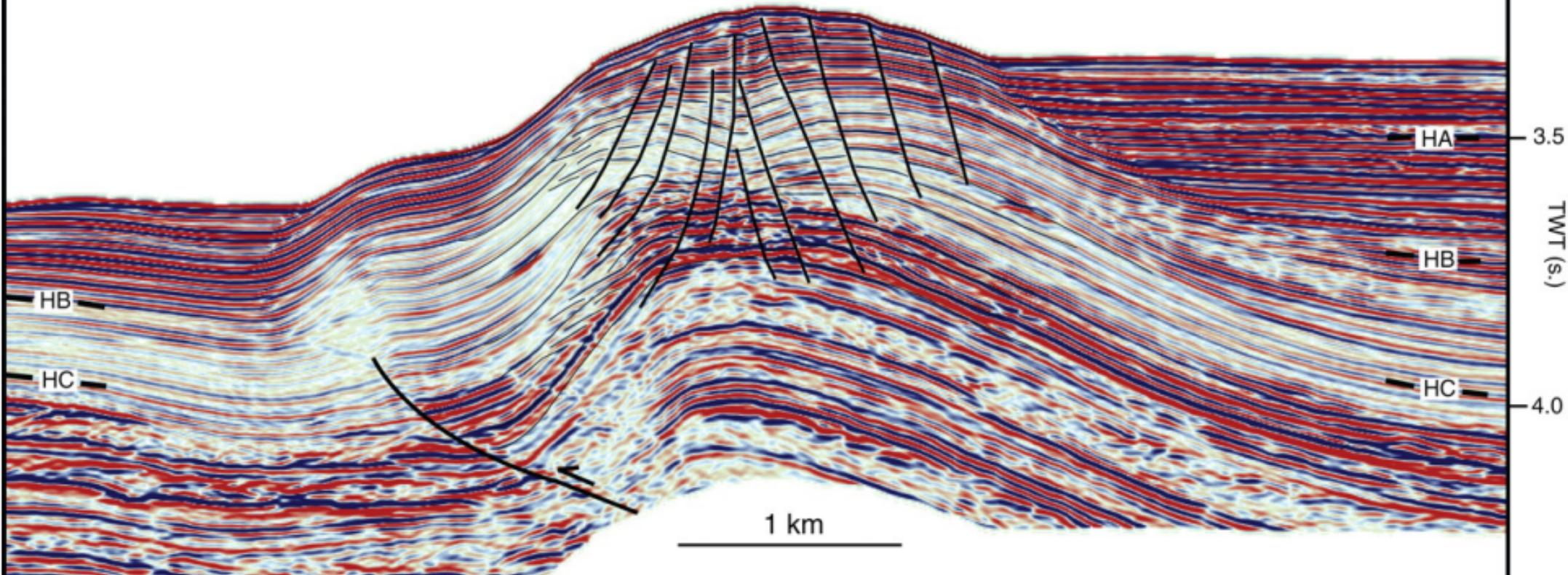
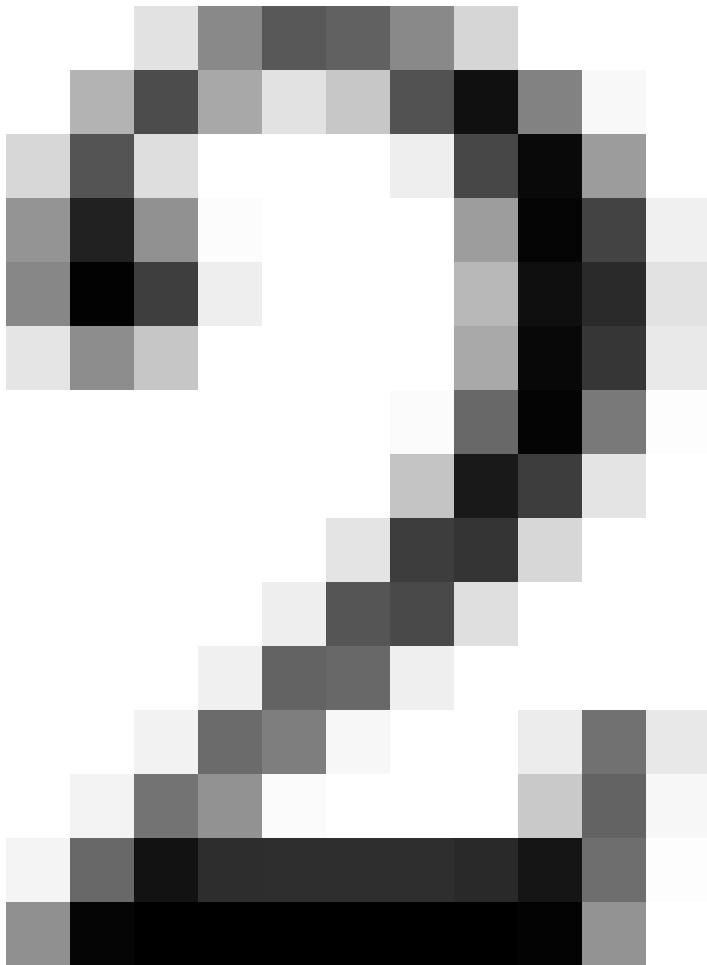


NW

Crestal normal faults

SE

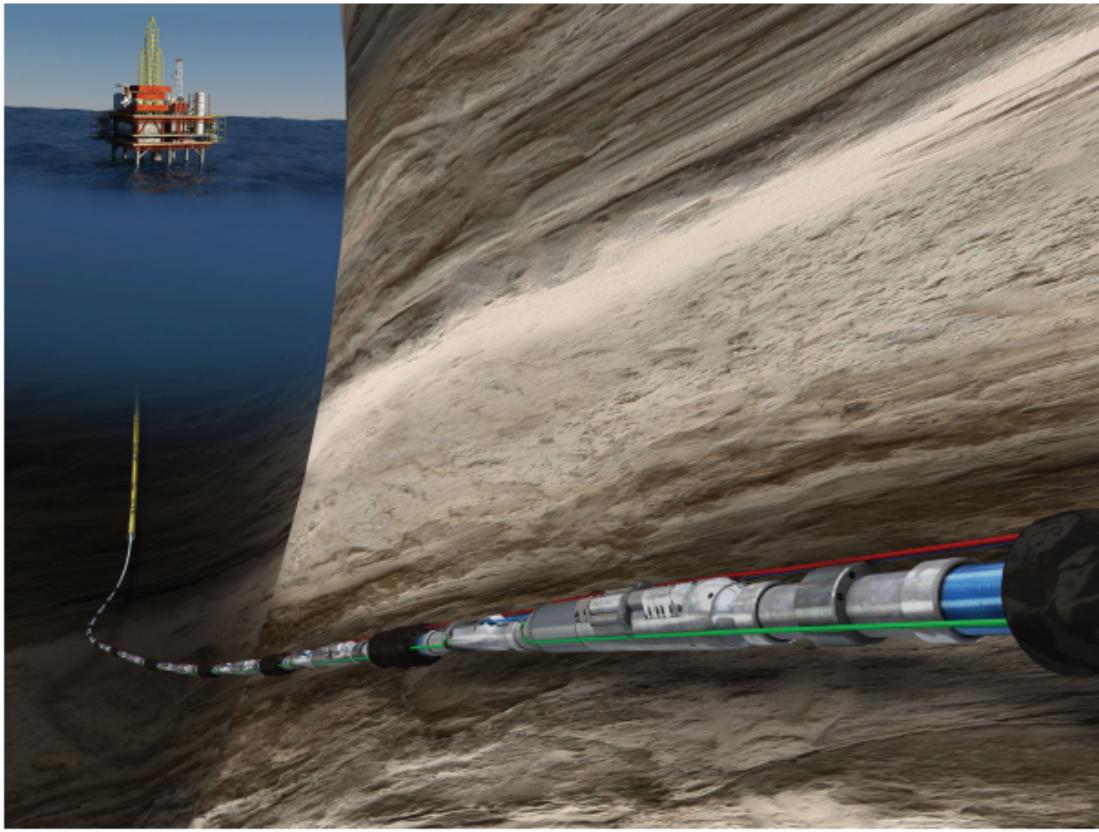


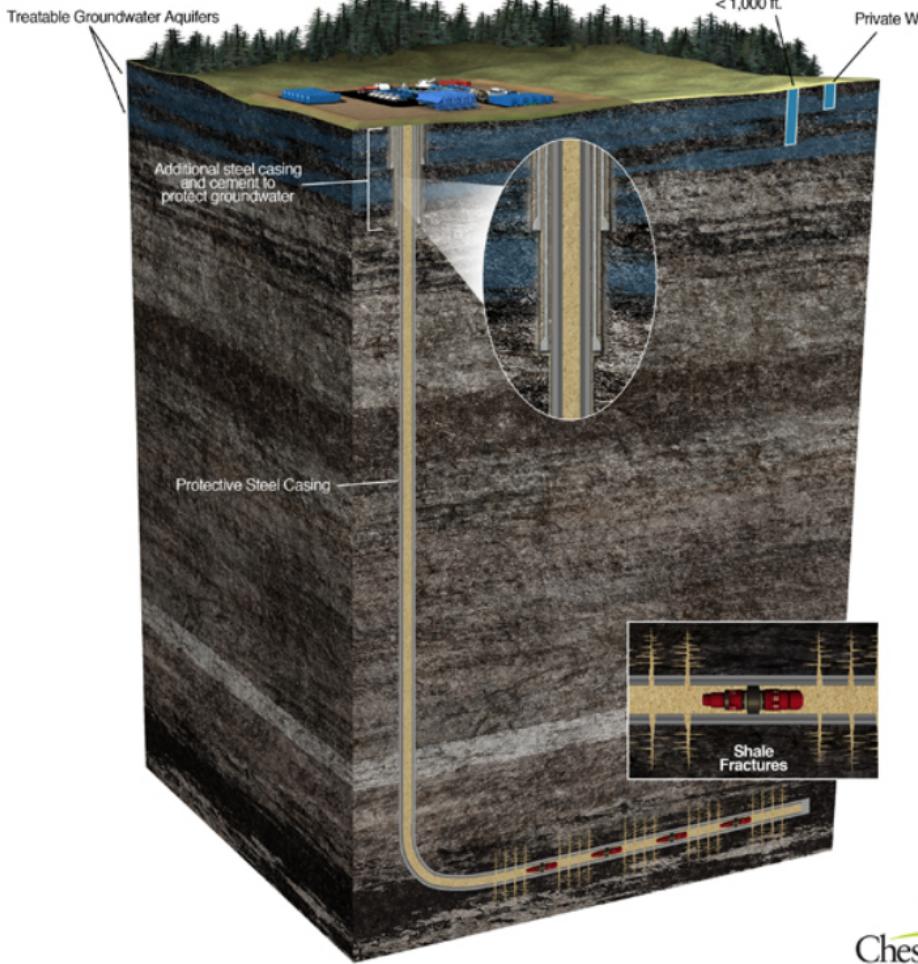


~1850



~2010





Risk-Based
Geomechanical
Screening

Stress Man

MUDLINE SUBSIDENCE



FAULT ACTIVATION

CASING CRUSHING

COMPACTION DRIVE

CASING
SHEAR

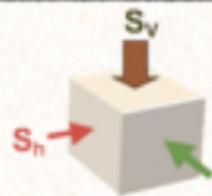
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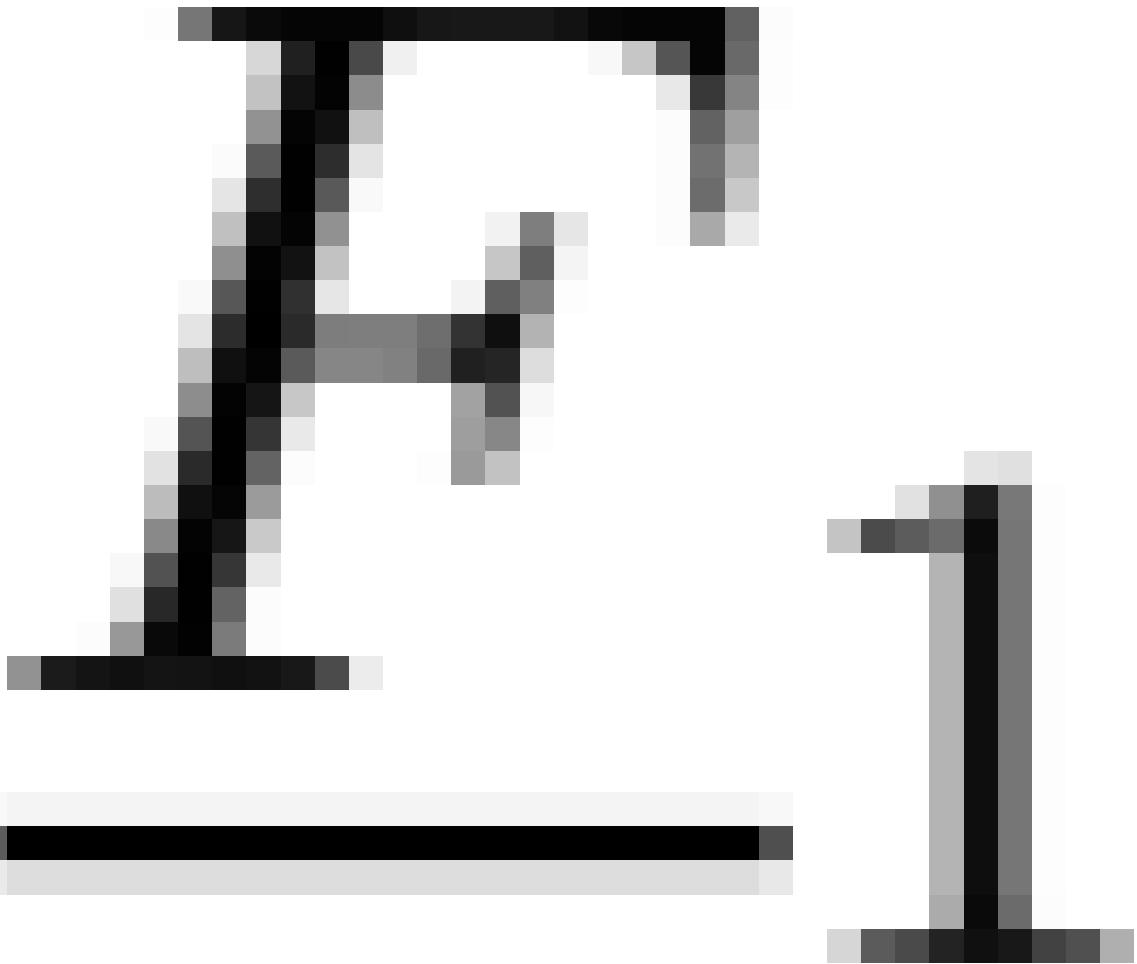


RESERVOIR YIELD

PERMEABILITY LOSS

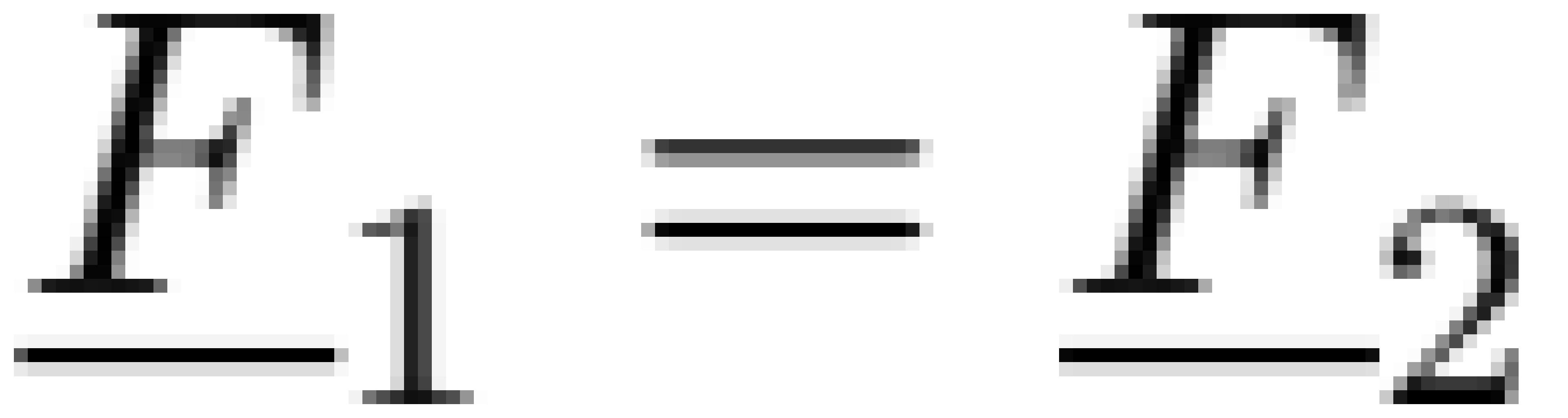
CASING BUCKLING



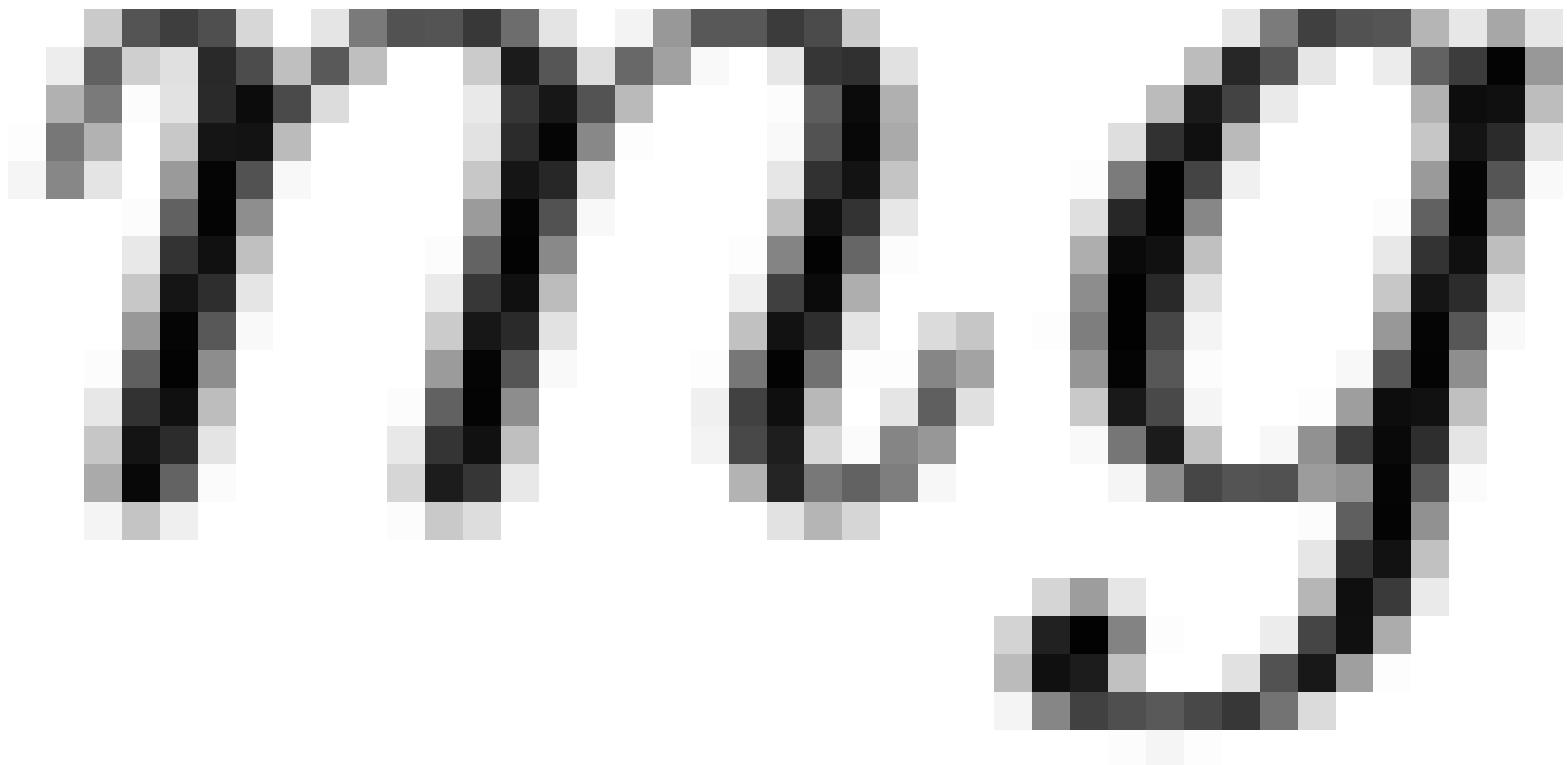


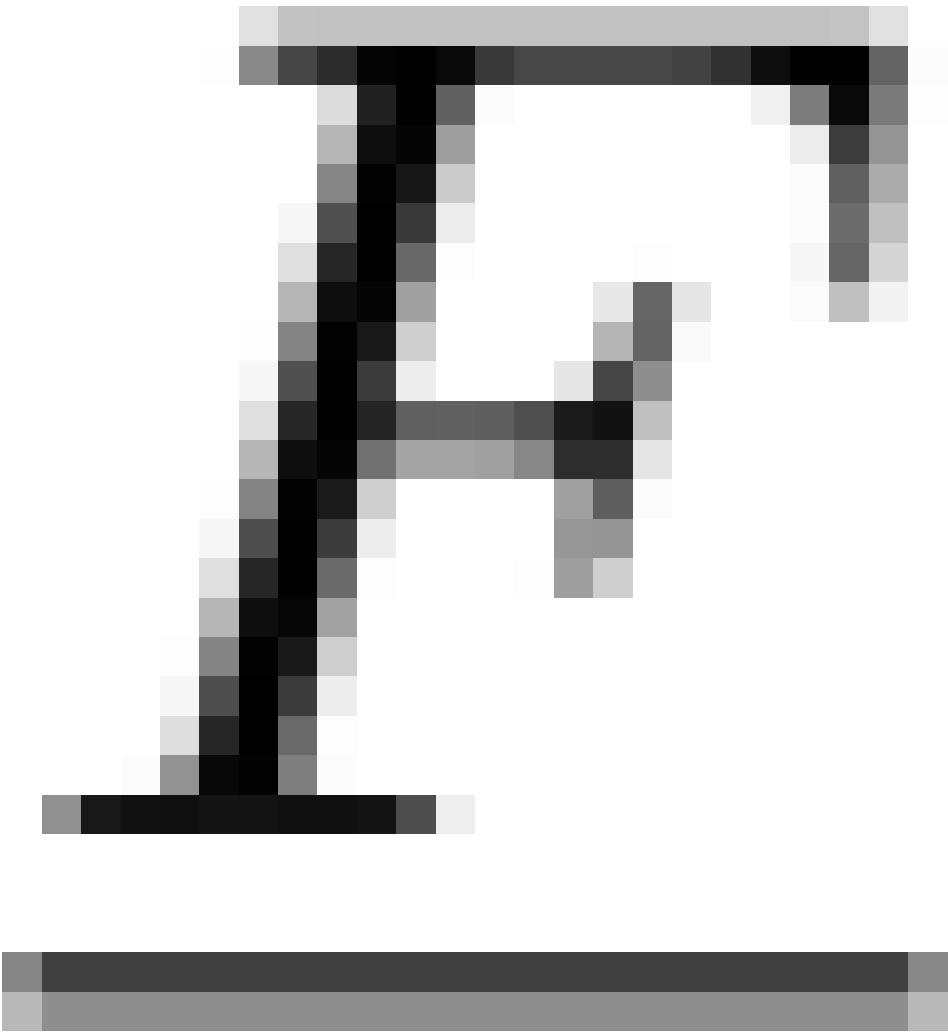


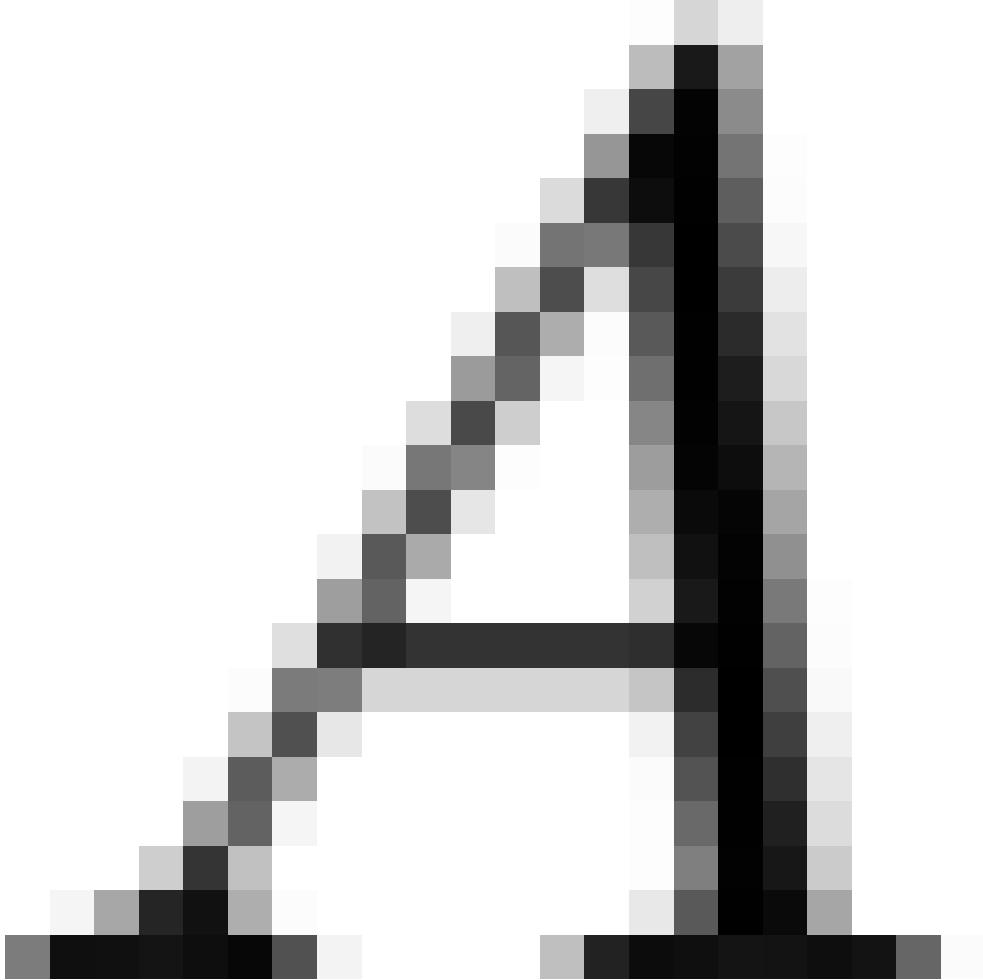












P

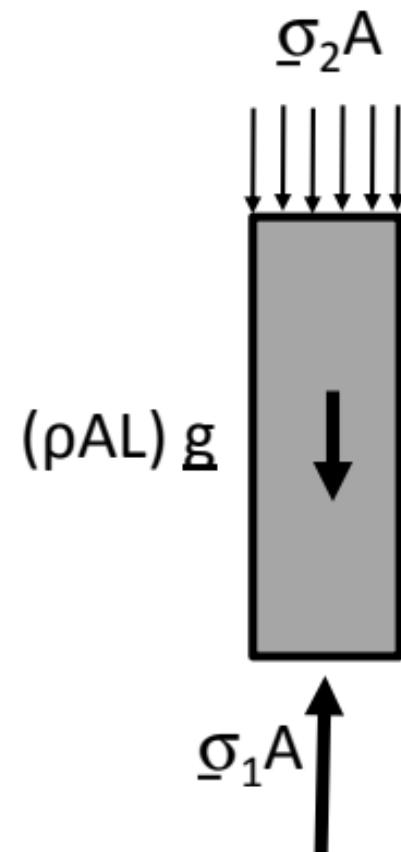
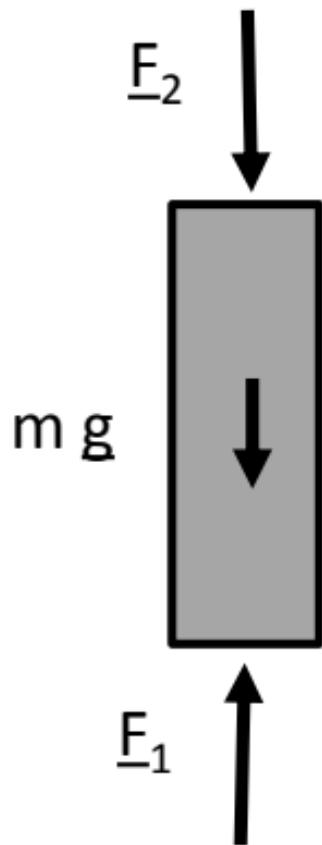
S

S

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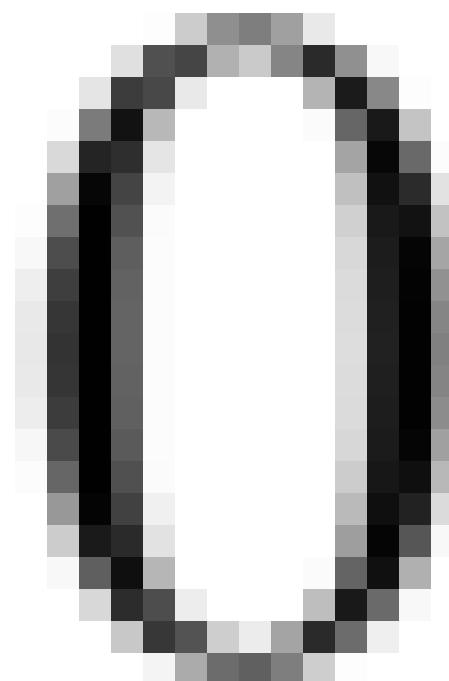
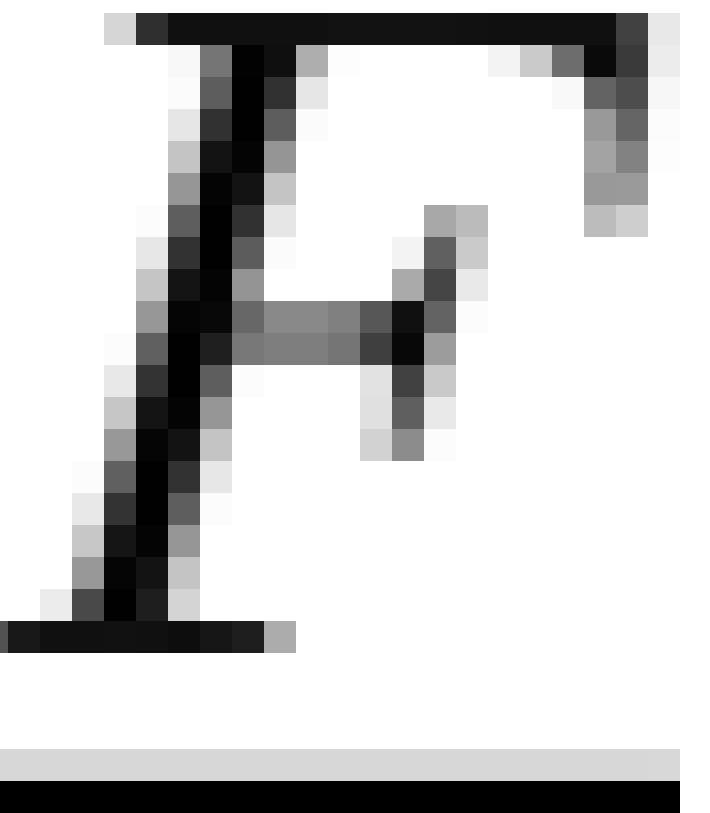
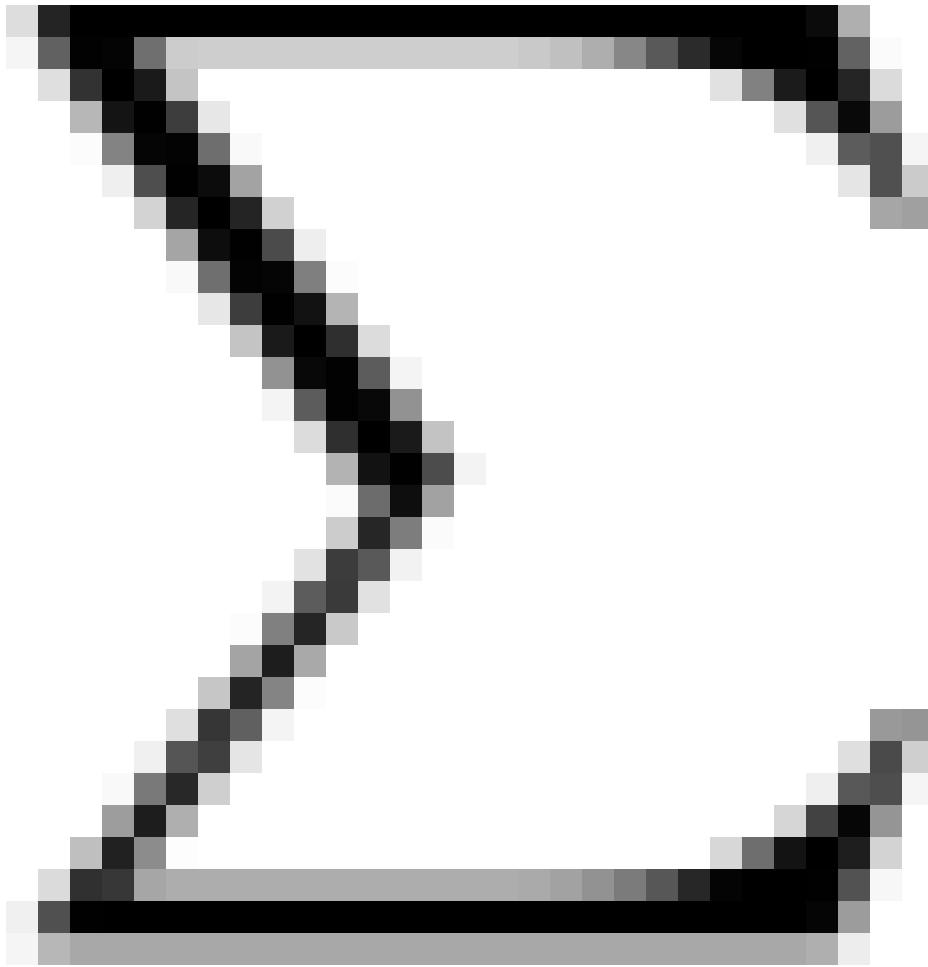


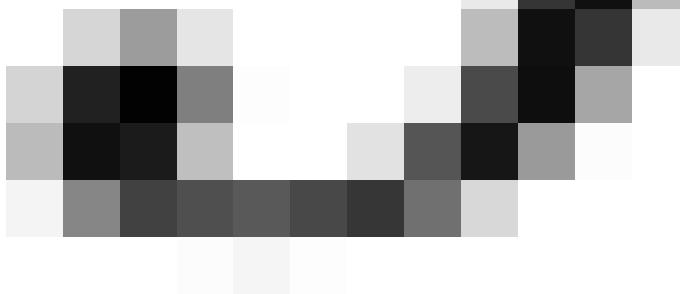
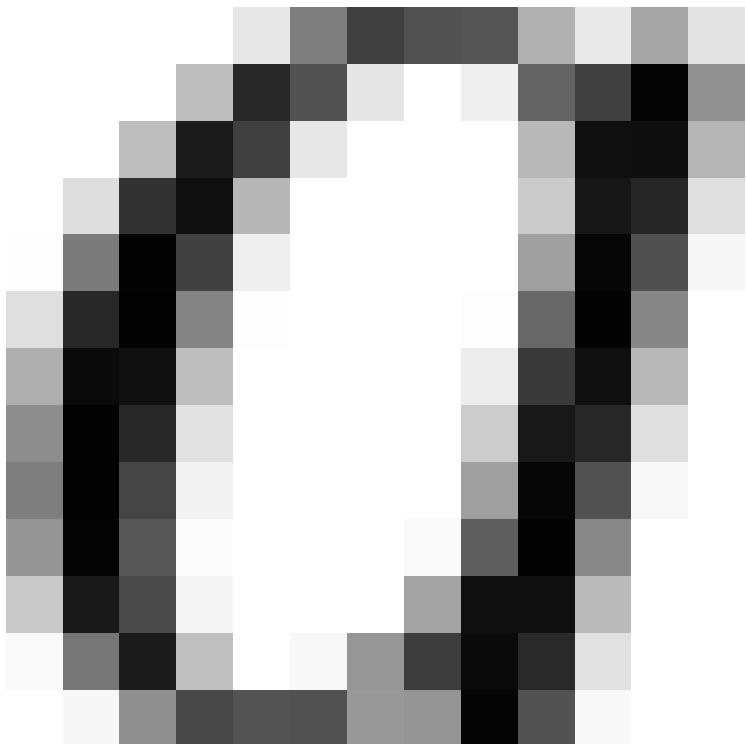
$$\Sigma F_z = +F_1 - F_2 = 0$$

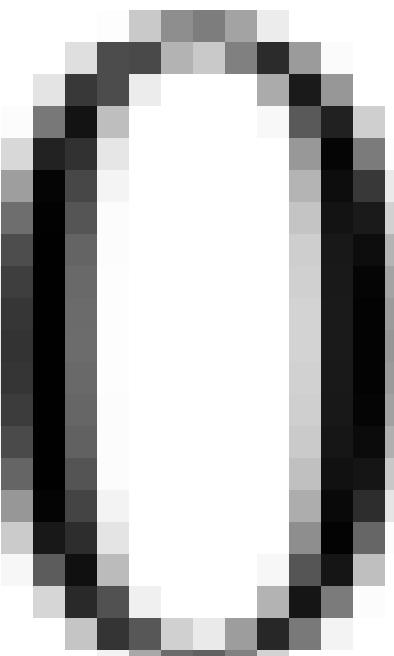
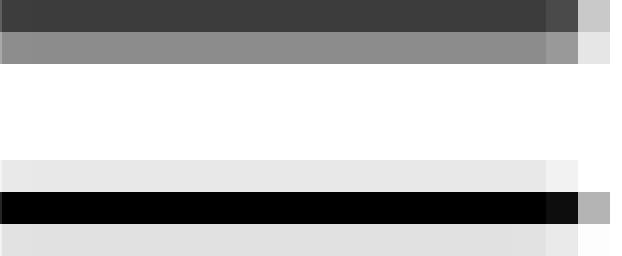
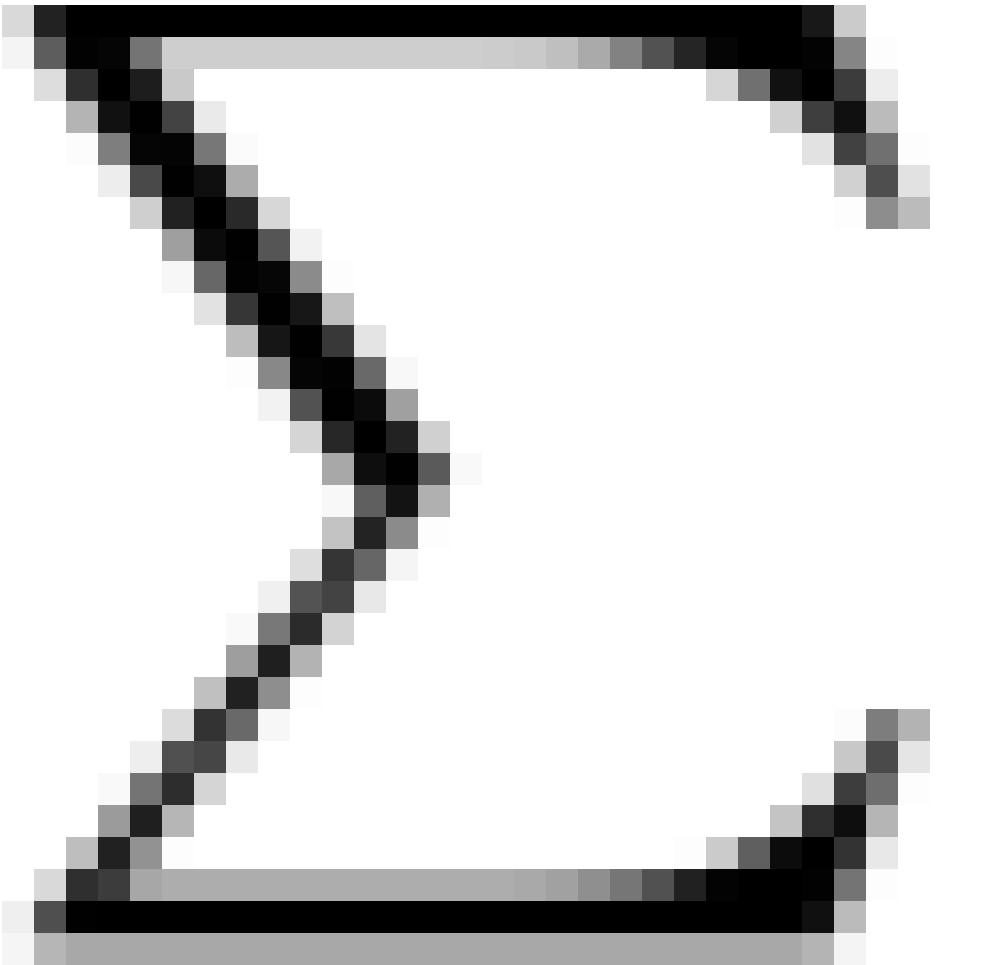
$$\Sigma F_z = +F_1 - m g - F_2$$

$$\Sigma F_z = +\sigma_1 A - (\rho A L)g - \sigma_2 A$$











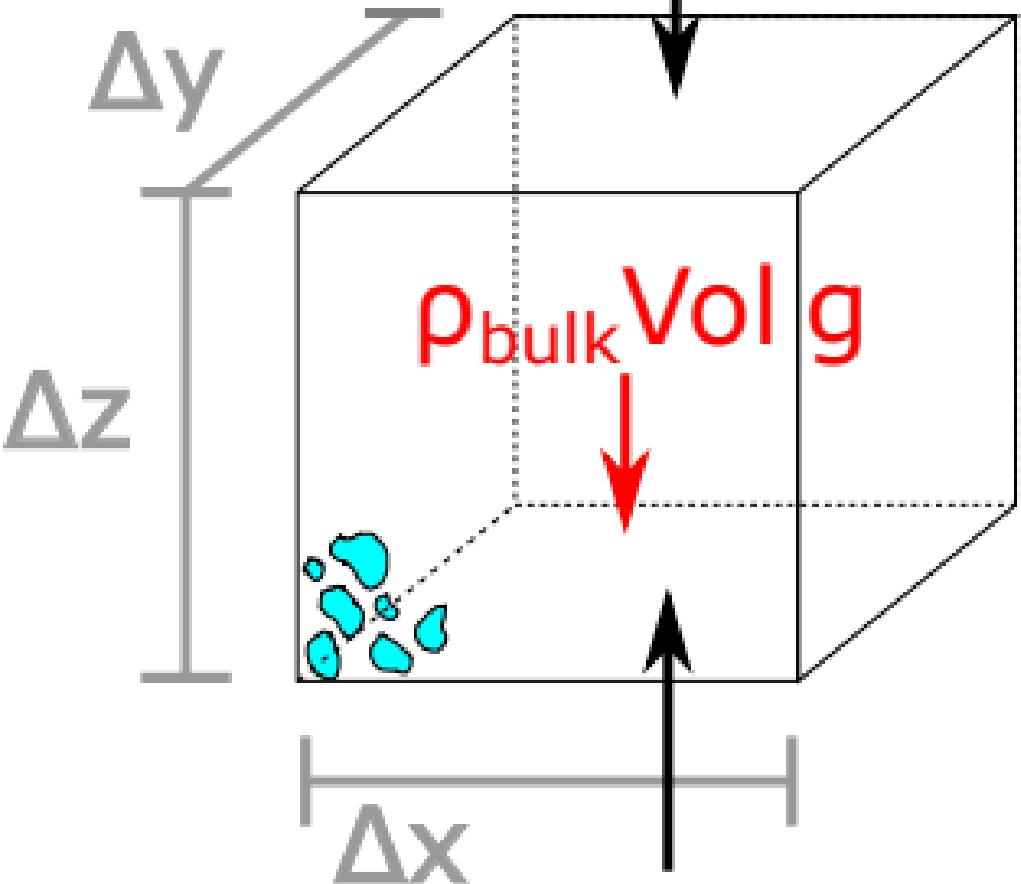


△ S₂

△ z

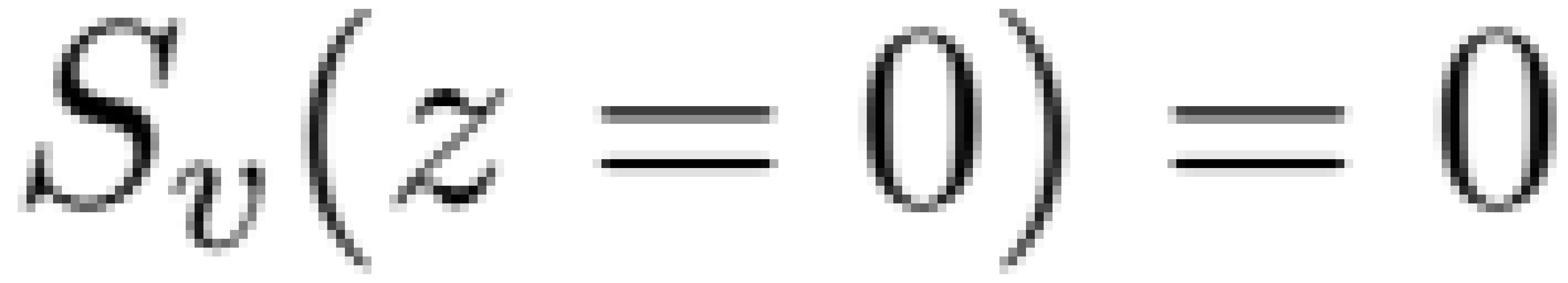
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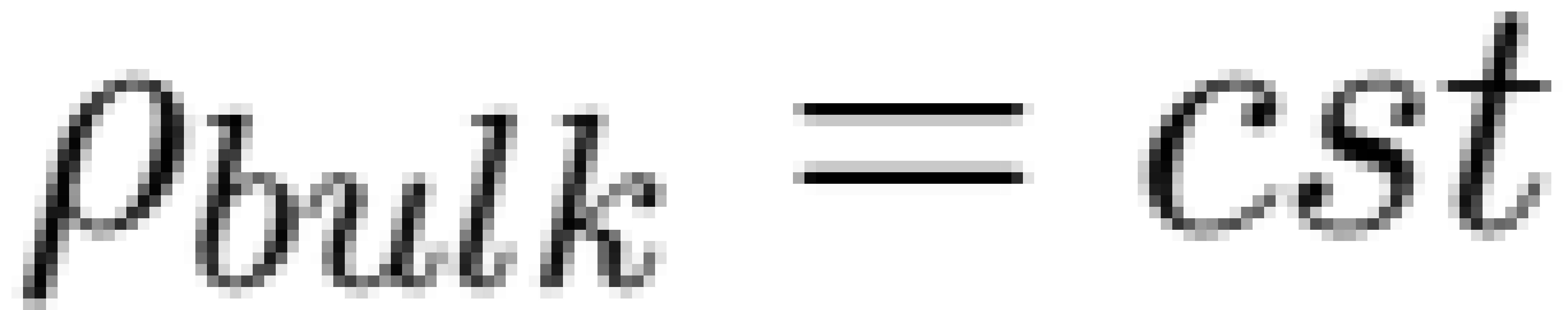
Observe 9

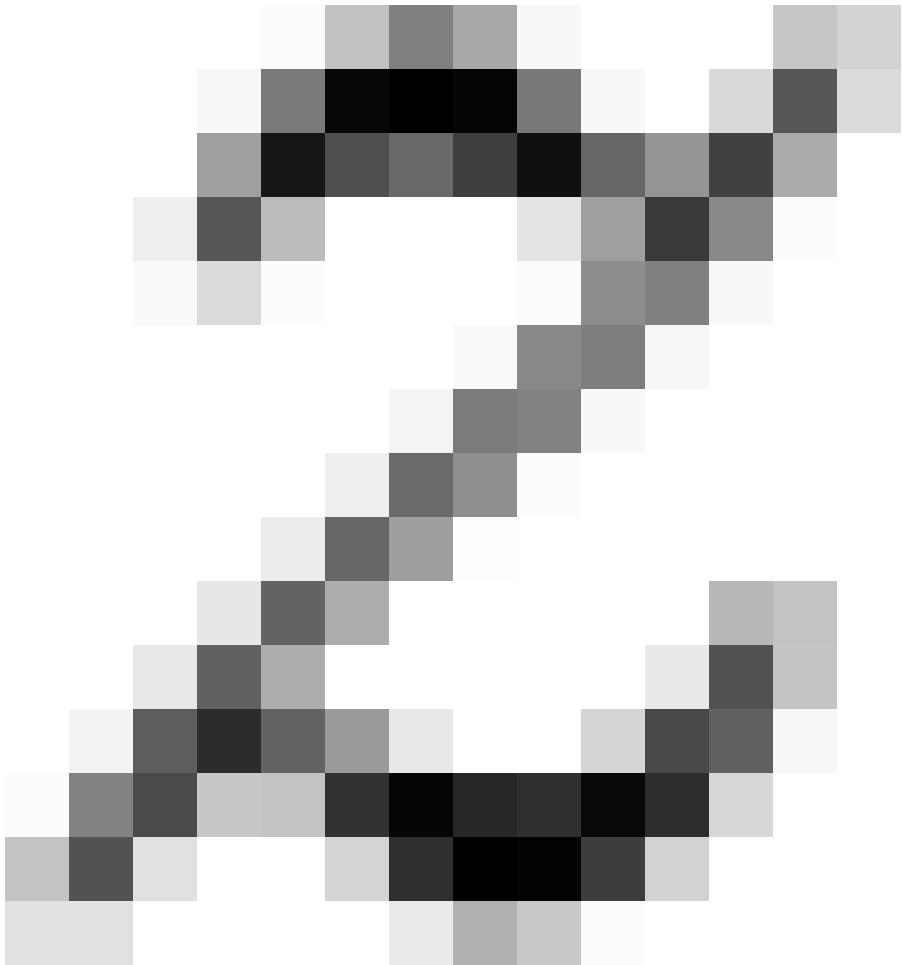
$S_v A$  $S_v A + \Delta S_v A$

$$\int_0^{S_w(z)} ds_w = \int_0^z \rho_{bulk}(z) g dz$$







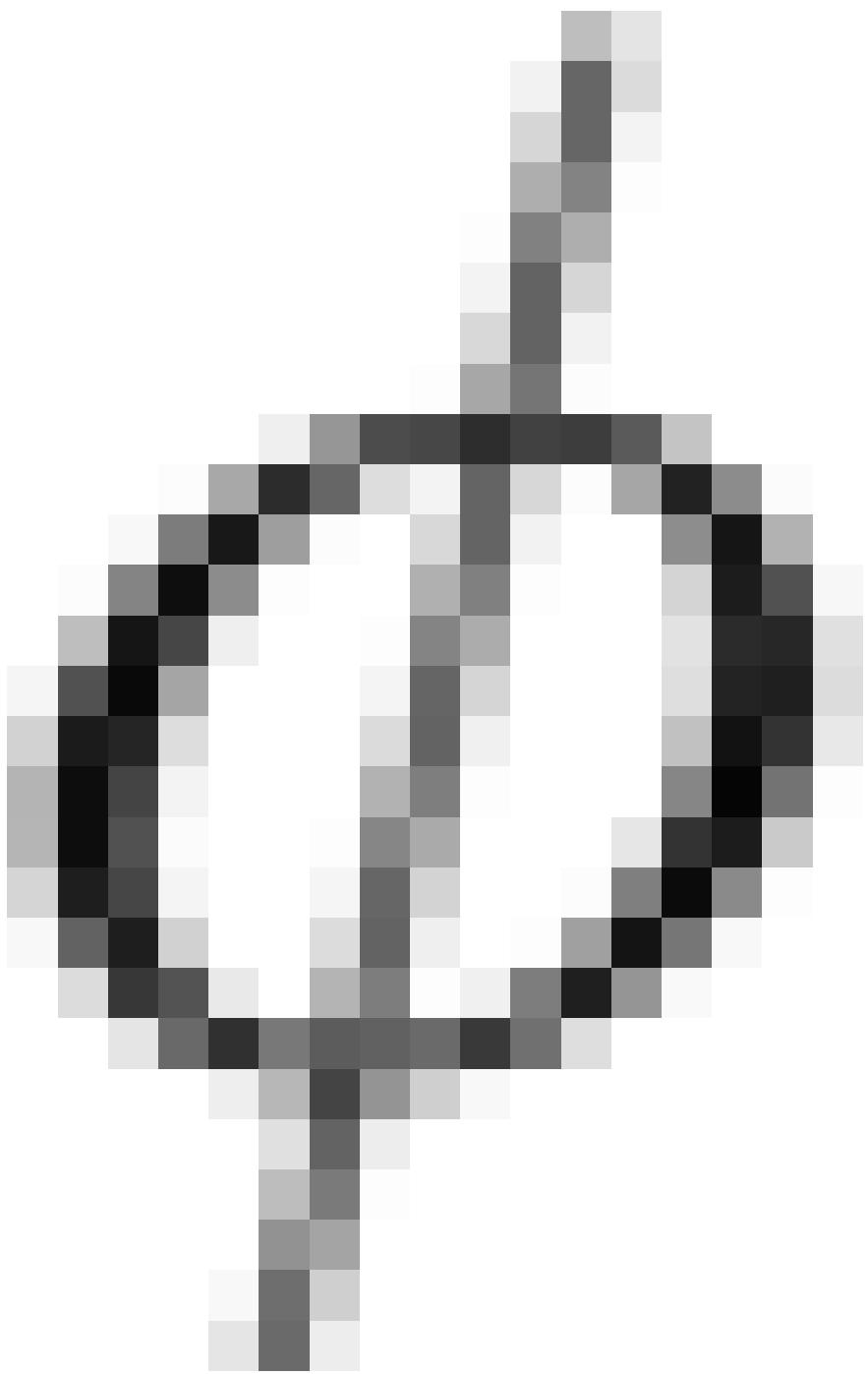


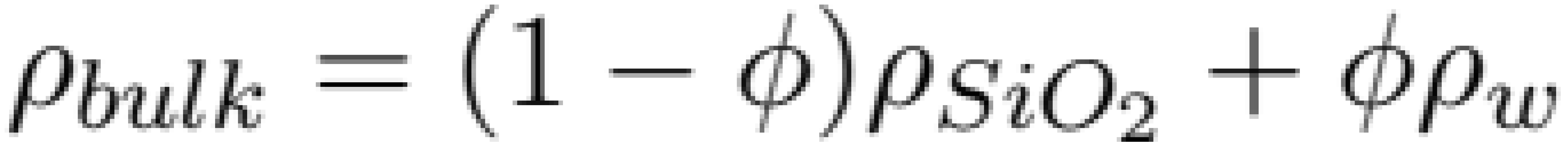




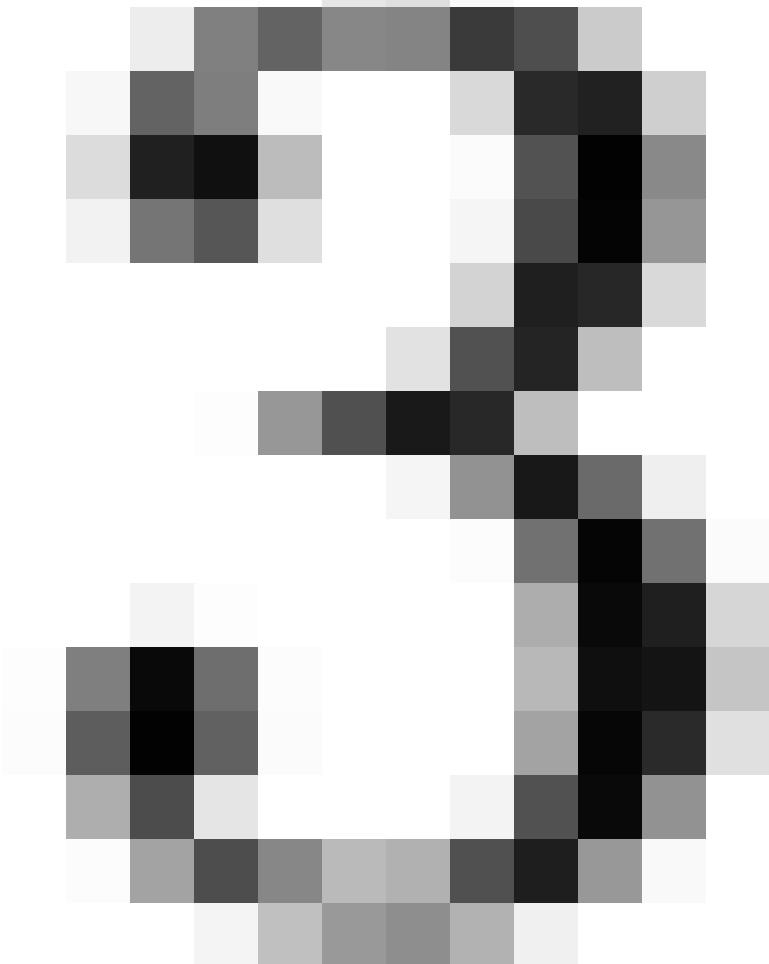


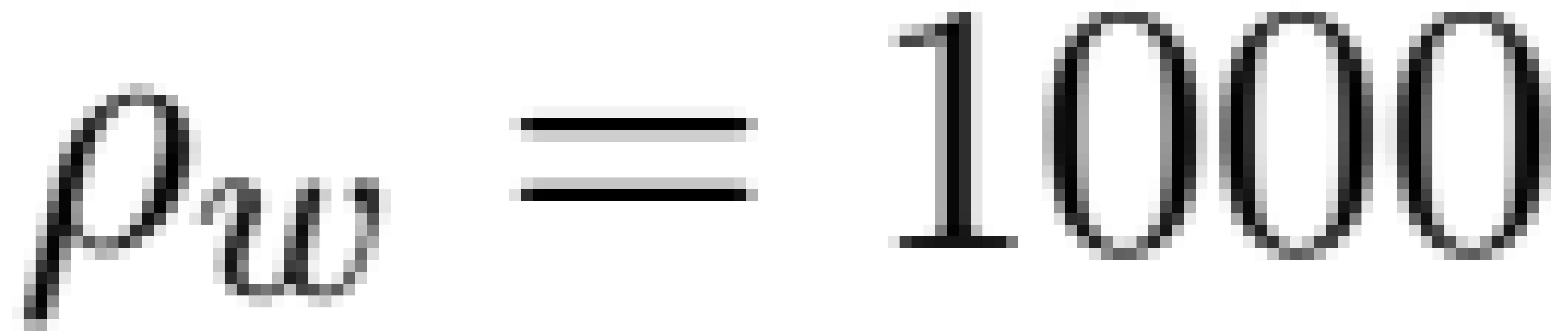


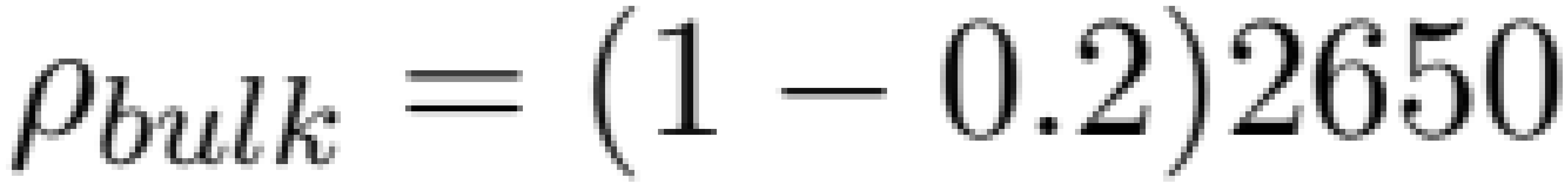


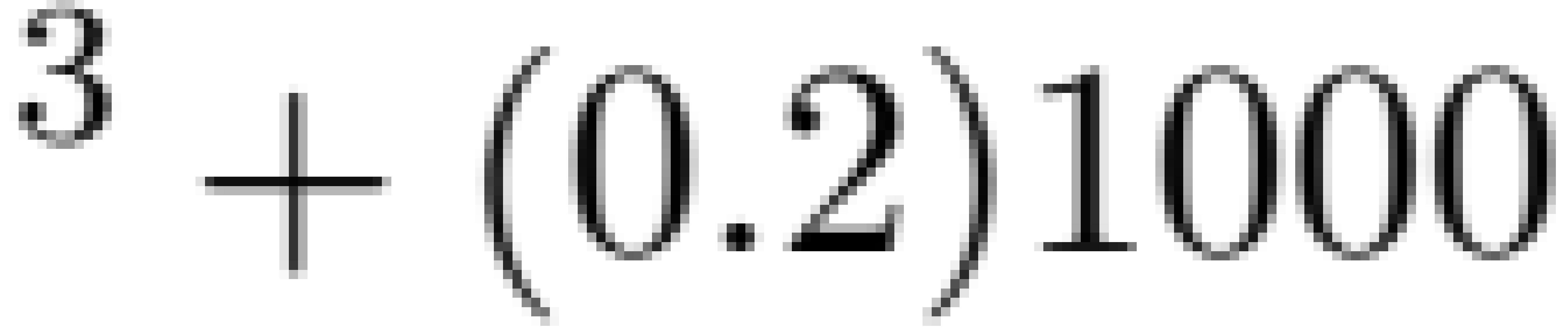




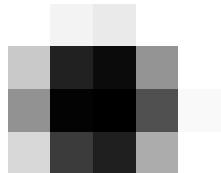
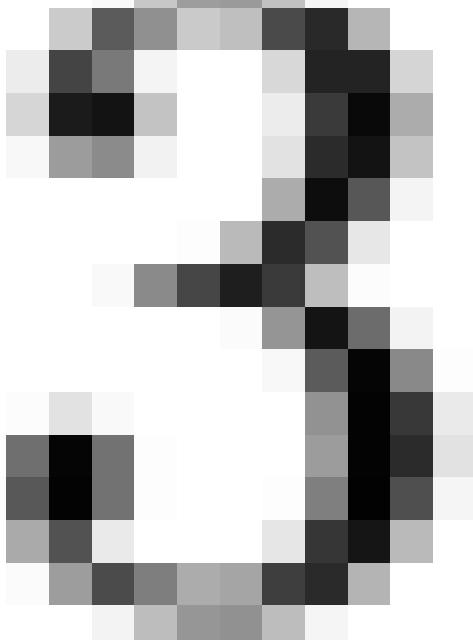


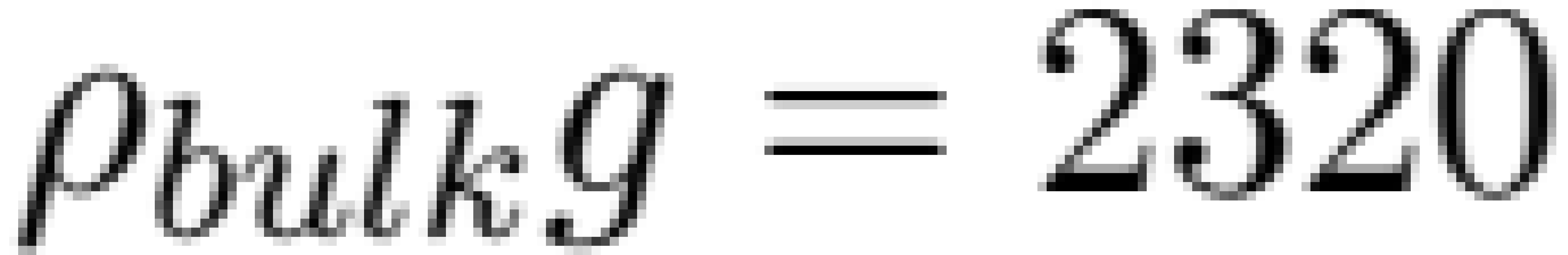




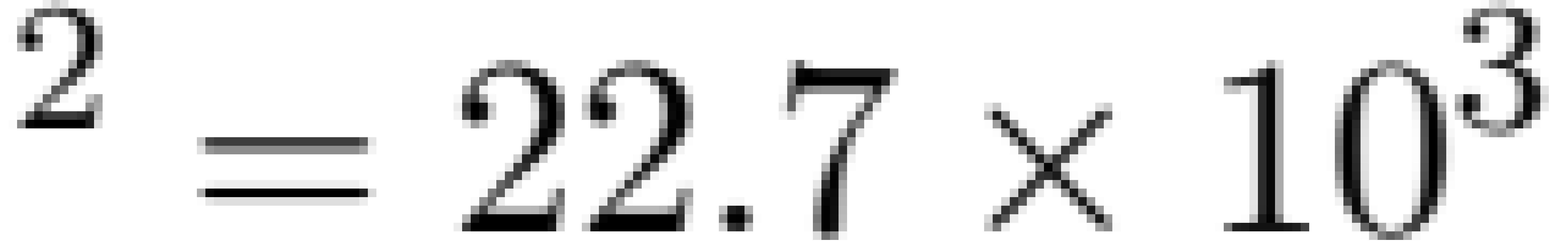


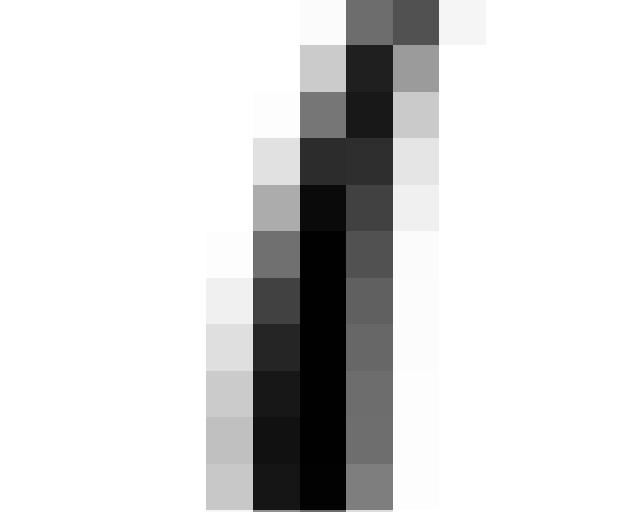
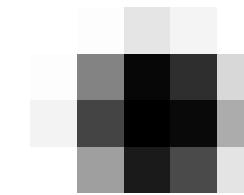
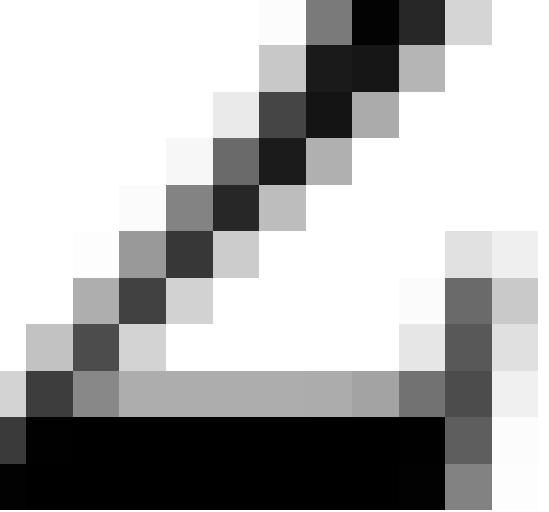
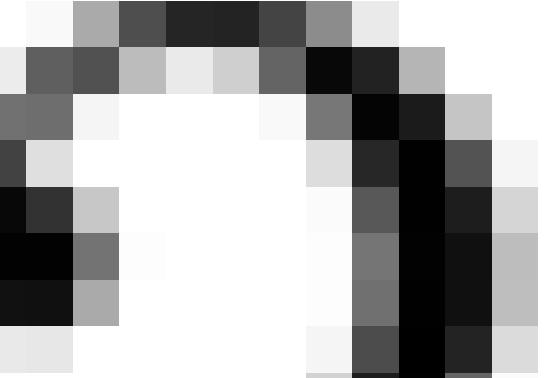
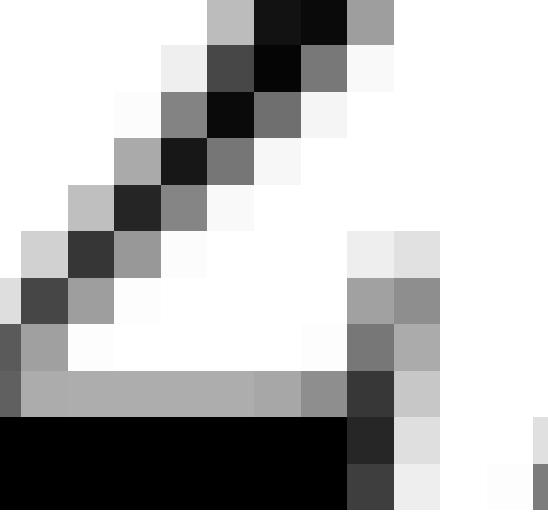
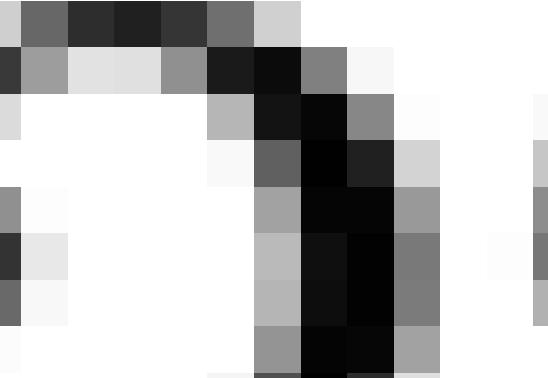


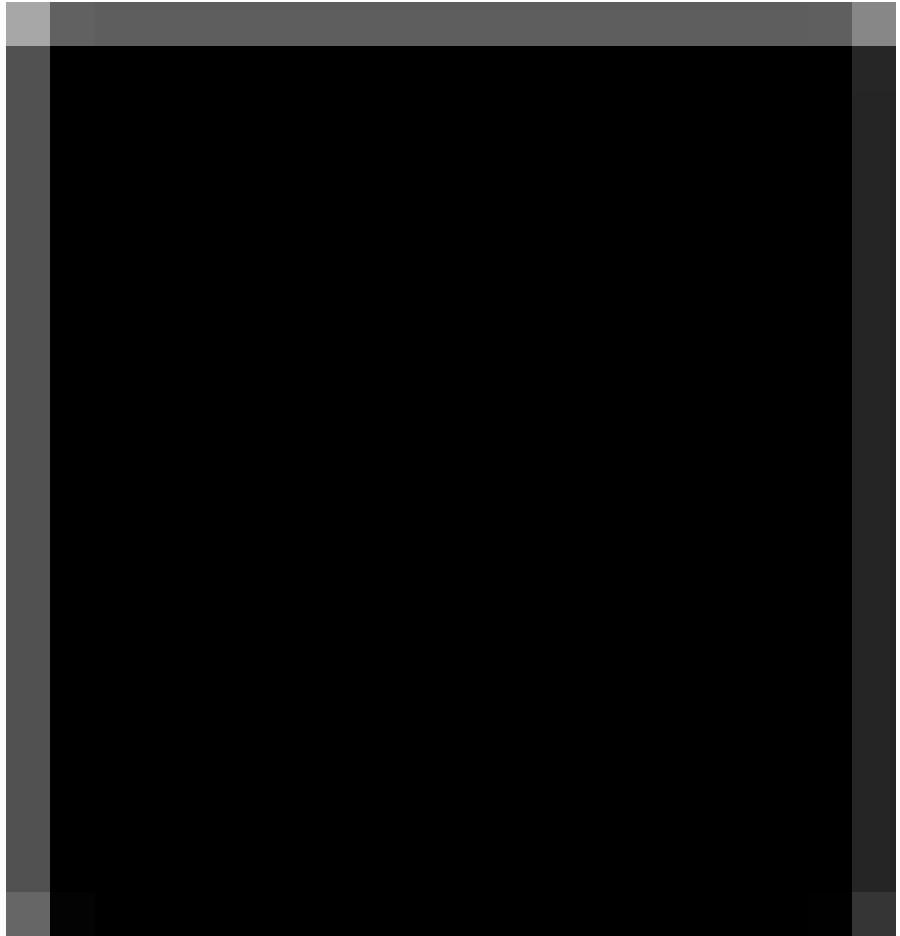
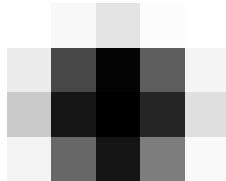


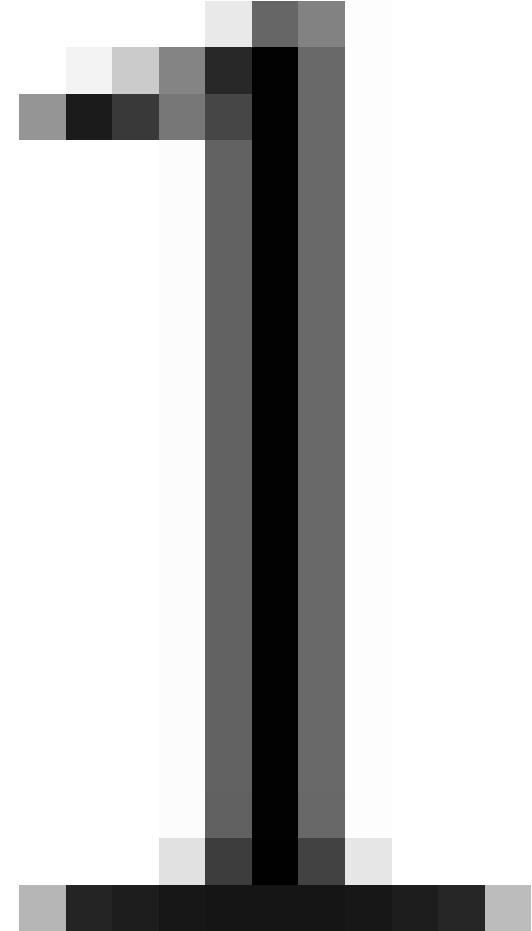
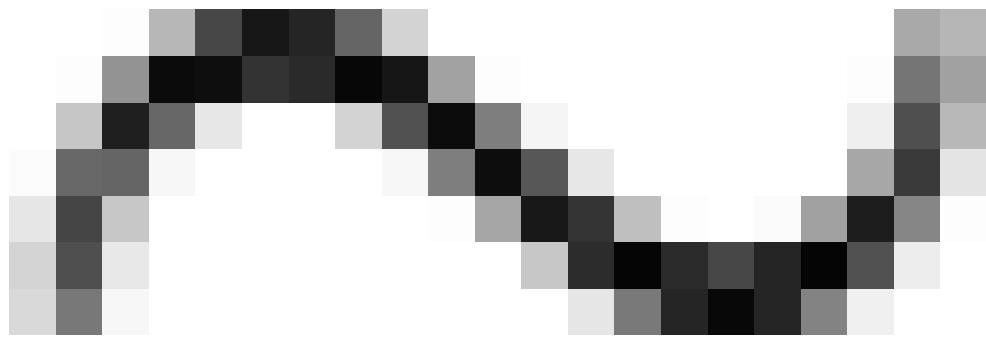


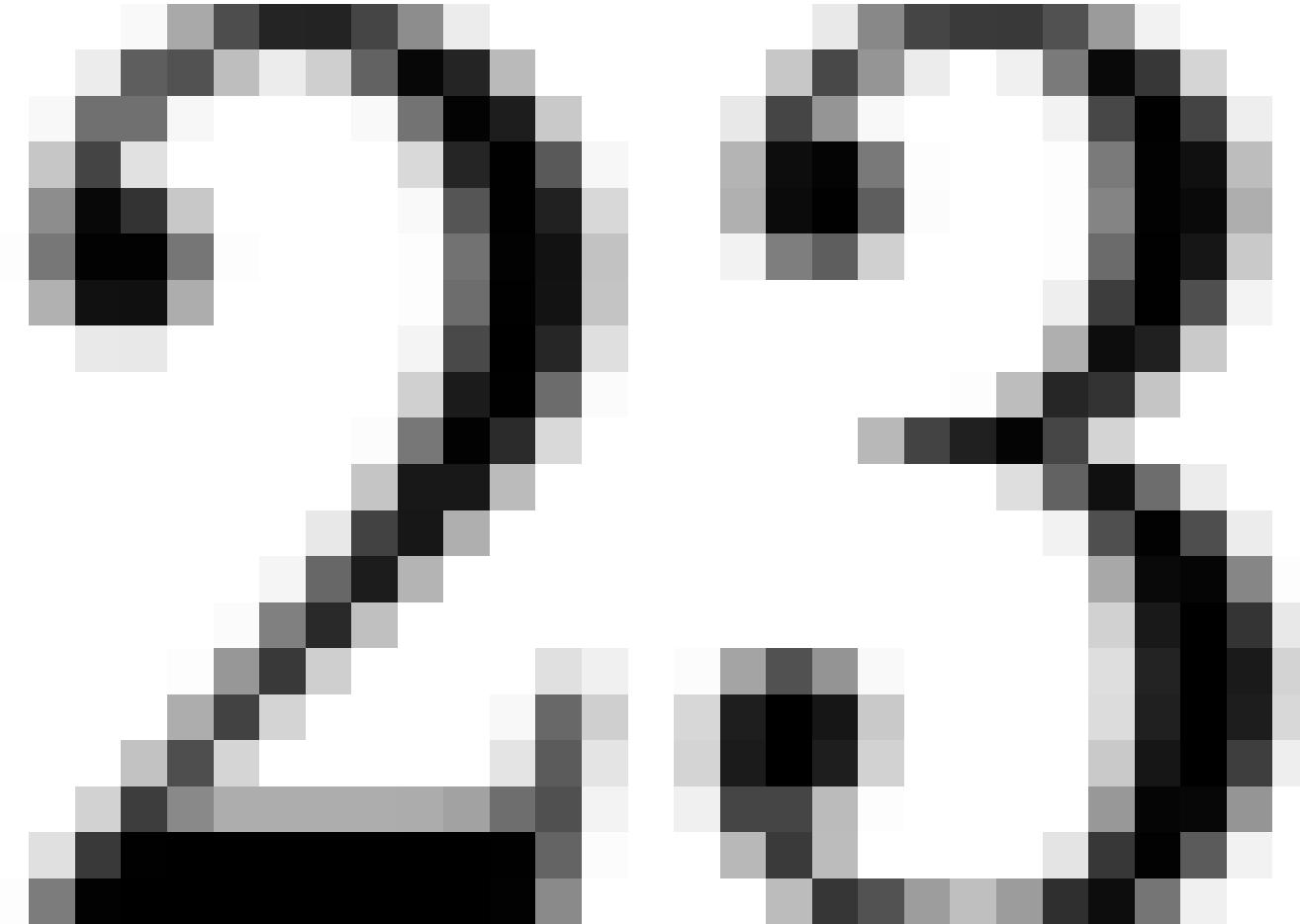
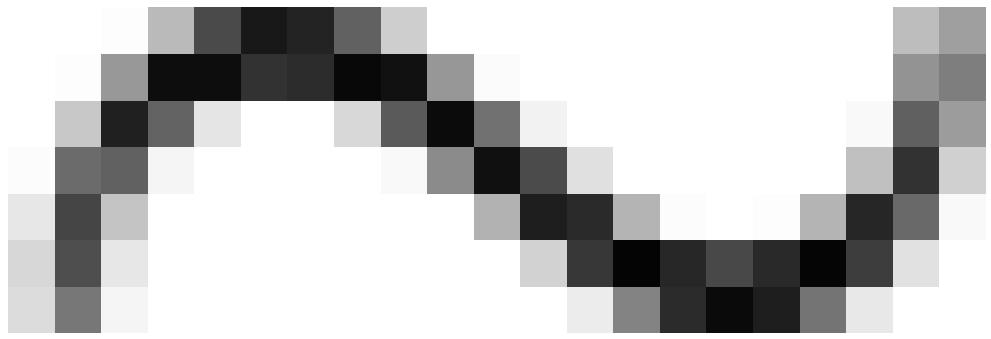


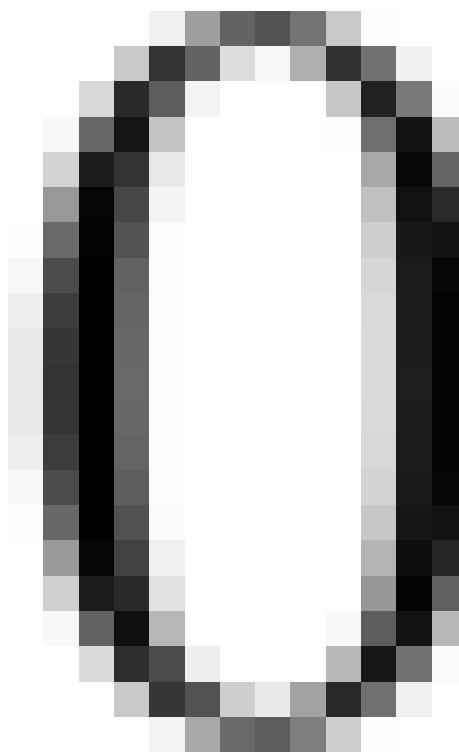
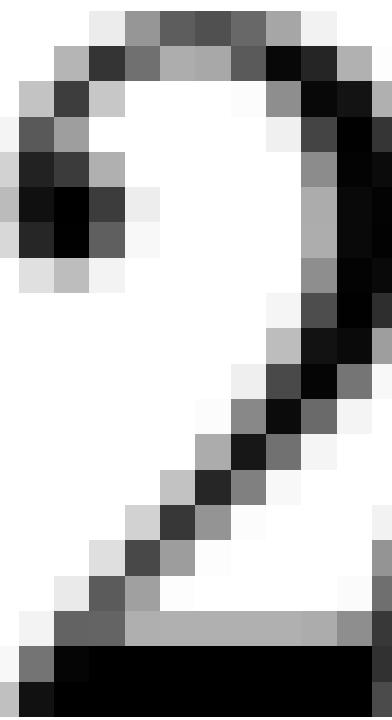
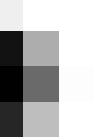
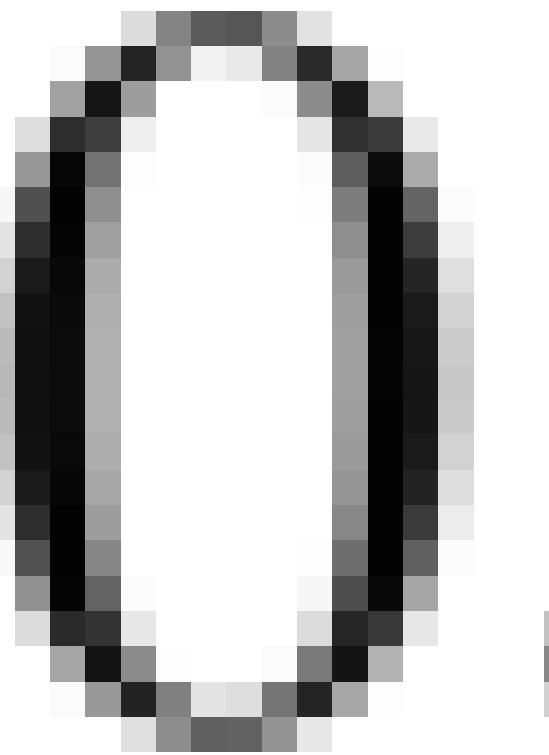
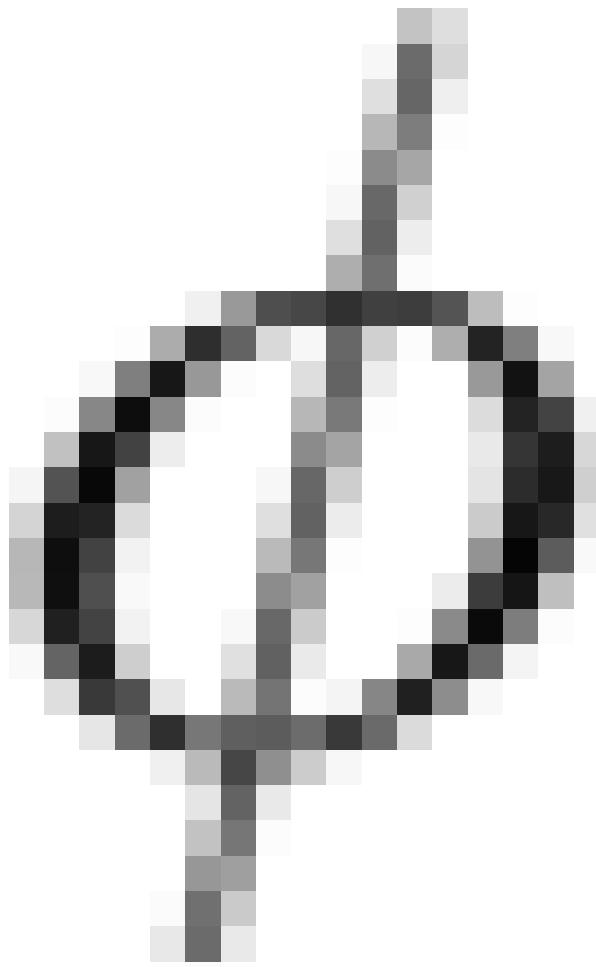


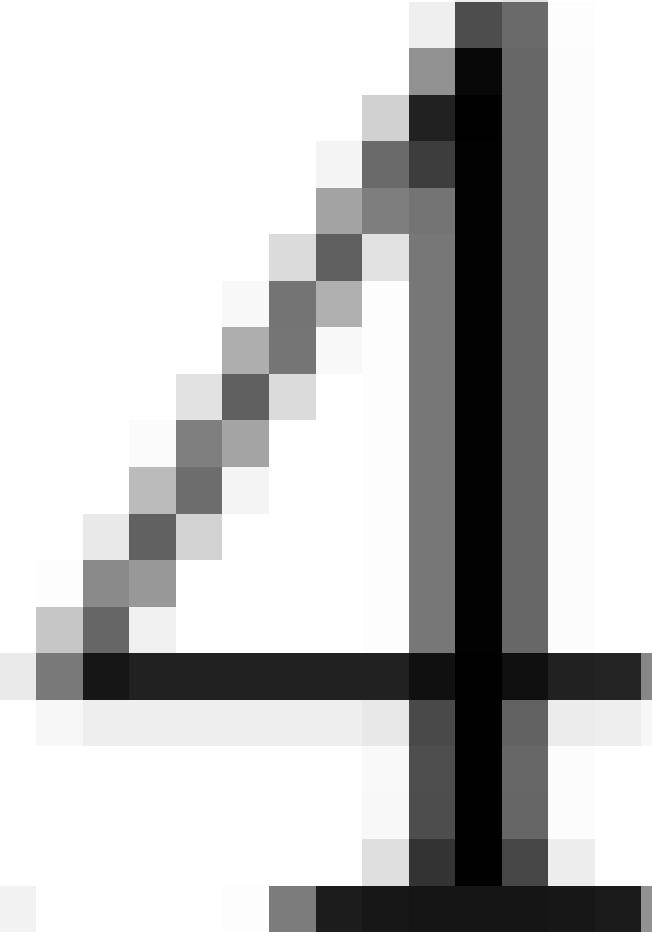
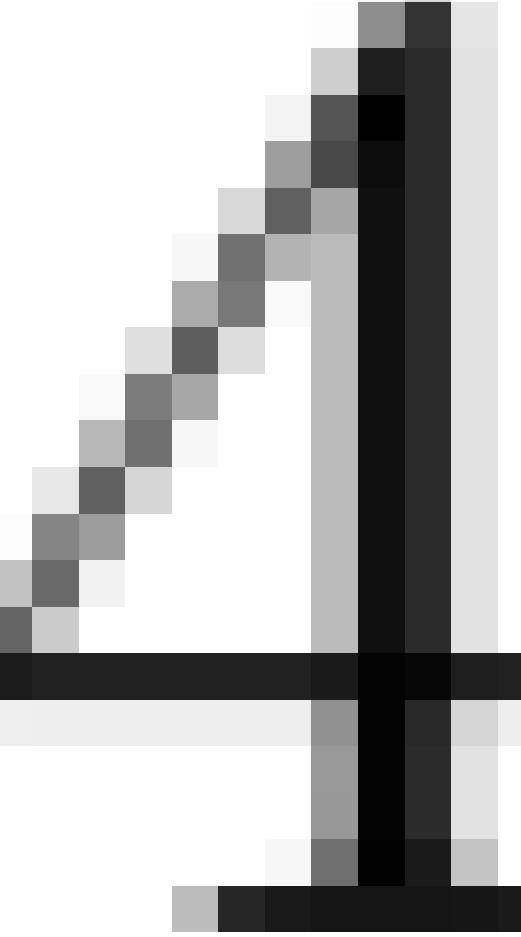
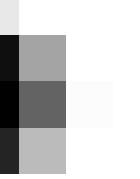
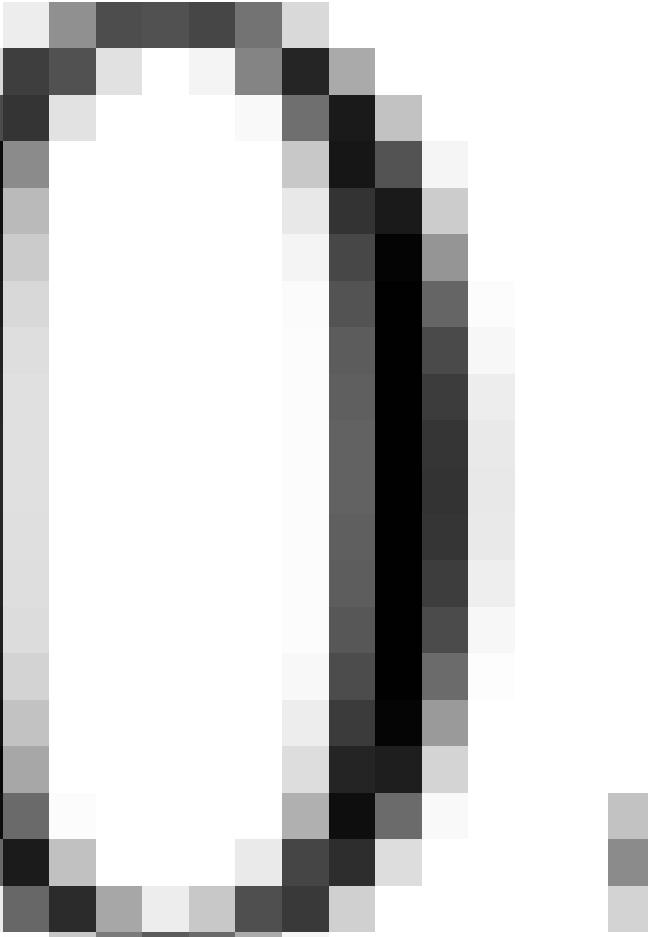
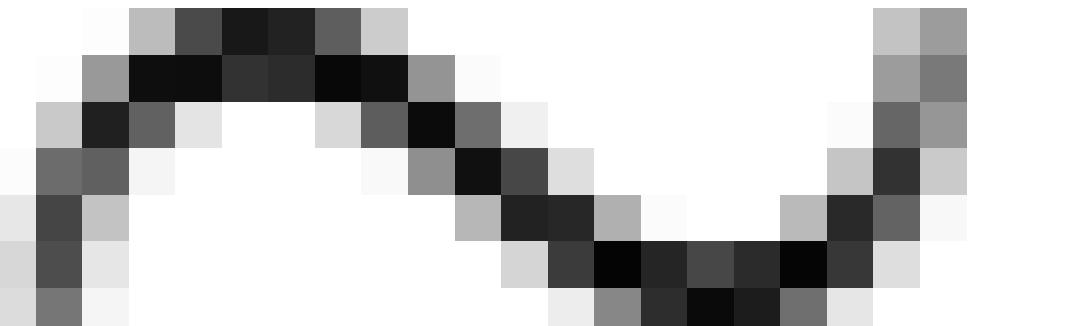


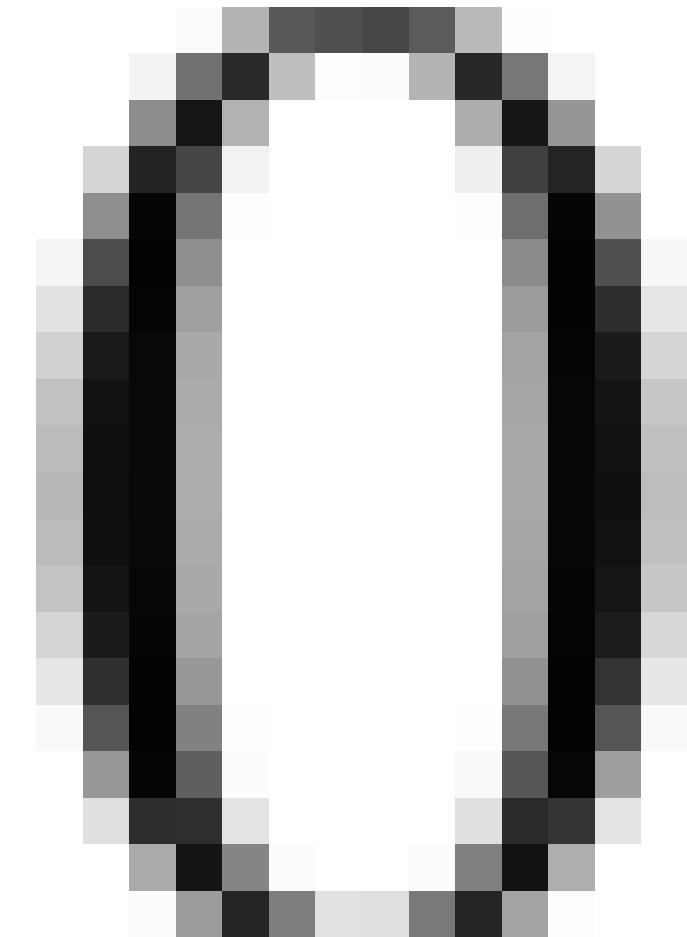
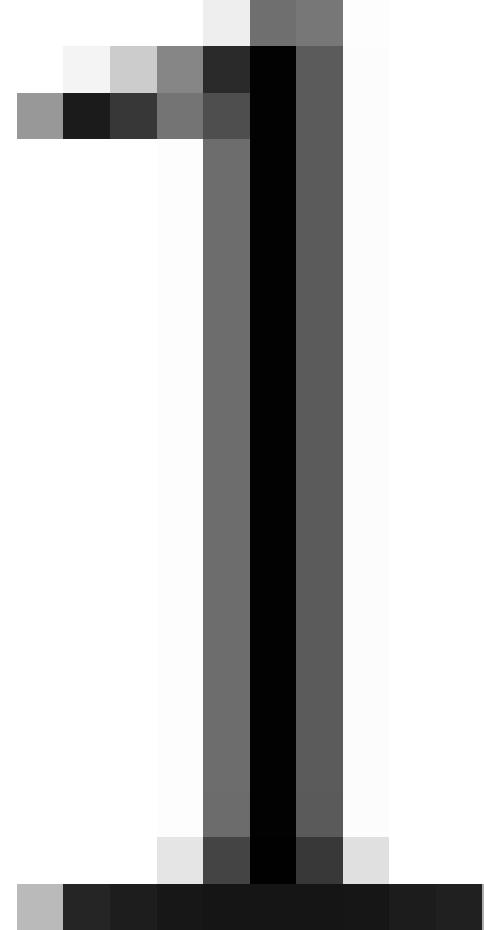
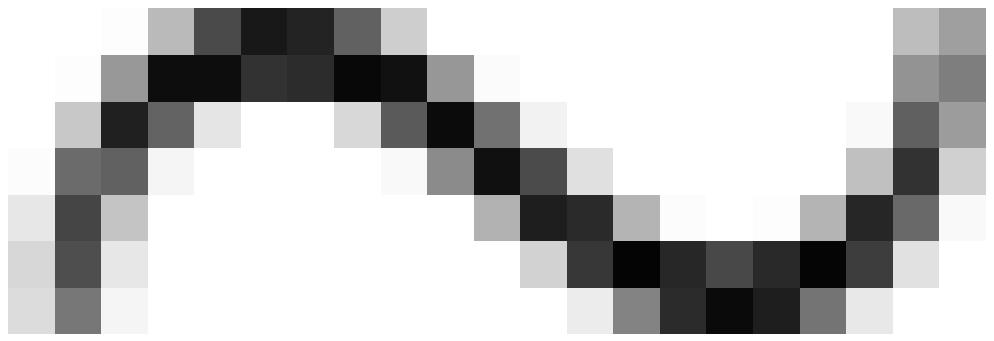




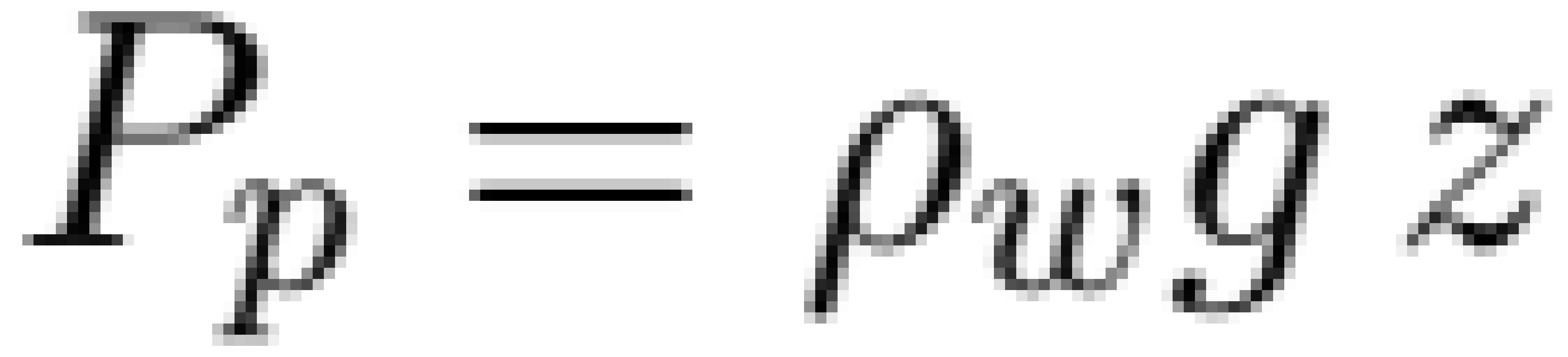


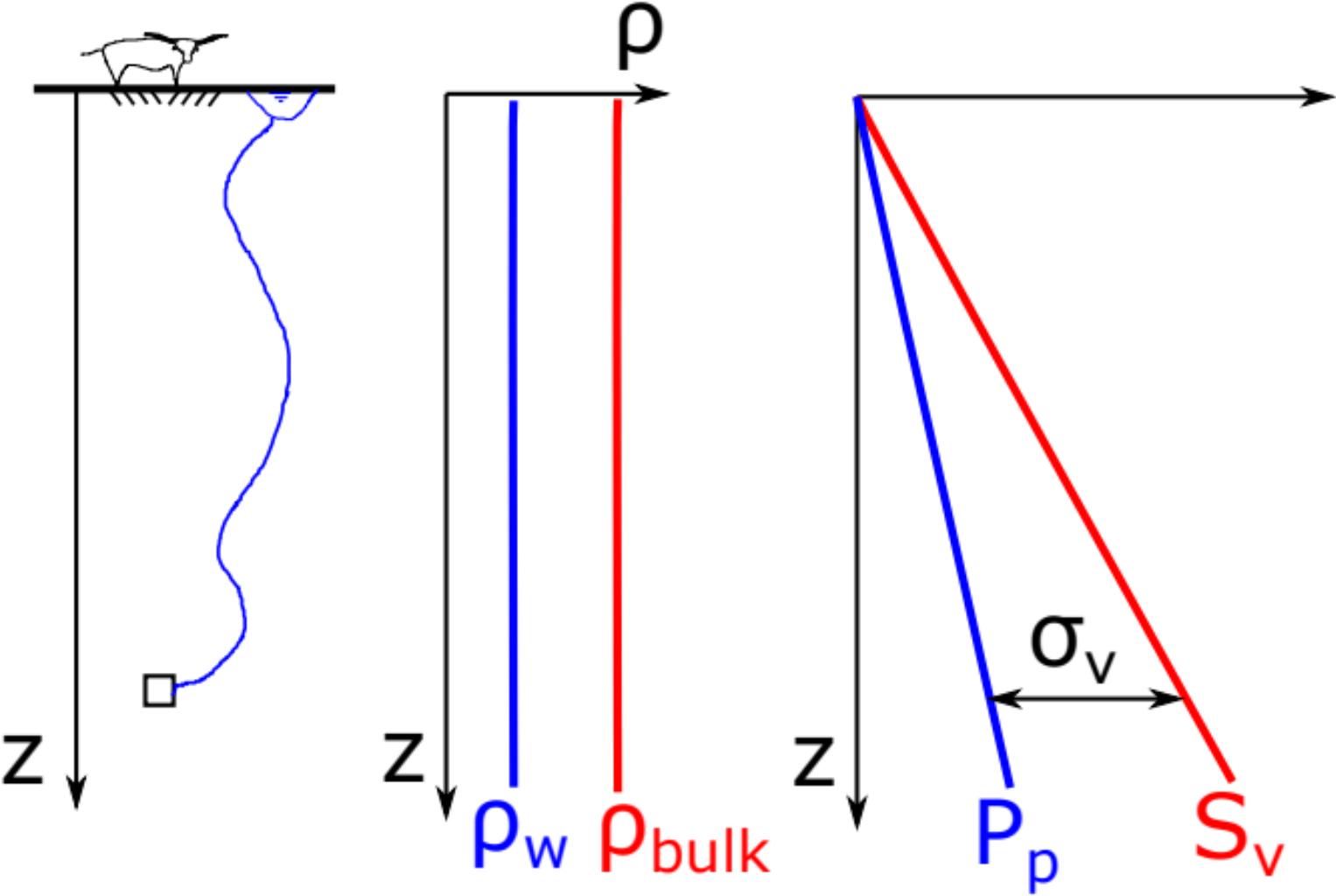




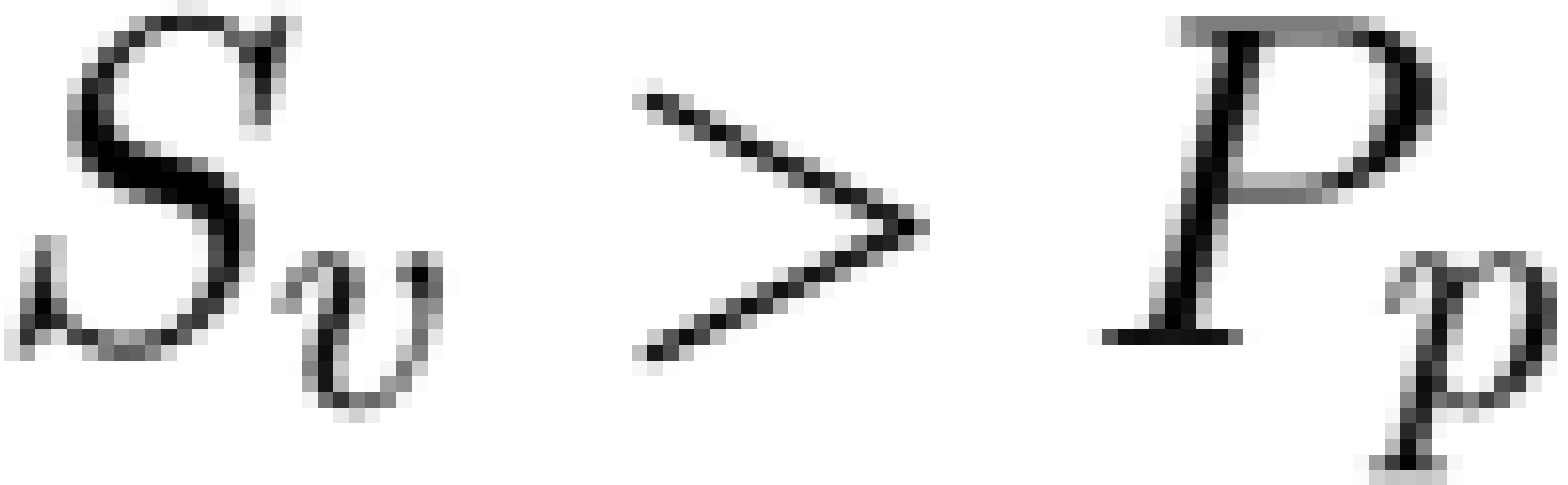








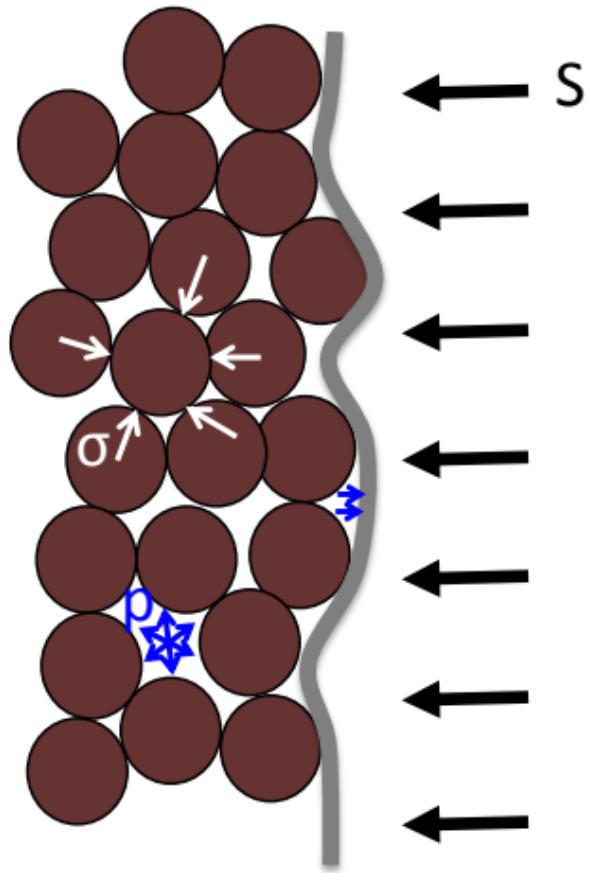




Effective stress =

Total stress – Pore pressure

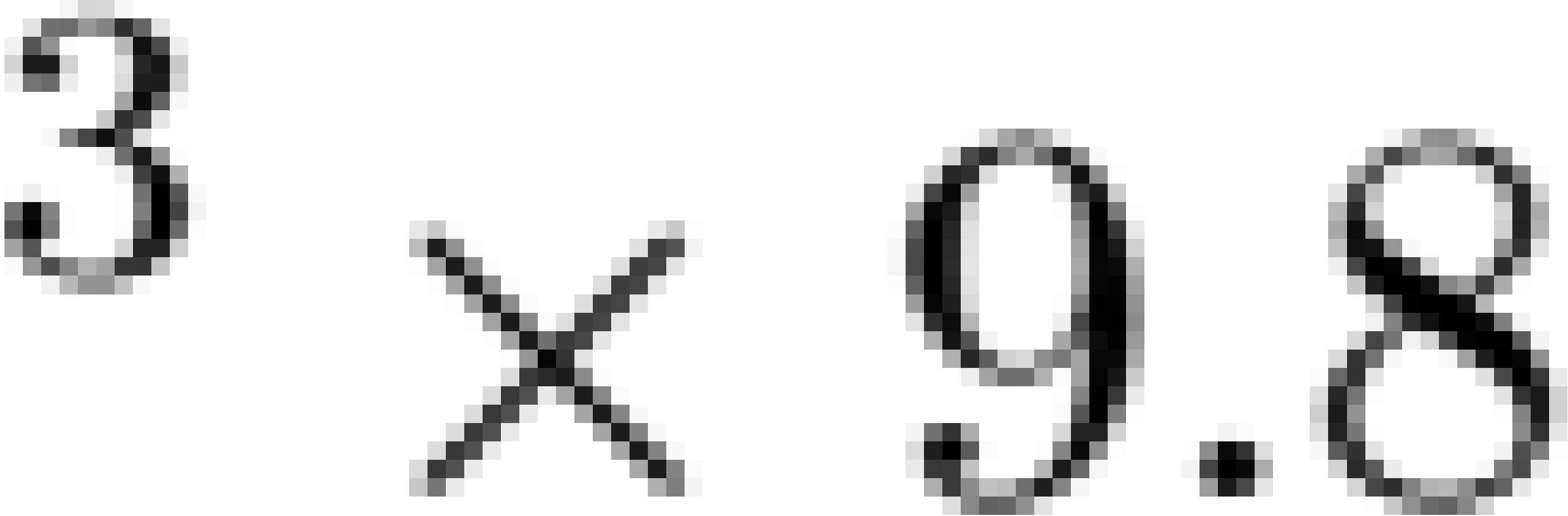
$$\sigma = S - p$$

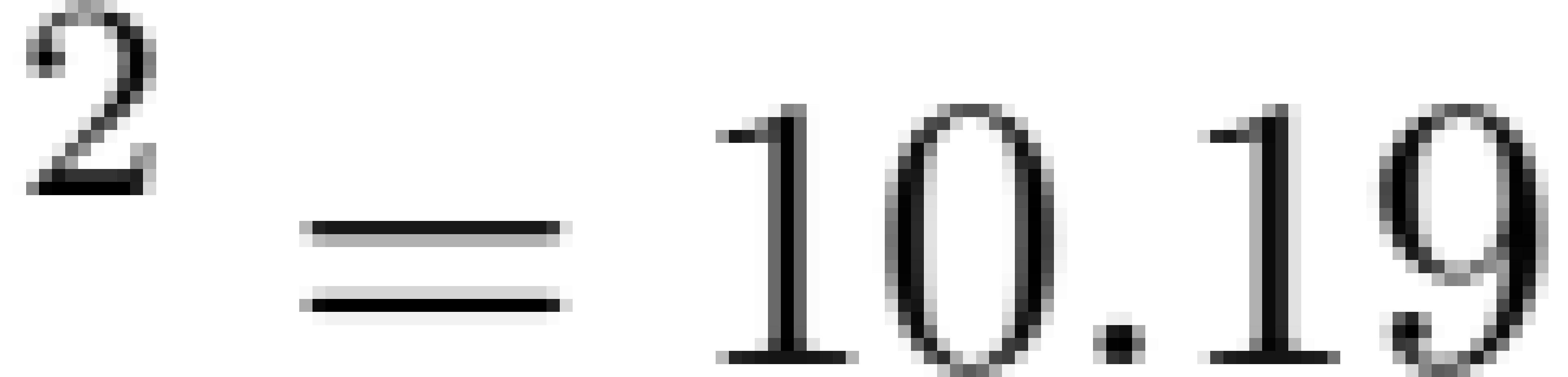




dP P \equiv $\rho_{w,9}$ \equiv 1040

dz





dsu = Paulk 2350

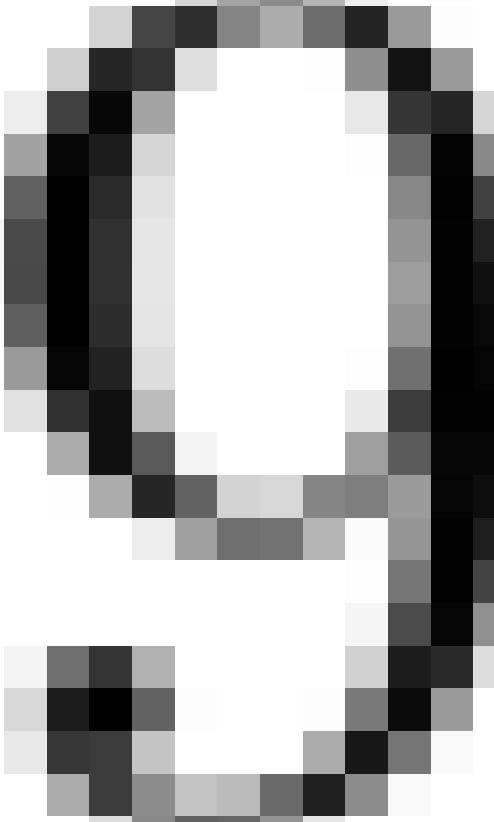
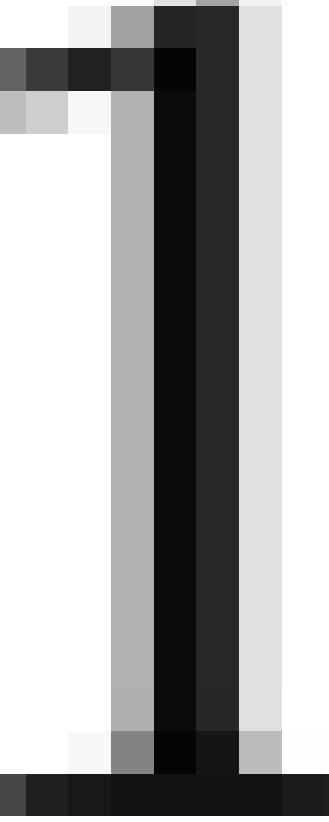
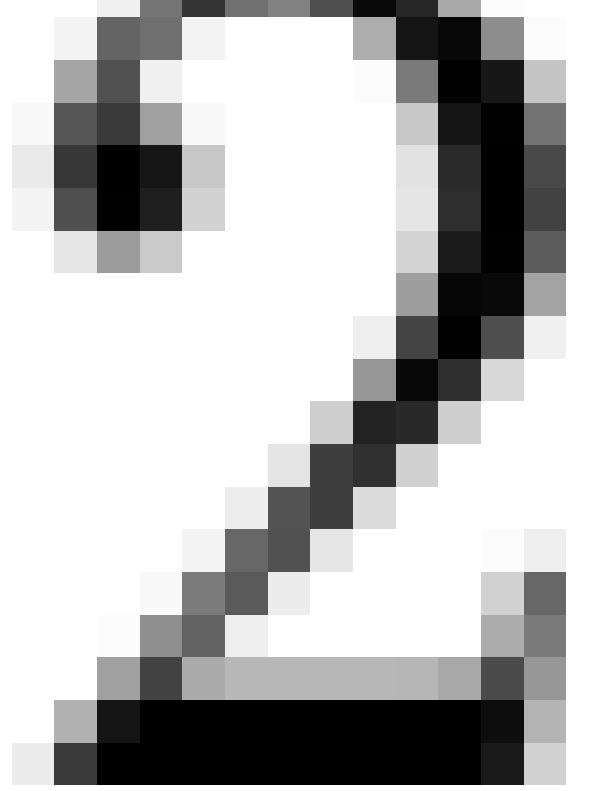
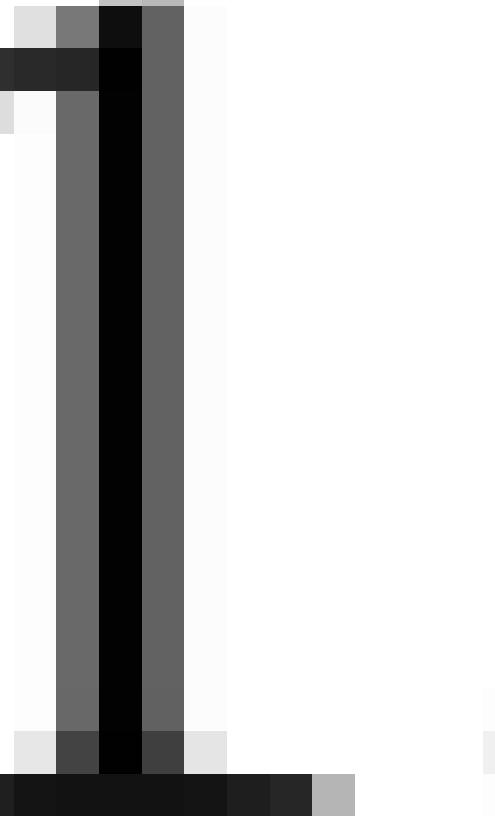
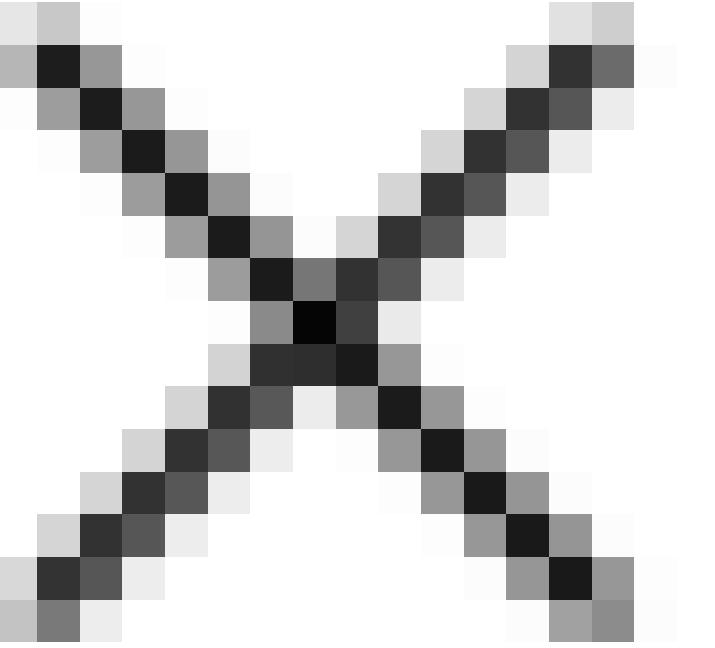


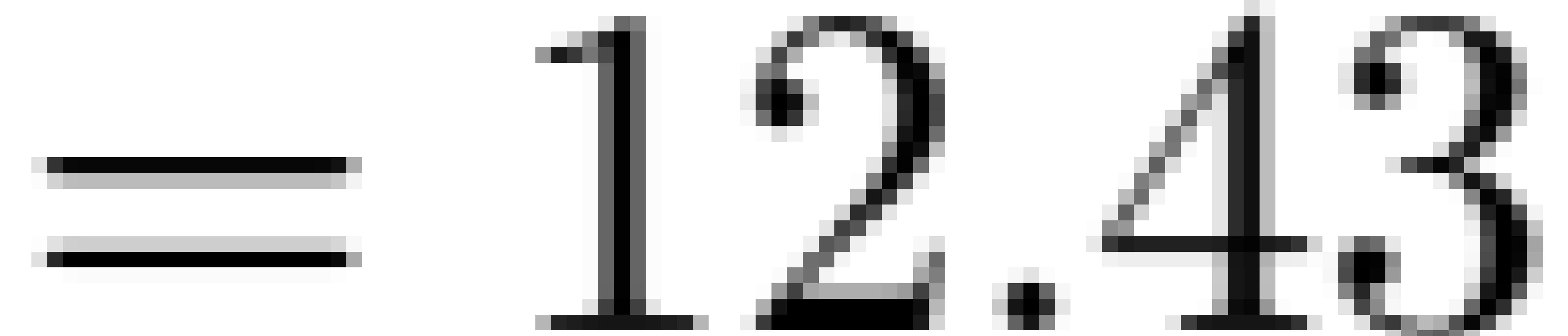
P
P

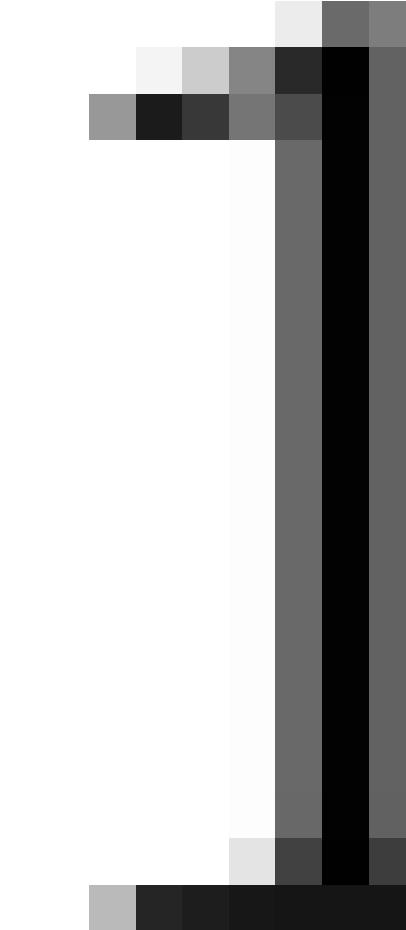
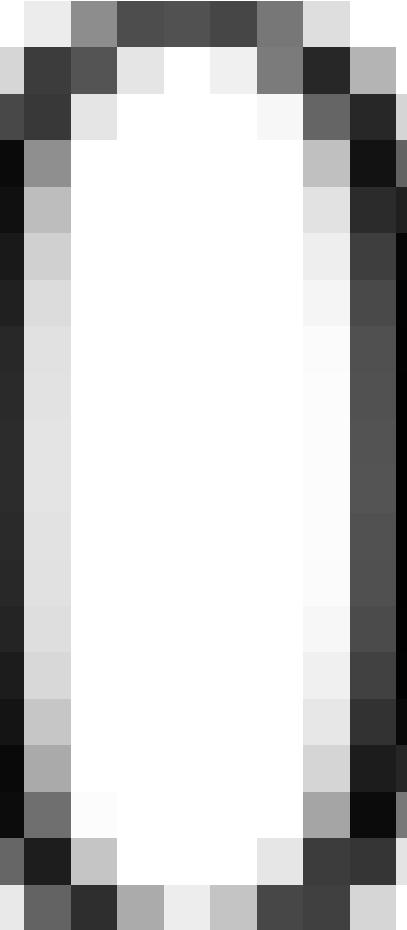
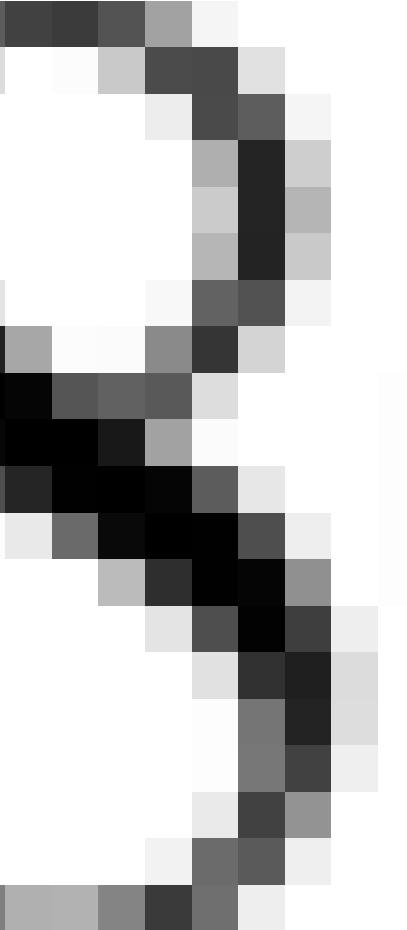
dP
dP
dZ
dZ

—
—

10.19







d₅

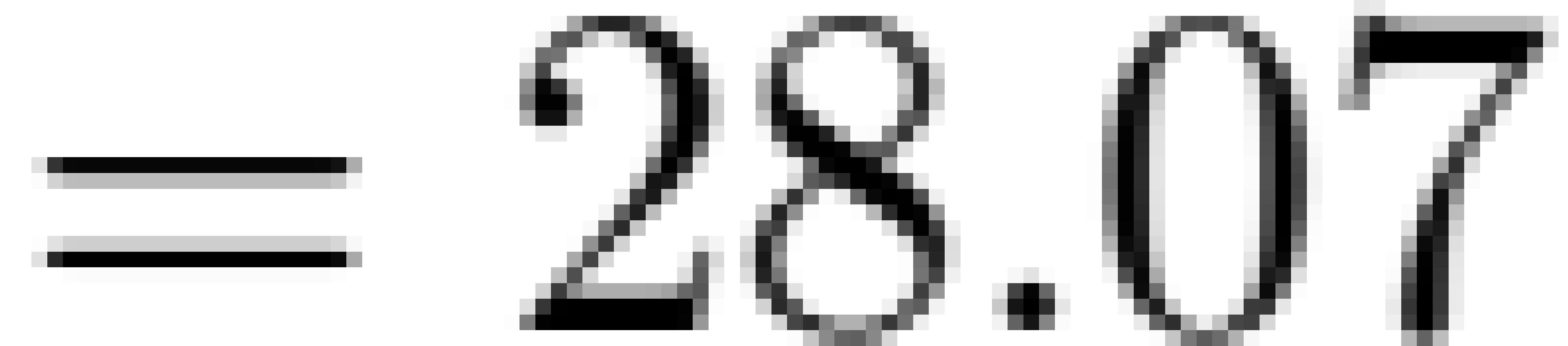
d₆

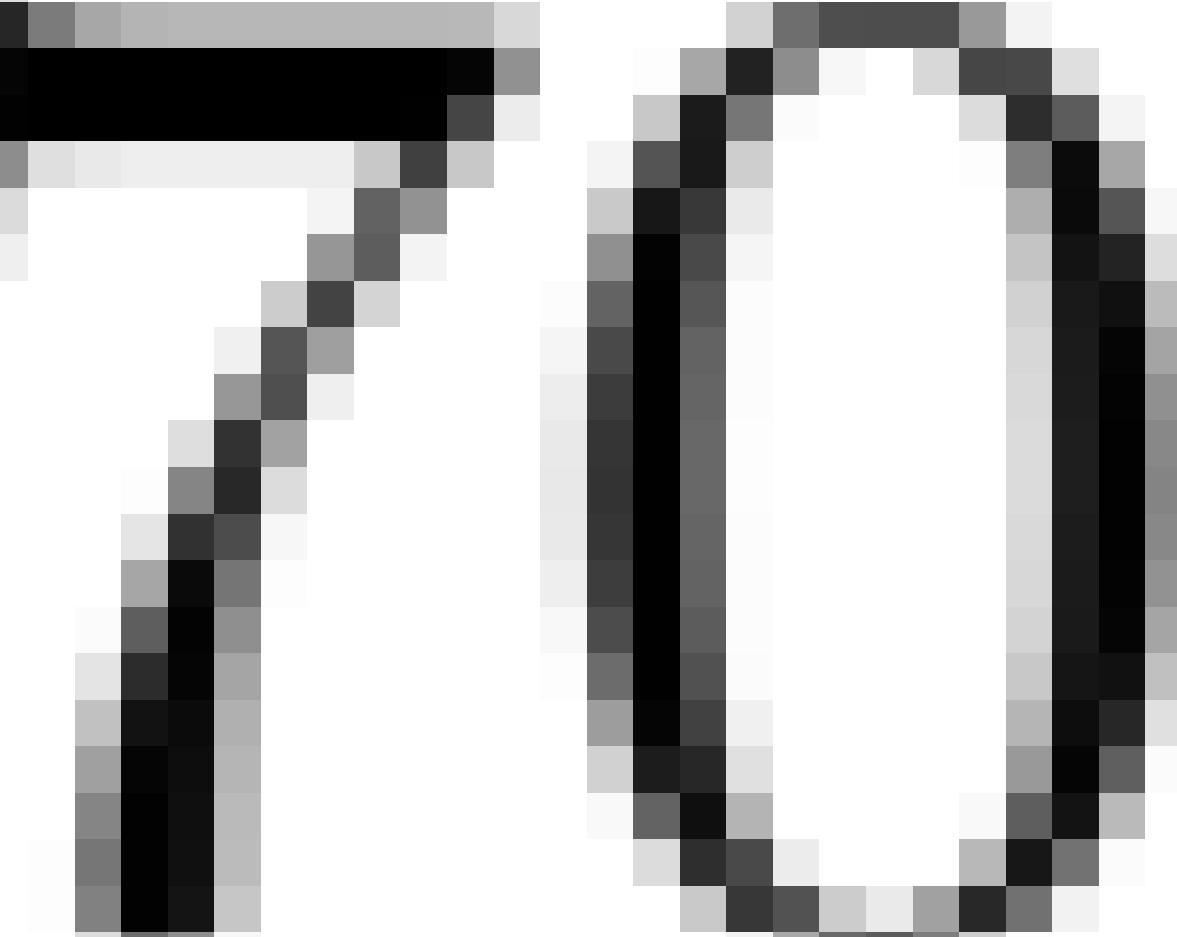
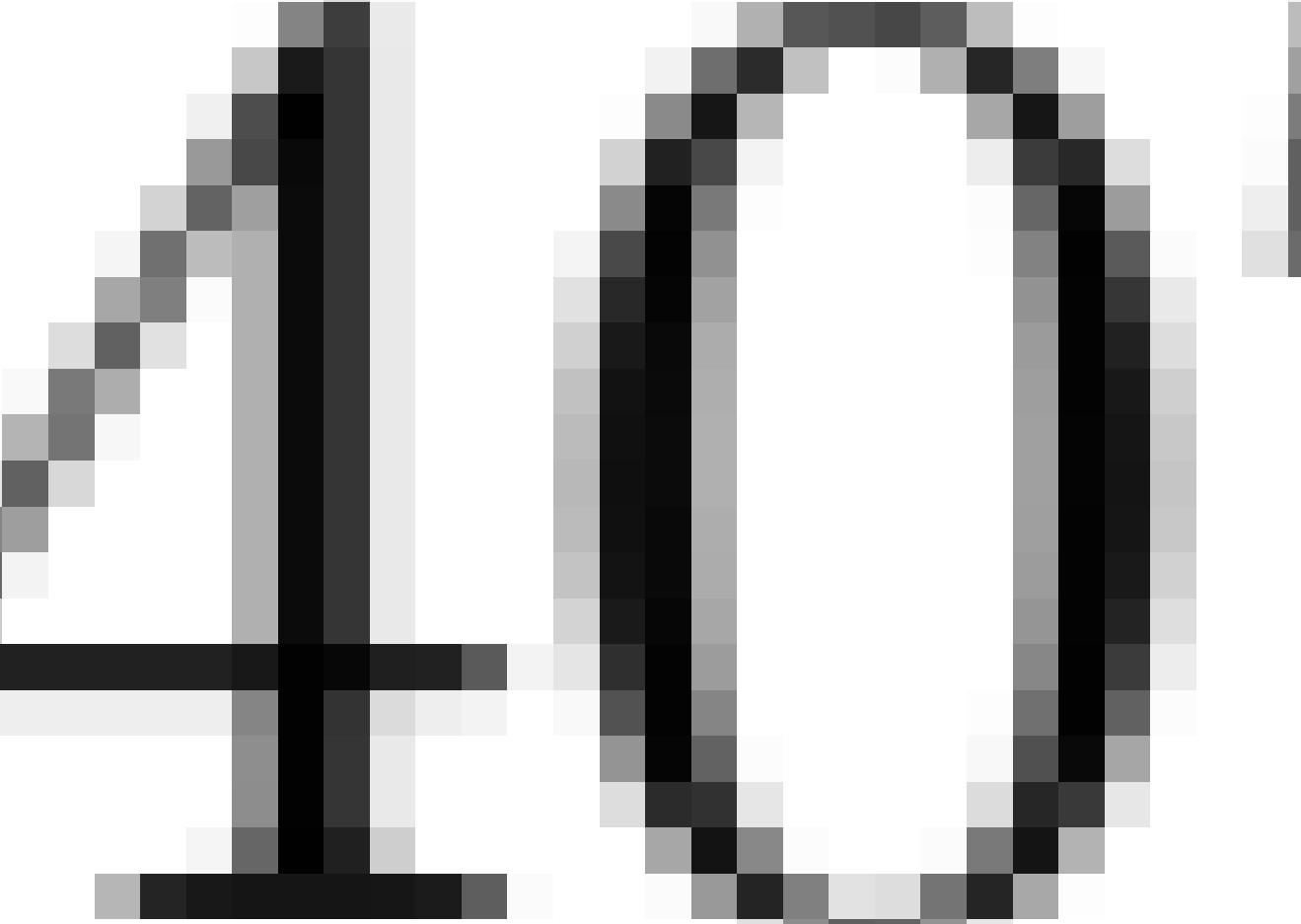
d₇

d₈

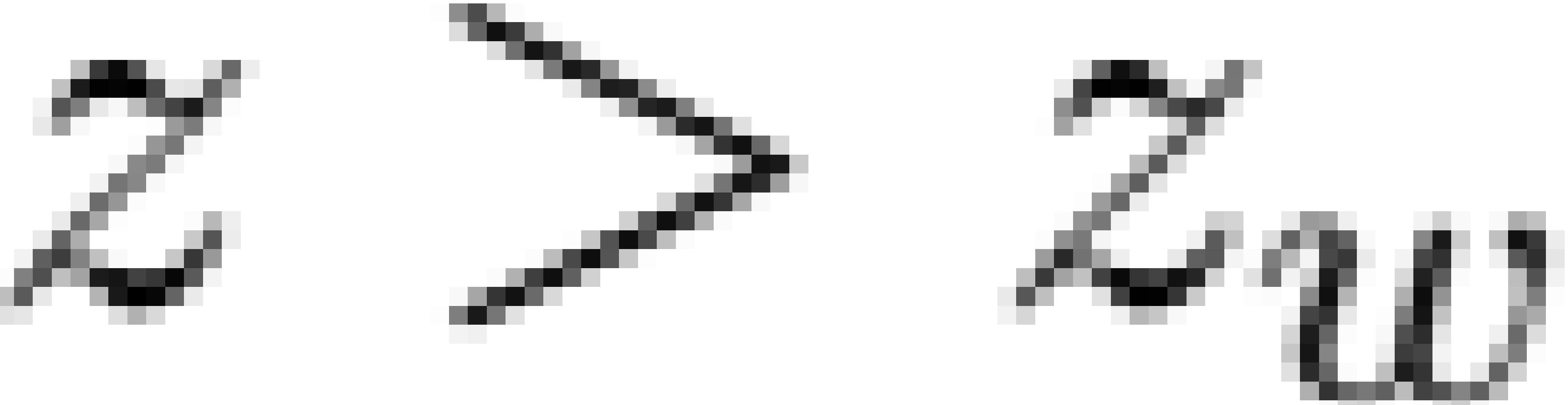
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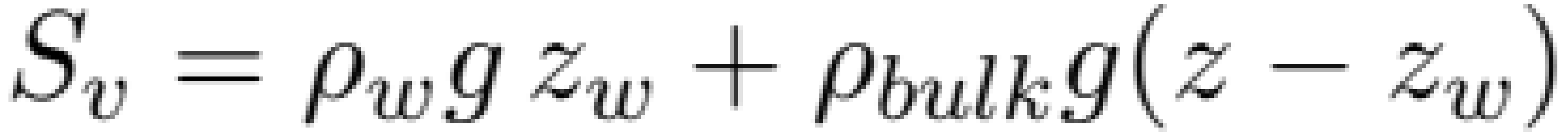
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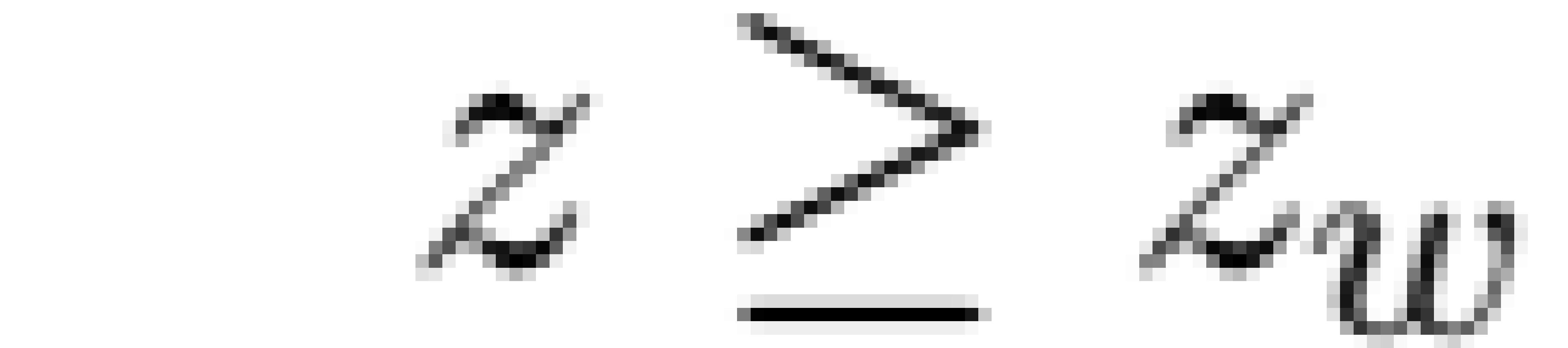


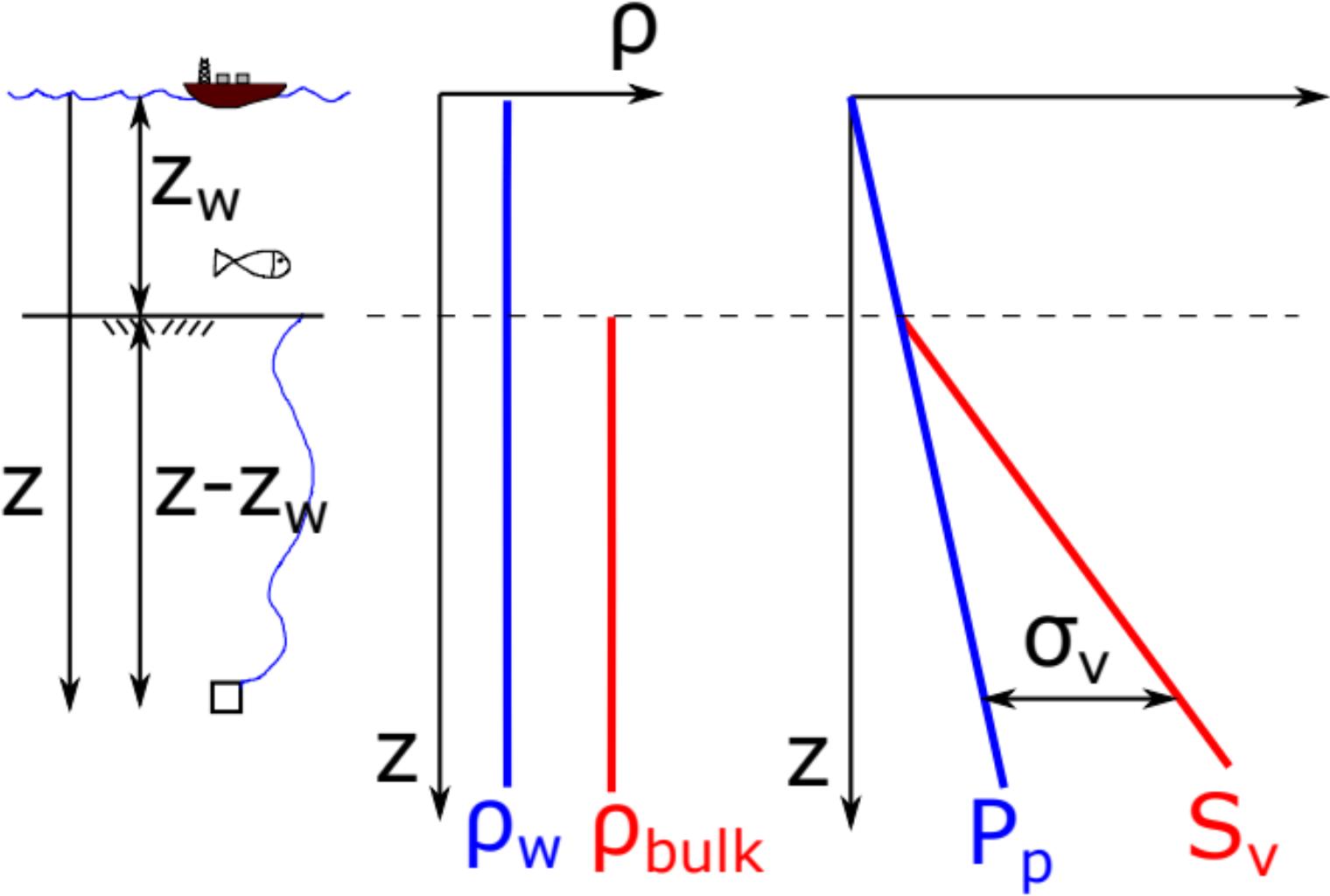




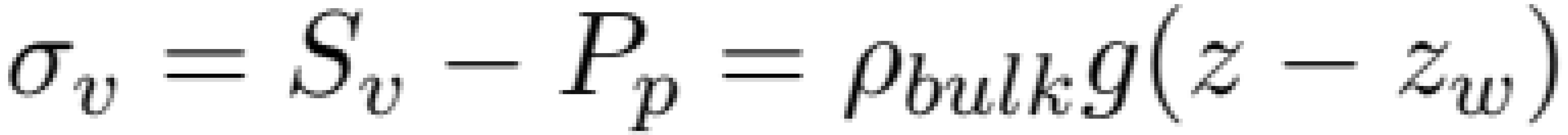




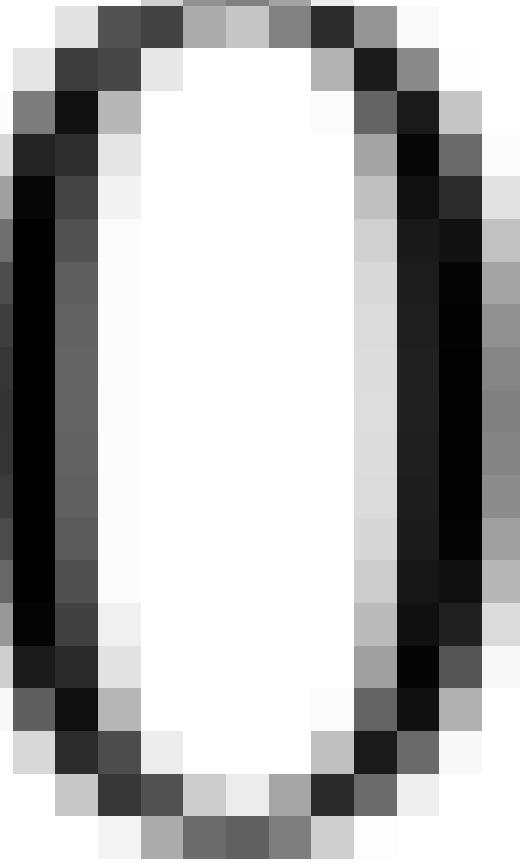
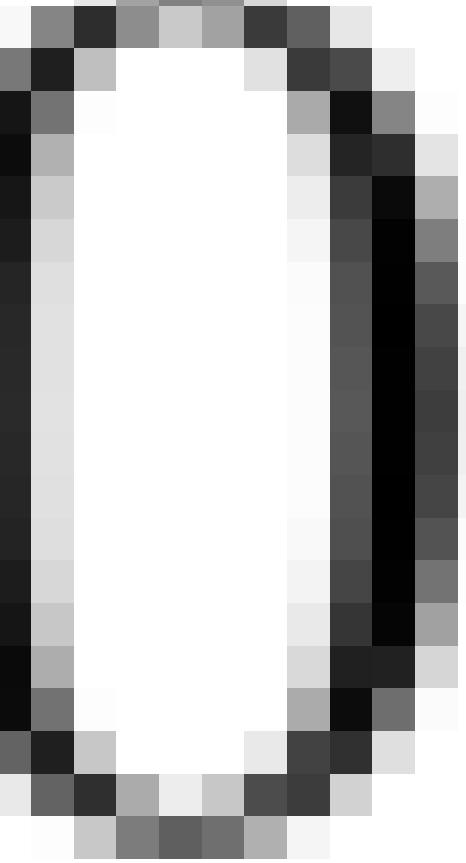
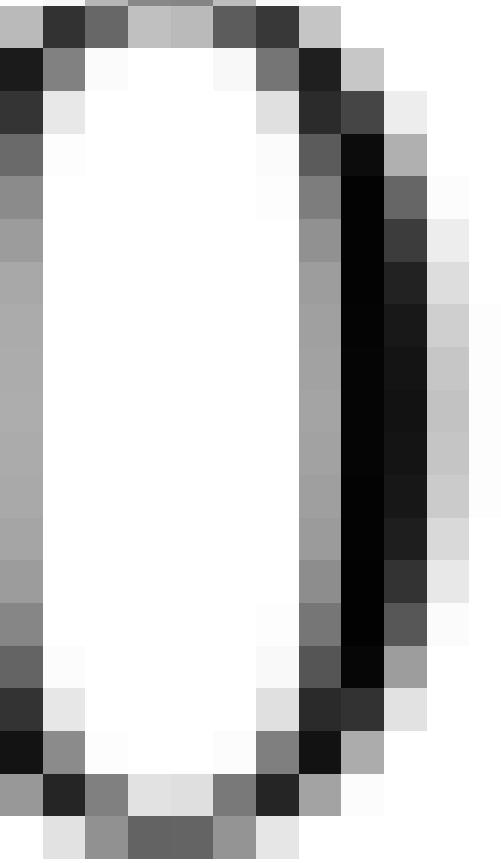
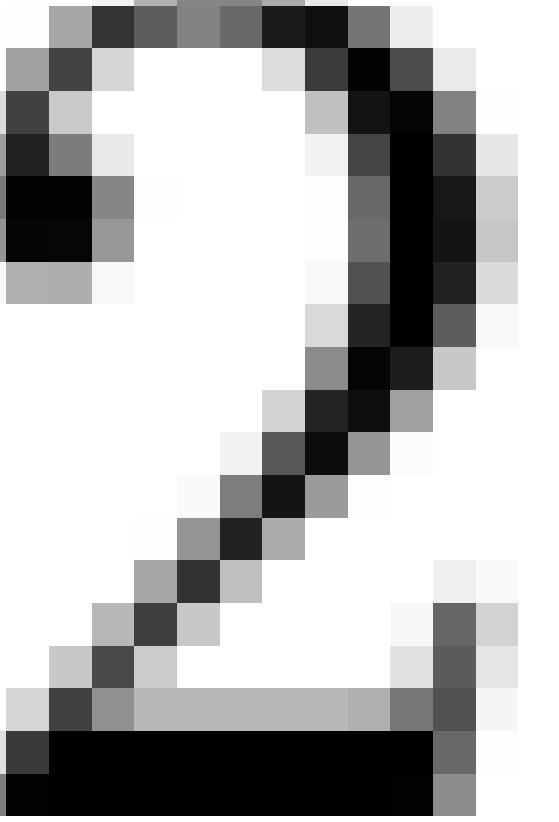
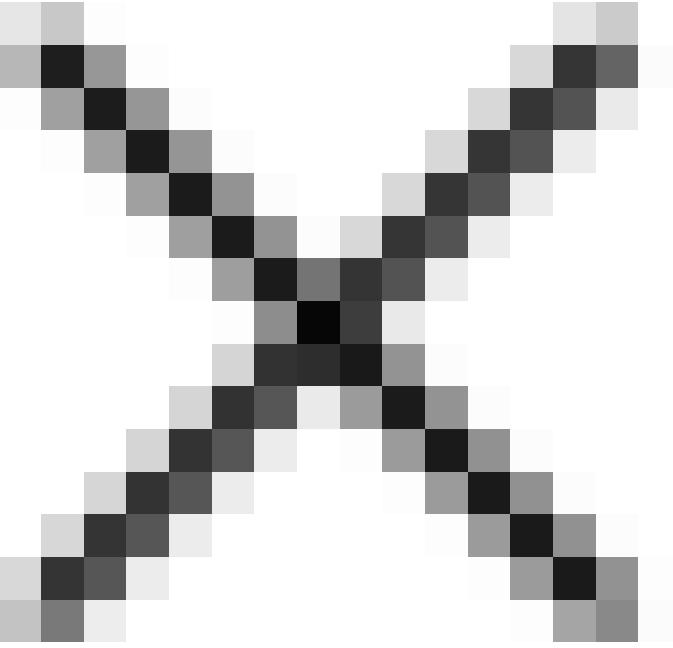


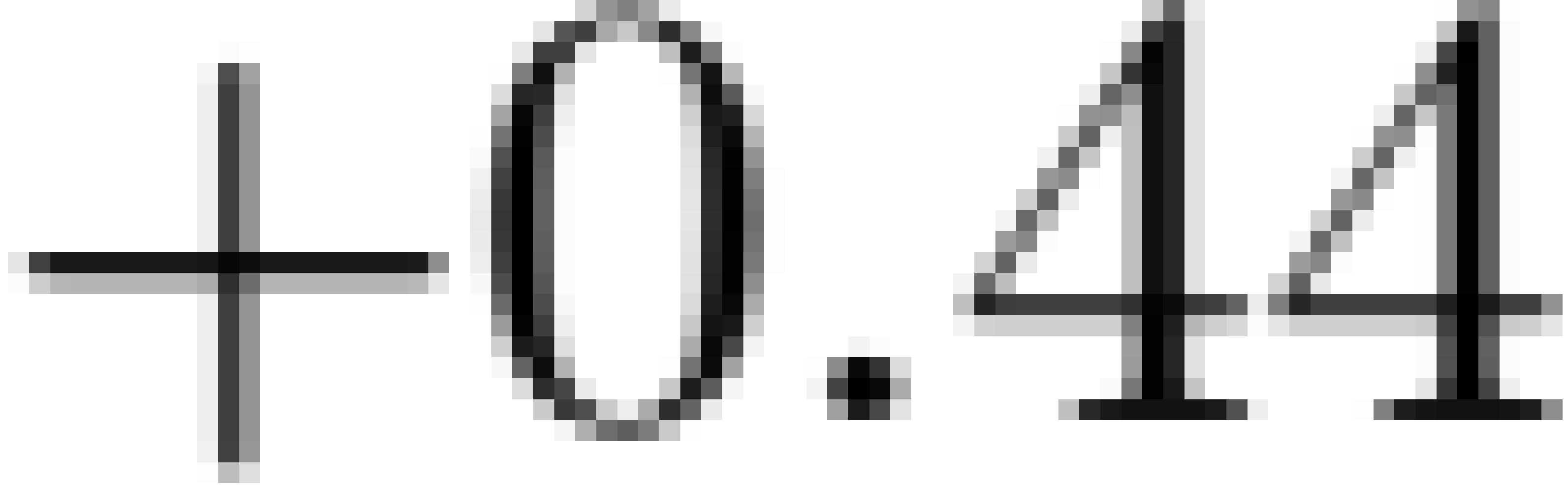


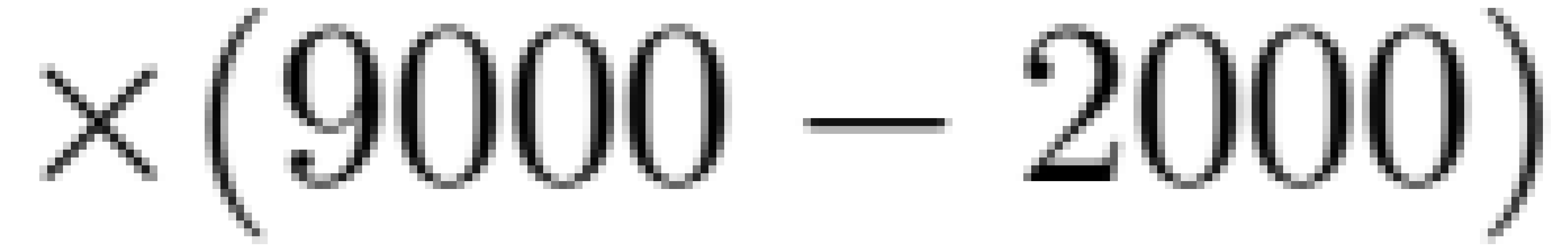


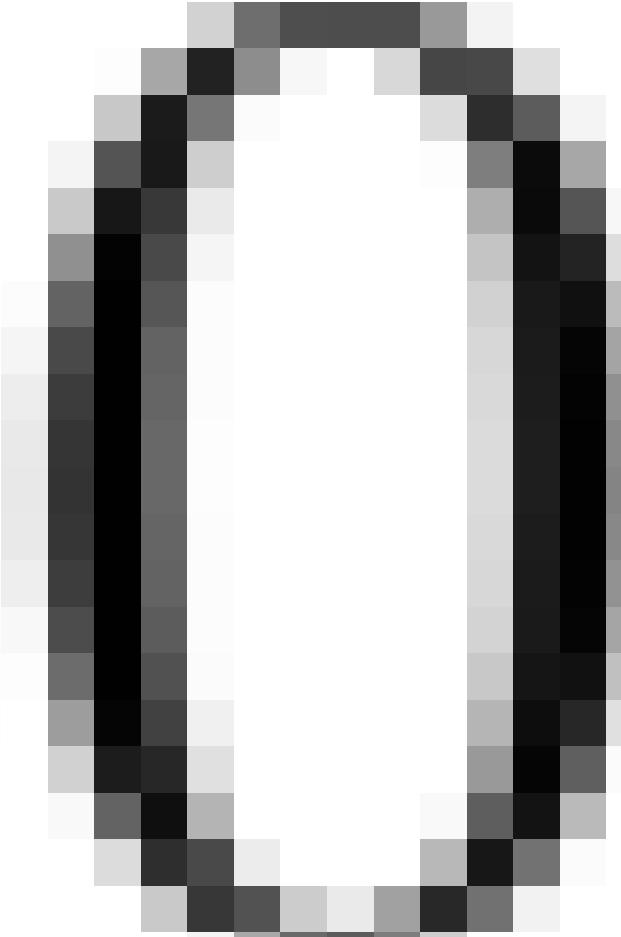
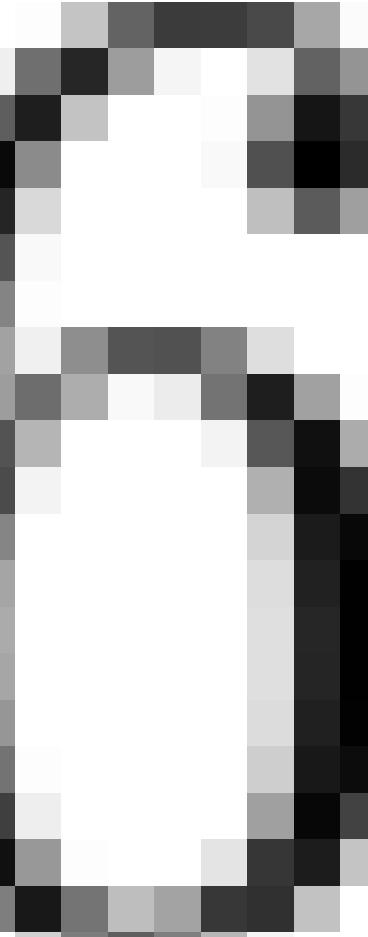
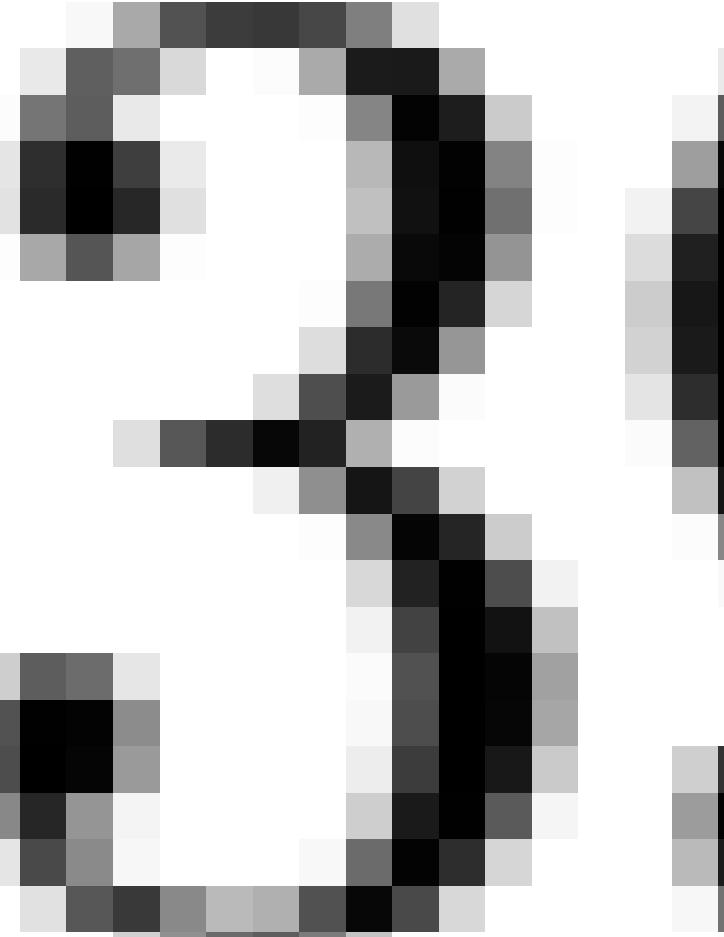


$$P_p = \rho w g z_w + \frac{dP}{dz} (z - z_w) = 0.44$$

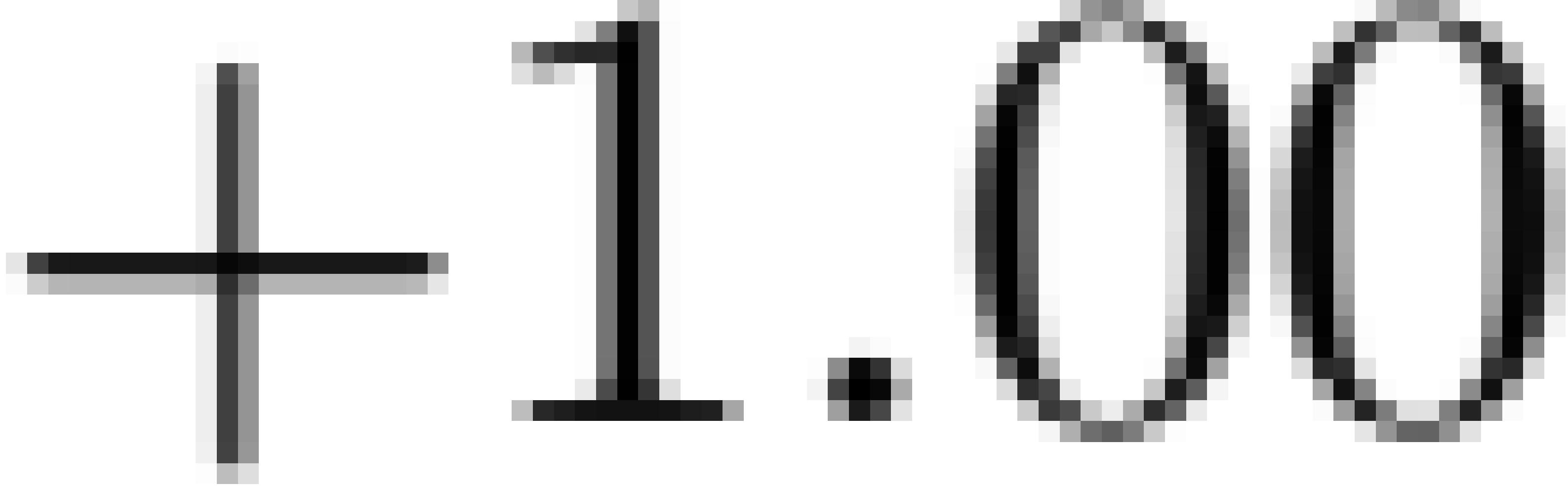


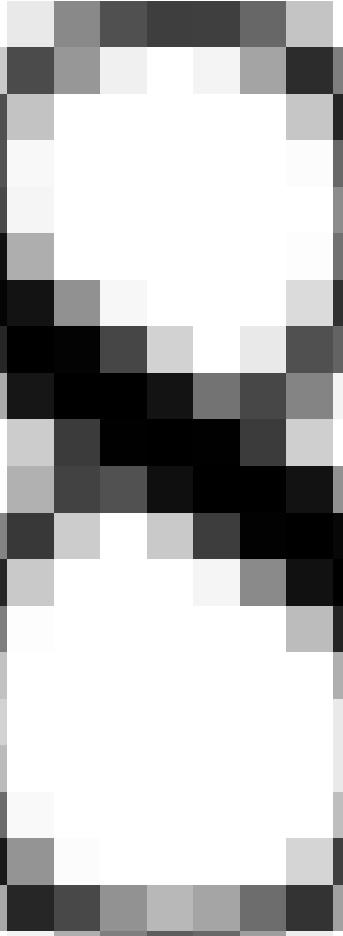
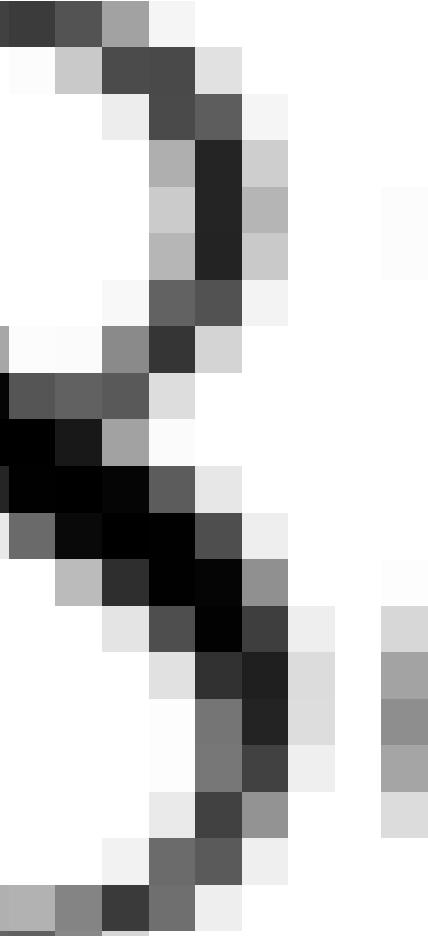
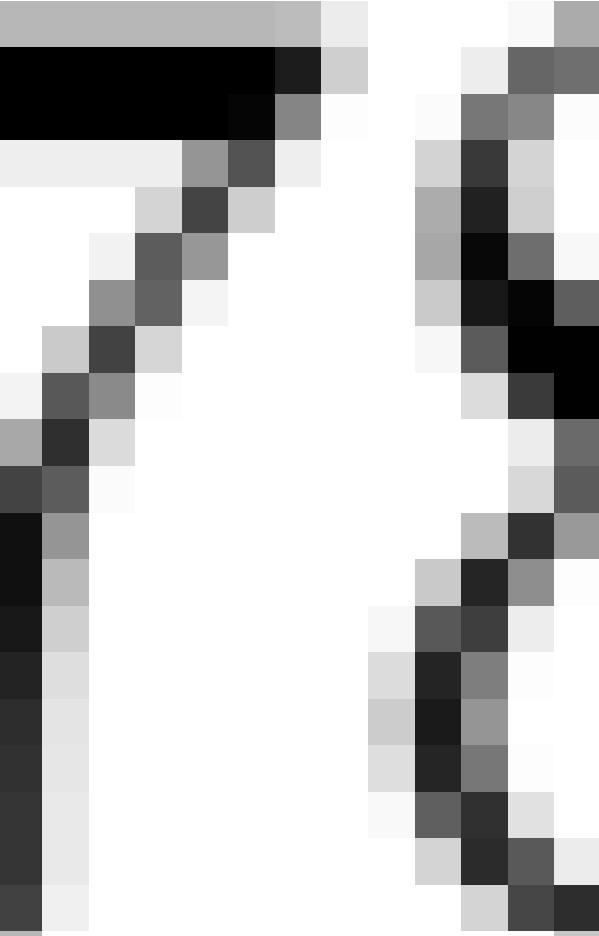


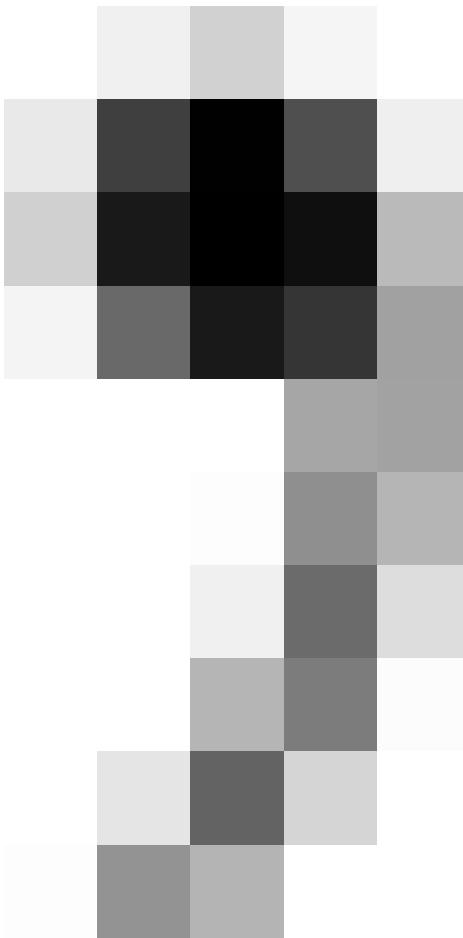


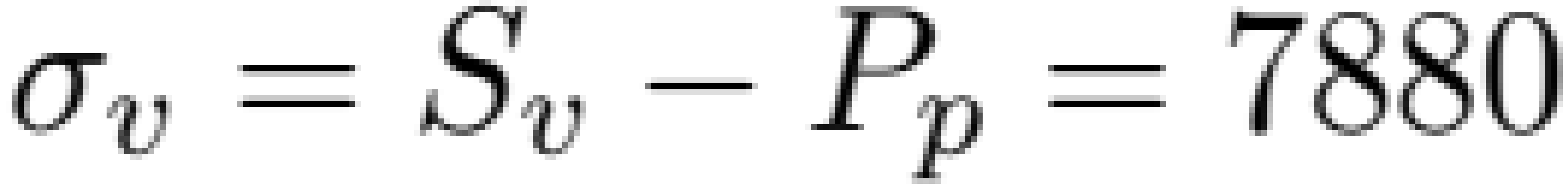


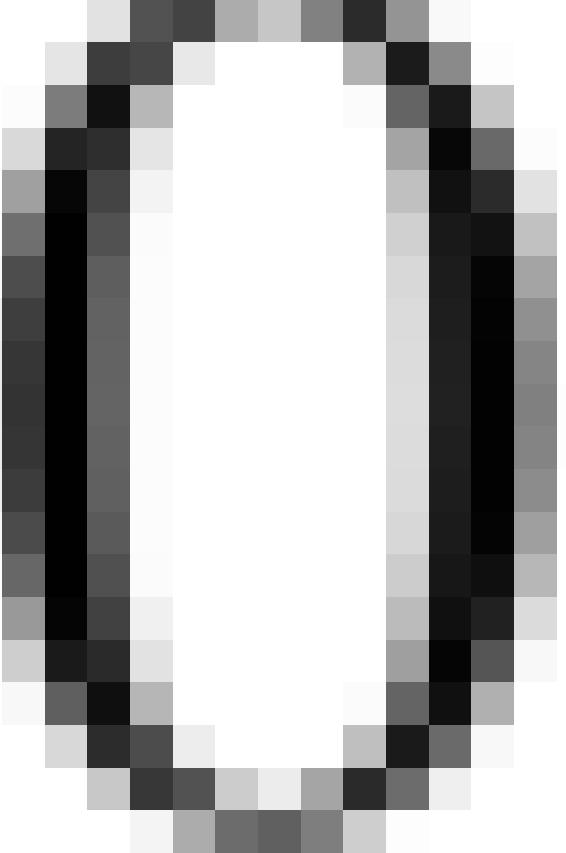
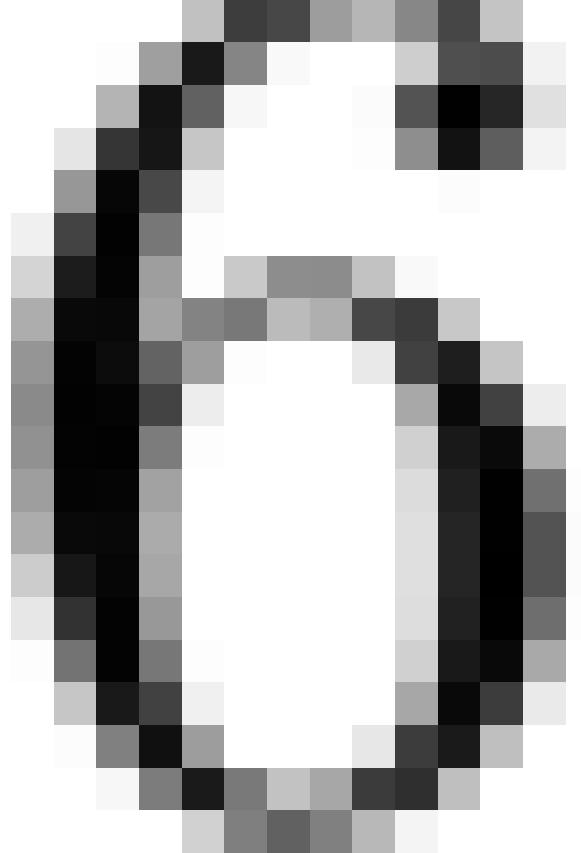
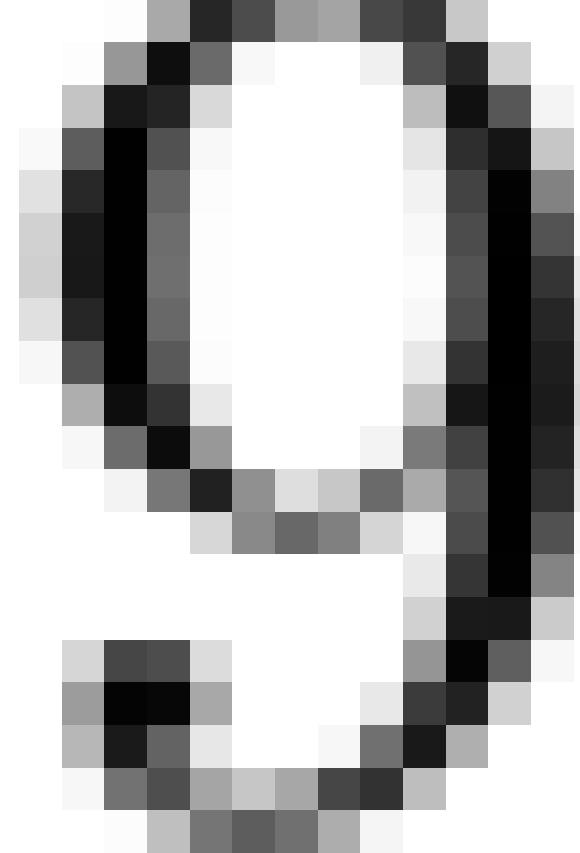
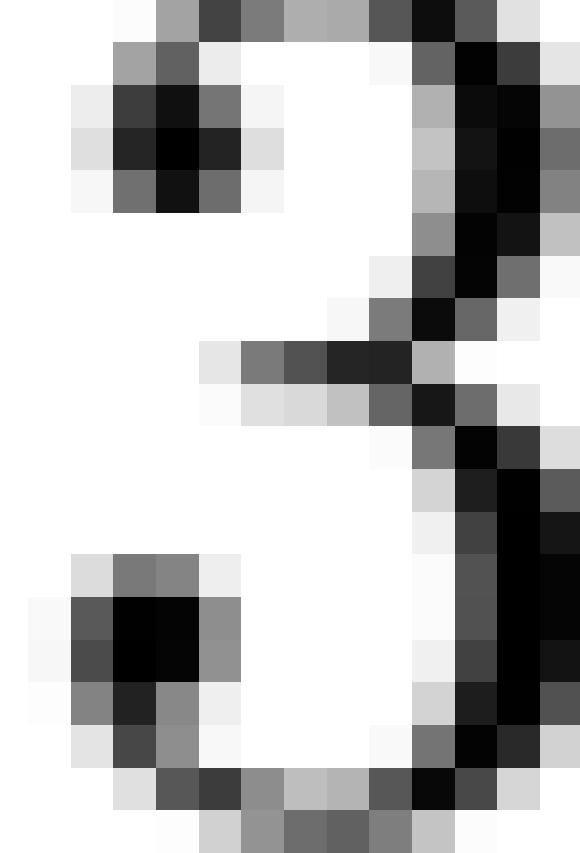
$$S_u = \rho_{u,g} z_u + \frac{dS_u}{dz} (z - z_u) = 0.44$$

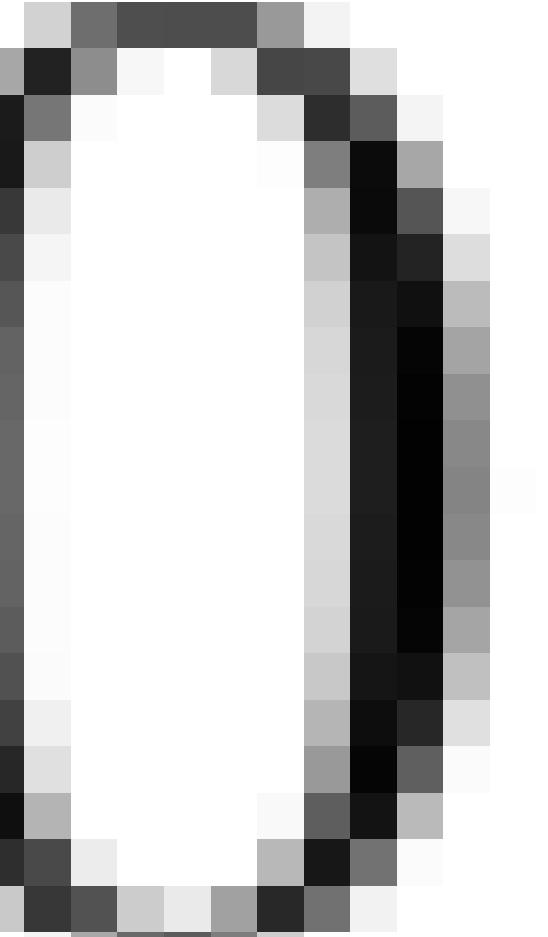
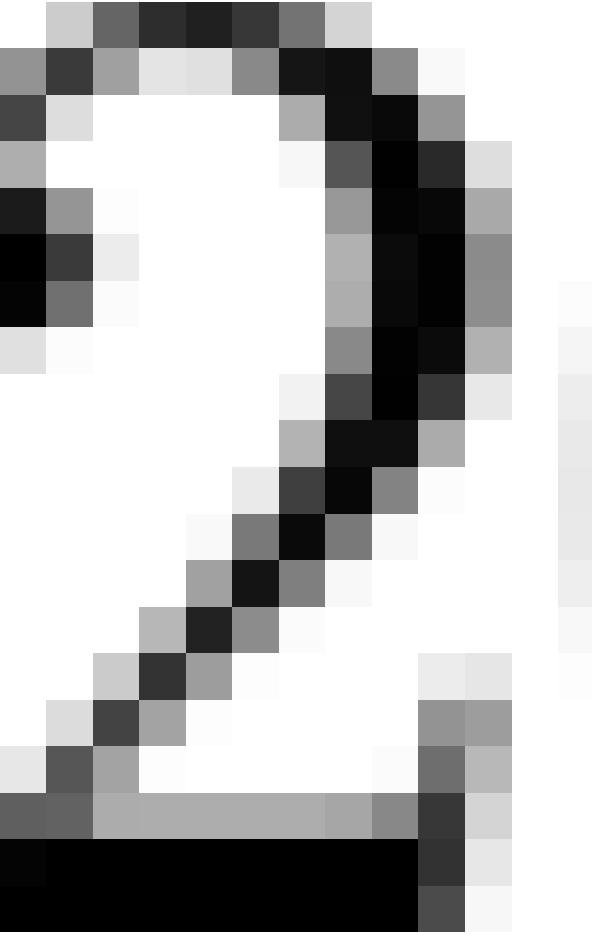
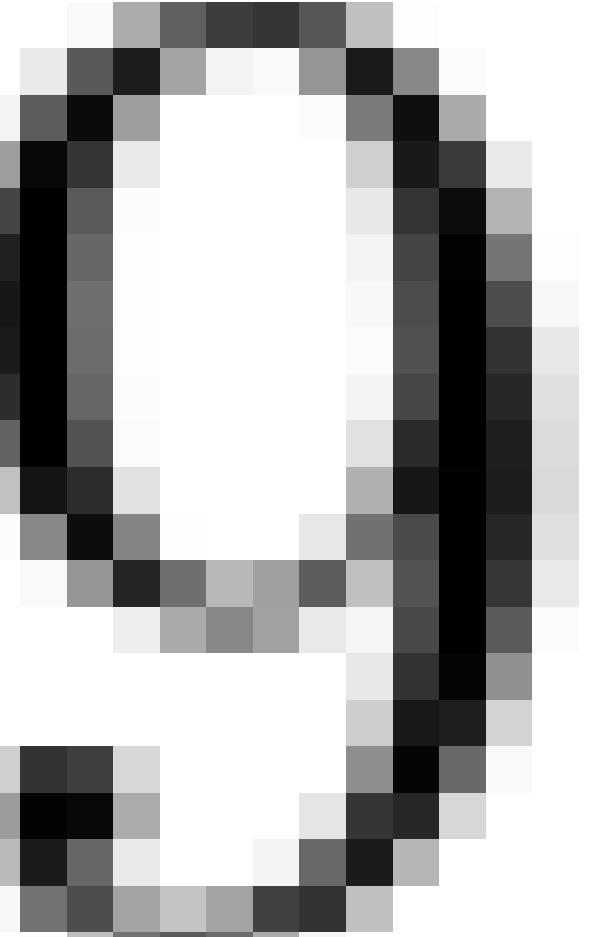
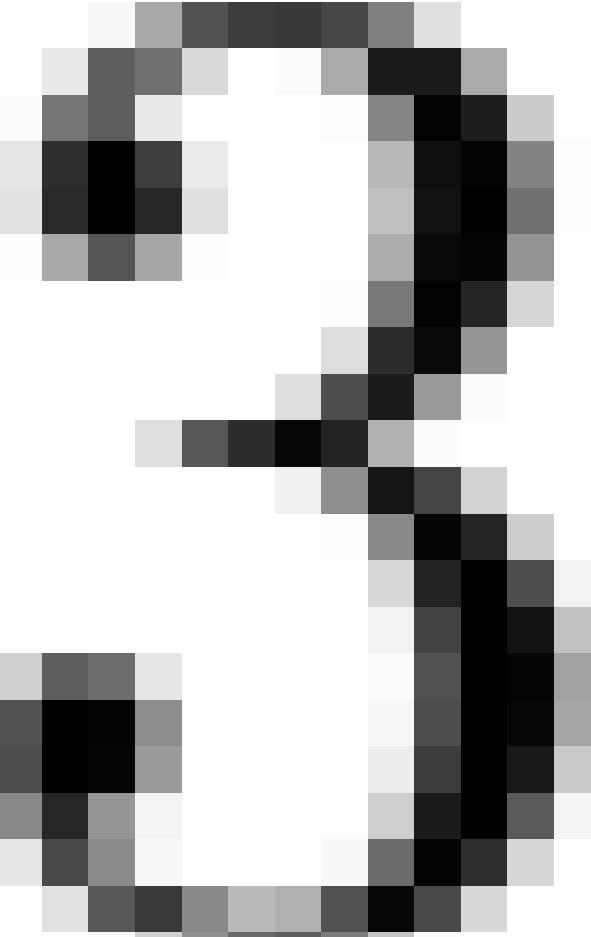


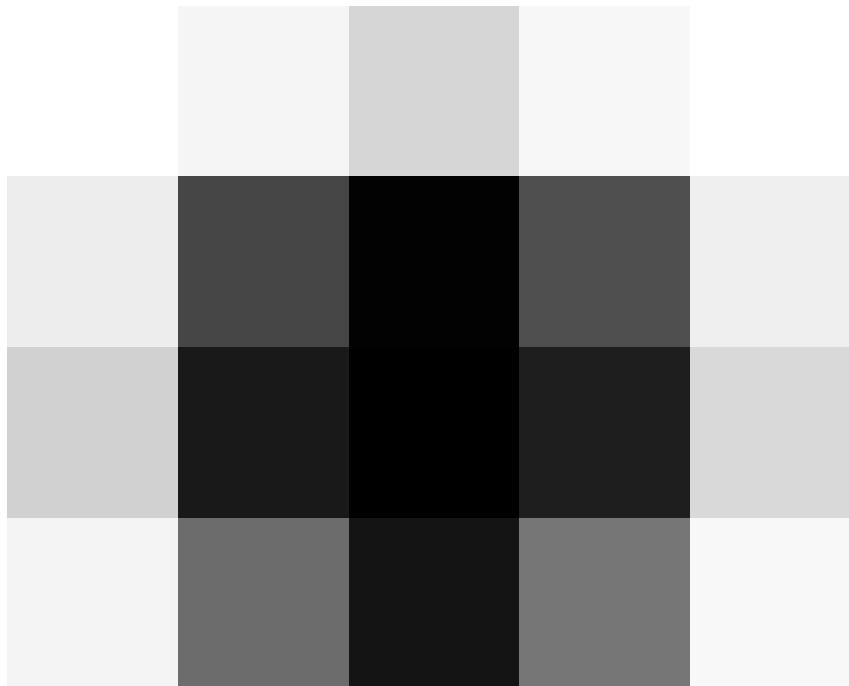


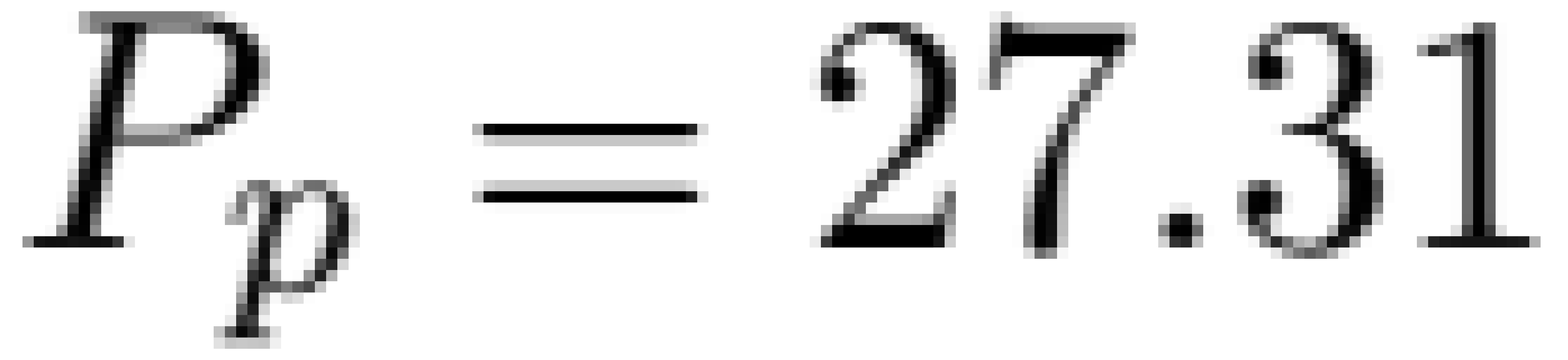






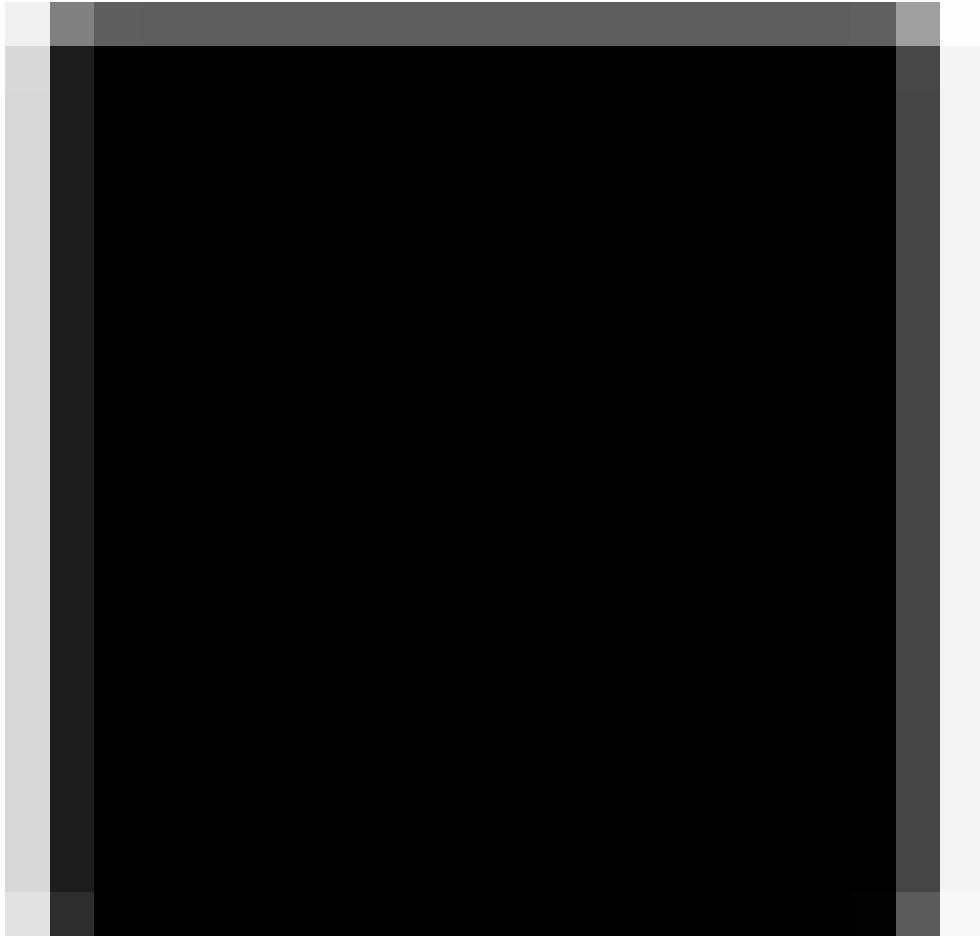








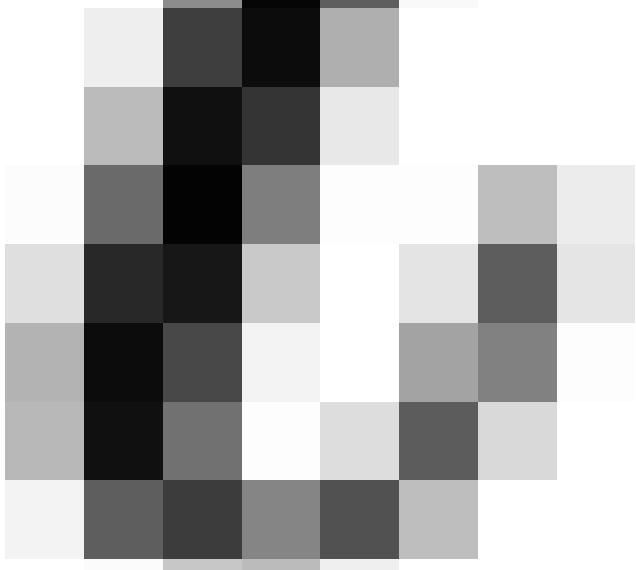
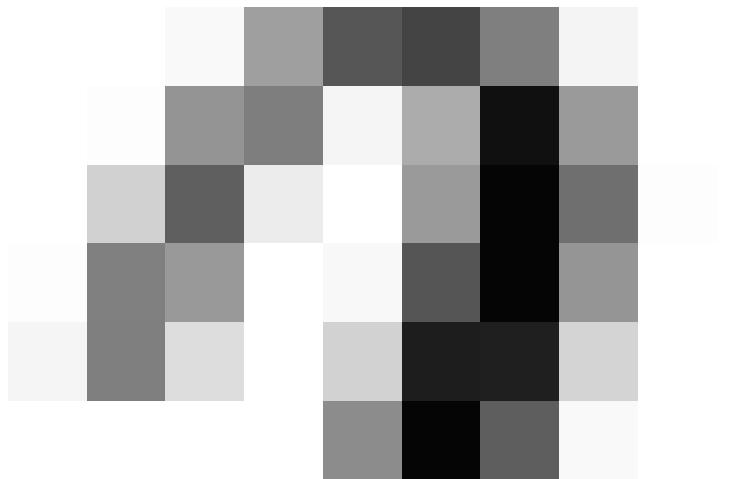
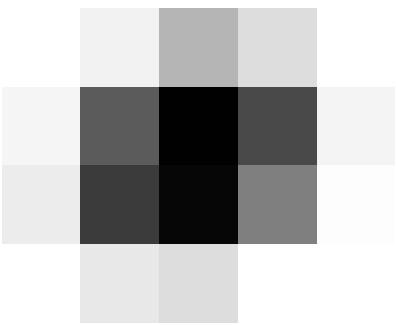


