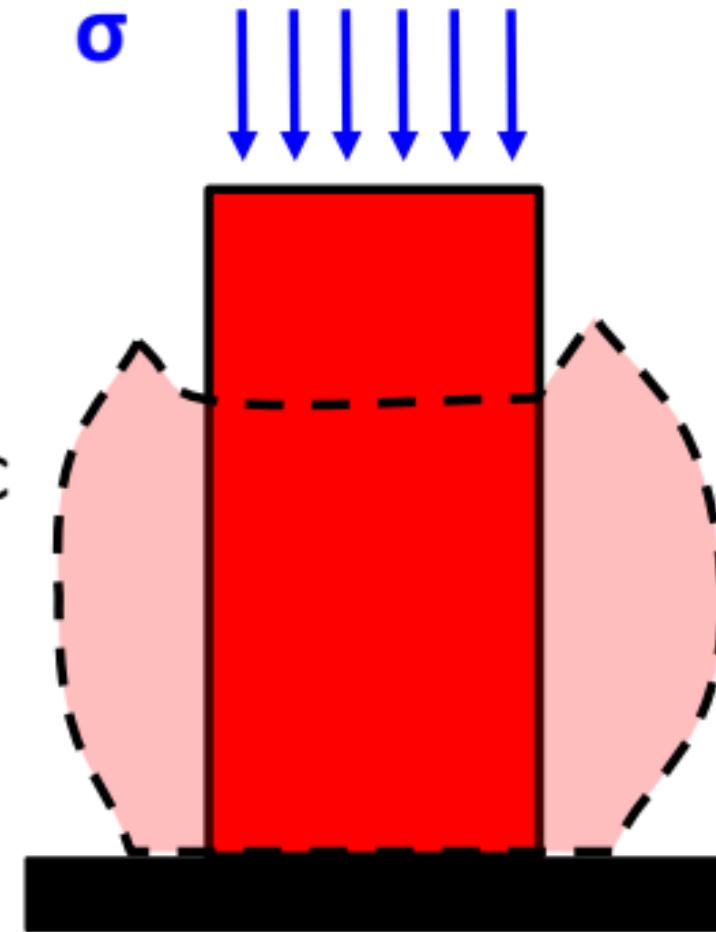
Hard
elastic
solid

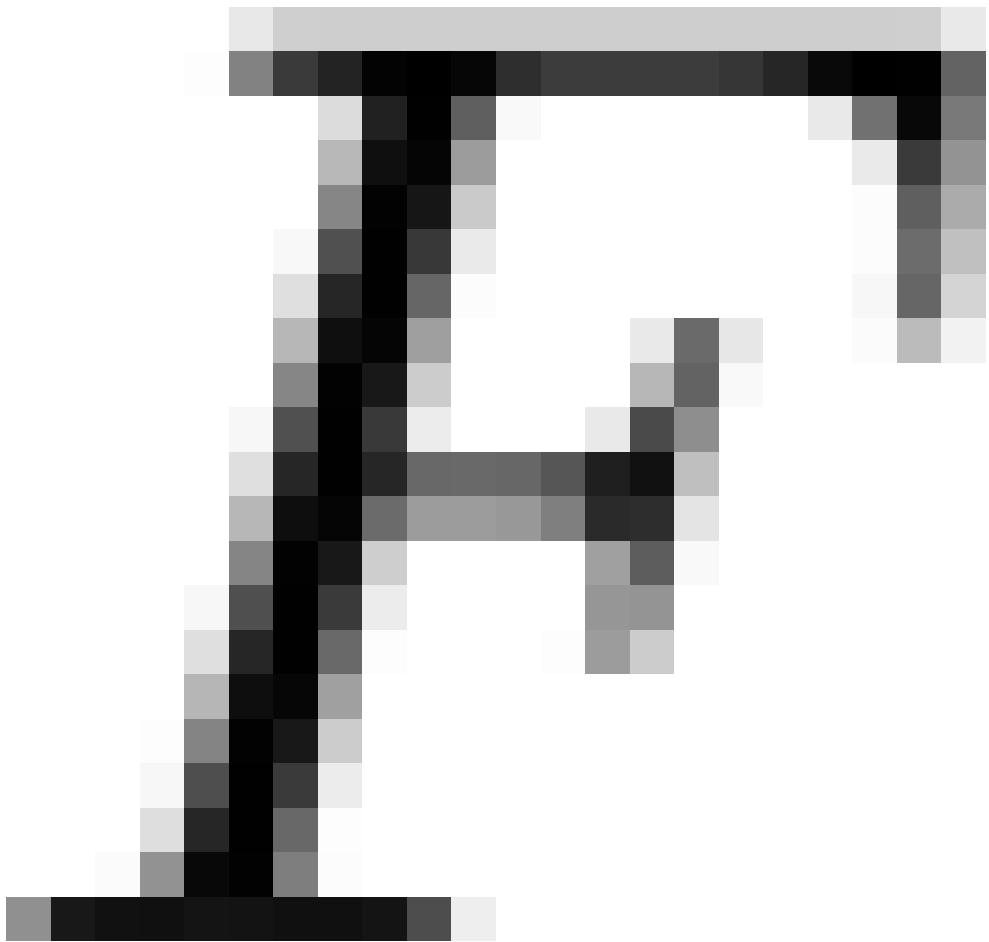


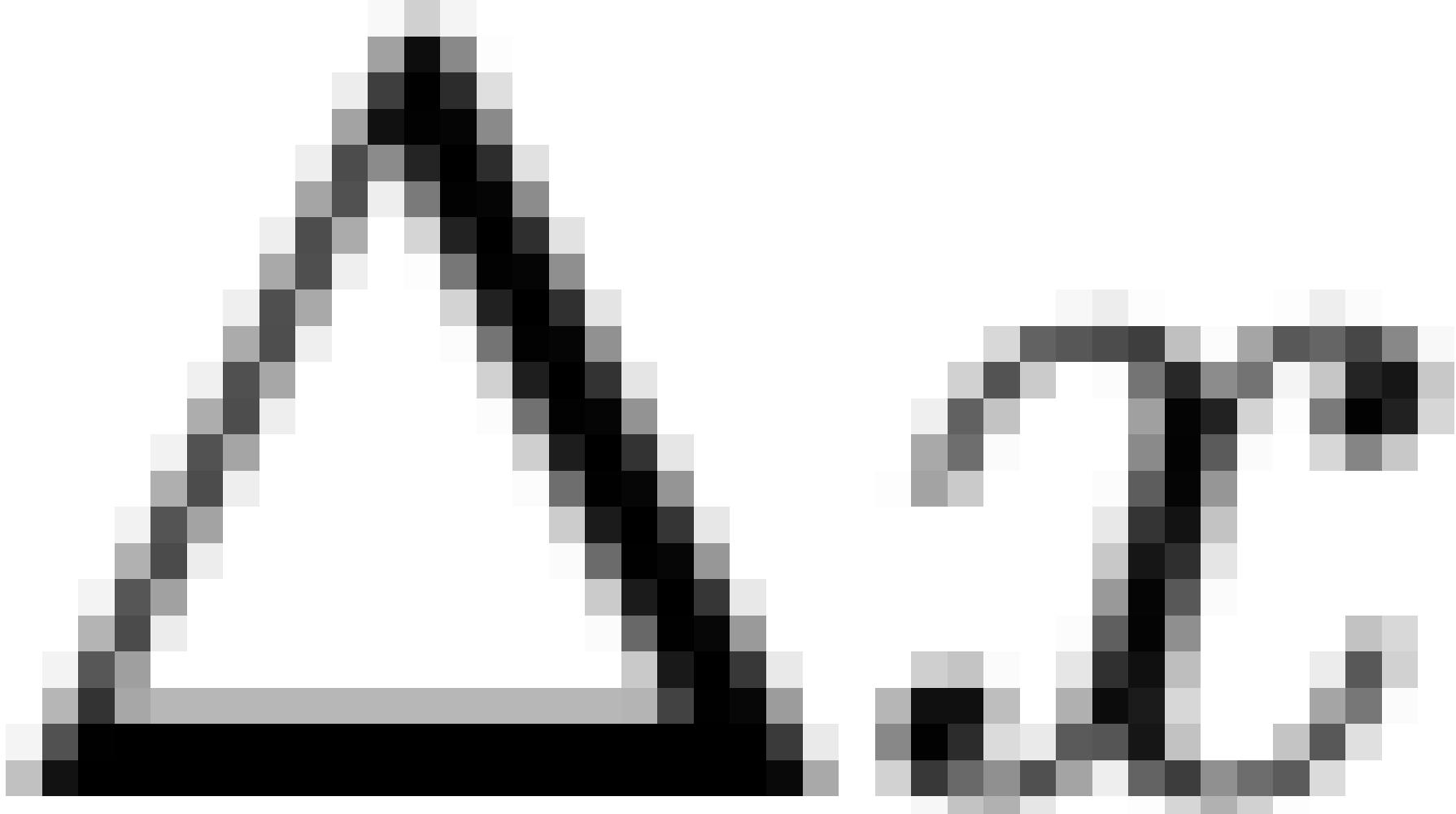
Soft
elastic
solid

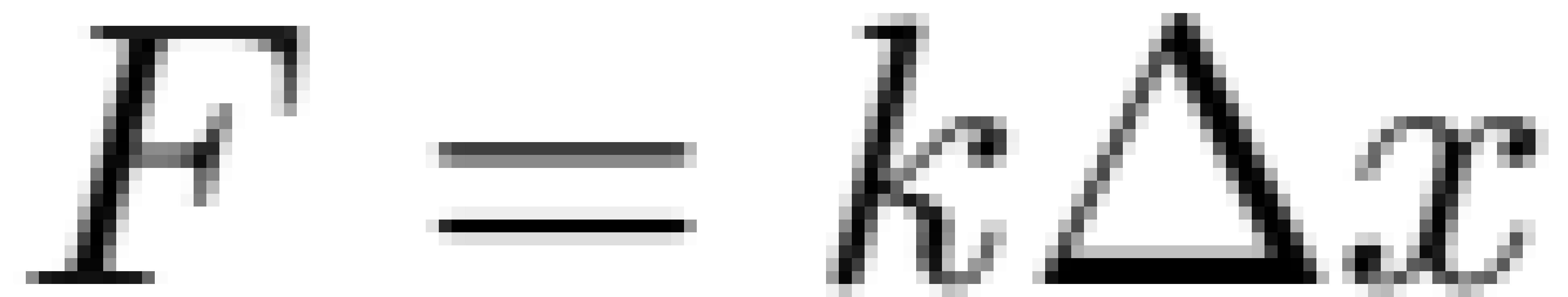


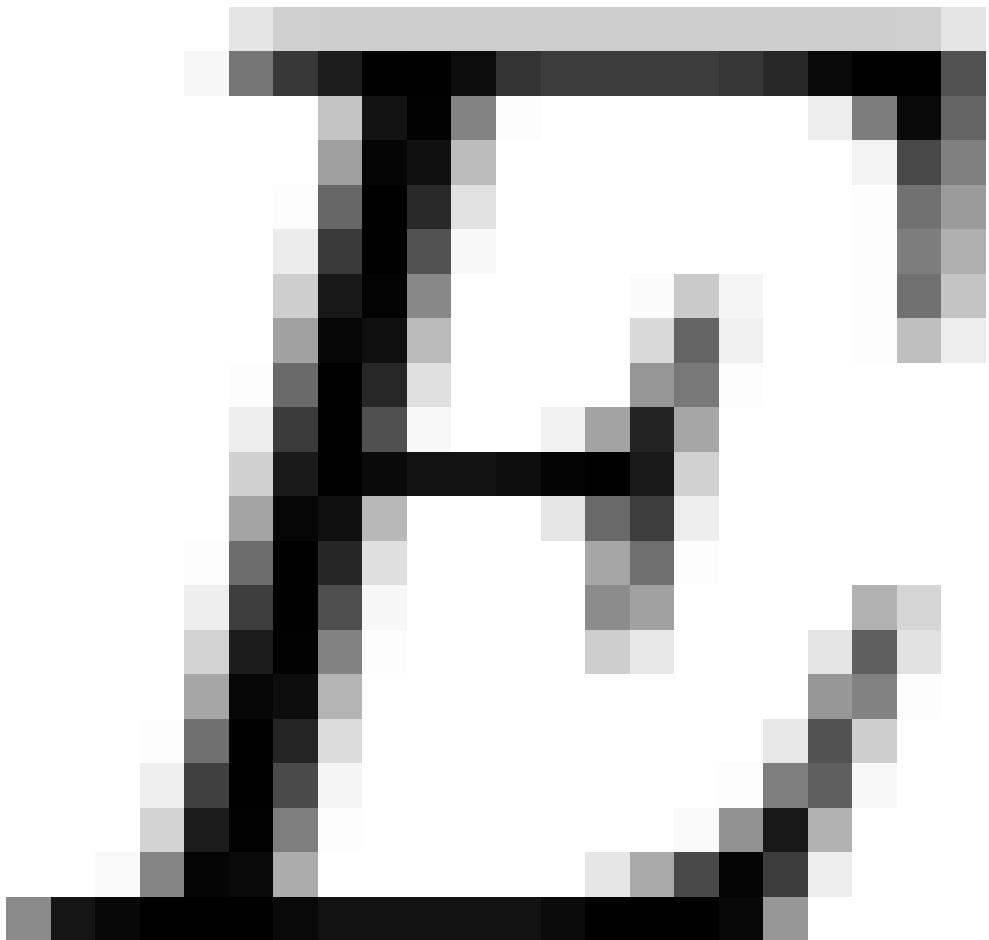
Soft
Visco-
plastic
solid











A

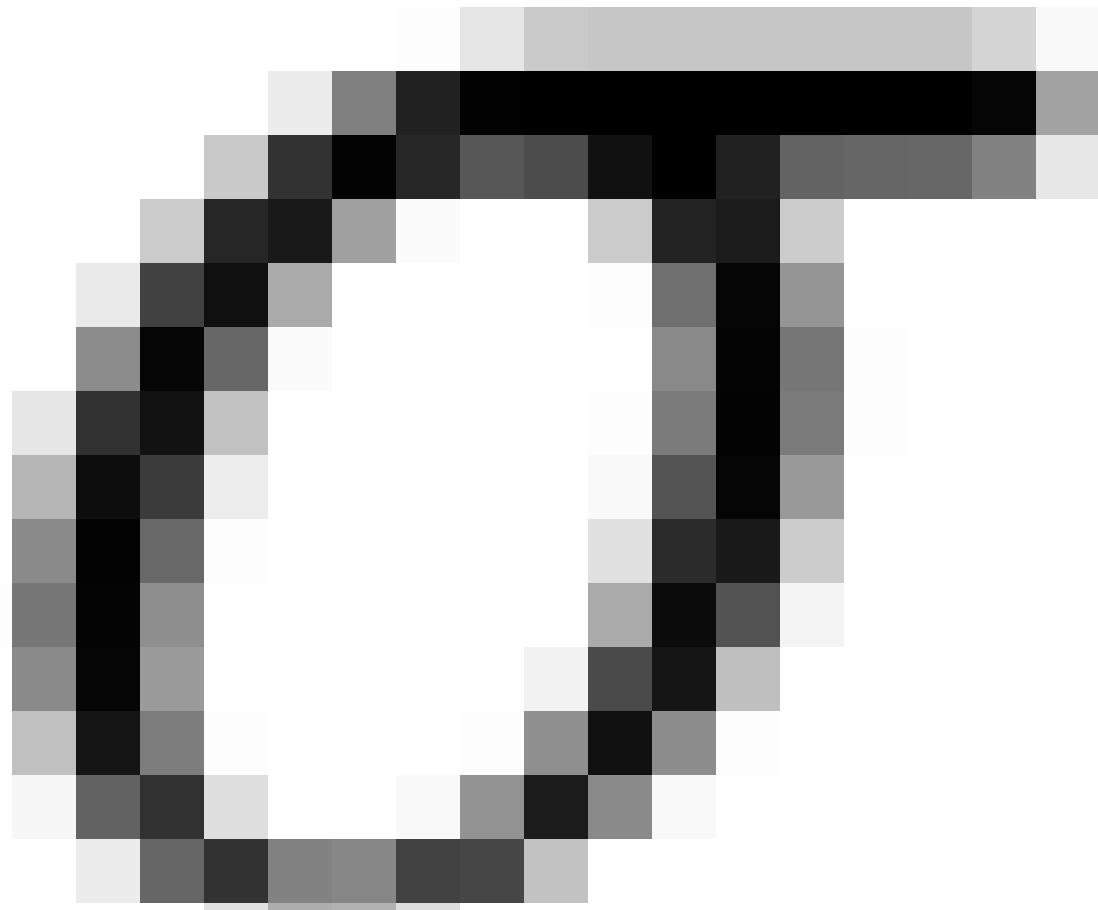
A

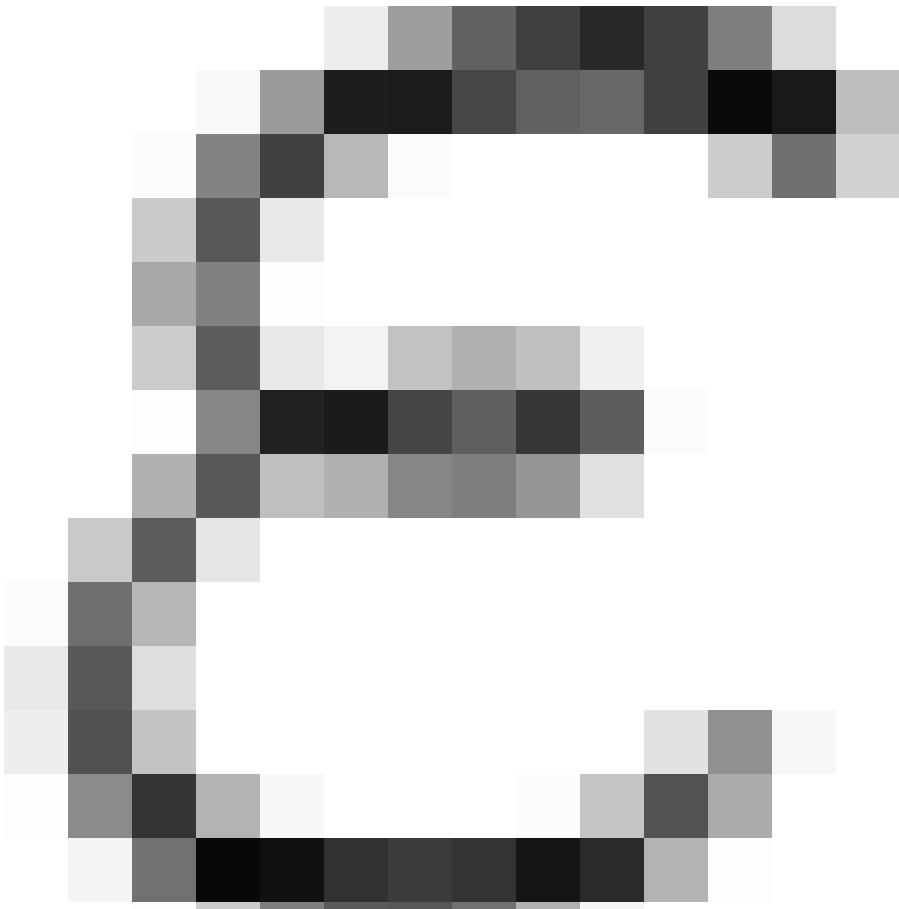
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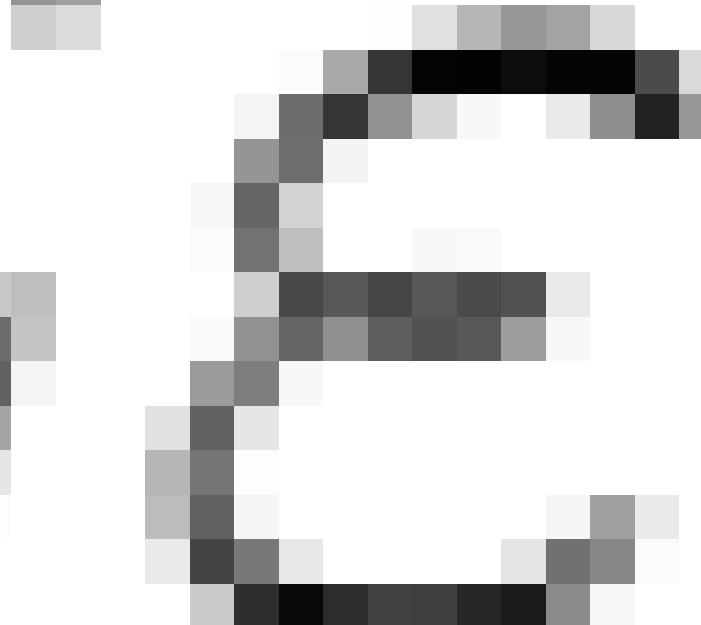
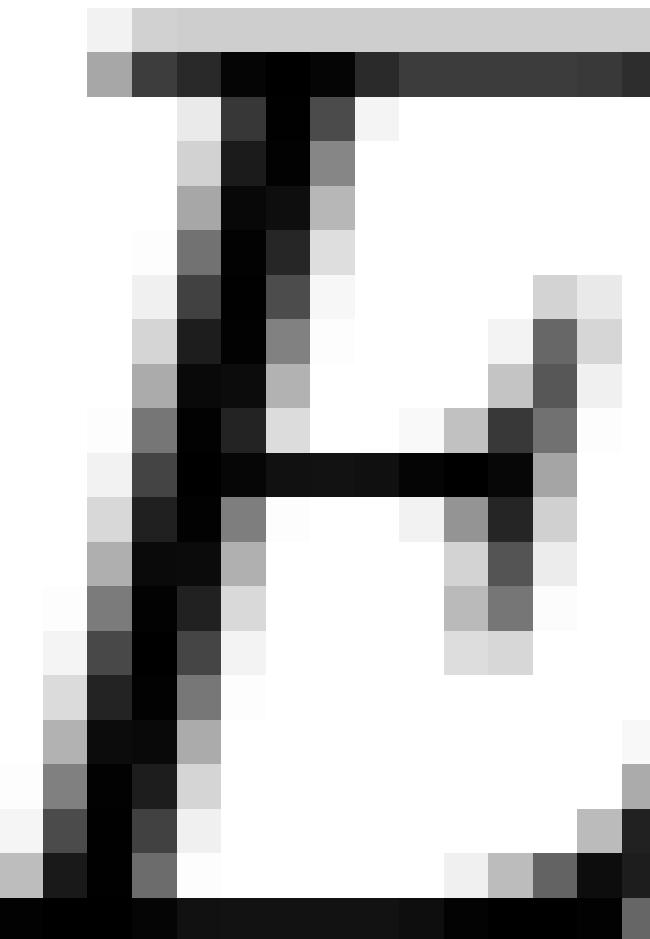
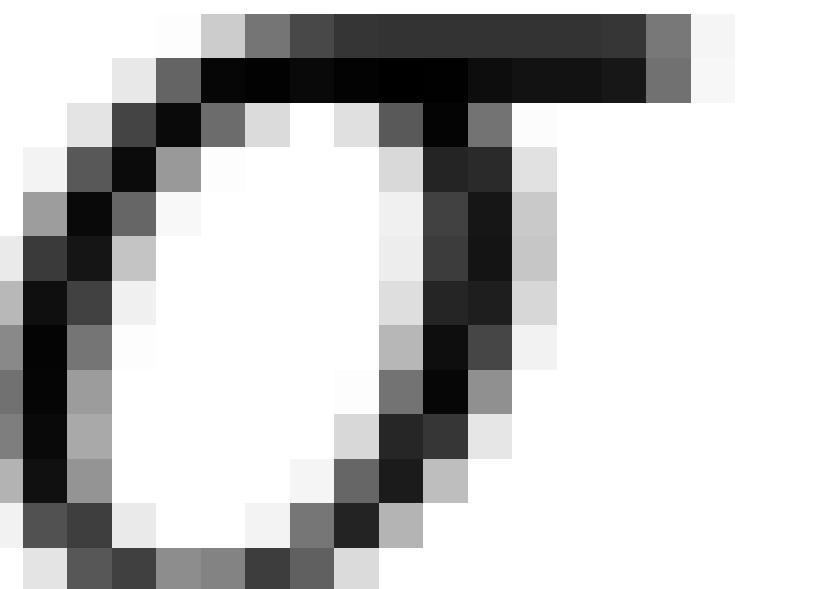
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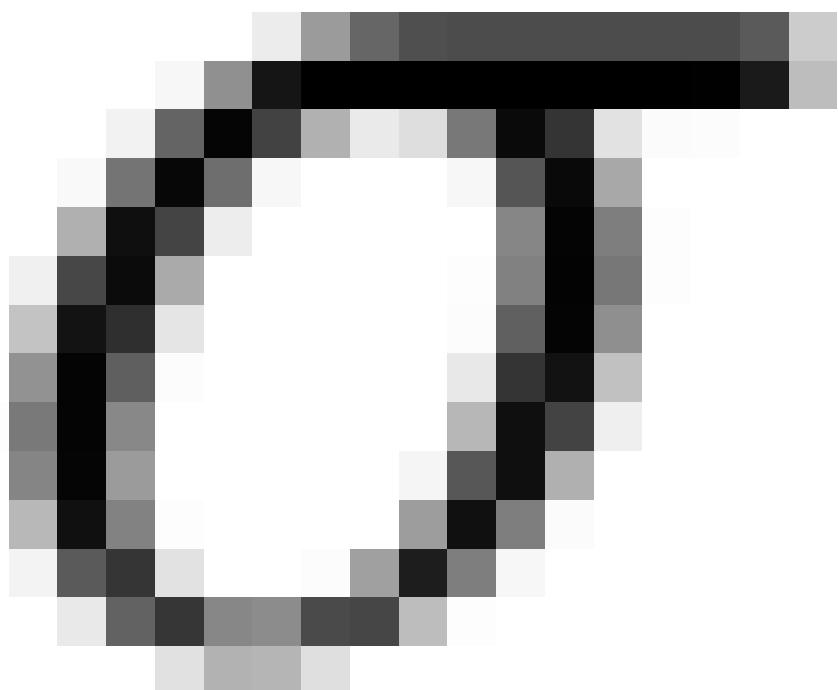
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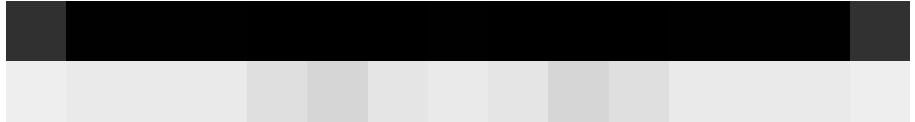
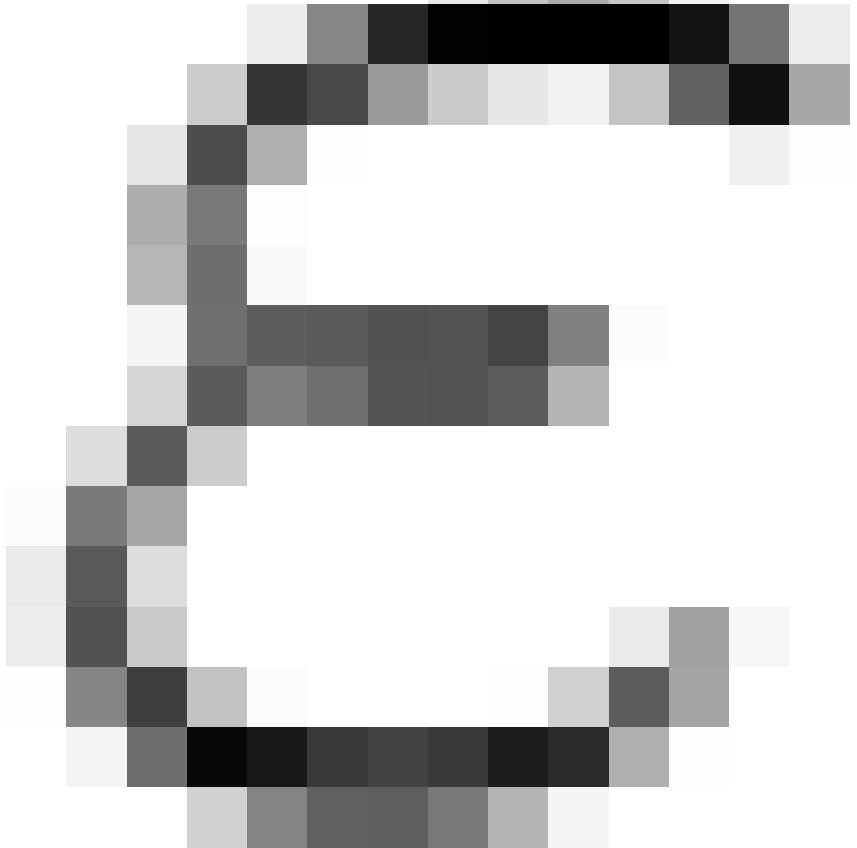
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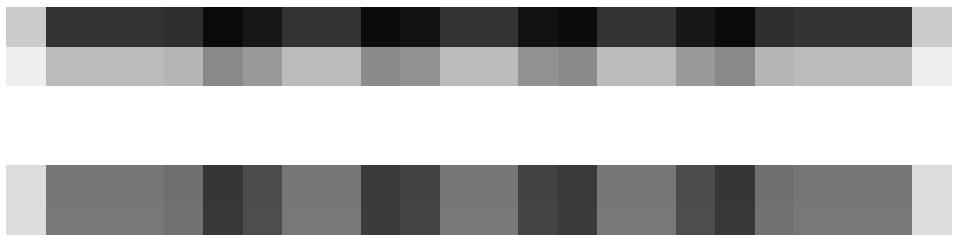
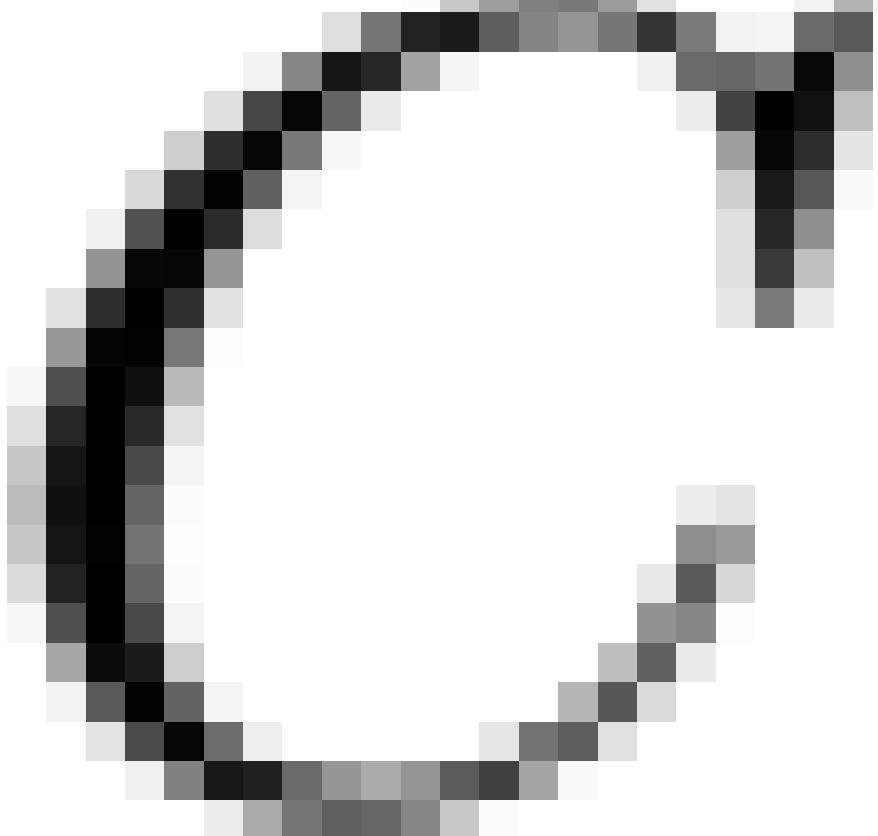


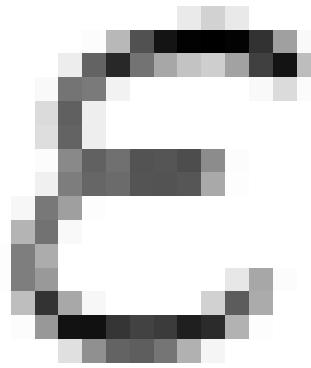
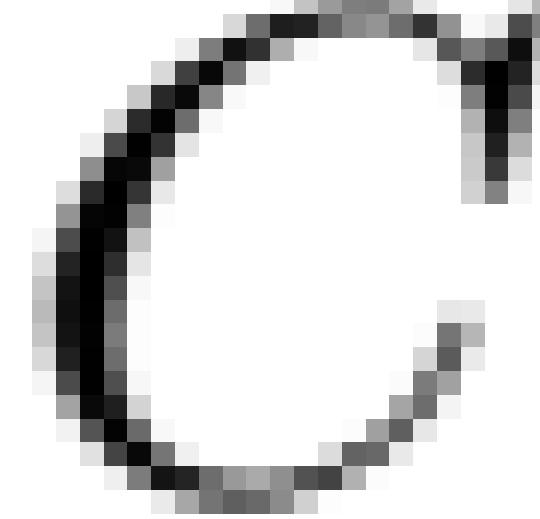
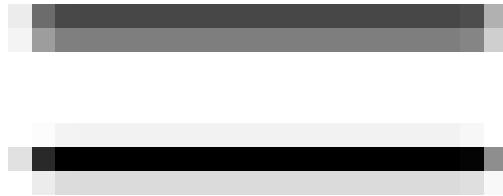
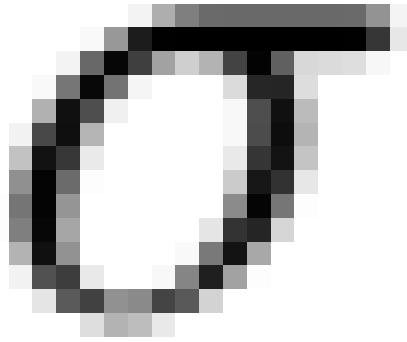






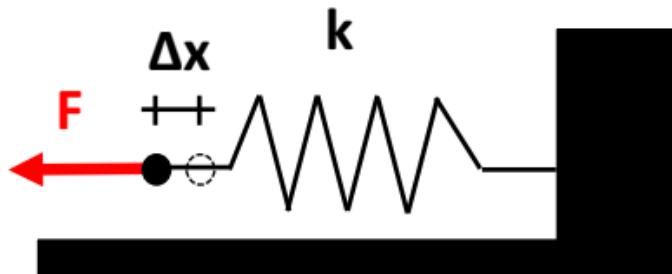






Hooke's law

$$F = k\Delta x$$

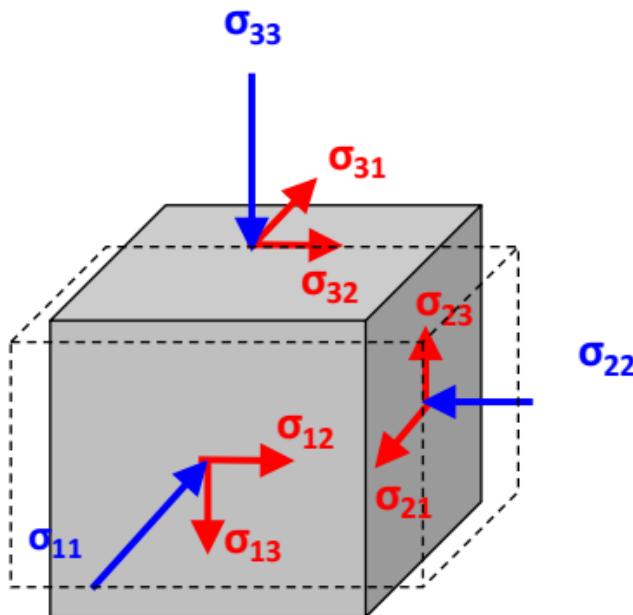


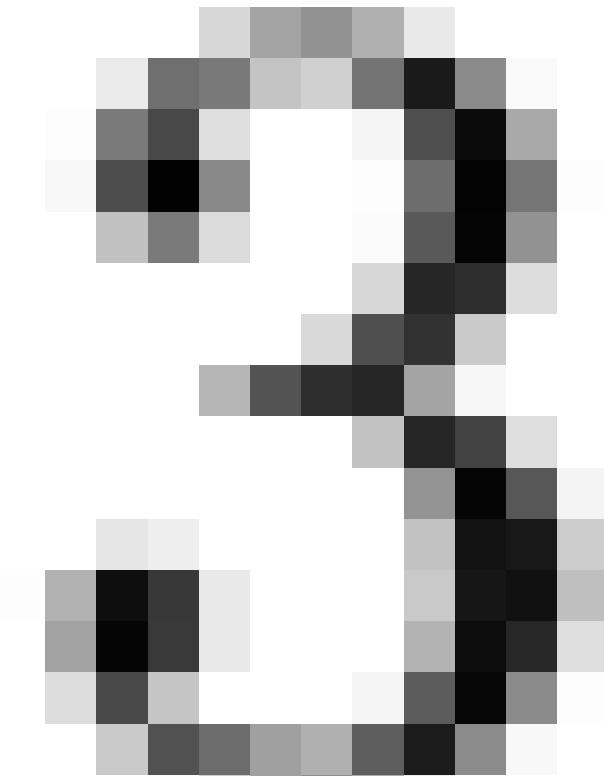
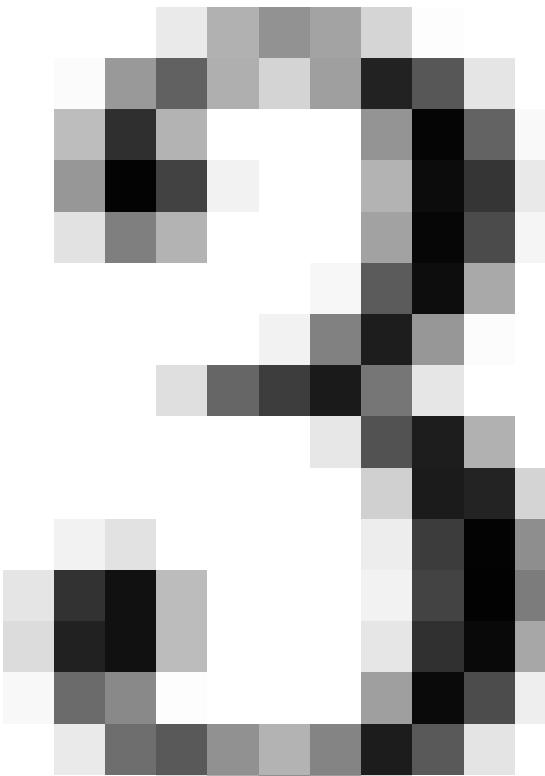
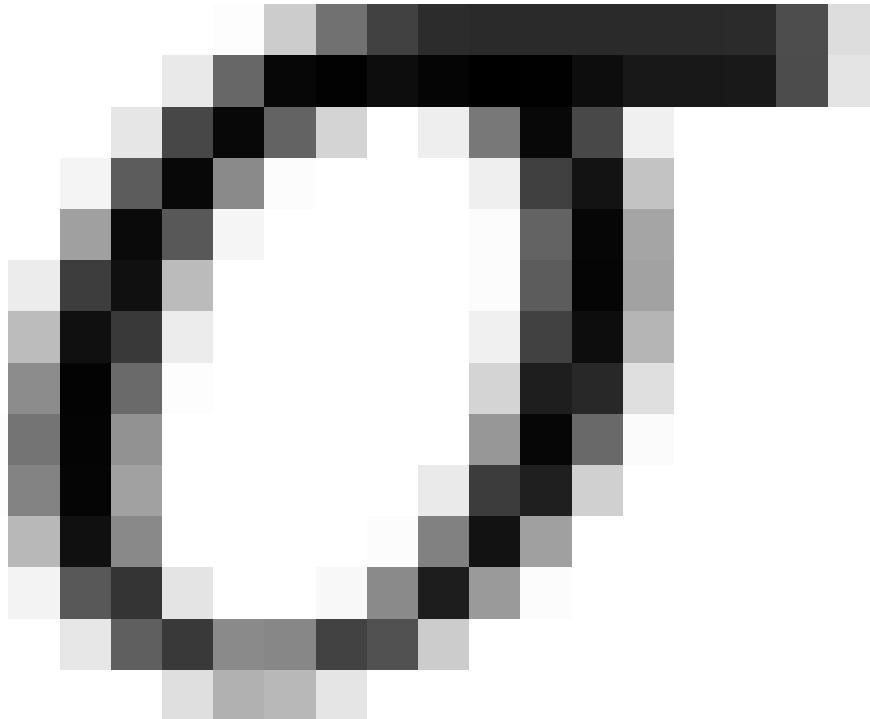
1-D (stress-strain) Hooke's law

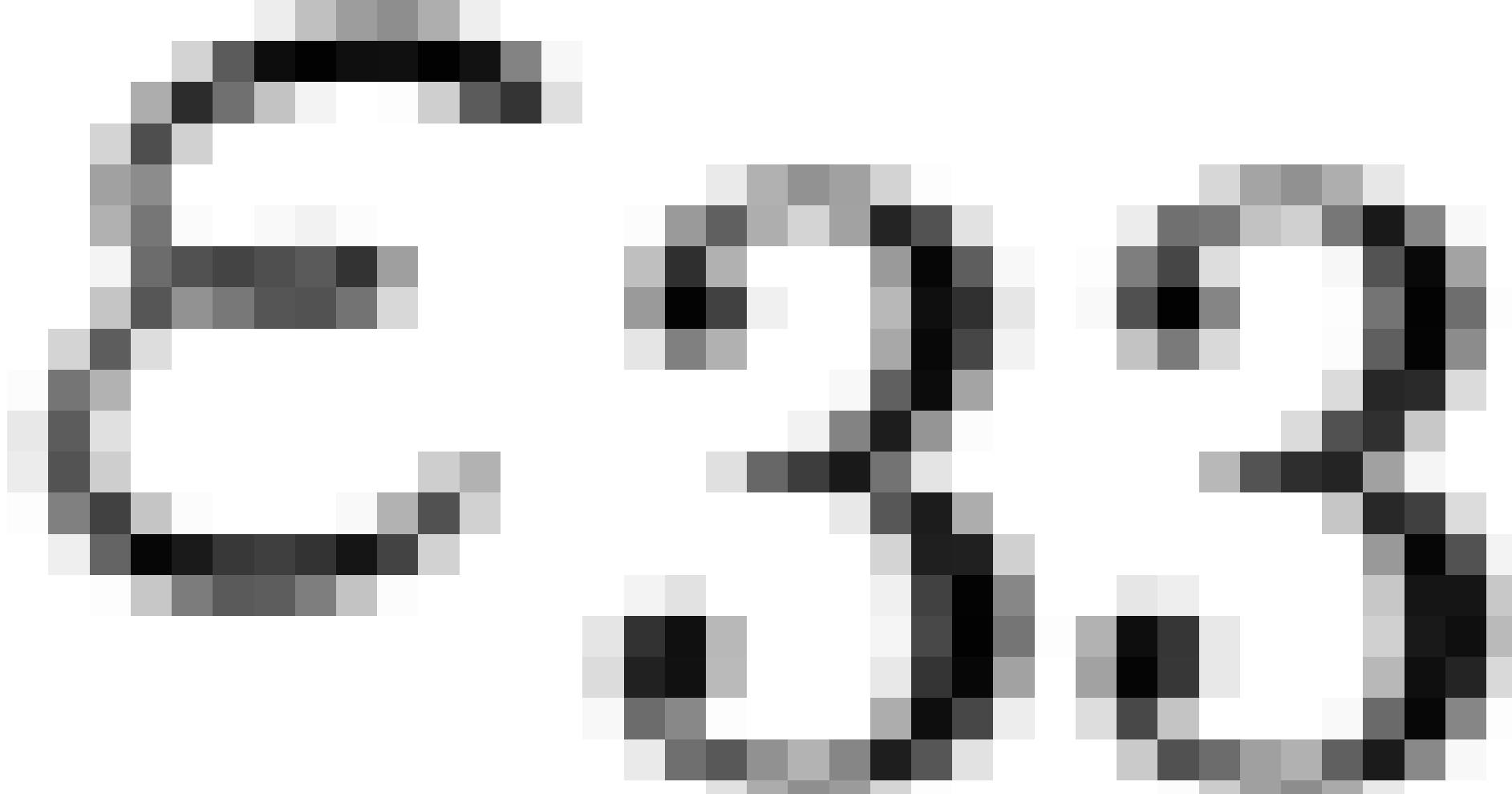
$$\sigma = E\varepsilon$$

Generalized Hooke's law

$$\underline{\sigma} = \underline{C}\underline{\varepsilon}$$







E

—
—

σ33

—

c33





611

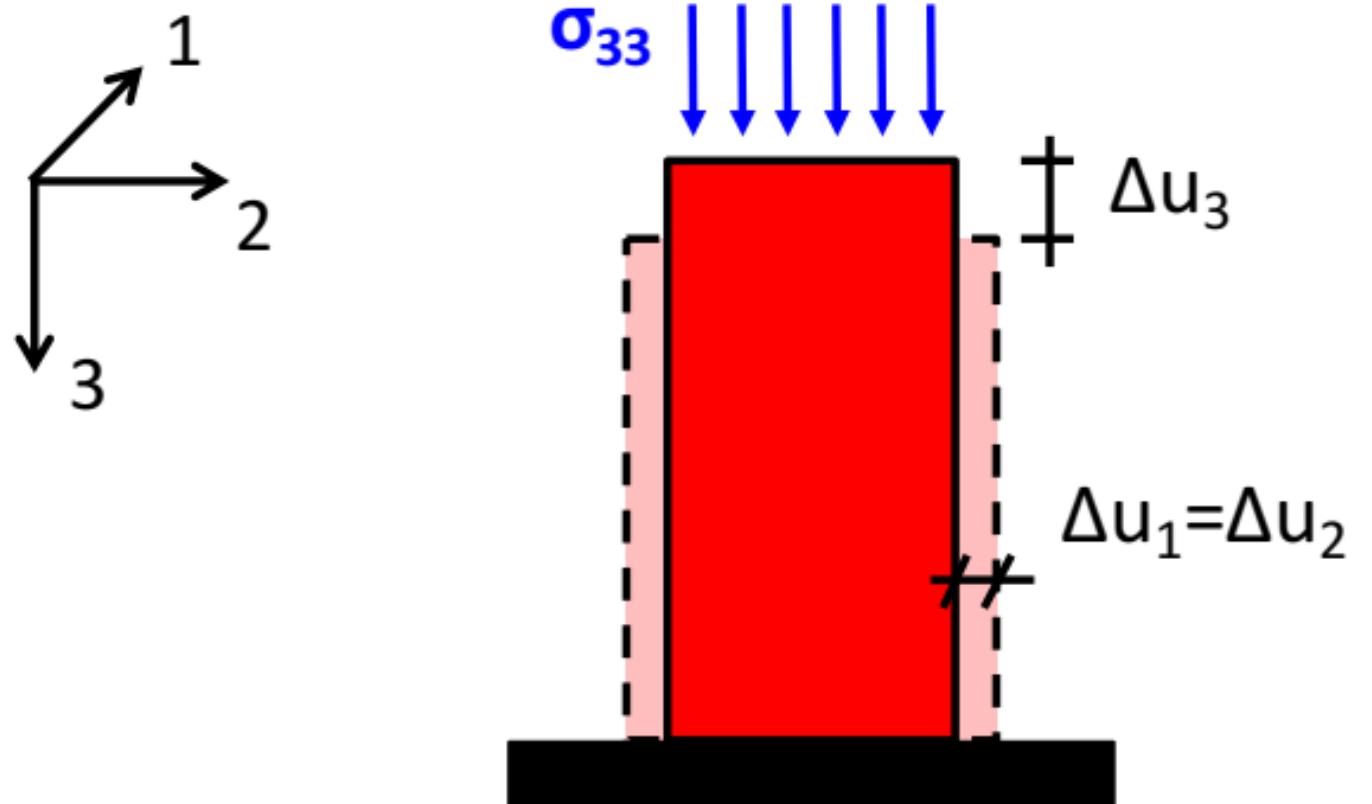
W

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633

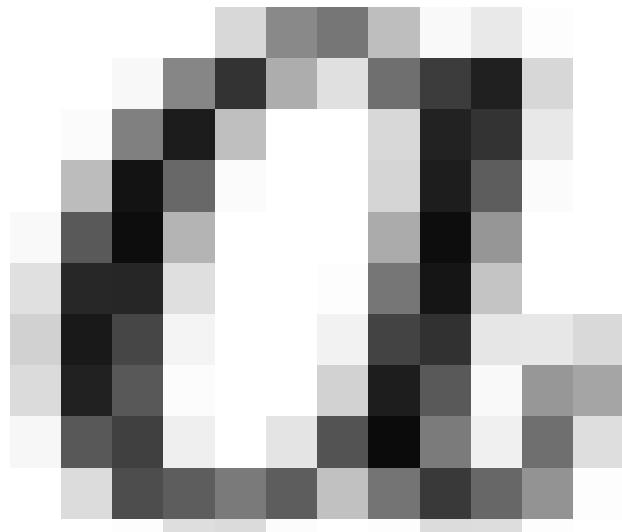
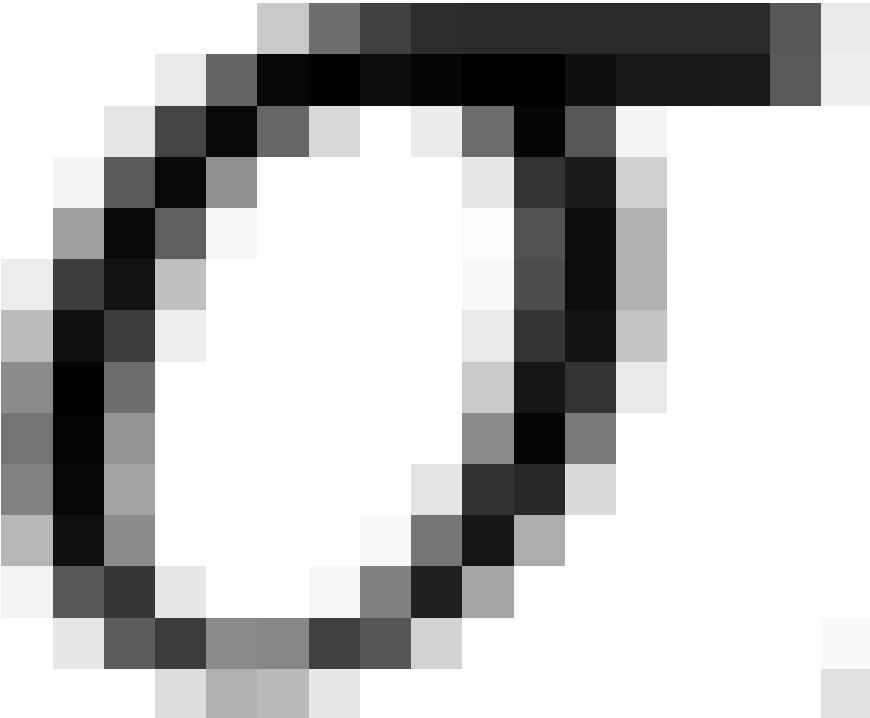


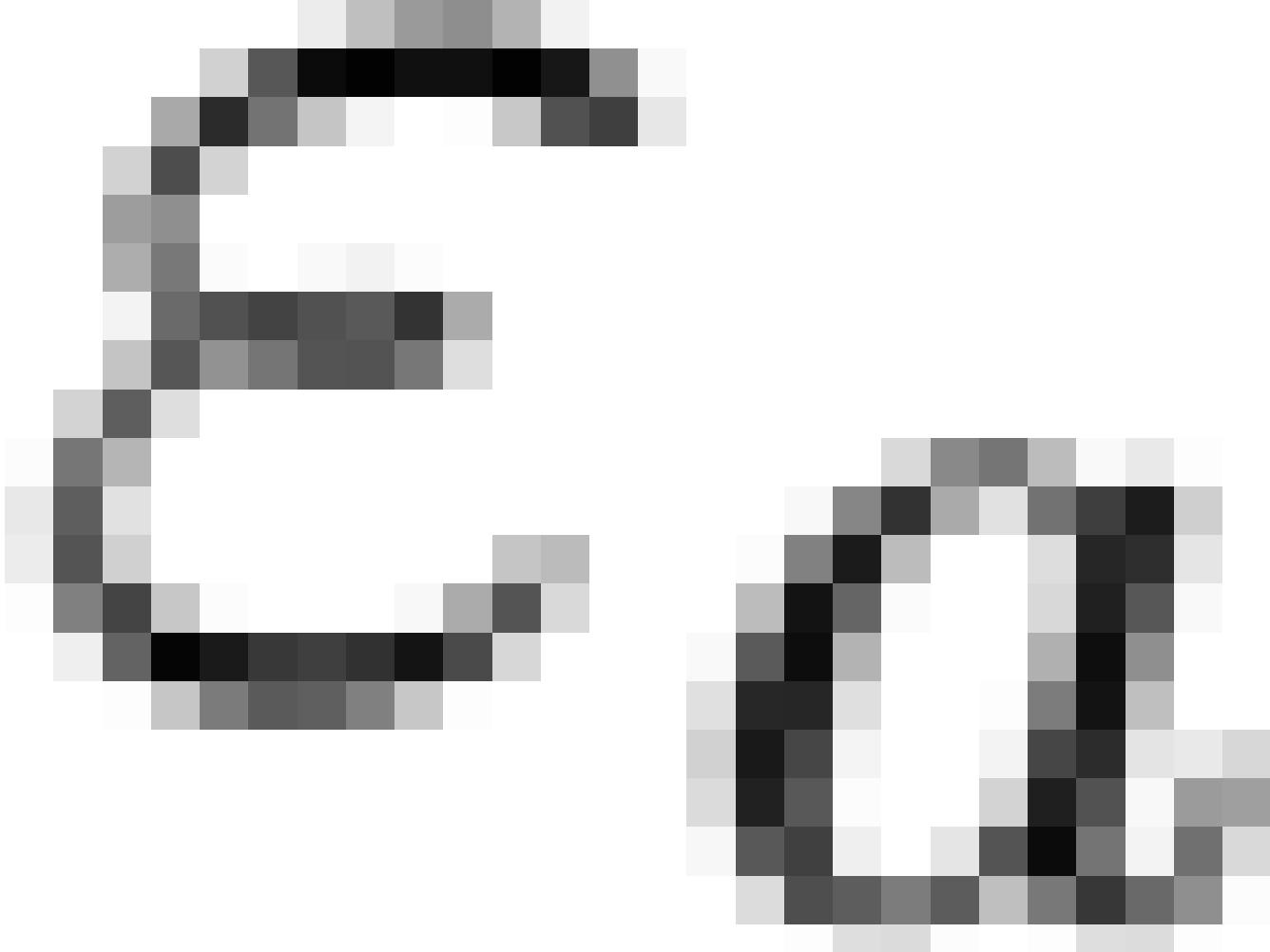
$$E = \frac{\sigma_{33}}{\epsilon_{33}}$$

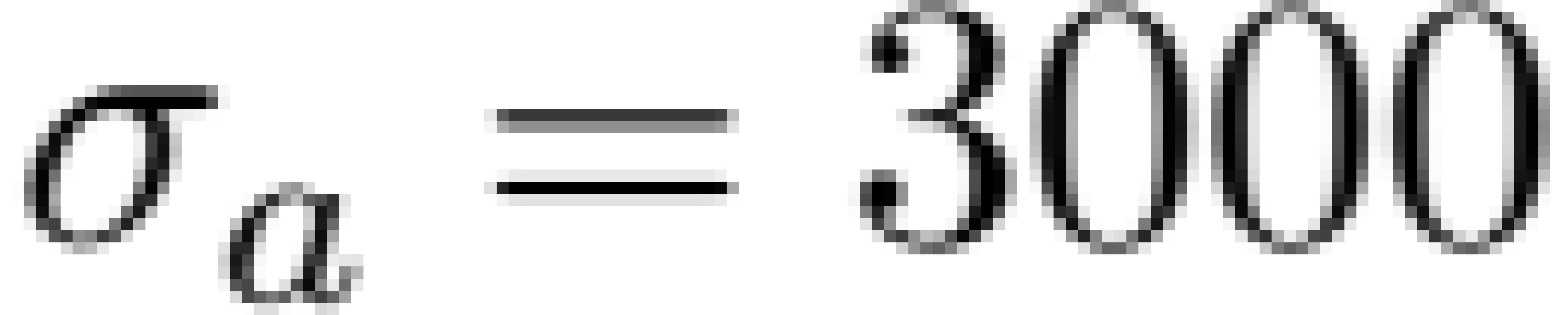
Young's Modulus

$$\nu = -\frac{\epsilon_{11}}{\epsilon_{33}}$$

Poisson's ratio (ν)







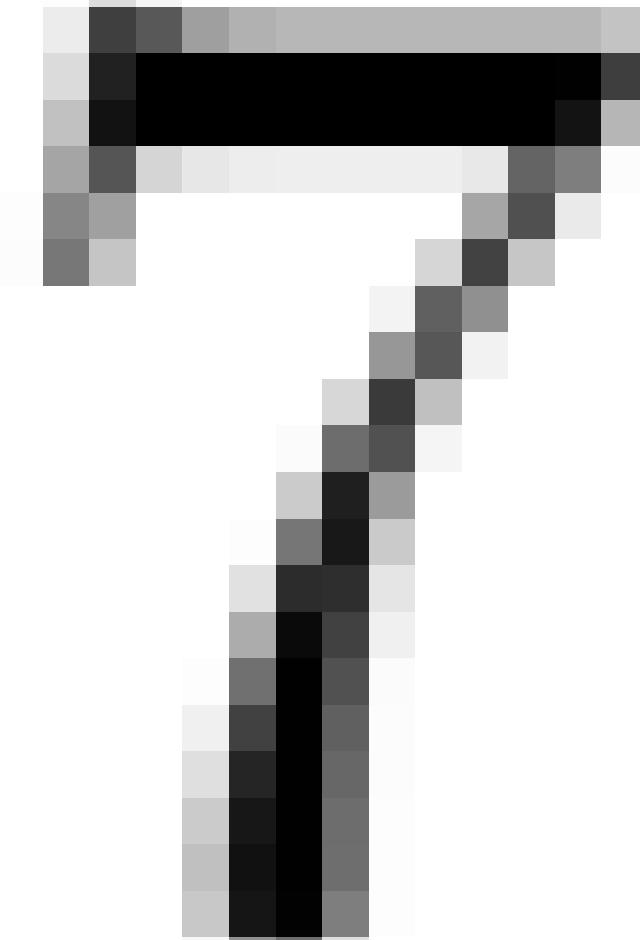
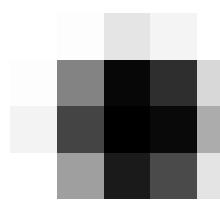
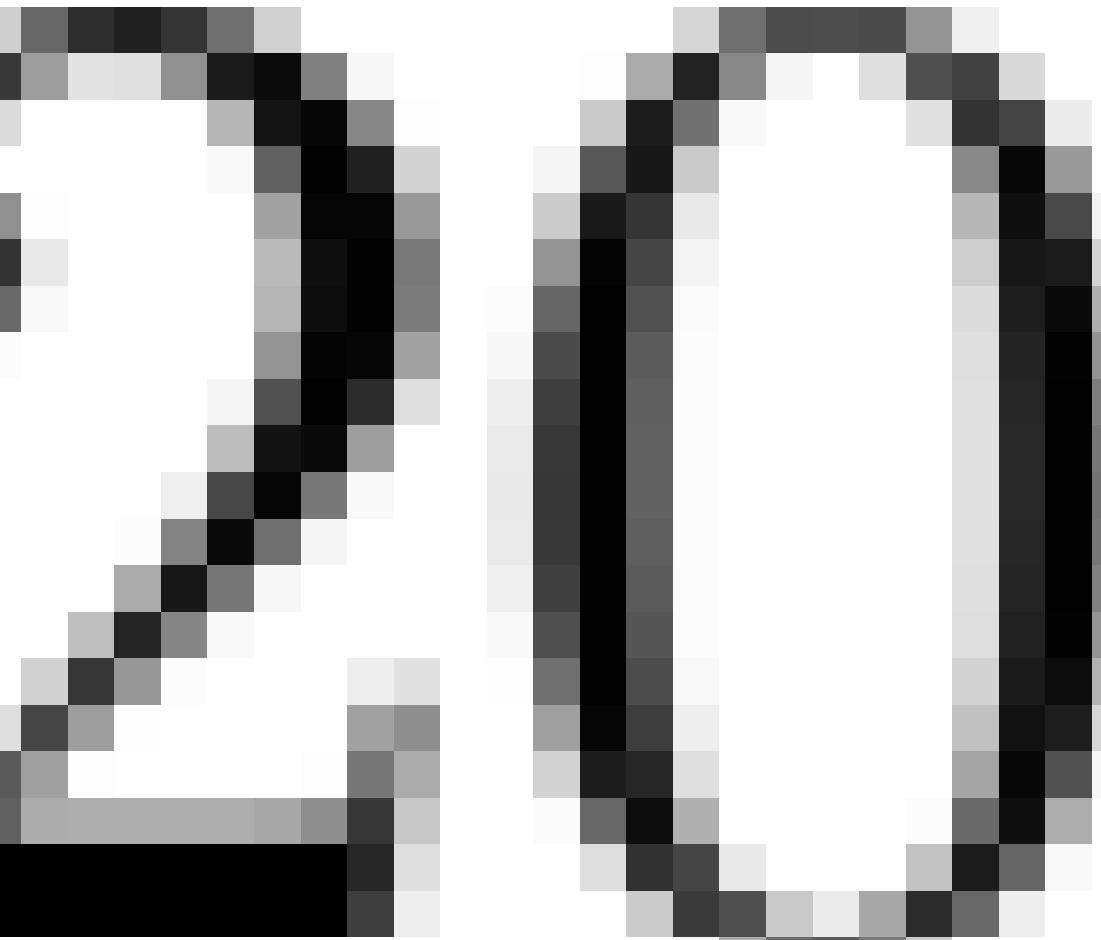
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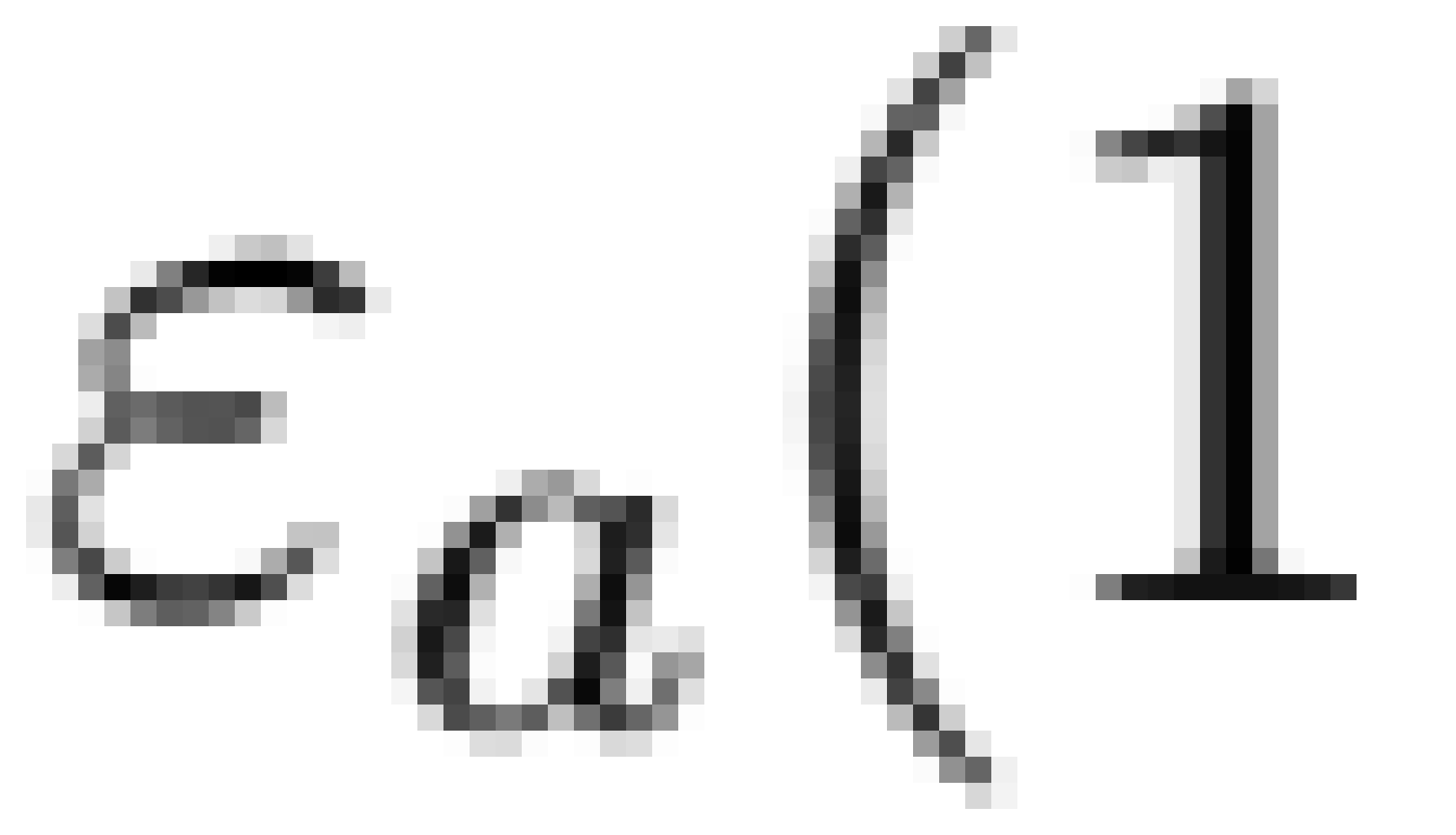
30000

145

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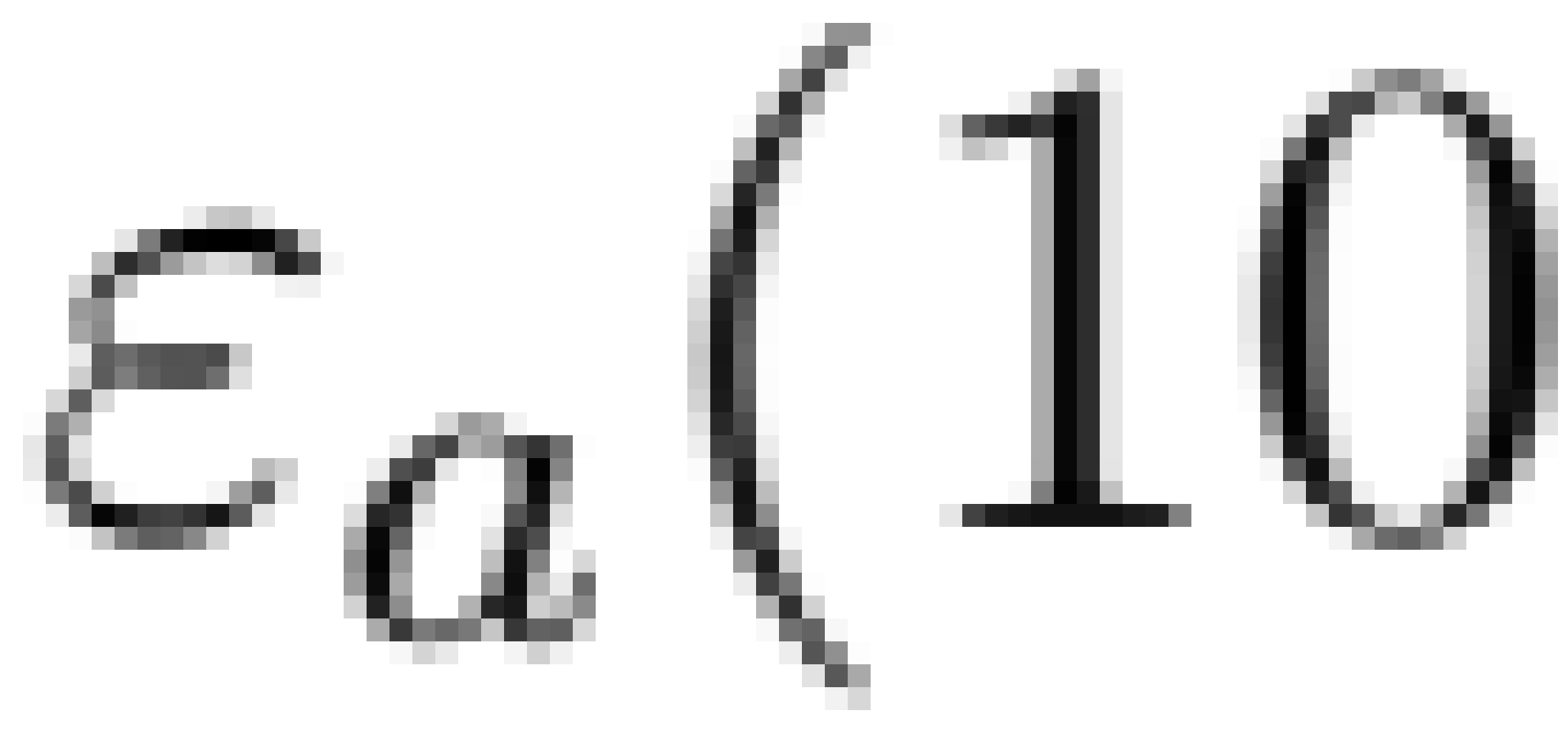
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$$\frac{\sigma_a}{E} = \frac{20.7}{100} = 0.207$$

$$0.207 \times 100 \text{ MPa} = 20.7 \text{ MPa}$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{100 \times 10^9 \text{ MPa}} = 0.207 \times 10^{-7}$$

$$= 0.00207 \times 10^{-7} = 0.00207 \%$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{50000 \text{ MPa}} = 0.00041$$

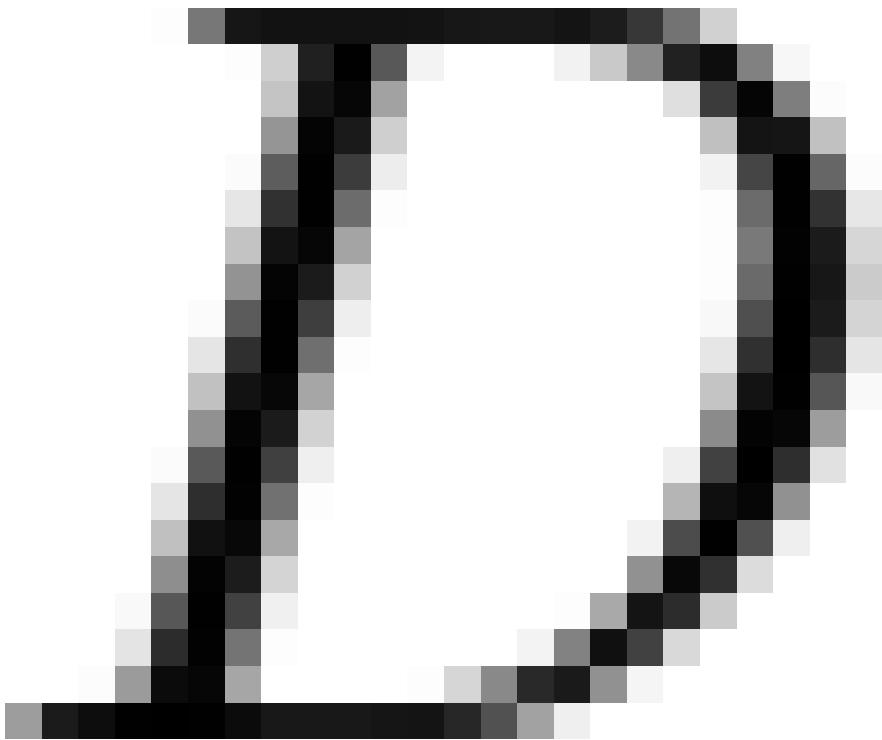
$$= 0.041\%$$

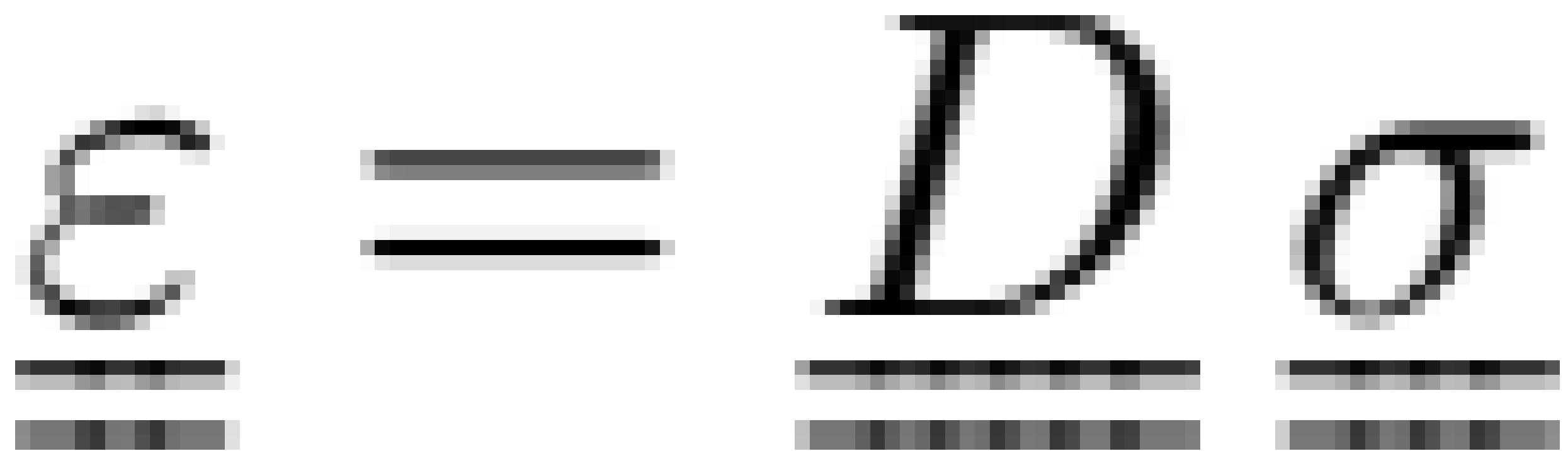
$$\left\{ \begin{array}{lcl} \epsilon_{11} & = & +\frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ & & \nu \quad \quad \quad 1 \quad \quad \quad \nu \\ \epsilon_{22} & = & -\frac{1}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{1}{E}\sigma_{33} \\ & & \nu \quad \quad \quad \nu \quad \quad \quad 1 \\ \epsilon_{33} & = & -\frac{1}{E}\sigma_{11} - \frac{1}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{array} \right.$$

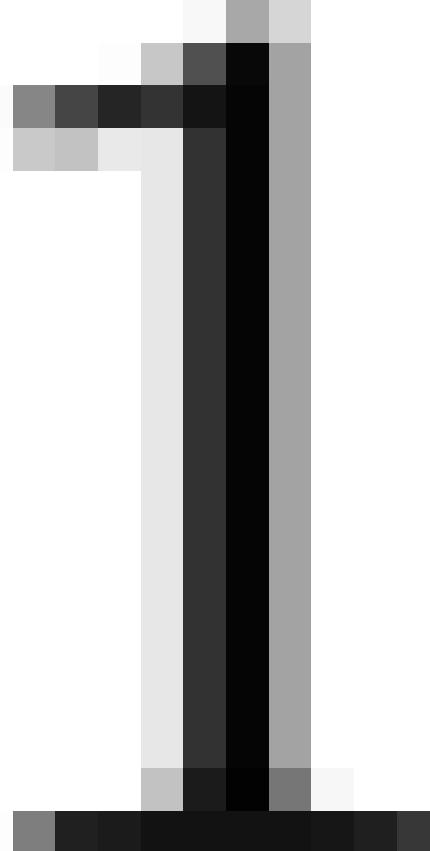
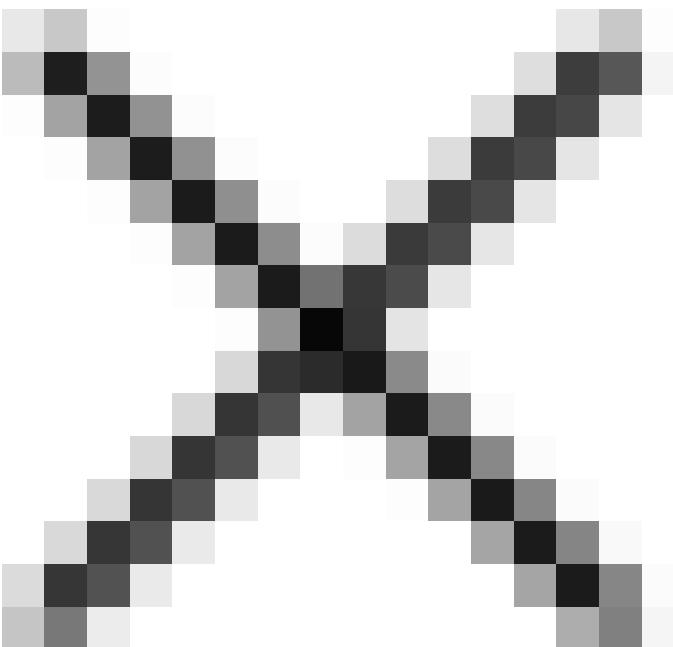
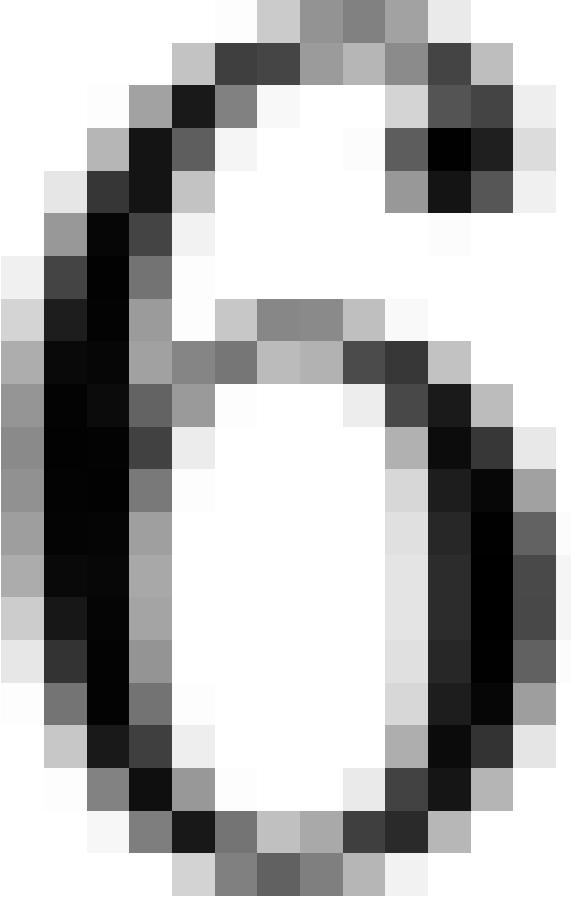


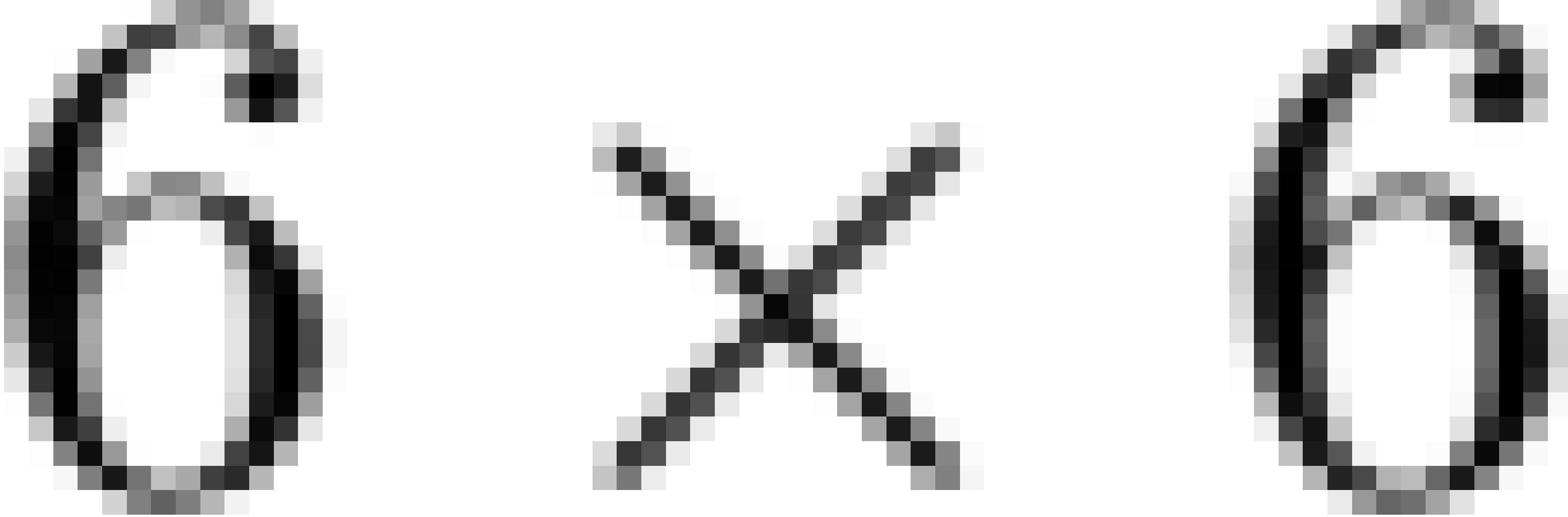


$$\left\{ \begin{array}{l} 2\varepsilon_{12} = \frac{1}{G} \sigma_{12} \\ \\ 2\varepsilon_{13} = \frac{1}{G} \sigma_{13} \\ \\ 2\varepsilon_{23} = \frac{1}{G} \sigma_{23} \end{array} \right.$$



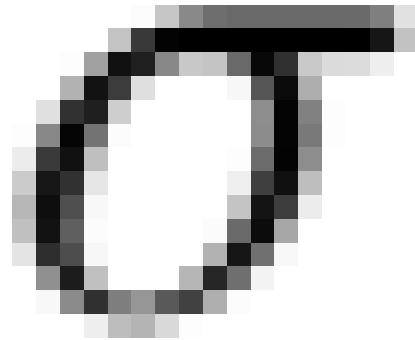
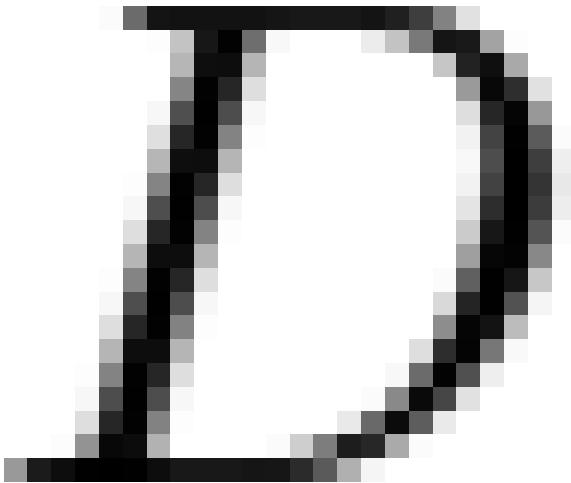




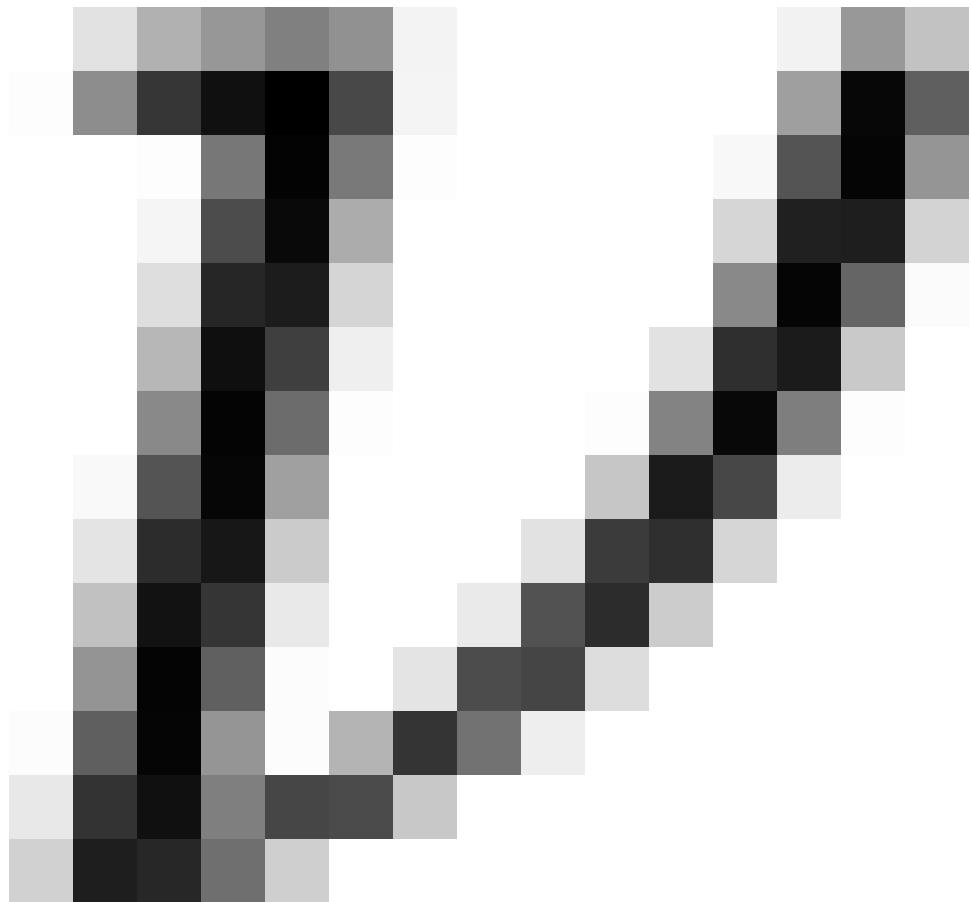


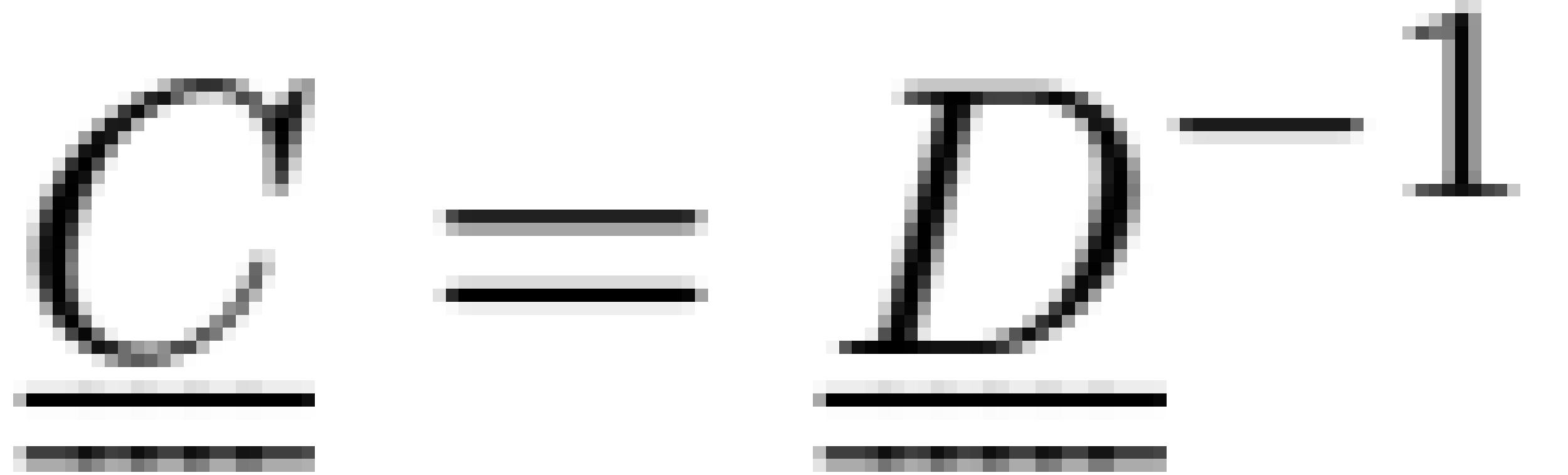
$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} = \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & 1 & \nu & 0 & 0 & 0 \\ -\frac{\nu}{E} & +\frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & \nu & 1 & 0 & 0 & 0 \\ -\frac{1}{E} & -\frac{\nu}{E} & +\frac{1}{E} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}$$



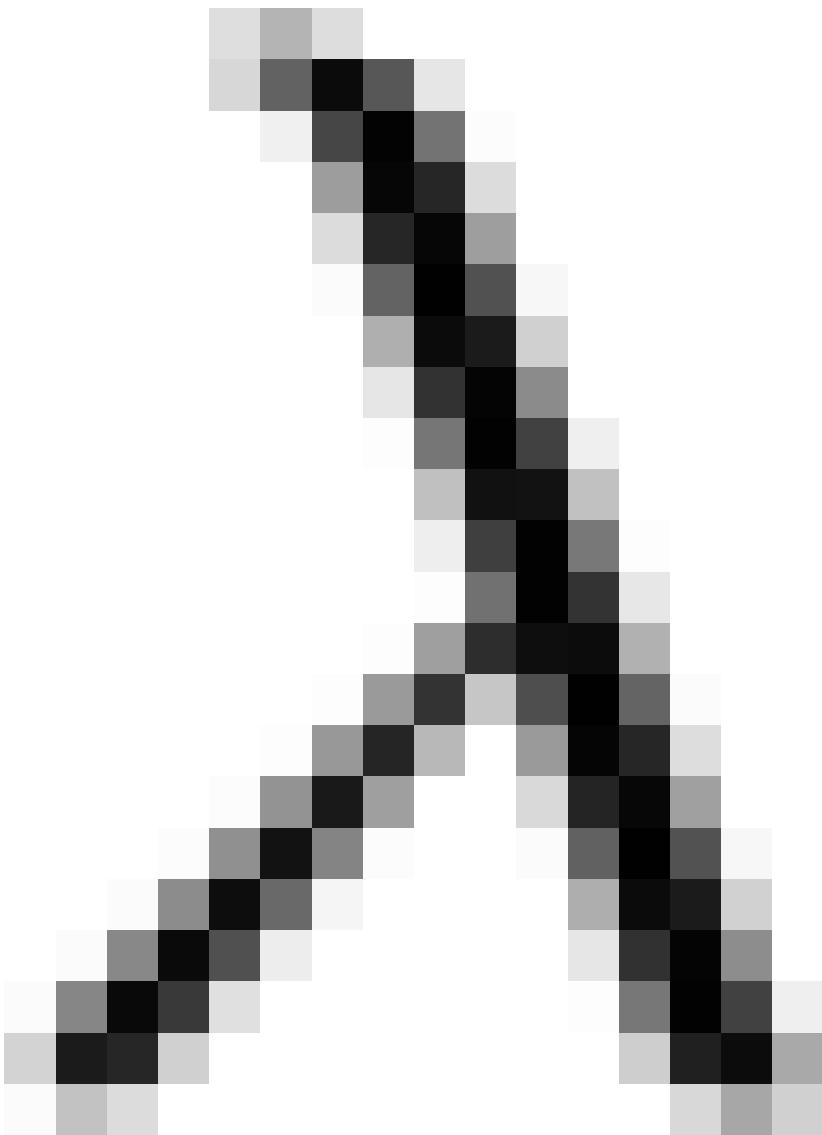


$$\underline{\underline{\epsilon}} = \begin{bmatrix} -\frac{v}{E}\sigma_{33}, -\frac{v}{E}\sigma_{33}, \frac{1}{E}\sigma_{33}, 0, 0, 0 \end{bmatrix}^T$$





$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$



$$\sigma_{11} = \frac{E}{(1+v)(1-2v)} [(1-v)e_{11} + v e_{22} + v e_{33}]$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + \frac{\nu E}{(1+\nu)(1-2\nu)} \left(\frac{1-\nu}{1+\nu} \epsilon_{11} - \frac{1+\nu}{1-\nu} \epsilon_{22} \right)$$

λ



$(1 + \nu)$

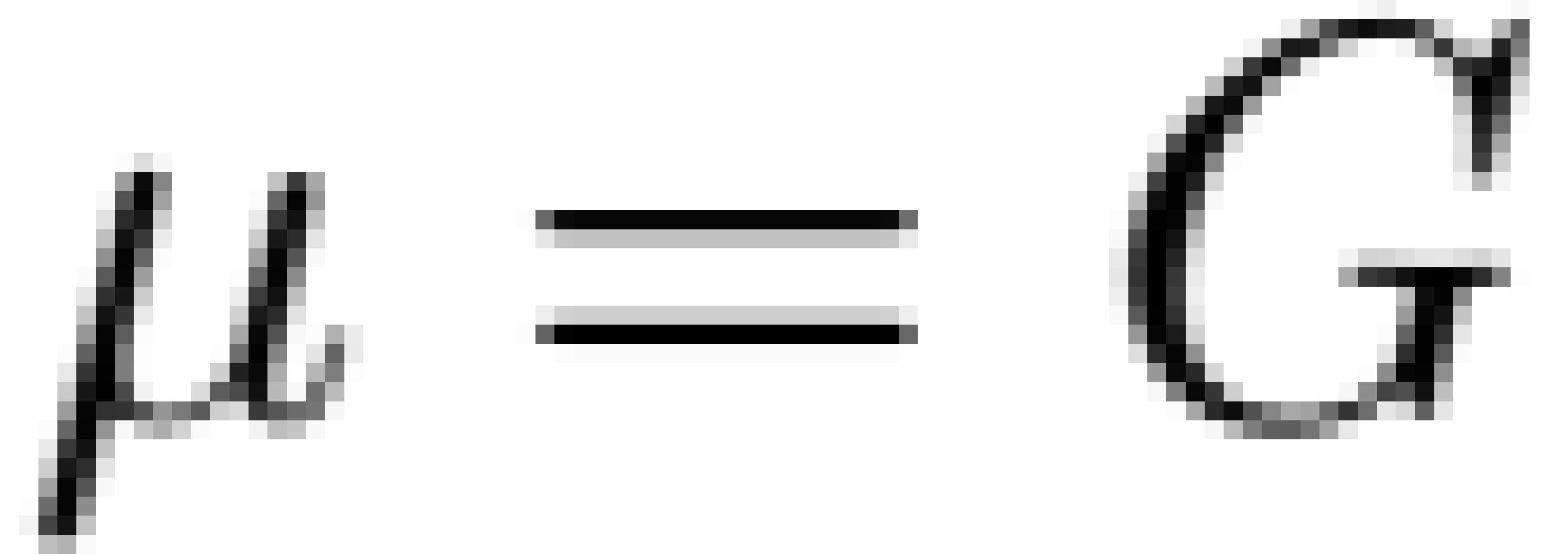
$(1 - \nu)$



(2ν)

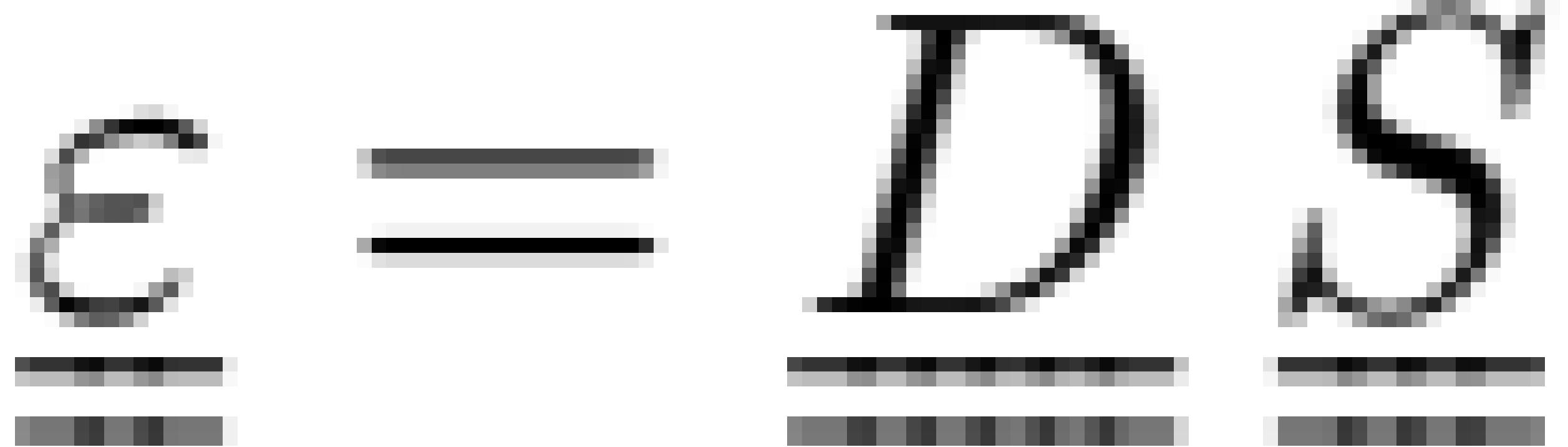
νE

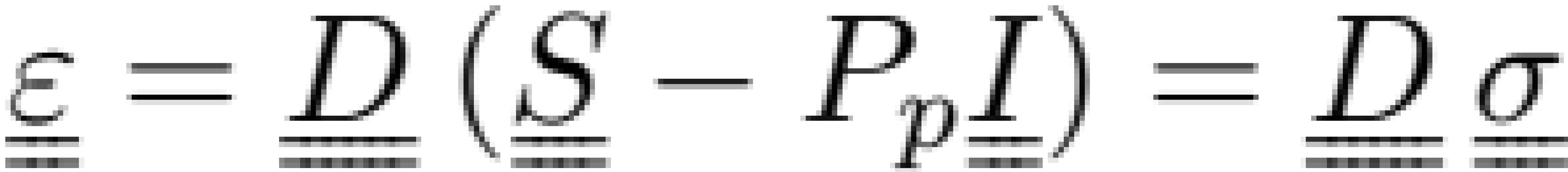
$$2\mu = \frac{\nu E}{(1+\nu)(1-2\nu)} = \frac{-1}{(1+\nu)} + \frac{1-\nu}{(1+\nu)}$$

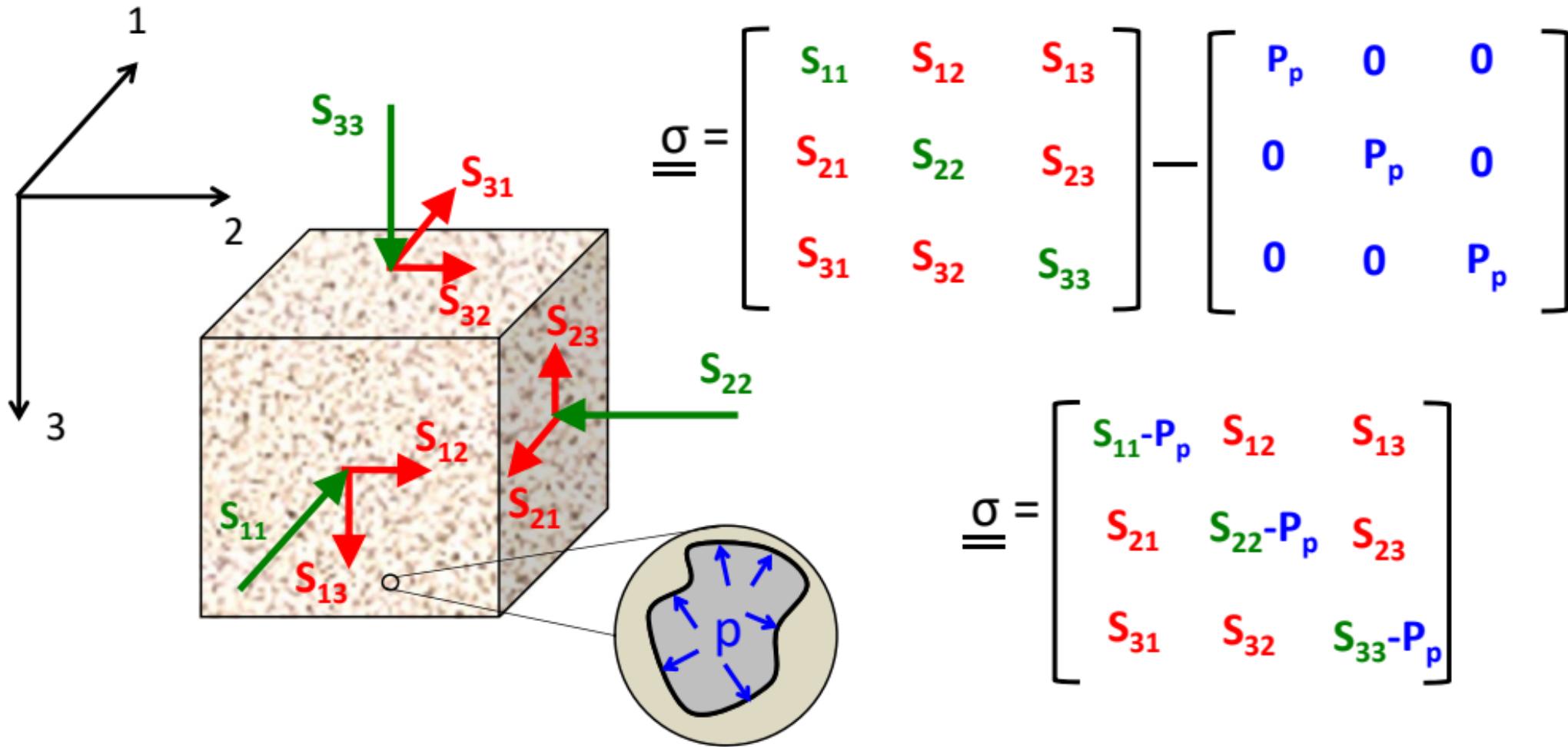


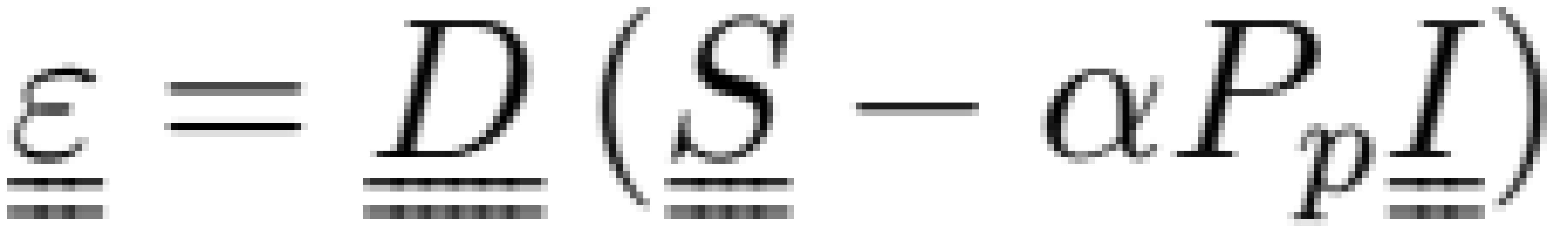
$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right. .$$

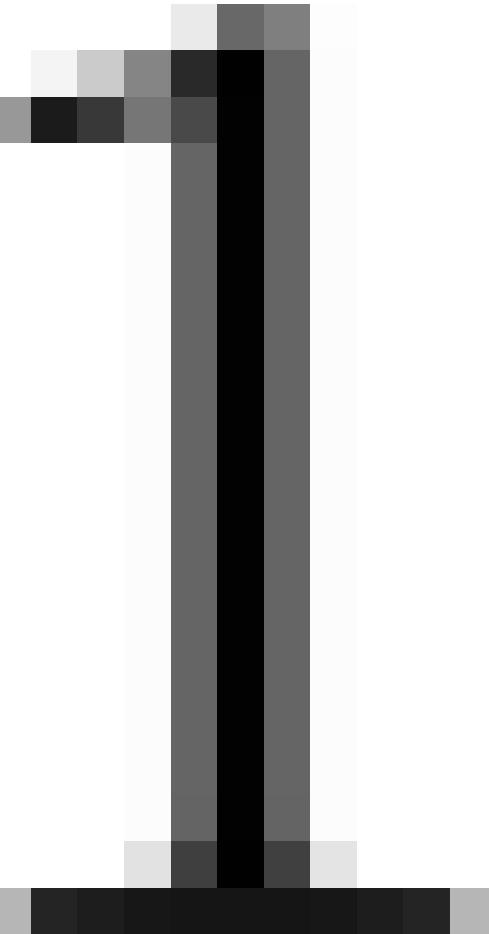
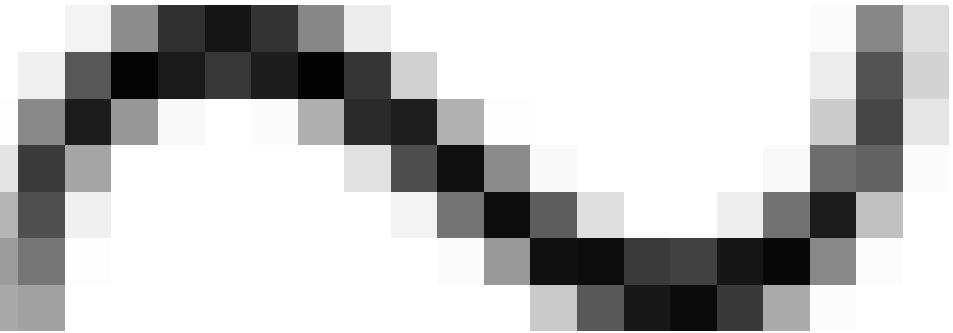
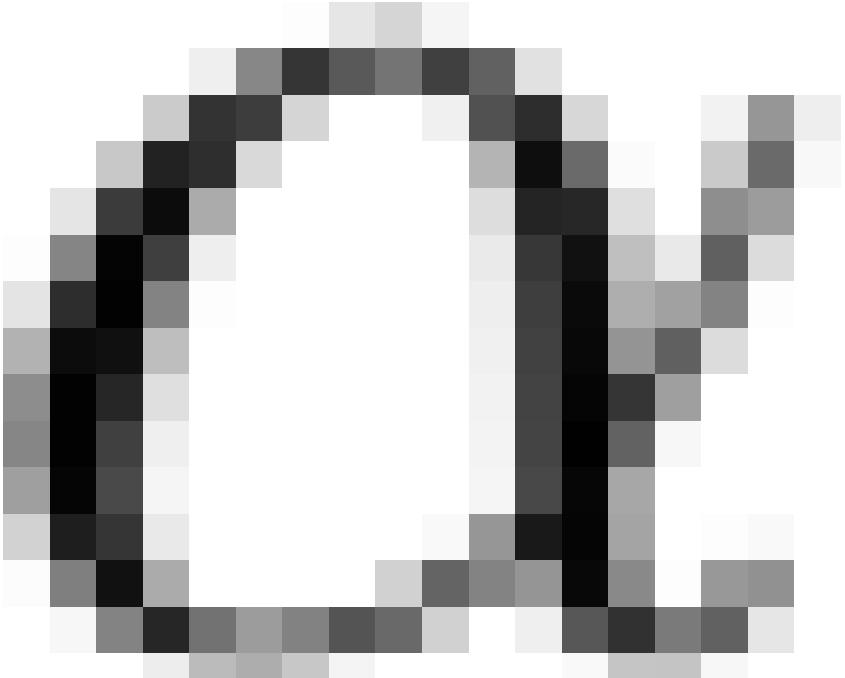
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$











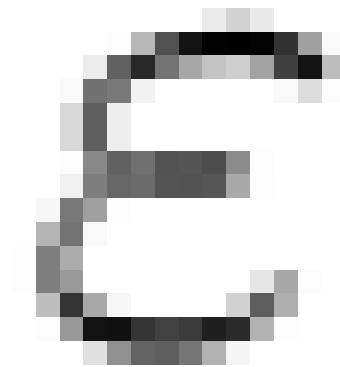
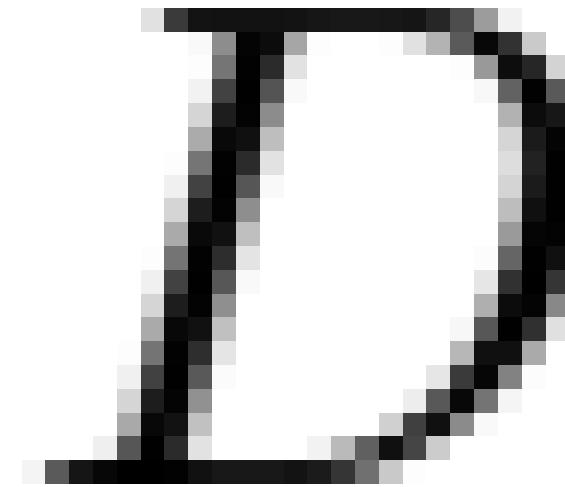
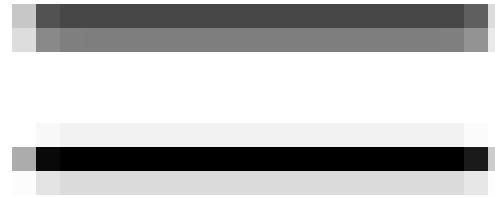
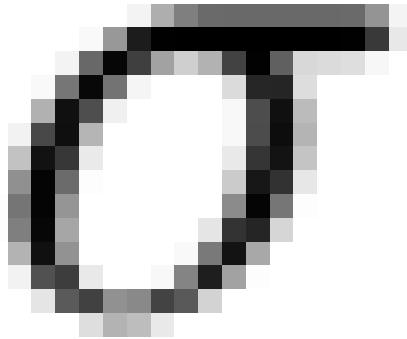




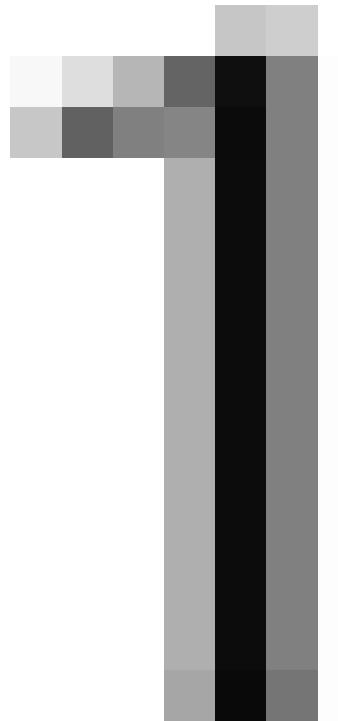
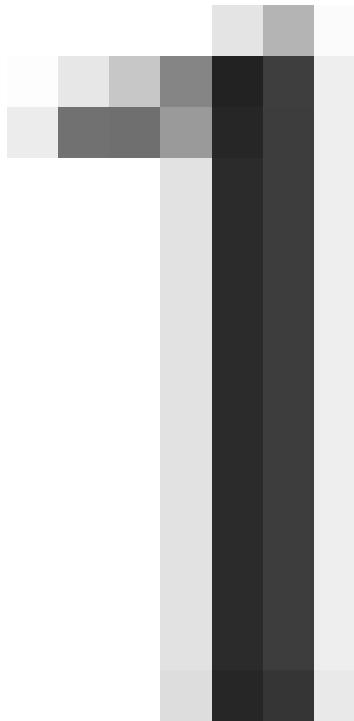
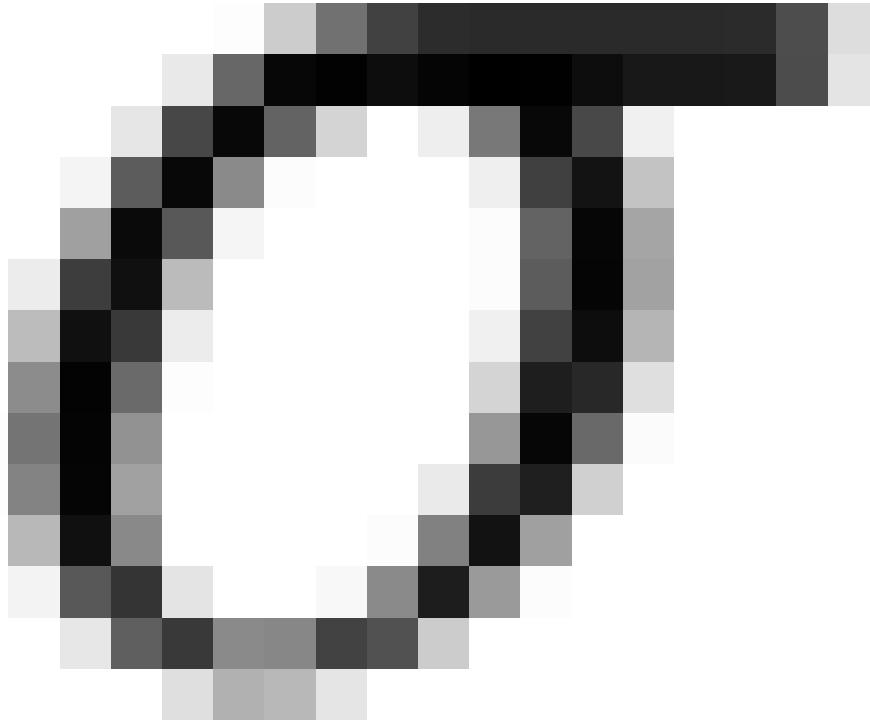


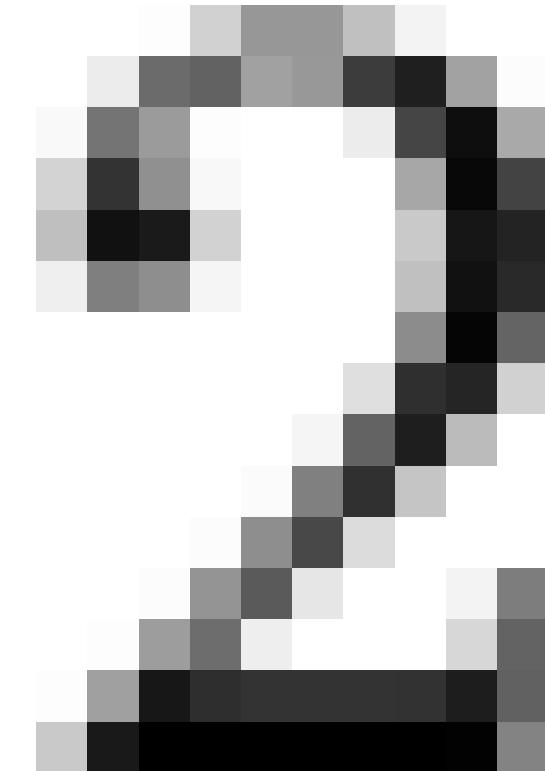
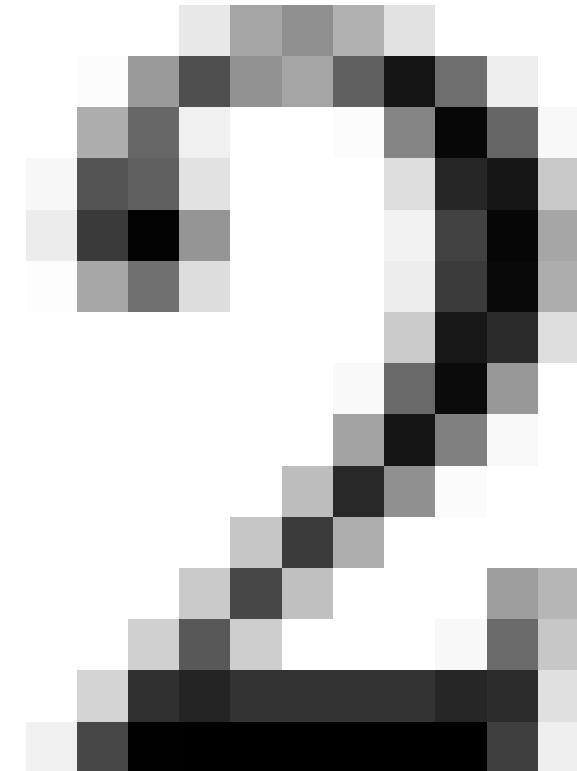
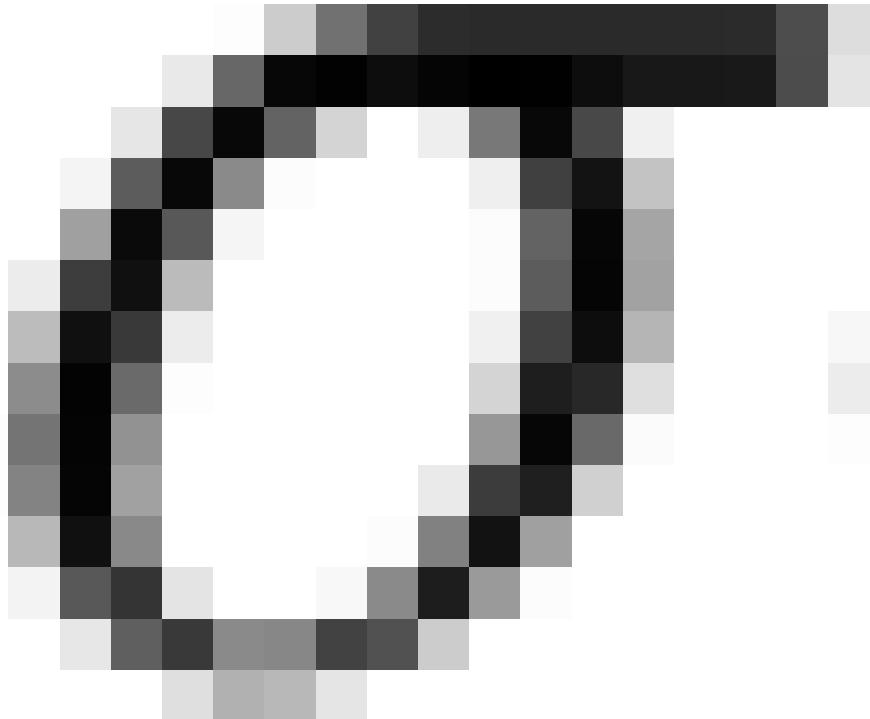






$$\left\{ \begin{array}{l} \sigma_{11} = \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)} \epsilon_{33} \\ \sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \epsilon_{33} \end{array} \right.$$





σ_{11}

$$= \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

 σ_{22}

$$= \begin{array}{c} \text{---} \\ | \end{array}$$

 σ_{33}

$$\nu$$

$$\nu$$

σ_b

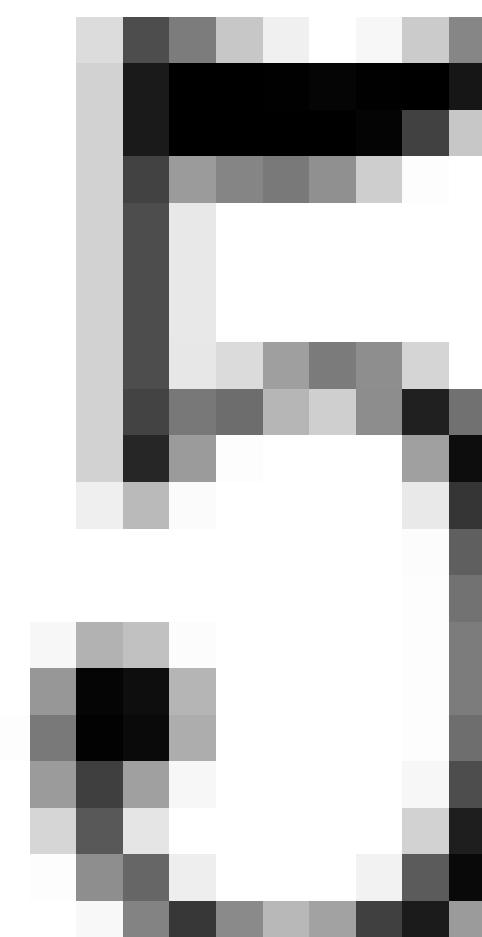
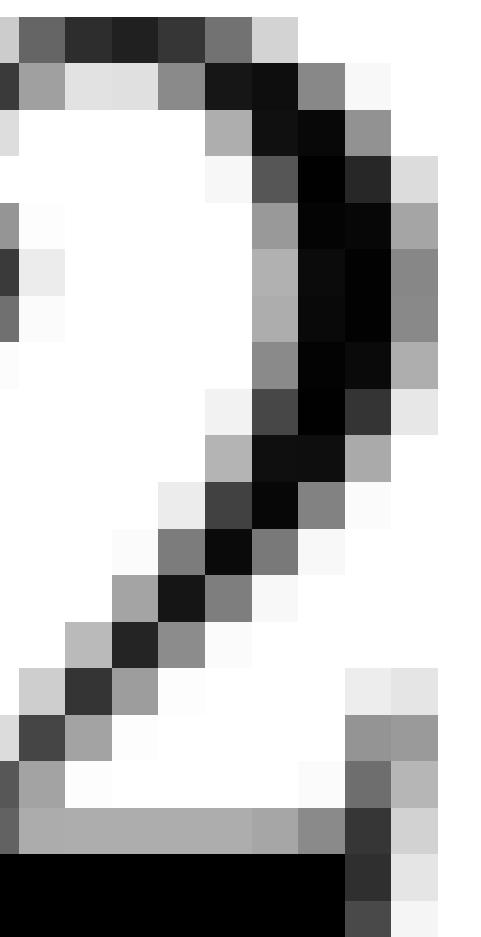
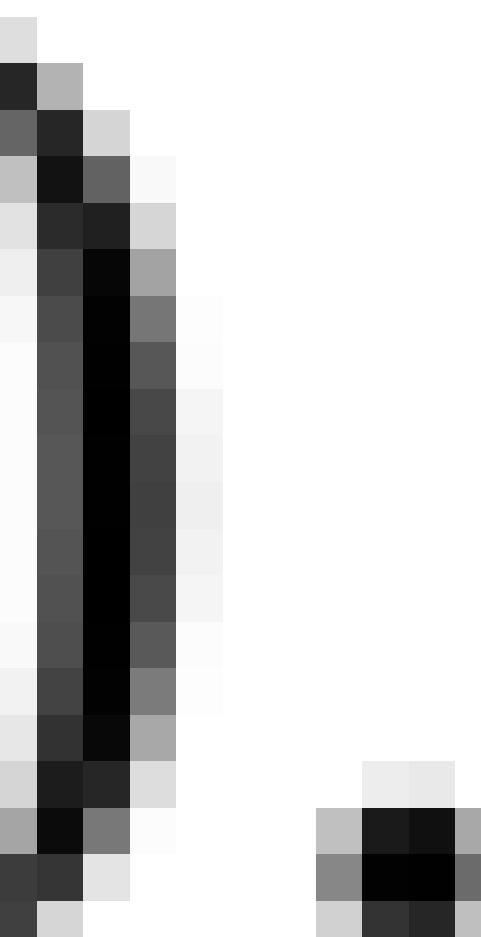
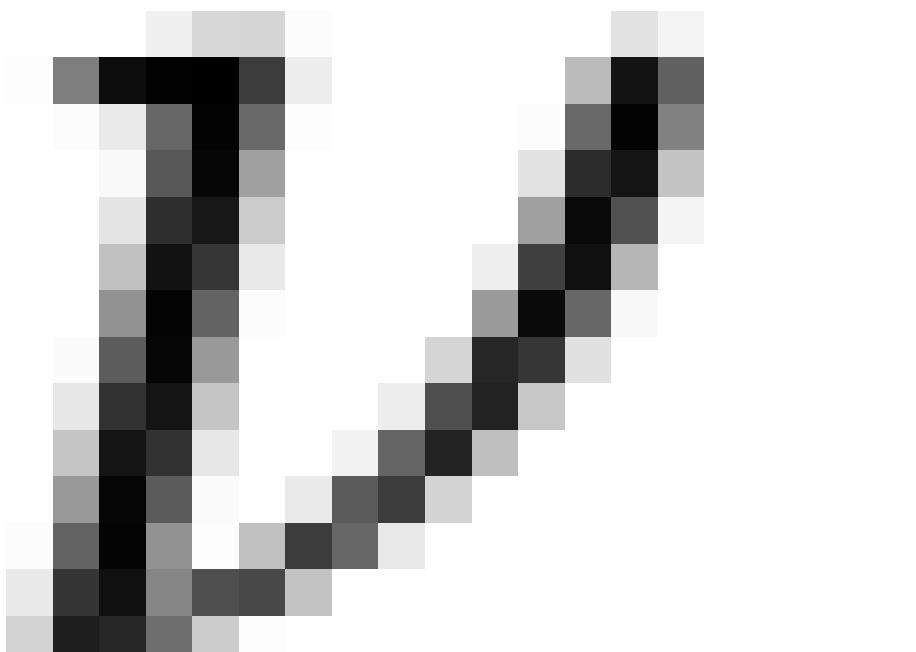
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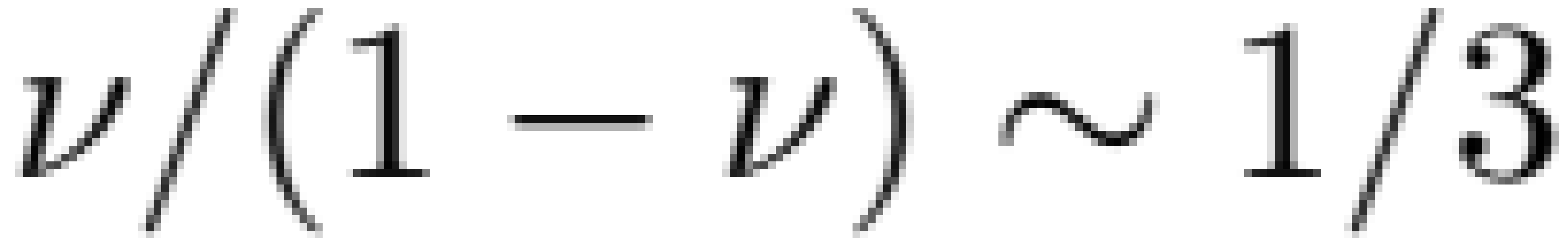
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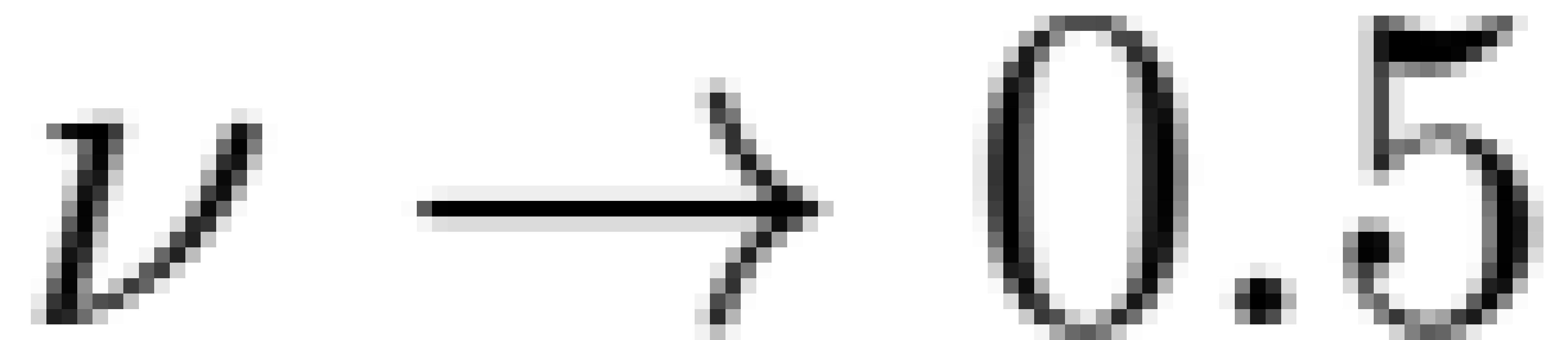
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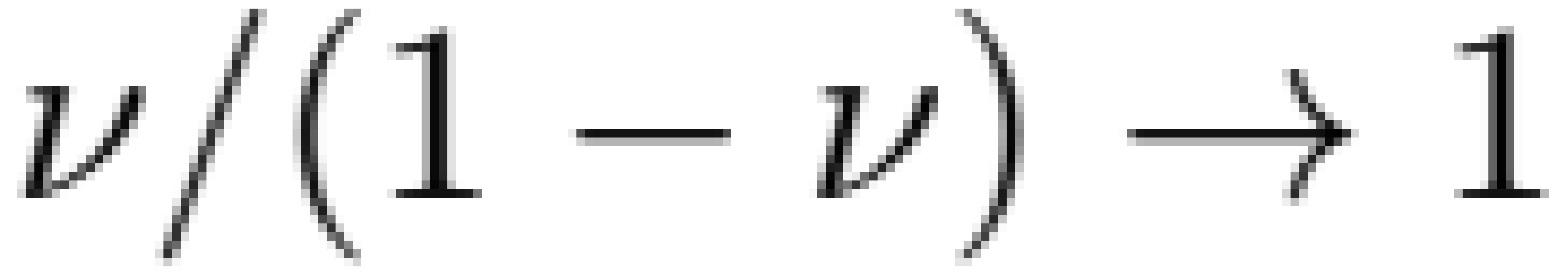
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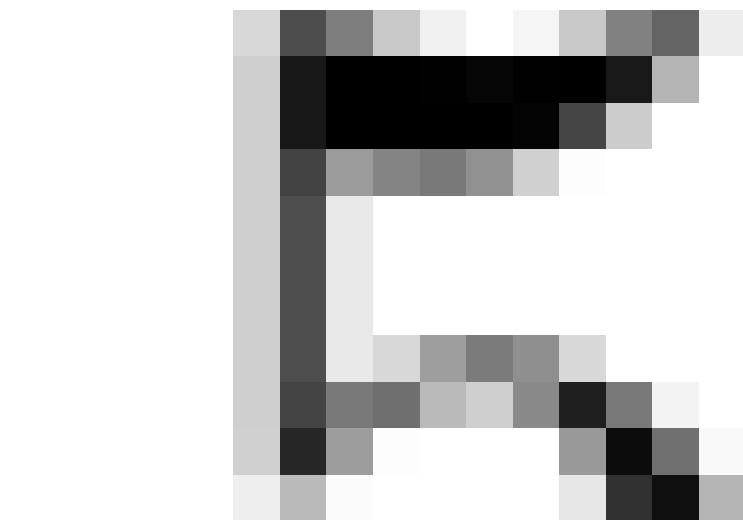
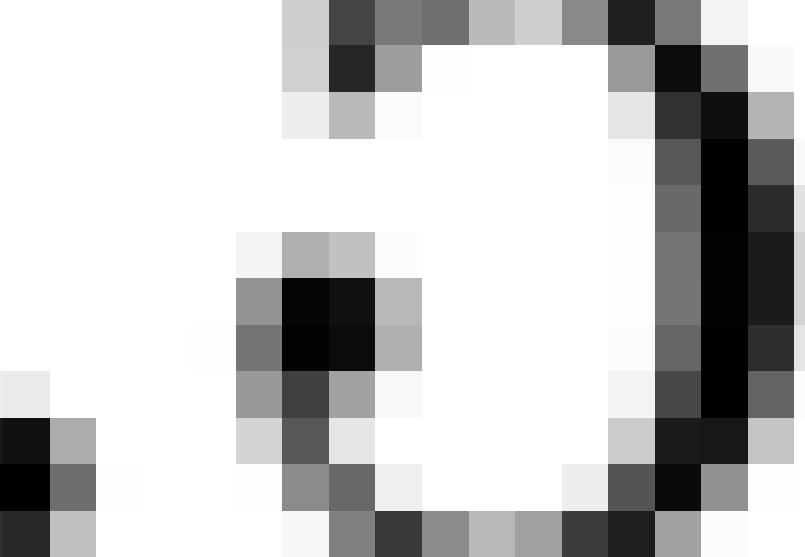
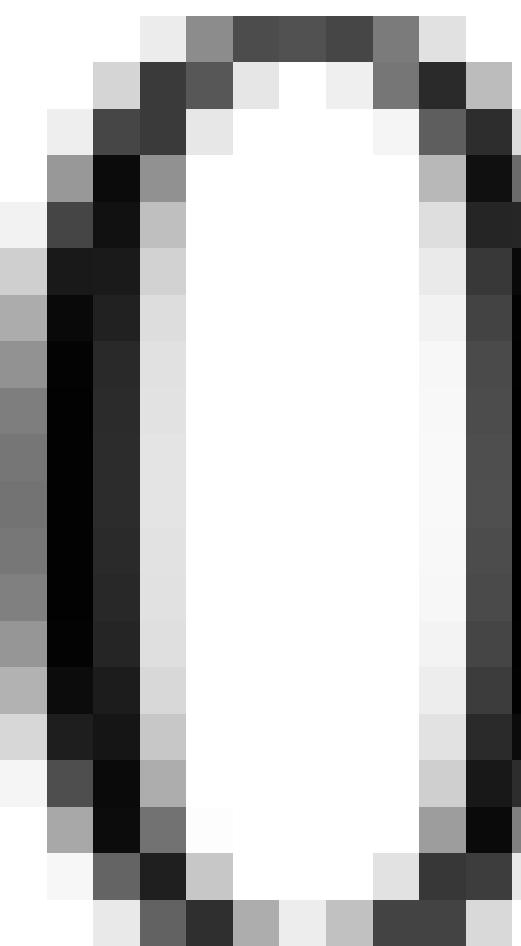
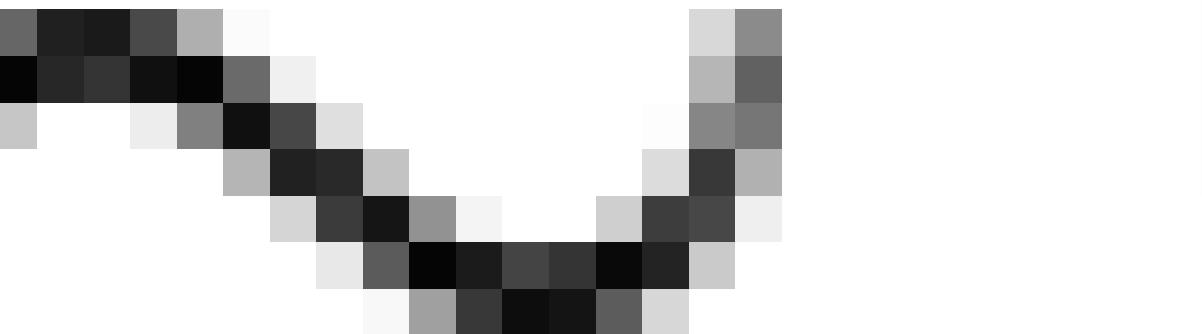
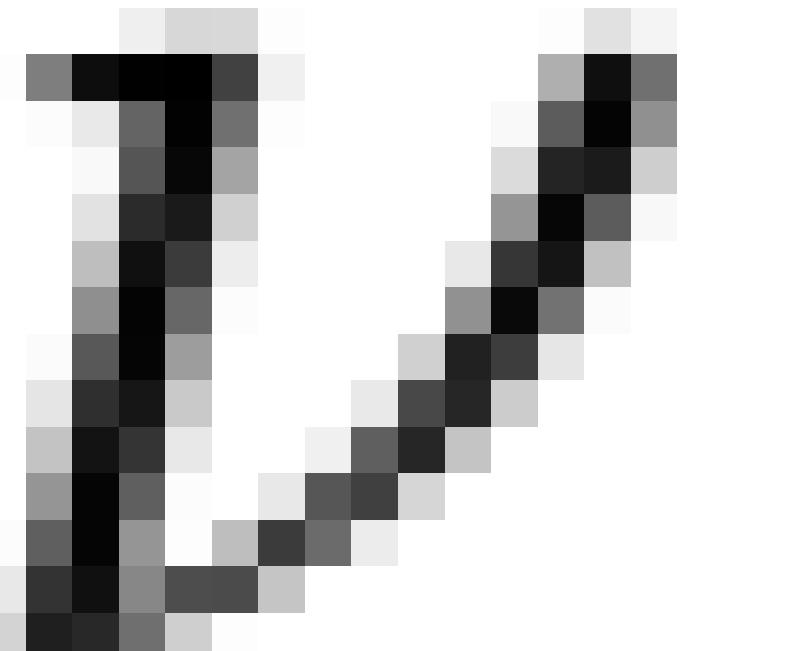
σ_w







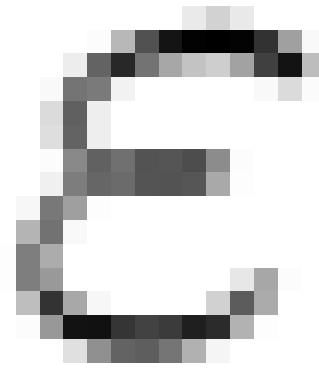
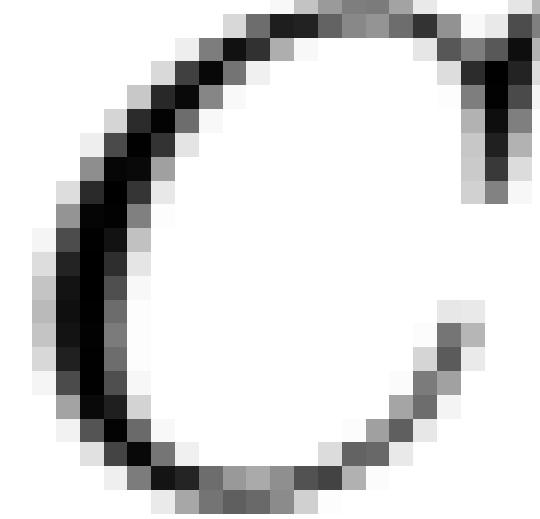
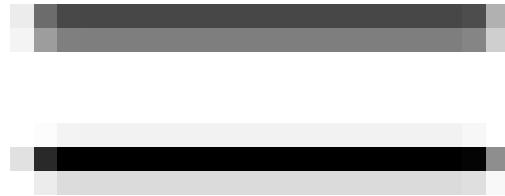
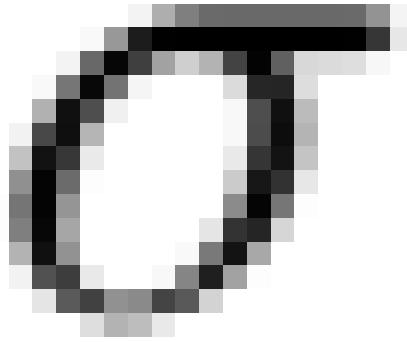


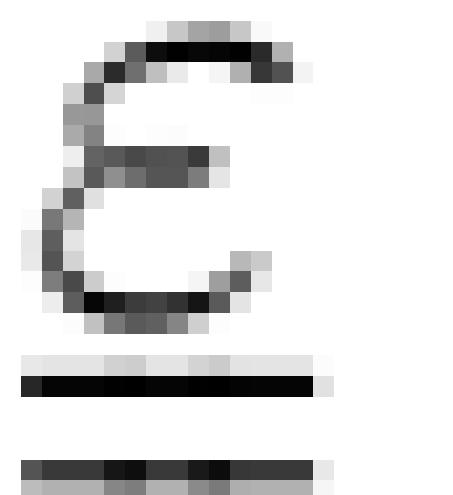




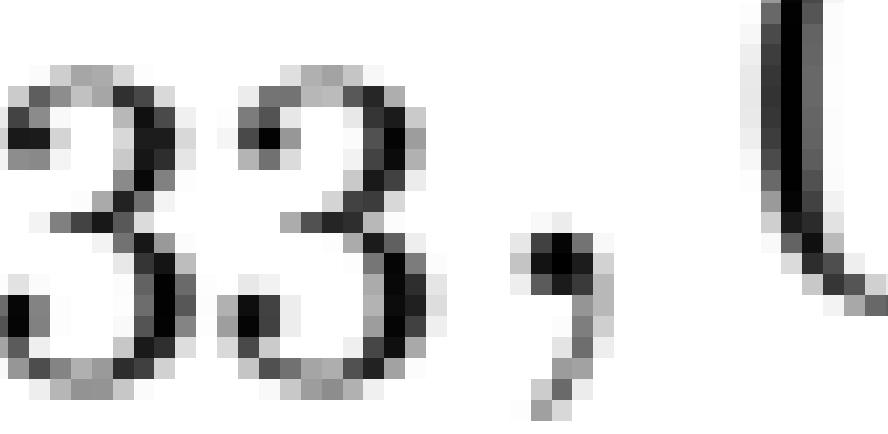




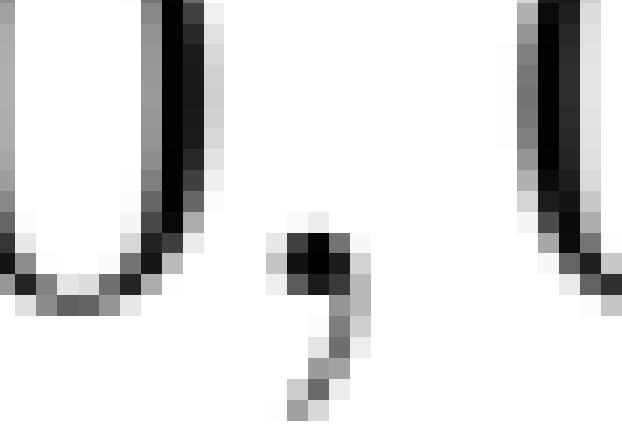




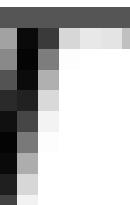
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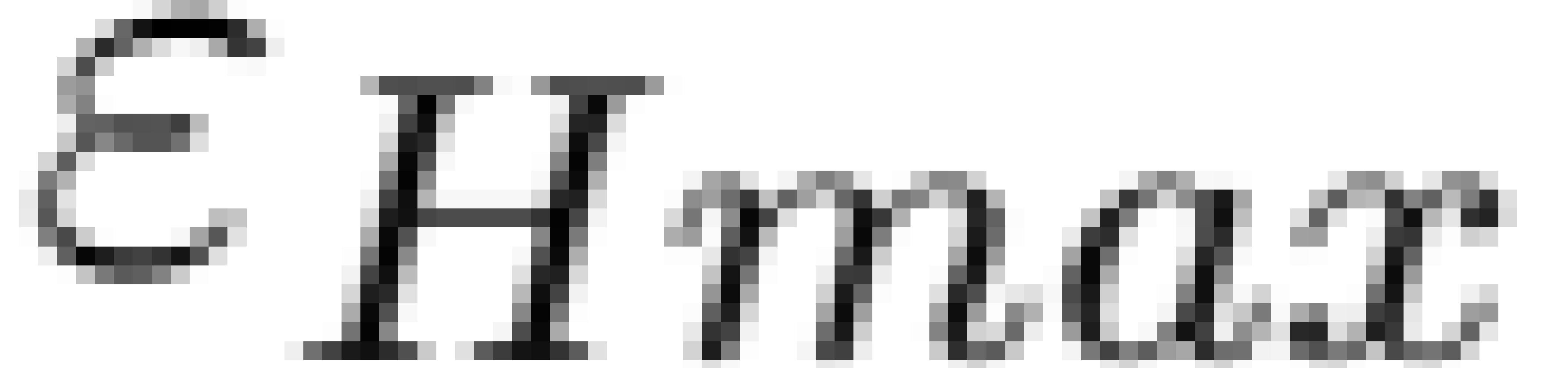


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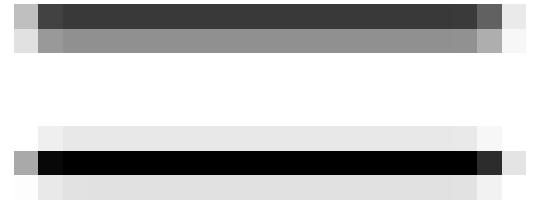
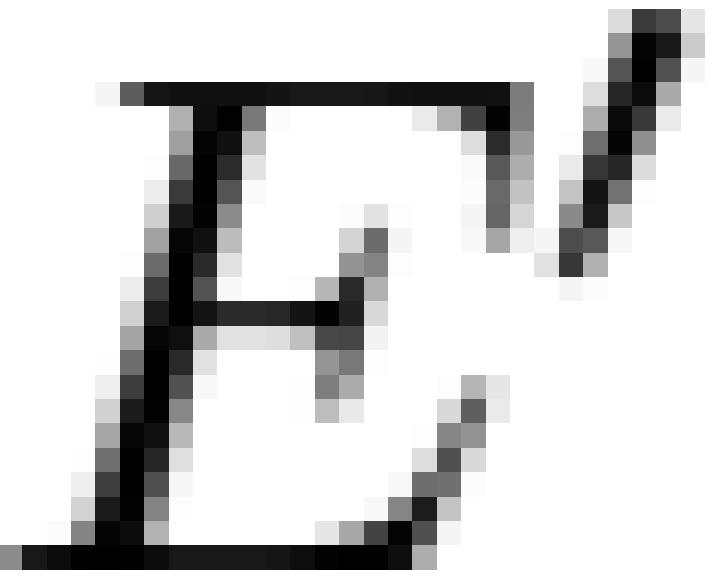
$$\left\{ \begin{array}{l} \sigma_{11} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{33} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \end{array} \right.$$

$$\left\{ \begin{array}{l} \sigma_{11} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{E}{1-\nu^2} \epsilon_{11} + \frac{\nu E}{1-\nu^2} \epsilon_{22} \\ \sigma_{22} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{\nu E}{1-\nu^2} \epsilon_{11} + \frac{E}{1-\nu^2} \epsilon_{22} \end{array} \right.$$

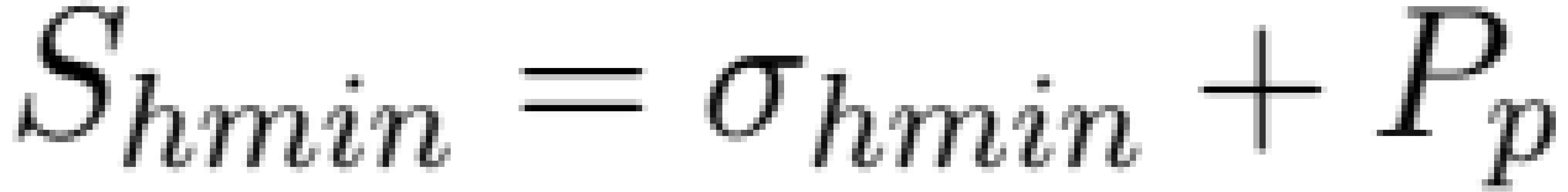




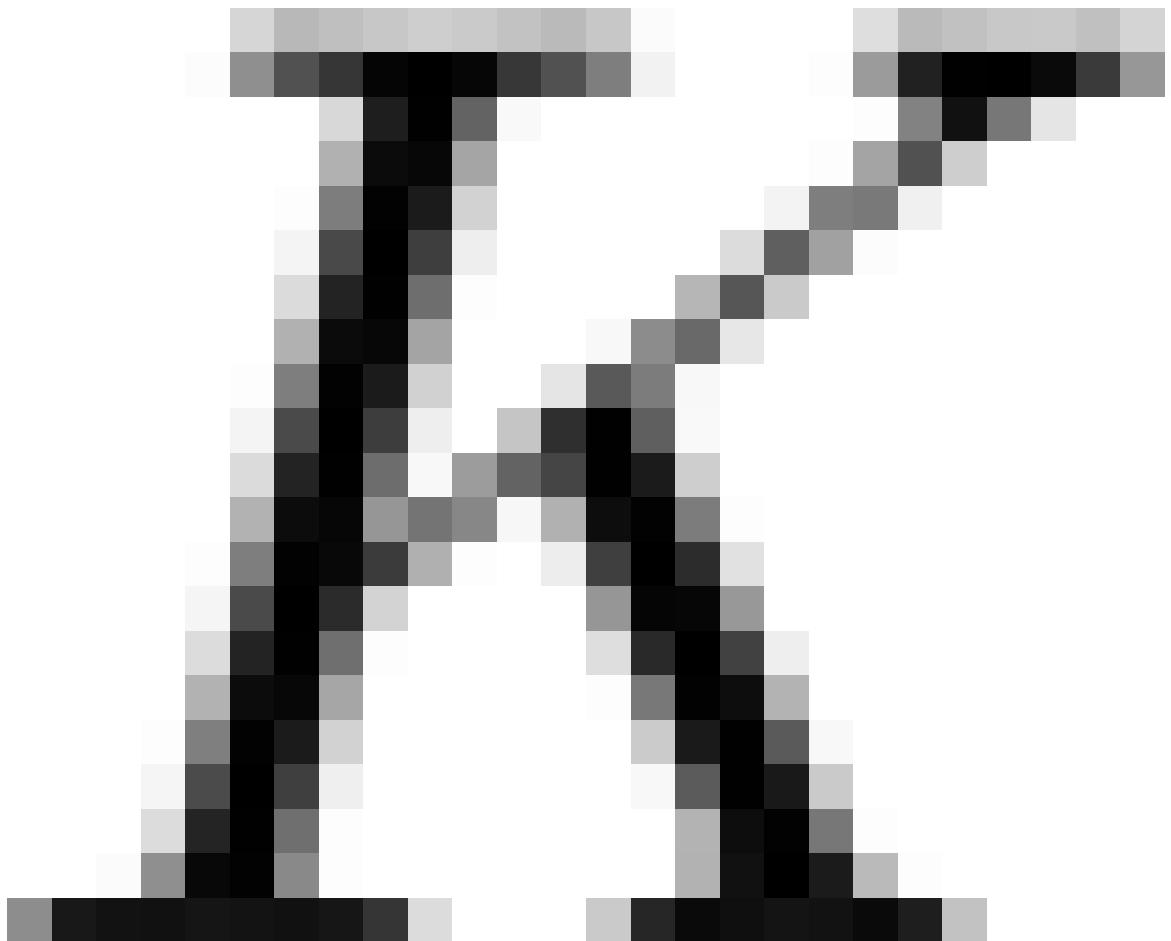
$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} + \nu E' \epsilon_{hmin} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + \nu E' \epsilon_{Hmax} + E' \epsilon_{hmin} \end{array} \right.$$

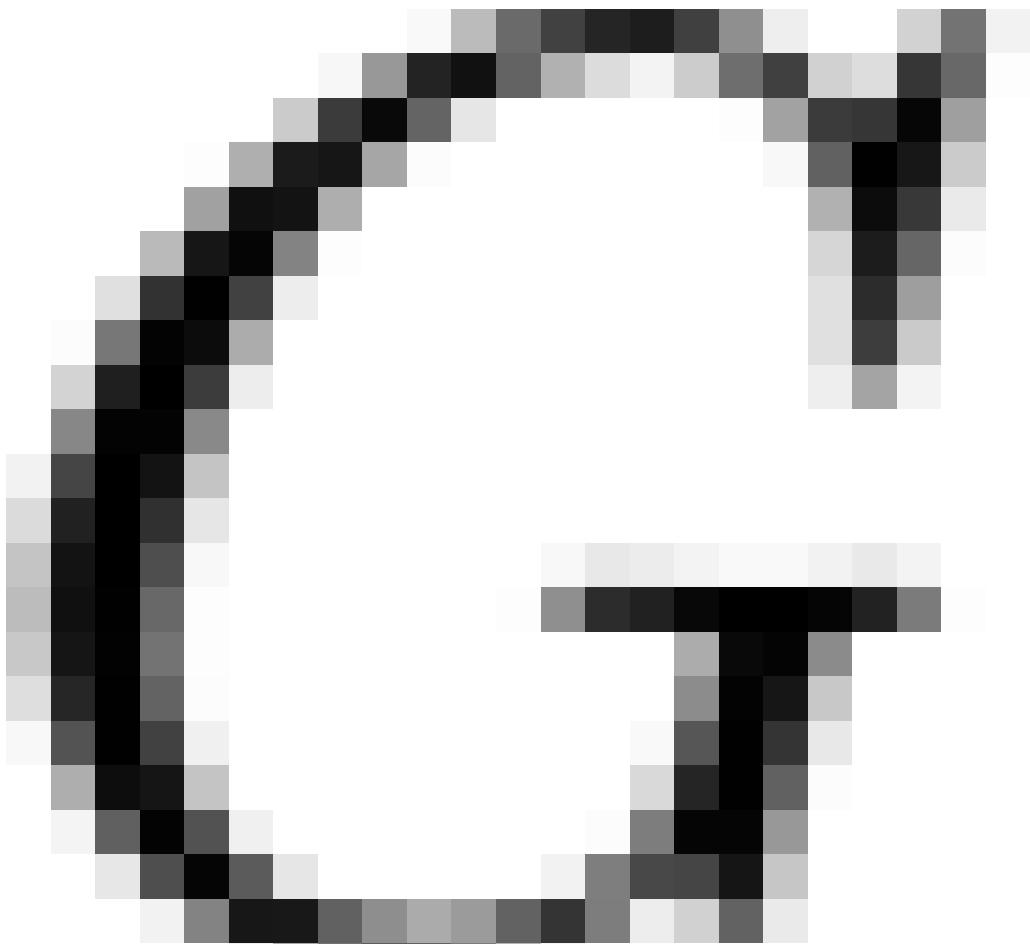




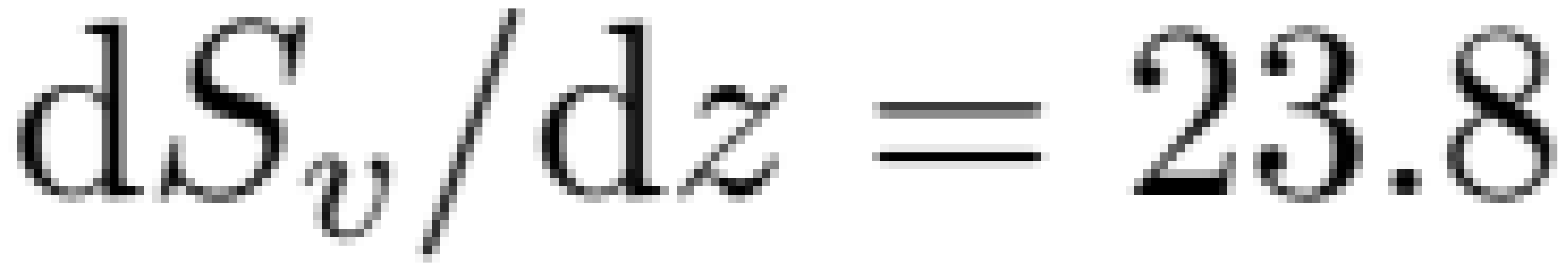


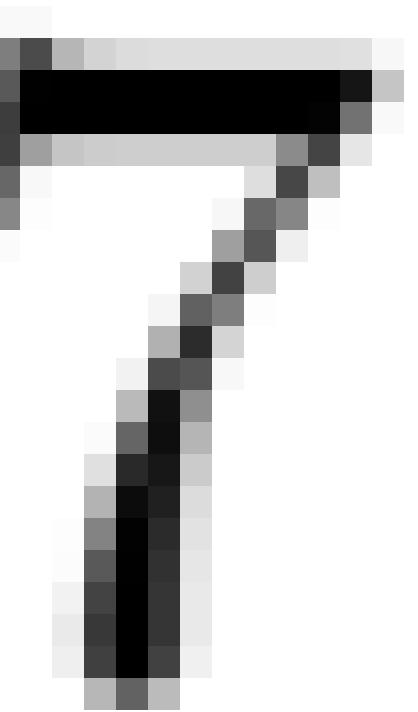
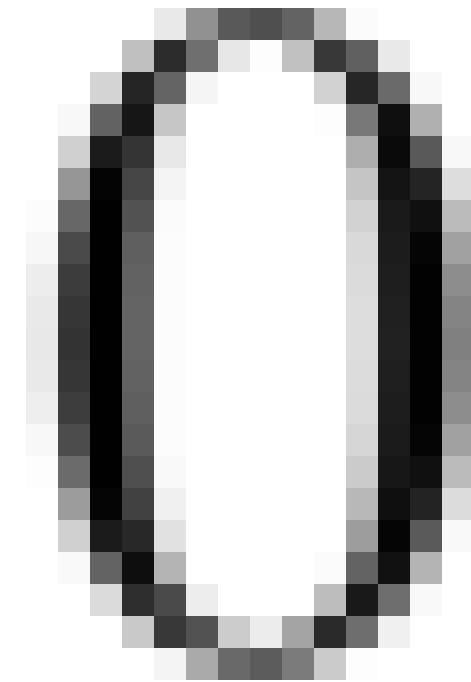


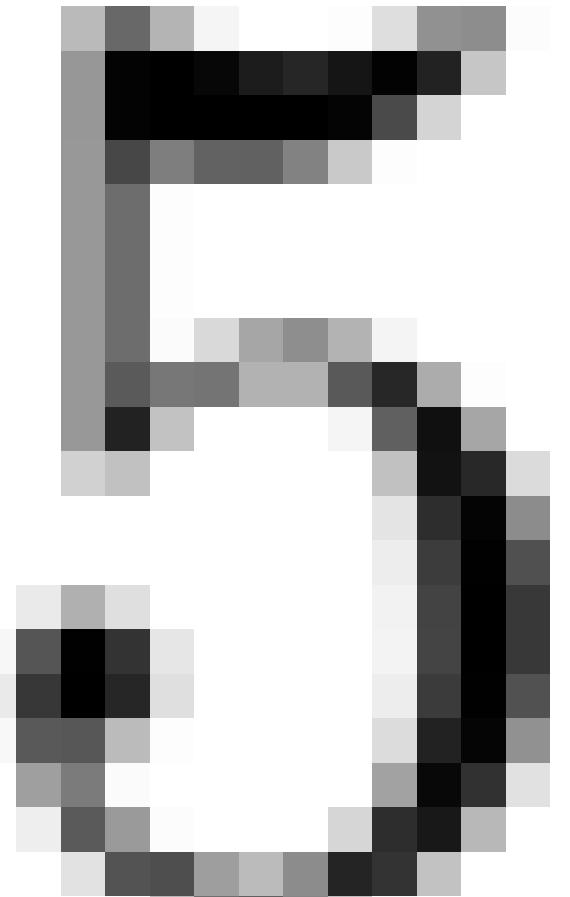
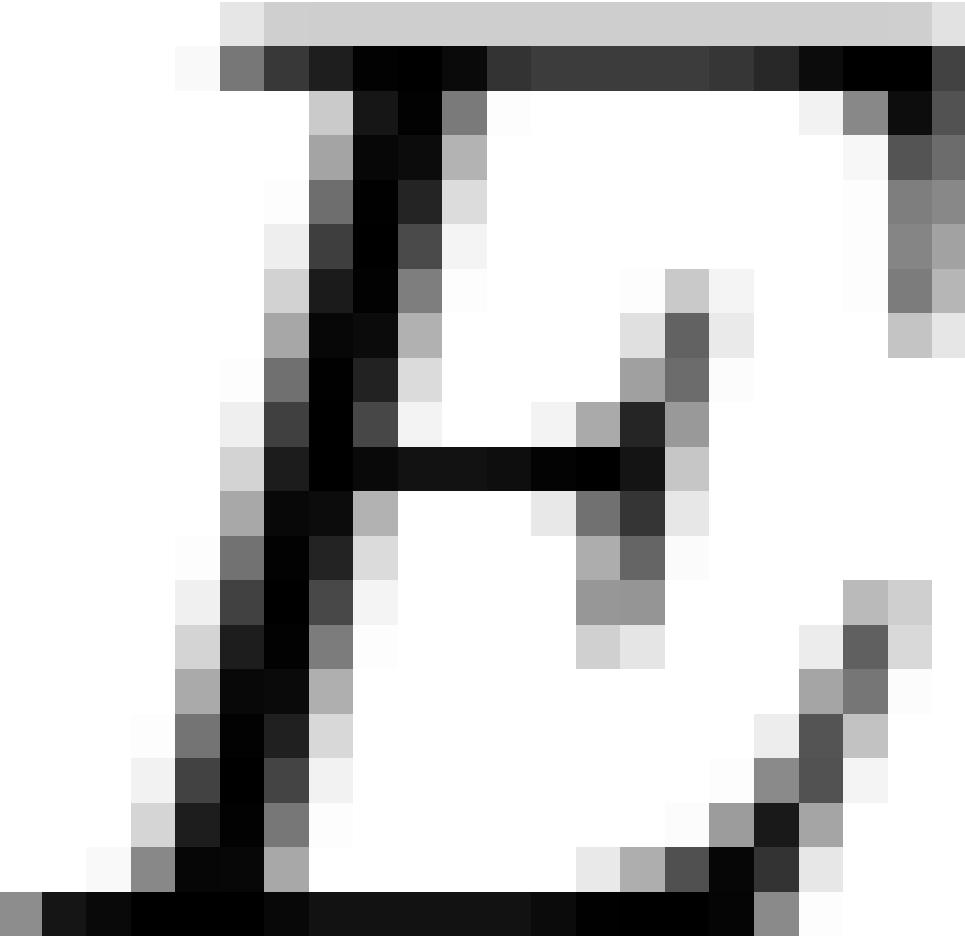


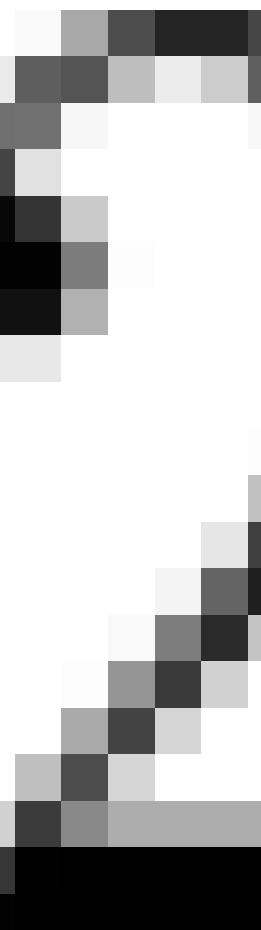
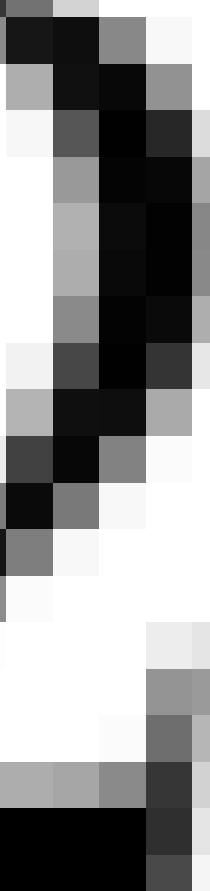
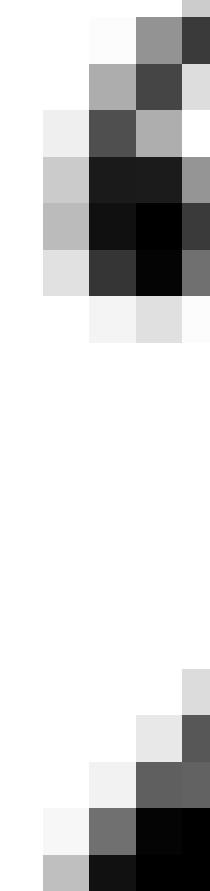
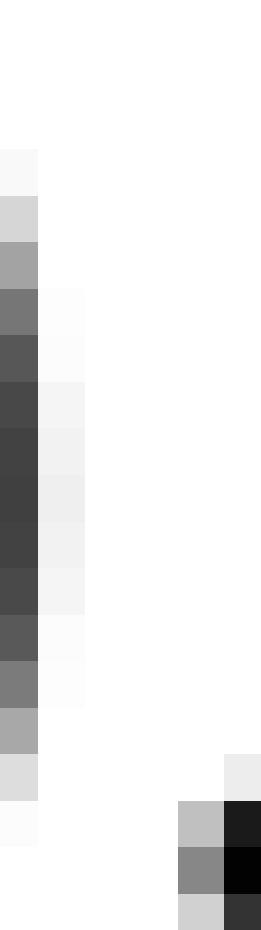
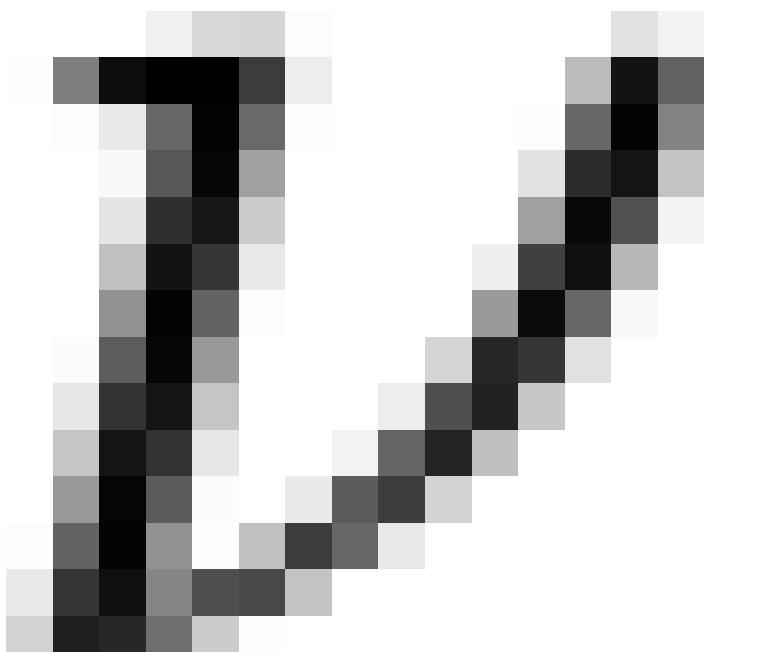


	(E, v)	(K, G)
$G =$	$\frac{E}{2(1+v)}$	<u>Shear modulus</u> (also noted as μ , S-wave)
$M =$	$\frac{(1-v)E}{(1+v)(1-2v)}$	<u>Constrained modulus</u> (uniaxial compaction, P-wave)
$\lambda =$	$\frac{vE}{(1+v)(1-2v)}$	<u>Lamé first parameter</u> (volumetric strain component)
$K =$	$\frac{E}{3(1-2v)}$	<u>Bulk modulus</u> (relates volumetric strain and isotropic stress)





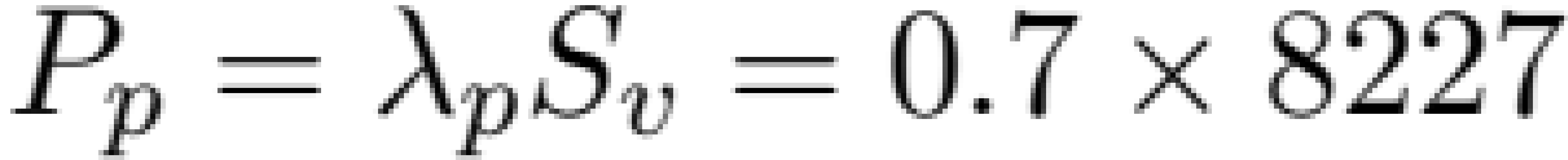




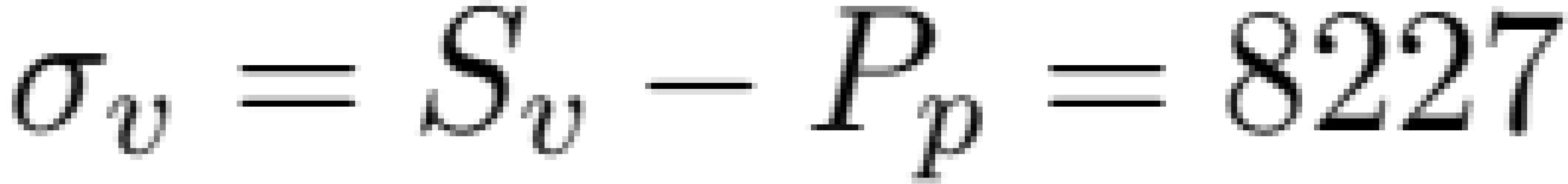




$$s_u = \frac{23.8 \text{ MPa}}{\text{km}} \times \frac{1 \frac{\text{psi}}{\text{ft}}}{\frac{\text{MPa}}{\text{km}}} \times 7950 \text{ ft} = 8227 \text{ psi}$$









$$\frac{E'}{1 - \nu^2} = \frac{E}{1 - 0.22^2} = \frac{5 \times 10^6 \text{ psi}}{5.25 \times 10^6 \text{ psi}}$$

$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} \times 0.0002 = 1745 \text{ psi} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{hmin} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} \times 0.0002 = 927 \text{ psi} \end{array} \right.$$

$$S_{H\max} = \sigma_{H\max} + P_p = 1745 \text{ psi} + 5759 \text{ psi} = 7504 \text{ psi}$$

$$S_{h\min} = \sigma_{h\min} + P_p = 927 \text{ psi} + 5759 \text{ psi} = 6686 \text{ psi}$$

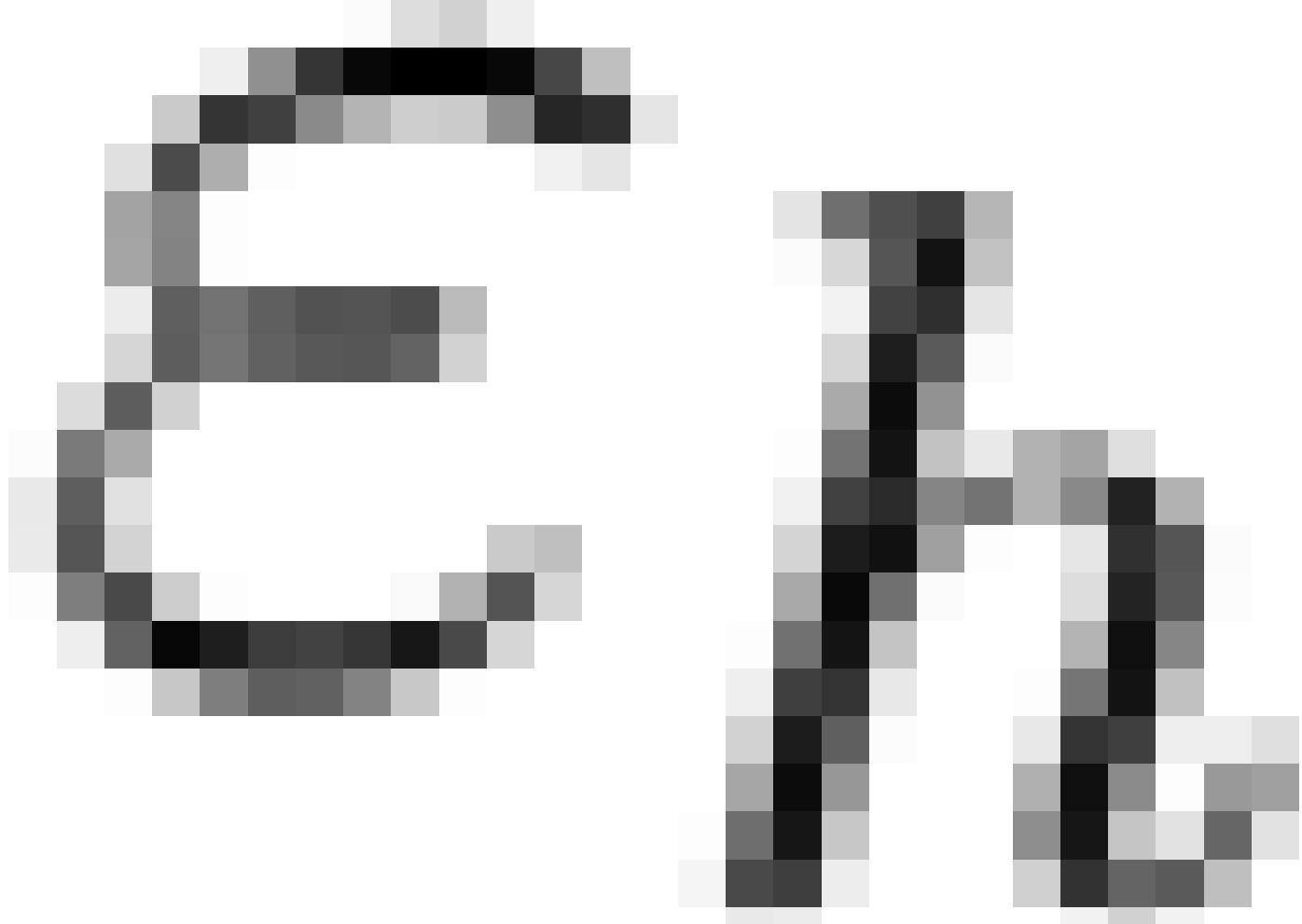




$$C_{pp} =$$

$$\frac{1}{V_p} \frac{dV_p}{dP_p}$$

S_u, ϵ_h



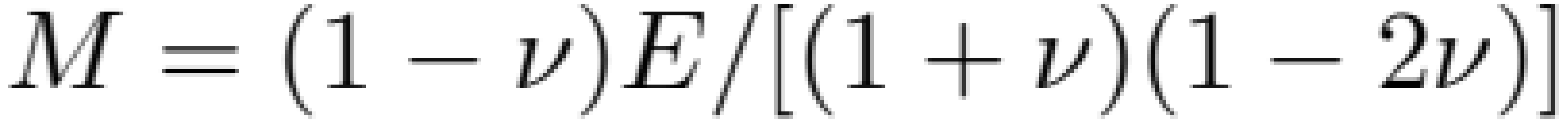
Op

$=$

Op

ϕ



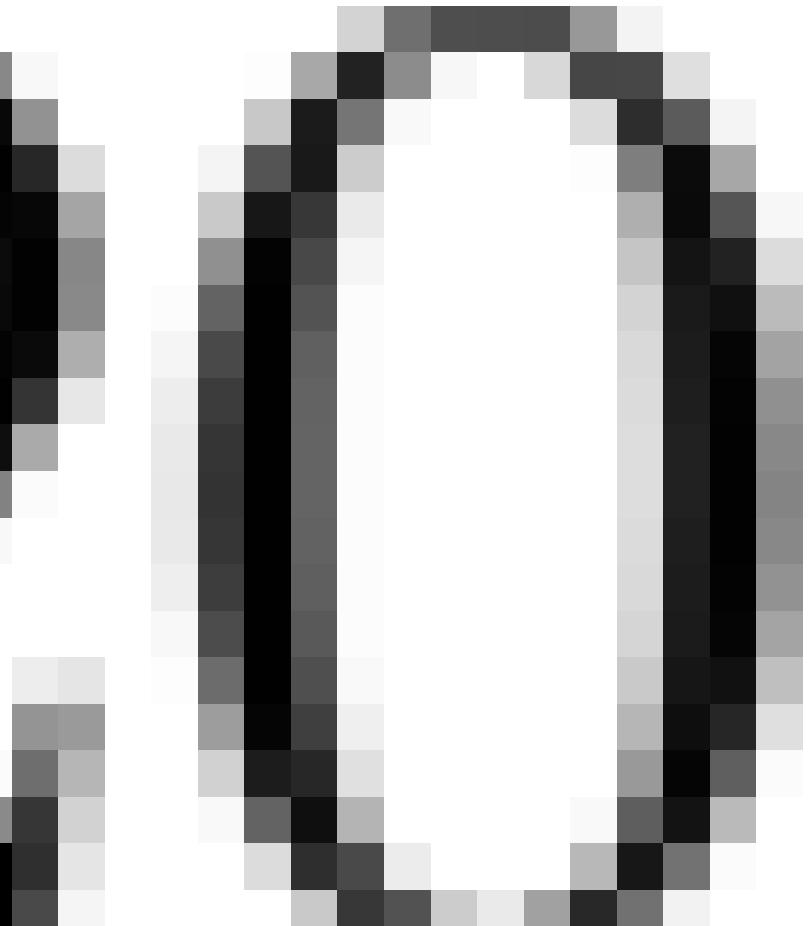
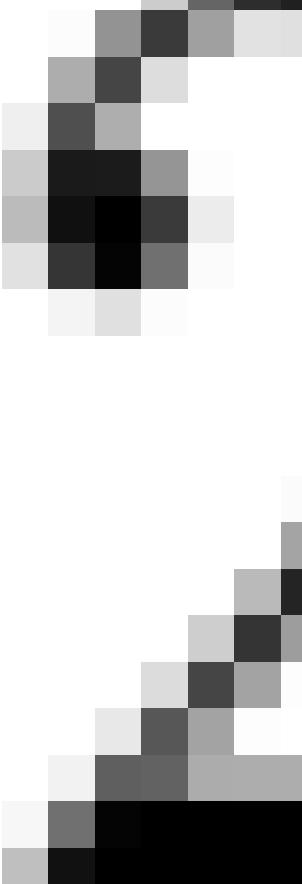
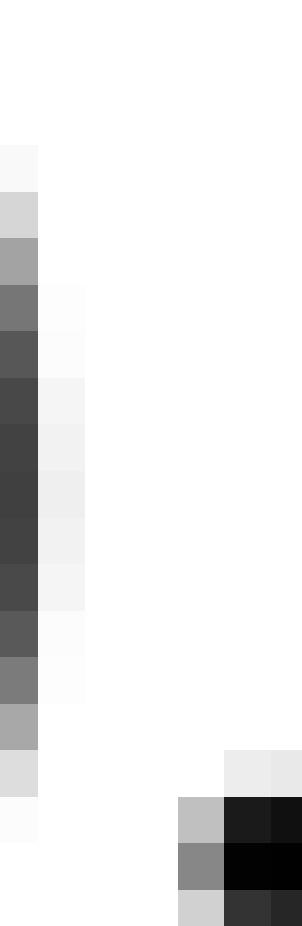
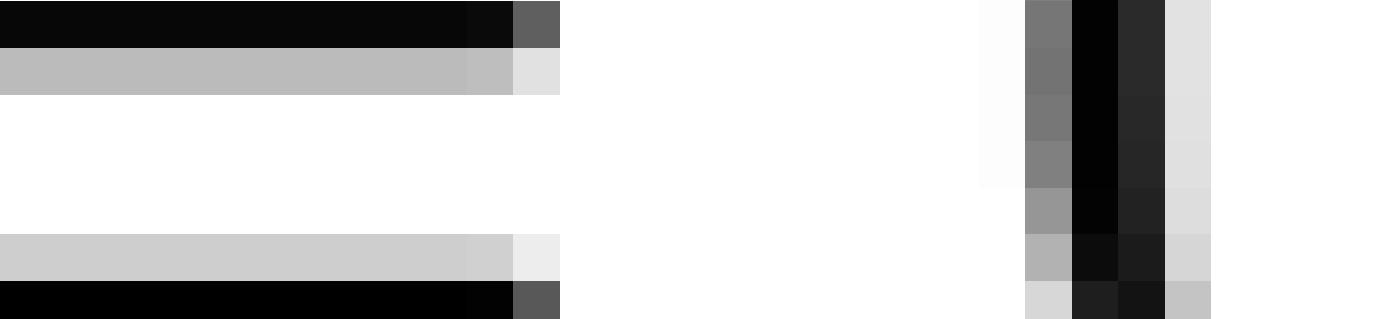
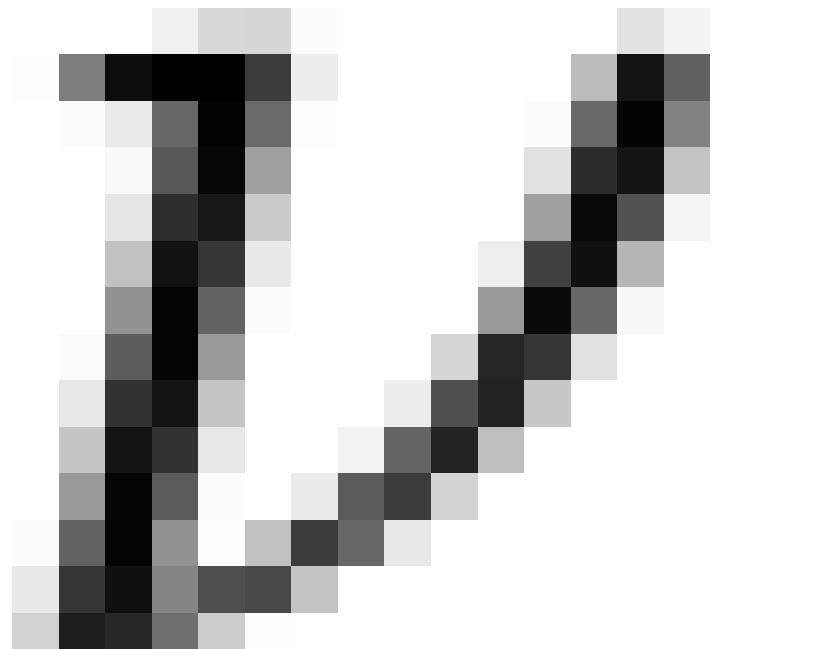


