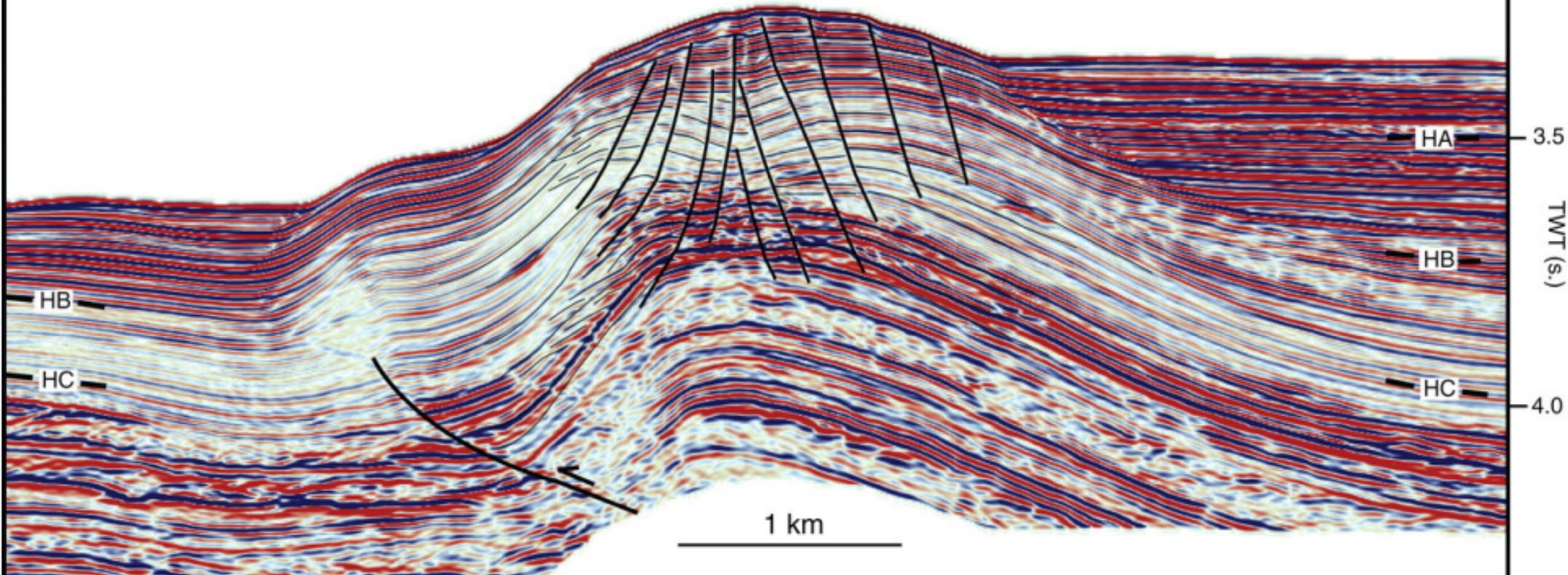


NW

Crestal normal faults

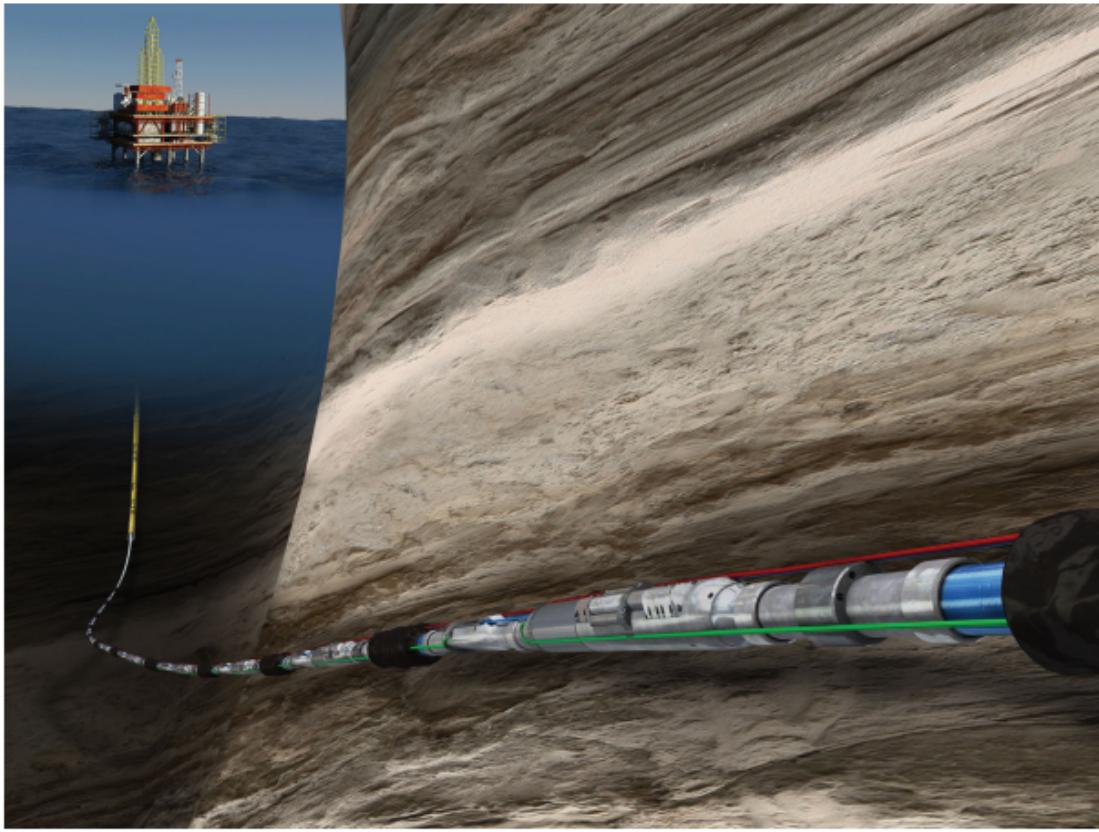
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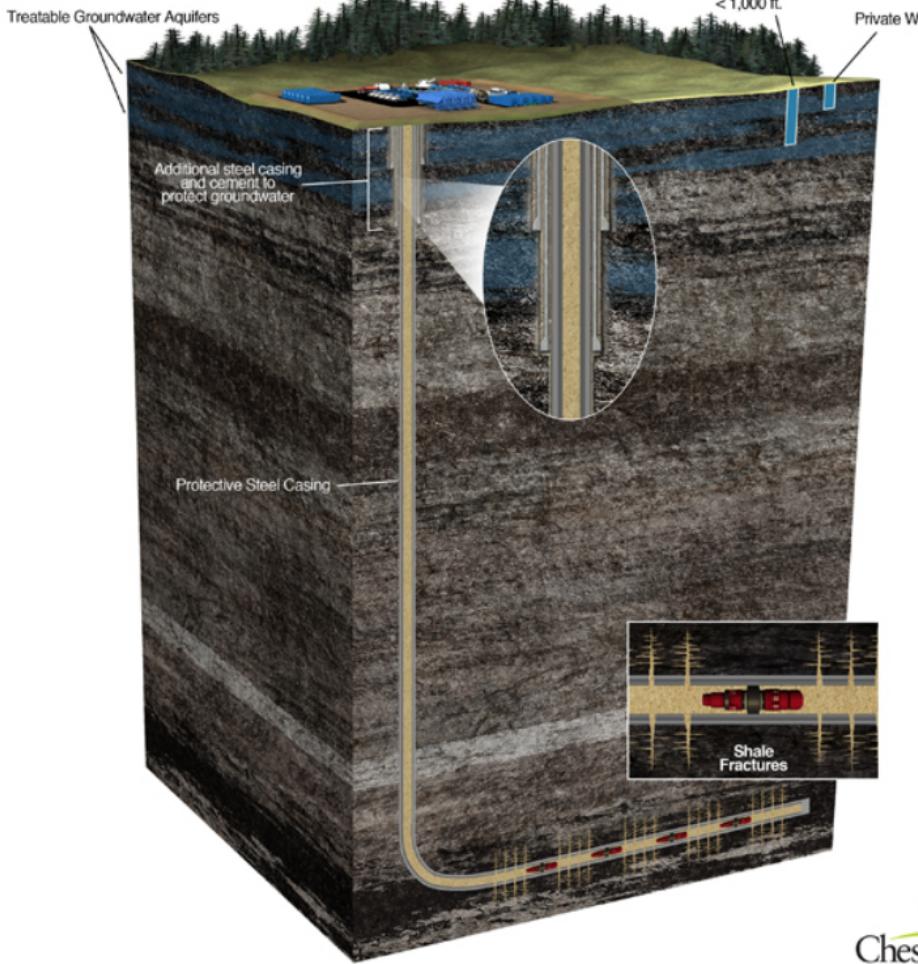


~1850



~2010





Risk-Based  
Geomechanical  
Screening

# Stress Man

MUDLINE SUBSIDENCE



FAULT ACTIVATION

CASING CRUSHING

COMPACTION DRIVE

CASING  
SHEAR

SAND PRODUCTION



RESERVOIR YIELD

PERMEABILITY LOSS

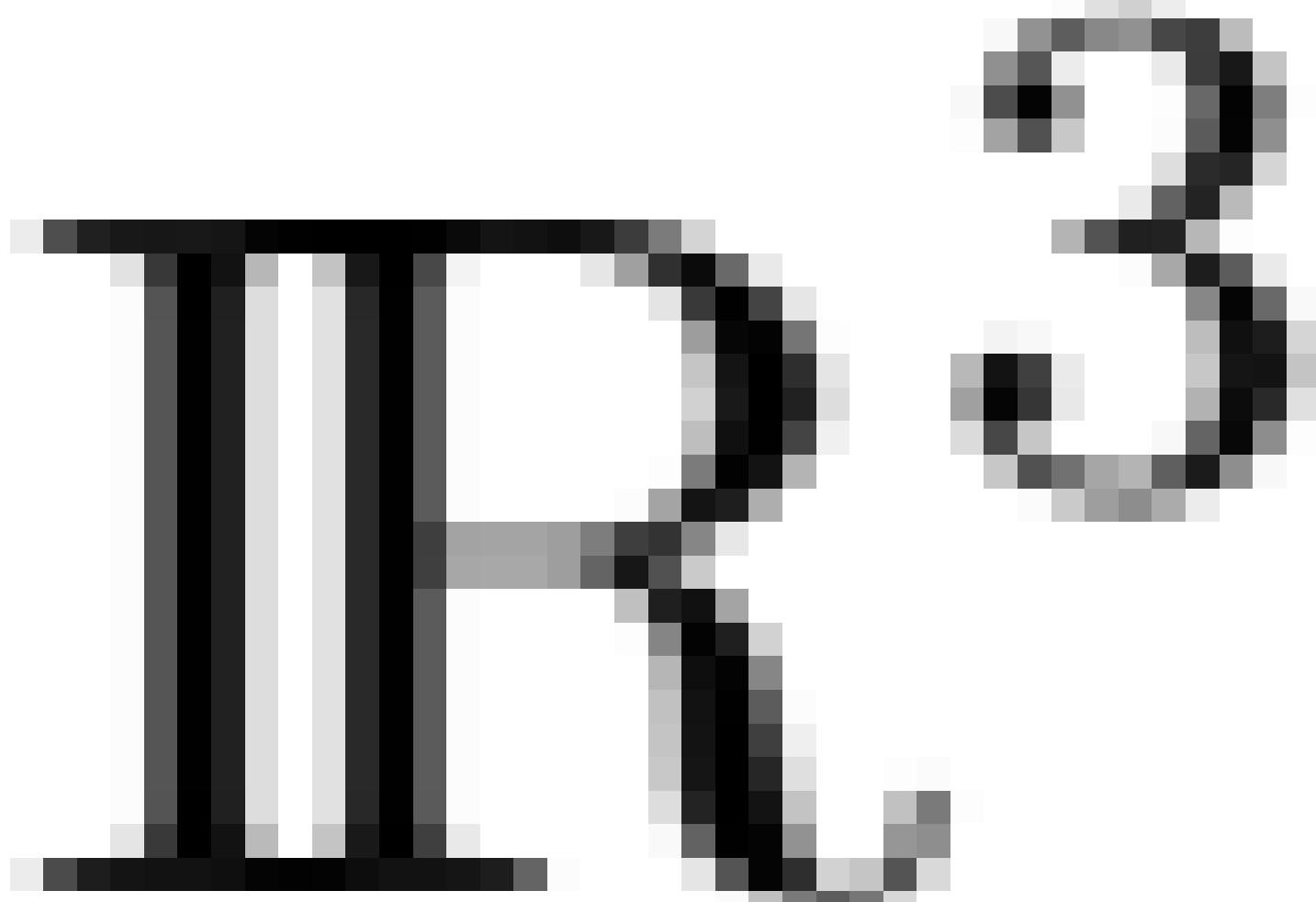
CASING BUCKLING

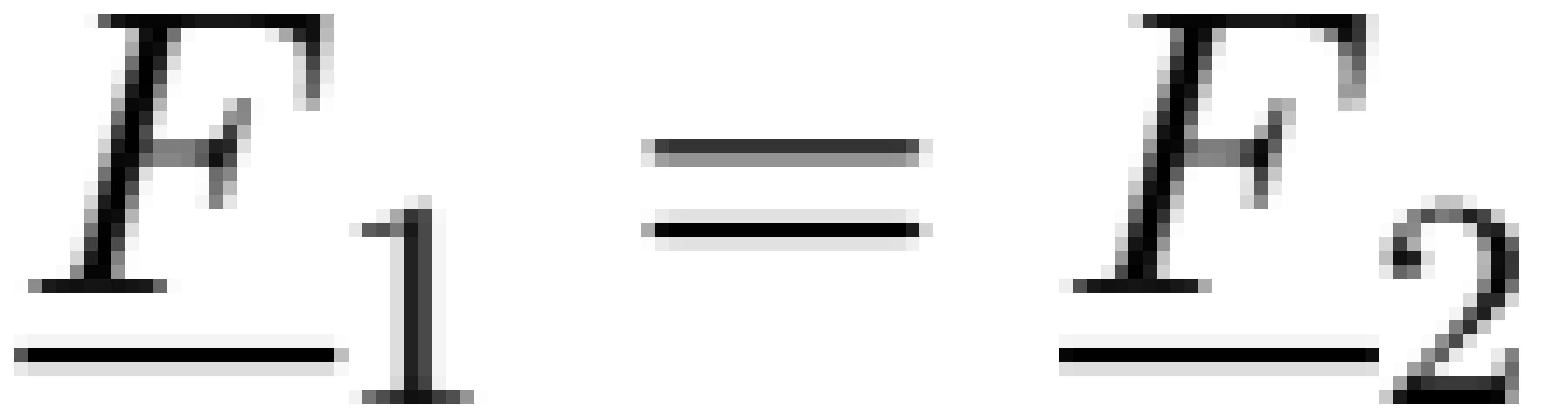
$S_V$



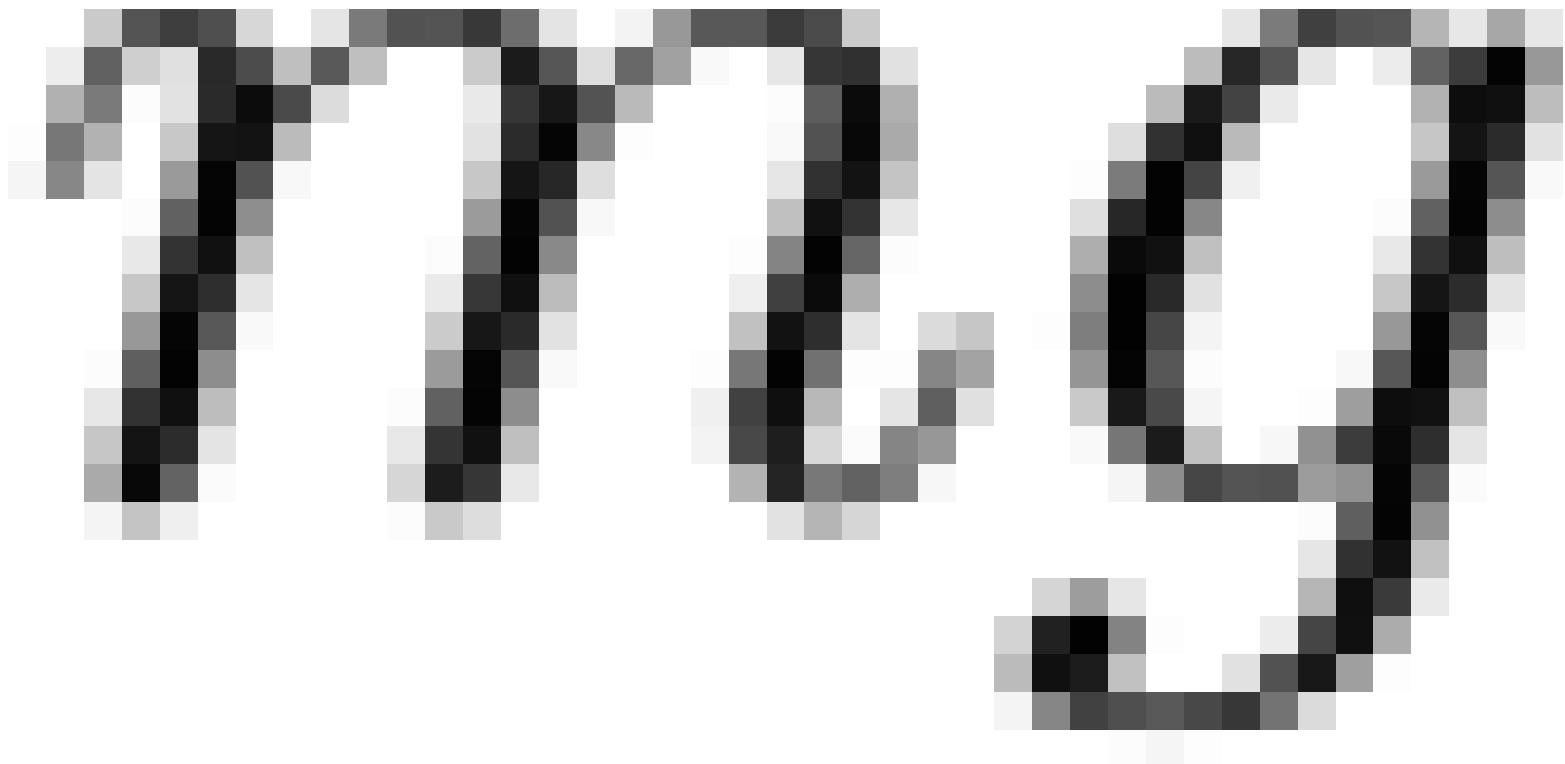


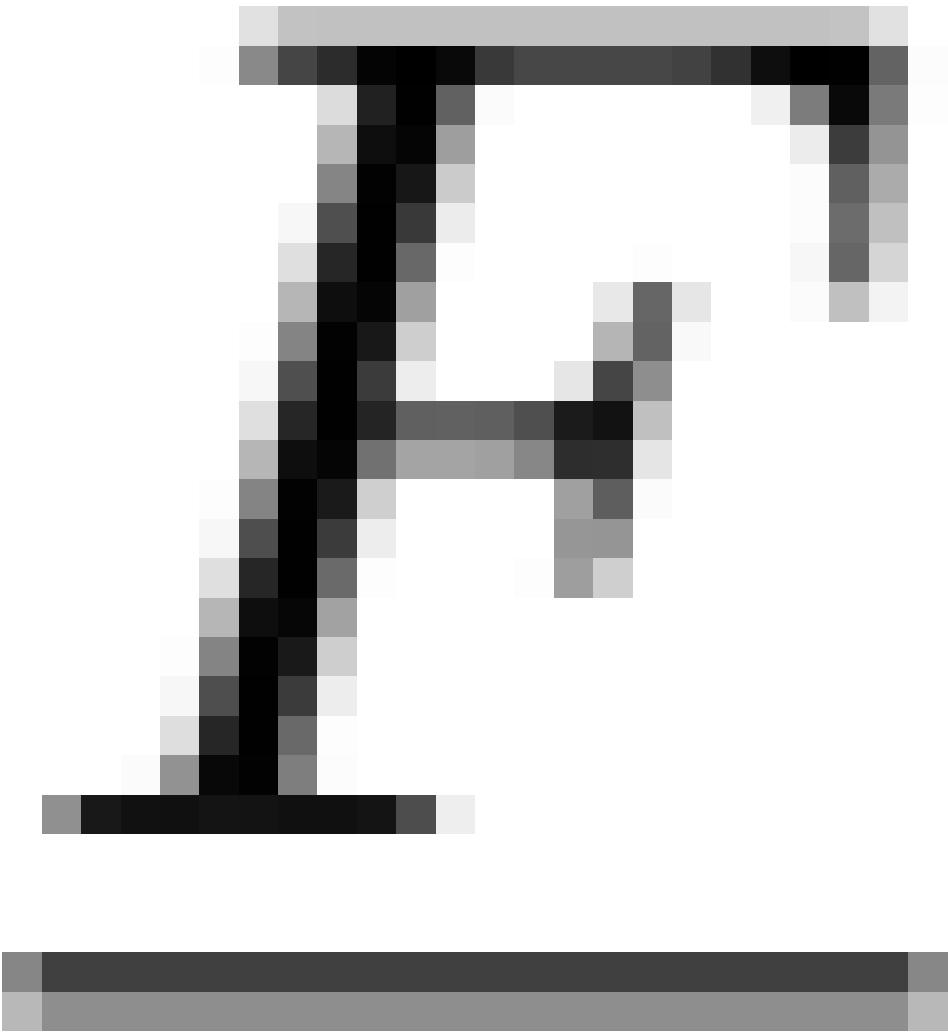


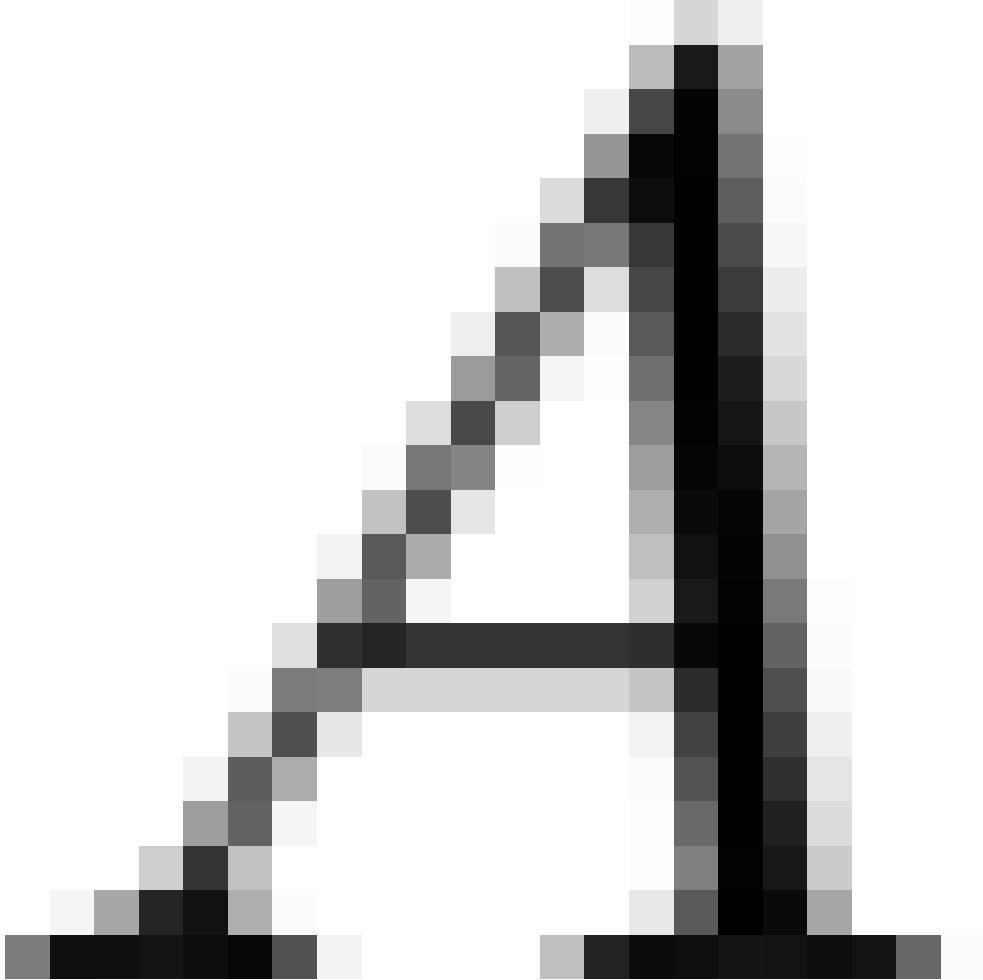












P

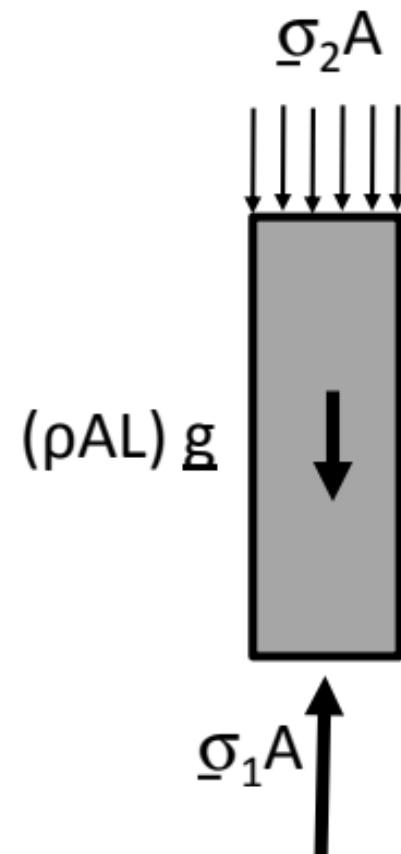
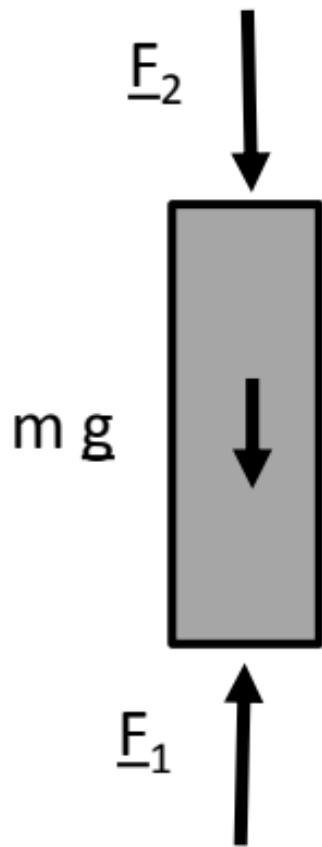
S

S

O

S

A

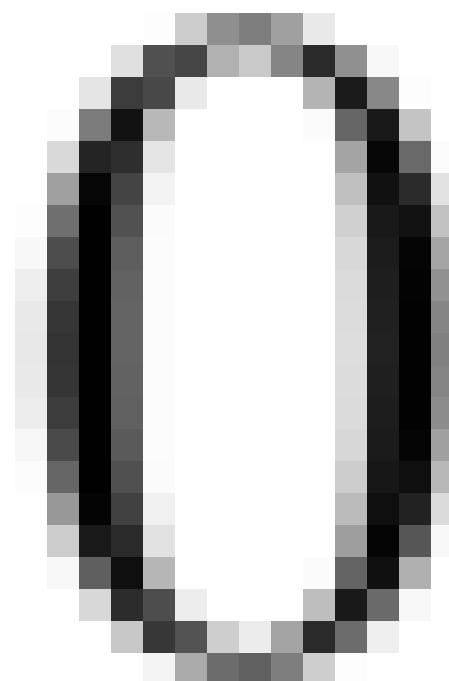
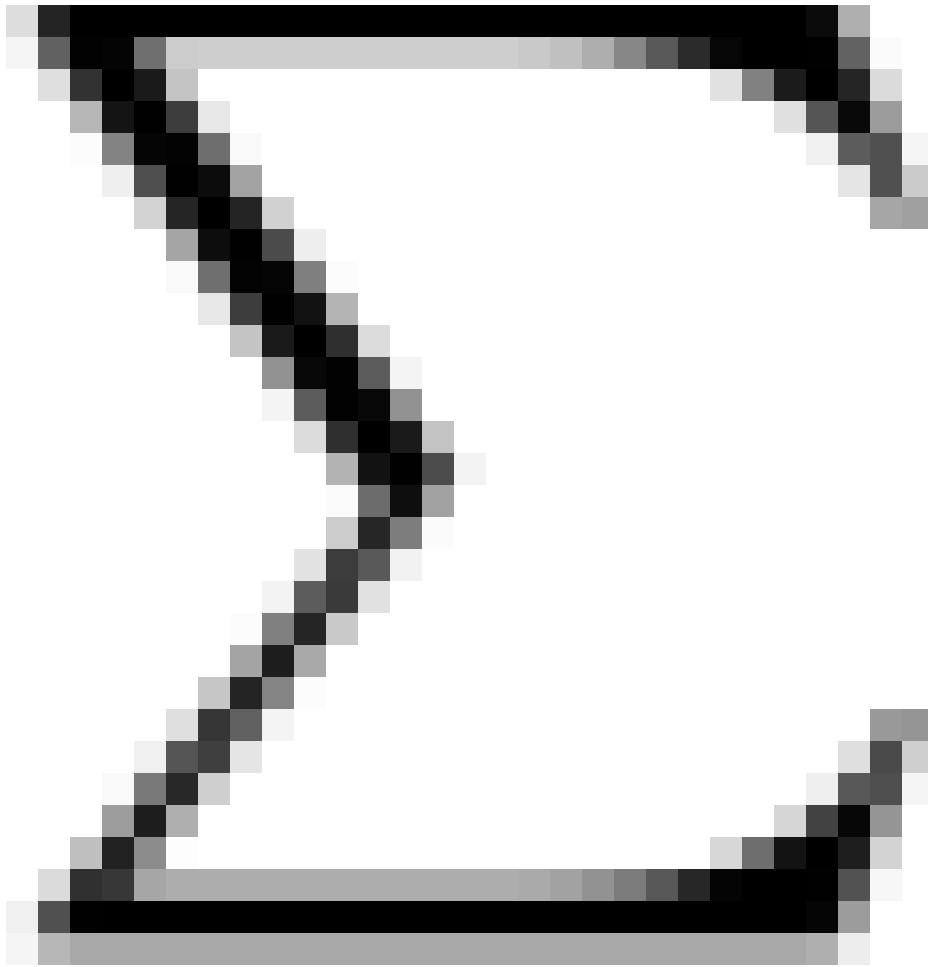


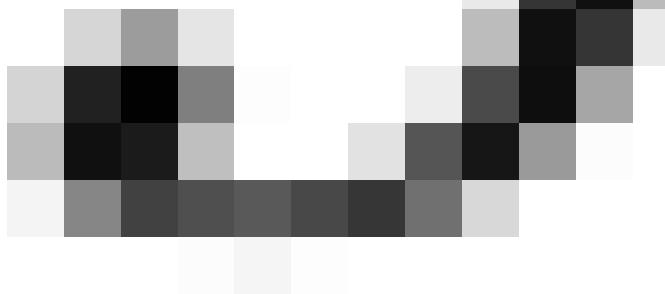
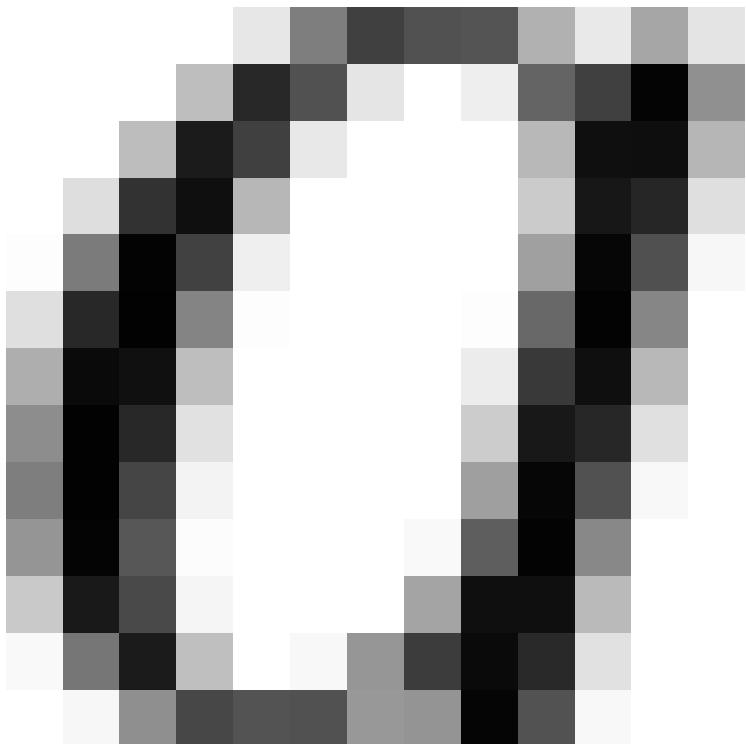
$$\Sigma F_z = +F_1 - F_2 = 0$$

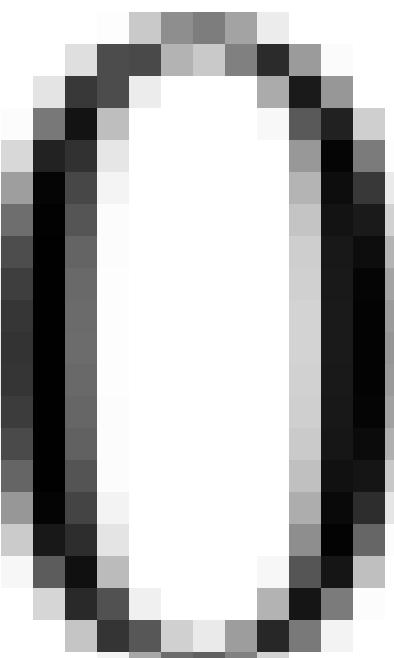
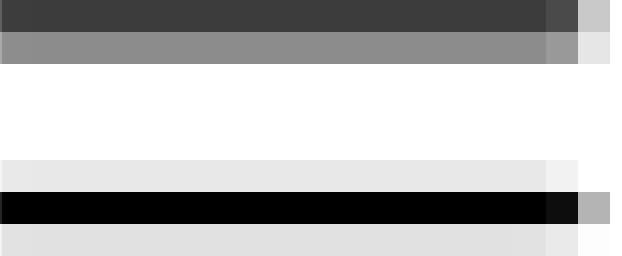
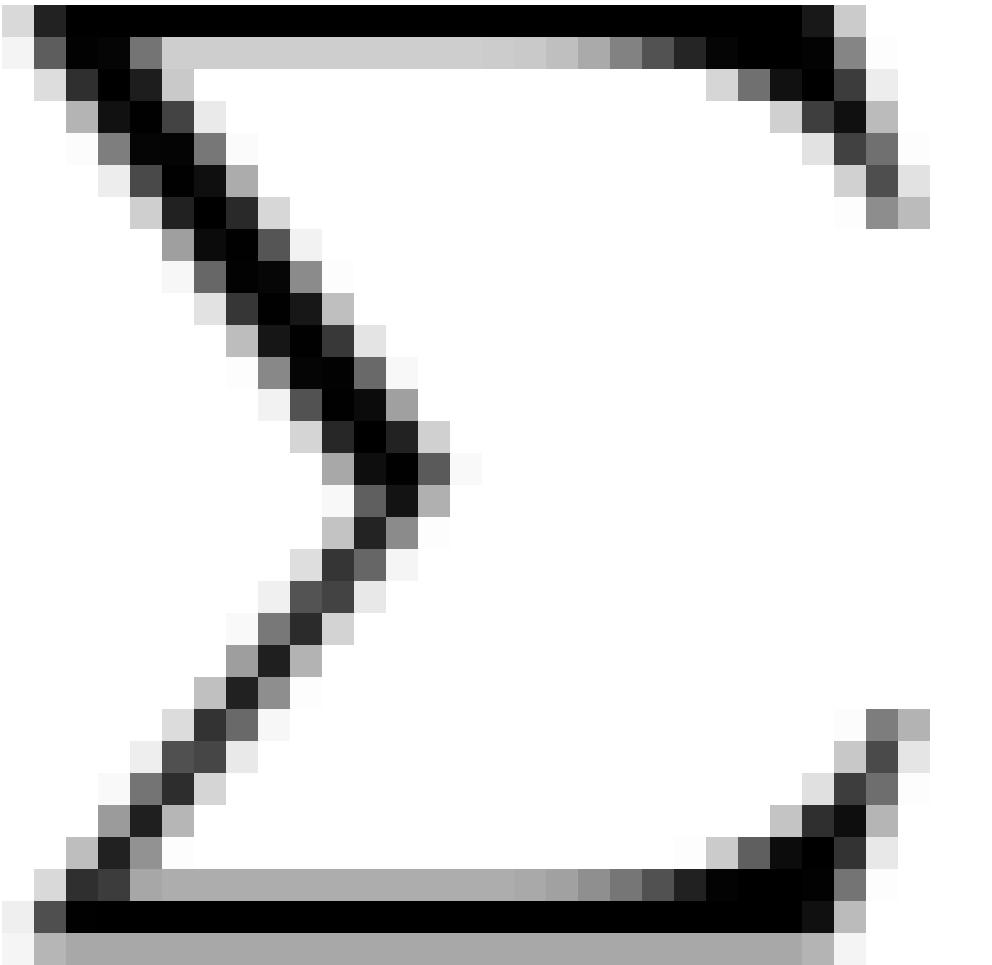
$$\Sigma F_z = +F_1 - m g - F_2$$

$$\Sigma F_z = +\sigma_1 A - (\rho A L)g - \sigma_2 A$$











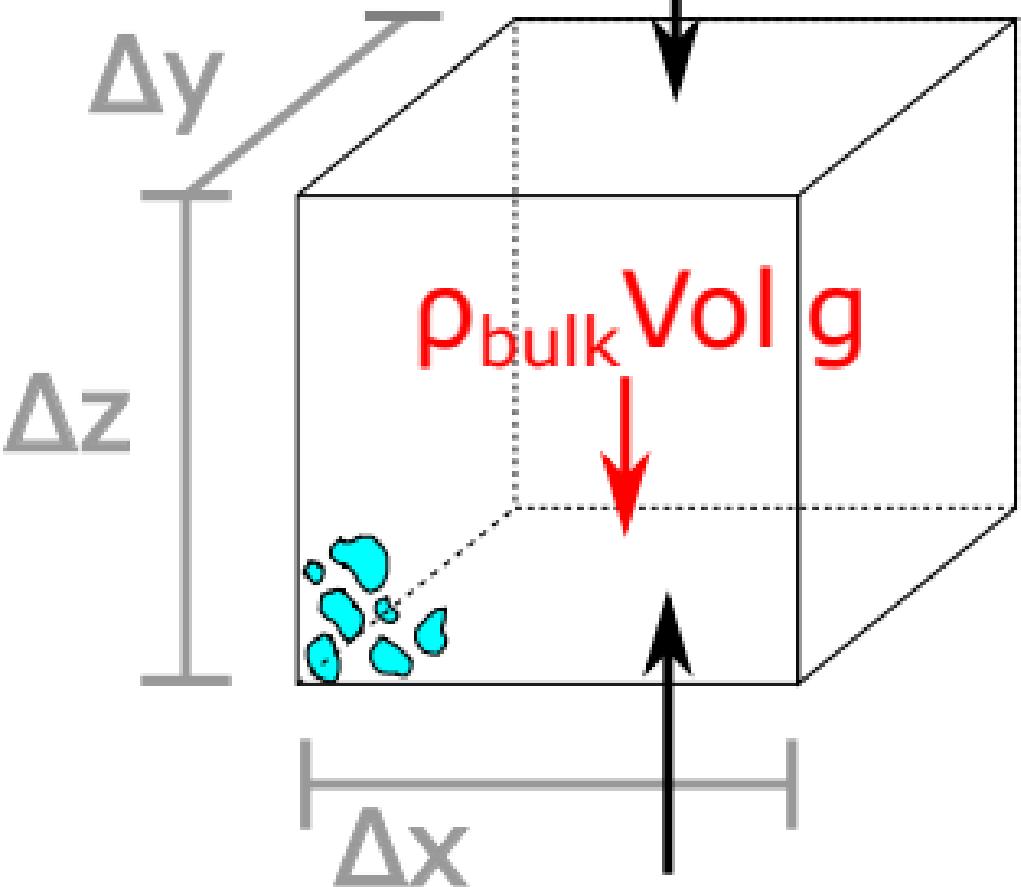


△ S<sub>2</sub>

△ z

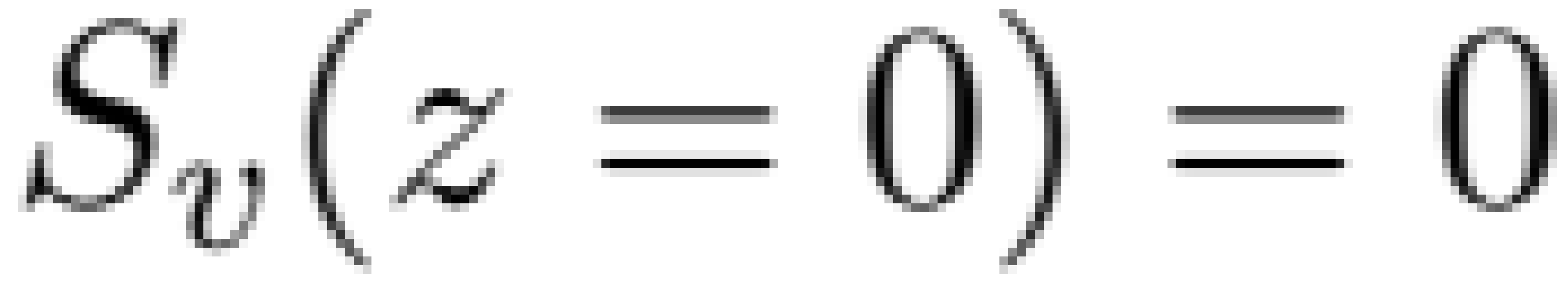
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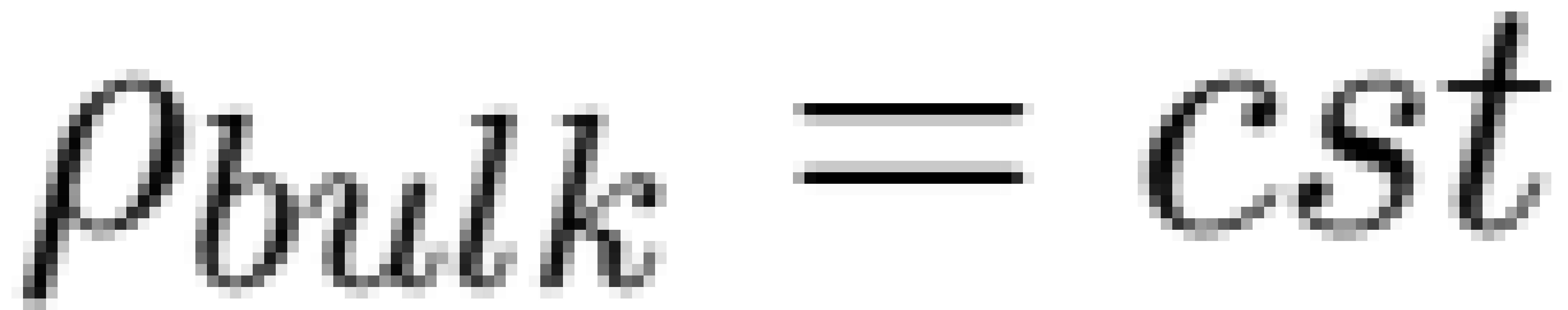
Observe 9

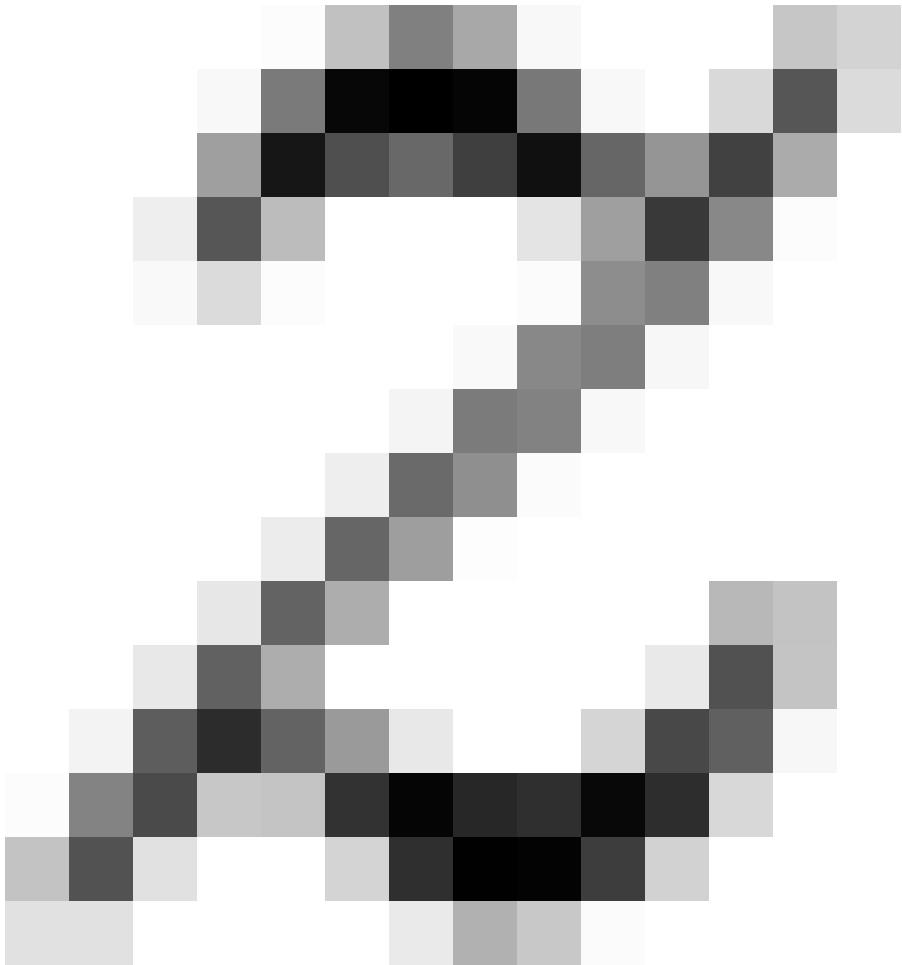
$S_v A$  $S_v A + \Delta S_v A$

$$\int_0^{S_w(z)} ds_w = \int_0^z \rho_{bulk}(z) g dz$$







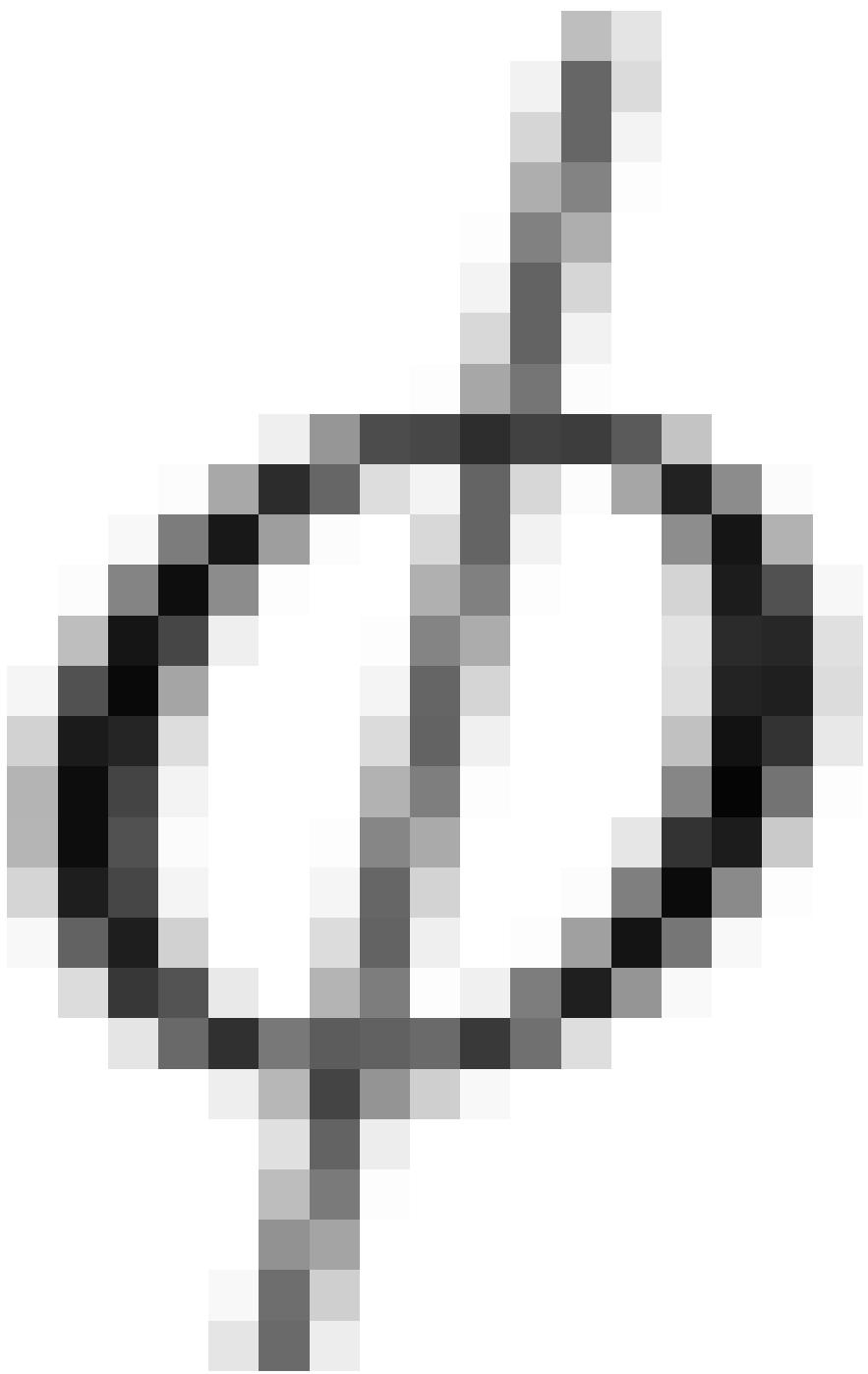


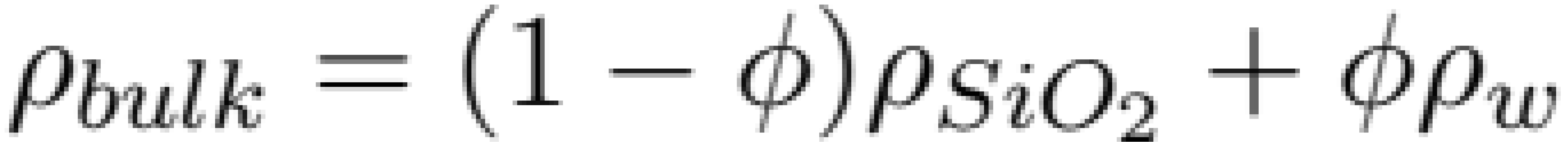




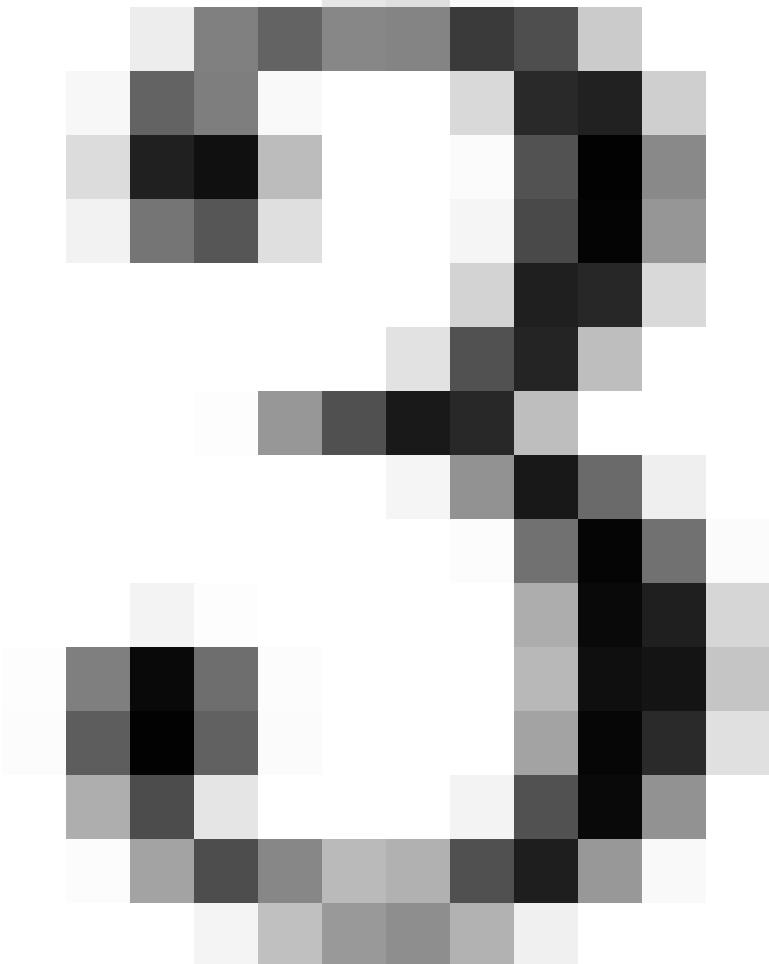


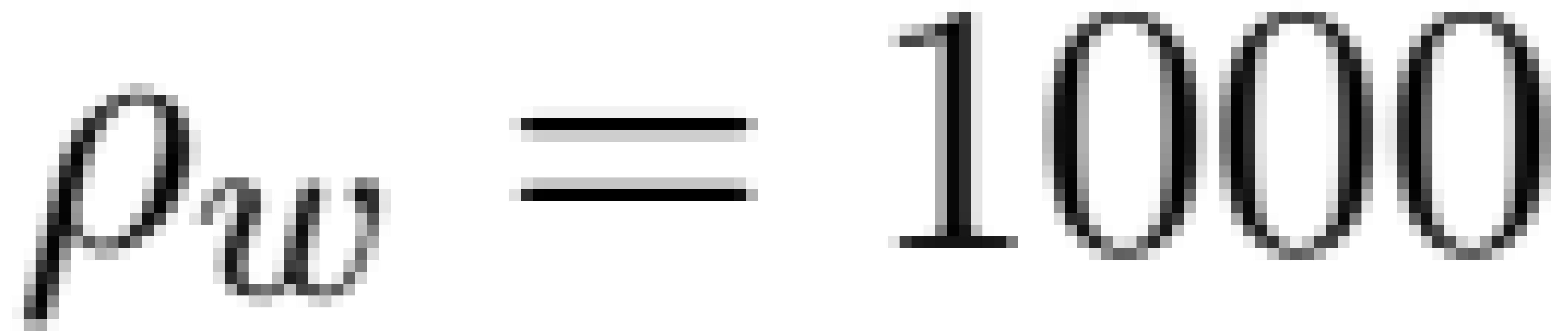


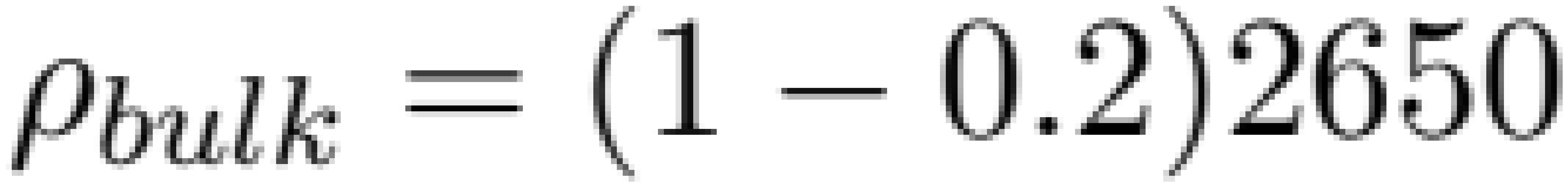


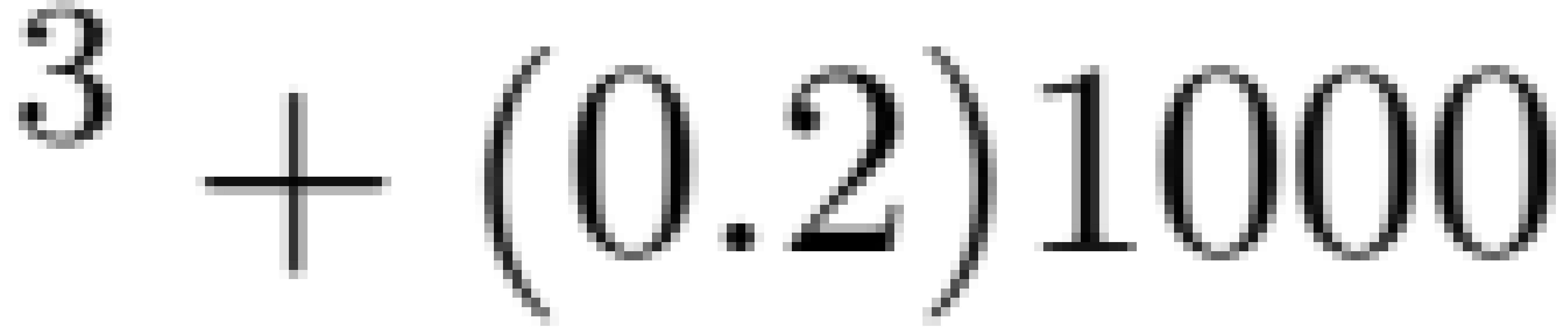




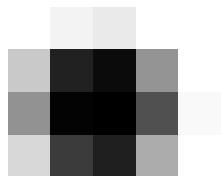
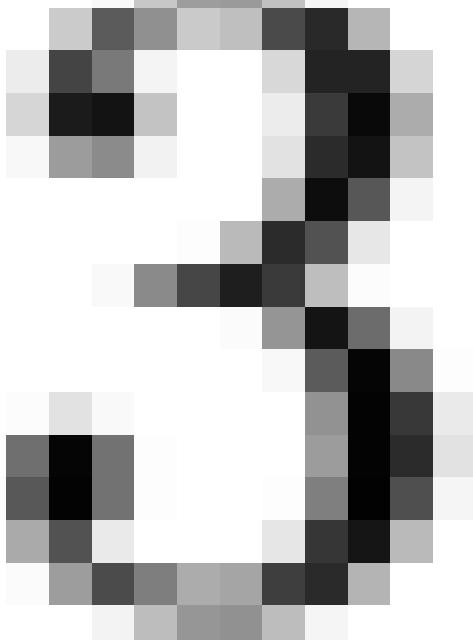


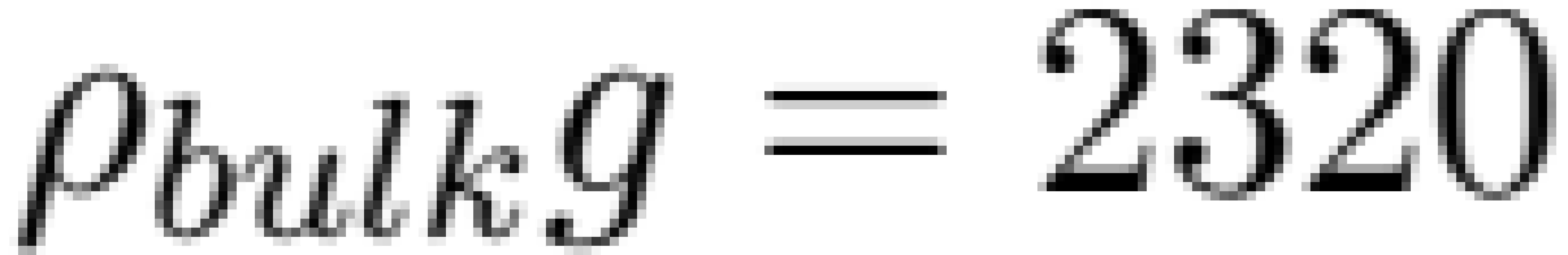




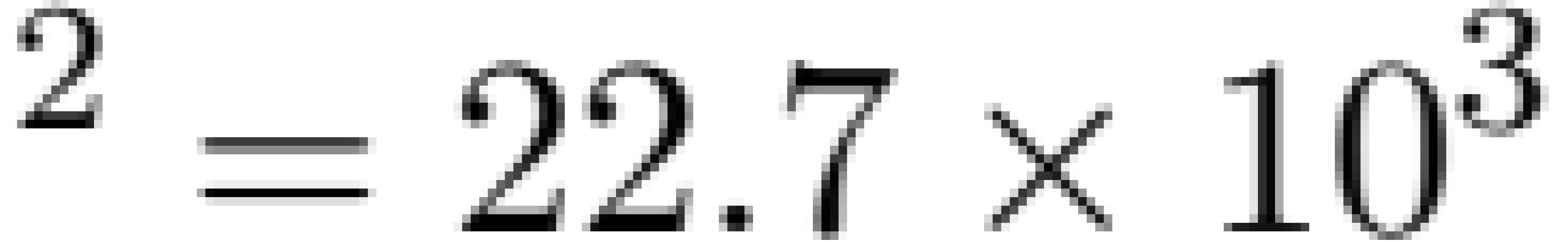


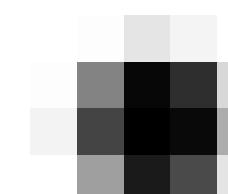
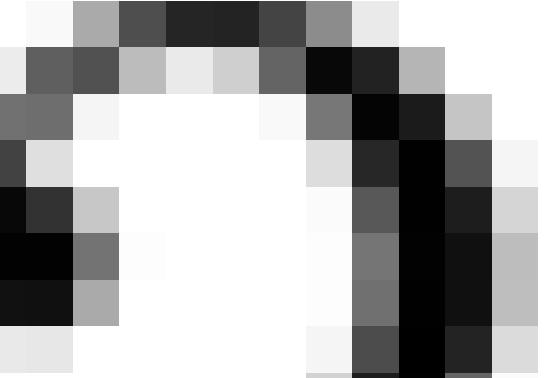
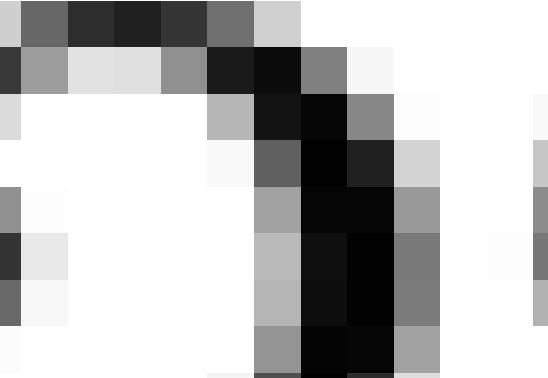


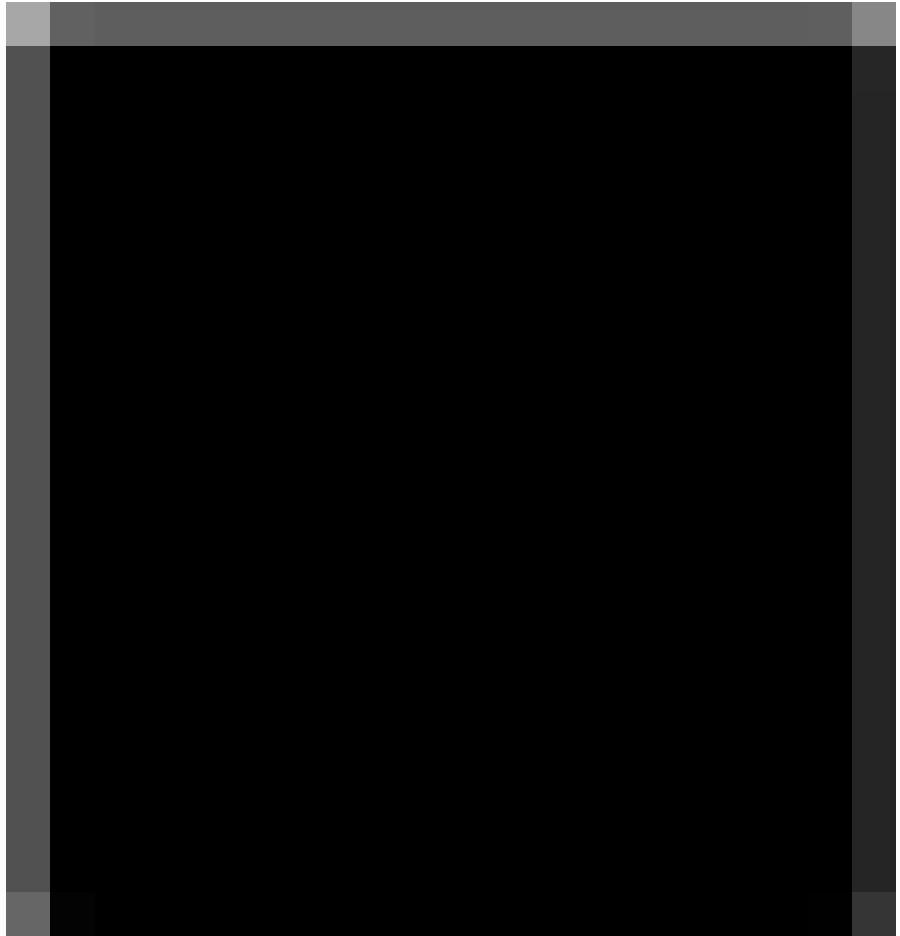
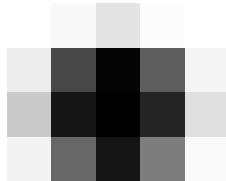


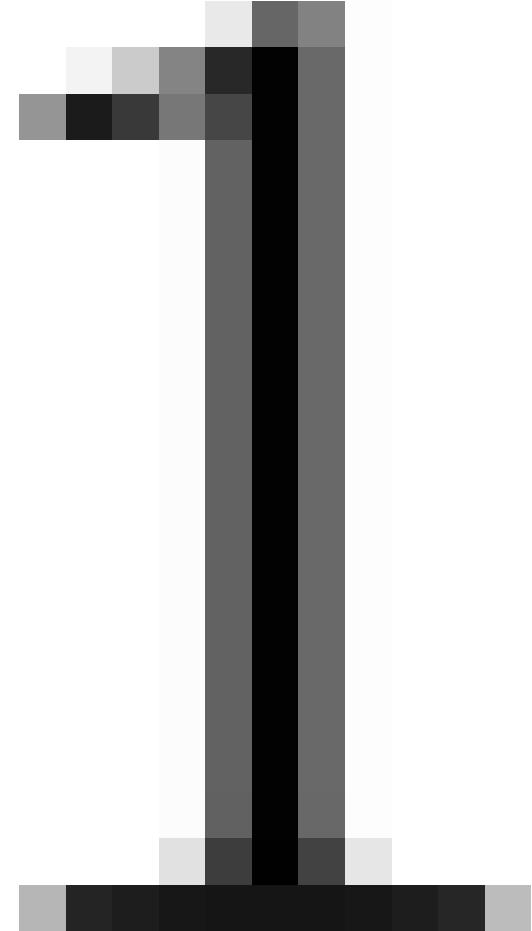
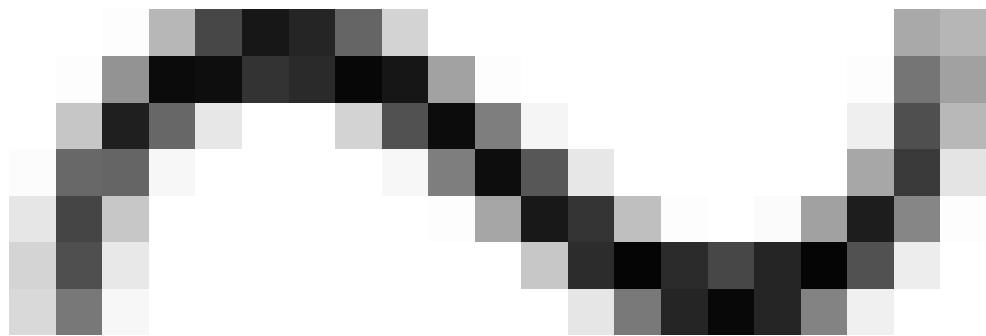


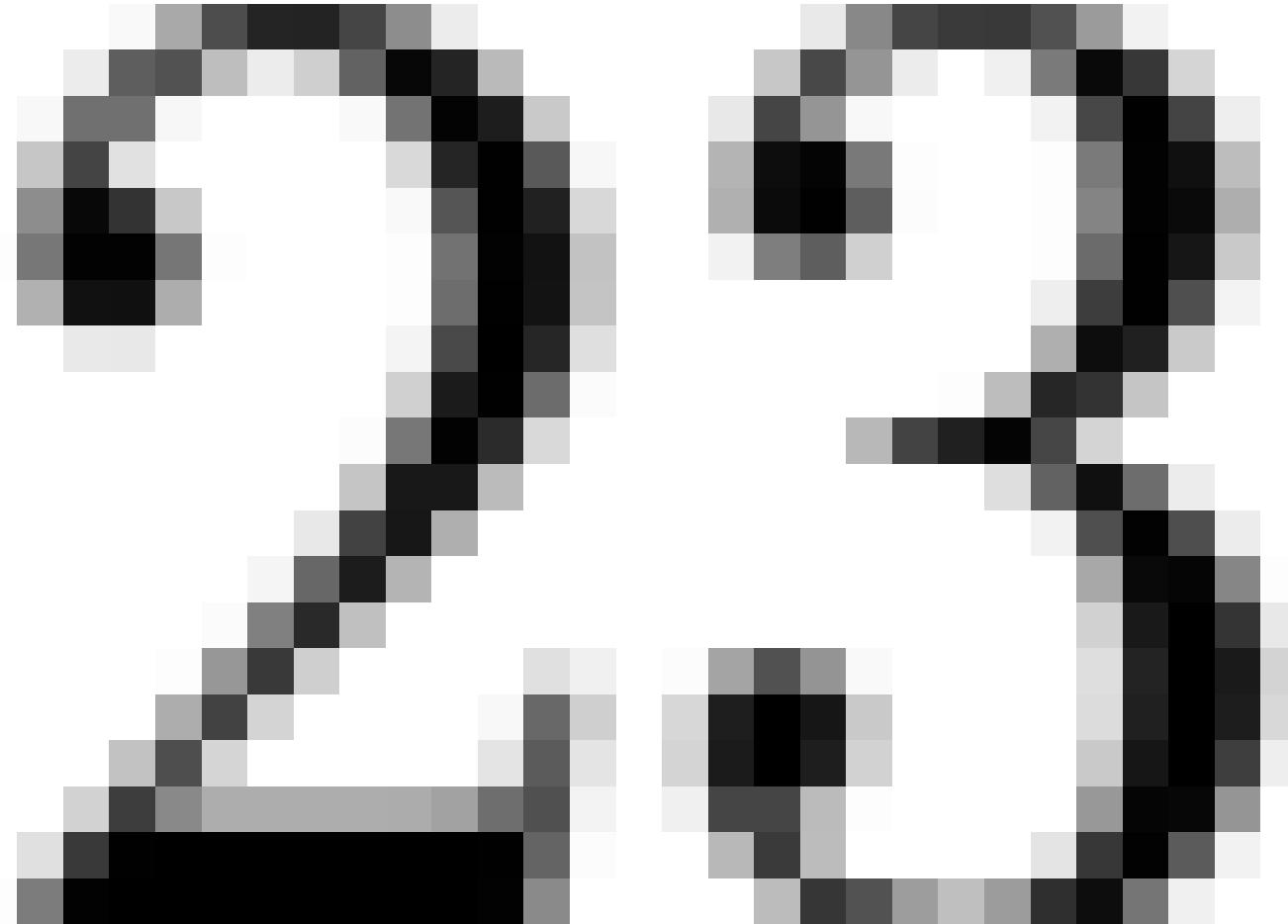
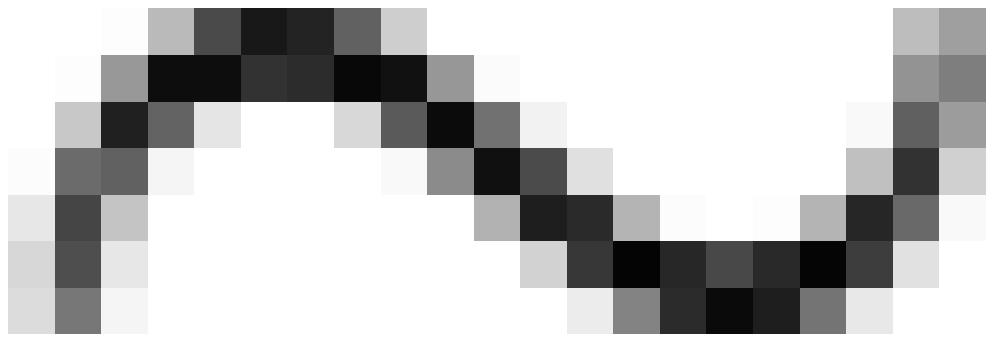


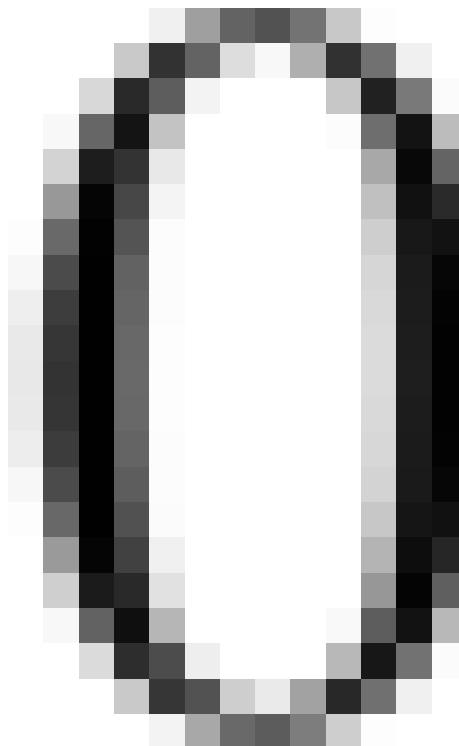
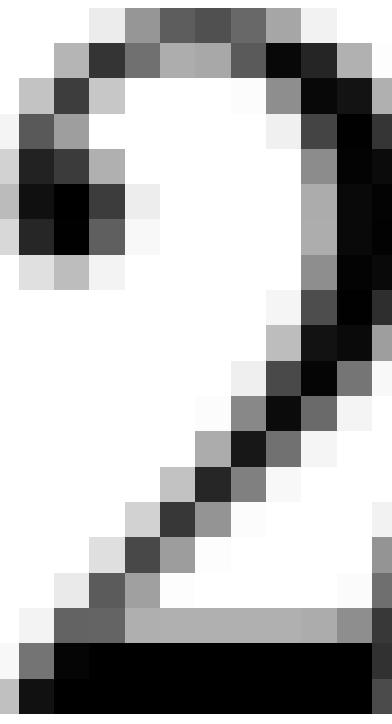
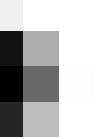
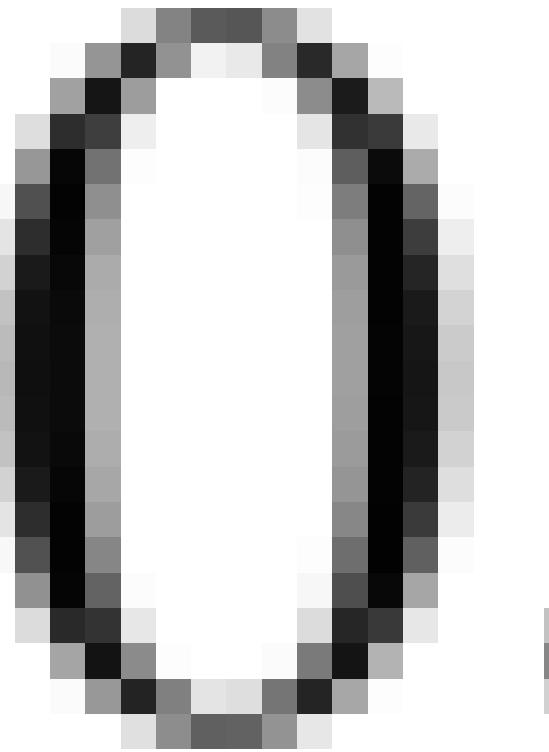
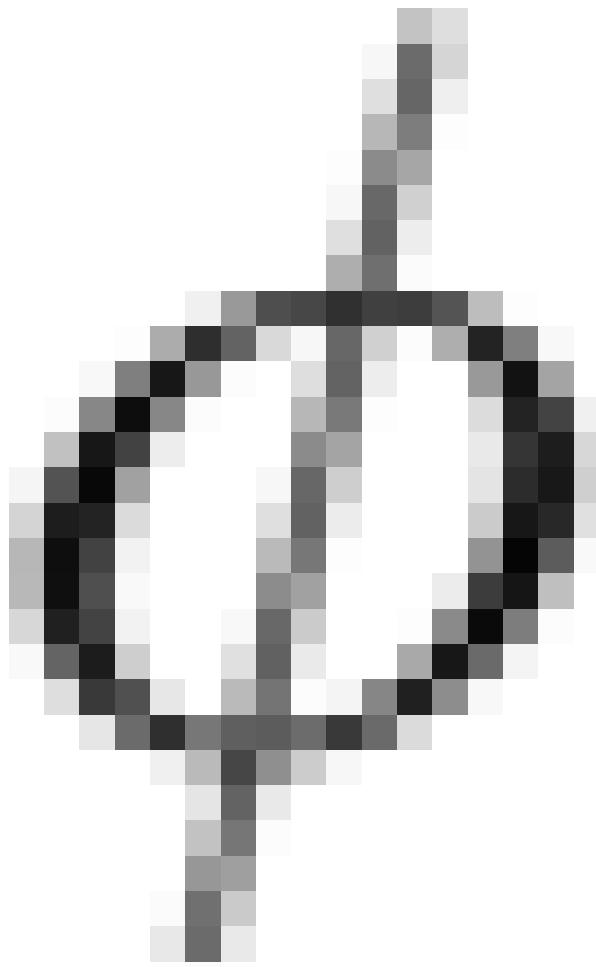


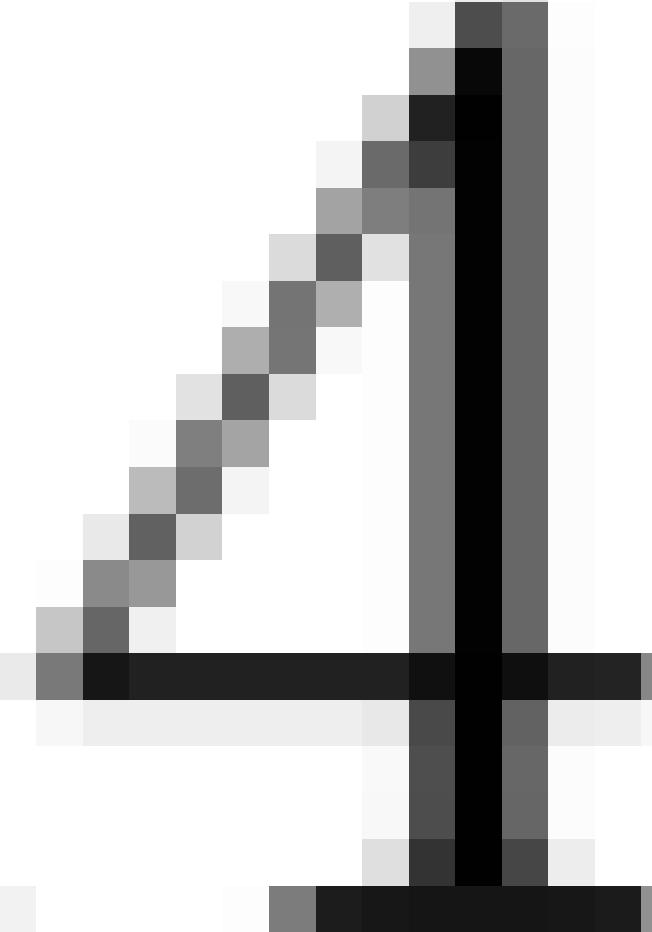
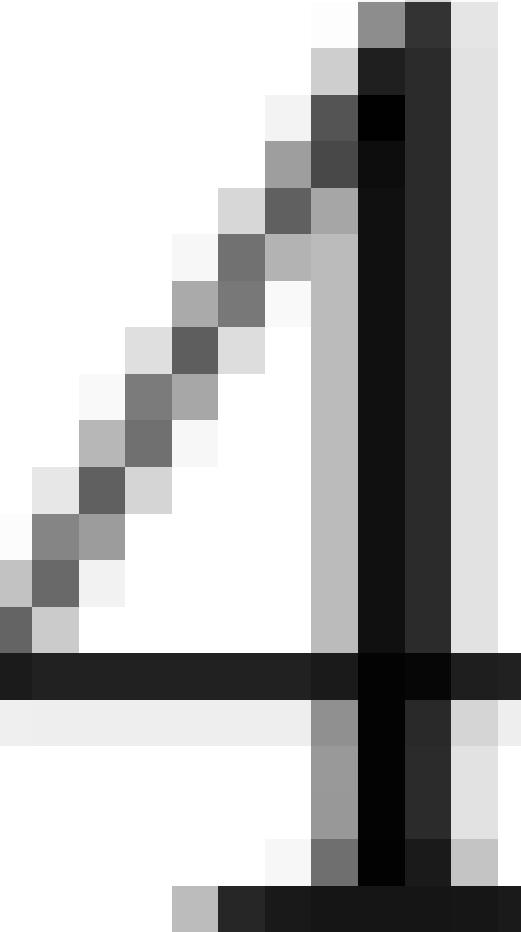
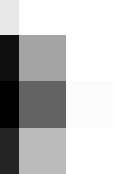
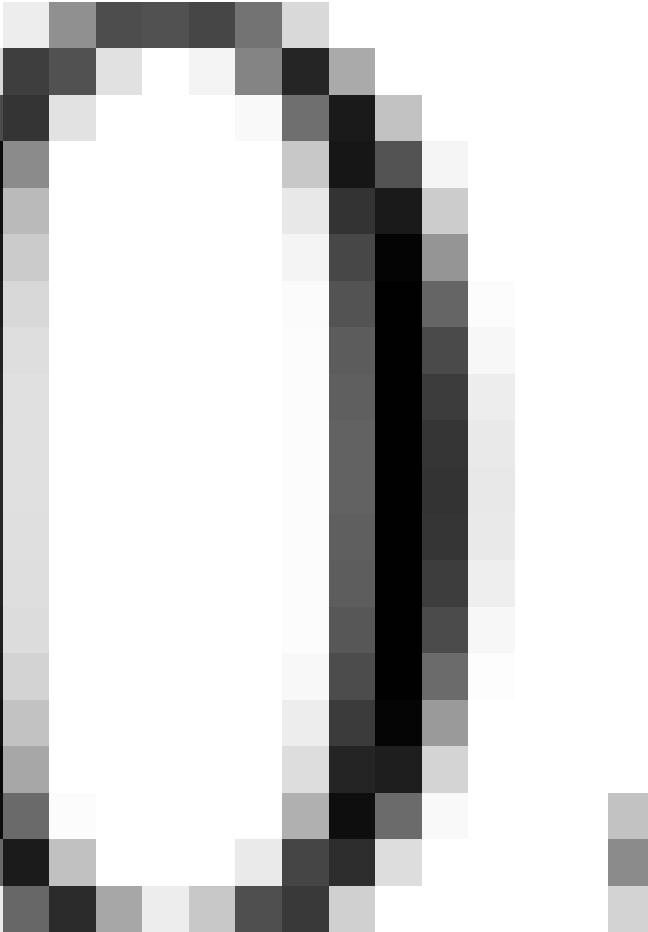
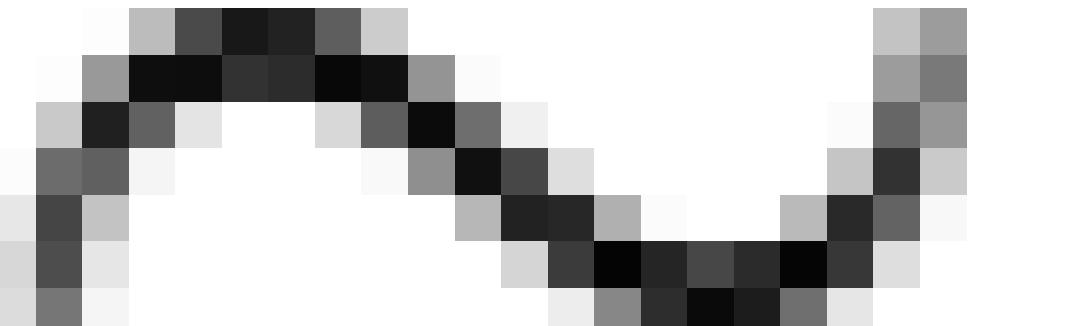


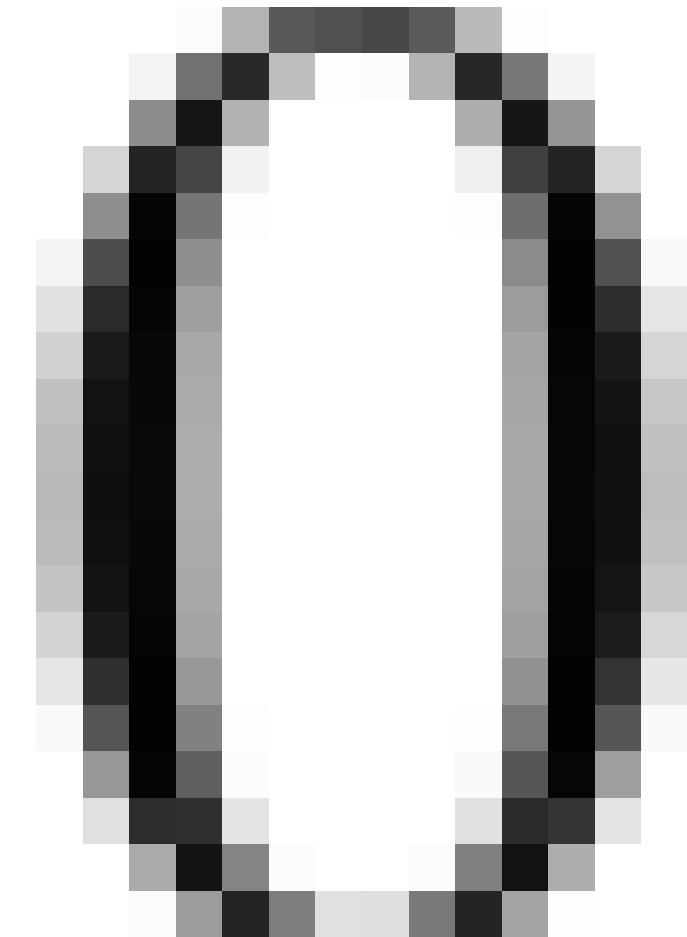
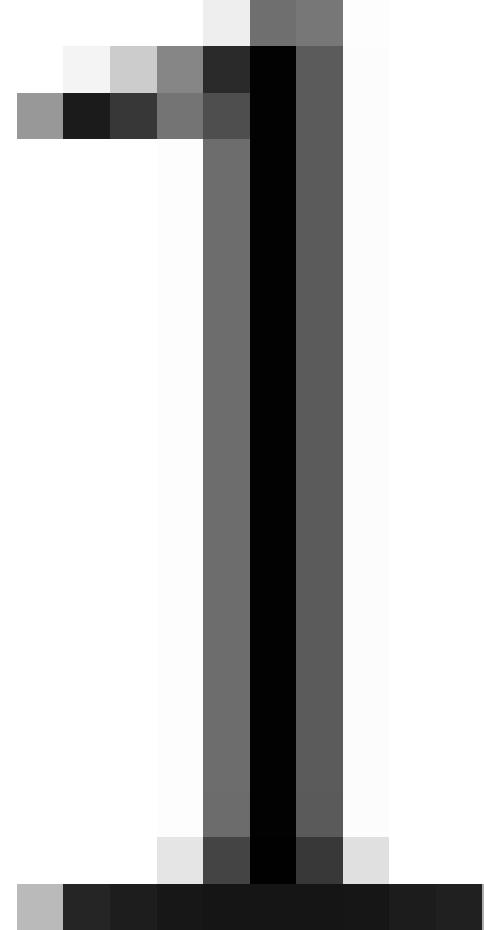
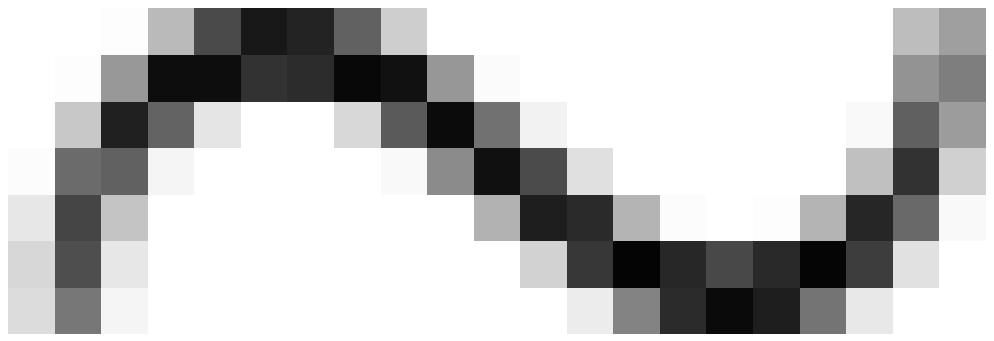