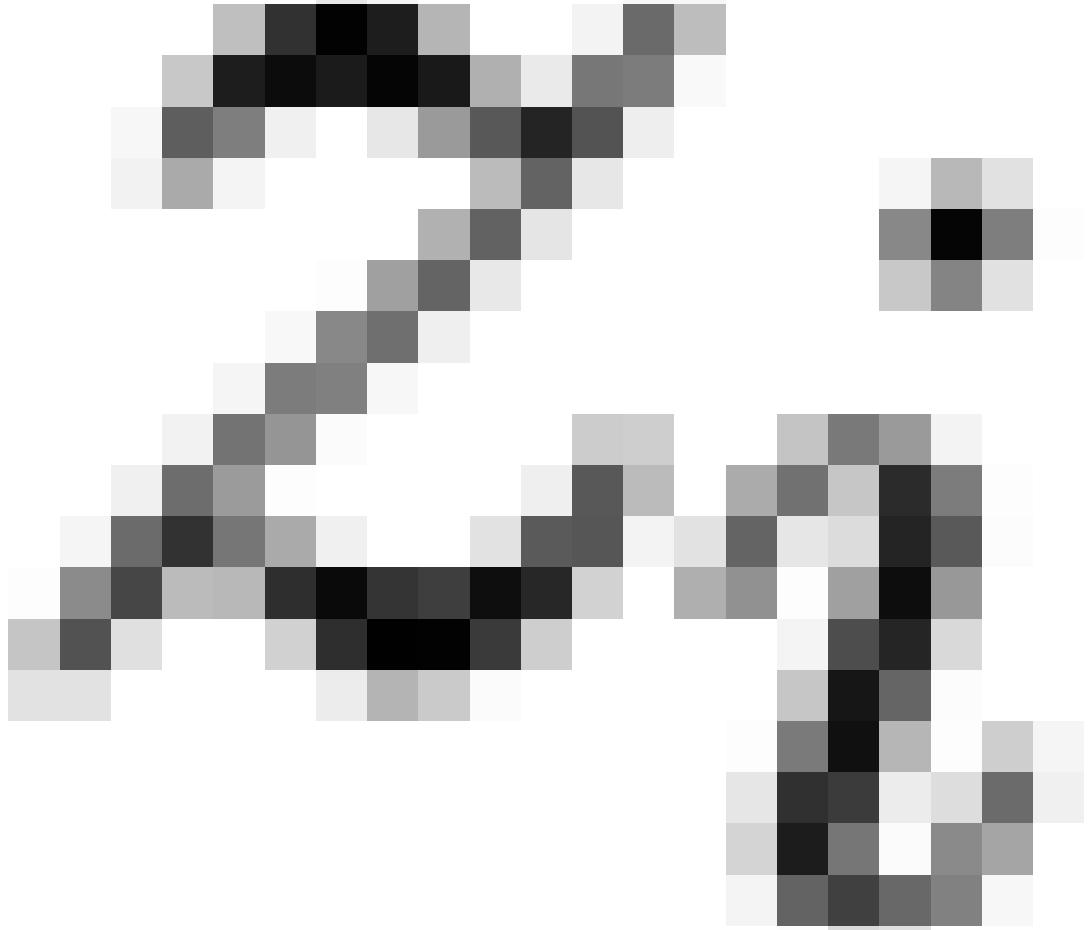


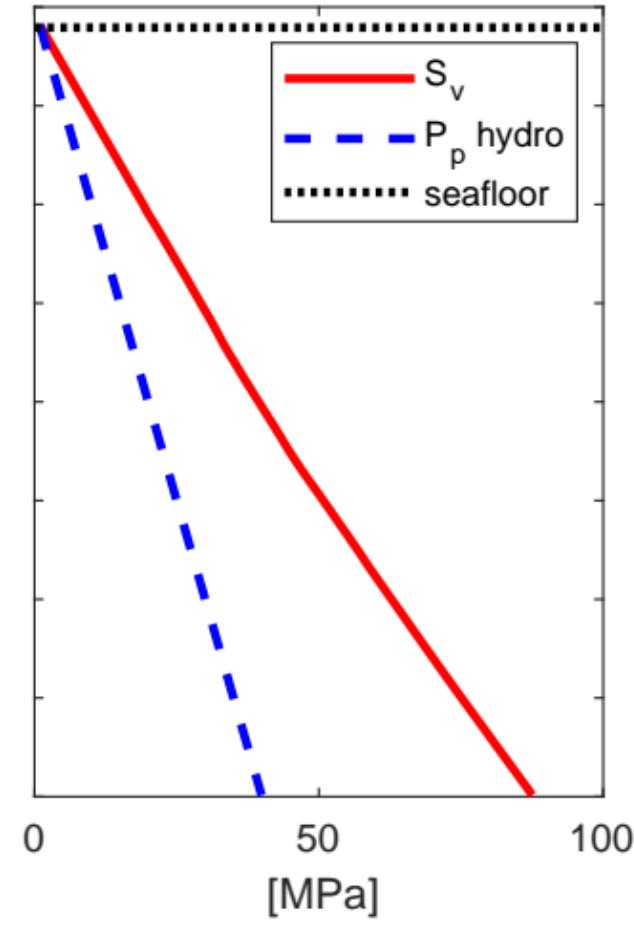
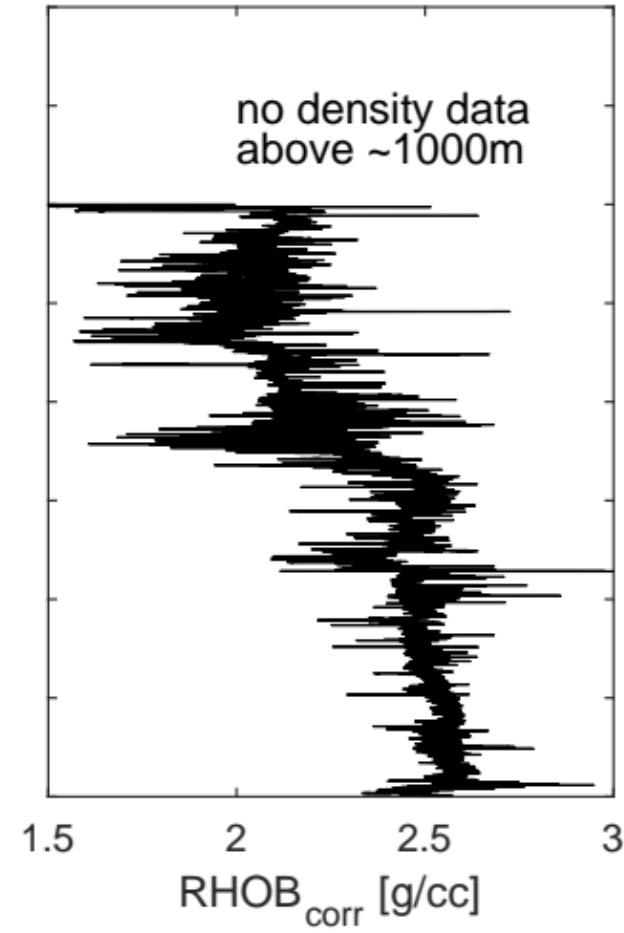
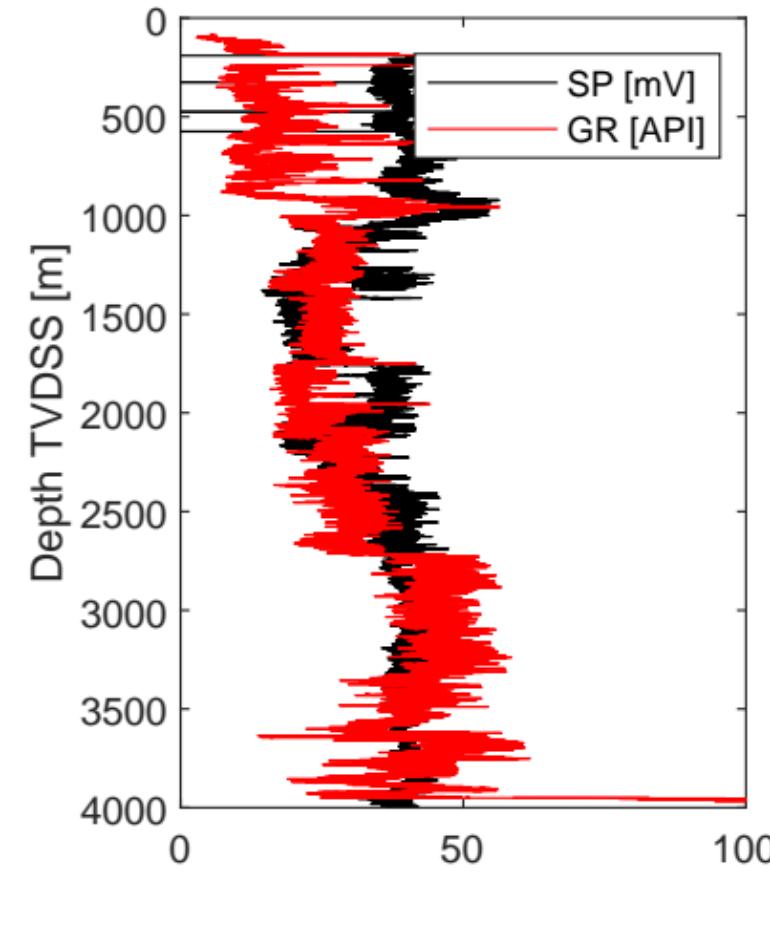
$$S_U(z) = \int_0^z \rho_{bulk}(z') dz'$$

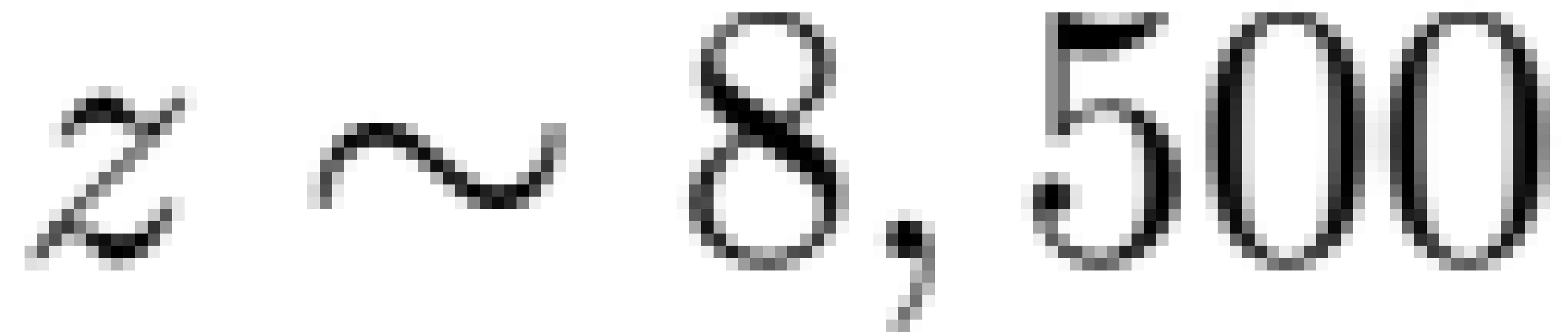


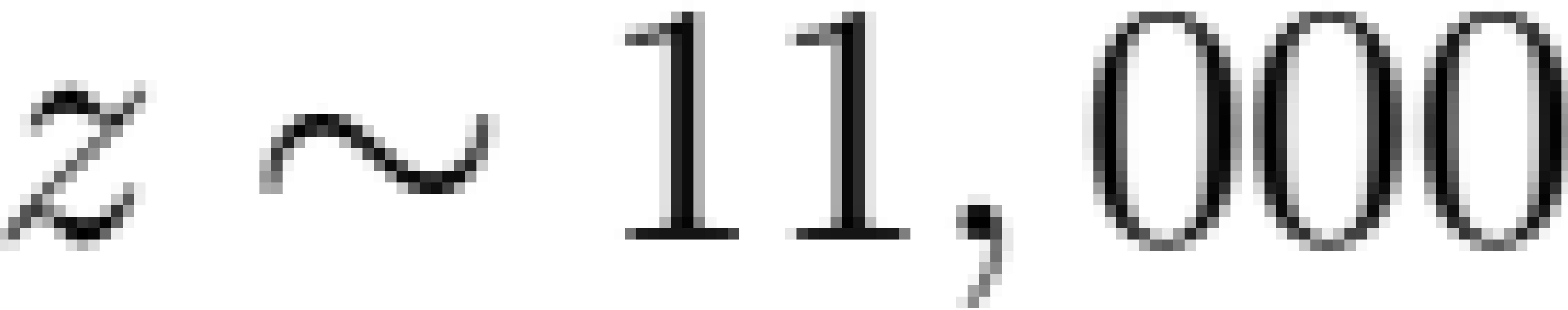
$$S_v(z_i) = \sum_{j=1}^i \rho_{bulk}(z_i) g\Delta z_i$$



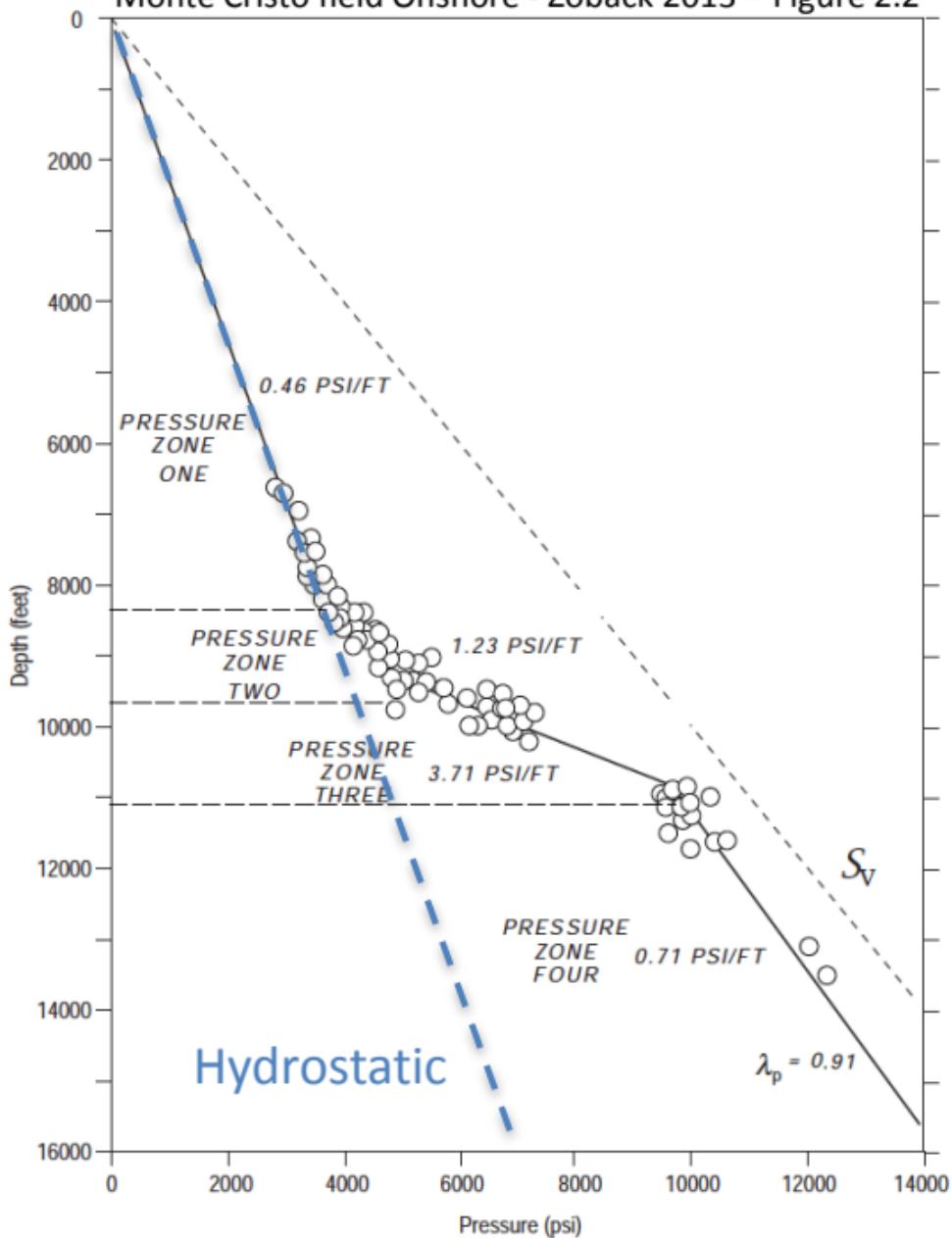








Monte Cristo field Onshore - Zoback 2013 – Figure 2.2

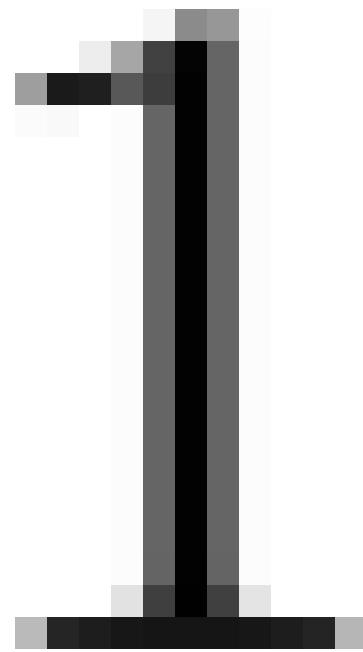
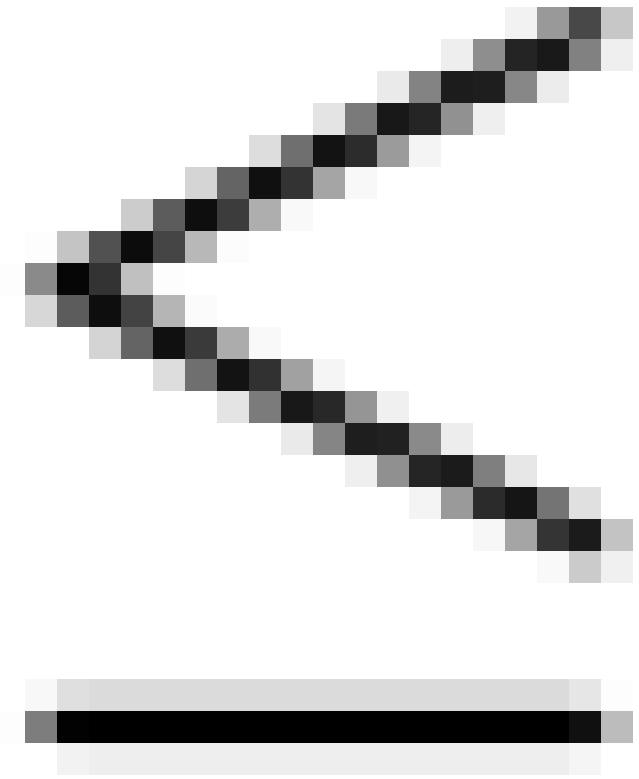


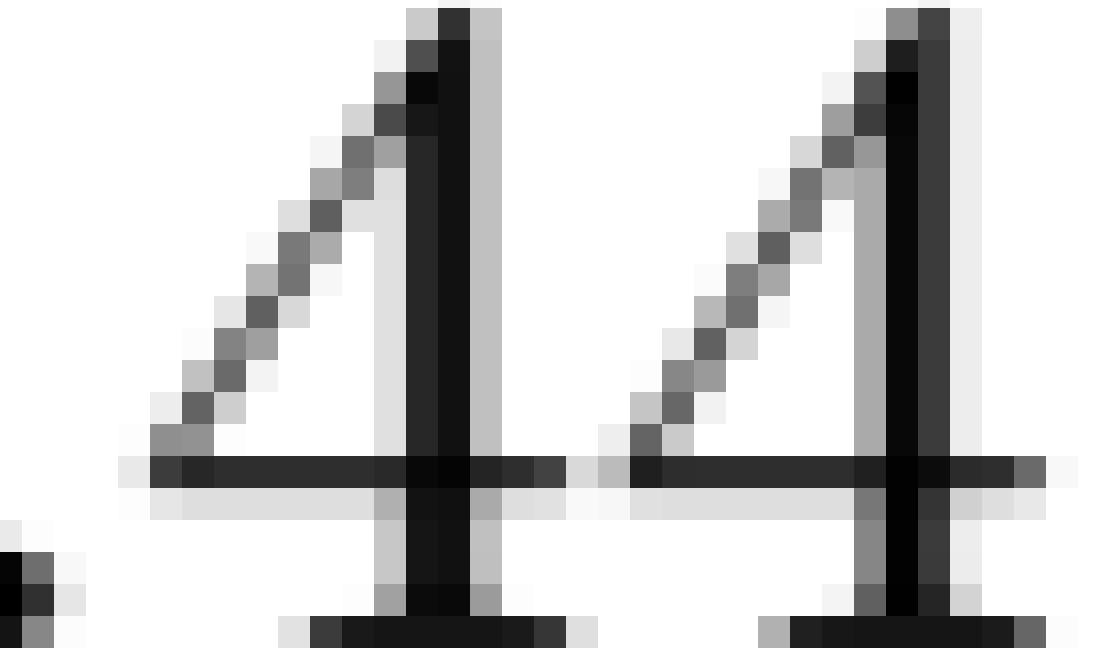
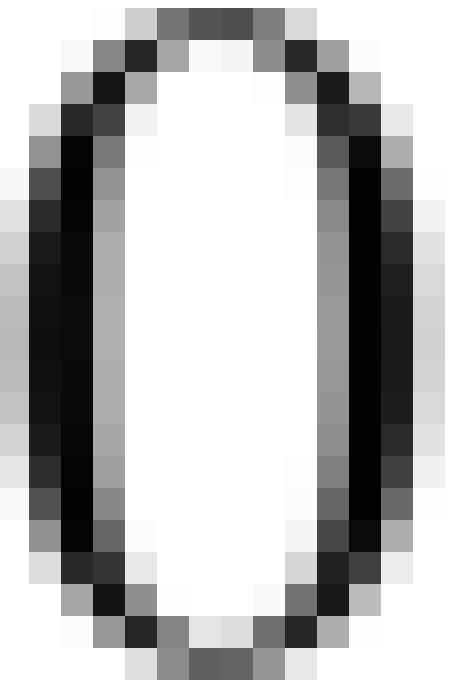
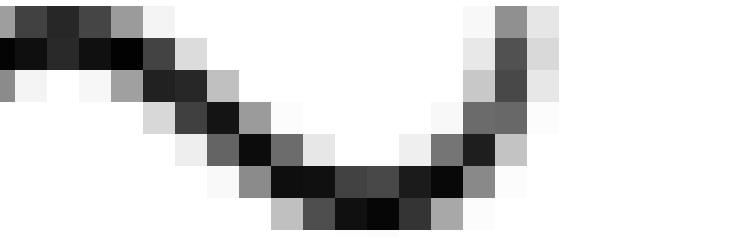


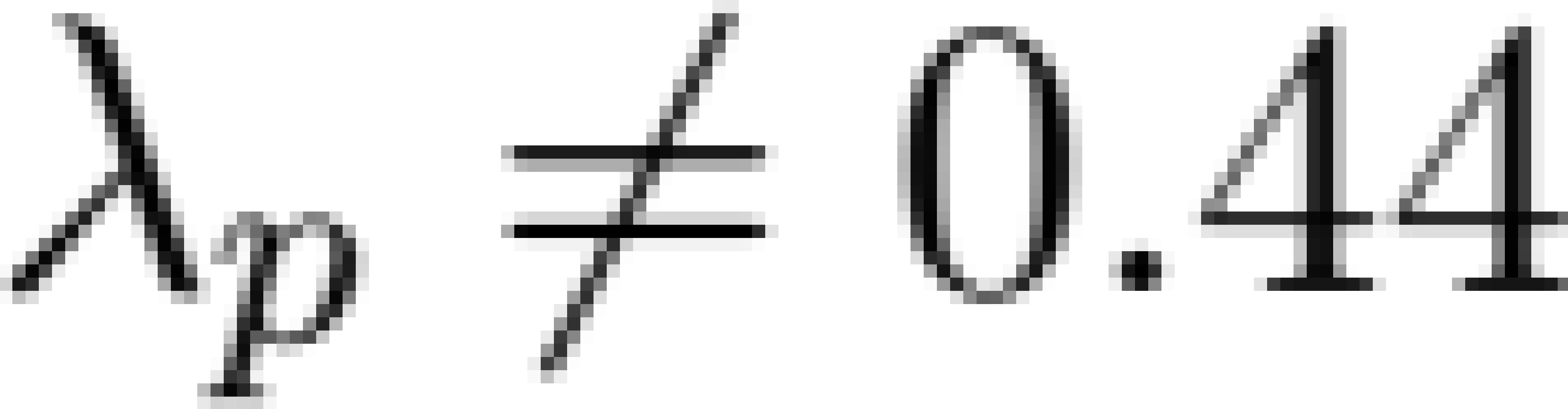
$p(z)$

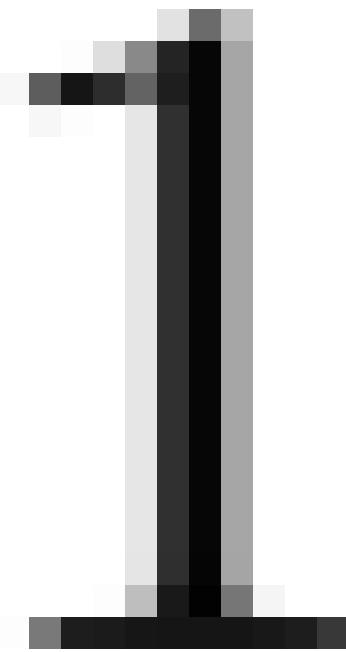
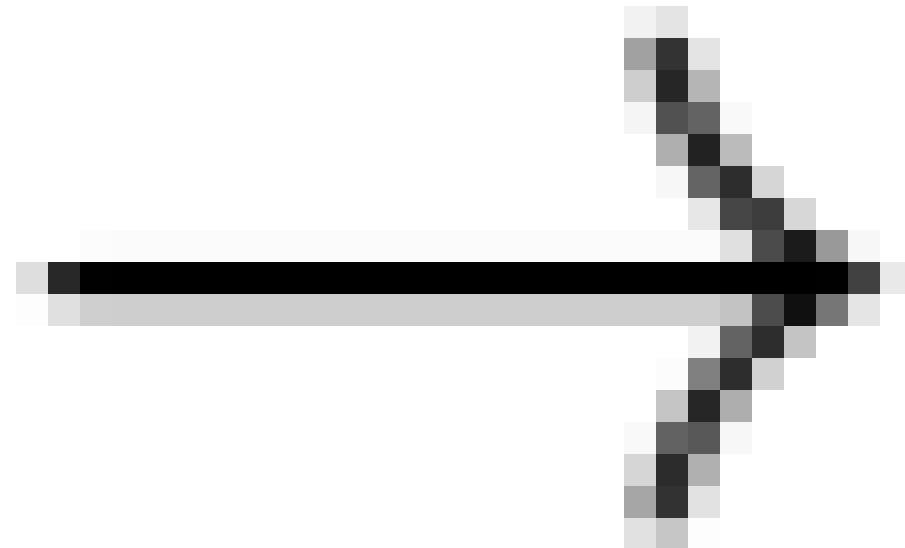


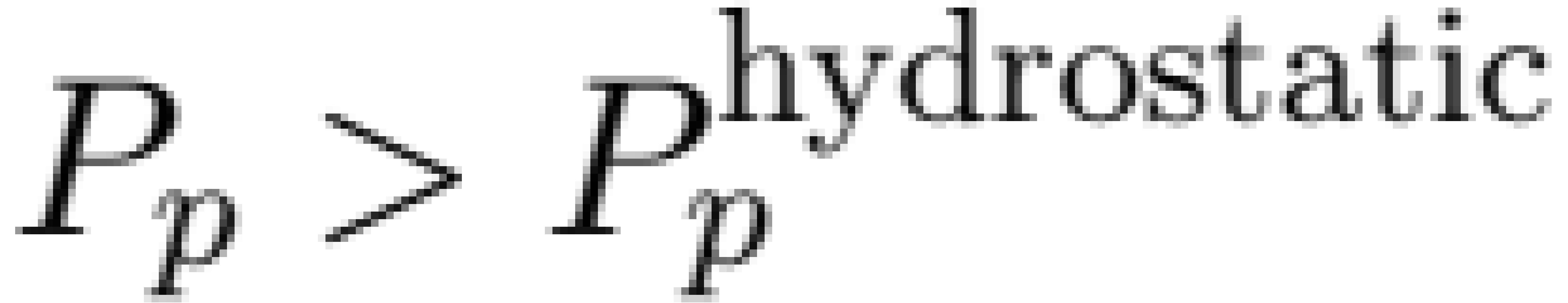
$P_p(z)$
 $S_q(z)$



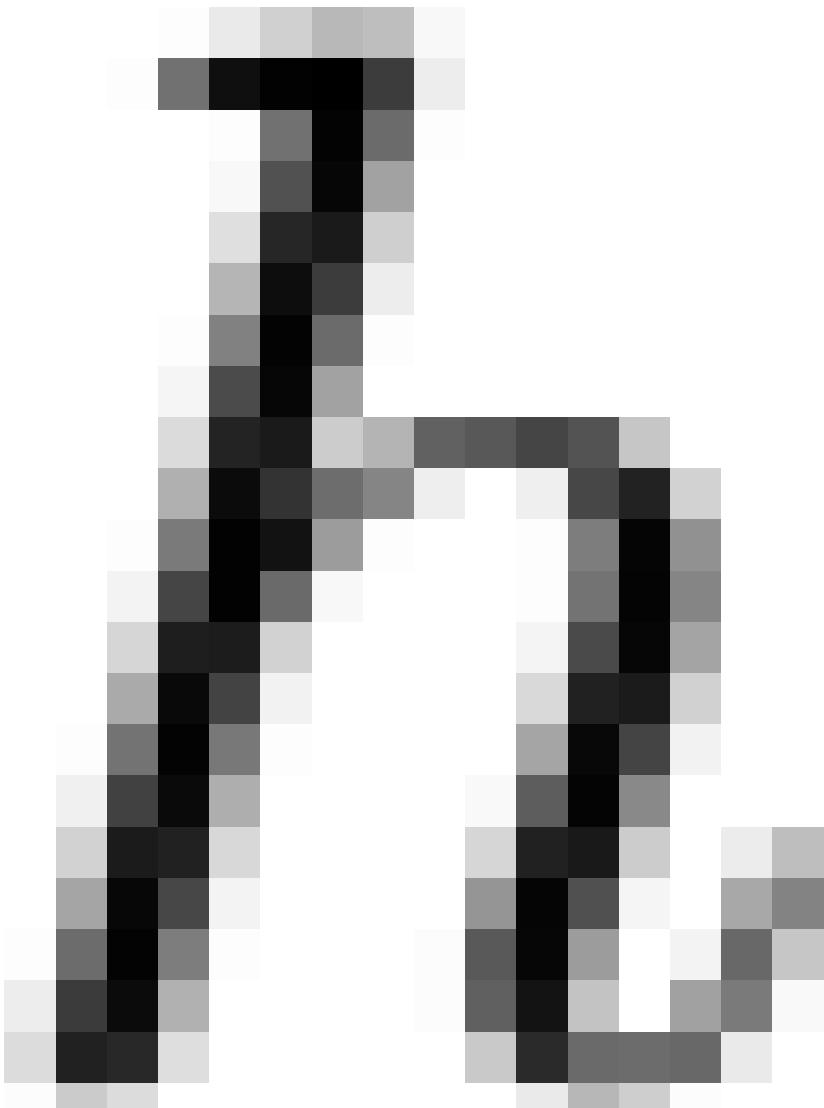














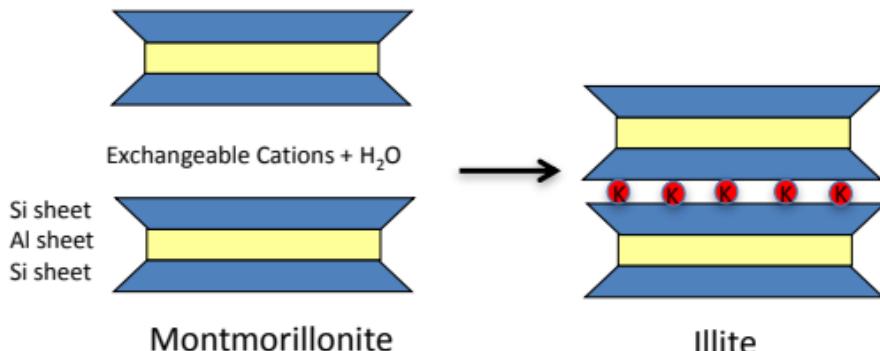


- **Aquathermal pressurization**

- $\Delta T \rightarrow \Delta P$

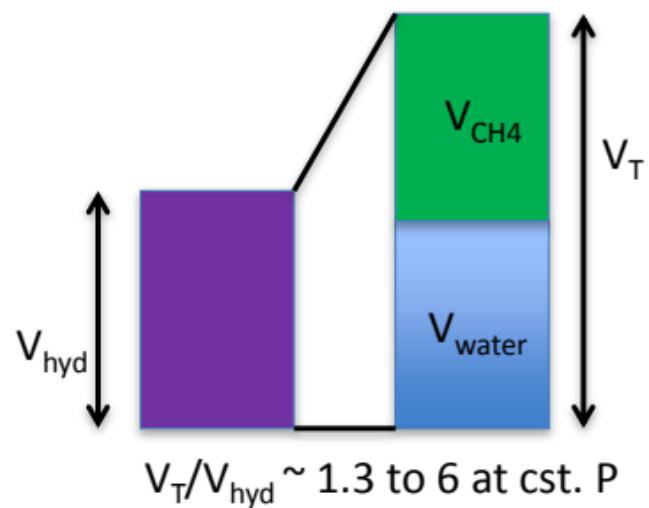
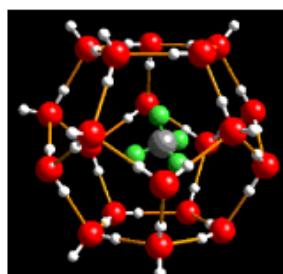
- **Dehydration reactions**

- $\Delta V \rightarrow \Delta P$
 - Montmorillonite to Illite (frees water)

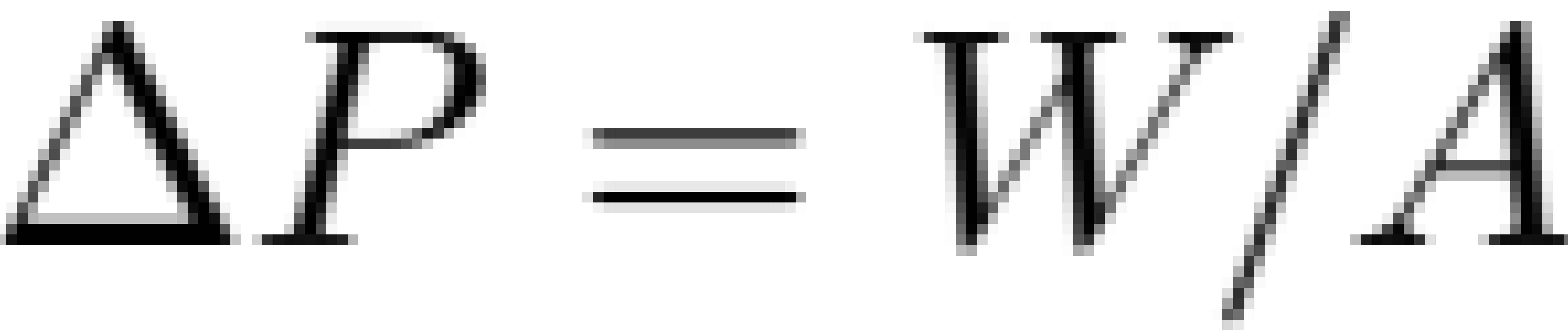


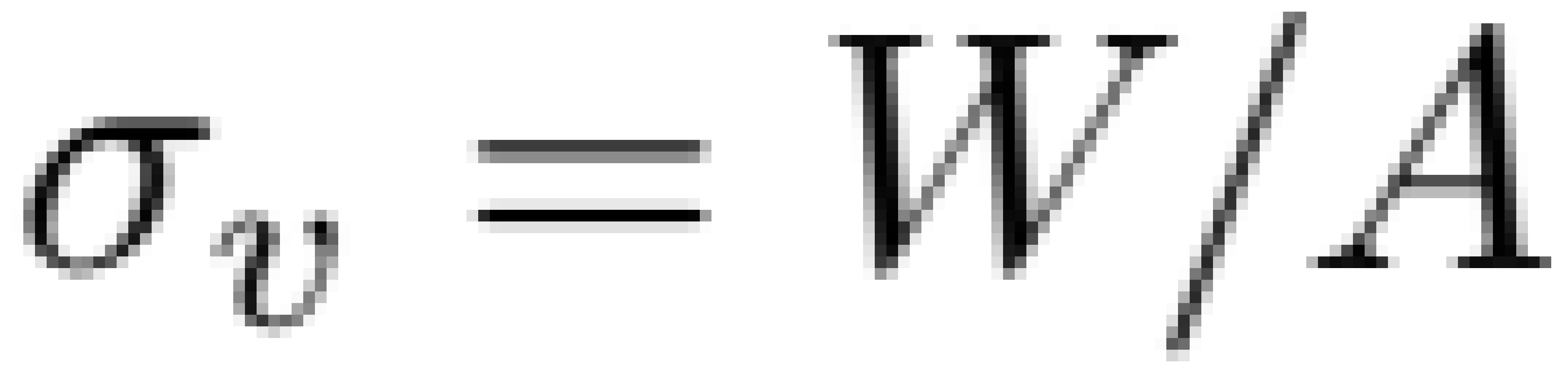
- **Hydrocarbon generation**

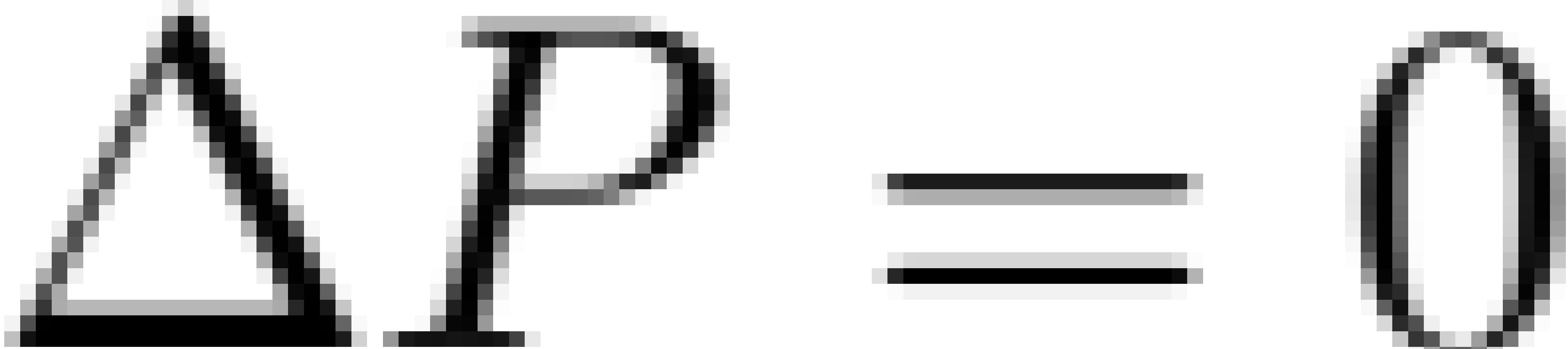
- $\Delta V \rightarrow \Delta P$



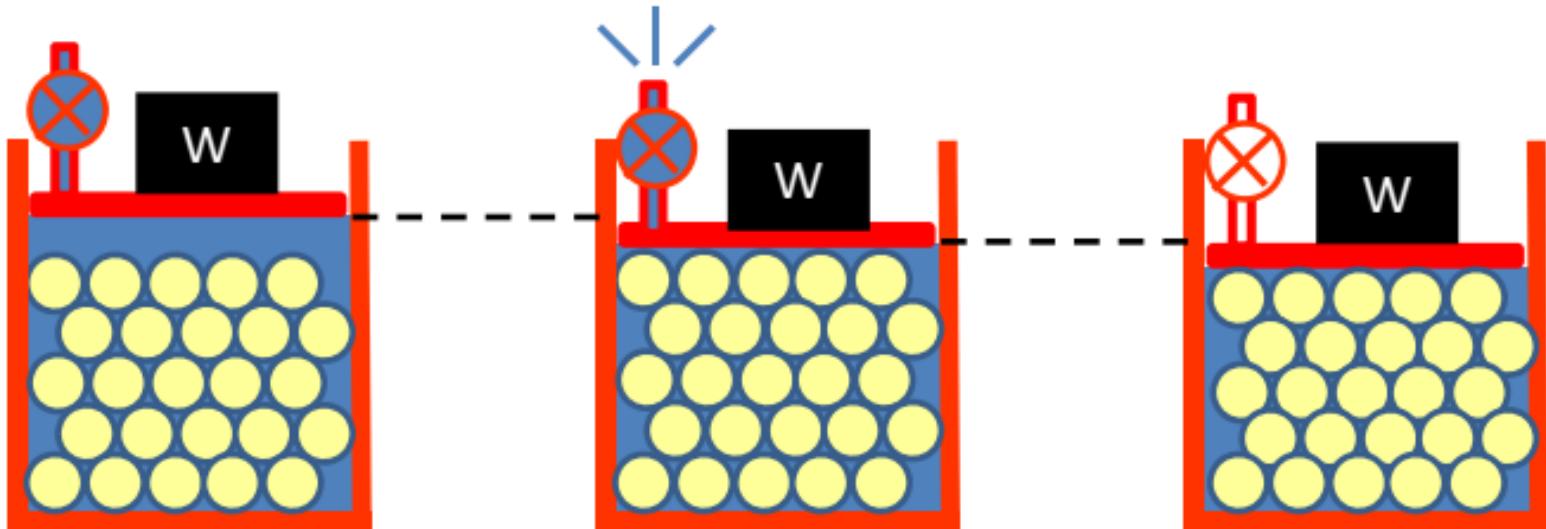




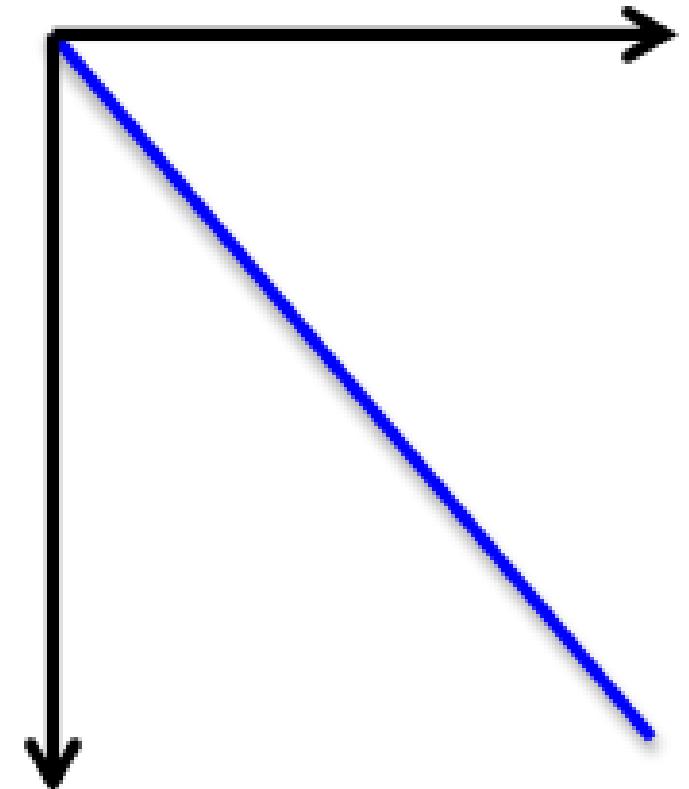
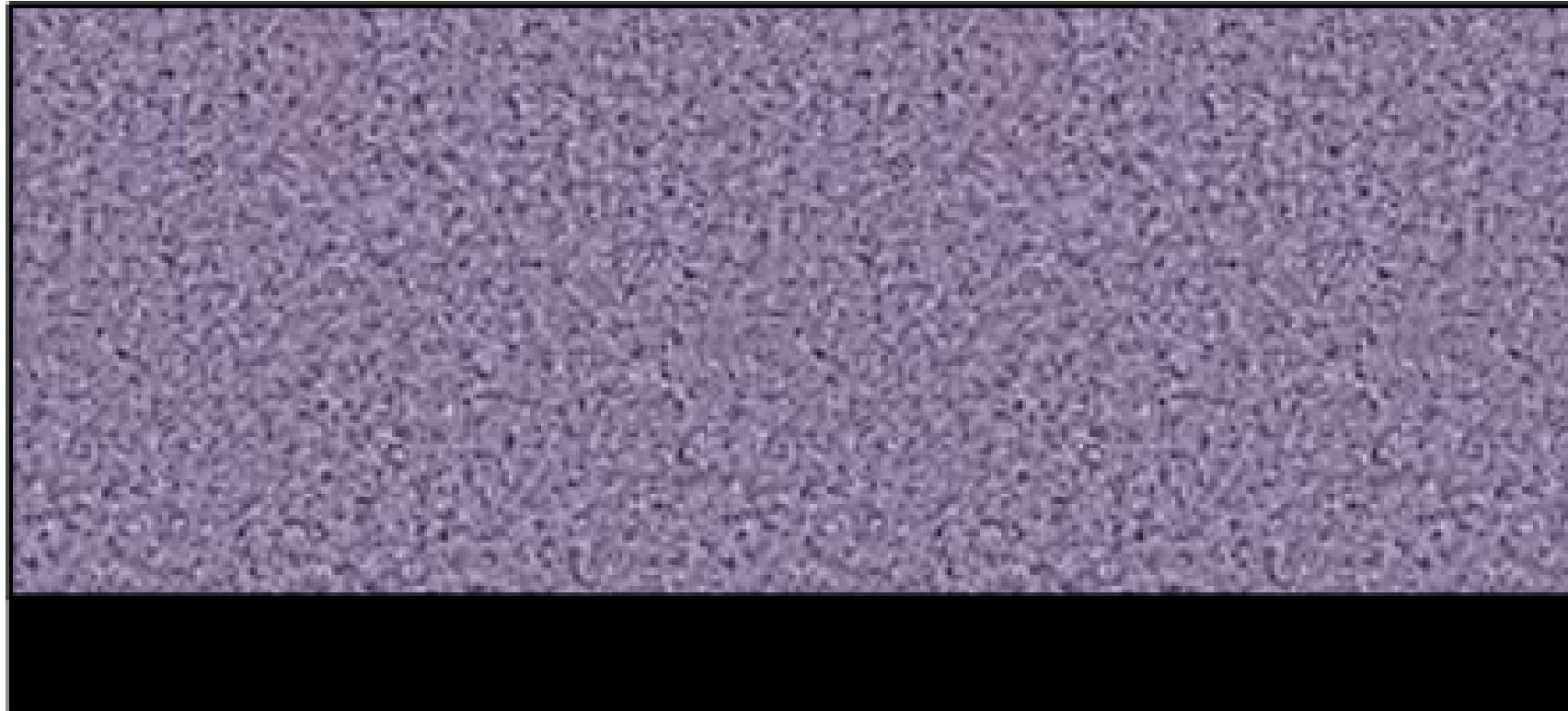


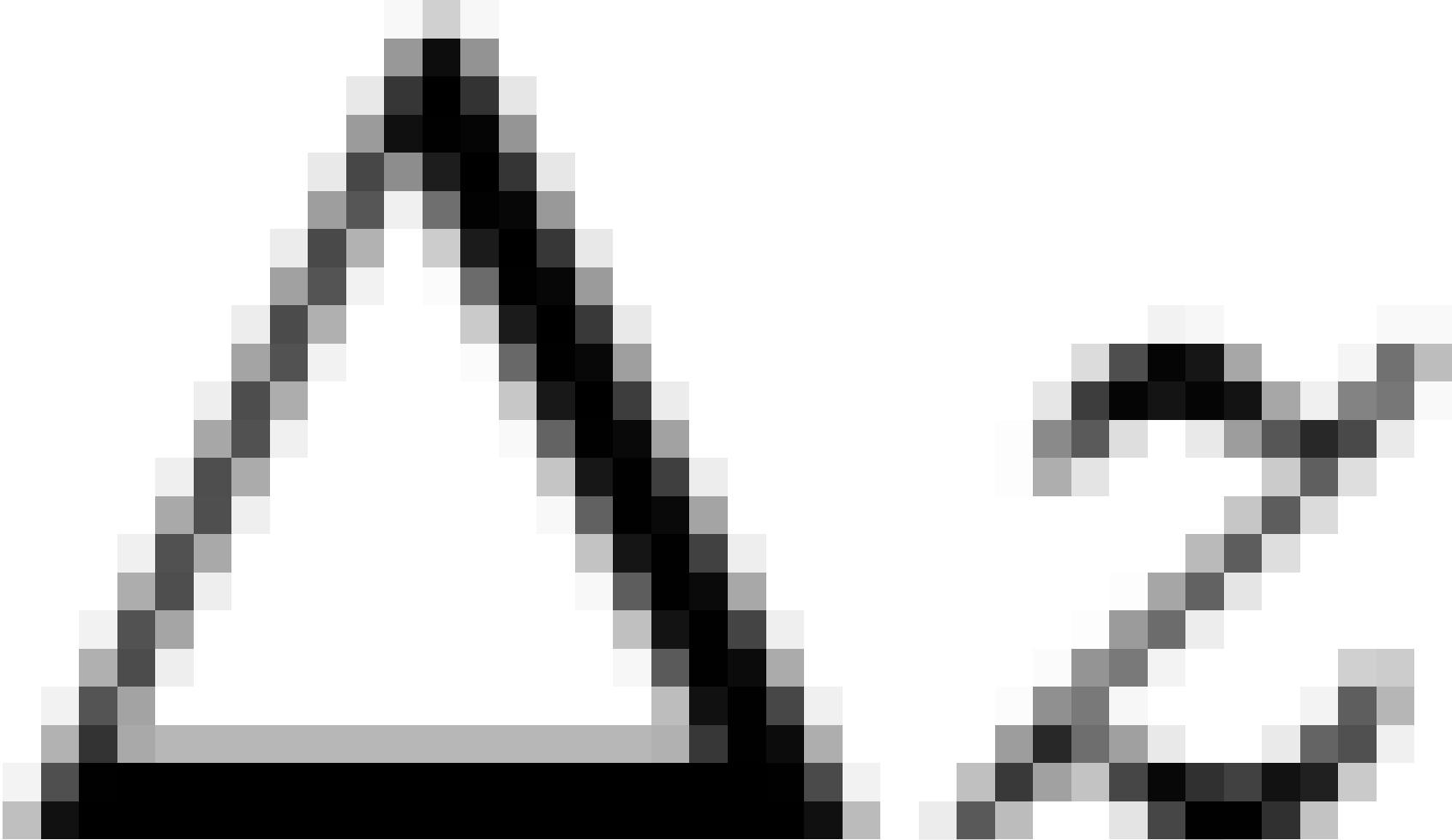


- **Disequilibrium compaction (Underconsolidation)**
 - $\Delta S \rightarrow \Delta P$ (Vertical)
- **Tectonic compression**
 - $\Delta S \rightarrow \Delta P$ (Horizontal)



Pressure water



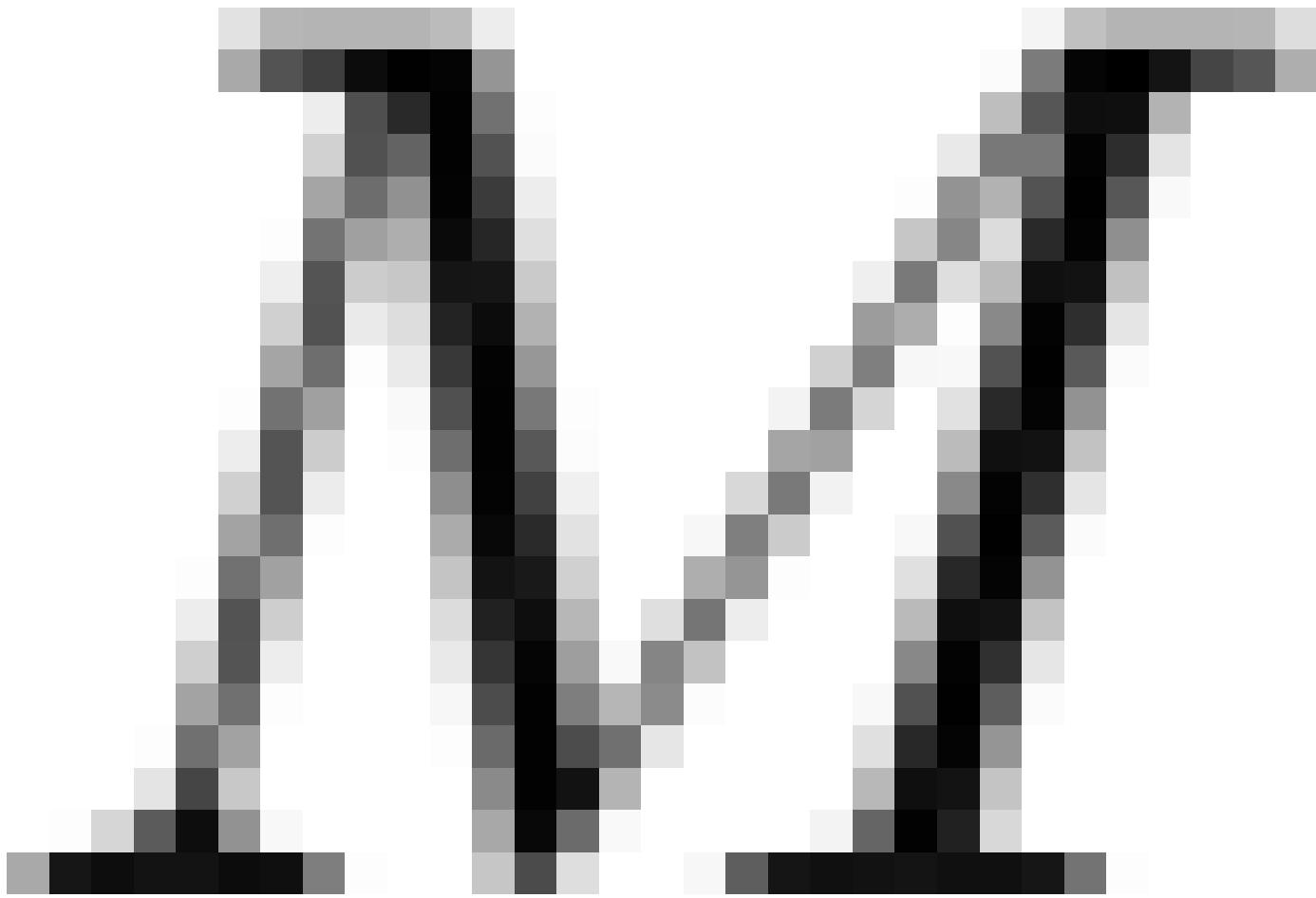


Dh

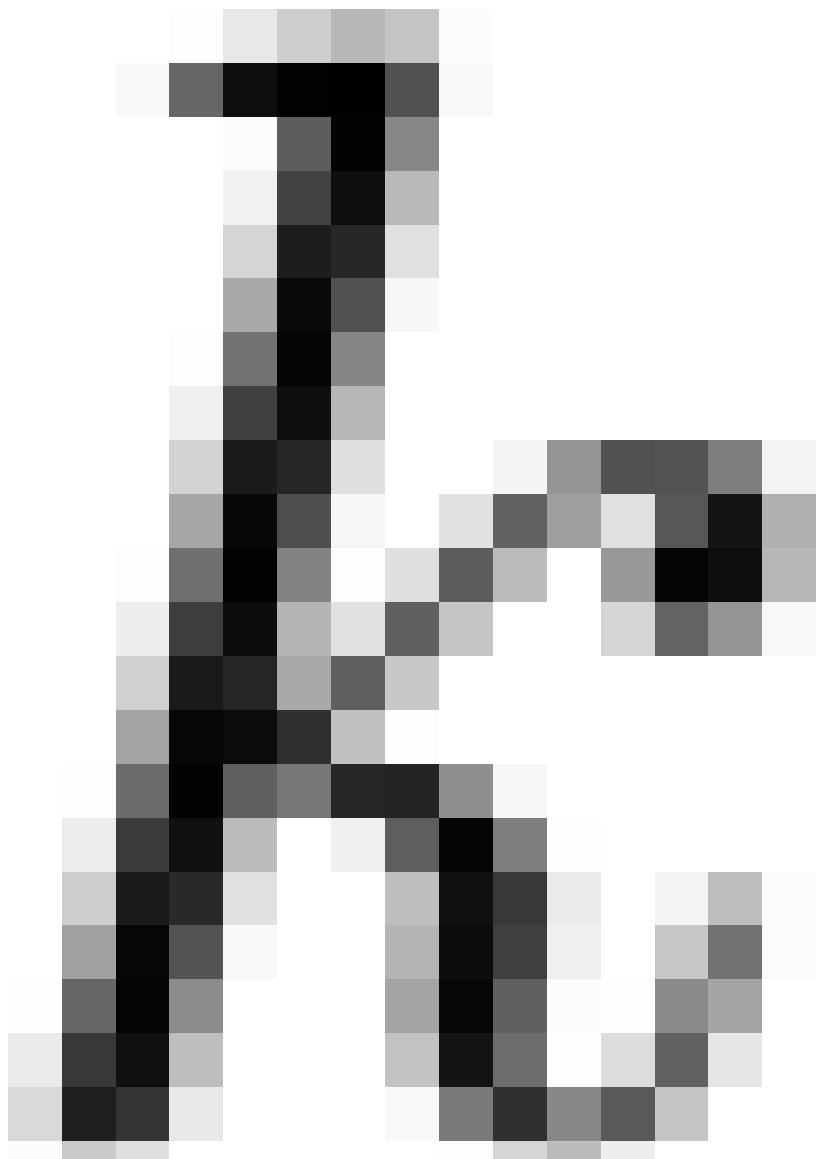
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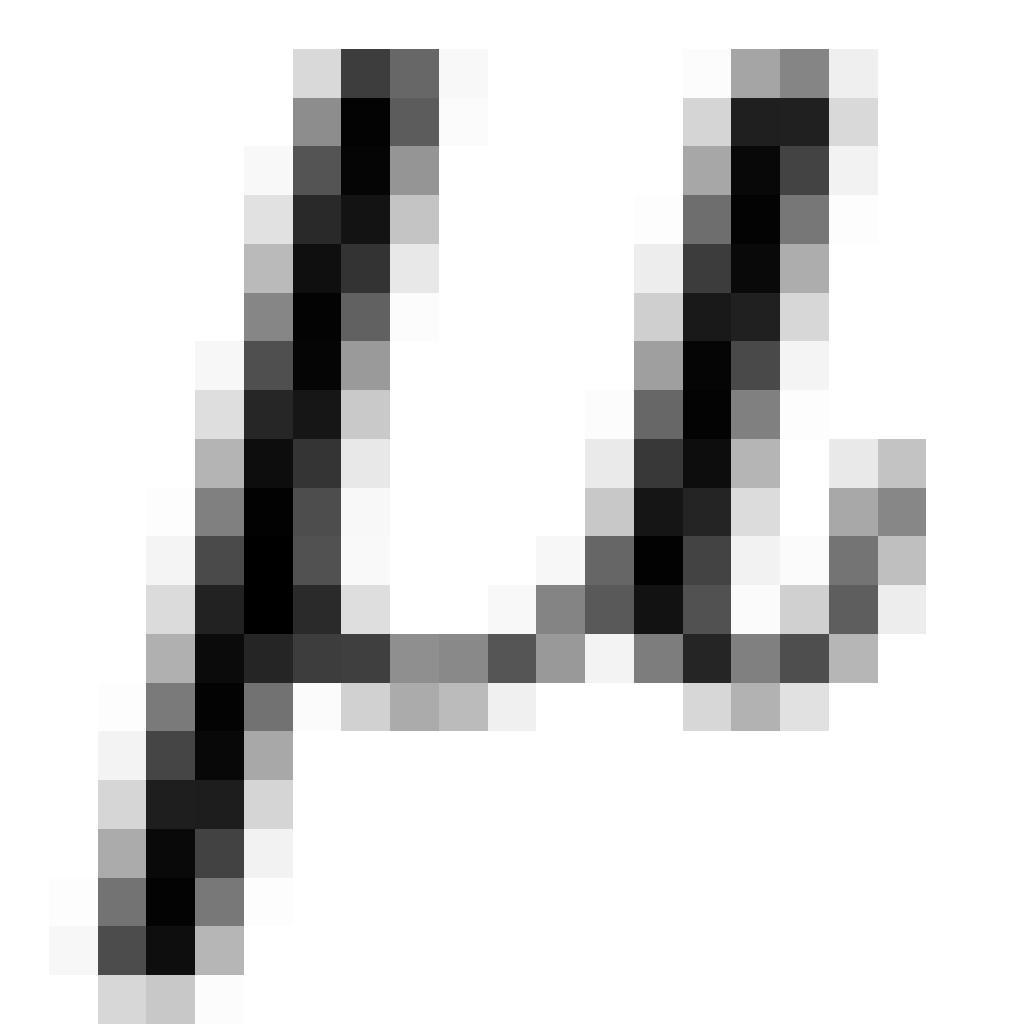
MK

ll









αP
 p



D_h

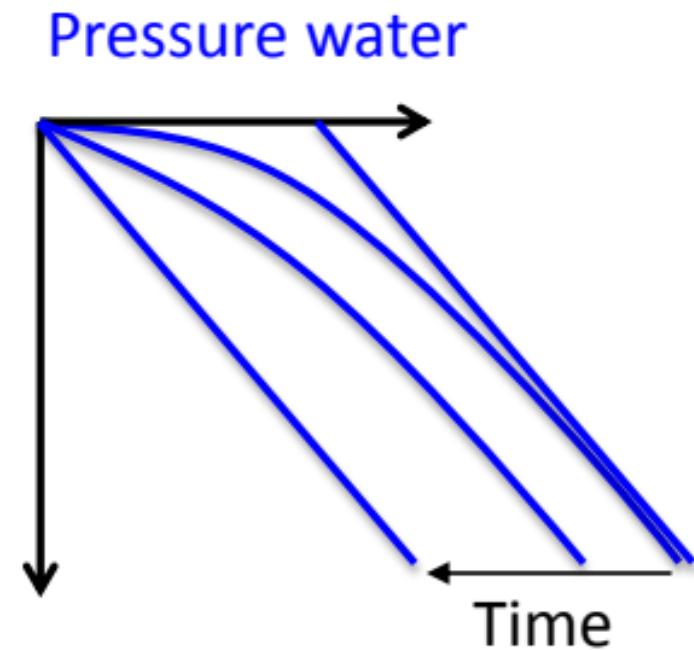
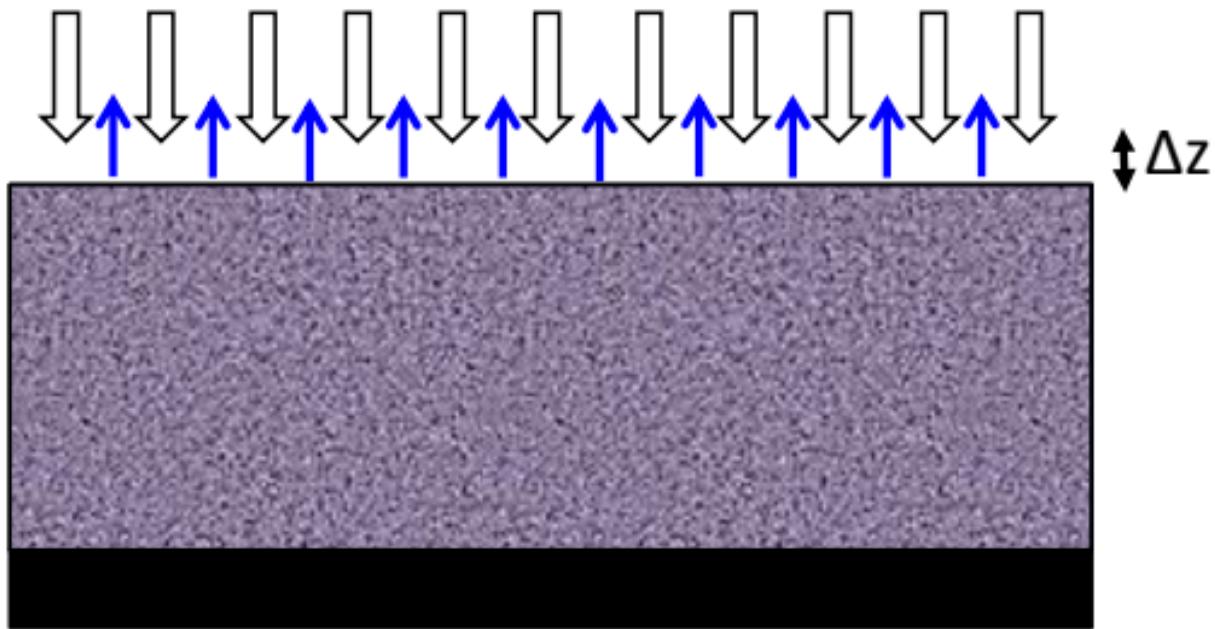
$d^2 P$
 p



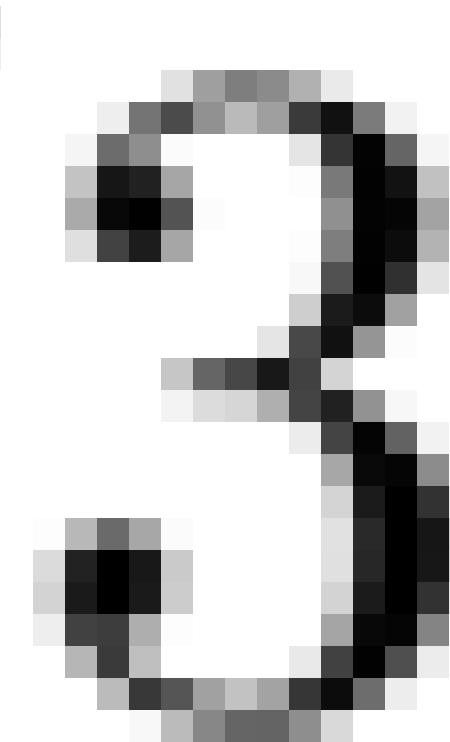
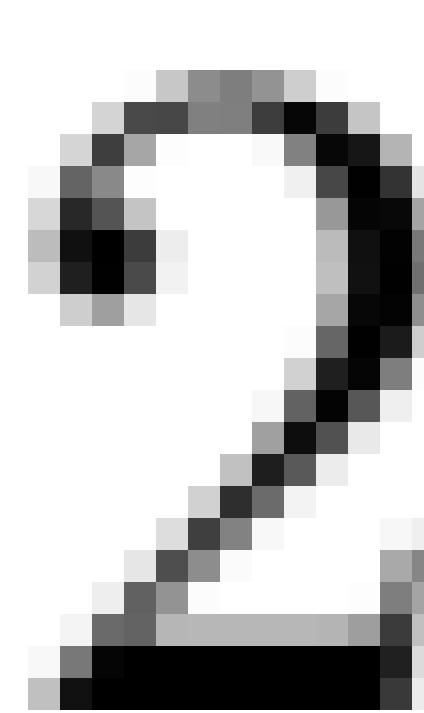
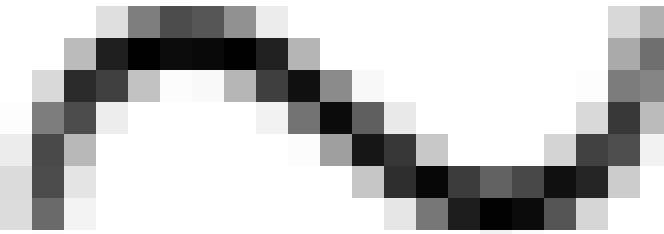
αt

$d z^2$

Rate of sedimentation (loading) and rate of fluid “escape”





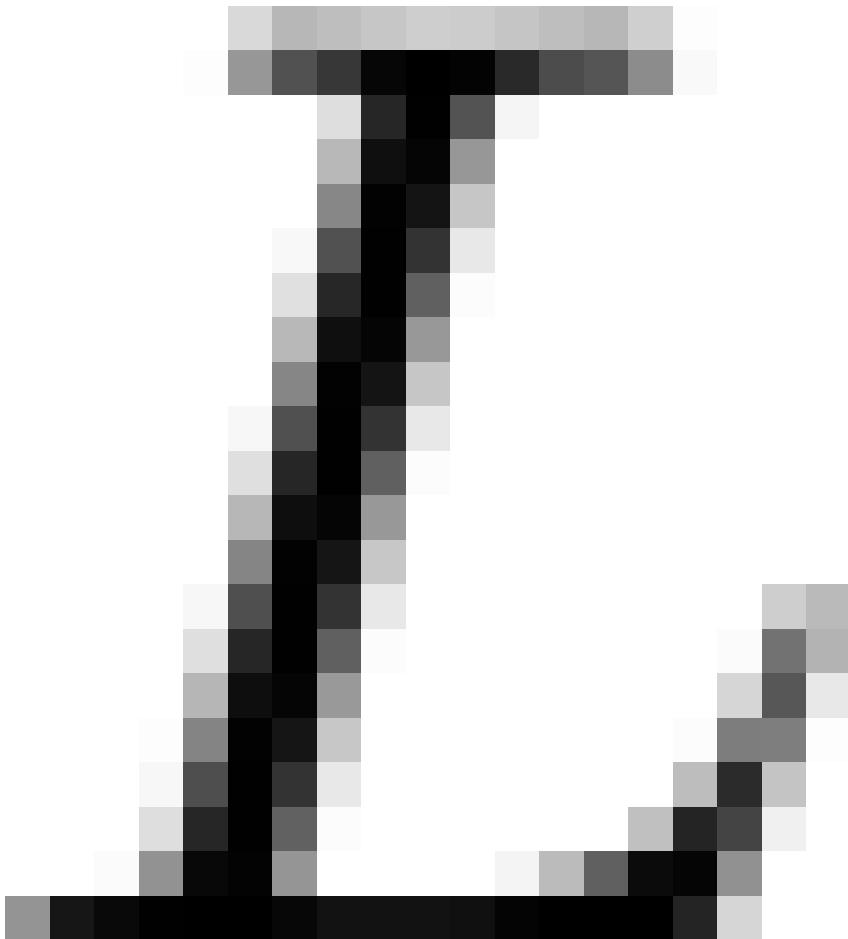


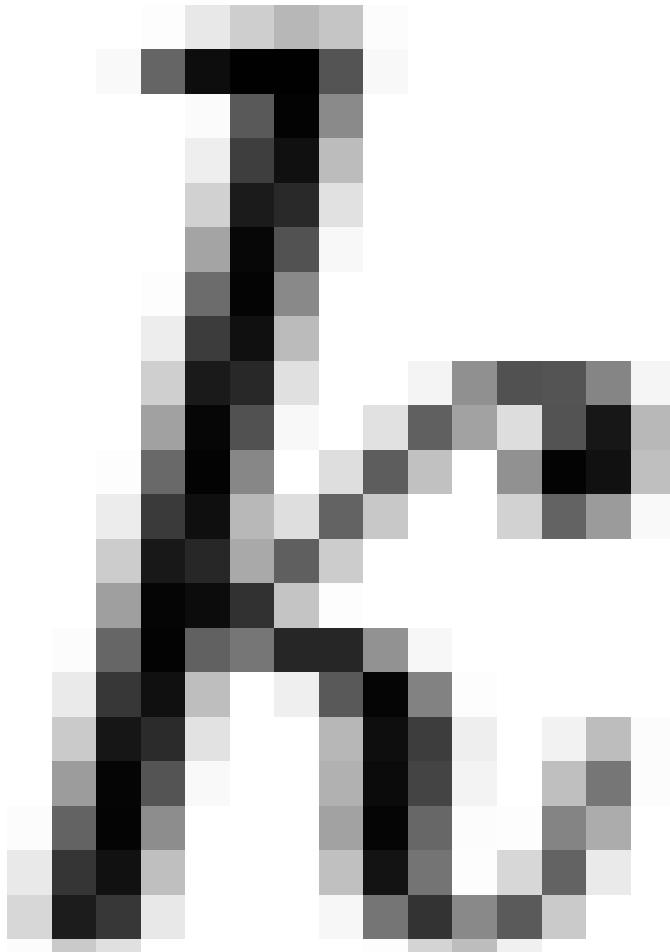
Γ_{ch}

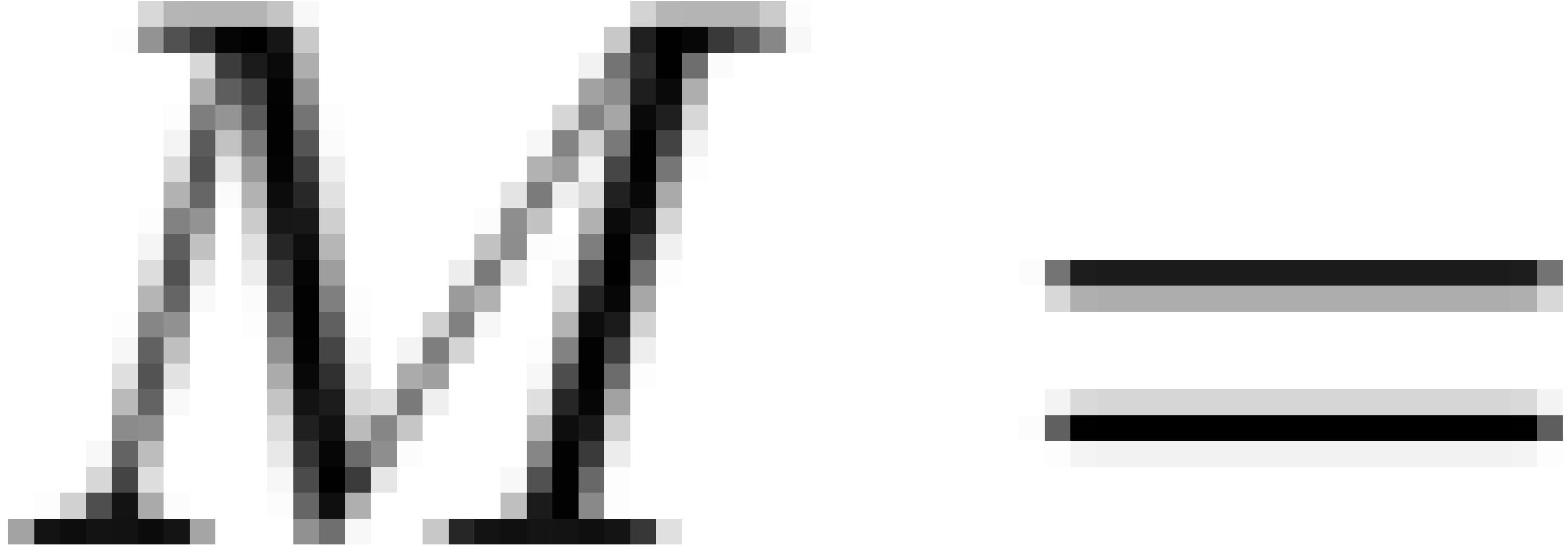


D_h

Γ^2

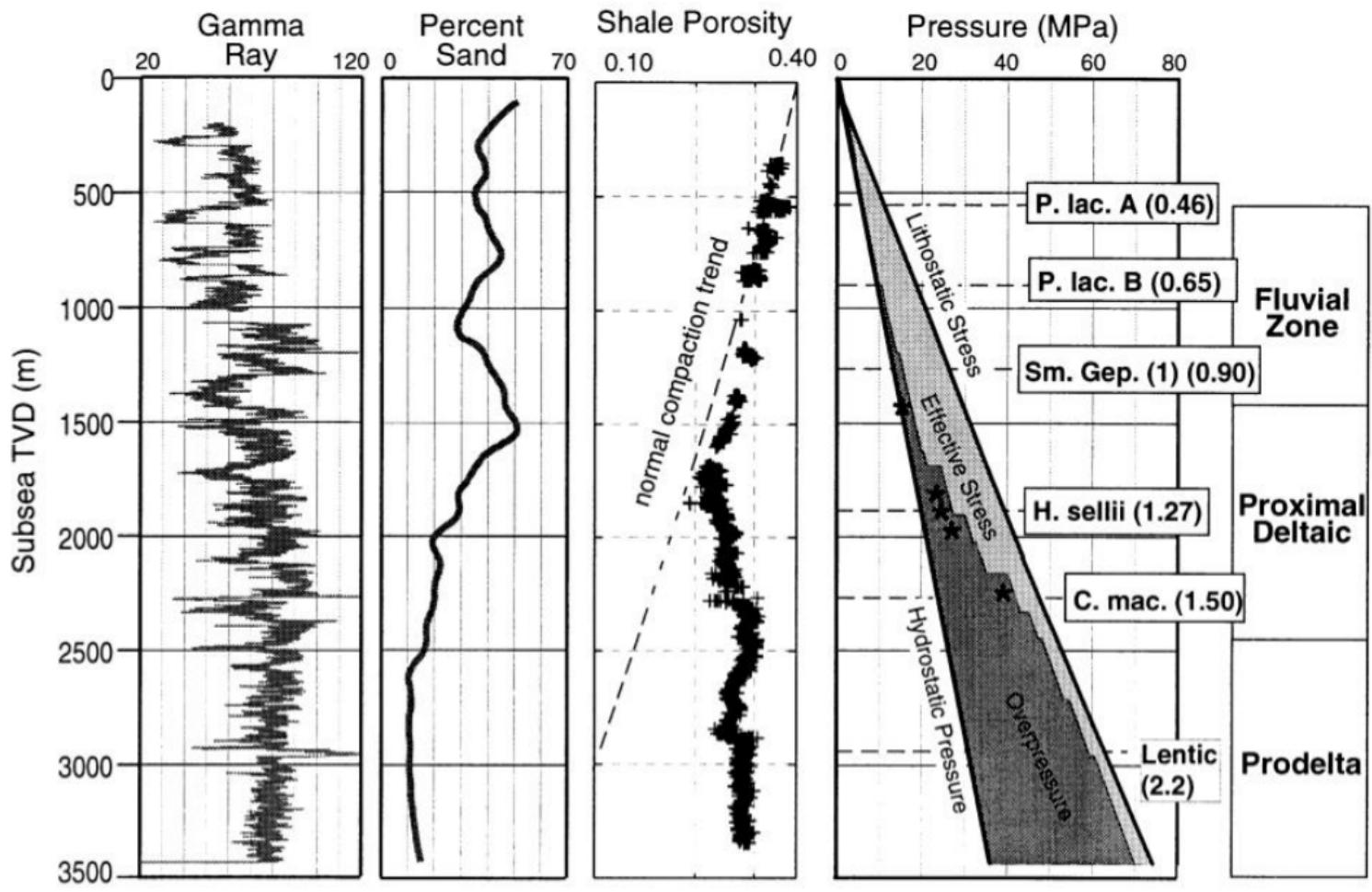




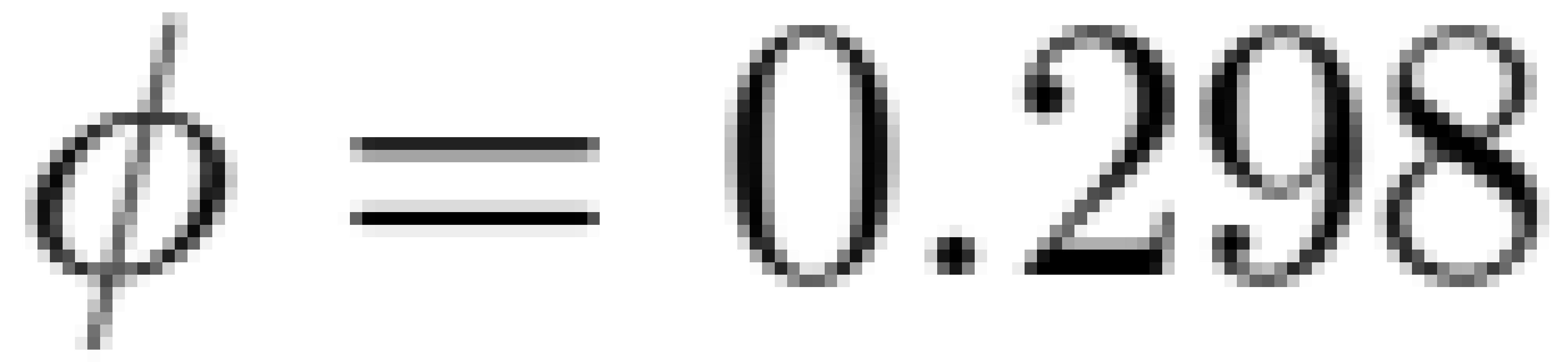


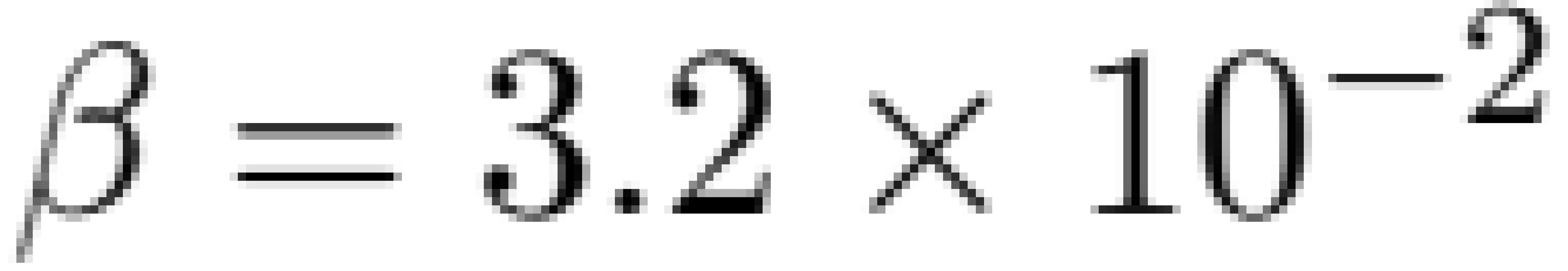


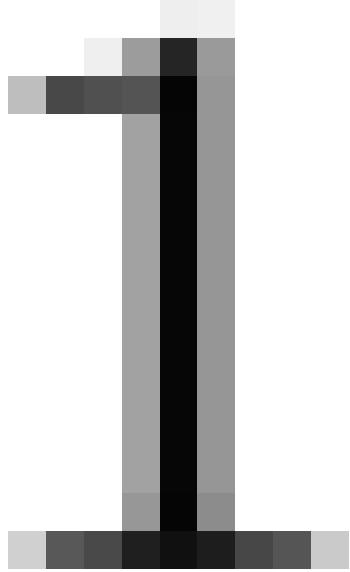


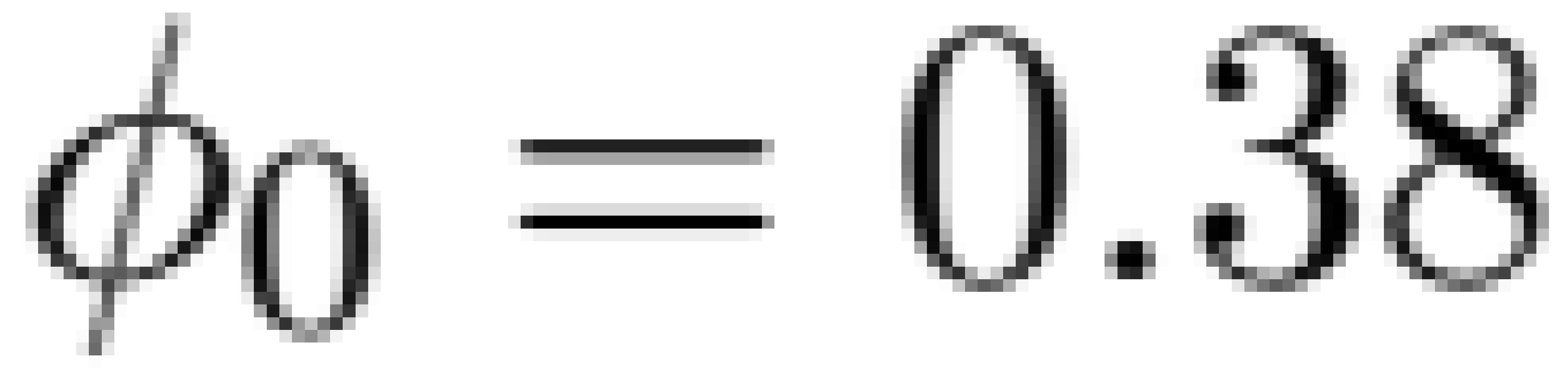


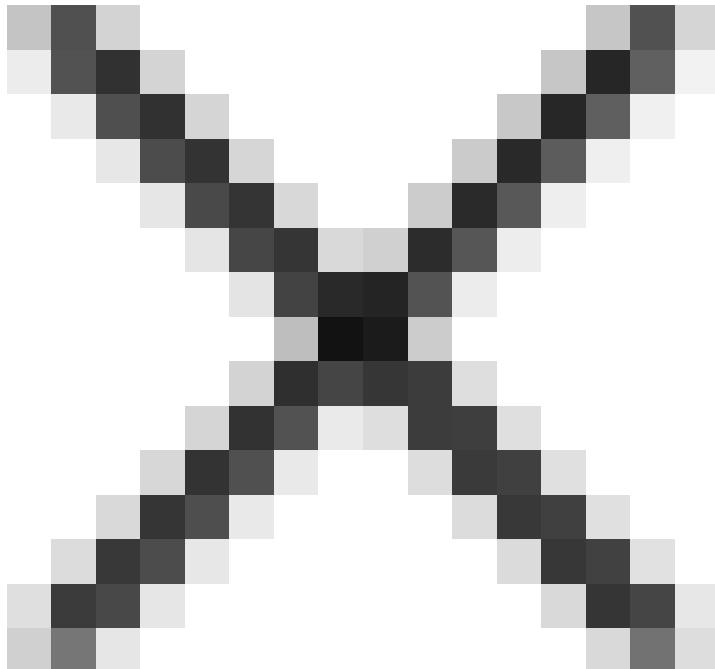
Off-shore Louisiana – Gordon and Flemings (1998) Basin Research



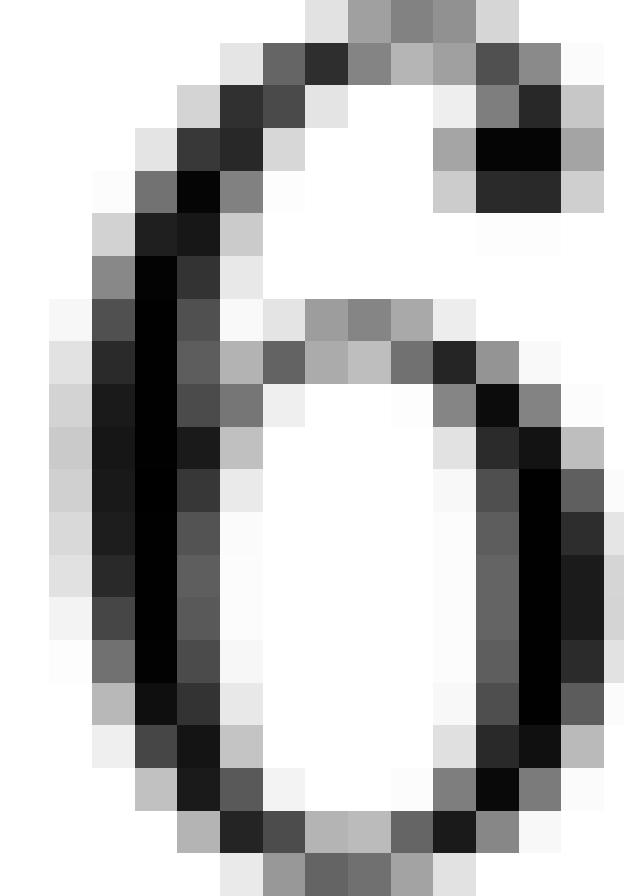
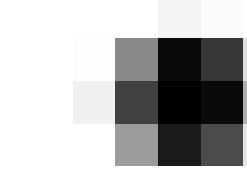
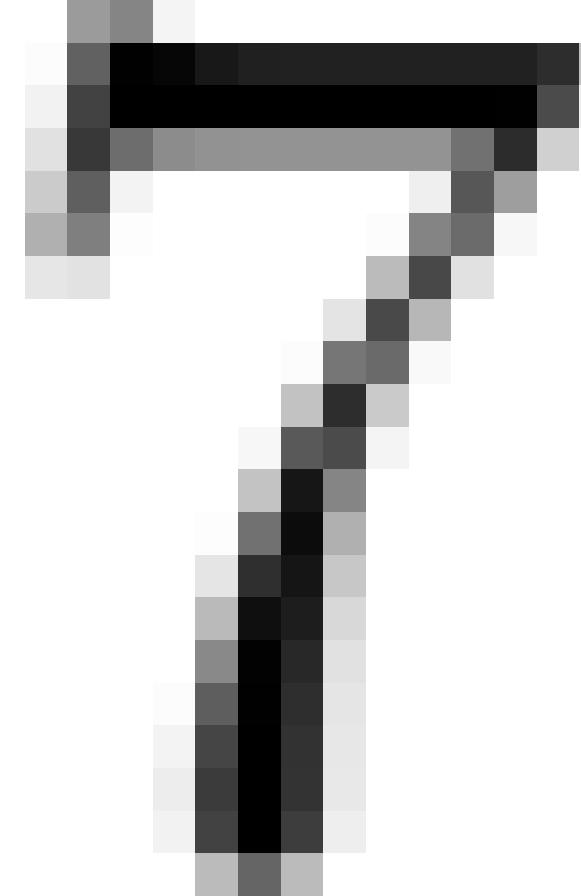


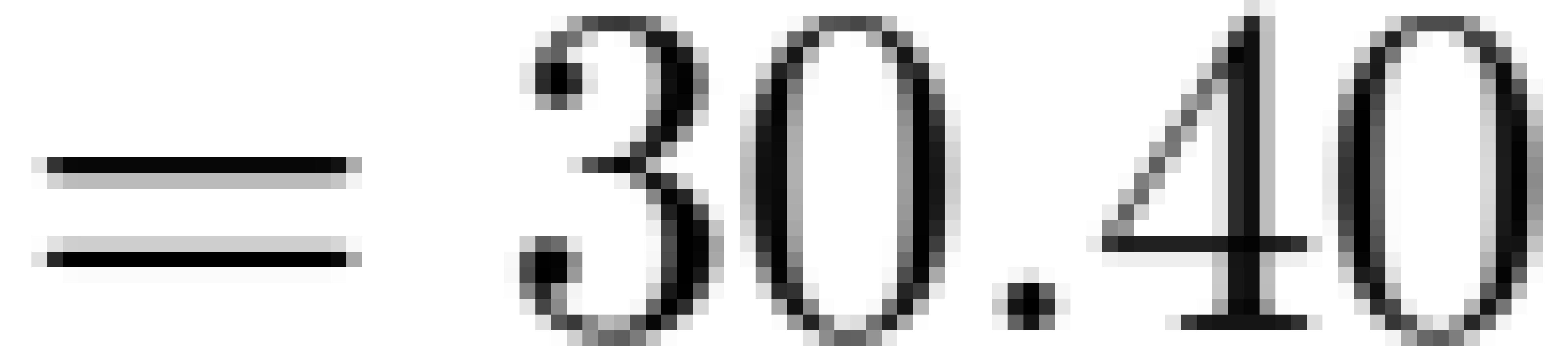




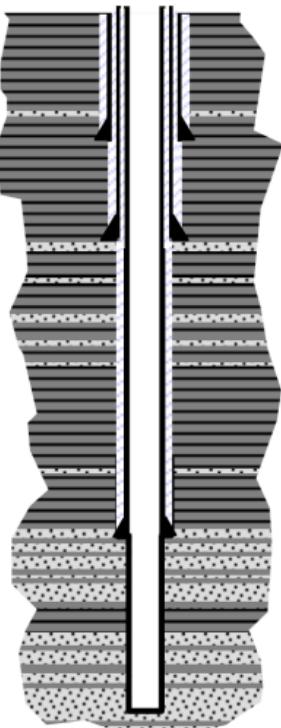
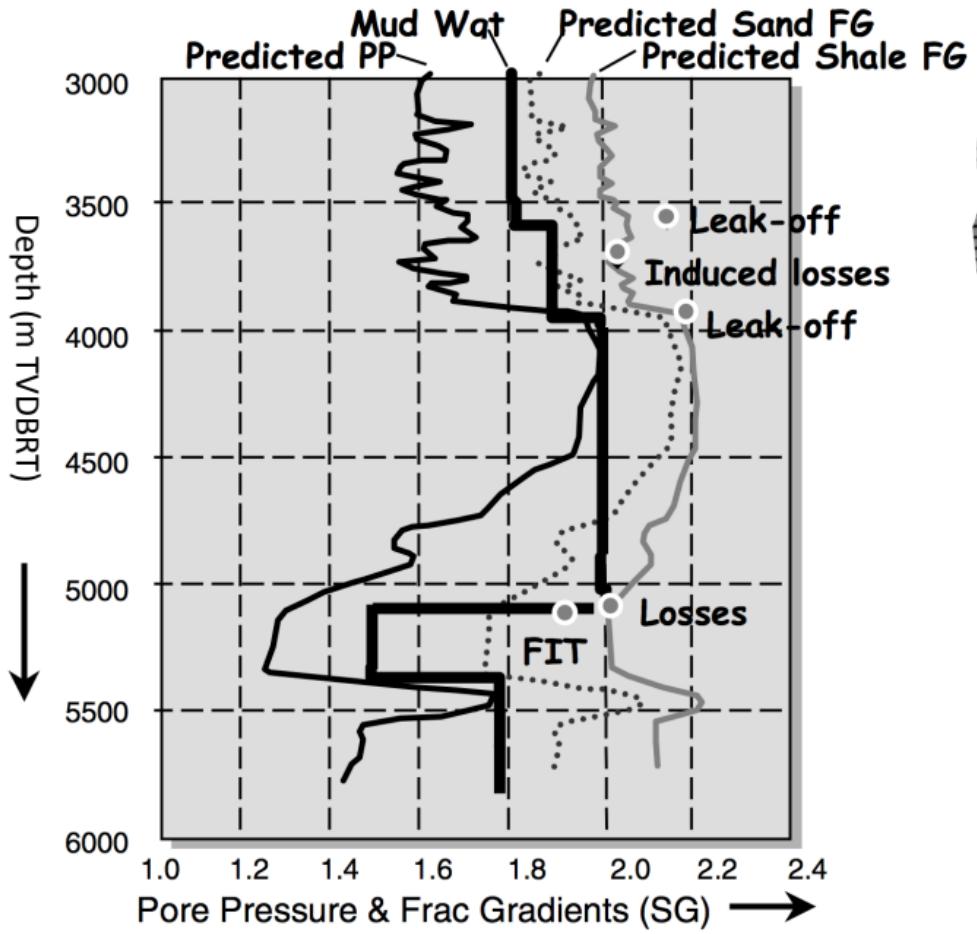


$$P_p = S_p + \ln(\phi/\phi_0) = 38$$





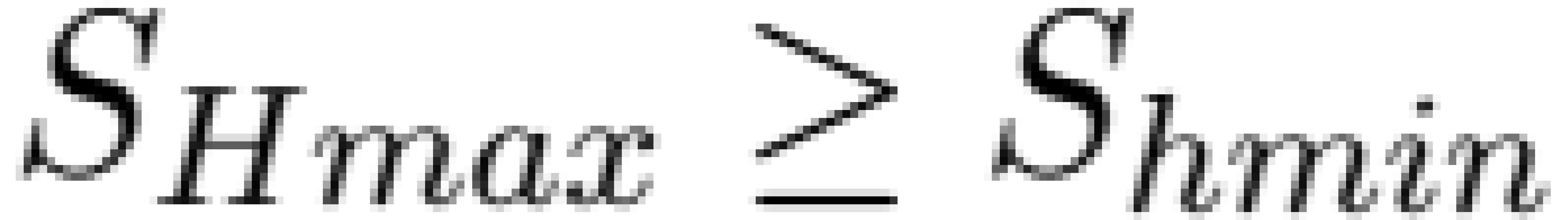
P = 30.40 MPa
 P = 0.8 MPa
 S_2 = 38 MPa

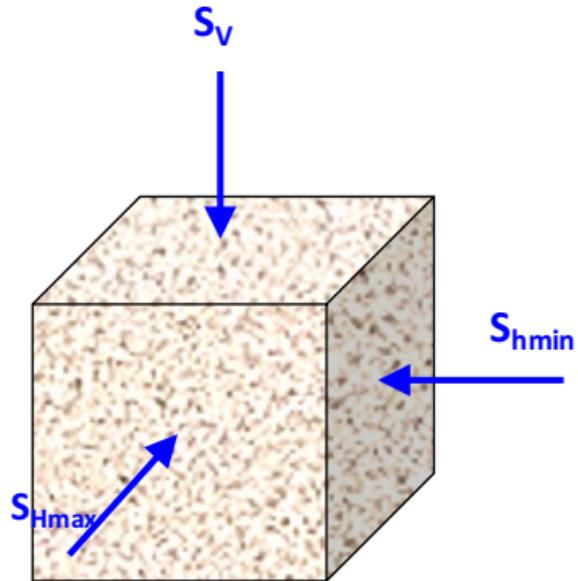
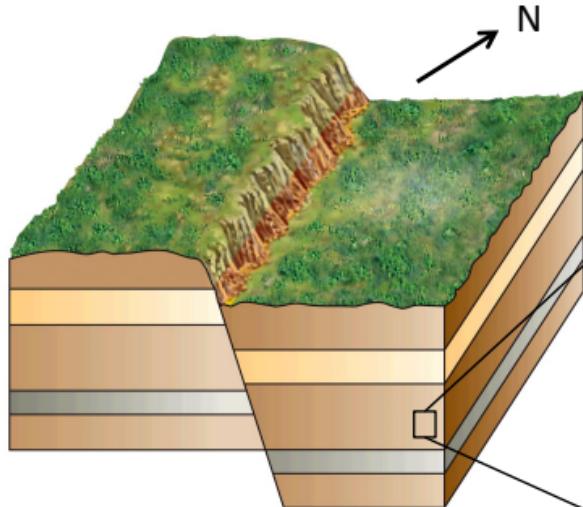


[Caspian Sea, Alberta and McLean – SPE67740]



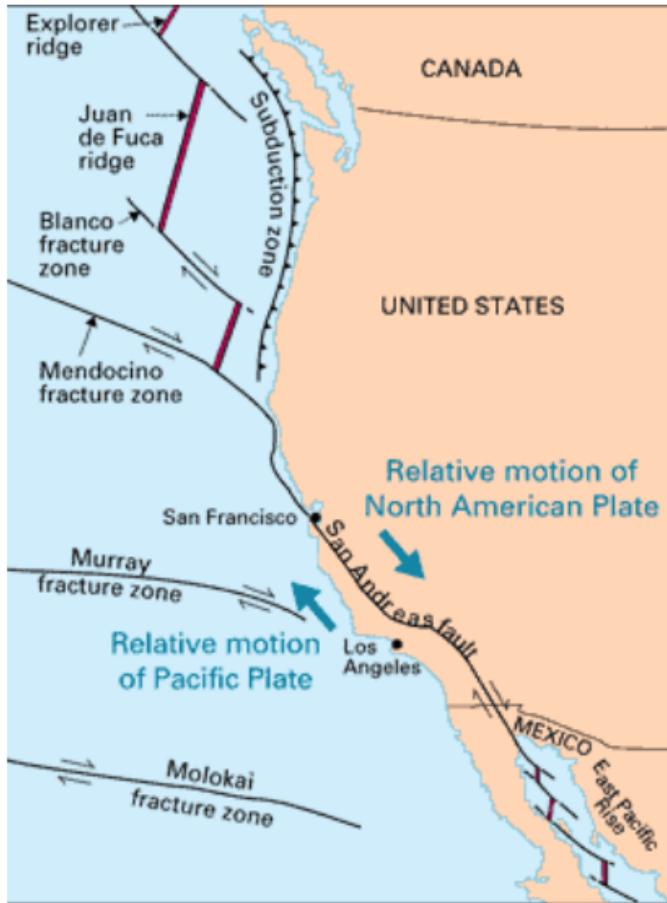






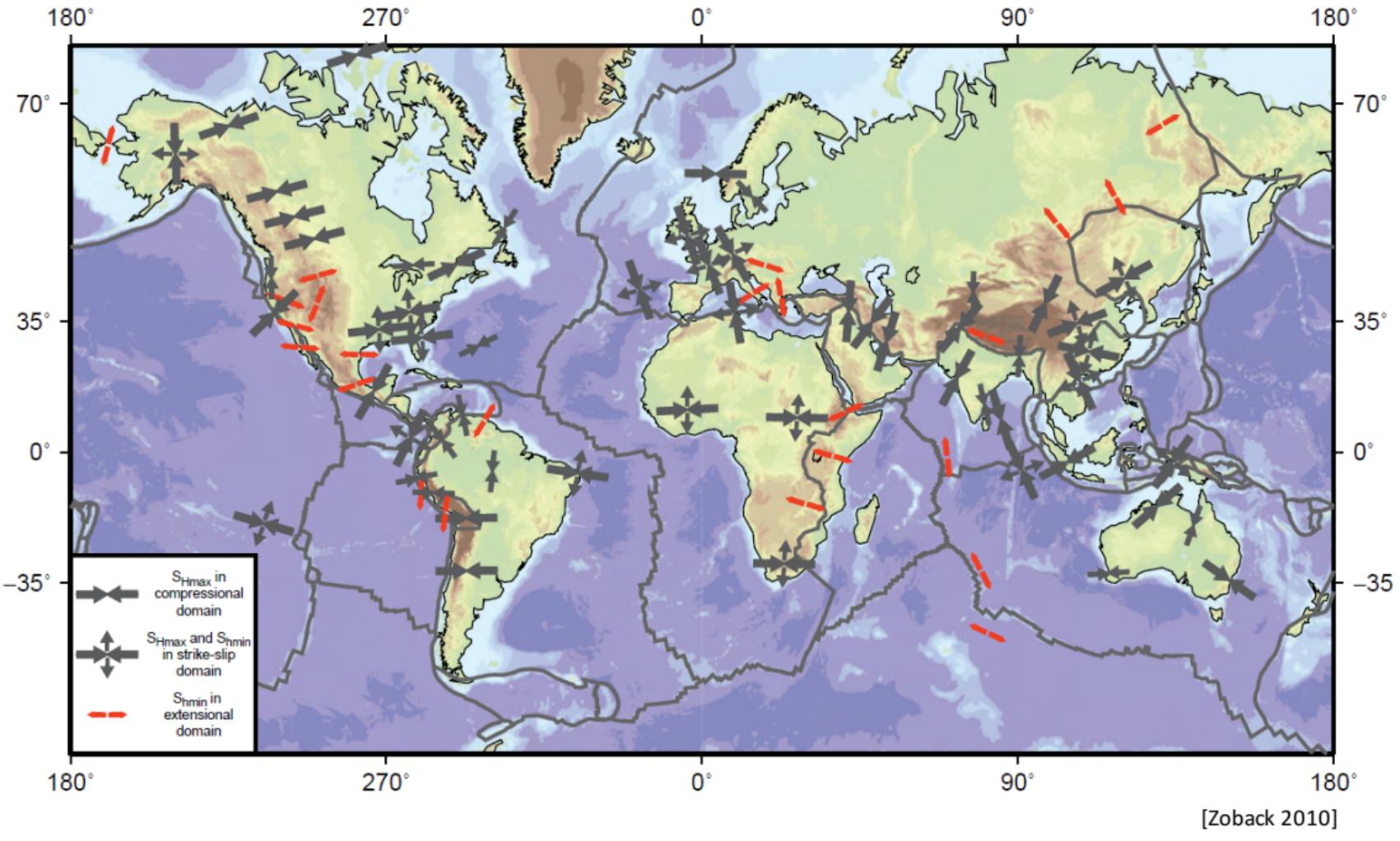
Self-test 9.8
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$$\underline{\underline{S}} = \begin{bmatrix} S_V & 0 & 0 \\ 0 & S_{H\max} & 0 \\ 0 & 0 & S_{h\min} \end{bmatrix}$$

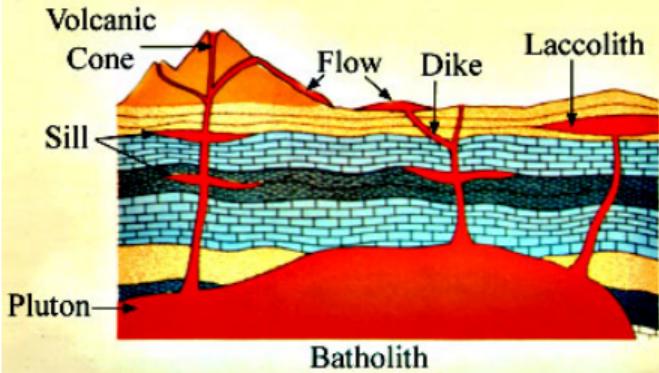


<http://pubs.usgs.gov/gip/dynamic/understanding.html#anchor5798673>

<http://en.wikipedia.org/wiki/File:Aerial-SanAndreas-CarrizoPlain.jpg>



PLUTONS & VOLCANIC LANDFORMS



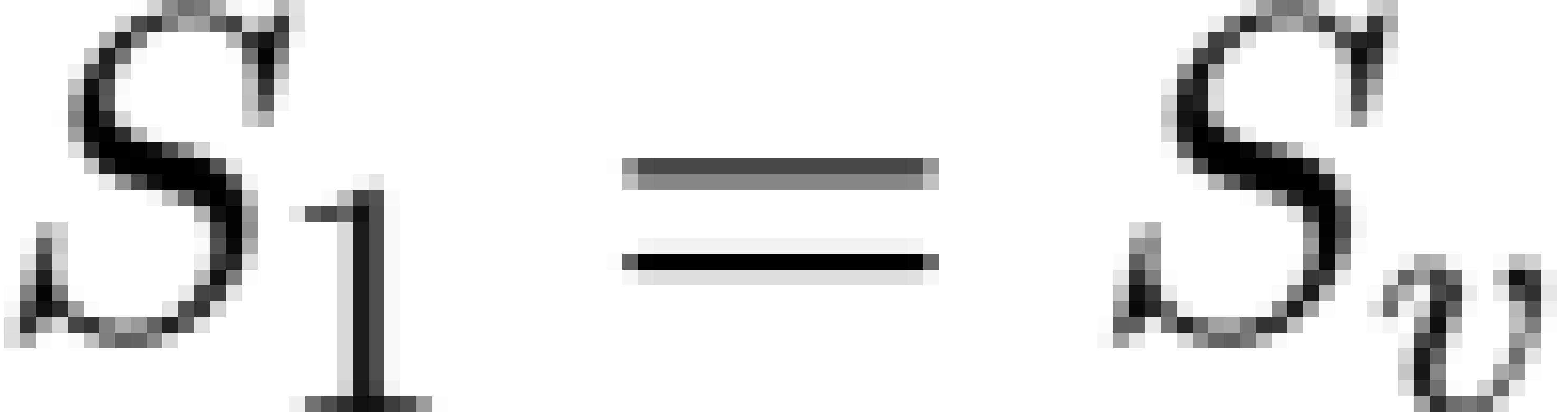
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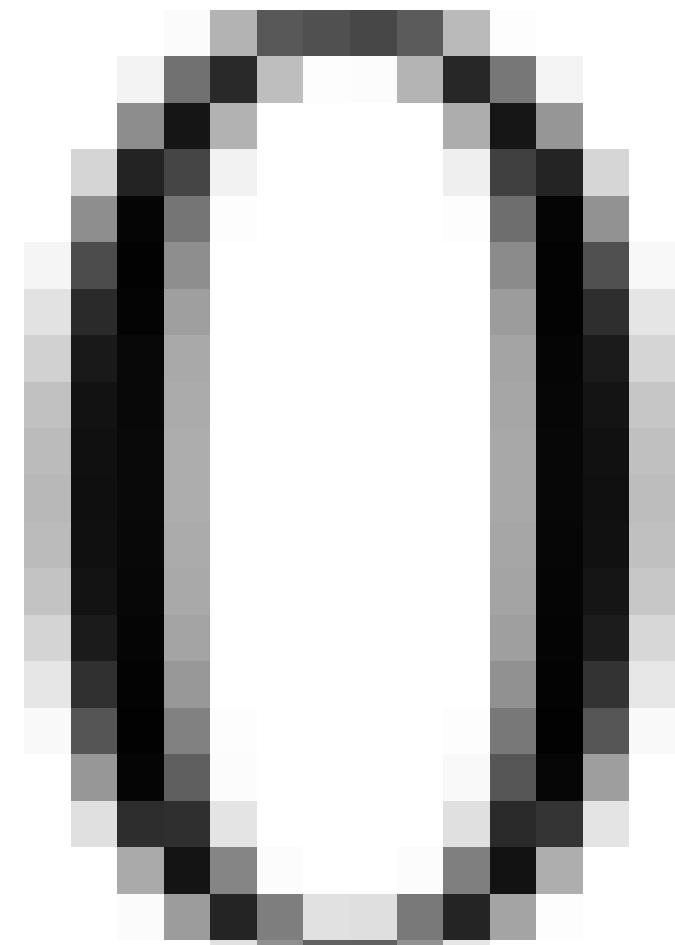
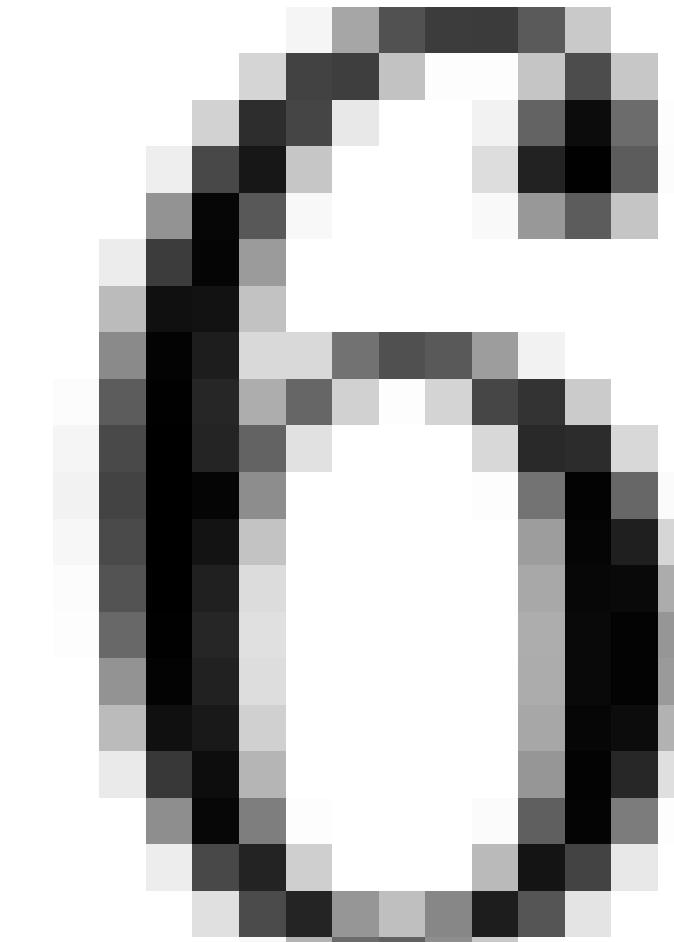
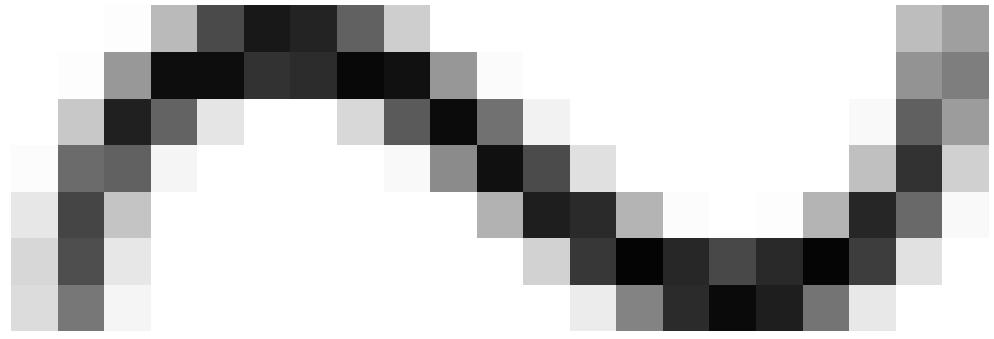








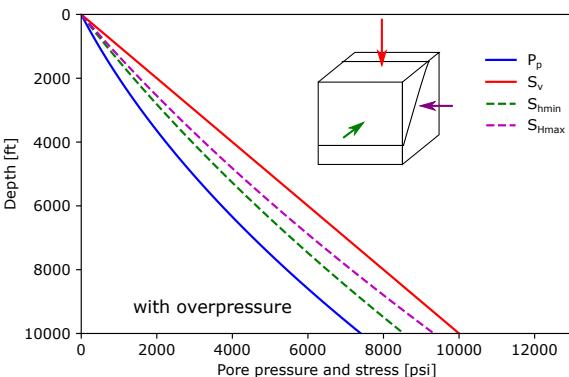
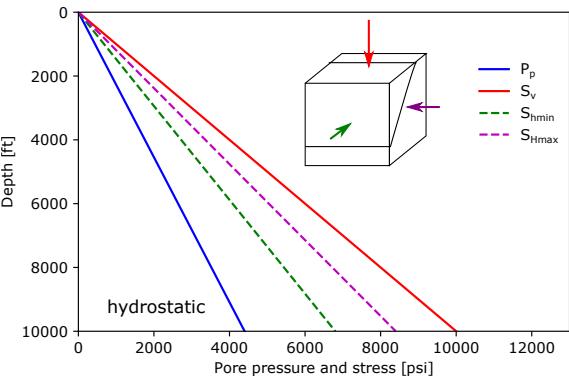




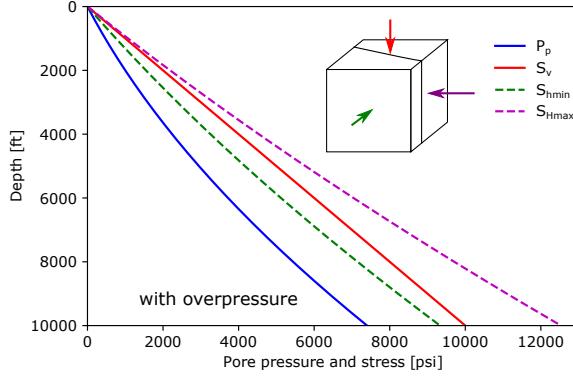
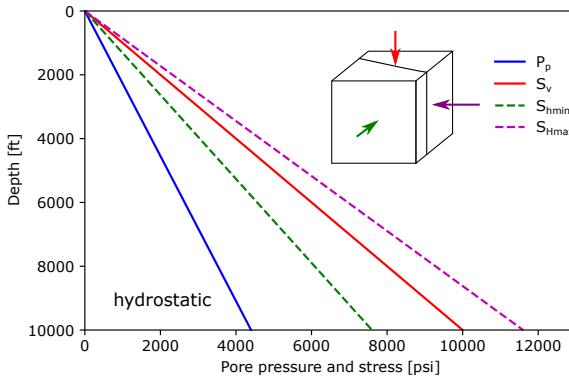




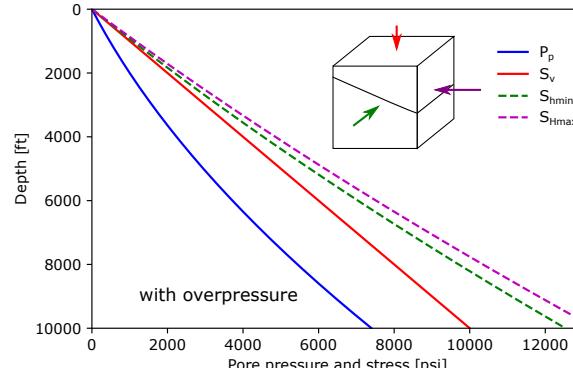
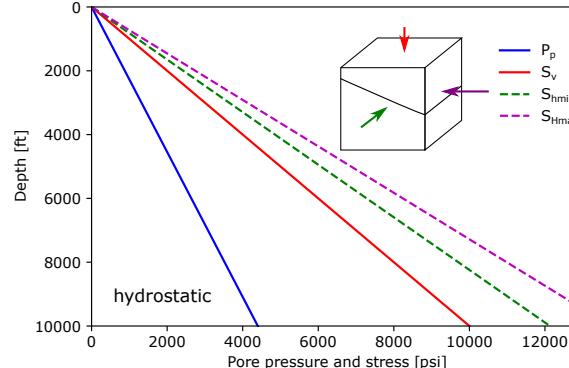
Normal faulting: $S_v > S_{H\max} > S_{h\min}$

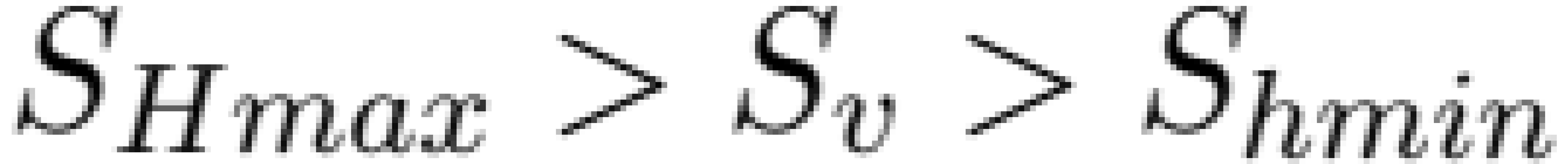


Strike slip faulting: $S_{H\max} > S_v > S_{h\min}$



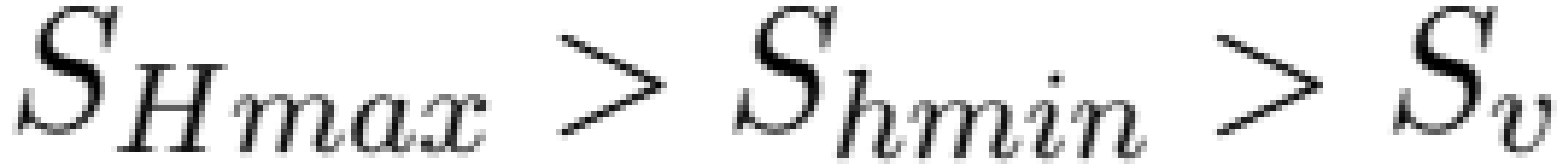
Reverse faulting: $S_{H\max} > S_{h\min} > S_v$





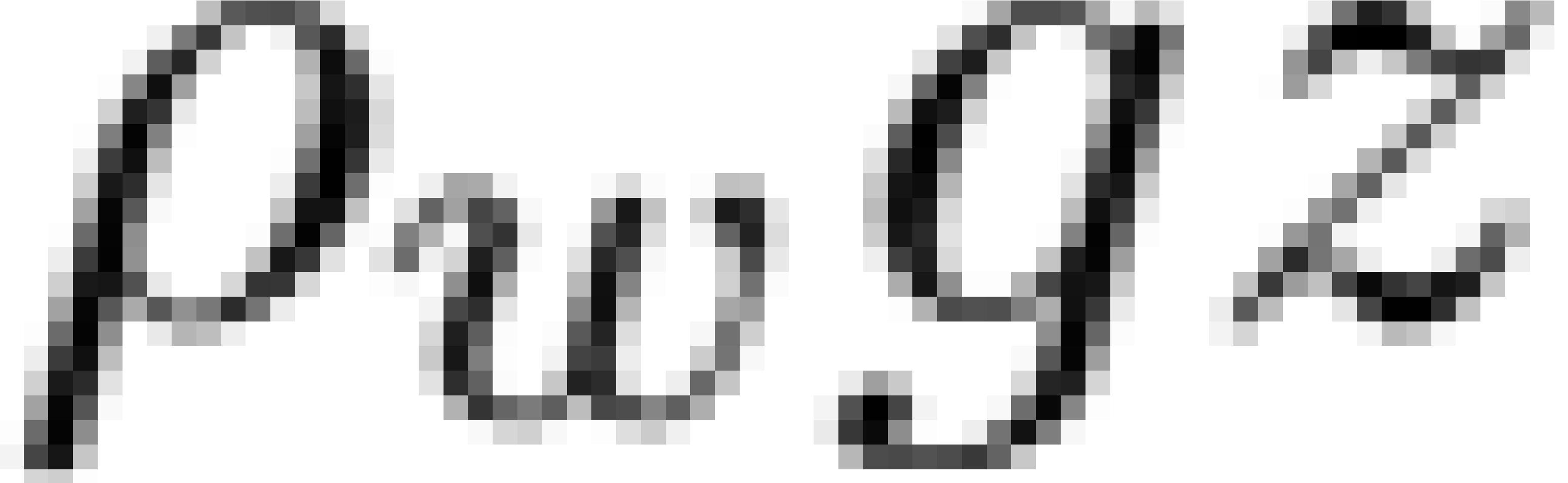










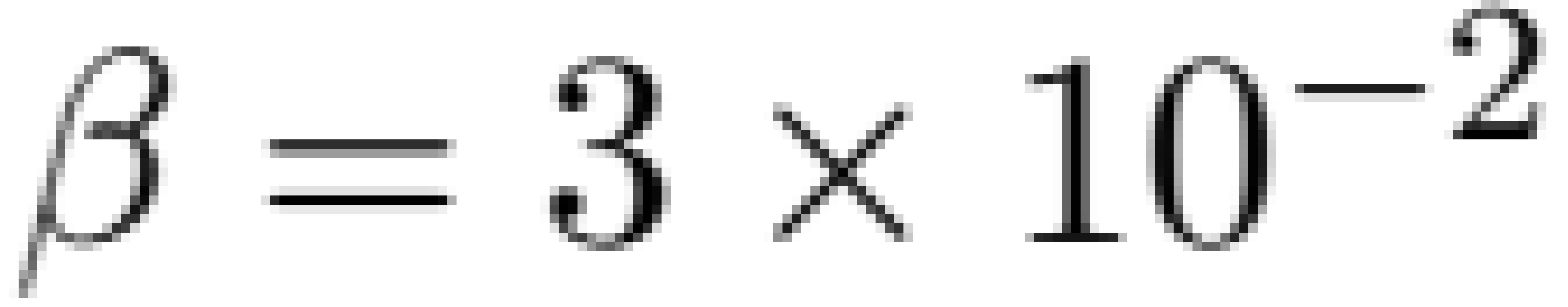


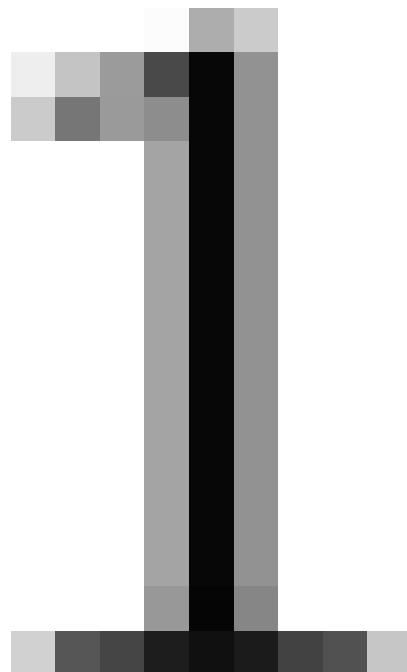
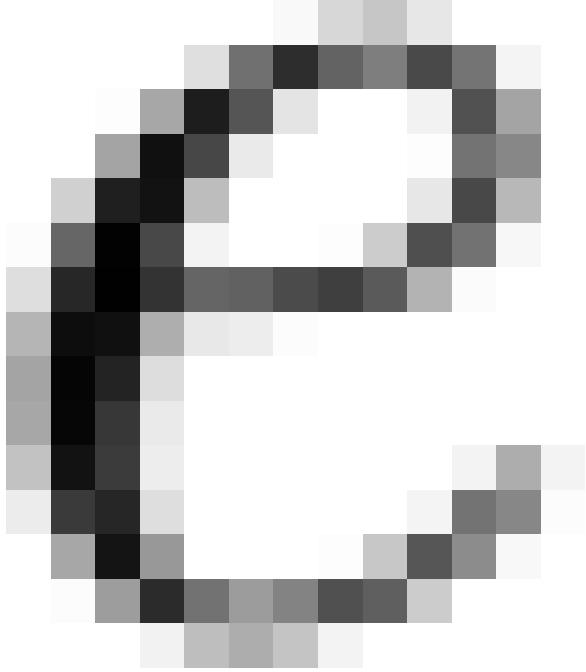


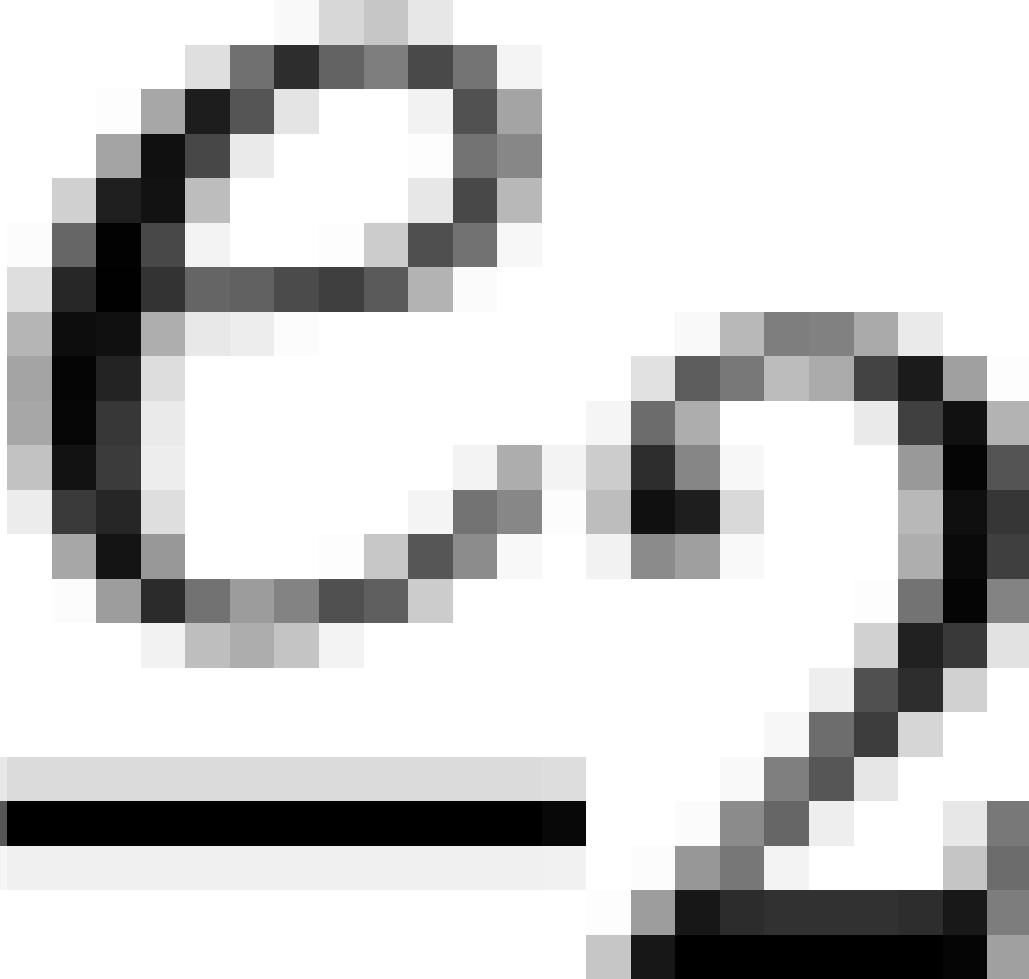


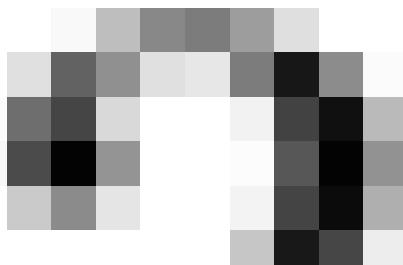
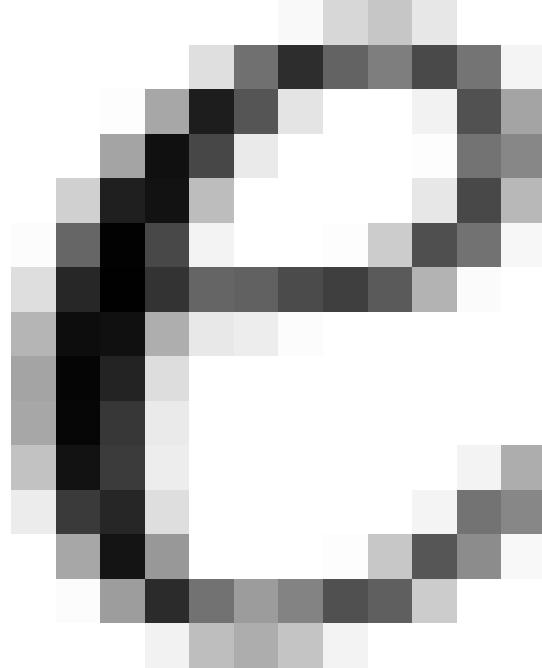


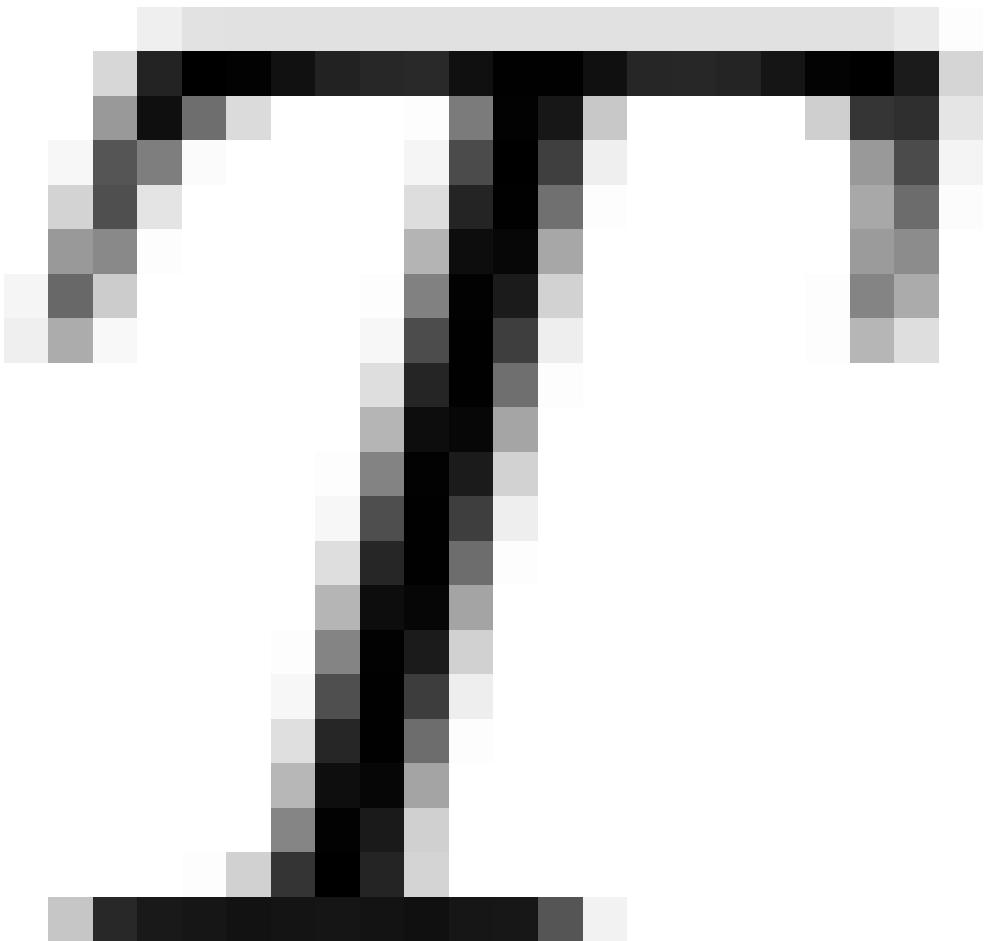




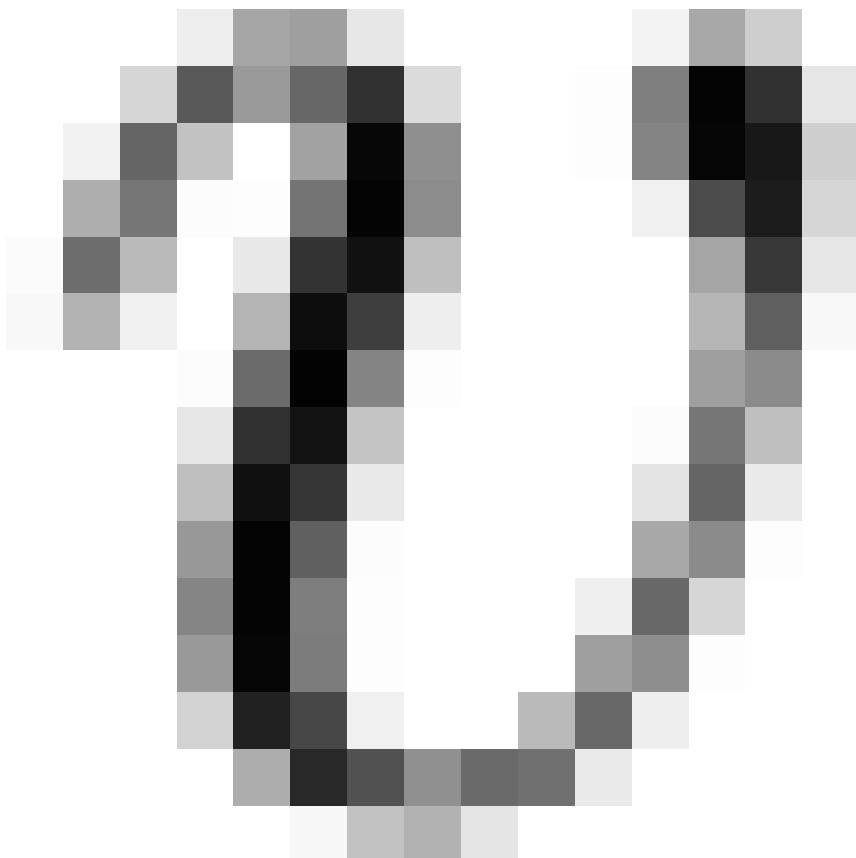




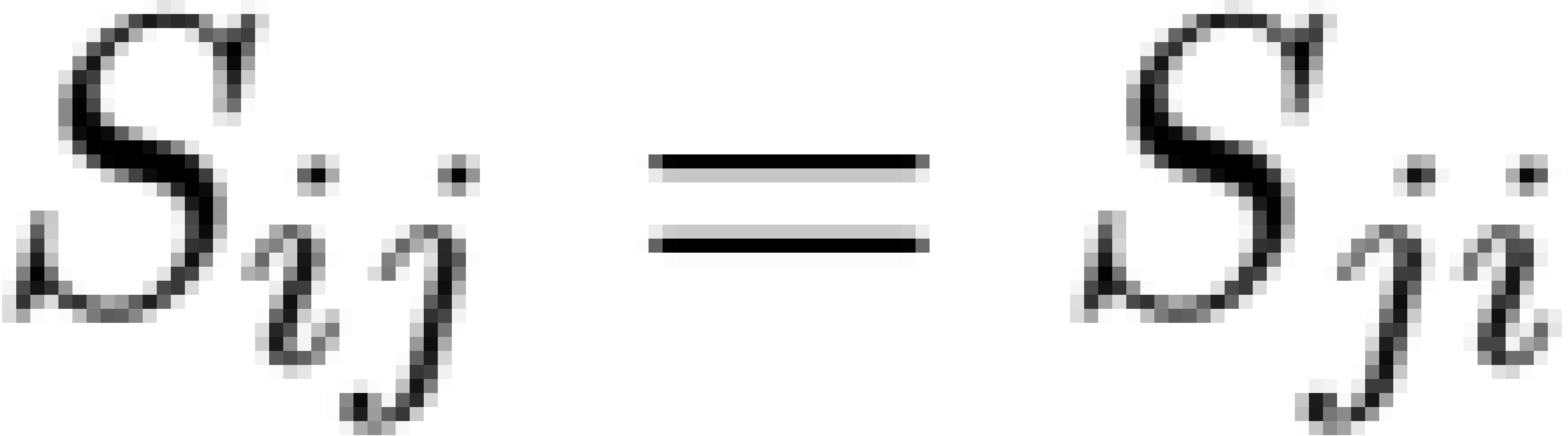


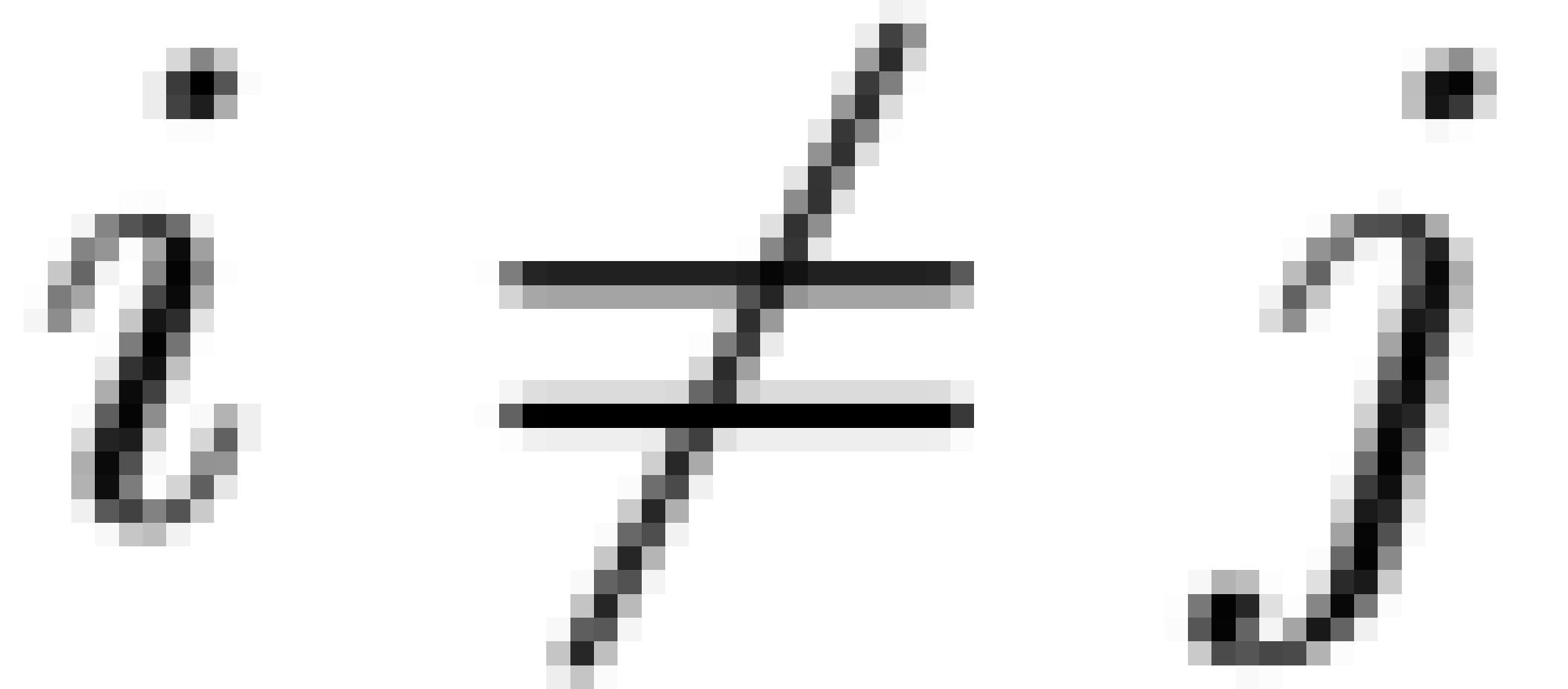


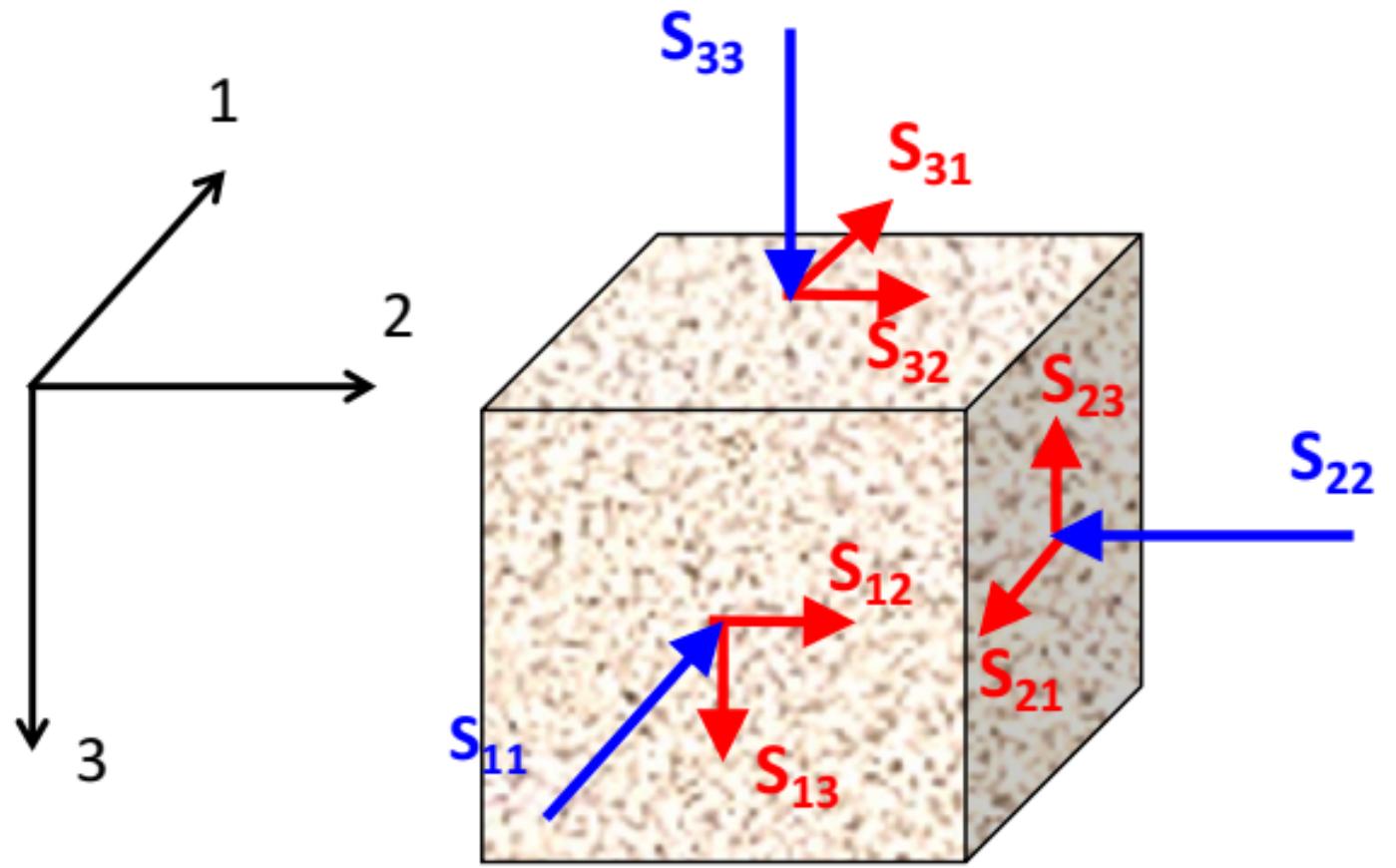






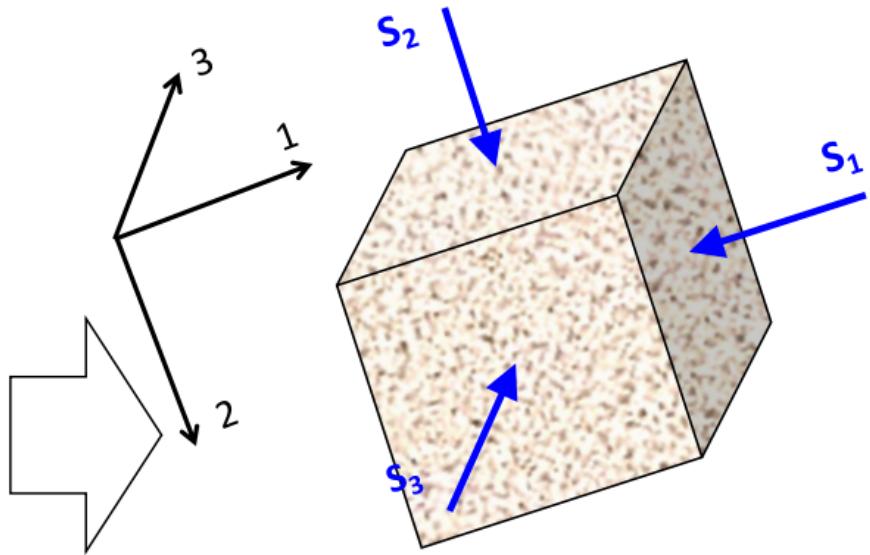
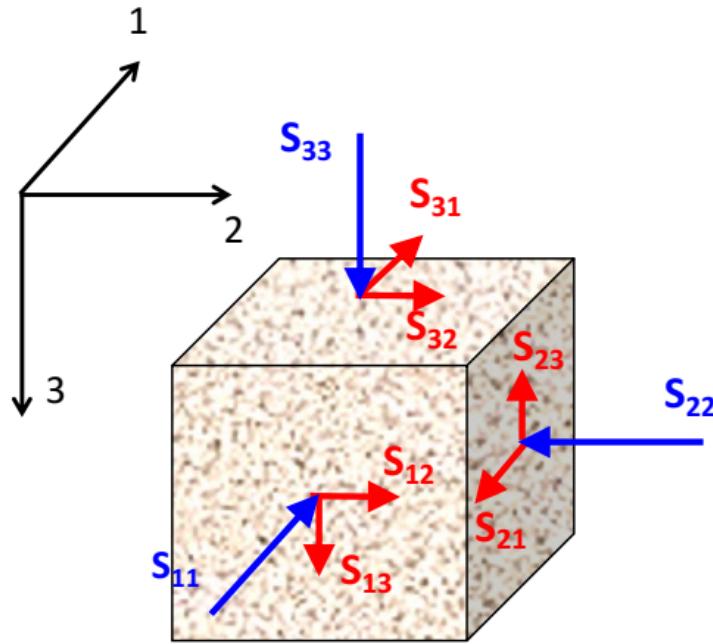






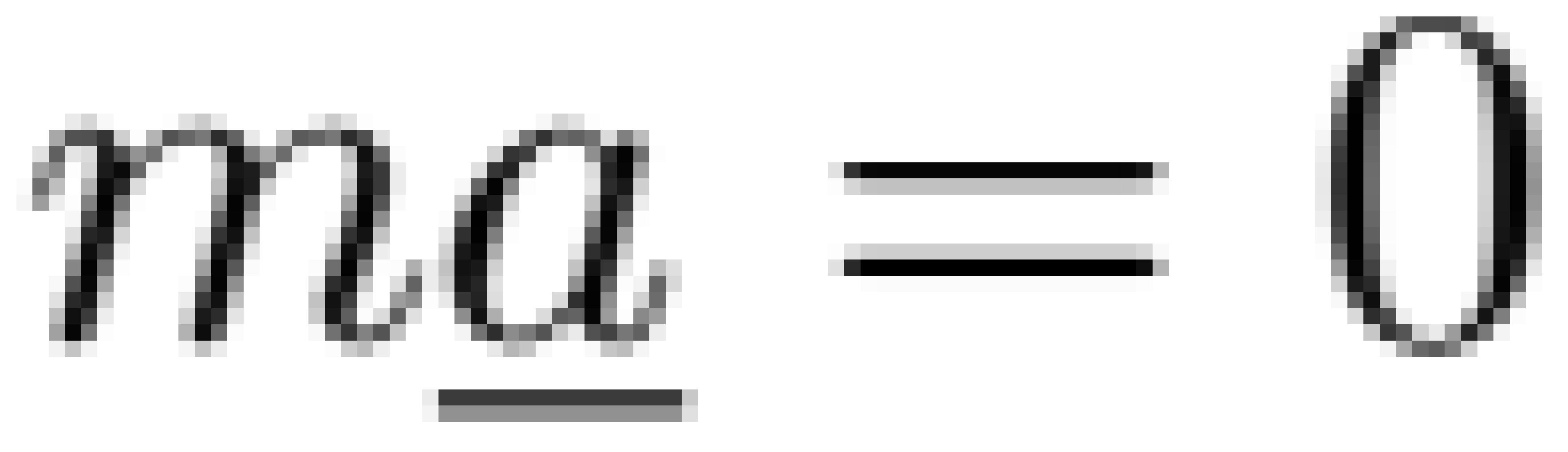
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$



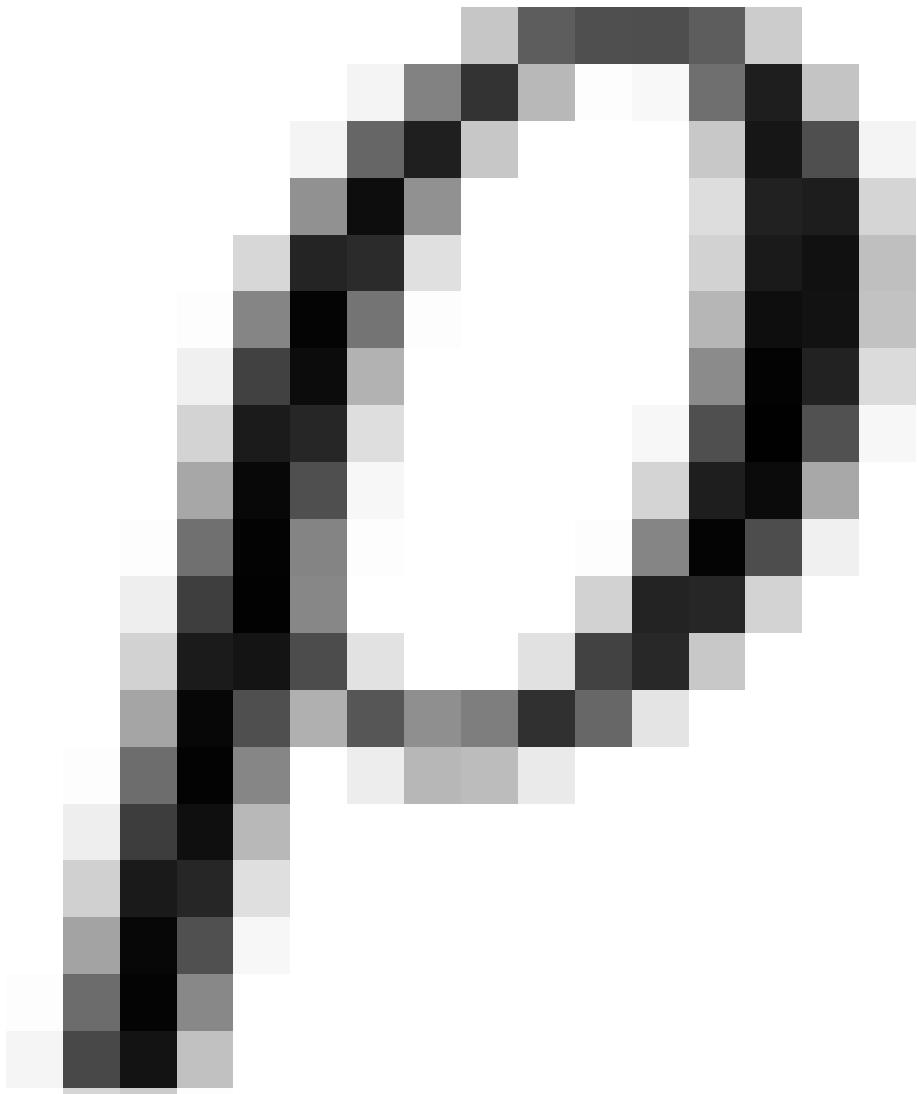


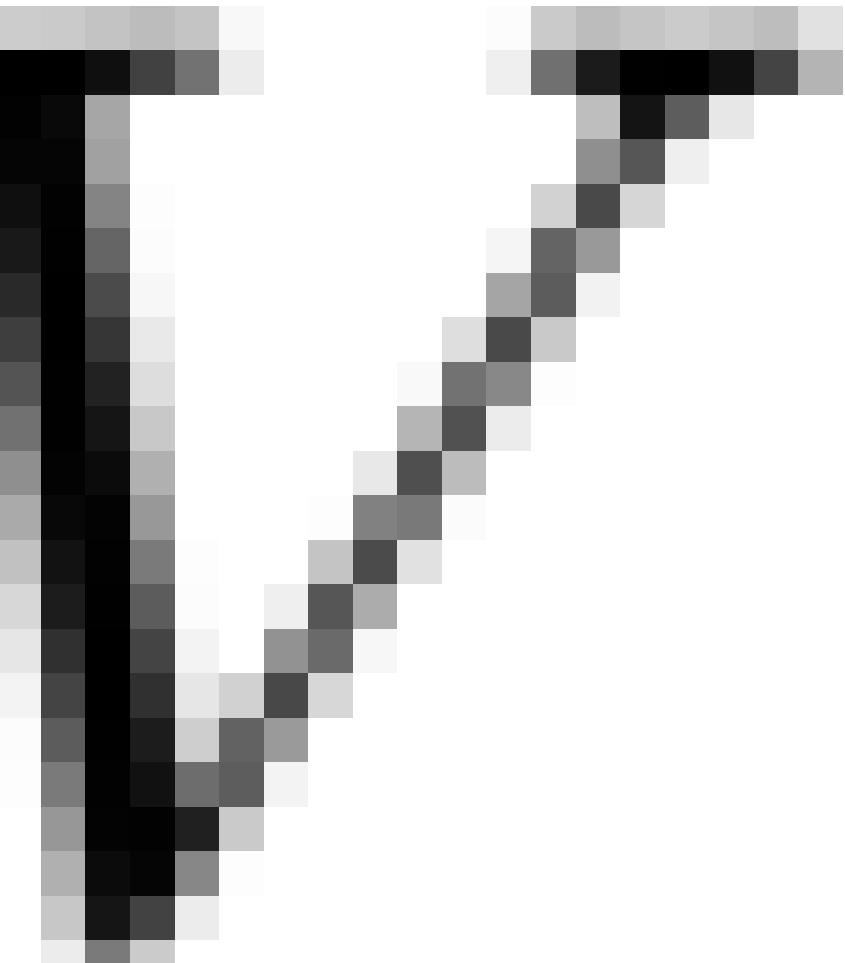
$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

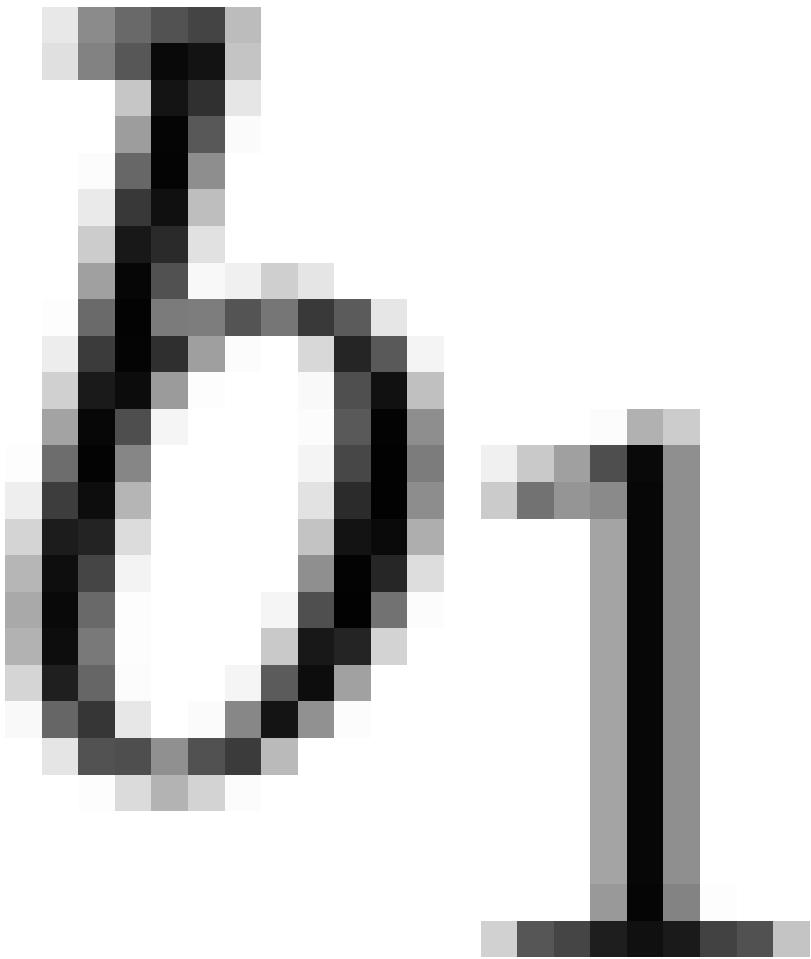
$$\underline{\underline{S}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$







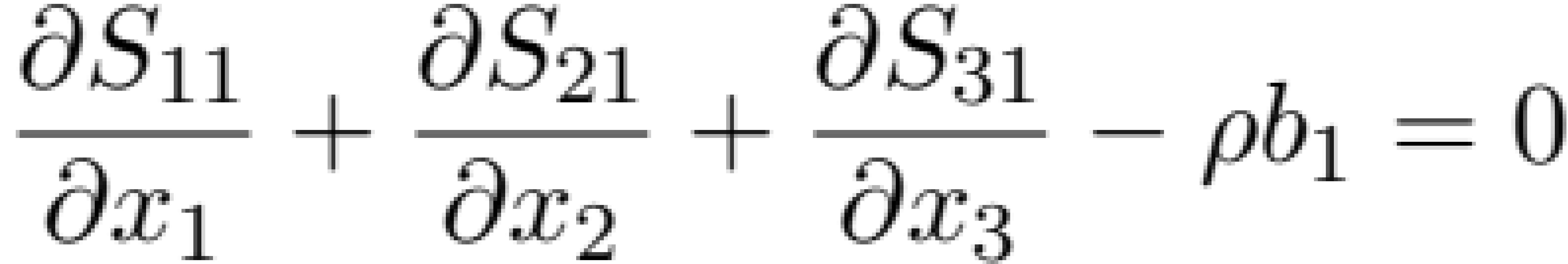


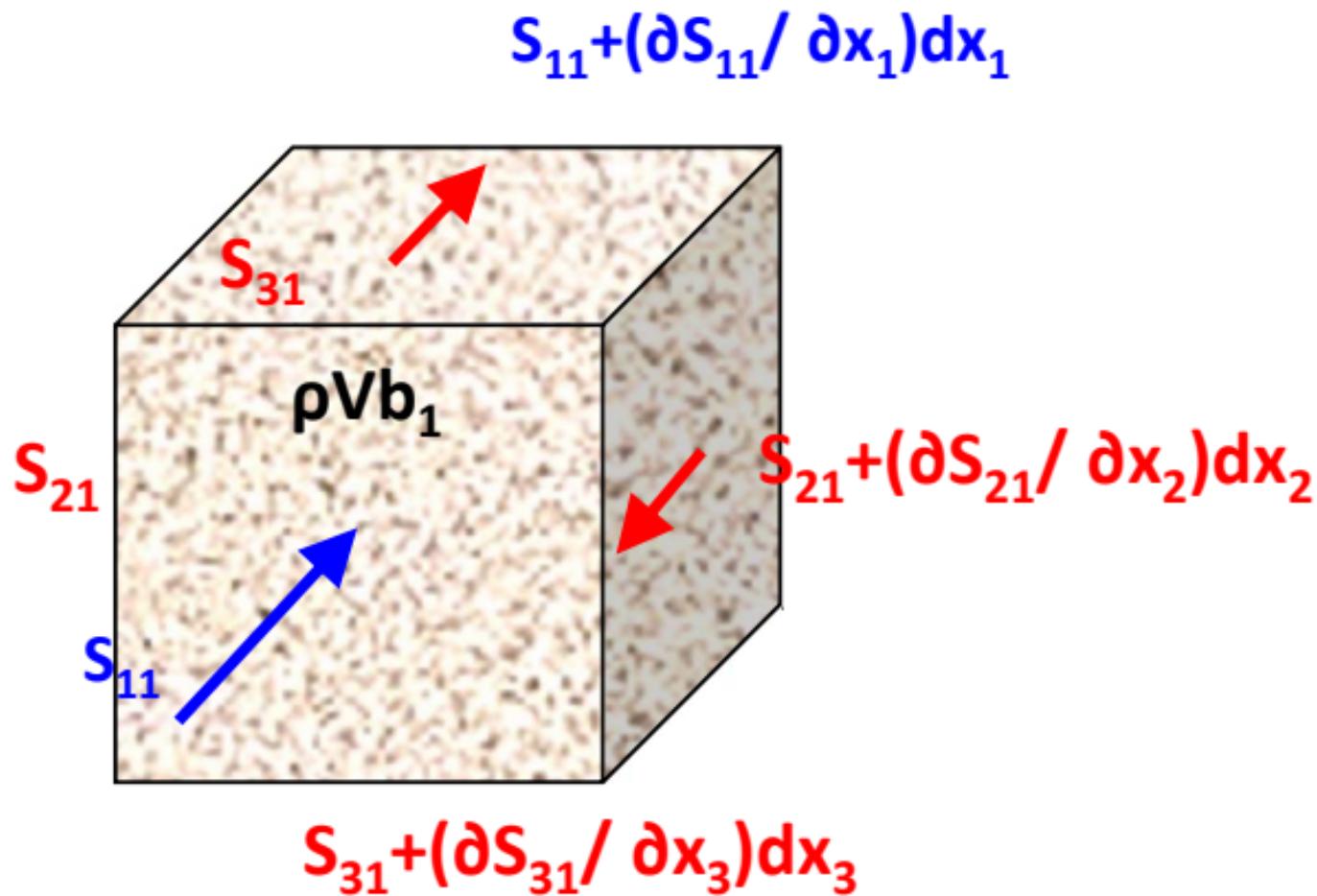
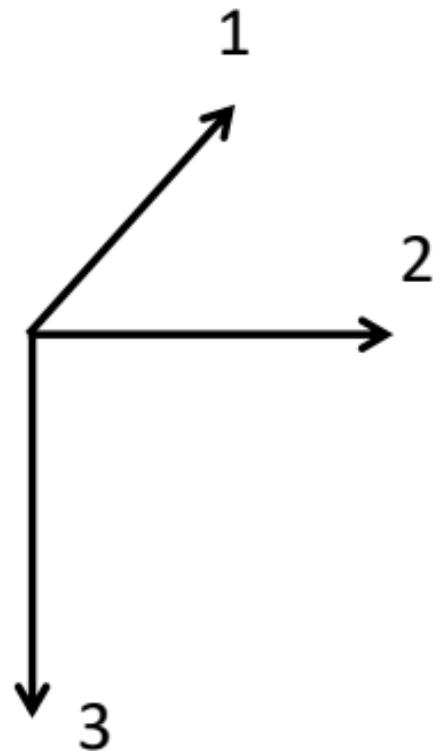


$$\sum F_1 = 0$$

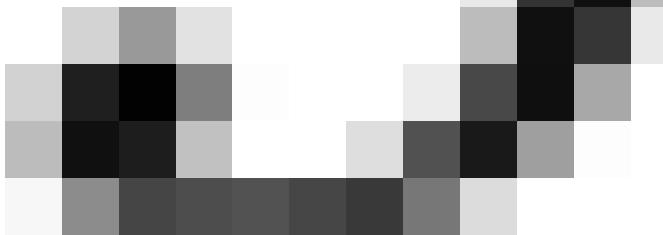
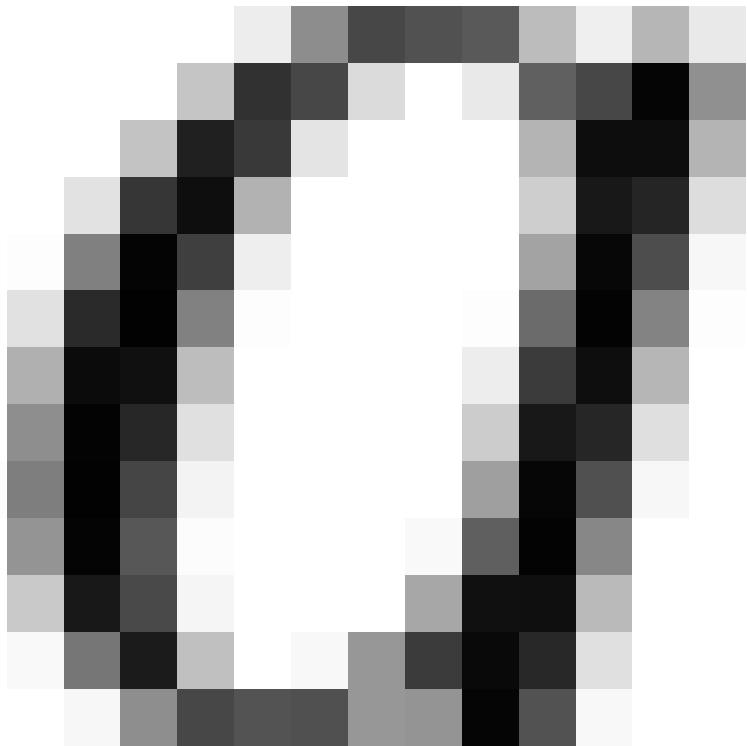
$$\begin{aligned}\sum F_1 &= +S_{11}dx_2dx_3 - \left[S_{11} + \left(\frac{\partial S_{11}}{\partial x_1} \right) dx_1 \right] dx_2dx_3 \\ &\quad + S_{21}dx_1dx_3 - \left[S_{21} + \left(\frac{\partial S_{21}}{\partial x_2} \right) dx_2 \right] dx_1dx_3 \\ &\quad + S_{31}dx_1dx_2 - \left[S_{31} + \left(\frac{\partial S_{31}}{\partial x_3} \right) dx_3 \right] dx_1dx_2 \\ &\quad - \rho(dx_1dx_2dx_3)b_1 = 0\end{aligned}$$

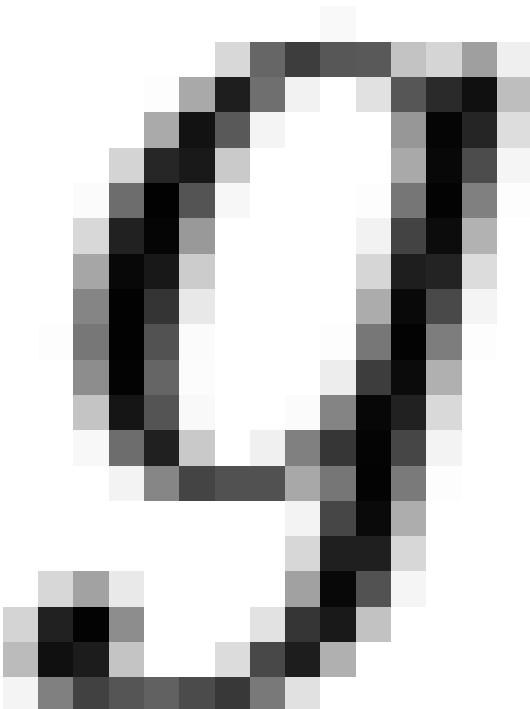
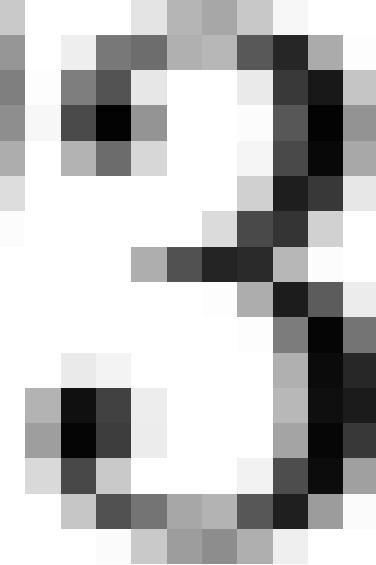
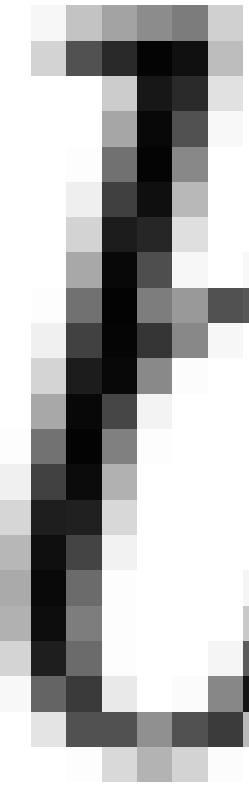






$$\left\{ \begin{array}{l} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} - \rho b_1 = 0 \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} - \rho b_2 = 0 \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} - \rho b_3 = 0 \end{array} \right.$$

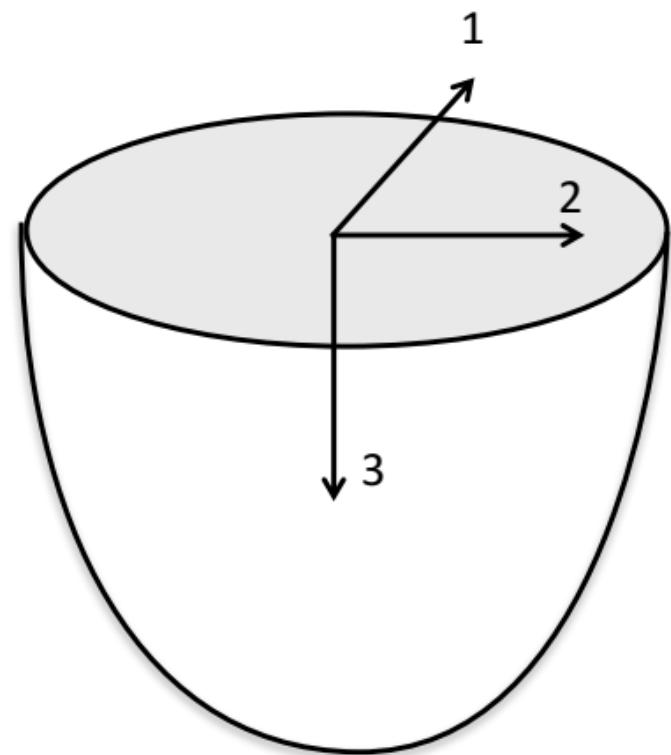








$$s_{33}(c_3) = \rho(c_3) \circ d_{c_3}$$



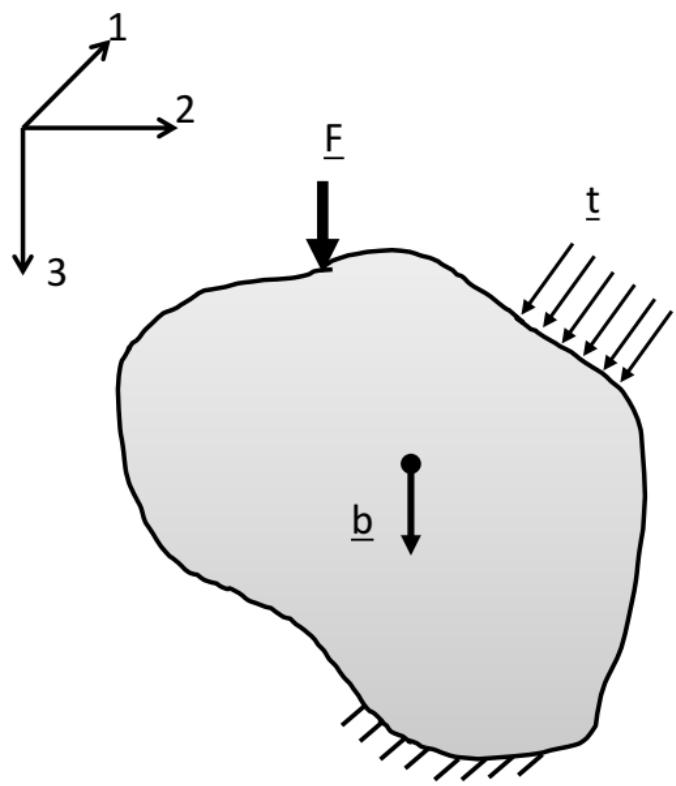
$$\left\{ \begin{array}{l} \cancel{\frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2}} \\ \cancel{\frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2}} \\ \cancel{\frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2}} \end{array} \right.$$

$$\frac{\partial S_{33}}{\partial x_3} - \rho(x_3)g = 0$$

$$S_{33} = \int_0^{x_3} \rho(x_3)g \, dx_3$$







Displacement condition

$$\begin{cases} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2} \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2} \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2} \end{cases}$$

And respect the boundary conditions:

- Displacement
- Boundary stresses
- Boundary Forces
- Body Forces

How do we relate stresses to displacements?