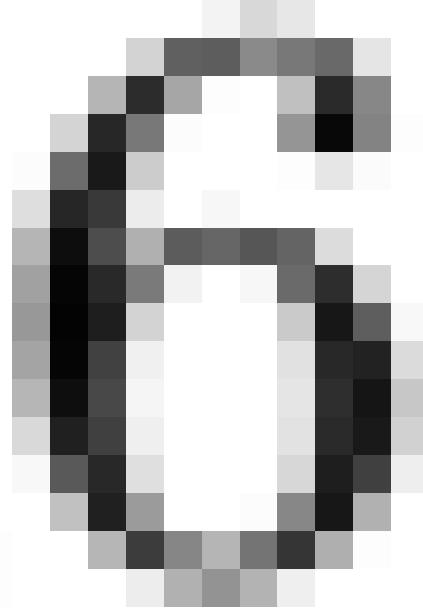
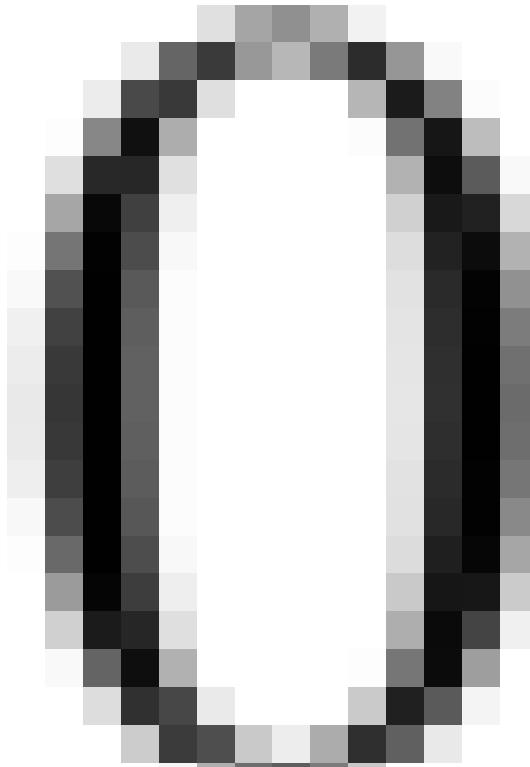
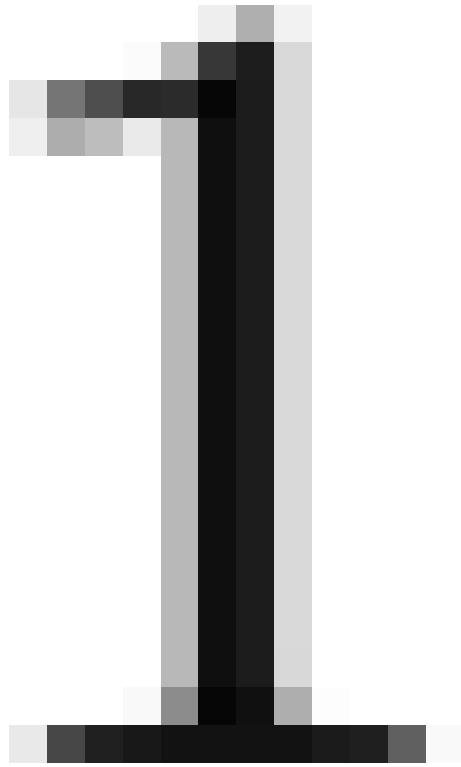
$\langle pp \rangle$

$$\frac{(1 + \nu)(1 - \nu)E\phi}{(1 - 2\nu)}$$

$\langle 1 - \nu \rangle E\phi$

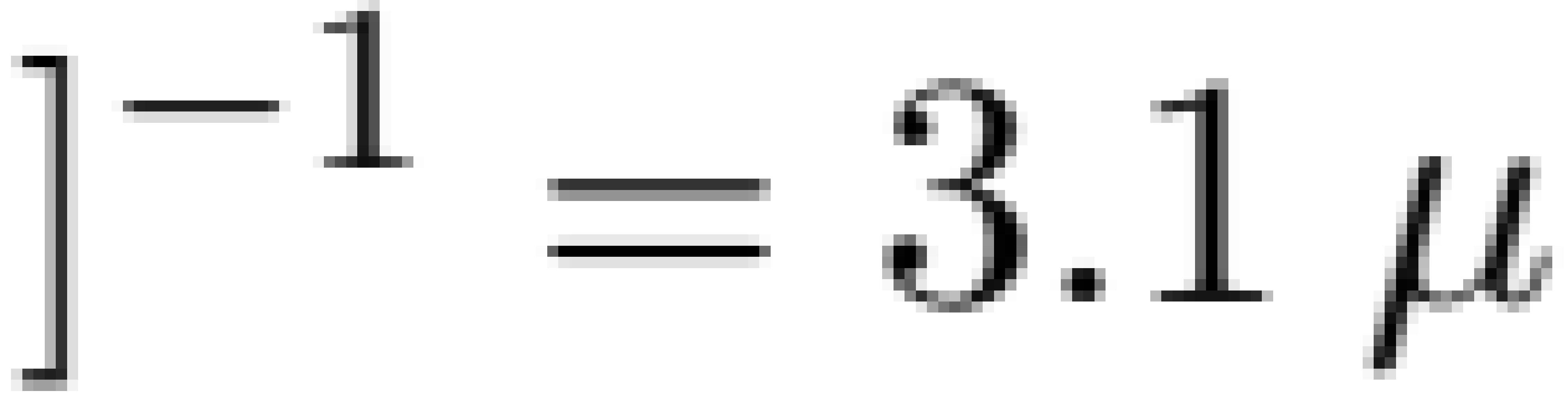


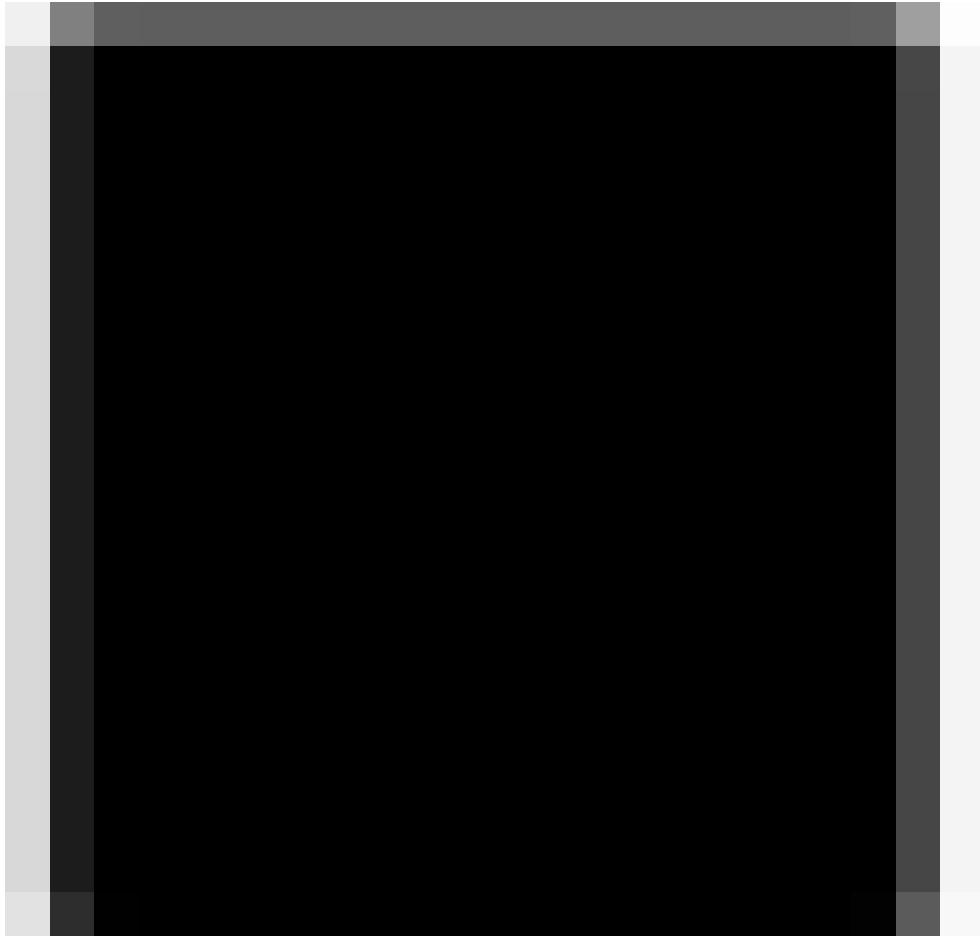


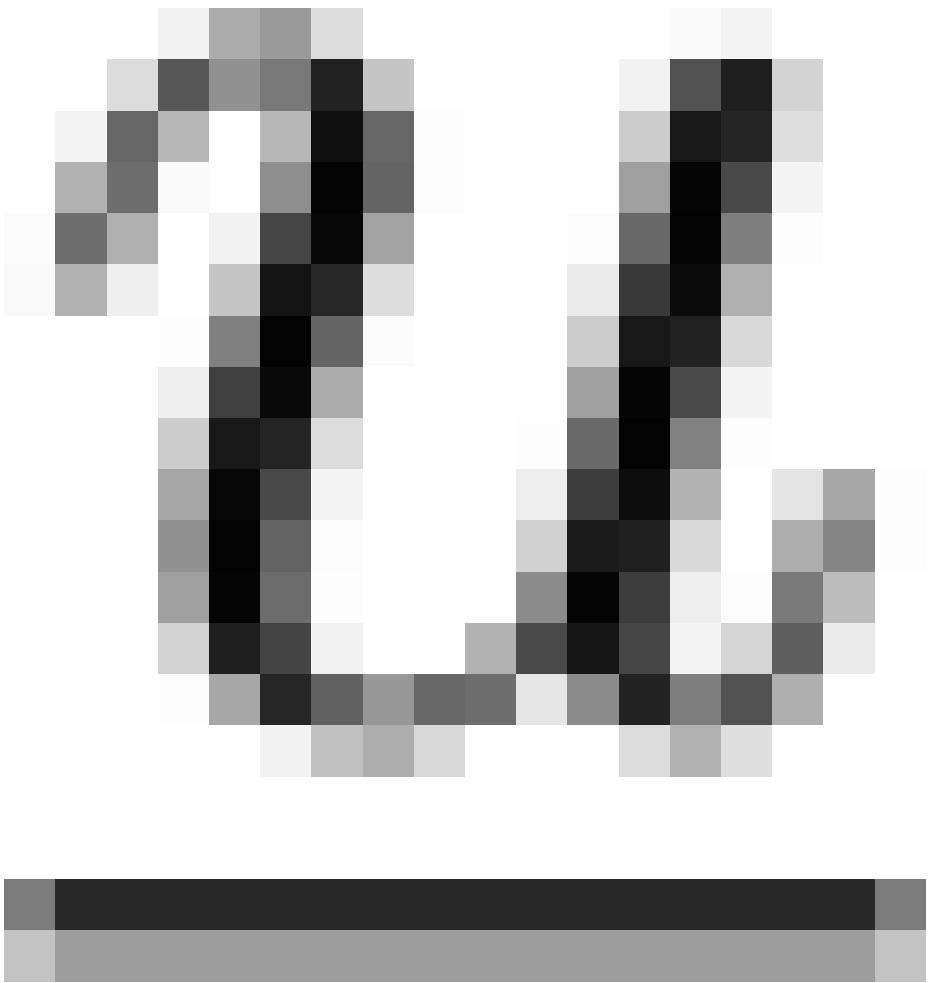


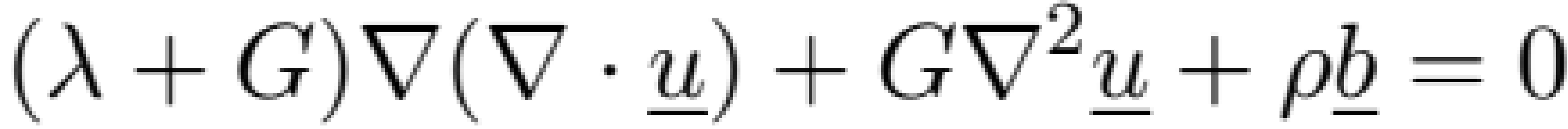
$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.20)10 \text{ GPa}}{1.6 \times 10^6 \text{ psi}} = \frac{11.11 \text{ GPa}}{(1+0.20)(1-2 \times 0.20)}$$

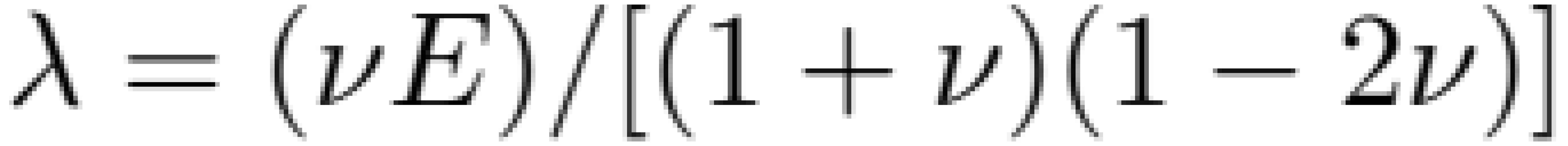
$$\frac{C_{pp}}{M_\phi} = \frac{1}{3.1 \times 10^6 \text{ psi} \times 0.20} = 1$$

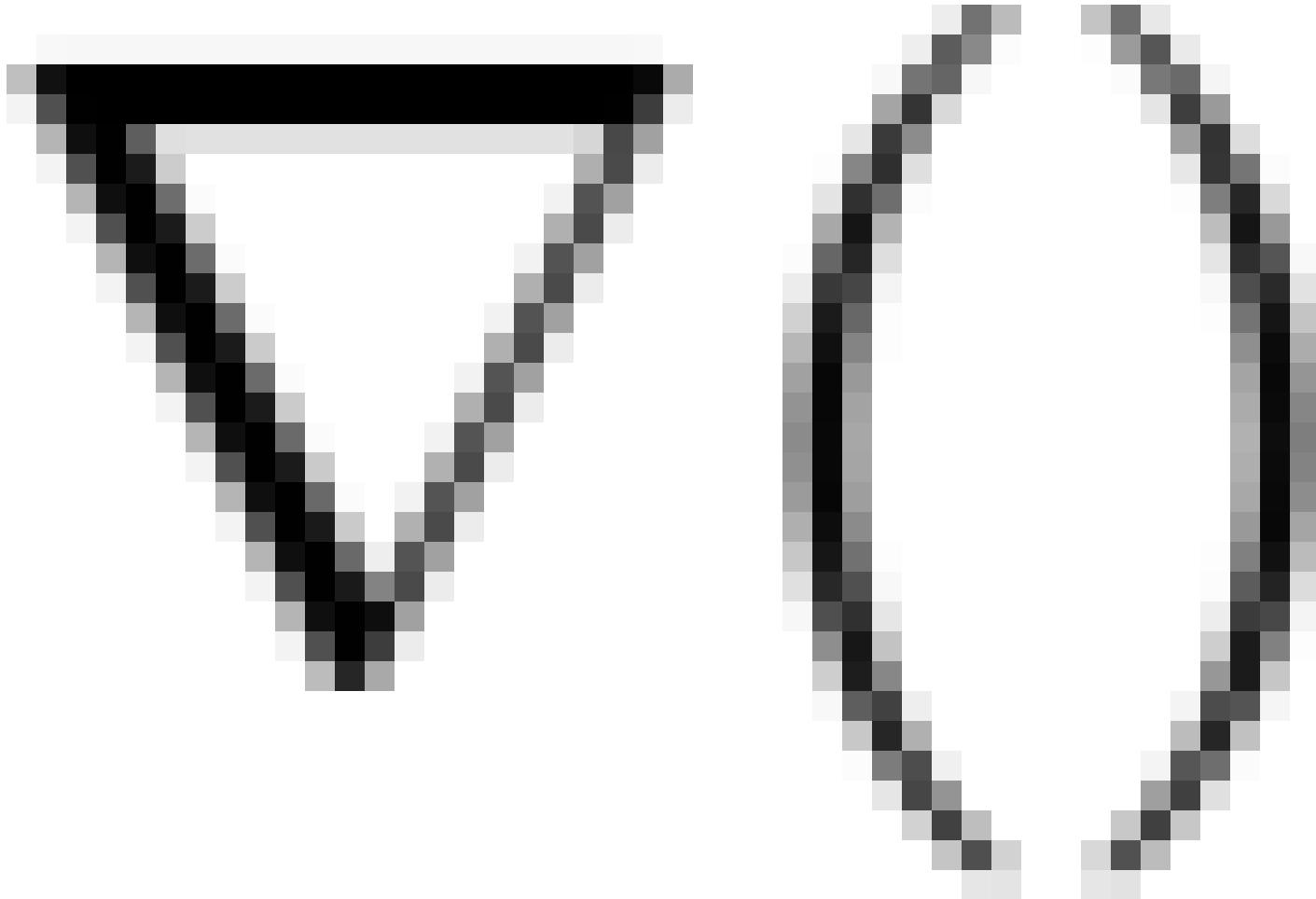


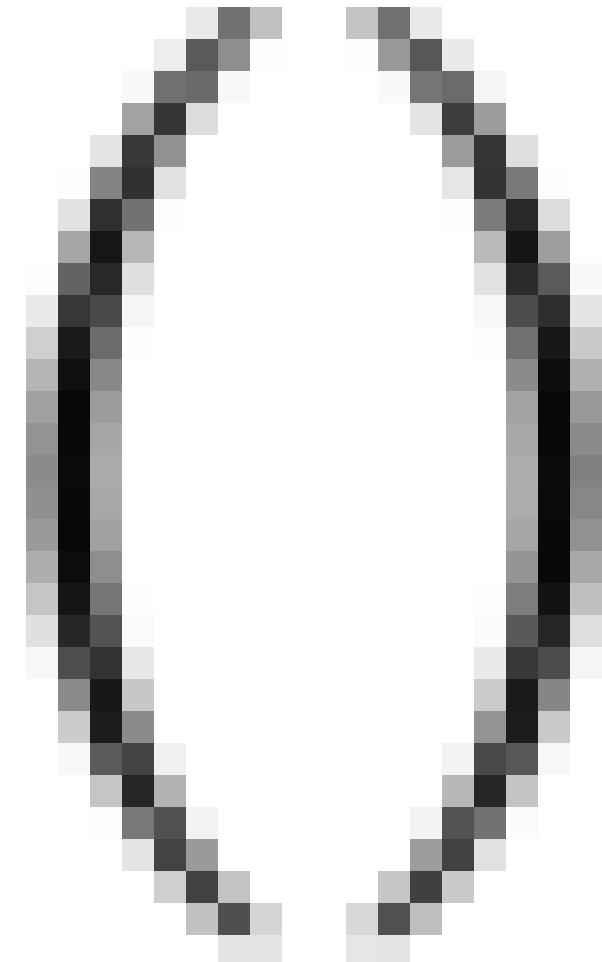
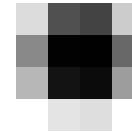
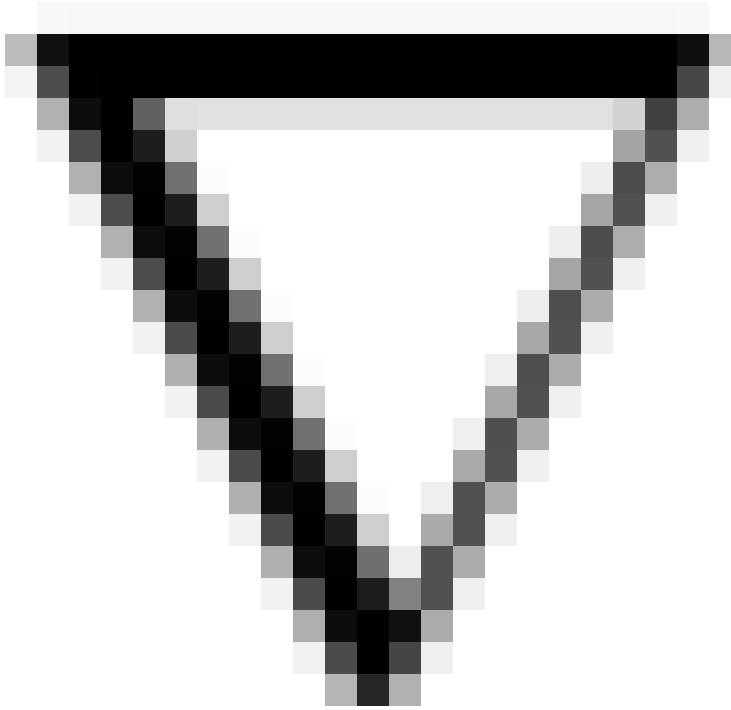


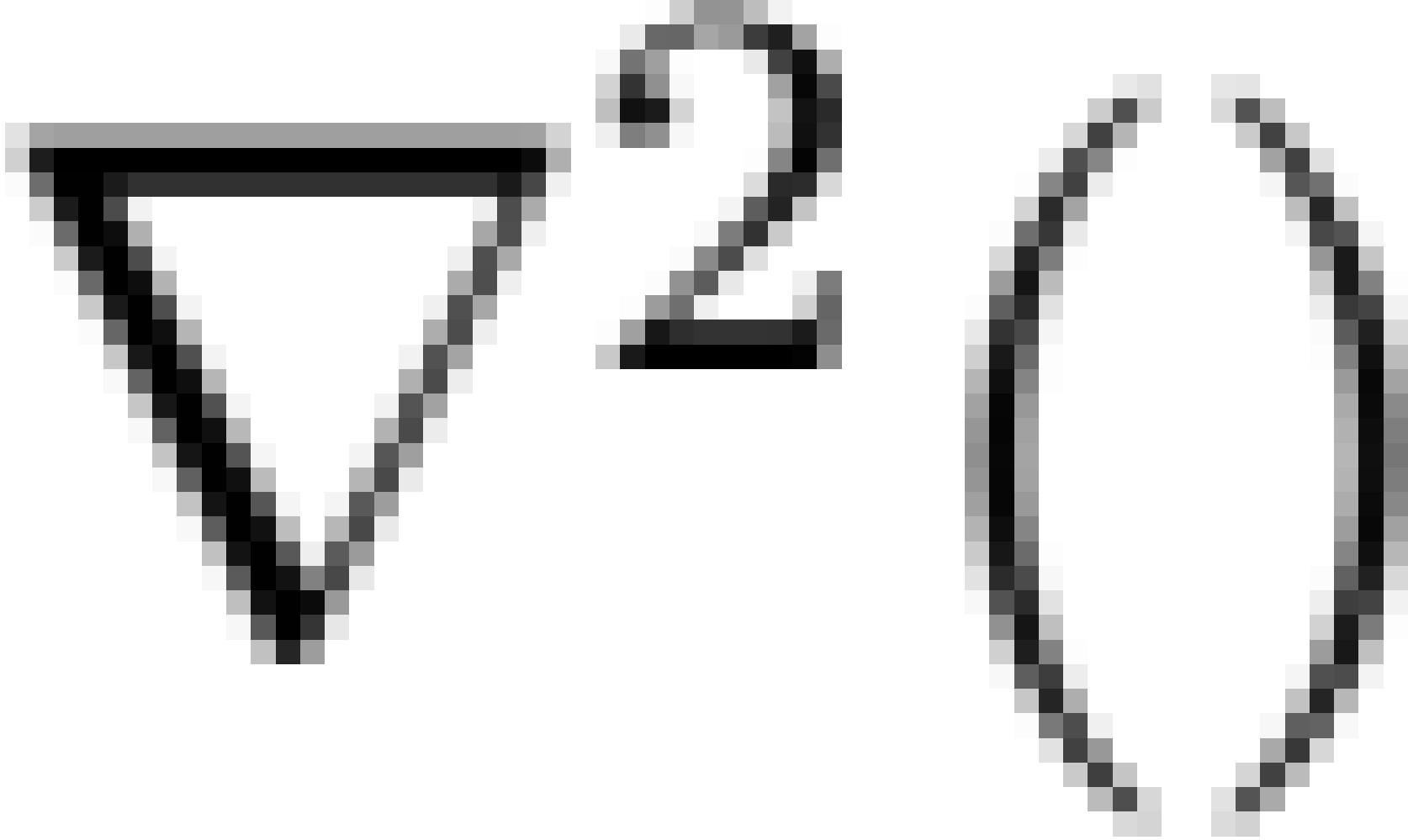




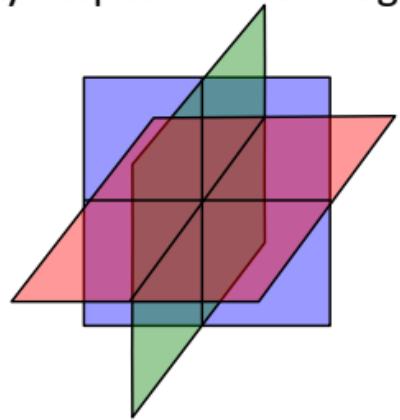








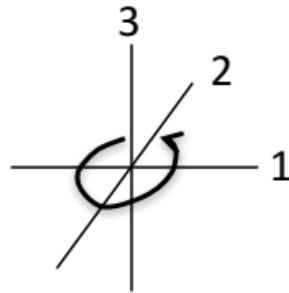
Orthorhombic symmetry
(symmetry respect to 3 orthogonal planes)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & 0 \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{55} & 0 \\ & 0 & 0 & 0 & & C_{66} \end{bmatrix}$$

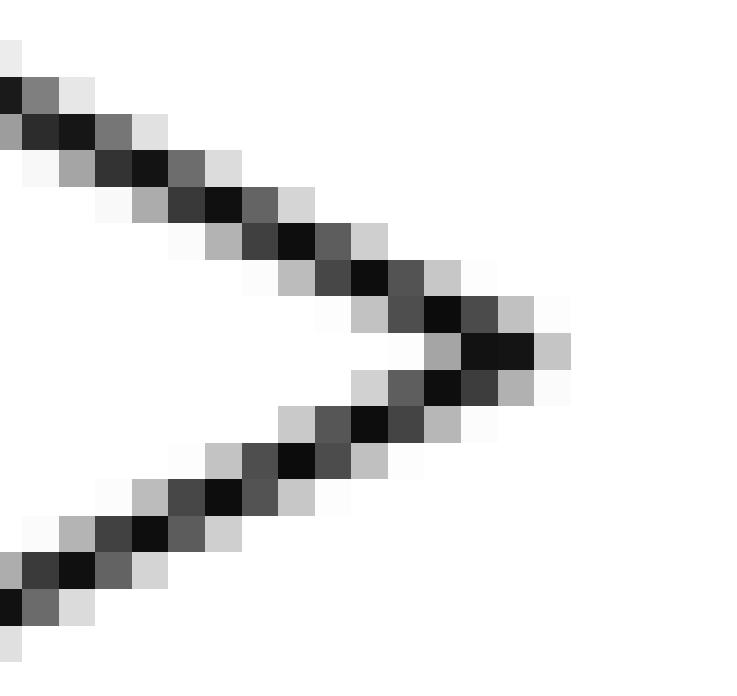
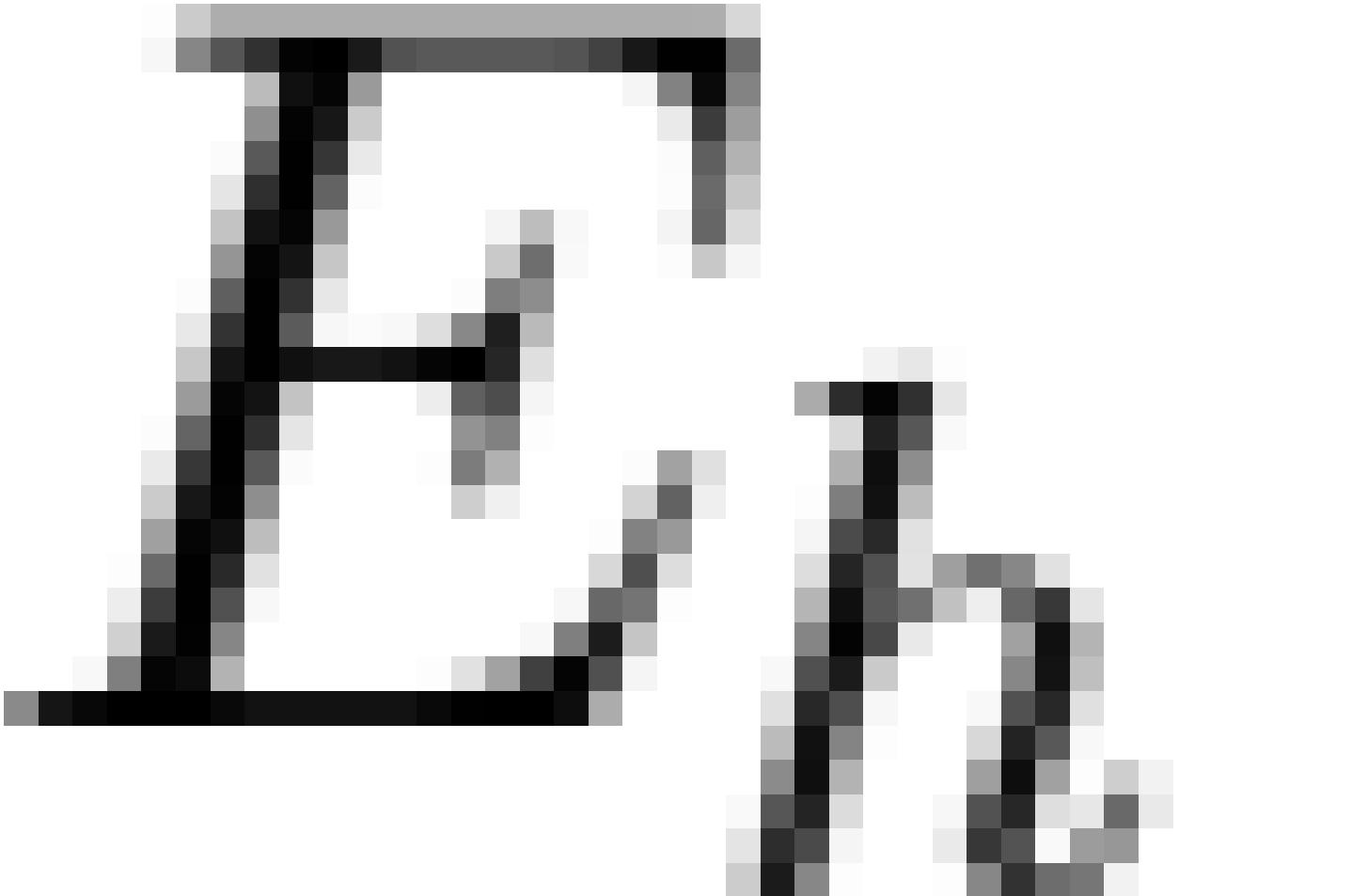
9 independent parameters

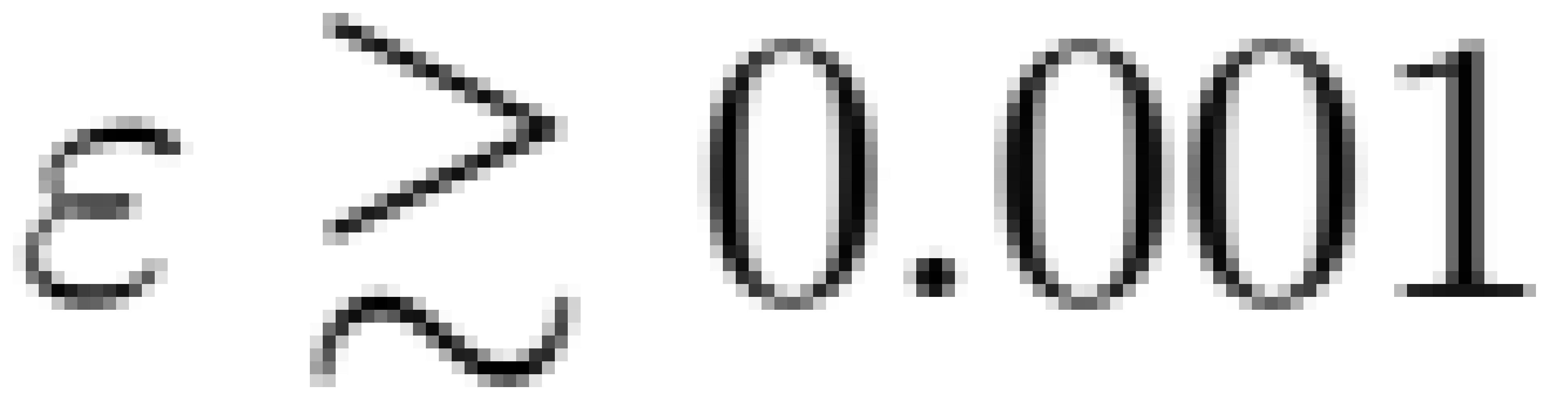
Transverse Isotropy
(symmetry respect to 1 axis)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & & 0 \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{44} & 0 \\ & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

5 independent parameters ($C_{12}=C_{11}-2C_{66}$)

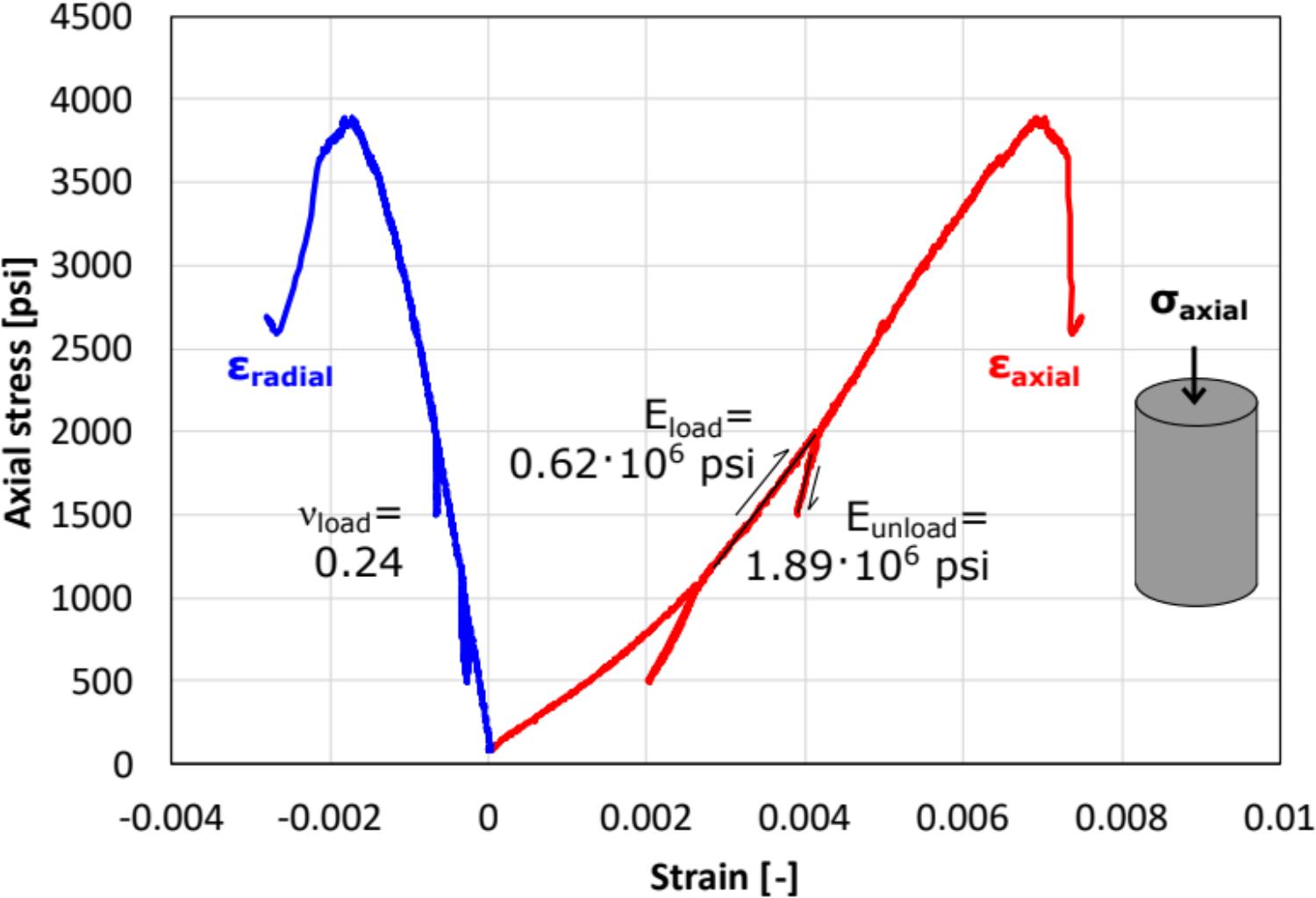




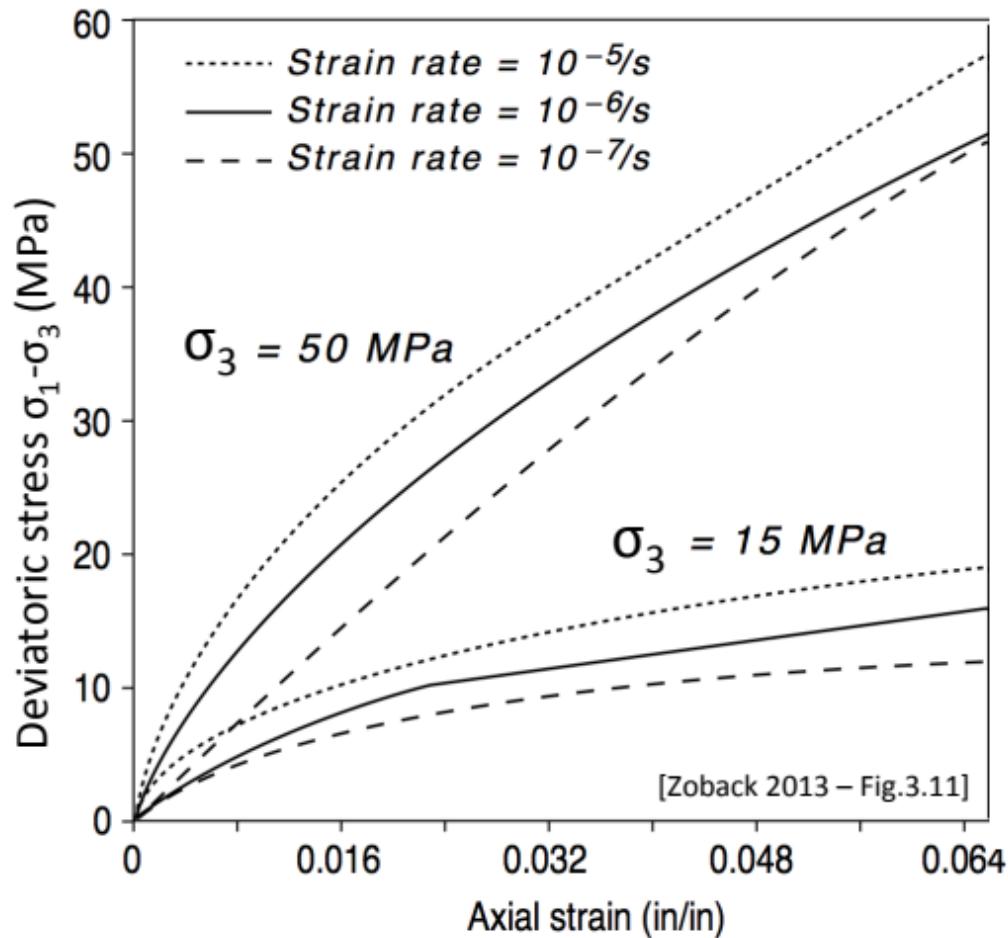
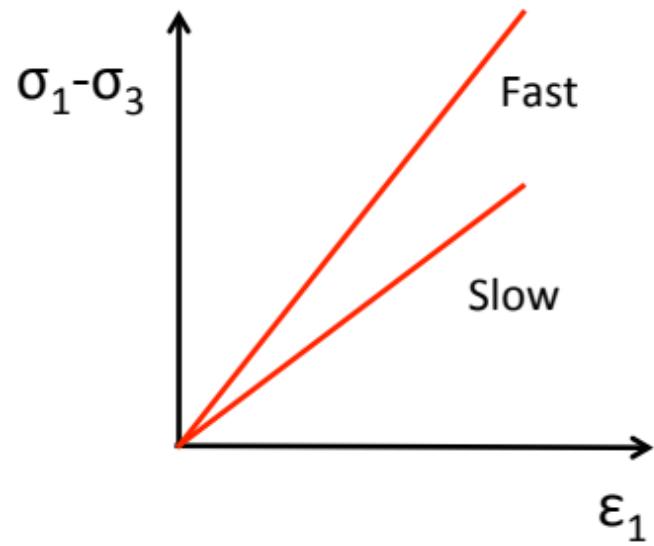




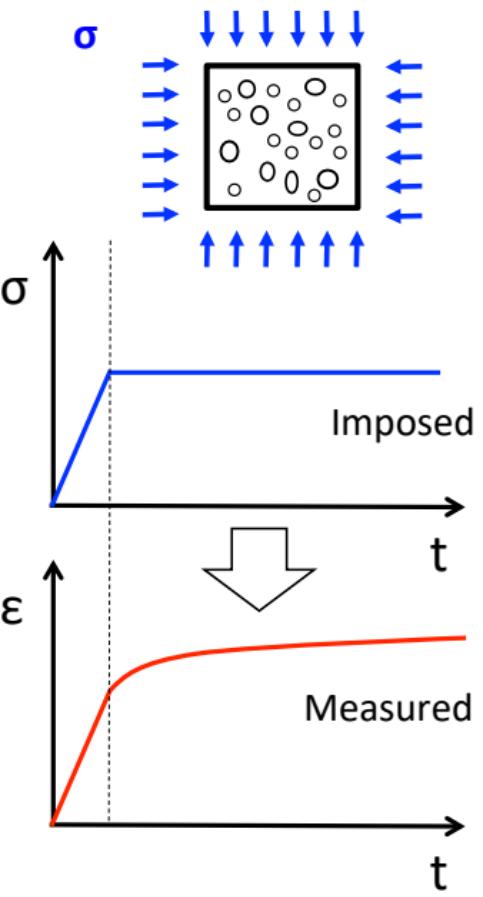




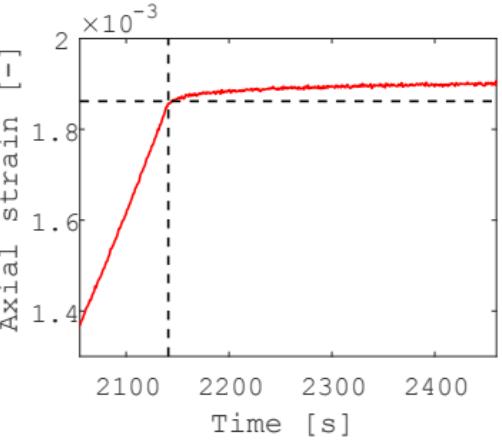
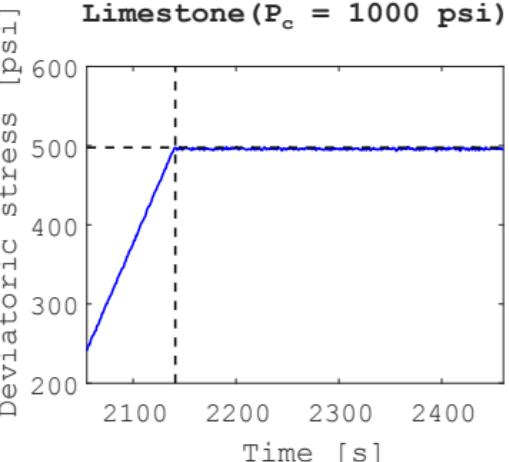
Strain rate hardening: The faster the loading, the stiffer the material



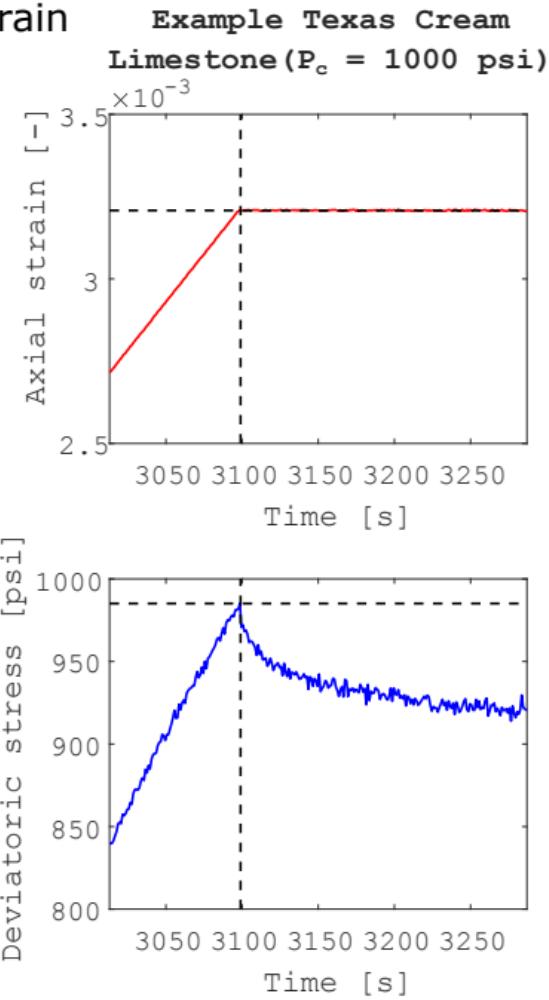
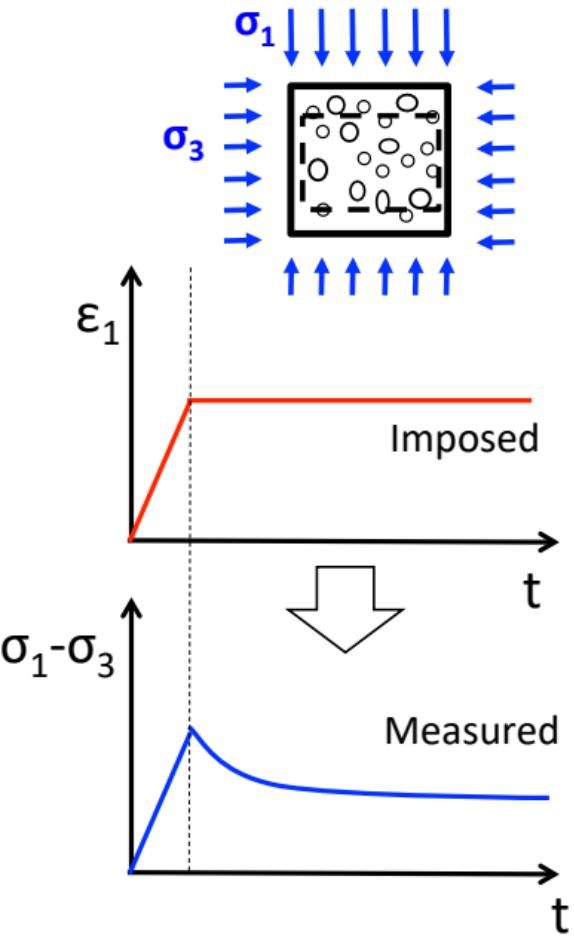
Creep strain at constant stress

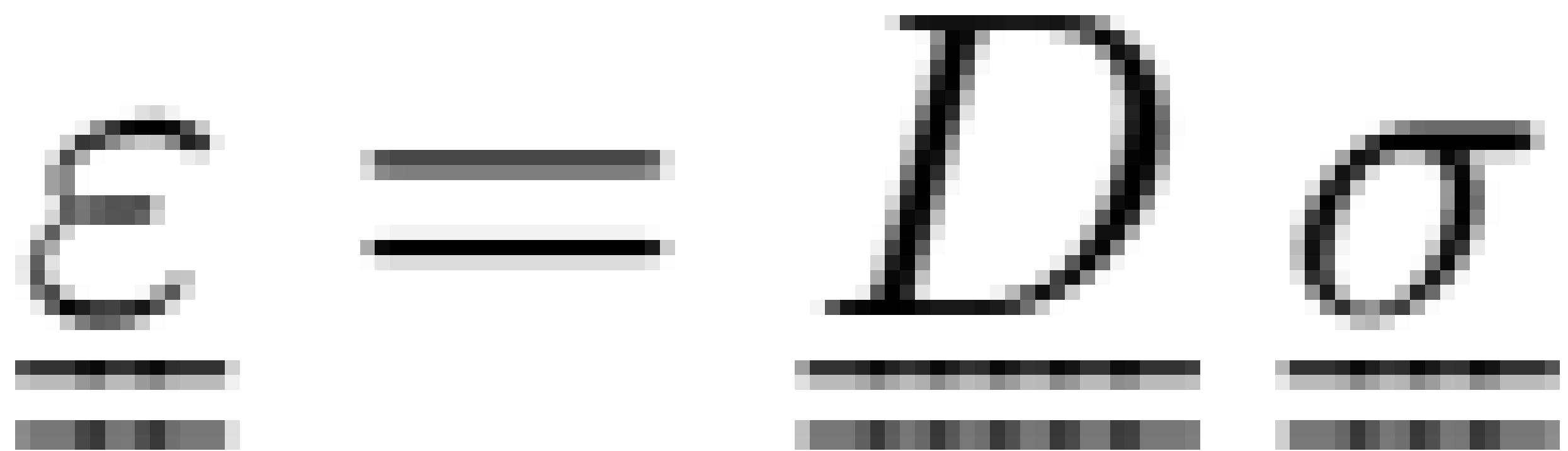


Example Texas Cream Limestone ($P_c = 1000$ psi)

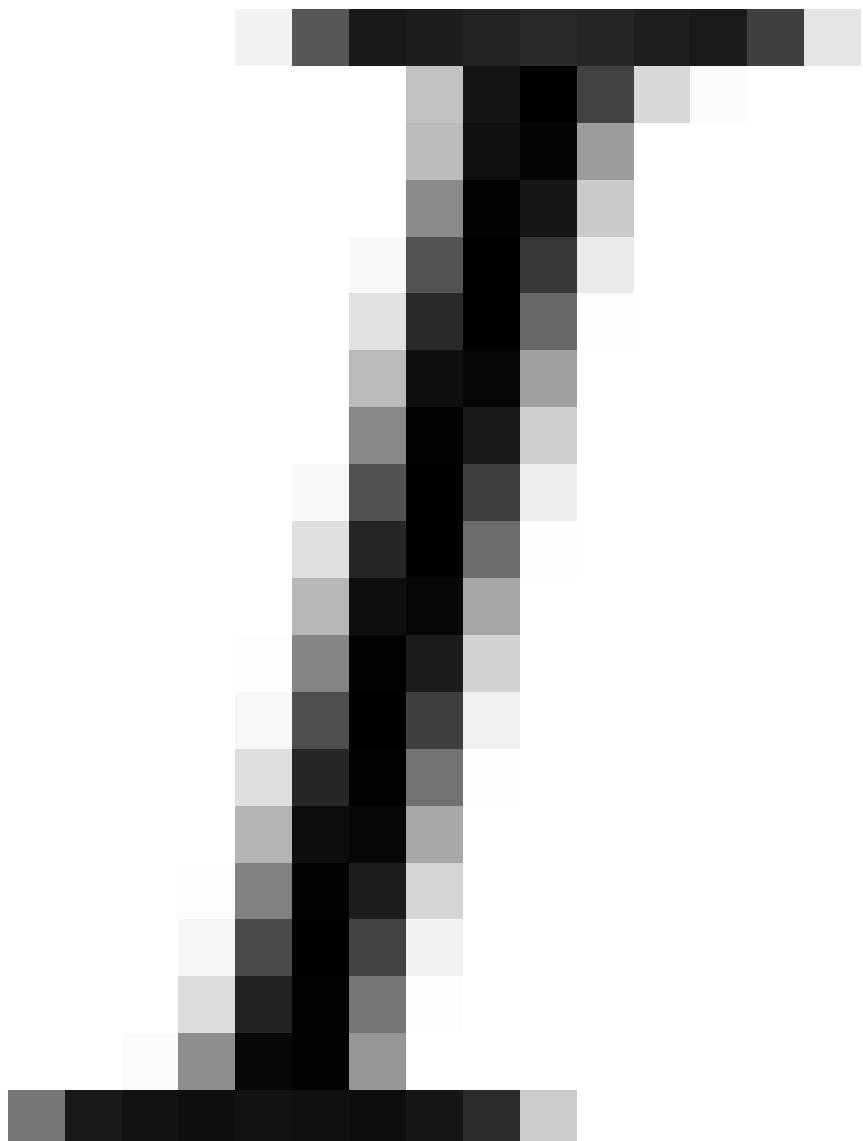


Stress relaxation at constant strain









α

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1

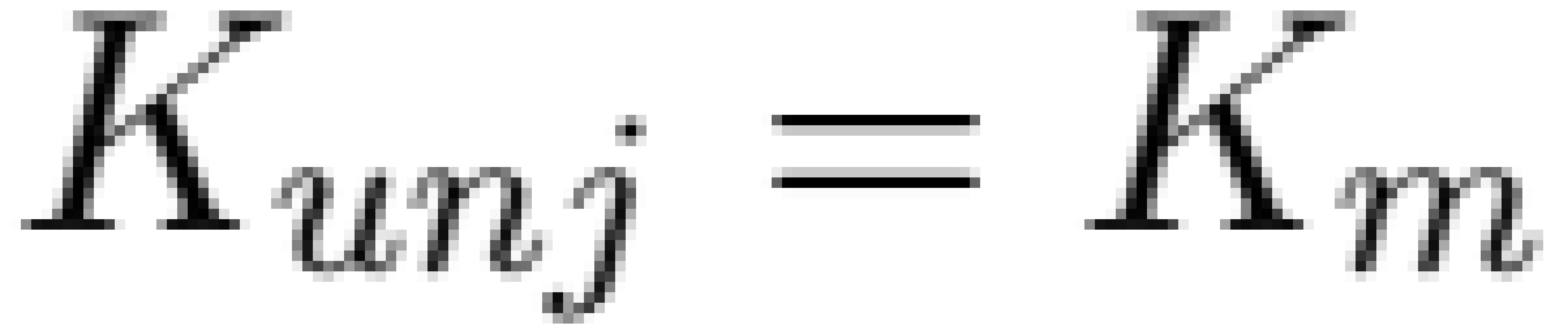
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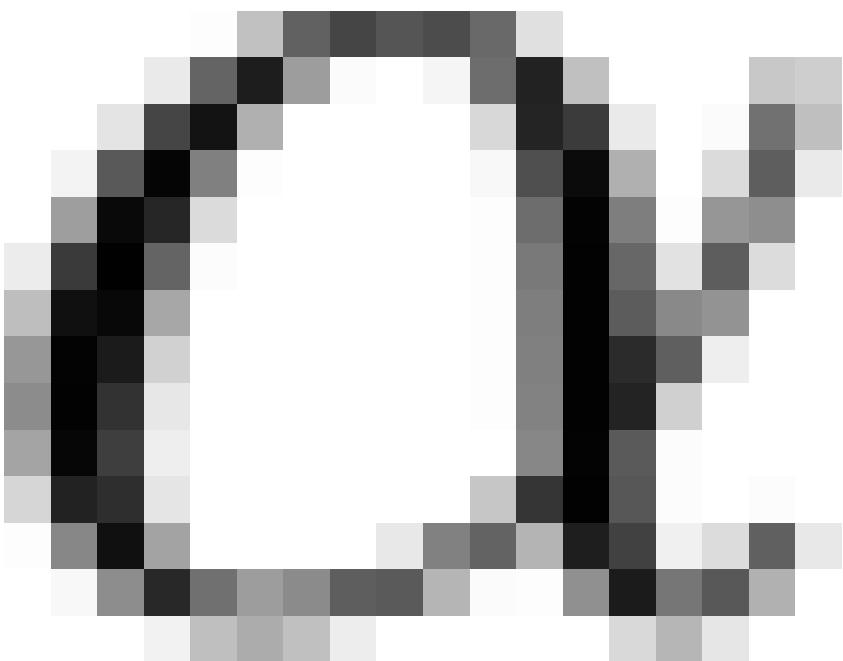
$K_{drained}$

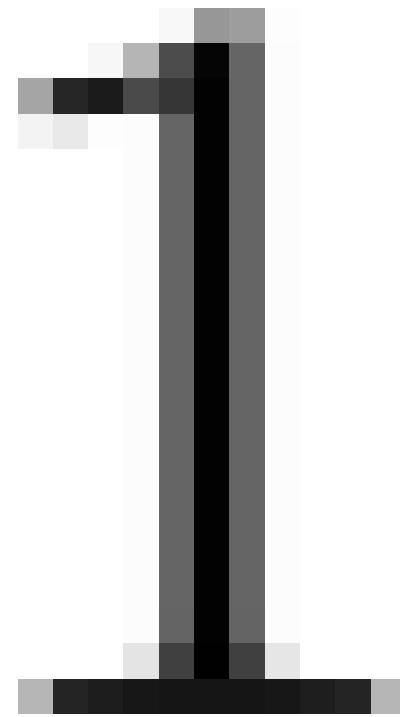
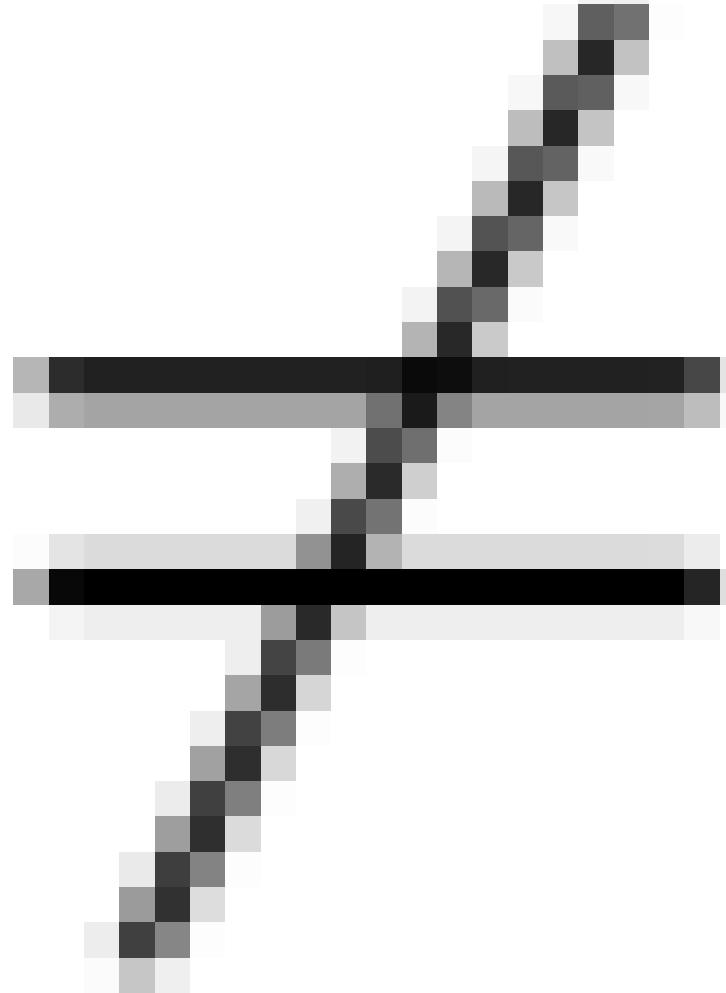
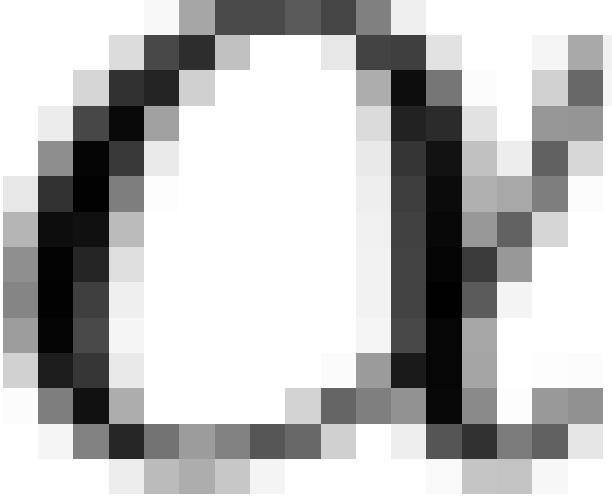
K_{unq} .

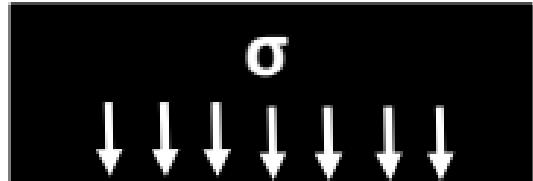
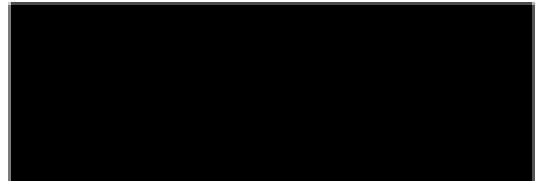




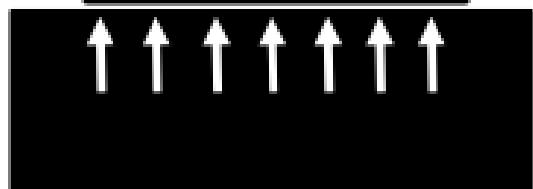
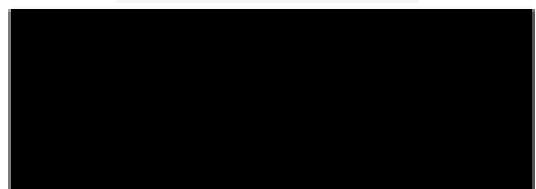


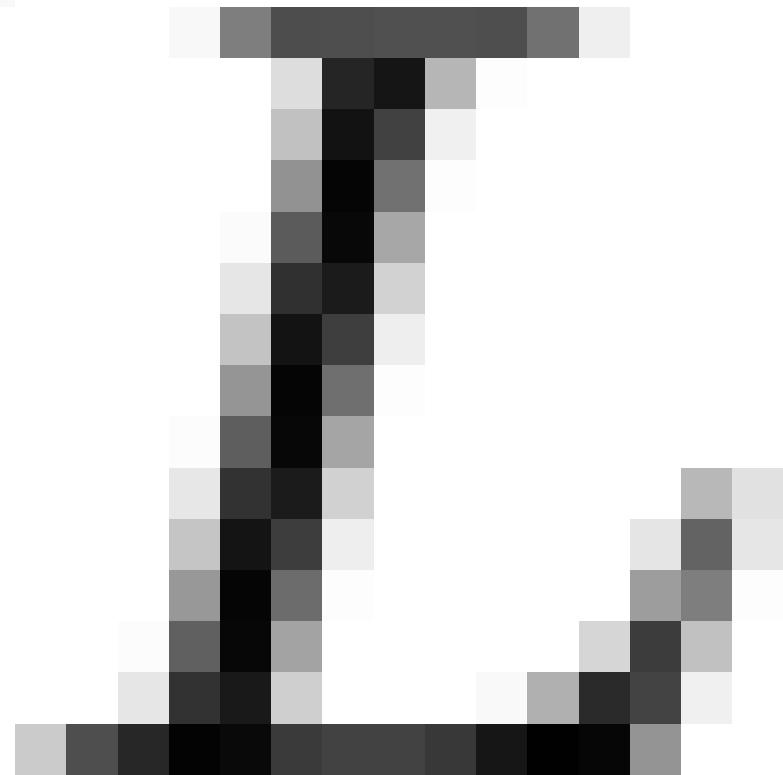
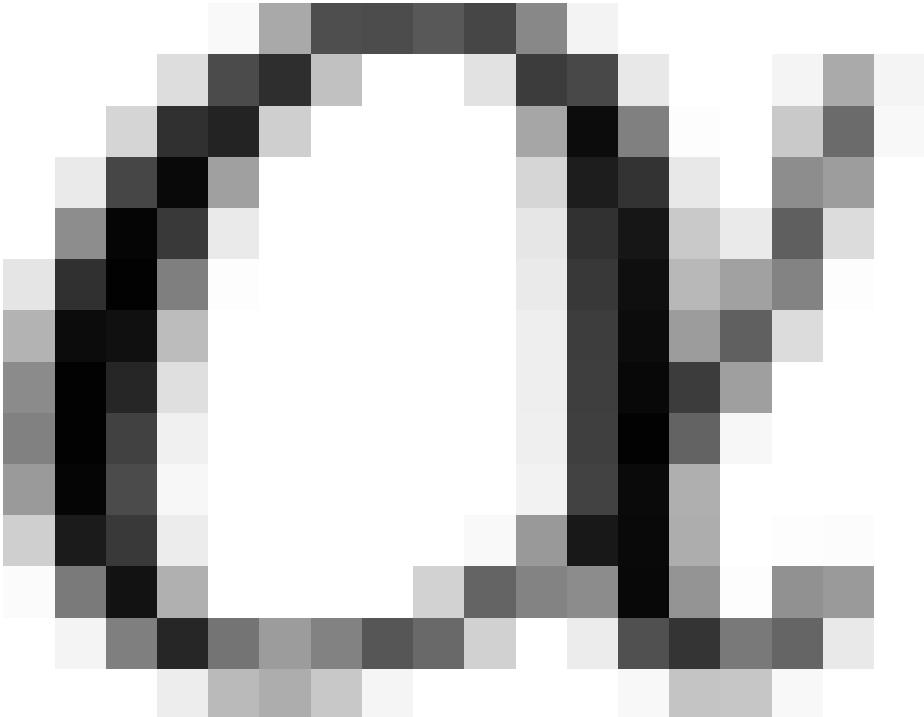


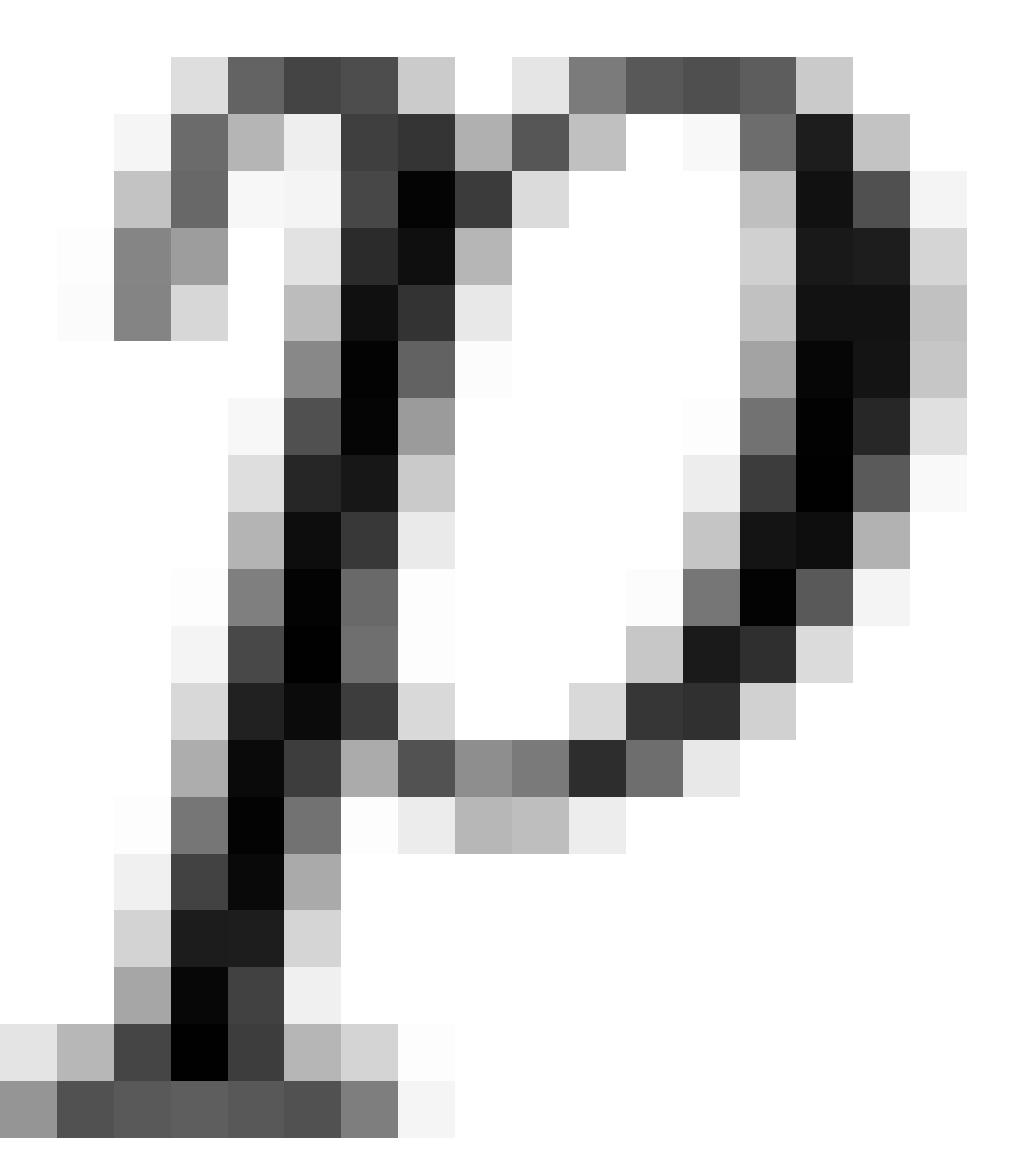




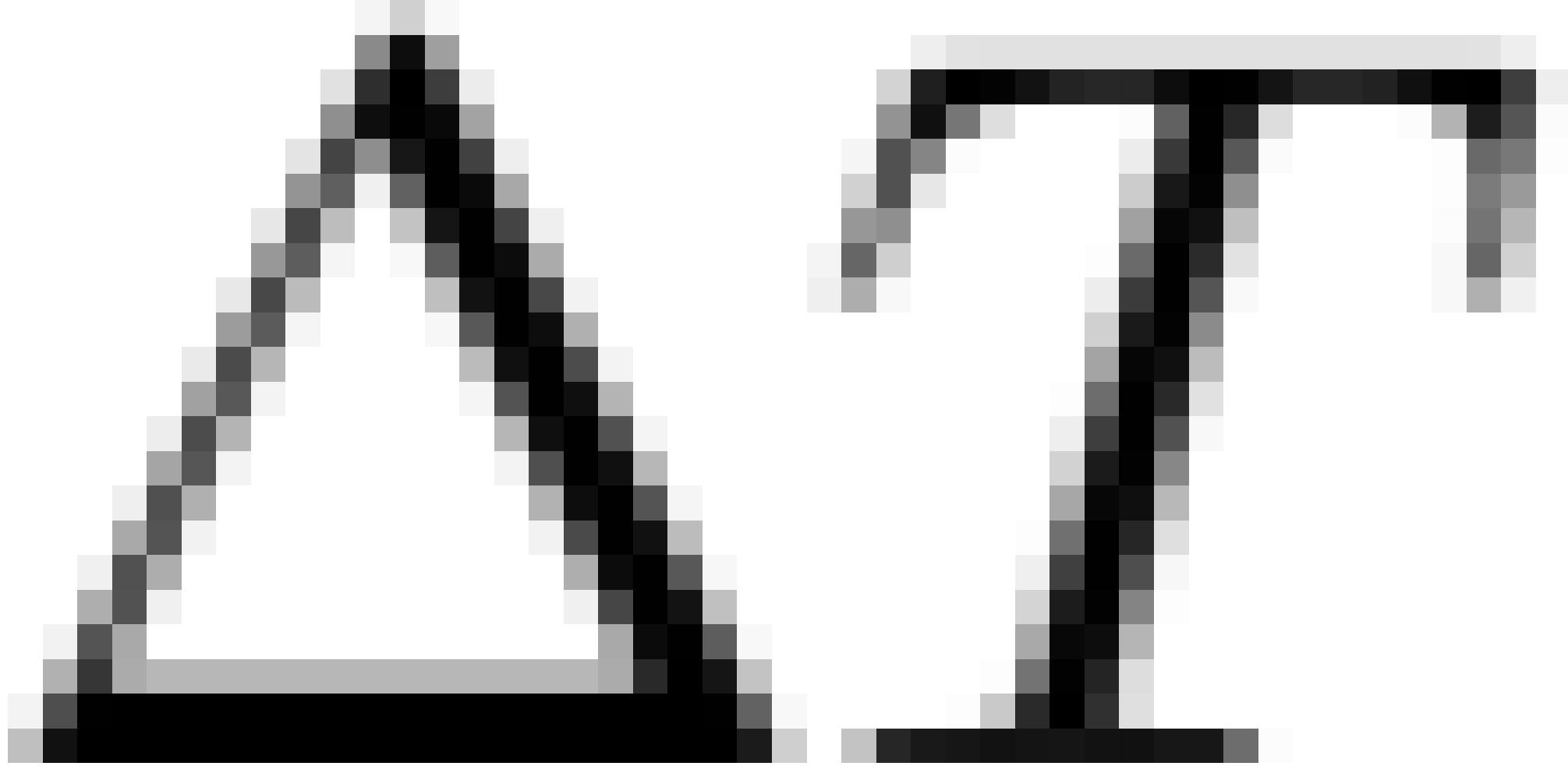
ΔT

A horizontal black arrow pointing to the right, positioned between the initial and final states, representing a change in temperature (ΔT).





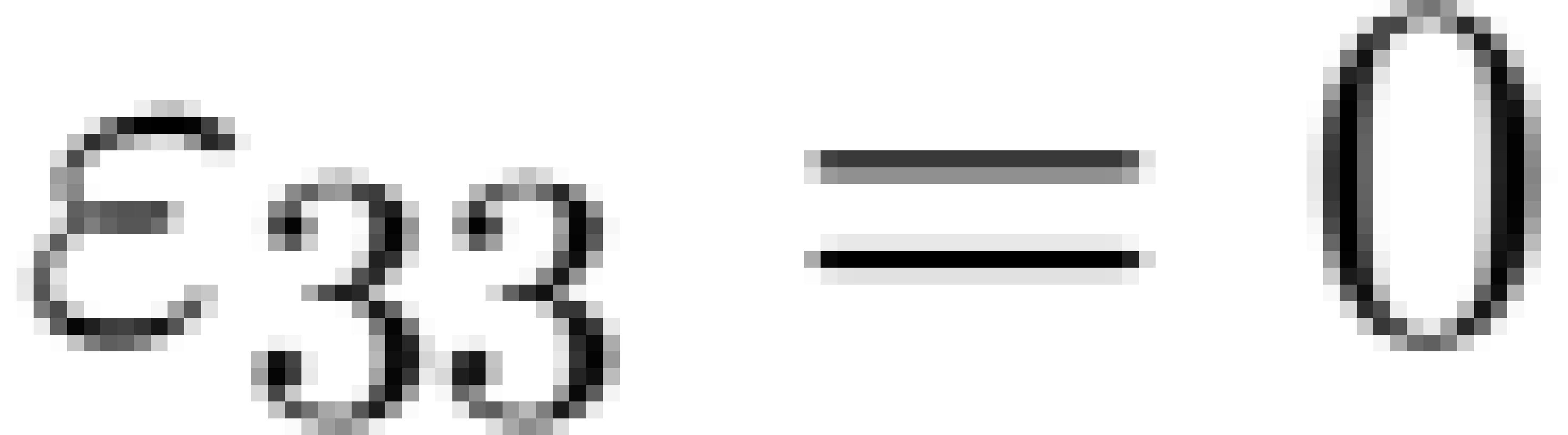
$$\alpha_L = \frac{1}{L} \frac{dL}{dT}$$

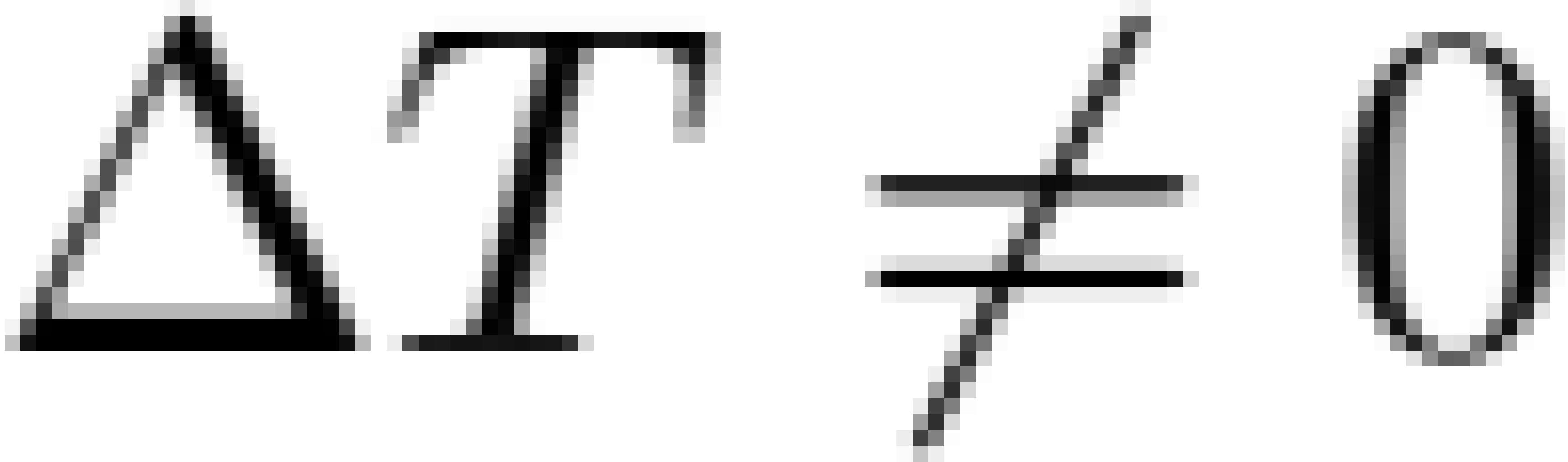


$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right.$$









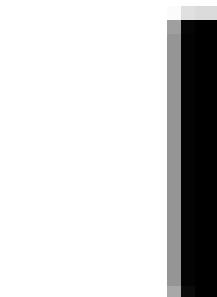
$$\begin{aligned} \sigma_{11}^0 &= (\lambda + 2\mu) \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \\ \sigma_{33} &= \lambda \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \end{aligned}$$

σ_{33}

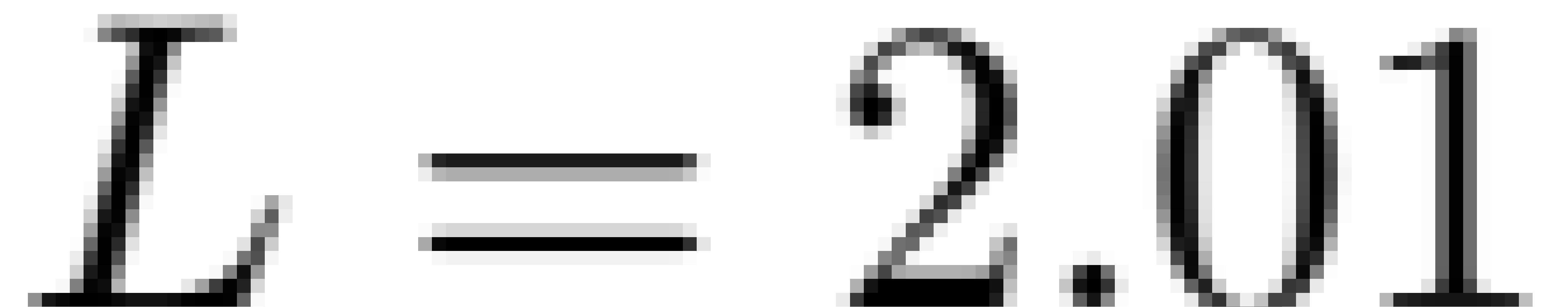
$=$

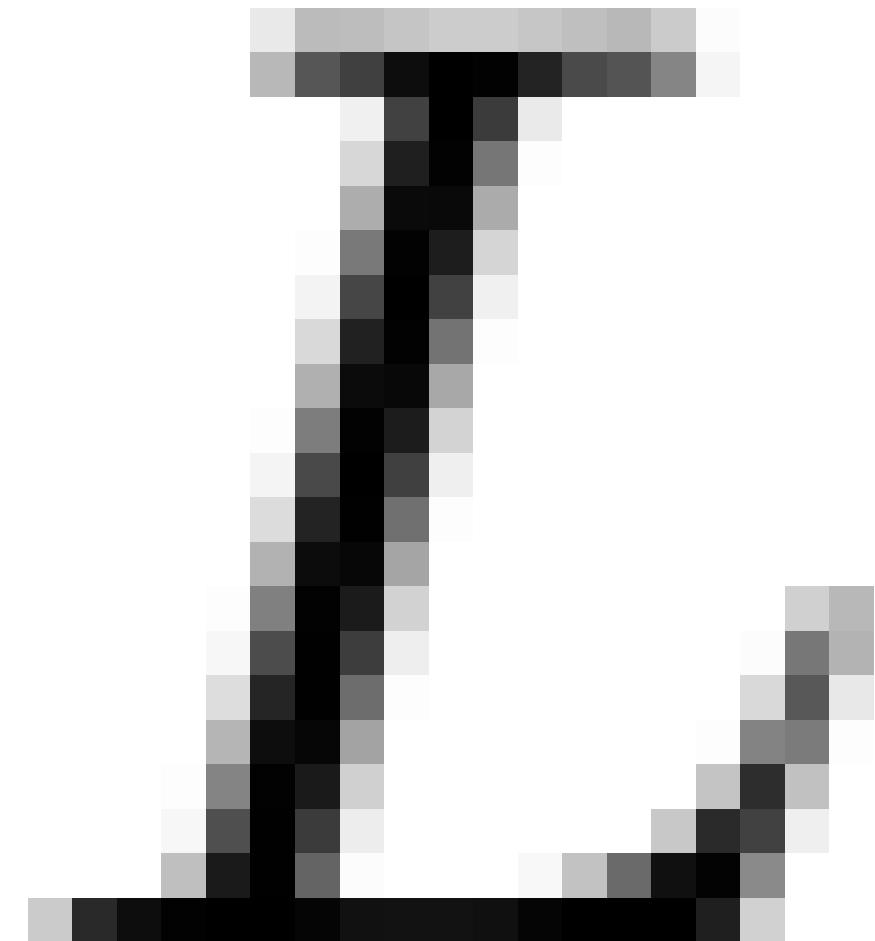
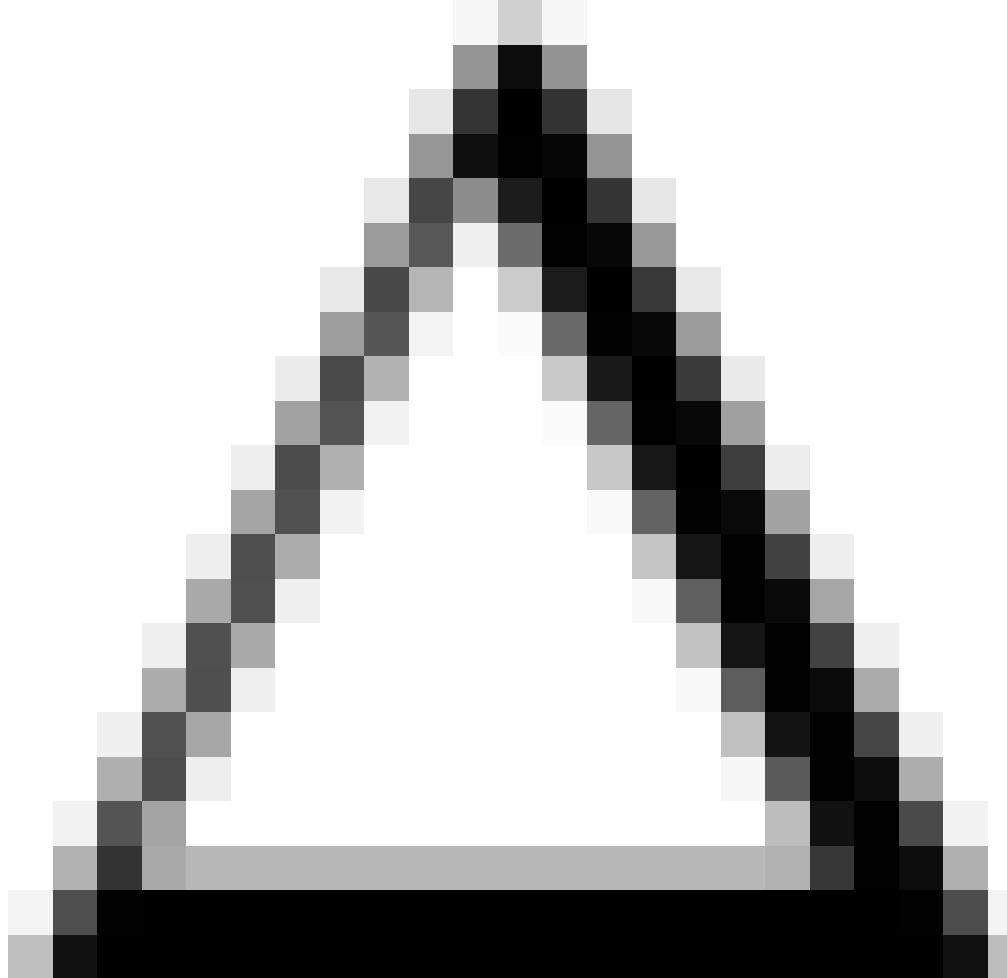
$$\frac{6}{3} \mu K$$

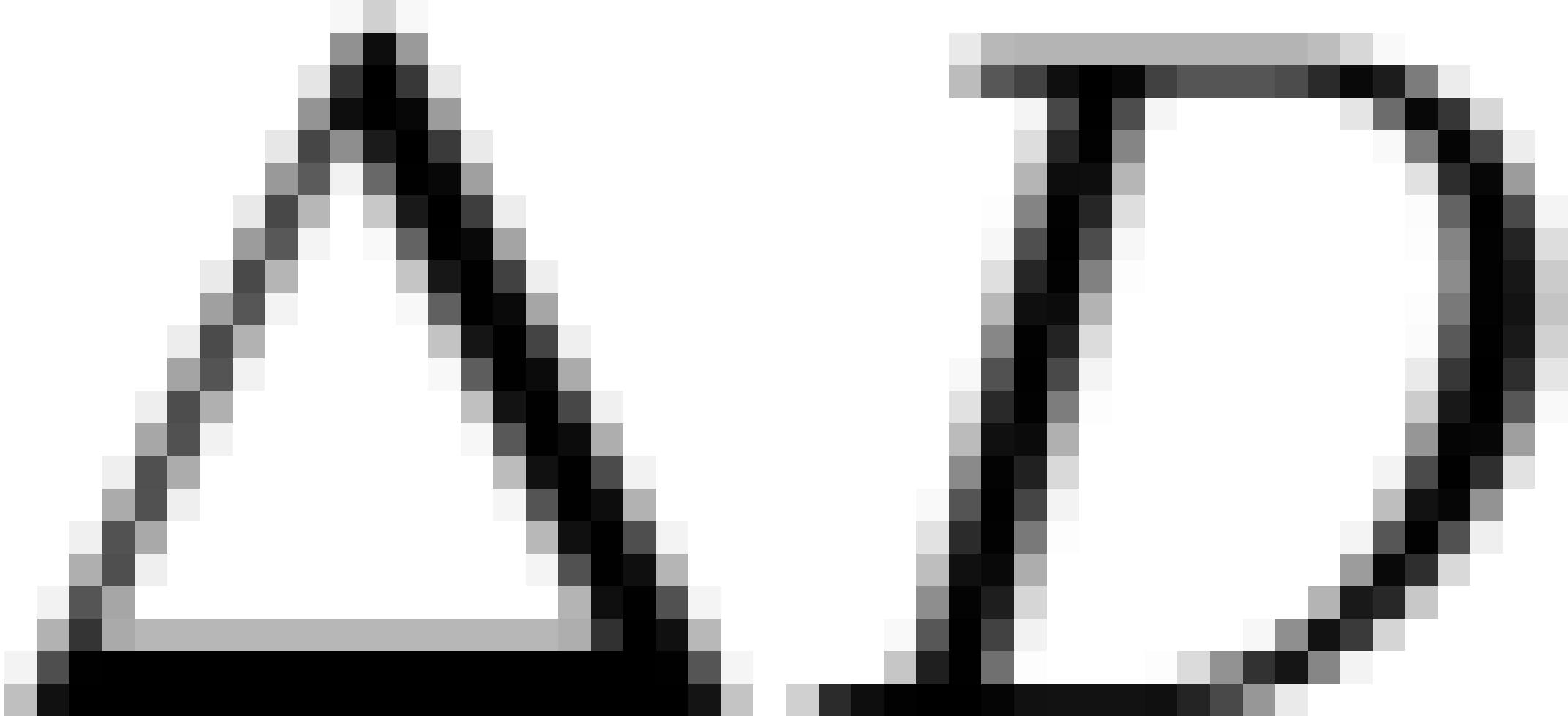
$K_{\Delta T}$

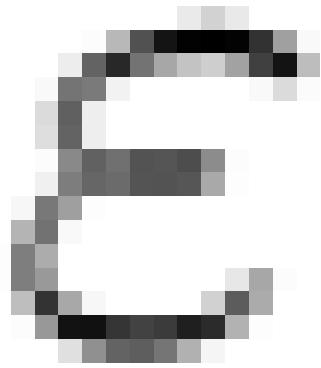
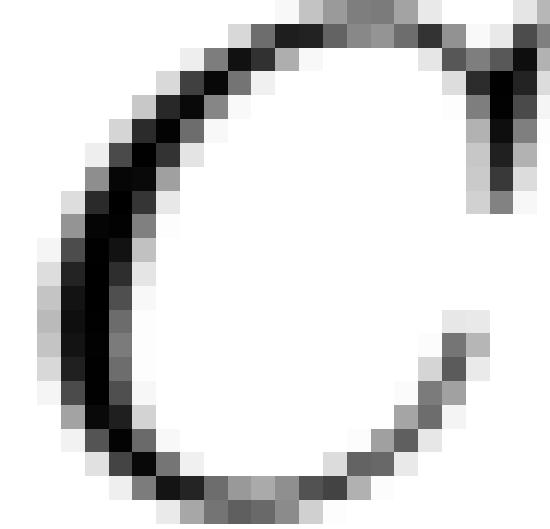
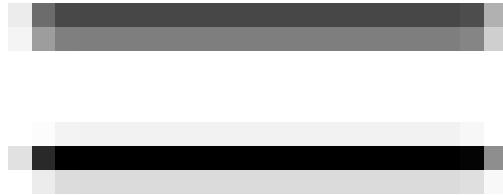
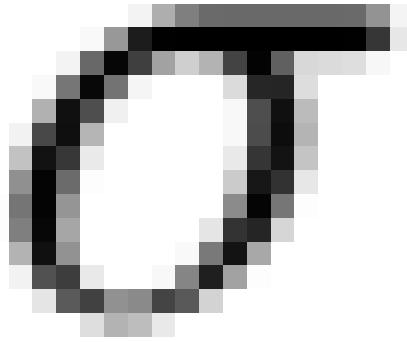


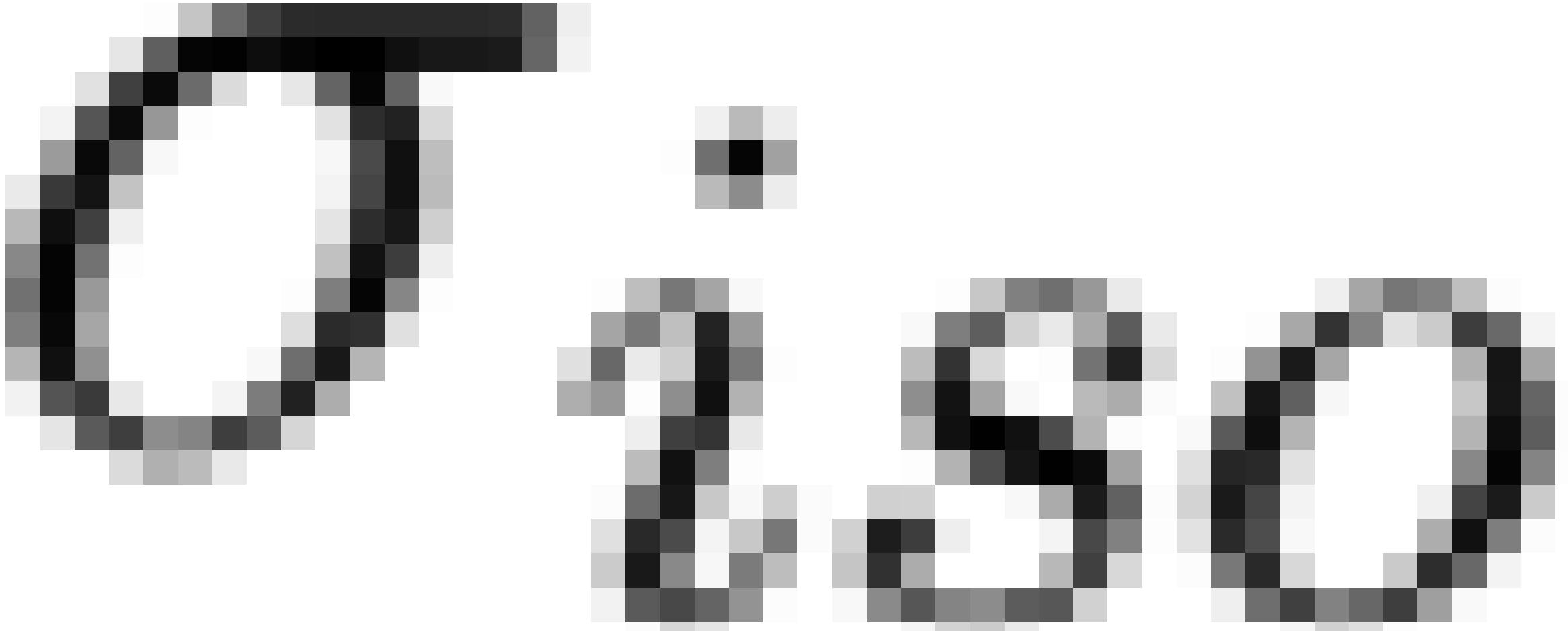






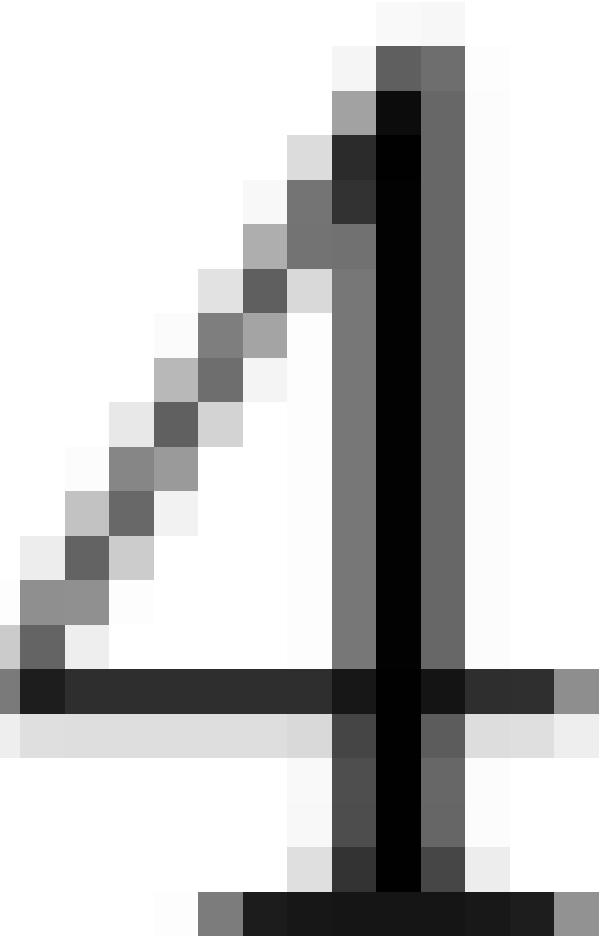
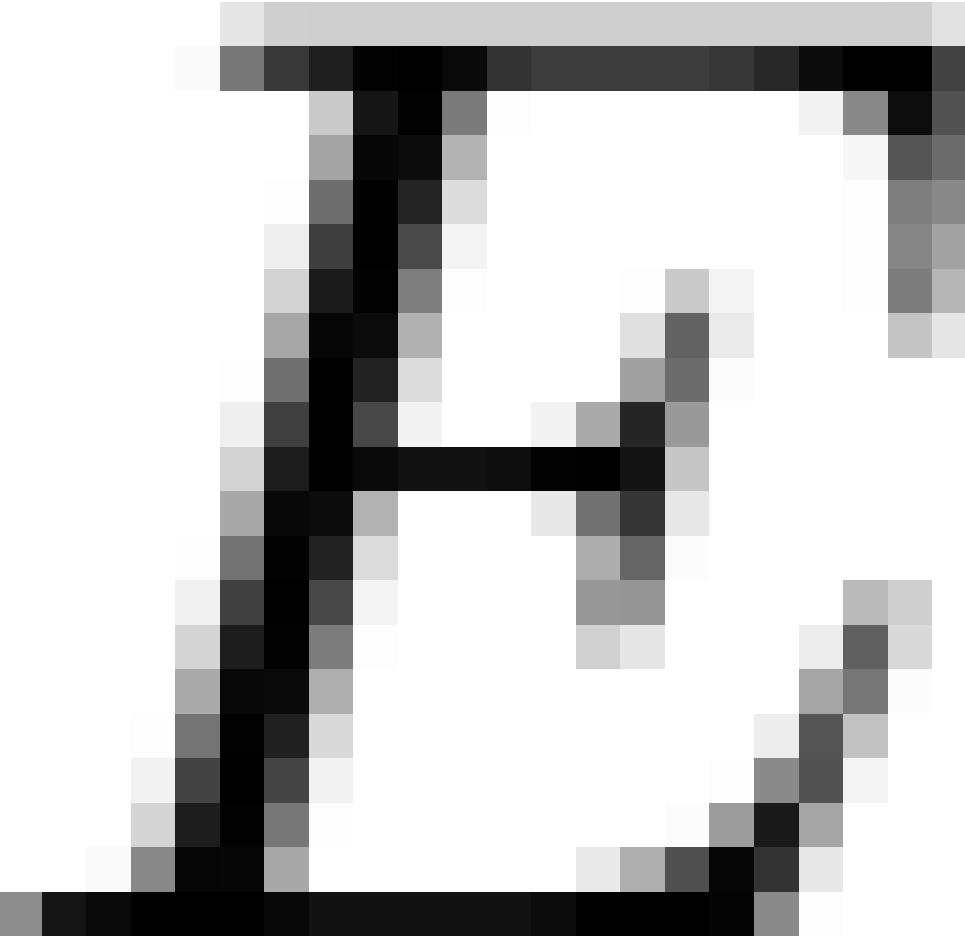












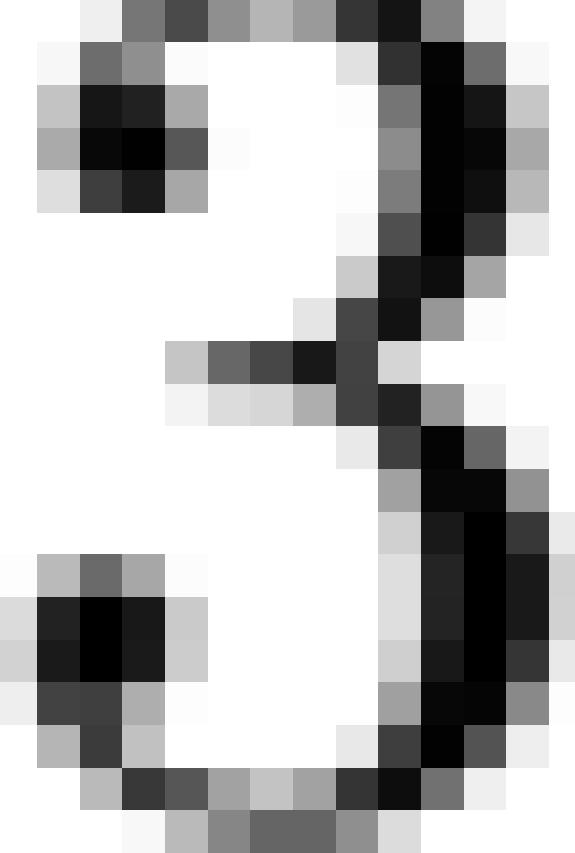
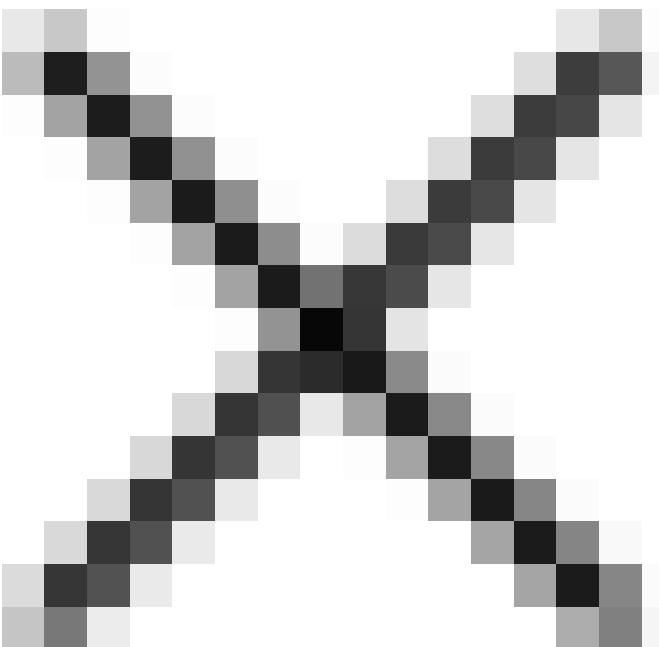
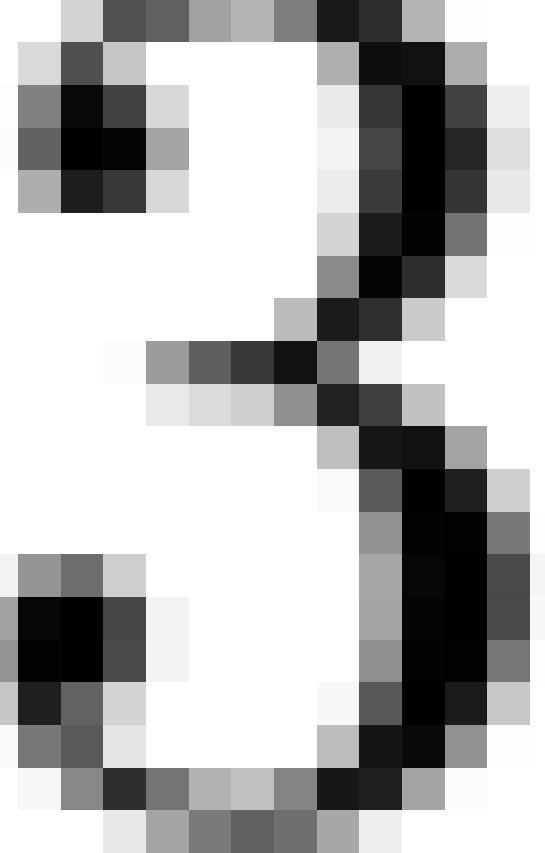


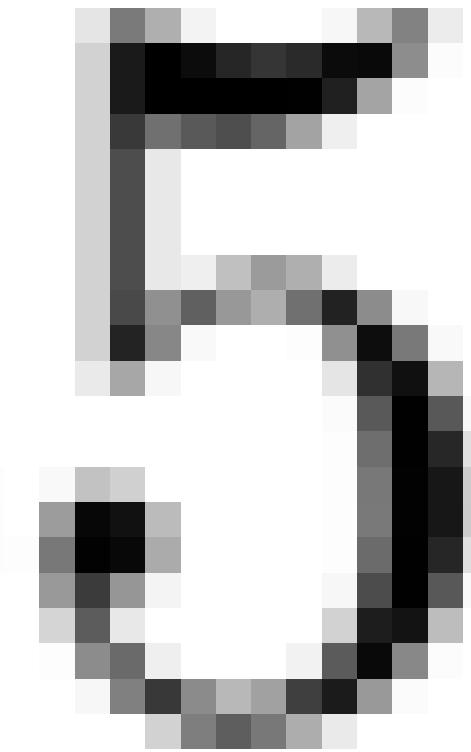
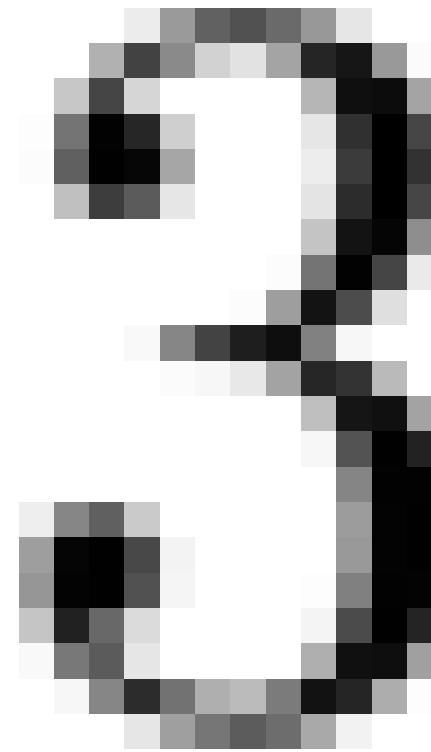
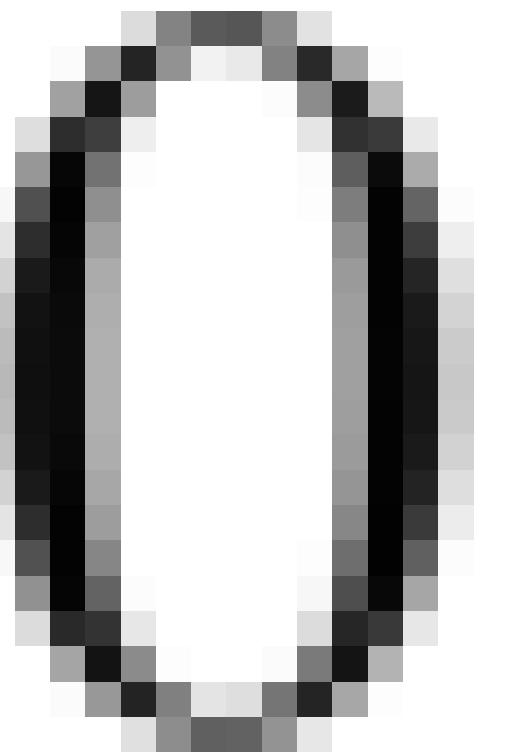
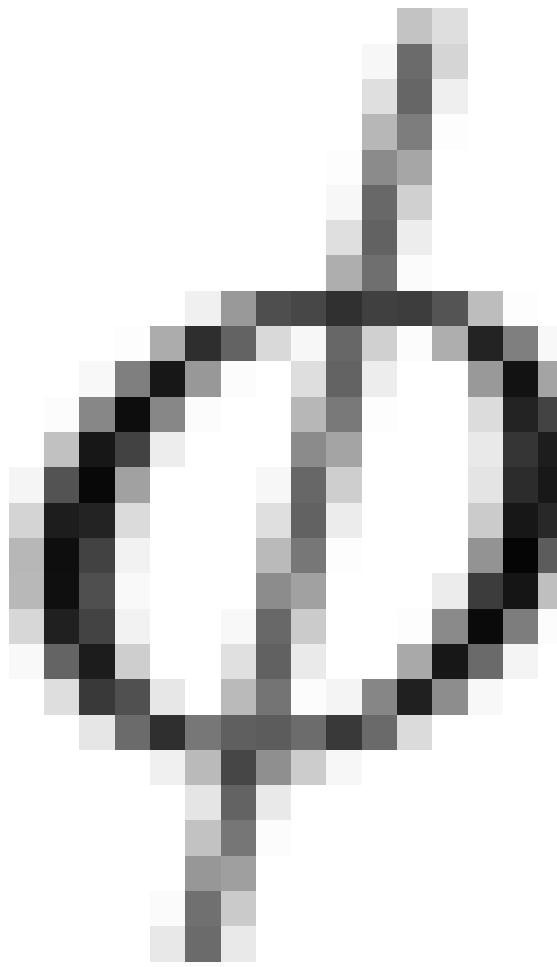


σ_{11}

$$= \frac{1}{2}$$

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\epsilon_{11}$$







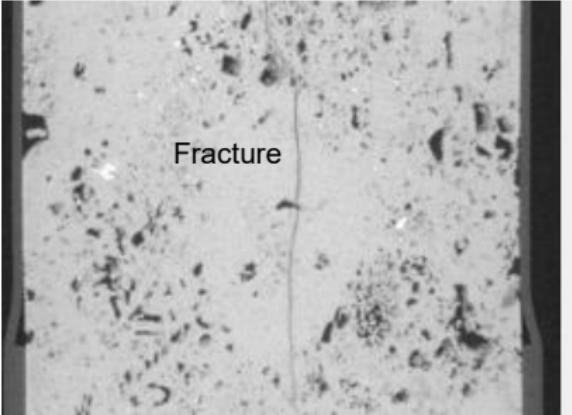
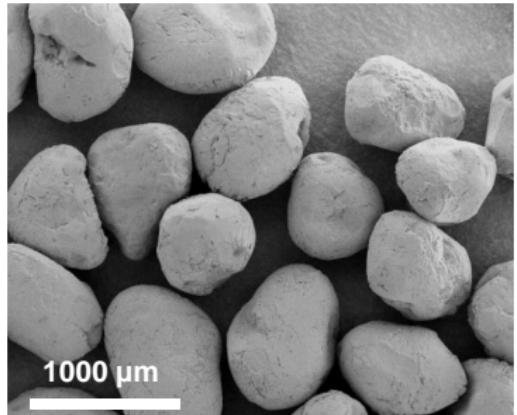
(a) Uncemented sand



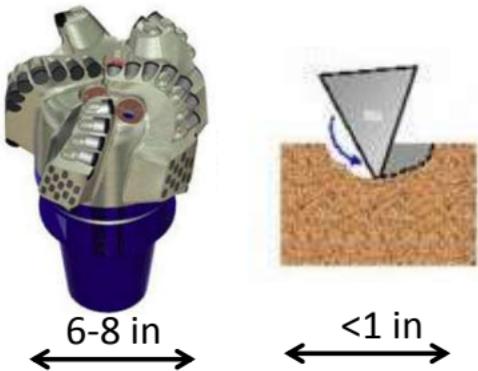
(b) Cemented sandstone



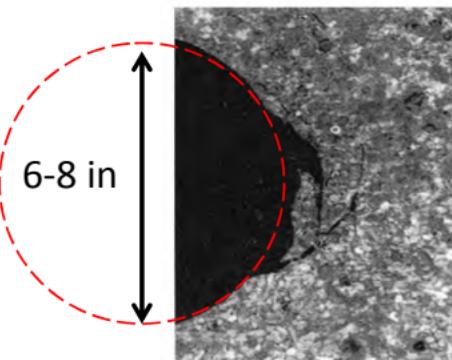
(c) Vuggy carbonate



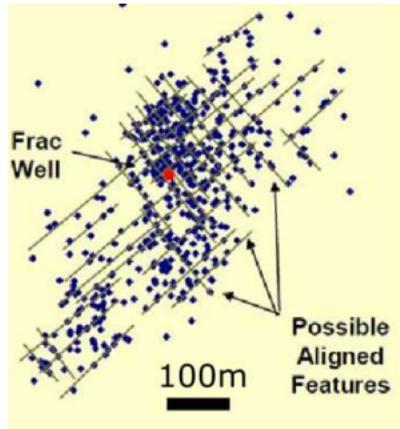
Rock cutting at the drill bit



Wellbore breakout



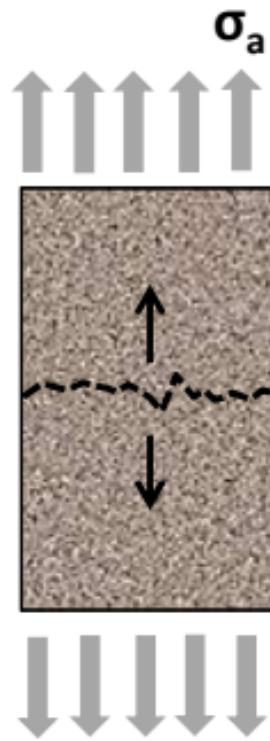
Shale hydraulic fracture



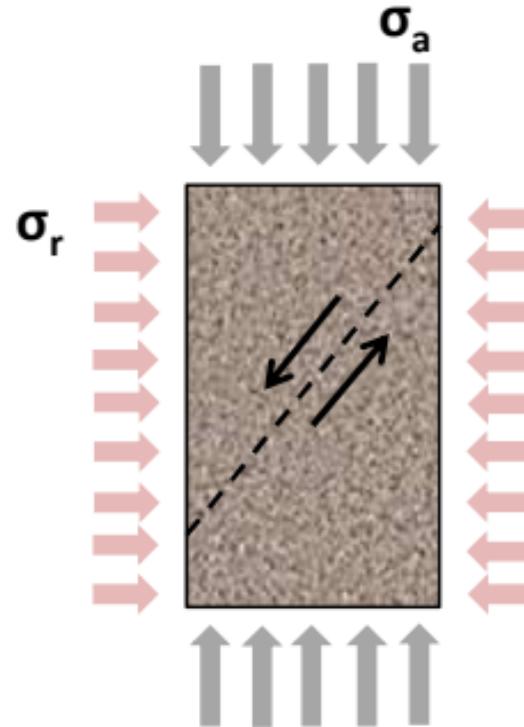
Reservoir depletion



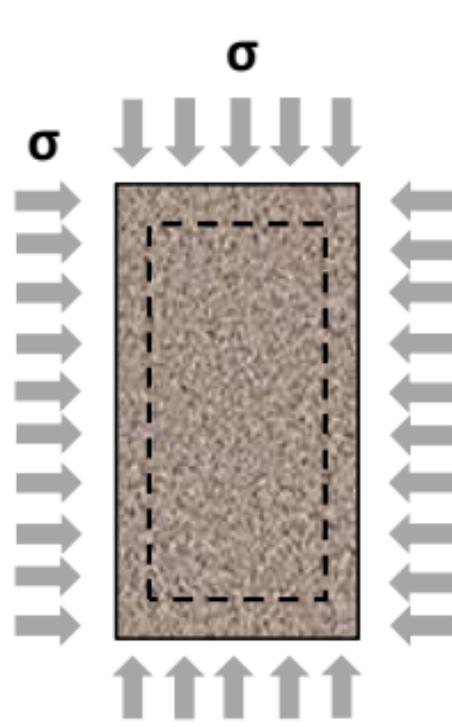
Images: Schlumberger/Terratek, Zoback 2013, Warpinski 2008, doe.gov



Tension
(bond breakage)
Ex: drilling-induced tens. fracs



Shear
(friction failure)
Ex: fault, breakout

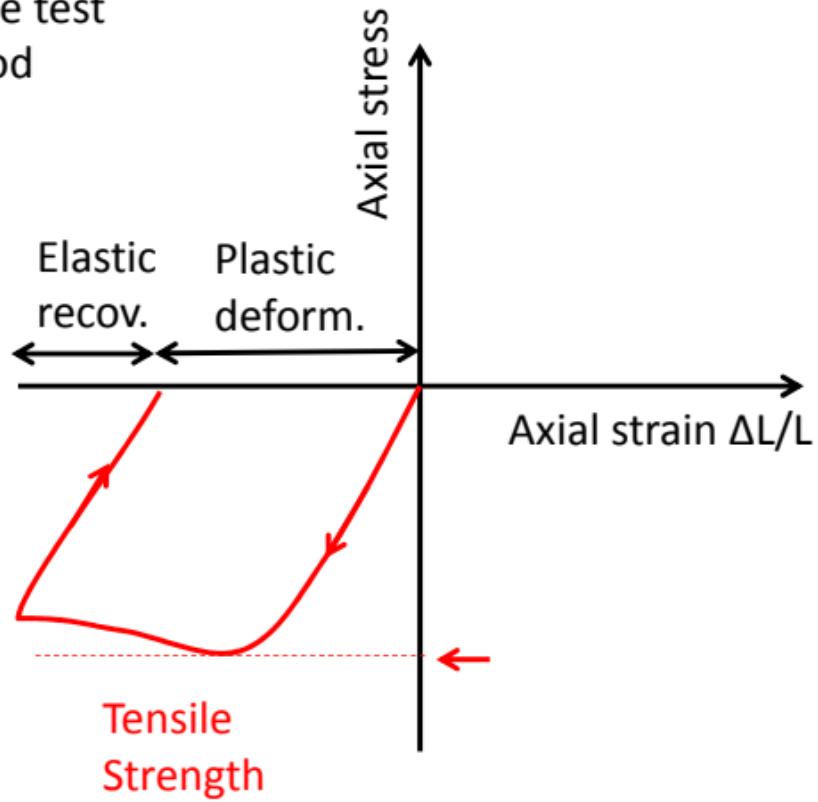


Compression
(pore collapse)
Ex: reservoir compaction

T



Typical tensile test
on a metal rod



Tensile test on a rock rod

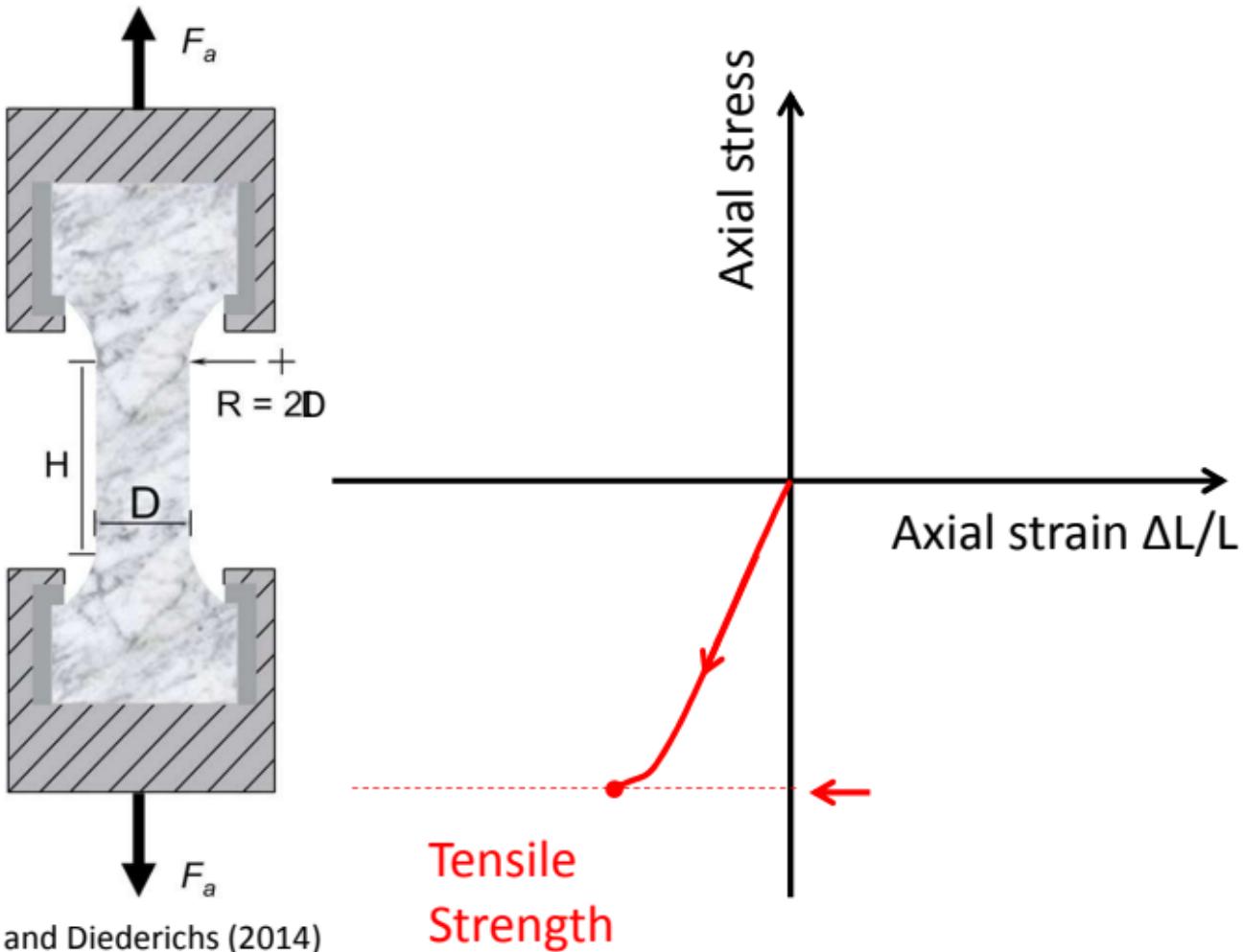
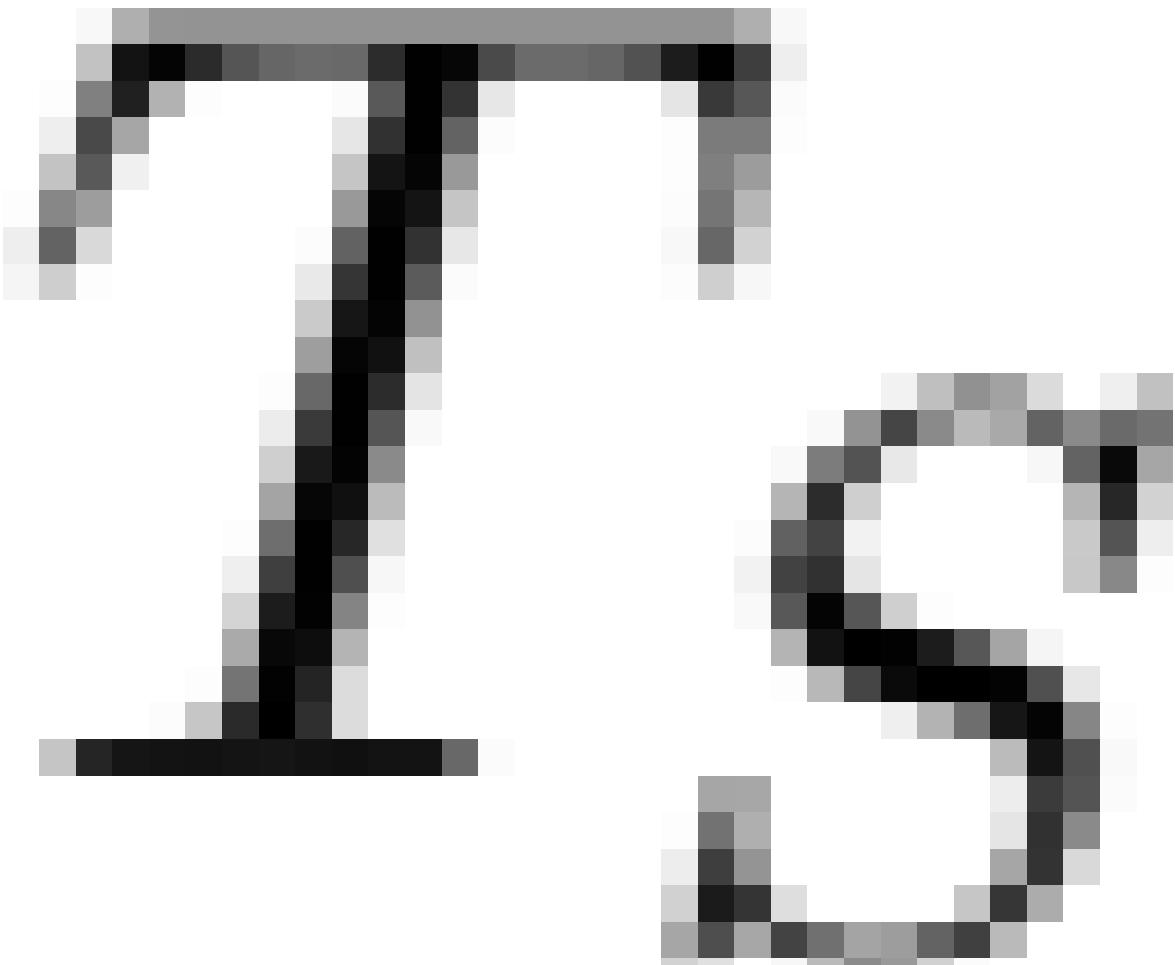


Figure direct tension: Perras and Diederichs (2014)



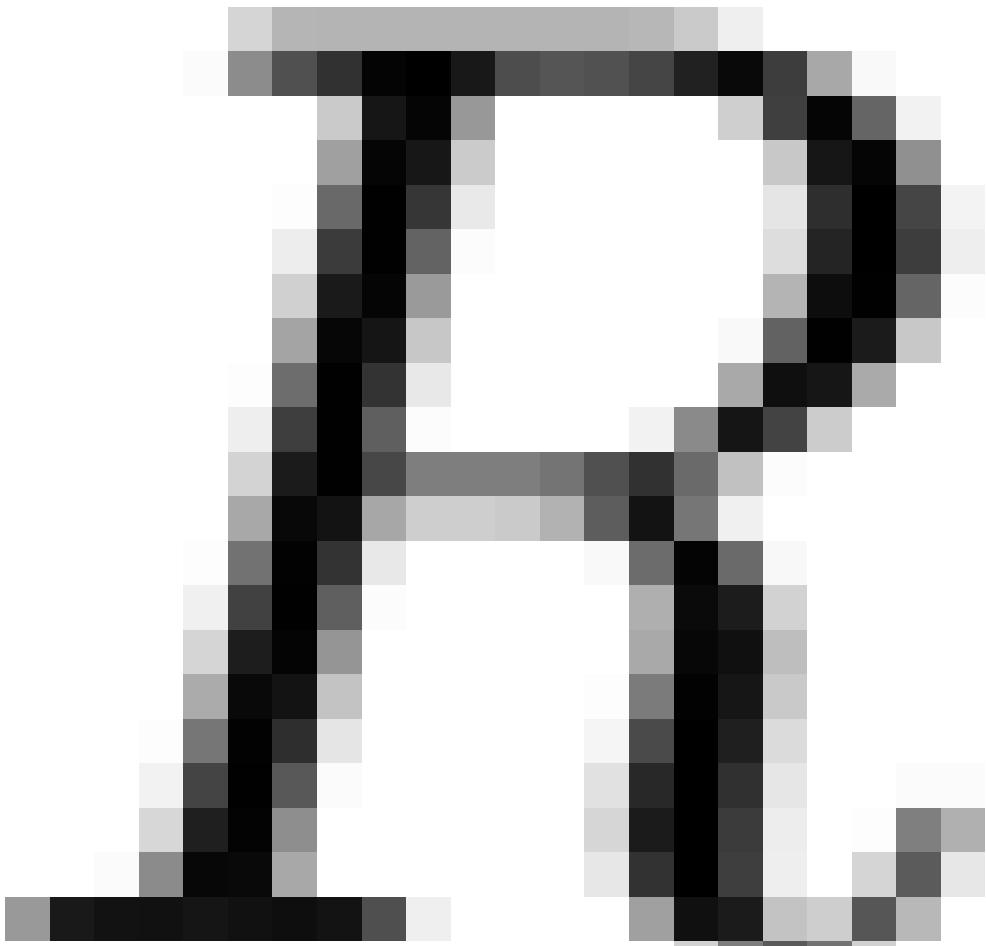
IS

—
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πLR

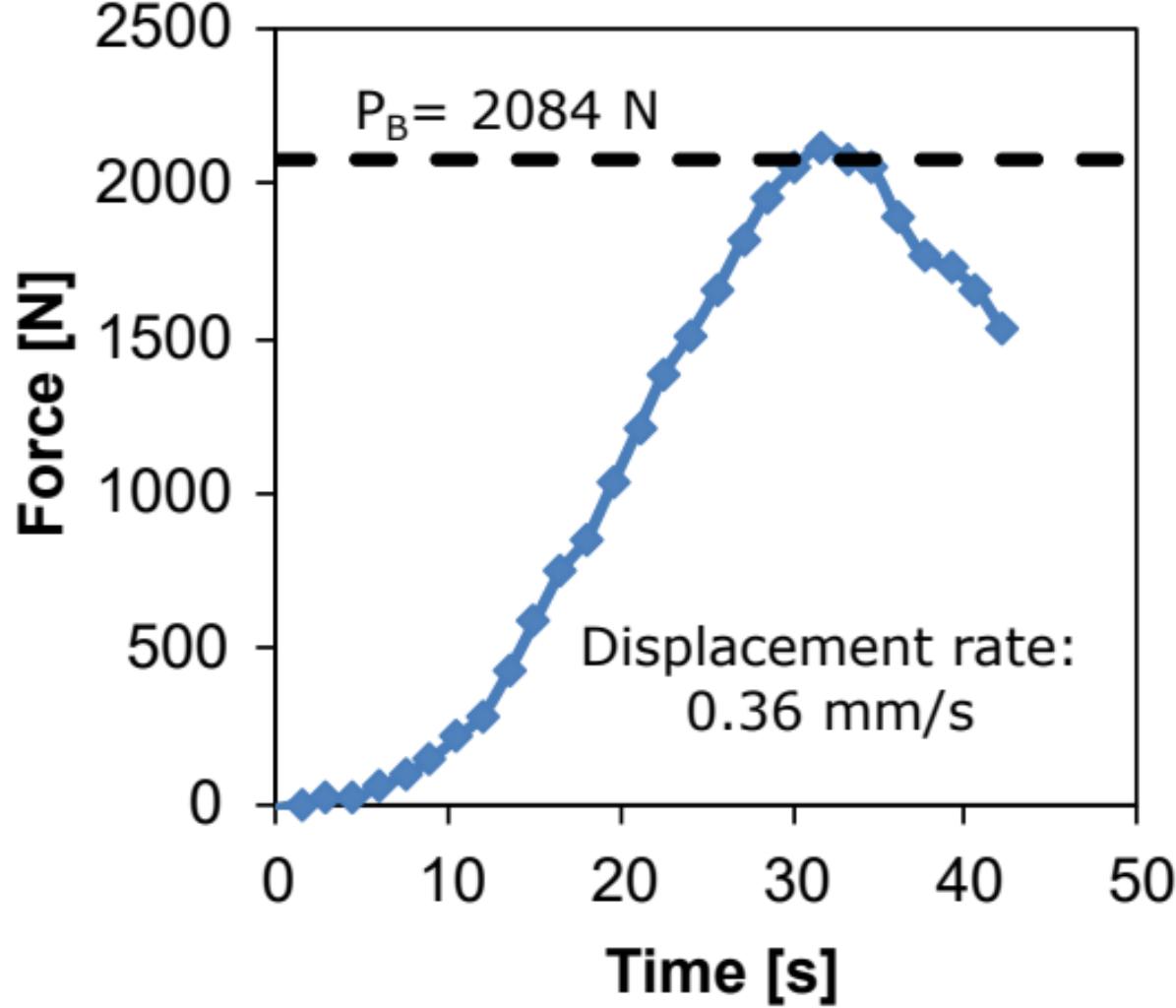
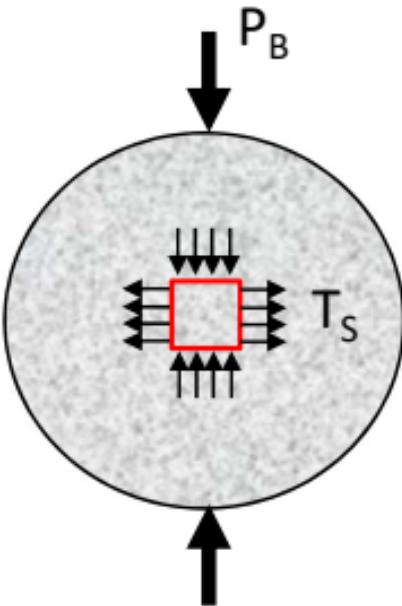
PB





Cylindrical sample:

- radius R
- length L

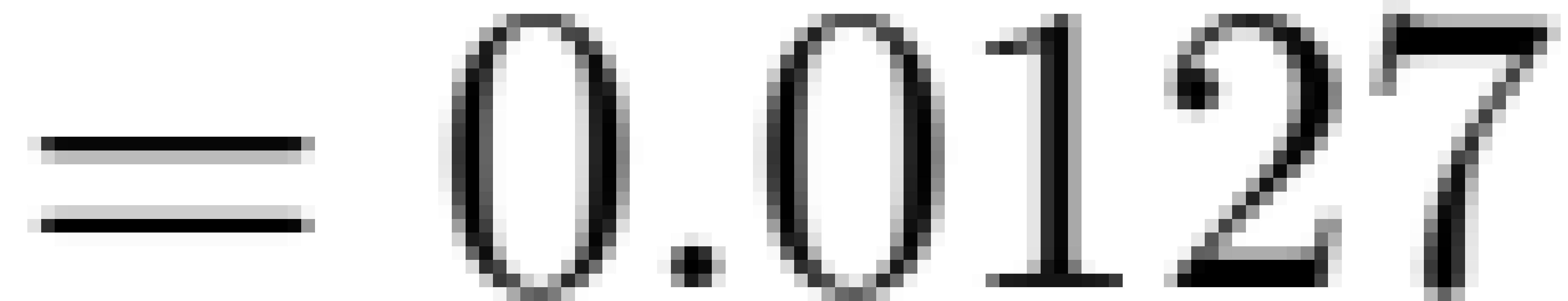


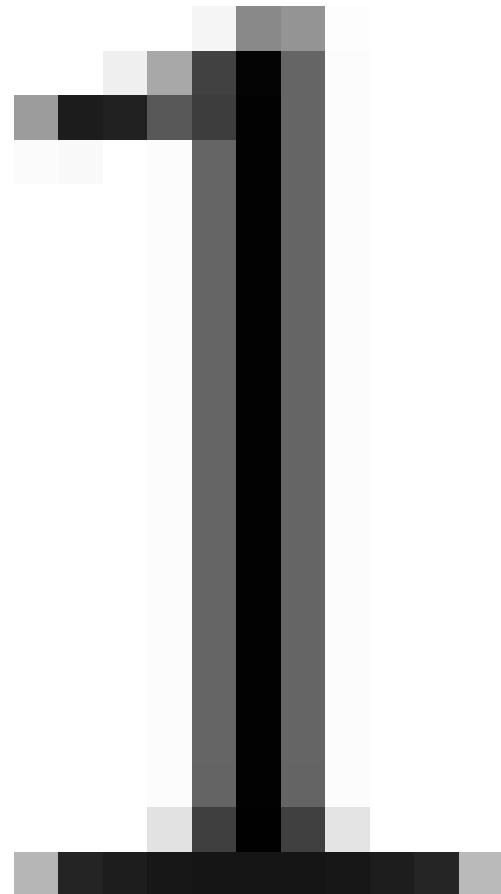
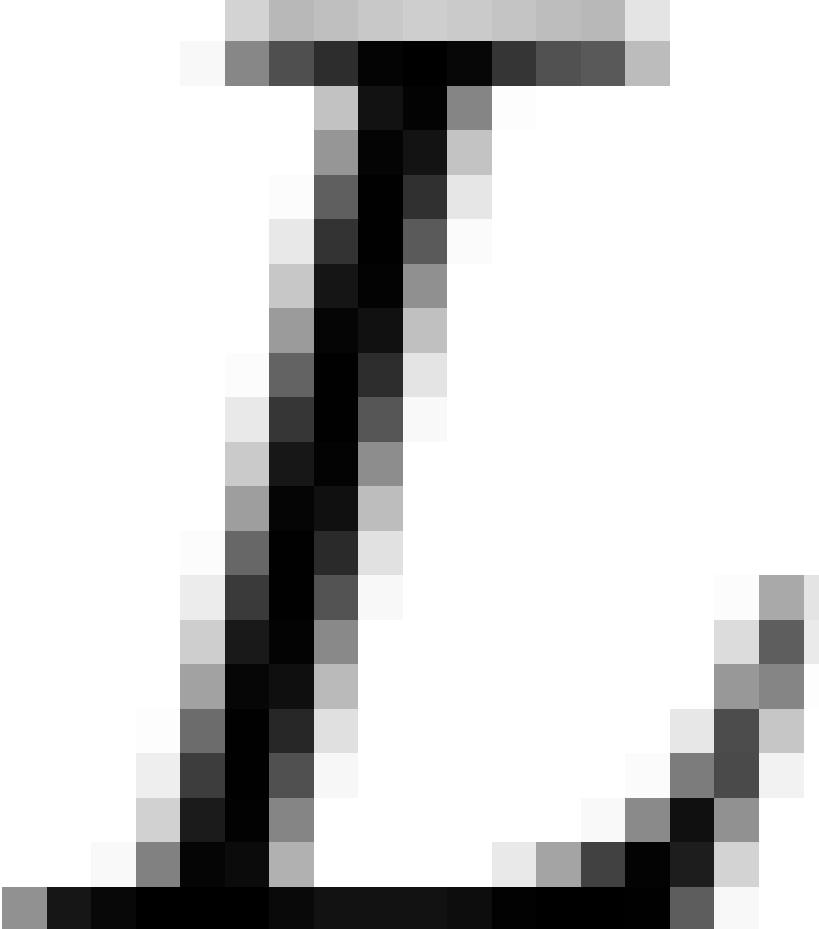
R

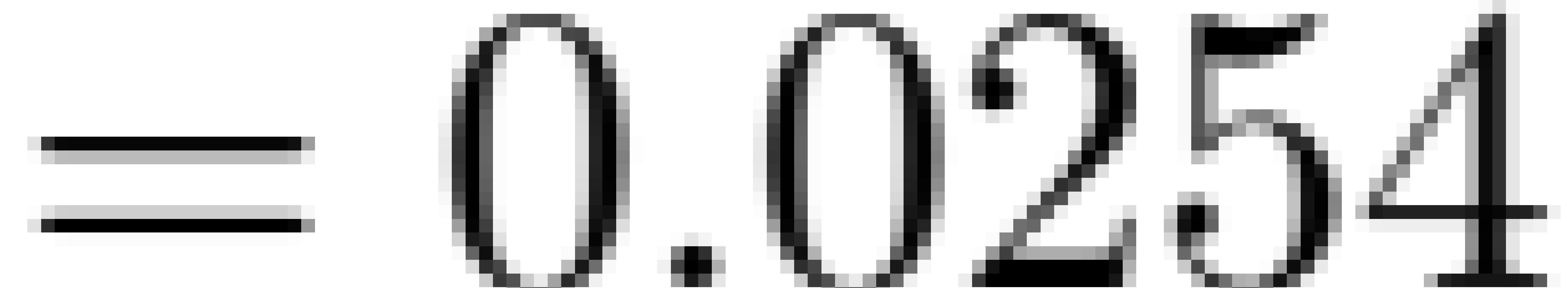
1
2

1
2

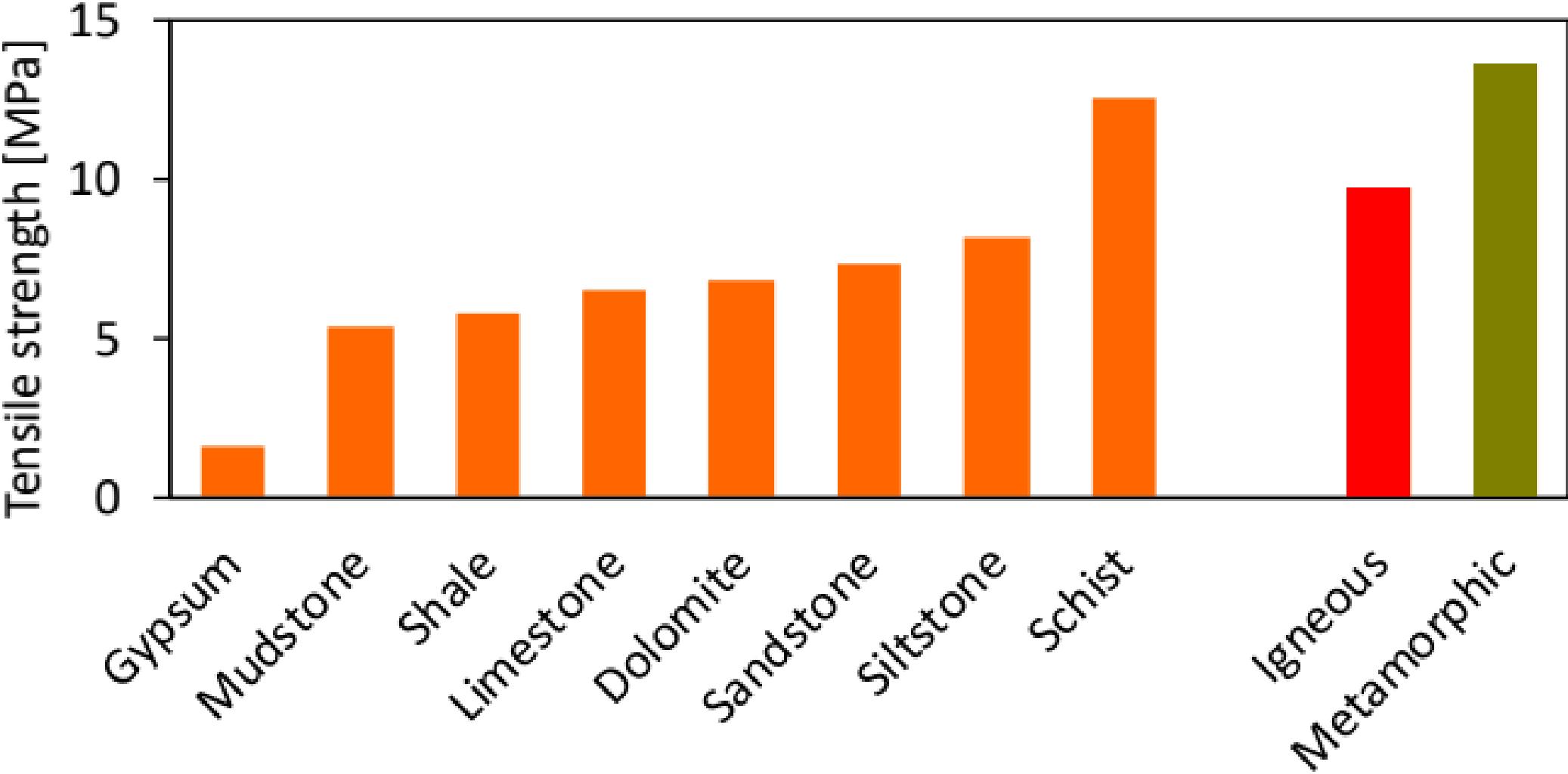
1
2

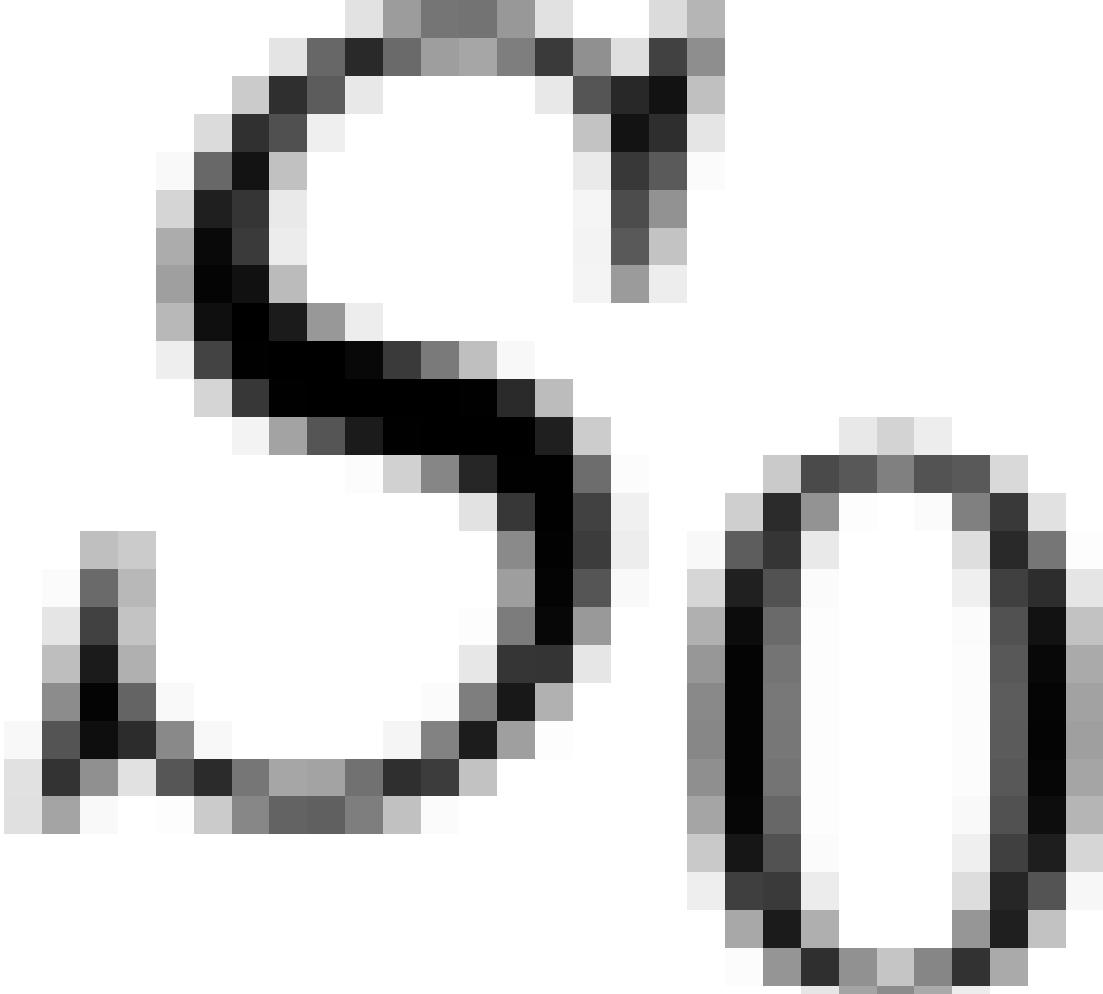


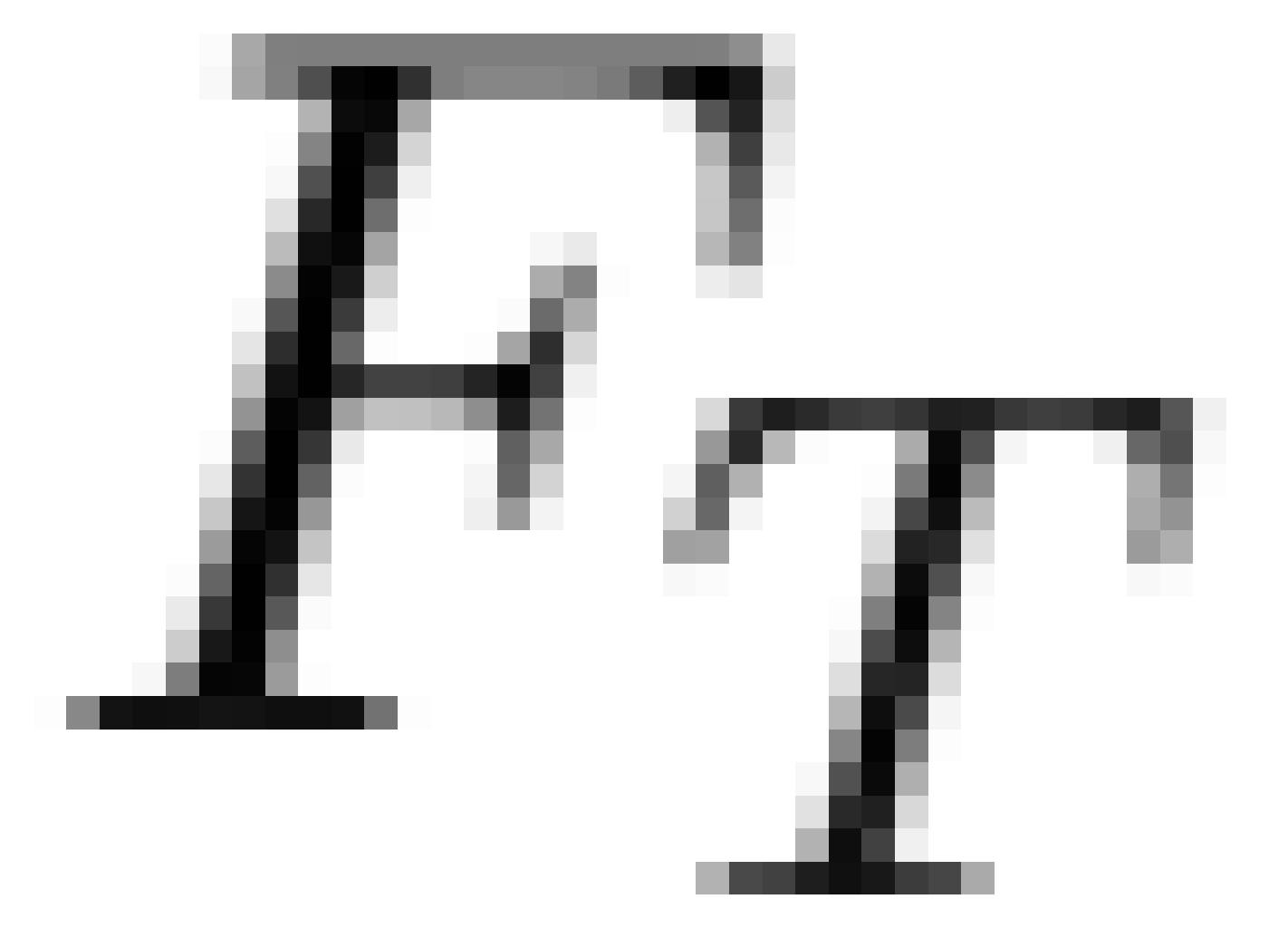




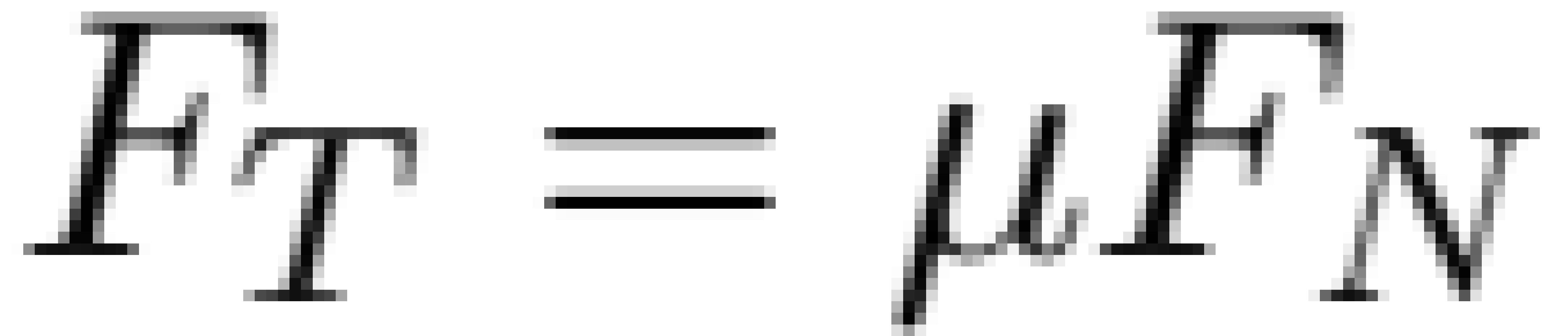
$$\frac{2084 \text{ N}}{\pi (0.0254 \text{ m})(0.0127 \text{ m})} = 2.06 \times 10^6 \text{ Pa} = 2.06 \text{ MPa}$$

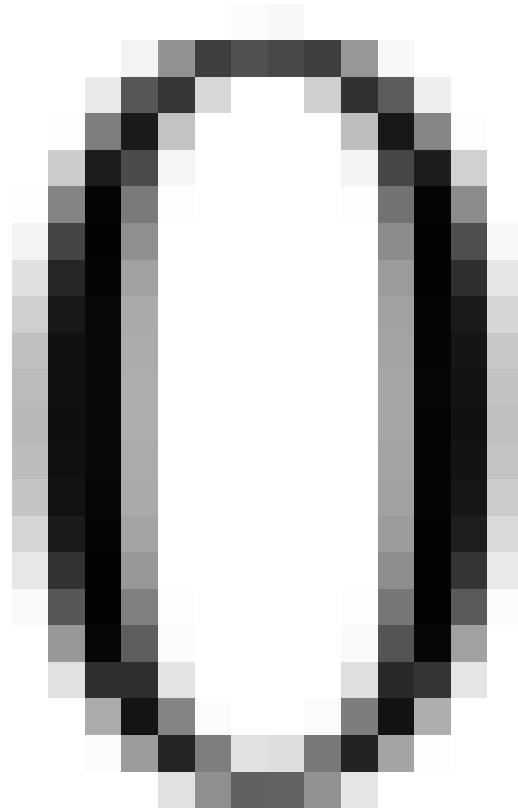


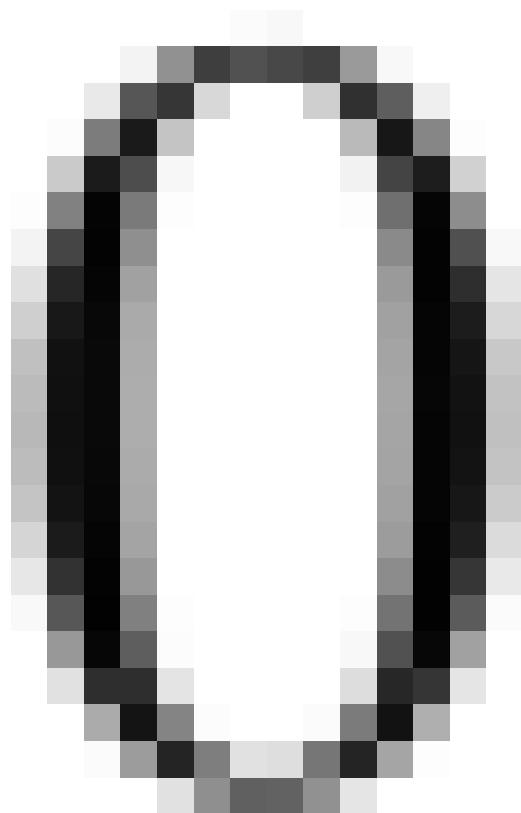
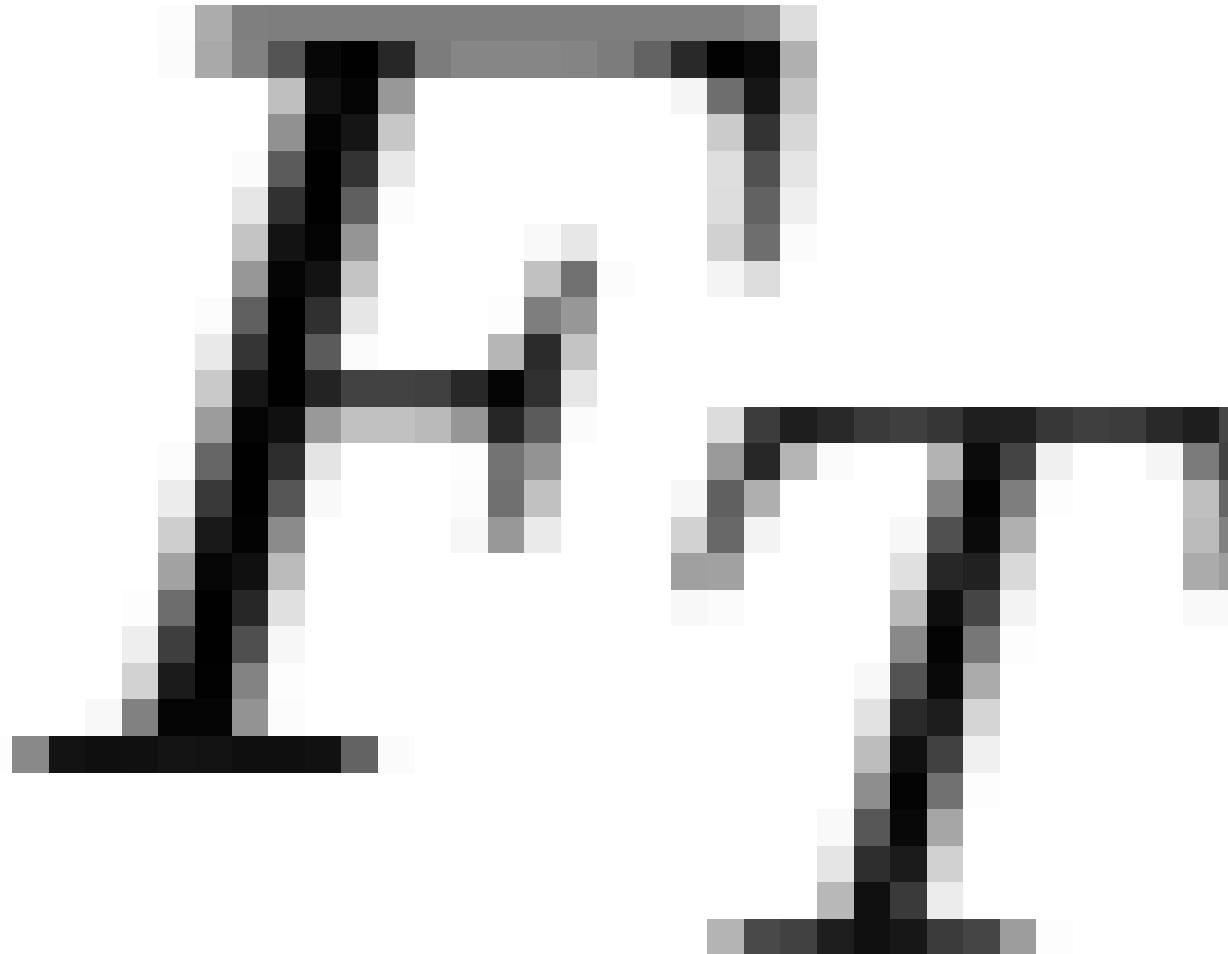


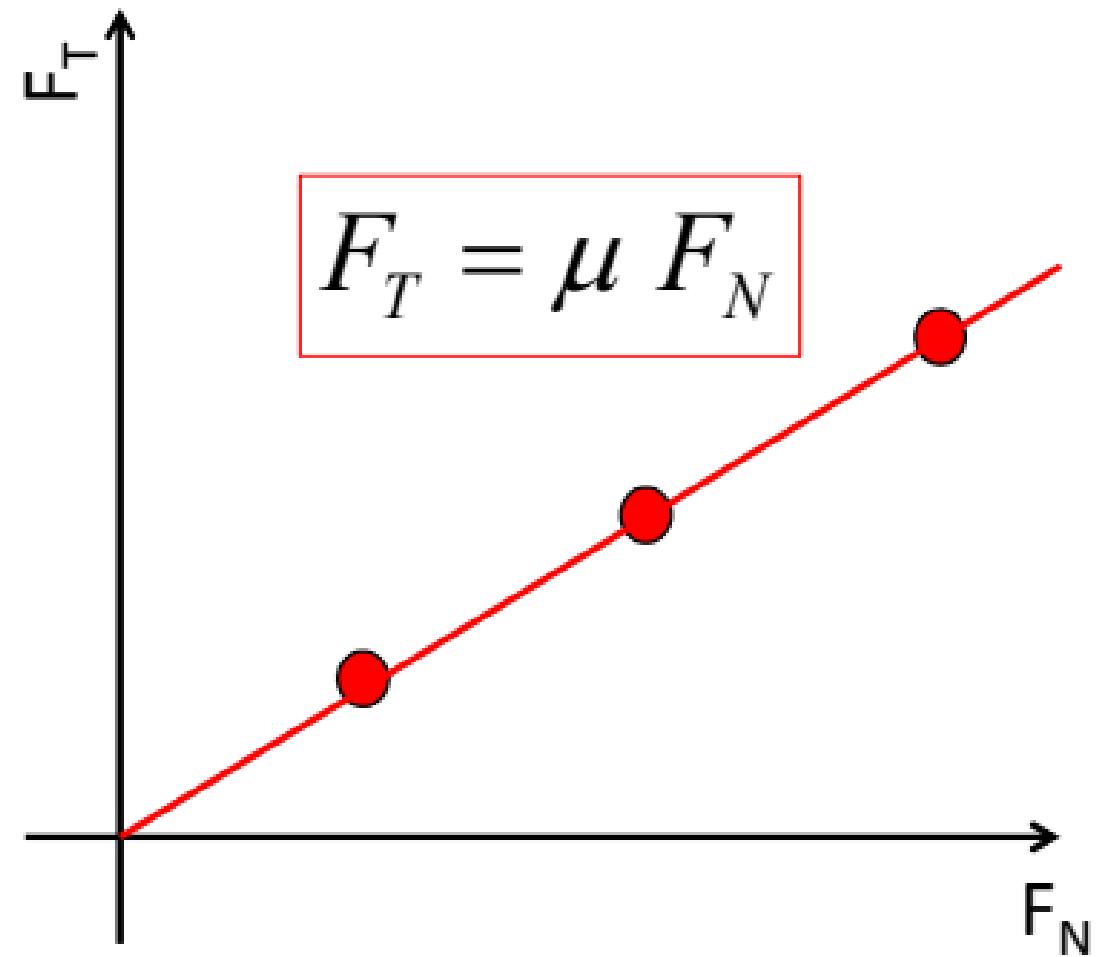
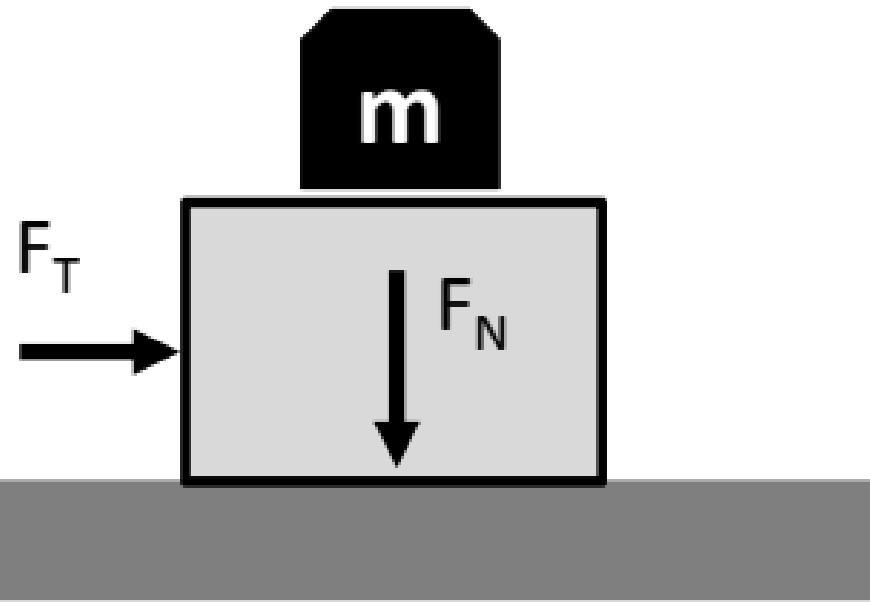


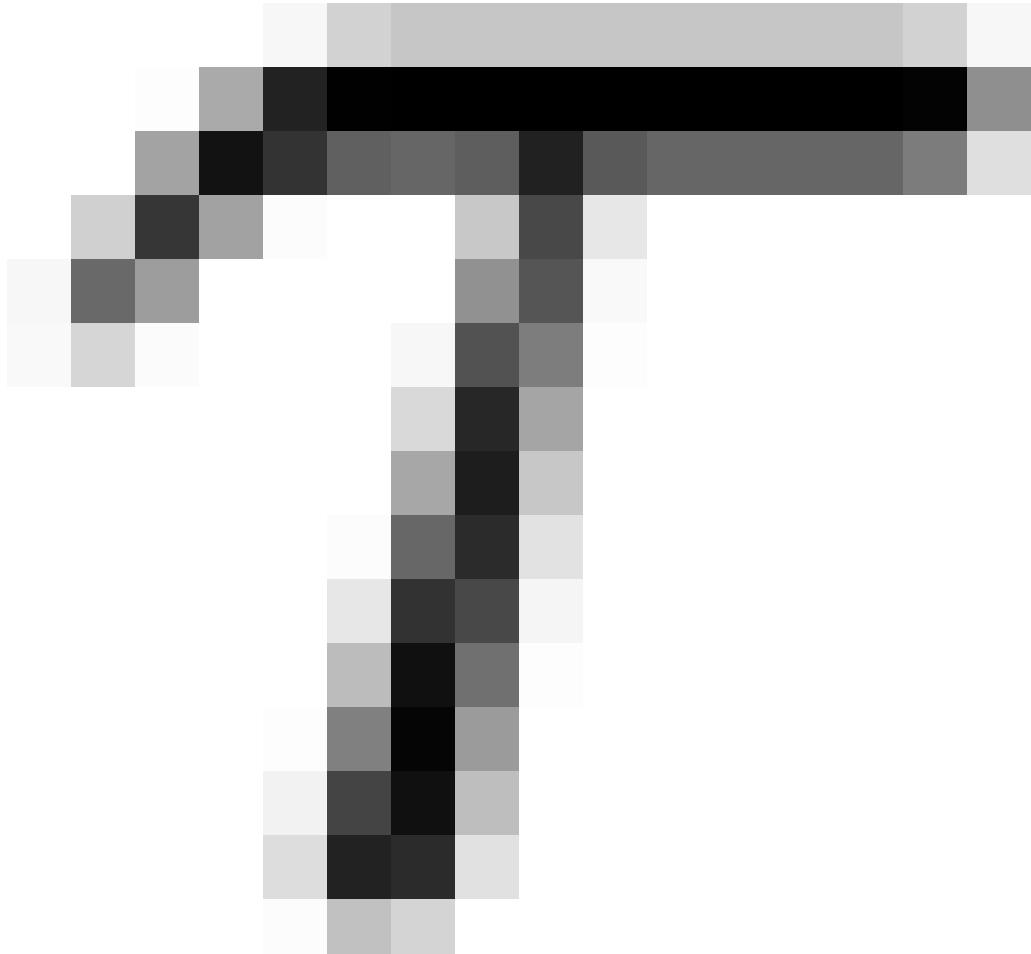


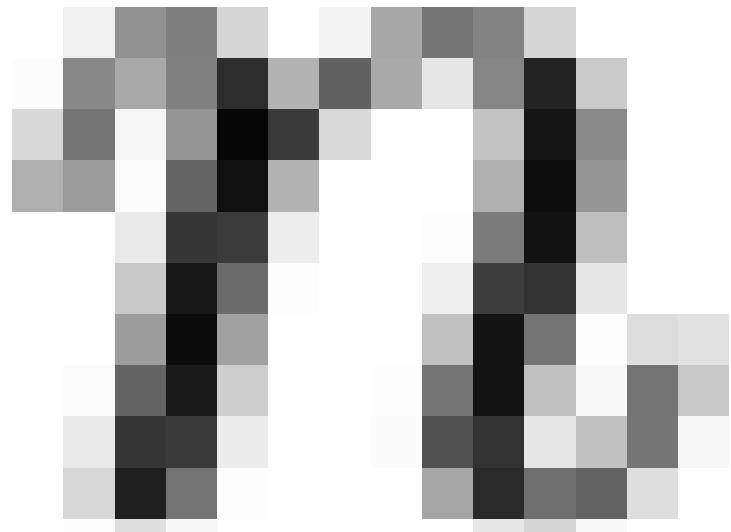
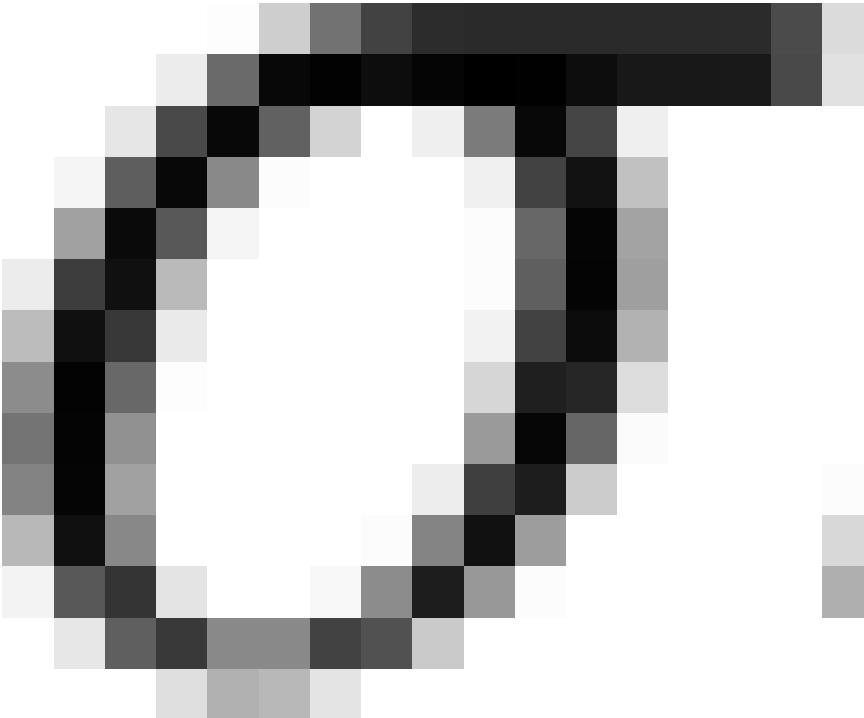




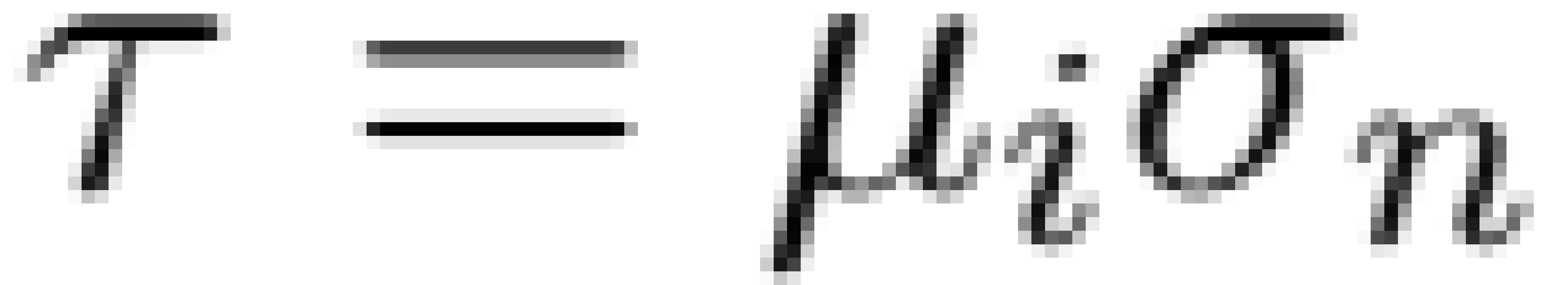


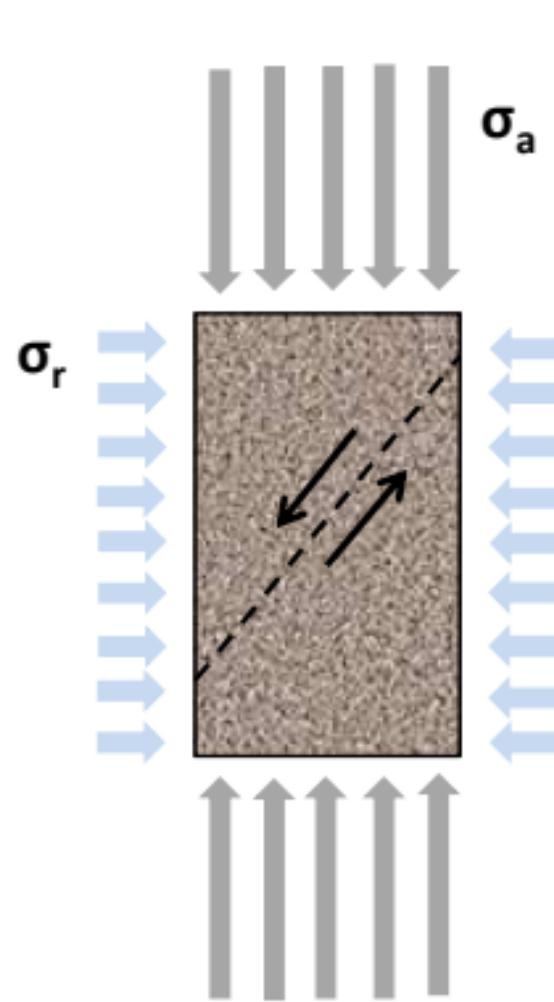




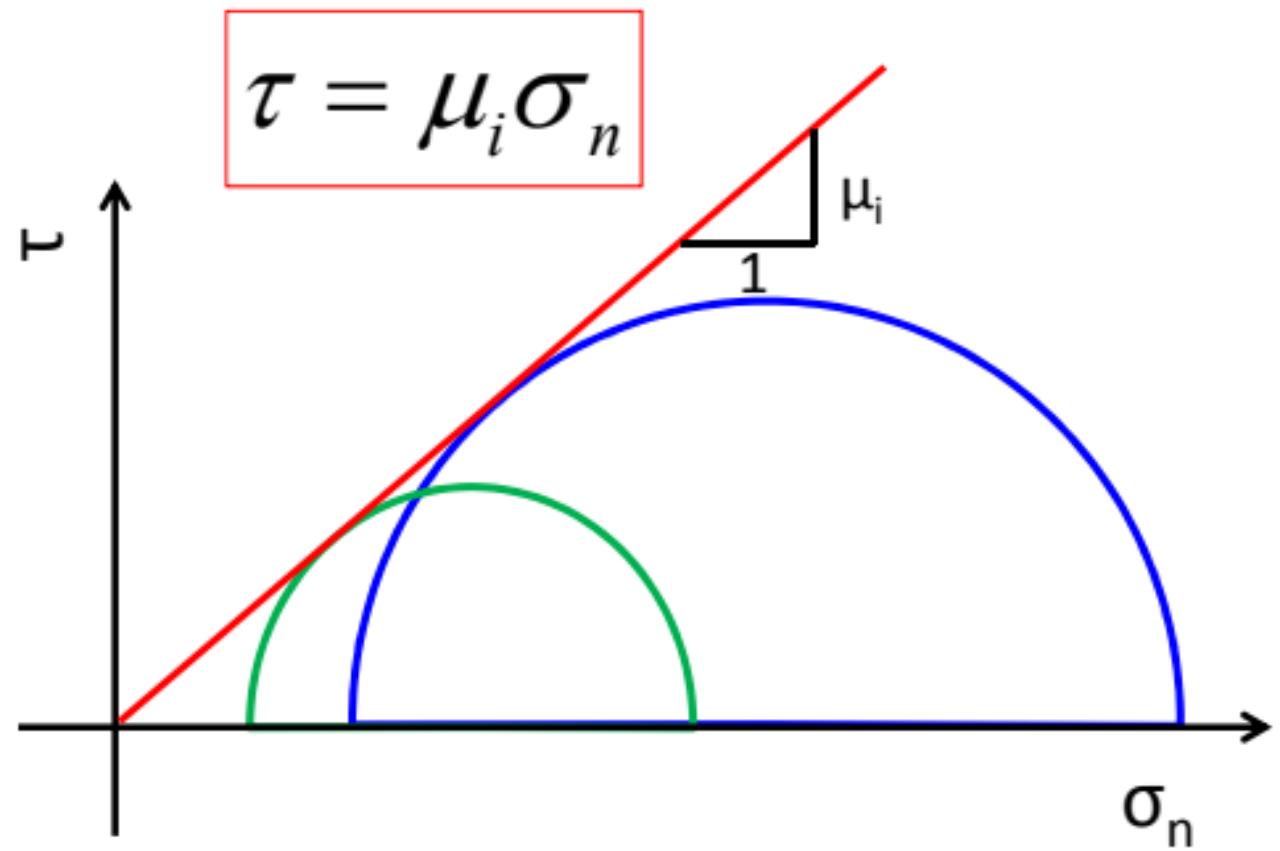


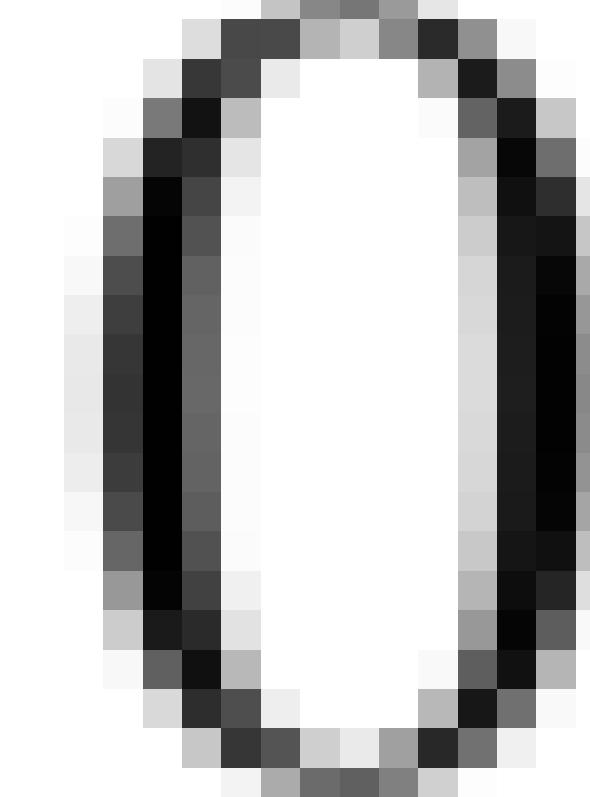
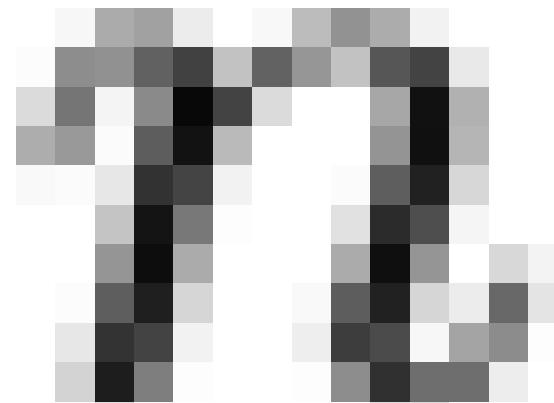
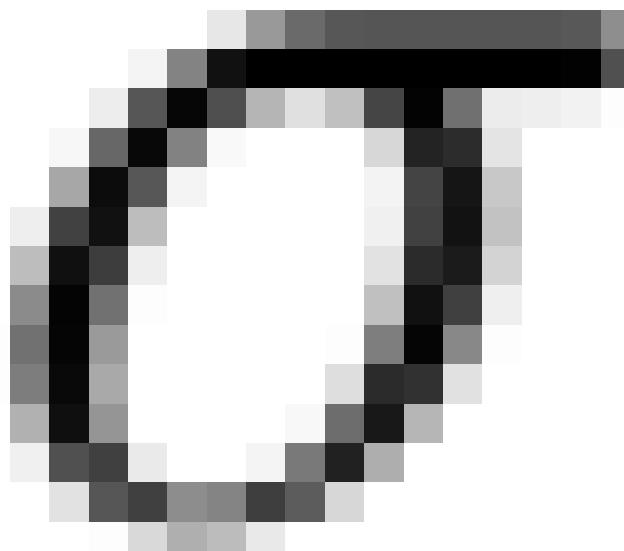