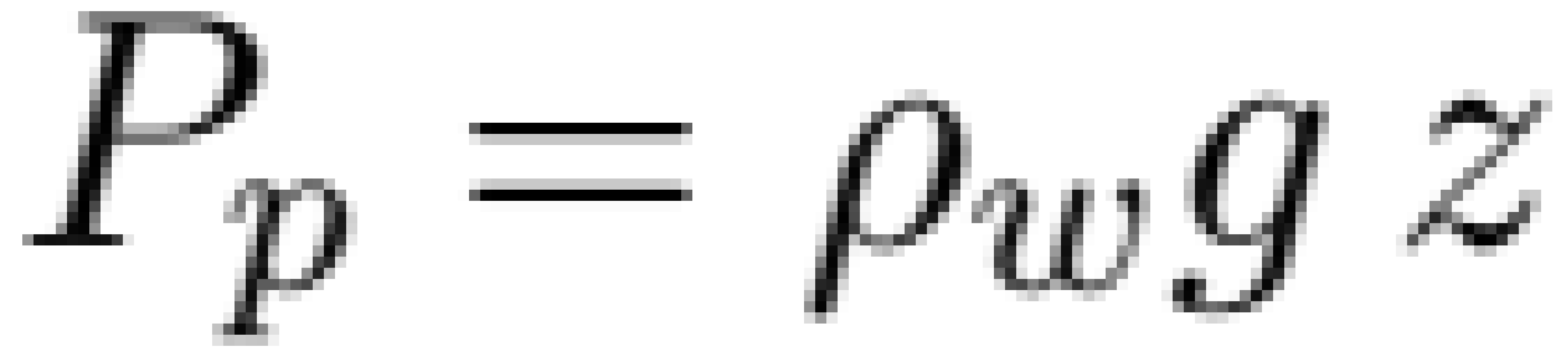


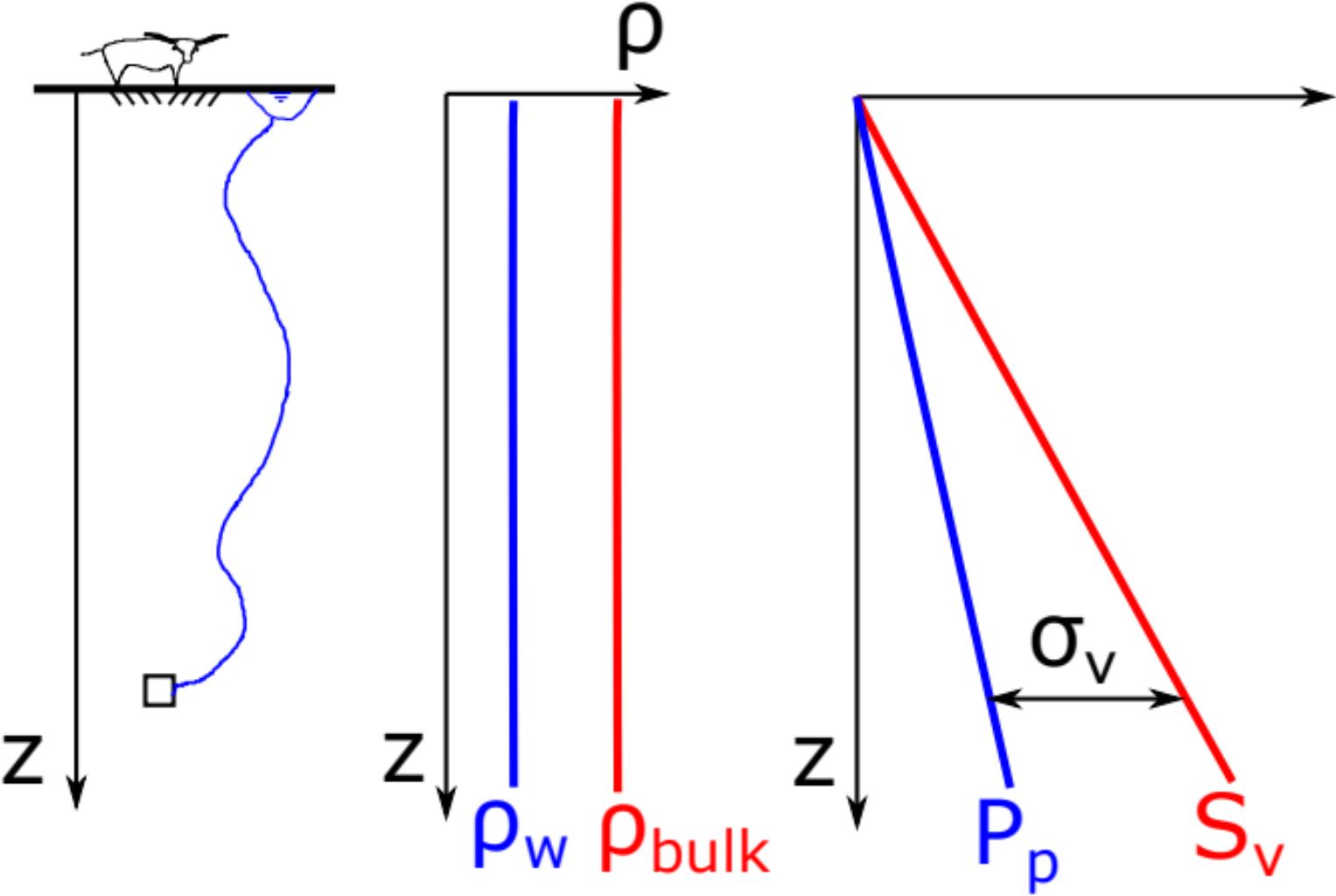




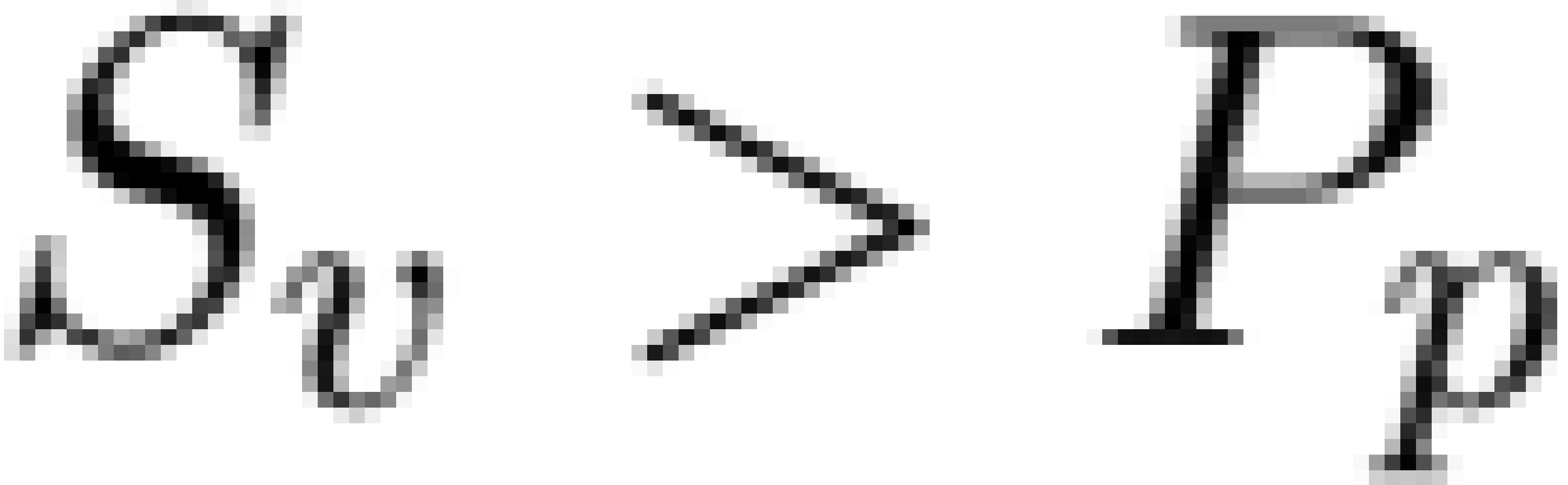








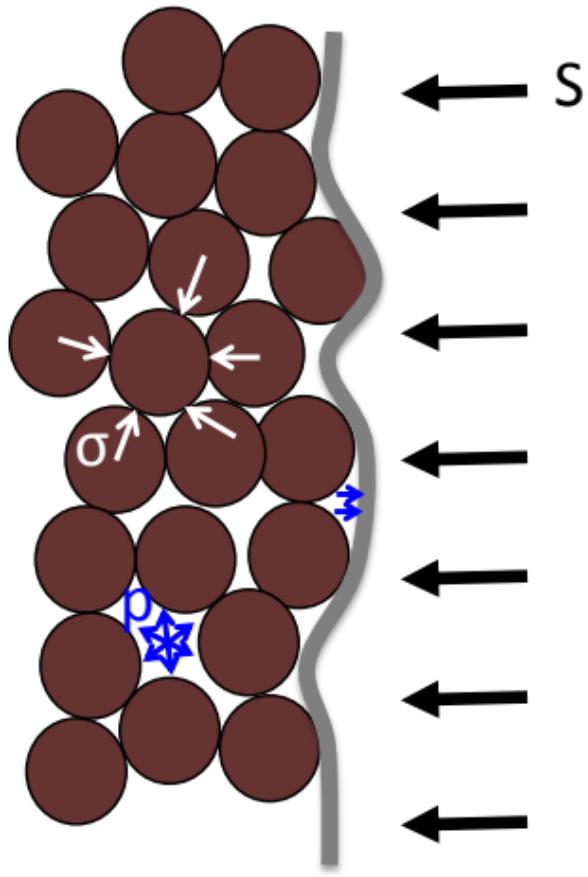




Effective stress =

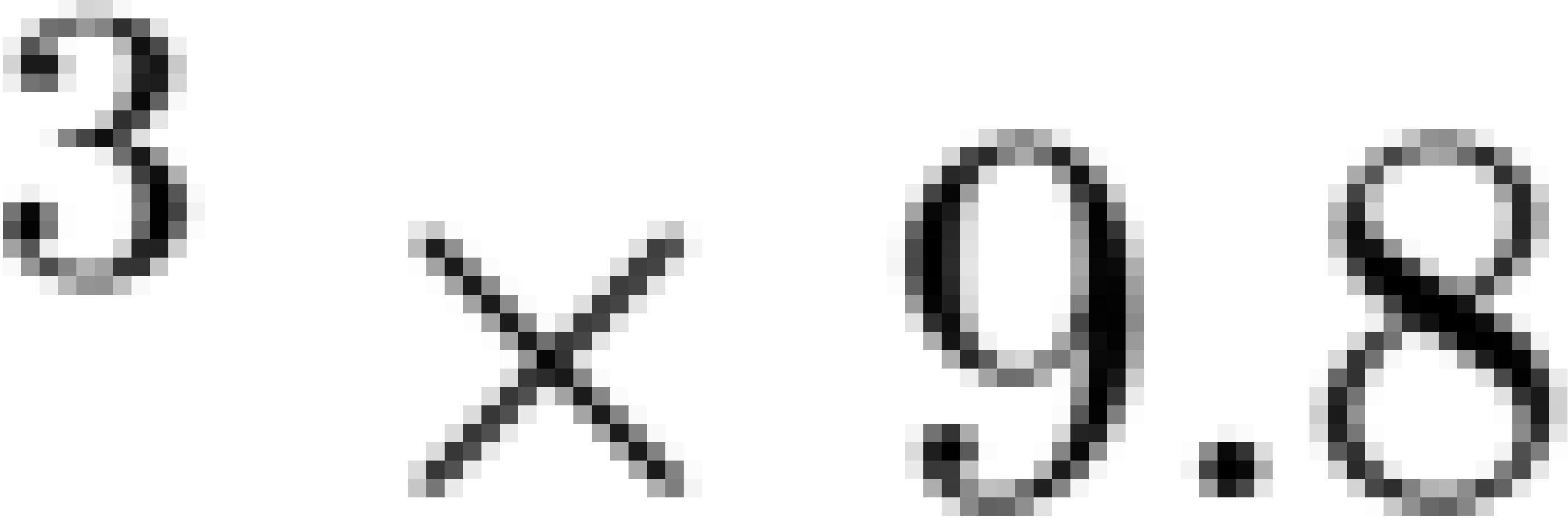
Total stress – Pore pressure

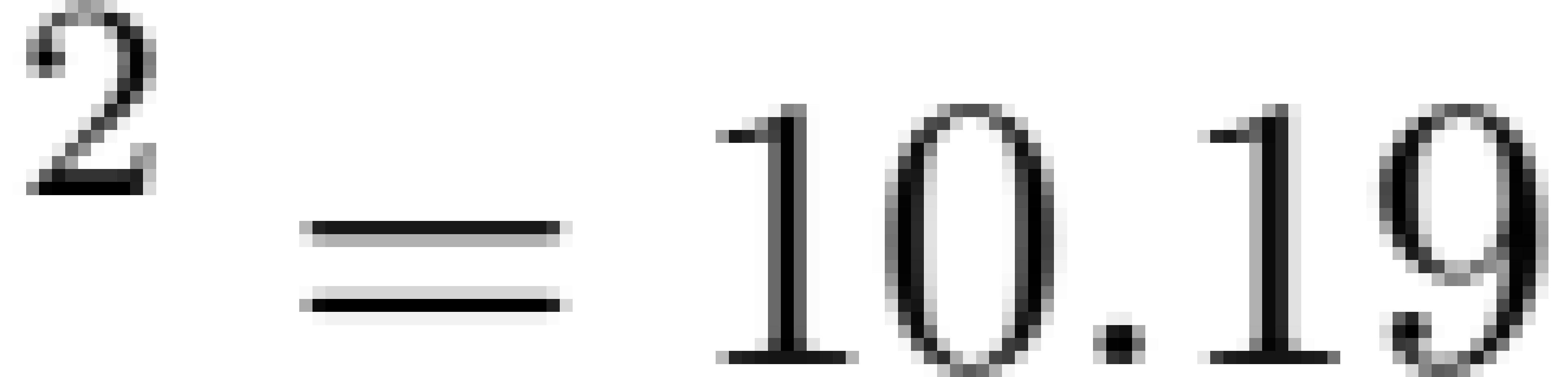
$$\sigma = S - p$$





$dP$   $P$   $\rho_{w,9}$   $1040$   
 $dz$





dsu = Paulk 2350

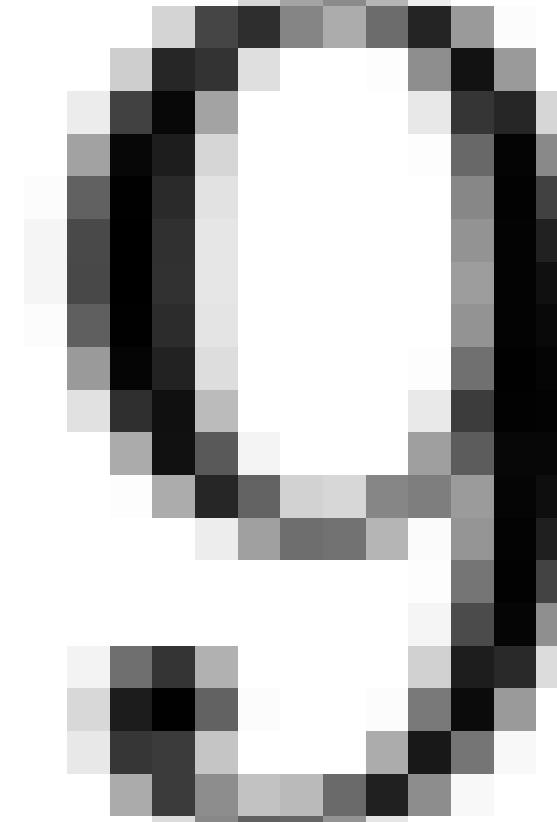
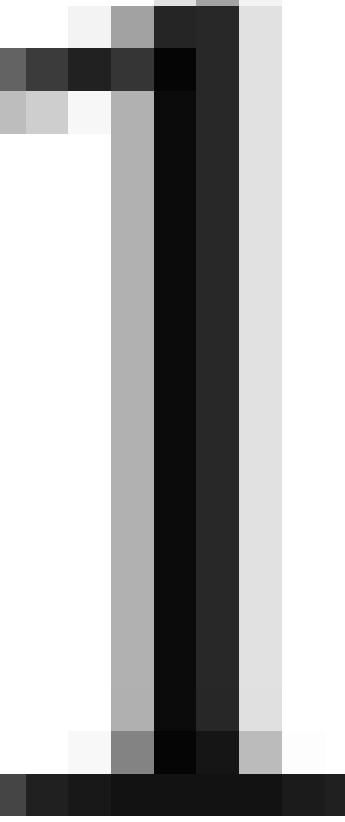
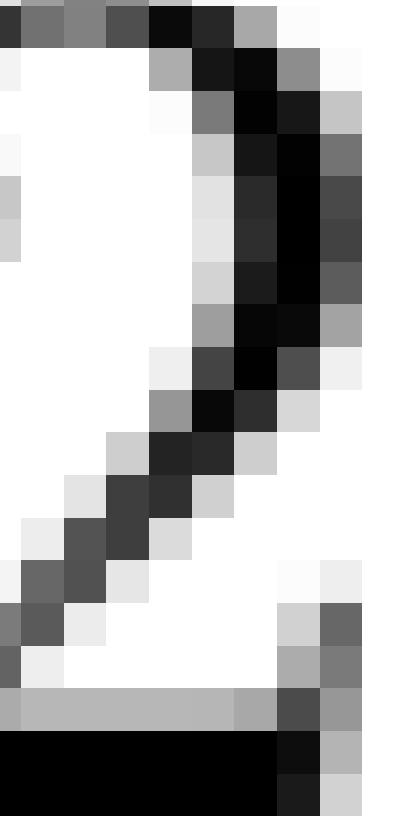
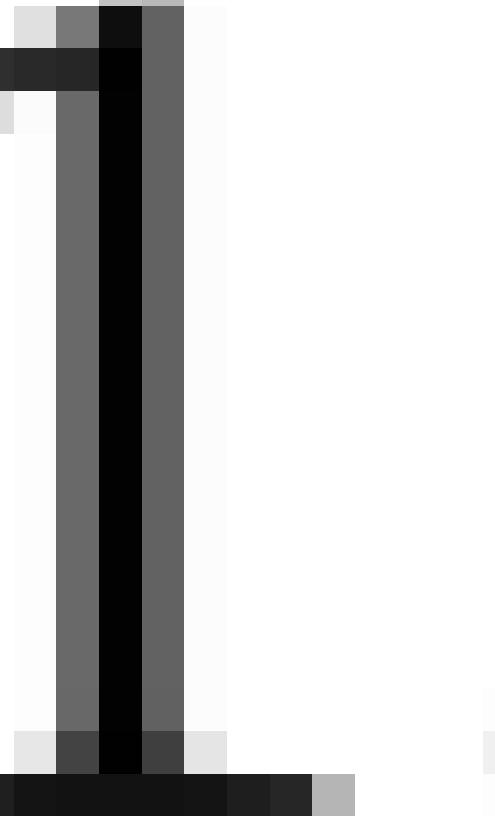
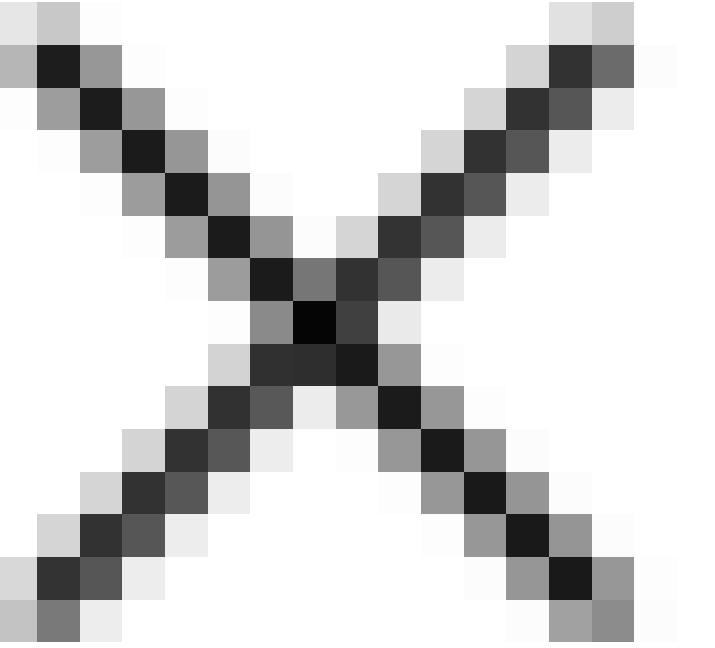


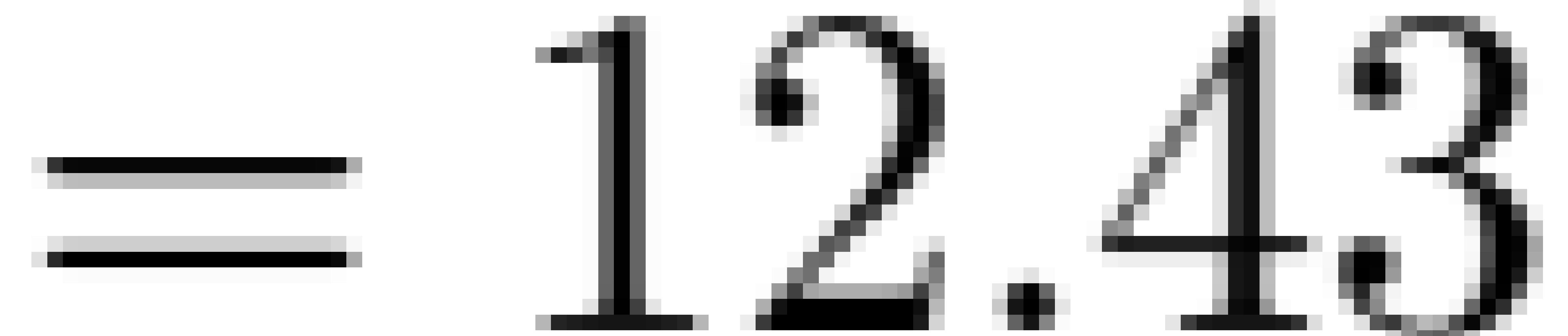
P  
P

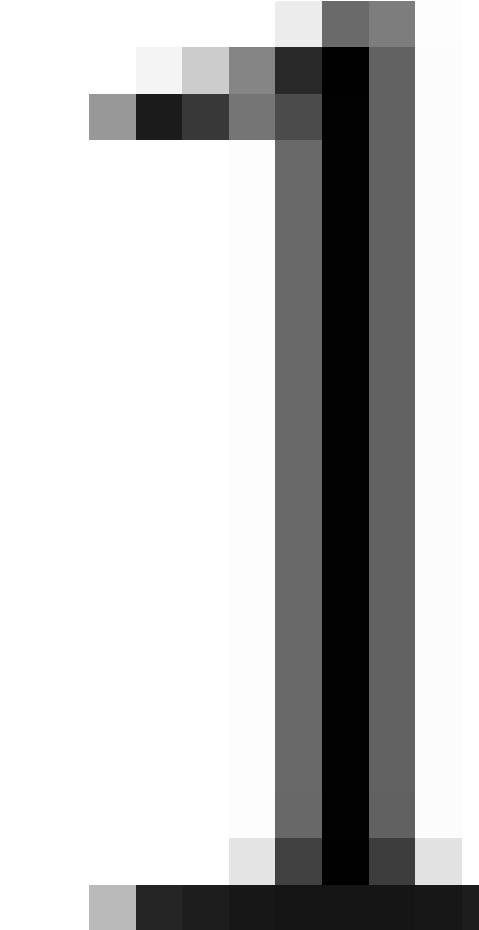
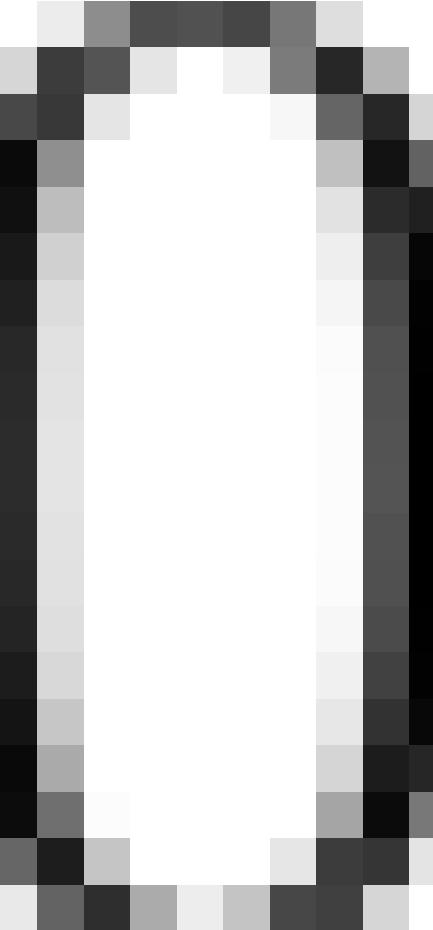
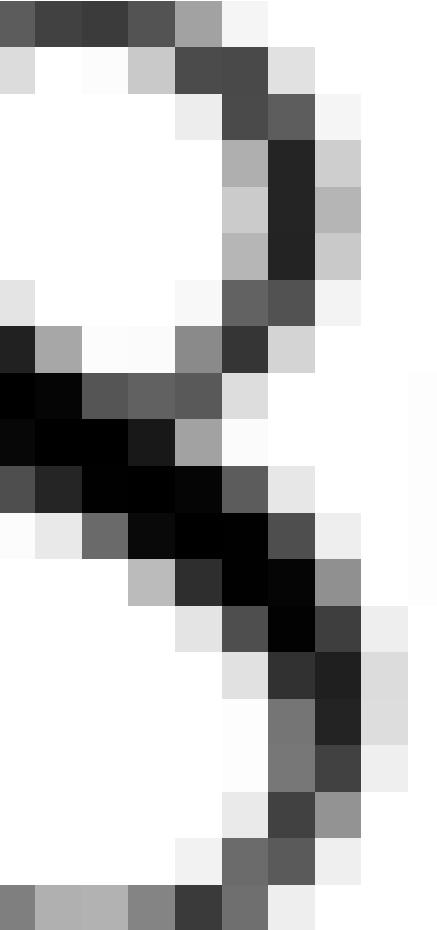
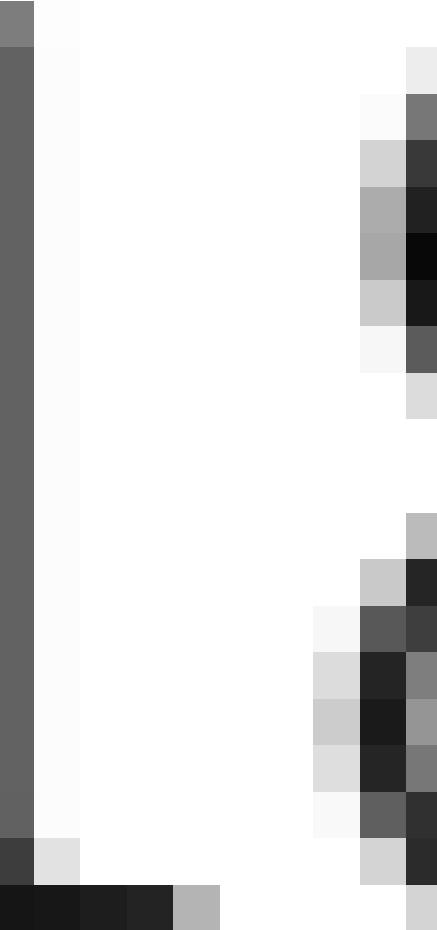
dP  
dP  
dZ  
dZ

—  
—

10.  
19







*d<sub>5</sub>*

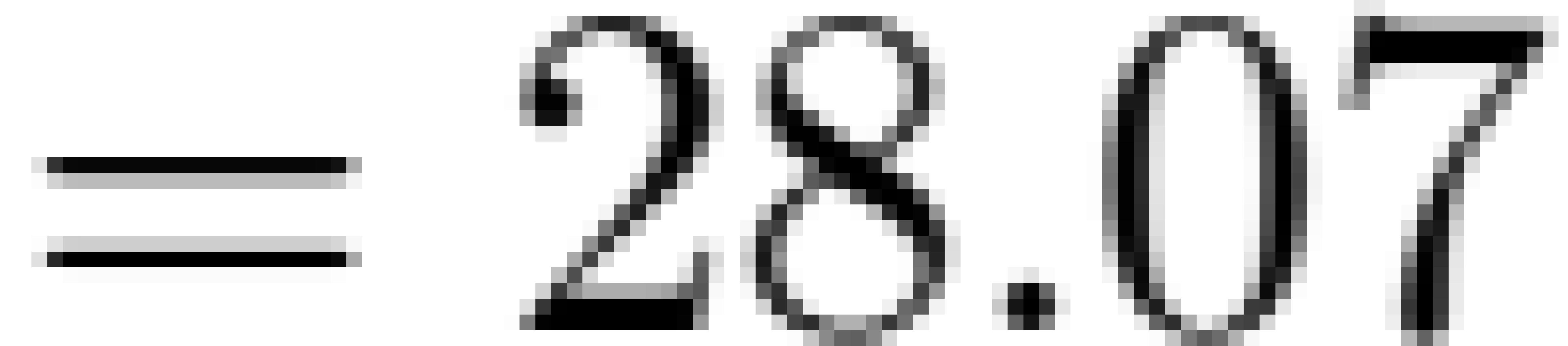
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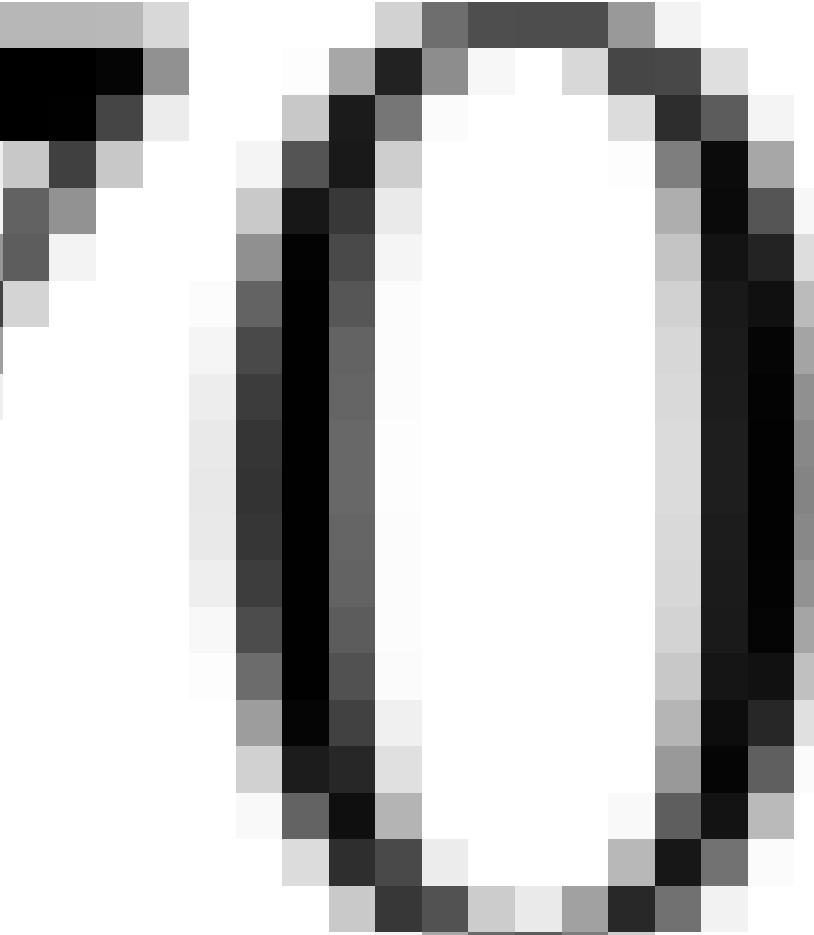
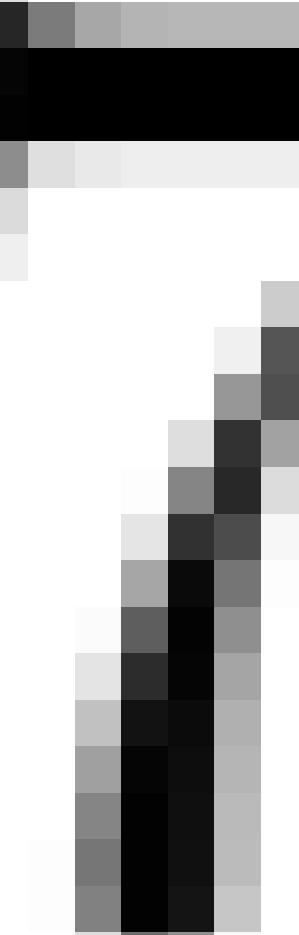
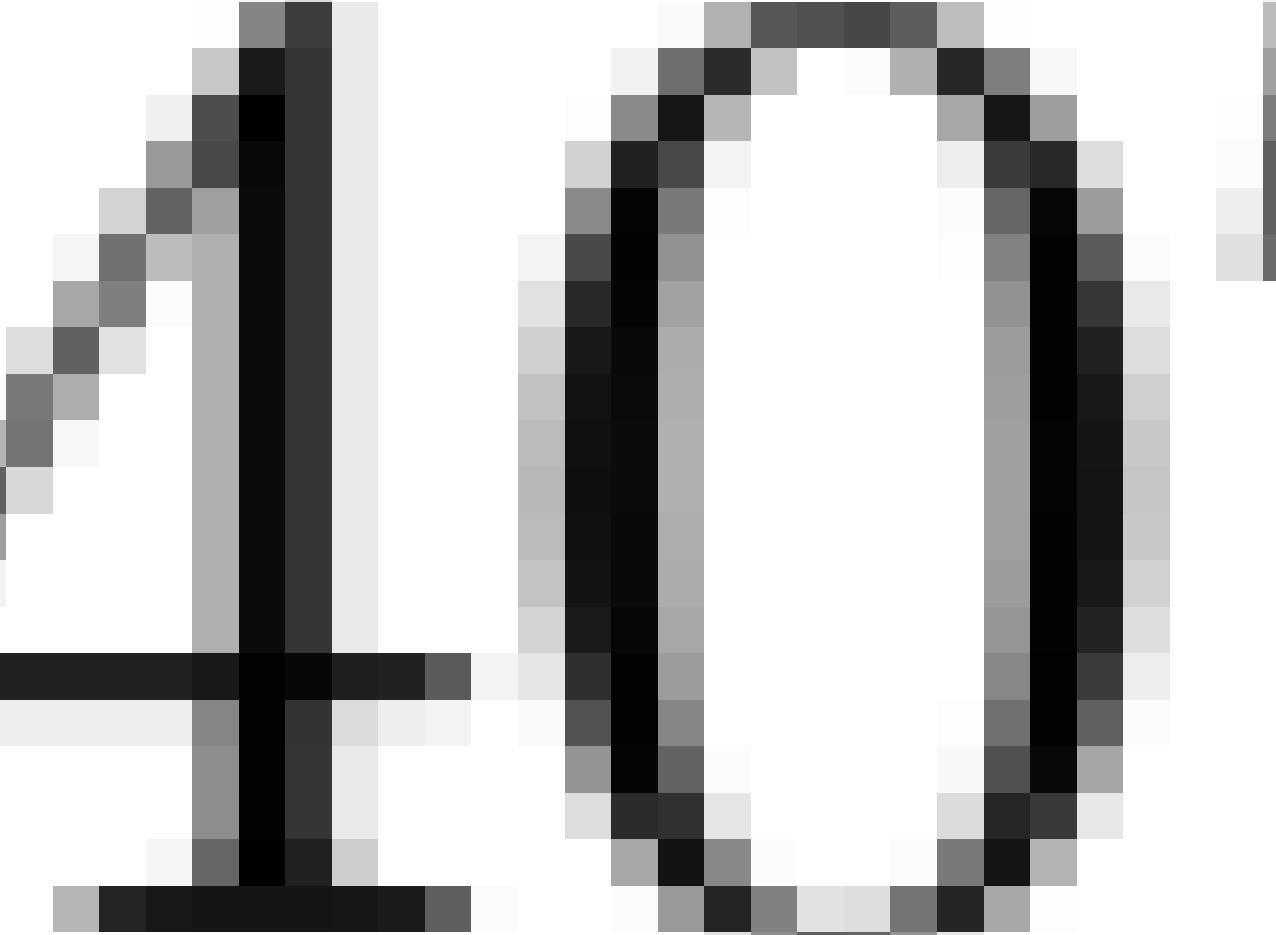
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*d<sub>8</sub>*

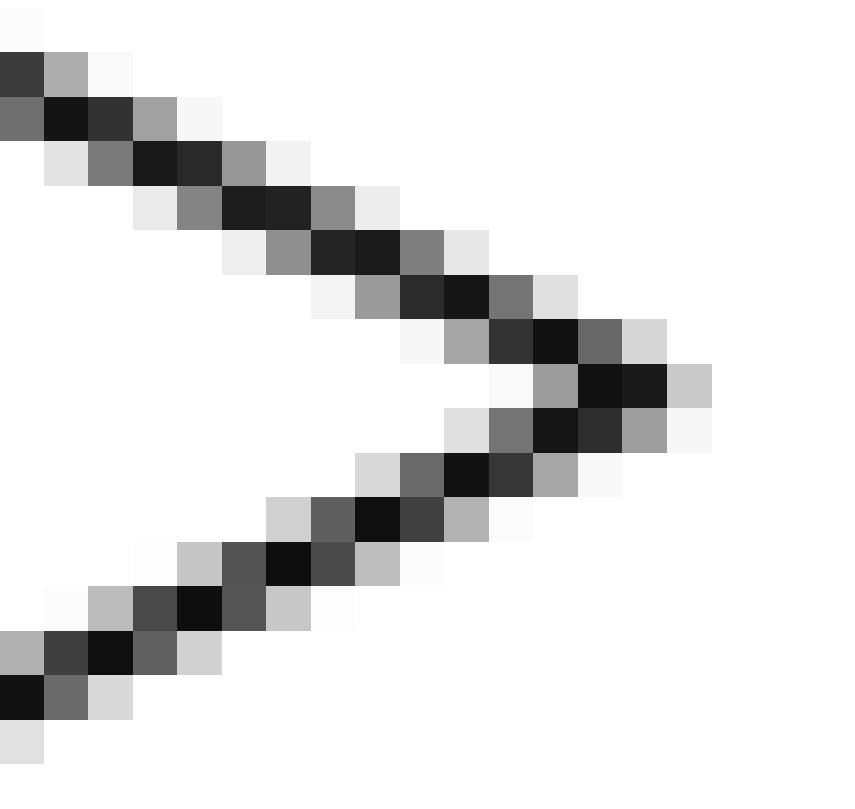
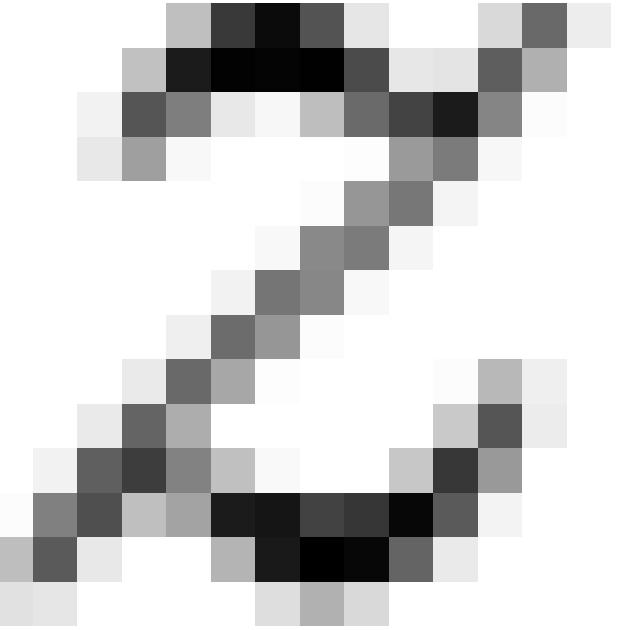
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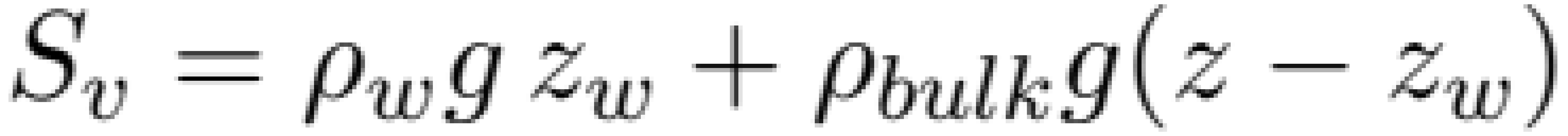
23.03

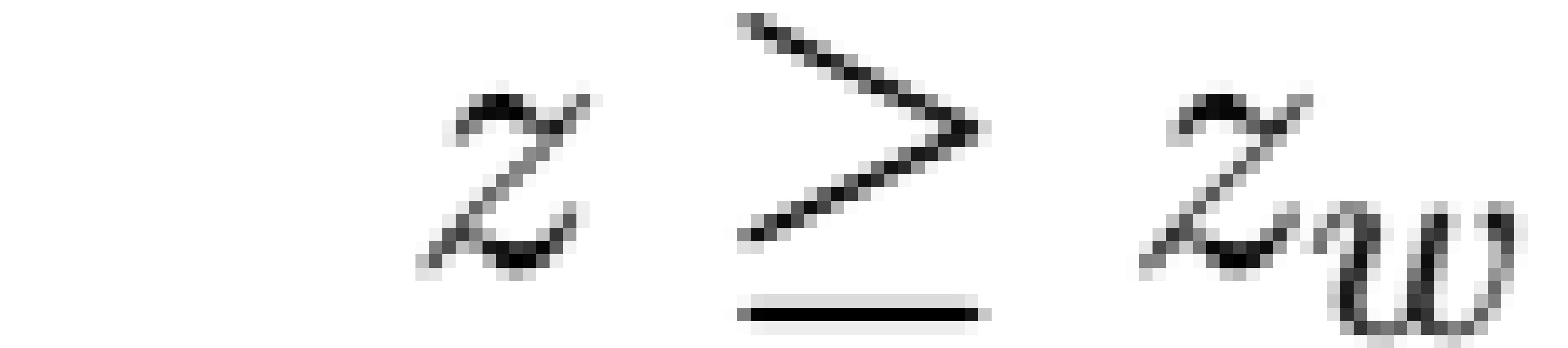


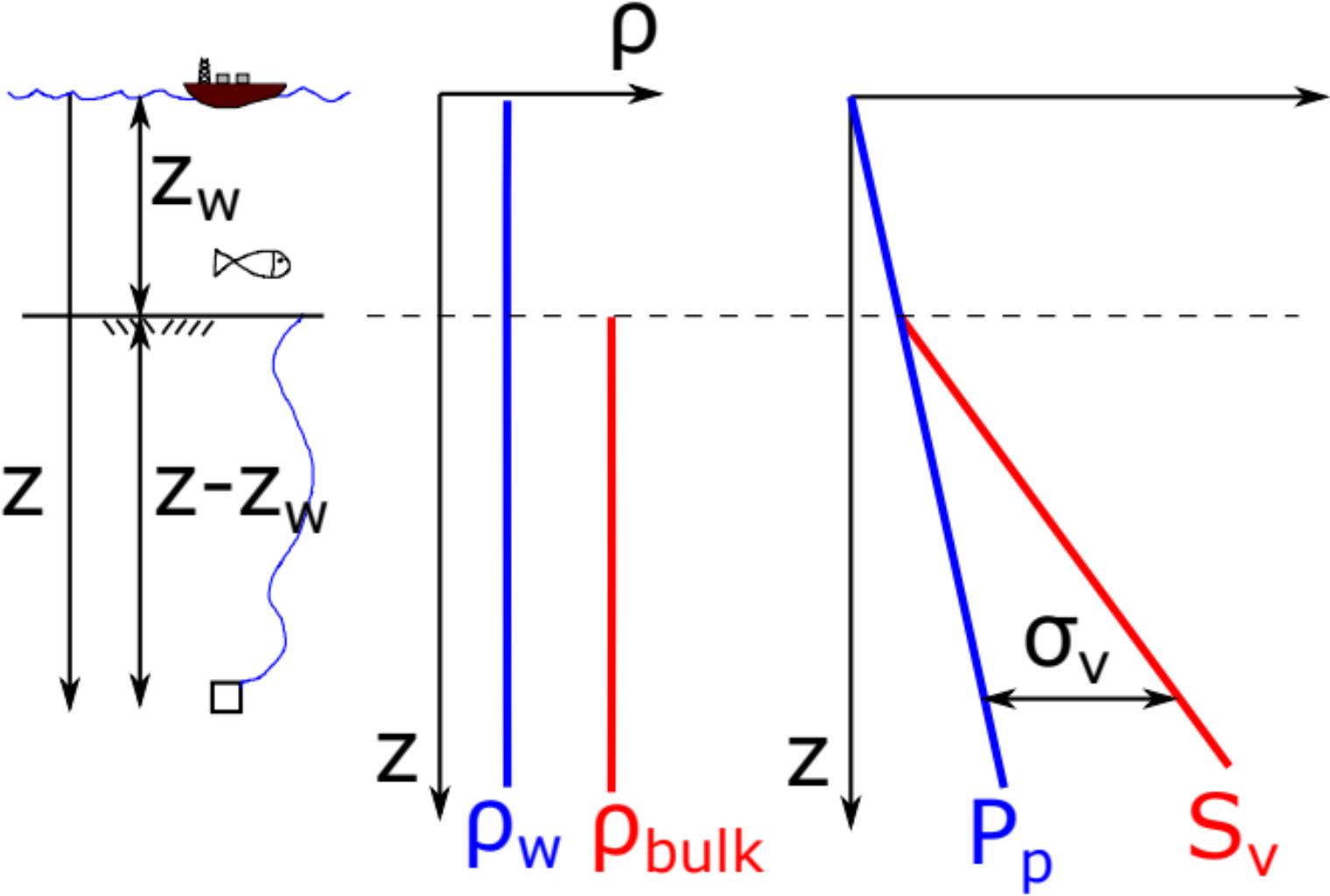




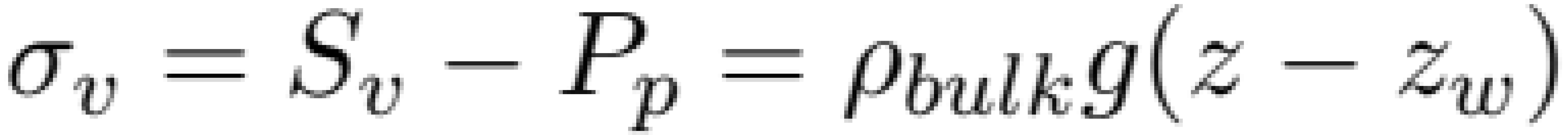




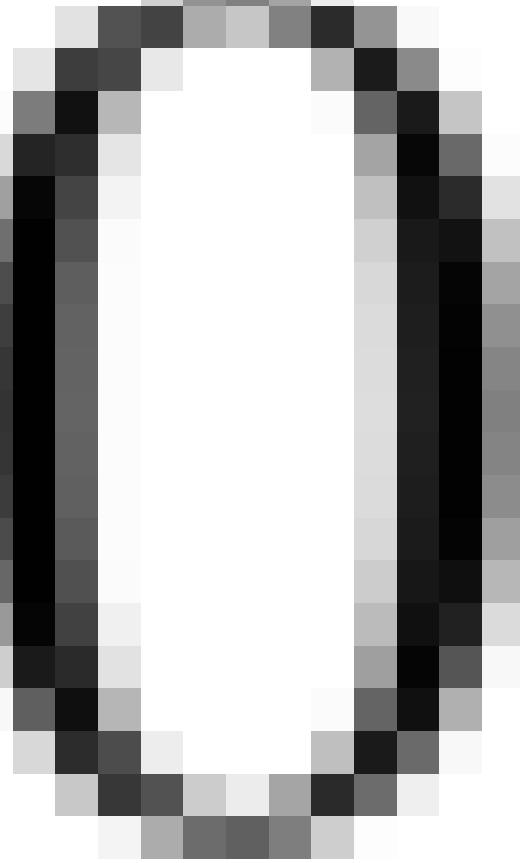
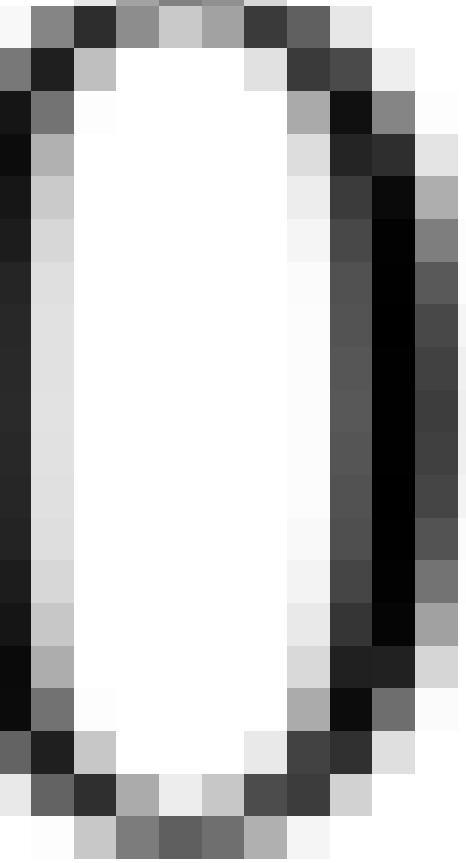
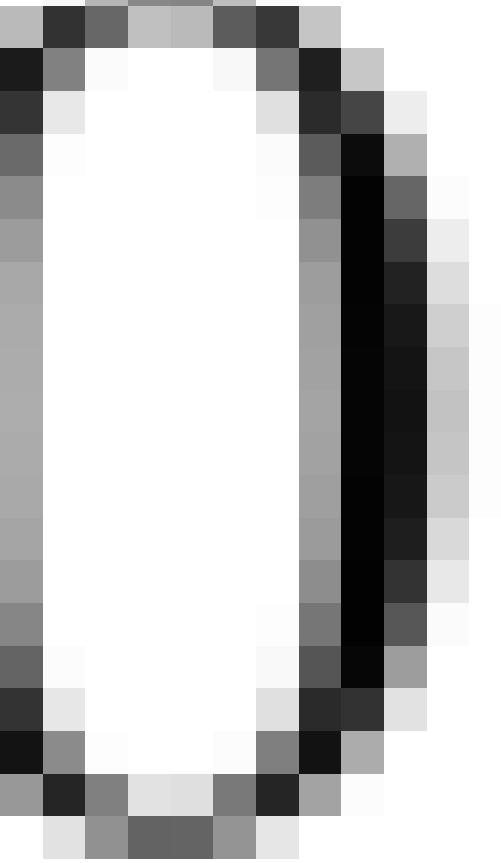
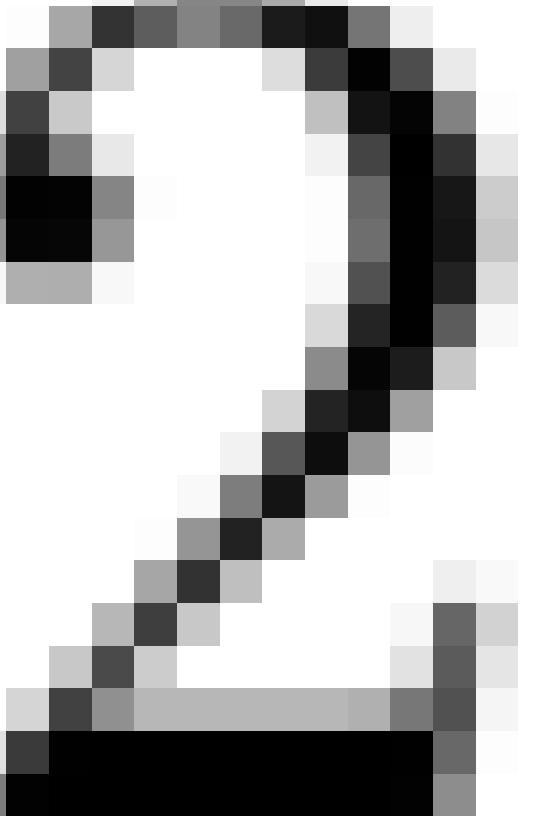
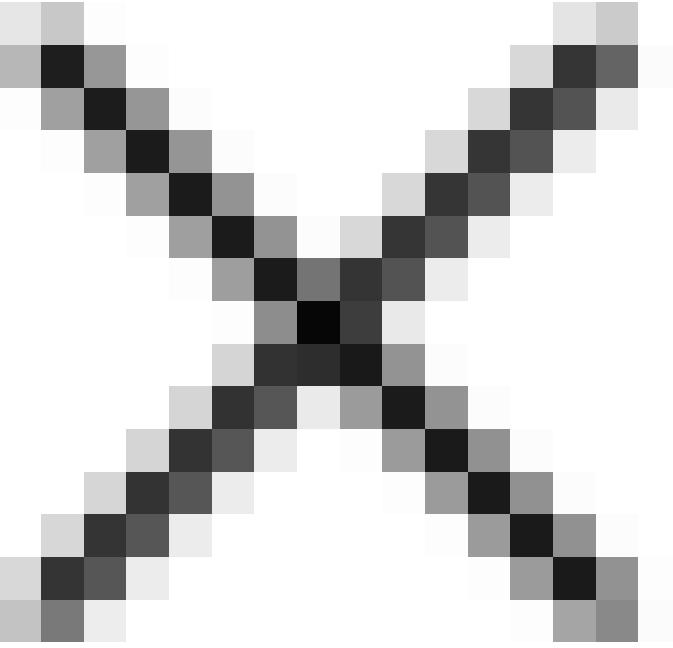


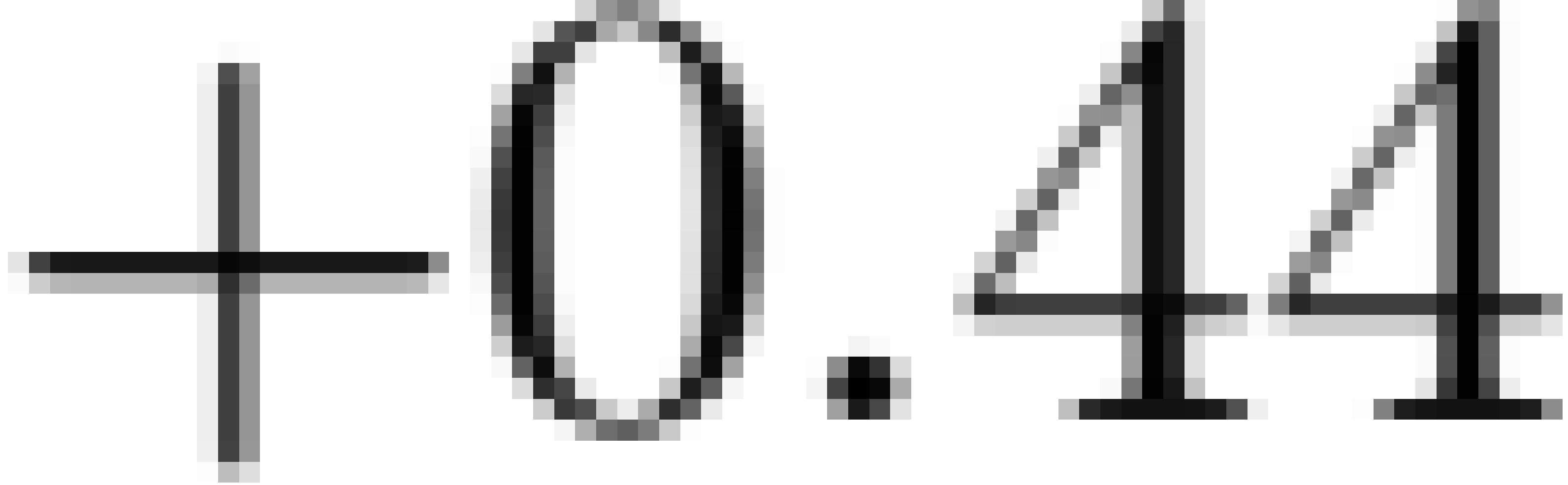


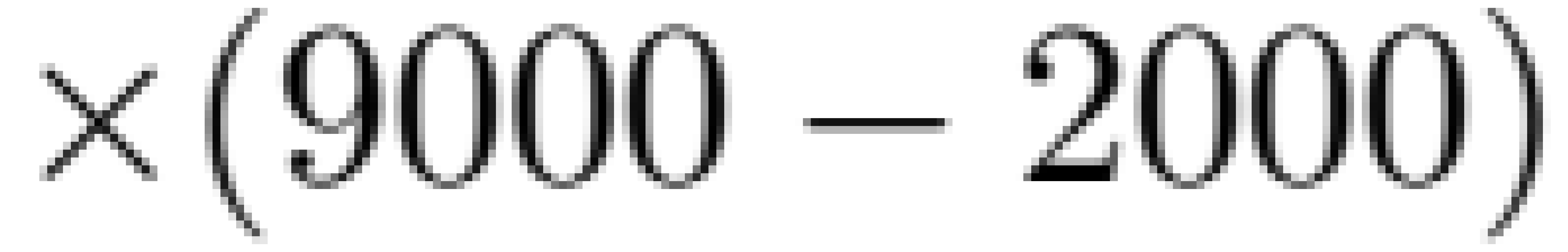


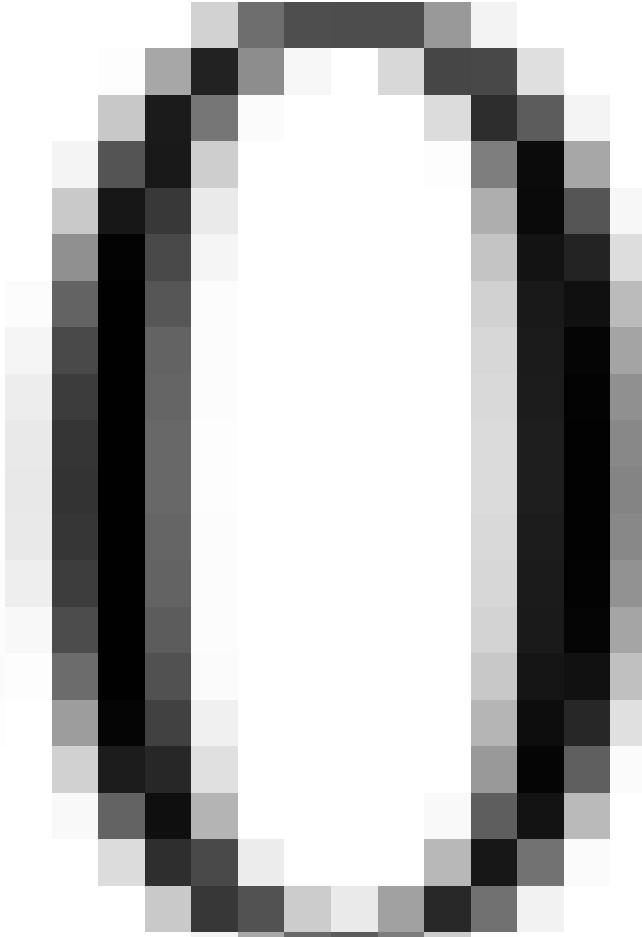
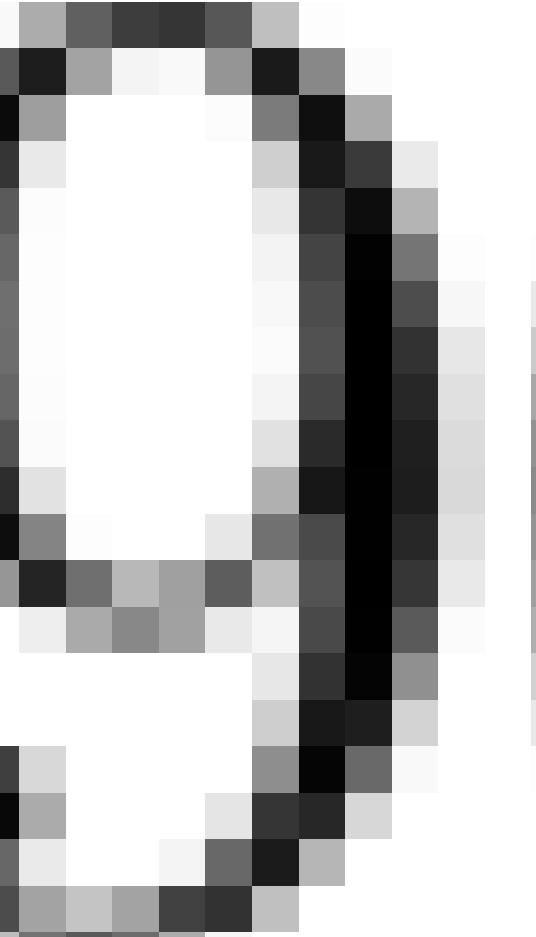
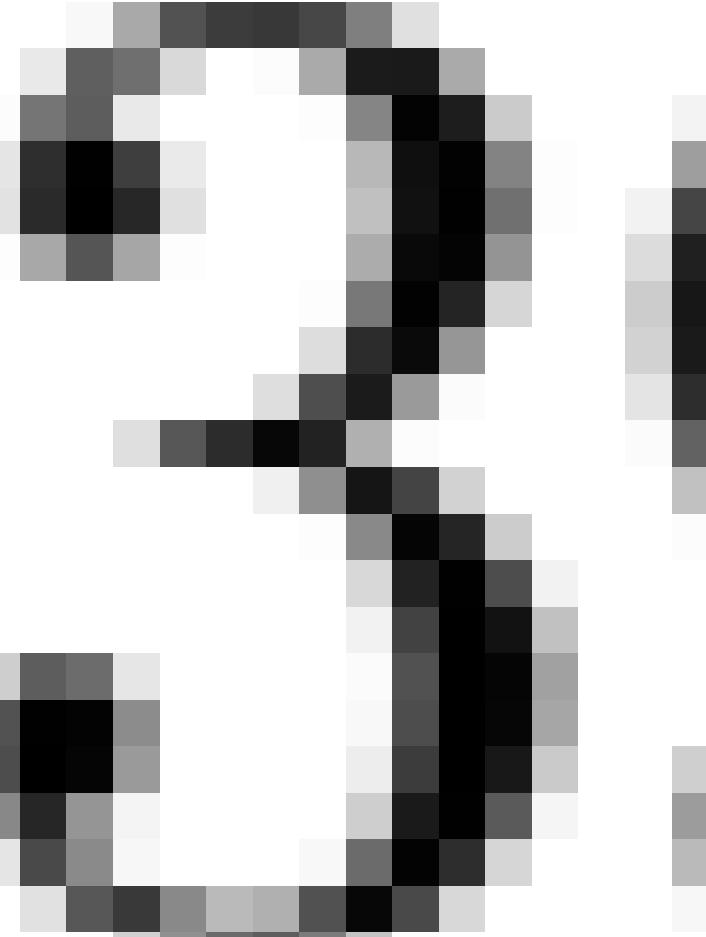


$$P_p = \rho w g z_w + \frac{dP}{dz} (z - z_w) = 0.44$$

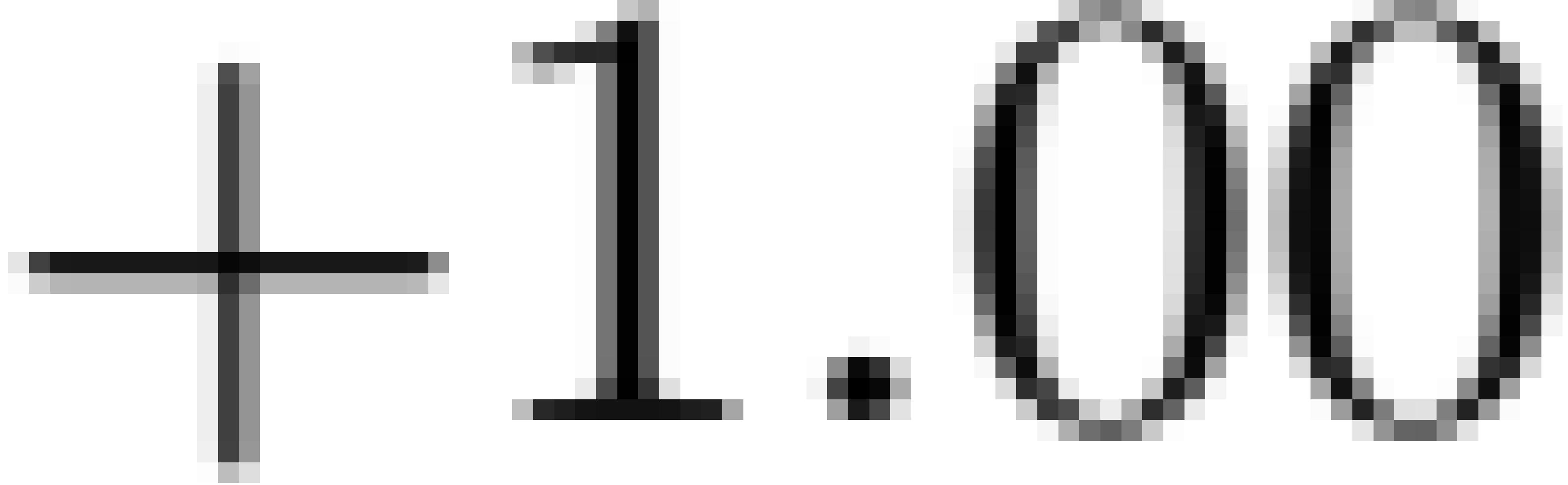


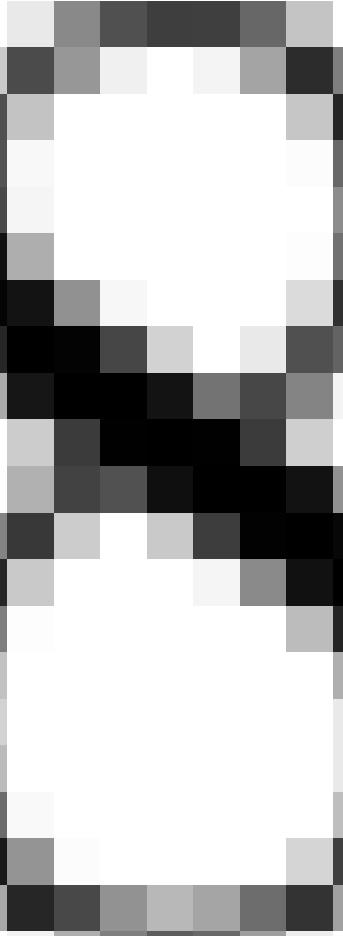
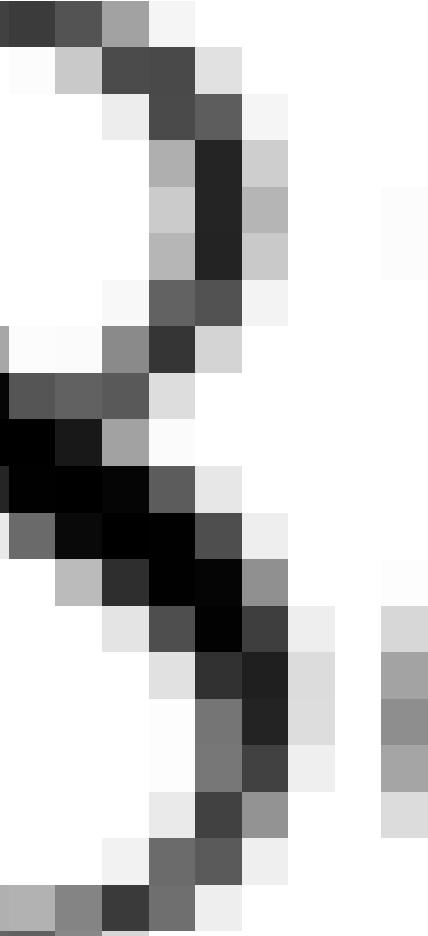
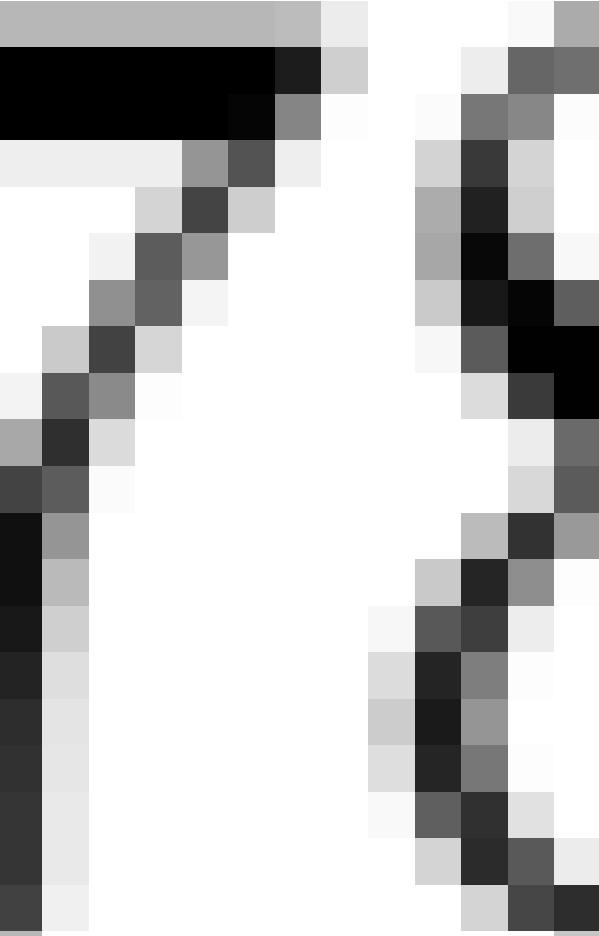


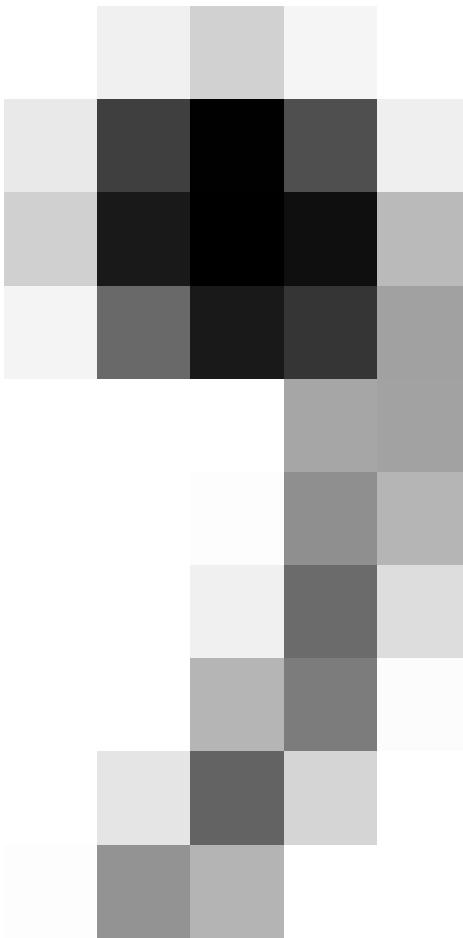


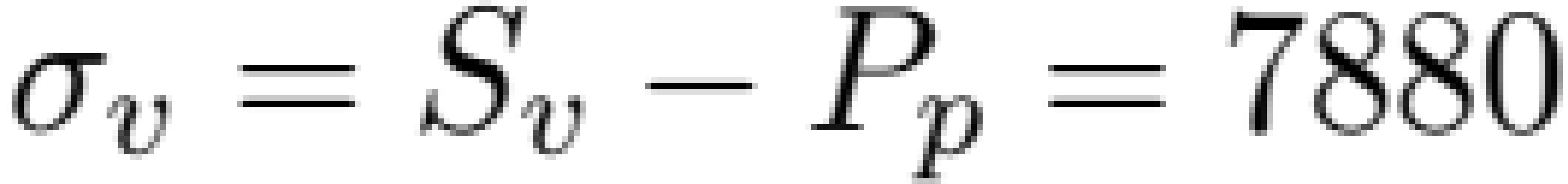


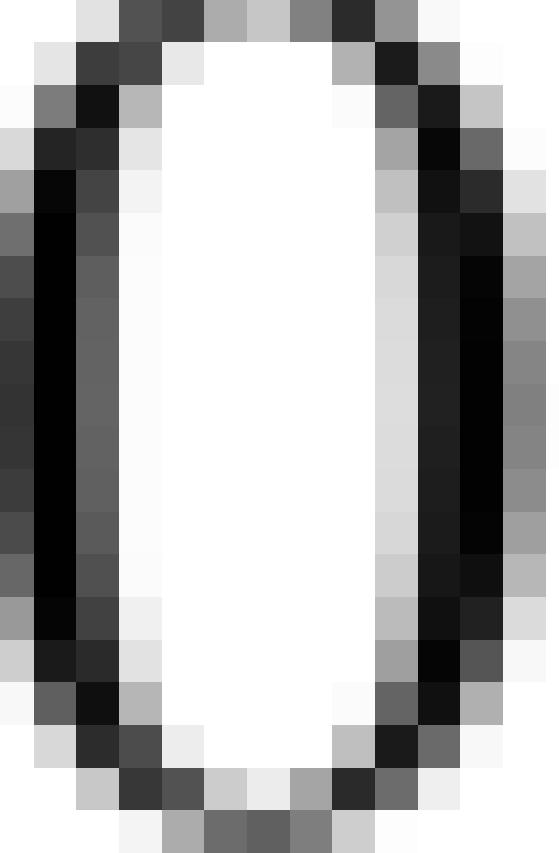
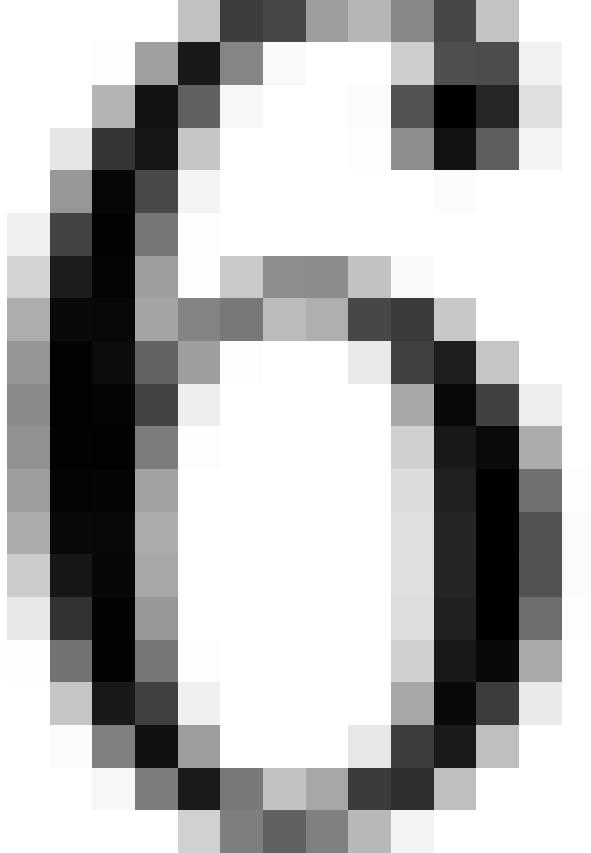
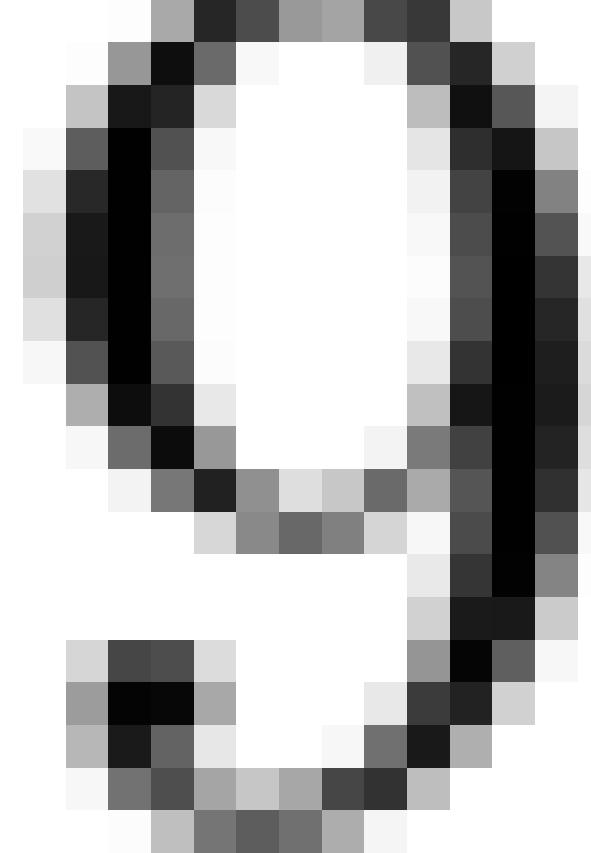
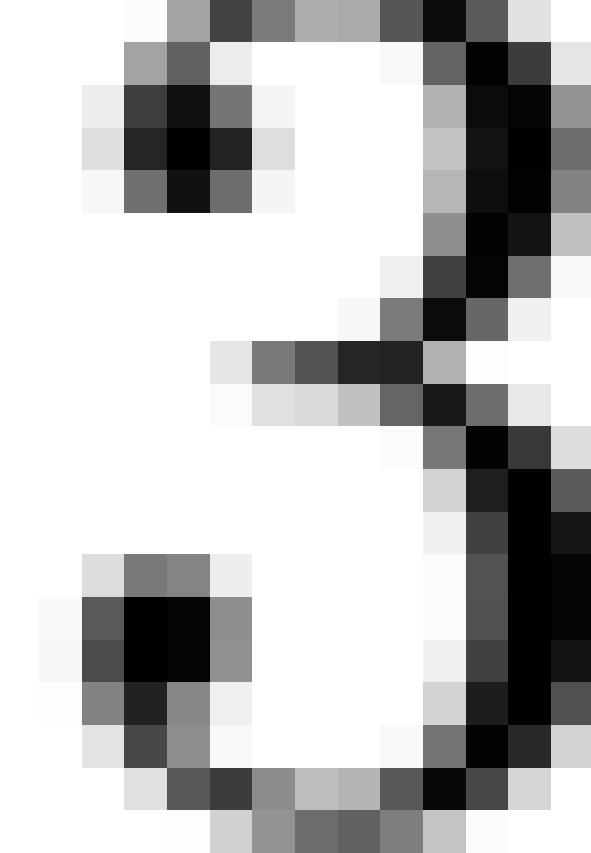
$$S_u = \rho_{u,g} z_u + \frac{dS_u}{dz} (z - z_u) = 0.44$$

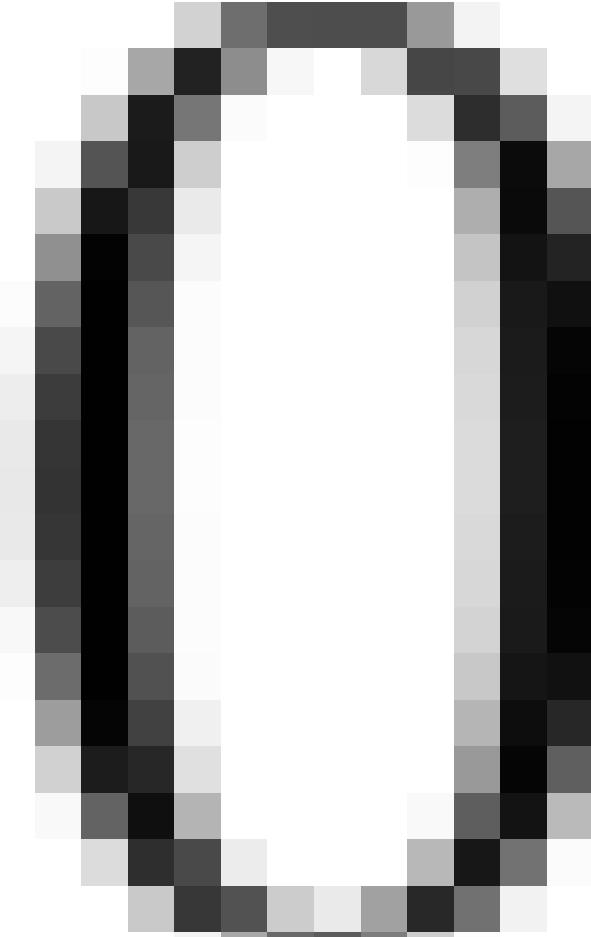
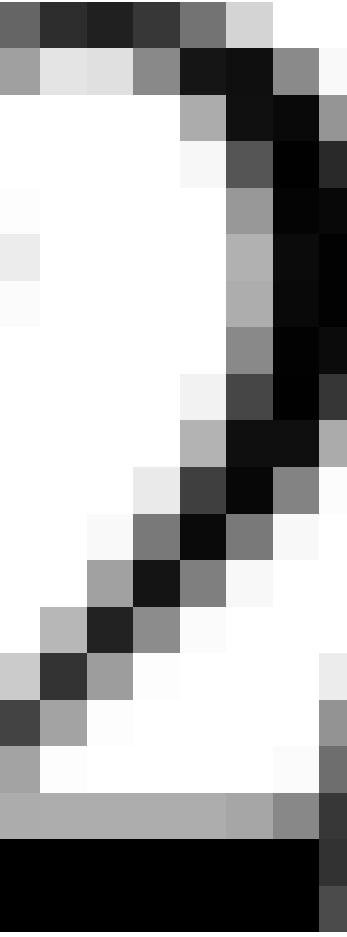
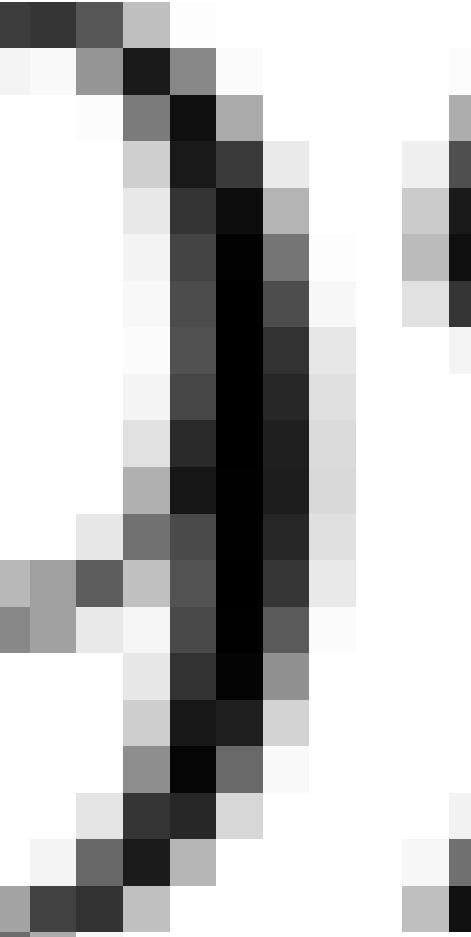
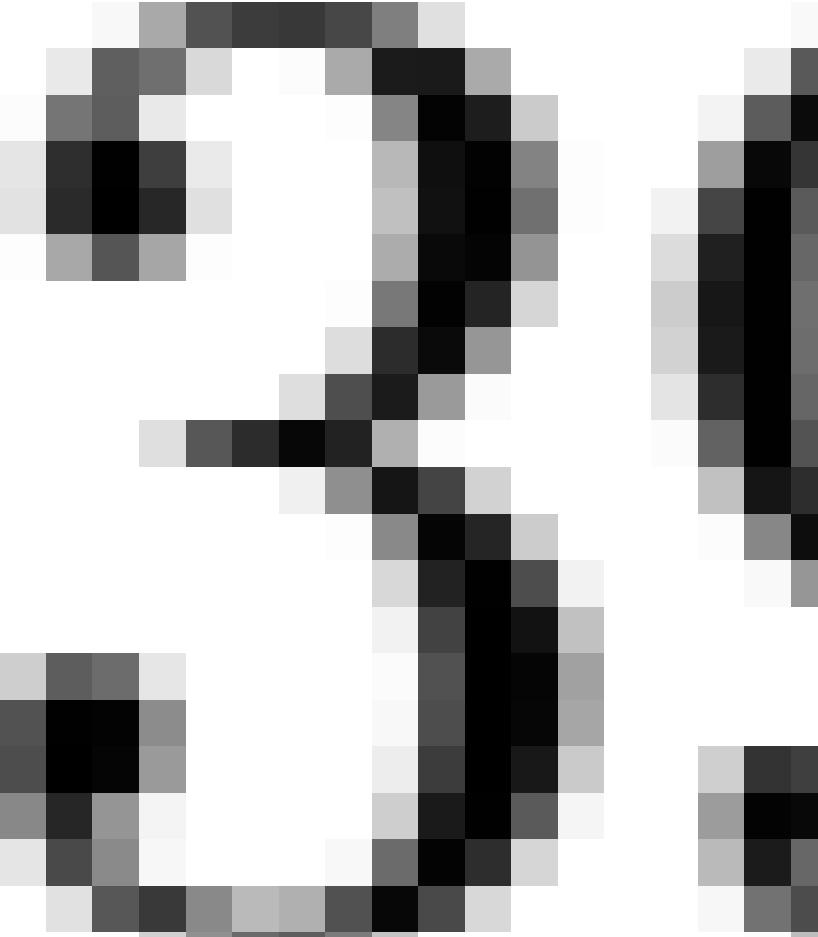


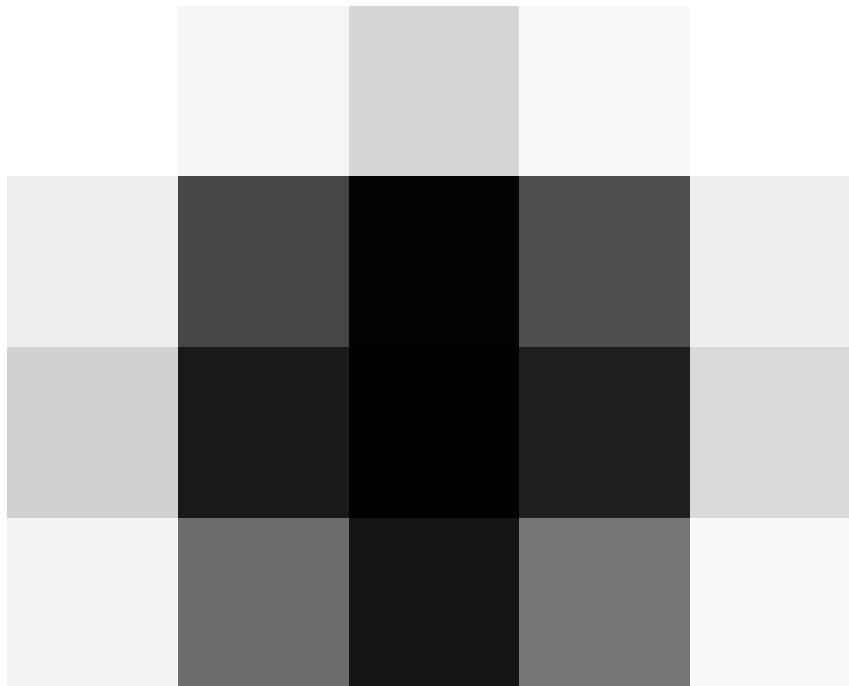


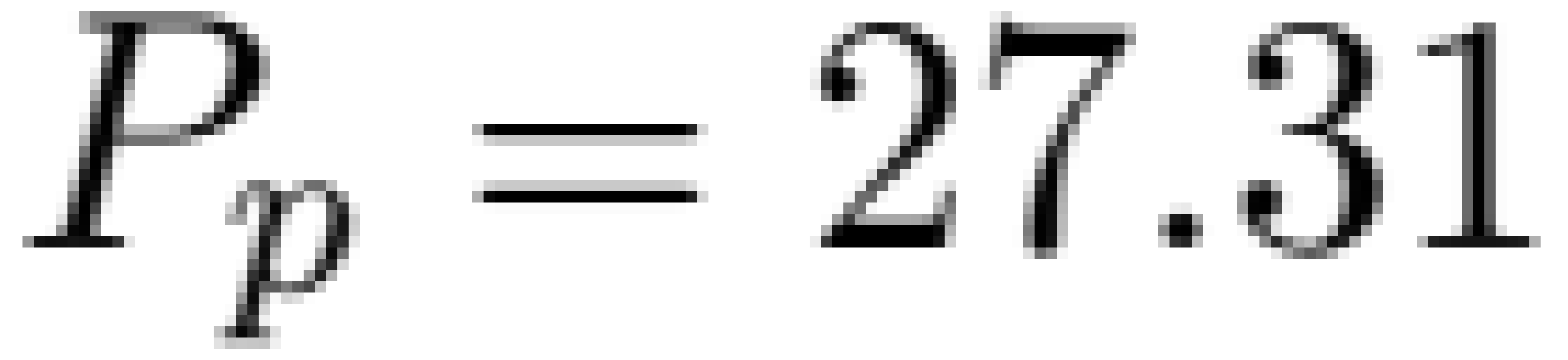






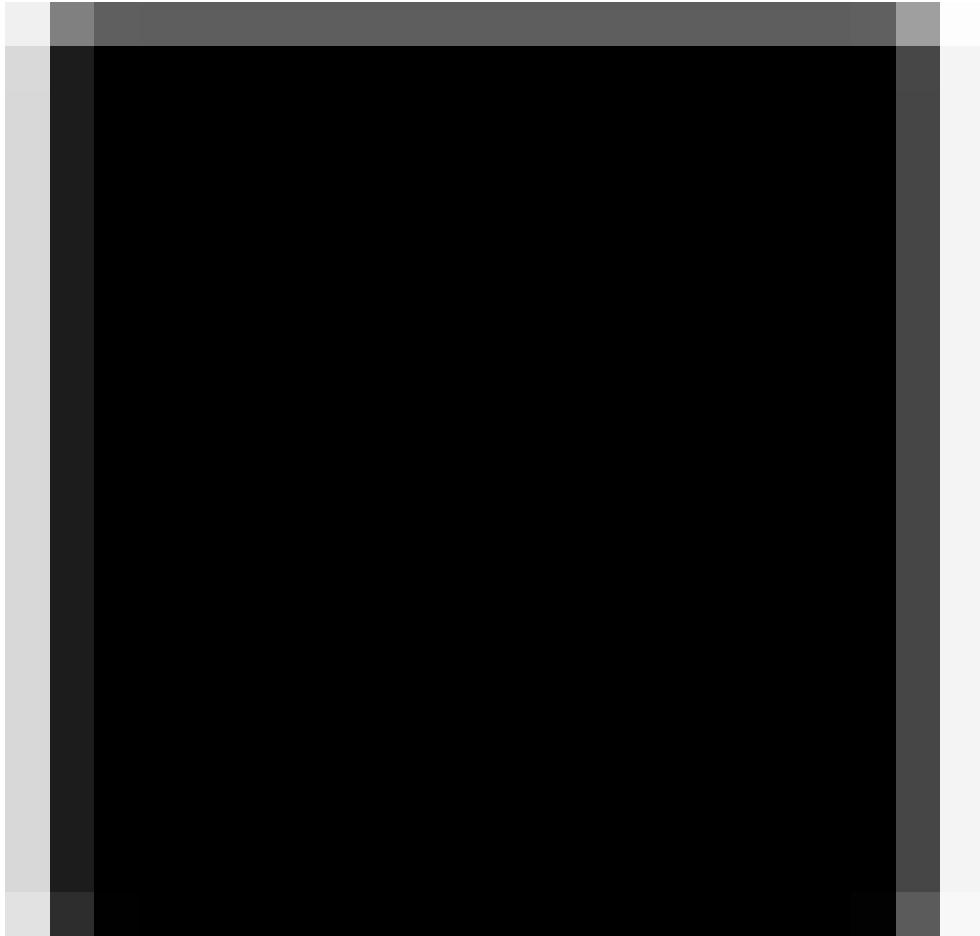








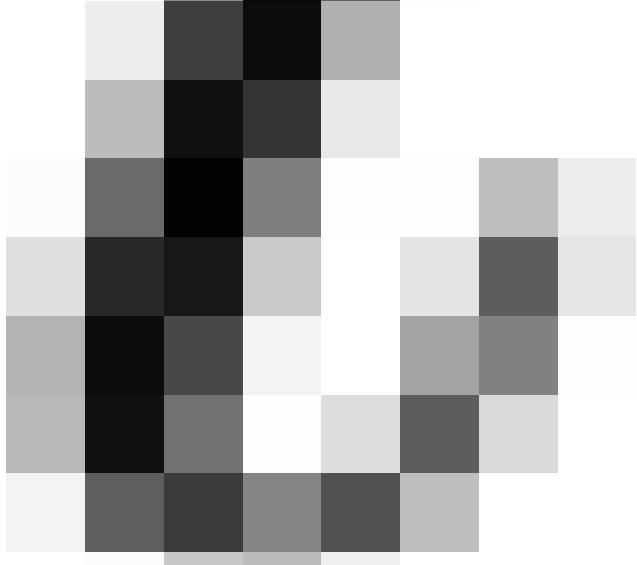
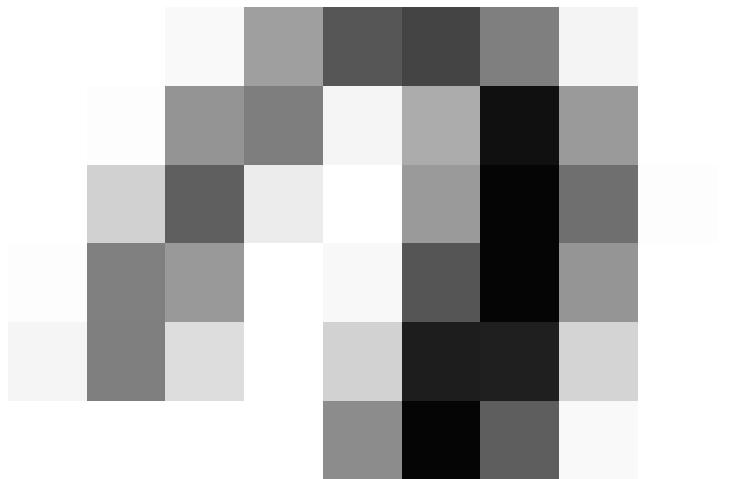
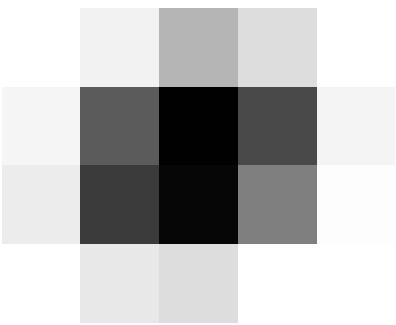