- Displacements → Strains (**Kinematic equations**)
- Strains → Stresses (**Constitutive equations**)



ϵ_{11}



α_1

α_1

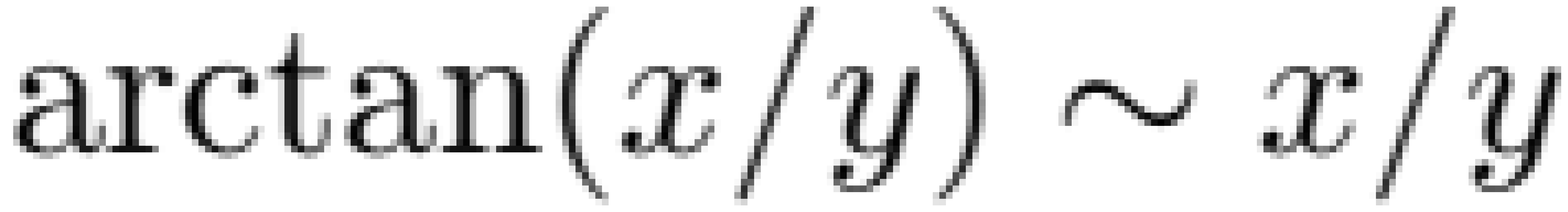
ϵ_{22}

$=$

$\triangle u_2$

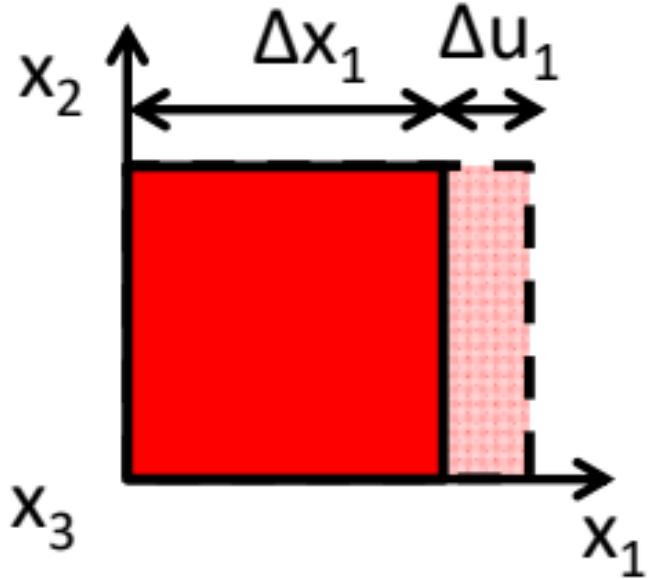
$\triangle c_2$

acc10(ut1) + acc10(ut2) + acc10(ut3)

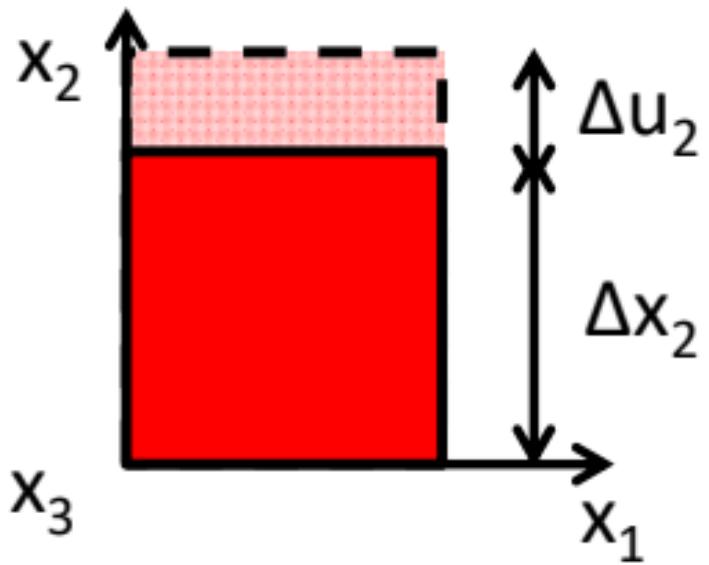


$\epsilon_{12} =$ $-\frac{1}{2}$

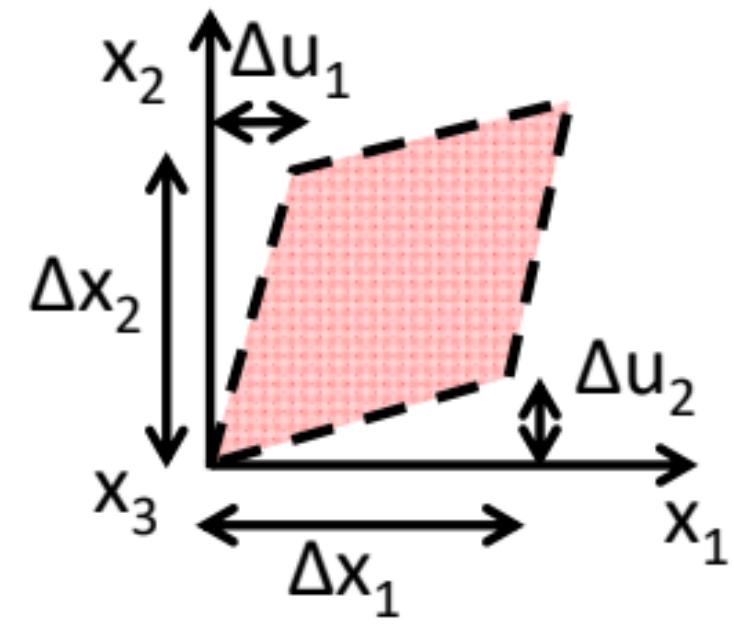
$$\frac{1}{2} \left(u_1 - u_2 + c_1 - c_2 \right)$$



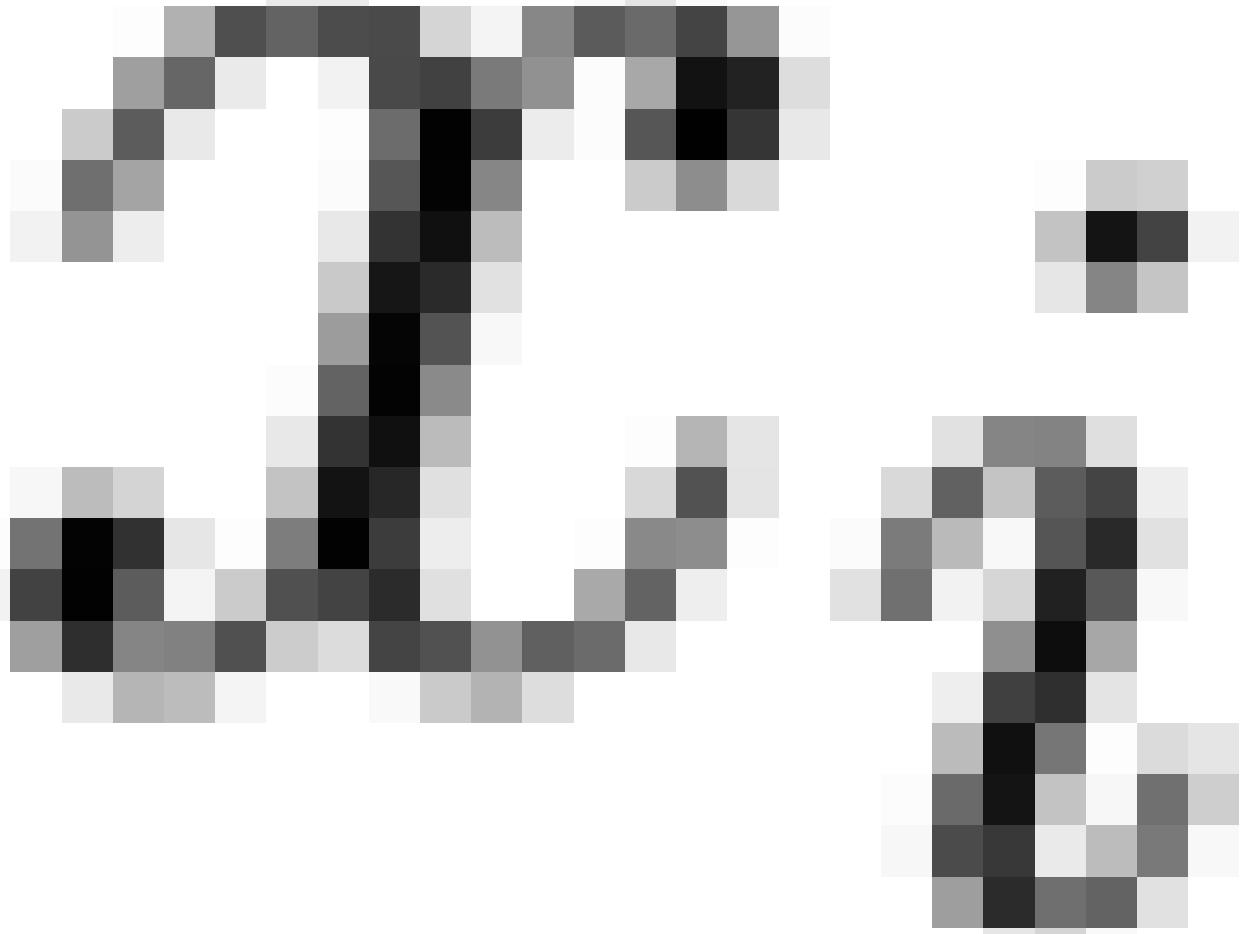
$$\varepsilon_{11} \approx \frac{\Delta u_1}{\Delta x_1}$$

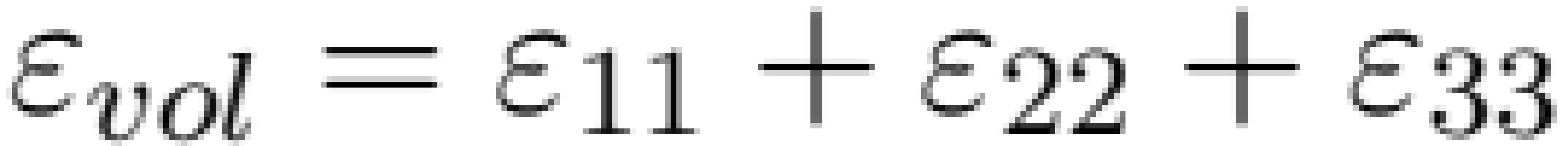


$$\varepsilon_{22} \approx \frac{\Delta u_2}{\Delta x_2}$$

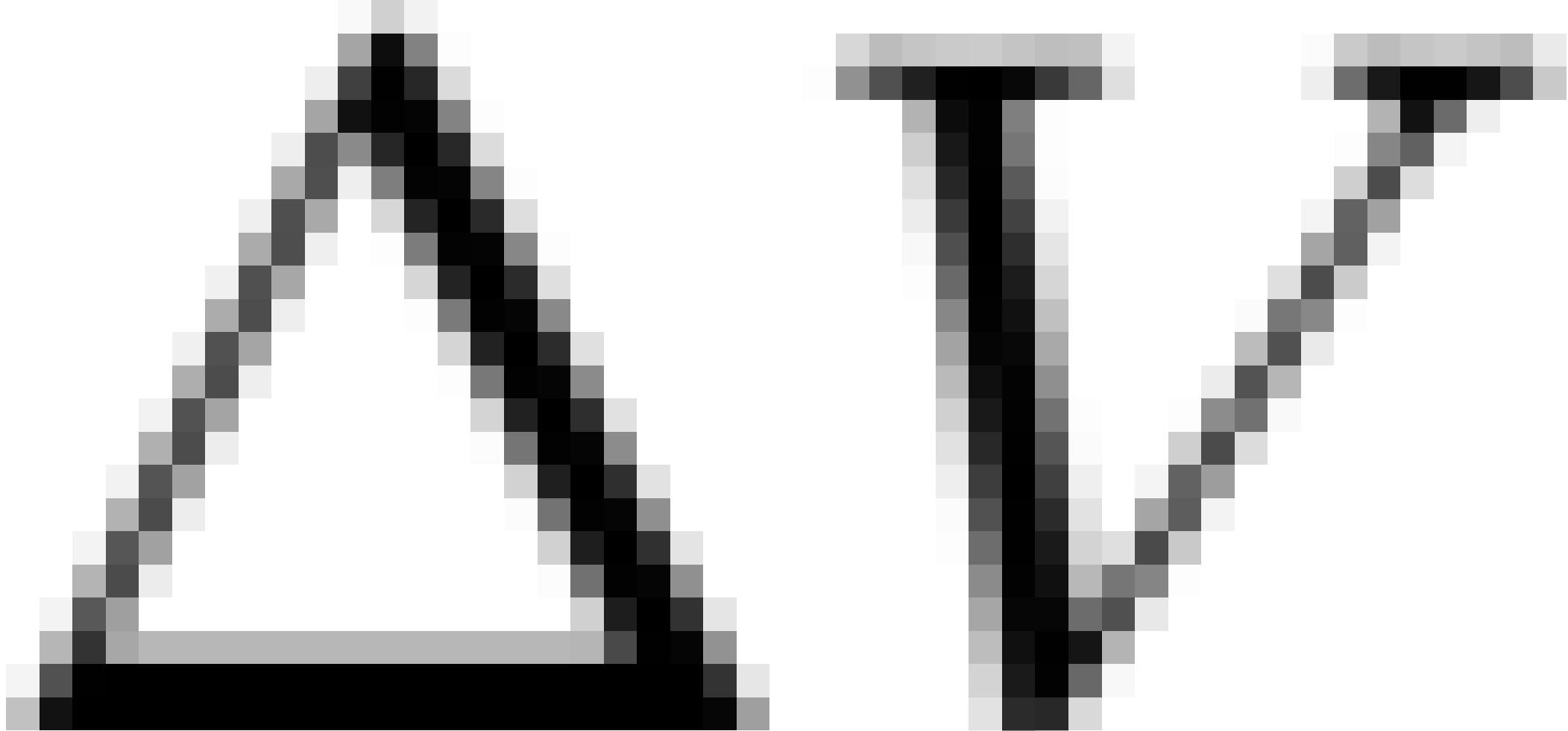


$$\varepsilon_{12} \approx \frac{1}{2} \left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1} \right)$$



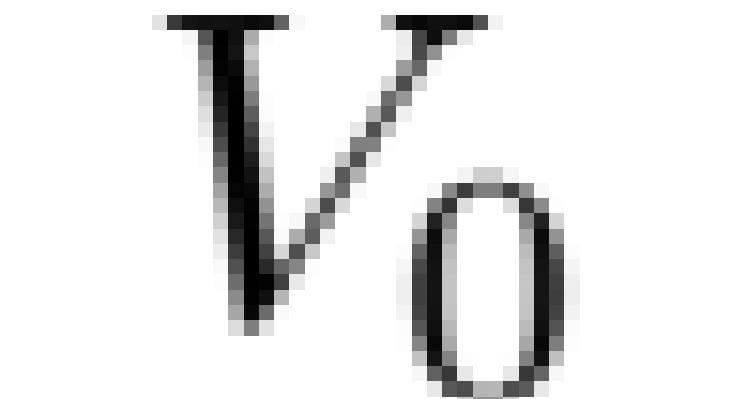
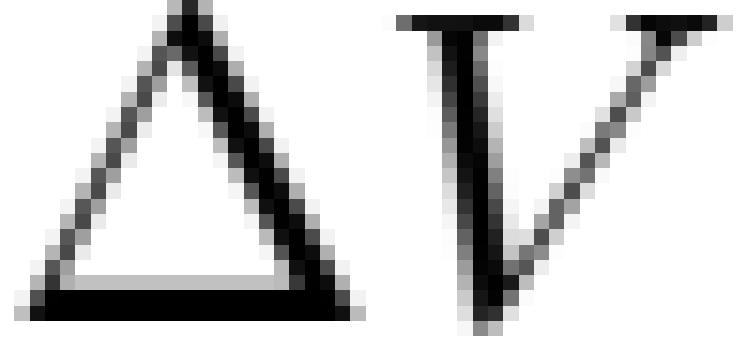




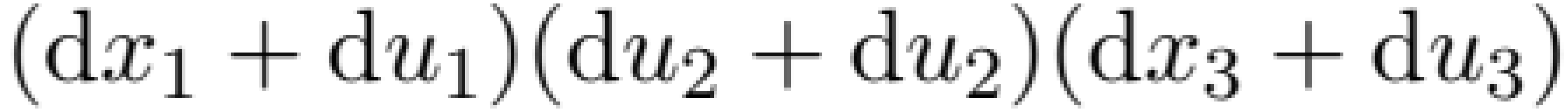




cool

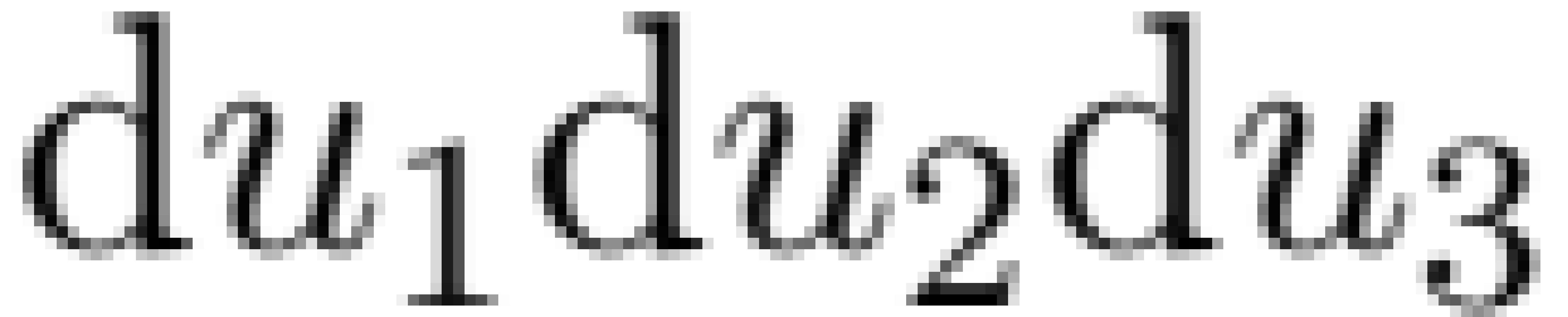


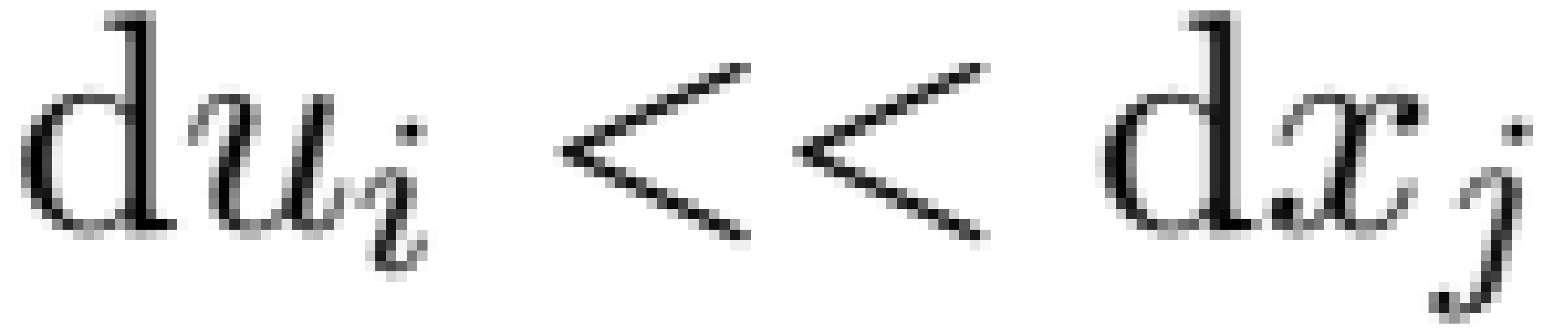




$$\epsilon_{vol} = \frac{[(dx_1 + du_1)(dx_2 + du_3) - (dx_1 dx_2 dx_3)]}{(dx_1 dx_2 dx_3)}$$

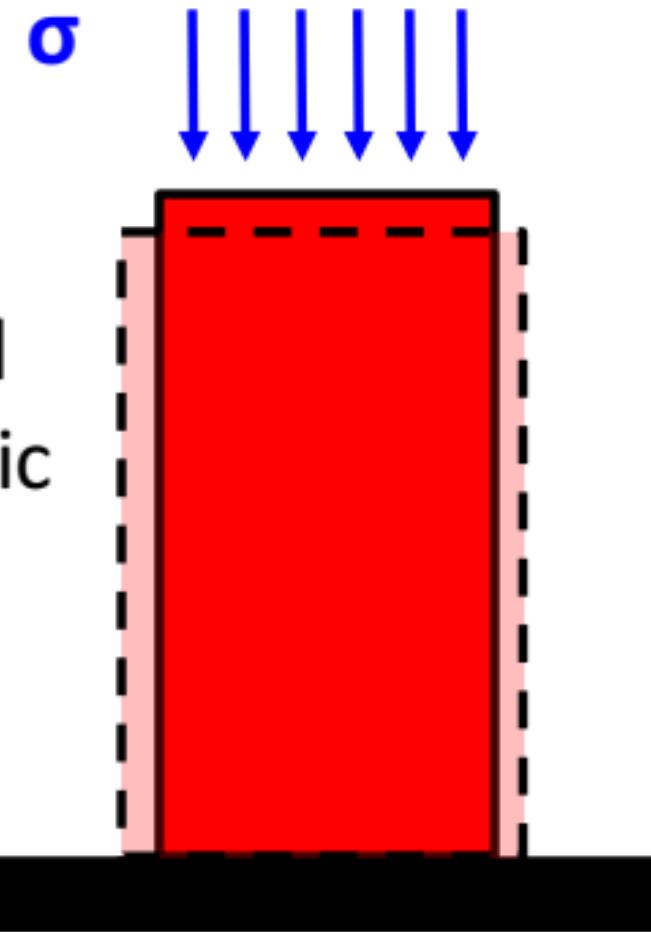




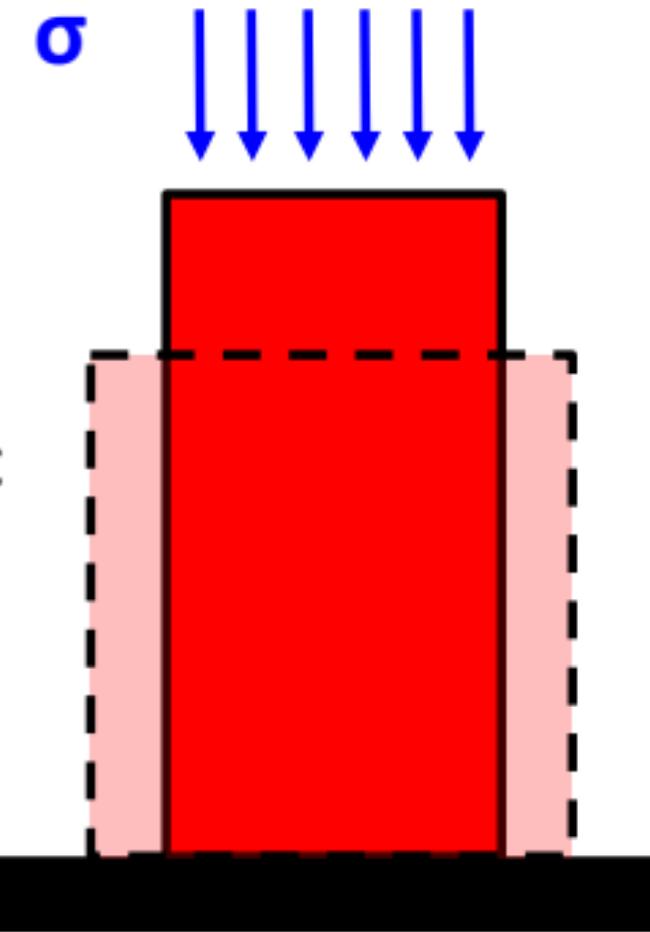


$$\text{vol}^2 \frac{(dx_1 dx_2 du_3 + dx_1 dx_3 du_2 + dx_2 dx_3 du_1)}{(dx_1 dx_2 dx_3)} = \frac{du_1}{\epsilon_{11}} + \frac{du_2}{\epsilon_{22}} + \frac{du_3}{\epsilon_{33}}$$

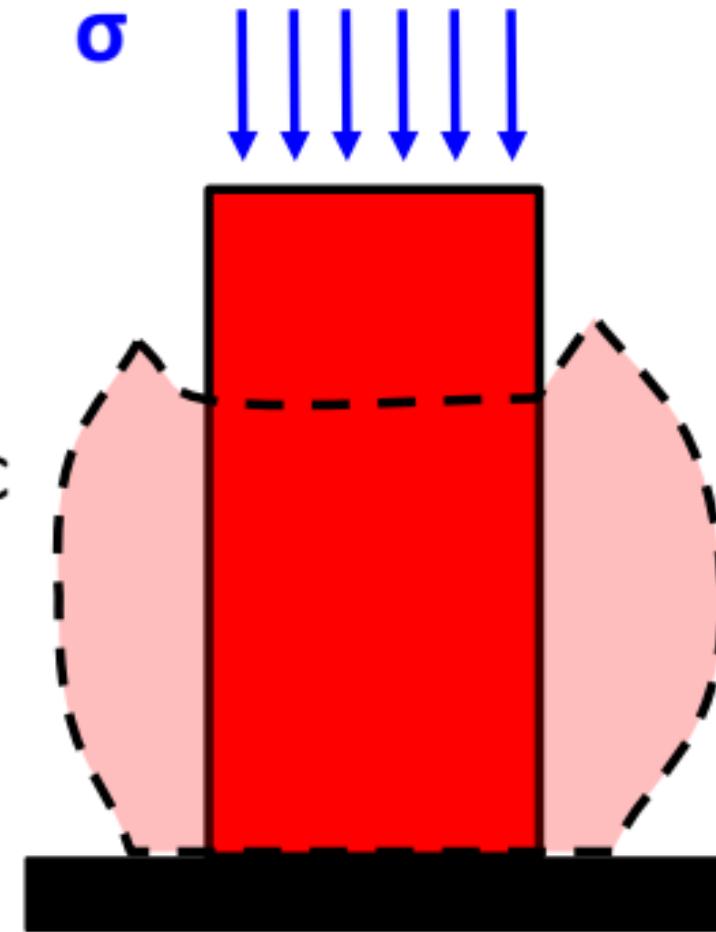
Hard
elastic
solid

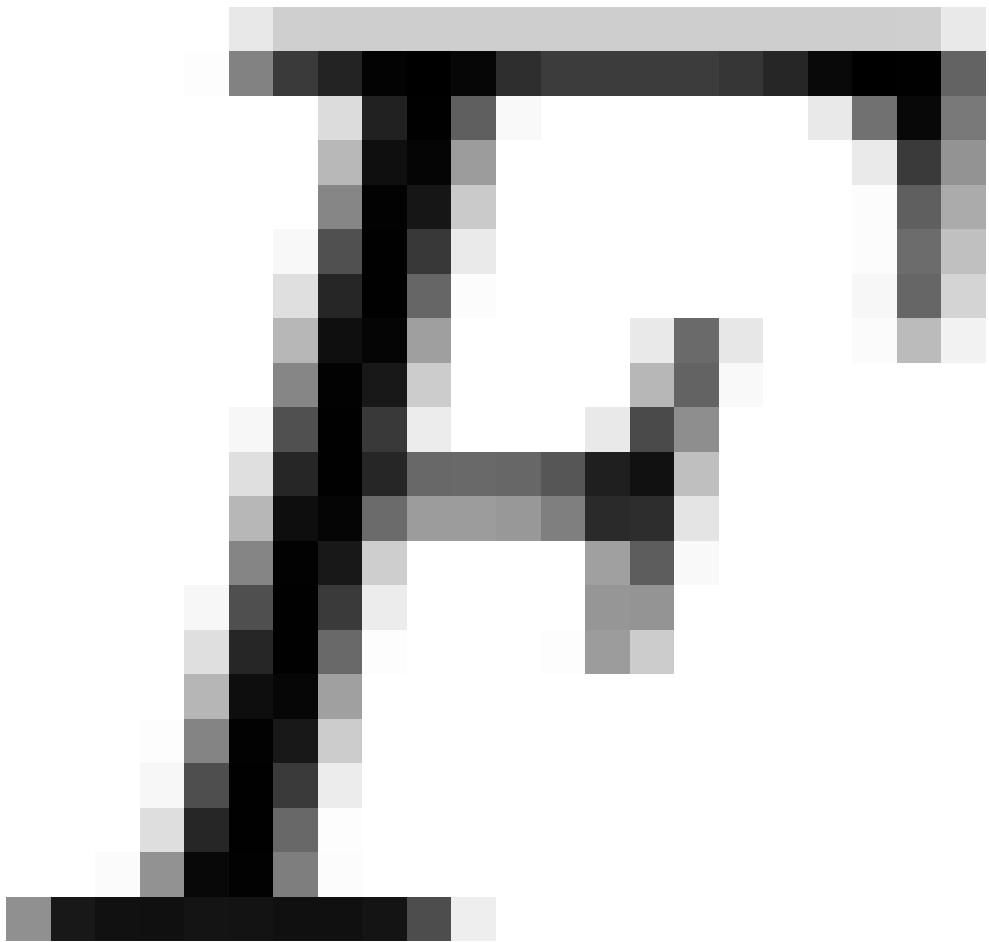


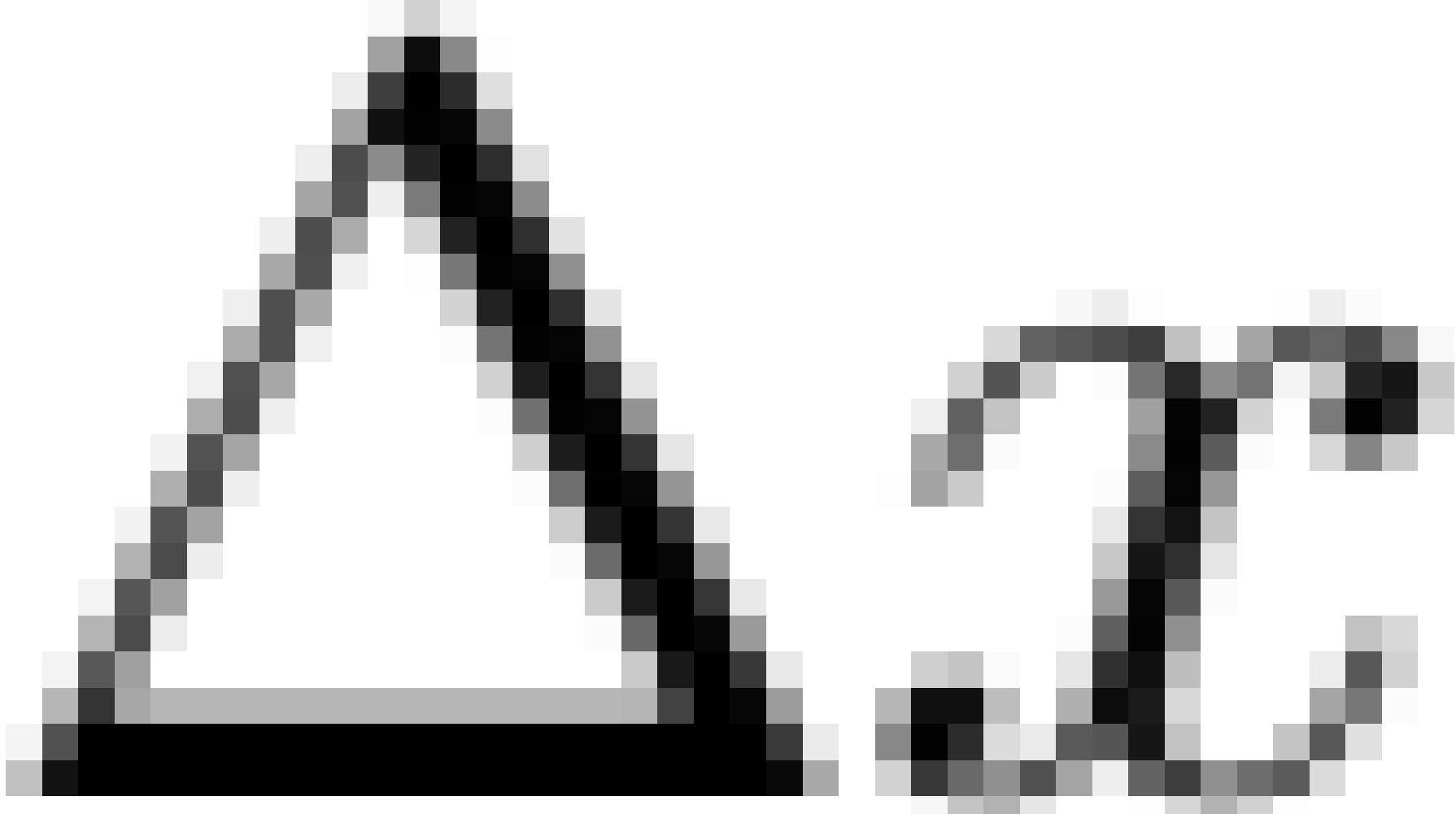
Soft
elastic
solid

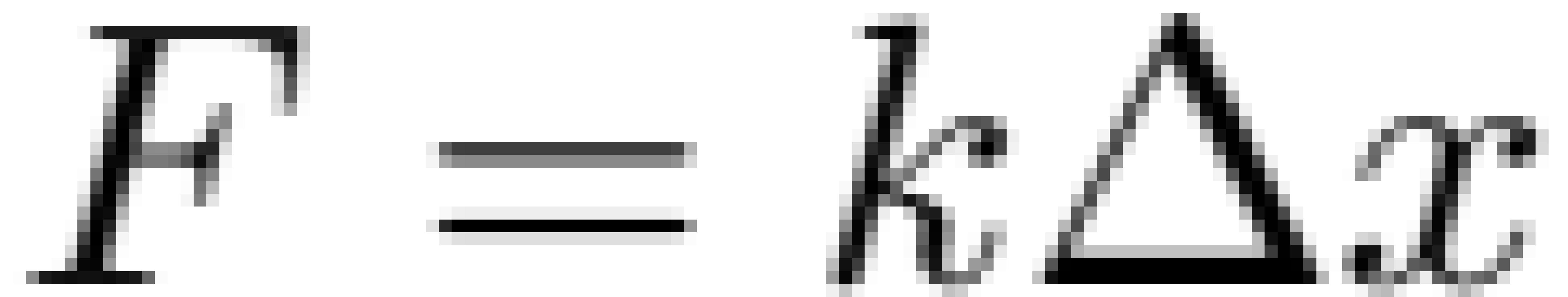


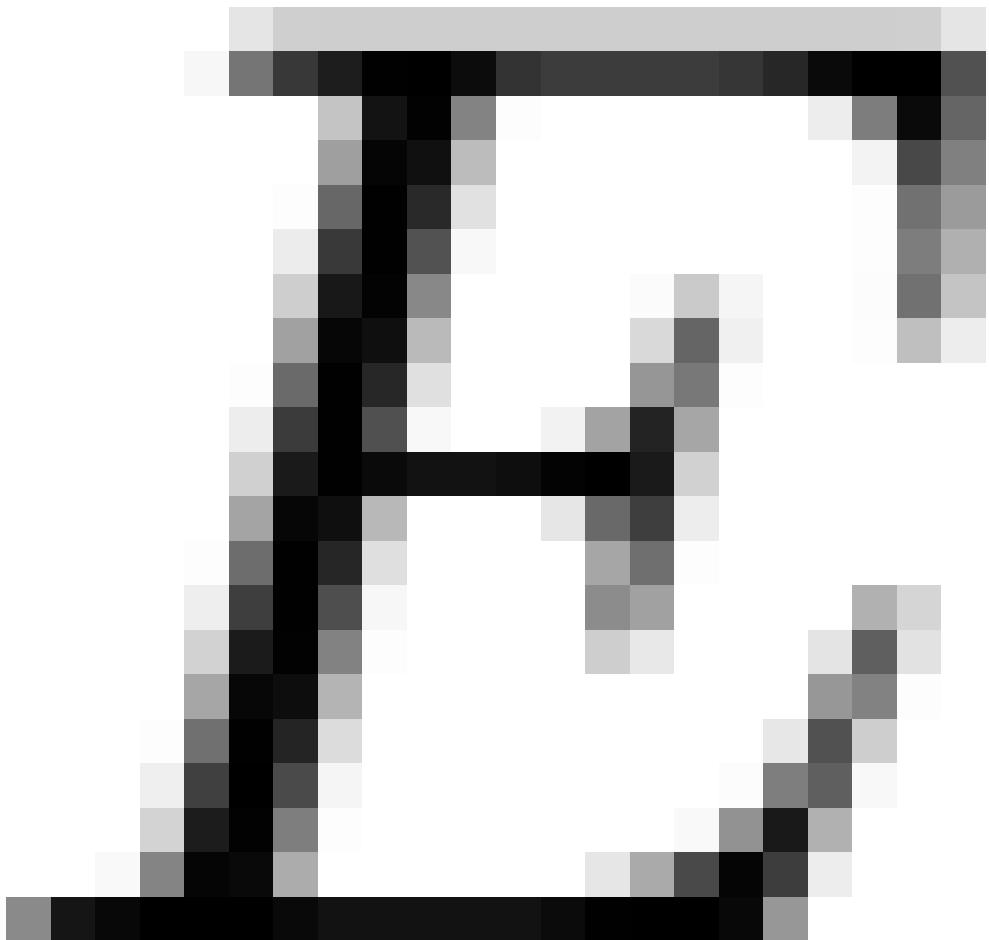
Soft
Visco-
plastic
solid











A

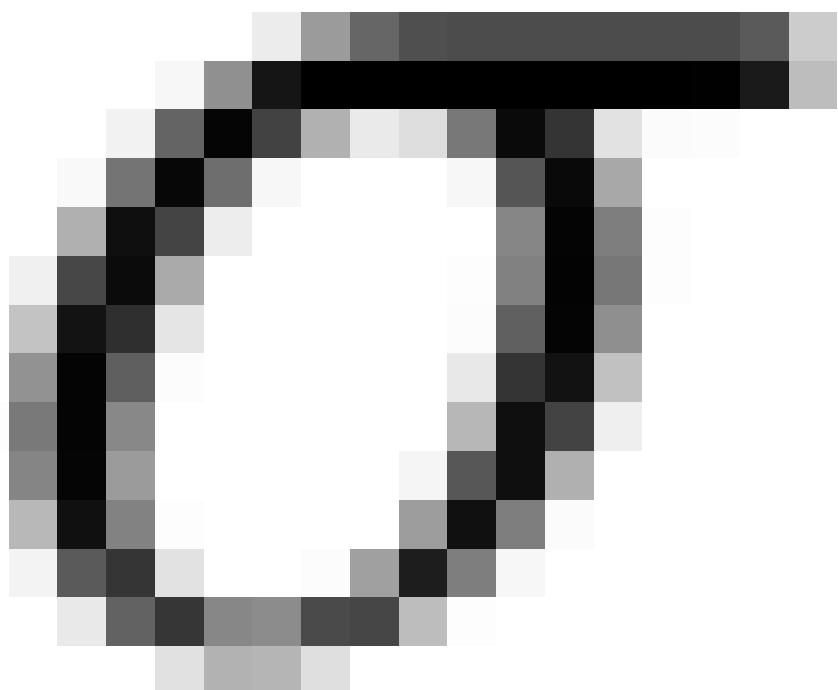
A

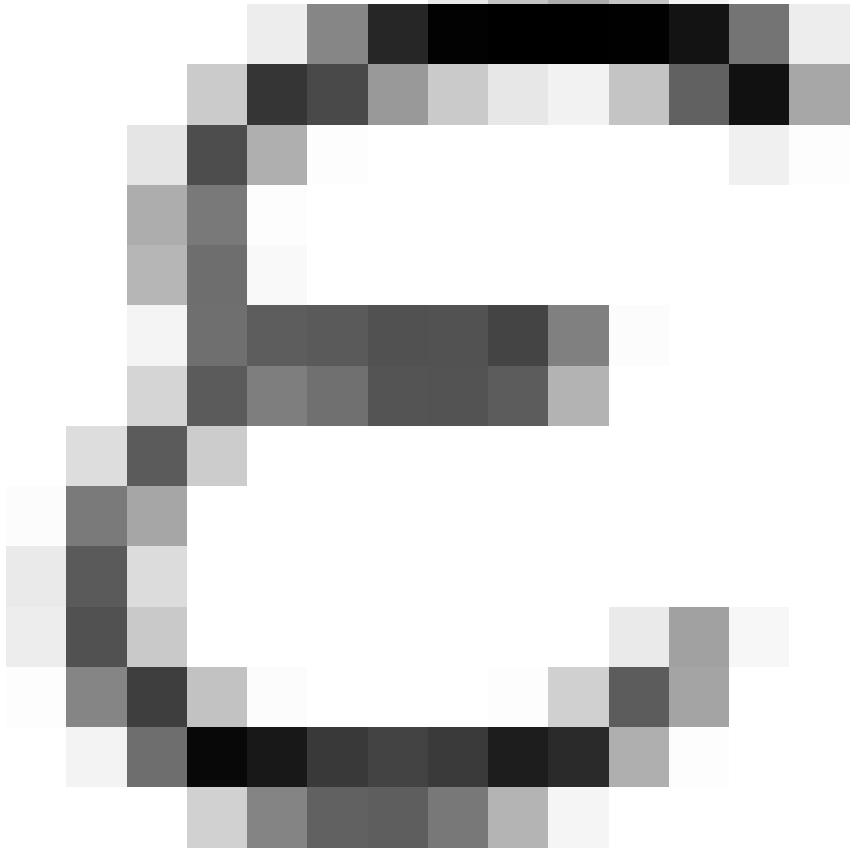
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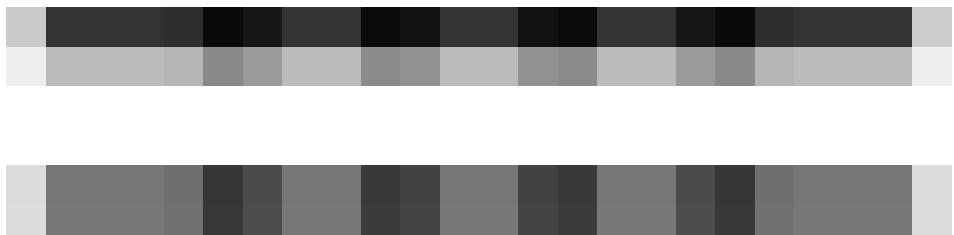
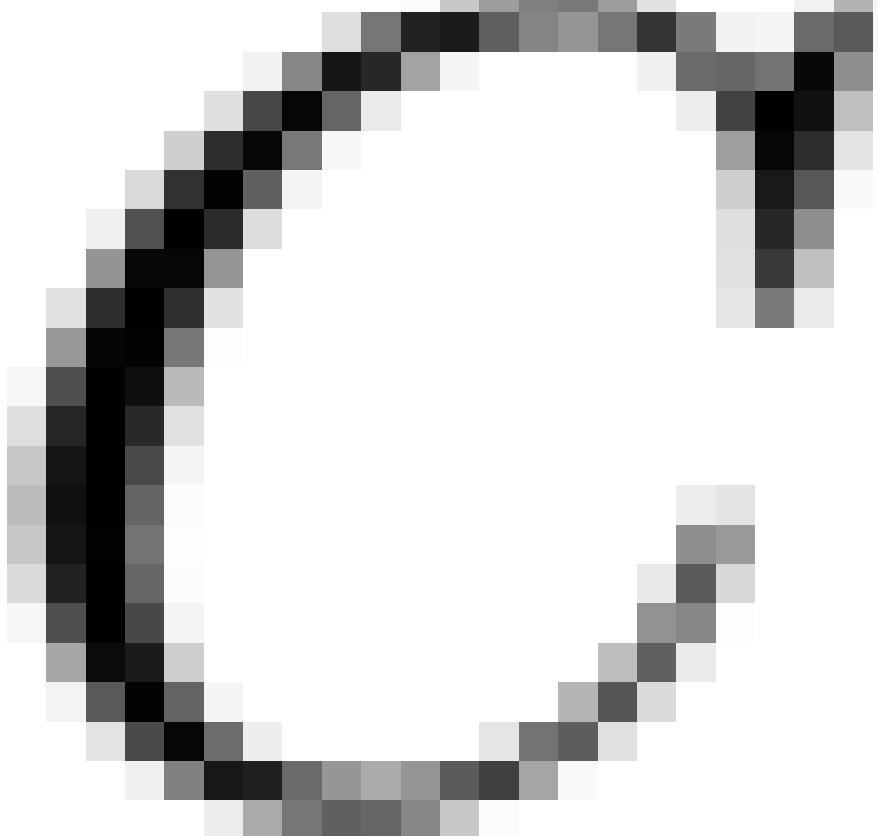
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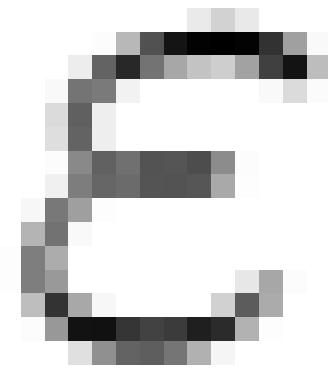
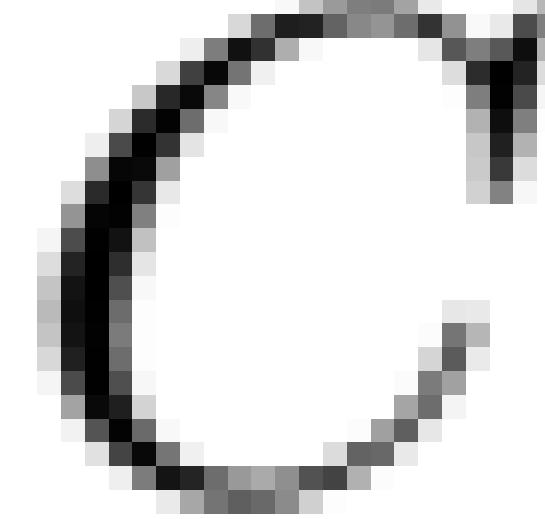
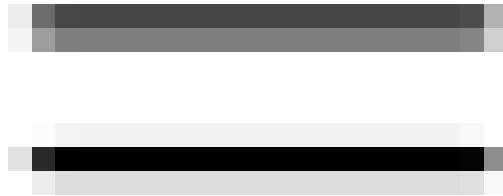
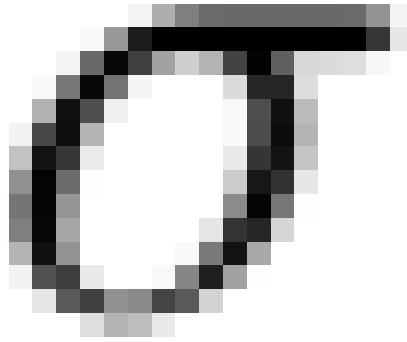
C

T



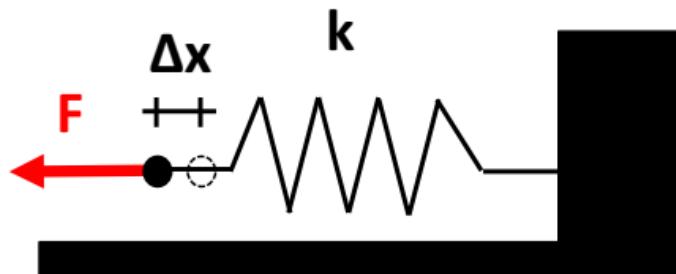






Hooke's law

$$F = k\Delta x$$

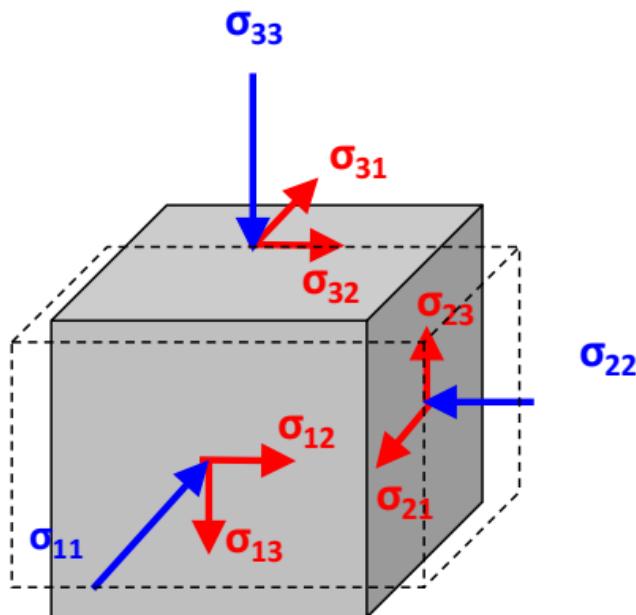


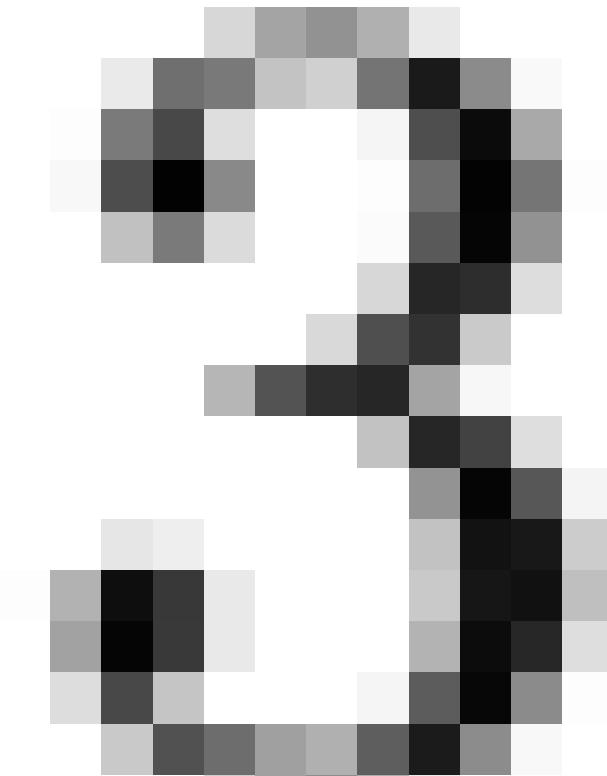
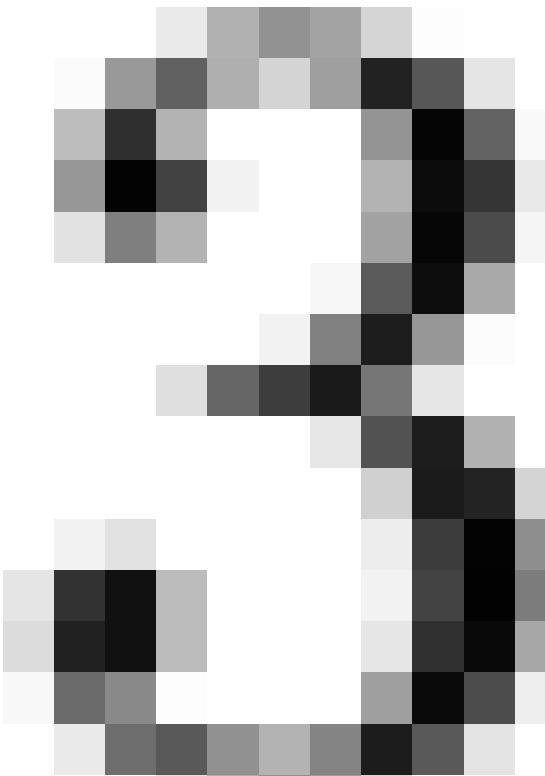
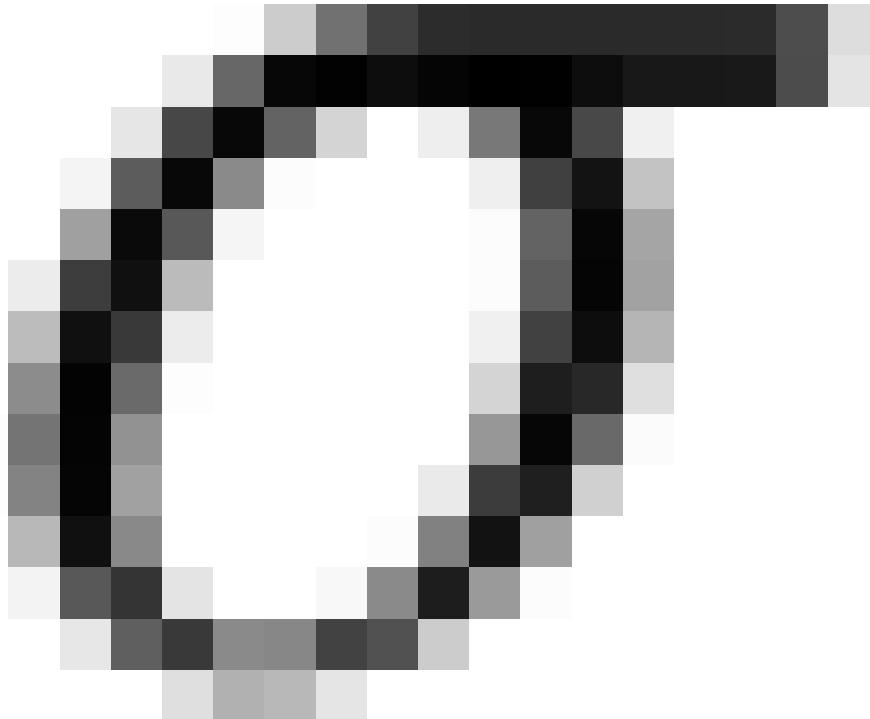
1-D (stress-strain) Hooke's law

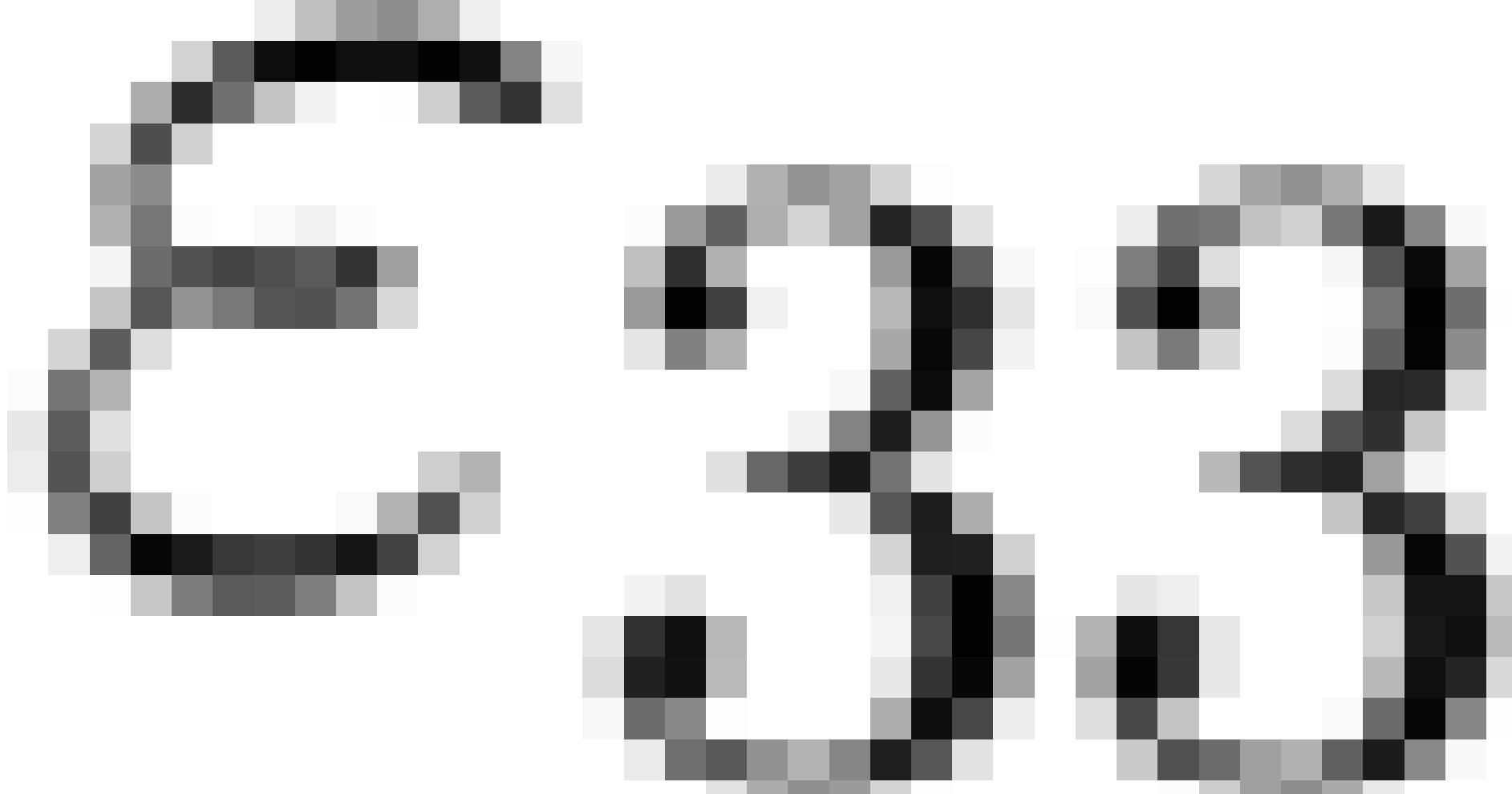
$$\sigma = E\varepsilon$$

Generalized Hooke's law

$$\underline{\sigma} = \underline{C}\underline{\varepsilon}$$







E

—
—

σ33

c33





611

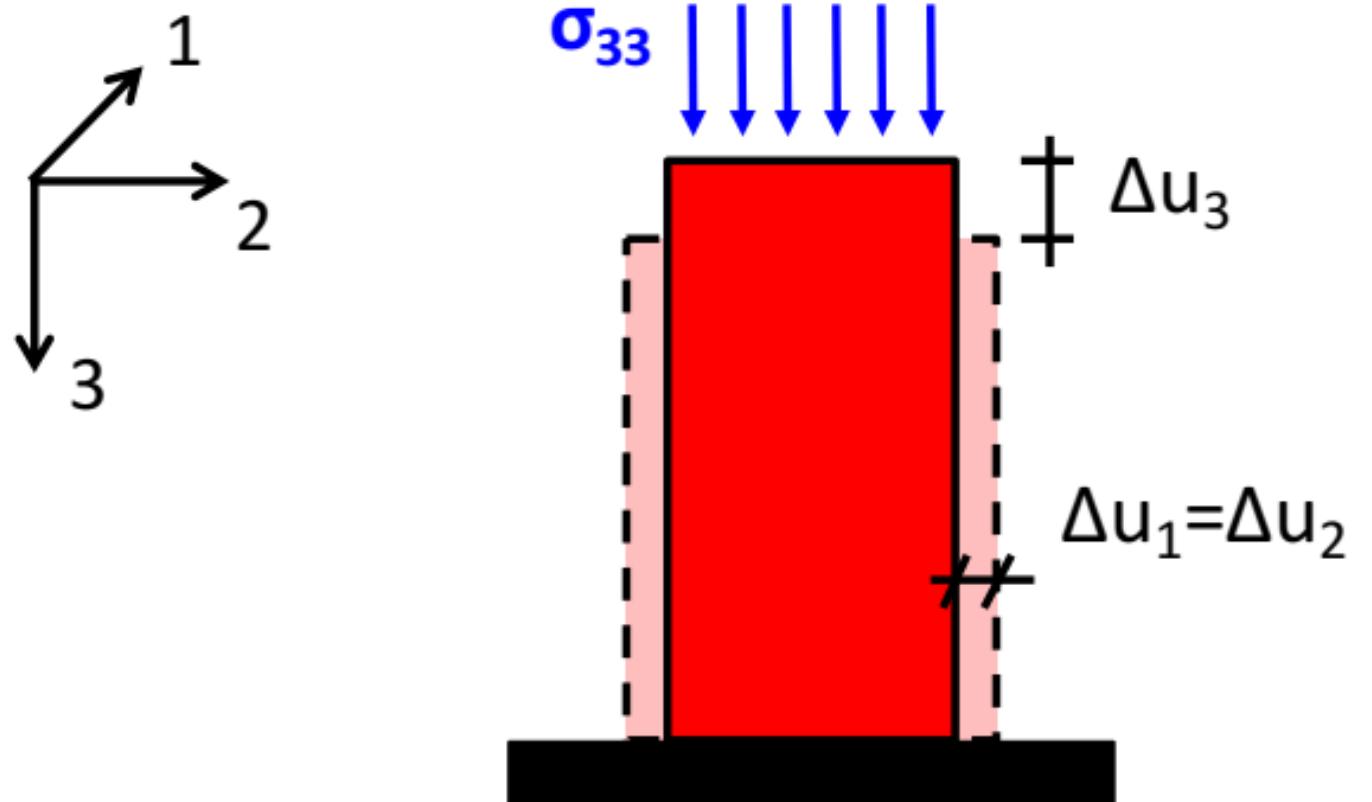
W

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633

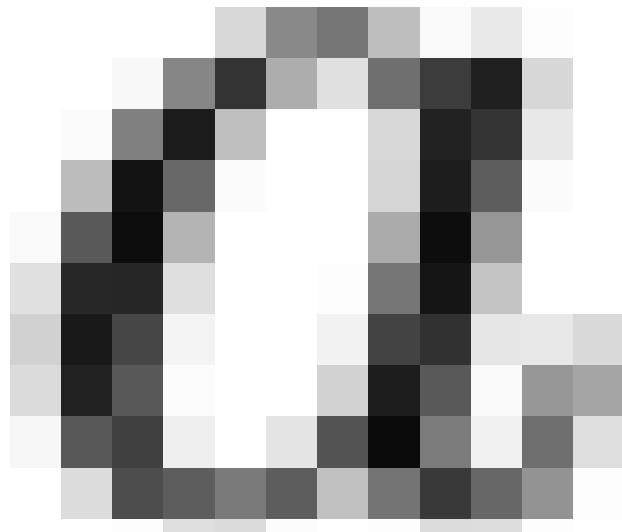
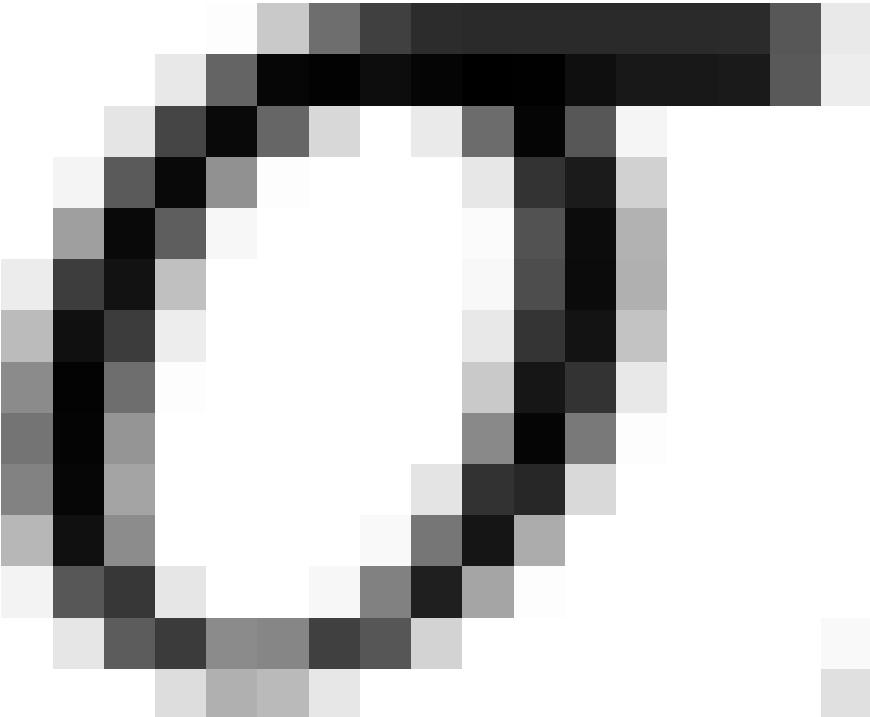


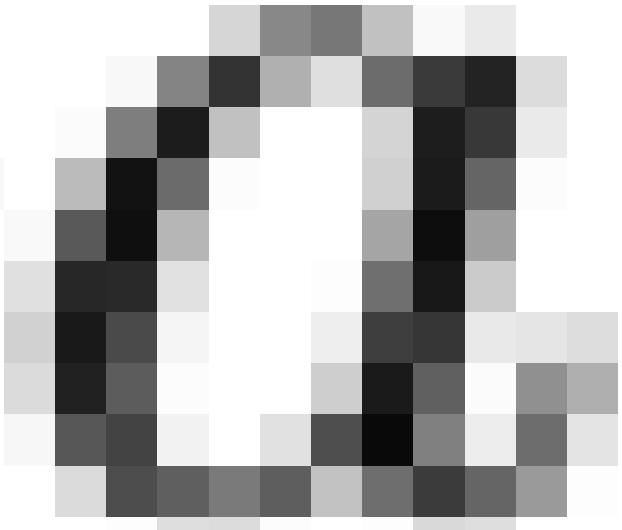
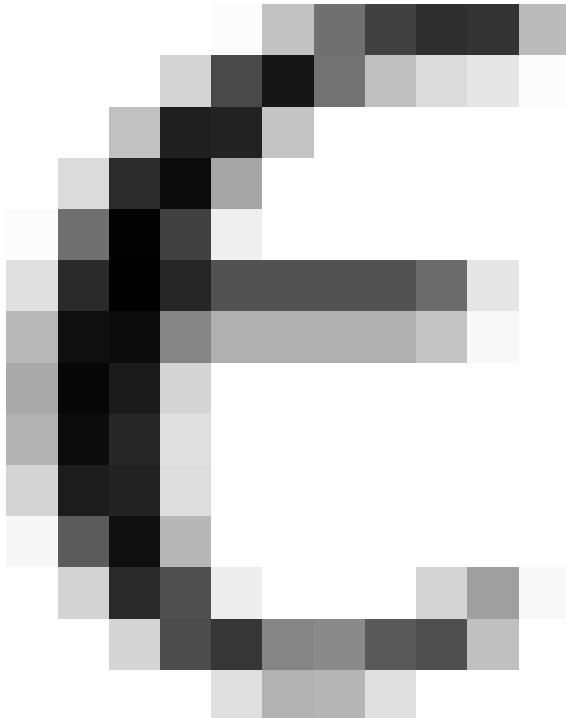
$$E = \frac{\sigma_{33}}{\epsilon_{33}}$$

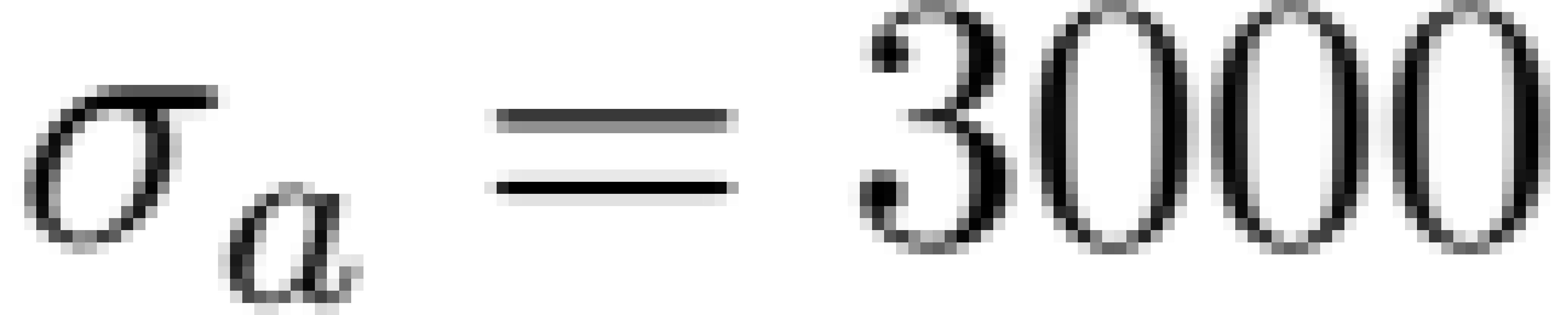
Young's Modulus

$$\nu = -\frac{\epsilon_{11}}{\epsilon_{33}}$$

Poisson's ratio (nu)





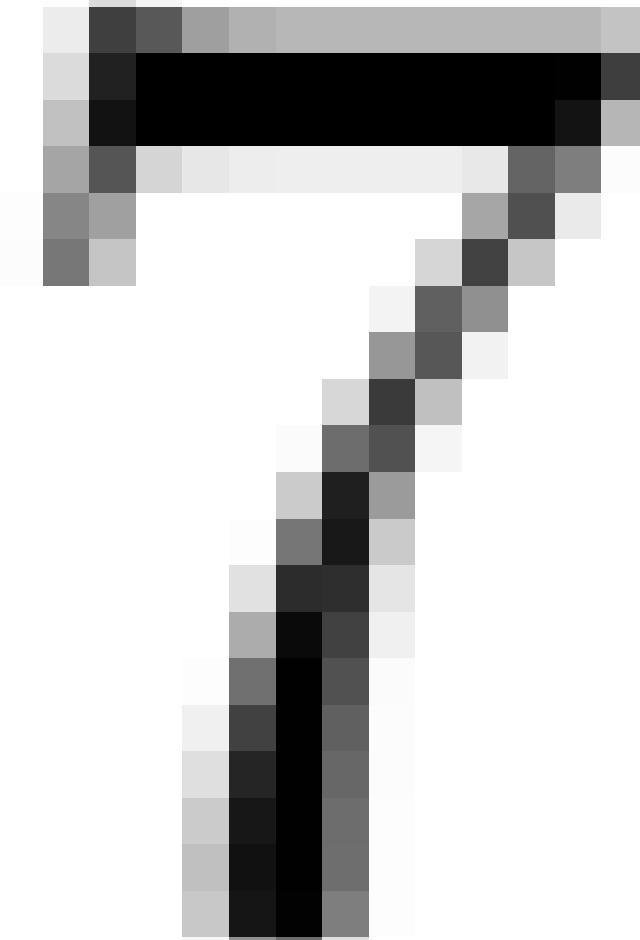
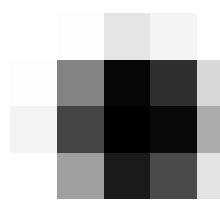
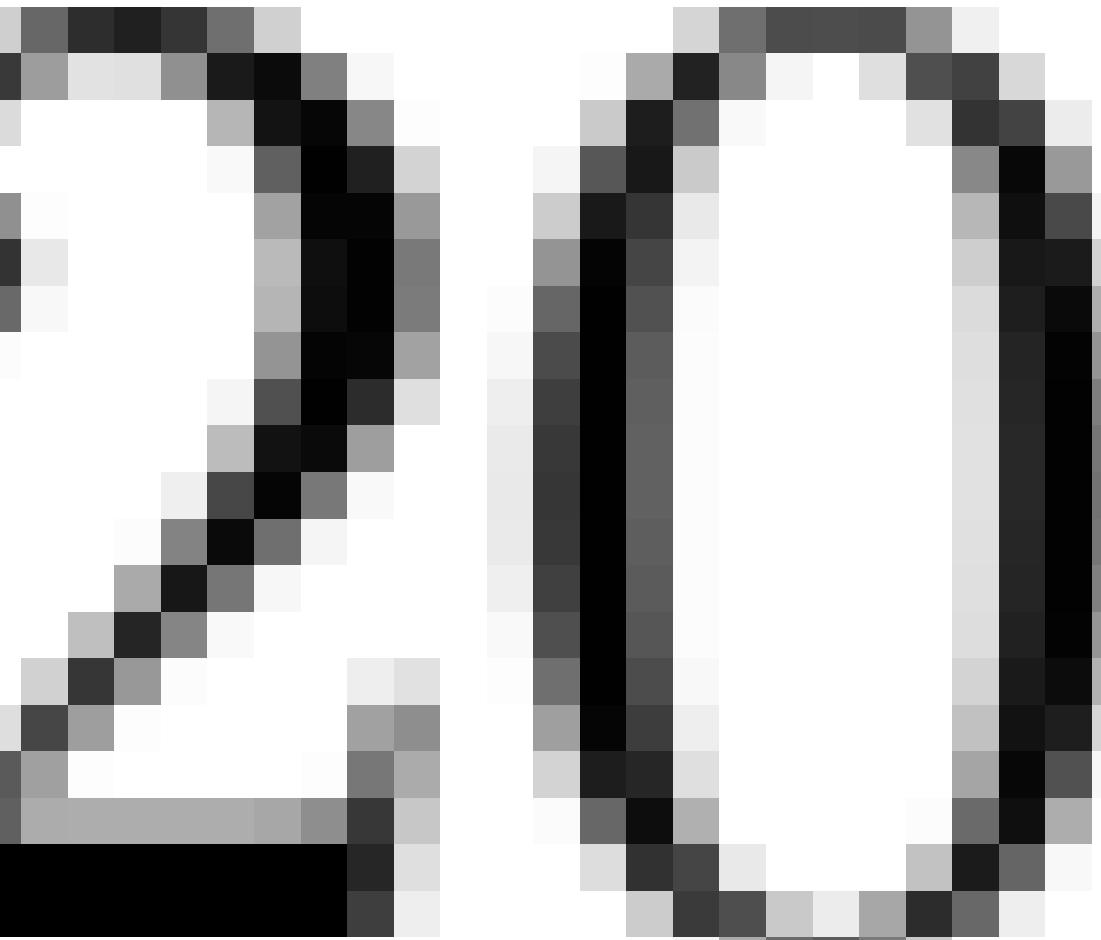


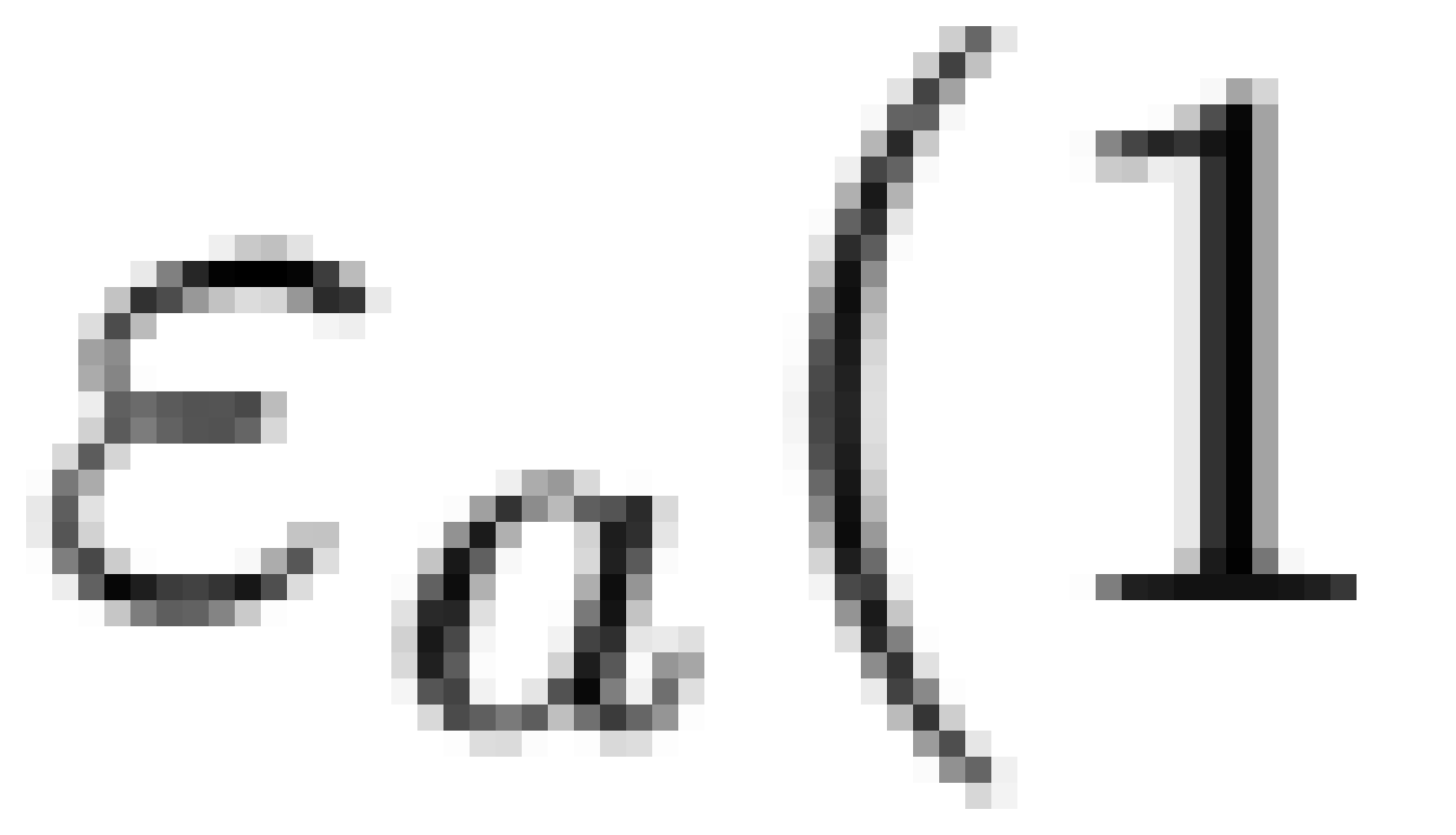
1

30000

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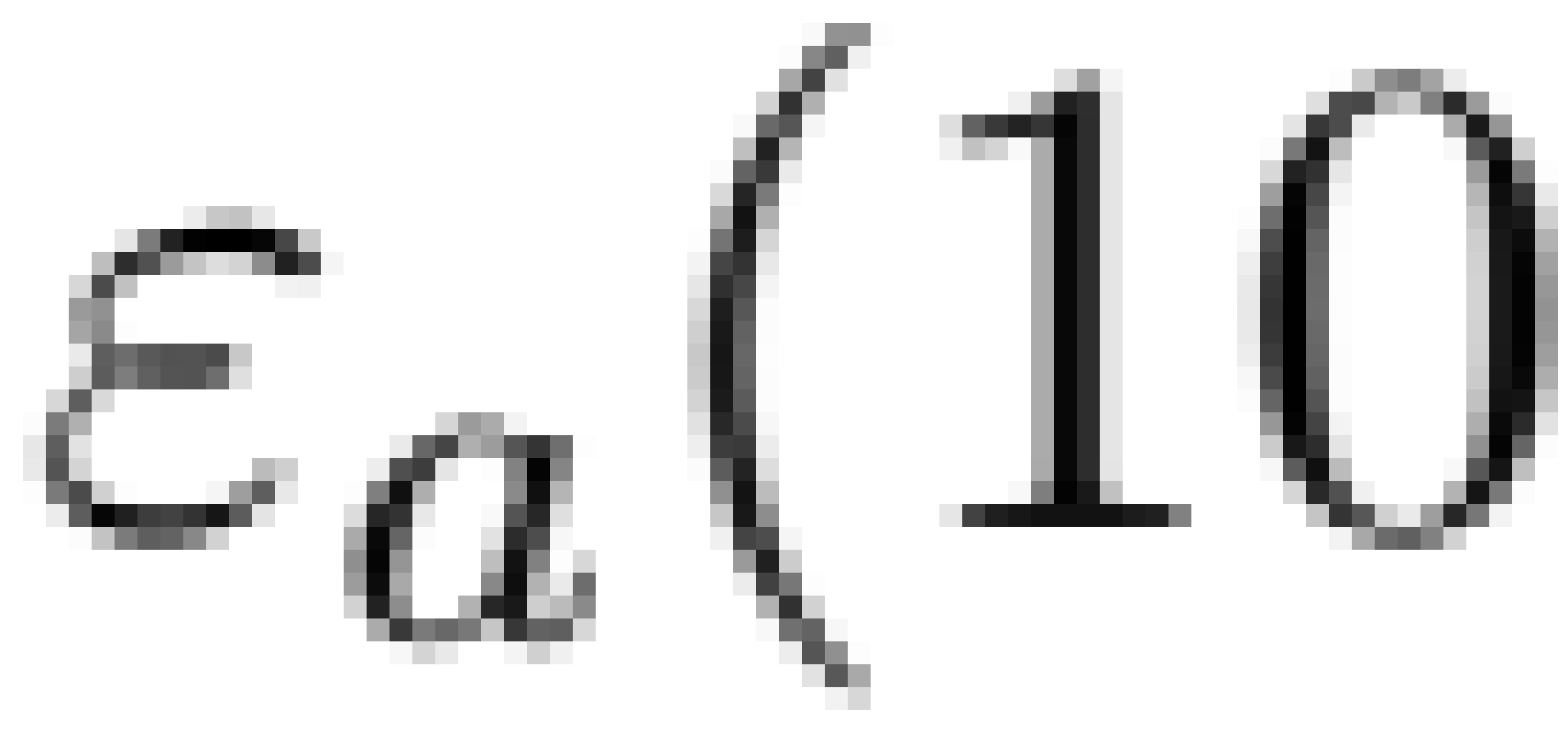
—





$$\frac{\sigma_a}{E} = \frac{20.7}{100} = 0.207$$

$$0.207 \times 100 \text{ MPa} = 20.7 \text{ MPa}$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{100 \times 10^9 \text{ MPa}} = 0.207\%$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{50000 \text{ MPa}} = 0.00041$$

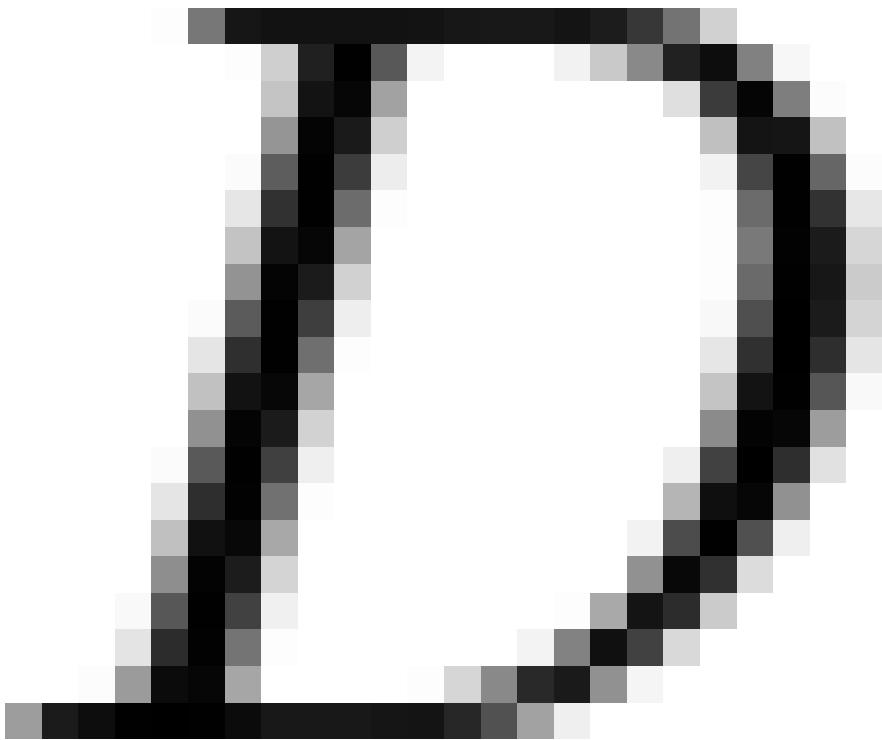
$$= 0.041\%$$

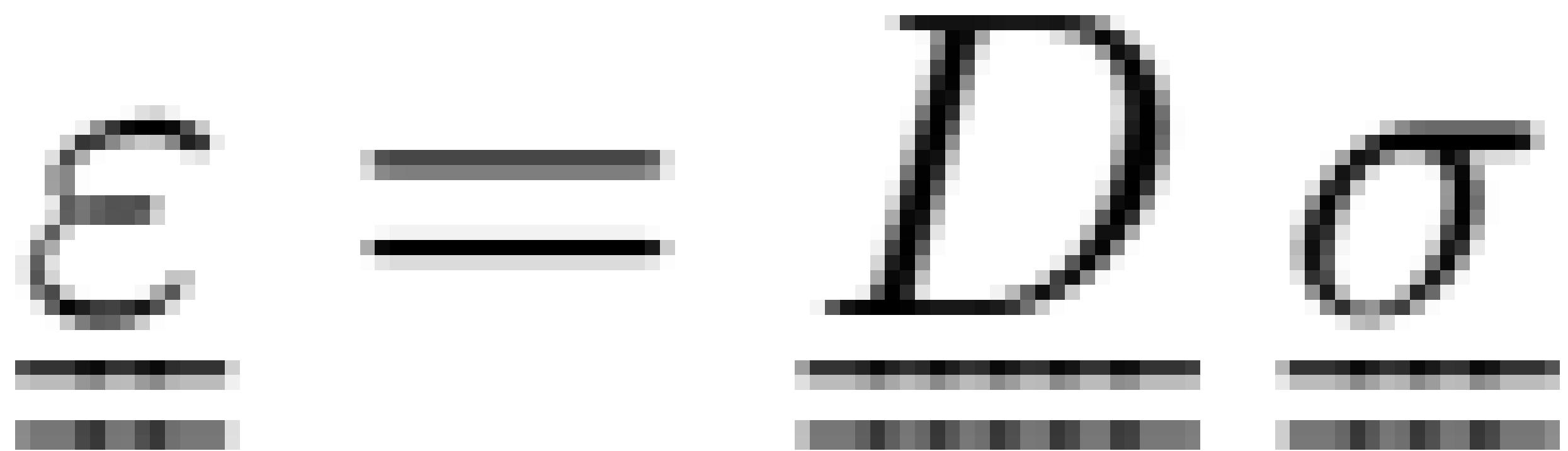
$$\left\{ \begin{array}{lcl} \epsilon_{11} & = & +\frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ & & \nu \quad \quad \quad 1 \quad \quad \quad \nu \\ \epsilon_{22} & = & -\frac{1}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{1}{E}\sigma_{33} \\ & & \nu \quad \quad \quad \nu \quad \quad \quad 1 \\ \epsilon_{33} & = & -\frac{1}{E}\sigma_{11} - \frac{1}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{array} \right.$$

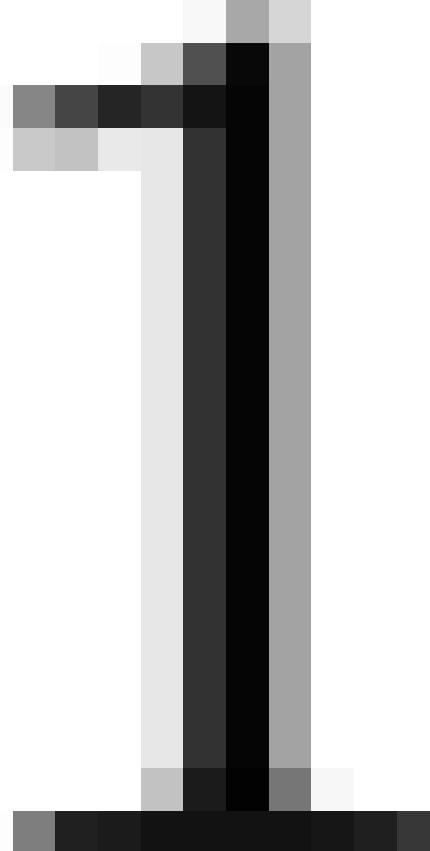
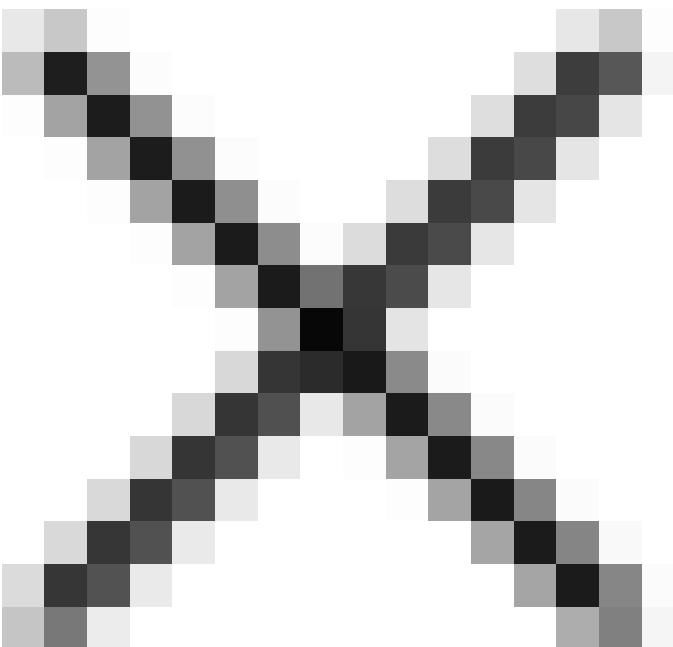
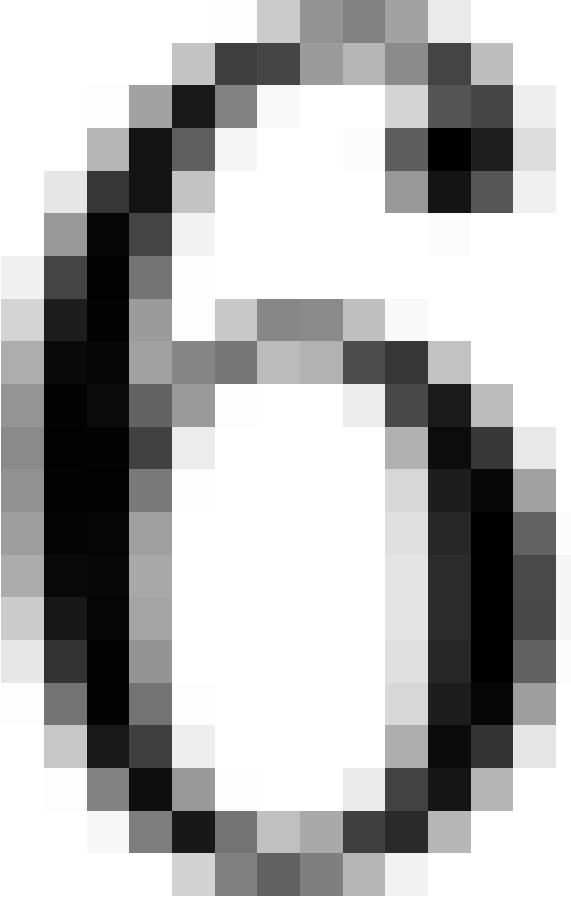


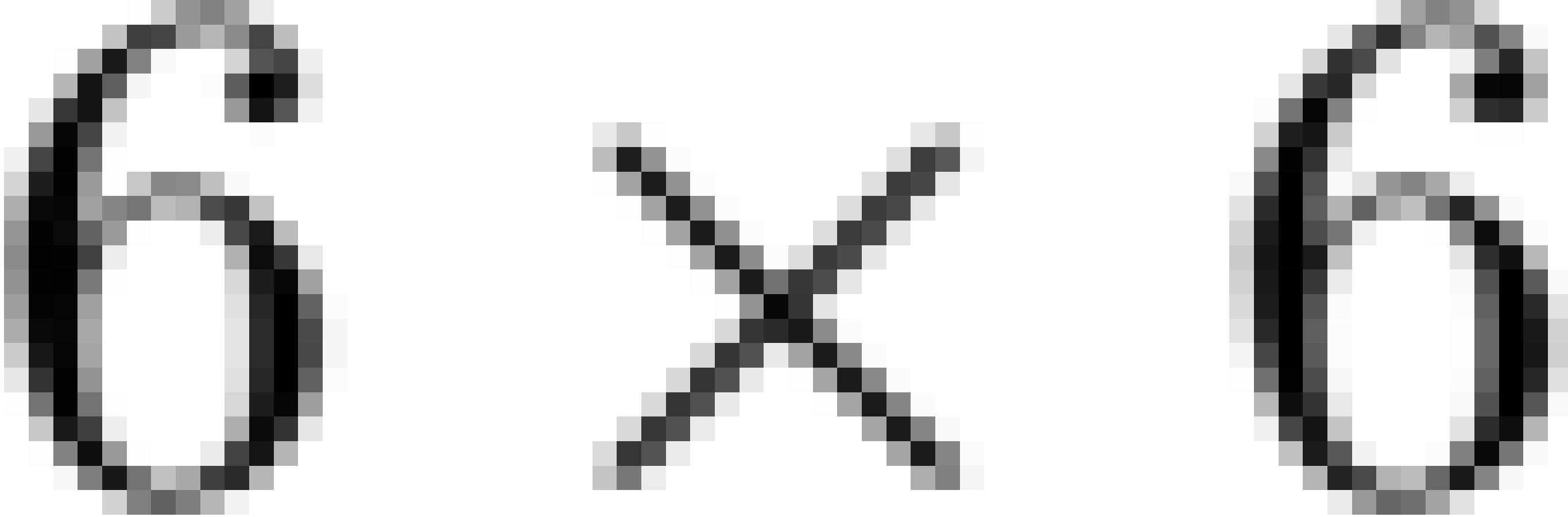


$$\left\{ \begin{array}{l} 2\epsilon_{12} = (1/G) \sigma_{12} \\ 2\epsilon_{13} = (1/G) \sigma_{13} \\ 2\epsilon_{23} = (1/G) \sigma_{23} \end{array} \right.$$



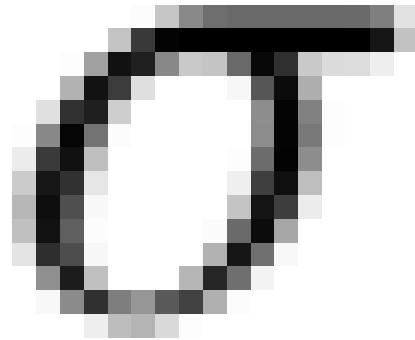
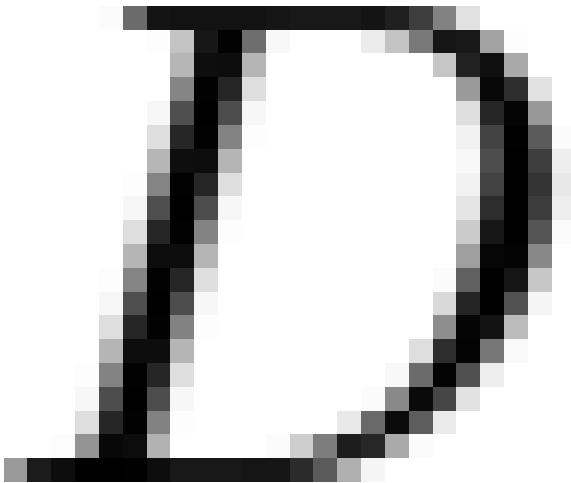




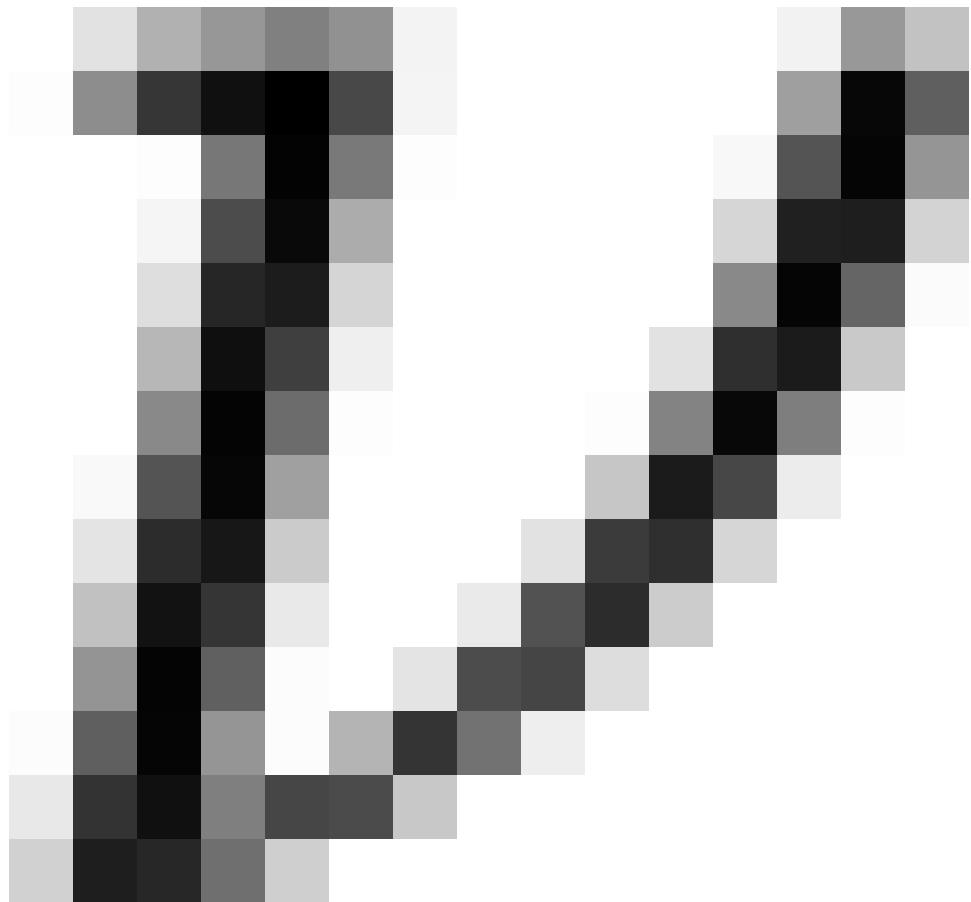


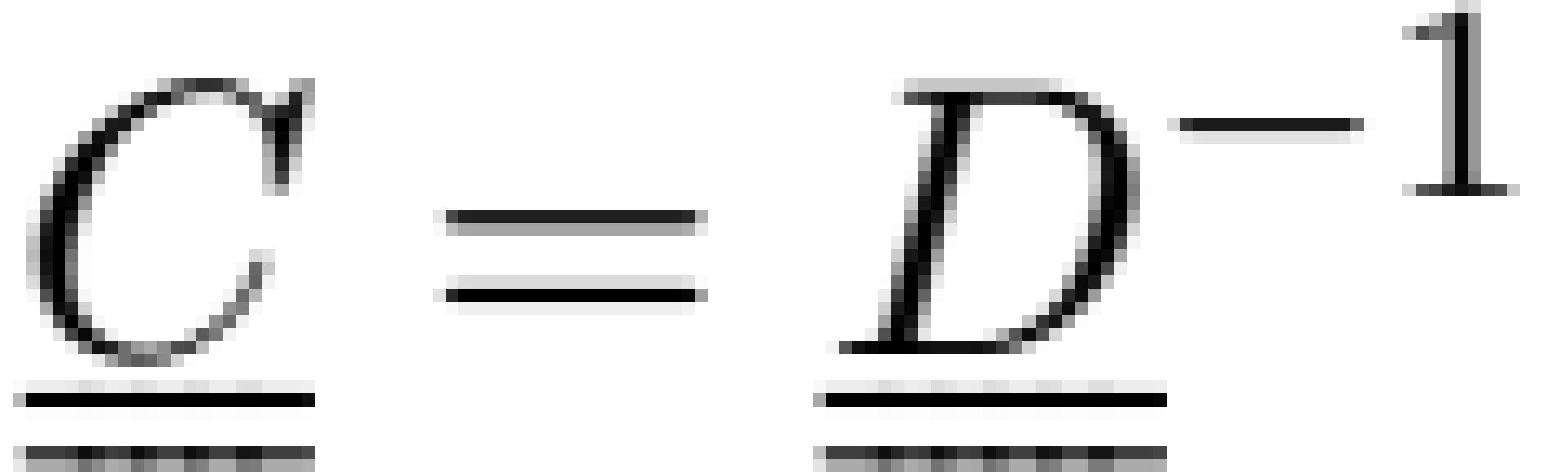
$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} = \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & 1 & \nu & 0 & 0 & 0 \\ -\frac{\nu}{E} & +\frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & \nu & 1 & 0 & 0 & 0 \\ -\frac{1}{E} & -\frac{\nu}{E} & +\frac{1}{E} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}$$



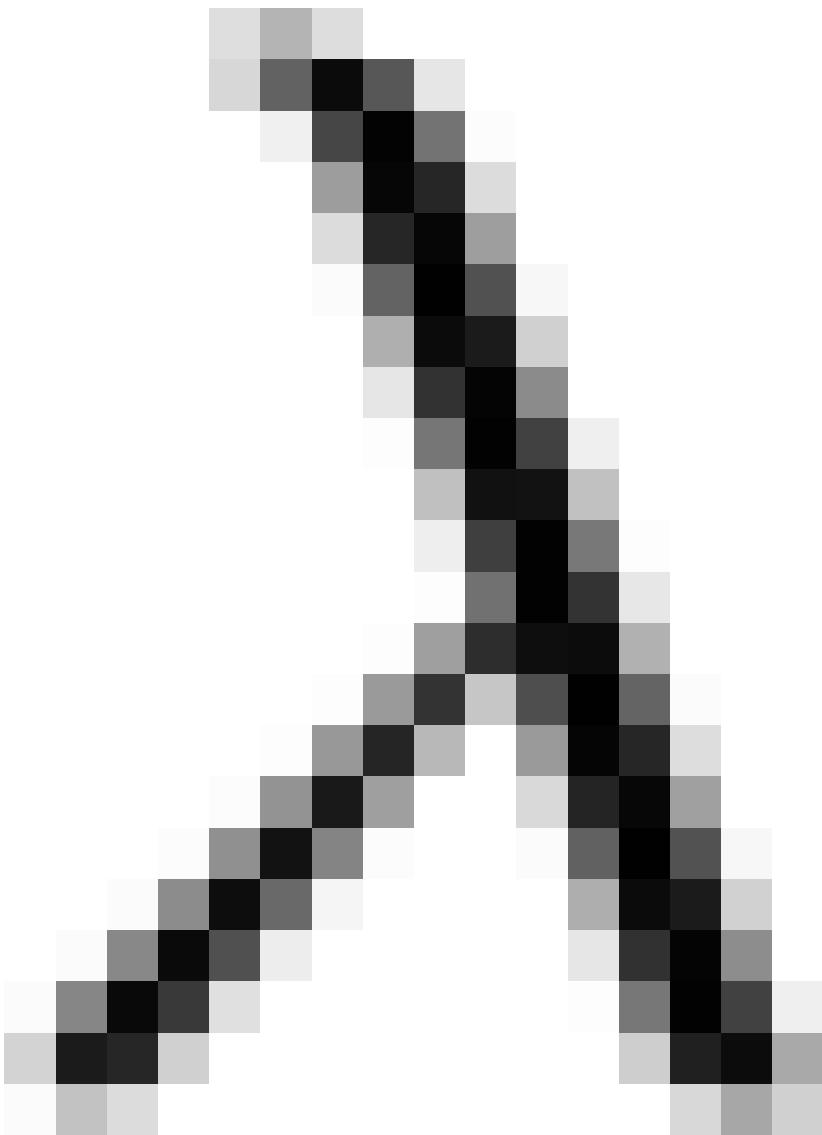


$$\underline{\underline{\epsilon}} = \begin{bmatrix} -\frac{v}{E}\sigma_{33}, -\frac{v}{E}\sigma_{33}, \frac{1}{E}\sigma_{33}, 0, 0, 0 \end{bmatrix}^T$$





$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$



$$\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)} [(1-\nu)e_{11} + \nu e_{22} + \nu e_{33}]$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + \frac{(1-2\nu)}{(1+\nu)(1-2\nu)} \epsilon_{11}$$

λ



$(1 + \nu)$

$(1 - \nu)$



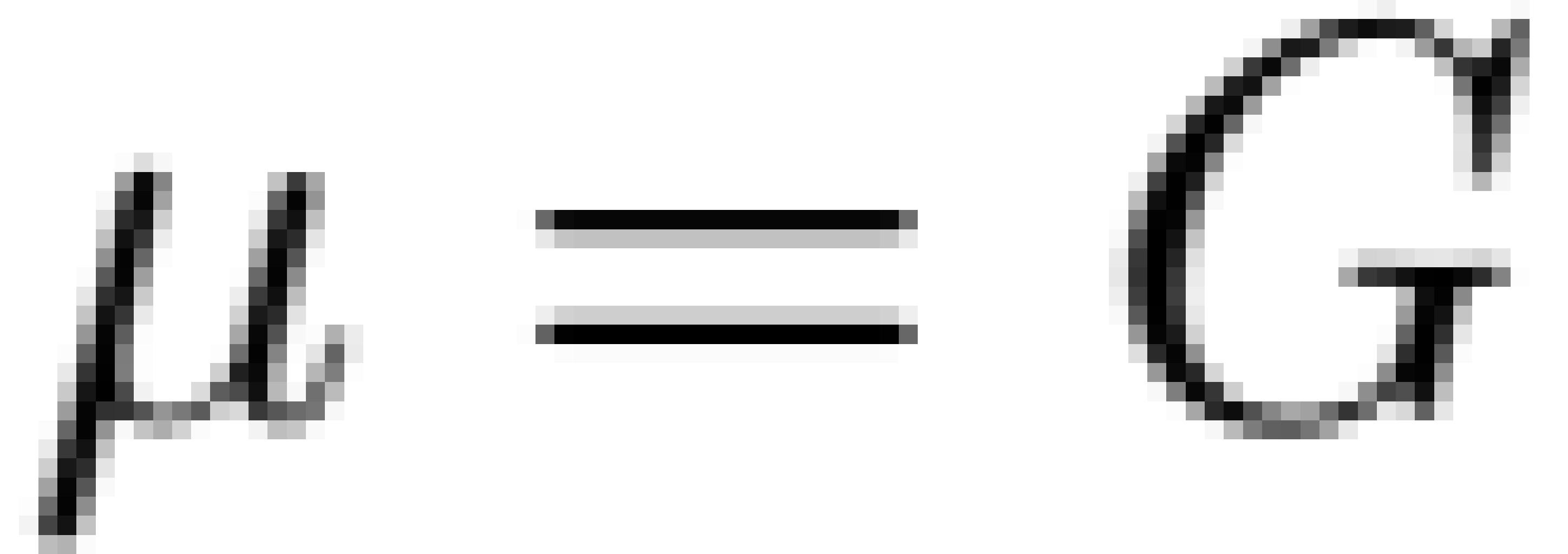
(2ν)

νE

$$2\mu =$$

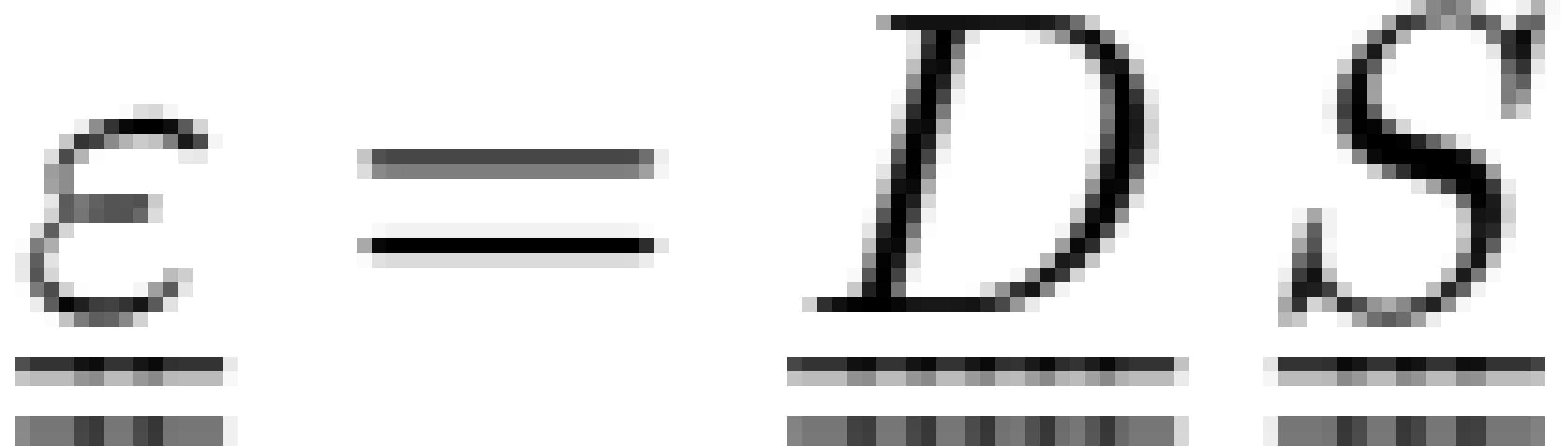
$$\frac{\nu E}{(1+\nu)(1-2\nu)} =$$

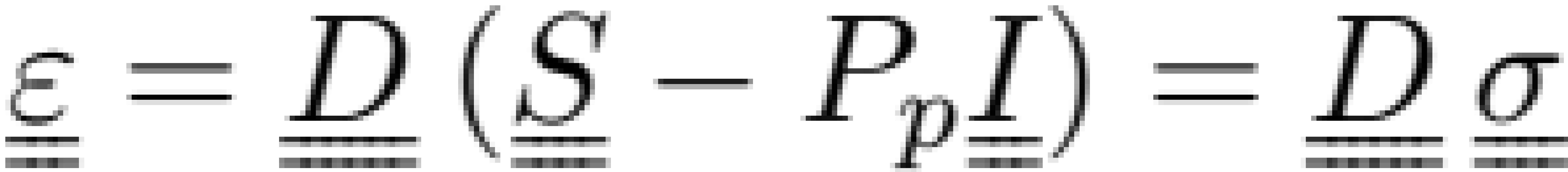
$$\frac{(1-2\nu)E}{\nu} = \frac{E}{(1+\nu)}$$

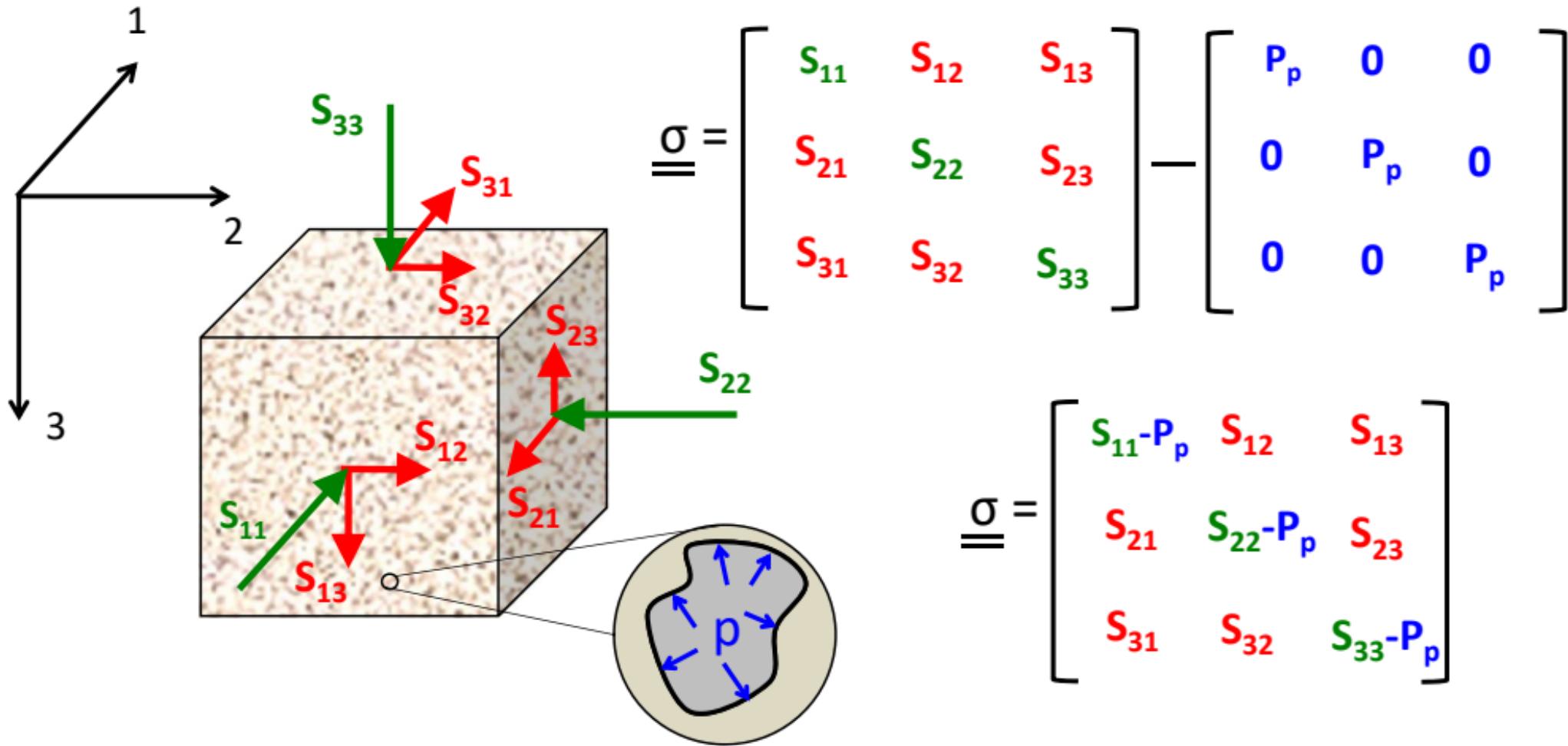


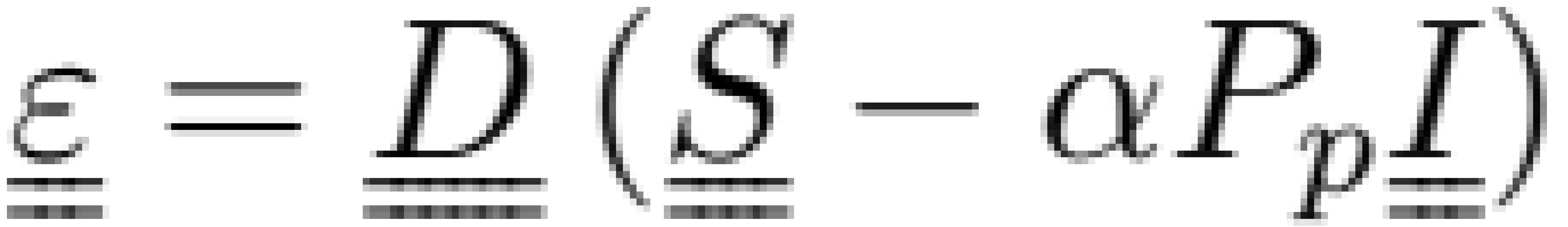
$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right. .$$

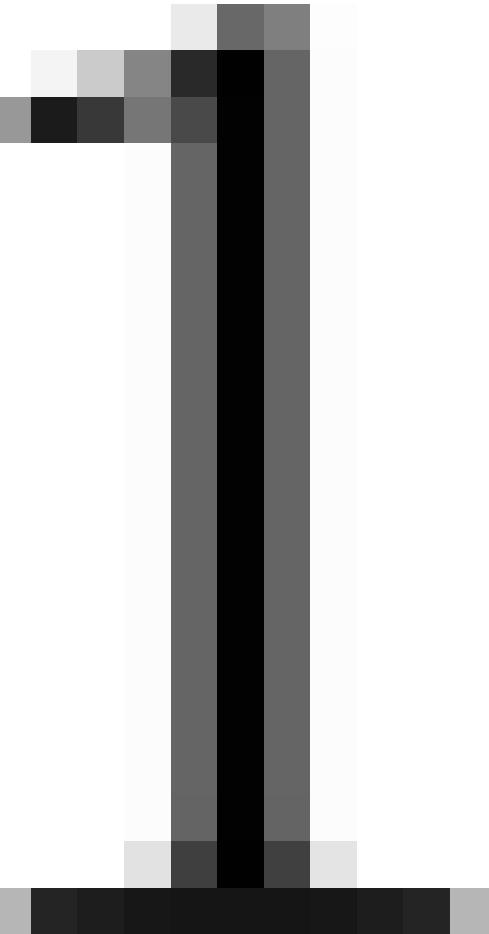
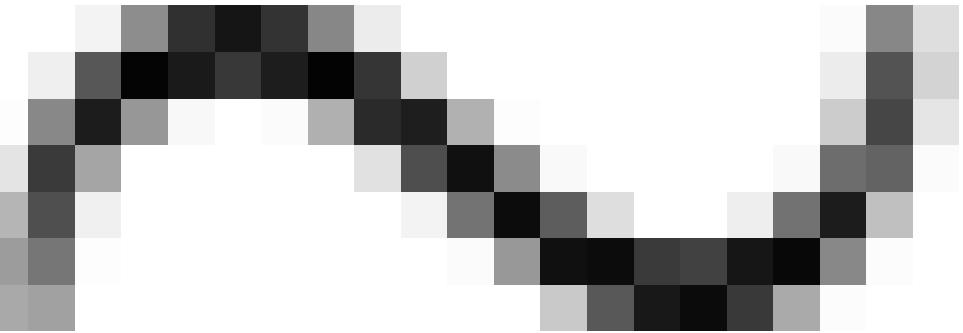
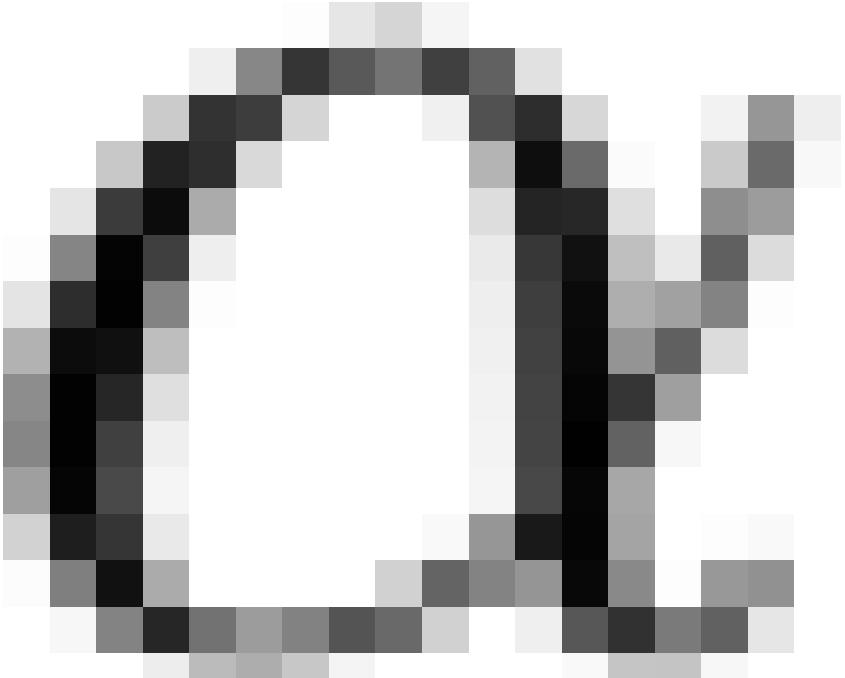
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$









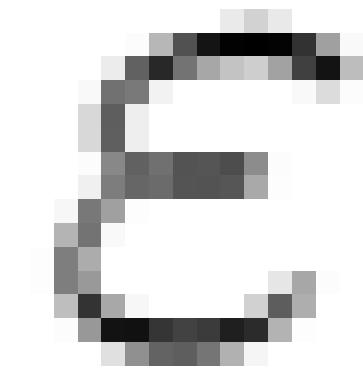
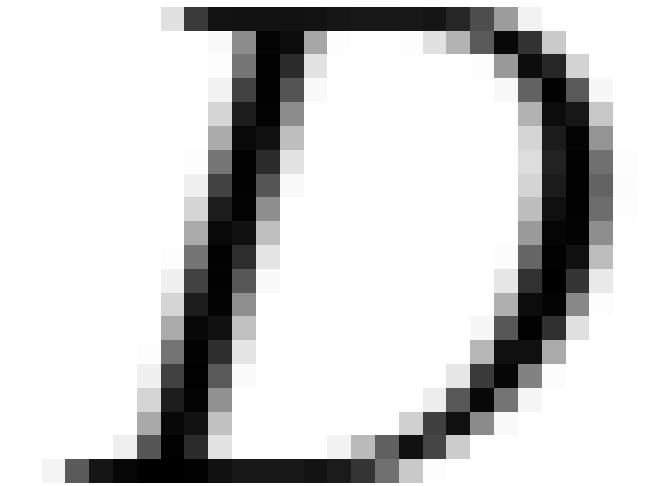
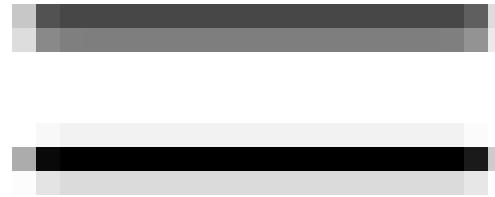
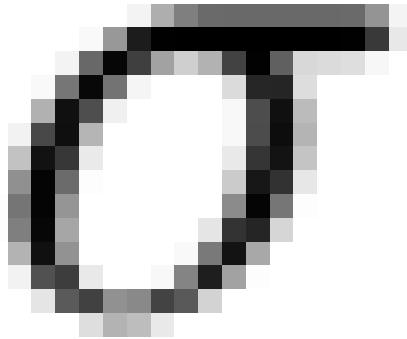














$$\left\{ \begin{array}{l} \sigma_{11} = \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)} \epsilon_{33} \\ \sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \epsilon_{33} \end{array} \right.$$

σ_{11}

$$= \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

 σ_{22}

$$= \begin{array}{c} \text{---} \\ | \end{array}$$

 σ_{33} ν ν

σ_b

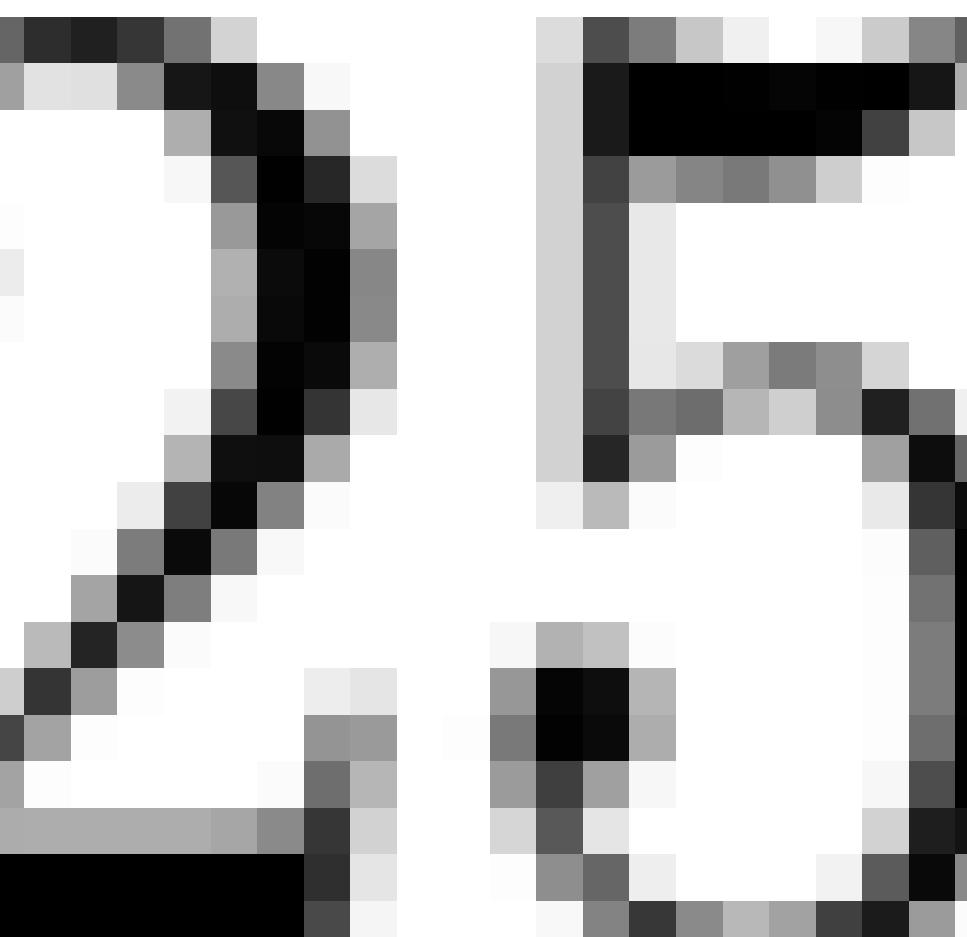
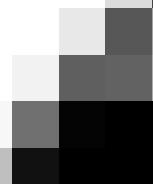
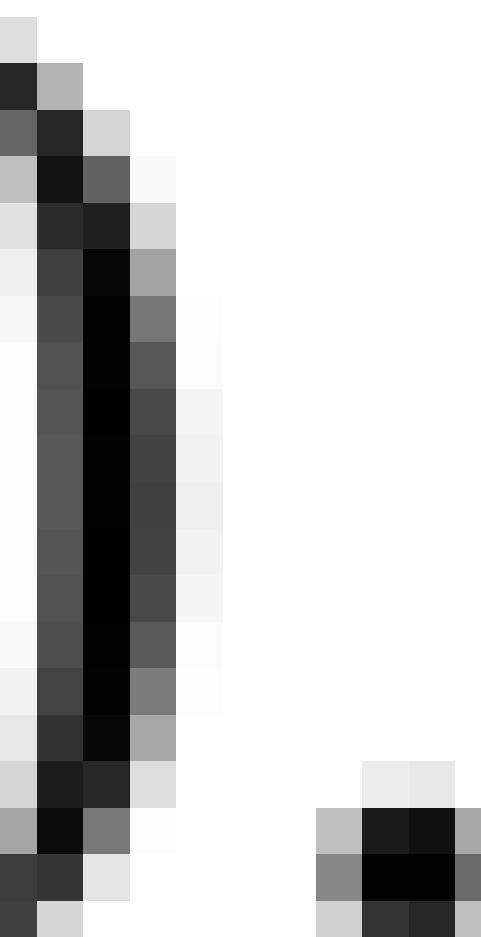
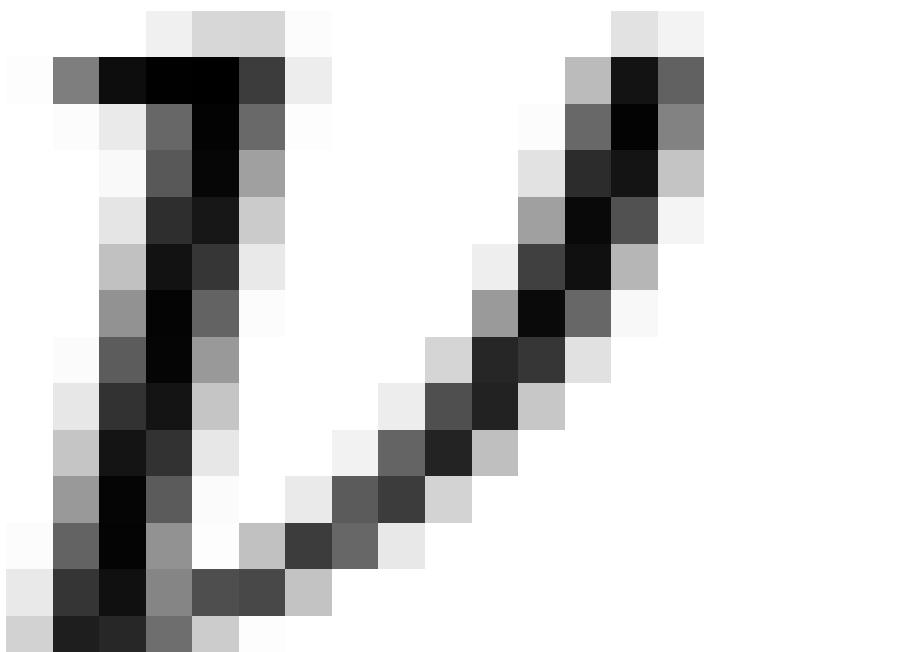
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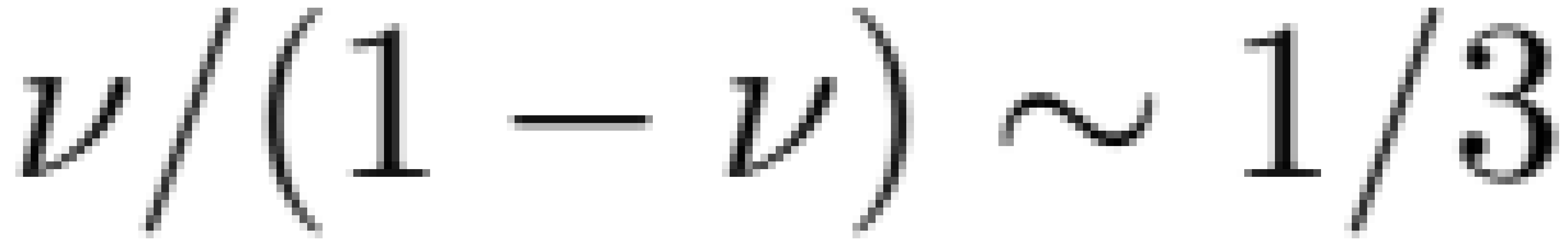
1

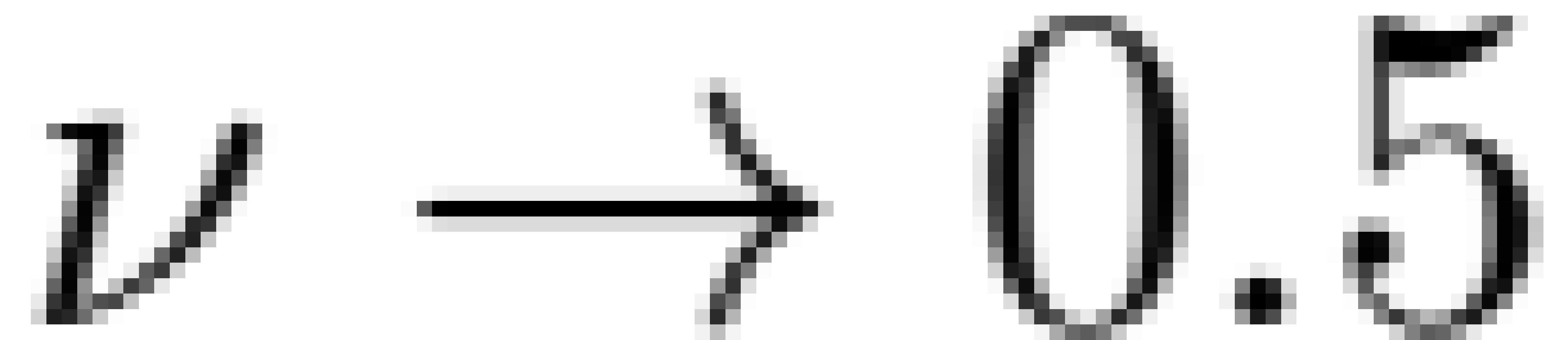
V

V

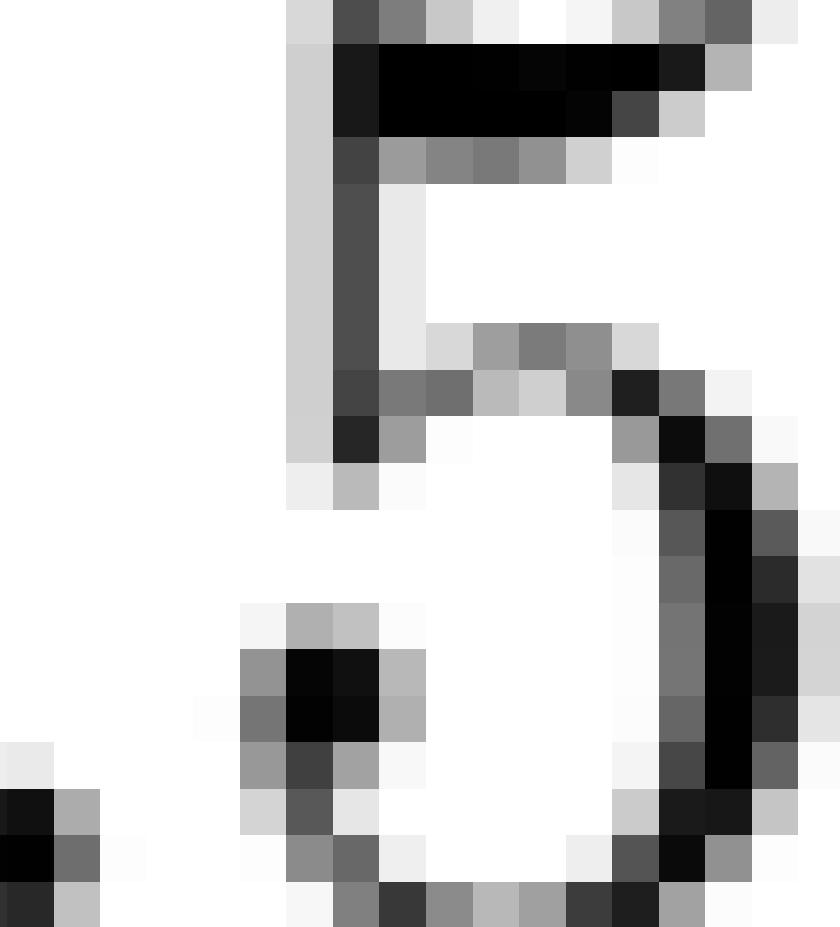
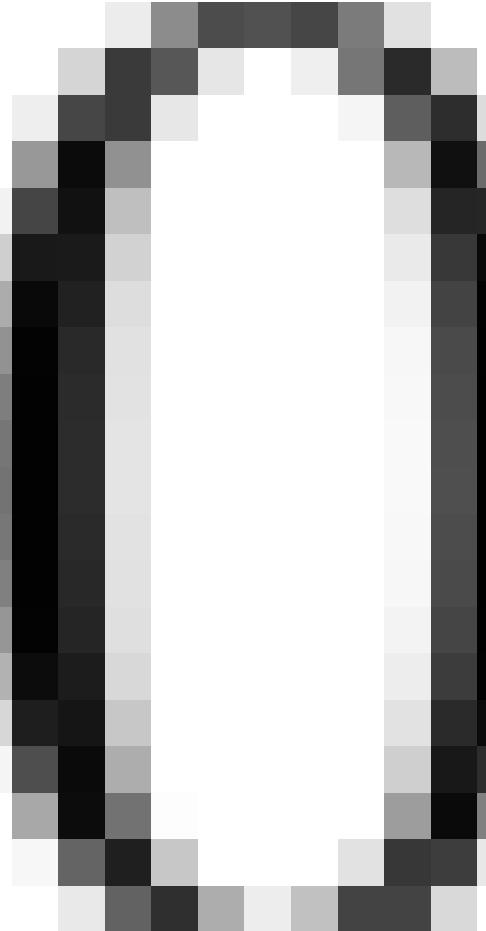
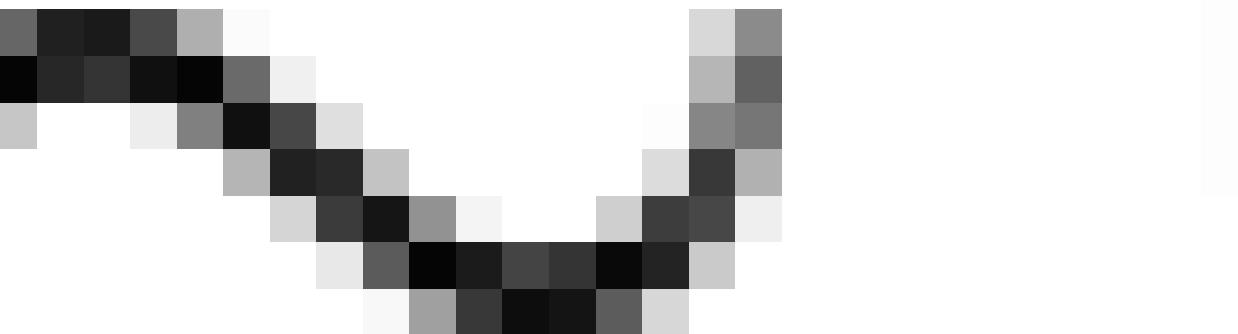
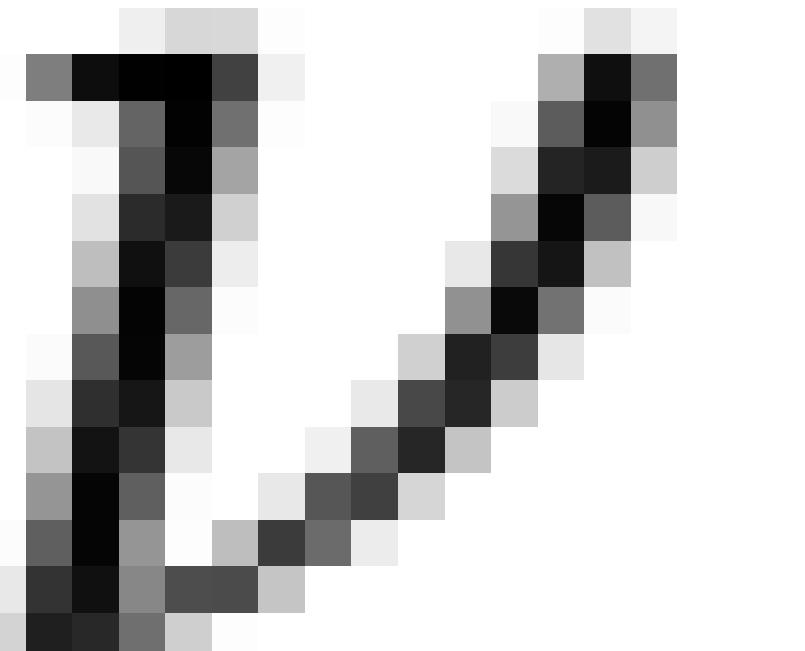
σ_w







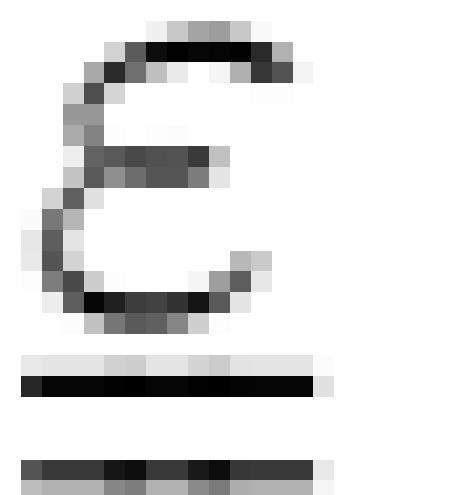




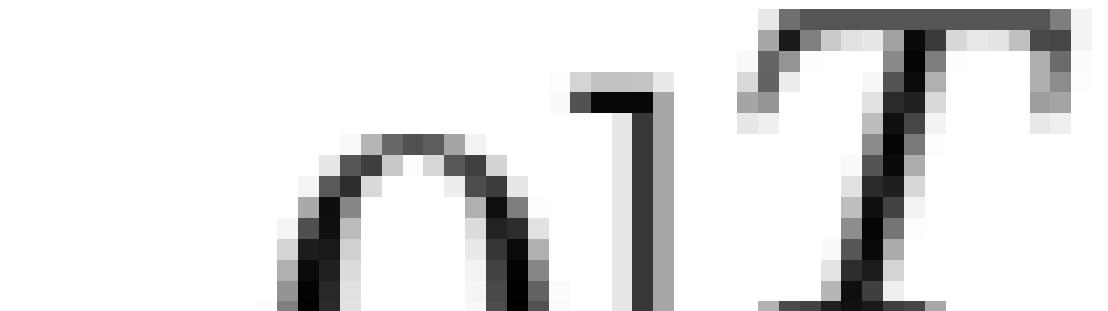
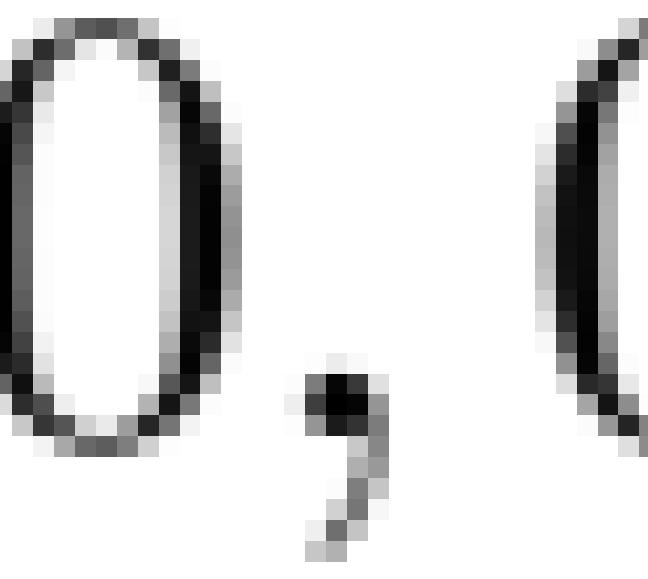
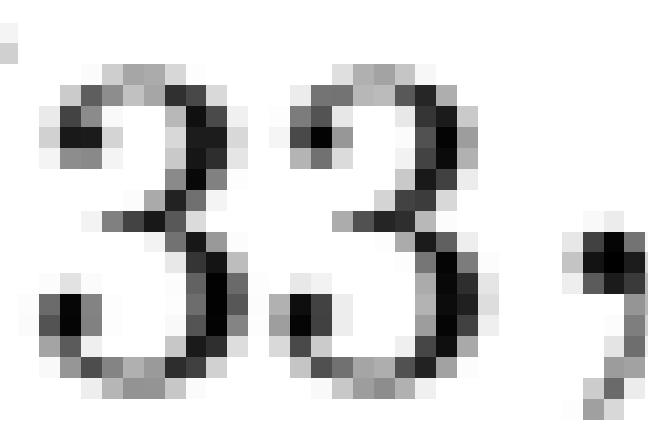




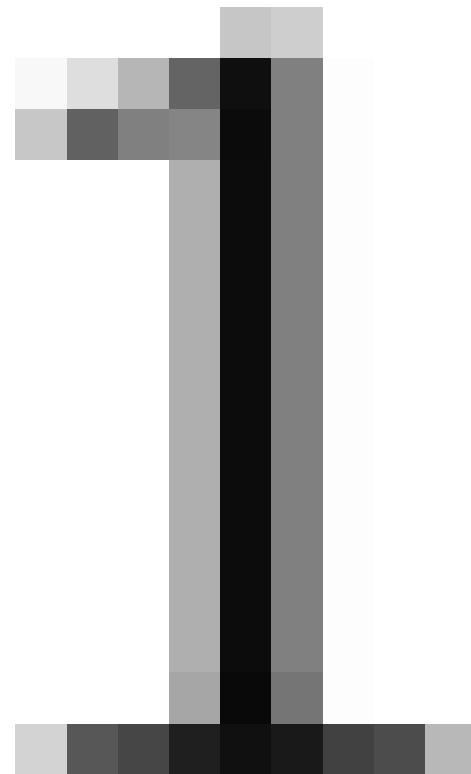
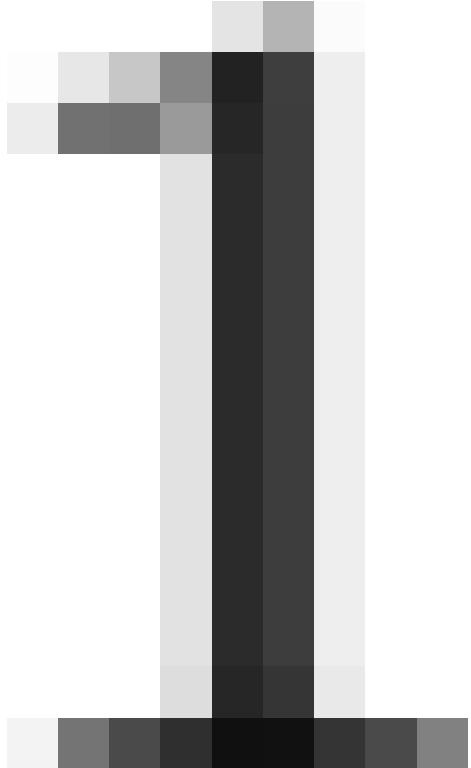
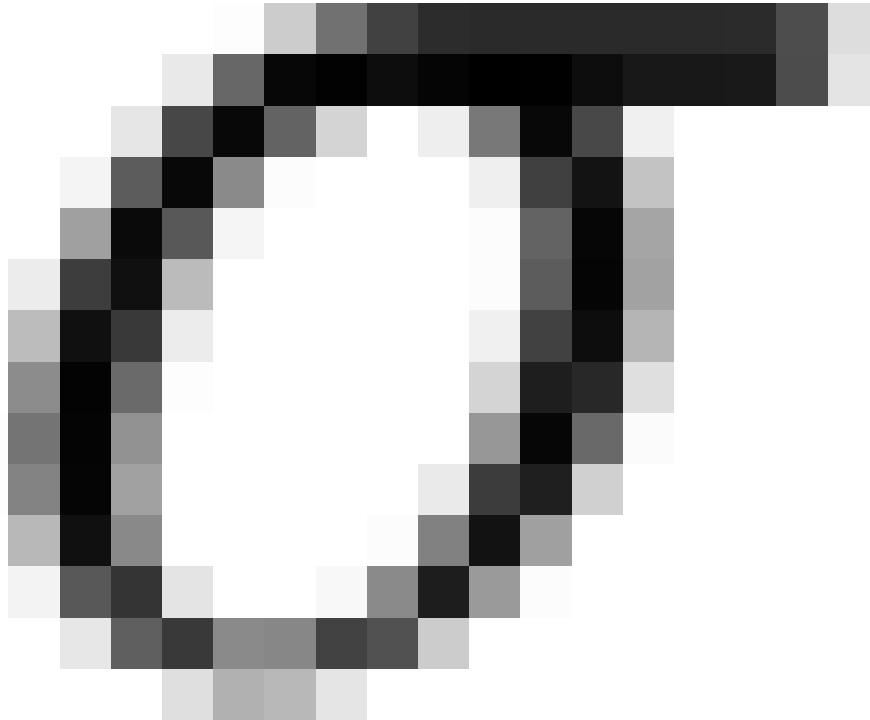


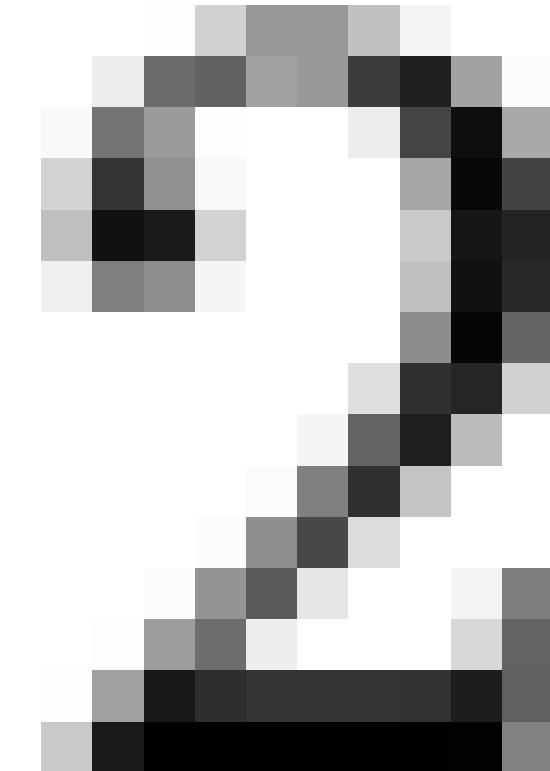
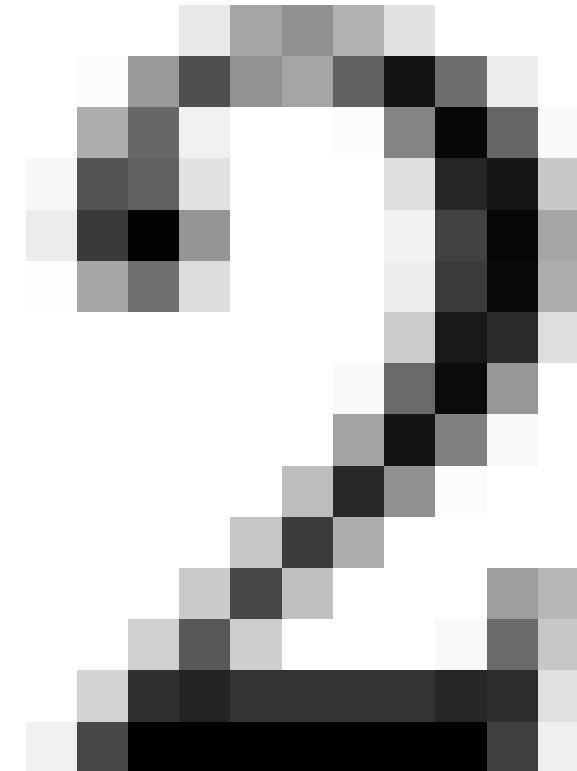
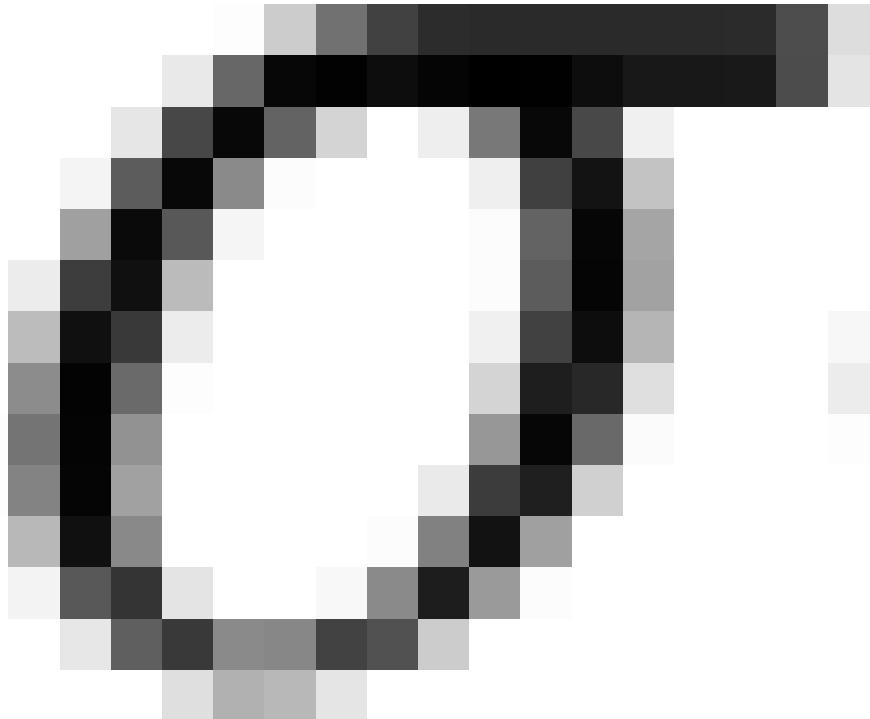


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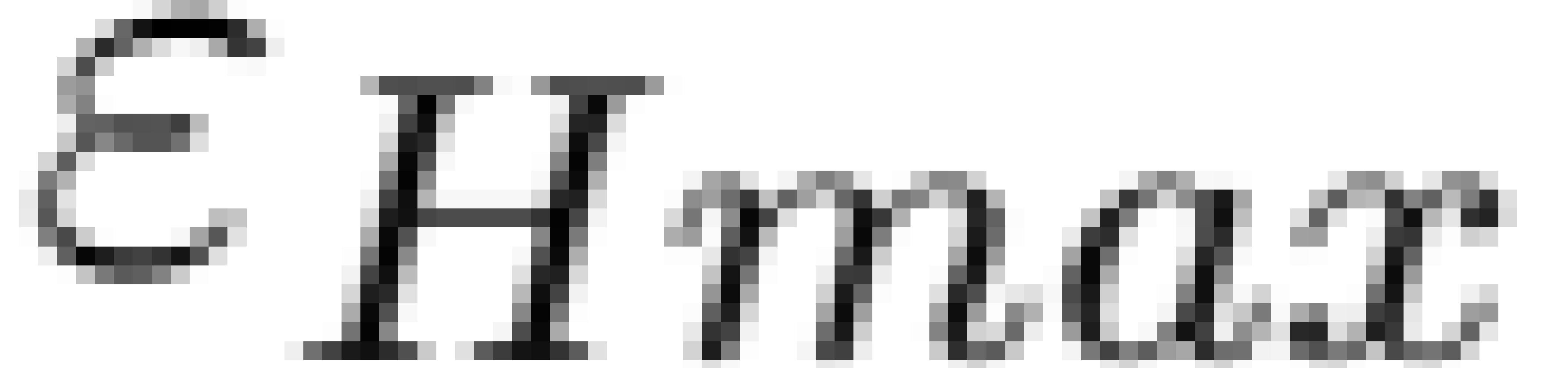


$$\left\{ \begin{array}{l} \sigma_{11} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{33} \\ \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{11} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{33} \\ \sigma_{33} = \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{22} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon_{33} \end{array} \right.$$



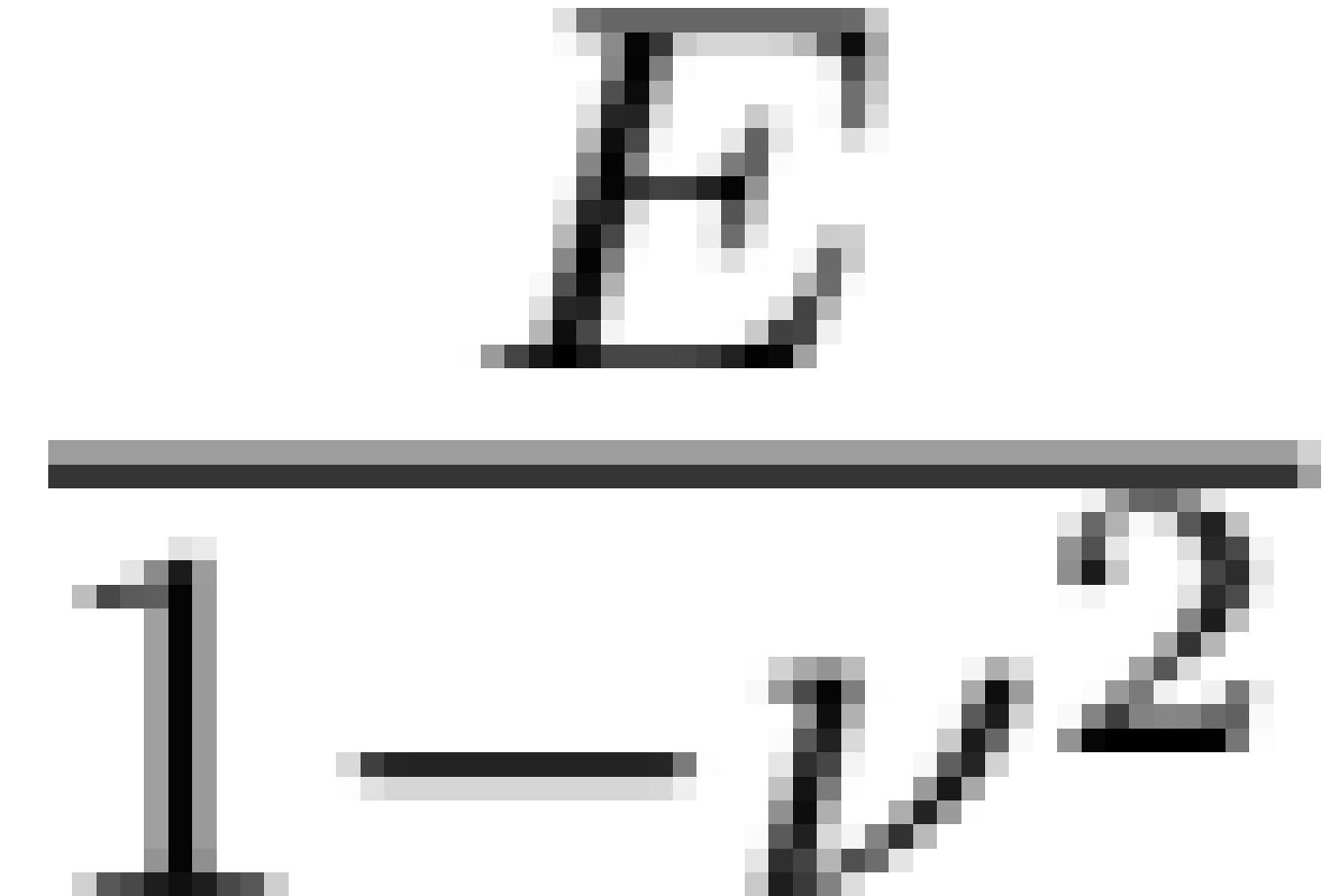
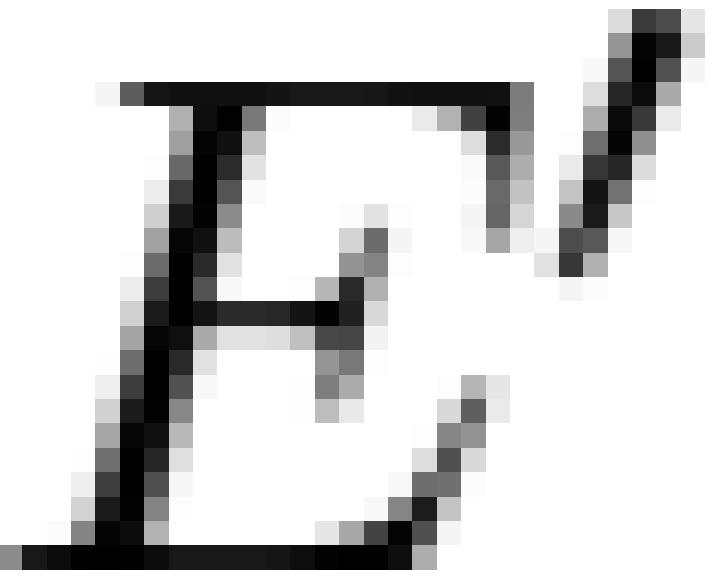


$$\left\{ \begin{array}{l} \sigma_{11} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{E}{1-\nu^2} \epsilon_{11} + \frac{\nu E}{1-\nu^2} \epsilon_{22} \\ \sigma_{22} = \frac{\nu}{1-\nu} \sigma_{33} \frac{\nu E}{1-\nu^2} \epsilon_{11} + \frac{E}{1-\nu^2} \epsilon_{22} \end{array} \right.$$

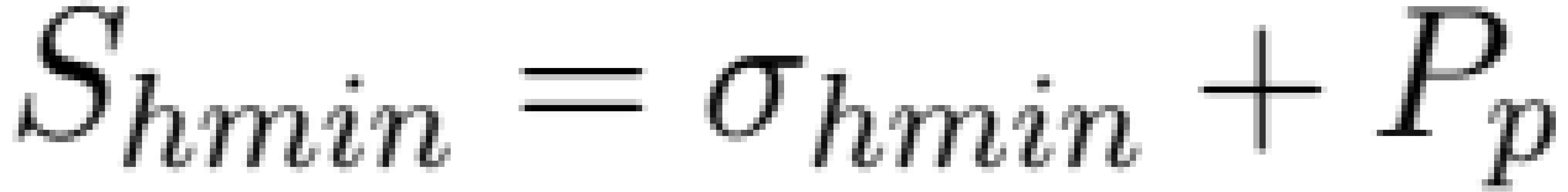




$$\left\{ \begin{array}{l} \sigma_{H\max} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{H\max} + \nu E' \epsilon_{hmin} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{H\max} + \nu E' \epsilon_{hmin} \end{array} \right.$$

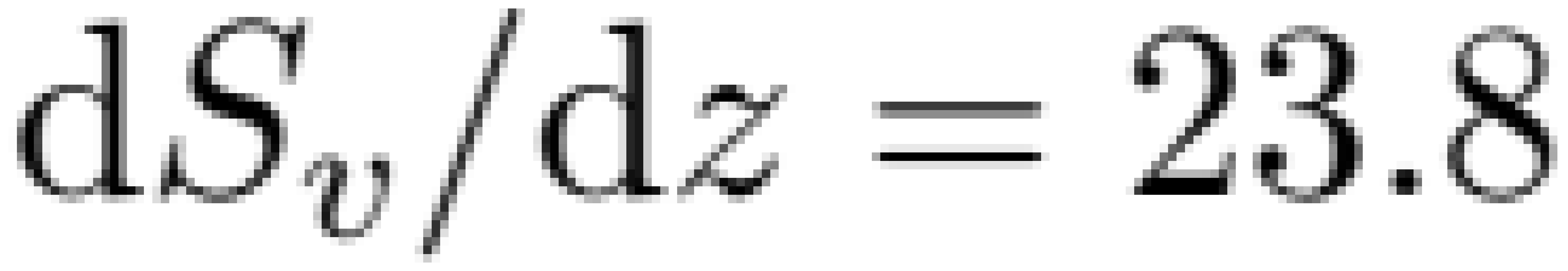


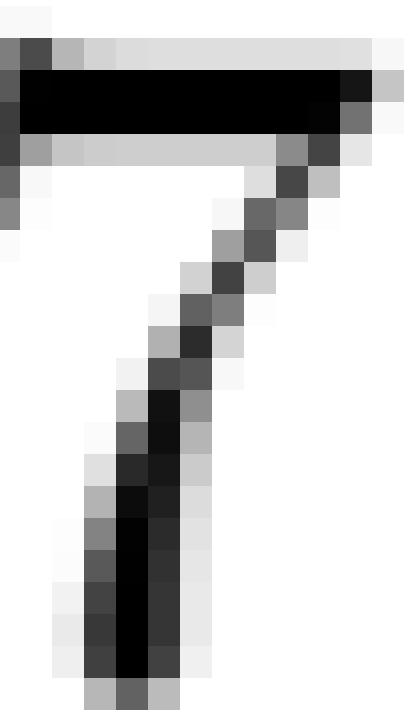
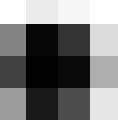
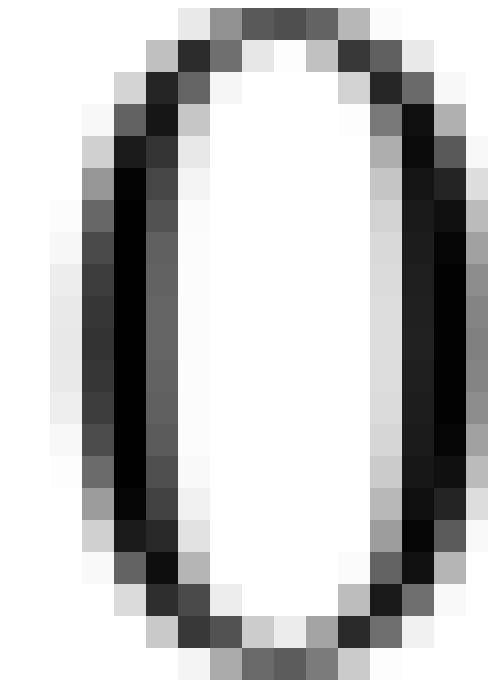


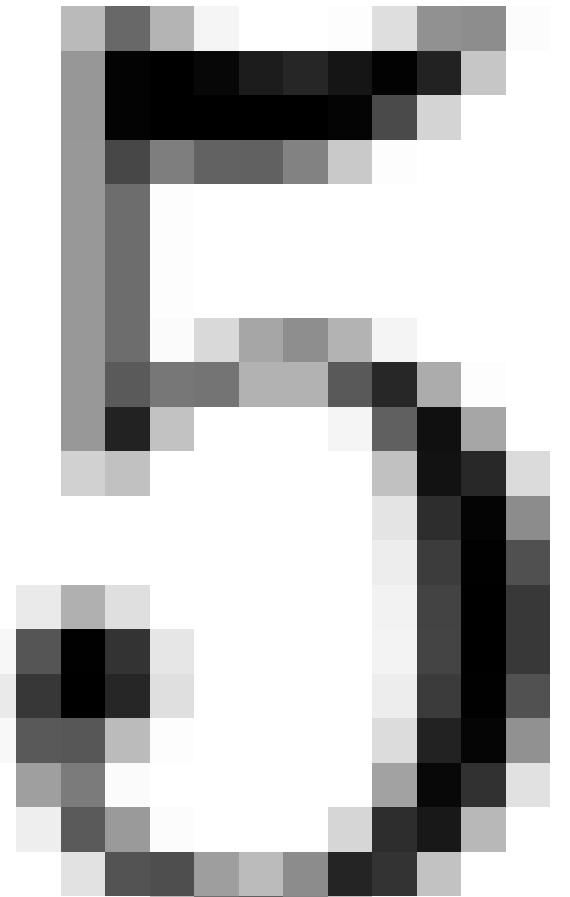
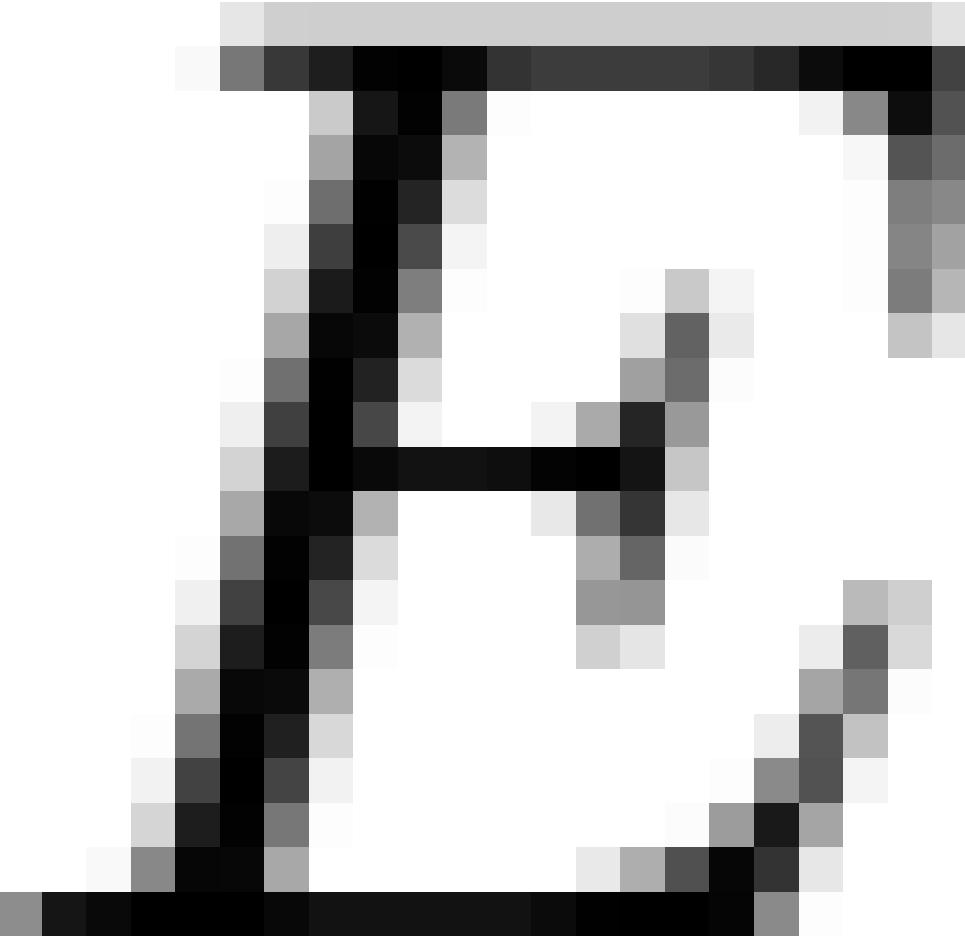


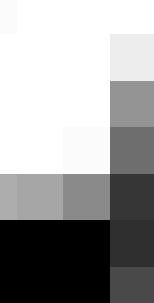
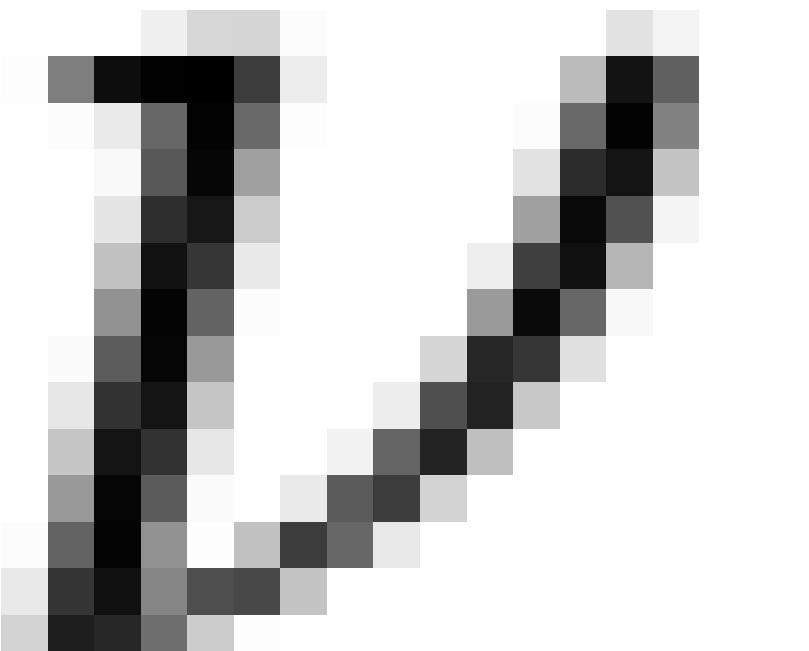


	(E, v)	(K, G)
$G =$	$\frac{E}{2(1+v)}$	<u>Shear modulus</u> (also noted as μ , S-wave)
$M =$	$\frac{(1-v)E}{(1+v)(1-2v)}$	<u>Constrained modulus</u> (uniaxial compaction, P-wave)
$\lambda =$	$\frac{vE}{(1+v)(1-2v)}$	<u>Lamé first parameter</u> (volumetric strain component)
$K =$	$\frac{E}{3(1-2v)}$	<u>Bulk modulus</u> (relates volumetric strain and isotropic stress)





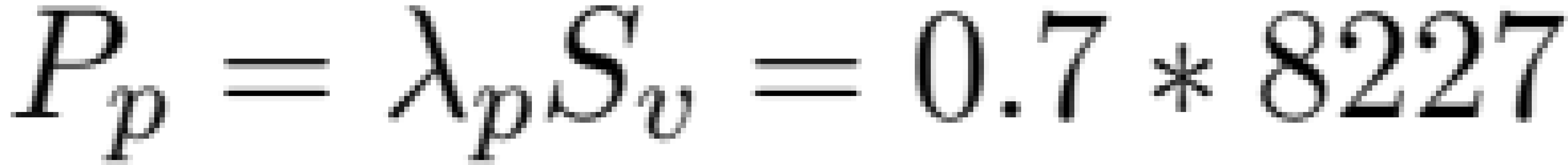




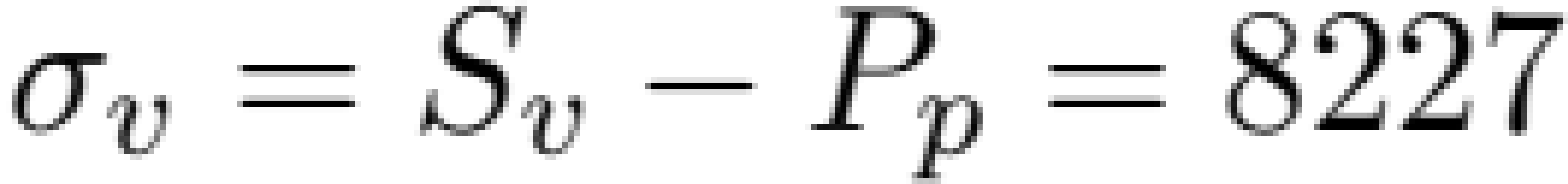




$$S_v = 23.8 \frac{\text{MPa}}{\text{km}} \cdot \frac{1 \frac{\text{psi}}{\text{ft}}}{\frac{\text{MPa}}{\text{km}}} \cdot 7950 \text{ ft} = 8227 \text{ psi}$$









$$\frac{E'}{1 - \nu^2} = \frac{E}{1 - 0.22^2} = \frac{5 \times 10^6 \text{ psi}}{5.25 \times 10^6 \text{ psi}}$$

$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} * 0.0002 = 1745 \text{ psi} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{hmin} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 0.22 * 5.25 \times 10^6 \text{ psi} * 0.0002 = 927 \text{ psi} \end{array} \right.$$

$$S_{H\max} = \sigma_{H\max} + P_p = 1745 \text{ psi} + 5759 \text{ psi} = 7504 \text{ psi}$$

$$S_{h\min} = \sigma_{h\min} + P_p = 927 \text{ psi} + 5759 \text{ psi} = 6686 \text{ psi}$$

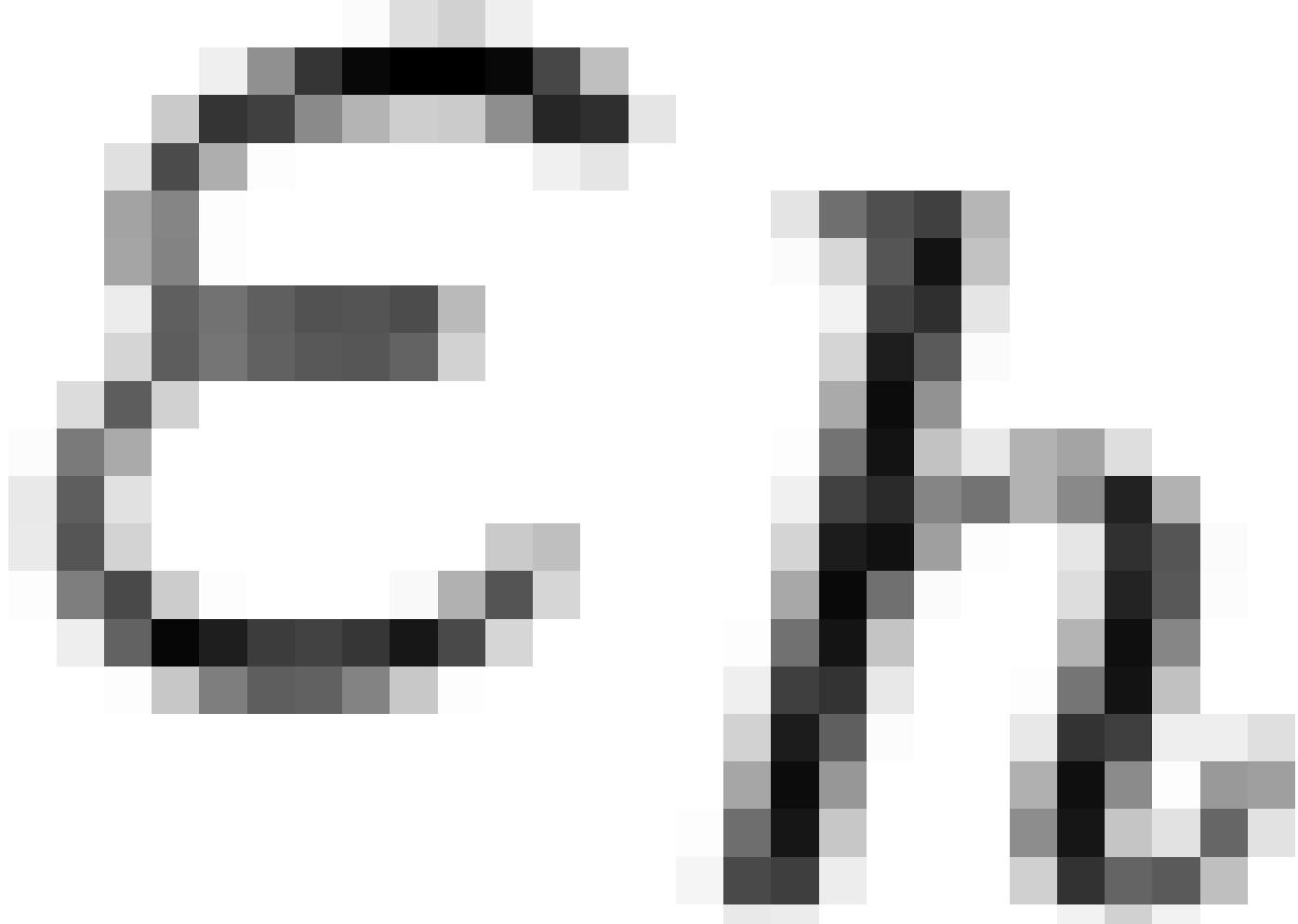




$$C_{pp} =$$

$$\frac{1}{V_p} \frac{dV_p}{dP_p}$$

S_u, ϵ_h



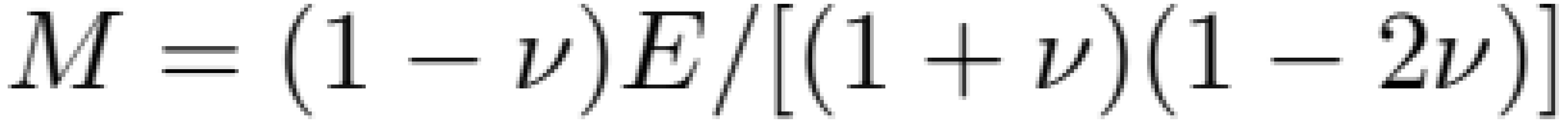
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Op

ϕ



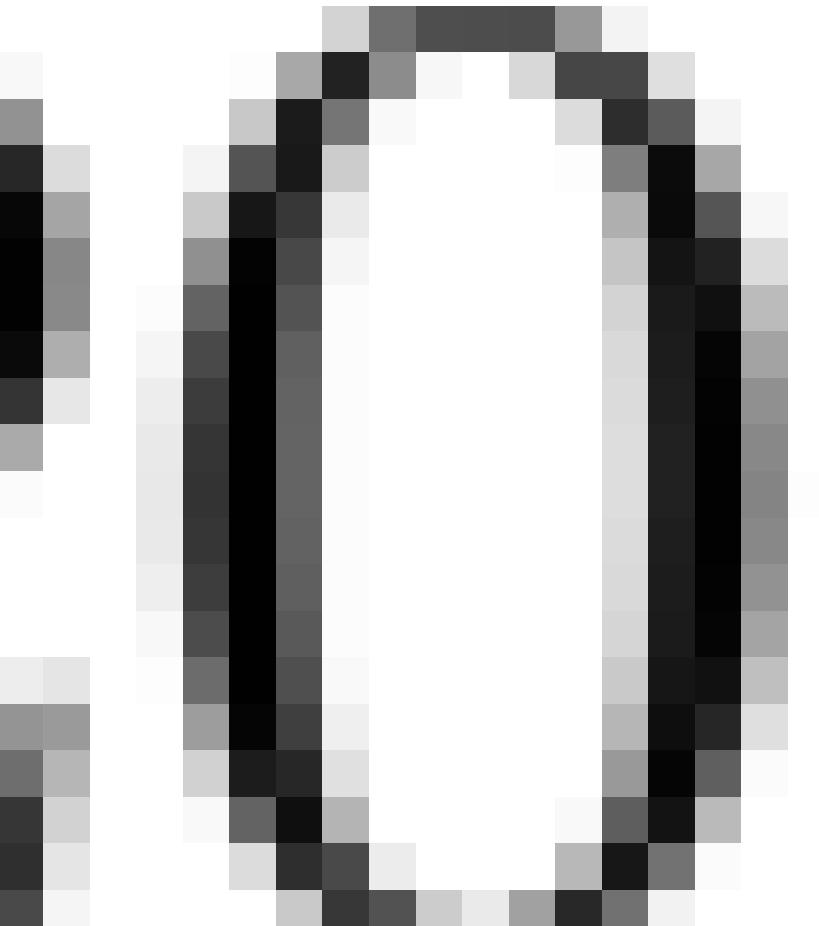
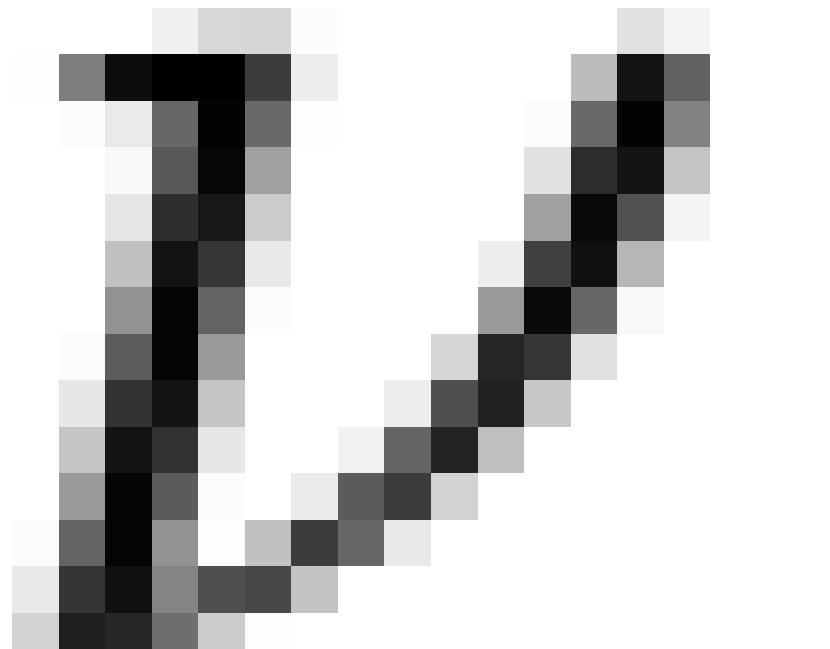


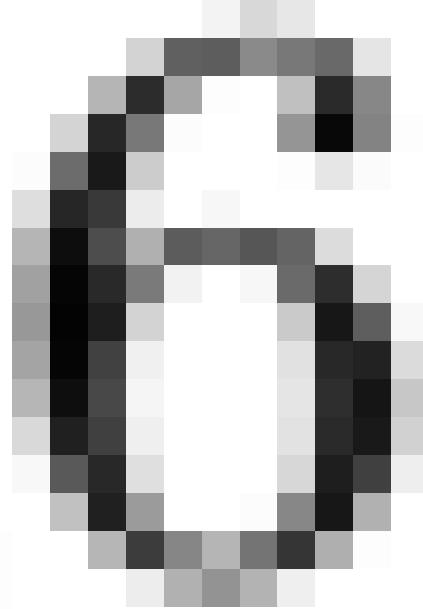
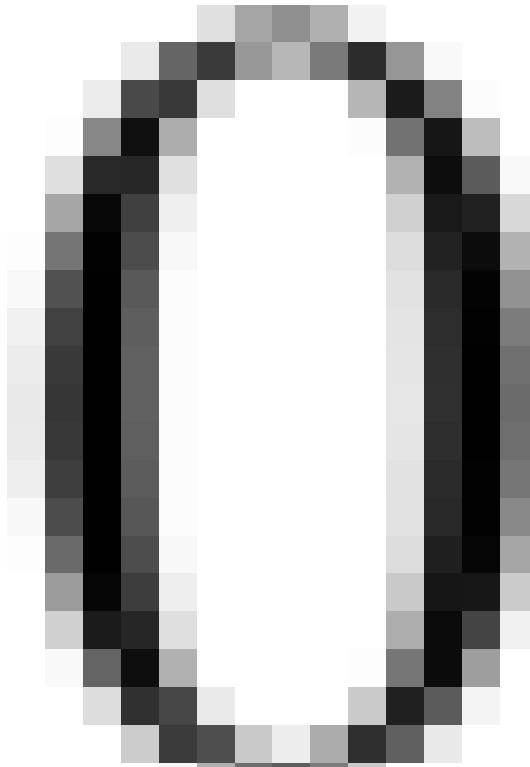
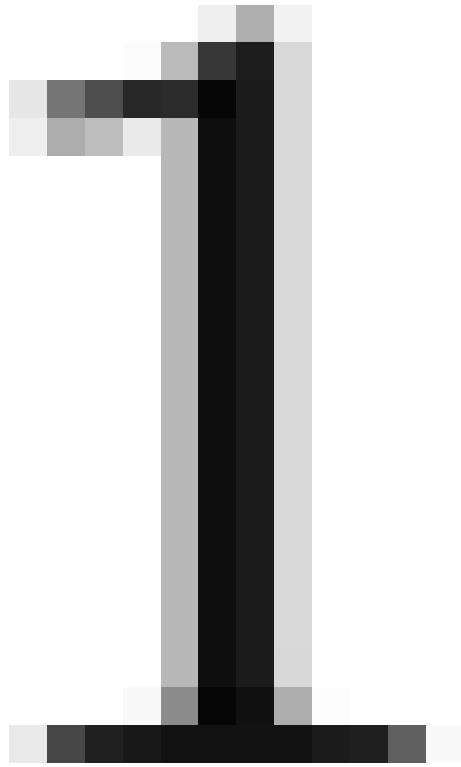
$\langle pp \rangle$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\frac{(1 + \nu)(1 - \nu)}{2\nu} E^\phi$$

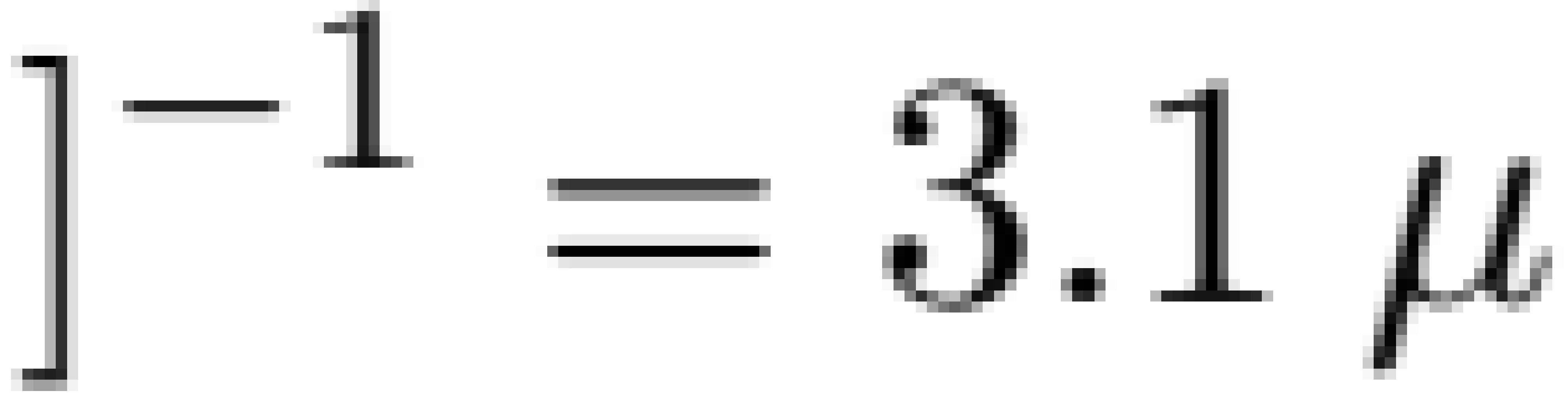


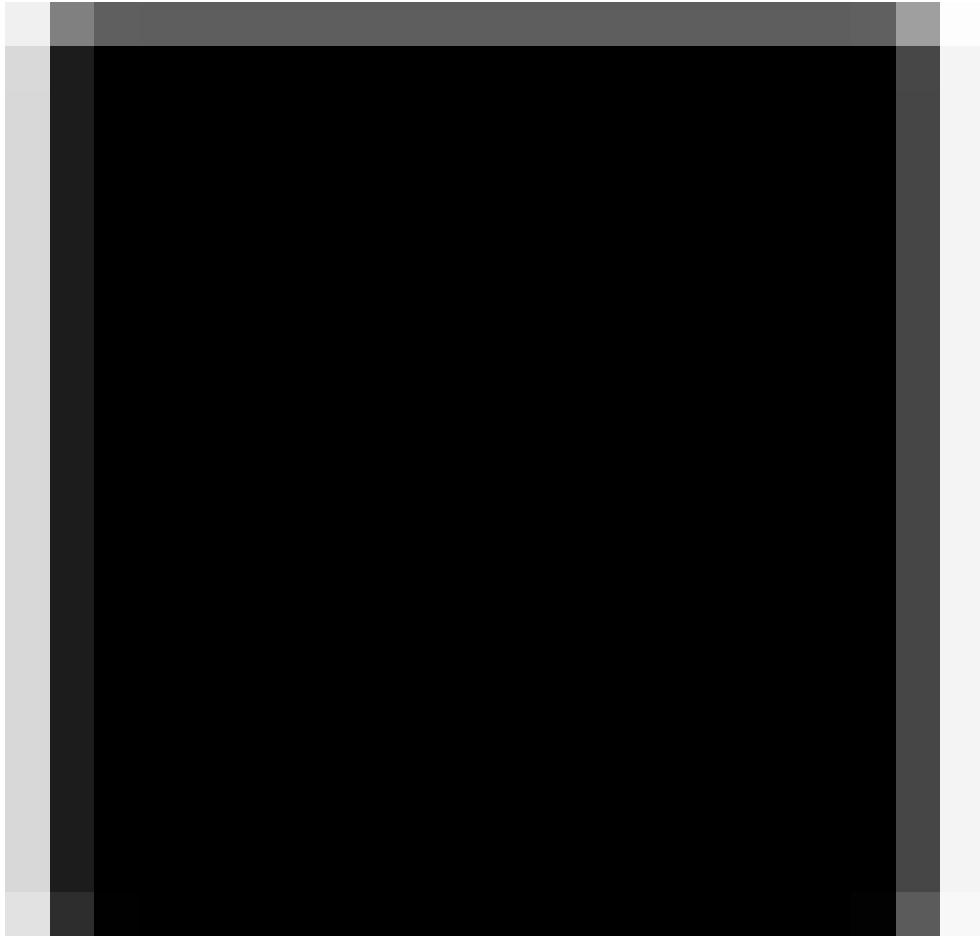


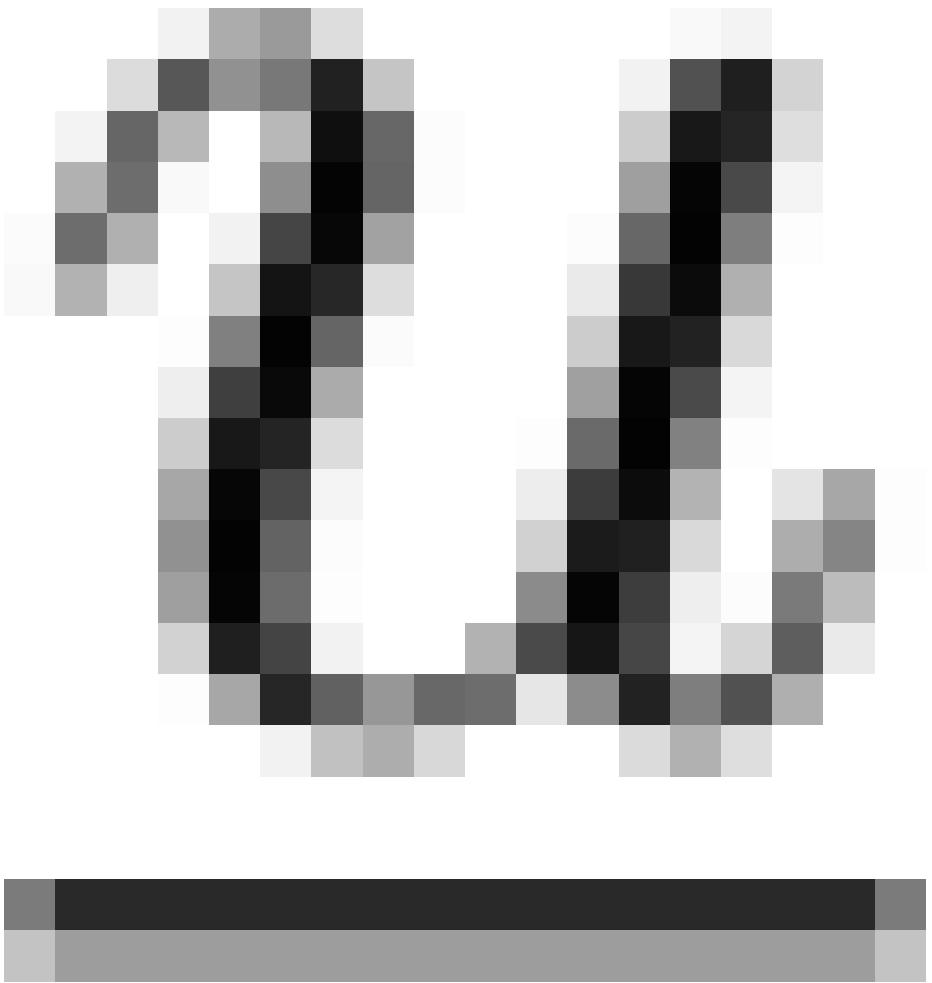


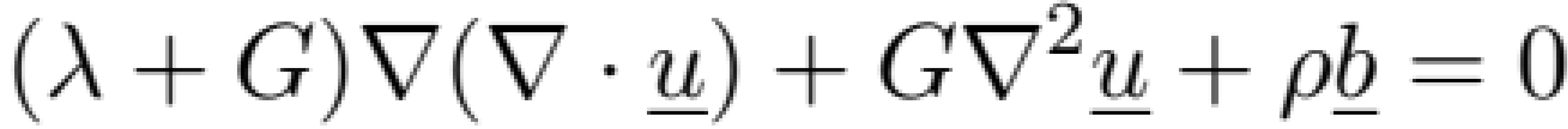
$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.20)10 \text{ GPa}}{1.6 \times 10^6 \text{ psi}} = \frac{11.11 \text{ GPa}}{(1+0.20)(1-2 \times 0.20)}$$

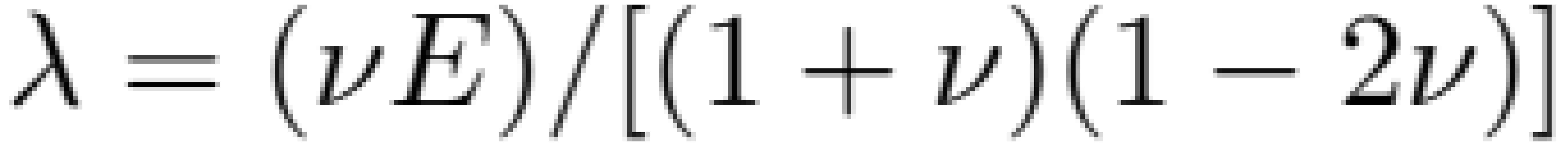
$$\frac{C_{pp}}{M_\phi} = \frac{1}{3.1 \times 10^6 \text{ psi} \times 0.20} = 1$$