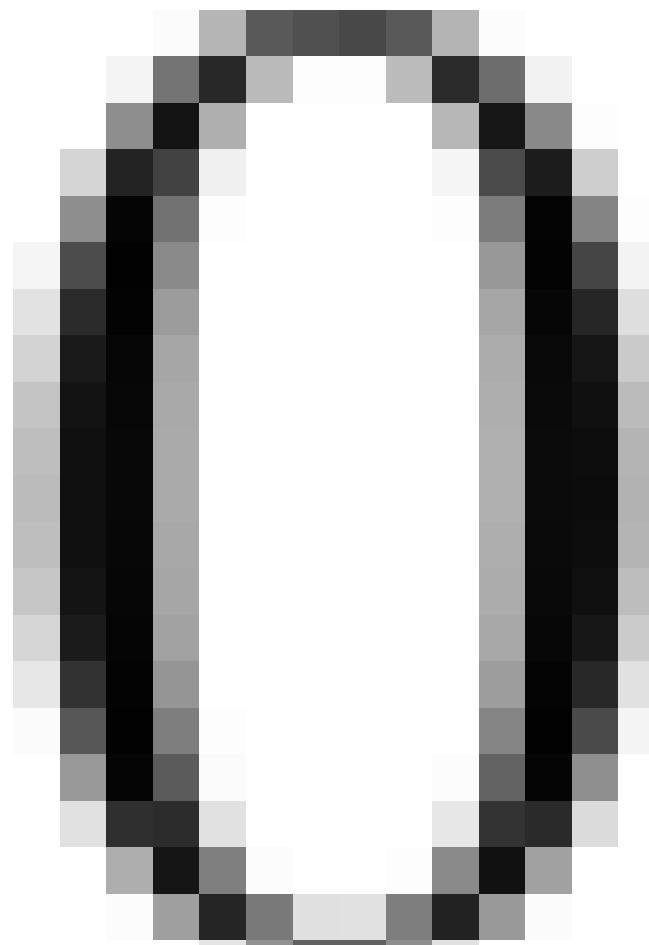
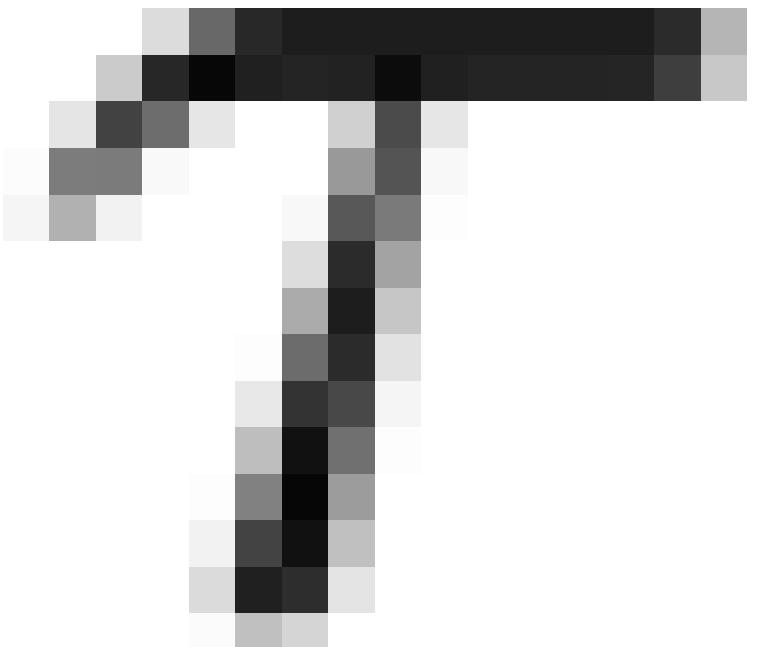


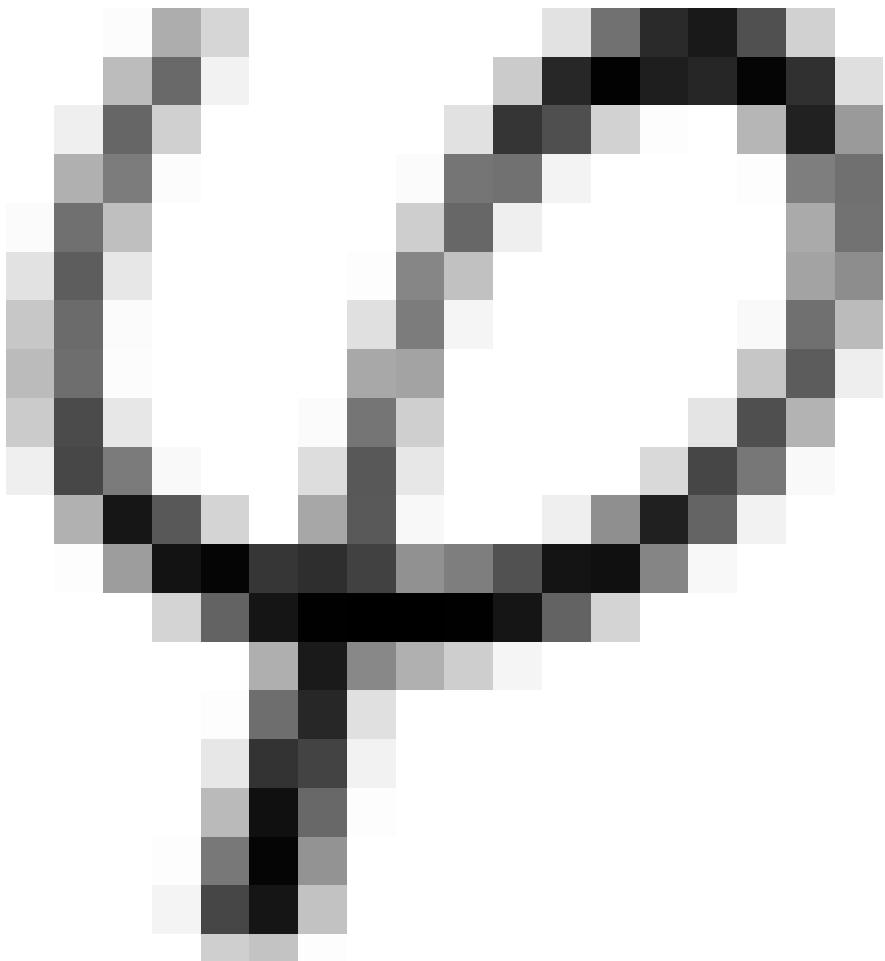


Unconsolidated Sand

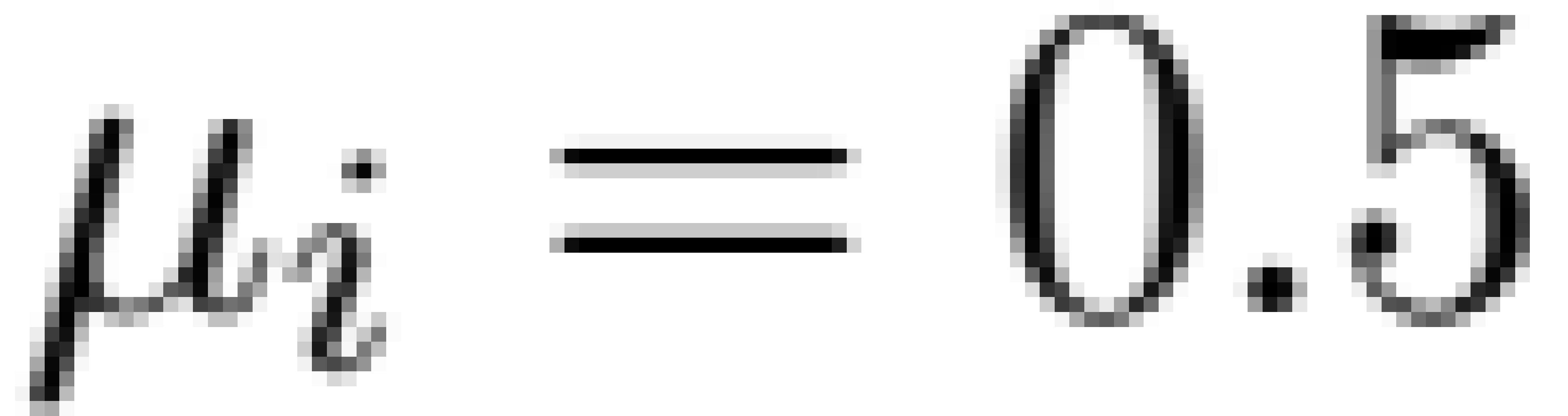


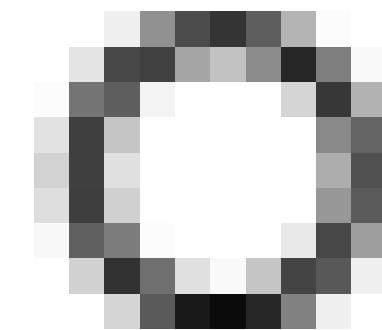
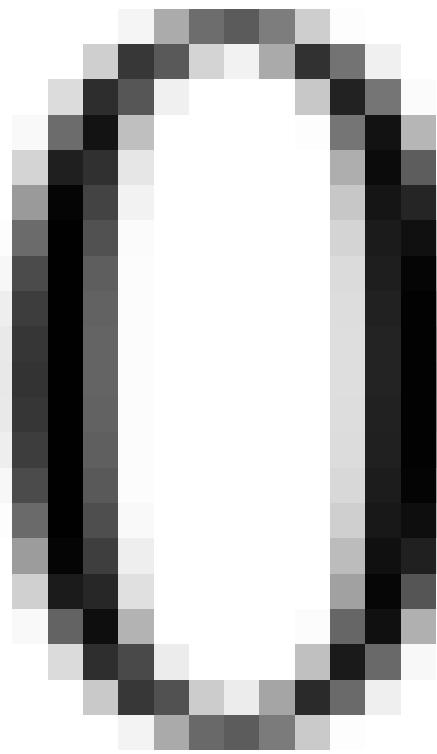
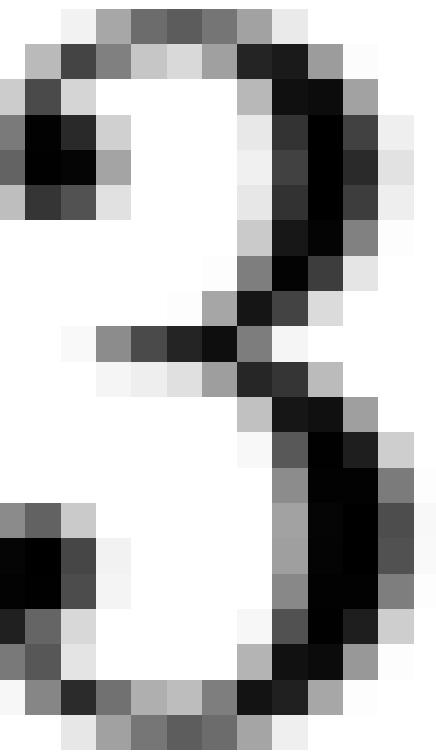
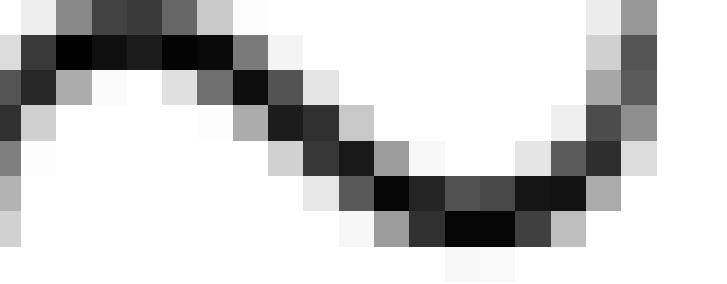
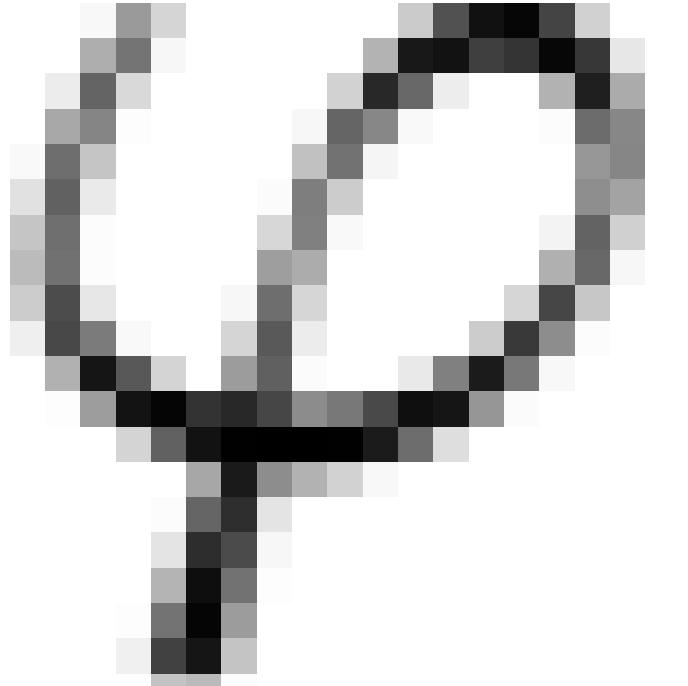


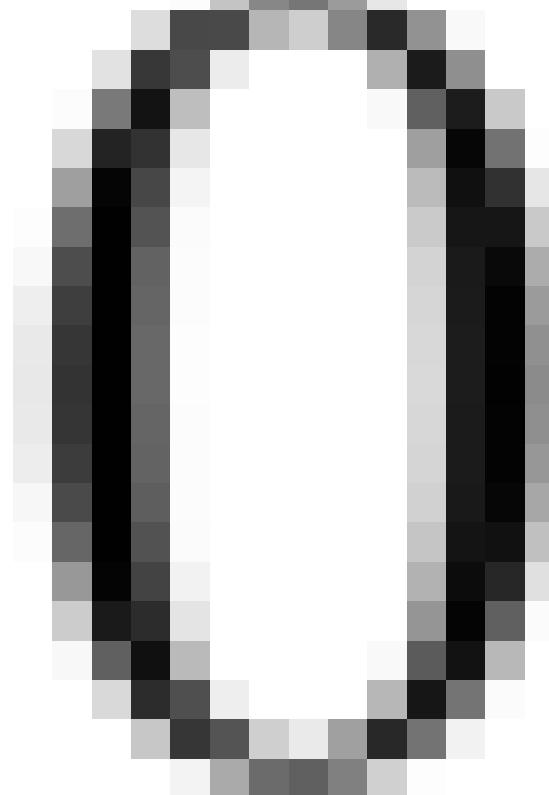
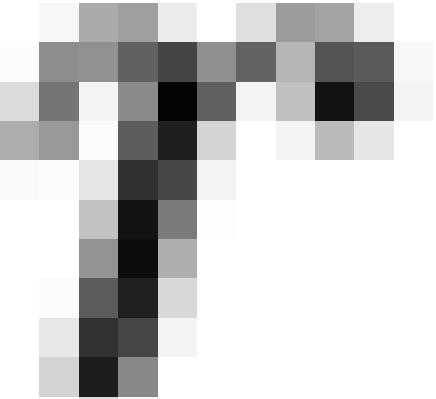
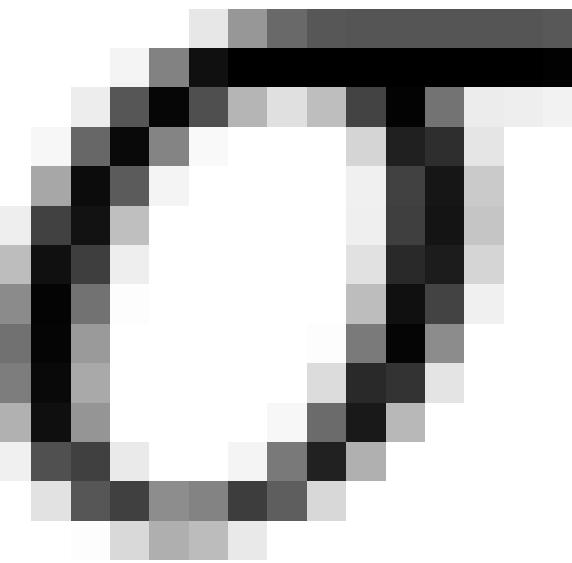


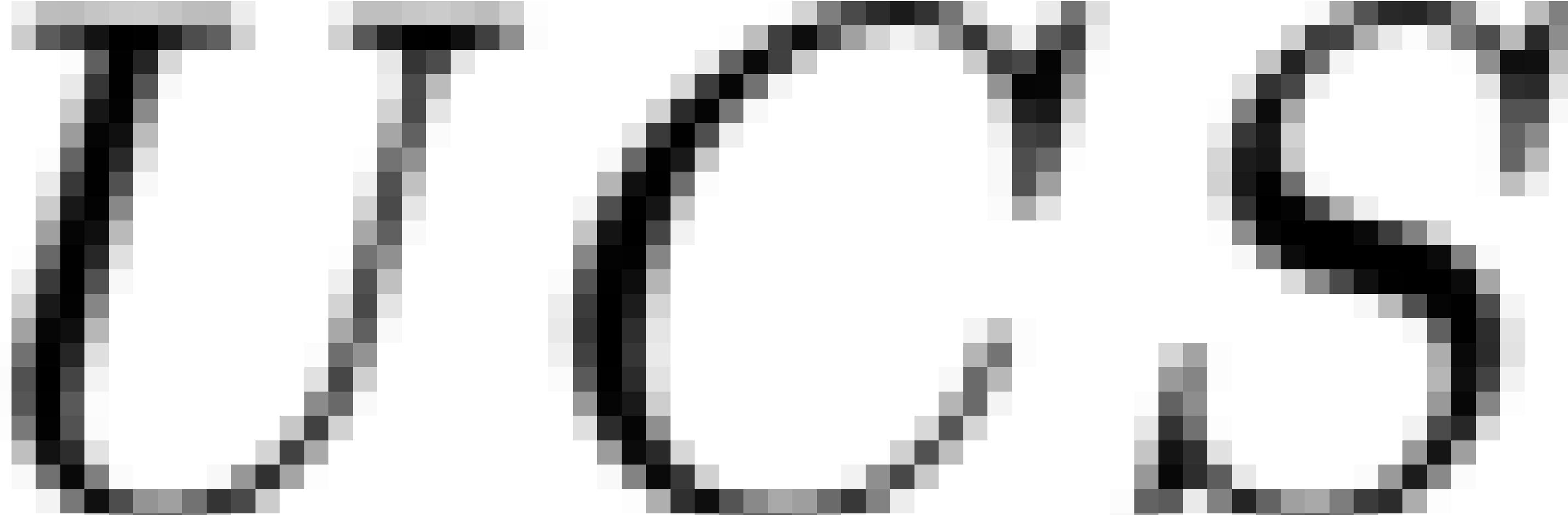




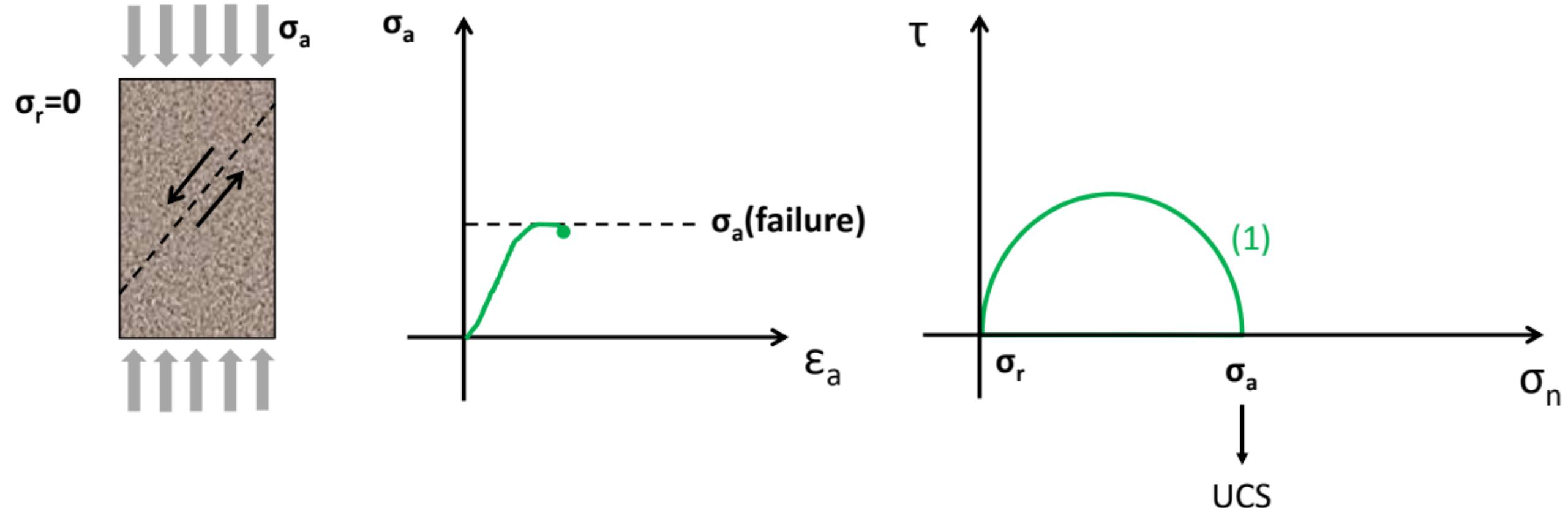


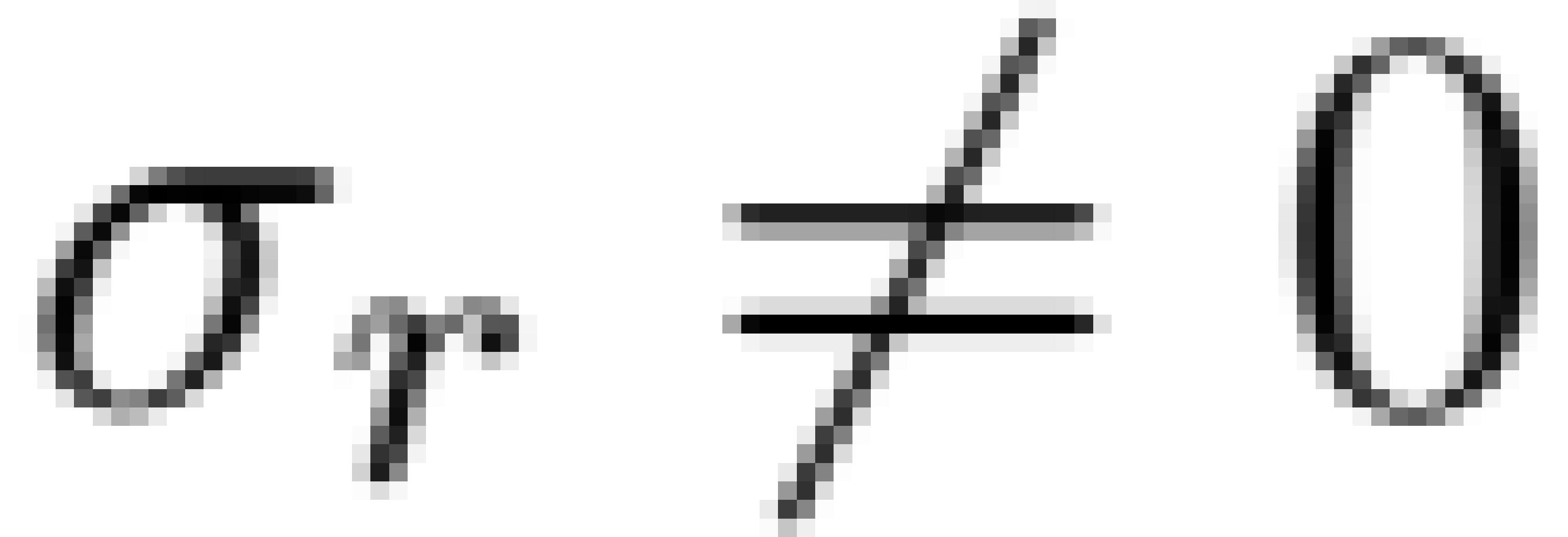


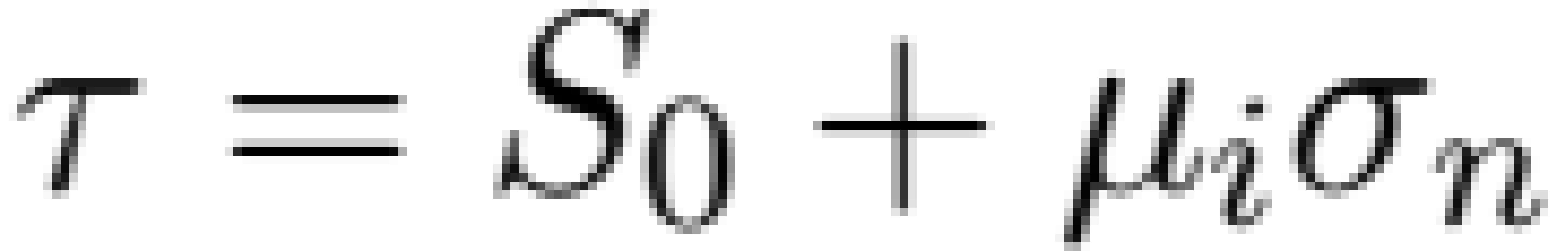




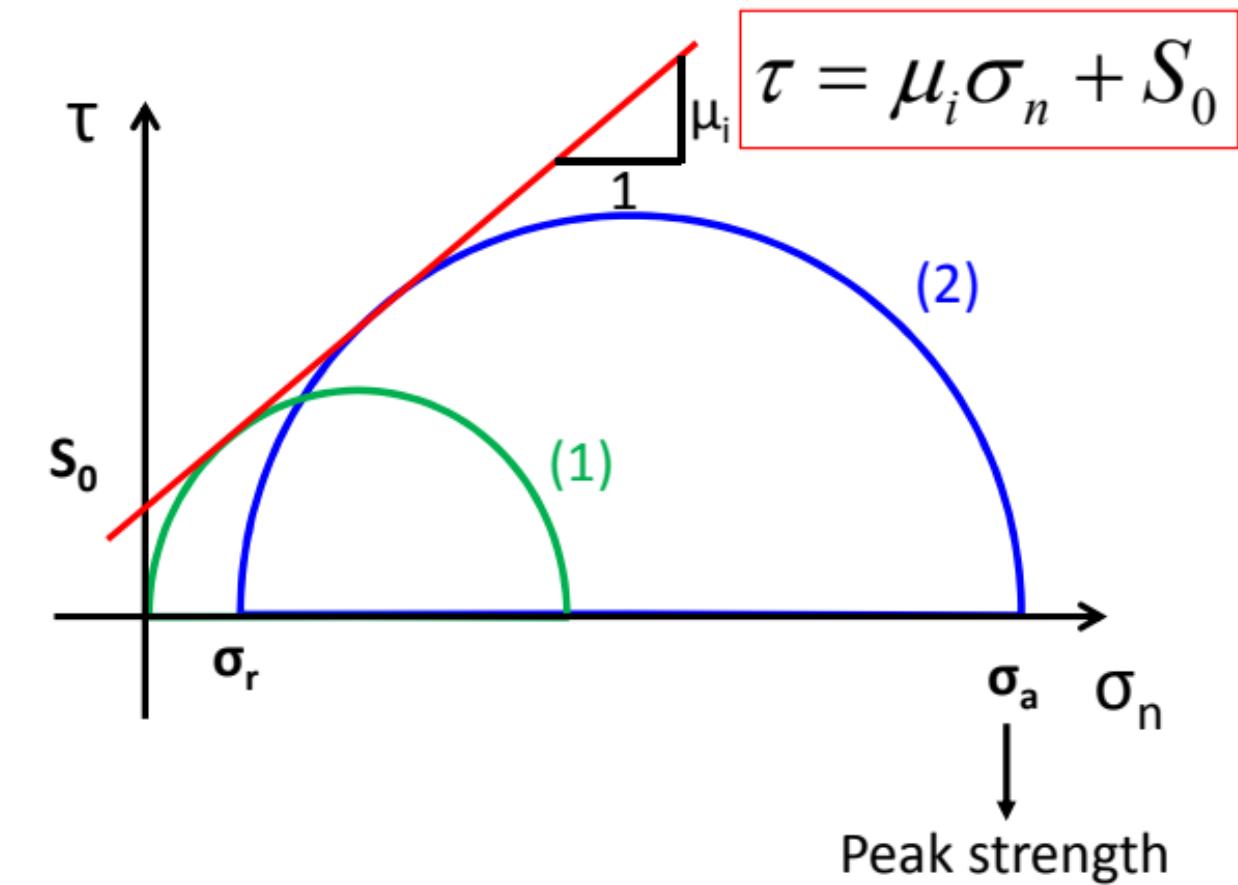
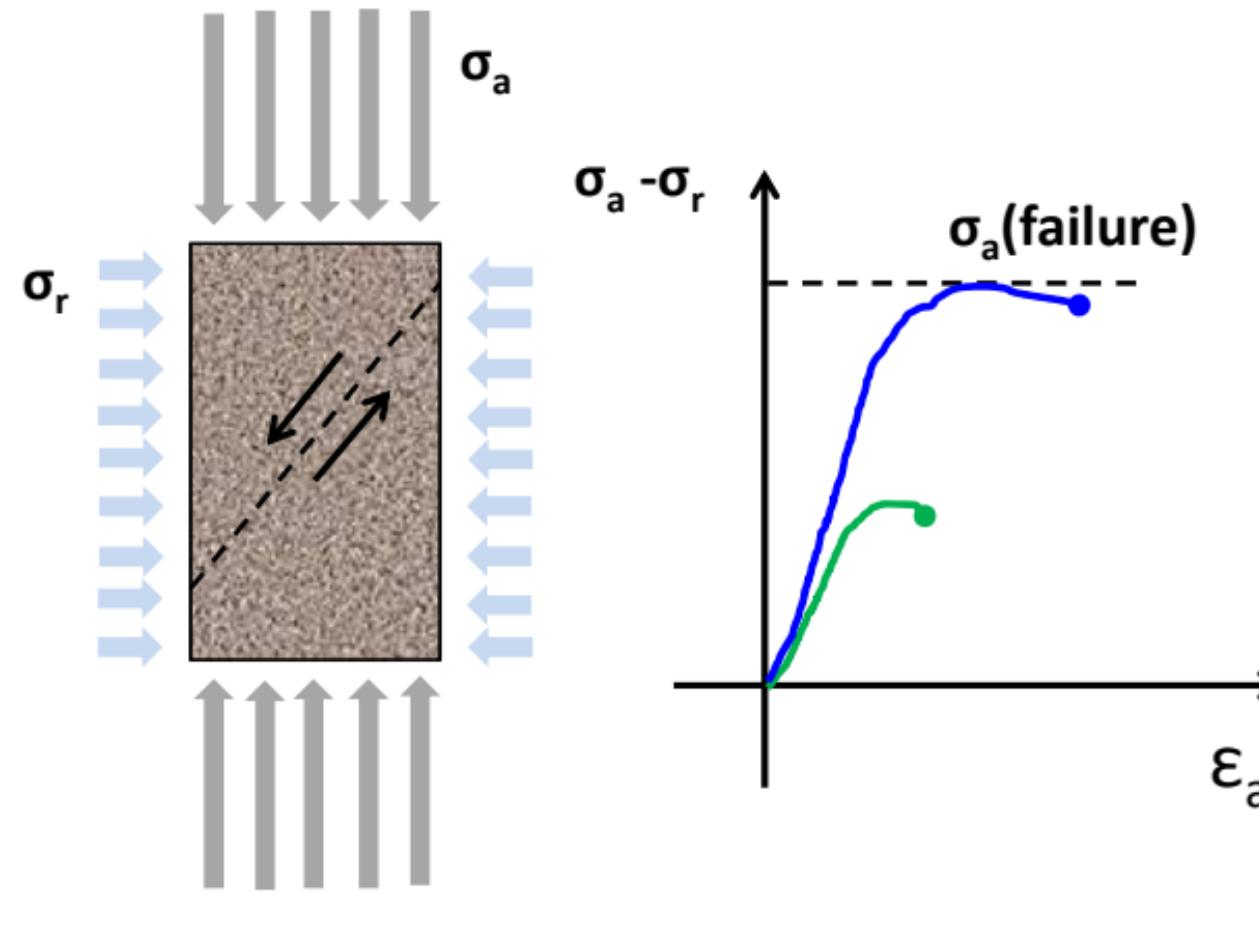
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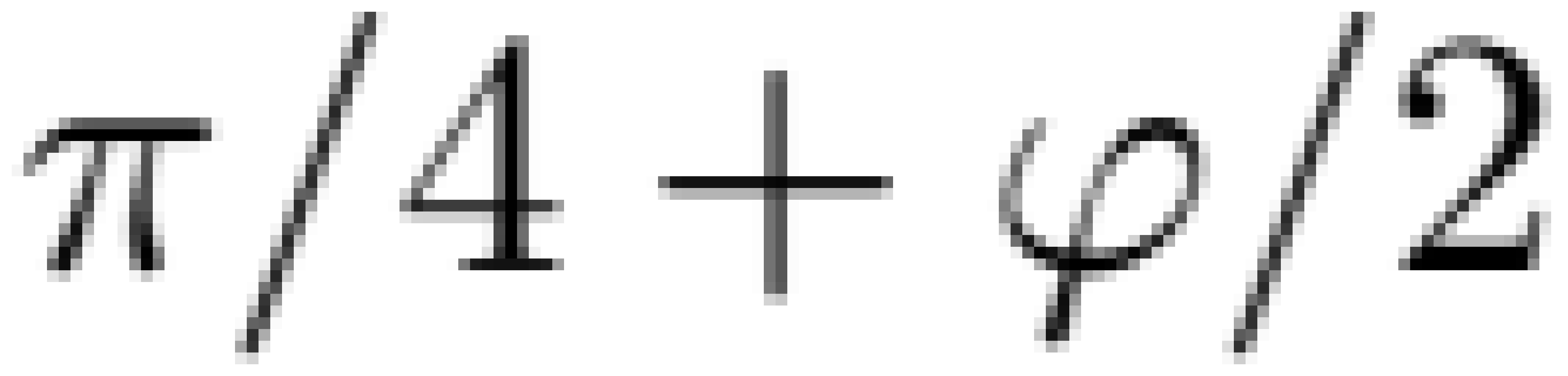






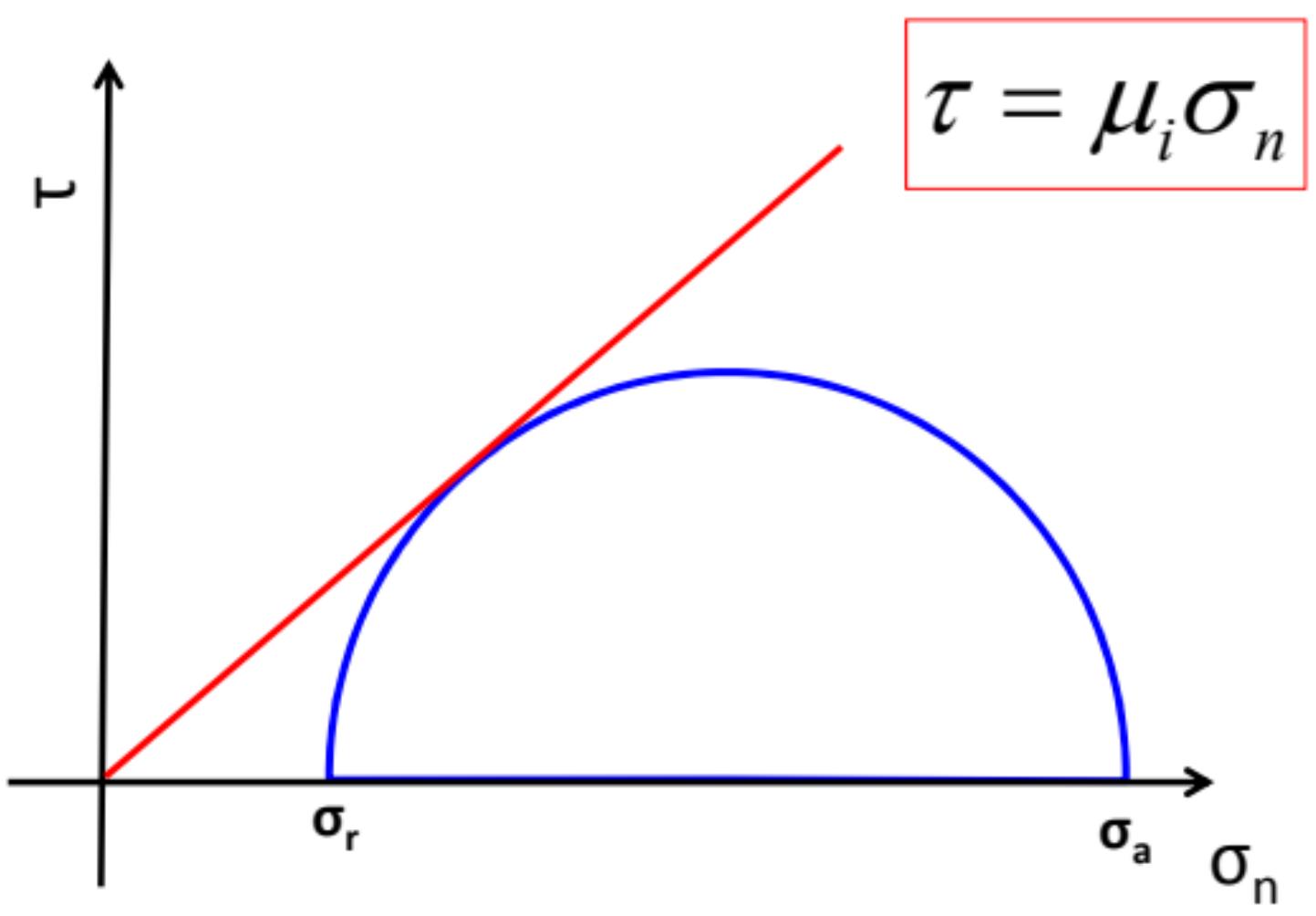
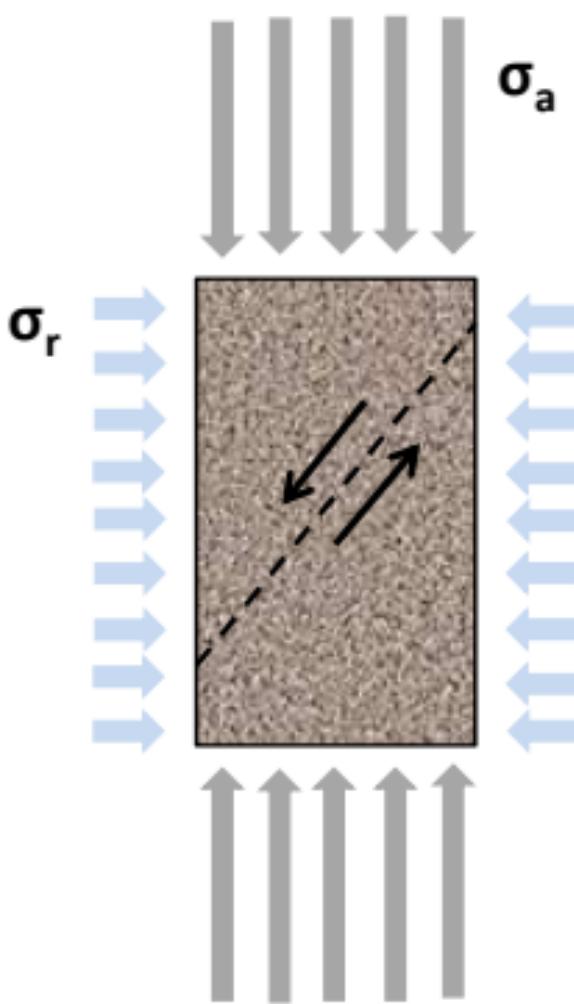
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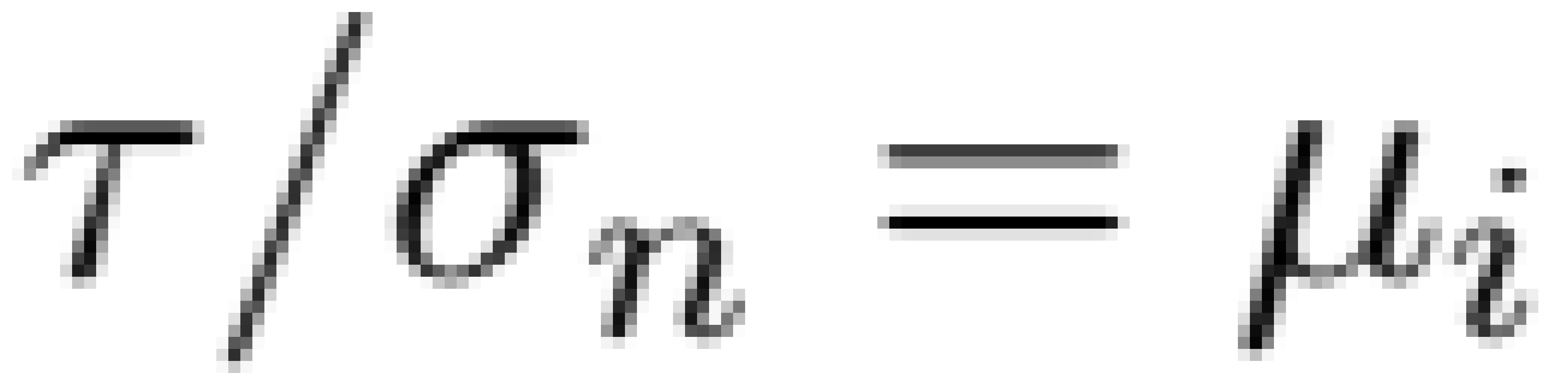


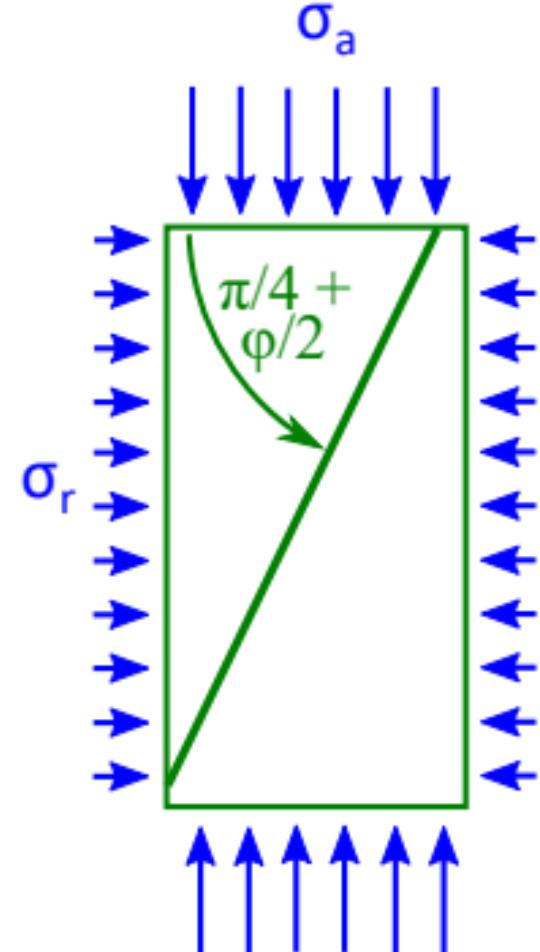
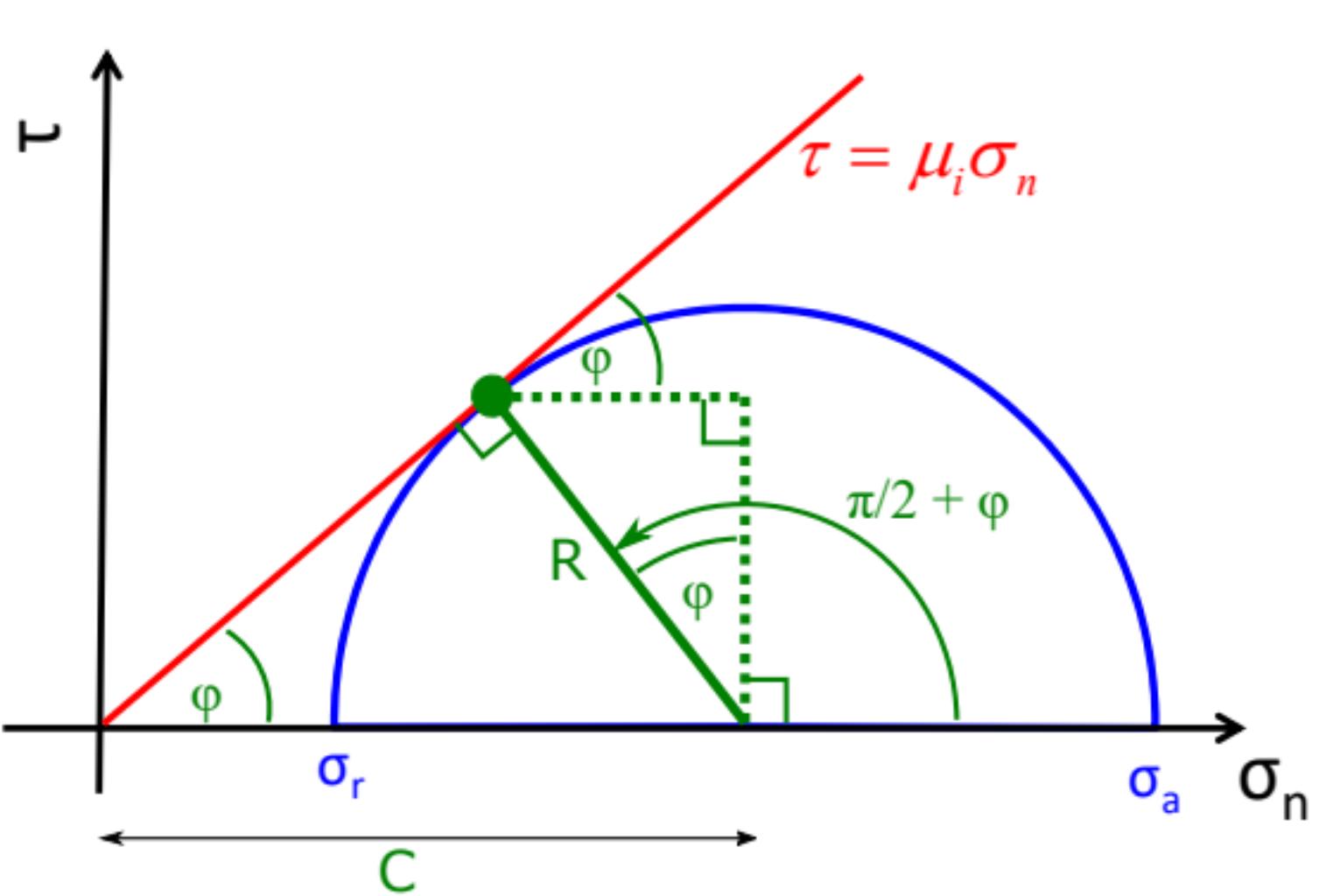








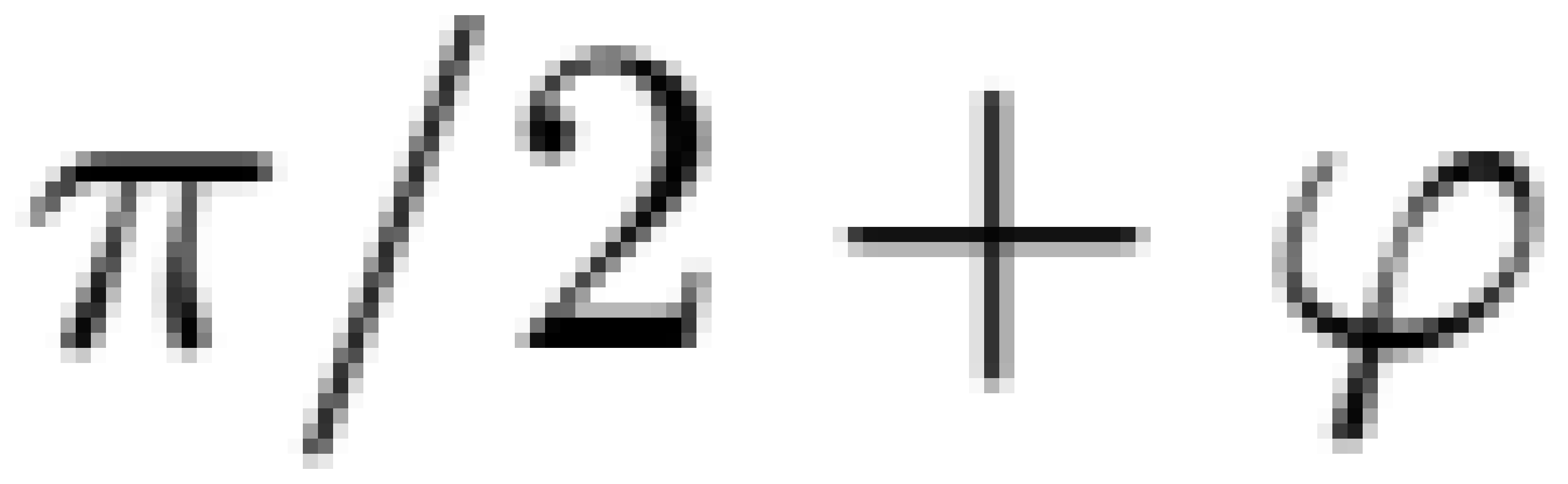


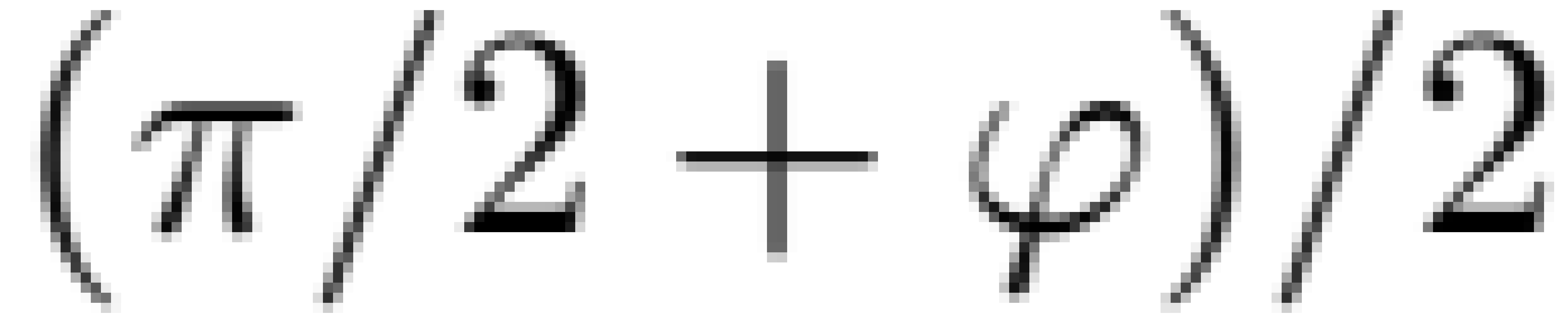


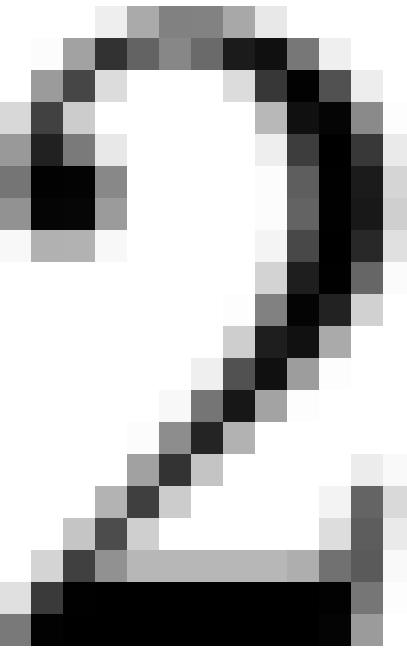
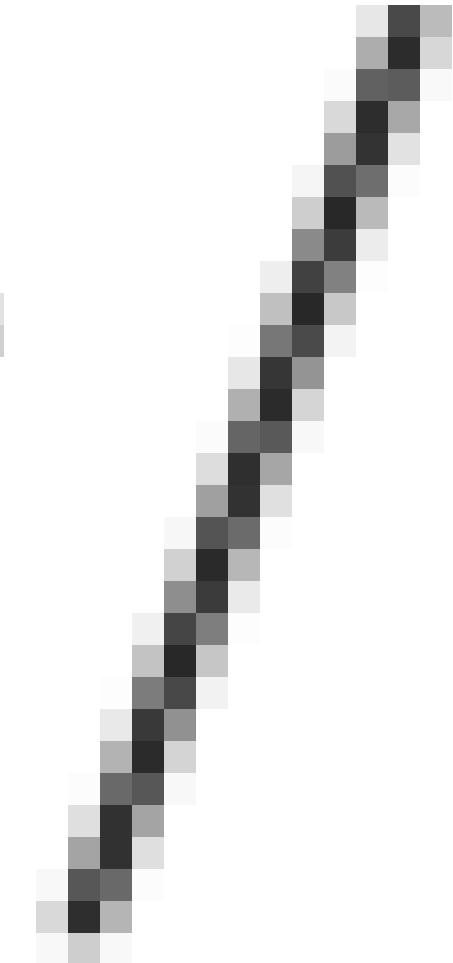
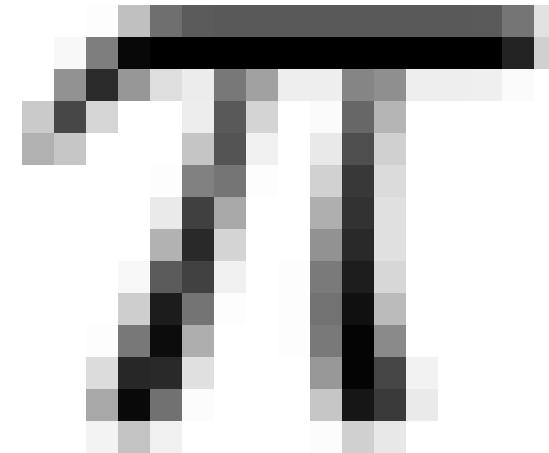
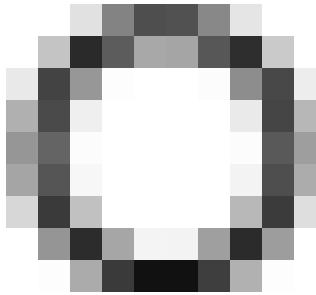


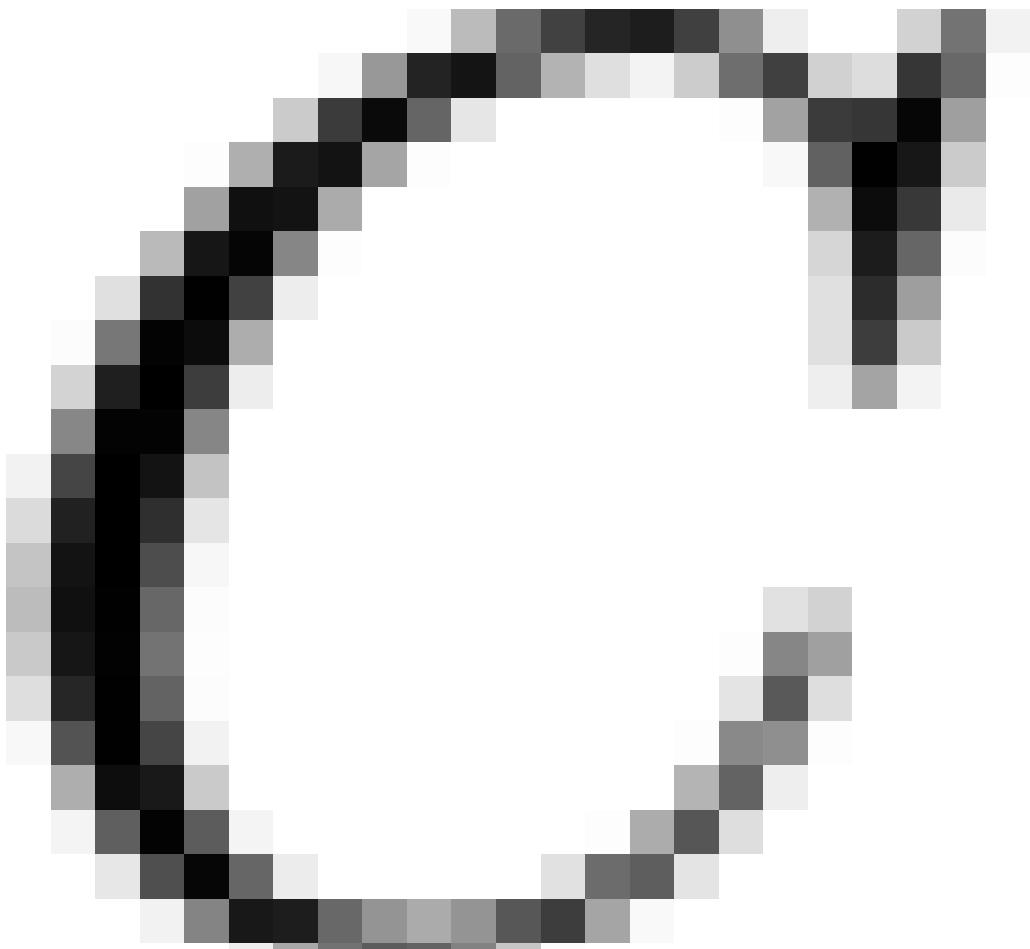








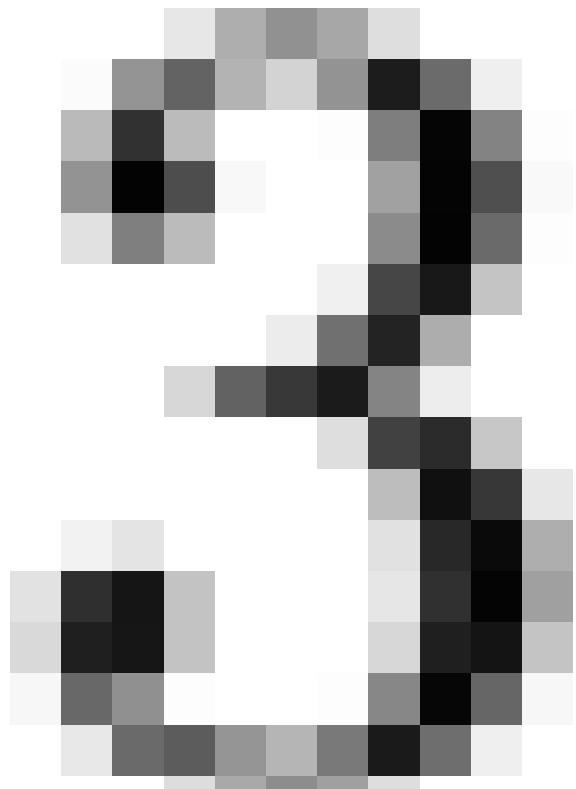
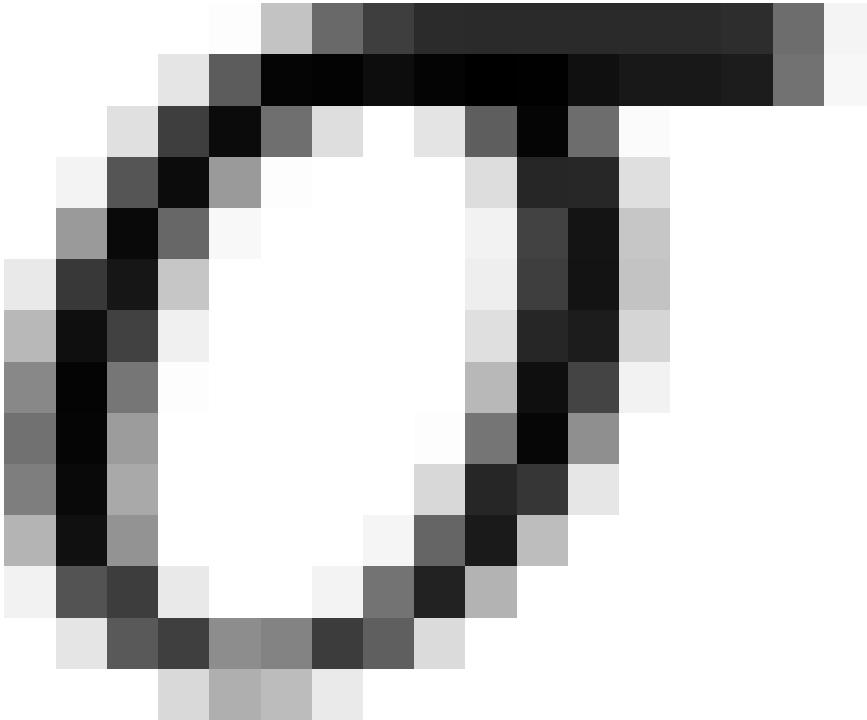


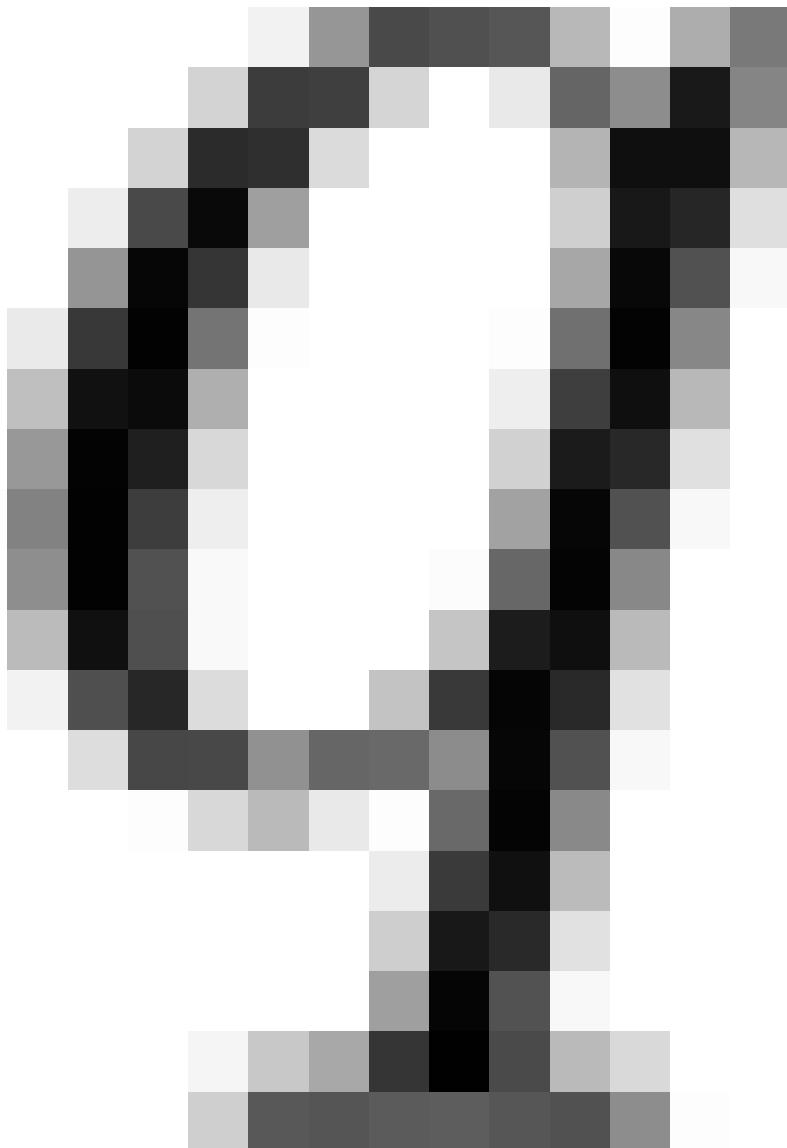


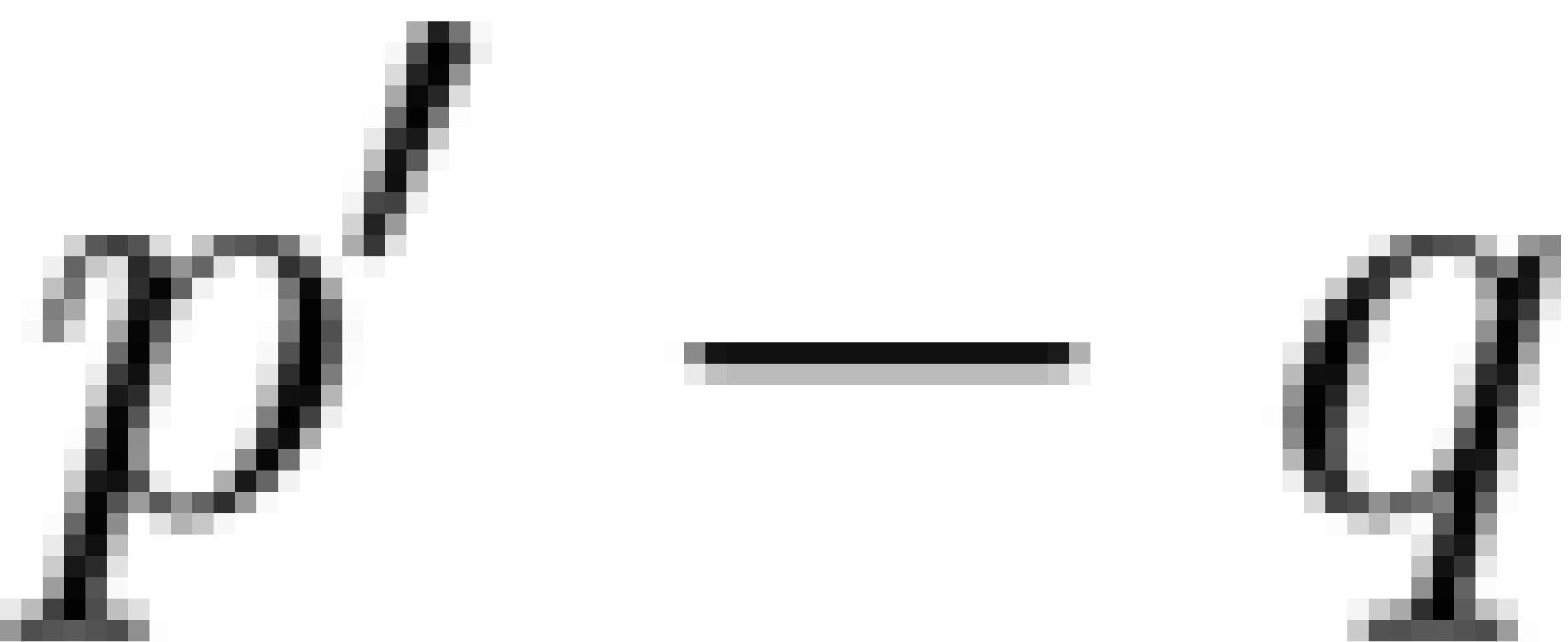
$$\begin{aligned} \sigma_a &= C + R \\ \sigma_r &= C - R \end{aligned}$$
$$\begin{aligned} C + \sin \varphi &= 1 + \sin \varphi \\ C - \sin \varphi &= 1 - \sin \varphi \end{aligned}$$

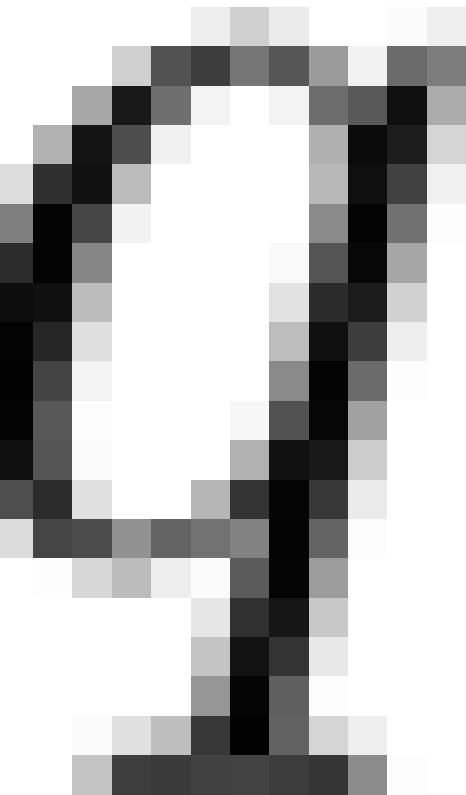
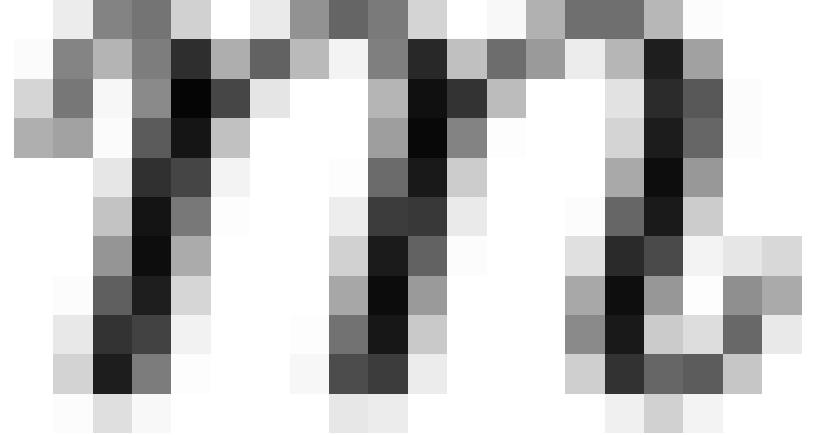
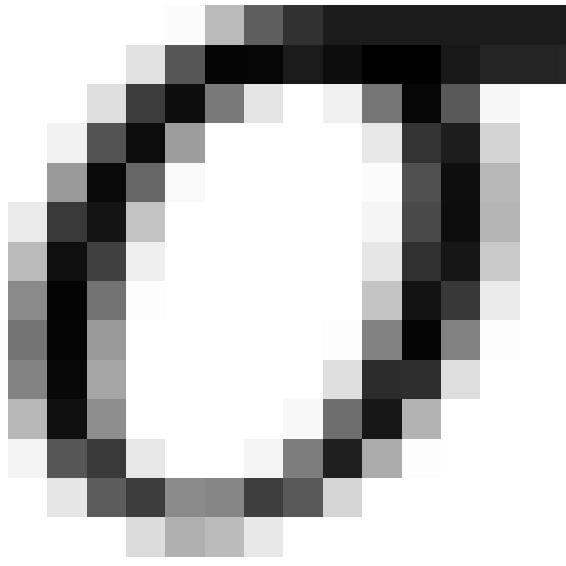






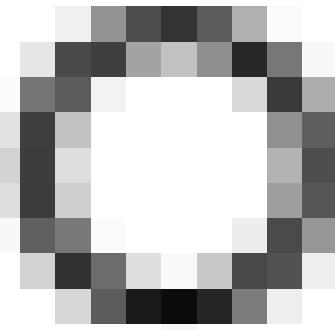
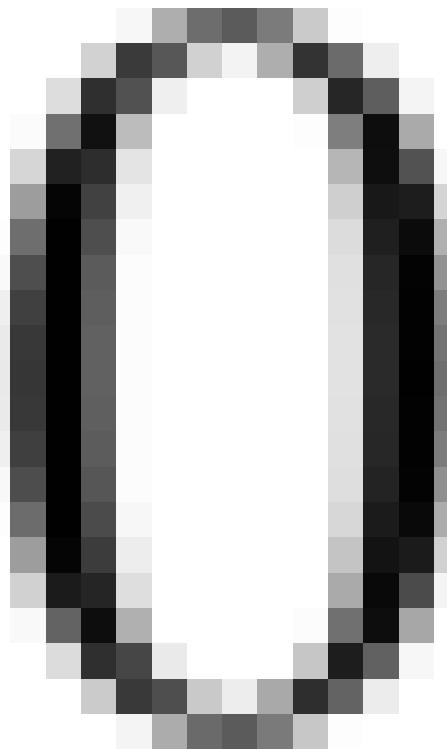
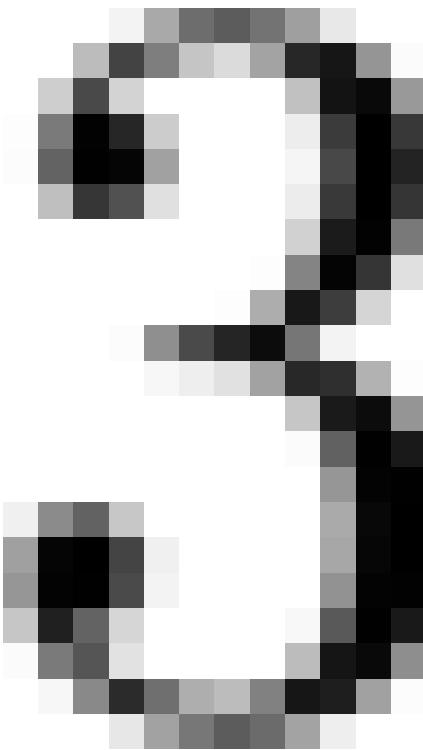
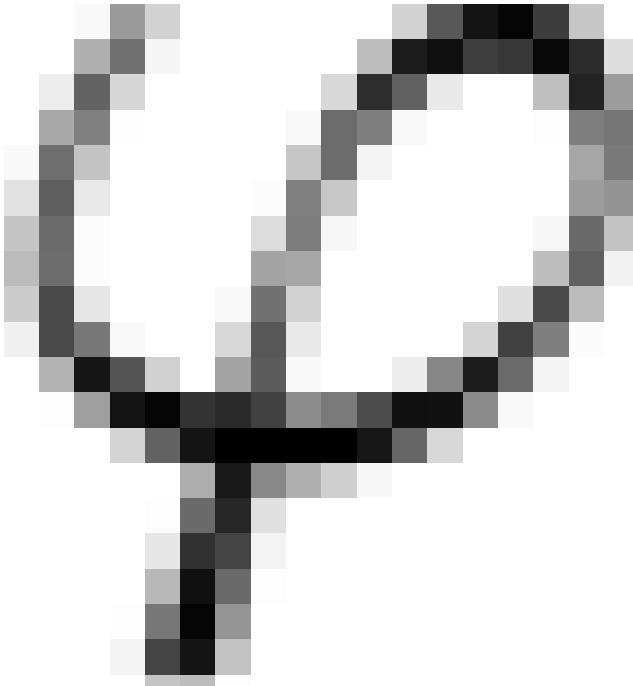


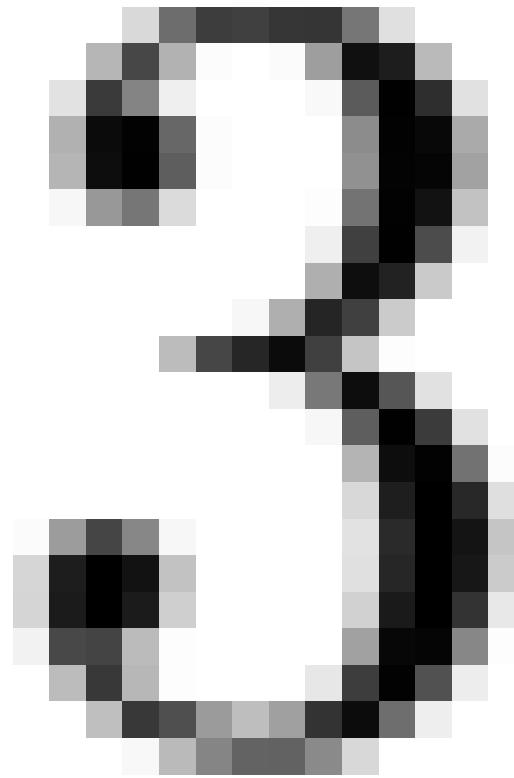
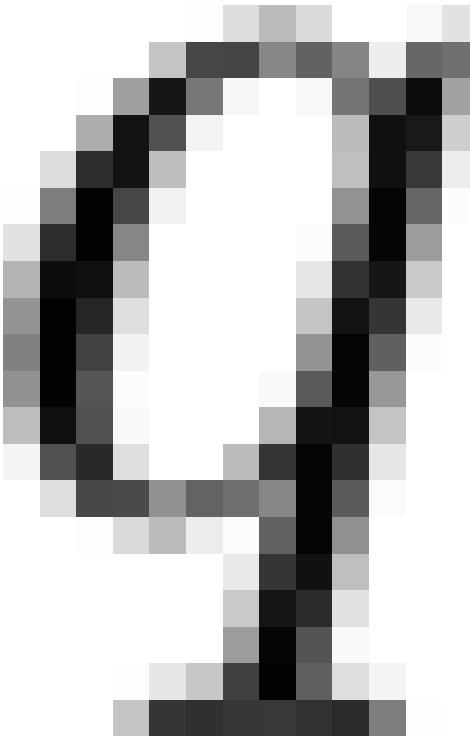




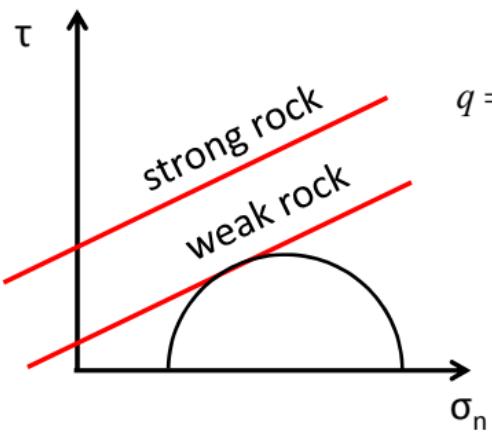
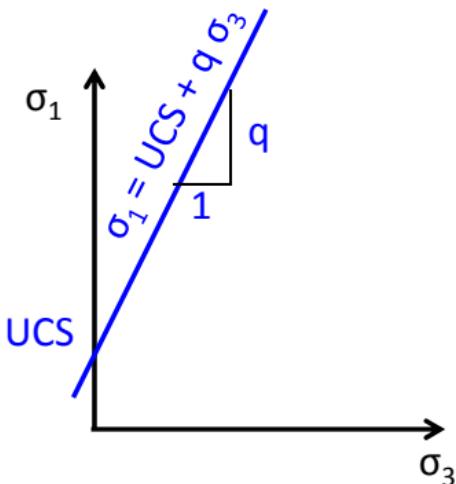
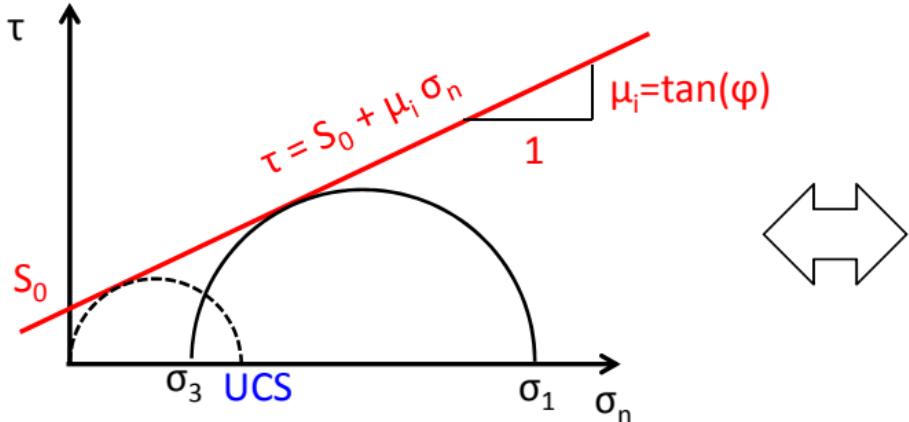
φ sin φ

φ sin φ



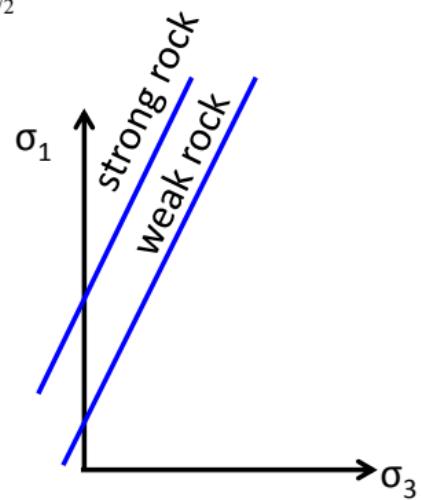


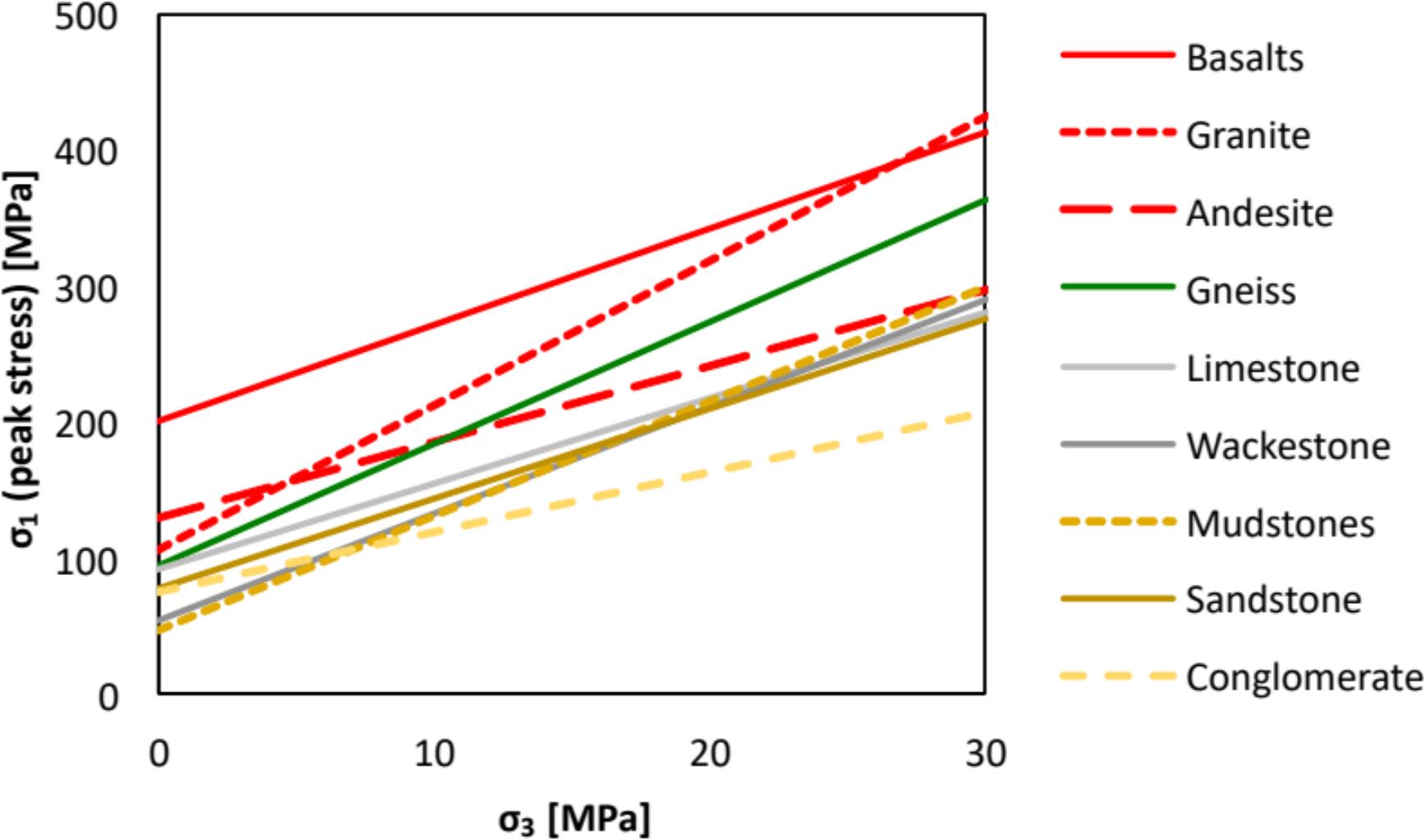
$$U_0 S = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} = 2S_0 \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi}}$$



$$UCS = 2S_0 \left(\sqrt{\mu_i^2 + 1} + \mu_i \right) = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2}$$

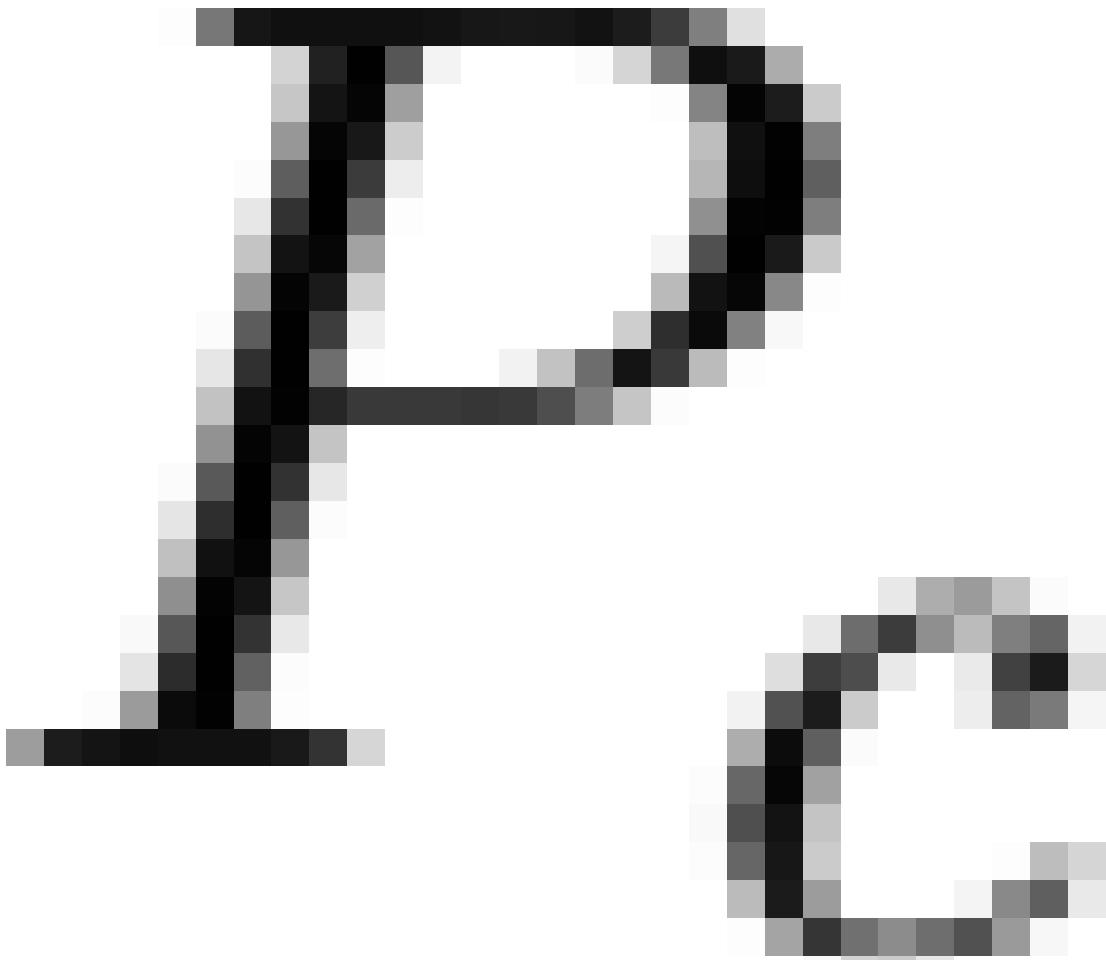
$$q = \left(\sqrt{\mu_i^2 + 1} + \mu_i \right)^2 = \frac{1 + \sin \varphi}{1 - \sin \varphi}$$

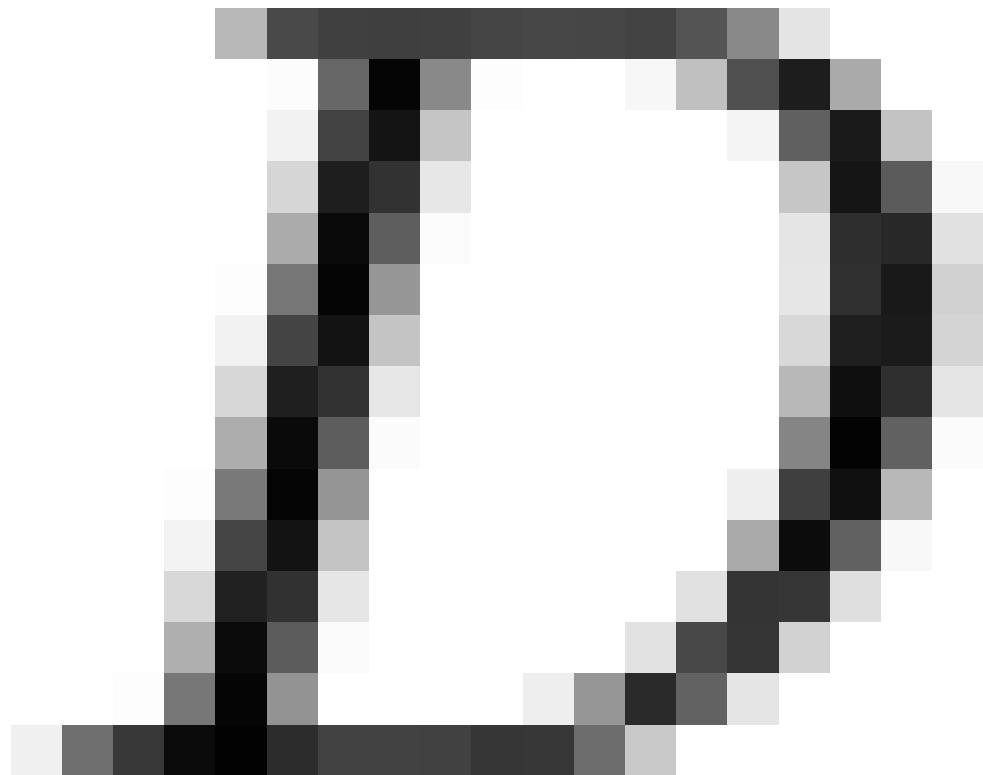
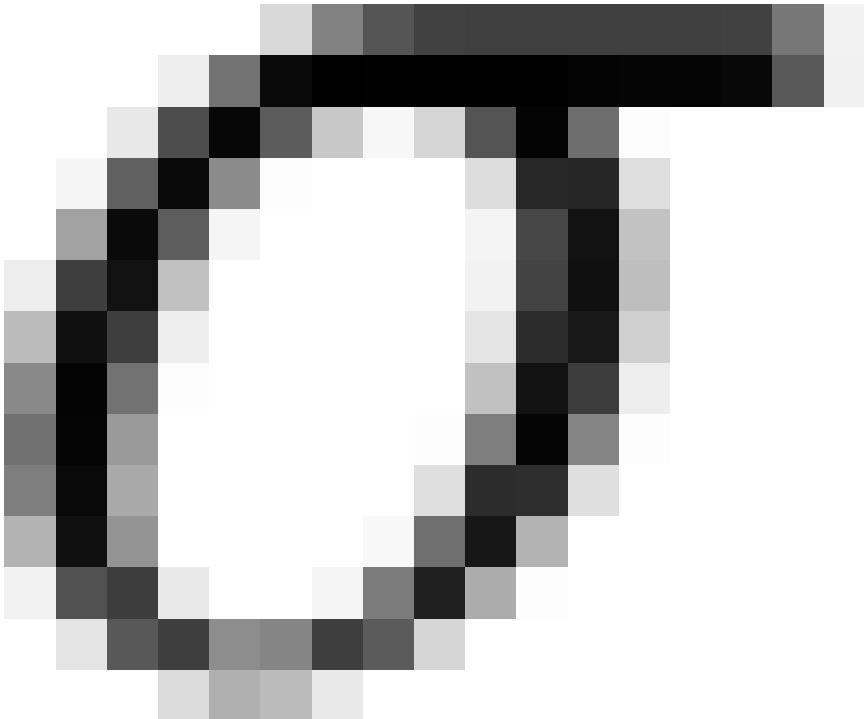




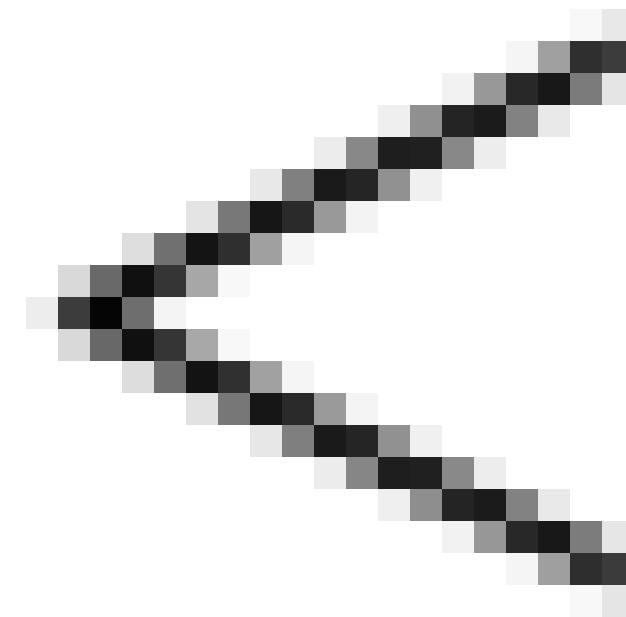












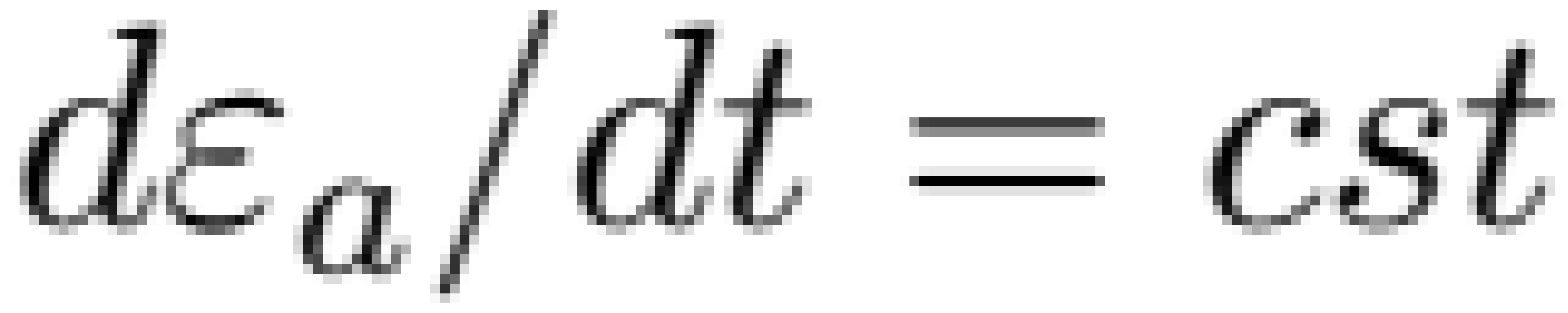


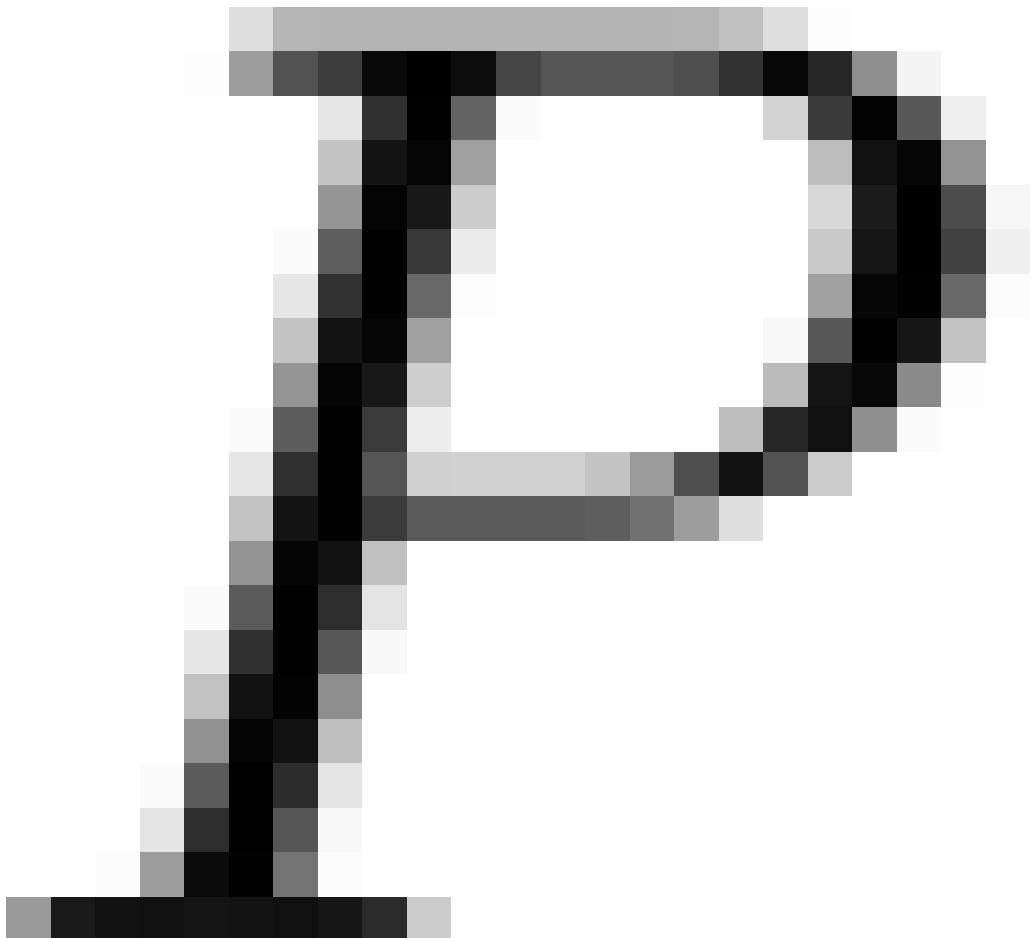










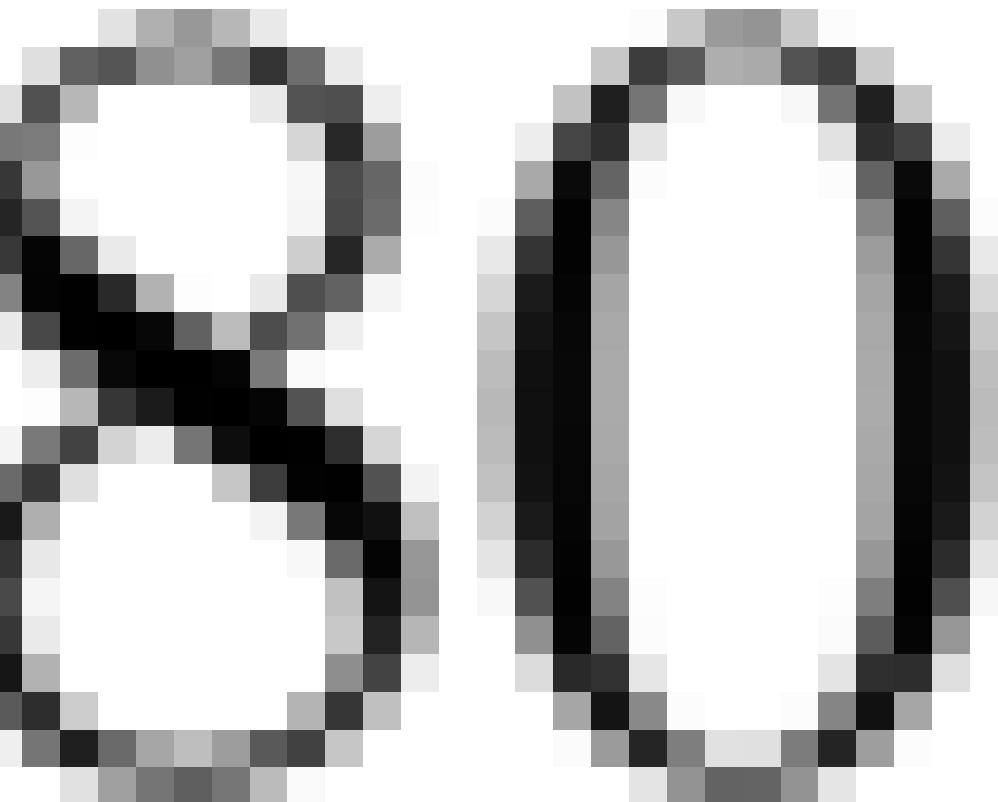


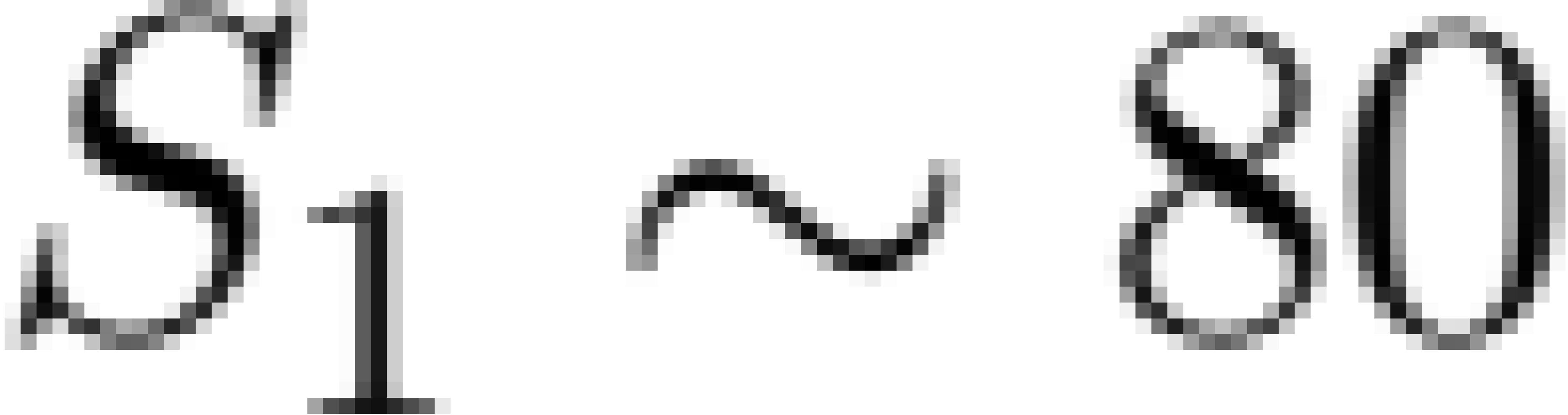


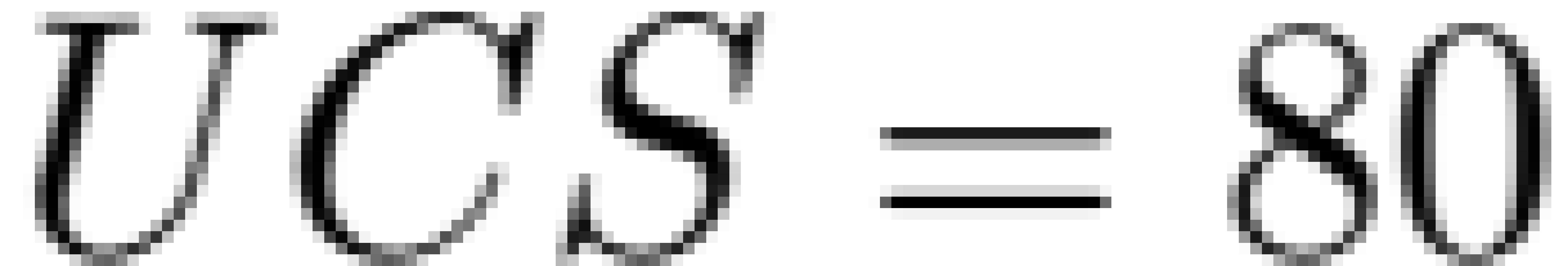


$$\varphi = \arctan$$

$$\left(\frac{q-1}{2\sqrt{q}} \right)$$







q

$=$

σ_1

$=$

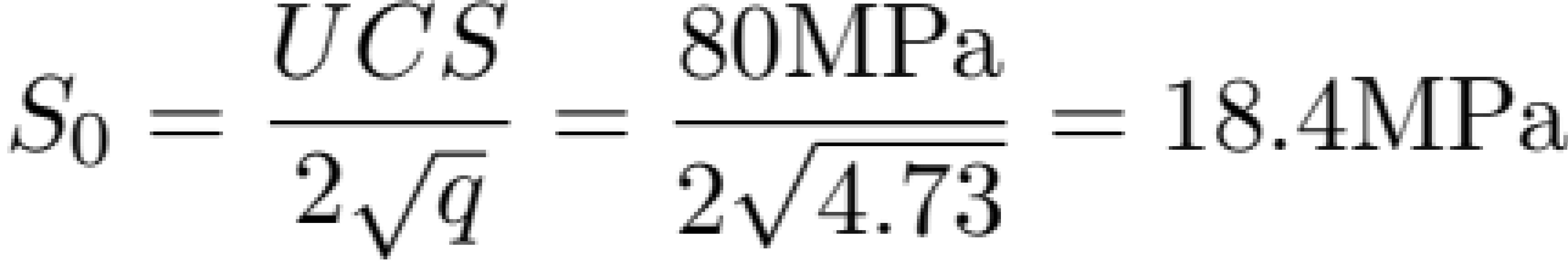
σ_3

110 MPa

520 MPa

MPa

4.73

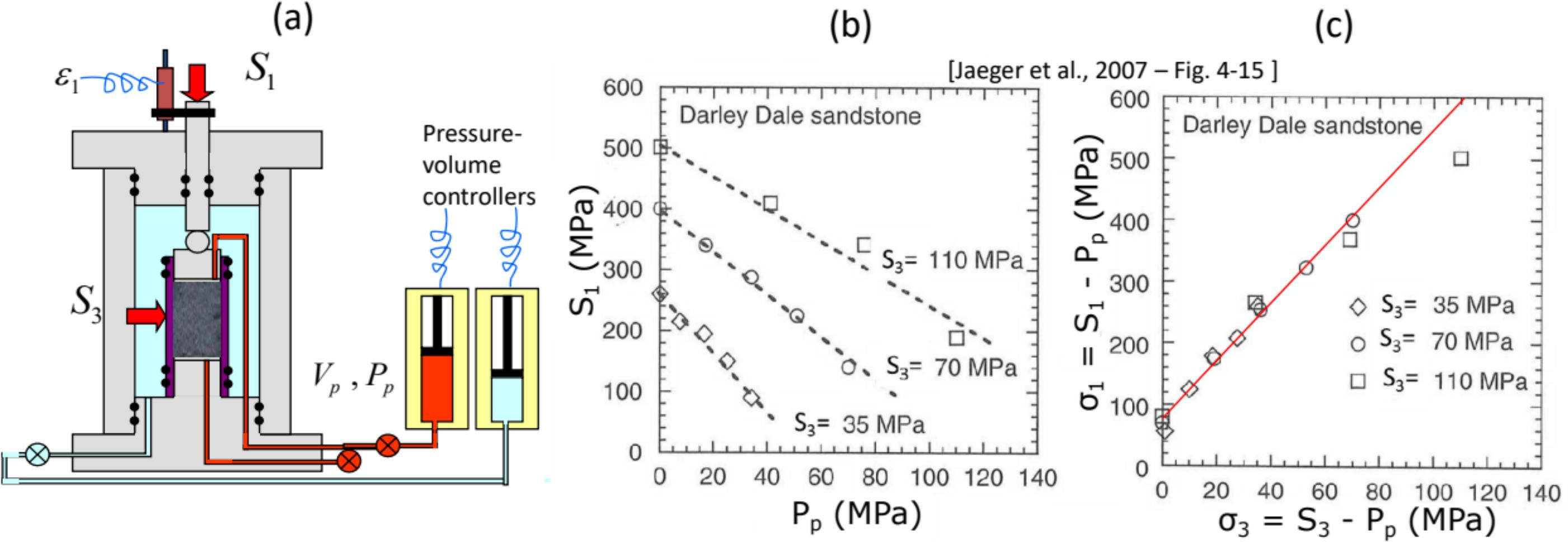


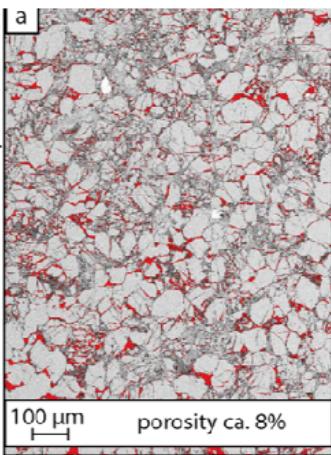
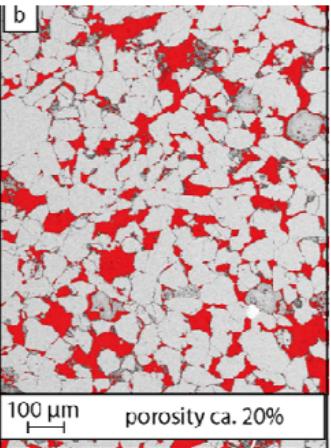
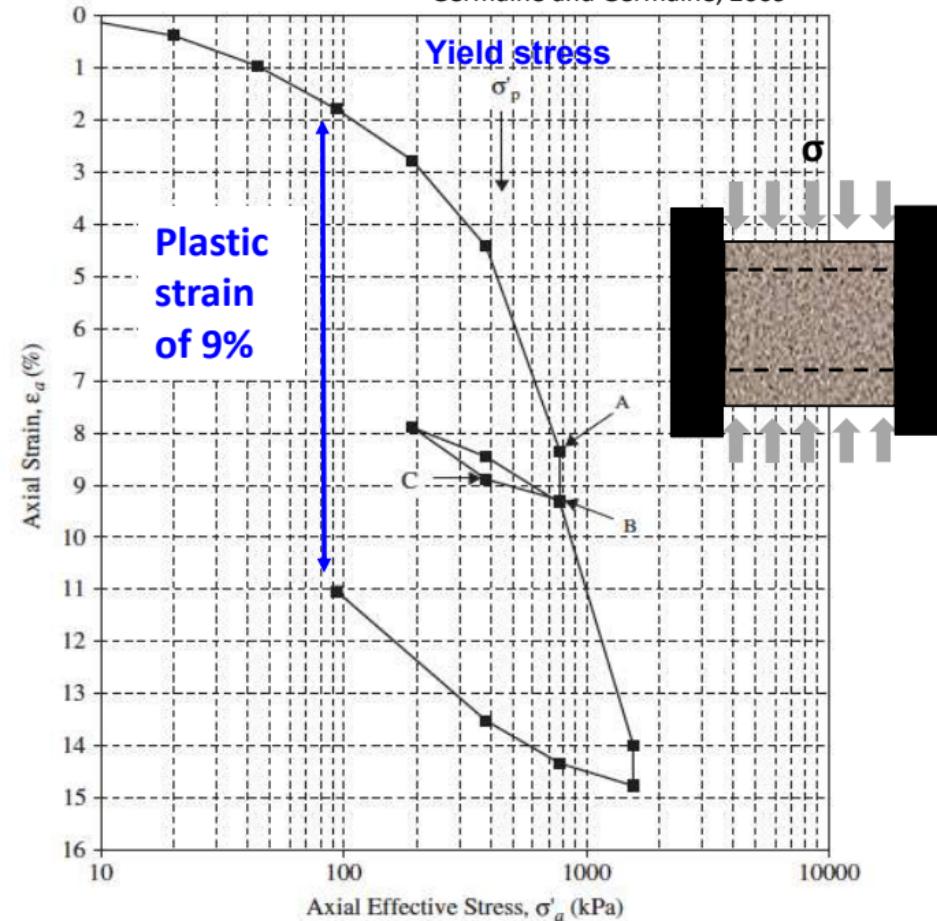
$$\varphi = \arctan \left(\frac{q-1}{2\sqrt{q}} \right) = \arctan \left(\frac{4.73 - 1}{2\sqrt{4.73}} \right) = 45.1^\circ$$

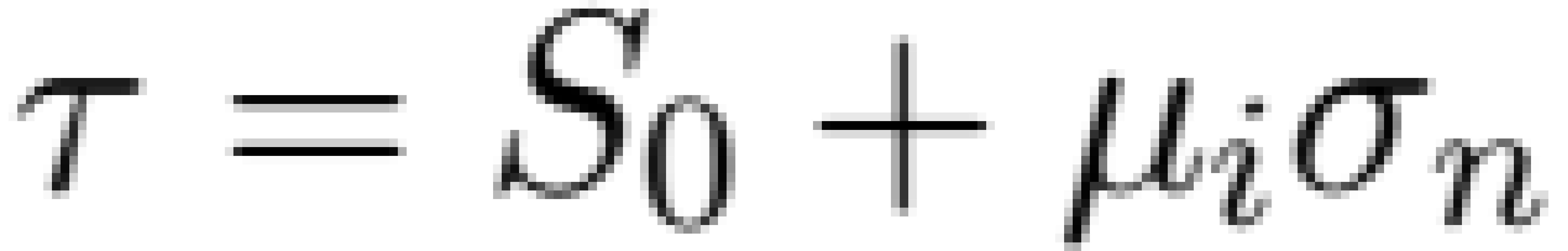


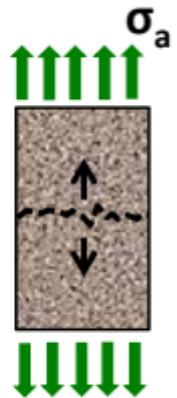








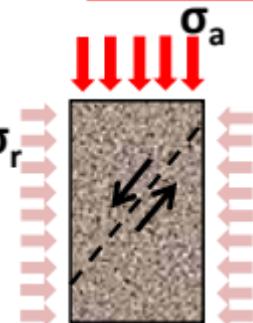


τ 

$$\sigma_n = T_s$$

 T_s

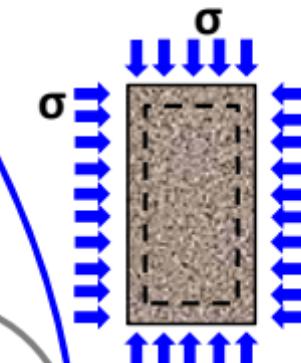
$$\boxed{\tau = \mu_i \sigma_n + S_0}$$



1

 μ_i

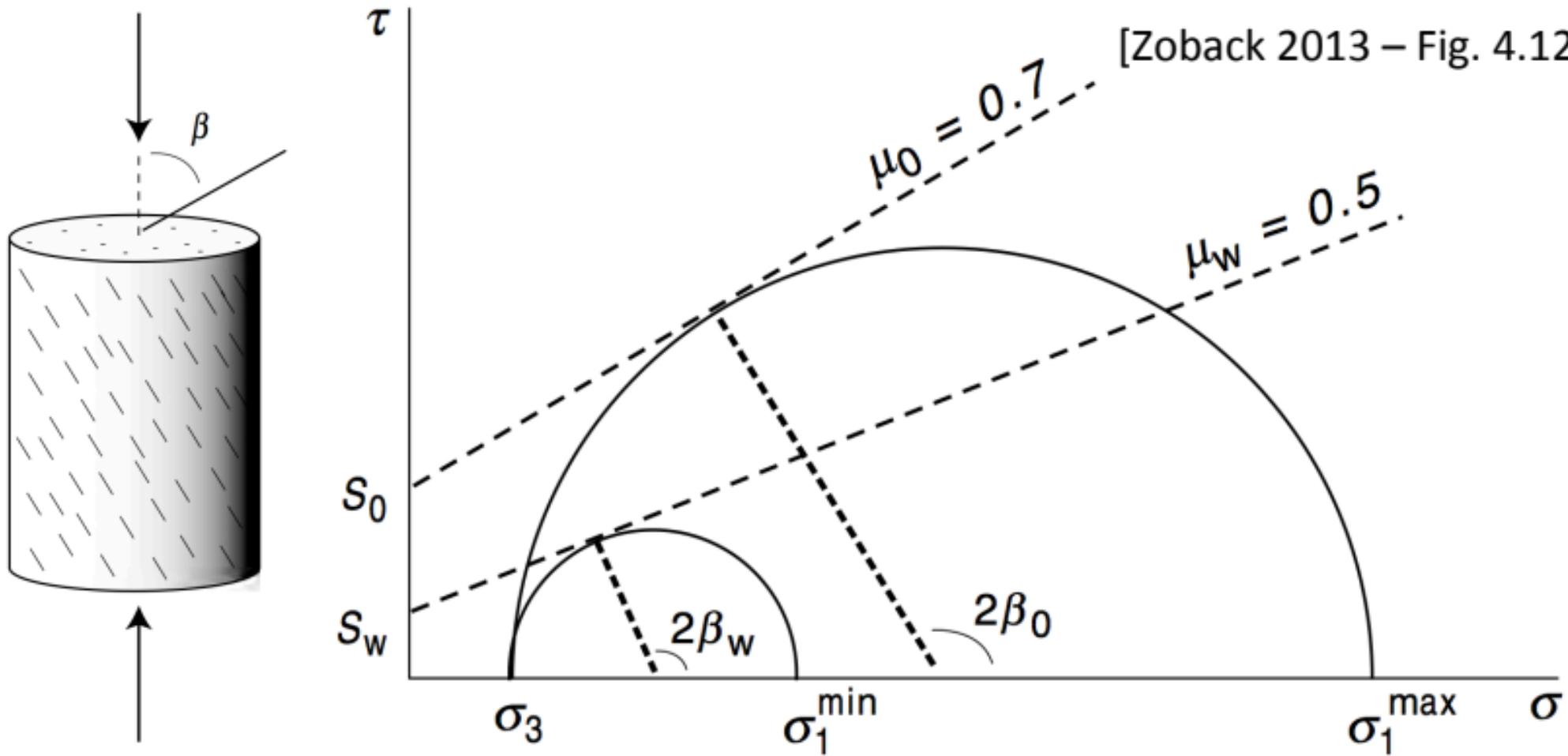
$$\boxed{\mathcal{E}_V^p = \mathcal{E}_{Vcrit}^p}$$

 σ_n





[Zoback 2013 – Fig. 4.12]

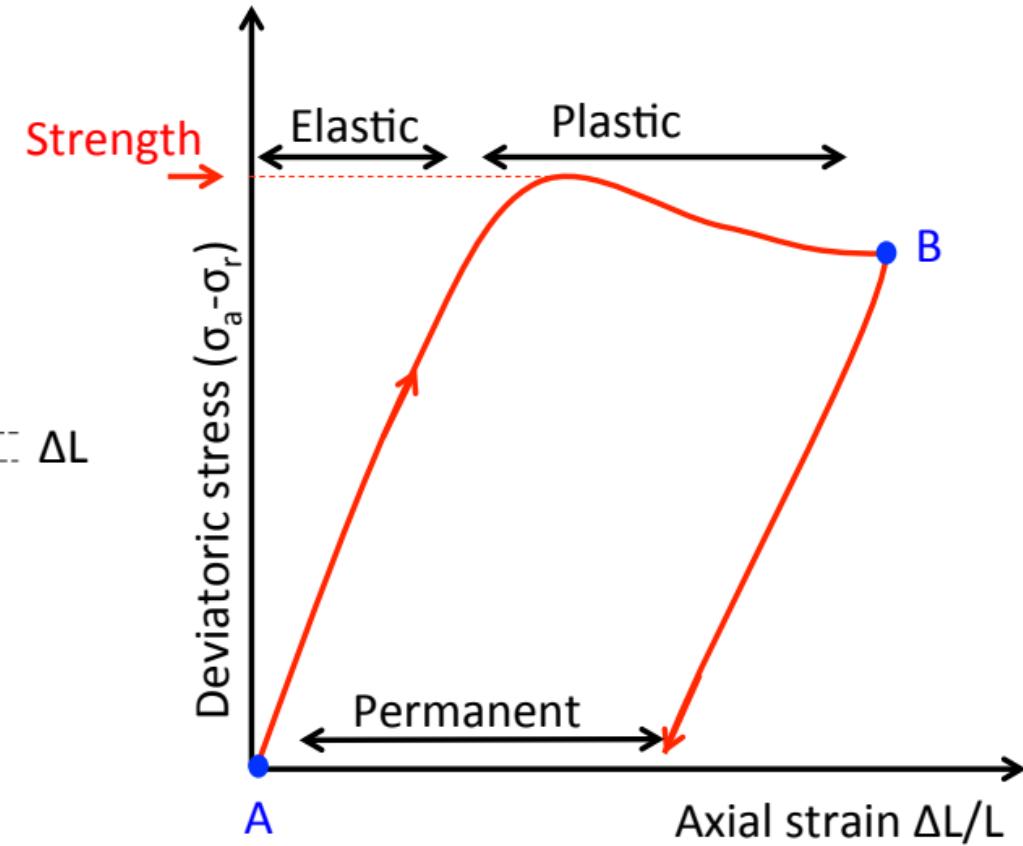
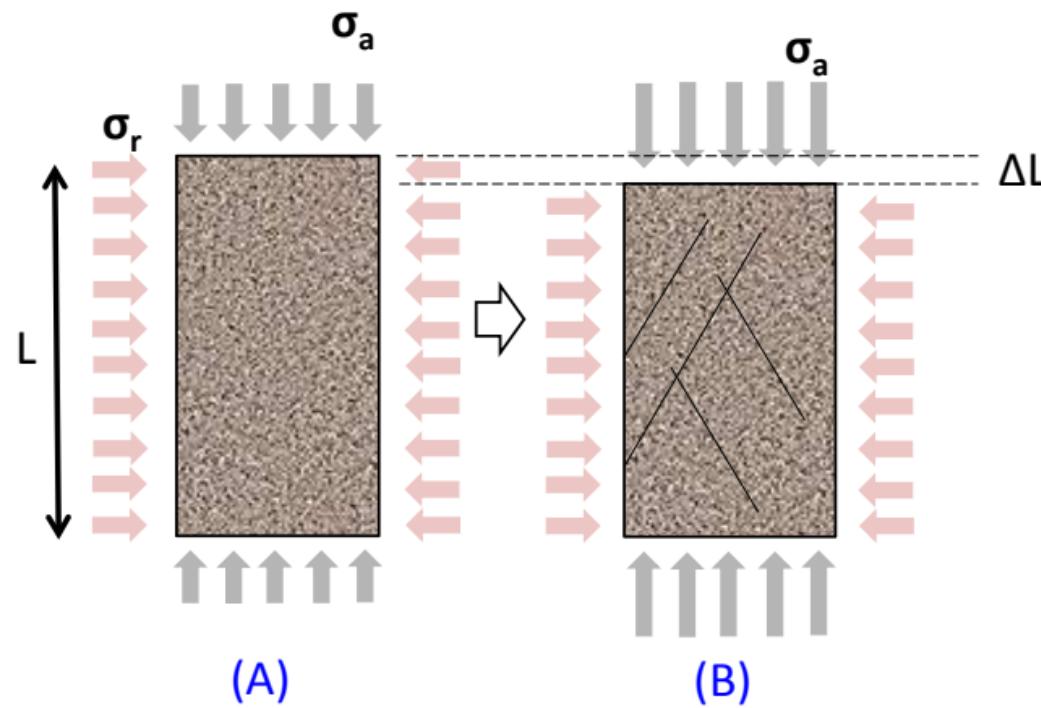


Rock deformation

Relation strain V.S. stress

Elastic (Young modulus)

Plastic (~Viscosity)



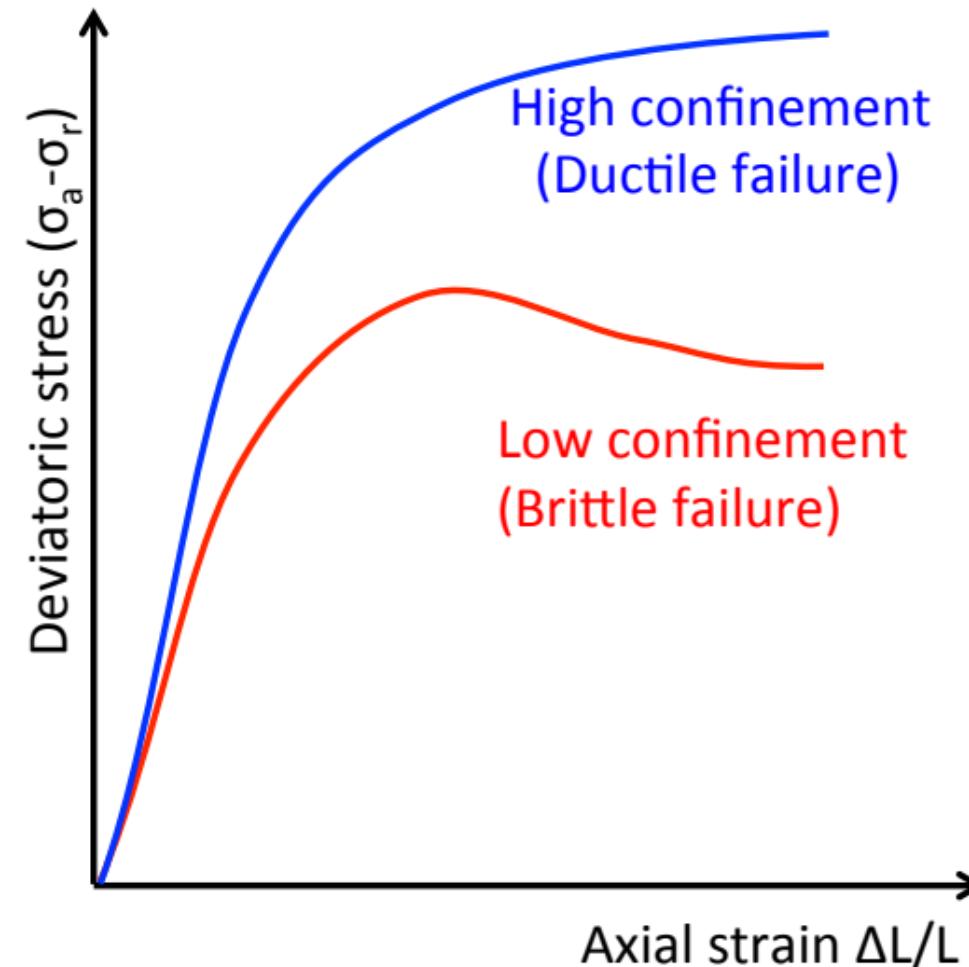
a. This is an undeformed cylinder of rock.



b. This cylinder was subjected to high confining pressure (uniform in all directions) and, at the same time, compression from above. It deformed in a ductile manner, becoming shorter and fatter.



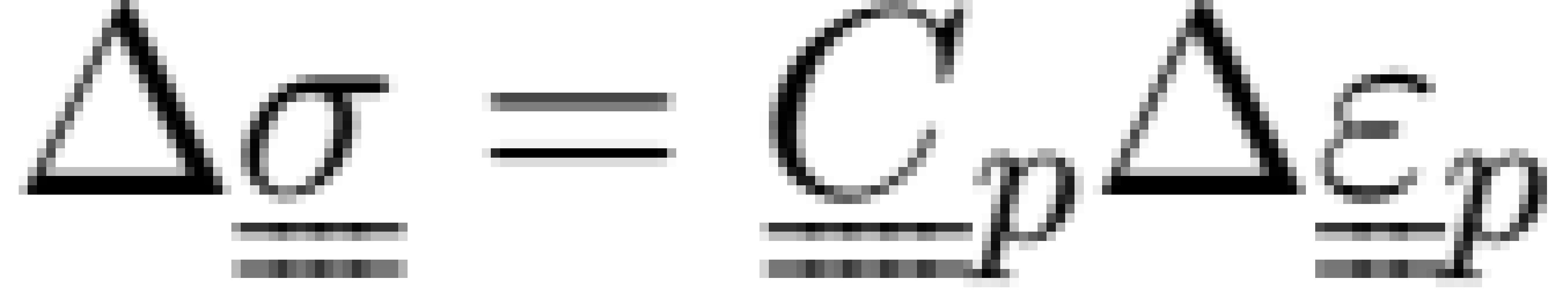
c. An identical cylinder was subjected to the same amount of compression from above, but this time with a lower confining pressure. It deformed in a brittle manner, with many large fractures.



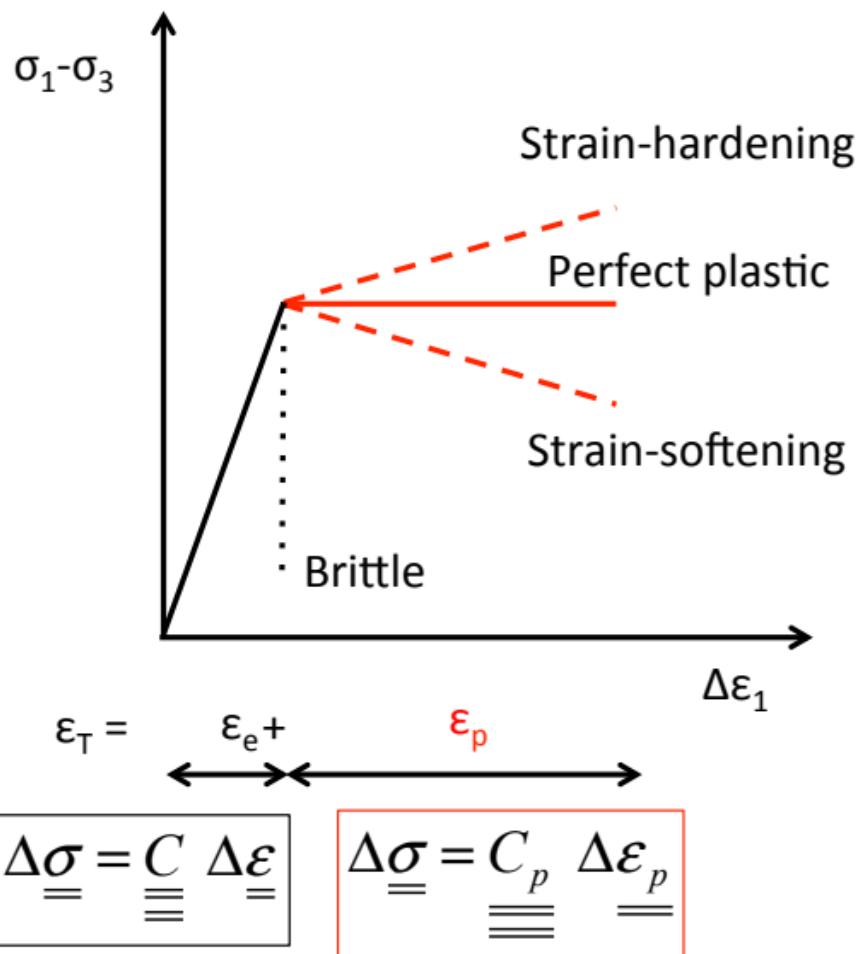
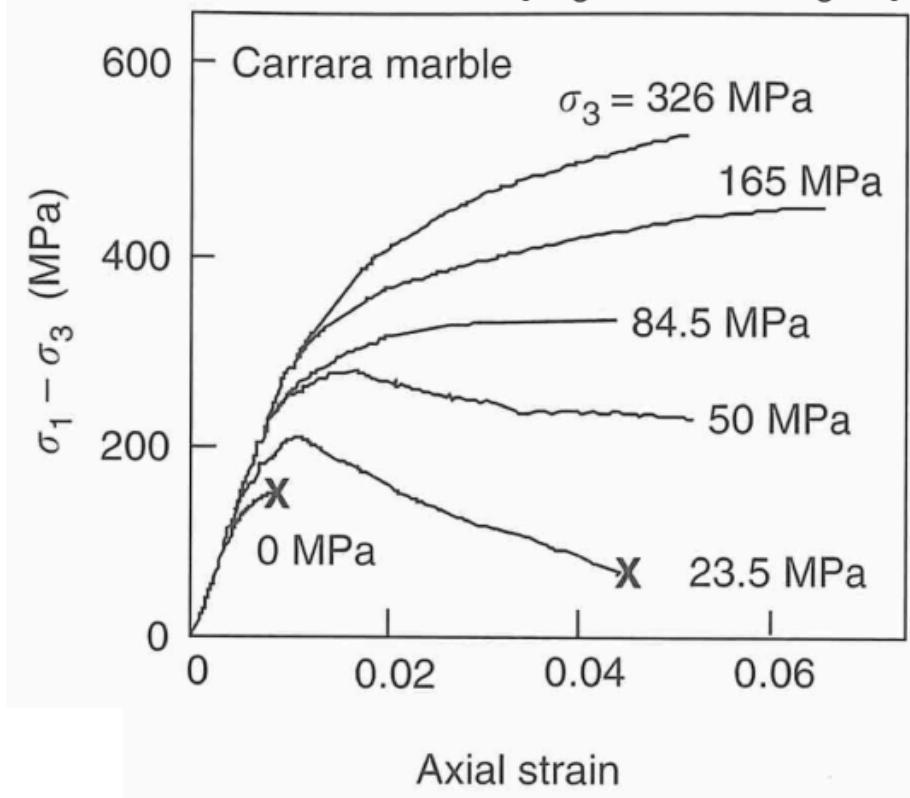




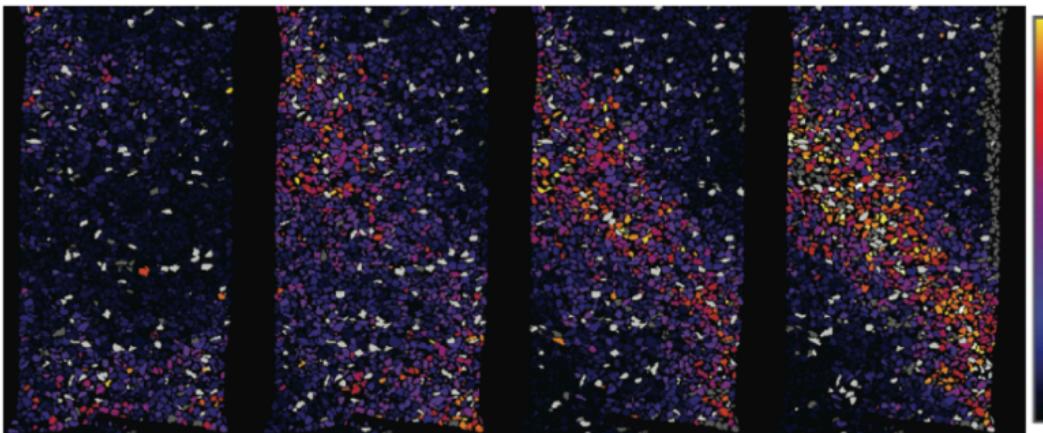
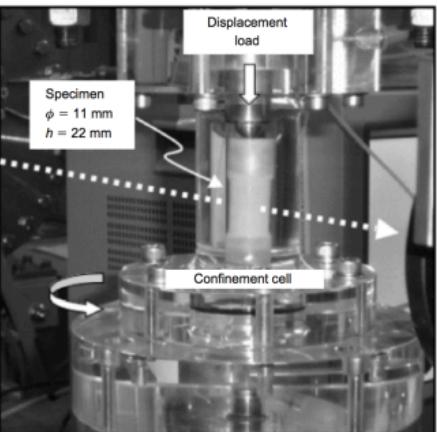




[Jaeger et al. 2007 – Fig. 4.5]



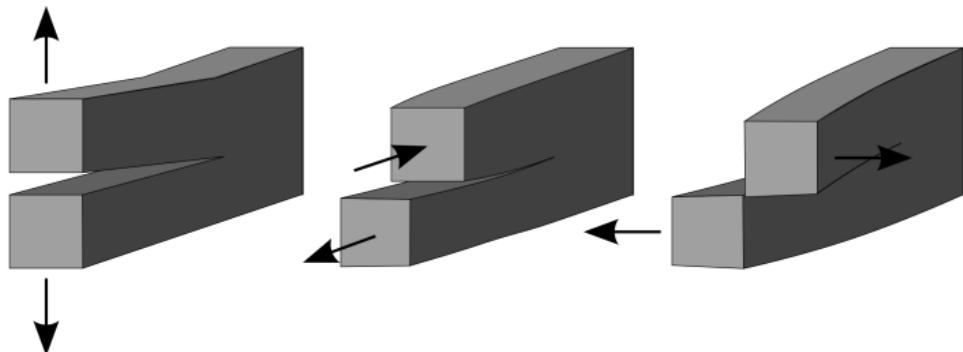
(a) Uncemented or poorly cemented rock →



grain friction, dilation, grain crushing/rotation

(b) Cemented rock →

Propagation of microfractures, grain friction/crushing

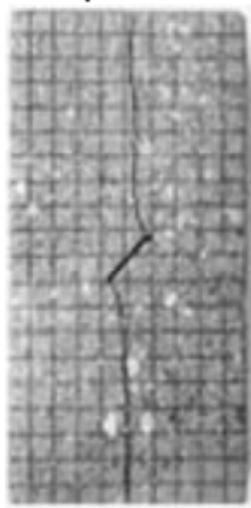


Stress intensification at
the tip of fractures

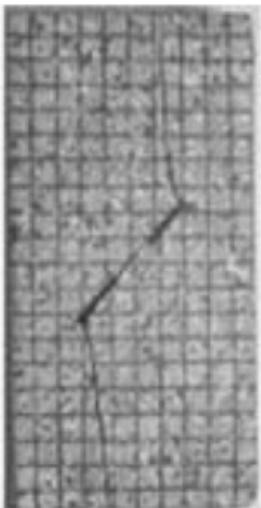
Propagation starts at
fracture tips

Napolitan Tuffo

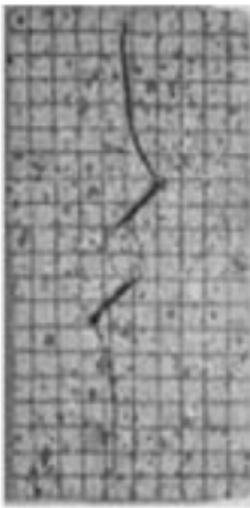
[Hall et al. 2006 – Pure Appl. Geophys.]



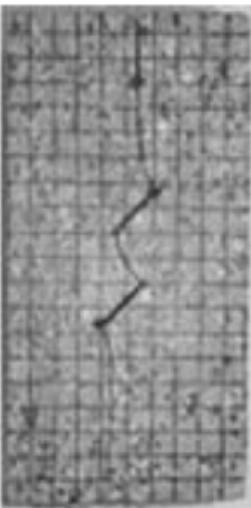
Single flaw



$\beta = 45^\circ$

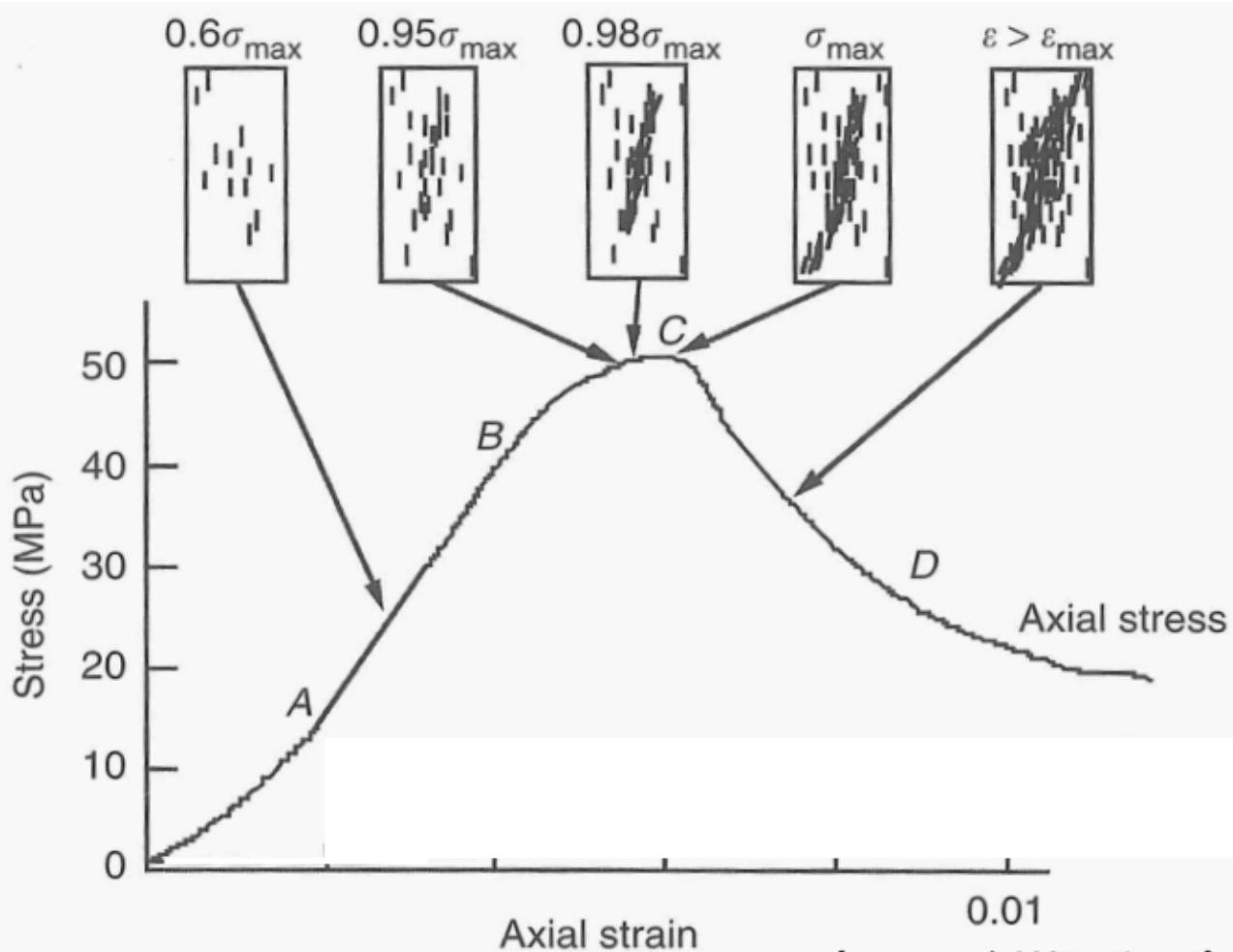


$\beta = 105^\circ$

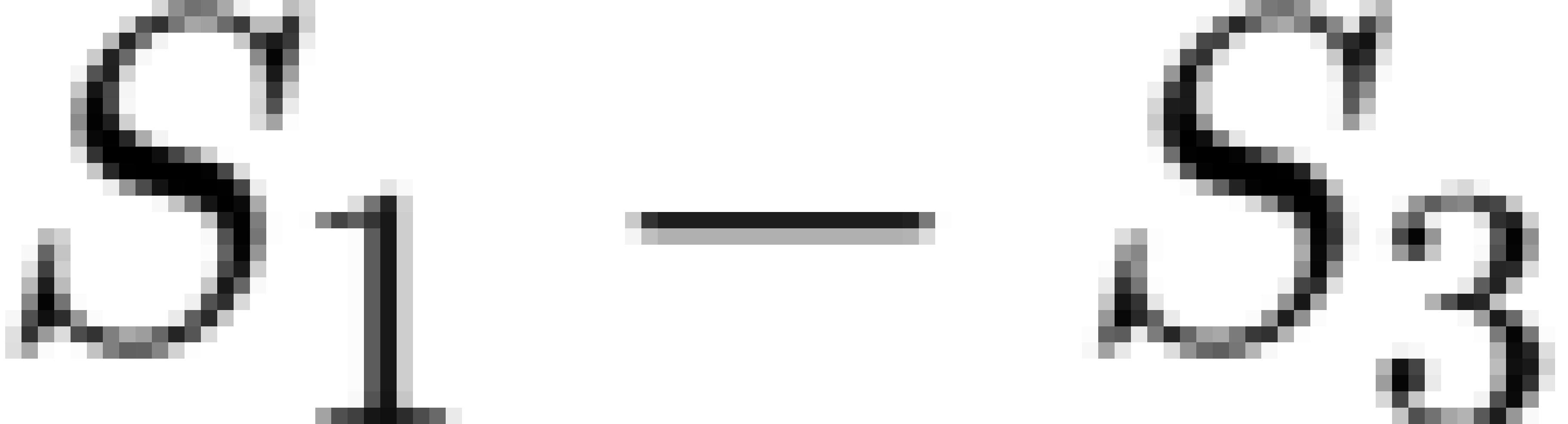


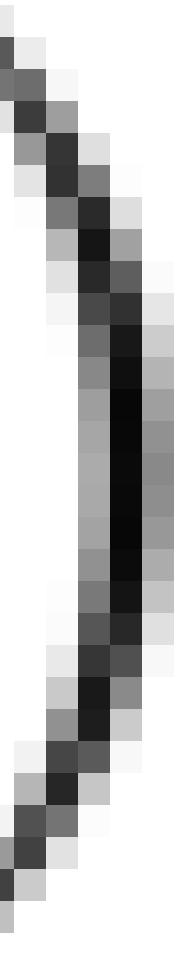
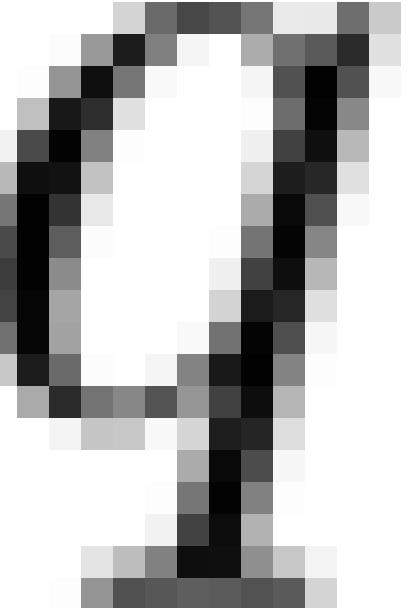
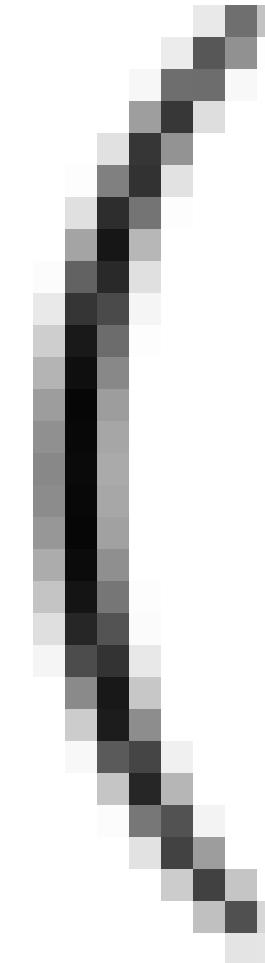
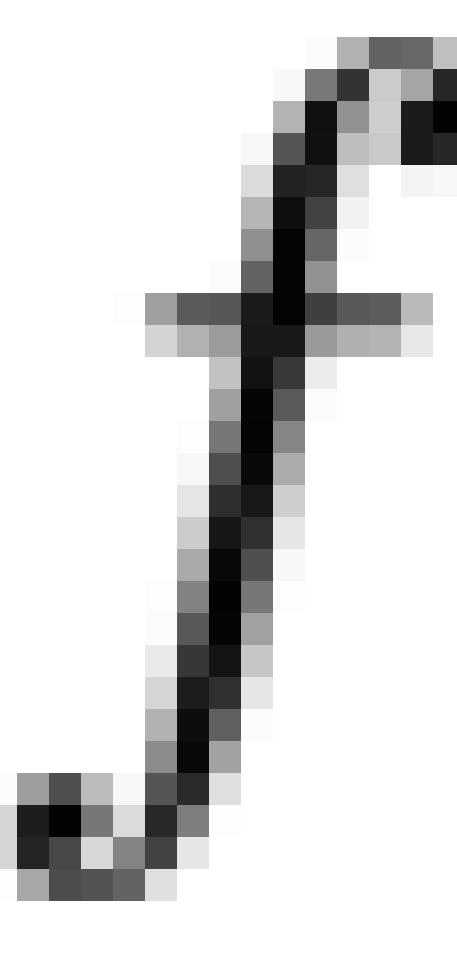
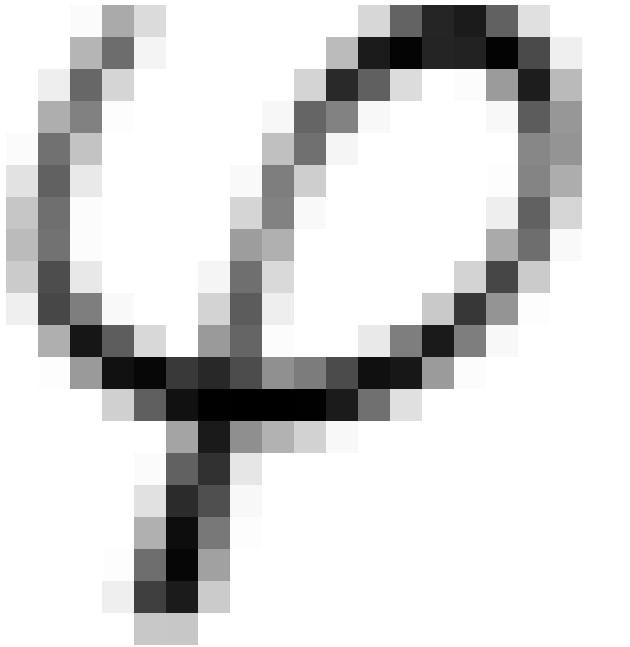
$\beta = 120^\circ$





[Jaeger et al. 2007 – Fig. 4.5]

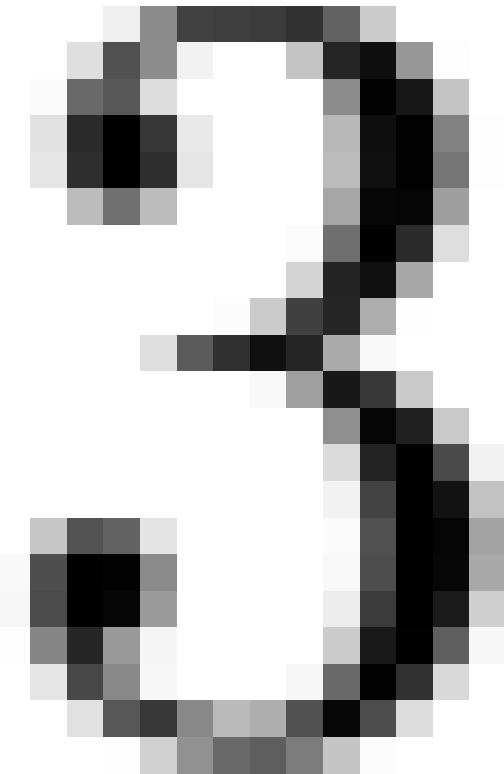
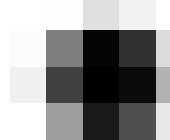
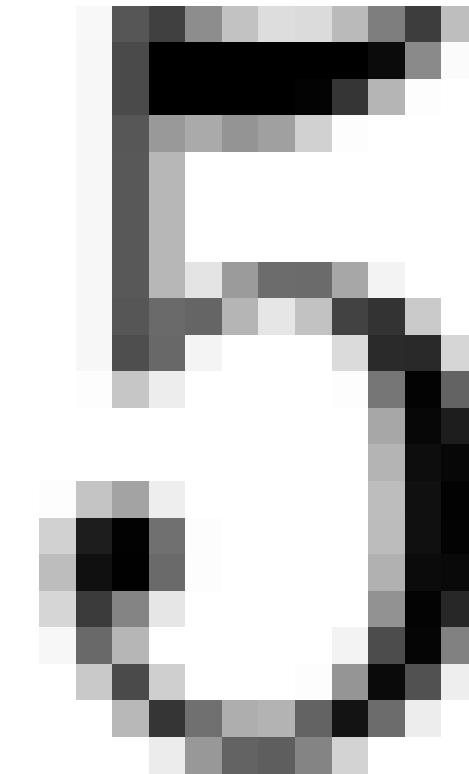
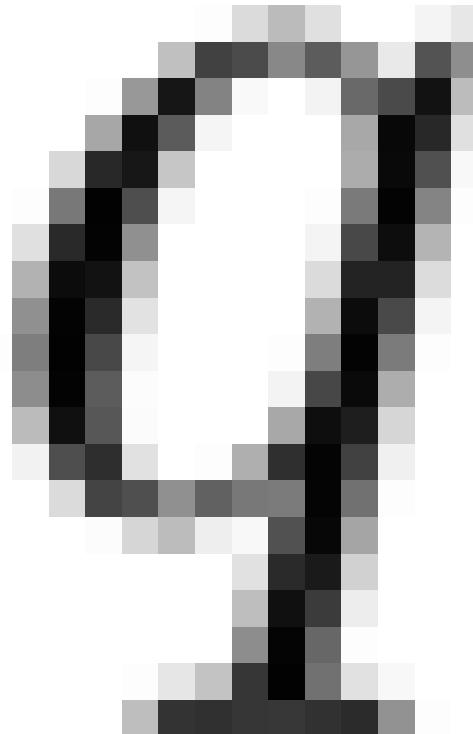












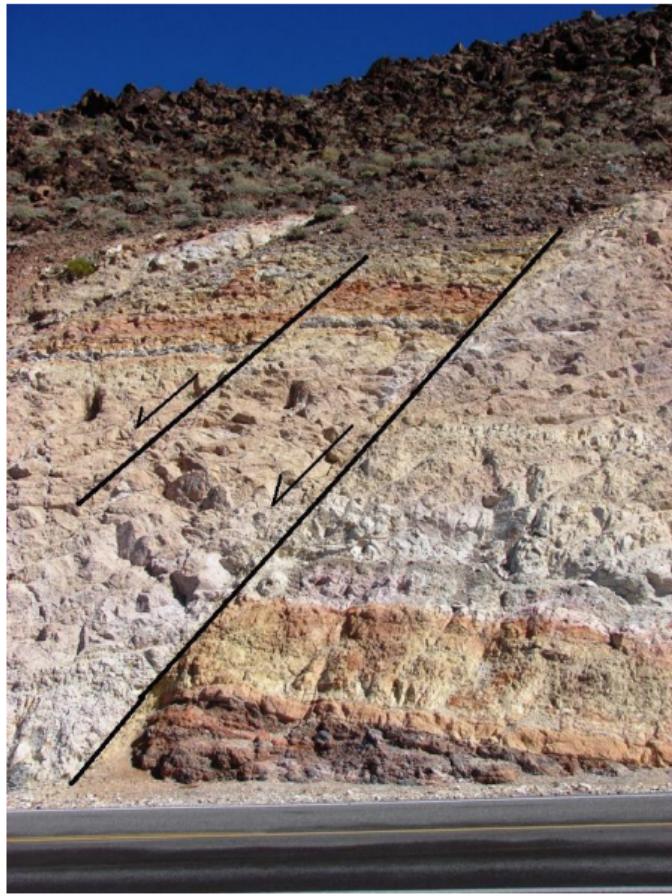






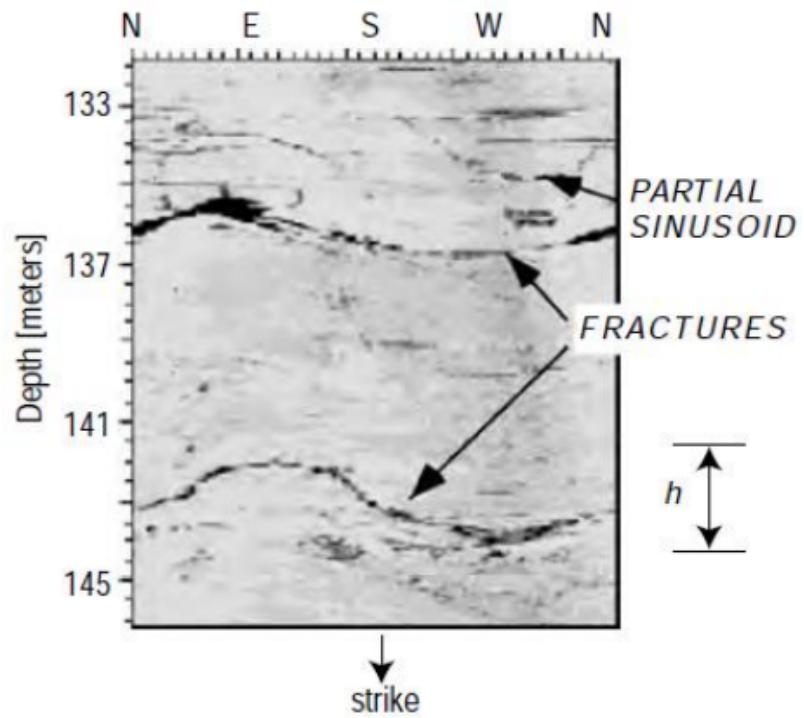
Outcrop of a normal fault in Split Mountain gorge

<http://geology.csupomona.edu/janourse/TectonicsFieldTrips.htm>

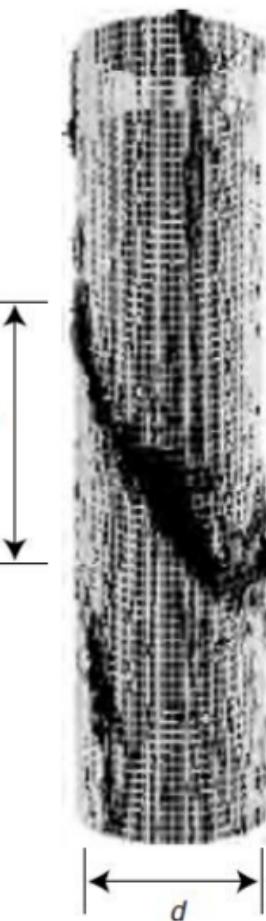


<http://geotripperimages.com/images/DSC03438%20Charlie%20Brown%20b.jpg>

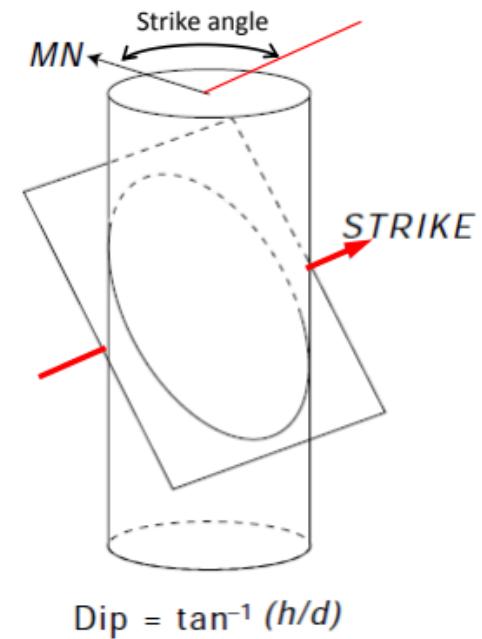
**(a) Un-wrapped image
(ultrasonic)**



(b) 3D-representation



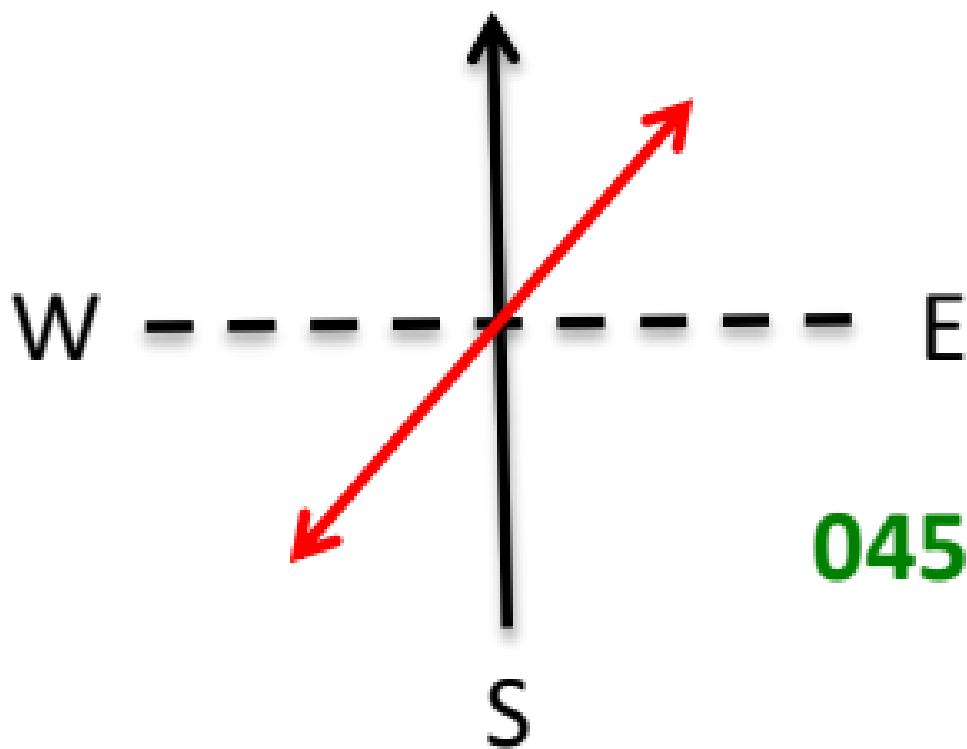
(c) Interpretation



[Zoback 2013 - Figure 5.3]



N45°E==S45°W



045° == 225°

Geologic map

Figure from Prof. Prodanovic

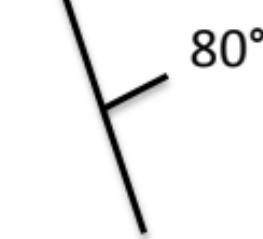


a)

$N30^\circ E$, $45^\circ NW$

or

030° , $45^\circ NW$



b)

$N10^\circ W$, $80^\circ NE$

or

350° , $80^\circ NE$



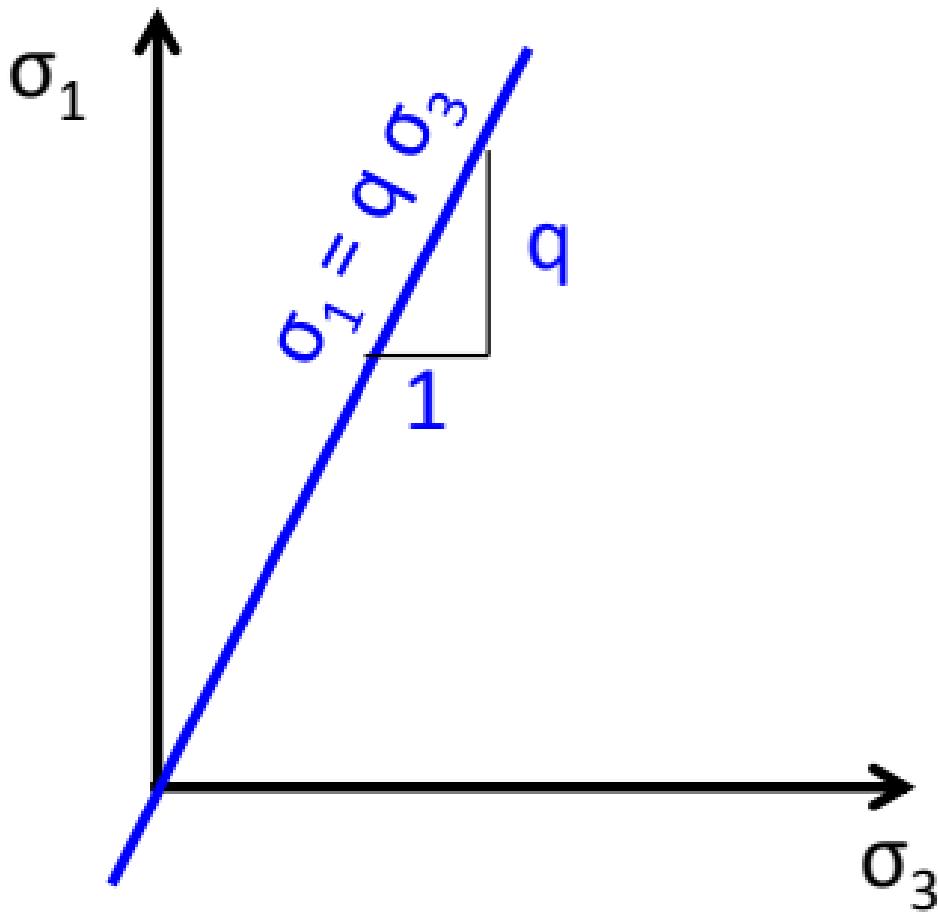
c)

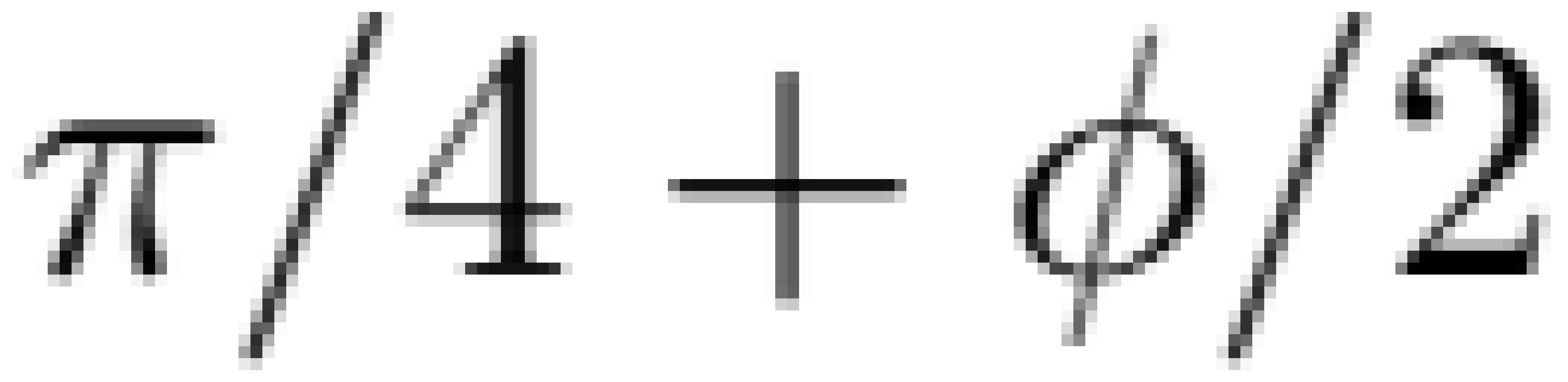
Symbols for
horizontal plane
and vertical plane

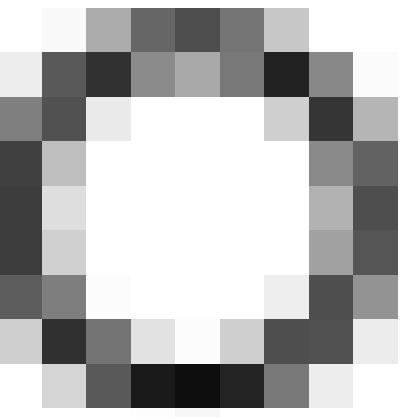
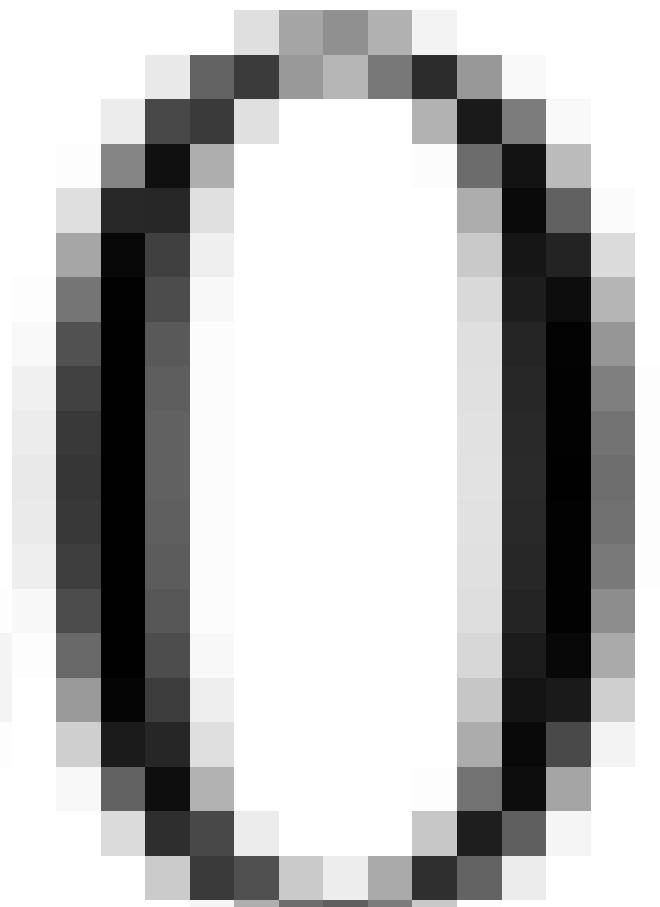
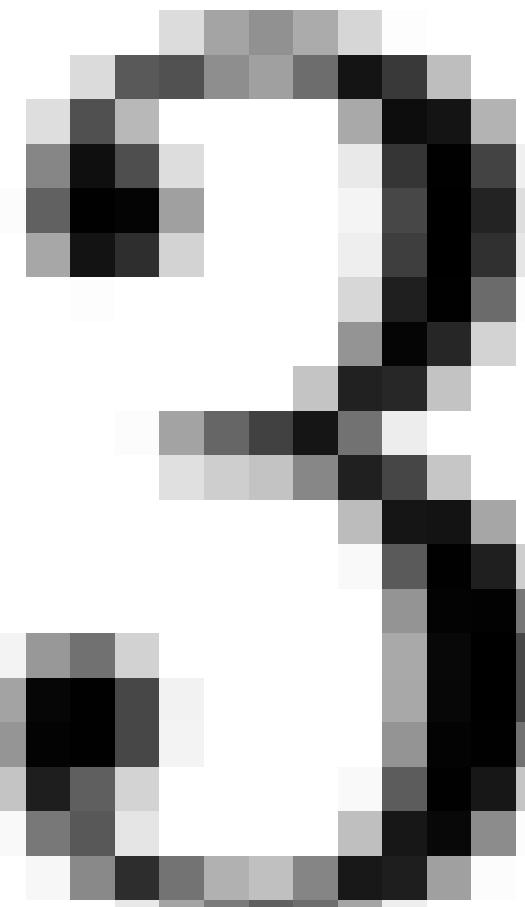
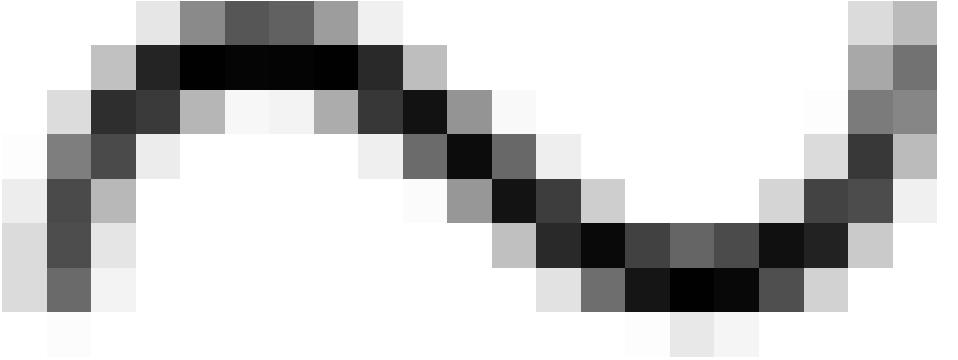


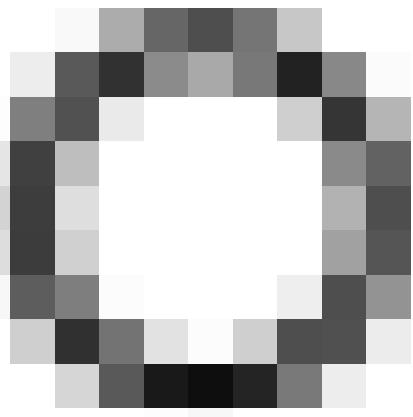
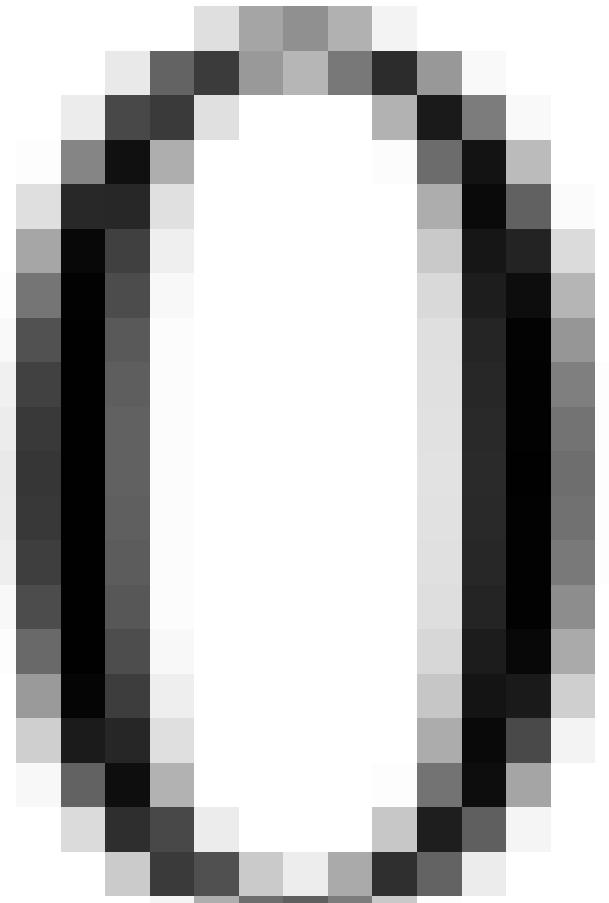
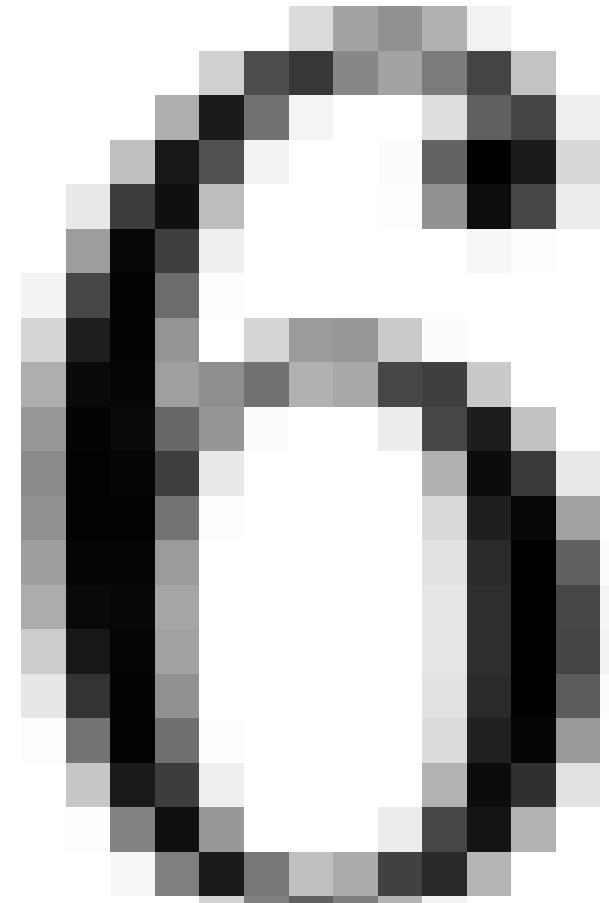
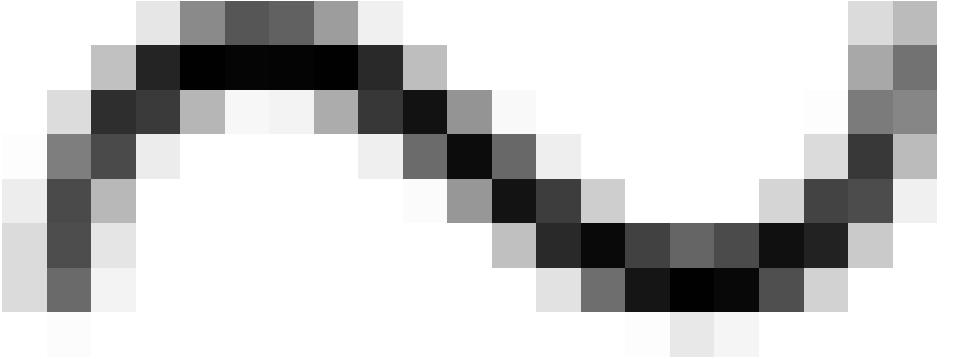
$$\begin{aligned} q &= \sin^2 \varphi + 1 \\ &= 1 - \sin^2 \varphi \end{aligned}$$