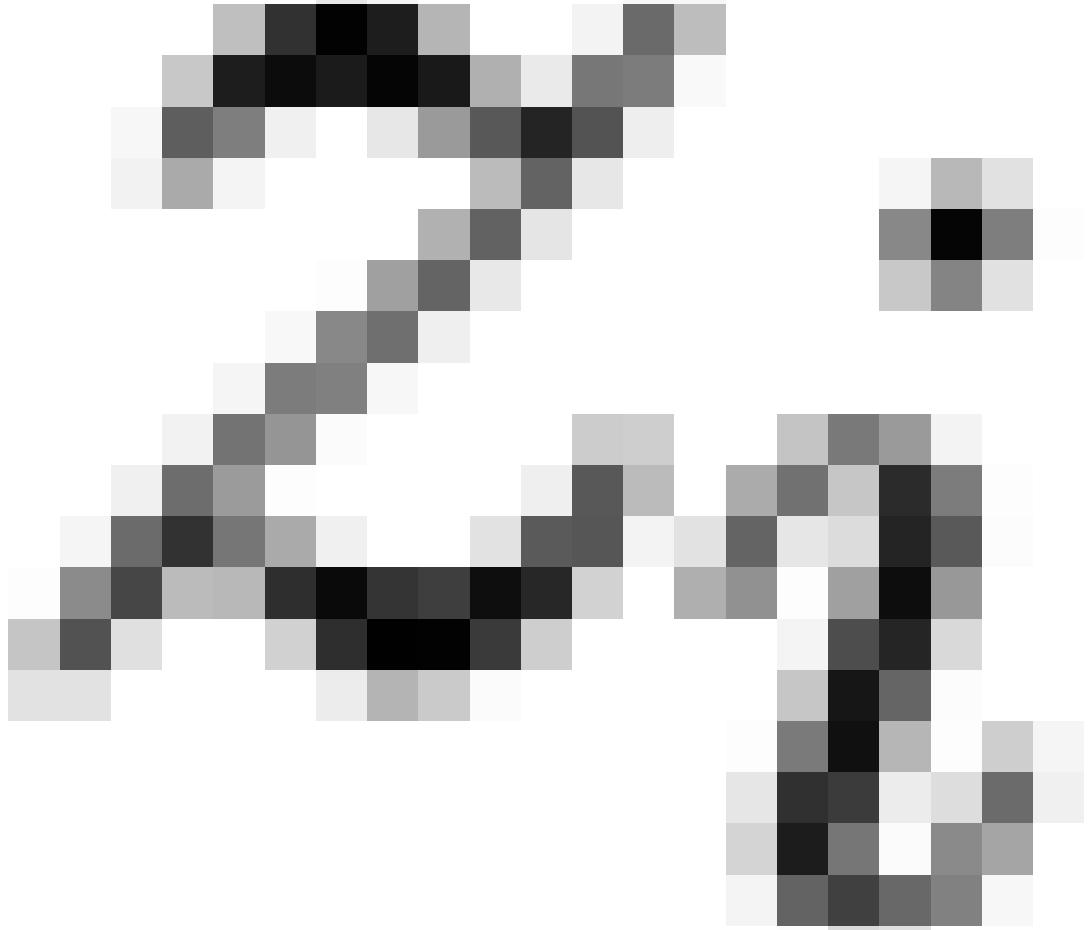


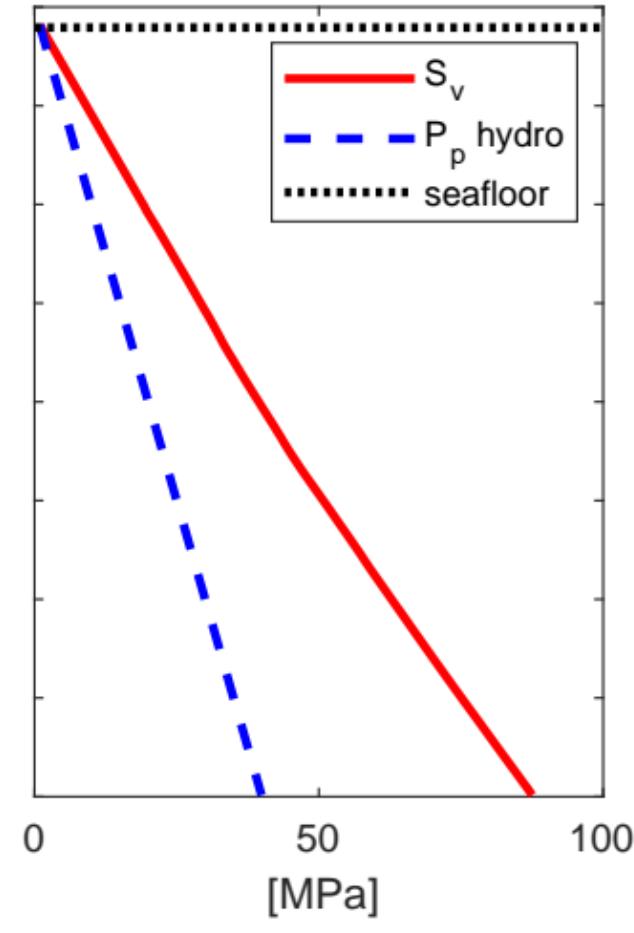
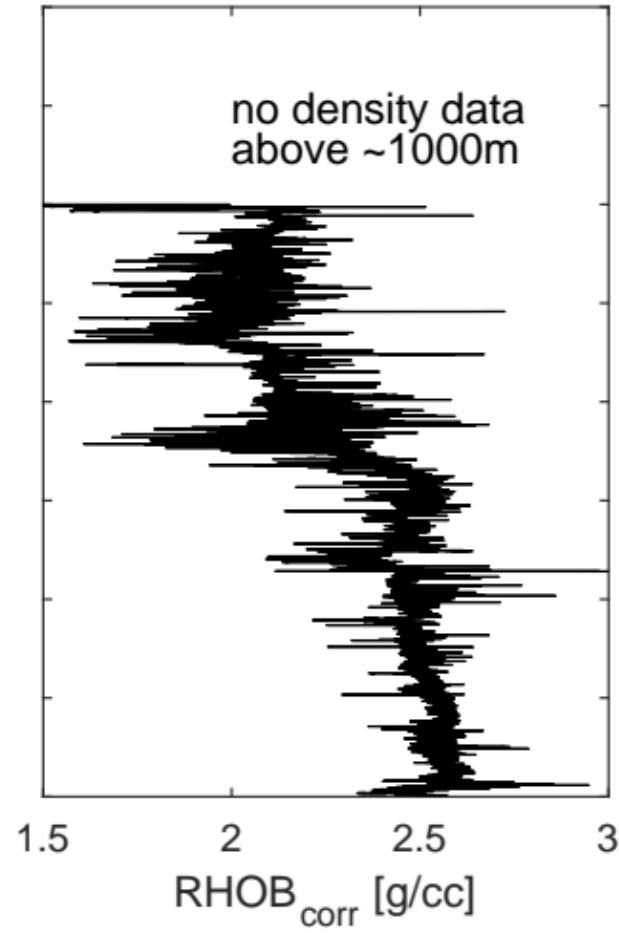
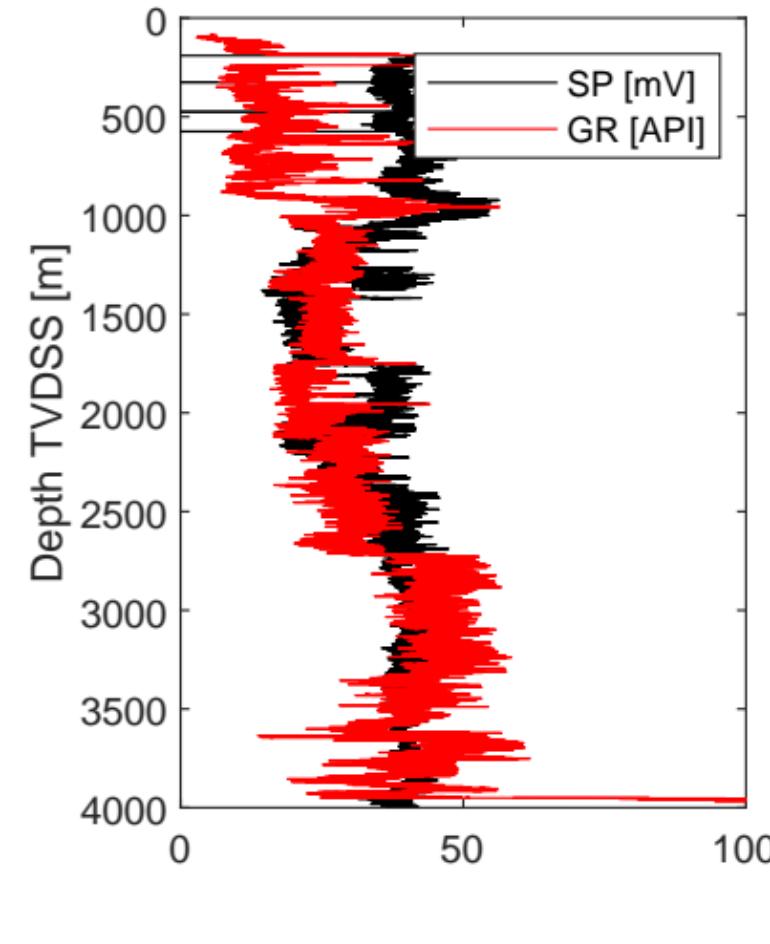
$$S_U(z) = \int_0^z \rho_{bulk}(z') dz'$$

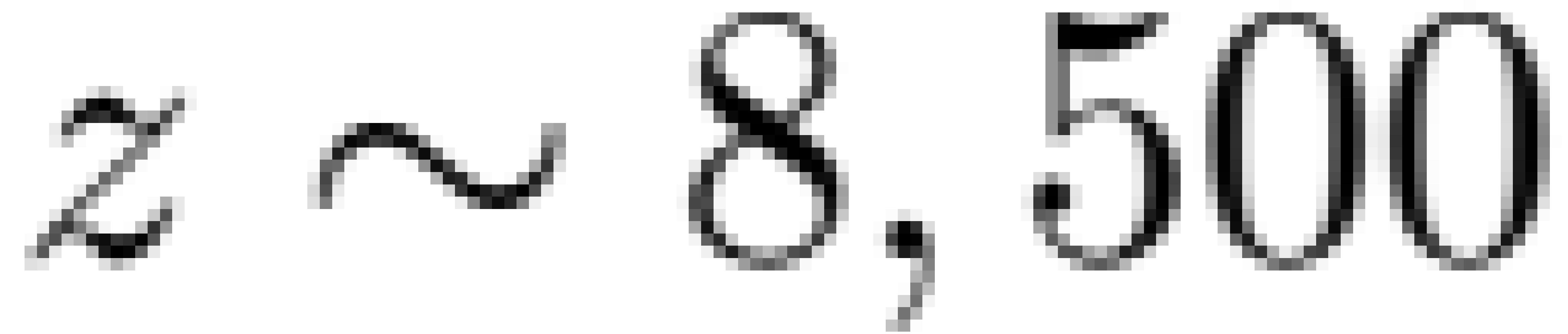


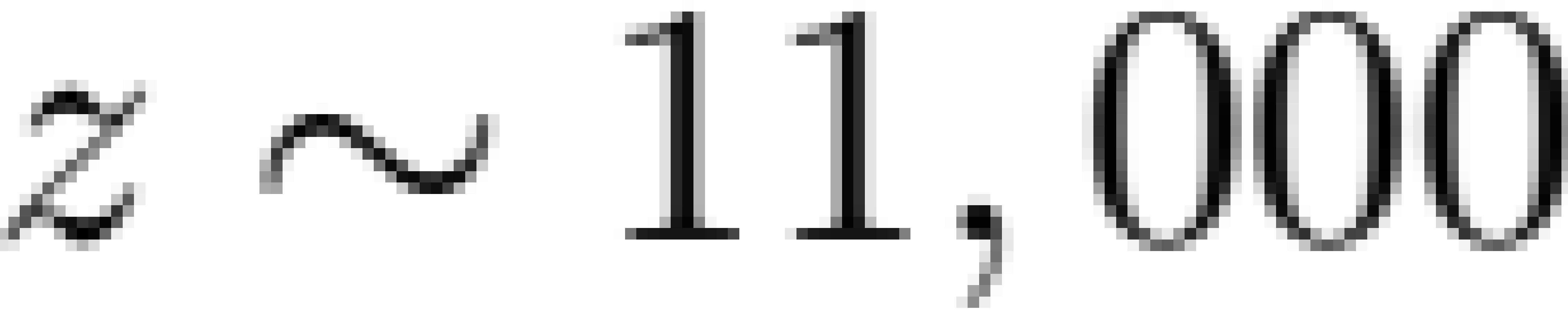
$$S_v(z_i) = \sum_{j=1}^i \rho_{bulk}(z_i) g \Delta z_i$$



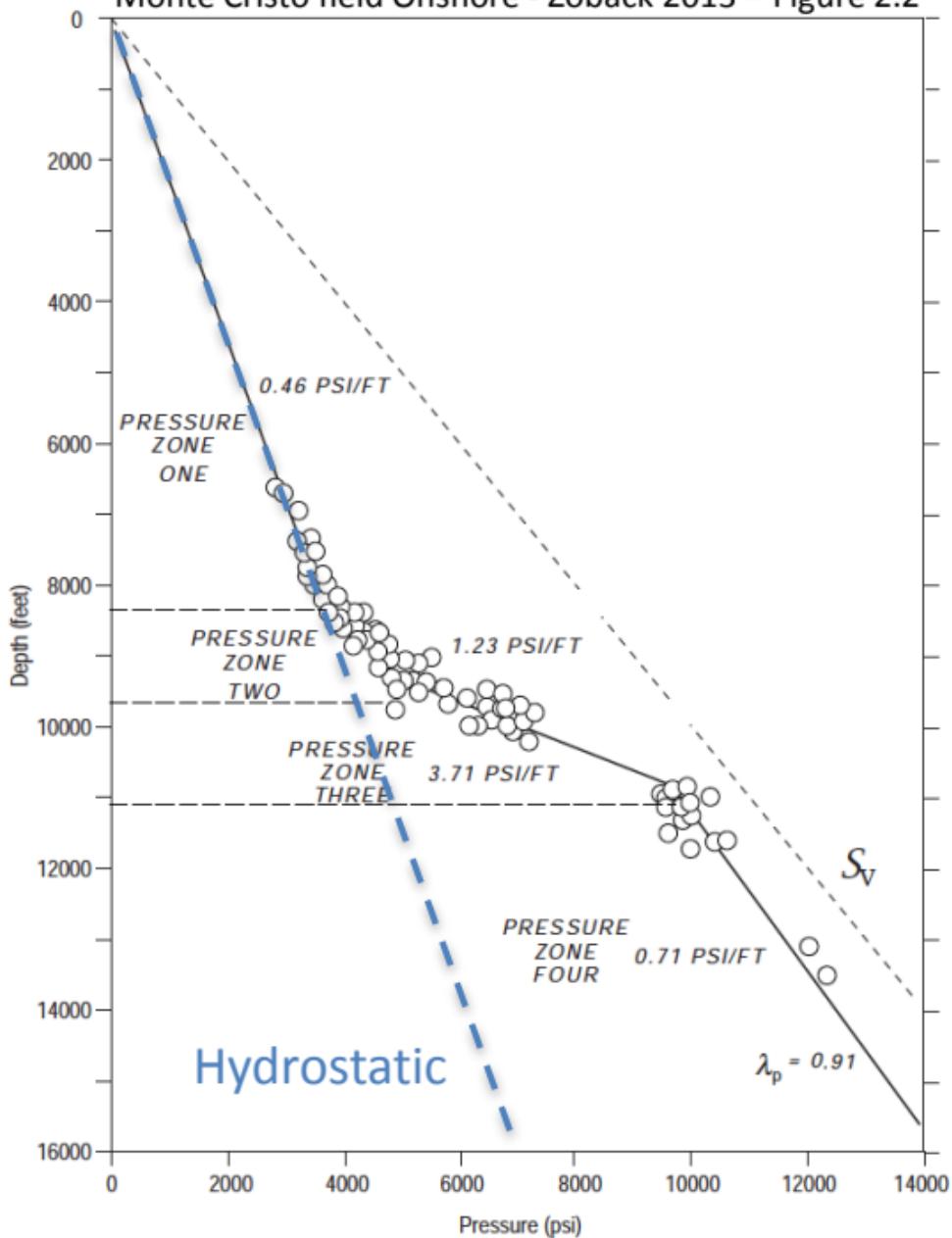








# Monte Cristo field Onshore - Zoback 2013 – Figure 2.2

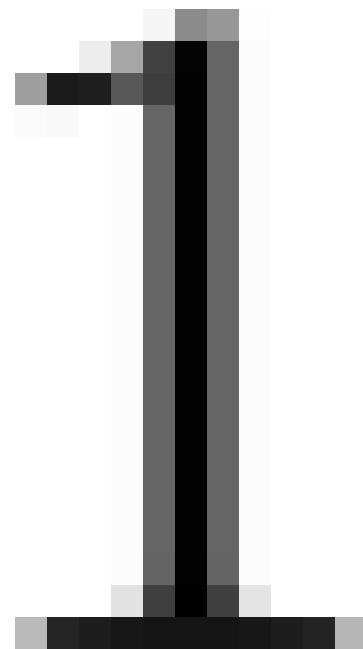
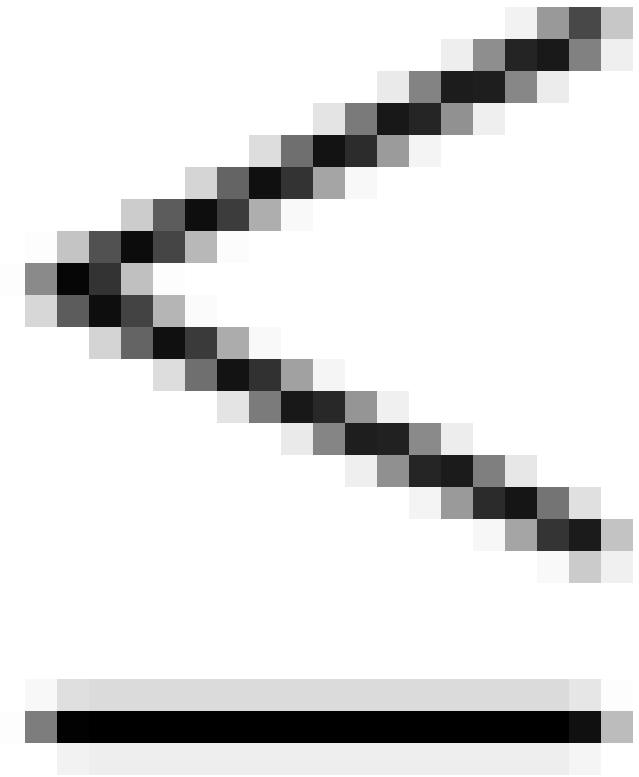


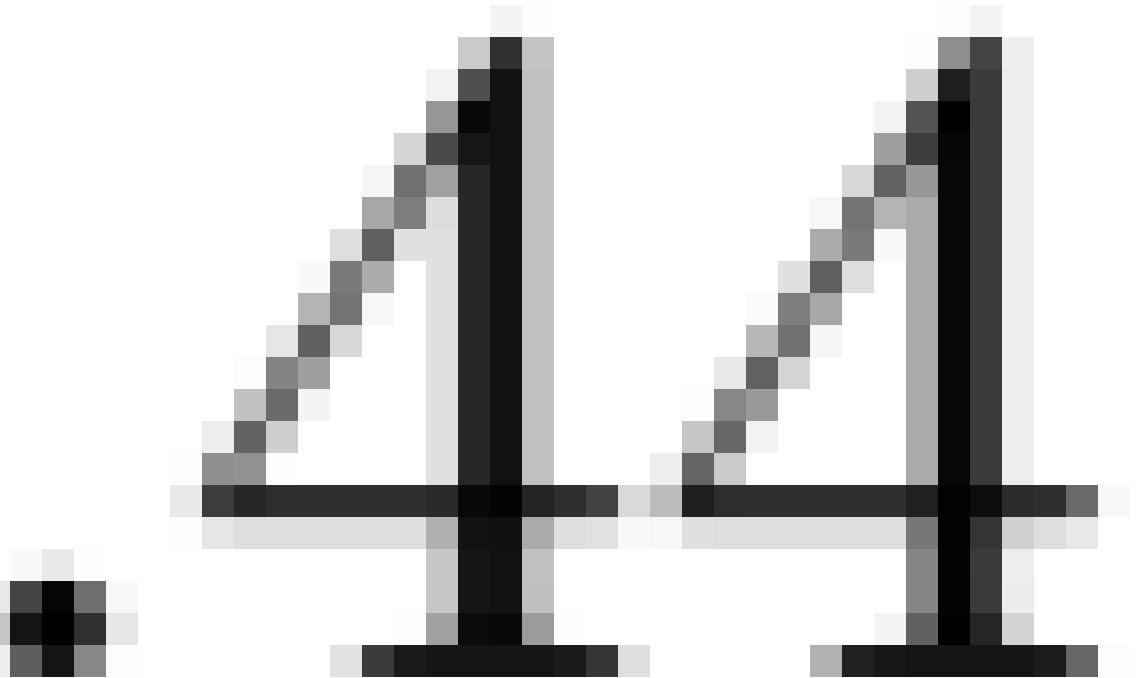
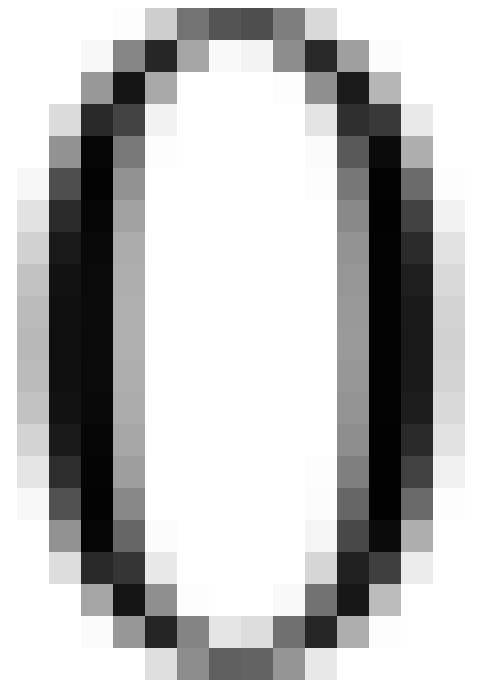
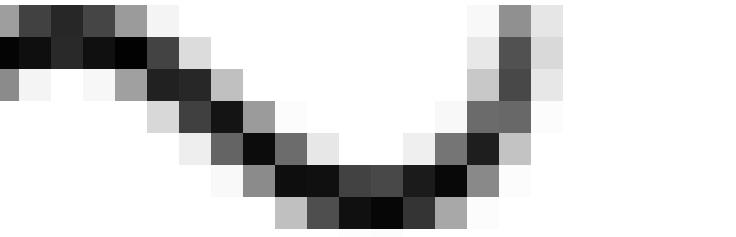


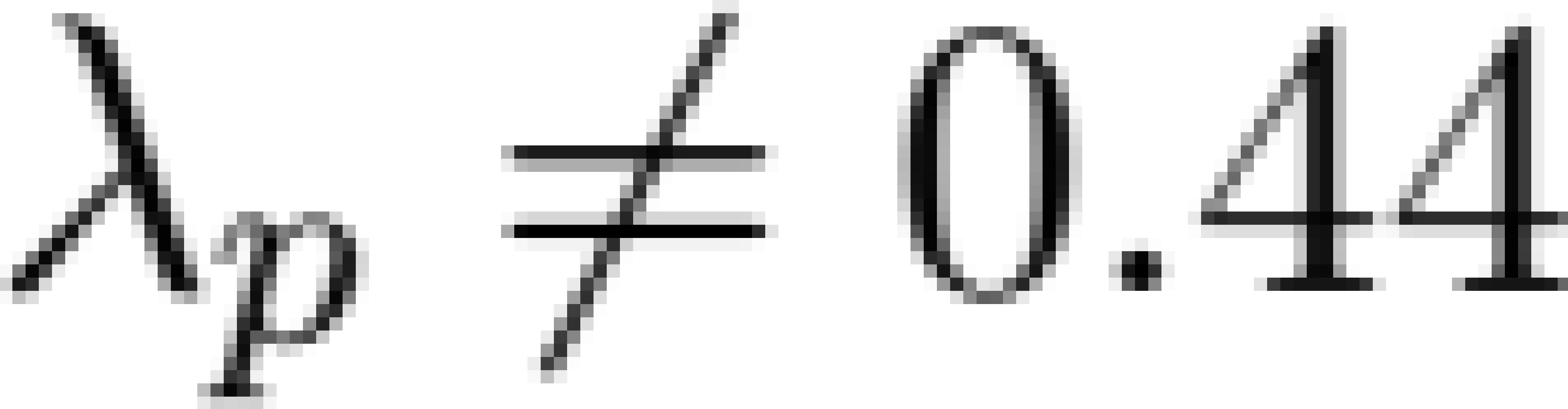
$p(z)$

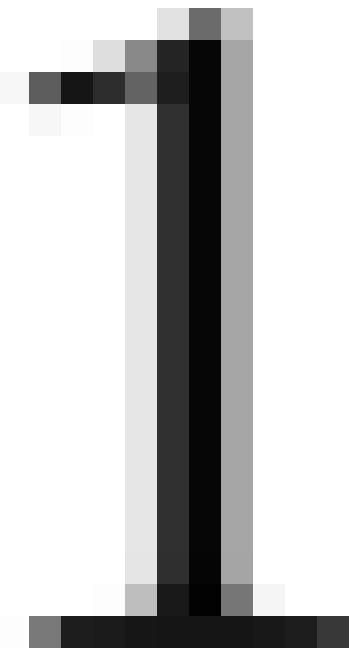
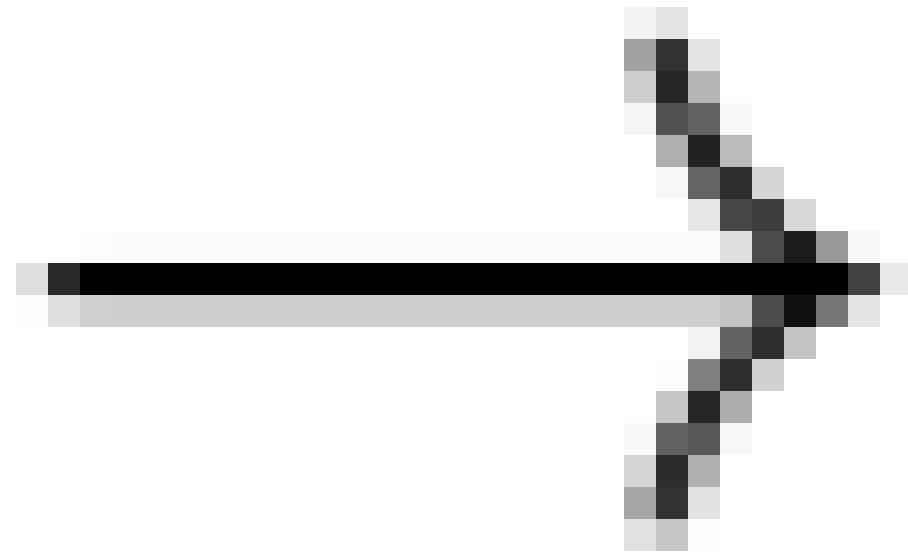


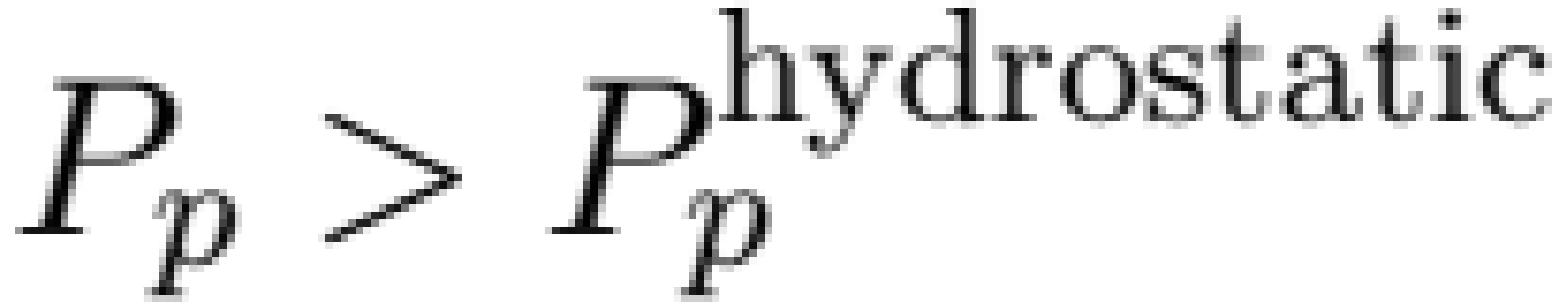
$P_p(z)$   
 $S_q(z)$



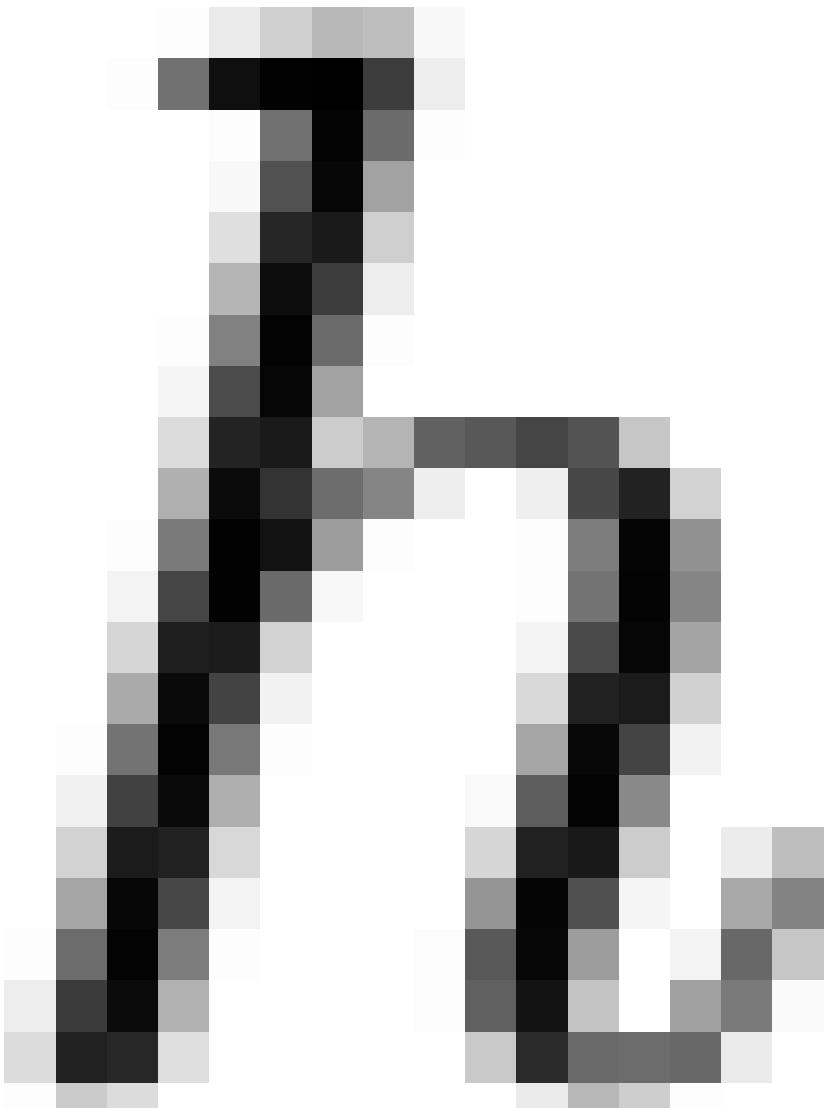














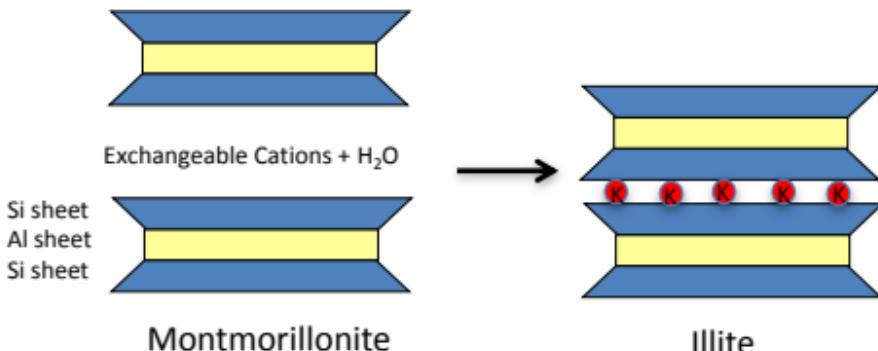


- **Aquathermal pressurization**

- $\Delta T \rightarrow \Delta P$

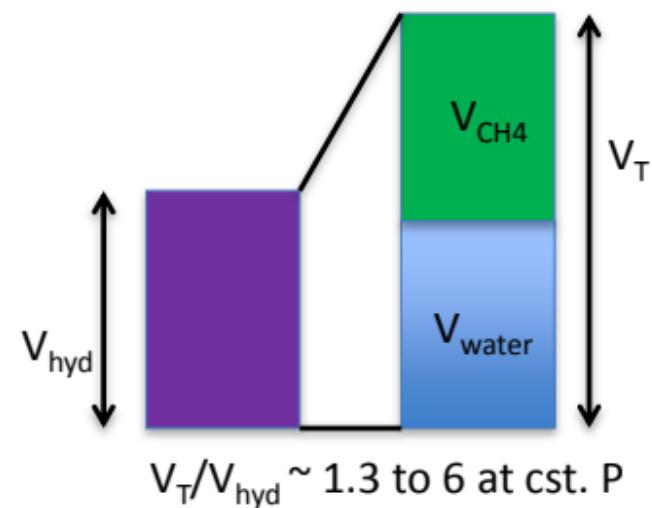
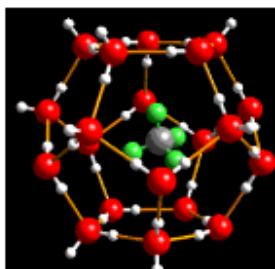
- **Dehydration reactions**

- $\Delta V \rightarrow \Delta P$
  - Montmorillonite to Illite (frees water)

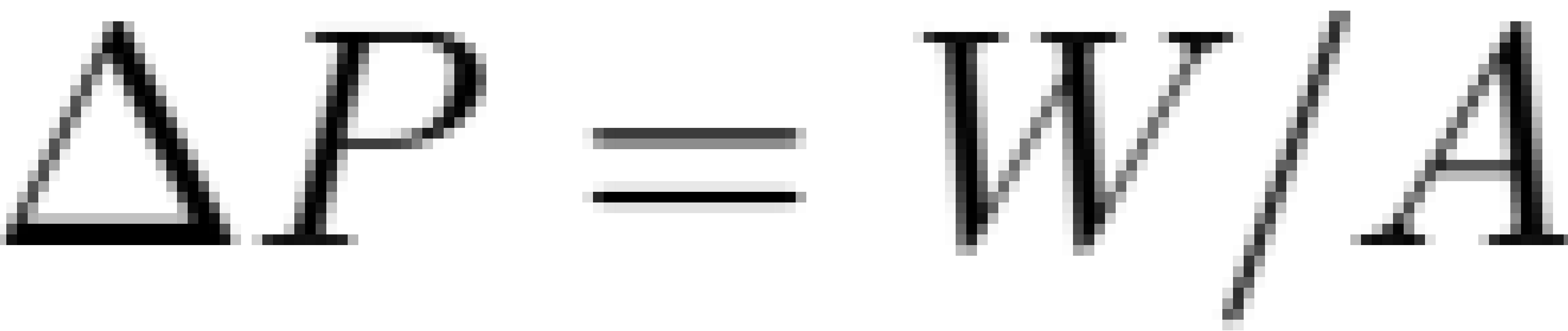


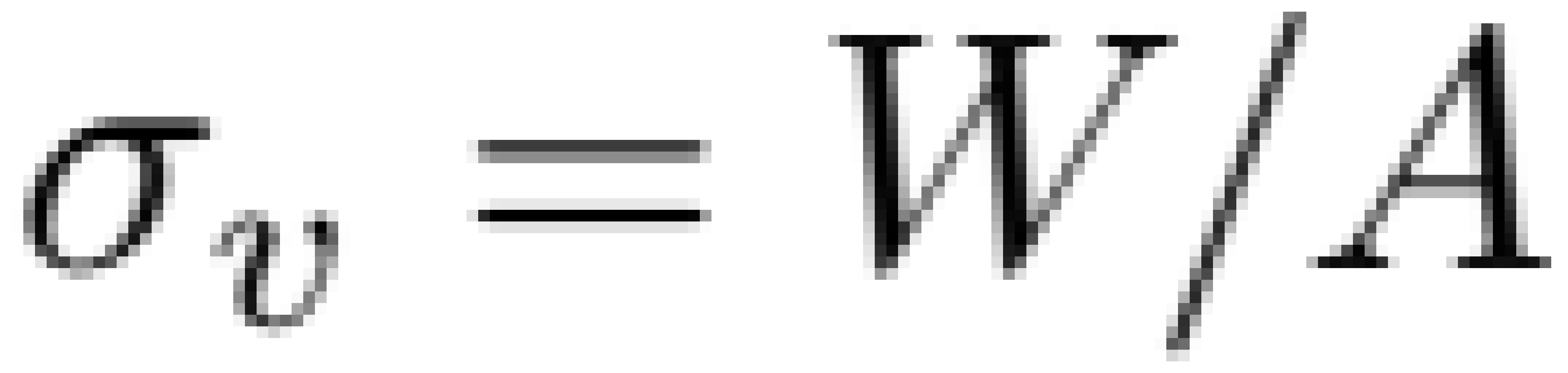
- **Hydrocarbon generation**

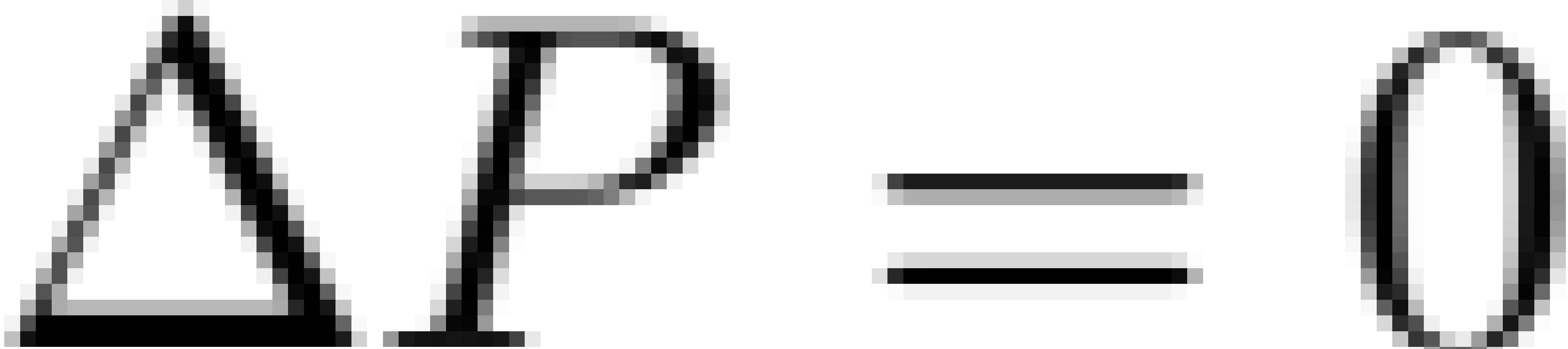
- $\Delta V \rightarrow \Delta P$



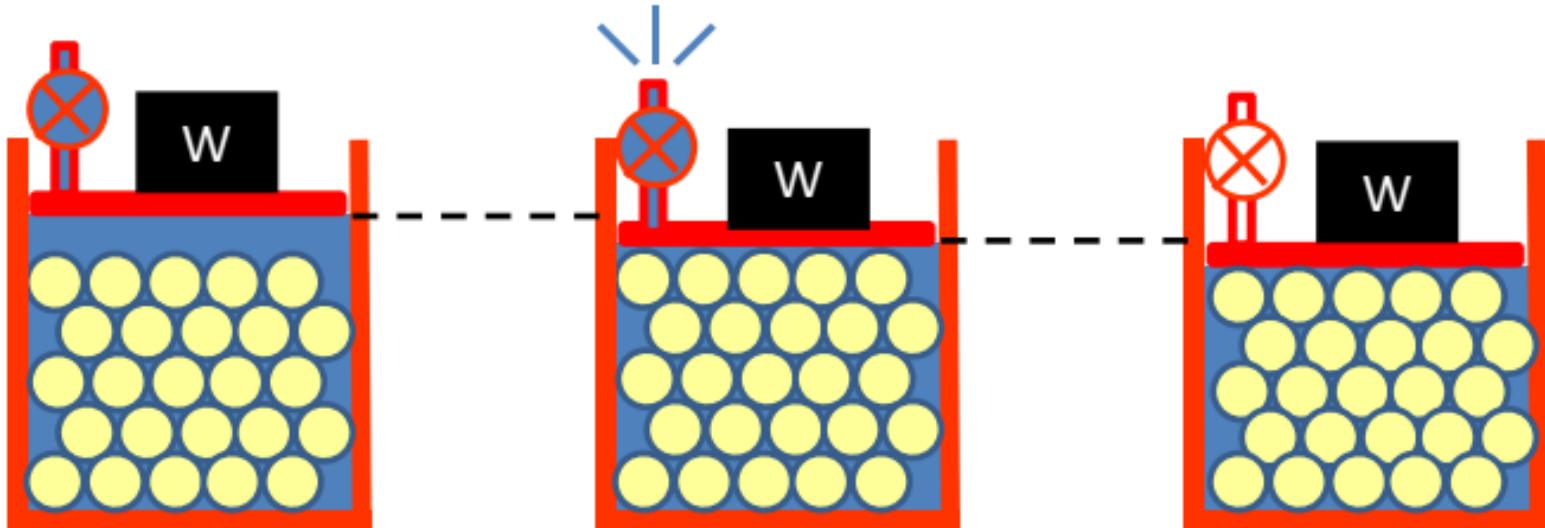




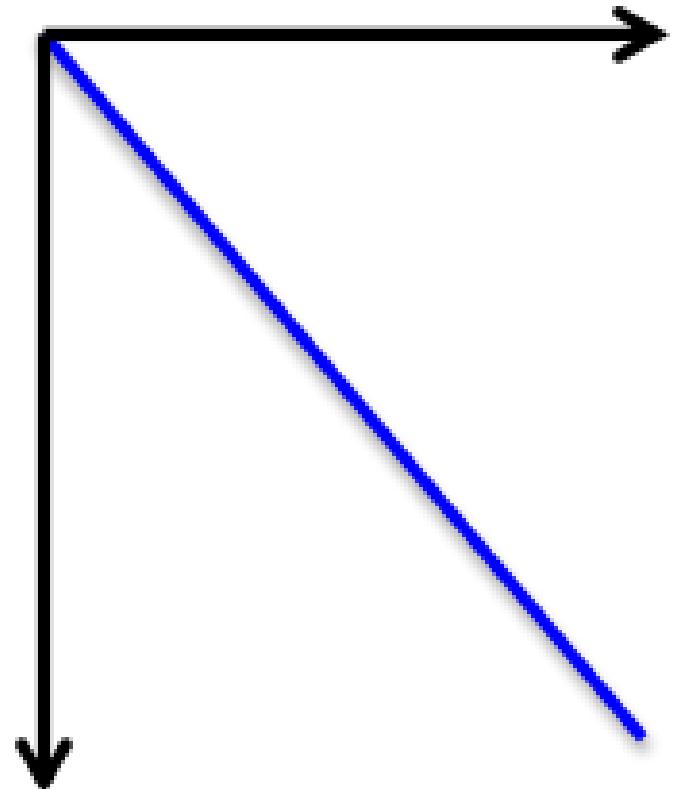
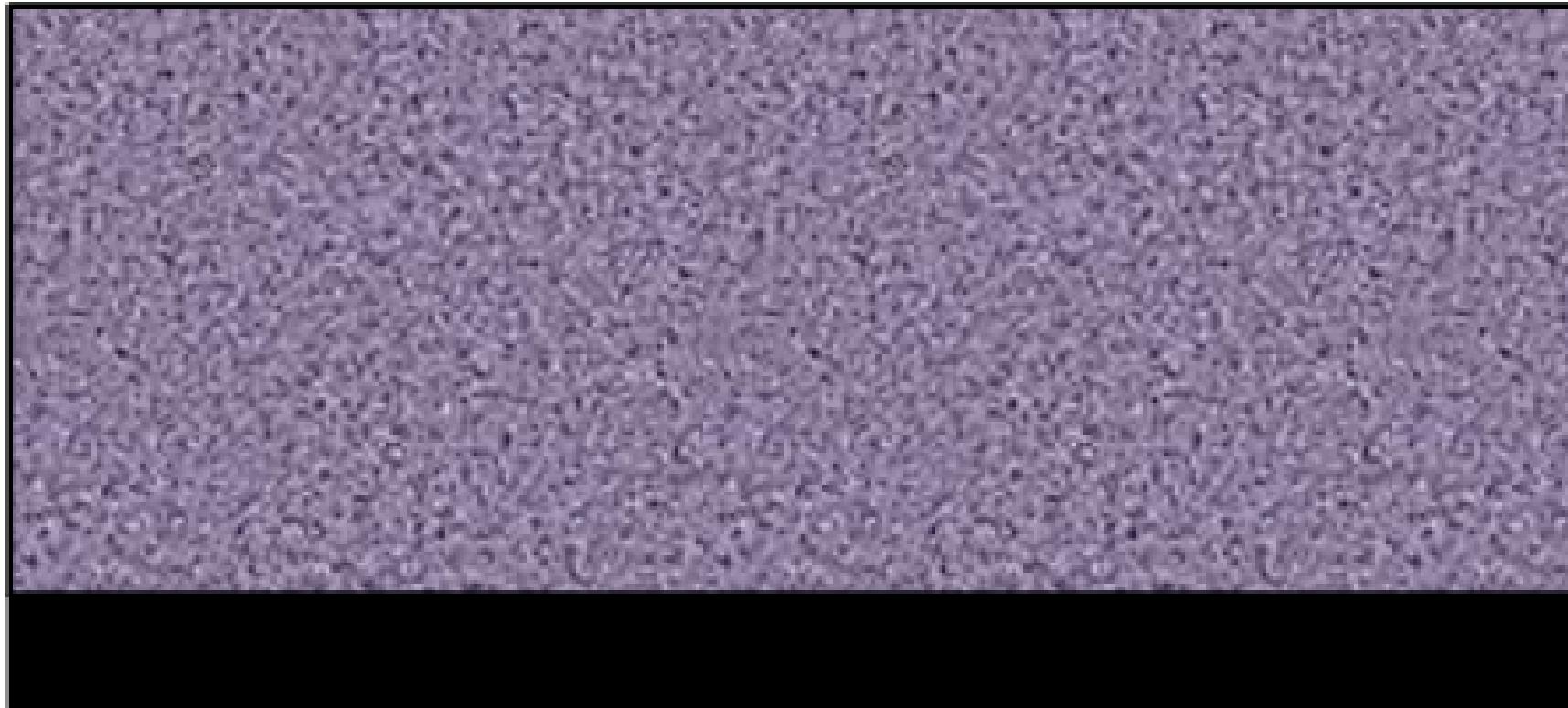


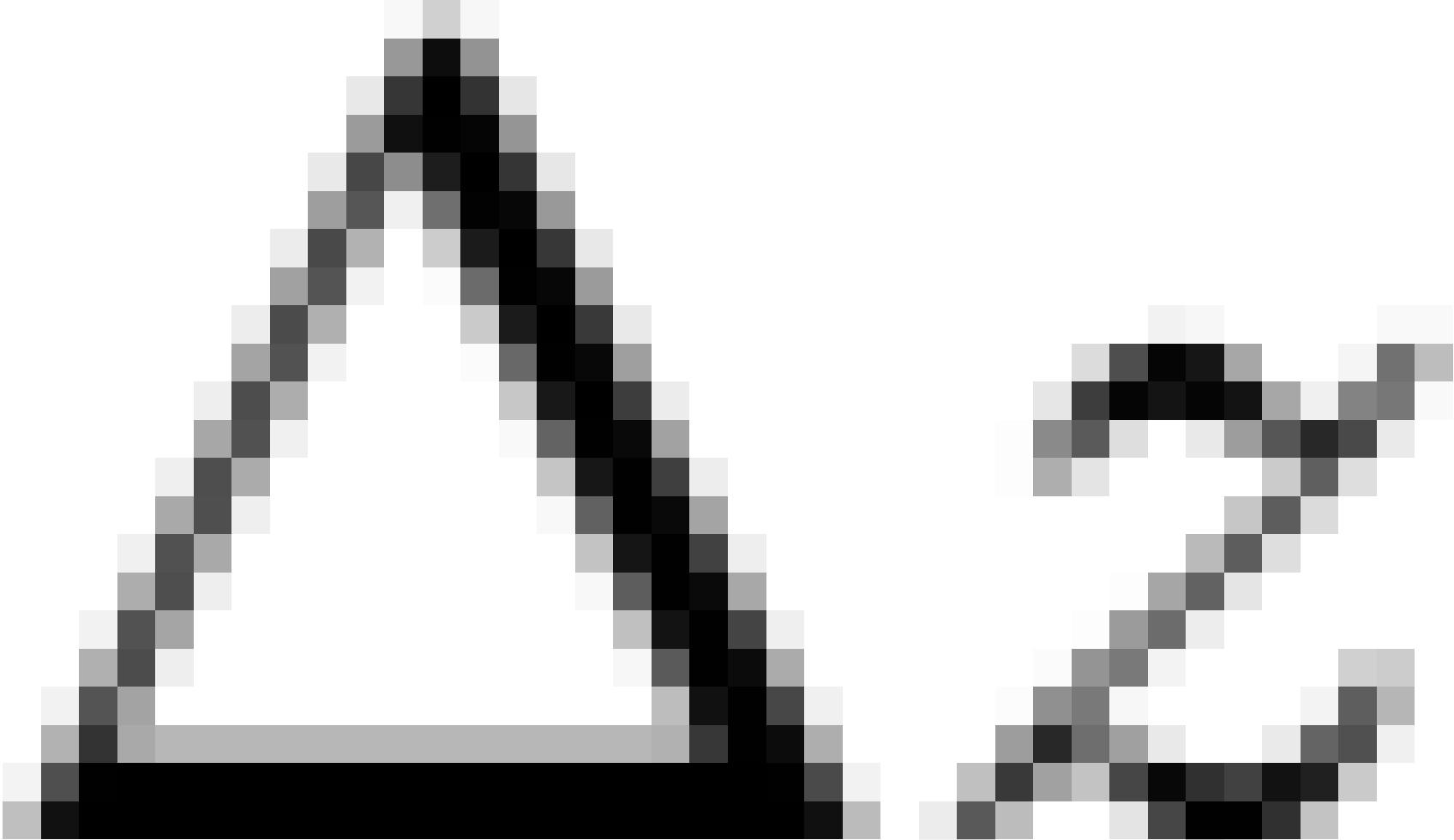


- **Disequilibrium compaction (Underconsolidation)**
  - $\Delta S \rightarrow \Delta P$  (Vertical)
- **Tectonic compression**
  - $\Delta S \rightarrow \Delta P$  (Horizontal)



Pressure water





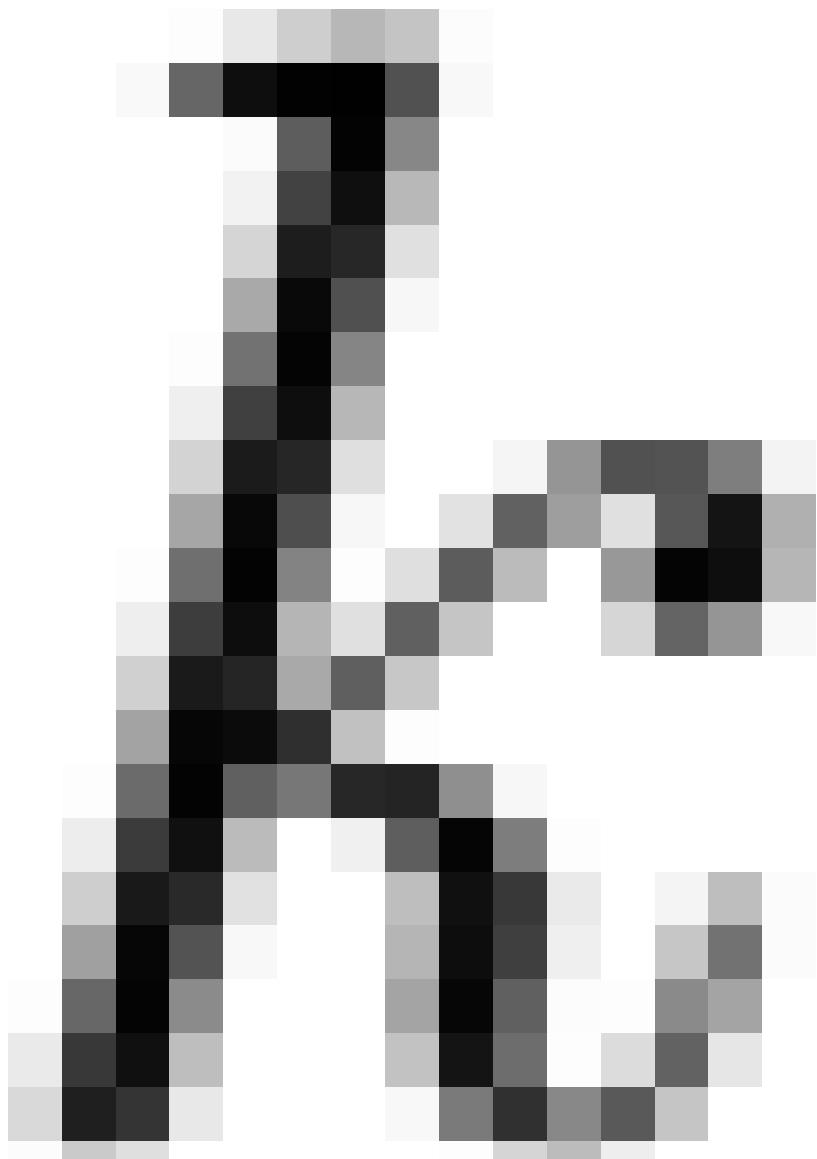
Dh

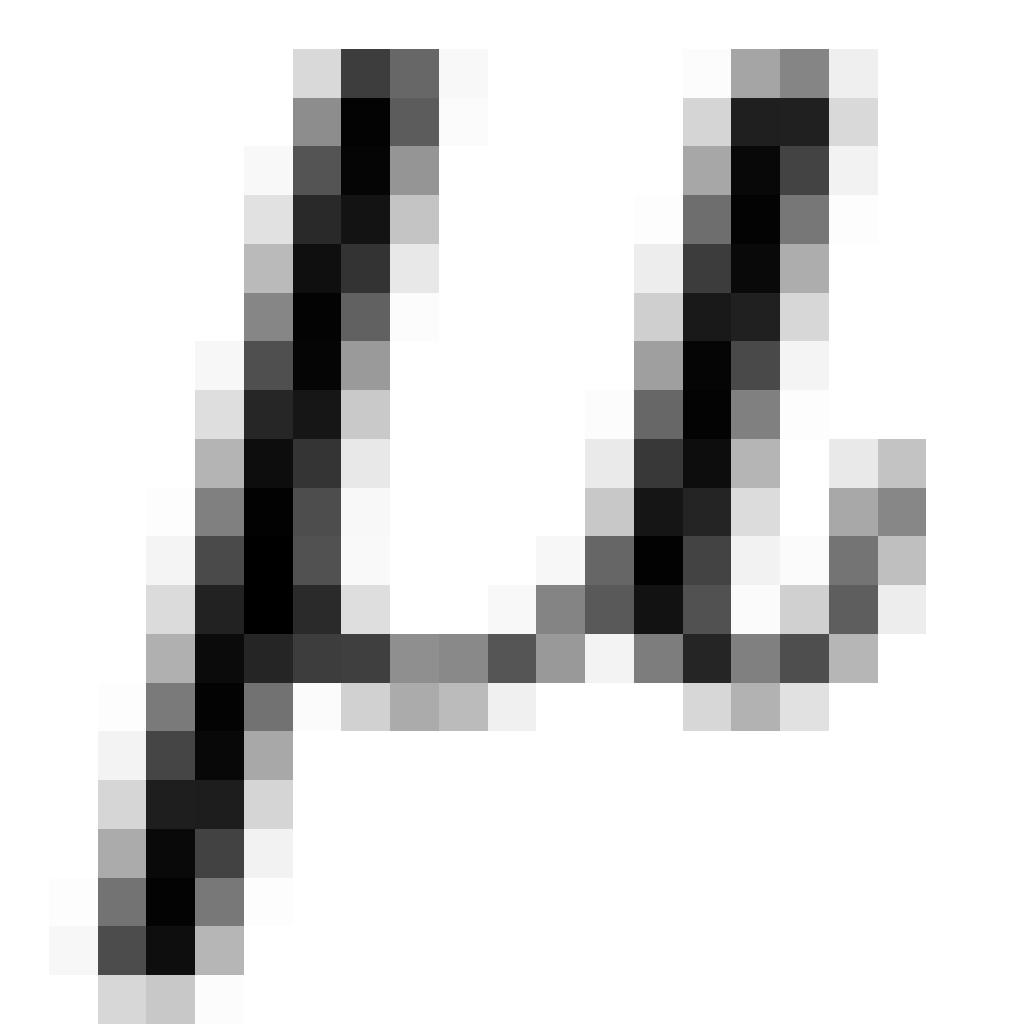
=

MK

ll







$\alpha p$

$\alpha t$

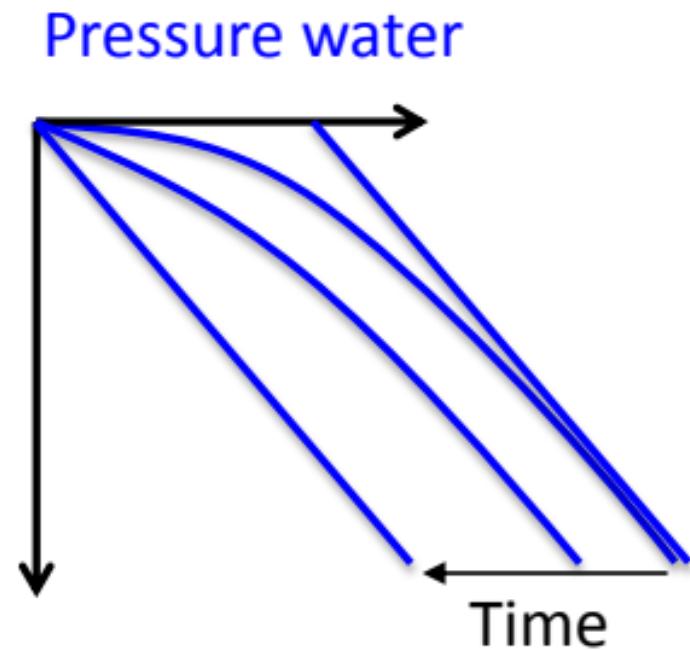
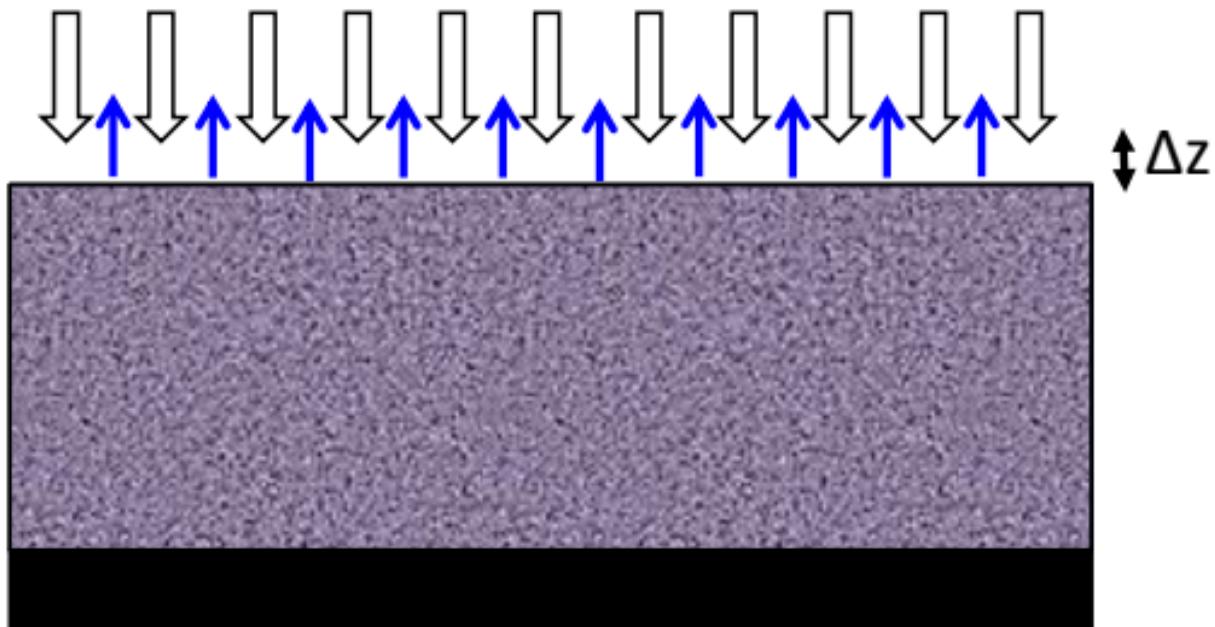
$=$

$Dh$

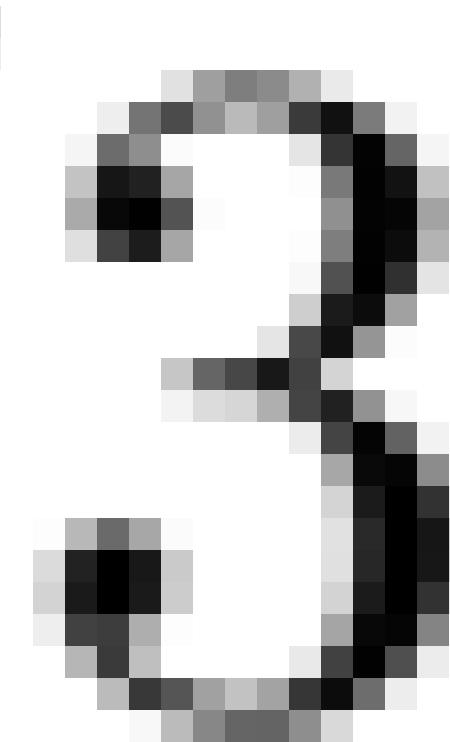
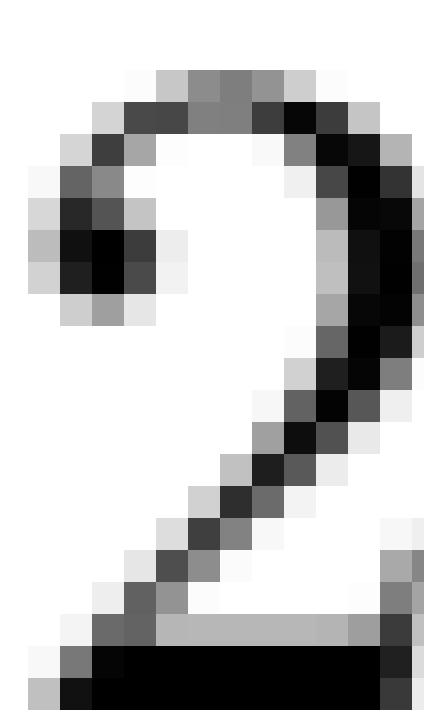
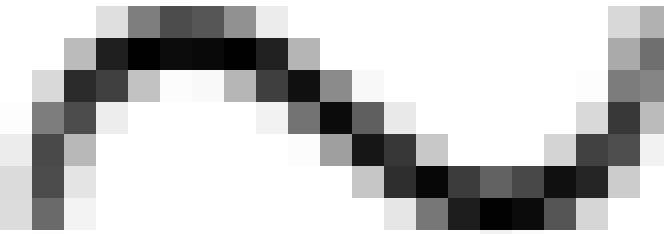
$d^2z^2$

$d^2p$

## Rate of sedimentation (loading) and rate of fluid “escape”





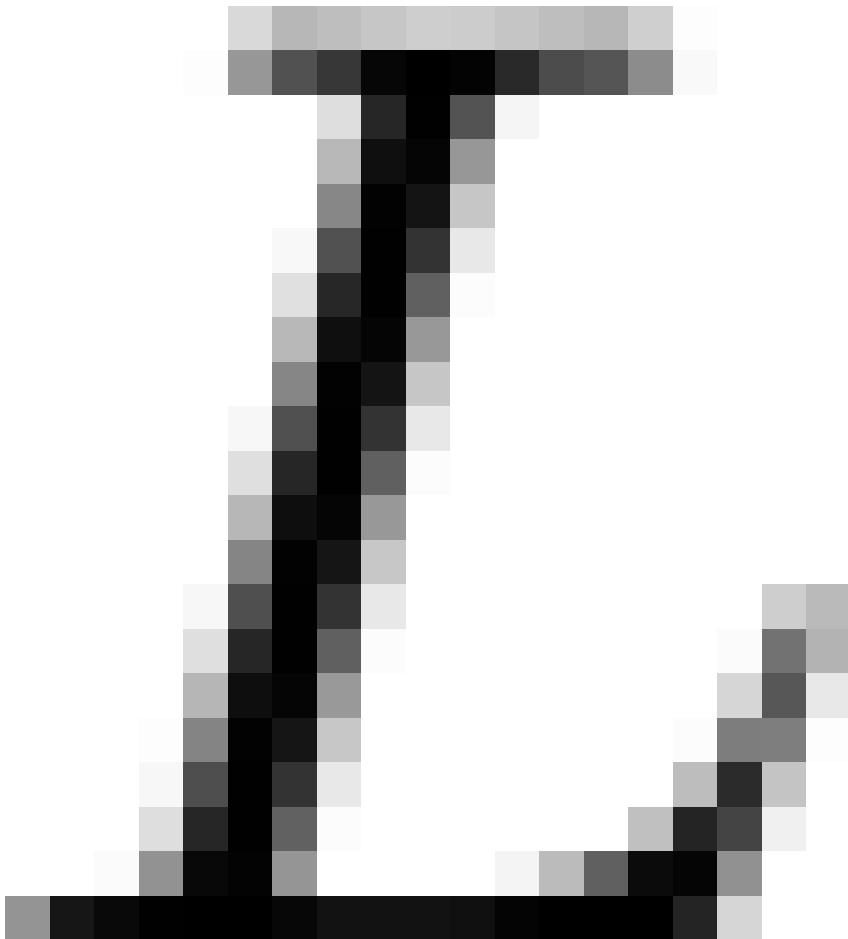


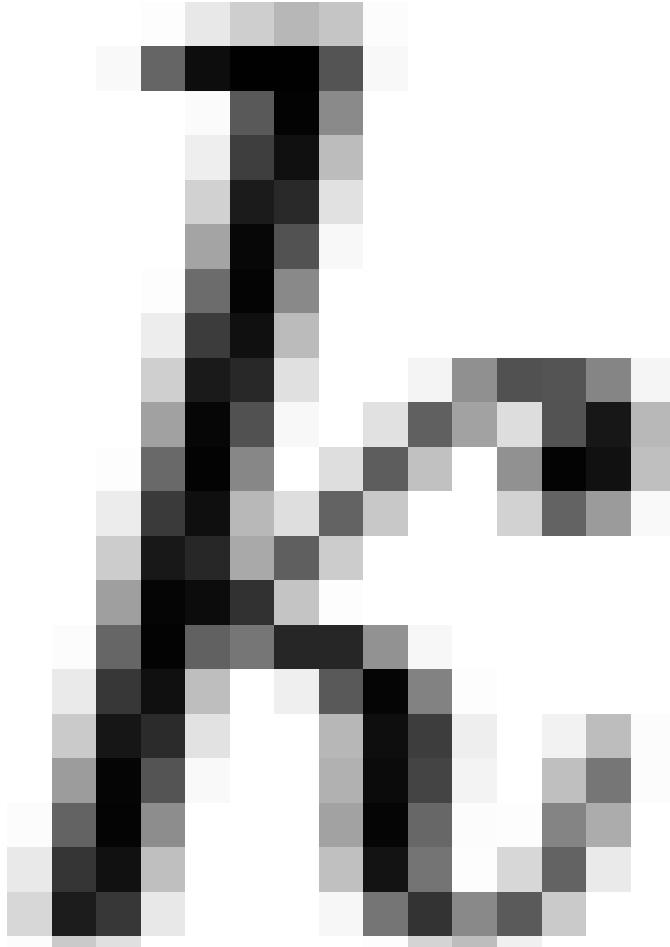
$\Gamma_{ch}$

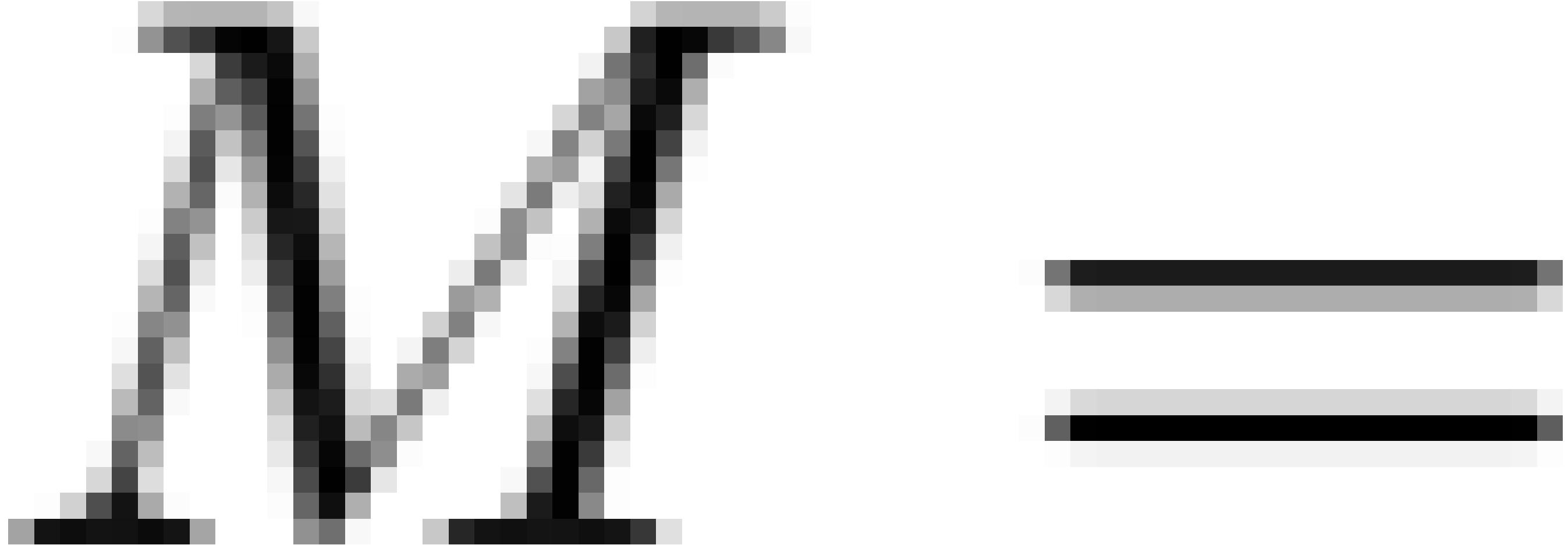


$D_h$

$\Gamma^2$

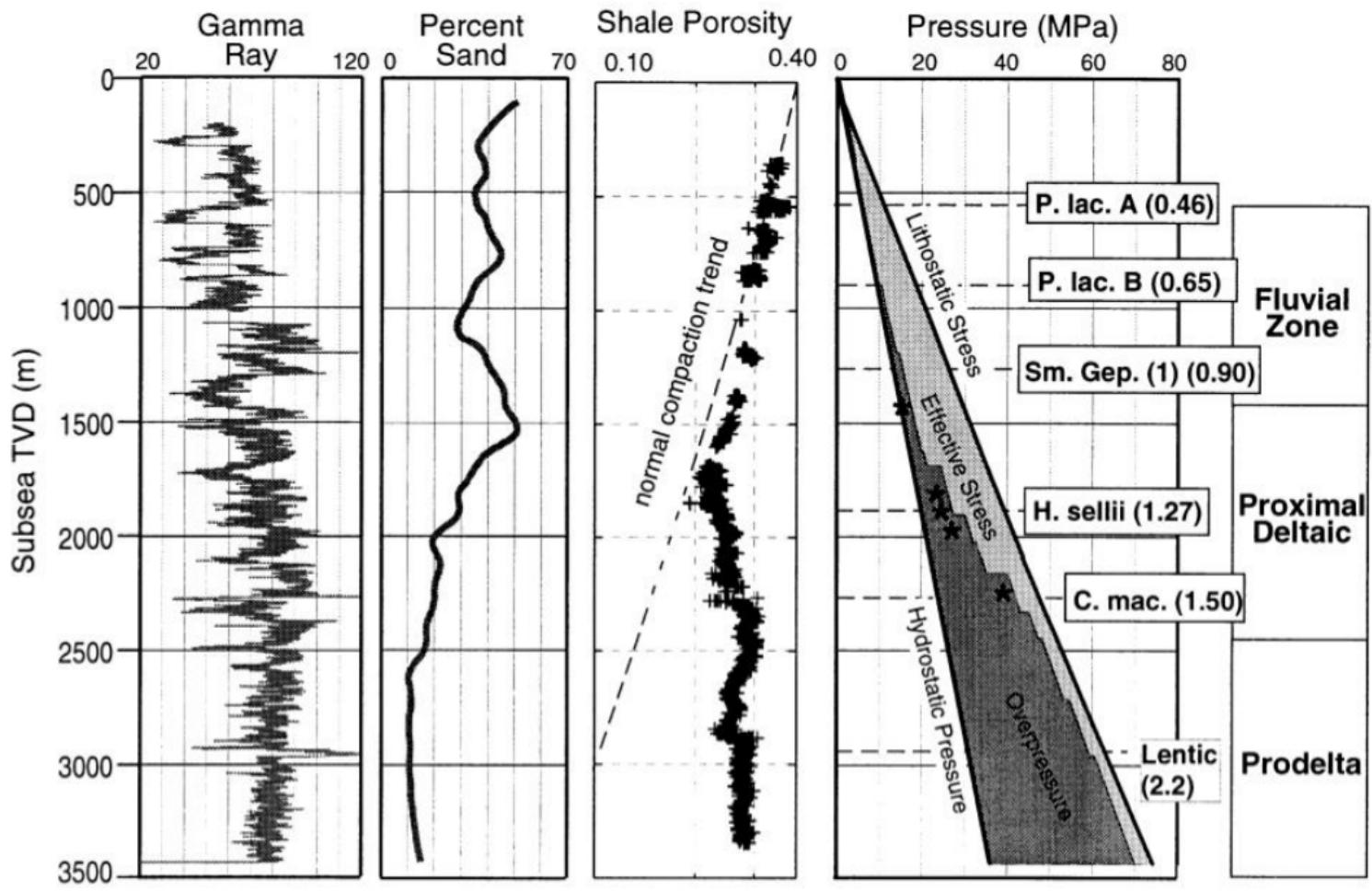




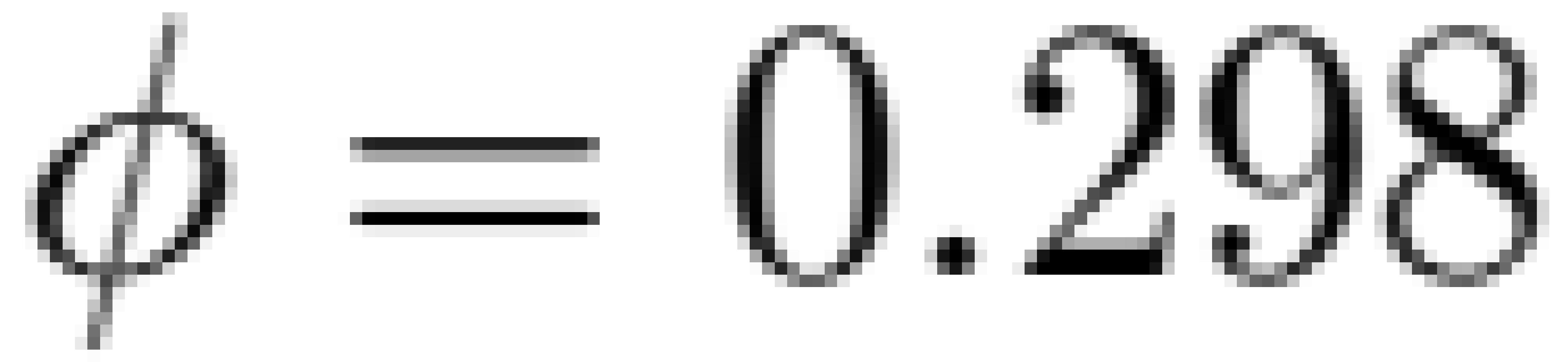


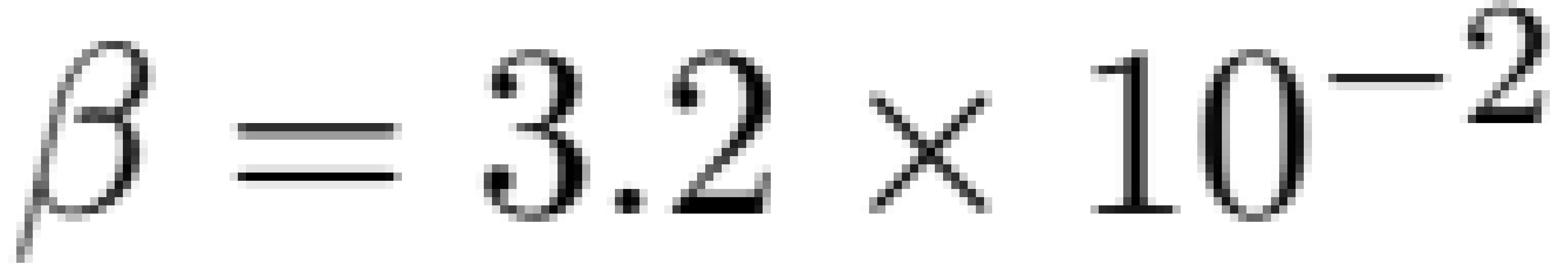


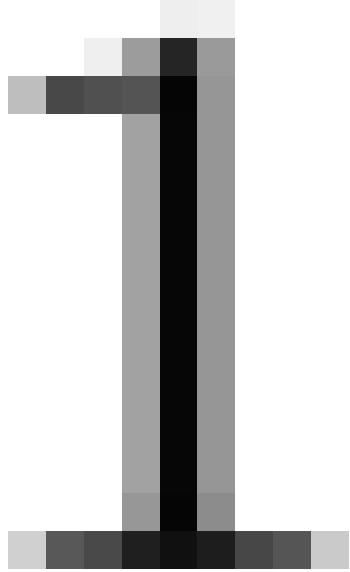


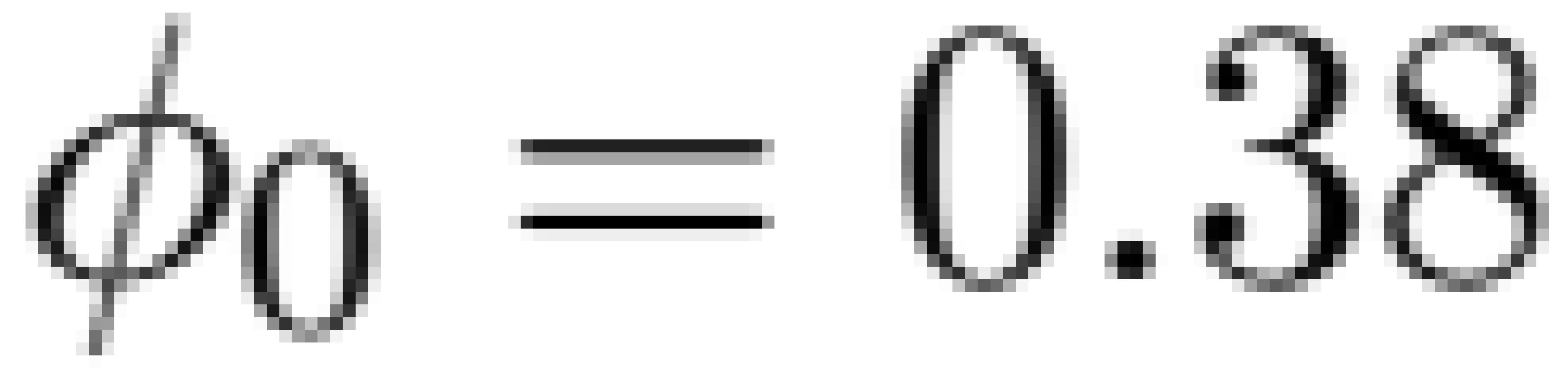


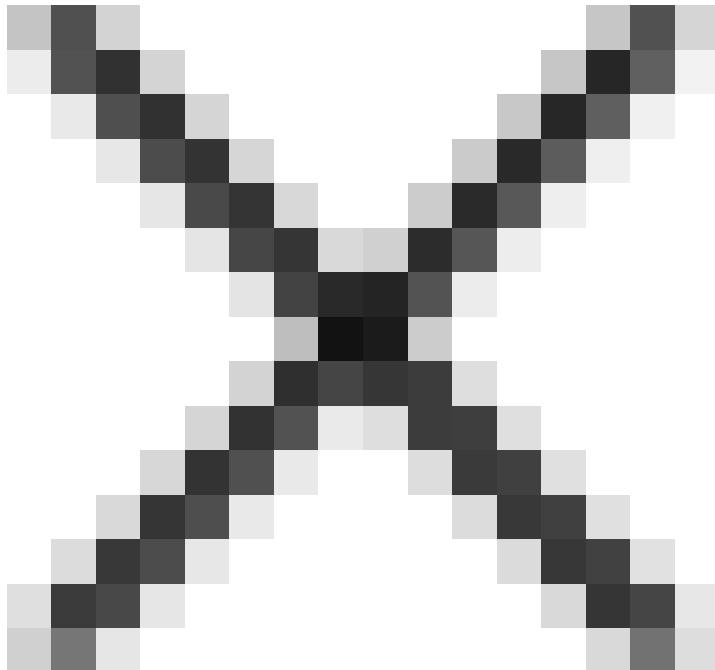
Off-shore Louisiana – Gordon and Flemings (1998) Basin Research



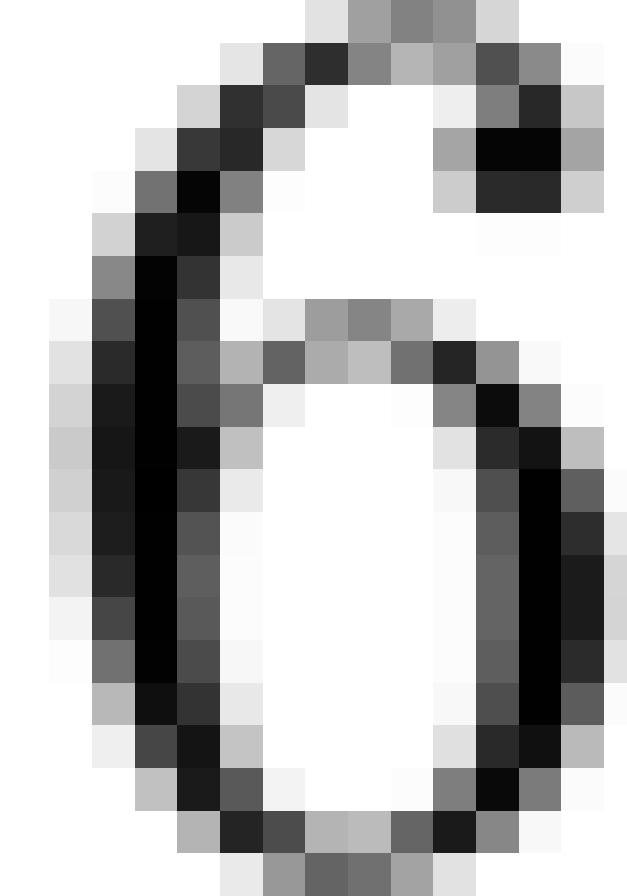
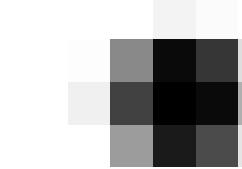
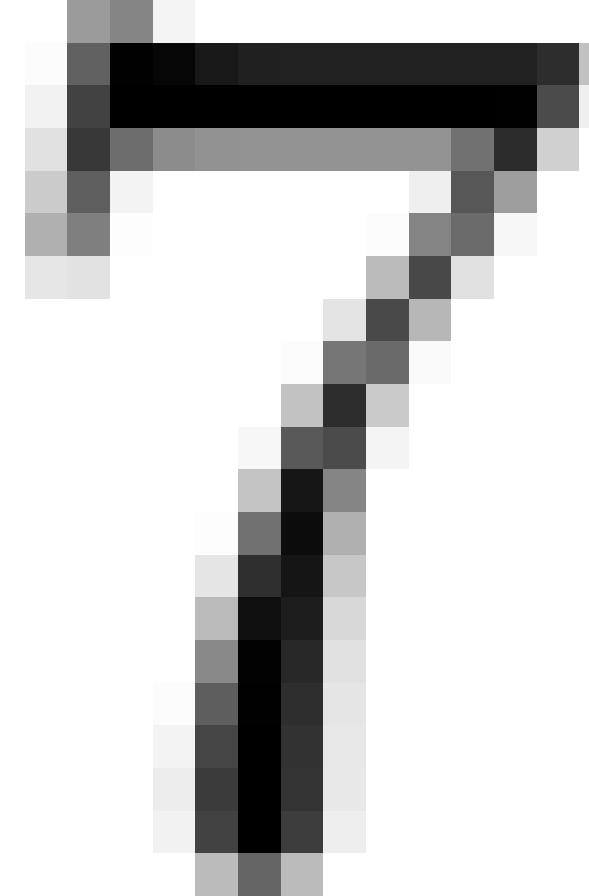


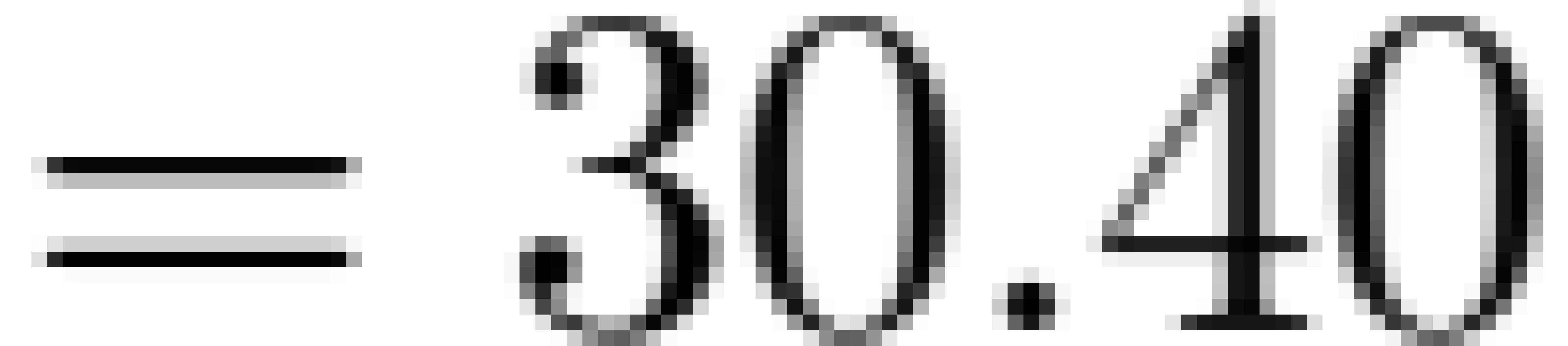




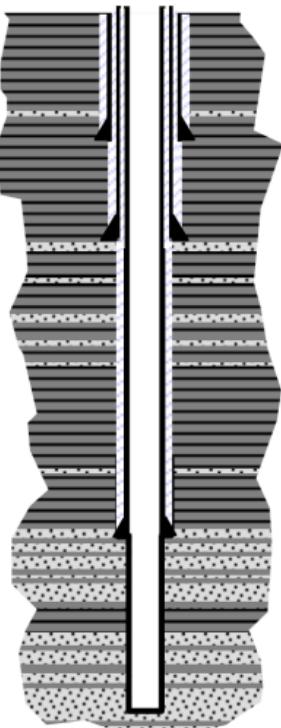
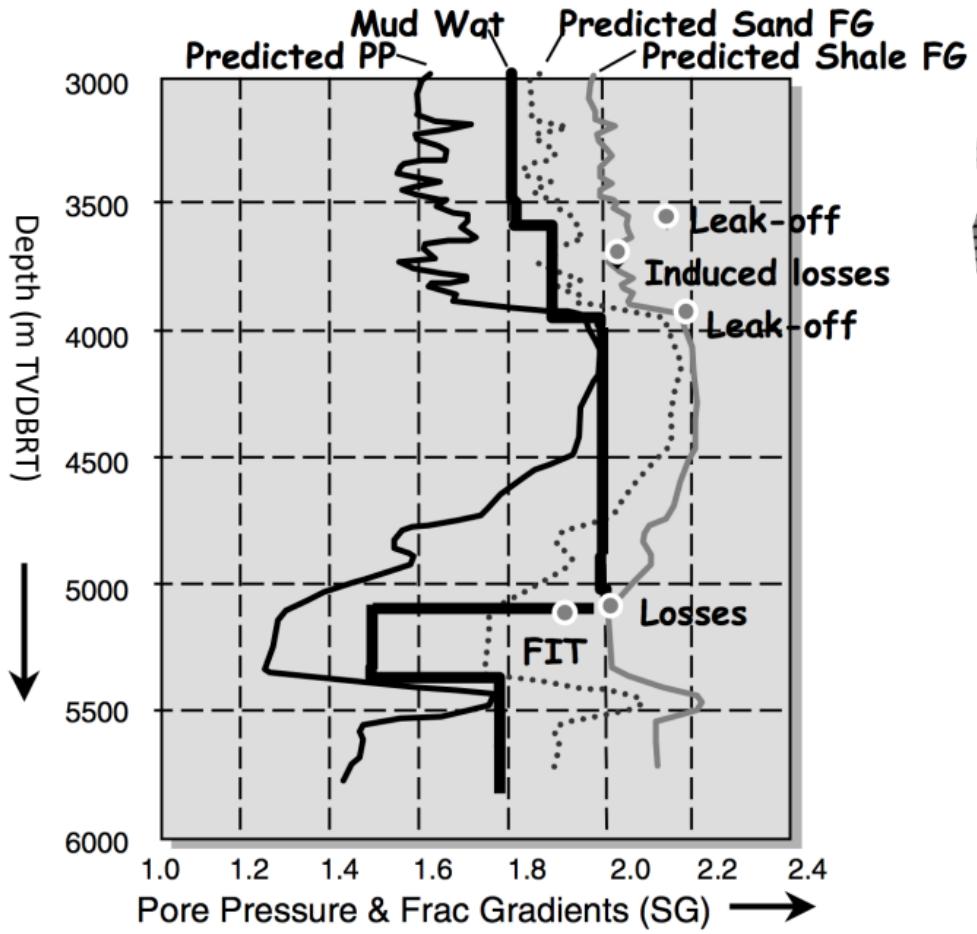


$$P_p = S_p + \ln(\phi/\phi_0) = 38$$

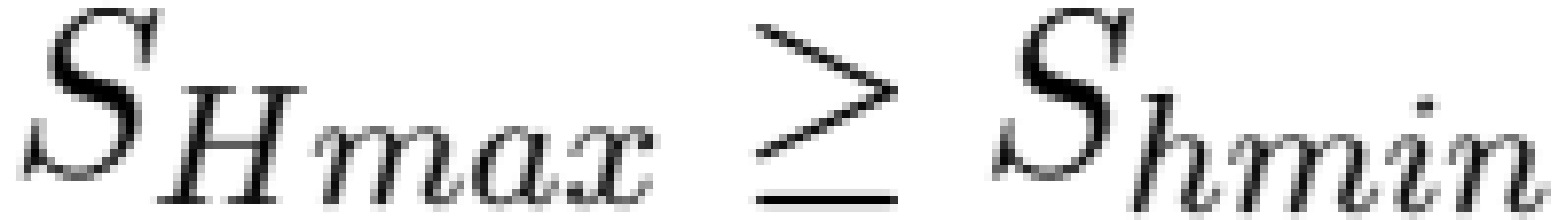


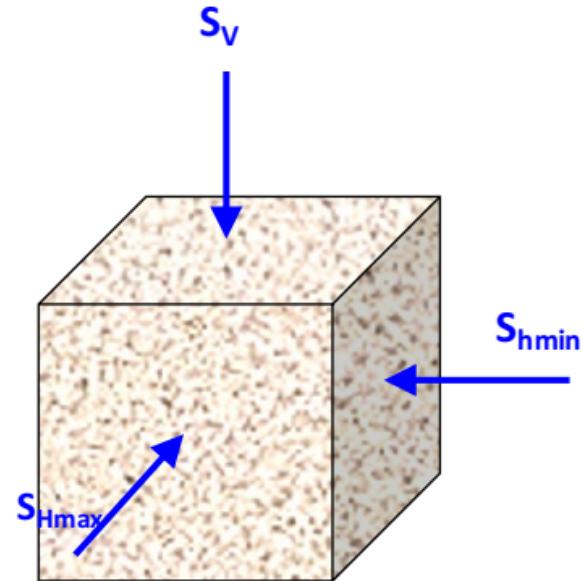
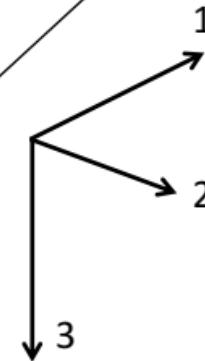
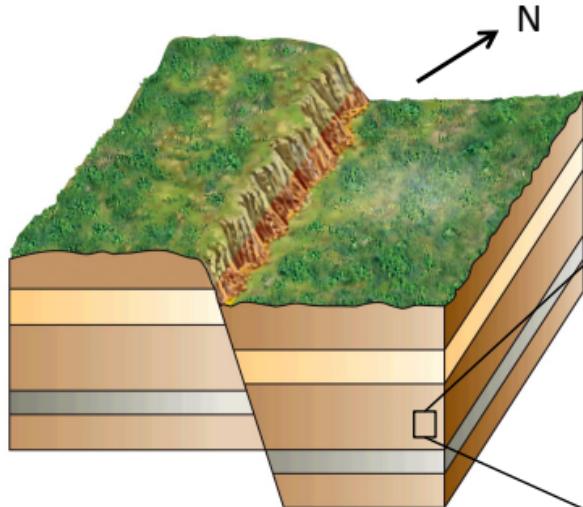