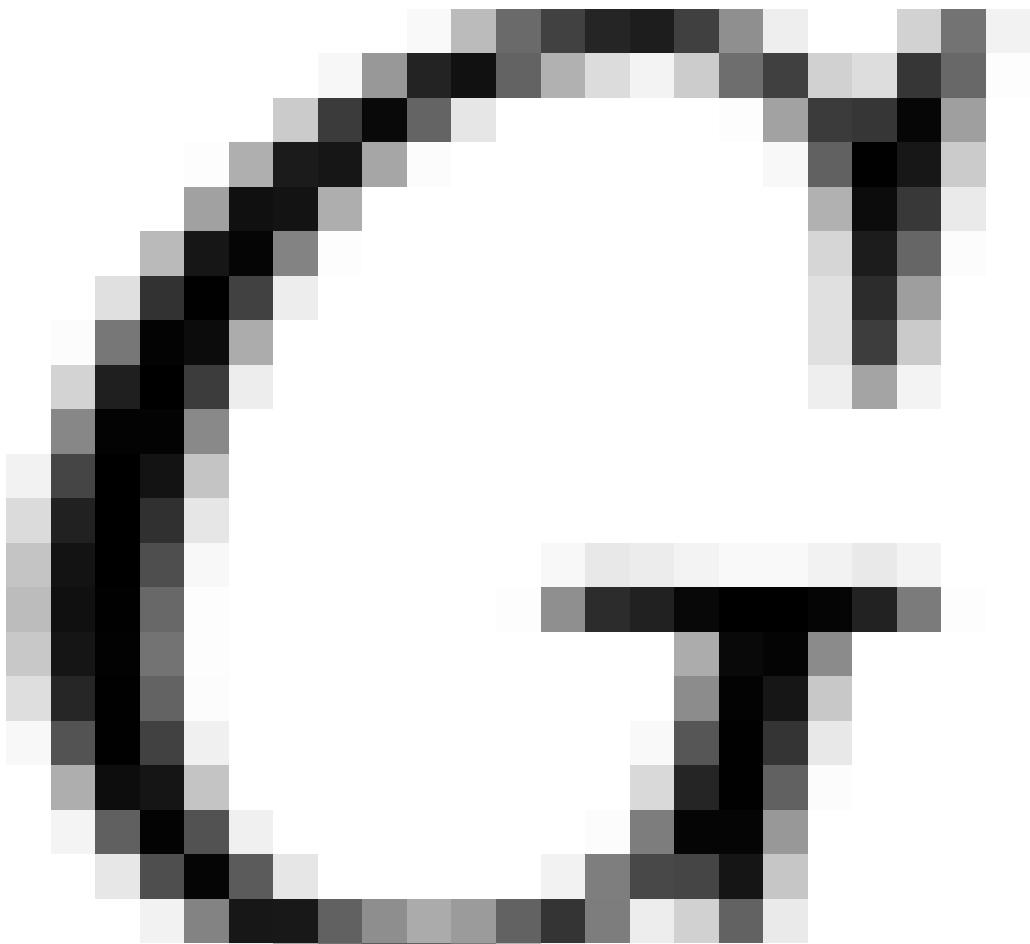


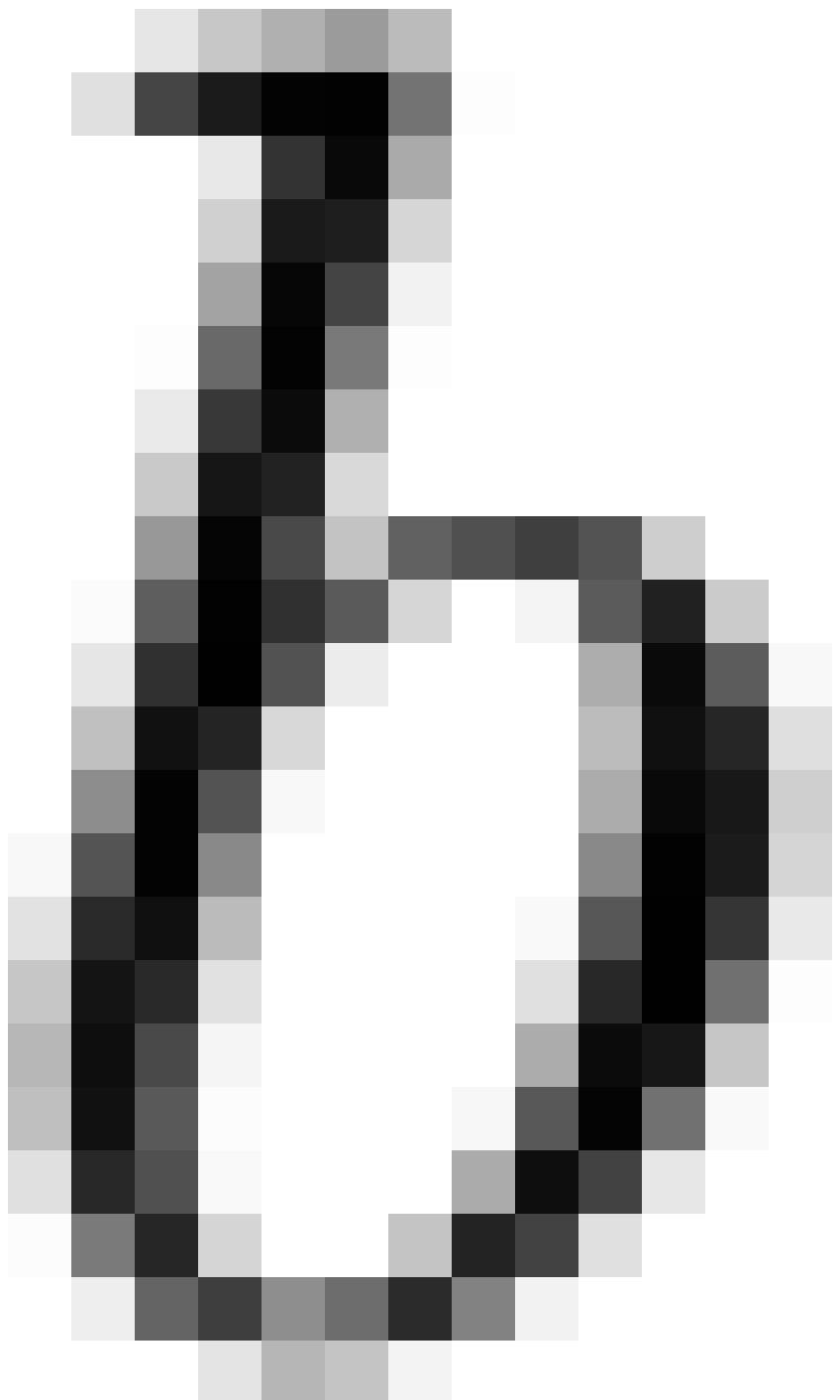




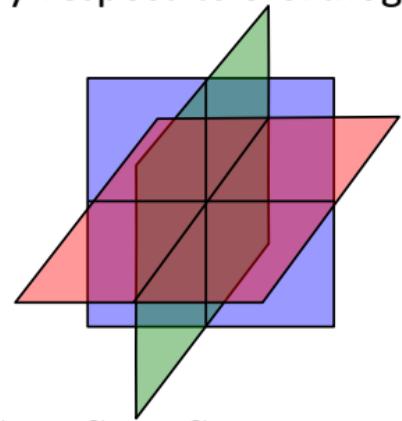








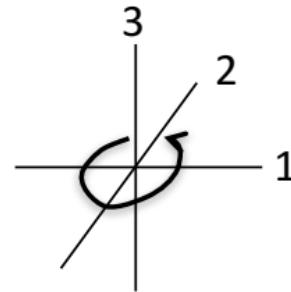
Orthorhombic symmetry
(symmetry respect to 3 orthogonal planes)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & 0 & & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{55} & 0 \\ & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

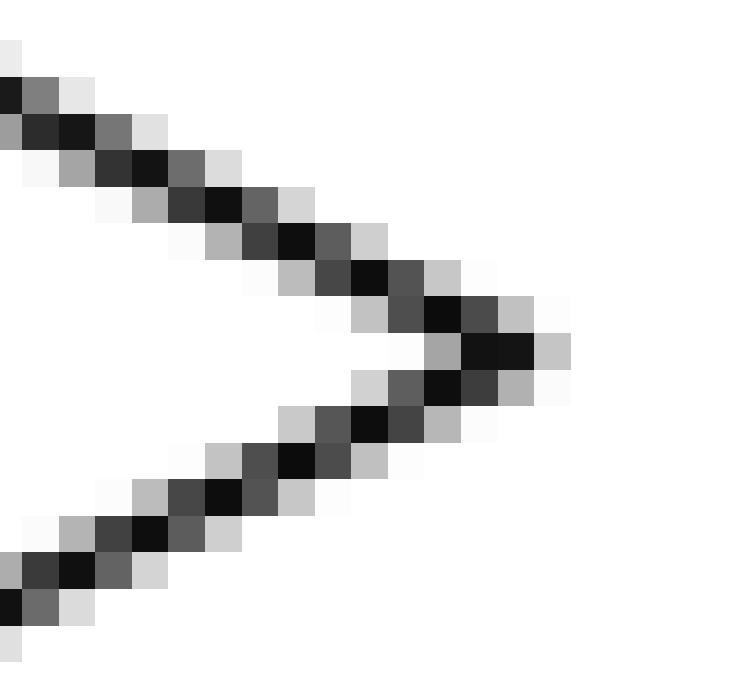
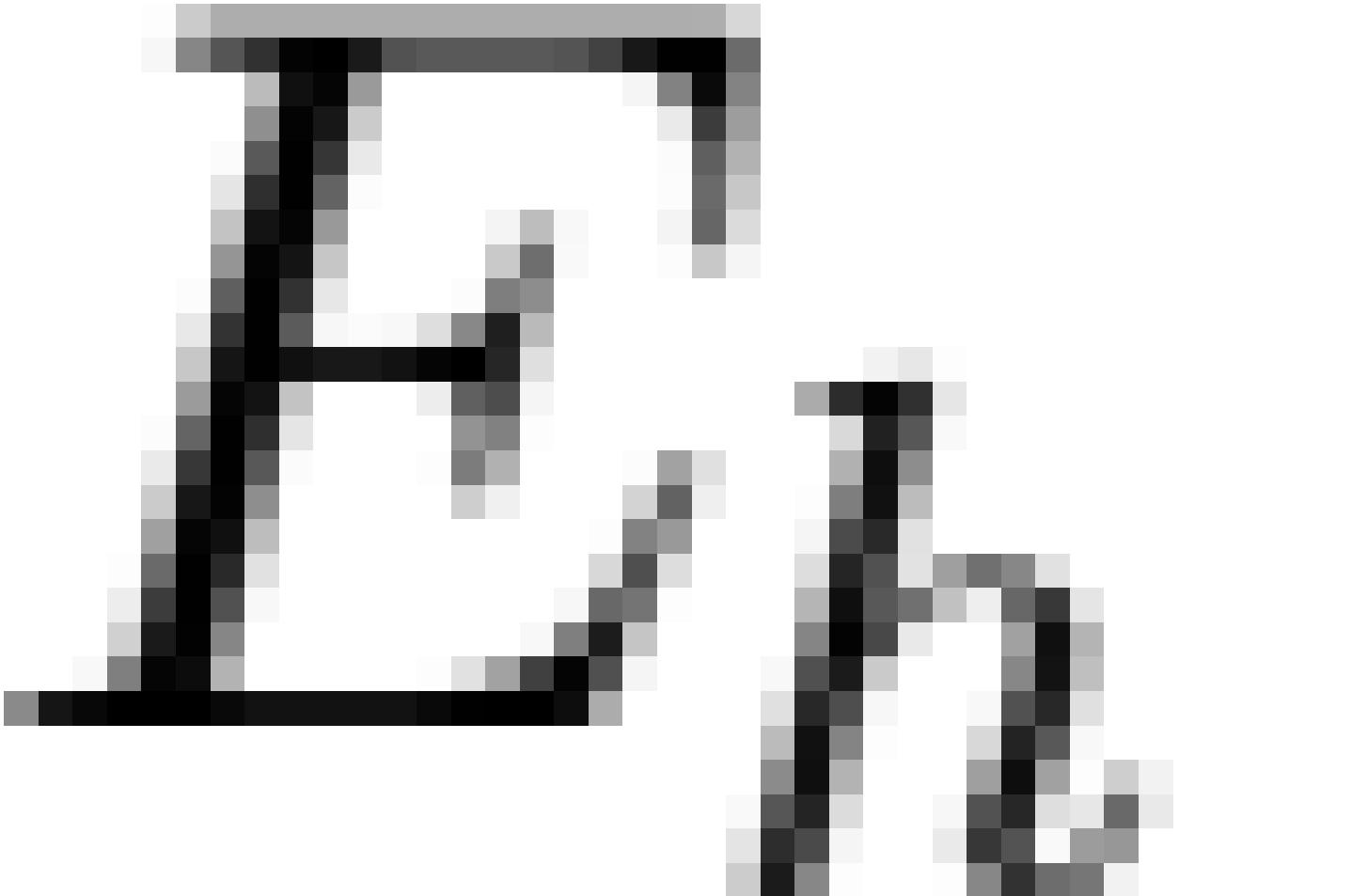
9 independent parameters

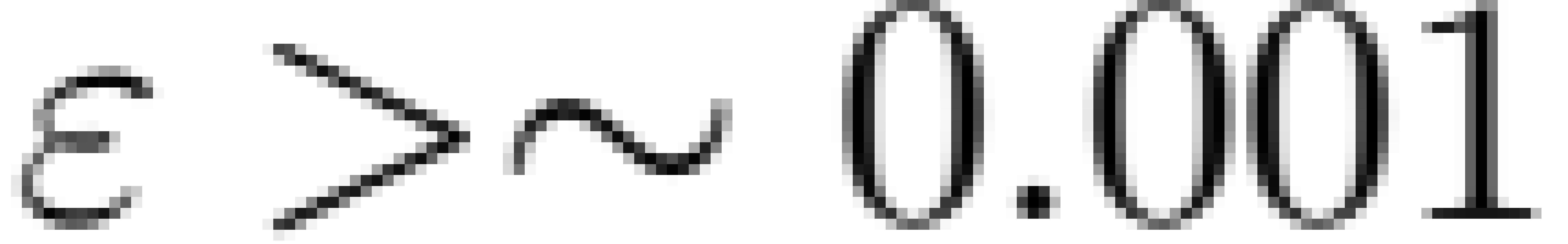
Transverse Isotropy
(symmetry respect to 1 axis)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & 0 & & \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{44} & 0 \\ & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

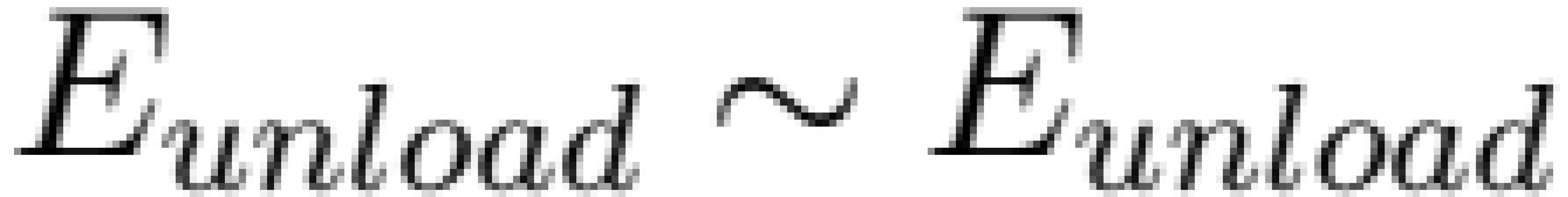
5 independent parameters ($C_{12}=C_{11}-2C_{66}$)



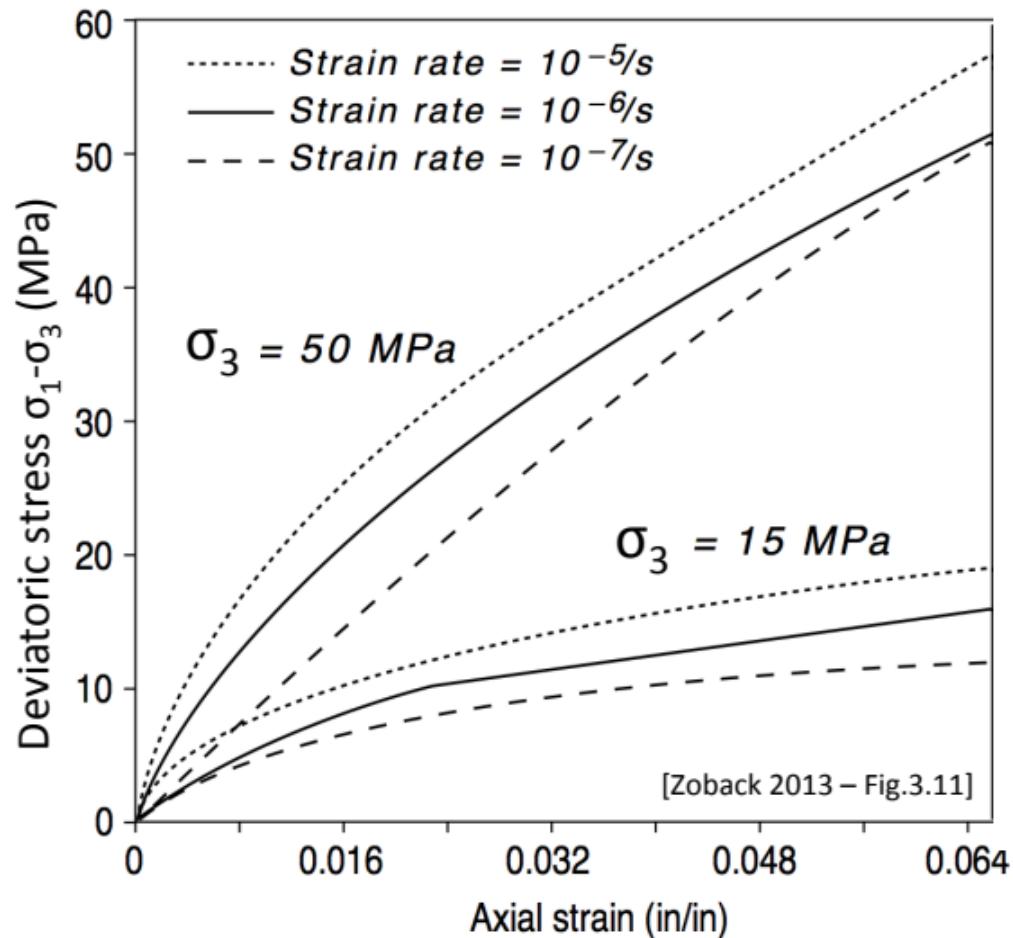
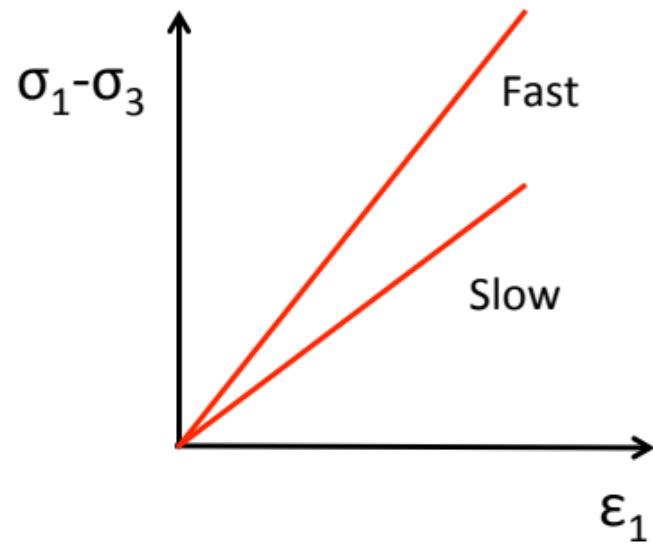




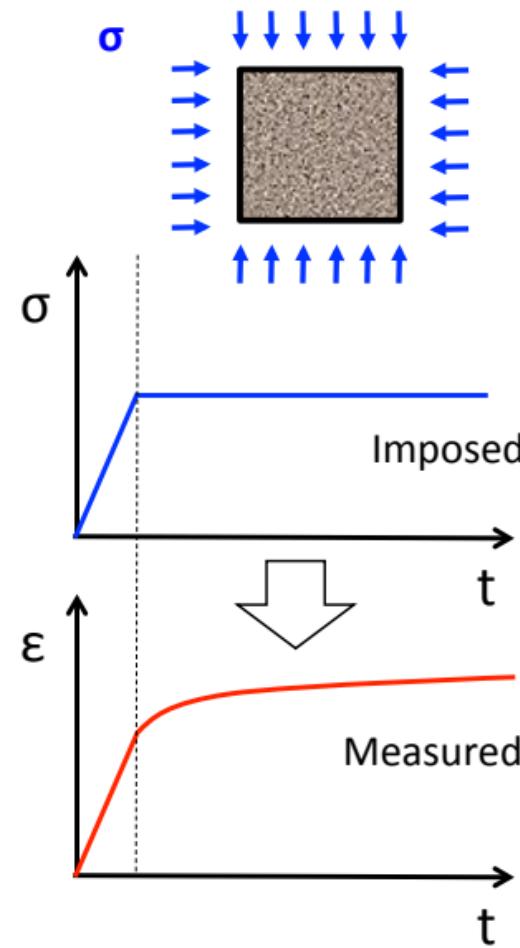




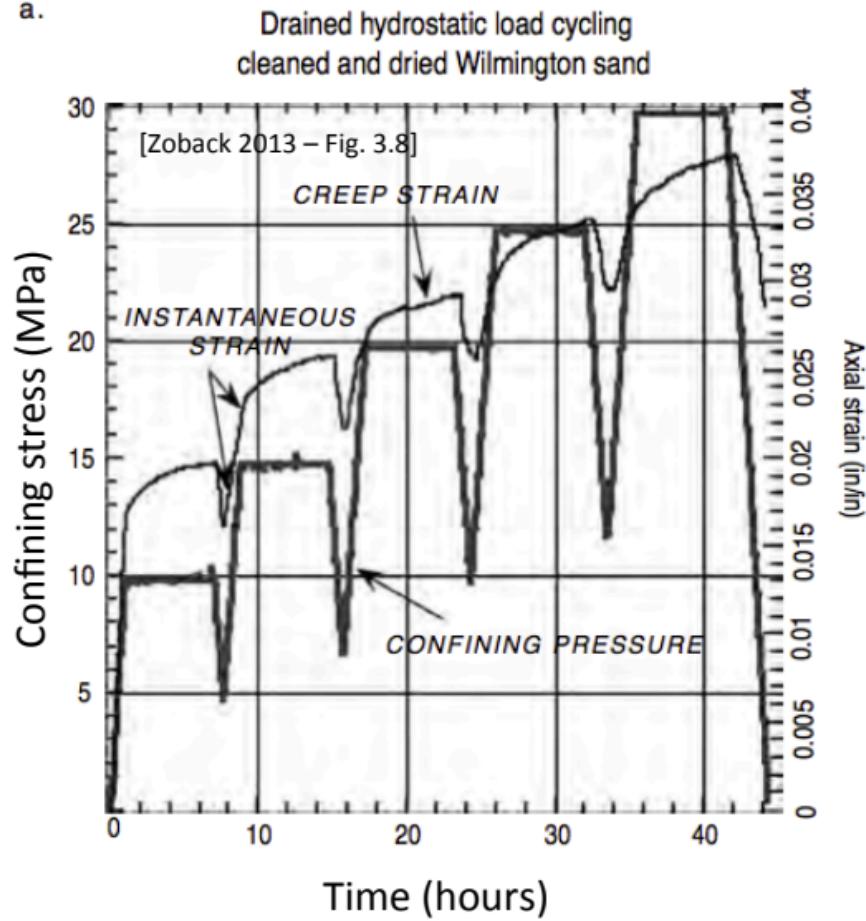
Strain rate hardening: The faster the loading, the stiffer the material



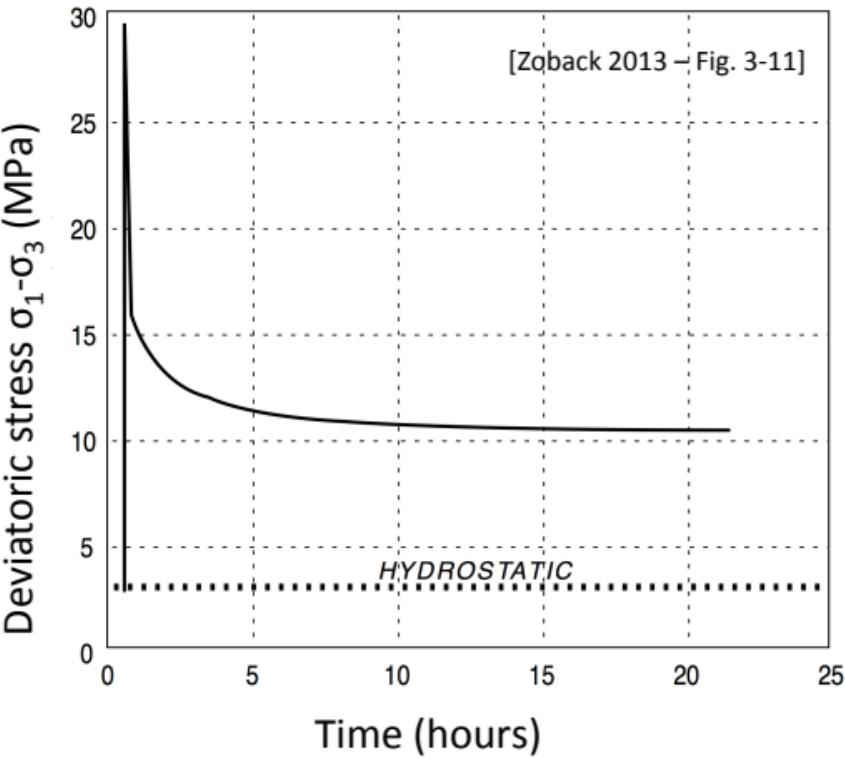
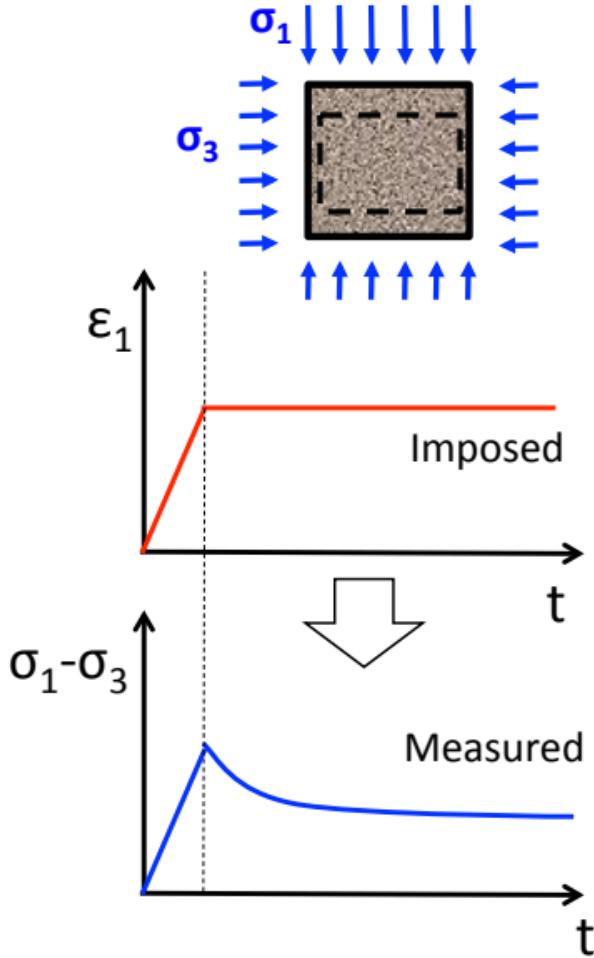
Creep strain at constant stress

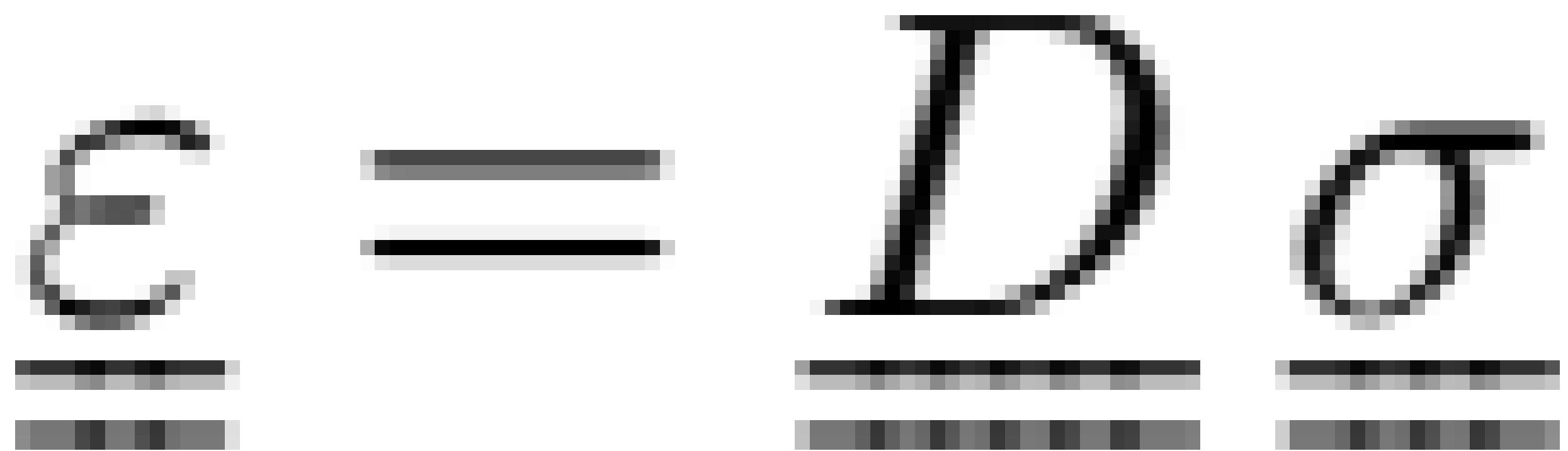


a.

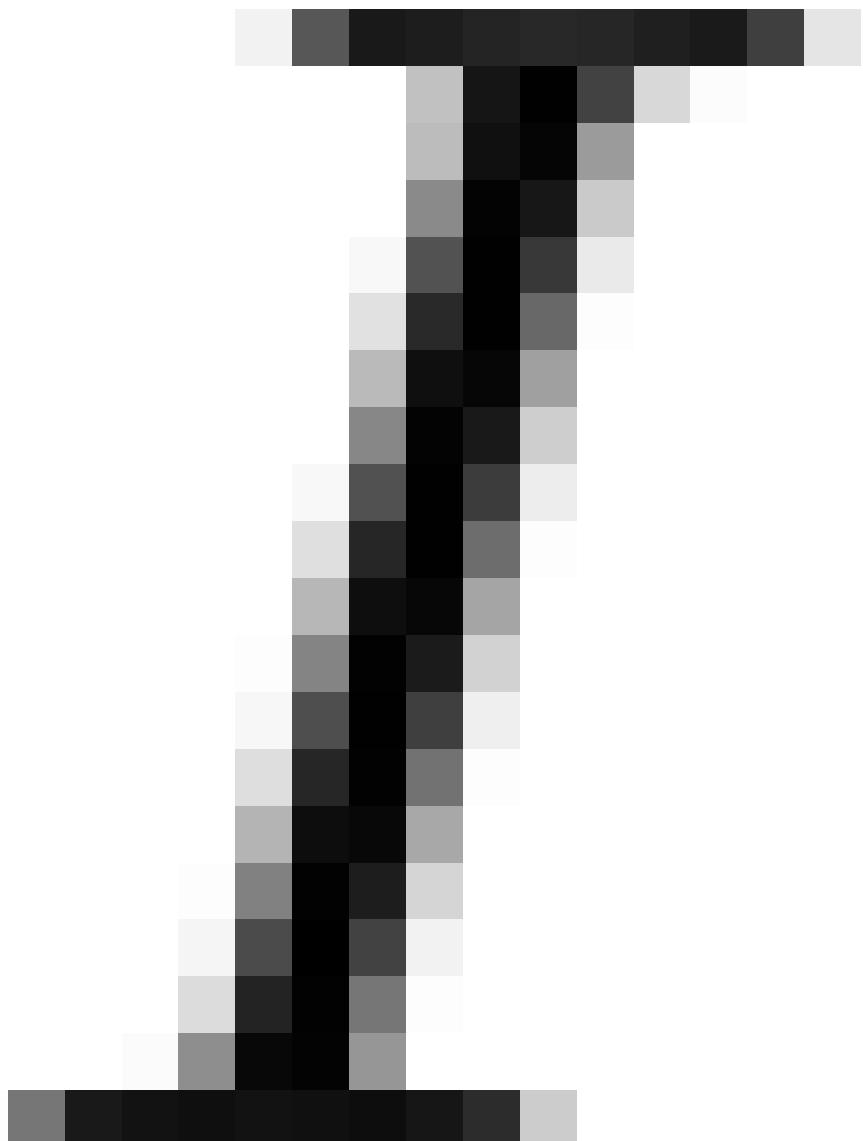


(2) Stress relaxation at constant strain









α

$=$

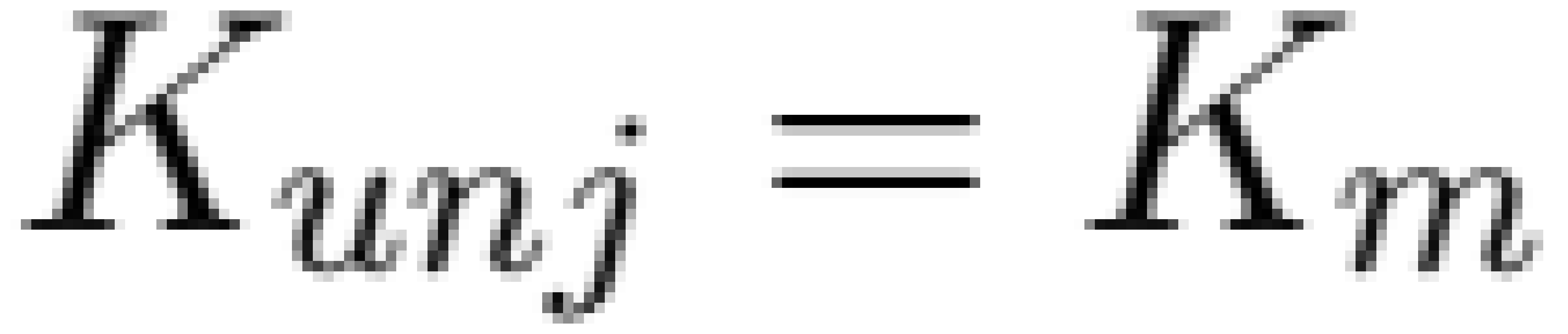
$1 - \frac{K_{drained}}{K_{unq}}$

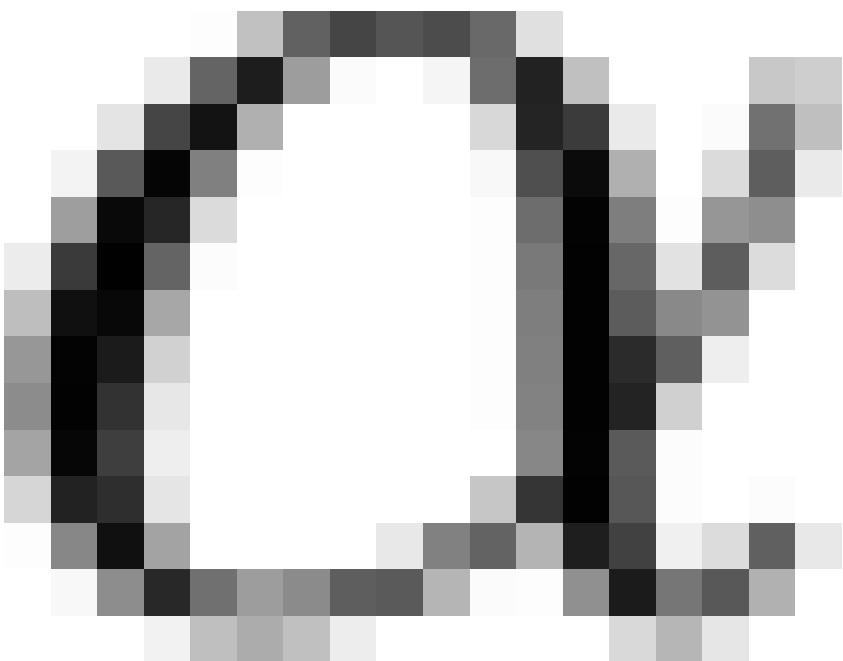
$K_{drained}$

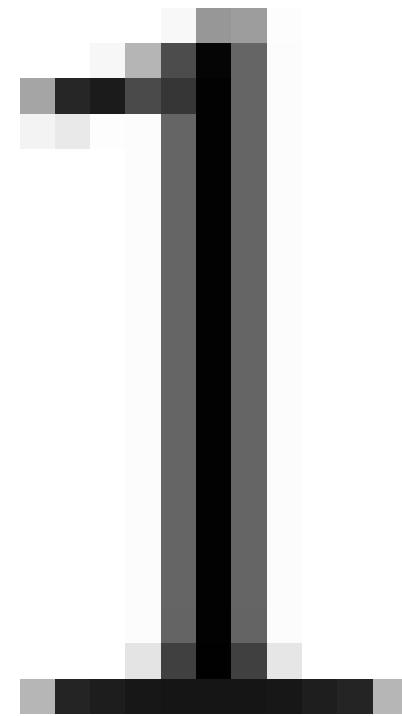
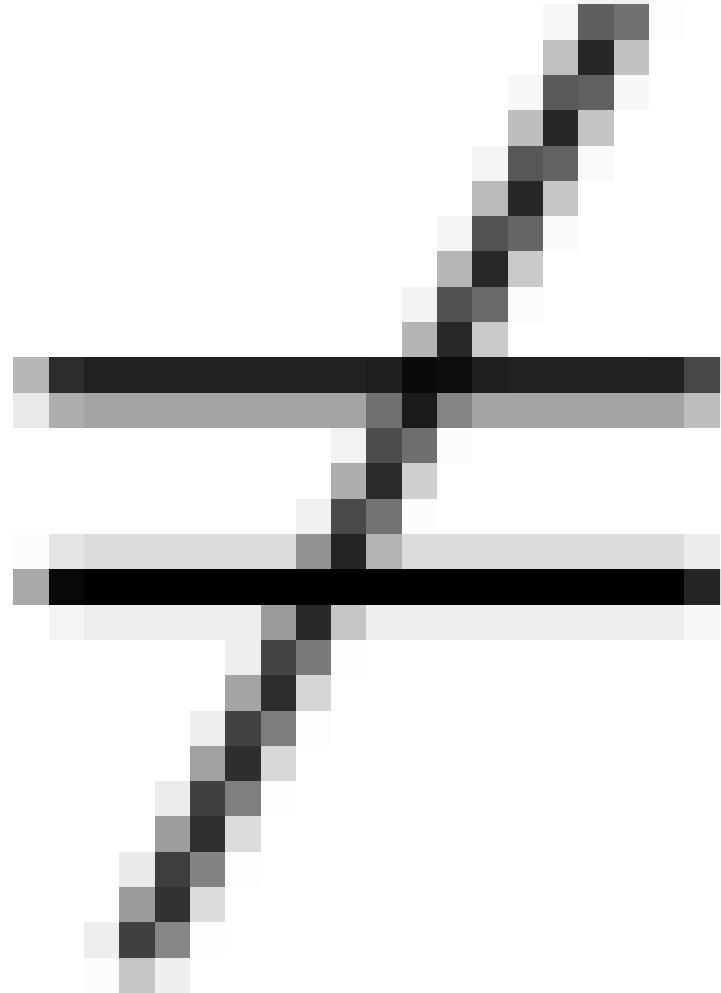
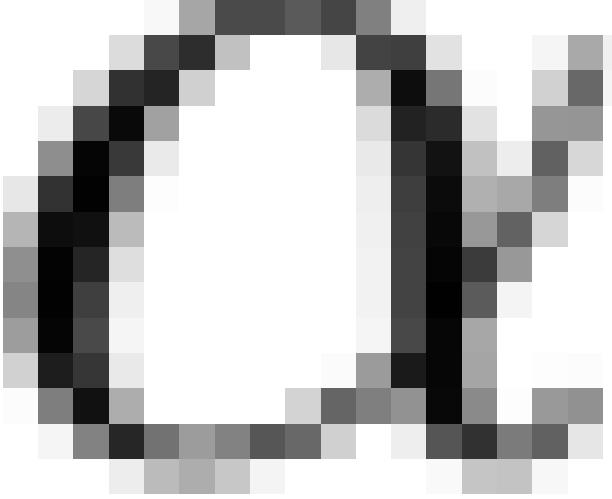
K_{unq}

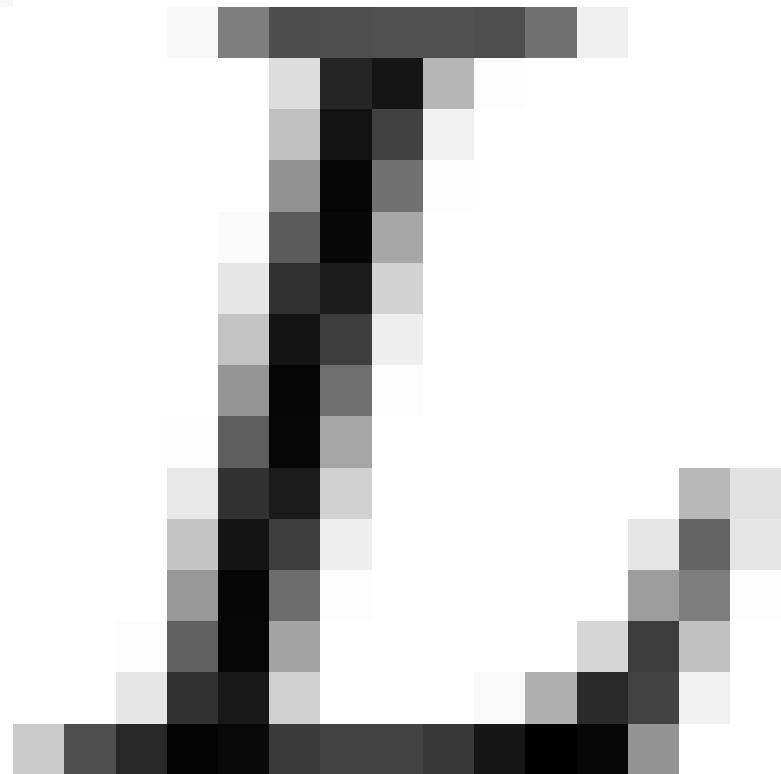
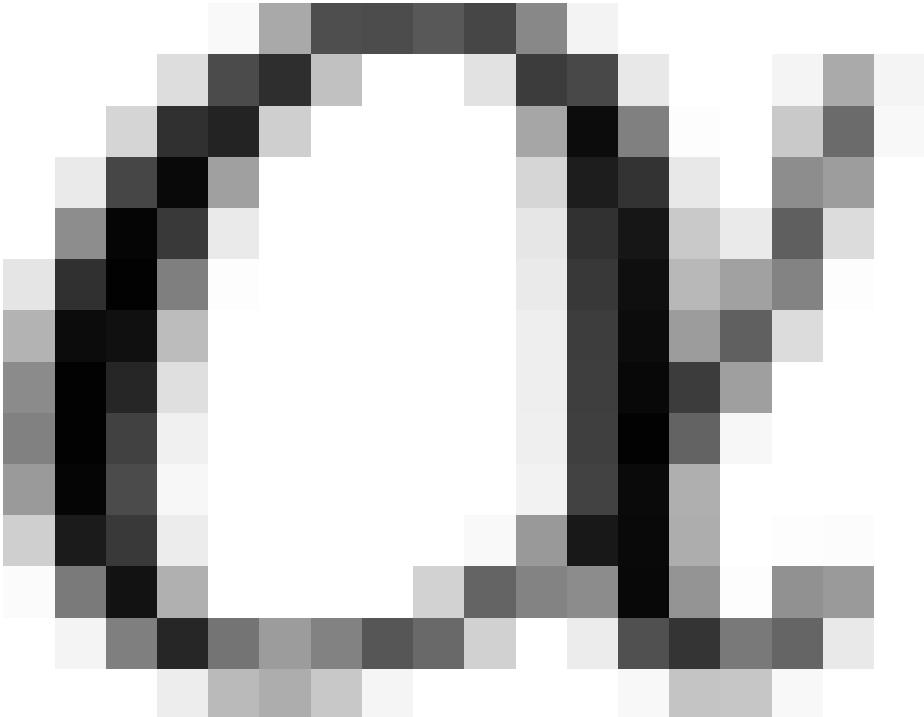


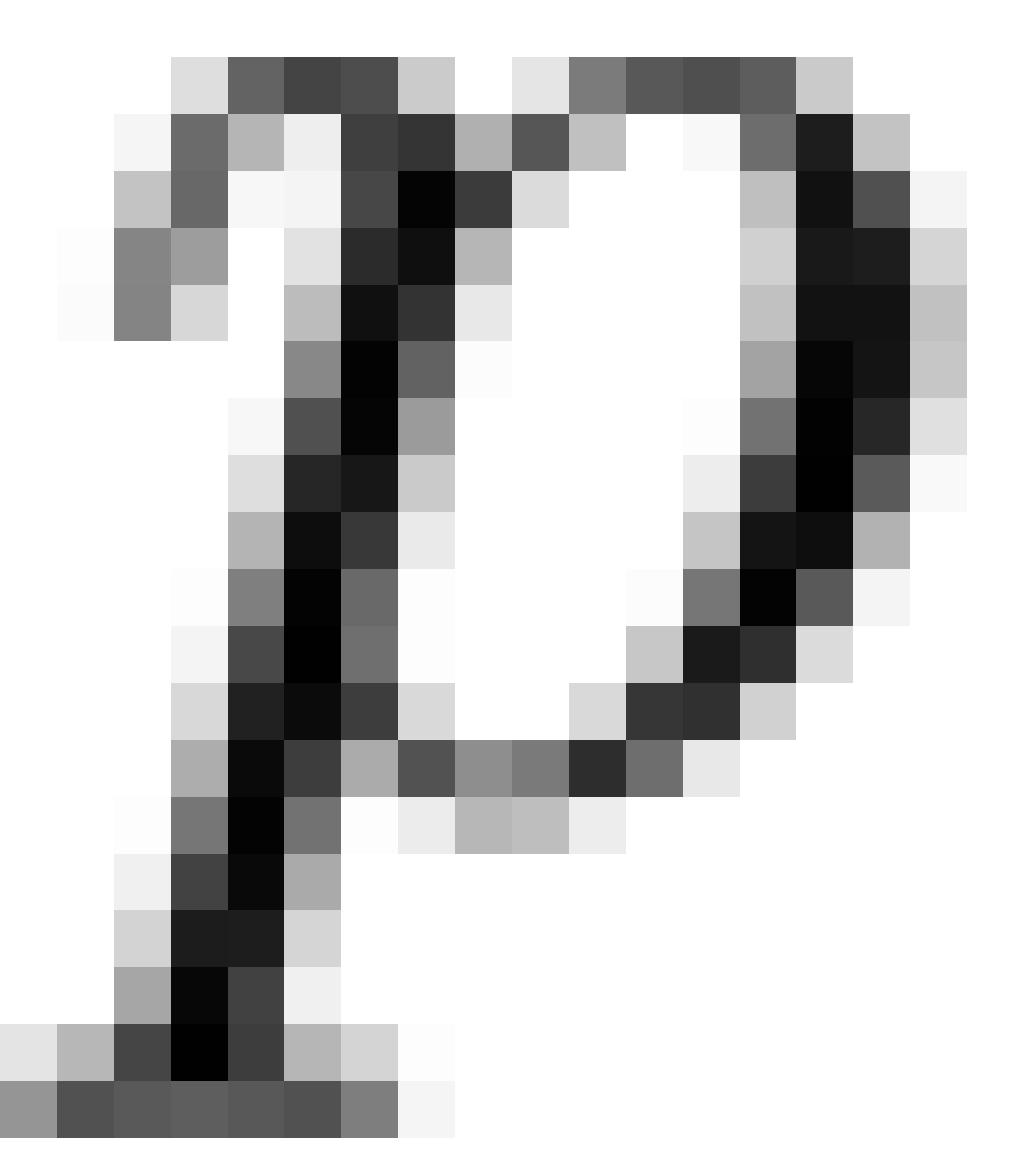




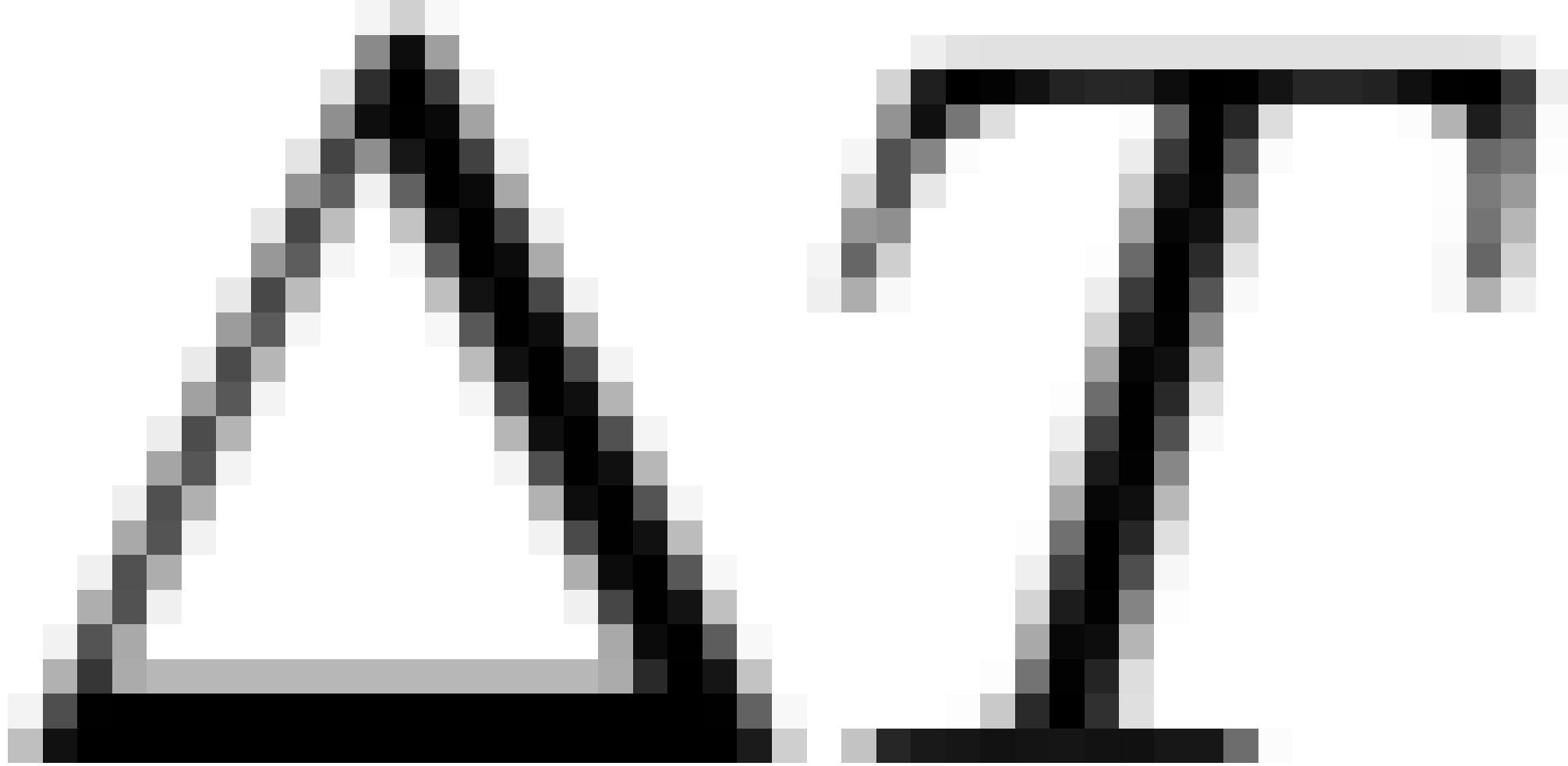




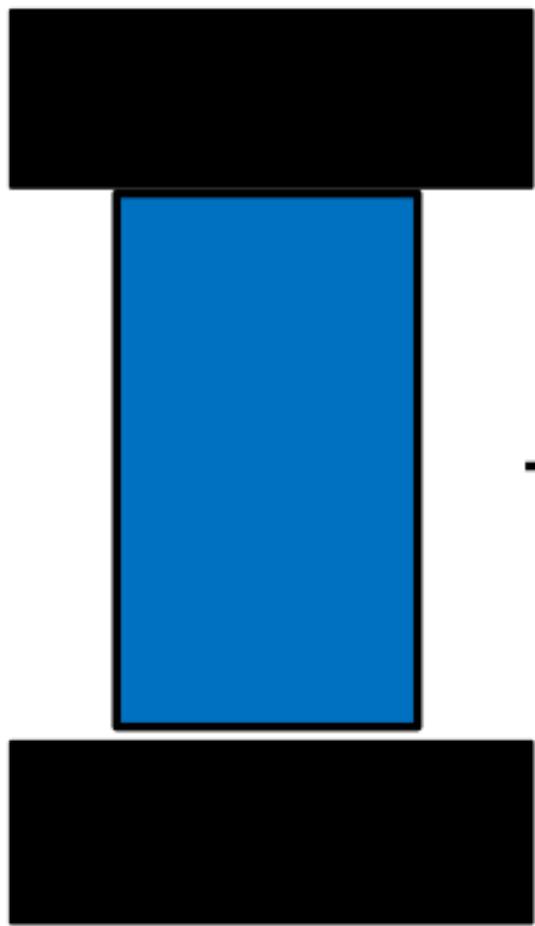




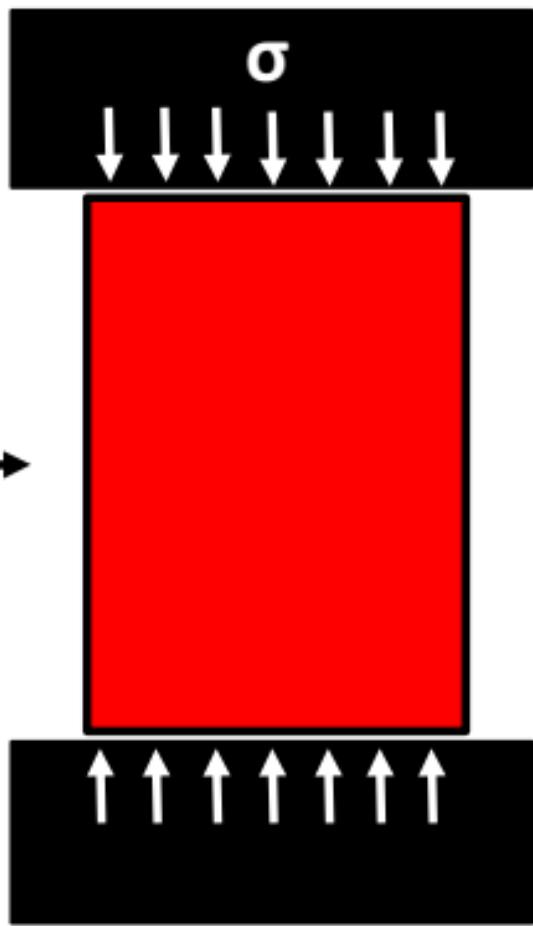
$$\alpha_L = \frac{1}{LDT} \frac{dL}{dp}$$



$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right.$$

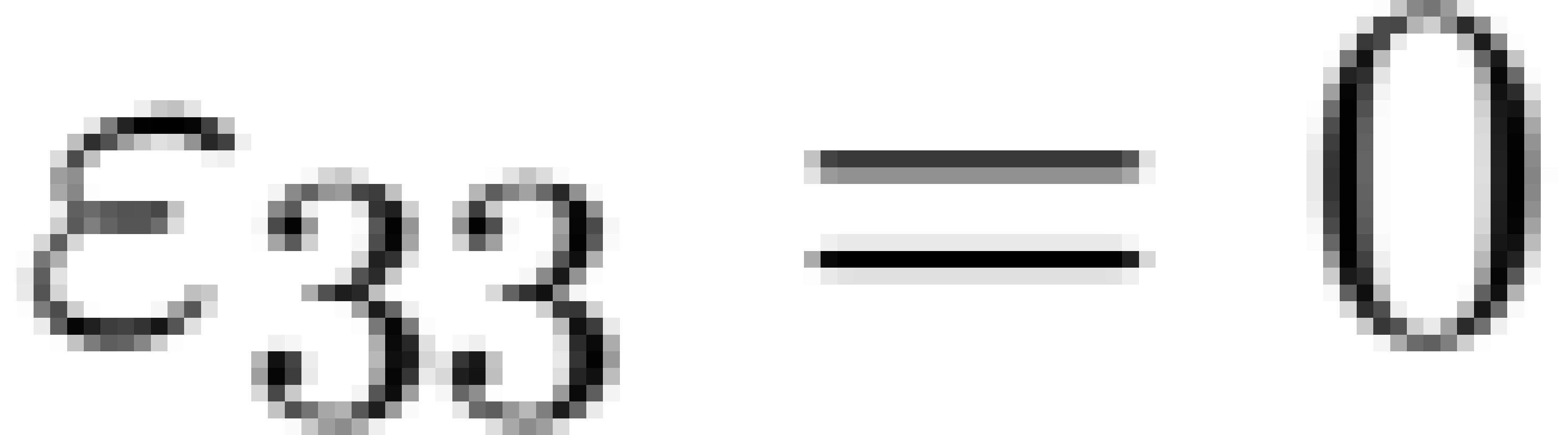


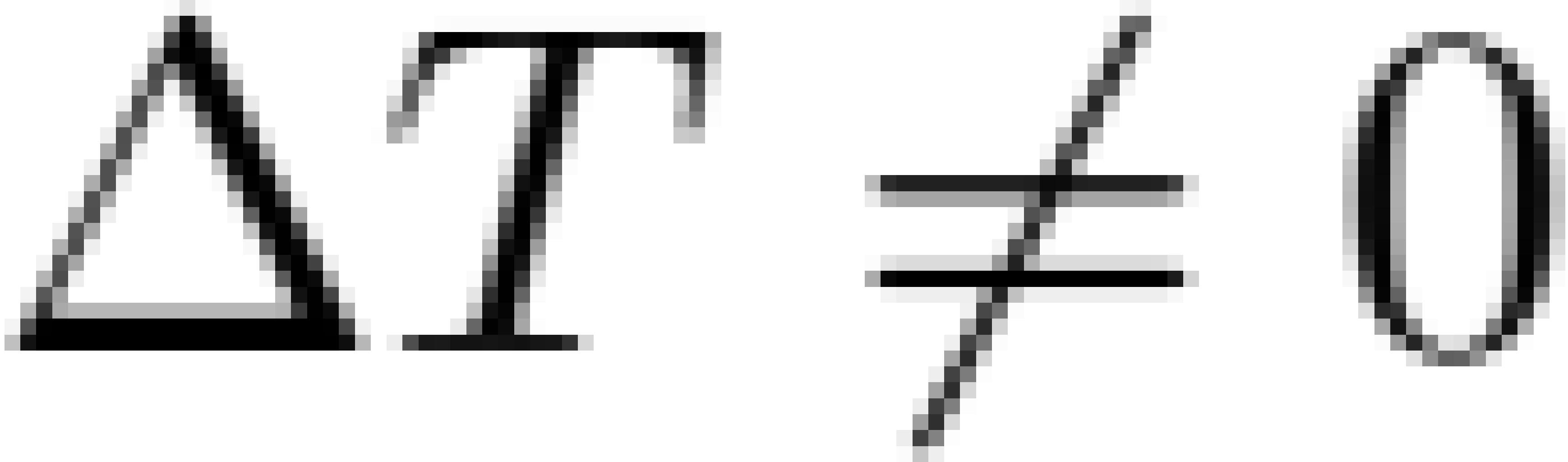
ΔT →











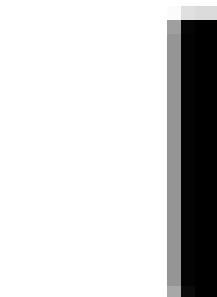
$$\begin{aligned} \sigma_{11}^0 &= (\lambda + 2\mu) \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \\ \sigma_{33} &= \lambda \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \end{aligned}$$

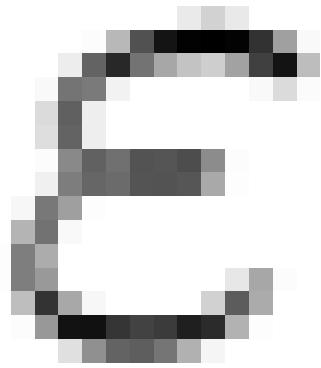
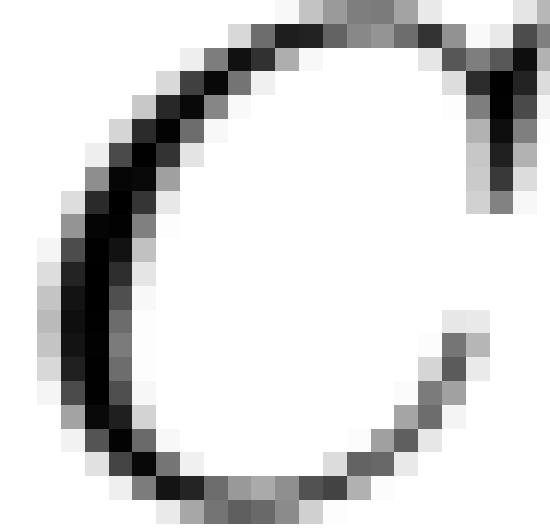
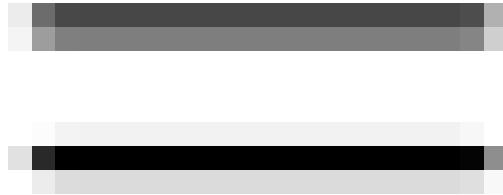
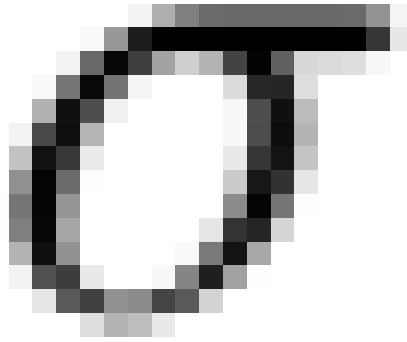
σ_{33}

$=$

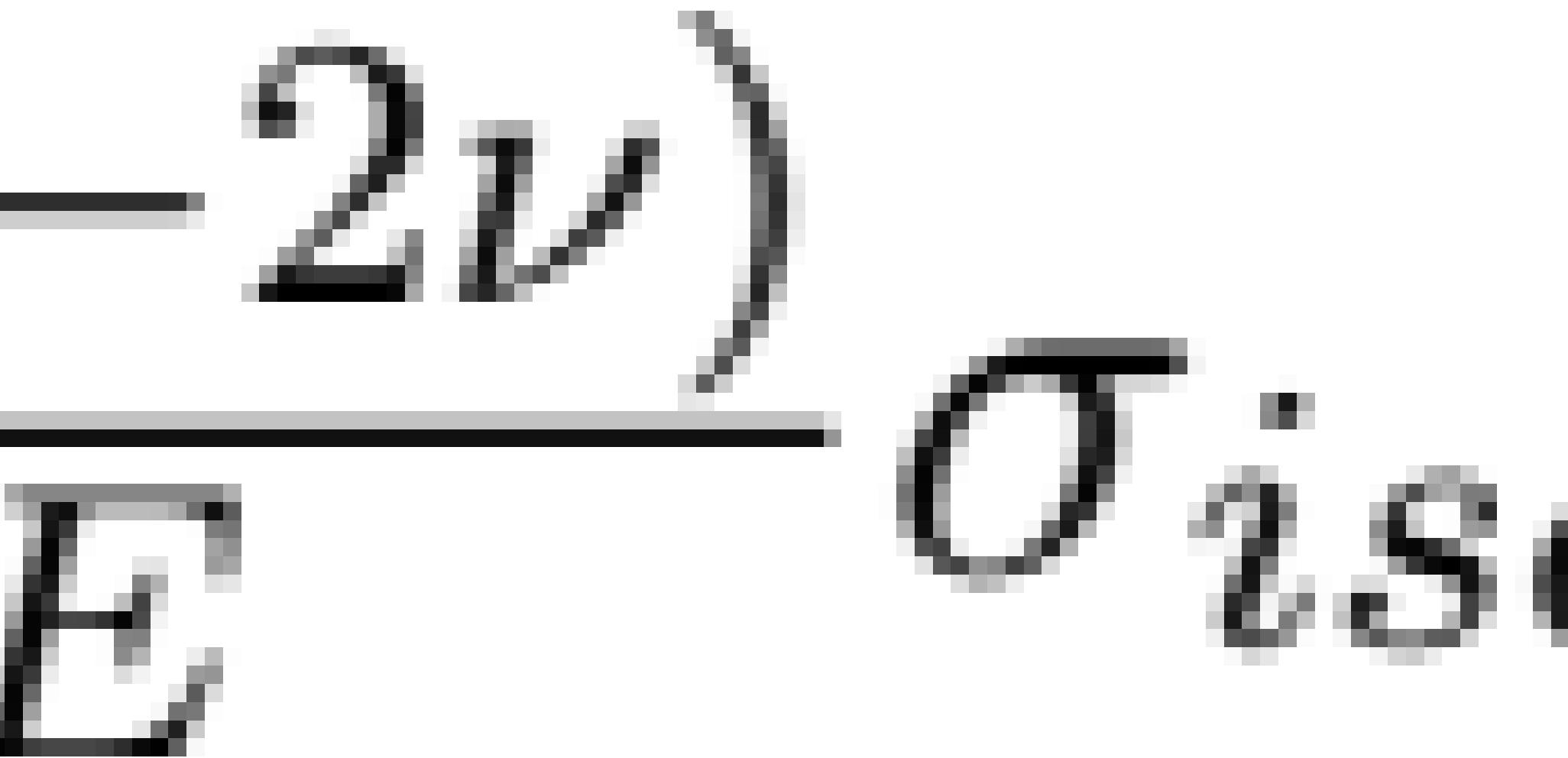
$$\frac{6}{3} \mu K$$

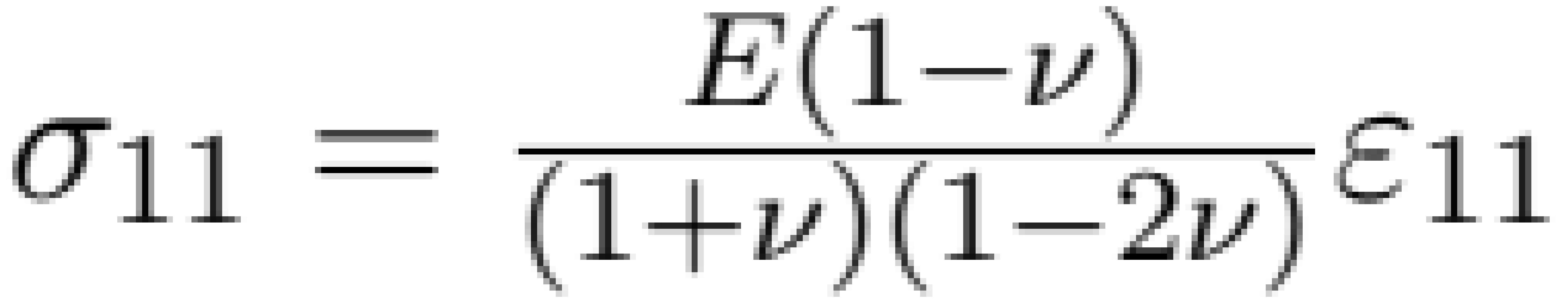
$K_{\Delta T}$













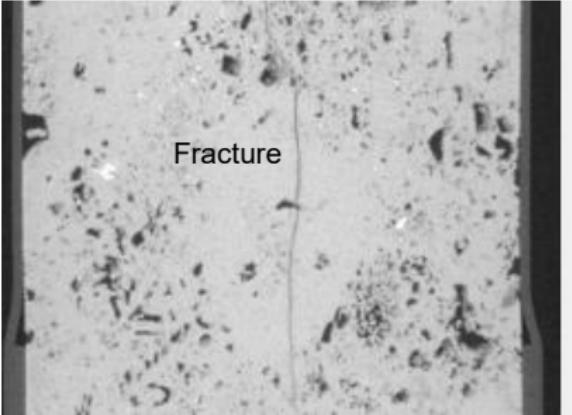
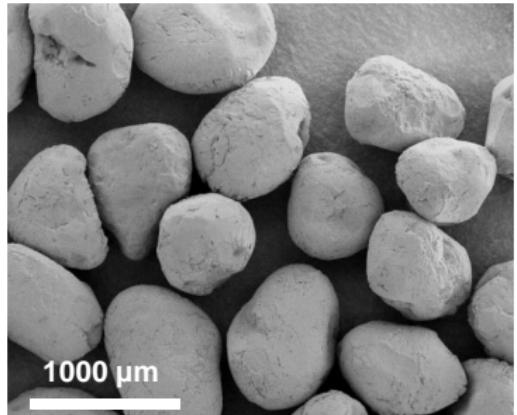
(a) Uncemented sand



(b) Cemented sandstone

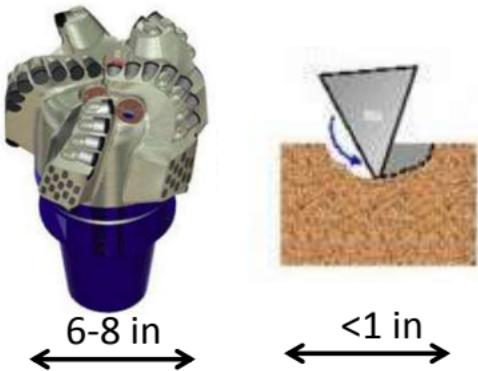


(c) Vuggy carbonate

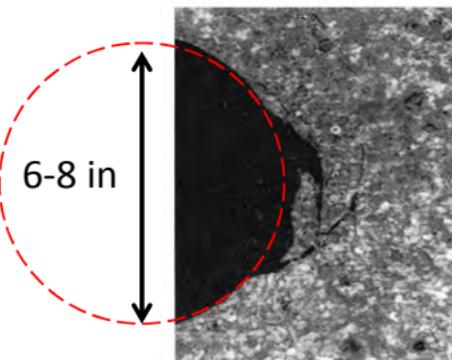


Fracture

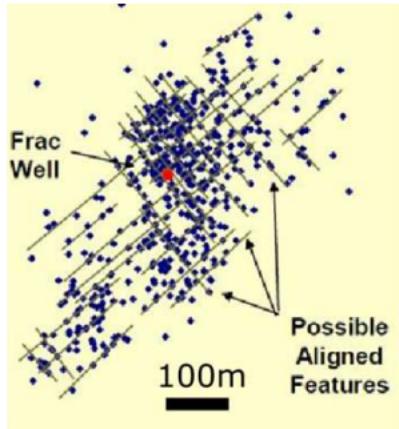
Rock cutting at the drill bit



Wellbore breakout



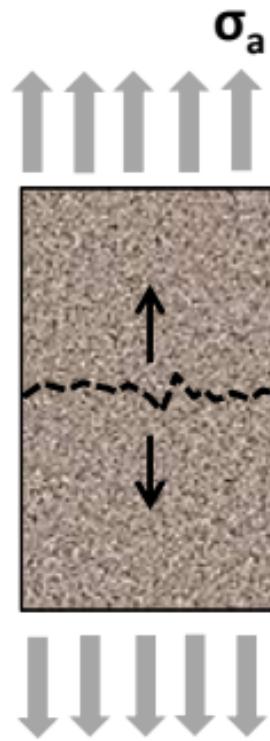
Shale hydraulic fracture



Reservoir depletion

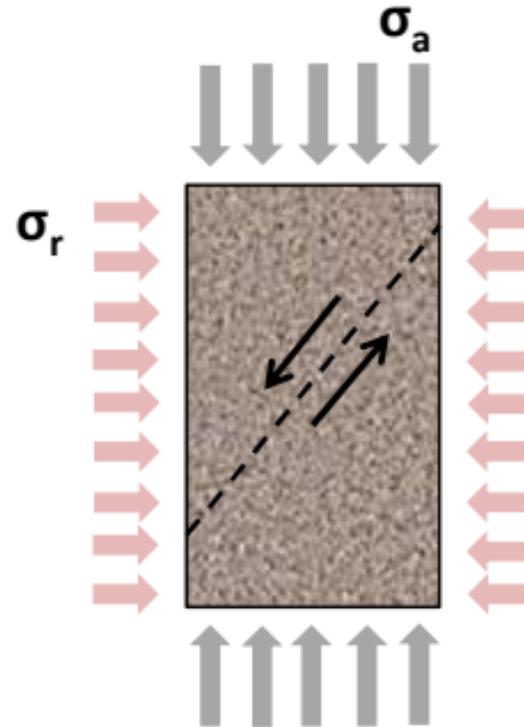


Images: Schlumberger/Terratek, Zoback 2013, Warpinski 2008, doe.gov

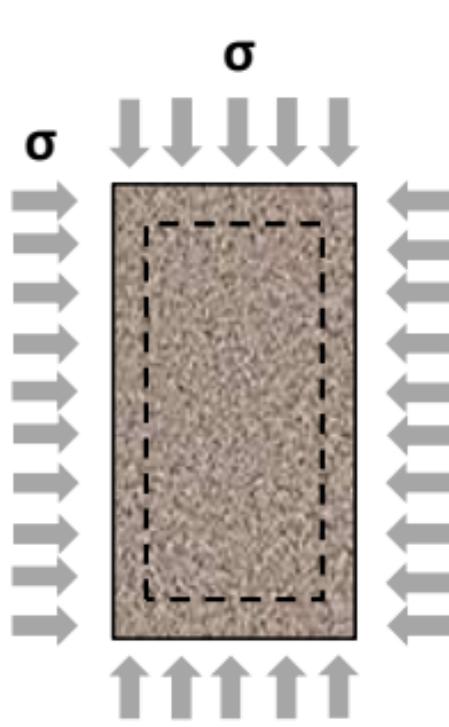


Tension
(bond breakage)

Ex: drilling-induced tens. fracs



Shear
(friction failure)
Ex: fault, breakout

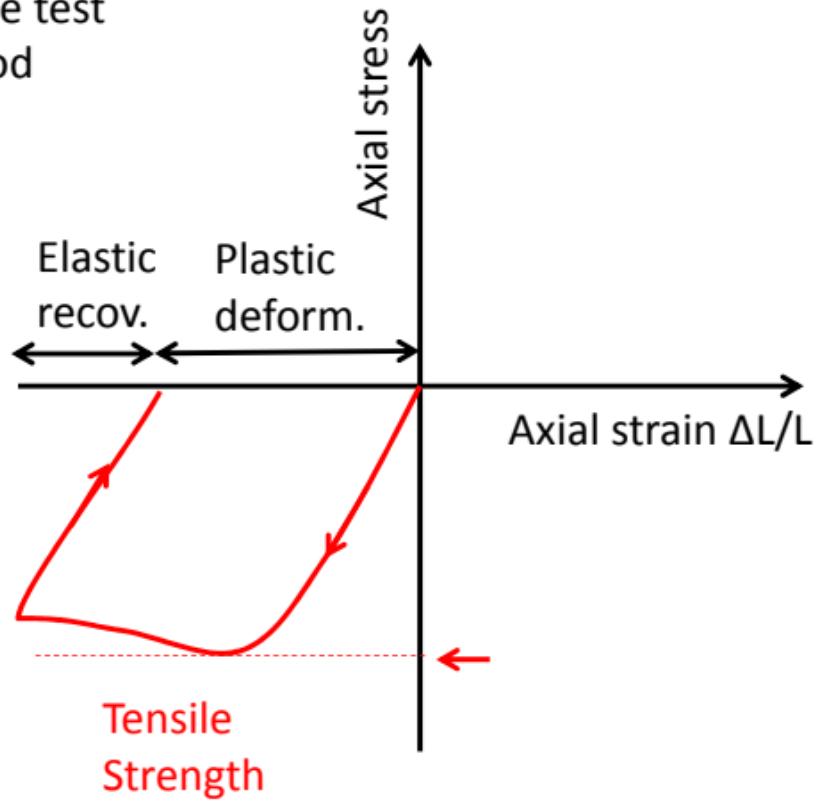


Compression
(pore collapse)
Ex: reservoir compaction

T



Typical tensile test
on a metal rod



Tensile test on a rock rod

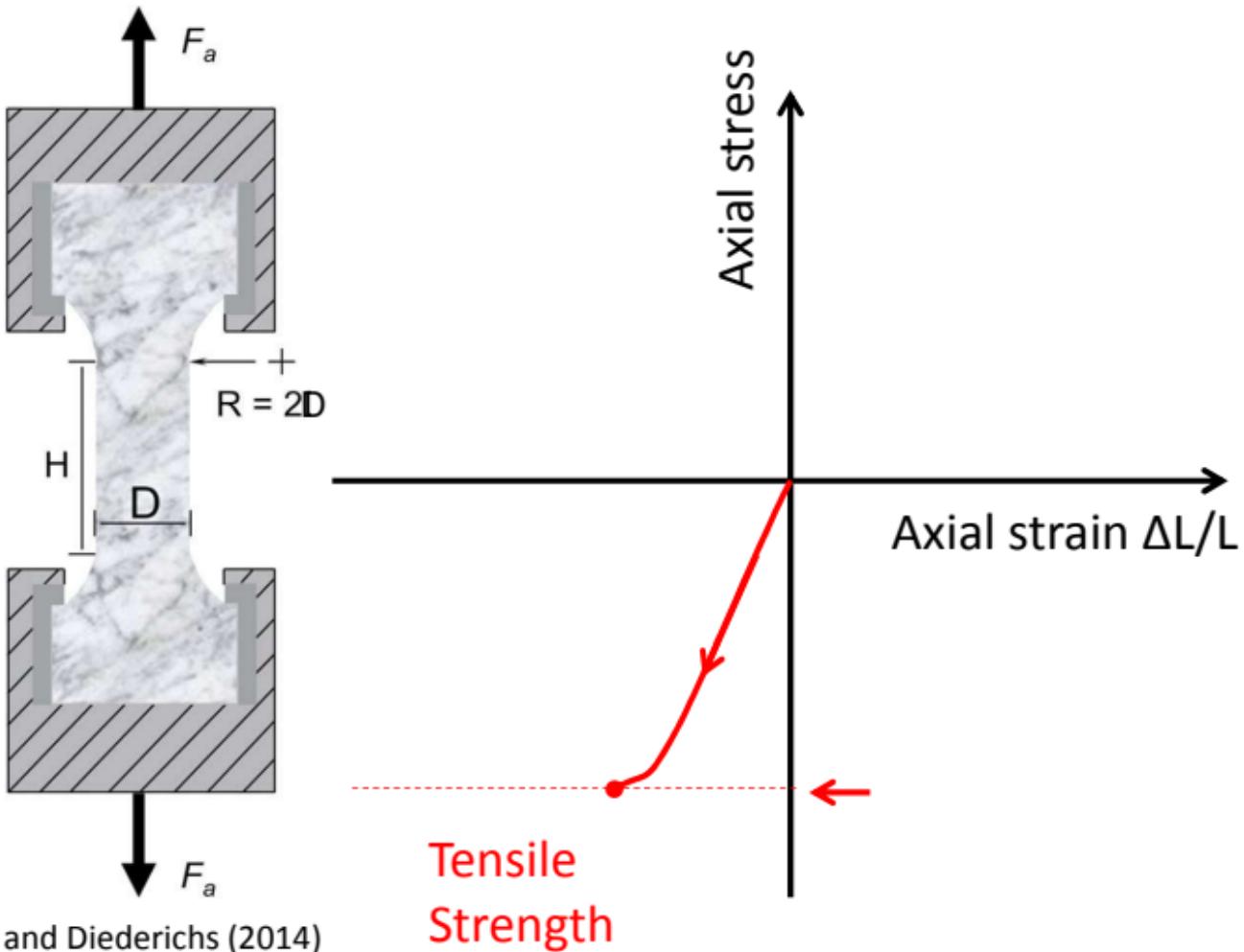


Figure direct tension: Perras and Diederichs (2014)

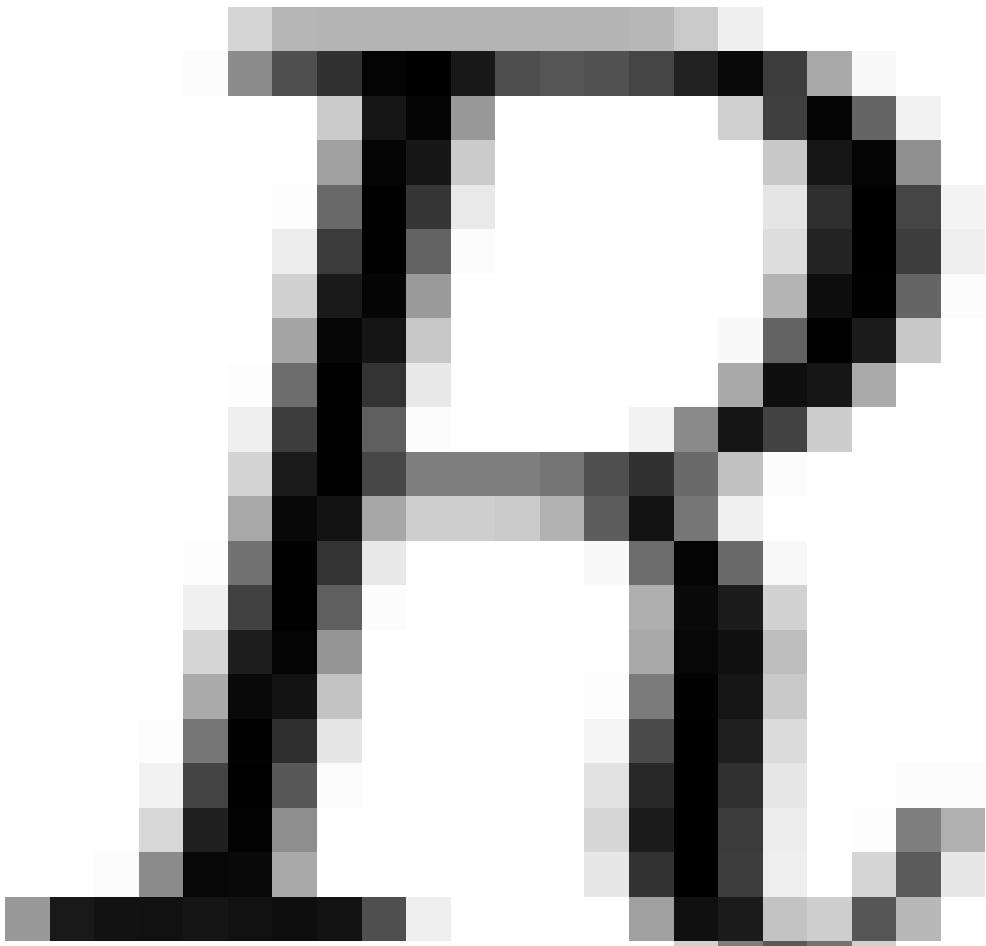
IS

—
—

πLR

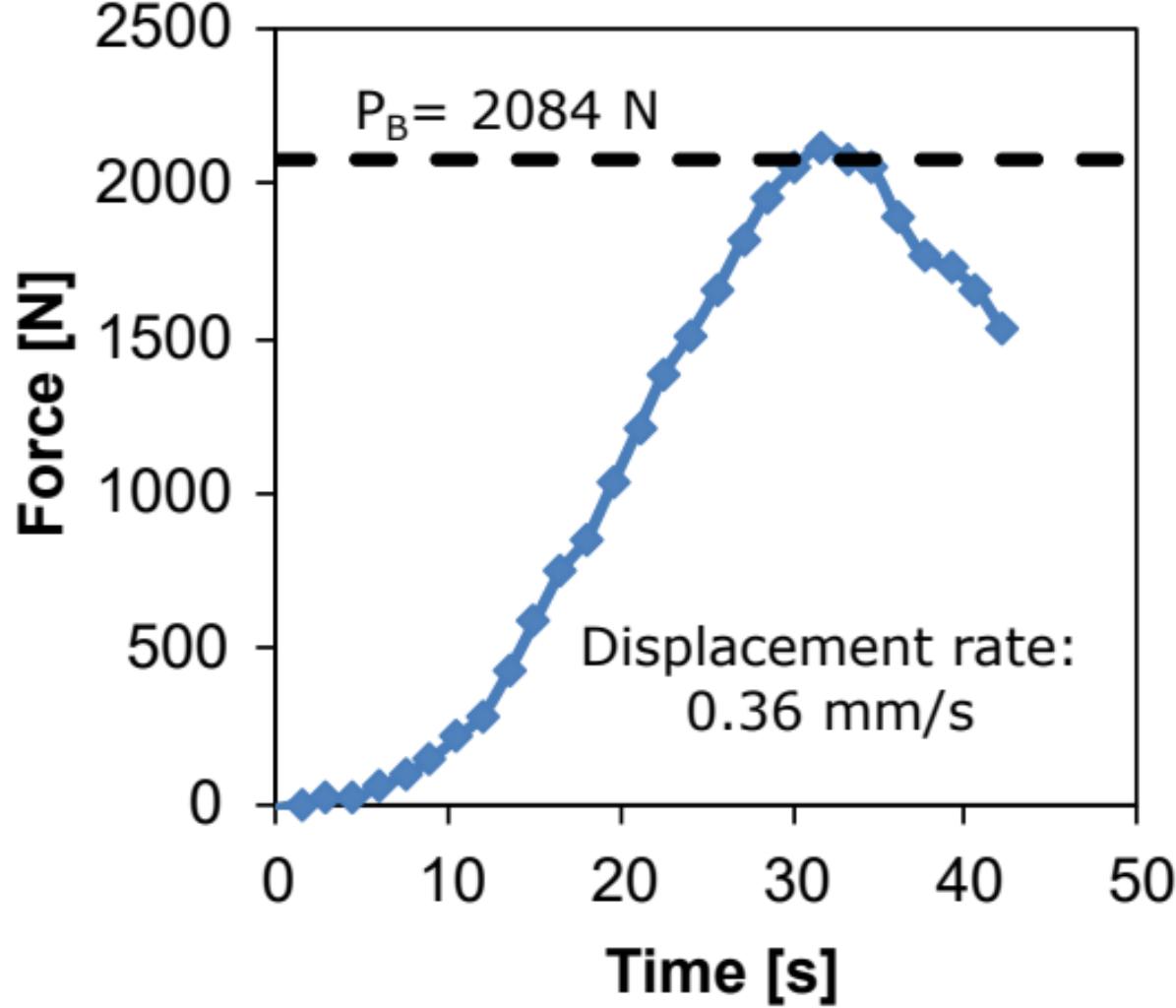
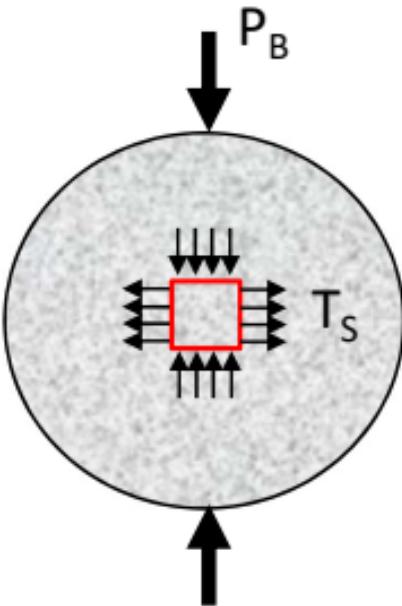
PB





Cylindrical sample:

- radius R
- length L

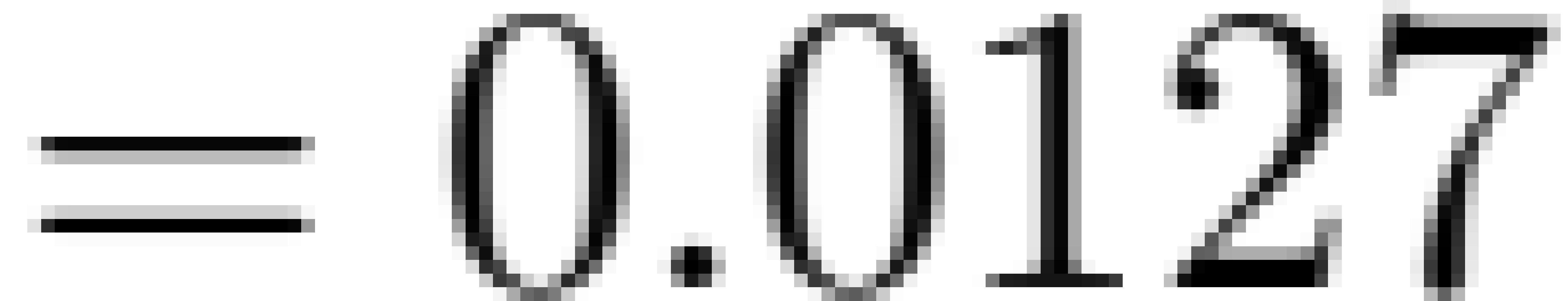


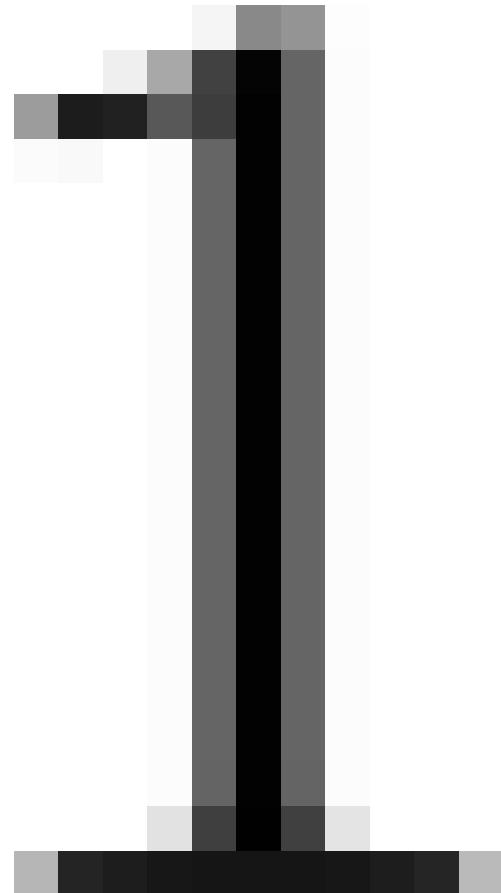
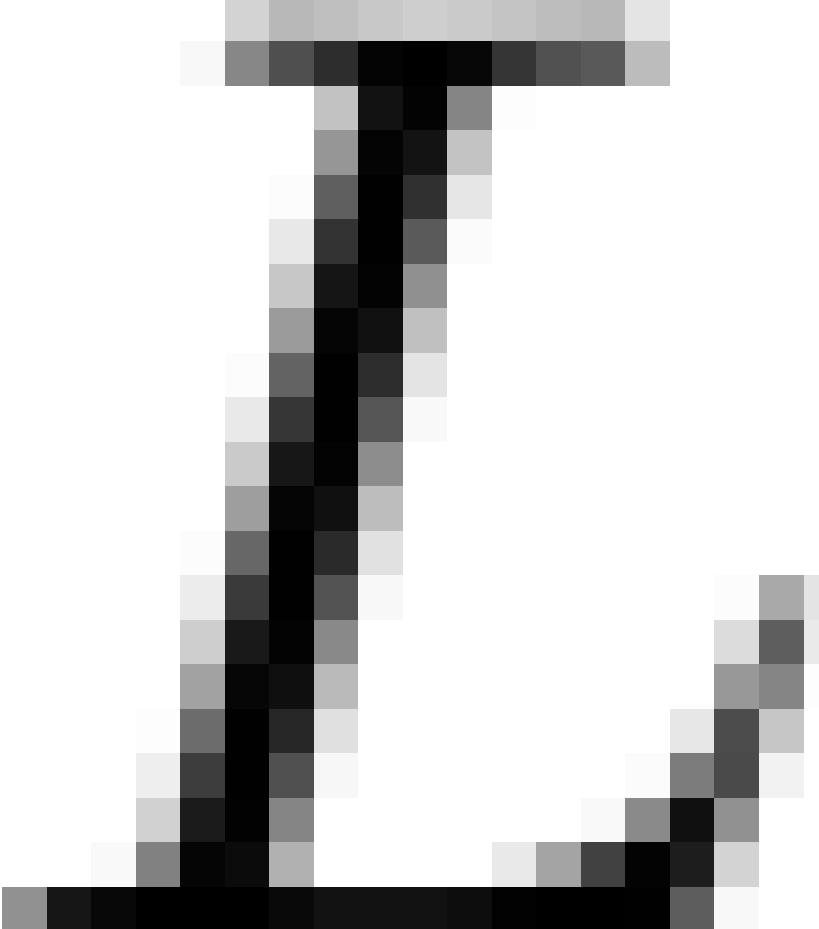
R

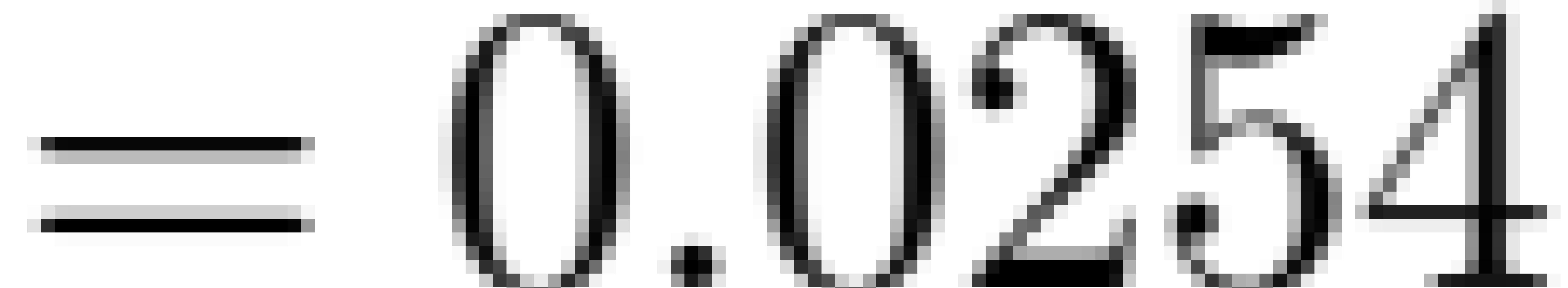
1
2

1
2

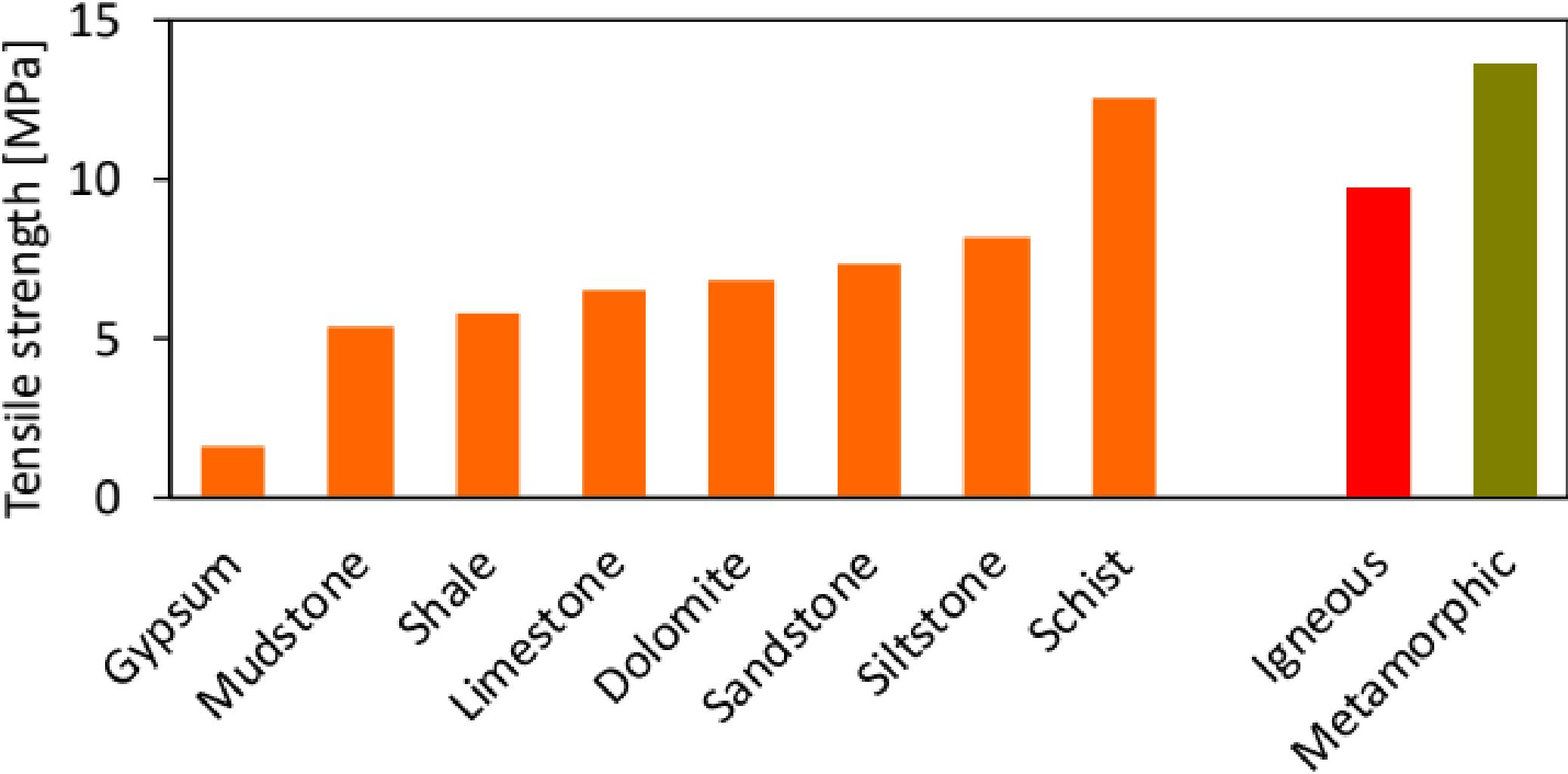
1
2

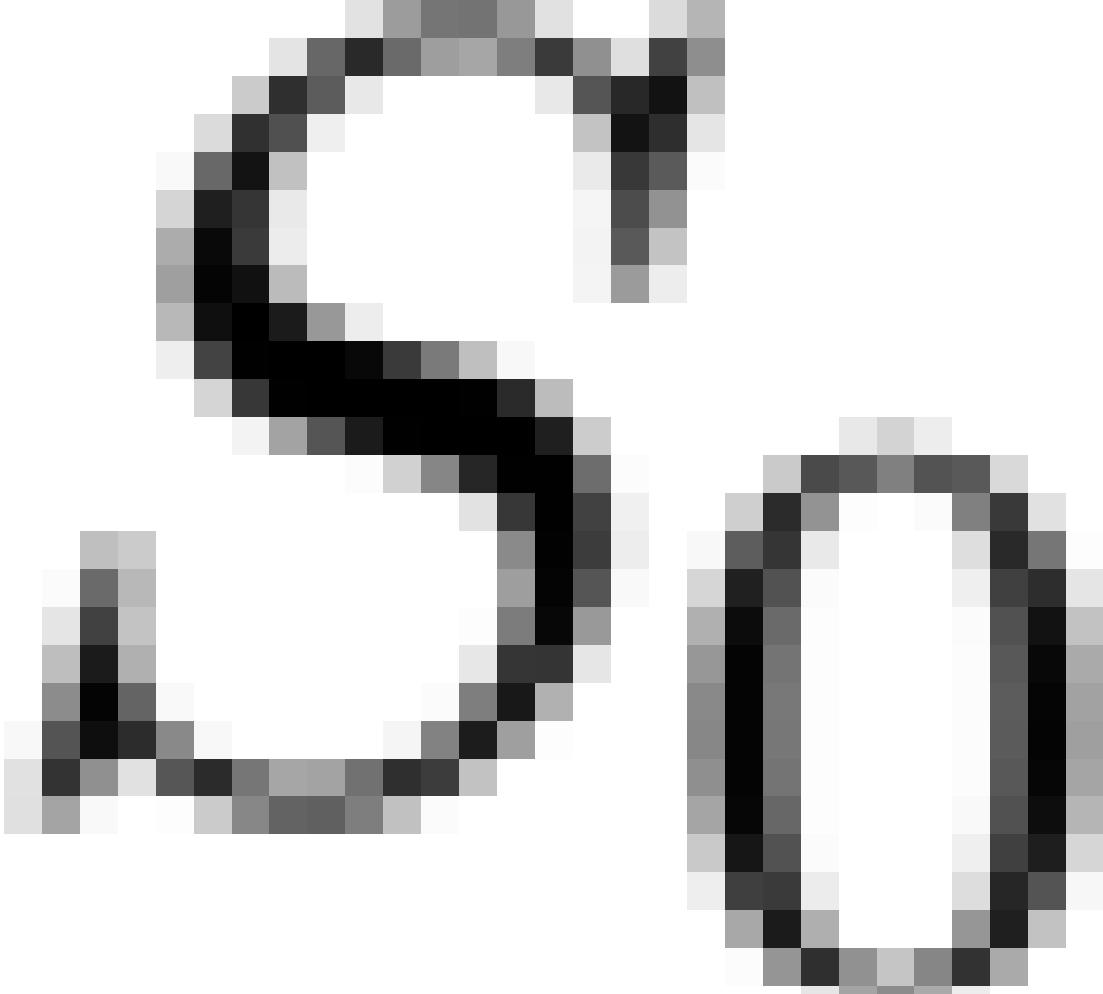


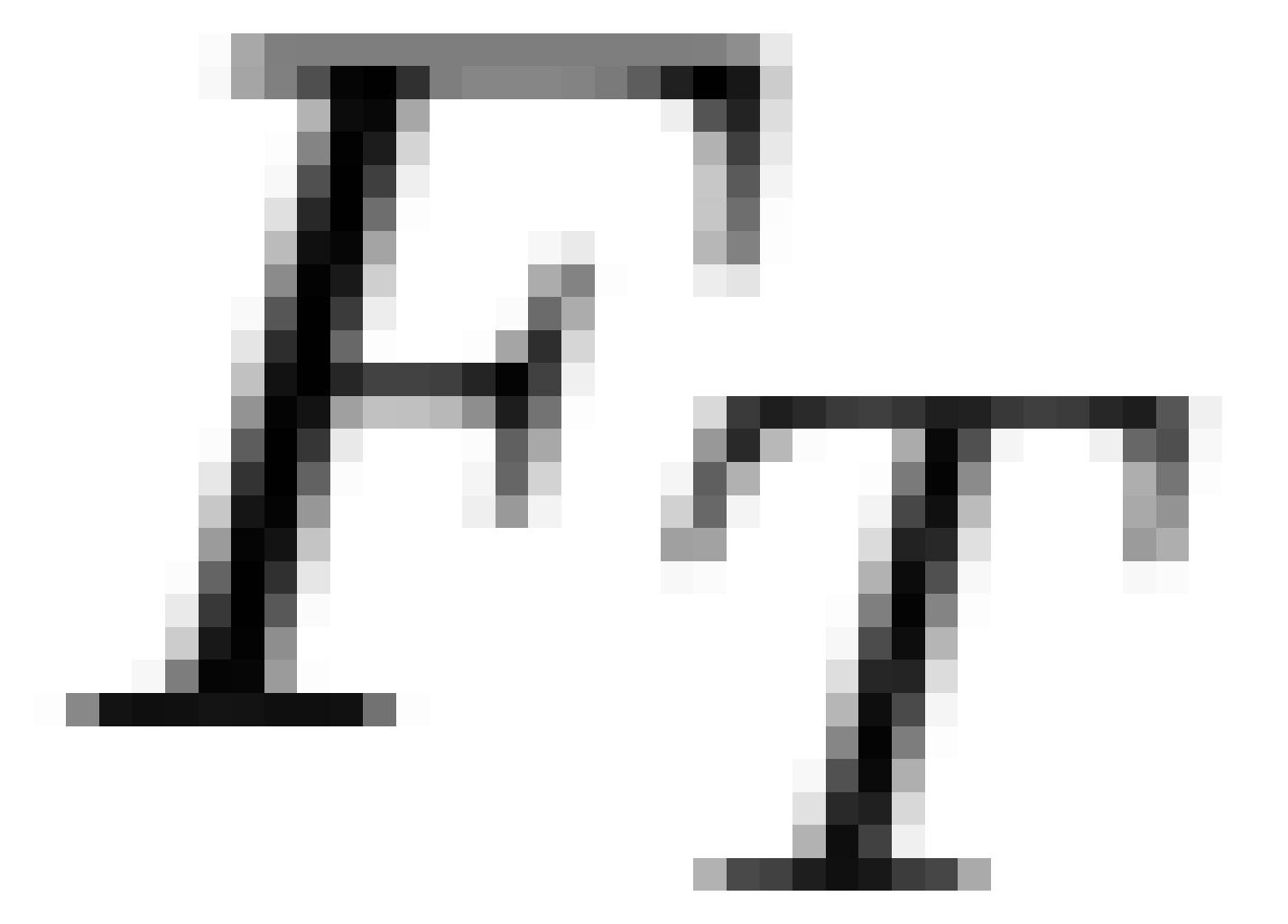




$$\frac{2084 \text{ N}}{\pi (0.0254 \text{ m})(0.0127 \text{ m})} = 2.06 \times 10^6 \text{ Pa} = 2.06 \text{ MPa}$$

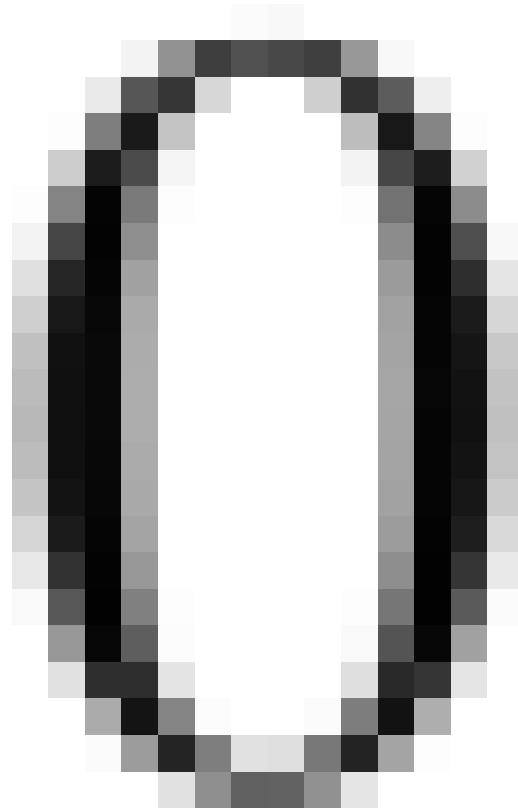


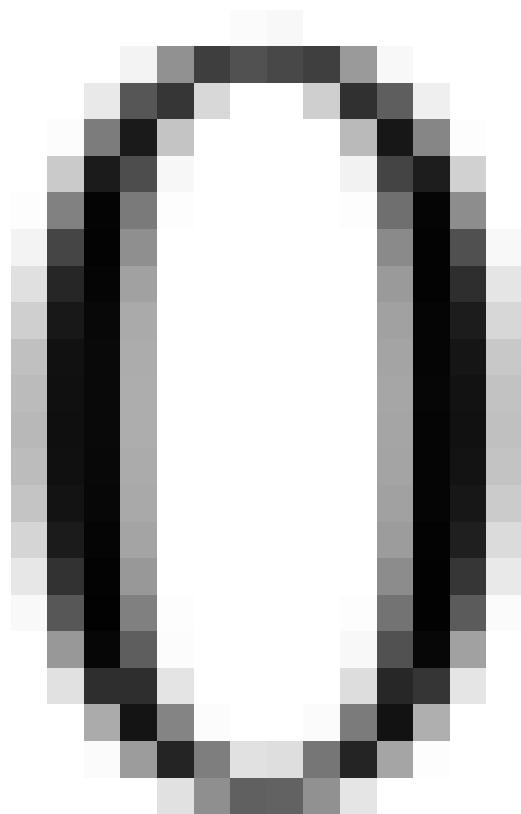


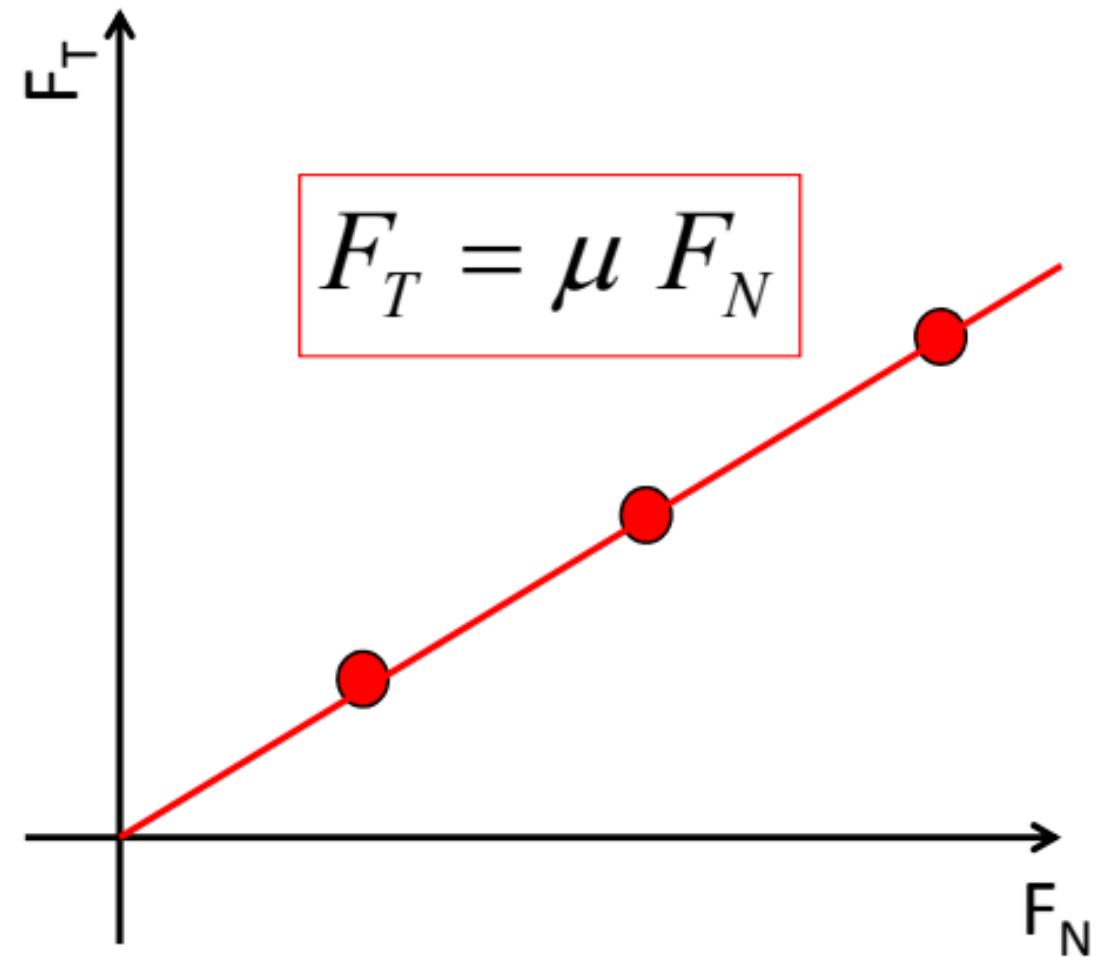
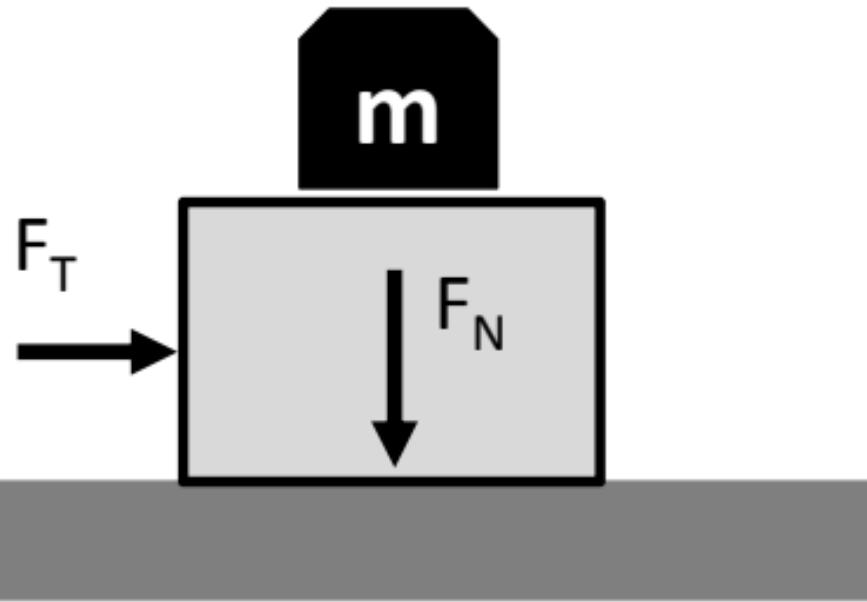


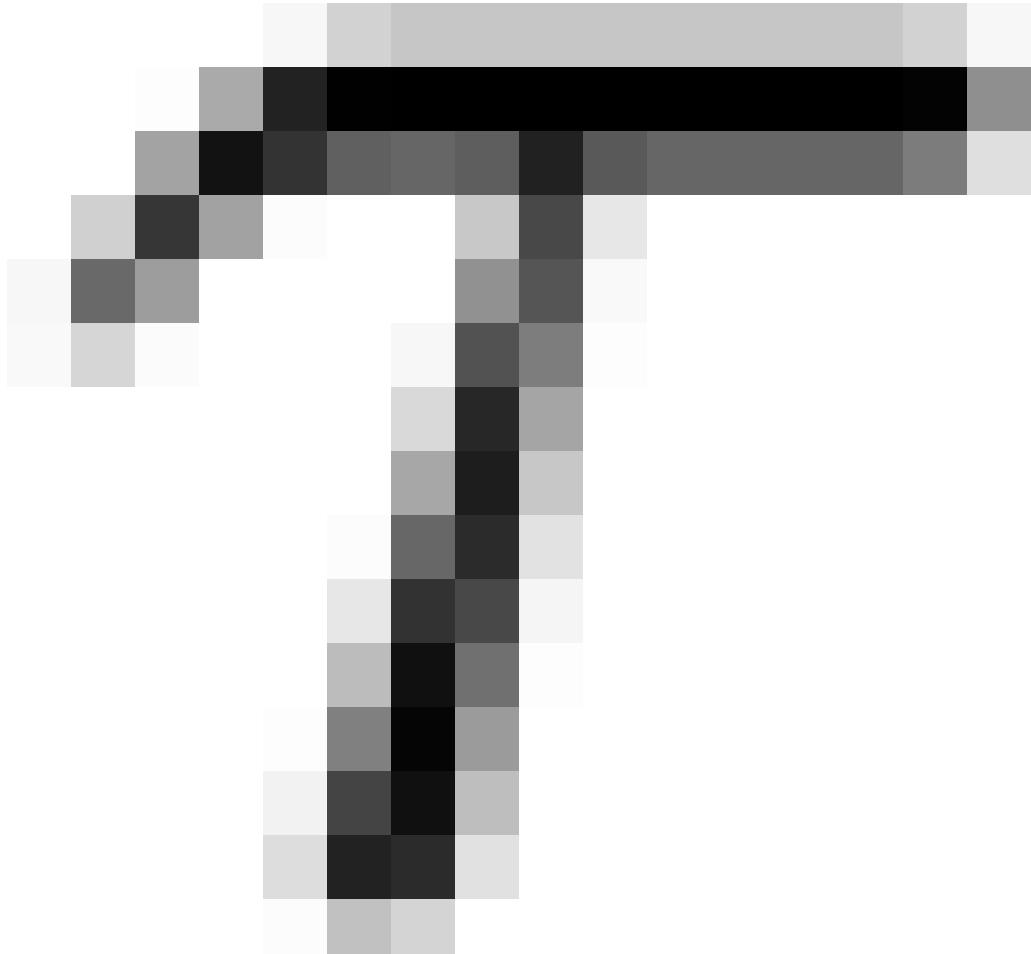


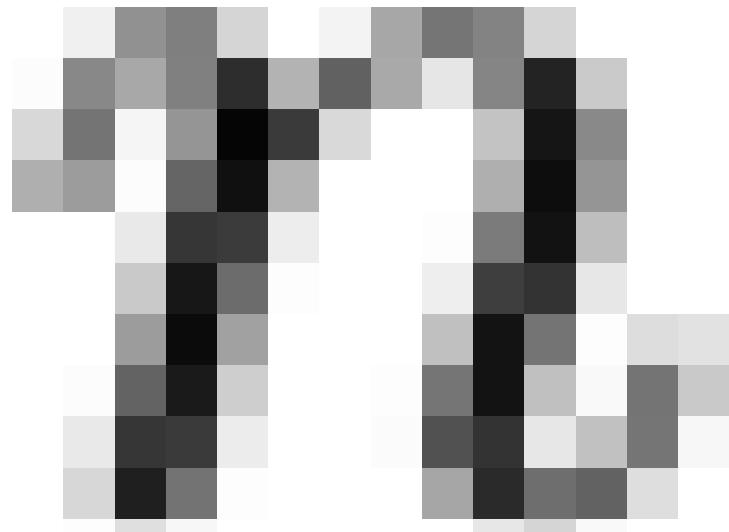
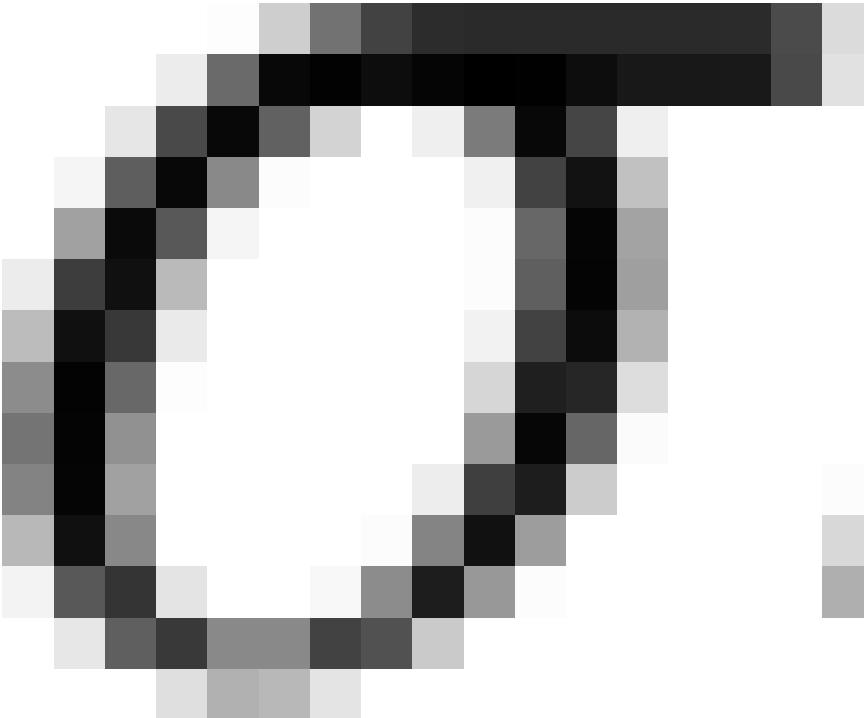




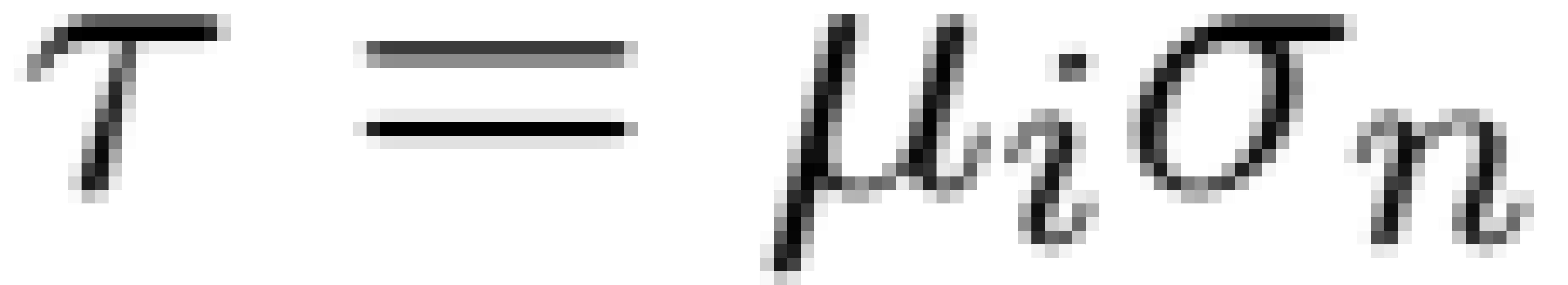


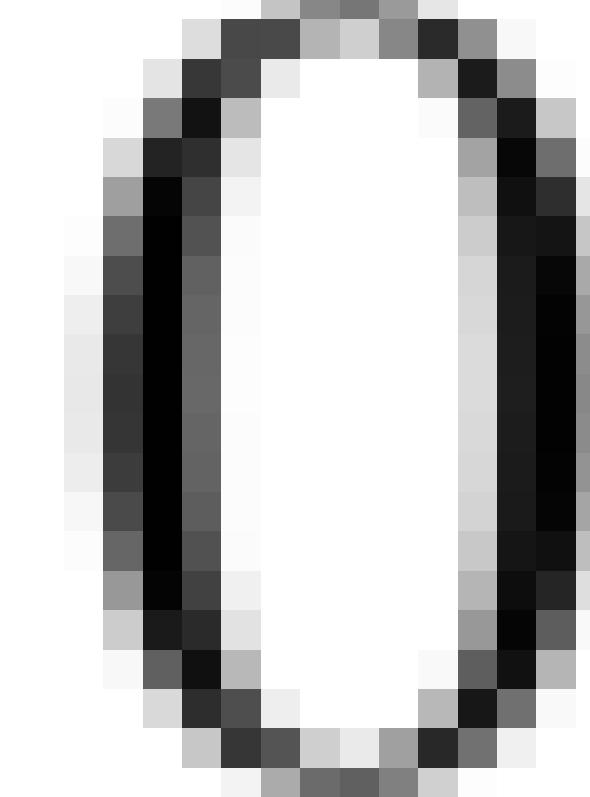
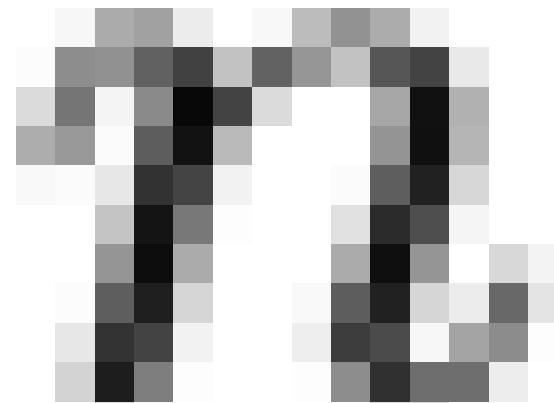
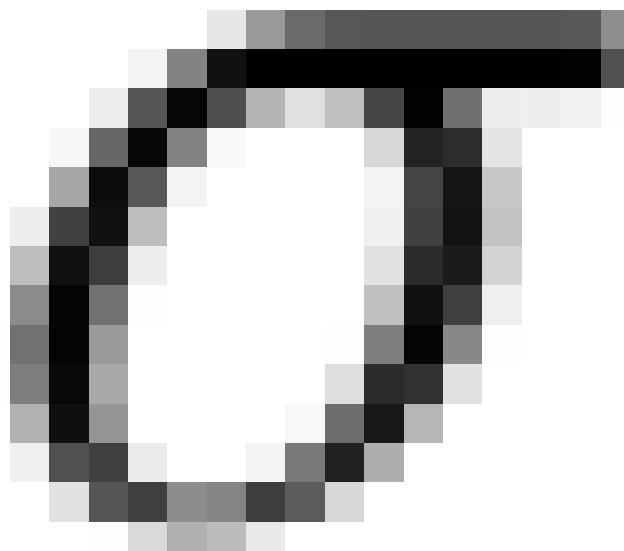


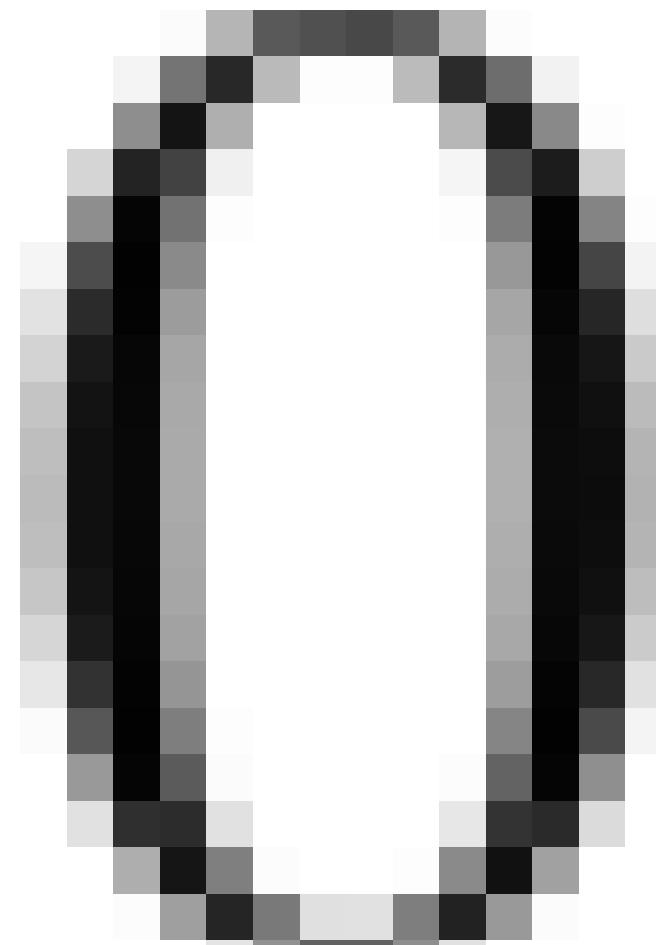
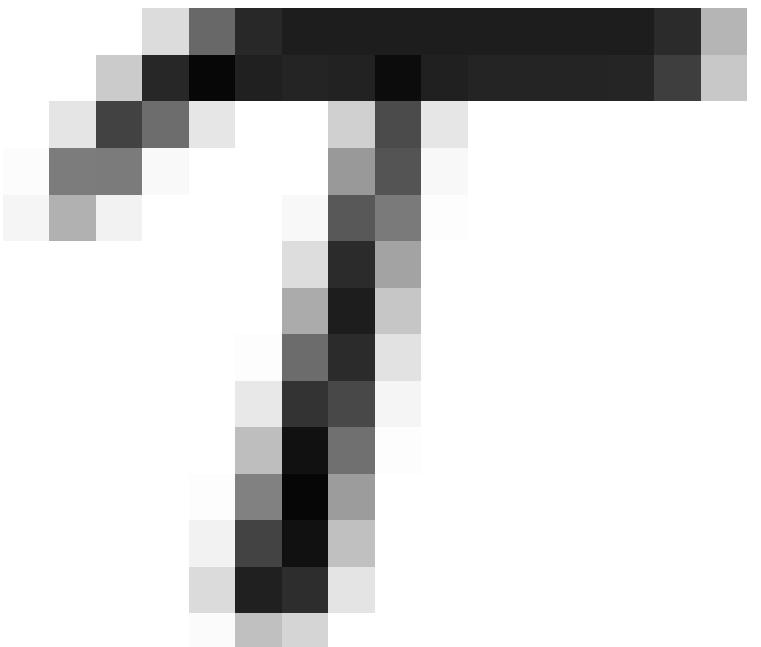


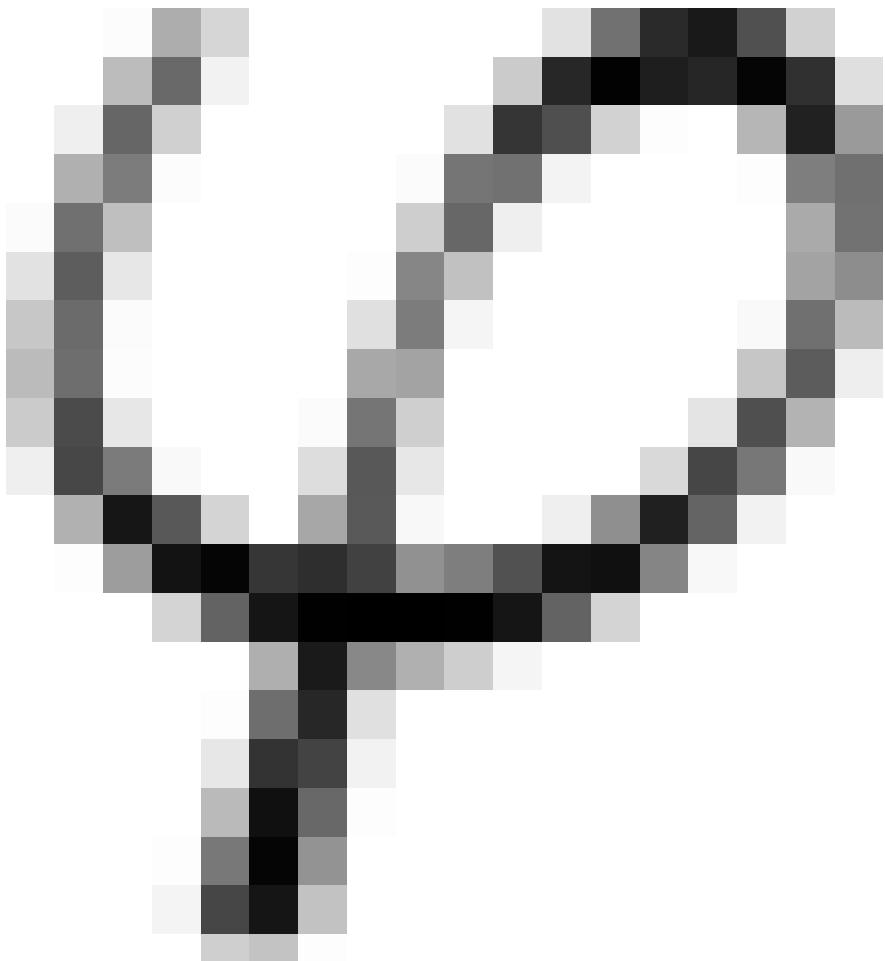




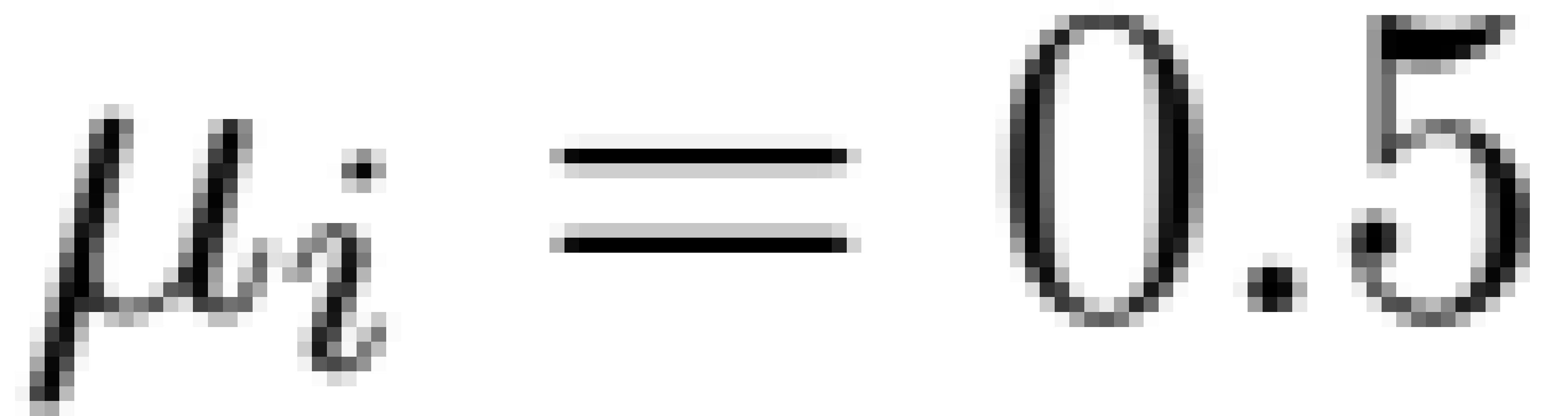


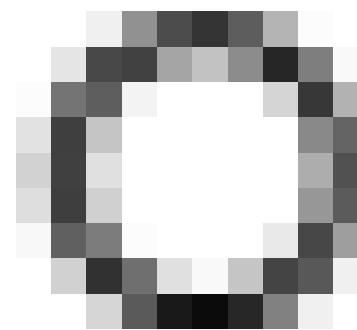
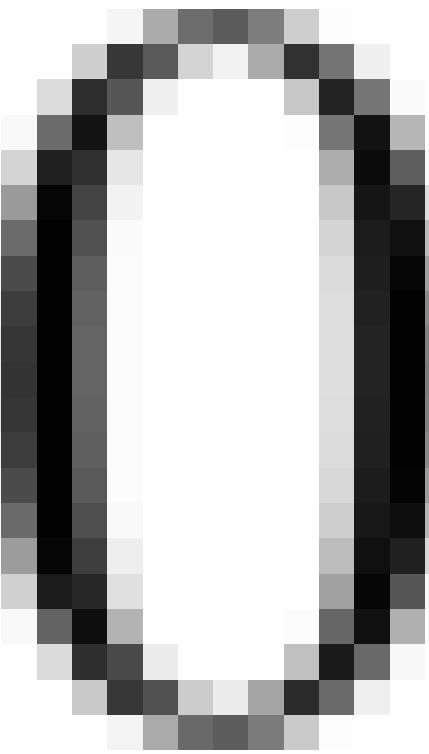
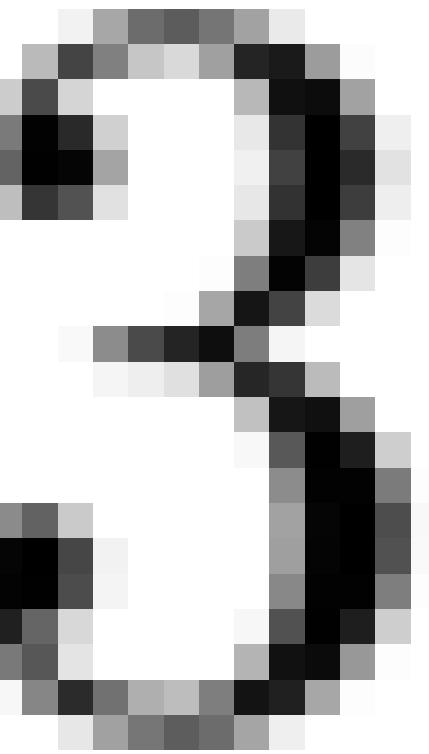
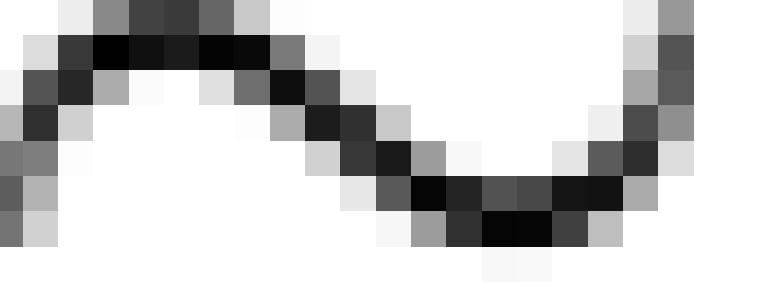
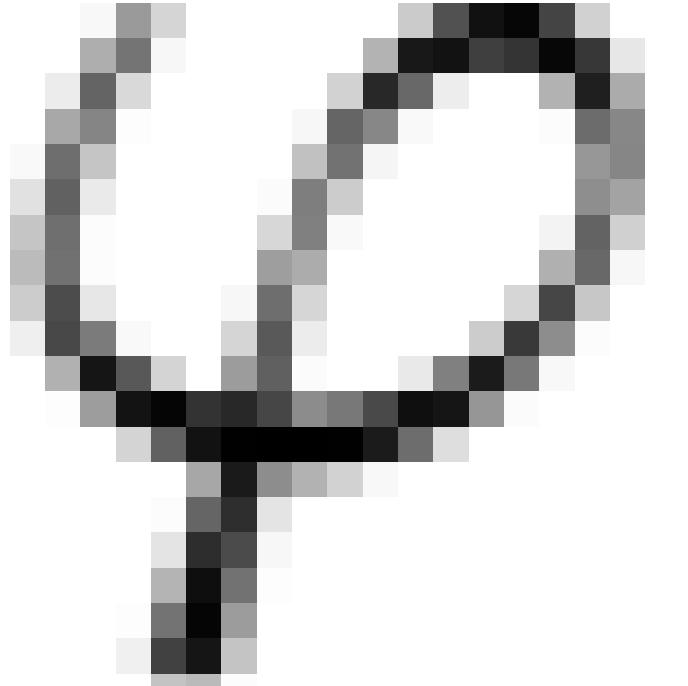


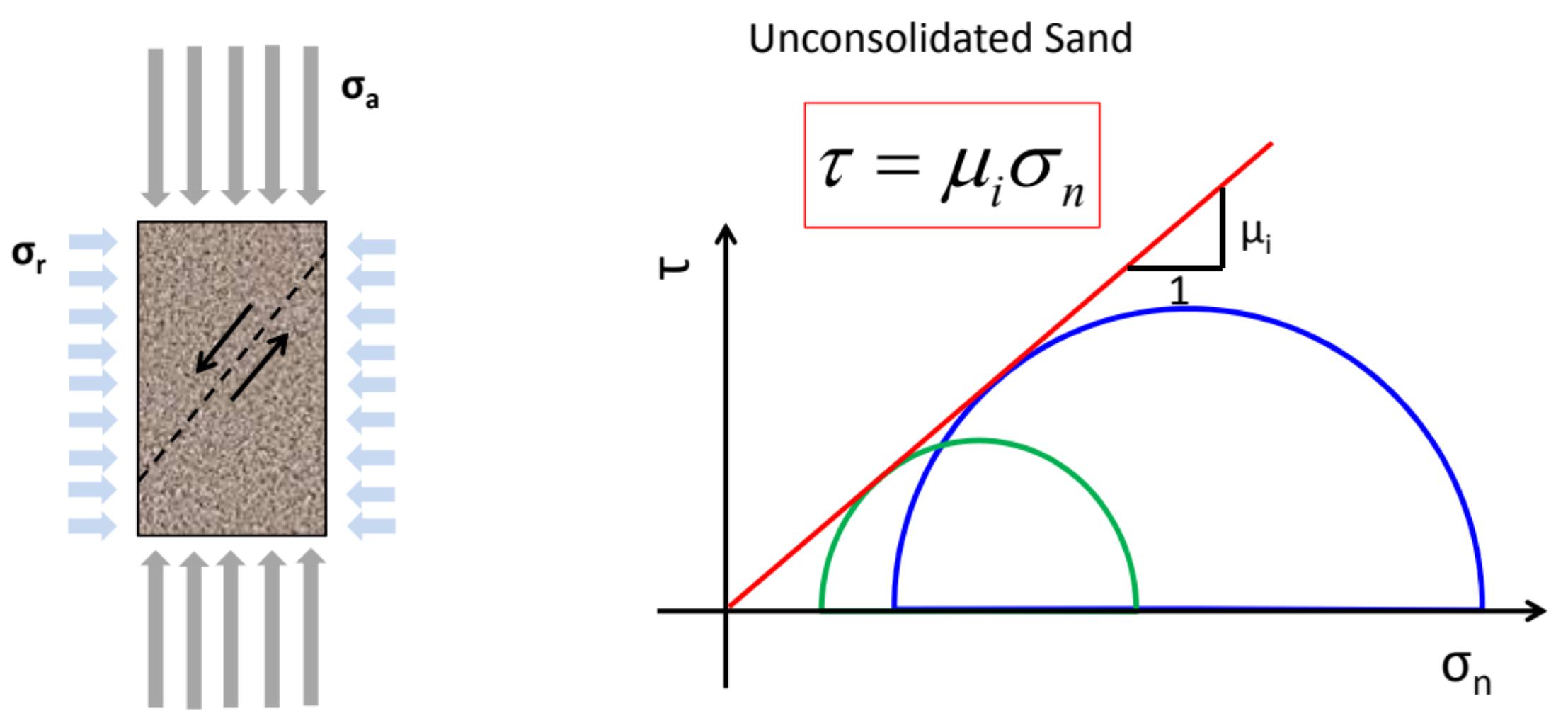


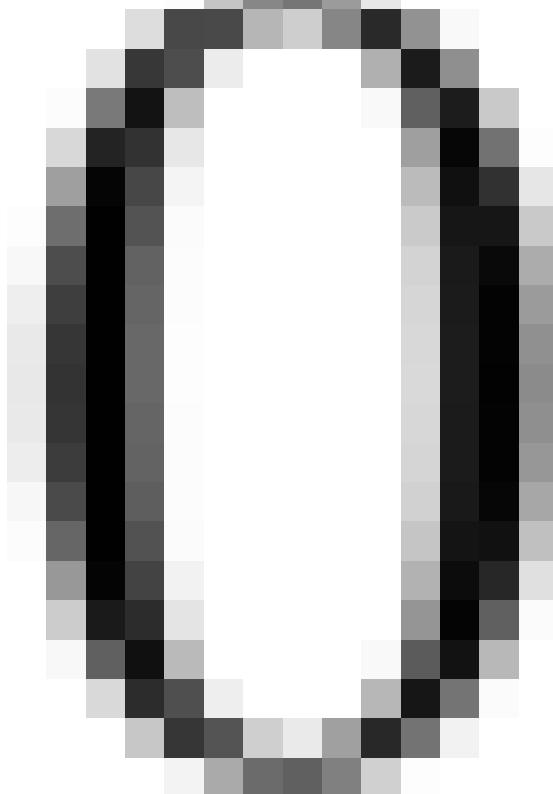
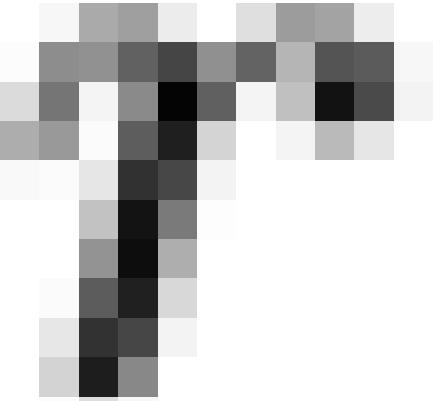
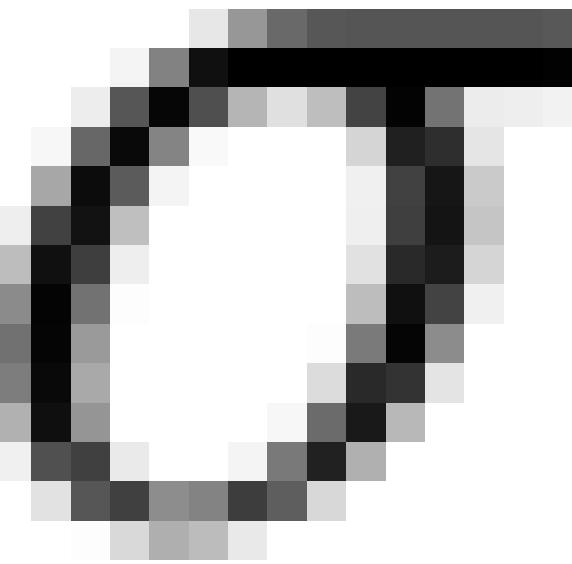


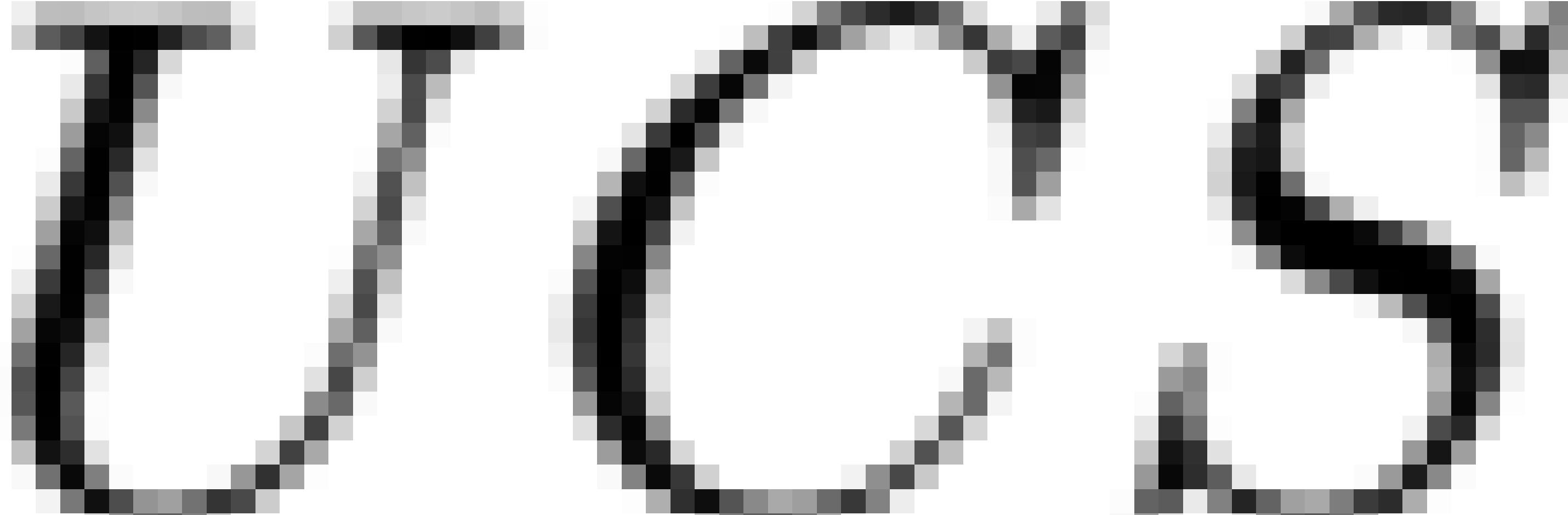




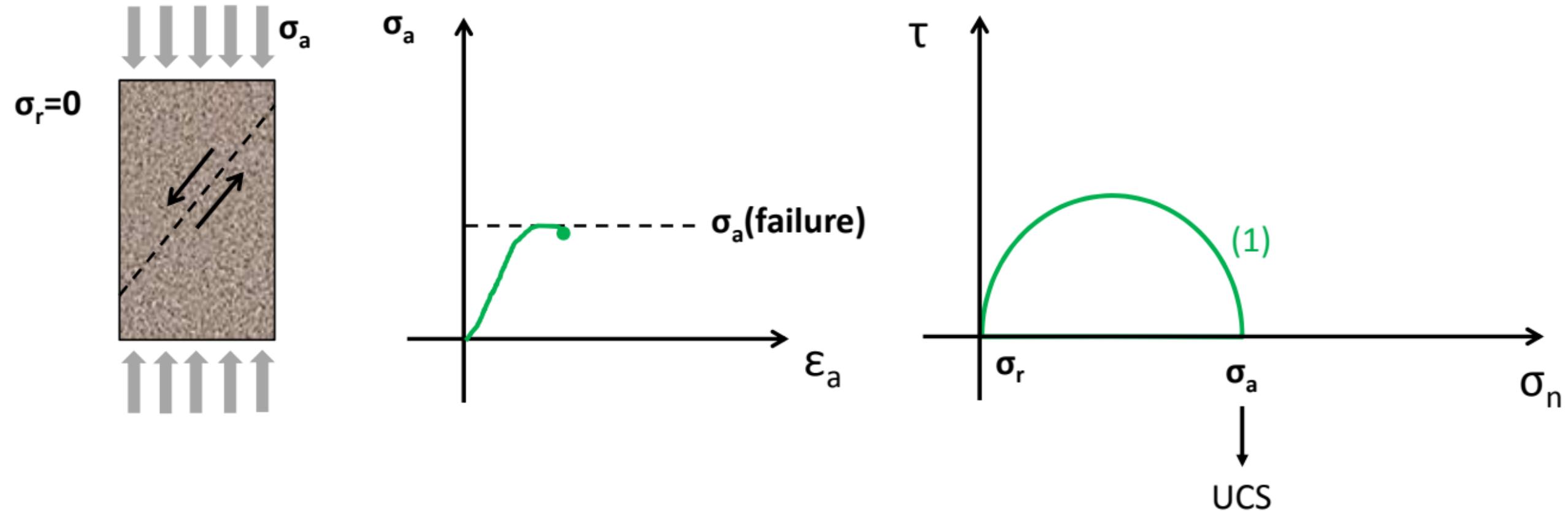


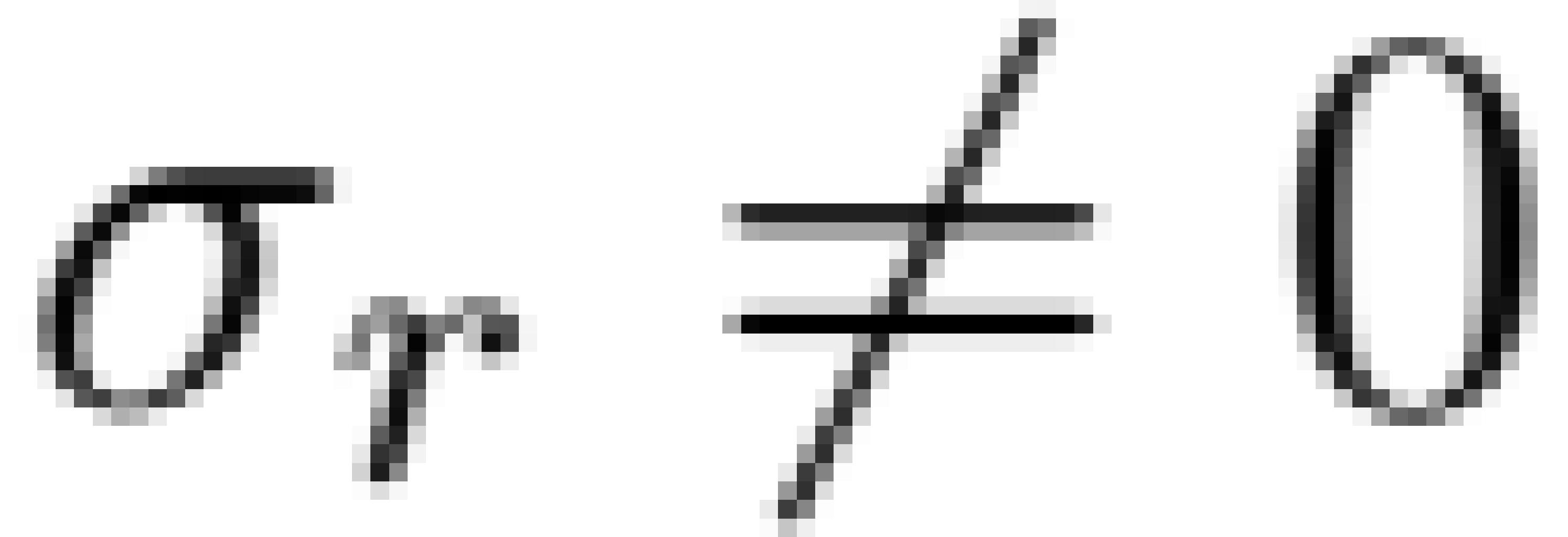


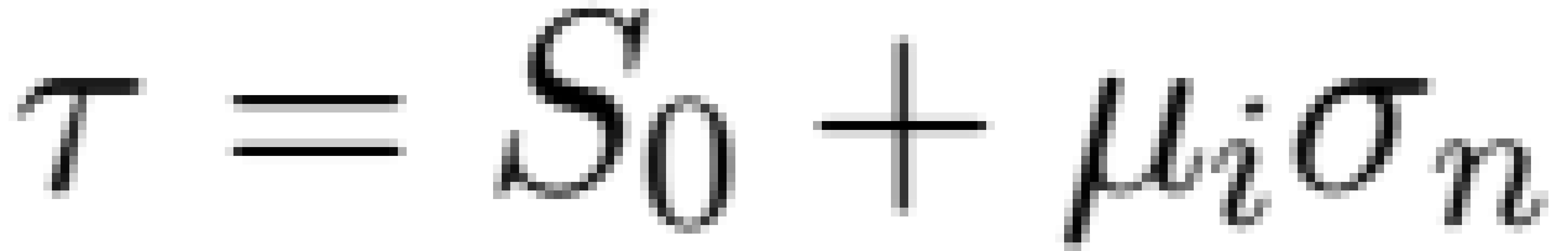




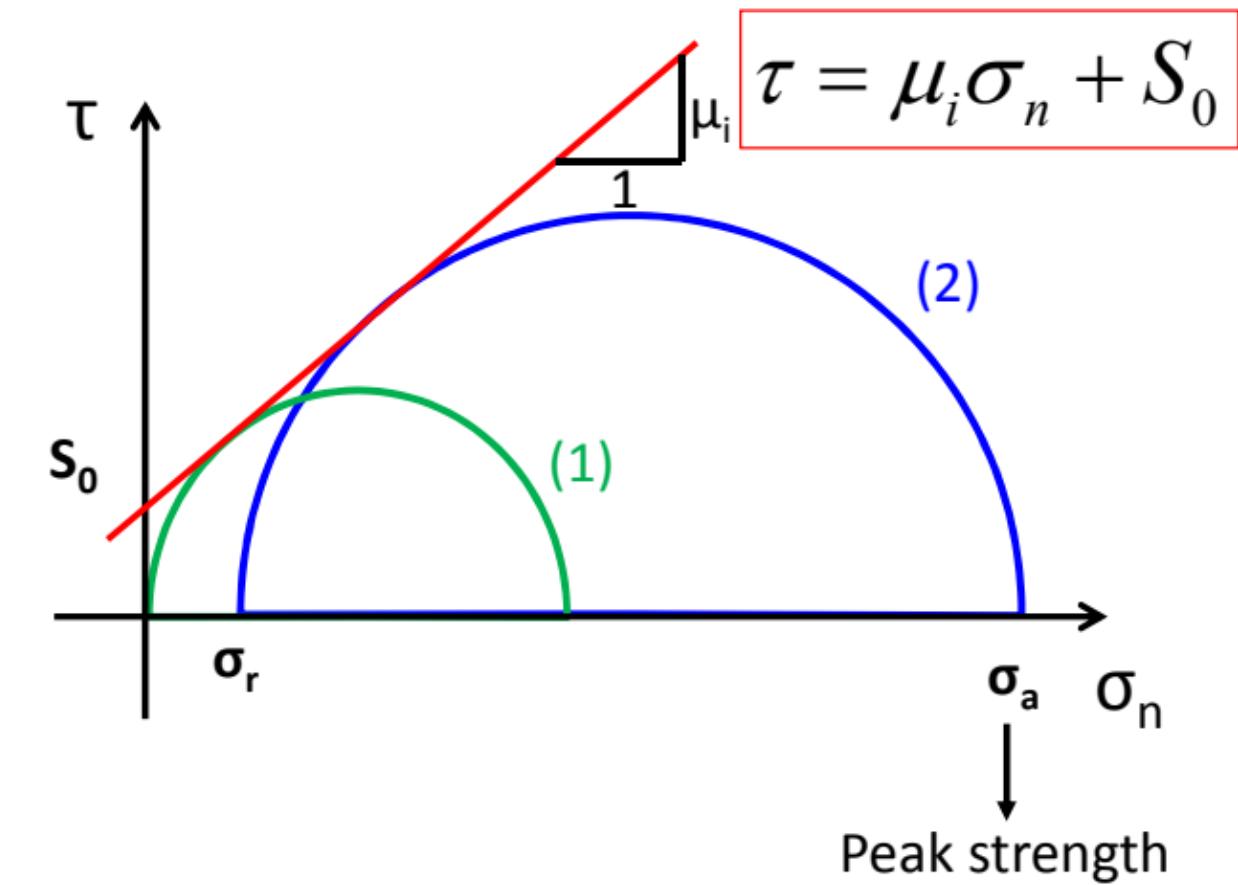
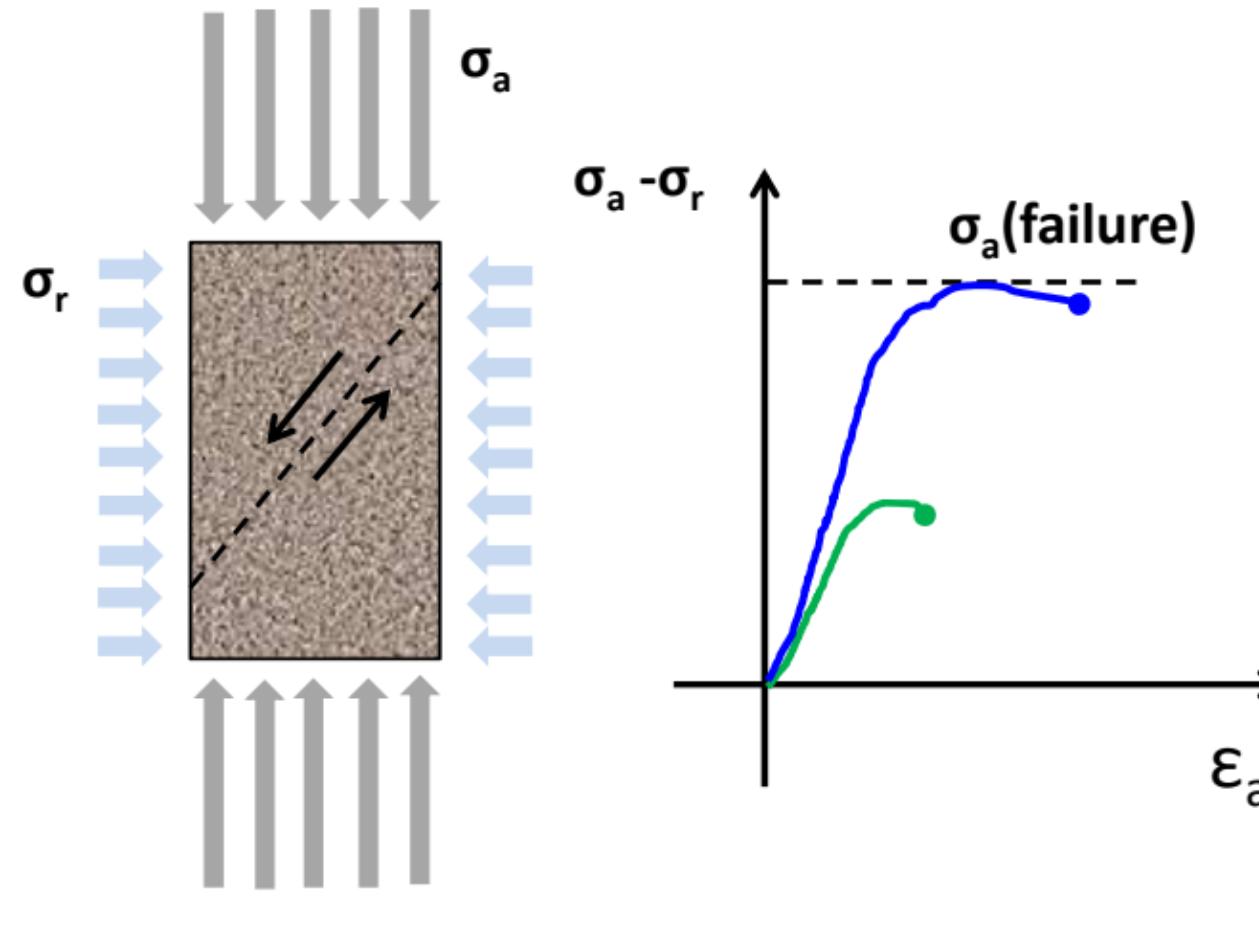
(1) Unconfined Loading

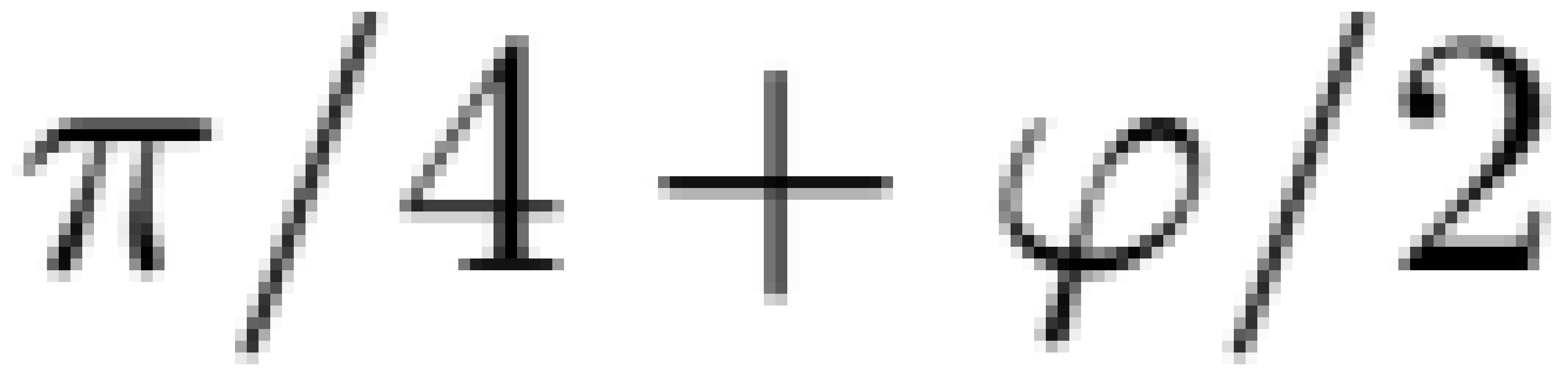






(2) Confined Loading

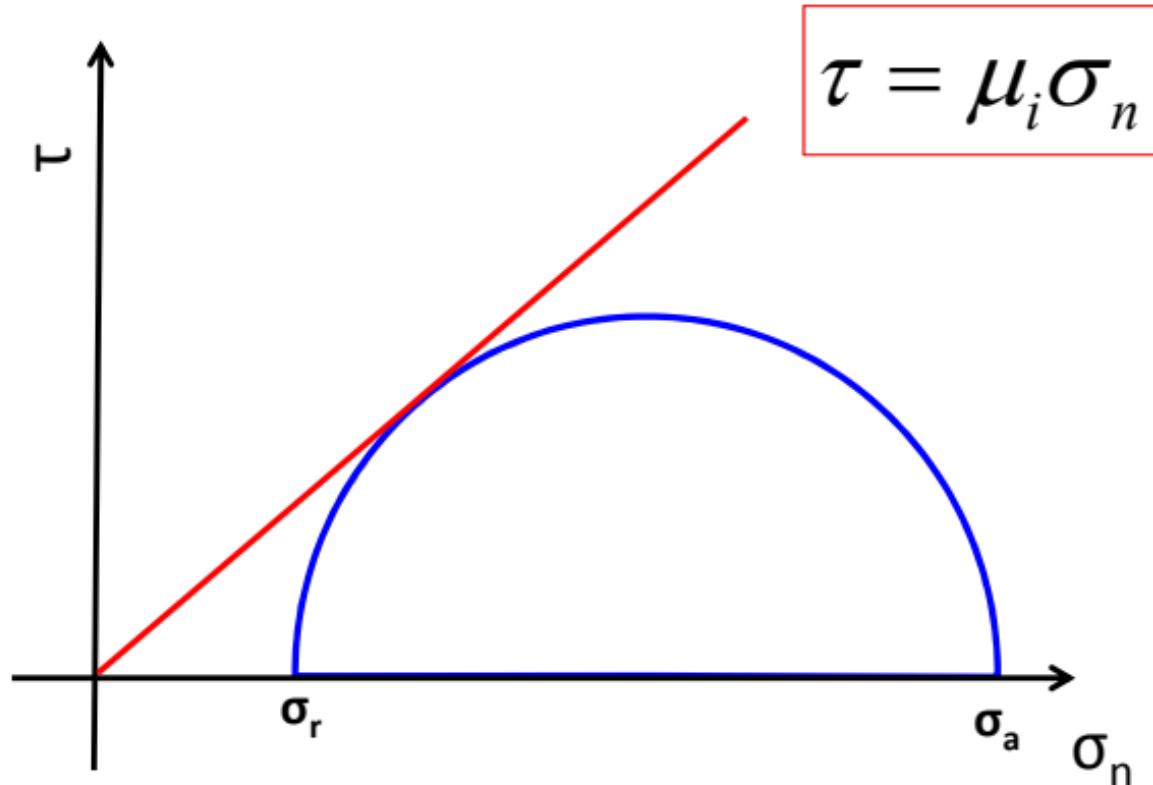
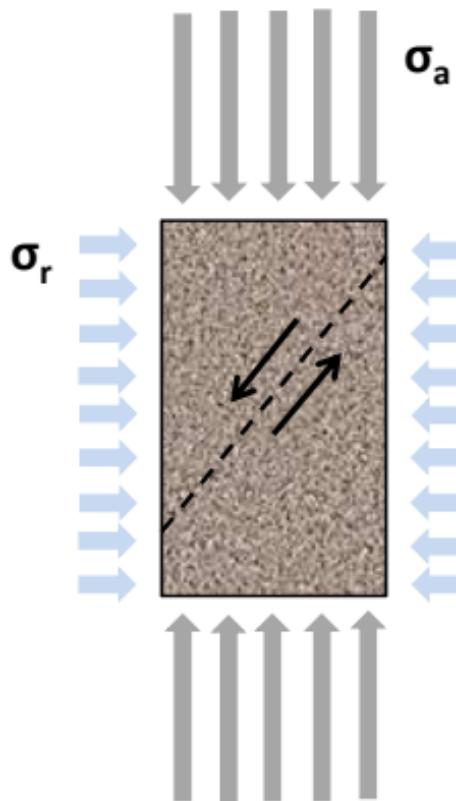




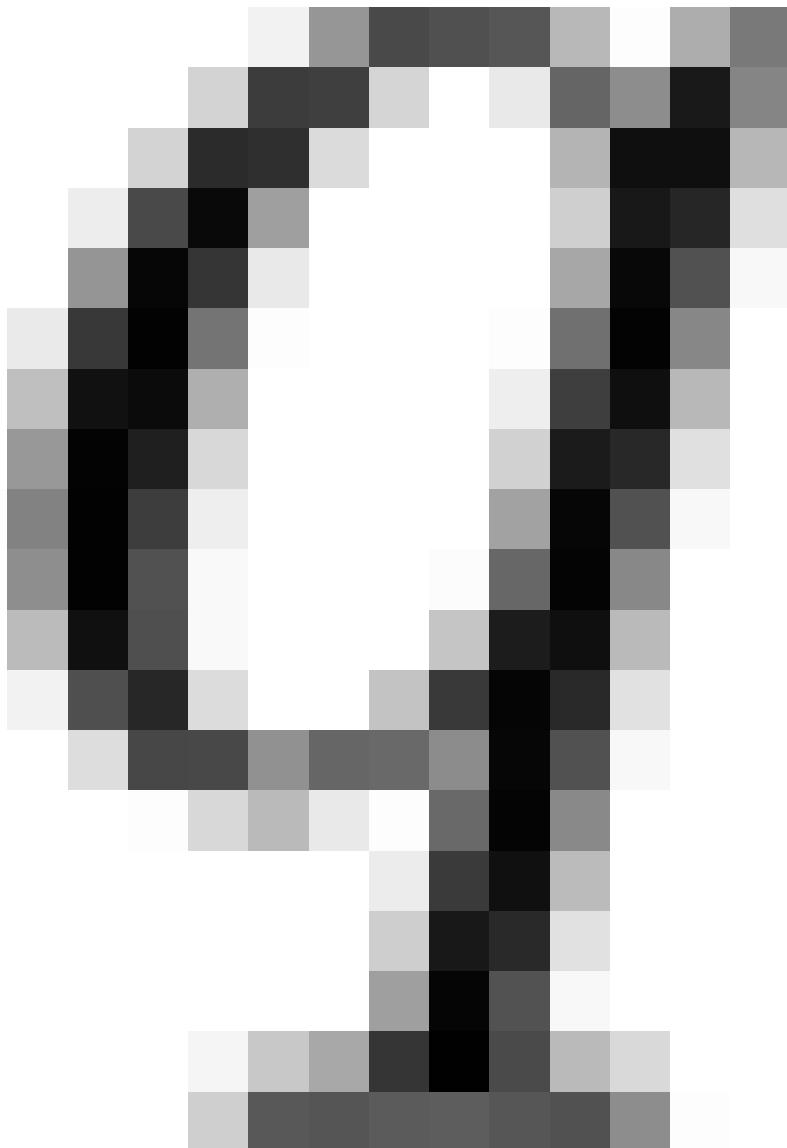


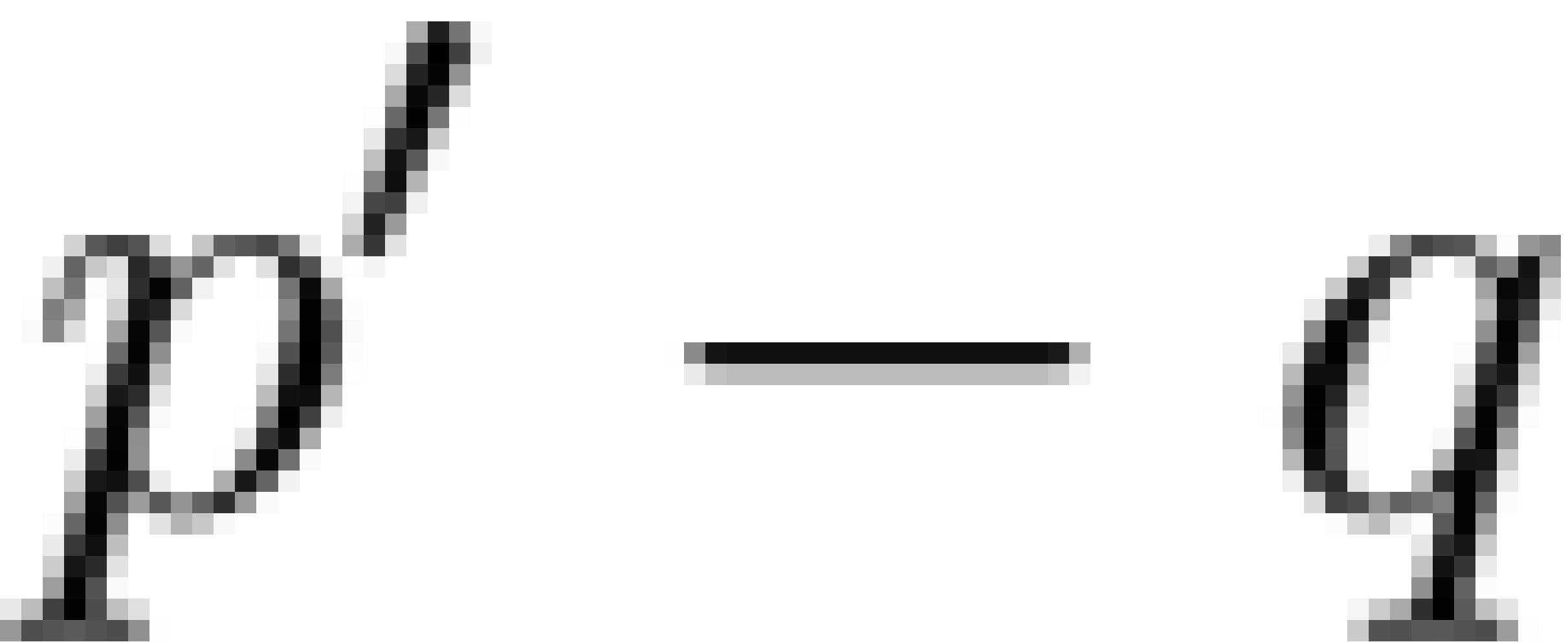
Which is point in the Mohr circle with maximum τ/σ_n ?

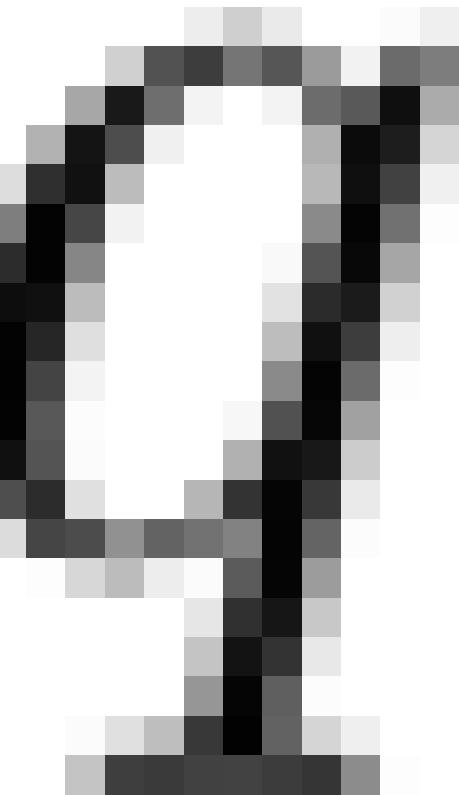
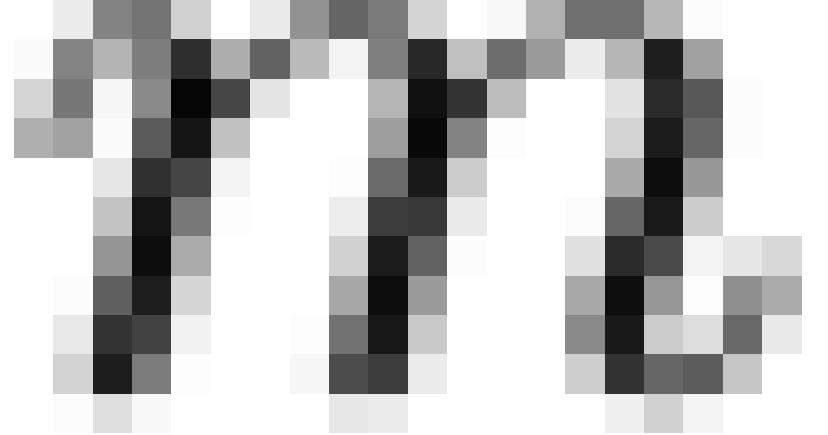
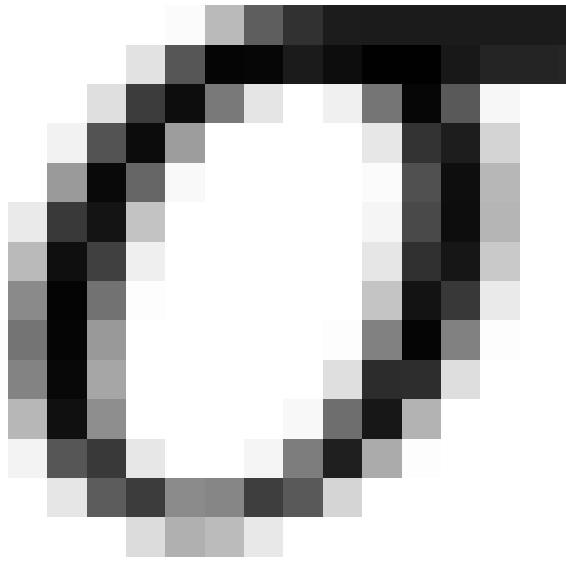
What is the angle of the failure plane?