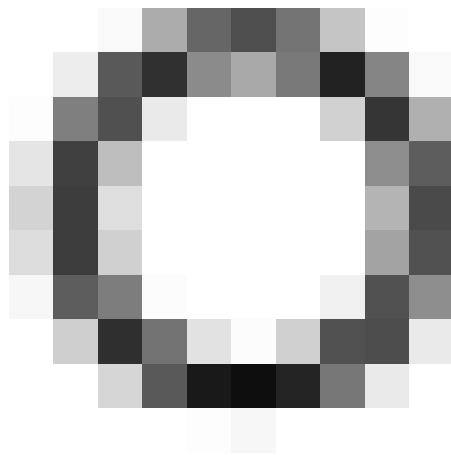
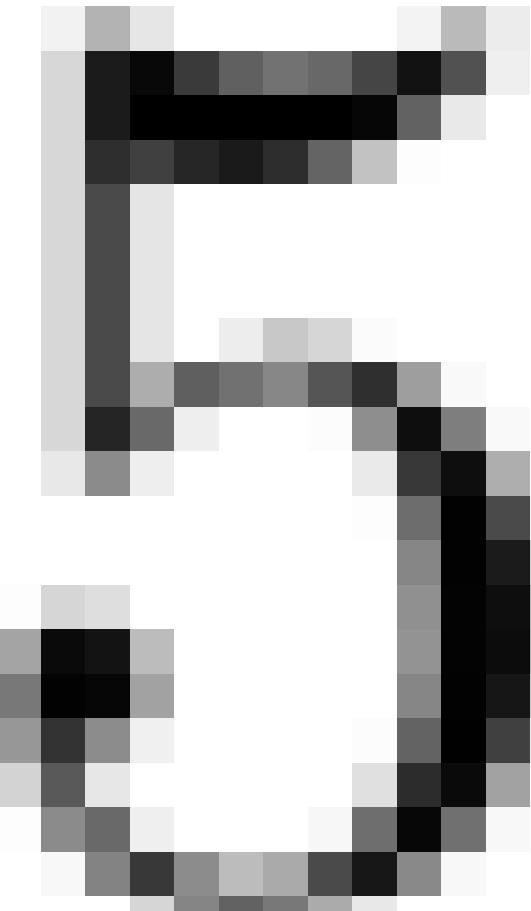
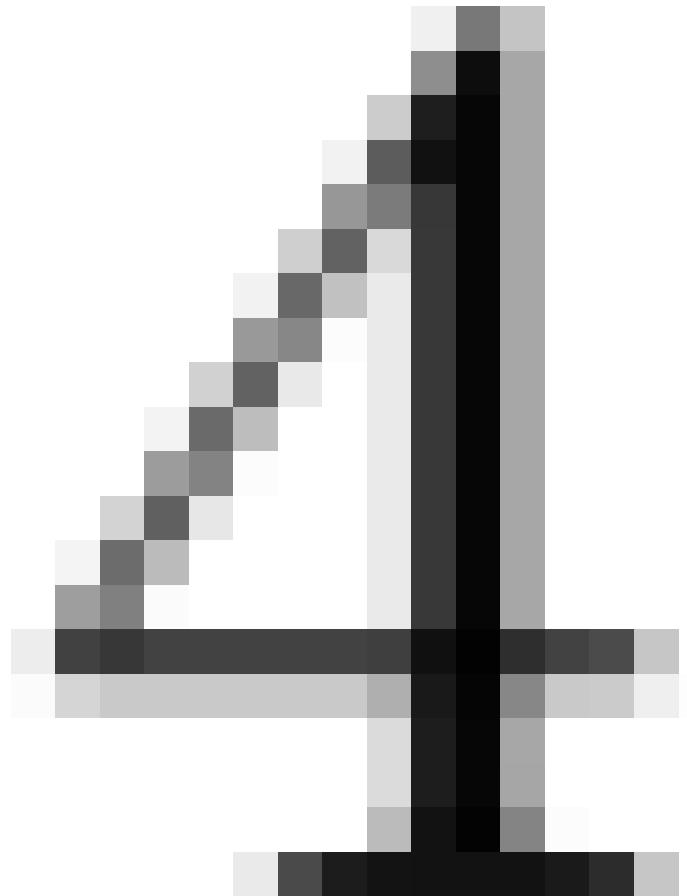


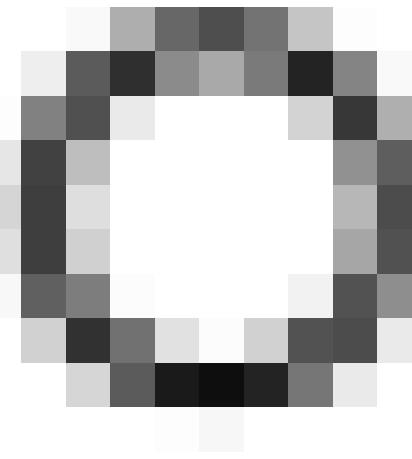
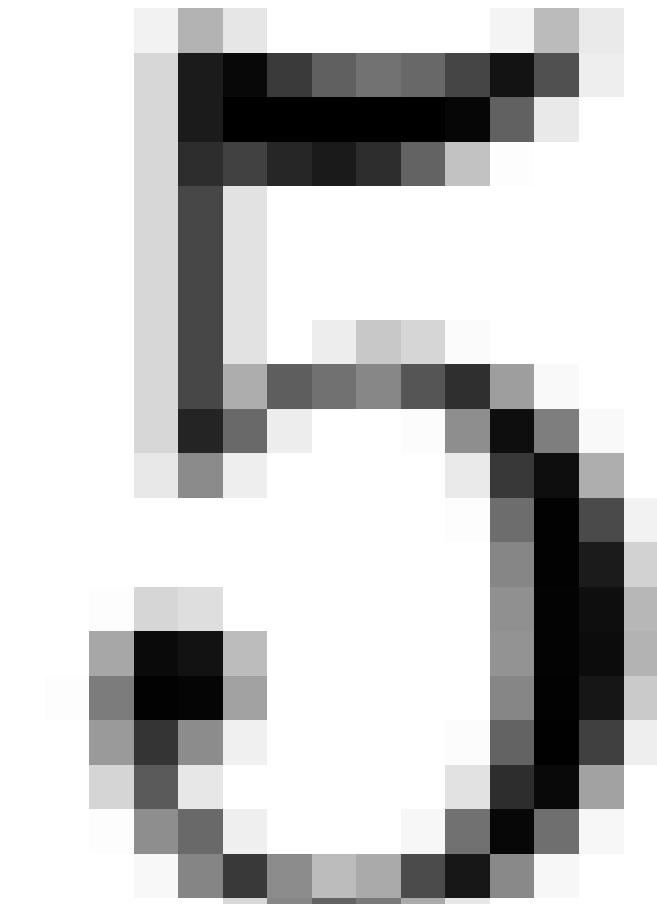
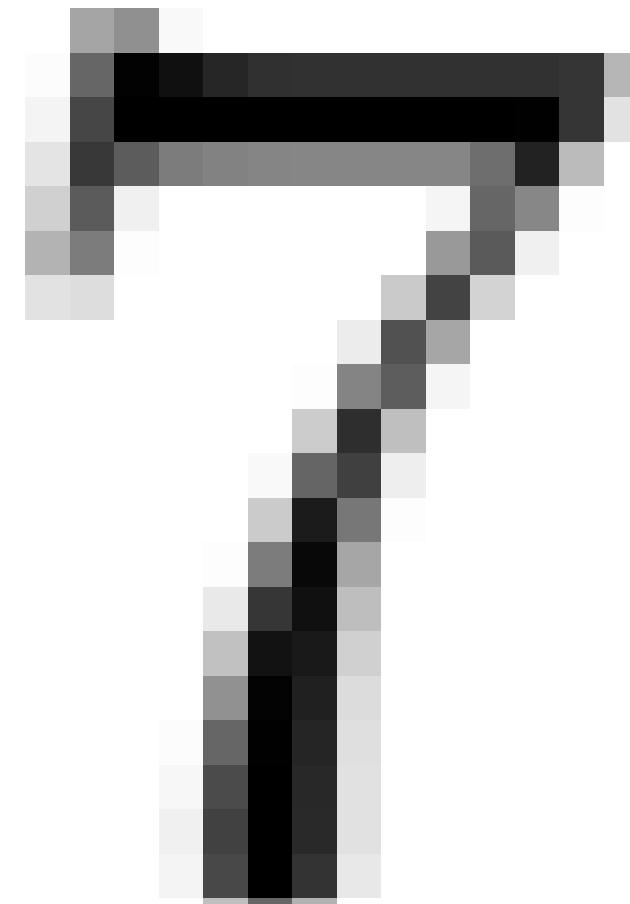
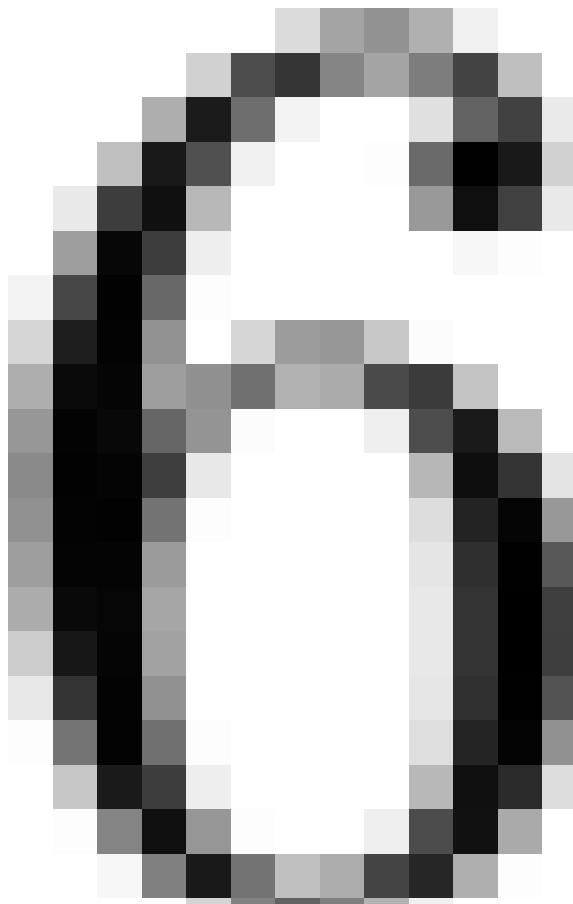


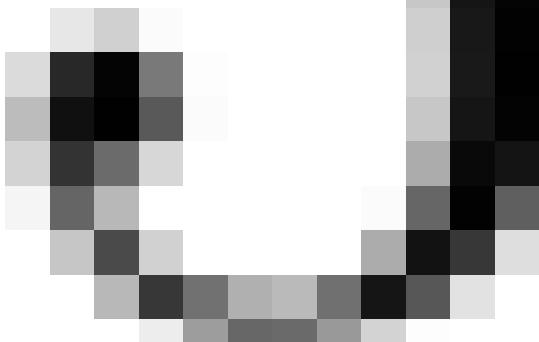
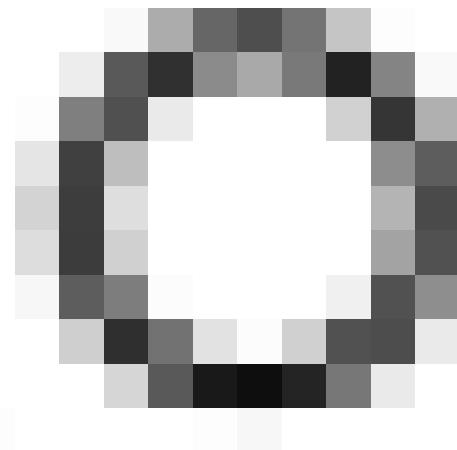
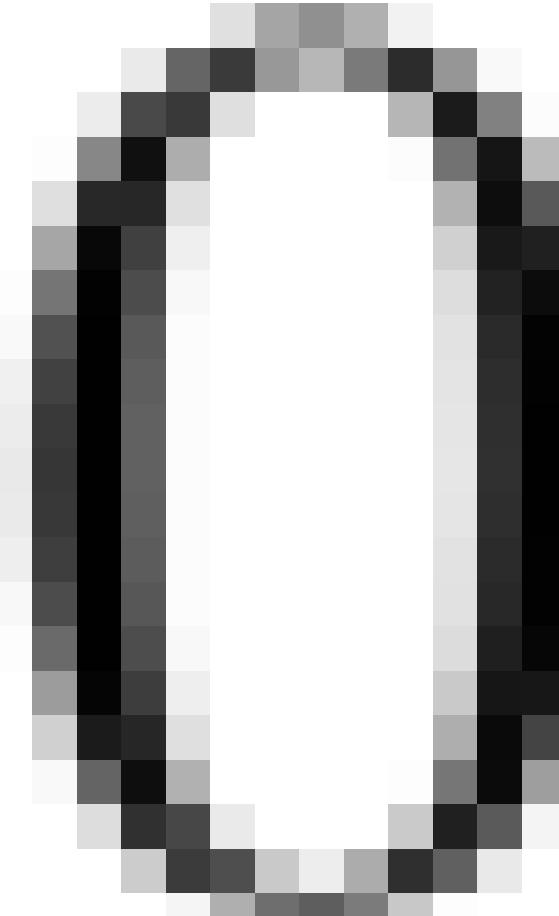
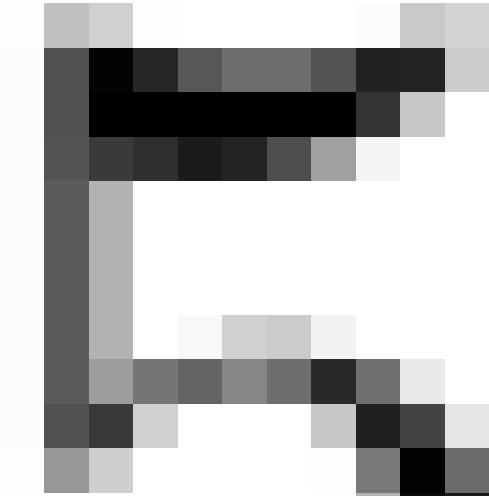


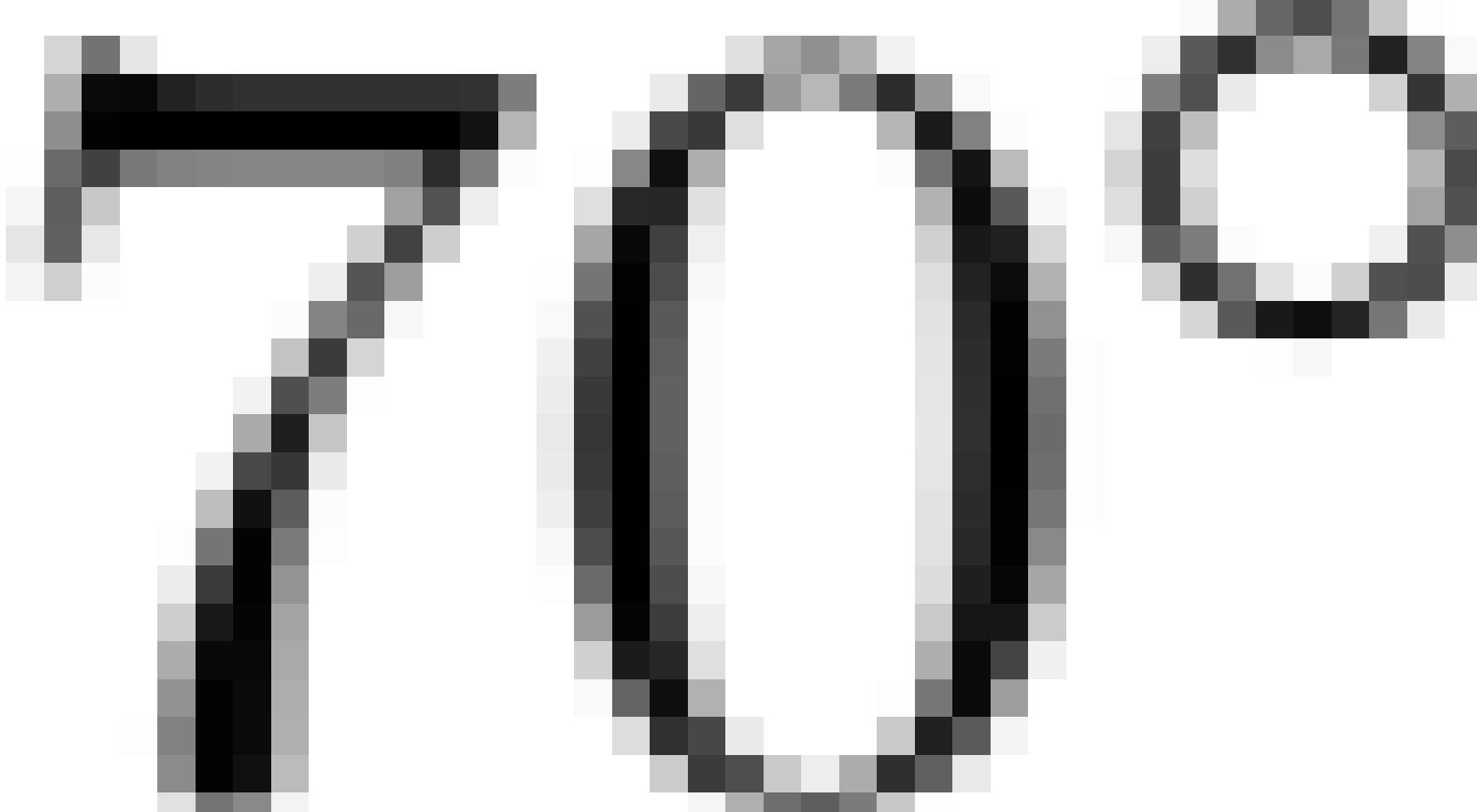


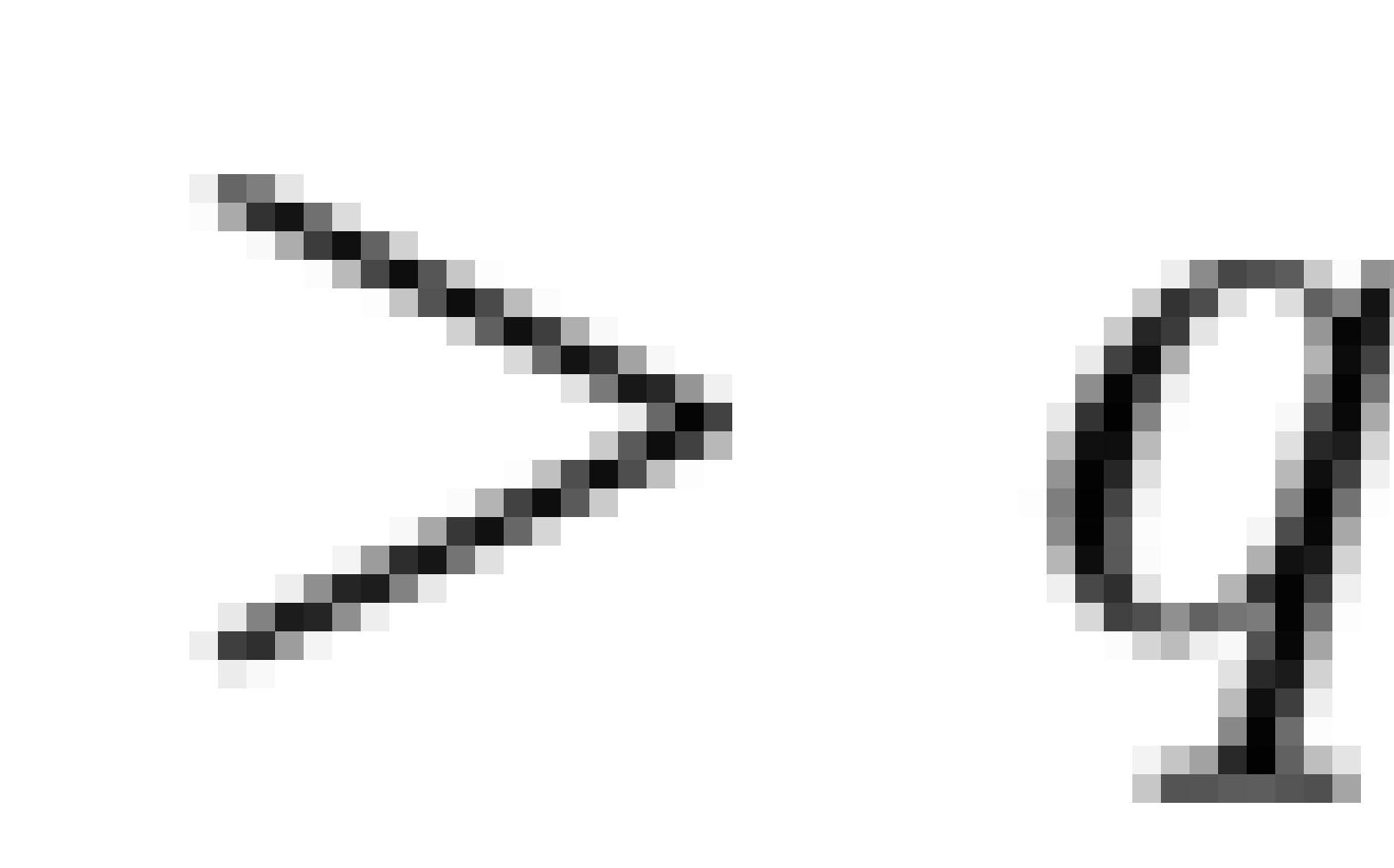














$$S_1 = s_v$$

$$S_2 = s_{H\max}$$

$$S_3 = s_{h\min}$$



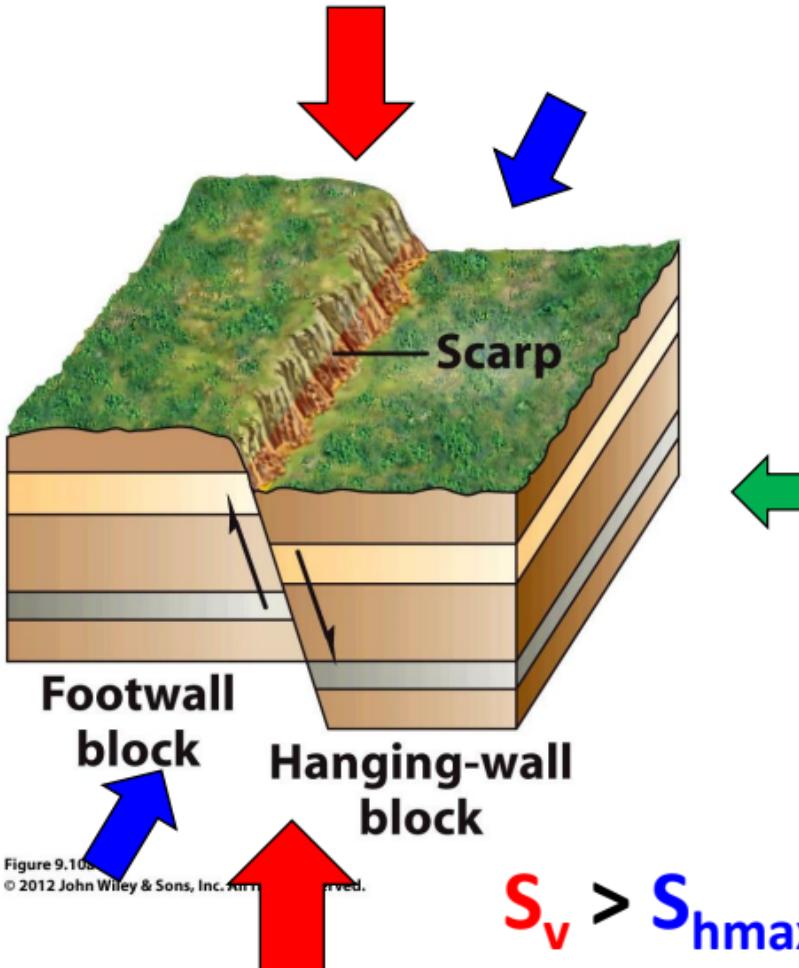
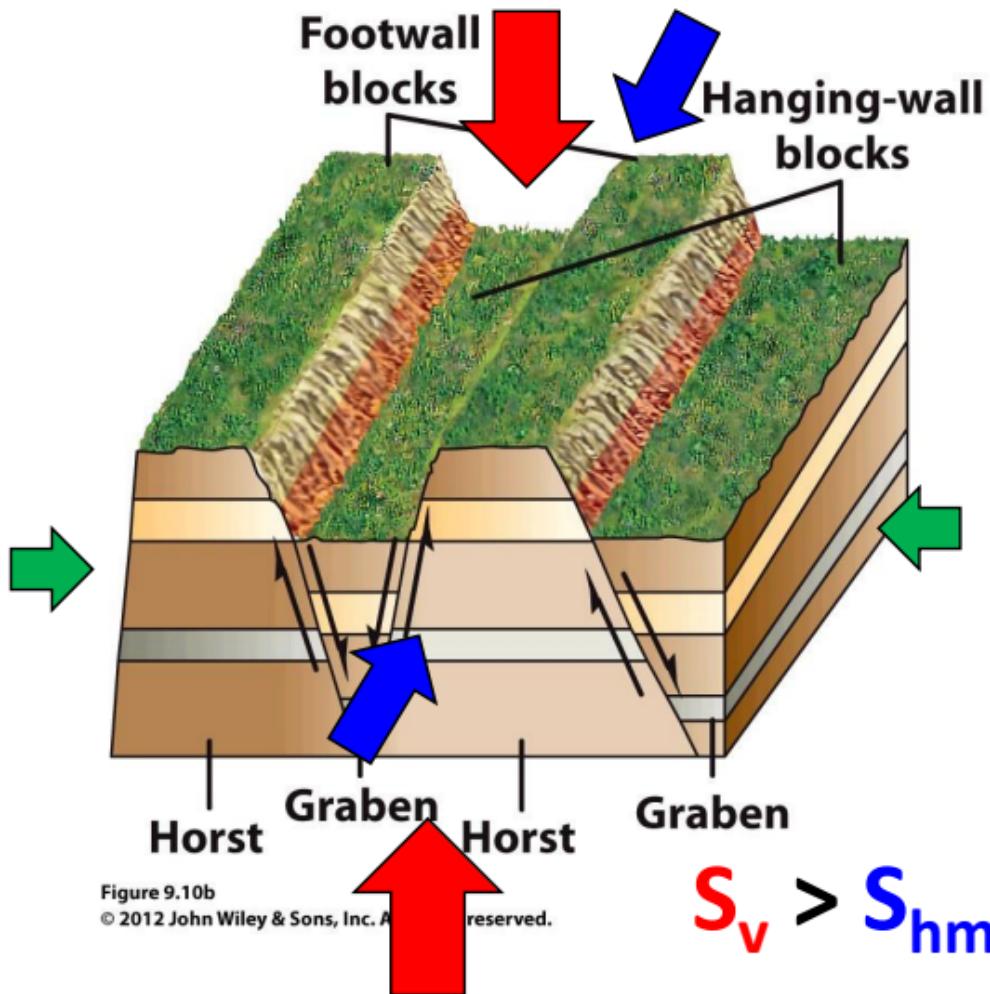


Figure 9.10b
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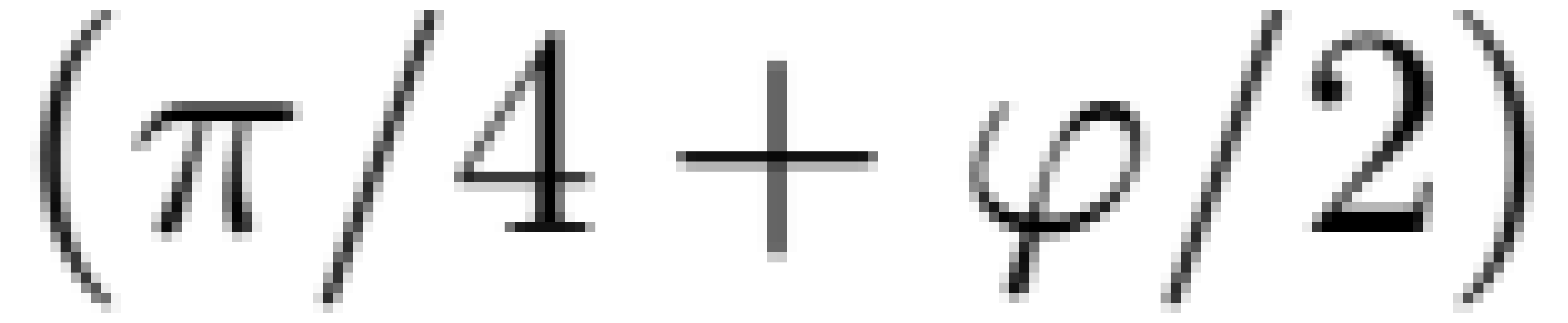


$$S_v > S_{h\max} > S_{h\min}$$

$$S_1 = S_{H\max}$$

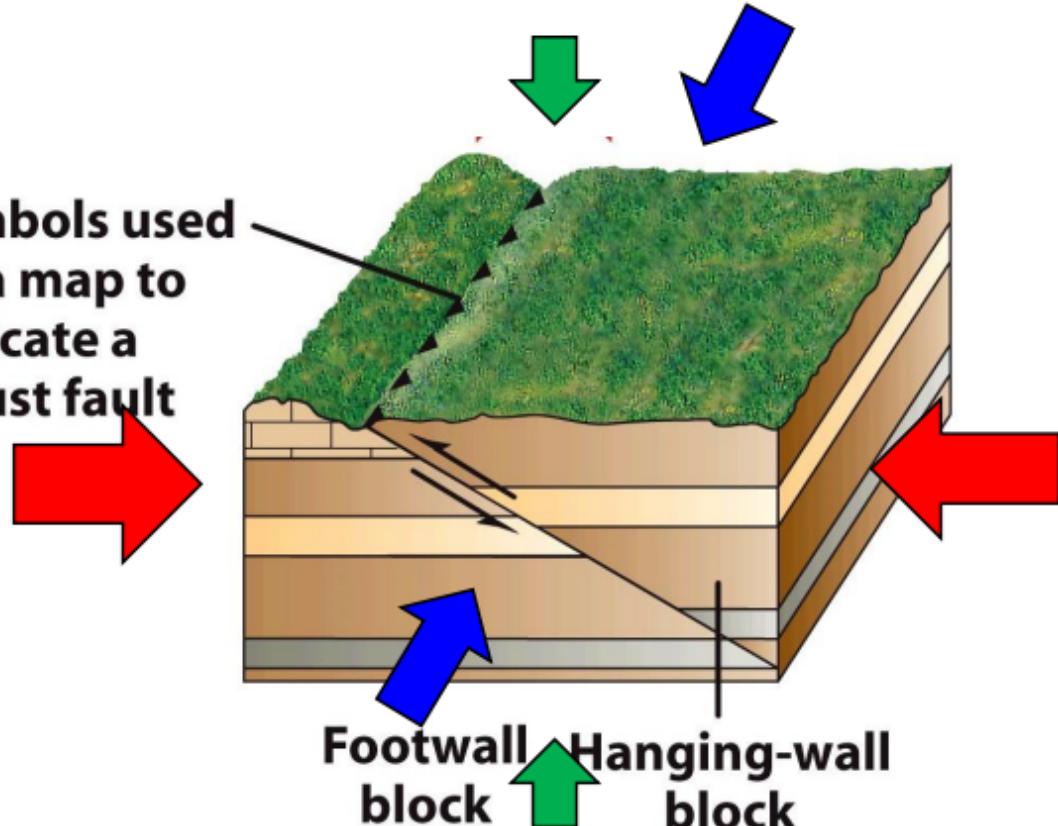
$$S_2 = S_{h\min}$$

$$S_3 = S_v$$





**Symbols used
on a map to
indicate a
thrust fault**



**Footwall
block** **Hanging-wall
block**

Figure 9.10d
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$$S_{H\max} > S_{h\min} > S_v$$

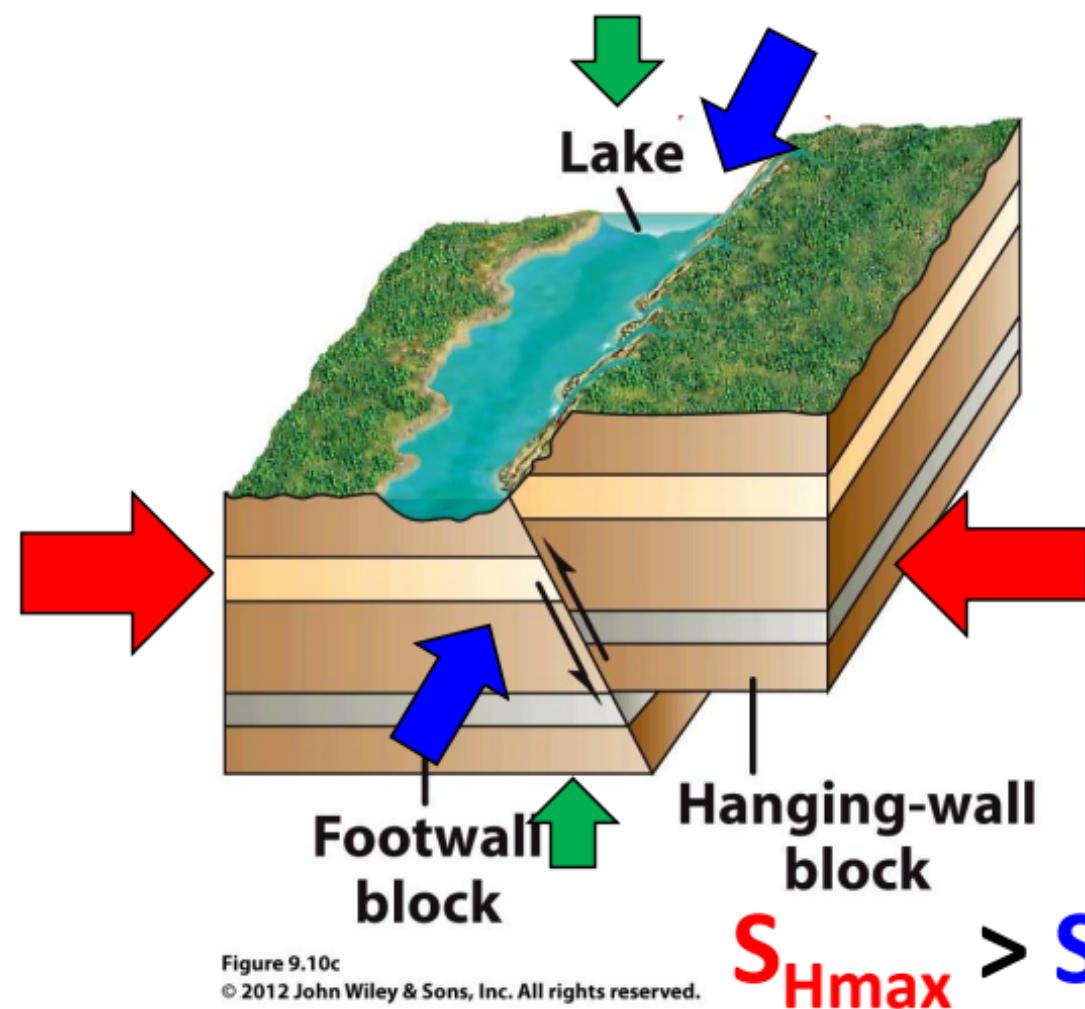


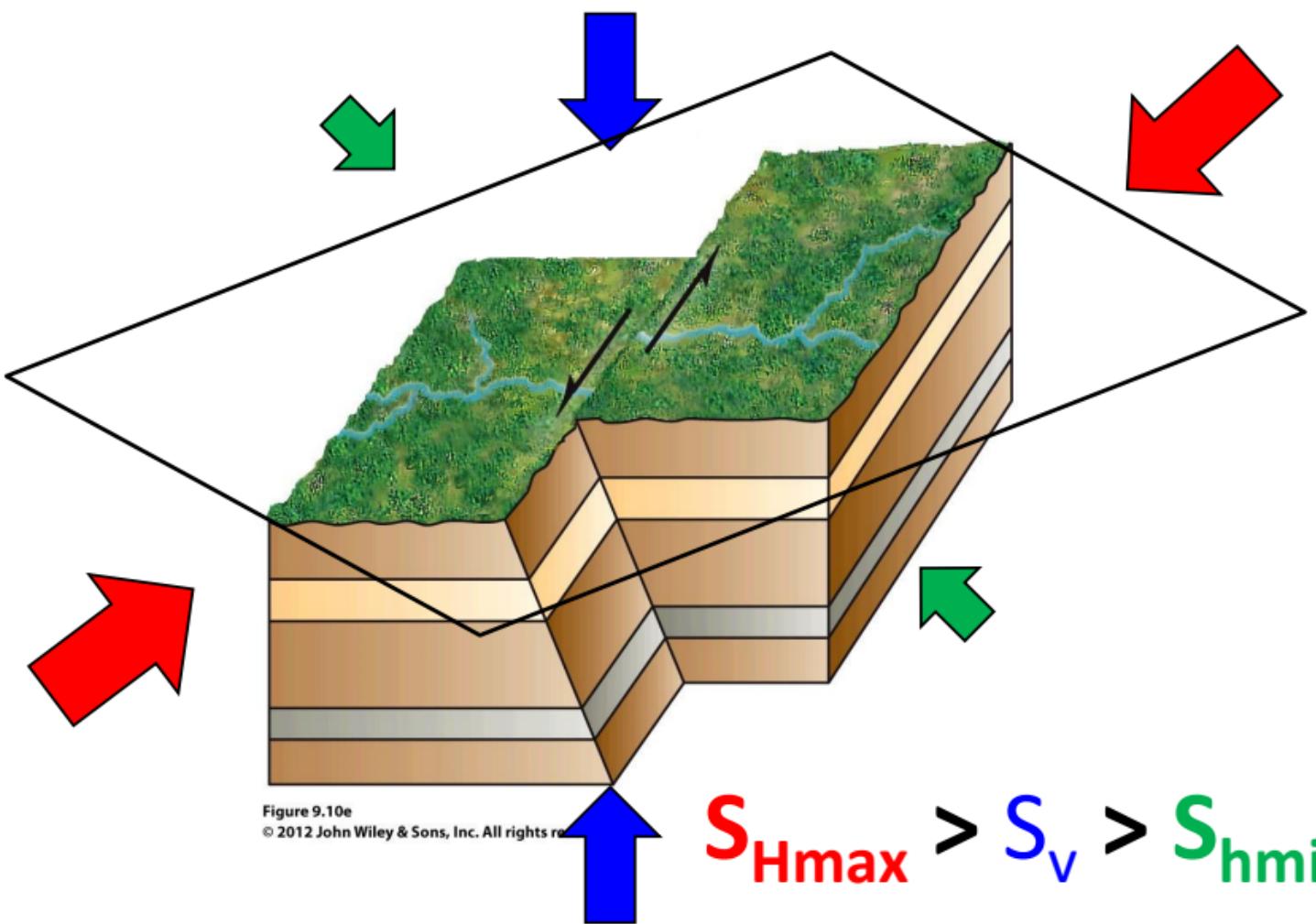
Figure 9.10c

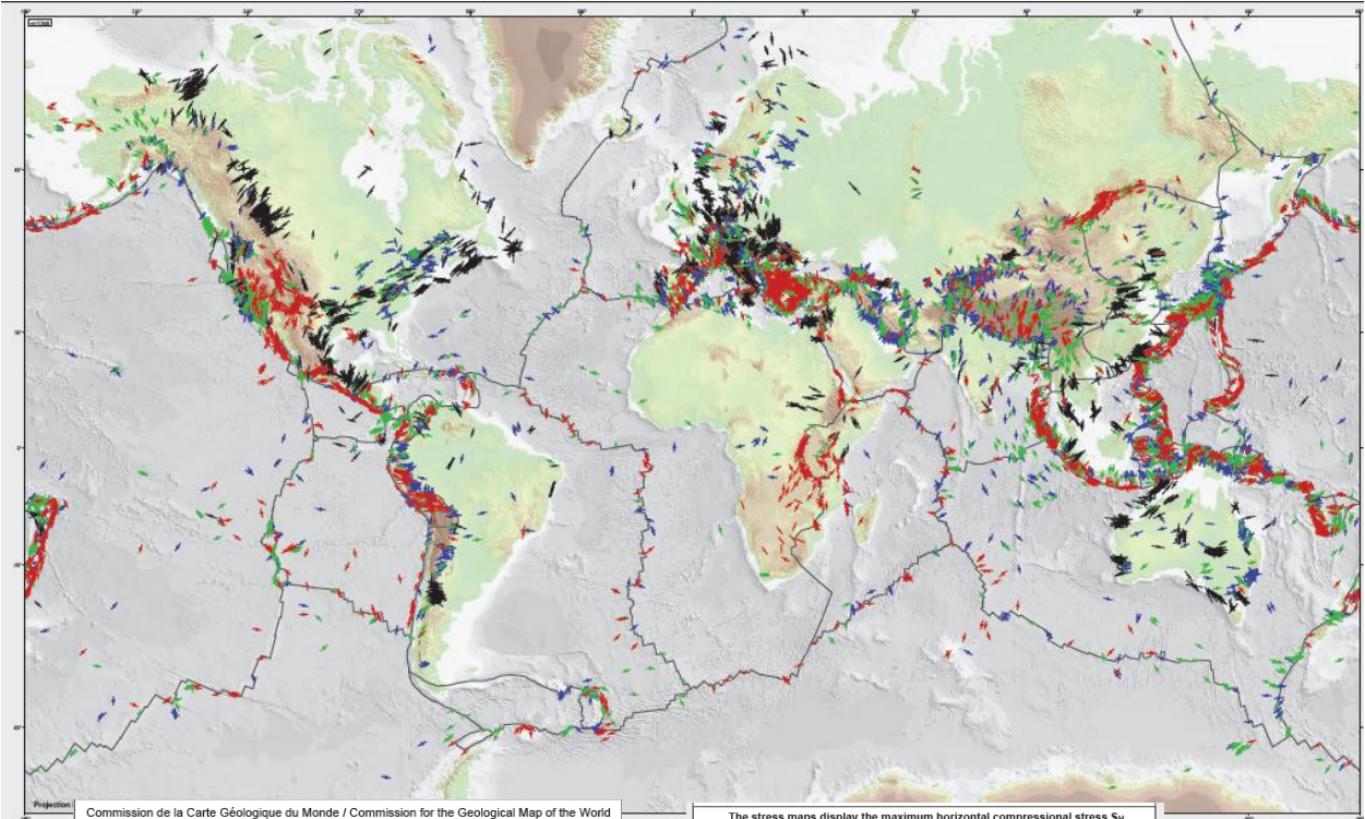
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$$S_1 = S_{H\max}$$

$$S_2 = S_v$$

$$S_3 = S_{hmin}$$





Commission de la Carte Géologique du Monde / Commission for the Geological Map of the World



WORLD STRESS MAP



2009 - 2nd edition, based on the WSM database release 2008

Helmholtz Centre Potsdam - GFZ German Research Centre for Geosciences

Authors

Oliver Heidbach, Mark Tingay, Andreas Barth, John Reinecker, Daniel Kurfürst, and Birgit Müller



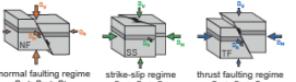
The stress maps display the maximum horizontal compressional stress S_H

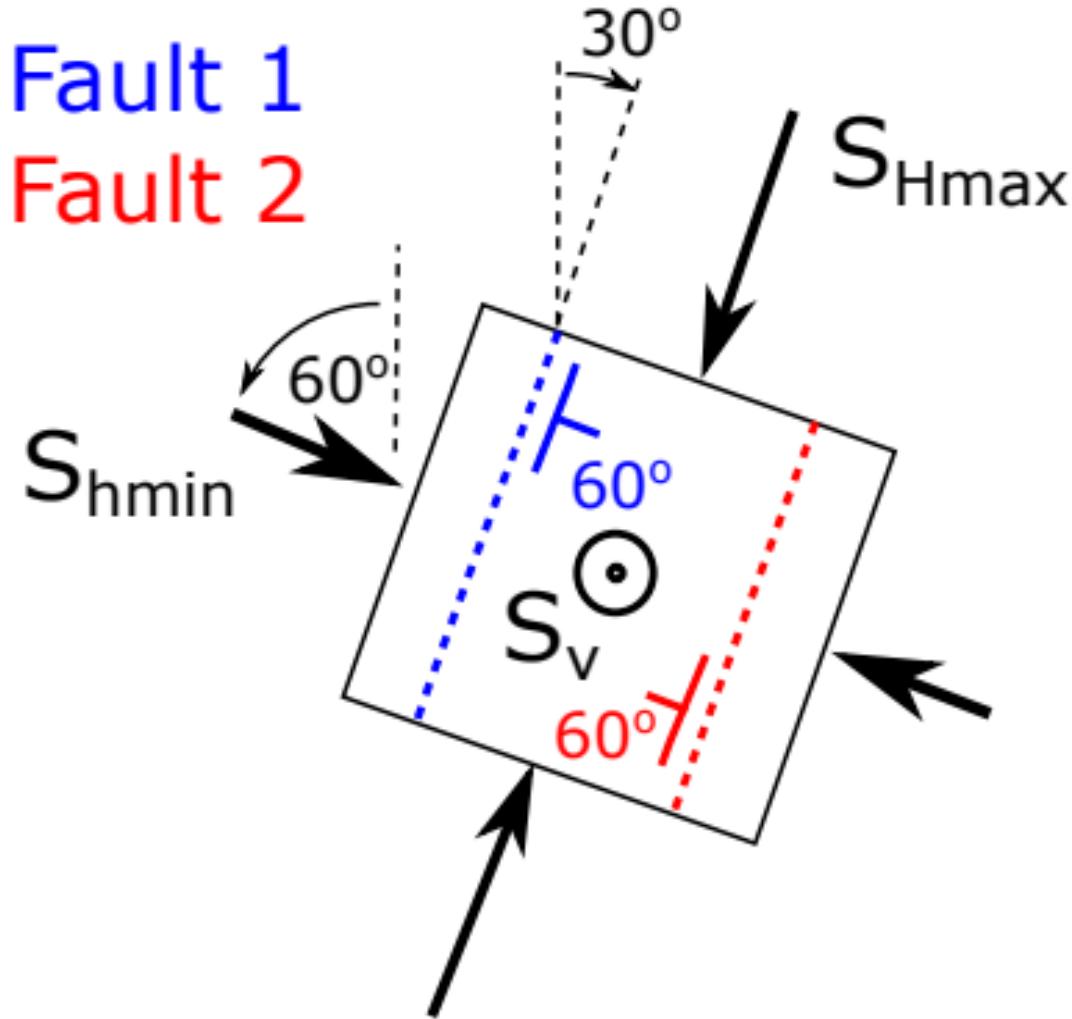
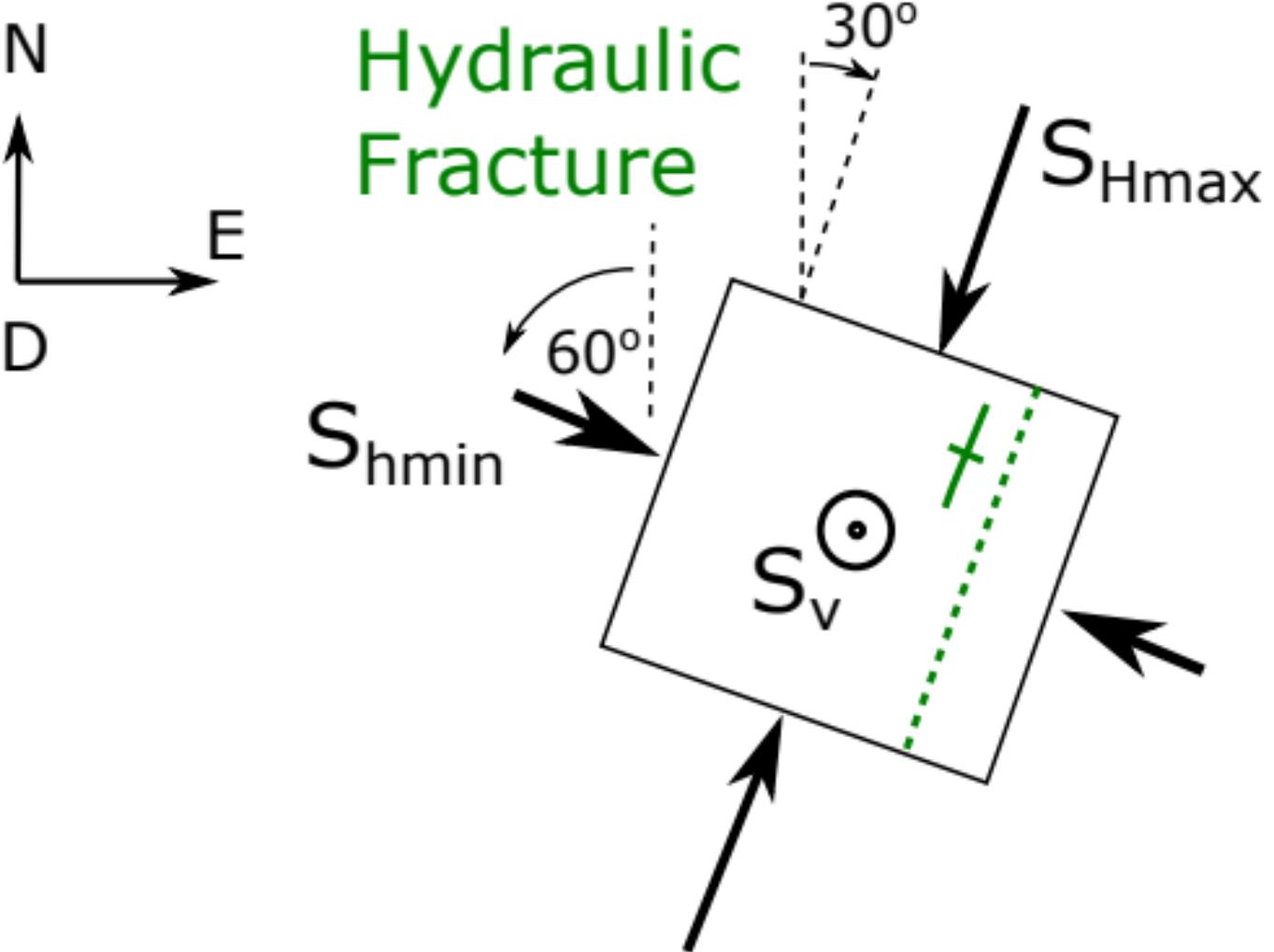
Method	Quality
focal mechanism	S_H is within $\pm 15^\circ$
breakouts	S_H is within $\pm 20^\circ$
drl. induced frac.	S_H is within $\pm 25^\circ$
overcoring	
hydro. fractures	
geol. indicators	

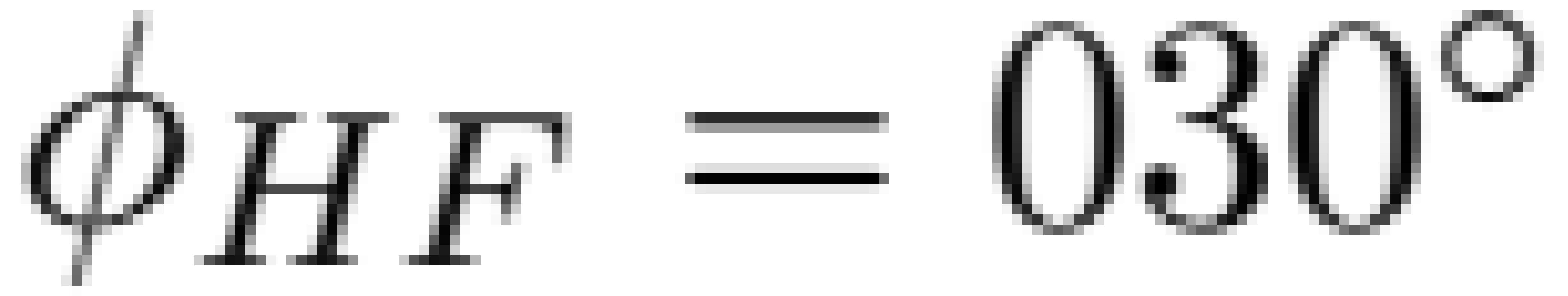
Data depth range

0-40 km

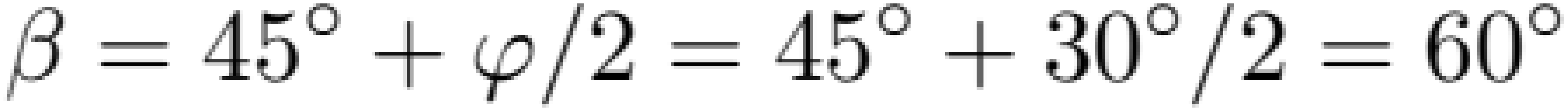
Stress Regime
Normal faulting
Strike-slip faulting
Thrust faulting
Unknown regime



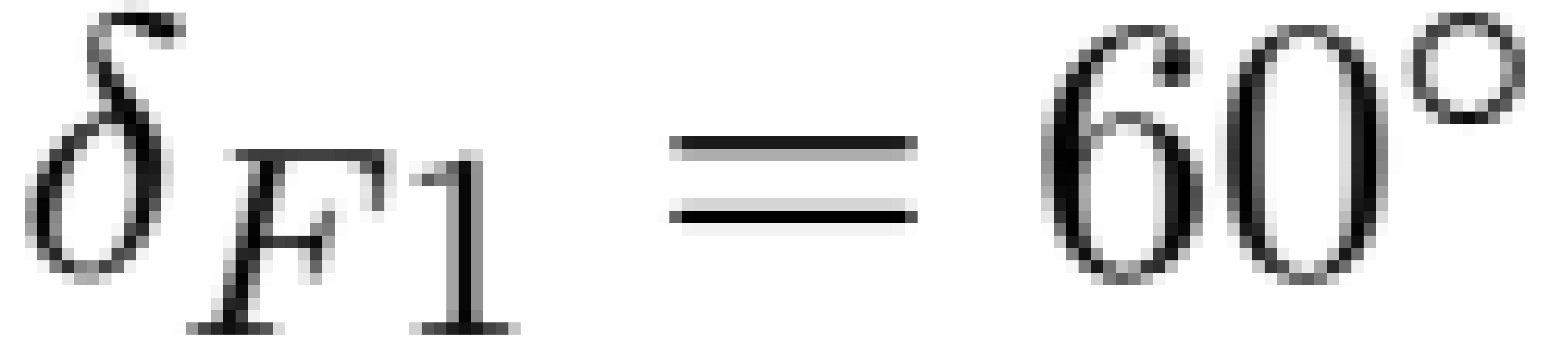


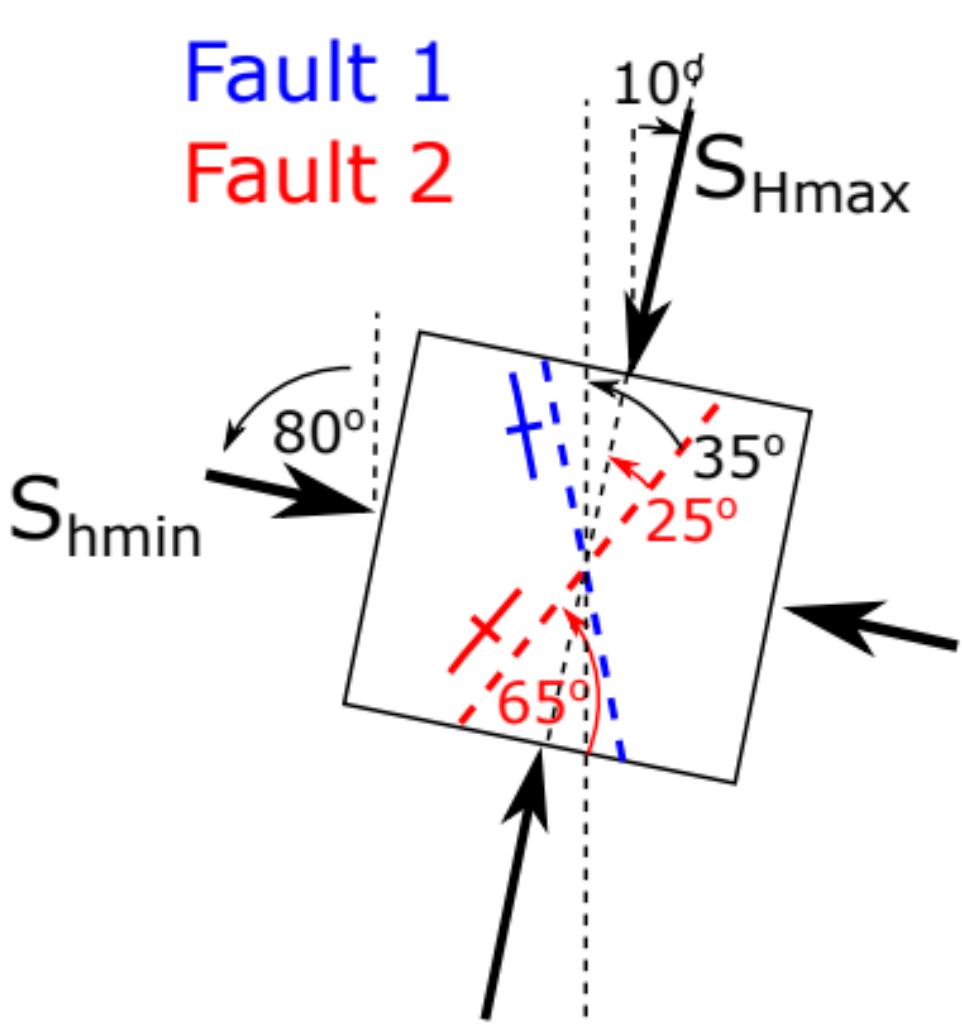
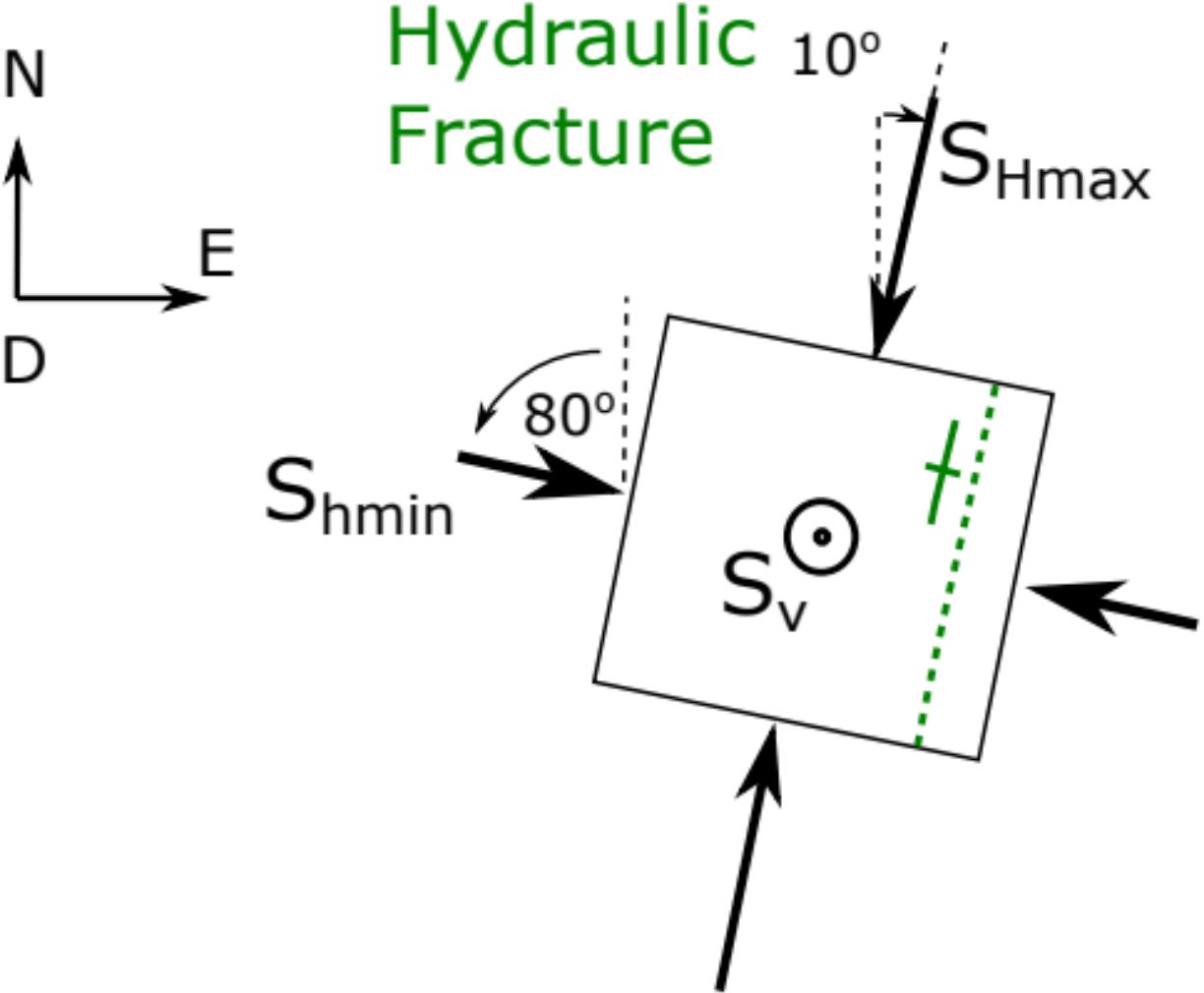


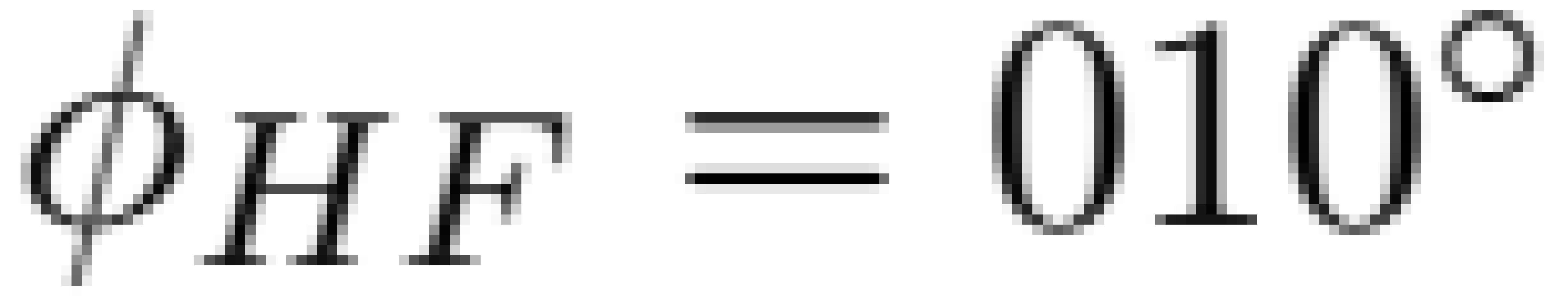


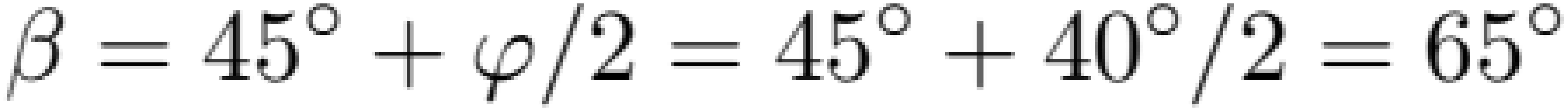


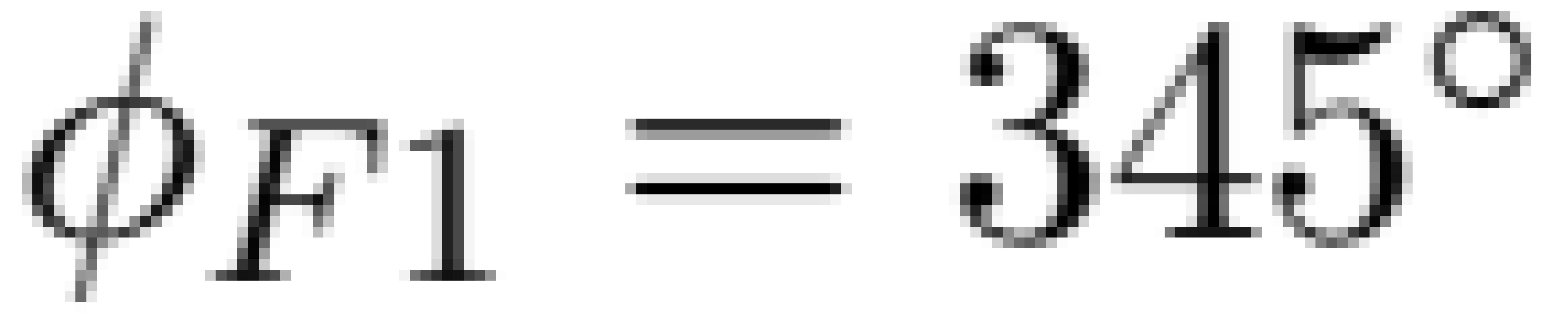


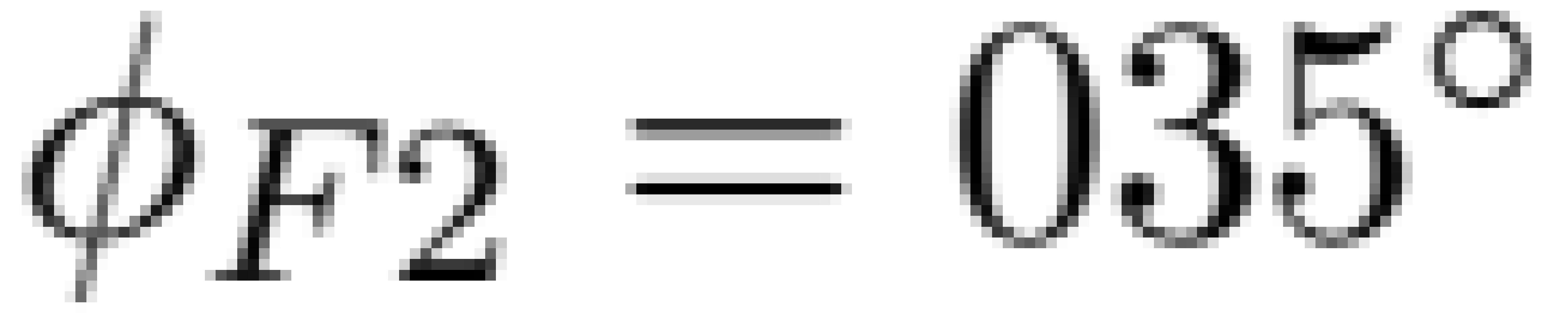




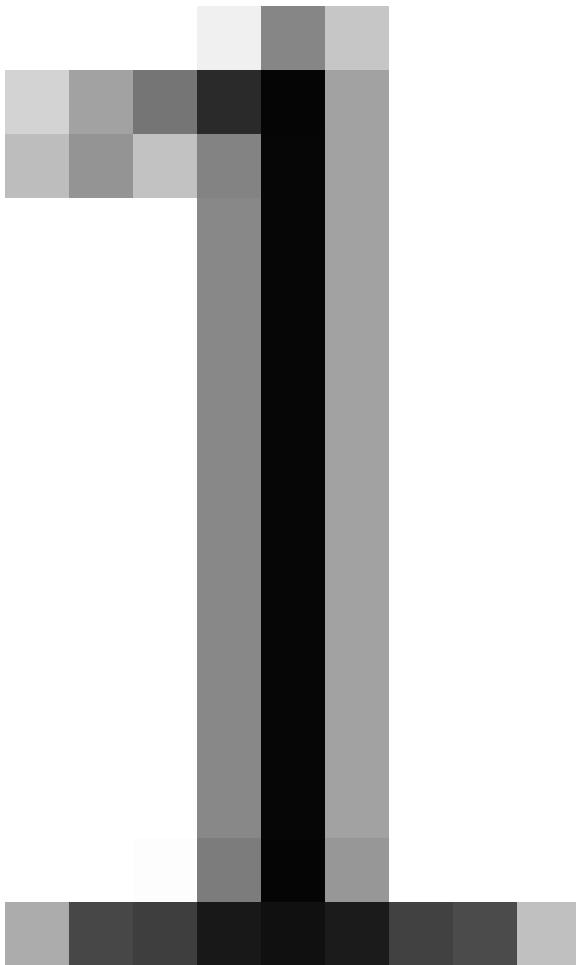




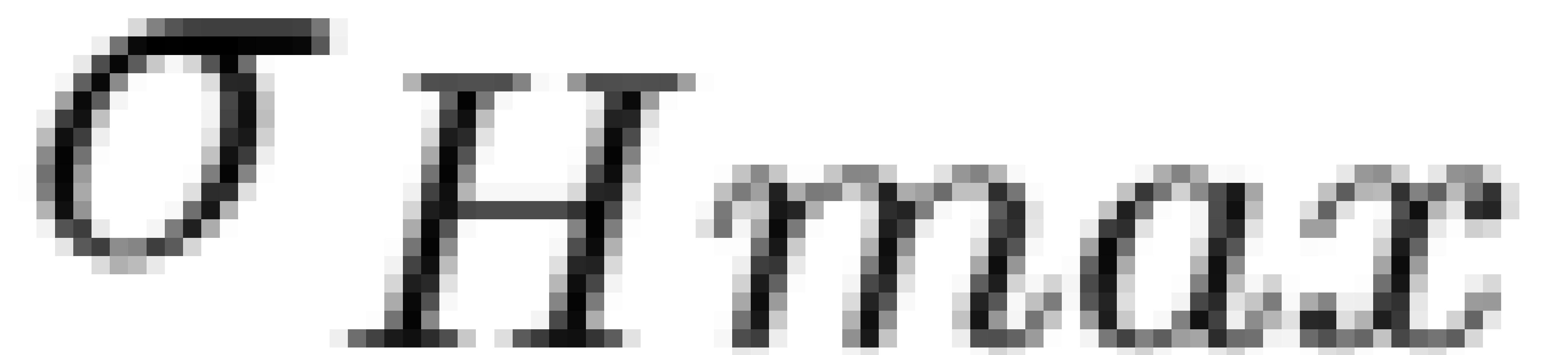




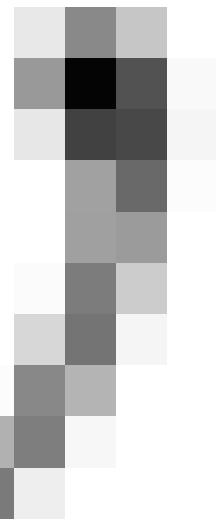
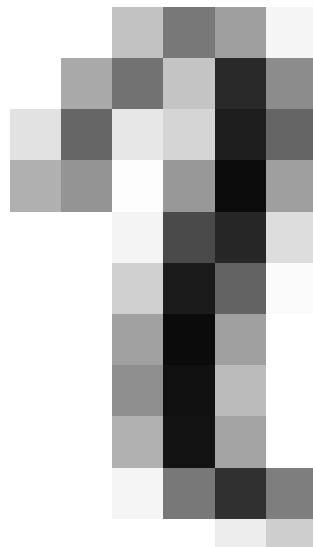
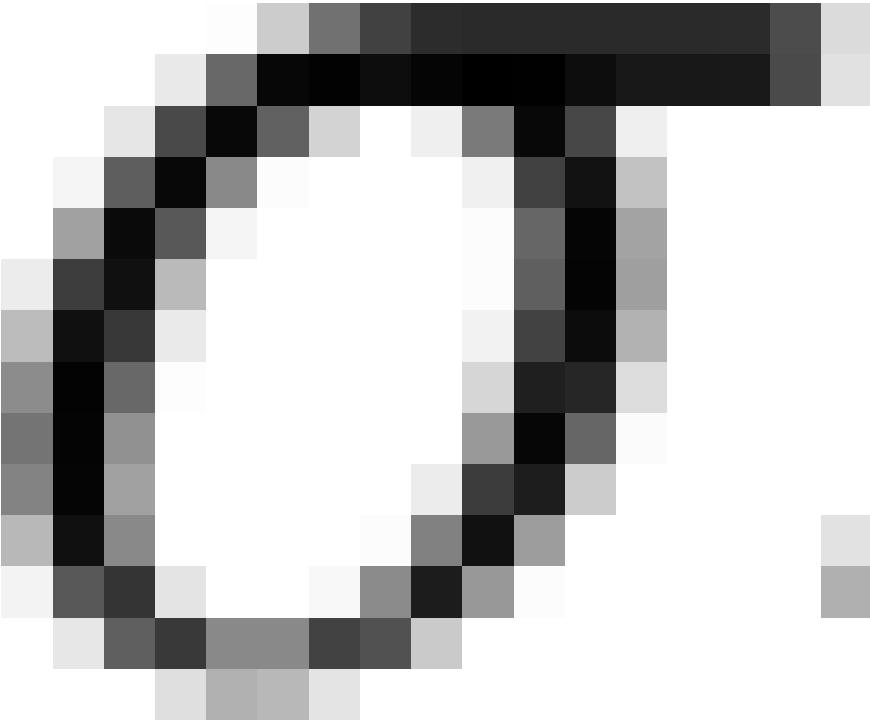




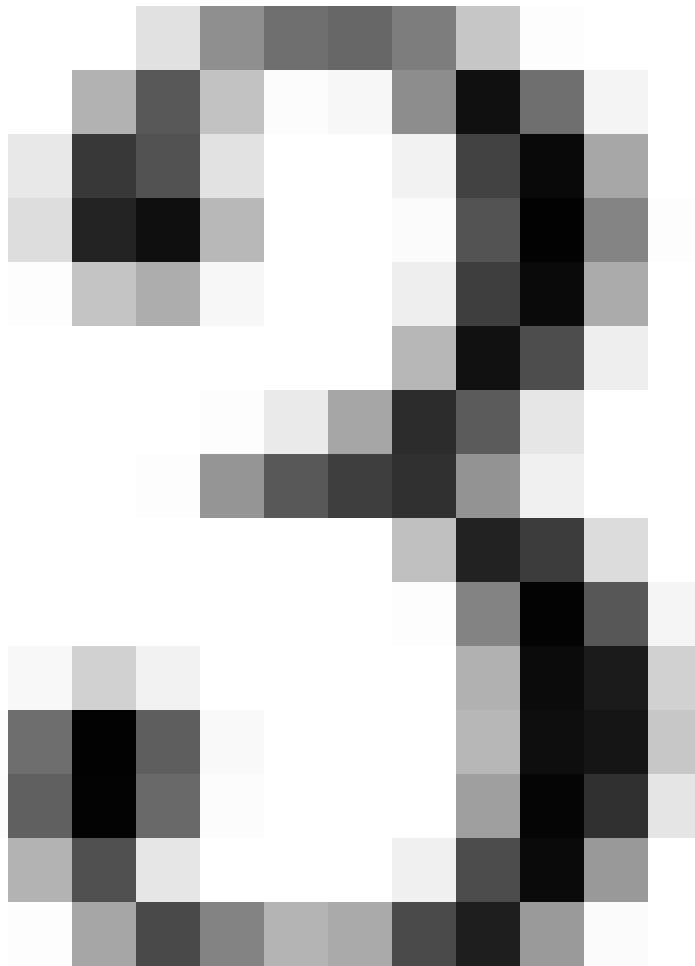




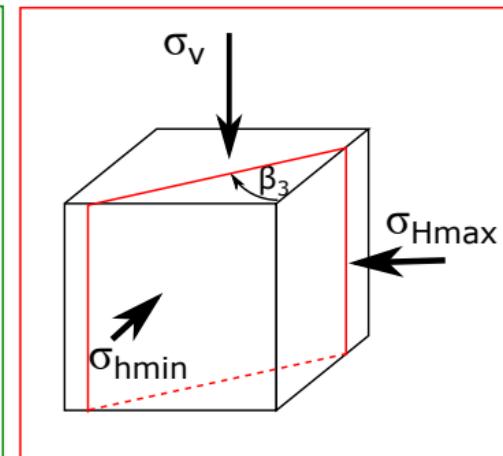
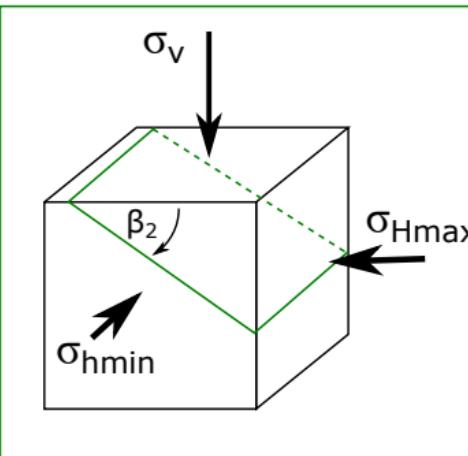
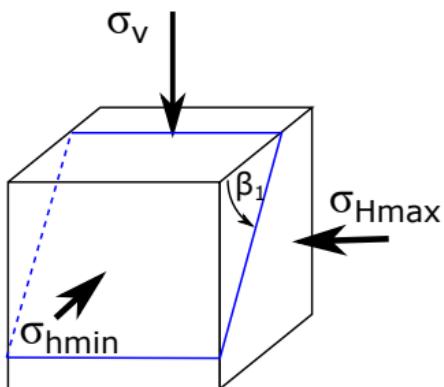
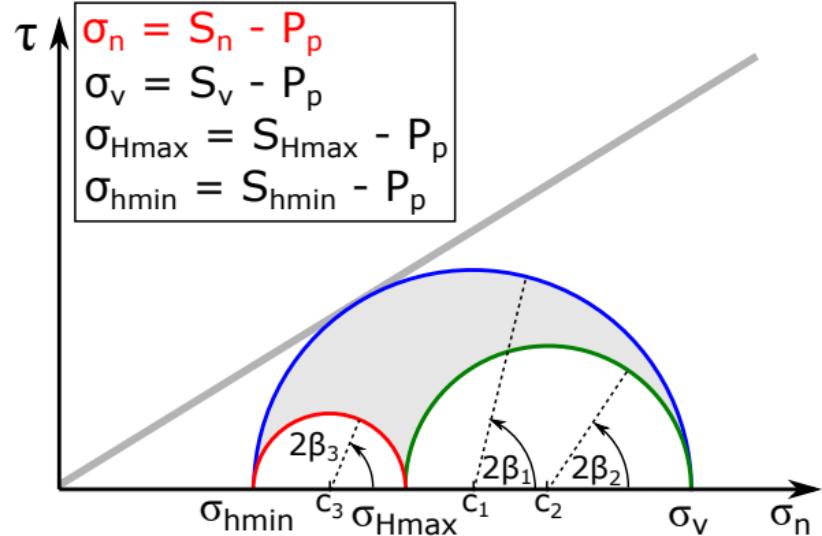
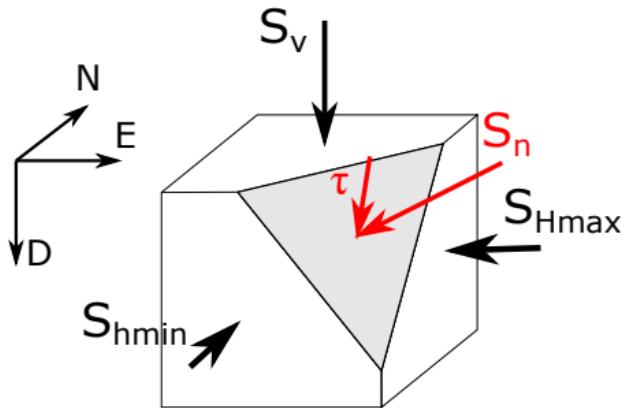


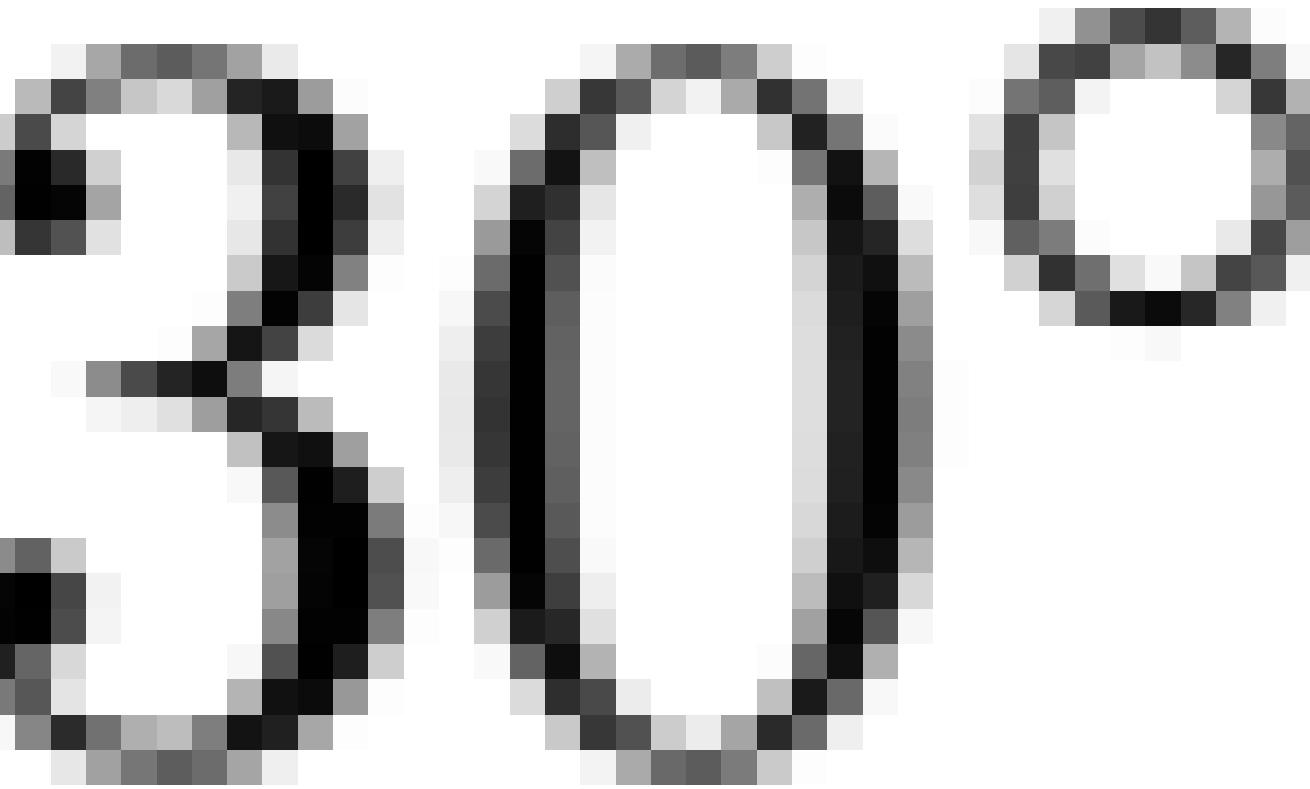
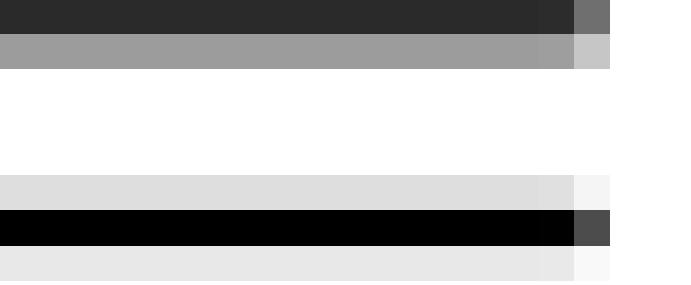
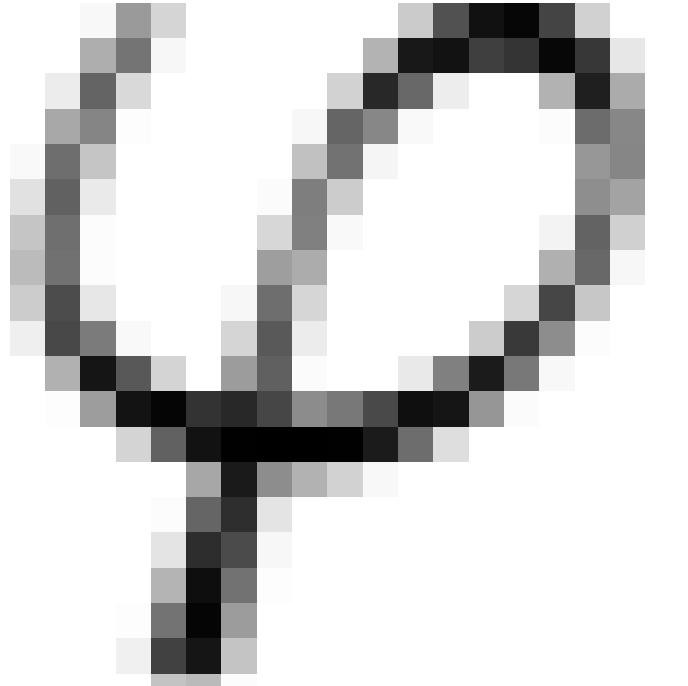




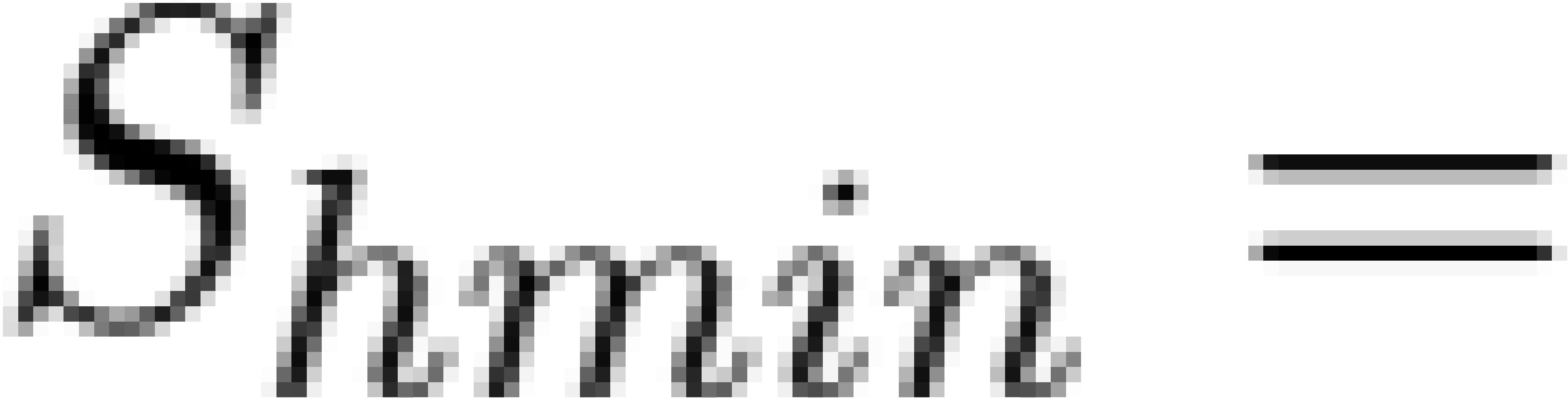




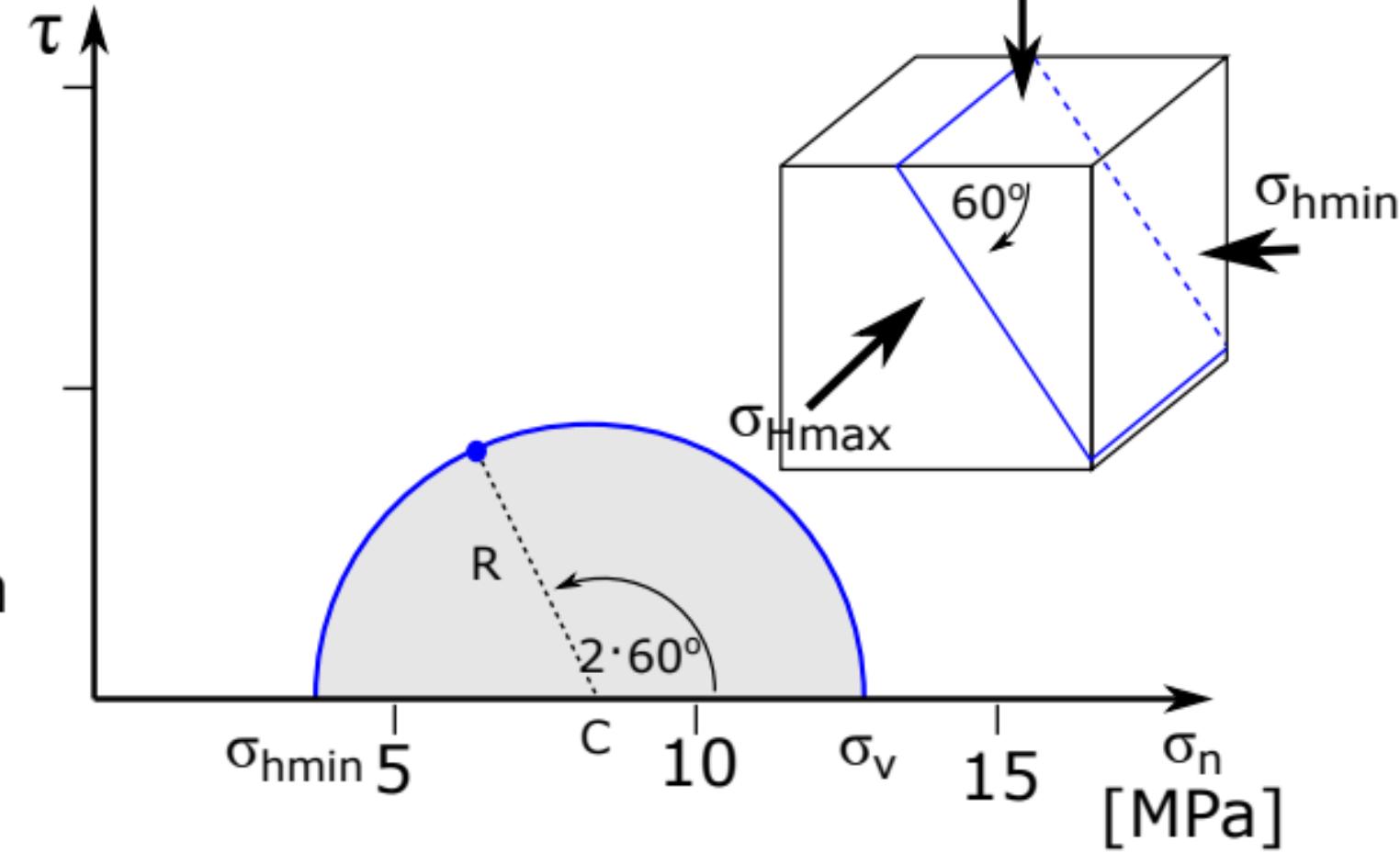
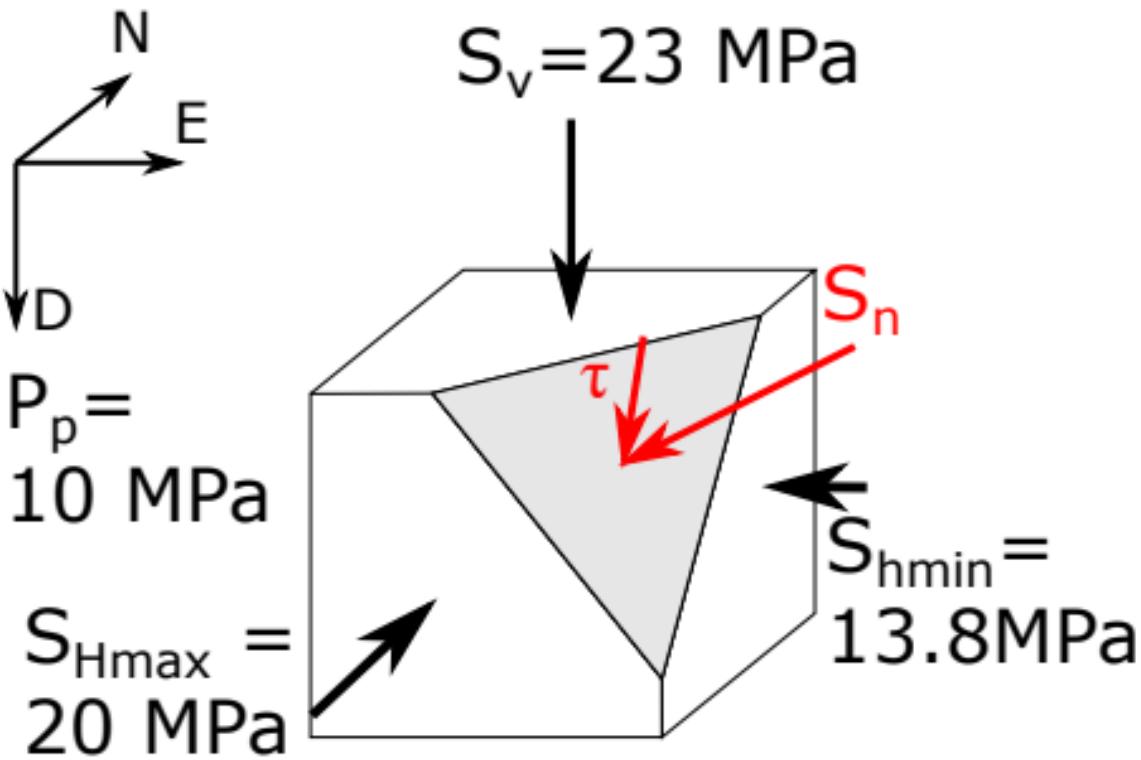


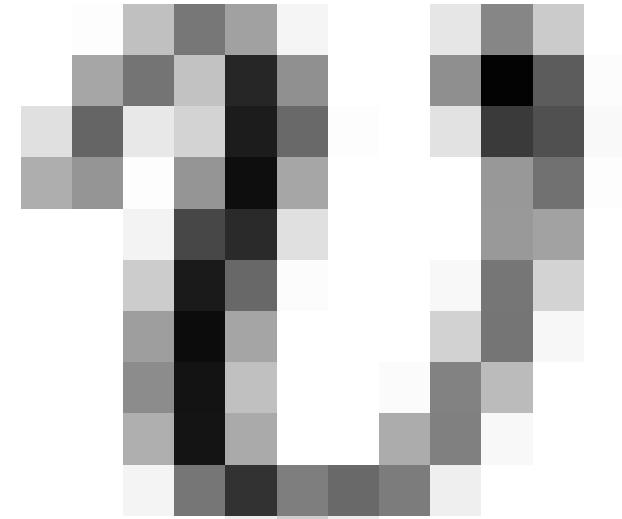
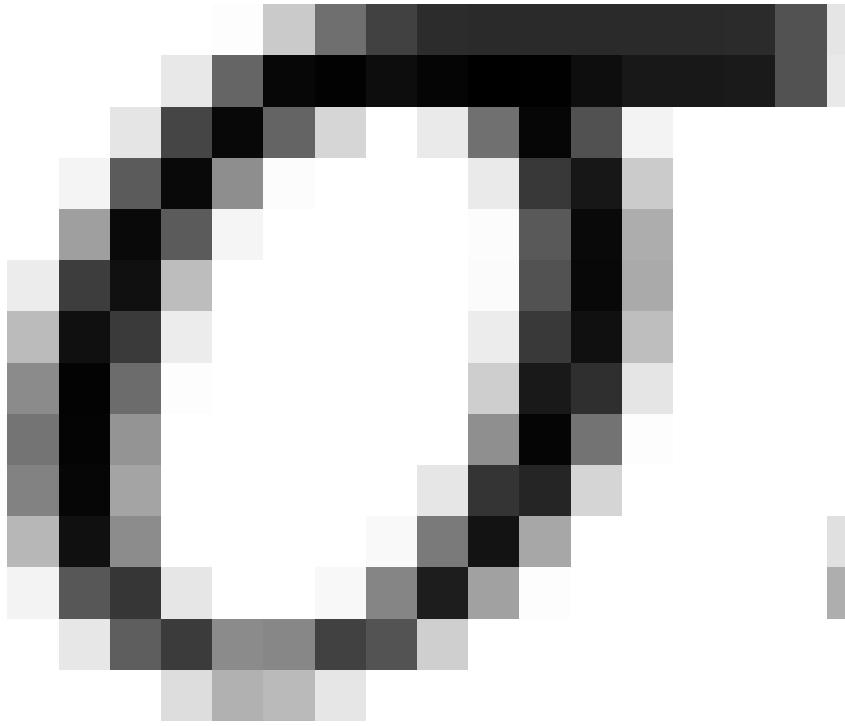




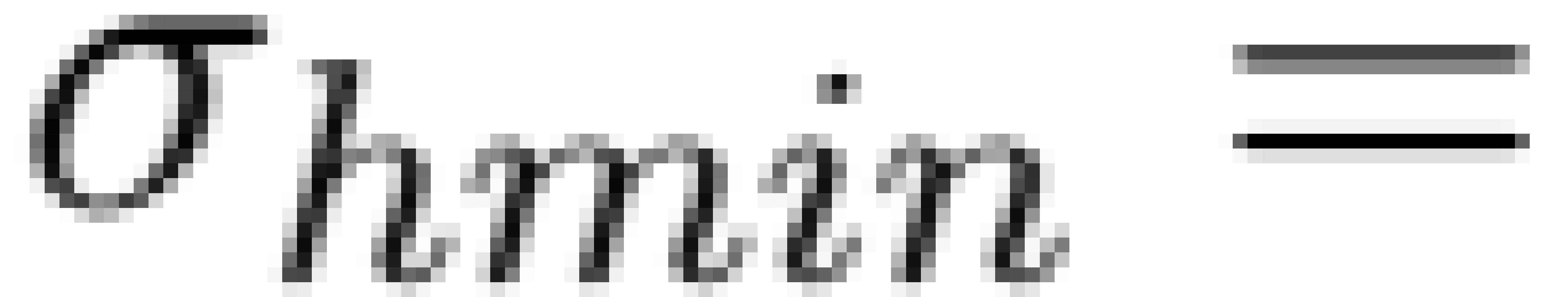








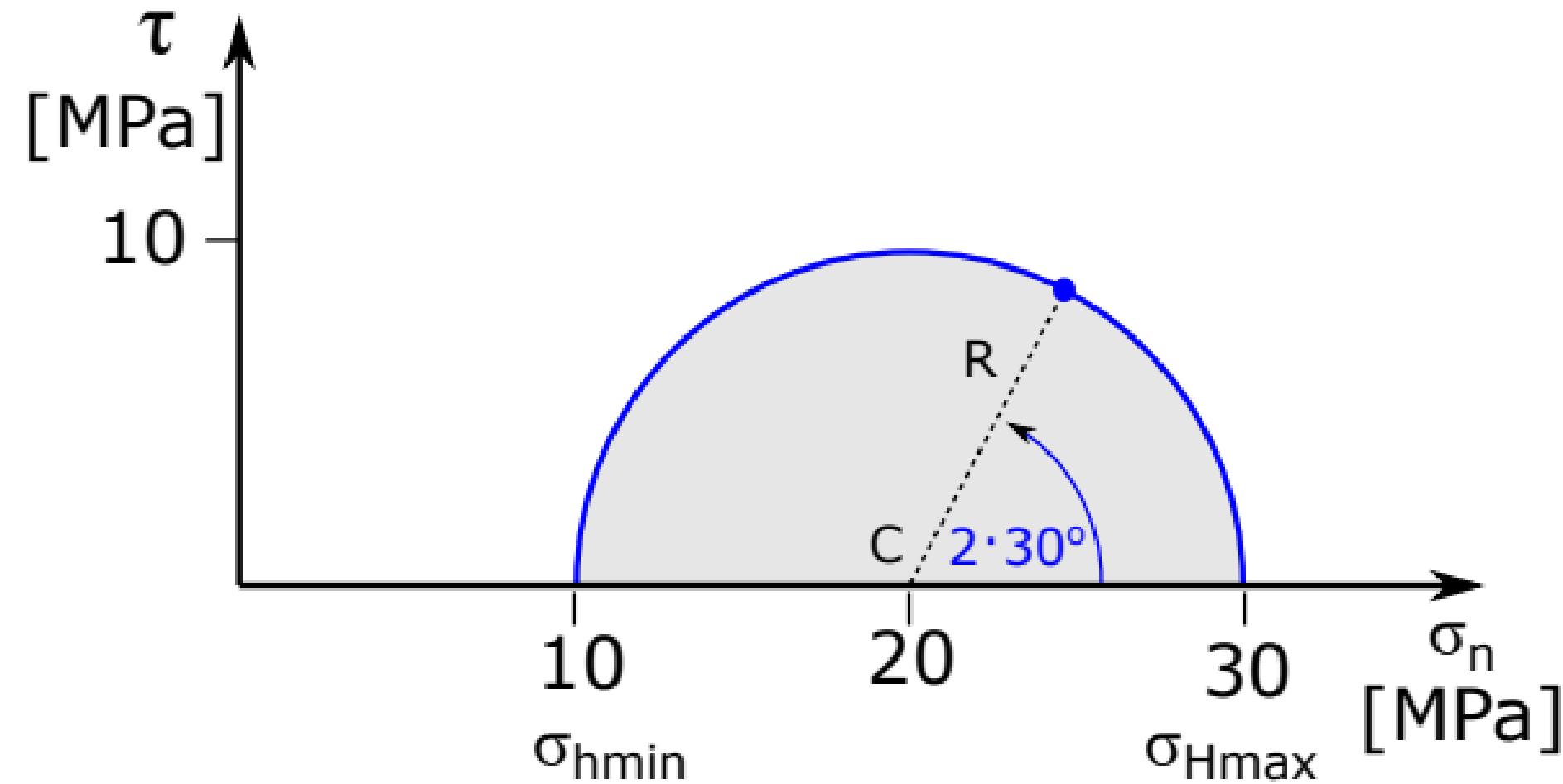
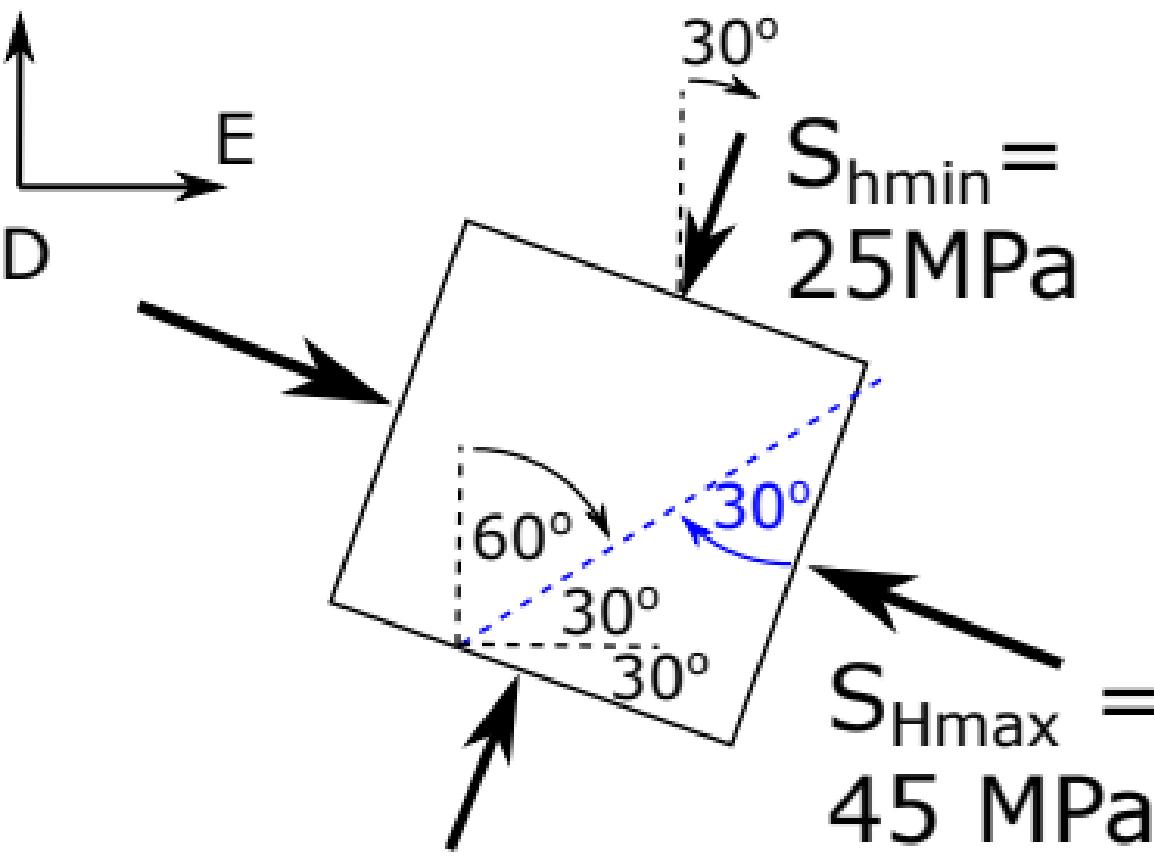




$$\sigma_n = \frac{(13 \text{ MPa} + 3.8 \text{ MPa})}{2} + \frac{(13 \text{ MPa} - 3.8 \text{ MPa}) \cos(2 \cdot 60^\circ)}{2} = 6.1 \text{ MPa}$$

$$\tau = \frac{(13 \text{ MPa} - 3.8 \text{ MPa})}{2} \sin(2 \cdot 60^\circ) = 4.0 \text{ MPa}$$

N $P_p = 15 \text{ MPa}$ $S_v = 30 \text{ MPa}$



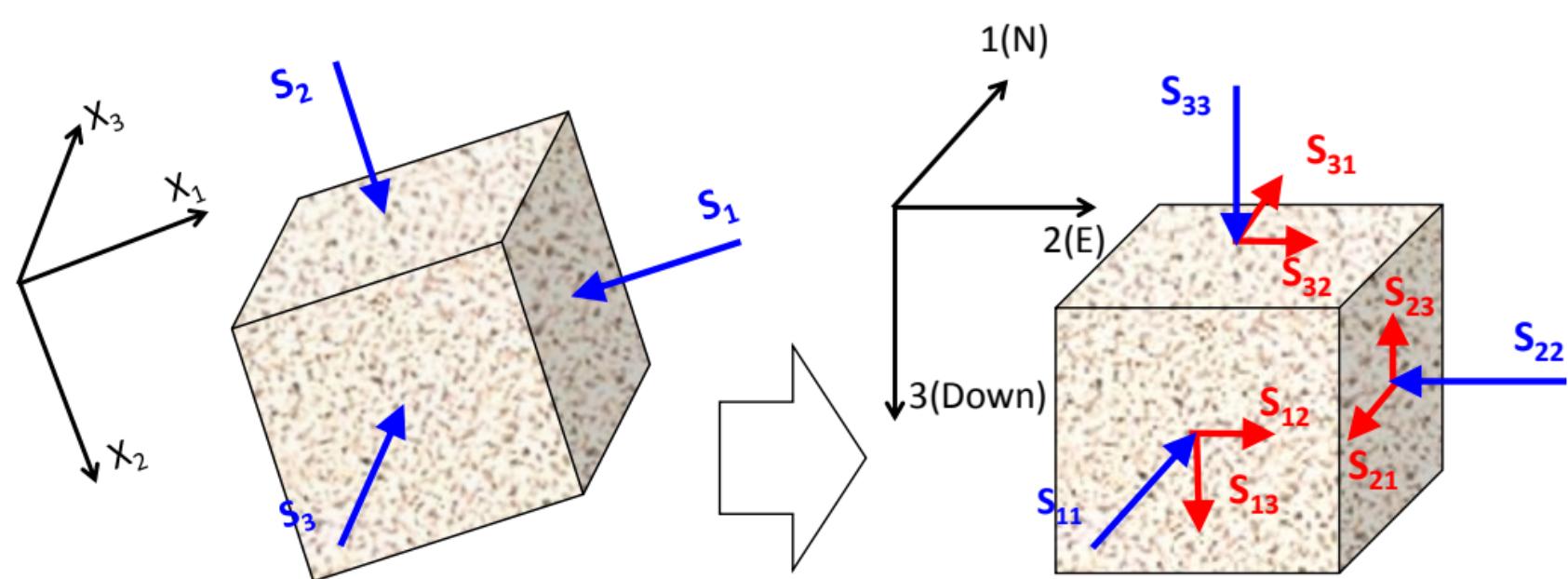
$$\sigma_n = \frac{(30 \text{ MPa} + 10 \text{ MPa})}{2} + \frac{(30 \text{ MPa} - 10 \text{ MPa}) \cos(2 \cdot 30^\circ)}{2} = 25 \text{ MPa}$$

$$\tau = \frac{(30 \text{ MPa} - 10 \text{ MPa})}{2} \sin(2 \cdot 30^\circ) = 8.7 \text{ MPa}$$



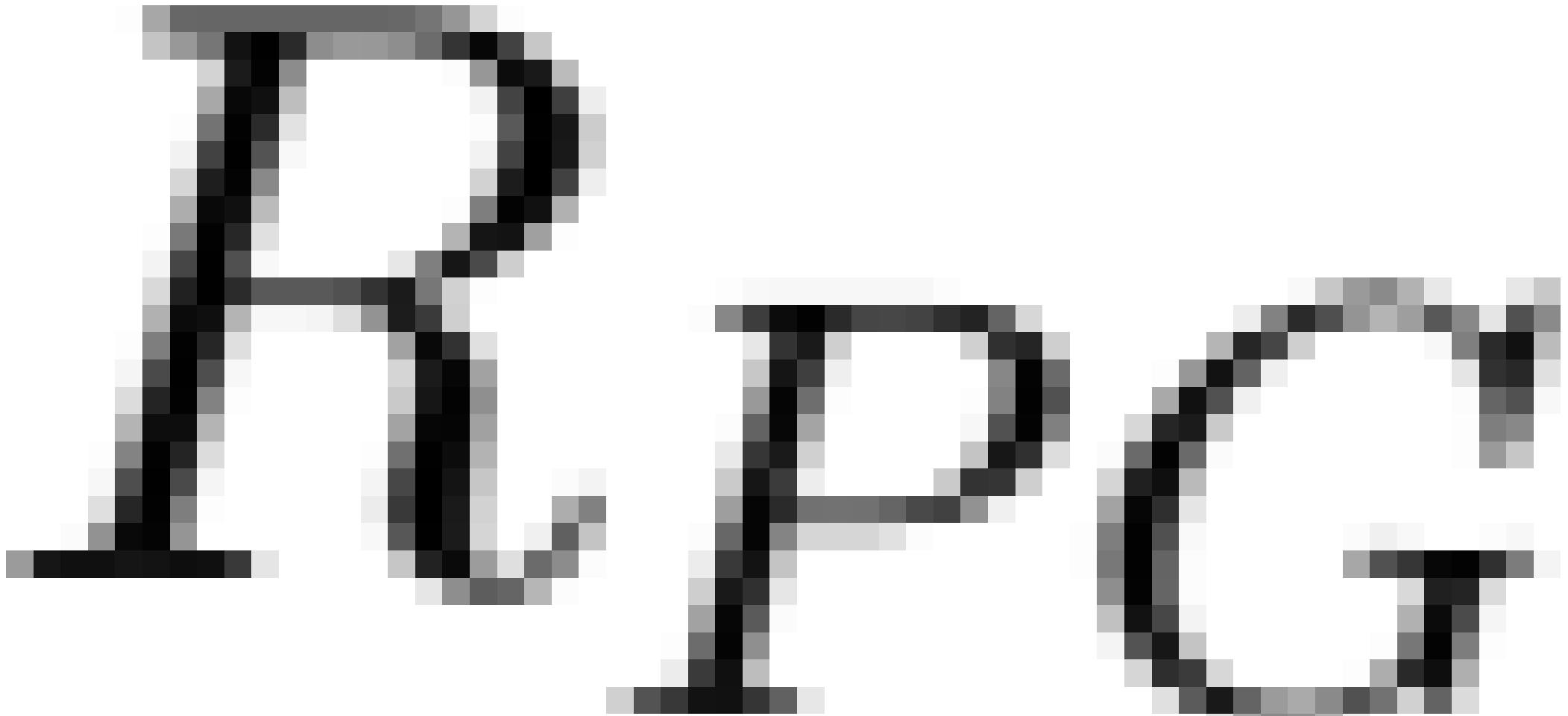


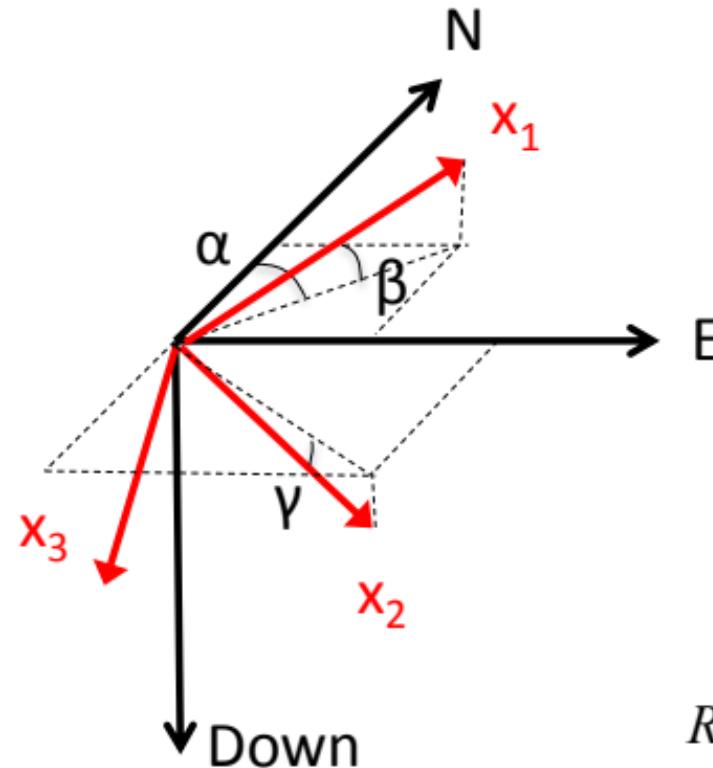




$$S_D = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$S_H = \begin{bmatrix} \frac{1}{3}(N) & 0 & 0 \\ 0 & \frac{1}{3}(N) & 0 \\ 0 & 0 & \frac{1}{3}(N) \end{bmatrix}$$





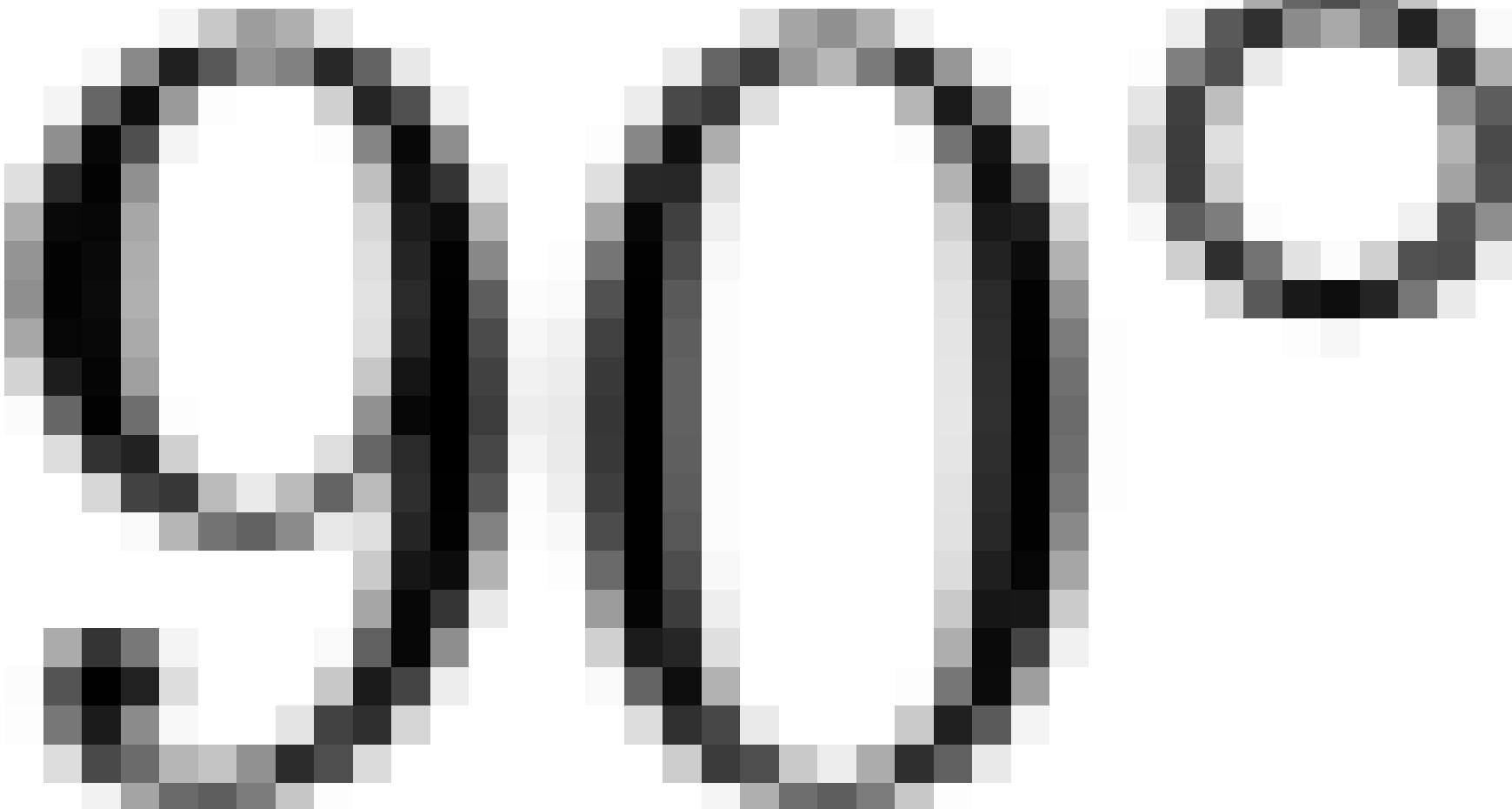
$$\underline{\underline{S}}' = \underline{\underline{A}} \underline{\underline{S}} \underline{\underline{A}}^T$$

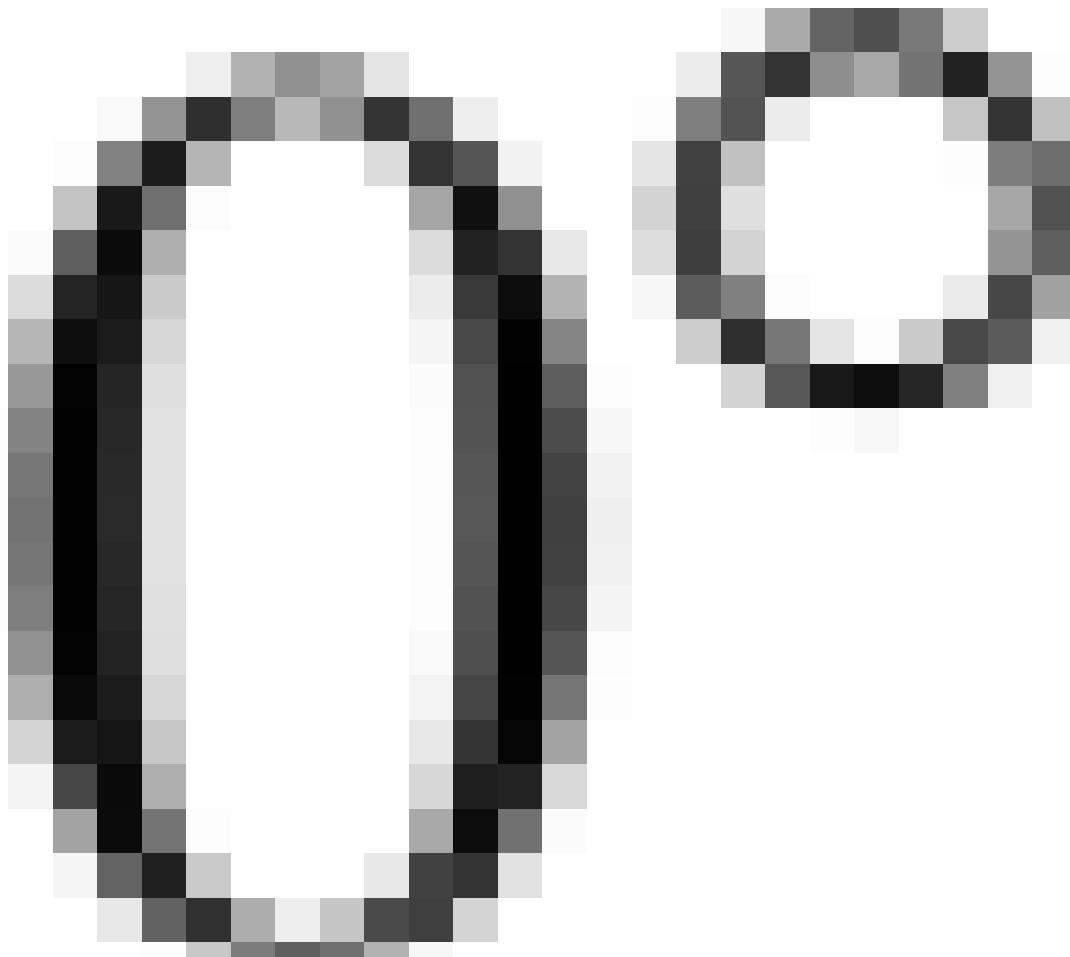
$$\underline{\underline{A}} = \begin{bmatrix} \underline{e}'_1 \cdot \underline{e}_1 & \underline{e}'_1 \cdot \underline{e}_2 & \underline{e}'_1 \cdot \underline{e}_3 \\ \underline{e}'_2 \cdot \underline{e}_1 & \underline{e}'_2 \cdot \underline{e}_2 & \underline{e}'_2 \cdot \underline{e}_3 \\ \underline{e}'_3 \cdot \underline{e}_1 & \underline{e}'_3 \cdot \underline{e}_2 & \underline{e}'_3 \cdot \underline{e}_3 \end{bmatrix}$$

where $\underline{\underline{A}}$ is the transformation matrix from the old base \underline{e}_i to base \underline{e}'_i and the components are the projection of the elements of the new base on the old base.

- Old system: N-E-D (Right-handed) Geographical system
- New system: 1-2-3 (Right-handed) Principal stress system

$$R_{PG} = \begin{bmatrix} \cos \alpha \cos b & \sin \alpha \cos b & -\sin b \\ \cos \alpha \sin b \sin g - \sin \alpha \cos g & \sin \alpha \sin b \sin g + \cos \alpha \cos g & \cos b \sin g \\ \cos \alpha \sin b \cos g + \sin \alpha \sin g & \sin \alpha \sin b \cos g - \cos \alpha \sin g & \cos b \cos g \end{bmatrix}$$







S

G

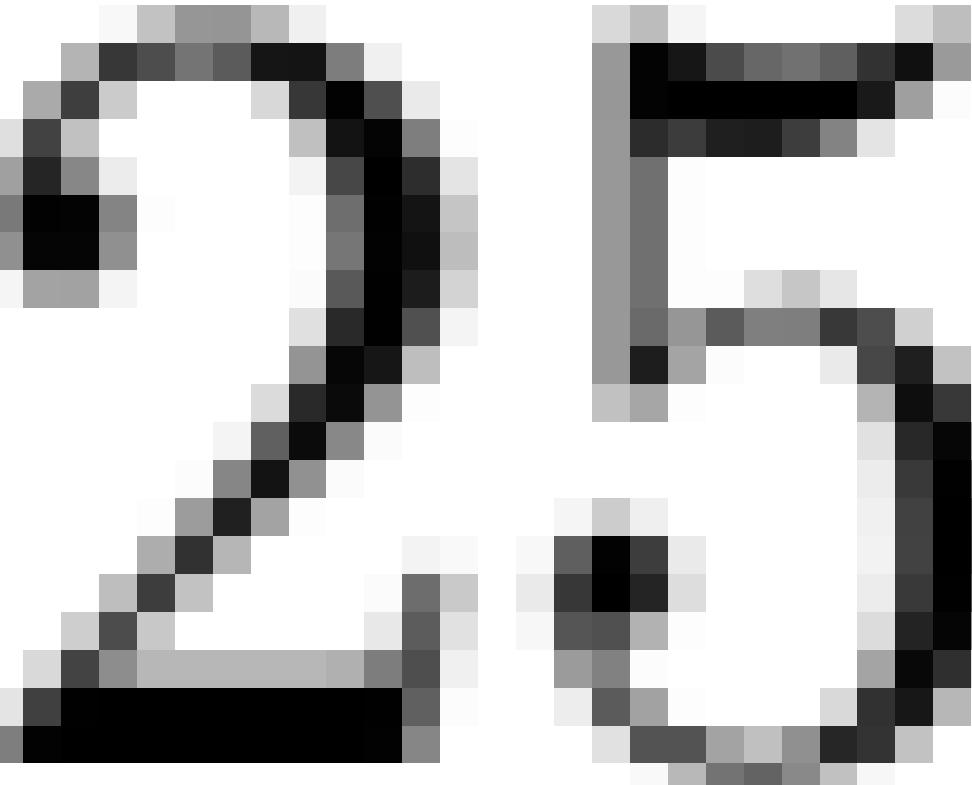
S

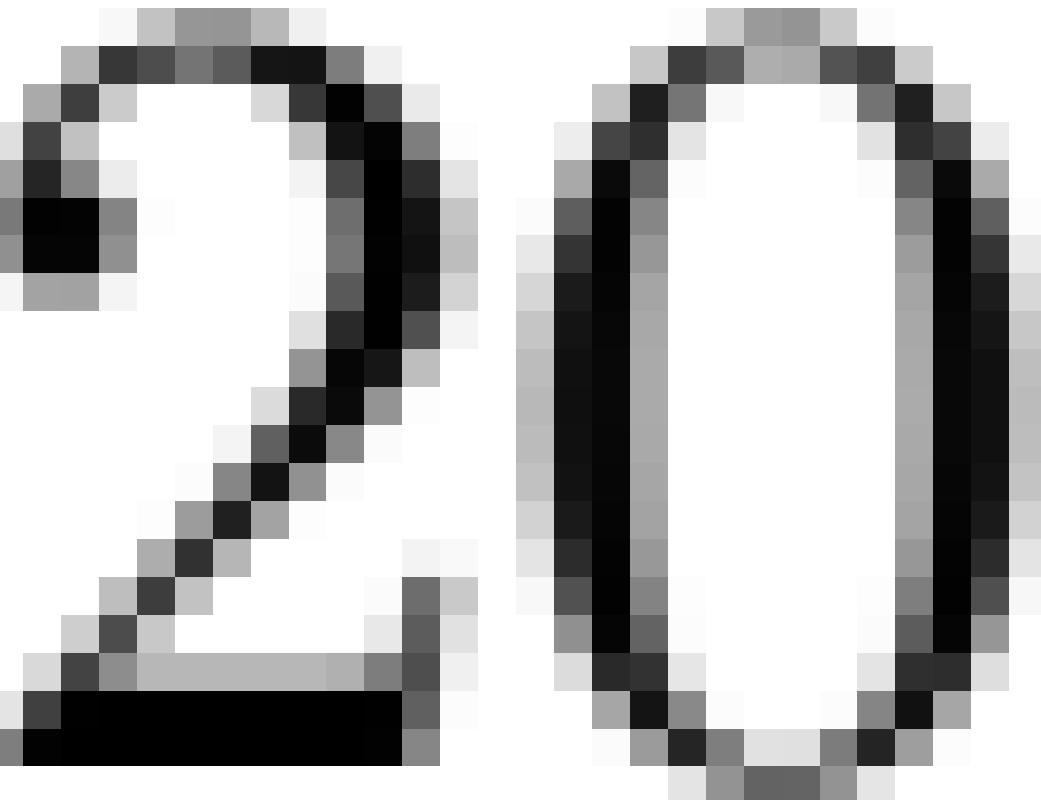
SP

S

SP

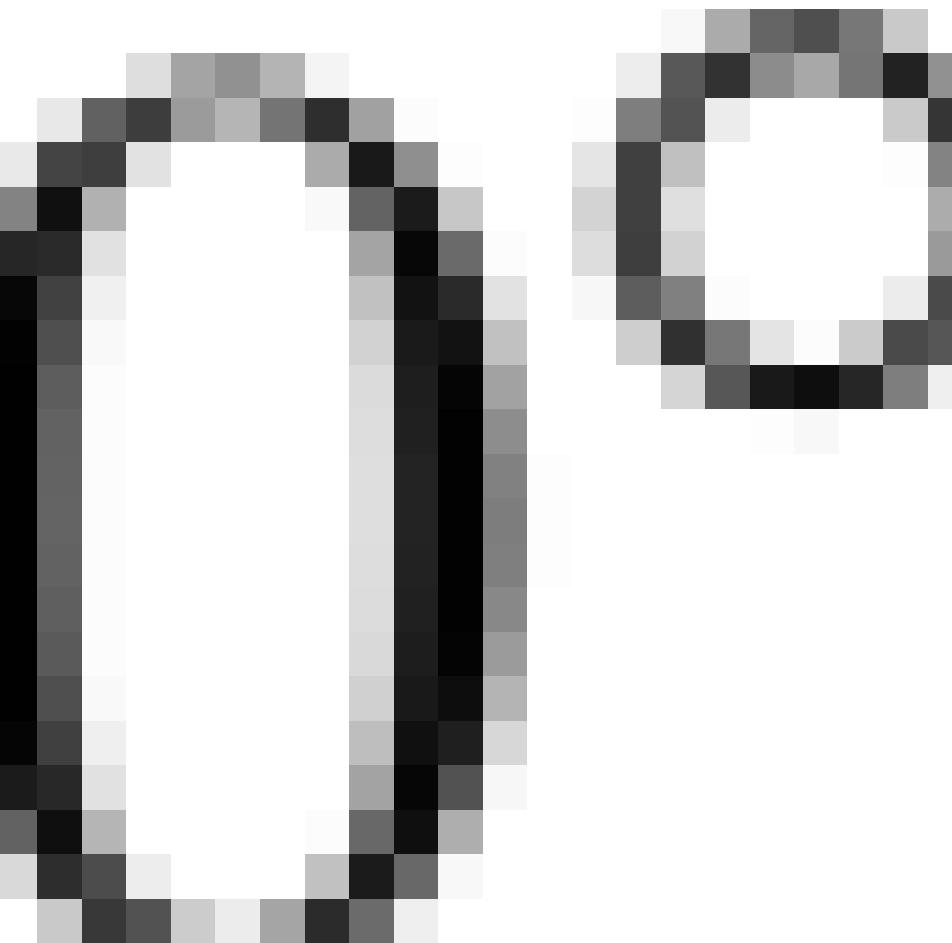
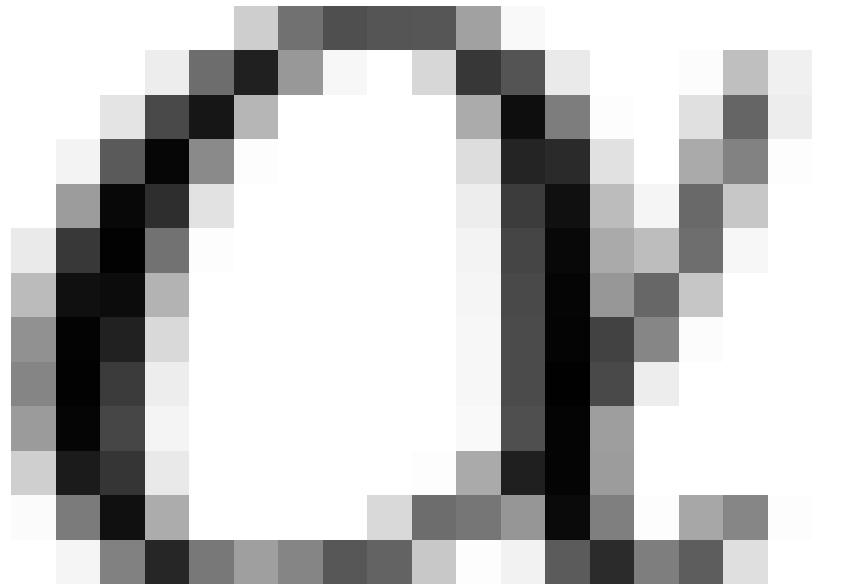


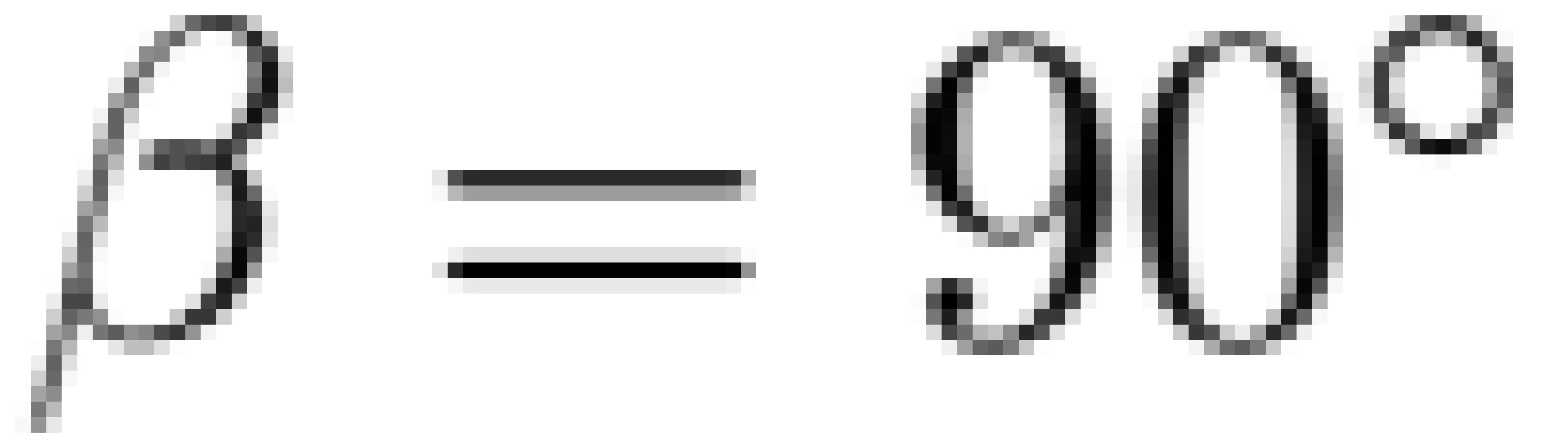




$$\underline{S}_P =$$

$$\begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$



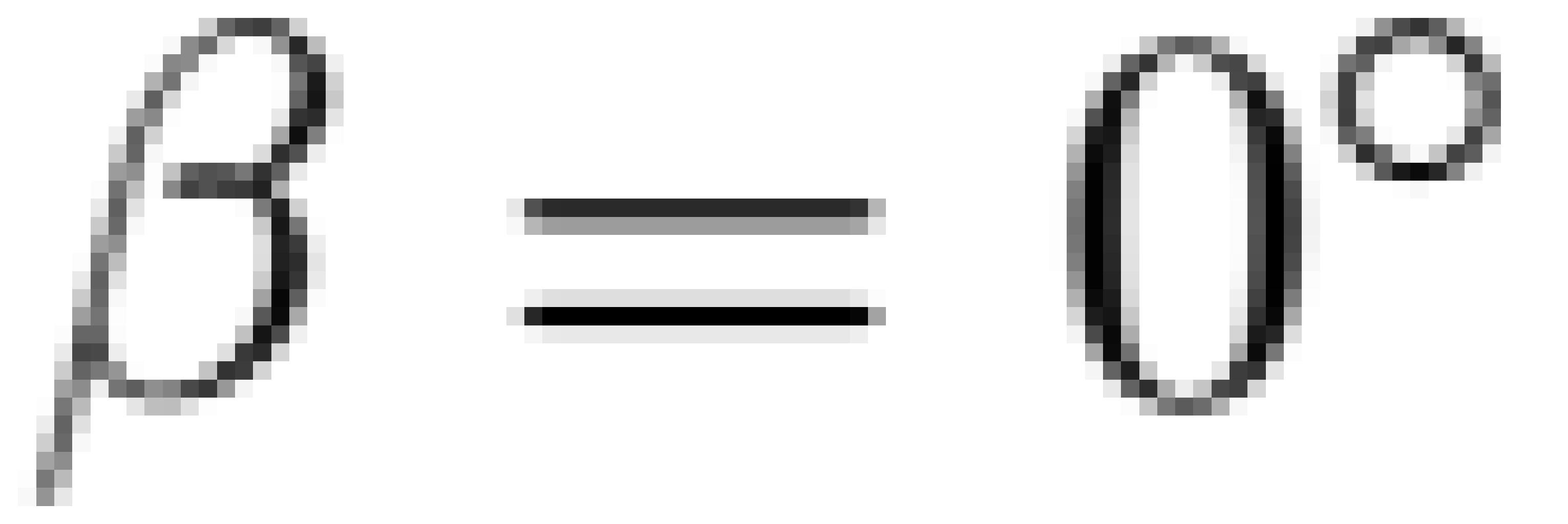


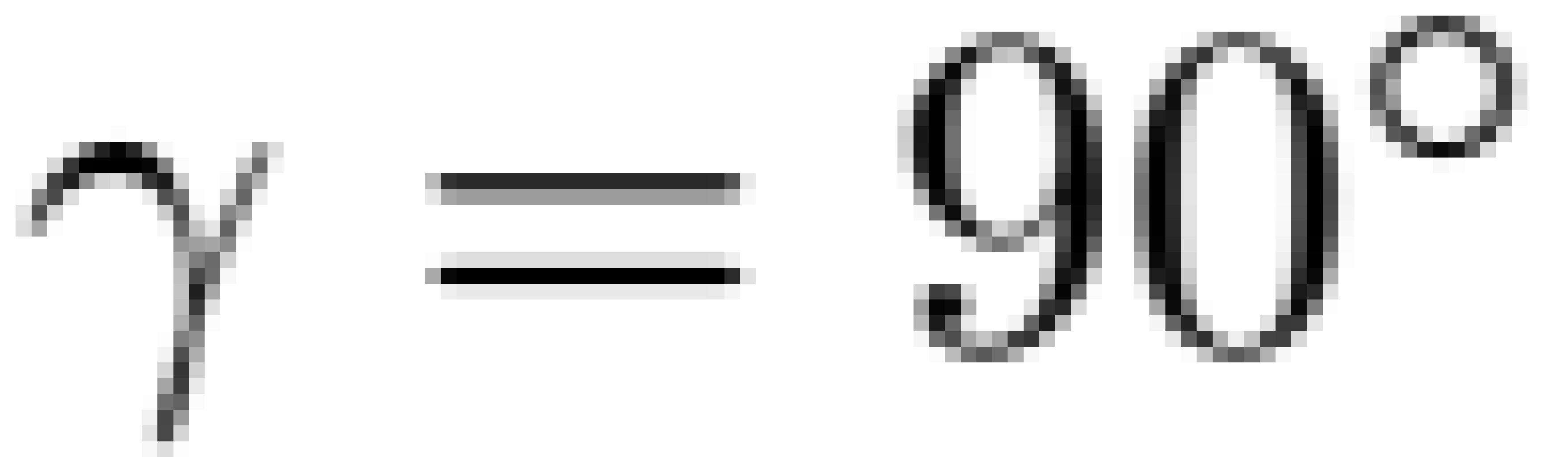


$$R_{PG} =$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 10 & 5 \\ 1 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 1 & 0 & 30 \end{bmatrix}$$





$$R_{PG} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 10 \\ 0 & -1 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & -1 & 25 \end{bmatrix}$$

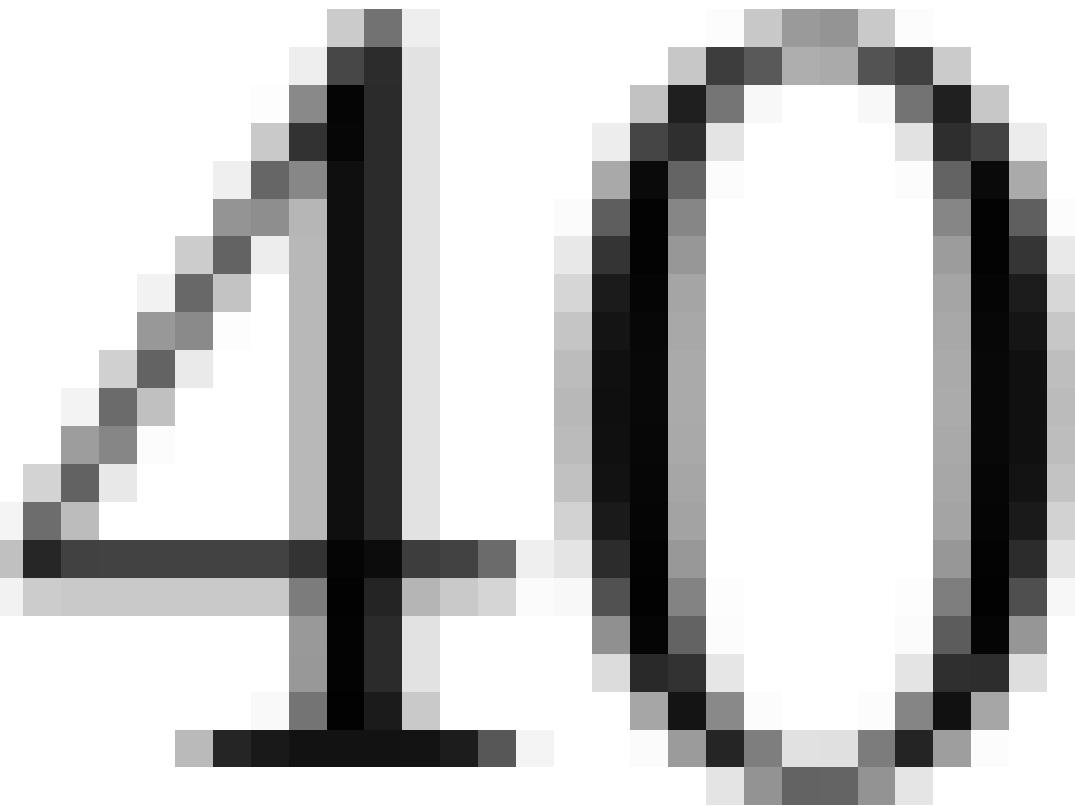


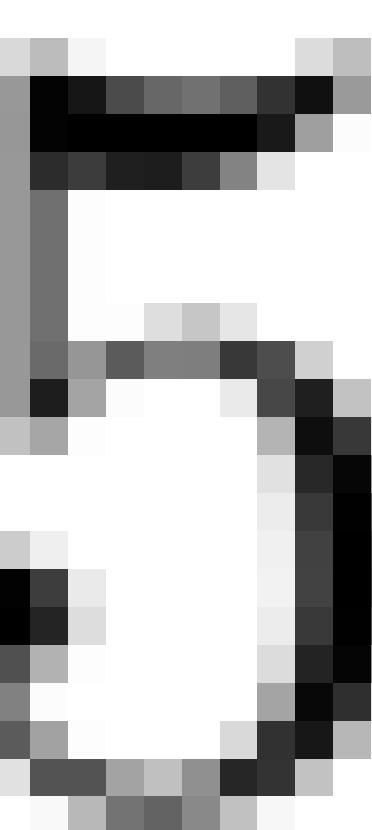
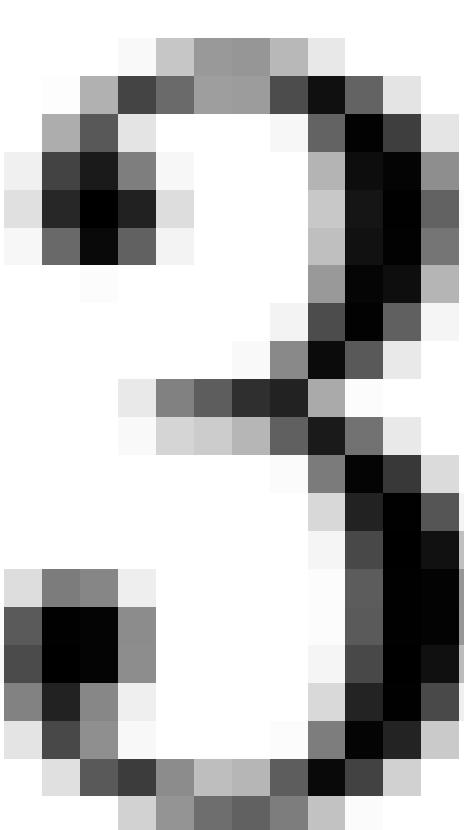
$$R_{PG} =$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 10 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

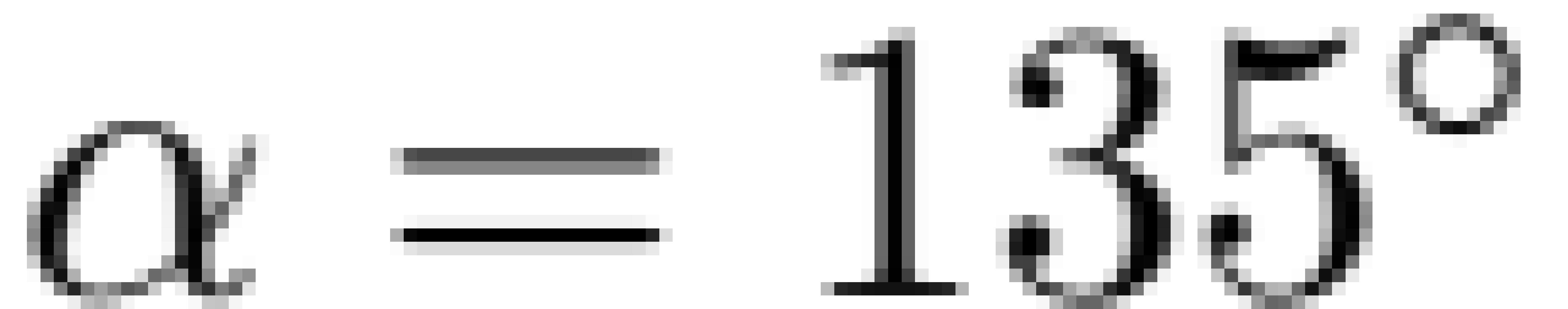






$$\underline{S}_P =$$

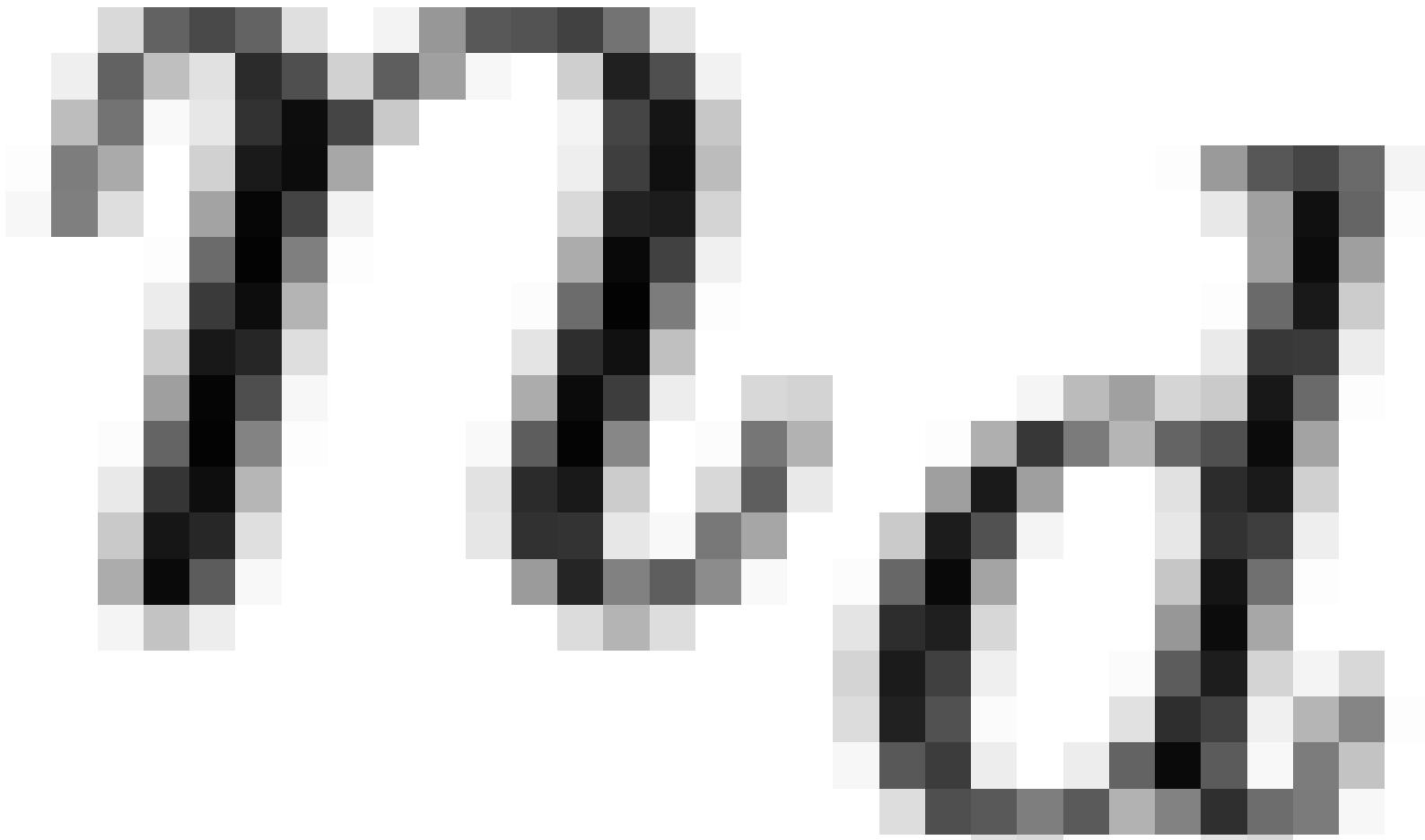
$$\begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

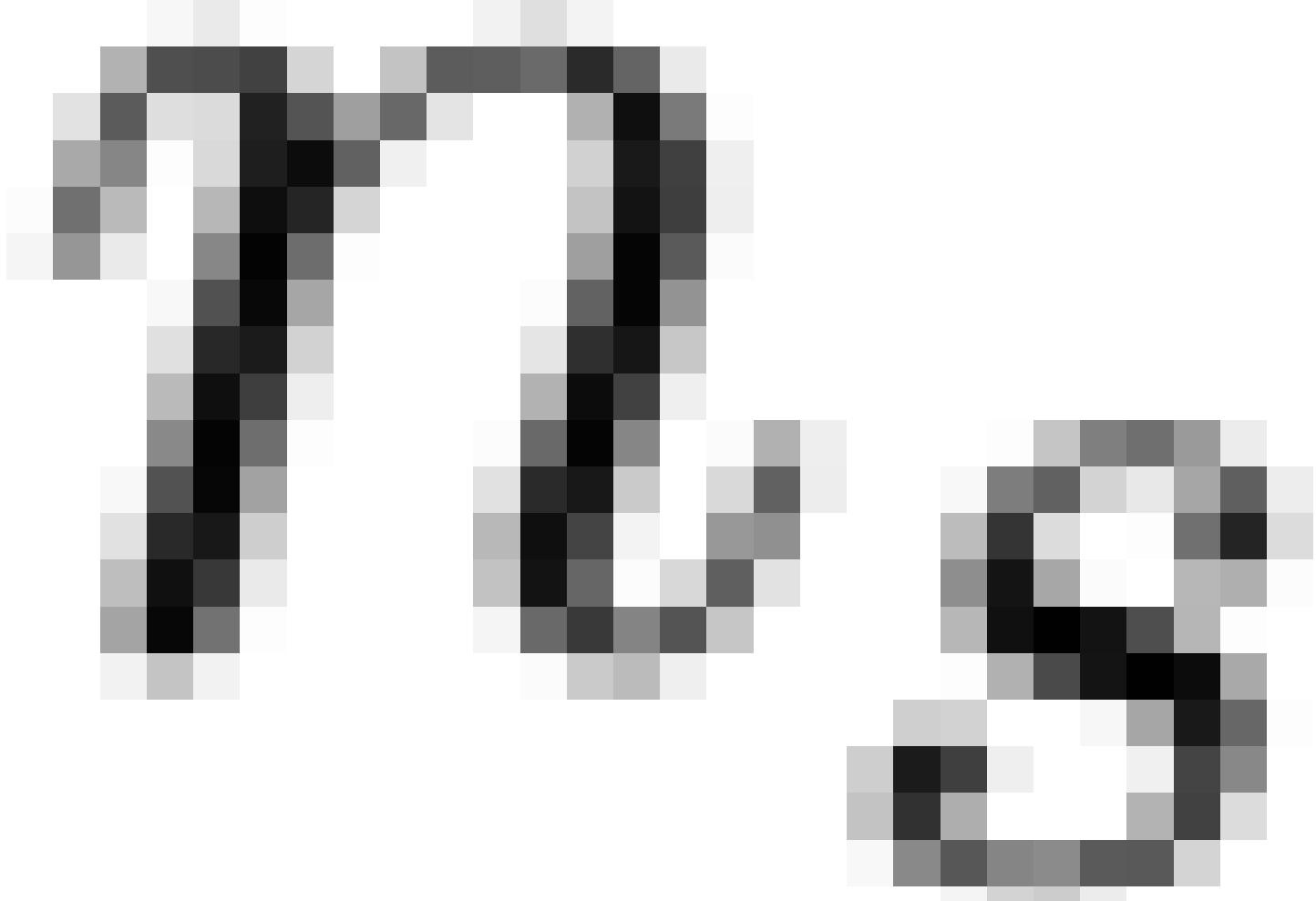


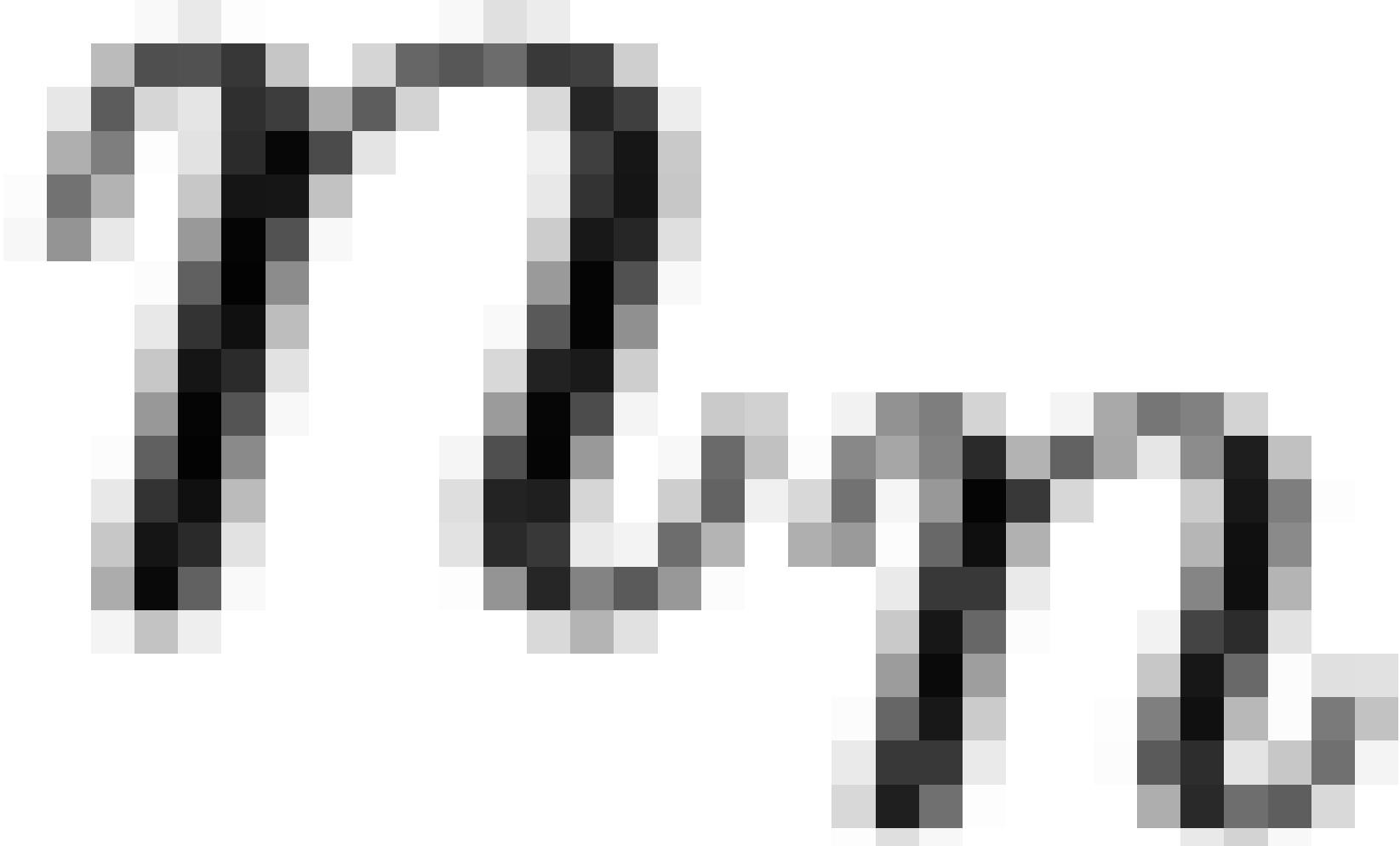
$$R_{PG} =$$

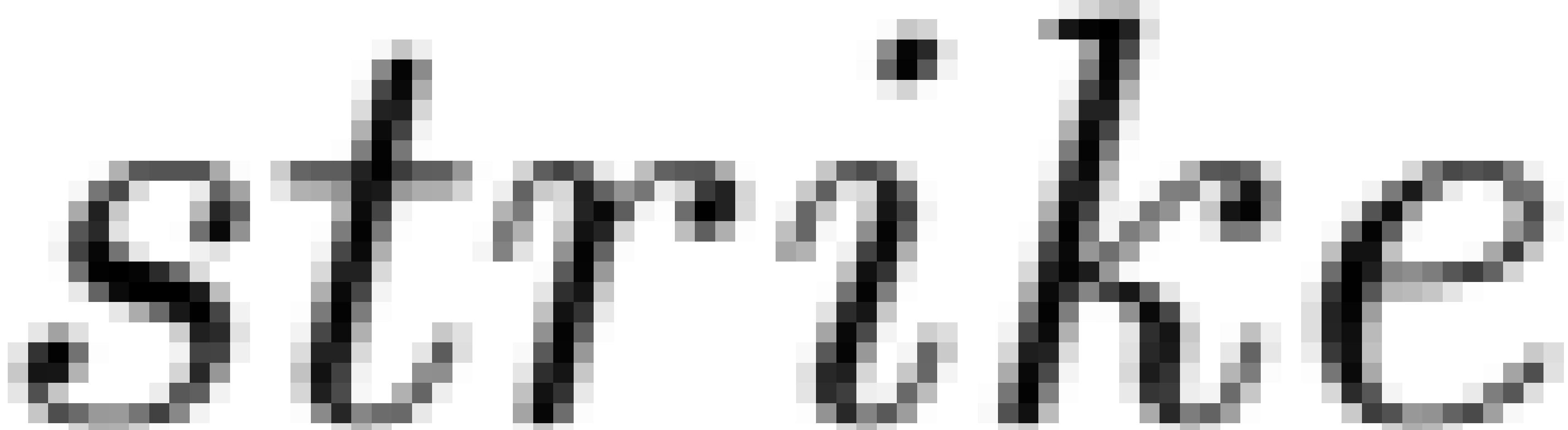
$$\begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}^T \begin{bmatrix} 60 & 0 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -0.707 & 47.5 & -12.5 \\ 0 & 40 & 0 \\ -12.5 & 47.5 & 0 \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 & 0 \\ 0 & 0 & 35 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$



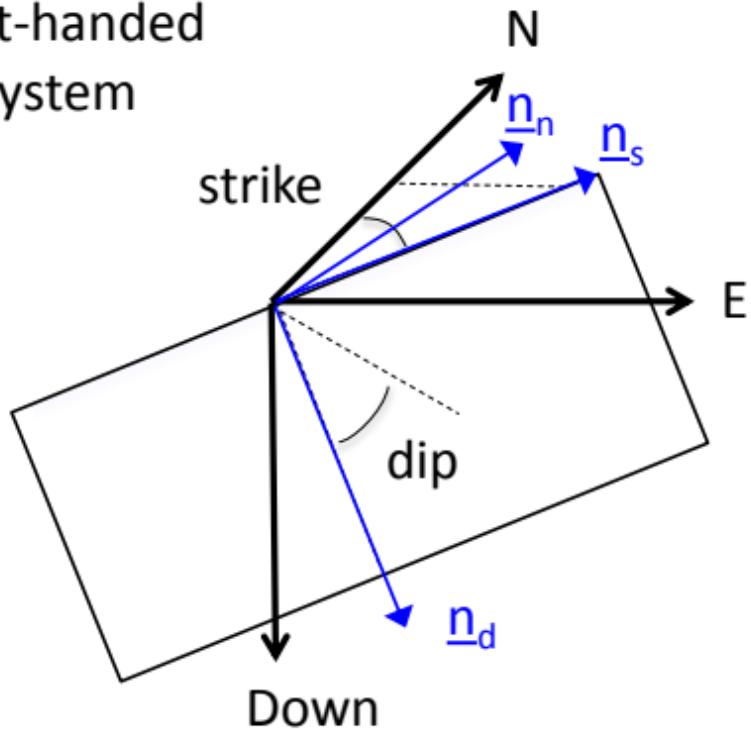






d-s-n

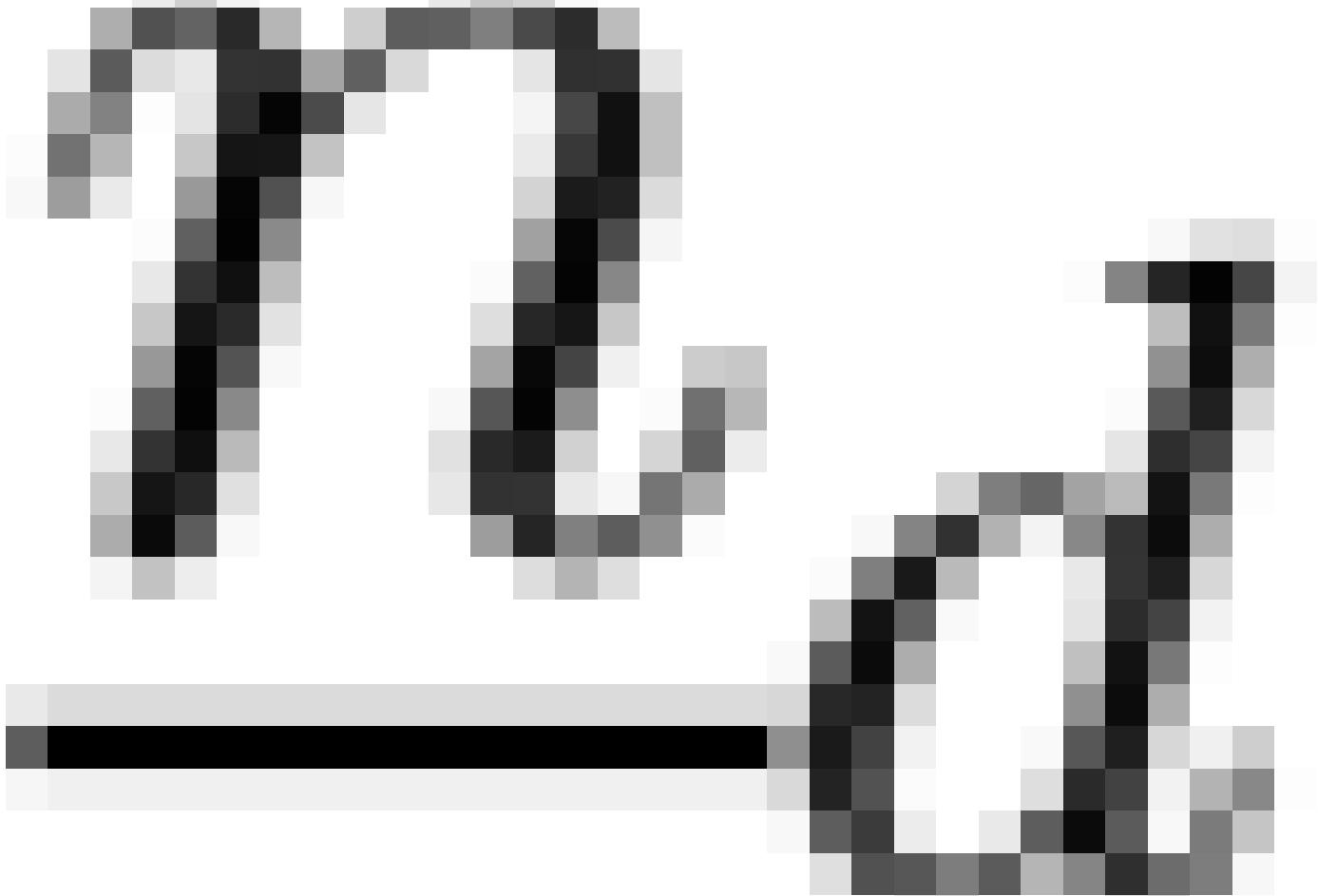
Right-handed
system

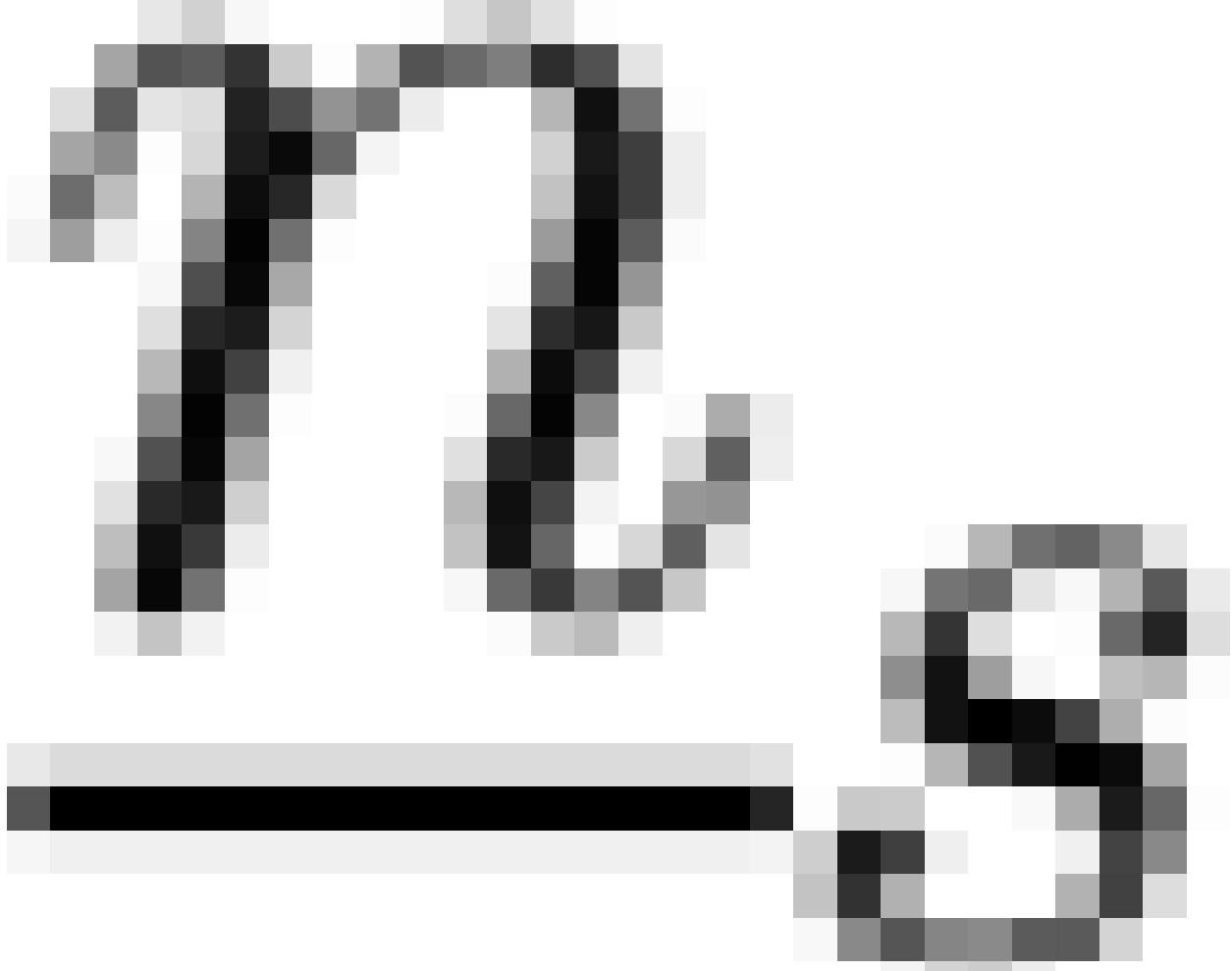


$$\underline{n}_n = \begin{bmatrix} -\sin(strike)\sin(dip) \\ \cos(strike)\sin(dip) \\ -\cos(dip) \end{bmatrix}$$

$$\underline{n}_s = \begin{bmatrix} \cos(strike) \\ \sin(strike) \\ 0 \end{bmatrix}$$

$$\underline{n}_d = \begin{bmatrix} -\sin(strike)\cos(dip) \\ \cos(strike)\cos(dip) \\ \sin(dip) \end{bmatrix}$$