$P$  = 30.40 MPa  
 $S_2$  = 0.8.  
 $P$  = 38 MPa

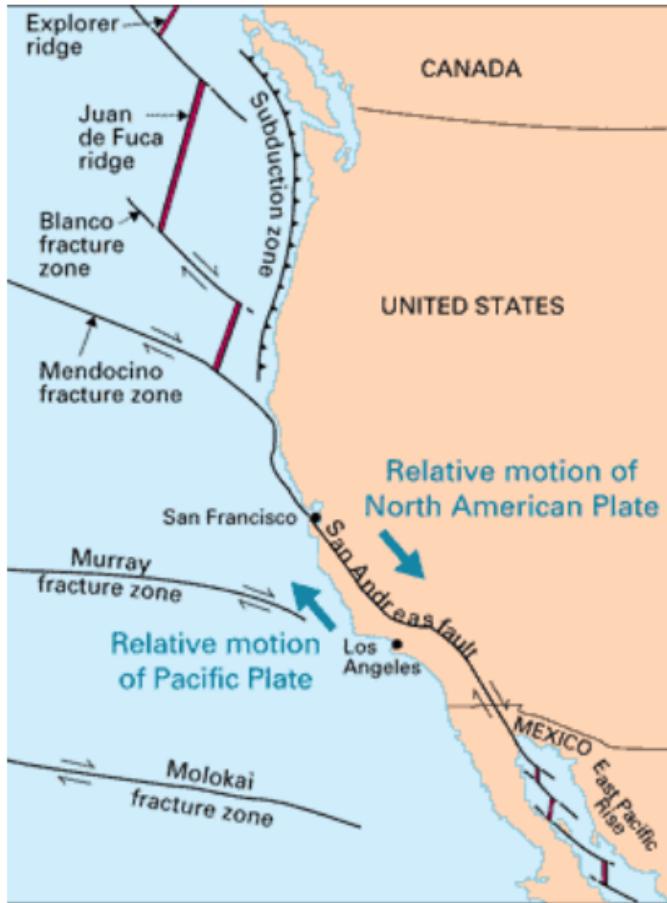


[Caspian Sea, Alberta and McLean – SPE67740]



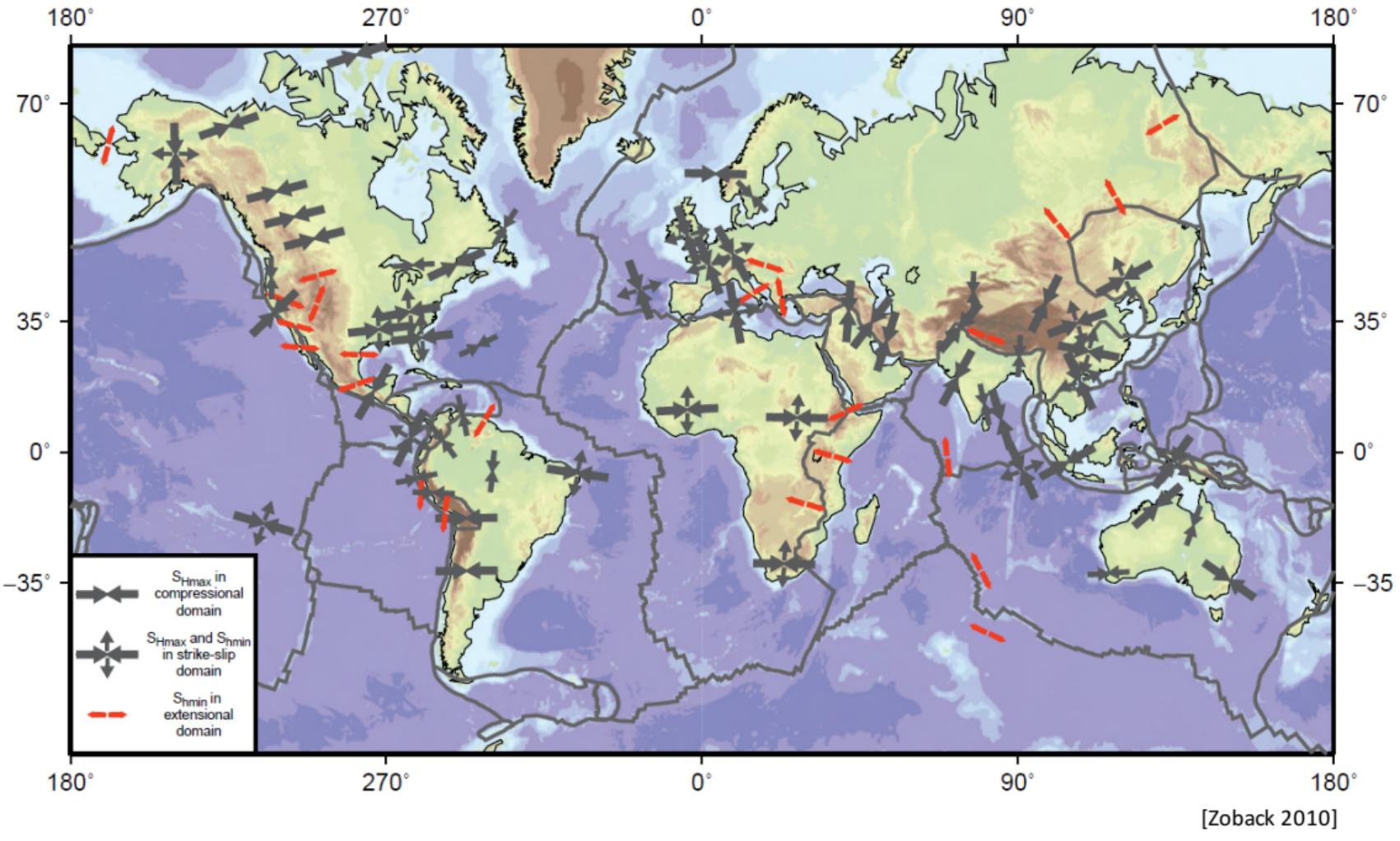


$$\underline{\underline{S}} = \begin{bmatrix} S_V & 0 & 0 \\ 0 & S_{H\max} & 0 \\ 0 & 0 & S_{h\min} \end{bmatrix}$$

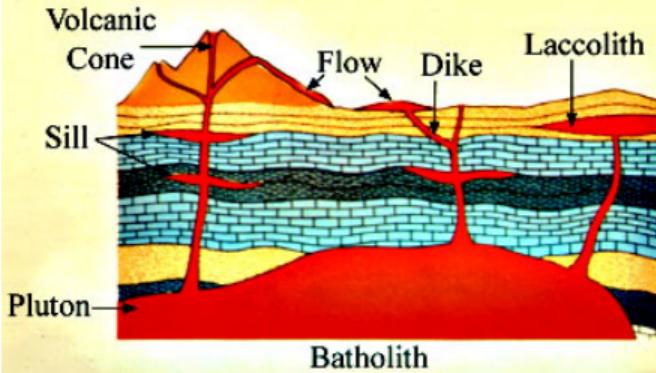


<http://pubs.usgs.gov/gip/dynamic/understanding.html#anchor5798673>

<http://en.wikipedia.org/wiki/File:Aerial-SanAndreas-CarrizoPlain.jpg>



## PLUTONS & VOLCANIC LANDFORMS



<http://www.indiana.edu/~geol105/1425chap5.htm>  
<http://geophysics.ou.edu/geol1114>





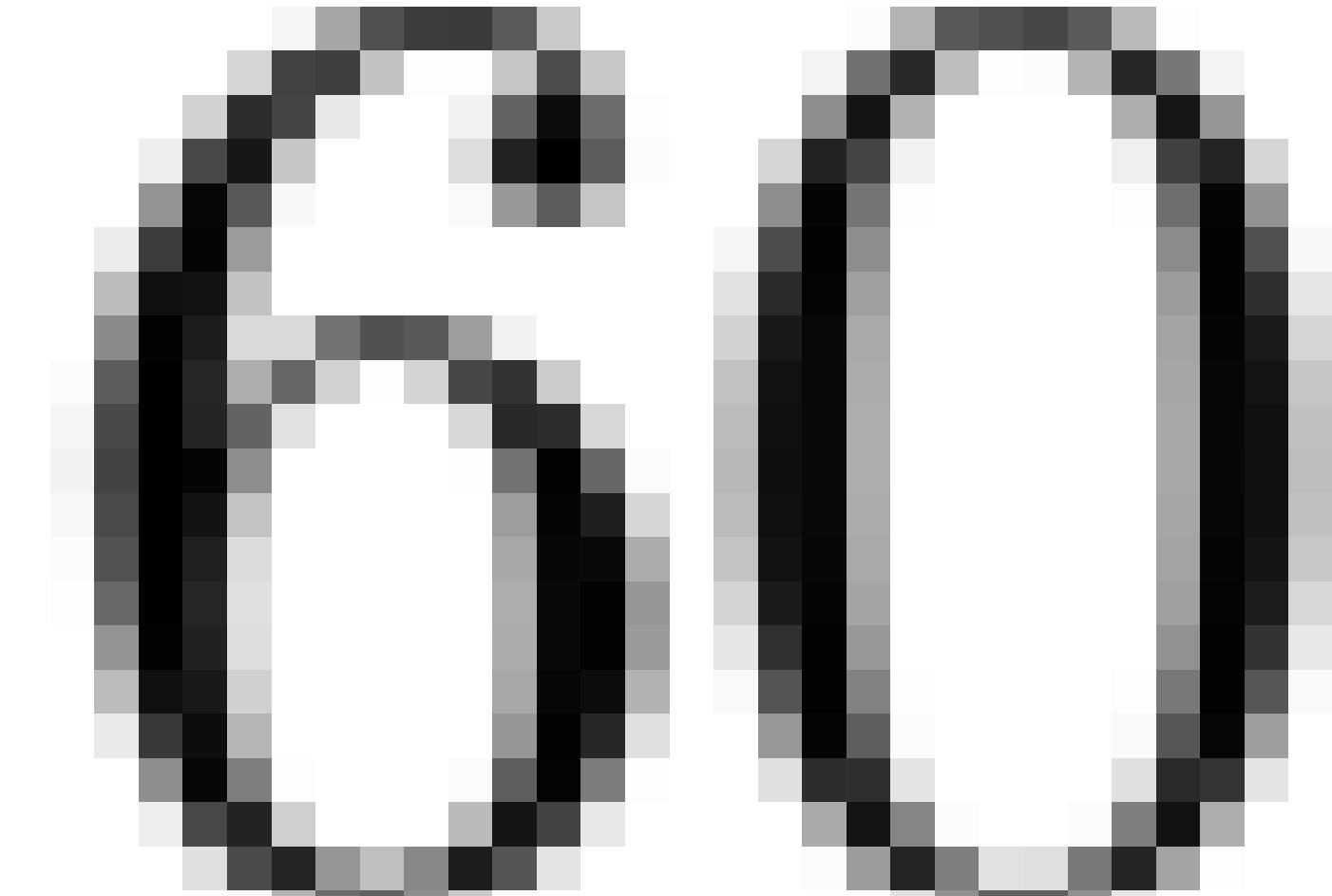
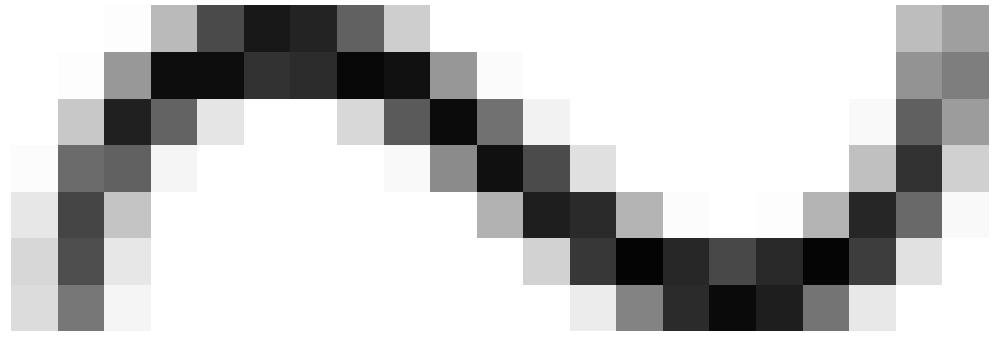








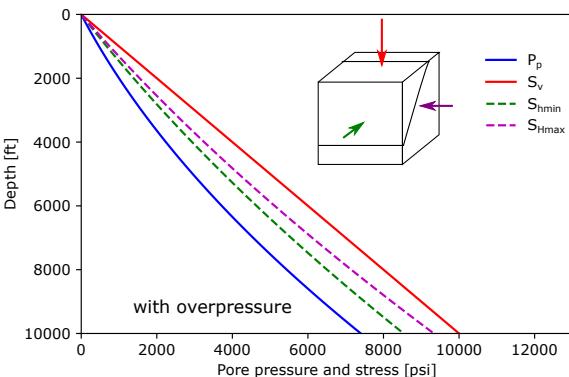
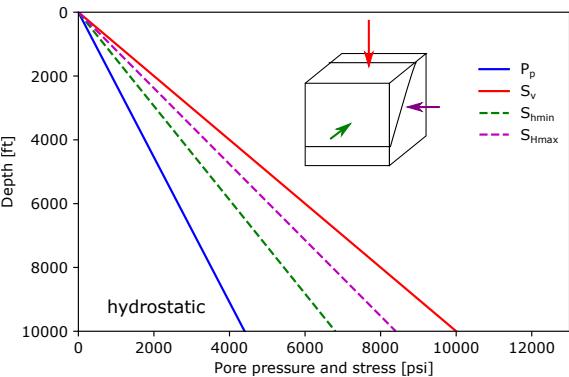




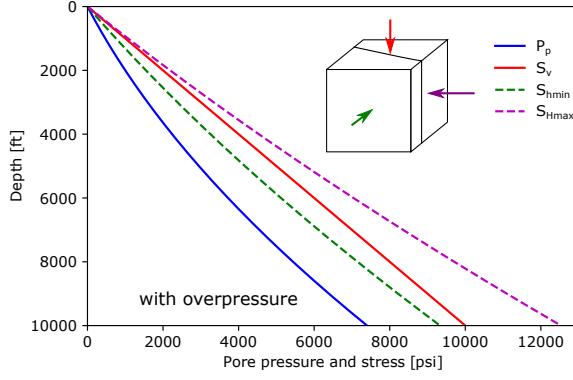
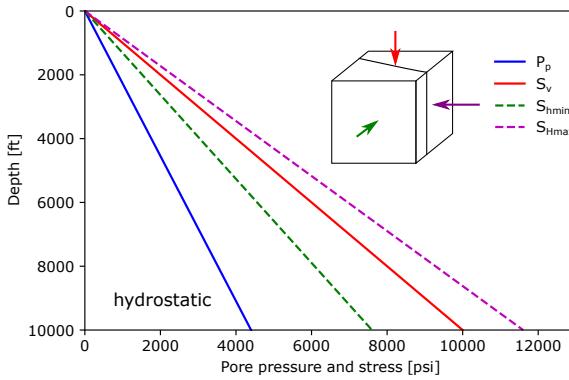




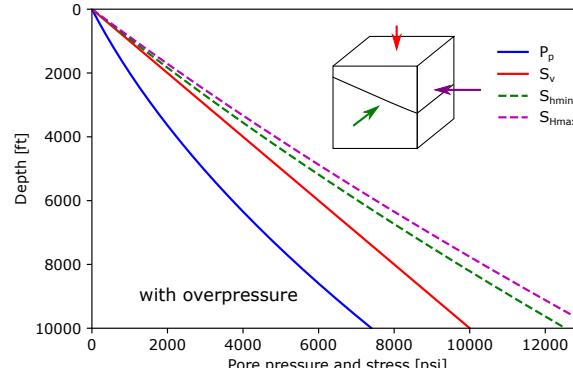
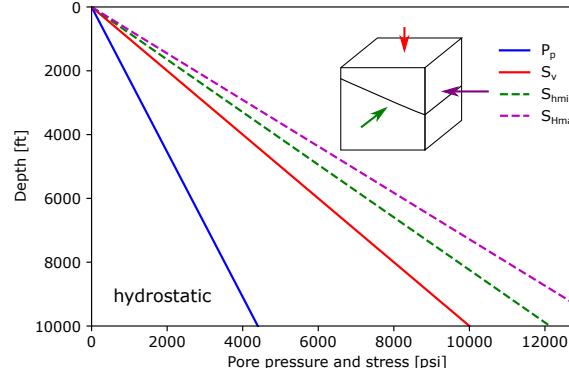
### Normal faulting: $S_v > S_{H\max} > S_{h\min}$

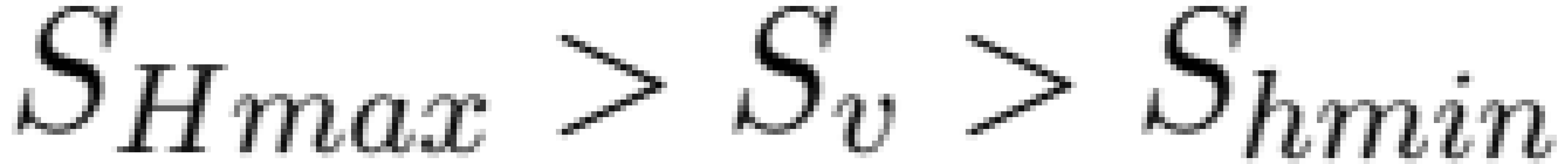


### Strike slip faulting: $S_{H\max} > S_v > S_{h\min}$



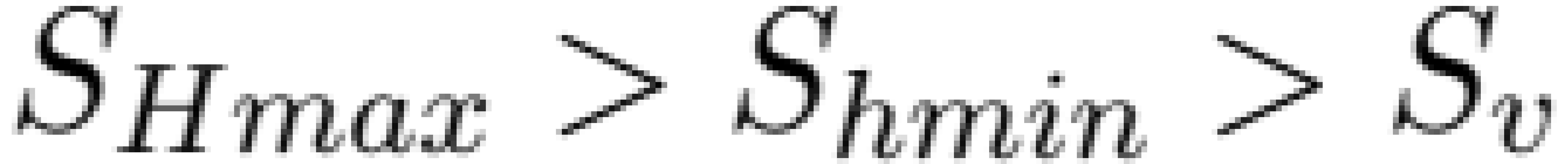
### Reverse faulting: $S_{H\max} > S_{h\min} > S_v$





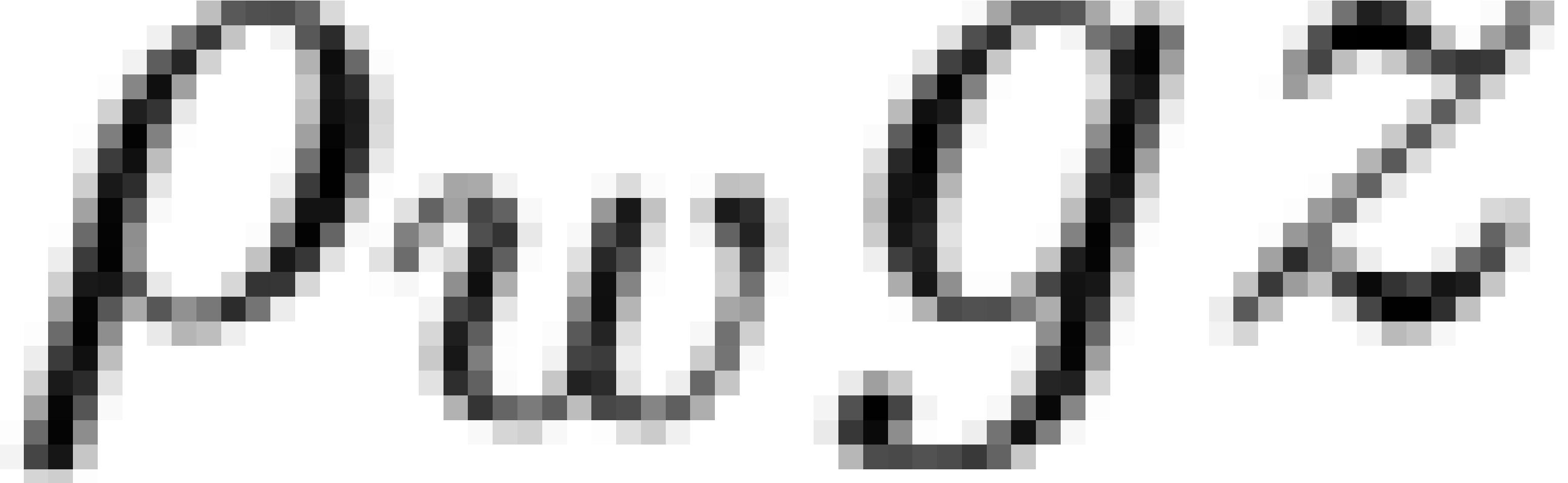










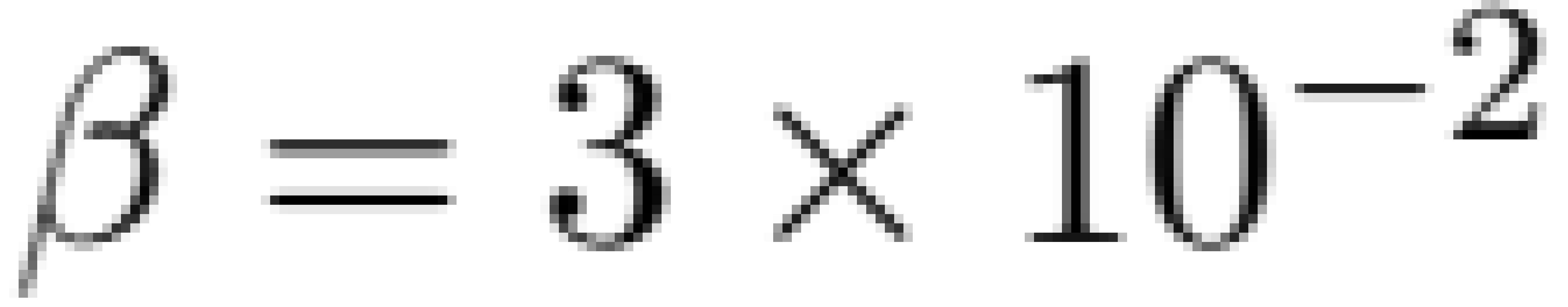


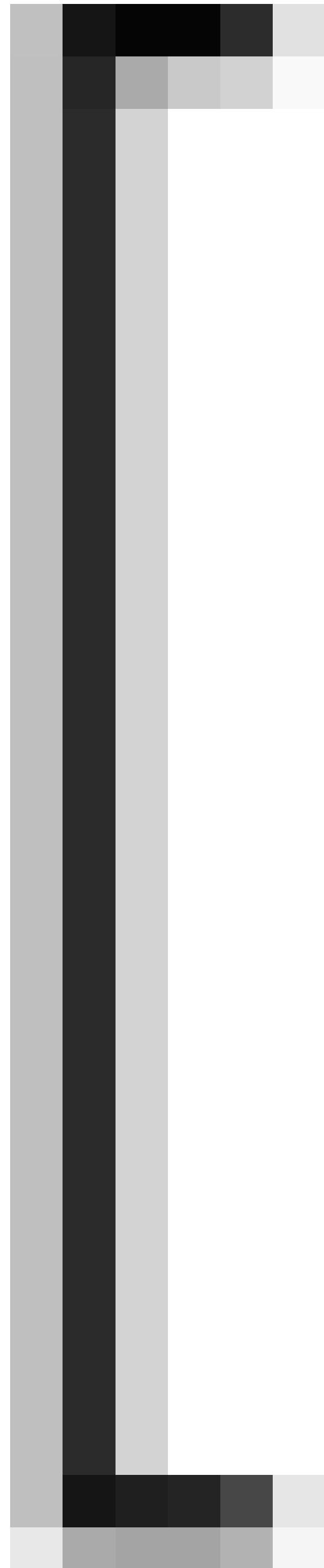


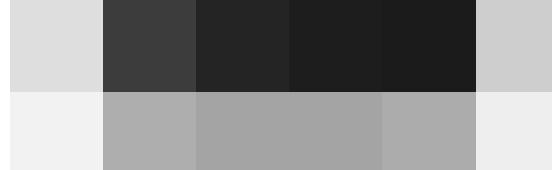
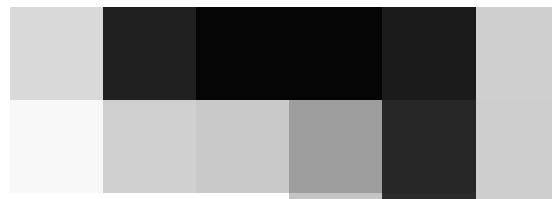


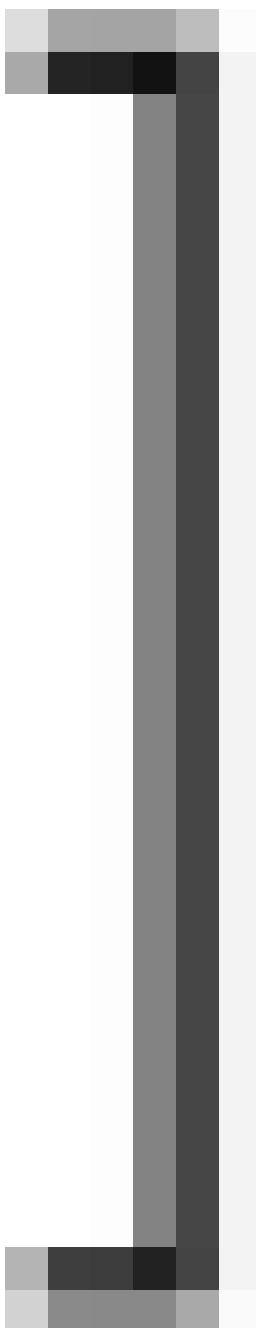
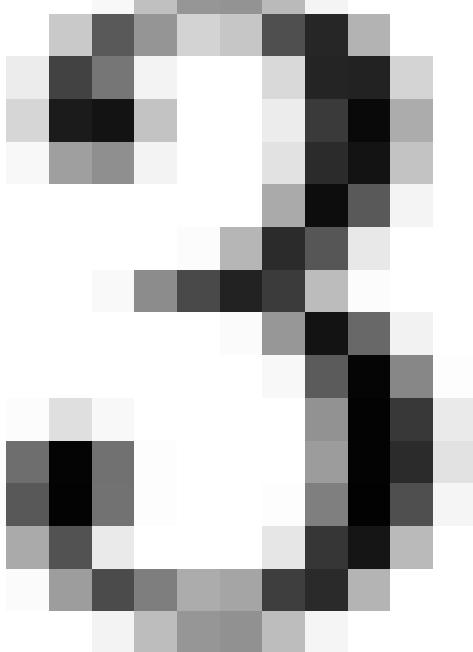


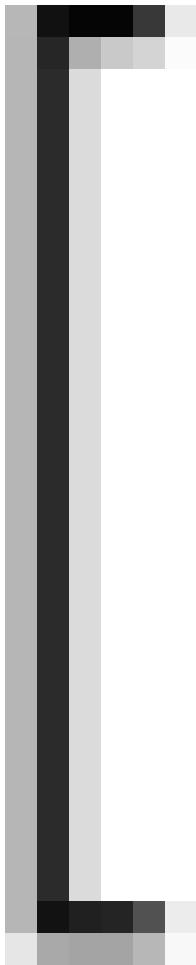


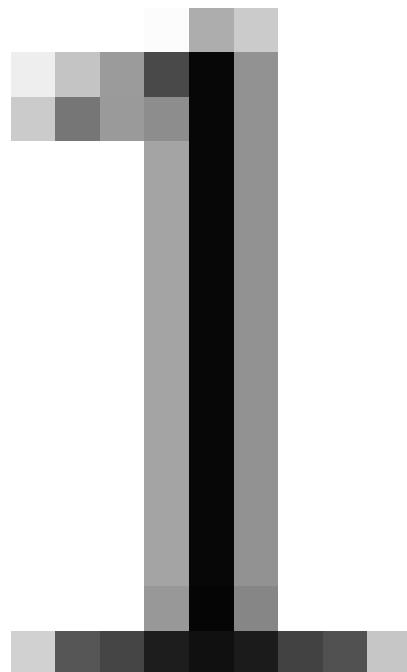
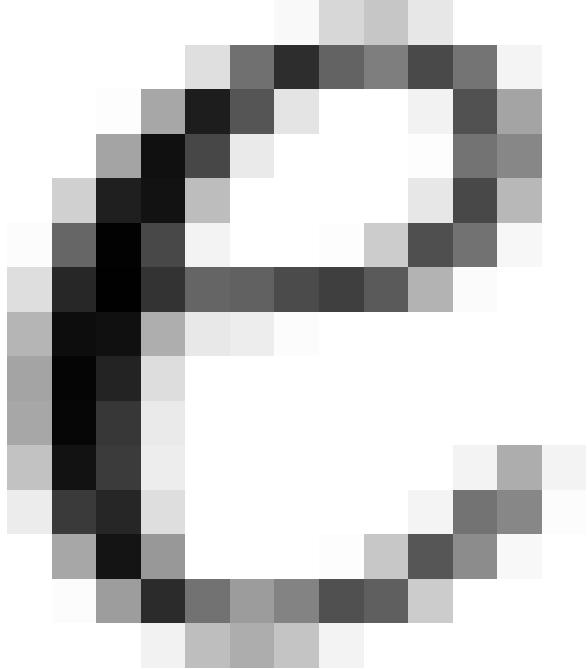


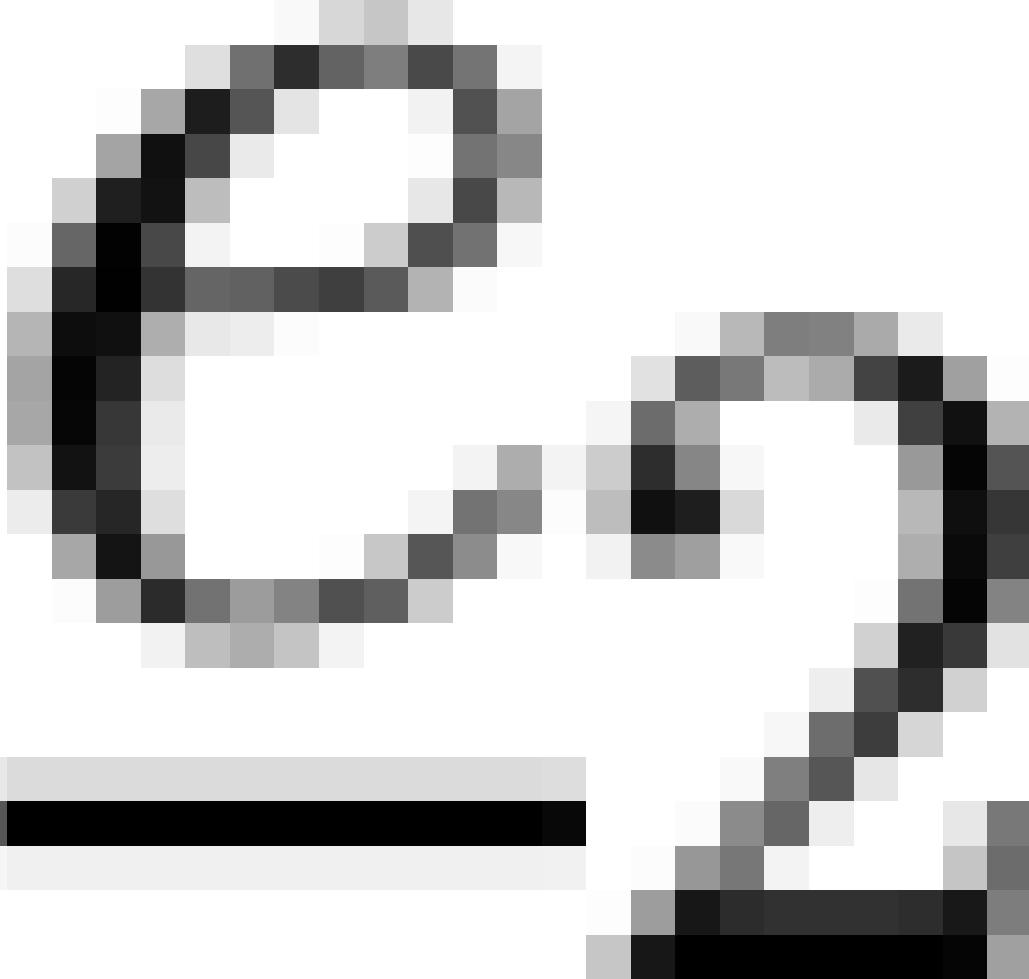


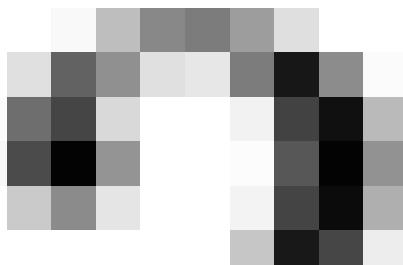
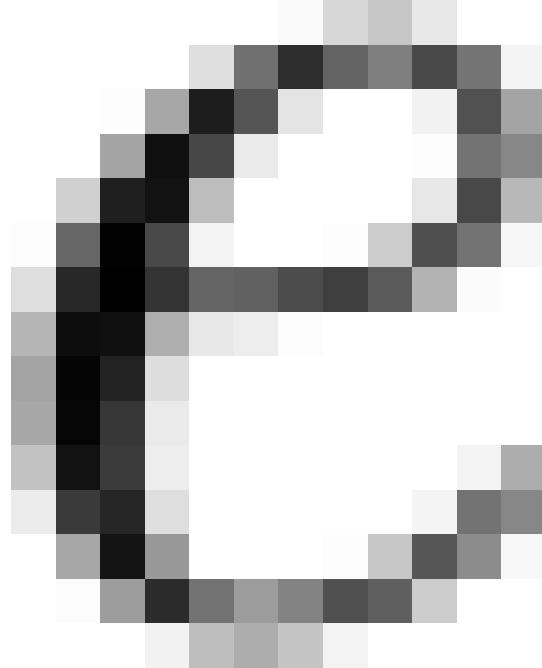


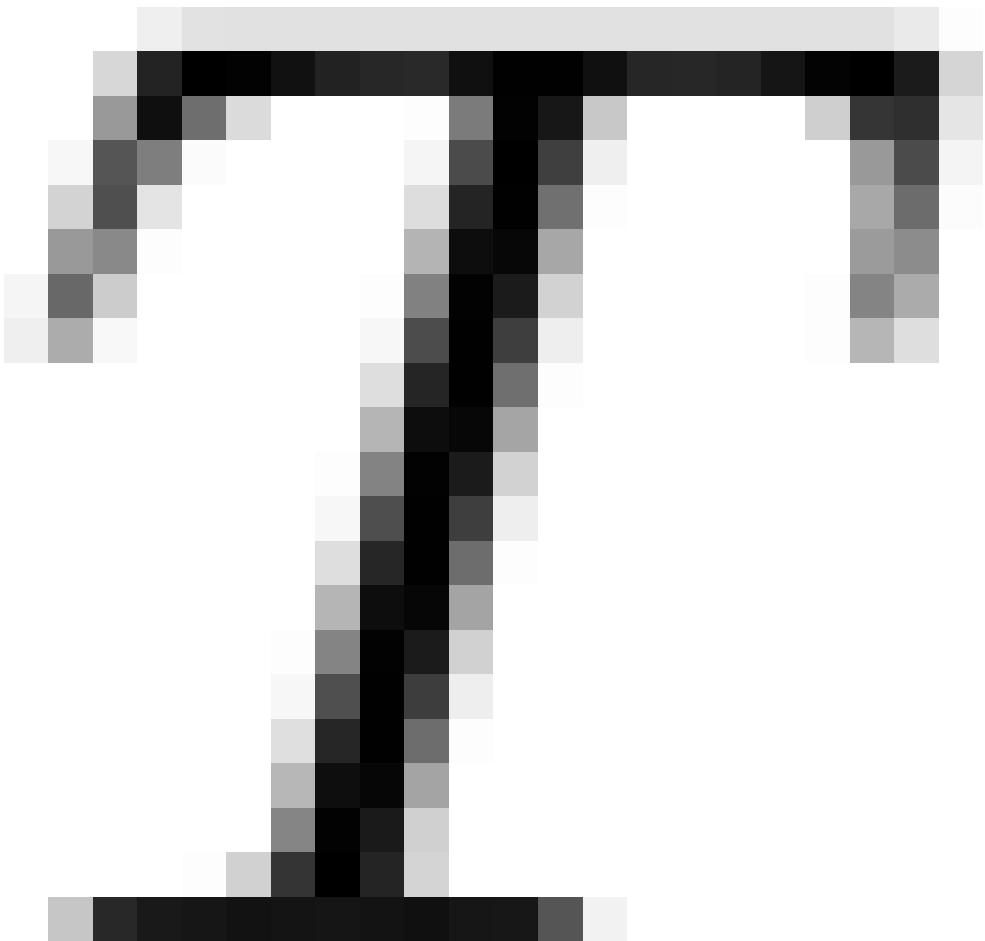


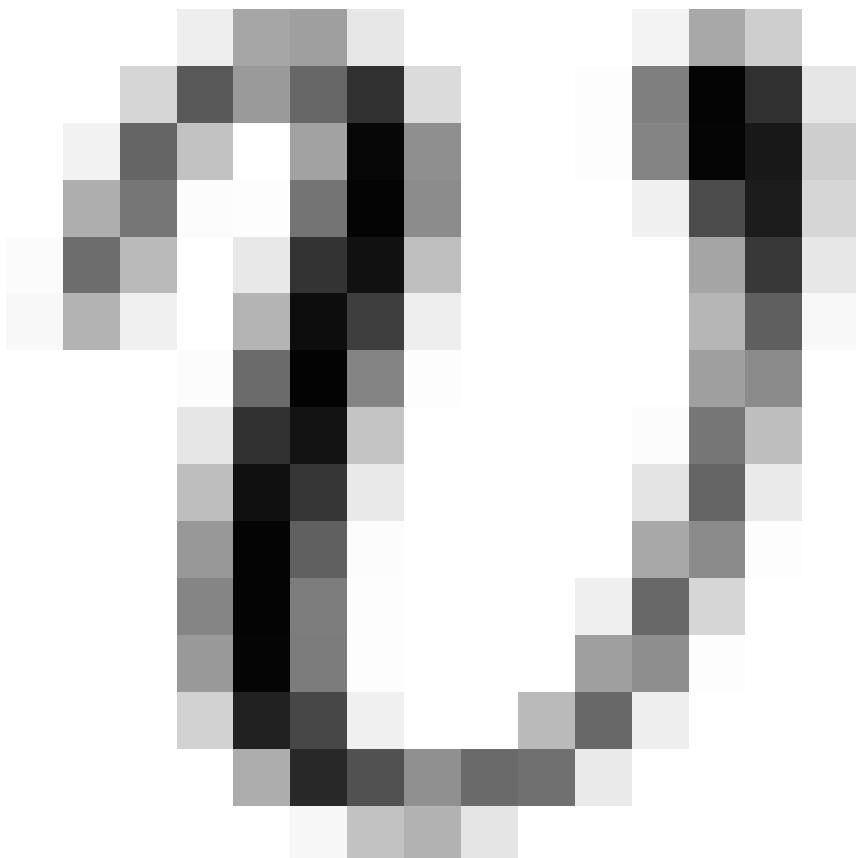


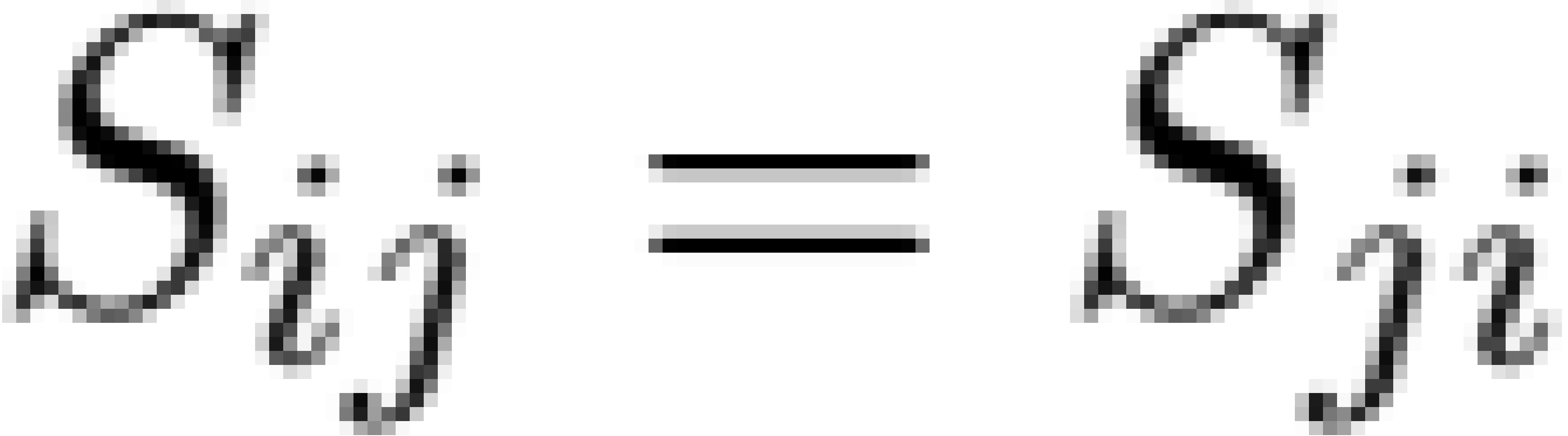


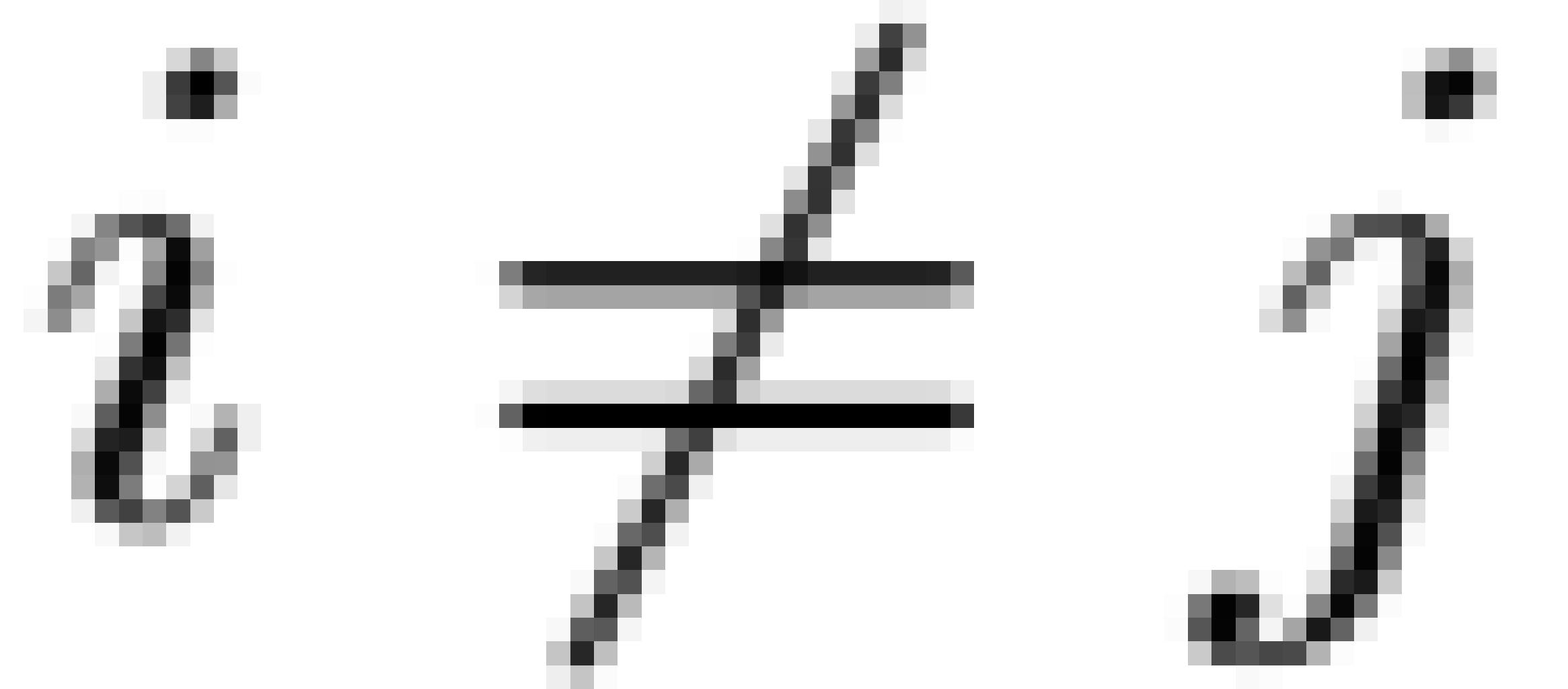


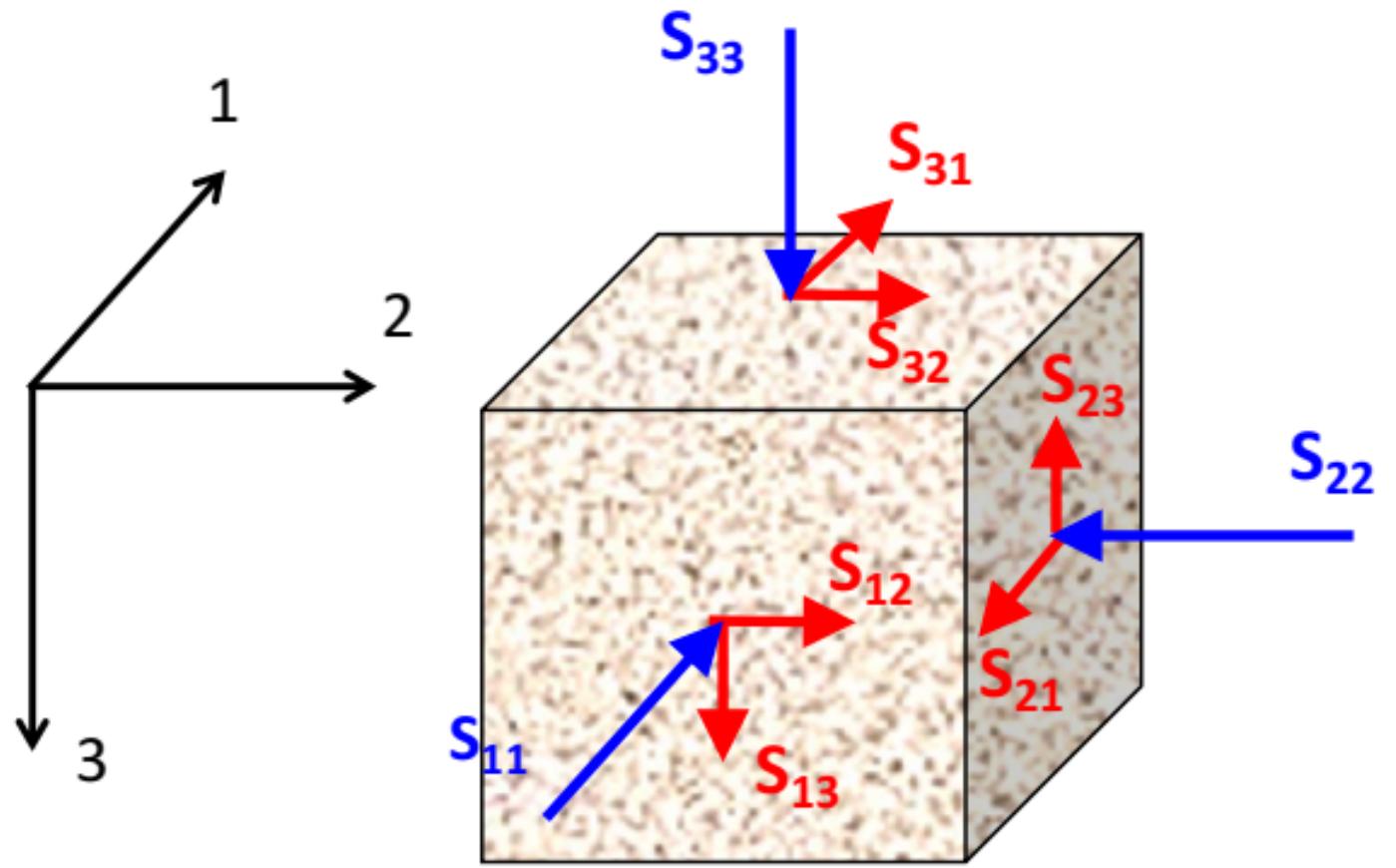






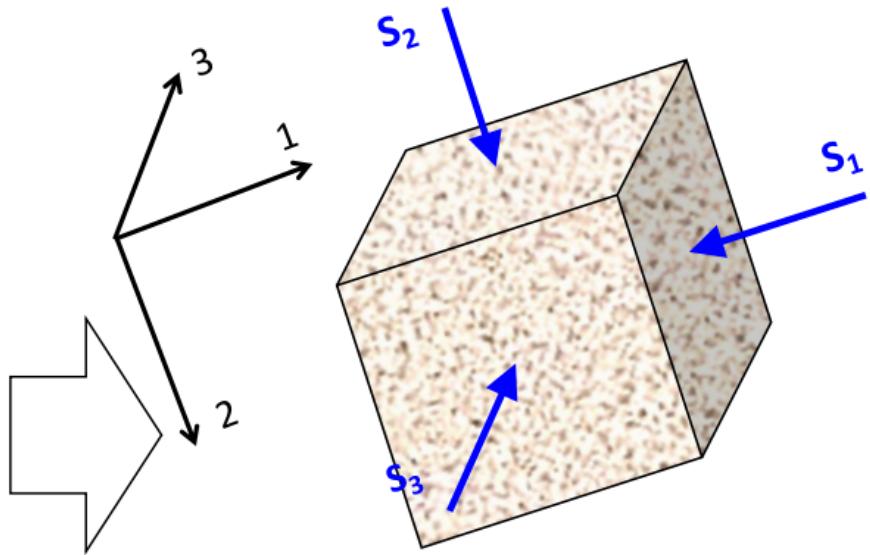
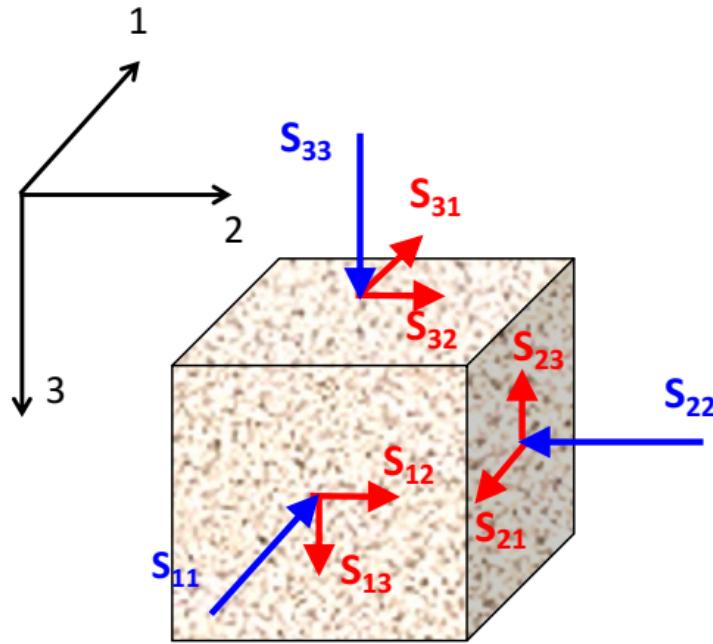






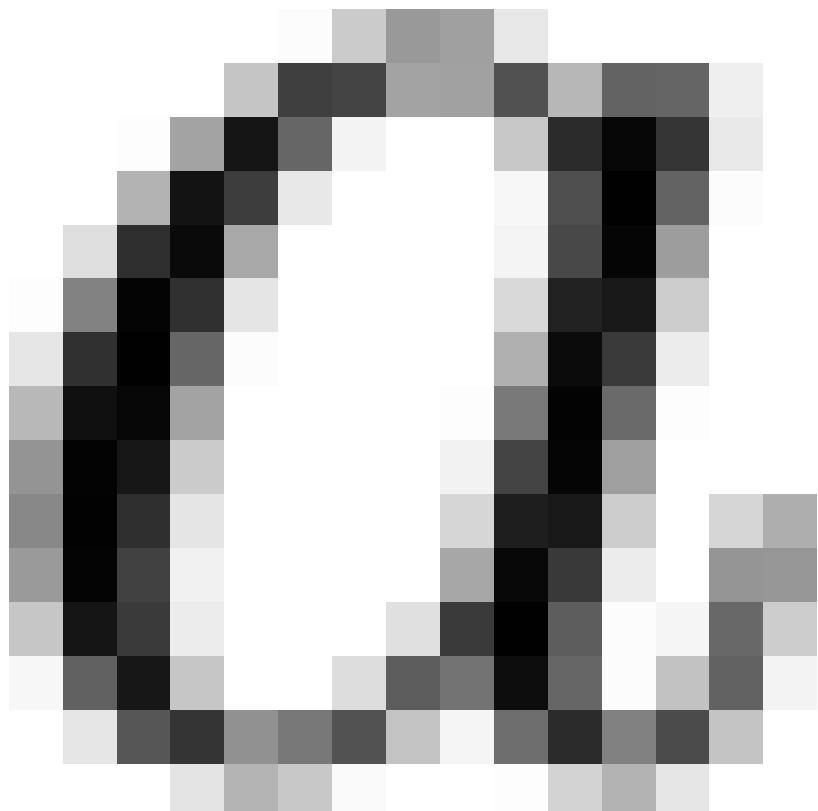
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

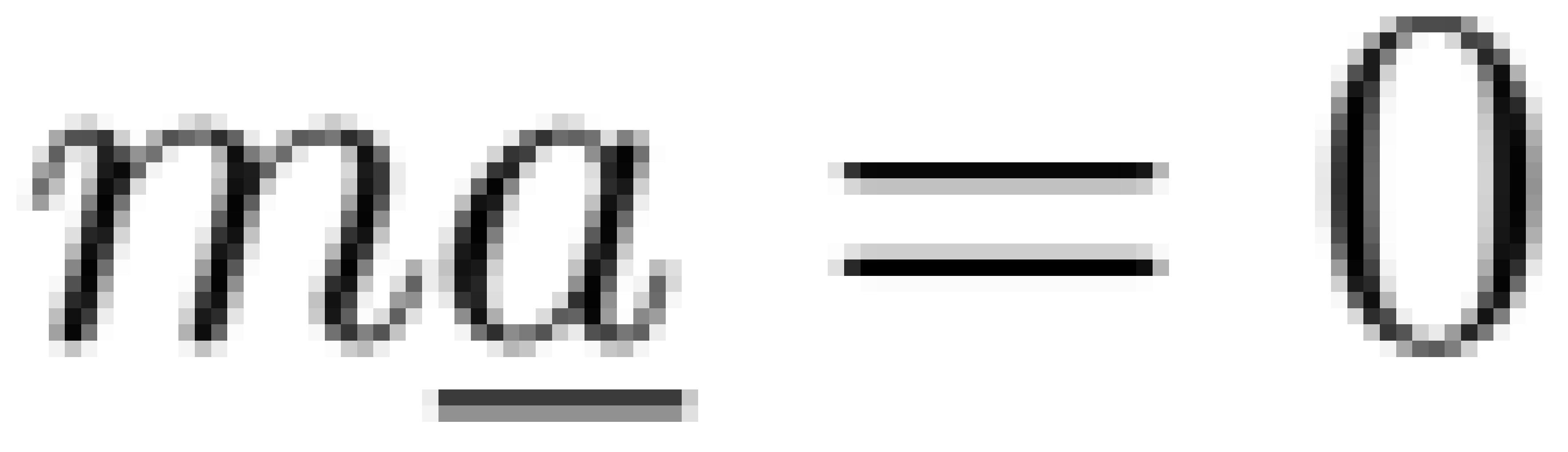




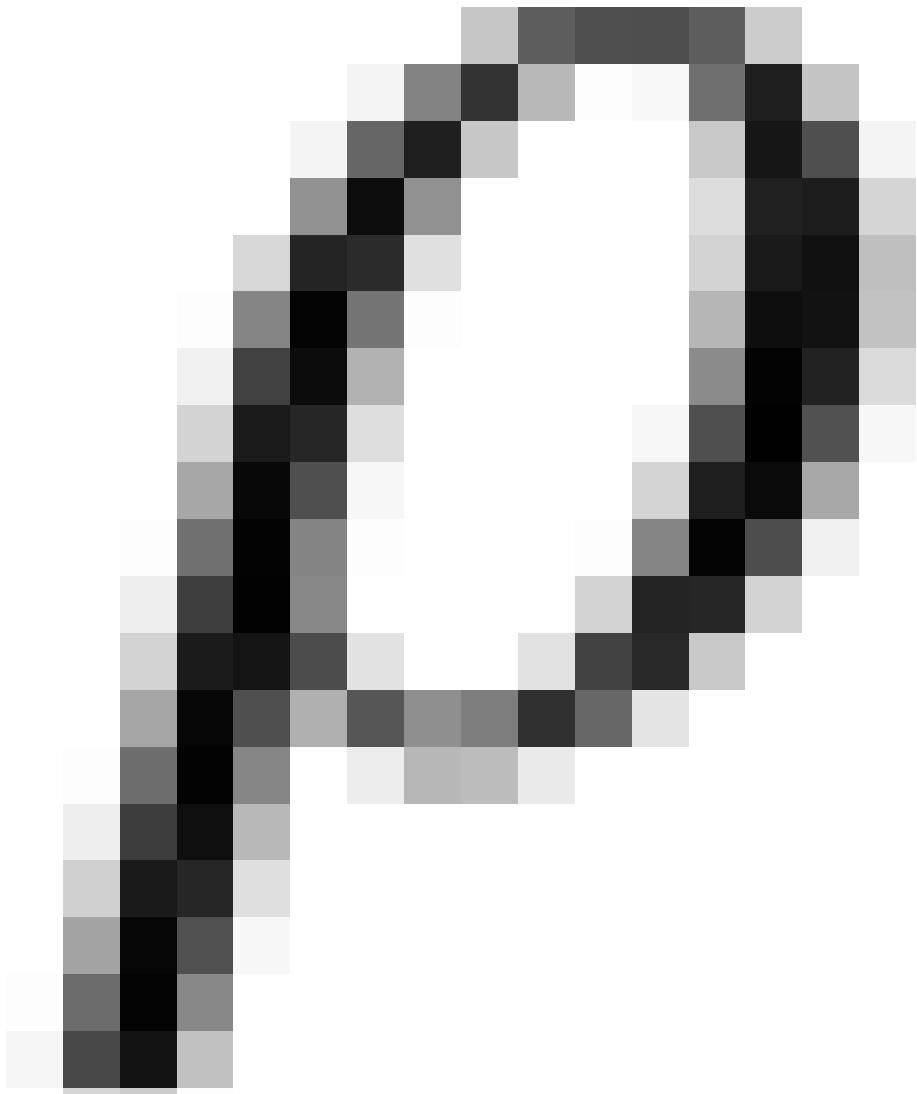
$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

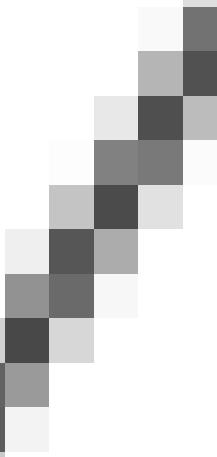
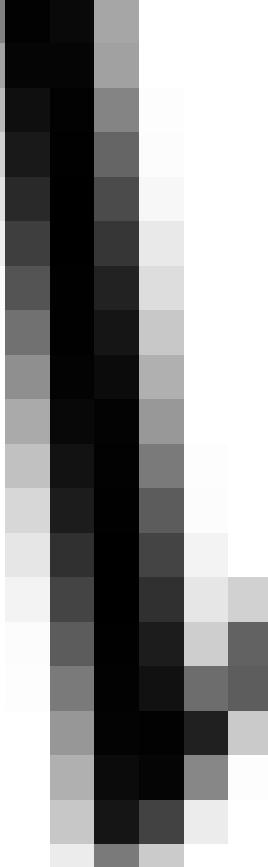
$$\underline{\underline{S}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

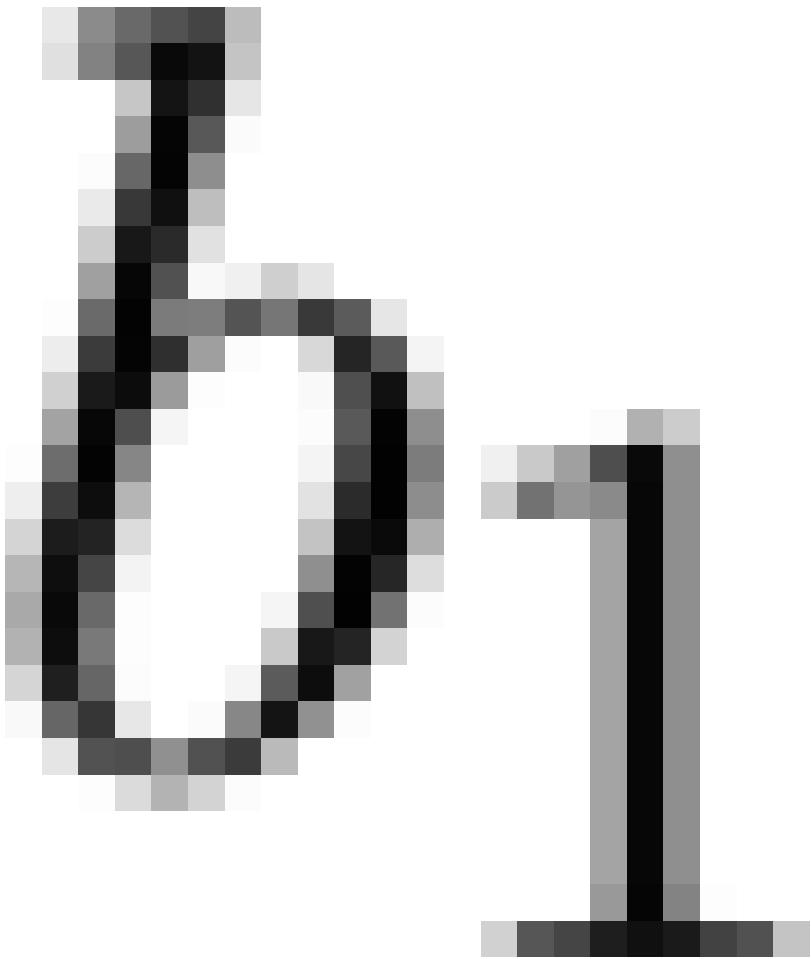








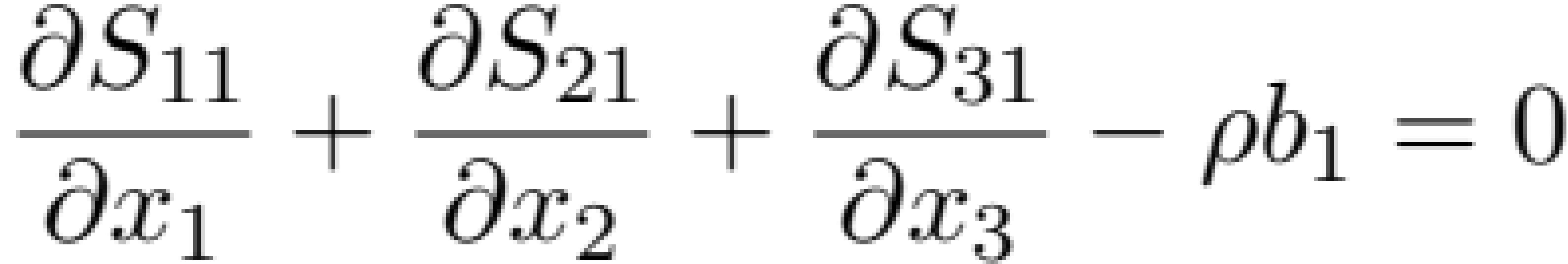


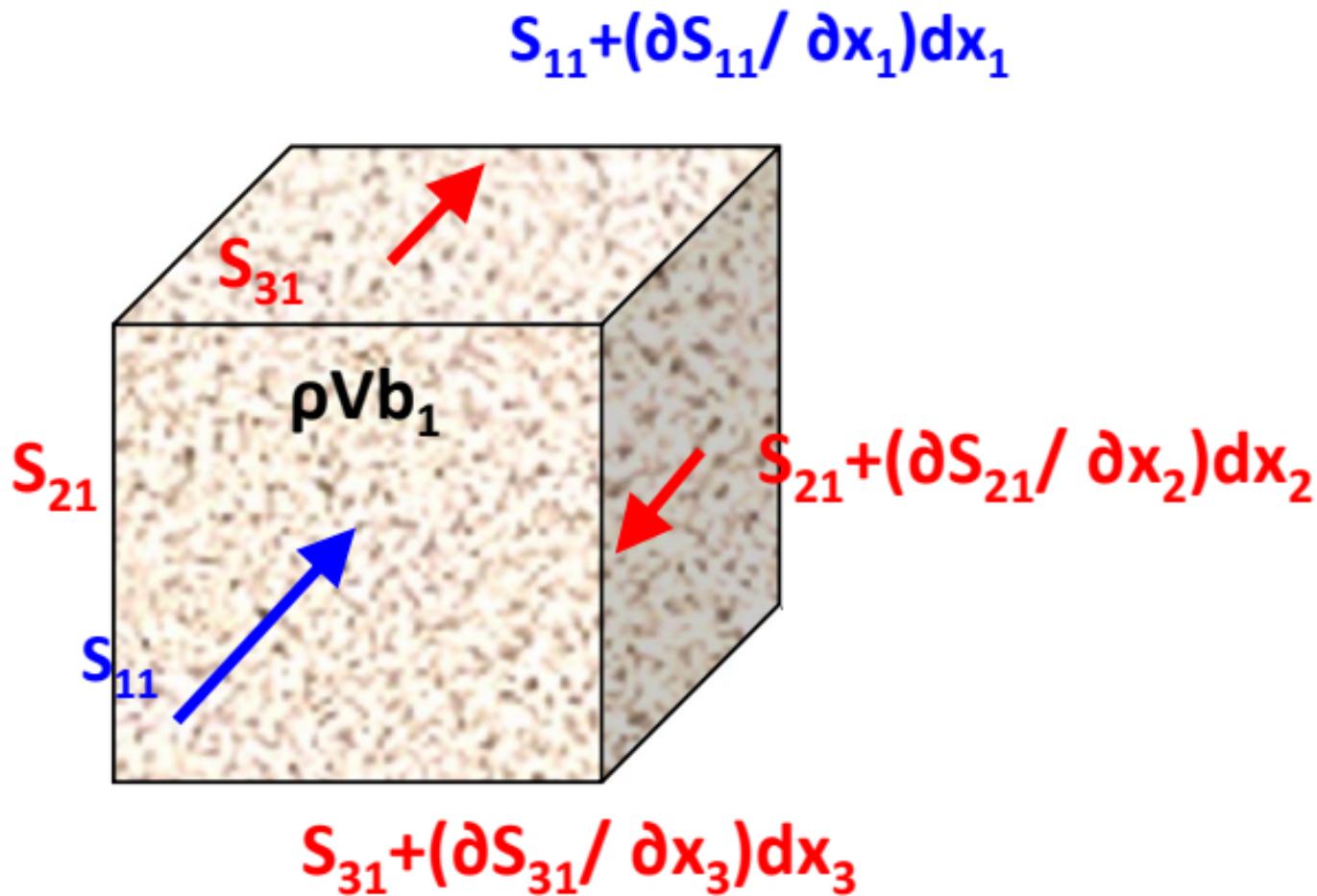
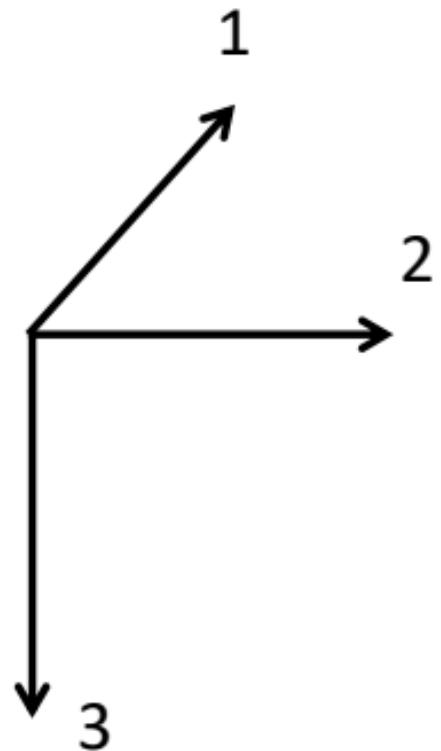


$$\sum F_1 = 0$$

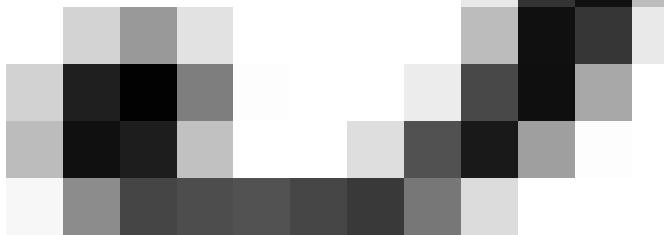
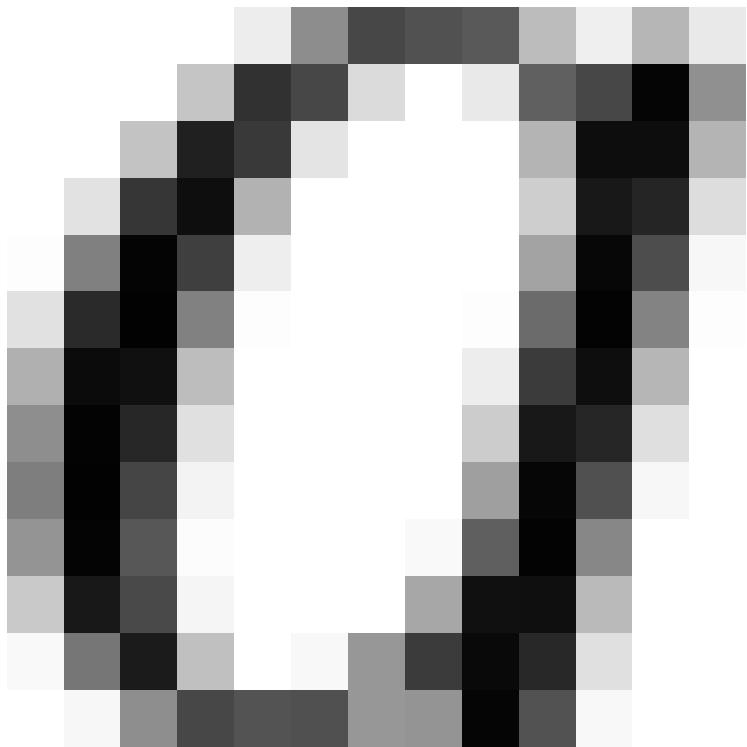
$$\begin{aligned}\sum F_1 &= +S_{11}dx_2dx_3 - \left[ S_{11} + \left( \frac{\partial S_{11}}{\partial x_1} \right) dx_1 \right] dx_2dx_3 \\ &\quad + S_{21}dx_1dx_3 - \left[ S_{21} + \left( \frac{\partial S_{21}}{\partial x_2} \right) dx_2 \right] dx_1dx_3 \\ &\quad + S_{31}dx_1dx_2 - \left[ S_{31} + \left( \frac{\partial S_{31}}{\partial x_3} \right) dx_3 \right] dx_1dx_2 \\ &\quad - \rho(dx_1dx_2dx_3)b_1 = 0\end{aligned}$$

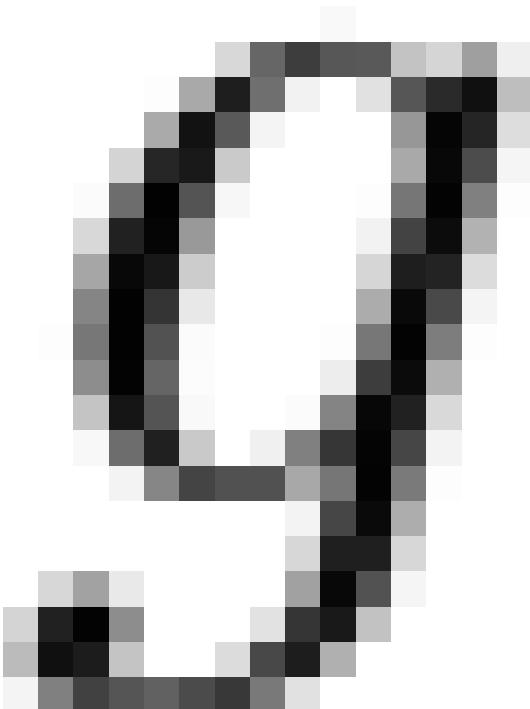
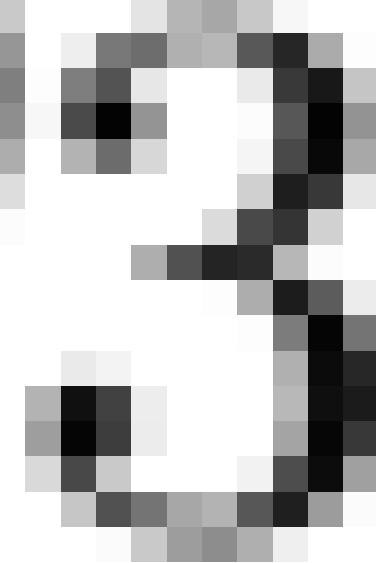
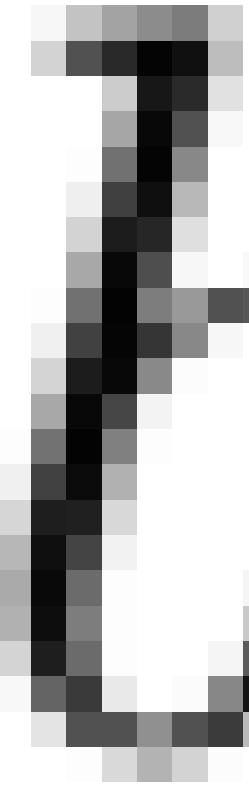






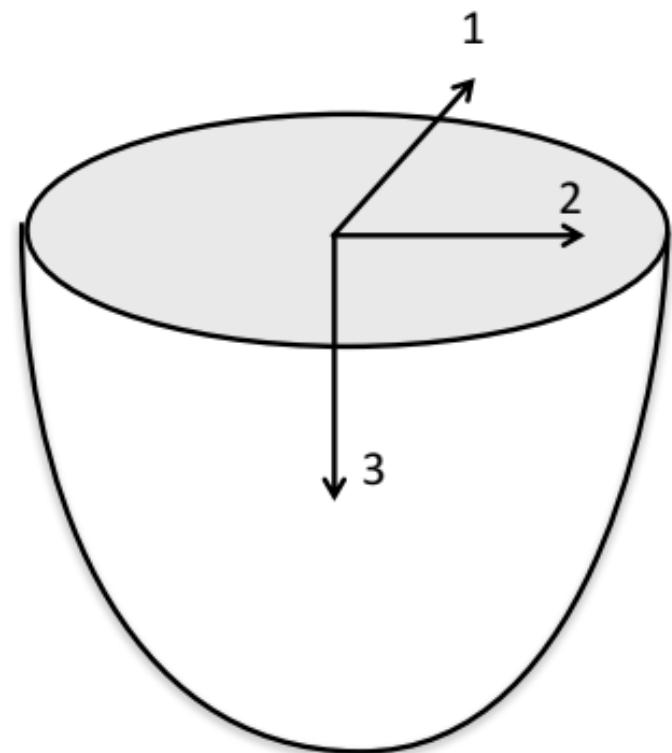
$$\left\{ \begin{array}{l} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} - \rho b_1 = 0 \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} - \rho b_2 = 0 \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} - \rho b_3 = 0 \end{array} \right.$$







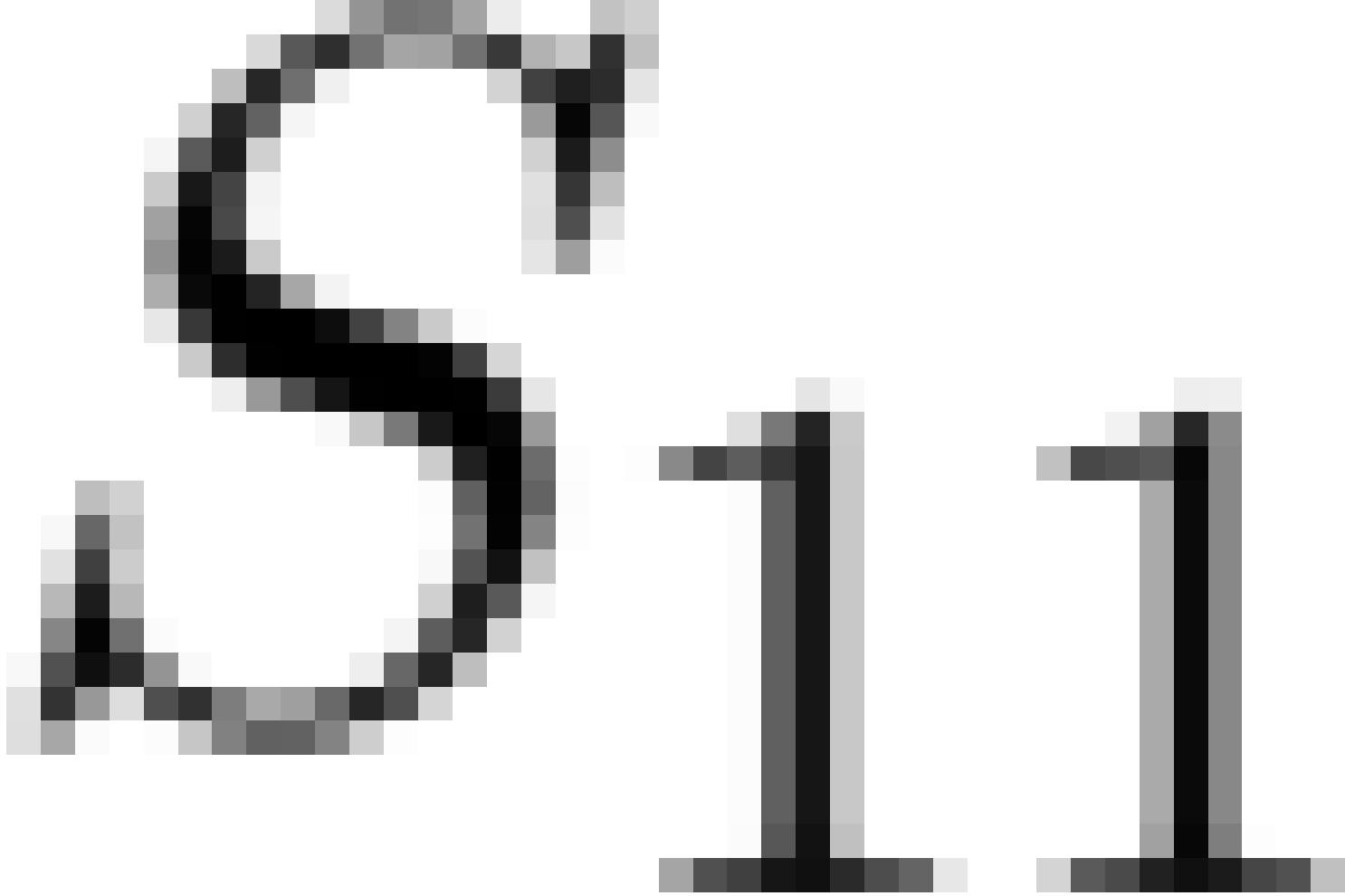
$$s_{33}(c_3) = \rho(c_3) \circ d_{c_3}$$



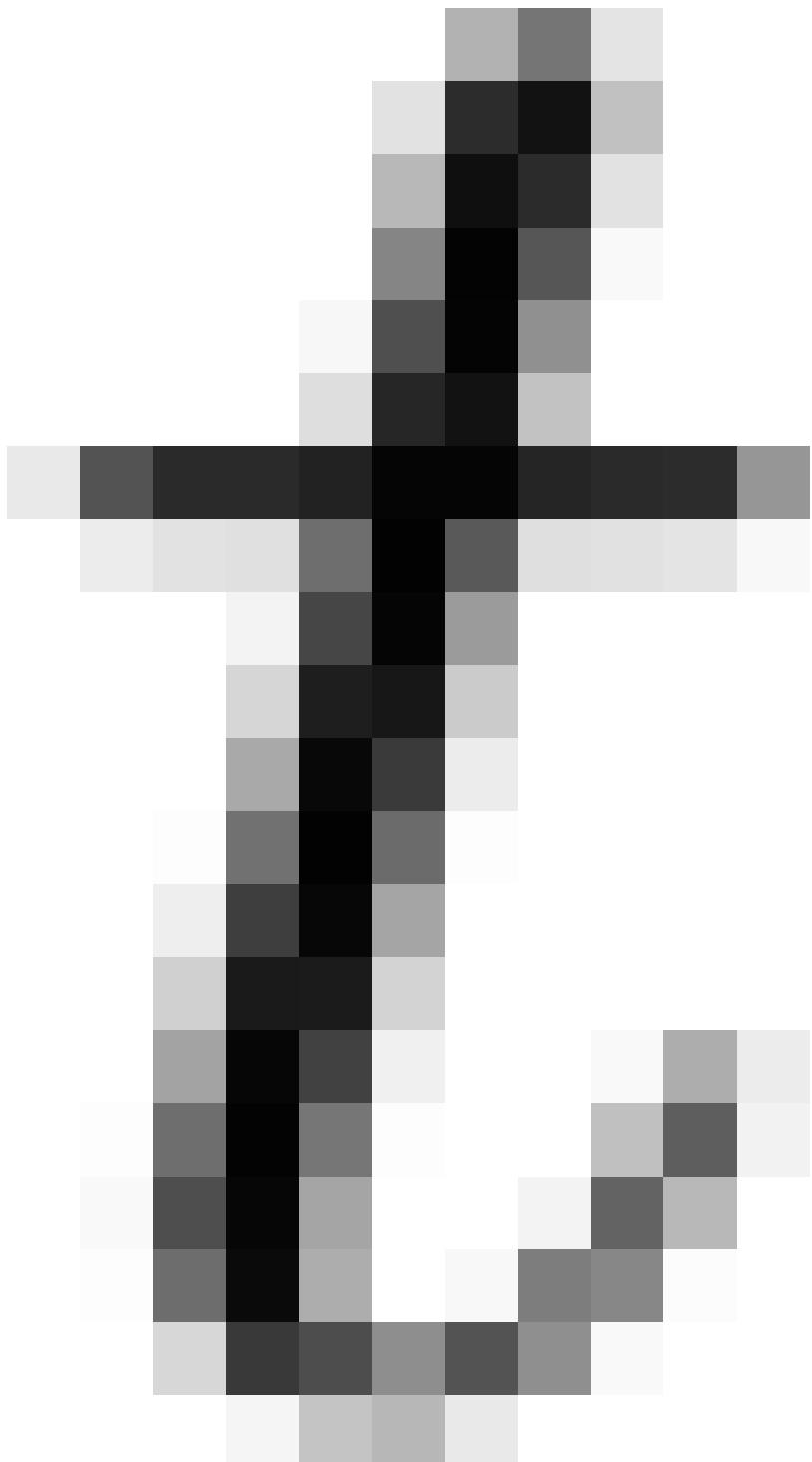
$$\left\{ \begin{array}{l} \cancel{\frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2}} \\ \cancel{\frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2}} \\ \cancel{\frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2}} \end{array} \right.$$

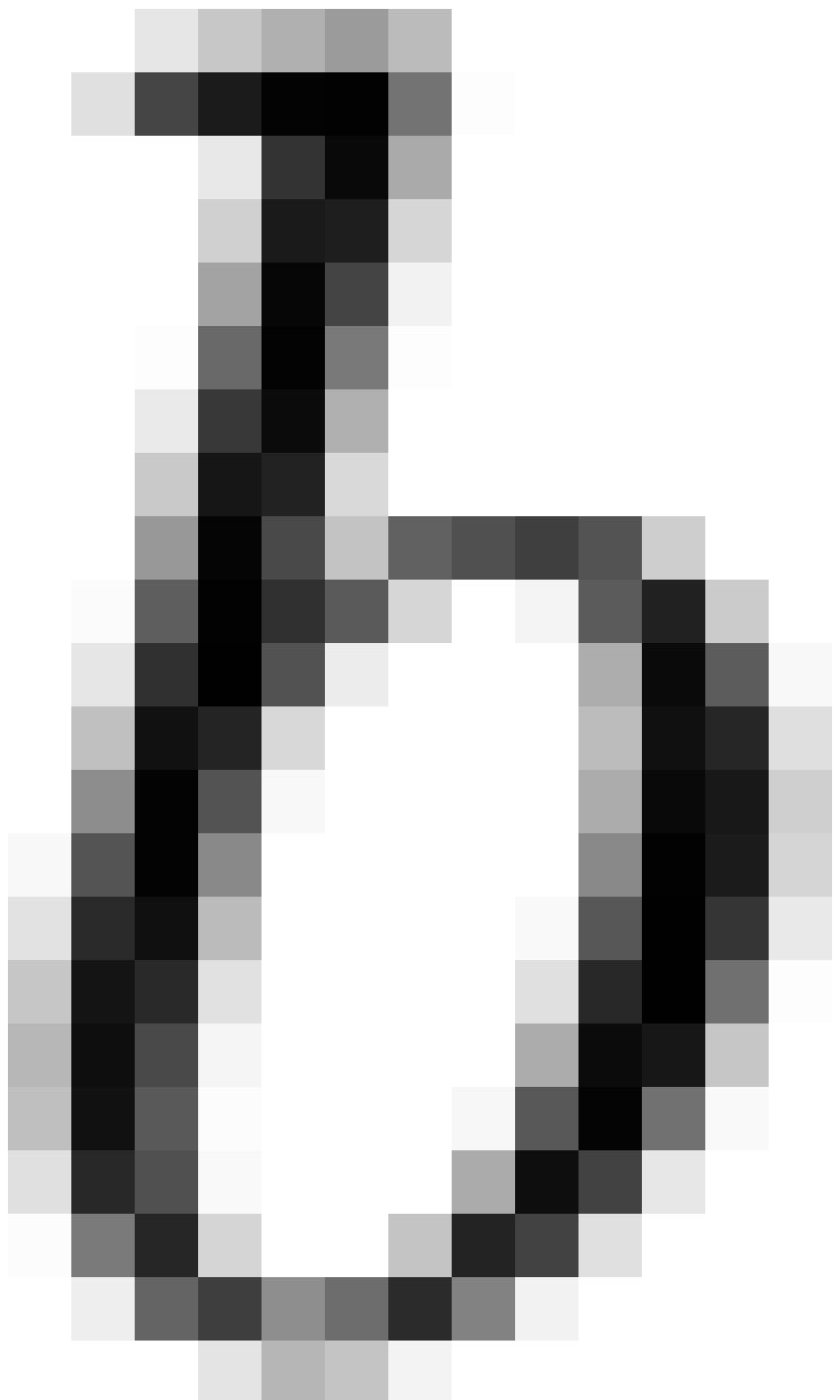
$$\frac{\partial S_{33}}{\partial x_3} - \rho(x_3)g = 0$$

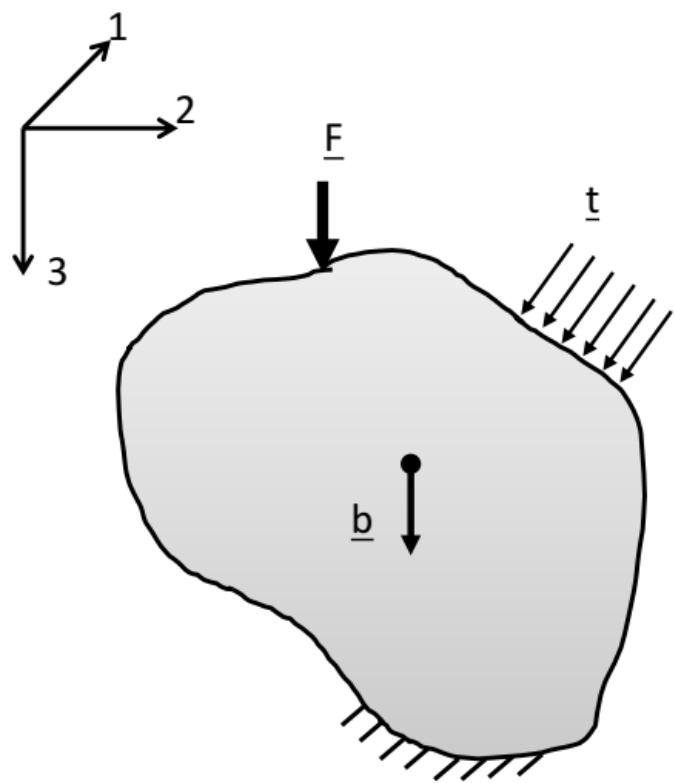
$$S_{33} = \int_0^{x_3} \rho(x_3)g \, dx_3$$











Displacement condition

$$\begin{cases} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2} \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2} \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2} \end{cases}$$

And respect the boundary conditions:

- Displacement
- Boundary stresses
- Boundary Forces
- Body Forces

How do we relate stresses to displacements?