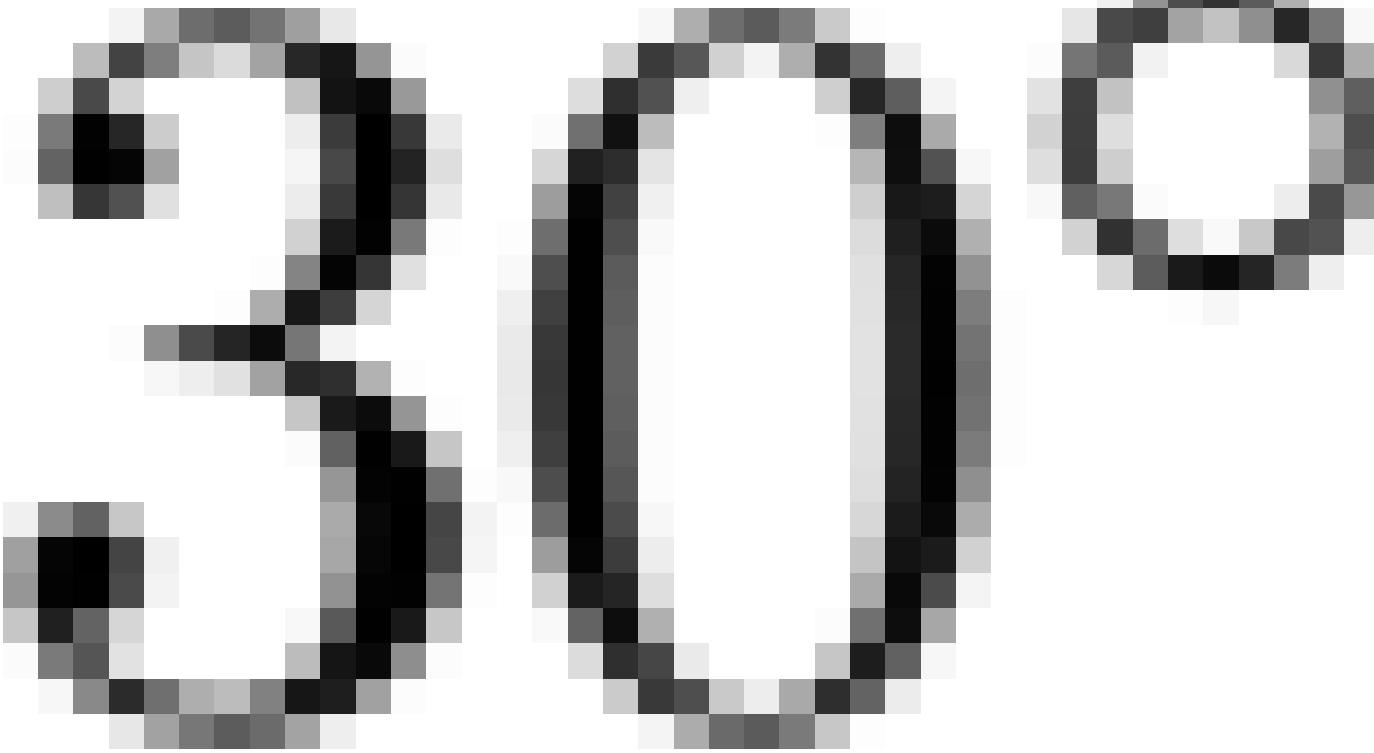
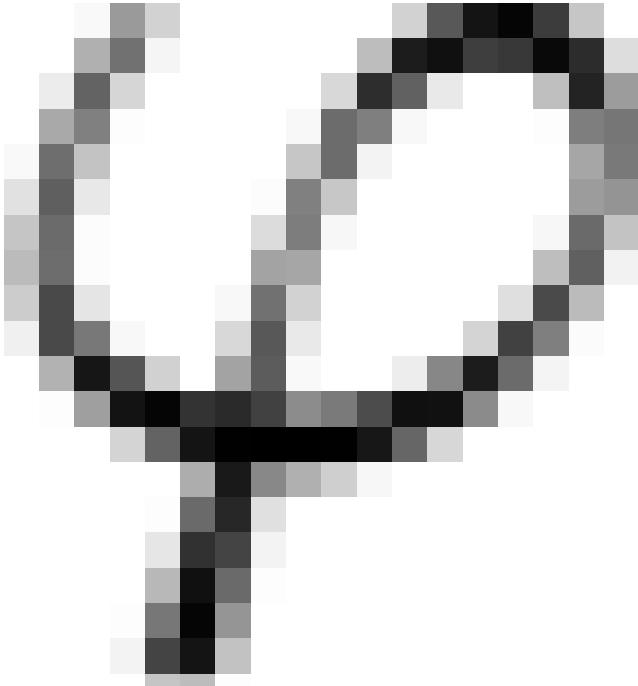


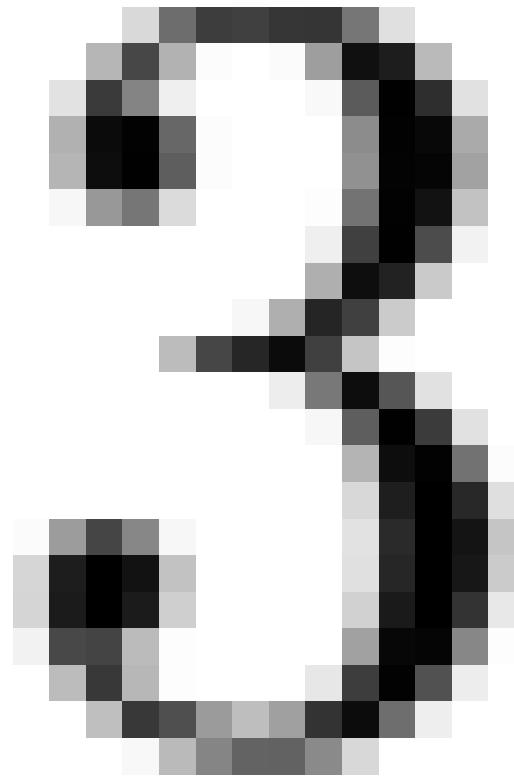
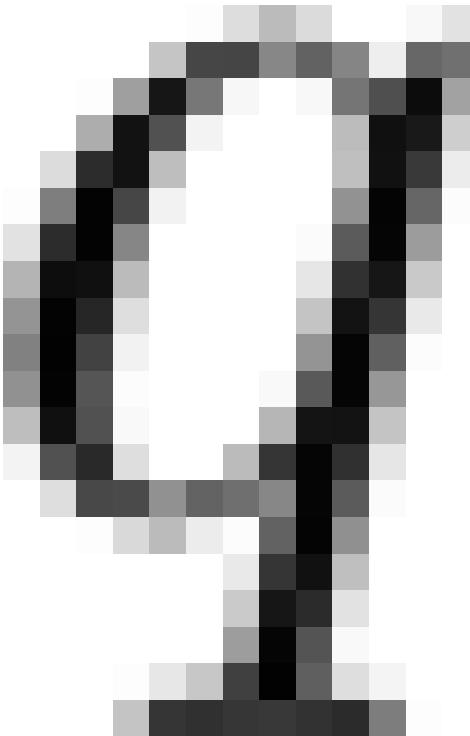




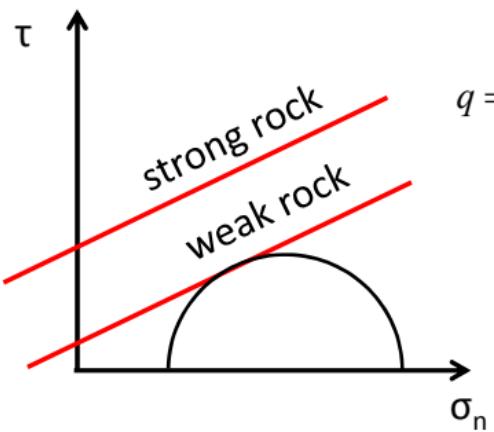
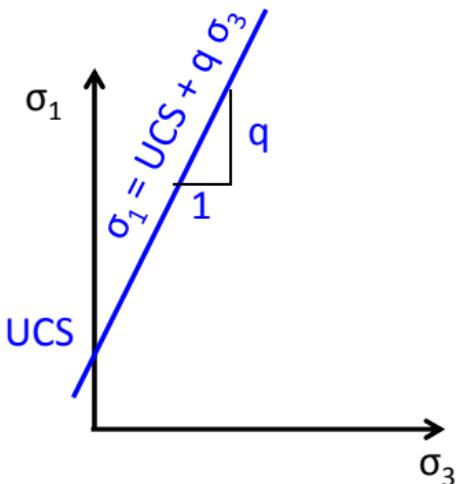
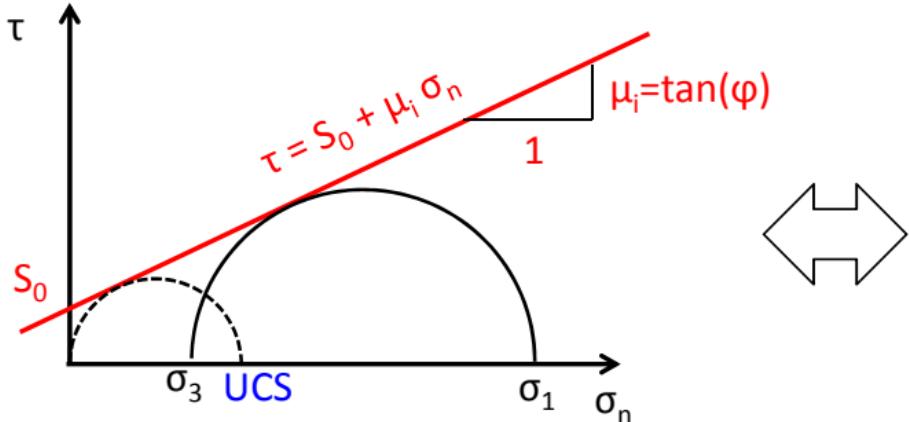
φ sin φ

φ sin φ



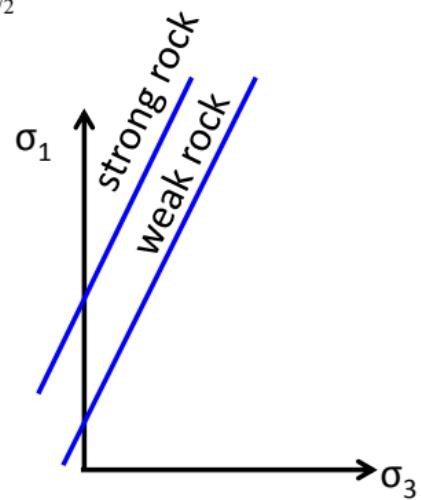


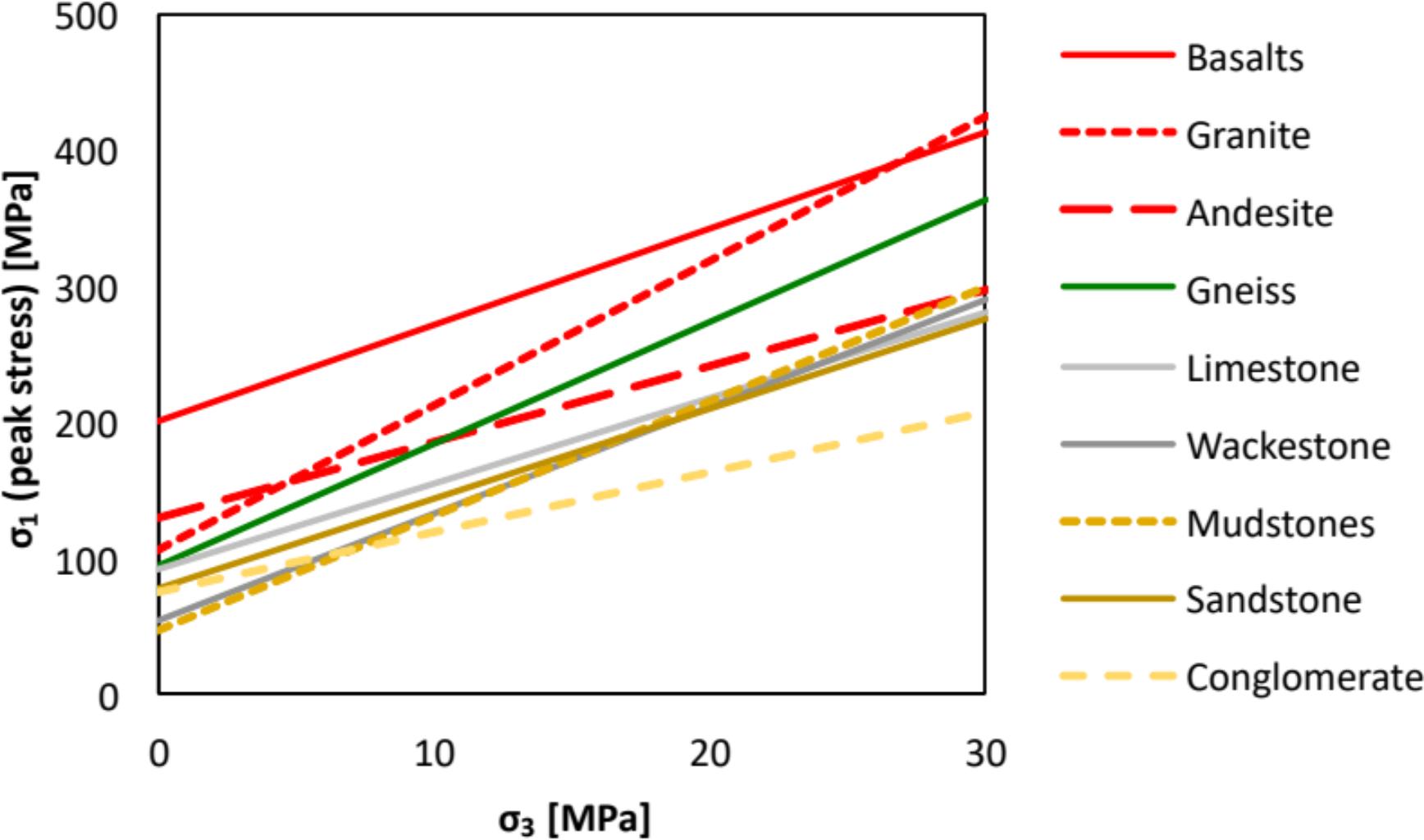
$$U_0 S = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} = 2S_0 \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi}}$$



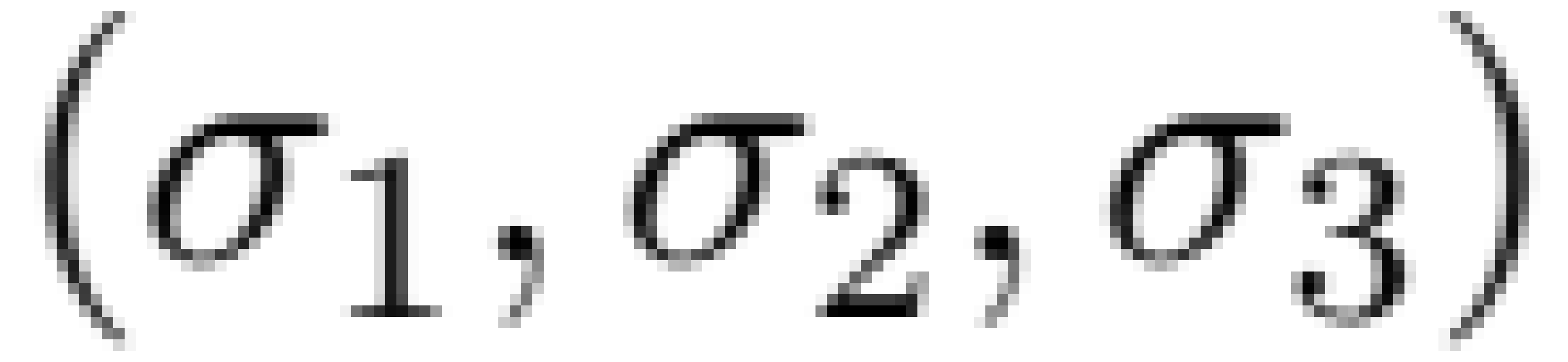
$$UCS = 2S_0 \left(\sqrt{\mu_i^2 + 1} + \mu_i \right) = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2}$$

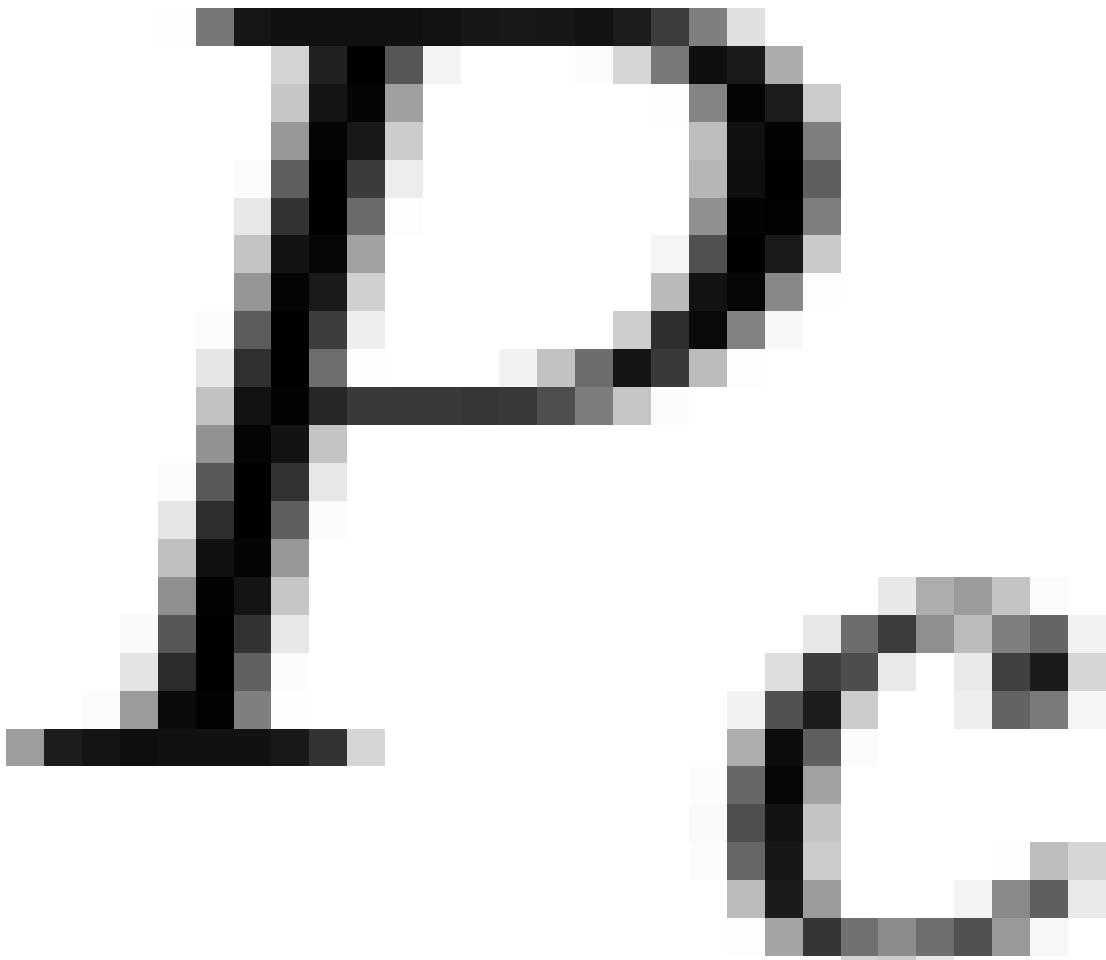
$$q = \left(\sqrt{\mu_i^2 + 1} + \mu_i \right)^2 = \frac{1 + \sin \varphi}{1 - \sin \varphi}$$

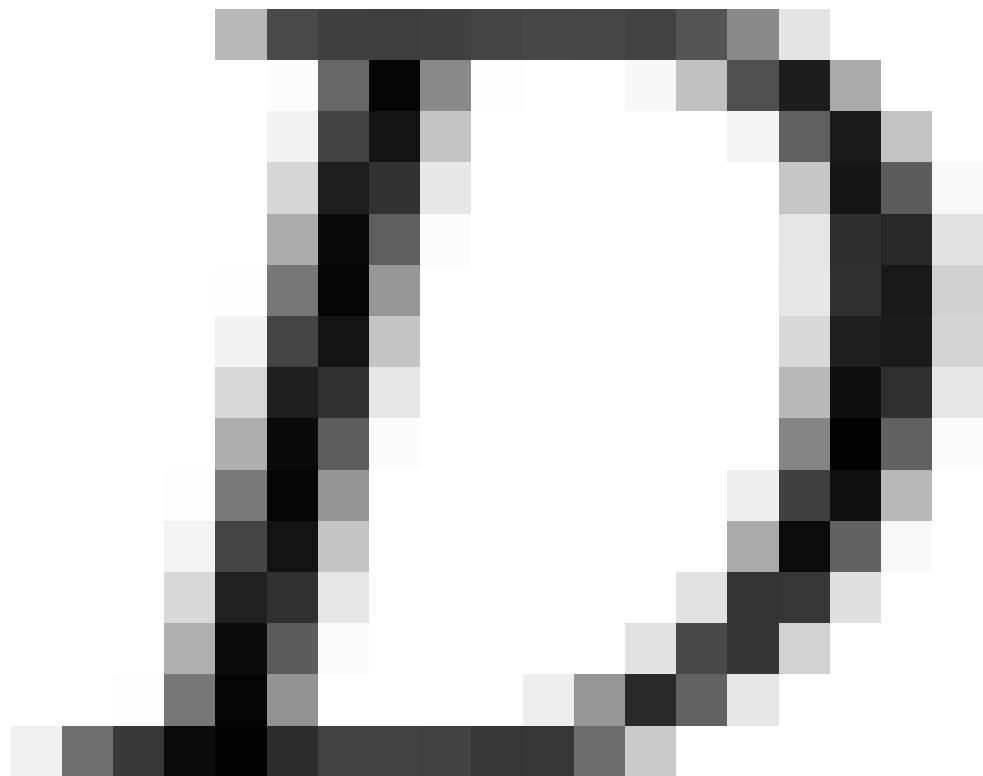
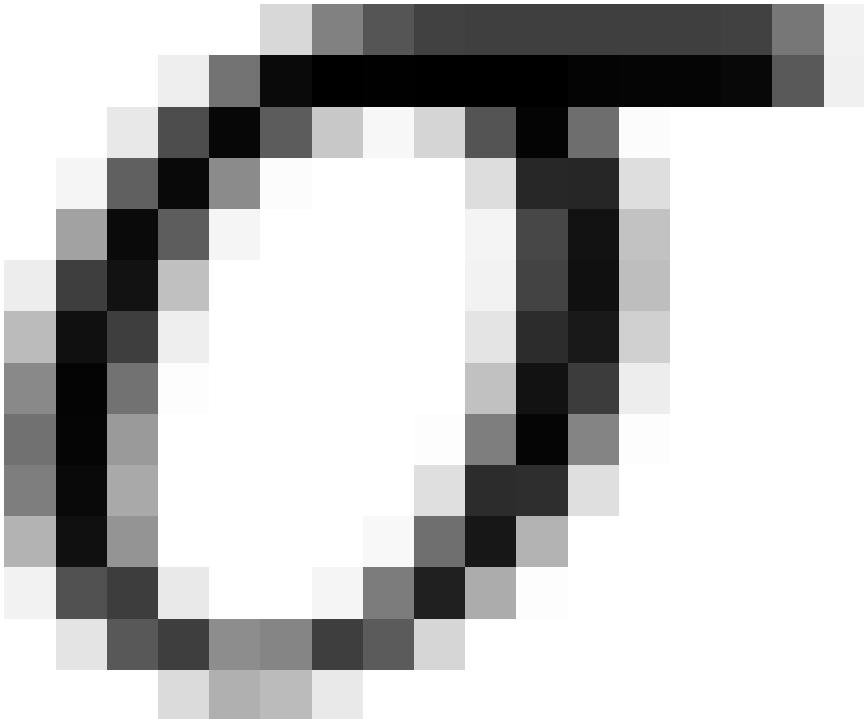


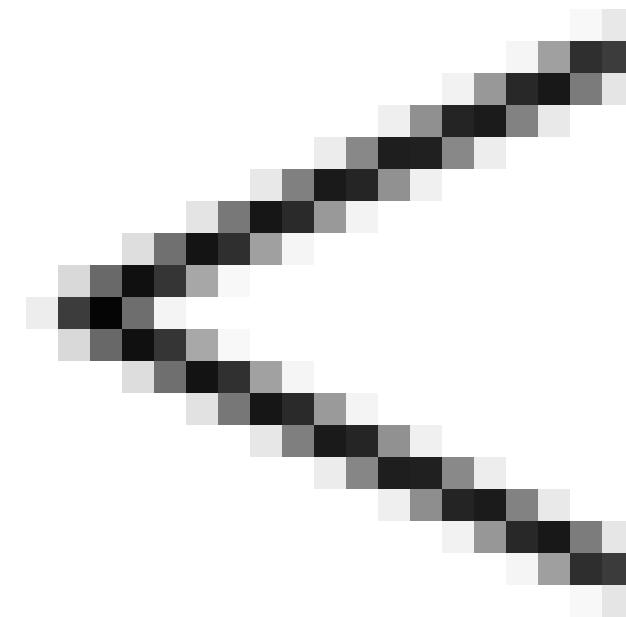
















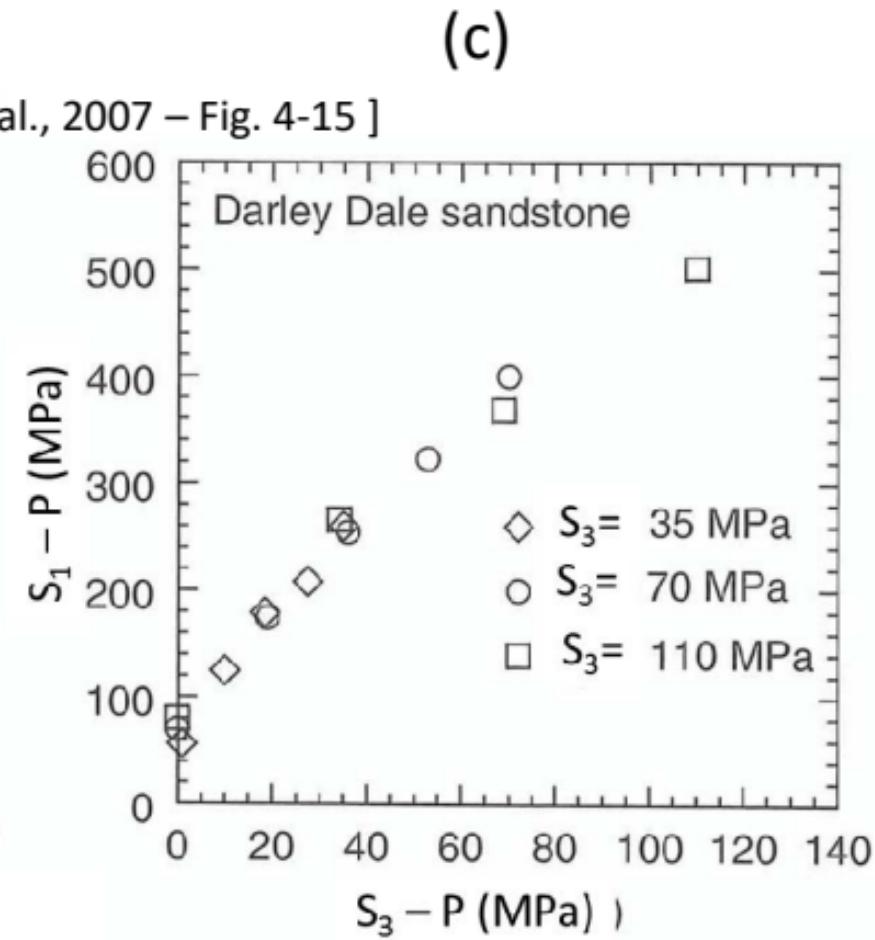
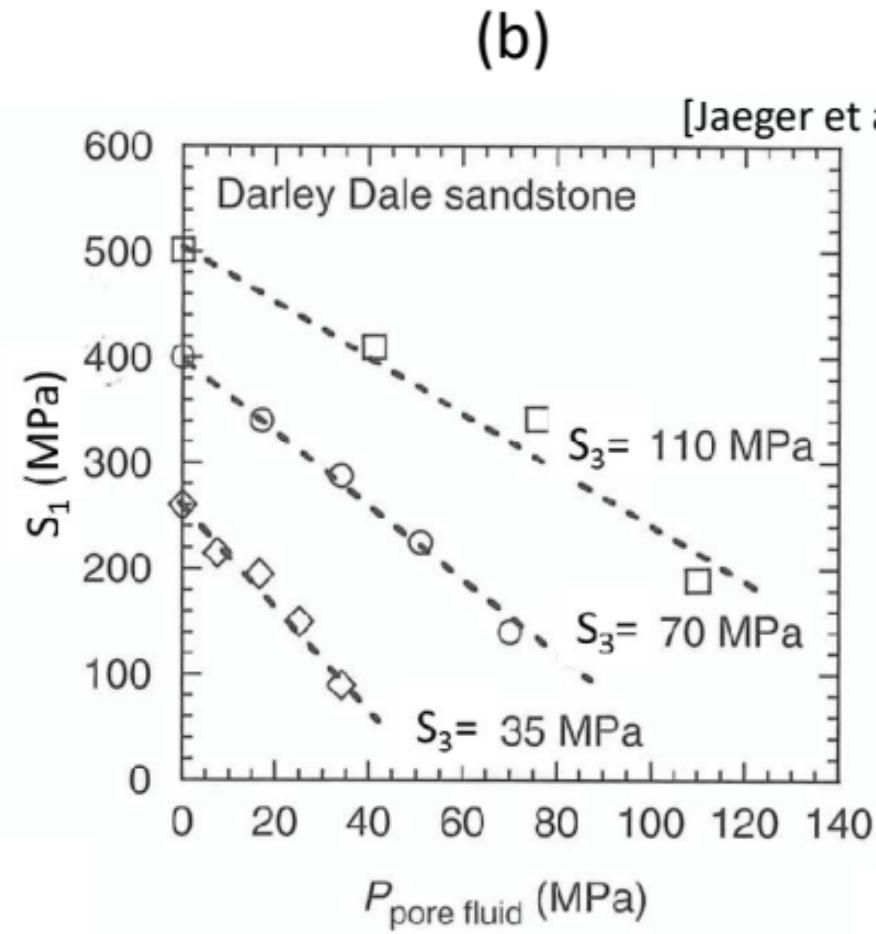
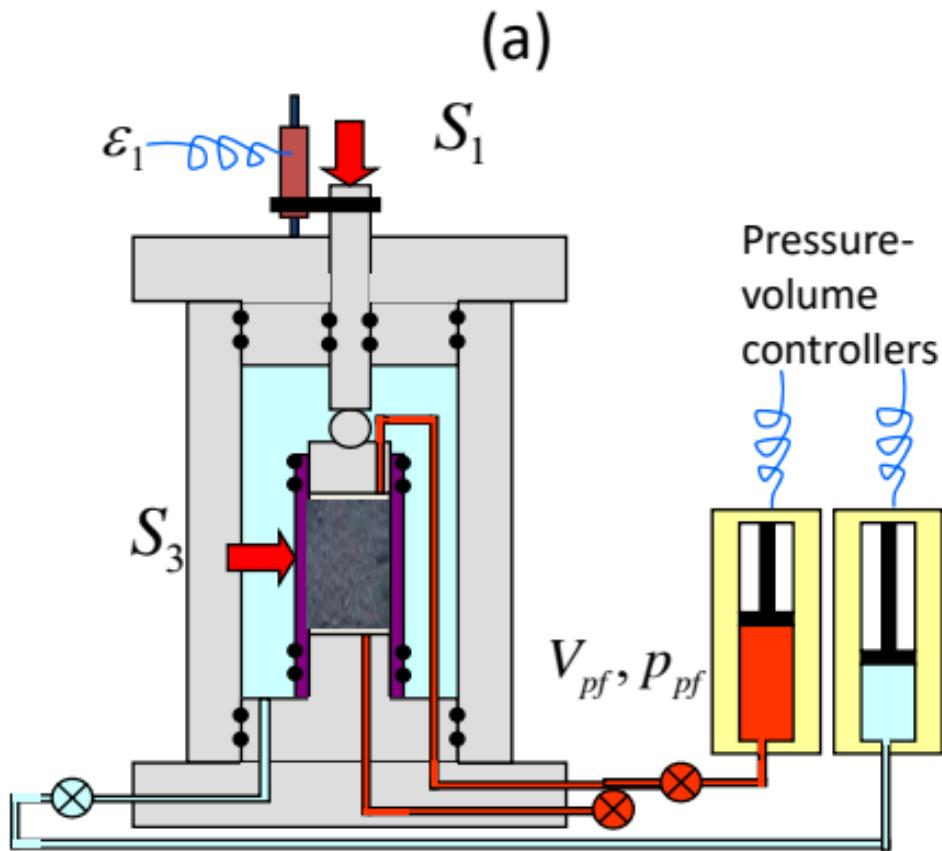


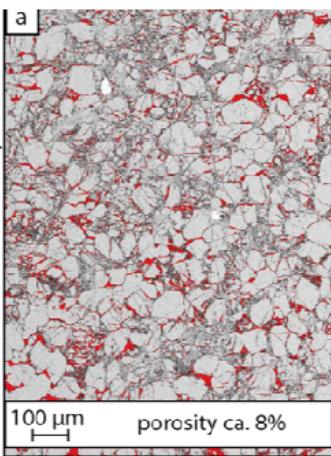
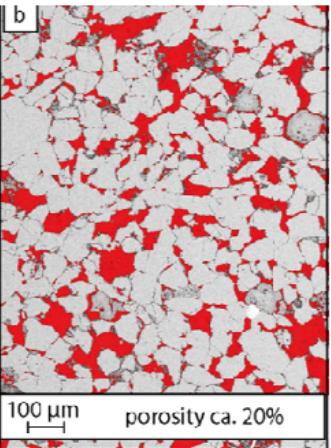
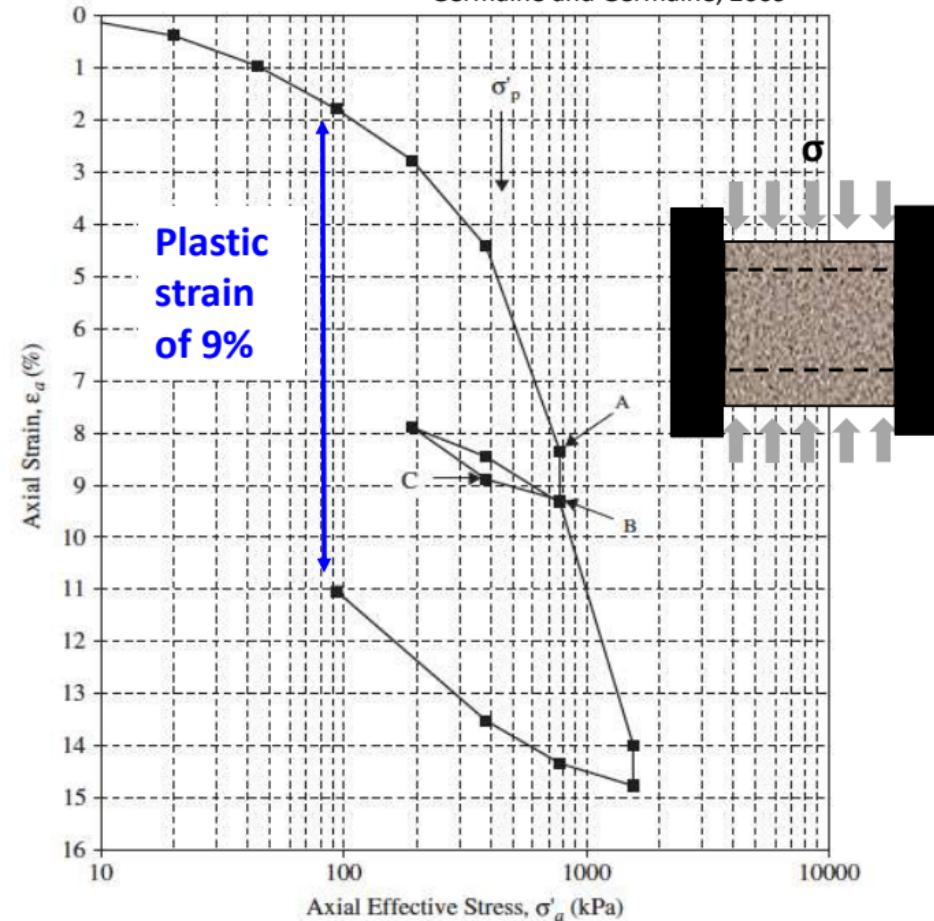




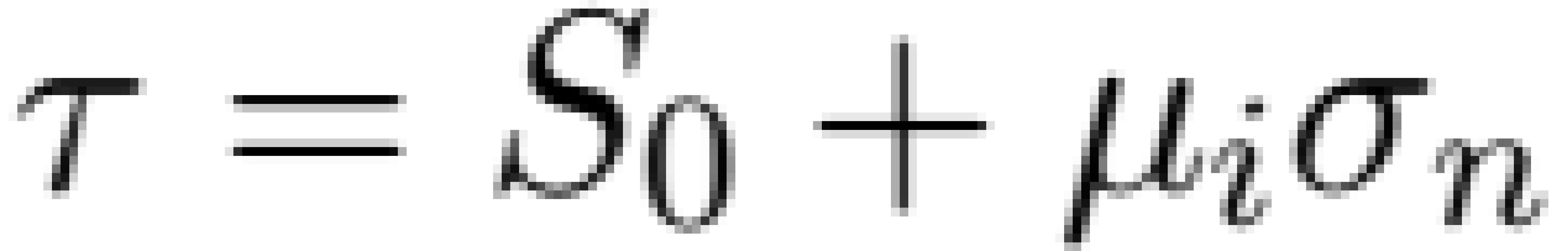


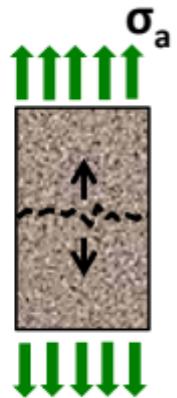








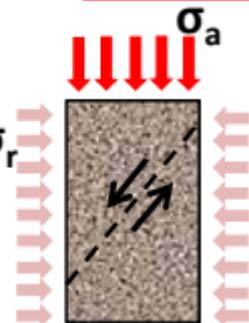


τ 

$$\sigma_n = T_s$$

 T_s

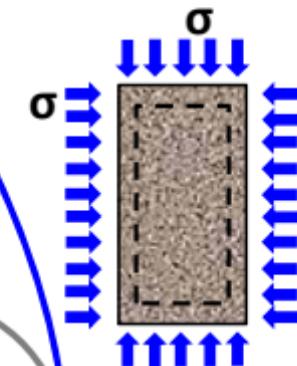
$$\boxed{\tau = \mu_i \sigma_n + S_0}$$



1

 μ_i

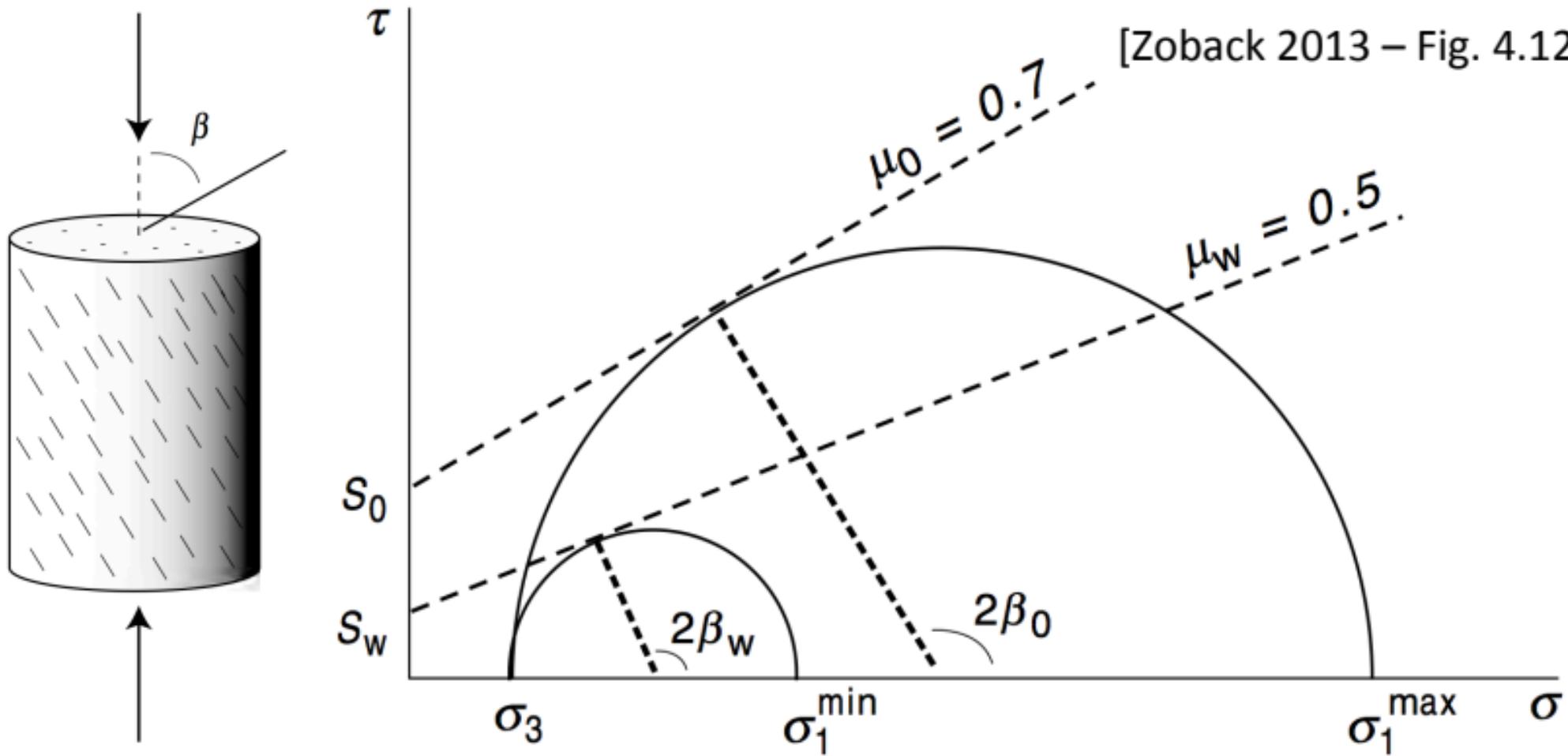
$$\boxed{\mathcal{E}_V^p = \mathcal{E}_{Vcrit}^p}$$

 σ_n





[Zoback 2013 – Fig. 4.12]

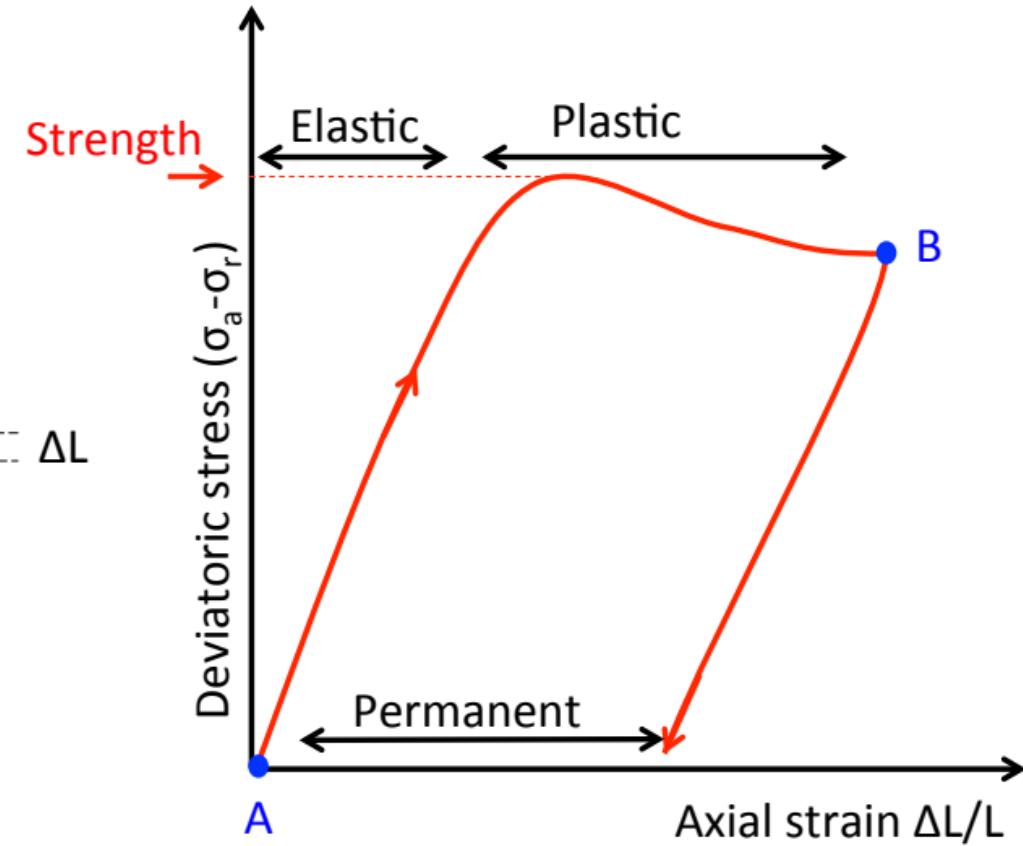
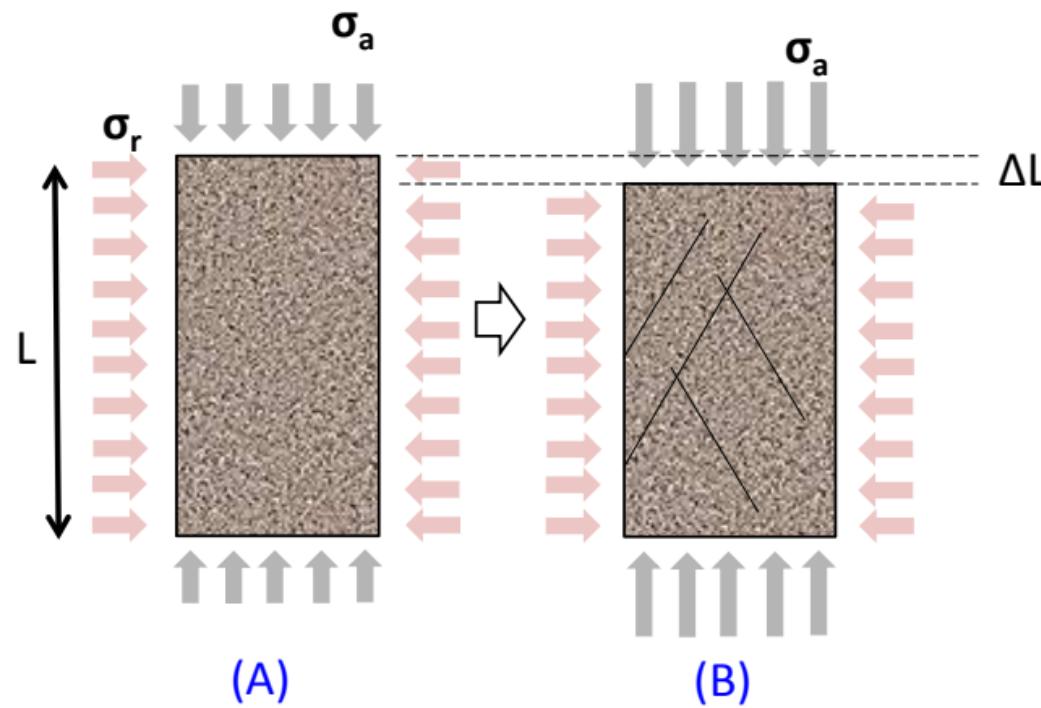


Rock deformation

Relation strain V.S. stress

Elastic (Young modulus)

Plastic (~Viscosity)



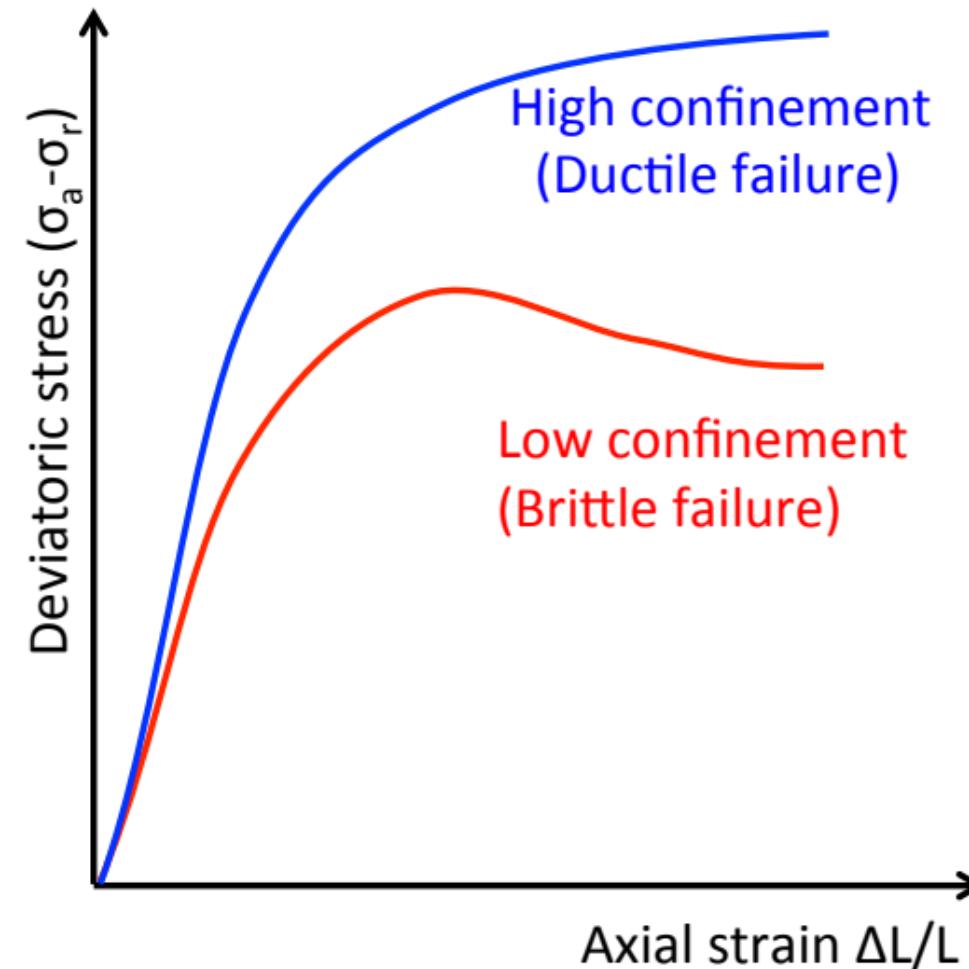
a. This is an undeformed cylinder of rock.

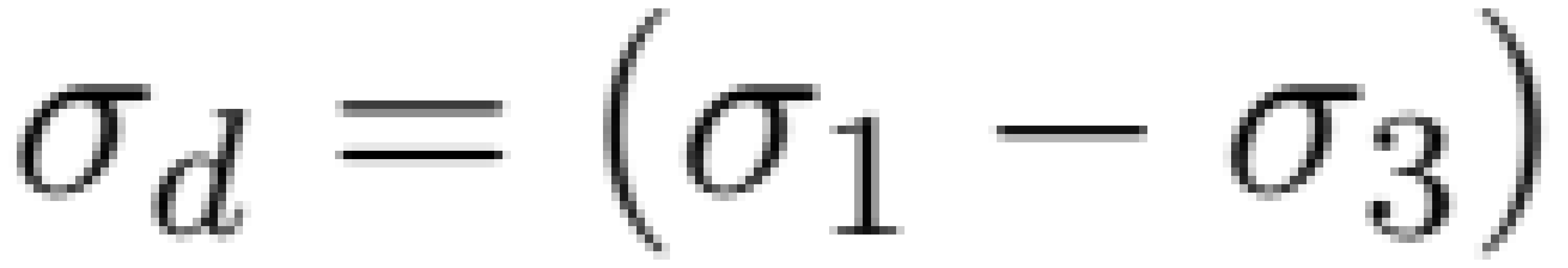


b. This cylinder was subjected to high confining pressure (uniform in all directions) and, at the same time, compression from above. It deformed in a ductile manner, becoming shorter and fatter.



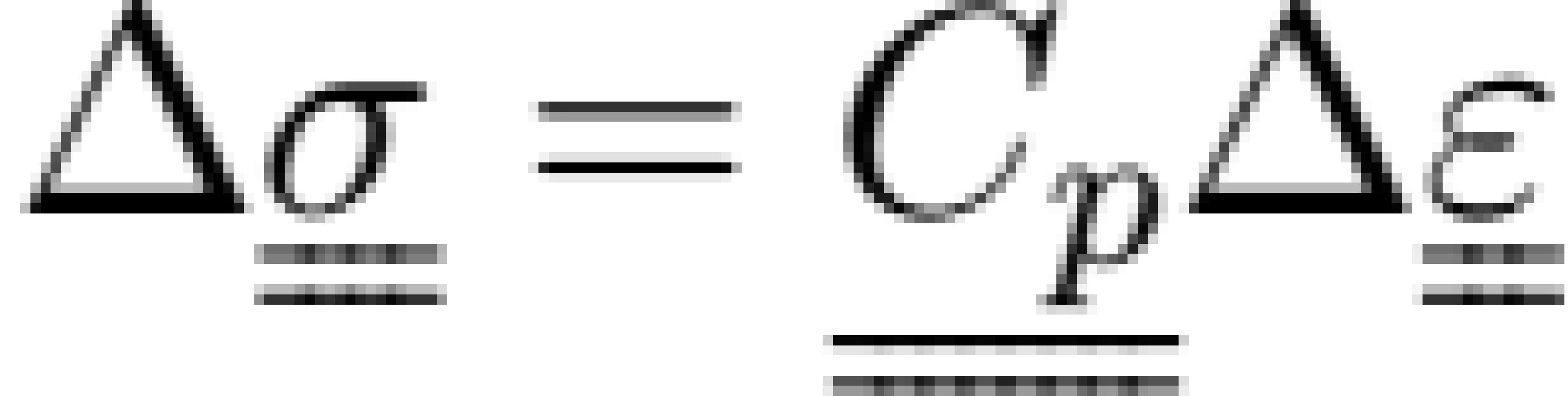
c. An identical cylinder was subjected to the same amount of compression from above, but this time with a lower confining pressure. It deformed in a brittle manner, with many large fractures.



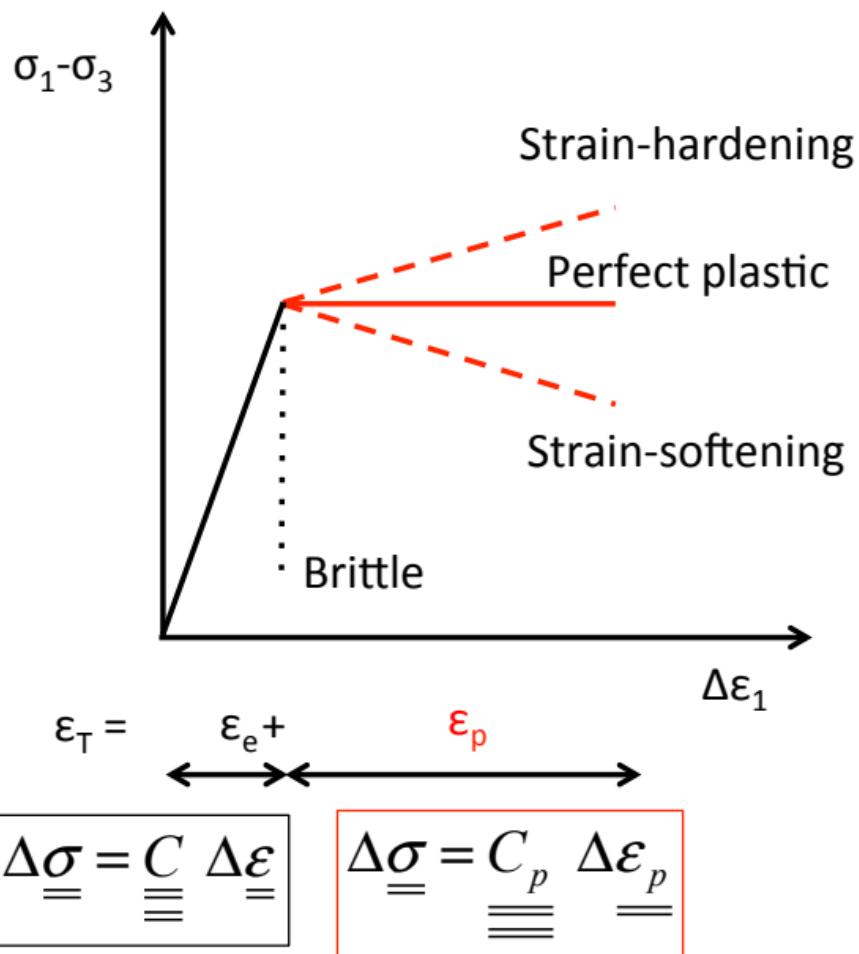
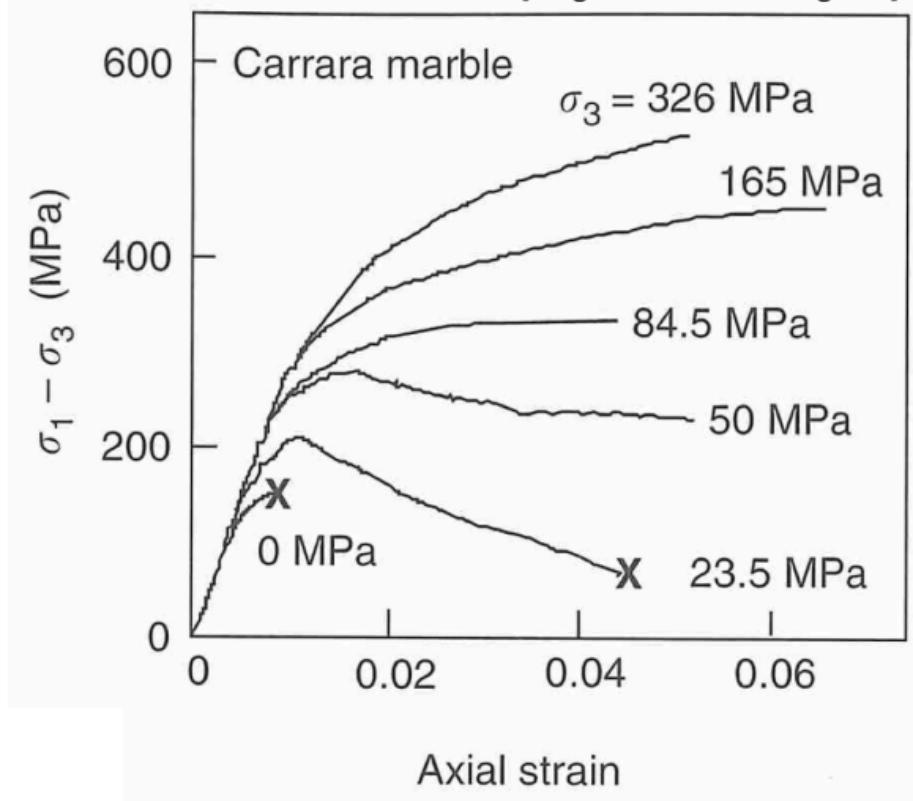




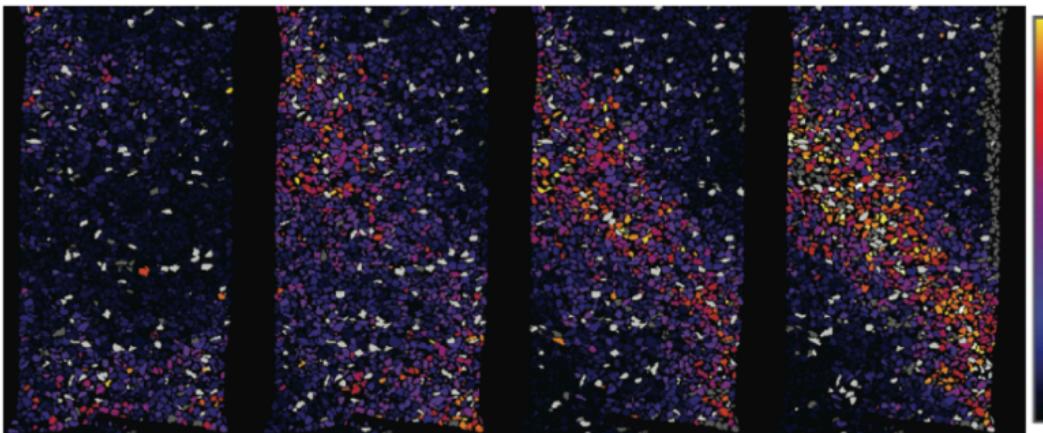
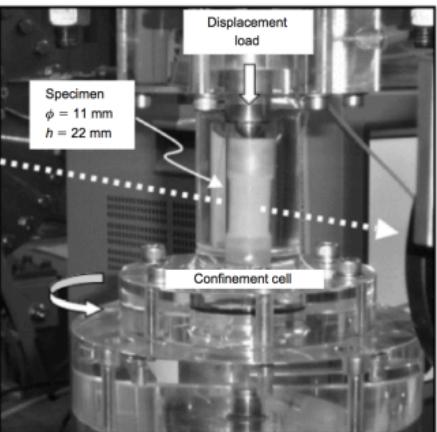




[Jaeger et al. 2007 – Fig. 4.5]



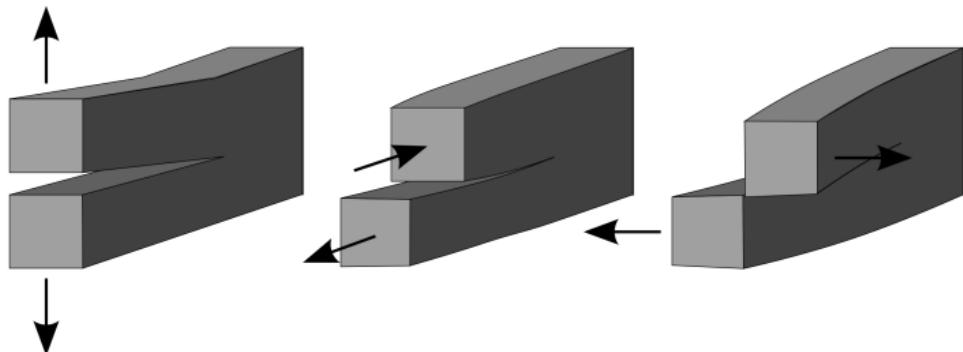
(a) Uncemented or poorly cemented rock →



grain friction, dilation, grain crushing/rotation

(b) Cemented rock →

Propagation of microfractures, grain friction/crushing

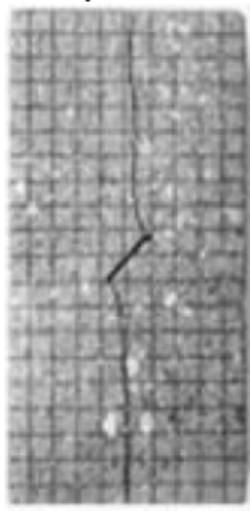


Stress intensification at
the tip of fractures

Propagation starts at
fracture tips

Napolitan Tuffo

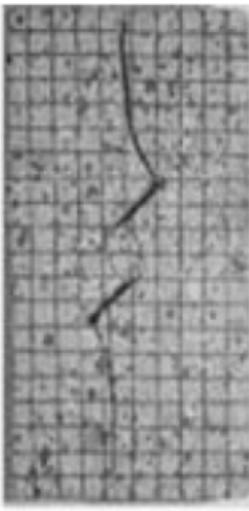
[Hall et al. 2006 – Pure Appl. Geophys.]



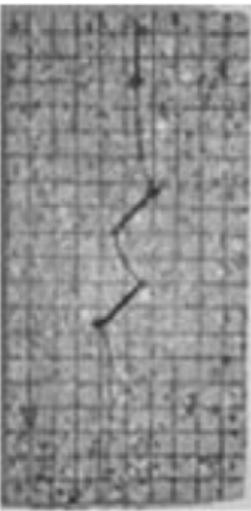
Single flaw



$\beta=45^\circ$

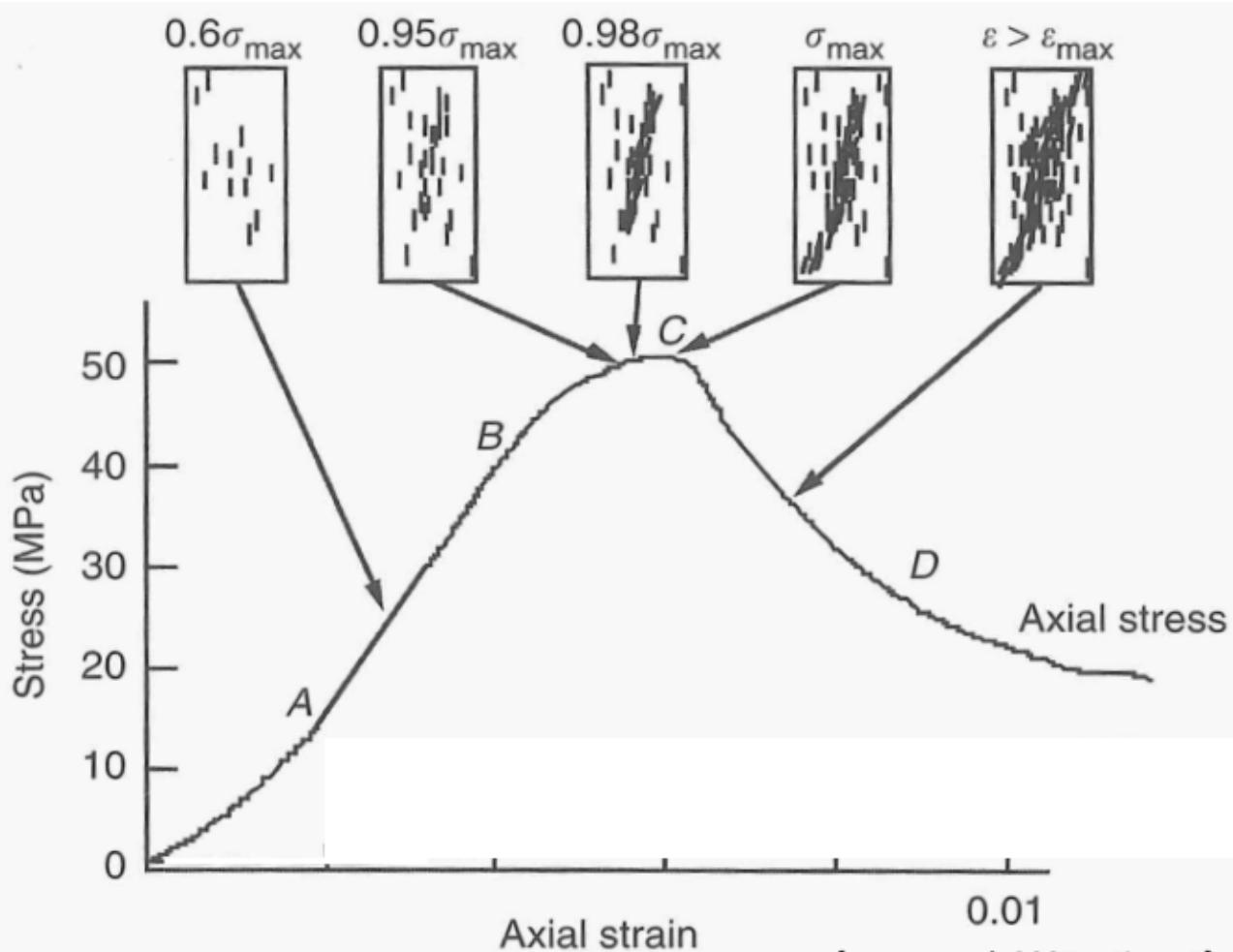


$\beta=105^\circ$



$\beta=120^\circ$

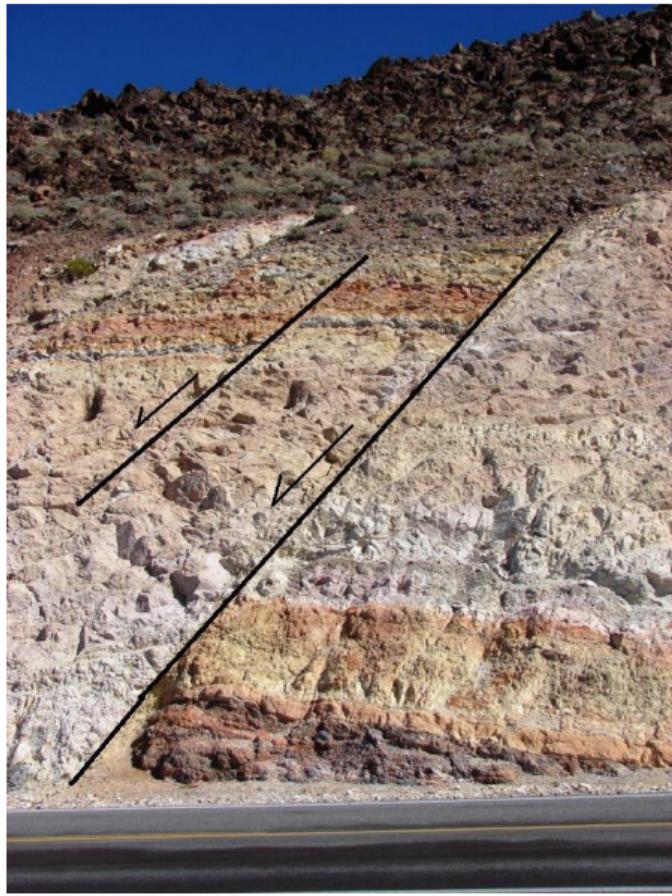






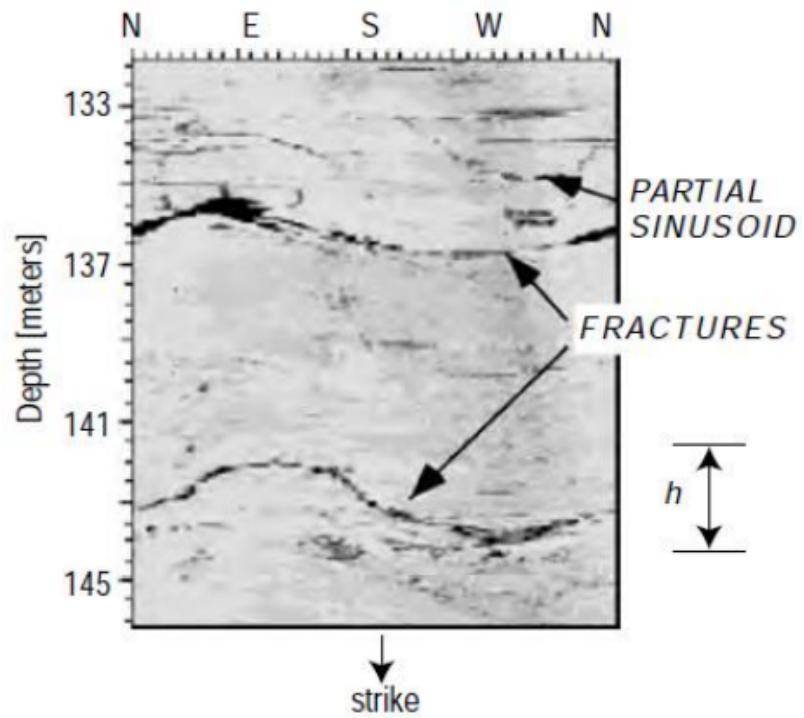
Outcrop of a normal fault in Split Mountain gorge

<http://geology.csupomona.edu/janourse/TectonicsFieldTrips.htm>

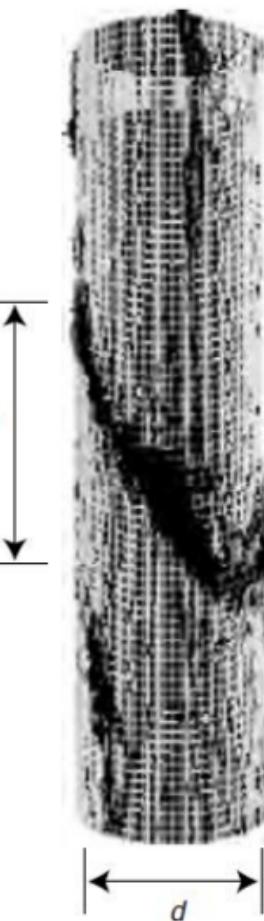


<http://geotripperimages.com/images/DSC03438%20Charlie%20Brown%20b.jpg>

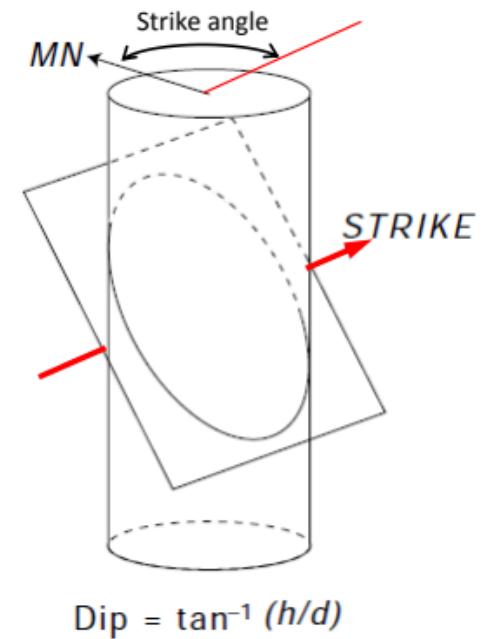
**(a) Un-wrapped image
(ultrasonic)**



(b) 3D-representation



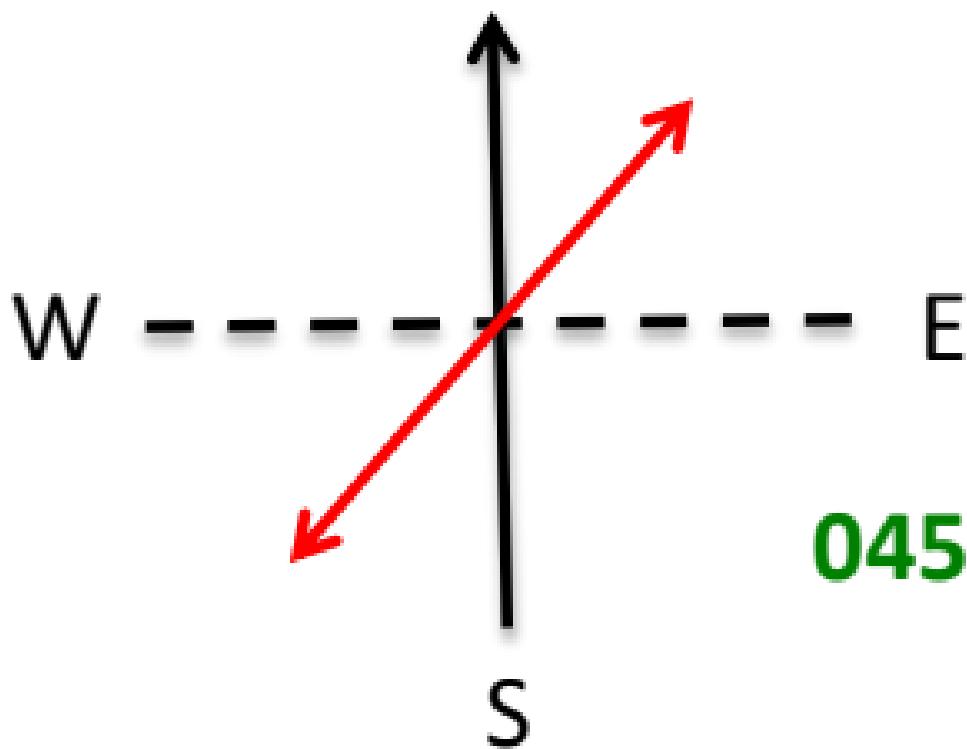
(c) Interpretation



[Zoback 2013 - Figure 5.3]



N45°E==S45°W



045° == 225°

Geologic map

Figure from Prof. Prodanovic



a)

$N30^\circ E$, $45^\circ NW$

or

030° , $45^\circ NW$



b)

$N10^\circ W$, $80^\circ NE$

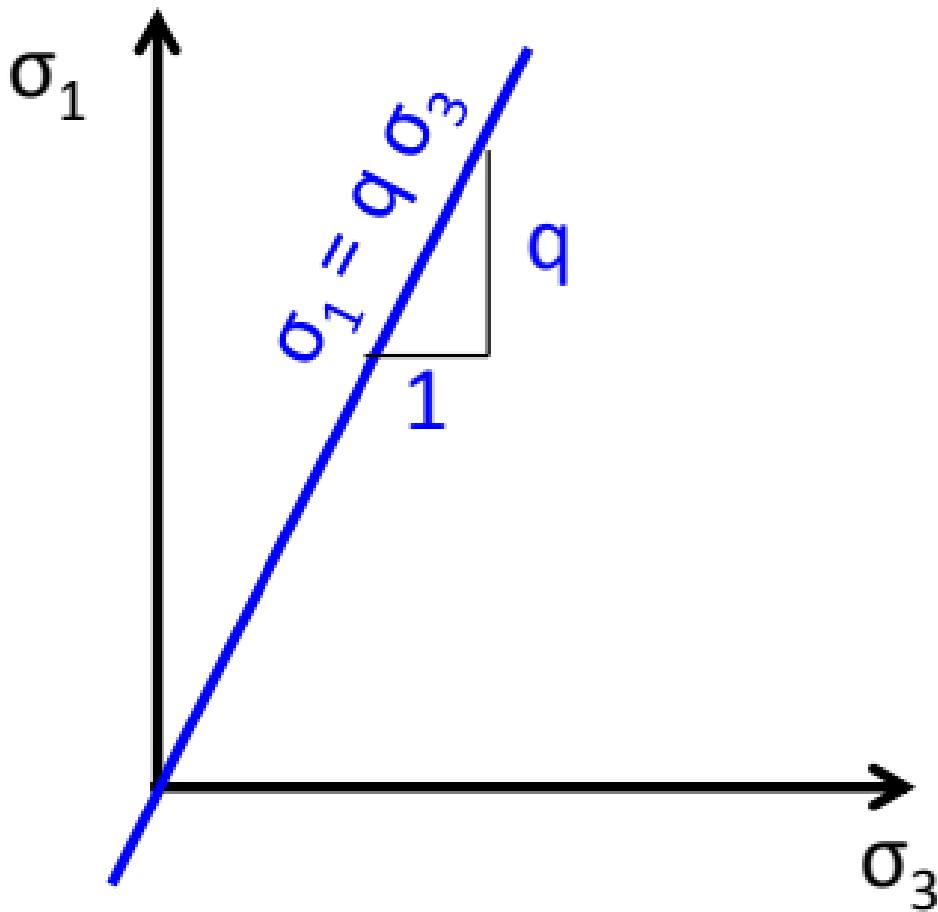
or

350° , $80^\circ NE$



c)

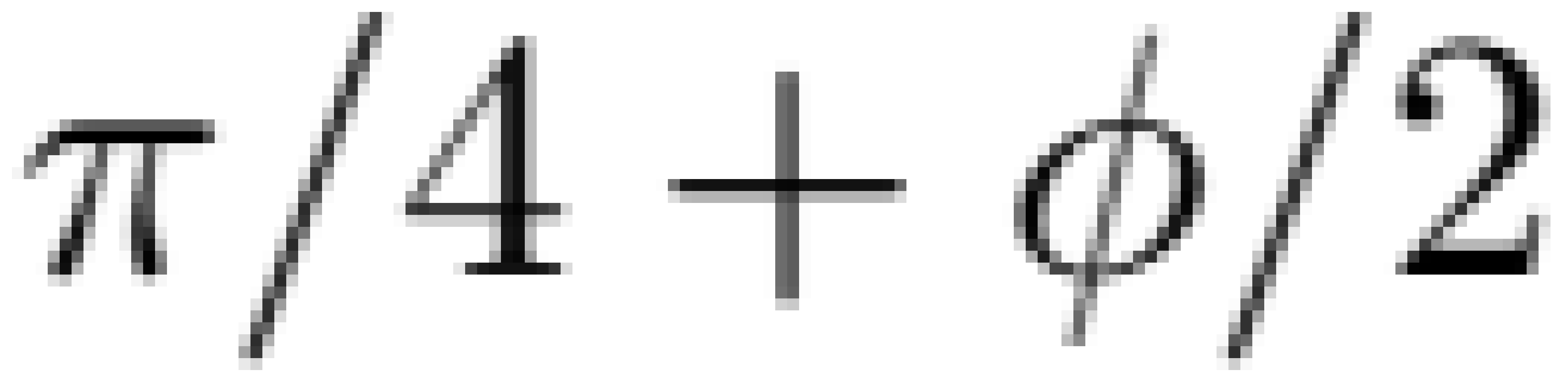
Symbols for
horizontal plane
and vertical plane

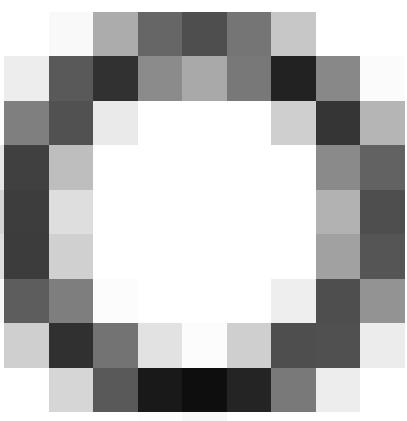
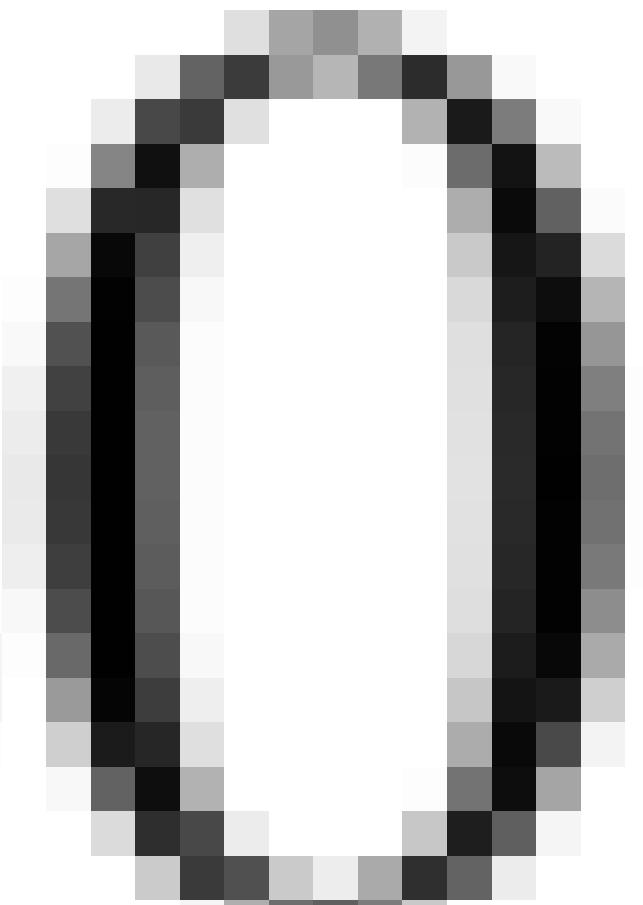
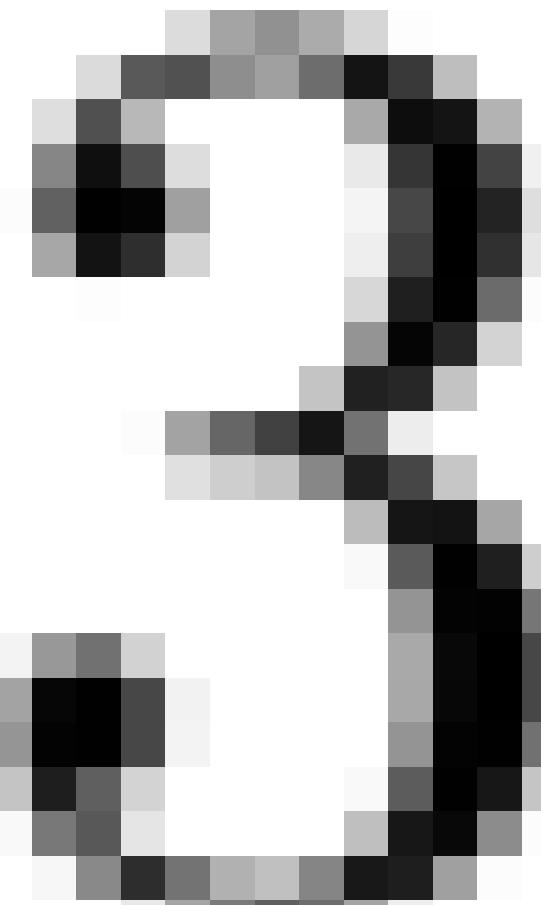
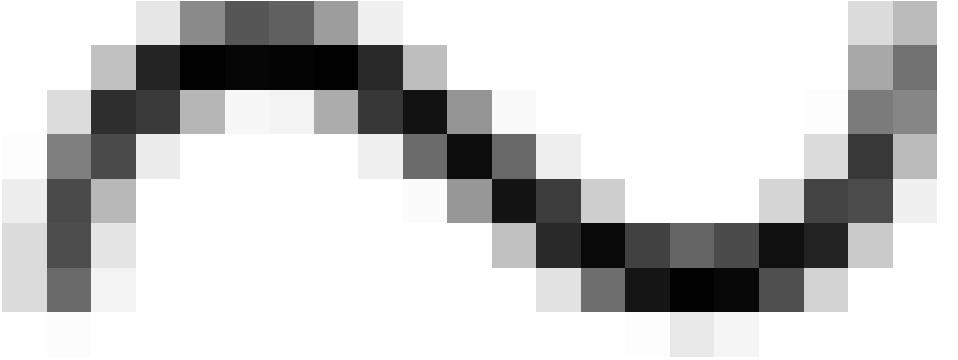


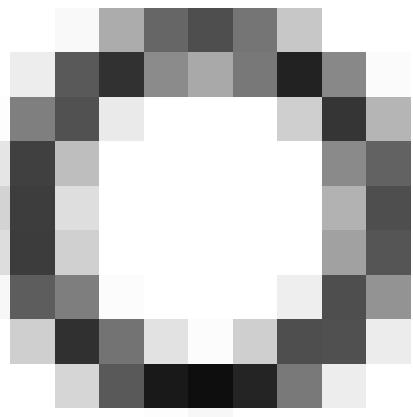
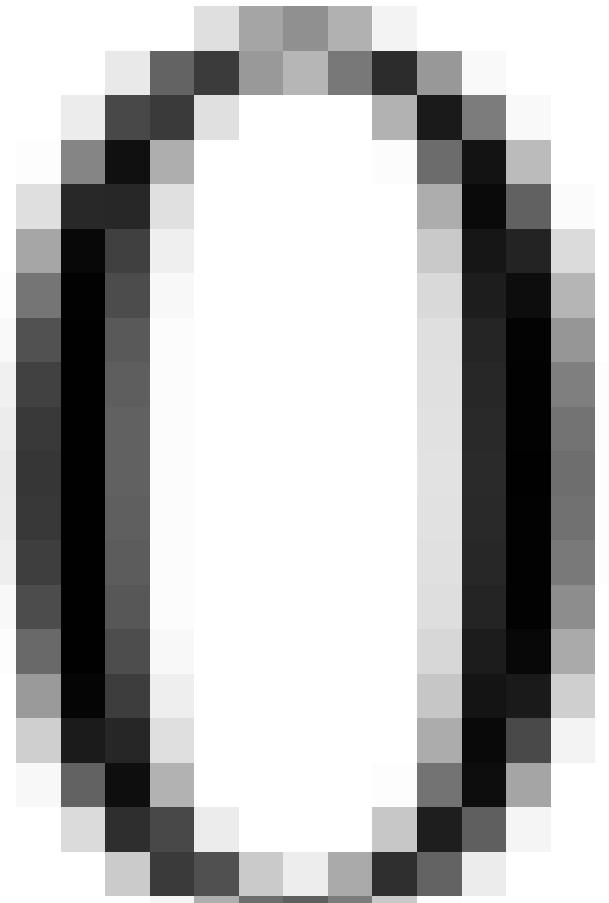
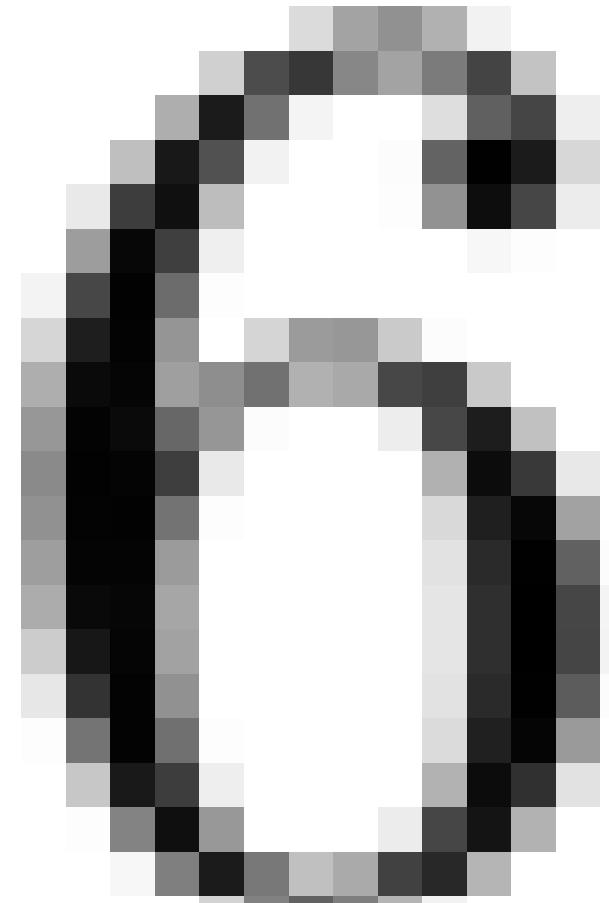
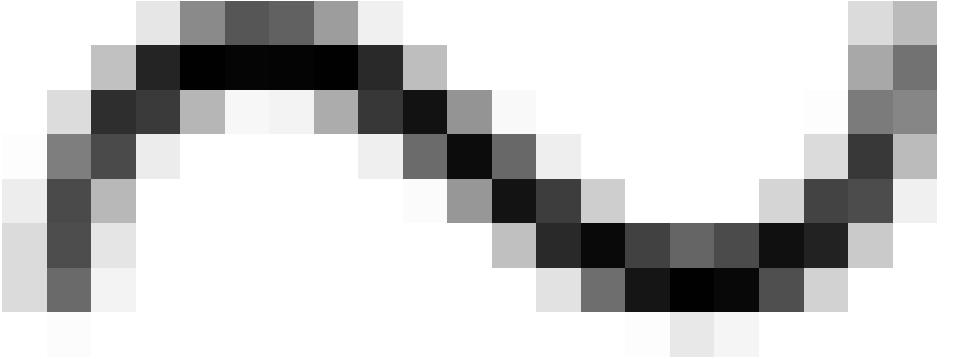


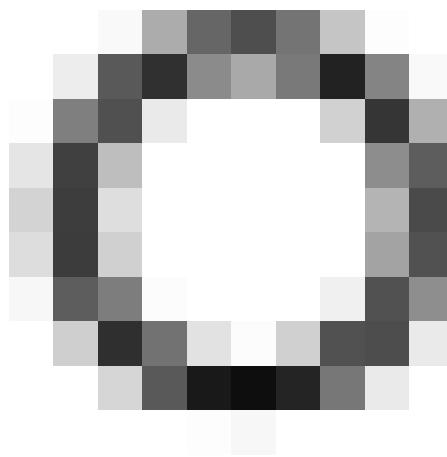
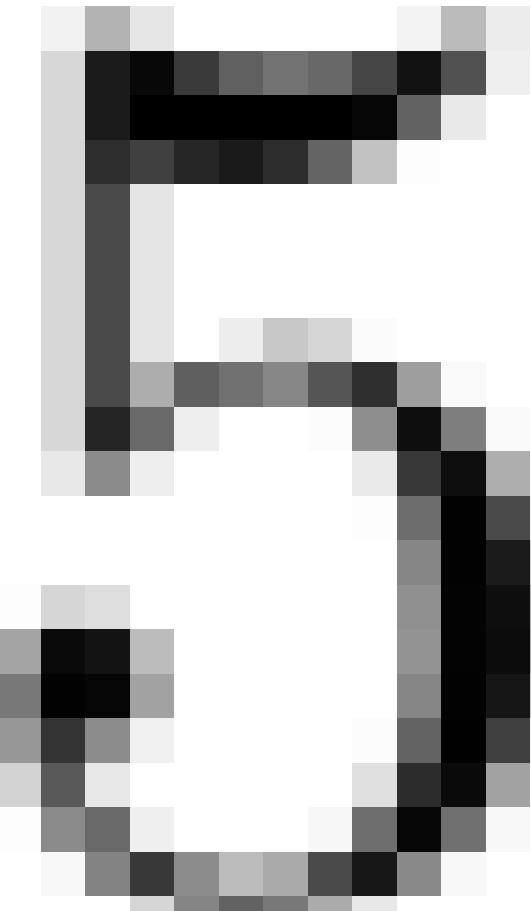
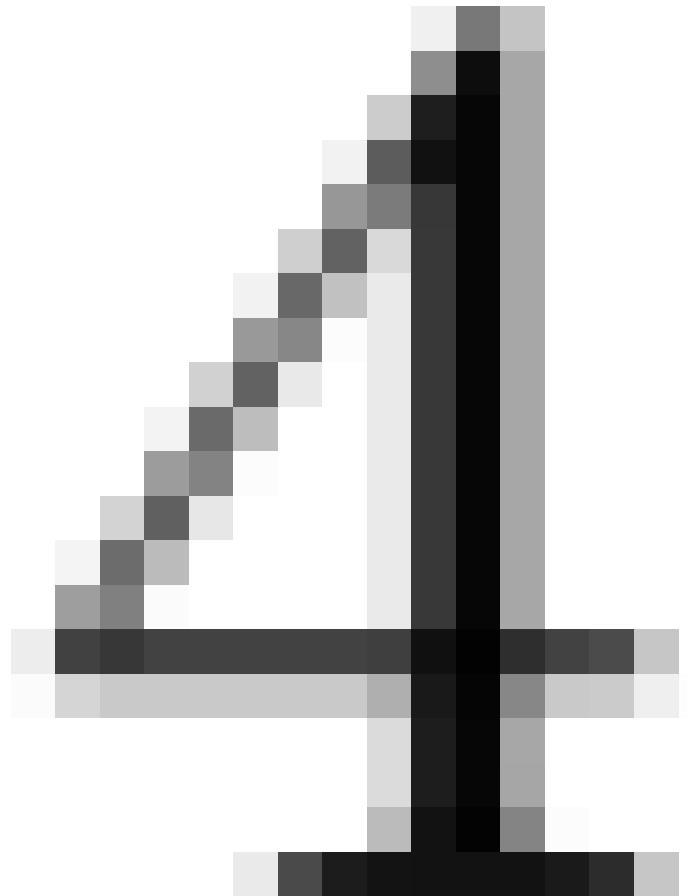
$$\begin{aligned} q &= \sin^2 \varphi + 1 \\ &= 1 - \sin^2 \varphi \end{aligned}$$

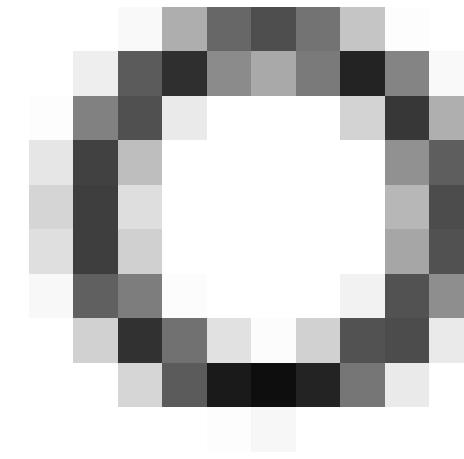
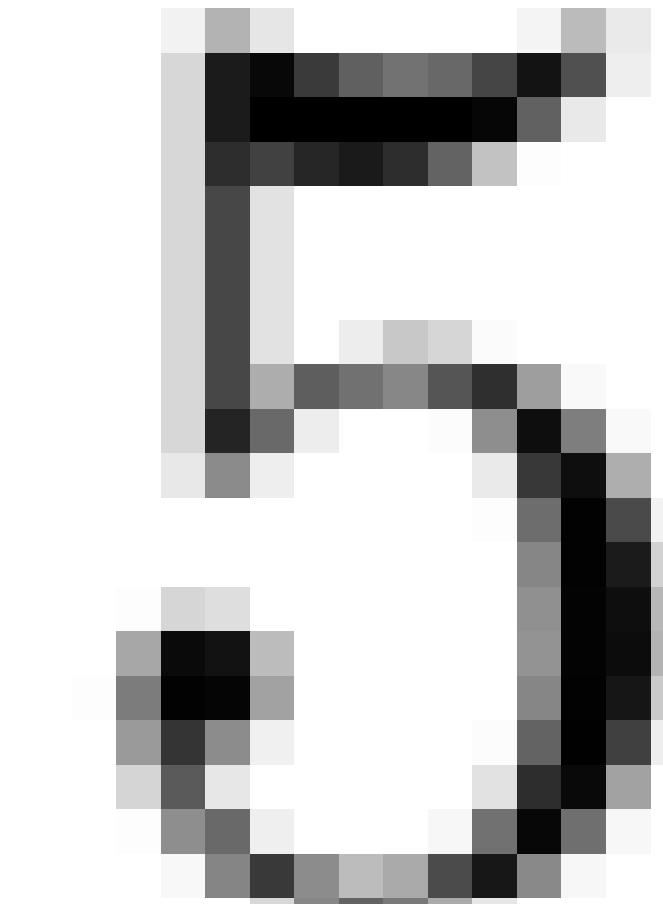
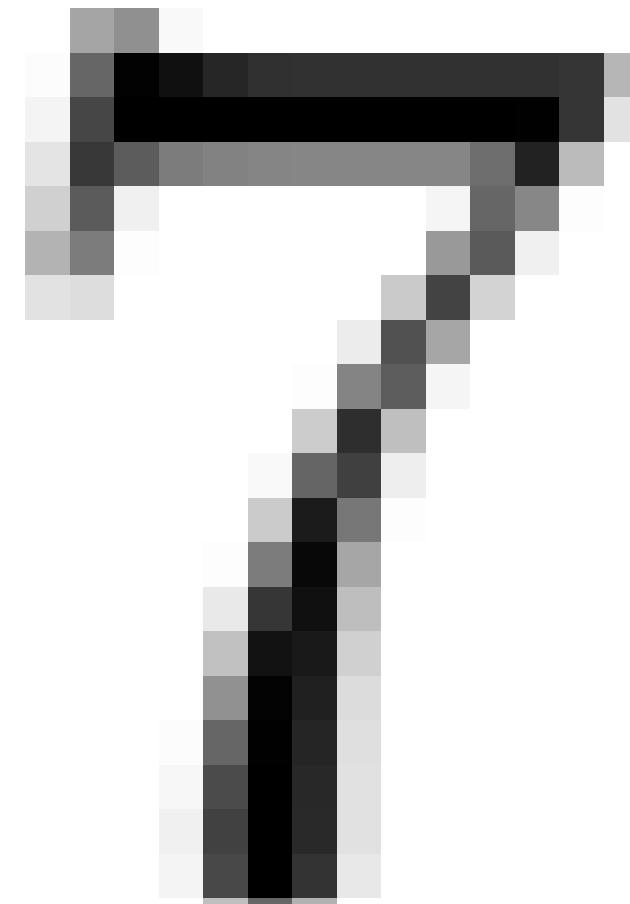
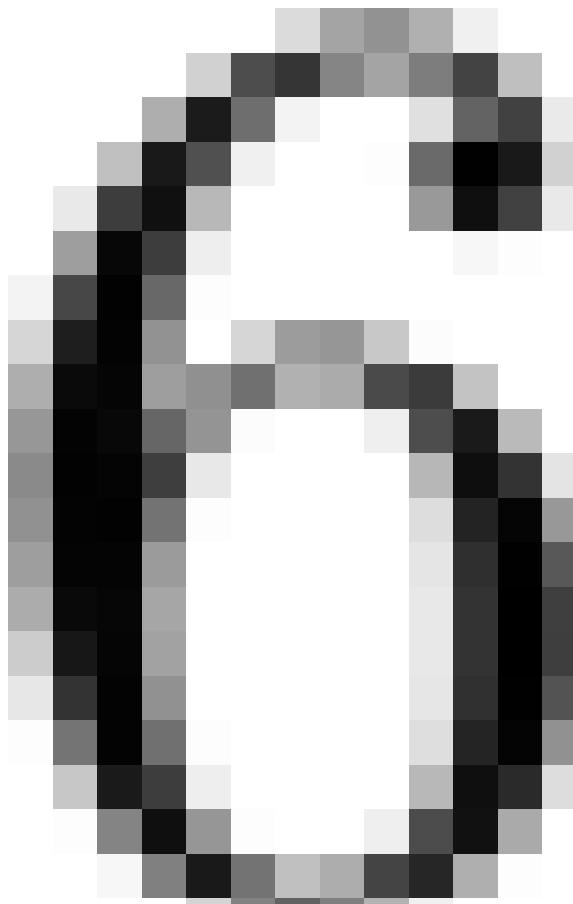


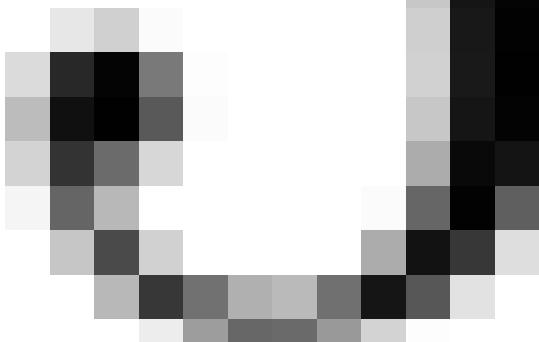
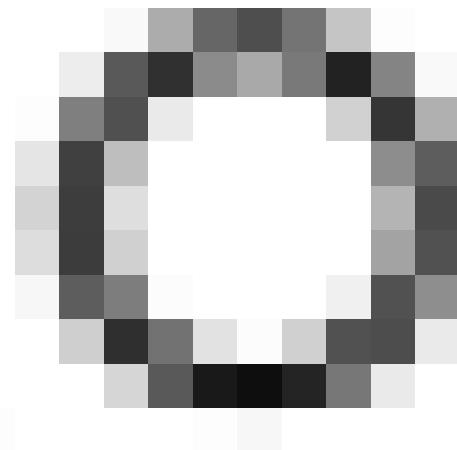
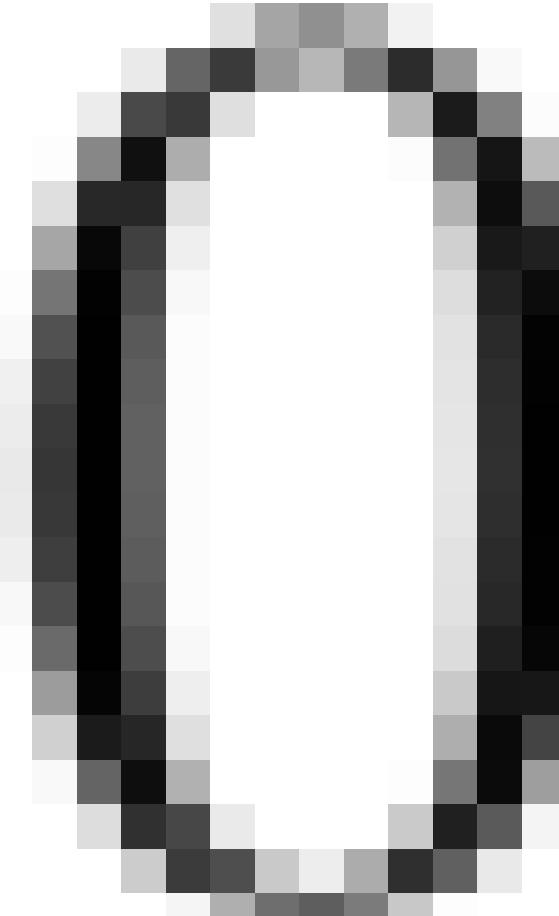
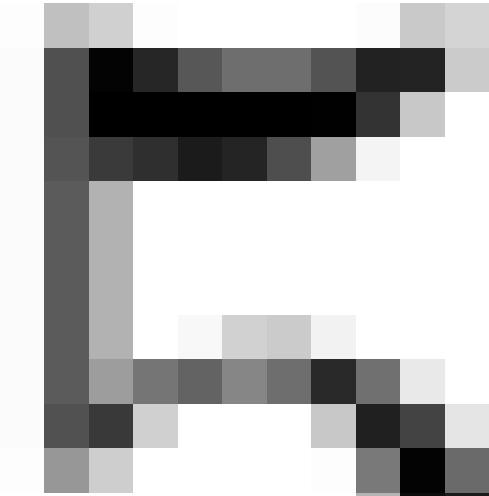


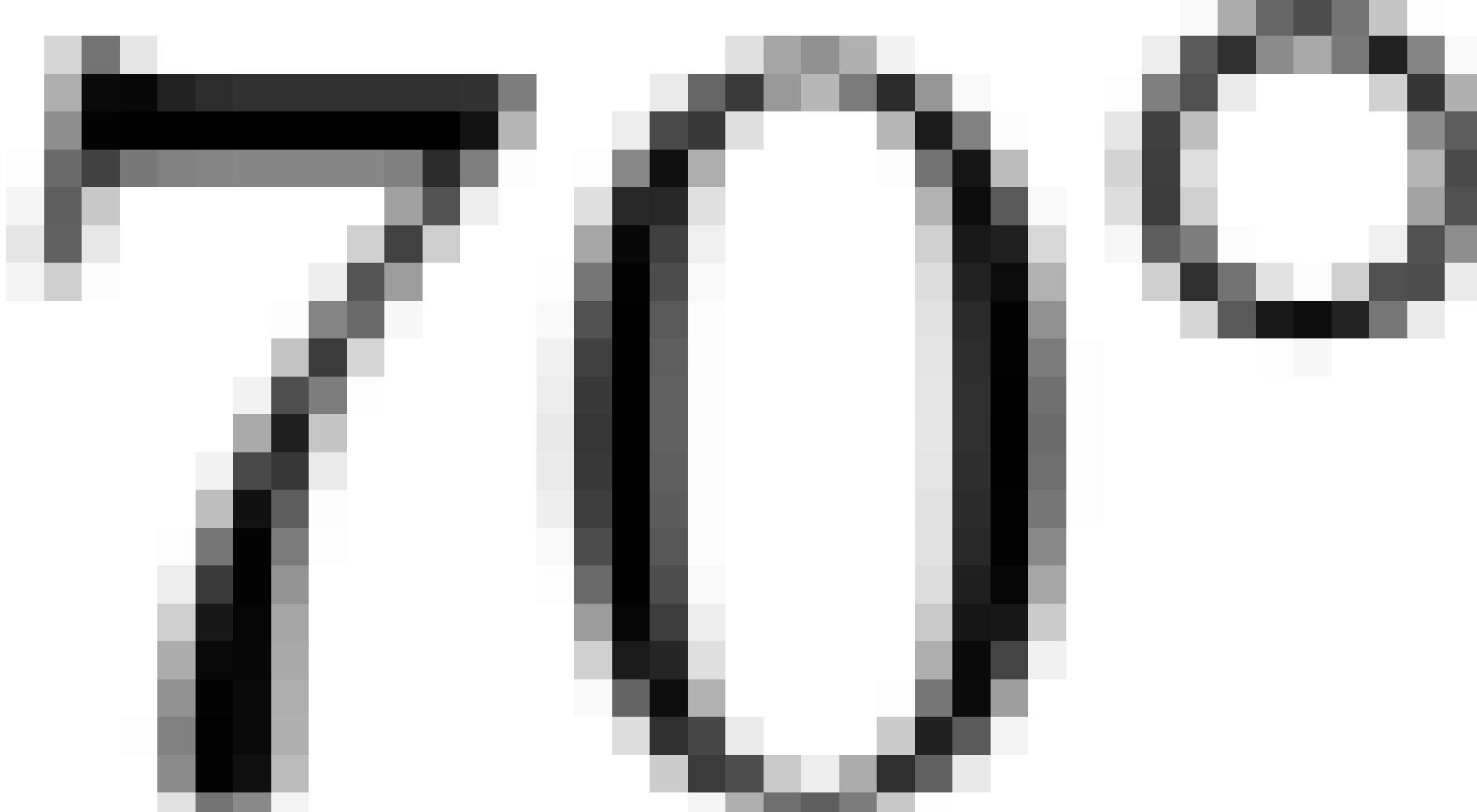


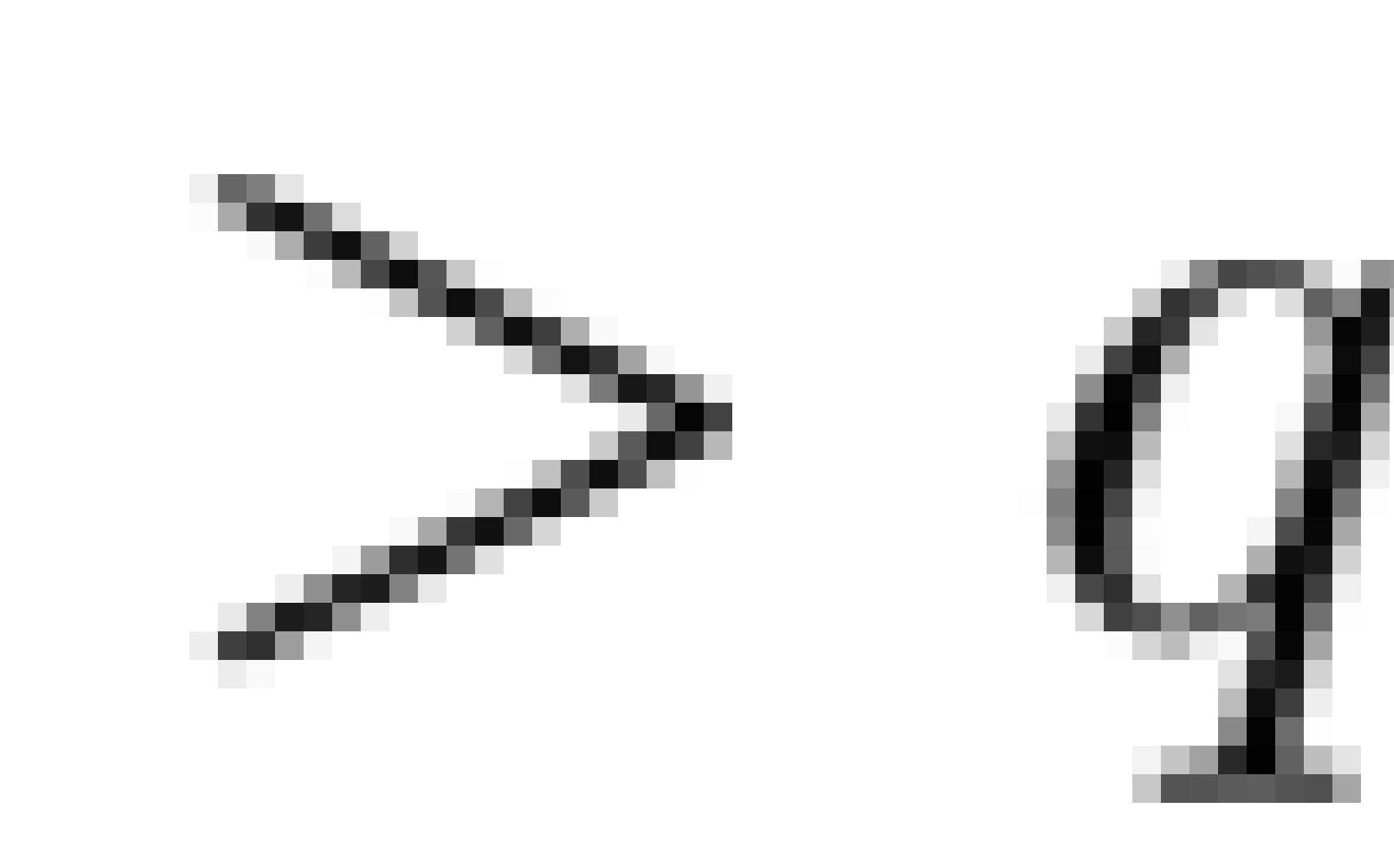












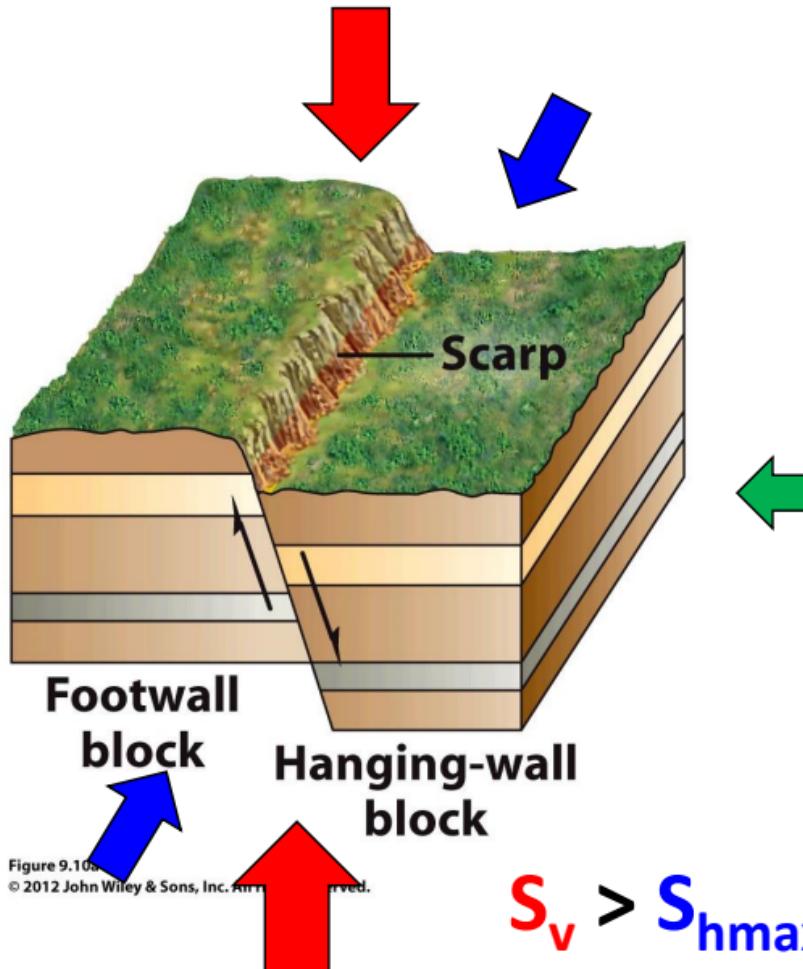


$$S_1 = s_v$$

$$S_2 = s_{H\max}$$

$$S_3 = s_{h\min}$$







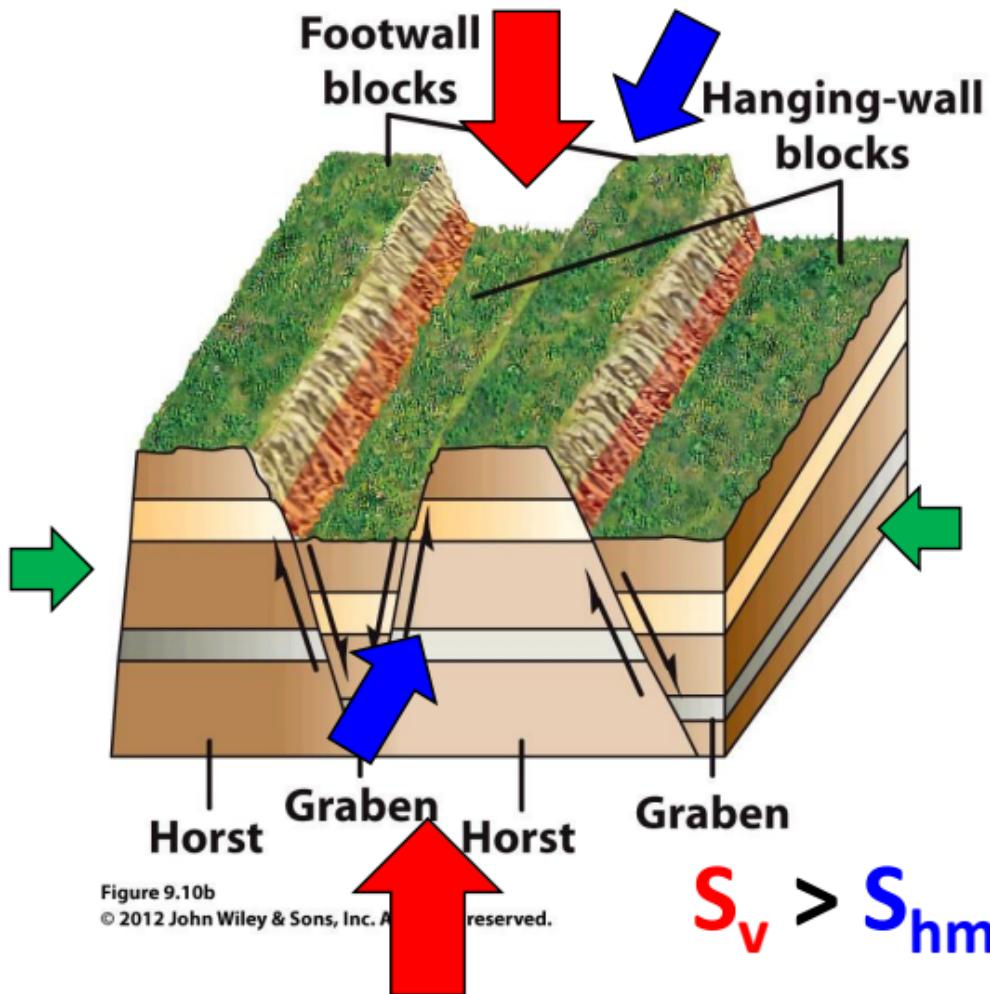


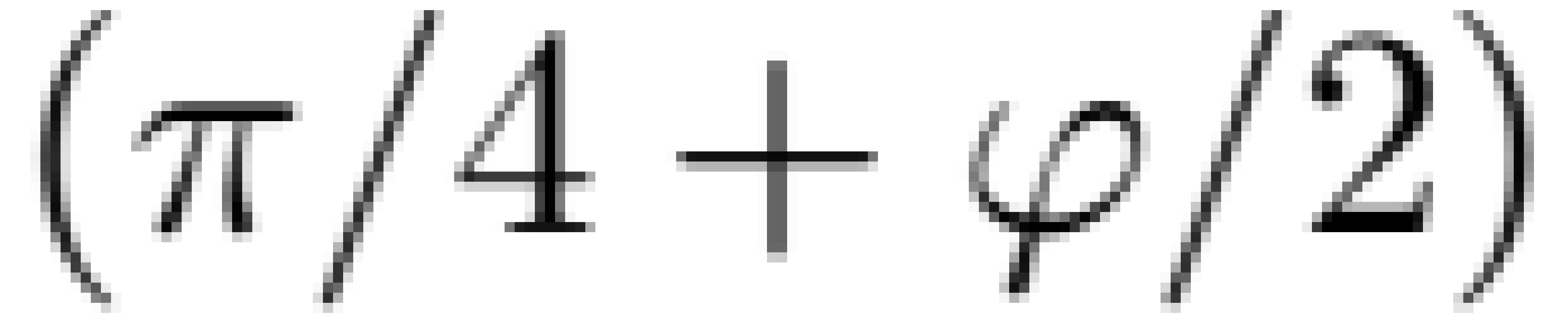
Figure 9.10b
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$$S_v > S_{h\max} > S_{h\min}$$

$$S_1 = S_{H\max}$$

$$S_2 = S_{h\min}$$

$$S_3 = S_v$$



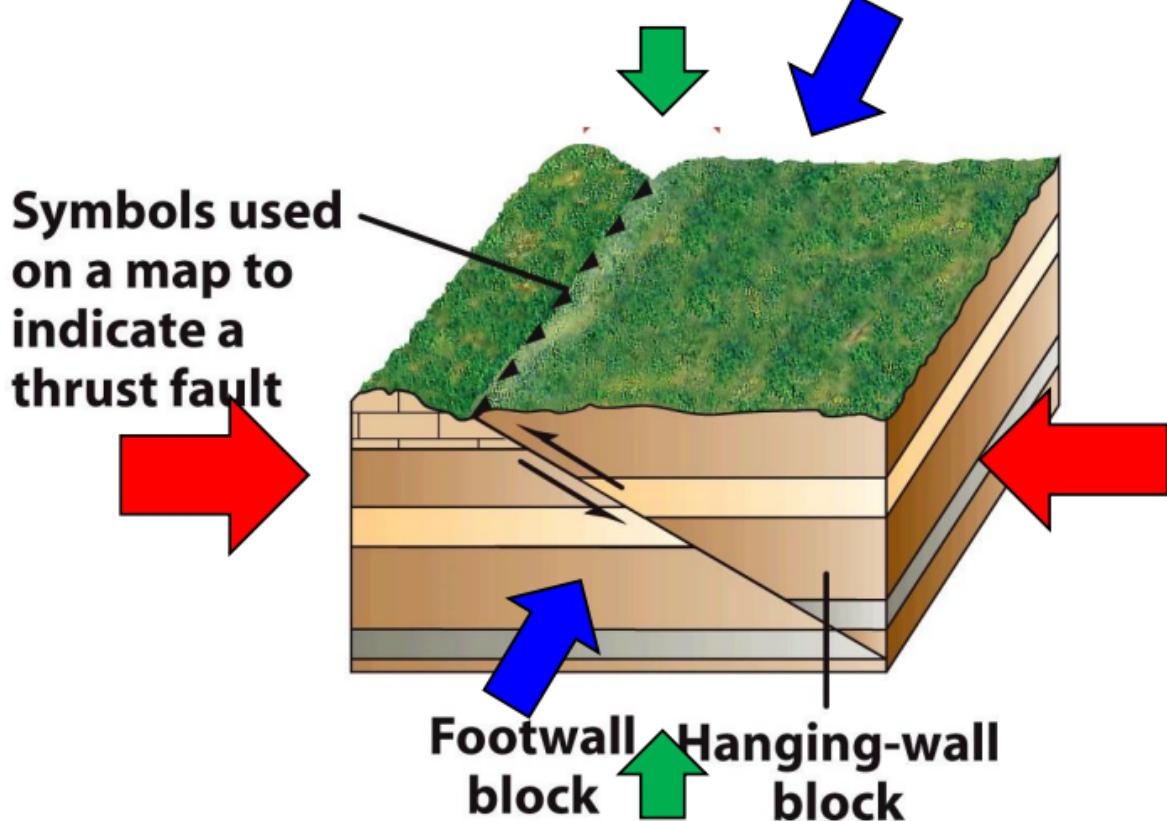


Figure 9.10d
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$$S_{H\max} > S_{h\min} > S_v$$

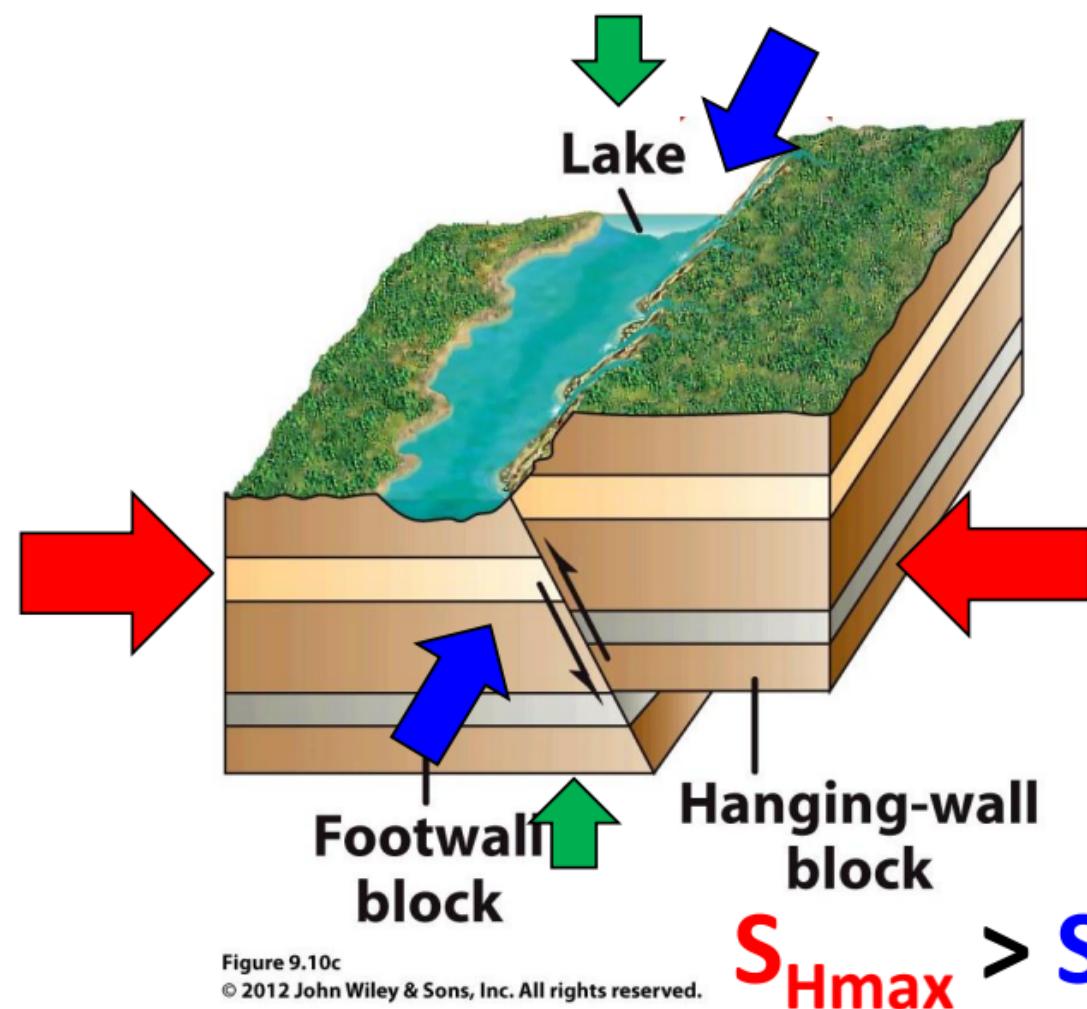


Figure 9.10c

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$$S_1 = S_{H\max}$$

$$S_2 = S_v$$

$$S_3 = S_{hmin}$$

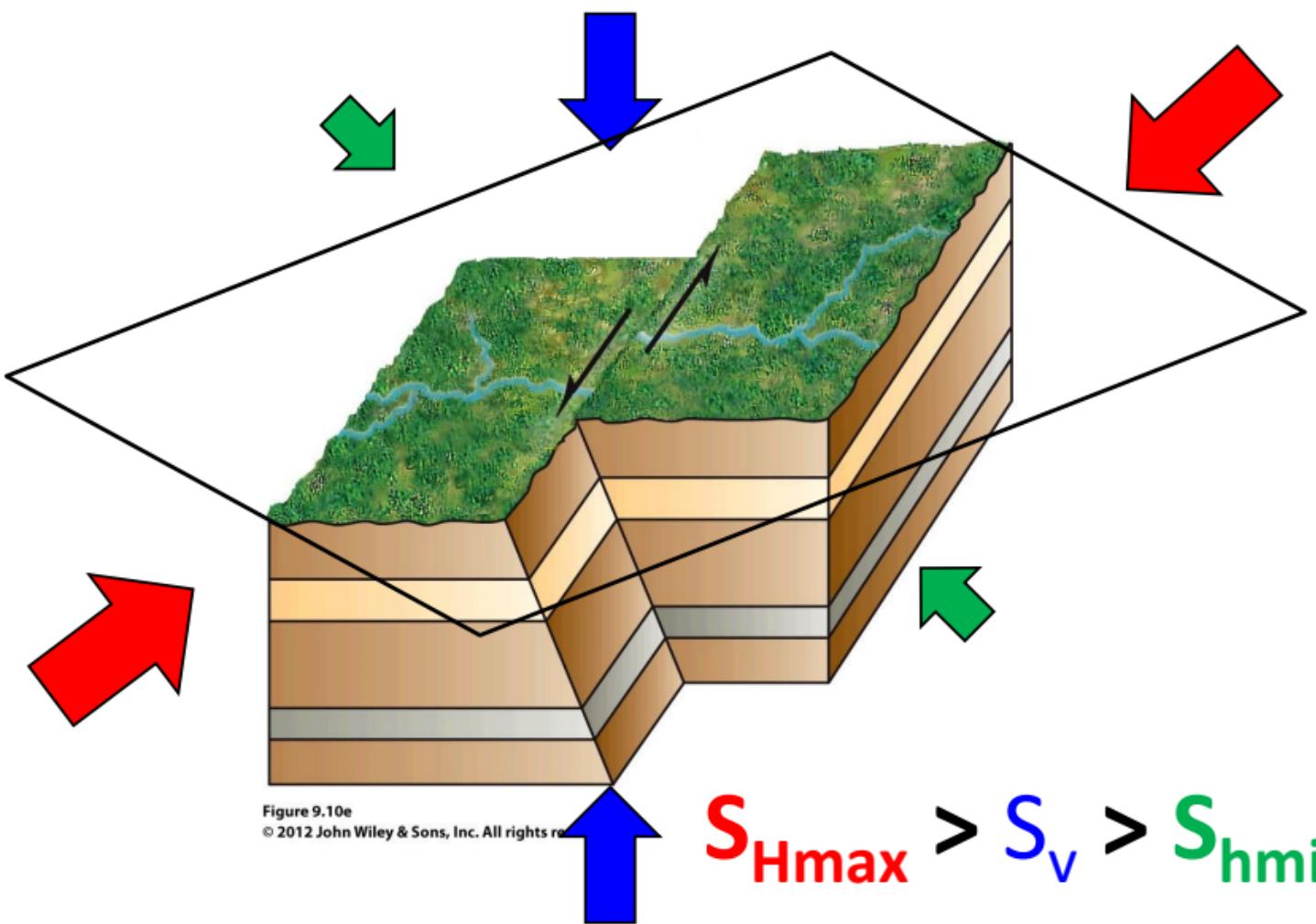
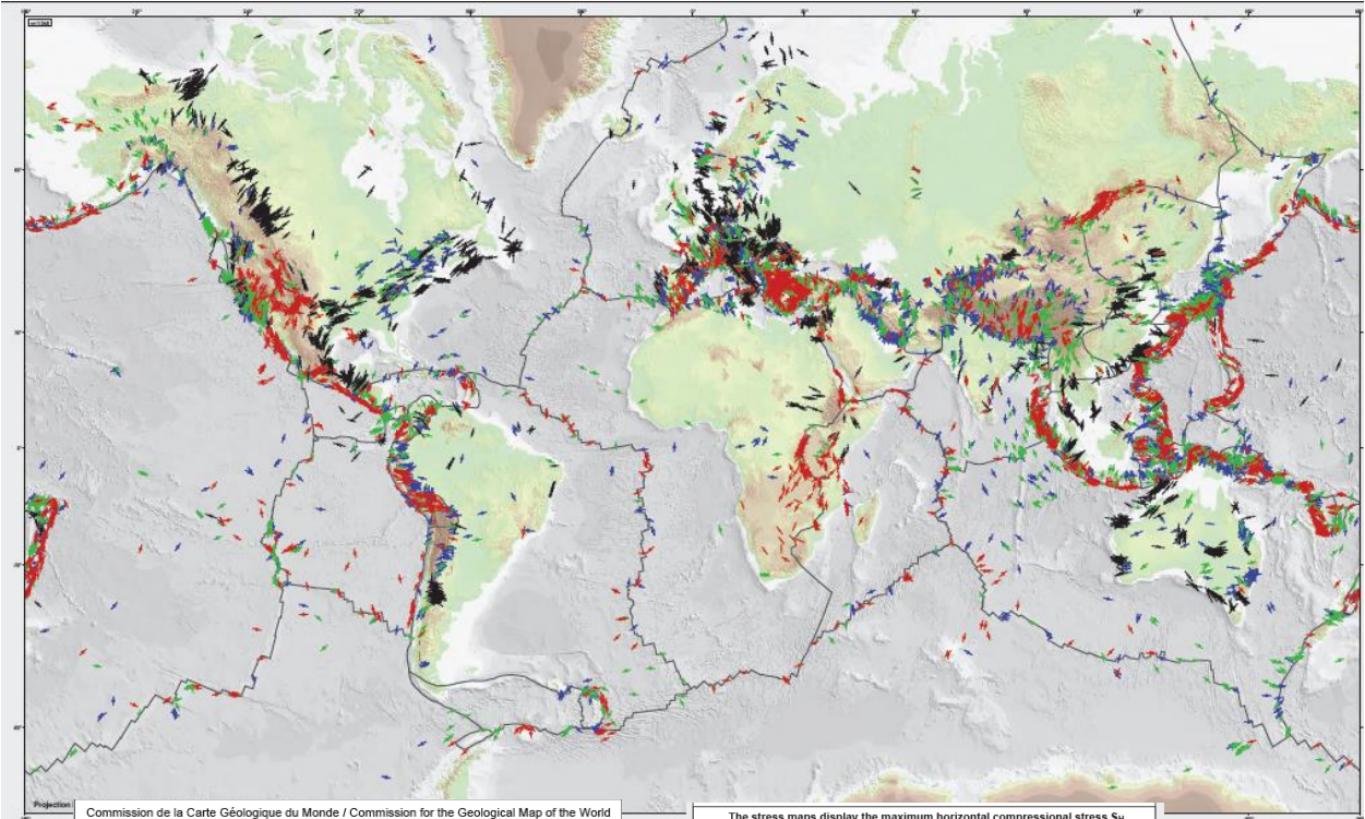


Figure 9.10e

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Commission de la Carte Géologique du Monde / Commission for the Geological Map of the World



WORLD STRESS MAP



2009 - 2nd edition, based on the WSM database release 2008

Helmholtz Centre Potsdam - GFZ German Research Centre for Geosciences

Authors

Oliver Heidbach, Mark Tingay, Andreas Barth, John Reinecker, Daniel Kurfürst, and Birgit Müller



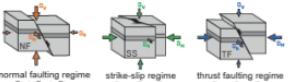
The stress maps display the maximum horizontal compressional stress S_H

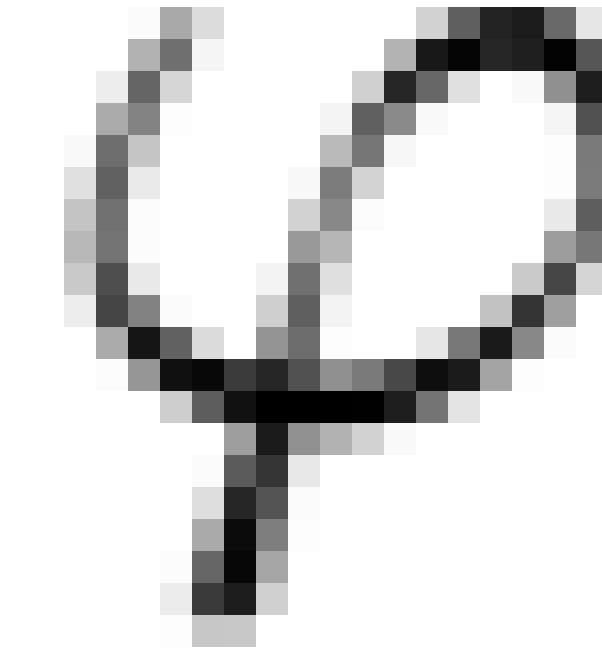
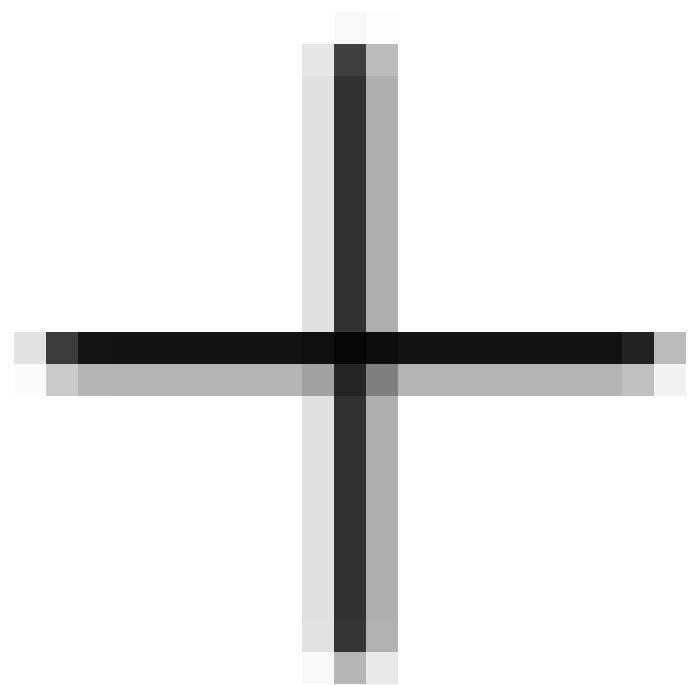
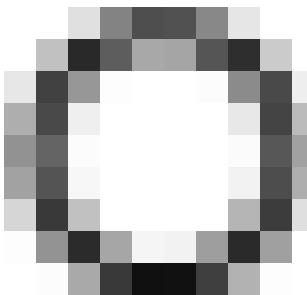
Method	Quality
focal mechanism	S_H is within $\pm 15^\circ$
breakouts	S_H is within $\pm 20^\circ$
drl. induced frac.	S_H is within $\pm 25^\circ$
overcoring	
hydro. fractures	
geol. indicators	

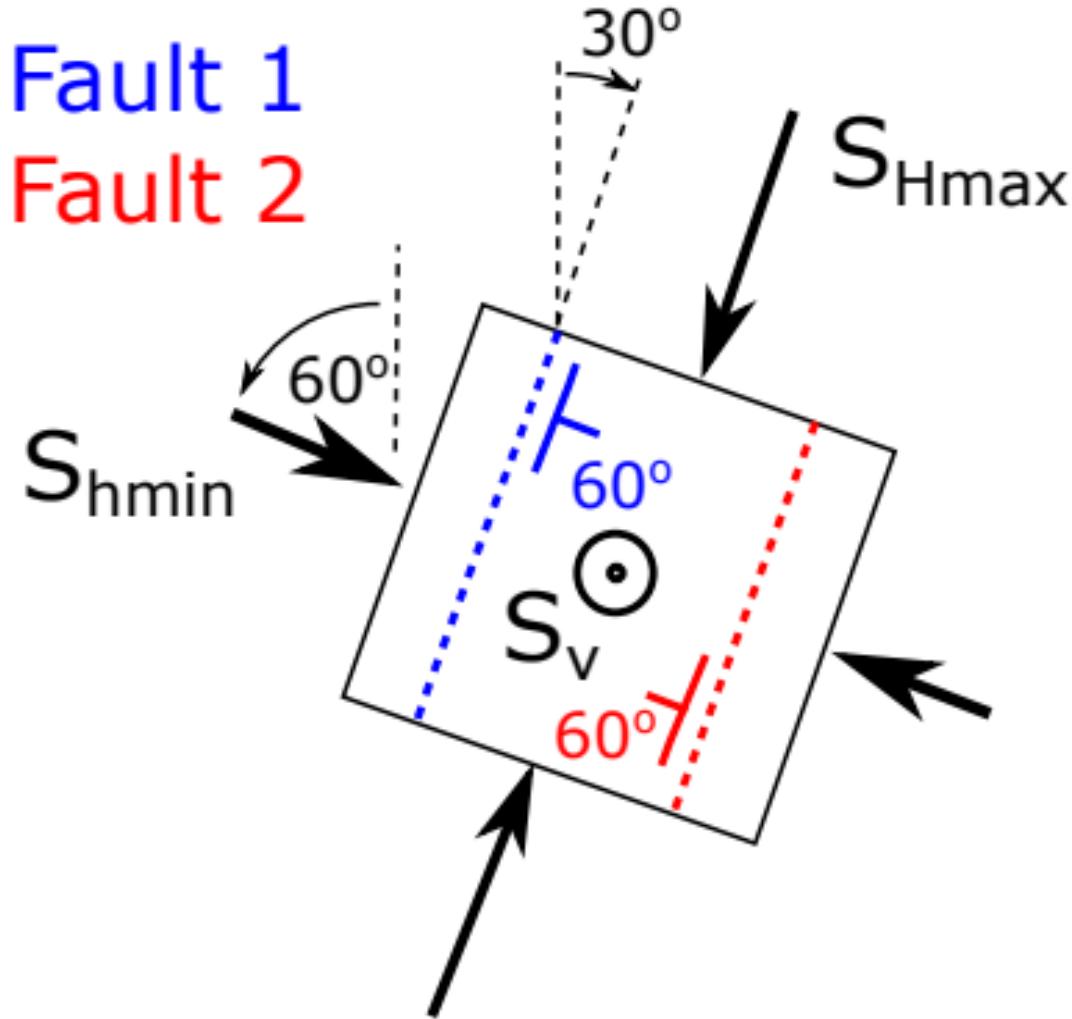
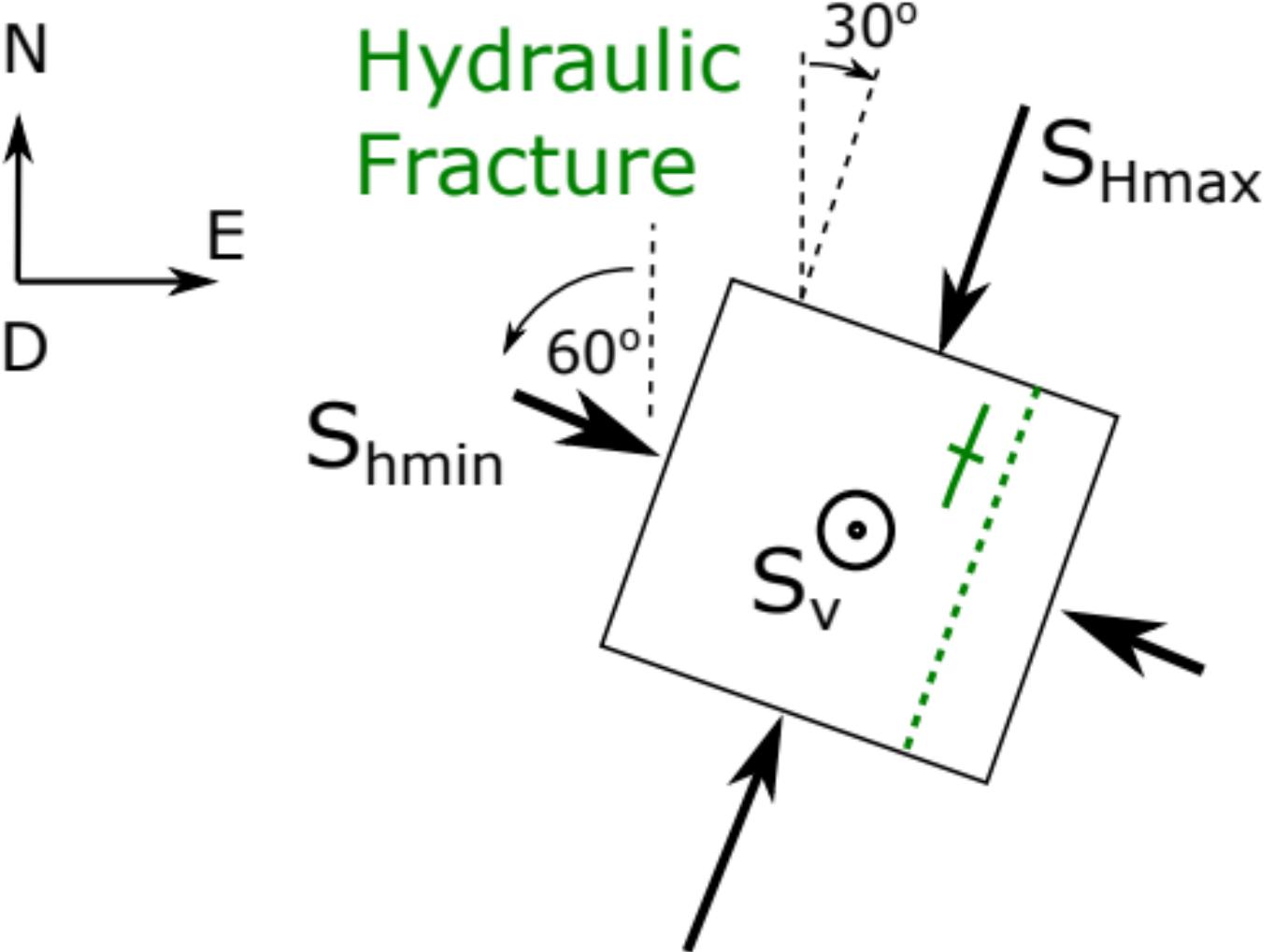
Data depth range

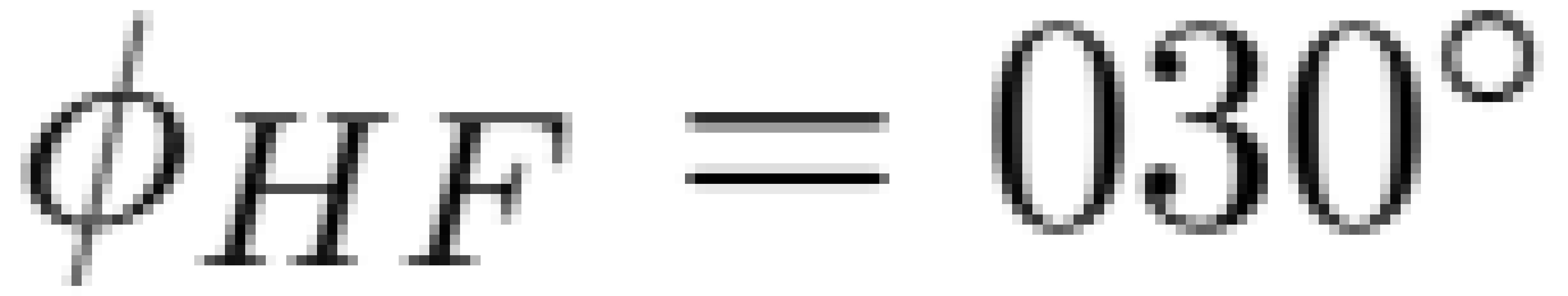
0-40 km

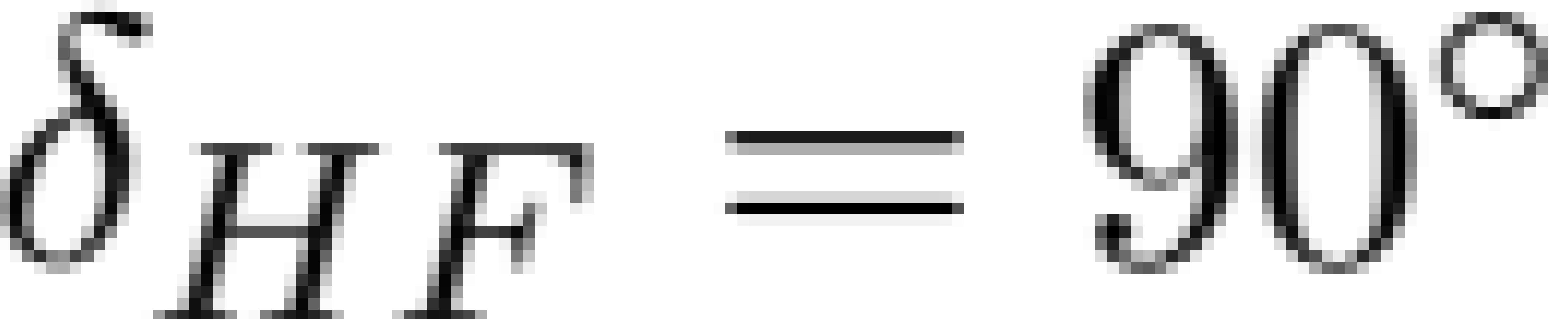
Stress Regime
Normal faulting
Strike-slip faulting
Thrust faulting
Unknown regime

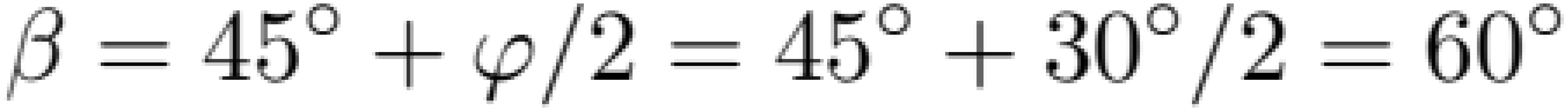




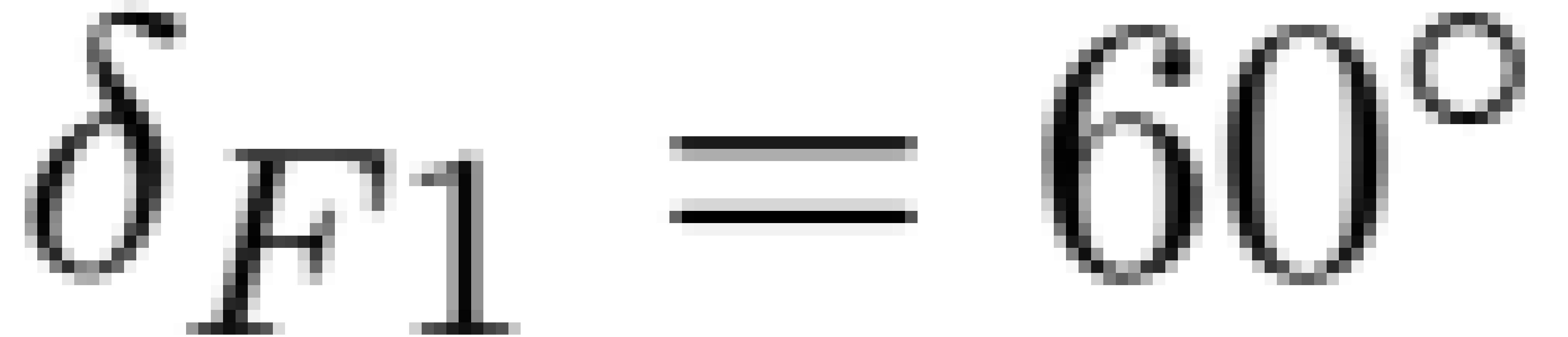


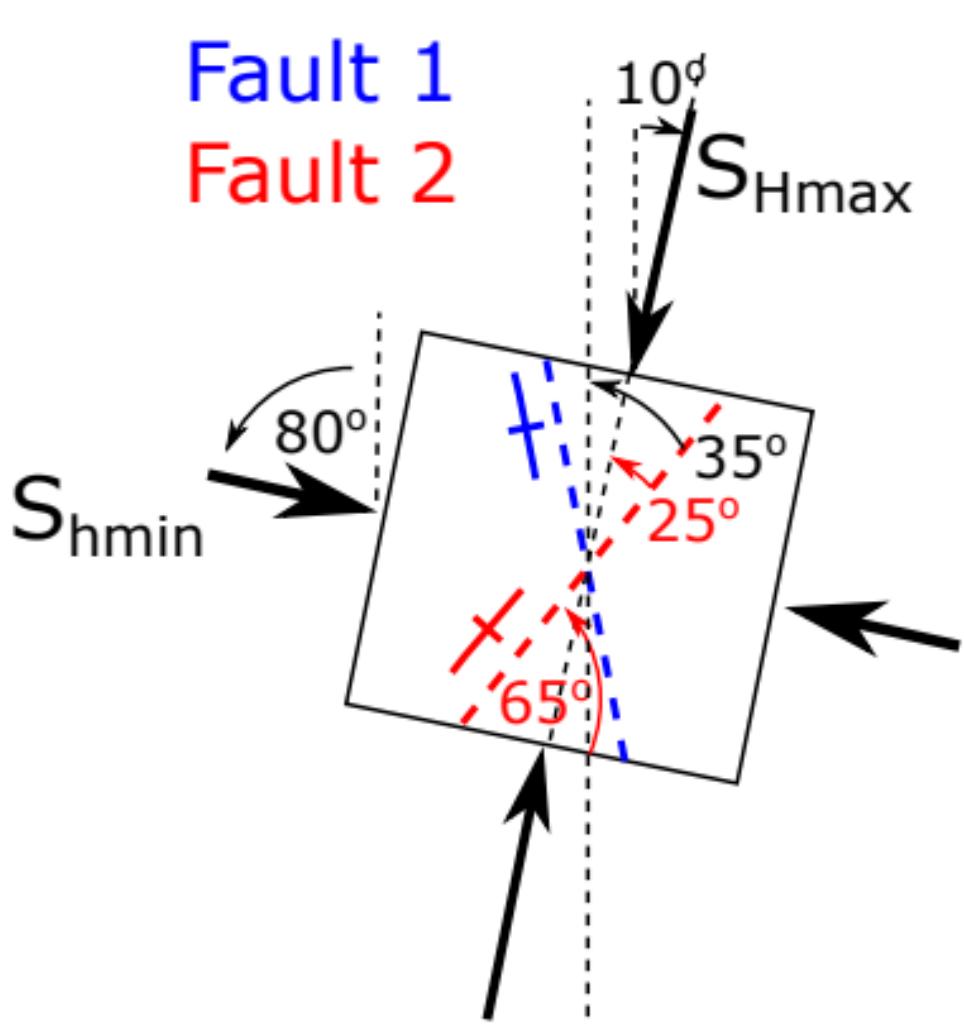
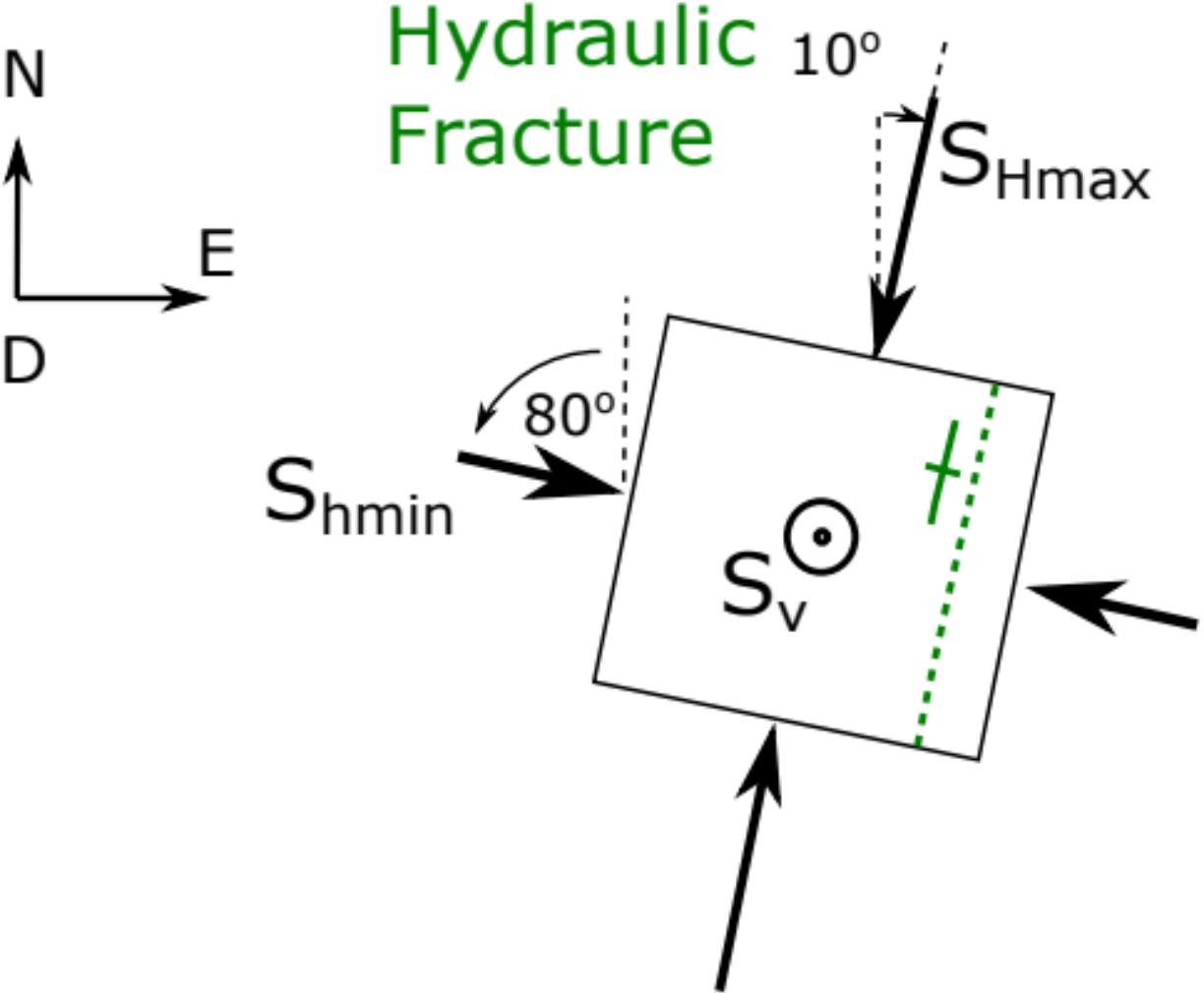


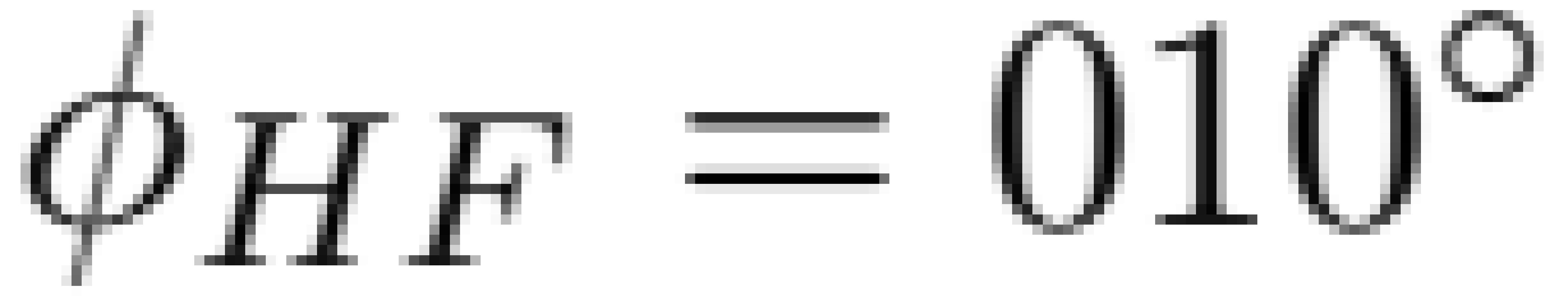


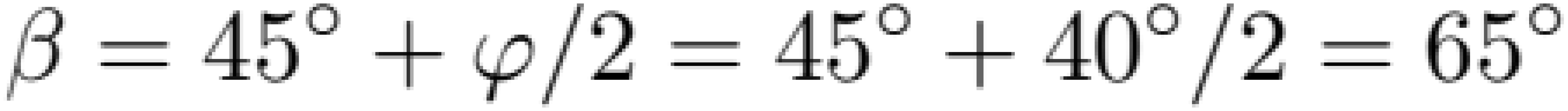


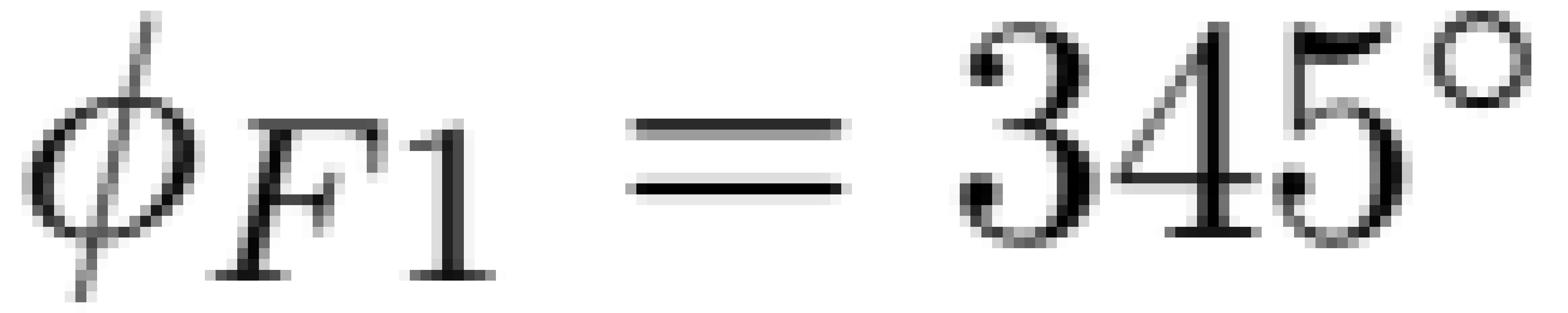


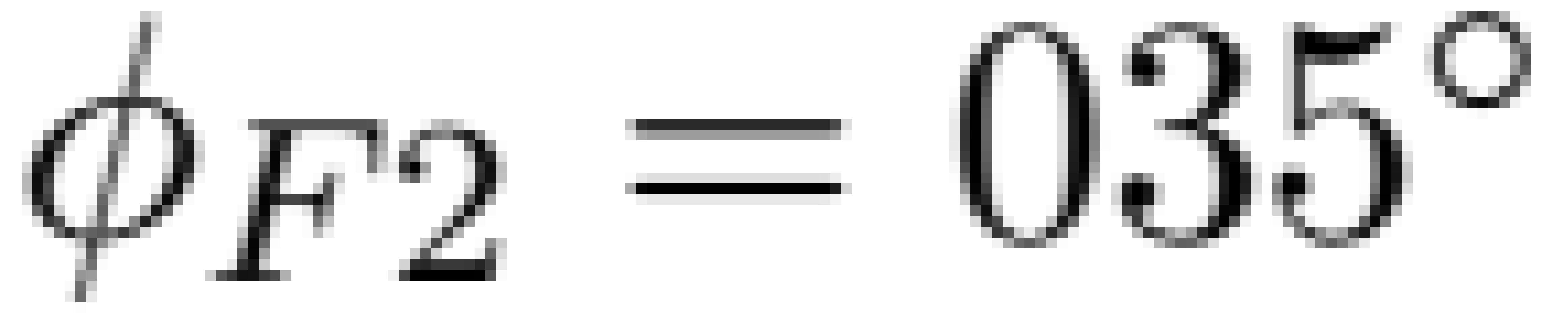






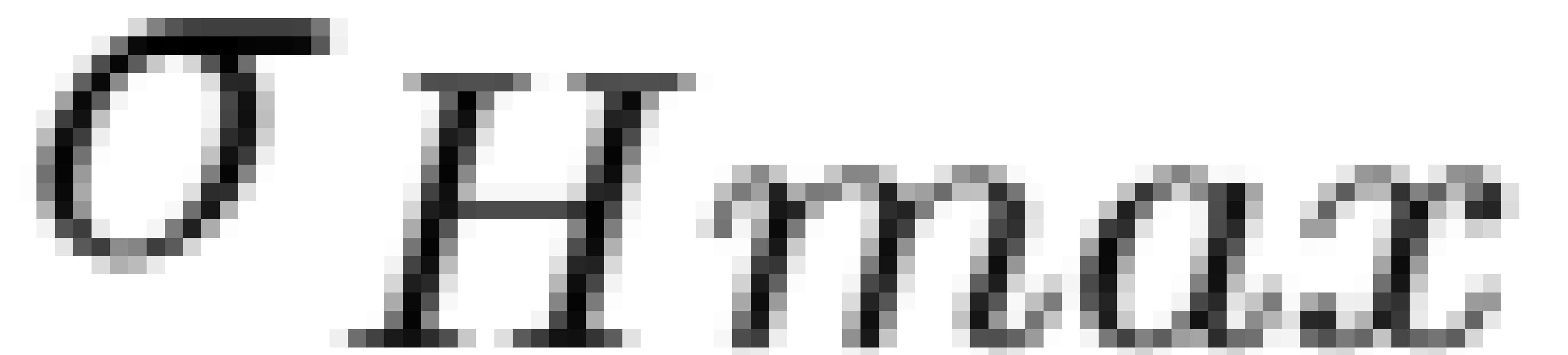




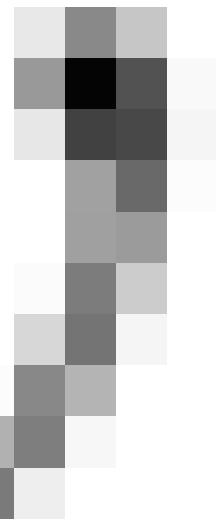
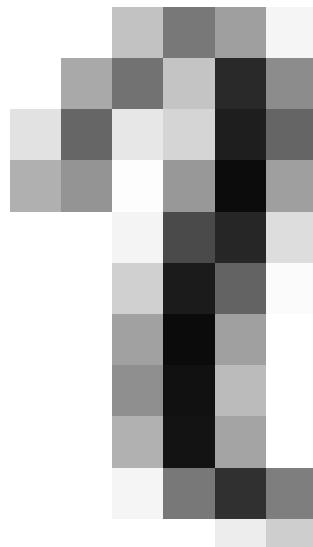
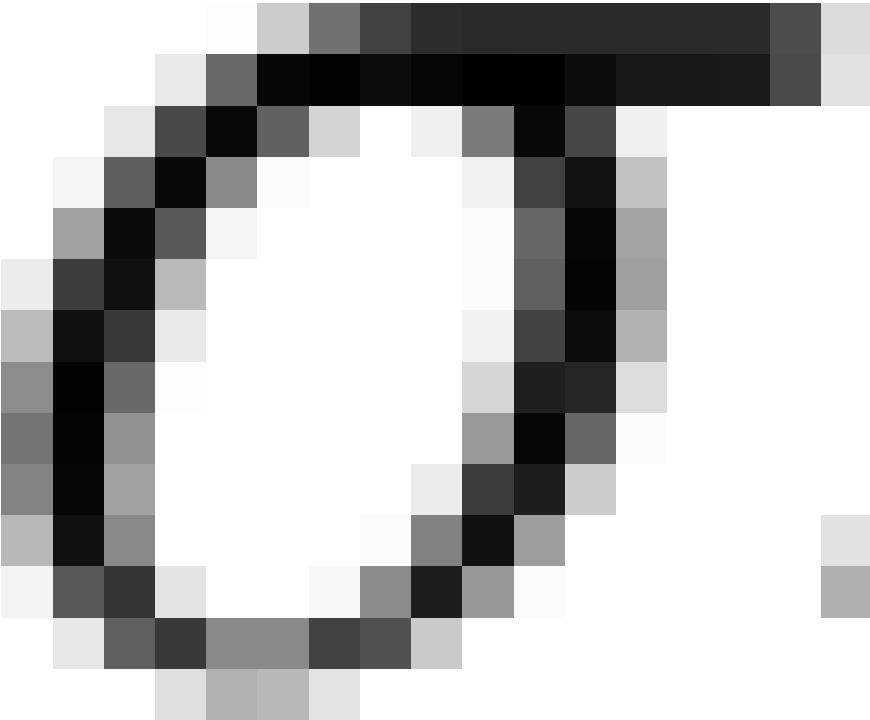






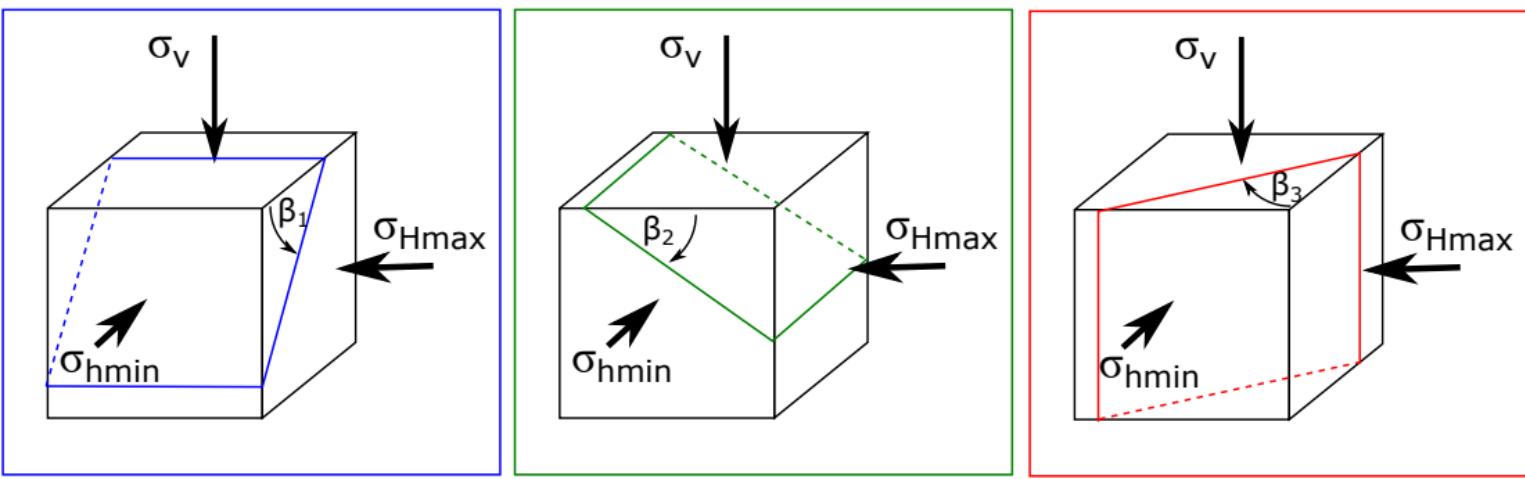
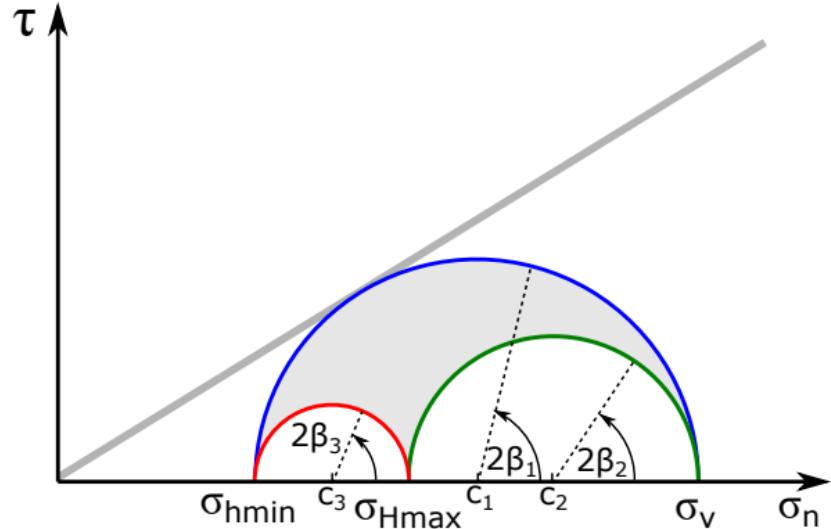
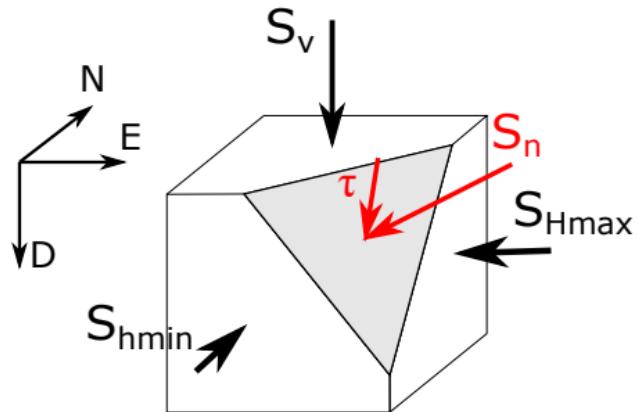


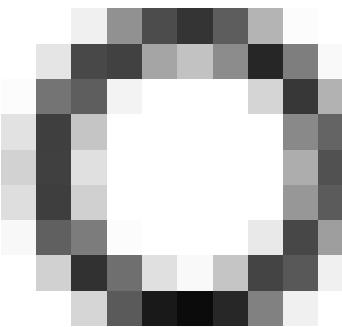
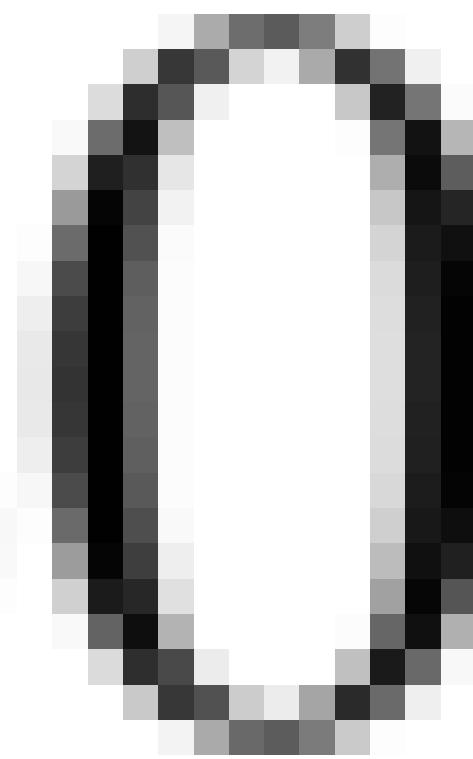
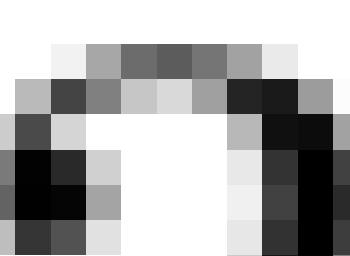
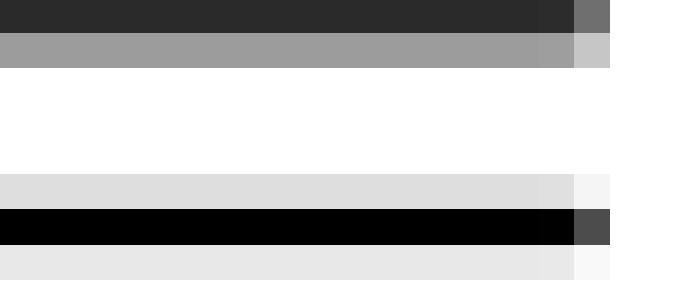
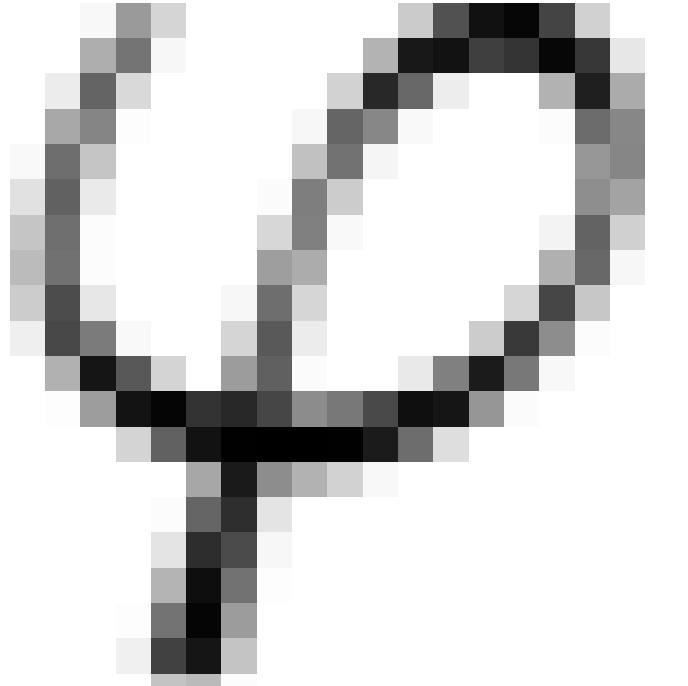


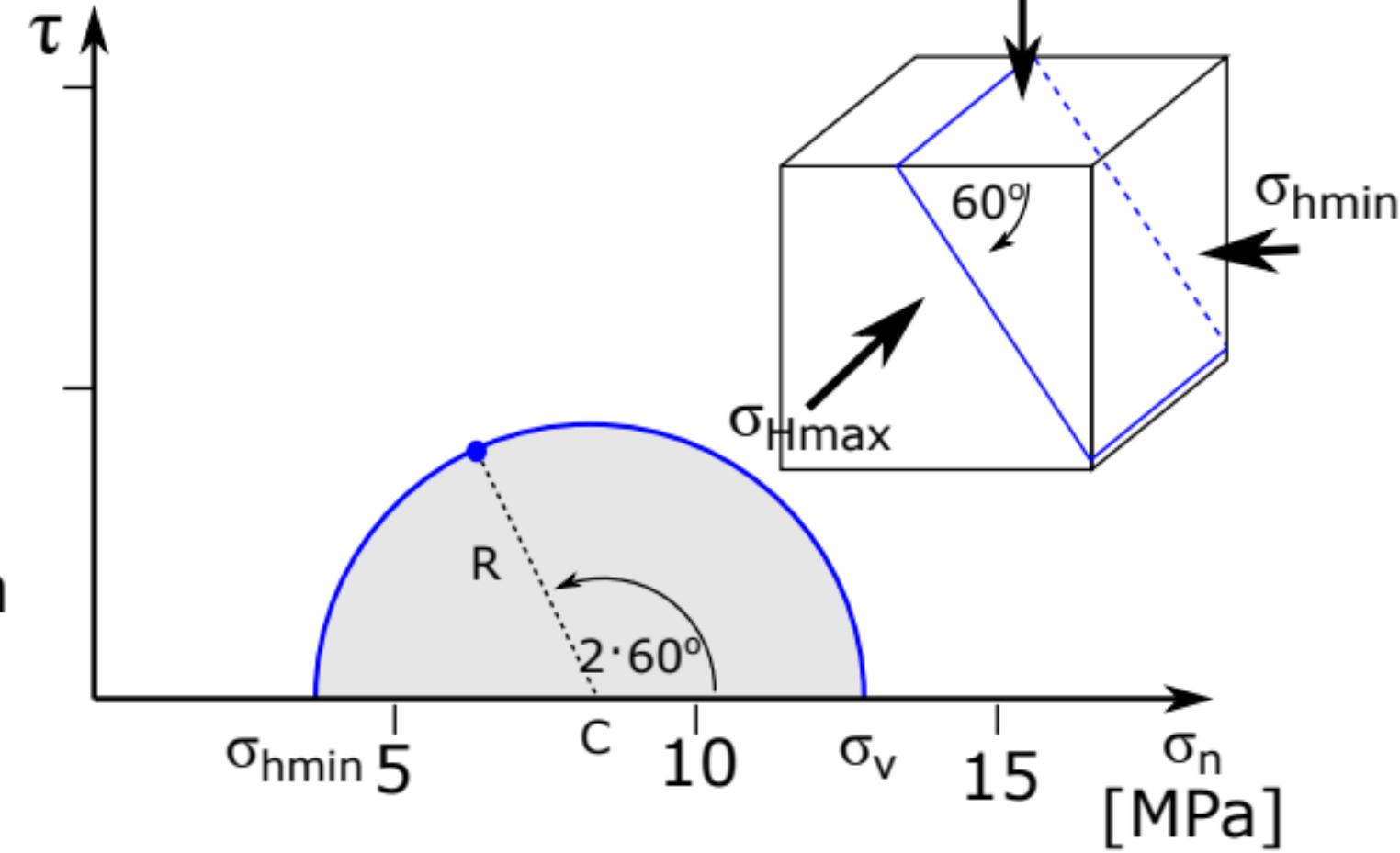
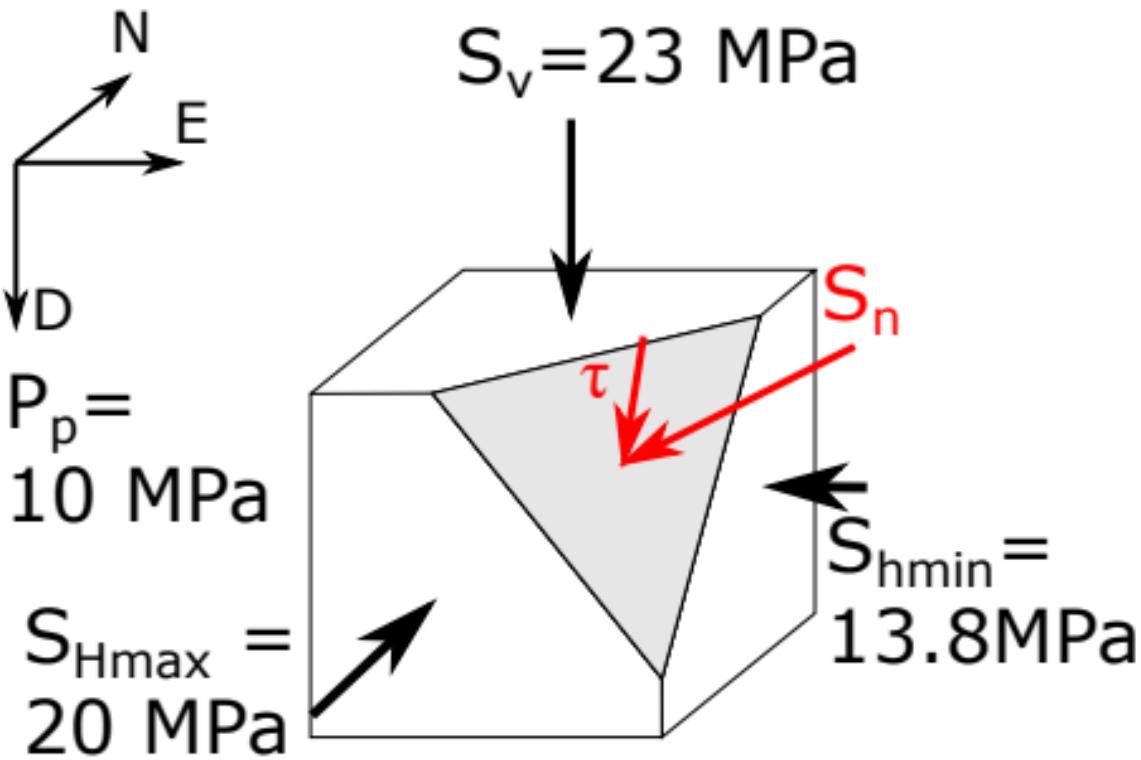


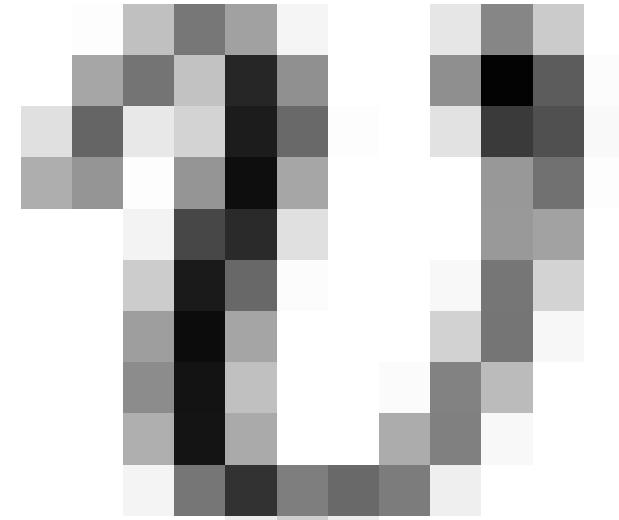
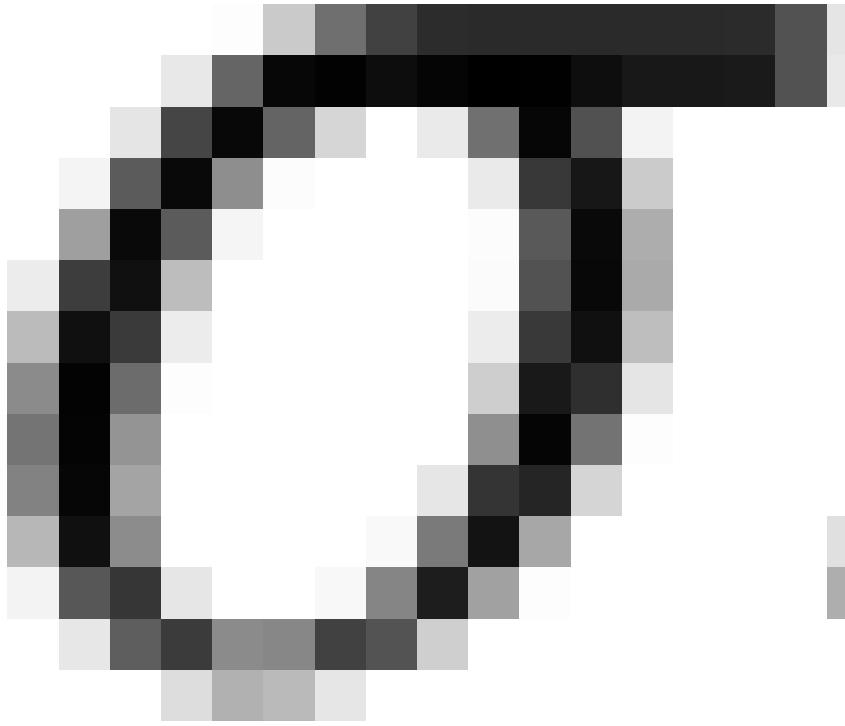




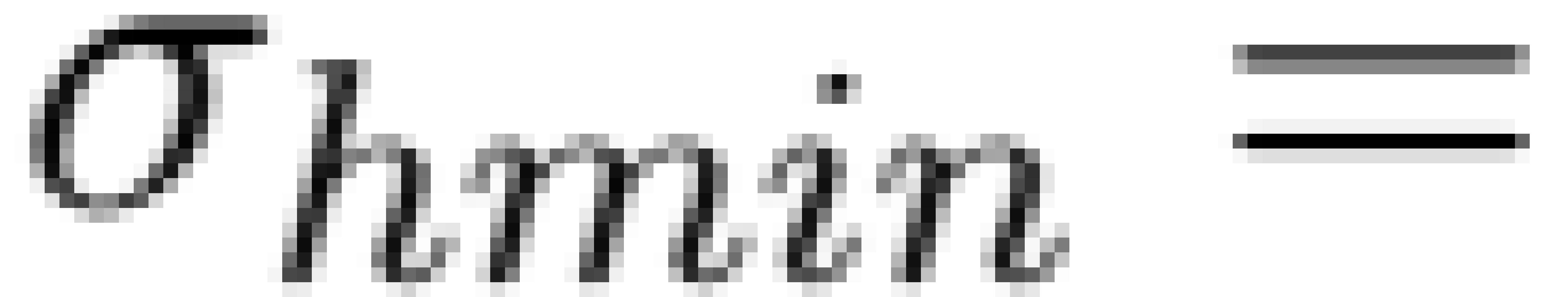








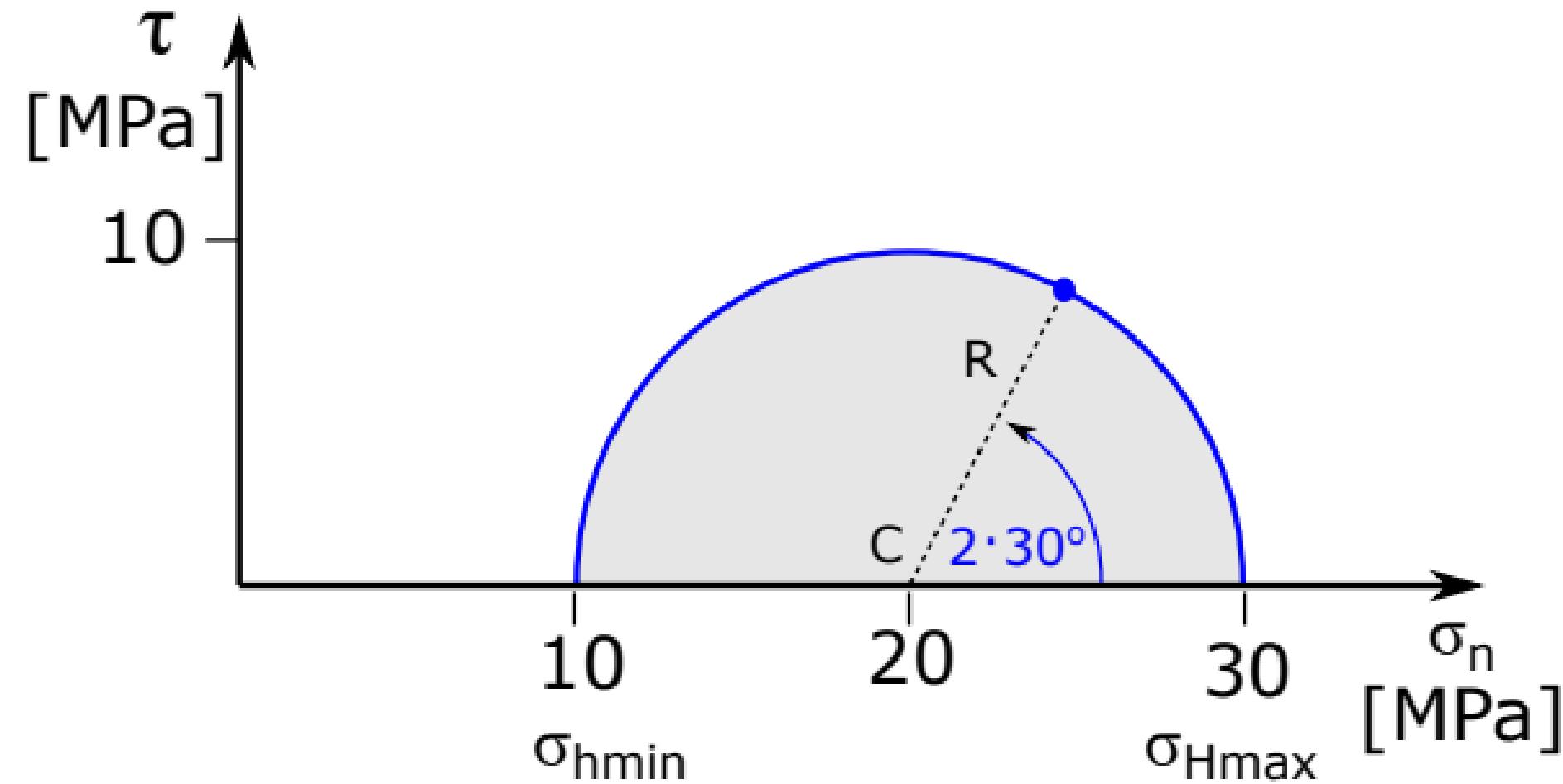
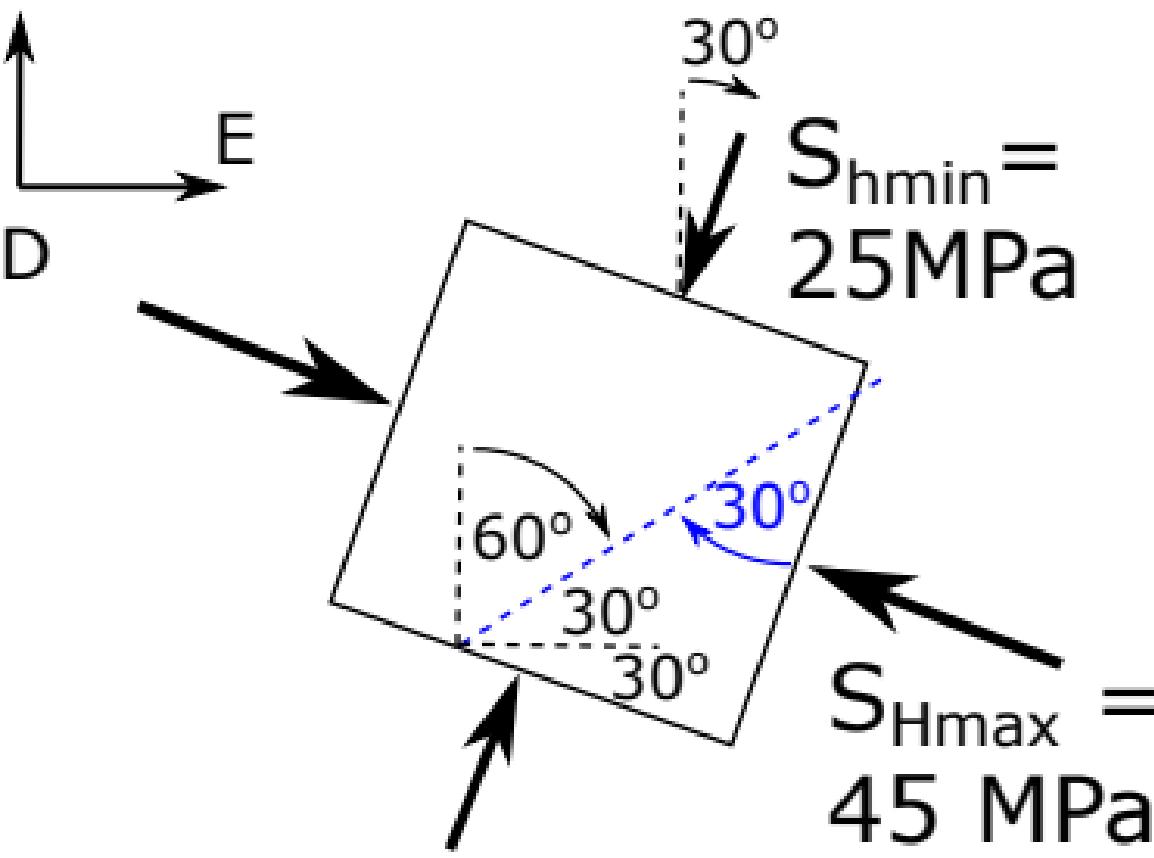




$$\sigma_n = \frac{(13 \text{ MPa} + 3.8 \text{ MPa})}{2} + \frac{(13 \text{ MPa} - 3.8 \text{ MPa}) \cos(2 \cdot 60^\circ)}{2} = 6.1 \text{ MPa}$$

$$\tau = \frac{(13 \text{ MPa} - 3.8 \text{ MPa})}{2} \sin(2 \cdot 60^\circ) = 4.0 \text{ MPa}$$

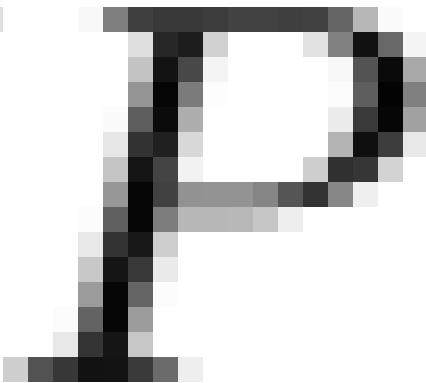
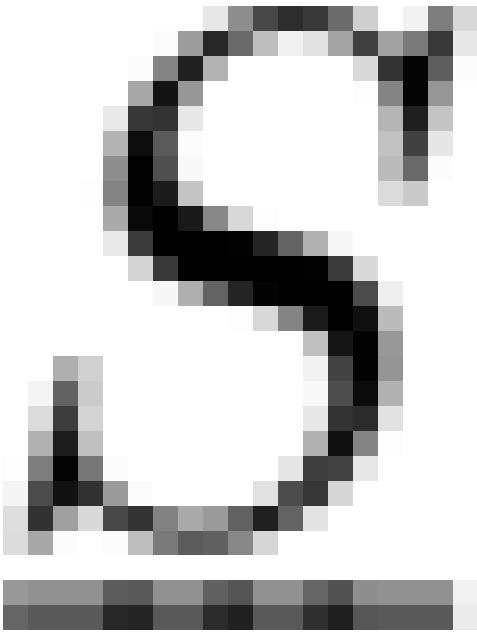
N $P_p = 15 \text{ MPa}$ $S_v = 30 \text{ MPa}$

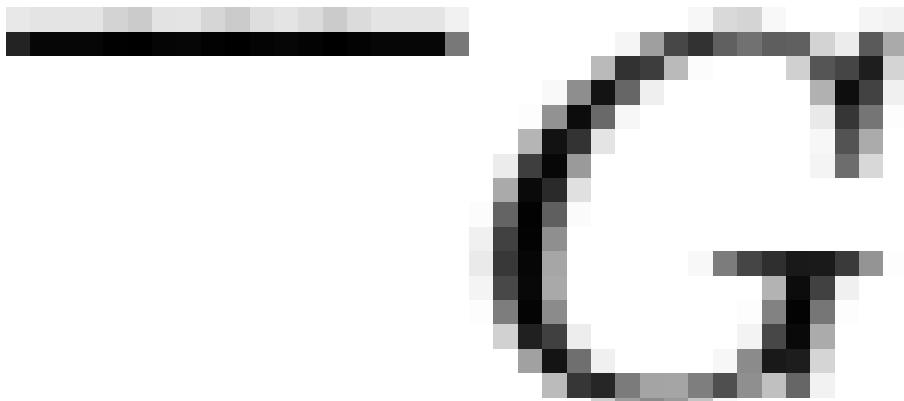
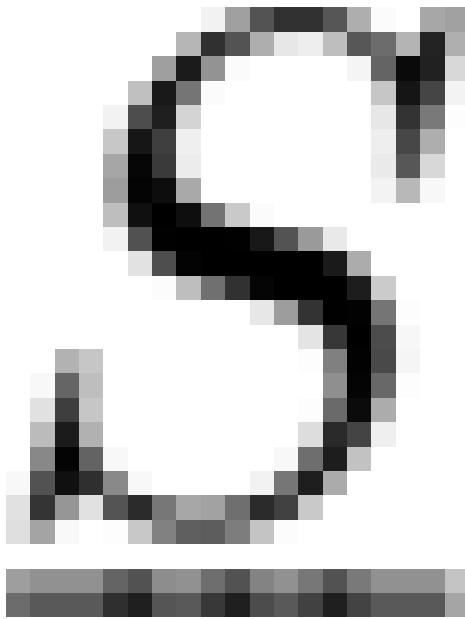


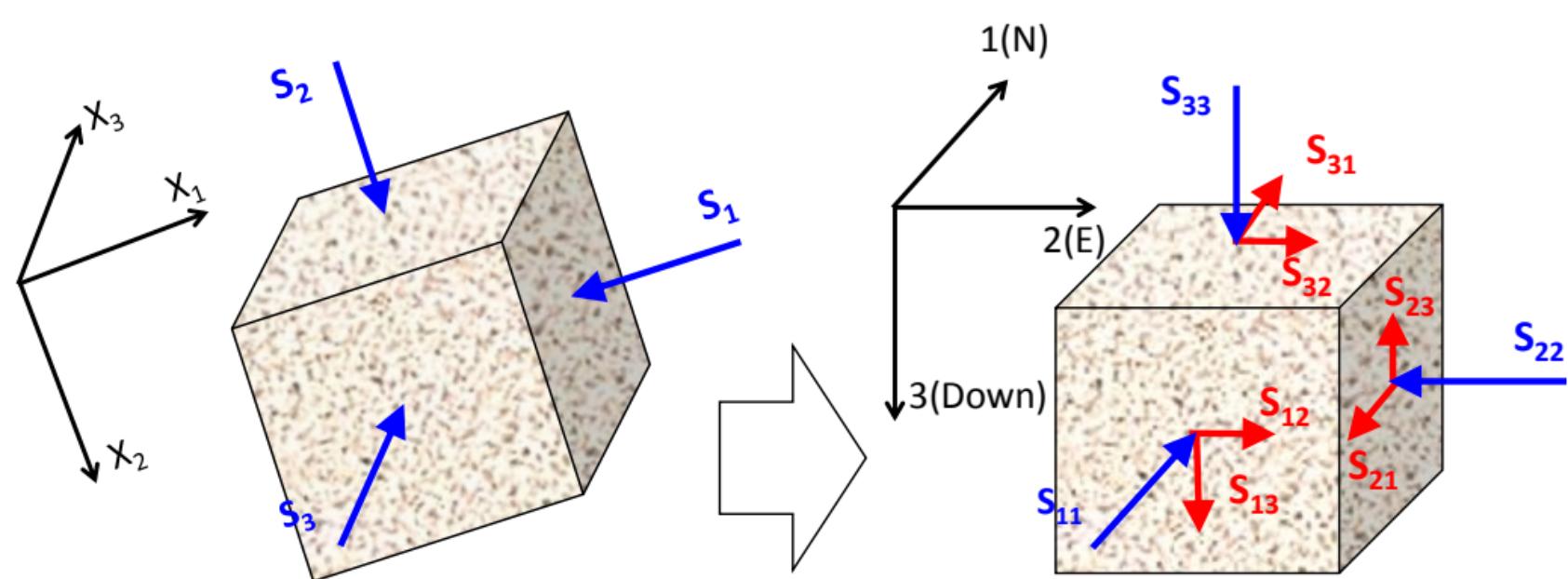
$$\sigma_n = \frac{(30 \text{ MPa} + 10 \text{ MPa})}{2} + \frac{(30 \text{ MPa} - 10 \text{ MPa}) \cos(2 \cdot 30^\circ)}{2} = 25 \text{ MPa}$$

$$\tau = \frac{(30 \text{ MPa} - 10 \text{ MPa})}{2} \sin(2 \cdot 30^\circ) = 8.7 \text{ MPa}$$





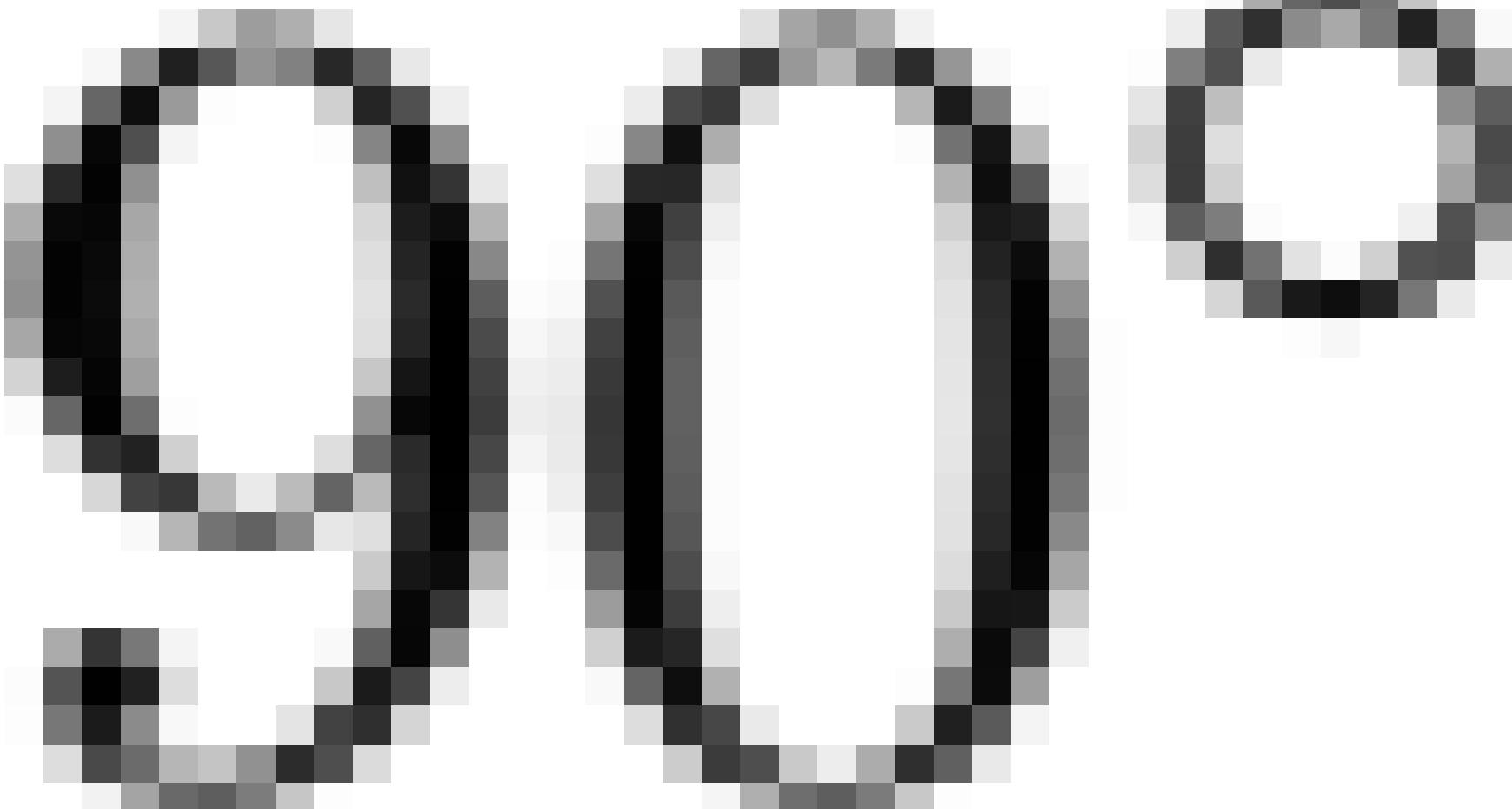


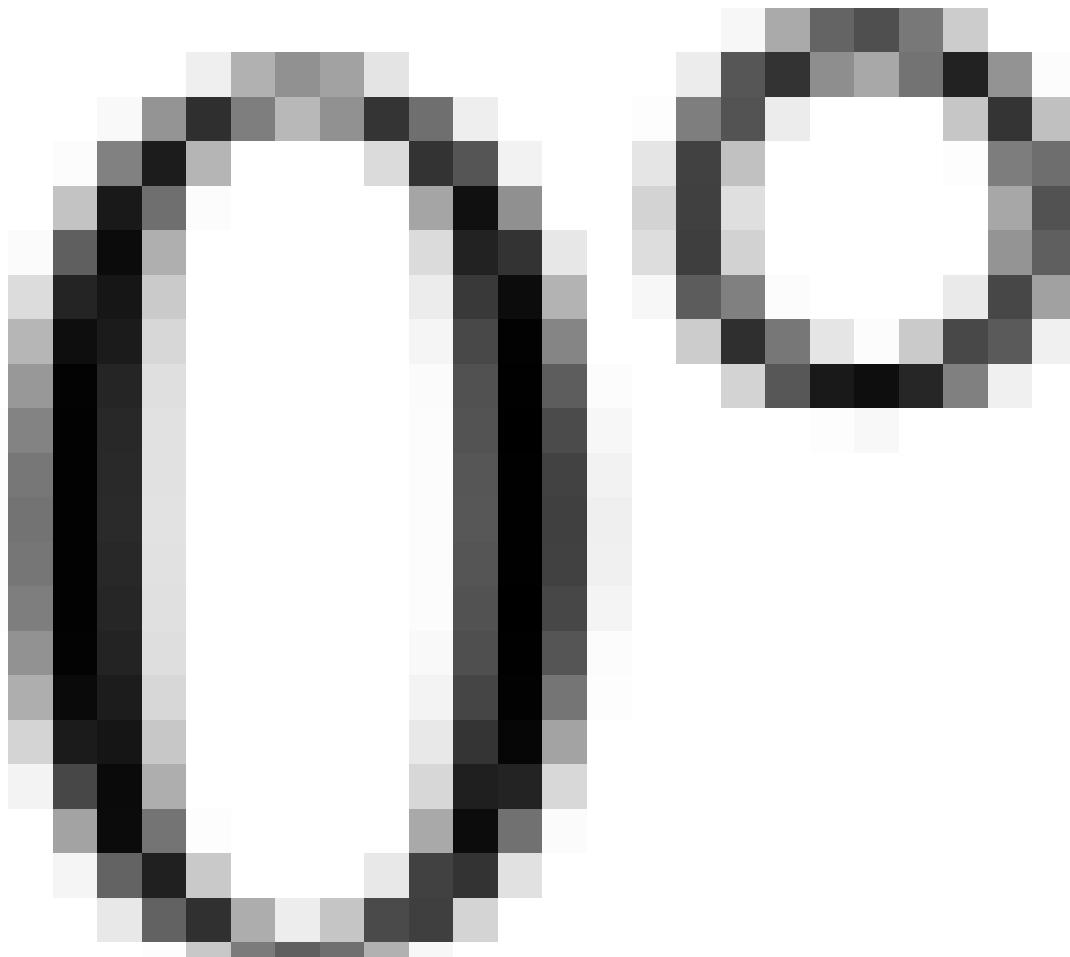


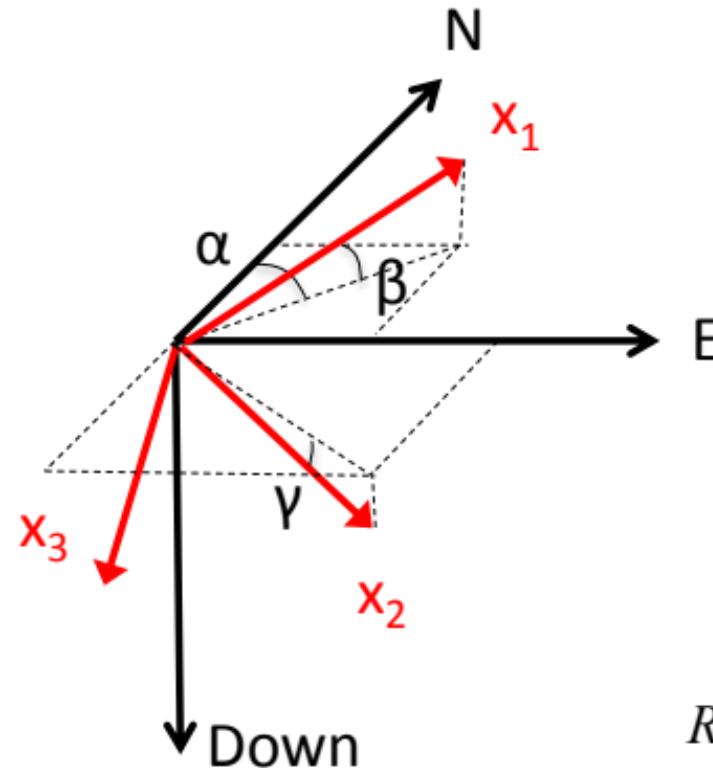
$$S_P = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$S_G = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$









$$\underline{\underline{S}}' = \underline{\underline{A}} \underline{\underline{S}} \underline{\underline{A}}^T$$

$$\underline{\underline{A}} = \begin{bmatrix} \underline{e}'_1 \cdot \underline{e}_1 & \underline{e}'_1 \cdot \underline{e}_2 & \underline{e}'_1 \cdot \underline{e}_3 \\ \underline{e}'_2 \cdot \underline{e}_1 & \underline{e}'_2 \cdot \underline{e}_2 & \underline{e}'_2 \cdot \underline{e}_3 \\ \underline{e}'_3 \cdot \underline{e}_1 & \underline{e}'_3 \cdot \underline{e}_2 & \underline{e}'_3 \cdot \underline{e}_3 \end{bmatrix}$$

where $\underline{\underline{A}}$ is the transformation matrix from the old base \underline{e}_i to base \underline{e}'_i and the components are the projection of the elements of the new base on the old base.