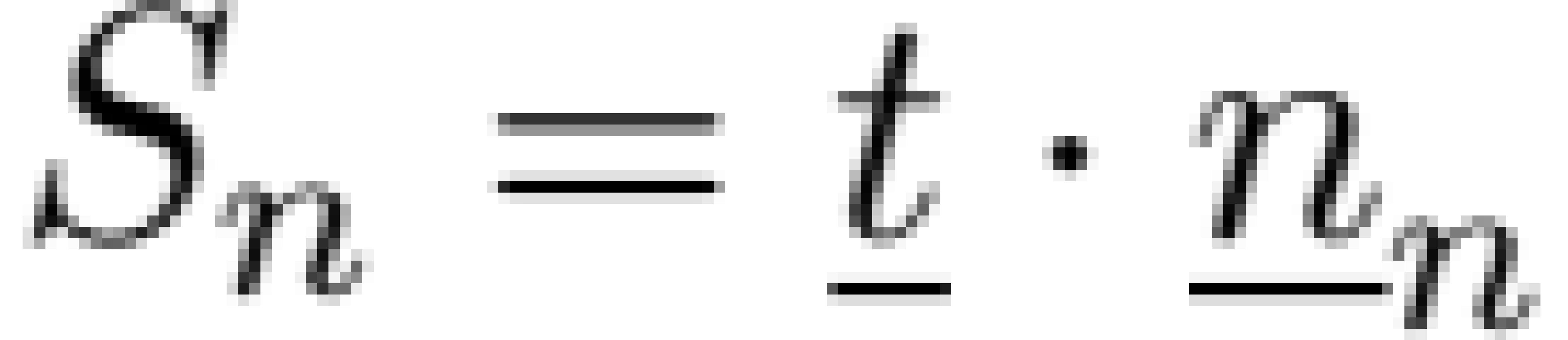








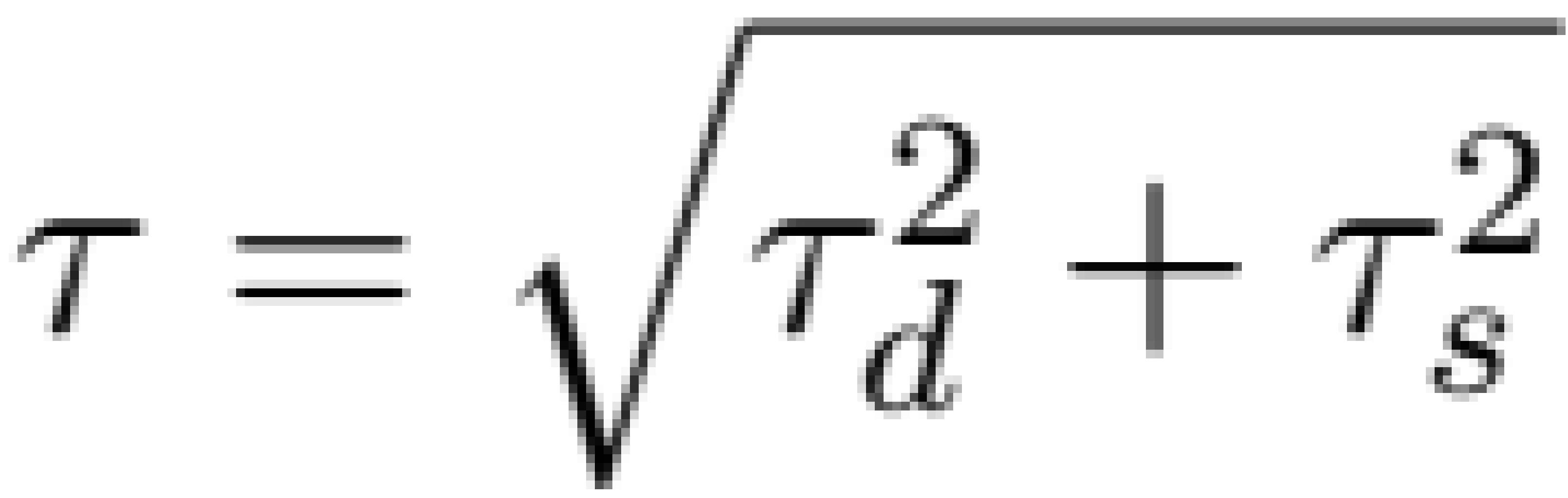


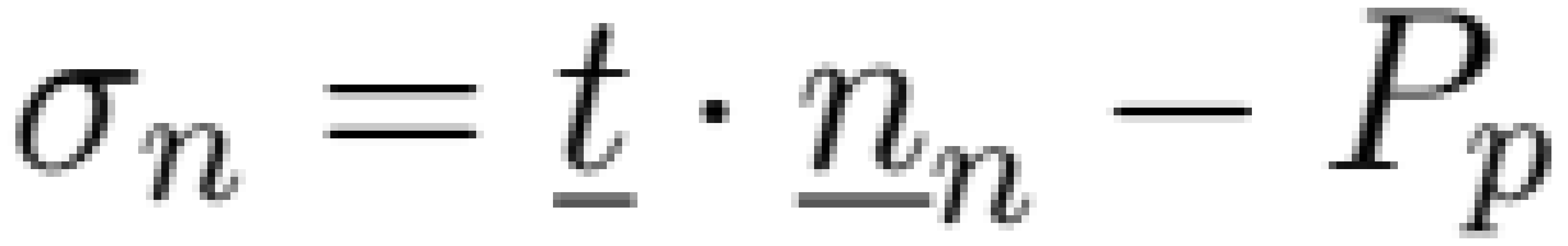


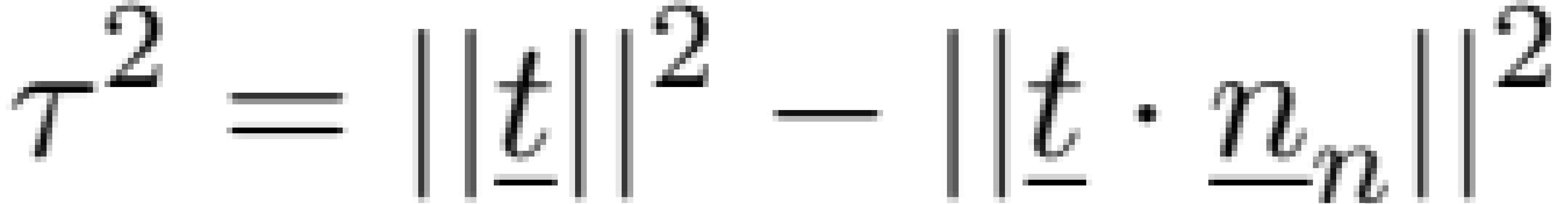


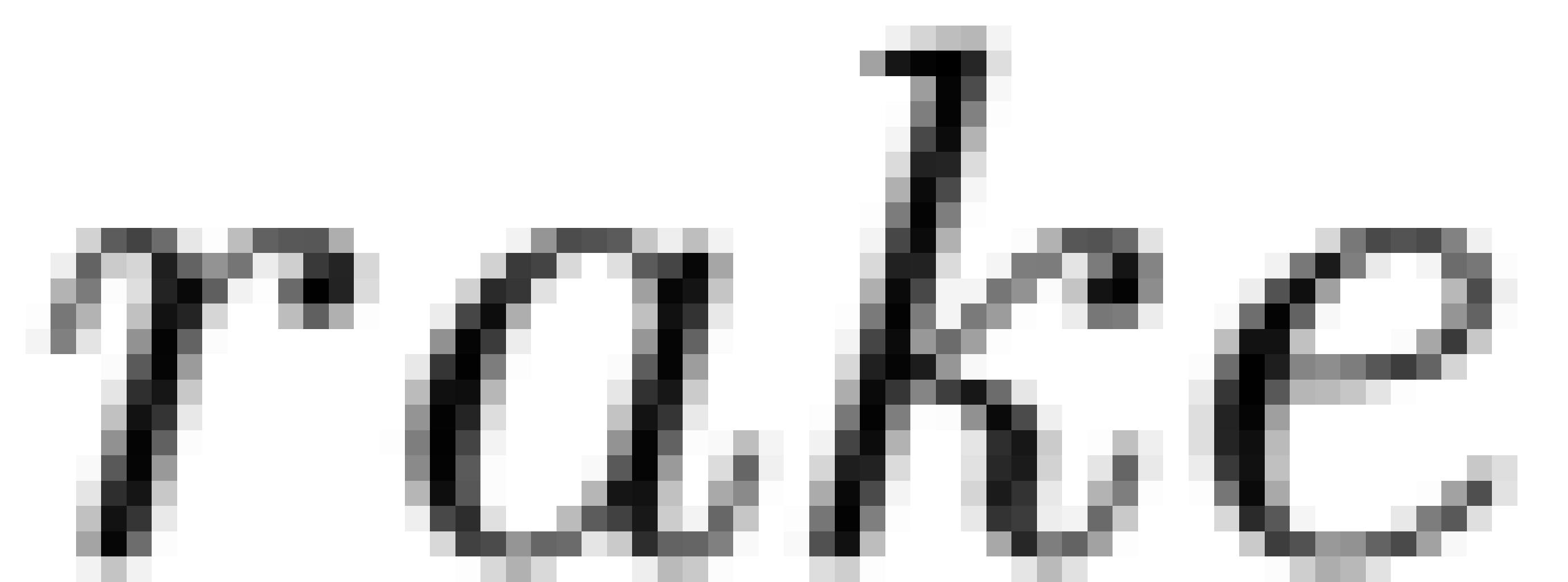
$$\tau_d = t \cdot \frac{r_d}{n_d}$$

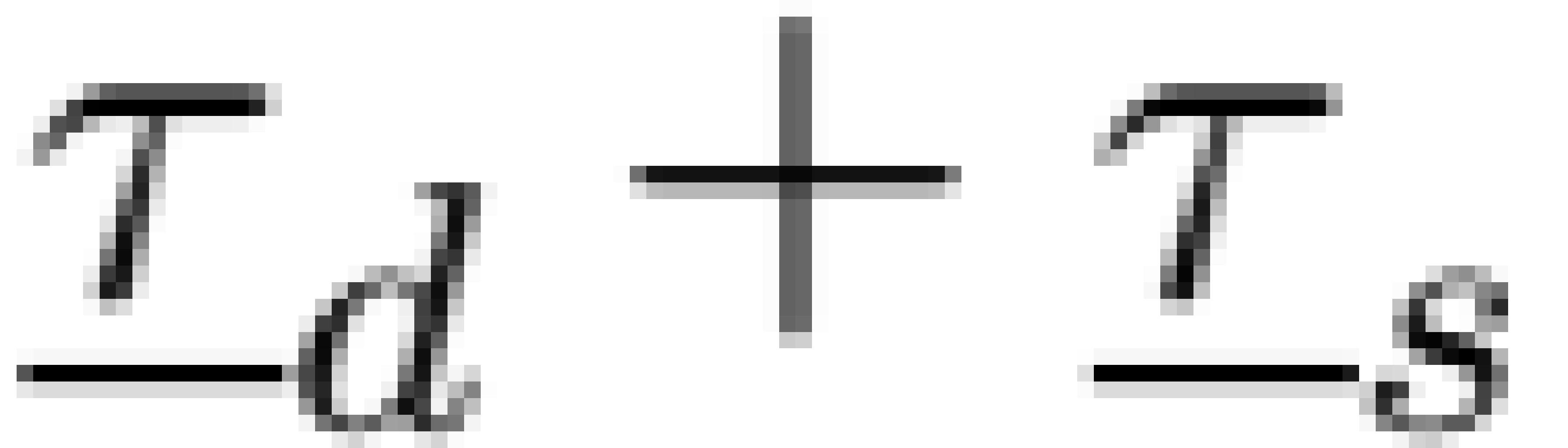
$$\tau_s = t \cdot \frac{r_s}{n_s}$$





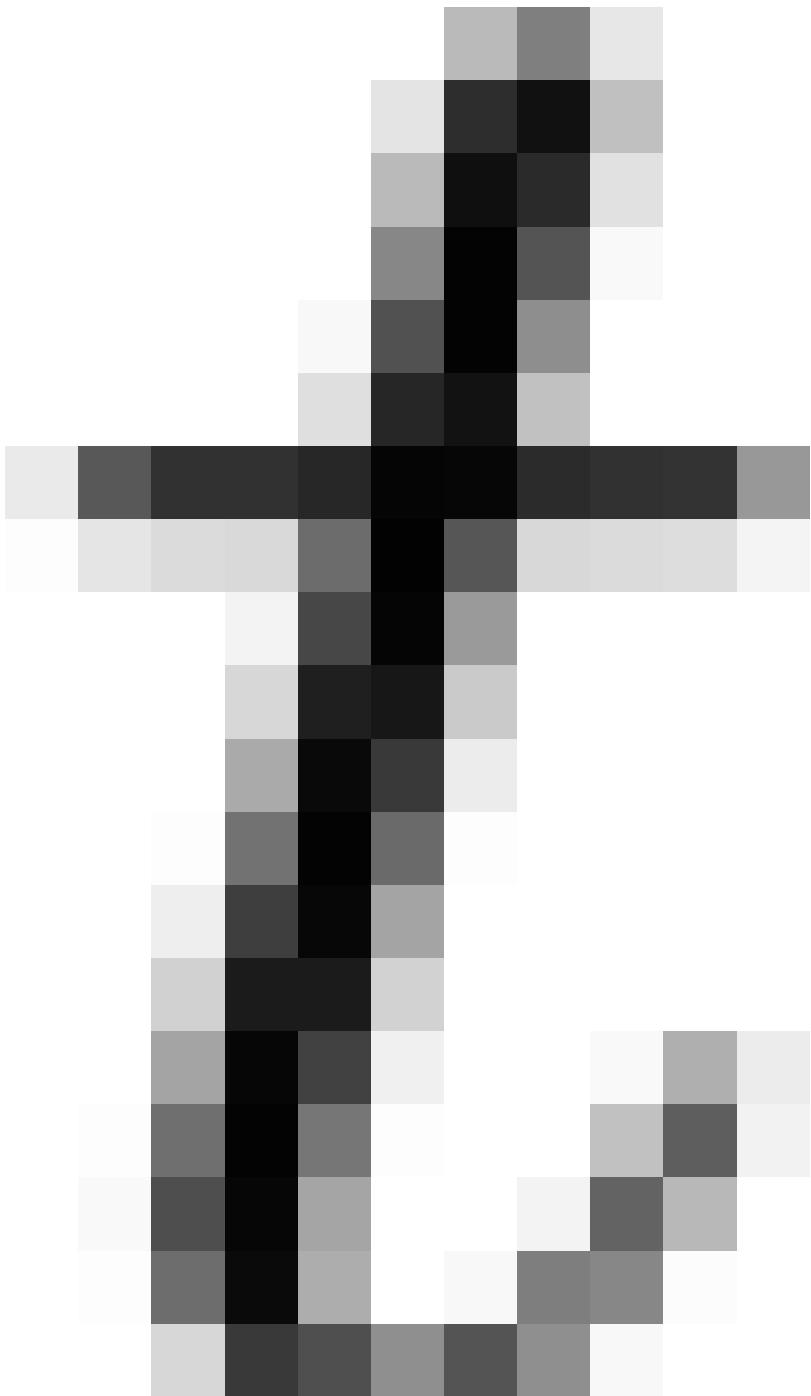


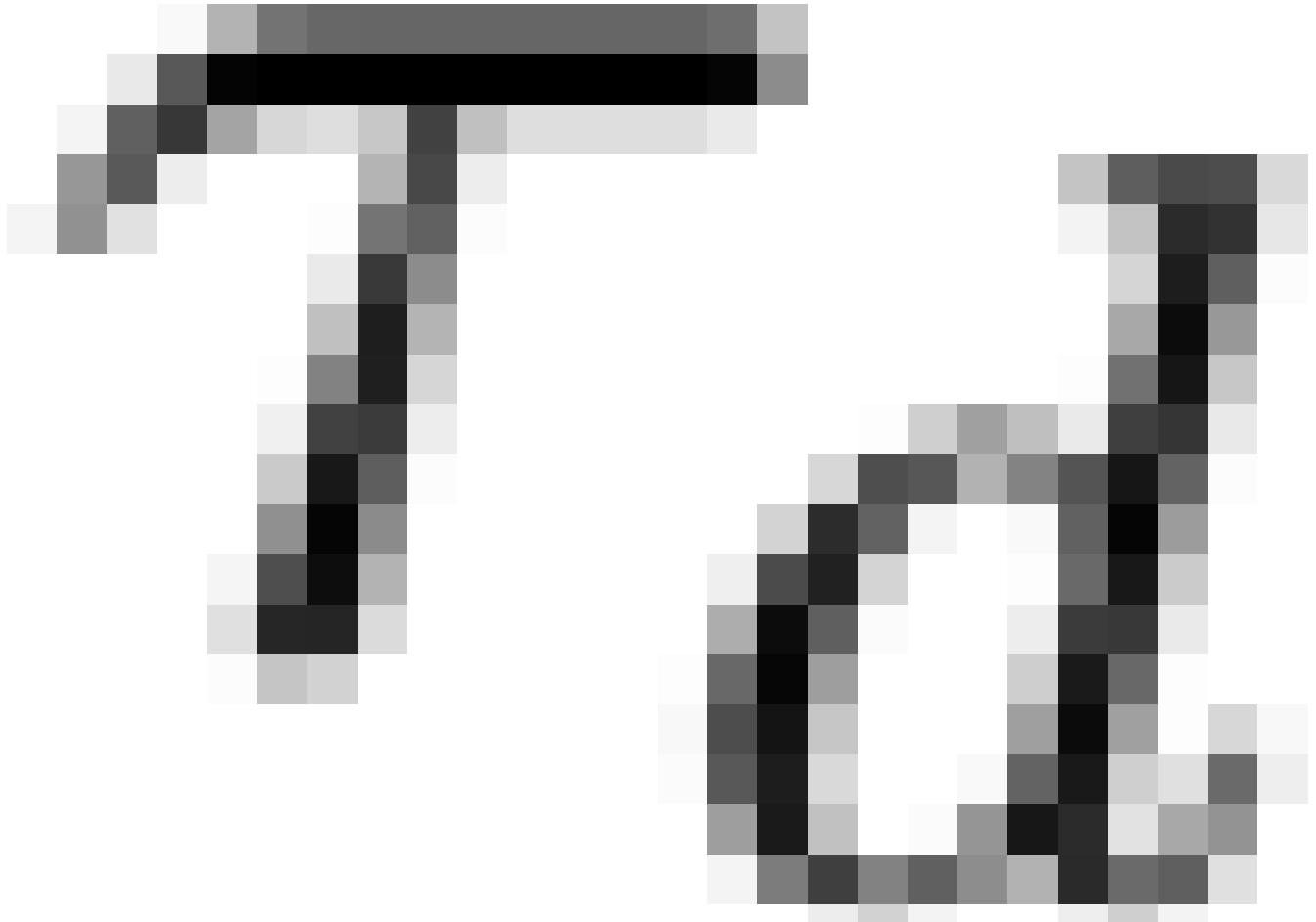


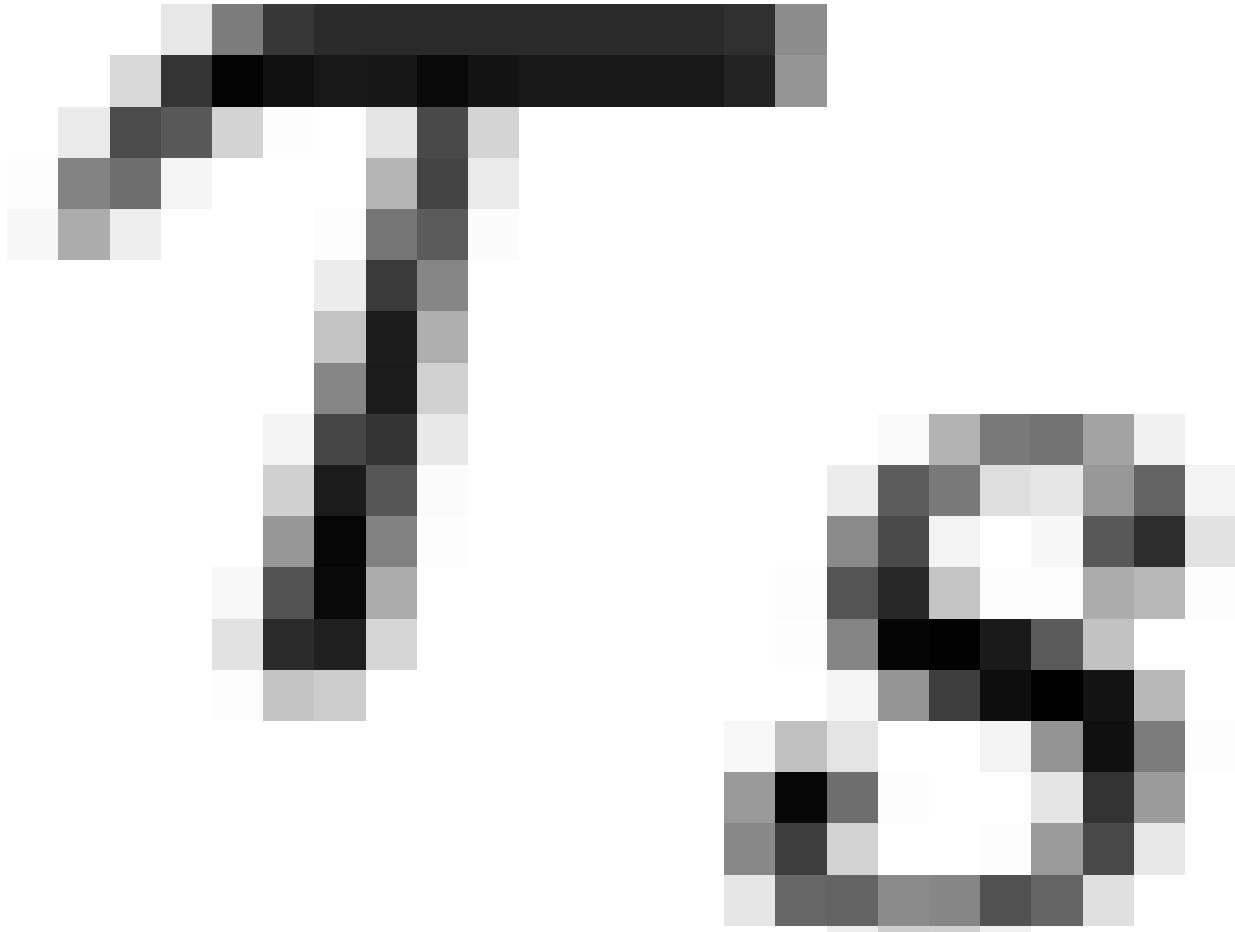


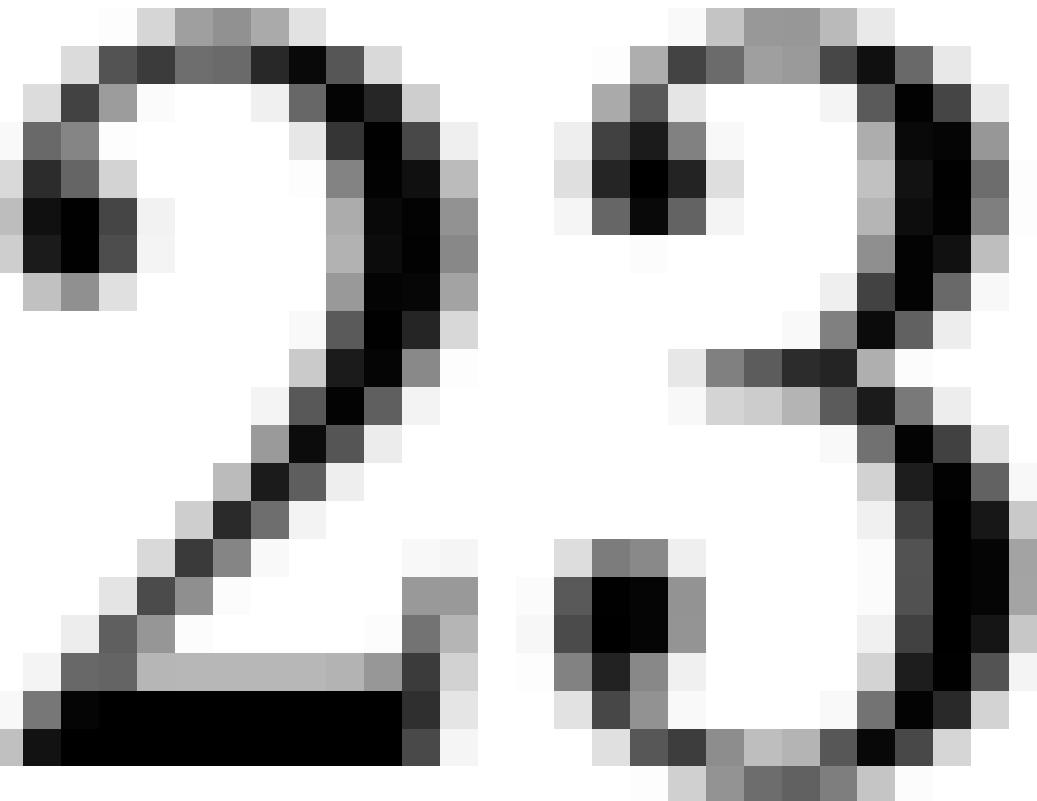
rake = arctan

$$\frac{\tau_d}{\tau_s}$$

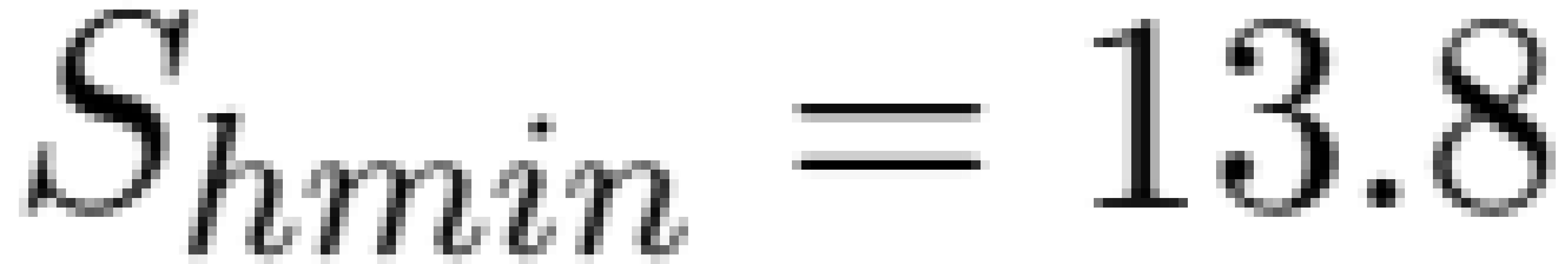


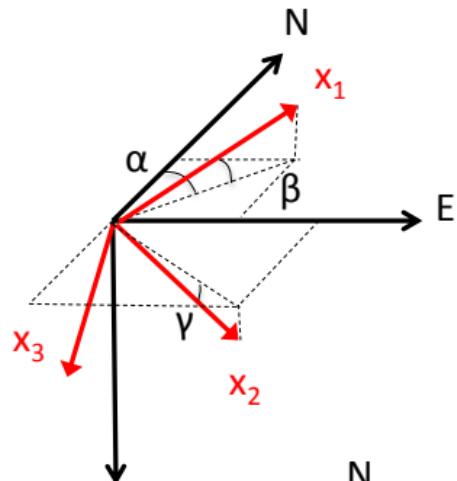










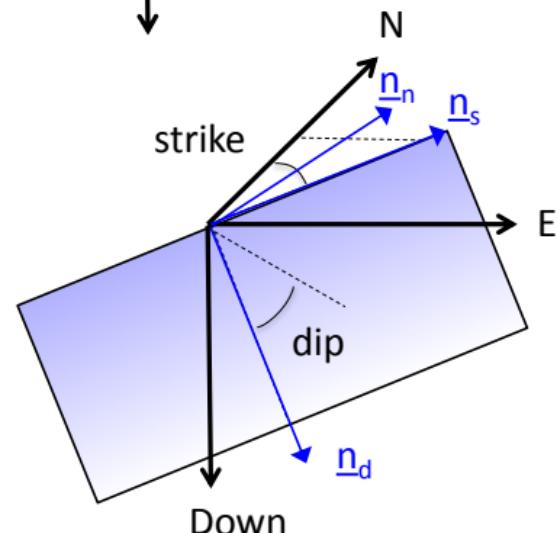


$$\underline{\underline{S}}_P = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 13.8 \end{bmatrix}$$

$a = 90^\circ$ Azimuth of $S_{h\min}$
 $b = 90^\circ$ $S_1 = S_V$
 $g = 0^\circ$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 13.8 & 0 \\ 0 & 0 & 23 \end{bmatrix}$$



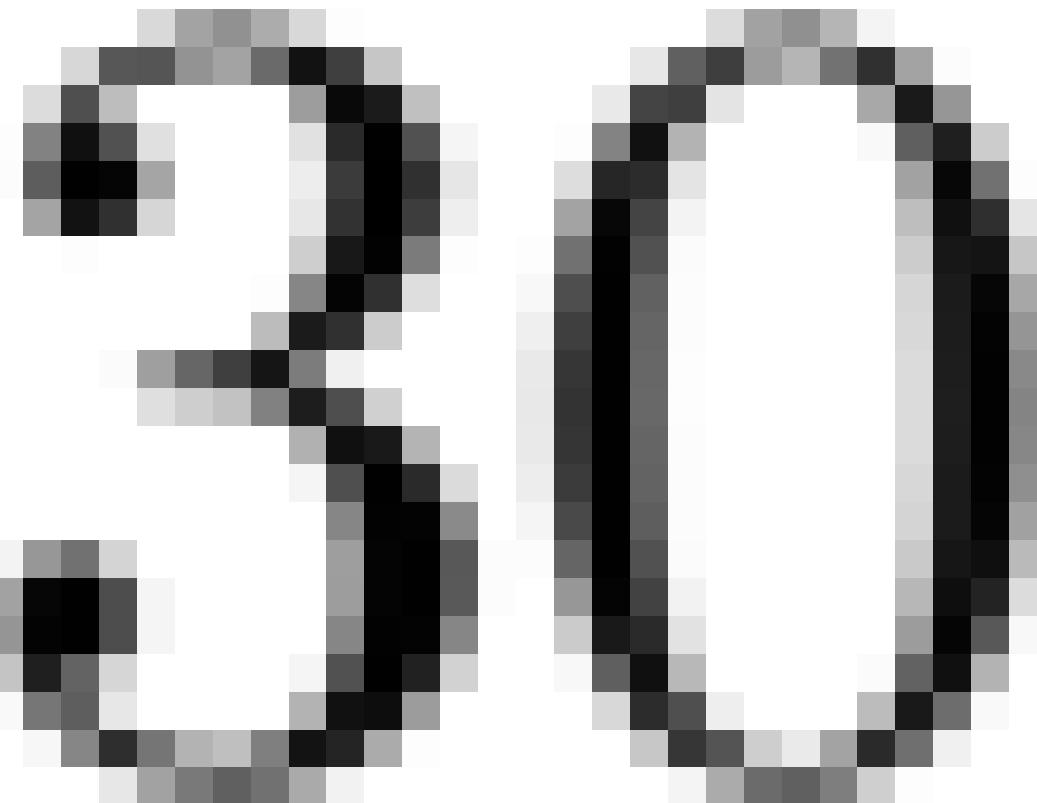
Fault geometry

Strike = 000°
 Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0 \\ 0.867 \\ -0.5 \end{bmatrix}$$

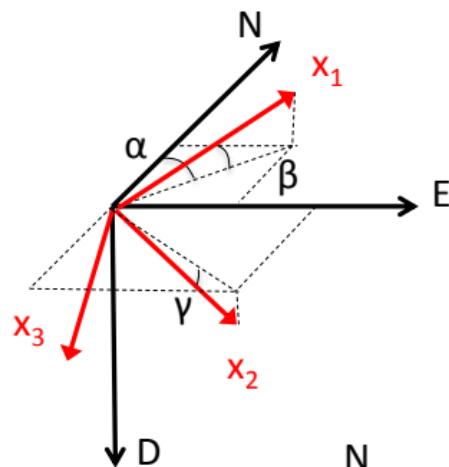
$\underline{t} = [0, 11.95, -11.50] \text{ MPa}$
 $S_n = 16.1 \text{ MPa}$
 $t_d = -3.98 \text{ MPa}$
 $t_s = 0 \text{ MPa}$
 rake = 90°

Actual stresses opposite to d-s-n



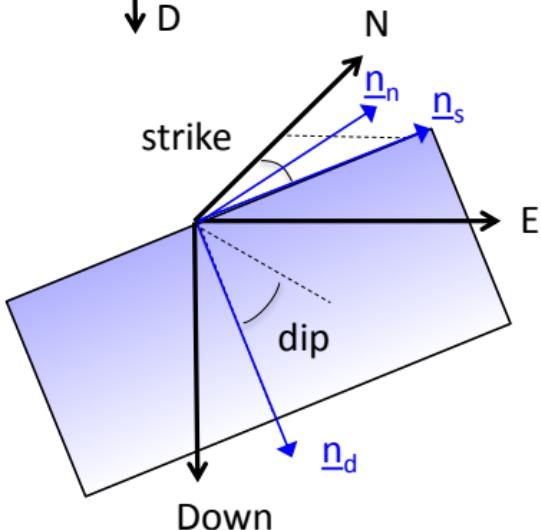






$$\underline{\underline{S}}_P = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix} \quad \begin{array}{ll} a = 120^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 90^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \\ 0.866 & 0.5 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$



Fault geometry

Strike = 060°

Dip = 90°

$$\underline{n}_n = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$\underline{t} = [-30.31, 27.5, 0] \text{ MPa}$$

$$S_n = 40 \text{ MPa}$$

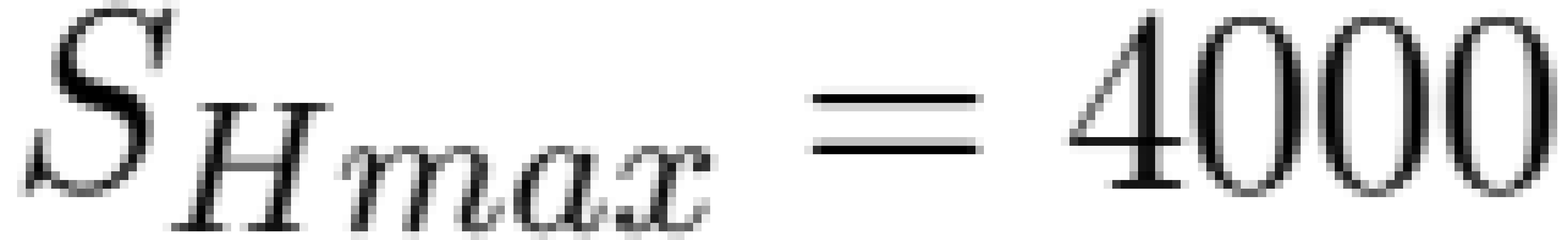
$$t_d = 0 \text{ MPa}$$

$$t_s = 8.66 \text{ MPa}$$

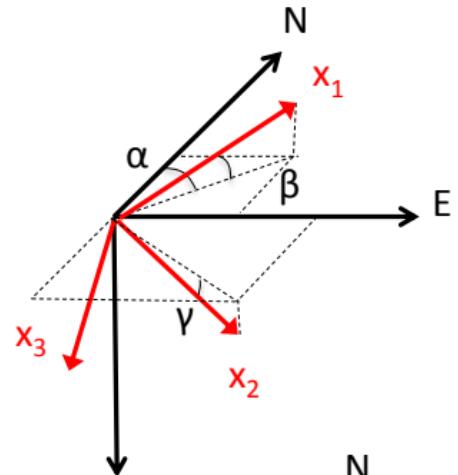
$$rake = 0^\circ$$

Actual stresses opposite to d-s-n



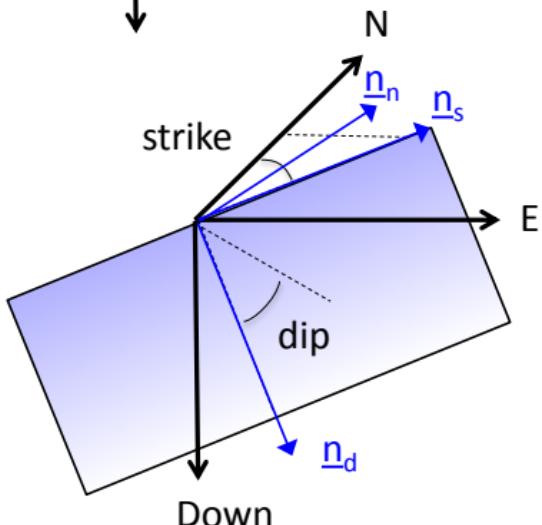






$$\underline{\underline{S}}_P = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \quad \begin{array}{ll} a = 90^\circ & \text{Azimuth of } S_{h\min} \\ b = 90^\circ & \\ g = 0^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$



Fault geometry

Strike = 045°

Dip = 60°

$$\underline{n}_n = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [-2450, 1840, -2500] \text{ psi}$$

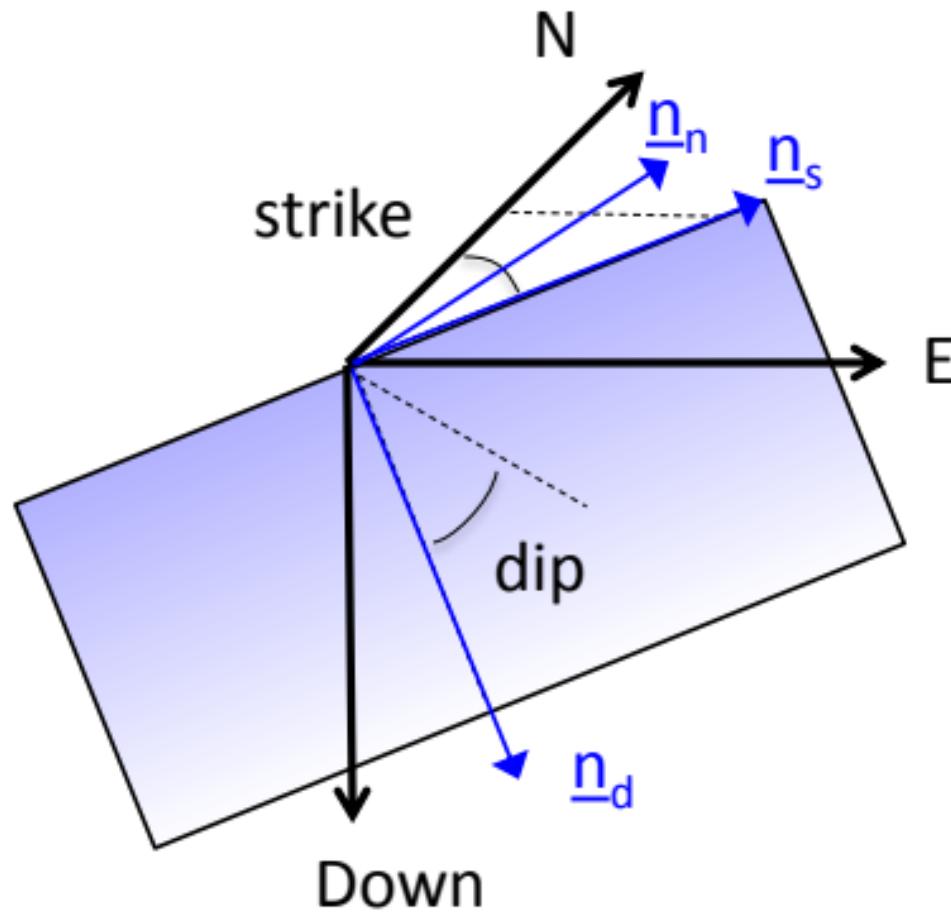
$$S_n = 3870 \text{ psi}$$

$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

$$\text{rake} = 56.3^\circ$$

Actual stresses opposite to d-s-n



Fault geometry

Strike = 225°

Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix}$$

$$t = [2450, -1840, -2500] \text{ psi}$$

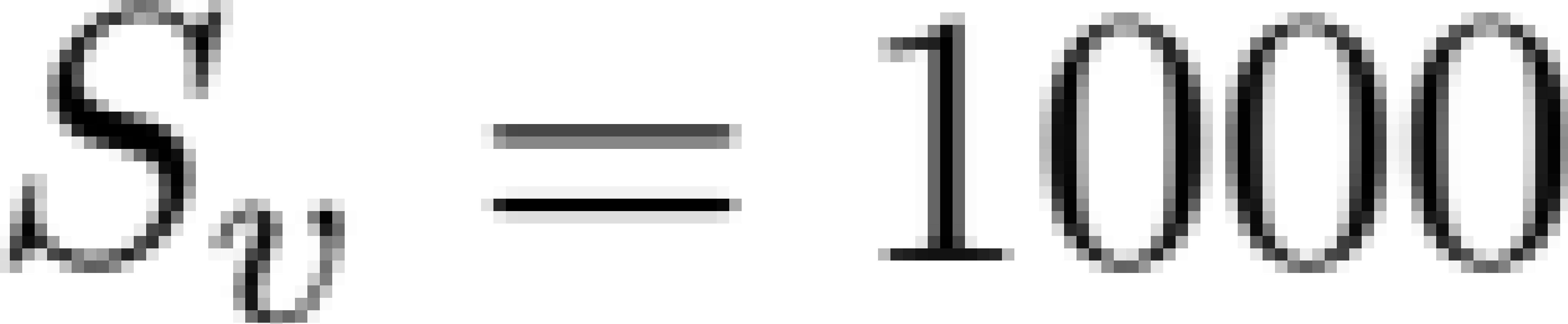
$$S_n = 3870 \text{ psi}$$

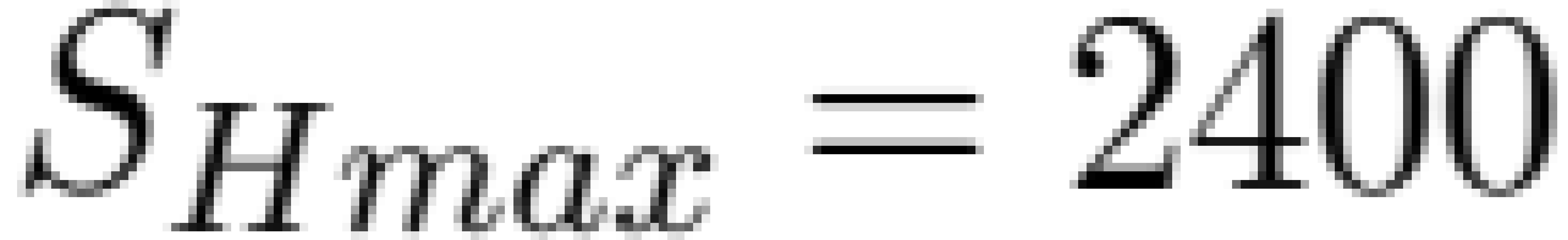
$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

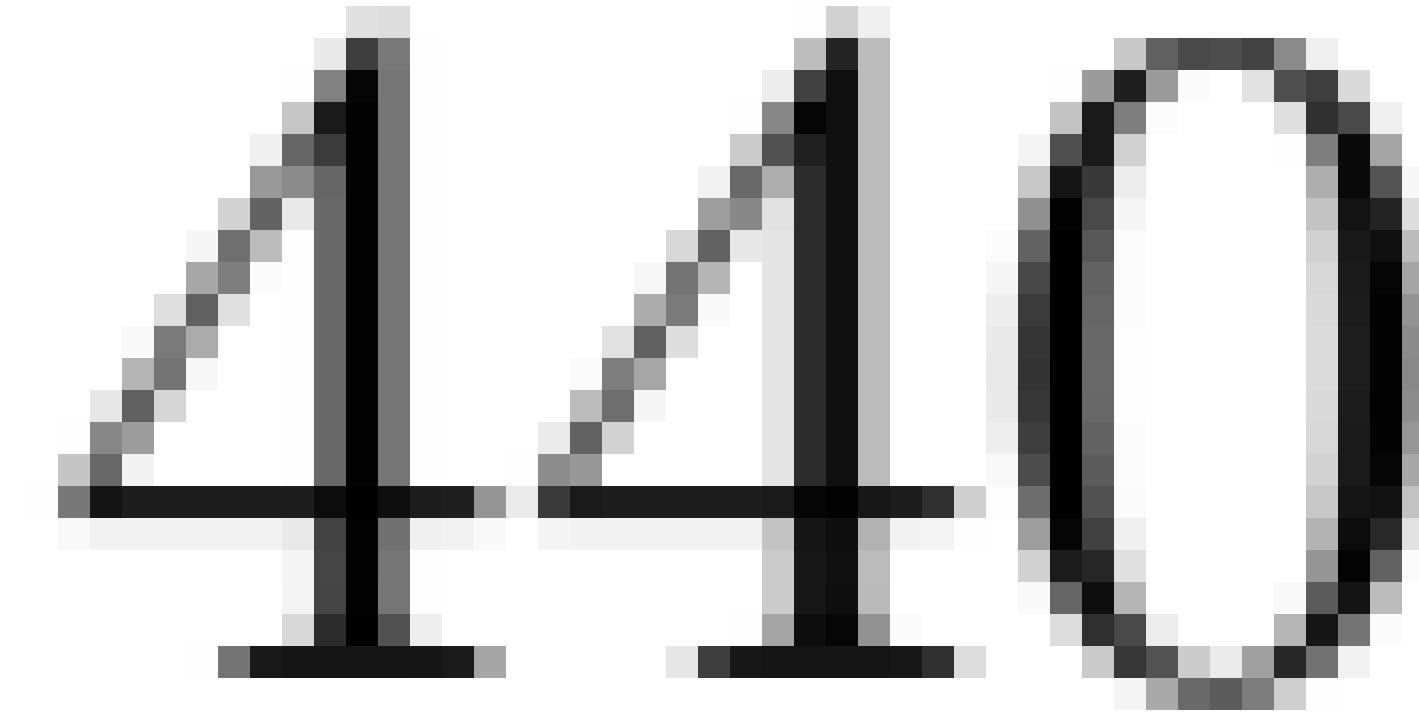
$$\text{rake} = 56.3^\circ$$

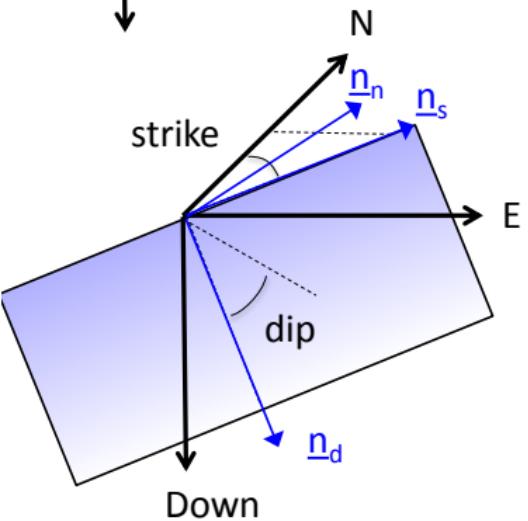
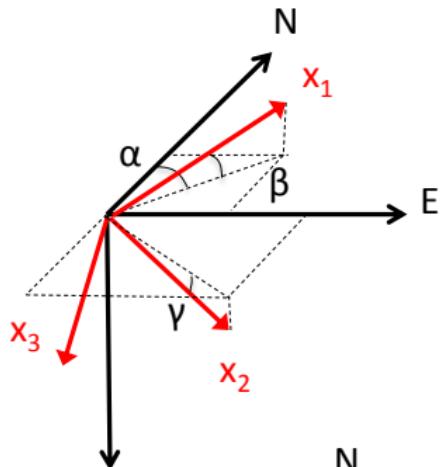
Actual stresses opposite to d-s-n











$$\underline{\underline{S}}_P = \begin{bmatrix} 2400 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \quad \begin{array}{ll} \alpha = 150^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 0^\circ & S_3 = S_V \end{array}$$

$$P_w = 440 \text{ psi}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.866 & 0.5 & 0 \\ -0.5 & -0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

Fault geometry

Strike = 120°

Dip = 70°

$$\underline{n}_n = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix}$$

$$\underline{t} = [-1465, -281, -342] \text{ psi}$$

$$S_n = 1441 \text{ psi}$$

$$t_d = 160 \text{ psi}$$

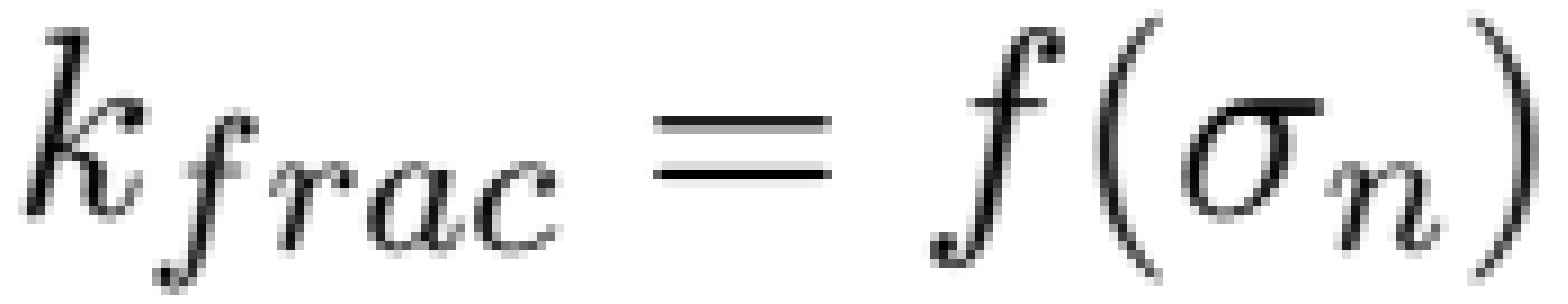
$$t_s = 488 \text{ psi}$$

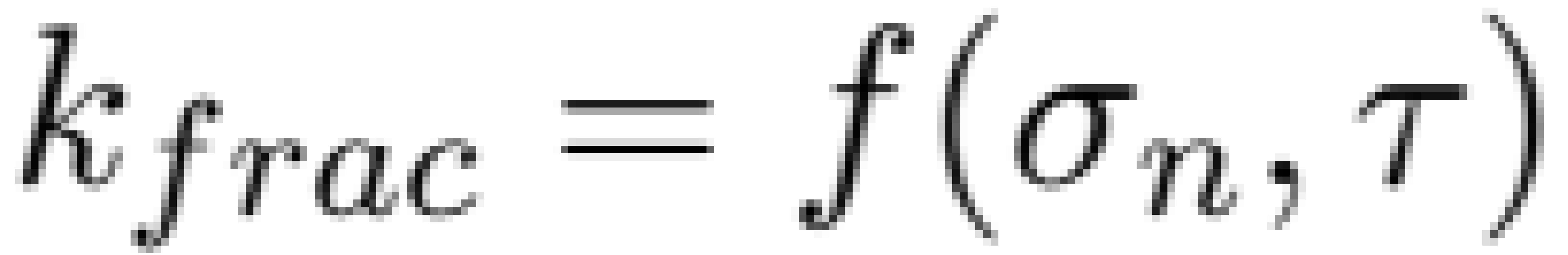
$$\text{rake} = 18.21^\circ$$

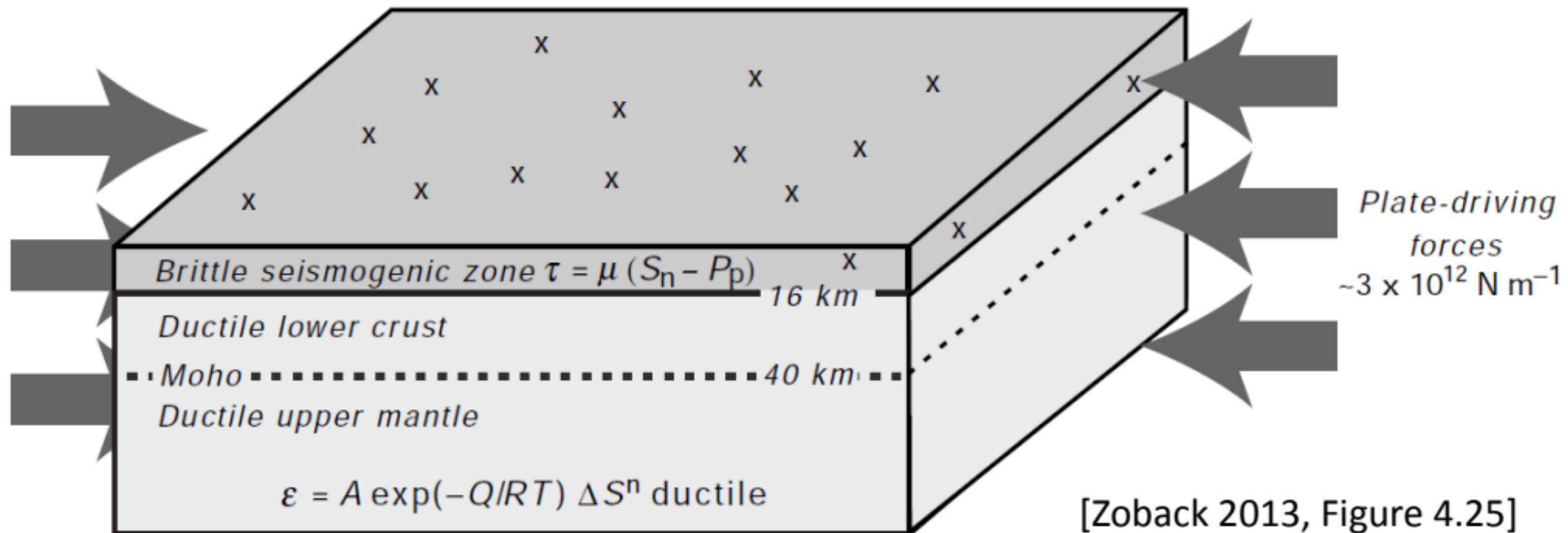
$$t / S_n = 0.51$$

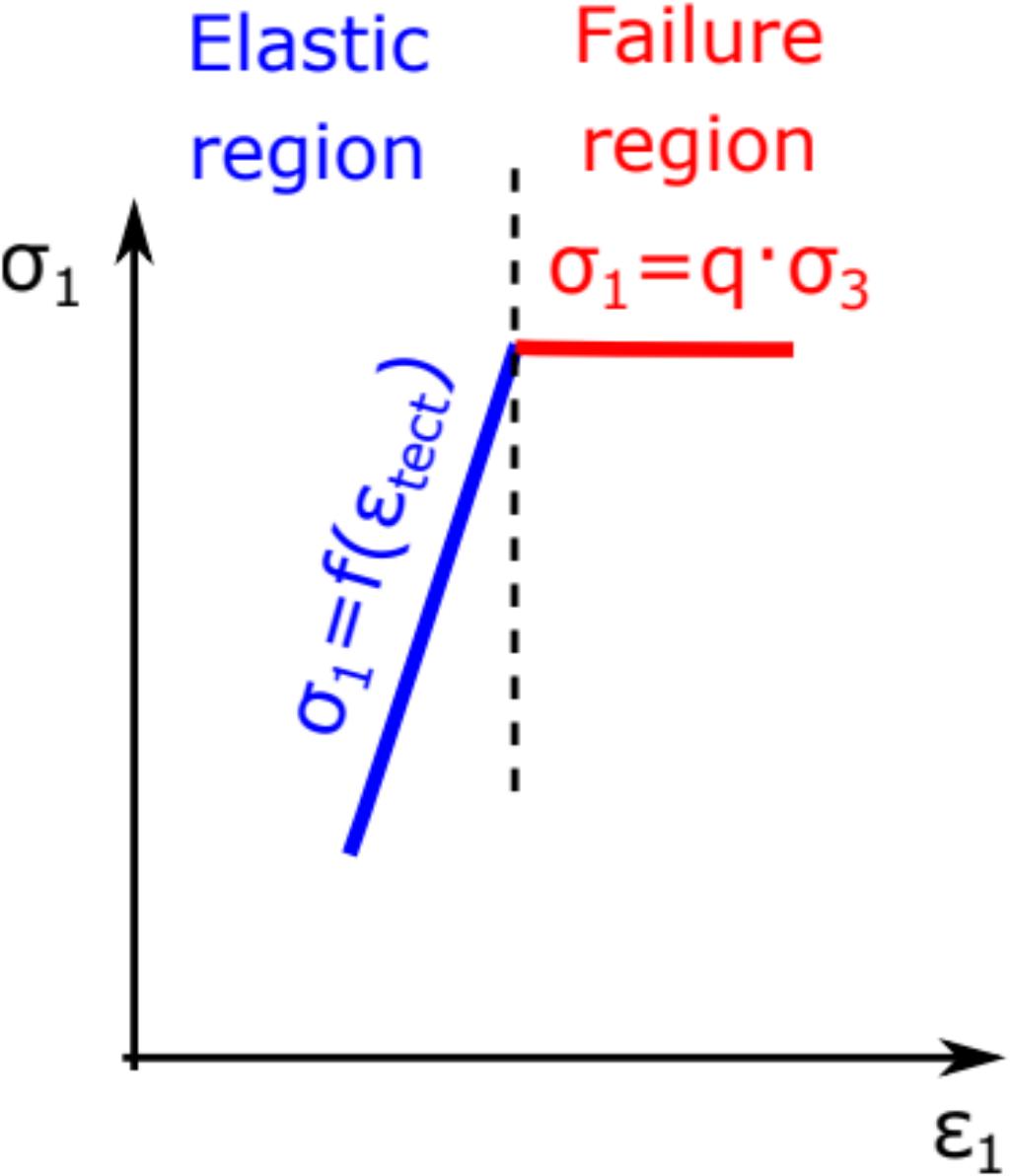
Actual stresses opposite to d-s-n





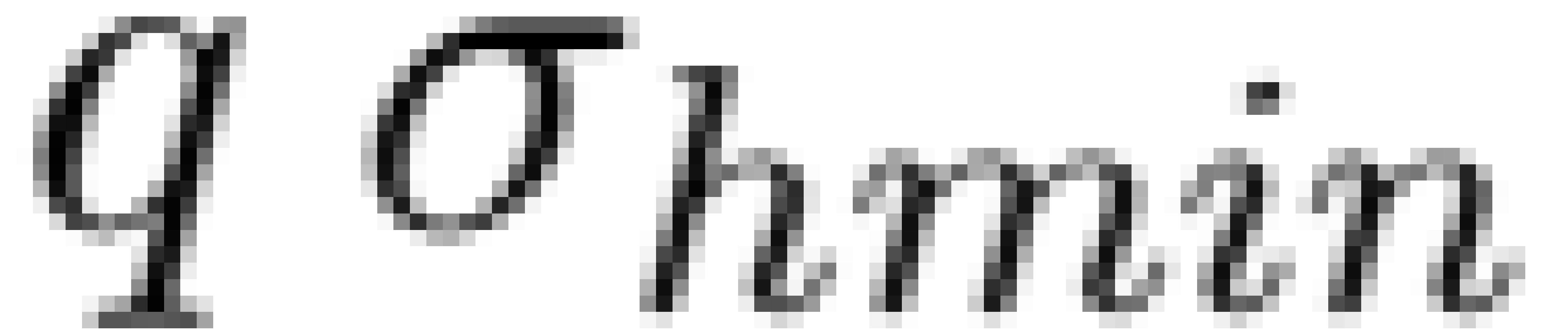


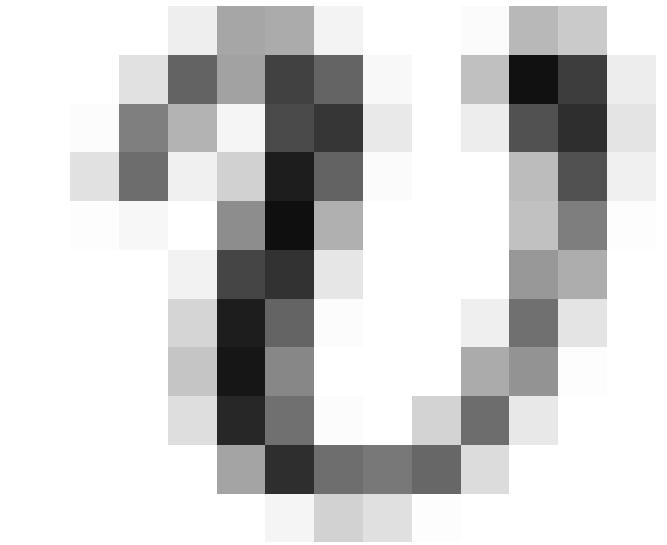
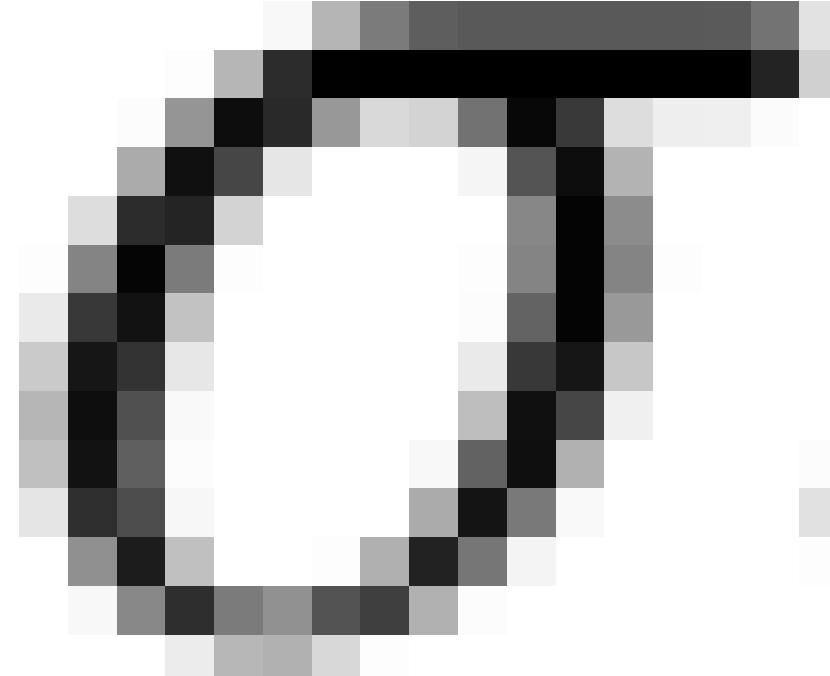
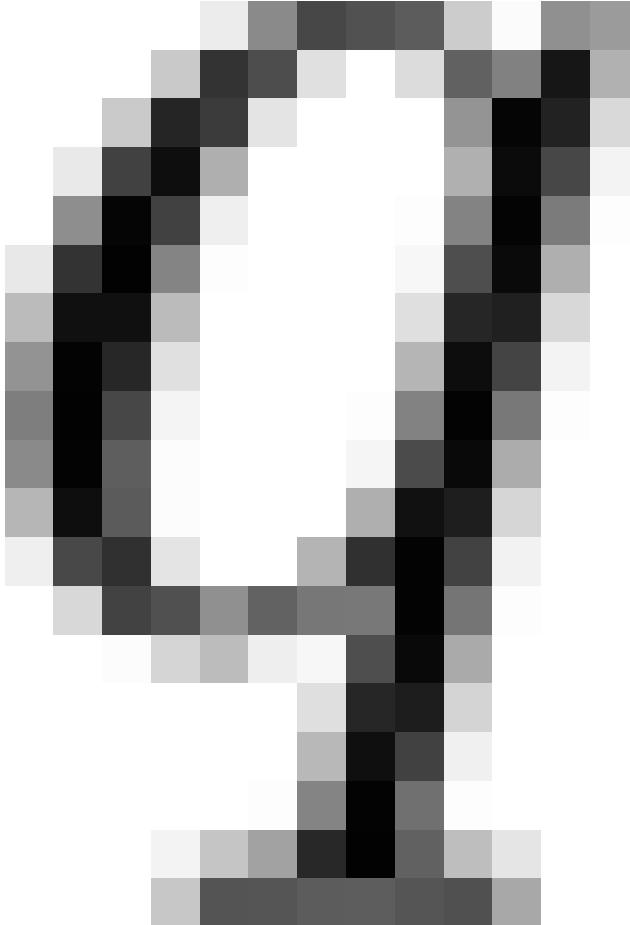


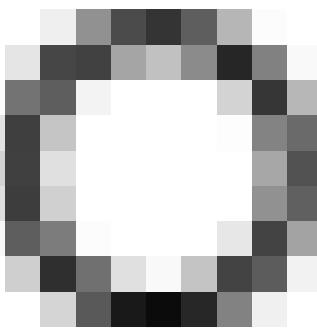
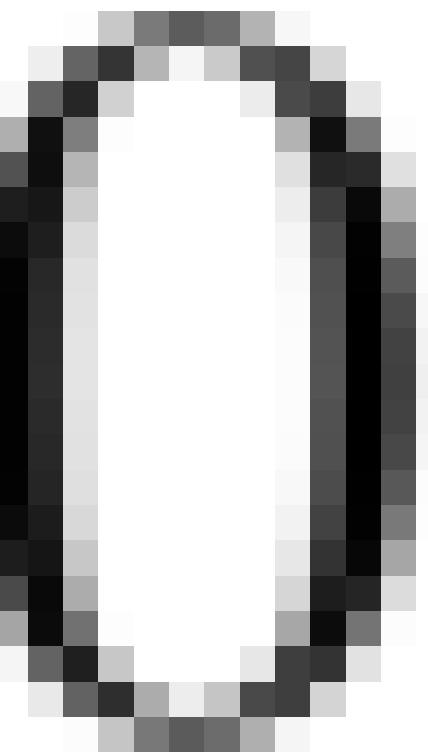
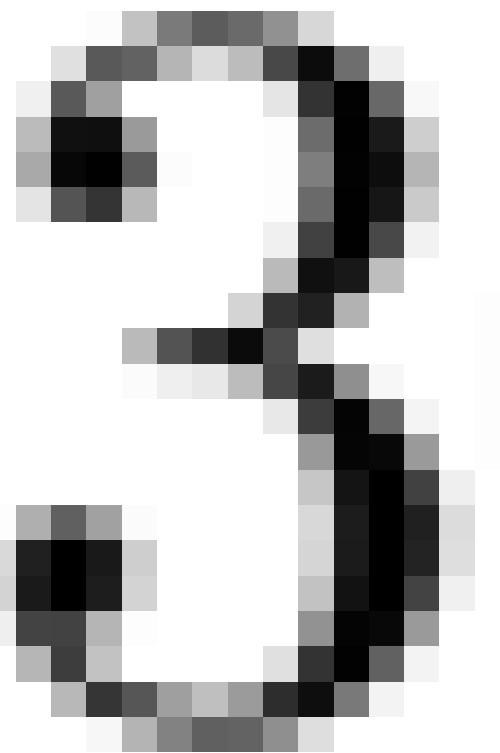
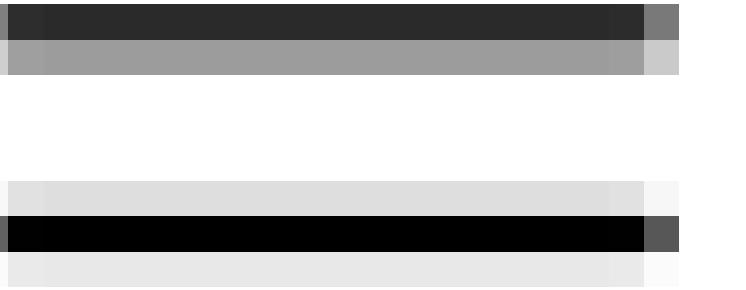
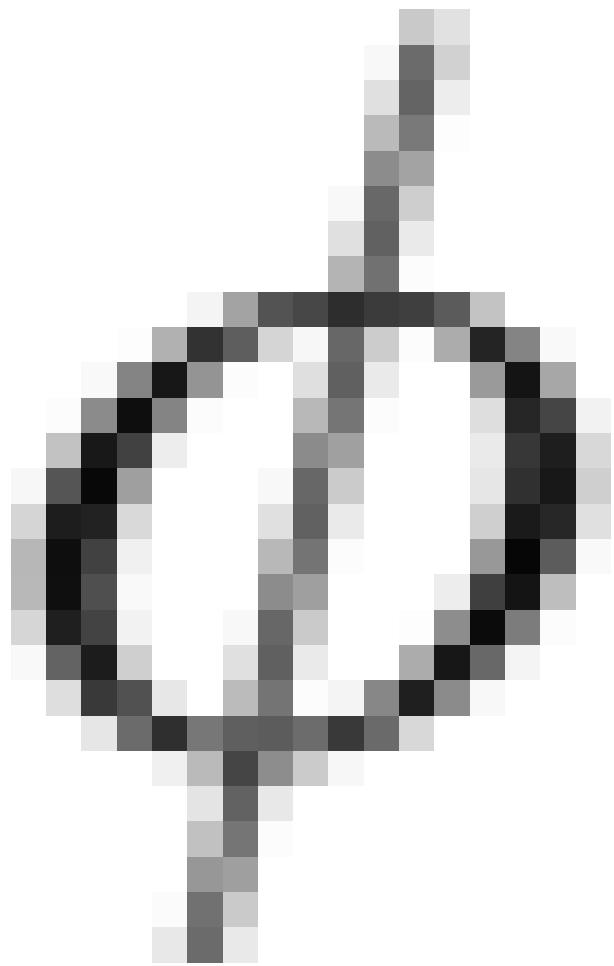


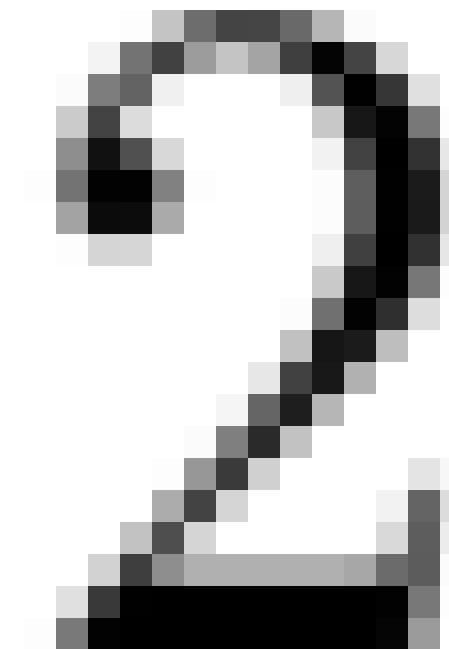
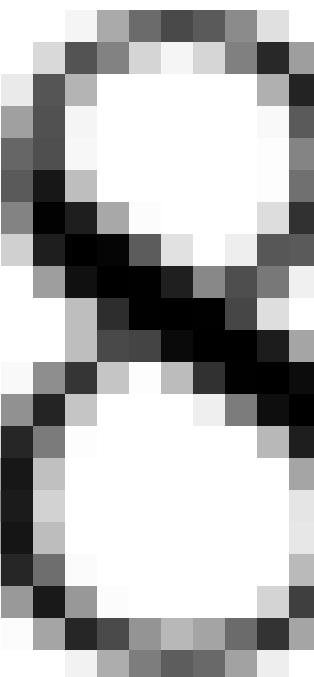
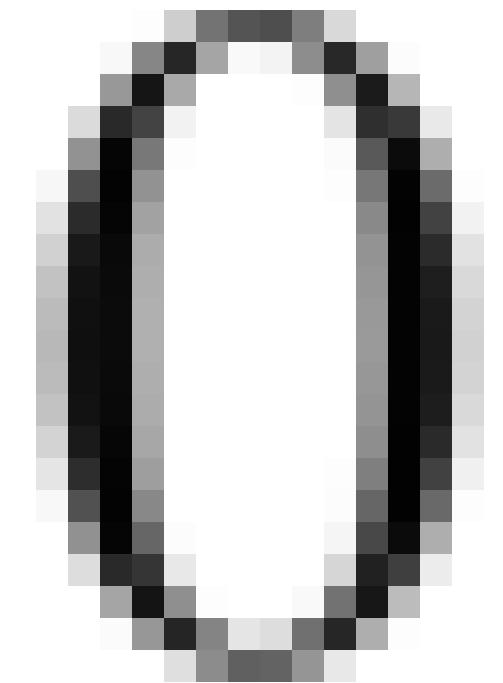


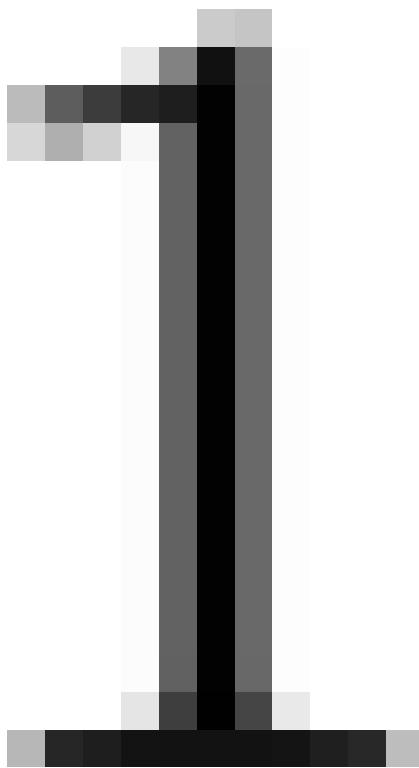


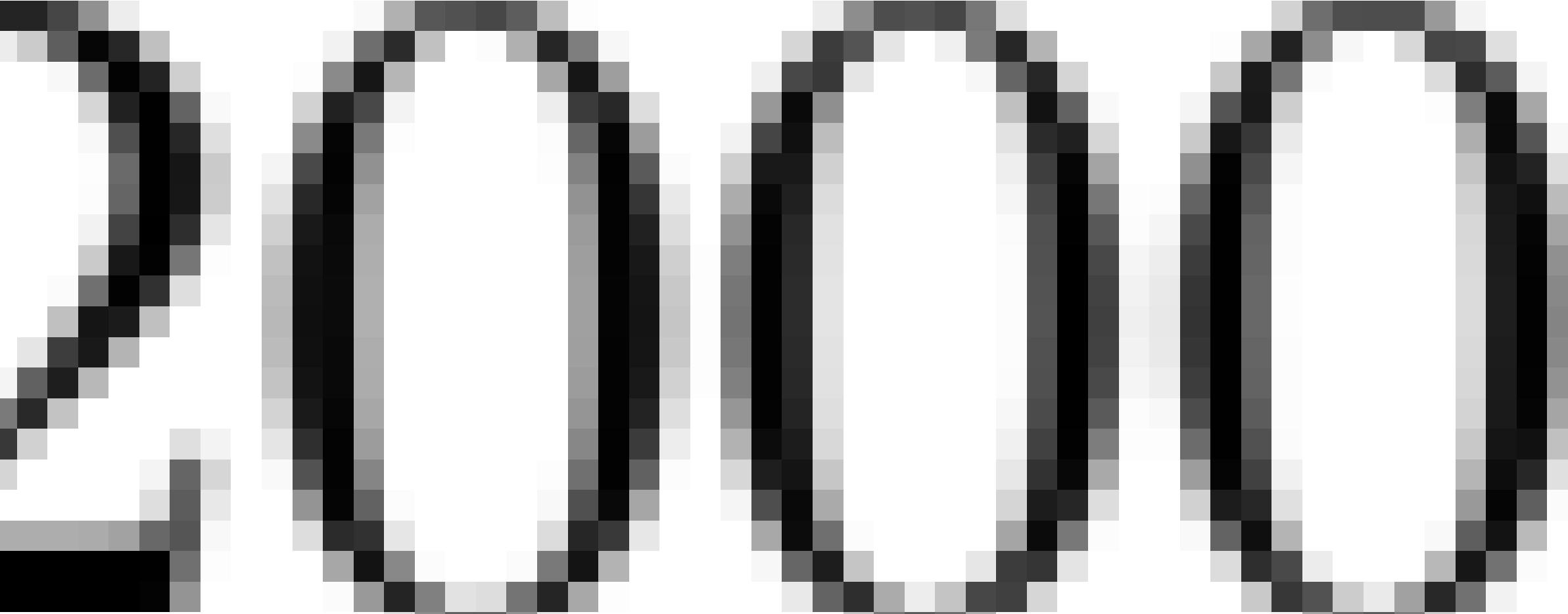


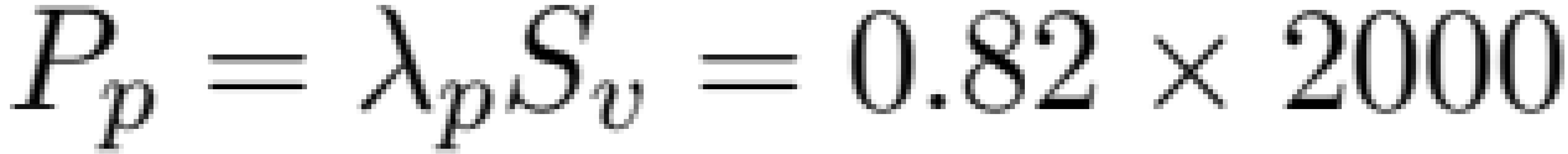


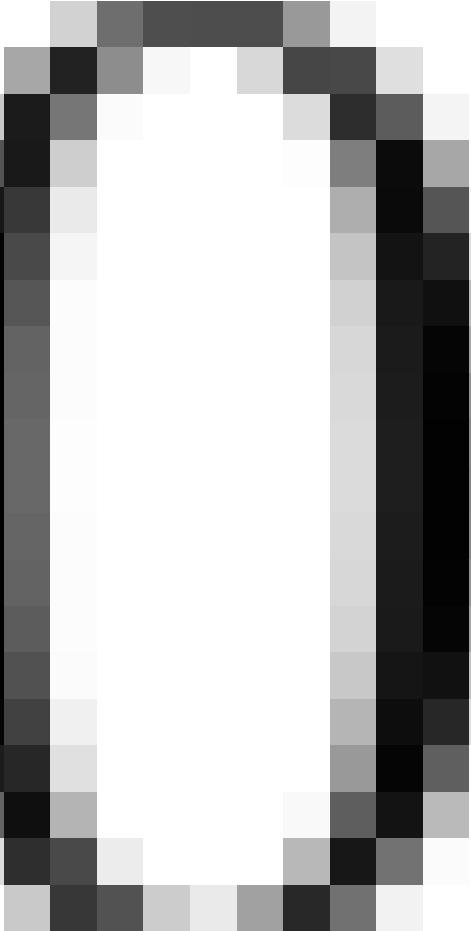
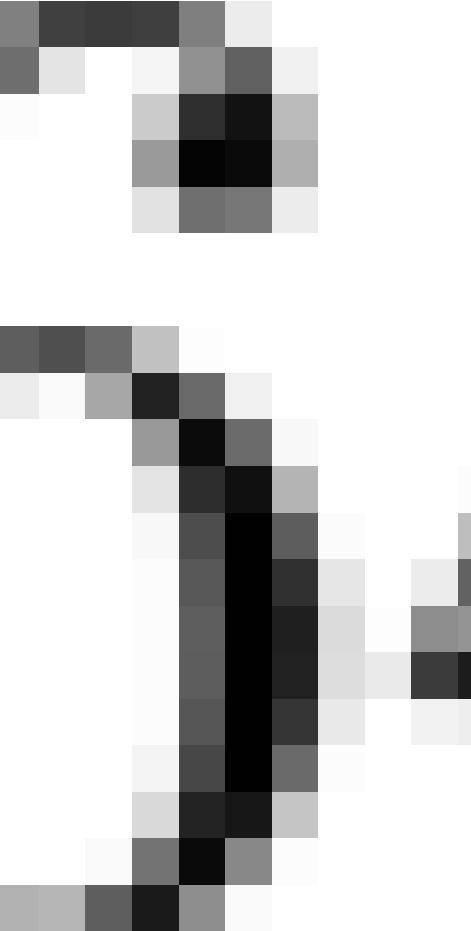
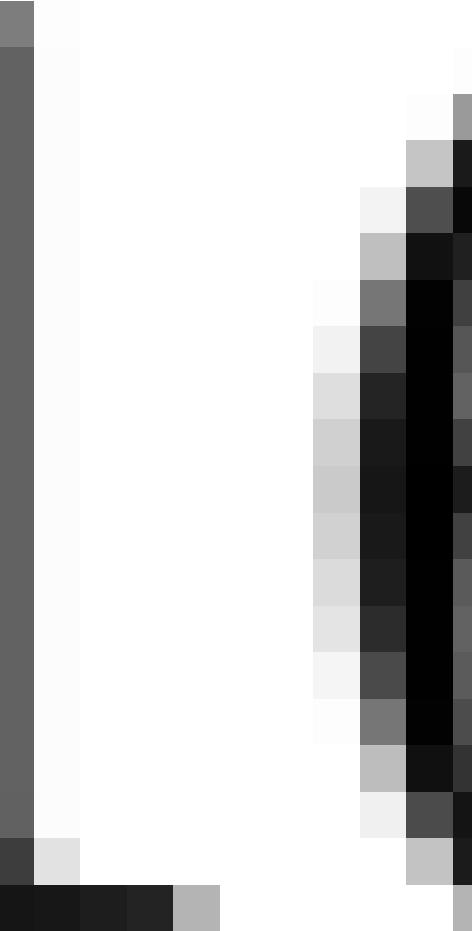


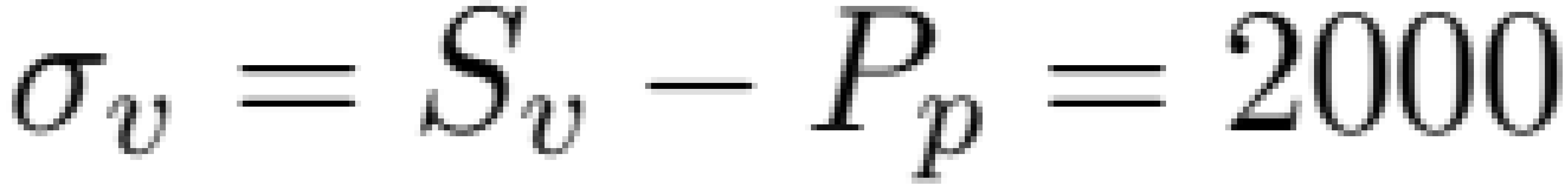


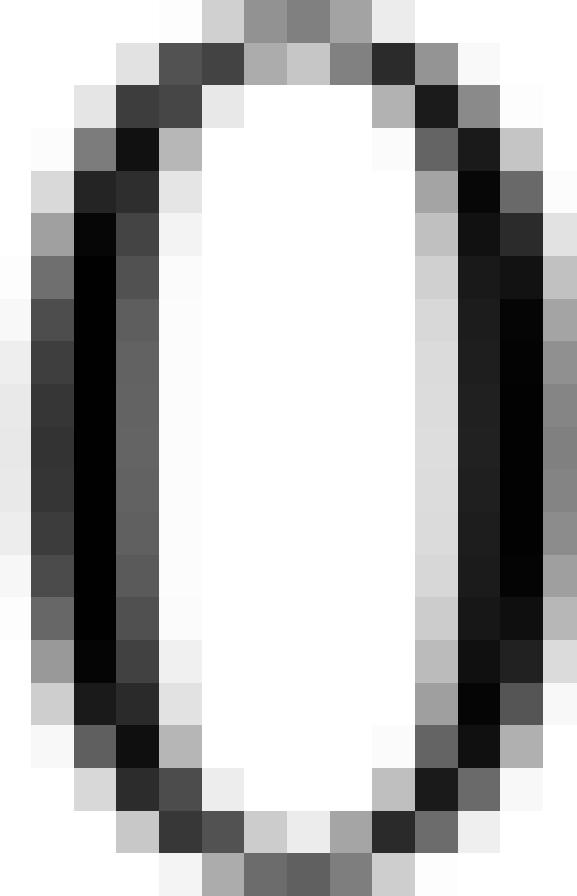
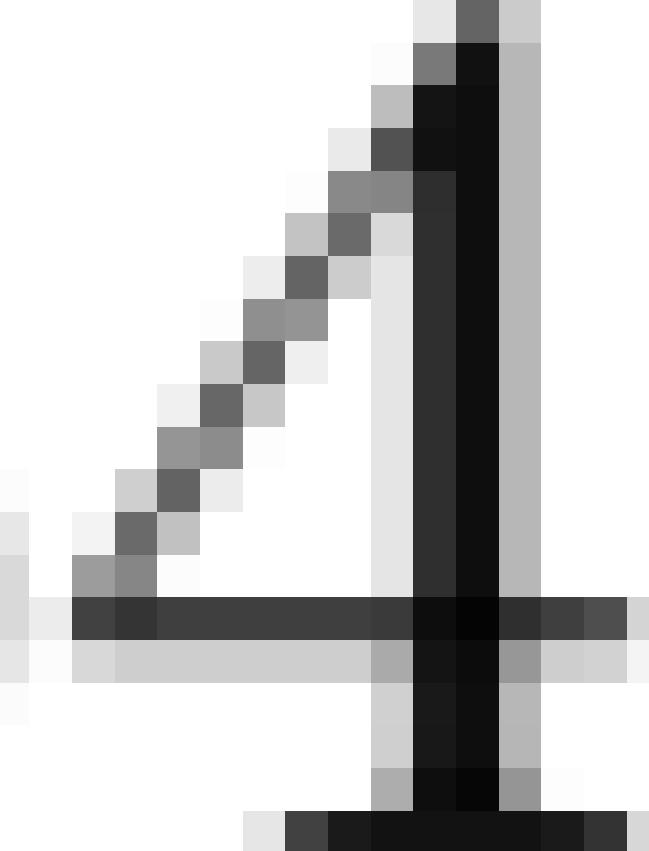
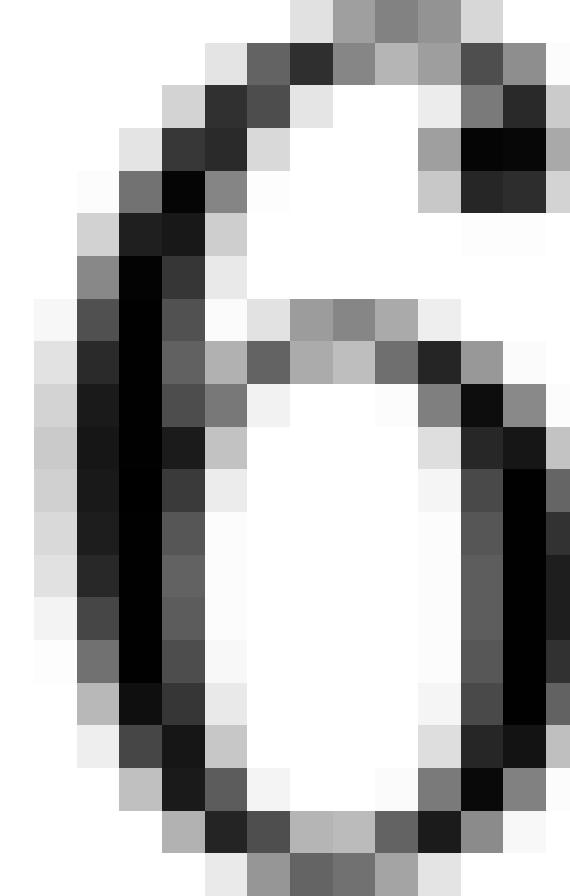
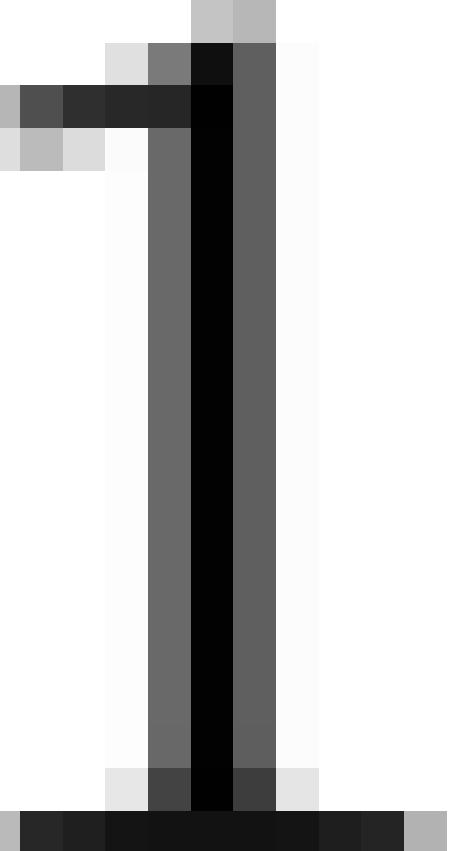


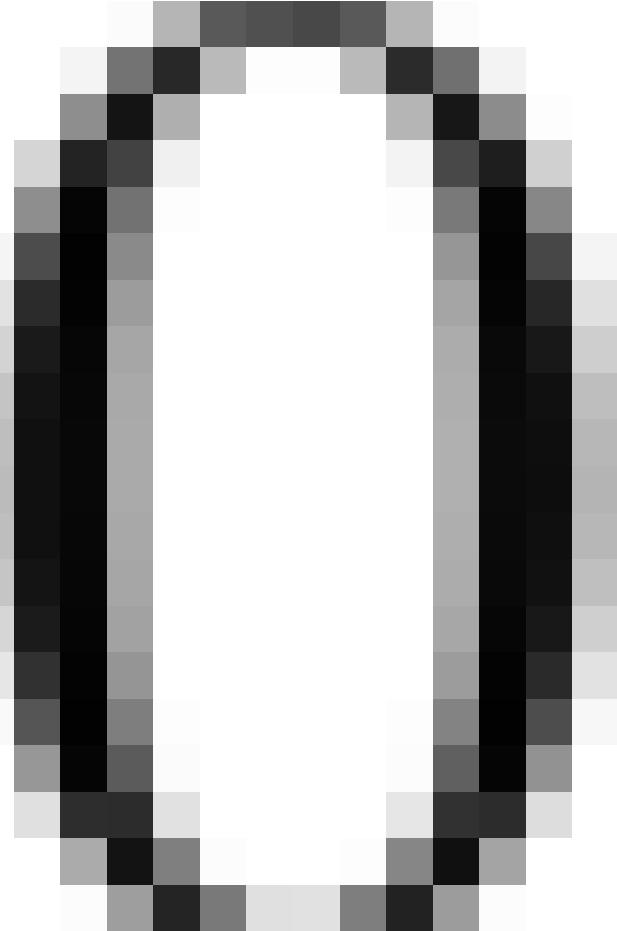
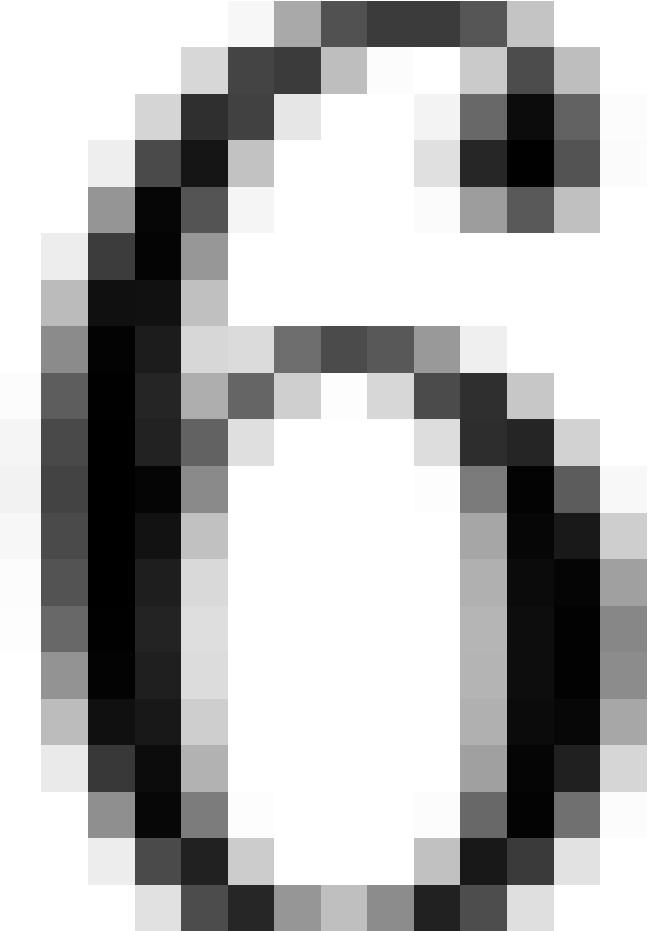
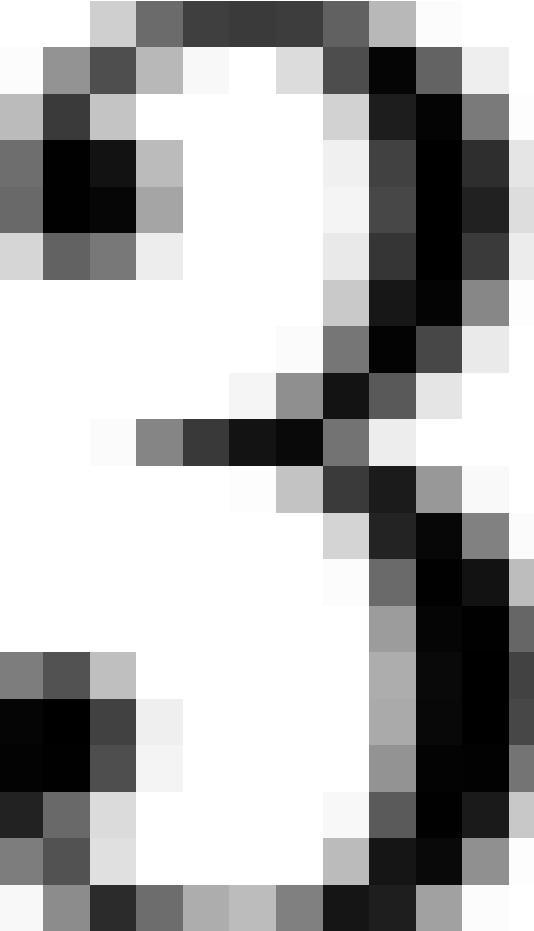








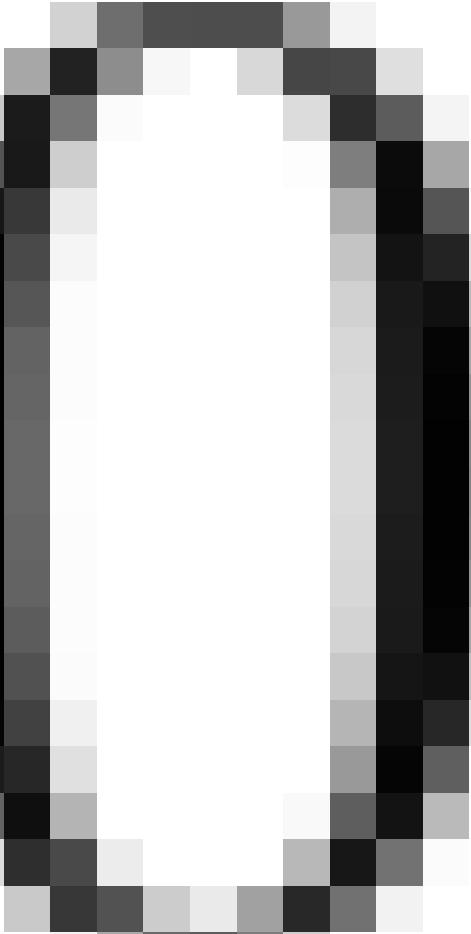
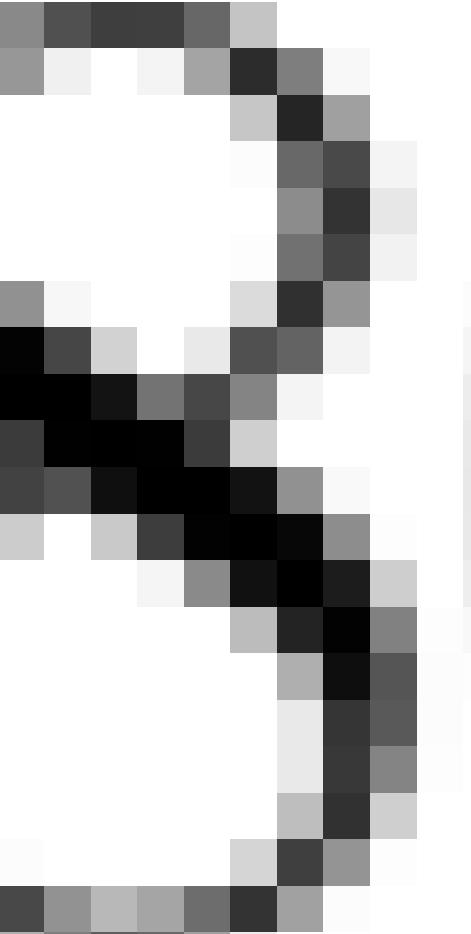
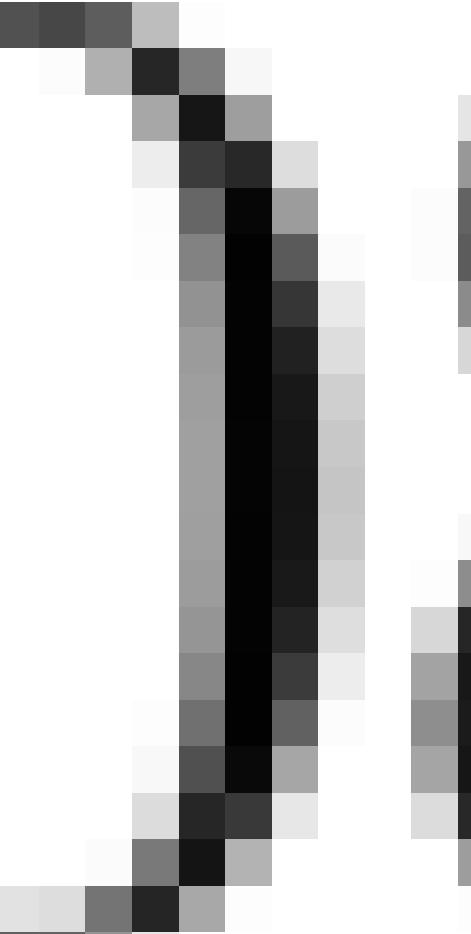
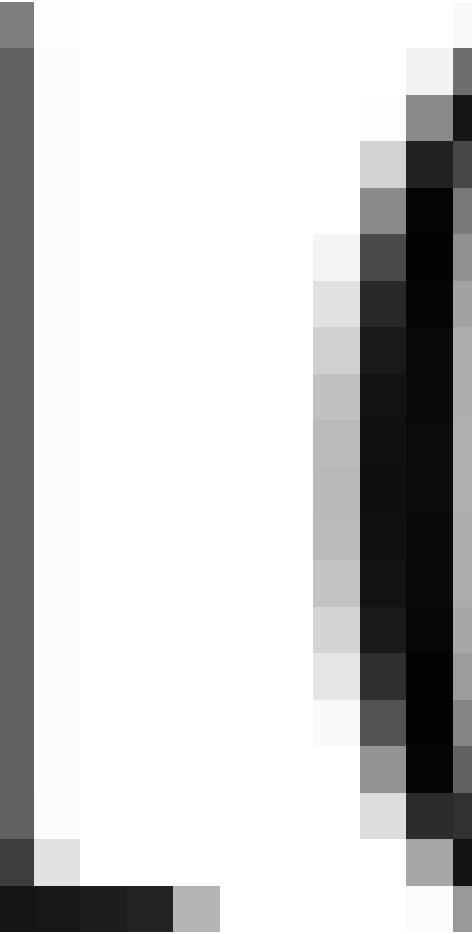


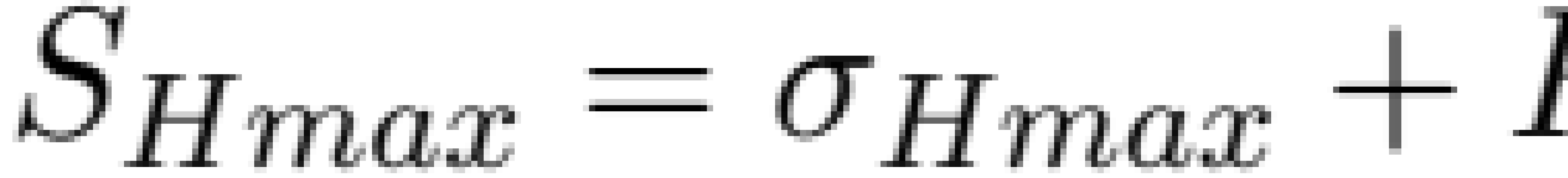


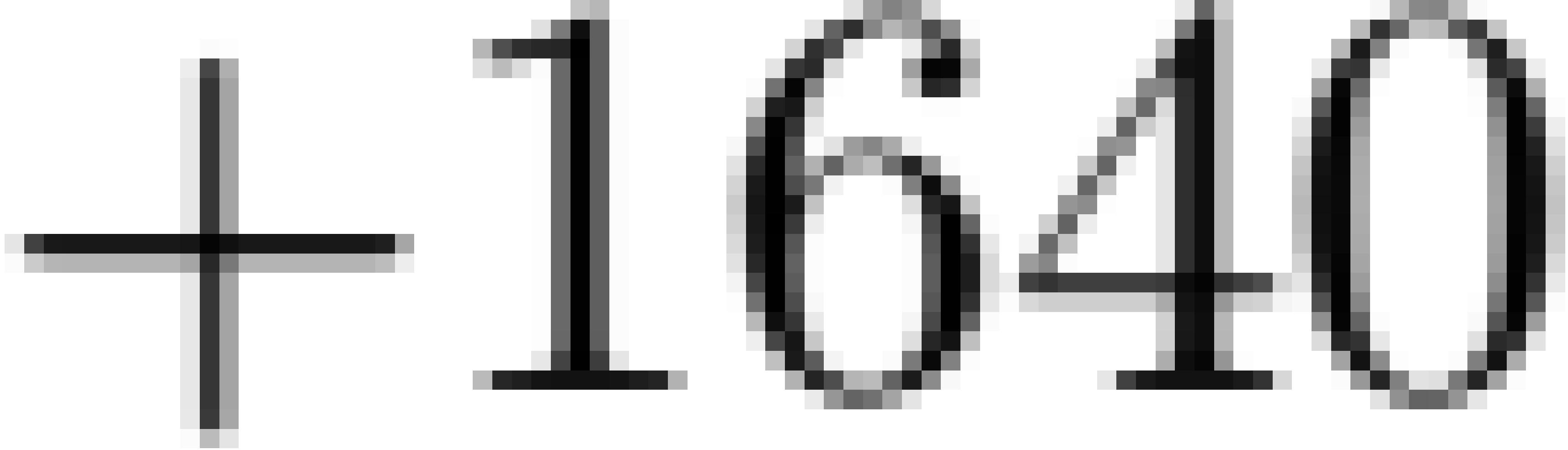
$$\sigma_{H\text{max}} =$$

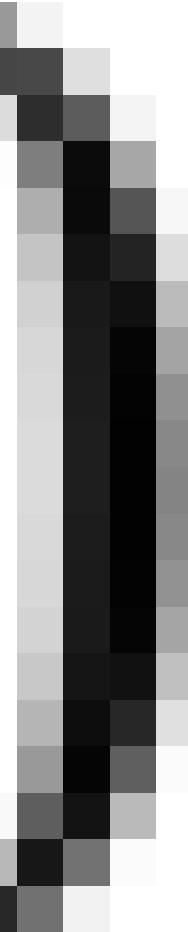
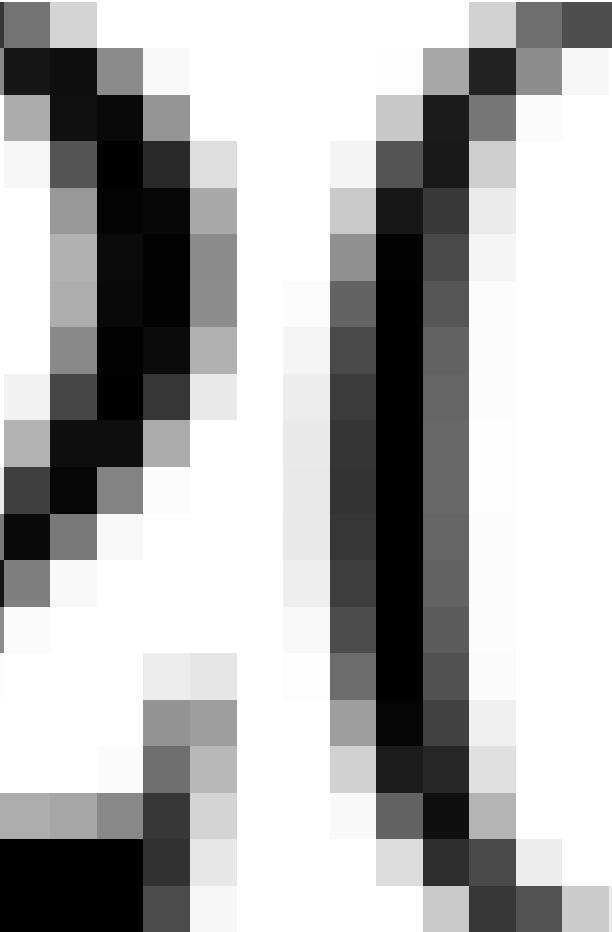
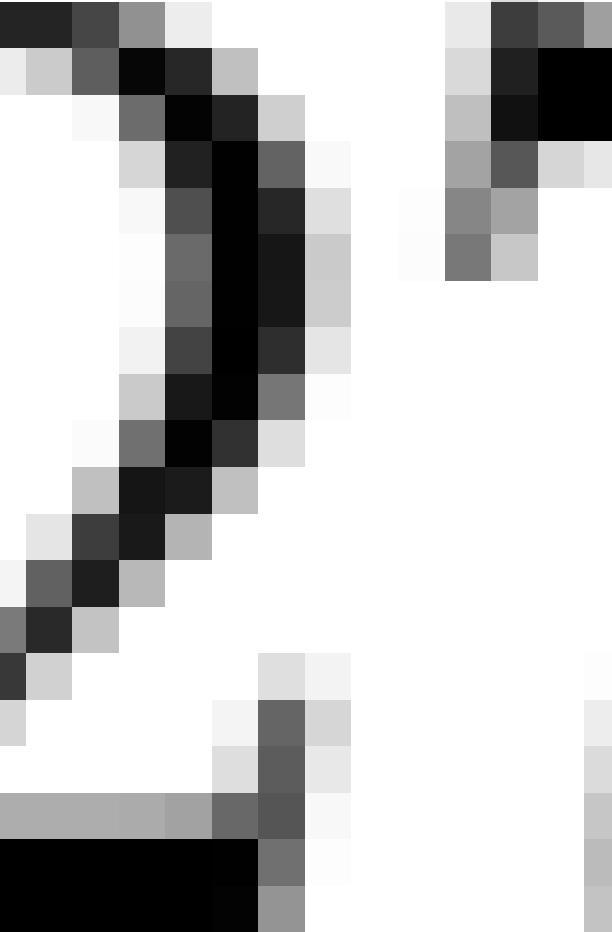
$$90^\circ =$$

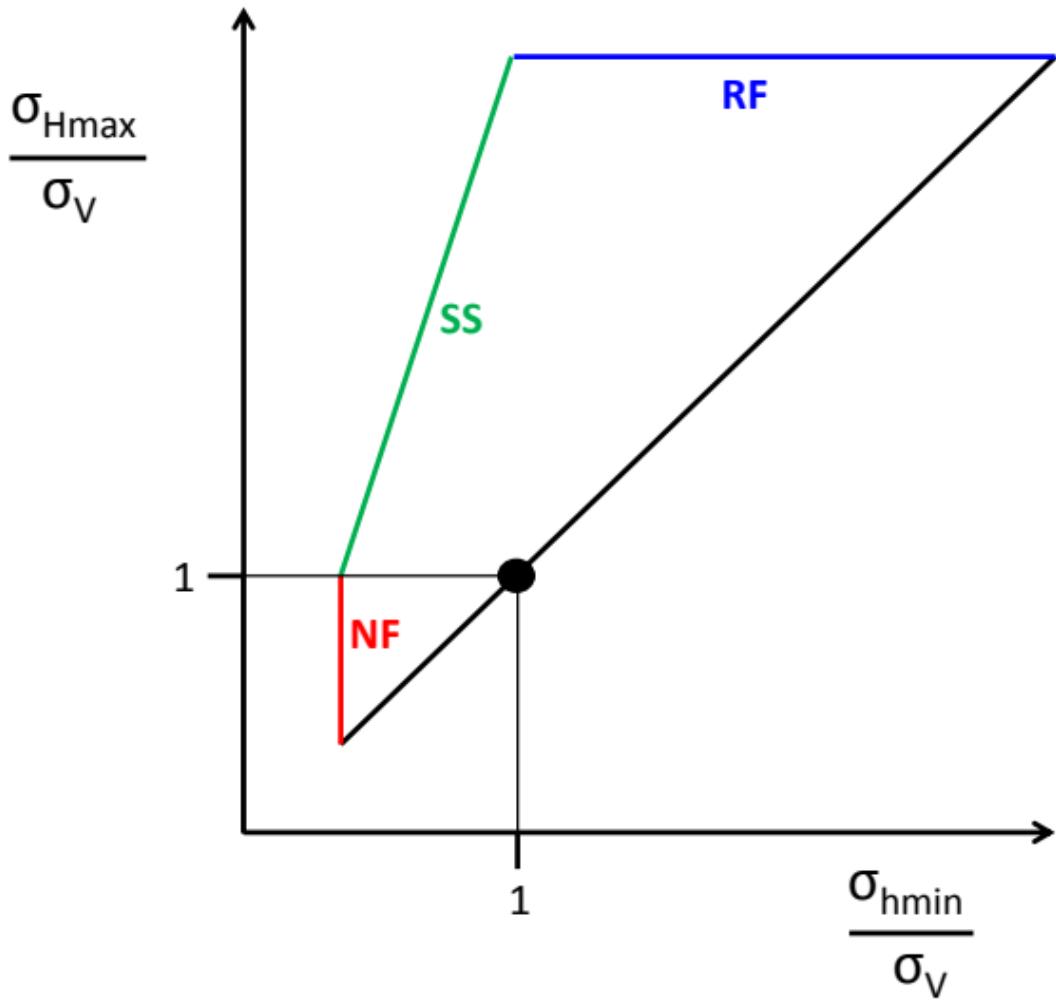
$$\frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} 360$$



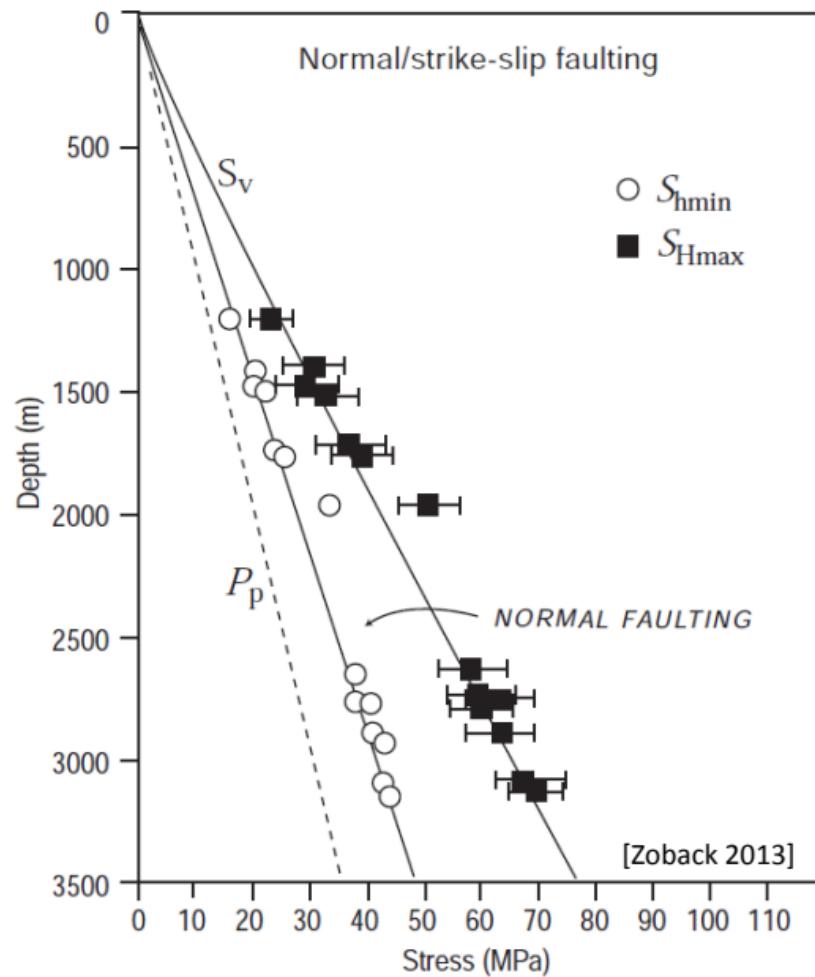
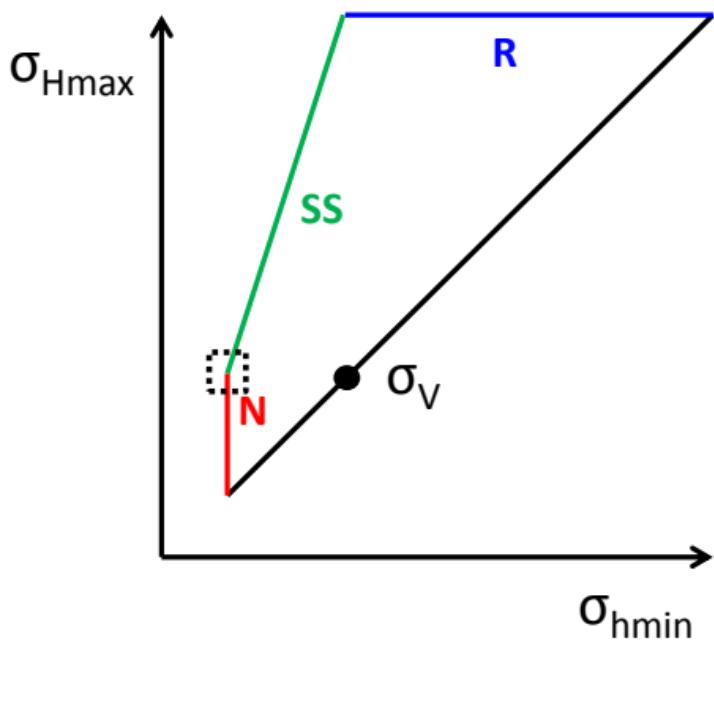


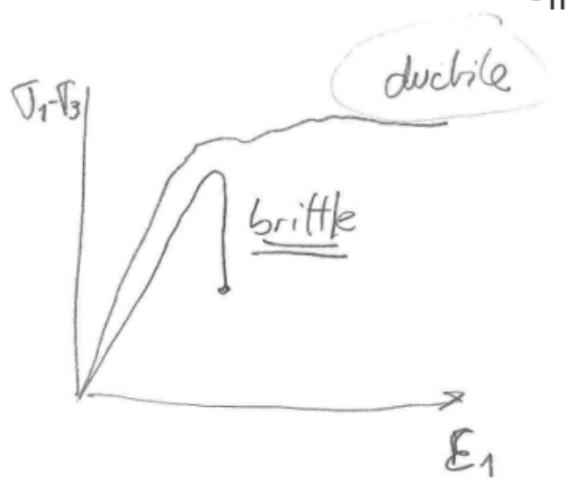
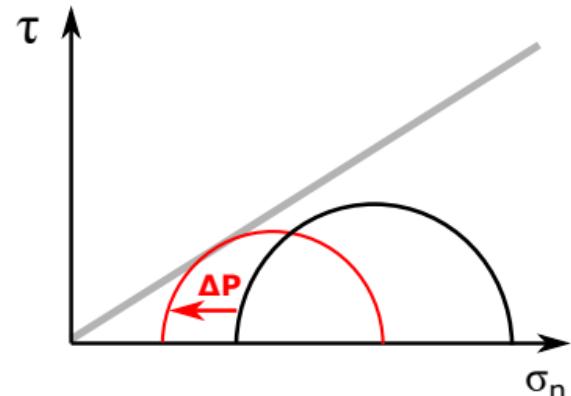
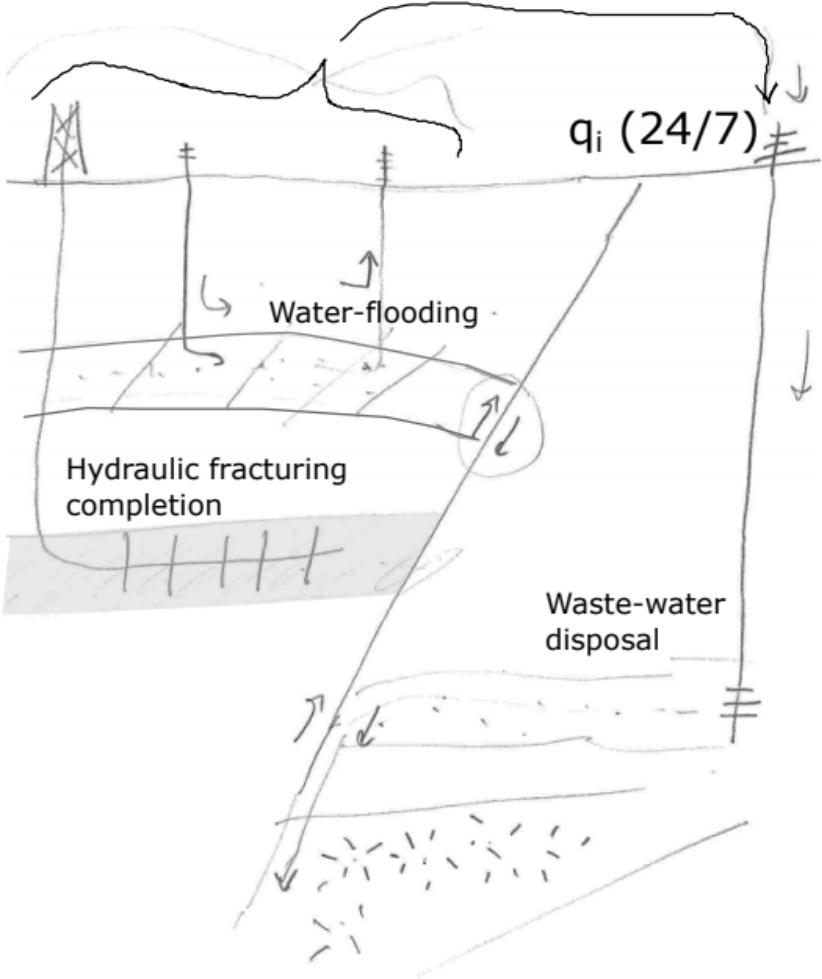






$$\sigma_{h\min} < \sigma_{H\max} \approx \sigma_v$$

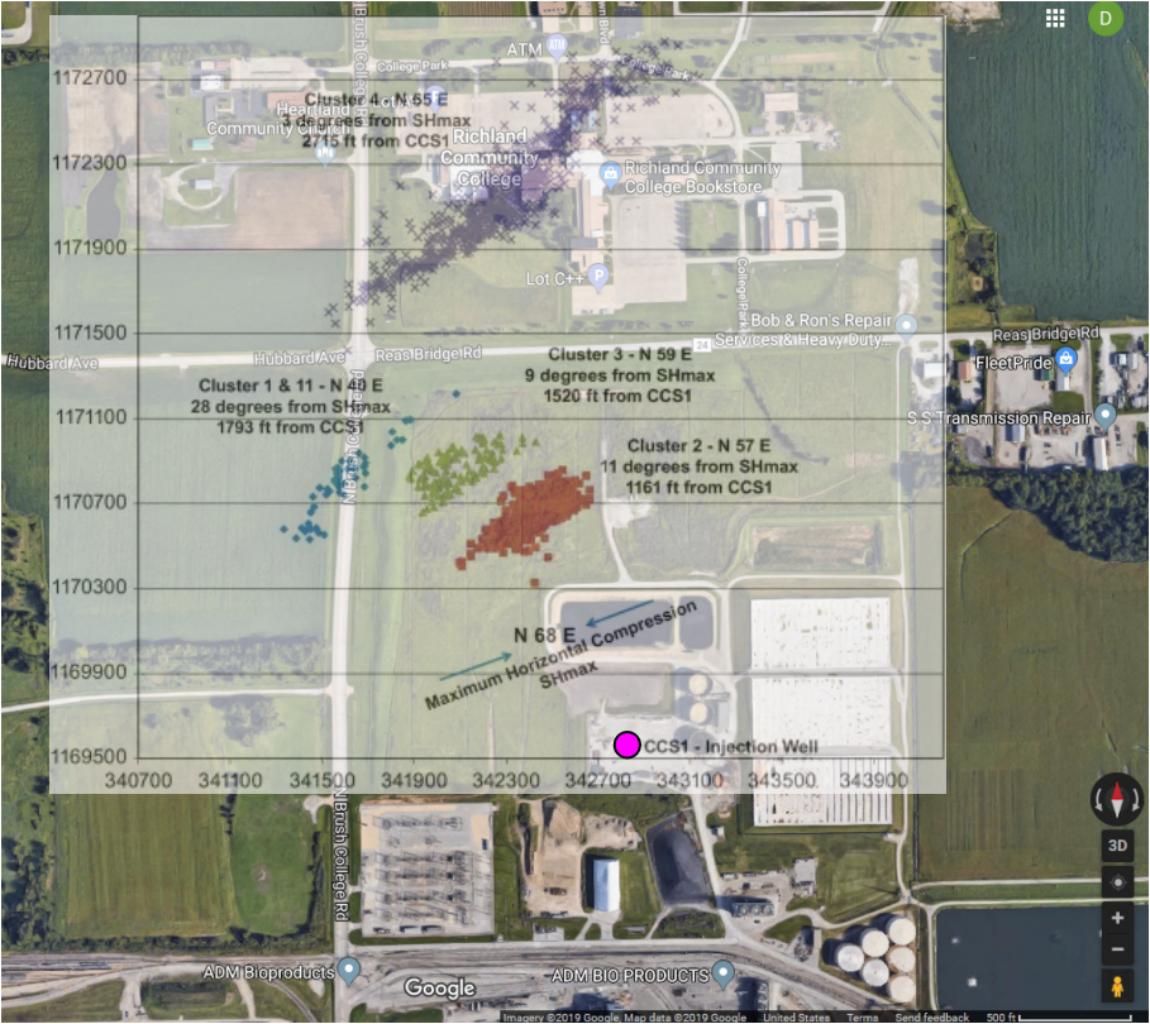


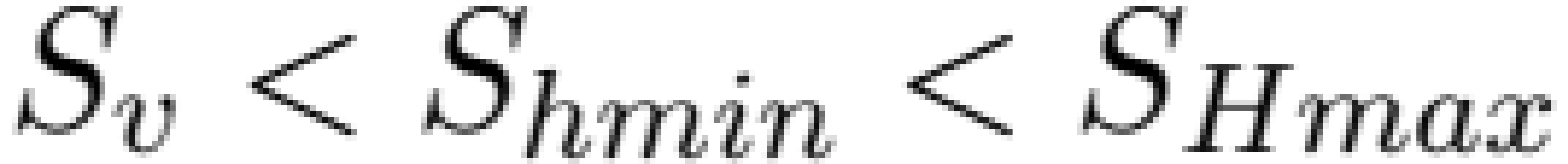


$$\Delta\sigma_a = -\Delta P_p$$

$$\Delta\sigma_{\text{Hermann}} \leq -\Delta P_p$$

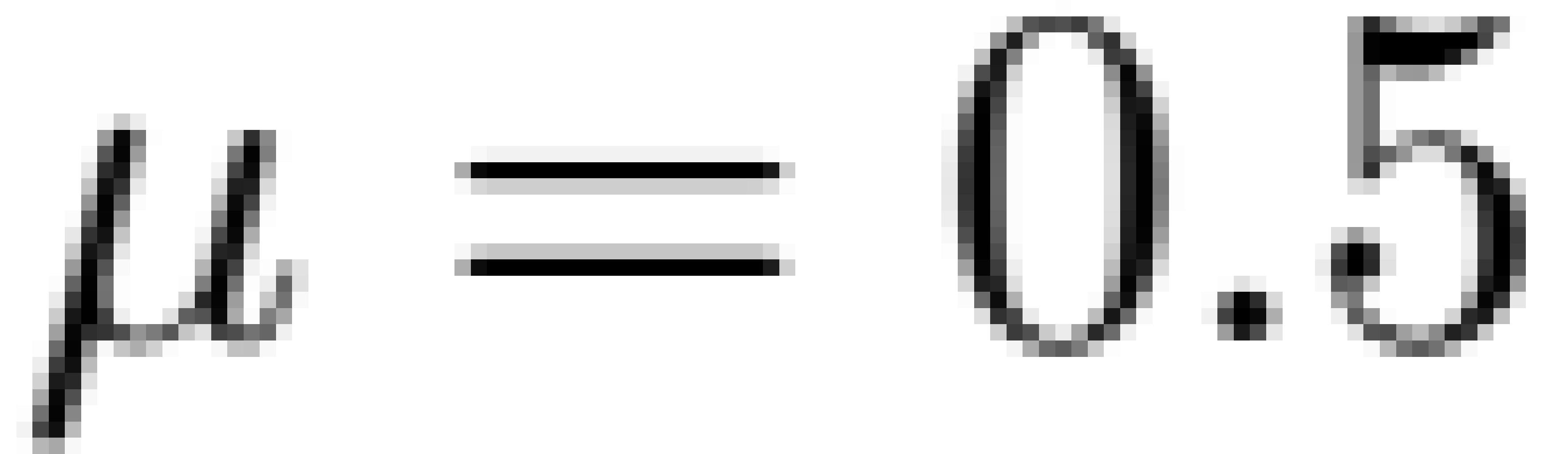


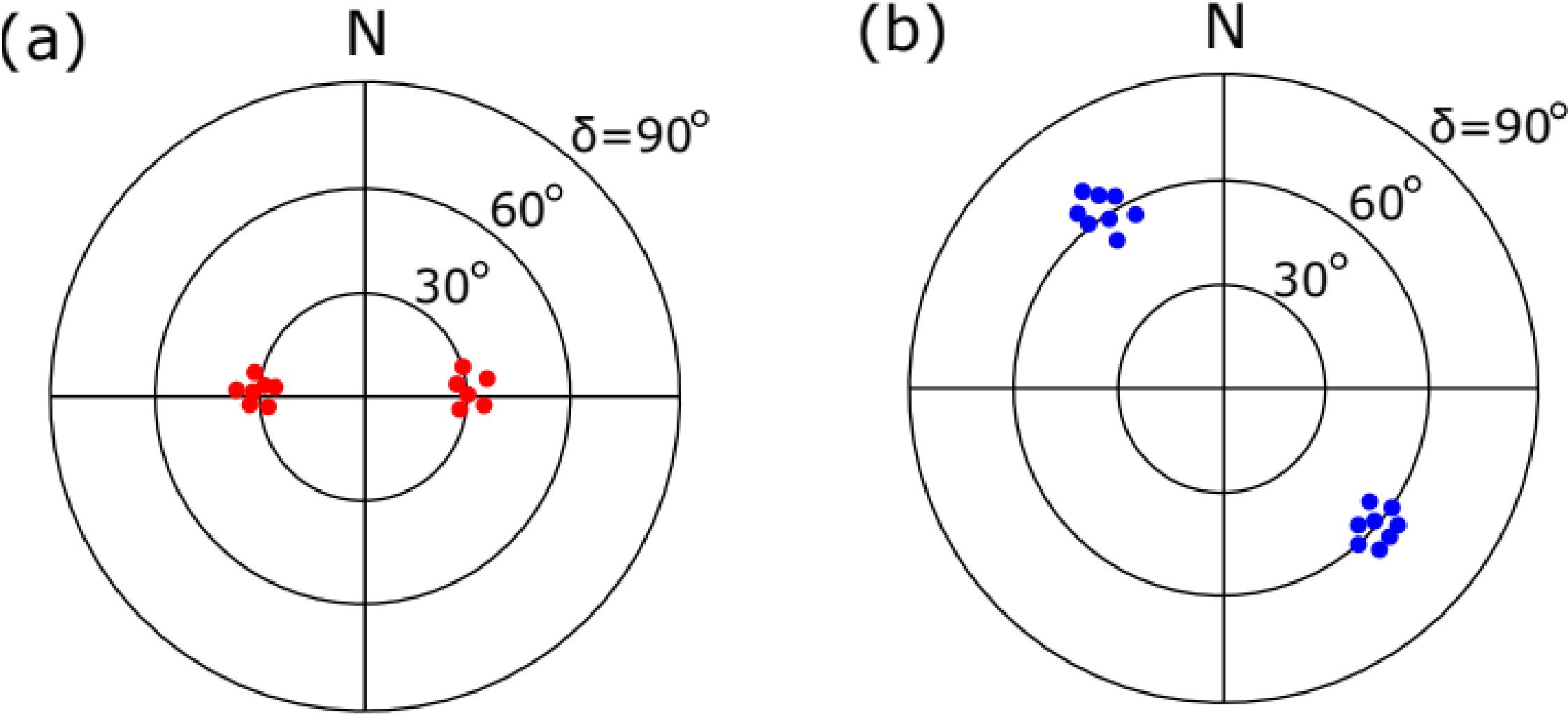




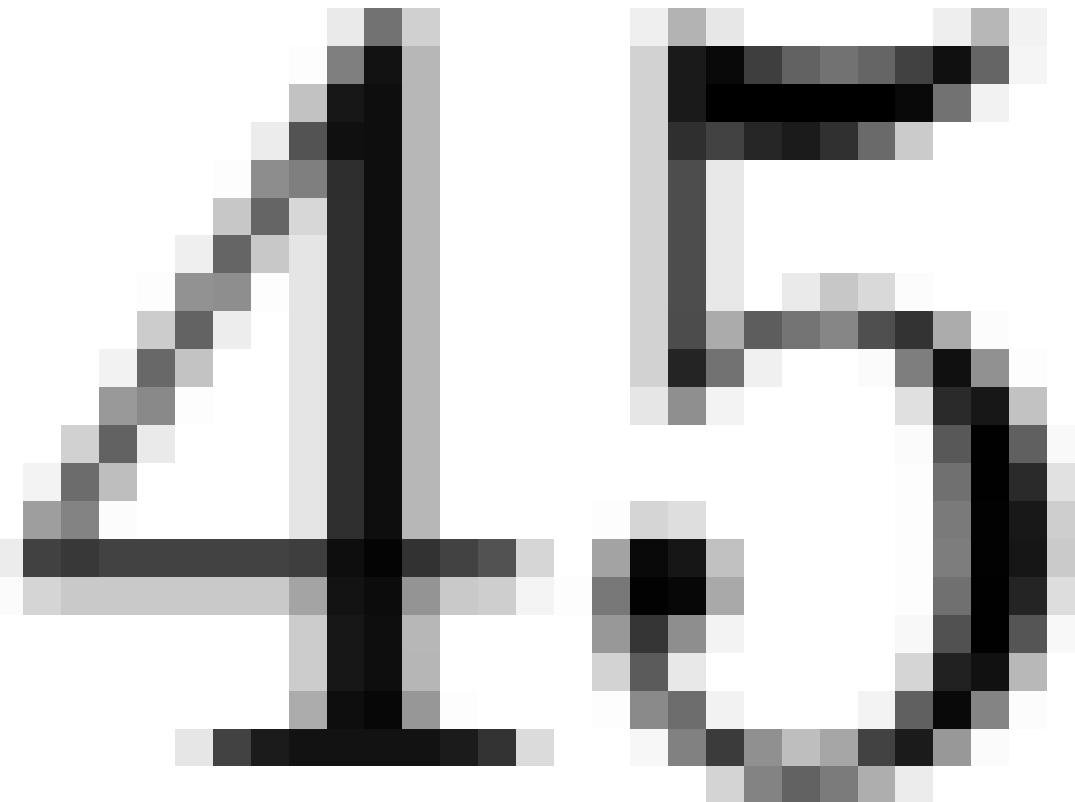


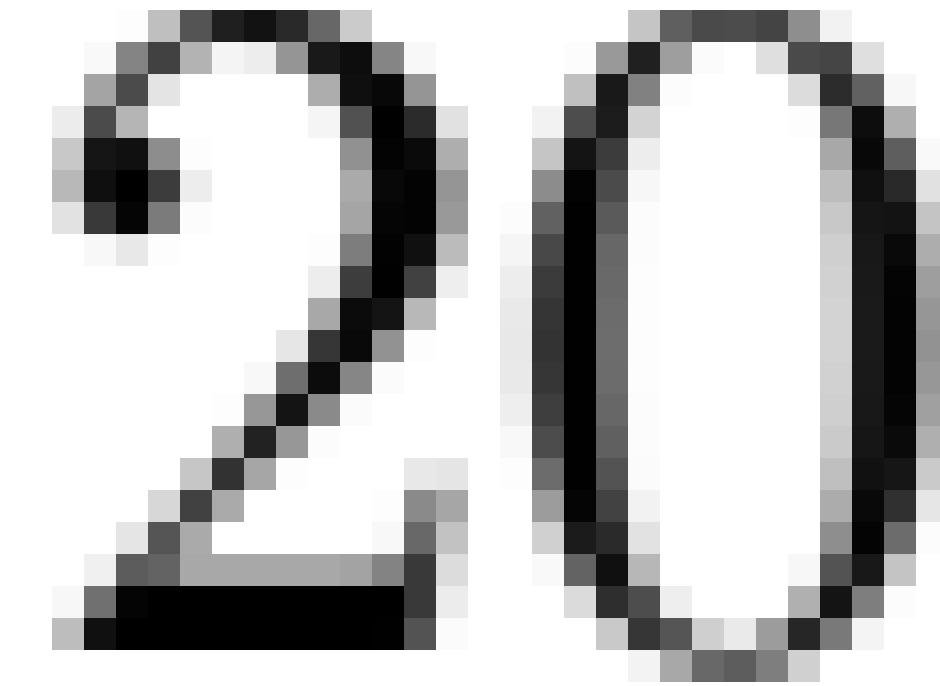


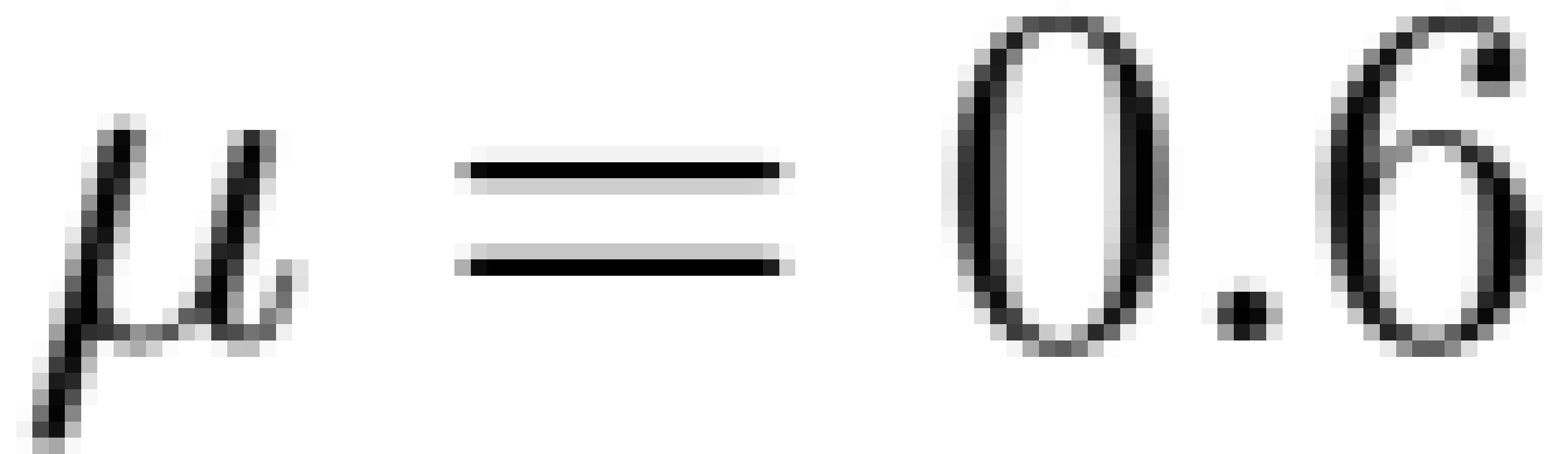






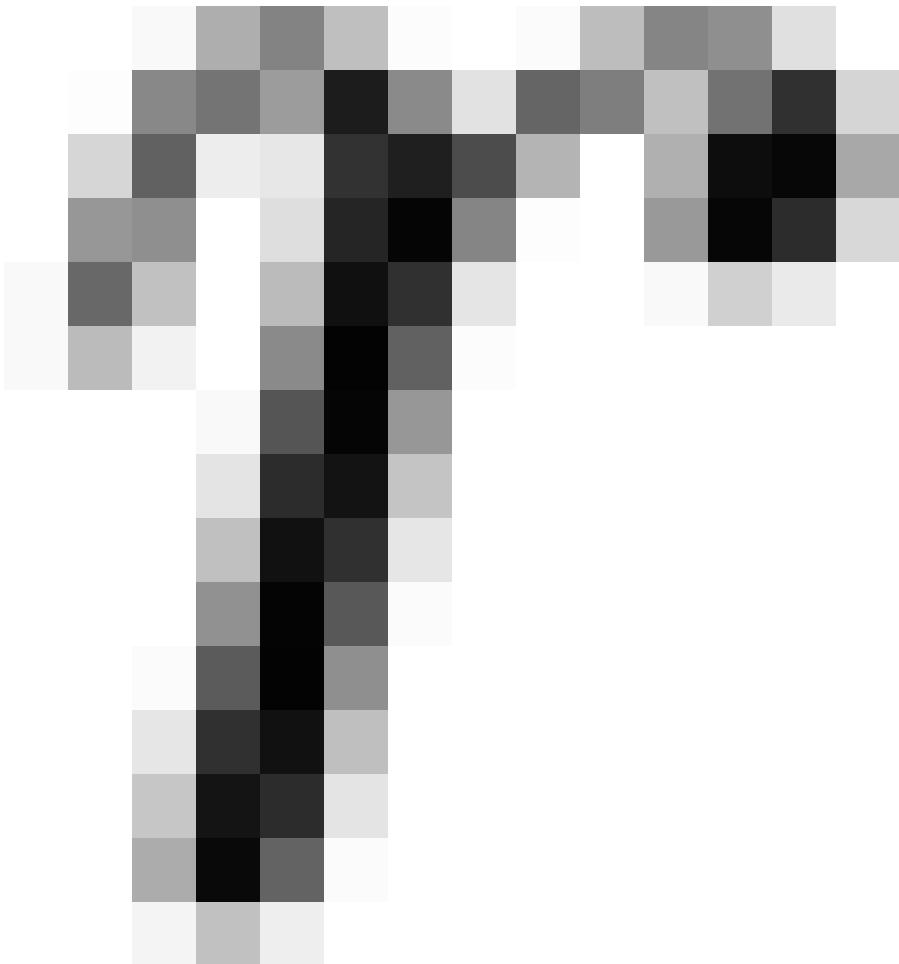


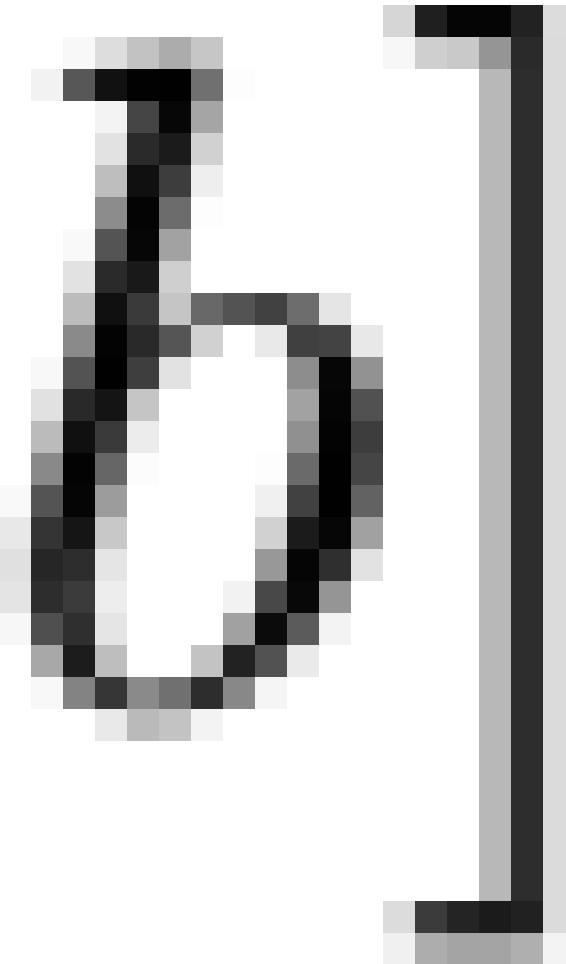
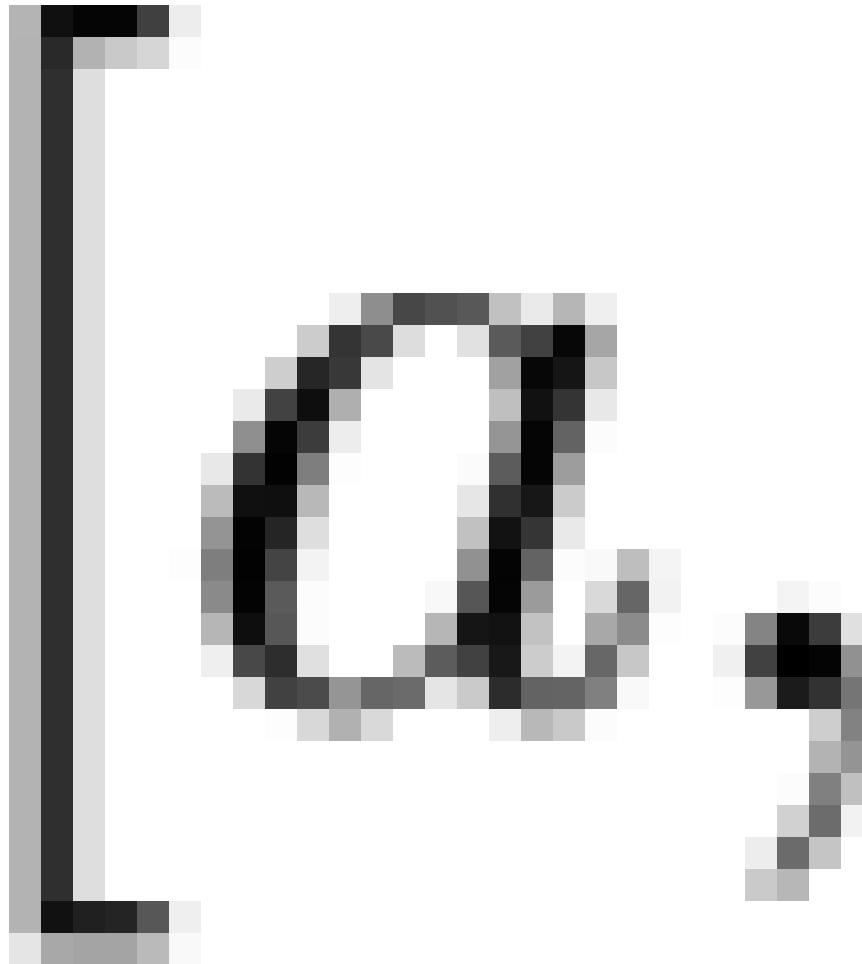


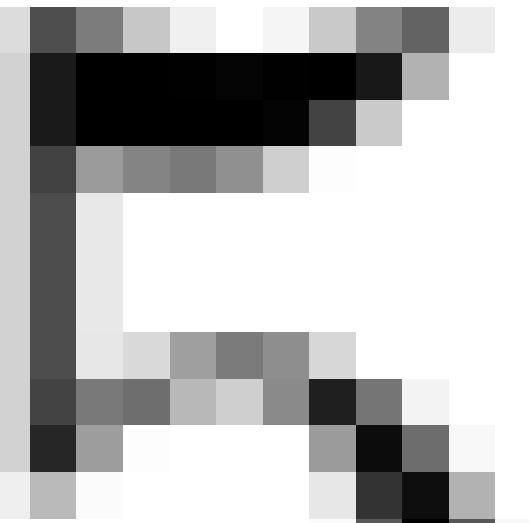
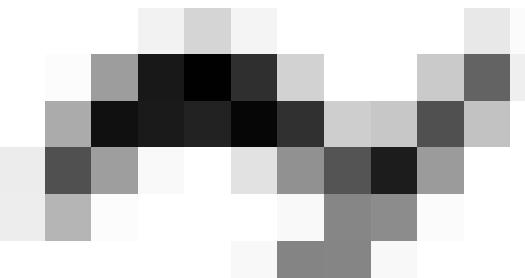
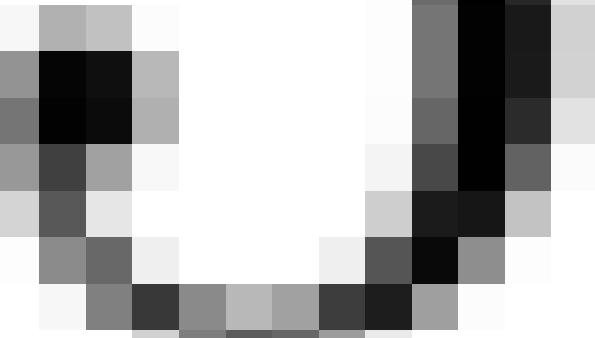
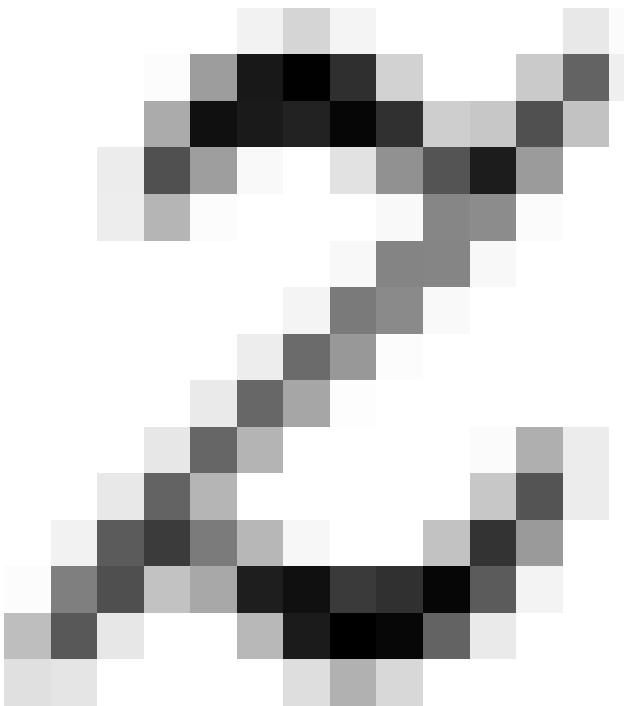




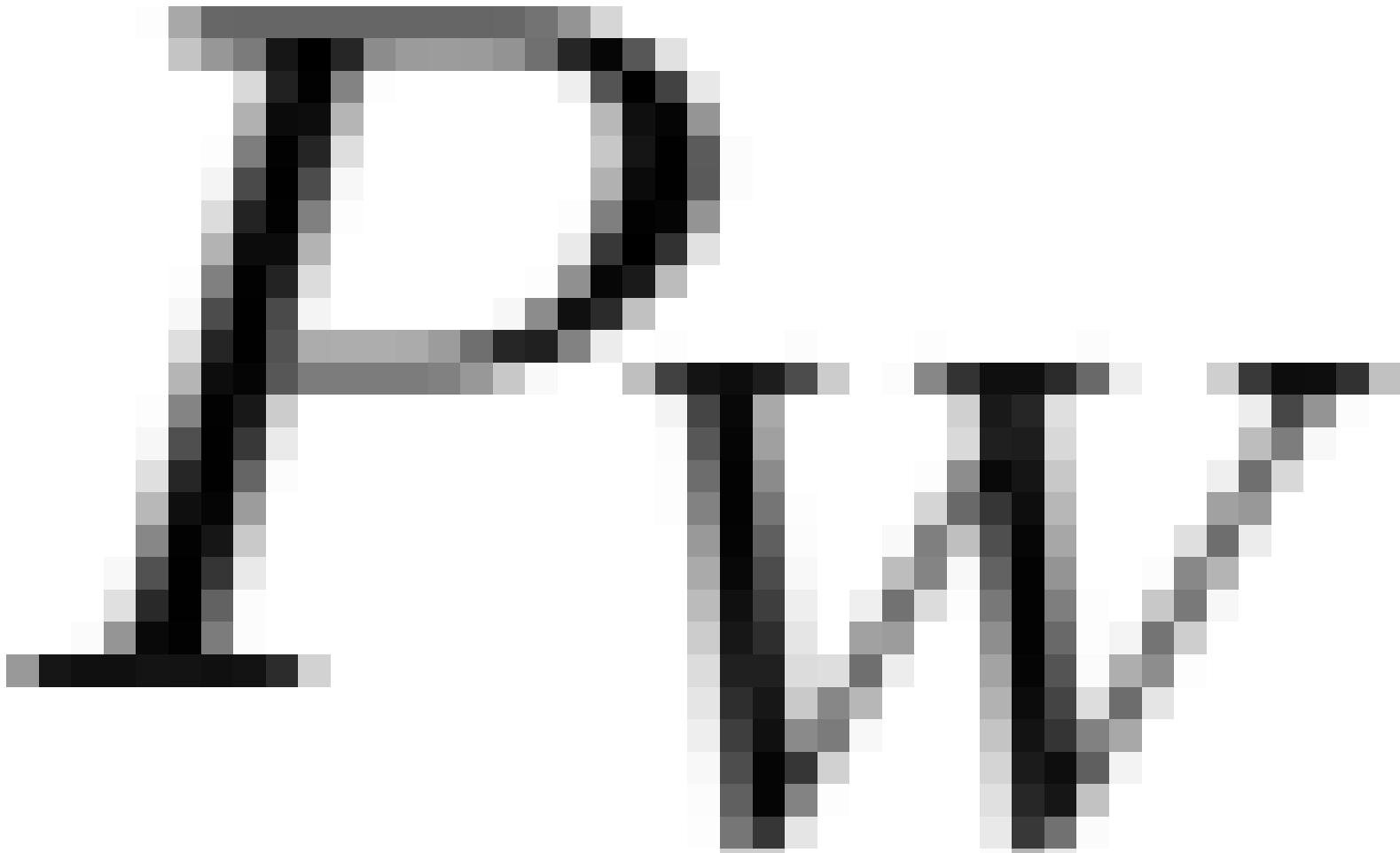




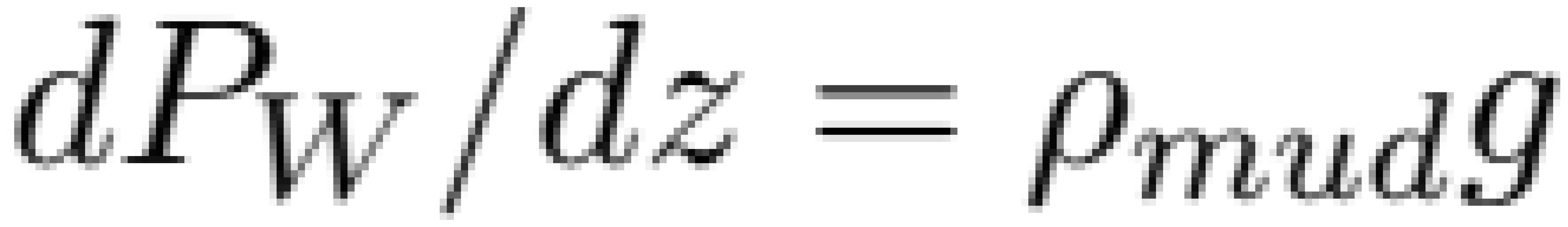


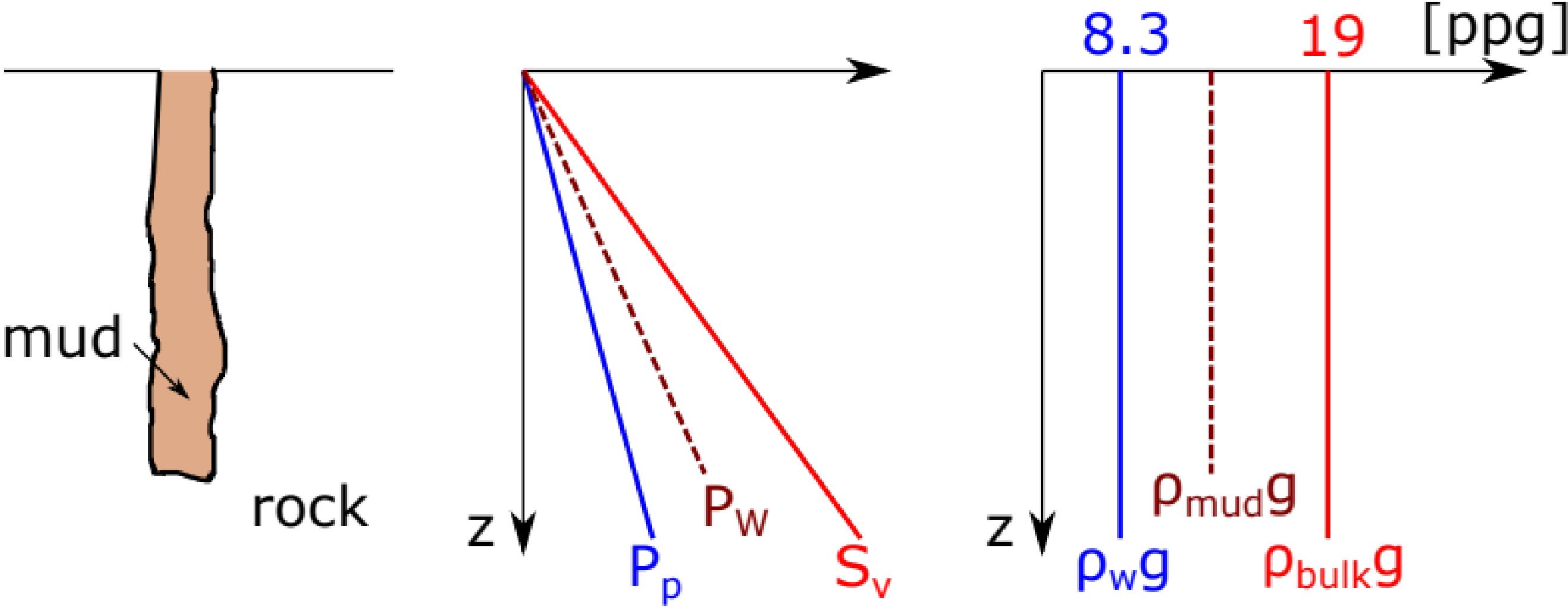


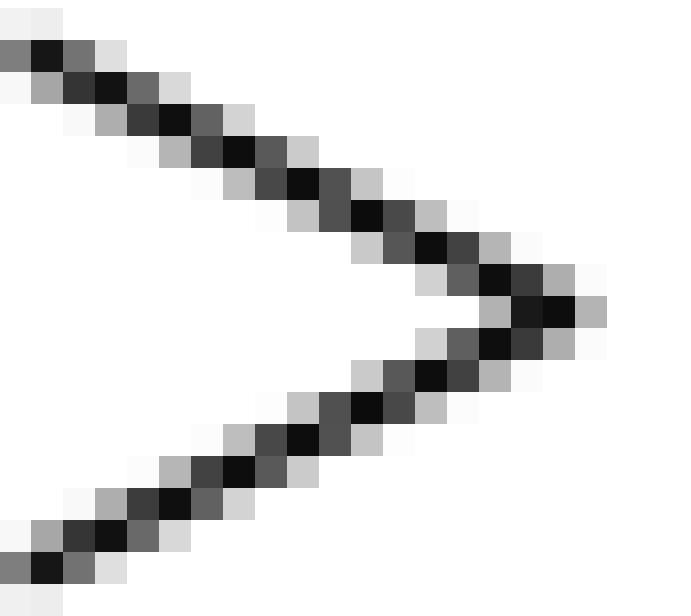




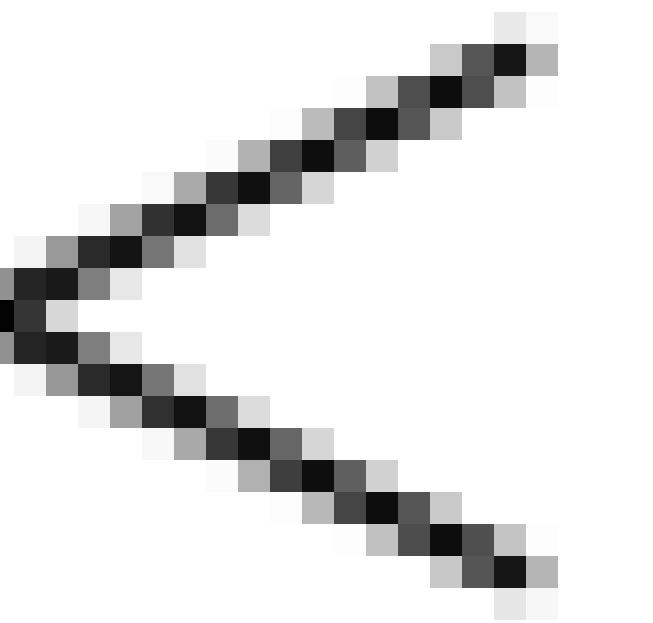
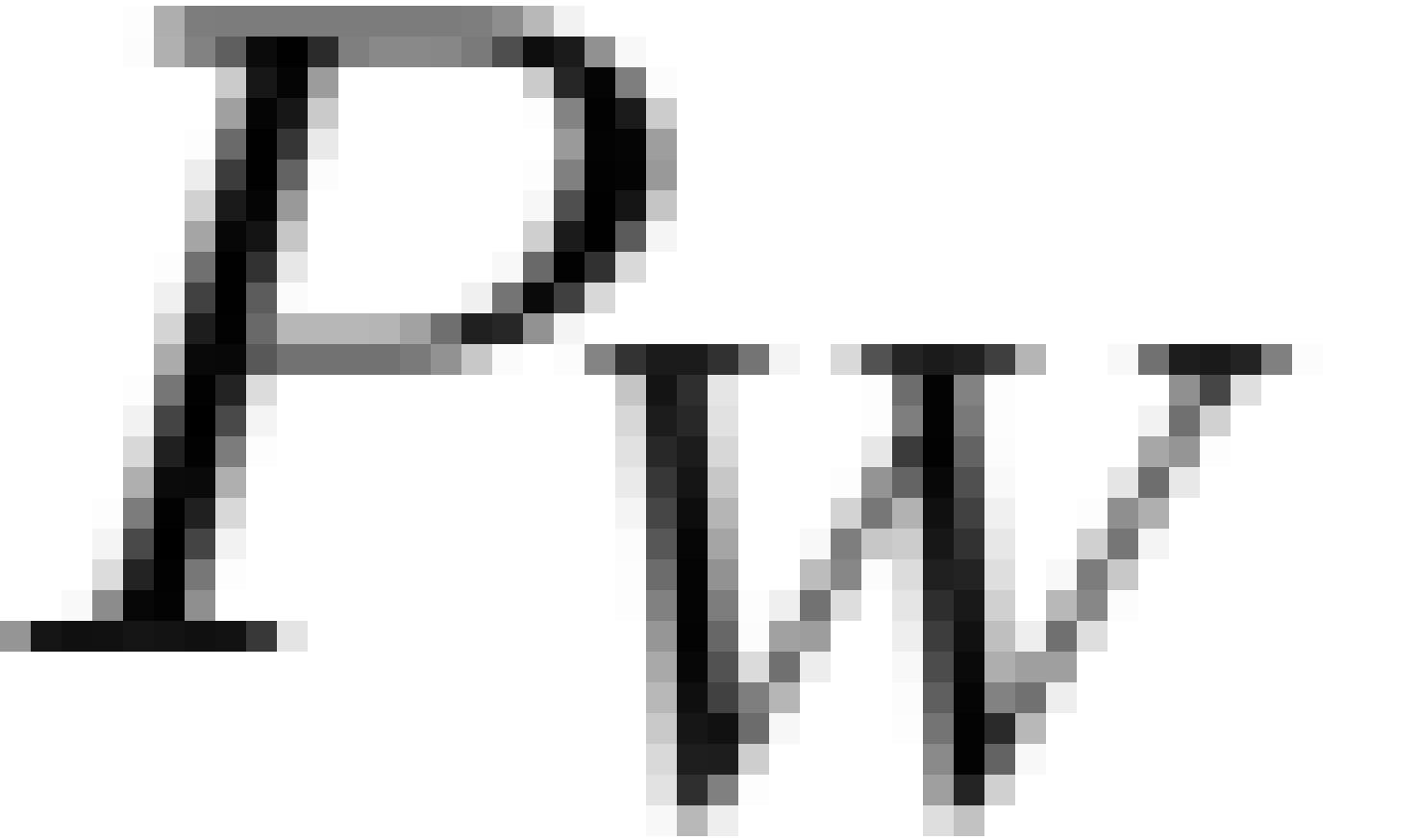


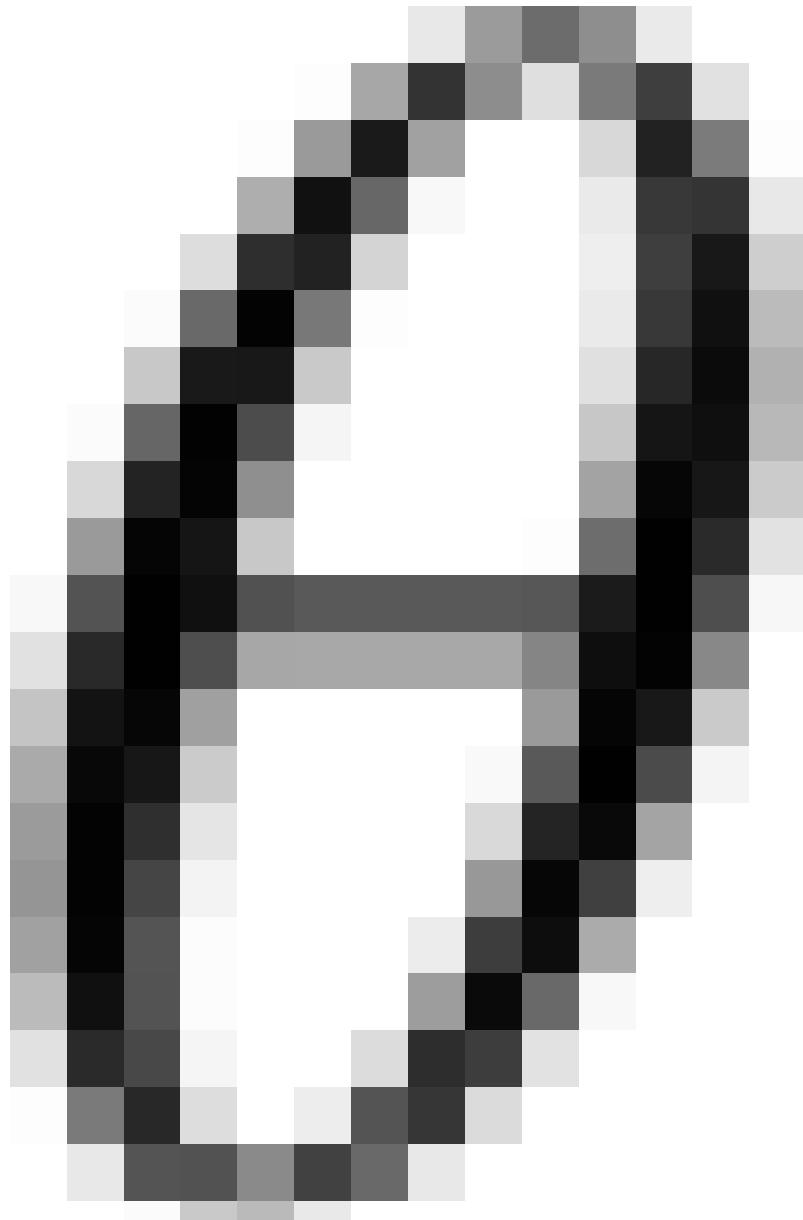


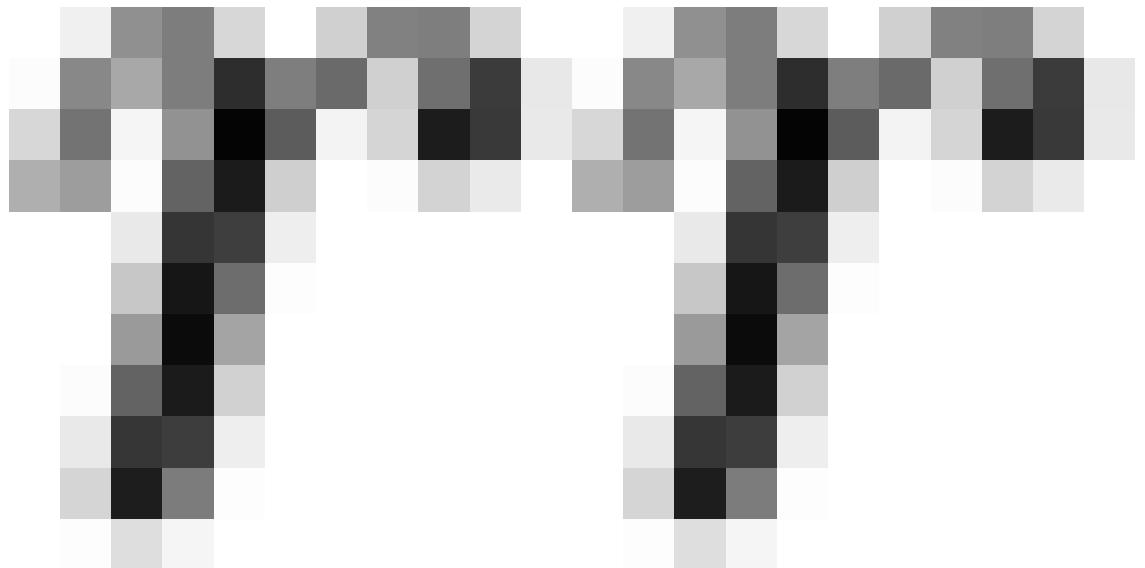
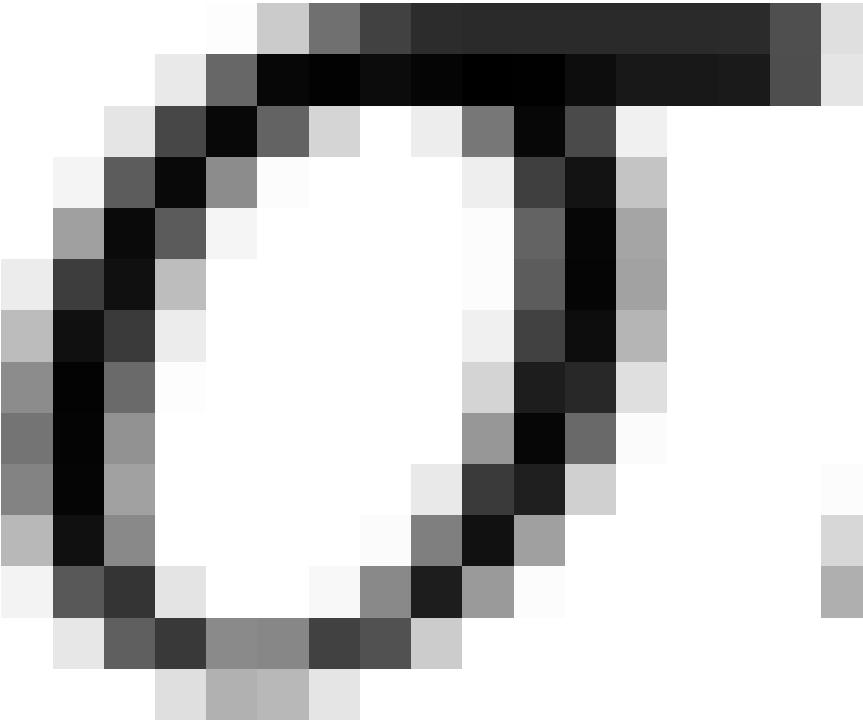


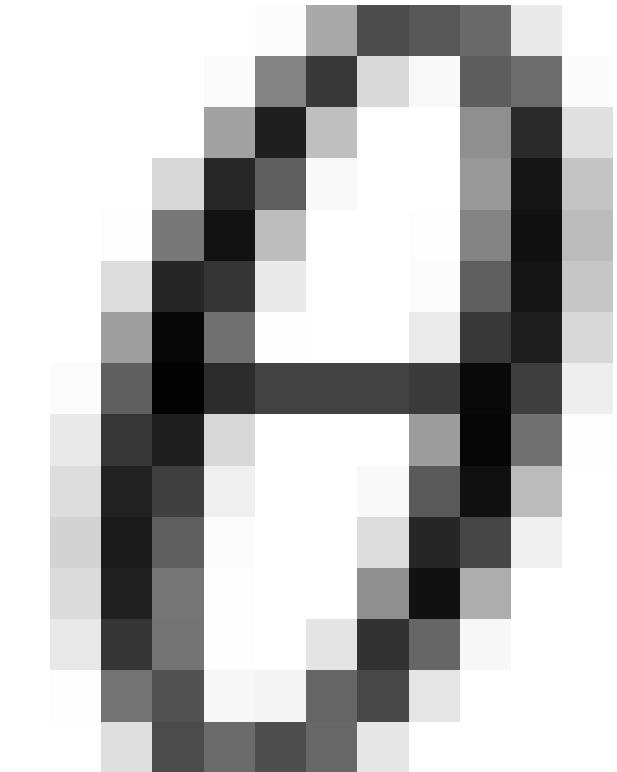
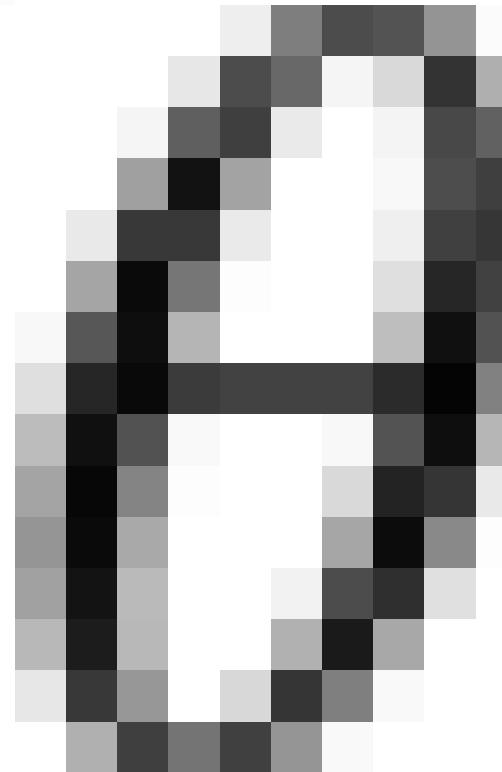
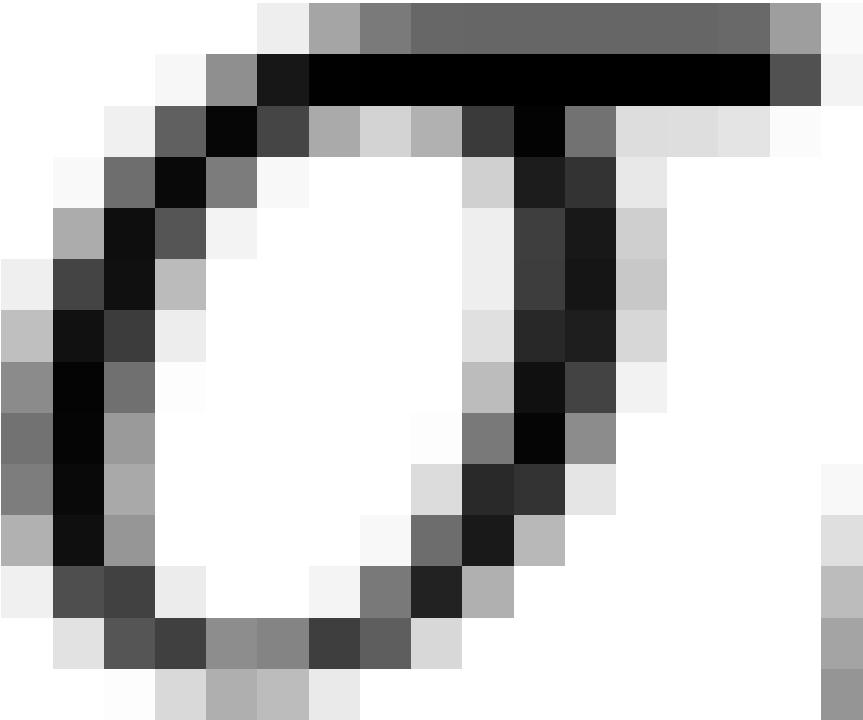


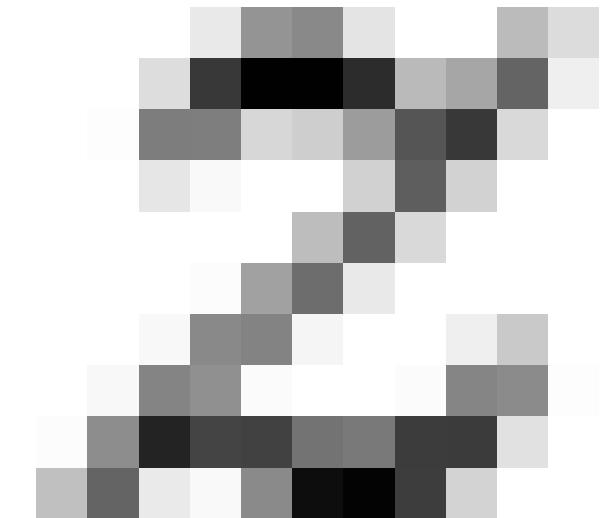
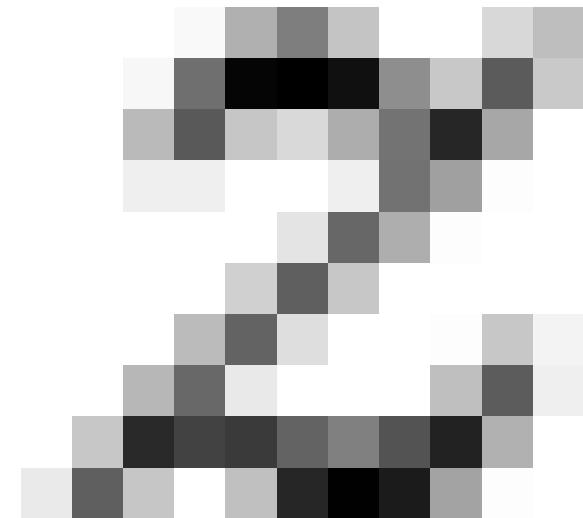
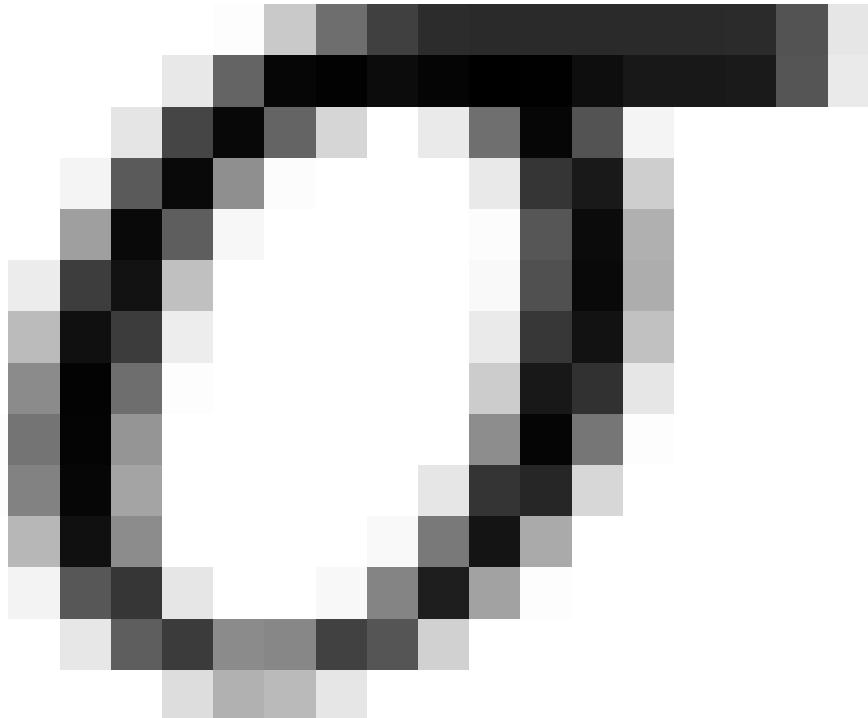


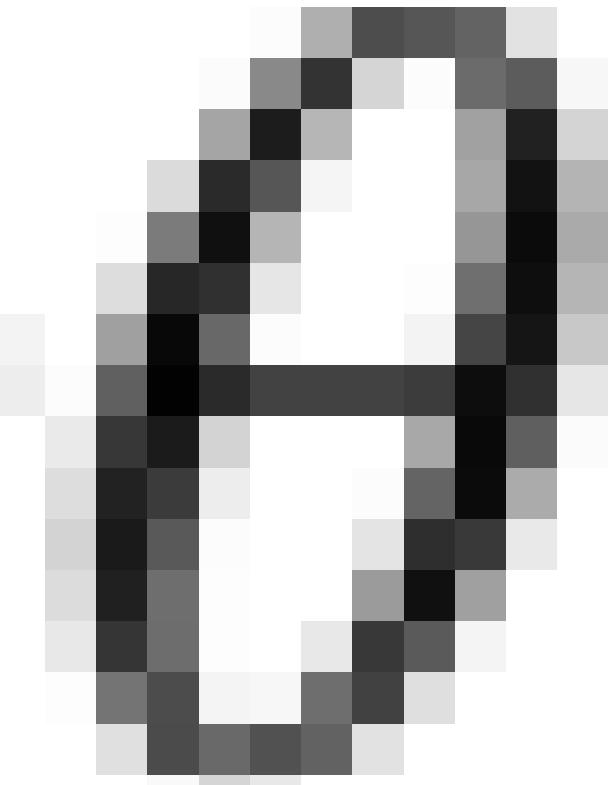
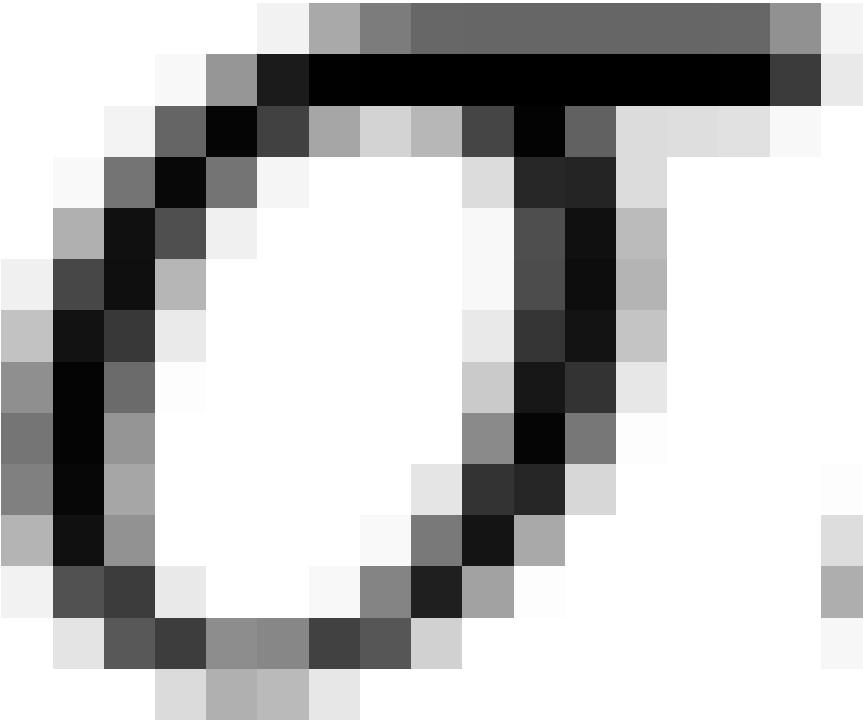


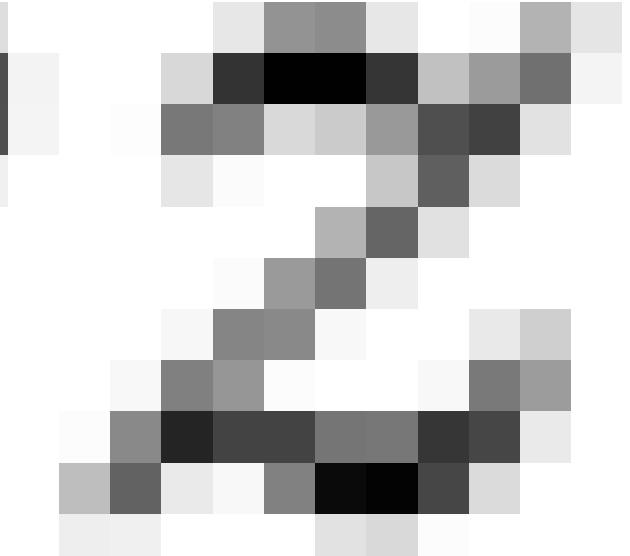
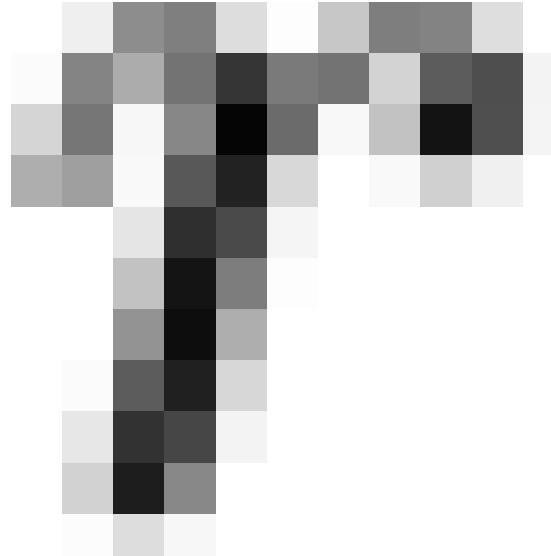
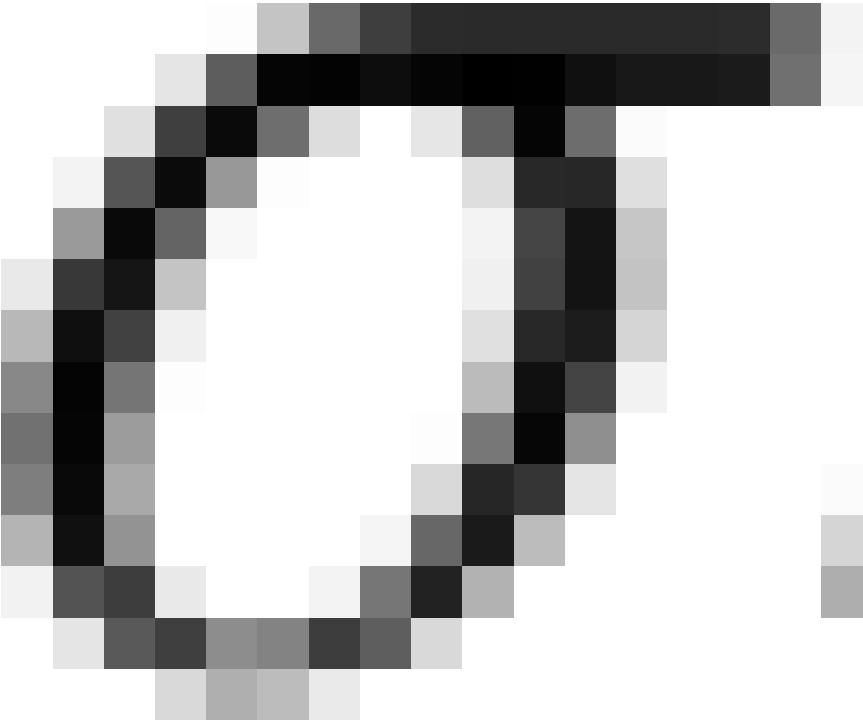