- Displacements → Strains (**Kinematic equations**)
- Strains → Stresses (**Constitutive equations**)



$\epsilon_{11}$



$\alpha_1$

$\alpha_1$

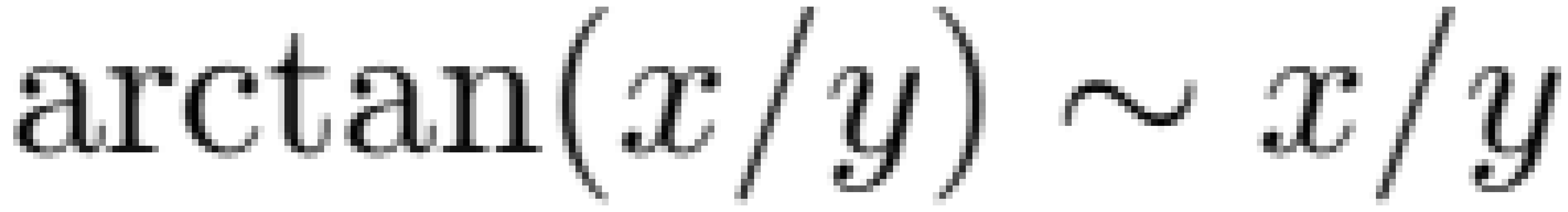
$\epsilon_{22}$

$=$

$\alpha_2$

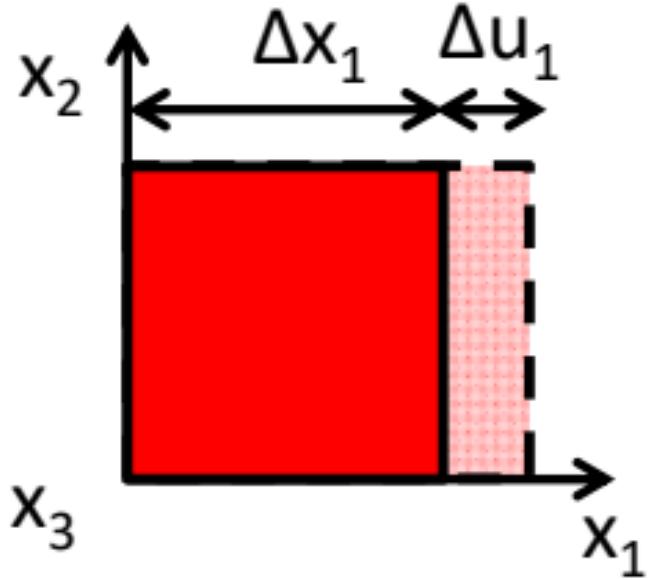
$\alpha_2$

acc10(01) + acc20(02) + acc30(03)

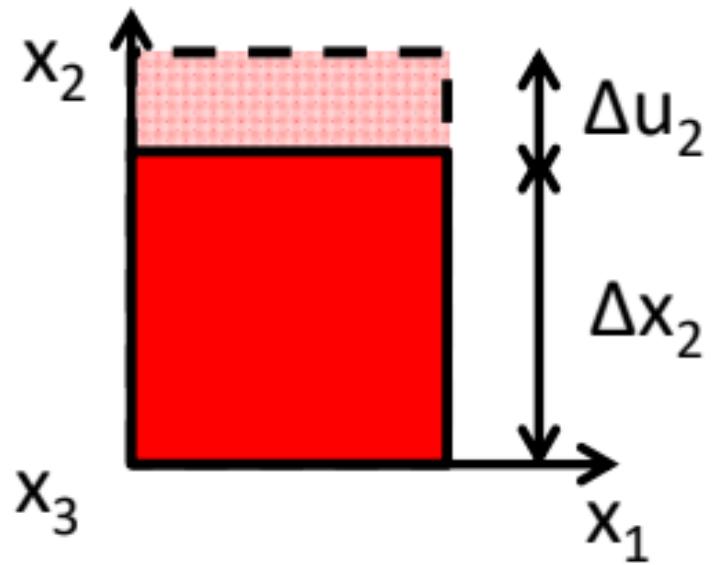


$\epsilon_{12} =$  $\frac{1}{2}$ 

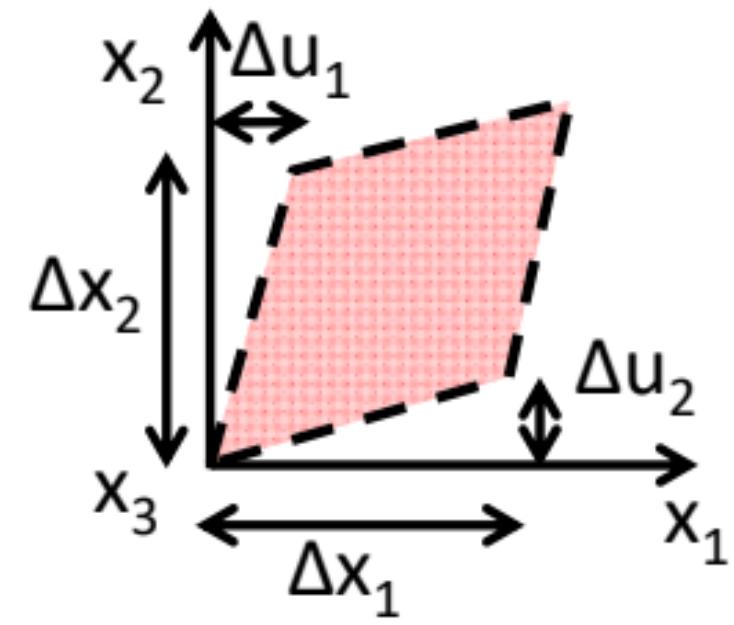
$$\frac{1}{2} \left( u_1 - u_2 + c_1 - c_2 \right)$$



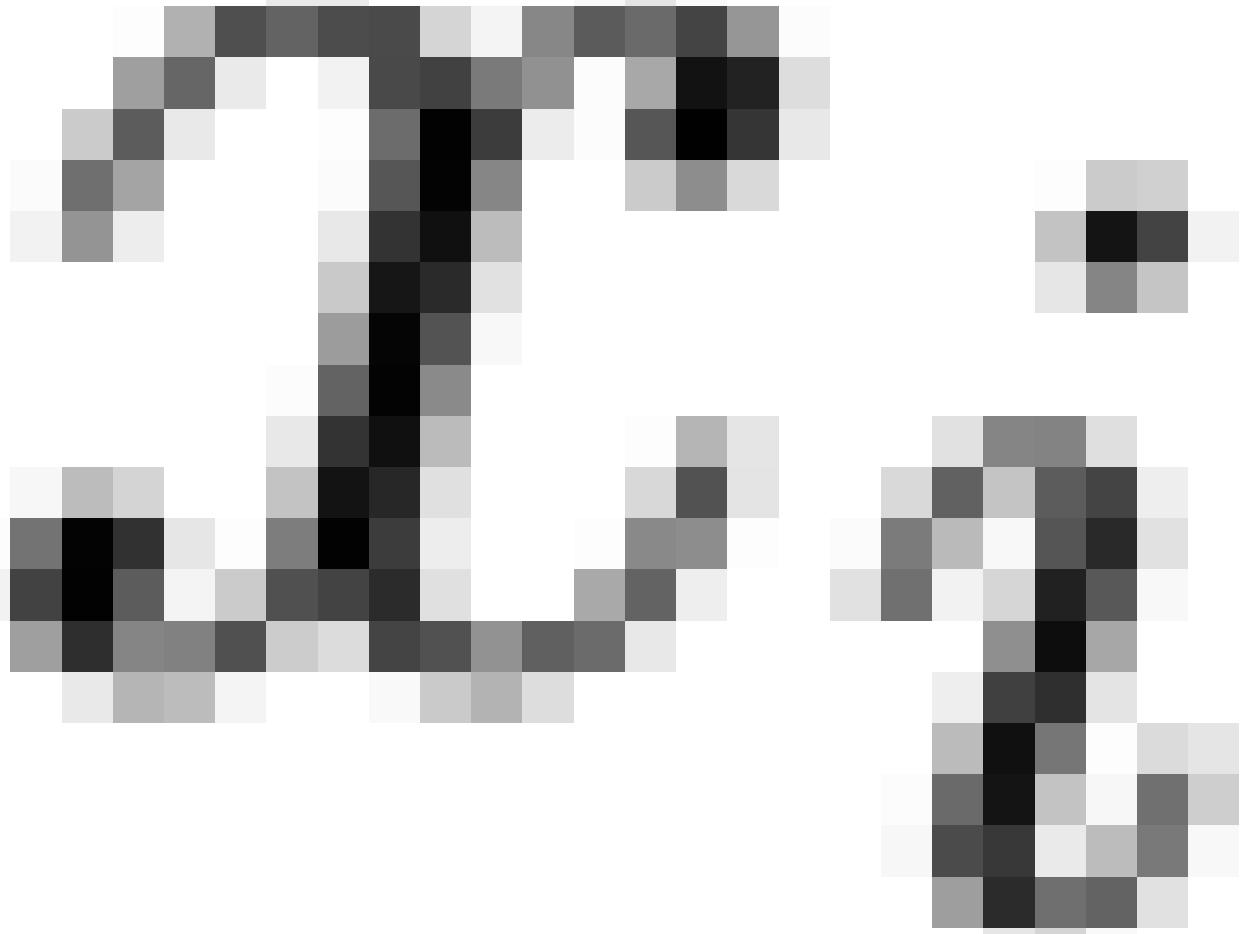
$$\varepsilon_{11} \approx \frac{\Delta u_1}{\Delta x_1}$$



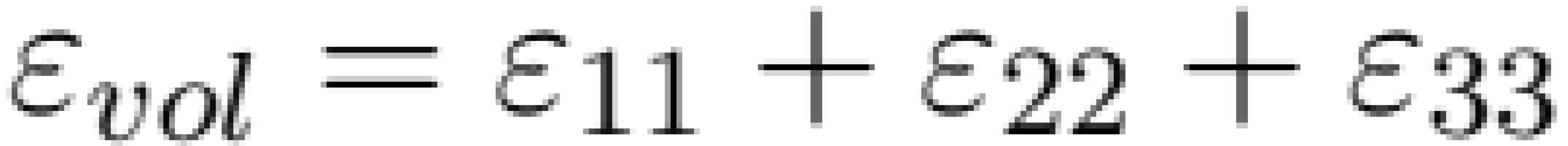
$$\varepsilon_{22} \approx \frac{\Delta u_2}{\Delta x_2}$$



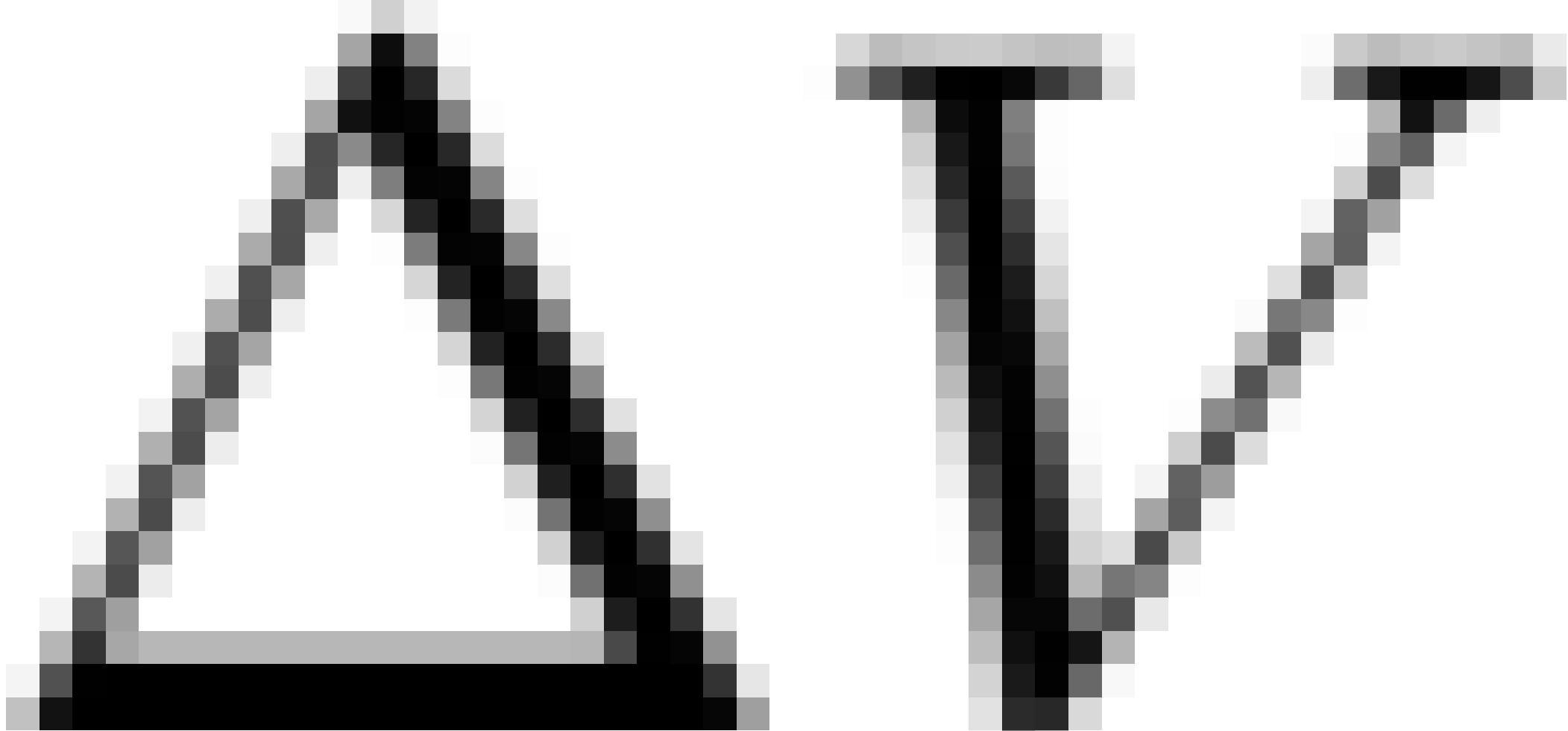
$$\varepsilon_{12} \approx \frac{1}{2} \left( \frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1} \right)$$

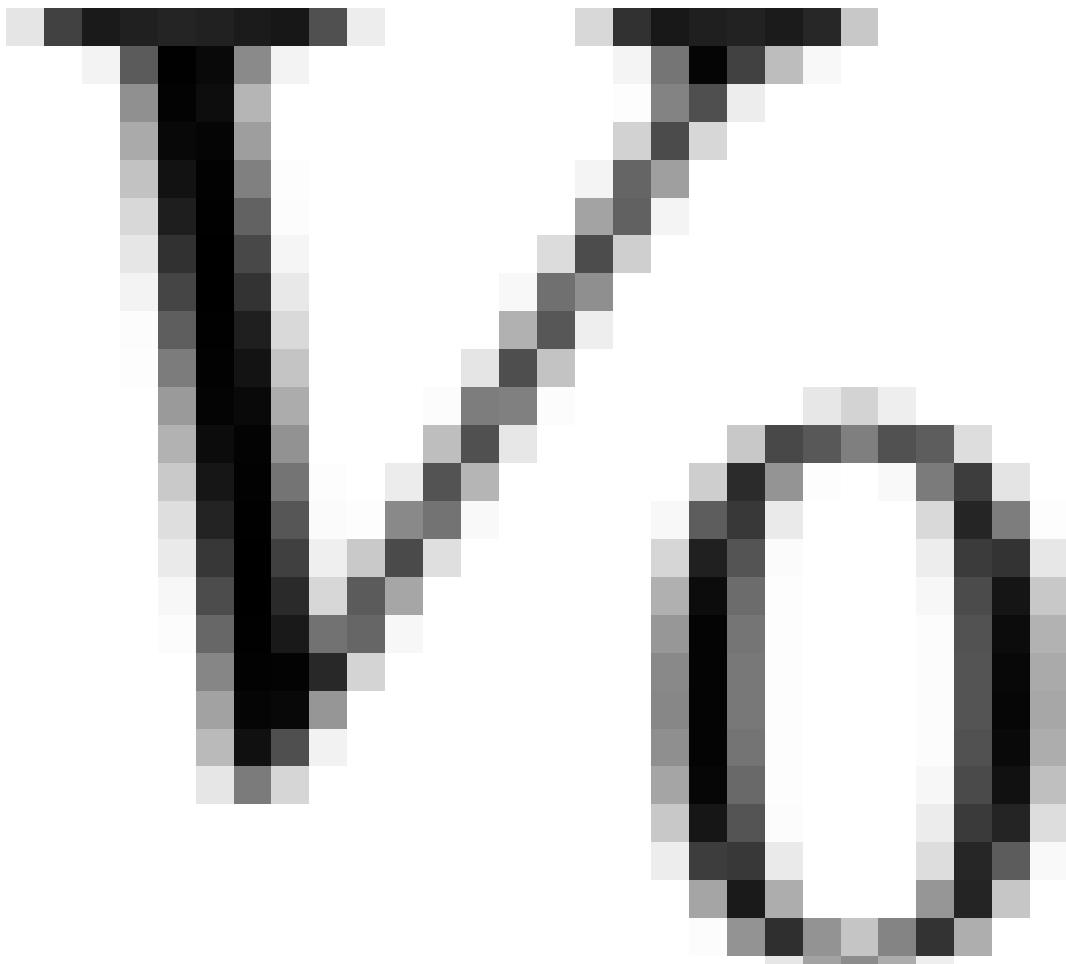




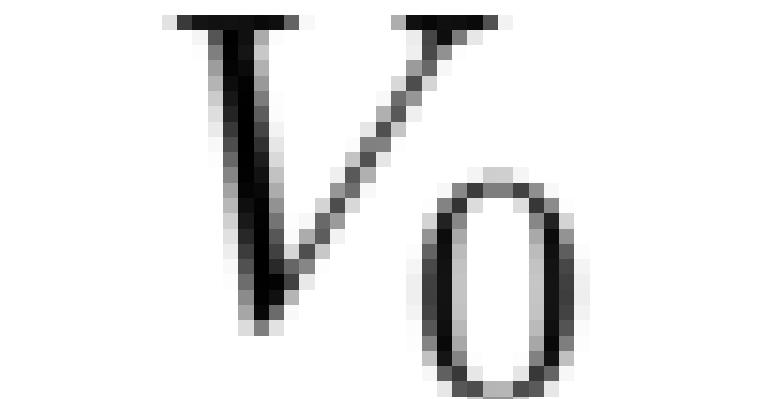
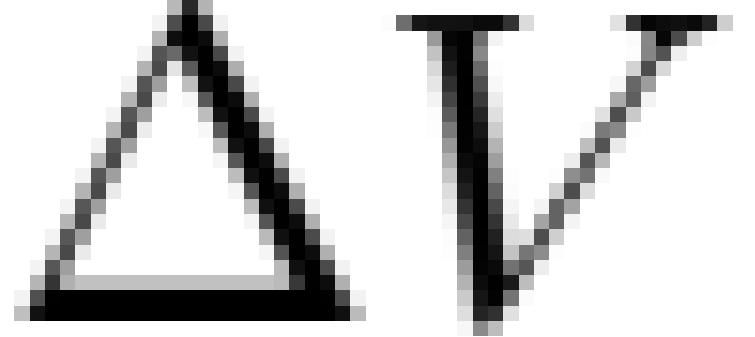




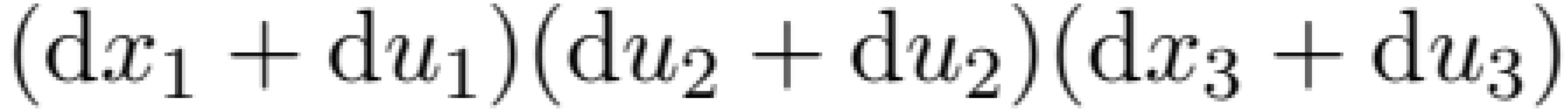




cool





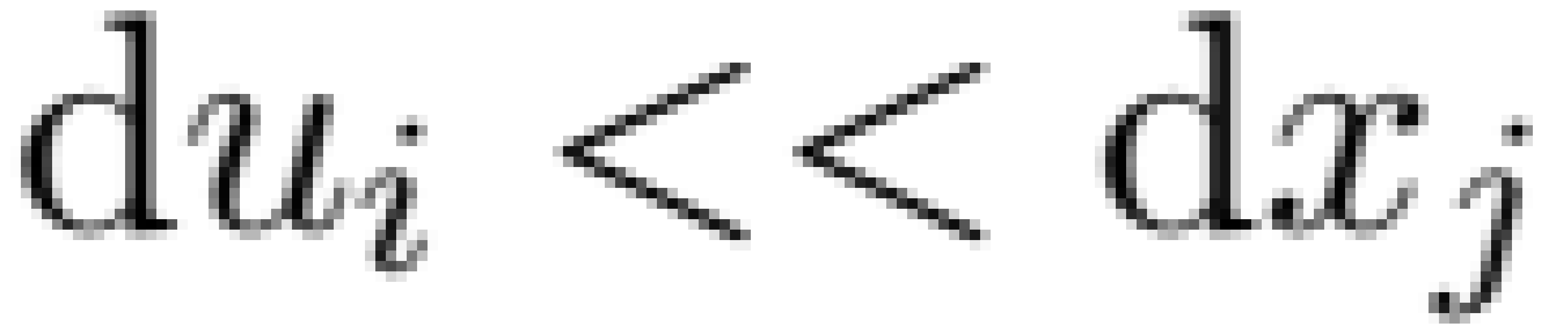


$$\epsilon_{vol} = \frac{[(dx_1 + du_1)(dx_2 + du_3) - (dx_1 dx_2 dx_3)]}{(dx_1 dx_2 dx_3)}$$



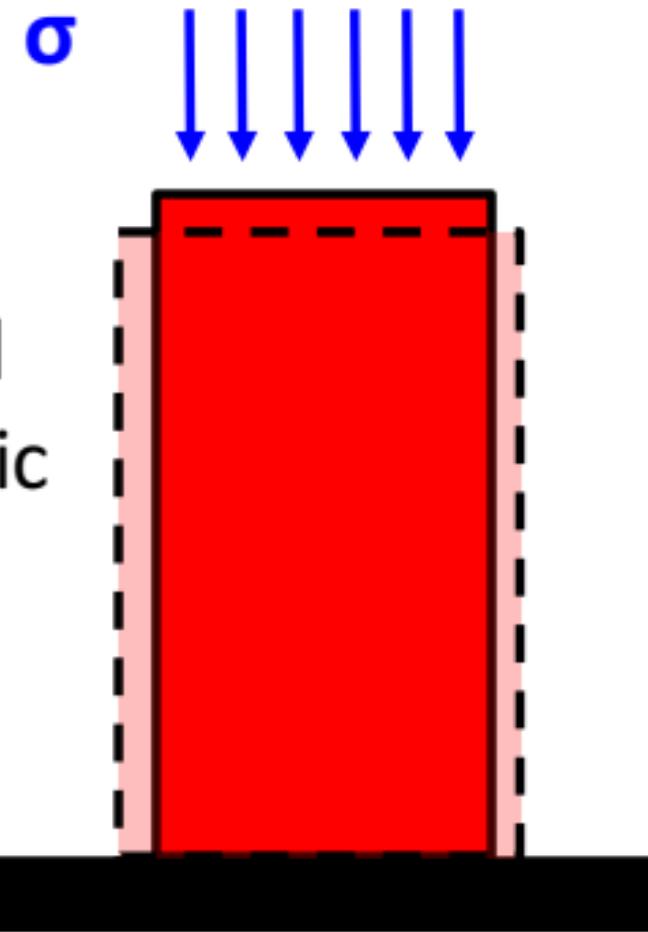




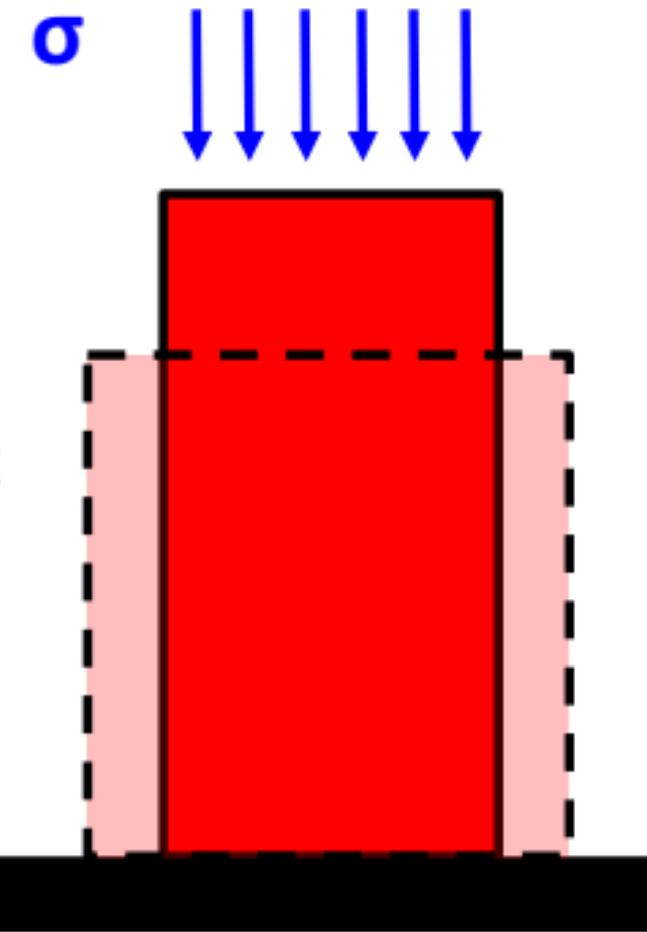


$$\text{vol}^2 \frac{(dx_1 dx_2 du_3 + dx_1 dx_3 du_2 + dx_2 dx_3 du_1)}{(dx_1 dx_2 dx_3)} = \frac{du_1}{\epsilon_{11}} + \frac{du_2}{\epsilon_{22}} + \frac{du_3}{\epsilon_{33}}$$

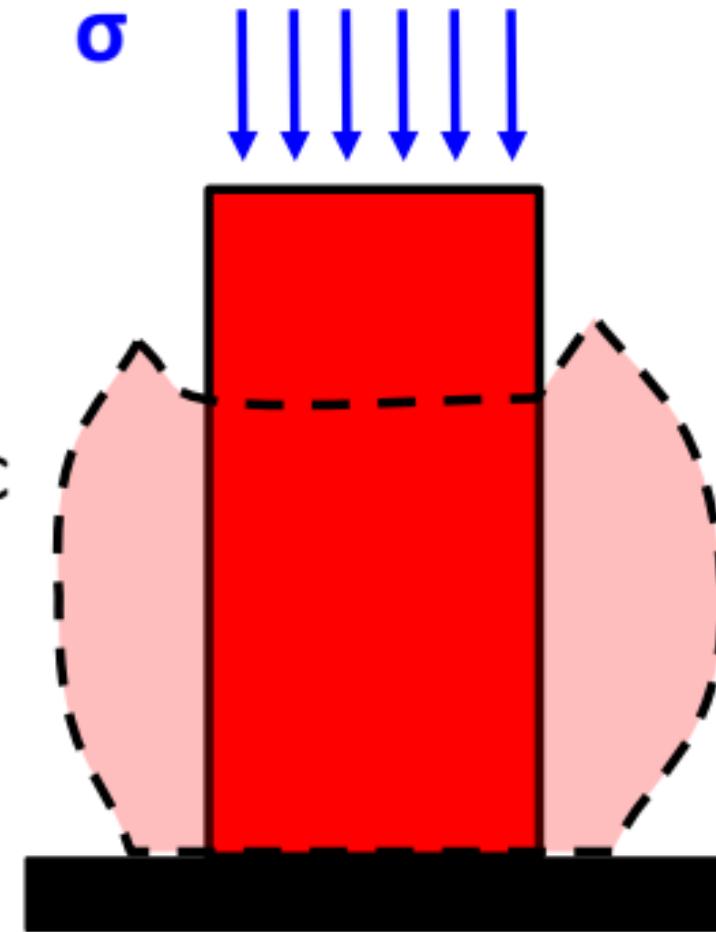
Hard  
elastic  
solid

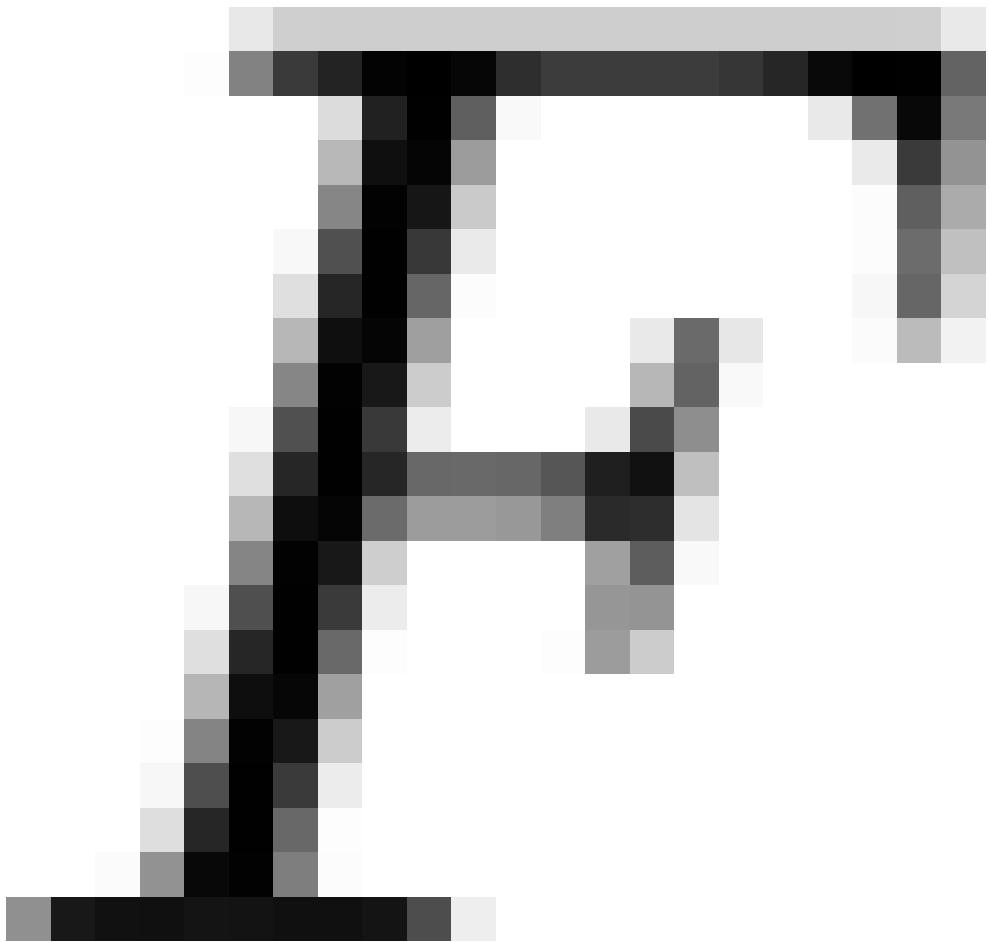


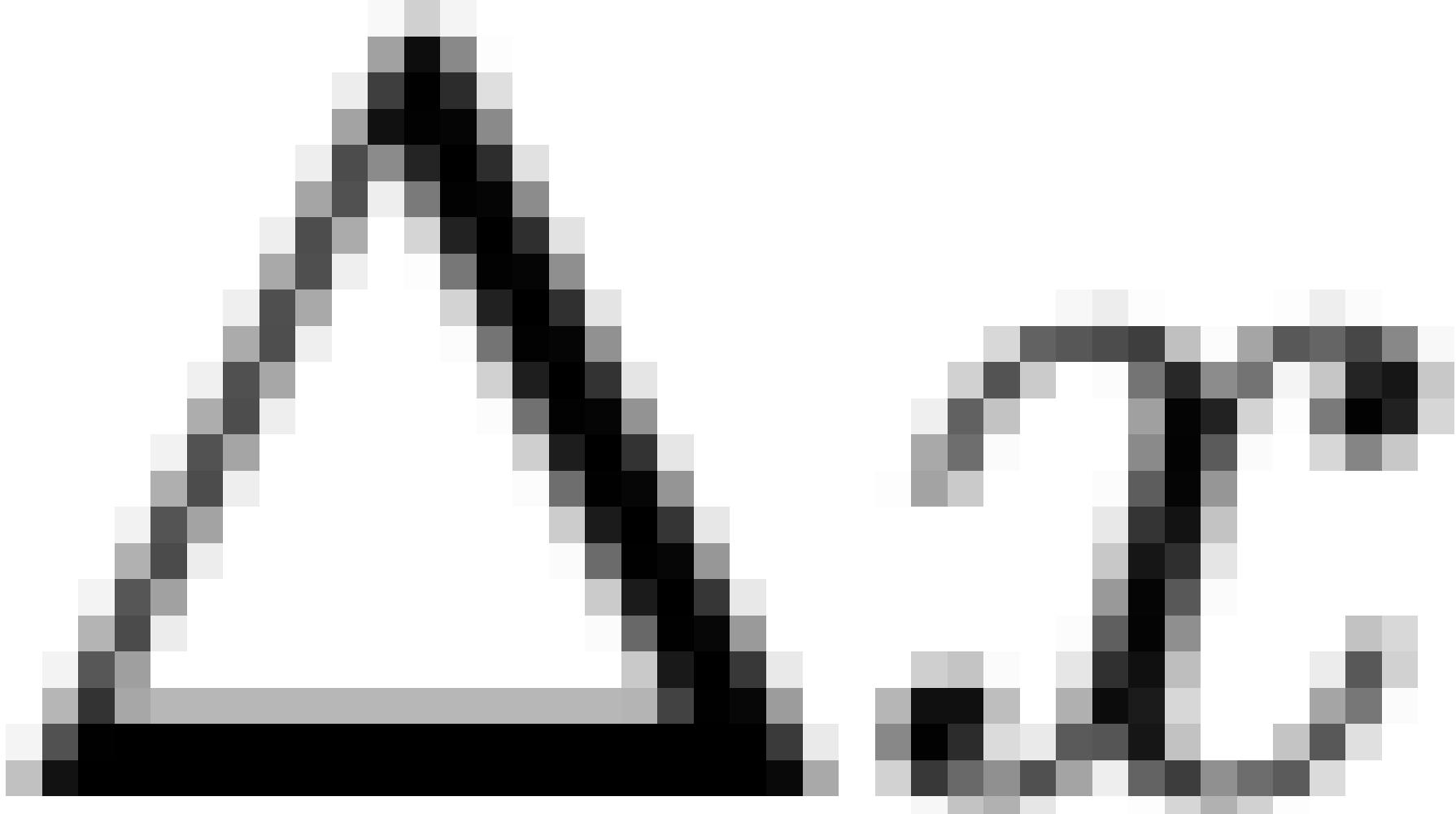
Soft  
elastic  
solid

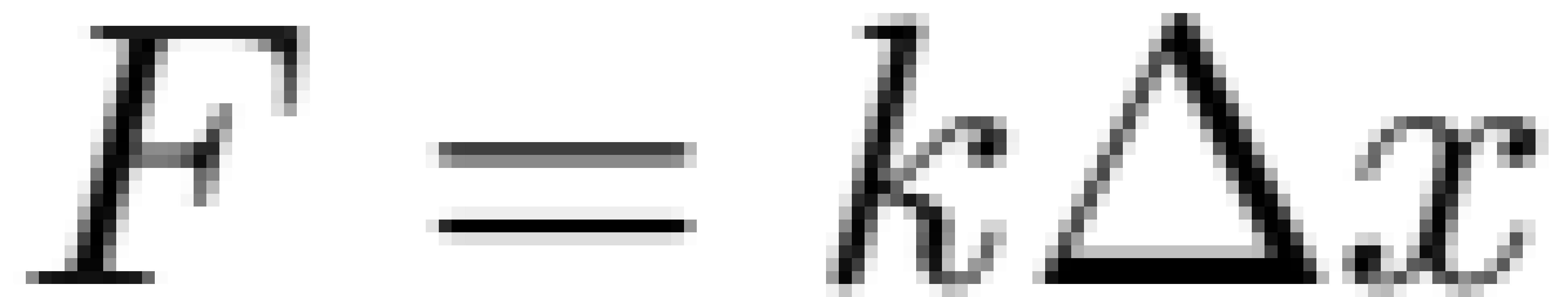


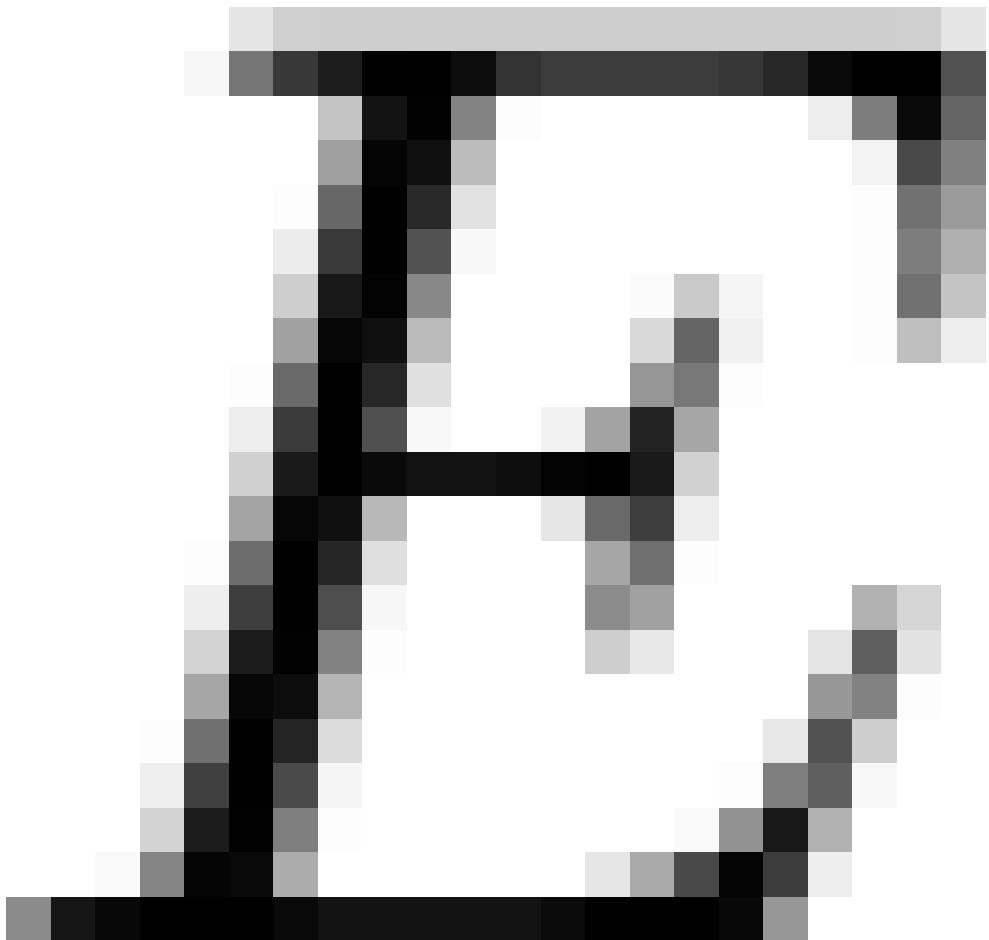
Soft  
Visco-  
plastic  
solid











A

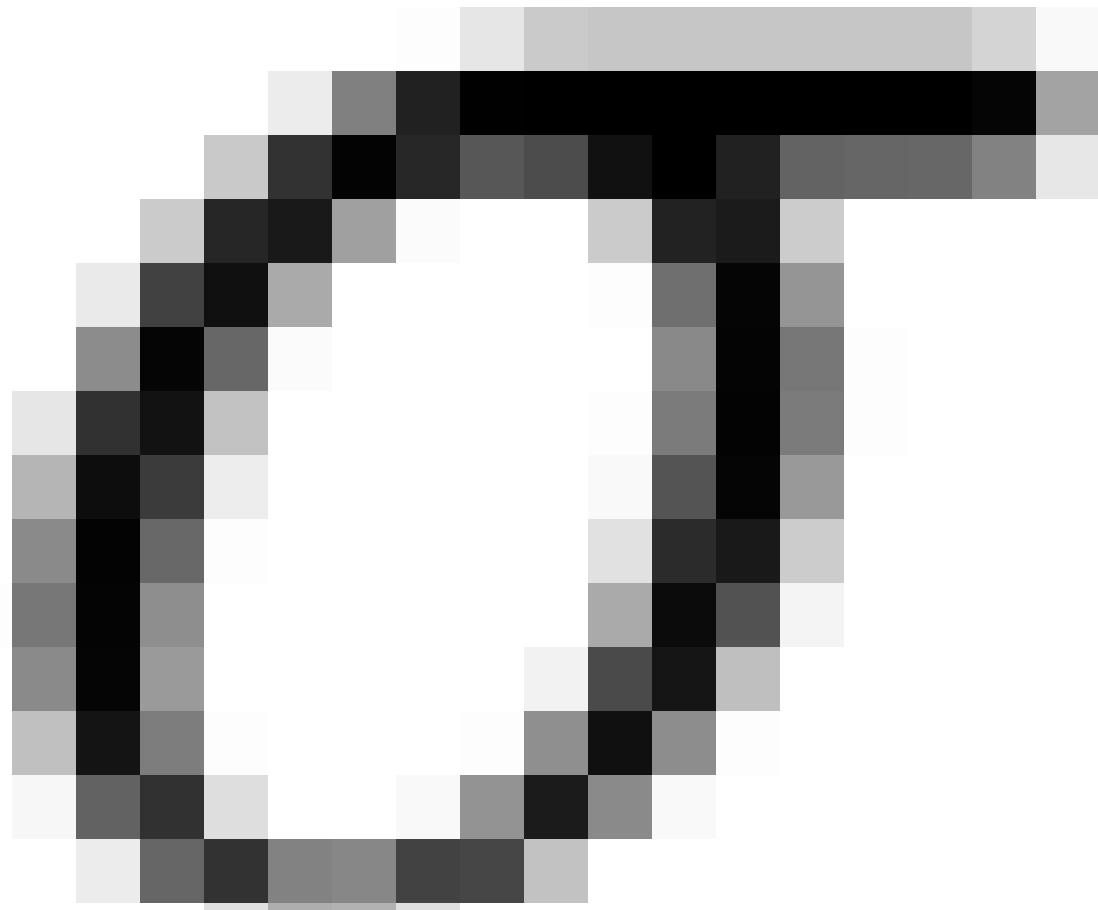
A

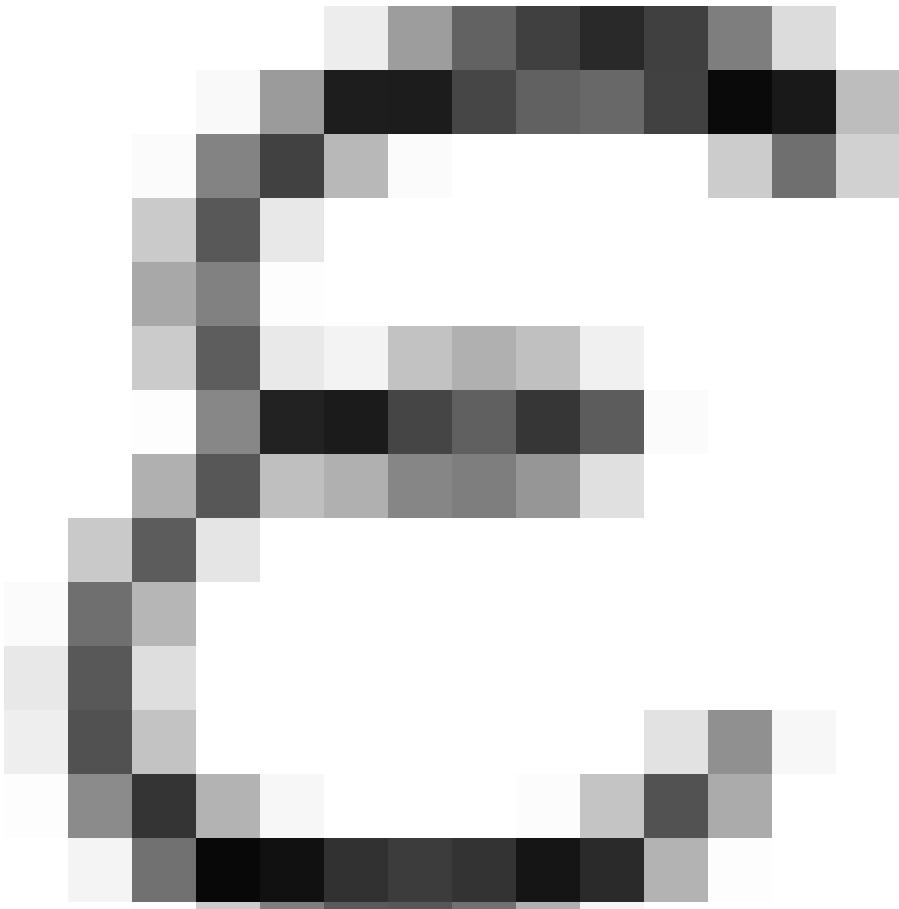
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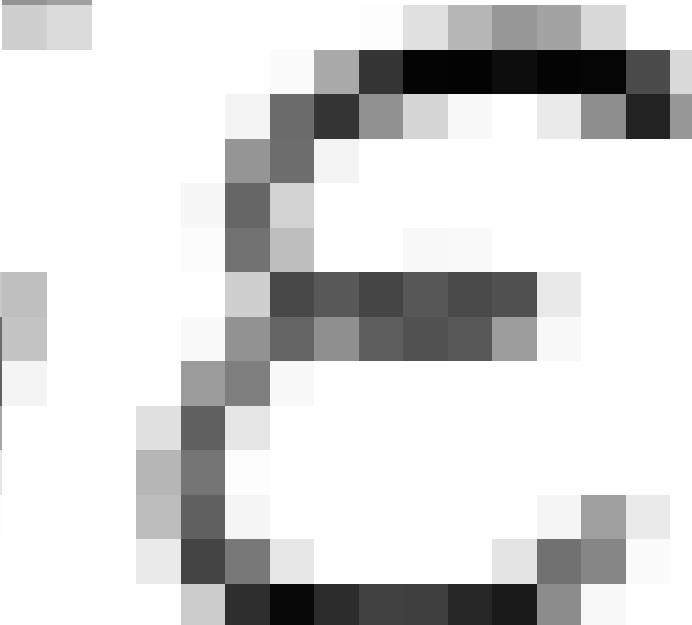
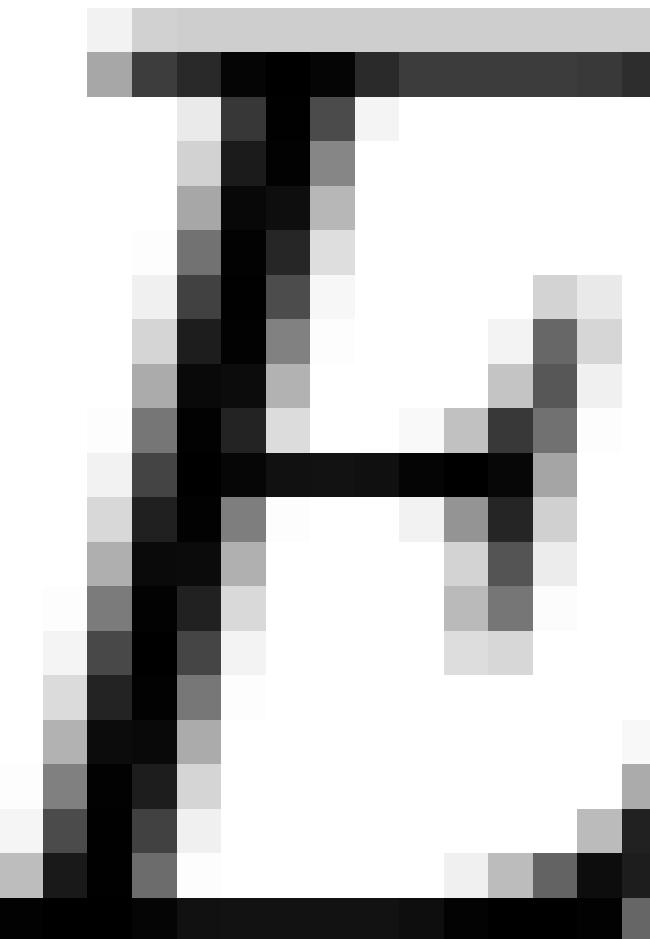
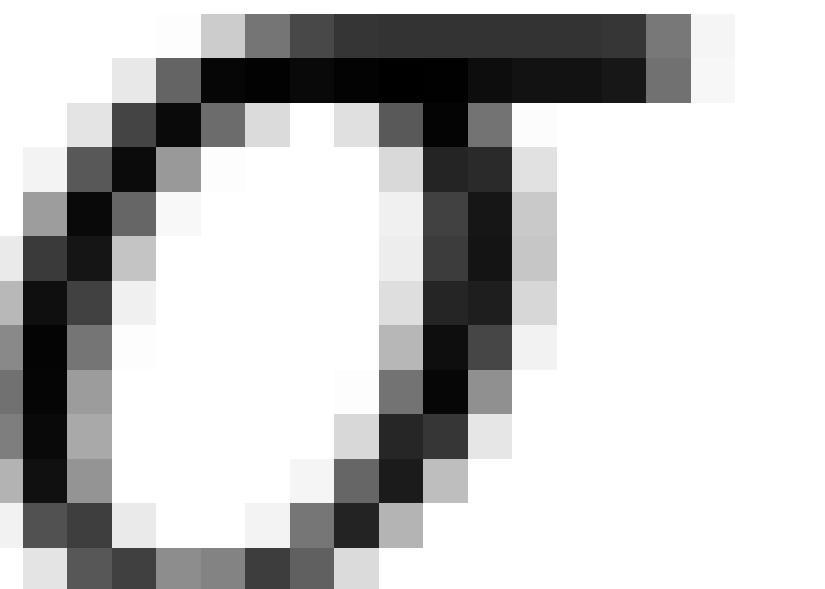
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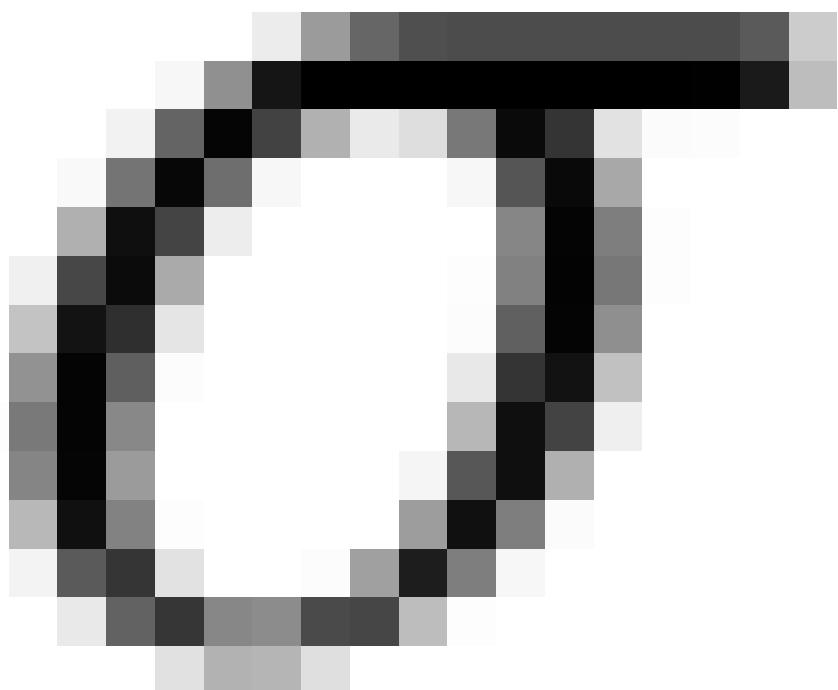
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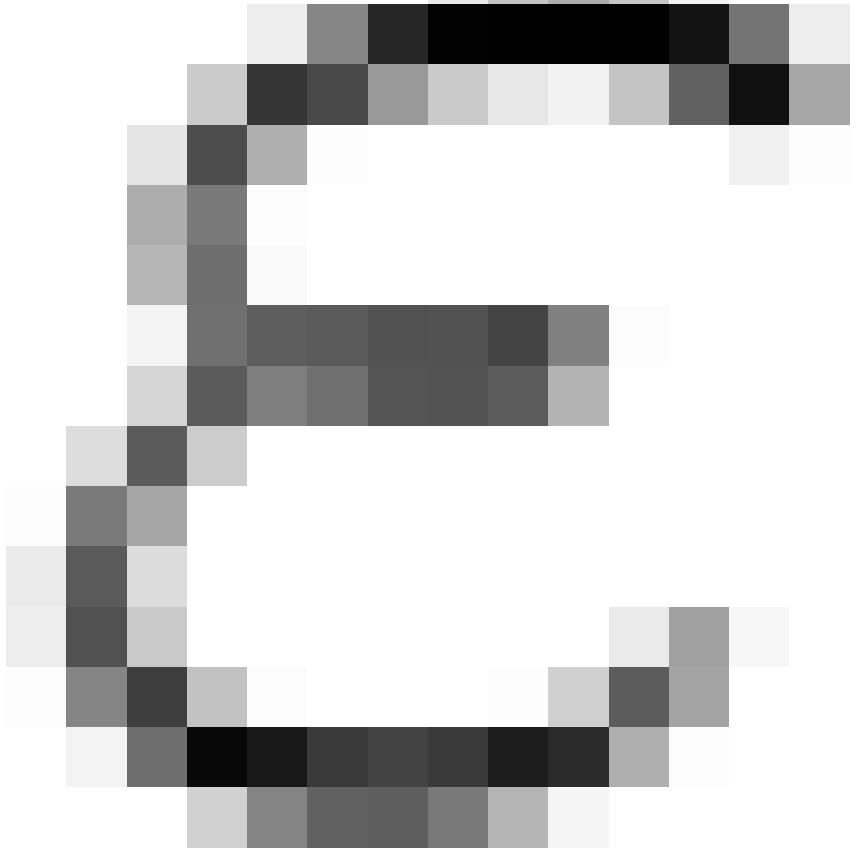
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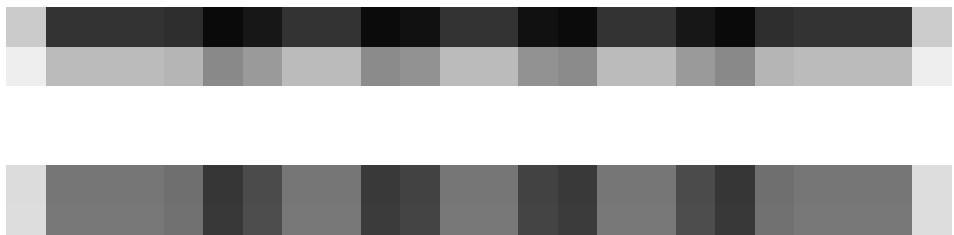
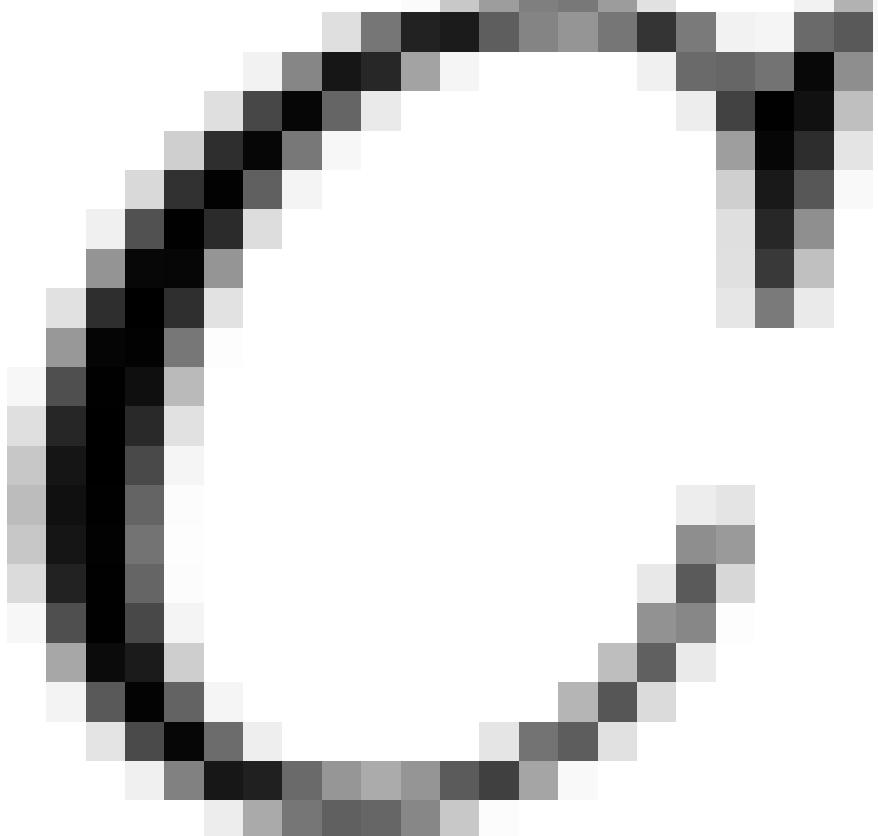


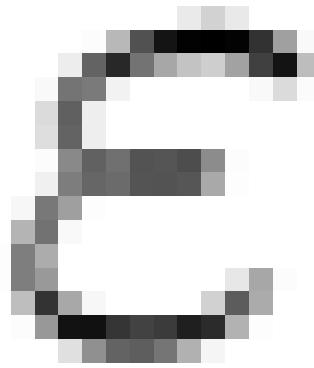
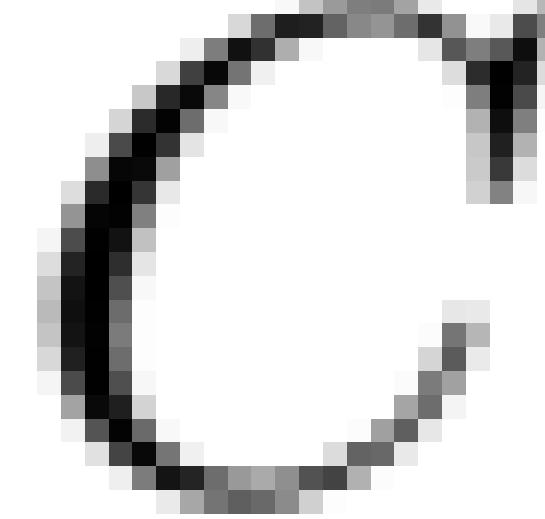
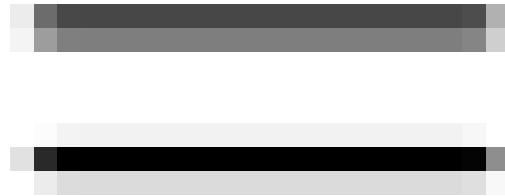
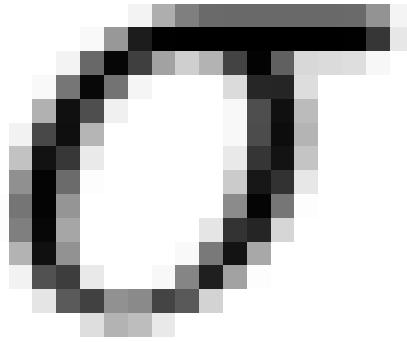






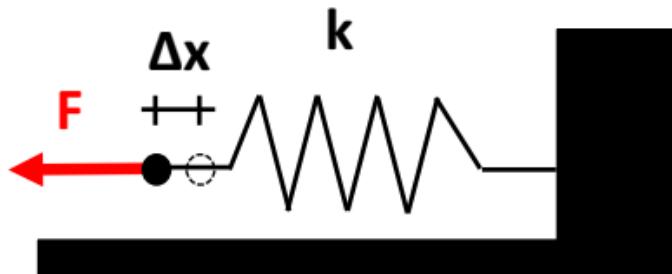






Hooke's law

$$F = k\Delta x$$

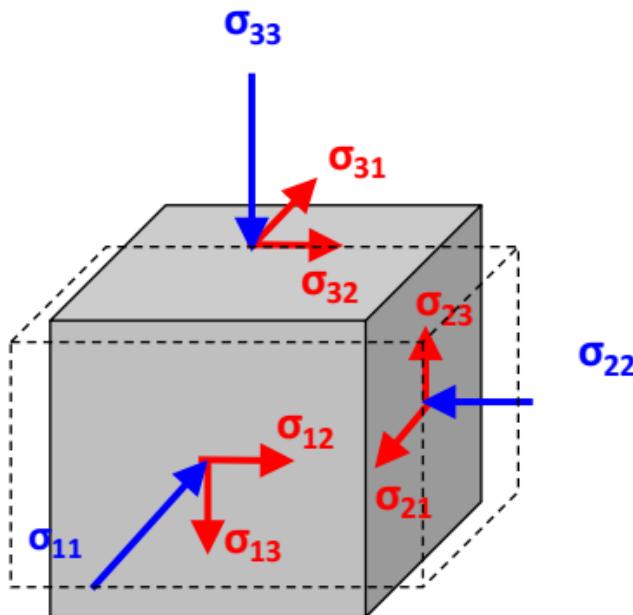


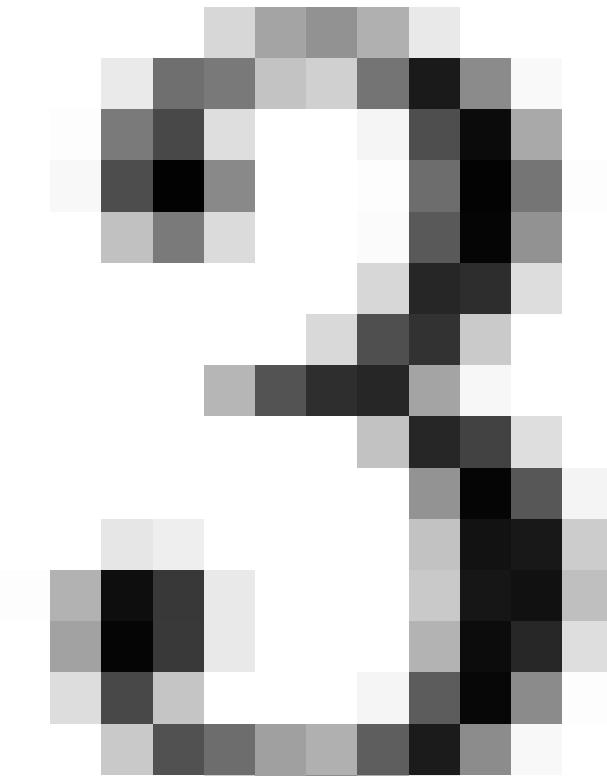
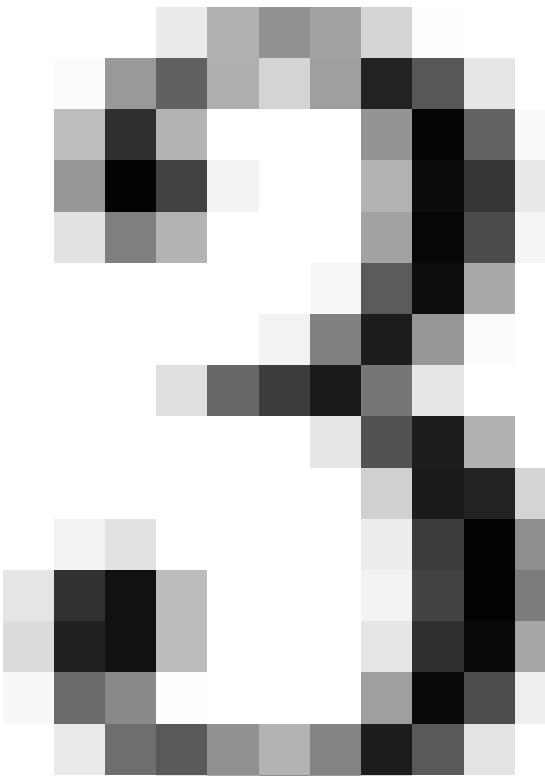
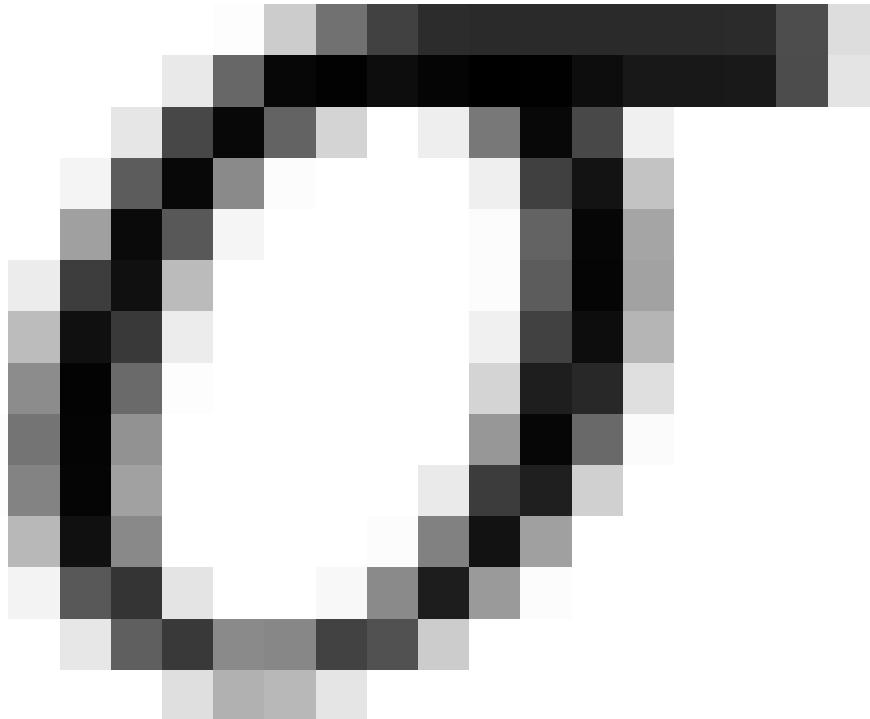
1-D (stress-strain) Hooke's law

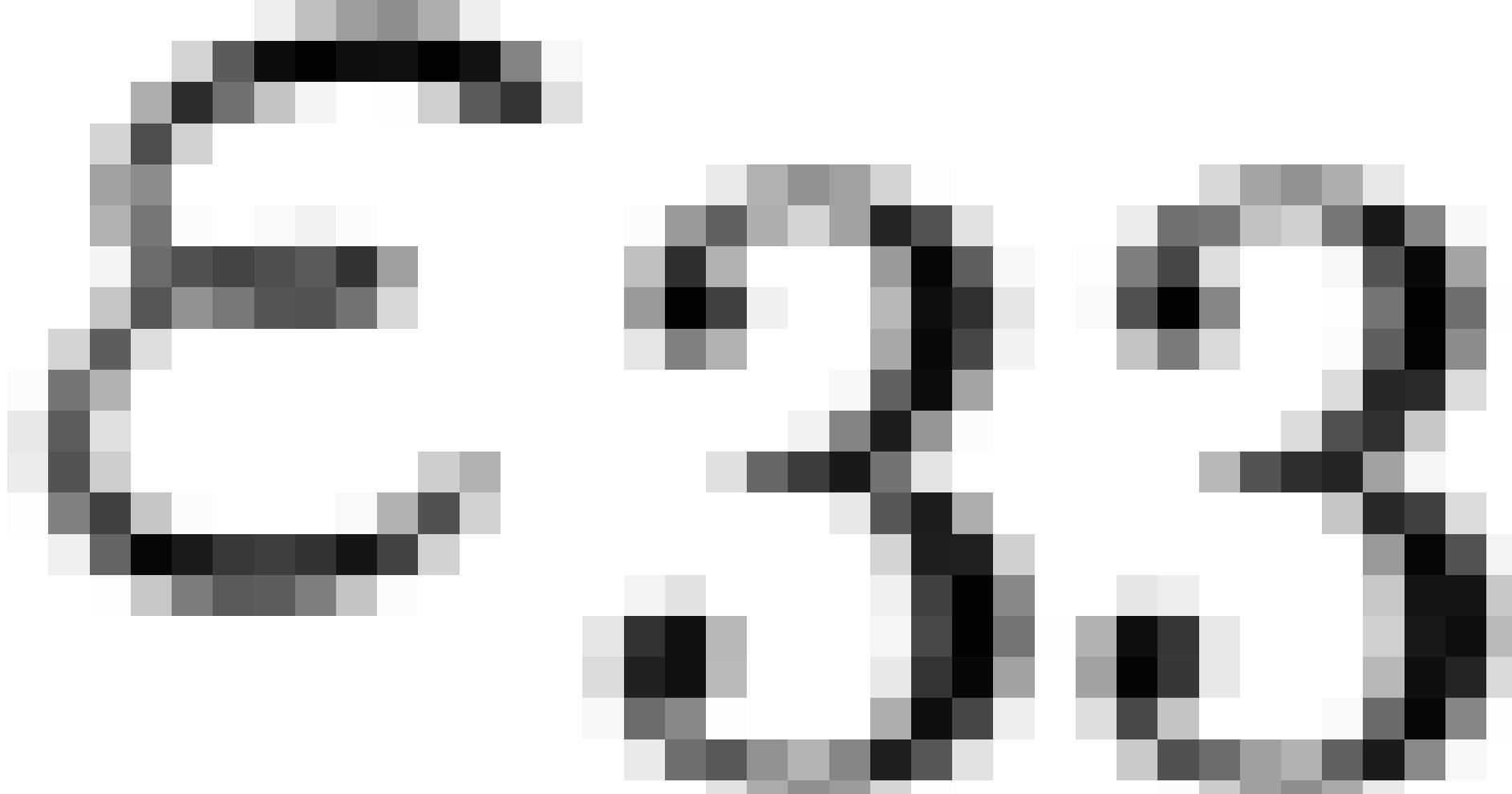
$$\sigma = E\varepsilon$$

Generalized Hooke's law

$$\underline{\sigma} = \underline{C}\underline{\varepsilon}$$







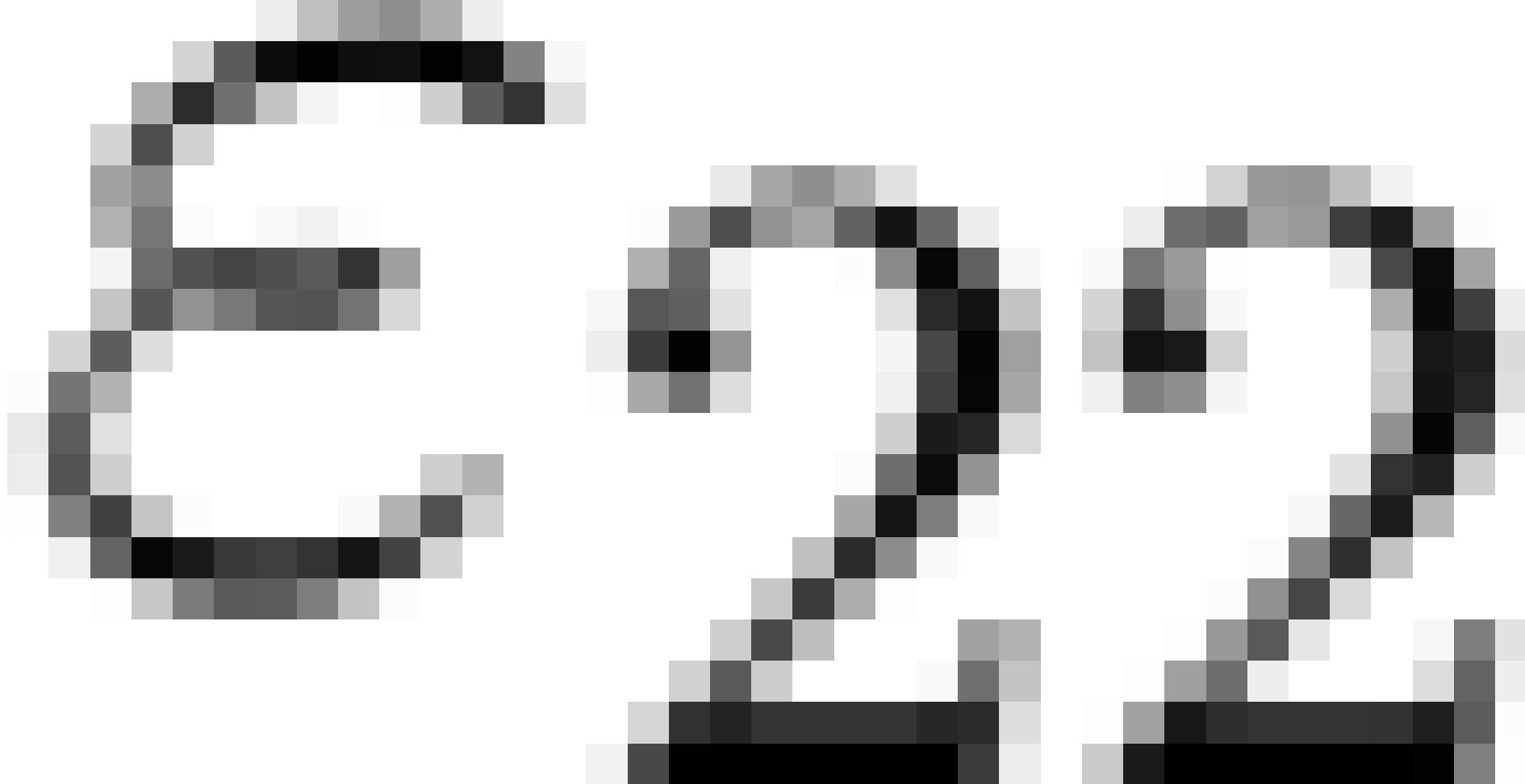
E

—  
—

σ33

c33





611

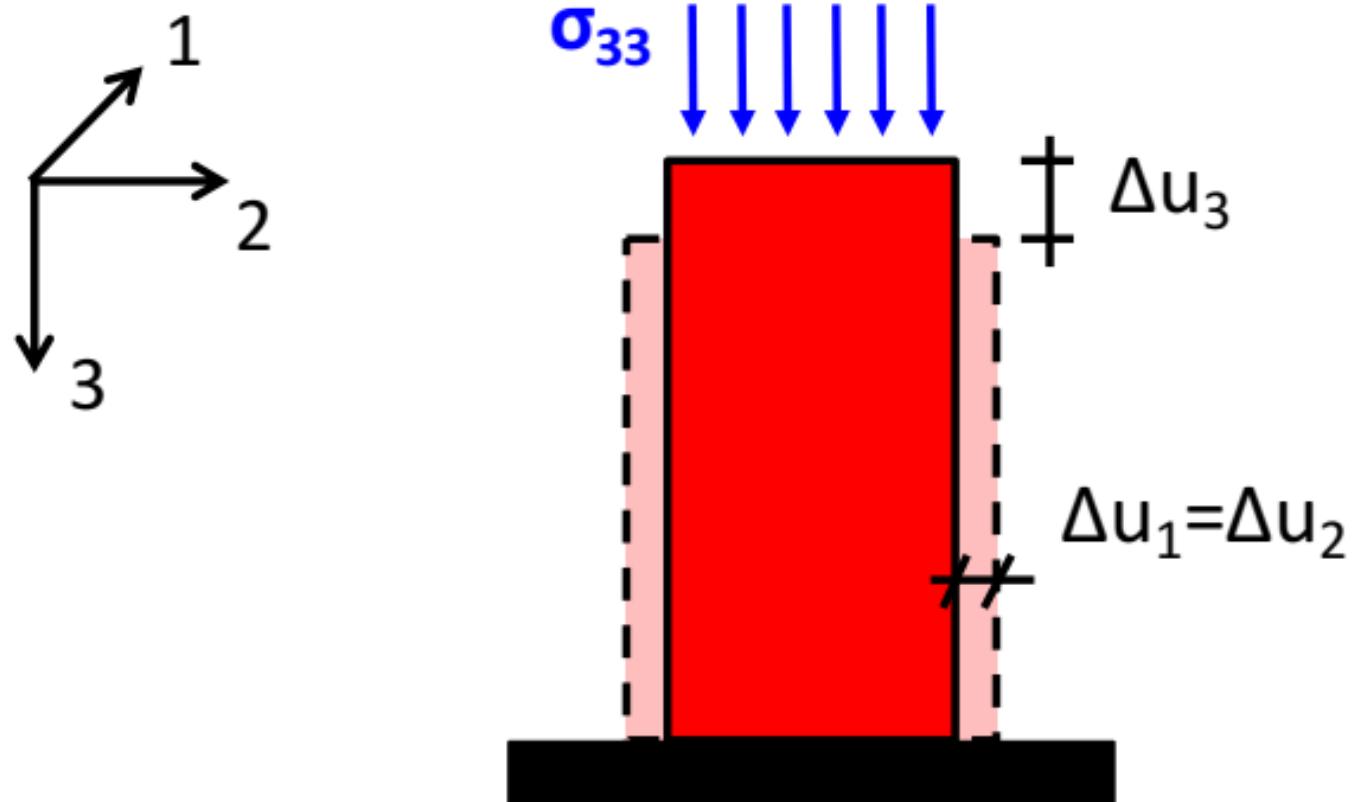
W

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633

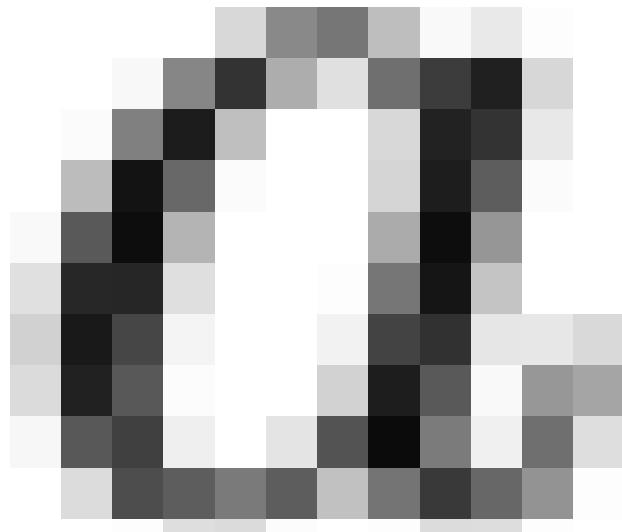
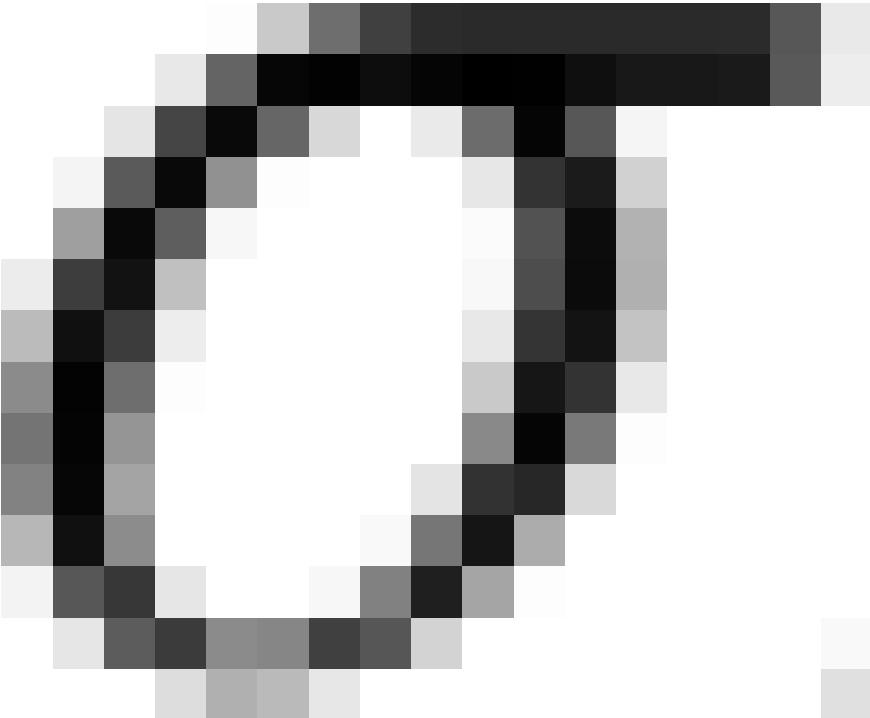


$$E = \frac{\sigma_{33}}{\epsilon_{33}}$$

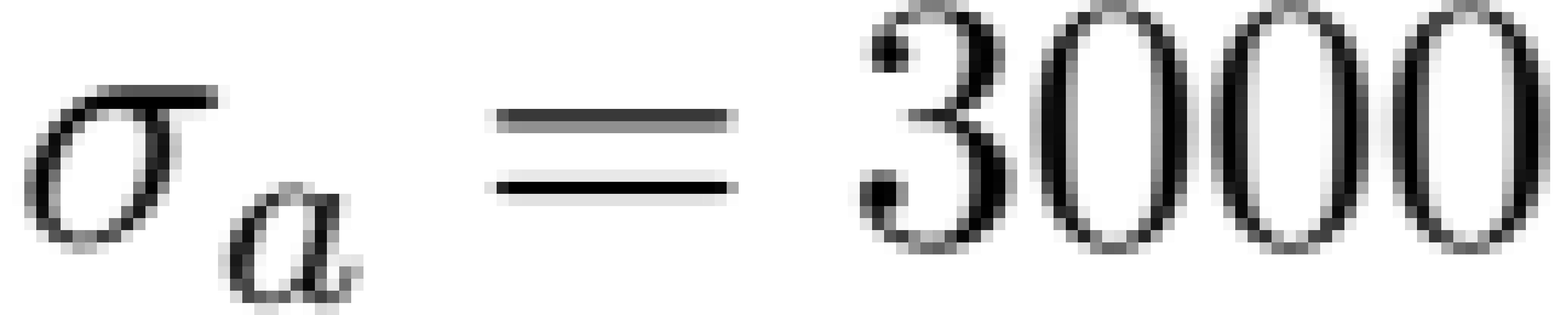
Young's Modulus

$$\nu = -\frac{\epsilon_{11}}{\epsilon_{33}}$$

Poisson's ratio (nu)







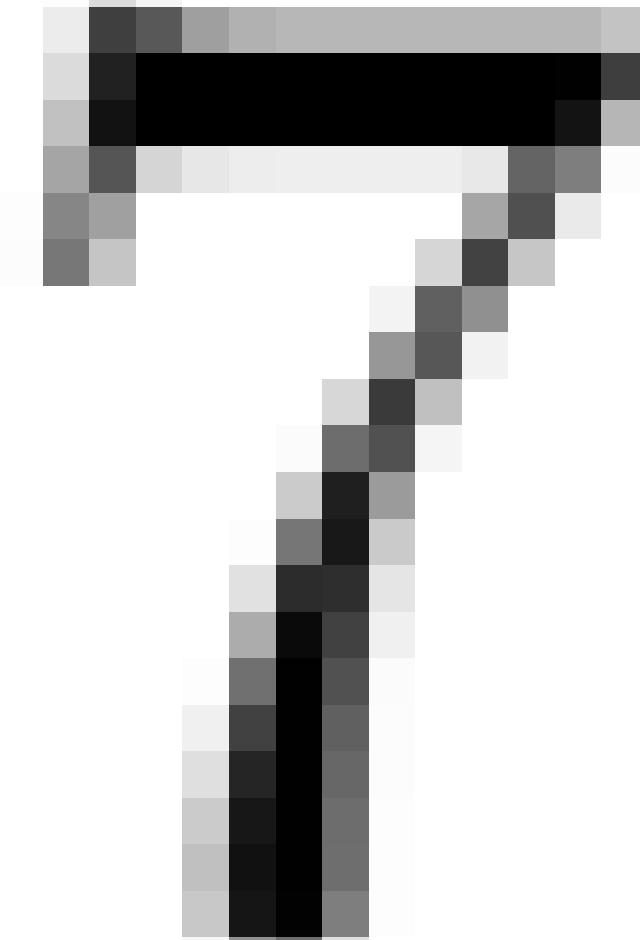
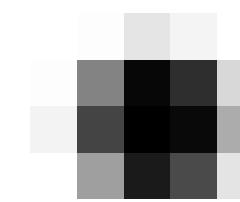
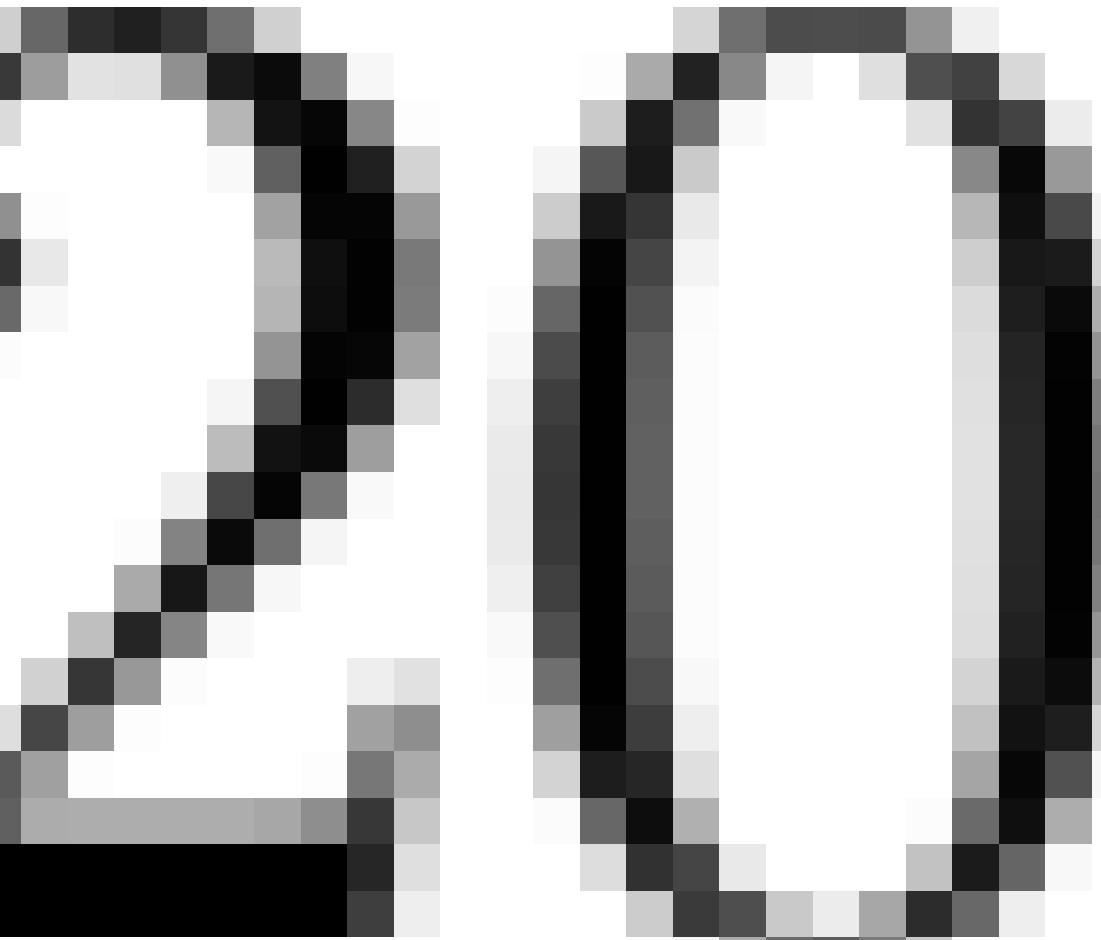
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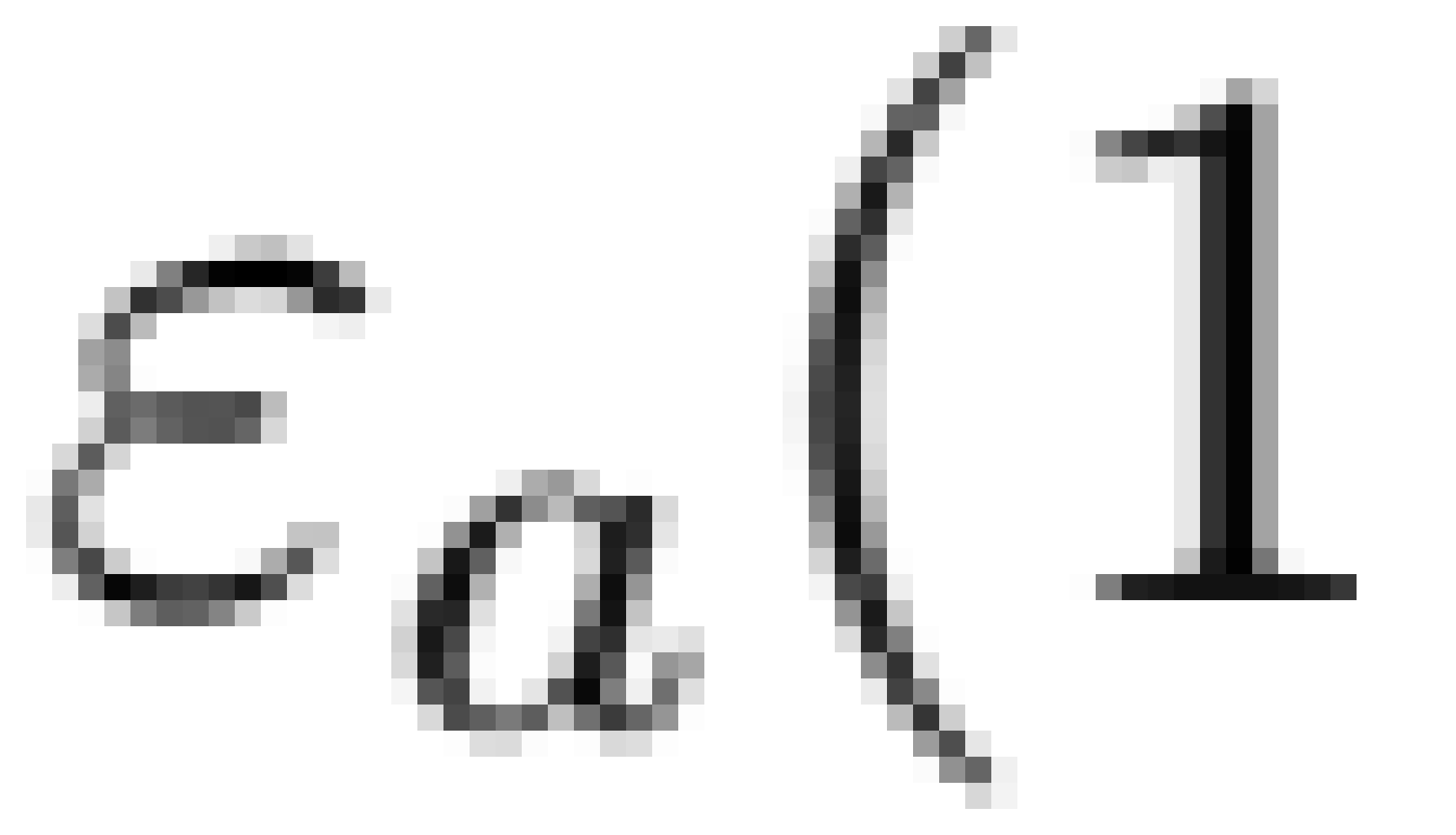
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145

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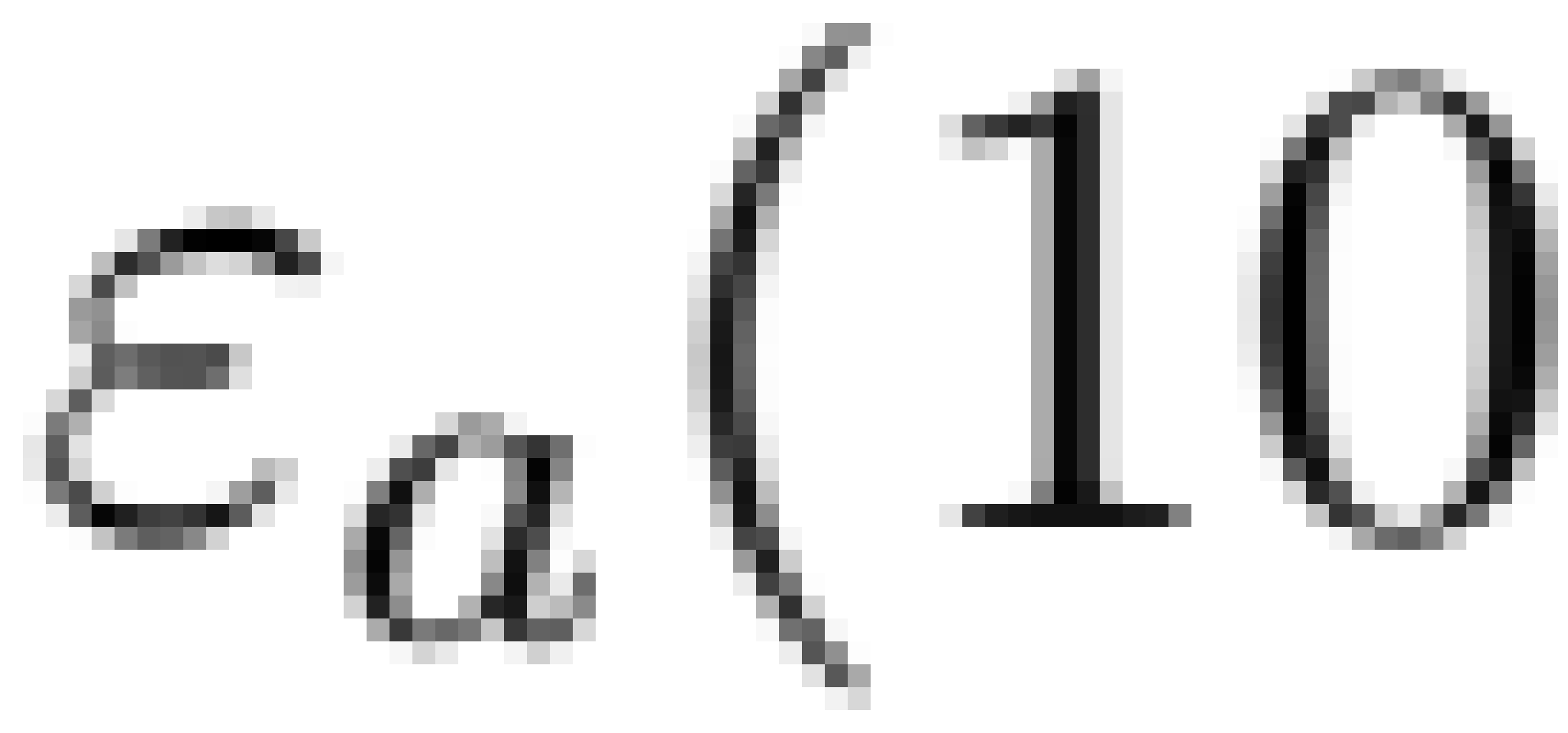
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$$\frac{\sigma_a}{E} = \frac{20.7}{100} = 0.207$$

$$0.207 \times 100 \text{ MPa} = 20.7 \text{ MPa}$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{100 \times 10^9 \text{ MPa}} = 0.207 \times 10^{-7}$$

$$= 0.00207 \times 10^{-7} = 0.00207 \%$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{50000 \text{ MPa}} = 0.00041$$