- Old system: N-E-D (Right-handed) Geographical system
- New system: 1-2-3 (Right-handed) Principal stress system

$$R_{PG} = \begin{bmatrix} \cos \alpha \cos b & \sin \alpha \cos b & -\sin b \\ \cos \alpha \sin b \sin g - \sin \alpha \cos g & \sin \alpha \sin b \sin g + \cos \alpha \cos g & \cos b \sin g \\ \cos \alpha \sin b \cos g + \sin \alpha \sin g & \sin \alpha \sin b \cos g - \cos \alpha \sin g & \cos b \cos g \end{bmatrix}$$

2

P

E

MPG

G

PG

Q

Q

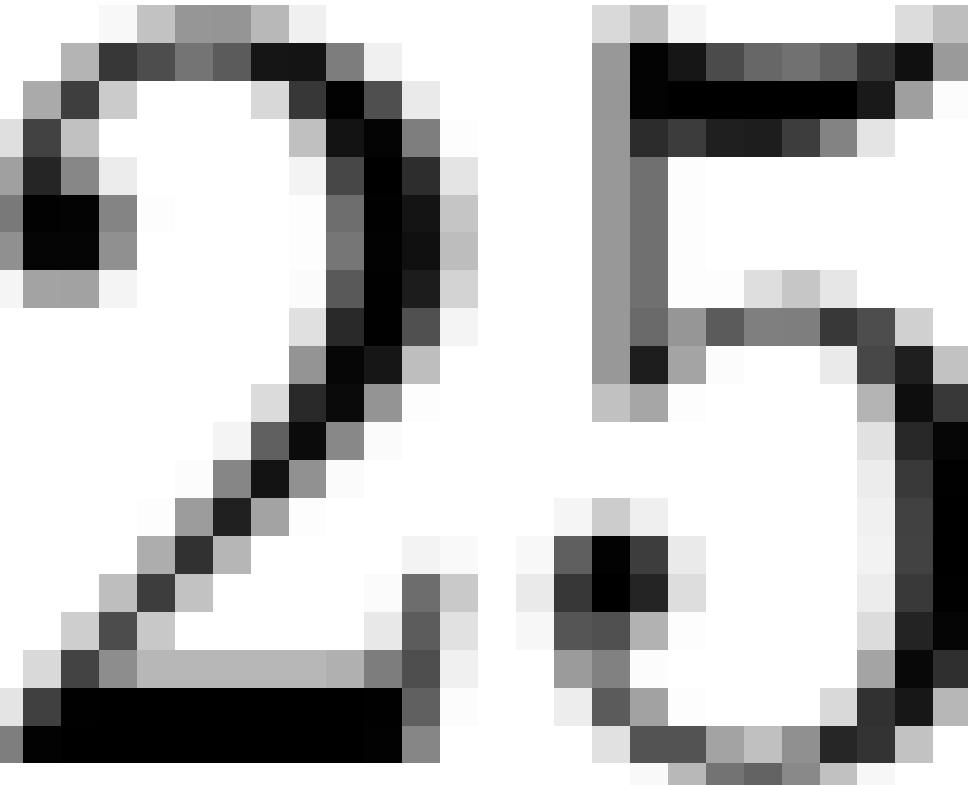
S

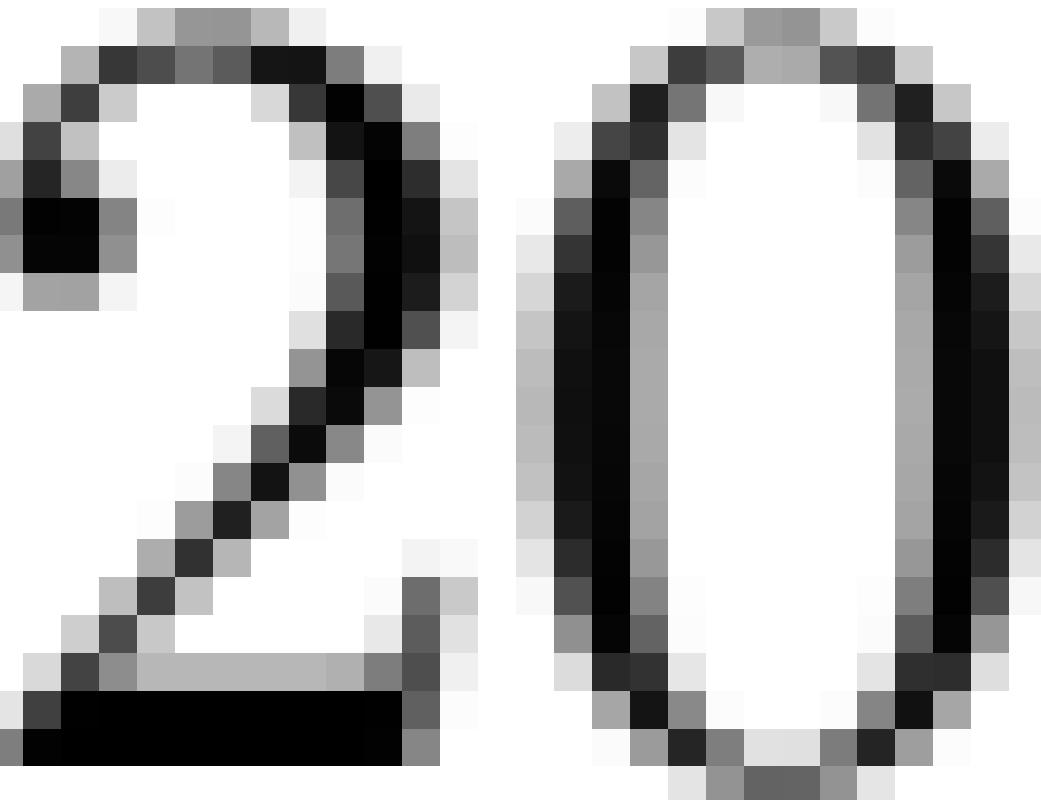
Q P Q S

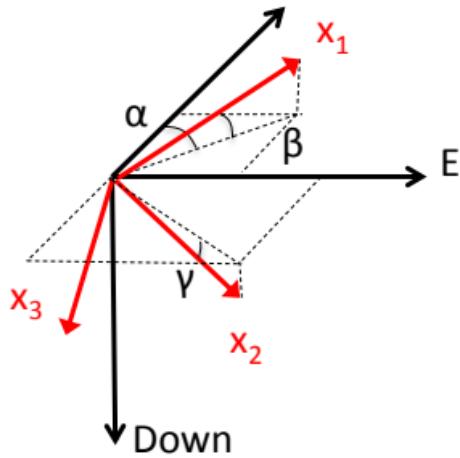
P

Q P Q









$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$a = 0^\circ$$

$$b = 90^\circ$$

$$g = 0^\circ$$

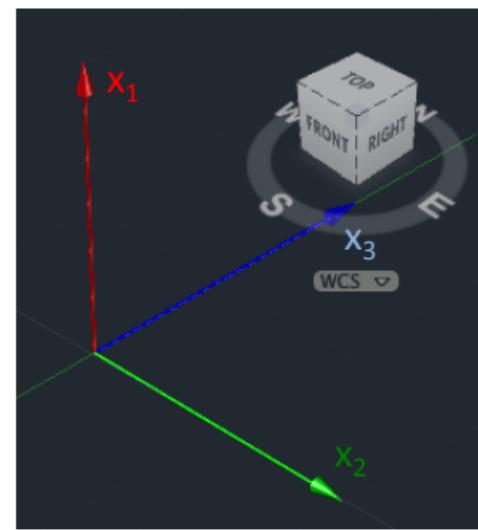
Azimuth of $S_{h\min}$

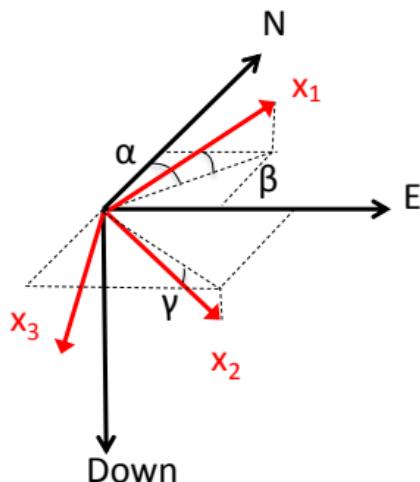
$$S_1 = S_V$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$





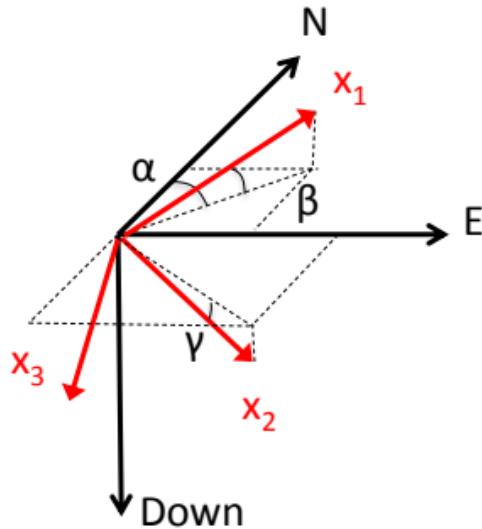
$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \quad \begin{array}{ll} a = 0^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & S_1 = S_{H\max} \\ g = 90^\circ & S_2 = S_V \end{array}$$



$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

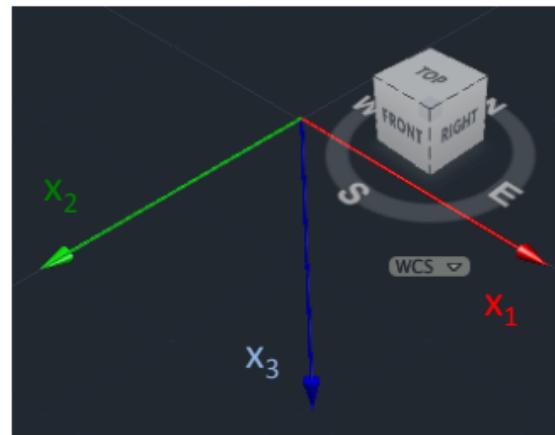
$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$



$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$a = 90^\circ$ Azimuth of $S_{H\max}$
 $b = 0^\circ$
 $g = 0^\circ$
 $S_1 = S_{H\max}$
 $S_2 = S_{h\min}$

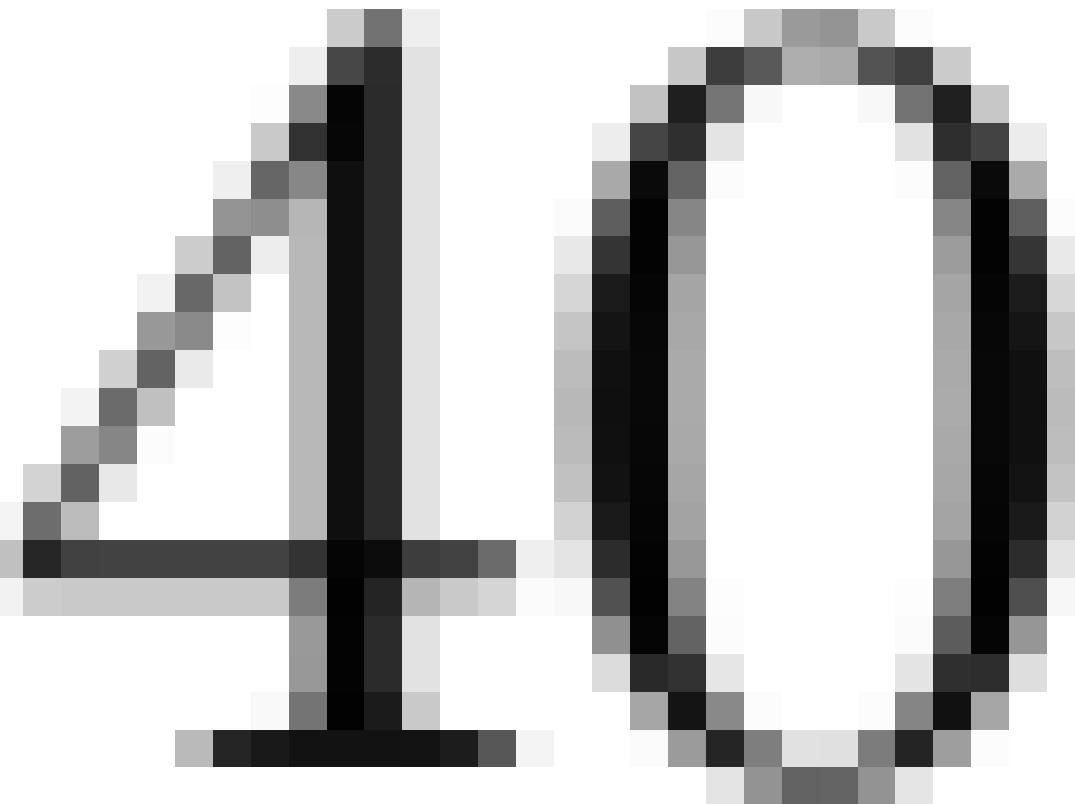


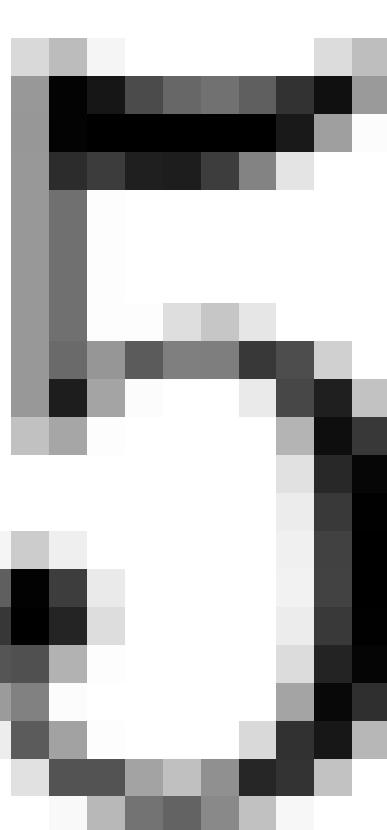
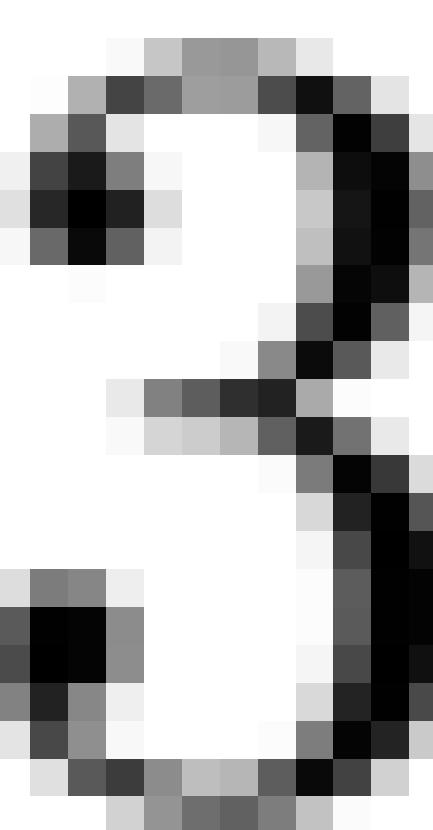
$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

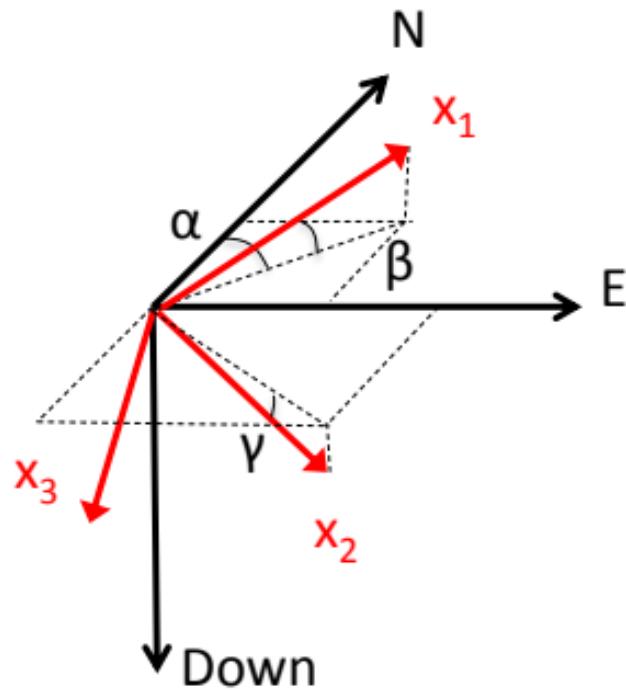
$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$







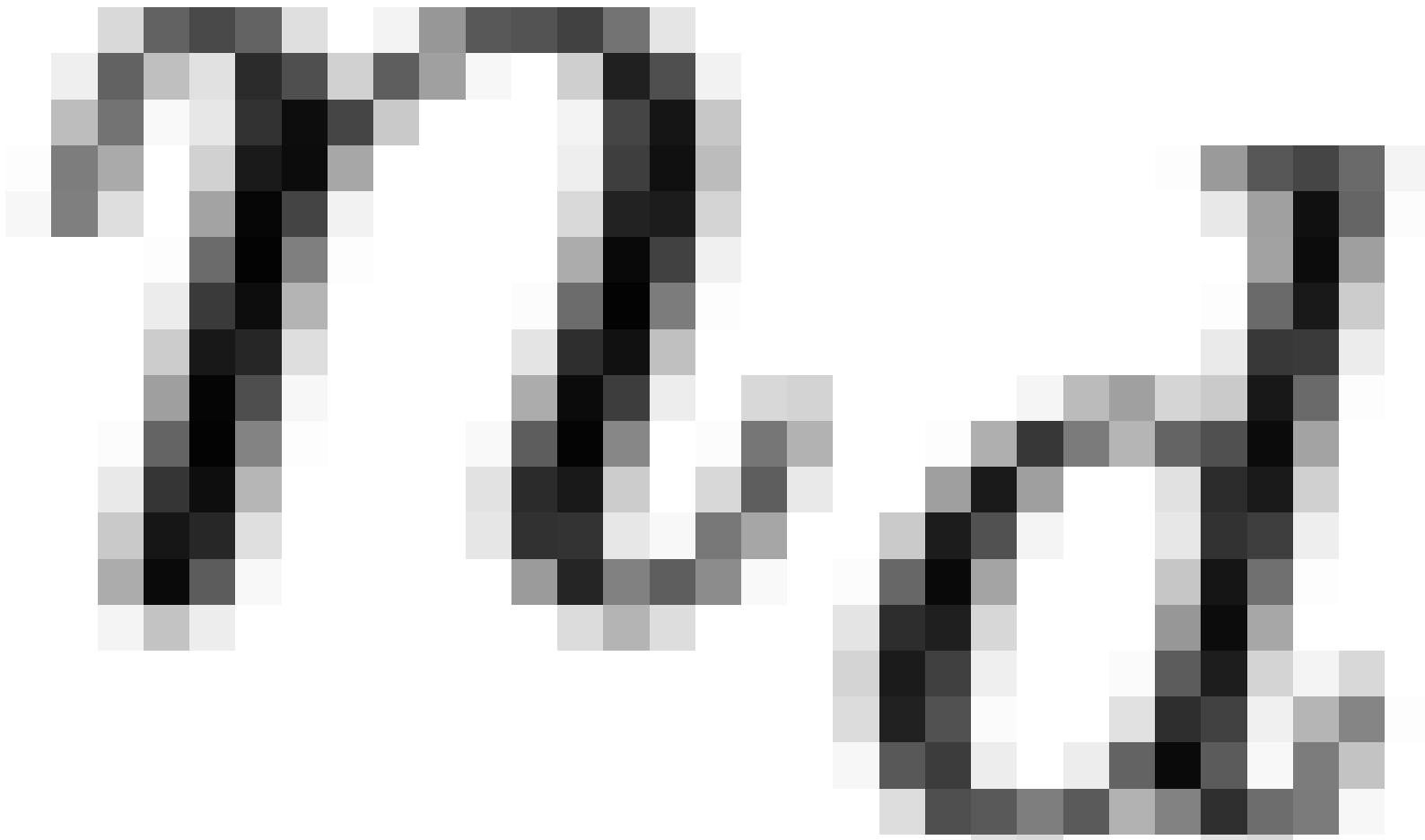


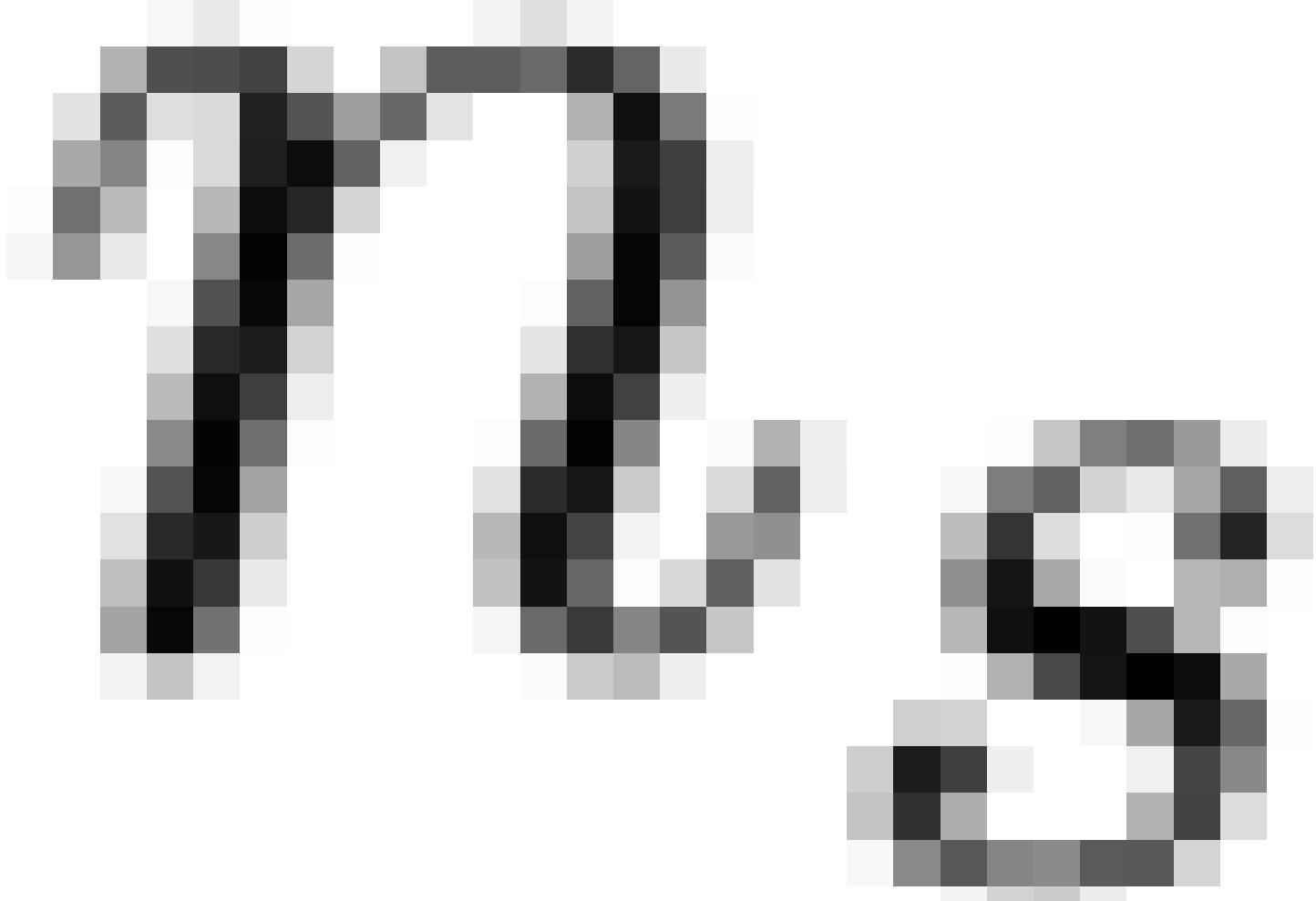
$$\underline{\underline{S}}_P = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix} \quad \begin{array}{ll} a = 135^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & S_1 = S_{H\max} \\ g = 90^\circ & S_2 = S_V \end{array}$$

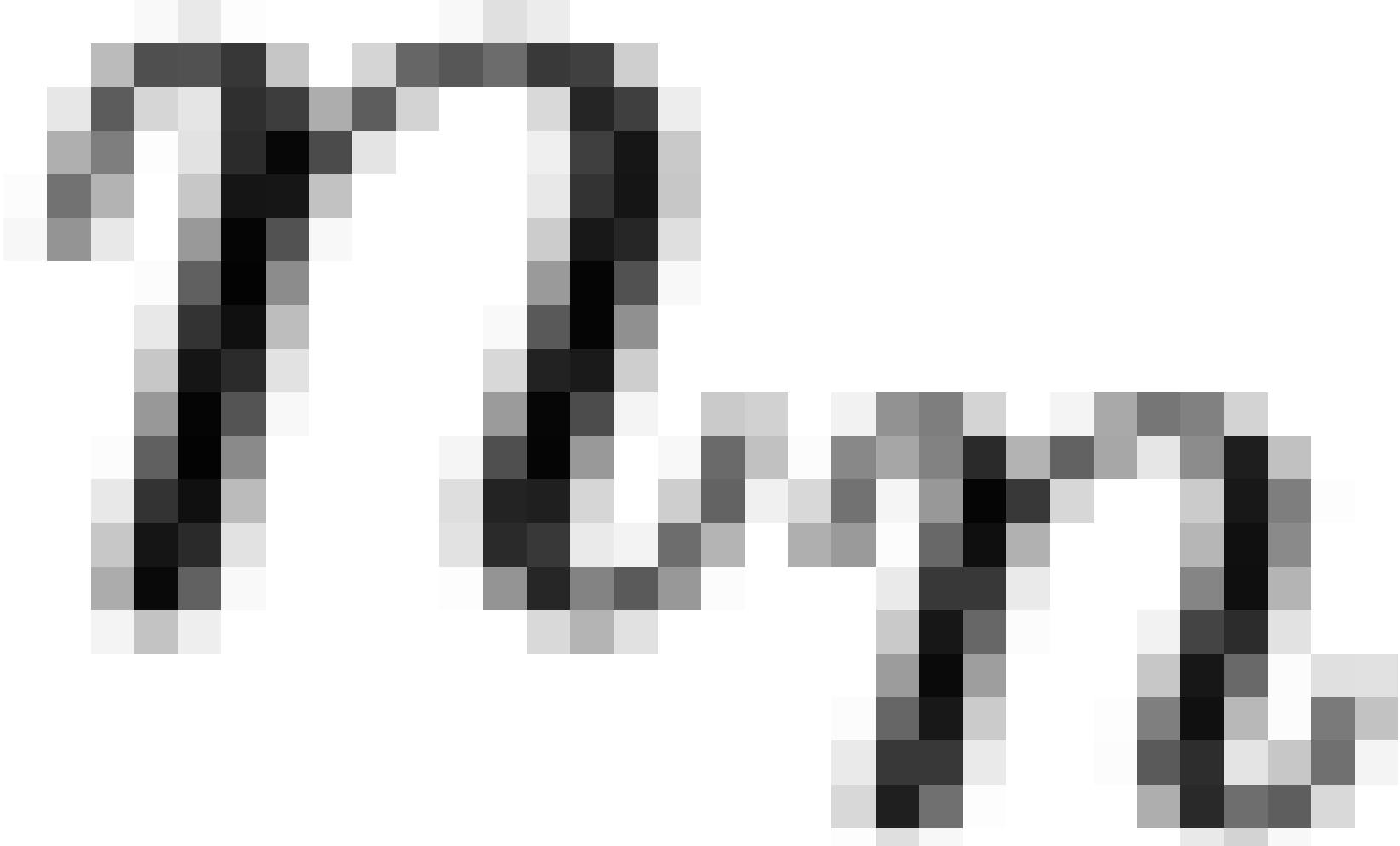
$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$

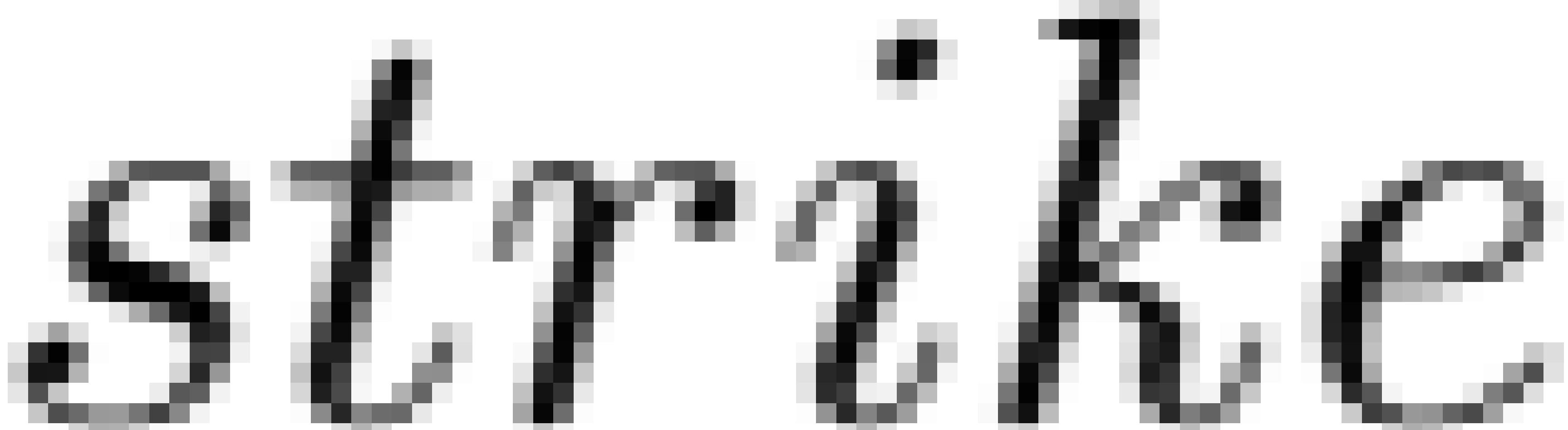
$$\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$



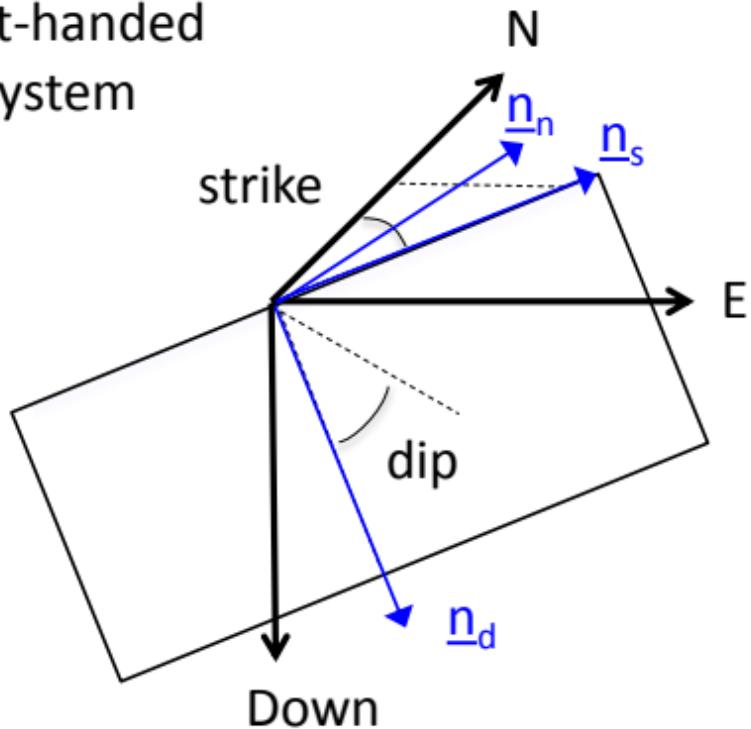






d-s-n

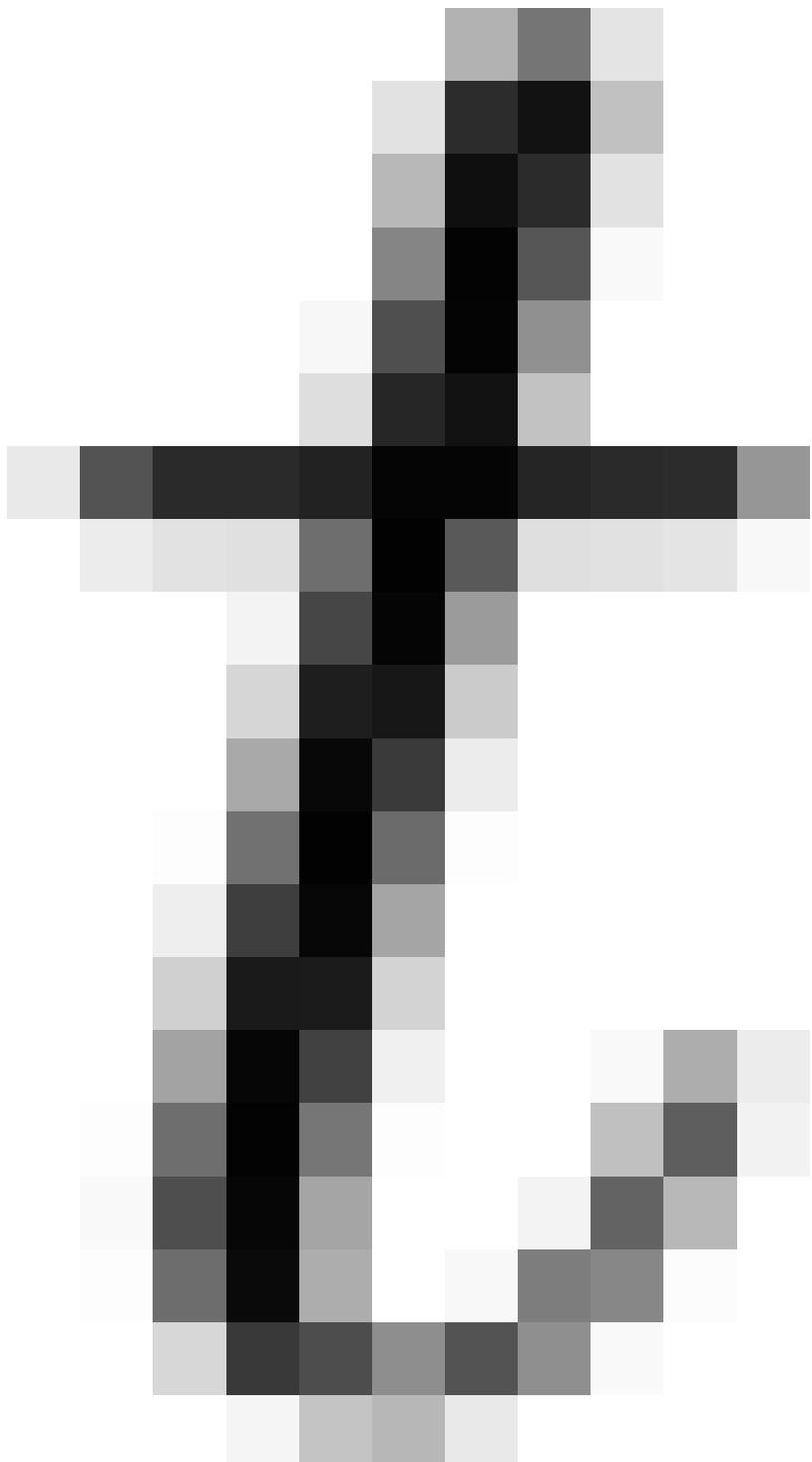
Right-handed
system

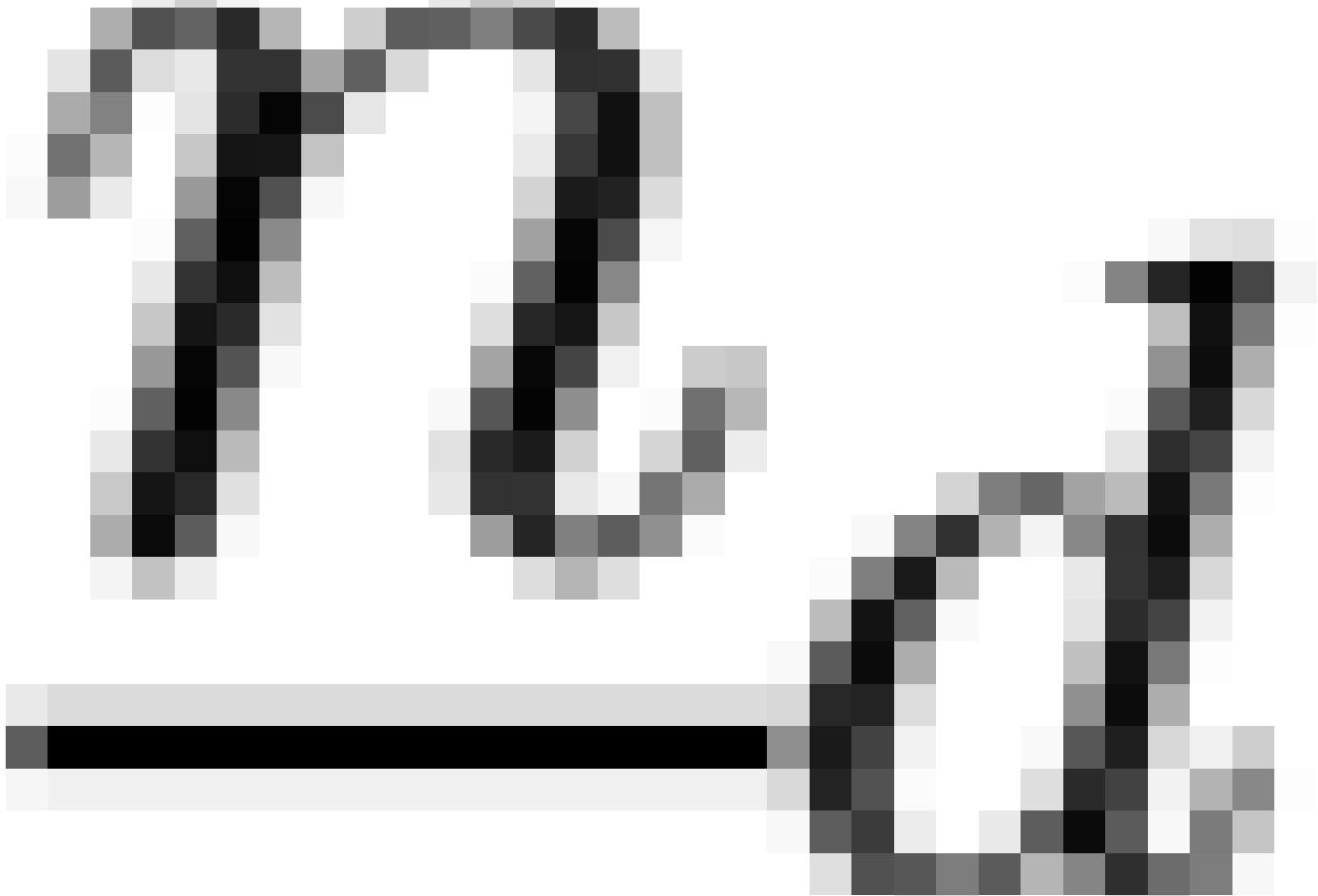


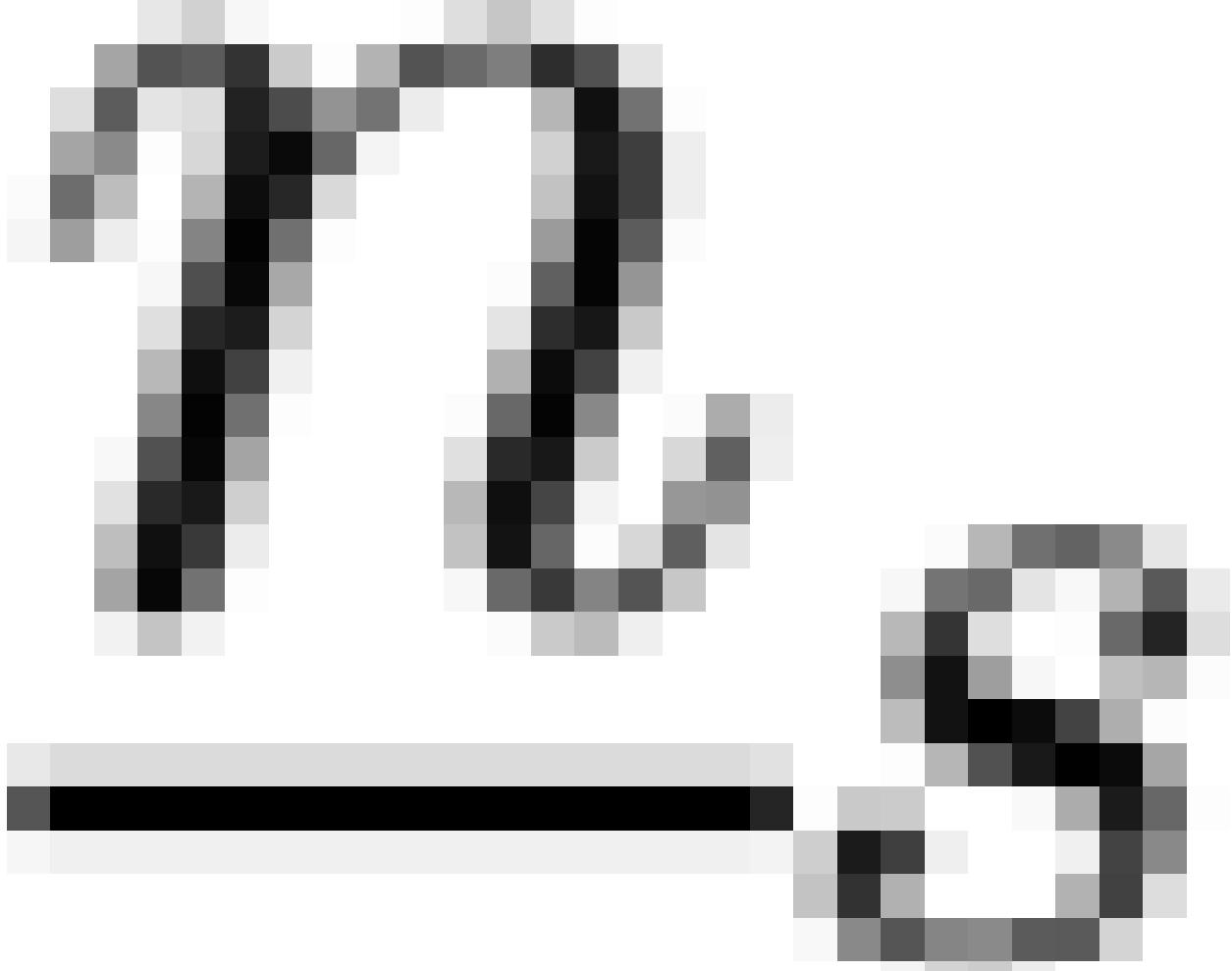
$$\underline{n}_n = \begin{bmatrix} -\sin(strike)\sin(dip) \\ \cos(strike)\sin(dip) \\ -\cos(dip) \end{bmatrix}$$

$$\underline{n}_s = \begin{bmatrix} \cos(strike) \\ \sin(strike) \\ 0 \end{bmatrix}$$

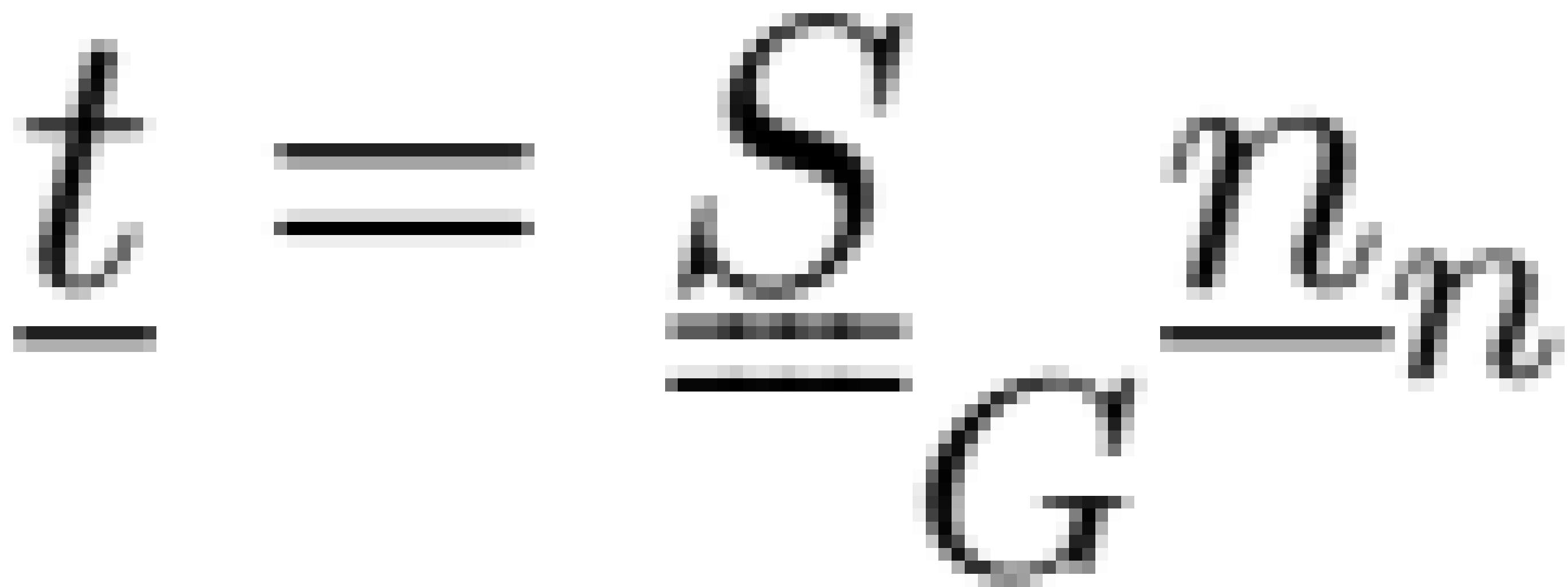
$$\underline{n}_d = \begin{bmatrix} -\sin(strike)\cos(dip) \\ \cos(strike)\cos(dip) \\ \sin(dip) \end{bmatrix}$$



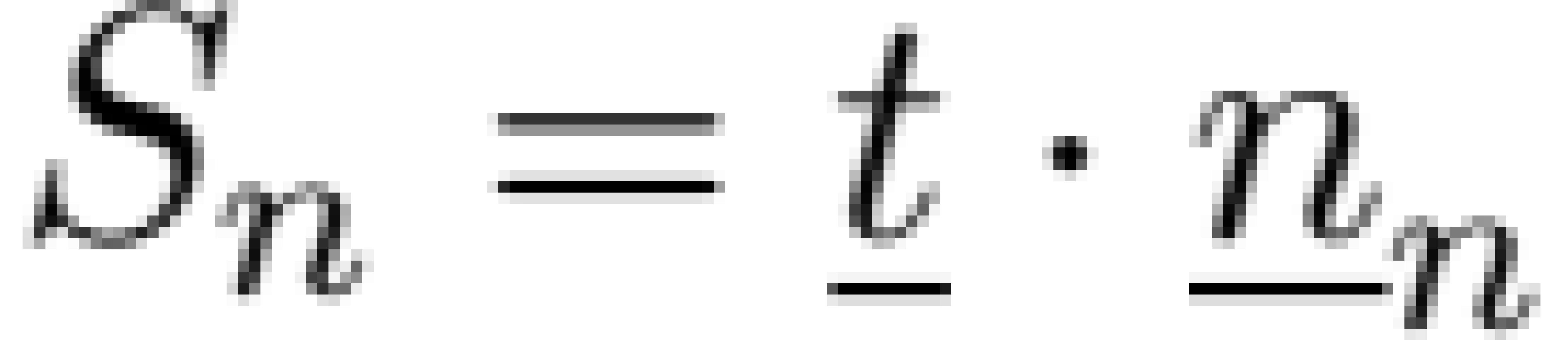








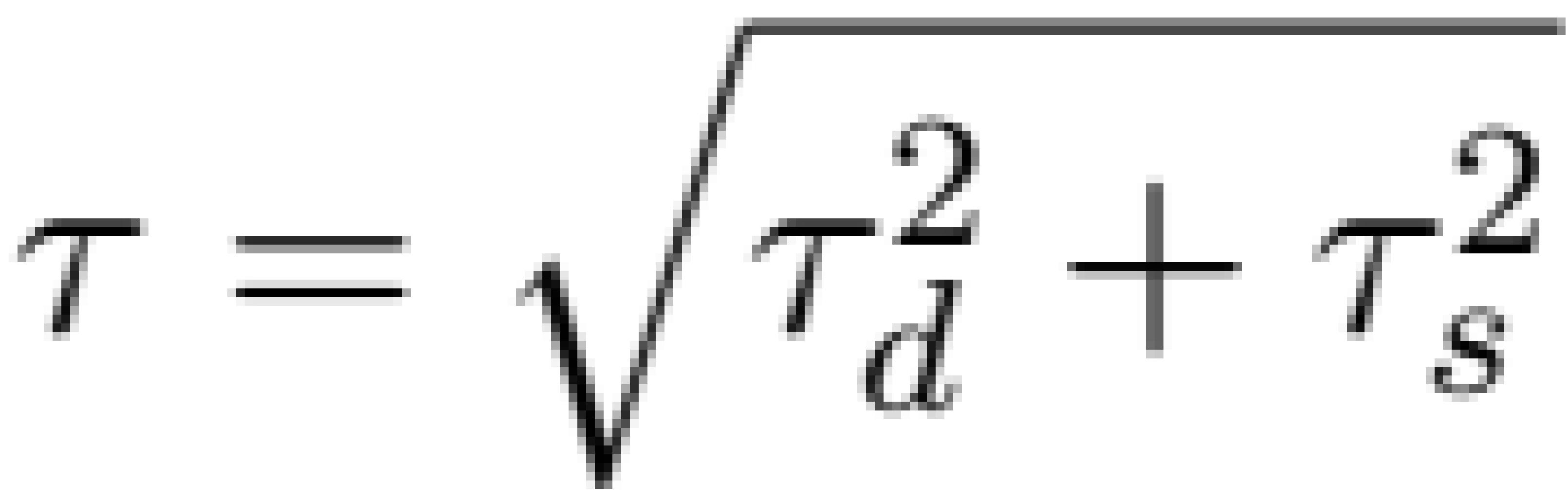


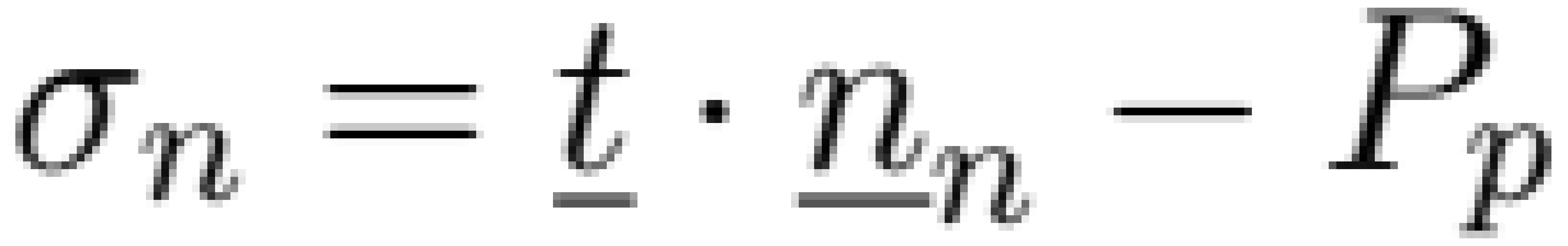


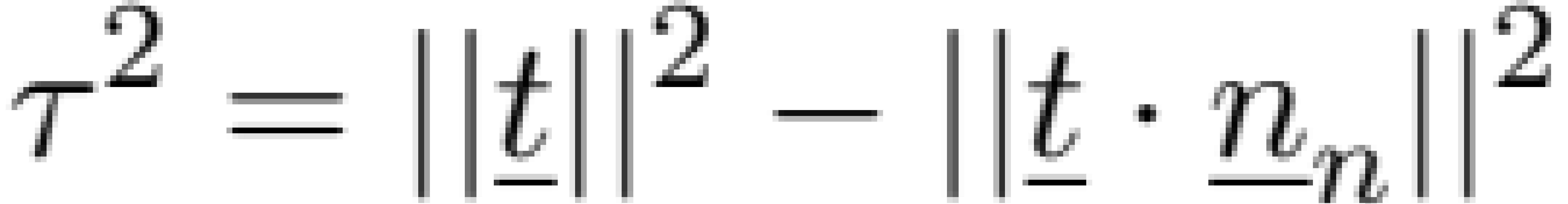


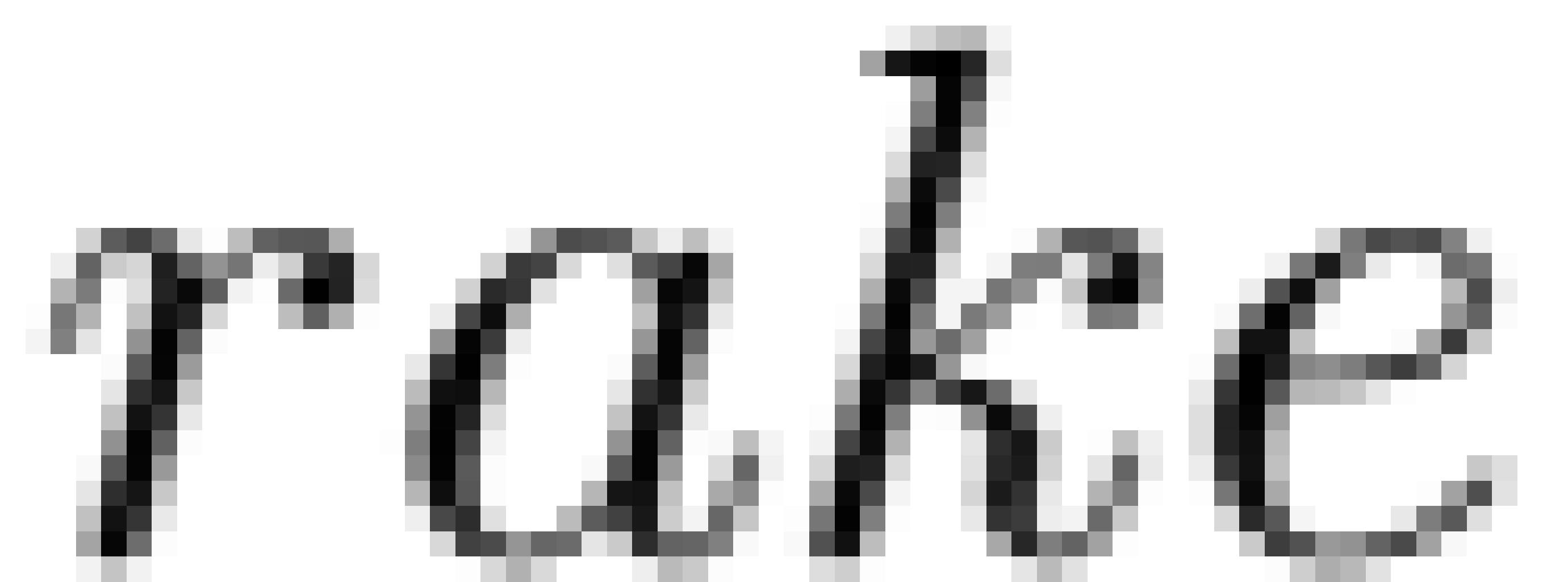
$$\tau_d = t \cdot \frac{r_d}{n_d}$$

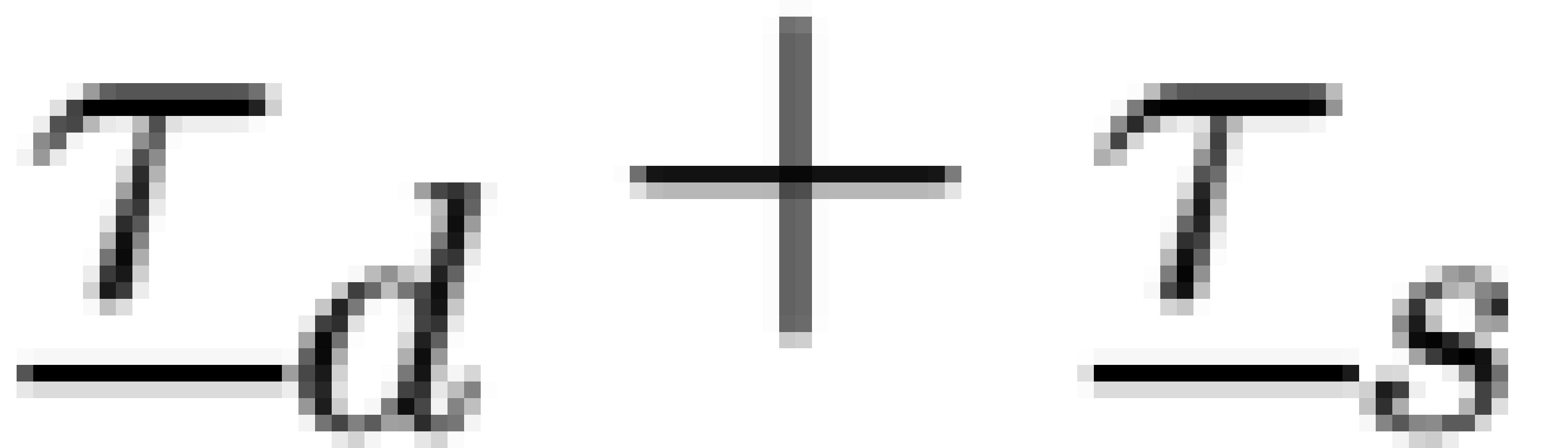
$$\tau_s = t \cdot \frac{r_s}{n_s}$$





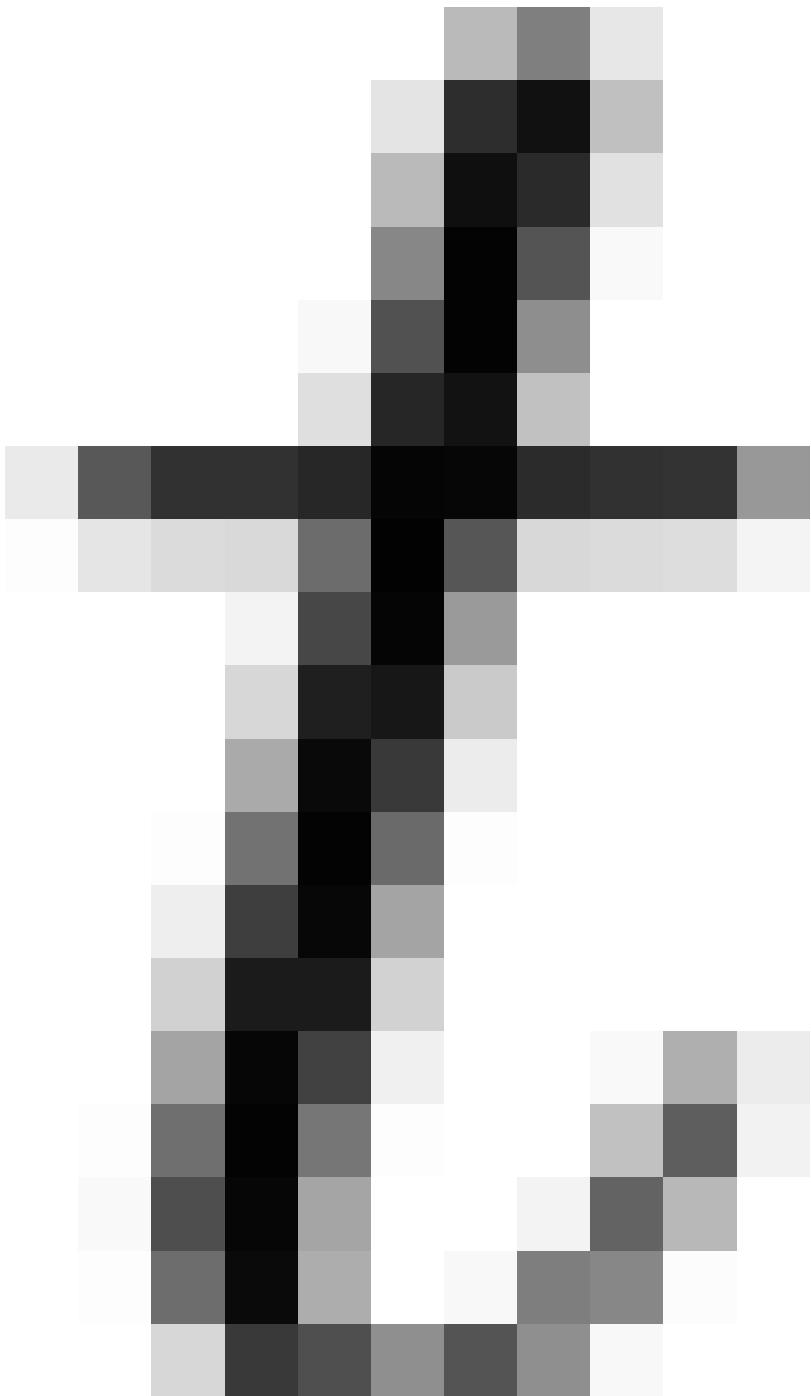


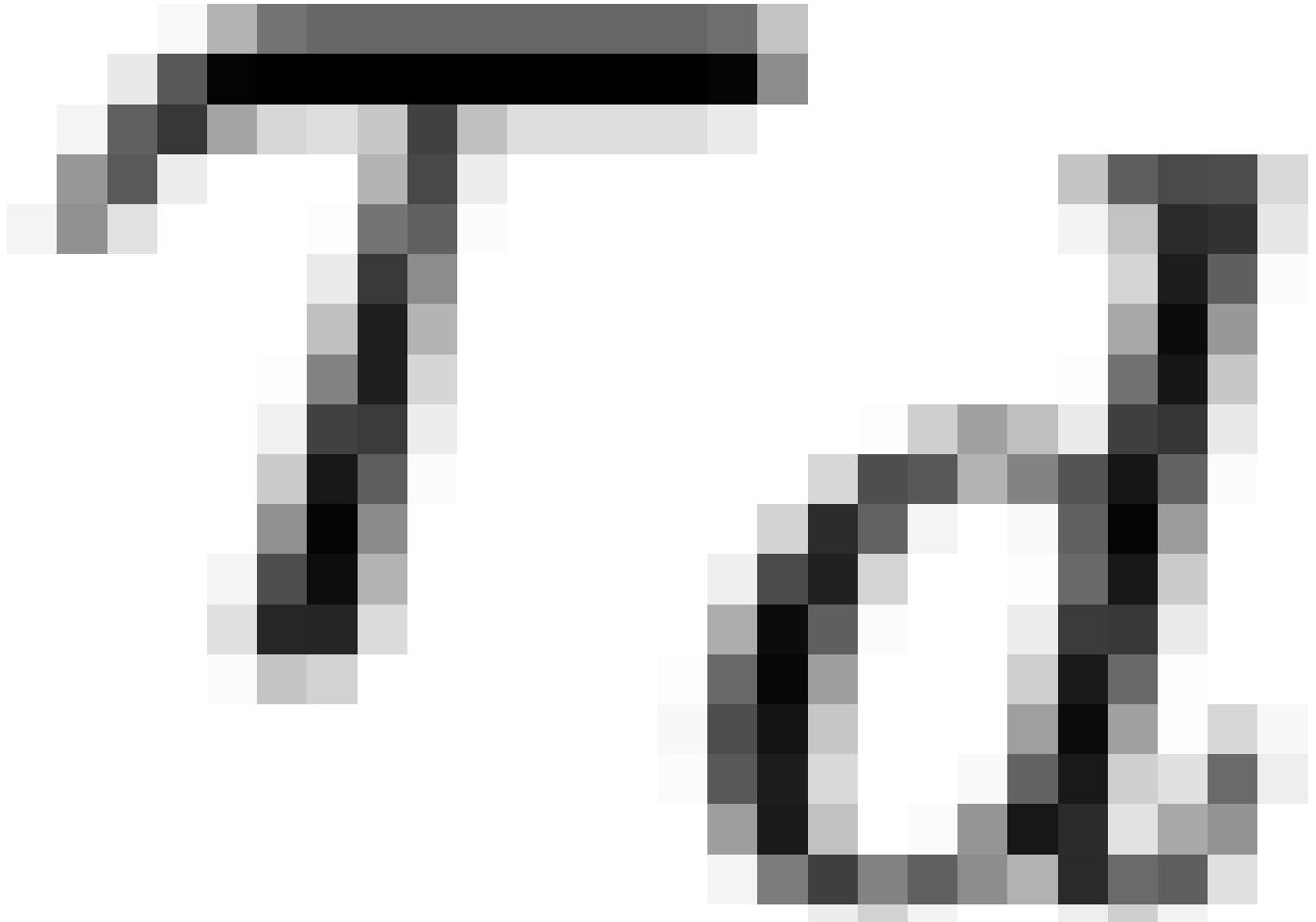


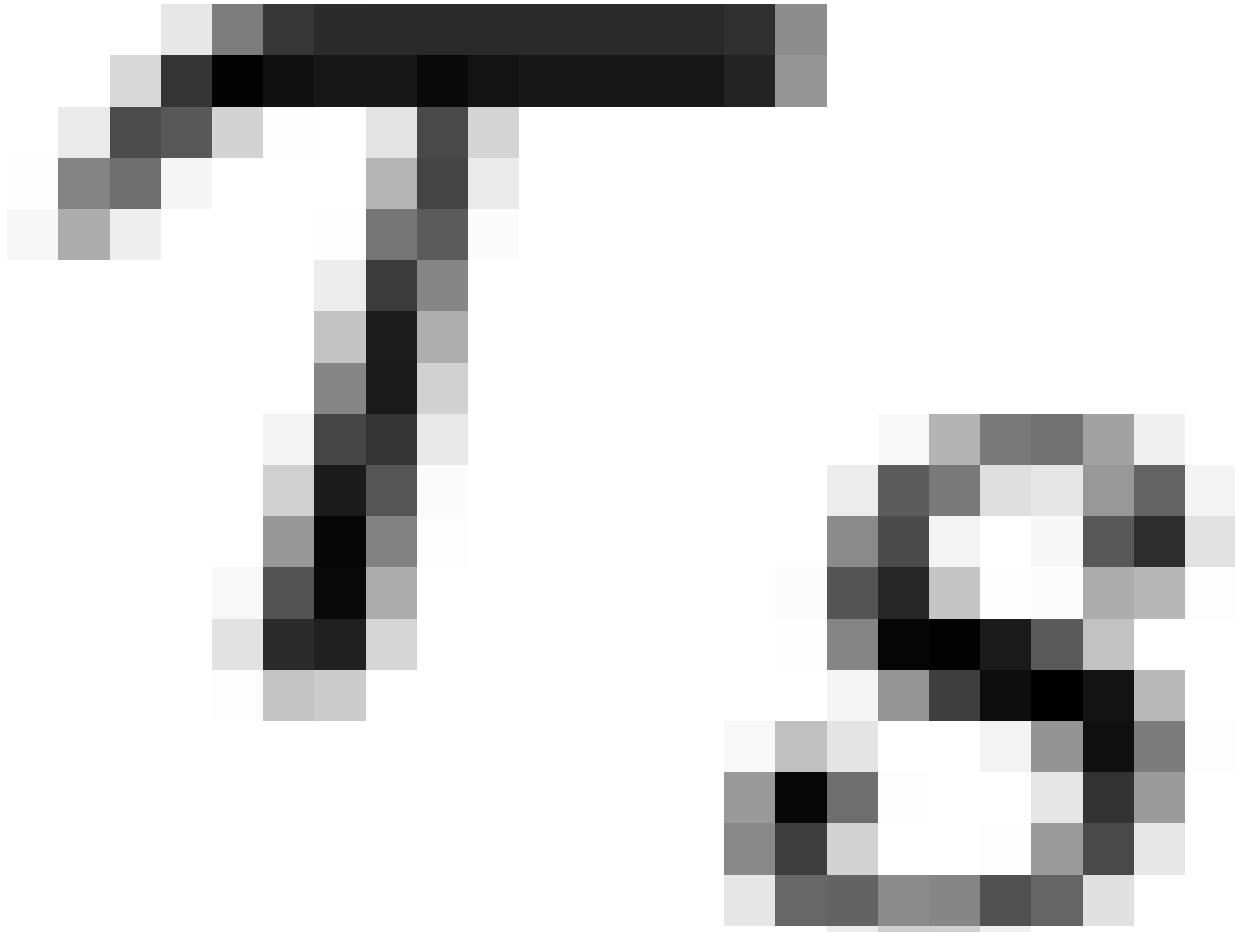


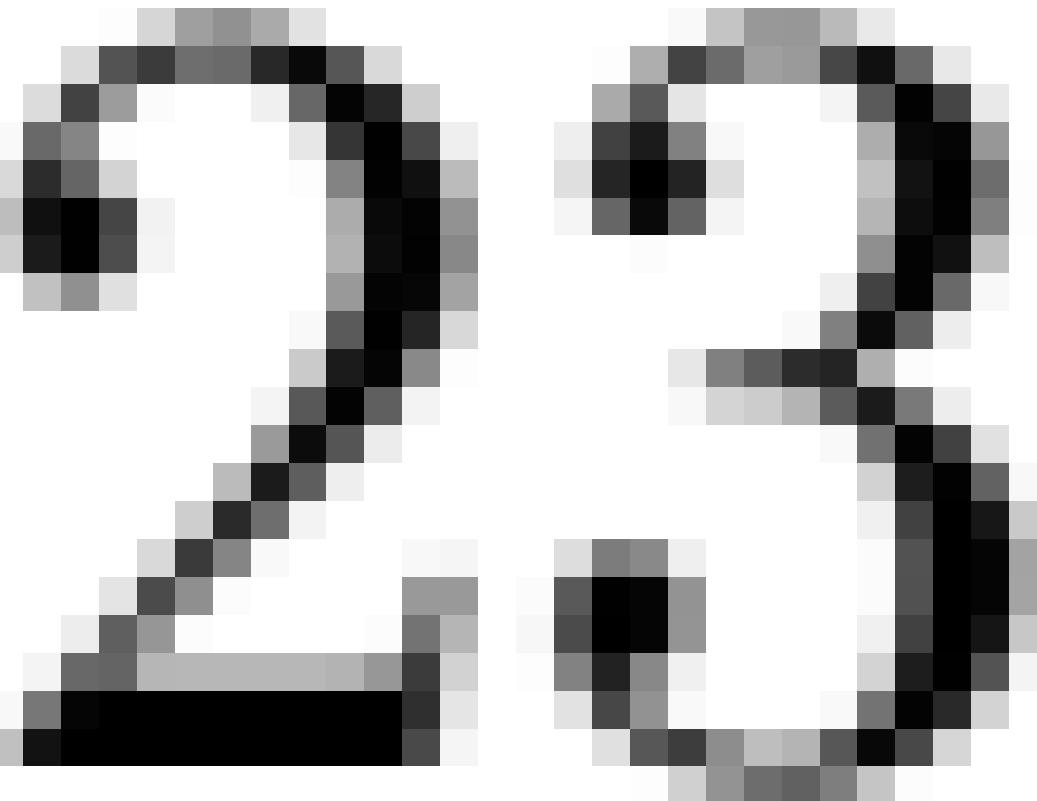
rake = arctan

$$\frac{\tau_d}{\tau_s}$$

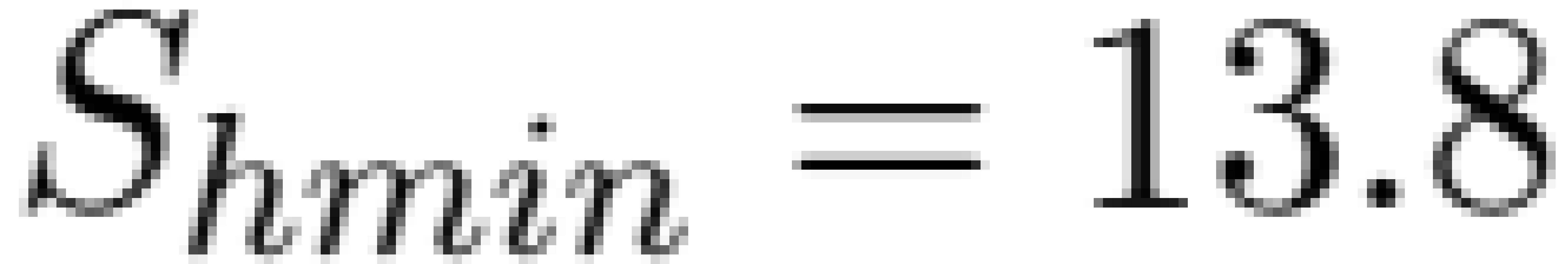


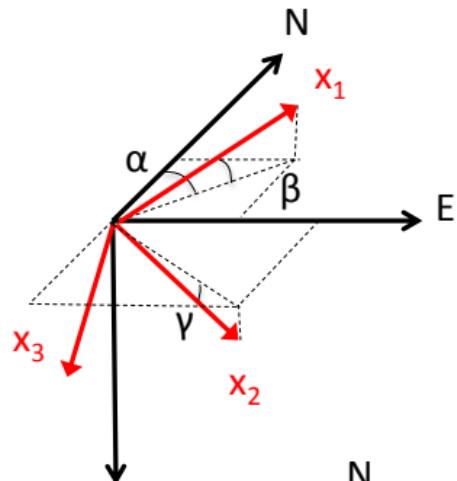










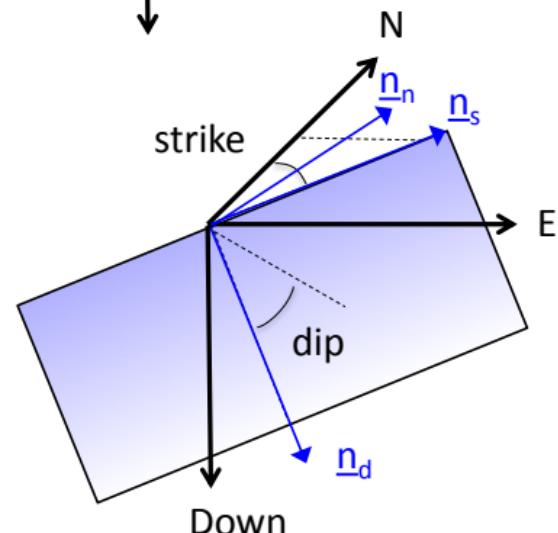


$$\underline{\underline{S}}_P = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 13.8 \end{bmatrix}$$

$a = 90^\circ$ Azimuth of $S_{h\min}$
 $b = 90^\circ$ $S_1 = S_V$
 $g = 0^\circ$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 13.8 & 0 \\ 0 & 0 & 23 \end{bmatrix}$$



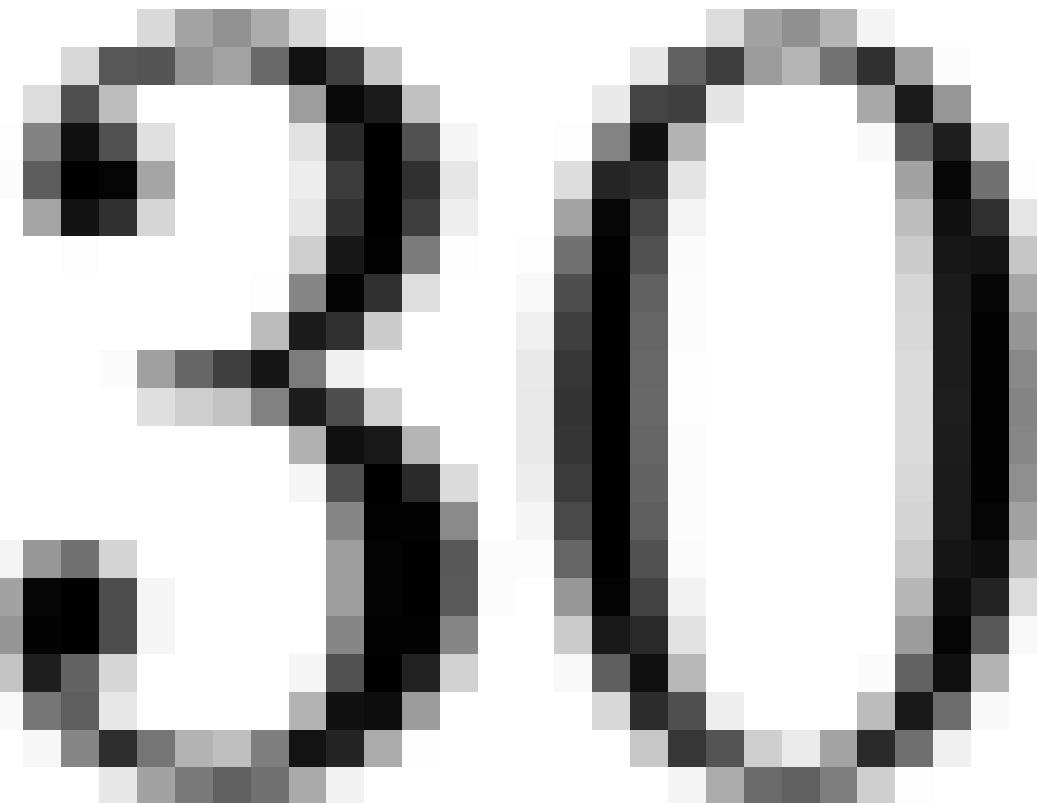
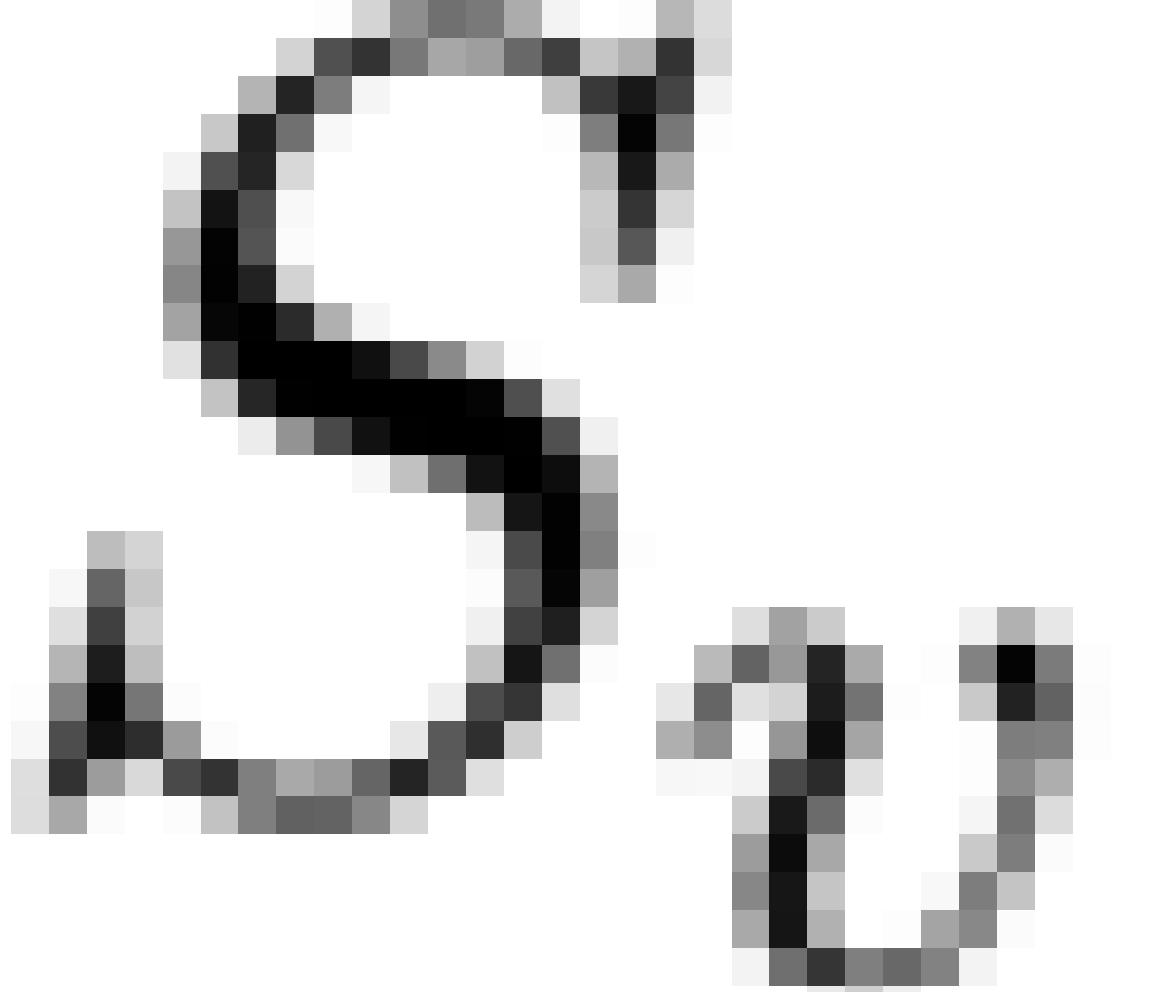
Fault geometry

Strike = 000°
 Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0 \\ 0.867 \\ -0.5 \end{bmatrix}$$

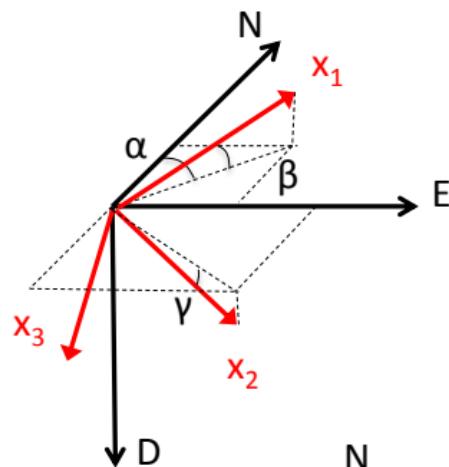
$\underline{t} = [0, 11.95, -11.50] \text{ MPa}$
 $S_n = 16.1 \text{ MPa}$
 $t_d = -3.98 \text{ MPa}$
 $t_s = 0 \text{ MPa}$
 rake = 90°

Actual stresses opposite to d-s-n



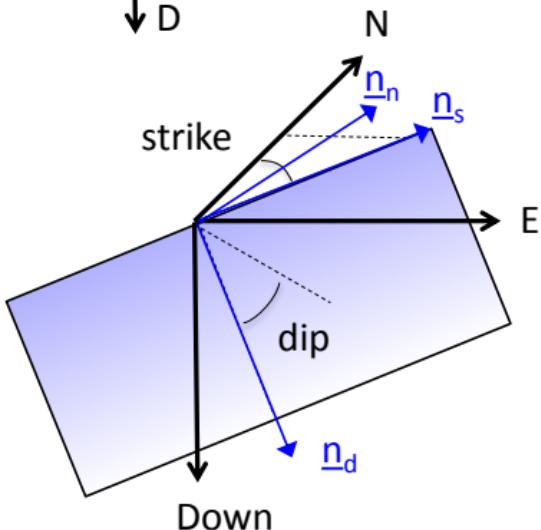






$$\underline{\underline{S}}_P = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix} \quad \begin{array}{ll} a = 120^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 90^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \\ 0.866 & 0.5 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$



Fault geometry

Strike = 060°

Dip = 90°

$$\underline{n}_n = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$\underline{t} = [-30.31, 27.5, 0] \text{ MPa}$$

$$S_n = 40 \text{ MPa}$$

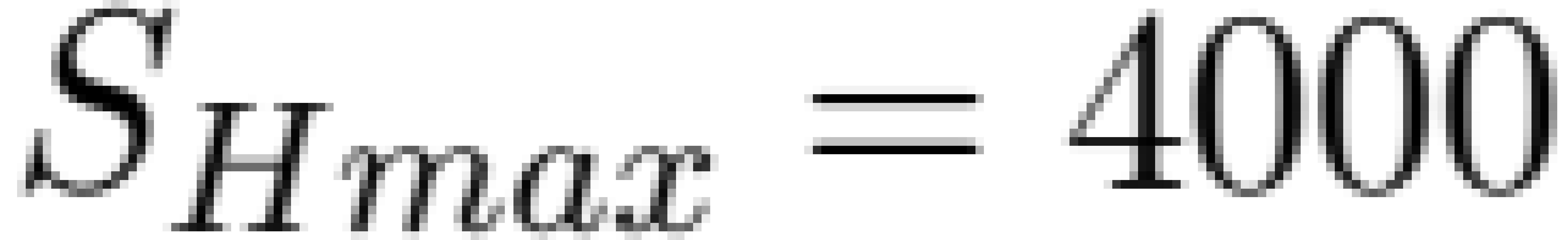
$$t_d = 0 \text{ MPa}$$

$$t_s = 8.66 \text{ MPa}$$

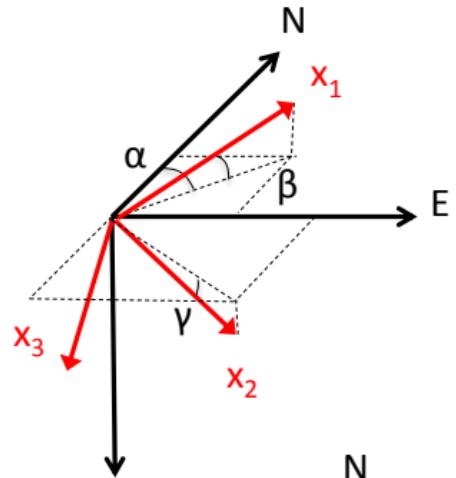
$$rake = 0^\circ$$

Actual stresses opposite to d-s-n



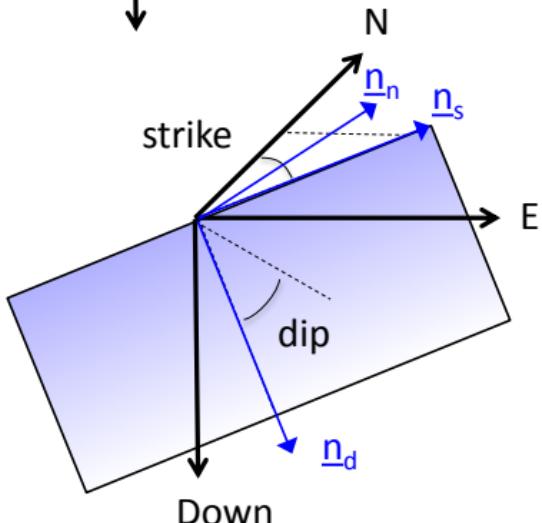






$$\underline{\underline{S}}_P = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \quad \begin{array}{ll} a = 90^\circ & \text{Azimuth of } S_{h\min} \\ b = 90^\circ & \\ g = 0^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$



Fault geometry

Strike = 045°

Dip = 60°

$$n_n = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [-2450, 1840, -2500] \text{ psi}$$

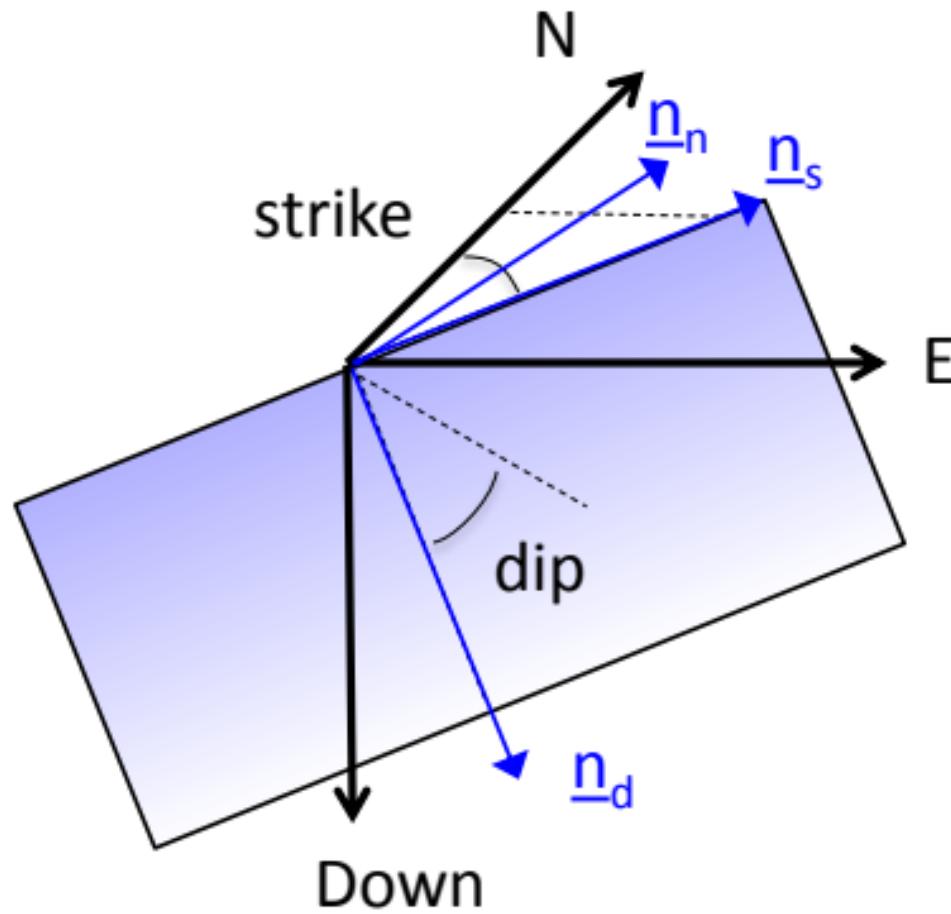
$$S_n = 3870 \text{ psi}$$

$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

$$\text{rake} = 56.3^\circ$$

Actual stresses opposite to d-s-n



Fault geometry

Strike = 225°

Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix}$$

$$t = [2450, -1840, -2500] \text{ psi}$$

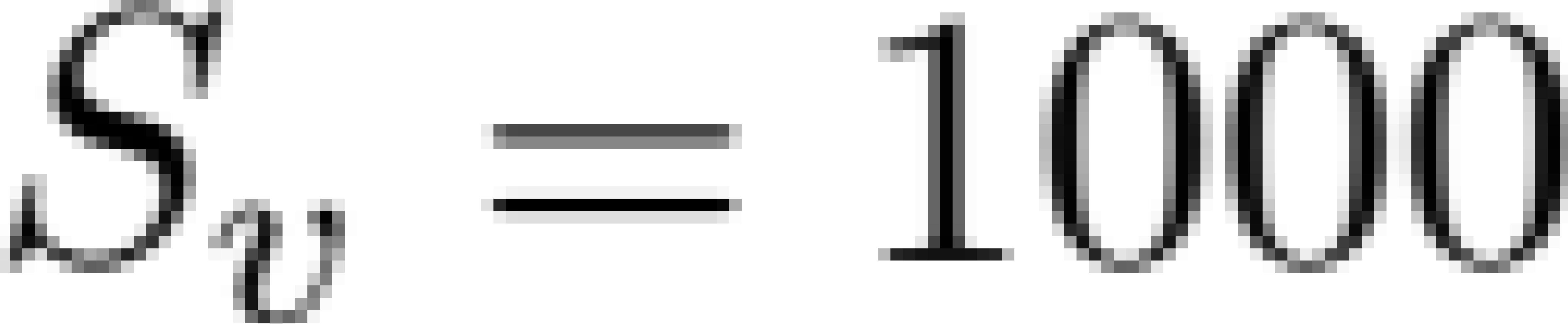
$$S_n = 3870 \text{ psi}$$

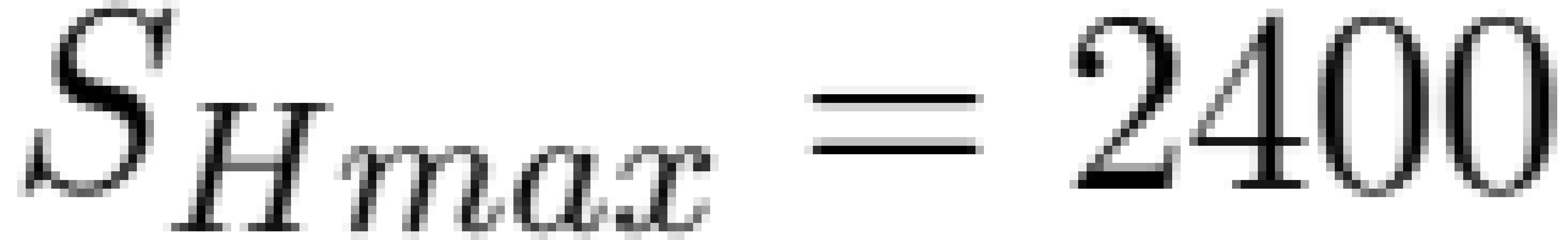
$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

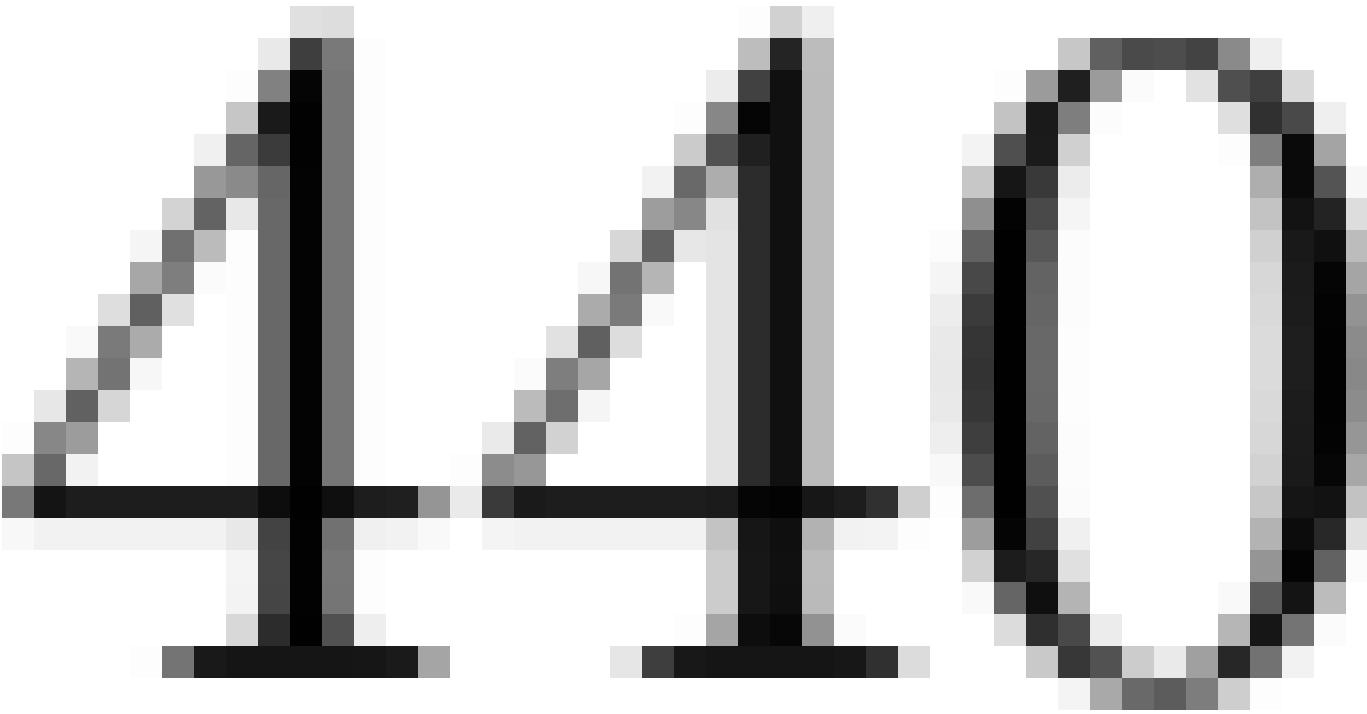
$$\text{rake} = 56.3^\circ$$

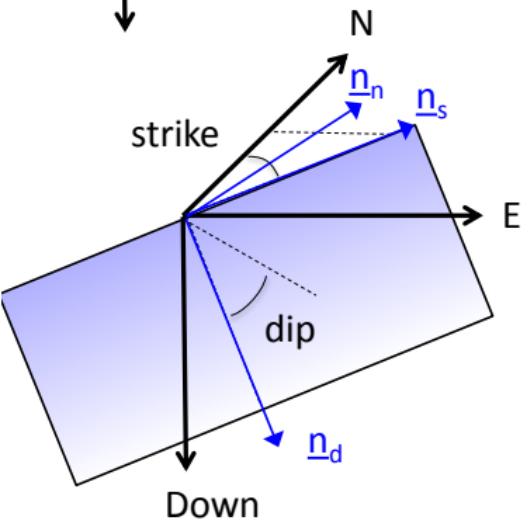
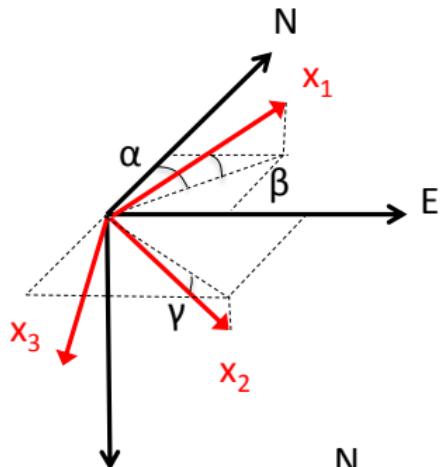
Actual stresses opposite to d-s-n











$$\underline{\underline{S}}_P = \begin{bmatrix} 2400 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \quad \begin{array}{ll} \alpha = 150^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 0^\circ & S_3 = S_V \end{array}$$

$$P_w = 440 \text{ psi}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.866 & 0.5 & 0 \\ -0.5 & -0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

Fault geometry

Strike = 120°

Dip = 70°

$$\underline{n}_n = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix}$$

$$\underline{t} = [-1465, -281, -342] \text{ psi}$$

$$S_n = 1441 \text{ psi}$$

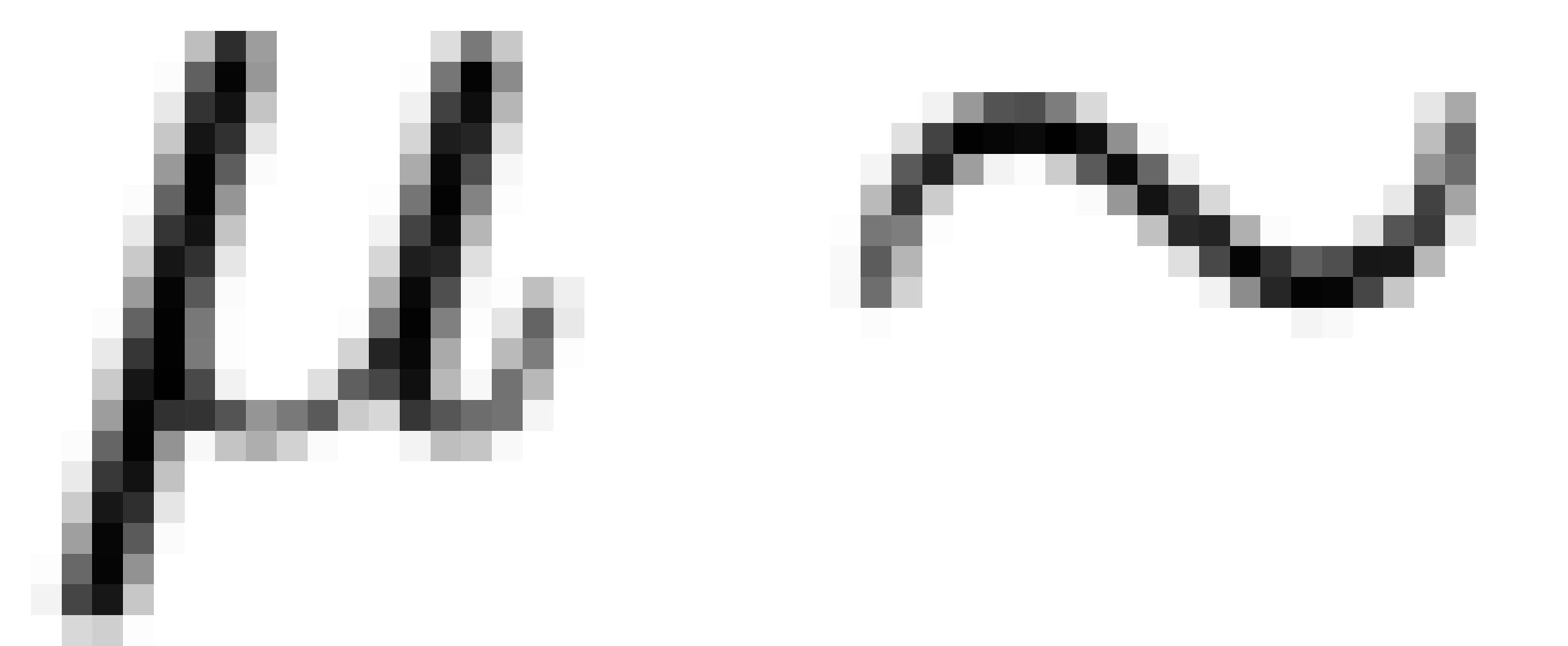
$$t_d = 160 \text{ psi}$$

$$t_s = 488 \text{ psi}$$

$$\text{rake} = 18.21^\circ$$

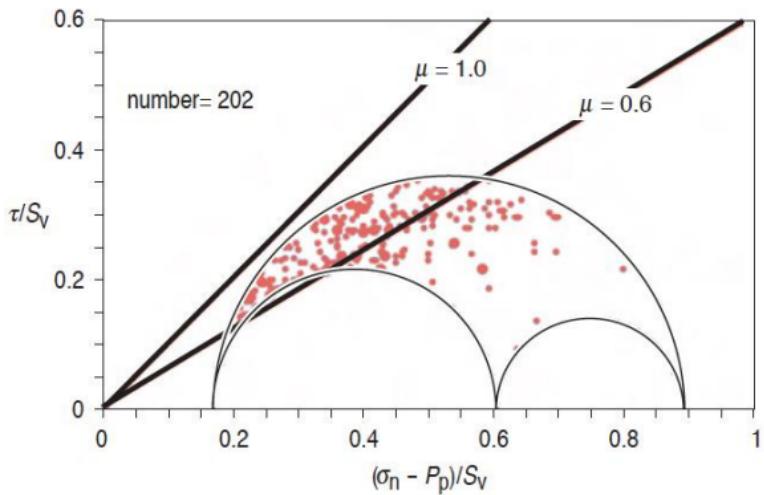
$$t / S_n = 0.51$$

Actual stresses opposite to d-s-n



Zoback 2013 - Figure 11.1-5.2

HYDRAULICALLY CONDUCTIVE FRACTURES

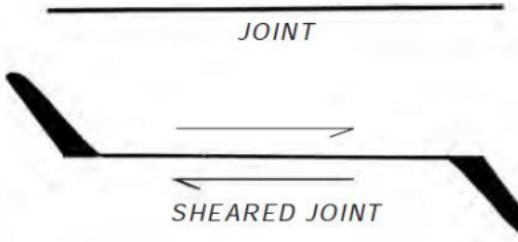
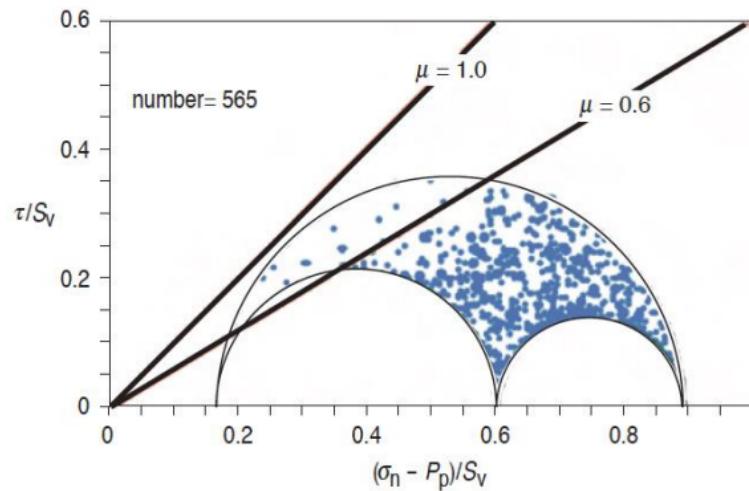


ONSET OF DAMAGE

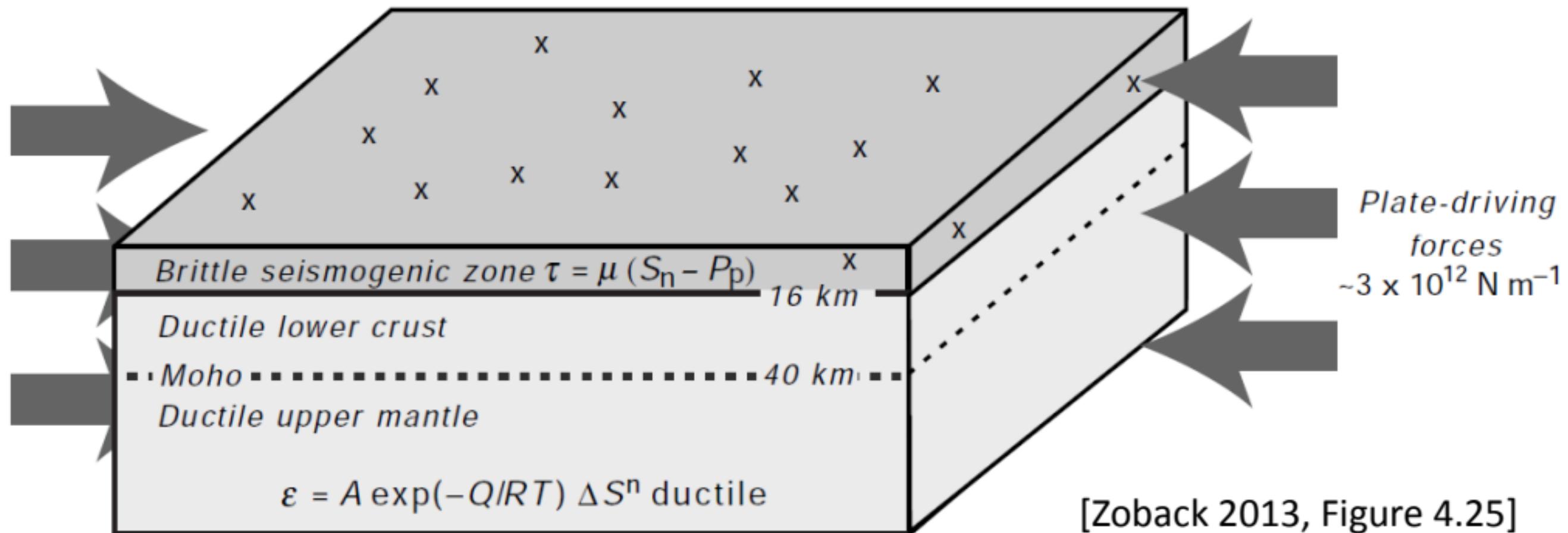


BRECCIAZION IN SHEAR ZONE

NON-HYDRAULICALLY CONDUCTIVE FRACTURES



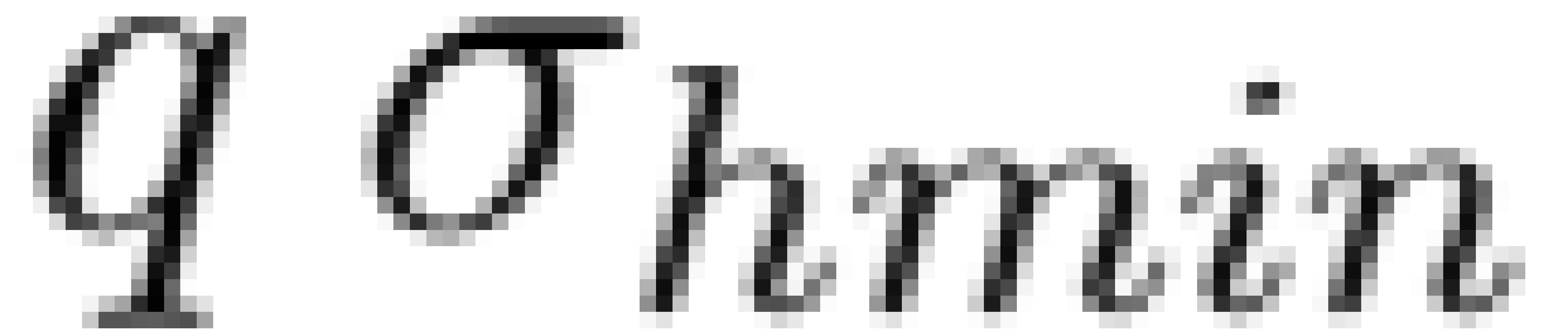
- + Mineral precipitation
- + Mineralization to clays

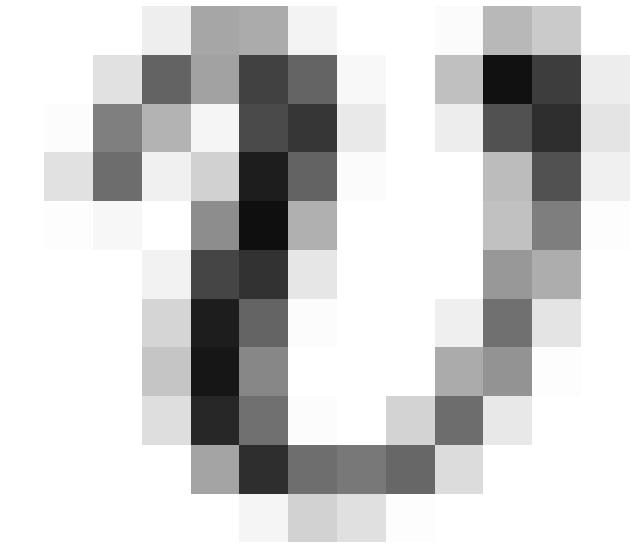
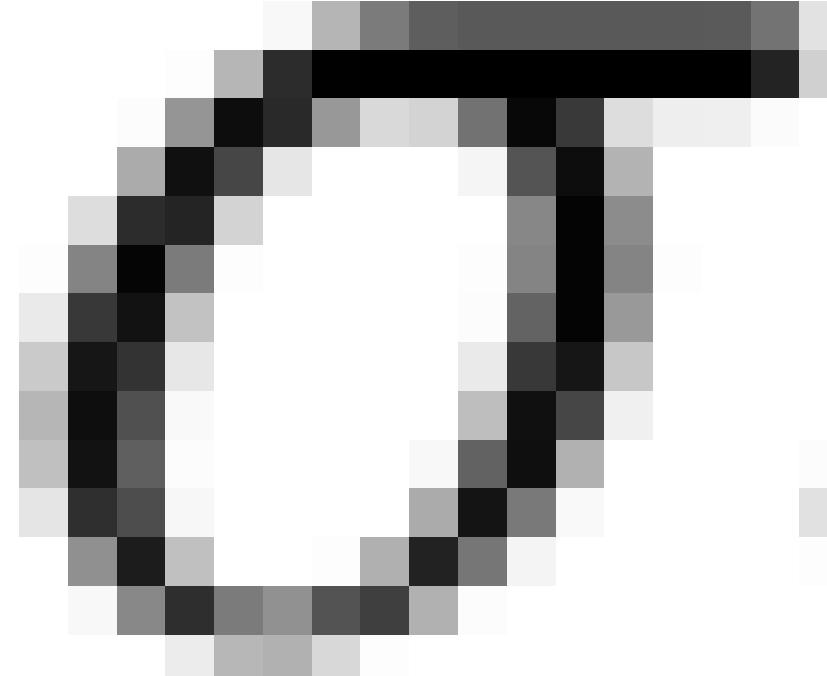
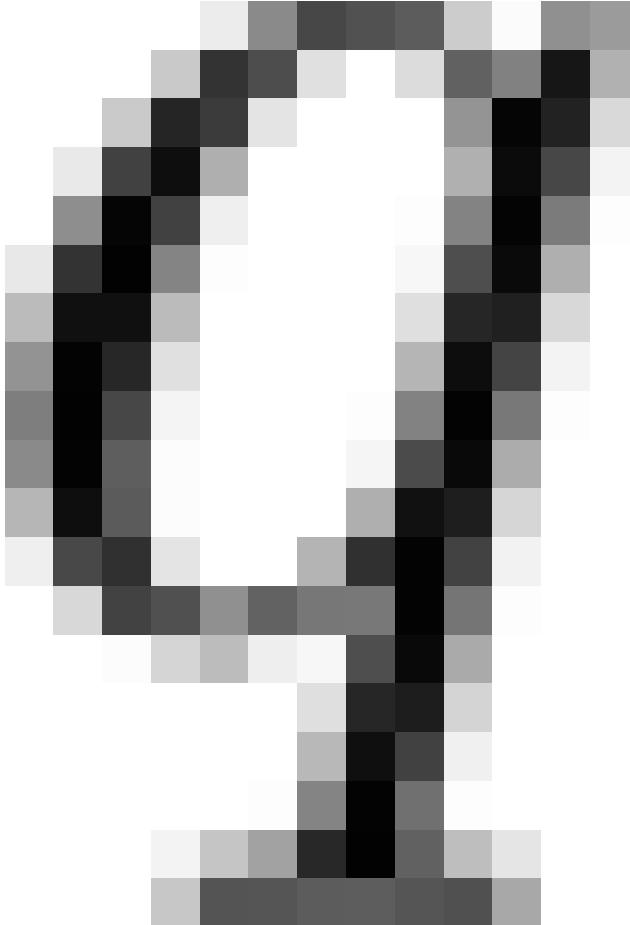


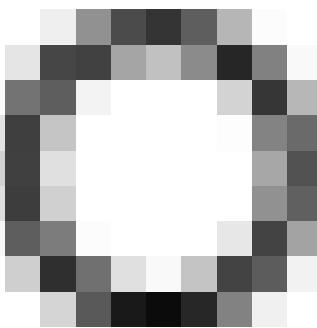
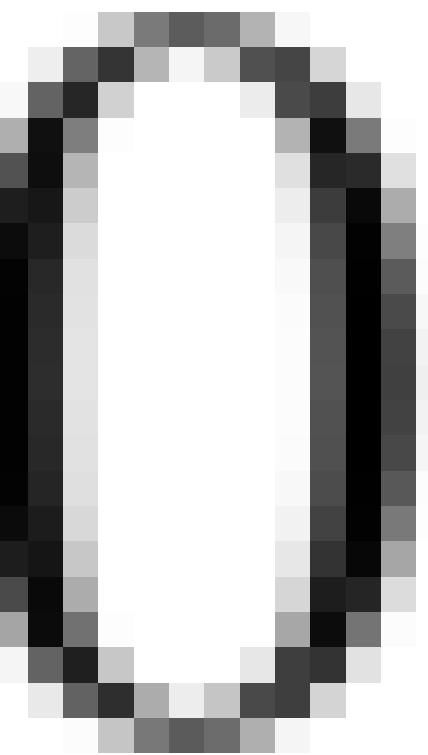
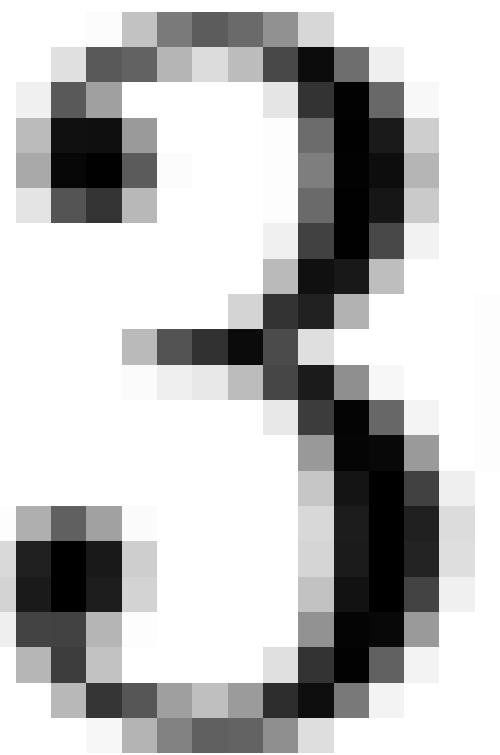
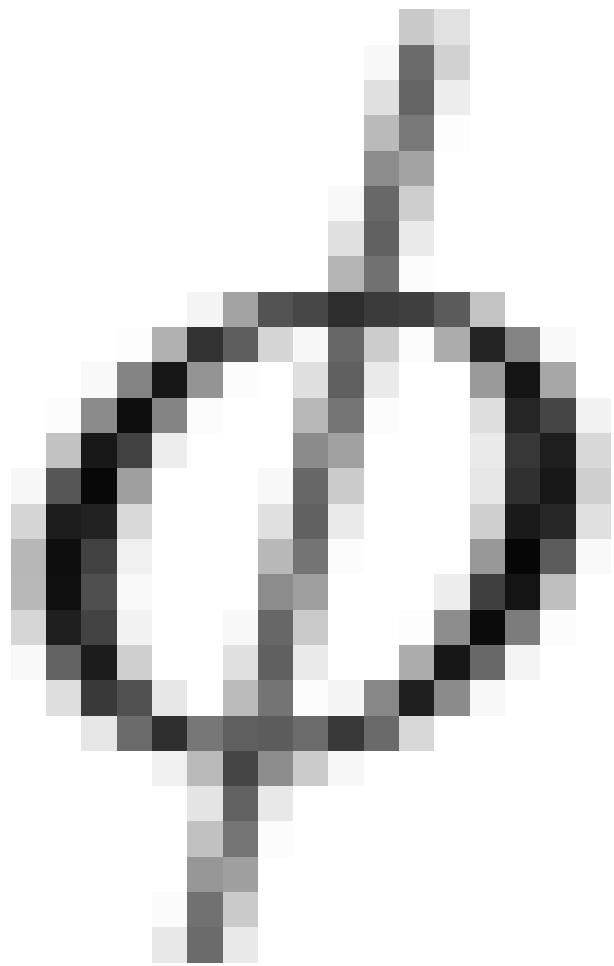


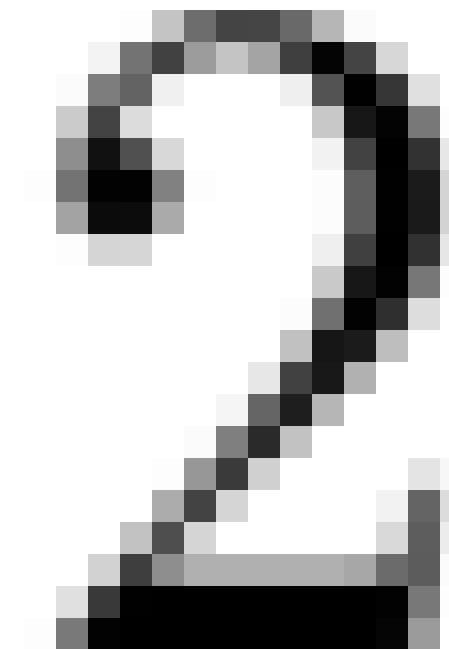
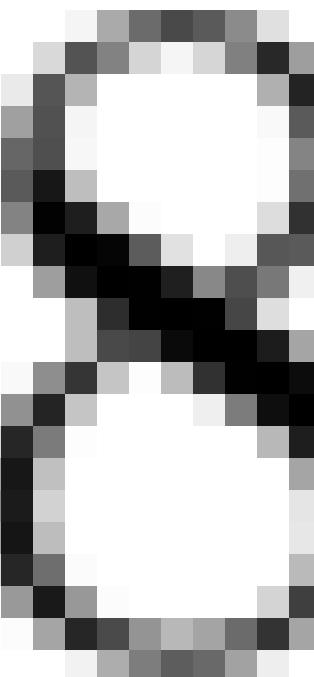
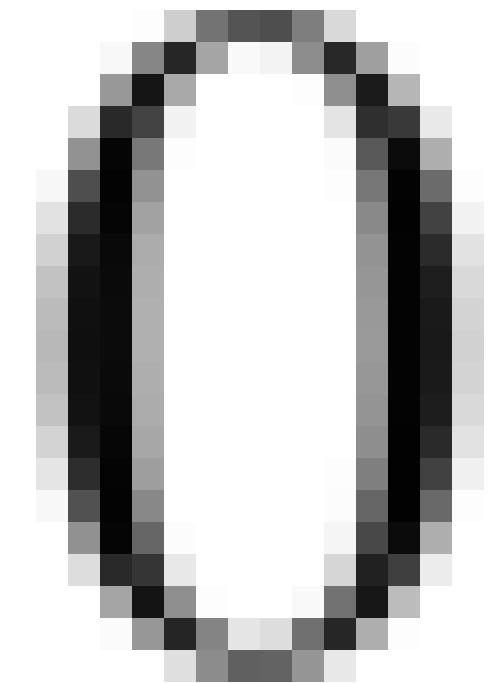


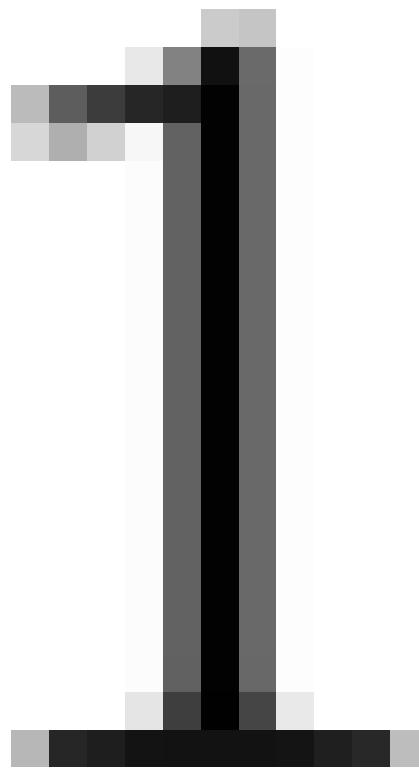


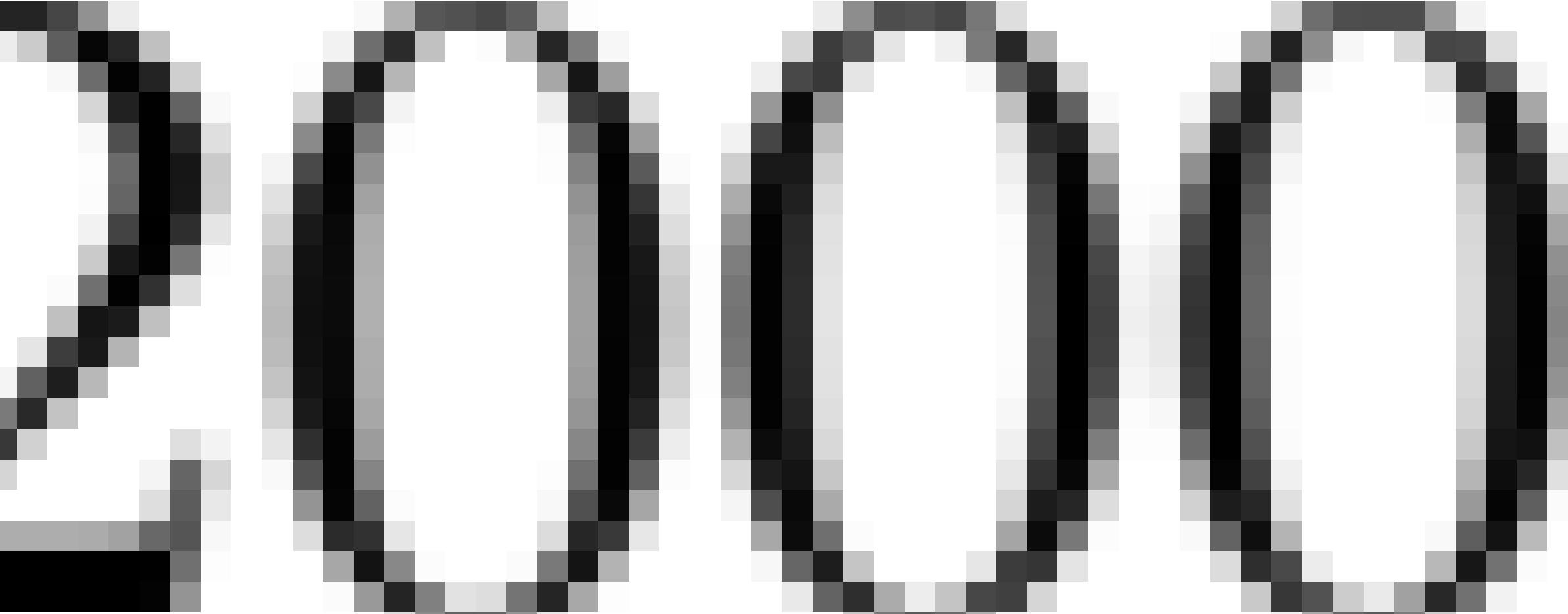


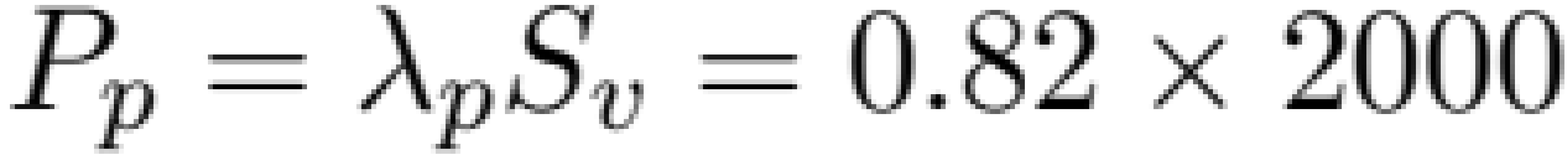


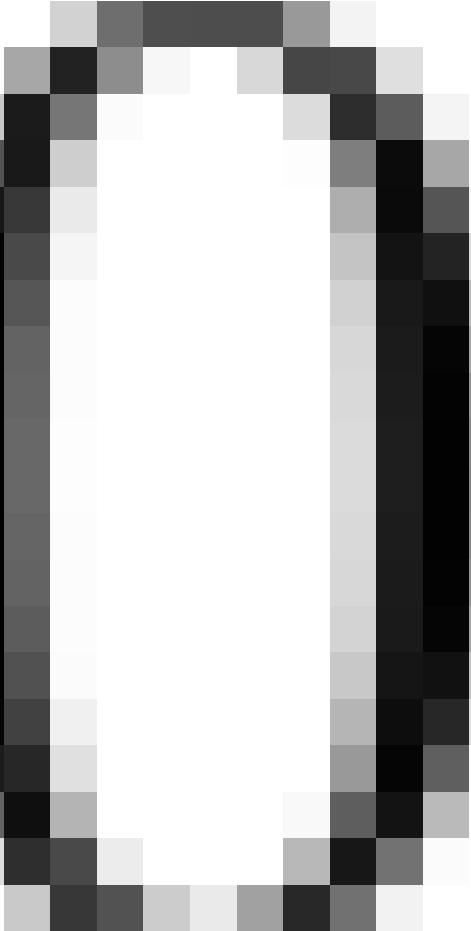
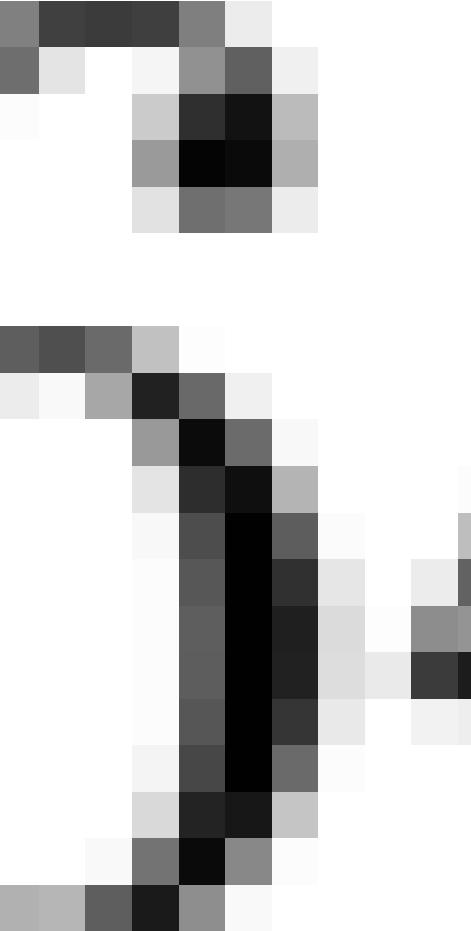
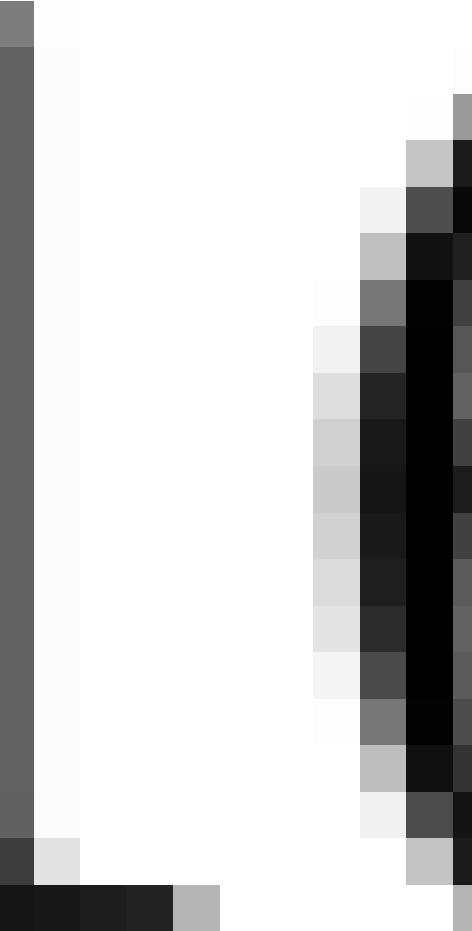


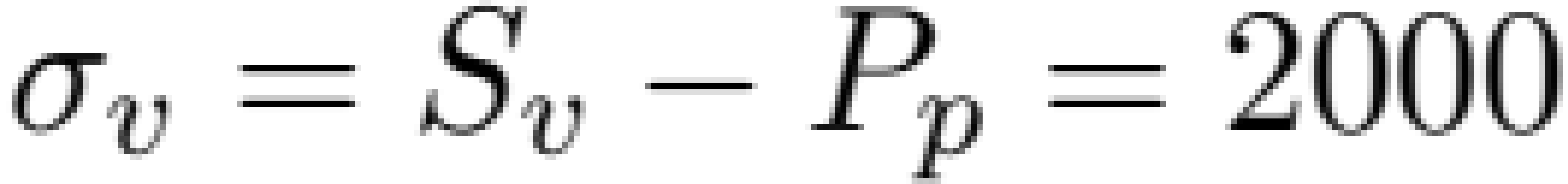


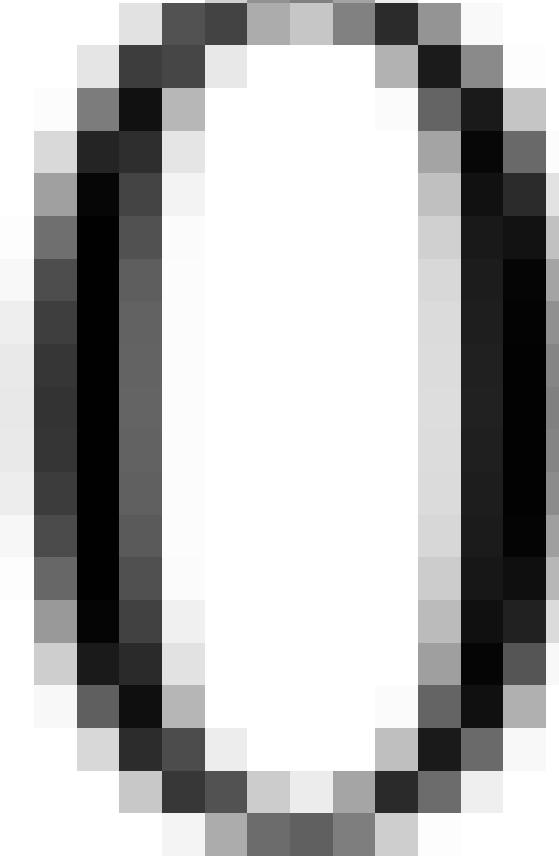
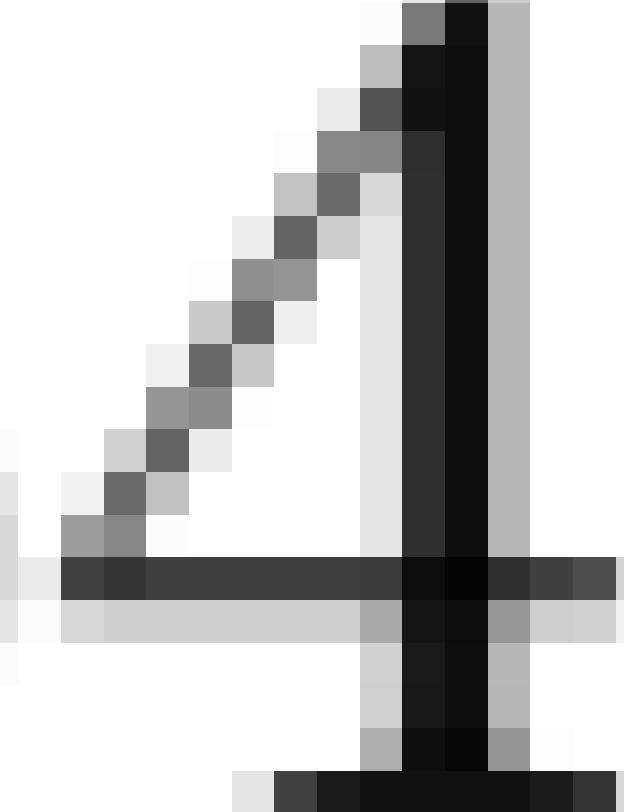
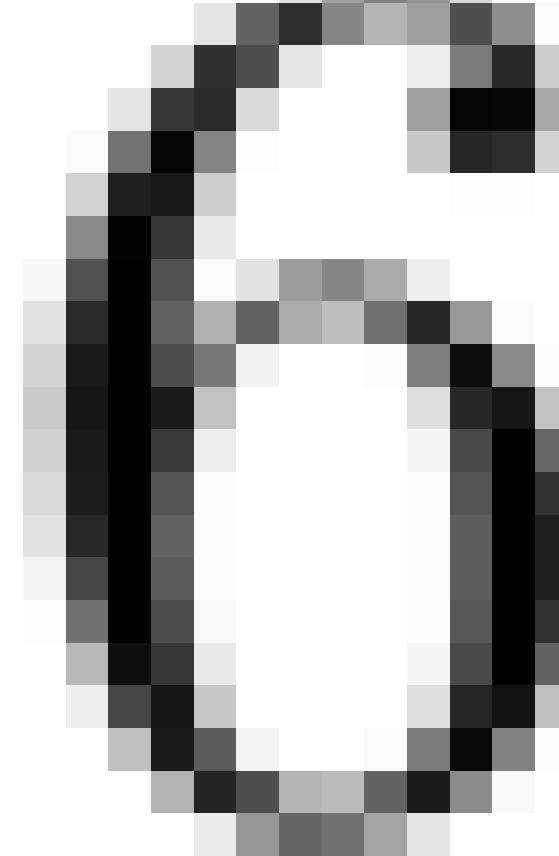
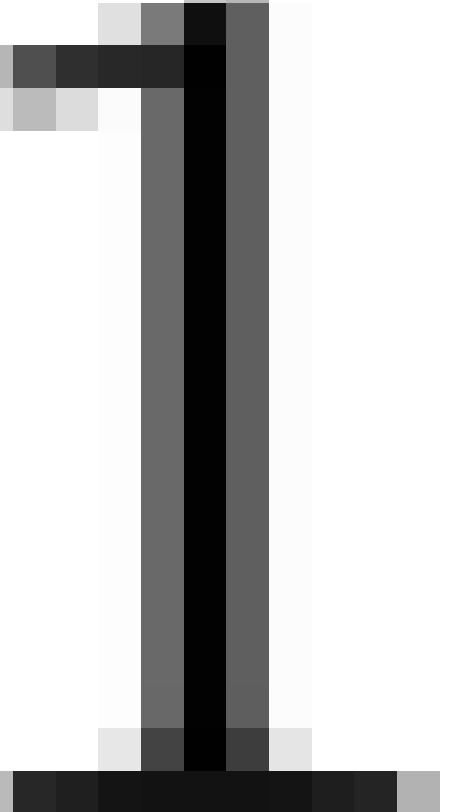


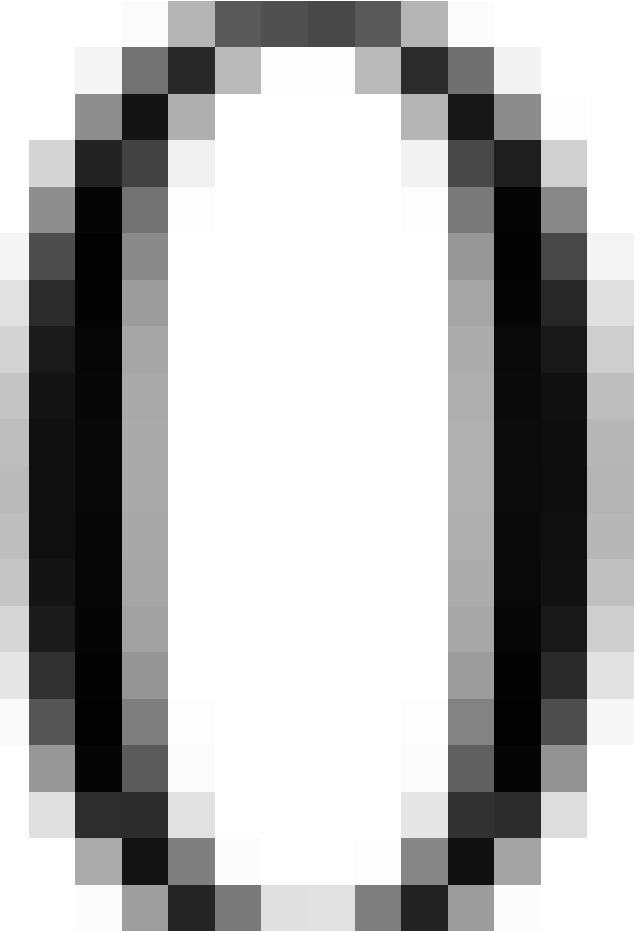
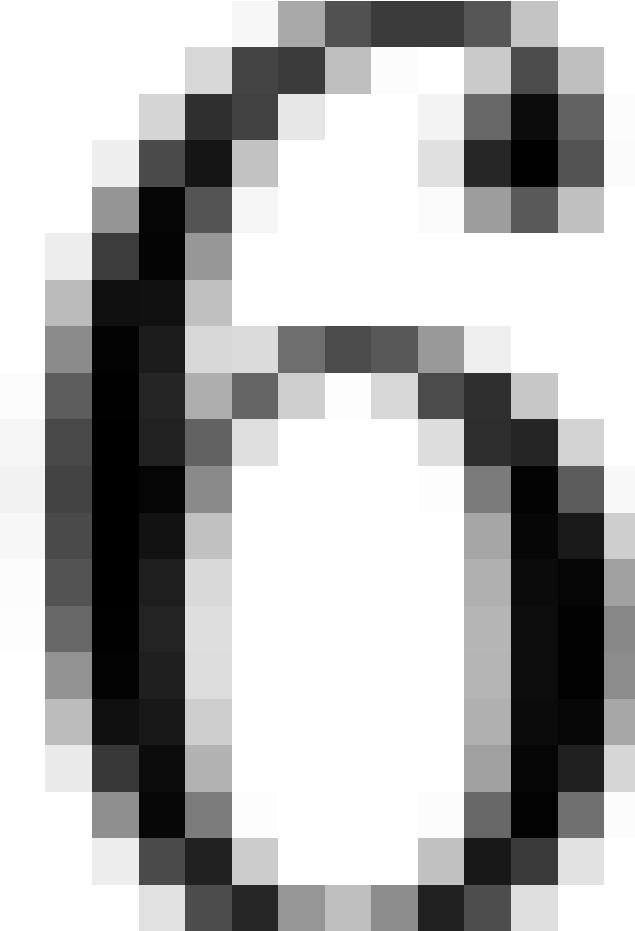
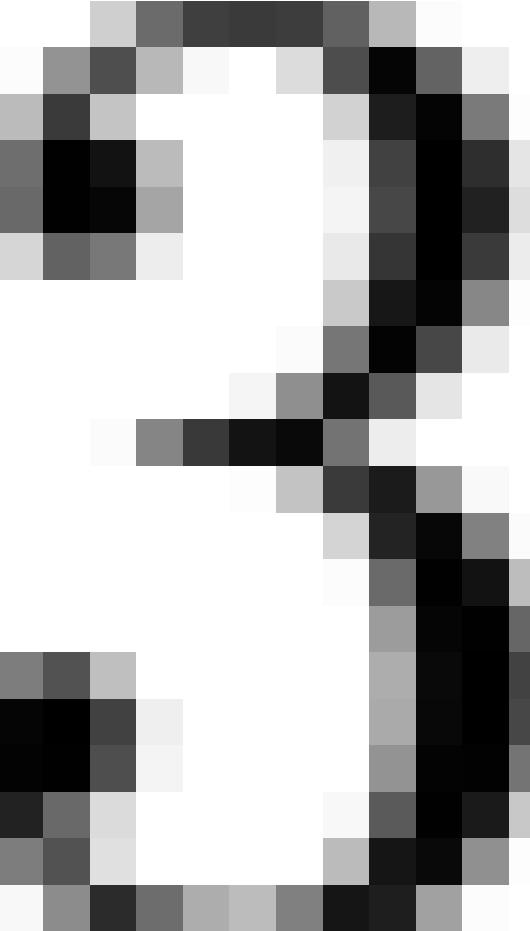








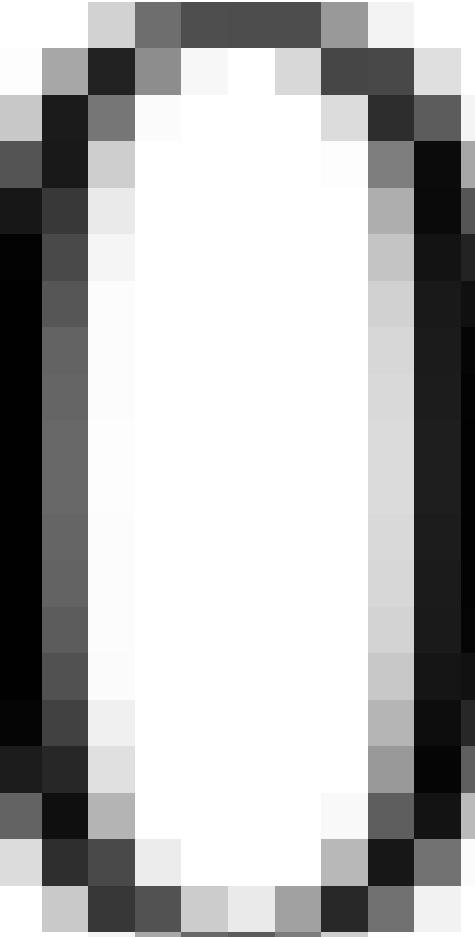
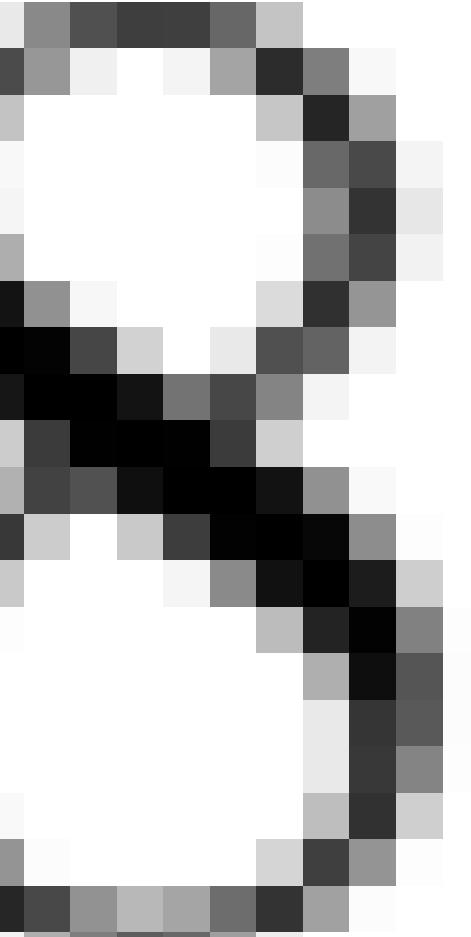
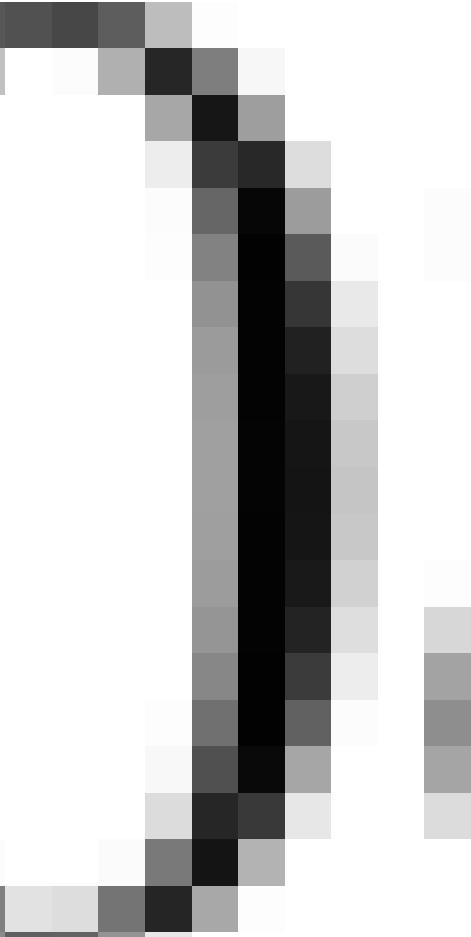
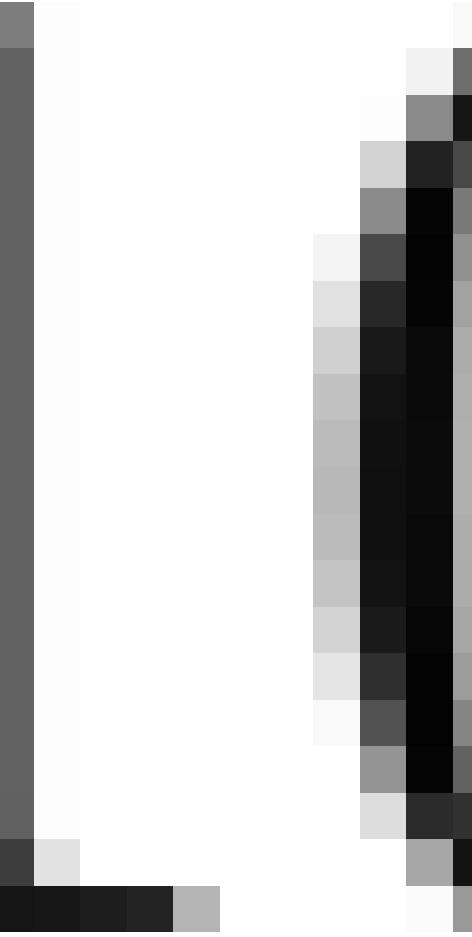


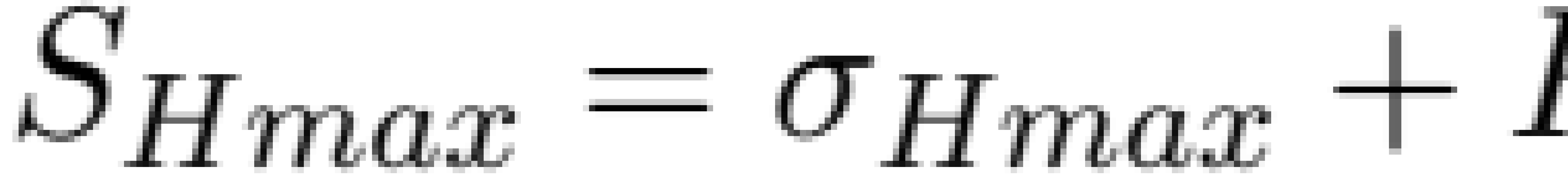


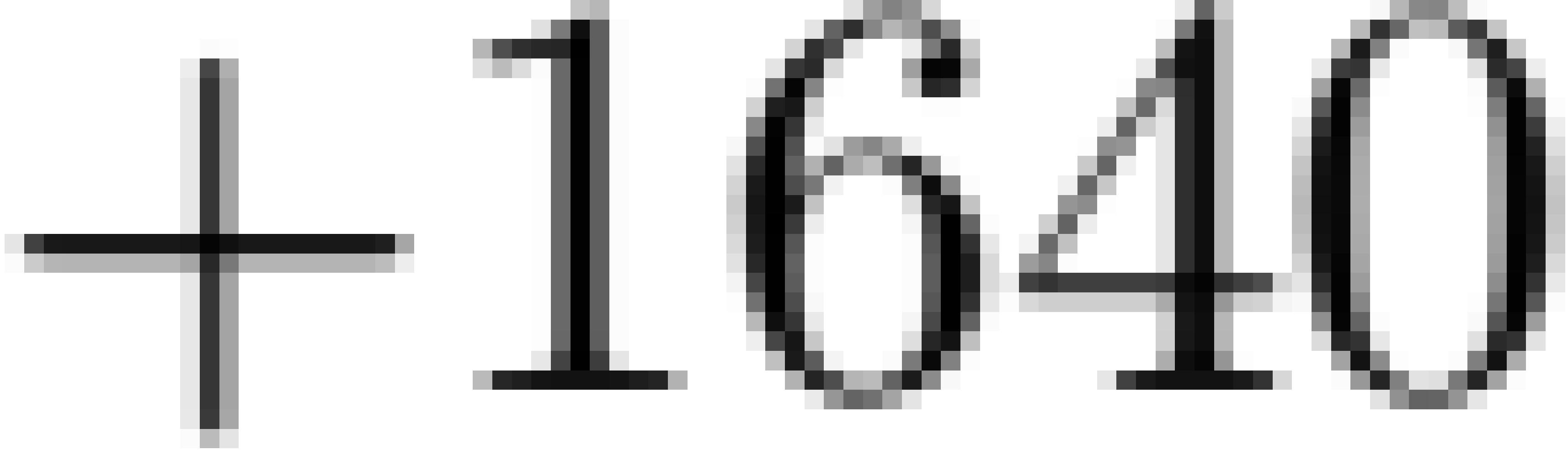
$$\sigma_{H\text{max}} =$$

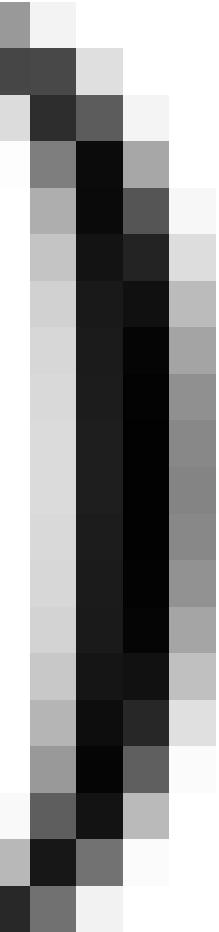
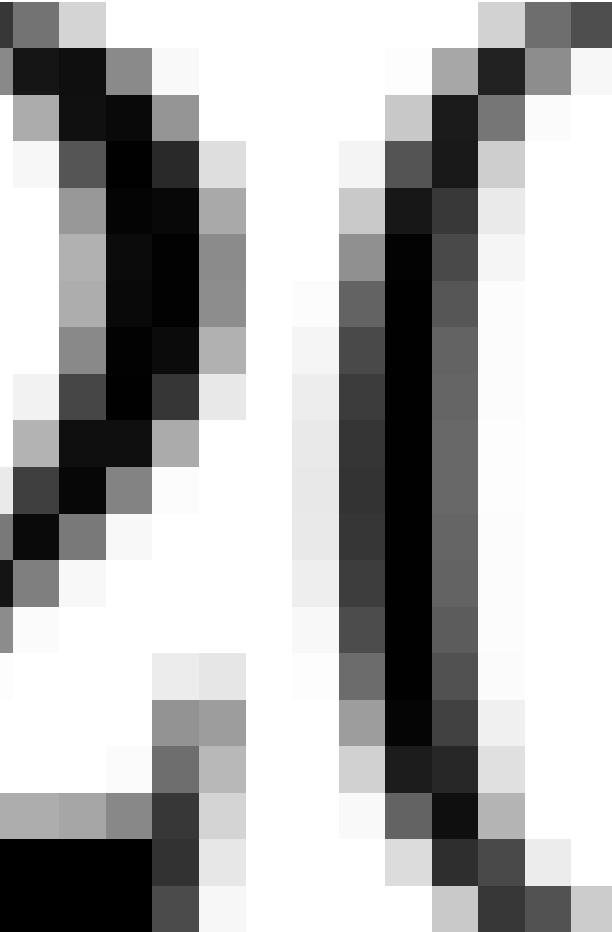
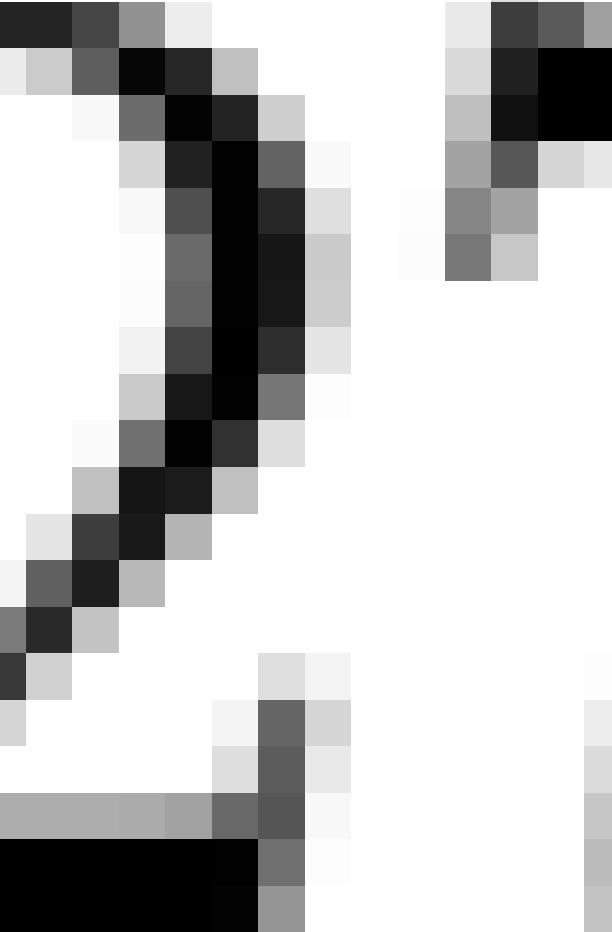
$$90^\circ =$$

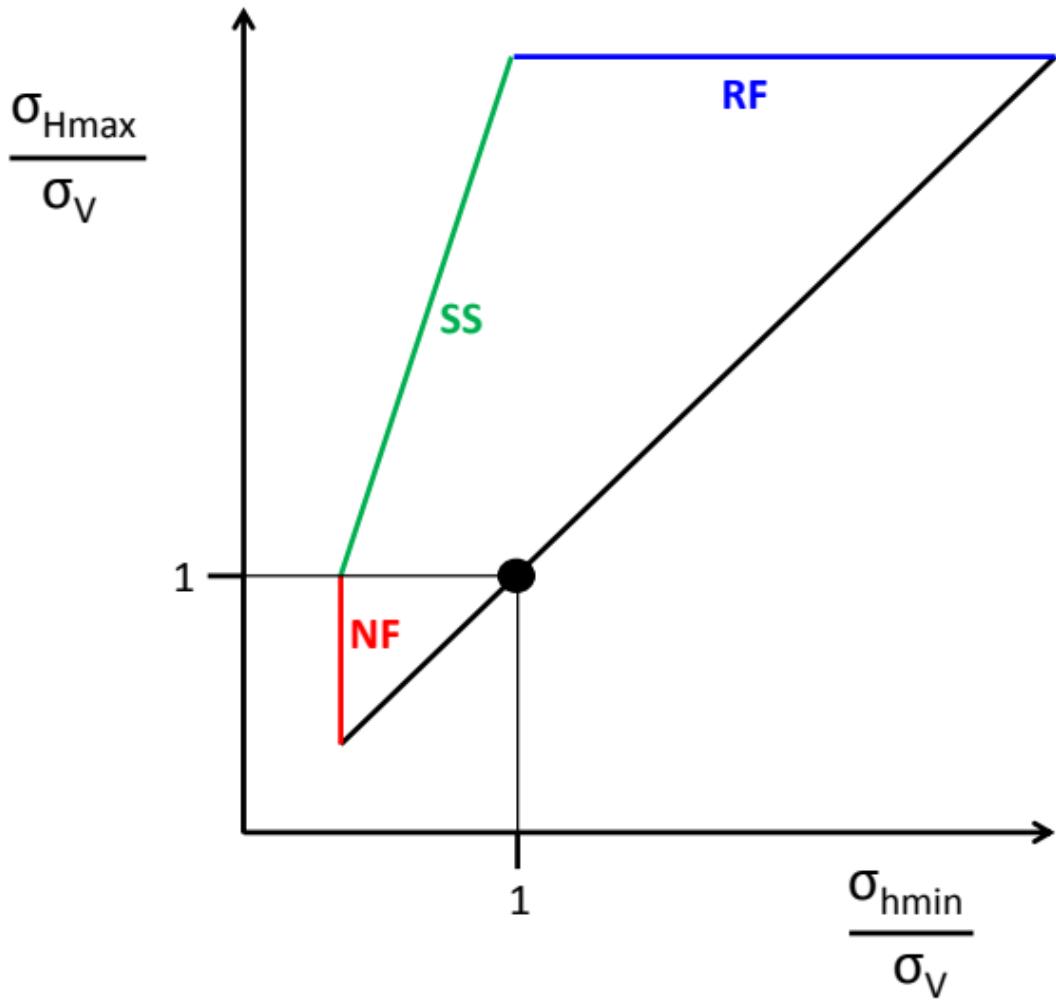
$$\frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} 360$$



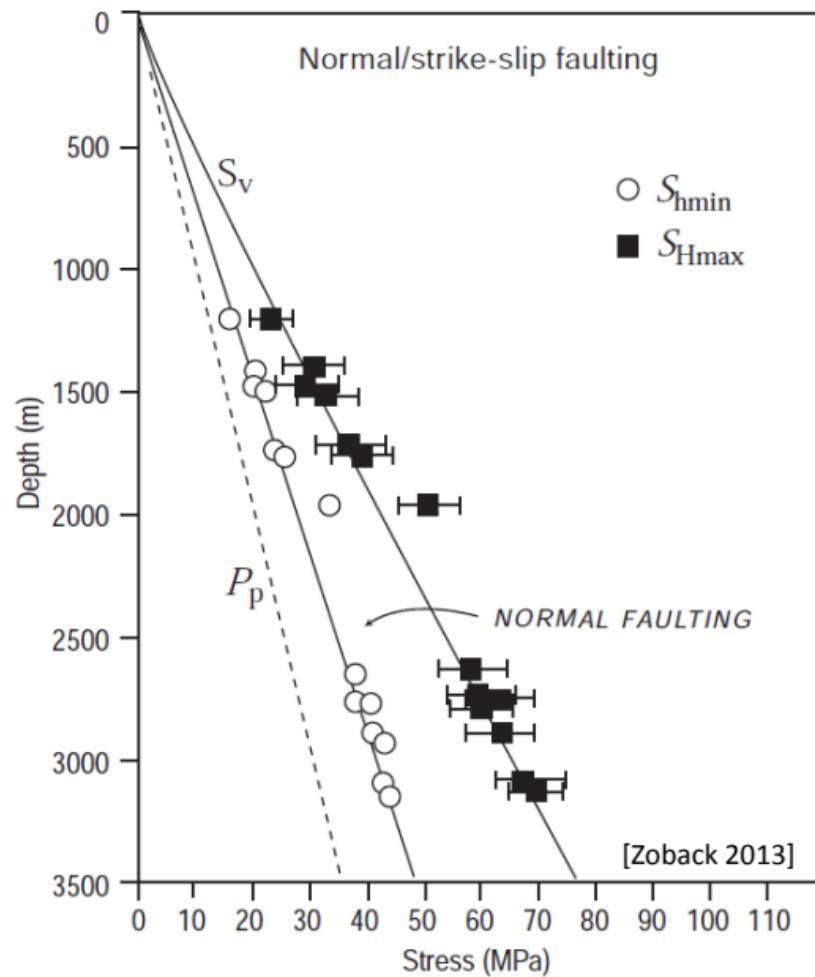
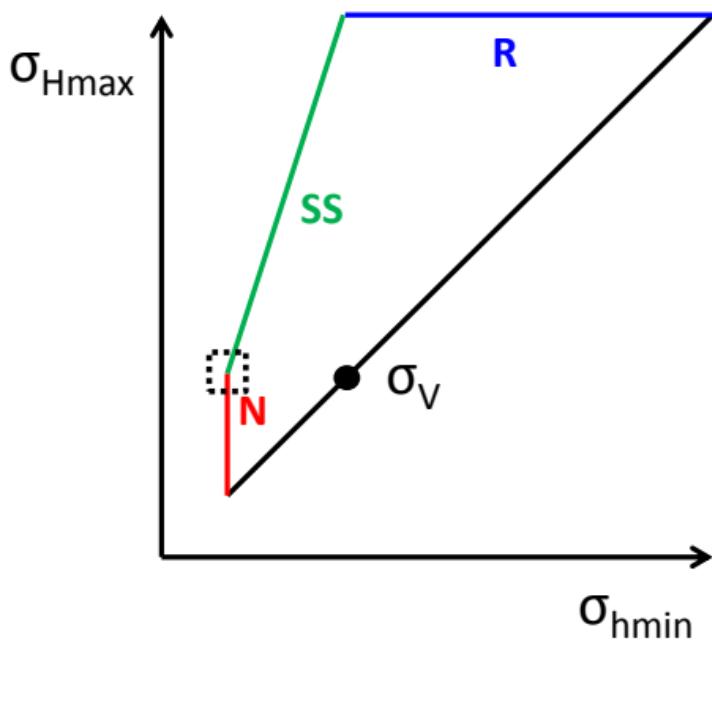


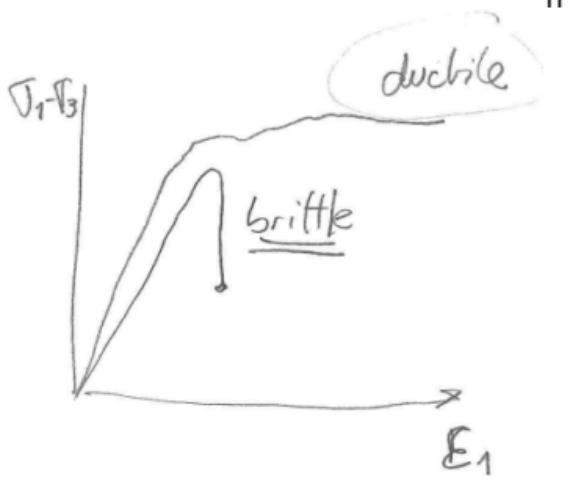
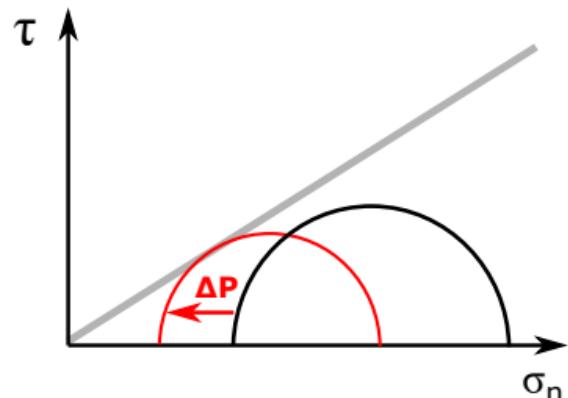
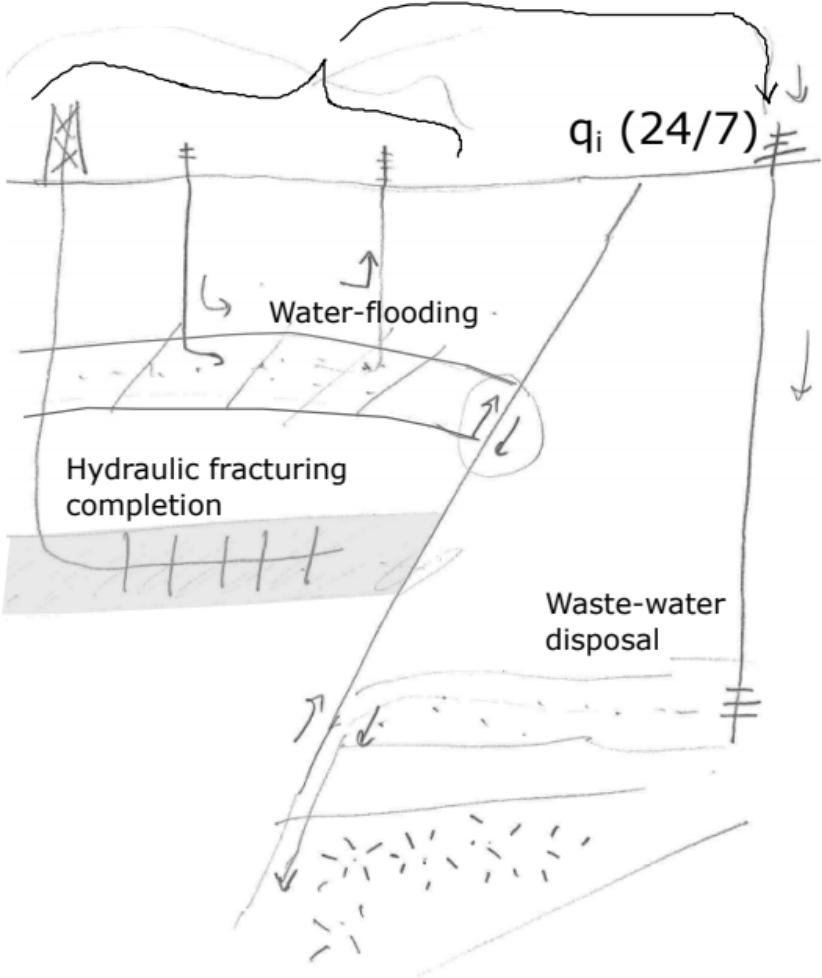






$$\sigma_{h\min} < \sigma_{H\max} \approx \sigma_v$$



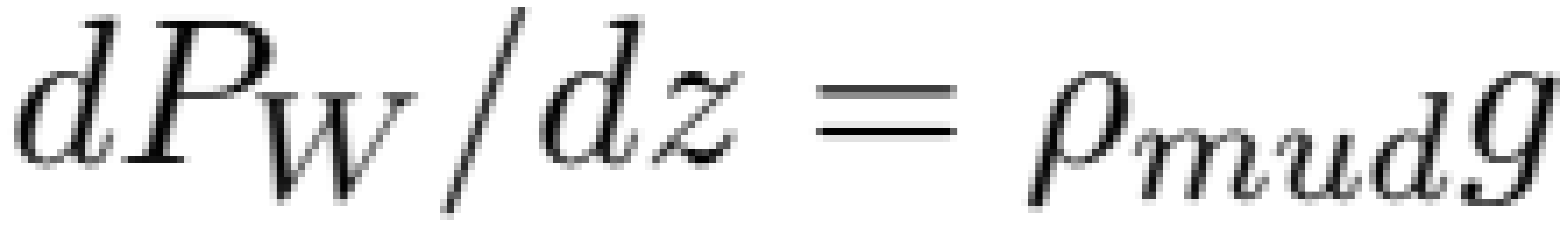


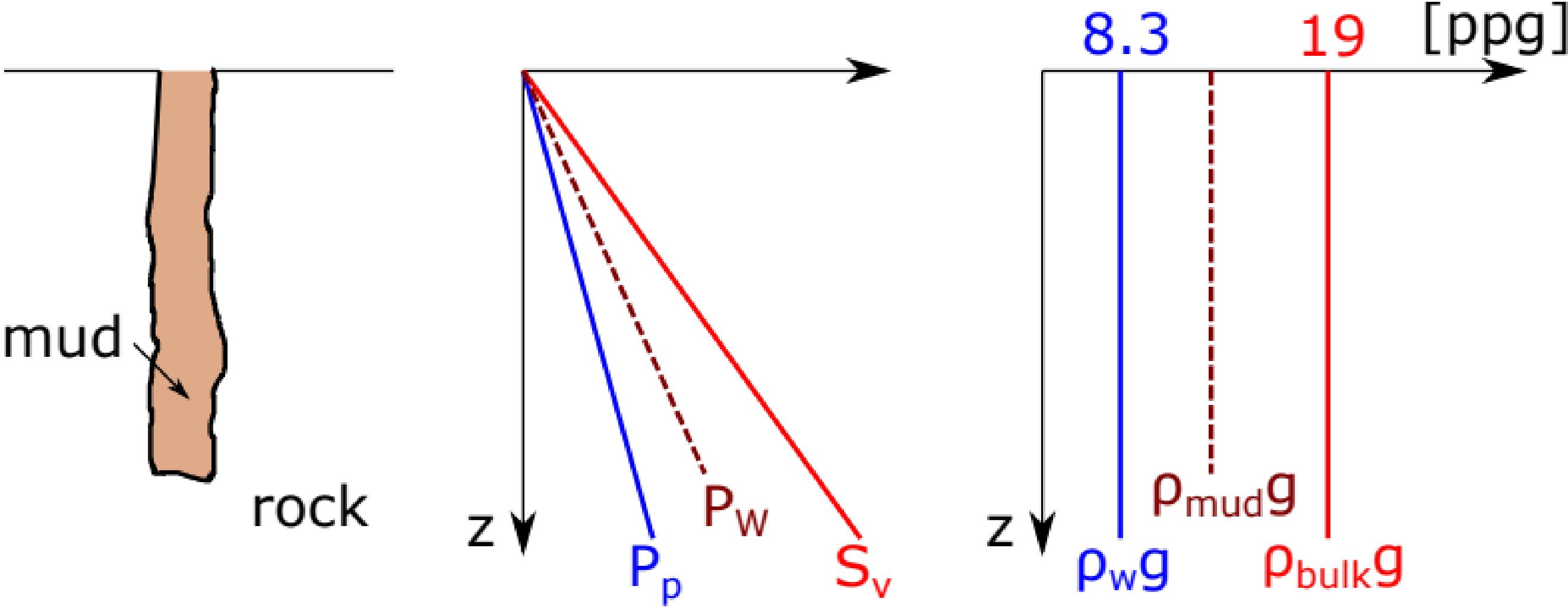
$$\Delta\sigma_a = -\Delta P_p$$

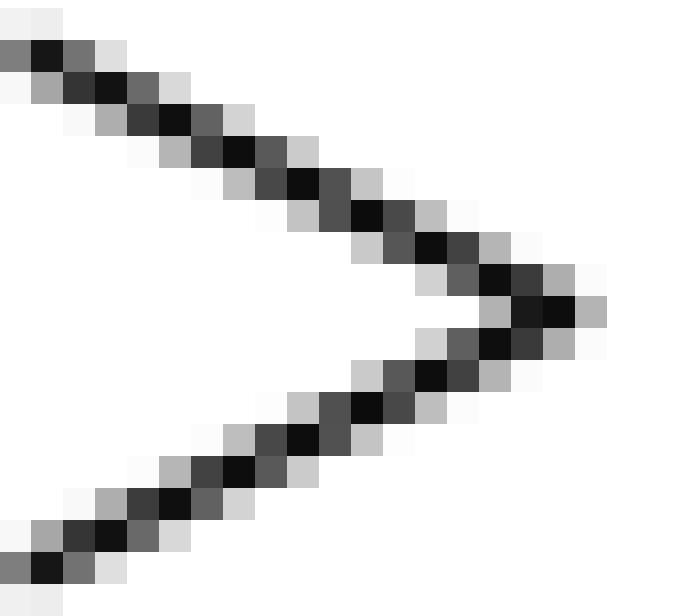
$$\Delta\sigma_{\text{Hermann}} \leq -\Delta P_p$$