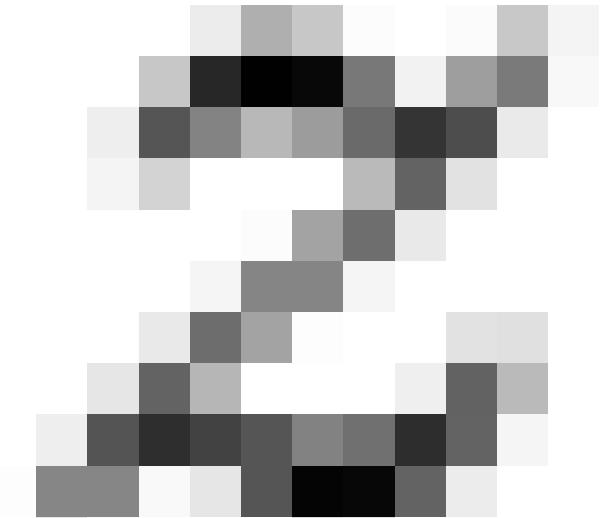
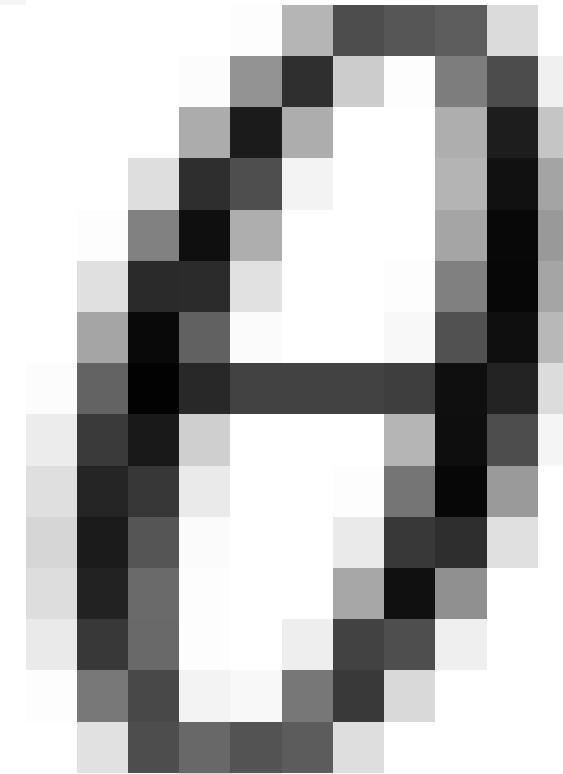
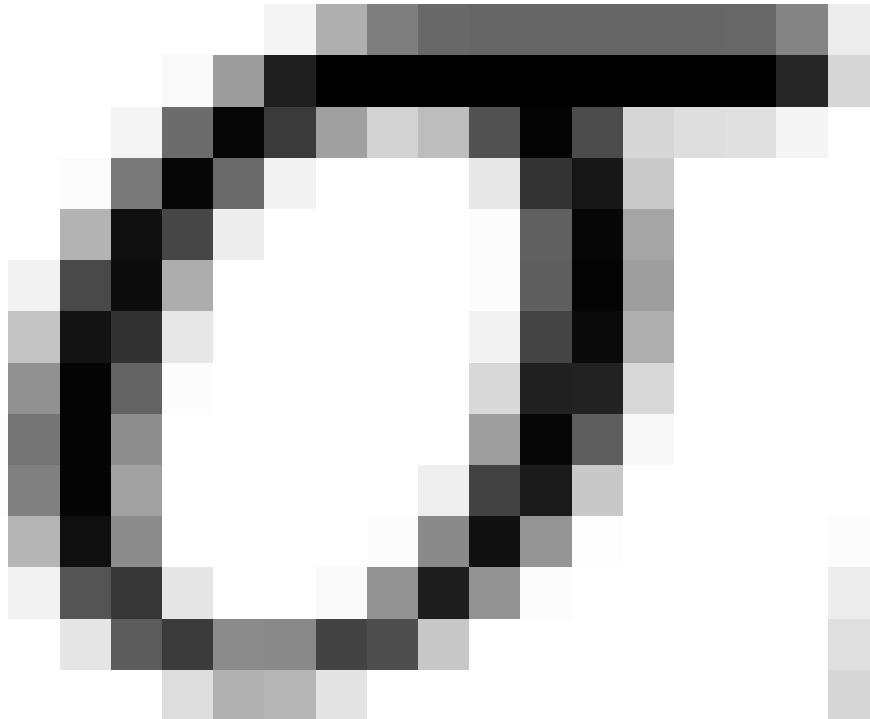


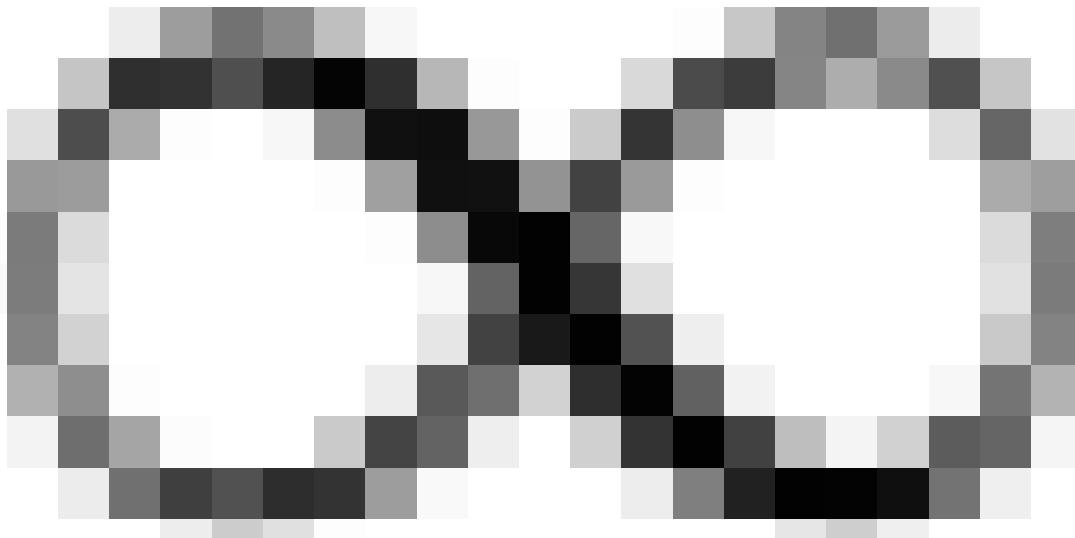
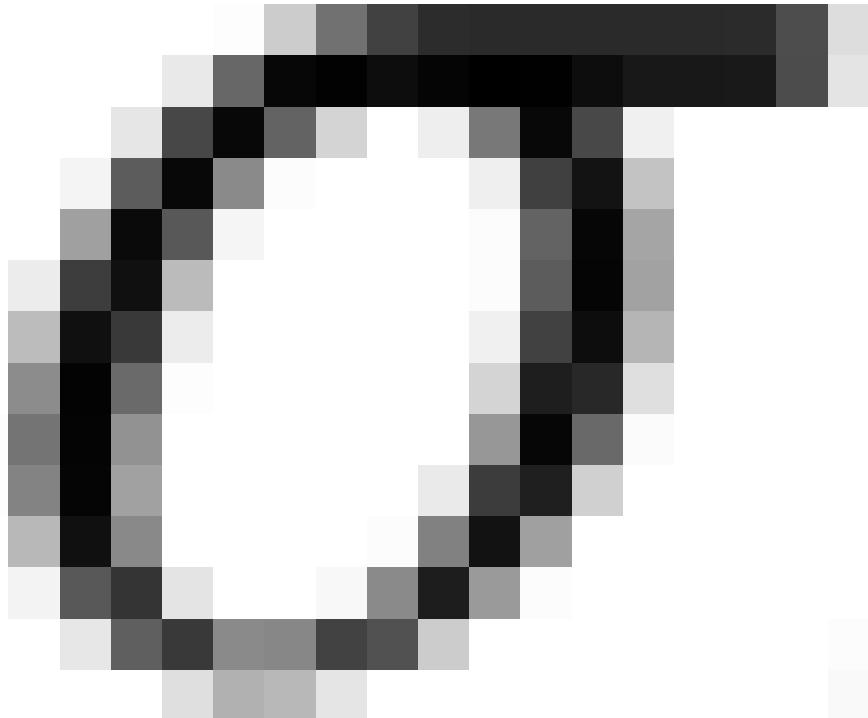




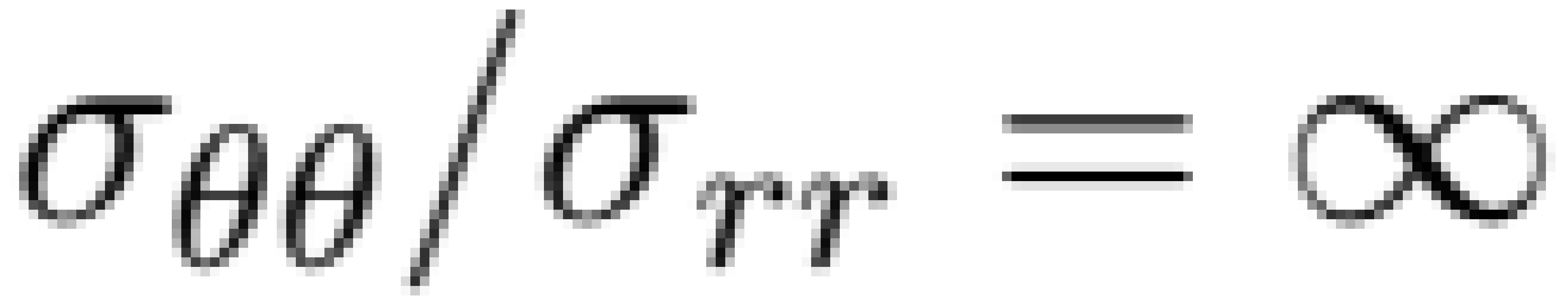


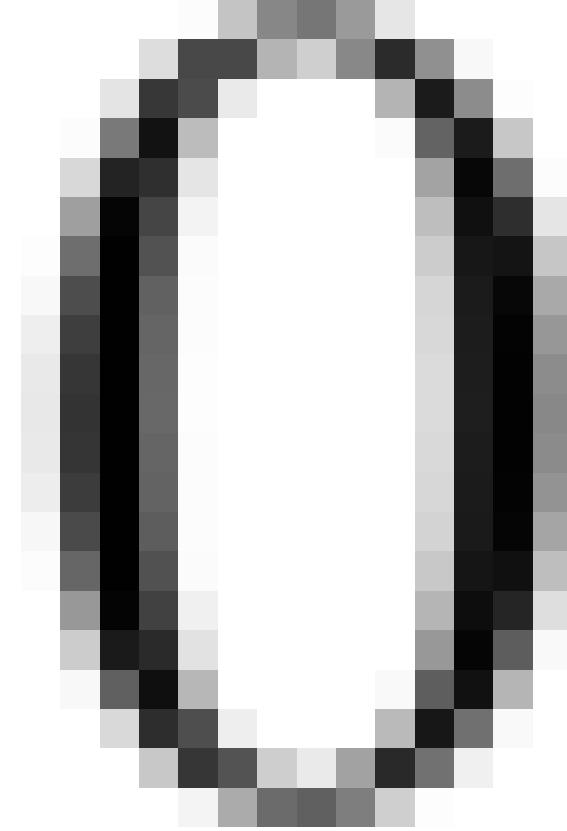
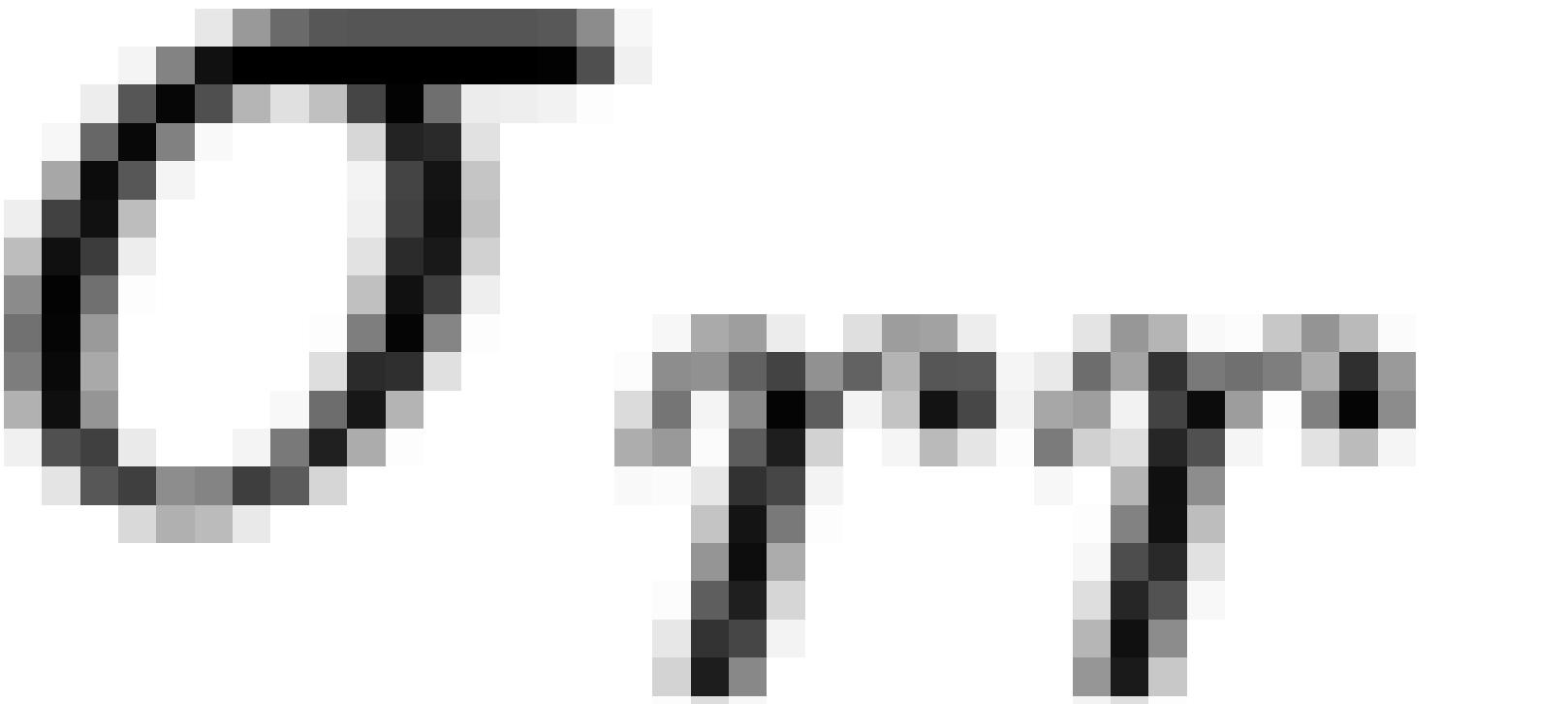




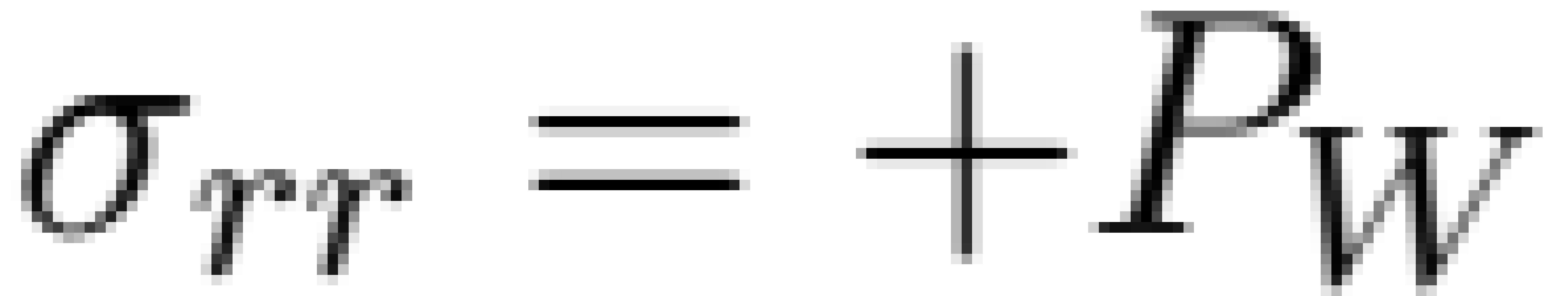


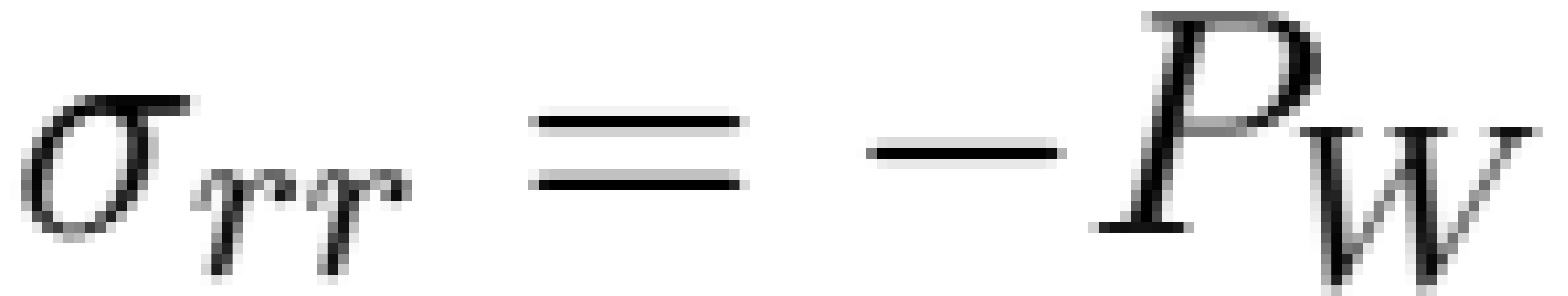


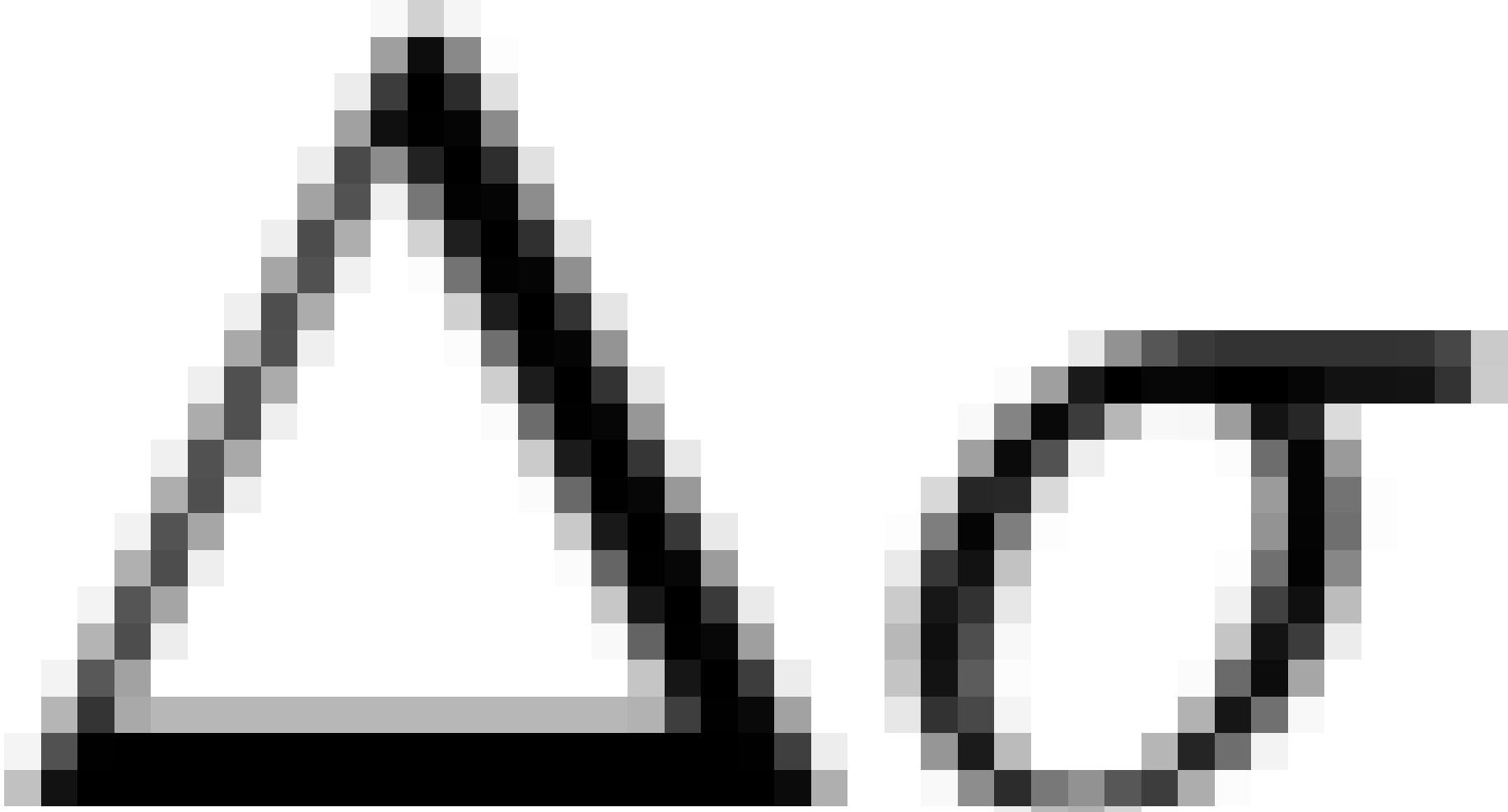


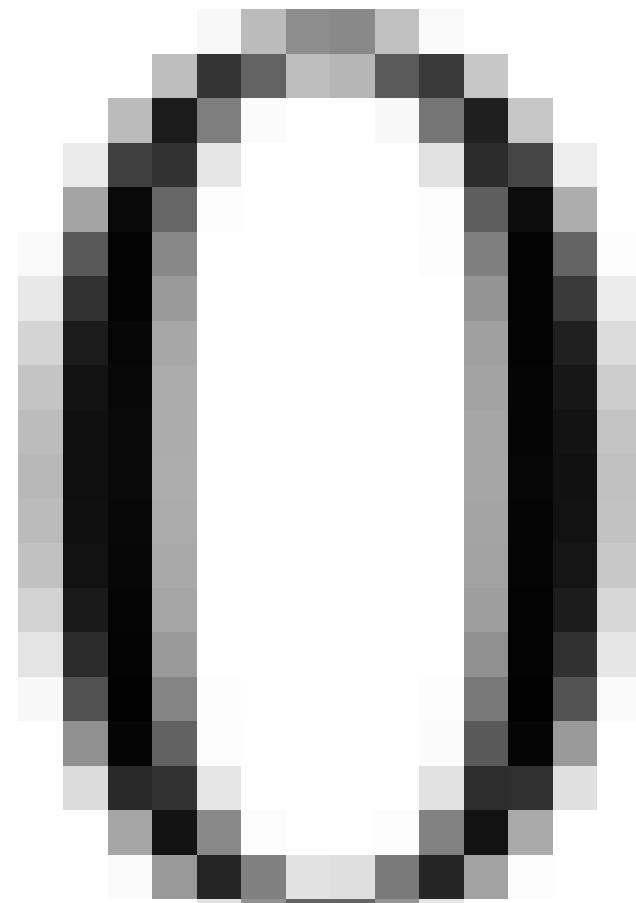
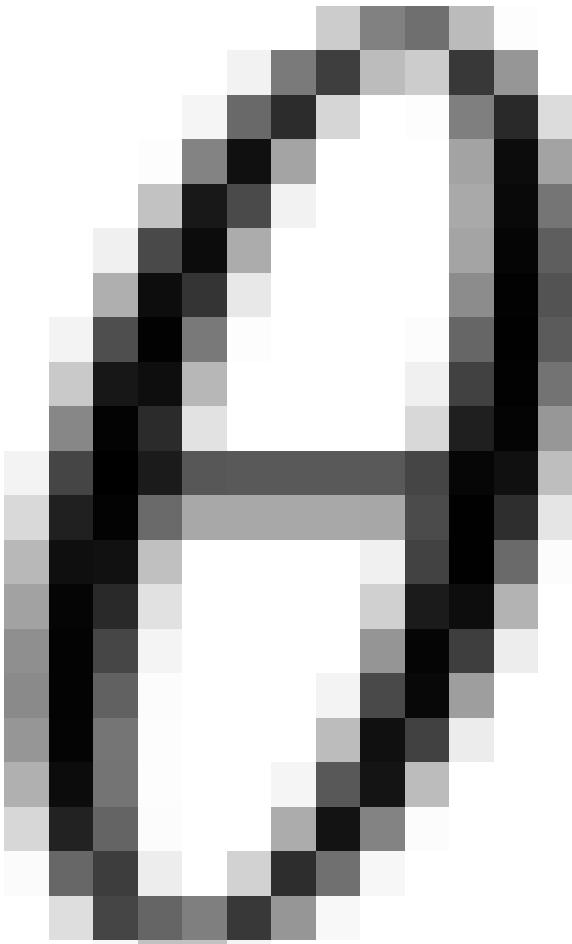


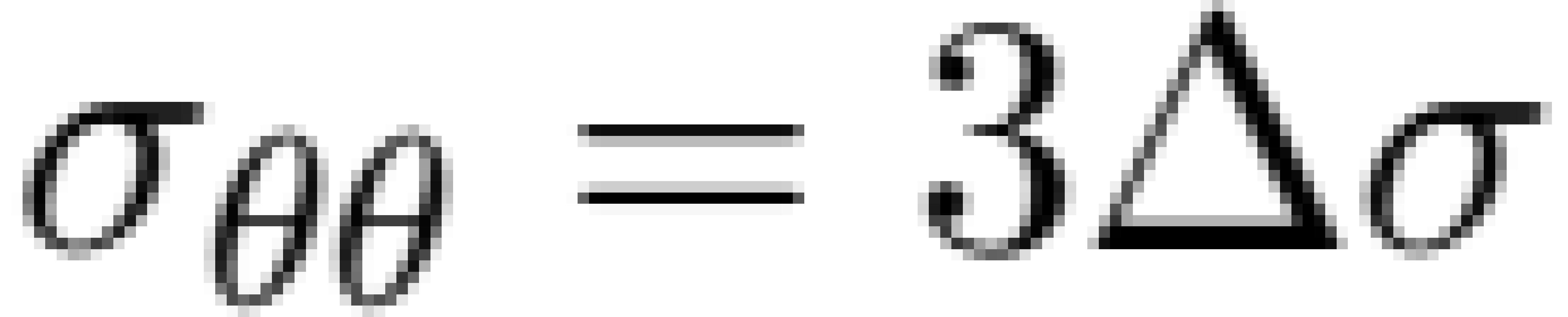


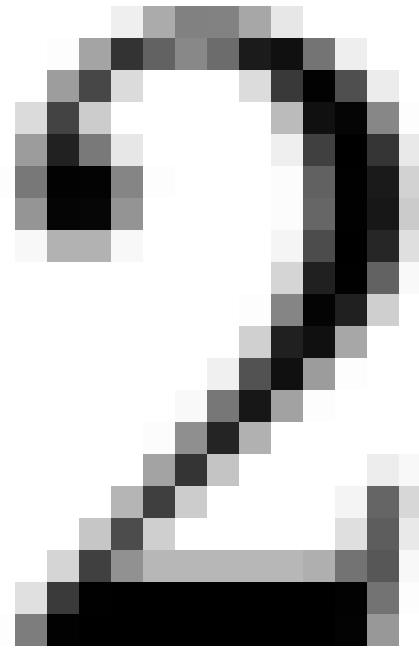
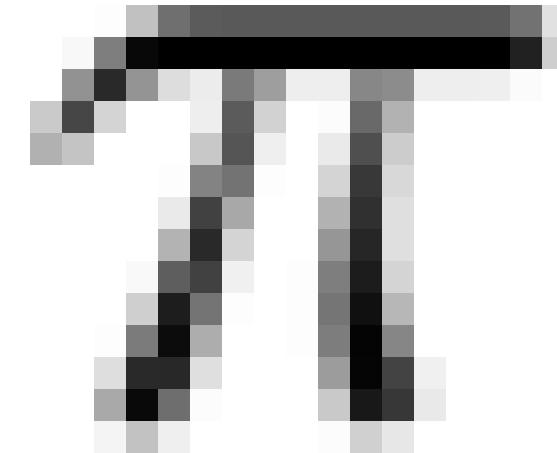
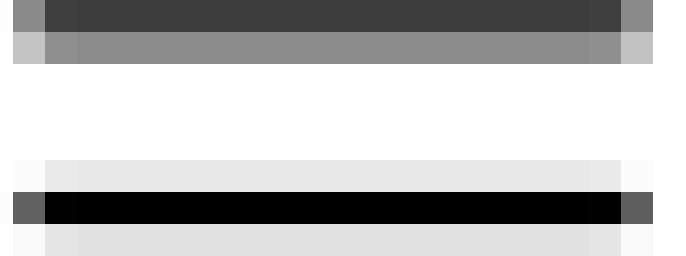
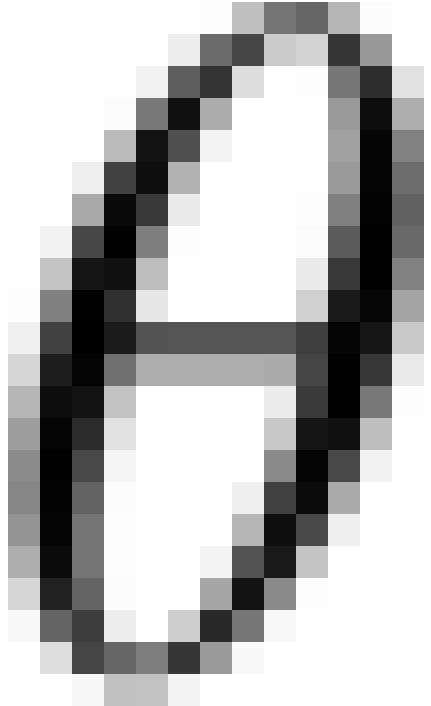


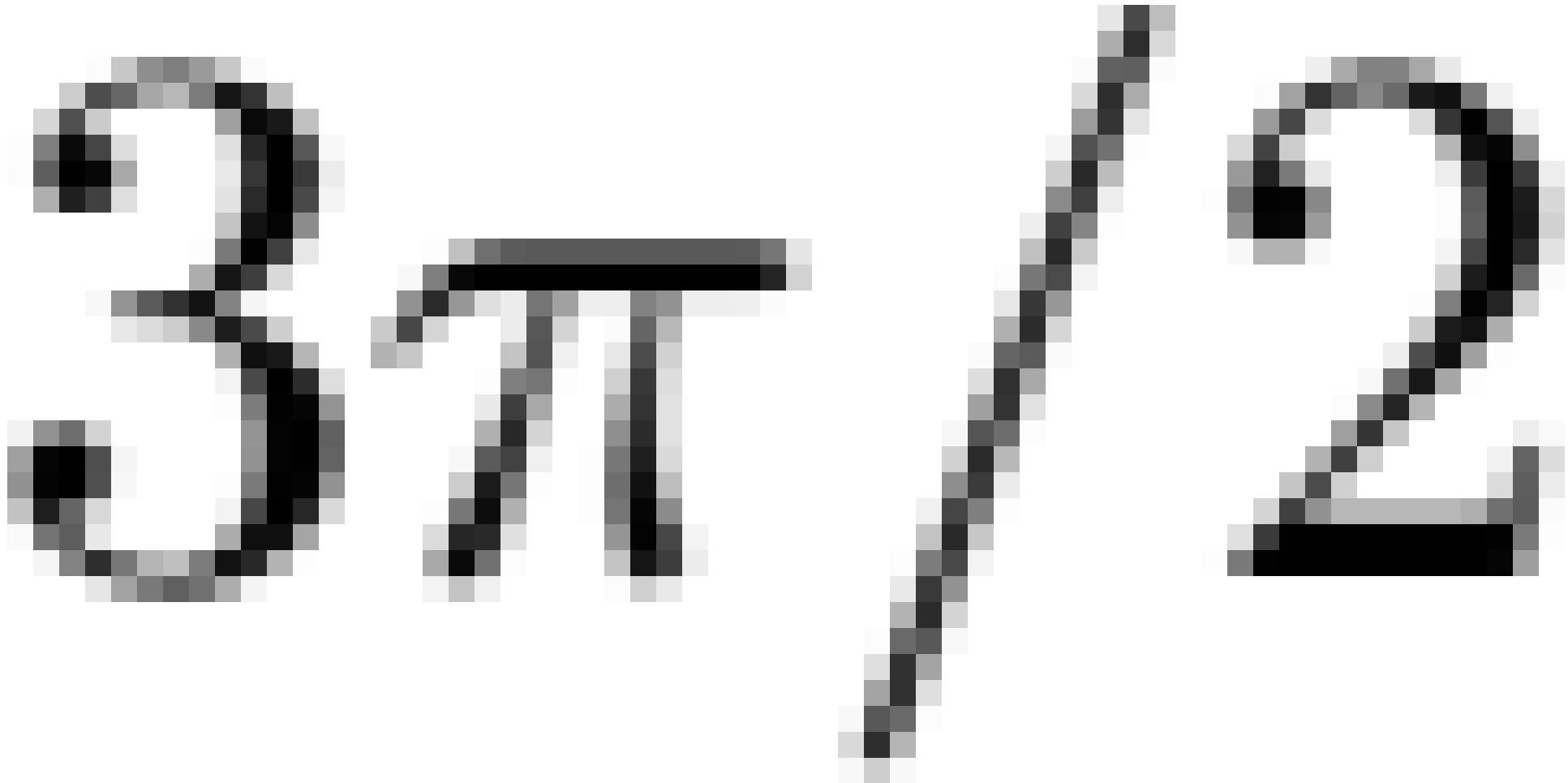


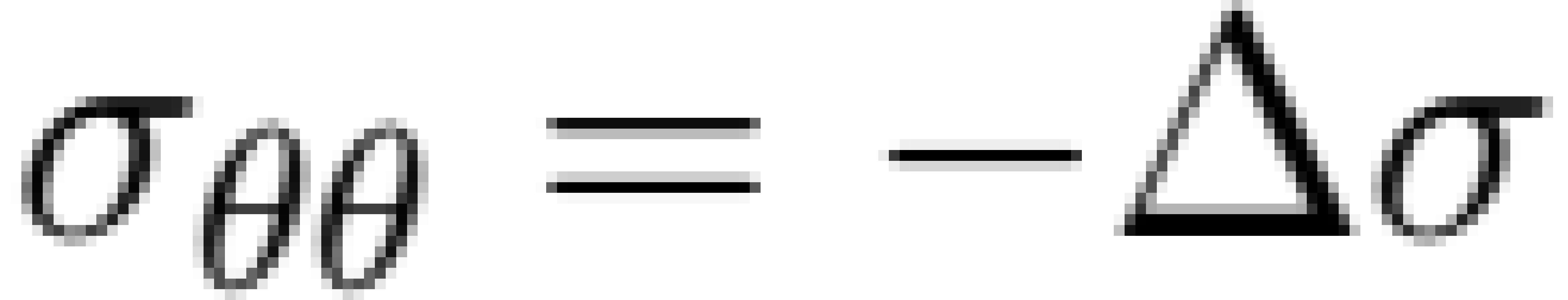


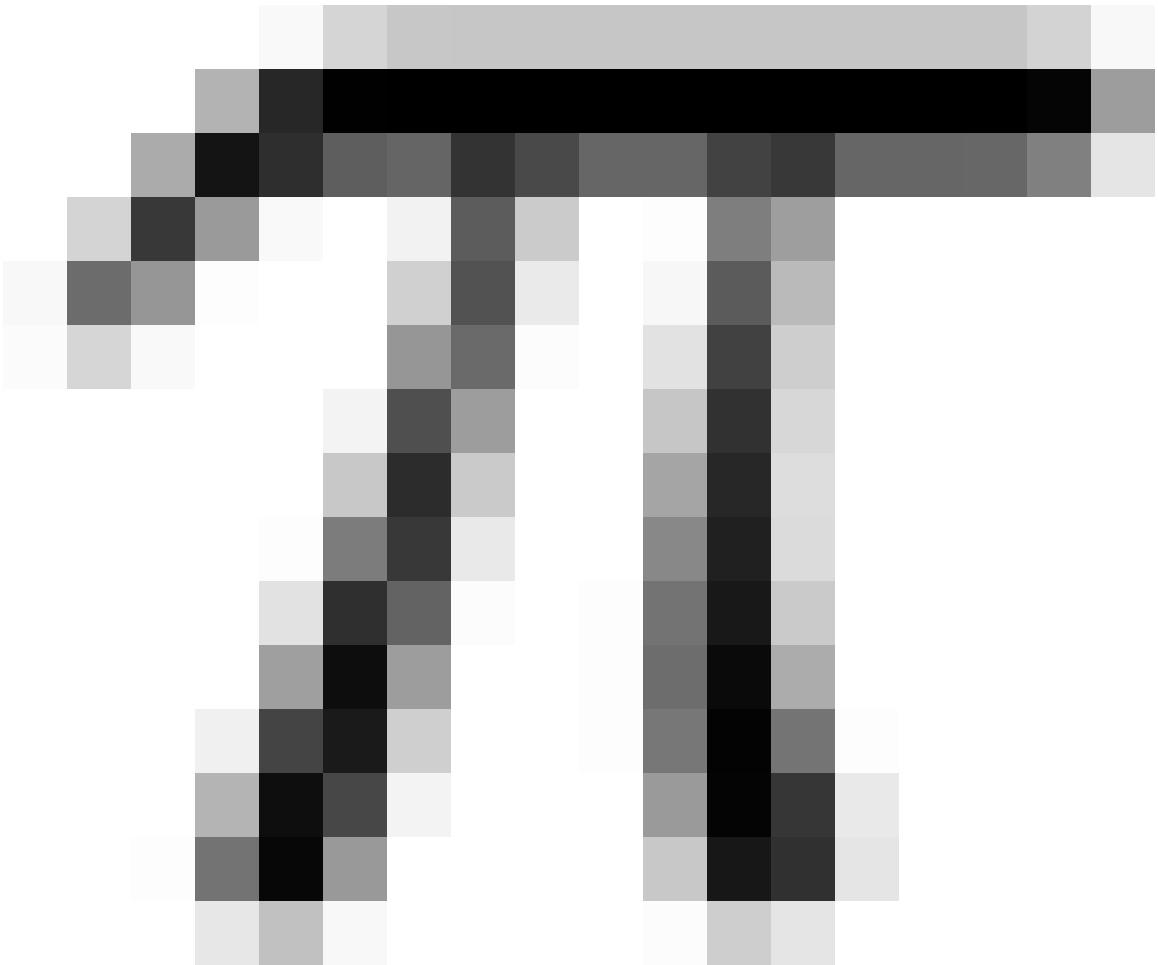




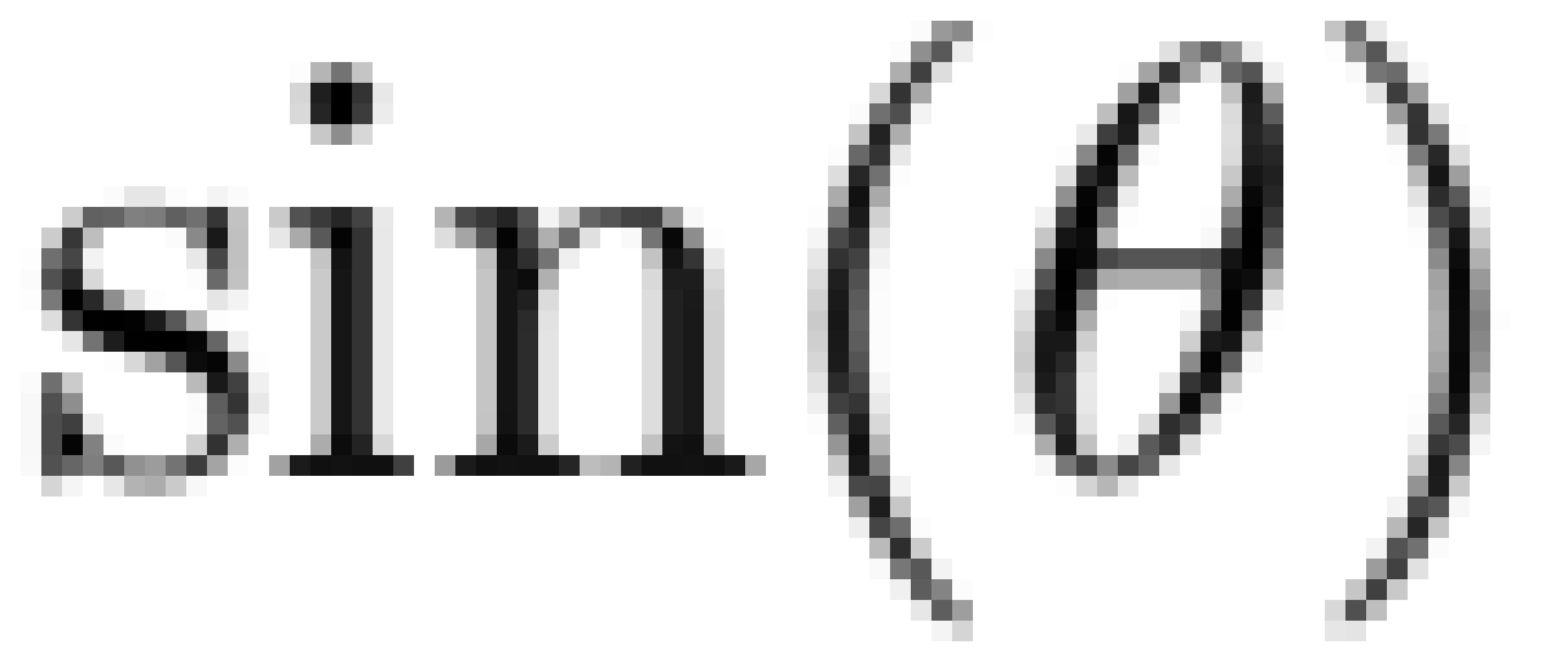




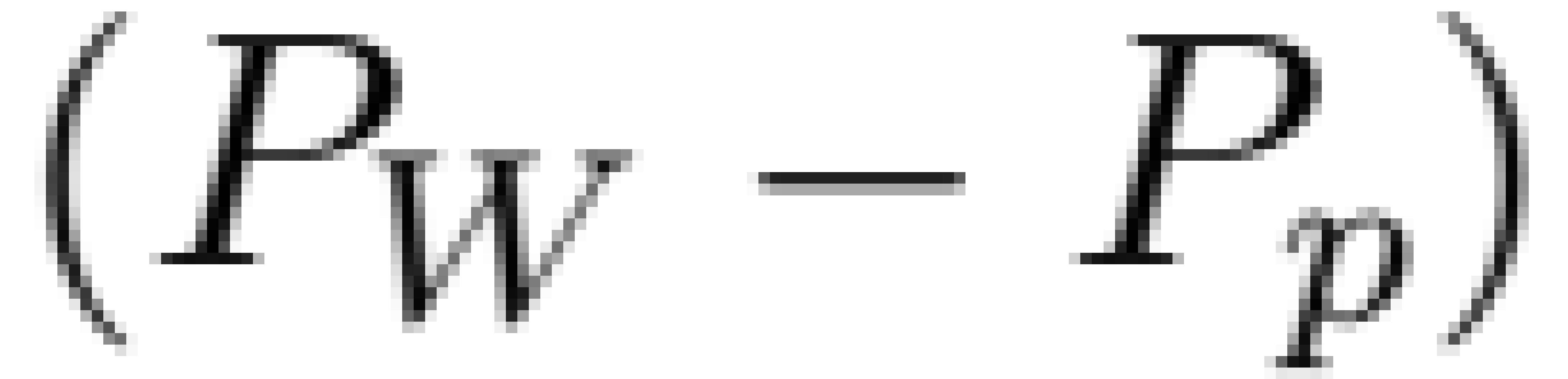


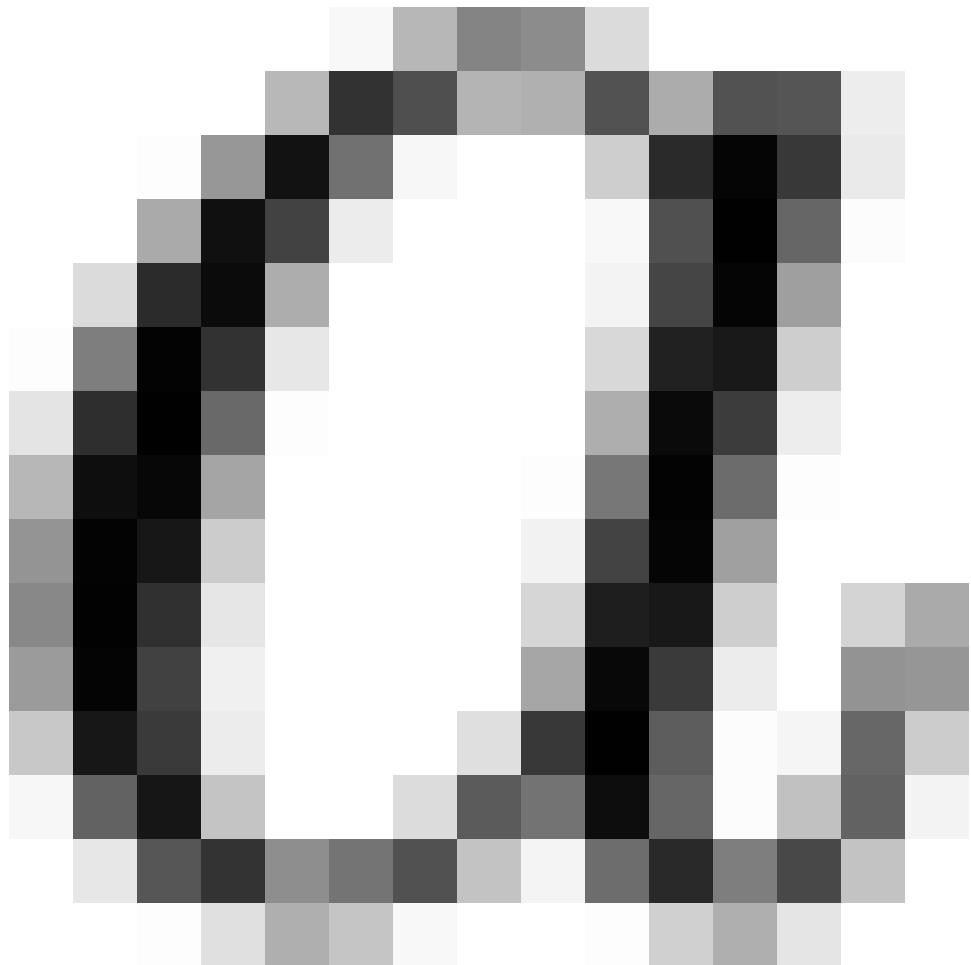




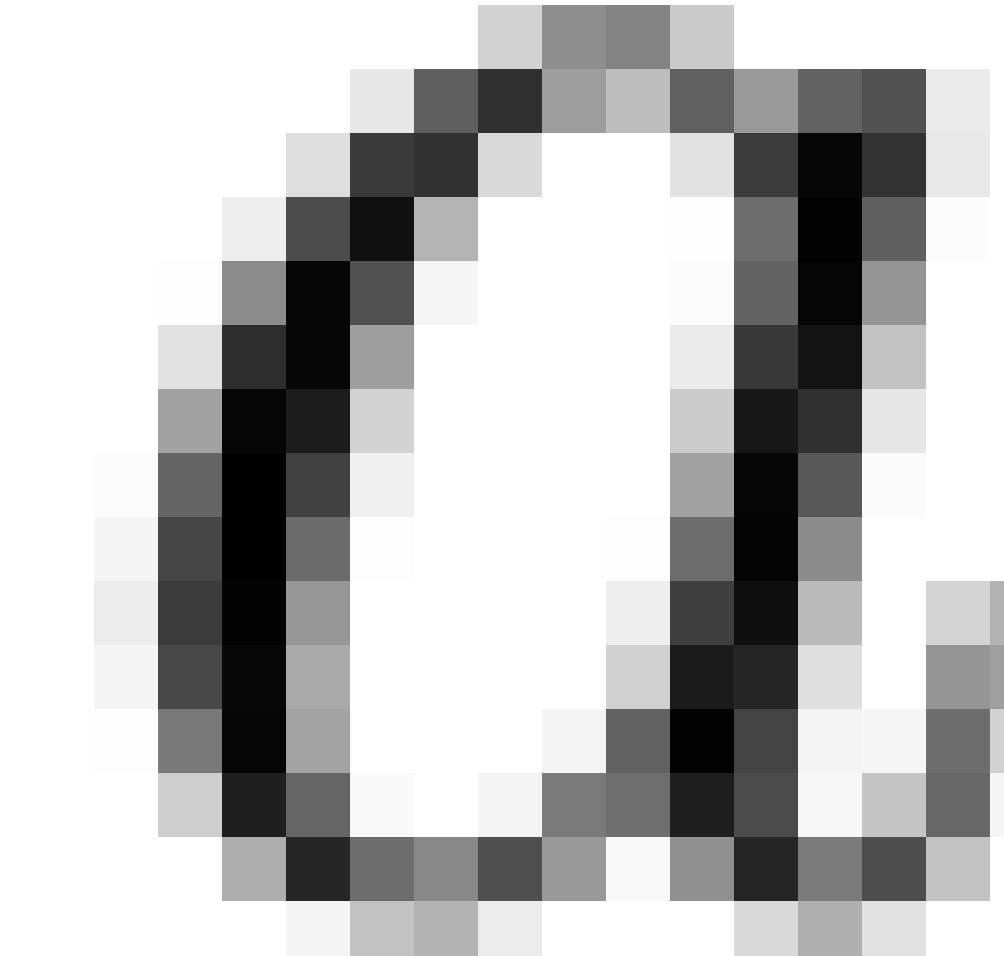
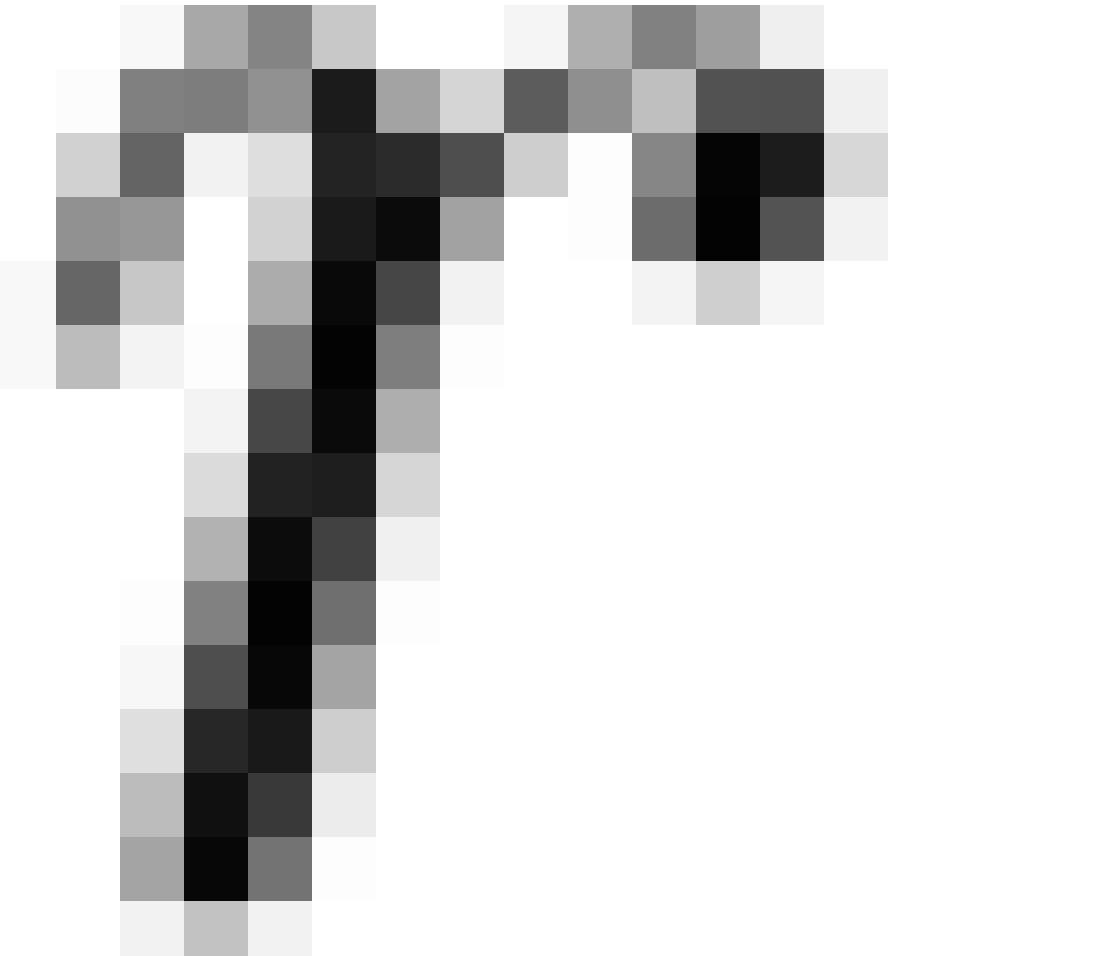






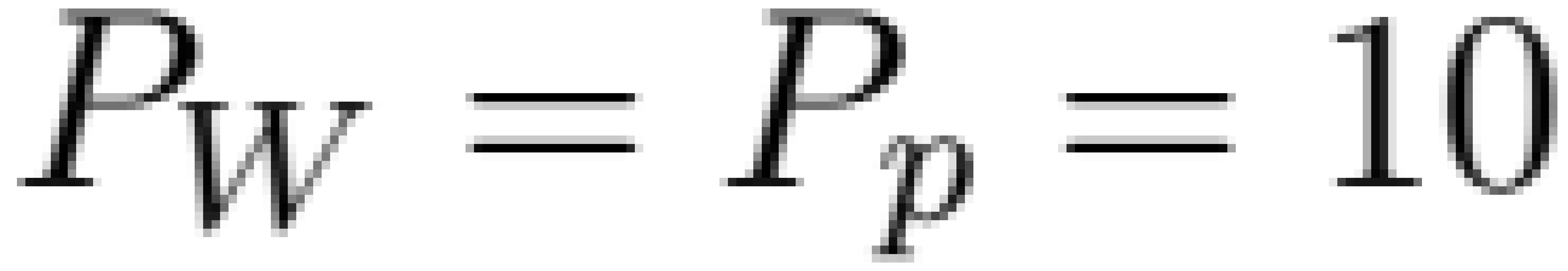


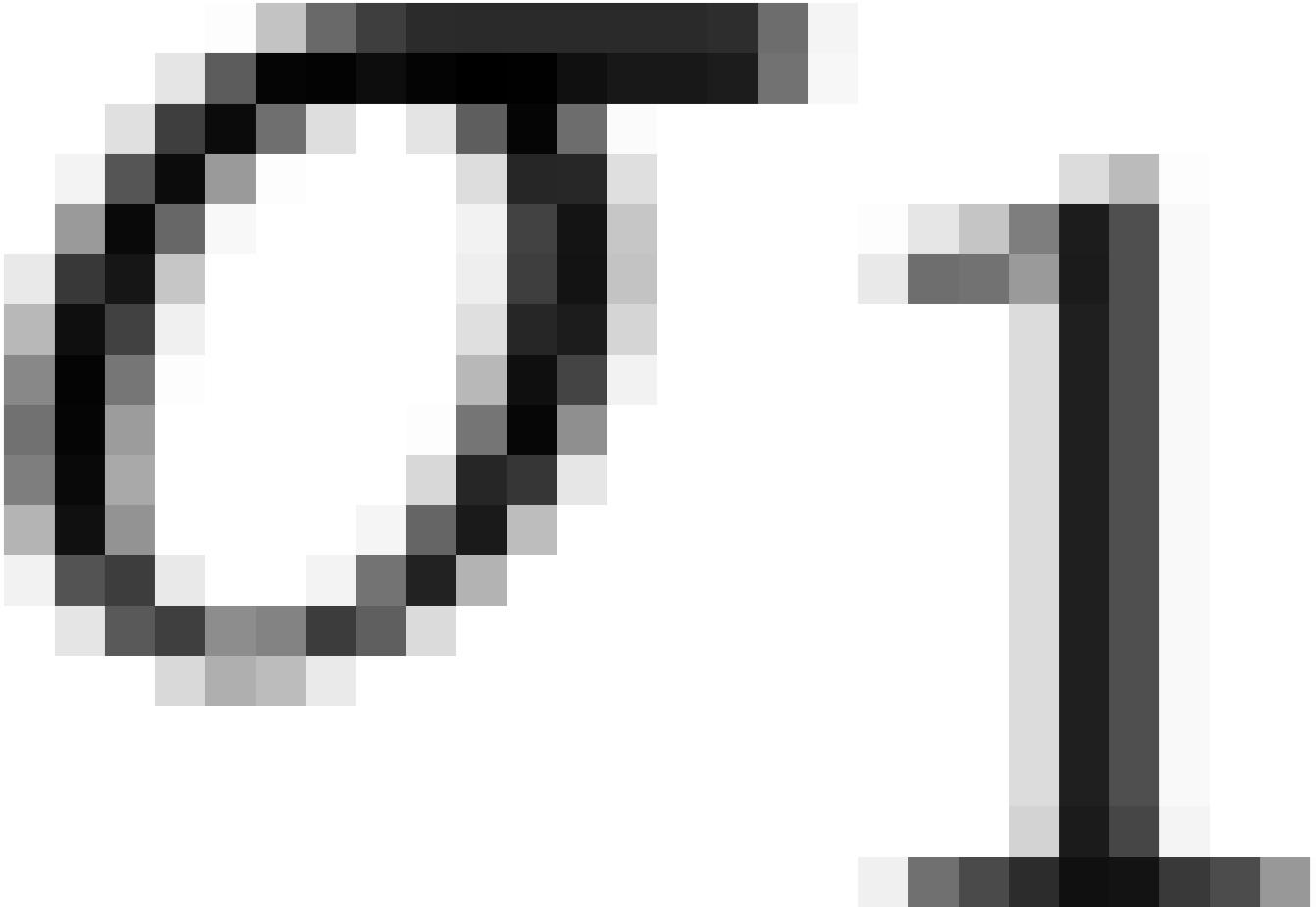
$$\left\{ \begin{array}{lcl} \sigma_{rr} & = & (P_W - P_p) \left(\frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left(1 - \frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{\theta\theta} & = & -(P_W - P_p) \left(\frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left(1 + \frac{a^2}{r^2} \right) - \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{r\theta} & = & \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 + 2 \frac{a^2}{r^2} - 3 \frac{a^4}{r^4} \right) \sin(2\theta) \\ \\ \sigma_{zz} & = & \sigma_v - 2\nu (\sigma_{Hmax} - \sigma_{hmin}) \left(\frac{a^2}{r^2} \right) \cos(2\theta) \end{array} \right.$$



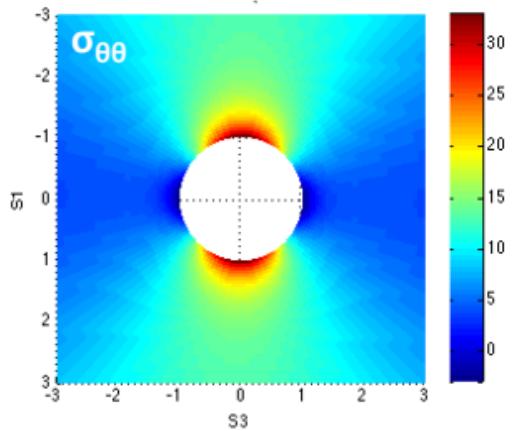
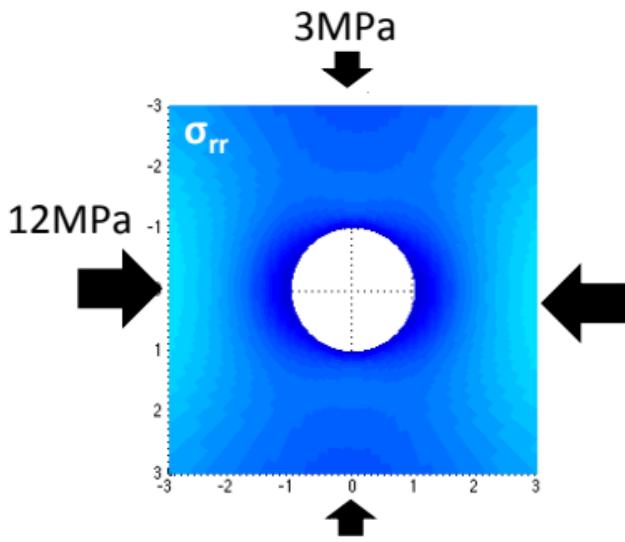




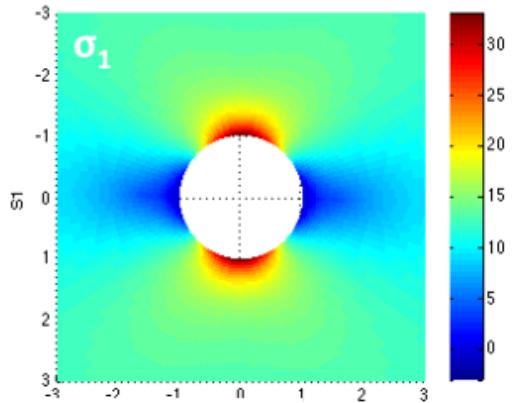
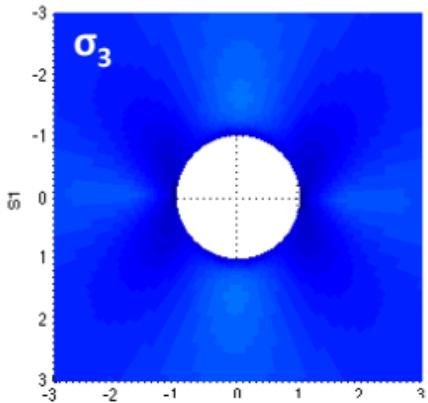


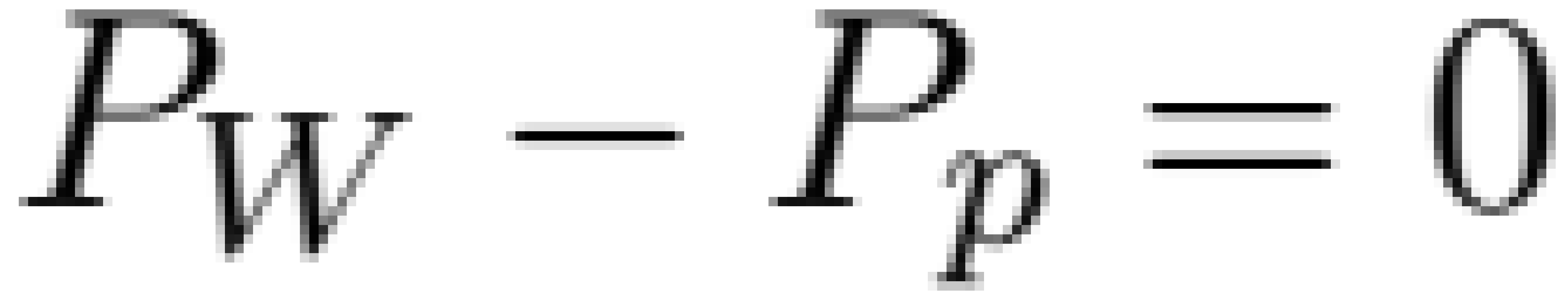


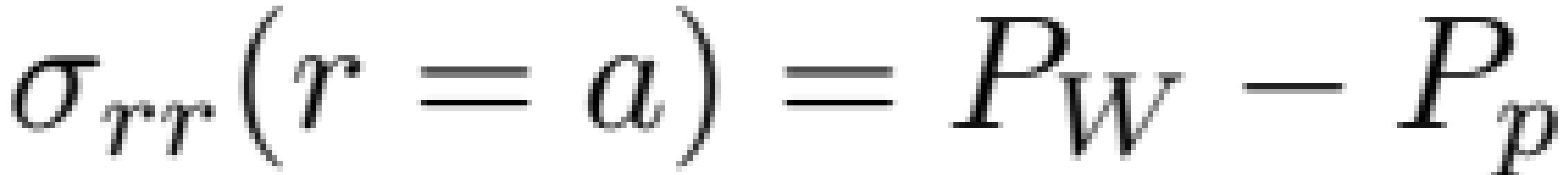
Stresses in cylindrical coordinates



Principal Stresses







$$\sigma_{\text{H}\alpha\alpha} = \sigma_{\text{H}\alpha\alpha}^{\text{max}} + \sigma_{\text{H}\alpha\alpha}^{\text{min}} \left(\frac{P_{\text{p}}}{P_{\text{p}} + P_{\text{d}}} \right)^2 \left(\frac{\sin(\theta)}{\sin(\theta + \phi)} \right)^2$$

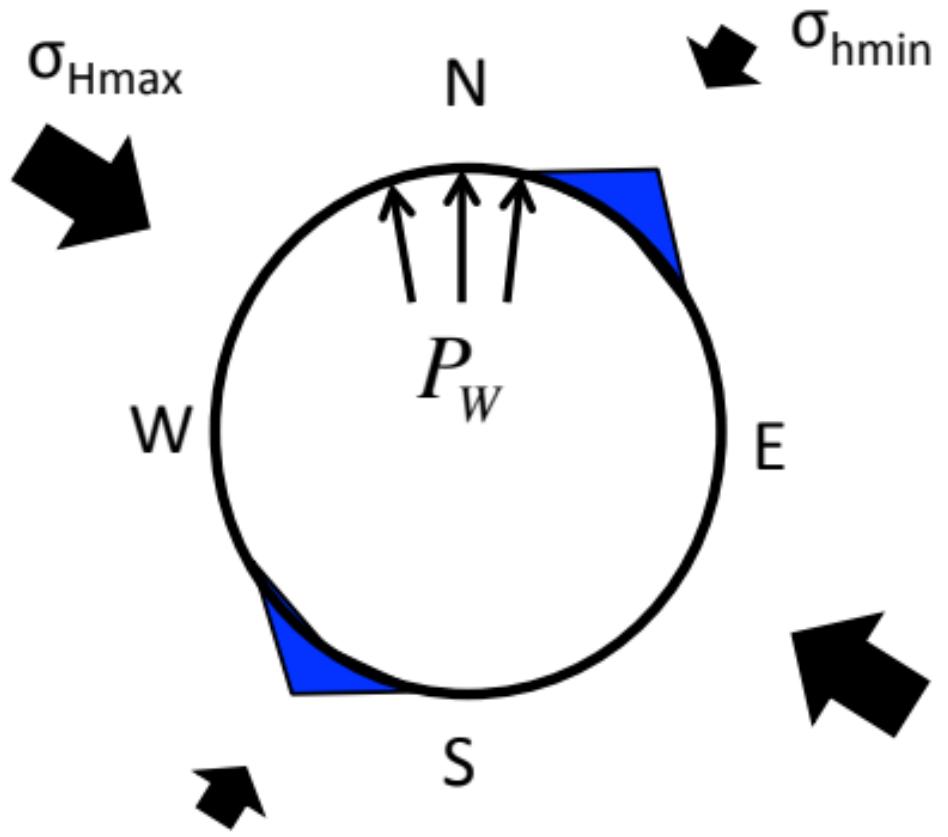
$$\begin{aligned} \sigma_{\theta\theta}(r=a, \theta=0) &= (P_W - P_p) + 3\sigma_{h\min} \\ \sigma_{\theta\theta}(r=a, \theta=\pi/2) &= -(P_W - P_p) + 3\sigma_{h\max} \end{aligned}$$







$$\left\{ \begin{array}{l} \sigma_1 = \sigma_{\theta\theta} = - (P_W - P_p) + 3 \sigma_{H\max} - \sigma_{hmin} \\ \sigma_3 = \sigma_{rr} = (P_W - P_p) \end{array} \right.$$



Stress anisotropy

$$P_{W\text{shear}} = P_p + \frac{3\sigma_{H\max} - \sigma_{h\min} - UCS}{1+q}$$

Pore pressure in the formation

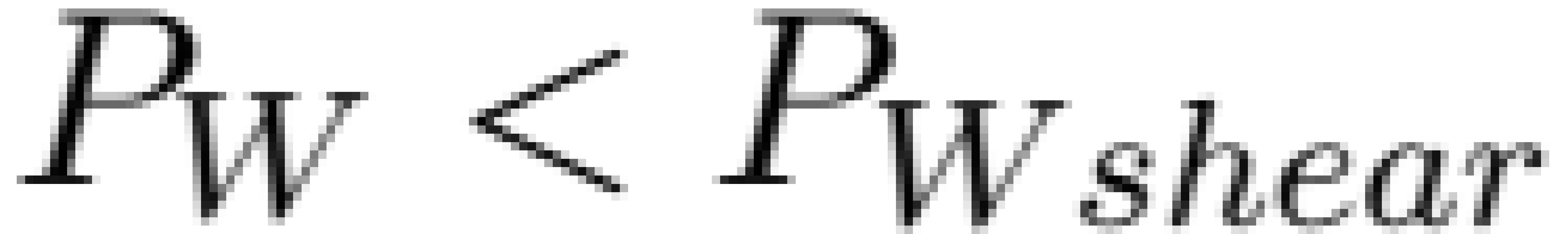
Shear strength

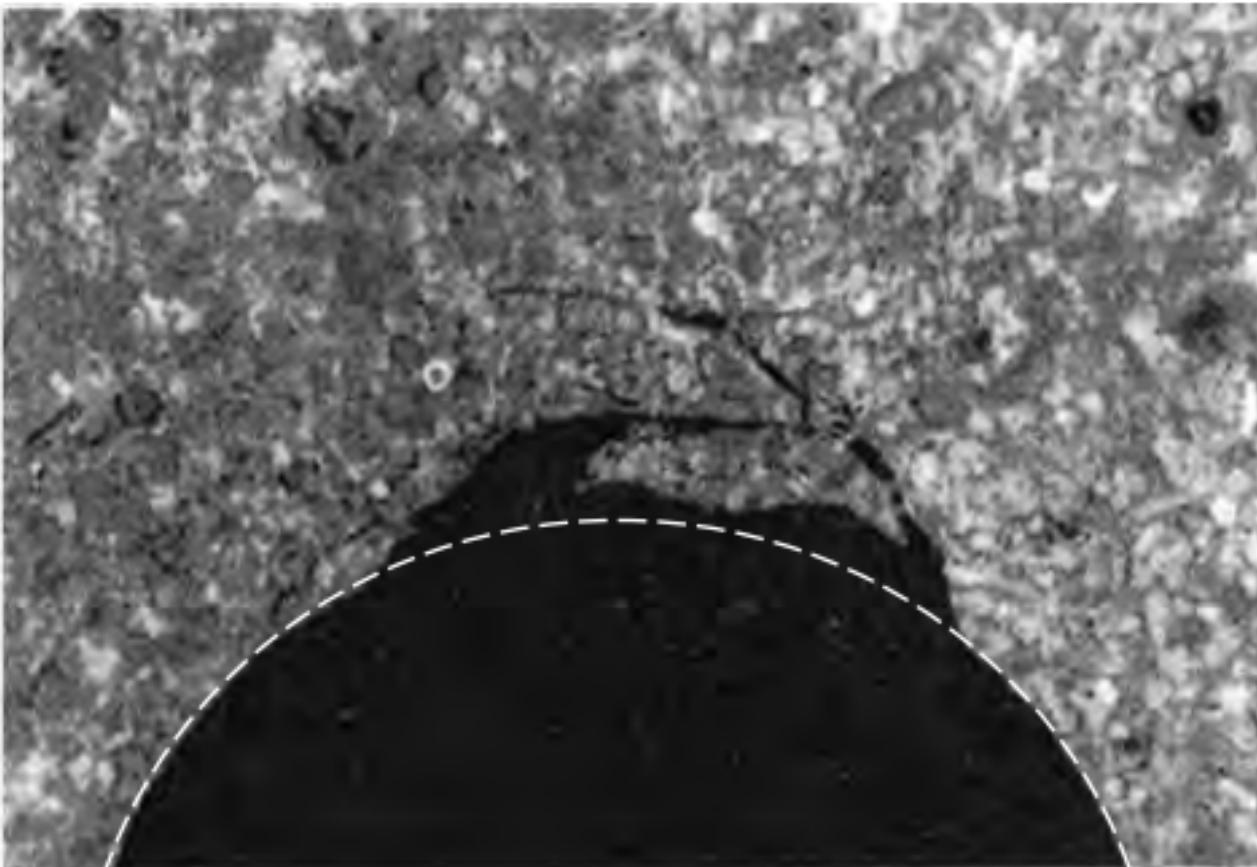
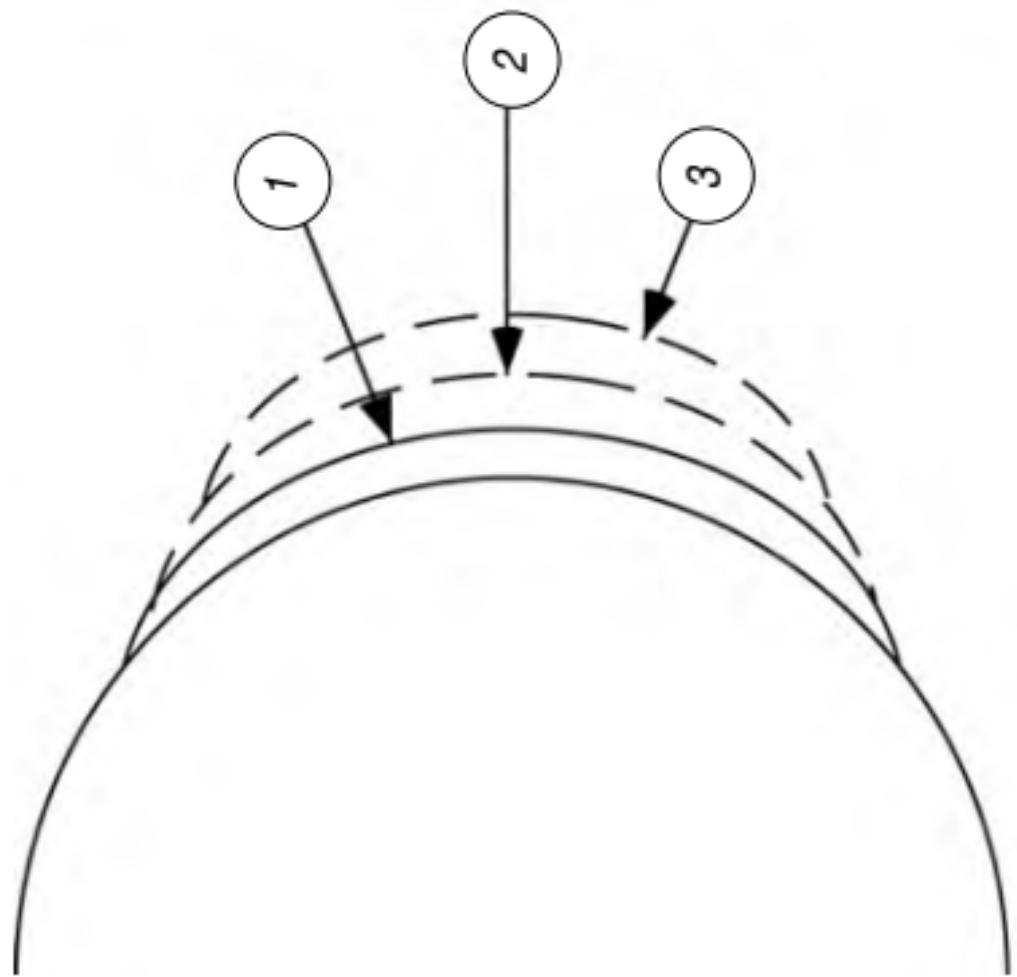
$P_W \leq P_{W\text{shear}}$ leads to breakouts

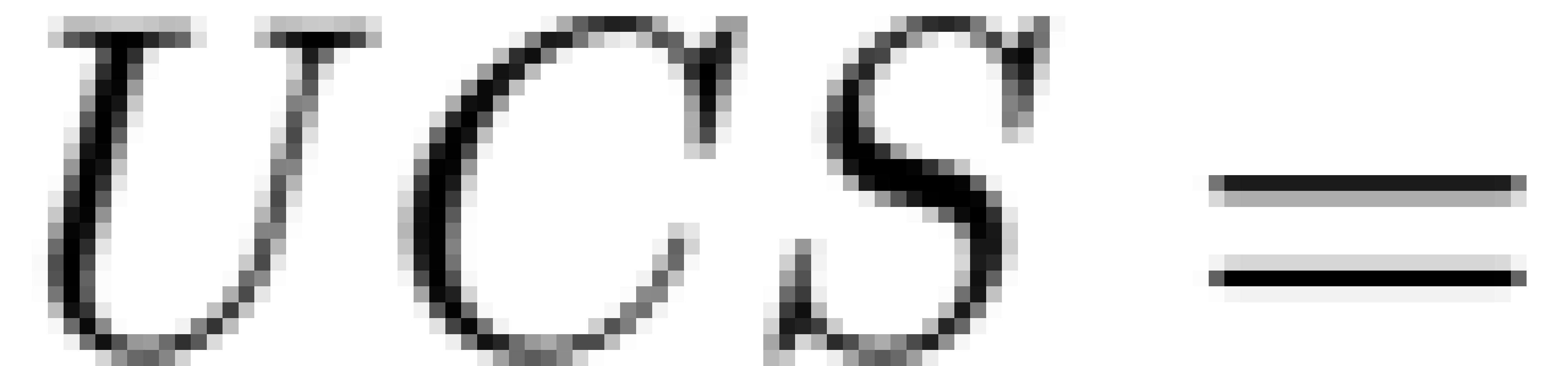


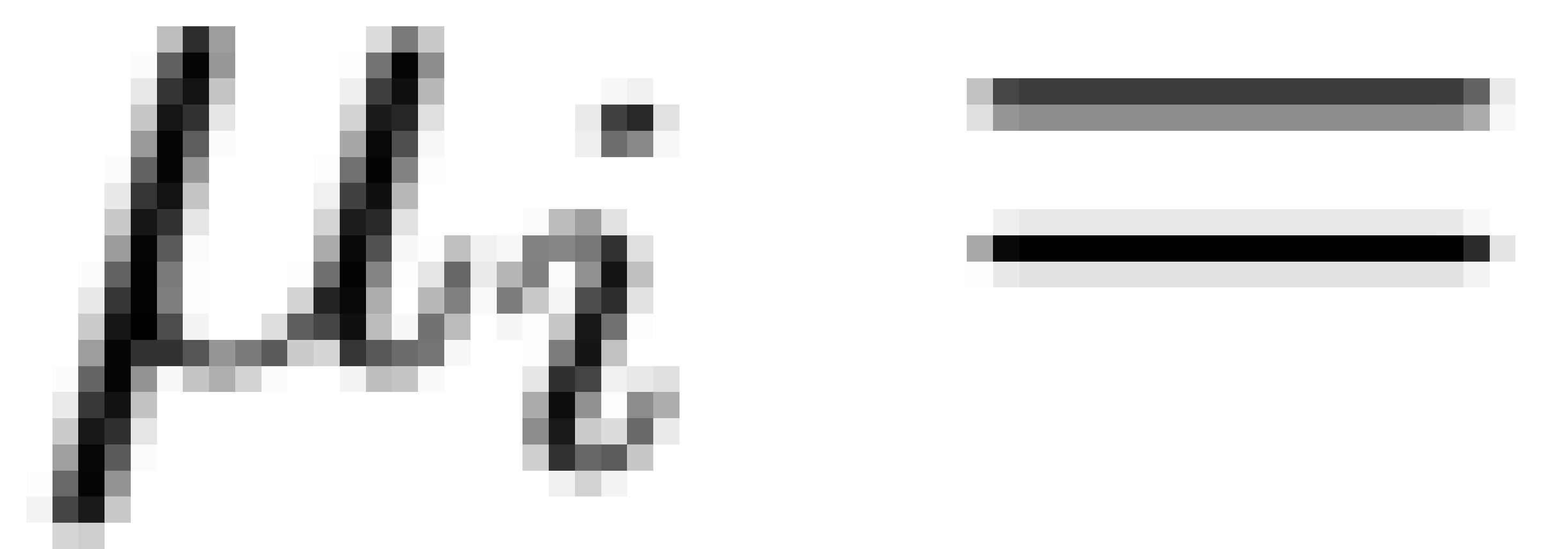
$\left[\frac{1}{\sqrt{2}}(P_+ + P_-) \right] = \frac{1}{\sqrt{2}}(S_x + iS_y)$

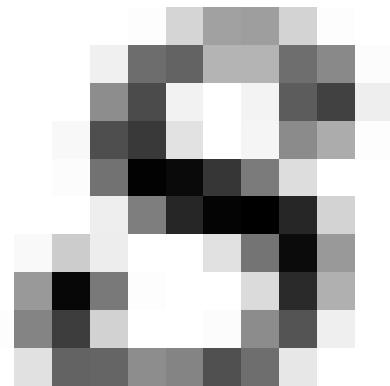
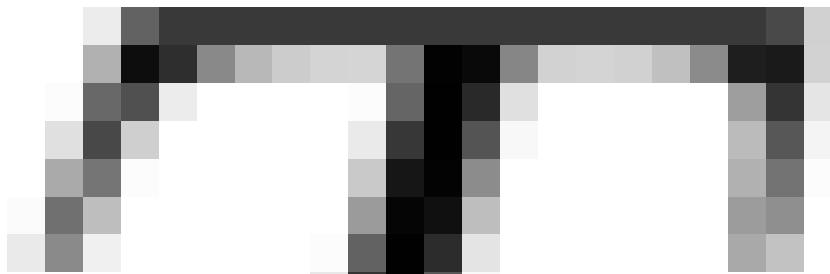
$$P_{W\text{ shear}} = \frac{3\sigma_H \max - \sigma_{hmin} - U_{CS}}{1 + q}$$

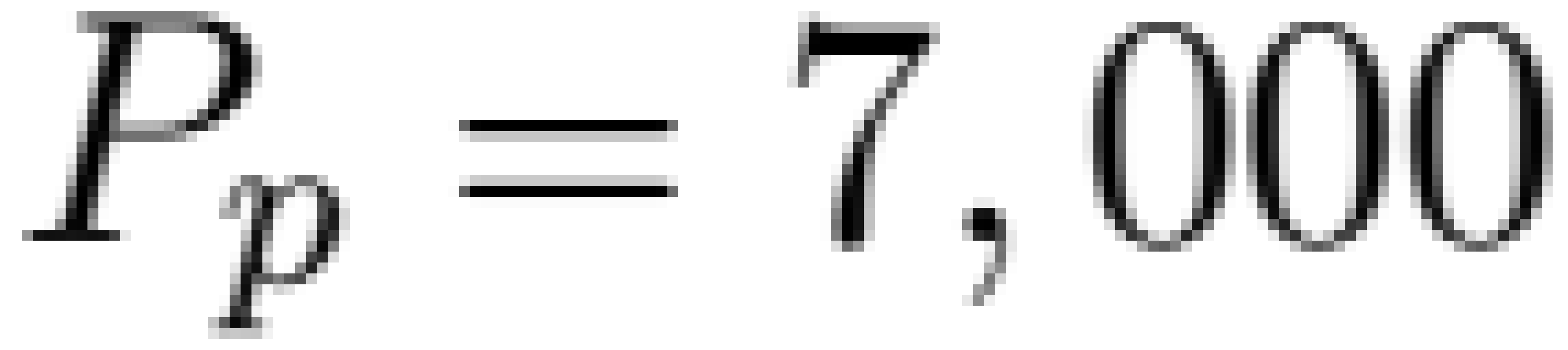


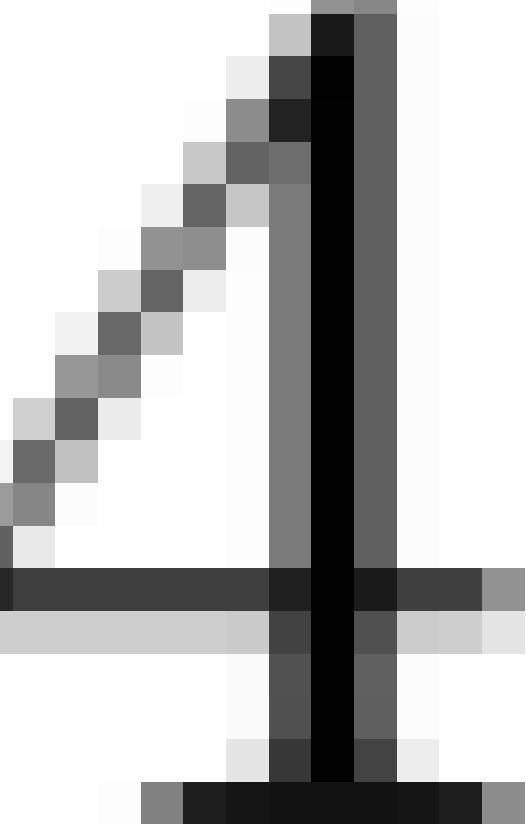
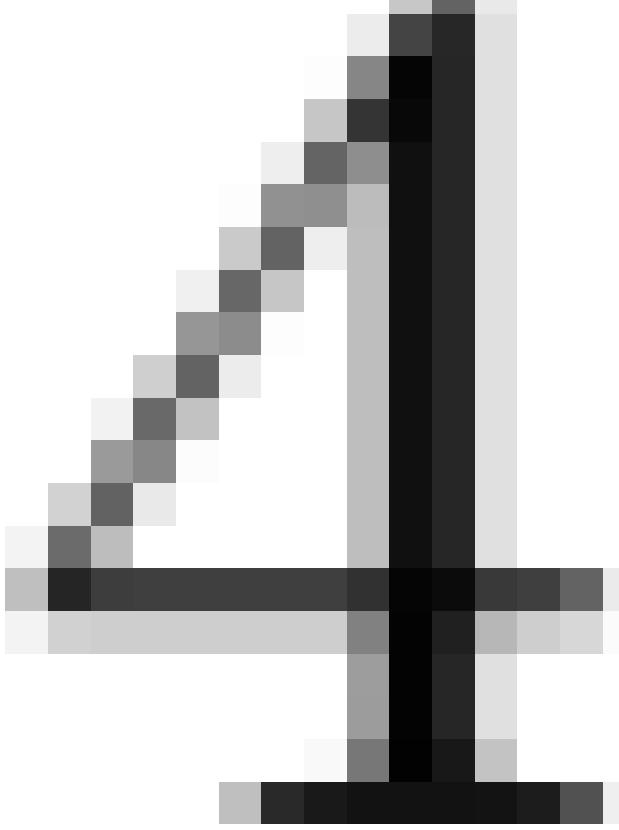
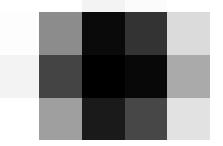
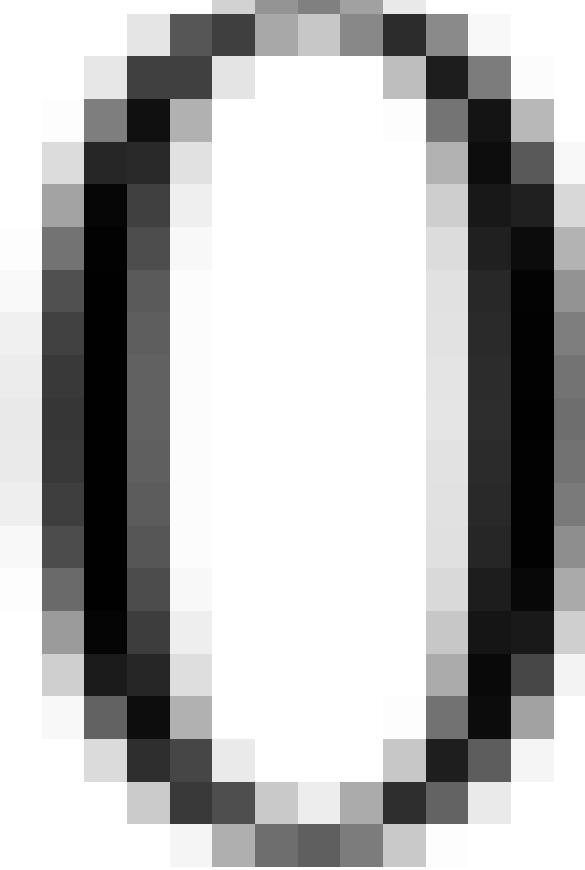
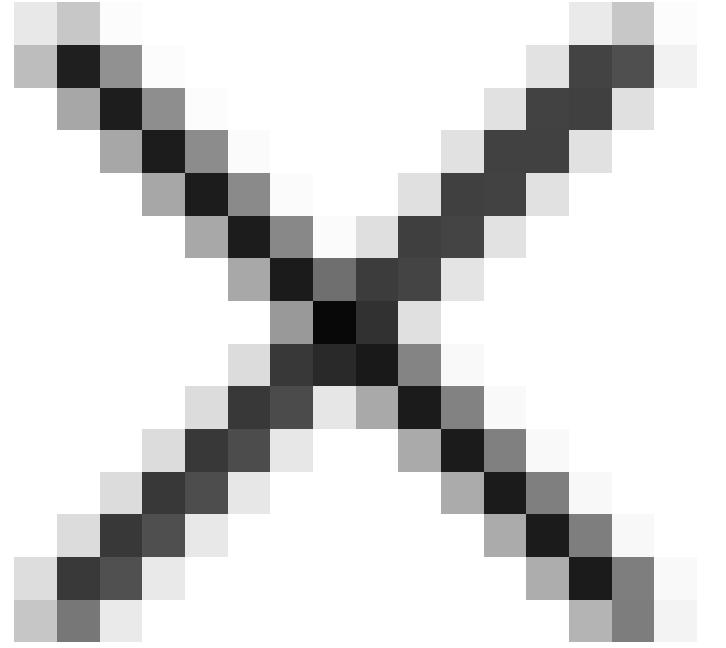


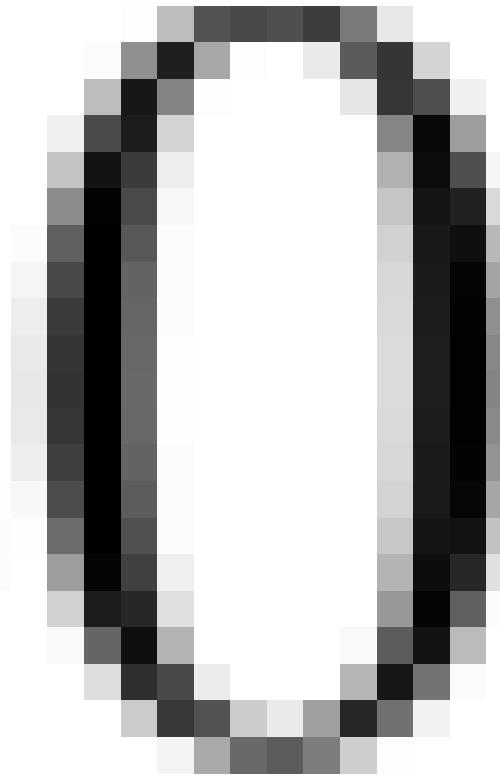
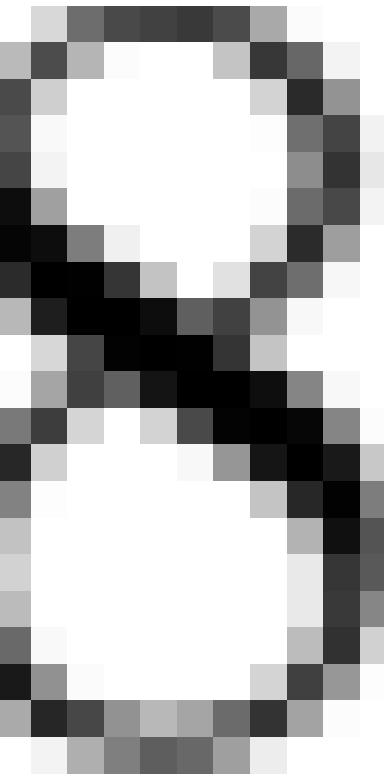
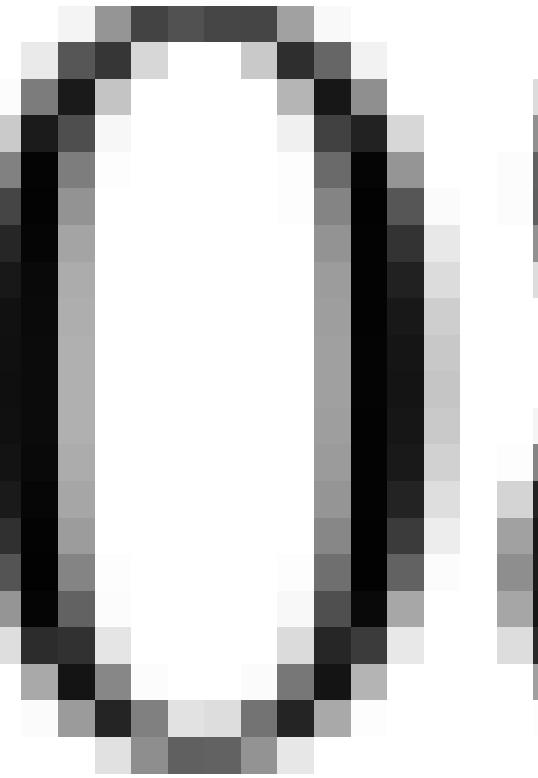
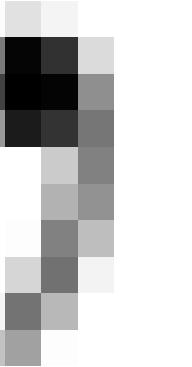
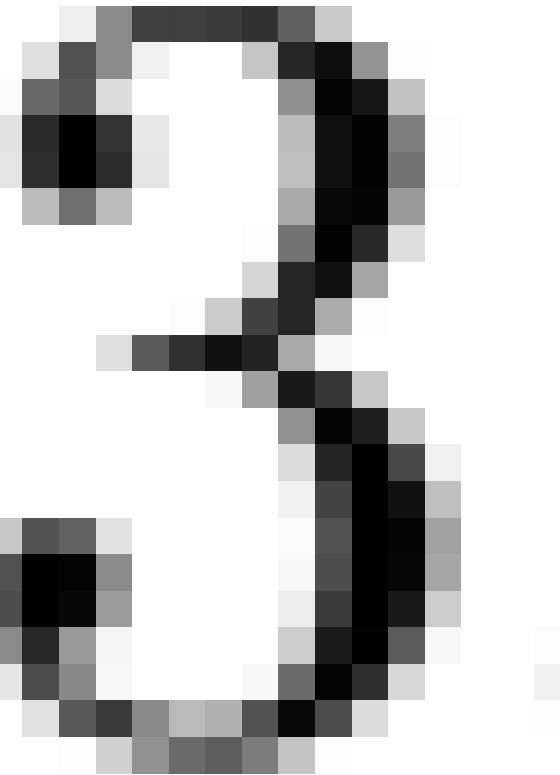


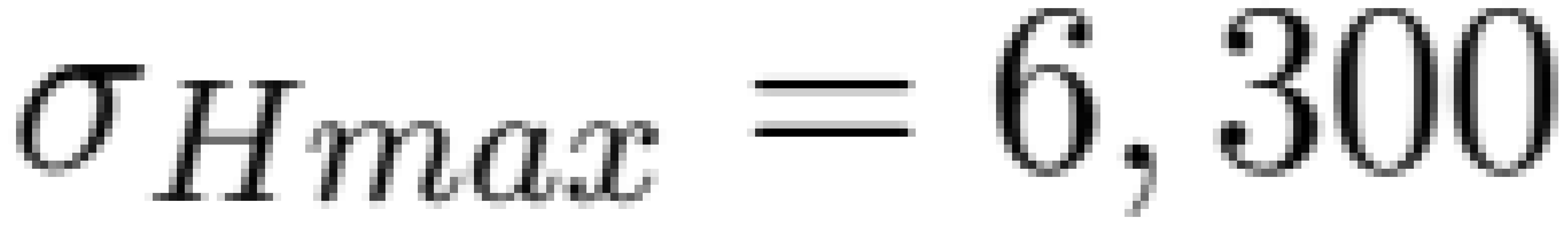


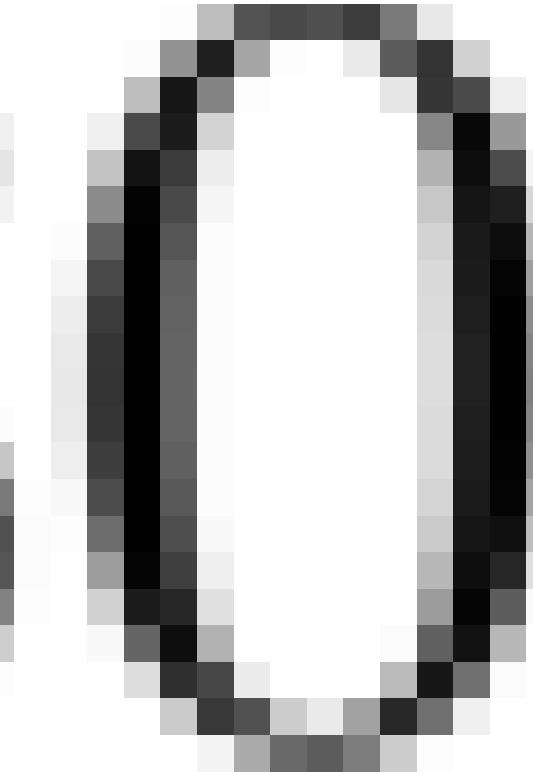
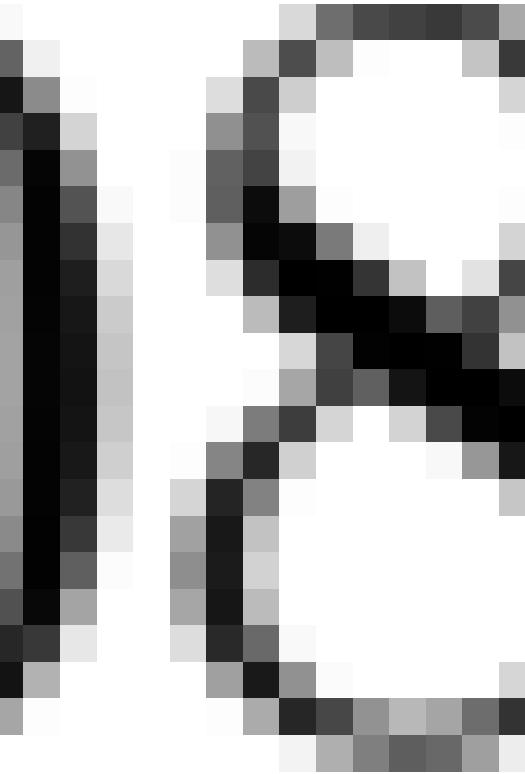
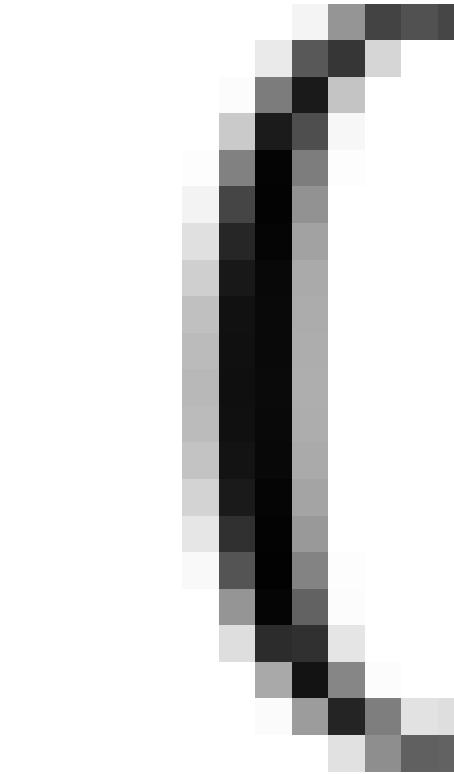
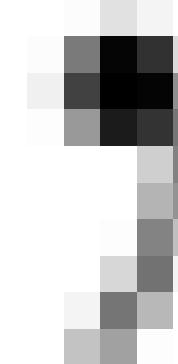
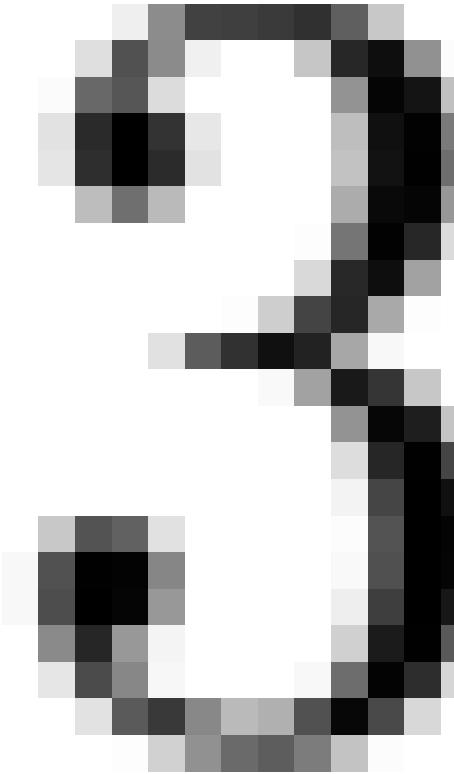


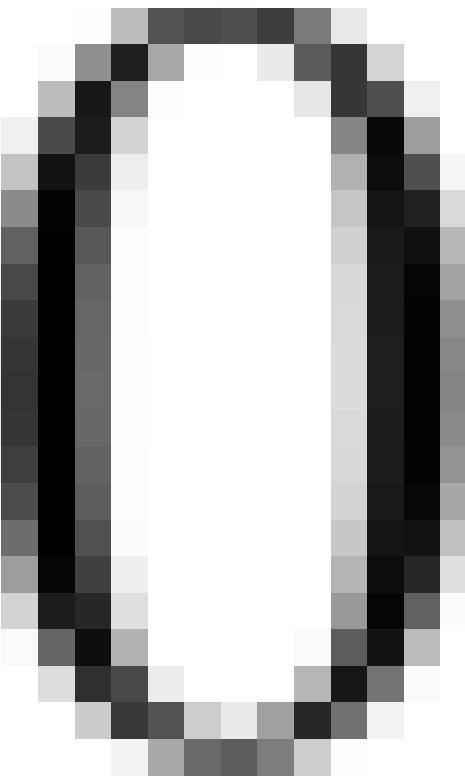
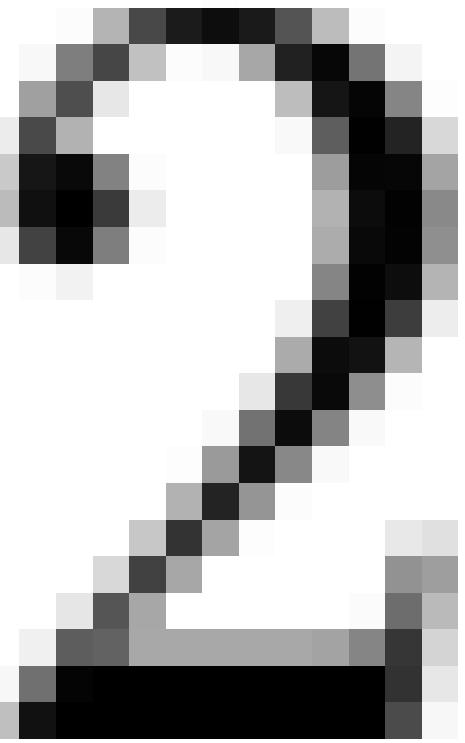
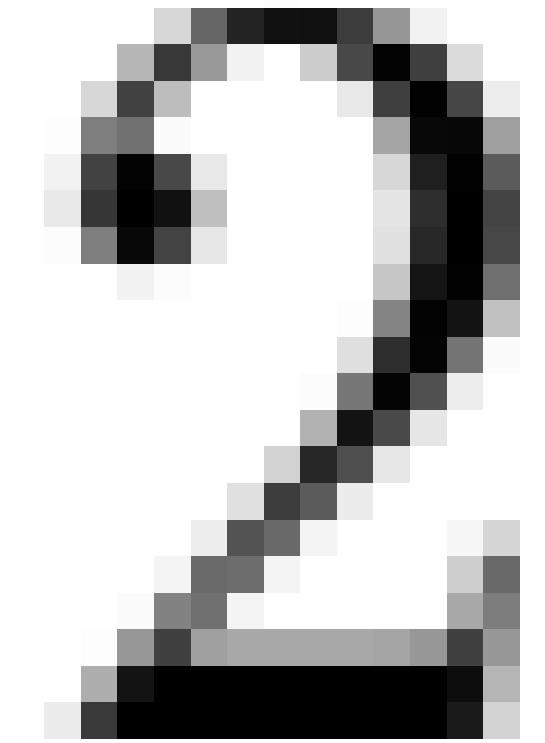
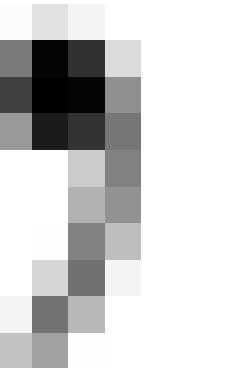
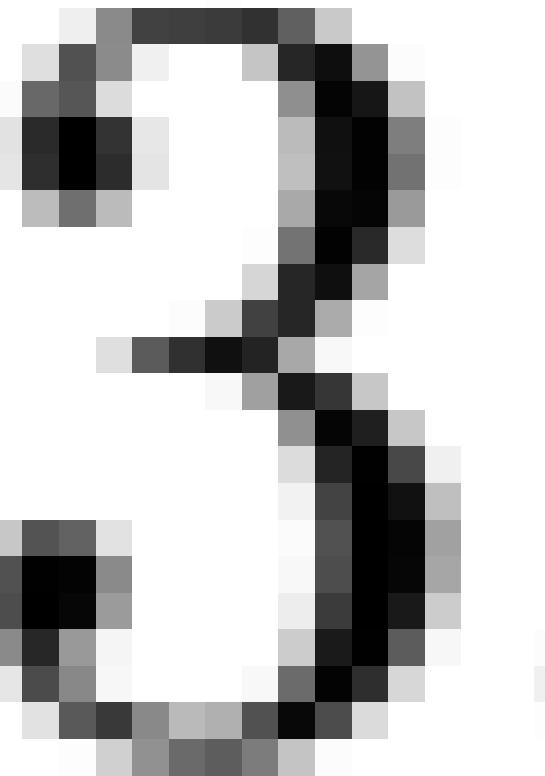


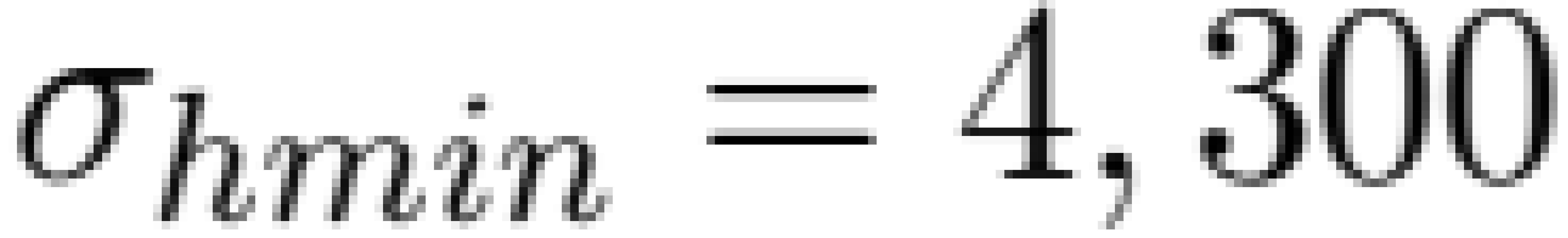


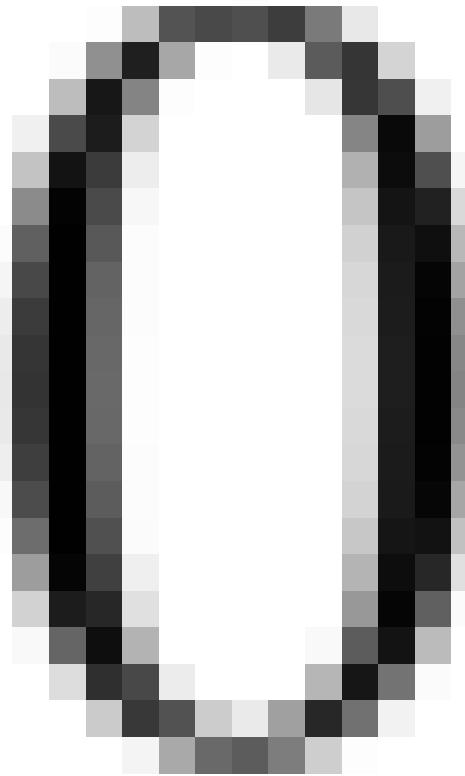
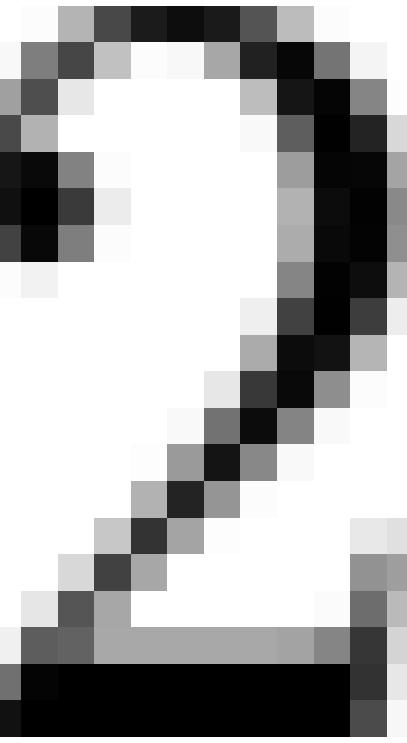
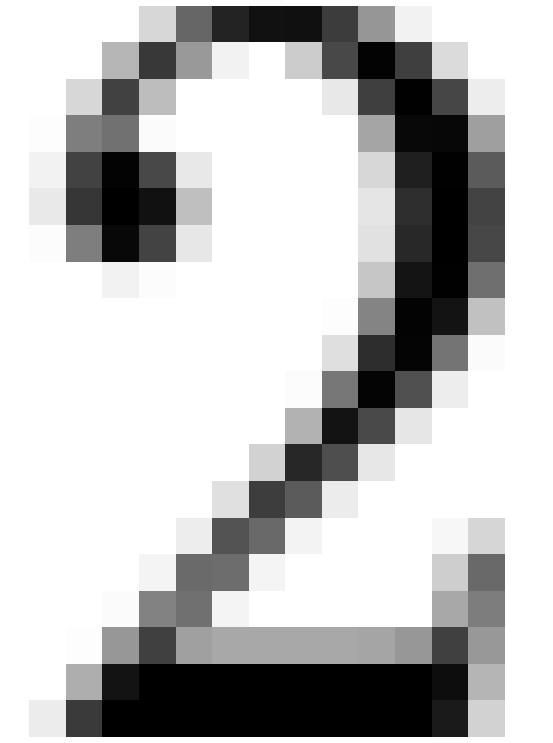
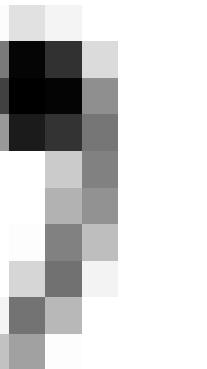
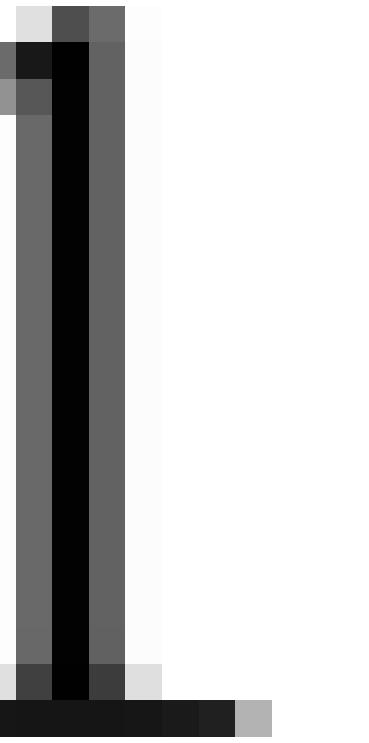














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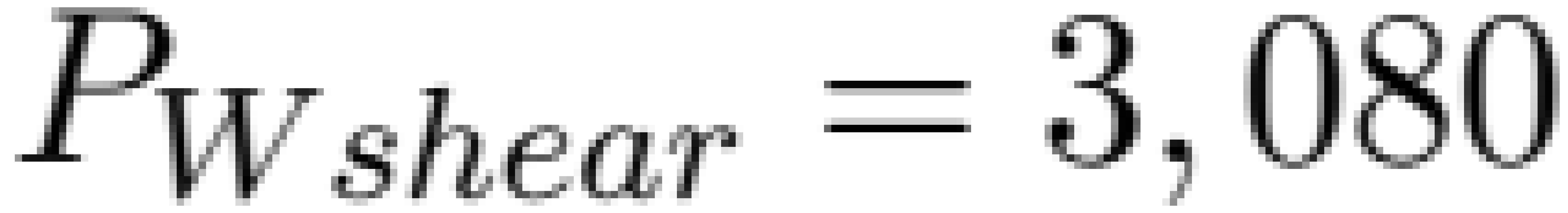
1

$$1 + \sin 30.96^\circ$$

$$- \sin 30.96^\circ$$

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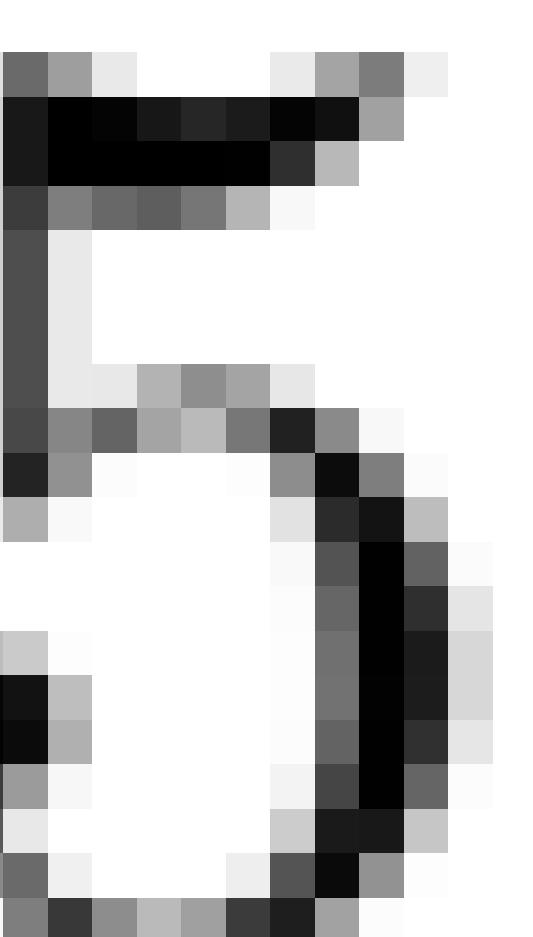
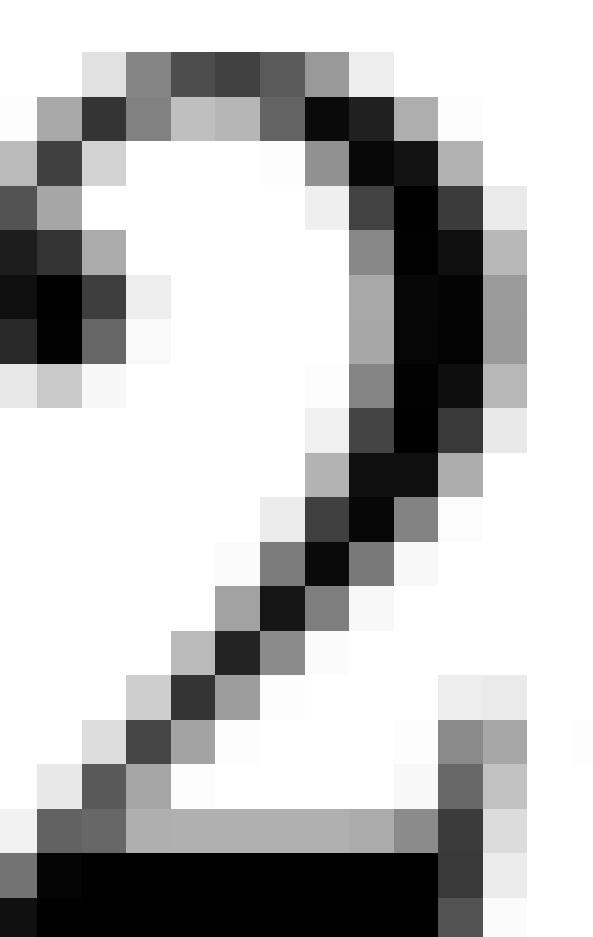
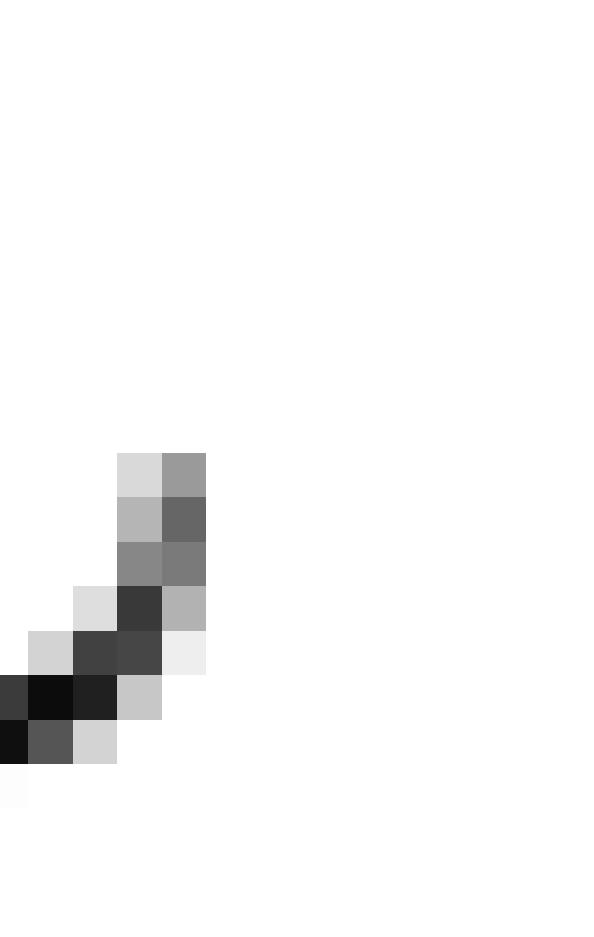
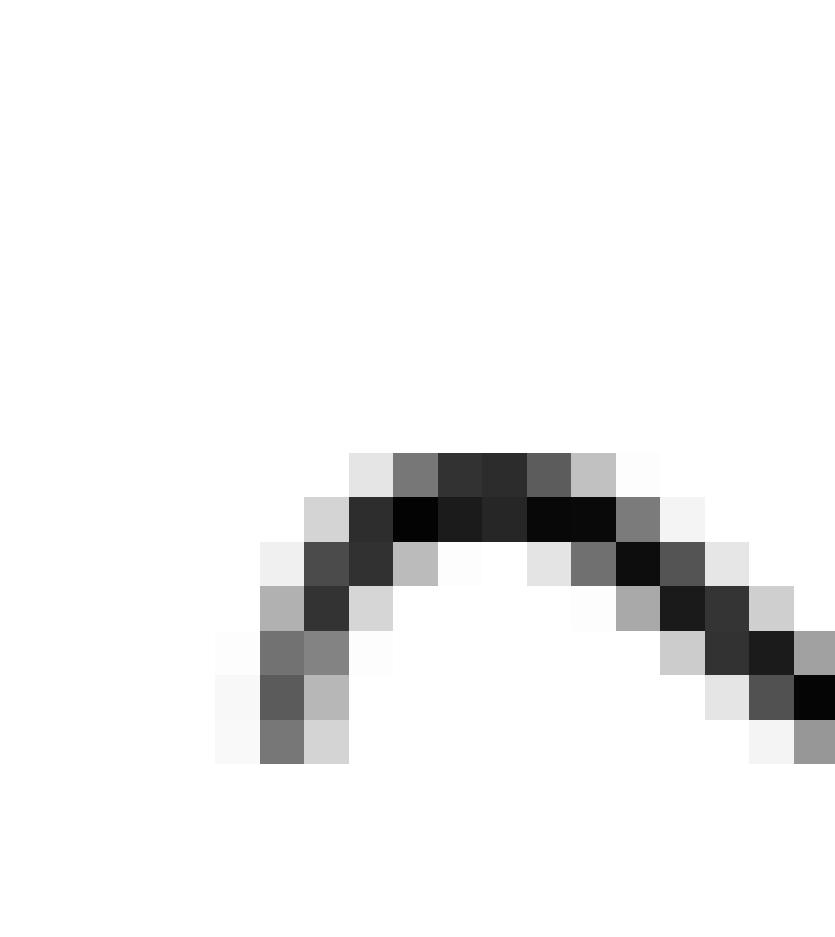
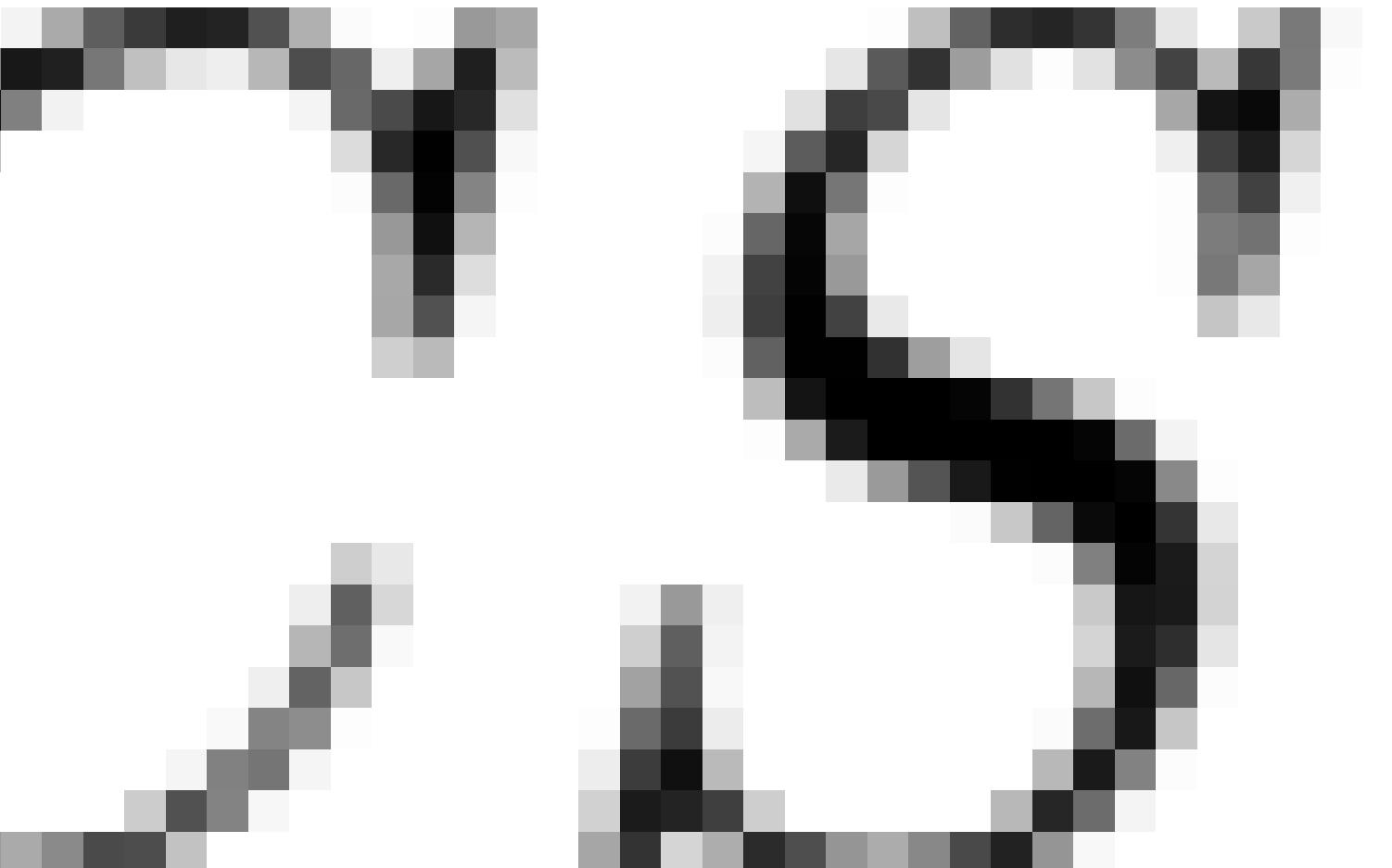
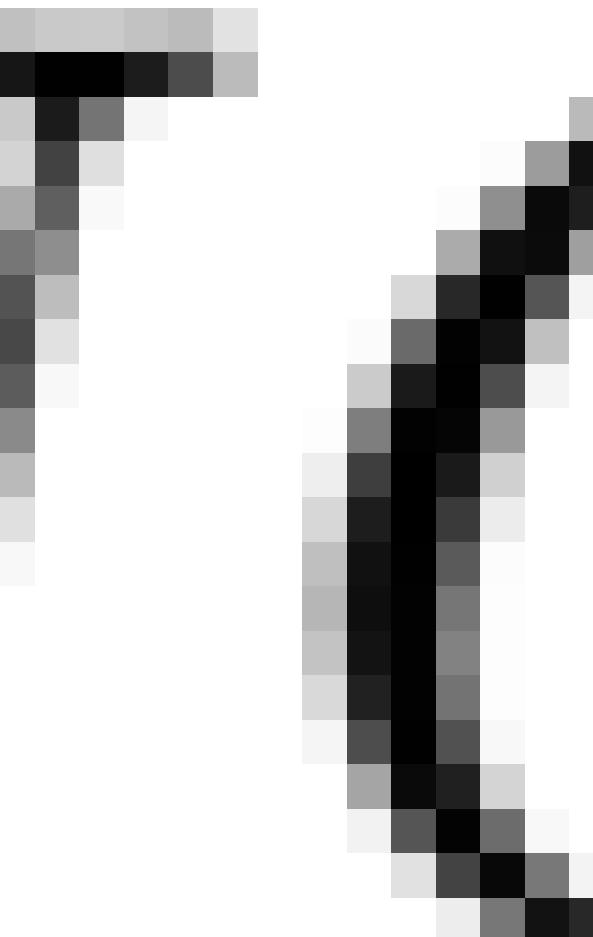
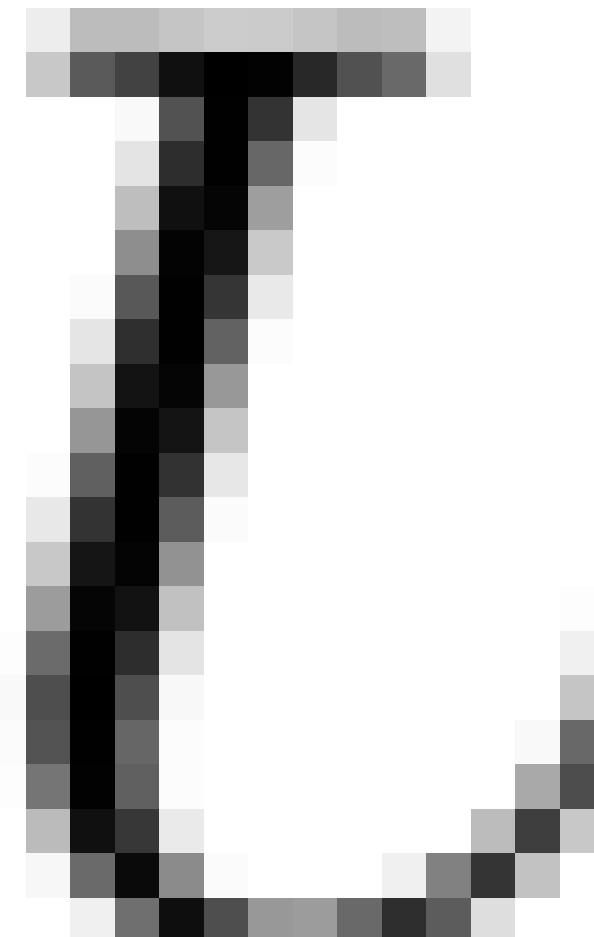
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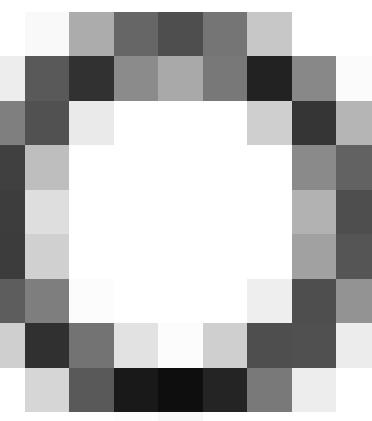
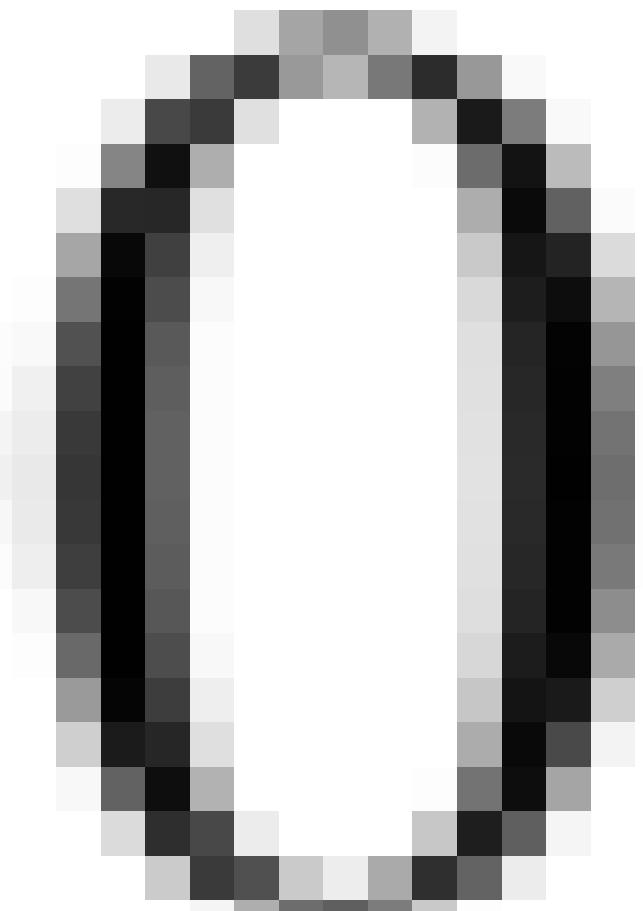
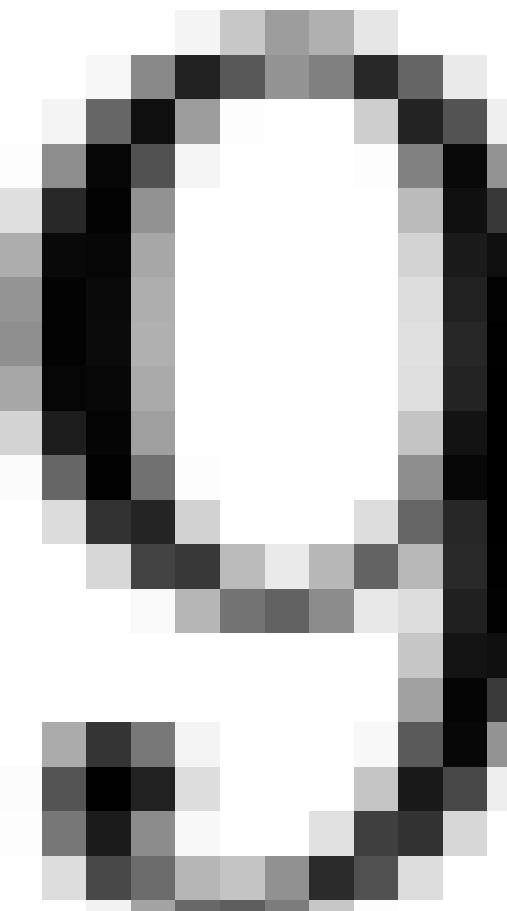
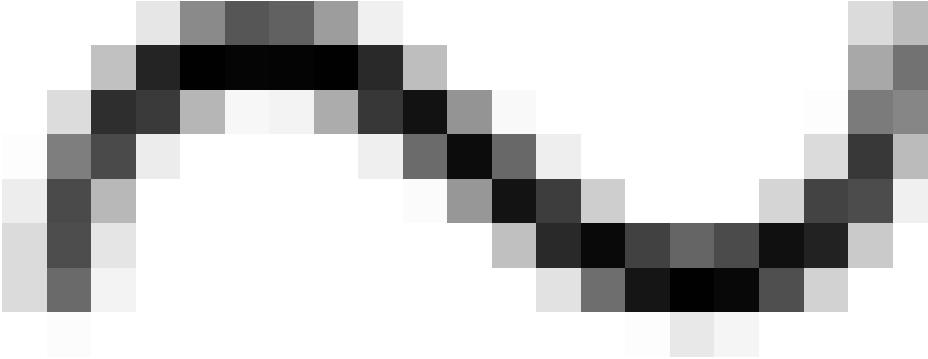


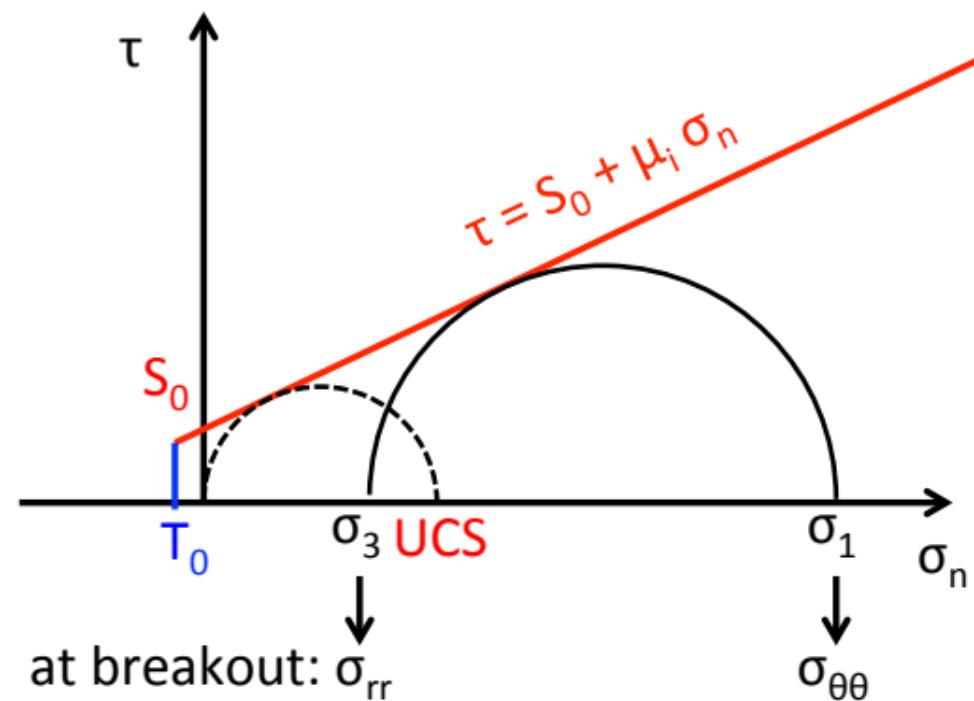
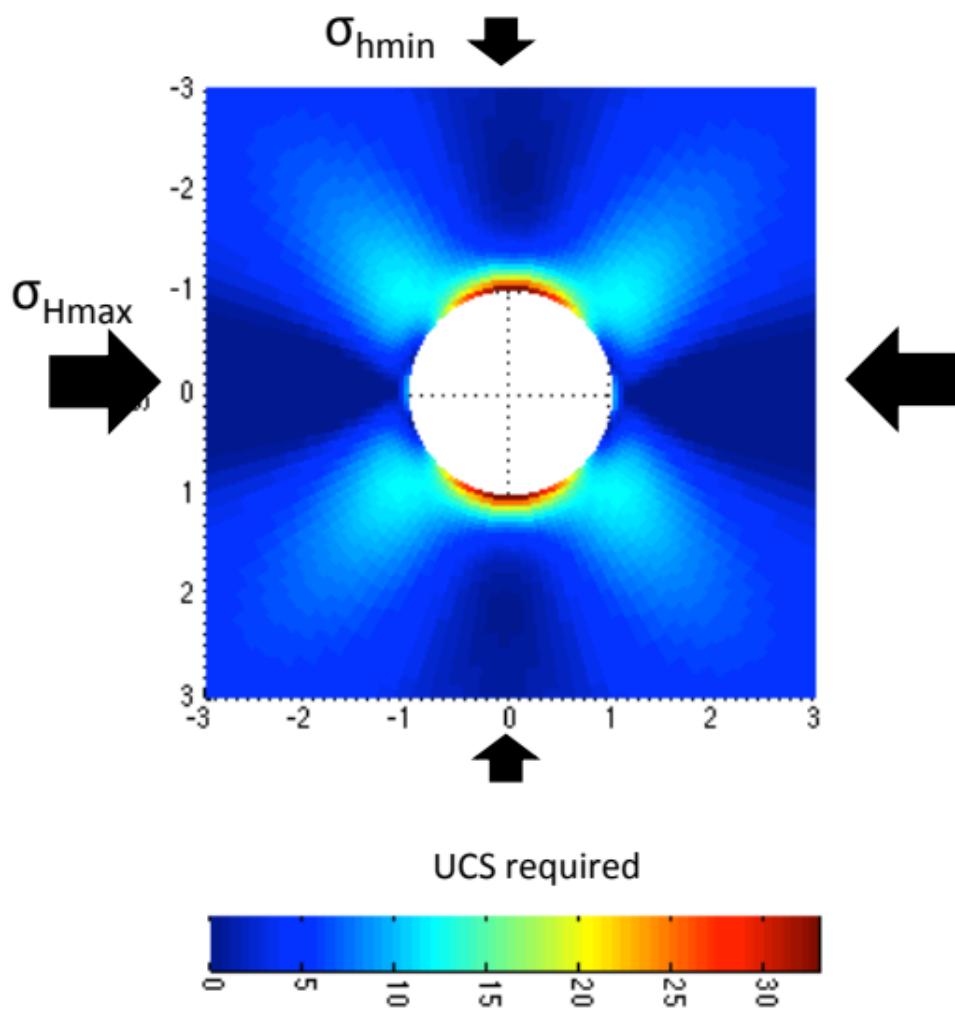
$$\frac{3 \times 3,220 \text{ psi} - 1,220 \text{ psi}}{1 + 3.12} = 4,279 \text{ psi}$$

~~4,279 psi~~ ~~7,000 ft~~ ~~0.44 psi/ft~~

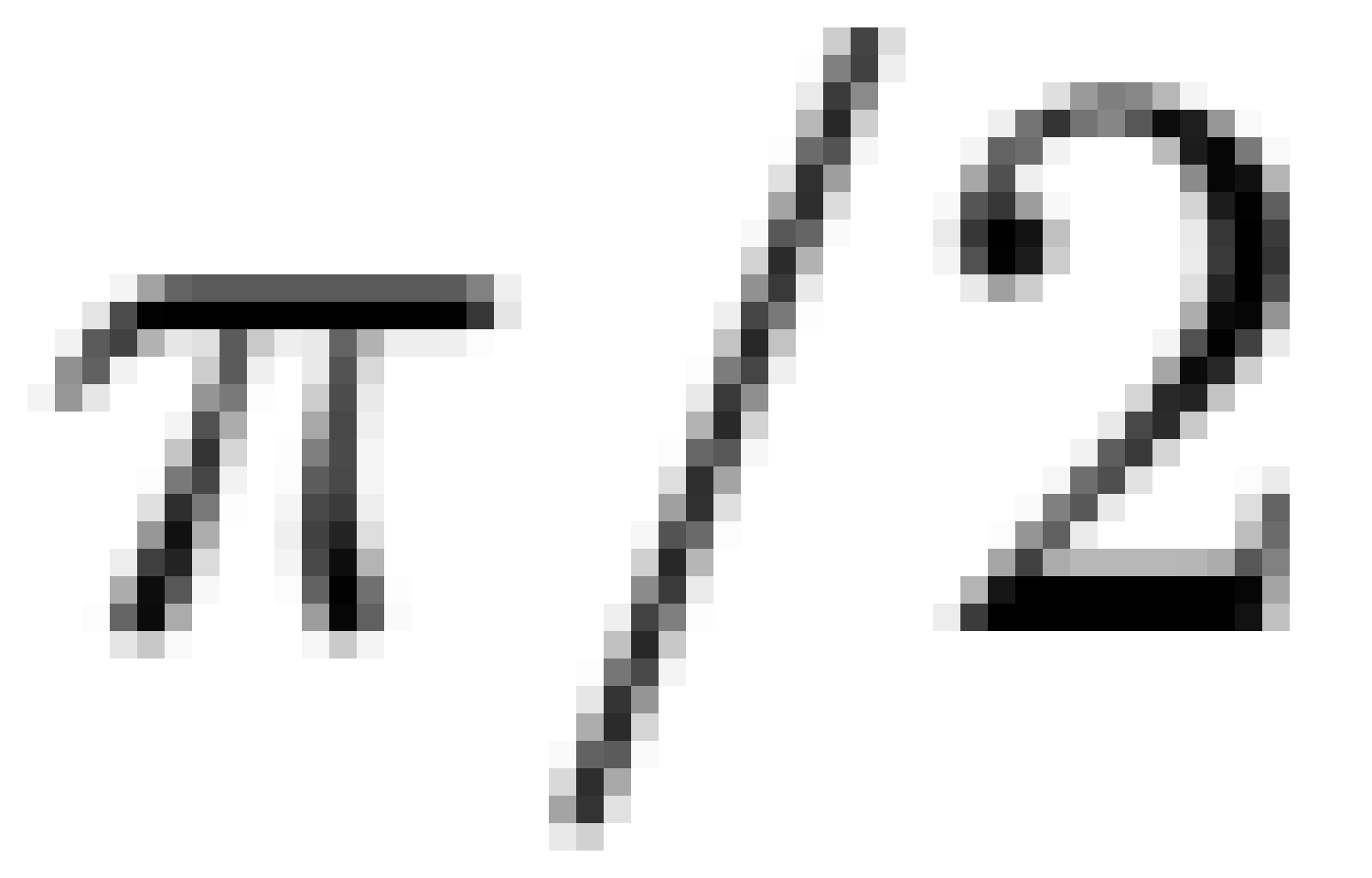
~~8.3 ppg~~ ~~11.57 ppg~~ ~~[redacted]~~

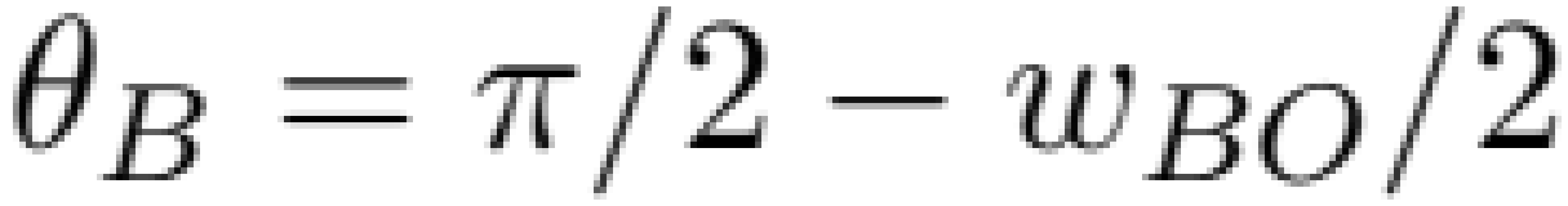




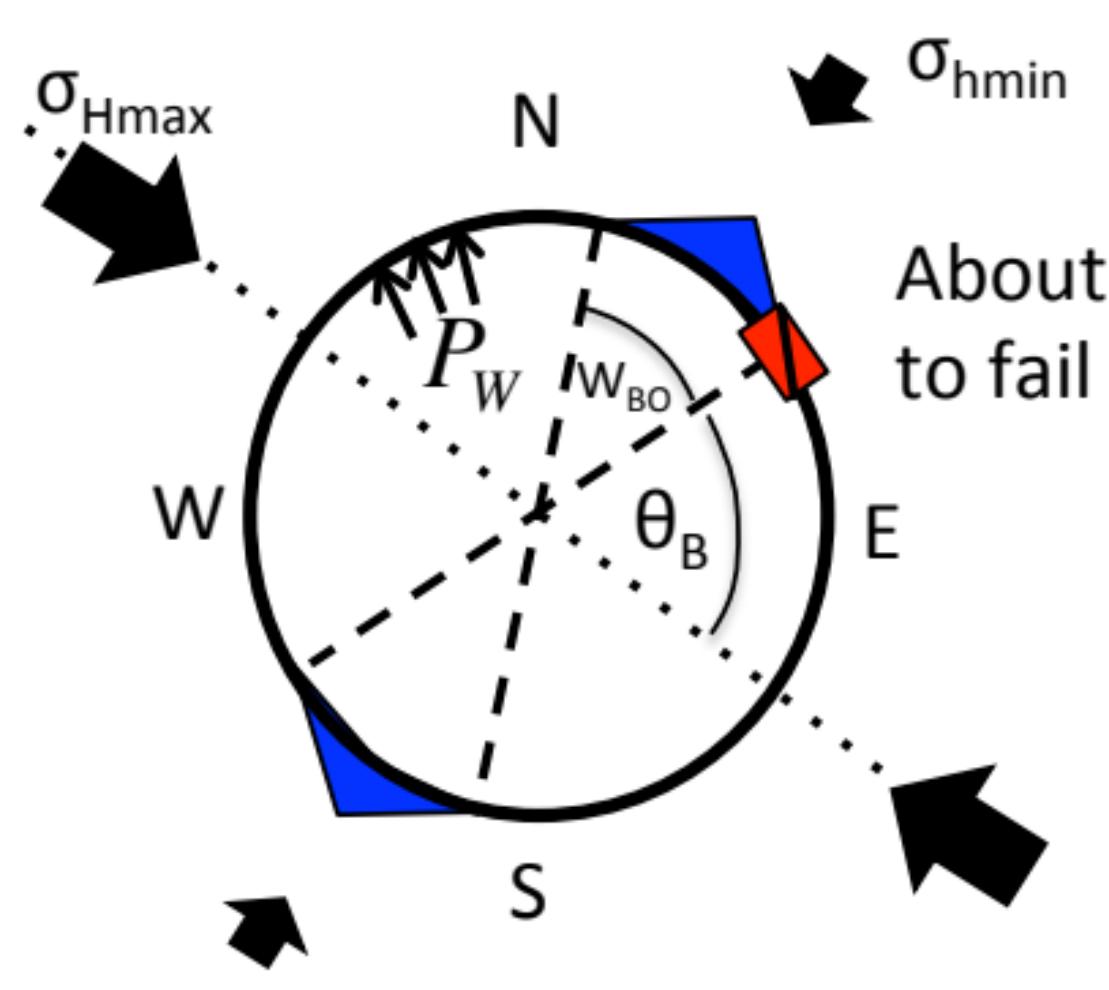








$$\begin{aligned} \sigma_{\theta\theta} &= (P_W - P_p) + 2(\sigma_{H\max} - \sigma_{h\min}) \cos(2\theta_B) \\ \sigma_{rr} &= + (P_W - P_p) \end{aligned}$$

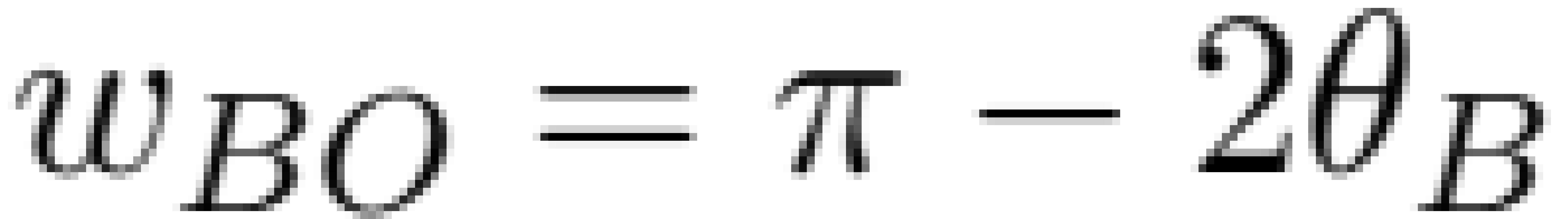


About
to fail



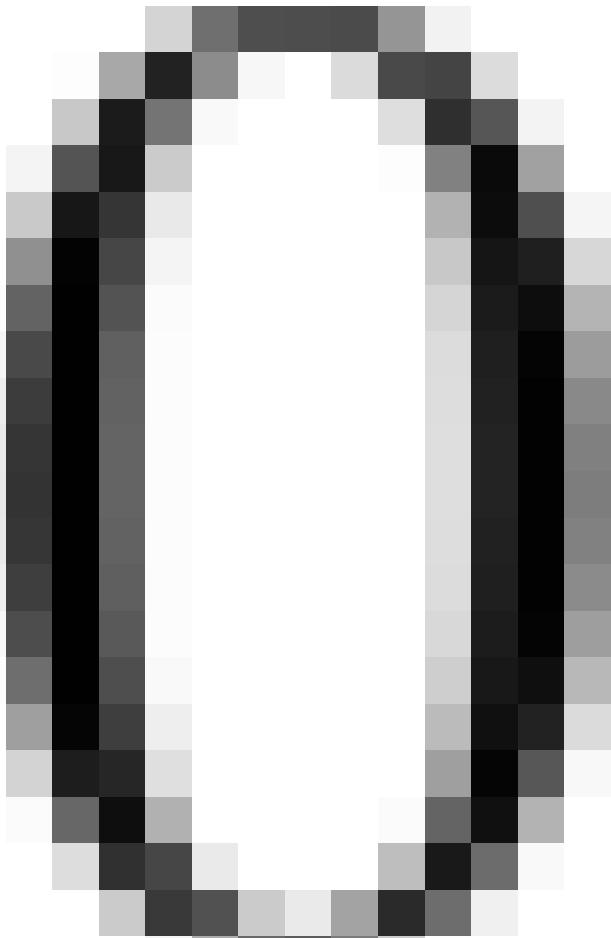
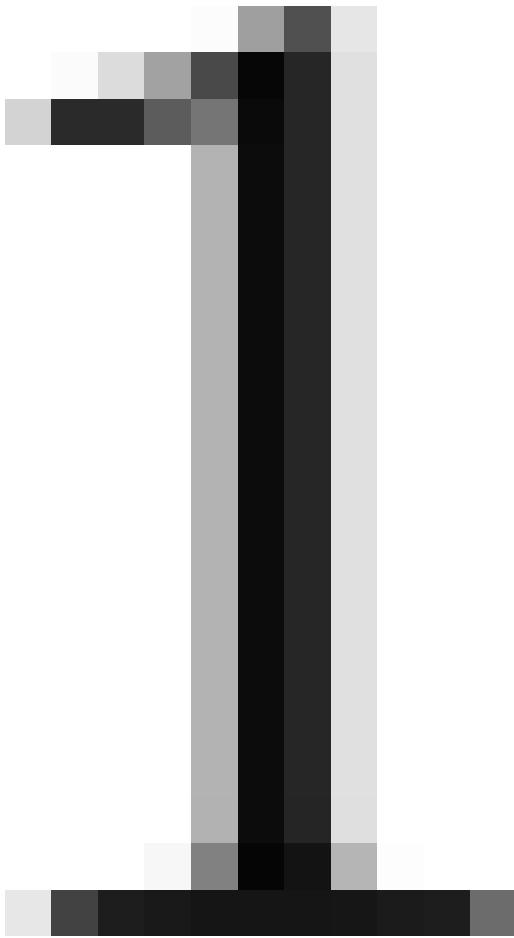
$$S_{\text{H}} = \frac{1}{2} \left[\sigma_{W_{\text{max}}}^2 + \sigma_{H_{\text{max}}}^2 \right] \cos(2\theta) + \sigma_{H_{\text{max}}}^2 \sin(2\theta) \cos(\phi_{H_{\text{max}}}) + \sigma_{W_{\text{max}}}^2 \sin(2\theta) \sin(\phi_{H_{\text{max}}})$$

$$2\theta_B = \arccos \left[\frac{\sigma_{Hmax} - \sigma_{hmin} - (1+q)(P_W - P_p)}{2(\sigma_{Hmax} - \sigma_{hmin})} \right]$$





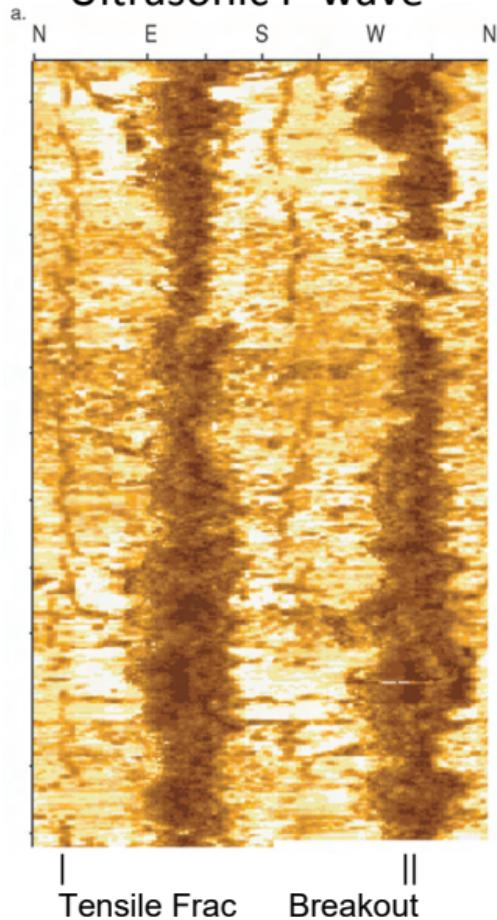
$$P_{WBO} = P + \frac{(\sigma_{Hmax} - \sigma_{hmin}) \cos(\pi - w_{BO}) - Ucs}{1 + q}$$



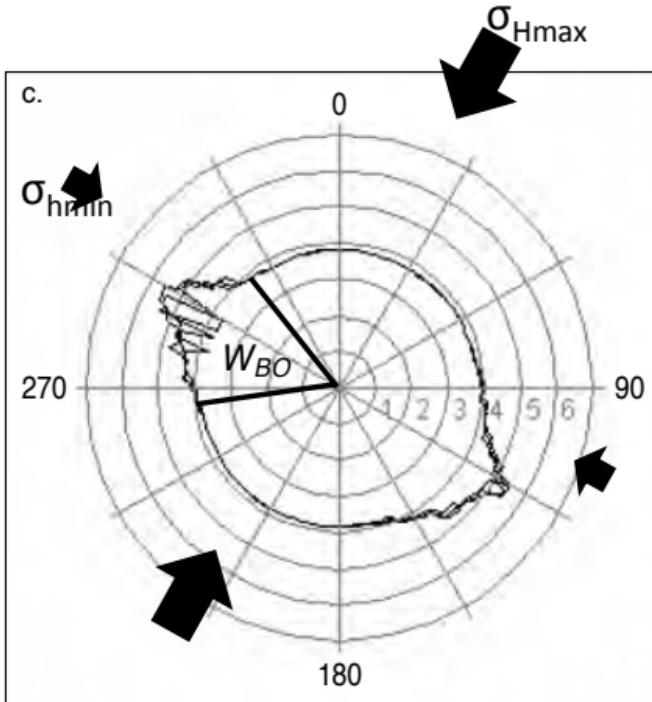
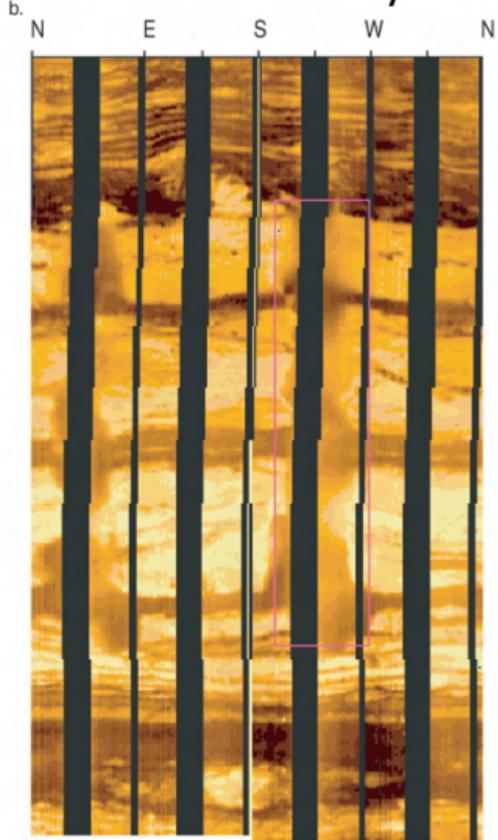
\times 0.44 psi/ft
 \times 8.3 ppg
 \times 7,000 psi = 3,710 psi

$$w_{BO} = \frac{180^\circ - \arccos \left[\frac{3,220 \text{ psi} + 1,220 \text{ psi} - (1 + 3.12)(3,710 \text{ psi} - 3,080 \text{ psi})}{2(3,220 \text{ psi} - 1,220 \text{ psi})} \right]}{66^\circ}$$

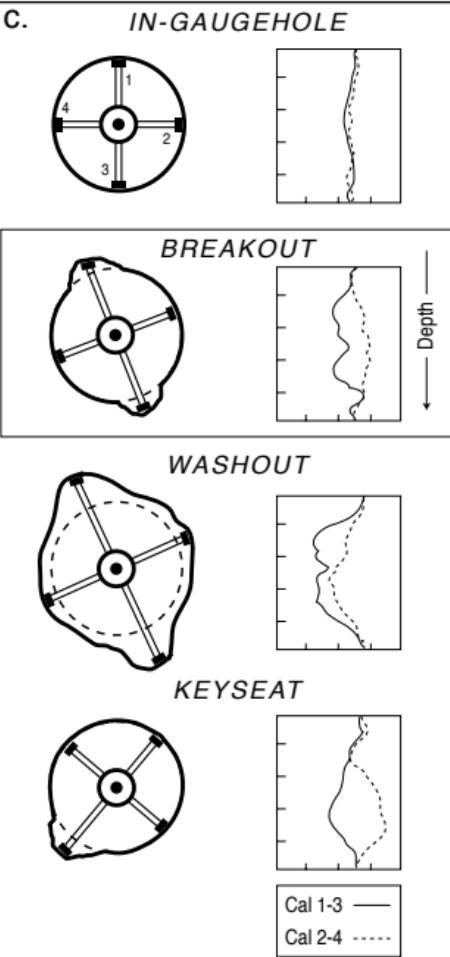
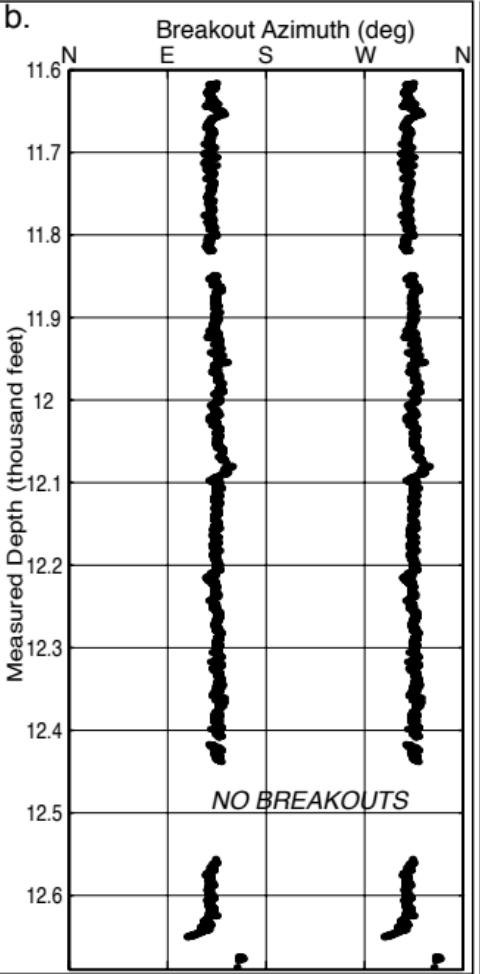
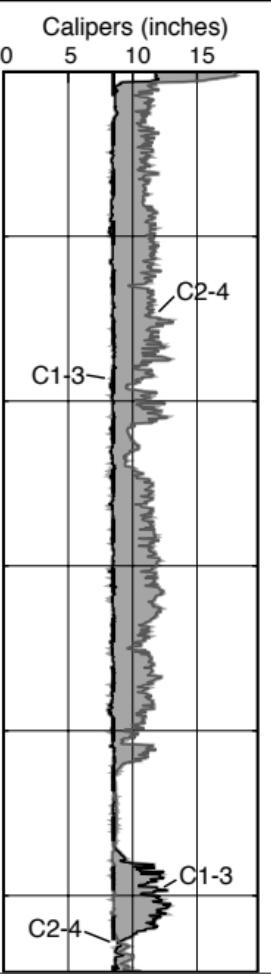
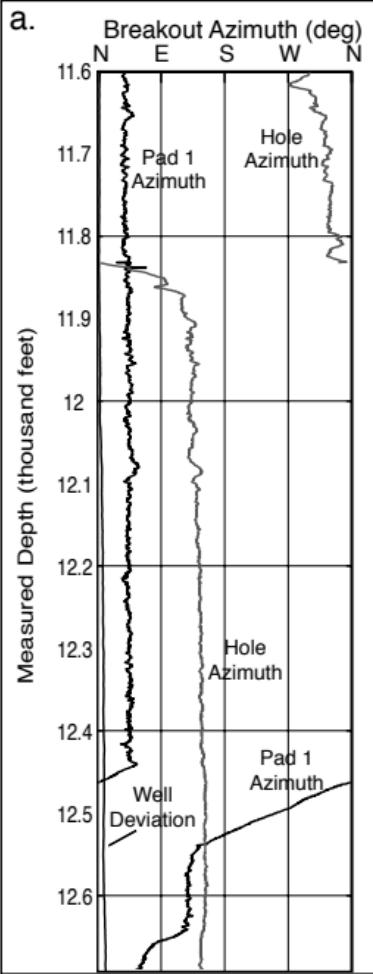
Ultrasonic P-wave



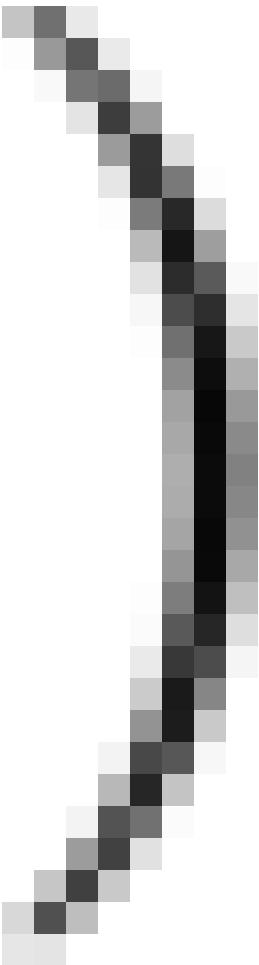
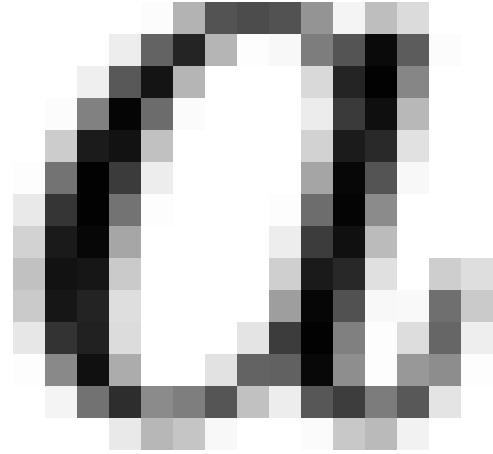
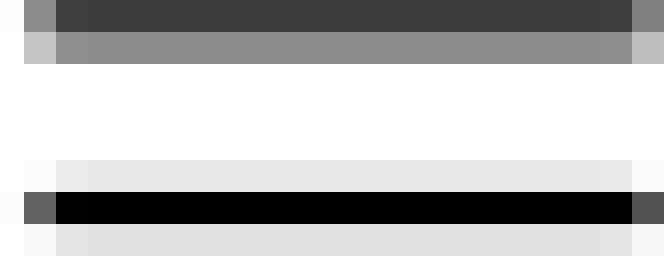
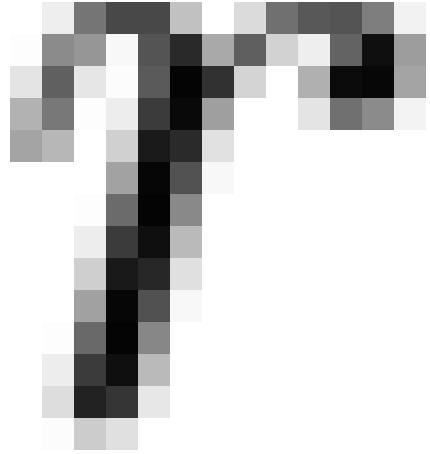
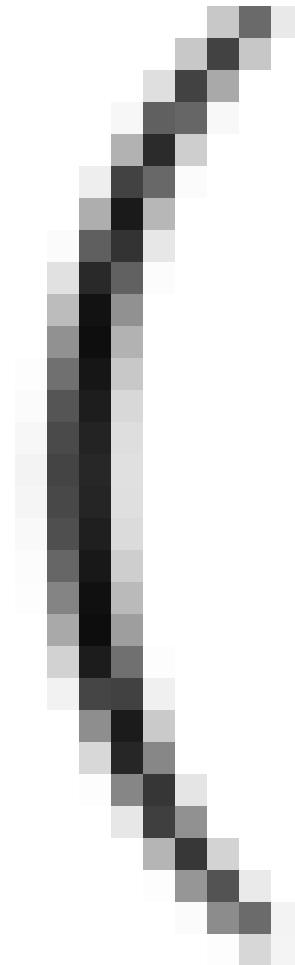
Electrical resistivity



[Zoback 2013 - Figure 6.4]

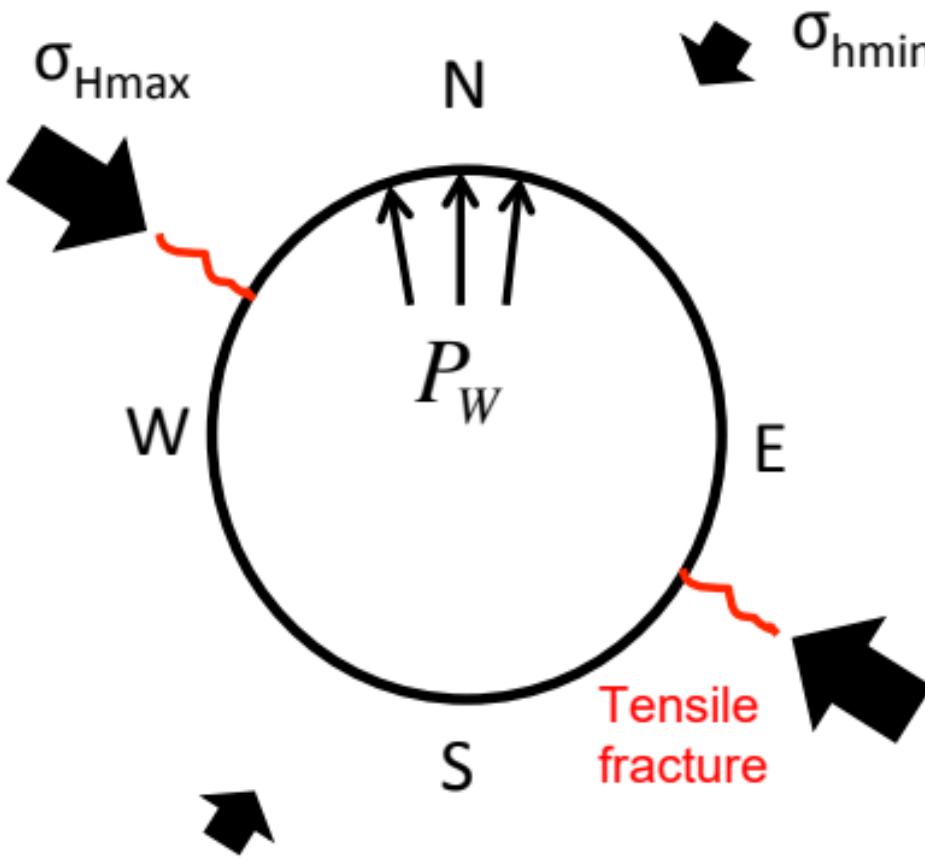


$$S_{H\max} = \frac{P}{P} + \frac{UCS + (1+q)(P_W - P_p) - \sigma_{hmin}[1 + 2\cos(\pi - w_{BO})]}{1 - 2\cos(\pi - w_{BO})}$$



σθαντικός + σούχος + στρατός



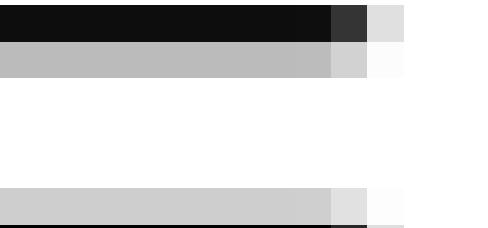
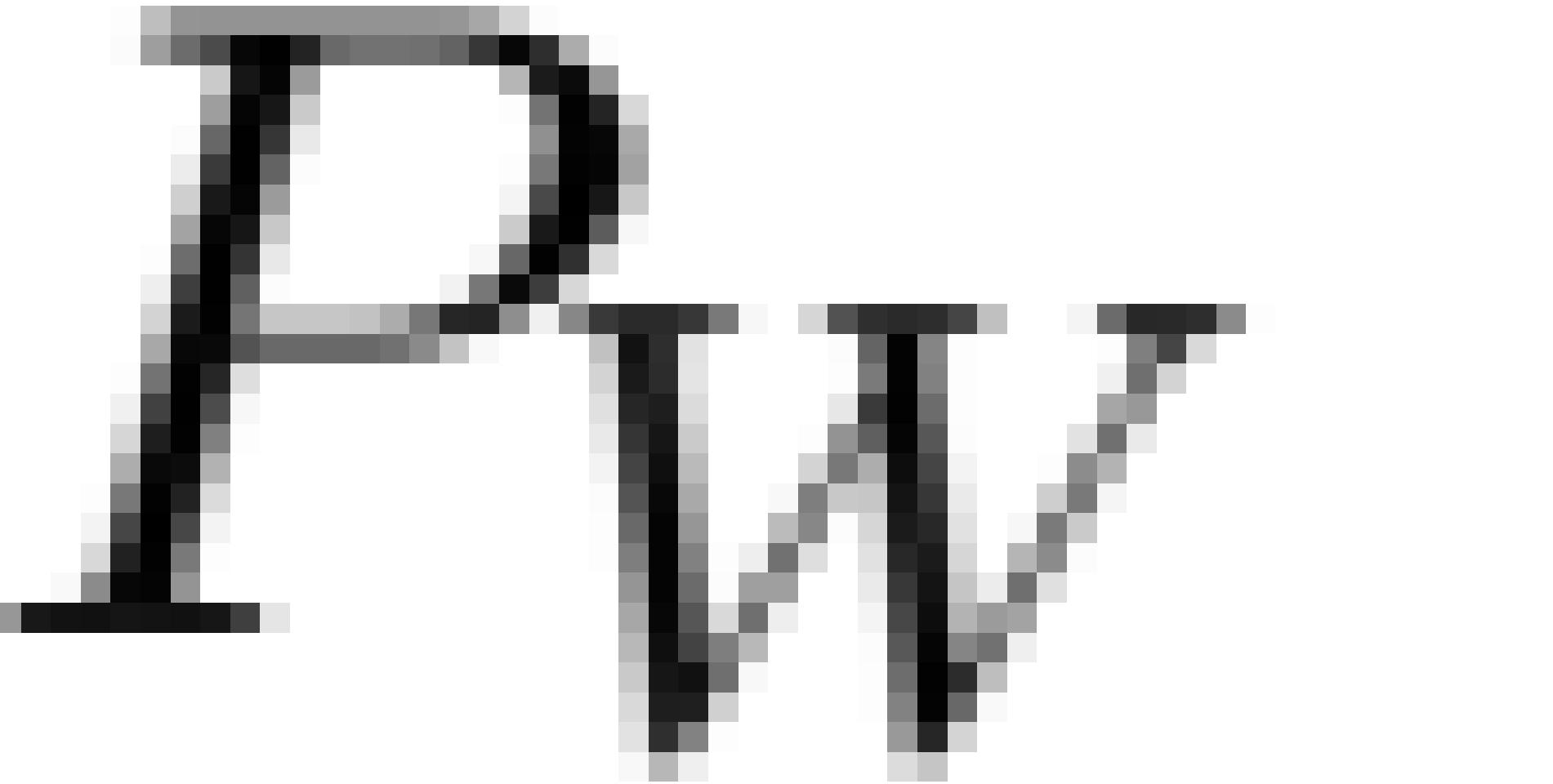


$$P_b = P_p + 3\sigma_{h\min} - \sigma_{H\max} + T_s + \sigma^{\Delta T}$$

Pore pressure
in the formation

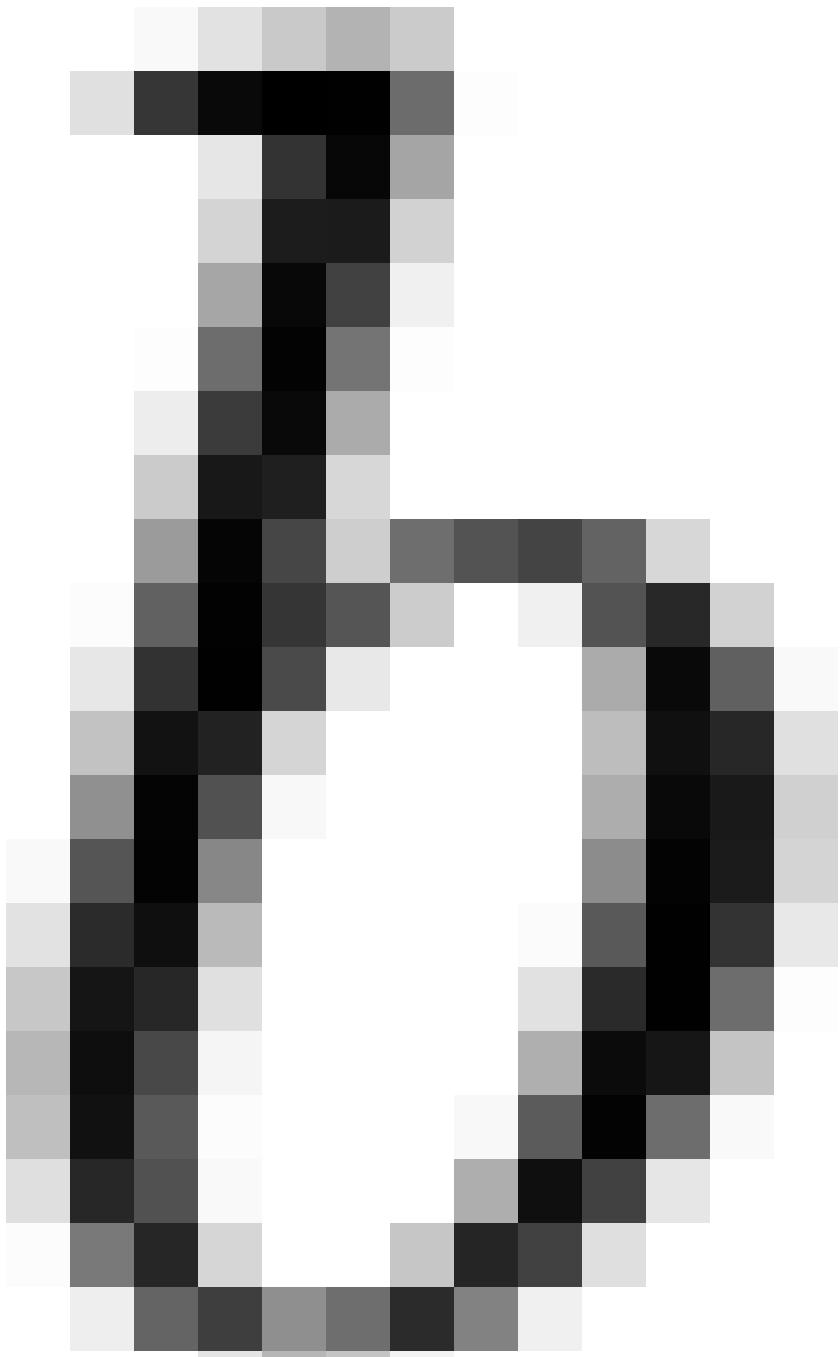
Stress anisotropy Tensile strength Cooling stress

$$P_b = 3S_{h\min} - S_{H\max} - P_p + T_s + \sigma^{\Delta T}$$

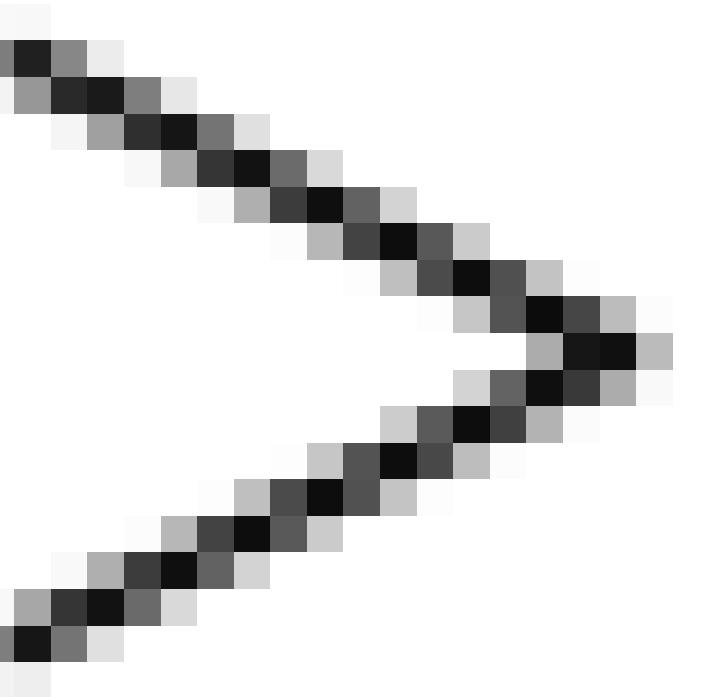
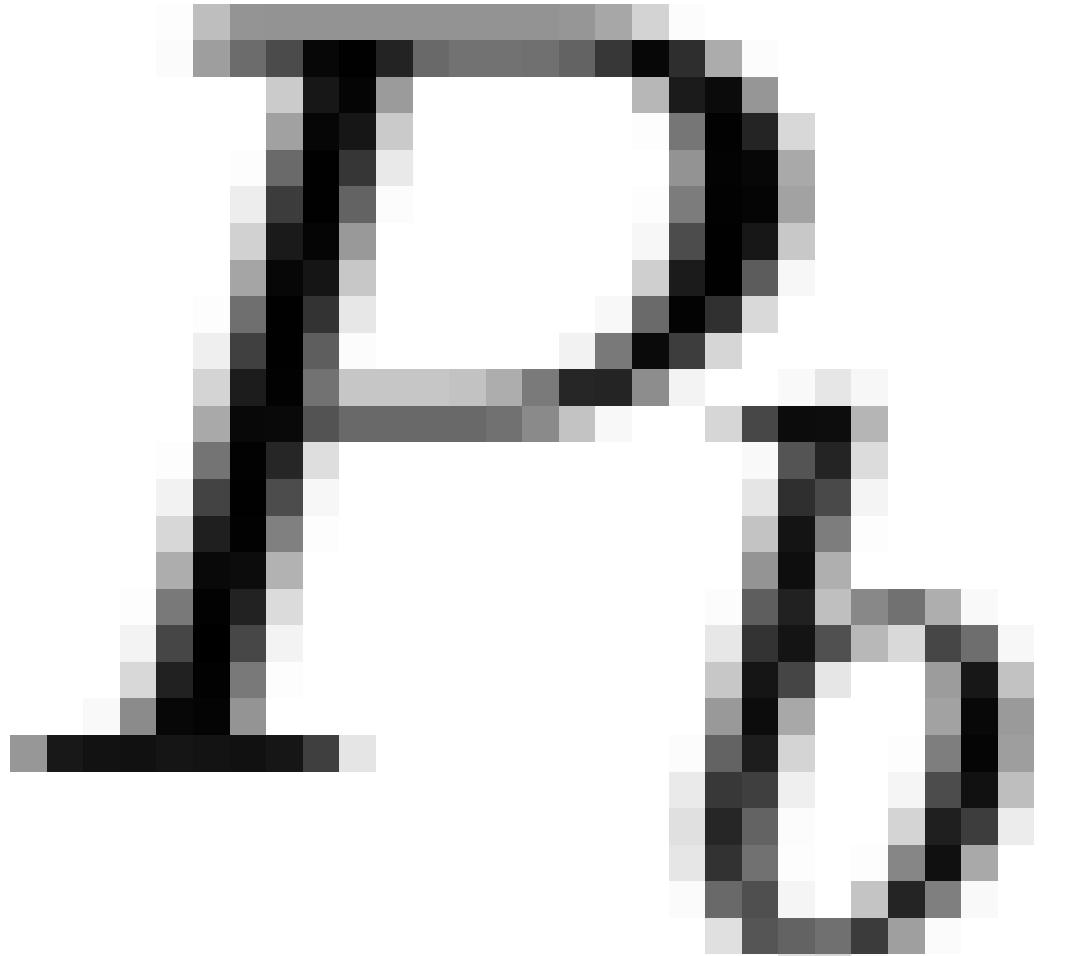