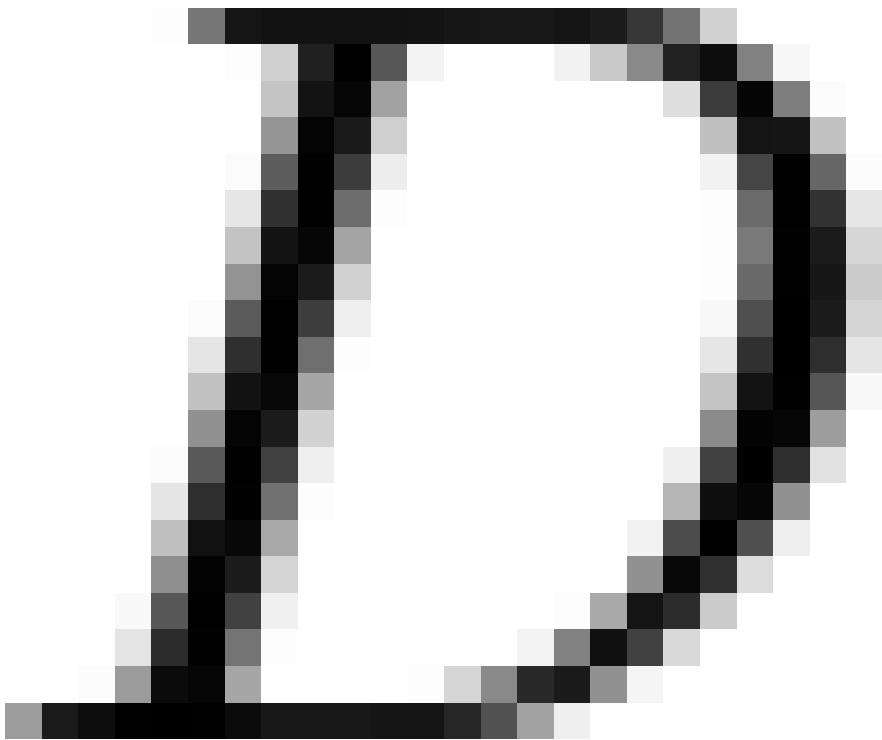
$$= 0.041\%$$

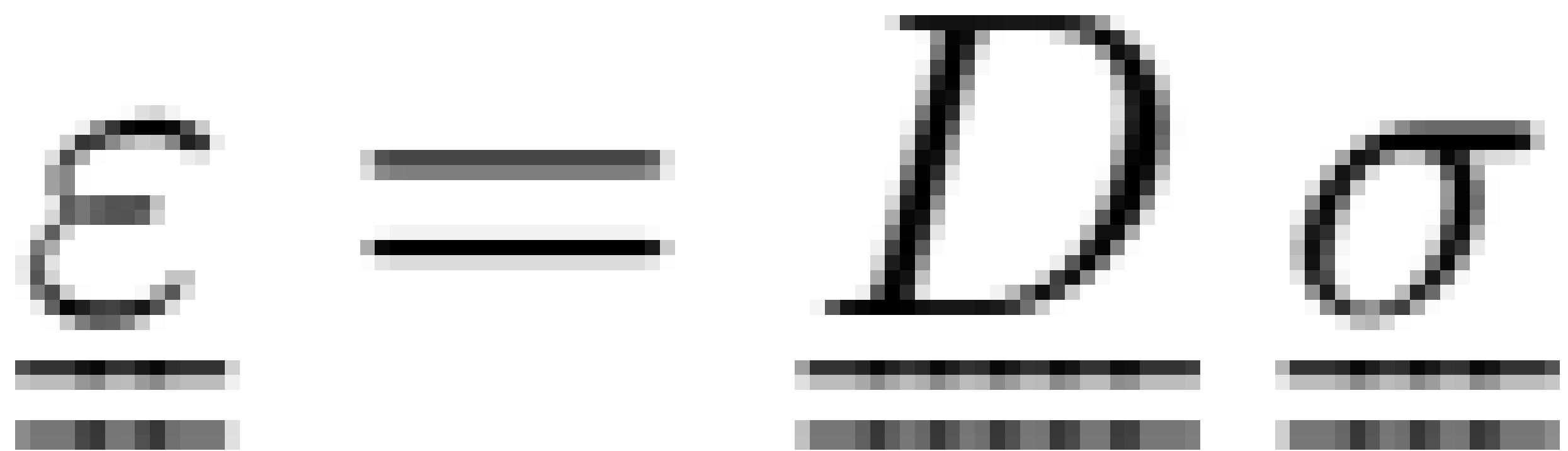
$$\left\{ \begin{array}{lcl} \epsilon_{11} & = & +\frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ & & \nu \quad \quad \quad 1 \quad \quad \quad \nu \\ \epsilon_{22} & = & -\frac{1}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{1}{E}\sigma_{33} \\ & & \nu \quad \quad \quad \nu \quad \quad \quad 1 \\ \epsilon_{33} & = & -\frac{1}{E}\sigma_{11} - \frac{1}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{array} \right.$$

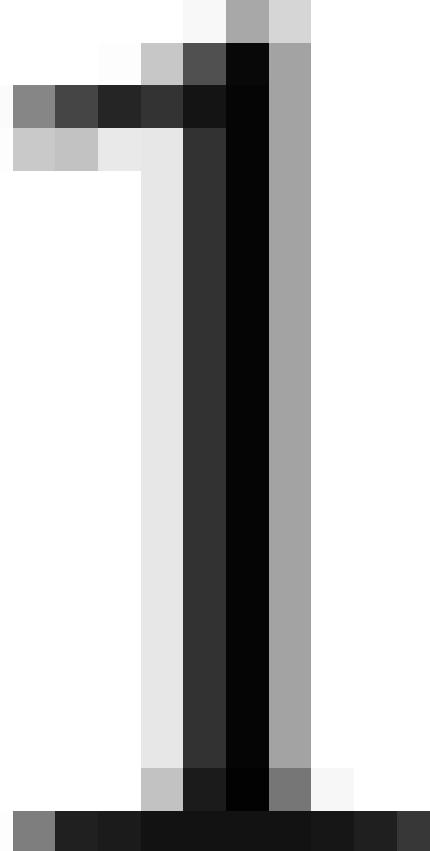
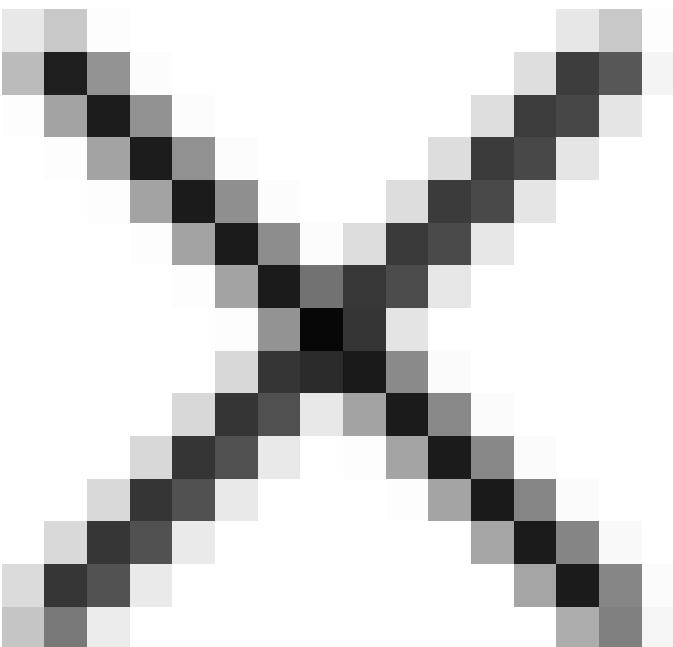
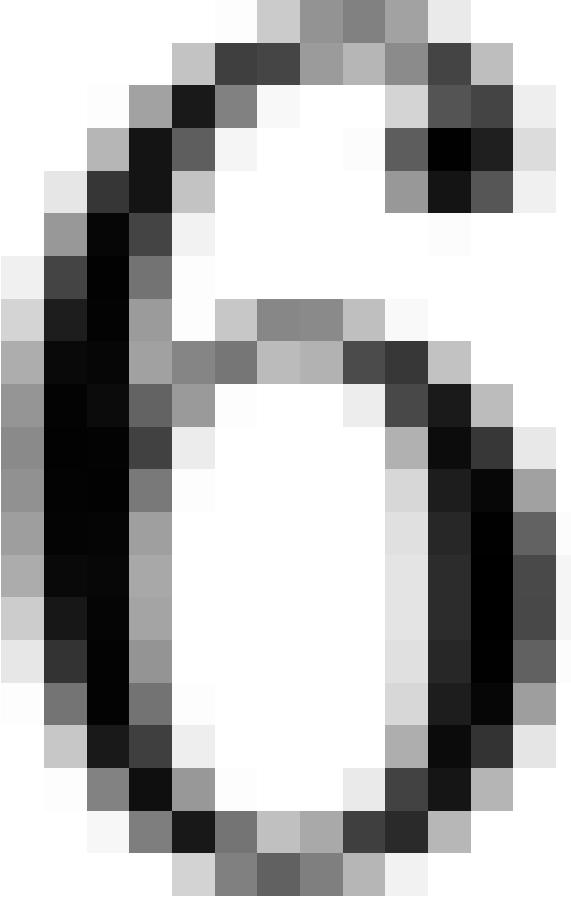


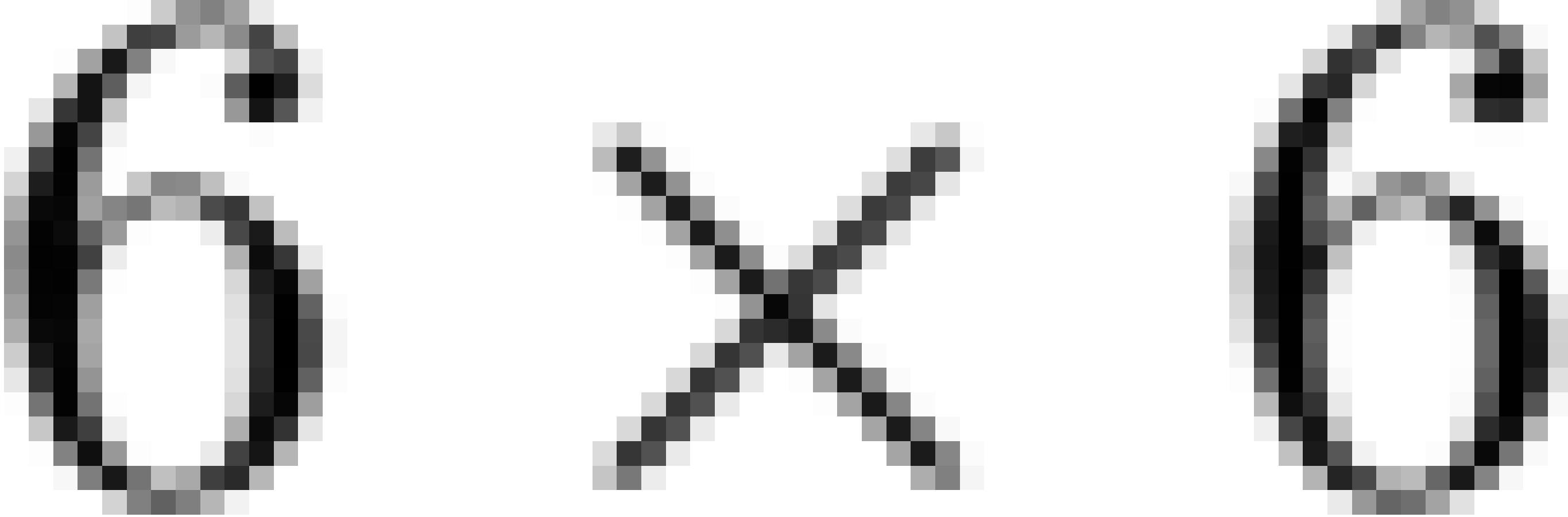


$$\left\{ \begin{array}{l} 2\varepsilon_{12} = \frac{1}{G} \sigma_{12} \\ \\ 2\varepsilon_{13} = \frac{1}{G} \sigma_{13} \\ \\ 2\varepsilon_{23} = \frac{1}{G} \sigma_{23} \end{array} \right.$$



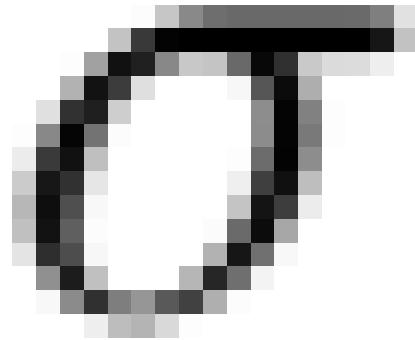
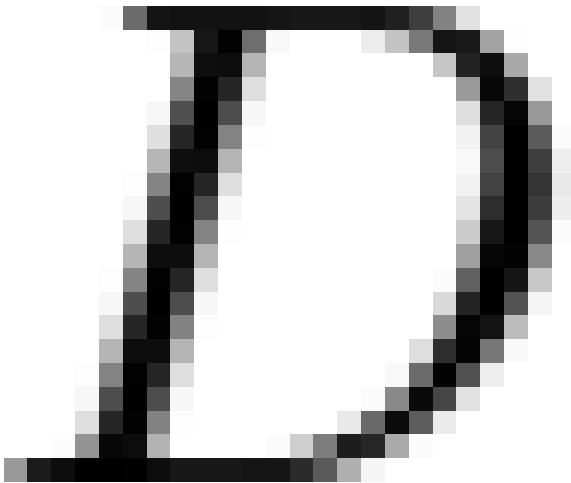




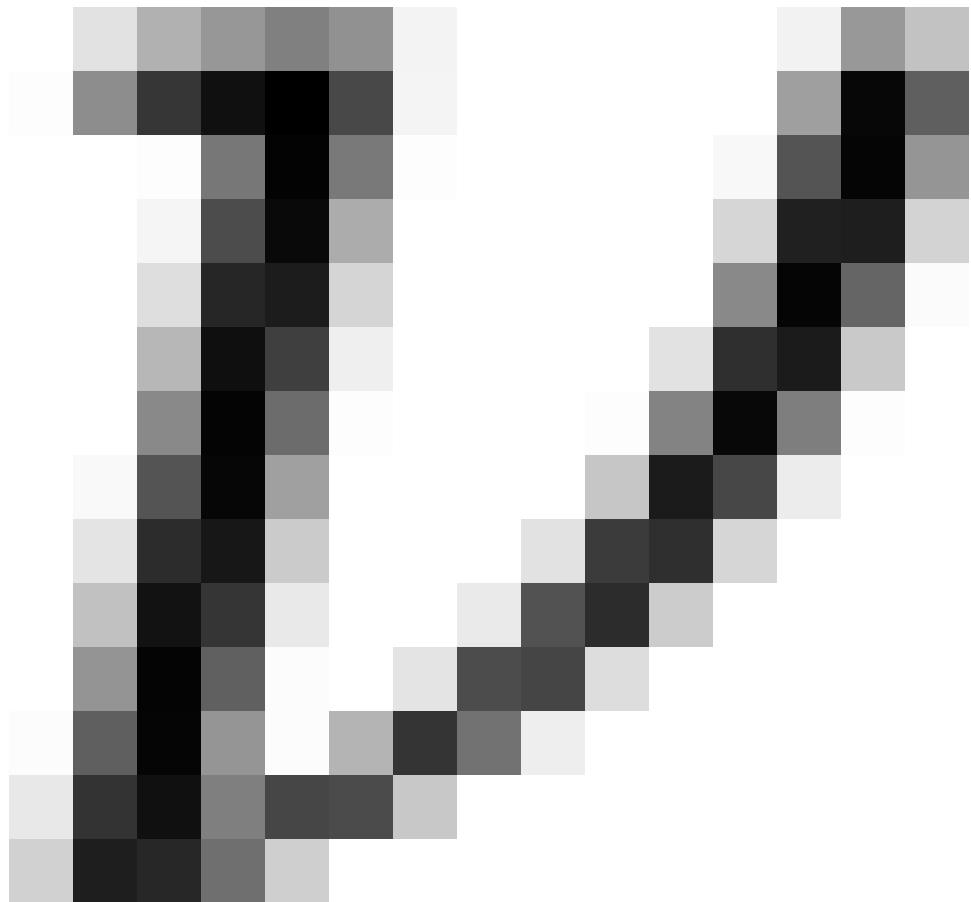


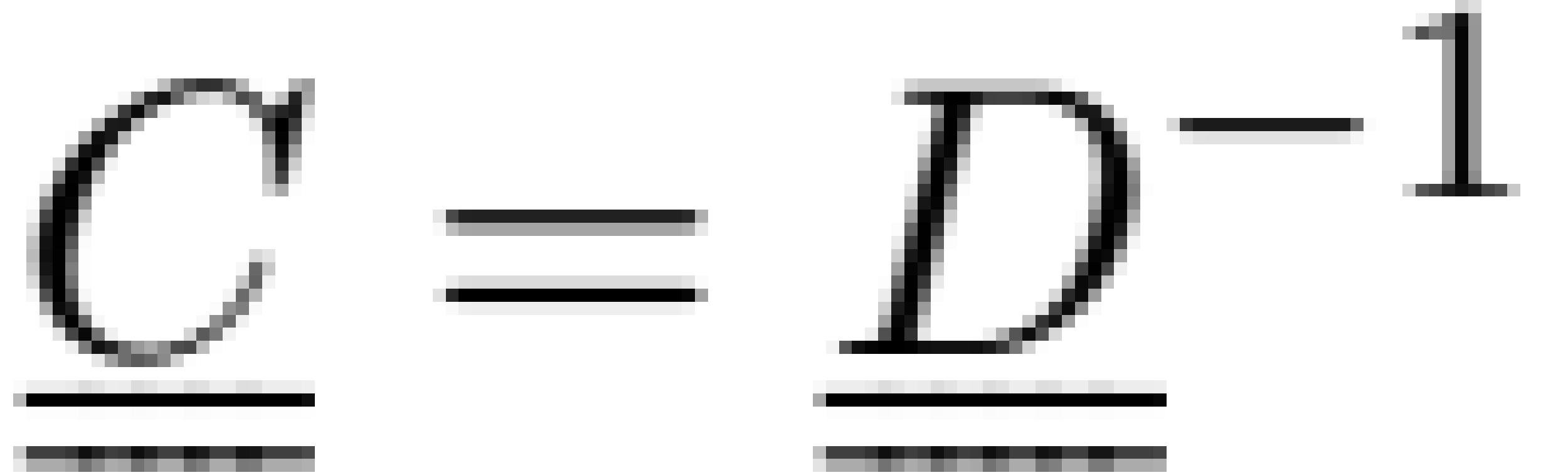
$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} = \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ +\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & +\frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & +\frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}$$



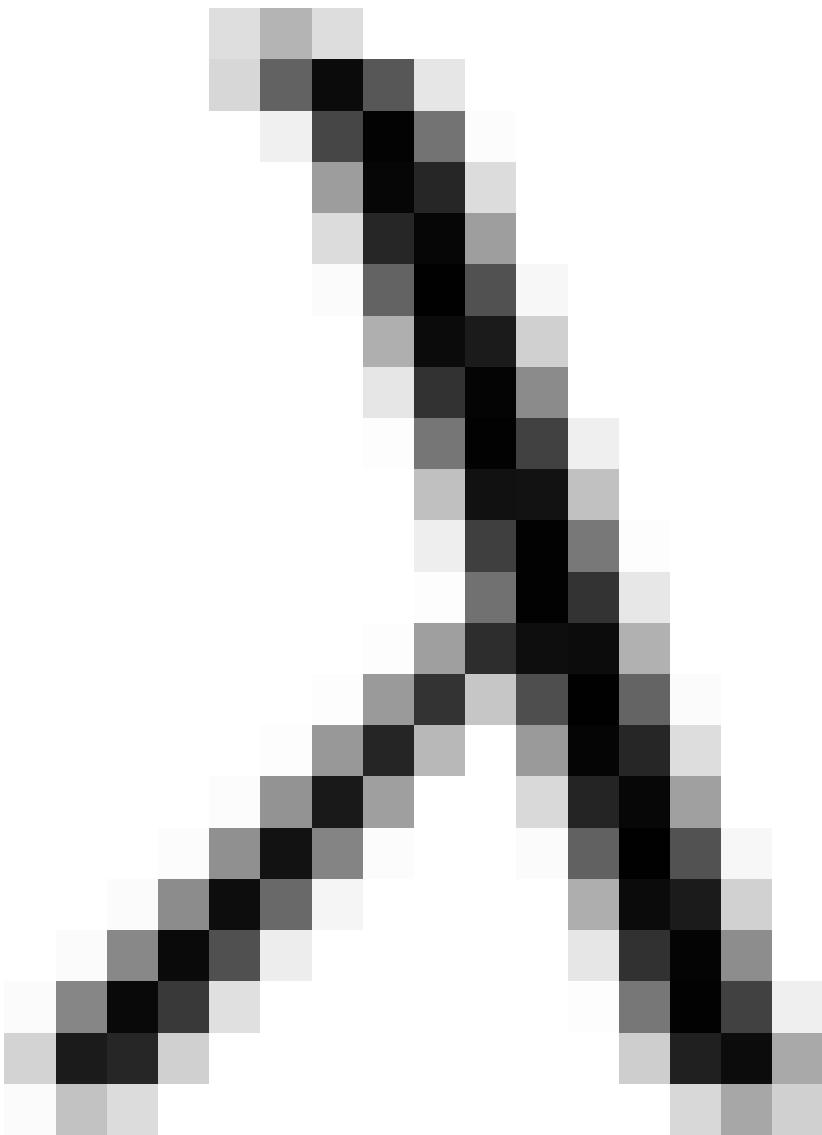


$$\underline{\underline{\epsilon}} = \begin{bmatrix} -\frac{v}{E}\sigma_{33}, -\frac{v}{E}\sigma_{33}, \frac{1}{E}\sigma_{33}, 0, 0, 0 \end{bmatrix}^T$$





$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$



$$\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)} [(1-\nu)e_{11} + \nu e_{22} + \nu e_{33}]$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + \frac{\nu E}{(1+\nu)(1-2\nu)} \left( \frac{1-\nu}{1+\nu} \epsilon_{11} - \frac{1+\nu}{1-\nu} \epsilon_{22} \right)$$

$\lambda$



$(1 + \nu)$

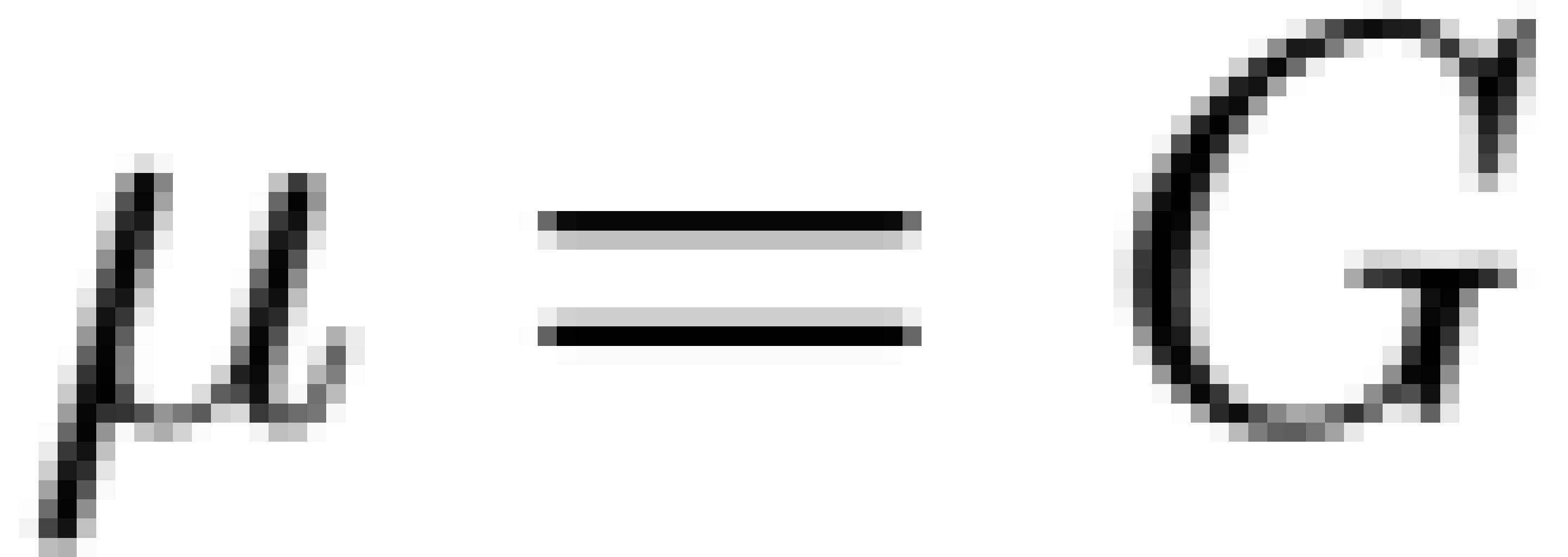
$(1 - \nu)$



$(2\nu)$

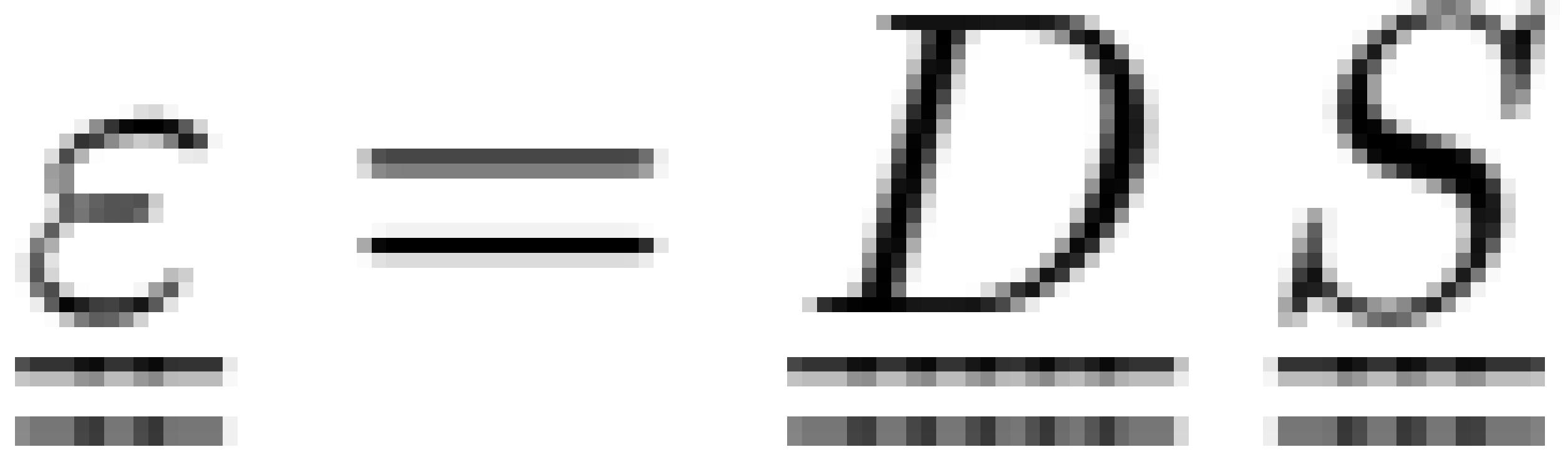
$\nu E$

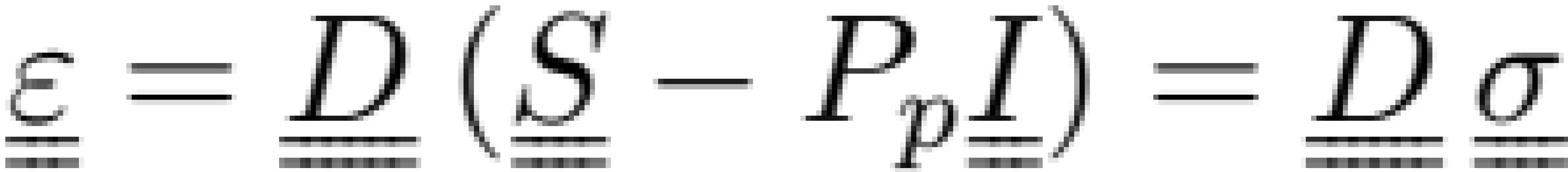
$$2\mu = \frac{\nu E}{(1+\nu)(1-2\nu)} = \frac{-1}{(1+\nu)} + \frac{1-\nu}{(1+\nu)}$$

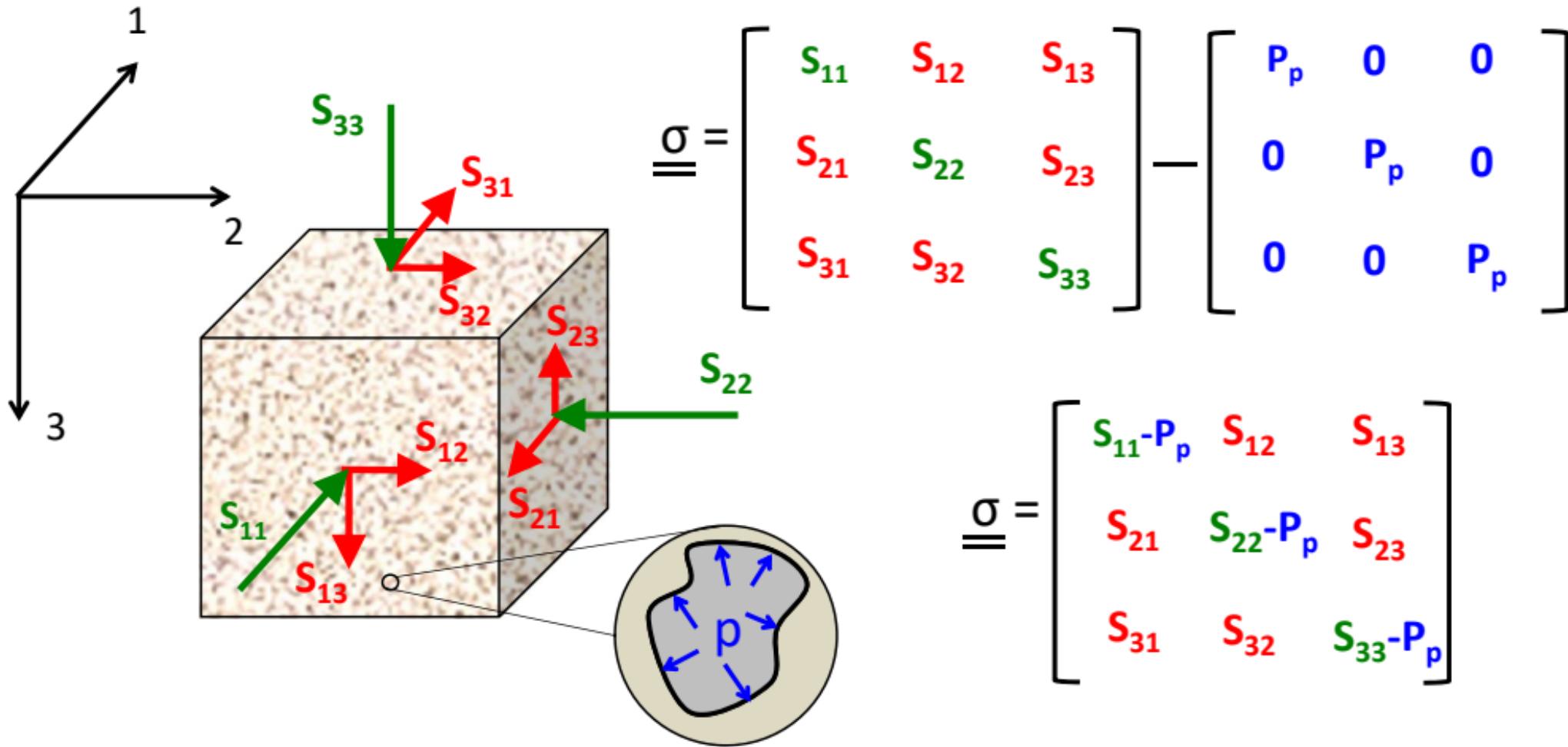


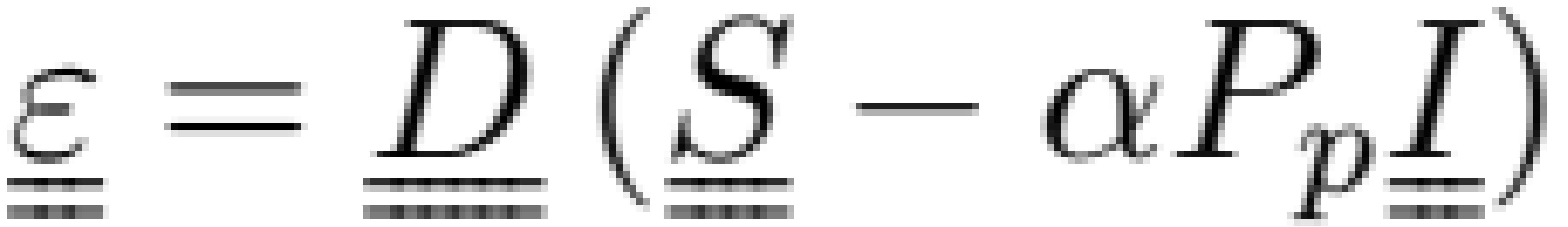
$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right. .$$

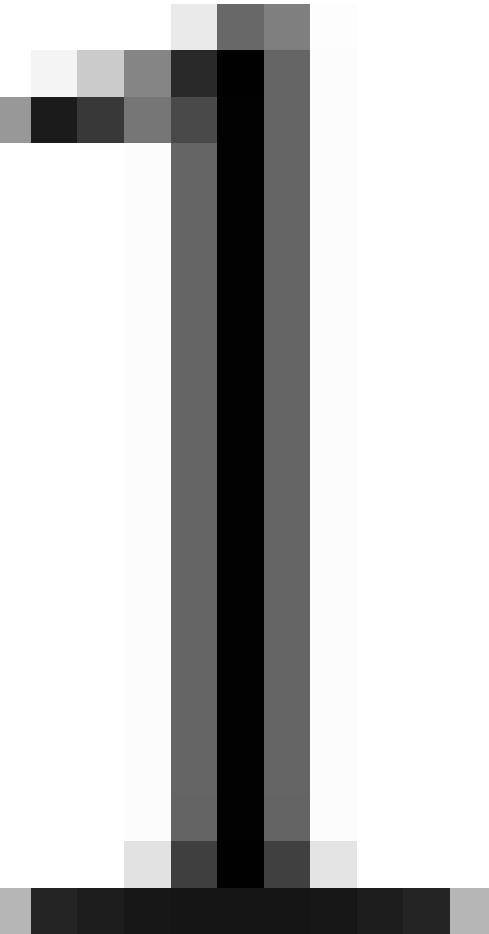
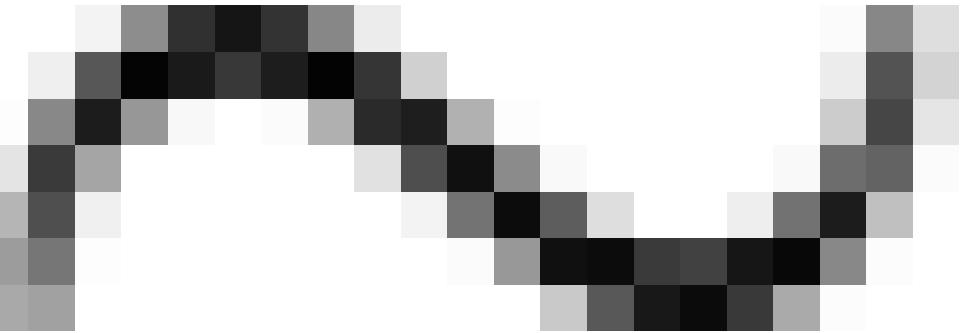
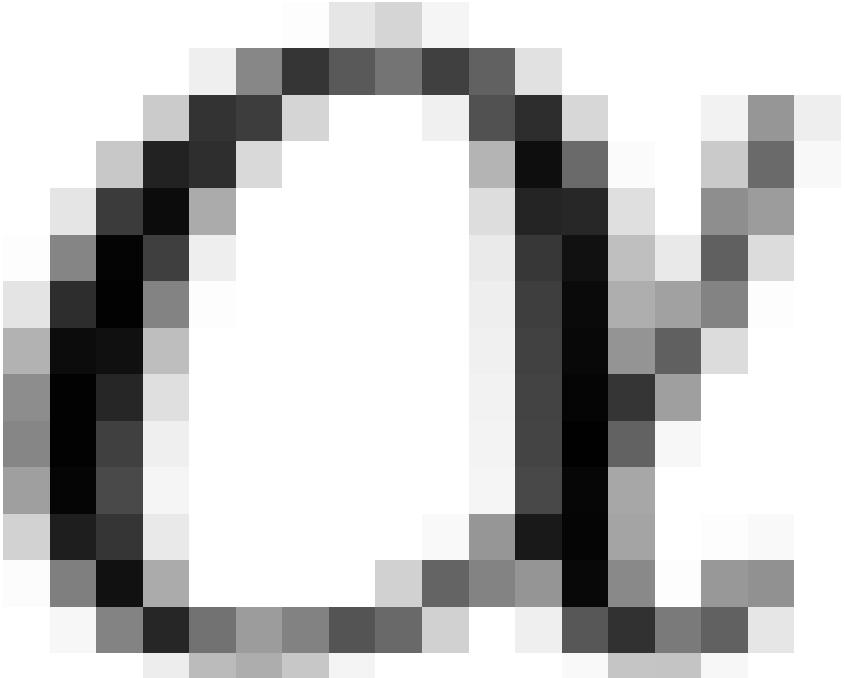
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$



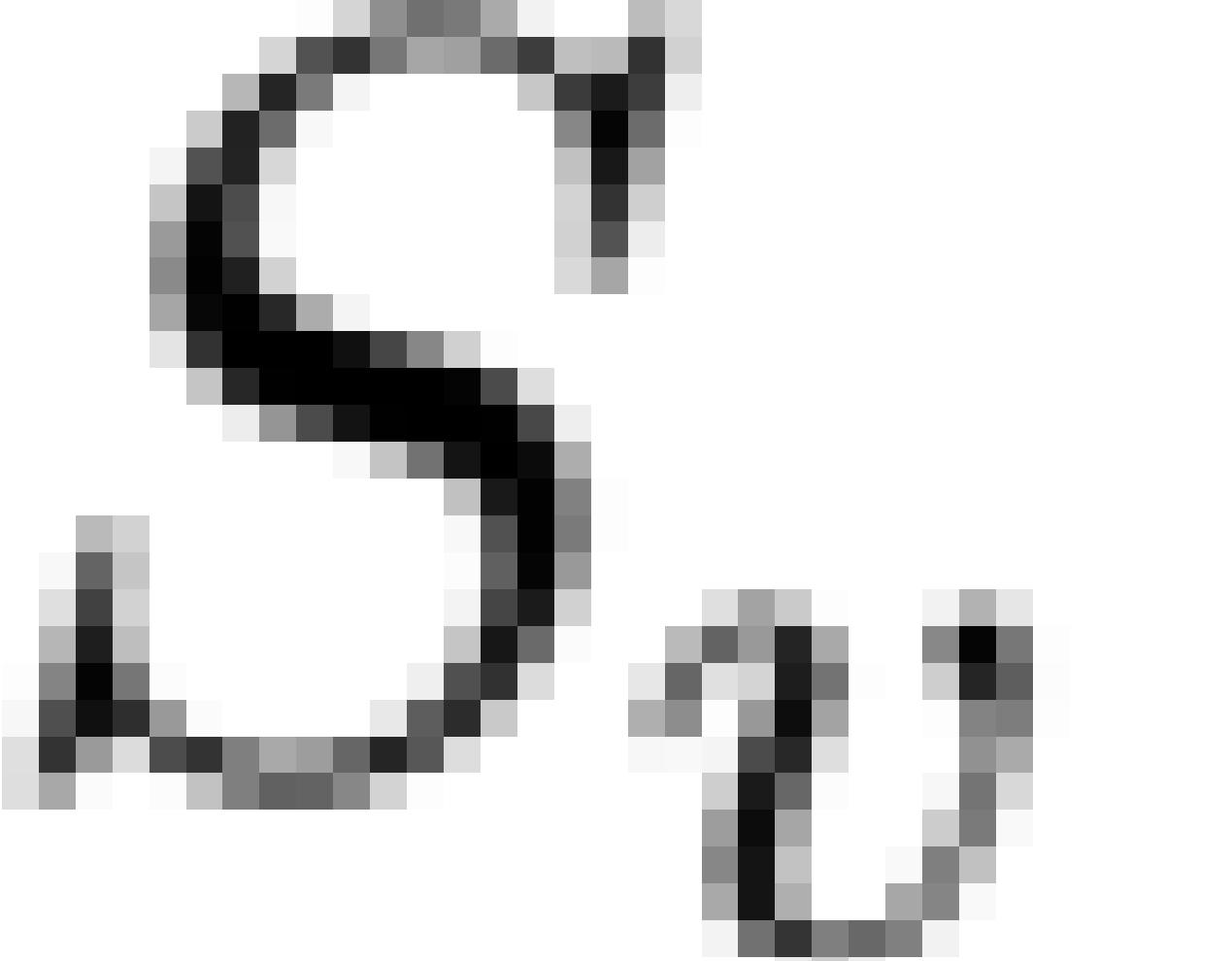








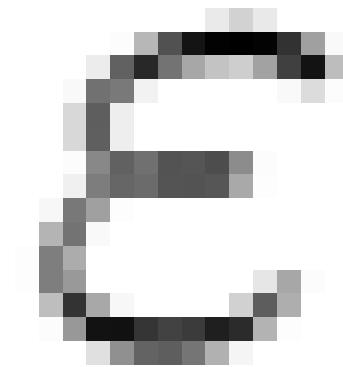
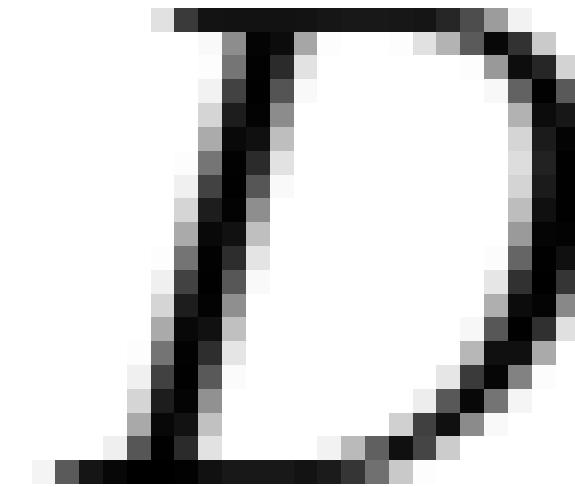
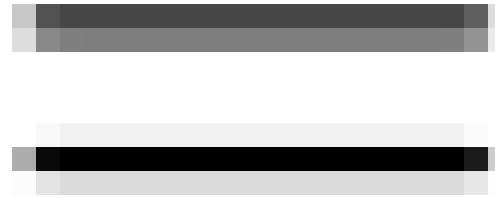
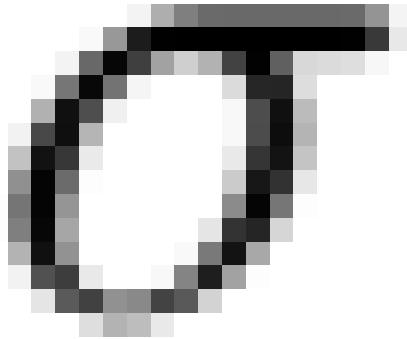




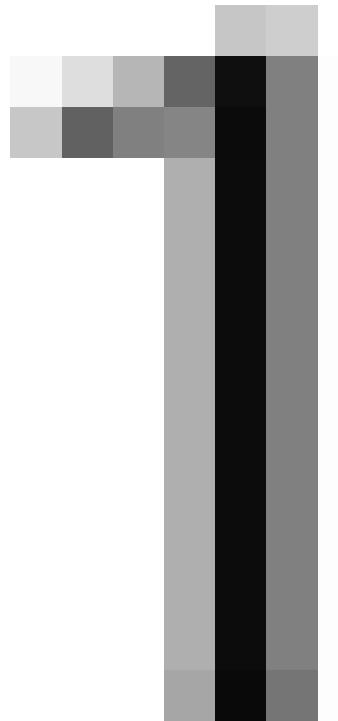
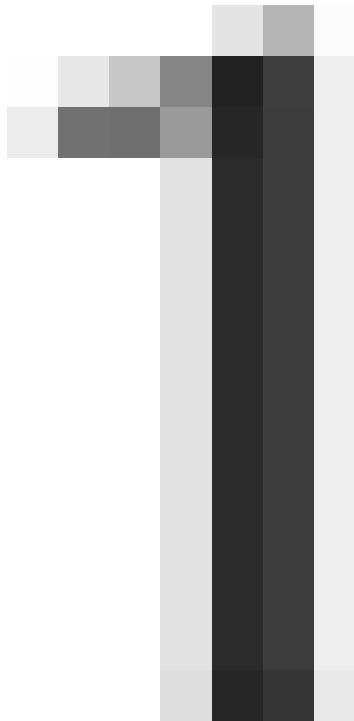
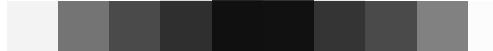
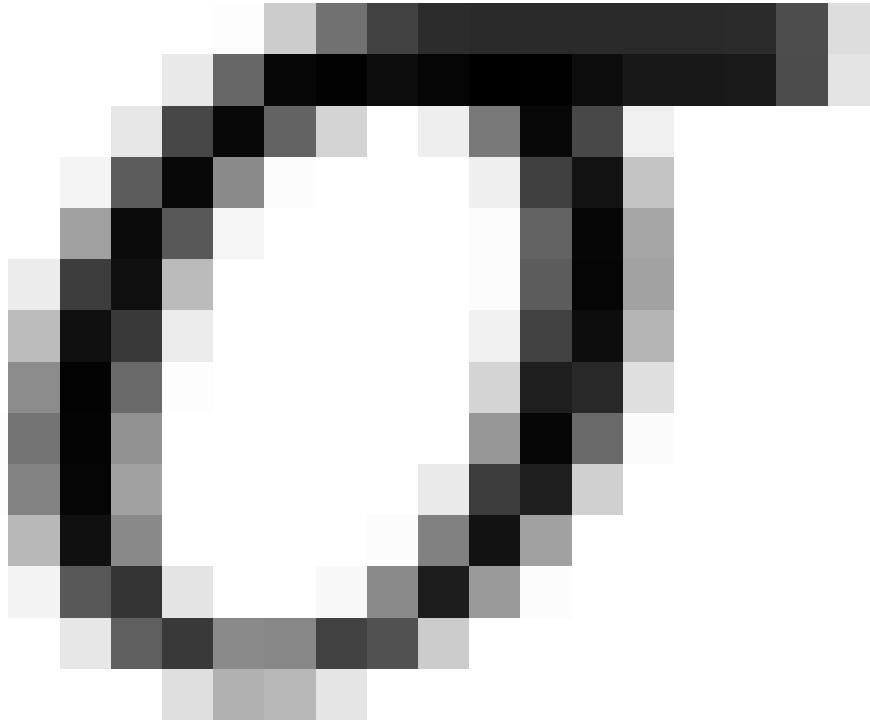


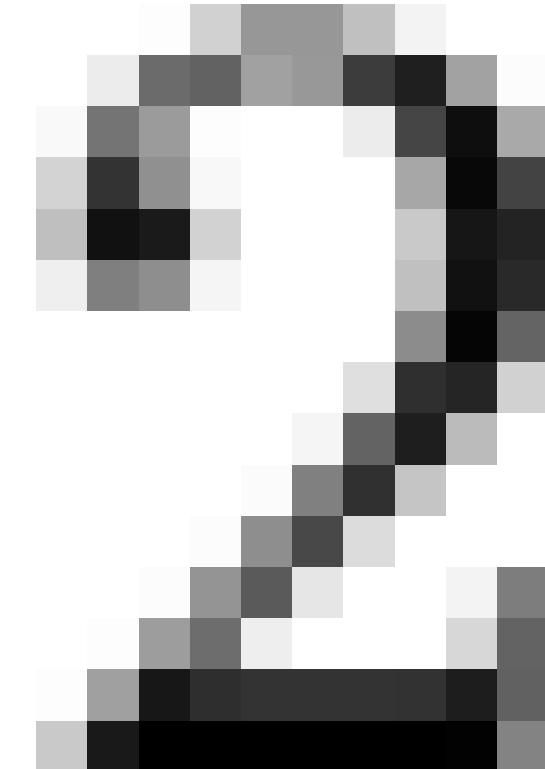
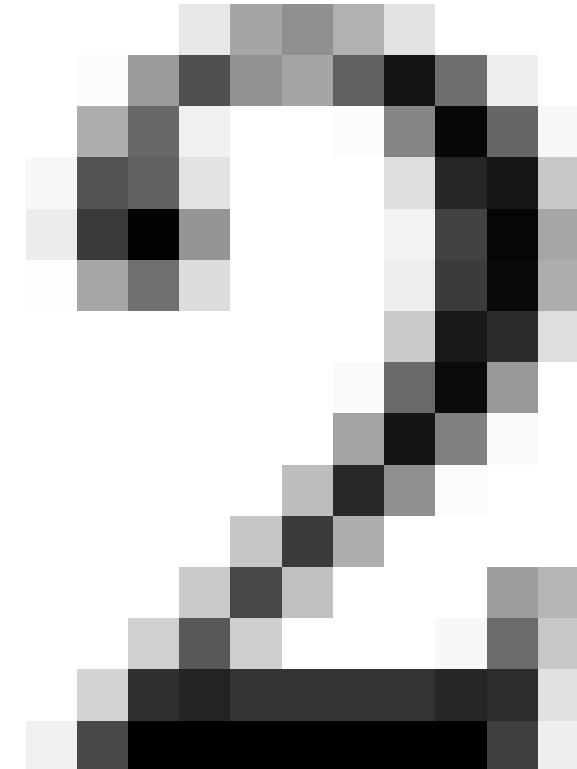
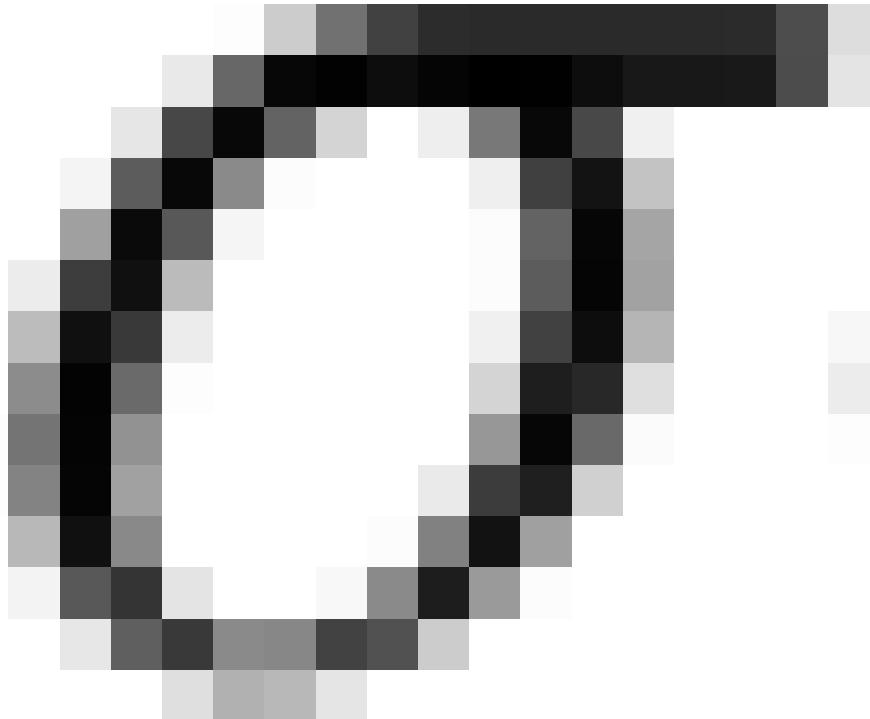






$$\left\{ \begin{array}{l} \sigma_{11} = \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)} \epsilon_{33} \\ \sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \epsilon_{33} \end{array} \right.$$





$\sigma_{11}$ 

$$= \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

 $\sigma_{22}$ 

$$= \begin{array}{c} \text{---} \\ | \end{array}$$

 $\sigma_{33}$ 

$$\begin{array}{c} \text{---} \\ \text{---} \end{array}$$

$$\begin{array}{c} \text{---} \\ \text{---} \end{array}$$

$\sigma_b$

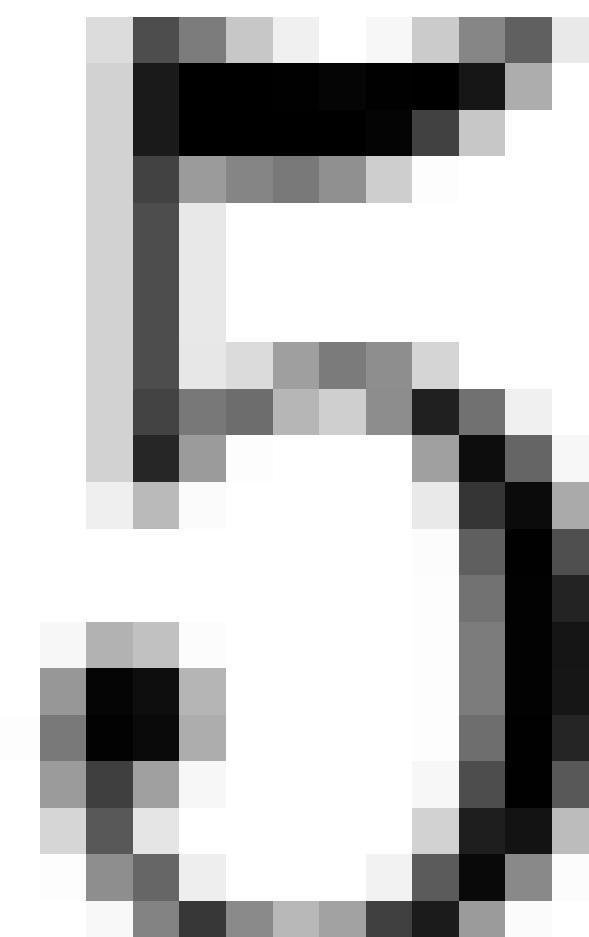
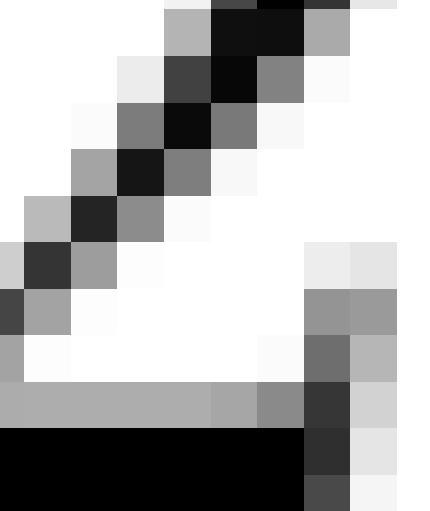
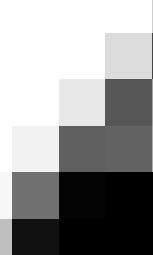
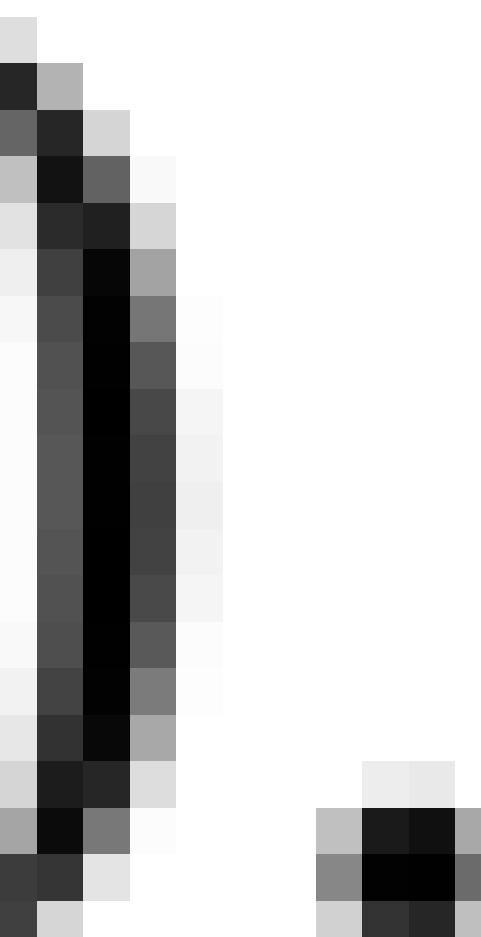
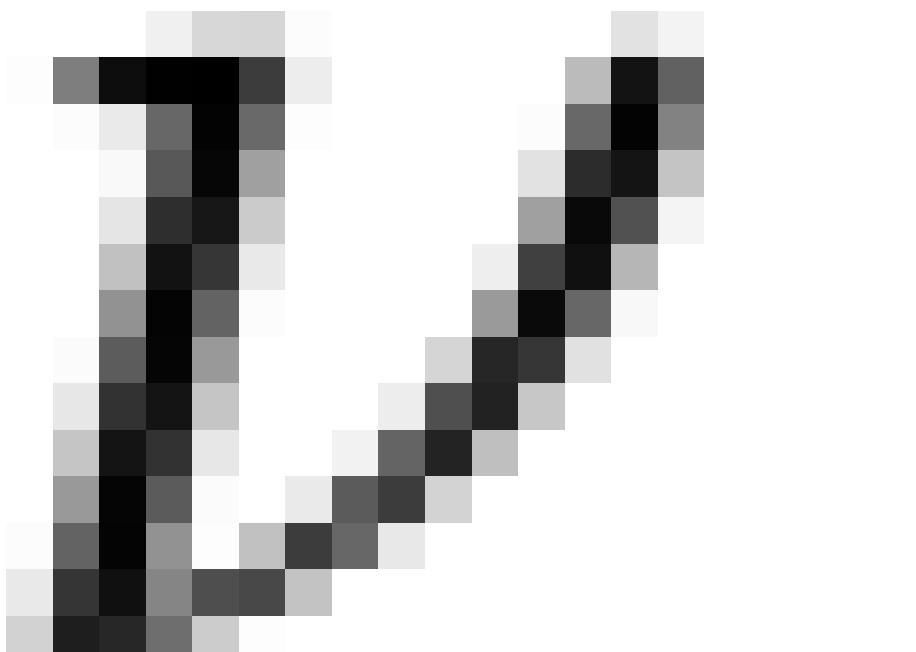
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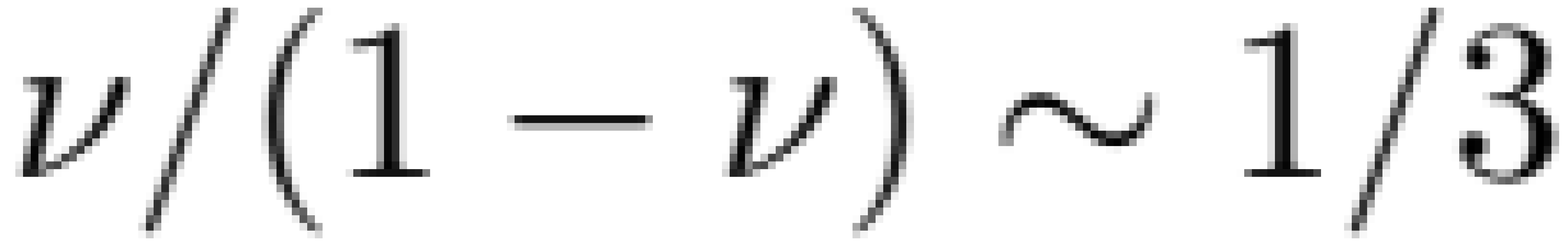
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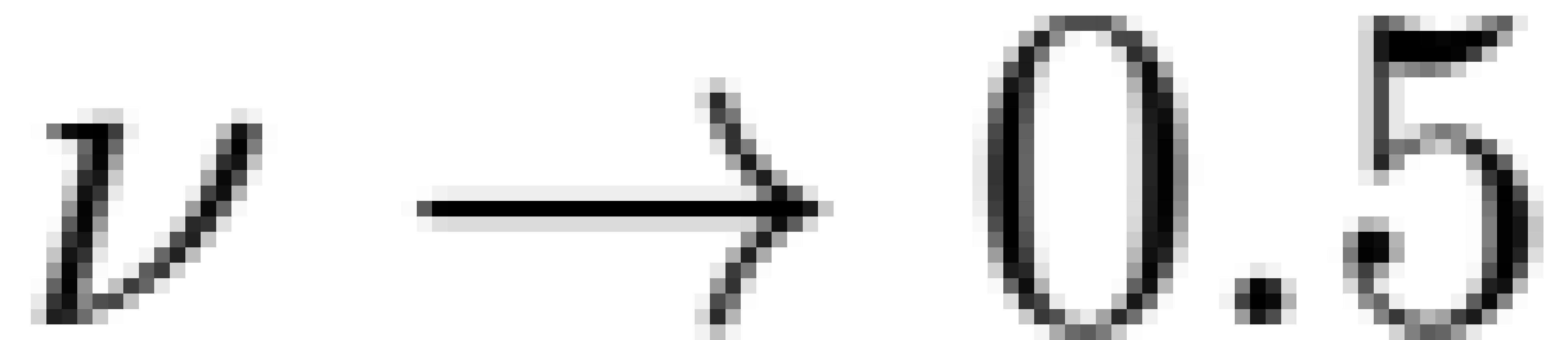
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V

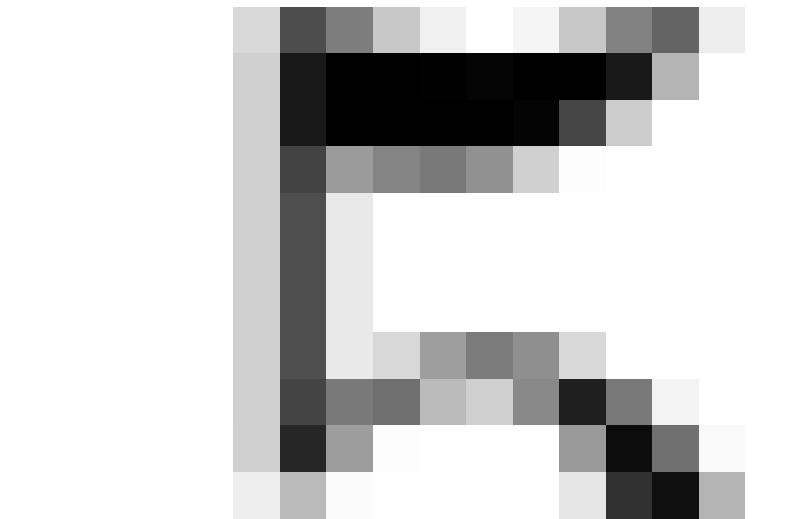
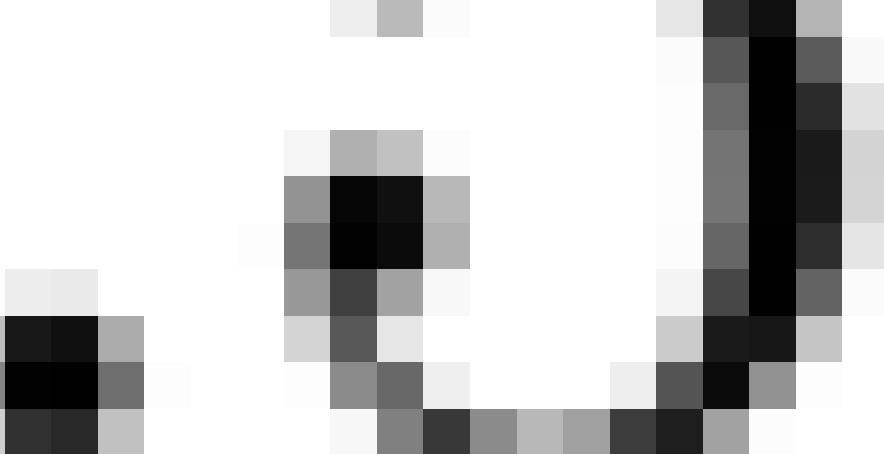
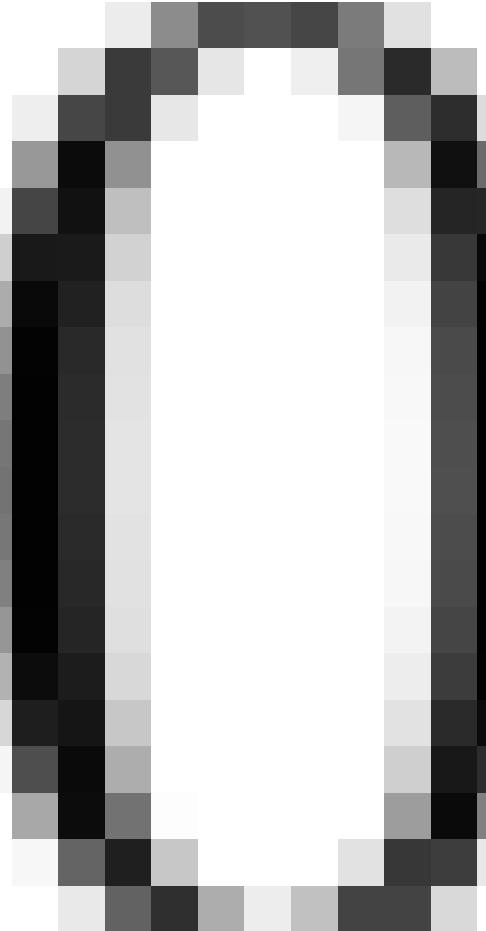
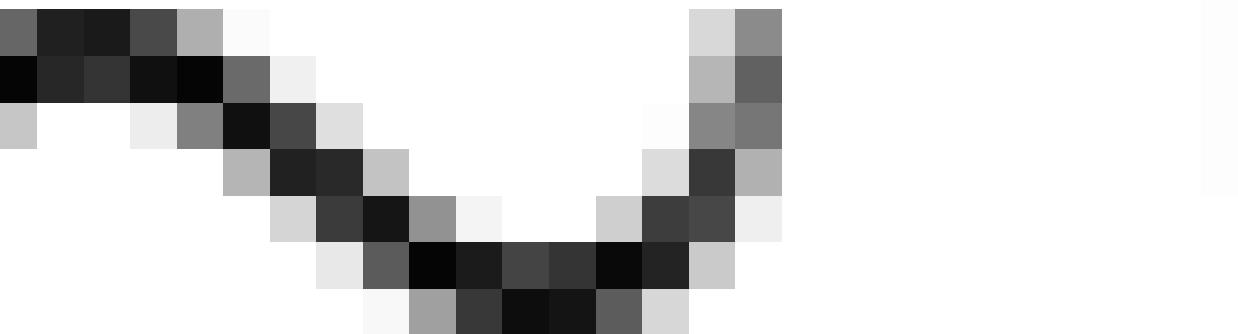
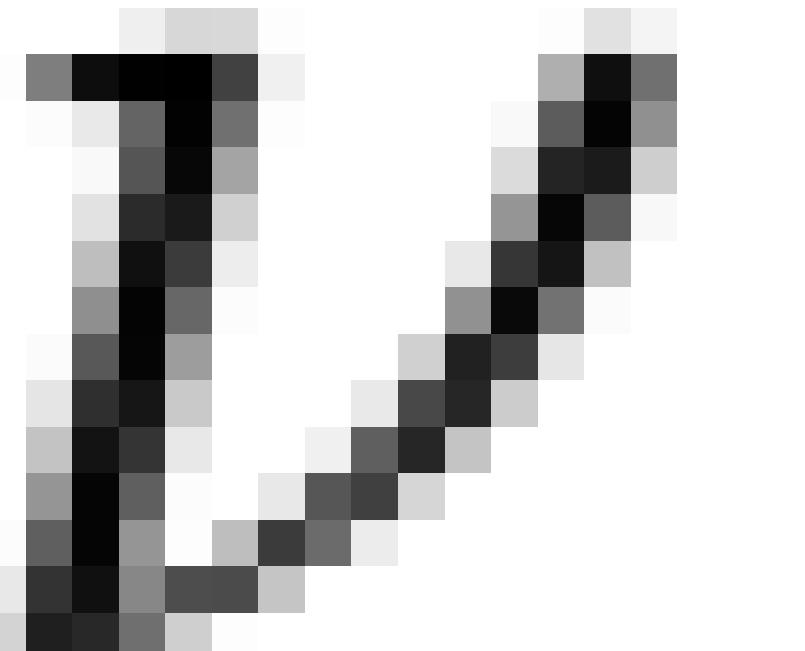
$\sigma_w$







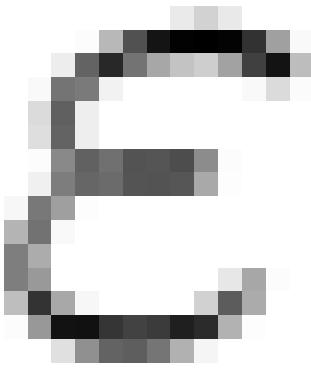
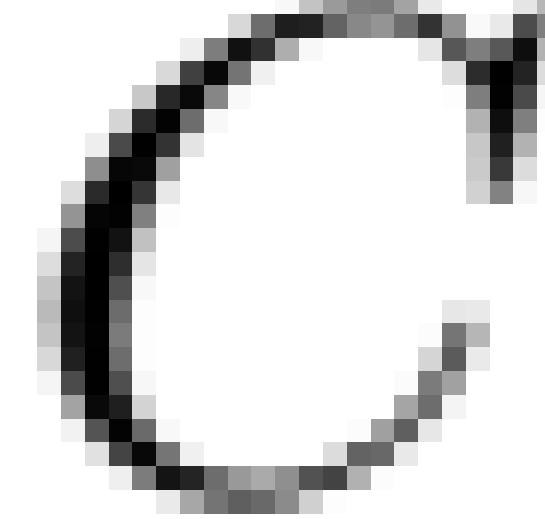
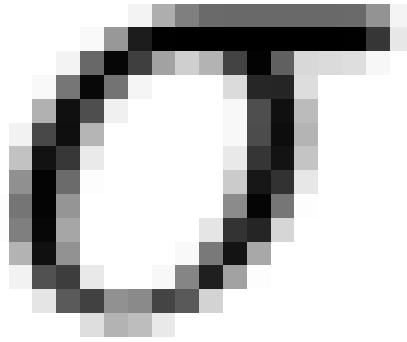


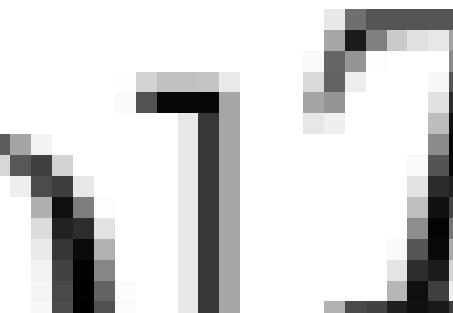
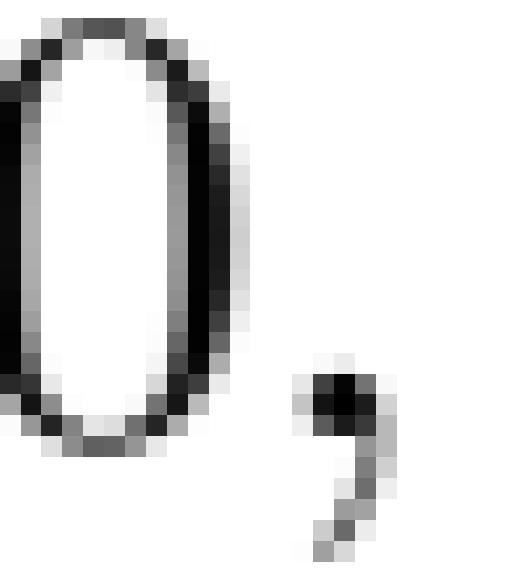
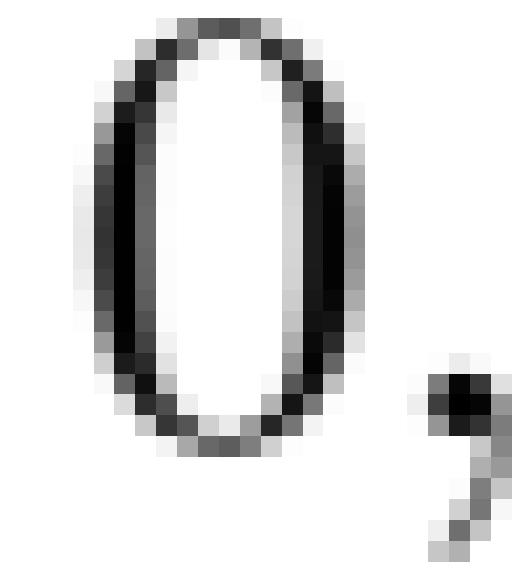
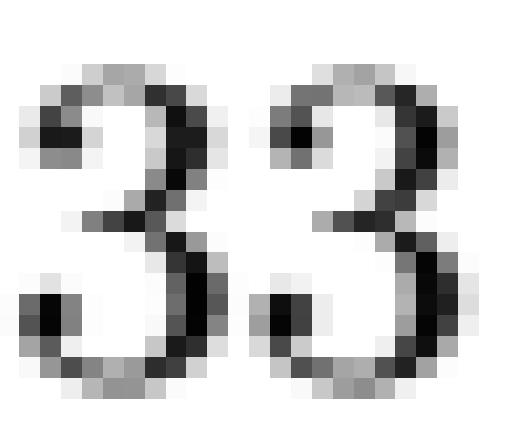
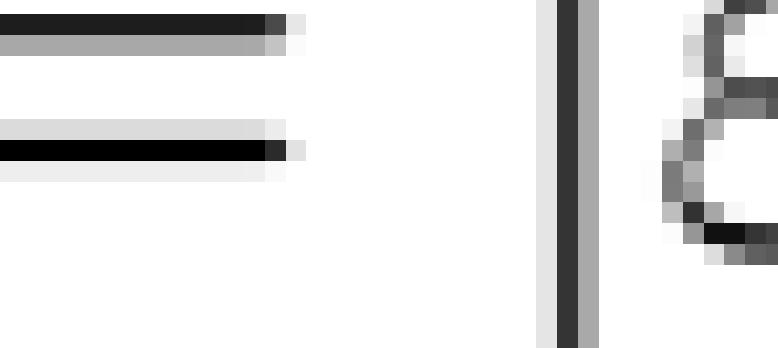
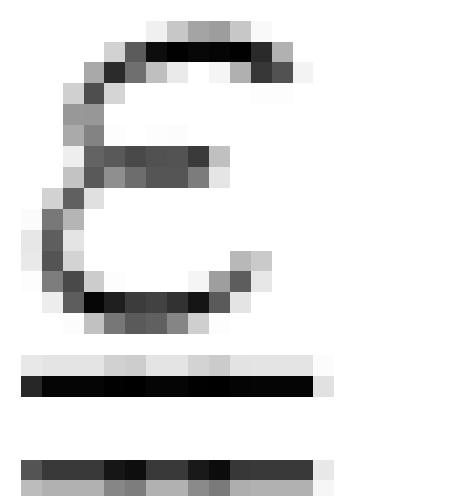












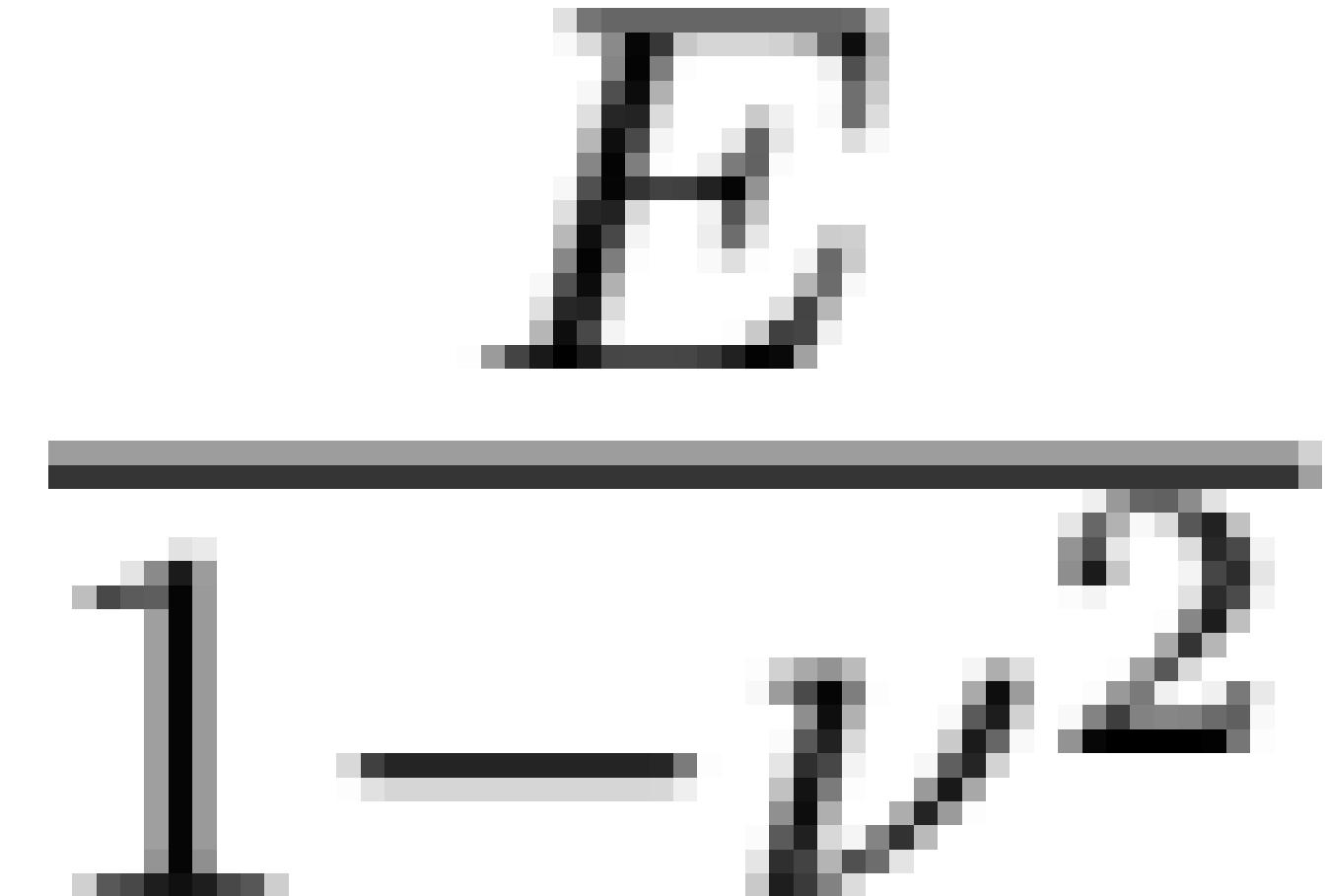
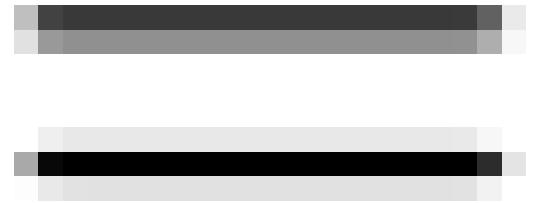
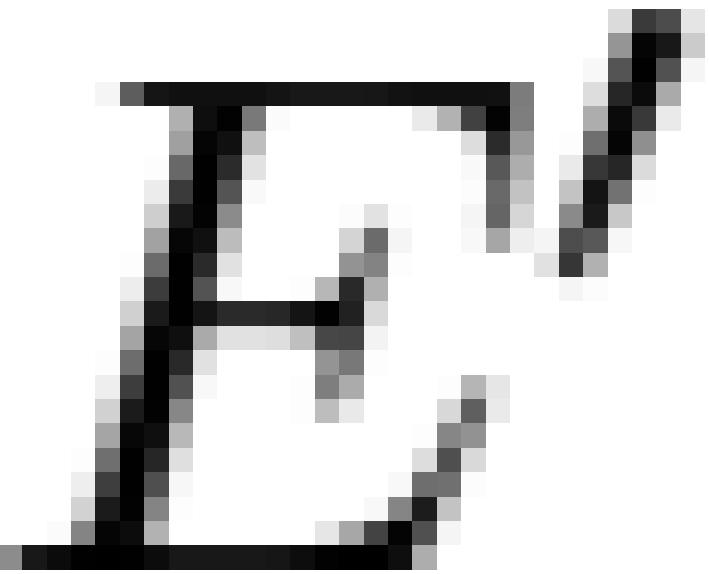
$$\left\{ \begin{array}{l} \sigma_{11} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{33} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \end{array} \right.$$

$$\left\{ \begin{array}{l} \sigma_{11} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{E}{1-\nu^2} \epsilon_{11} + \frac{\nu E}{1-\nu^2} \epsilon_{22} \\ \sigma_{22} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{\nu E}{1-\nu^2} \epsilon_{11} + \frac{E}{1-\nu^2} \epsilon_{22} \end{array} \right.$$

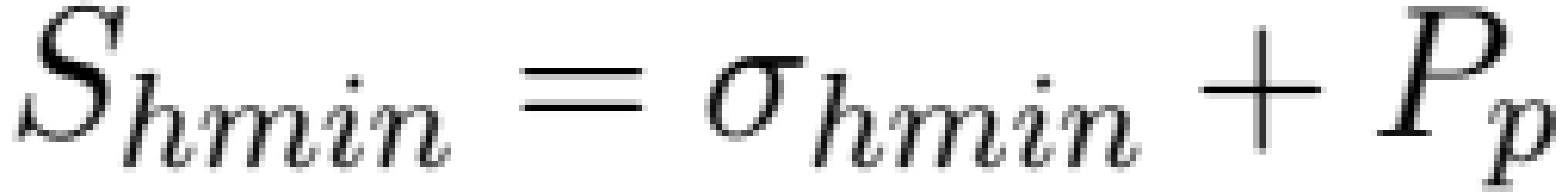




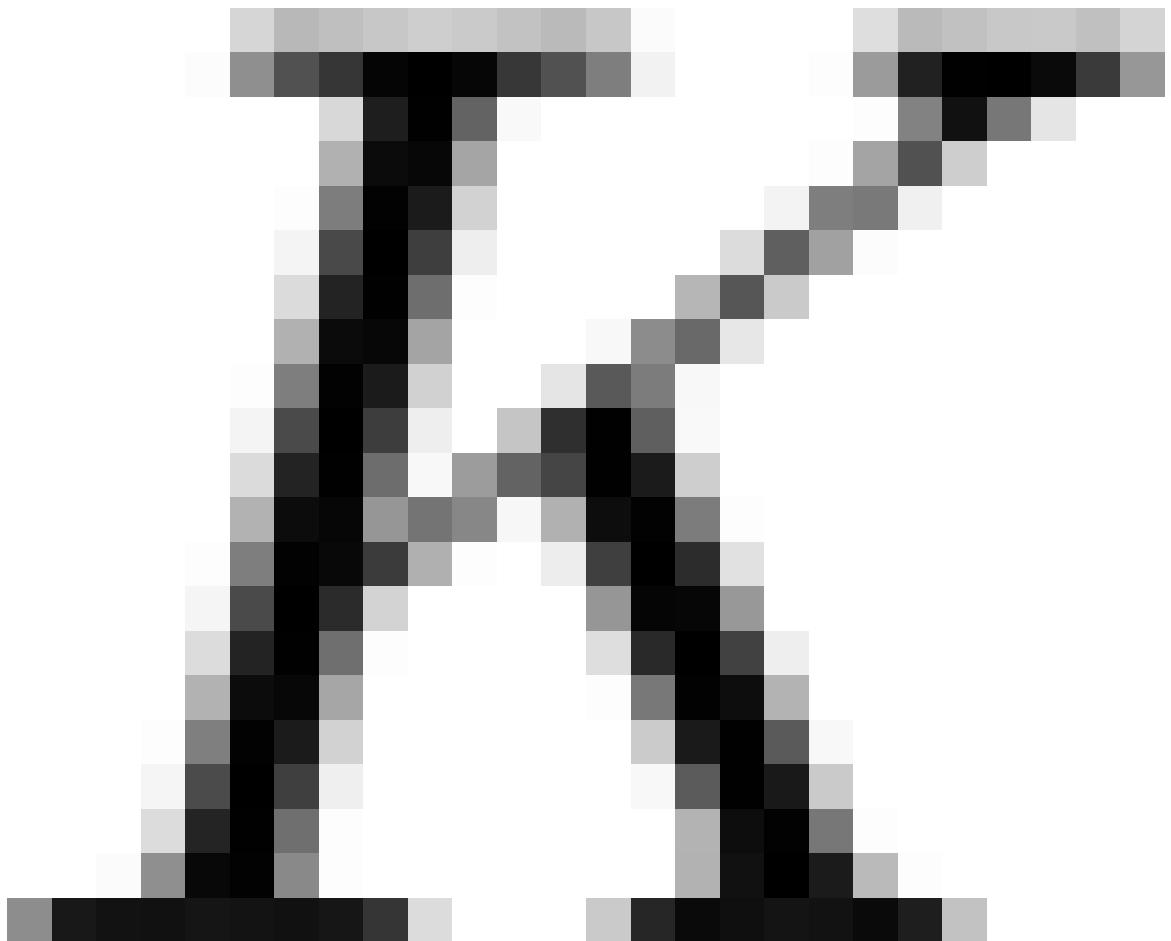
$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} + \nu E' \epsilon_{hmin} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + \nu E' \epsilon_{Hmax} + E' \epsilon_{hmin} \end{array} \right.$$

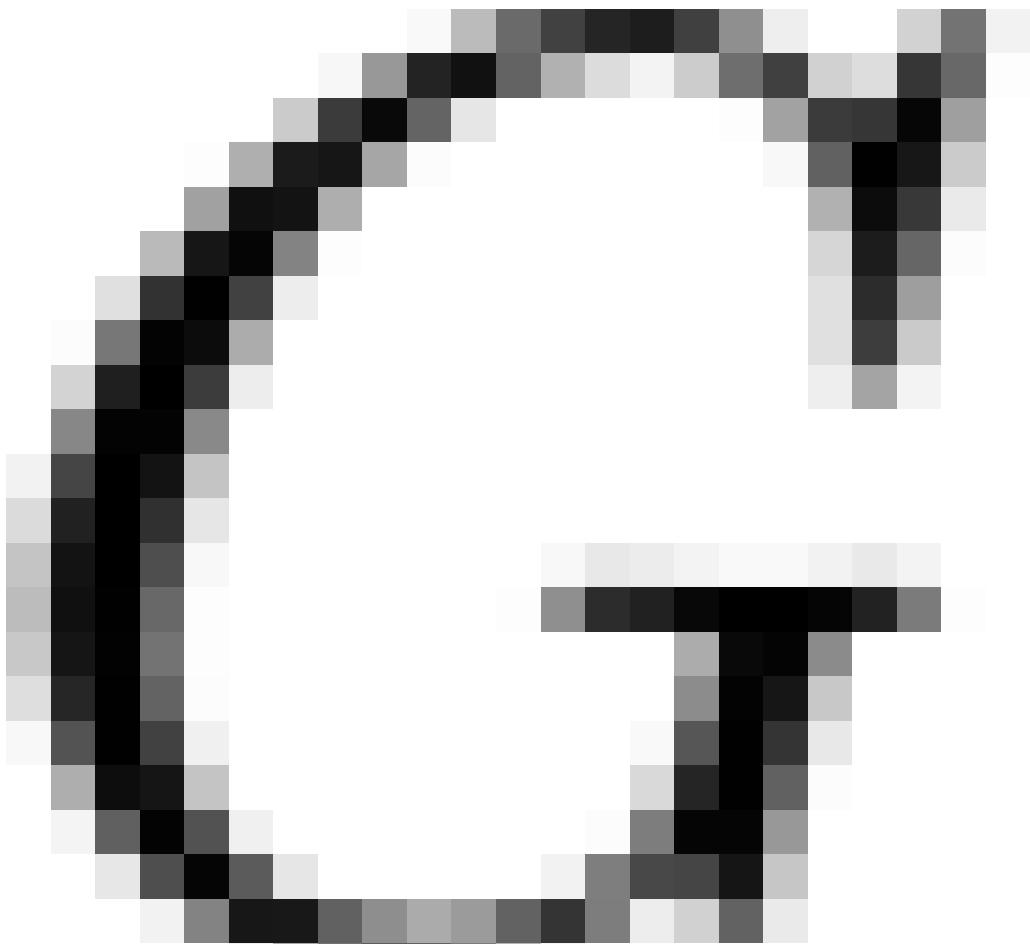




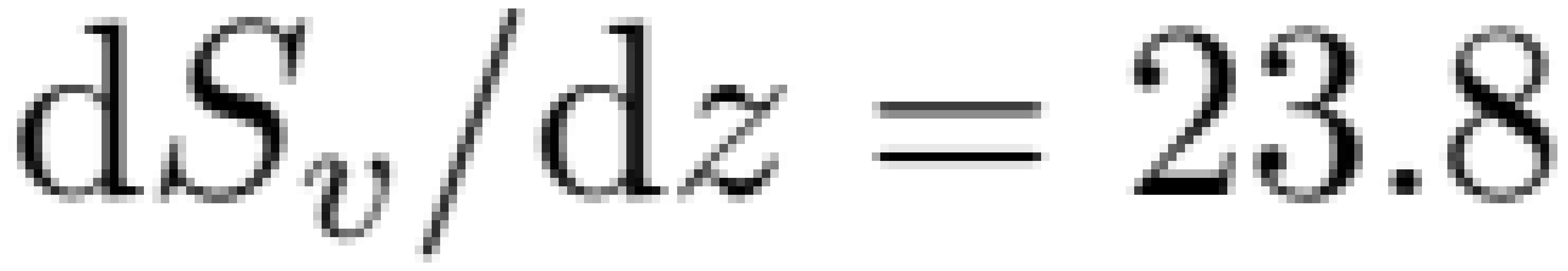


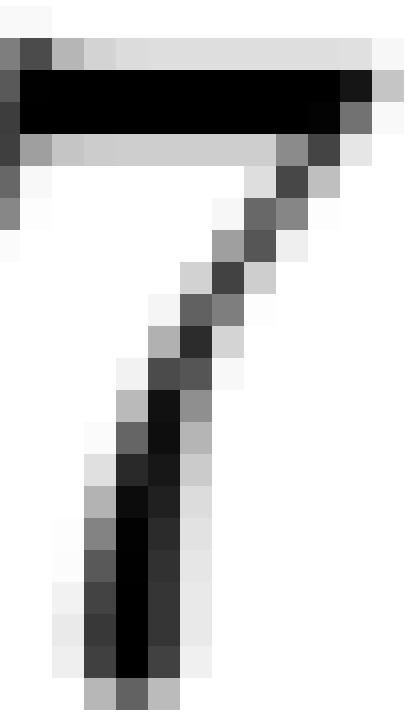
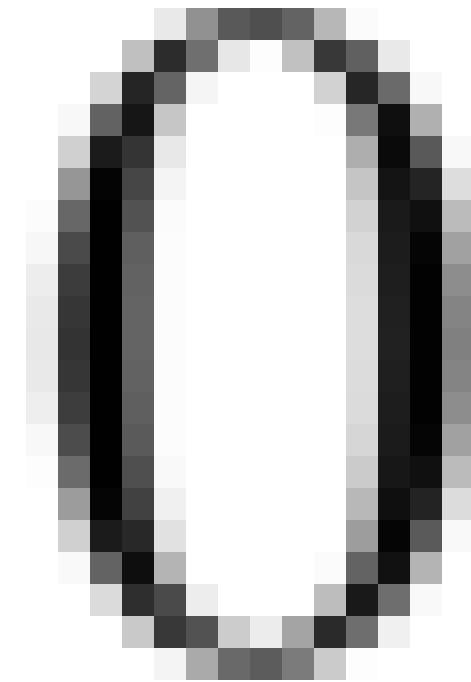