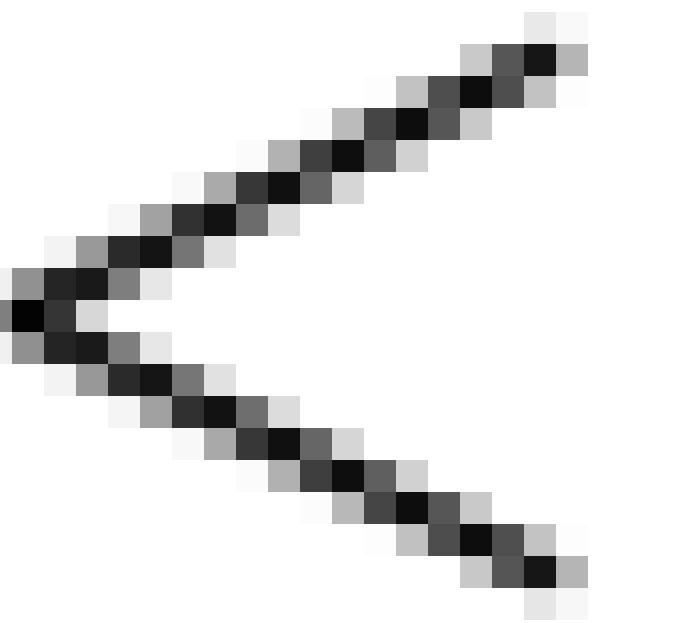


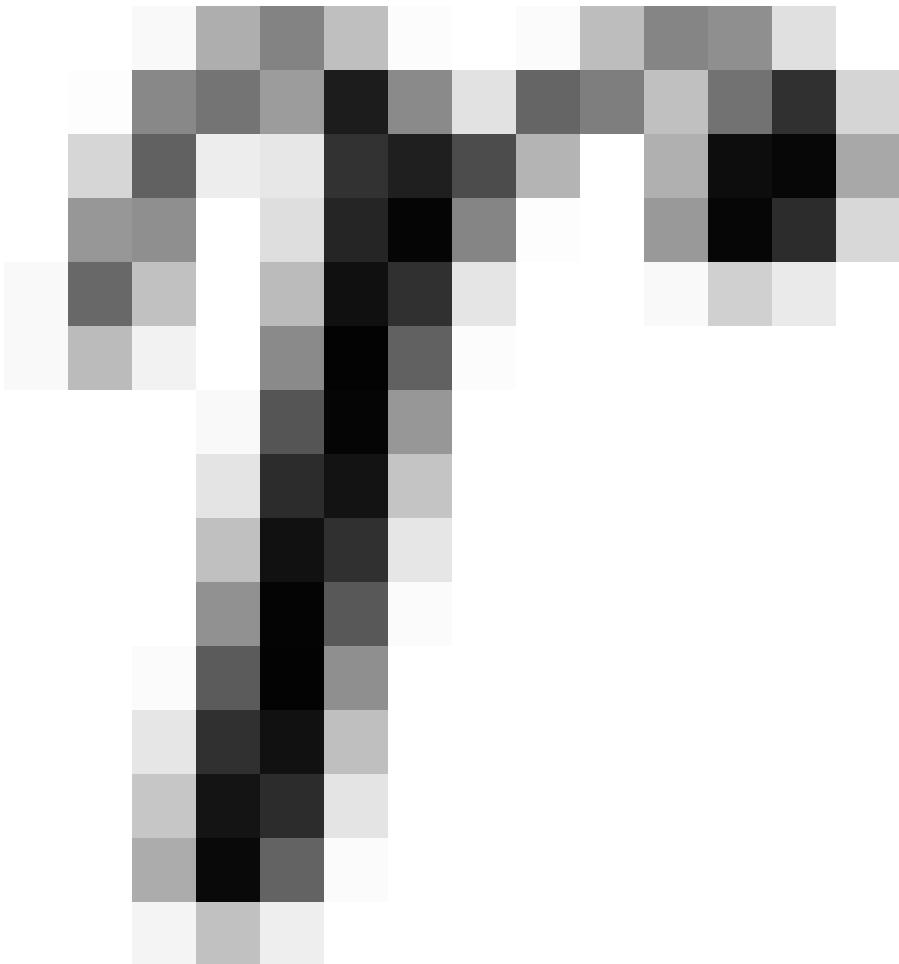


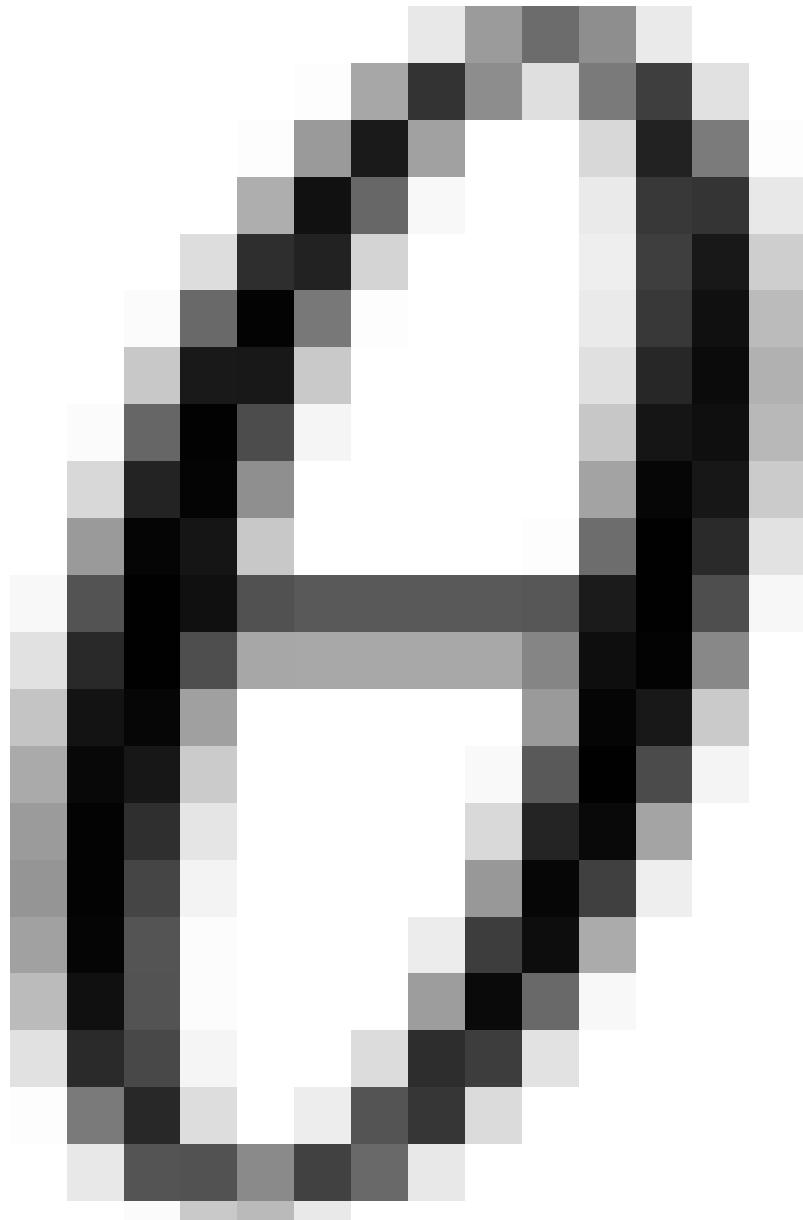


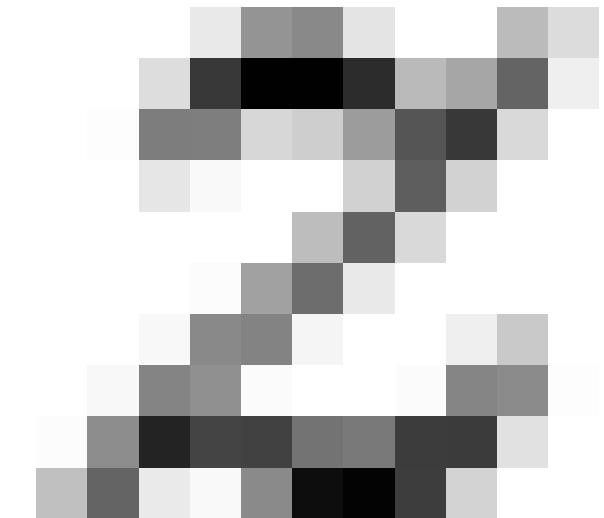
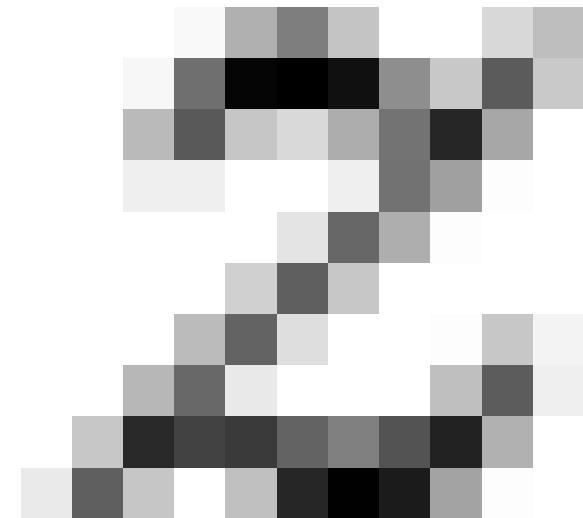
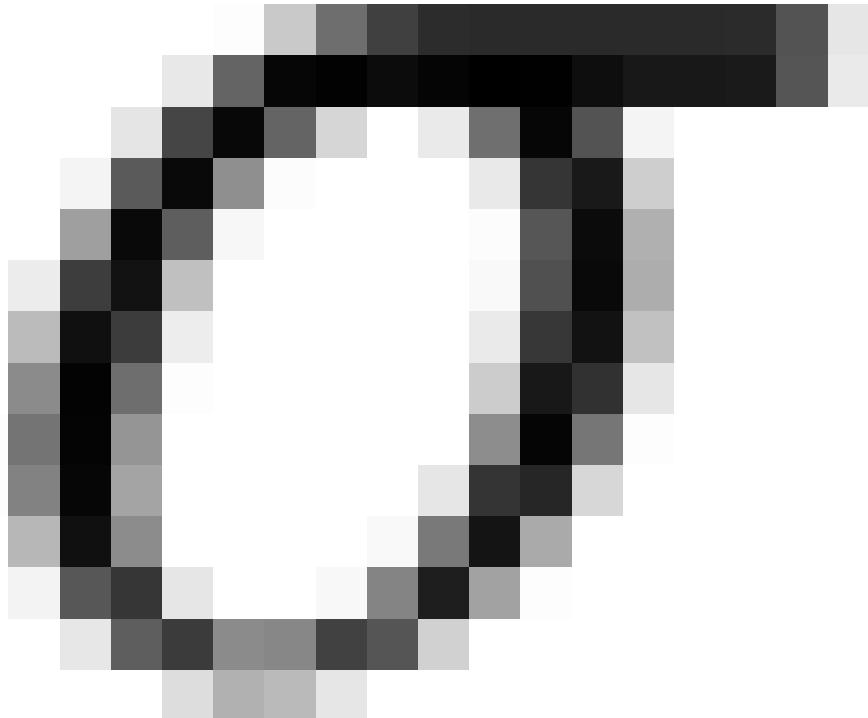


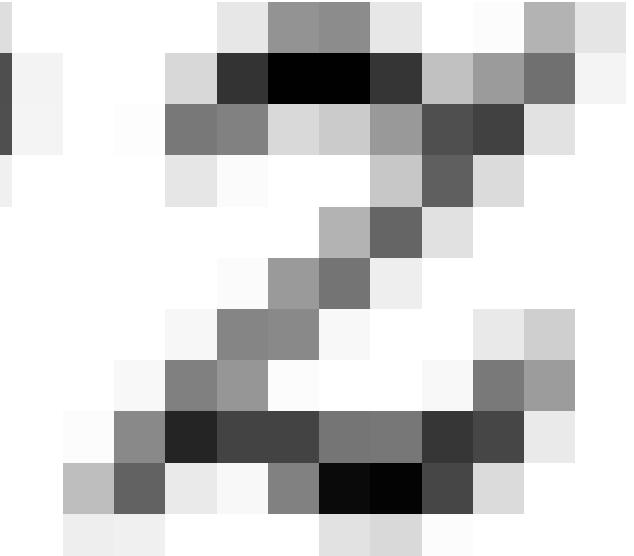
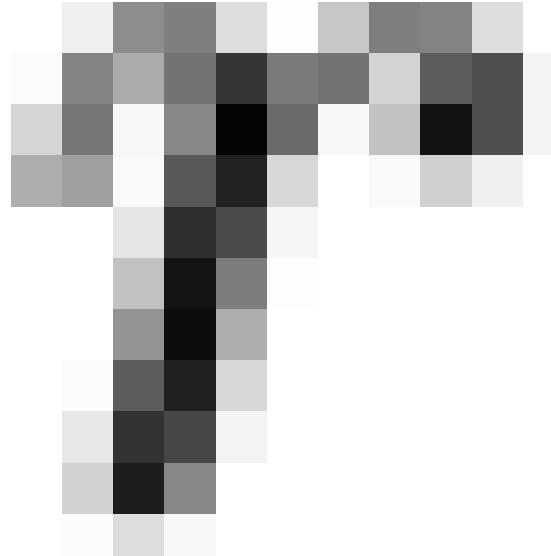
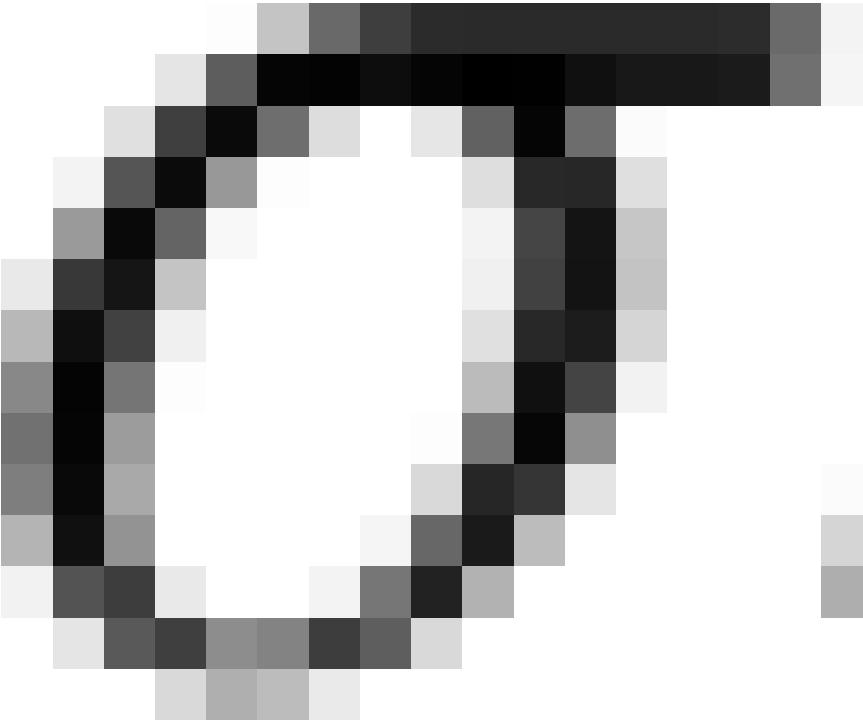


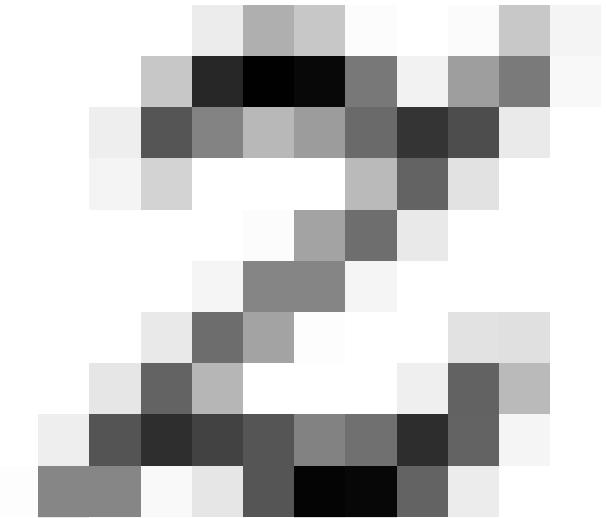
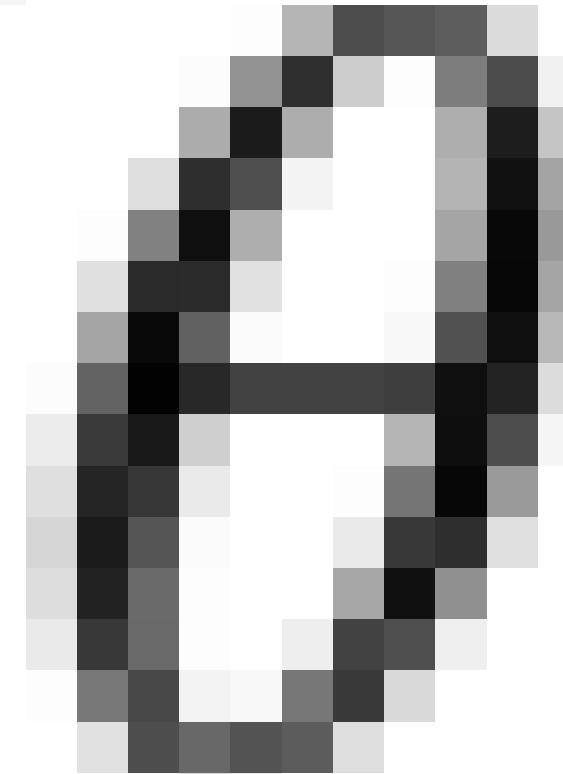
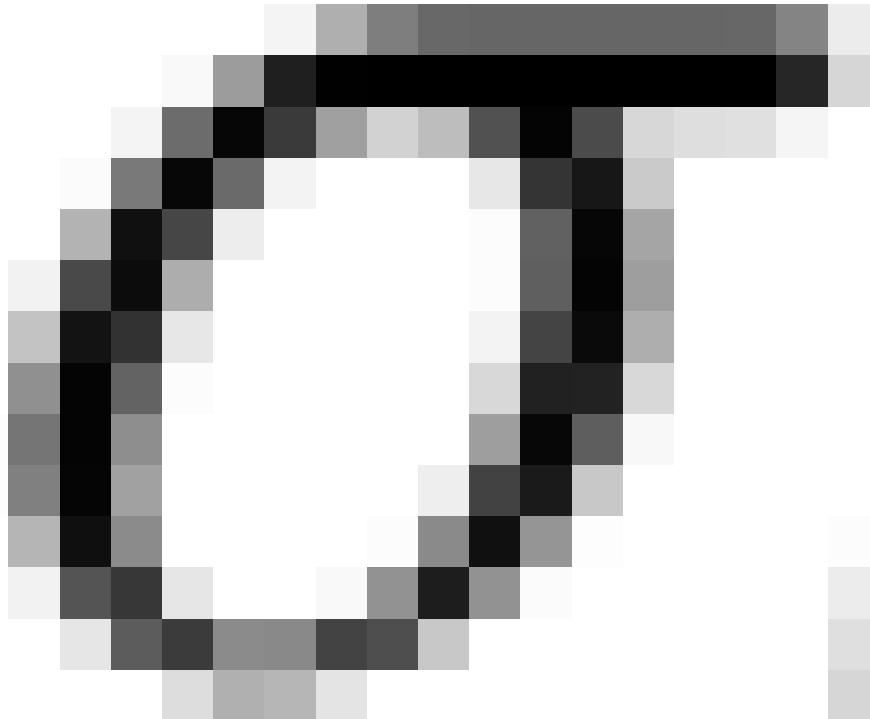




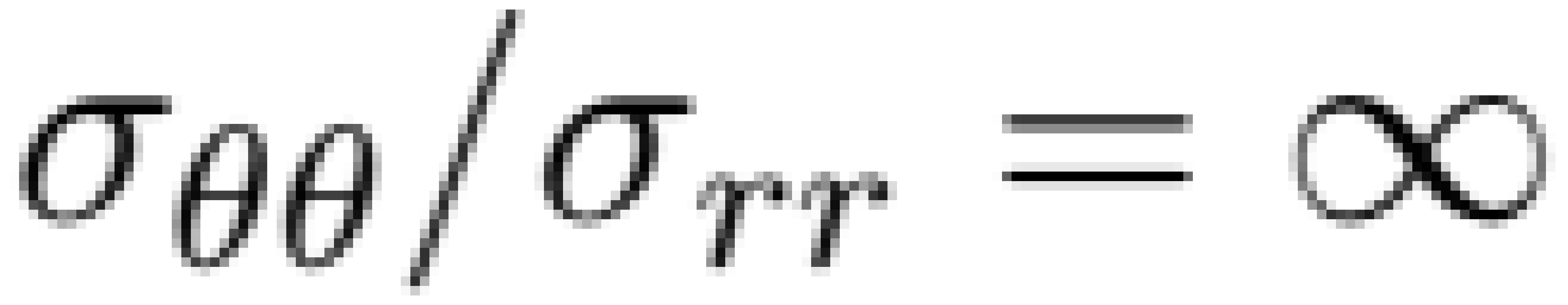




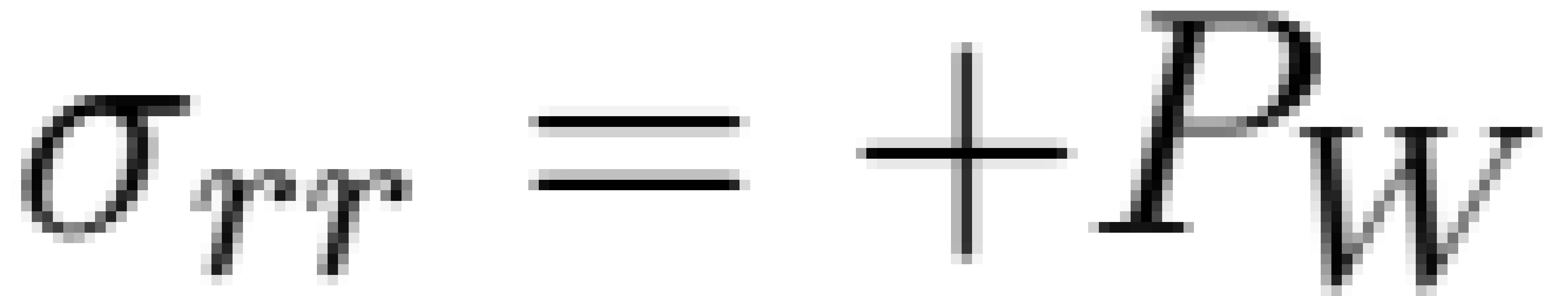


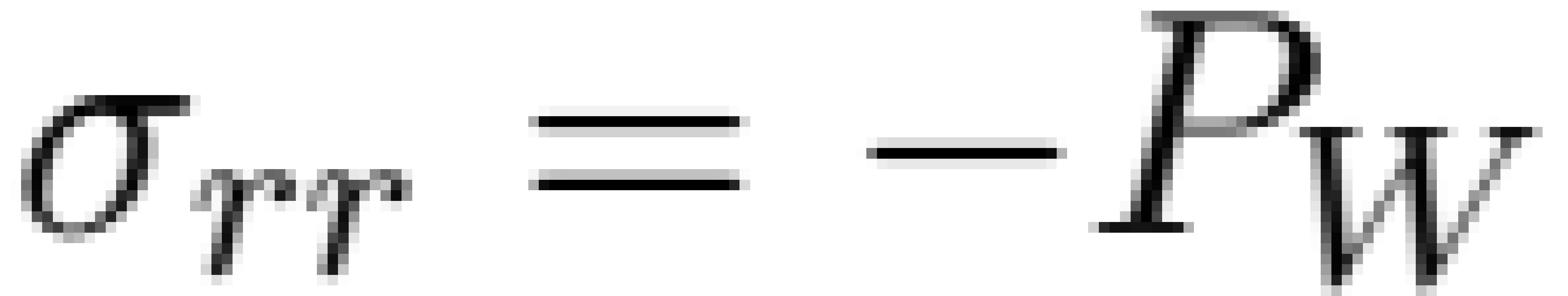


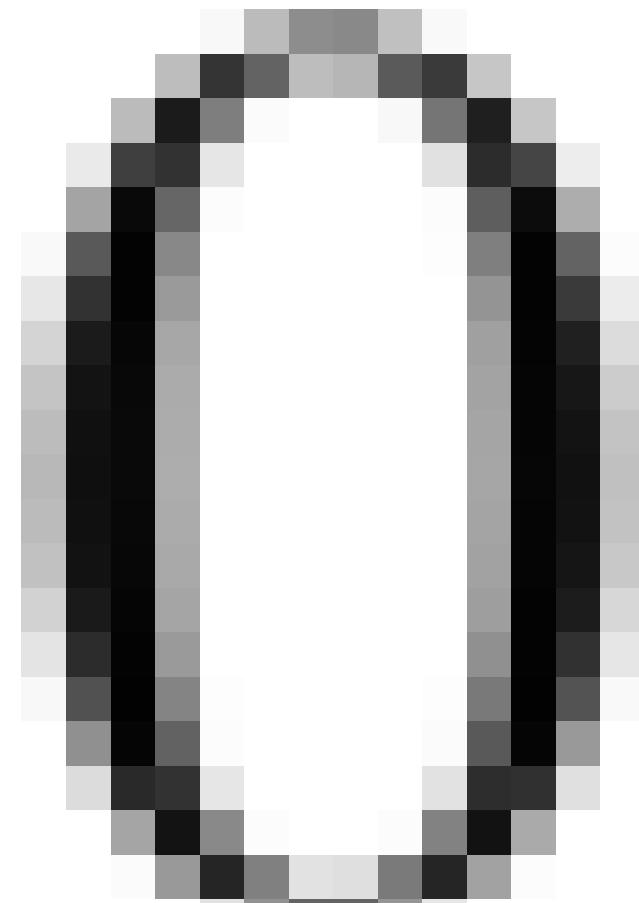
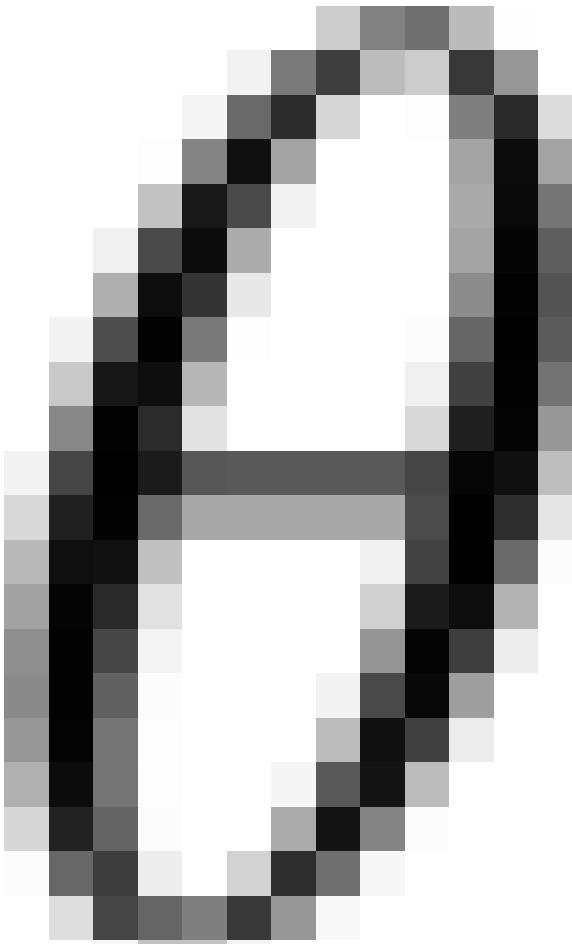


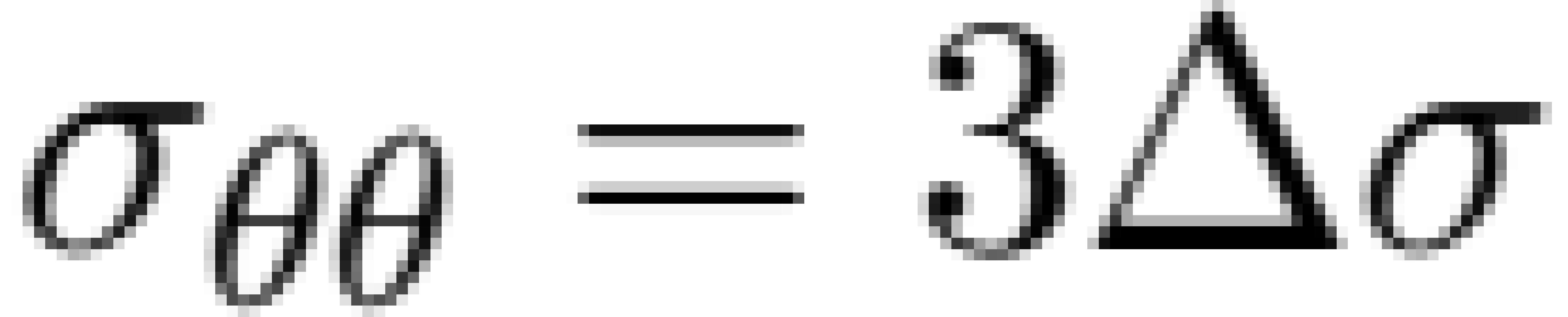


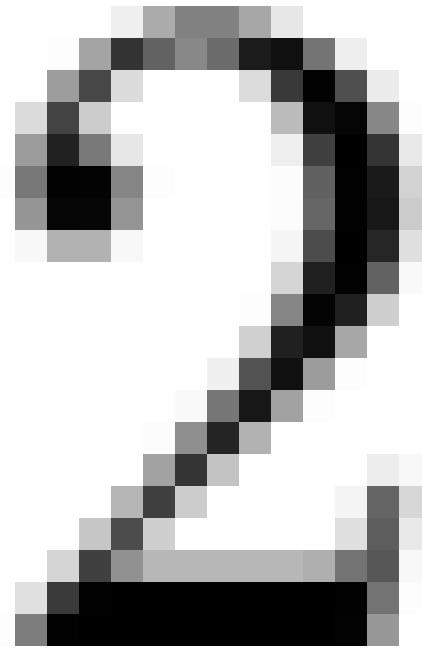
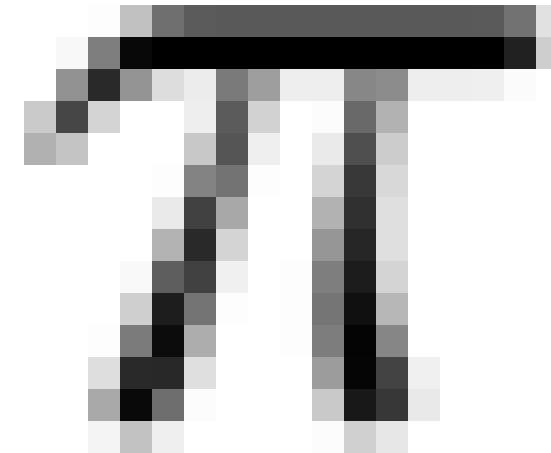
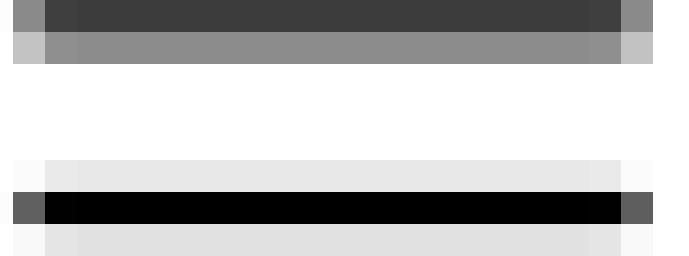
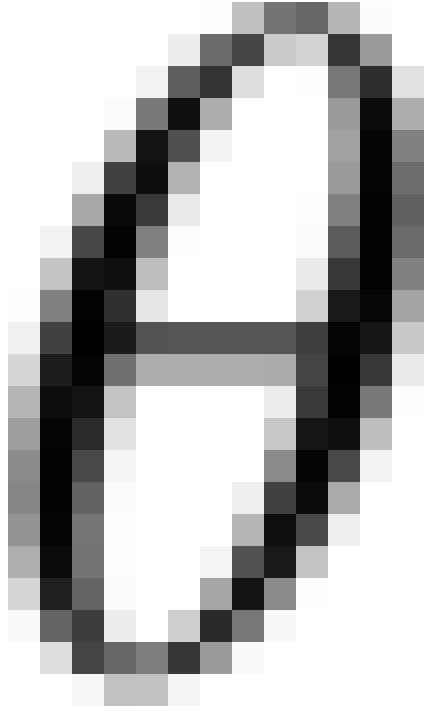


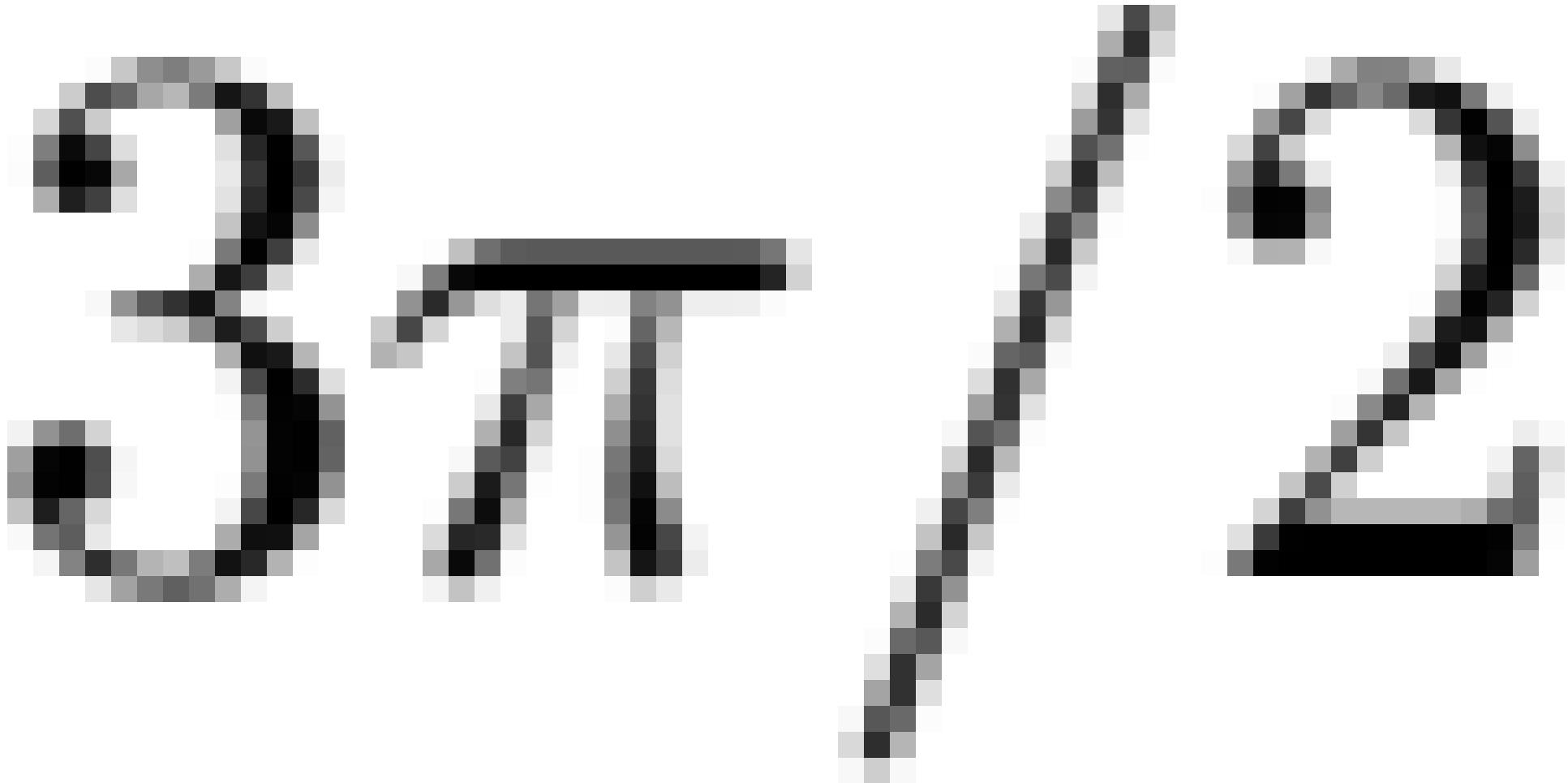


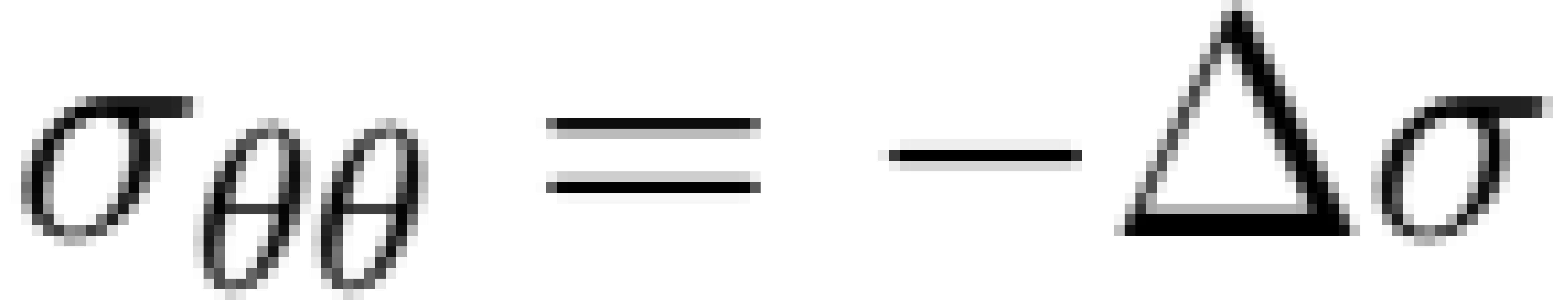


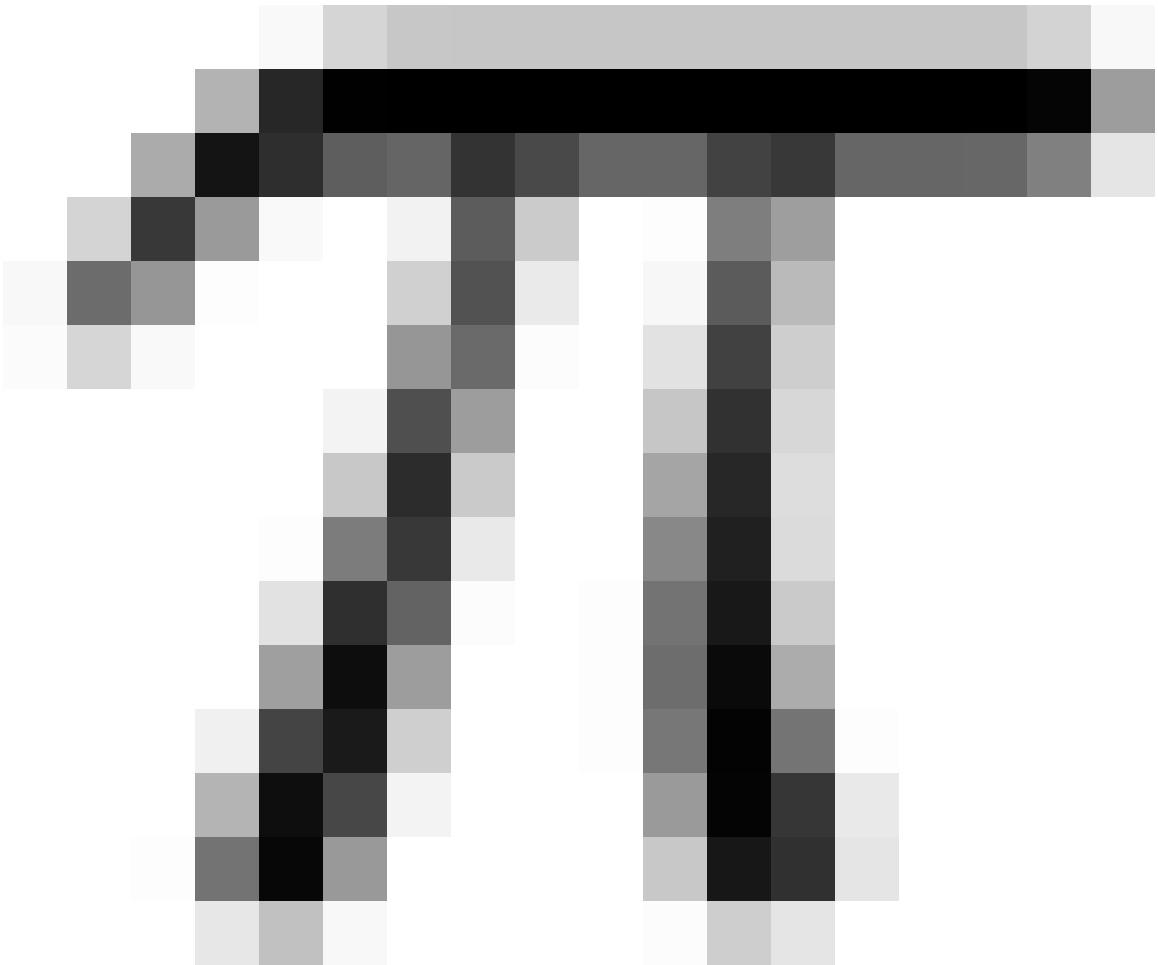




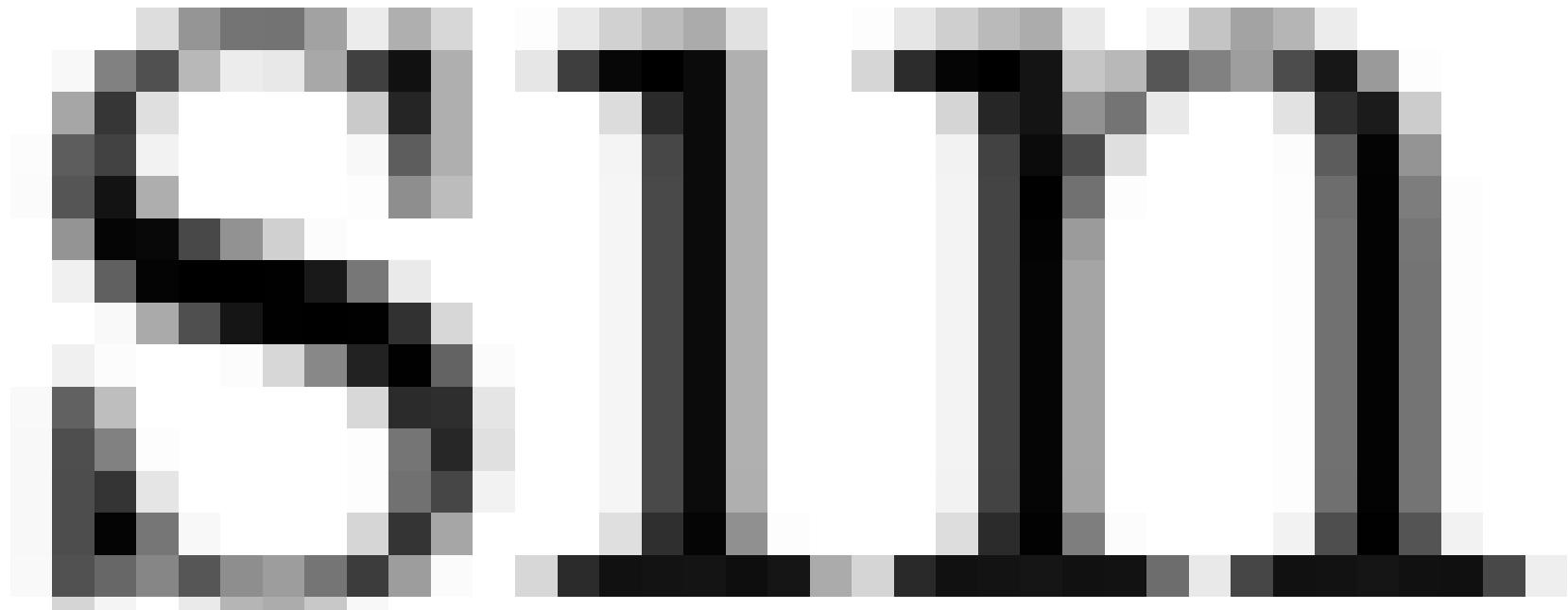
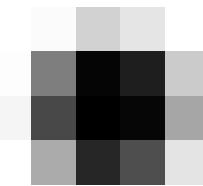


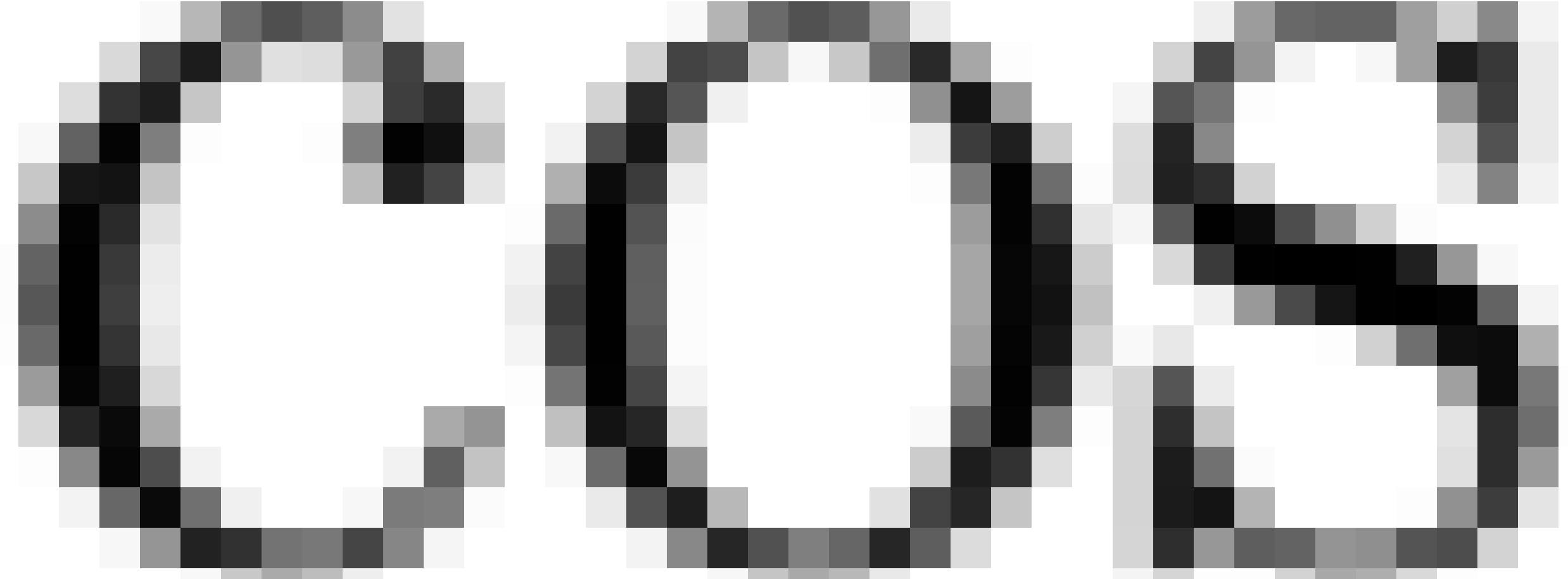


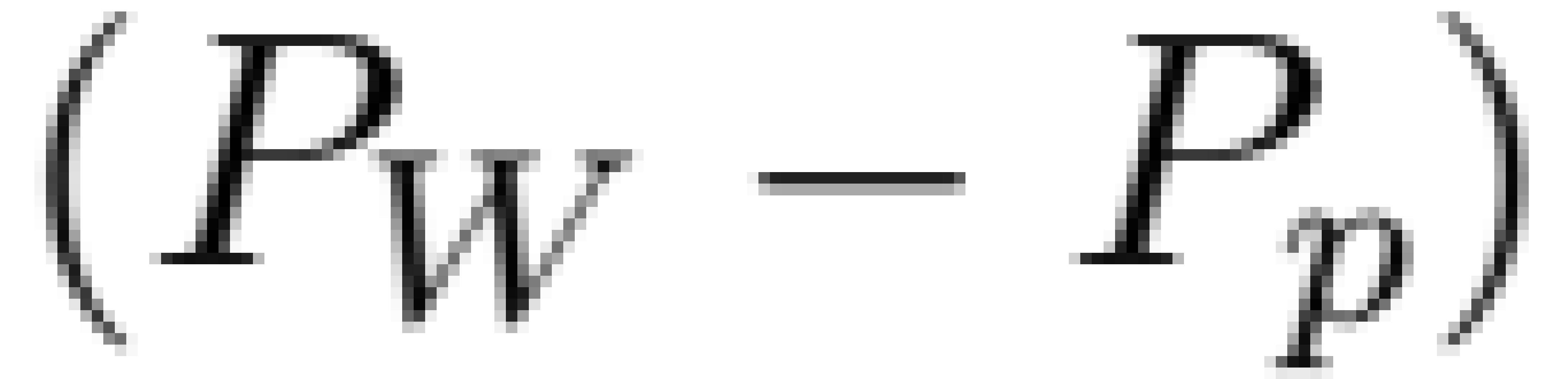




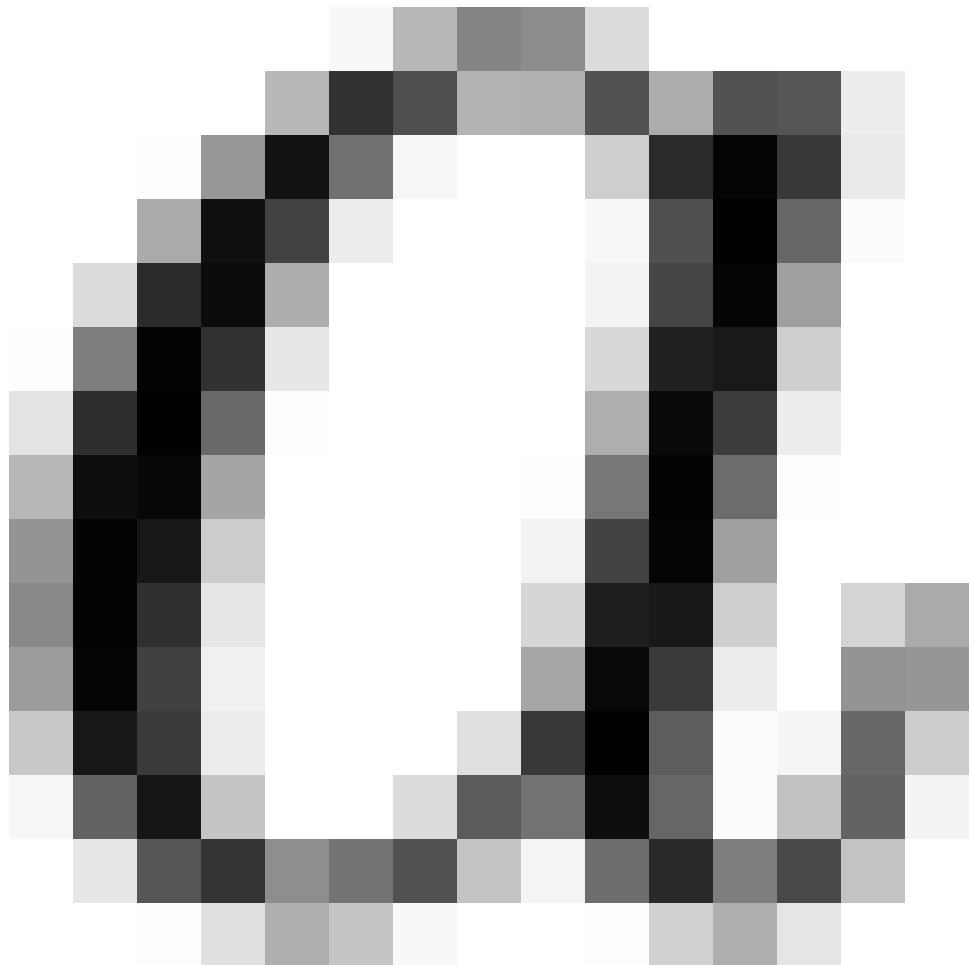


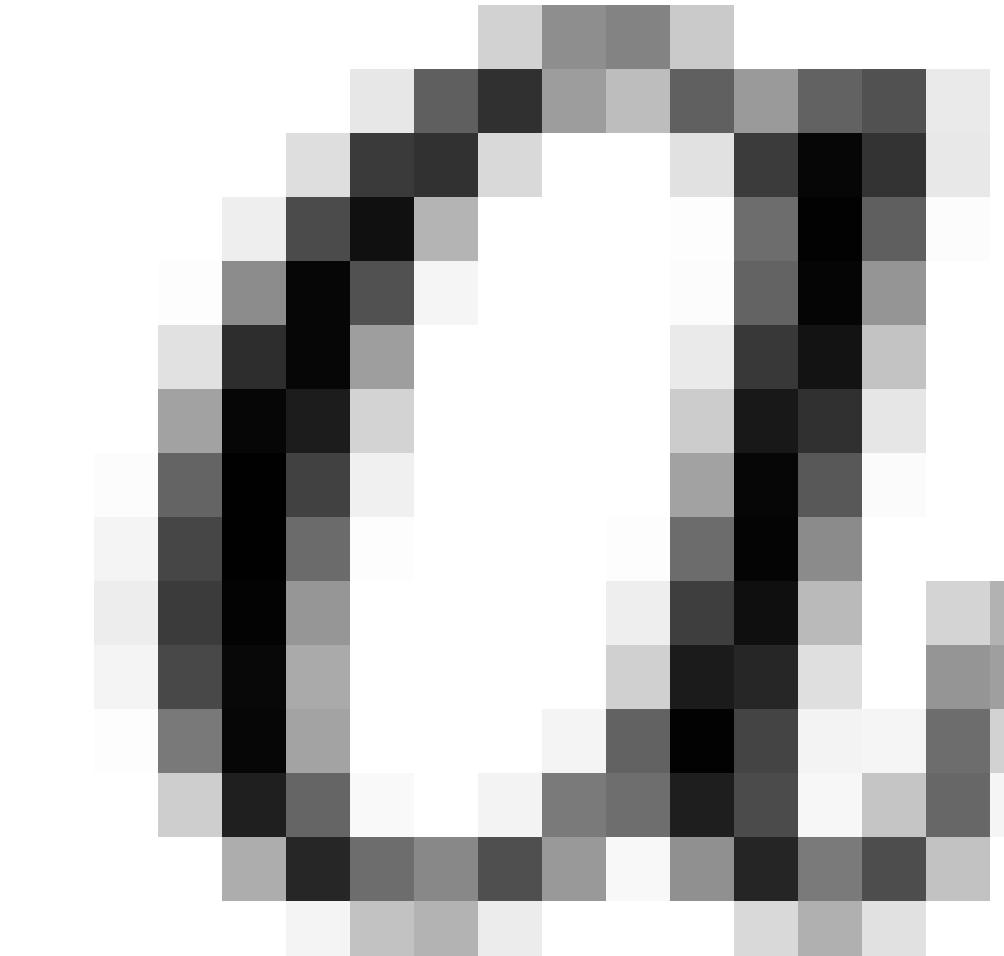
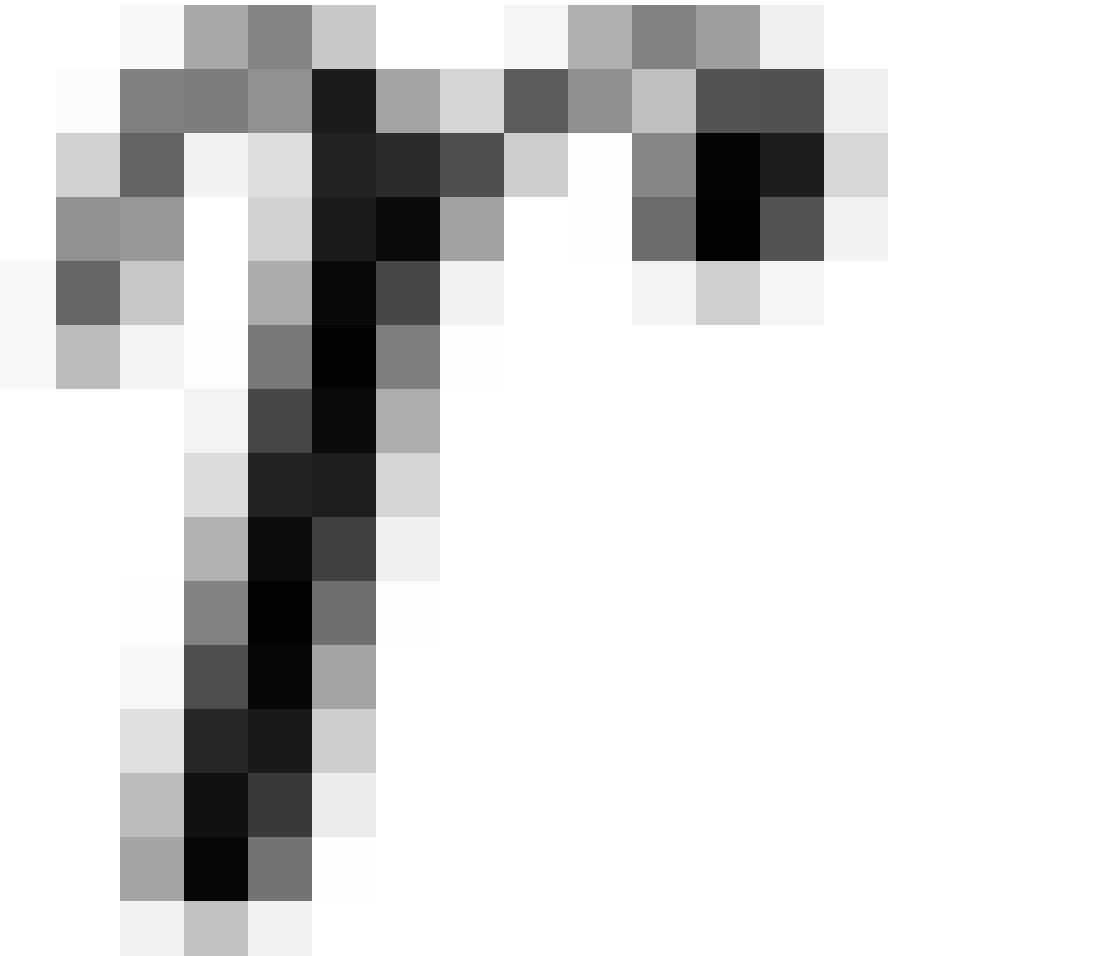






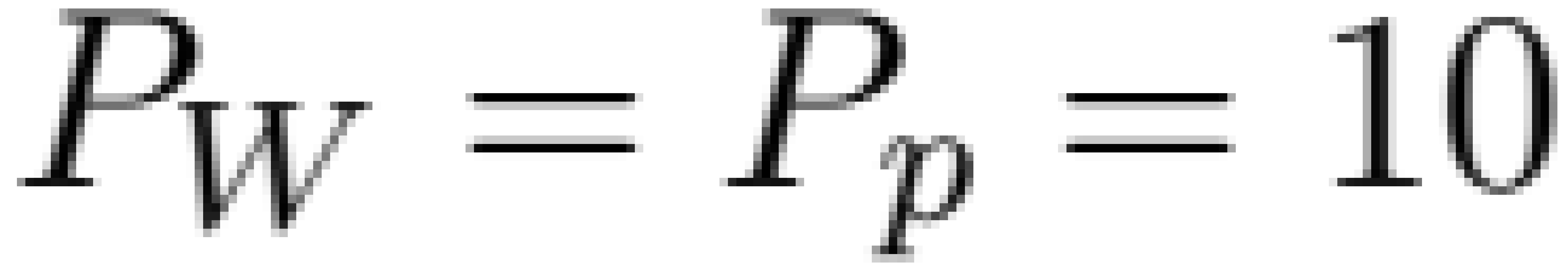
$$\left\{ \begin{array}{lcl} \sigma_{rr} & = & (P_W - P_p) \left(\frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left(1 - \frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{\theta\theta} & = & -(P_W - P_p) \left(\frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left(1 + \frac{a^2}{r^2} \right) - \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{r\theta} & = & \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 + 2 \frac{a^2}{r^2} - 3 \frac{a^4}{r^4} \right) \sin(2\theta) \\ \\ \sigma_{zz} & = & \sigma_v - 2\nu (\sigma_{Hmax} - \sigma_{hmin}) \left(\frac{a^2}{r^2} \right) \cos(2\theta) \end{array} \right.$$





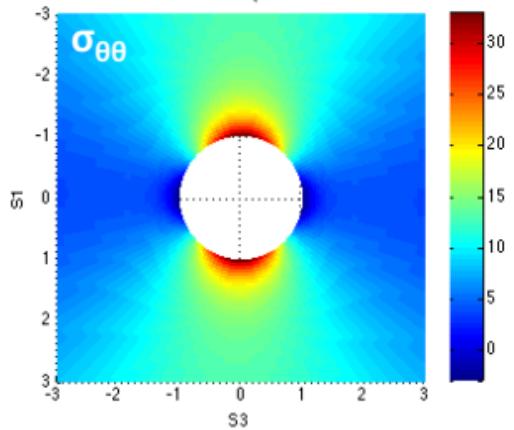
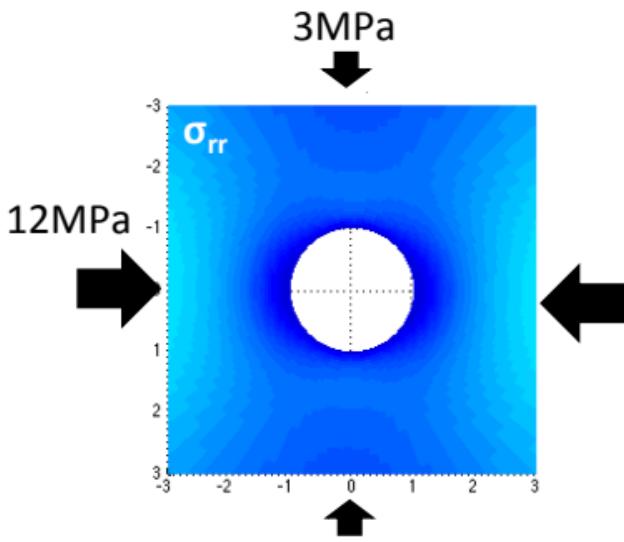




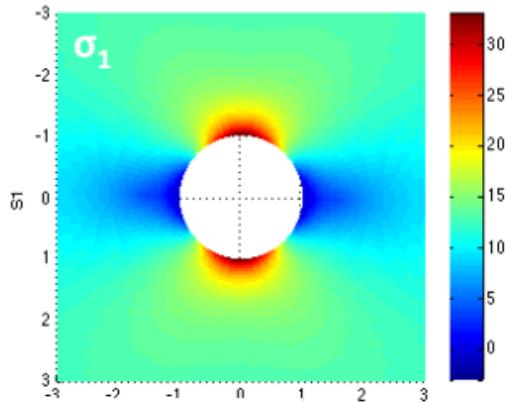
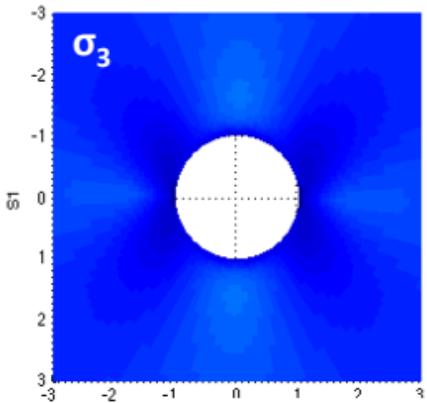


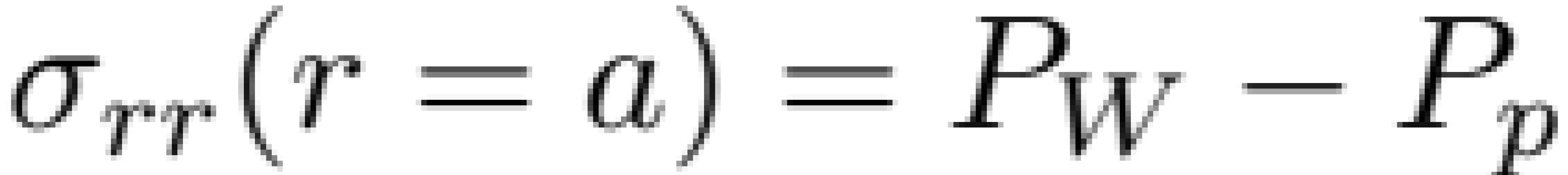


Stresses in cylindrical coordinates



Principal Stresses





$$\sigma_{\text{H}\alpha\alpha} = \sigma_{\text{H}\alpha\alpha}^{\text{max}} + \sigma_{\text{H}\alpha\alpha}^{\text{min}} \left(\frac{P_{\text{H}\alpha\alpha}}{P_{\text{H}\alpha\alpha}^{\text{max}}} - 1 \right)^2$$

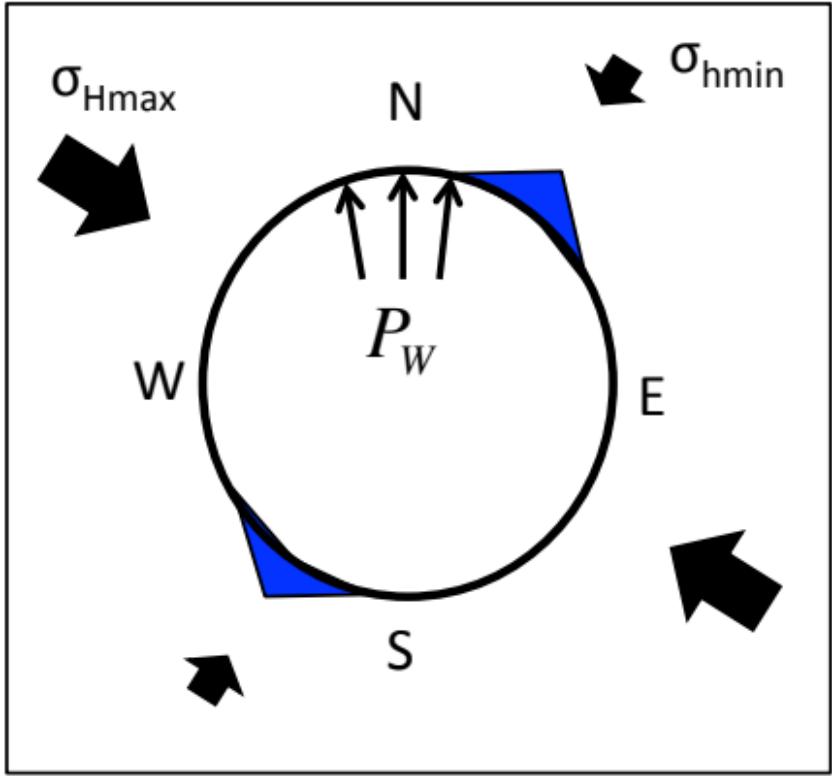
$$\begin{aligned} & \left(\sigma_{\theta\theta}(r=a, \theta=0) - \sigma_{H\max} + 3\sigma_{h\min} \right) = (P_W - P_p) \\ & \left(\sigma_{\theta\theta}(r=a, \theta=\pi/2) + \sigma_{H\max} - 3\sigma_{h\min} \right) = (P_W - P_p) \end{aligned}$$







$$\left\{ \begin{array}{l} \sigma_1 = \sigma_{\theta\theta} = -(P_W - P_p) + 3\sigma_{H\max} - \sigma_{\text{mean}} \\ \sigma_3 = \sigma_{rr} = (P_W - P_p) \end{array} \right.$$



$$P_{W\text{shear}} = P_p + \frac{3\sigma_{H\max} - \sigma_{h\min} - UCS}{1+q}$$

Stress anisotropy
Pore pressure
in the formation
Shear
strength

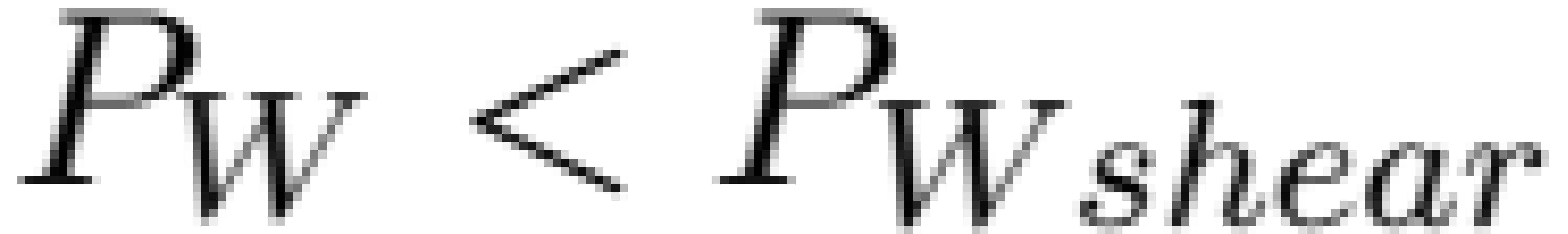
$$P_{W\text{shear}} = \frac{3S_{H\max} - S_{h\min} - UCS + (q-1)P_p}{1+q}$$

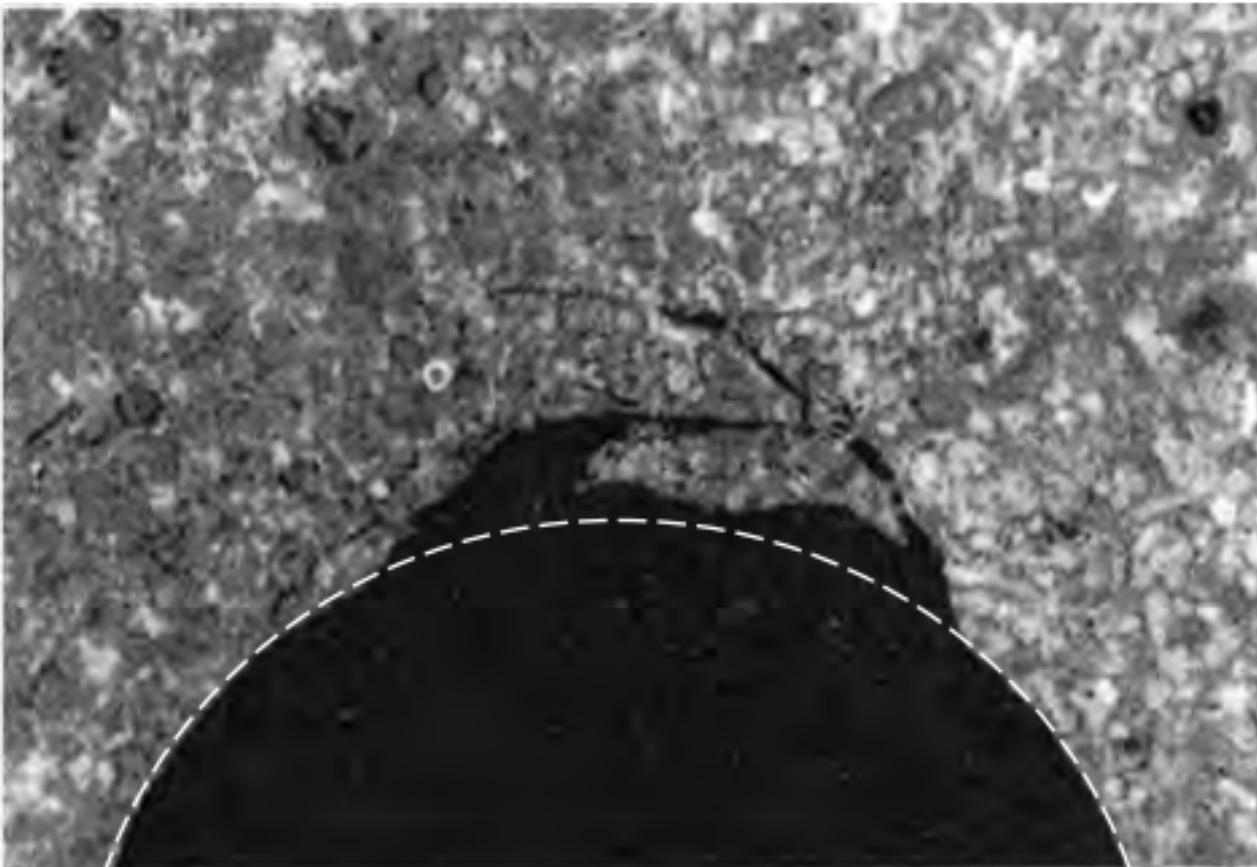
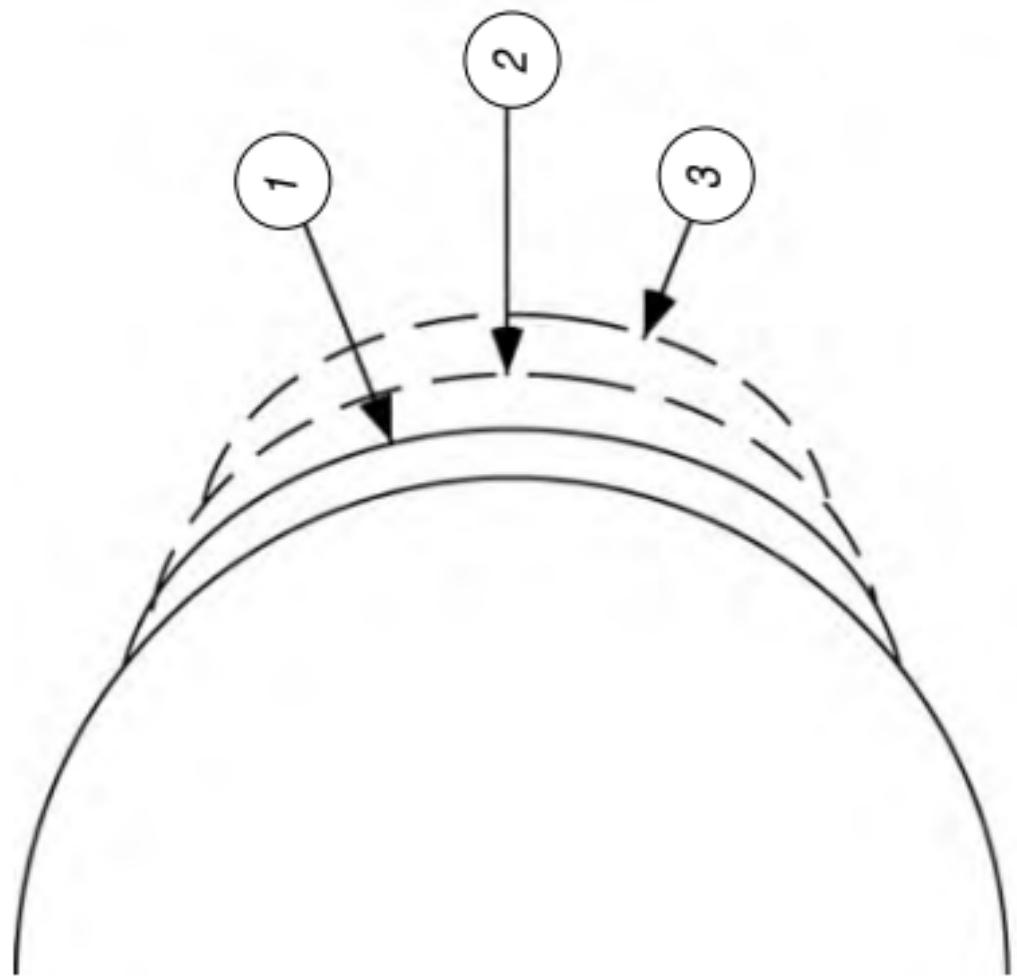
$P_W \leq P_{W\text{shear}}$ leads to breakouts and potential wellbore collapse

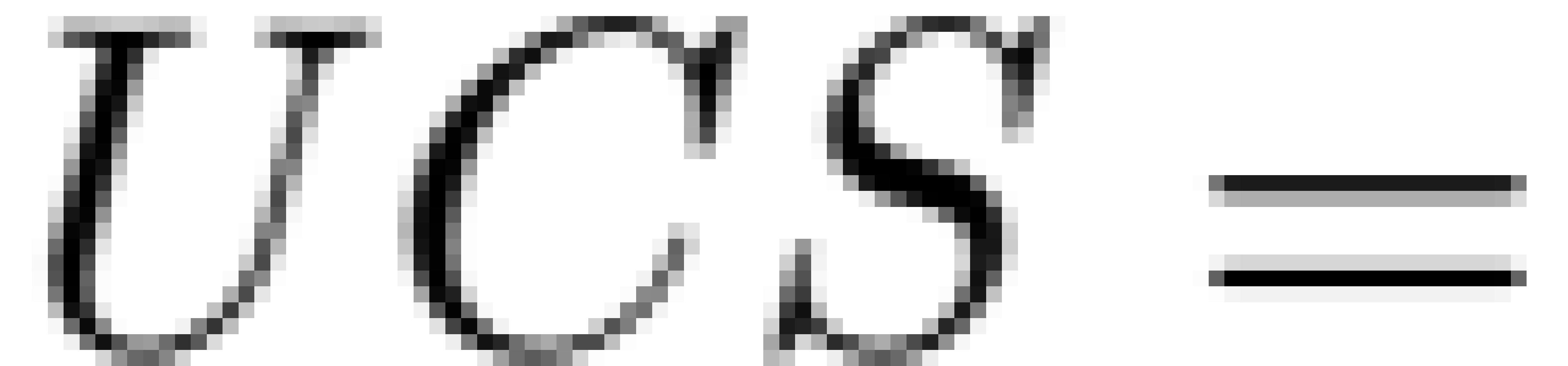


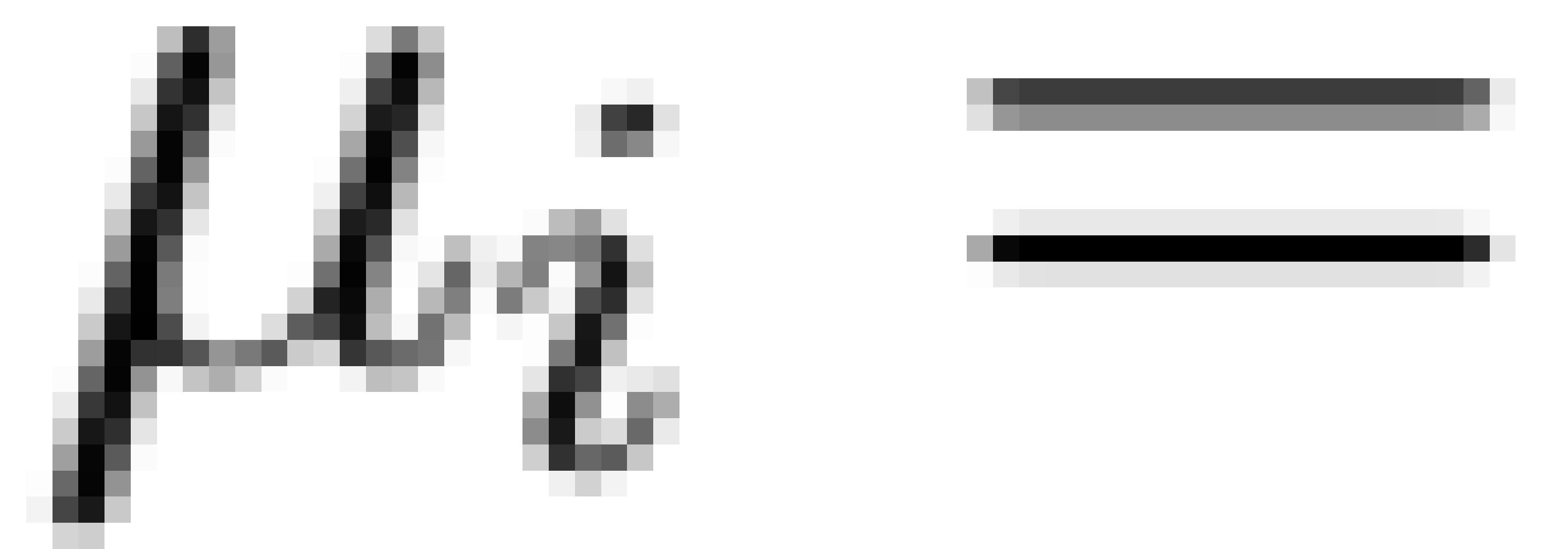
[P]M₅P + 3[O]H₂O → [P]M₃O₂H₂O + 9[H]₂O

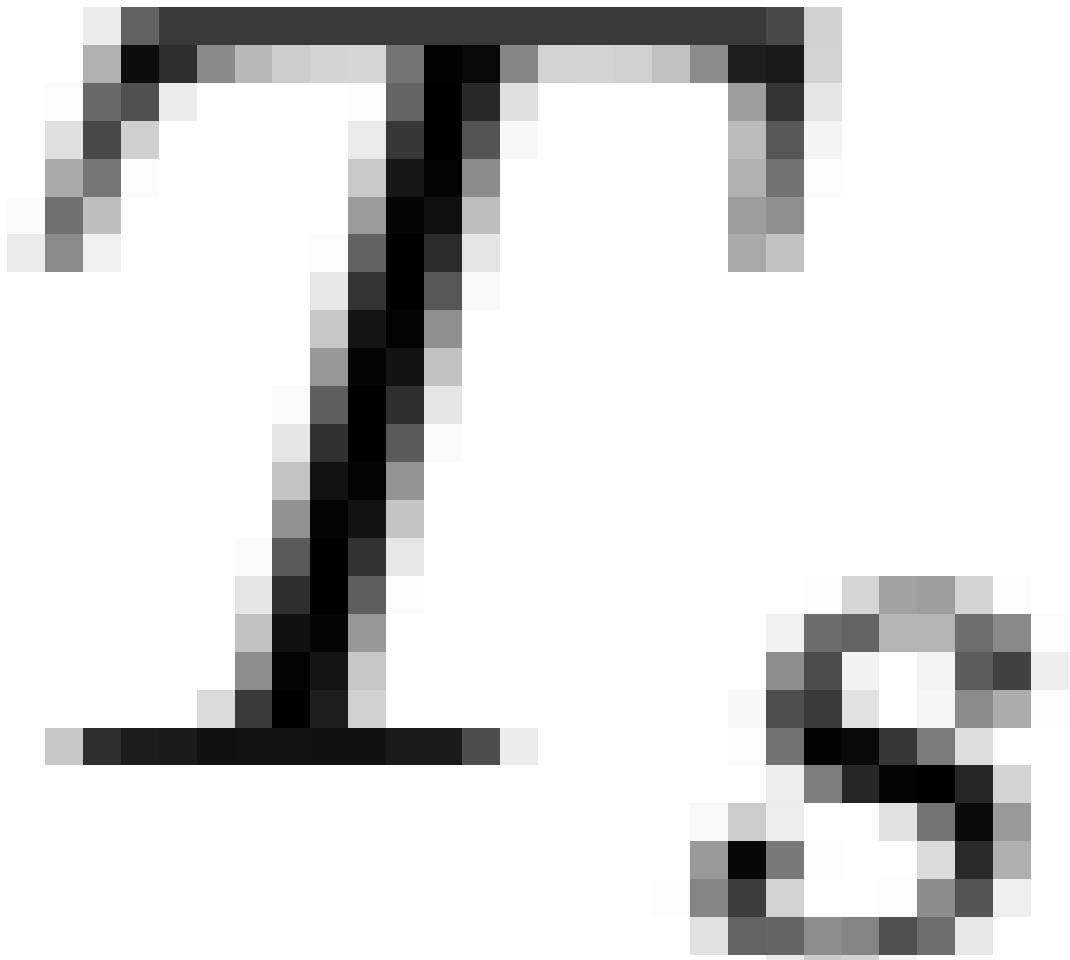
$$P_{W\text{ shear}} = \frac{3\sigma_H \max - \sigma_{hmin} - U_{CS}}{1 + q}$$

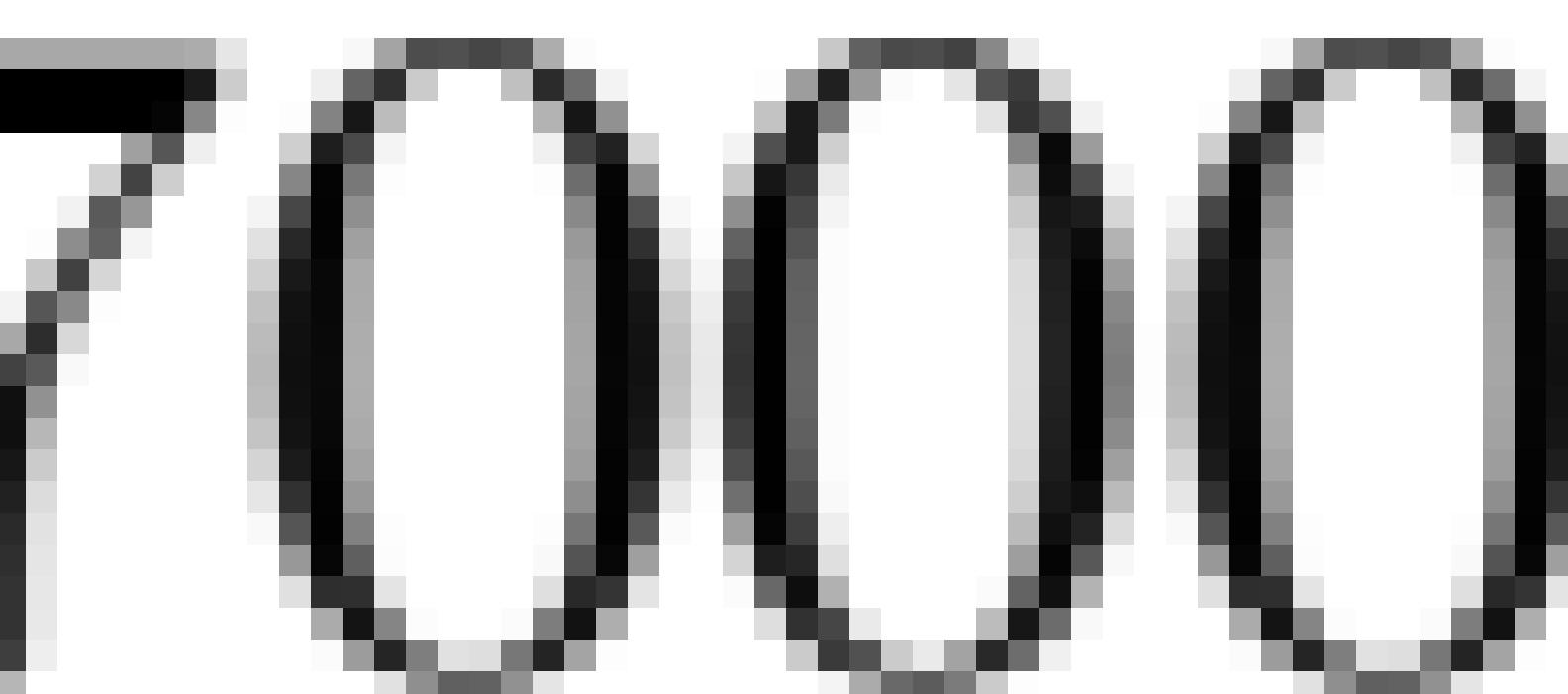


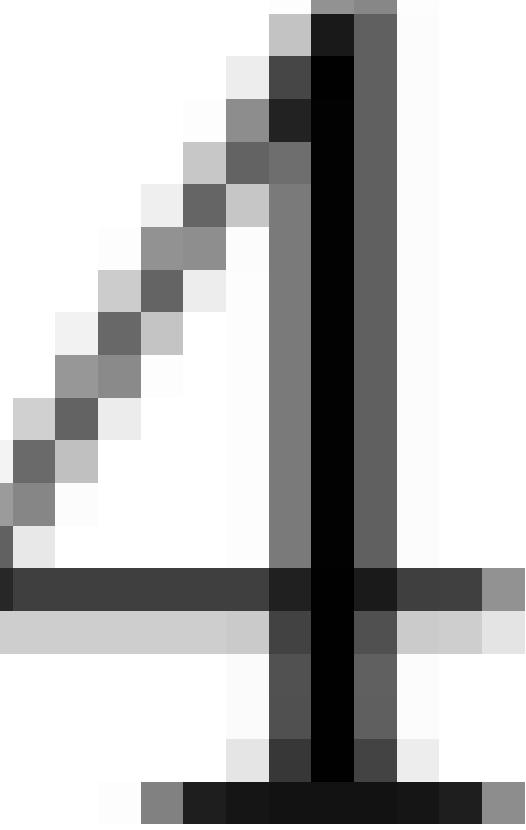
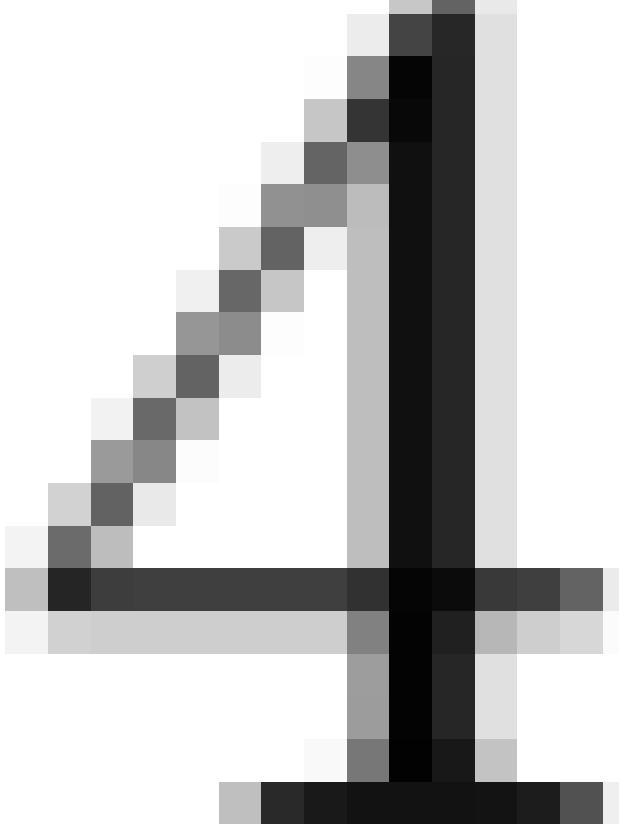
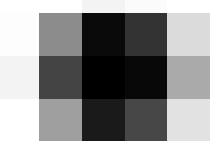
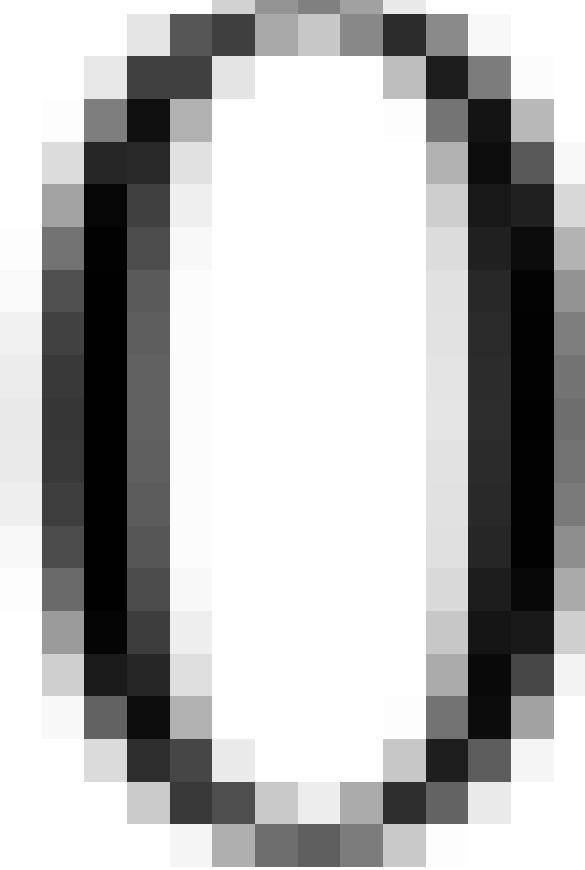
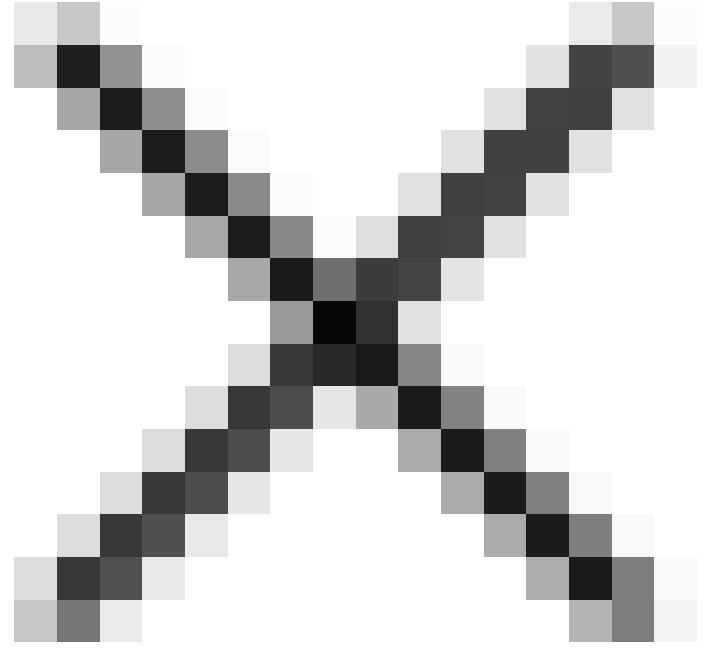


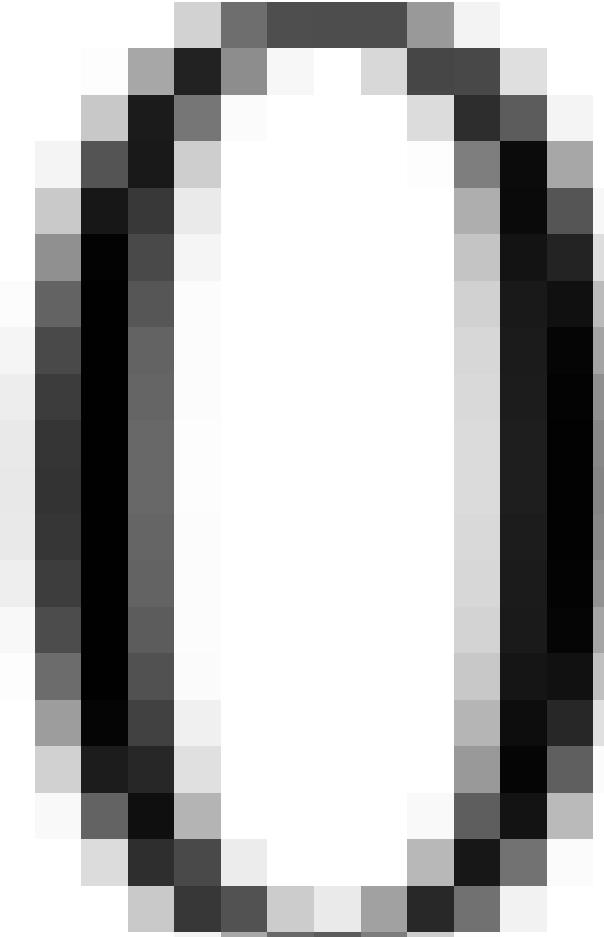
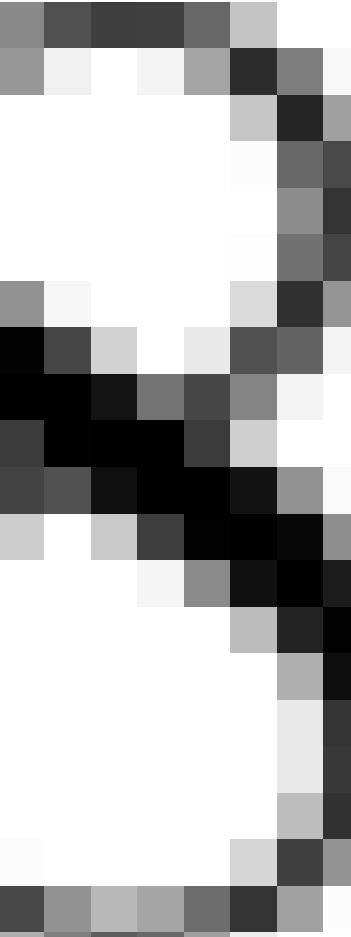
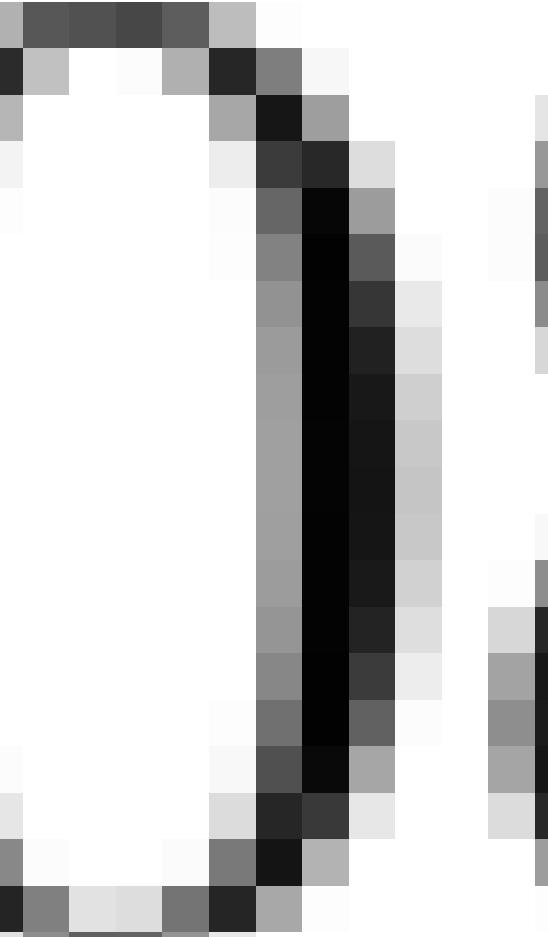
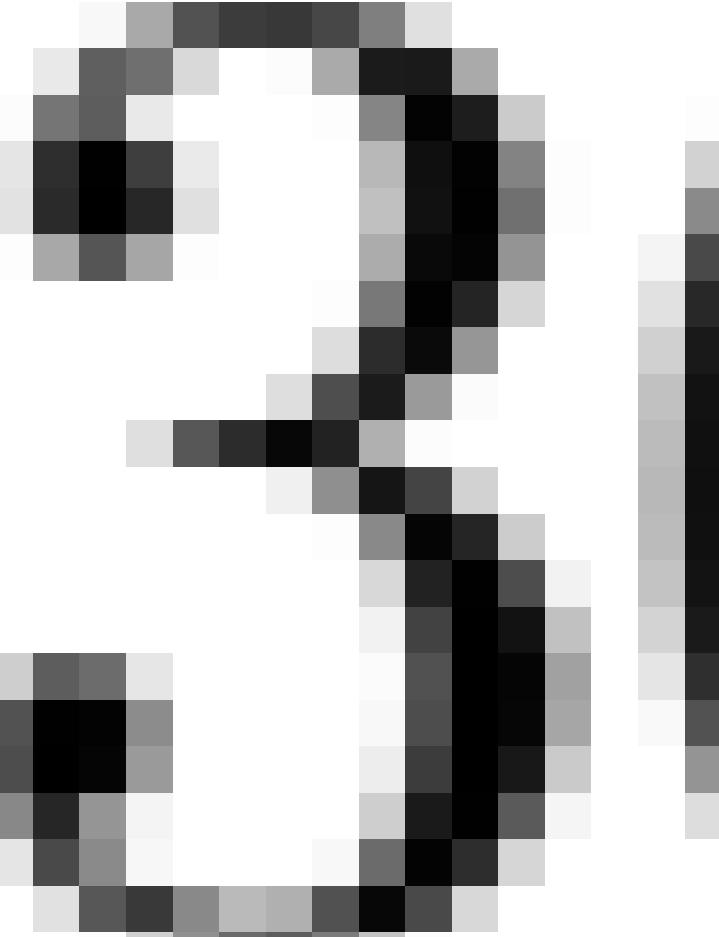




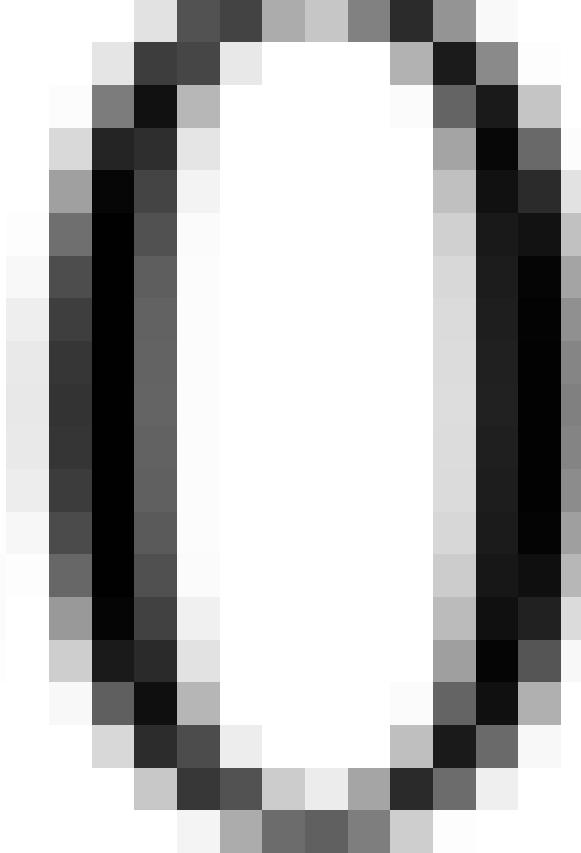
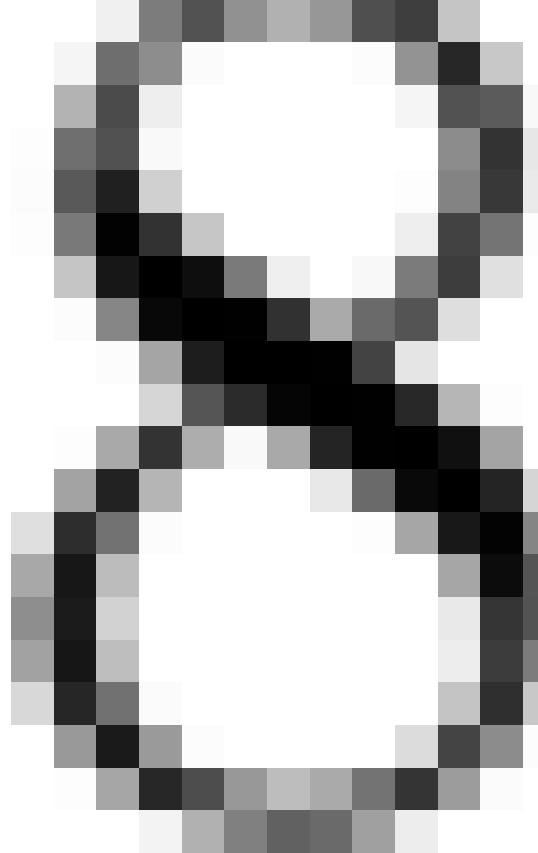
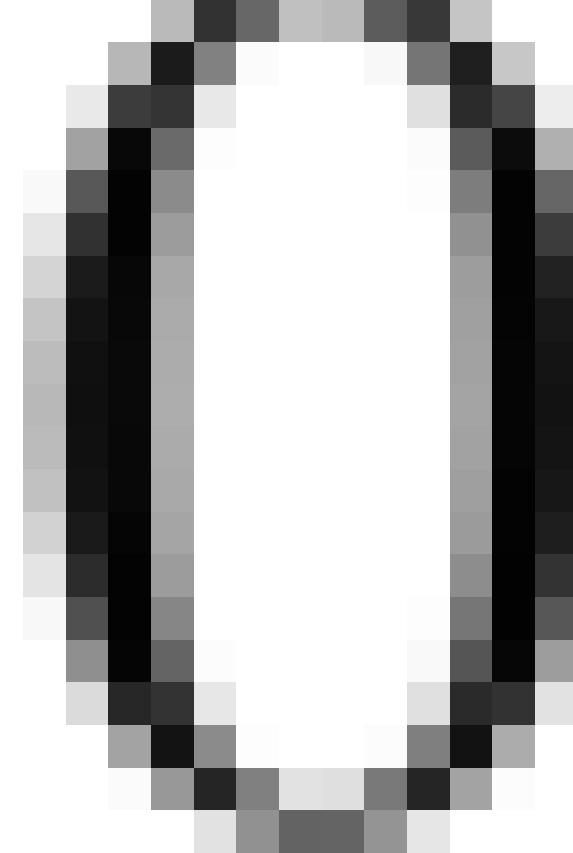
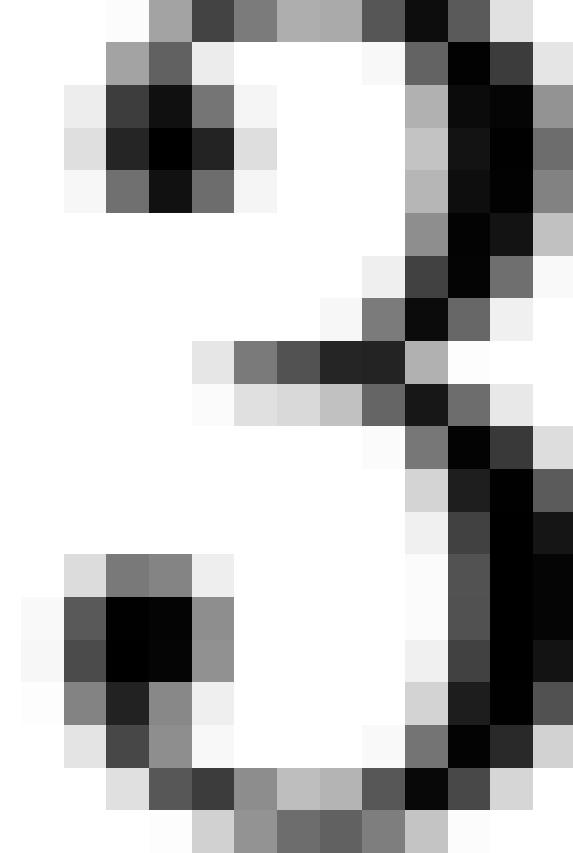


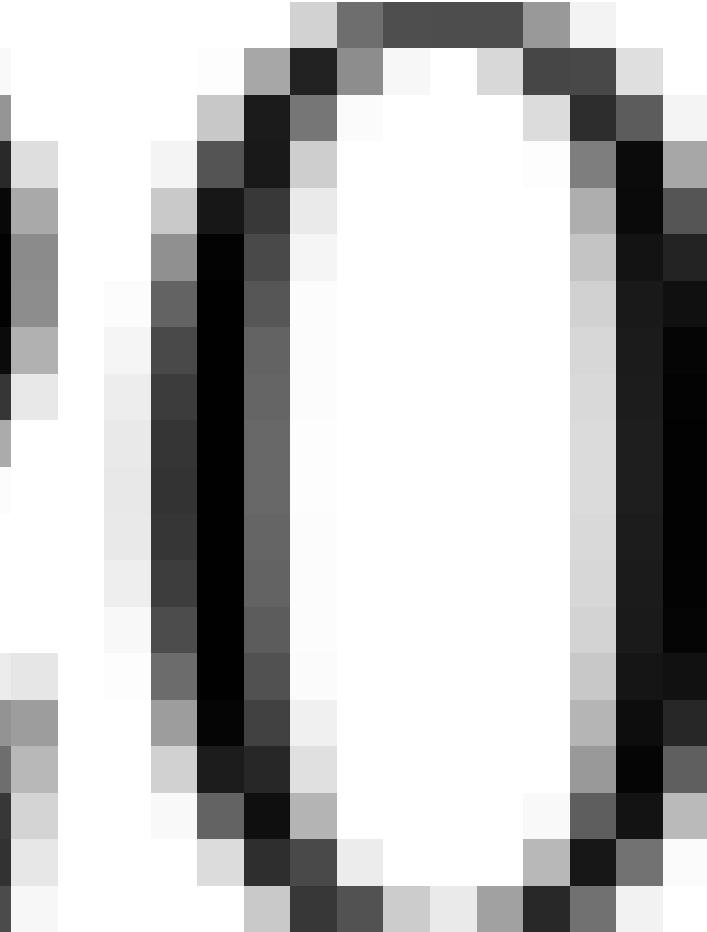
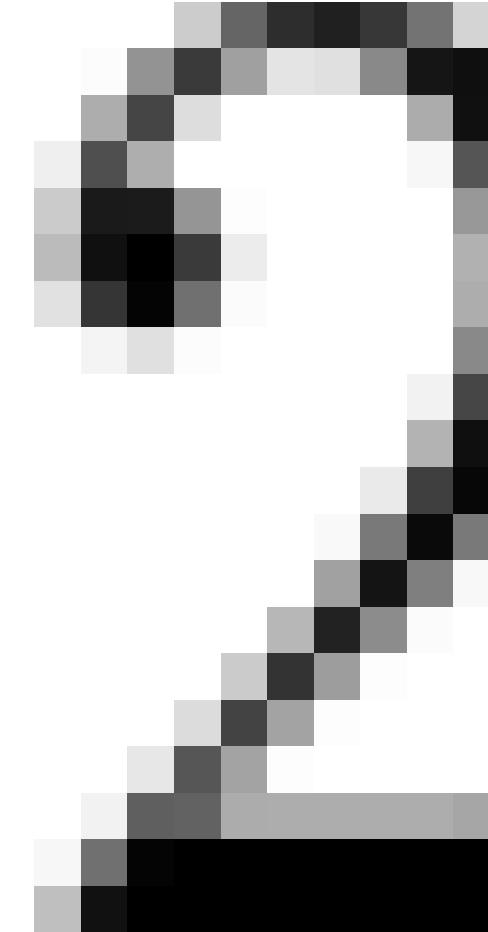
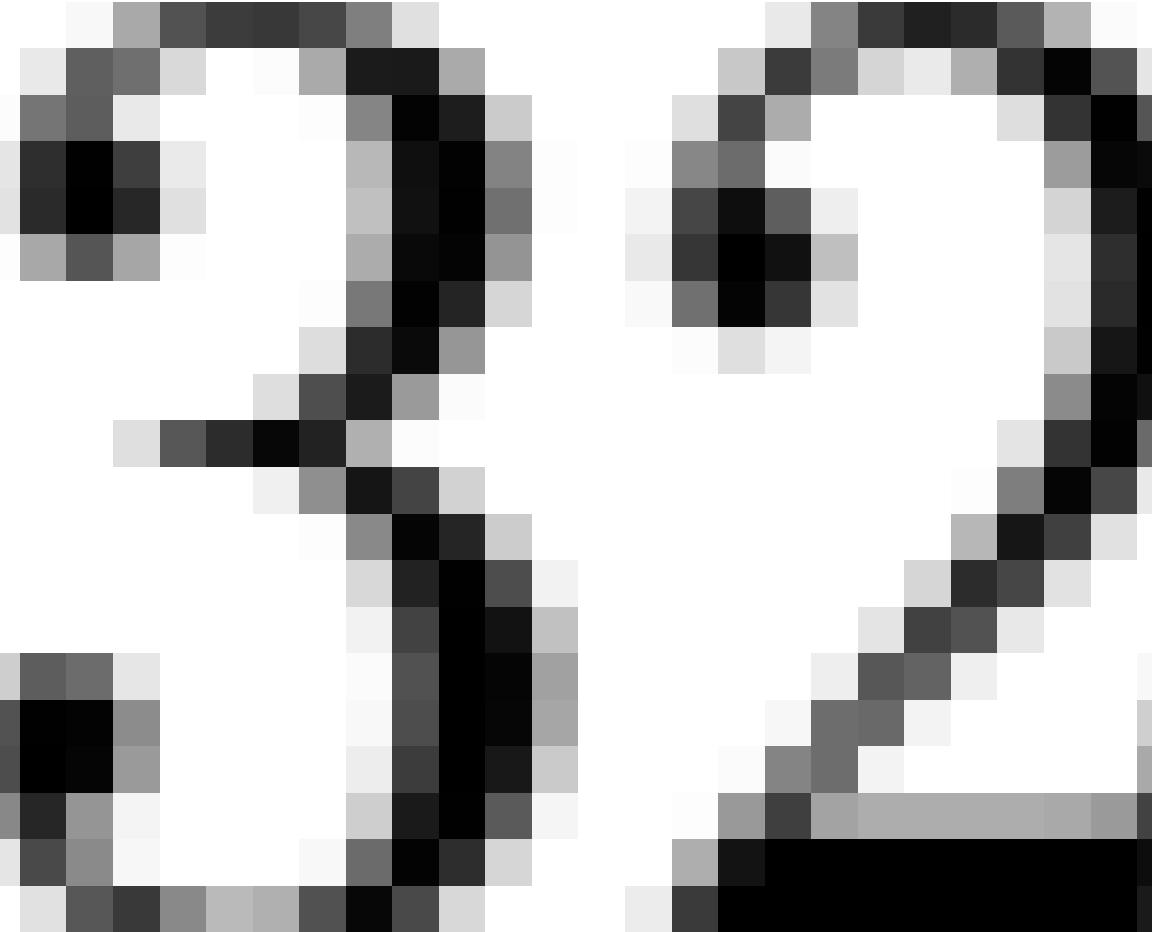


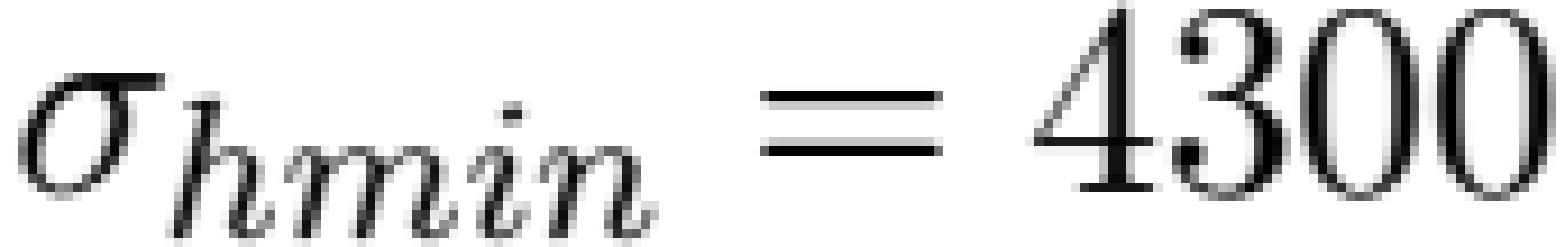


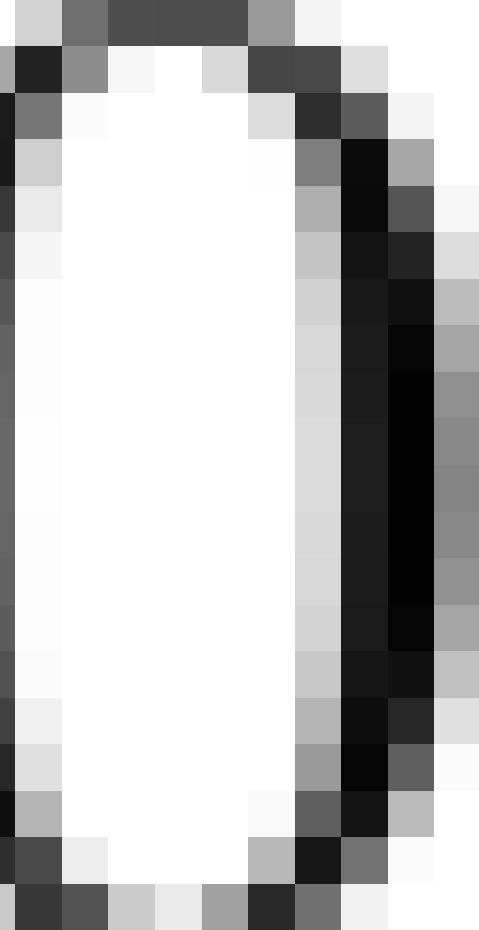
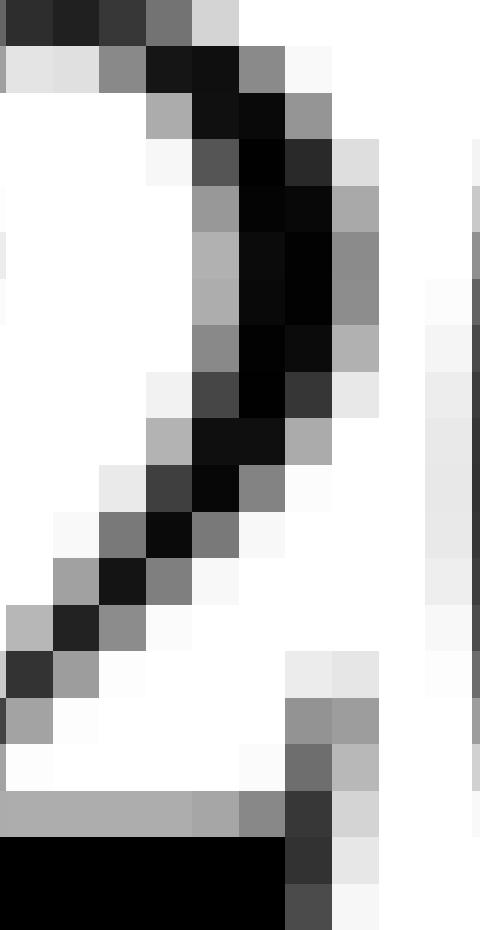
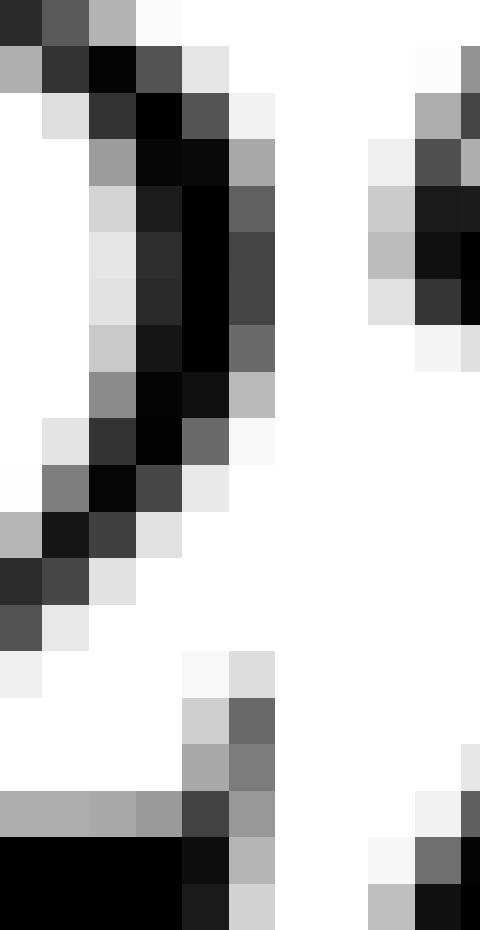














9 2

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$$1 + \sin 30.96^\circ$$

$$- \sin 30.96^\circ$$

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3.12



$$\frac{3 \times 3220 \text{ psi} - 1220 \text{ psi}}{1 + 3.12} = 4279 \text{ psi}$$

4279 psi

7000 ft

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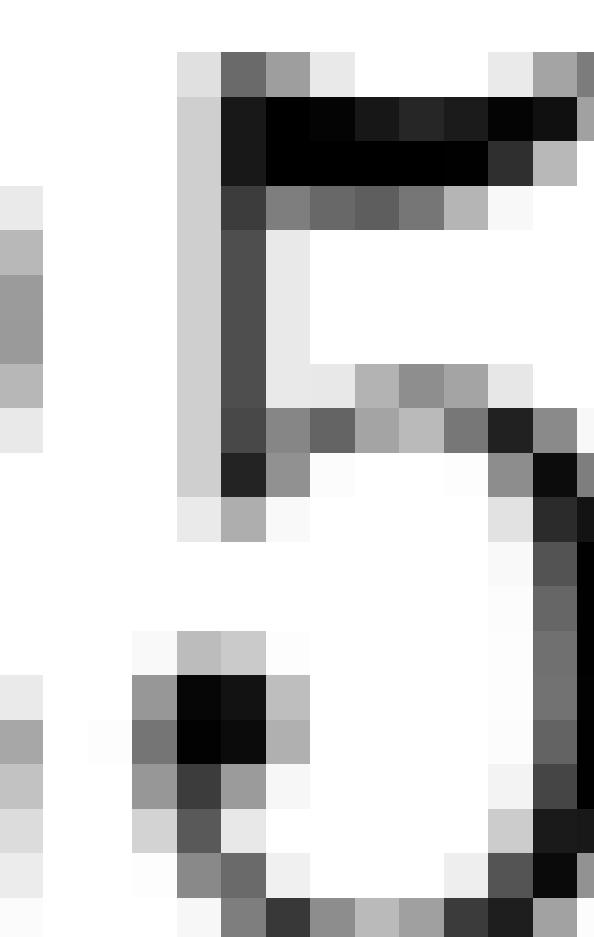
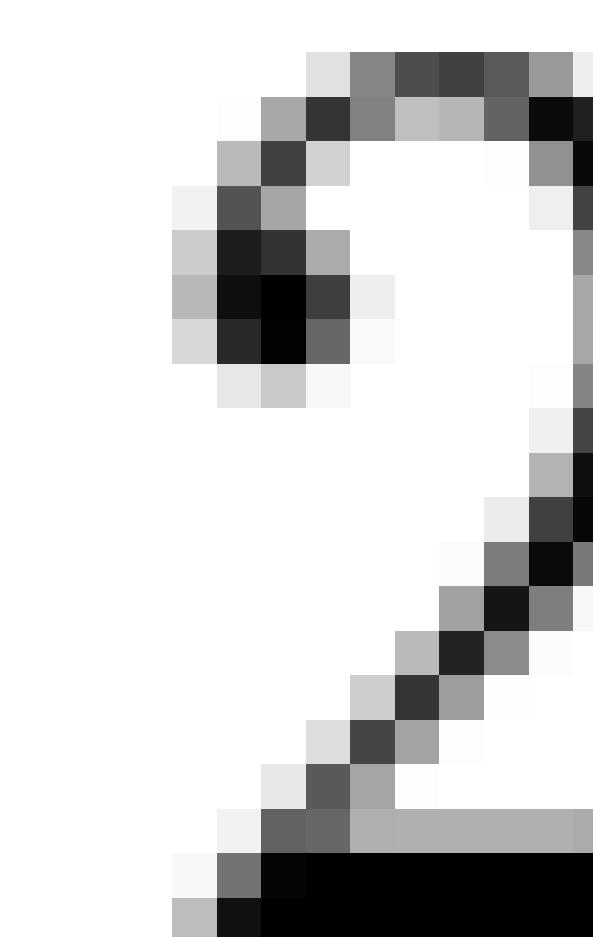
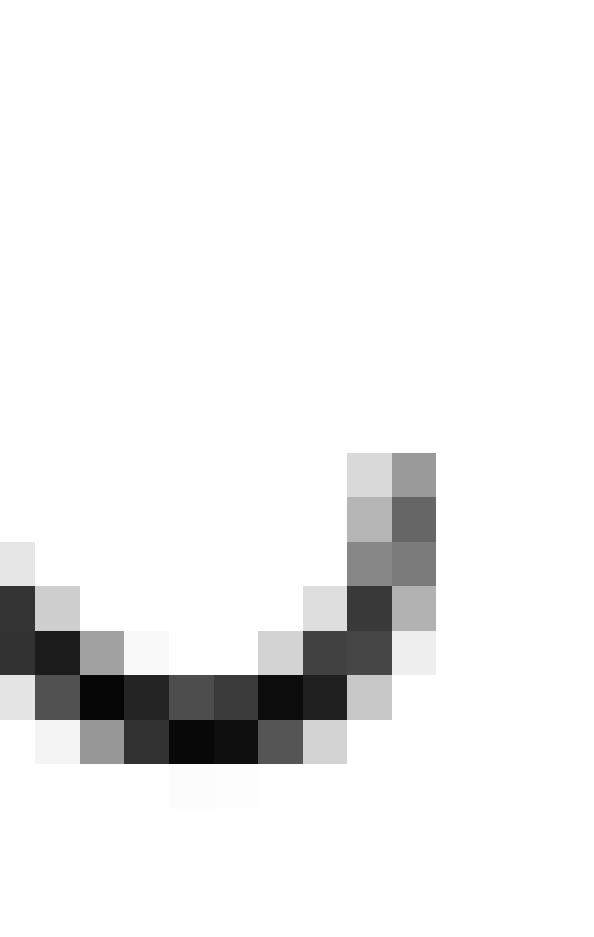
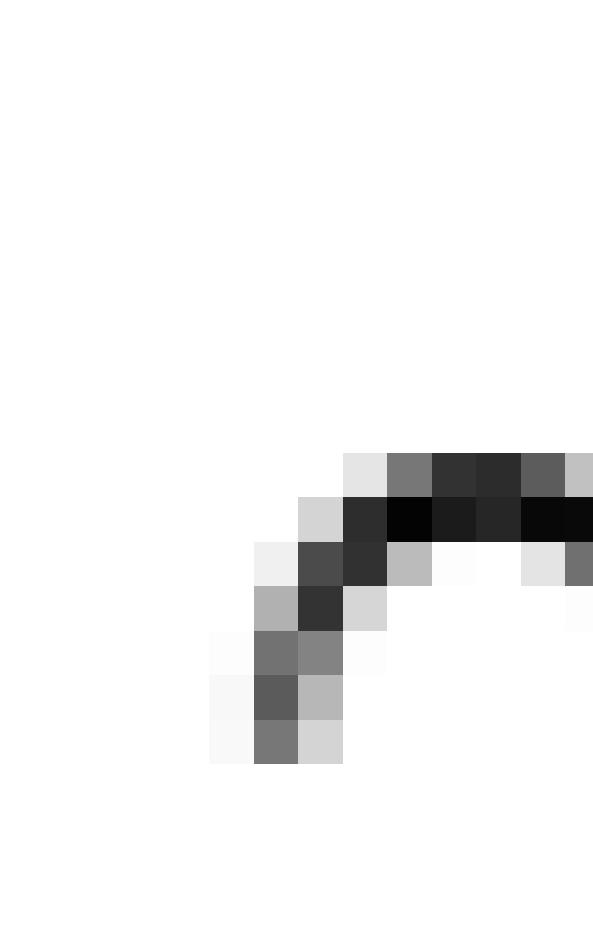
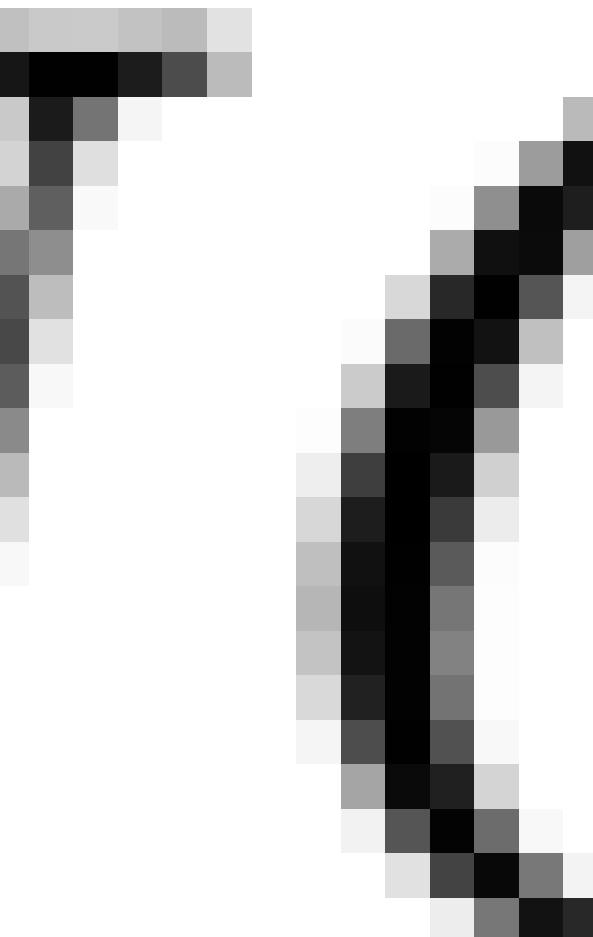
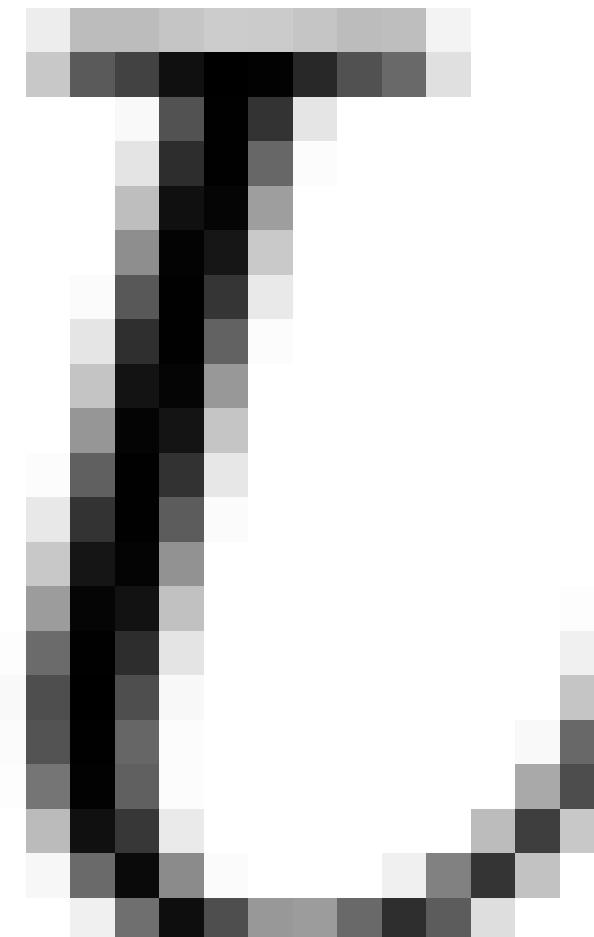
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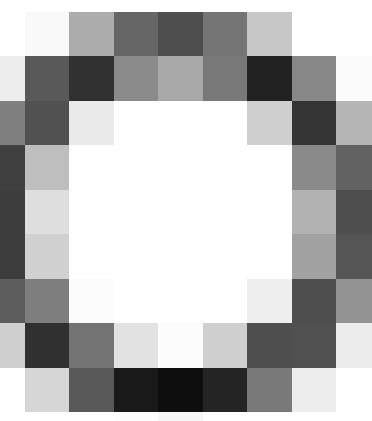
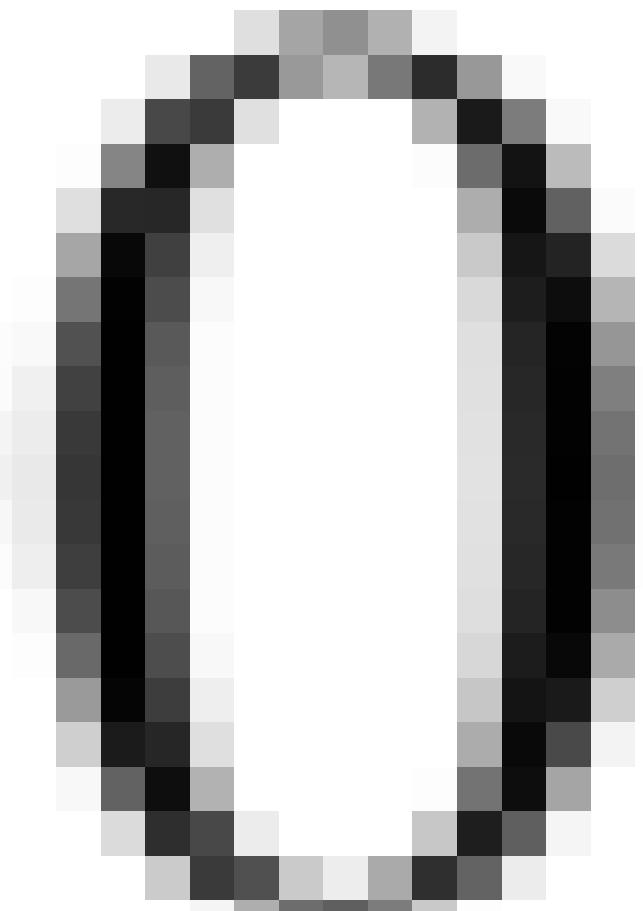
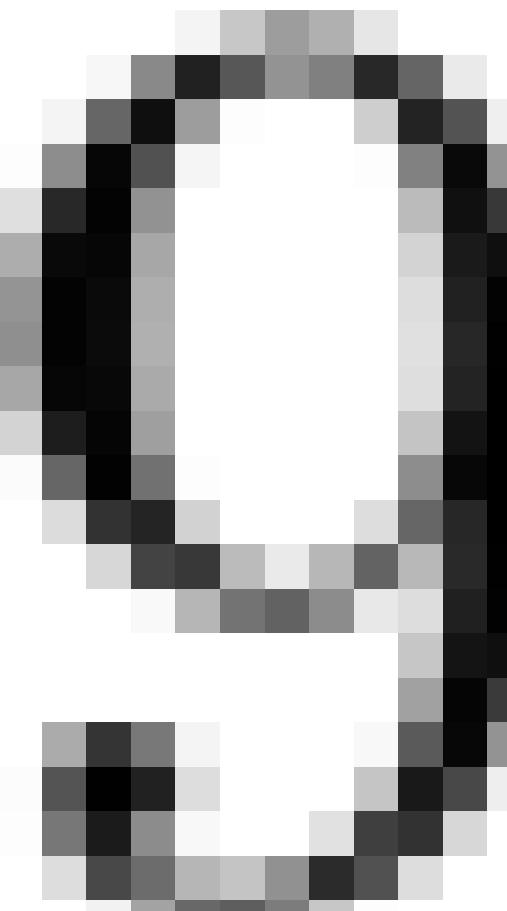
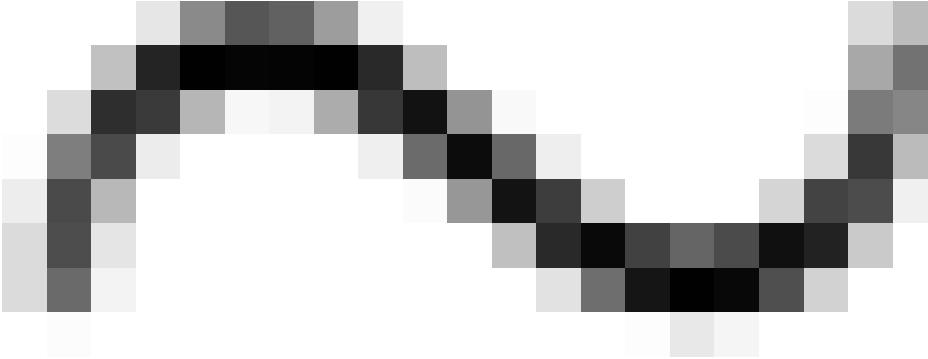
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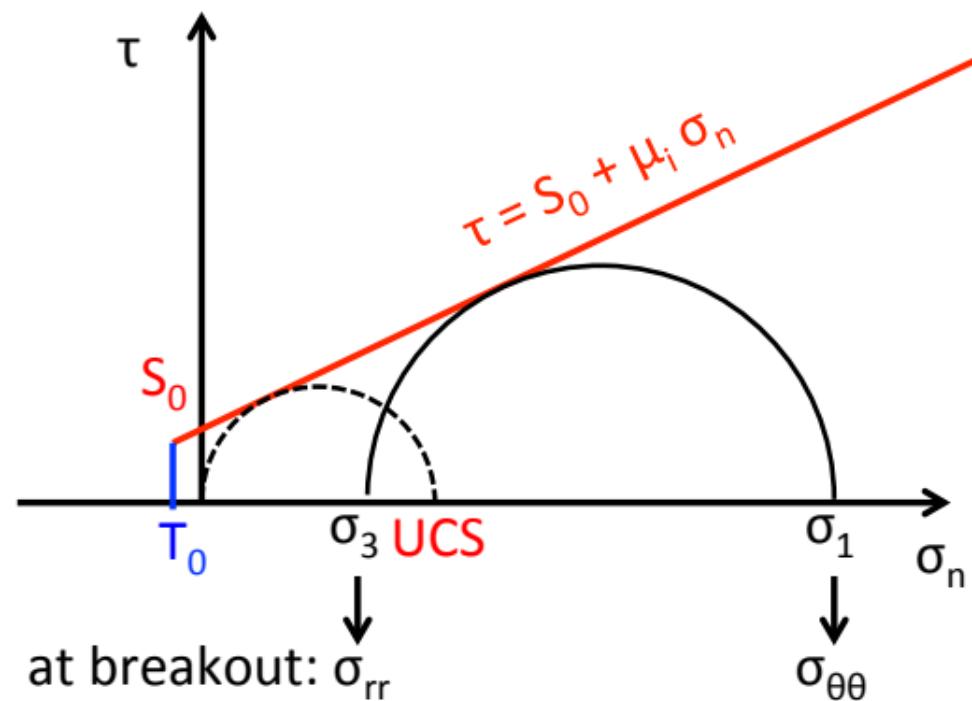
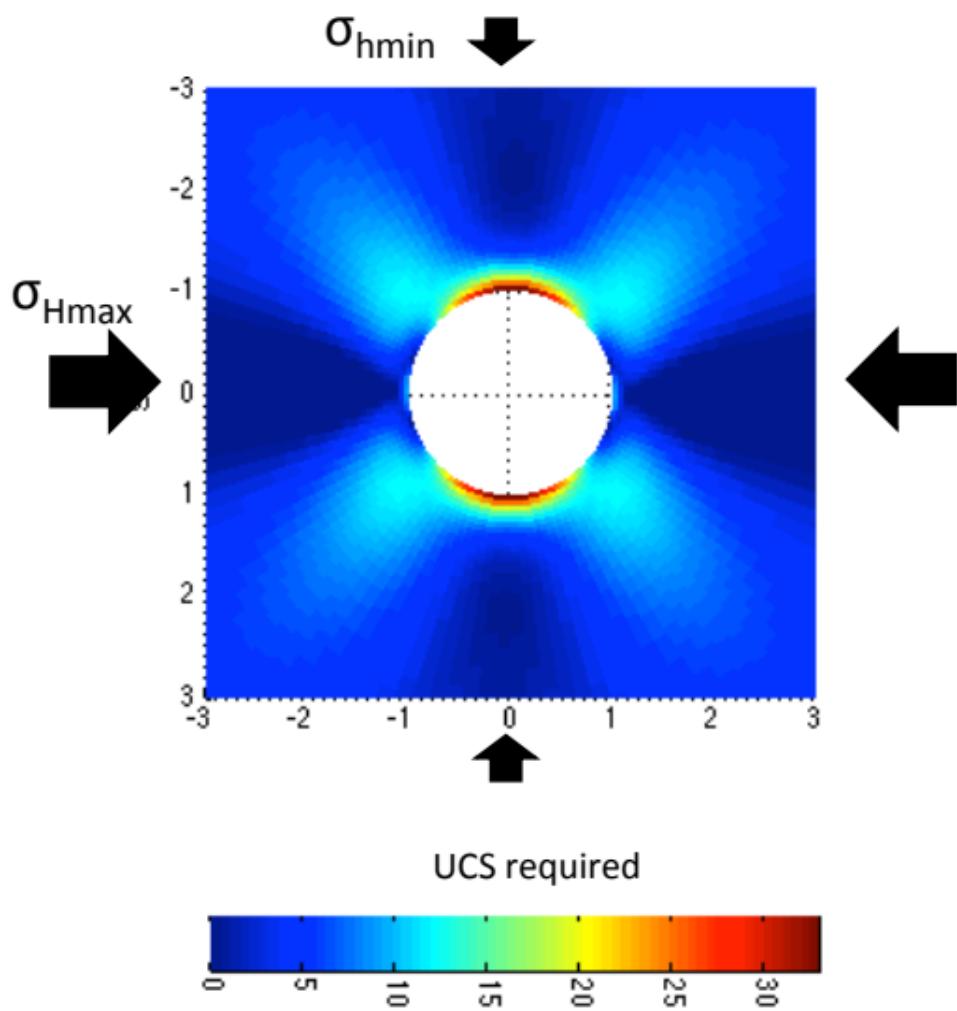
8.3 ppg

0.44 psi/ft

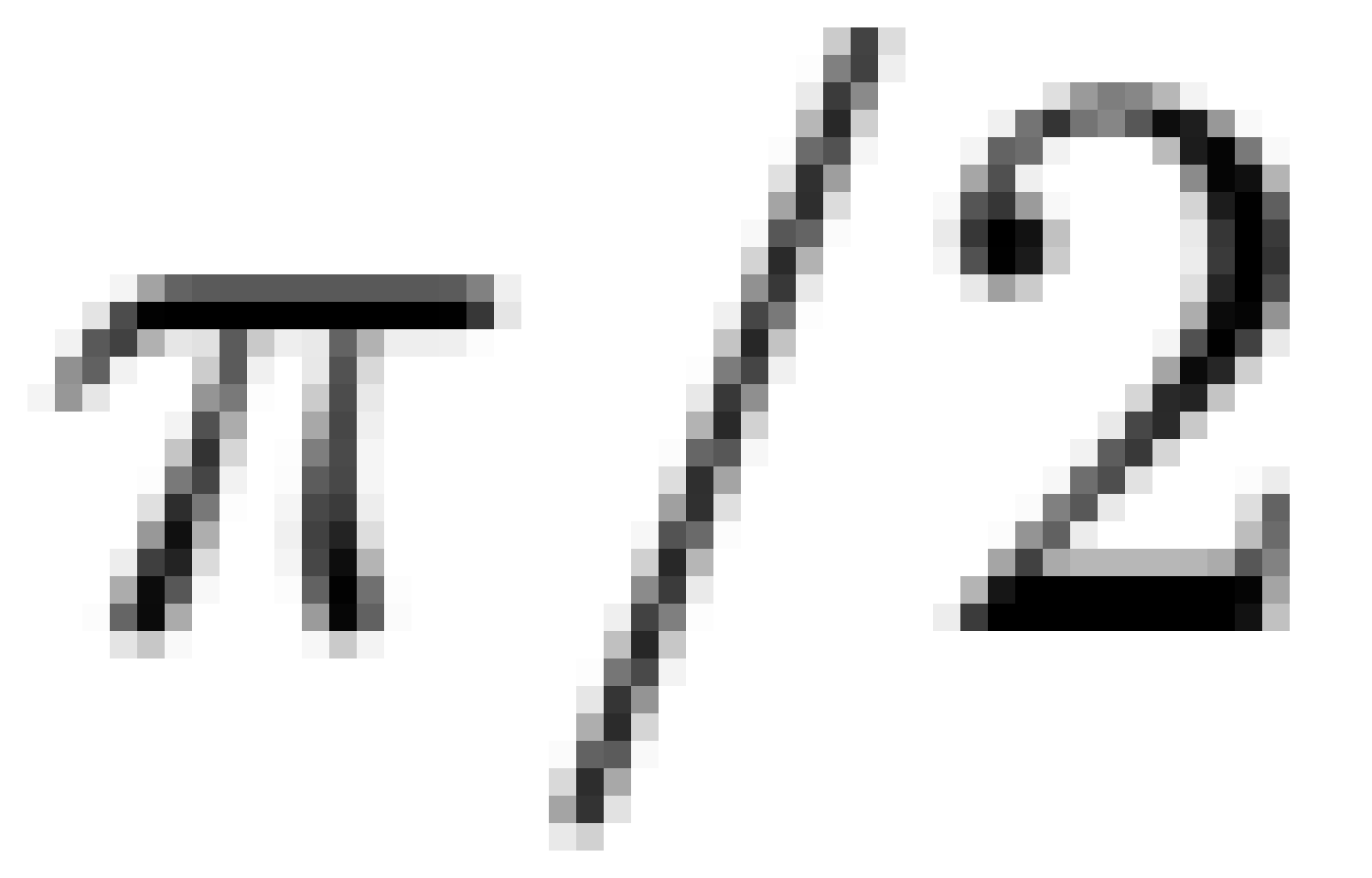
11.57 ppg

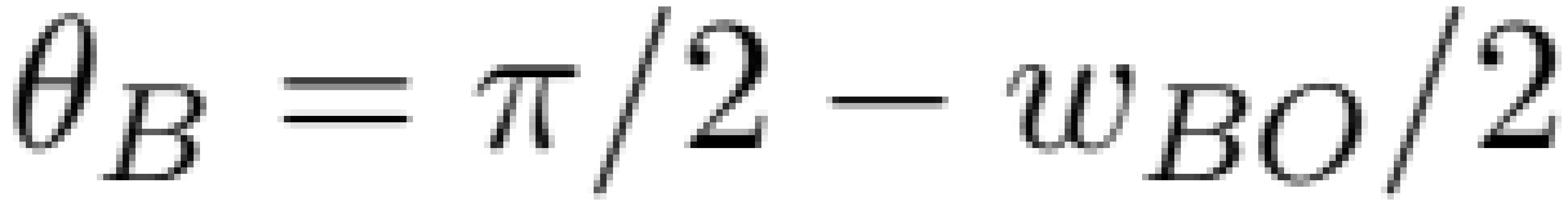




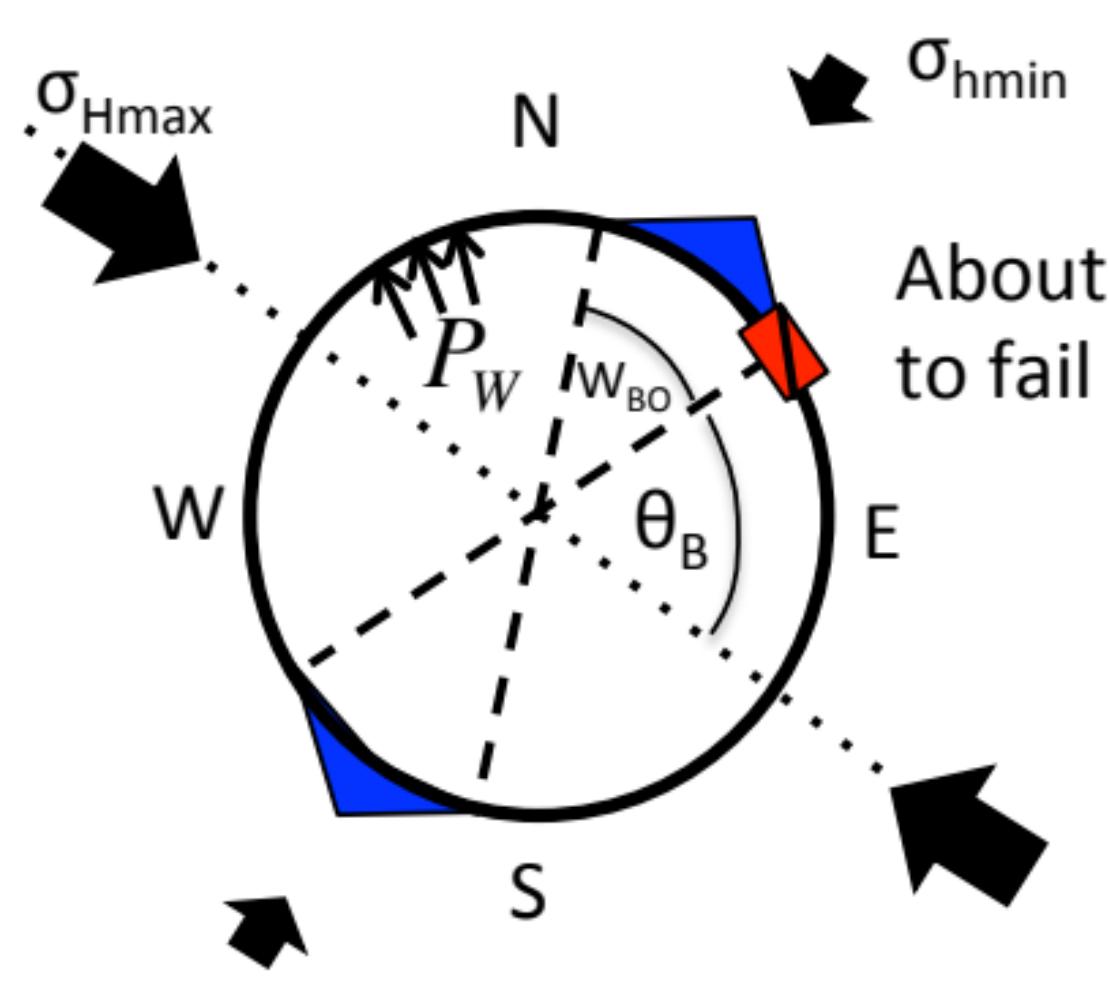








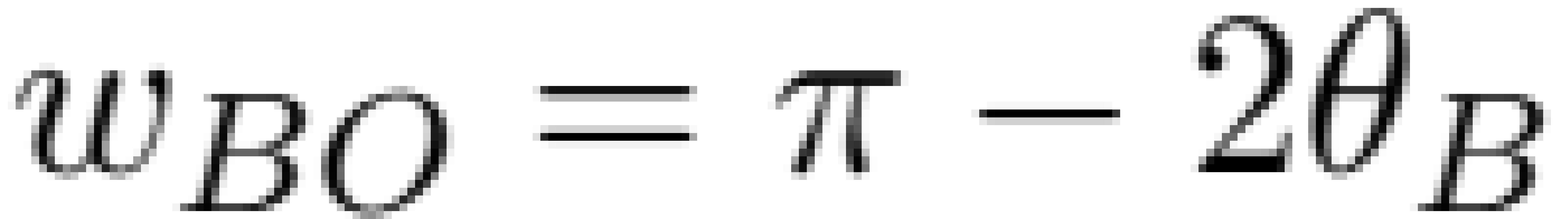
$$\begin{aligned} \sigma_{\theta\theta} &= (P_W - P_p) + 2(\sigma_{H\max} - \sigma_{h\min}) \cos(2\theta_B) \\ \sigma_{rr} &= + (P_W - P_p) \end{aligned}$$

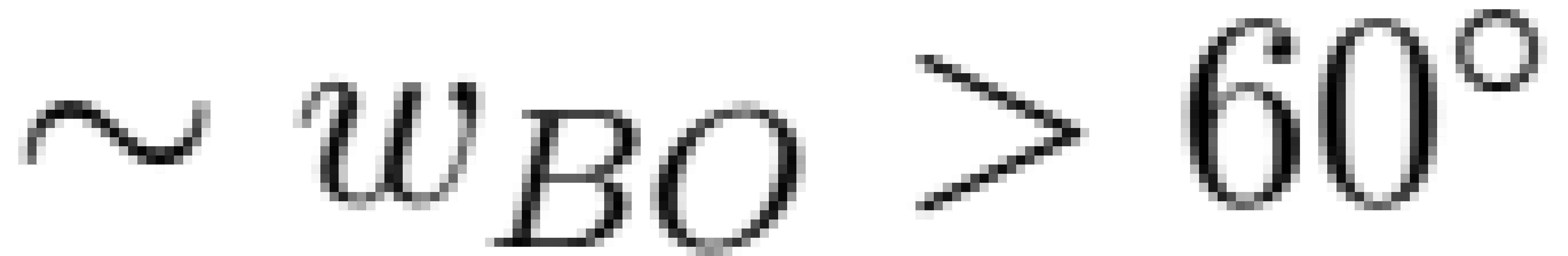




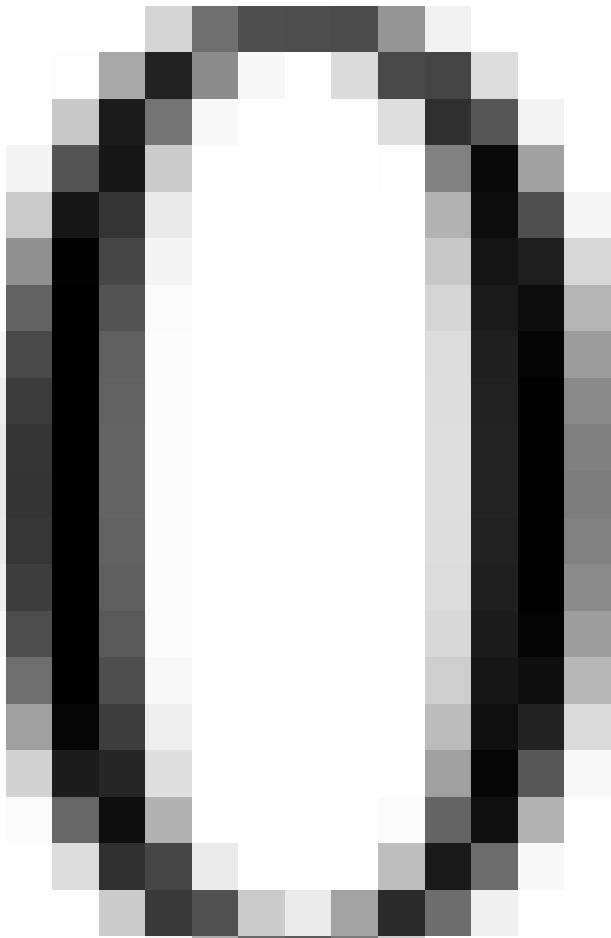
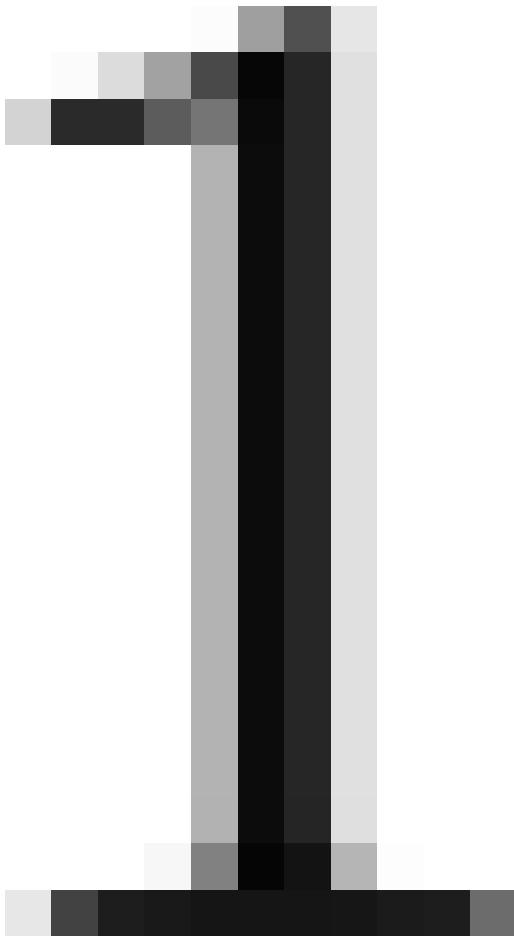
$$S_{\text{H}} = \frac{1}{2} \left[\sigma_{W_{\text{max}}}^2 + \sigma_{H_{\text{max}}}^2 \right] \cos(2\theta) + \sigma_{H_{\text{max}}}^2 \sin(2\theta) \cos(\phi_{H_{\text{max}}}) + \sigma_{W_{\text{max}}}^2 \sin(2\theta) \sin(\phi_{H_{\text{max}}})$$

$$2\theta_B = \arccos \left[\frac{\sigma_{Hmax} - \sigma_{hmin} - (1+q)(P_W - P_p)}{2(\sigma_{Hmax} - \sigma_{hmin})} \right]$$





$$P_{WBO} = P + \frac{(\sigma_{Hmax} - \sigma_{hmin}) \cos(\pi - w_{BO}) - Ucs}{1 + q}$$

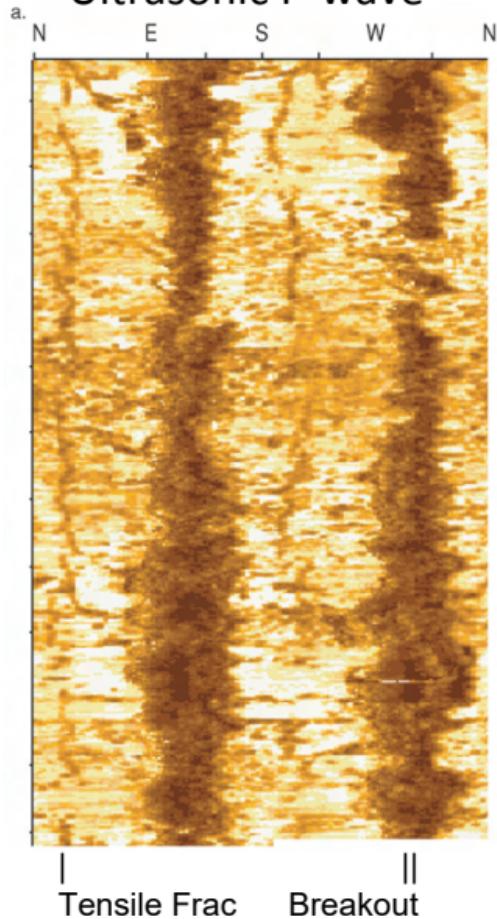


$$0.44 \text{ psi/ft} \times 8.3 \text{ DPG}$$

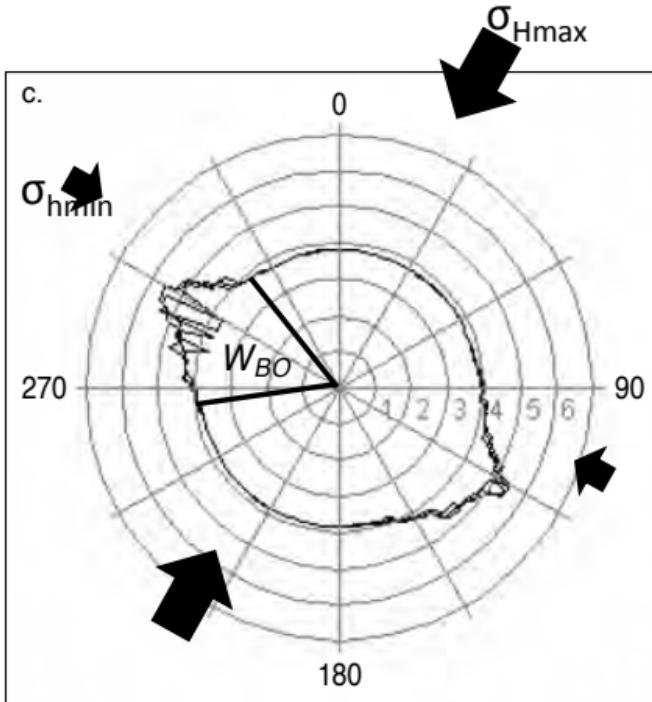
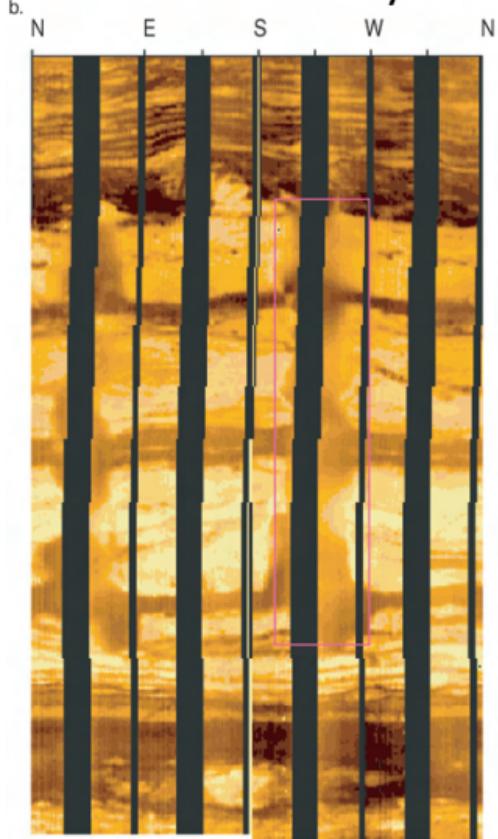
$$\times 7000 \text{ psi} = 3710 \text{ psi}$$

$$w_{BO}^{\circ} = 180^{\circ} - \arccos \frac{[3220 \text{ psi} + 1220 \text{ psi} - (1 + 3.12)(3710 \text{ psi} - 3080 \text{ psi})]}{2(3220 \text{ psi} - 1220 \text{ psi})} = 66^{\circ}$$

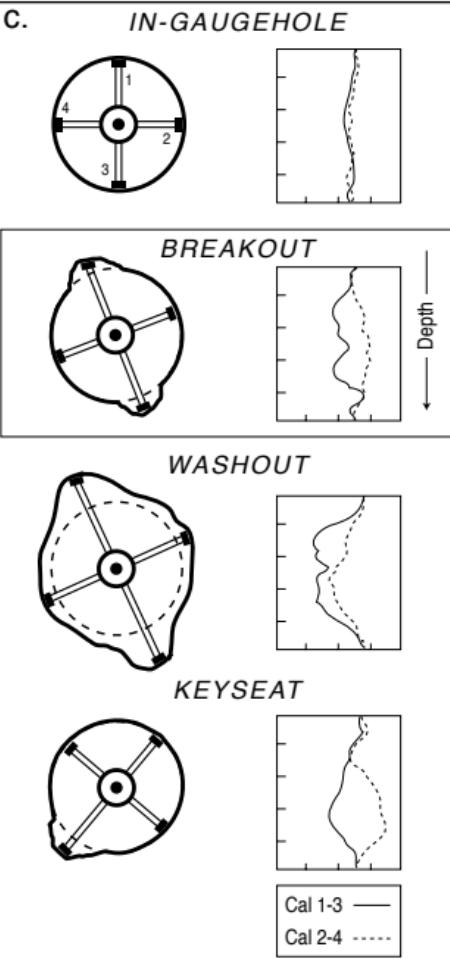
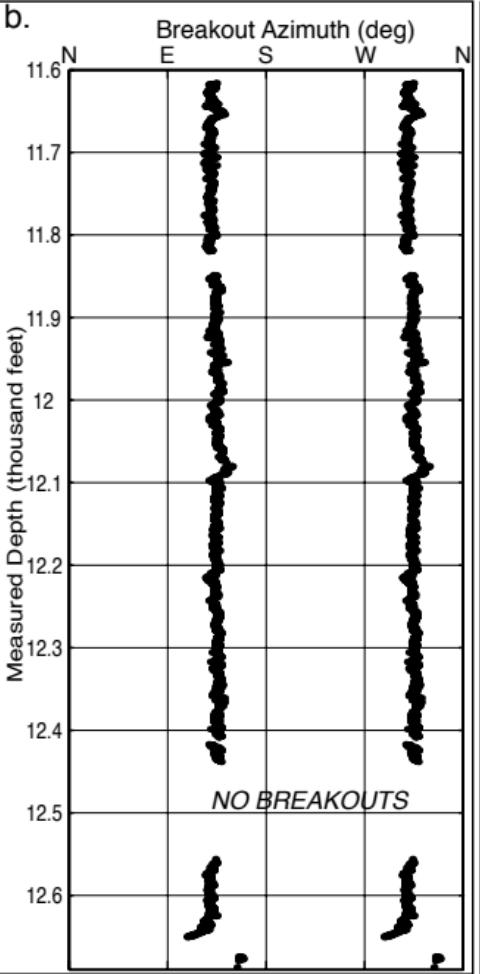
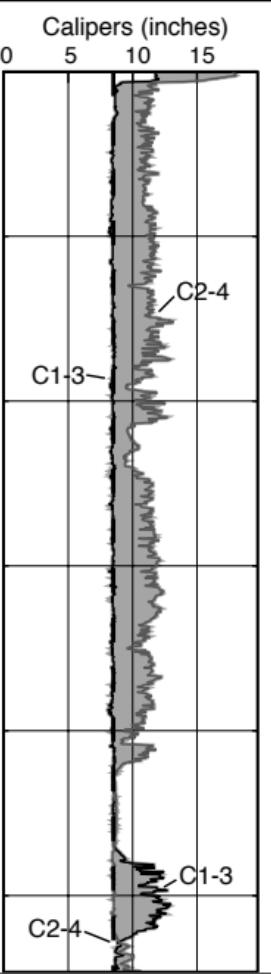
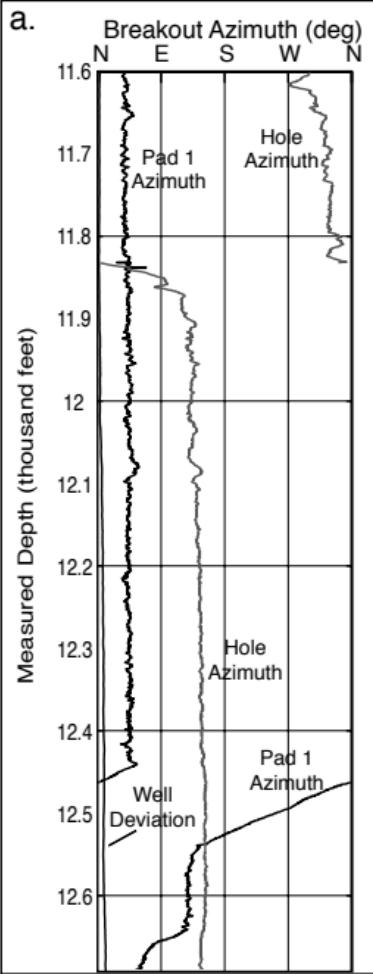
Ultrasonic P-wave



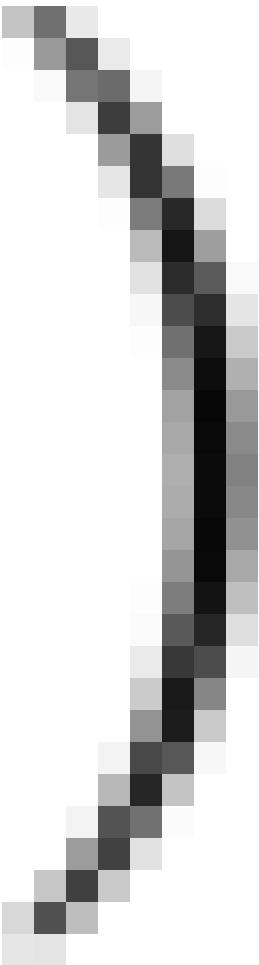
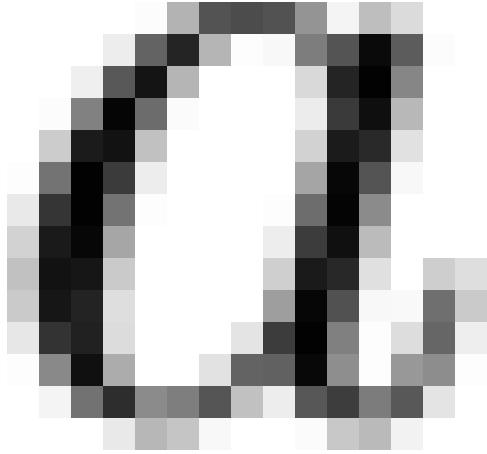
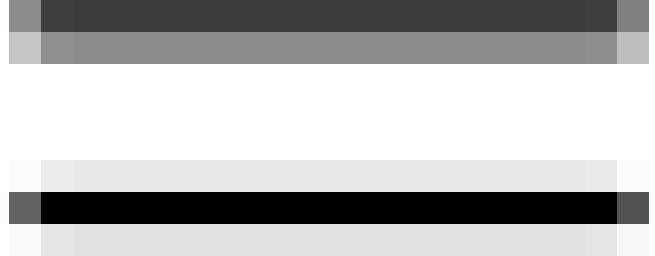
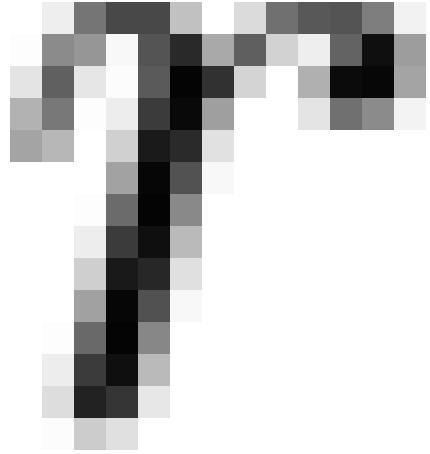
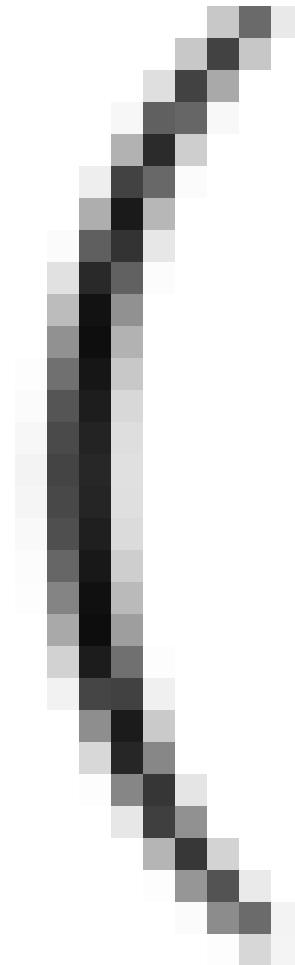
Electrical resistivity



[Zoback 2013 - Figure 6.4]

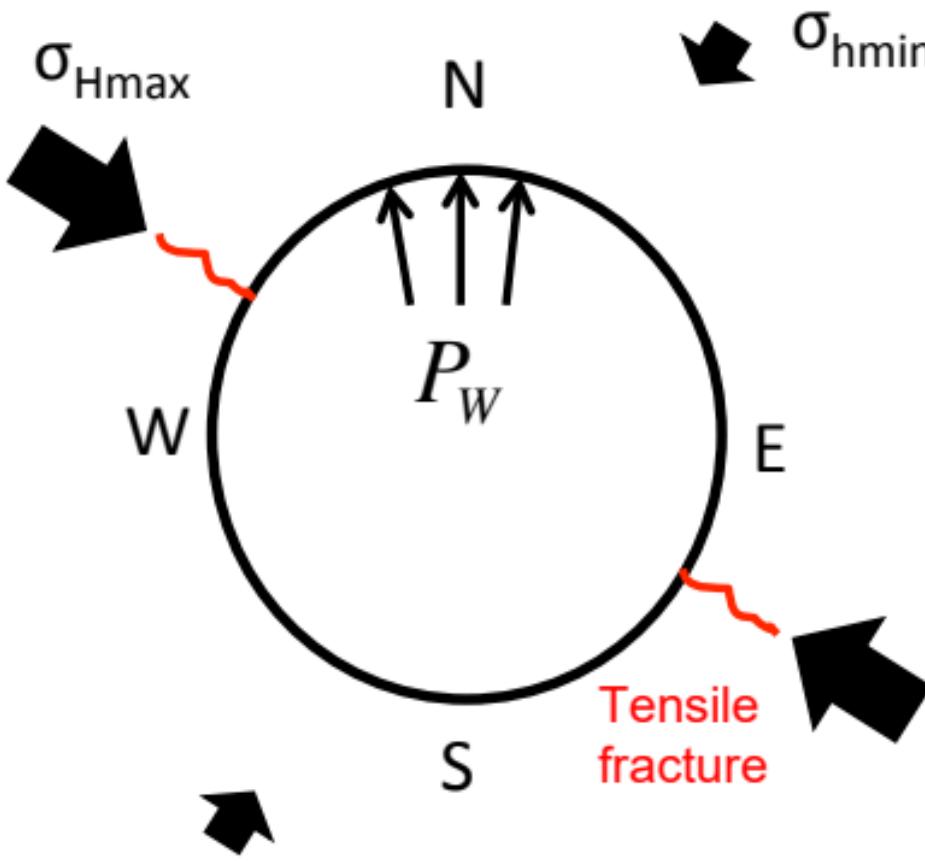


$$S_{H\max} = \frac{P}{P} + \frac{UCS + (1+q)(P_W - P_p) - \sigma_{hmin}[1 + 2\cos(\pi - w_{BO})]}{1 - 2\cos(\pi - w_{BO})}$$



σθαντικός + σούχος + στ



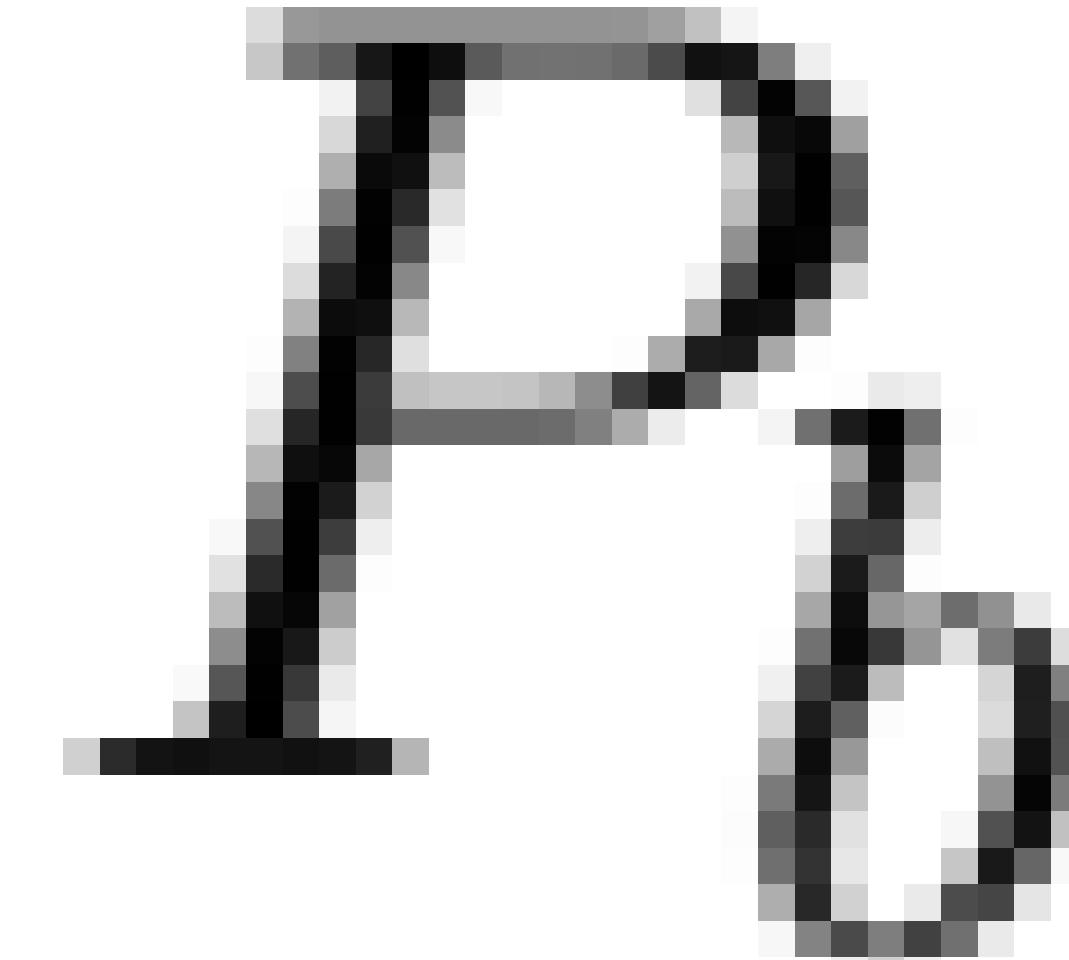
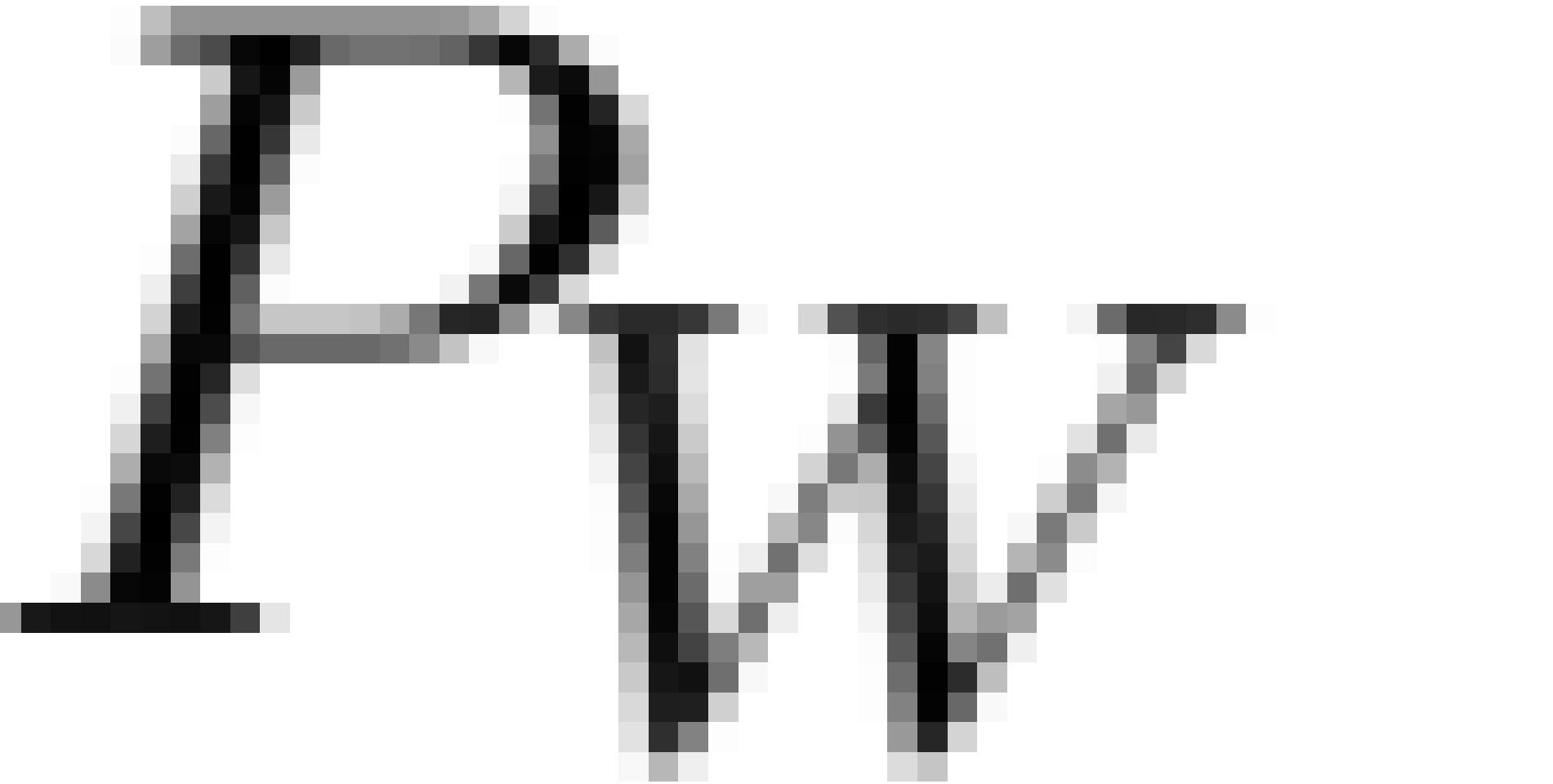


$$P_b = P_p + 3\sigma_{h\min} - \sigma_{H\max} + T_s + \sigma^{\Delta T}$$

Pore pressure
in the formation

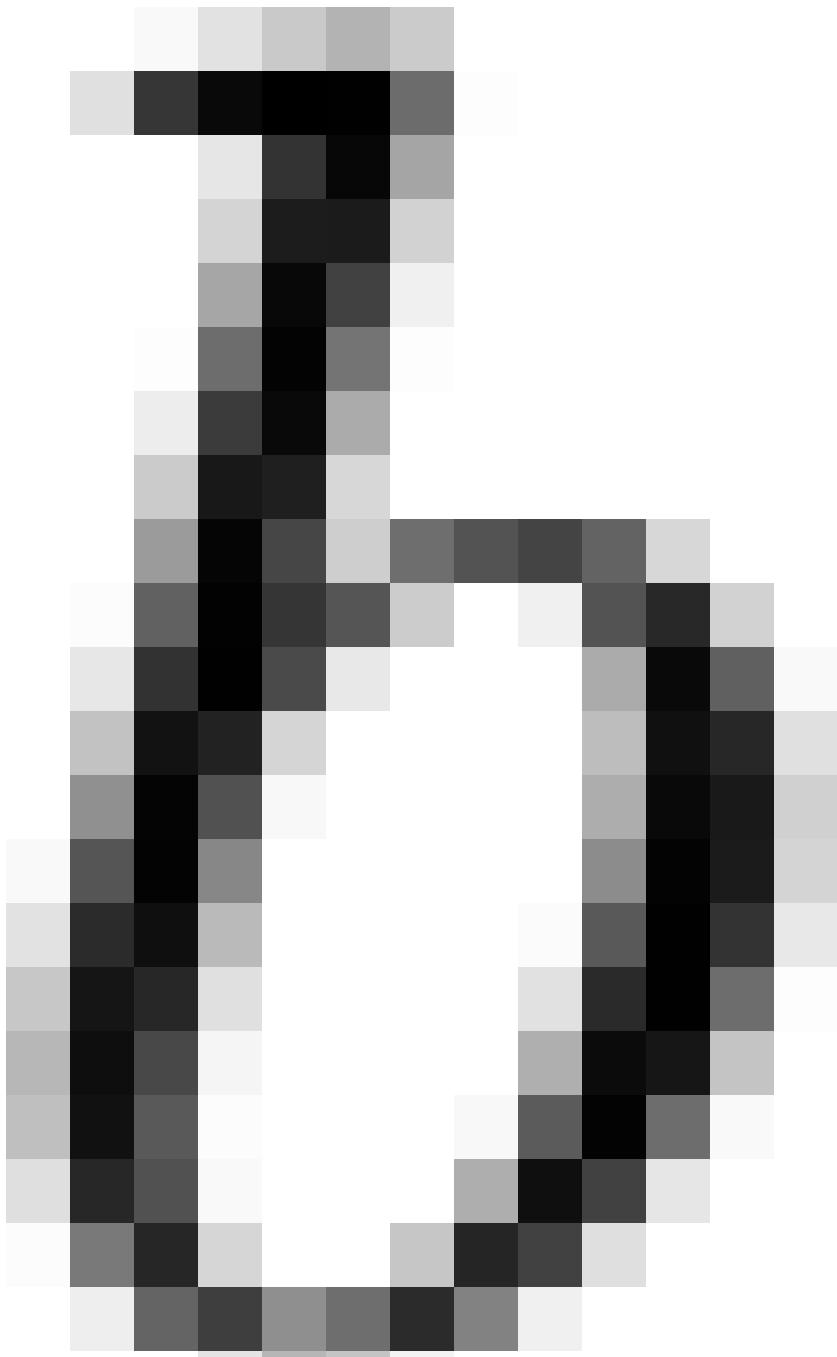
Stress anisotropy Tensile strength Cooling stress