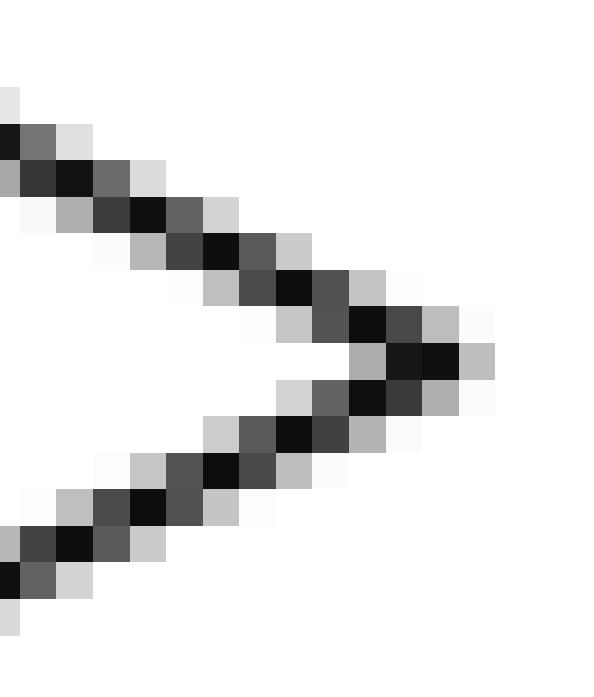
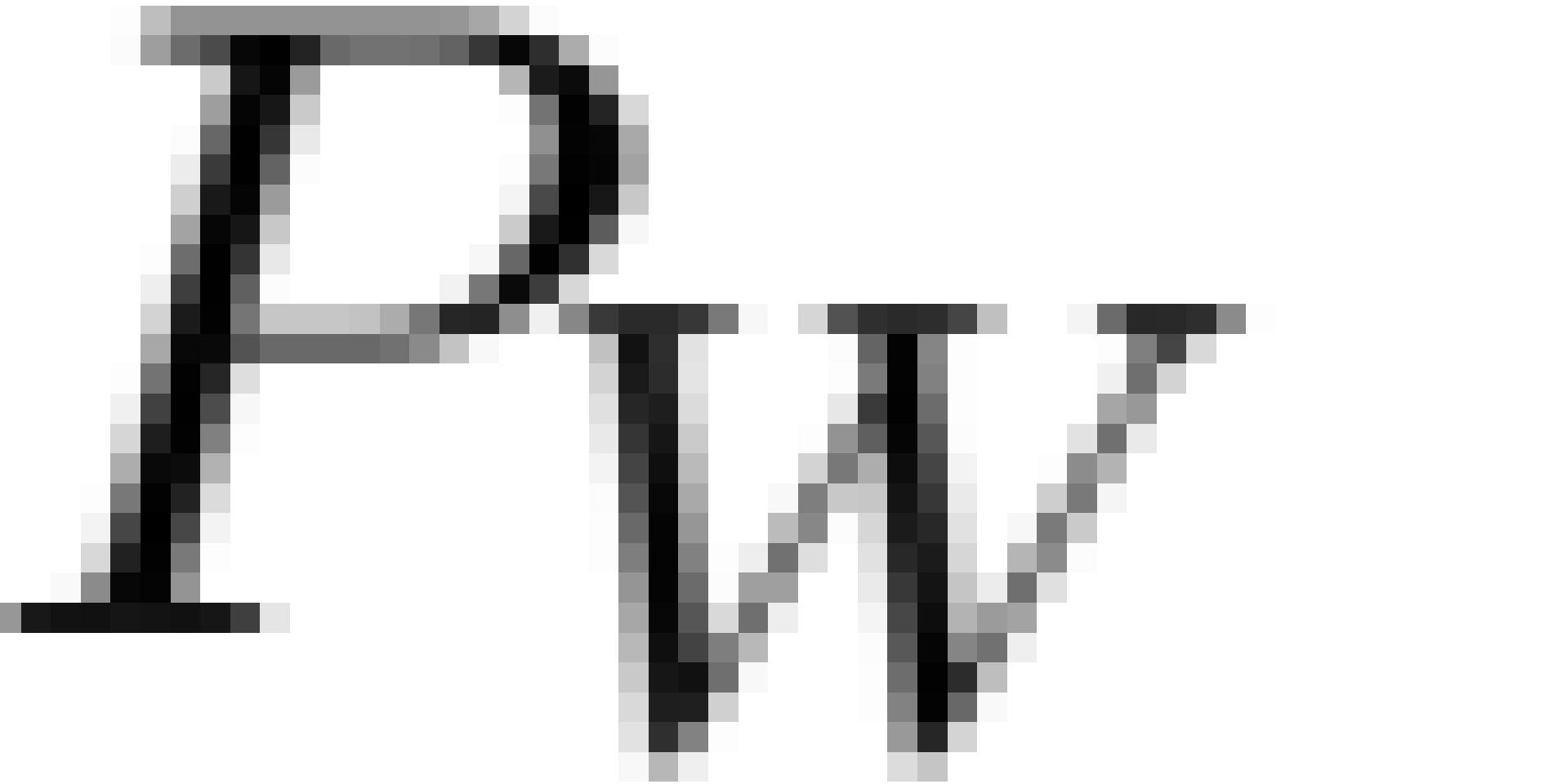
$\sigma_{\text{Hannac}} + \sigma_{\text{Hannac}}$

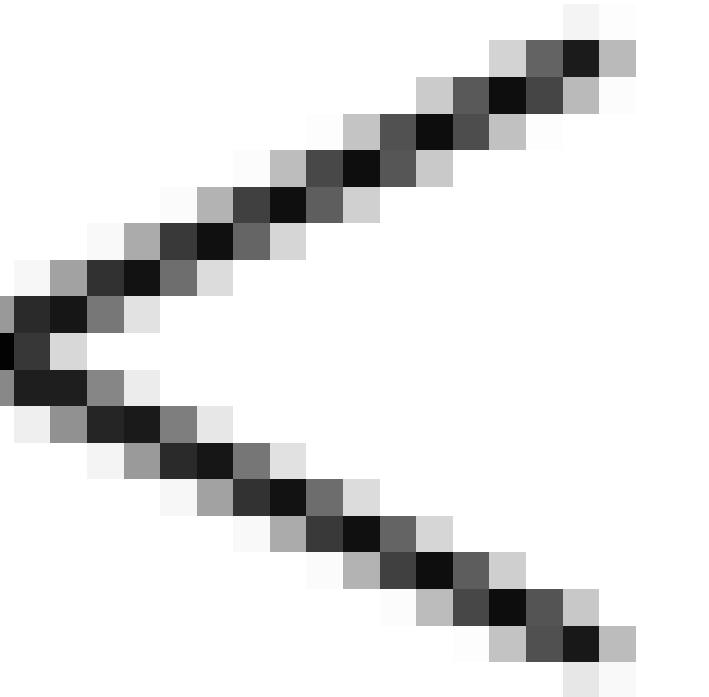
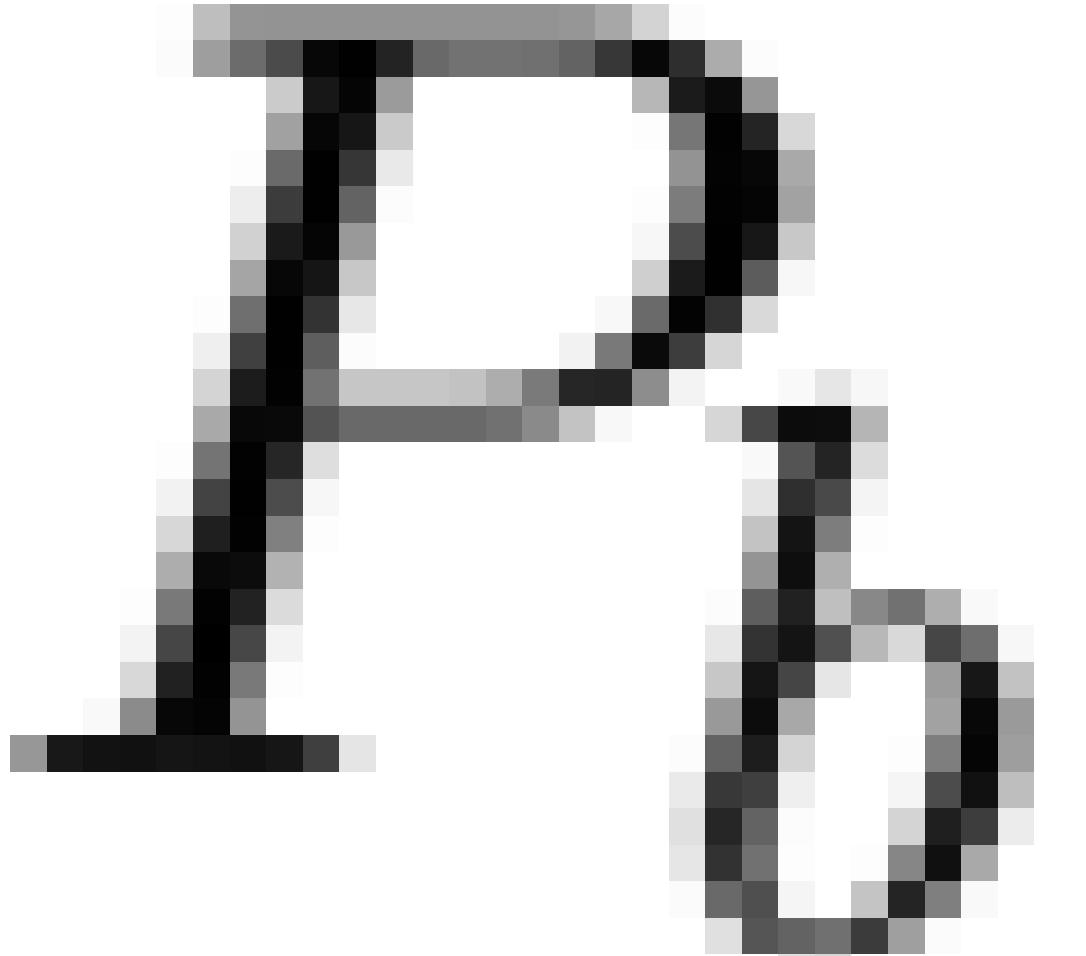
Bob Smith + 30 hours



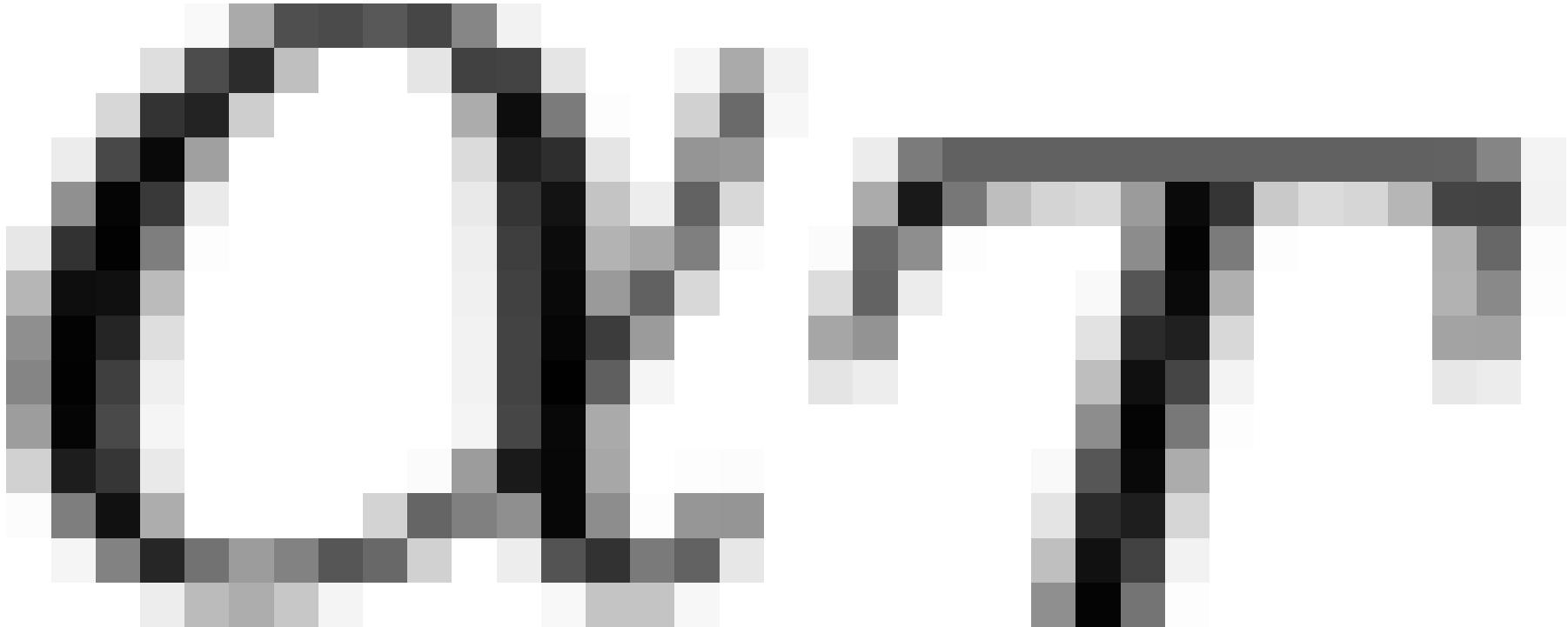


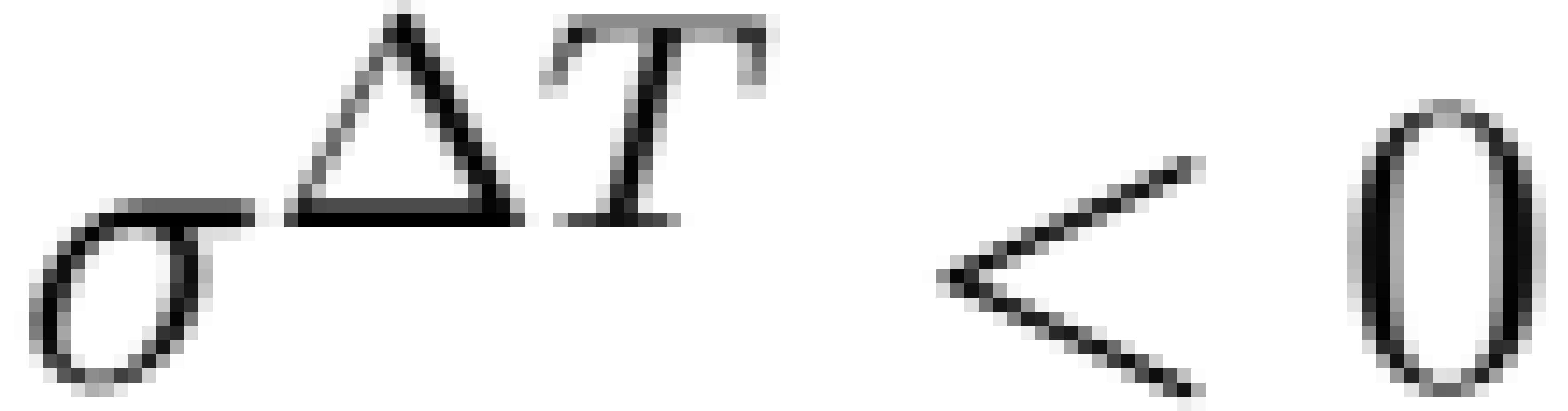


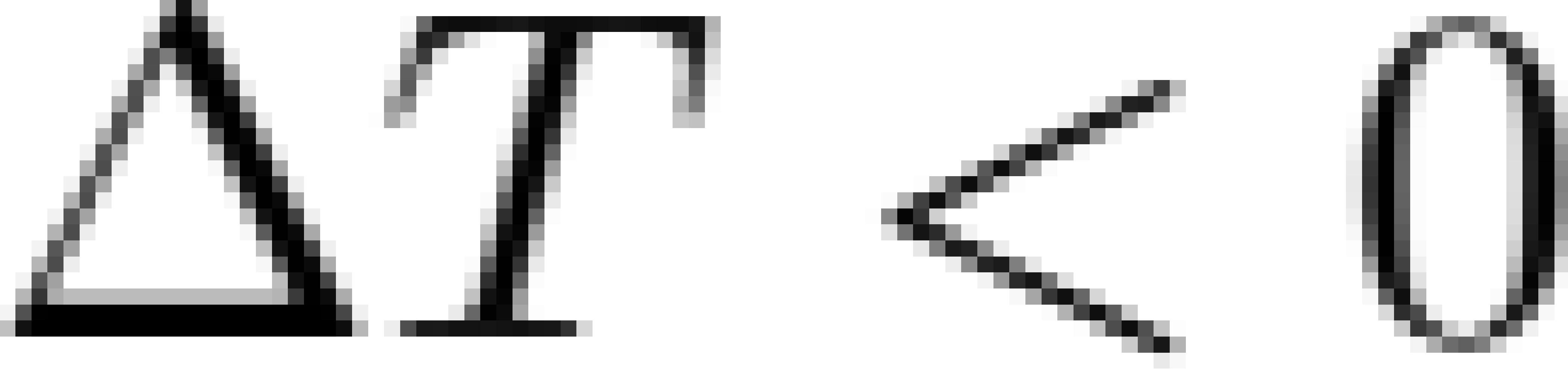


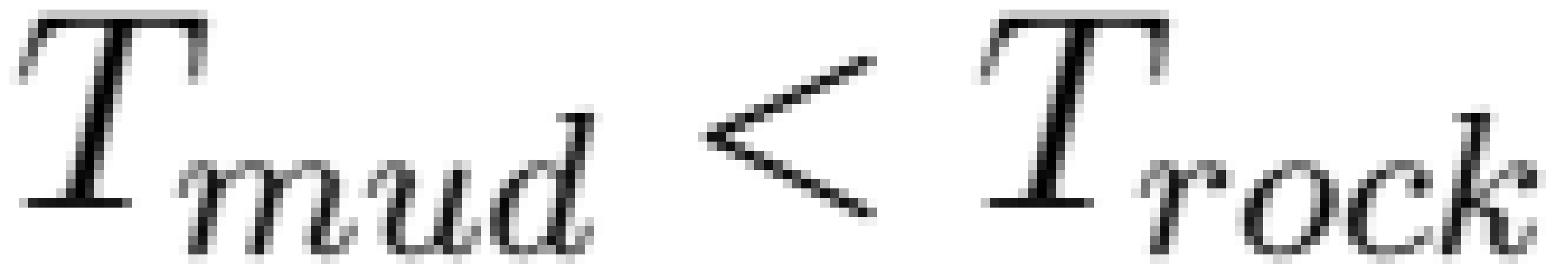


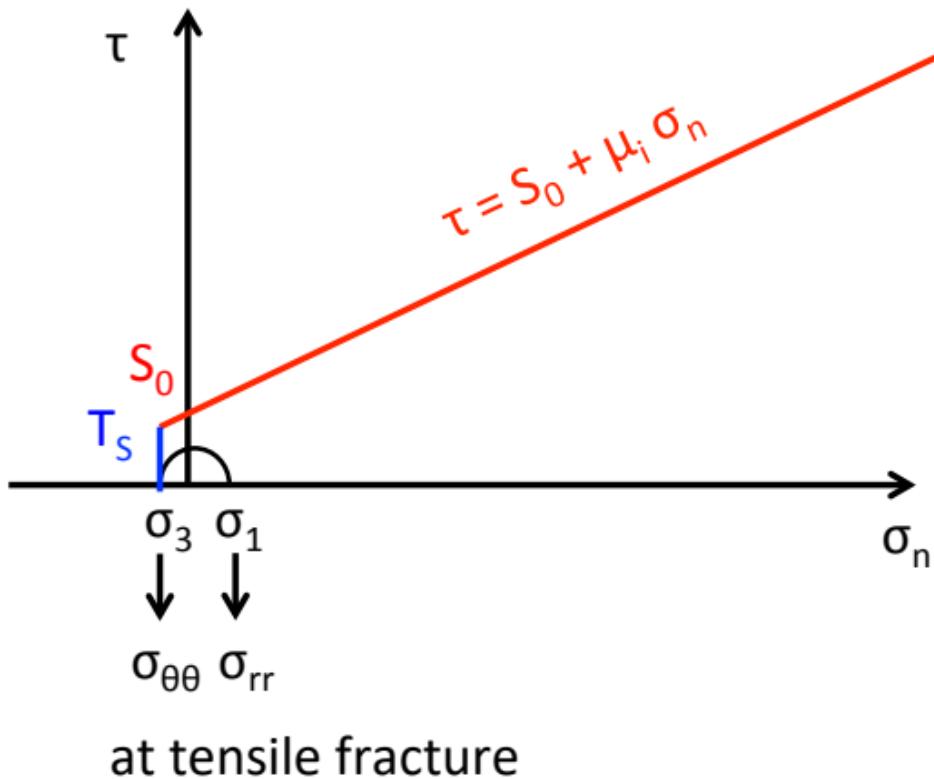
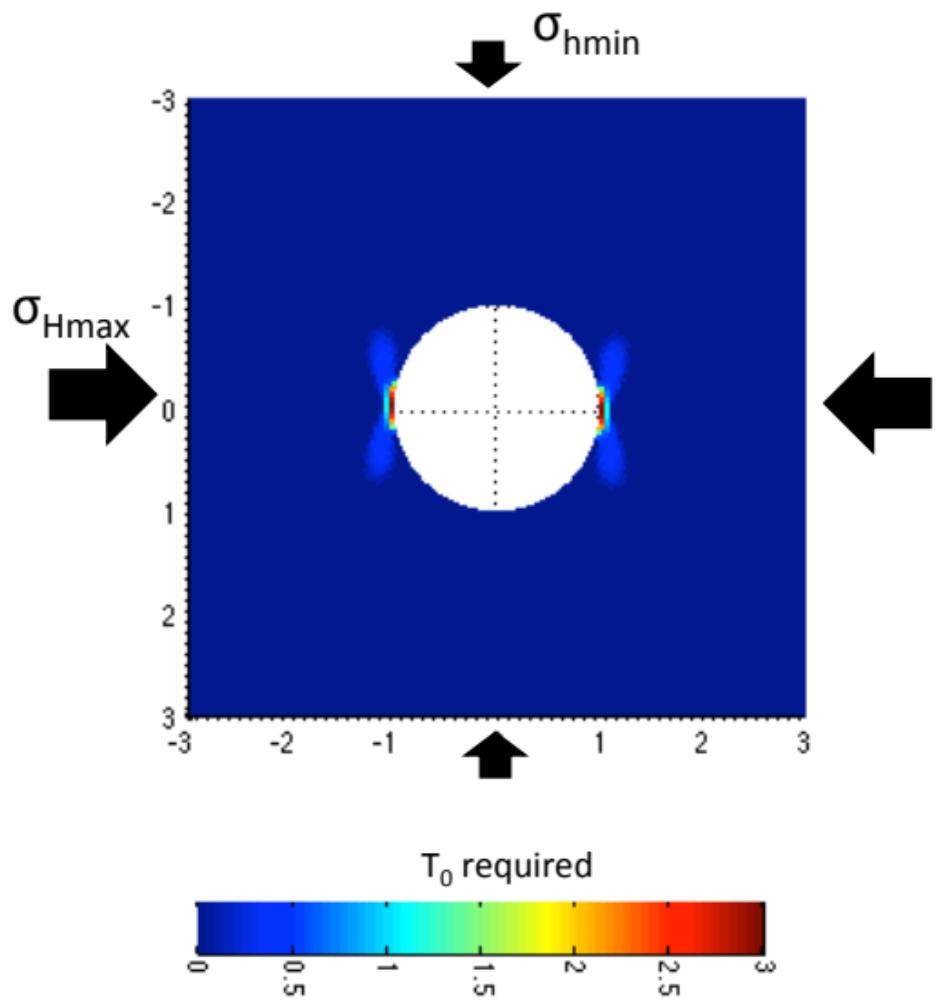


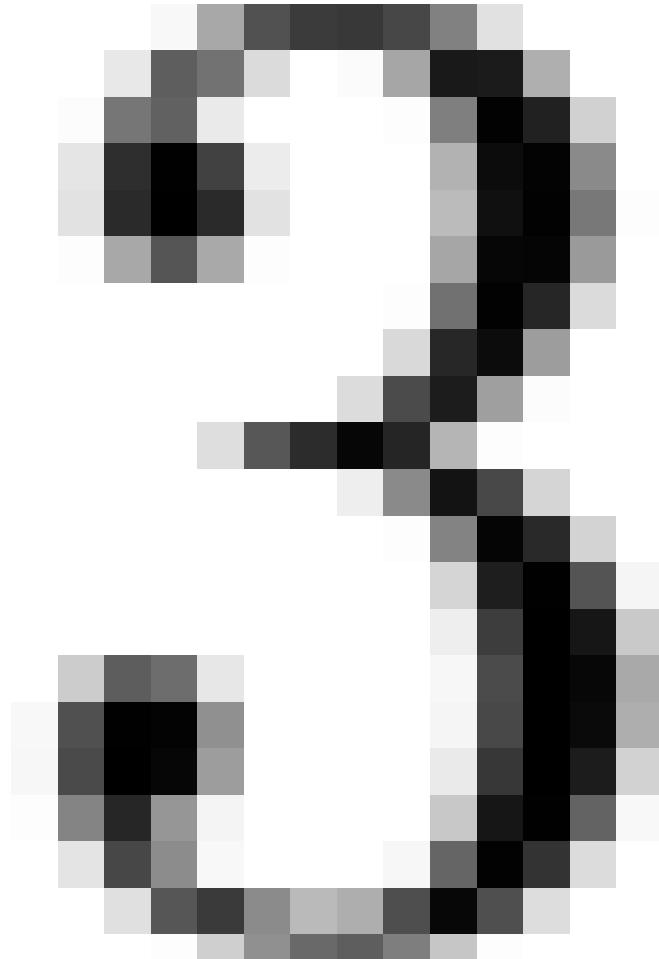
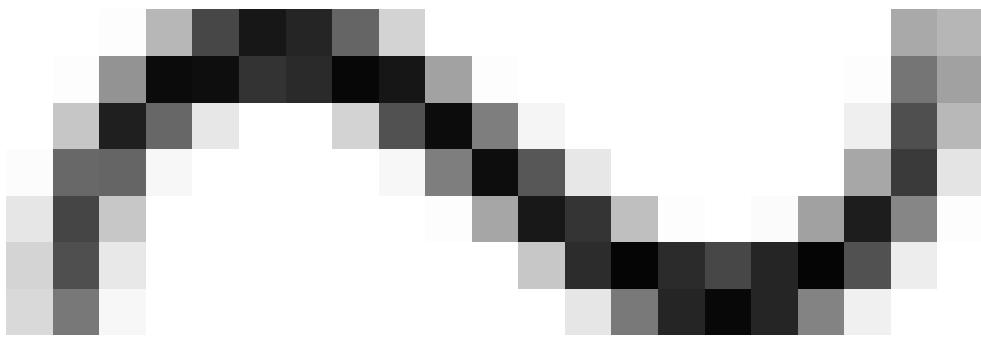


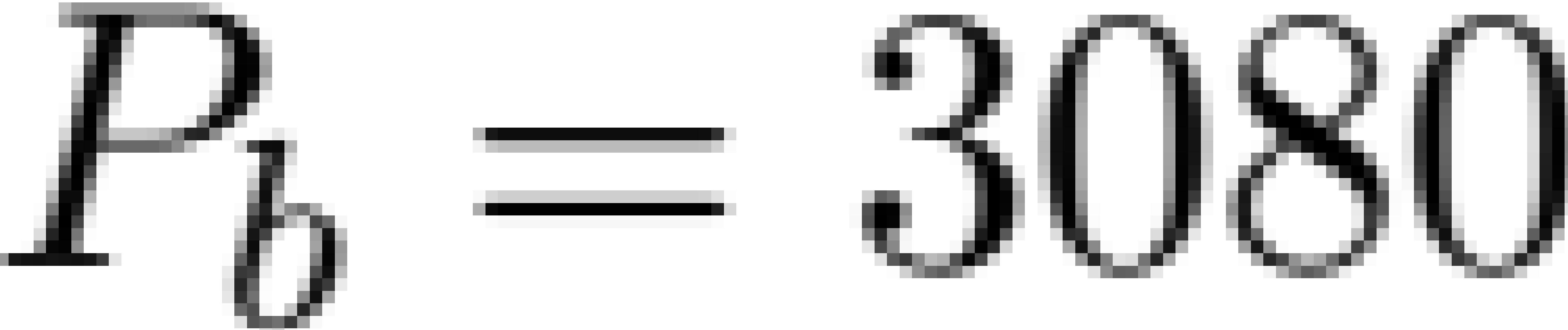


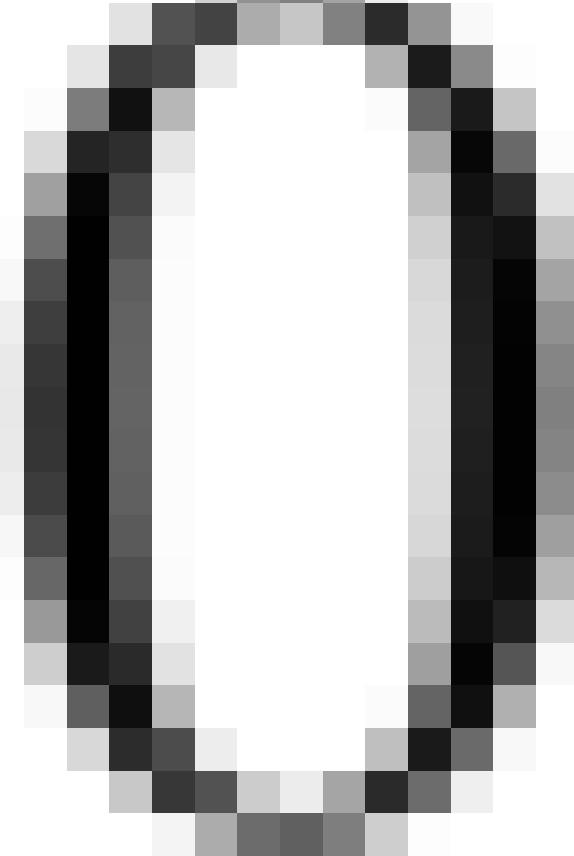
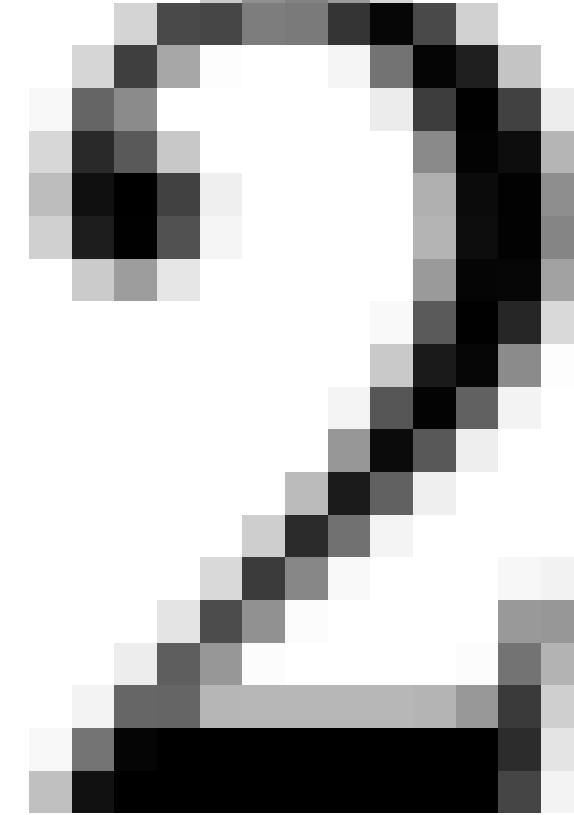
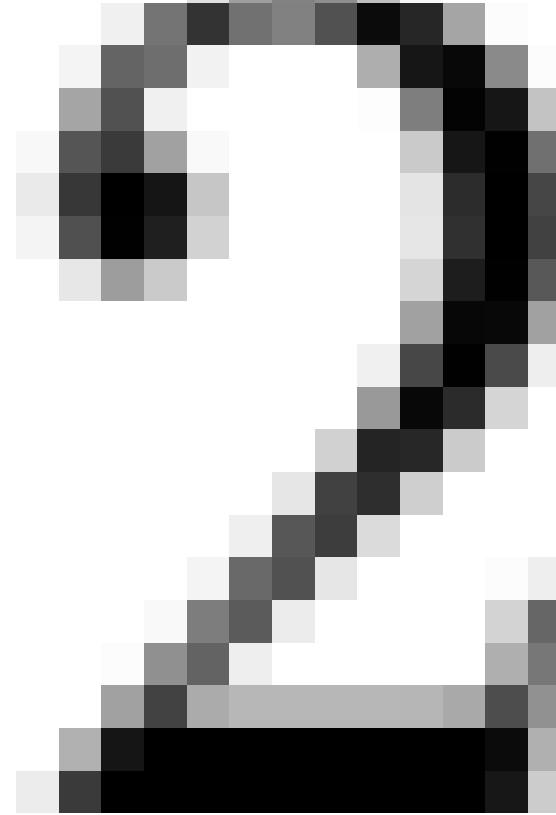
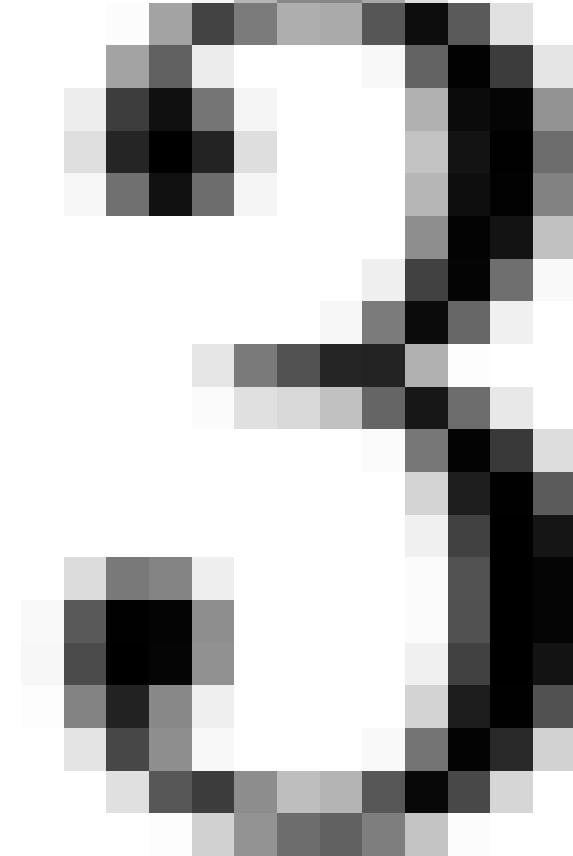


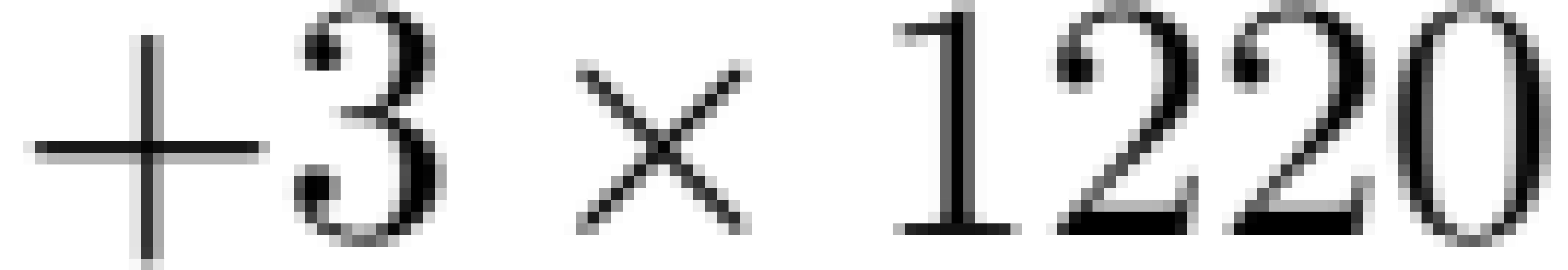


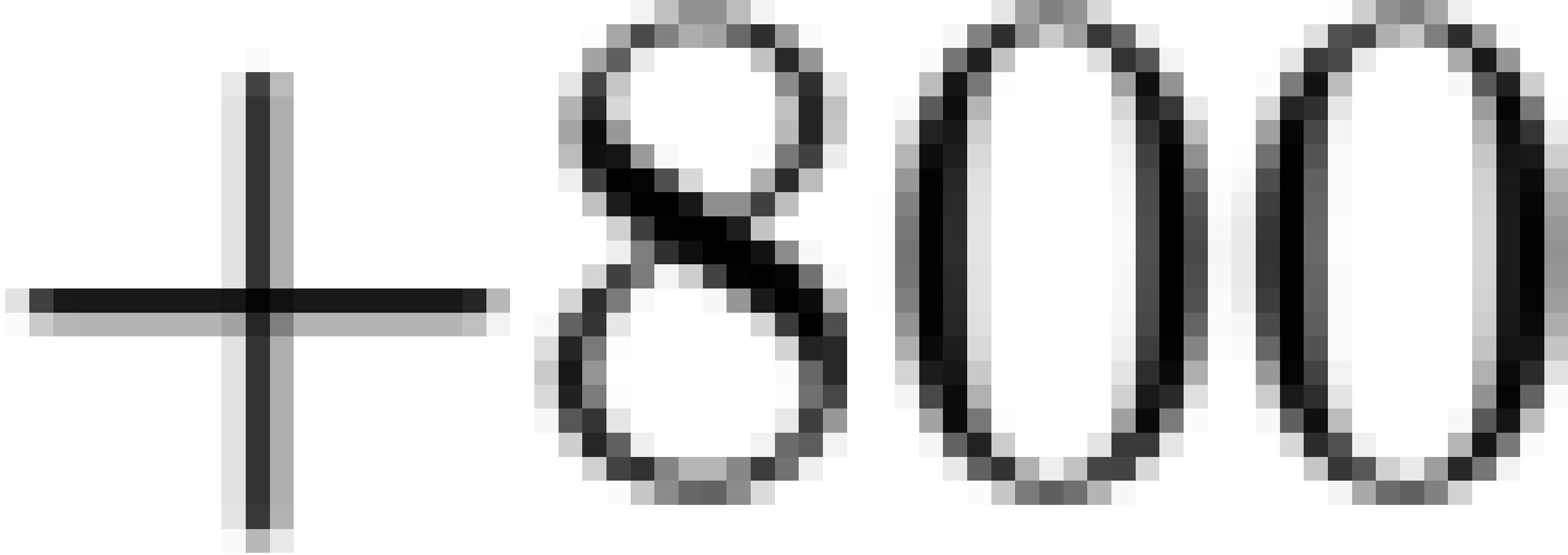


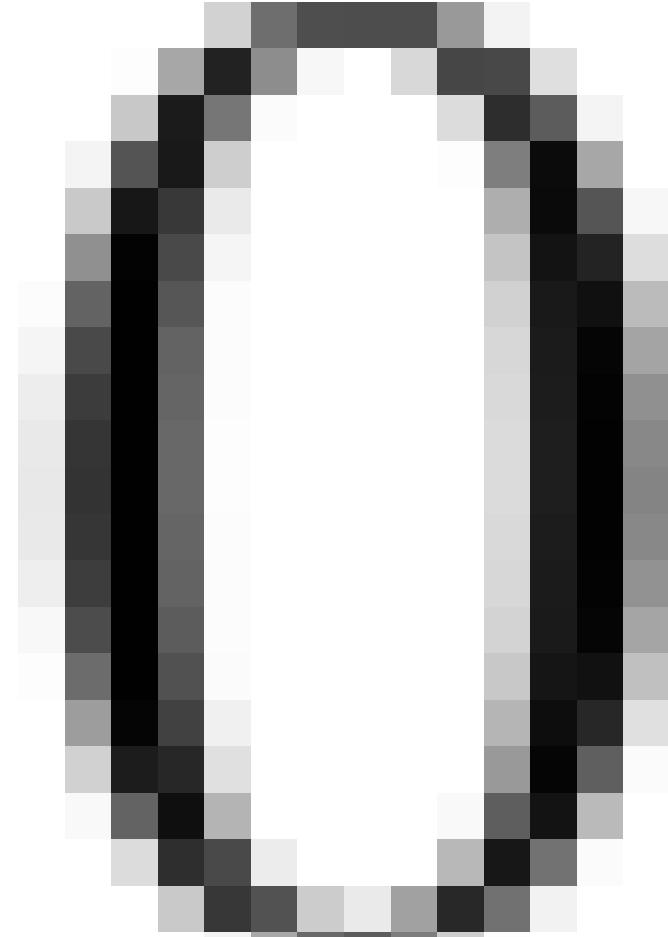
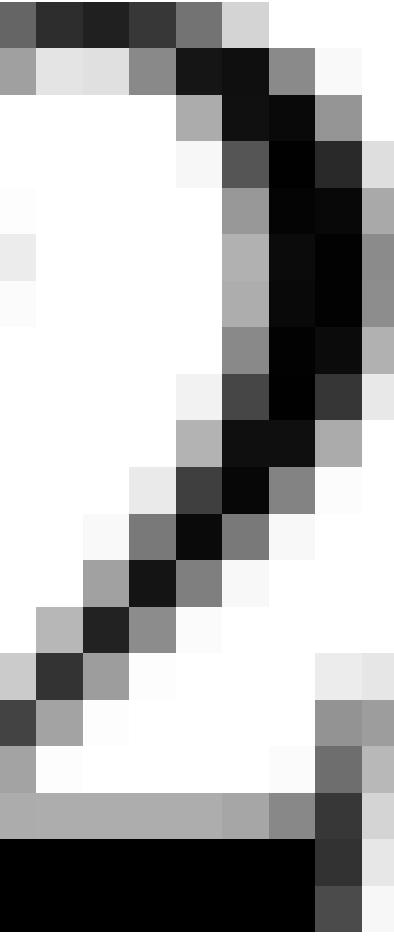
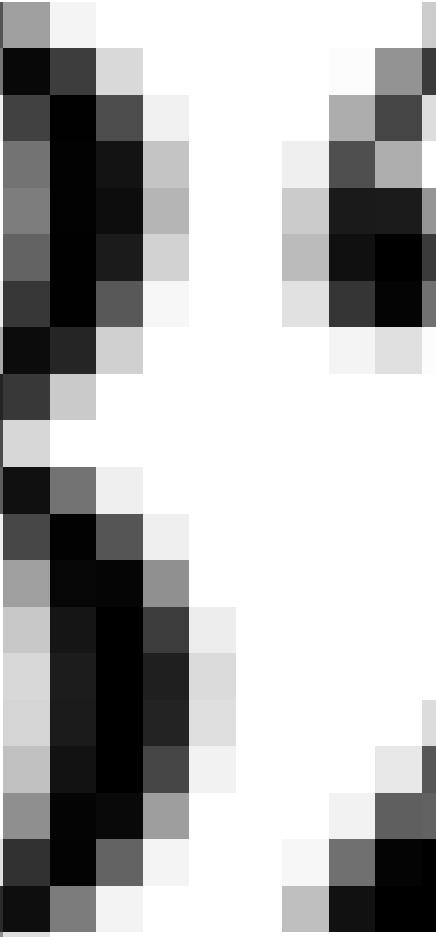
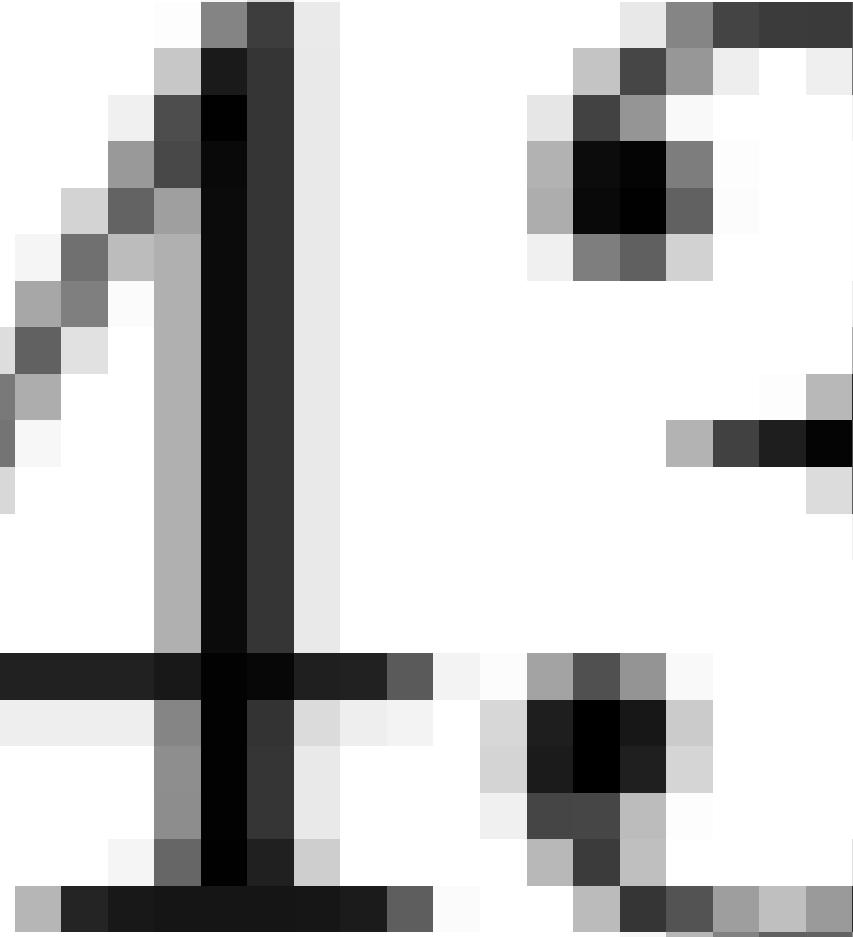












4320 psi

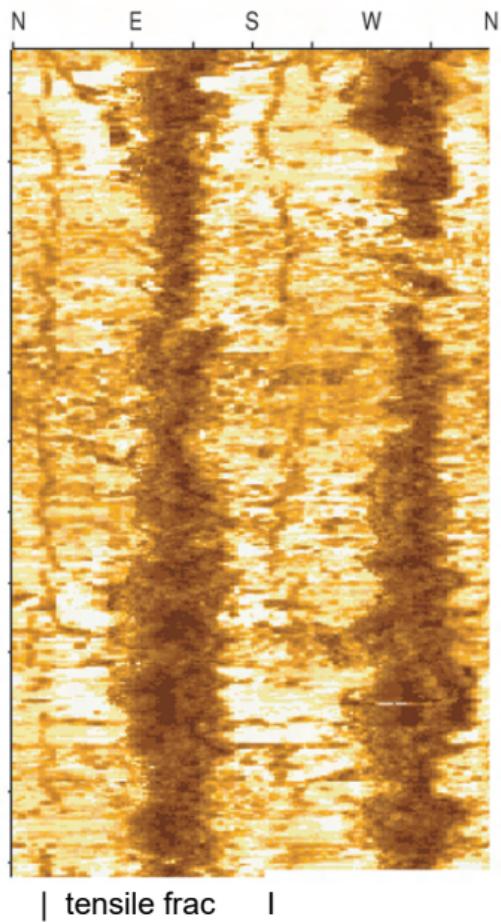
7000 ft

8.33 ppg

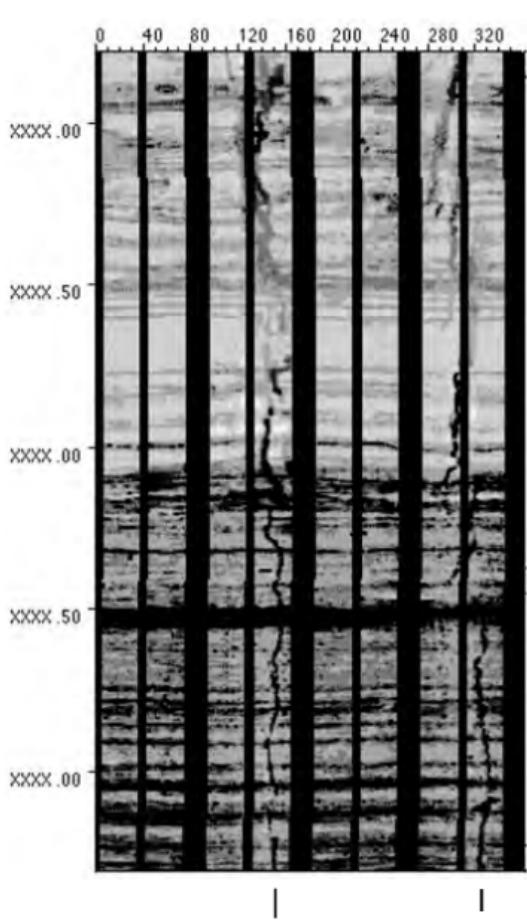
0.44 psi/ft

11.68 ppg

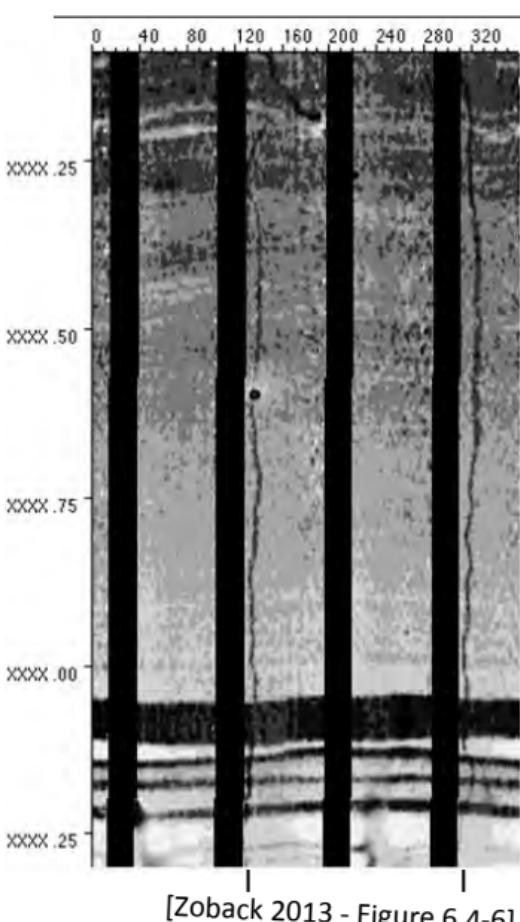
Ultrasonic P-wave

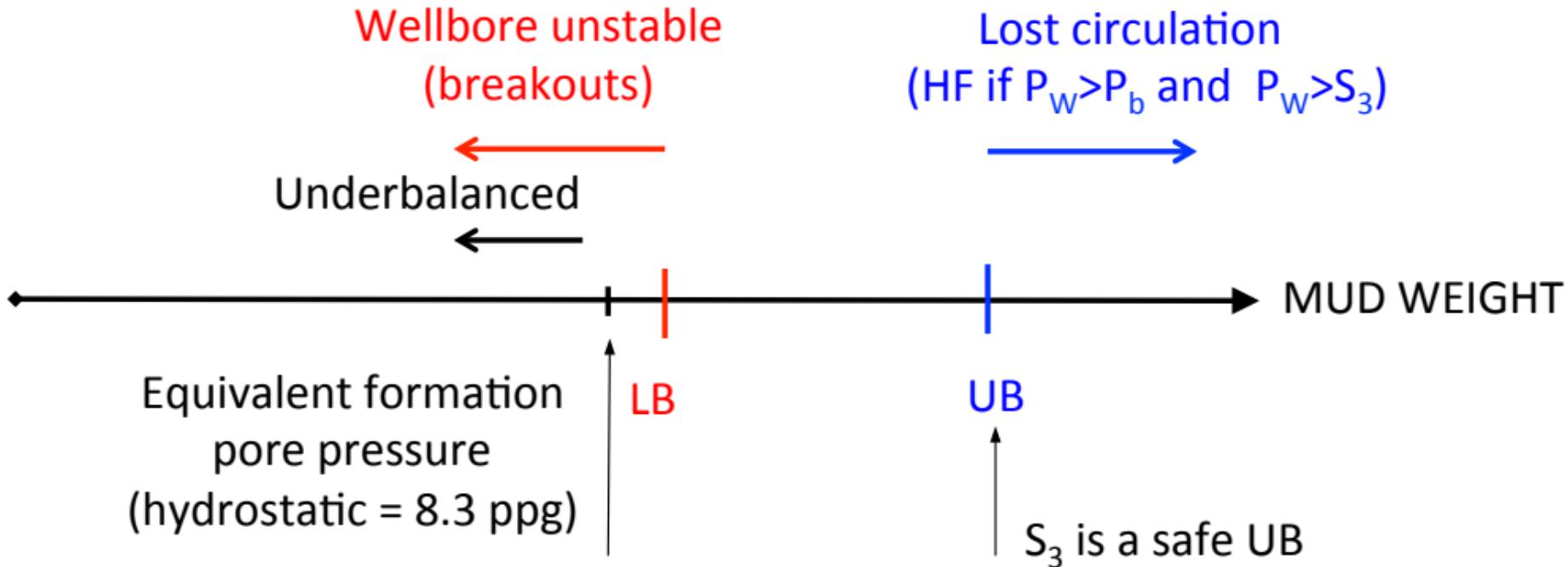


Electrical resistivity



Electrical resistivity



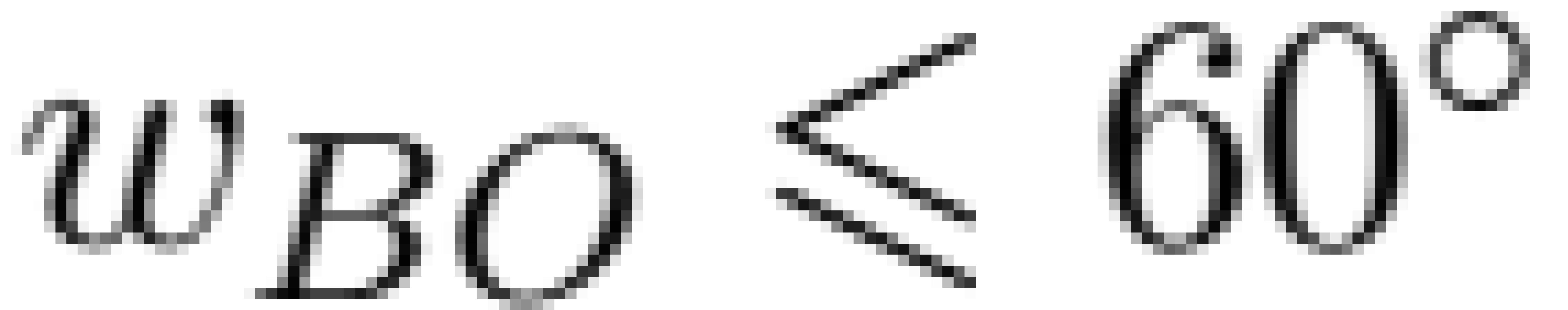


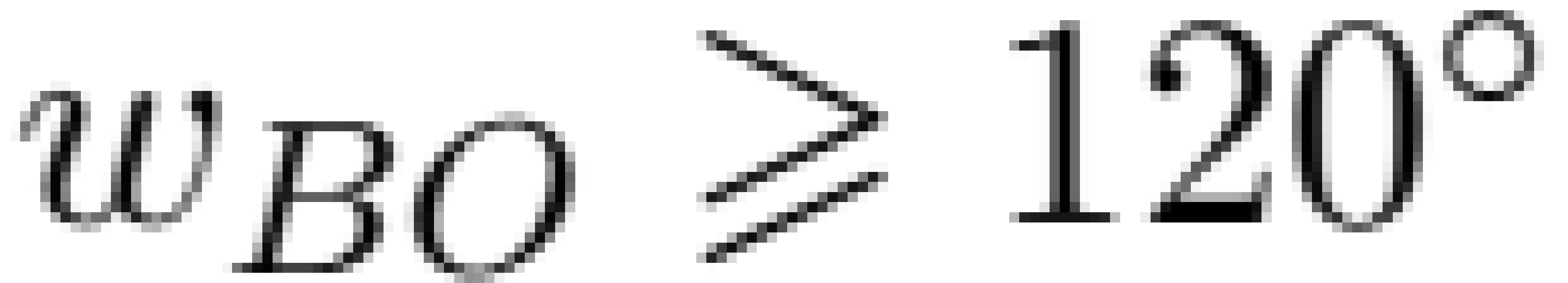
LIGHT MUDS

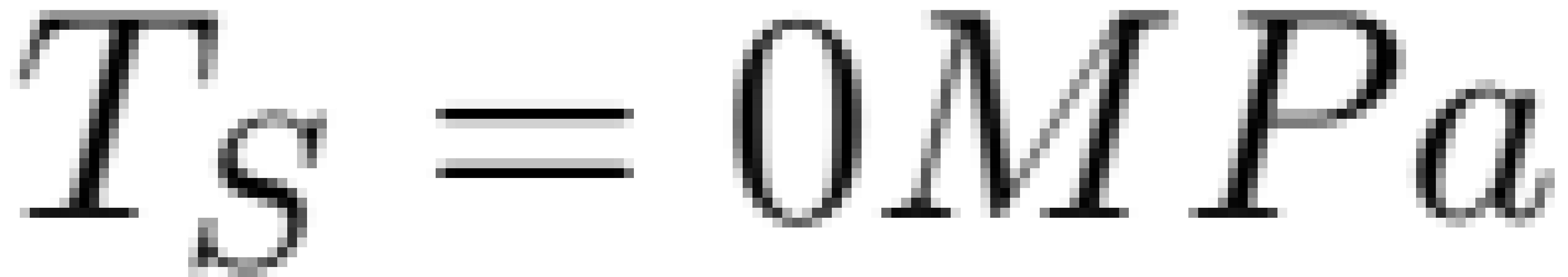
- May promote water production
- Compromise wellbore stability

HEAVY MUDS

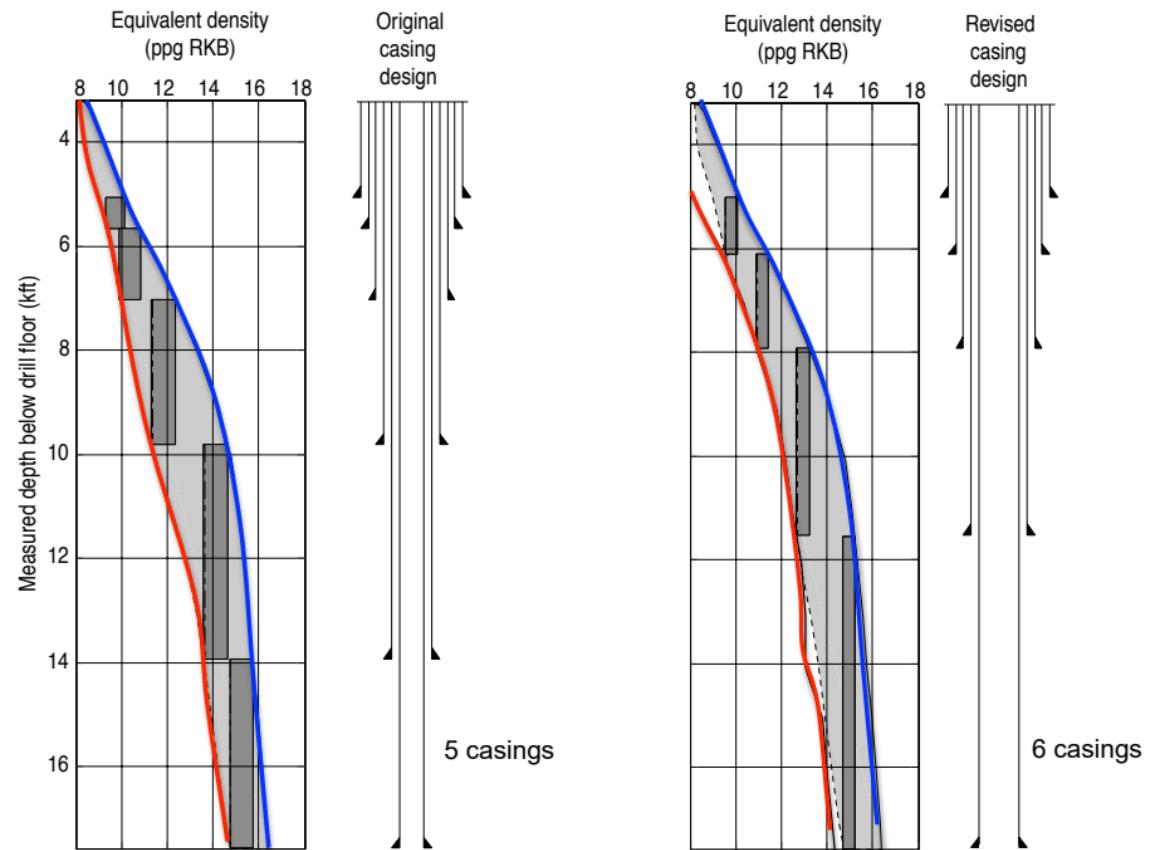
- Damage formation (k) by mud infiltration
- Promote mud losses in permeable strata
- Low ROP because of stronger rock





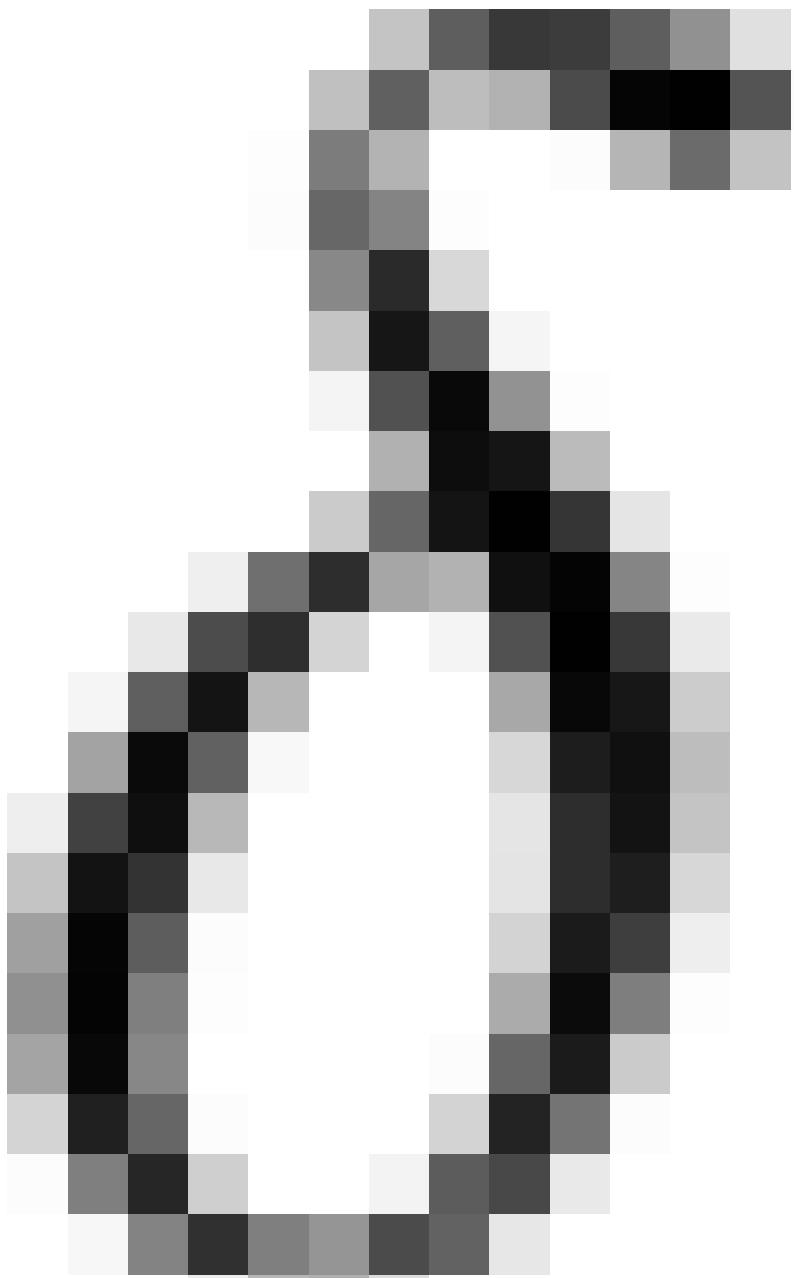


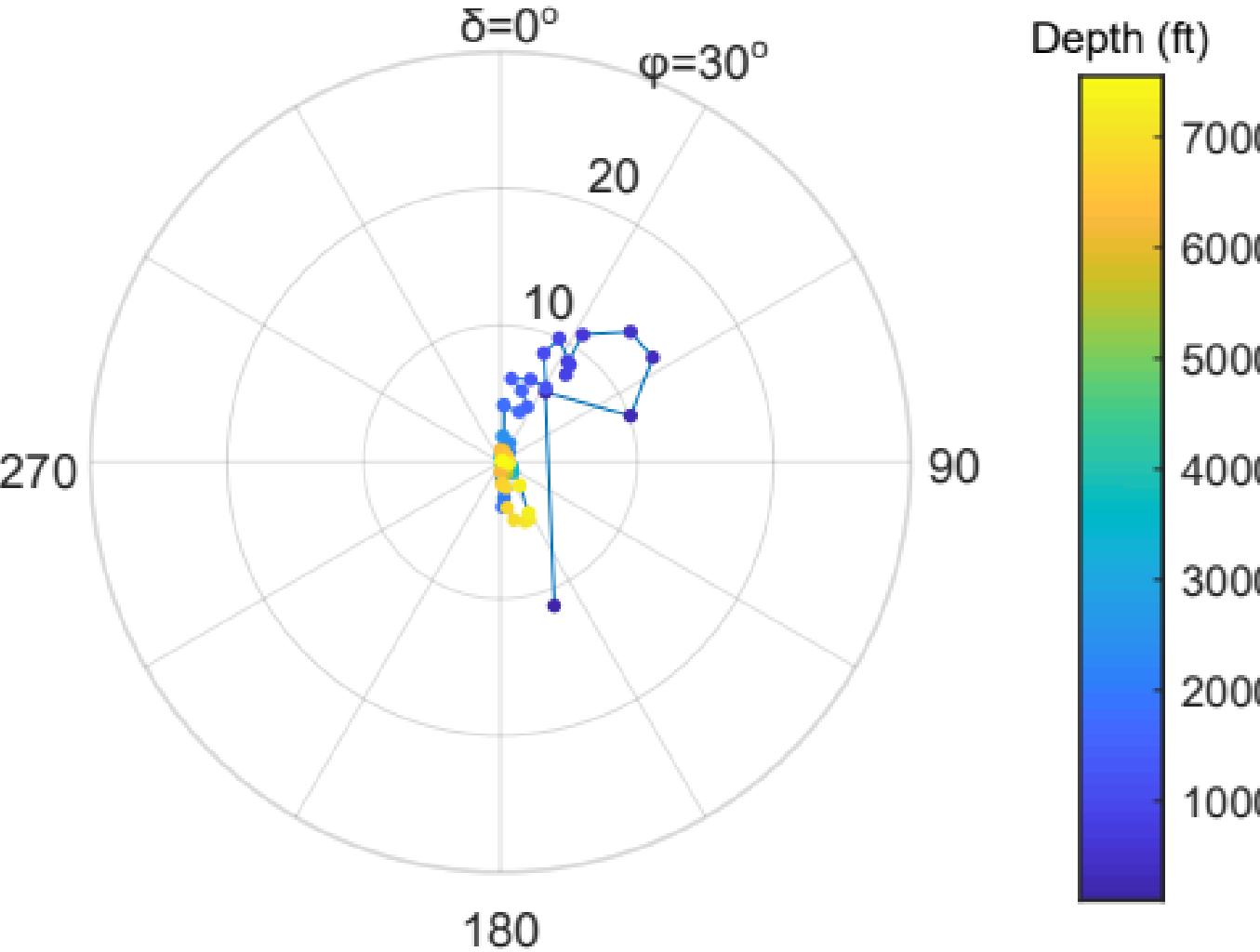
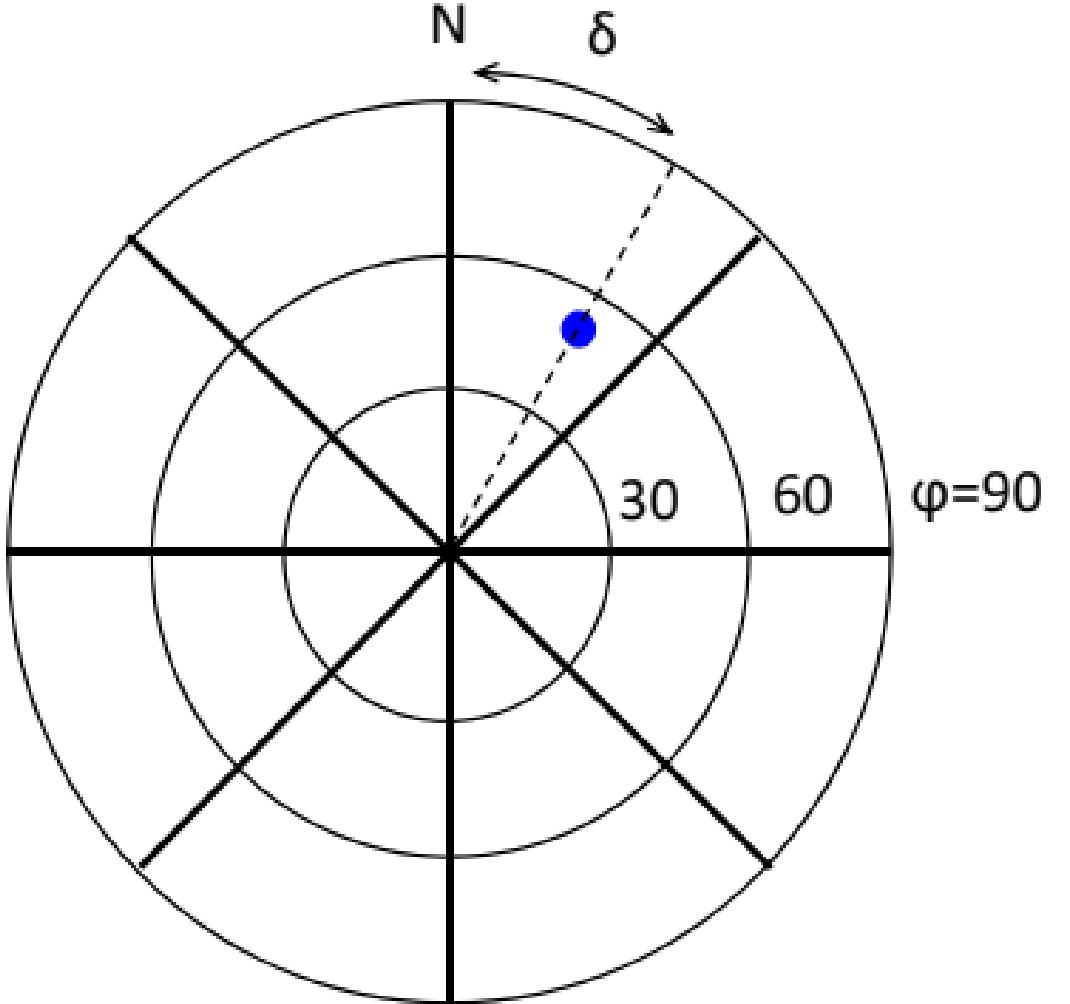
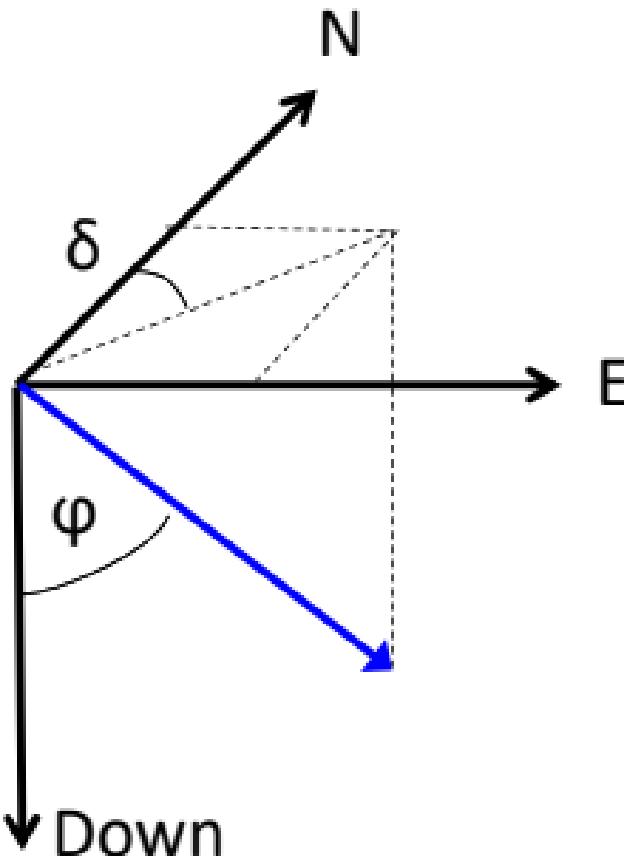


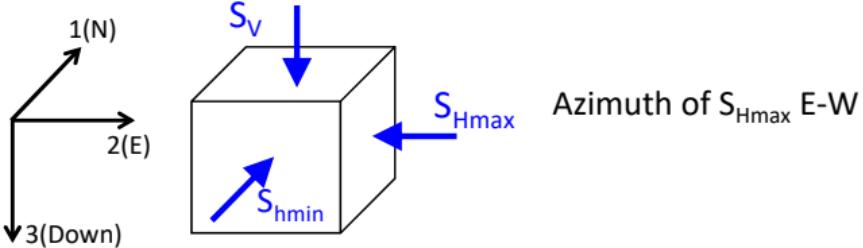


LB: Pore pressure
UB: Frac gradient

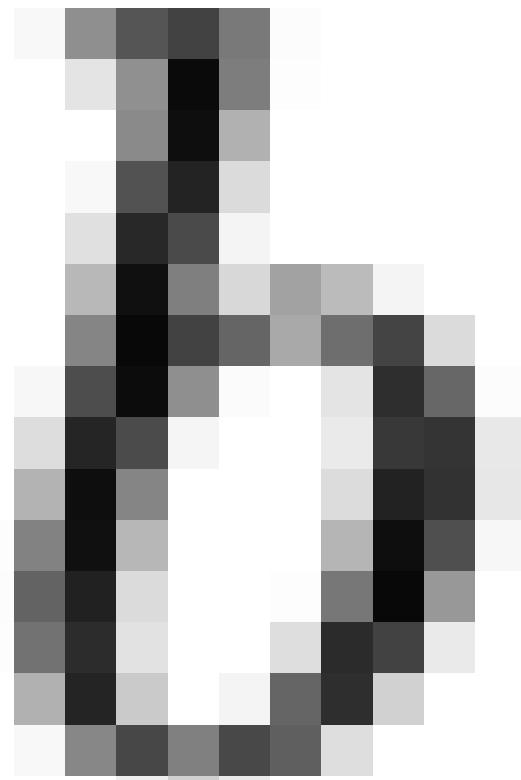
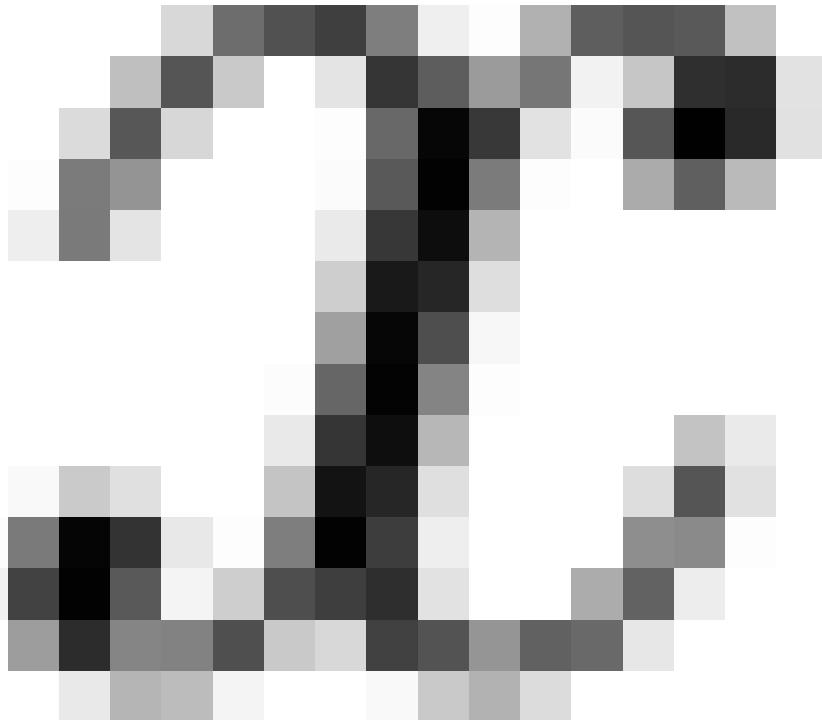
LB: Collapse pressure
UB: Frac gradient

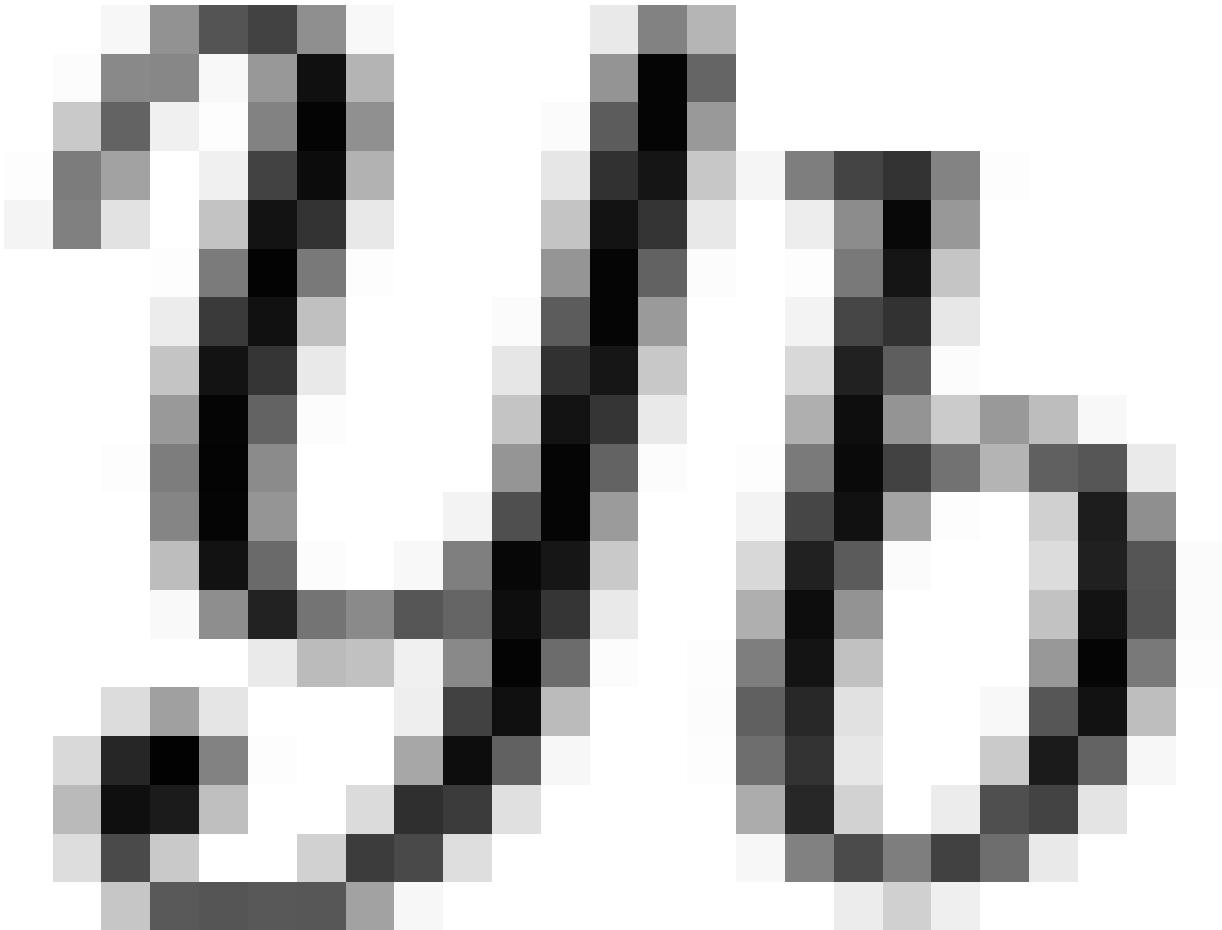


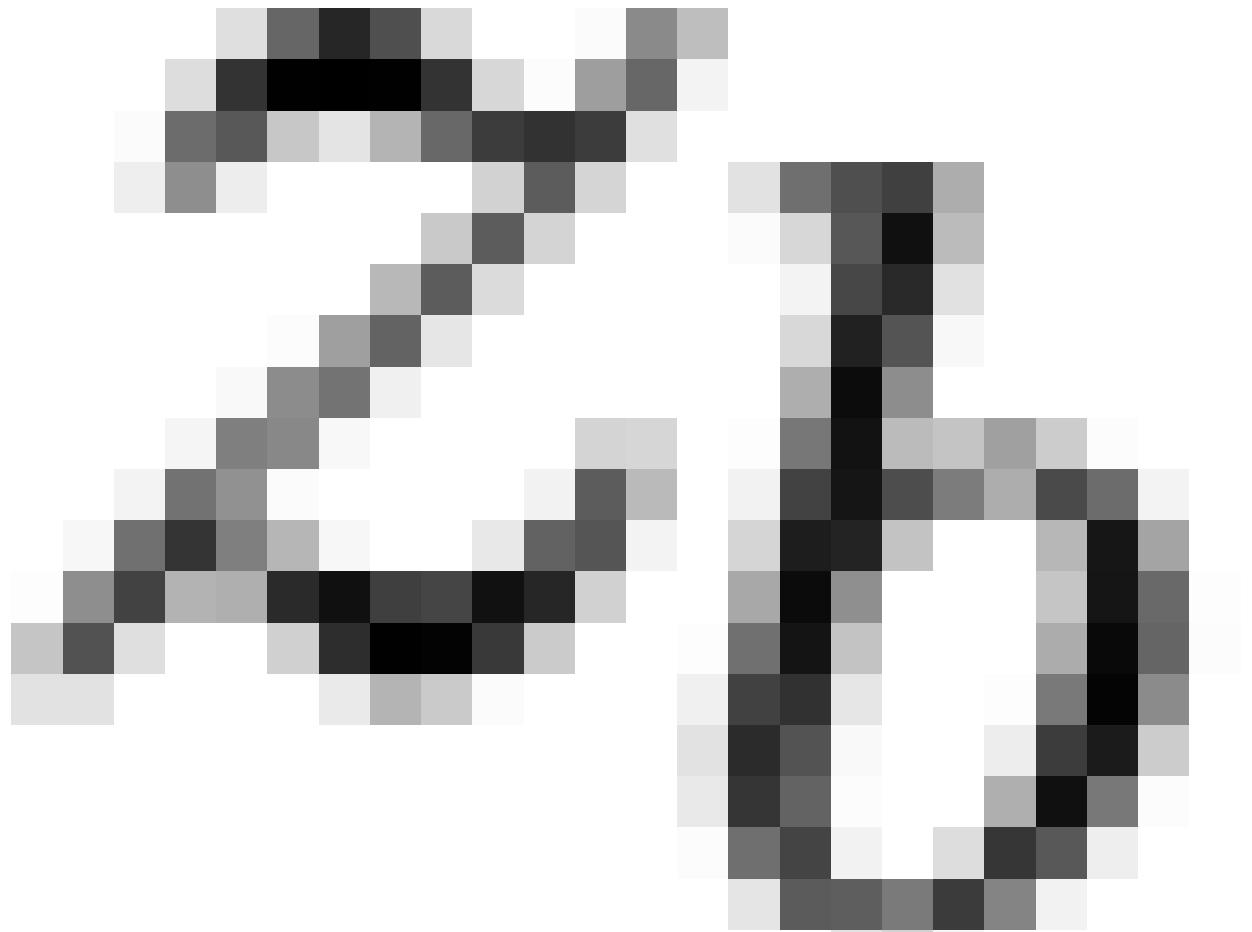


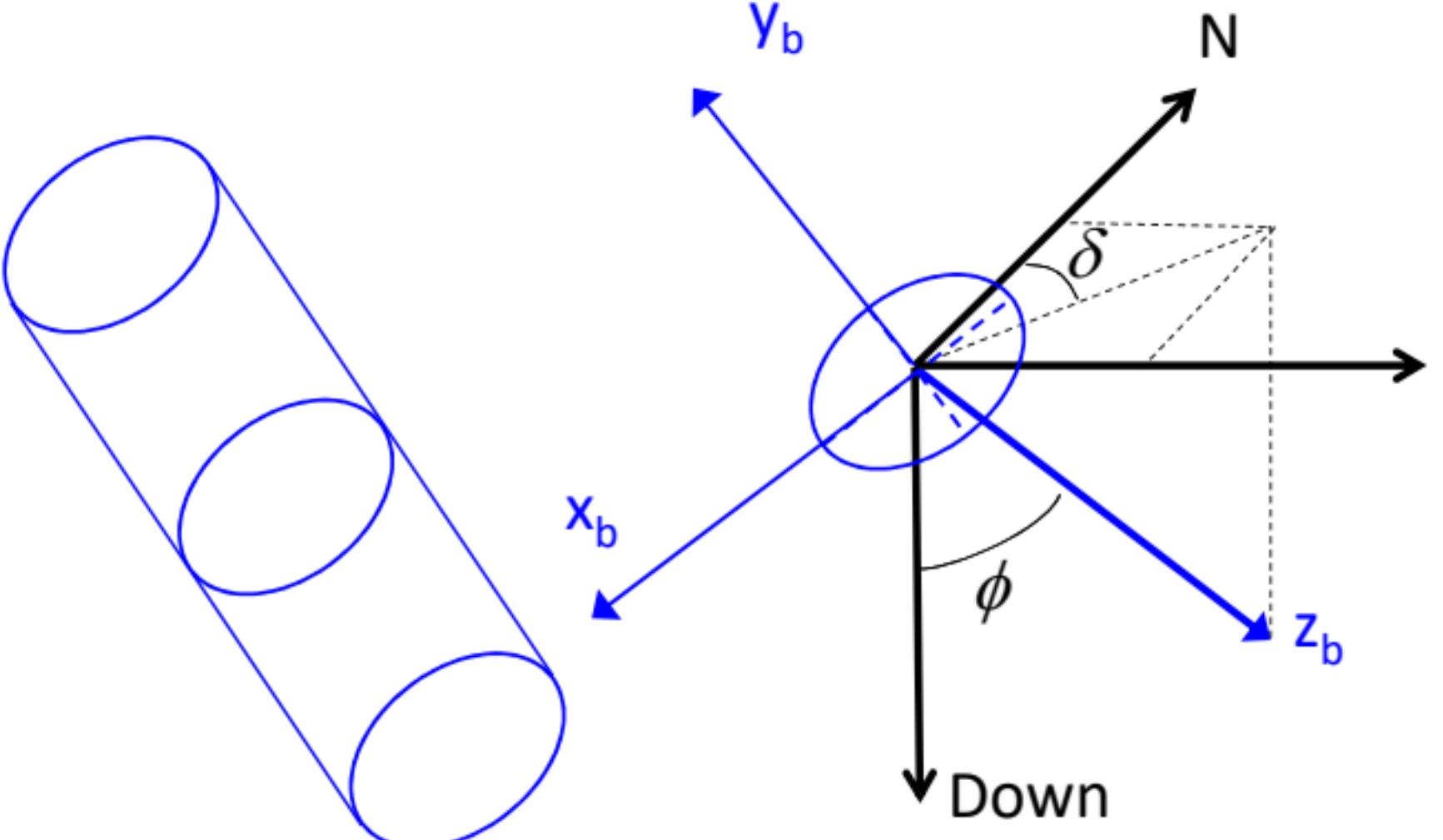


Stress Environment	NF	SS	RF
Plane with highest stress anisotropy	<p>Diagram of a stress element in the Normal Faulting (NF) environment. A red circle at the bottom indicates a plane of highest stress anisotropy. The vertical stress S_V acts downwards, and the minimum horizontal stress $S_{h\text{min}}$ acts upwards.</p>	<p>Diagram of a stress element in the Shear Stress (SS) environment. A red circle is located near the top edge. The vertical stress S_V acts downwards, and the maximum horizontal stress $S_{H\text{max}}$ acts to the left.</p>	<p>Diagram of a stress element in the Reverse Faulting (RF) environment. A red circle is located near the bottom edge. The vertical stress S_V acts downwards, and the maximum horizontal stress $S_{H\text{max}}$ acts to the left.</p>
Narrower drilling window and Breakouts orientation	<p>Diagram of a drill hole cross-section for the NF environment, oriented North (N).</p>	<p>Diagram of a drill hole cross-section for the SS environment, oriented North (N).</p>	<p>Diagram of a drill hole cross-section for the RF environment, oriented North (N).</p>

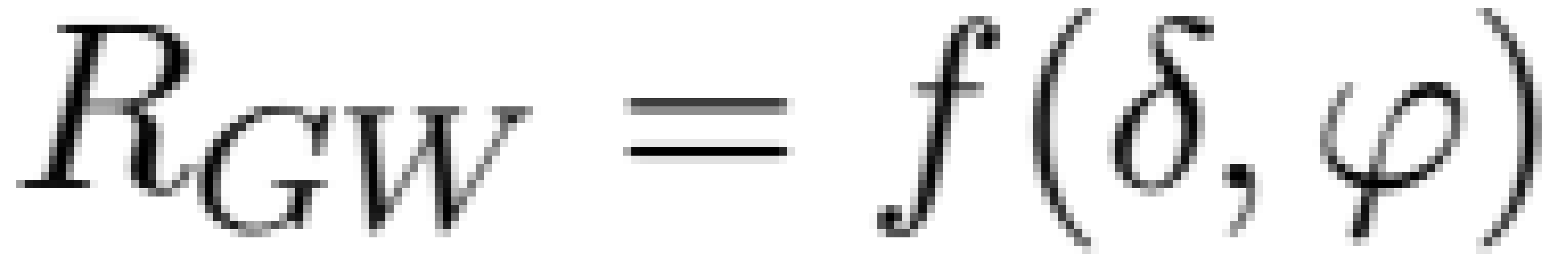








$$R_{GW} = \begin{bmatrix} -\cos\delta\cos\phi & -\sin\delta\cos\phi & \sin\phi \\ \sin\delta & -\cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix}$$









$$\underline{S}_W =$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$





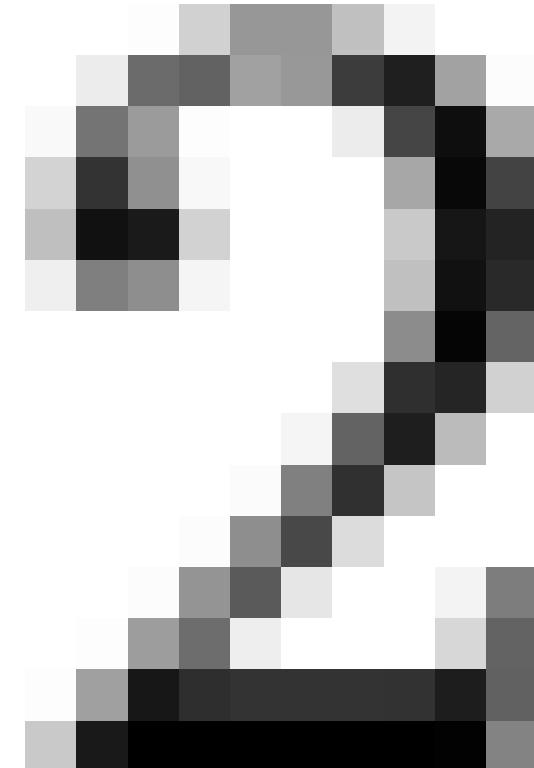
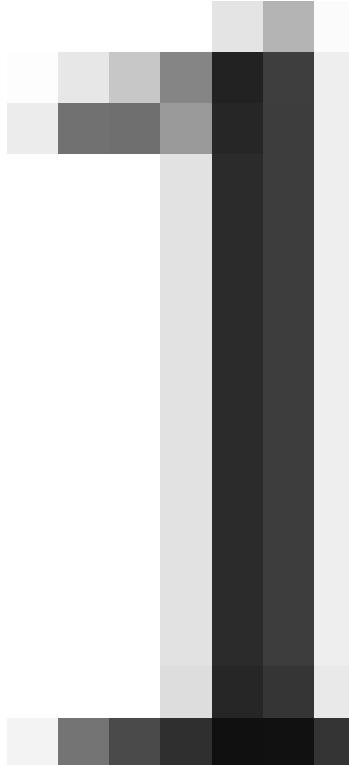
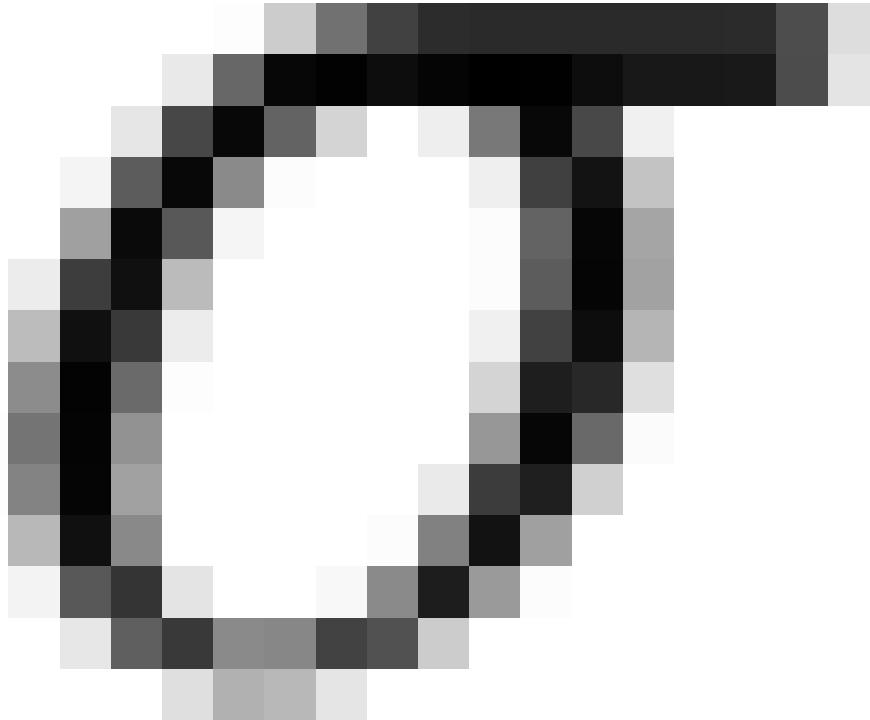


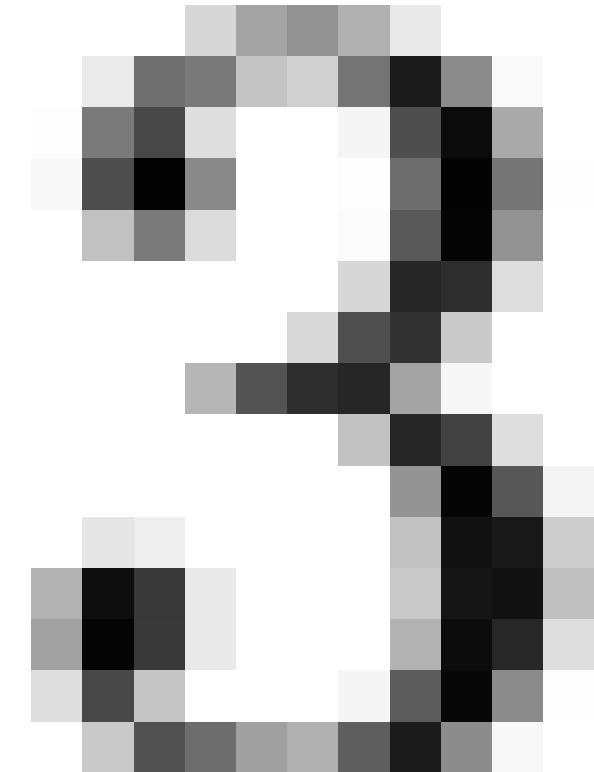
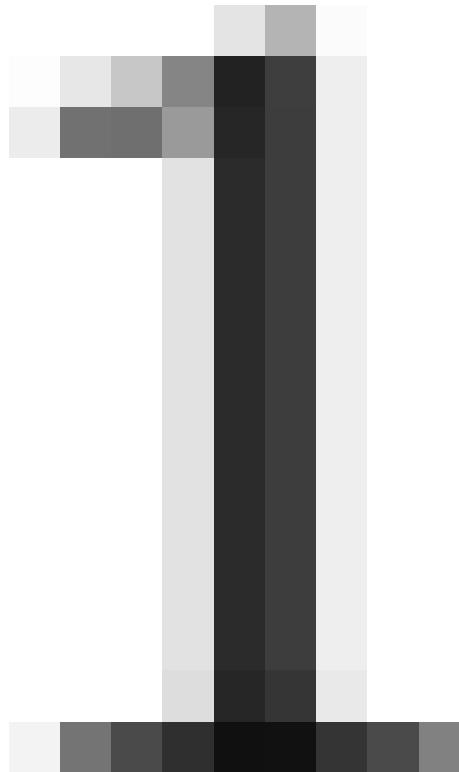
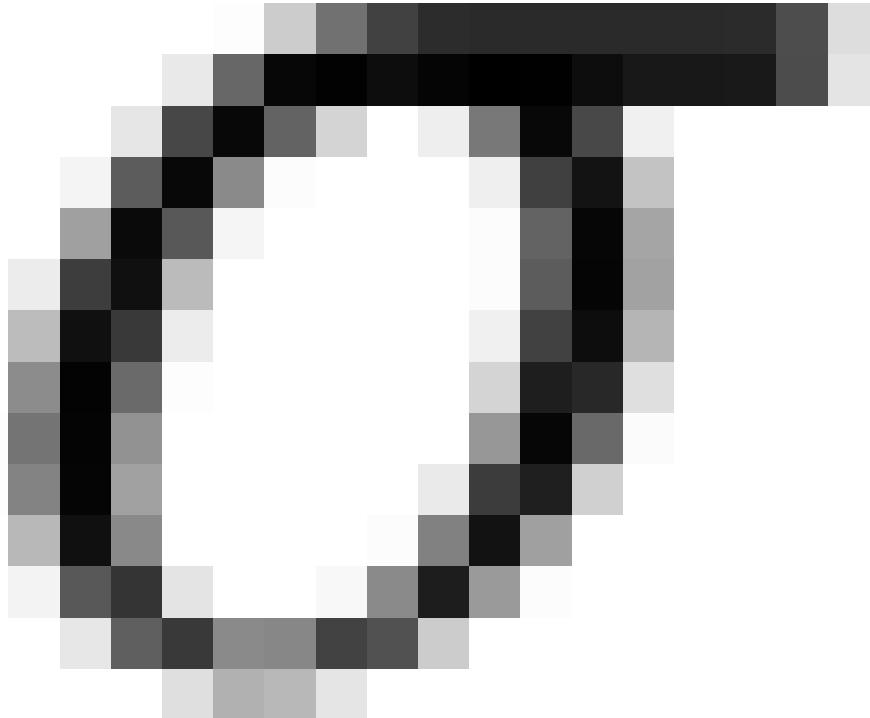


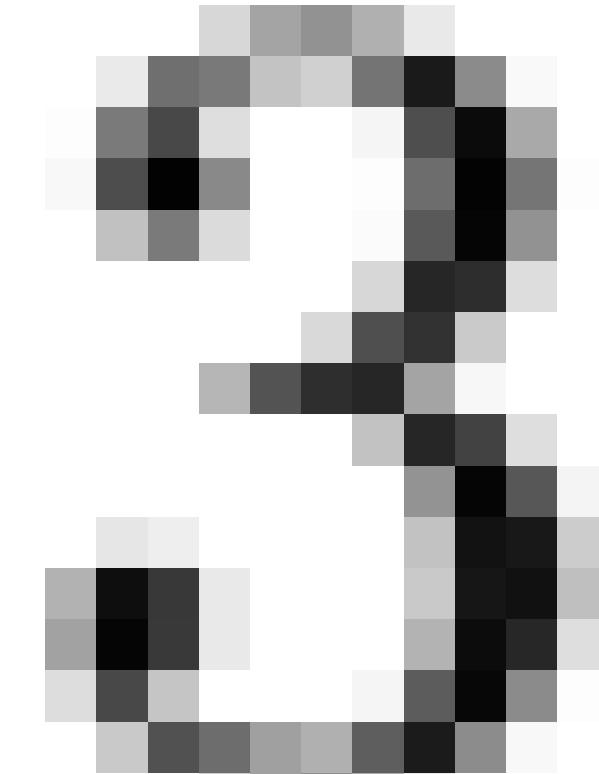
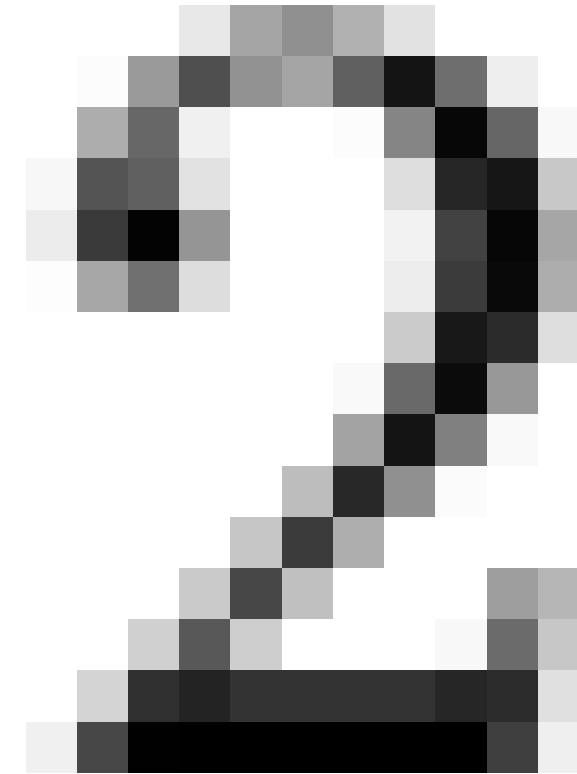
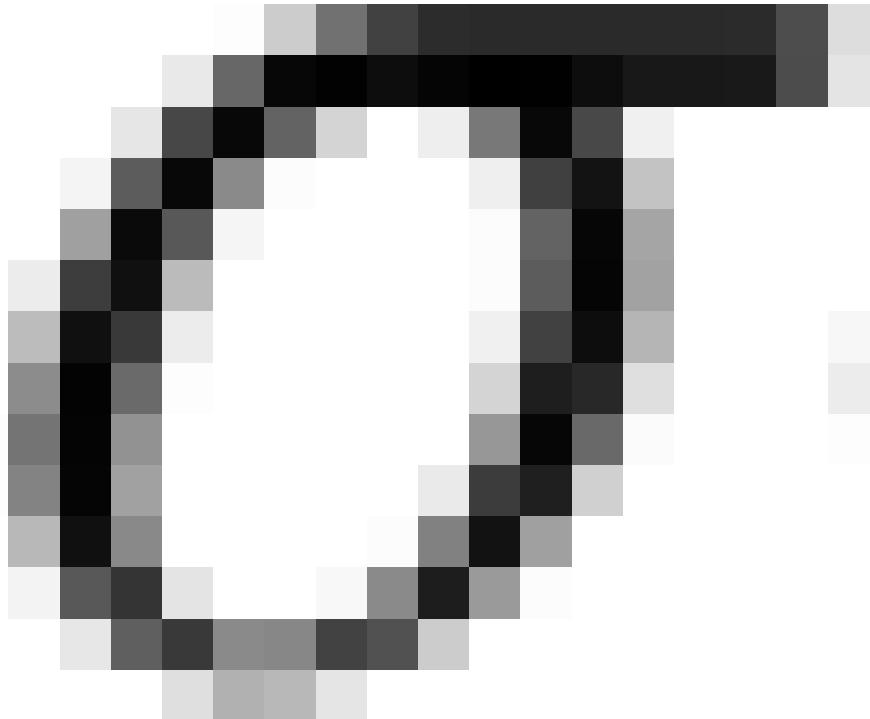


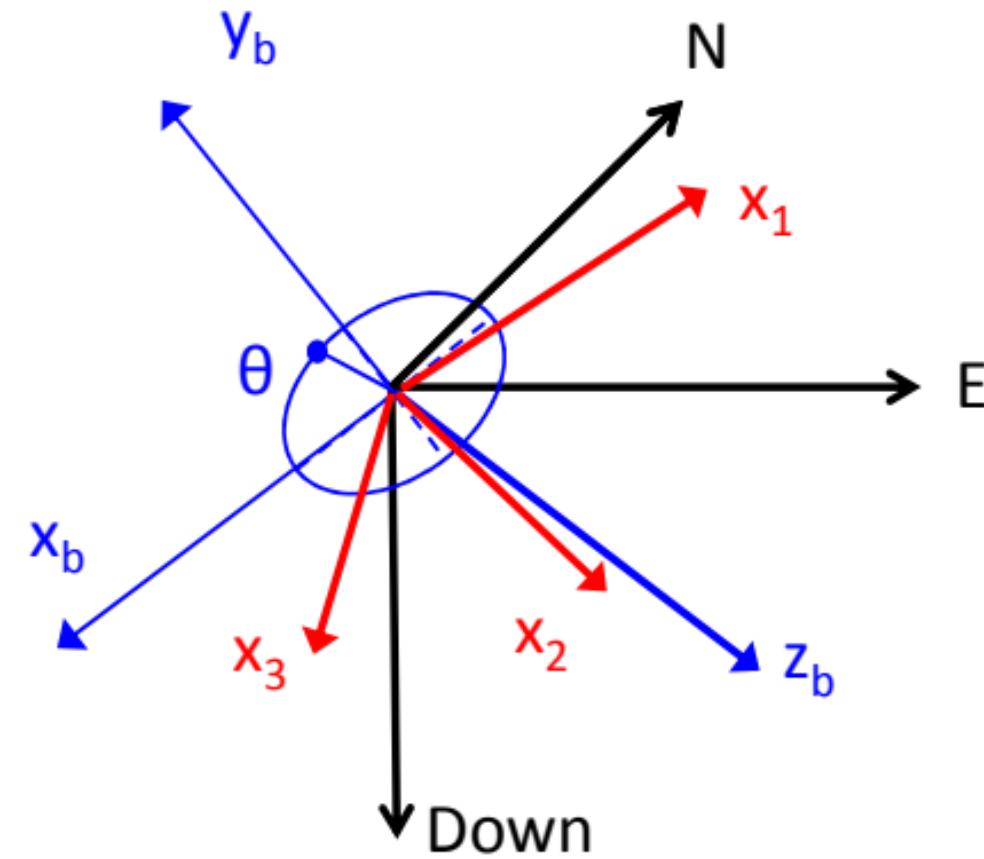








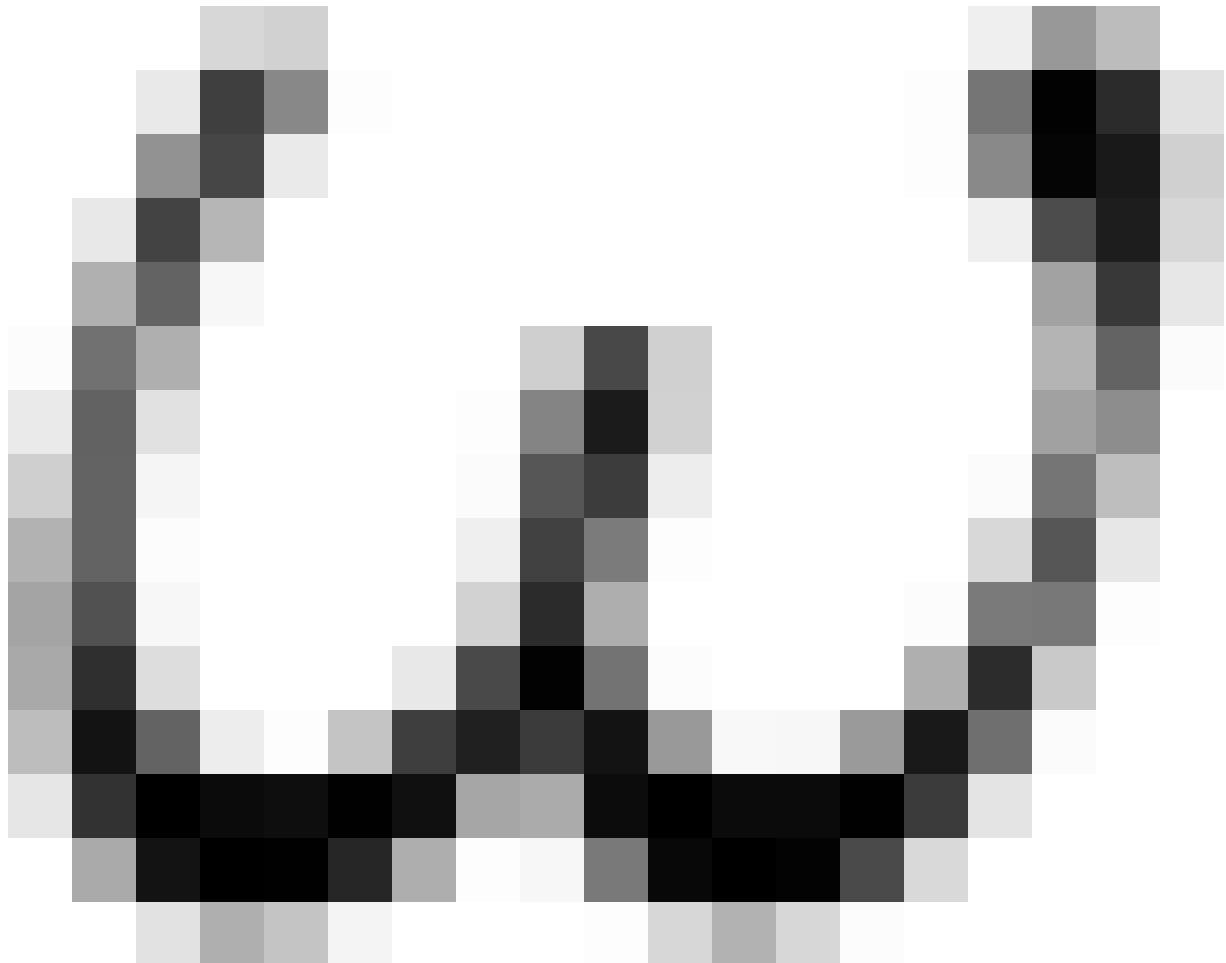


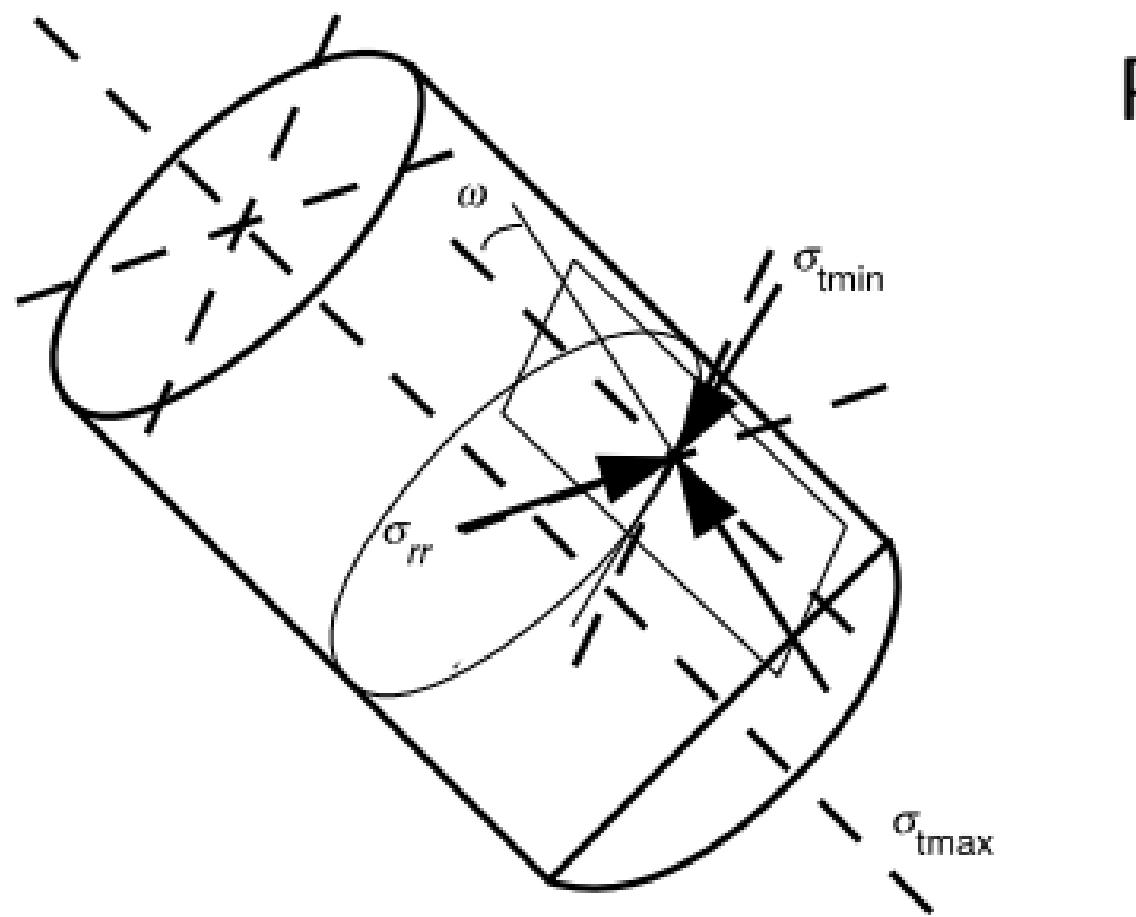


Stresses at the wellbore wall (Kirsch[PP]+Kirsch[S])

$$\begin{cases} \sigma_{rr} = \Delta P \\ \sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\sigma_{12}\sin 2\theta - \Delta P \\ \tau_{\theta z} = 2(\sigma_{23}\cos\theta - \sigma_{13}\sin\theta) \\ \sigma_{zz} = \sigma_{33} - 2\nu(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\nu\sigma_{12}\sin 2\theta \end{cases}$$

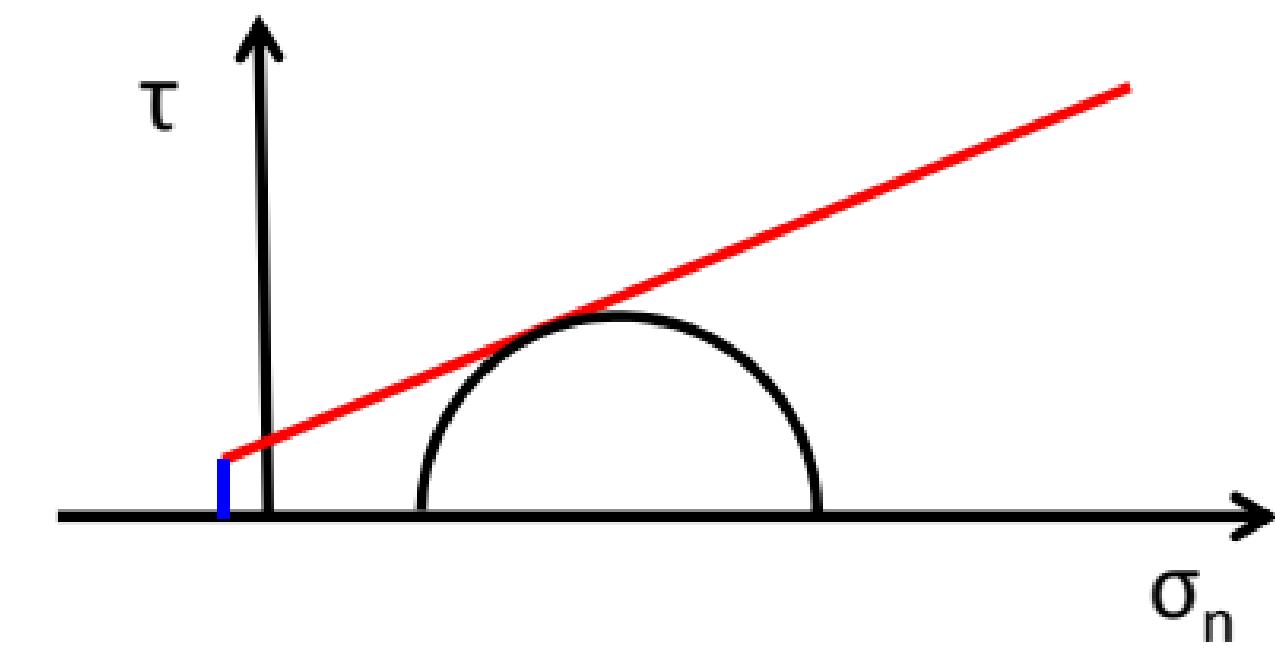


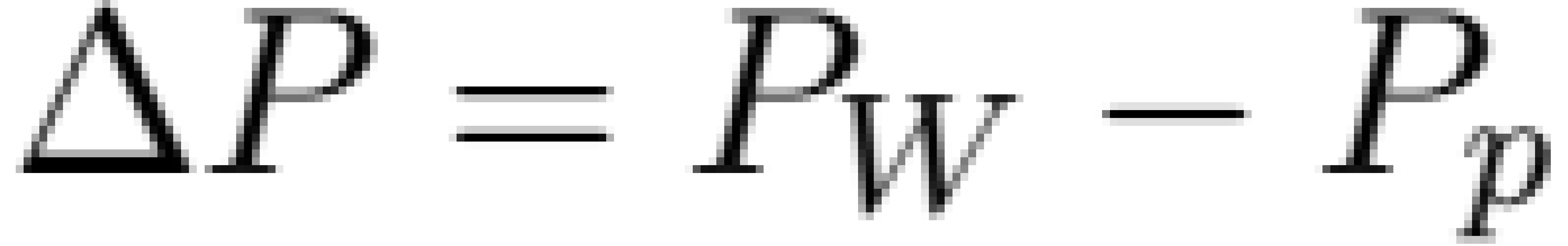




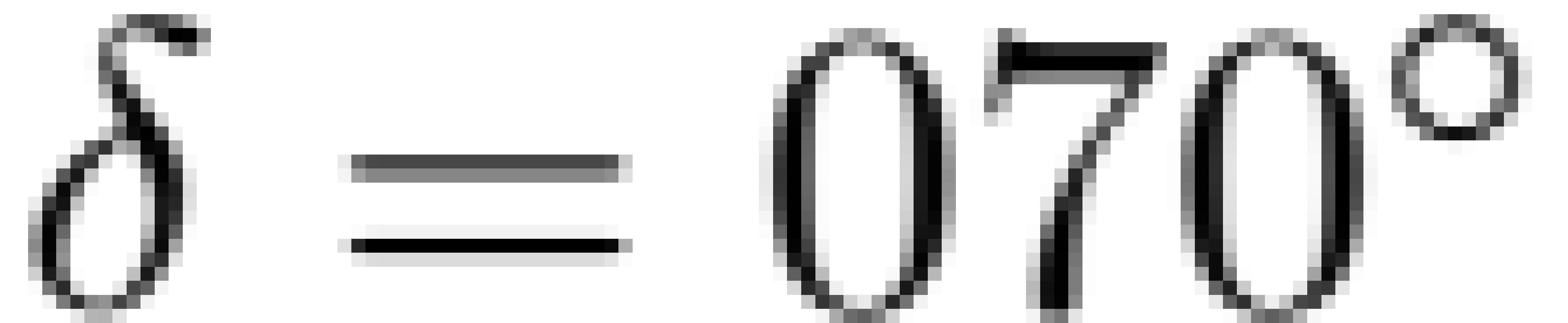
Principal stresses at the wellbore wall

$$\left\{ \begin{array}{l} \sigma_{t\max} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} + \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \\ \sigma_{t\min} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} - \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \end{array} \right.$$

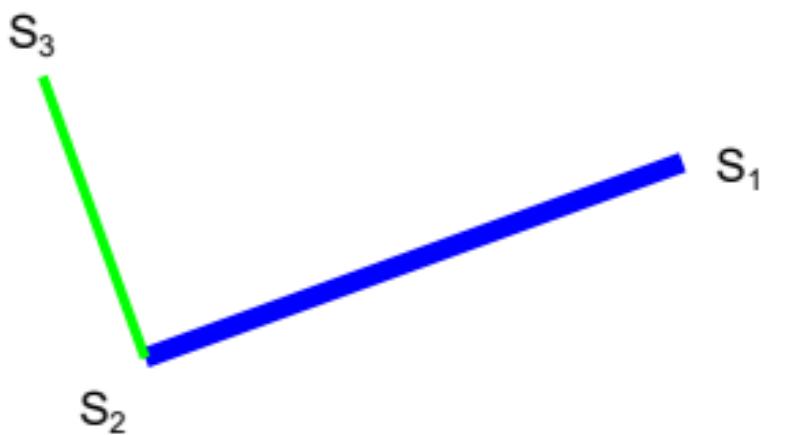




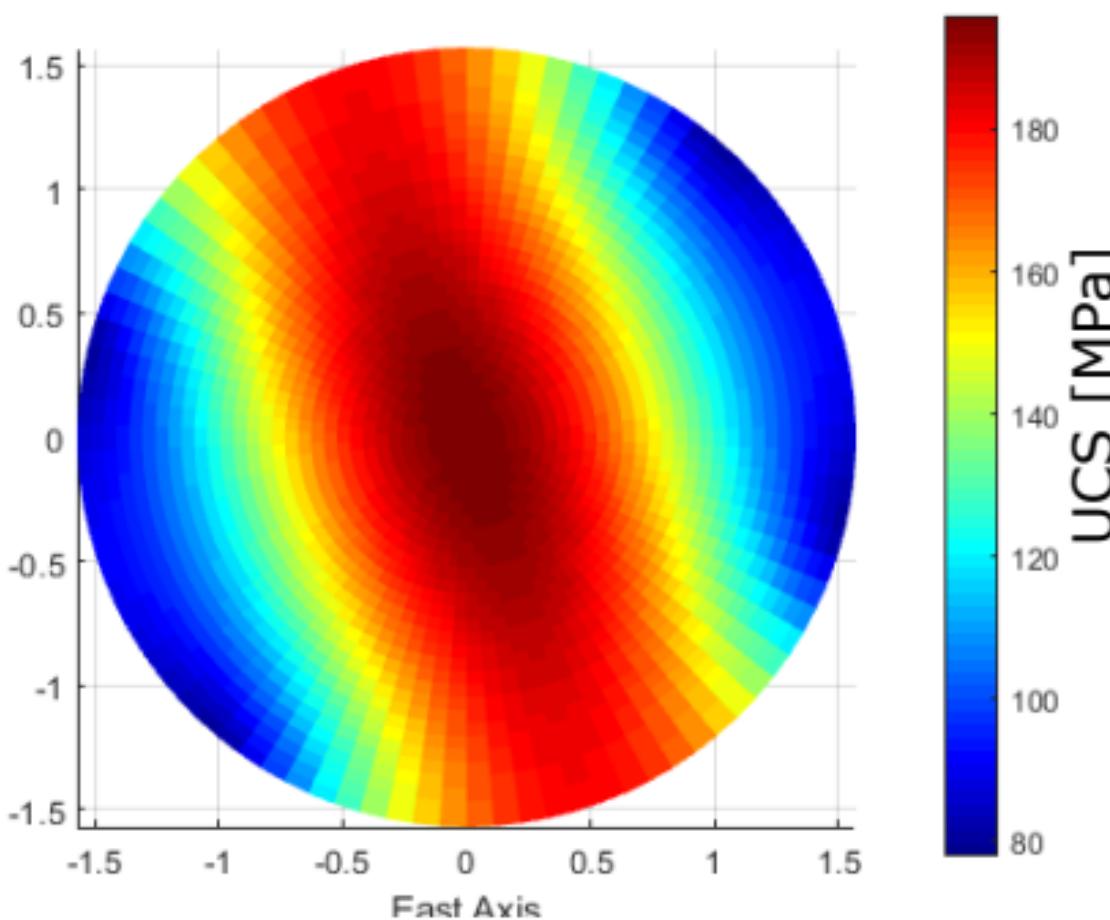




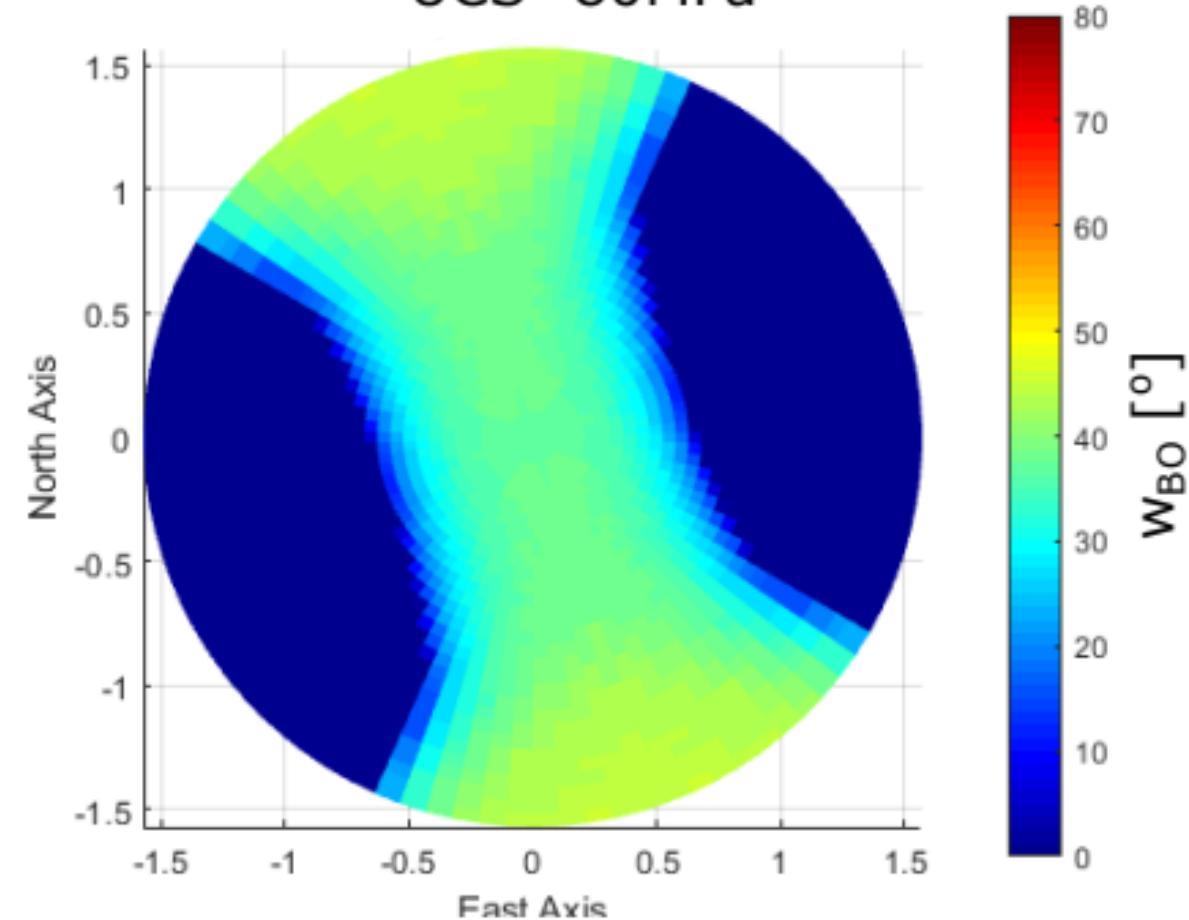
Geographical principal stresses

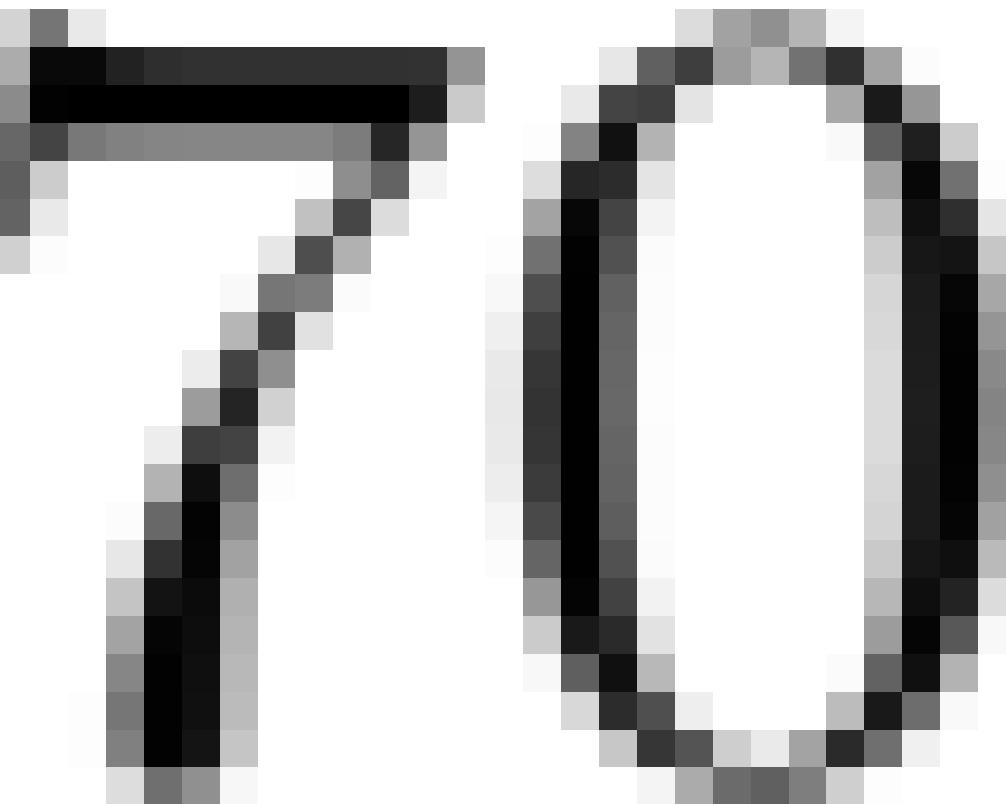


Required UCS ($P_w = P_p$)



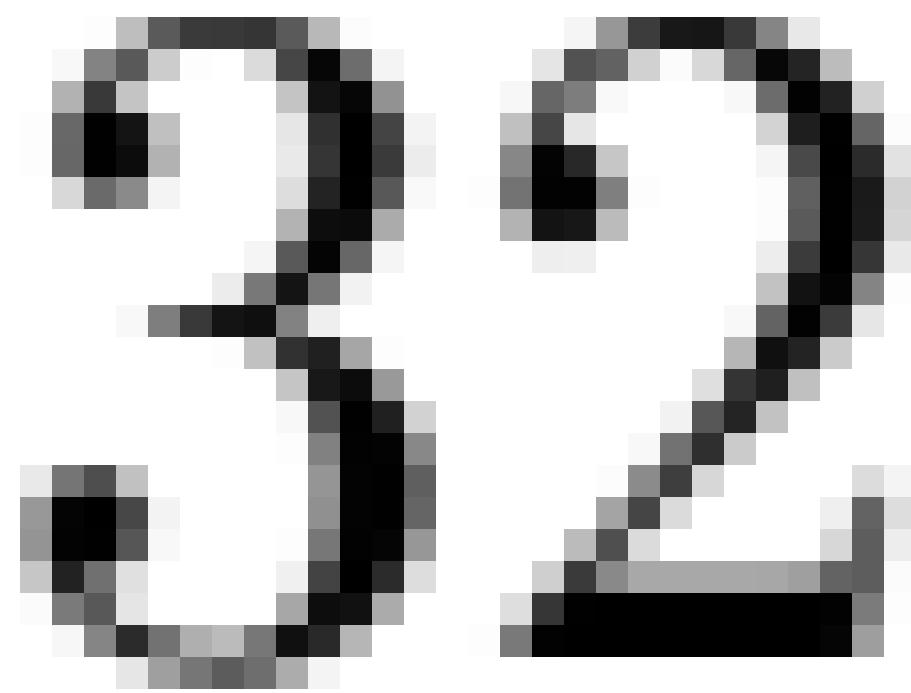
Breakout angle by Lade - $P_w = 45\text{MPa}$ UCS=80MPa

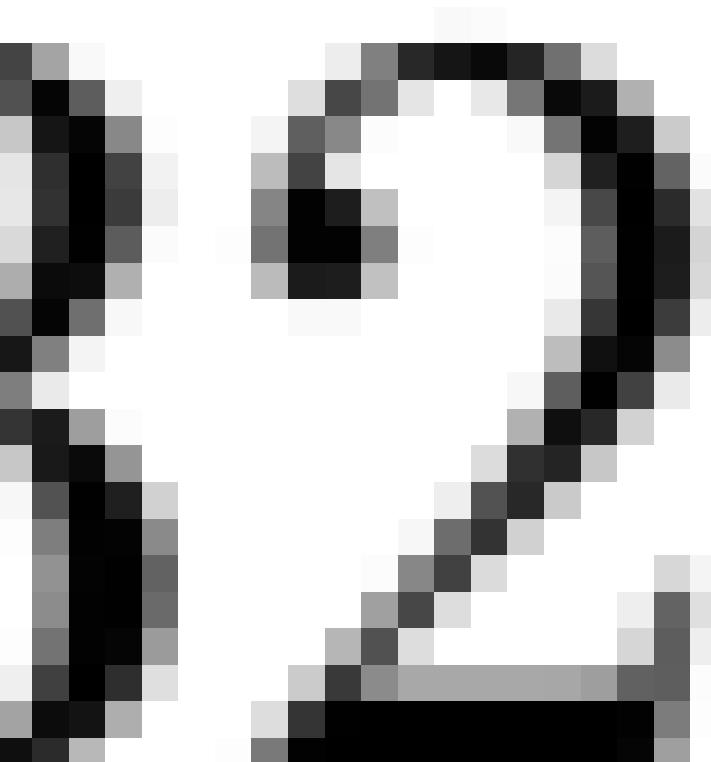
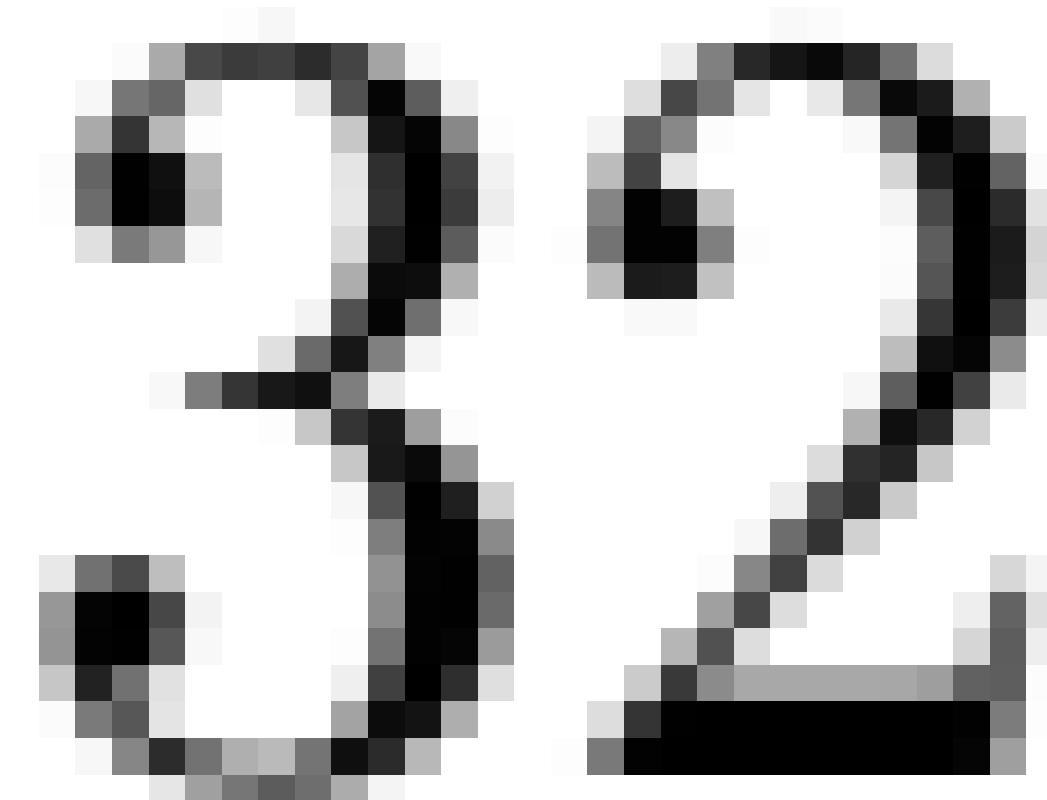


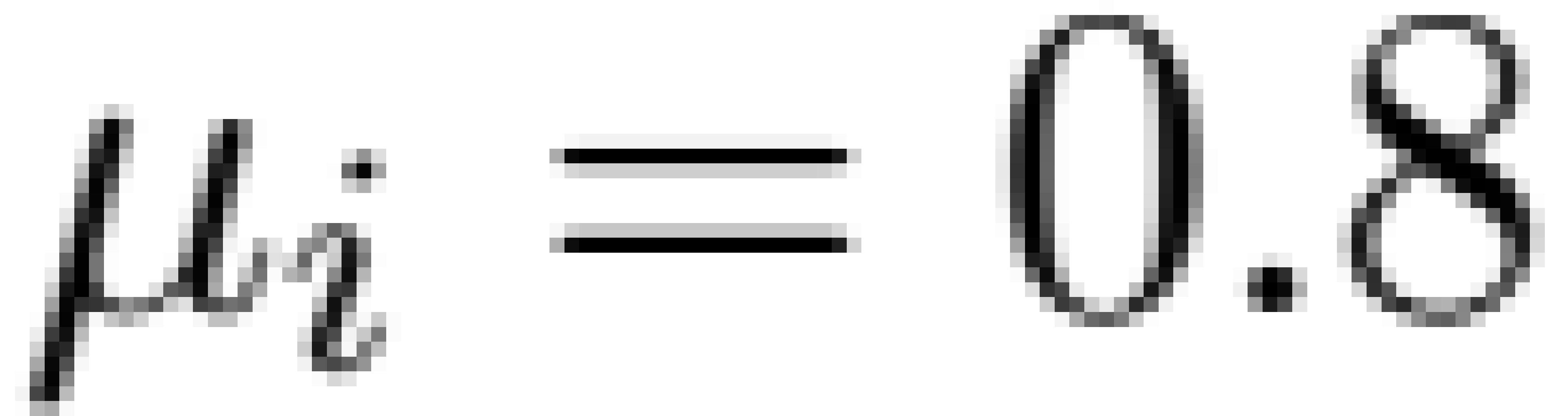




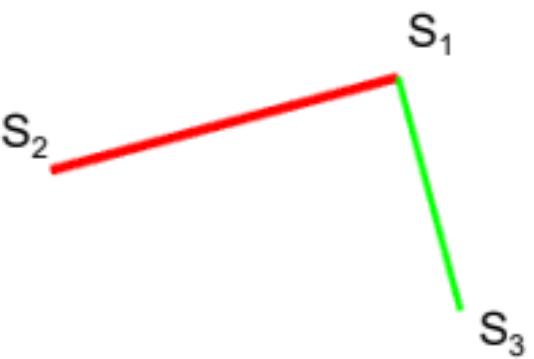




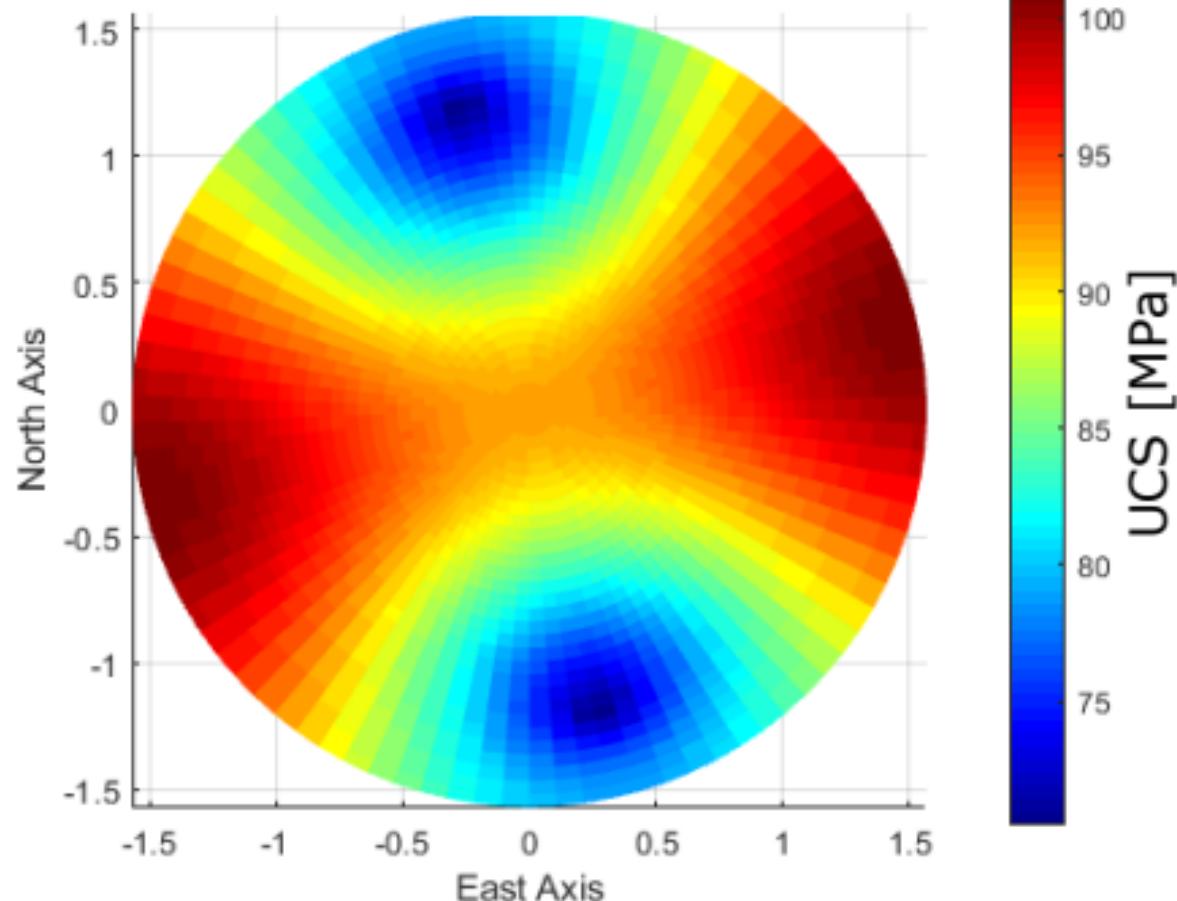




Principal stresses



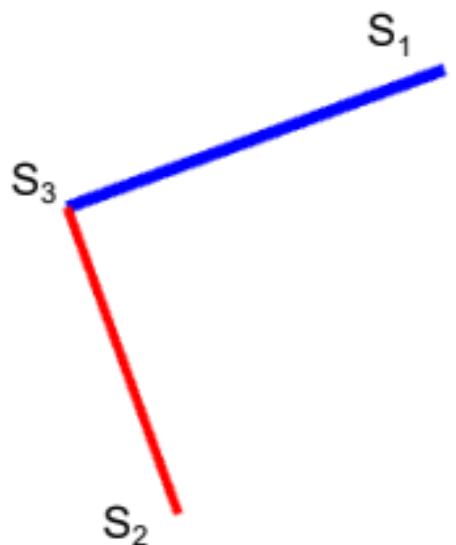
Required UCS ($P_w = P_p$)



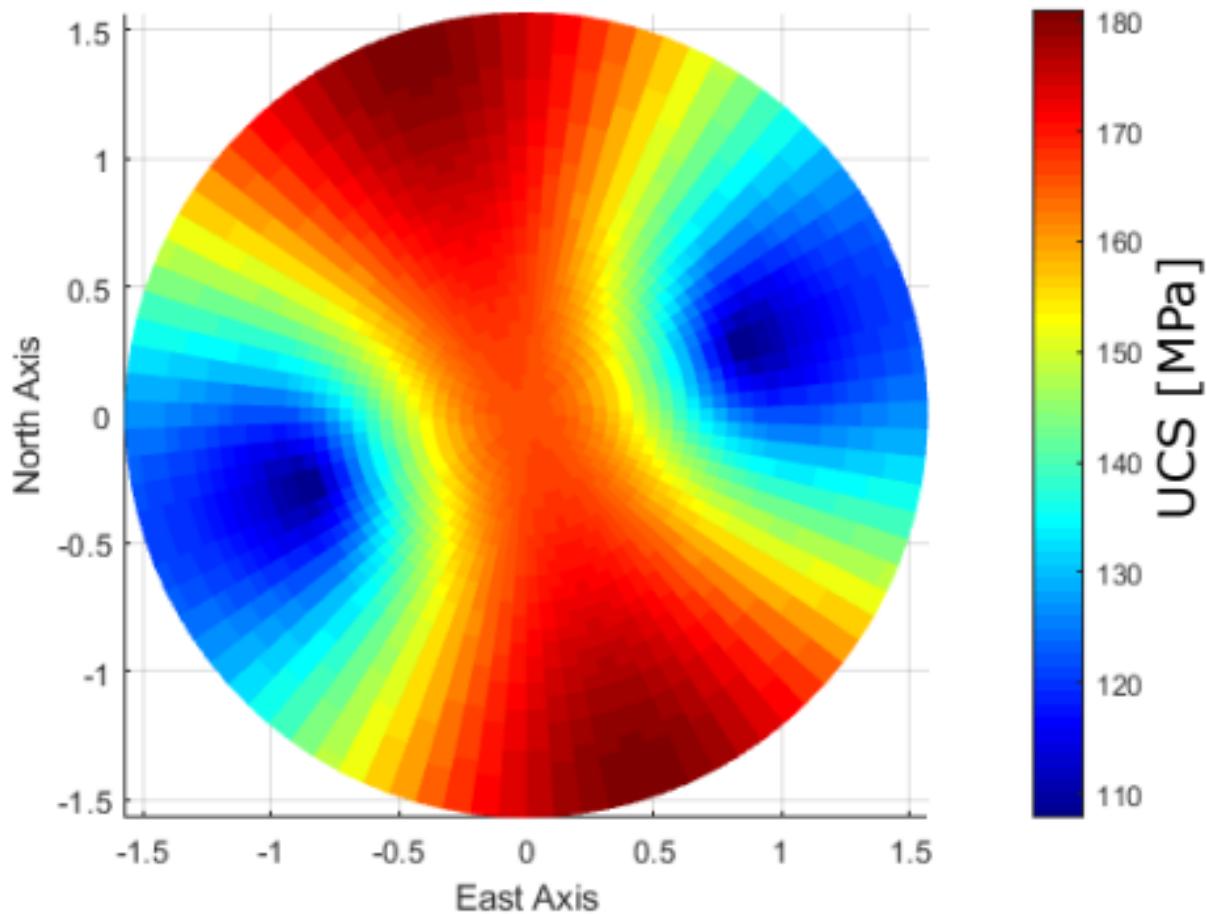




Principal stresses

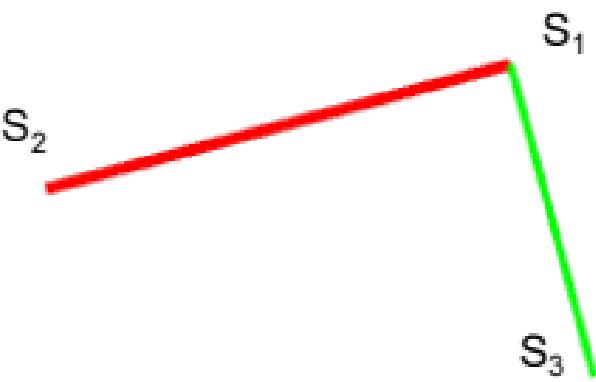


Required UCS ($P_w = P_p$)

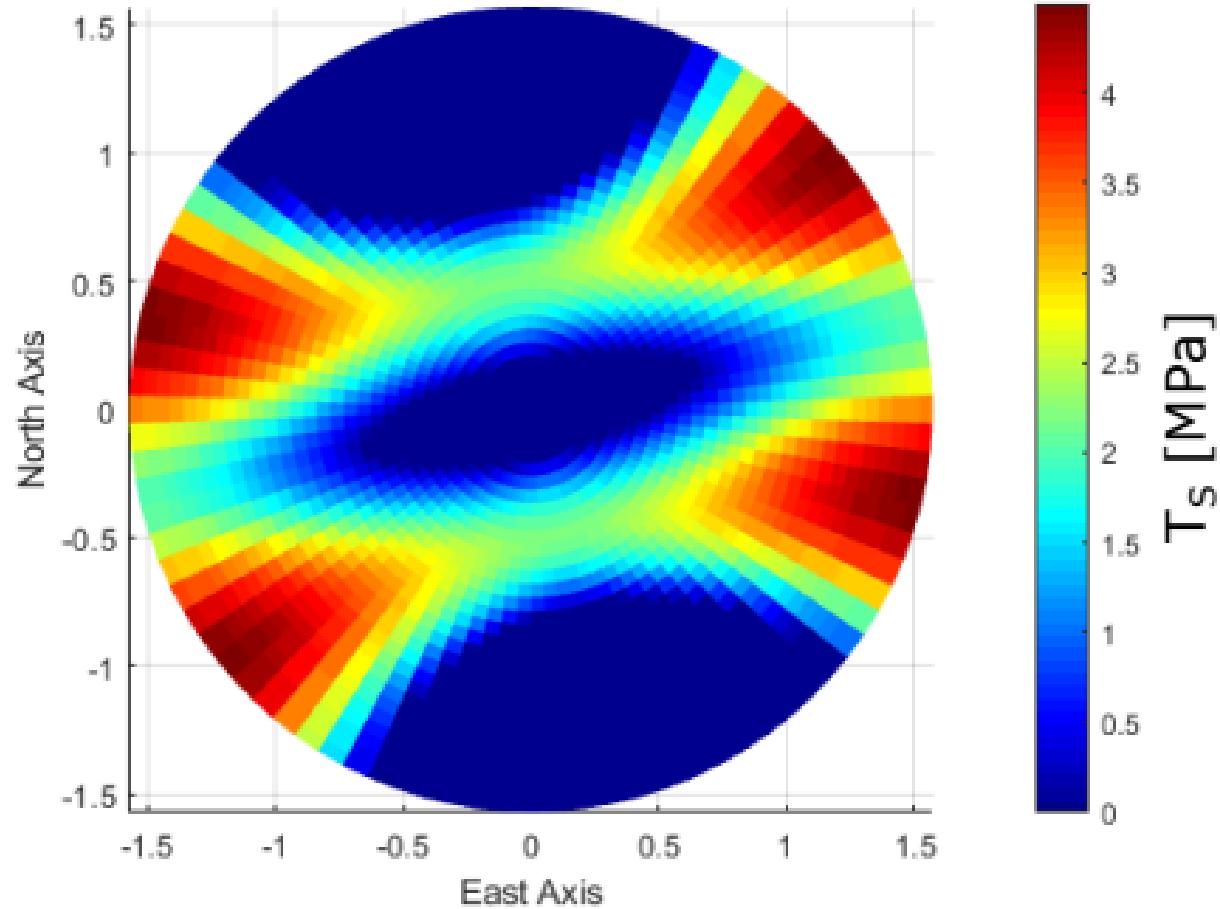


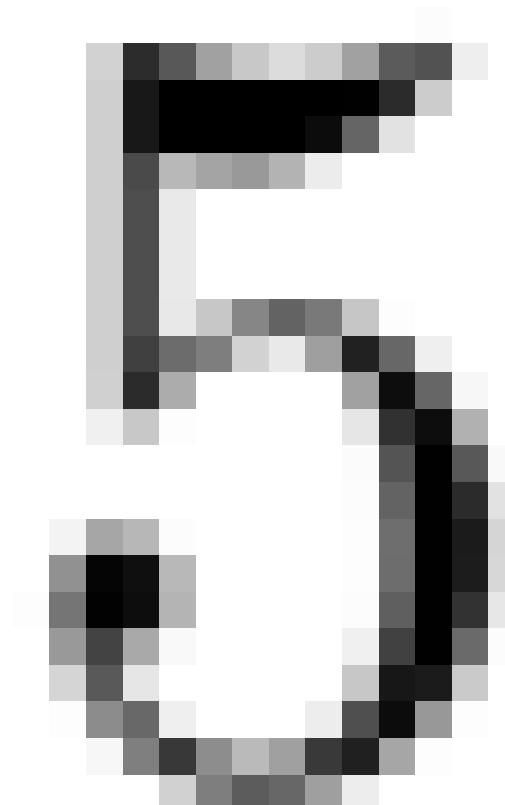
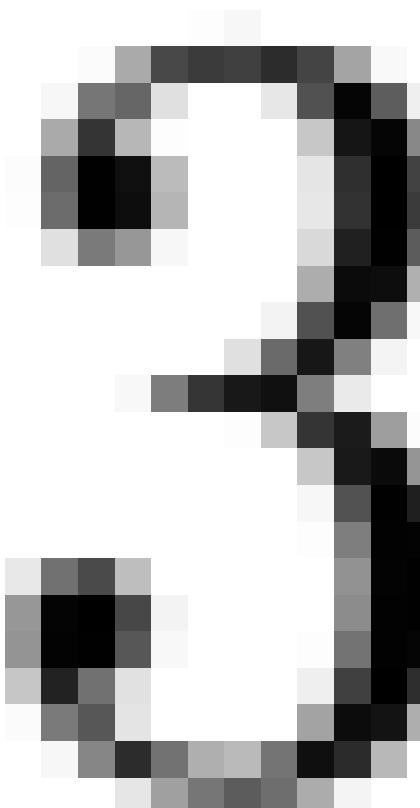


Principal stresses



Required T_s ($P_w=35\text{MPa}$)









$\delta\Omega$

Δ

$\delta\Omega$

Δ

$\Delta\Omega$

$\Delta\Omega$

$\Delta\Omega$

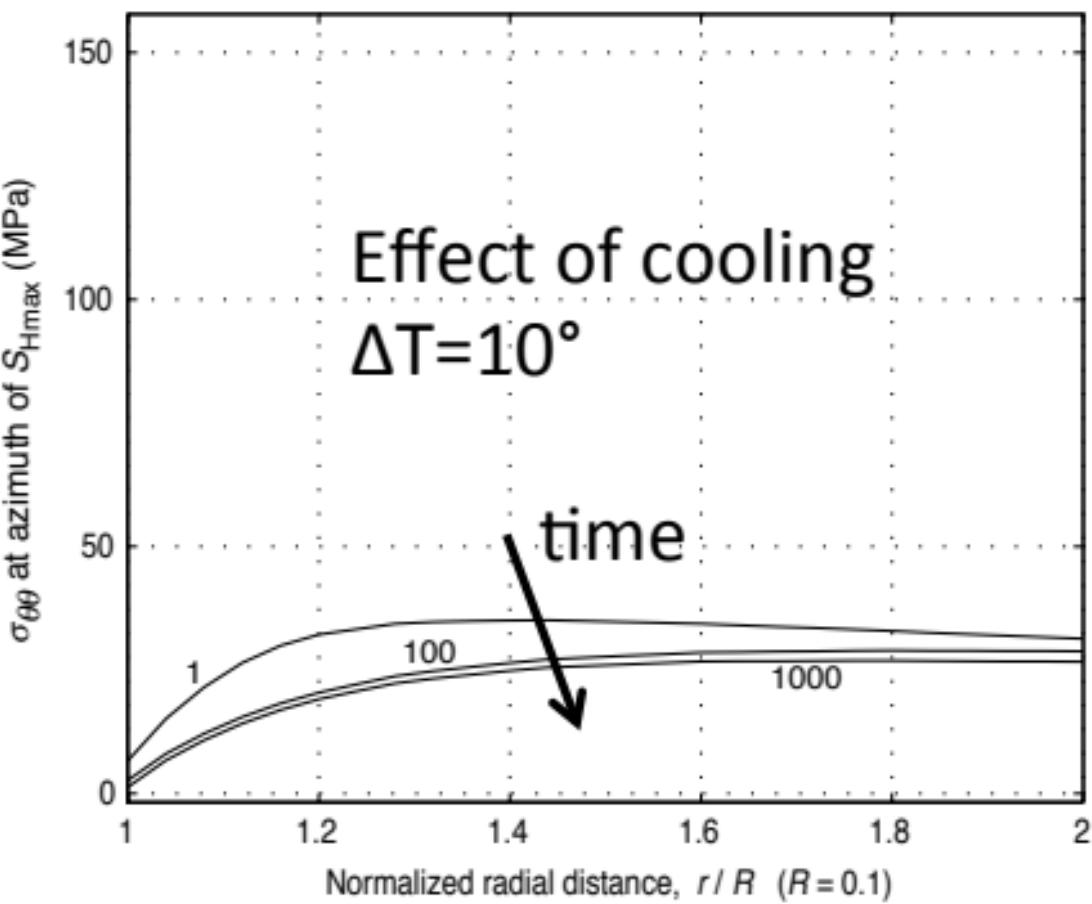
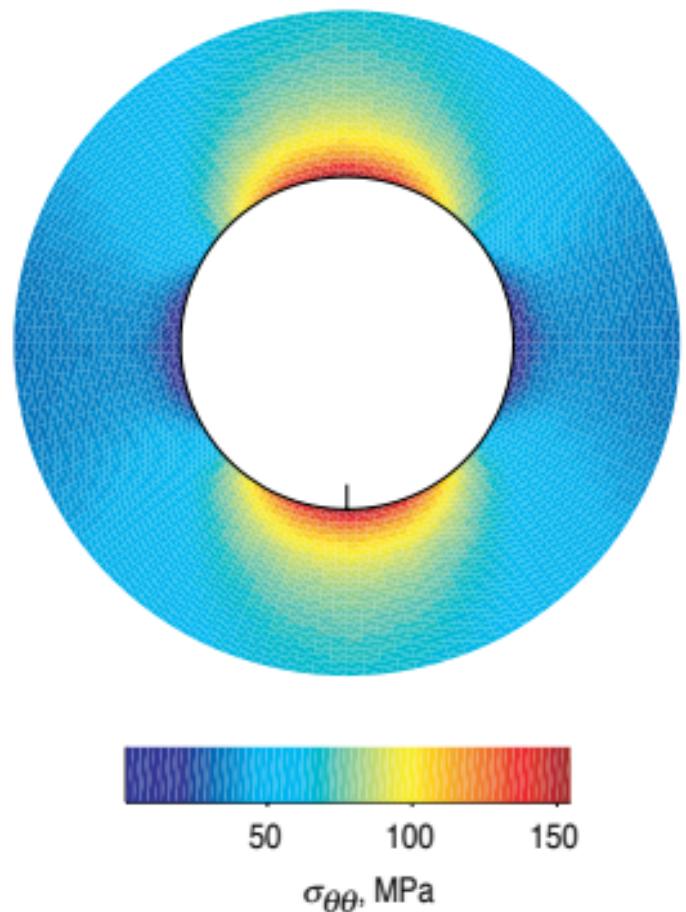








[Zoback 2013 - Figure 6.14]



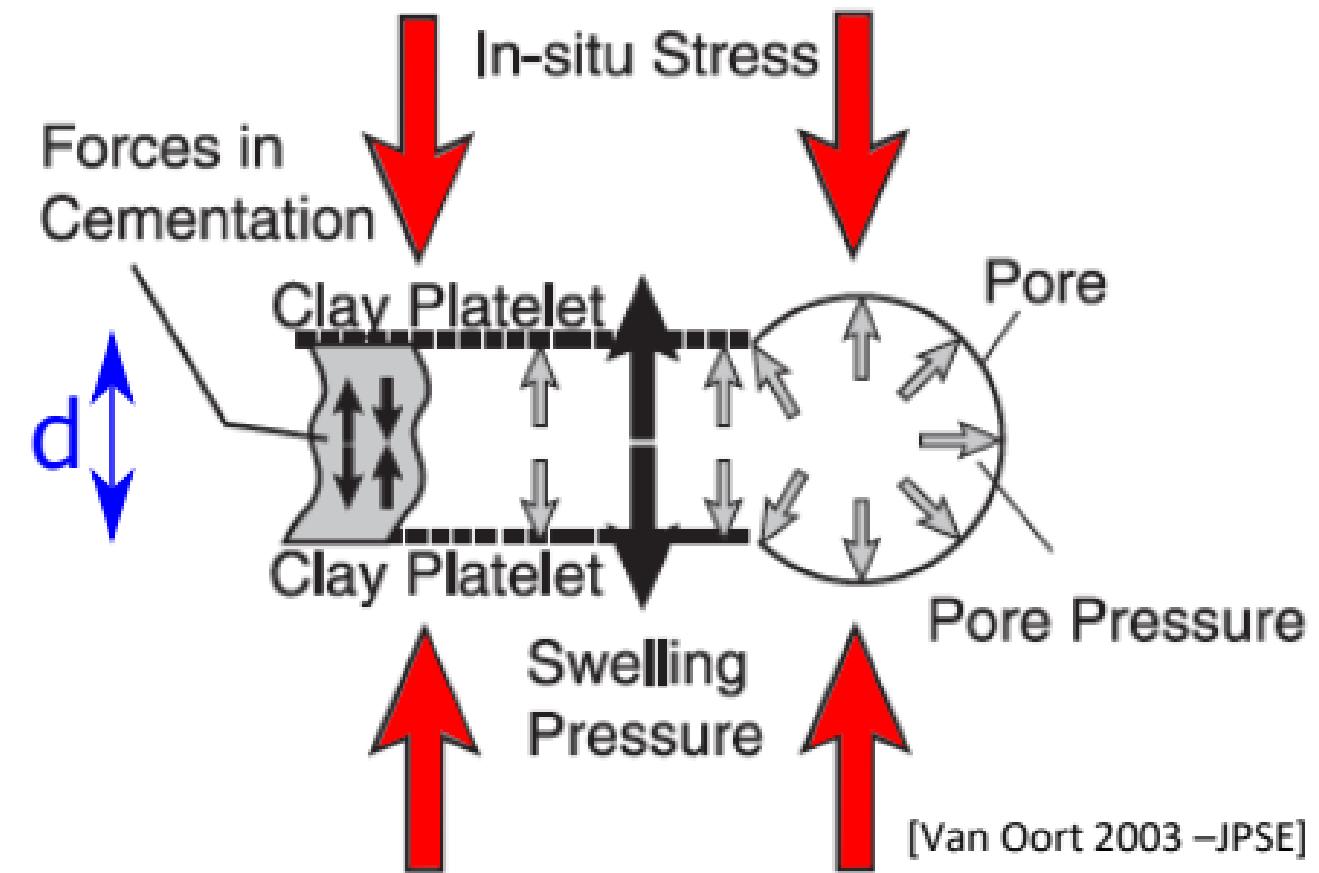
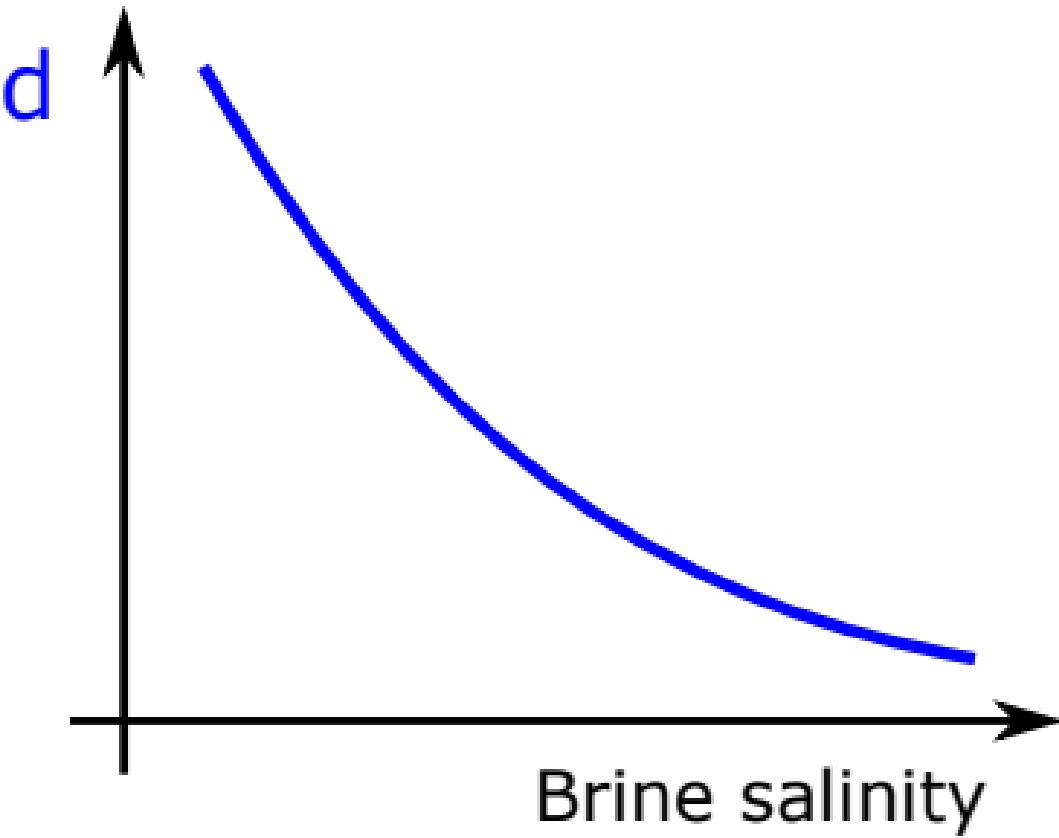


Δ $\sigma_{\theta\theta}$

$=$

$\alpha \Gamma B \Delta T$

$1 - \sqrt{ }$



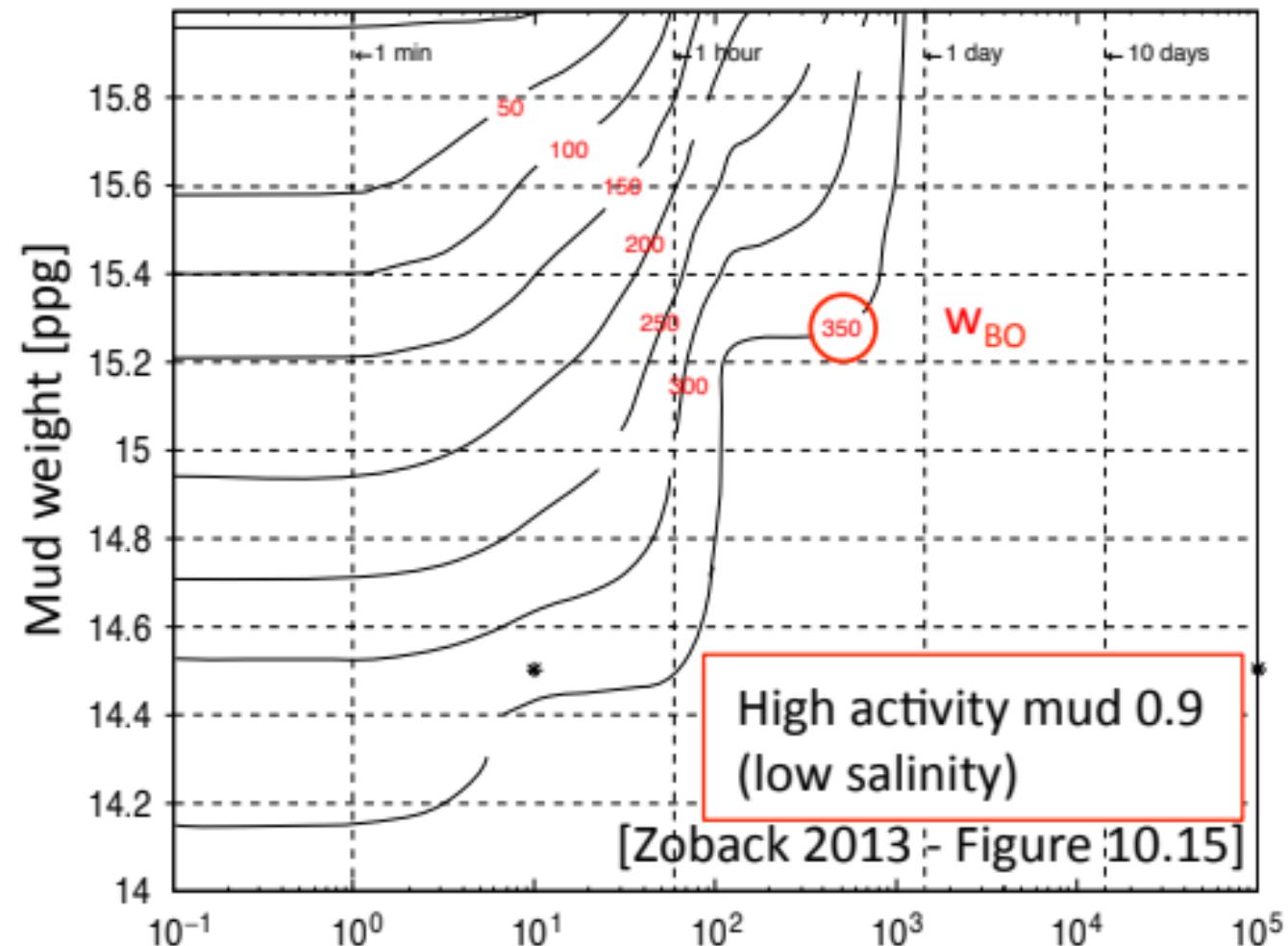
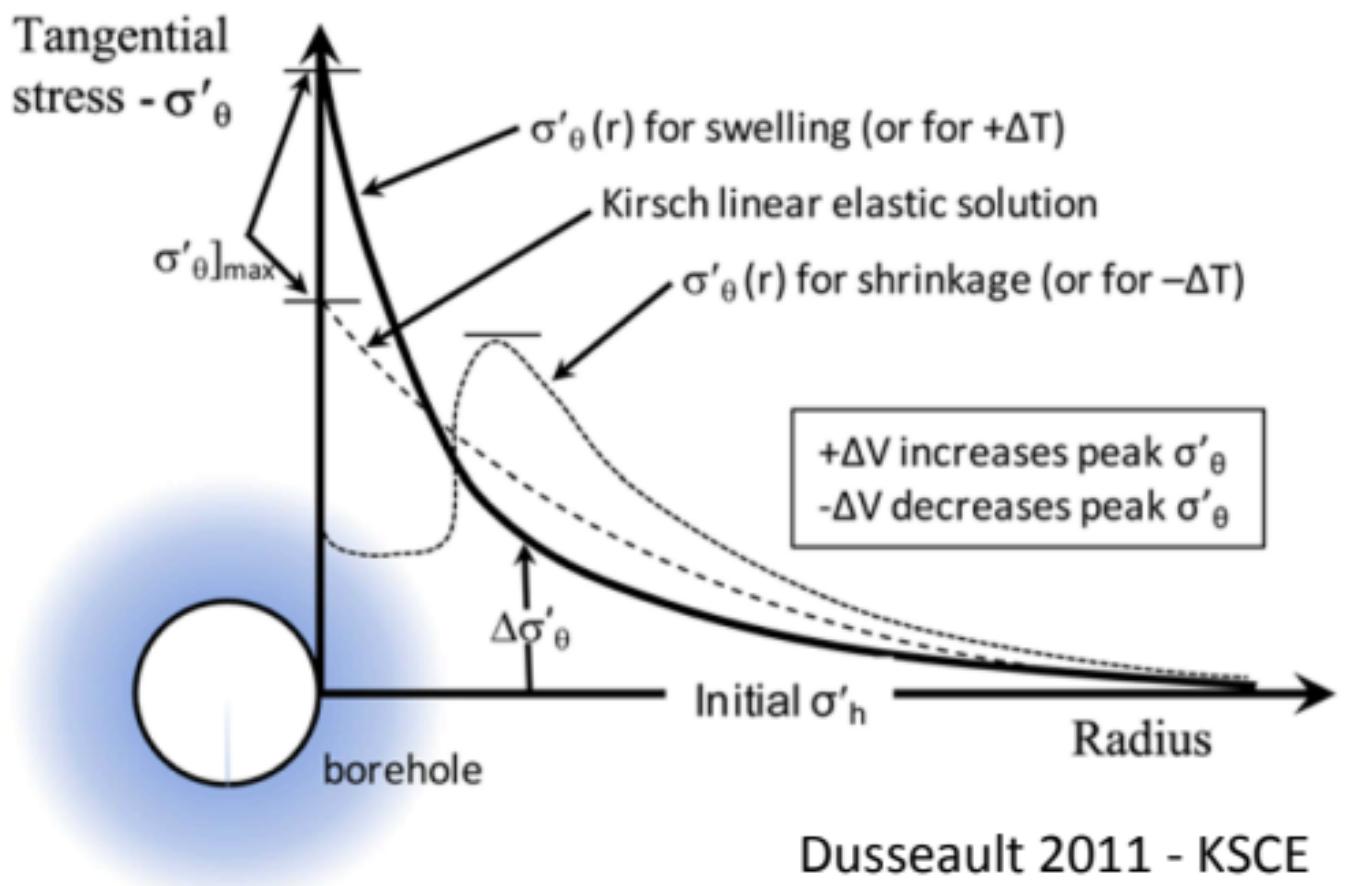
Norway shale