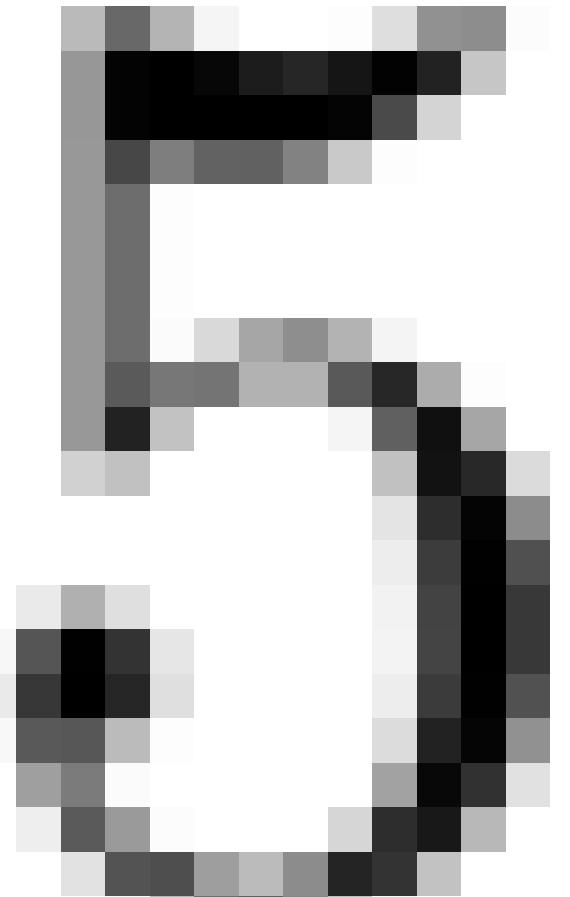
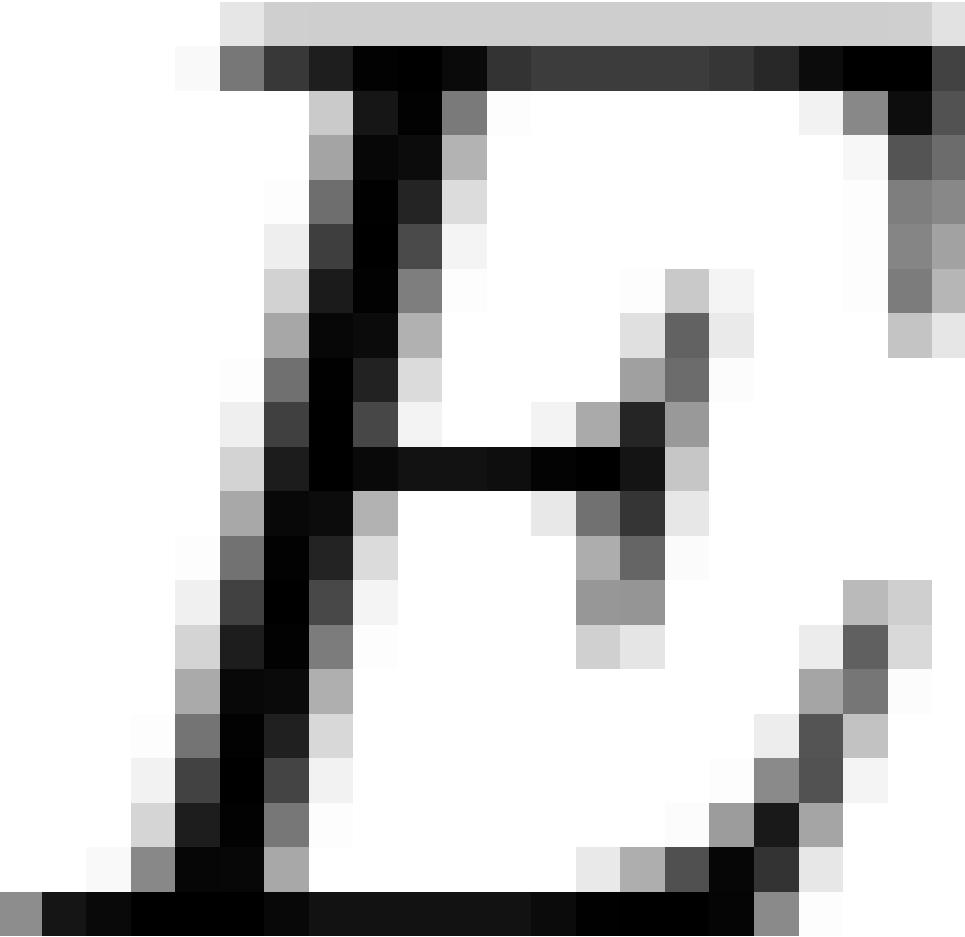


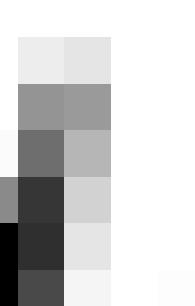
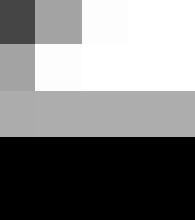
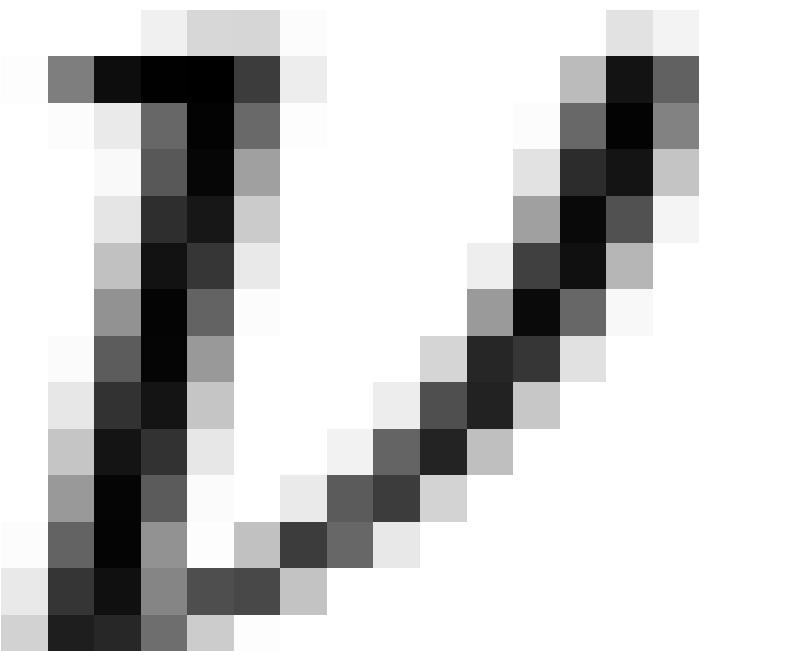


	$(E, v)$	$(K, G)$
$G =$	$\frac{E}{2(1+v)}$	<u>Shear modulus</u> (also noted as $\mu$ , S-wave)
$M =$	$\frac{(1-v)E}{(1+v)(1-2v)}$	<u>Constrained modulus</u> (uniaxial compaction, P-wave)
$\lambda =$	$\frac{vE}{(1+v)(1-2v)}$	<u>Lamé first parameter</u> (volumetric strain component)
$K =$	$\frac{E}{3(1-2v)}$	<u>Bulk modulus</u> (relates volumetric strain and isotropic stress)





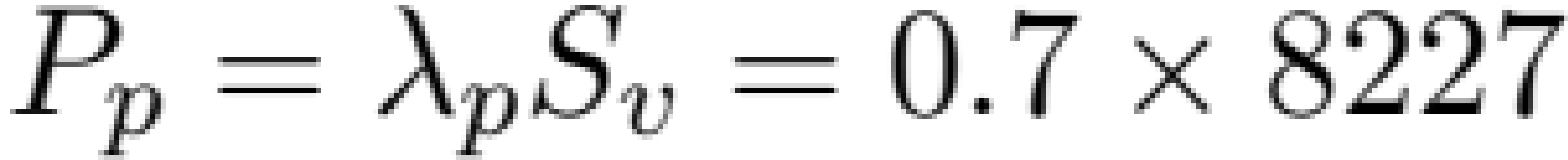




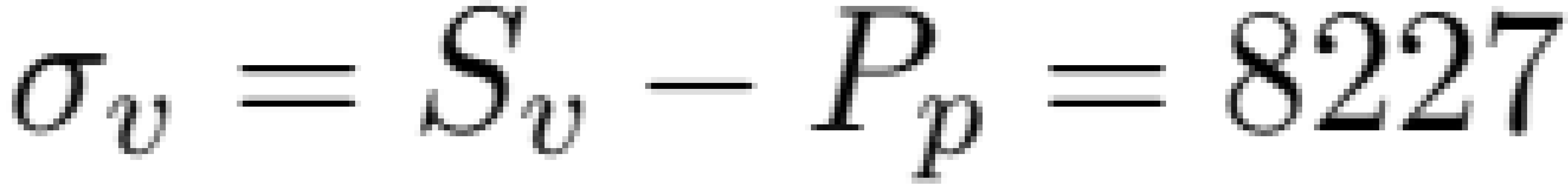




$$s_u = \frac{23.8 \text{ MPa}}{\text{km}} \times \frac{1 \frac{\text{psi}}{\text{ft}}}{\frac{\text{MPa}}{\text{km}}} \times 7950 \text{ ft} = 8227 \text{ psi}$$









$$\frac{E'}{1 - \nu^2} = \frac{E}{1 - 0.22^2} = \frac{5 \times 10^6 \text{ psi}}{5.25 \times 10^6 \text{ psi}}$$

$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} \times 0.0002 = 1745 \text{ psi} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{hmin} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} \times 0.0002 = 927 \text{ psi} \end{array} \right.$$

$$S_{H\max} = \sigma_{H\max} + P_p = 1745 \text{ psi} + 5759 \text{ psi} = 7504 \text{ psi}$$

$$S_{h\min} = \sigma_{h\min} + P_p = 927 \text{ psi} + 5759 \text{ psi} = 6686 \text{ psi}$$

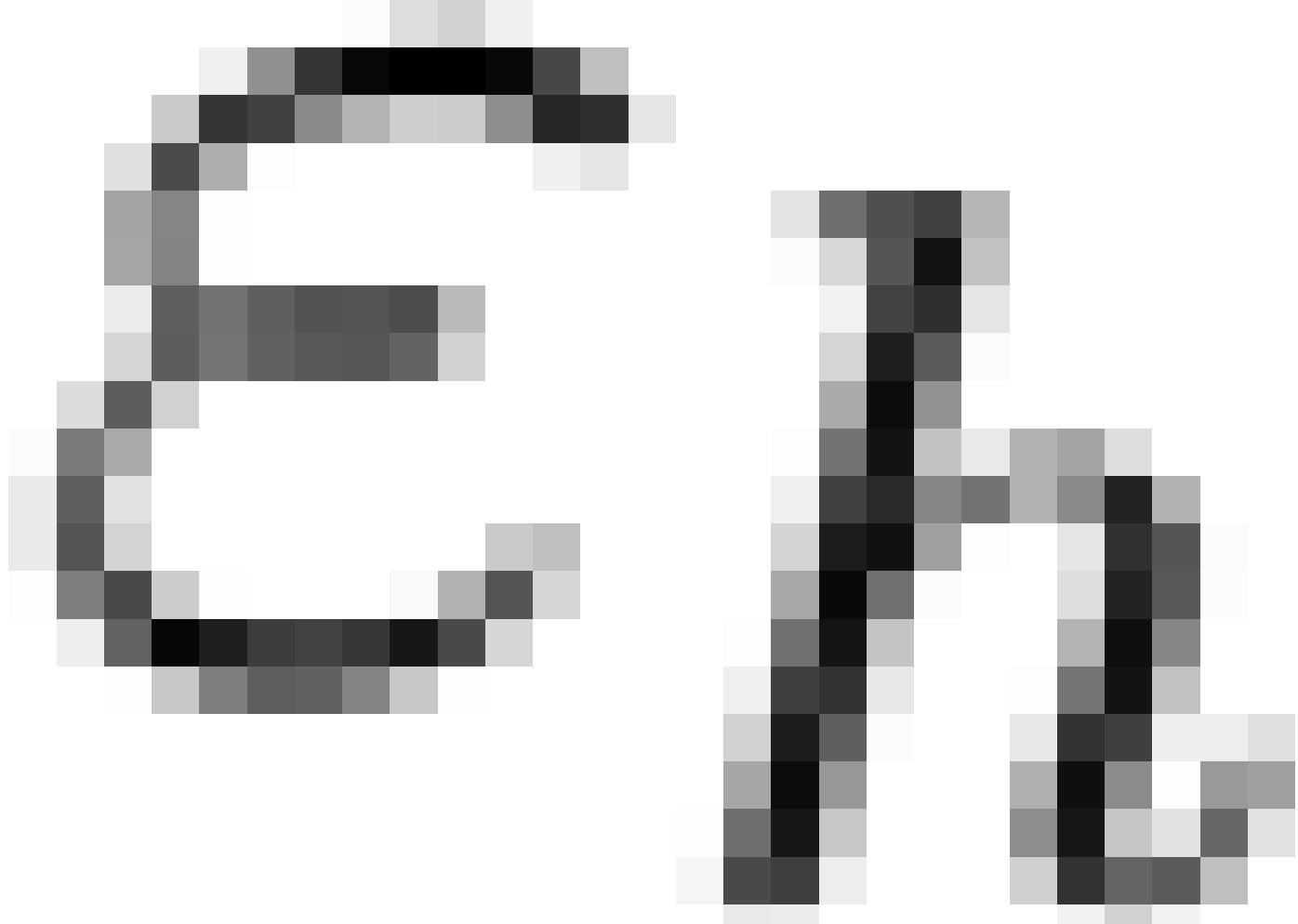




$$C_{pp} =$$

$$\frac{1}{V_p} \frac{dV_p}{dP_p}$$

$S_u, \epsilon_h$



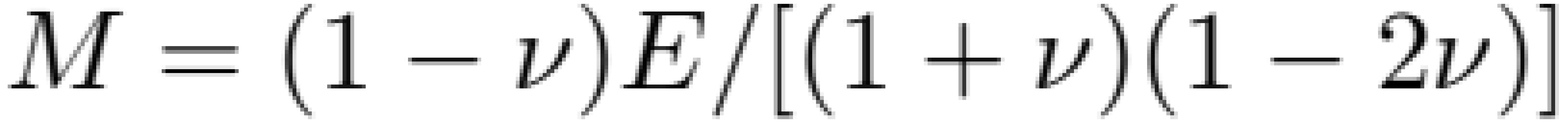
$\text{Op}$

$=$

$\text{Op}$

$\phi$





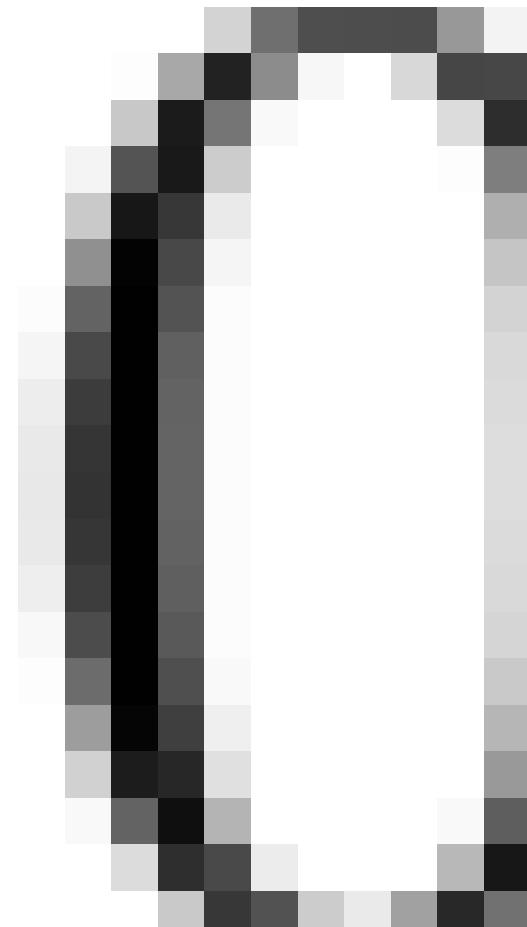
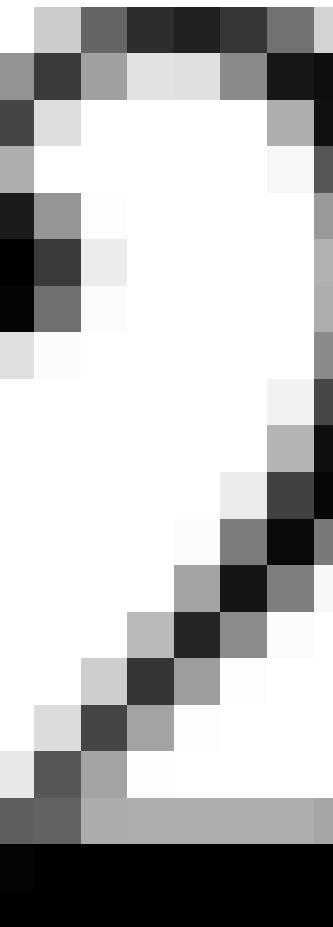
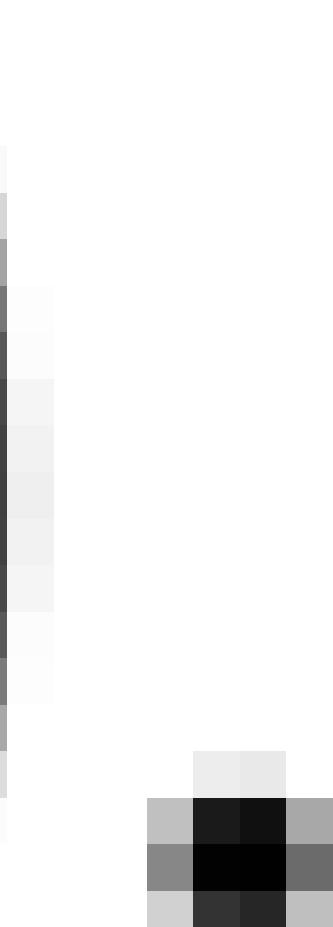
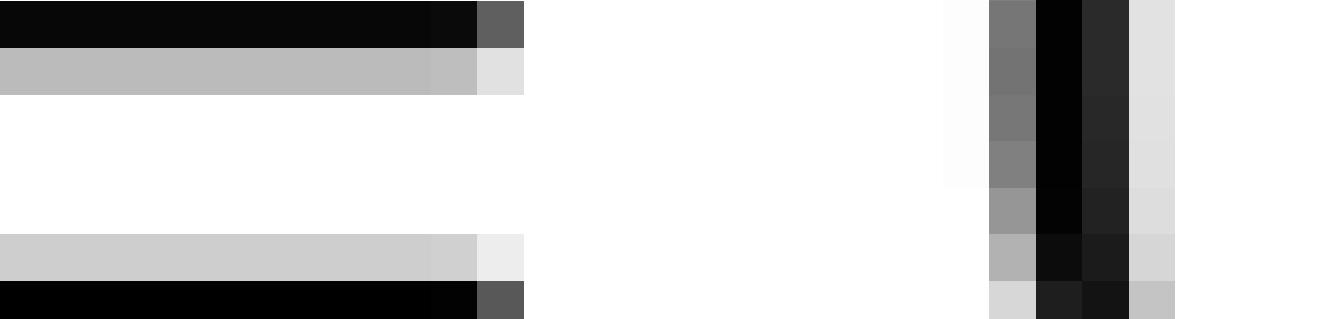
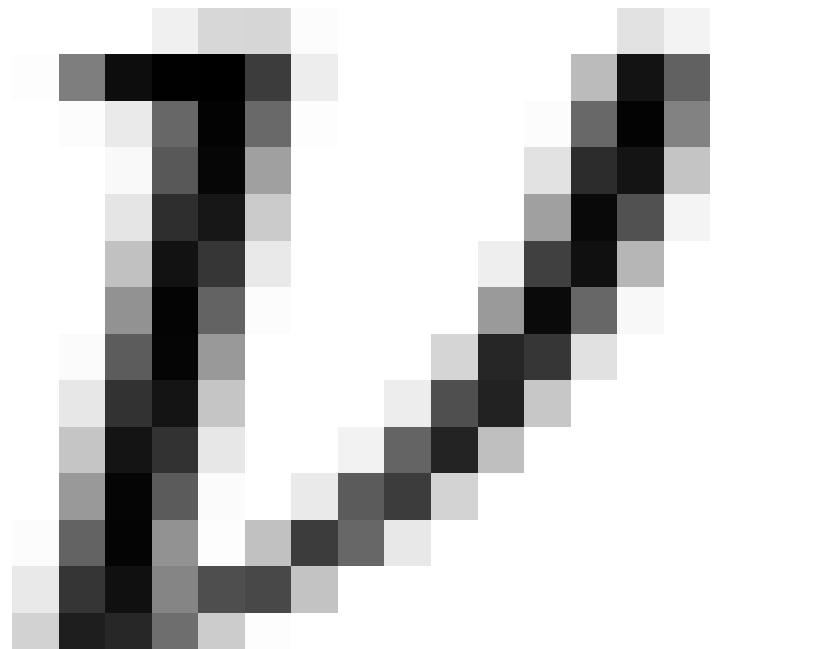
$\langle pp \rangle$

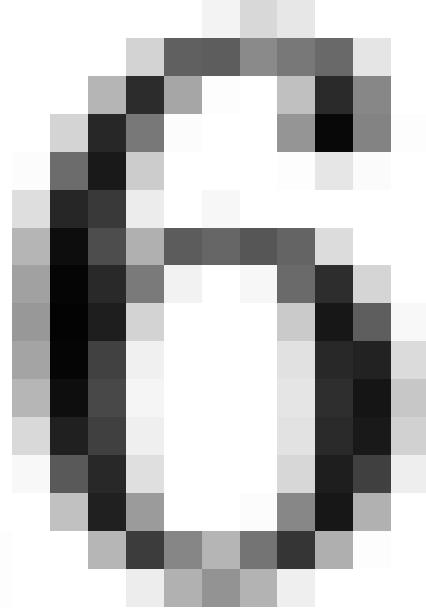
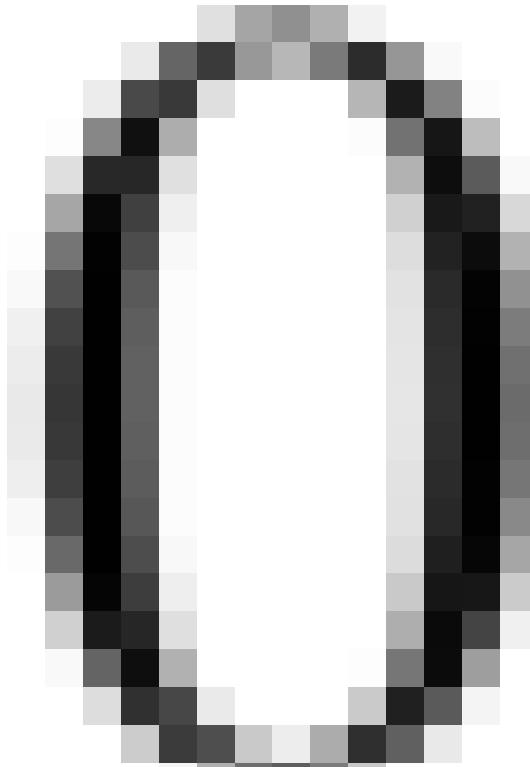
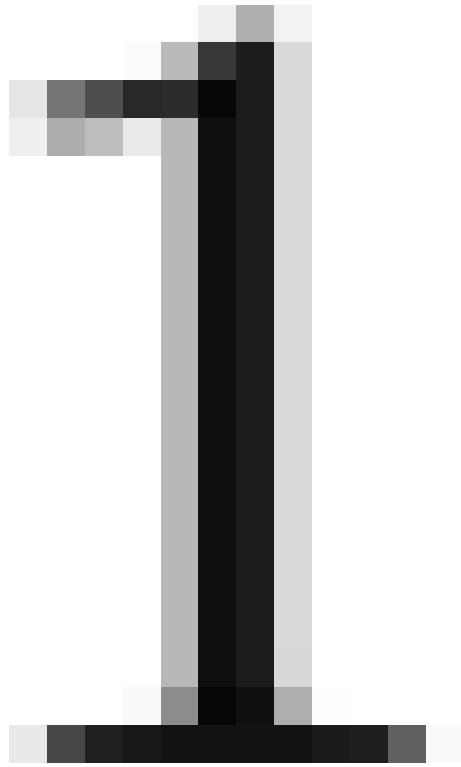
$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\frac{(1 + \nu)(1 - \nu)}{2\nu} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\langle E\phi \rangle$

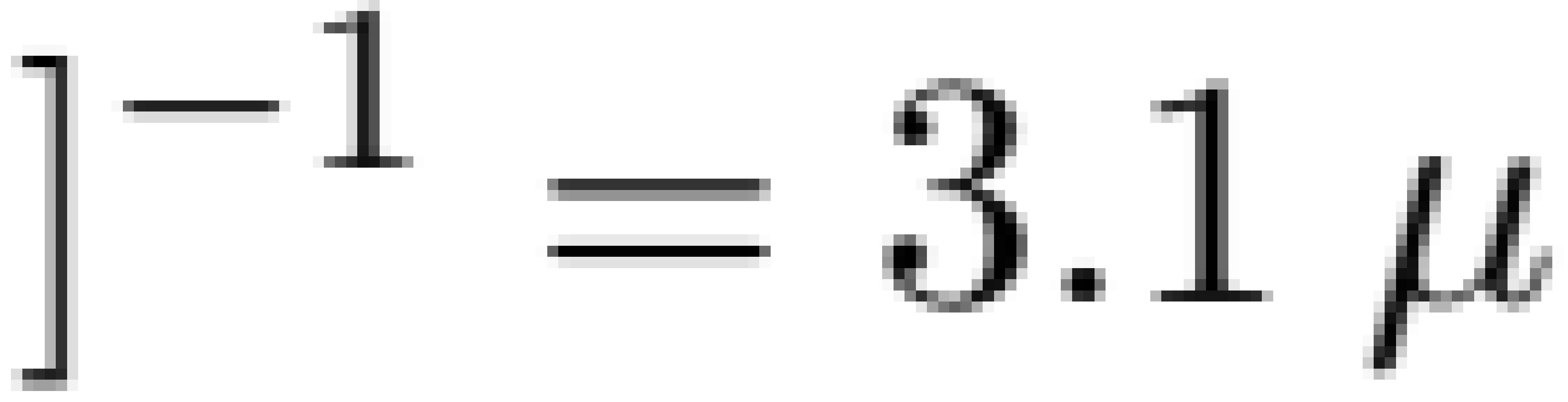


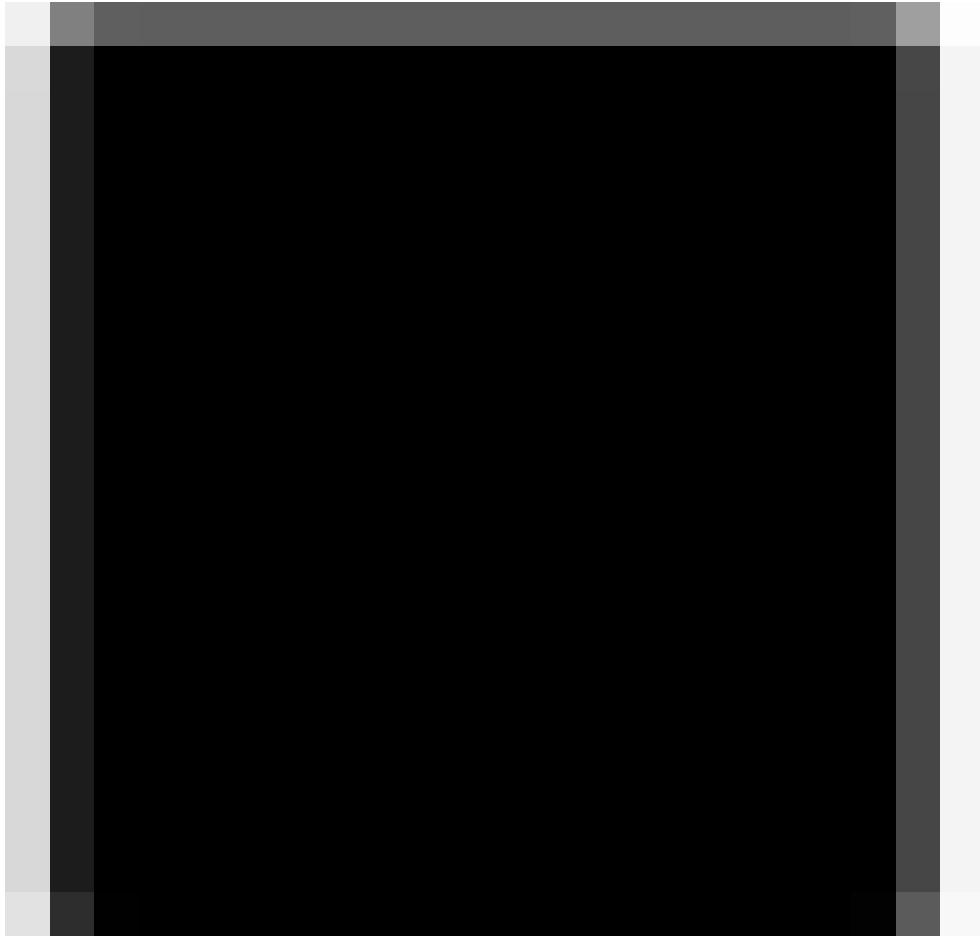


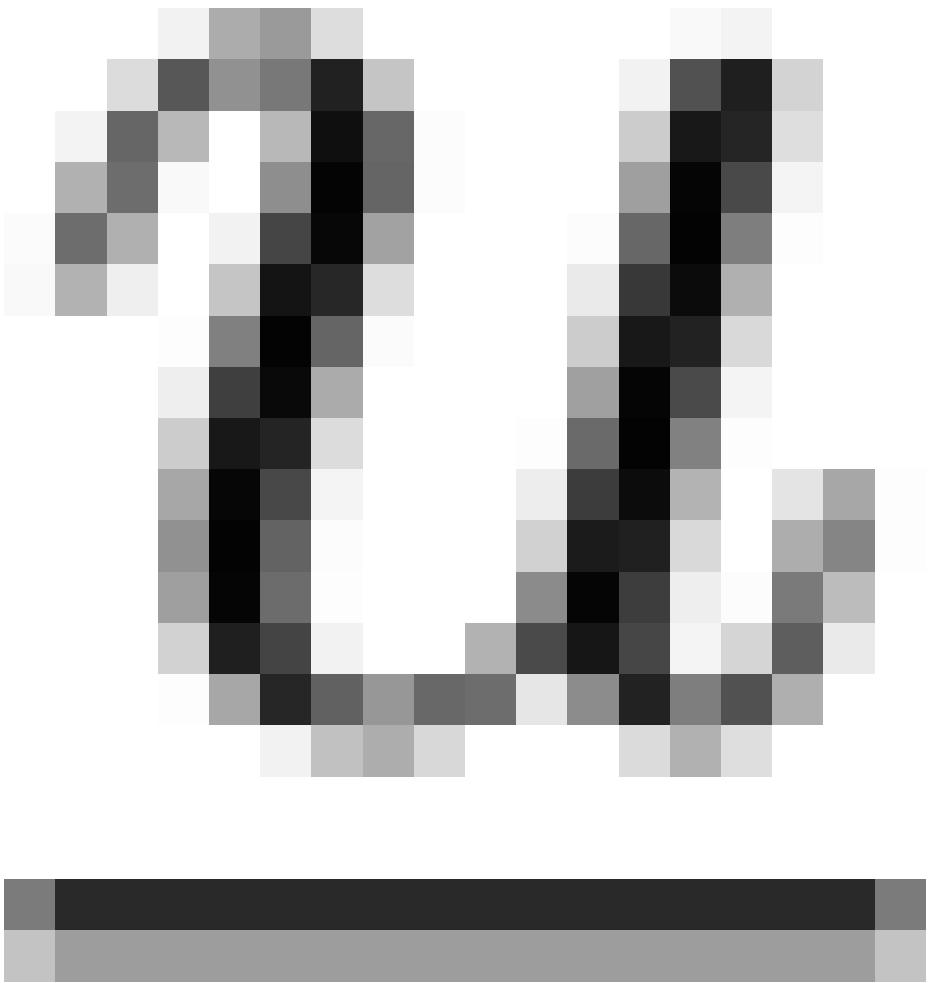


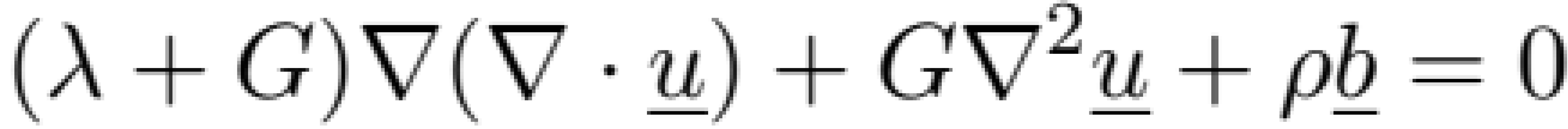
$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.20)10 \text{ GPa}}{1.6 \times 10^6 \text{ psi}} = \frac{11.11 \text{ GPa}}{(1+0.20)(1-2 \times 0.20)}$$

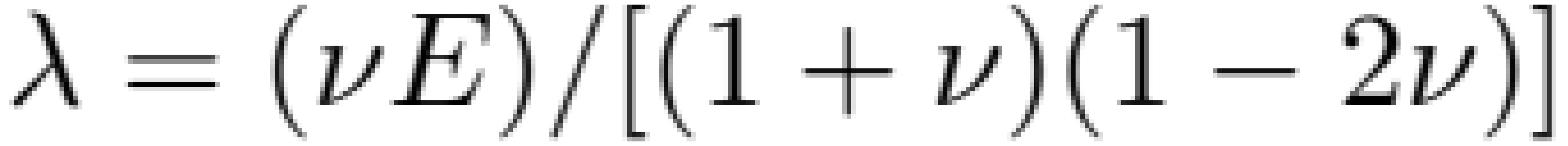
$$\frac{C_{pp}}{M_\phi} = \frac{1}{3.1 \times 10^6 \text{ psi} \times 0.20} = \frac{1}{1.6 \times 10^6}$$

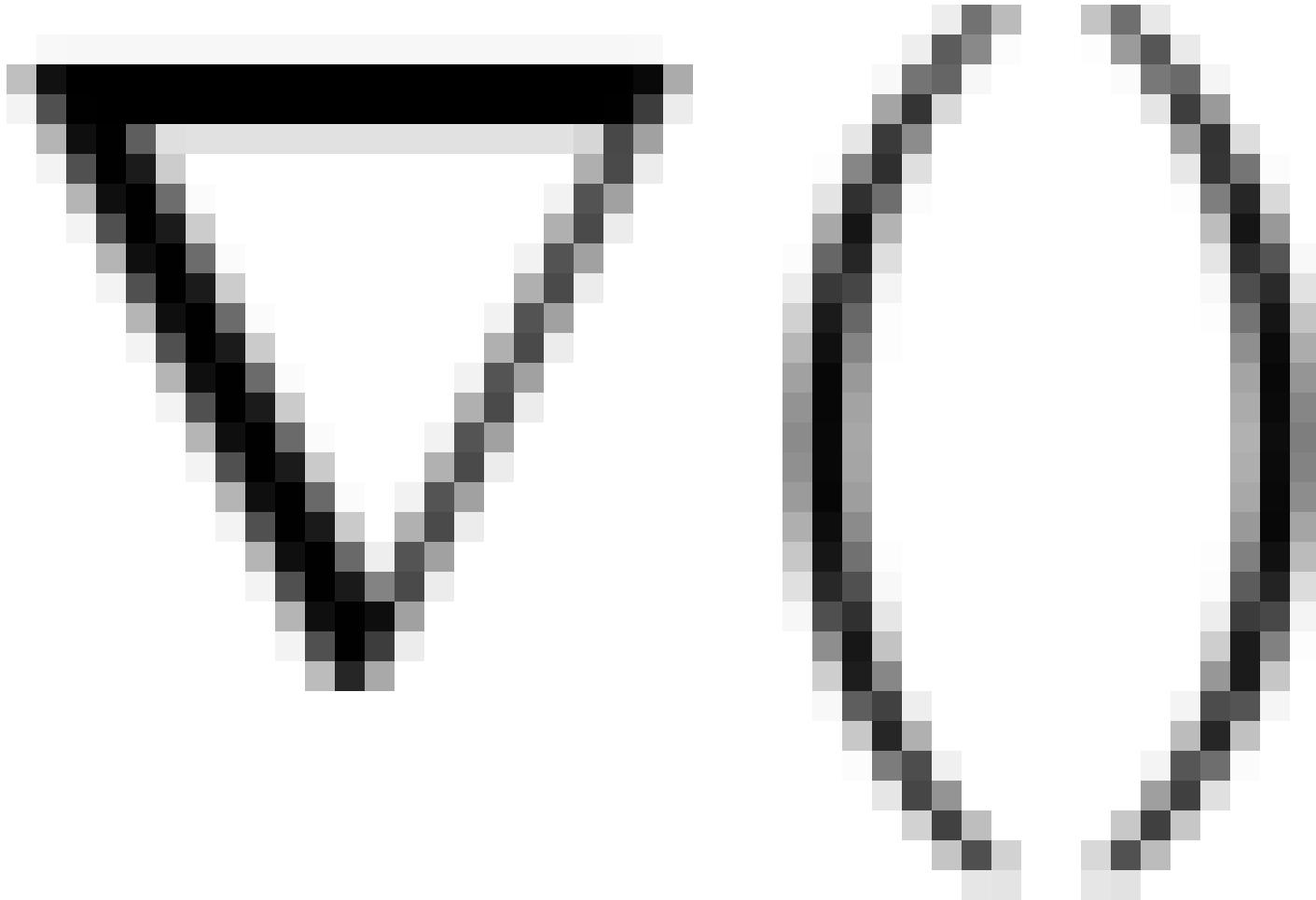


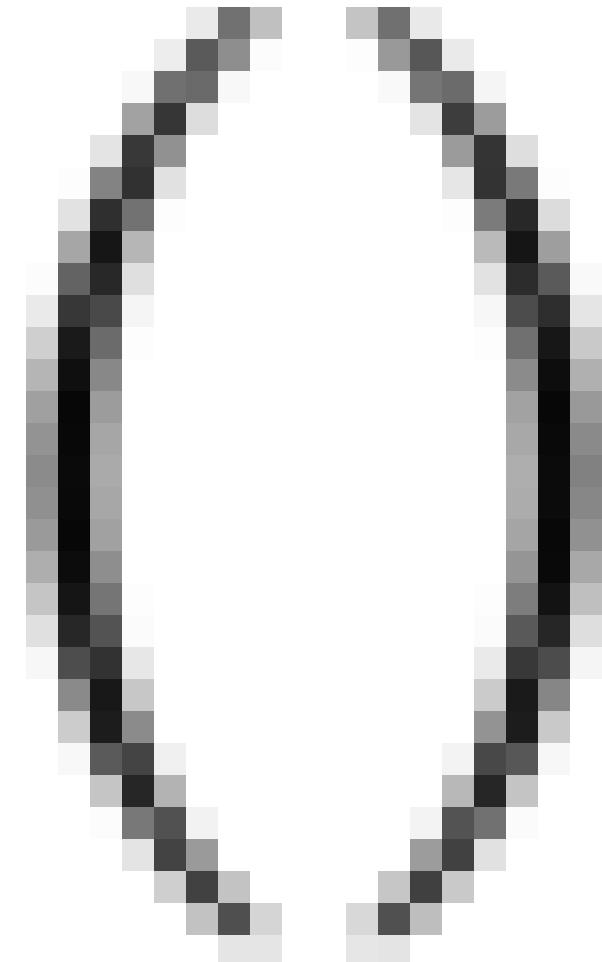
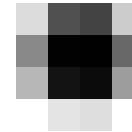
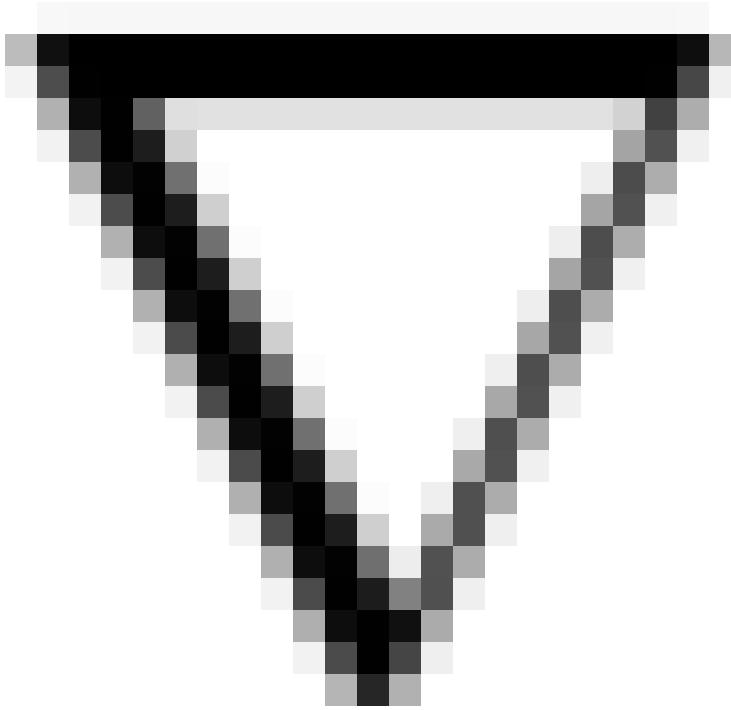






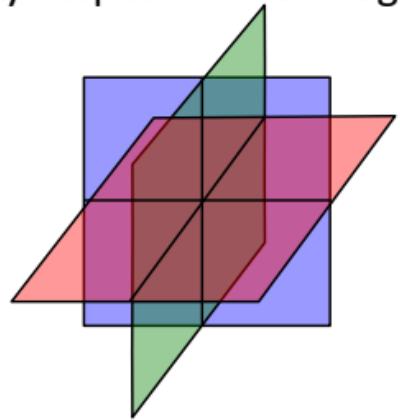








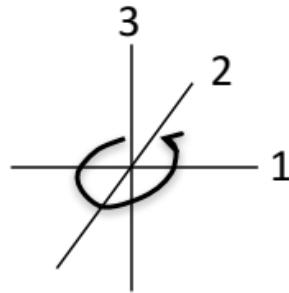
Orthorhombic symmetry  
(symmetry respect to 3 orthogonal planes)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & 0 \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{55} & 0 \\ & 0 & 0 & 0 & & C_{66} \end{bmatrix}$$

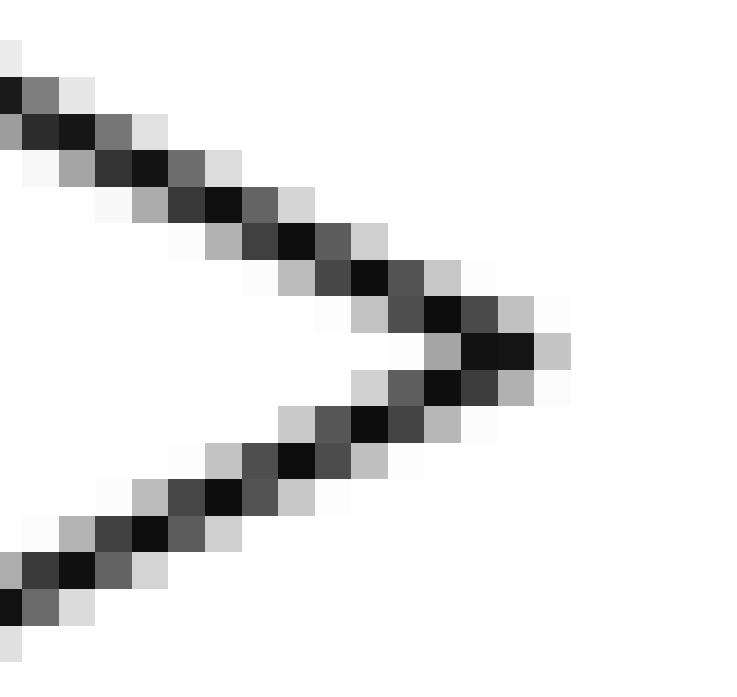
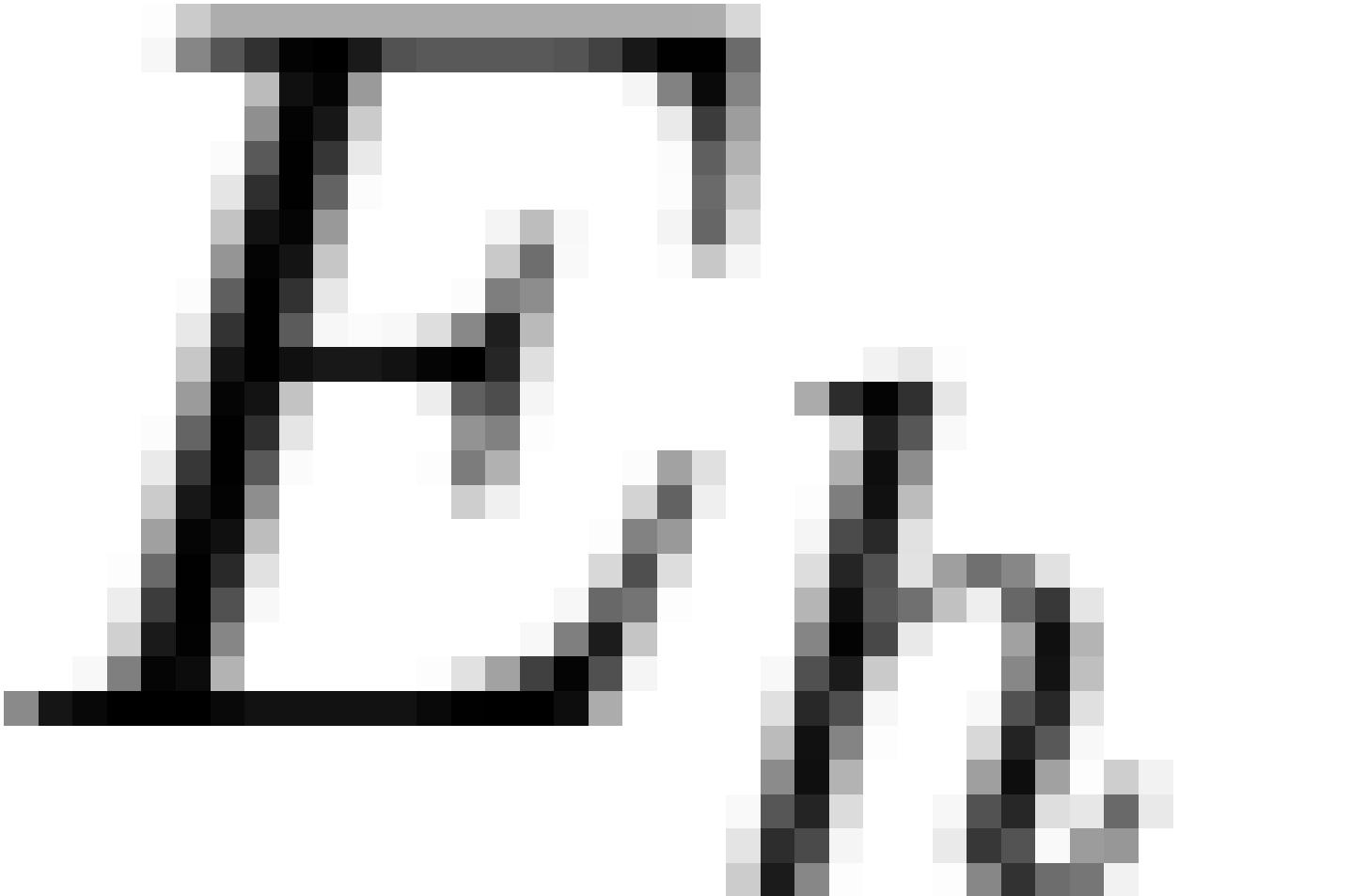
9 independent parameters

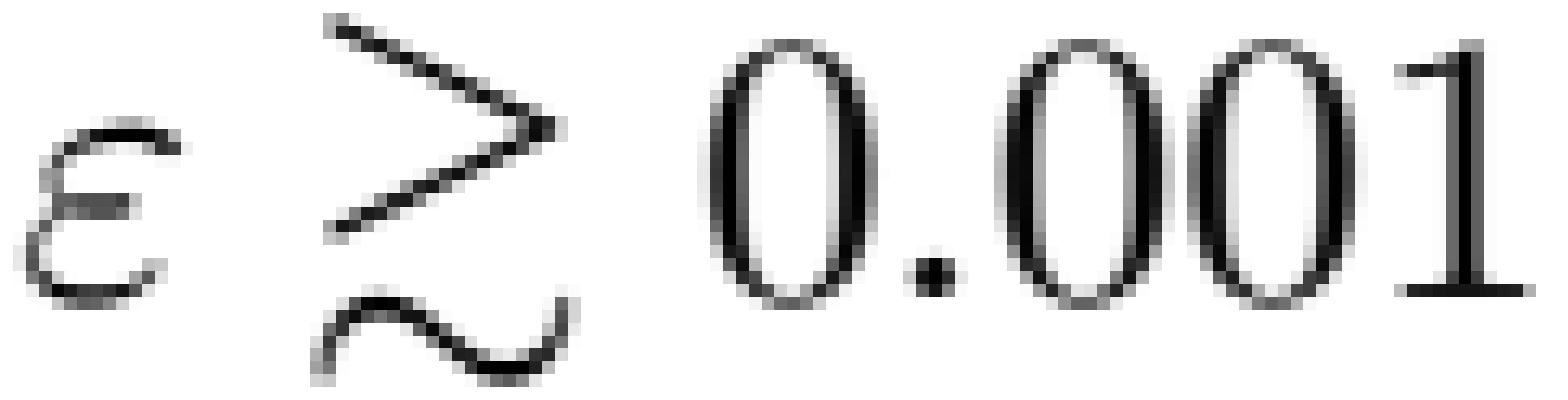
Transverse Isotropy  
(symmetry respect to 1 axis)

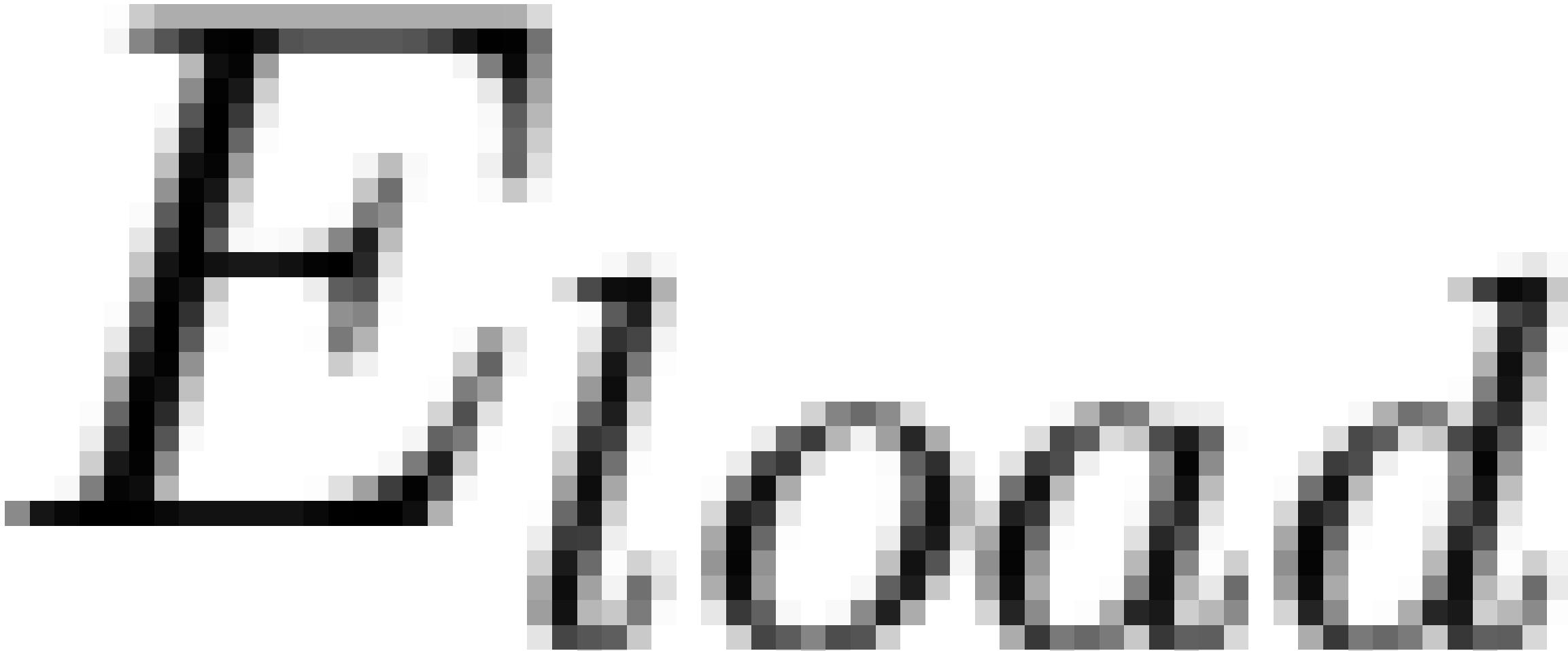


$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & & 0 \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{44} & 0 \\ & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

5 independent parameters ( $C_{12}=C_{11}-2C_{66}$ )

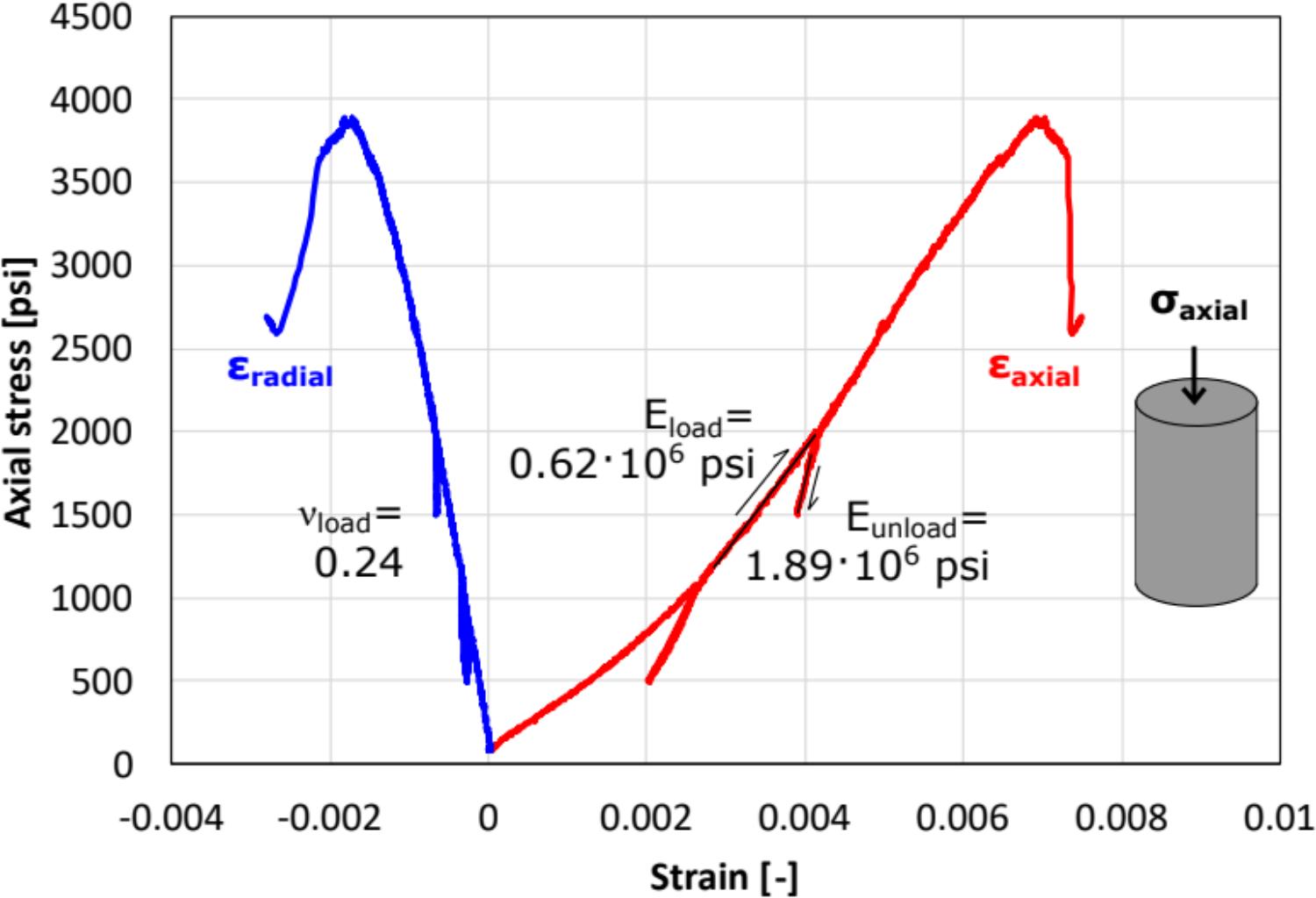




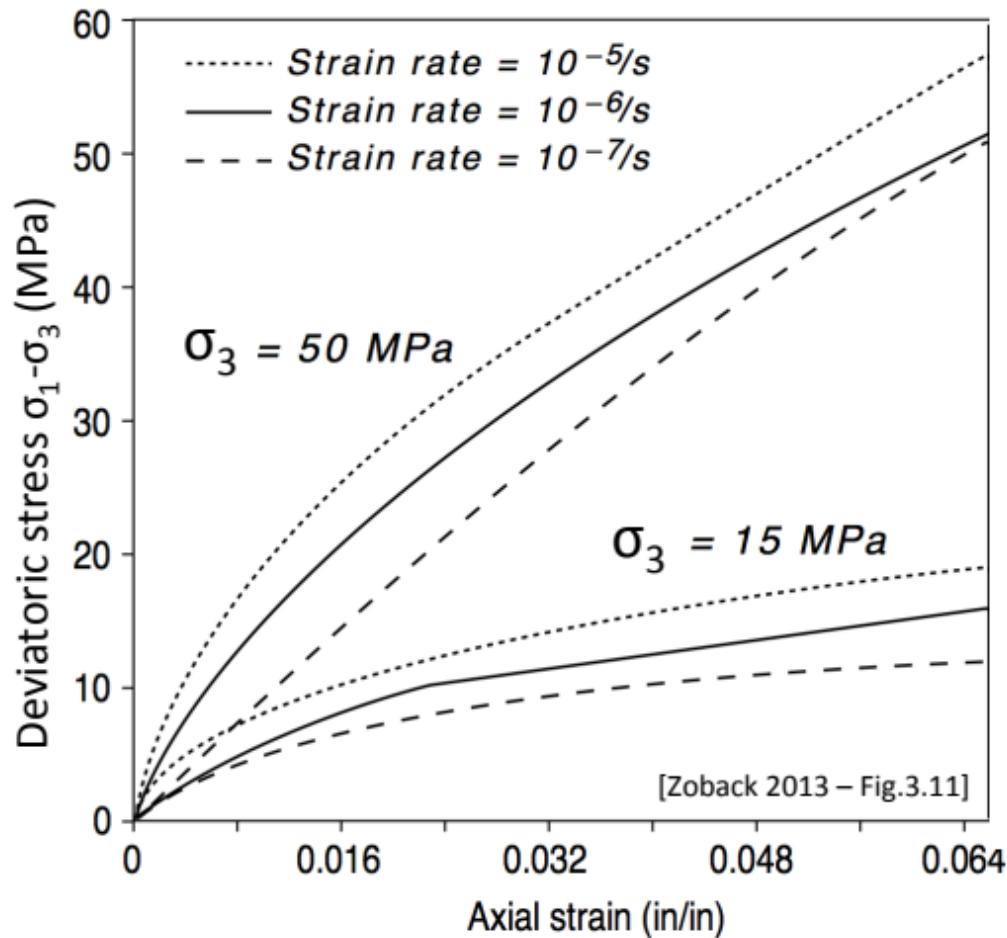
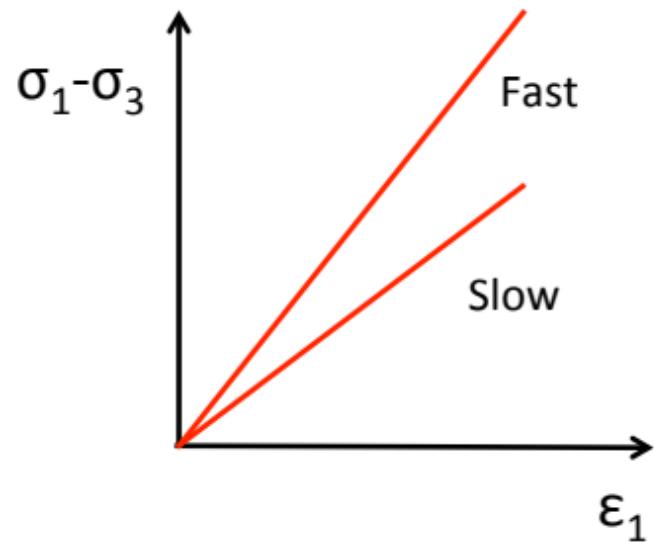




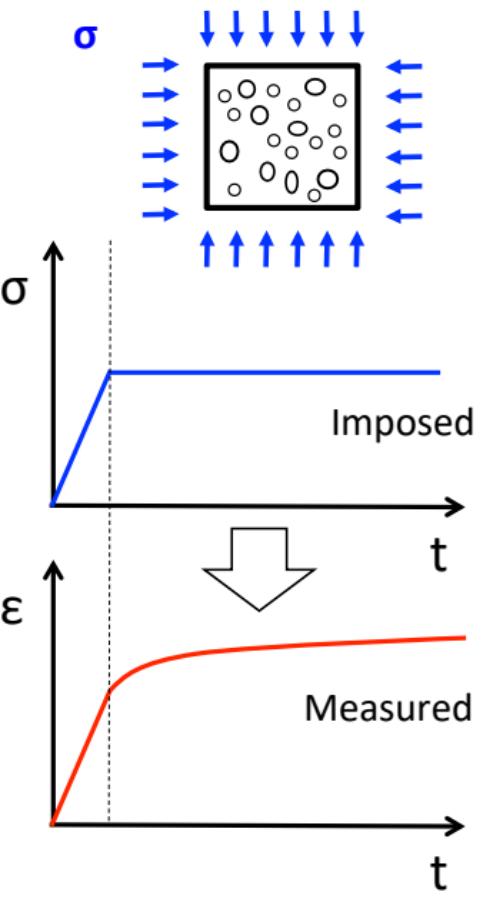




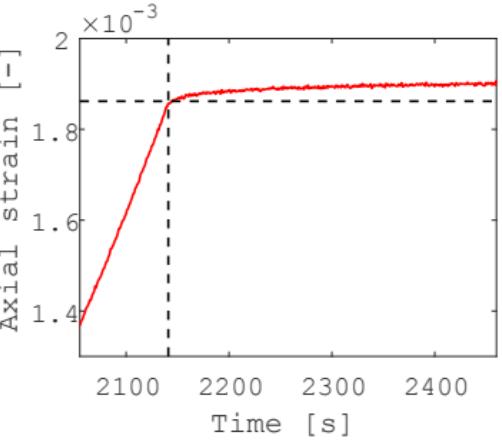
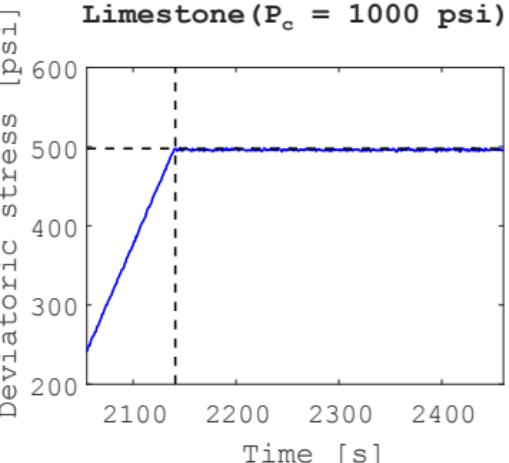
Strain rate hardening: The faster the loading, the stiffer the material



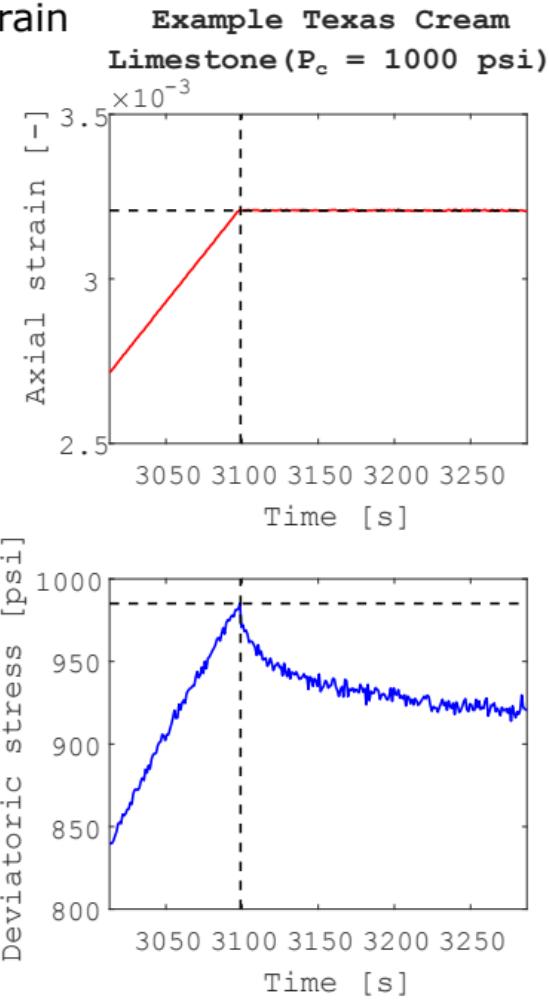
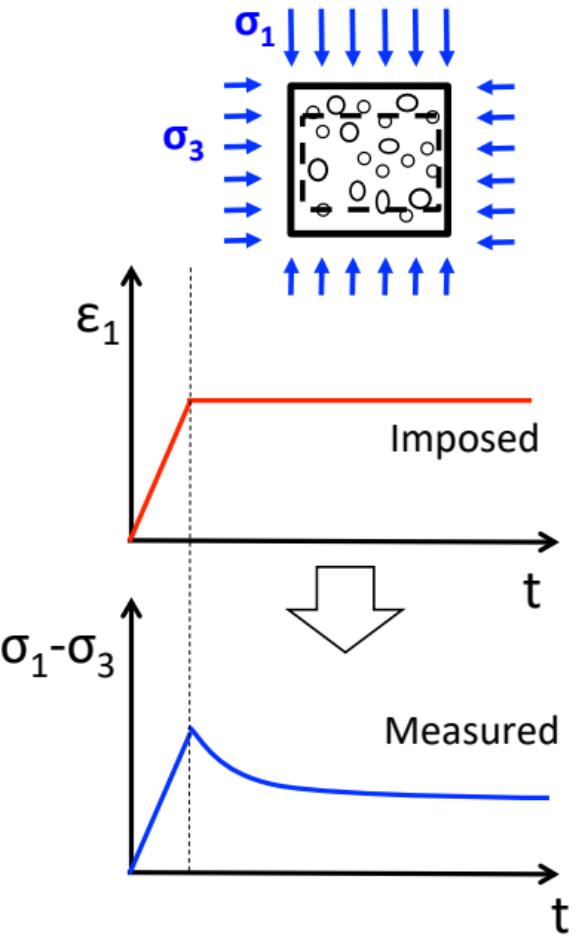
## Creep strain at constant stress

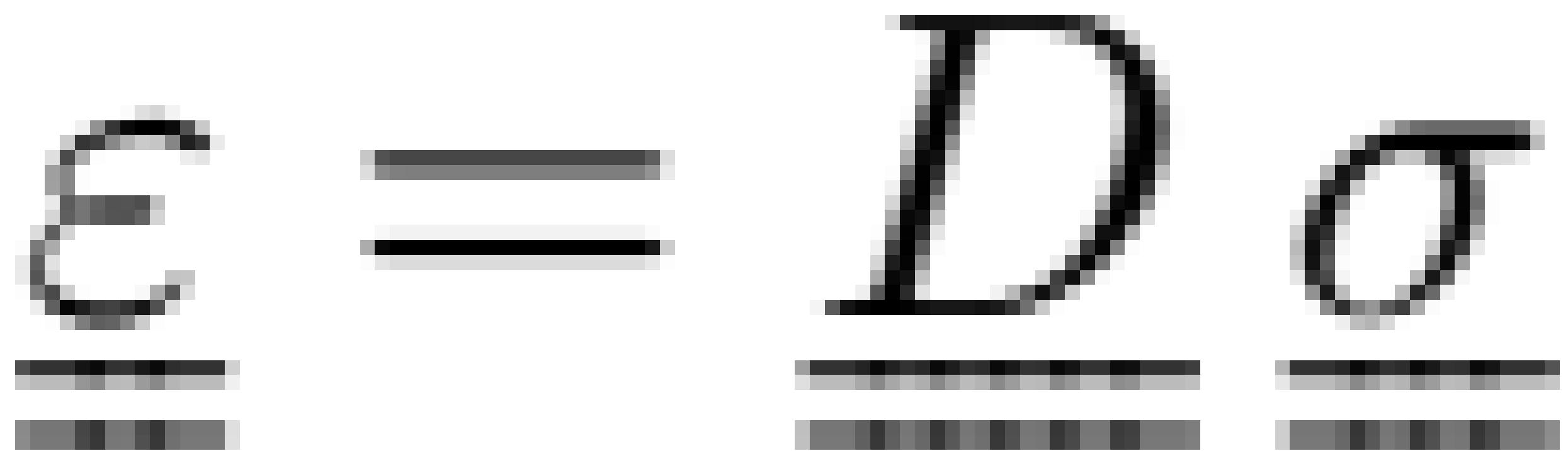


Example Texas Cream Limestone ( $P_c = 1000$  psi)

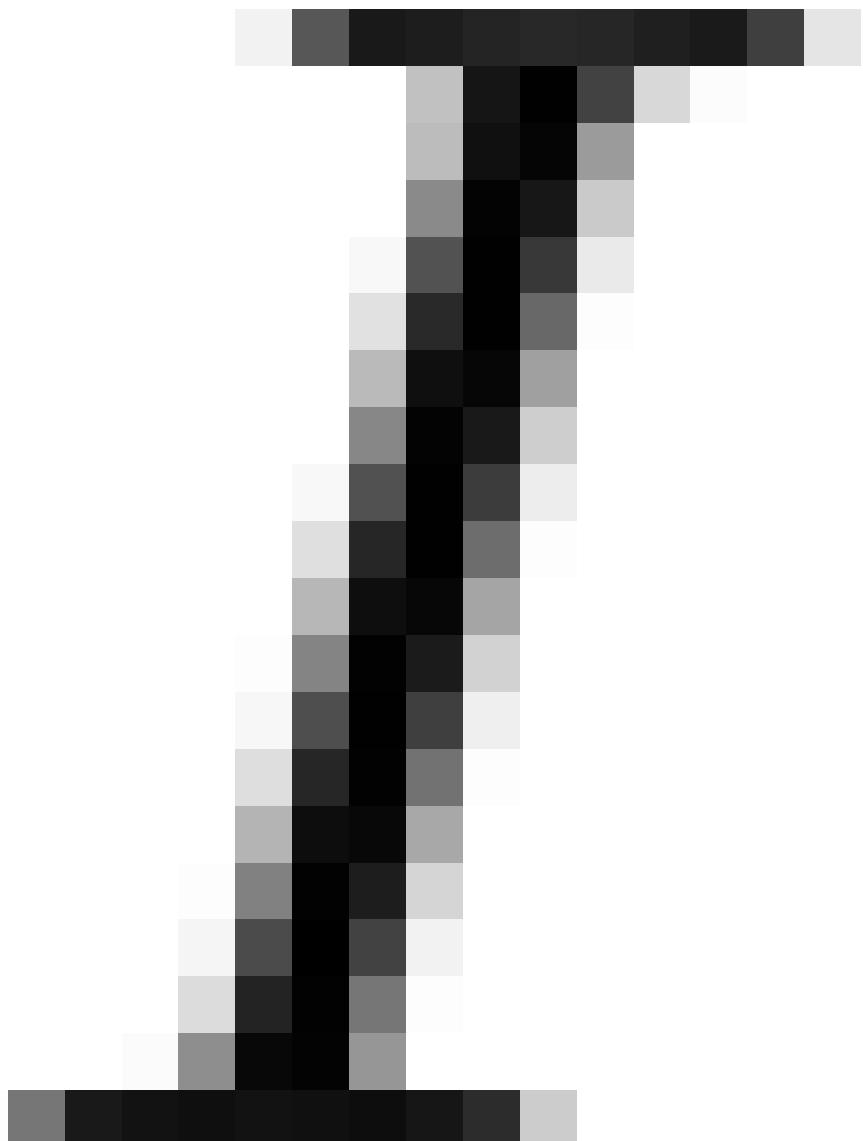


# Stress relaxation at constant strain









$\alpha$

=

1

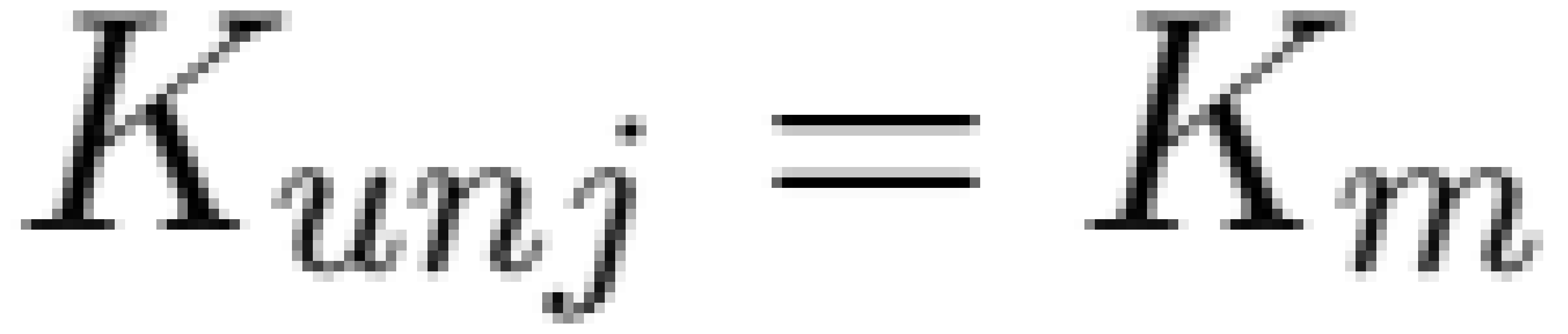
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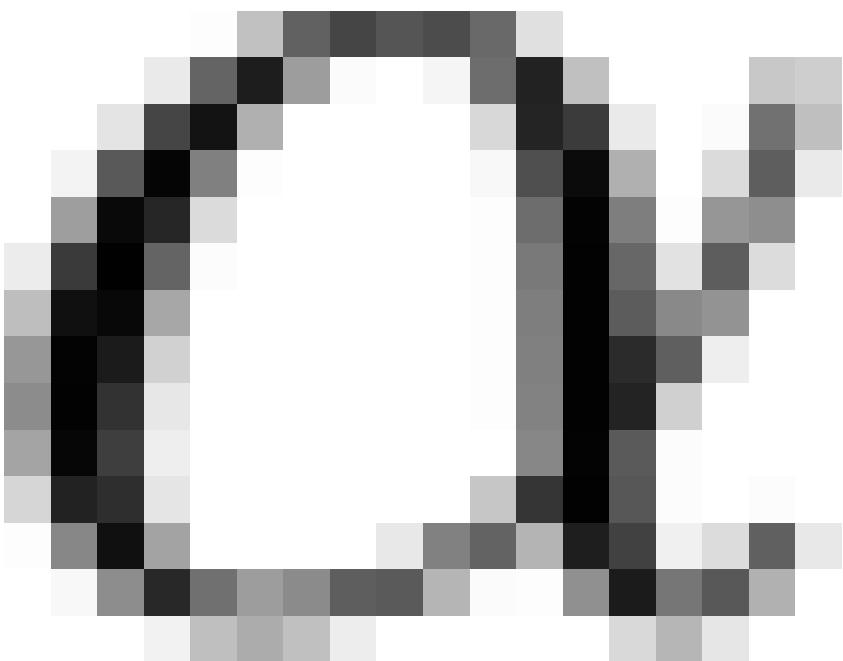
$K_{drained}$

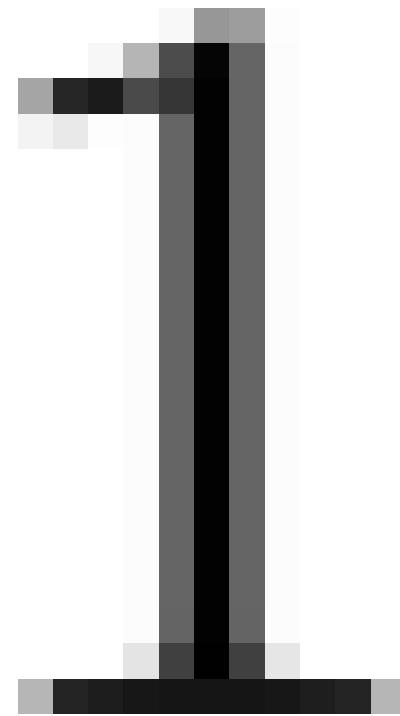
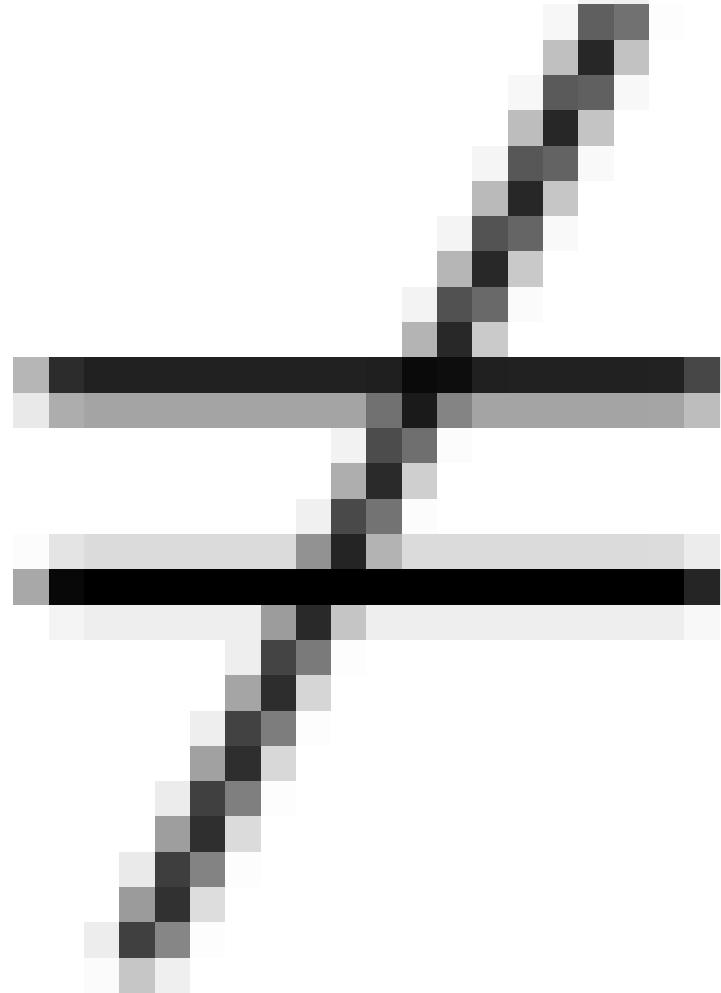
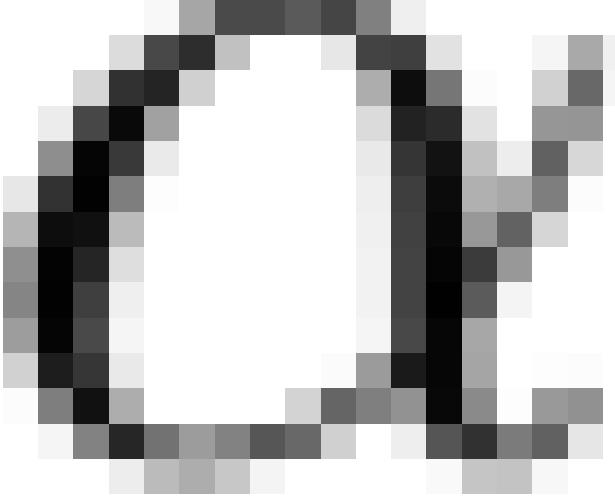
$K_{unq}$ .