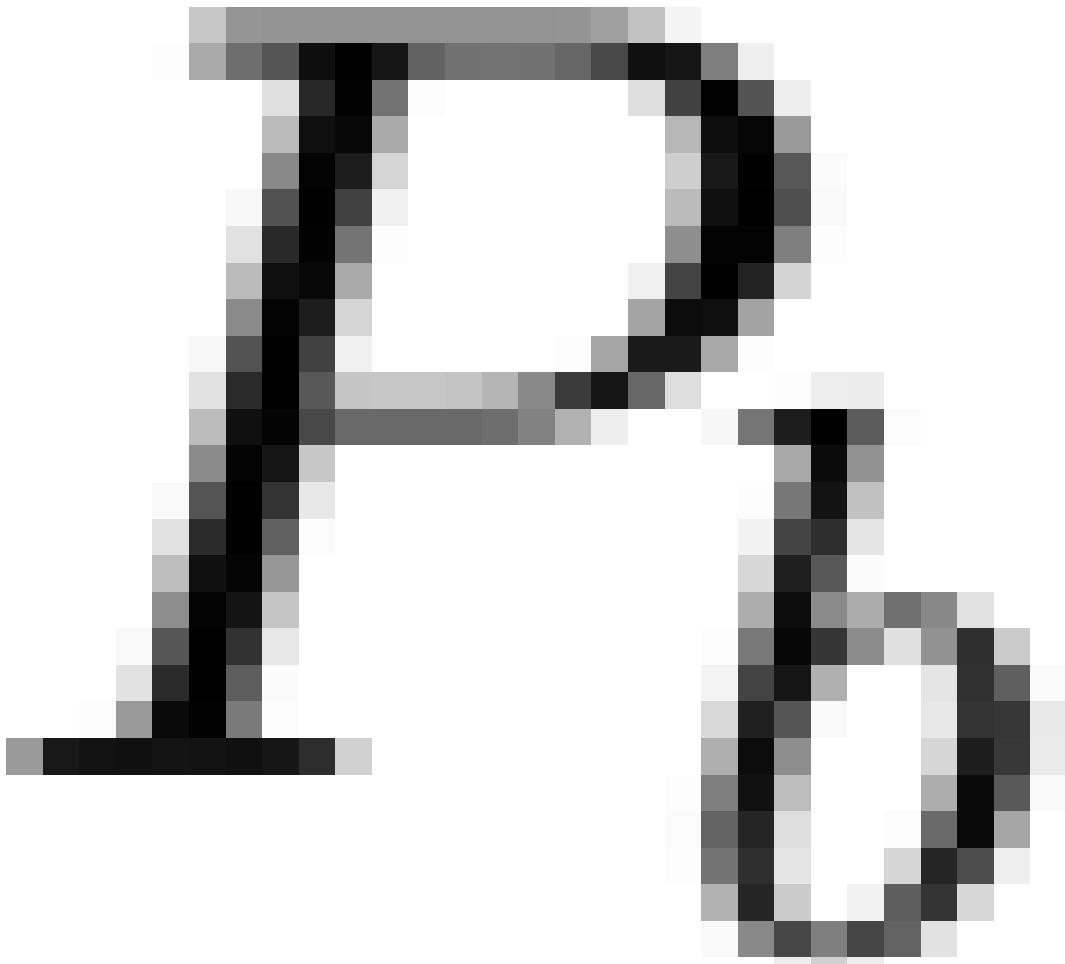
$$P_b = 3S_{h\min} - S_{H\max} - P_p + T_s + \sigma^{\Delta T}$$

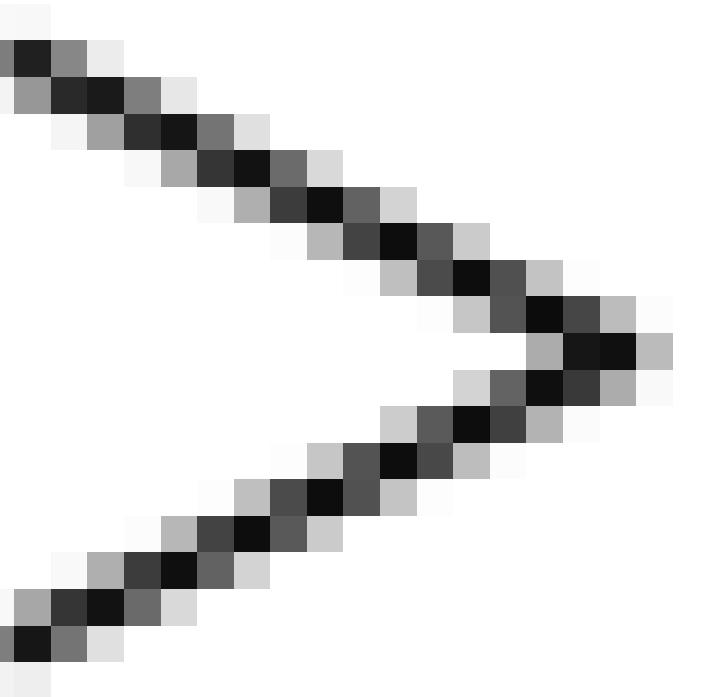
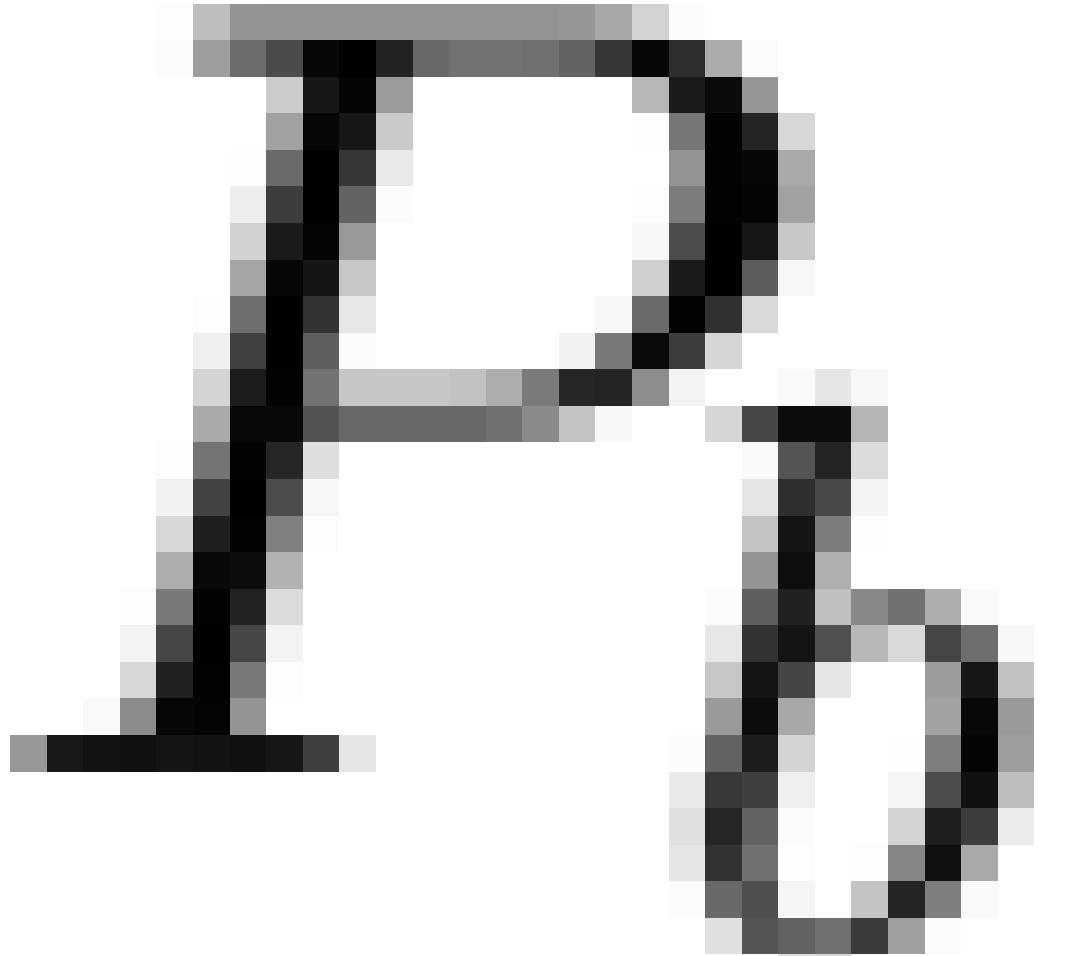


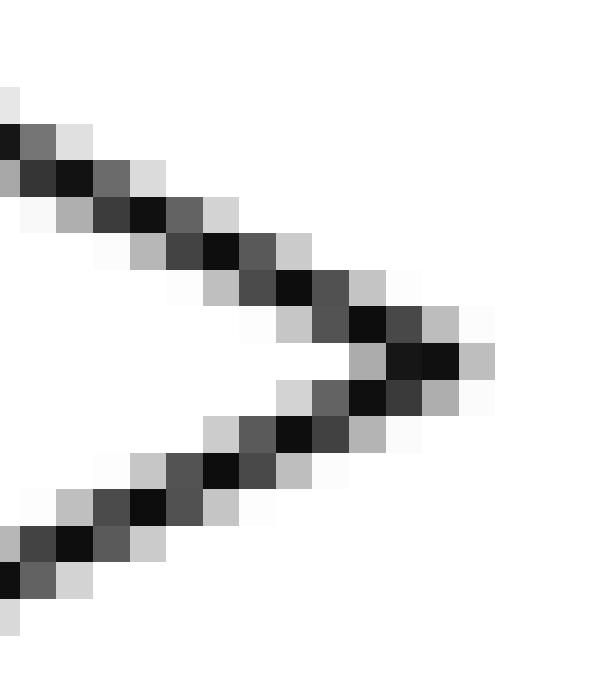
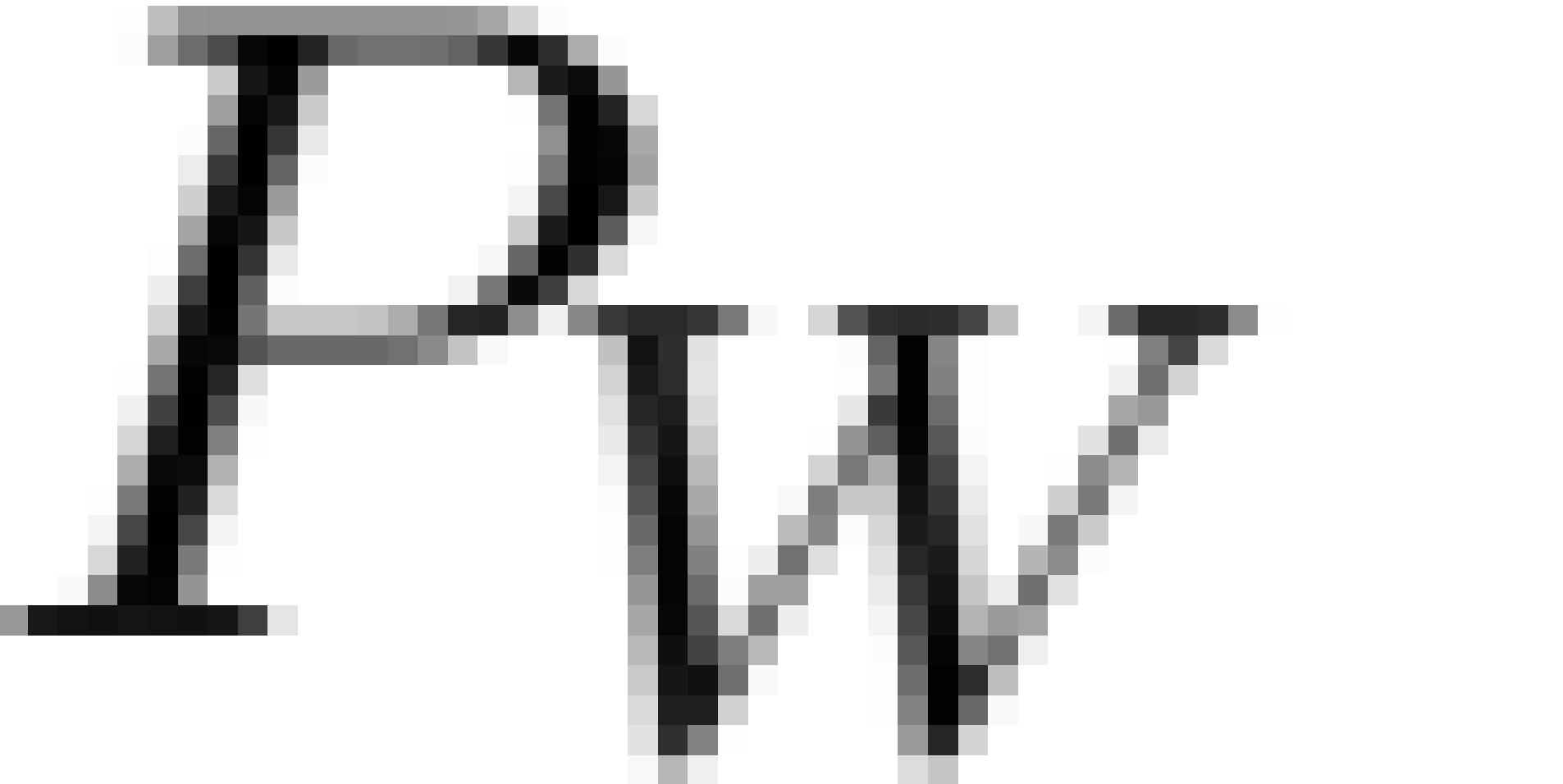
$\sigma_{\text{Hannac}} + \sigma_{\text{Hannac}}$

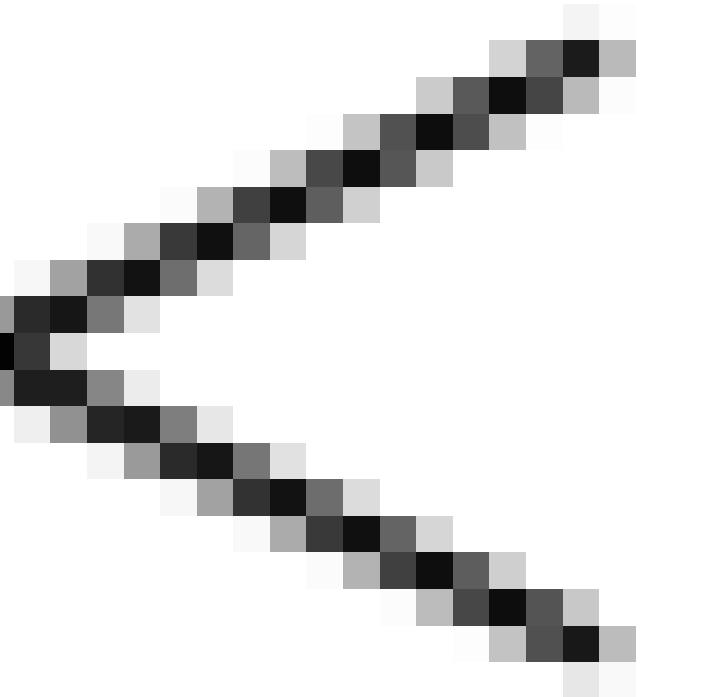
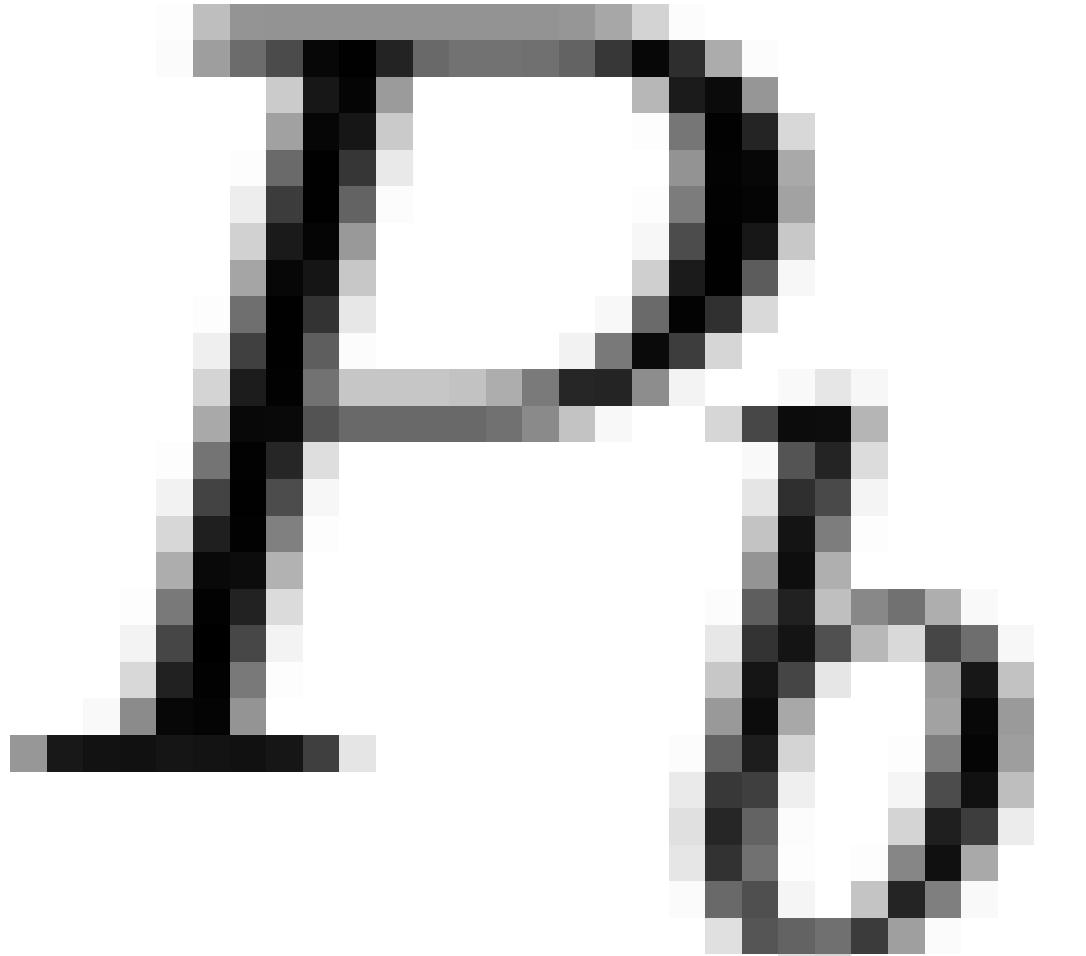
Bob Smith + 30 hours



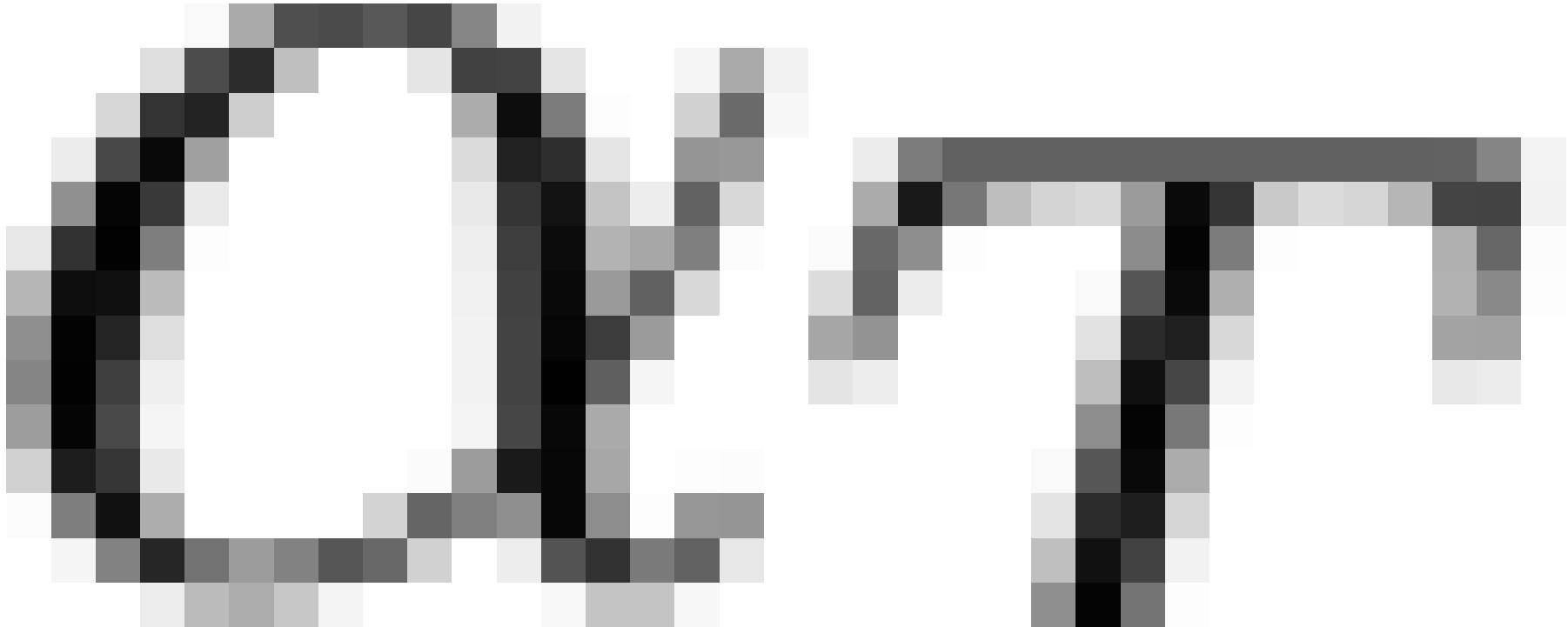


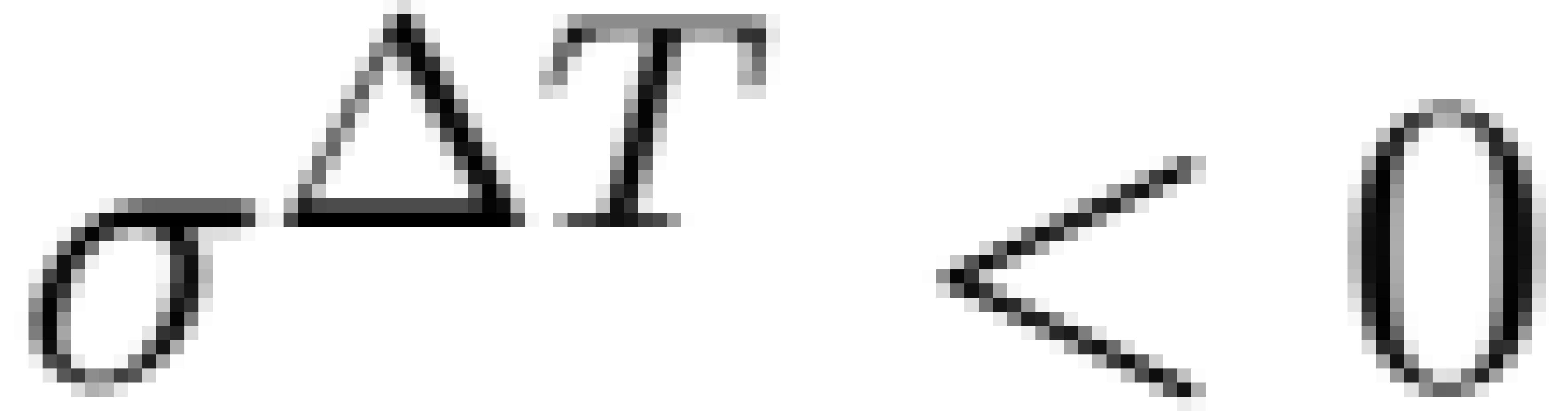


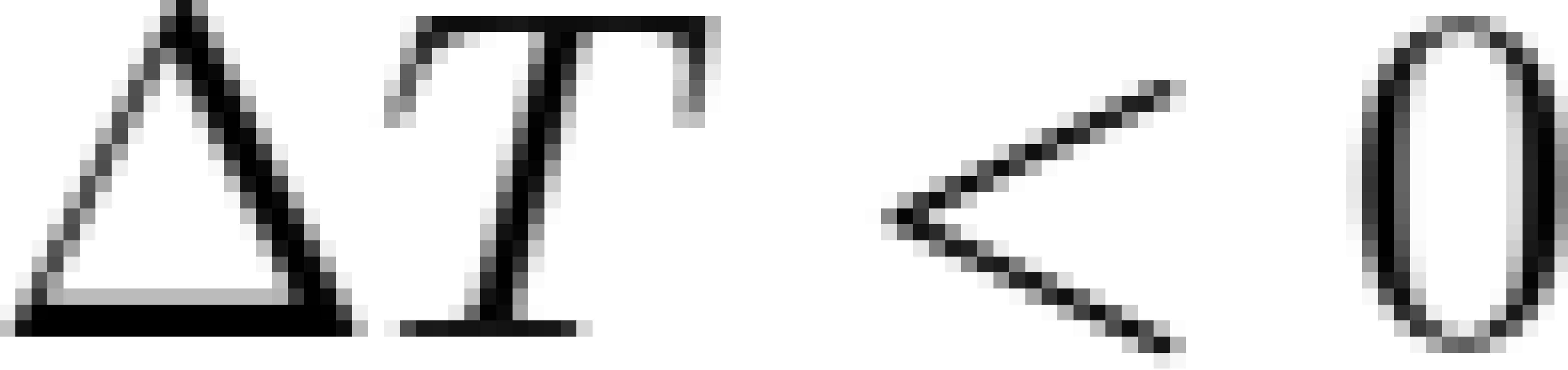


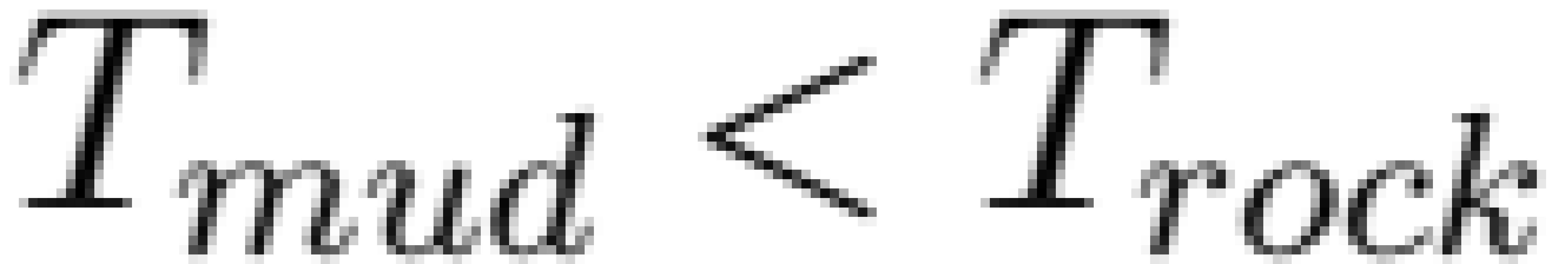


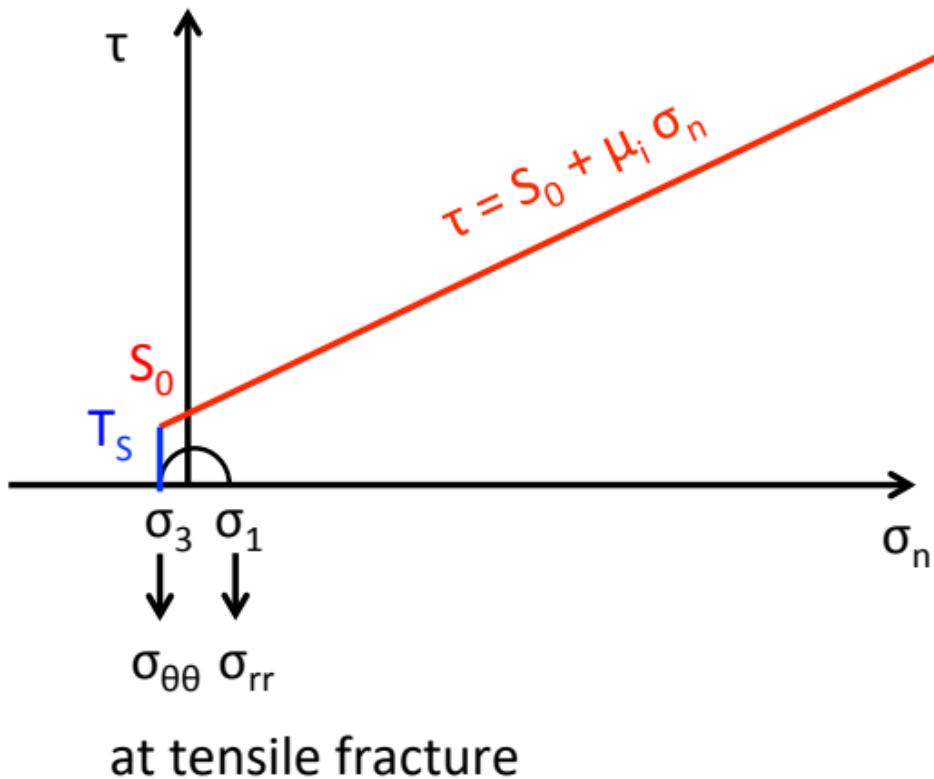
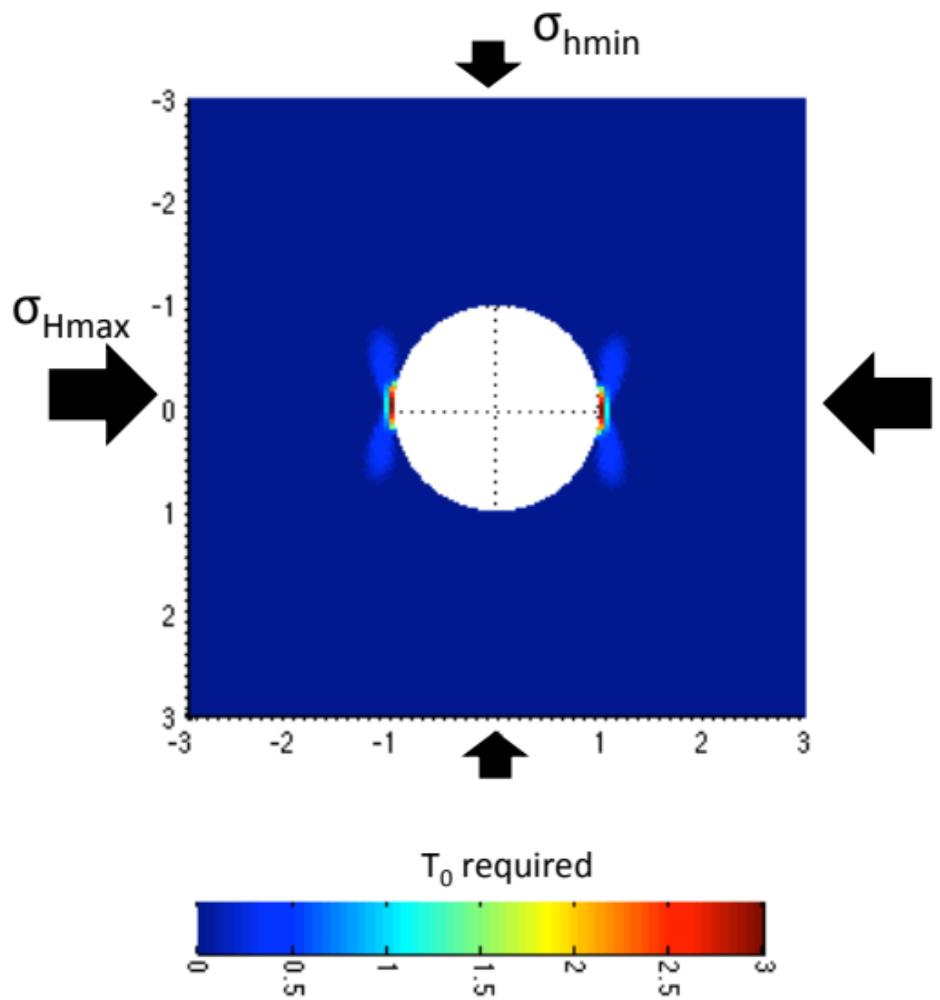


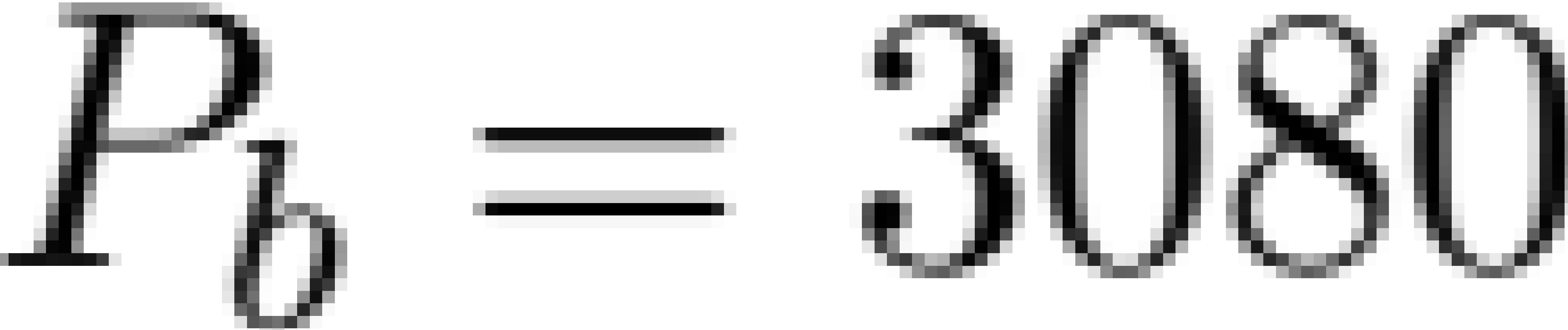


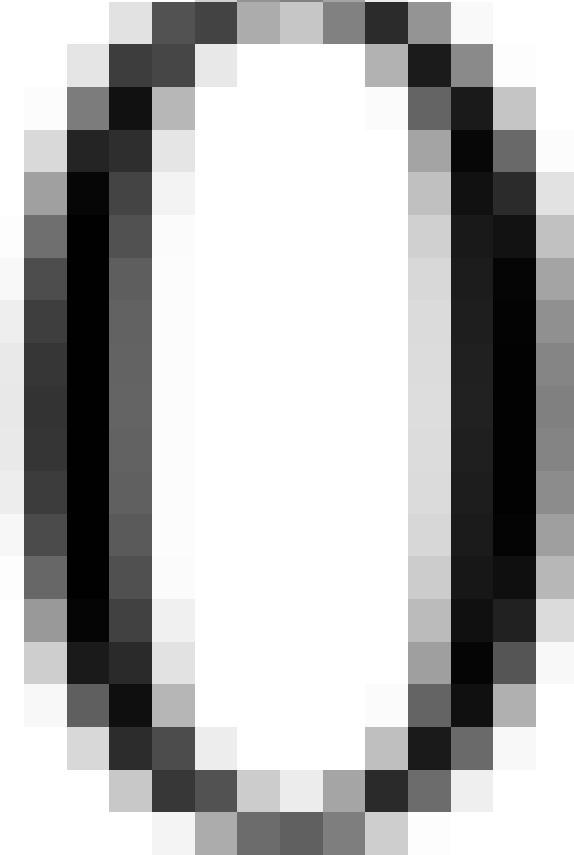
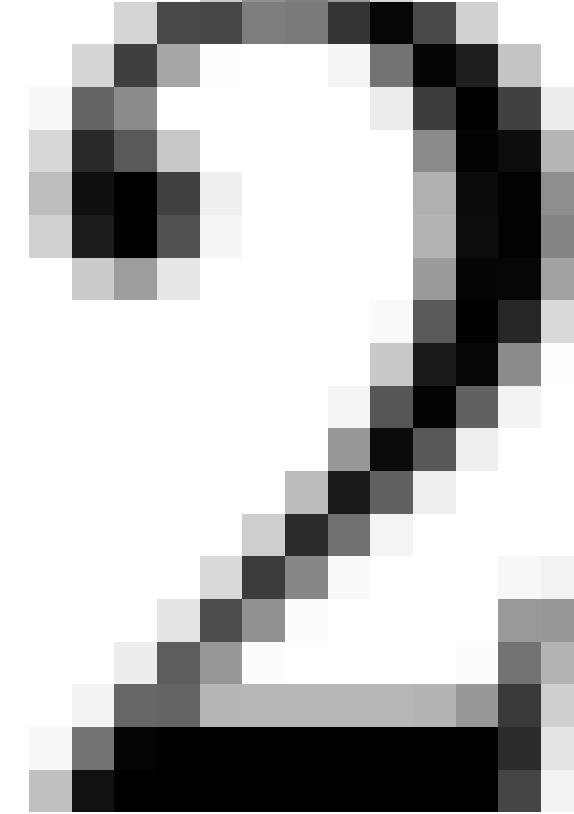
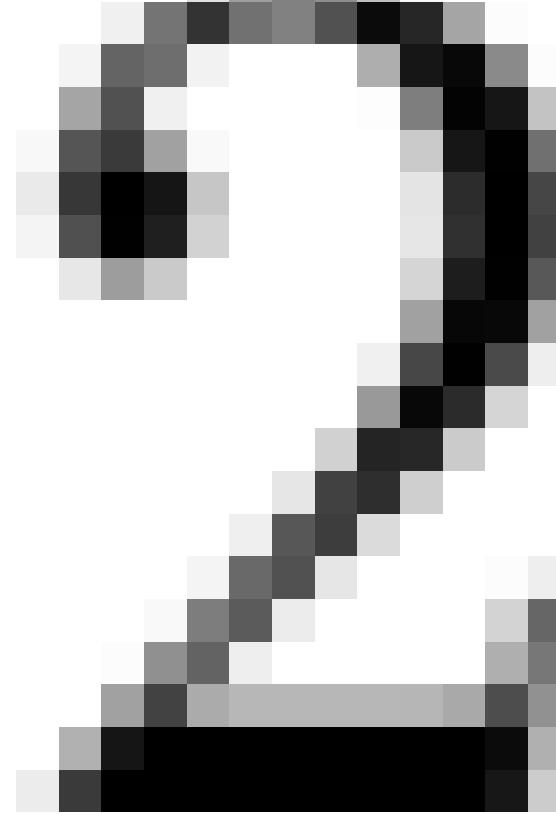
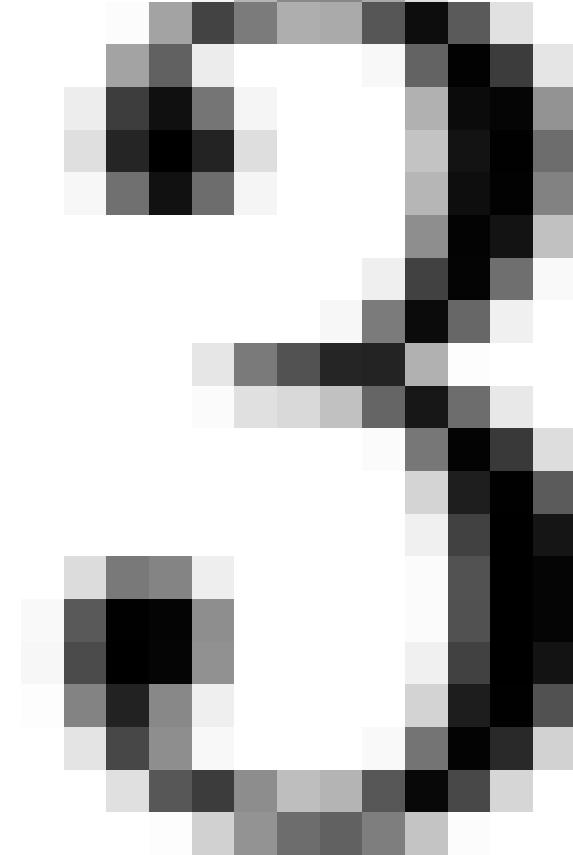


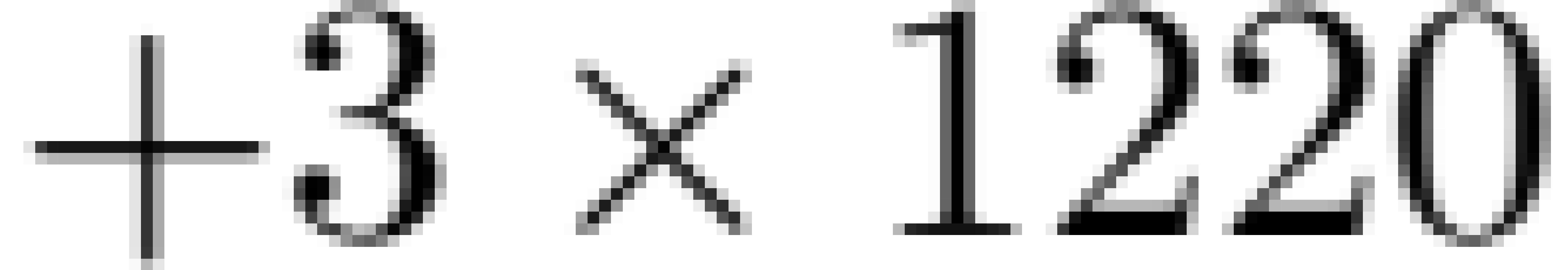


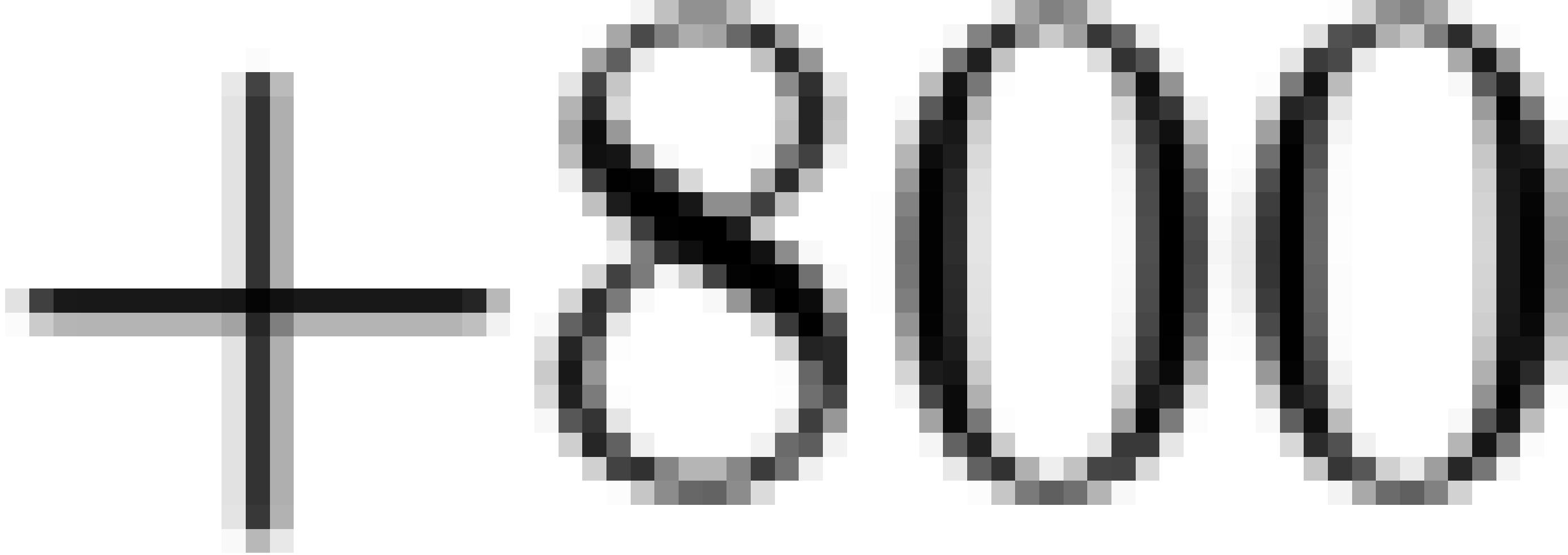


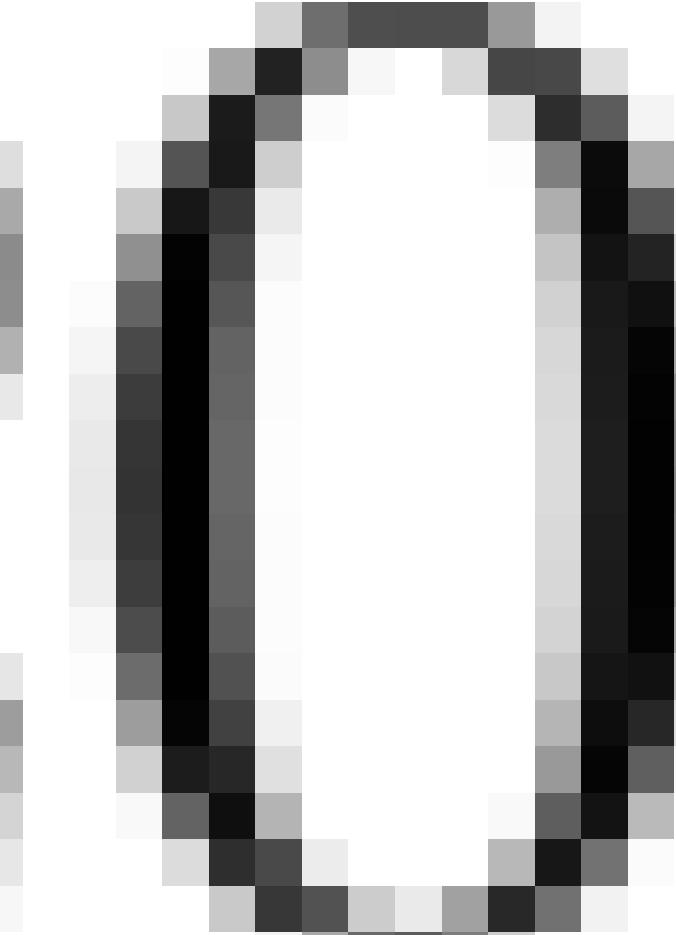
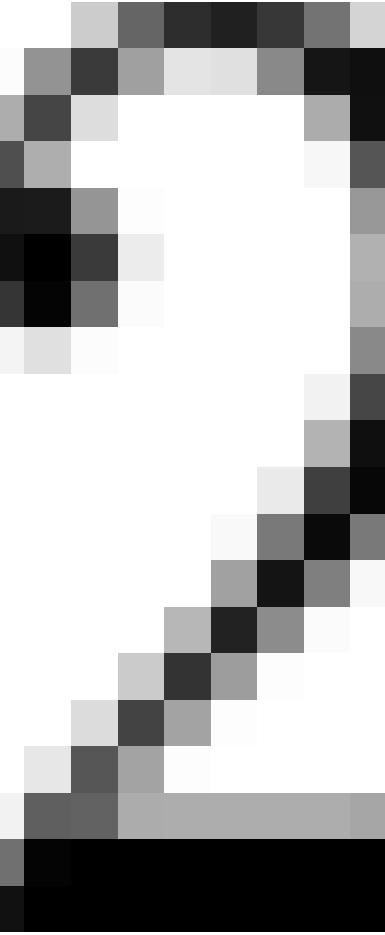
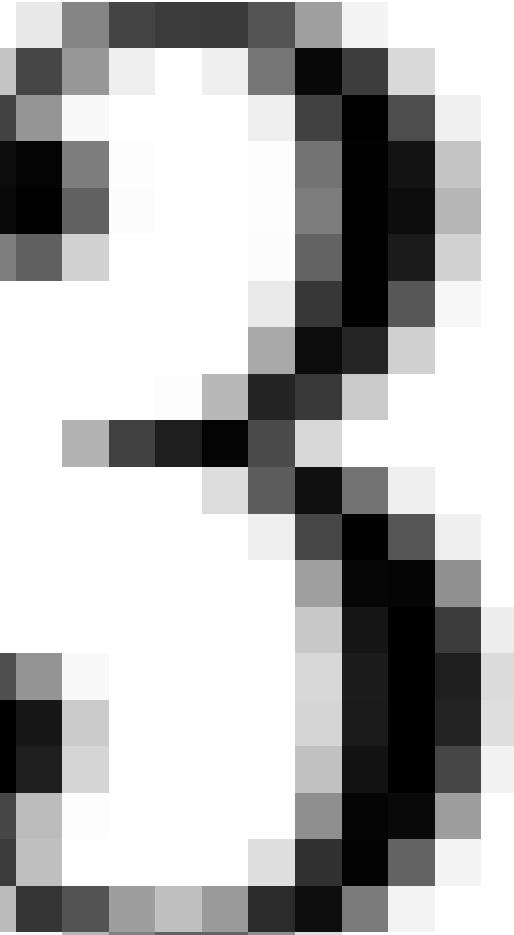
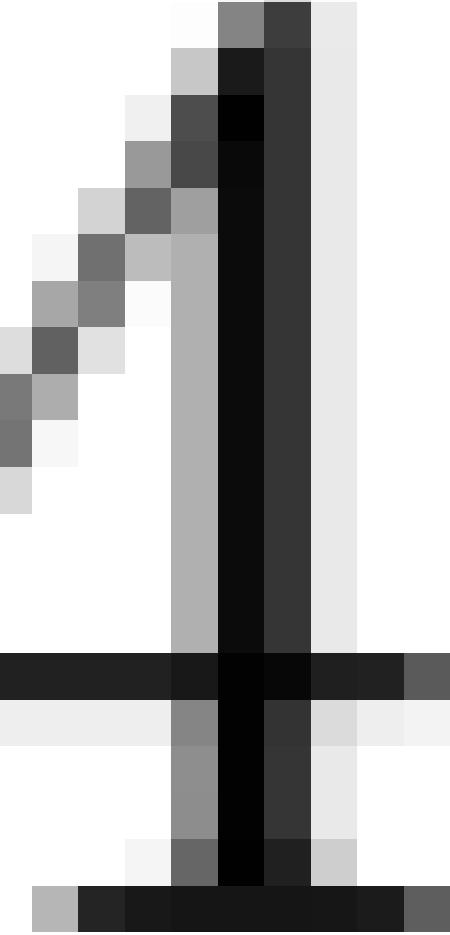












4320 psi

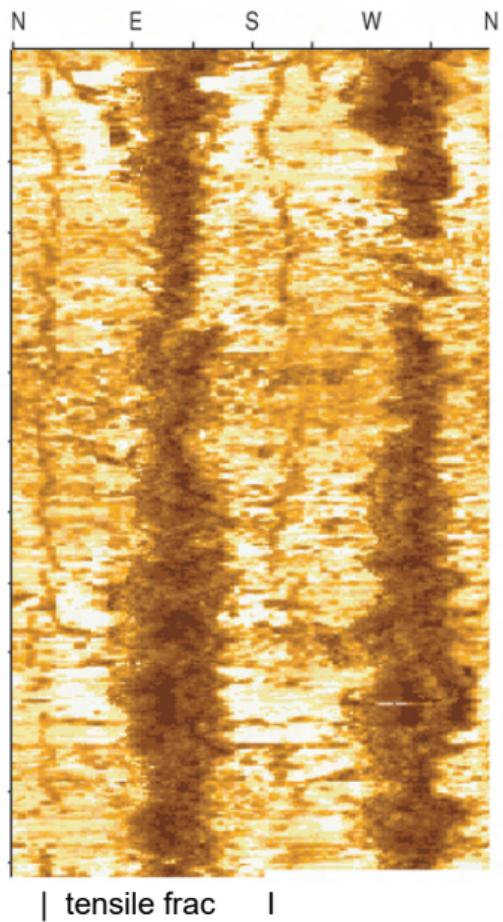
7000 ft

8.33 ppg

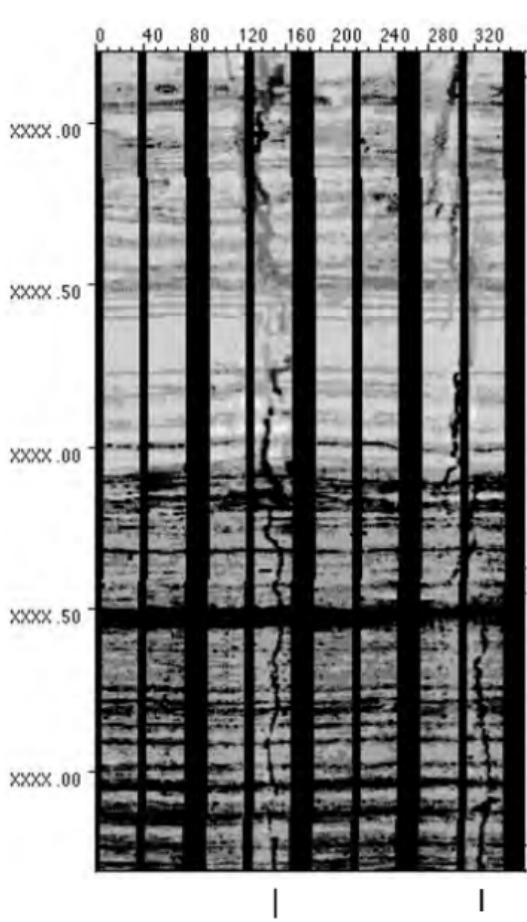
0.44 psi/ft

11.68 ppg

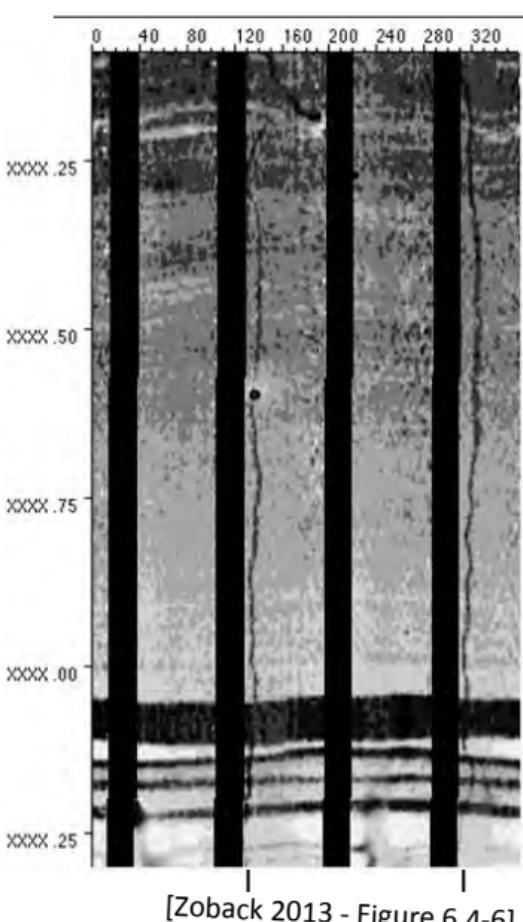
Ultrasonic P-wave

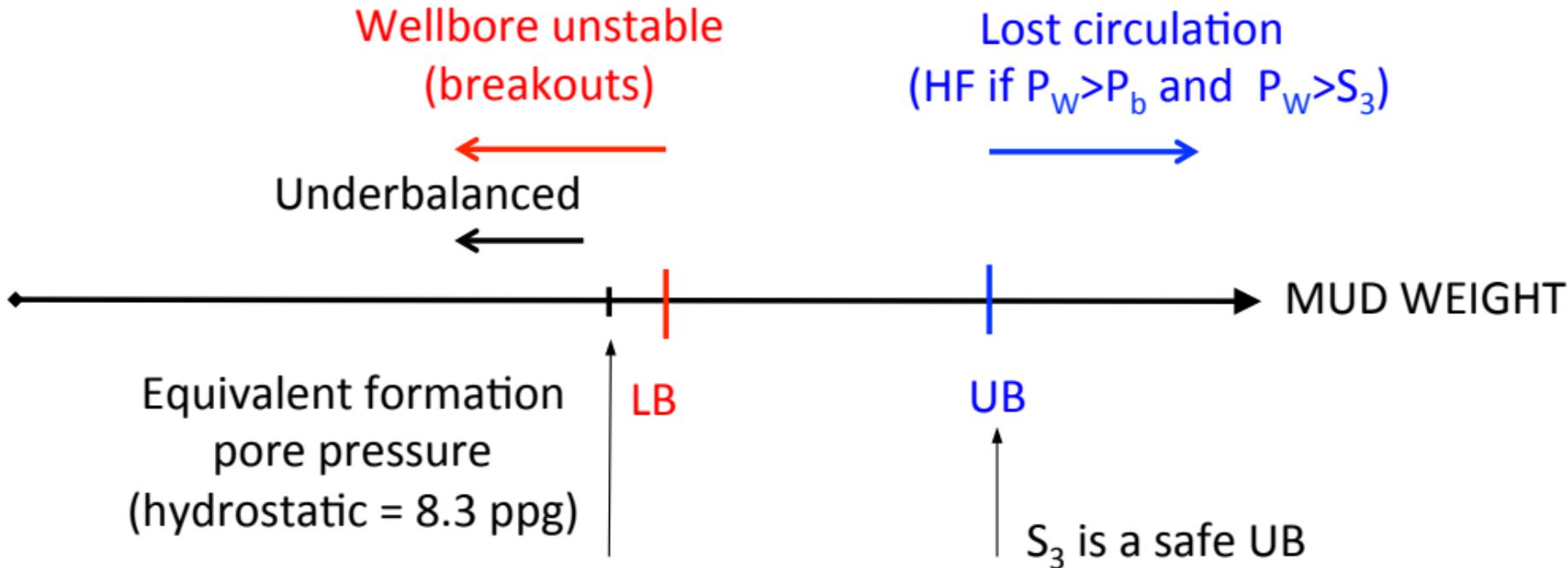


Electrical resistivity



Electrical resistivity



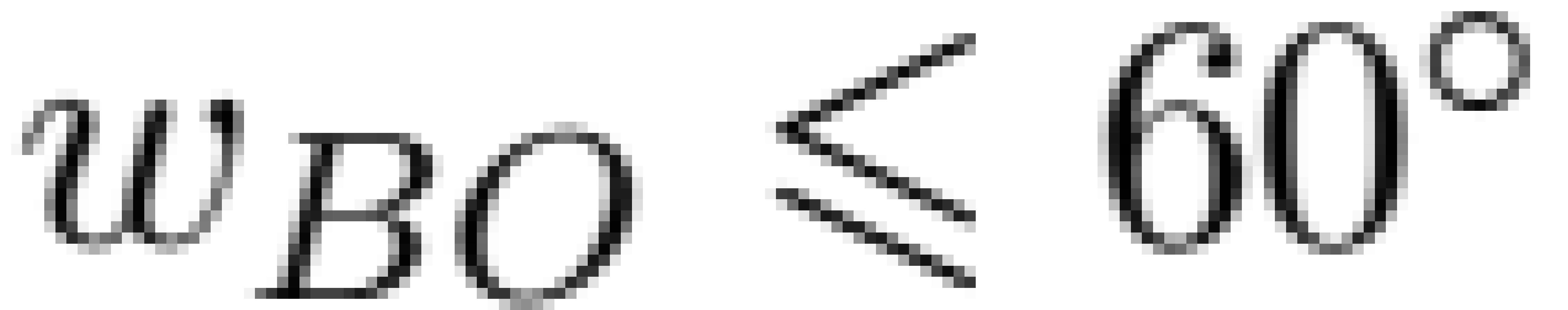


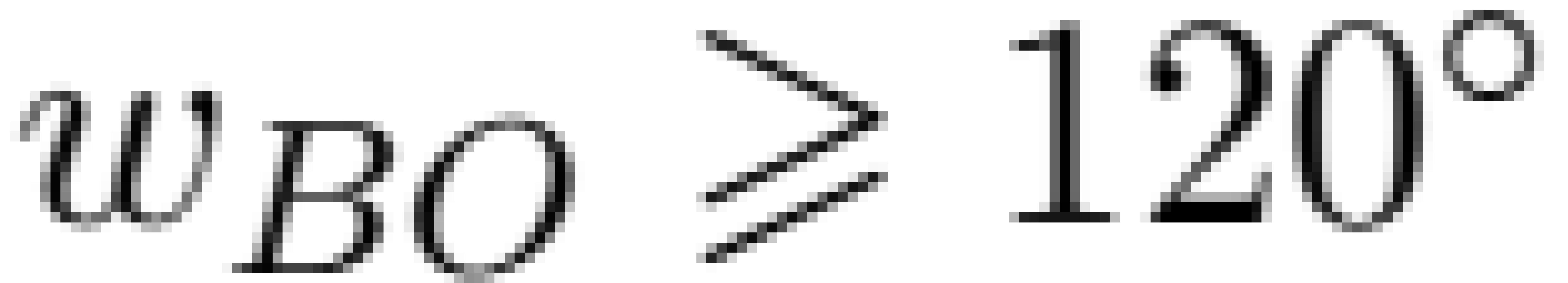
LIGHT MUDS

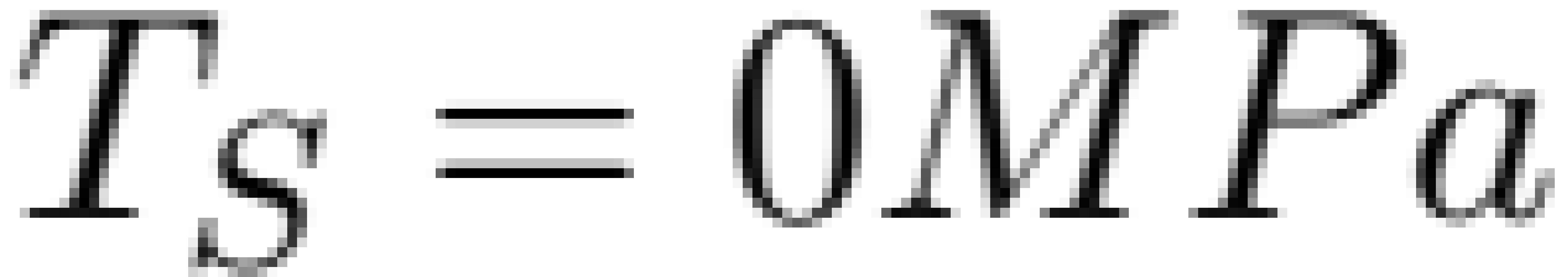
- May promote water production
- Compromise wellbore stability

HEAVY MUDS

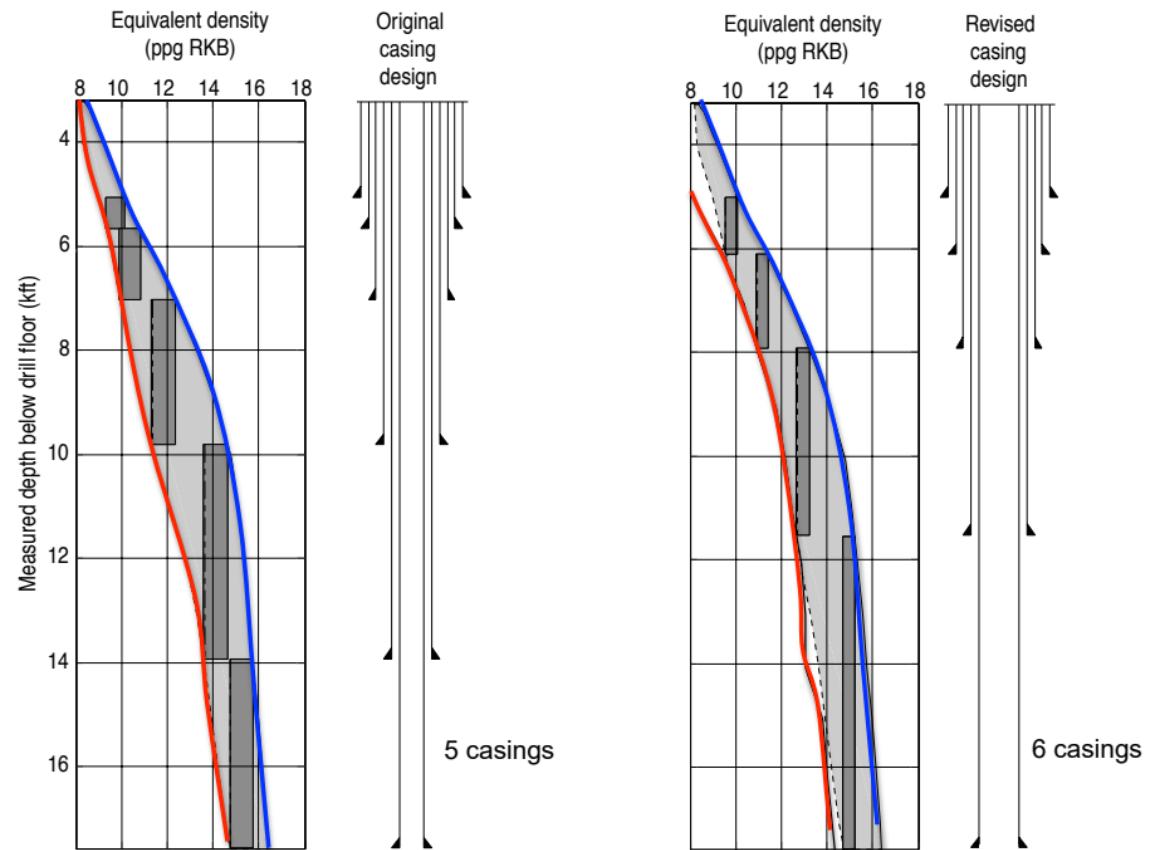
- Damage formation (k) by mud infiltration
- Promote mud losses in permeable strata
- Low ROP because of stronger rock



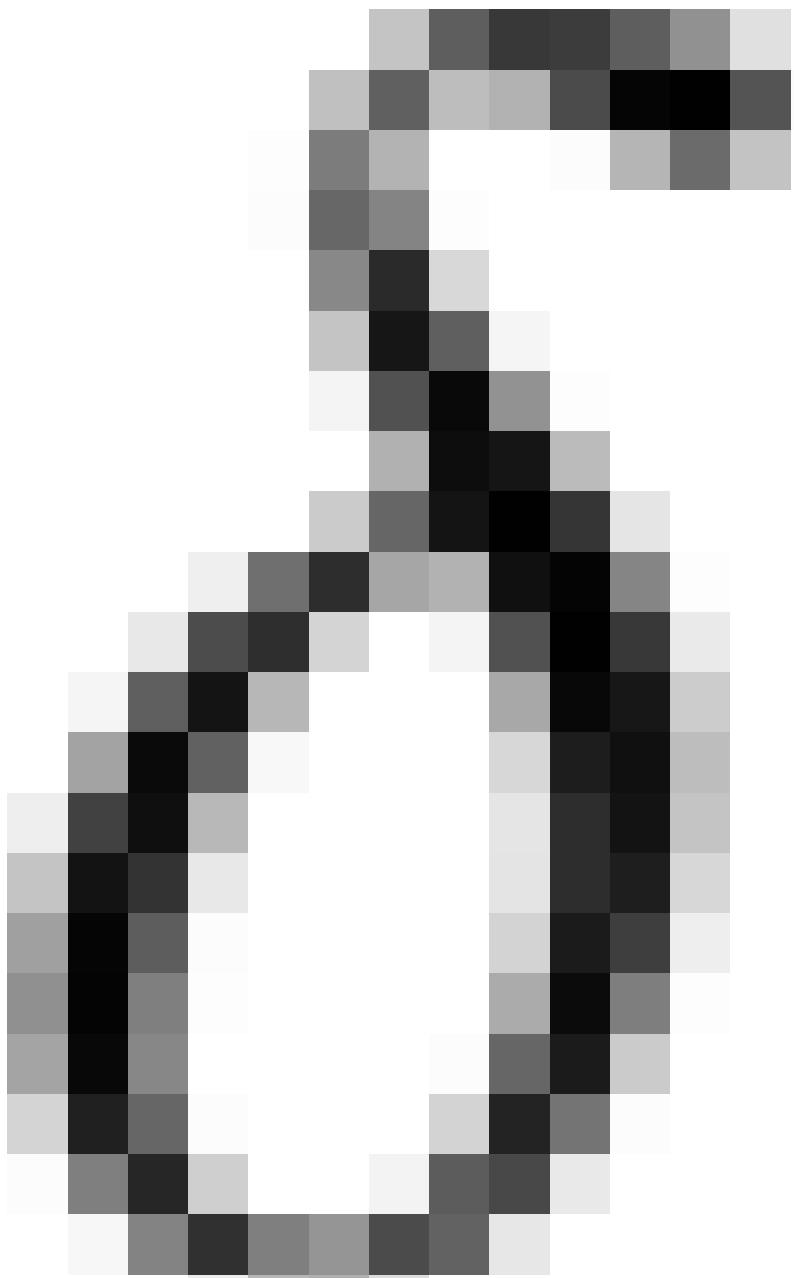


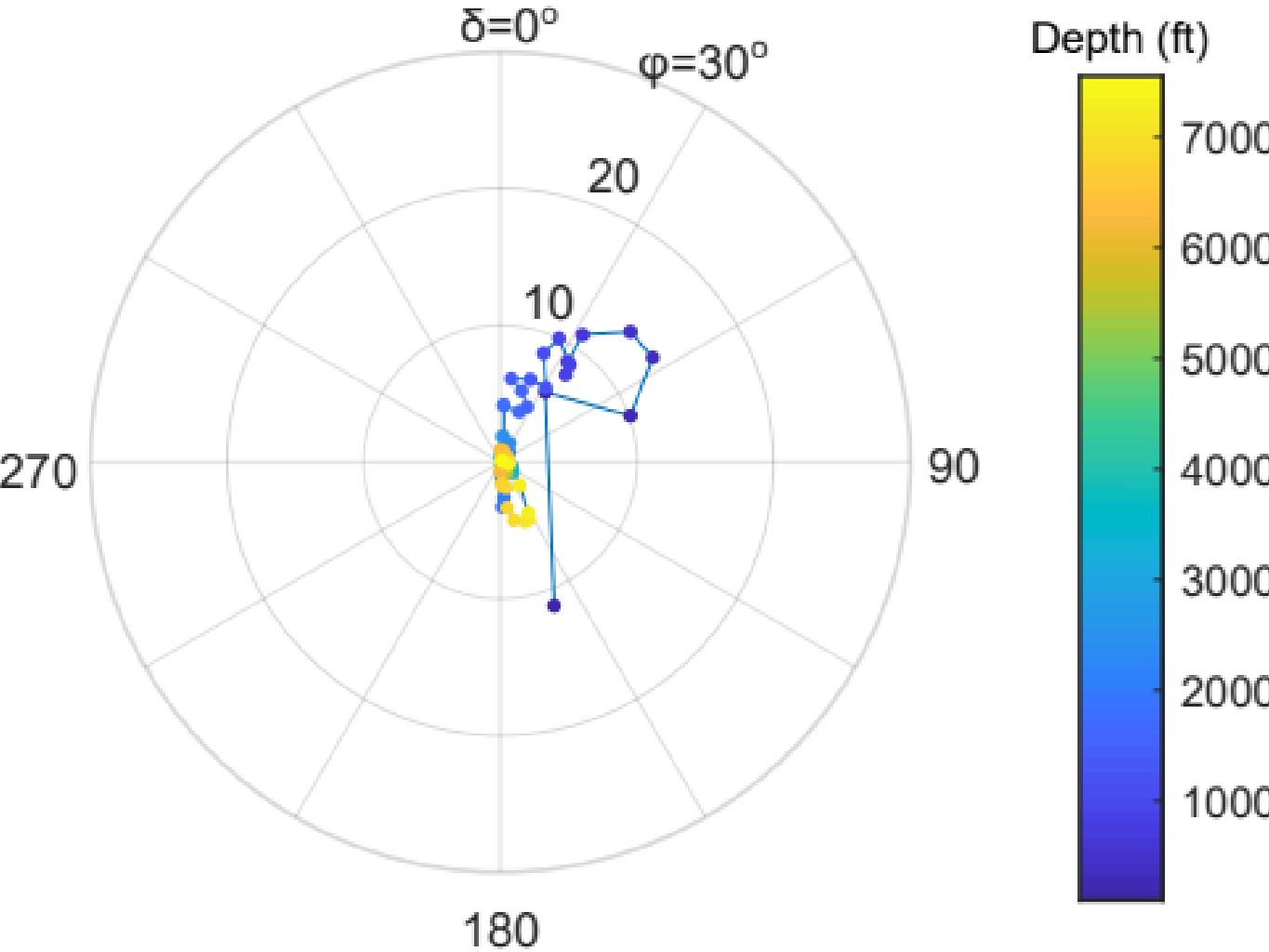
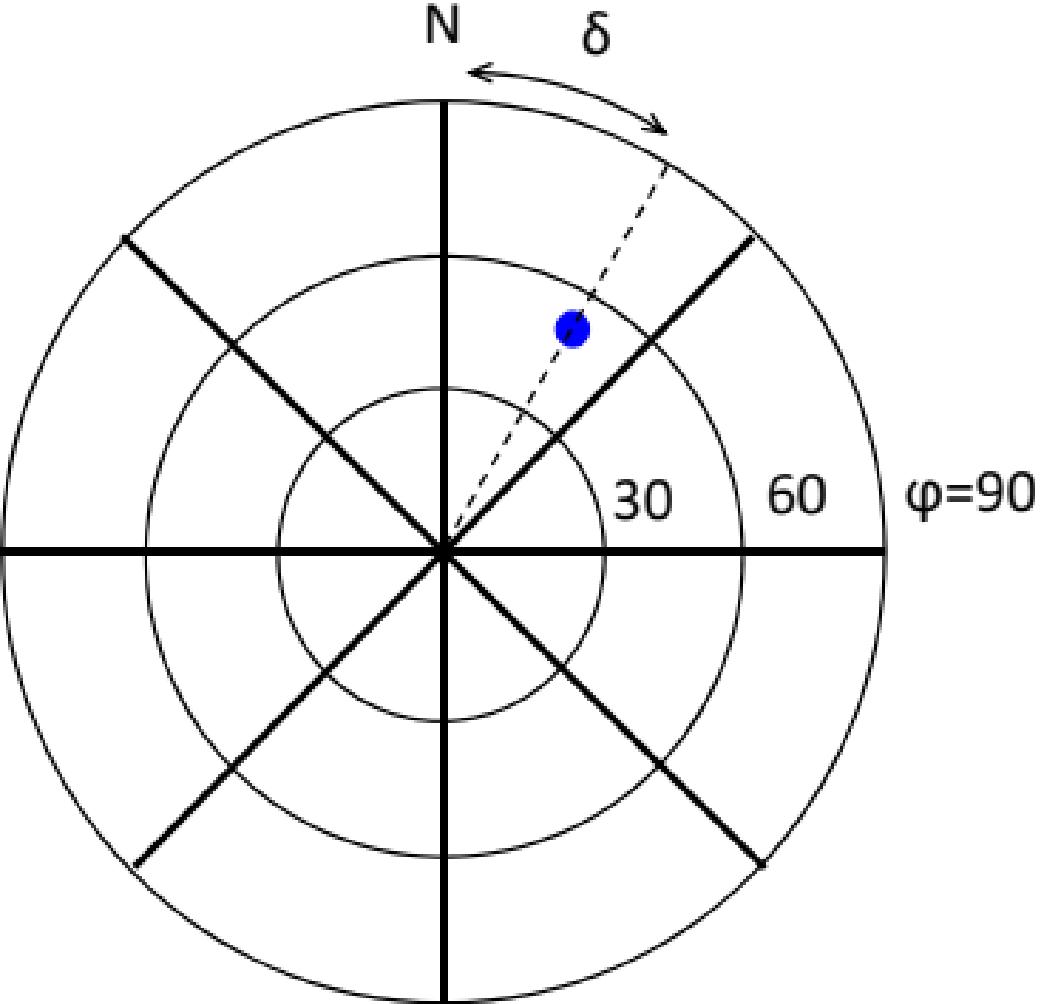
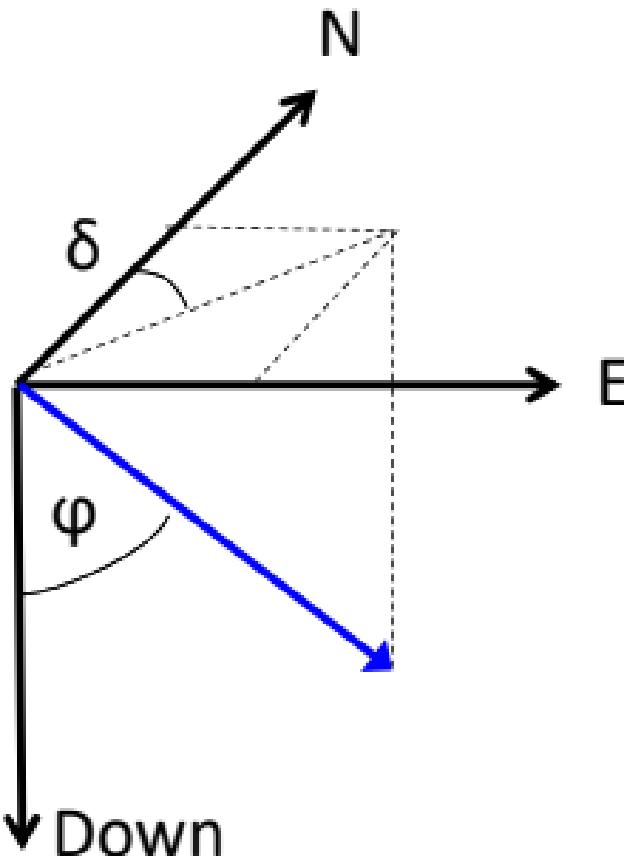


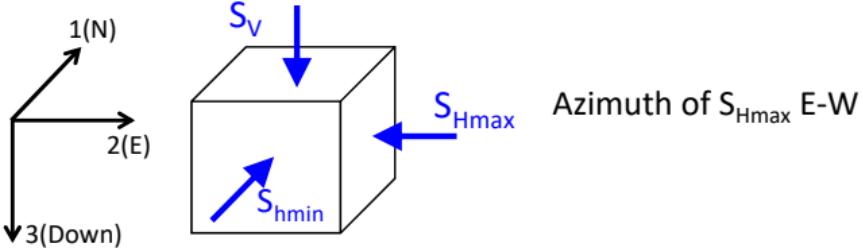




[Zoback 2013 – Figure 10.3]

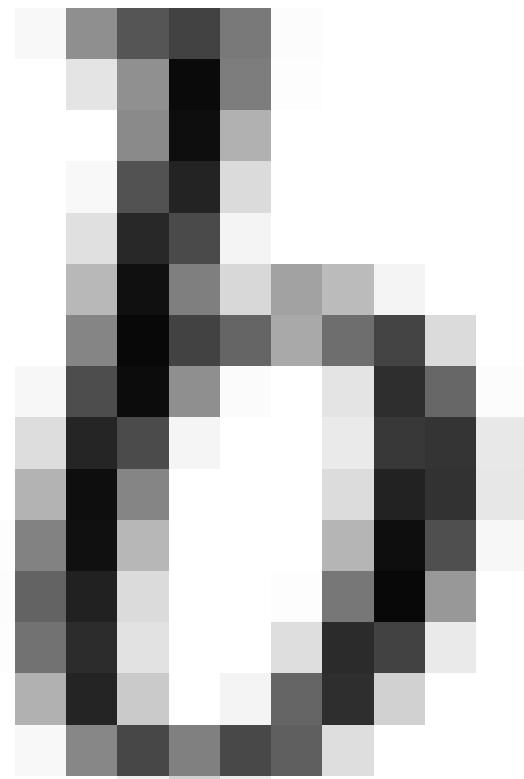
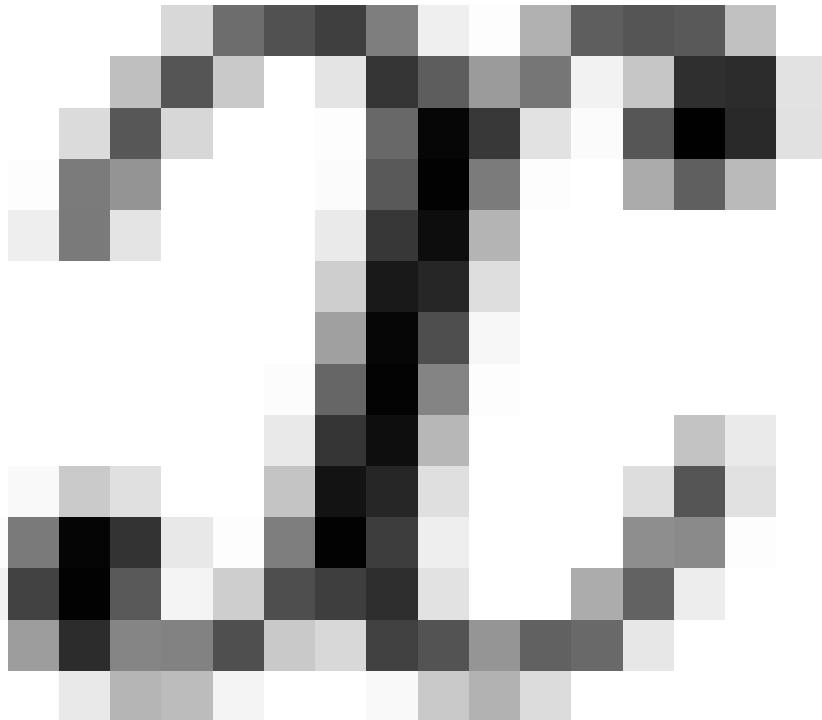


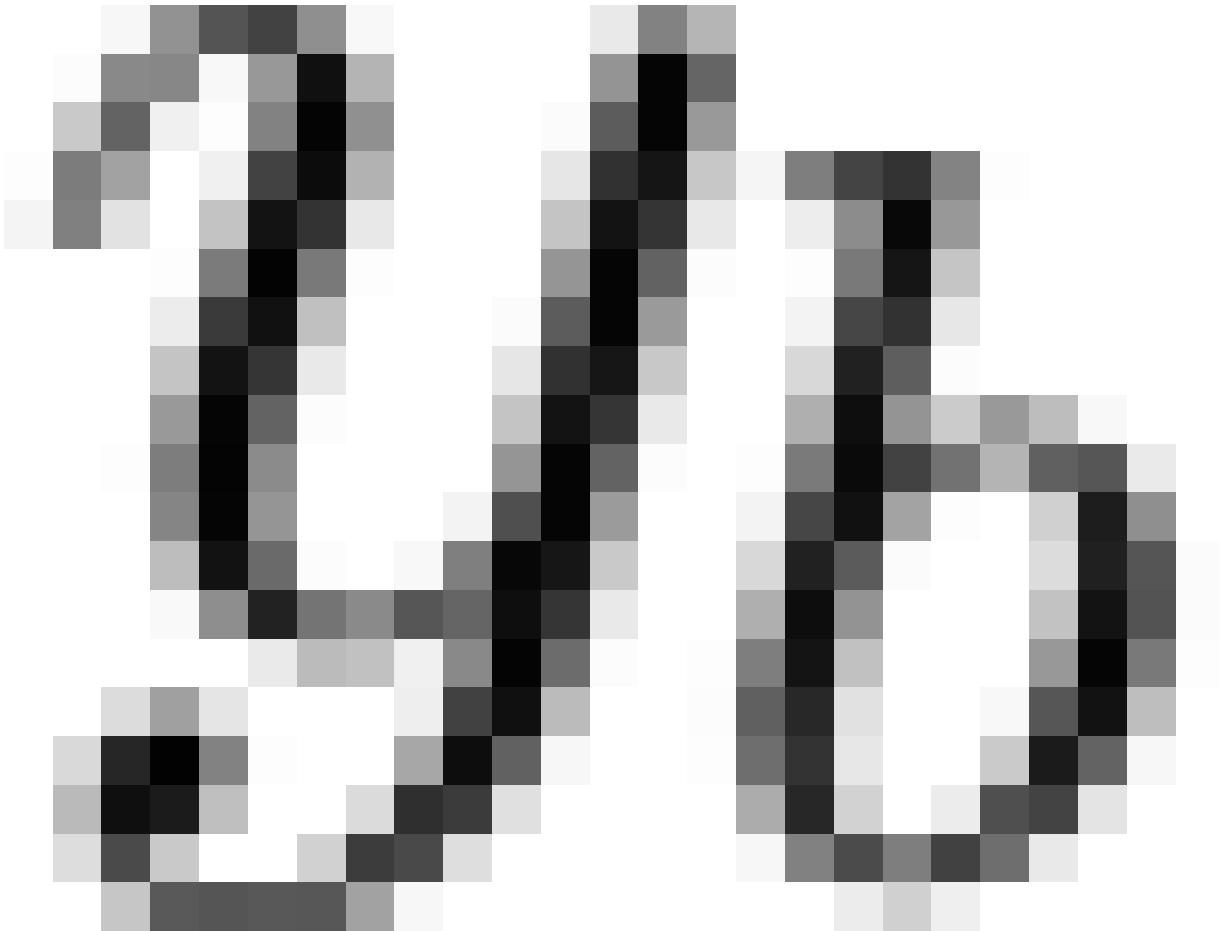


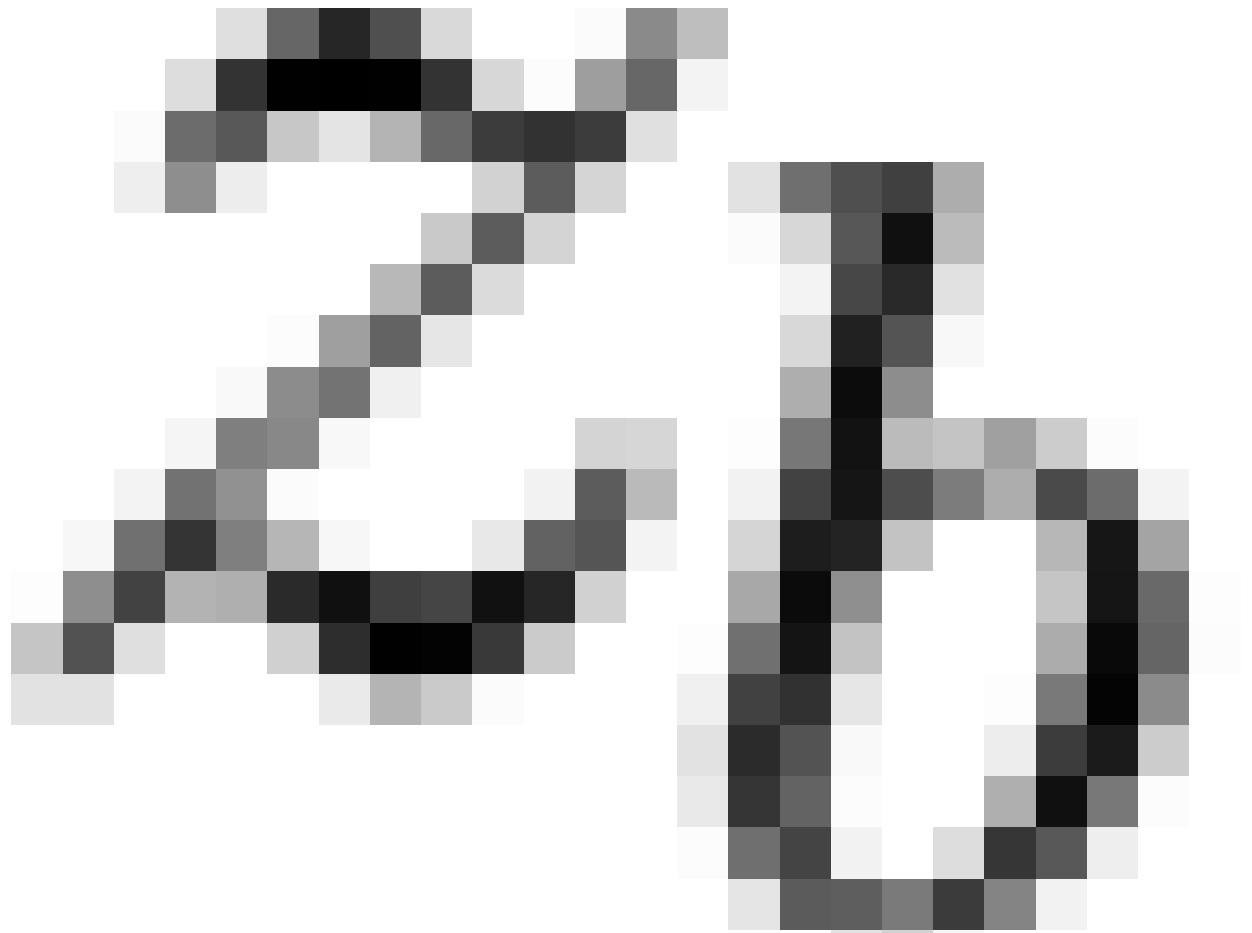


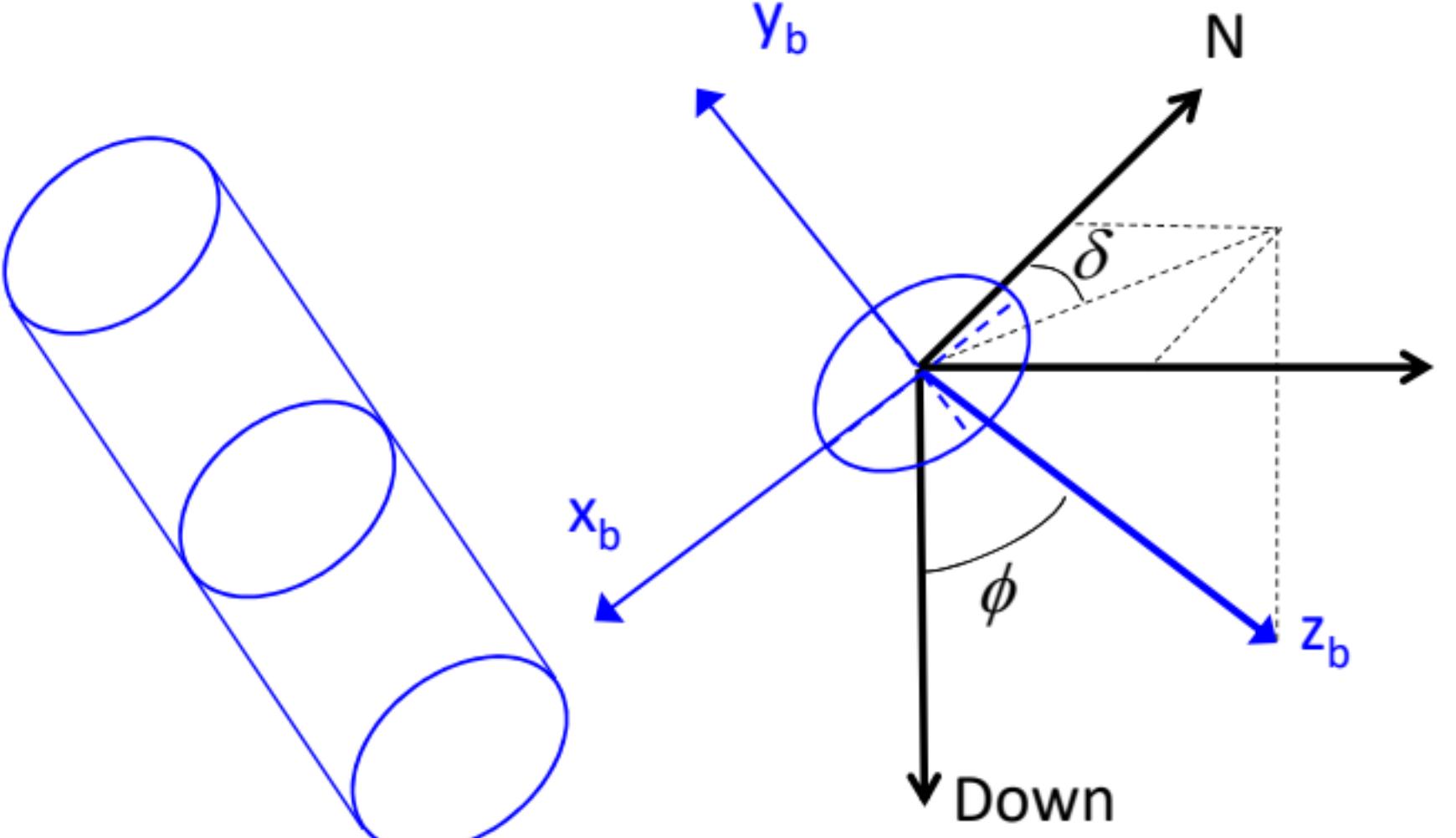
Azimuth of S_{Hmax} E-W

Stress Environment	NF	SS	RF
Plane with highest stress anisotropy	<p>A 3D diagram of a cube. A vertical blue arrow labeled S_V points downwards. A horizontal blue arrow labeled S_{Hmax} points to the right. A diagonal blue arrow labeled S_{hmin} points upwards and to the left. A red dot is located in the bottom-right corner of the front face of the cube.</p>	<p>A 3D diagram of a cube. A vertical blue arrow labeled S_V points downwards. A horizontal blue arrow labeled S_{Hmax} points to the right. A diagonal blue arrow labeled S_{hmin} points upwards and to the left. A red dot is located in the top-center of the front face of the cube.</p>	<p>A 3D diagram of a cube. A vertical blue arrow labeled S_V points downwards. A horizontal blue arrow labeled S_{Hmax} points to the right. A diagonal blue arrow labeled S_{hmin} points upwards and to the left. A red dot is located in the bottom-center of the front face of the cube.</p>
Narrower drilling window and Breakouts orientation	<p>A circle representing a cross-section. A red dot is located on the right side of the circle's circumference.</p>	<p>A circle representing a cross-section. A red dot is located in the center of the circle.</p>	<p>A circle representing a cross-section. A red dot is located on the bottom side of the circle's circumference.</p>

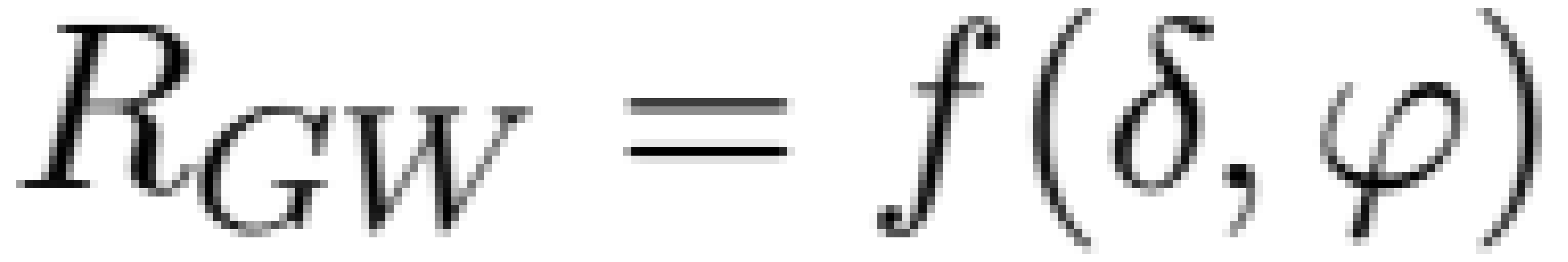








$$R_{GW} = \begin{bmatrix} -\cos\delta\cos\phi & -\sin\delta\cos\phi & \sin\phi \\ \sin\delta & -\cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix}$$











$$\underline{S}_W =$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$





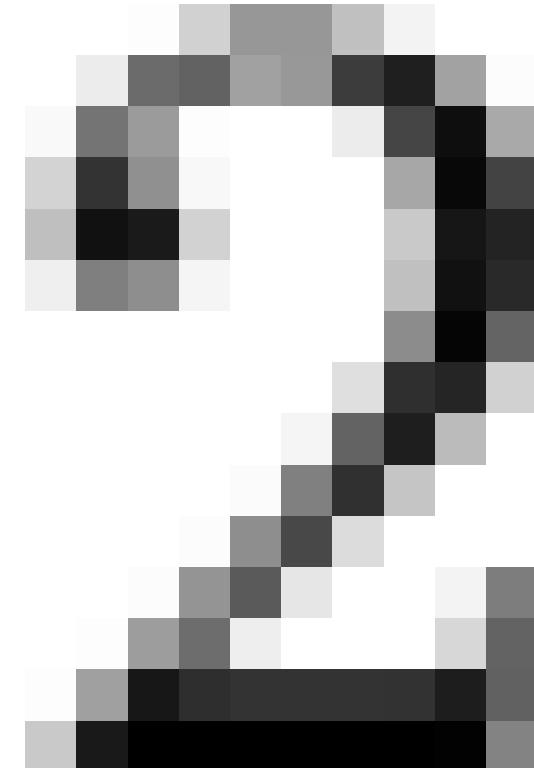
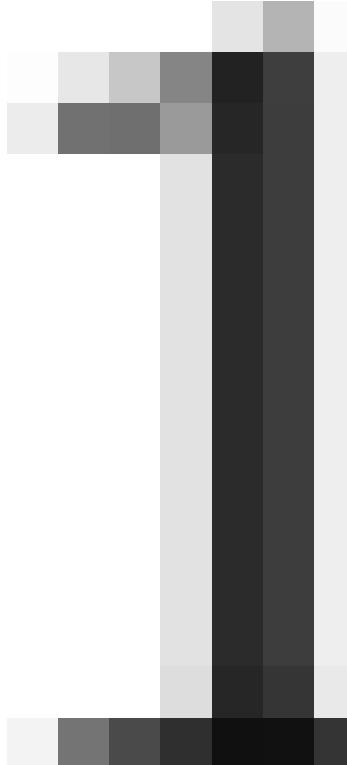
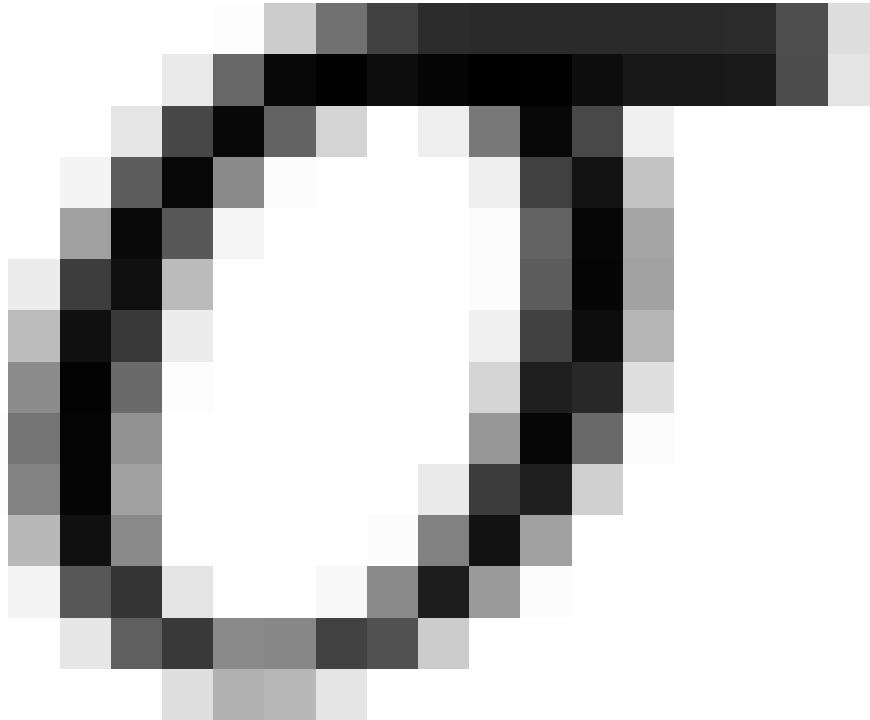


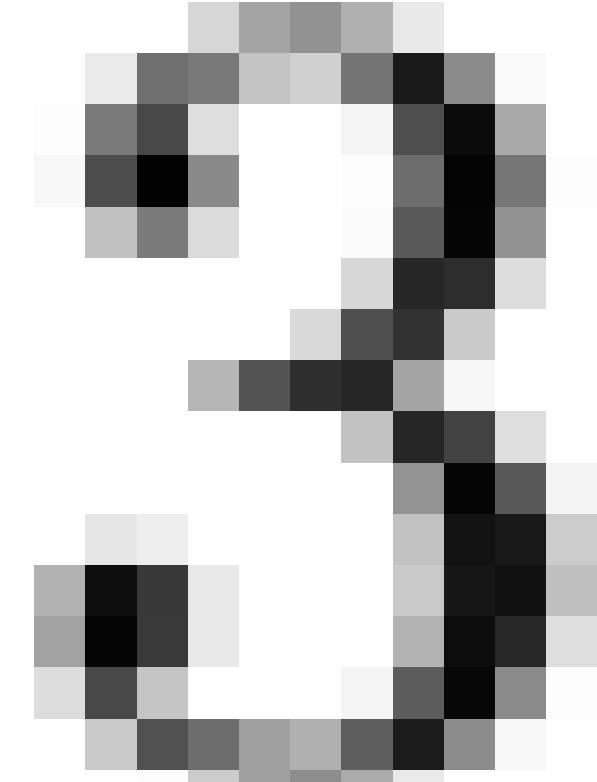
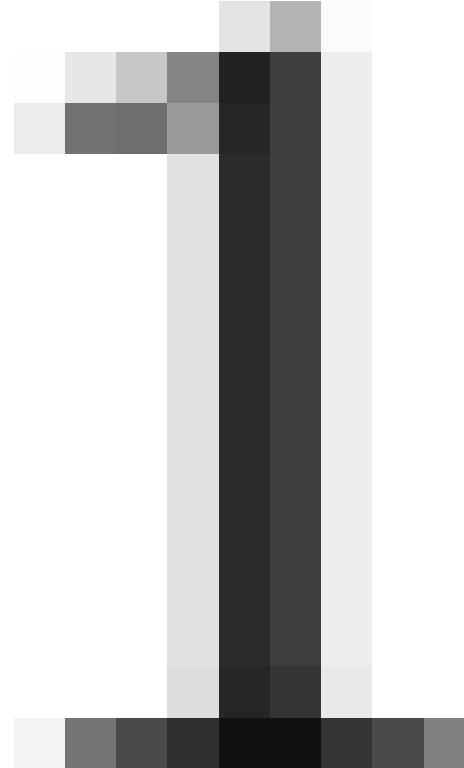
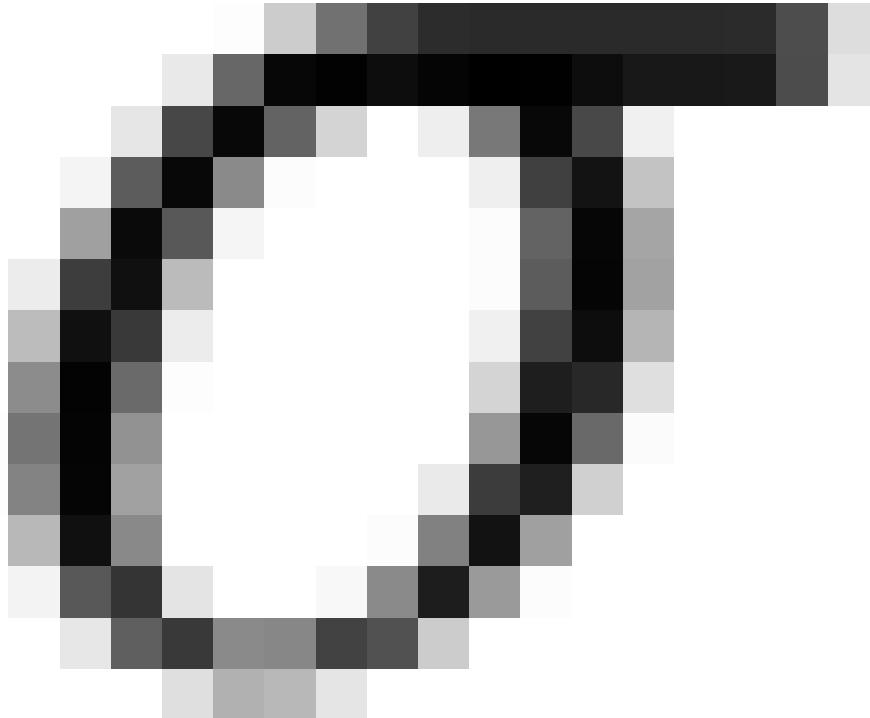


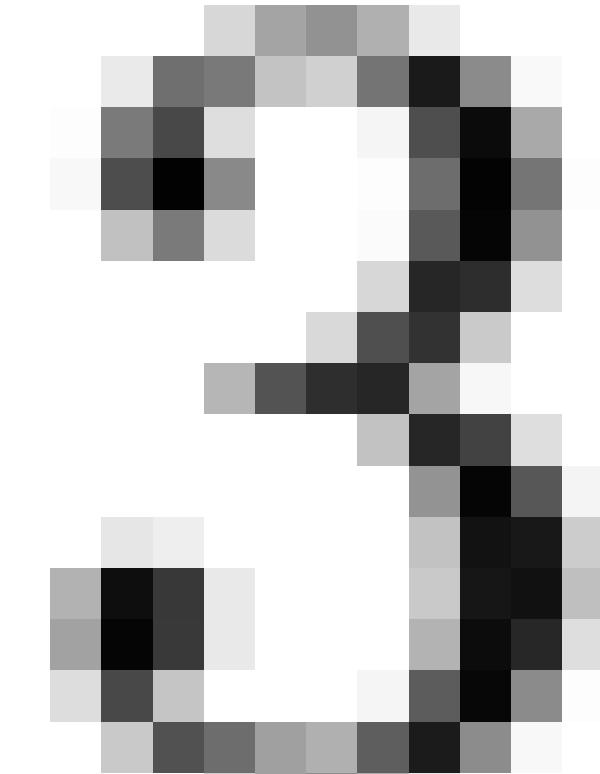
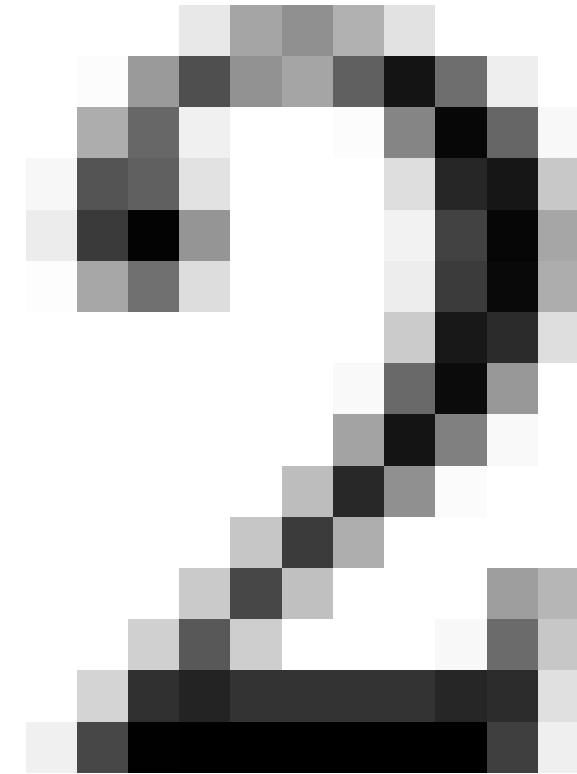
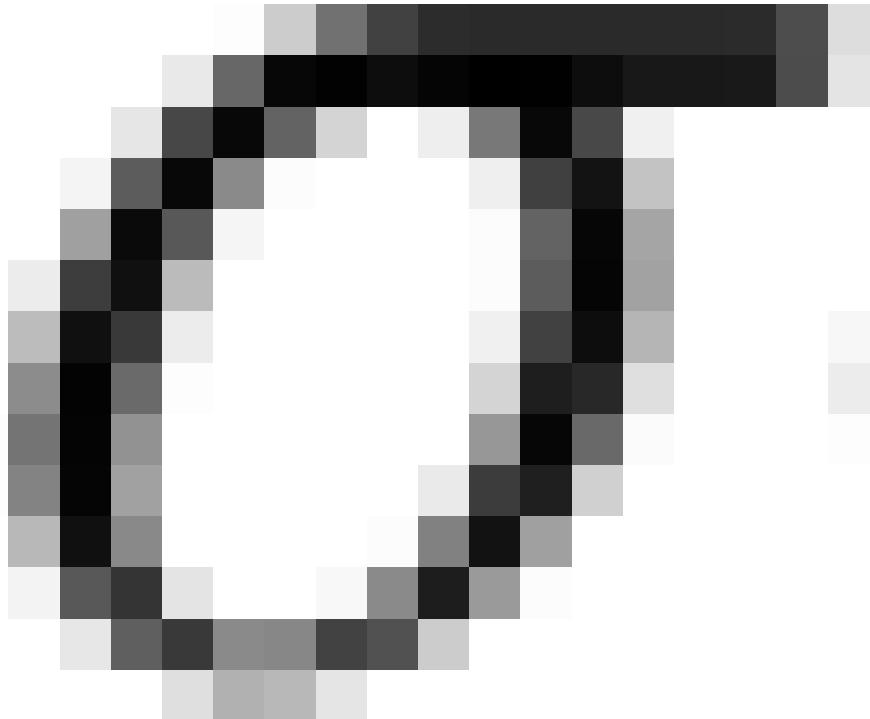


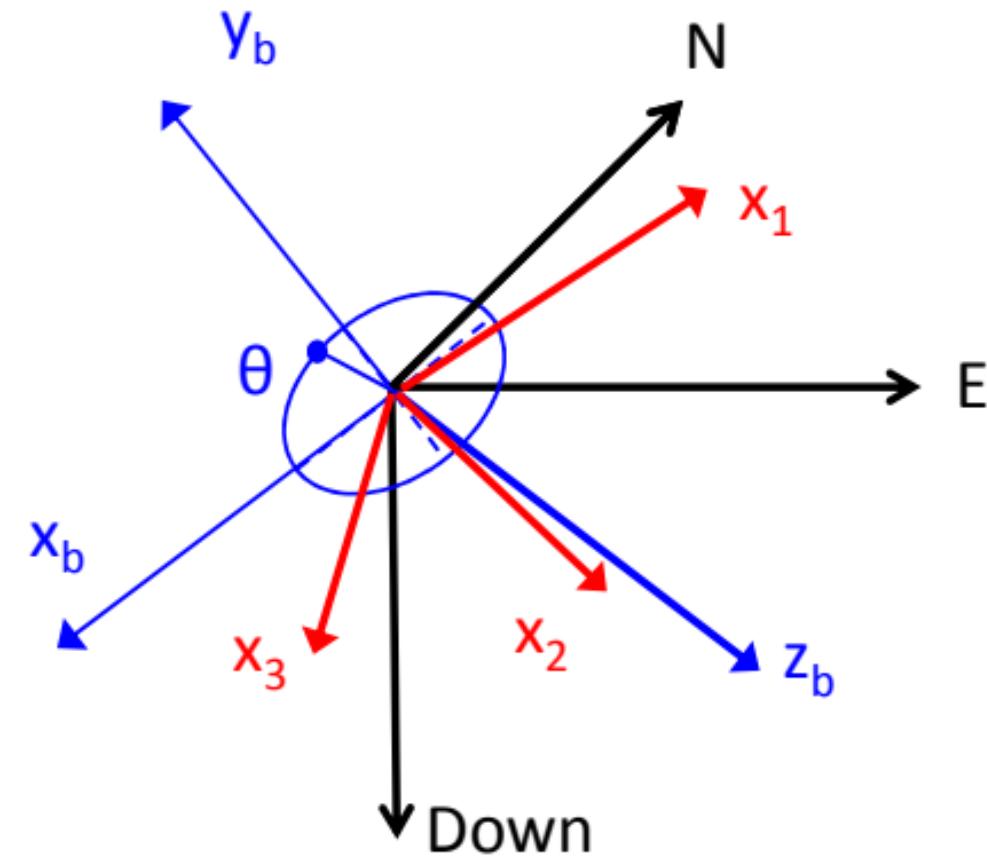








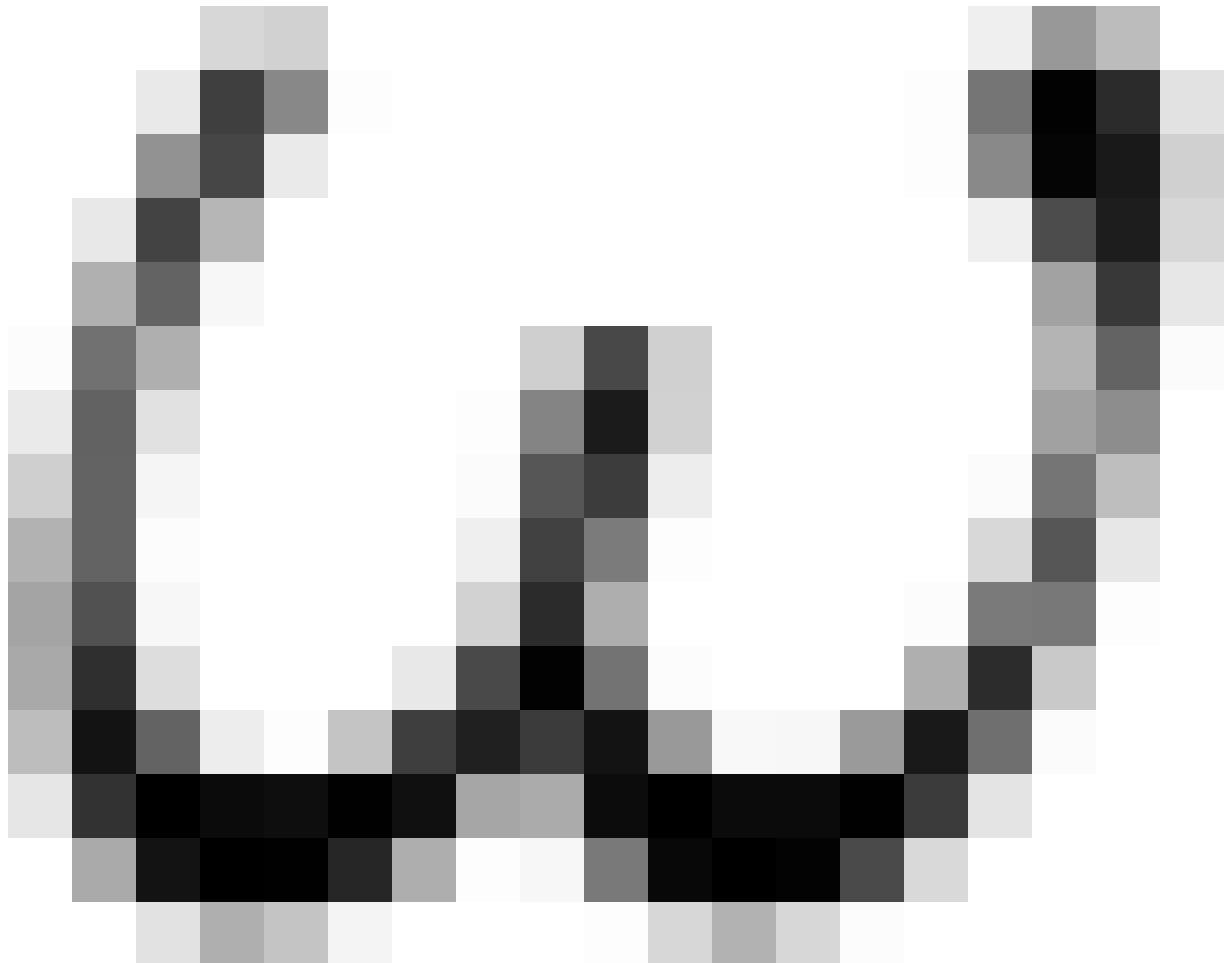


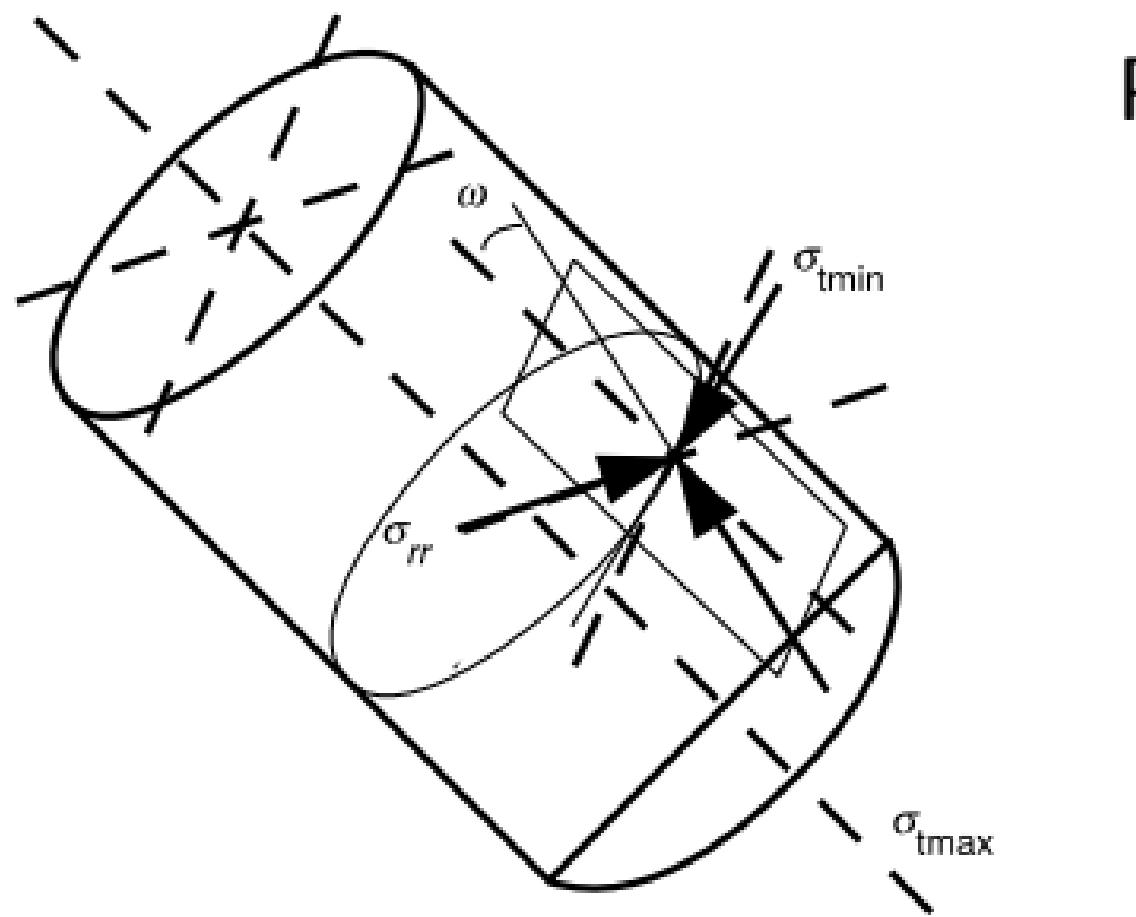


Stresses at the wellbore wall (Kirsch[PP]+Kirsch[S])

$$\begin{cases} \sigma_{rr} = \Delta P \\ \sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\sigma_{12}\sin 2\theta - \Delta P \\ \tau_{\theta z} = 2(\sigma_{23}\cos\theta - \sigma_{13}\sin\theta) \\ \sigma_{zz} = \sigma_{33} - 2\nu(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\nu\sigma_{12}\sin 2\theta \end{cases}$$

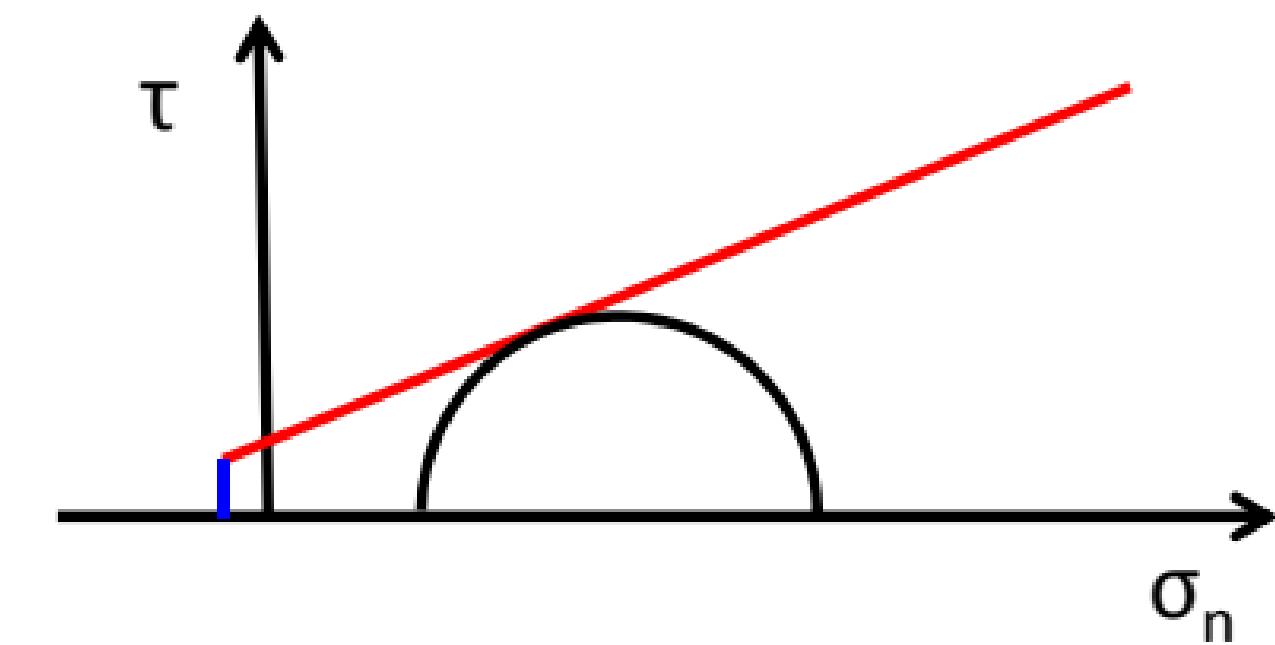




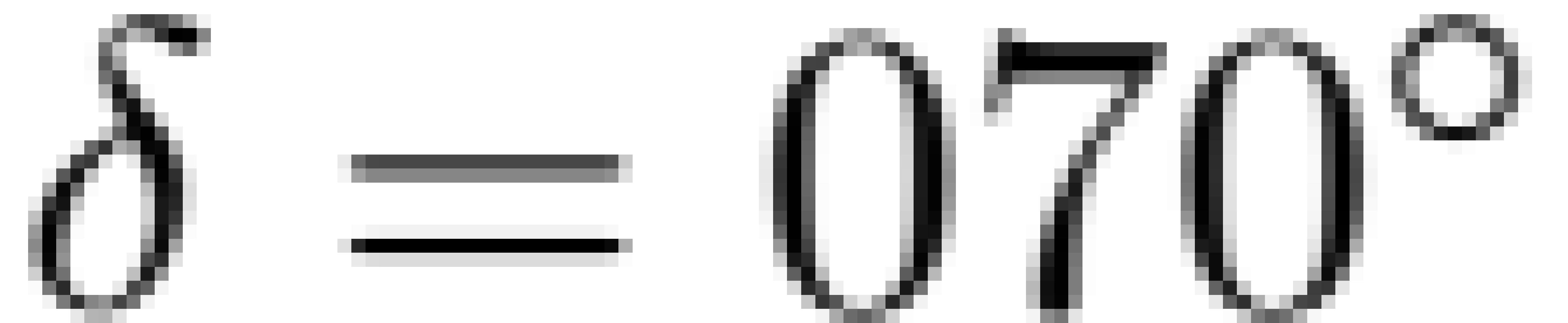


Principal stresses at the wellbore wall

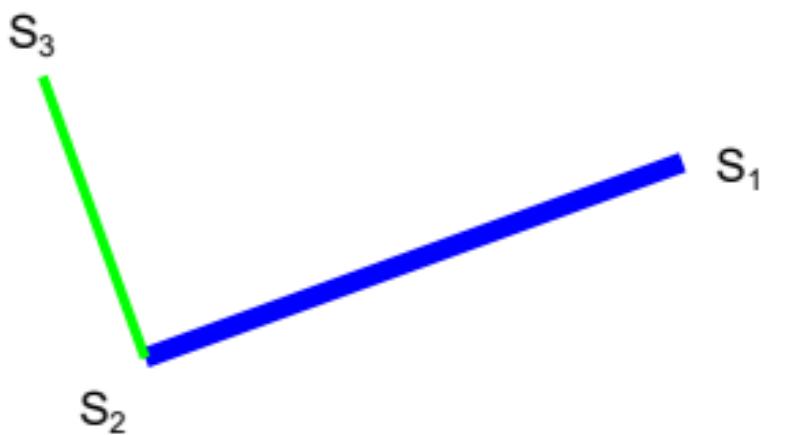
$$\left\{ \begin{array}{l} \sigma_{t\max} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} + \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \\ \sigma_{t\min} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} - \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \end{array} \right.$$



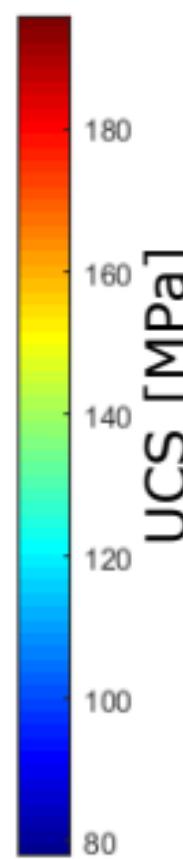
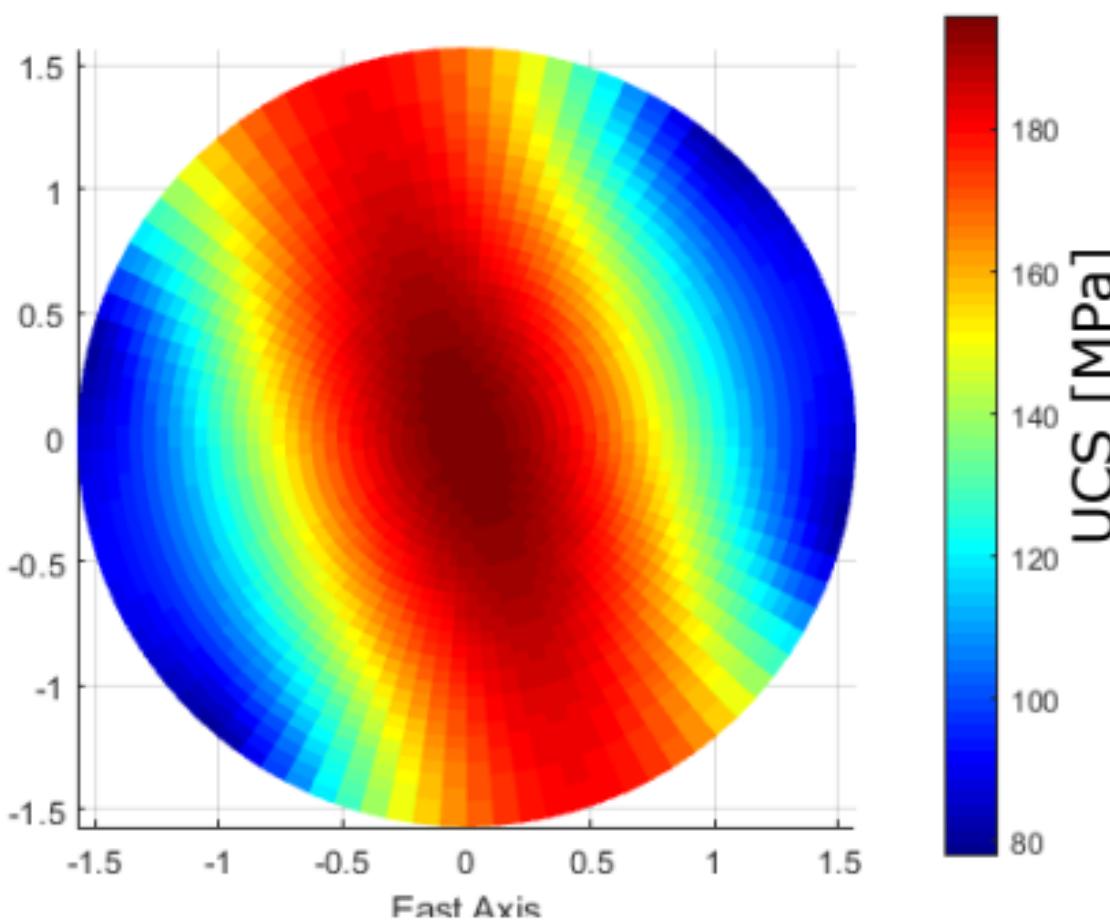




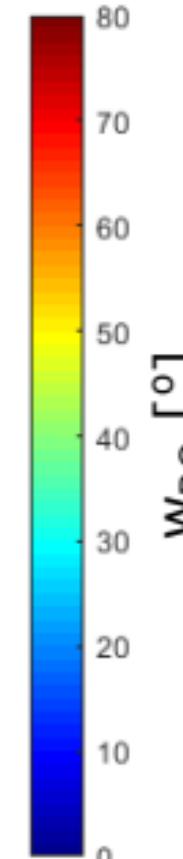
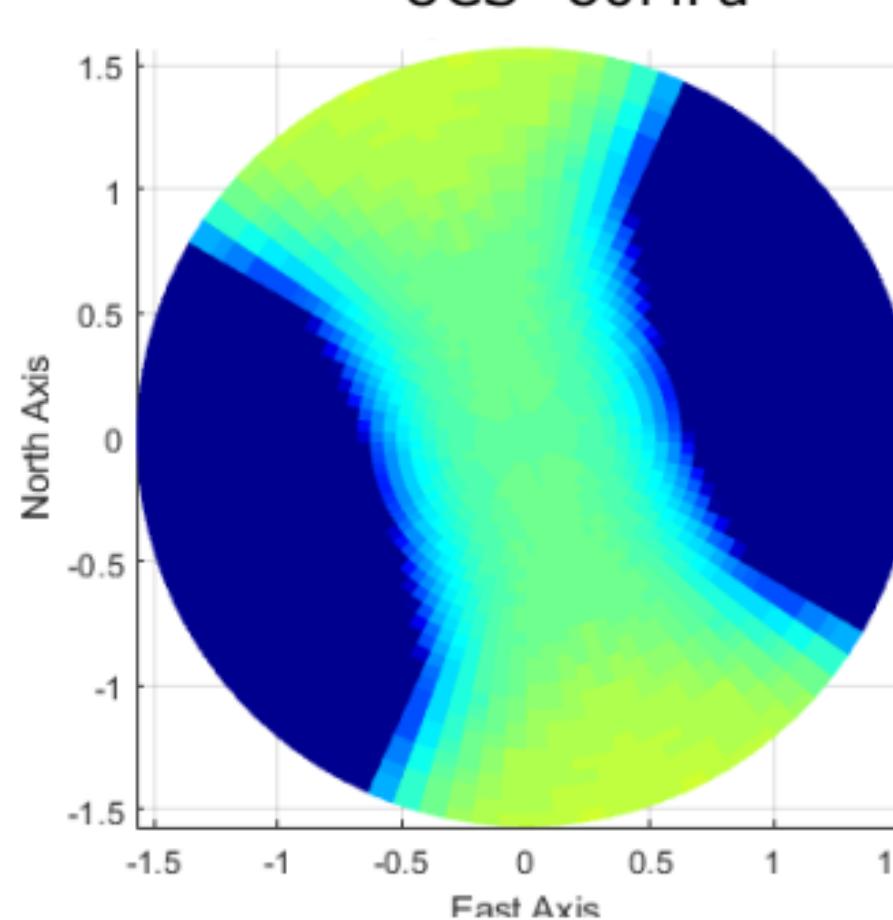
Geographical principal stresses

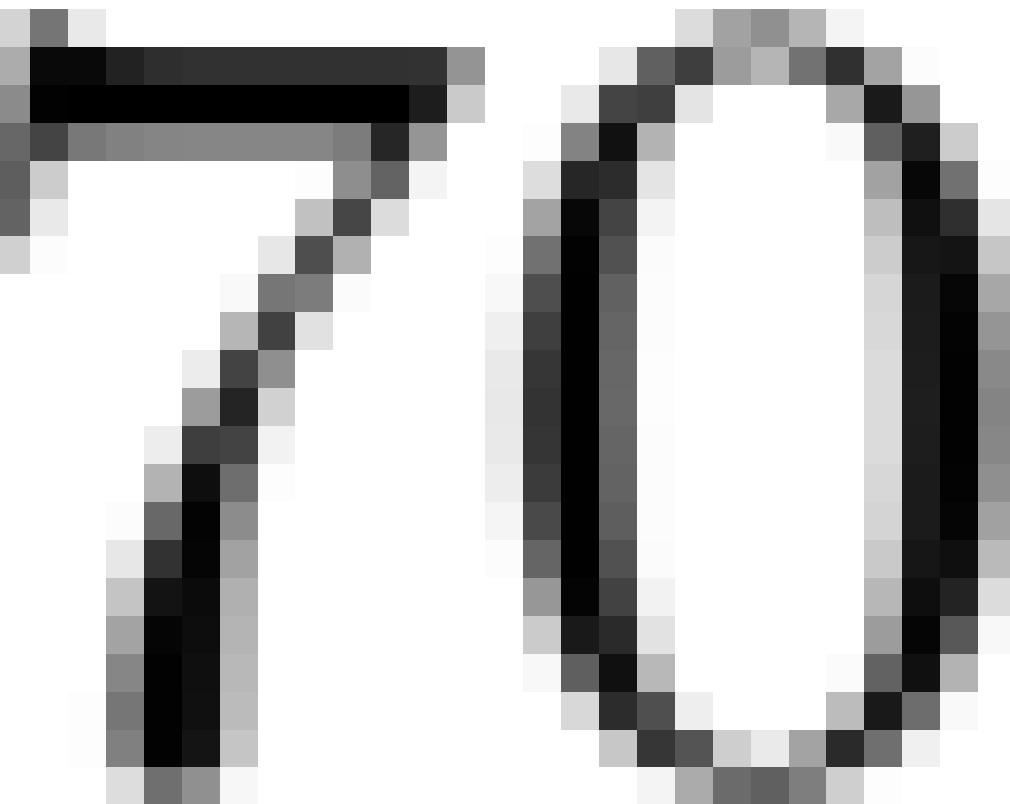


Required UCS ($P_w = P_p$)



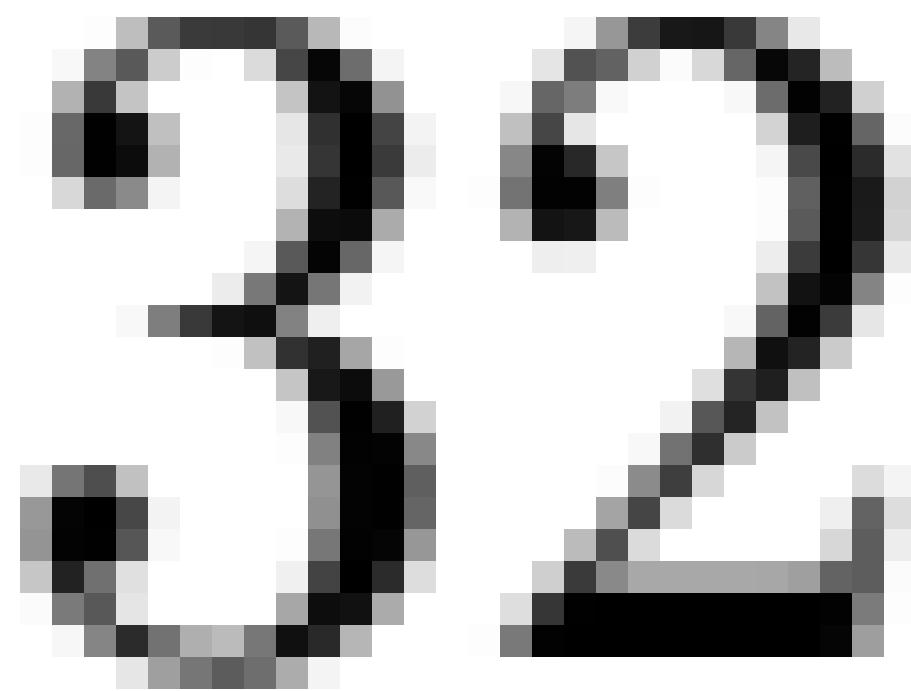
Breakout angle by Lade - $P_w = 45\text{ MPa}$ UCS=80MPa

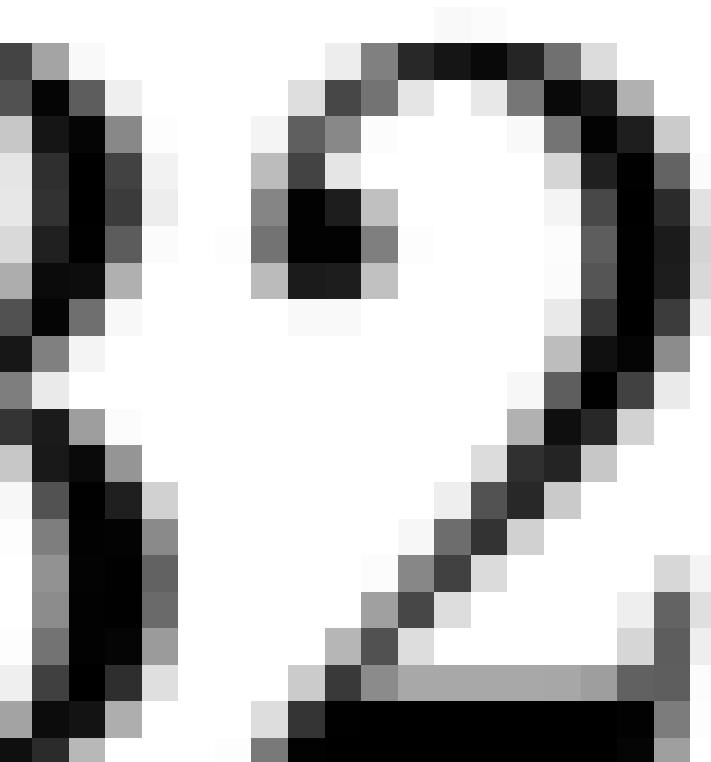
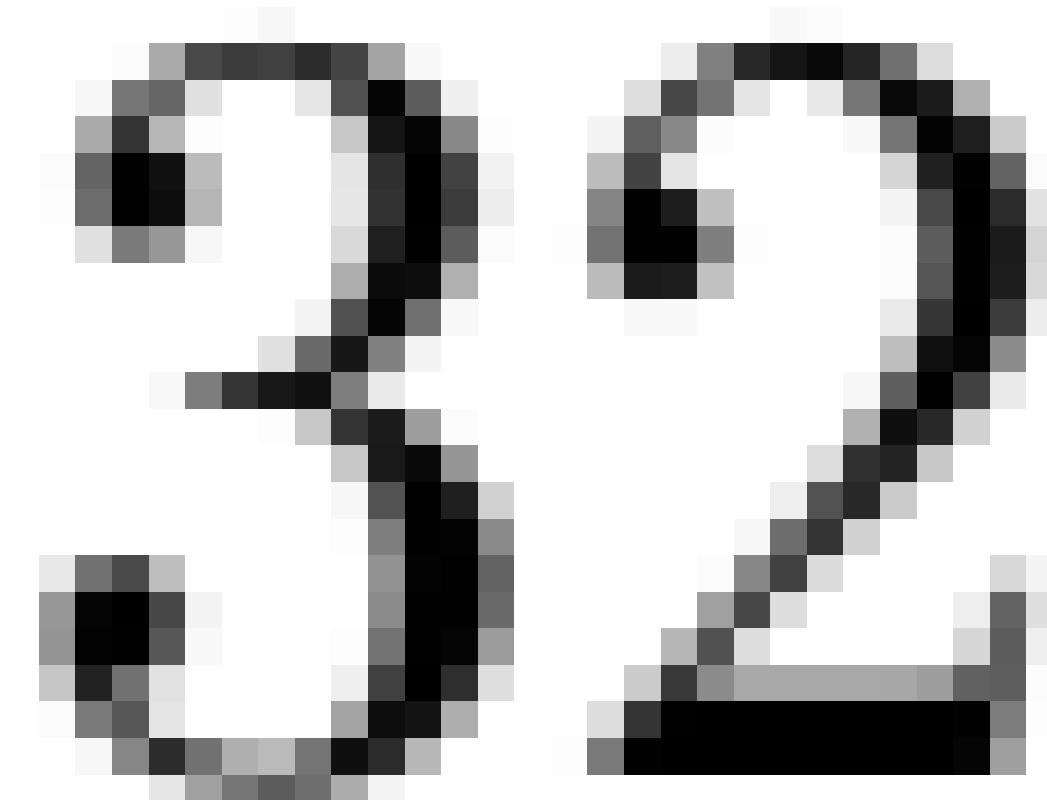
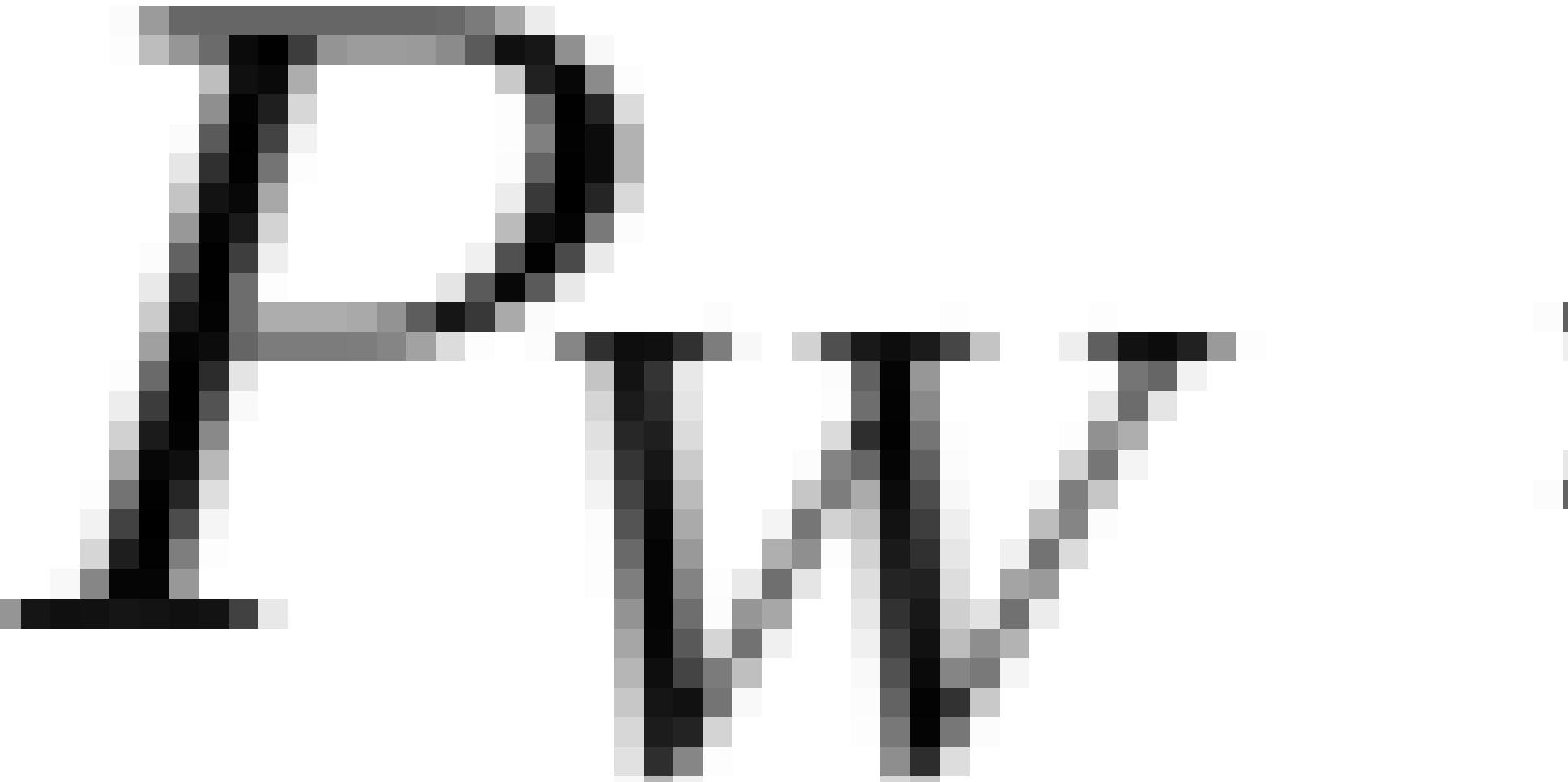


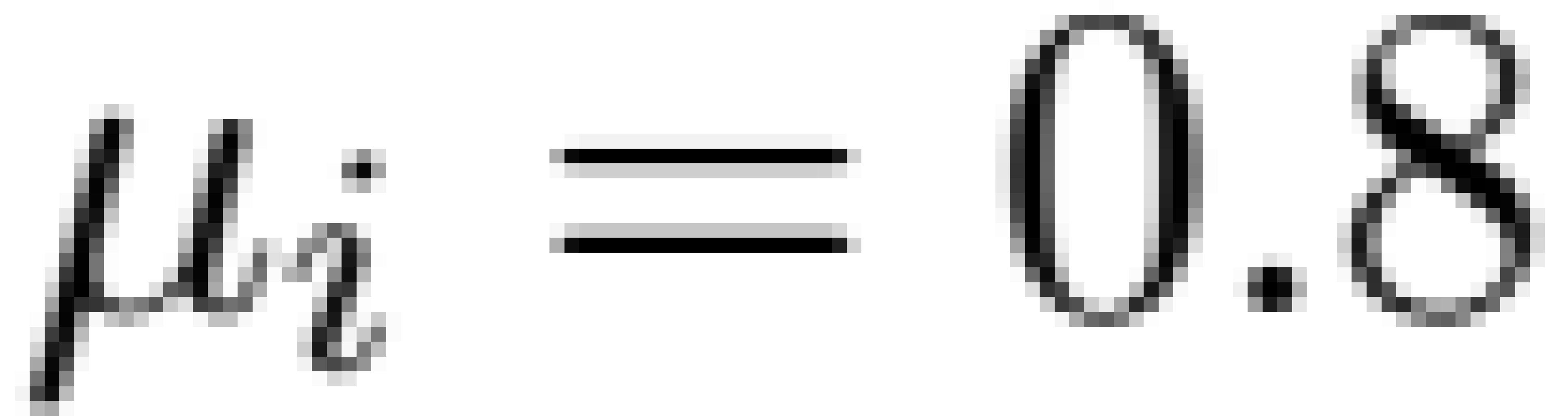




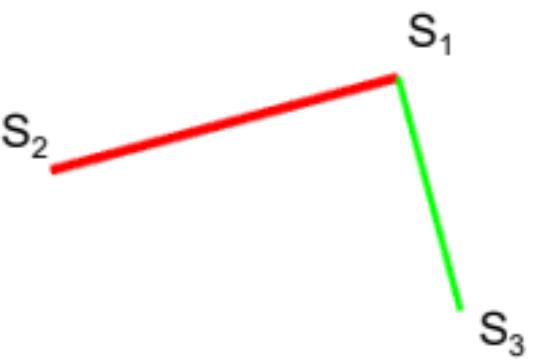




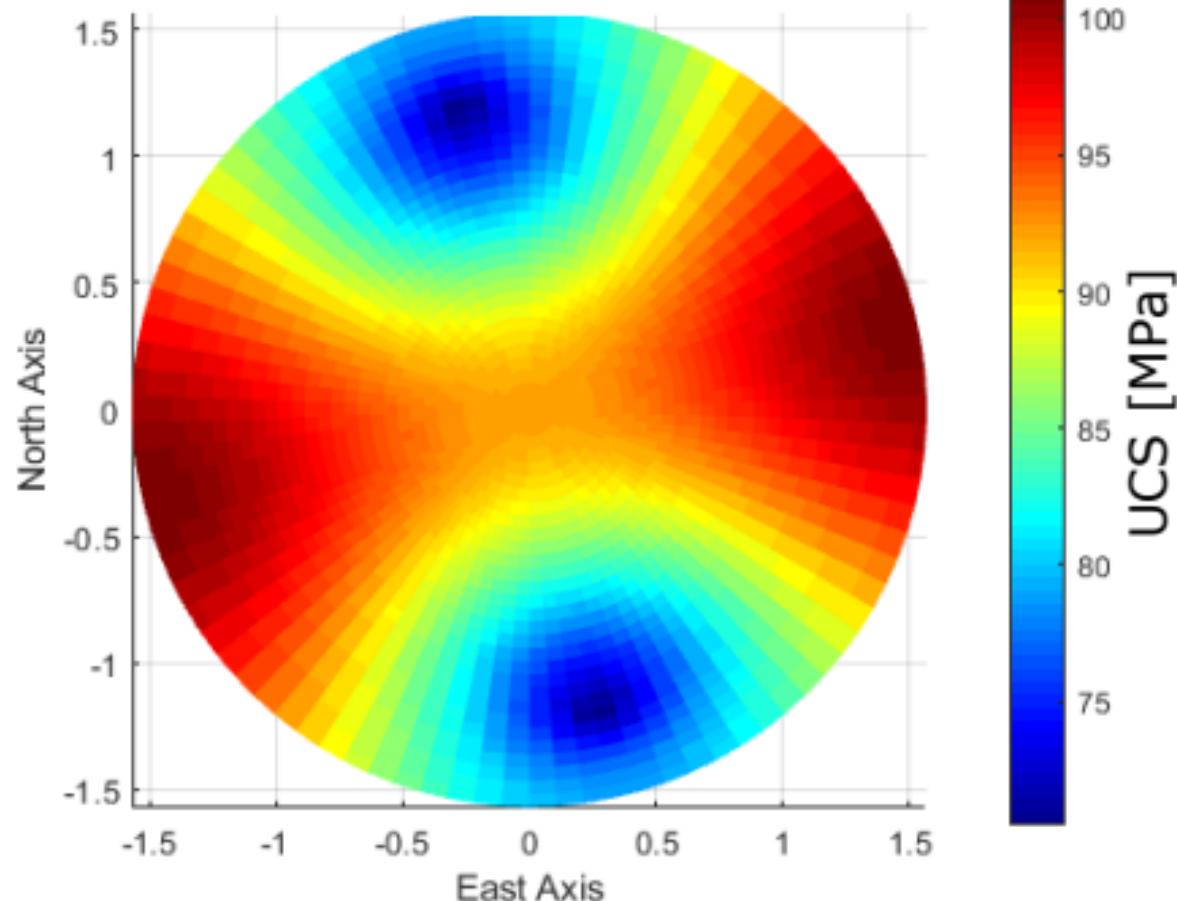




Principal stresses



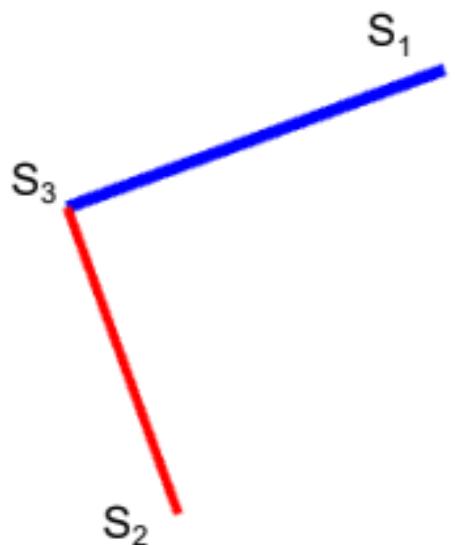
Required UCS ($P_w = P_p$)



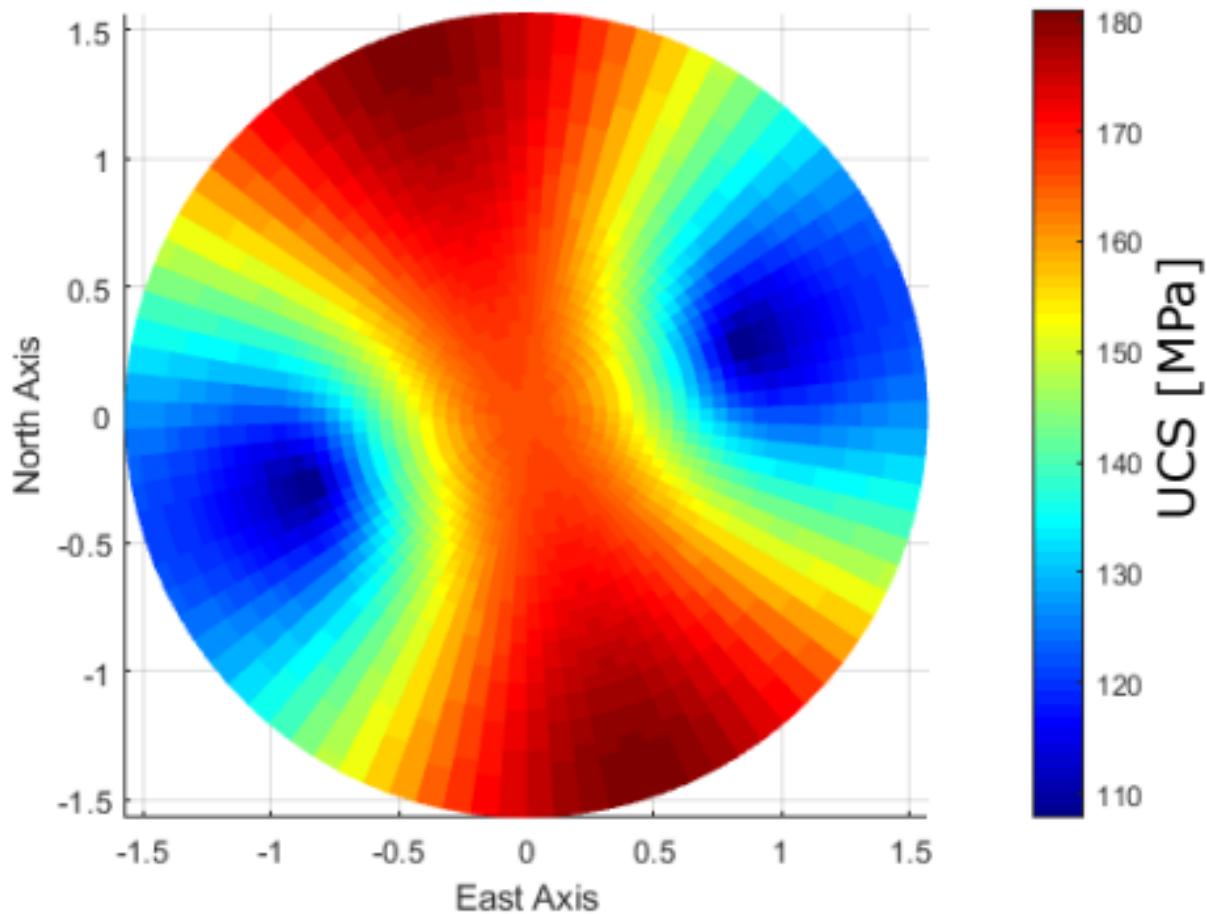


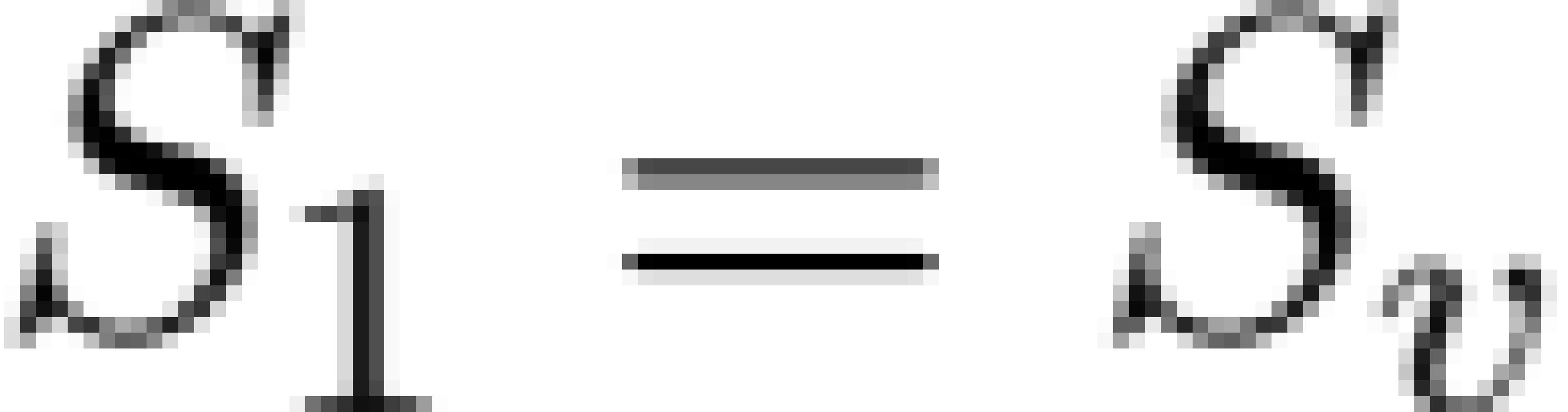


Principal stresses

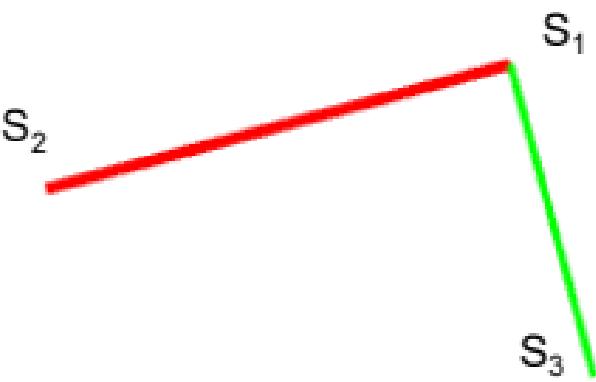


Required UCS ($P_w = P_p$)

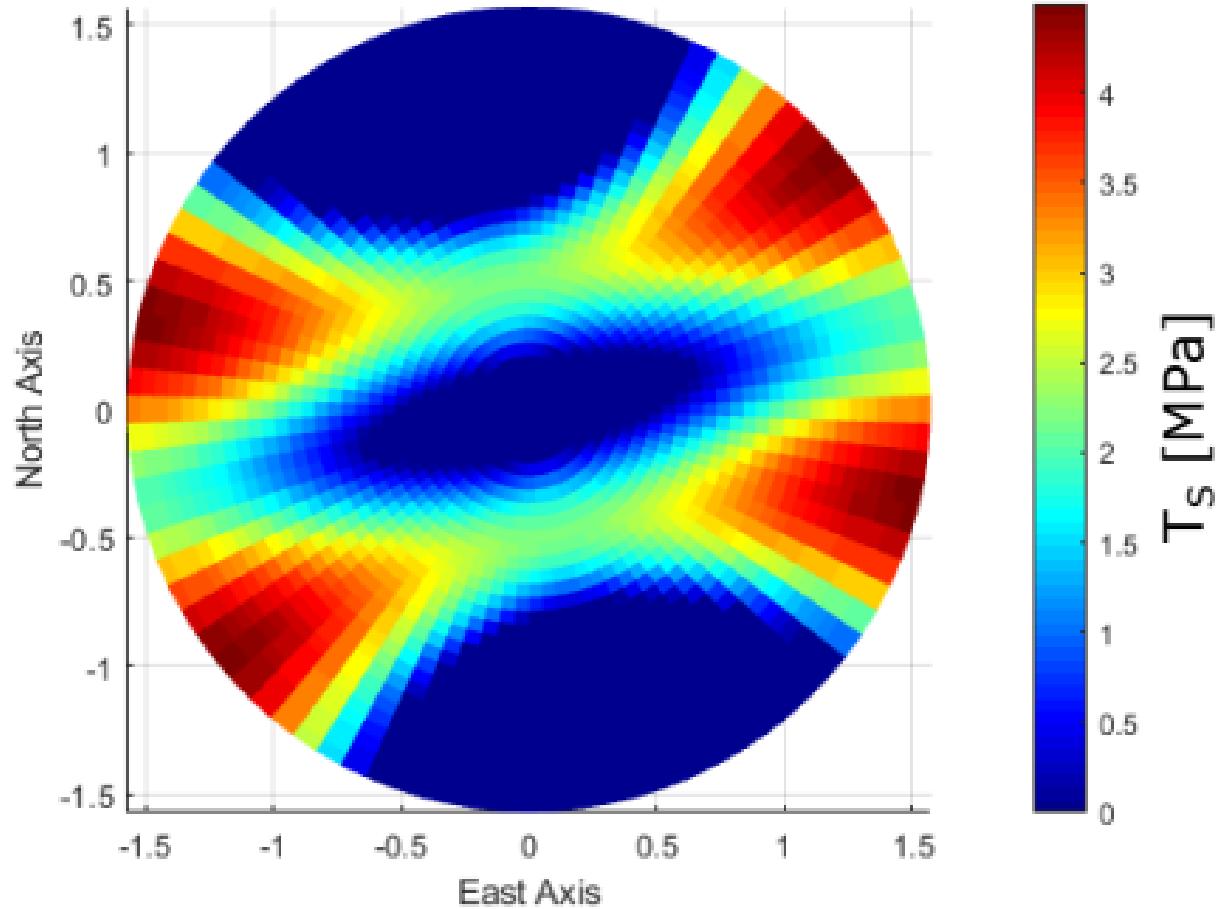




Principal stresses



Required T_s ($P_w=35\text{MPa}$)







$\delta\Omega$

Δ

$\delta\Omega$

Δ

$\Delta\Omega$

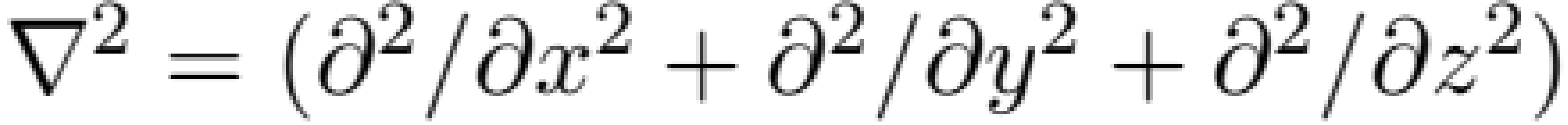
$\Delta\Omega$

$\Delta\Omega$

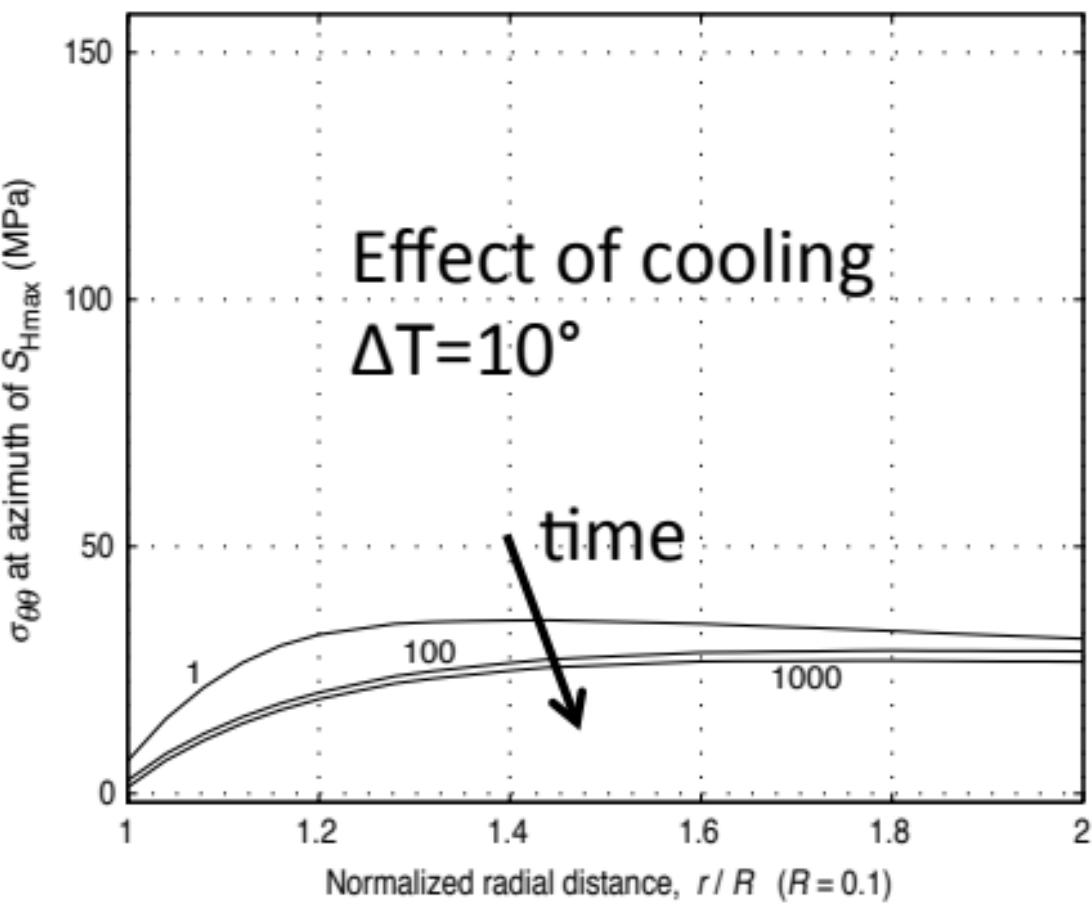
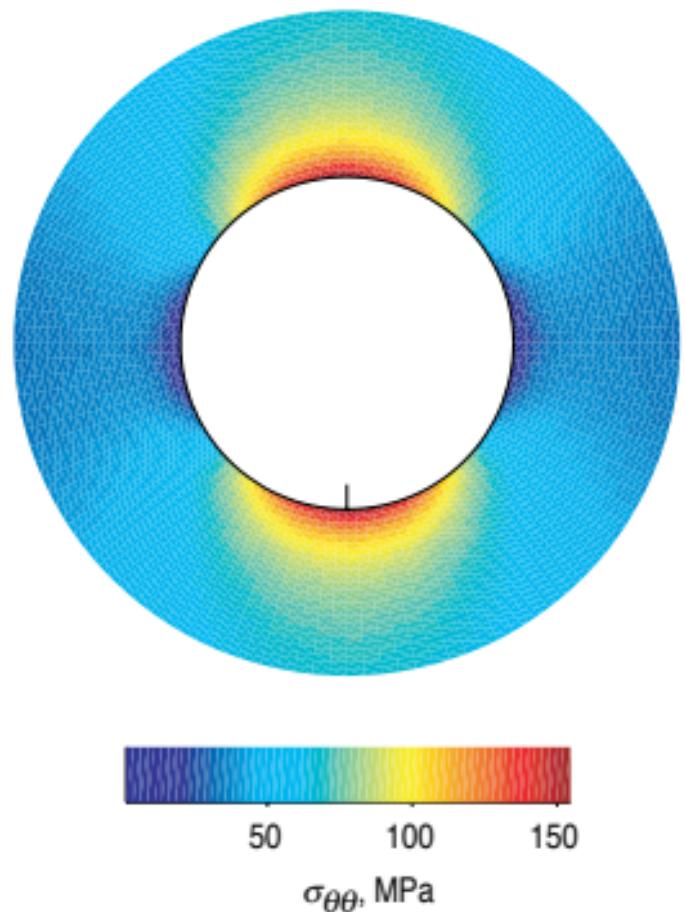








[Zoback 2013 - Figure 6.14]



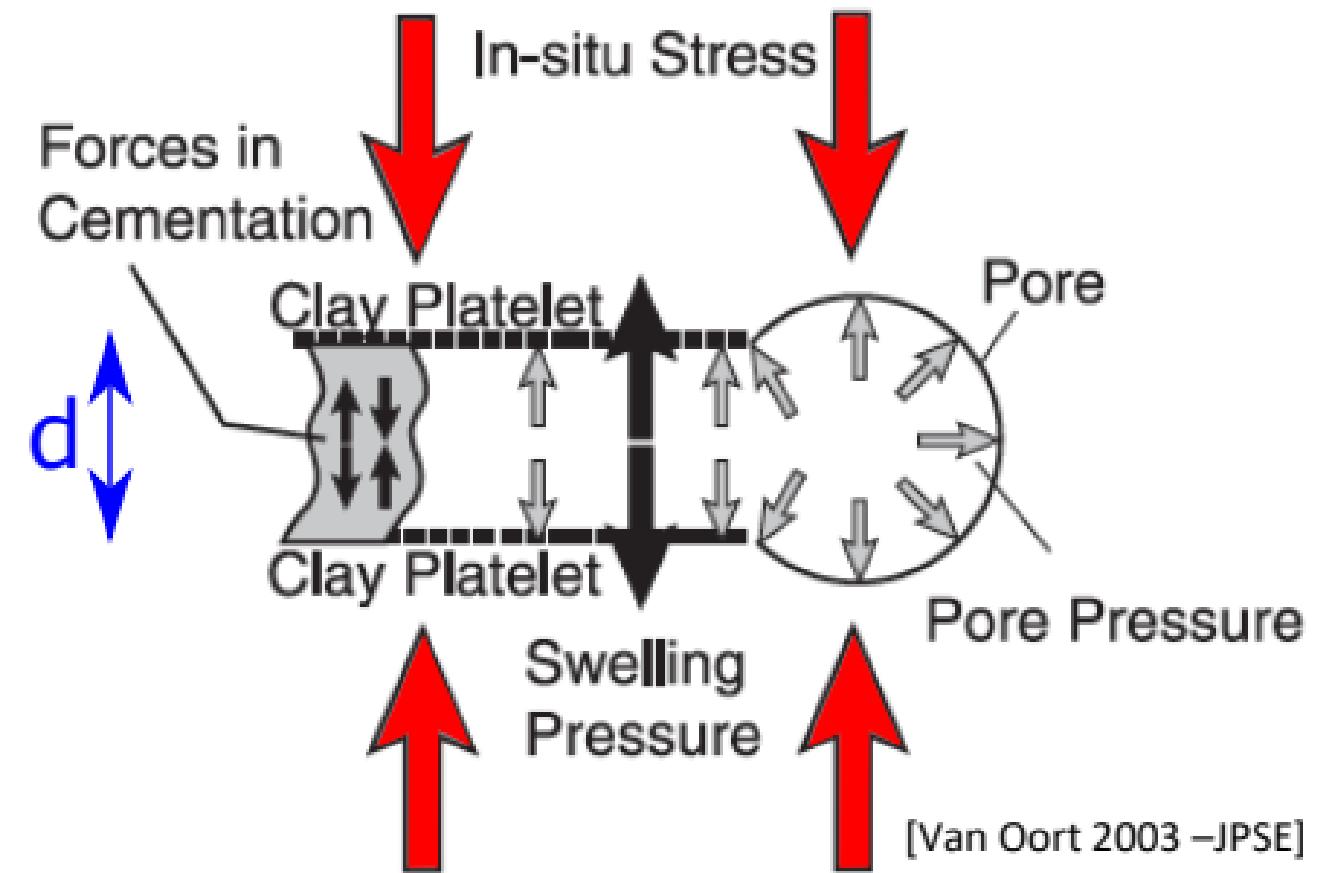
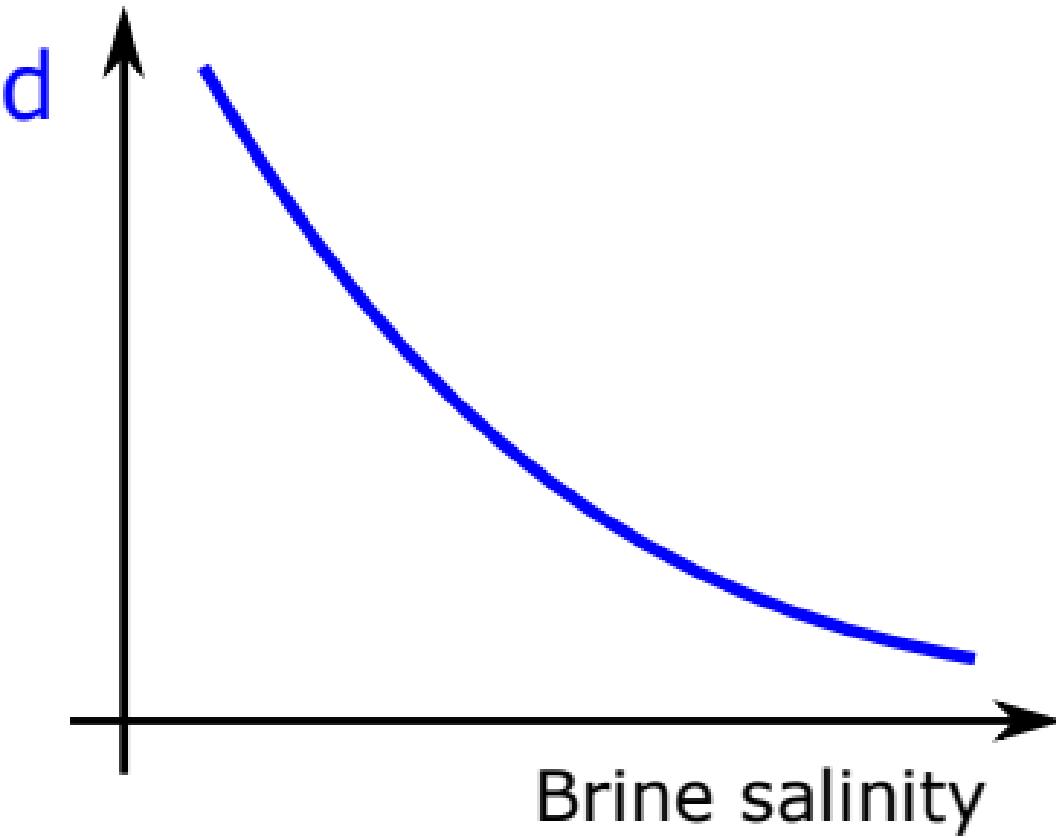


Δ $\sigma_{\theta\theta}$

$=$

$\alpha \Gamma B \Delta T$

$1 - \sqrt{ }$



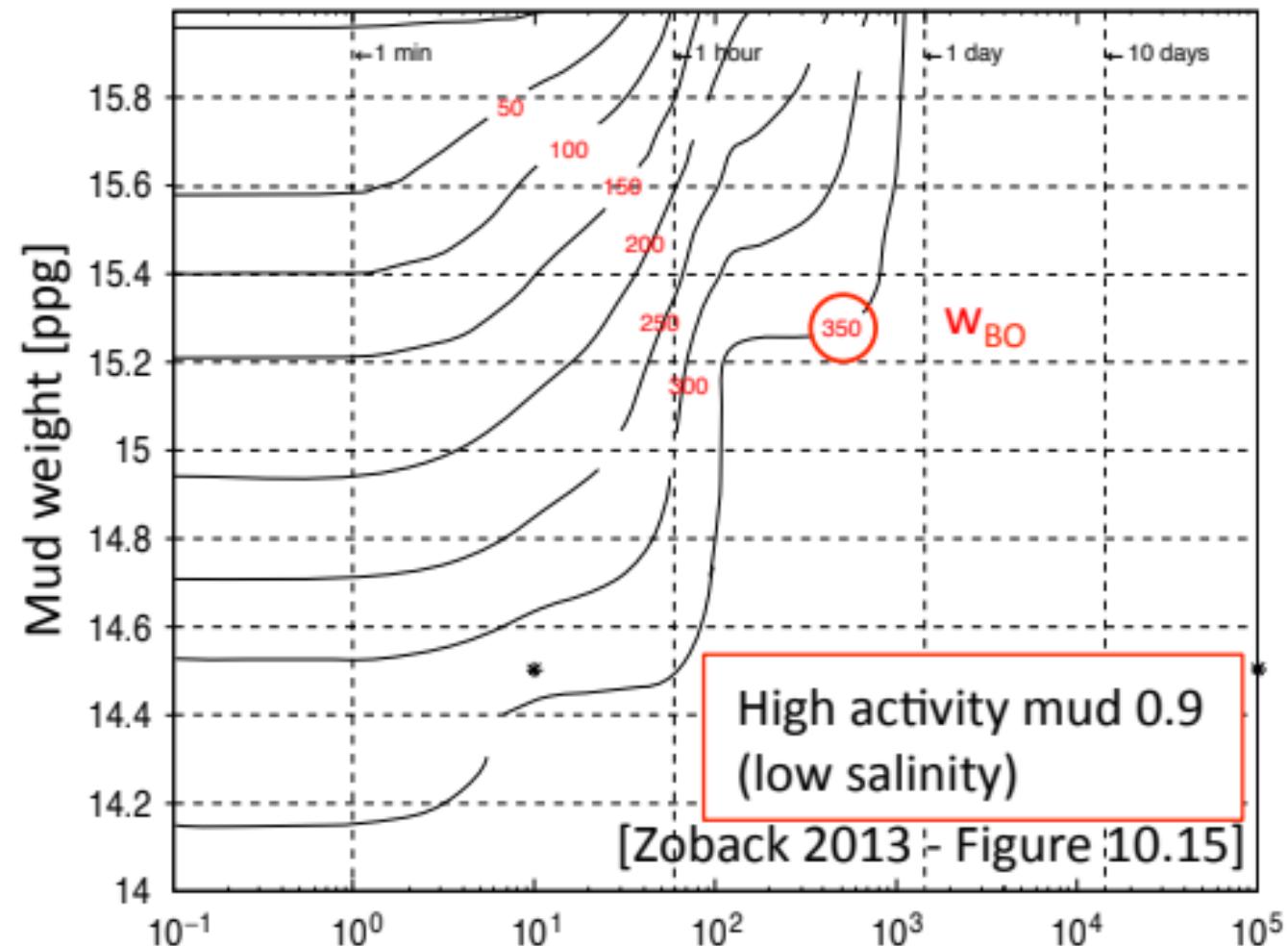
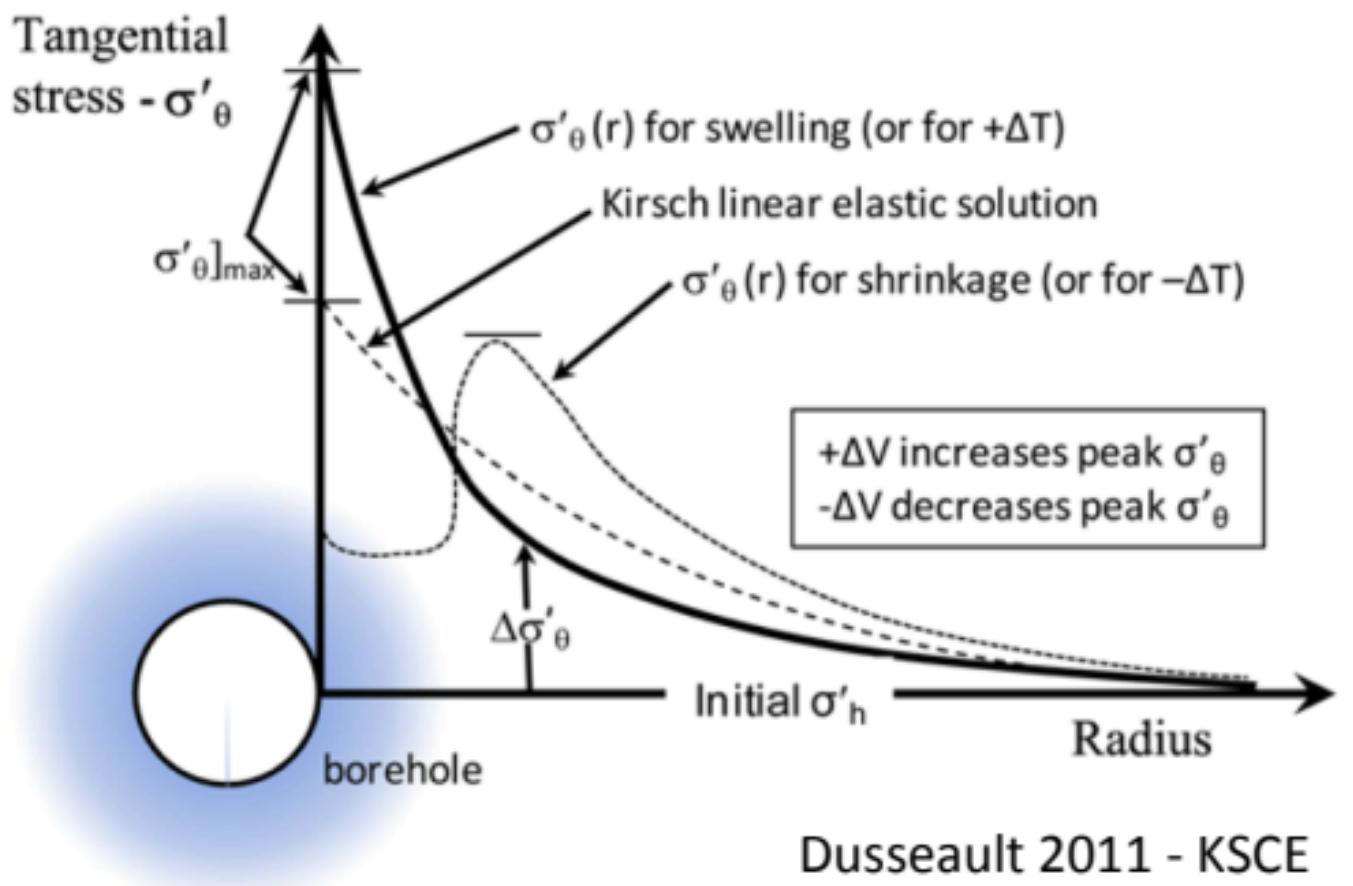
Norway shale



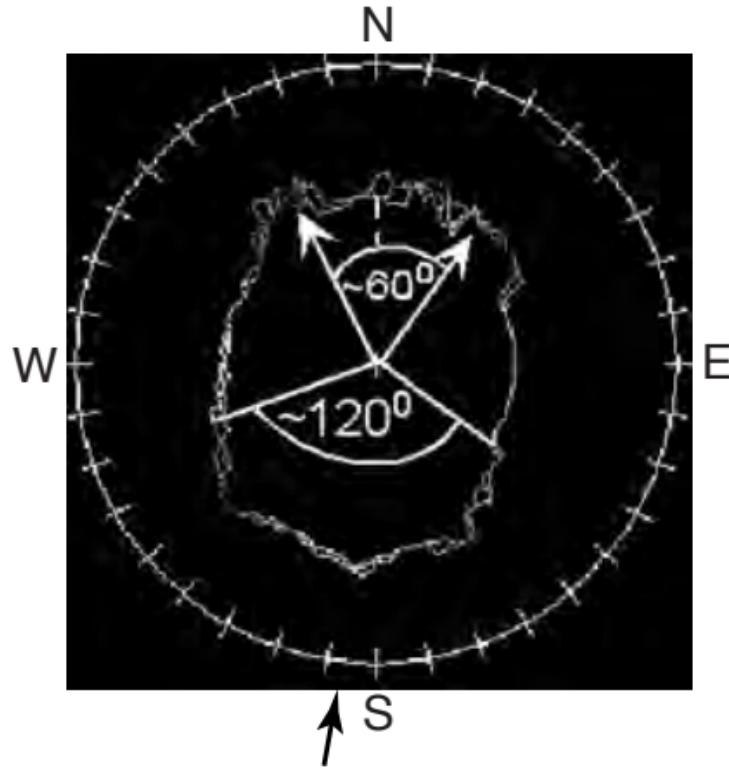
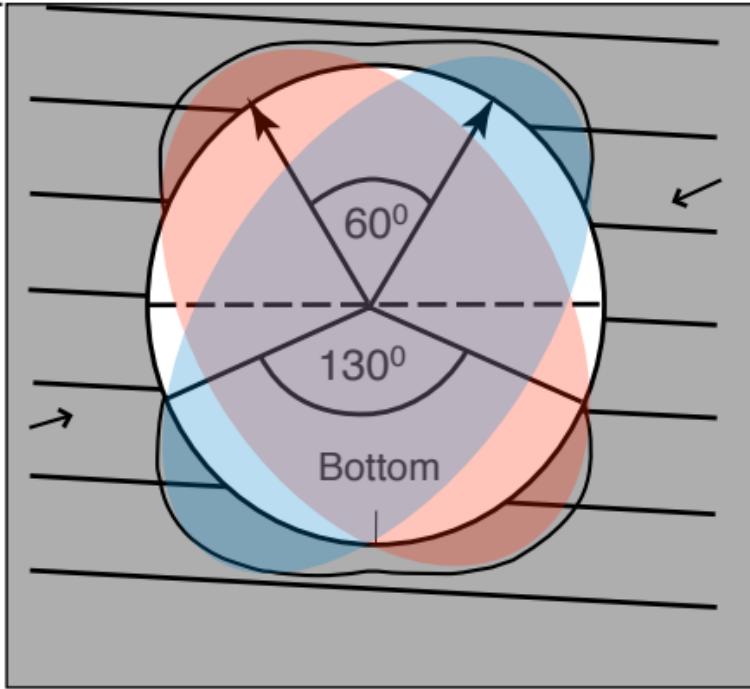
After 2hs exposition



CPGE – M. Chenevert







Shear failure intact rock

Shear failure at weak interfaces
(usually weak bedding planes)