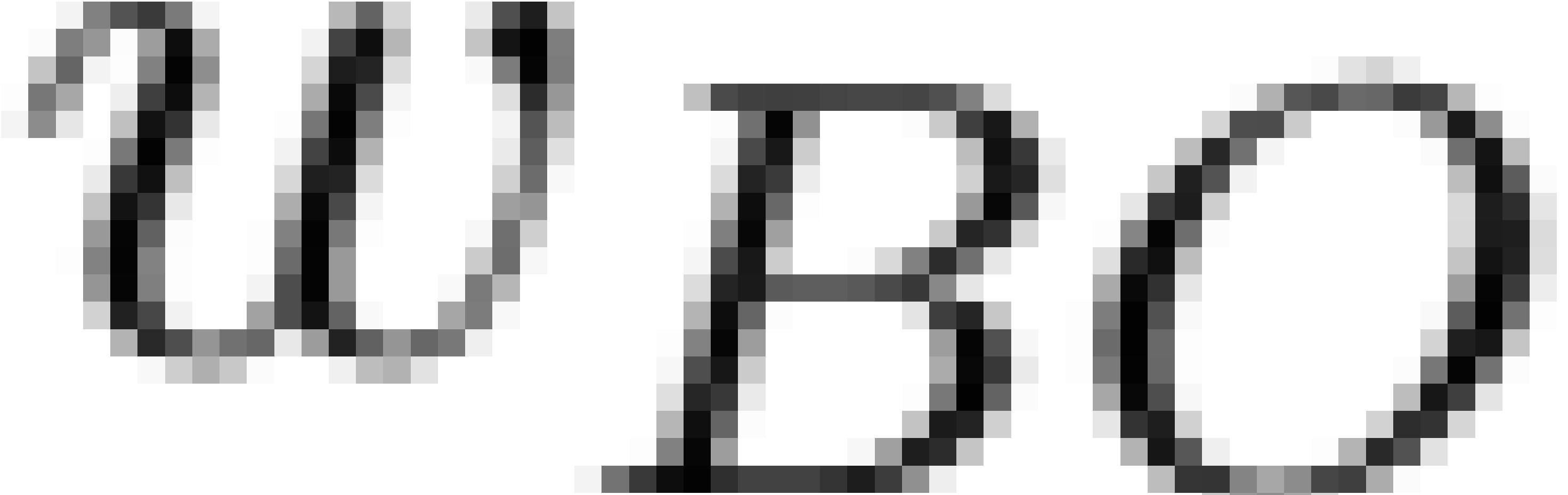


After 2hs exposition

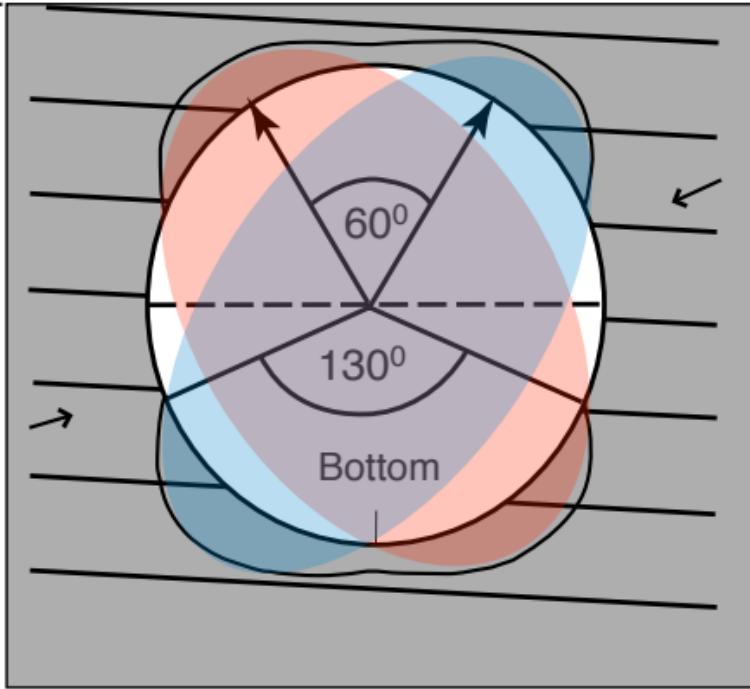


CPGE – M. Chenevert



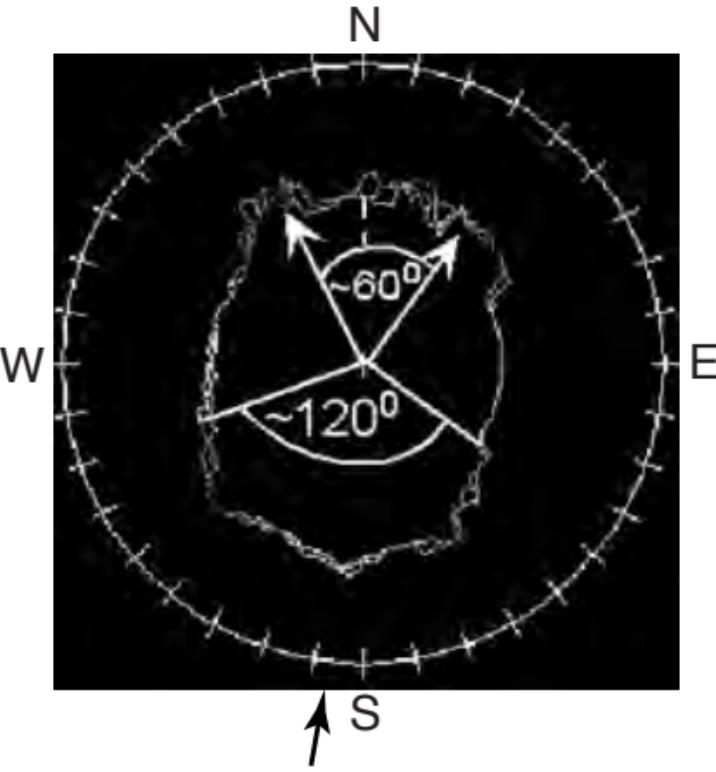


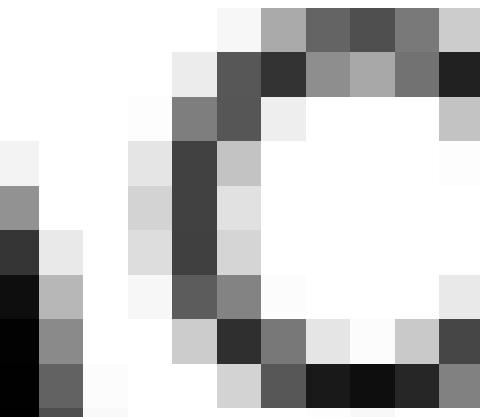
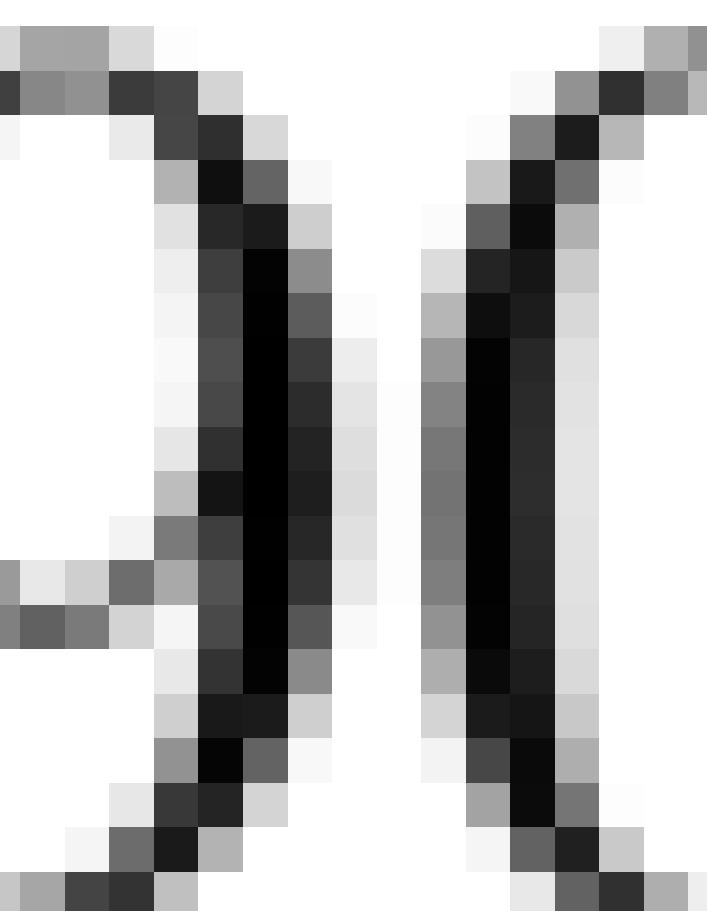
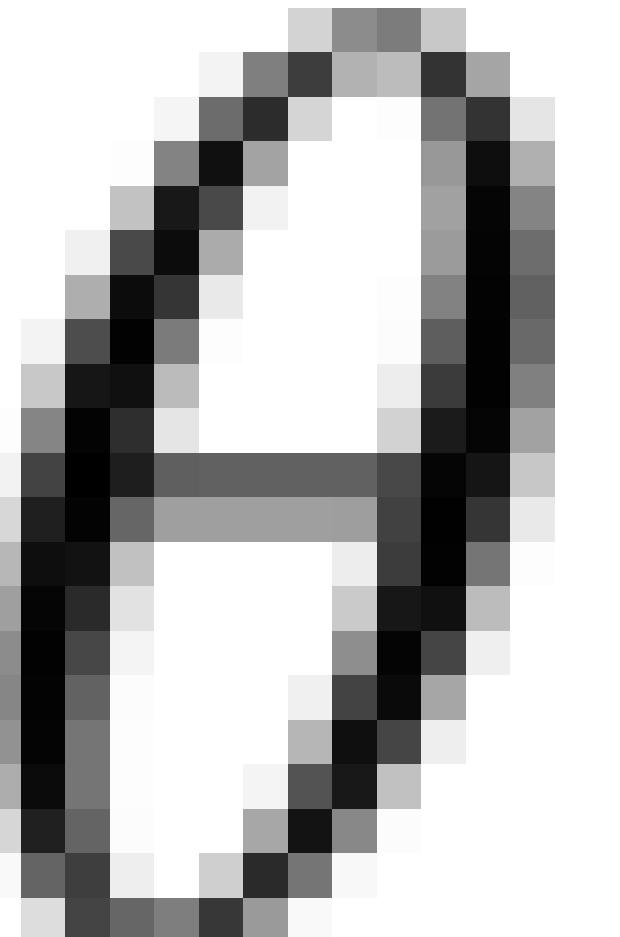


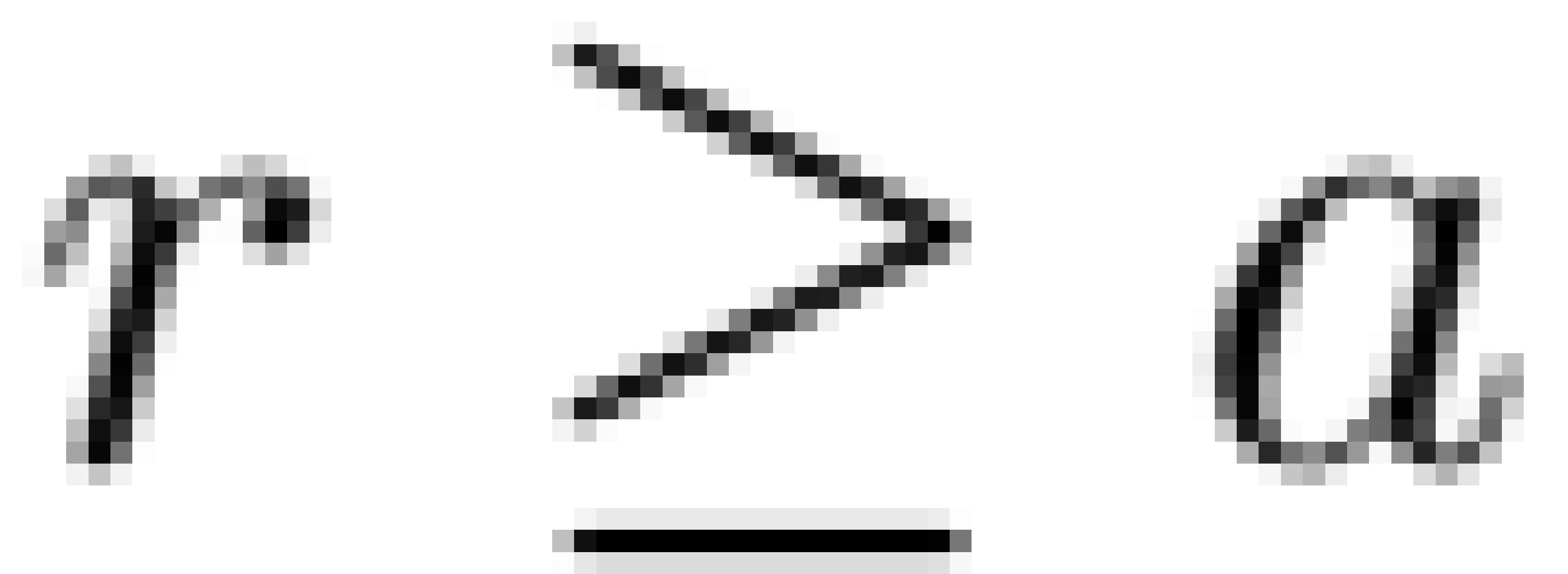


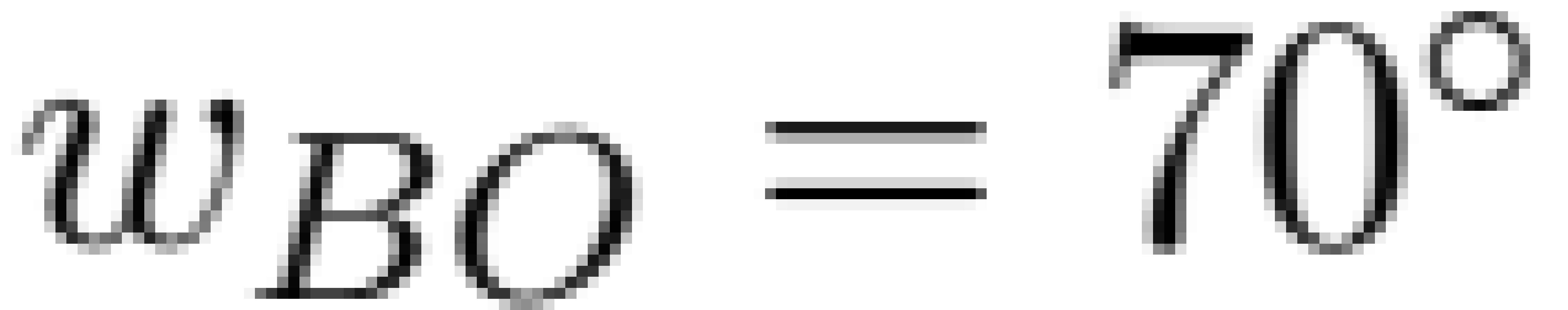
Shear failure intact rock

Shear failure at weak interfaces
(usually weak bedding planes)

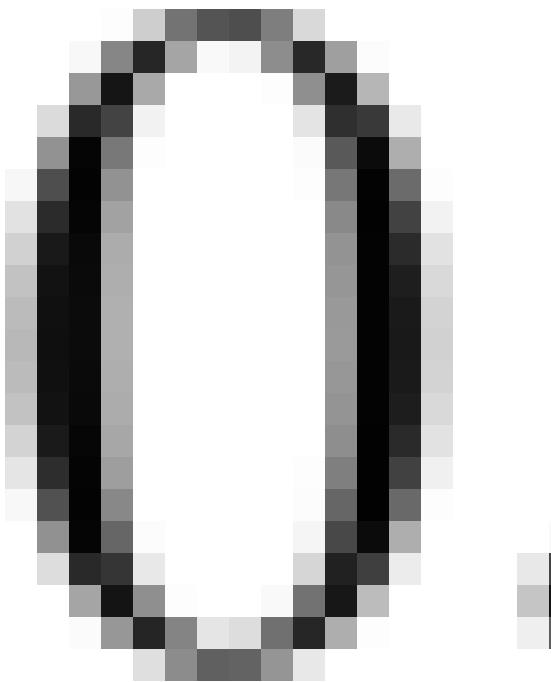


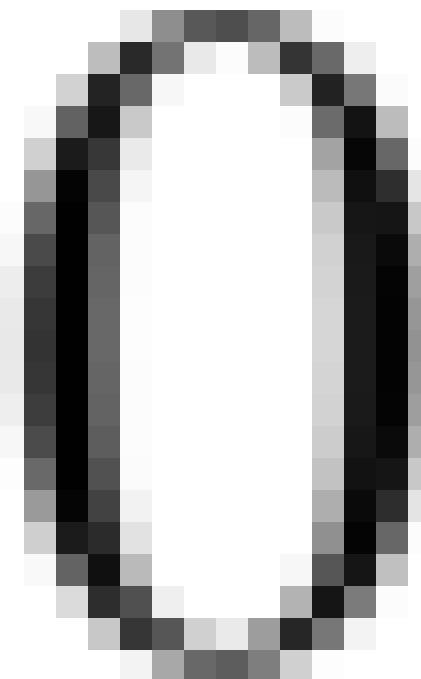
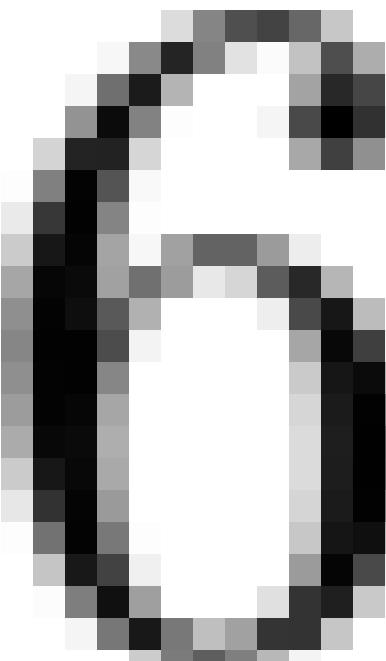
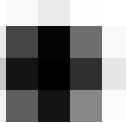
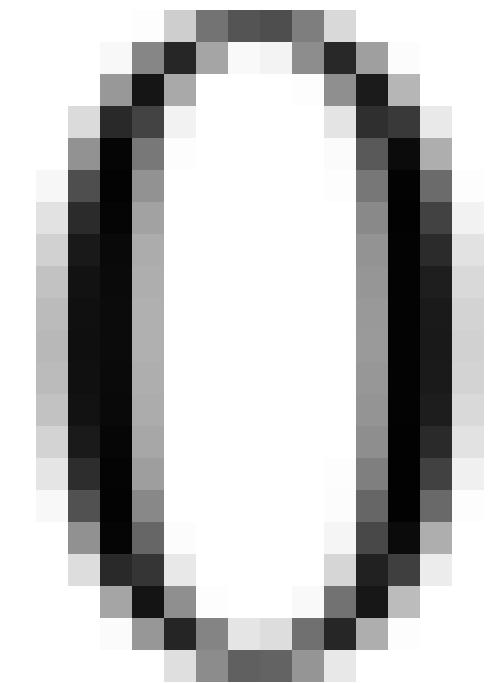


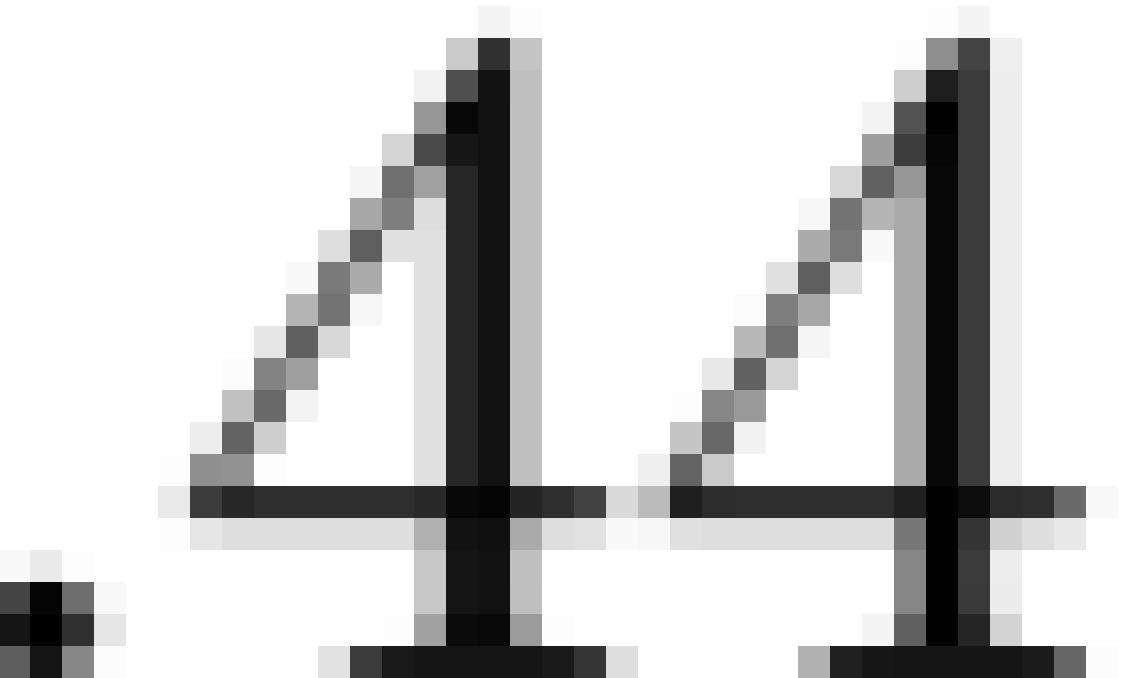
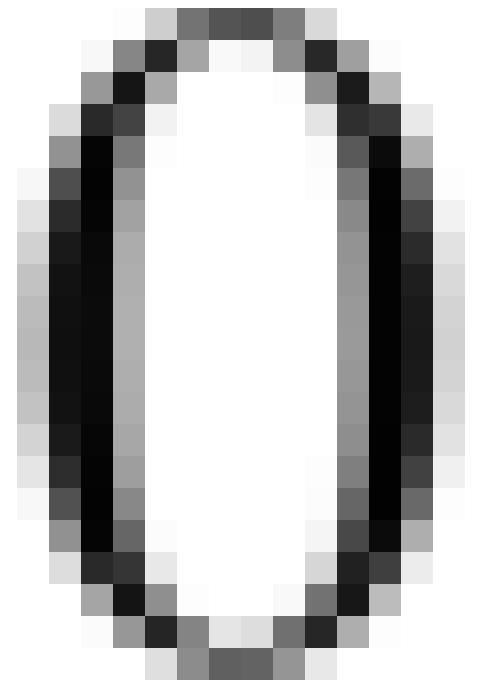


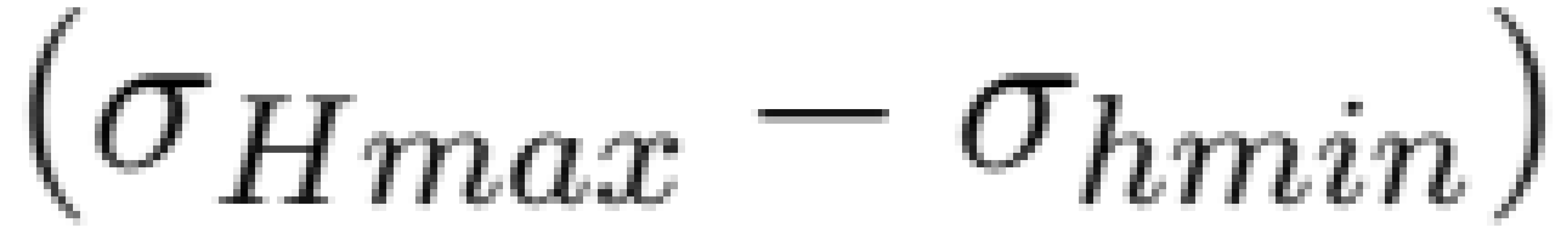


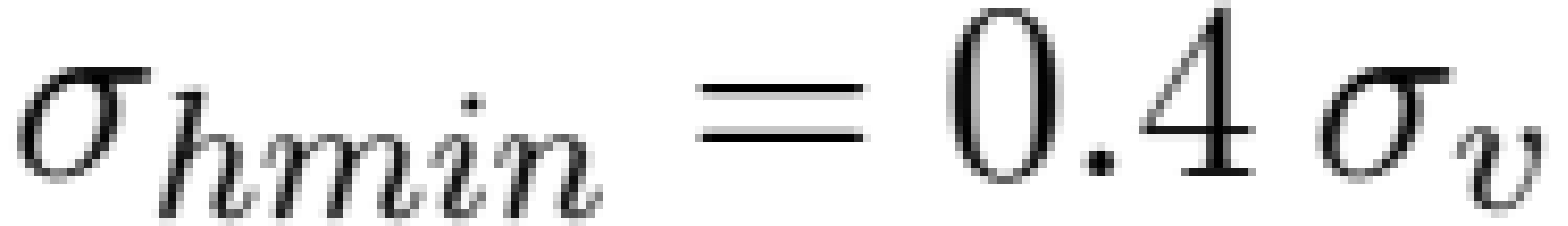


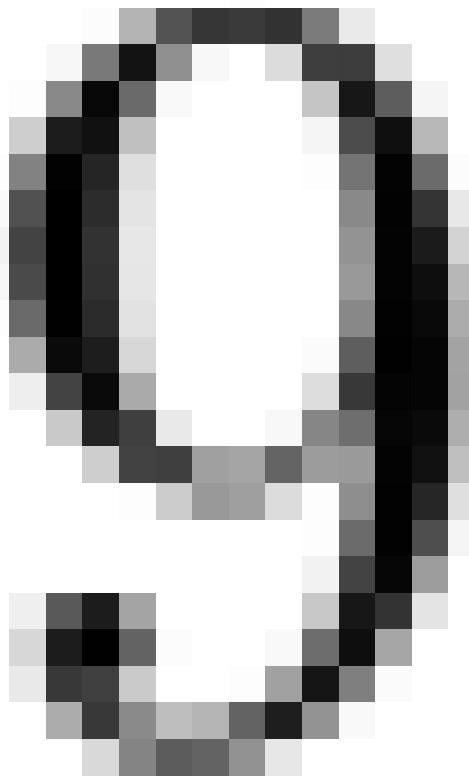
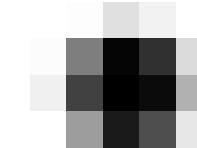
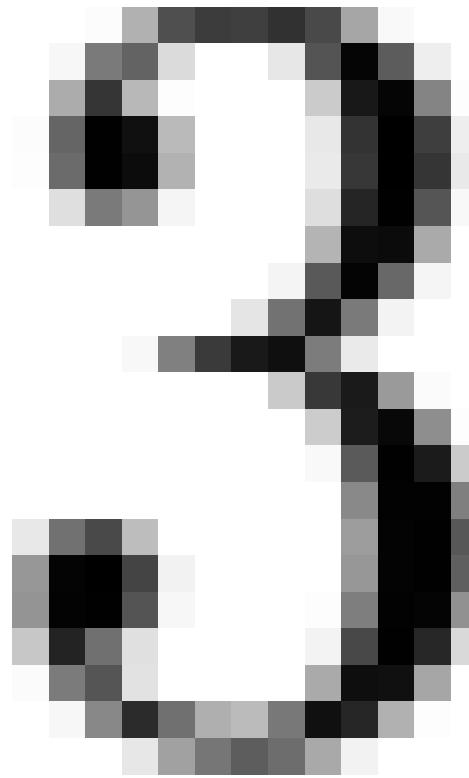
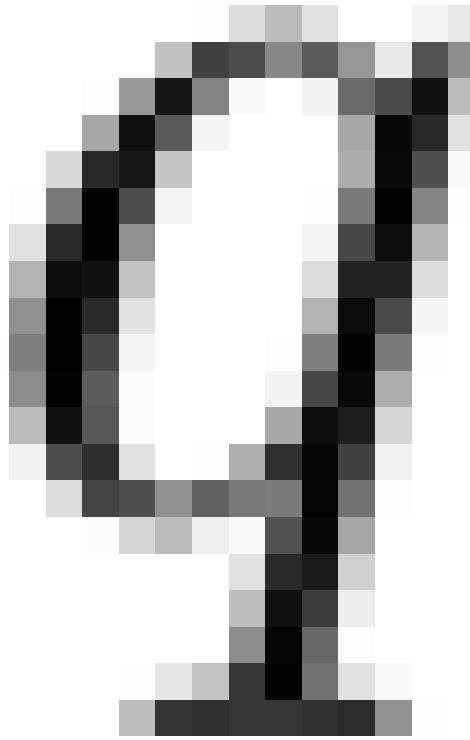


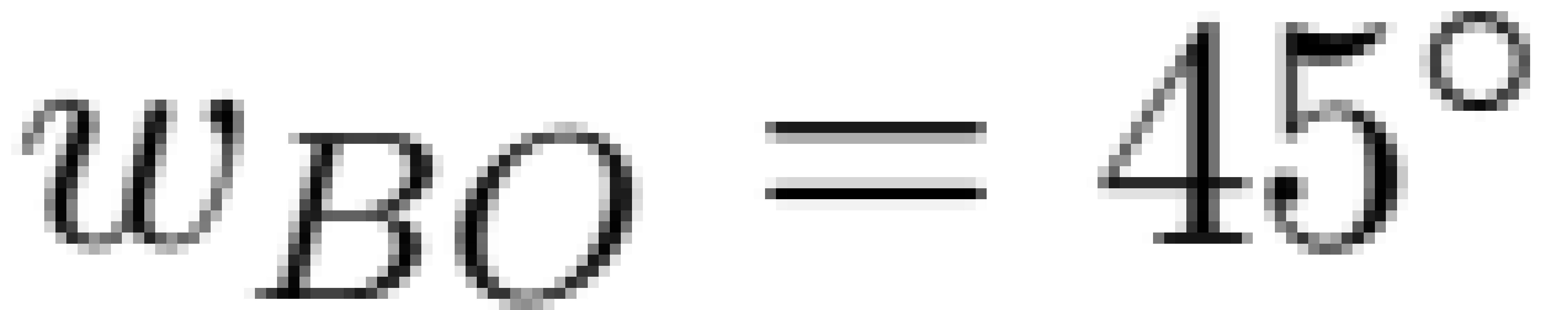








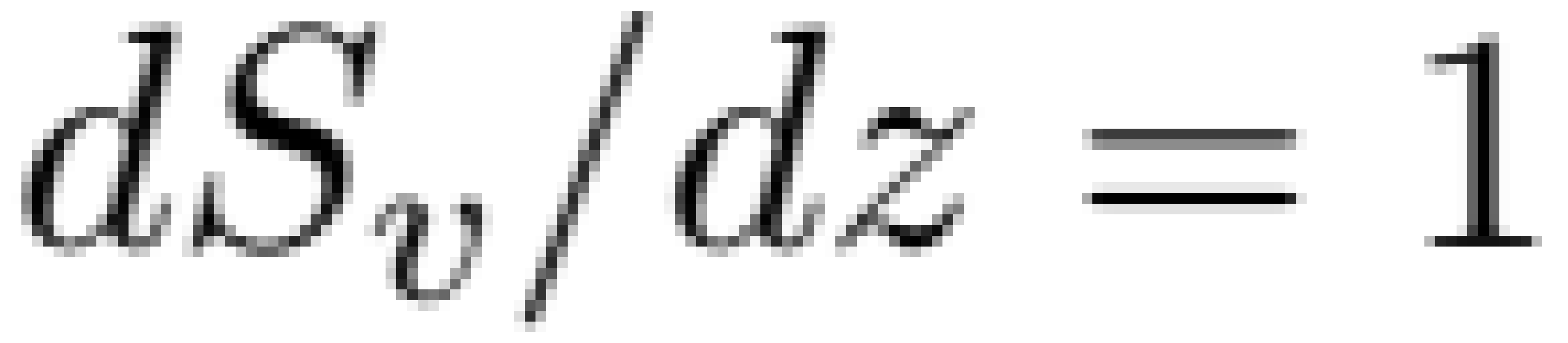




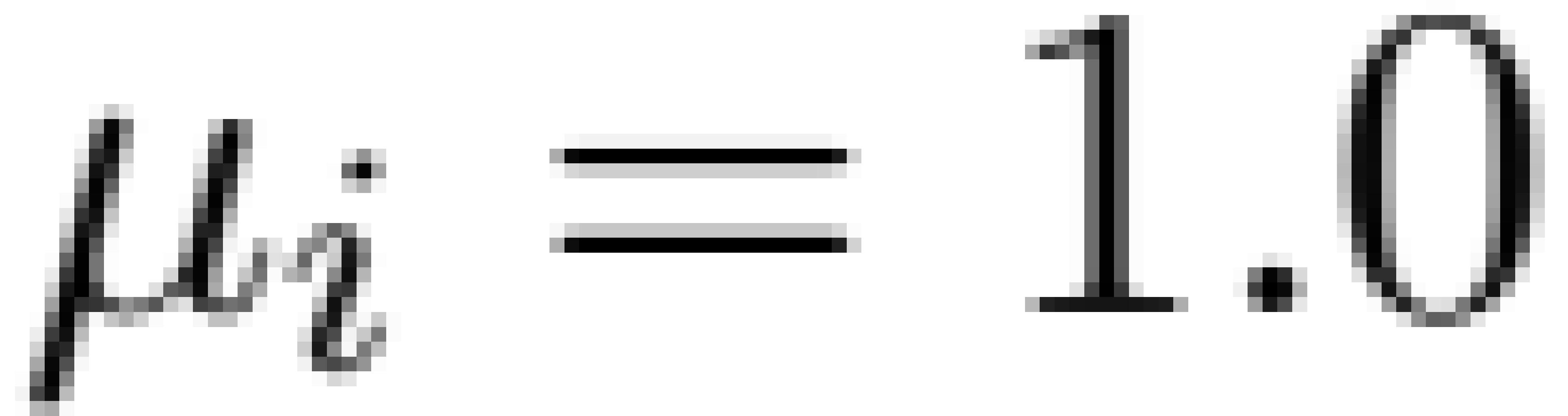




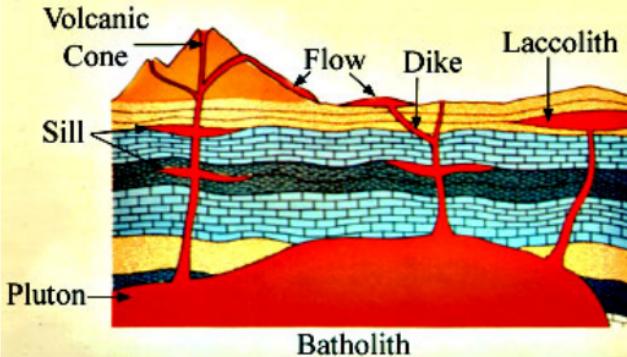




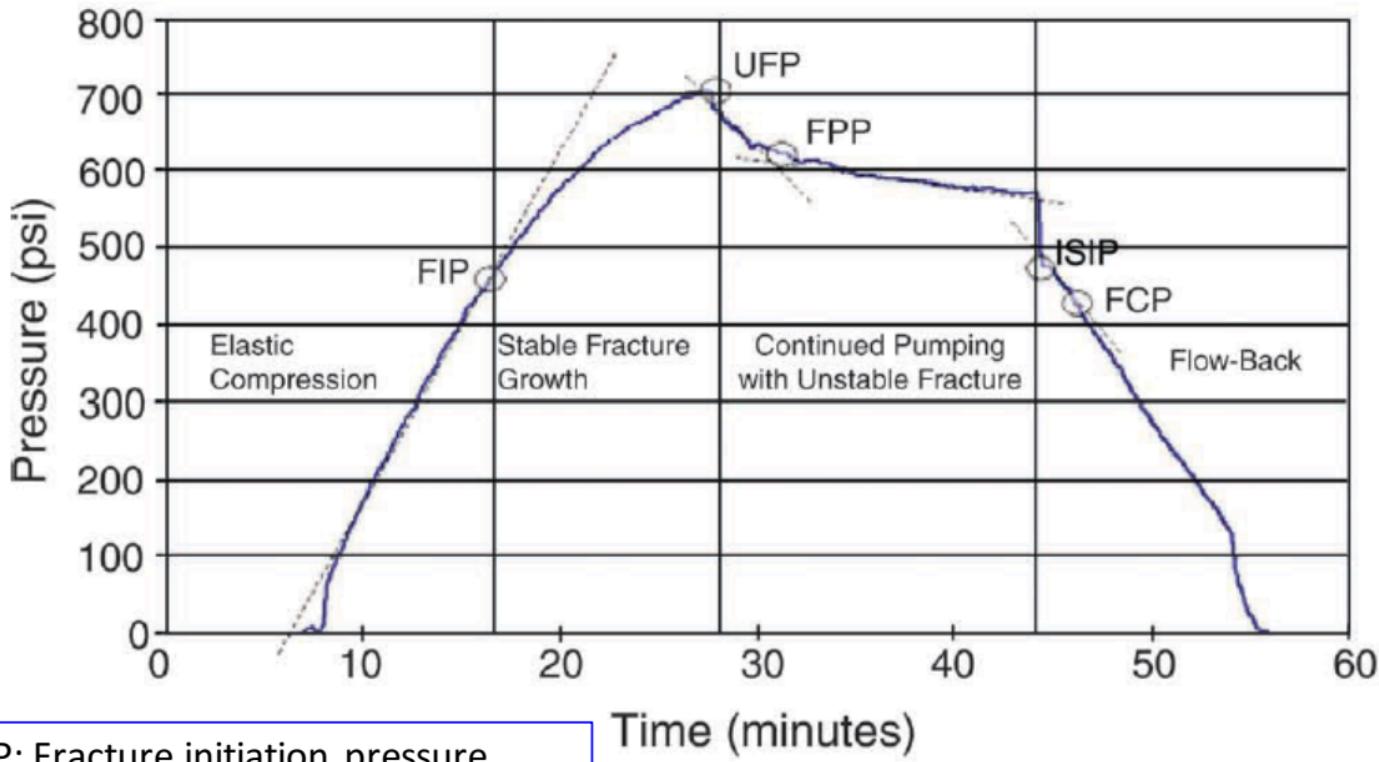




PLUTONS & VOLCANIC LANDFORMS







FIP: Fracture initiation pressure

UFP: Unstable fracture pressure

FPP: Fracture propagation pressure

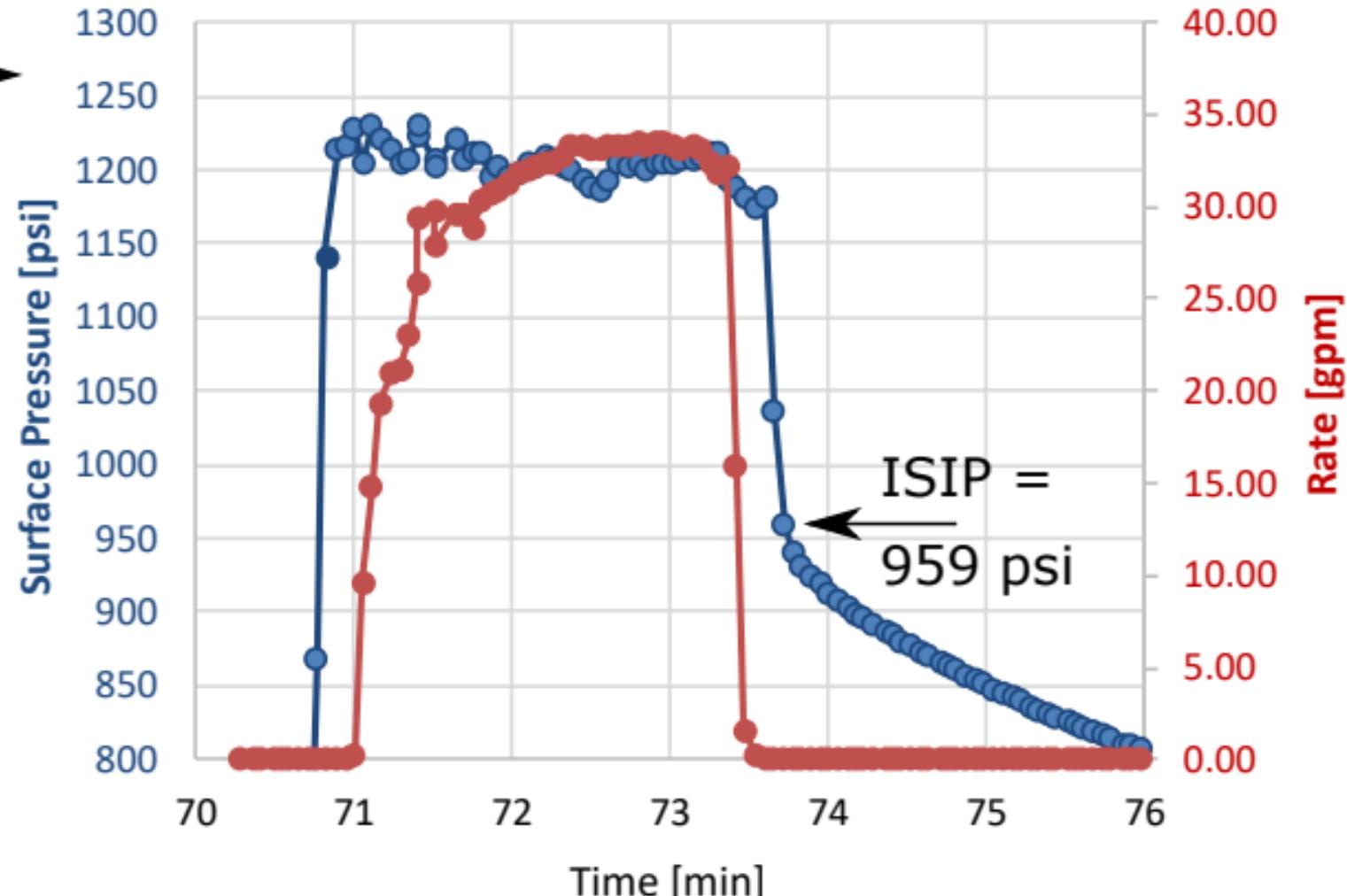
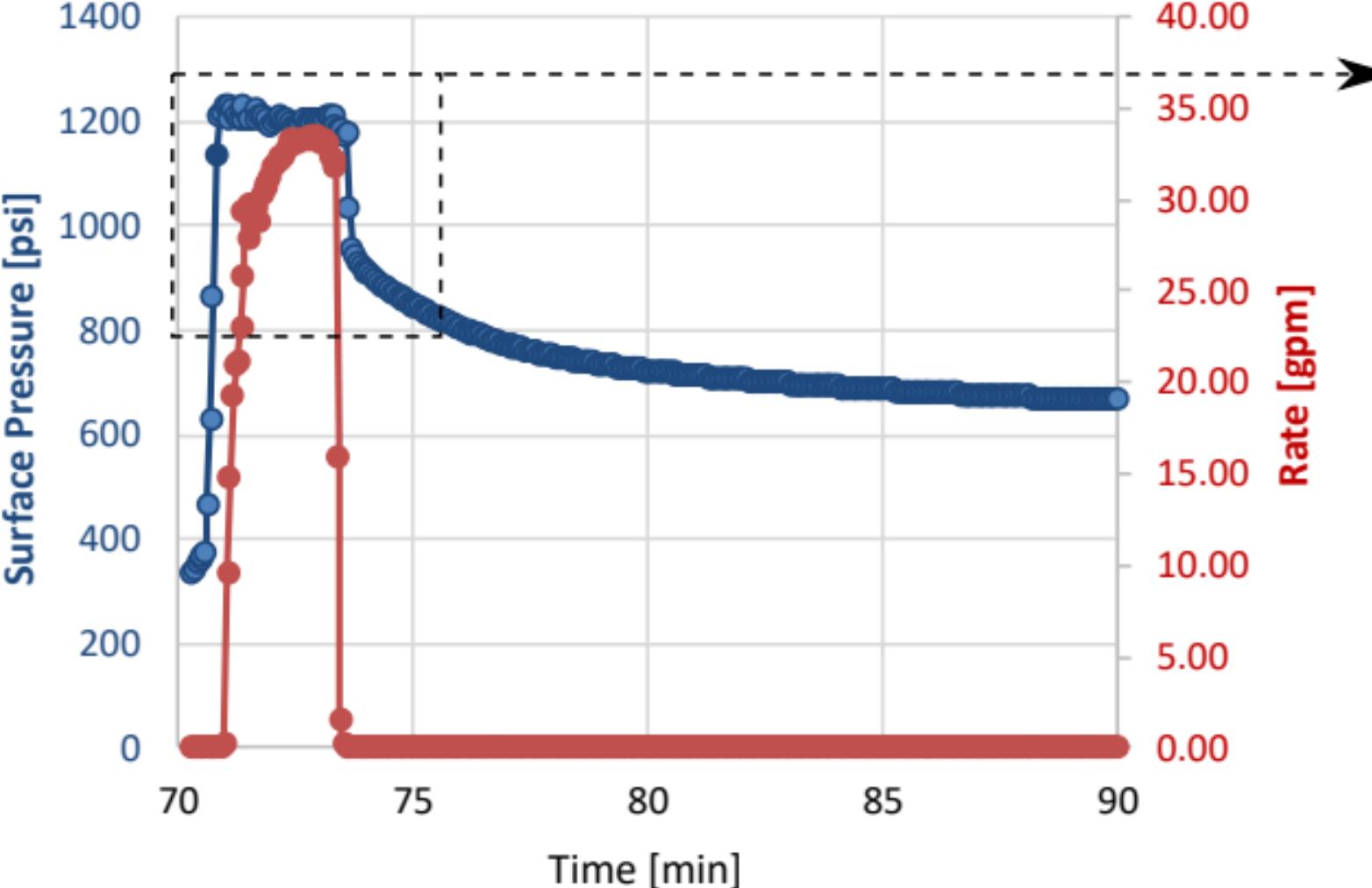
ISIP: Instantaneous shut-in pressure

FCP: Fracture closure pressure

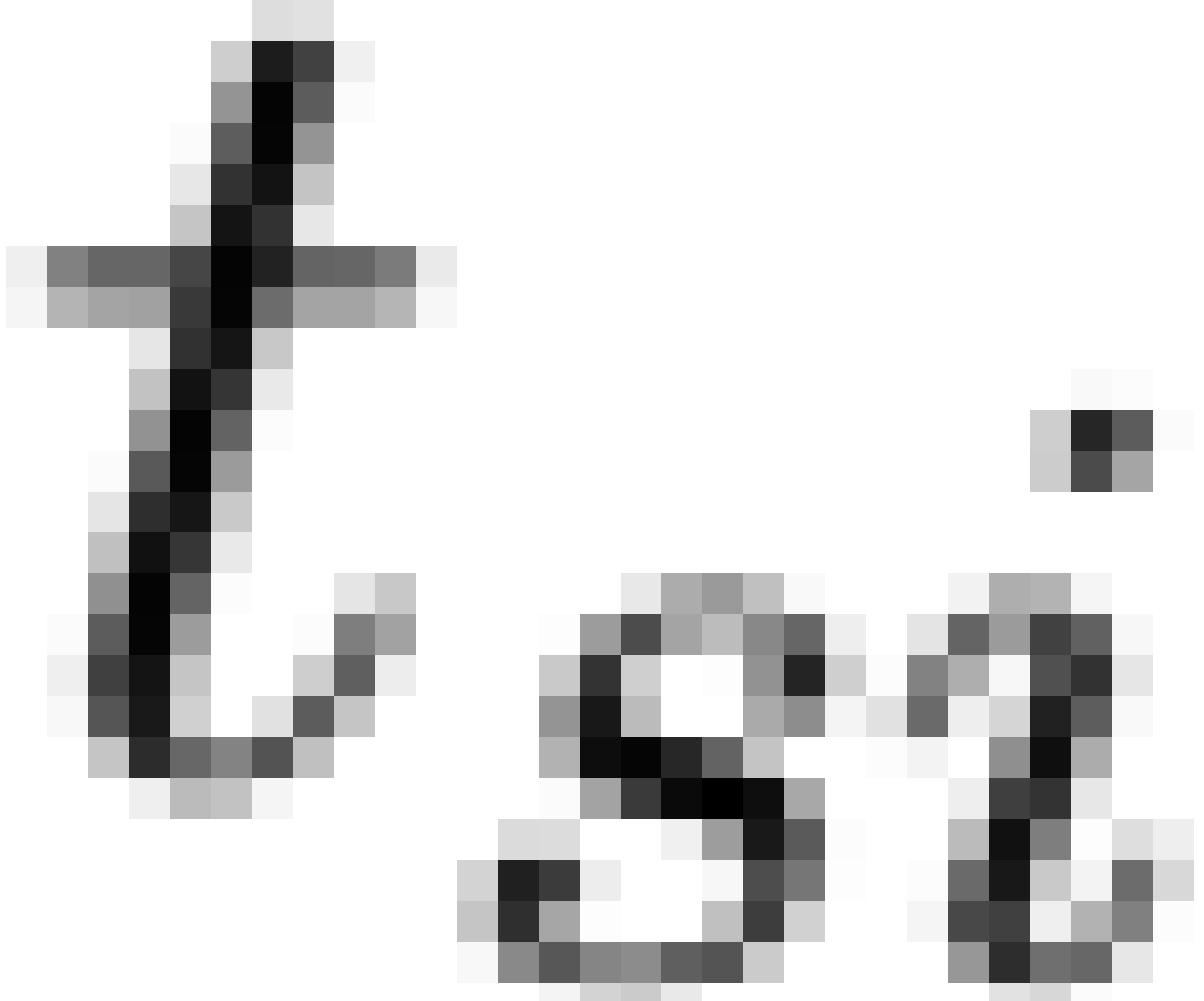
Time (minutes)

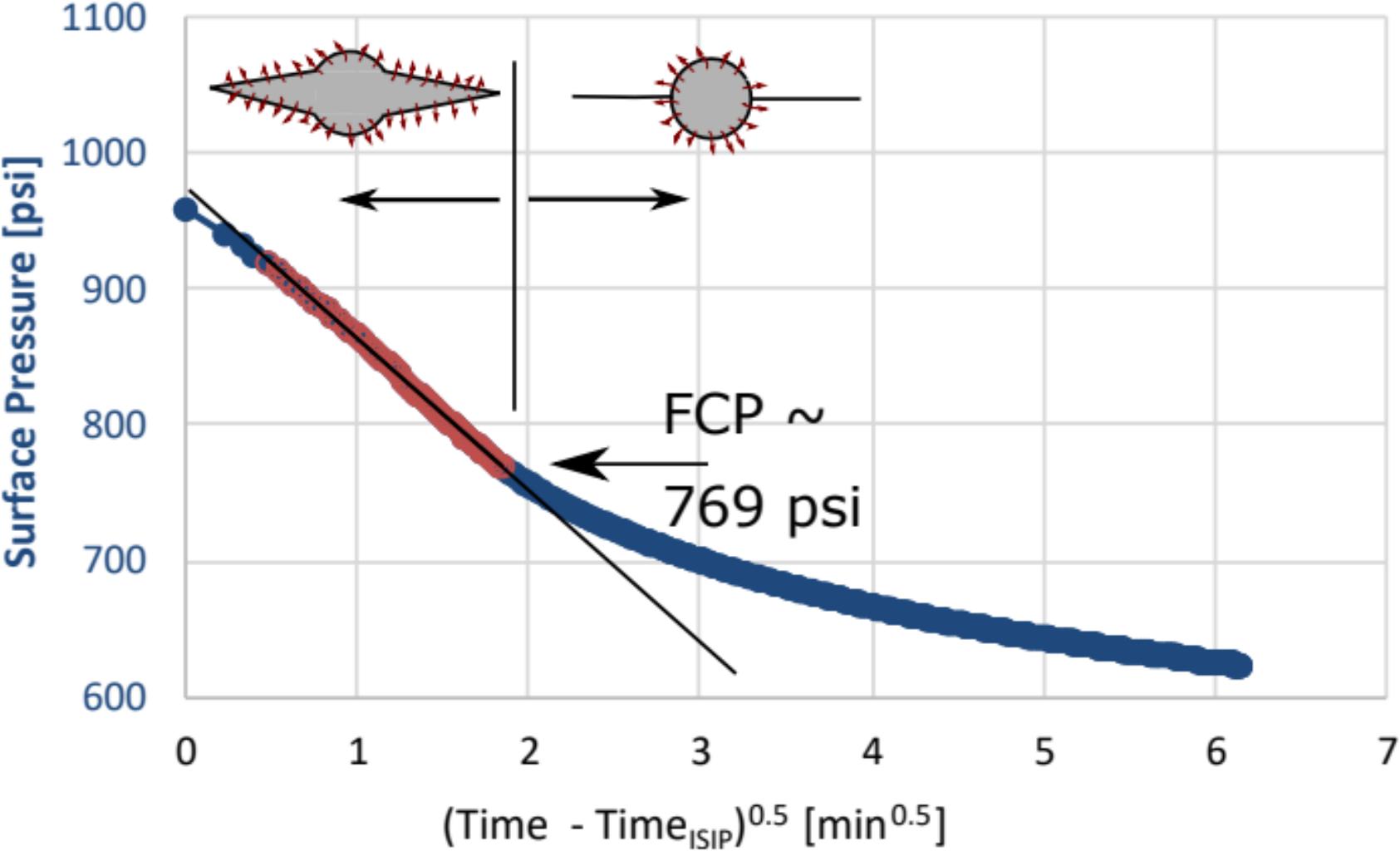
$$\Delta V = q \Delta t$$

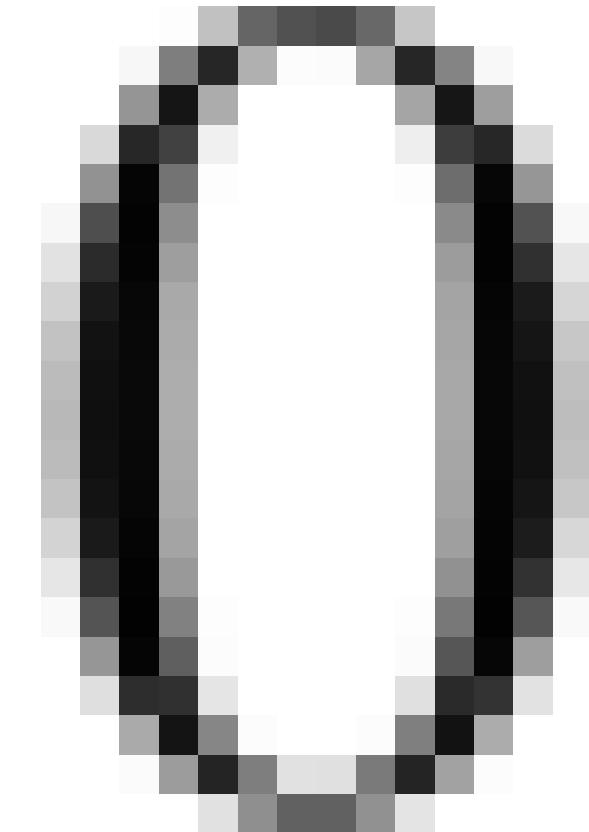
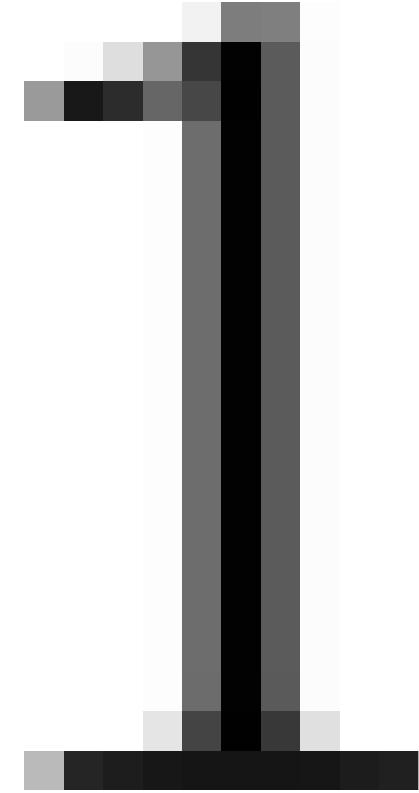
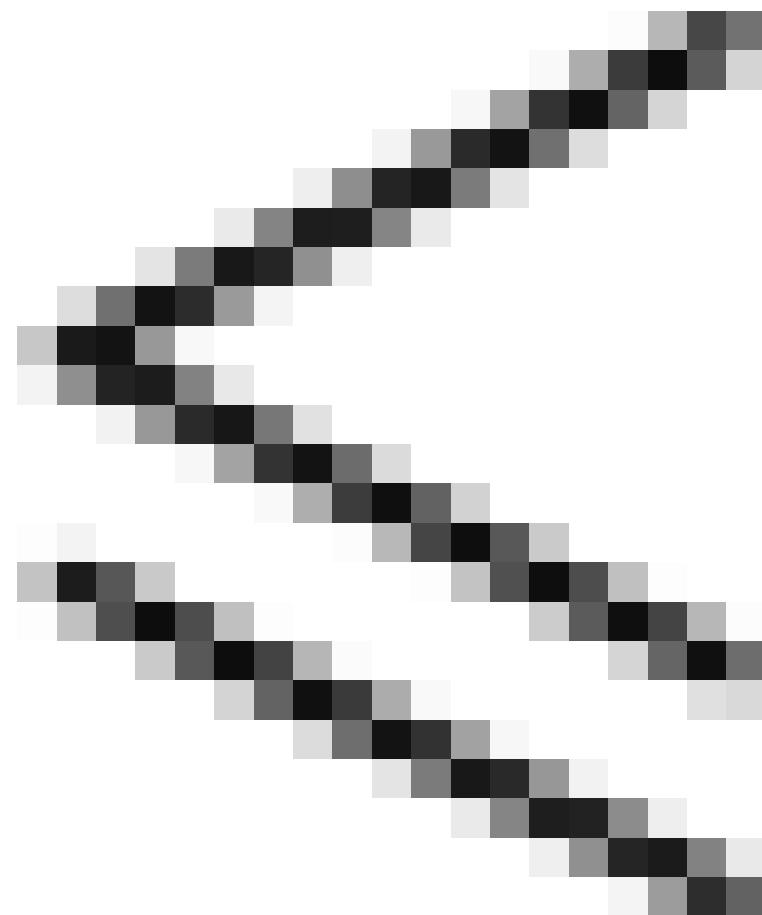
[van Oort and Vargo, 2008]

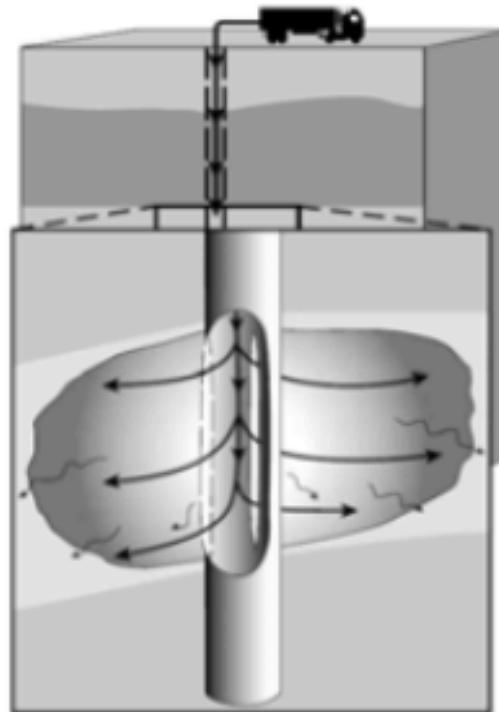




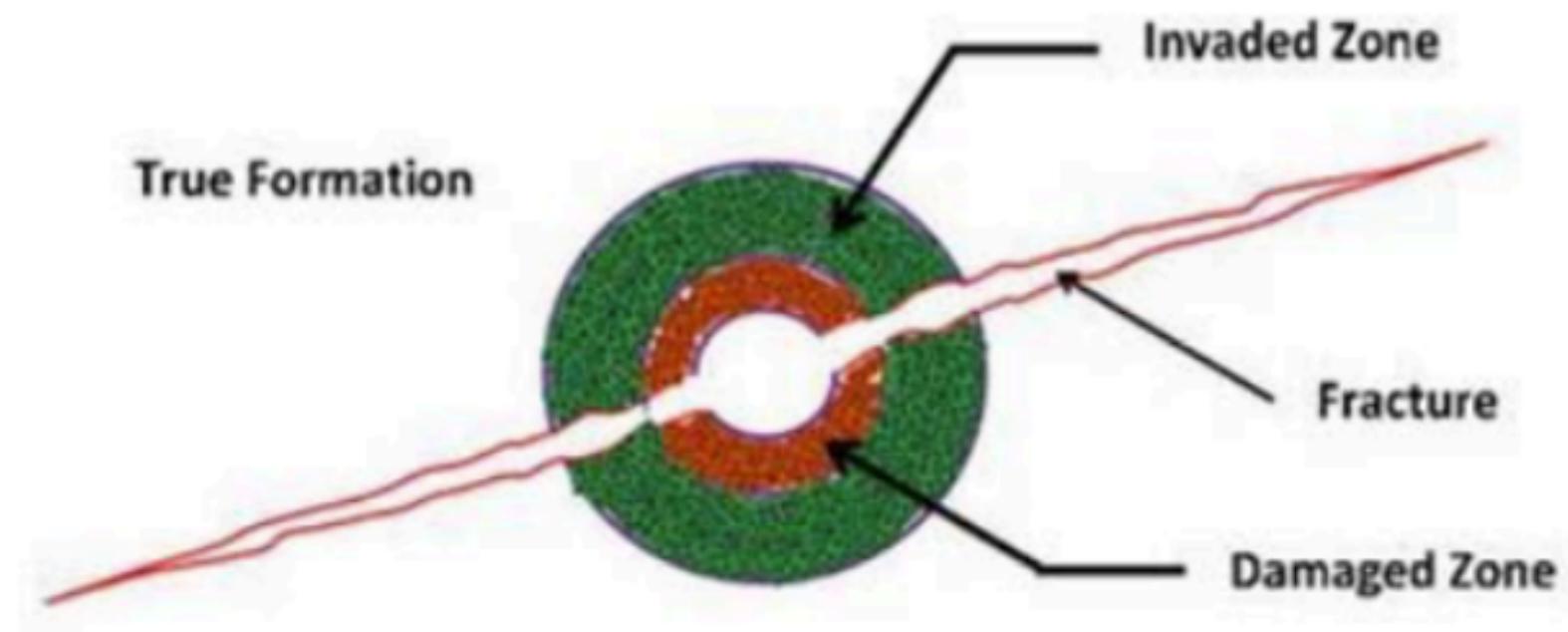








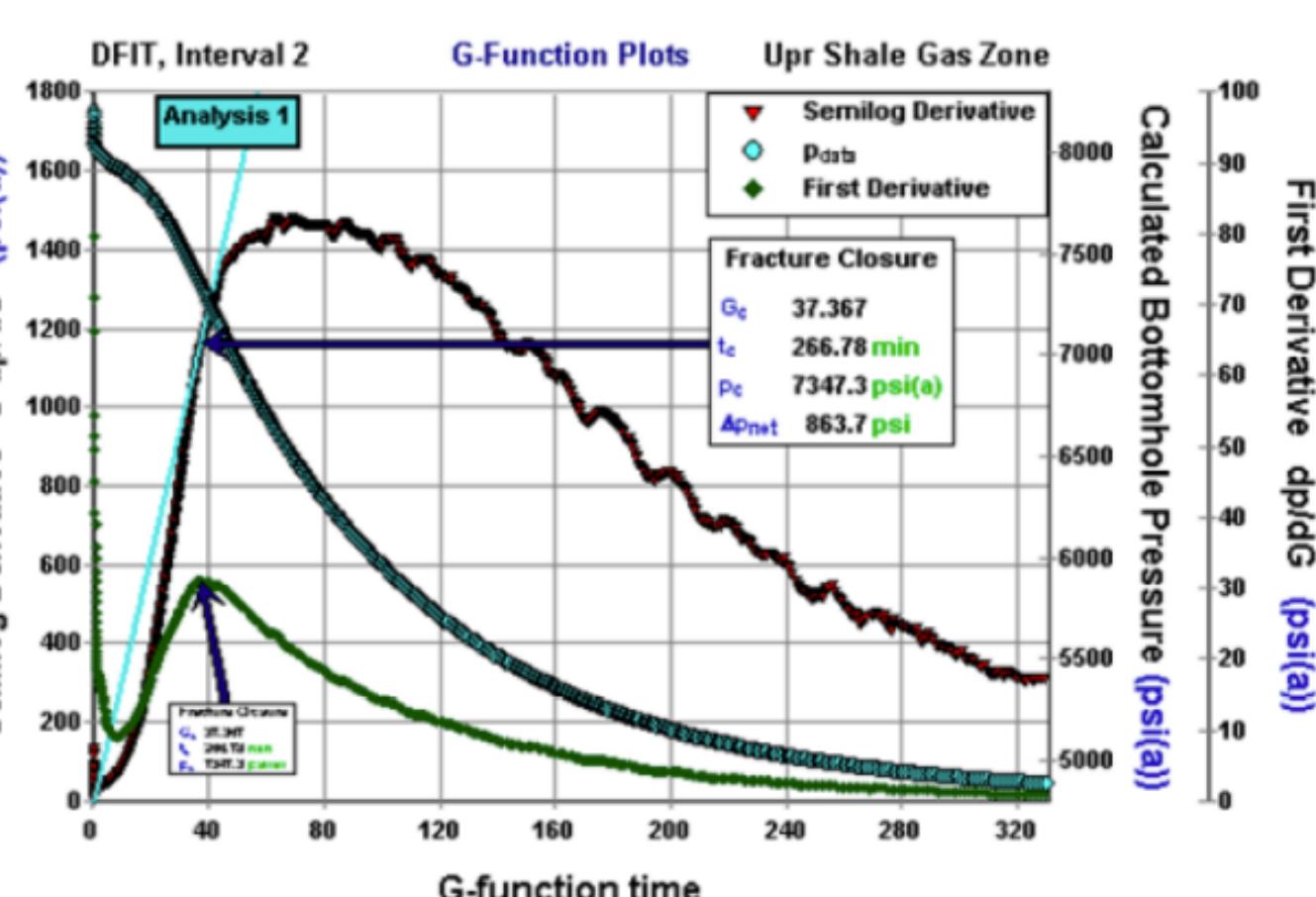
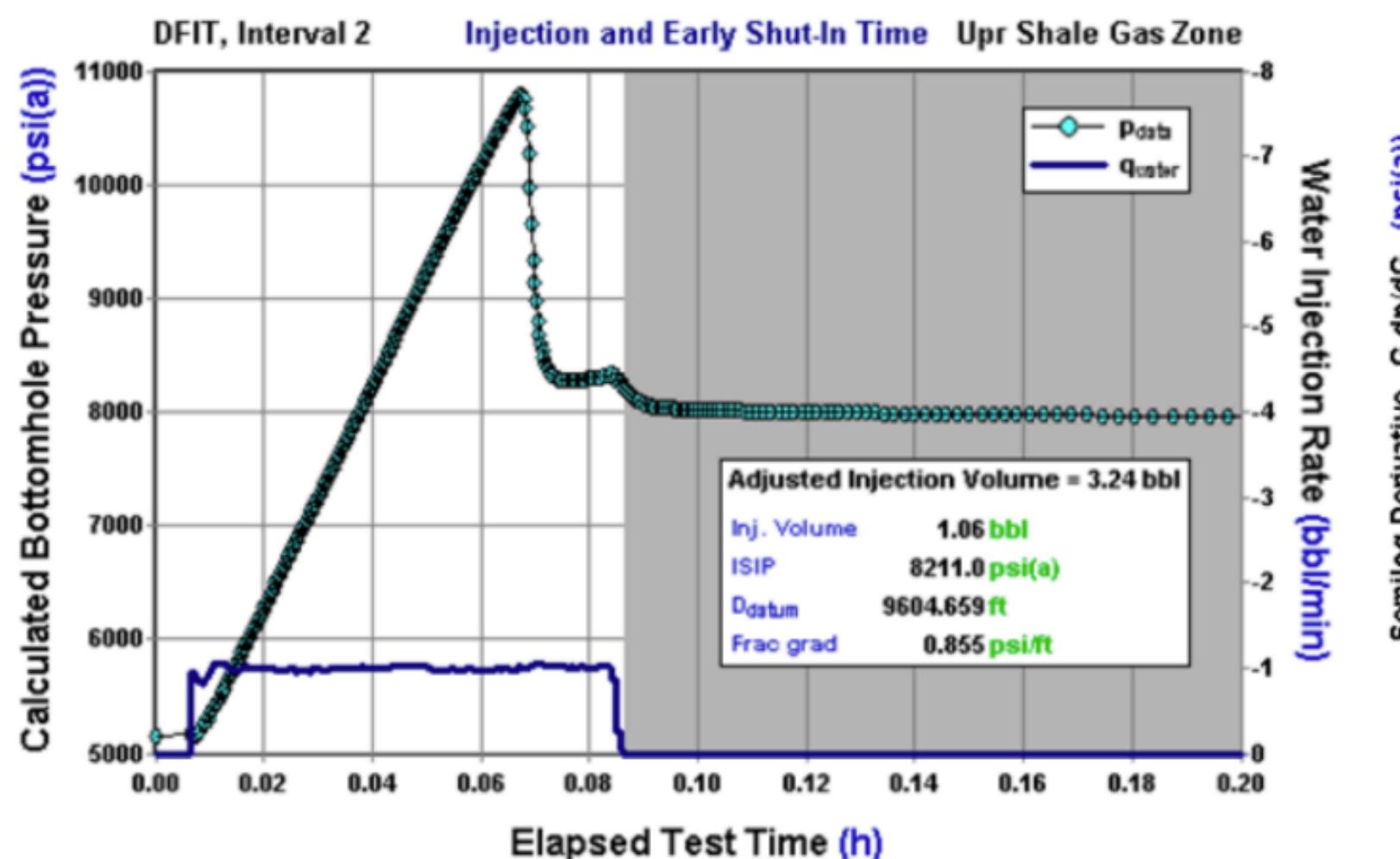
Side View



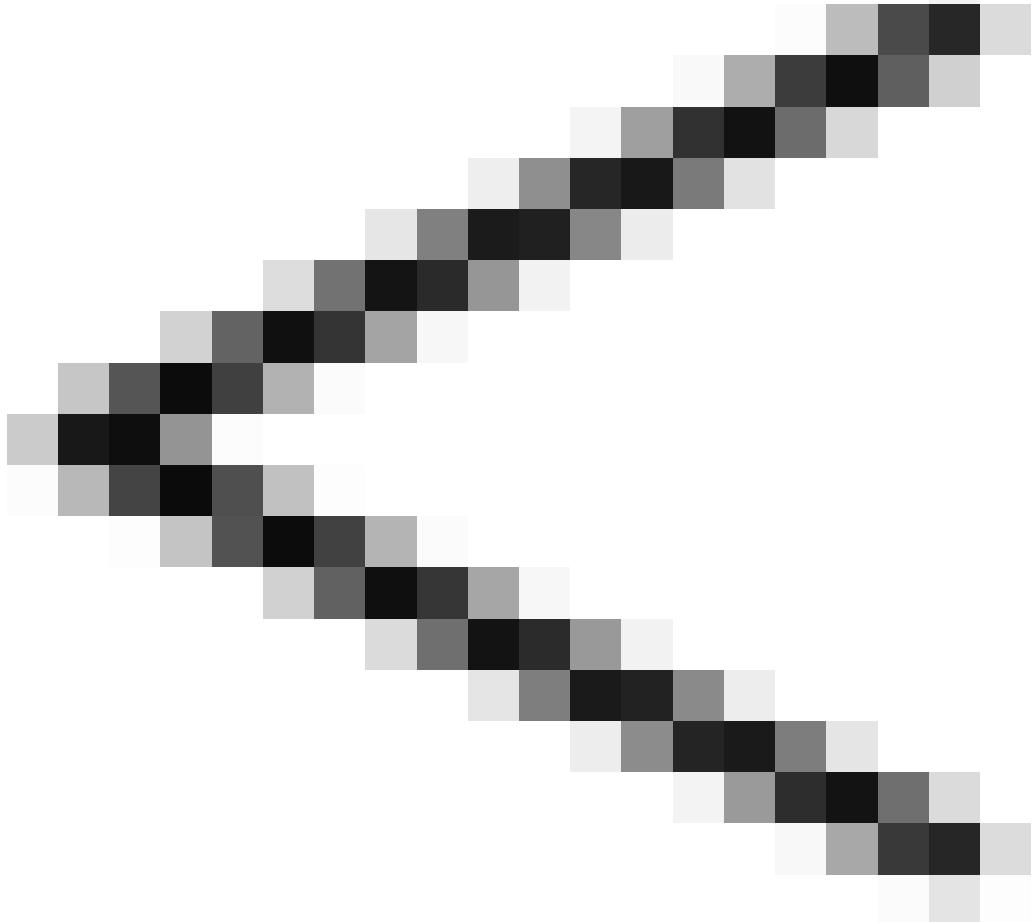
Plan View

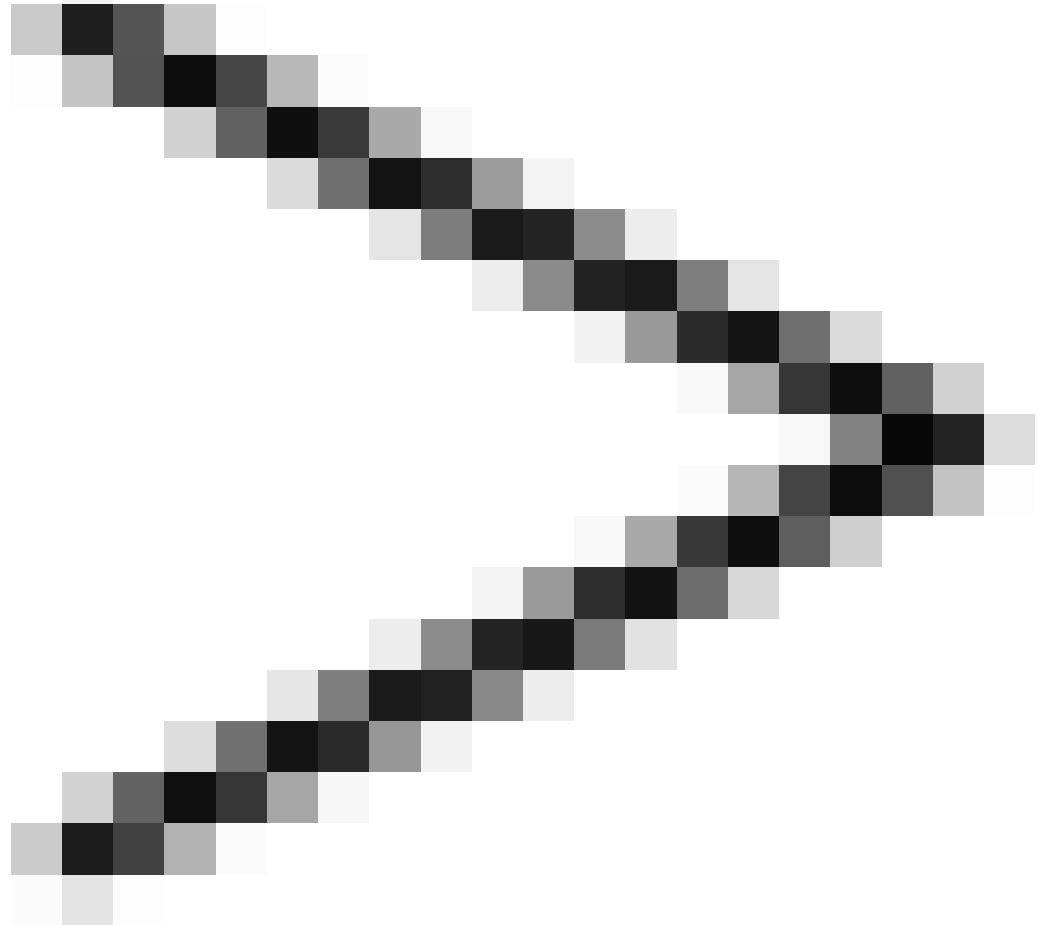
Fekete, 2011

[SPE 163863]



[SPE 163863]





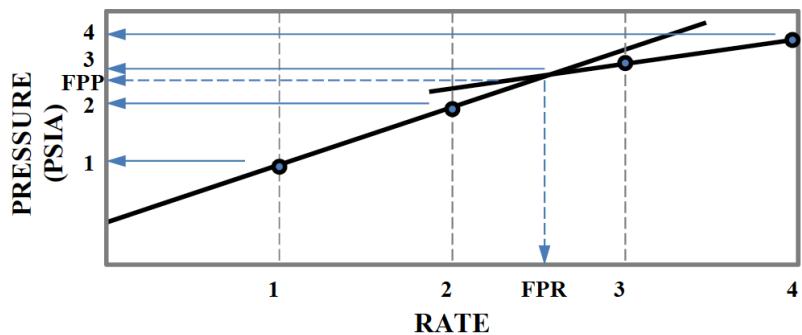
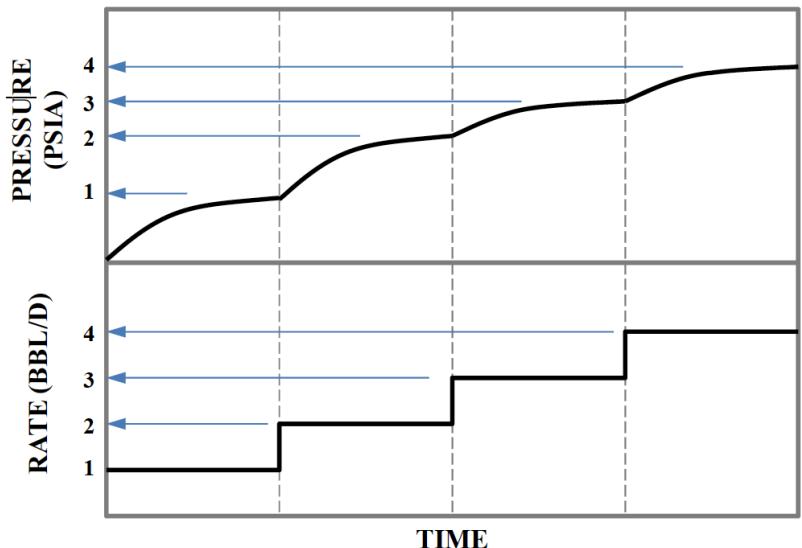
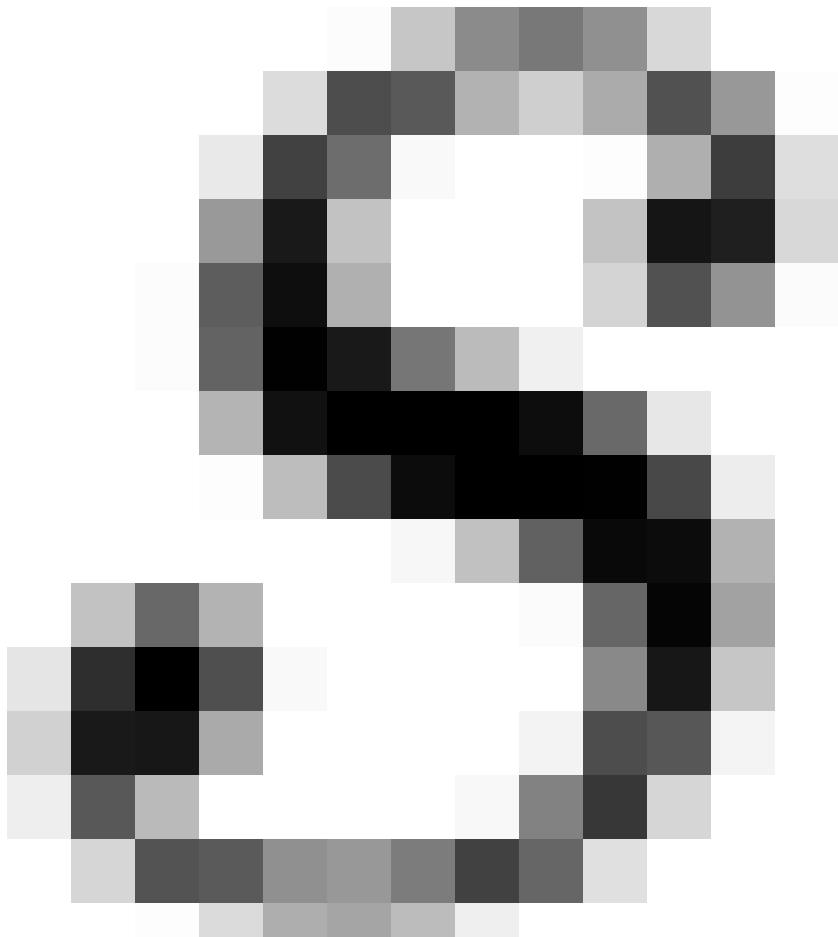
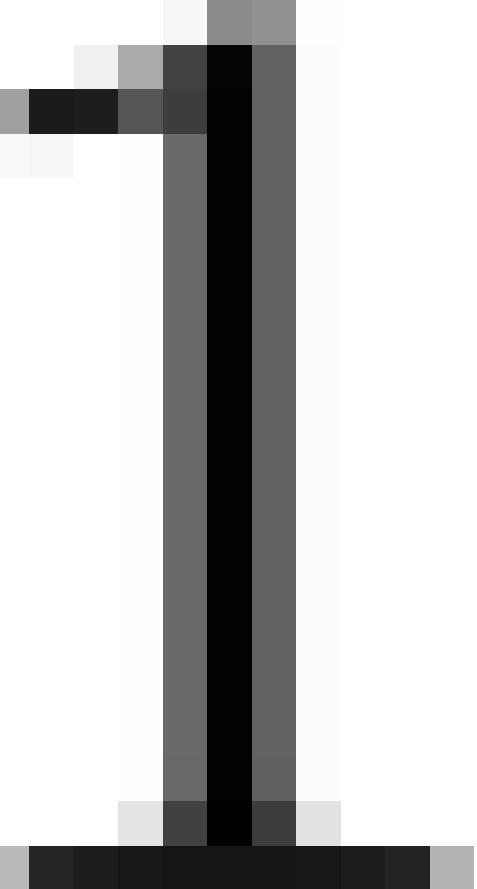
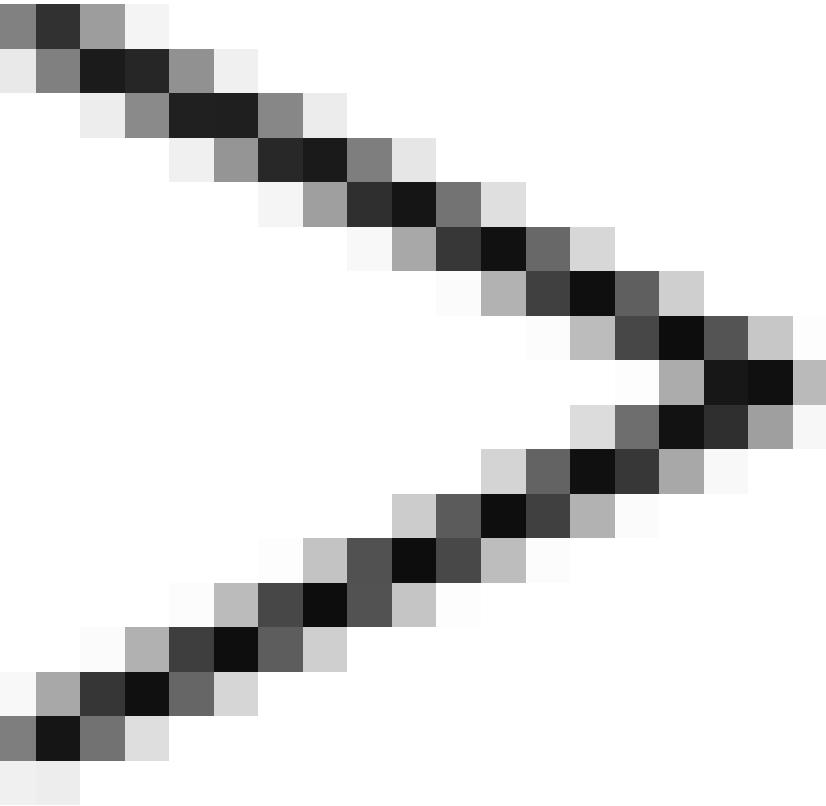
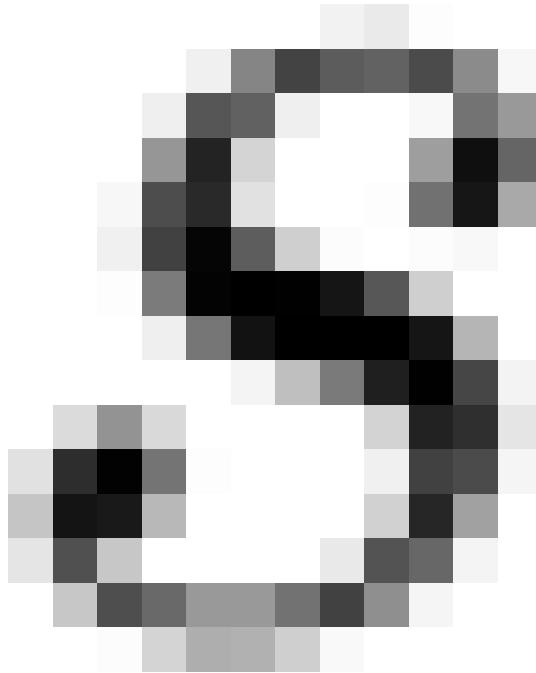
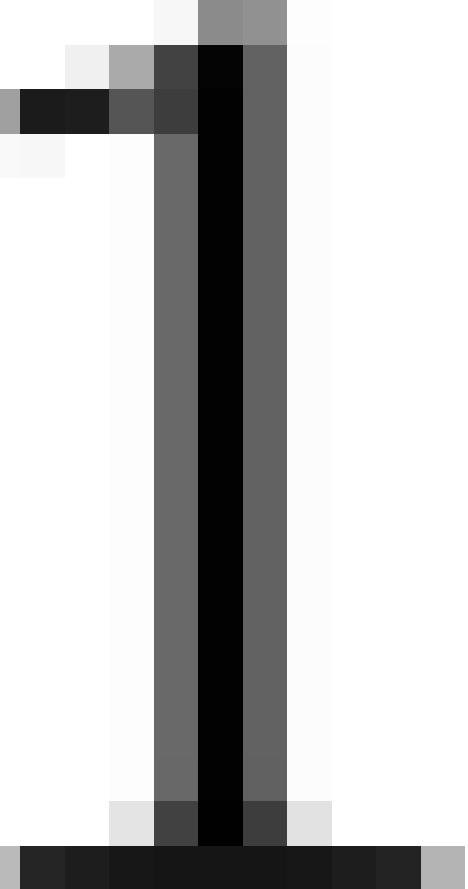
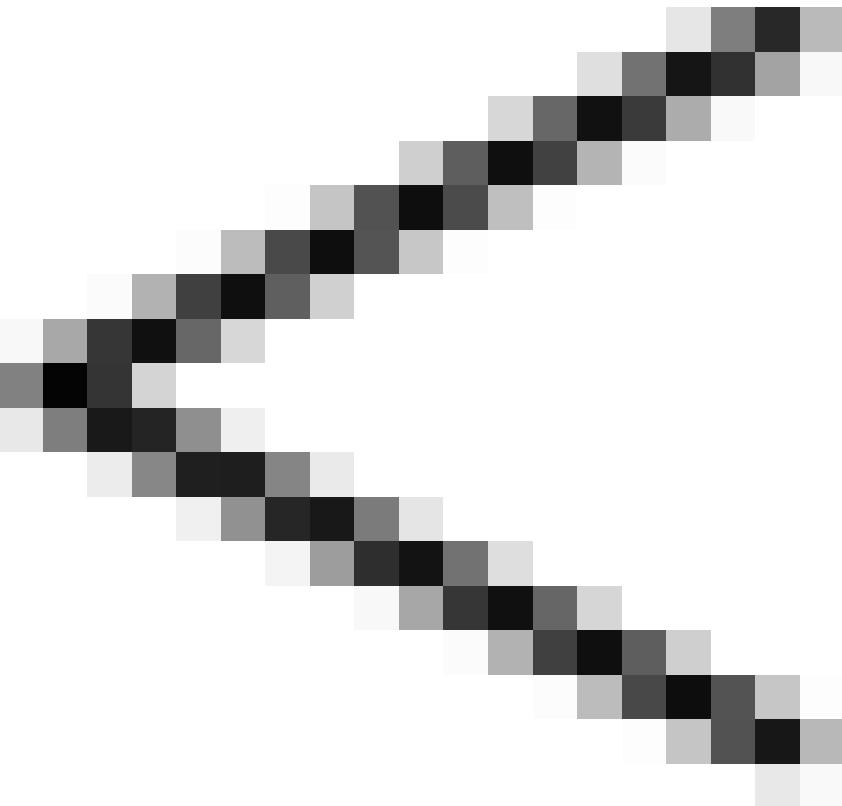
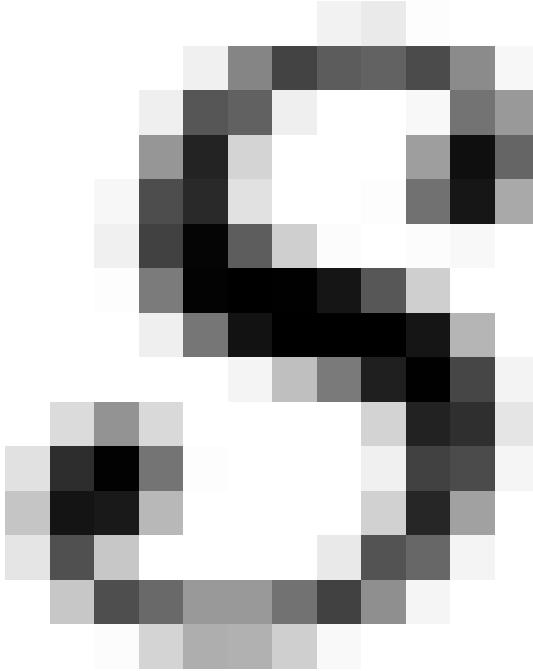


Figure 1. Schematic of a Generalized SRT





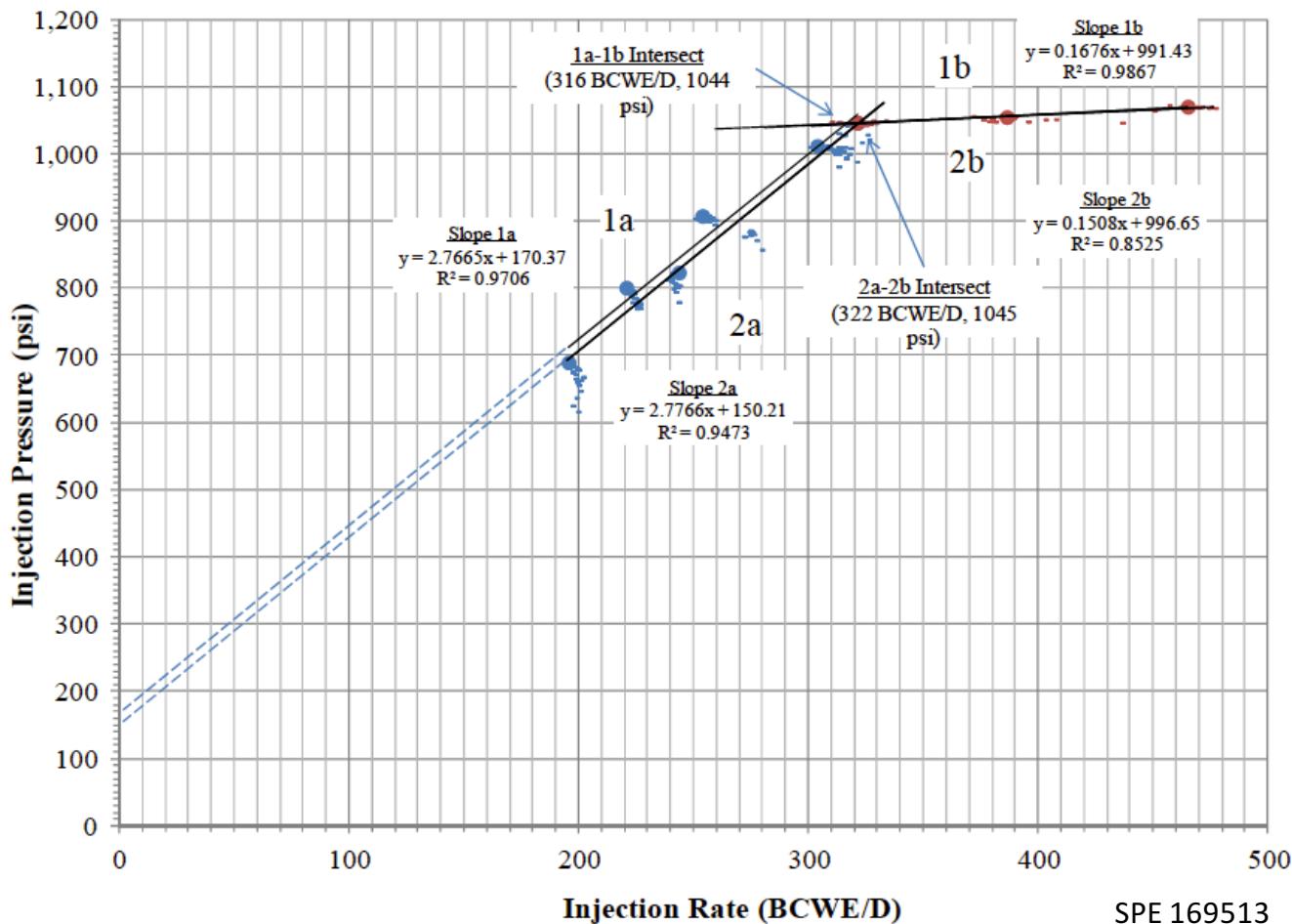


q

$$2\pi\hbar k$$

μ

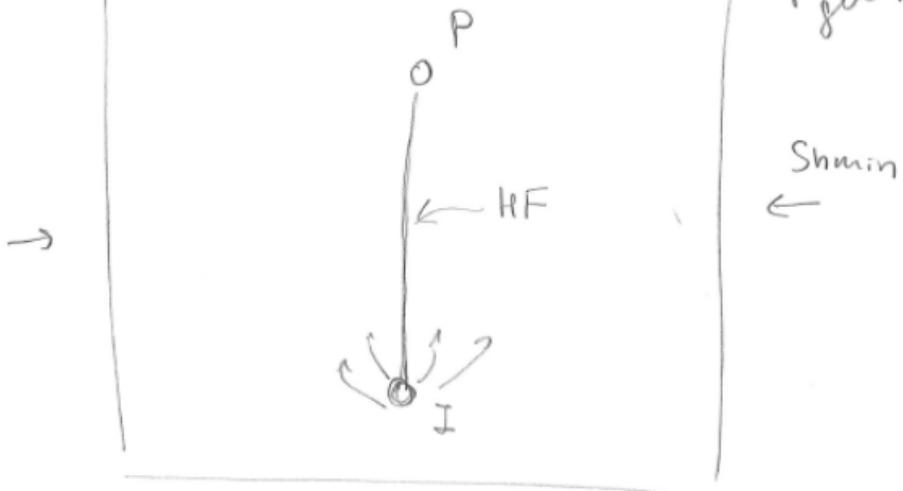
$$\frac{P_e - P_w}{\ln\left(\frac{\tau_e}{\tau_w}\right) + s}$$



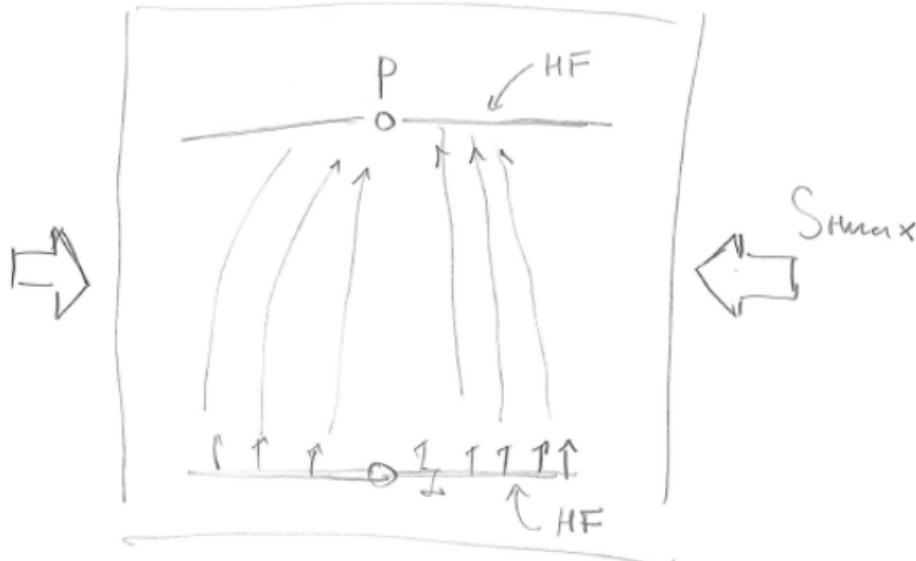
(NF)

$\downarrow S_{\text{max}}$

Not good

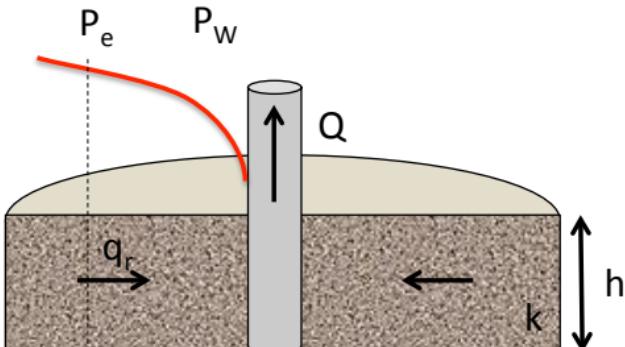


$\downarrow S_{\text{min}}$





Intact

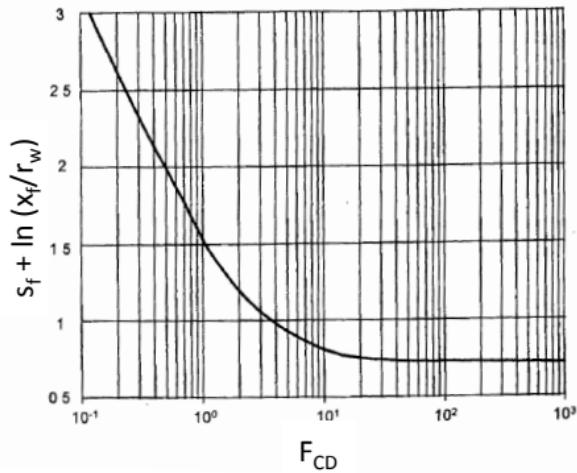
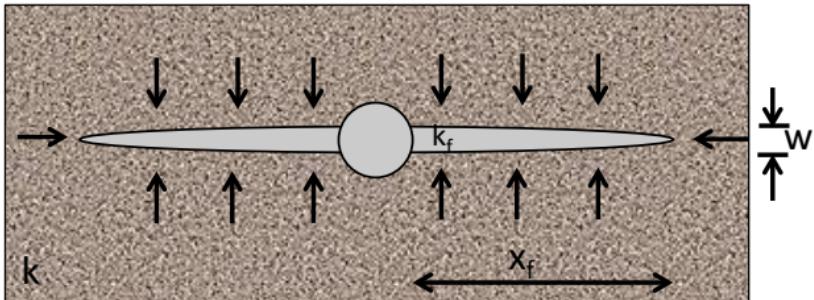


$$Q = -\frac{2\pi kh}{\mu} \frac{(P_e - P_w)}{p_D + s}$$

p_D : Dimensionless pressure
 $= \ln(r_e/r_w)$ if steady state

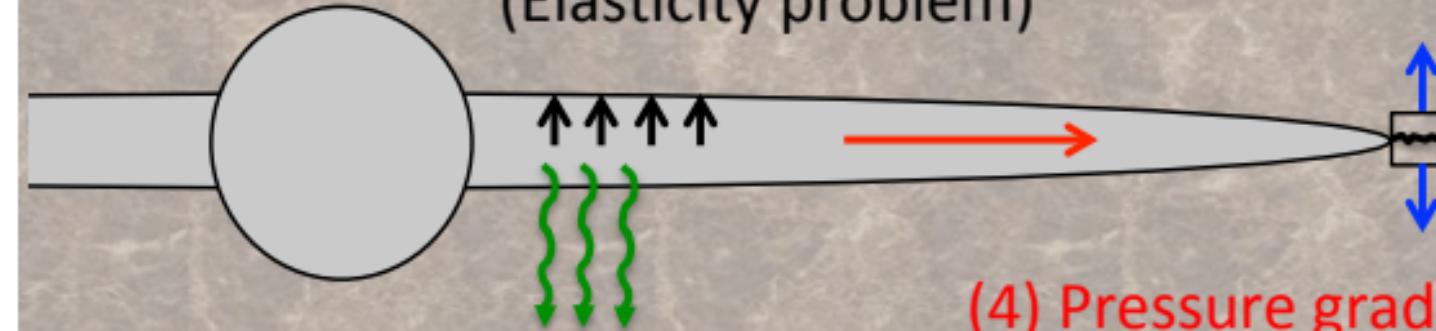
s : skin factor
 $> 1 \rightarrow$ damage
 $< 1 \rightarrow$ stimulation

Fractured



$$w, x_f, k_f \rightarrow F_{CD} = (k_f w) / (k x_f)$$

$$x_f, r_w, F_{CD} \rightarrow s_f \text{ from above plot}$$



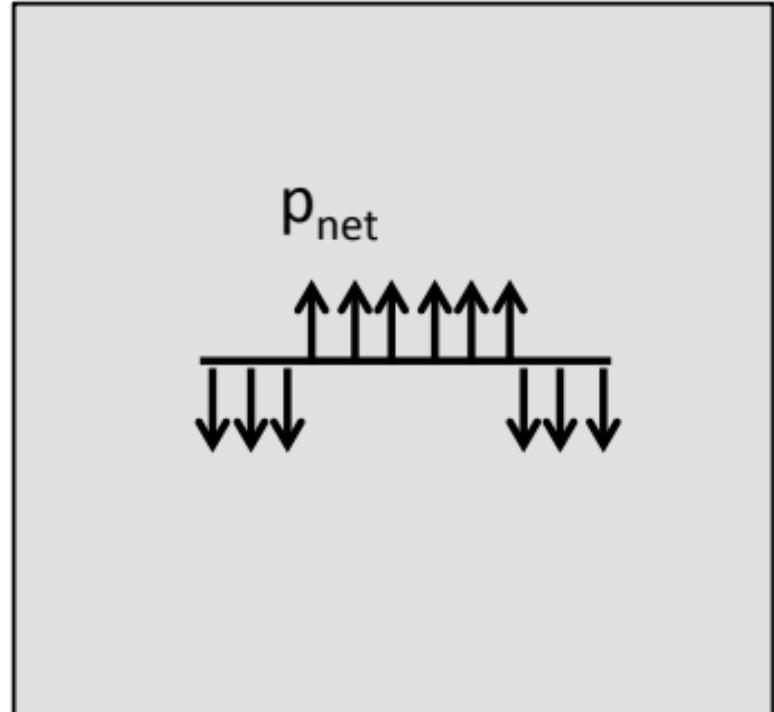
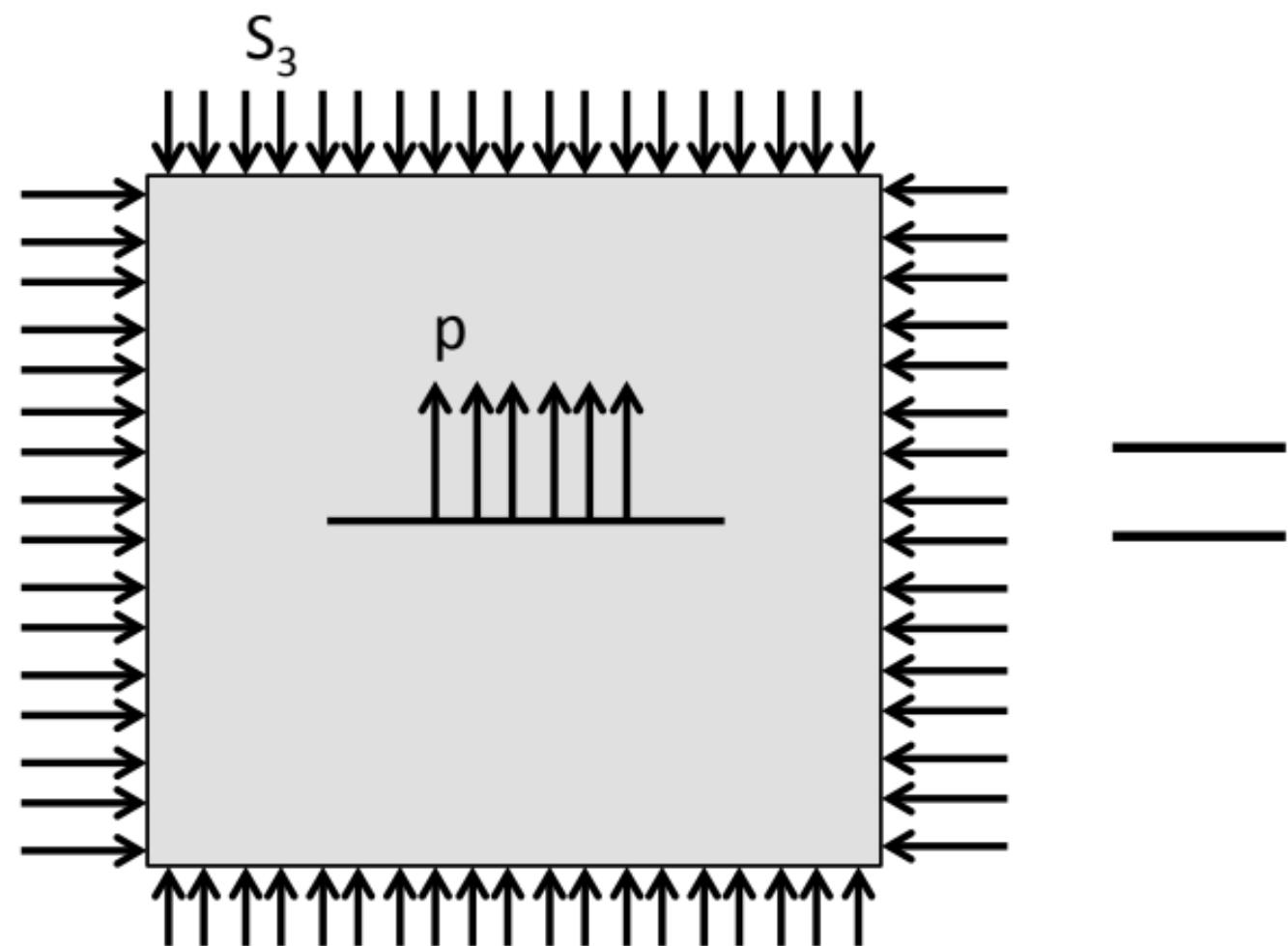
(1) Pressure on the fracture deforms adjacent rock
(Elasticity problem)

(2) Fracture propagates if the “stress intensity factor” is higher than what the rock can resist “rock toughness”
(Fracture mechanics problem)

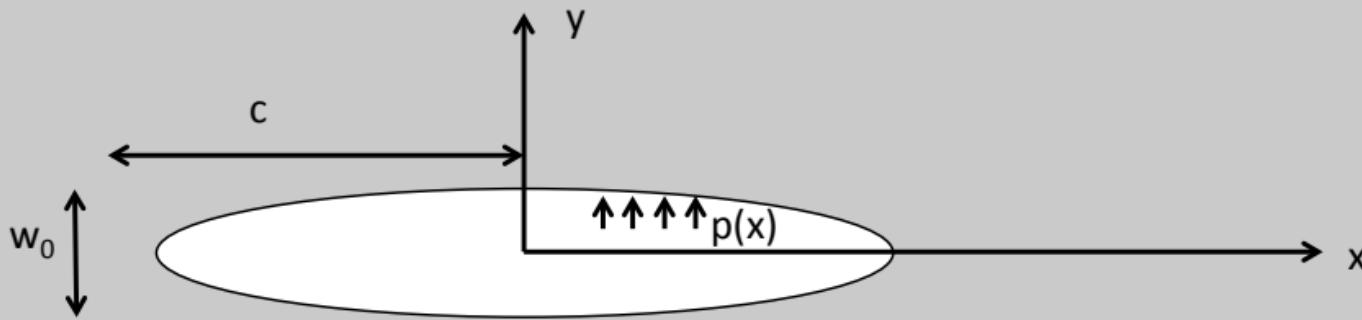
(3) Fracturing fluids can leak off to the formation
(Mud-cake design)

(4) Pressure gradient leads to flow of fracturing fluid through the fracture
(Lubrication problem)

Net pressure
 $p_{\text{net}} = p - S_3$



Griffith crack problem



$$(E, v)$$
$$E' = E / (1 - v^2)$$

Boundary conditions

$$\begin{cases} \sigma_{yy}(x,0) = -p(x) & \text{for } 0 \leq x \leq c \\ u_y(x,0) = 0 & \text{for } x > c \end{cases}$$

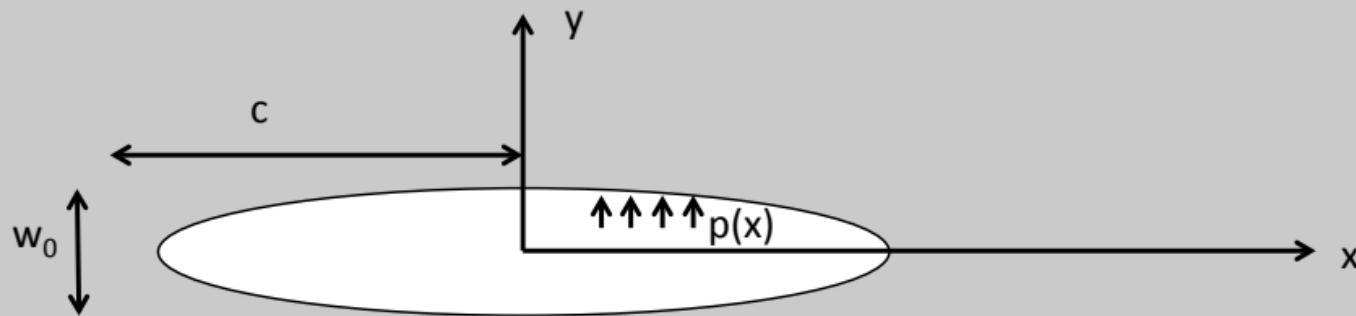
- Linear elastic and homogeneous solid
- Constant pressure p_0
- No failure

$$\begin{cases} \sigma_{yy}(x,0) = p_0[x/(x^2-c^2)^{1/2} - 1] \\ u_y(x,0) = 2p_0/E' * (c^2-x^2)^{1/2} \end{cases}$$



- σ_{yy} : tension for $x > c$; $\sigma_{yy} = -p_0$ at $1.15c$
- $w_0 = 4 \cdot c \cdot p_0 / E'$

Griffith crack problem



$$(E, v)$$
$$E' = E / (1 - v^2)$$

$$\sigma_{yy}(x,0) = p_0[x/(x^2-c^2)^{1/2} - 1]$$

- $\sigma_{yy} \rightarrow \infty$ at $x=c$!

A more convenient amount to compare is the stress intensity factor

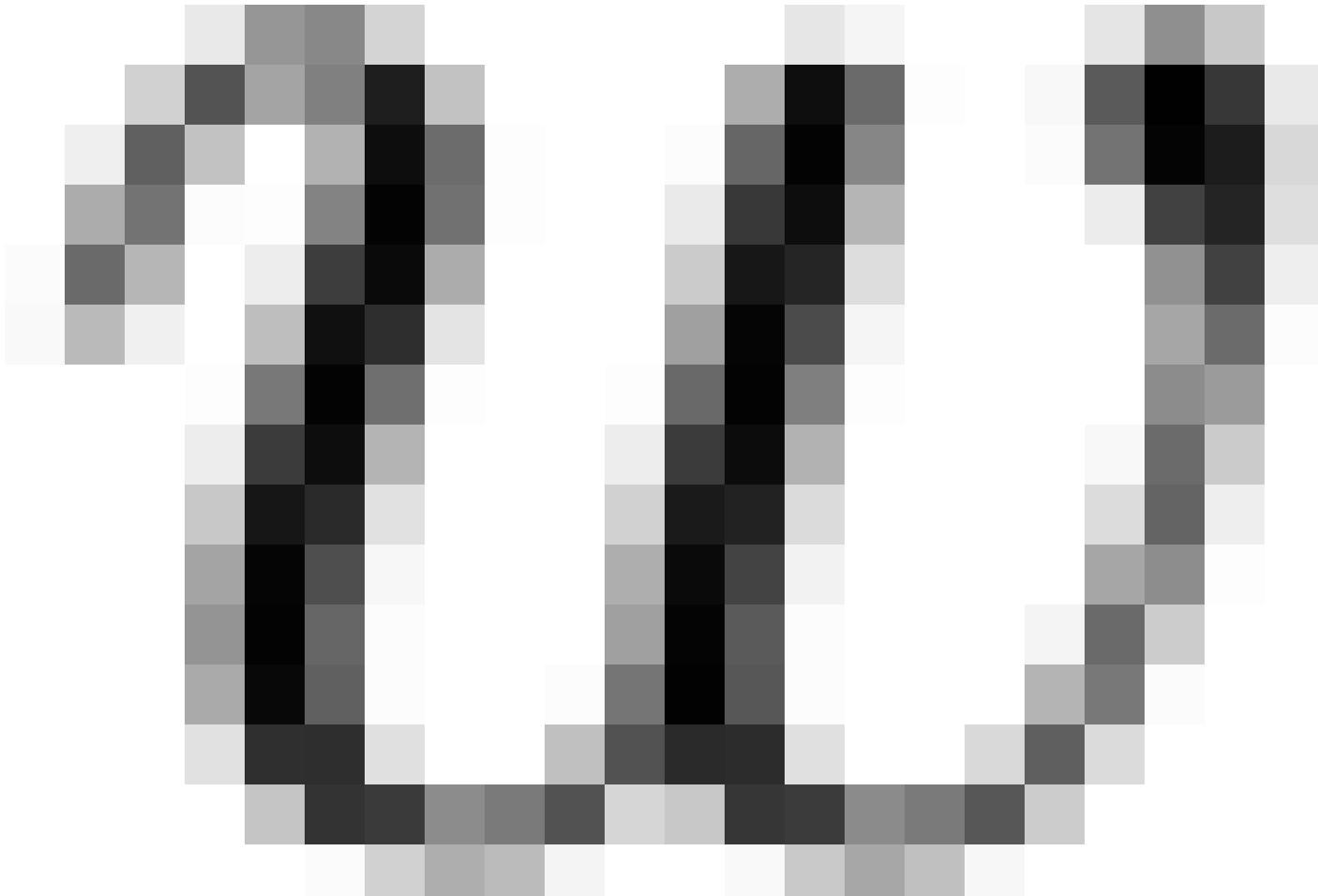
$$K_I = \lim_{r \rightarrow 0^+} [2\pi r^{1/2} \sigma_{yy}(x = c + r, y = 0)]$$

Constant pressure crack:

$$K_I = p_0 (\pi c)^{1/2}$$

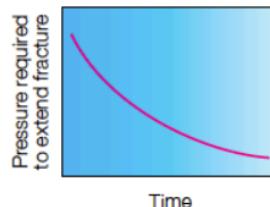
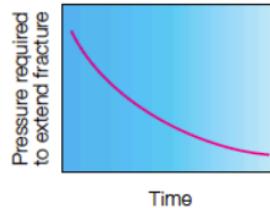
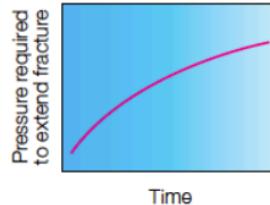
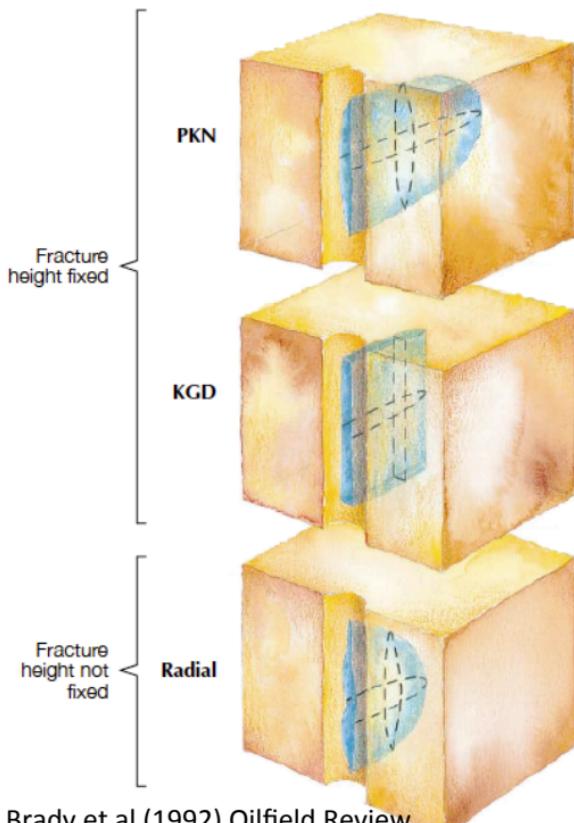
If $K_I > K_{IC}$ (Fracture Toughness)
→ Fracture propagates







2D Fracture Models



- Elliptical cross section
- Width \propto height
- Width < KGD; length > KGD
- More appropriate when fracture length > height

- Rectangular cross section
- Width \propto length
- More appropriate when fracture length < height

- Appropriate when fracture length = height

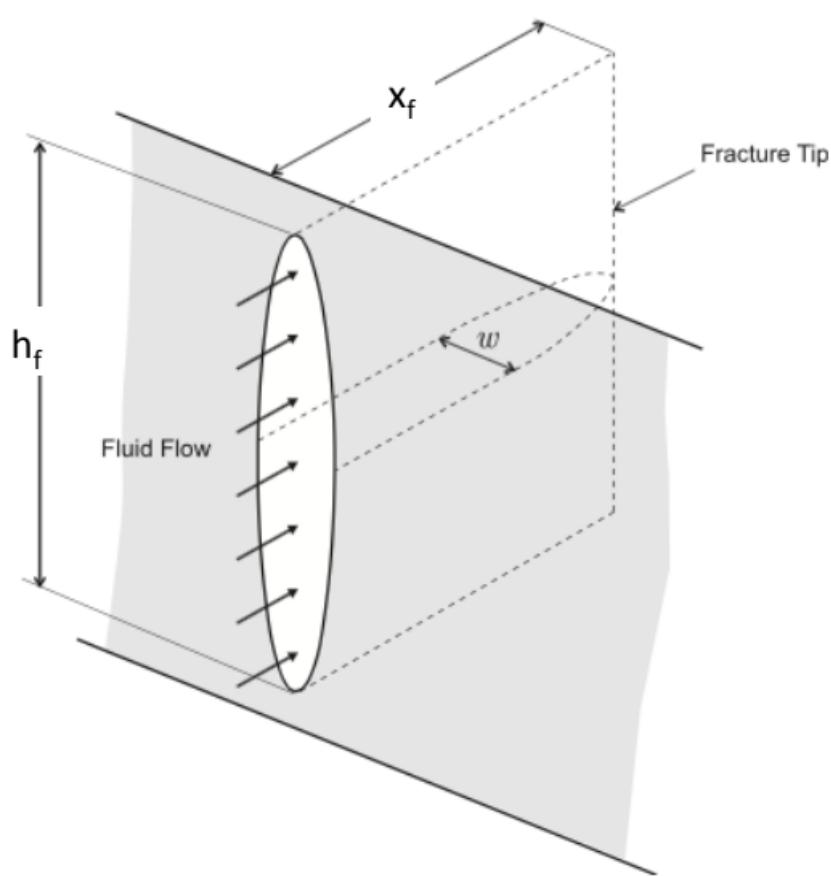


Fig. 1. Schematic showing PKN fracture geometry.

Figure: Adachi et al. 2007

- (1) Linear elasticity
- (2) Negligible Toughness
- (3) No leak-off
- (4) Laminar flow, constant injection rate

- Fracture half-length

$$x_f = 0.524 \left(\frac{i^3 E'}{\mu h_f^4} \right)^{1/5} (t)^{4/5}$$

- Maximum width at the wellbore

$$w_{w,0} = 3.04 \left(\frac{i^2 \mu}{E' h_f} \right)^{1/5} (t)^{1/5}$$

- Net Pressure at the wellbore

$$p_{n,w} = 1.52 \left(\frac{E'^4 i^2 \mu}{h_f^6} \right)^{1/5} (t)^{1/5}$$

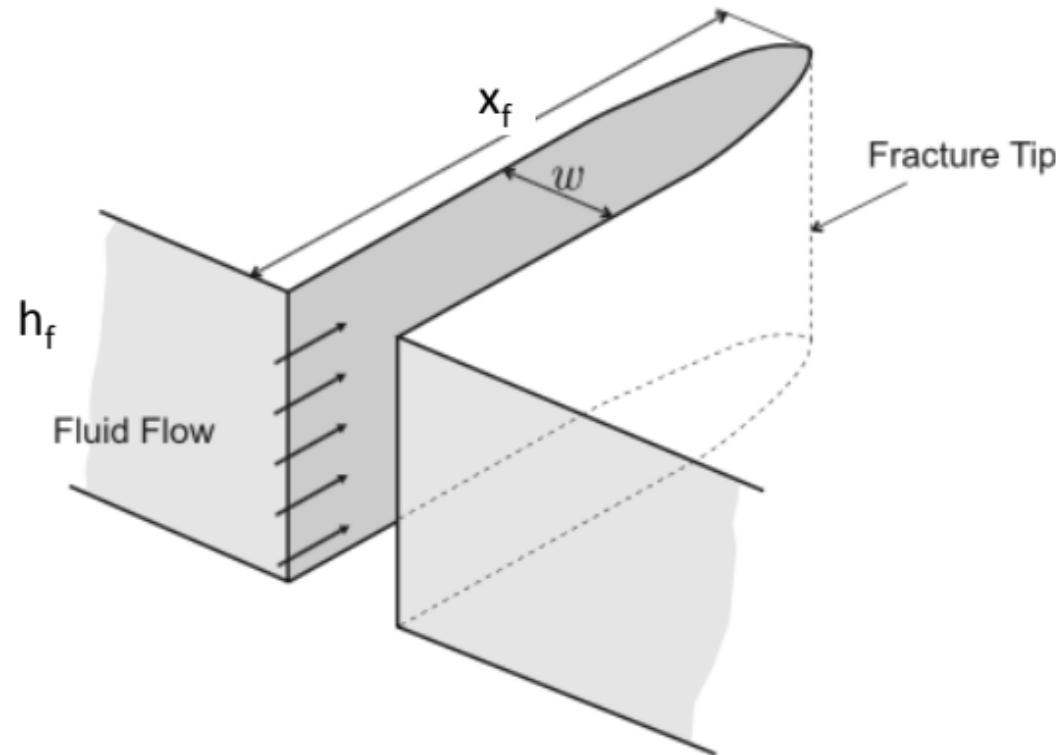


Fig. 2. Schematic showing KGD fracture geometry.

Figure: Adachi et al. 2007

- Fracture half-length

$$x_f = 0.539 \left(\frac{i^3 E'}{\mu h_f^3} \right)^{1/5} (t)^{2/3}$$

- Width at the wellbore

$$w_w = 2.36 \left(\frac{i^3 \mu}{E' h_f^3} \right)^{1/6} (t)^{1/3}$$

- Net Pressure at the wellbore

$$p_{n,w} = 1.09 (E'^2 \mu)^{1/3} (t)^{-1/3}$$

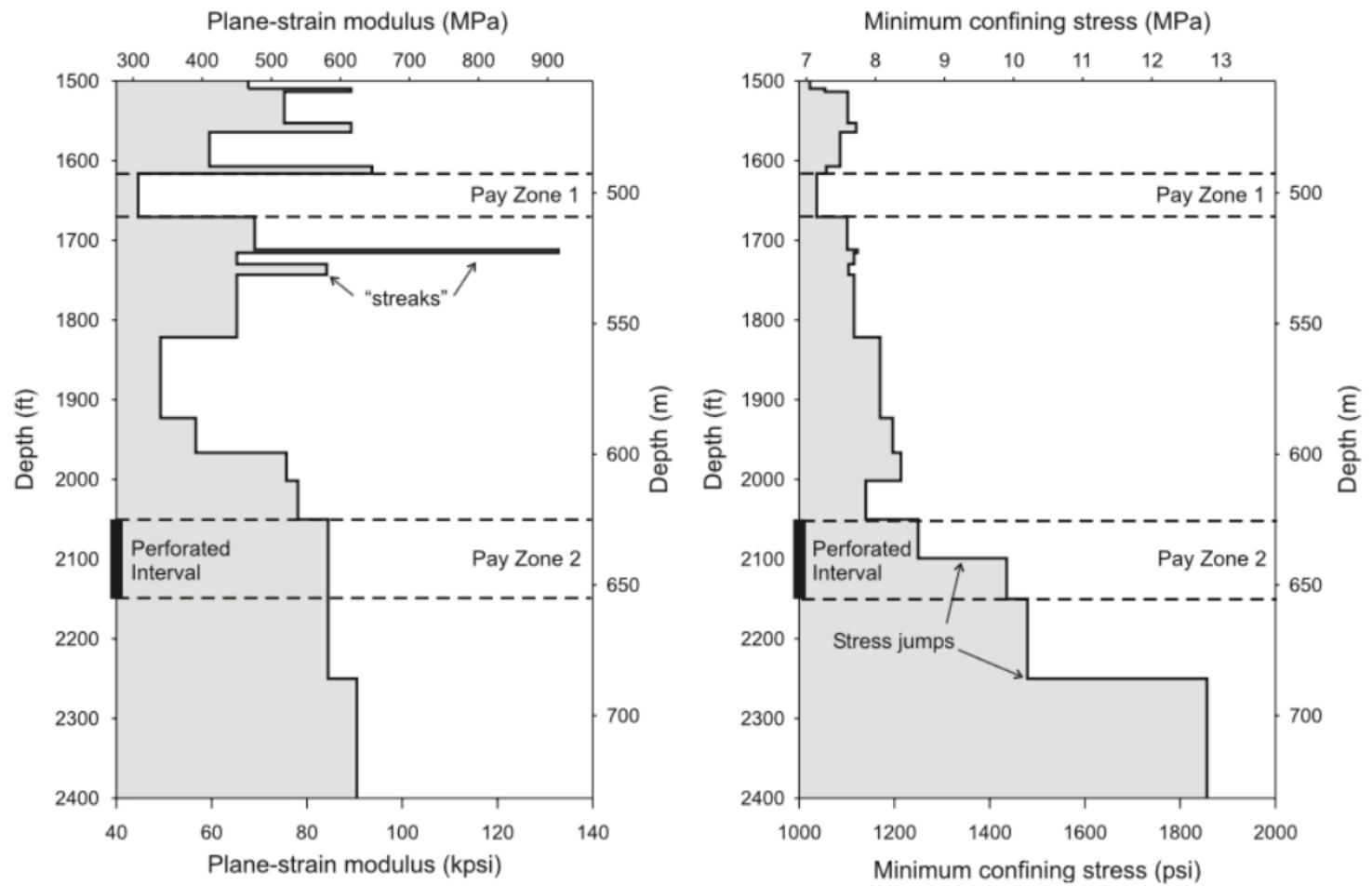
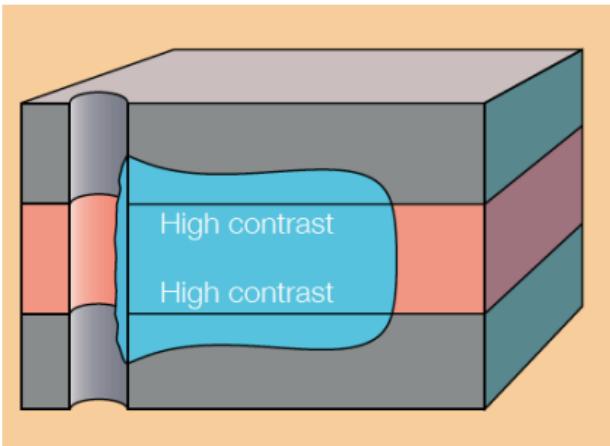
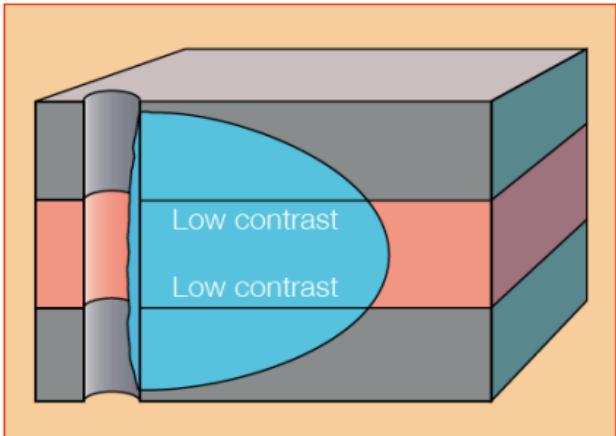
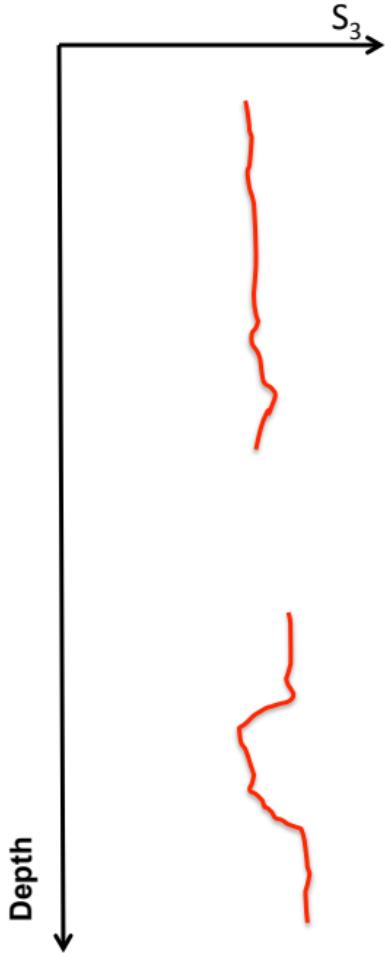
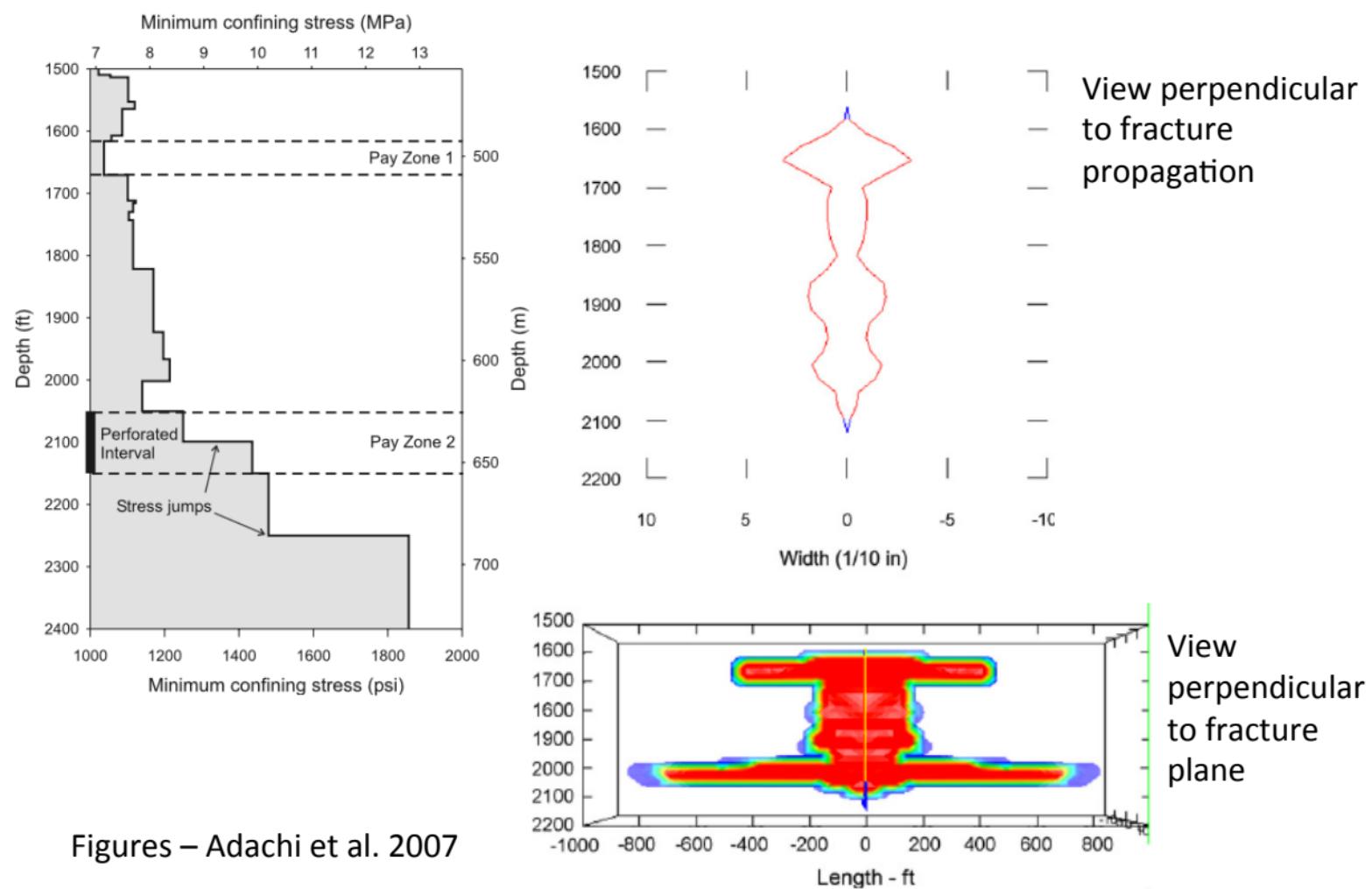
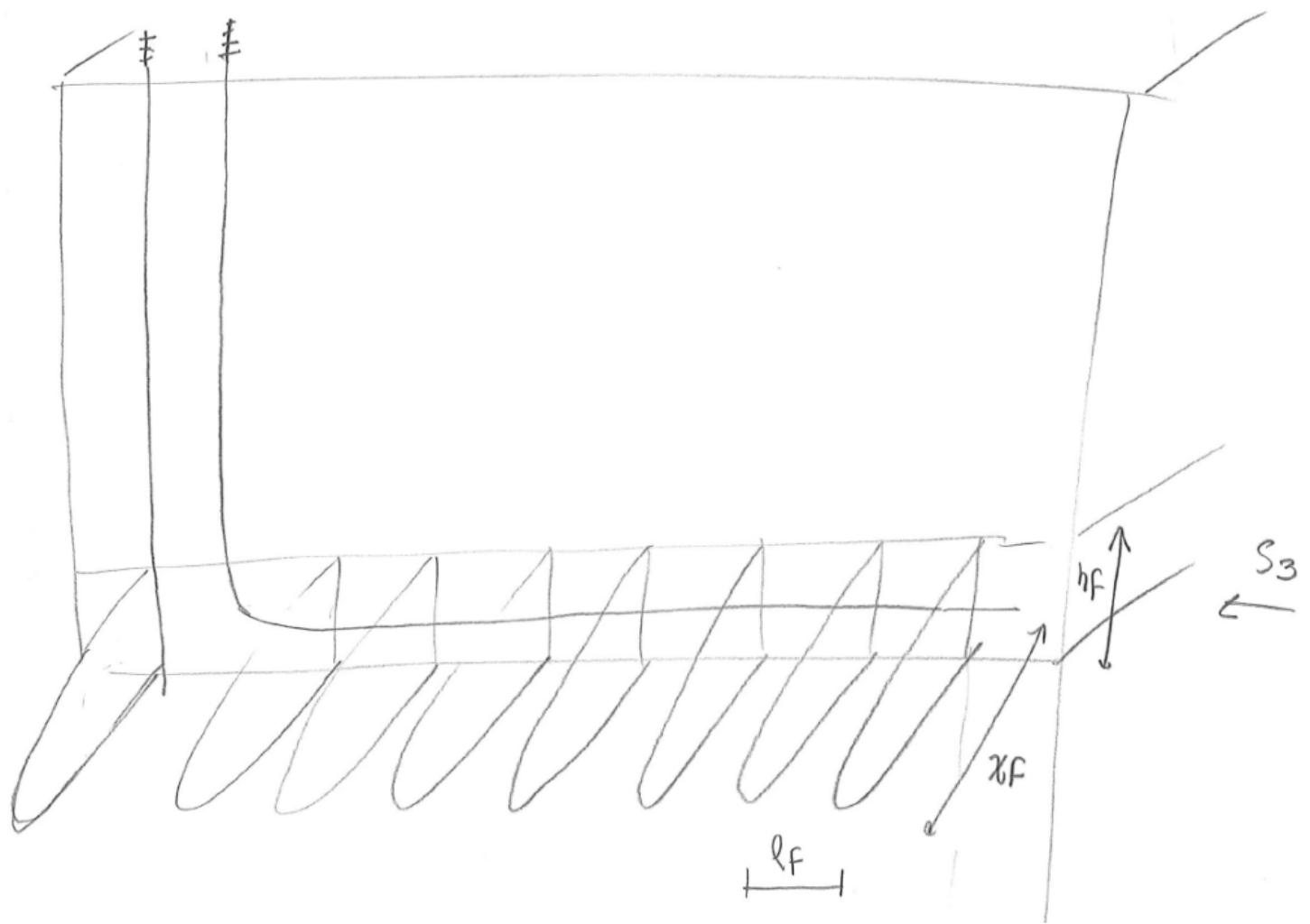
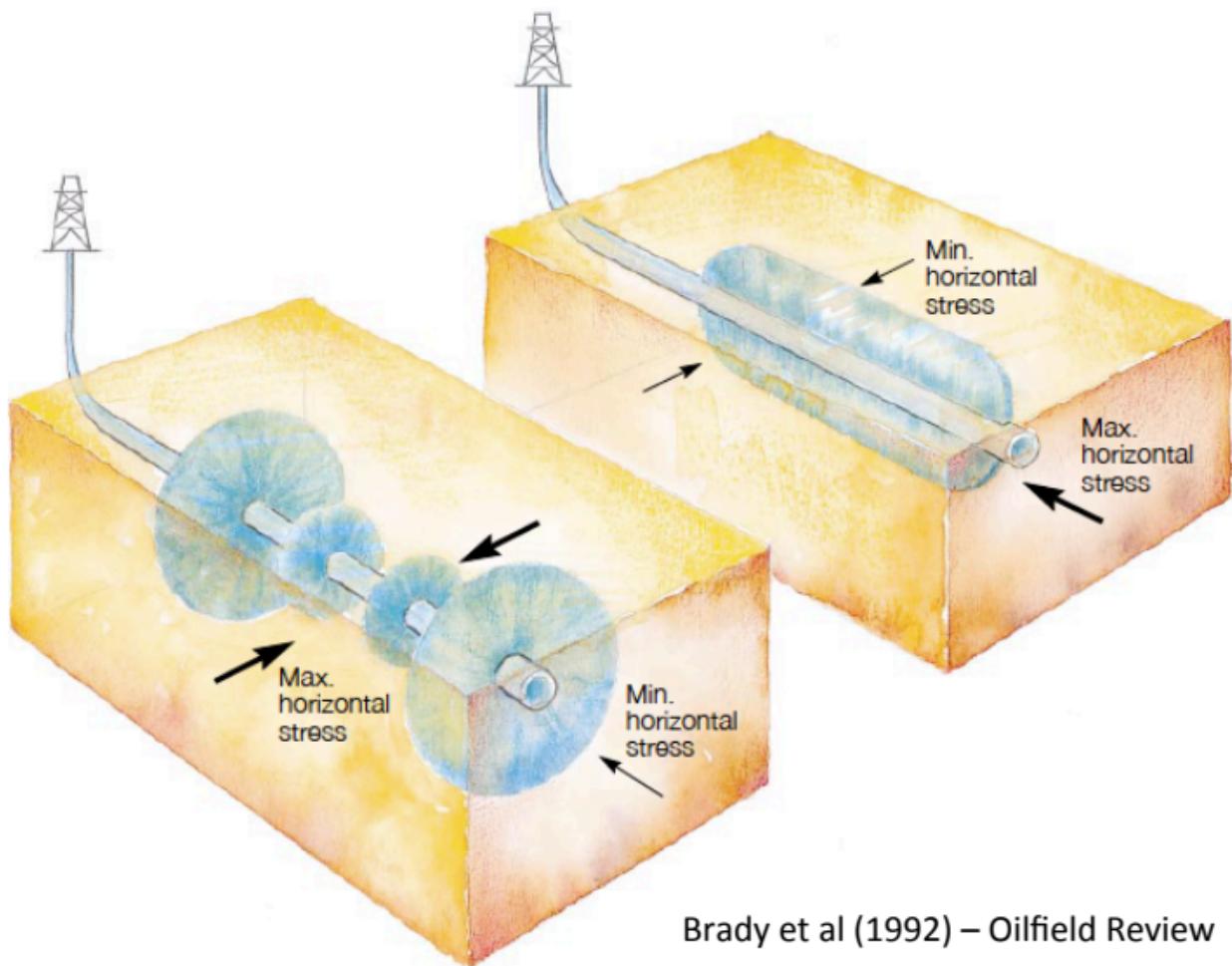


Fig. 9. Profiles of plane-strain modulus (left) and minimum *in situ* confining stress (right) versus depth.









Brady et al (1992) – Oilfield Review

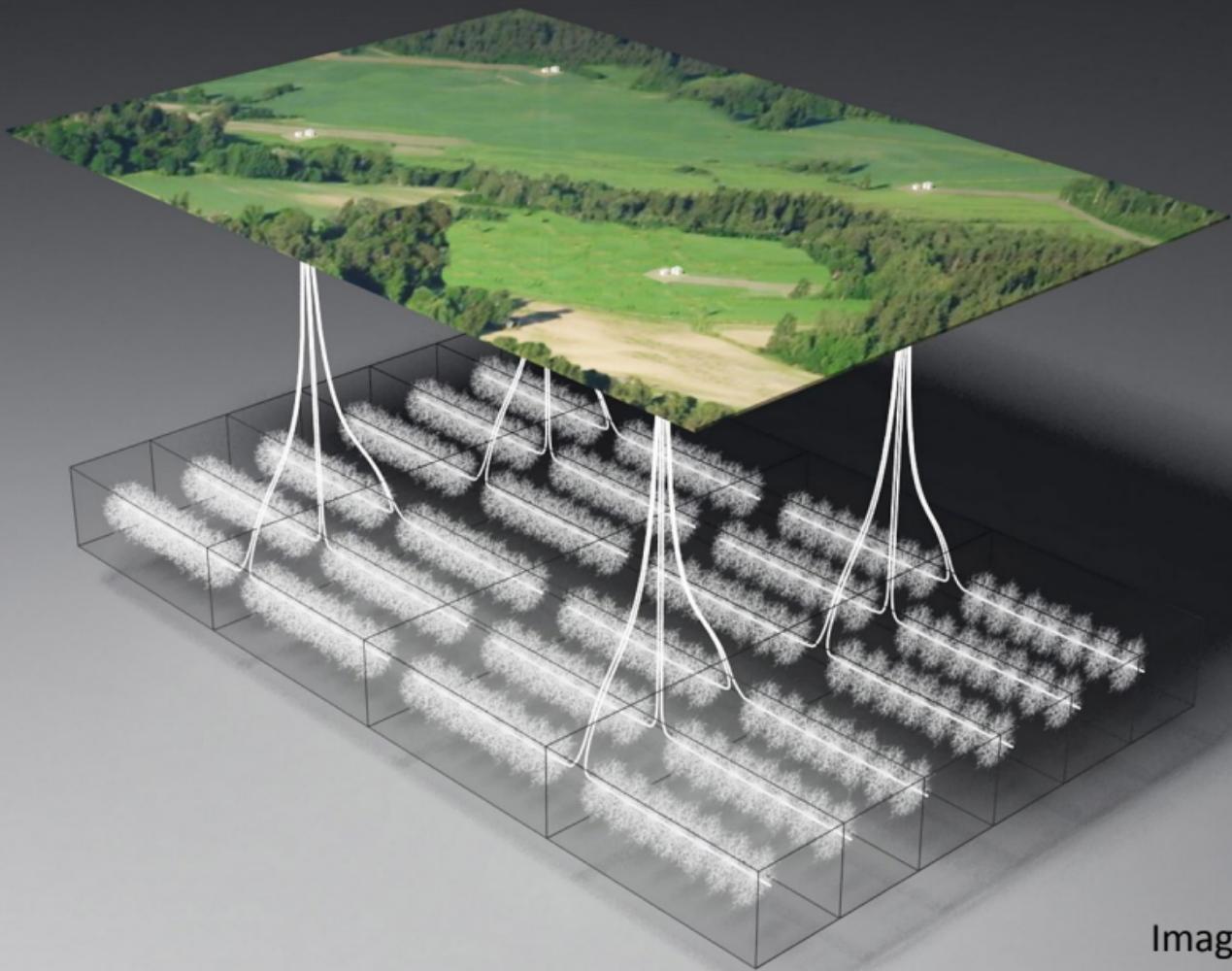
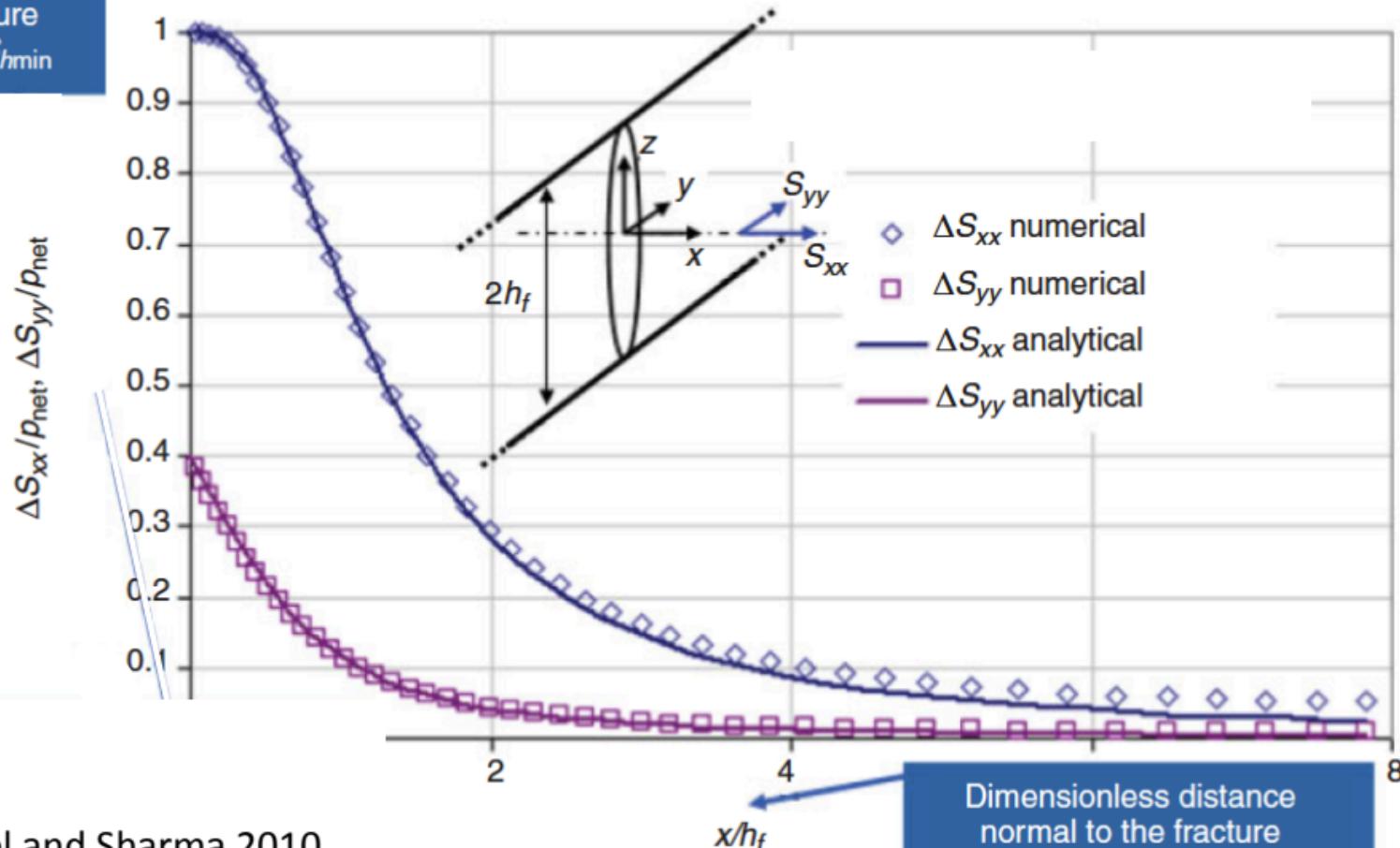


Image: Statoil

Net extension pressure
 $= p_f - S_{h\min}$



TOP VIEW

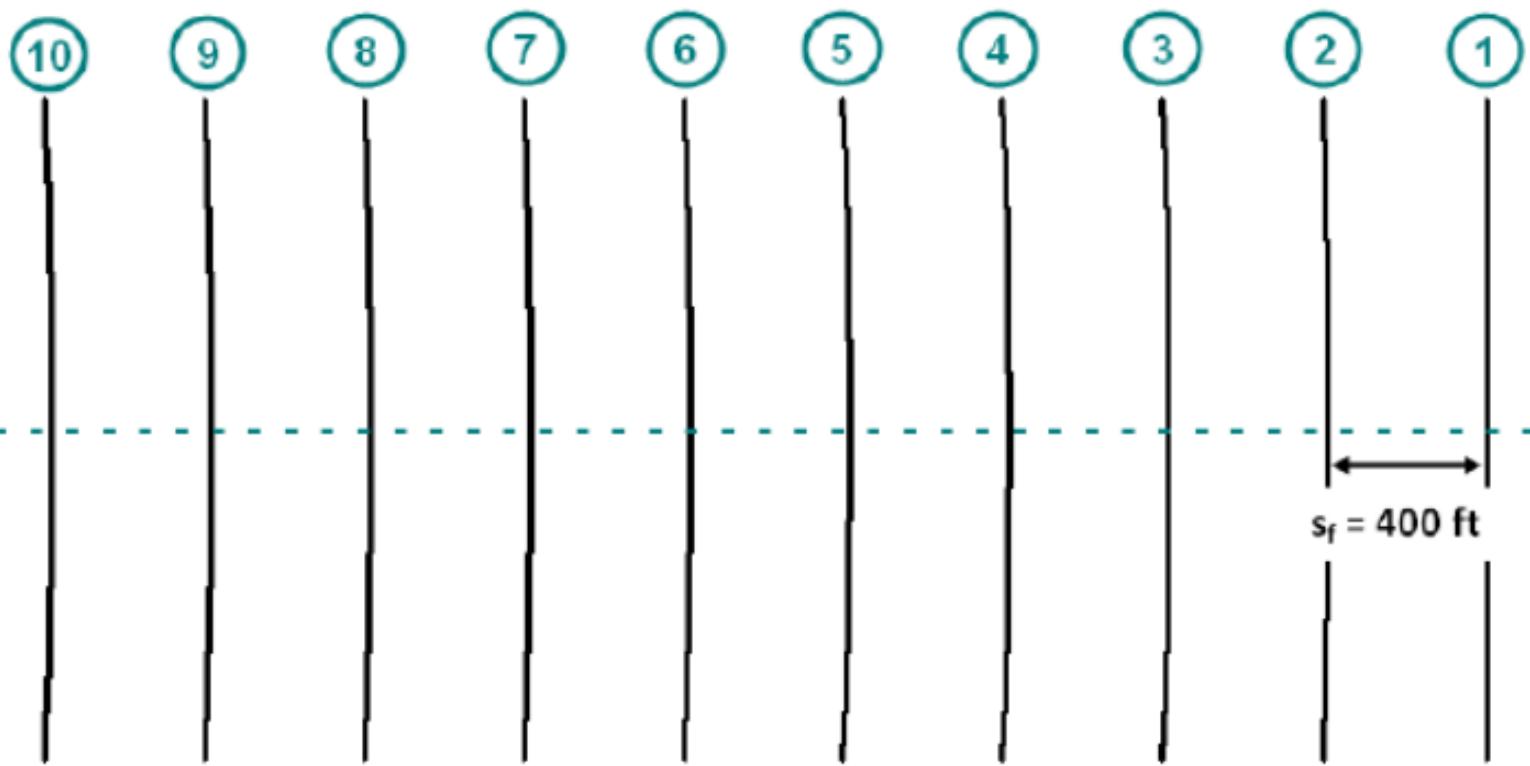


Fig. 7 – Trajectory of multiple consecutive transverse fractures spaced 400 ft apart

TOP VIEW

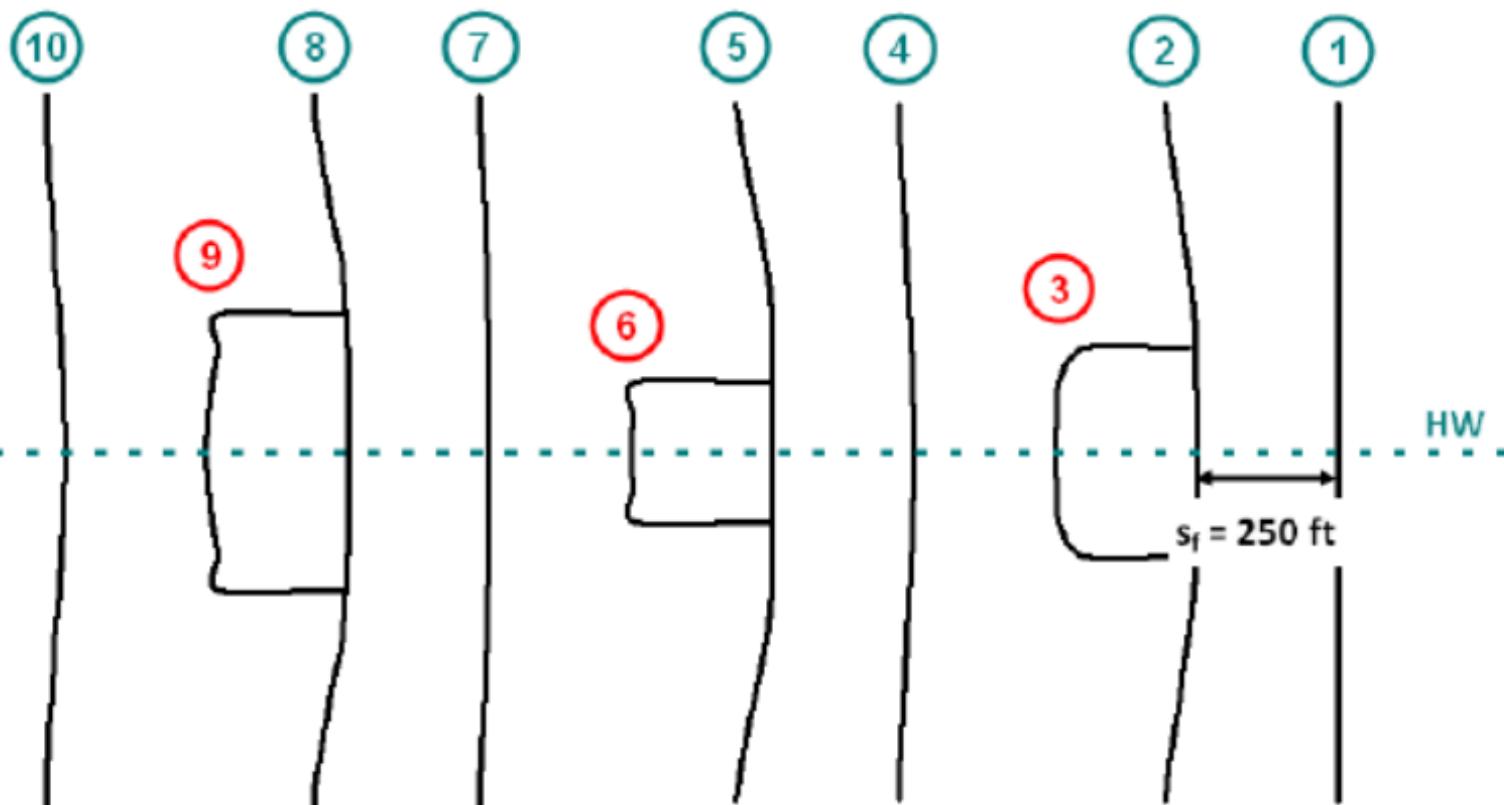
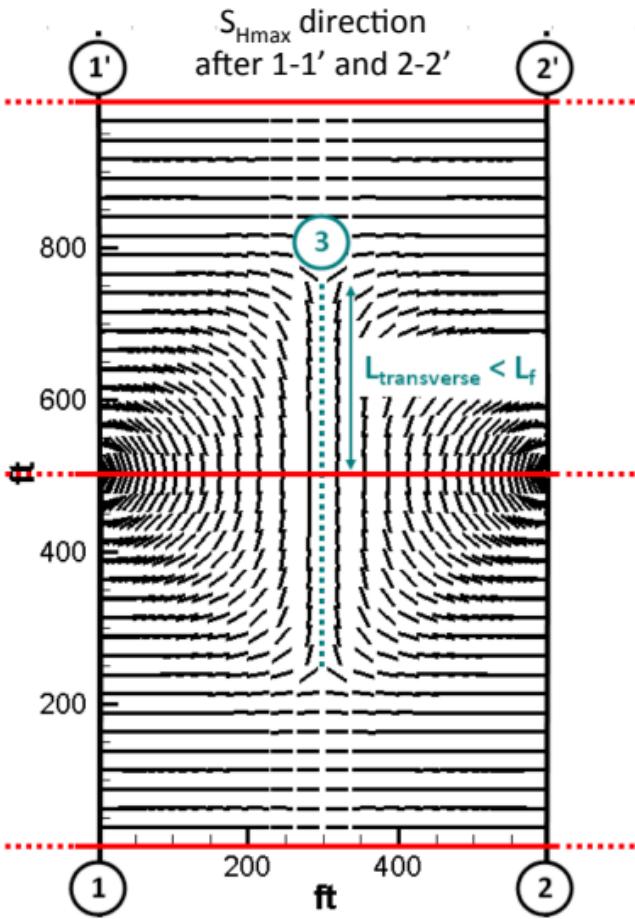
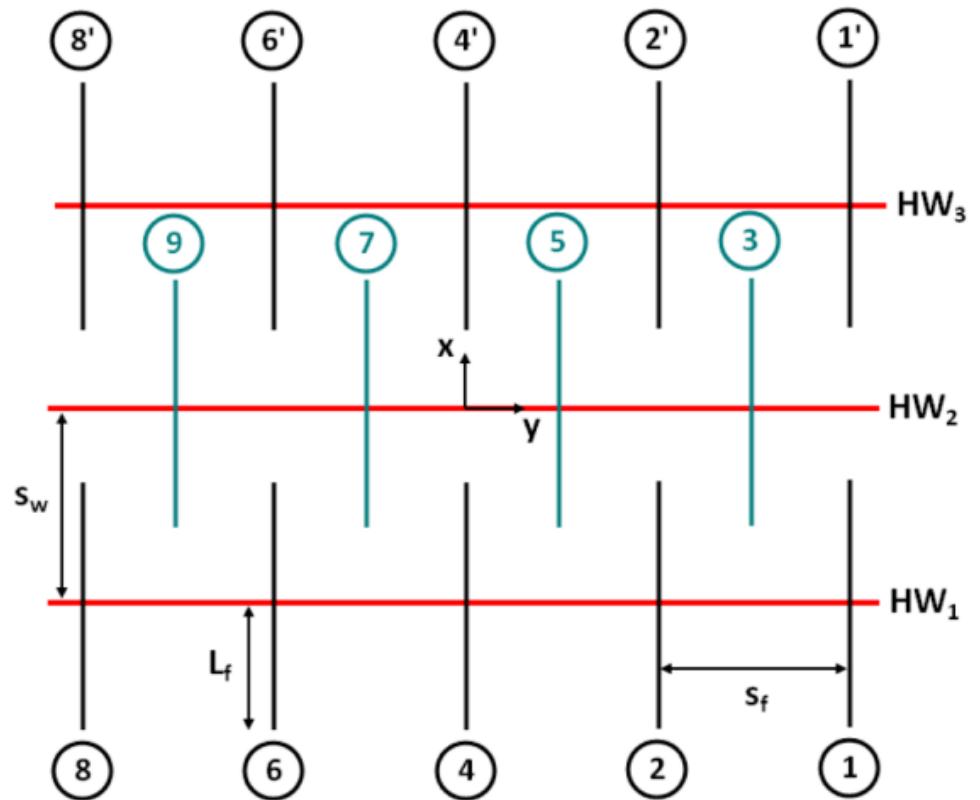
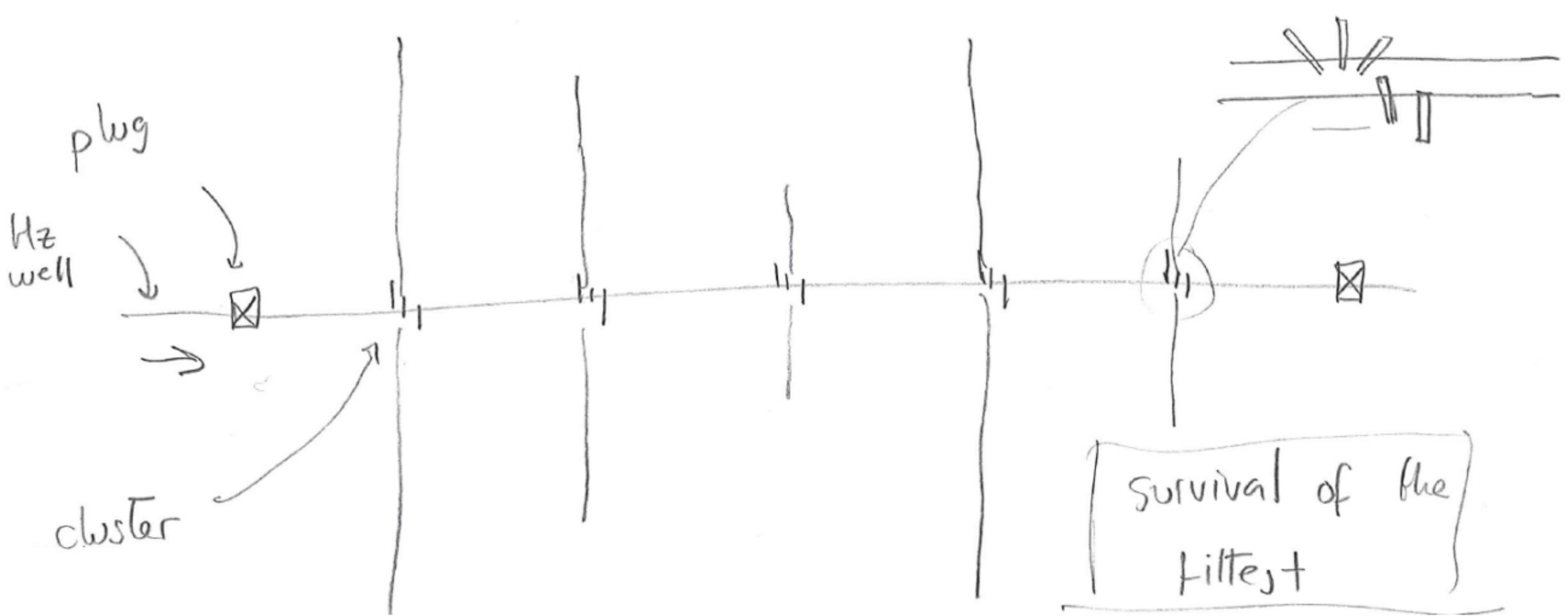
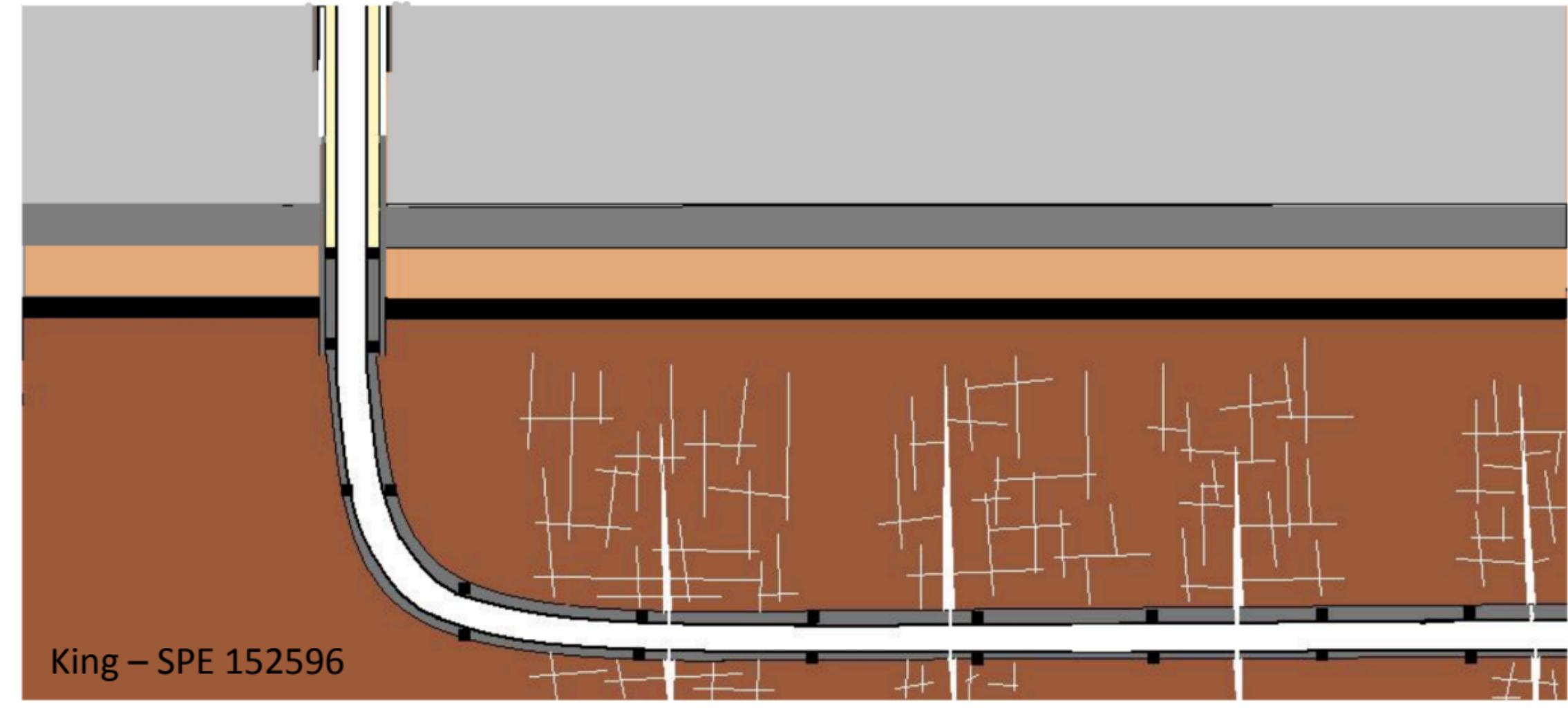


Fig. 10 – Trajectory of multiple consecutive transverse fractures spaced 250 ft apart

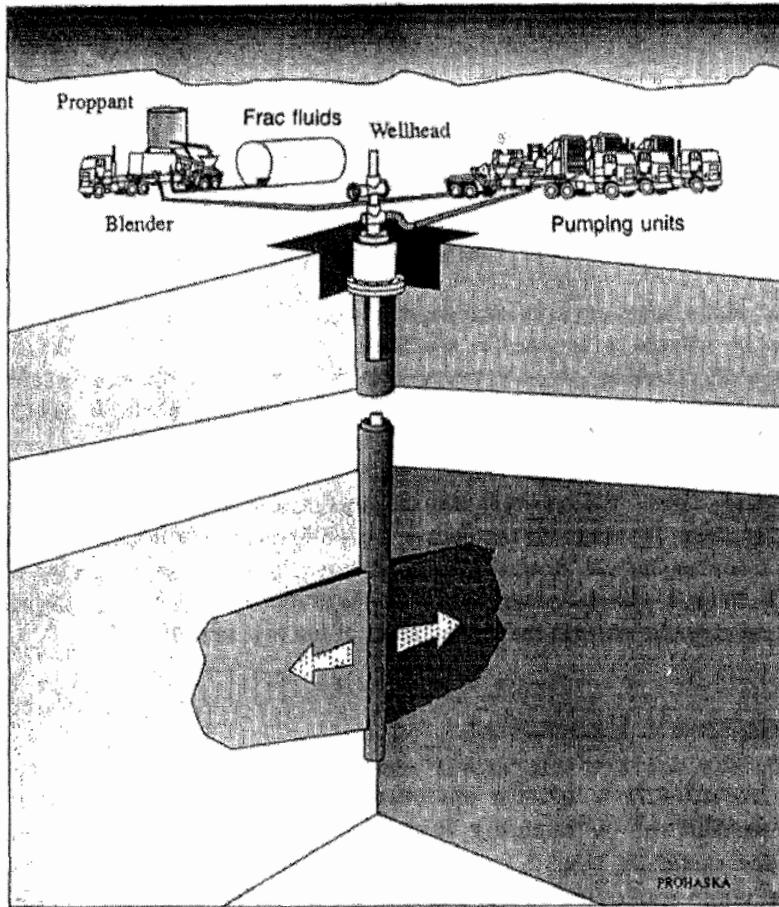




King – SPE 152596

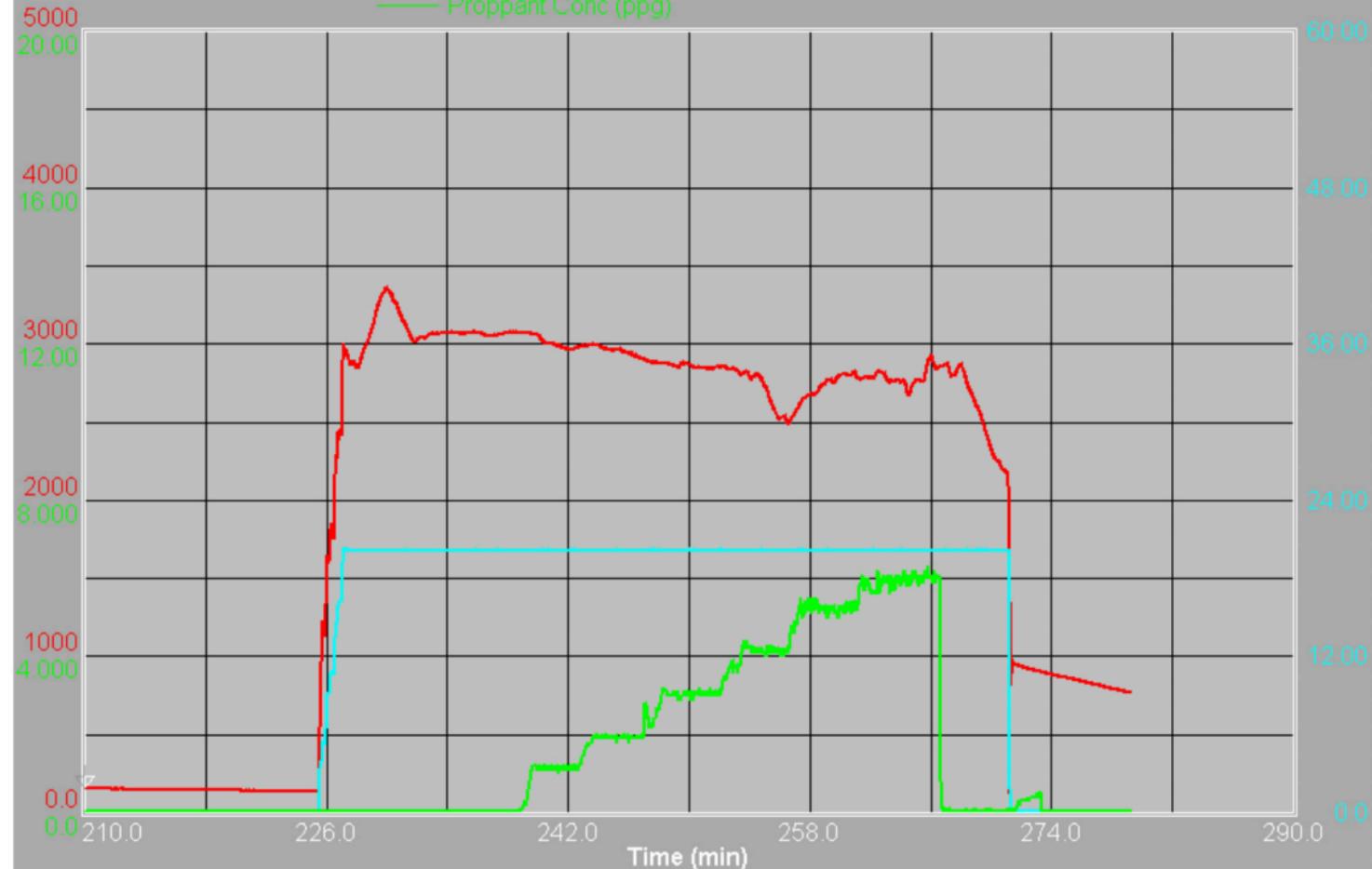


... / Fig



From
Valko and Economides, 1996

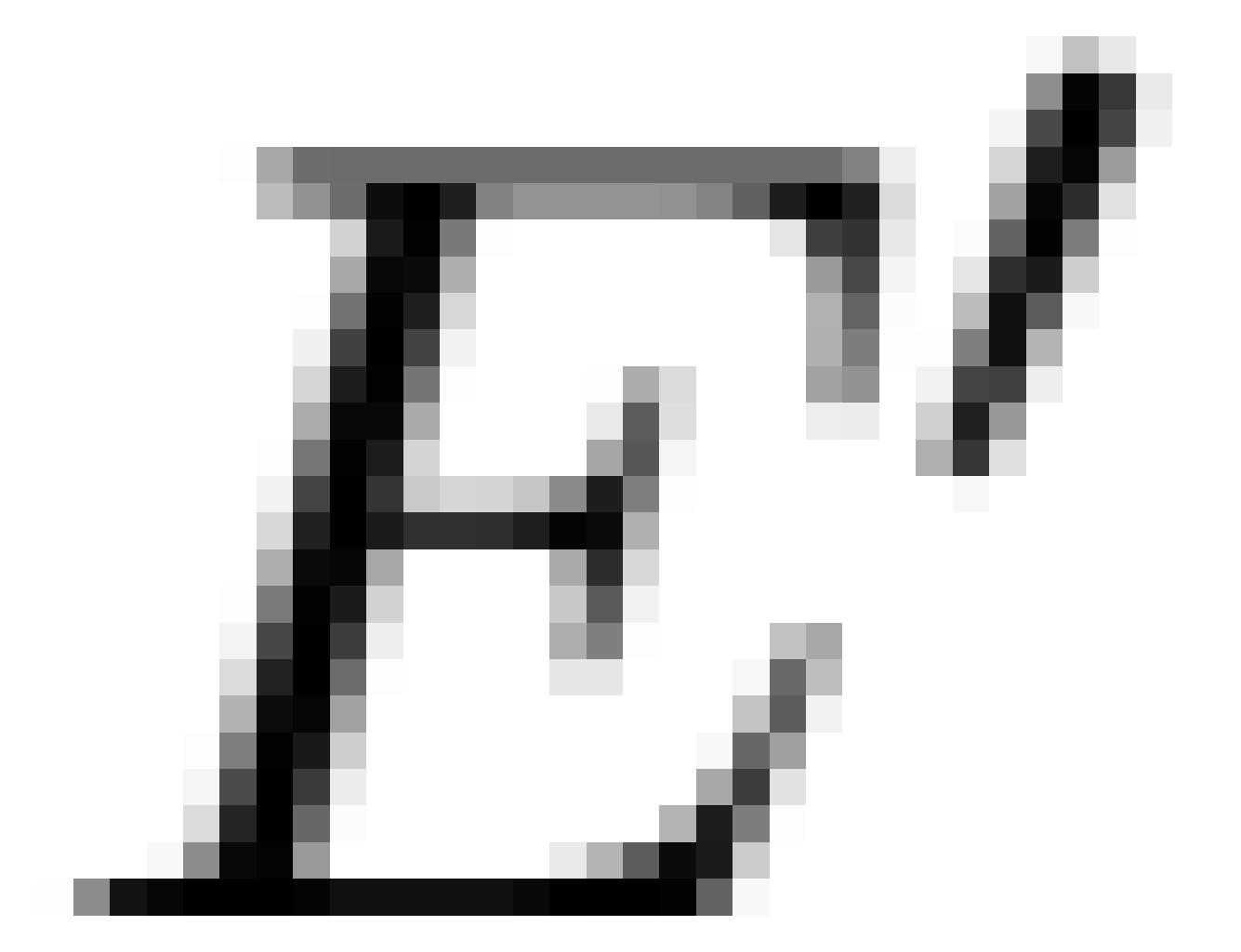
Surf Press [Tbg] (psi) Slurry Flow Rate (bpm)
Proppant Conc (ppg)













• . / E

• . / E

• . / E

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• . / E

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