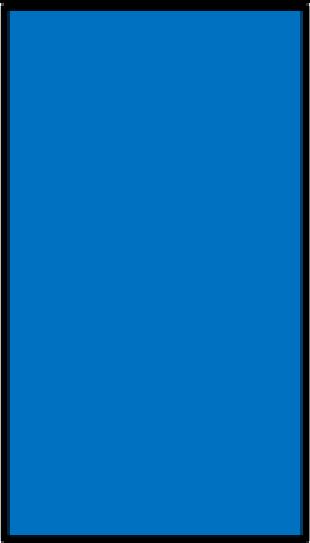
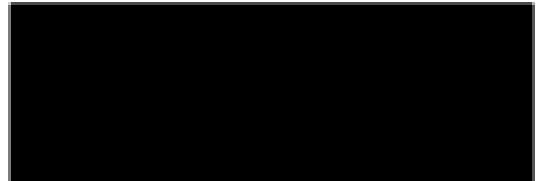
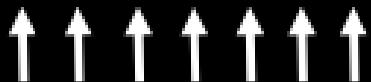
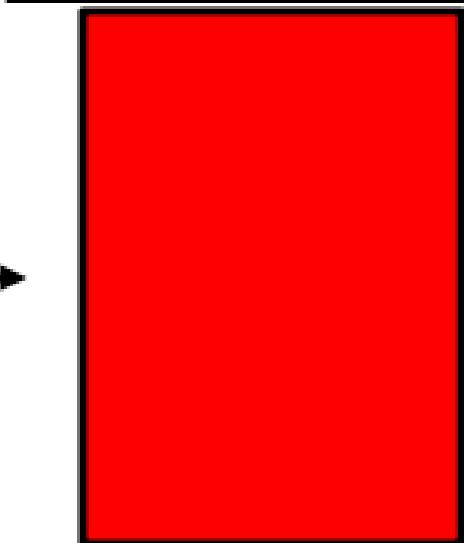
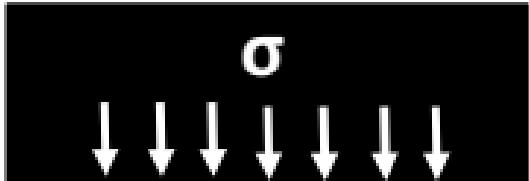


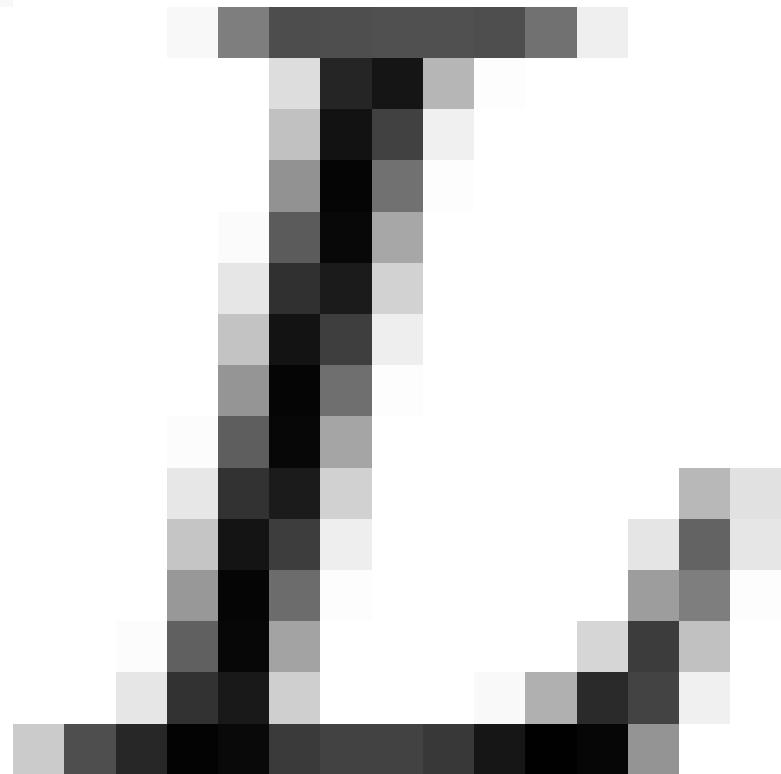
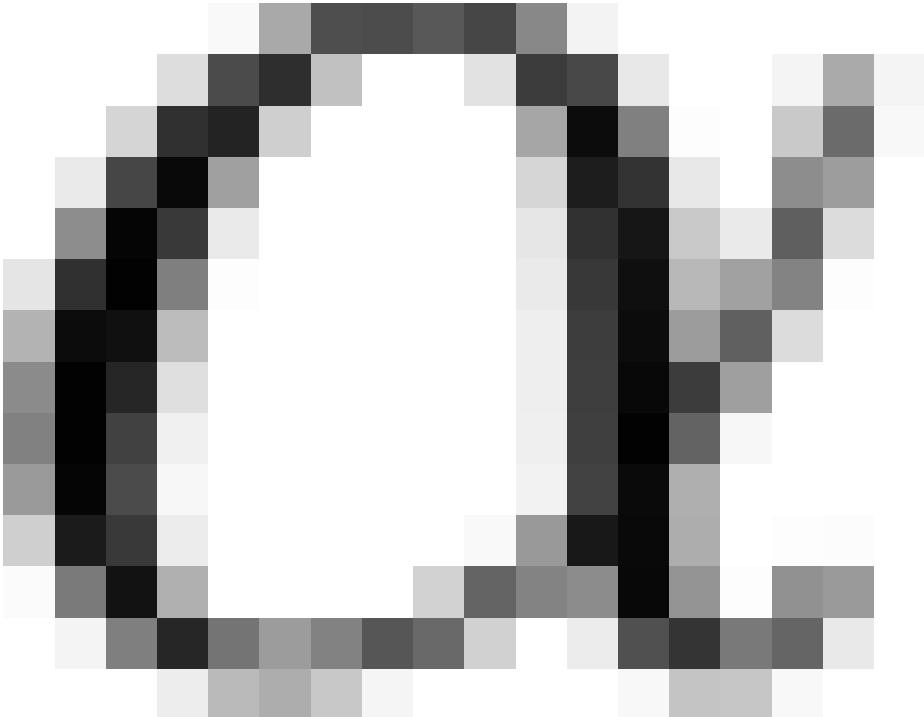


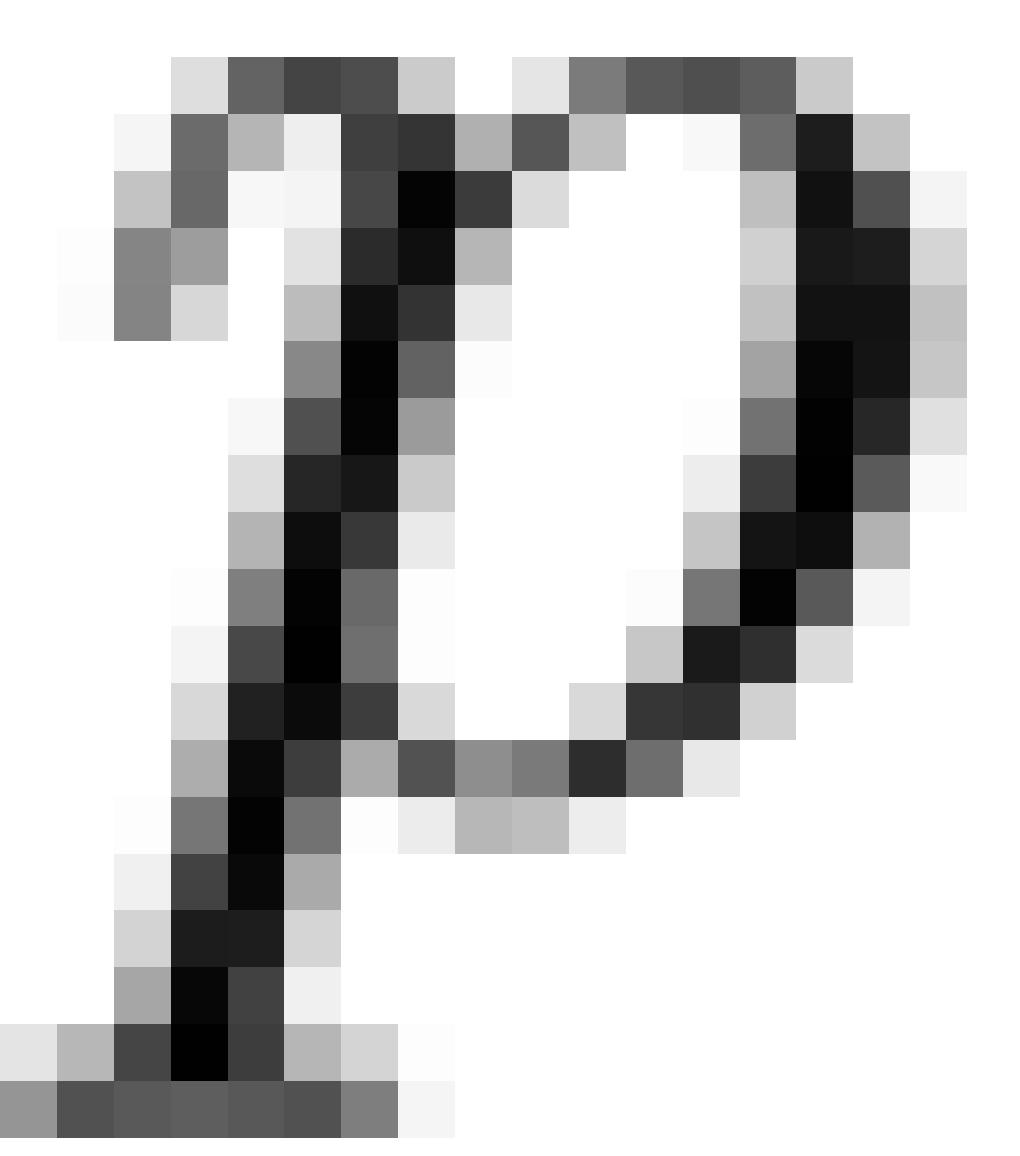




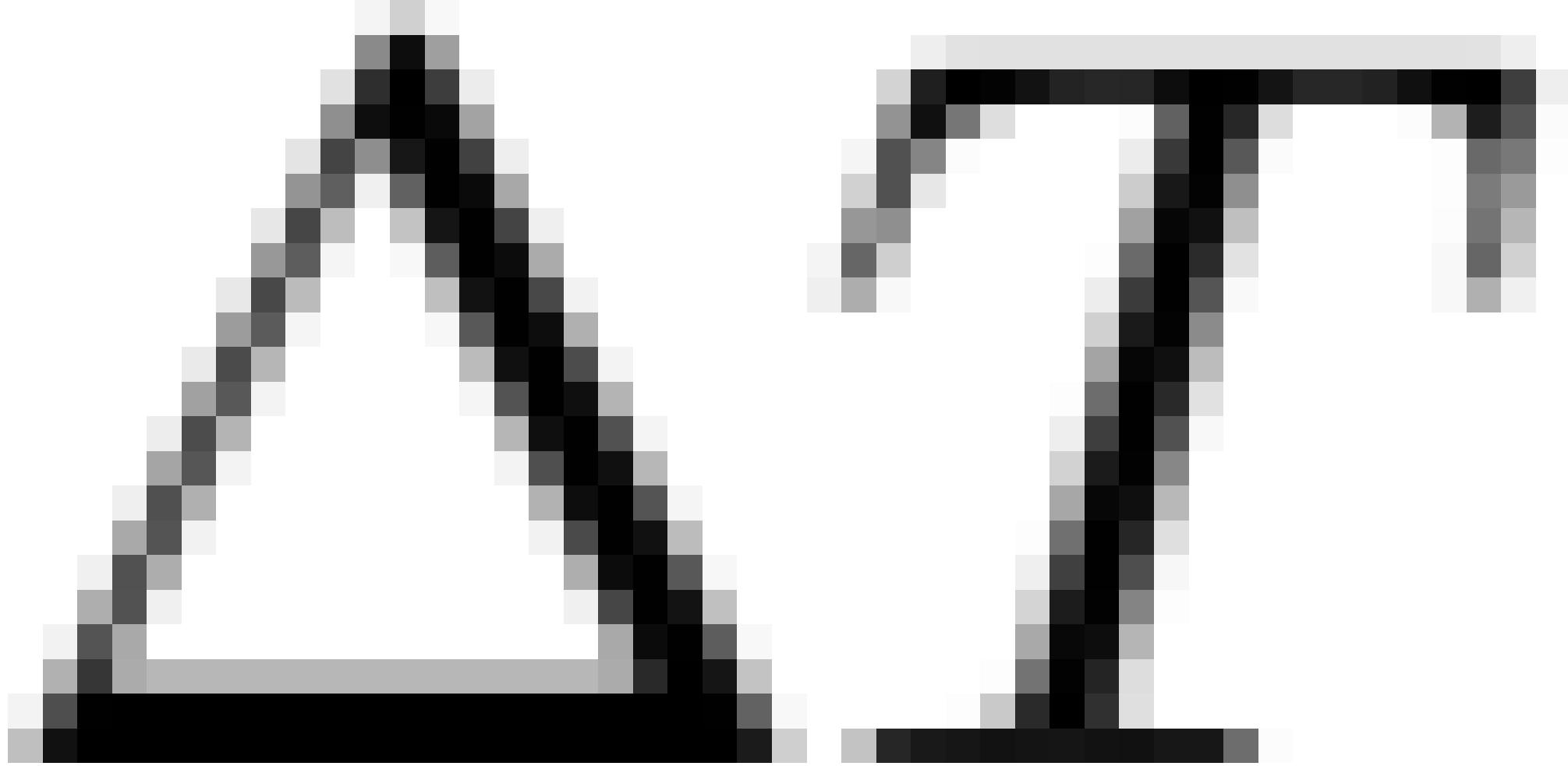


 $\Delta T$ 





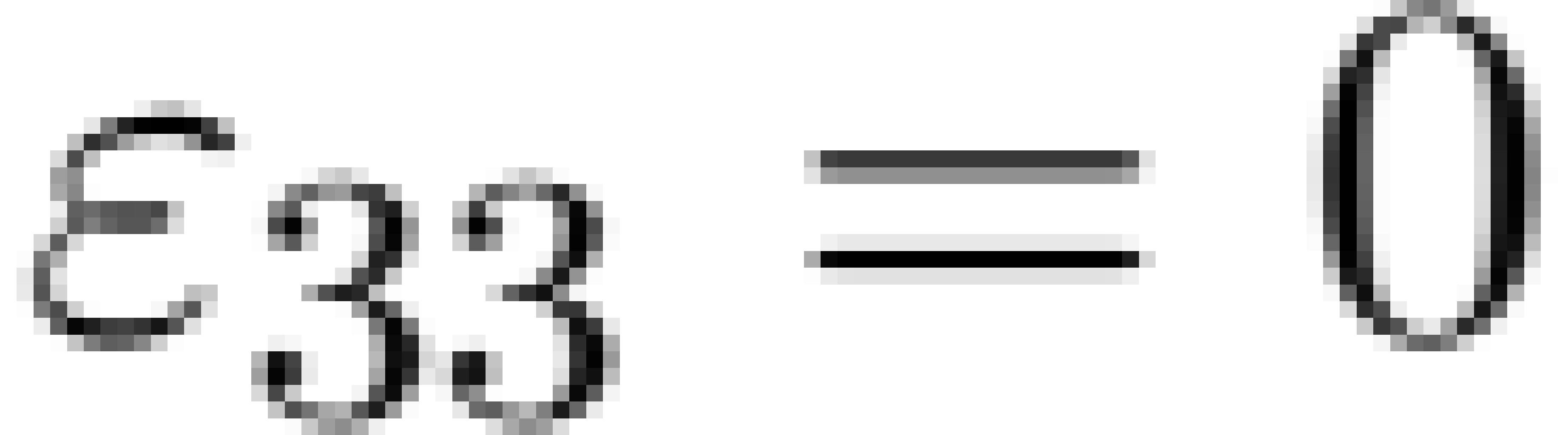
$$\alpha_L = \frac{1}{L} \frac{dL}{dT}$$

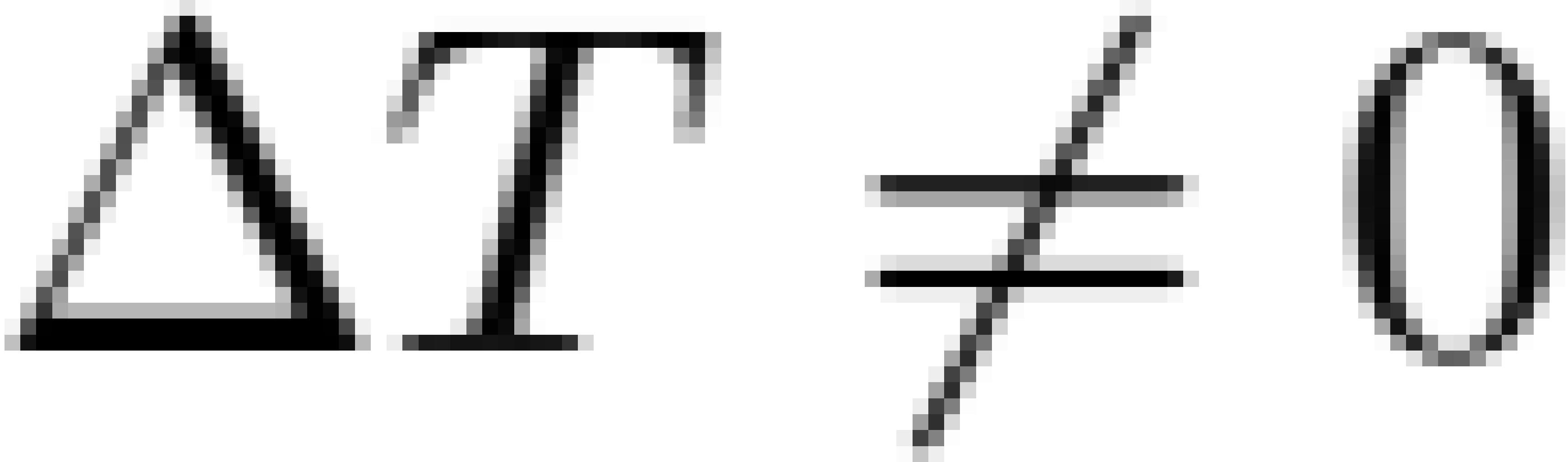


$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right.$$









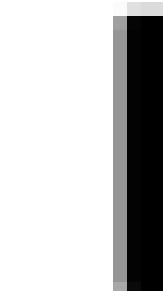
$$\begin{aligned} \sigma_{11}^0 &= (\lambda + 2\mu) \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \\ \sigma_{33} &= \lambda \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \end{aligned}$$

$\sigma_{33}$

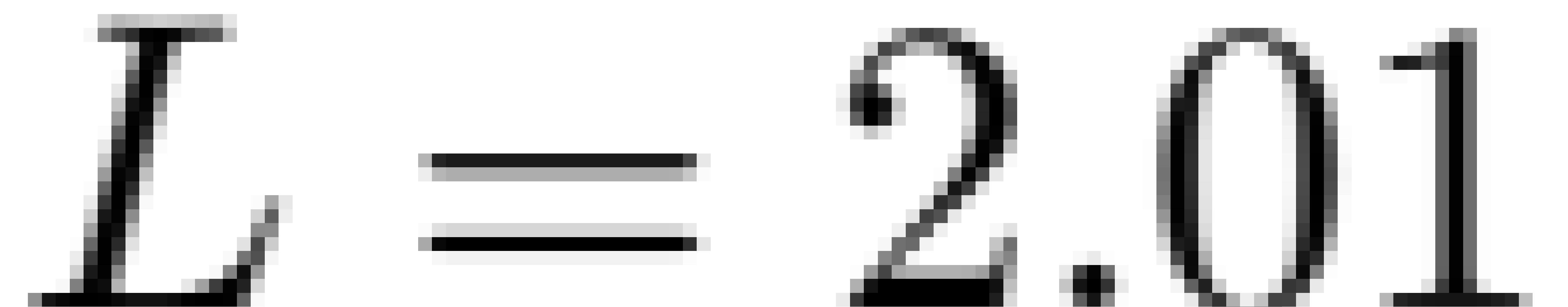
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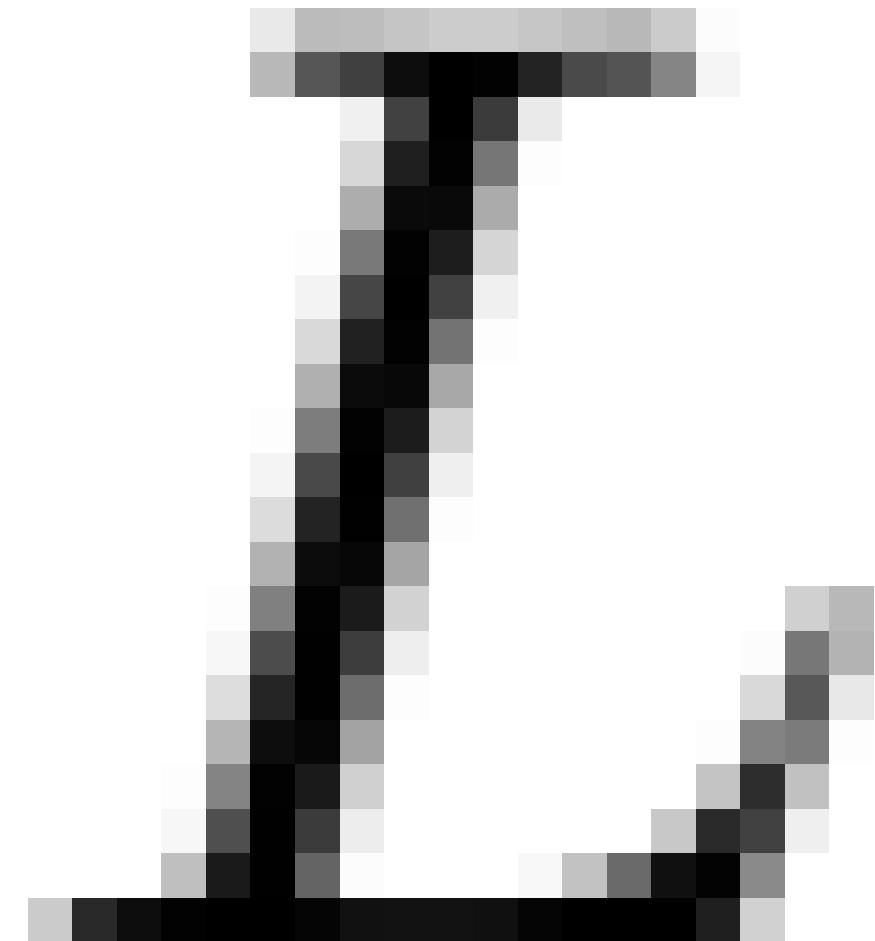
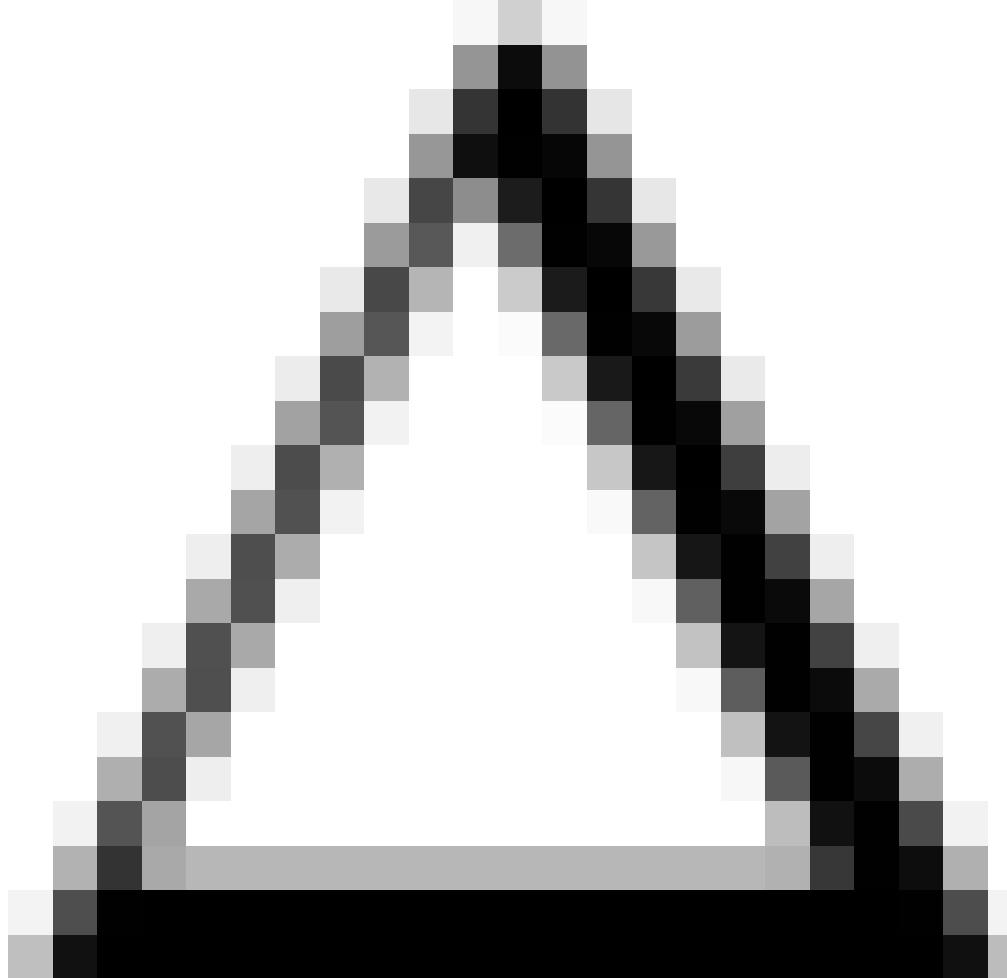
$$\frac{6}{3} \mu K$$

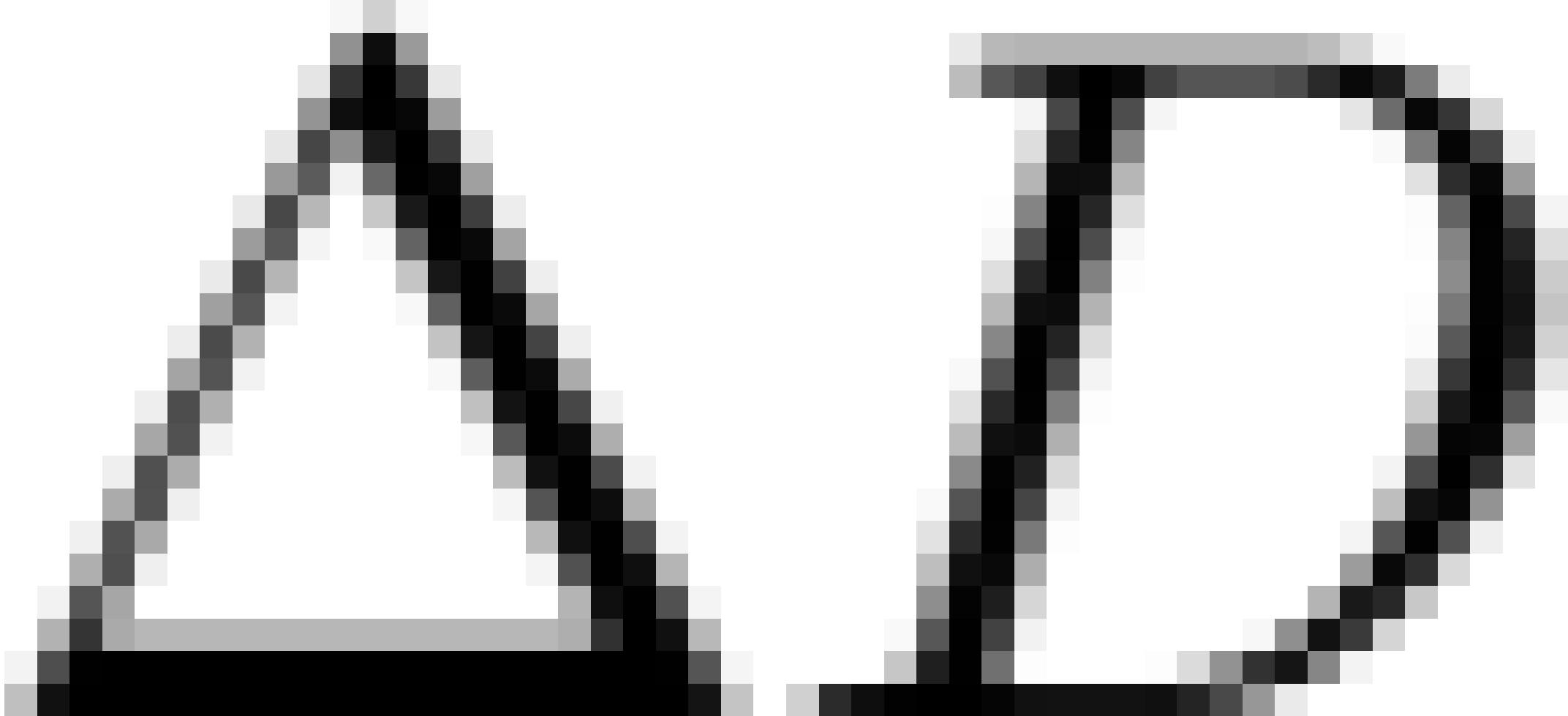
$K_{\Delta T}$

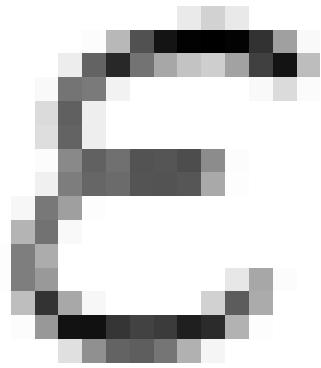
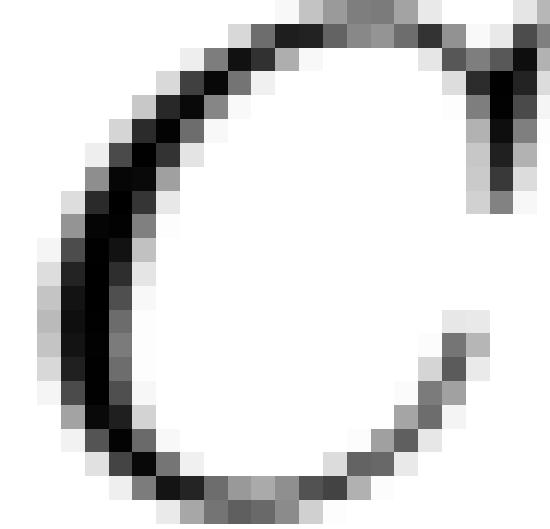
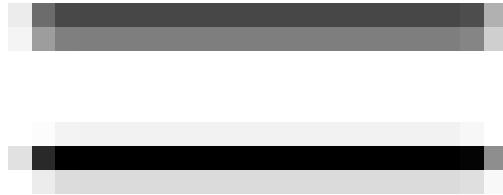
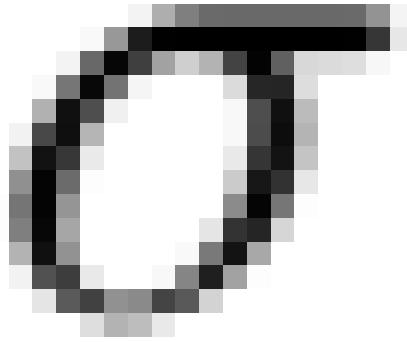


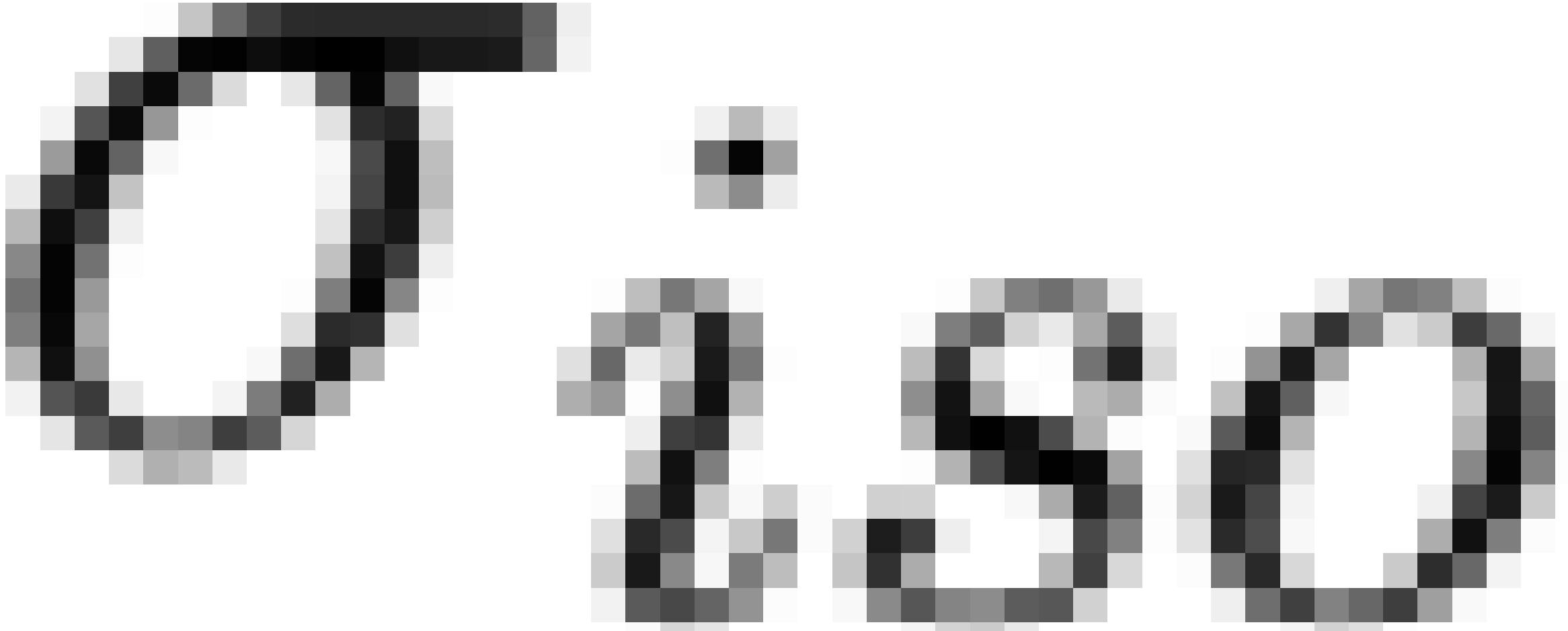


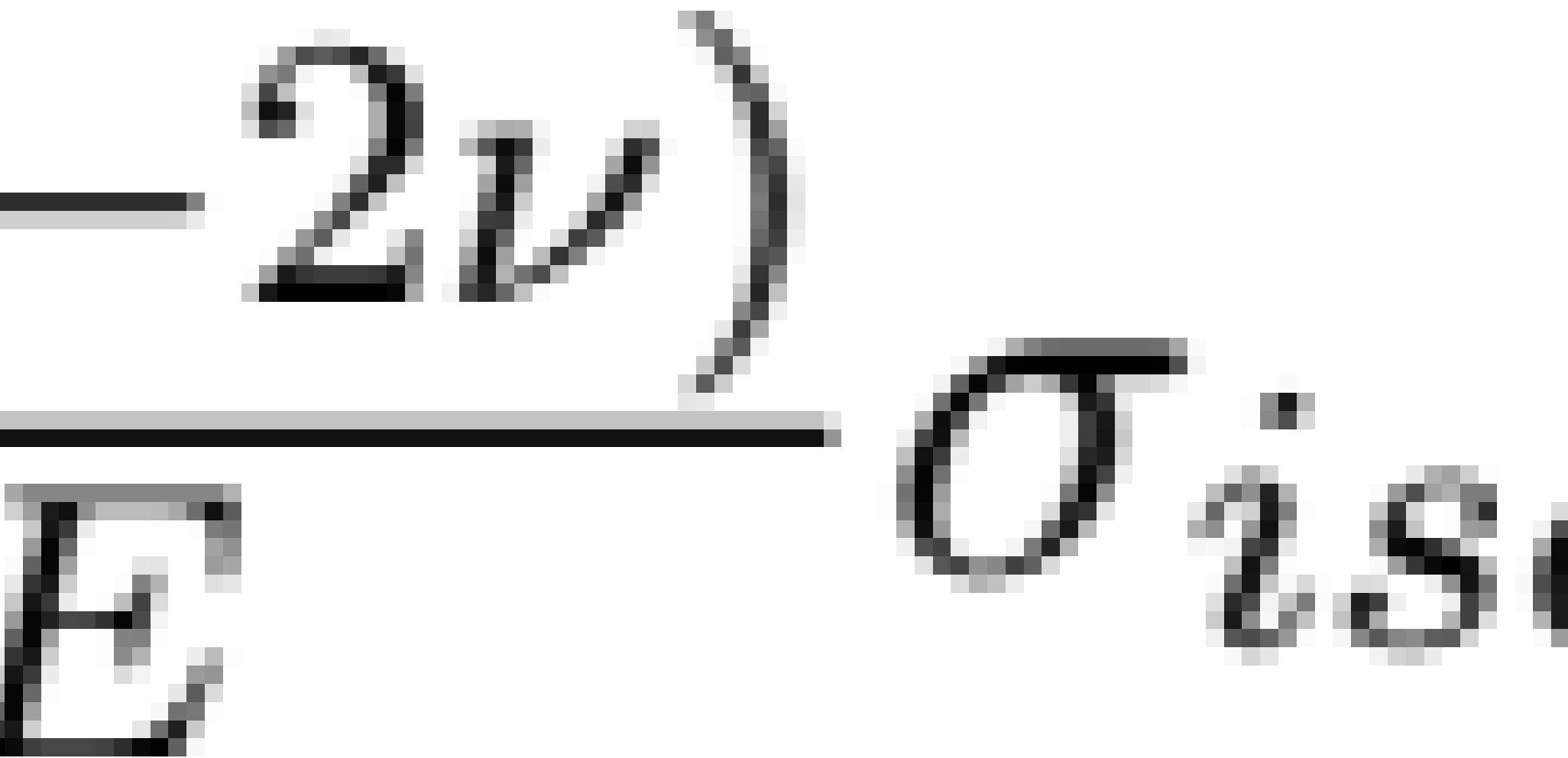




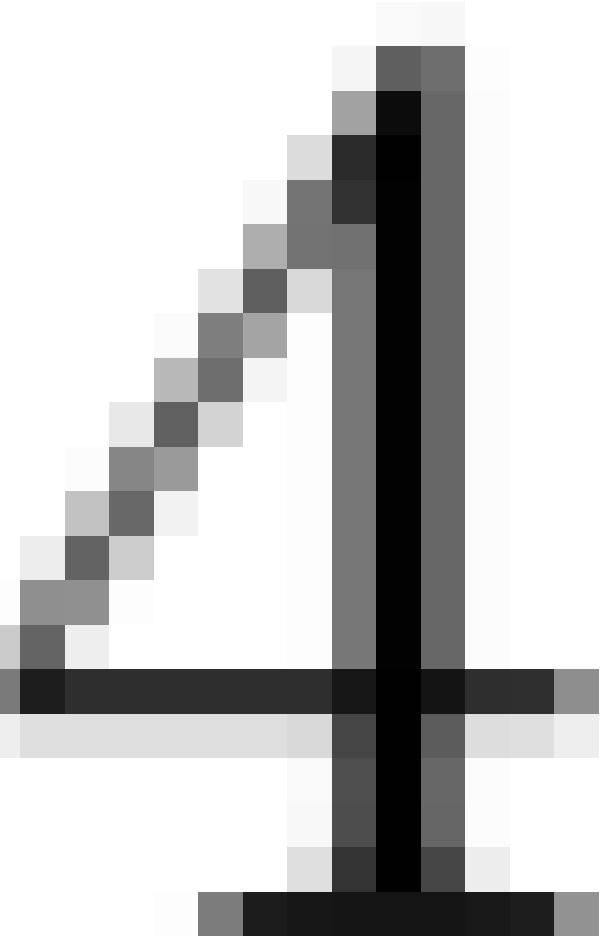
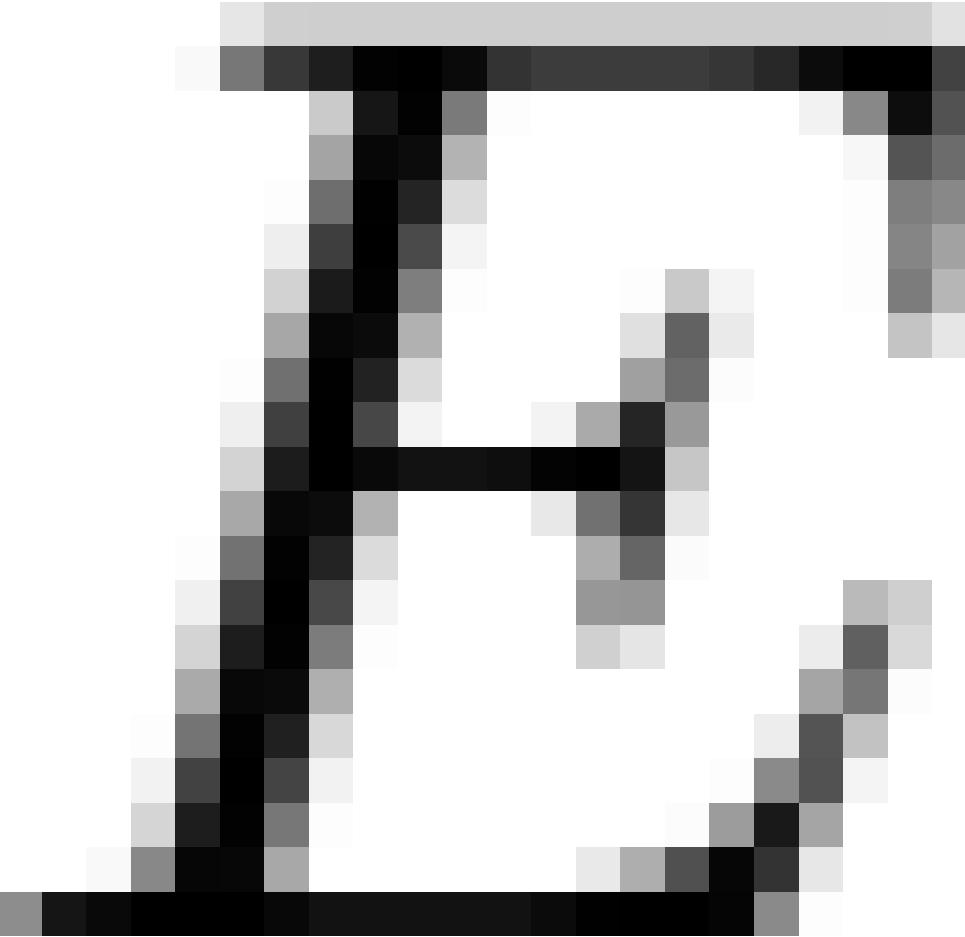












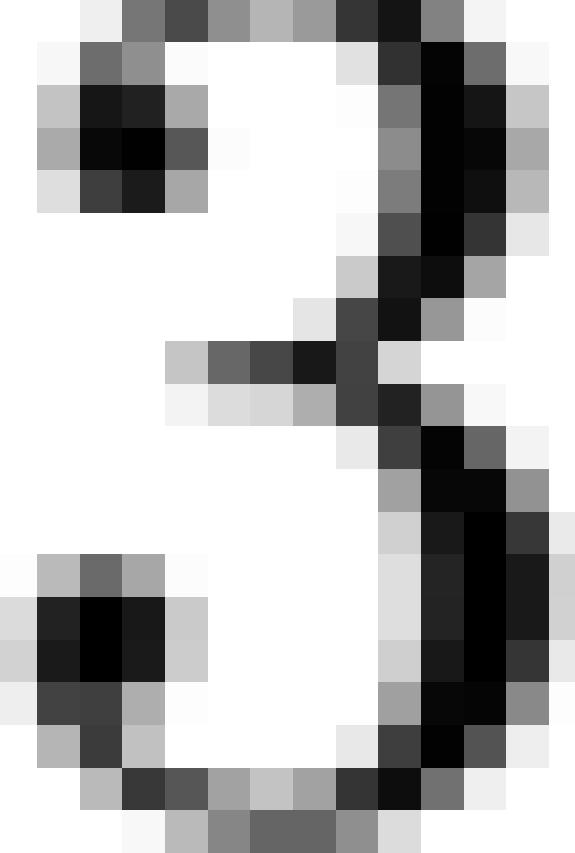
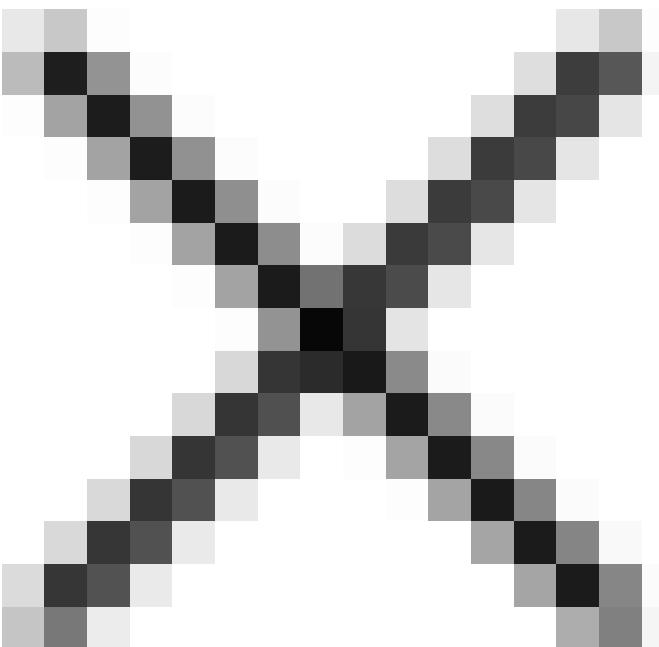
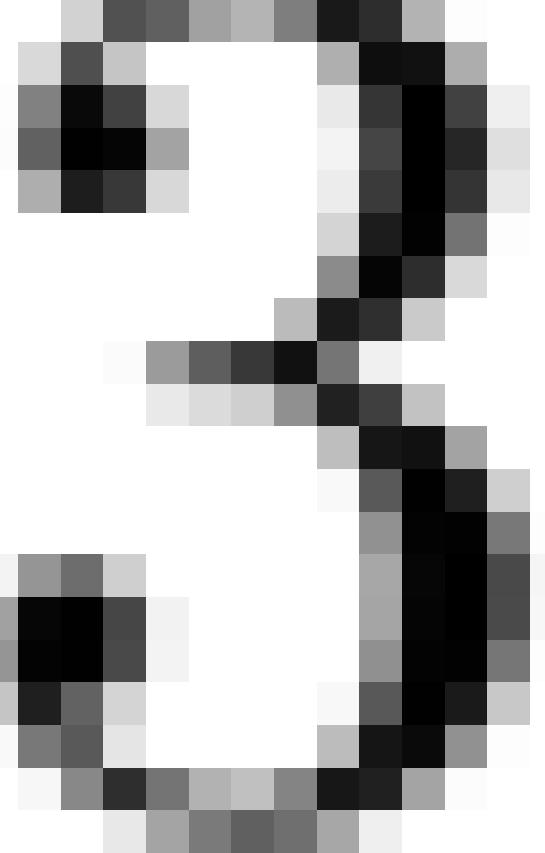


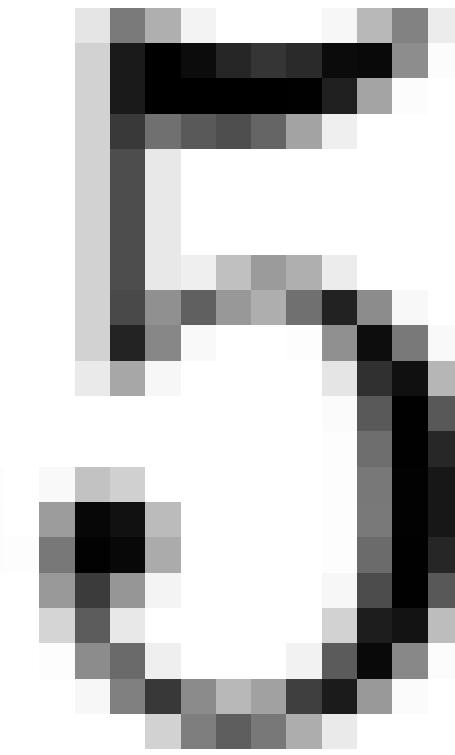
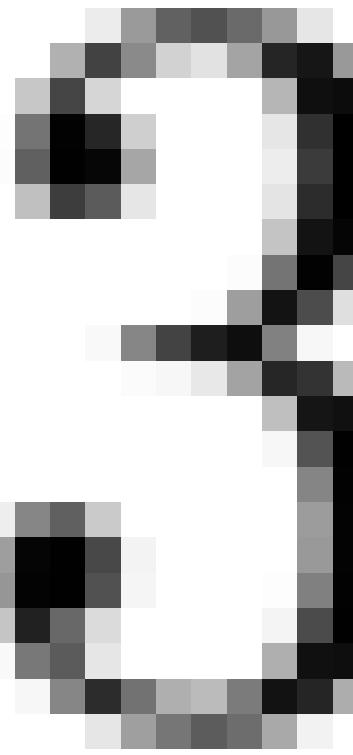
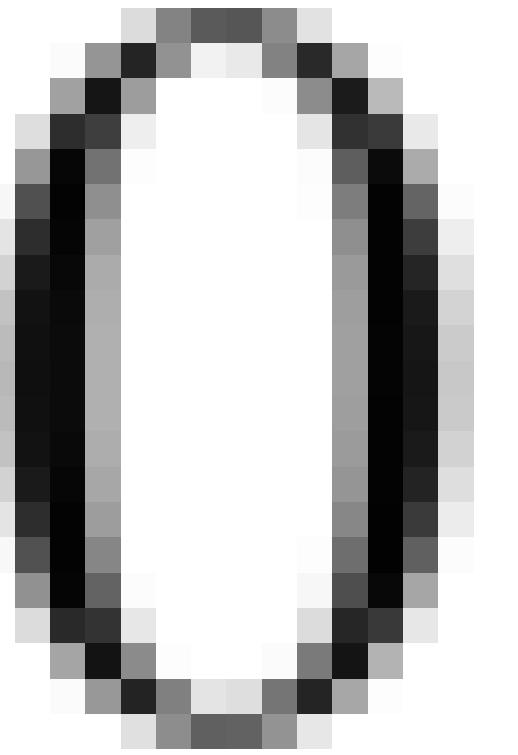
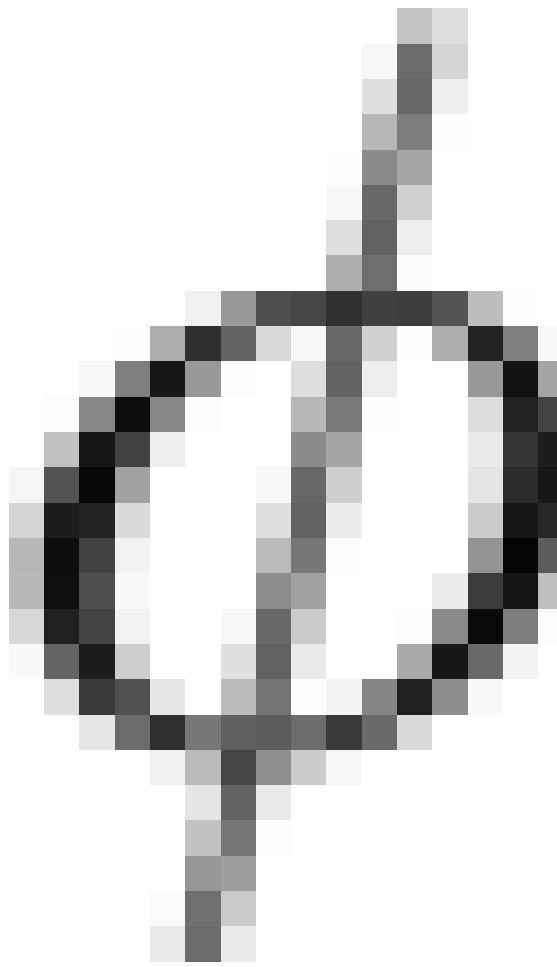


$\sigma_{11}$ 

$$= \frac{1}{2}$$

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\epsilon_{11}$$







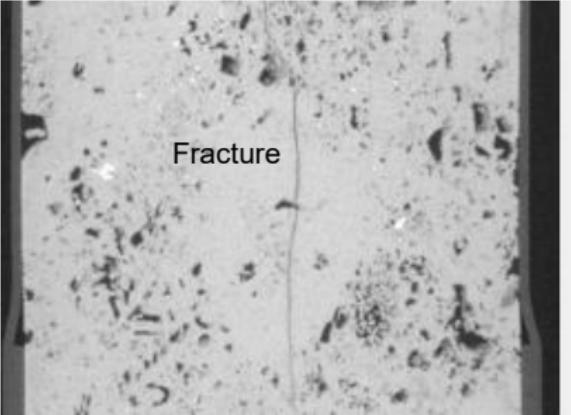
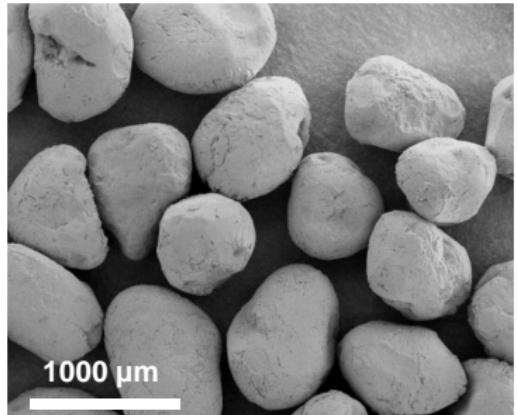
(a) Uncemented sand



(b) Cemented sandstone

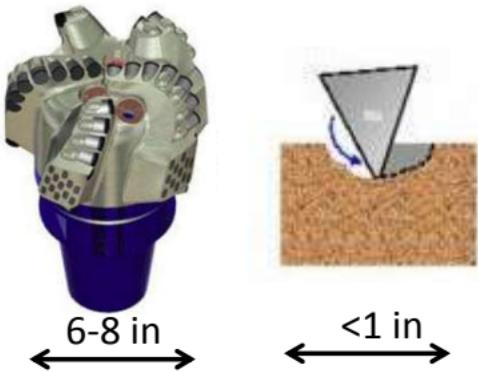


(c) Vuggy carbonate

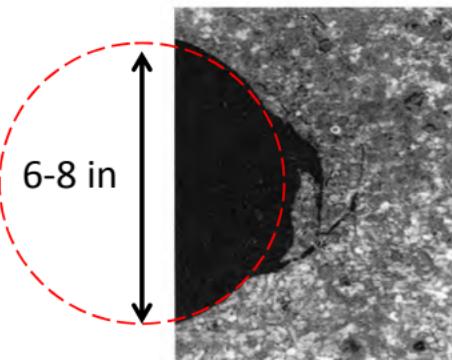


Fracture

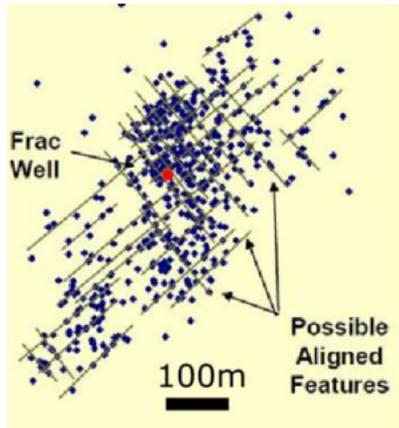
## Rock cutting at the drill bit



## Wellbore breakout



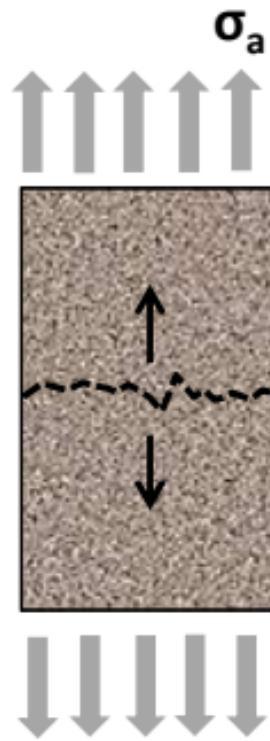
## Shale hydraulic fracture



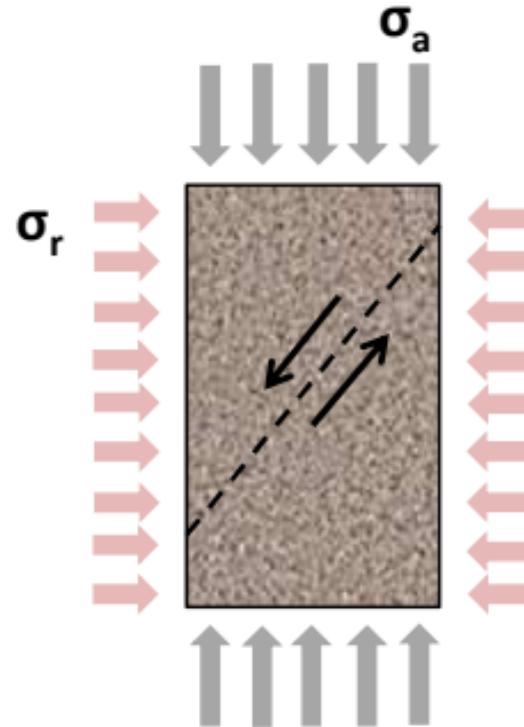
## Reservoir depletion



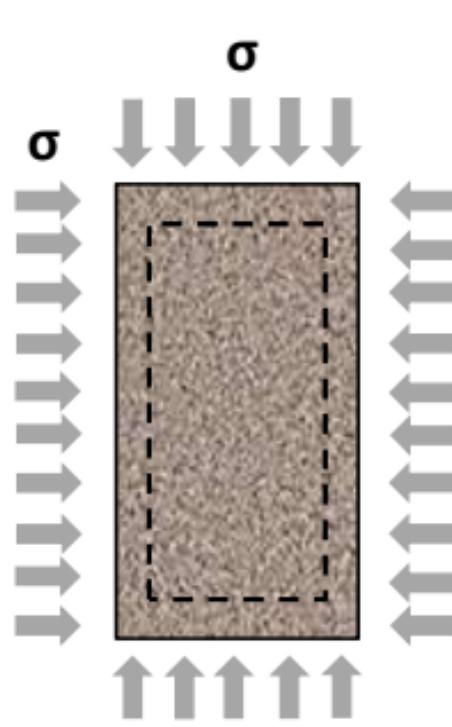
Images: Schlumberger/Terratek, Zoback 2013, Warpinski 2008, doe.gov



Tension  
(bond breakage)  
Ex: drilling-induced tens. fracs



Shear  
(friction failure)  
Ex: fault, breakout

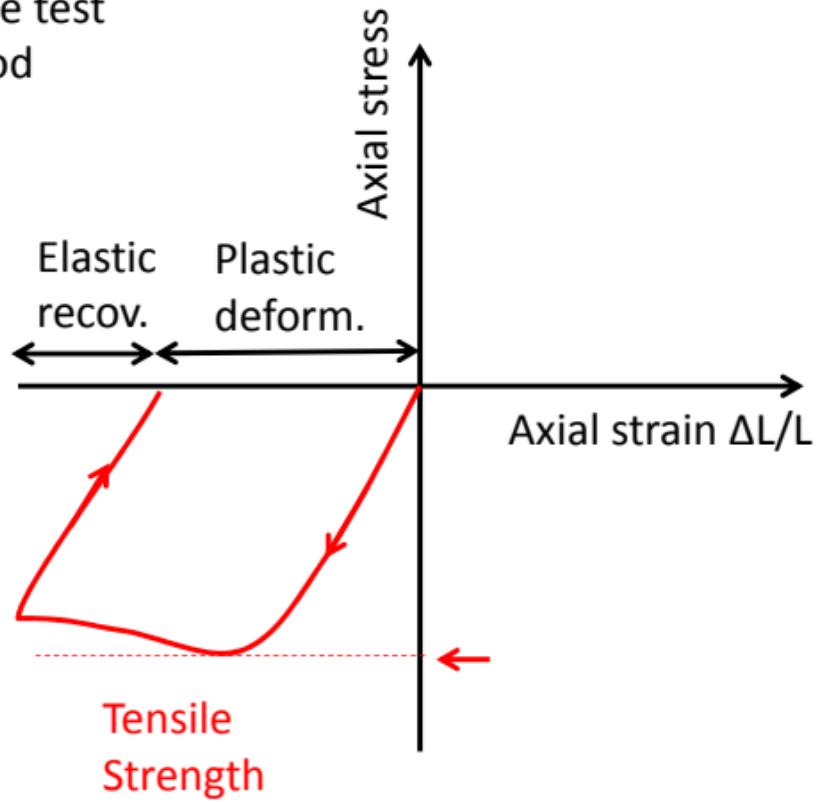


Compression  
(pore collapse)  
Ex: reservoir compaction

T



Typical tensile test  
on a metal rod



Tensile test on a rock rod

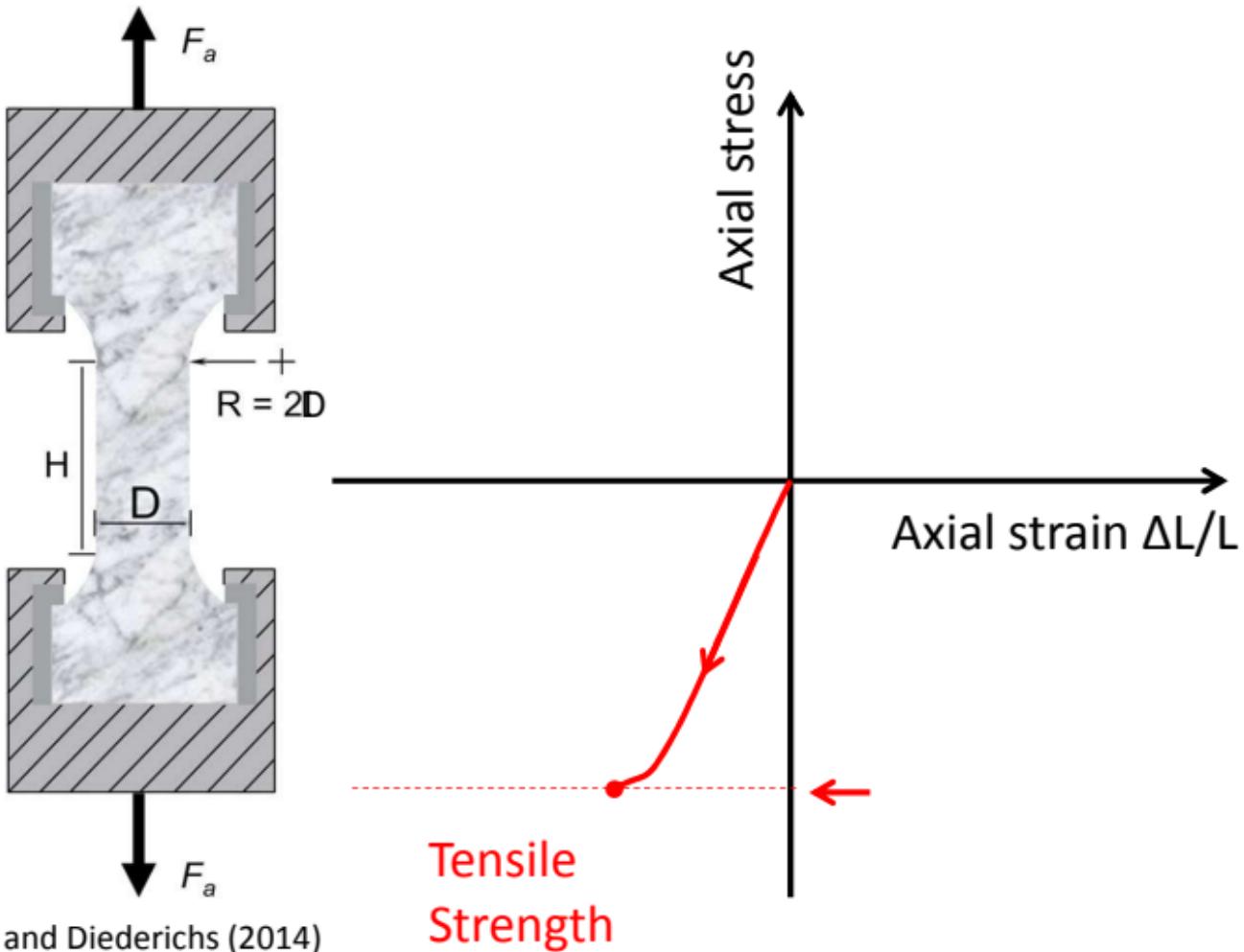


Figure direct tension: Perras and Diederichs (2014)

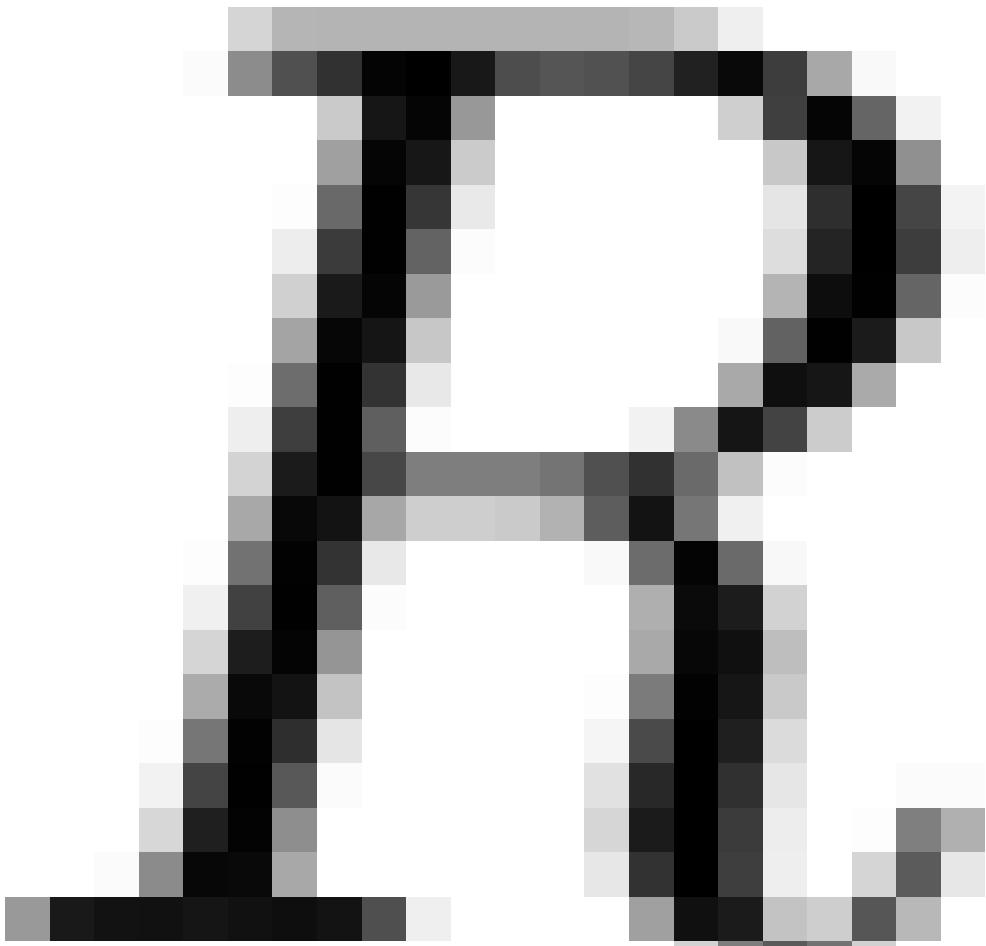
IS

—  
—

πLR

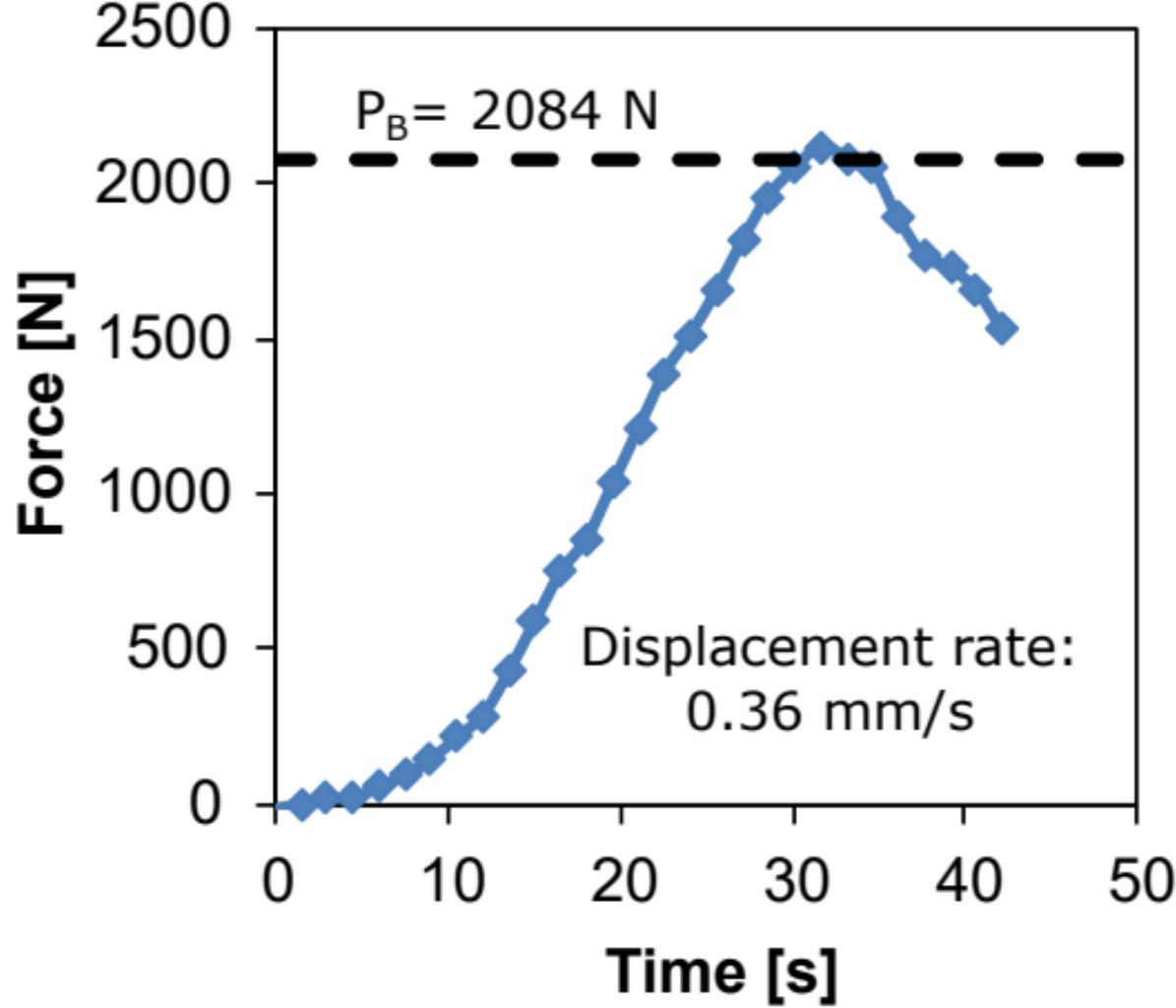
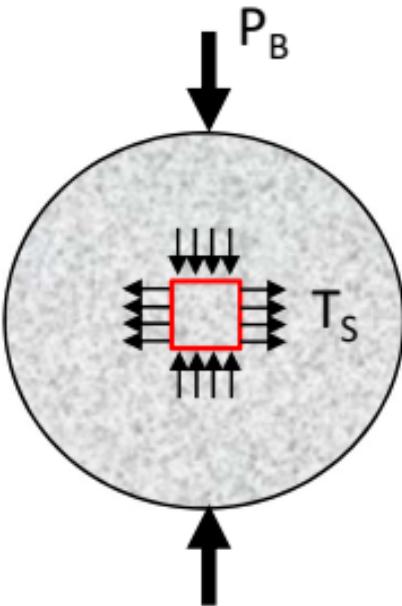
PB





Cylindrical sample:

- radius R
- length L

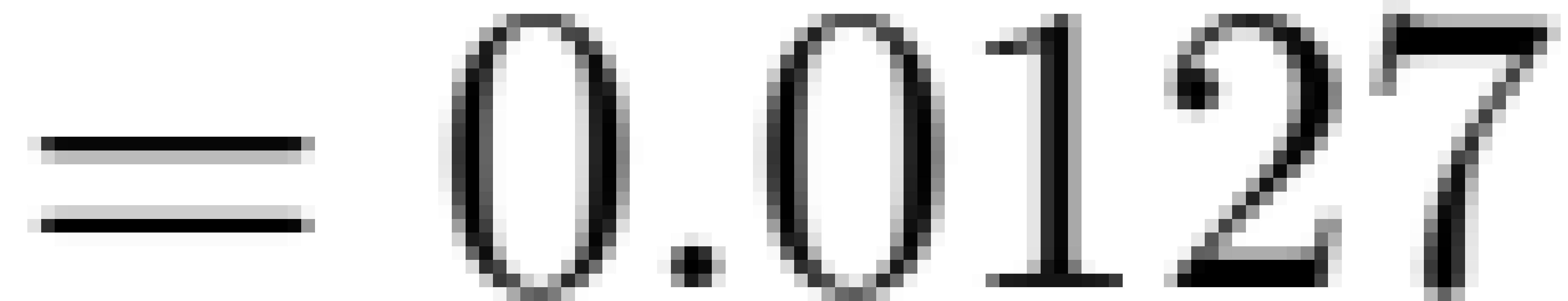


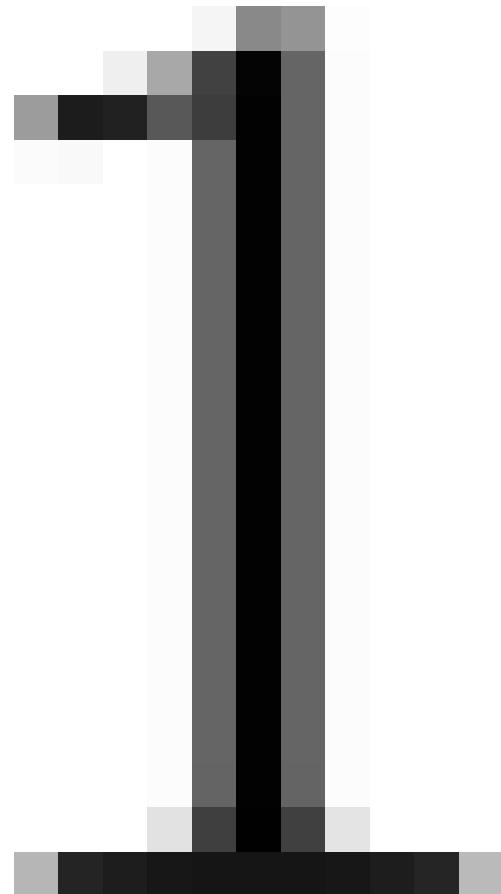
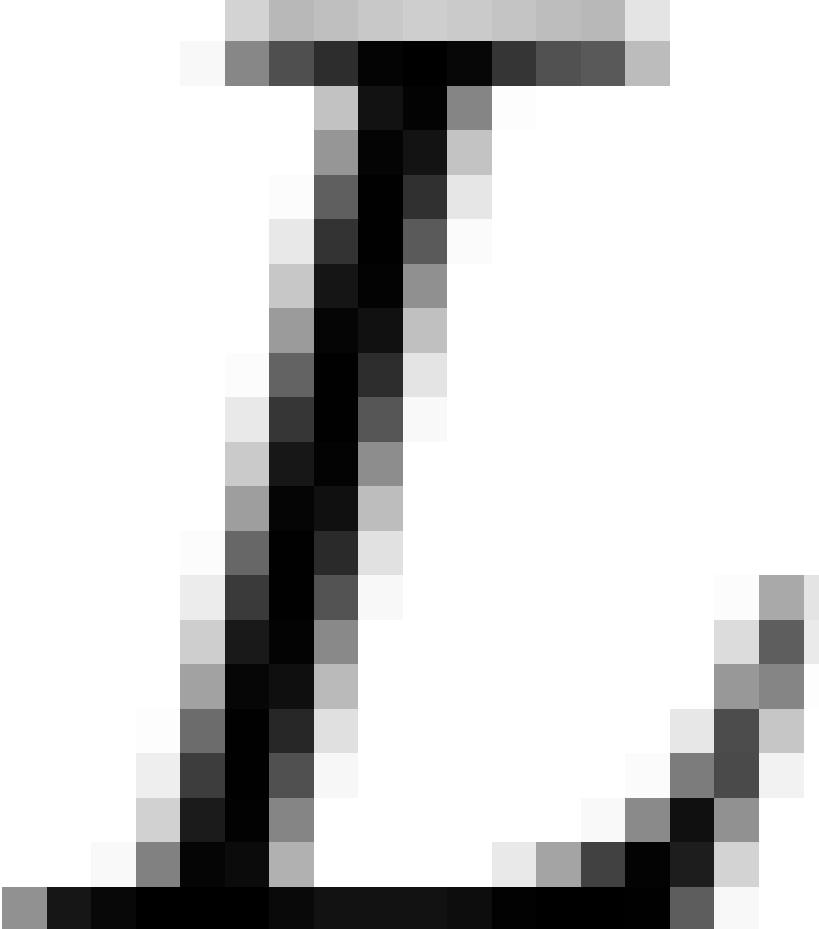
R

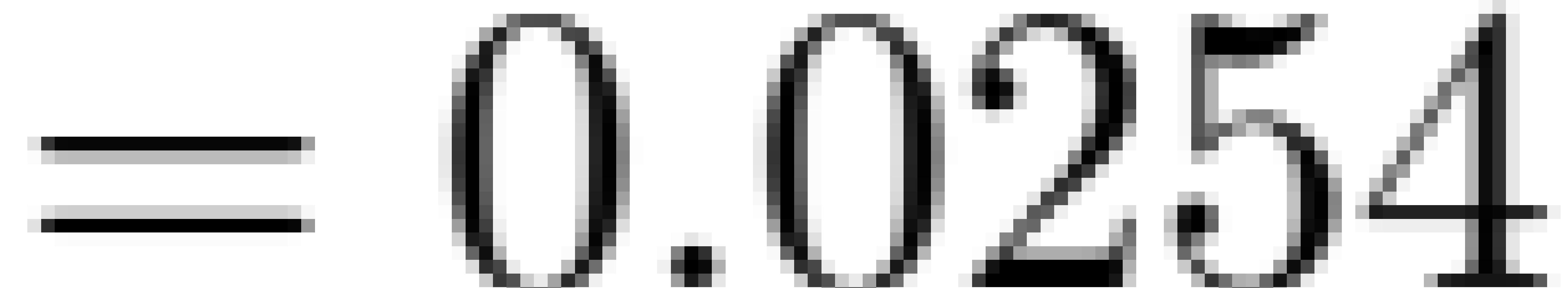
1  
2

1  
2

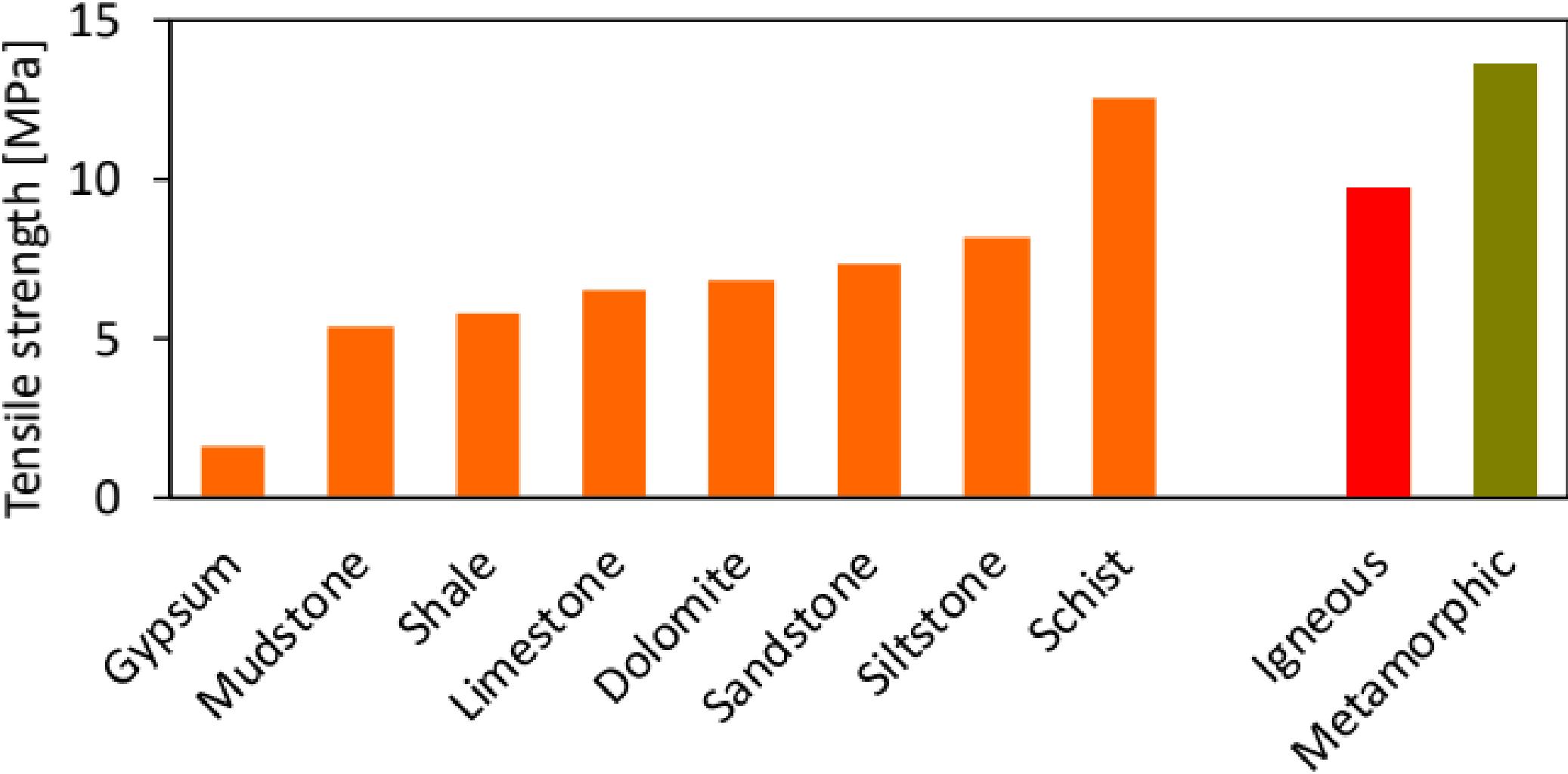
1  
2

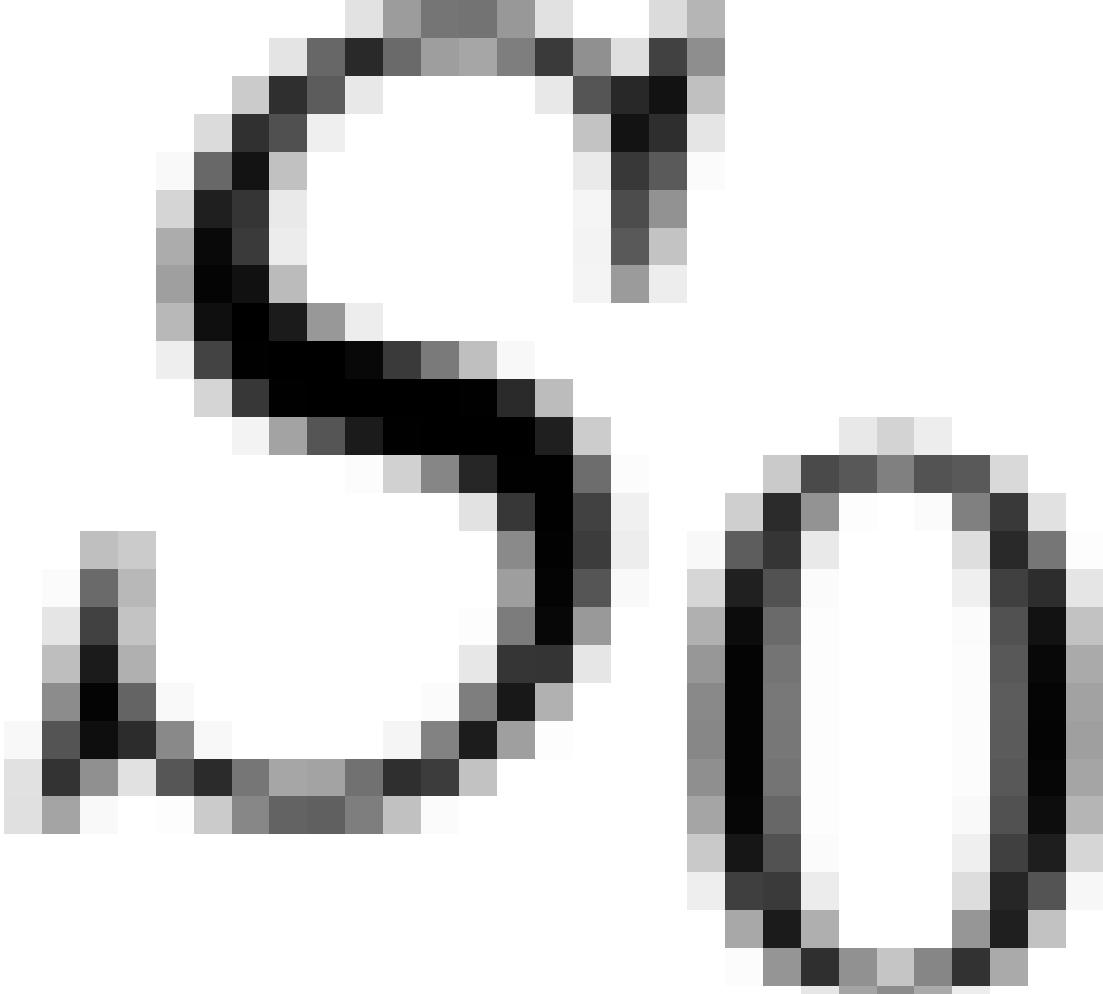


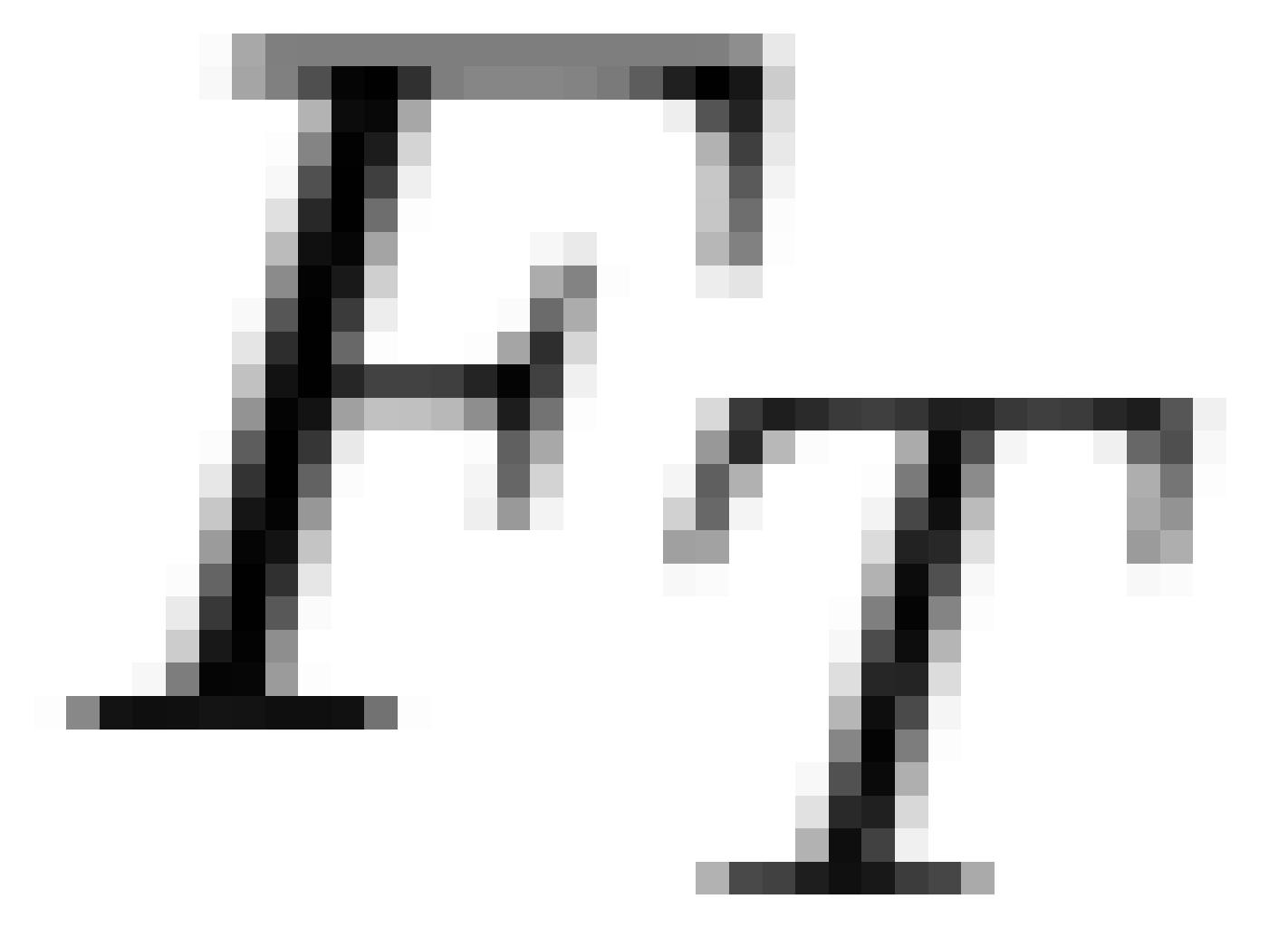




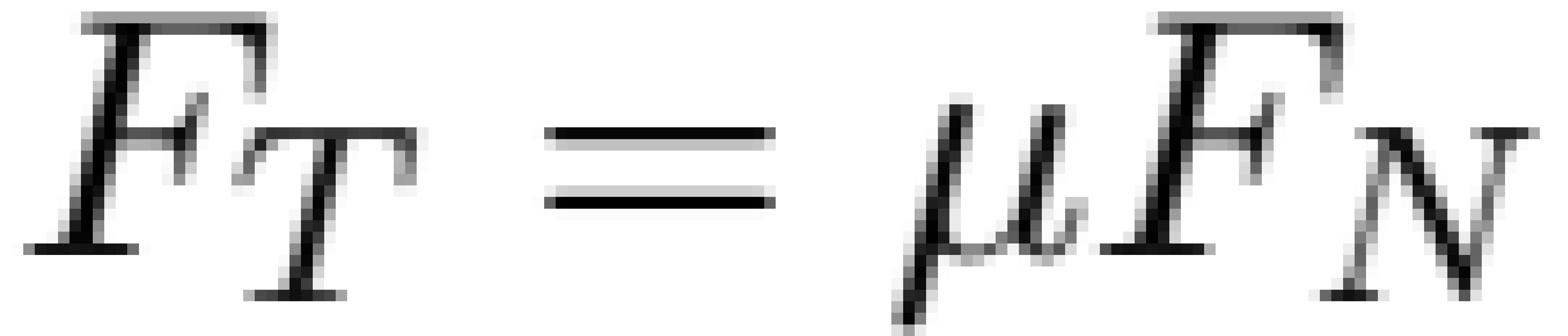
$$\frac{2084 \text{ N}}{\pi (0.0254 \text{ m})(0.0127 \text{ m})} = 2.06 \times 10^6 \text{ Pa} = 2.06 \text{ MPa}$$

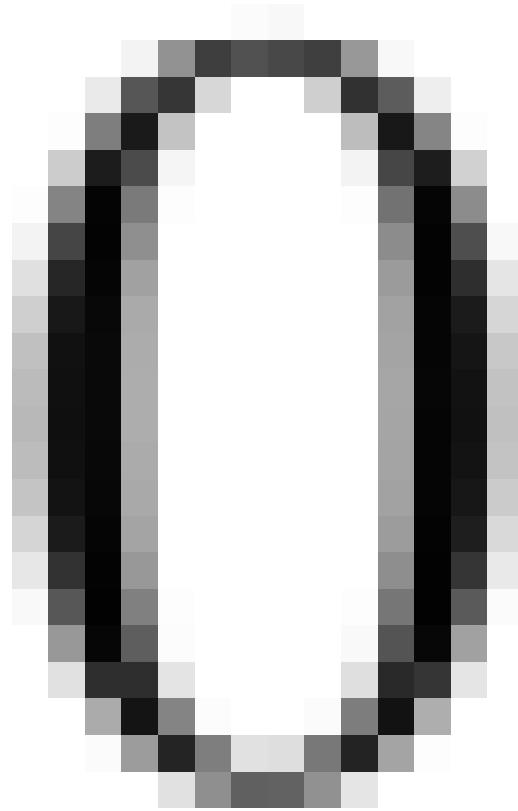


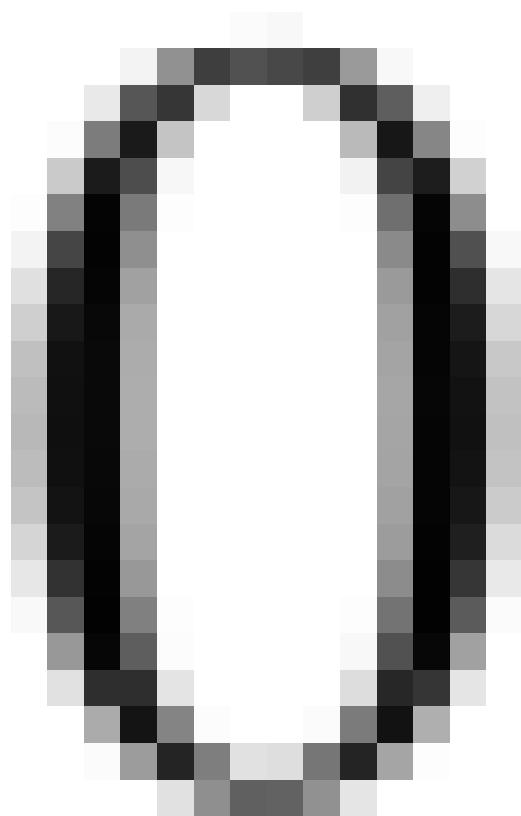
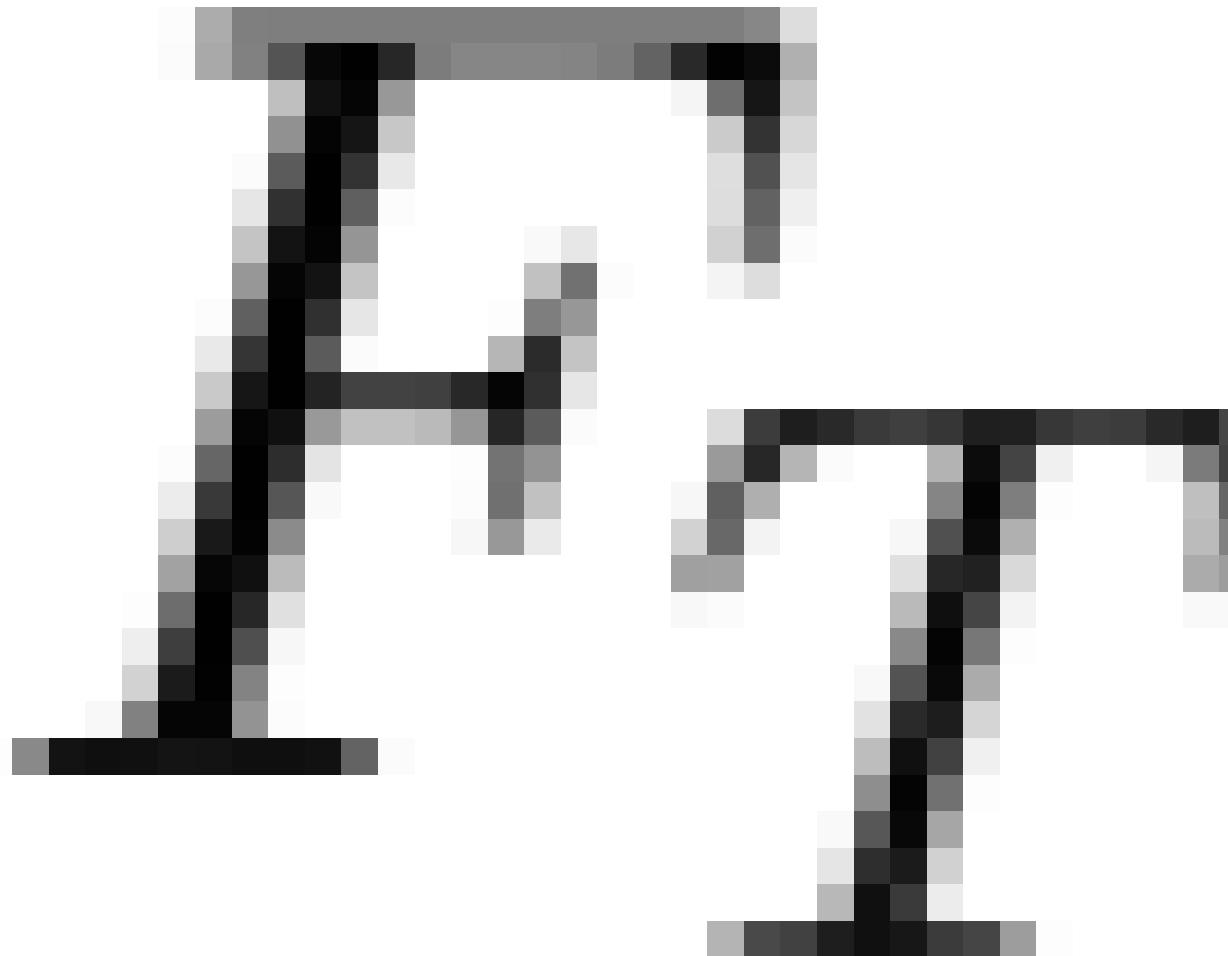


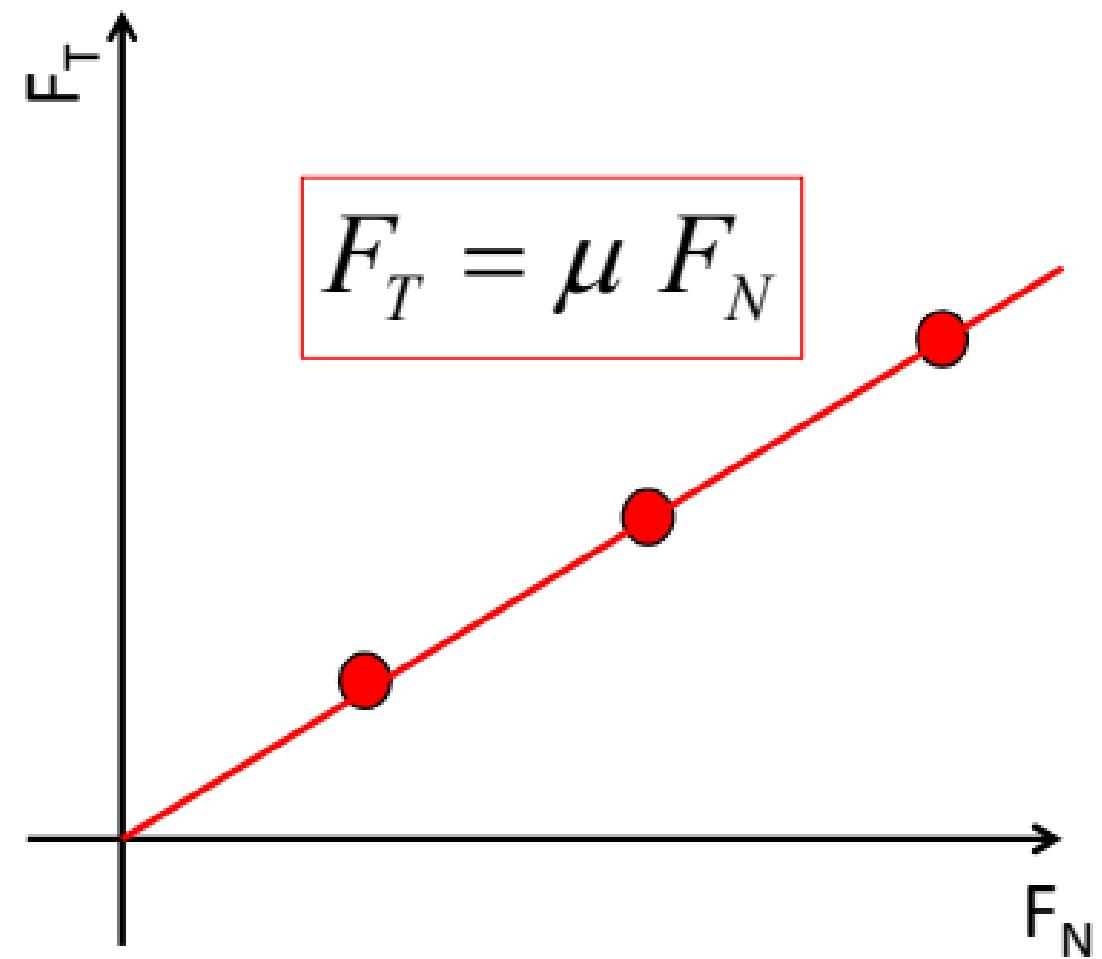
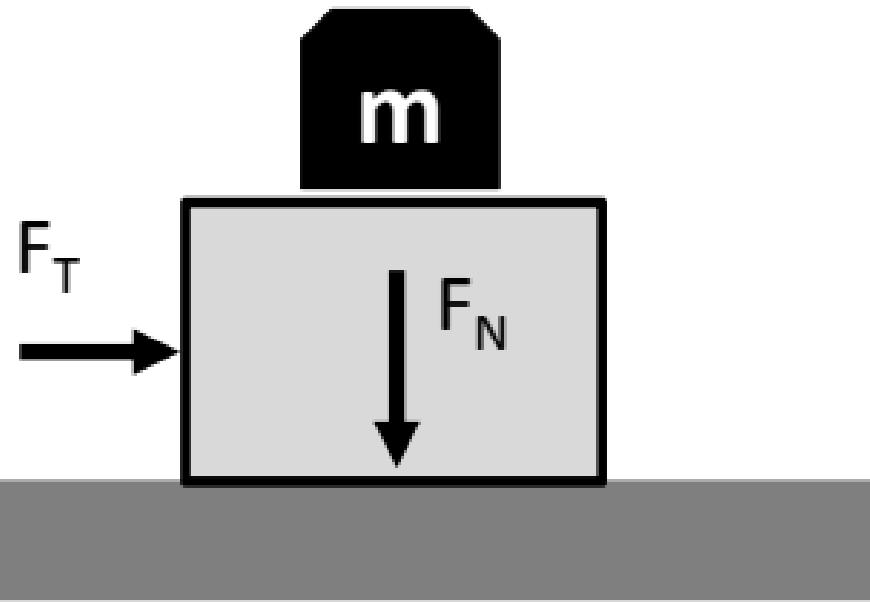


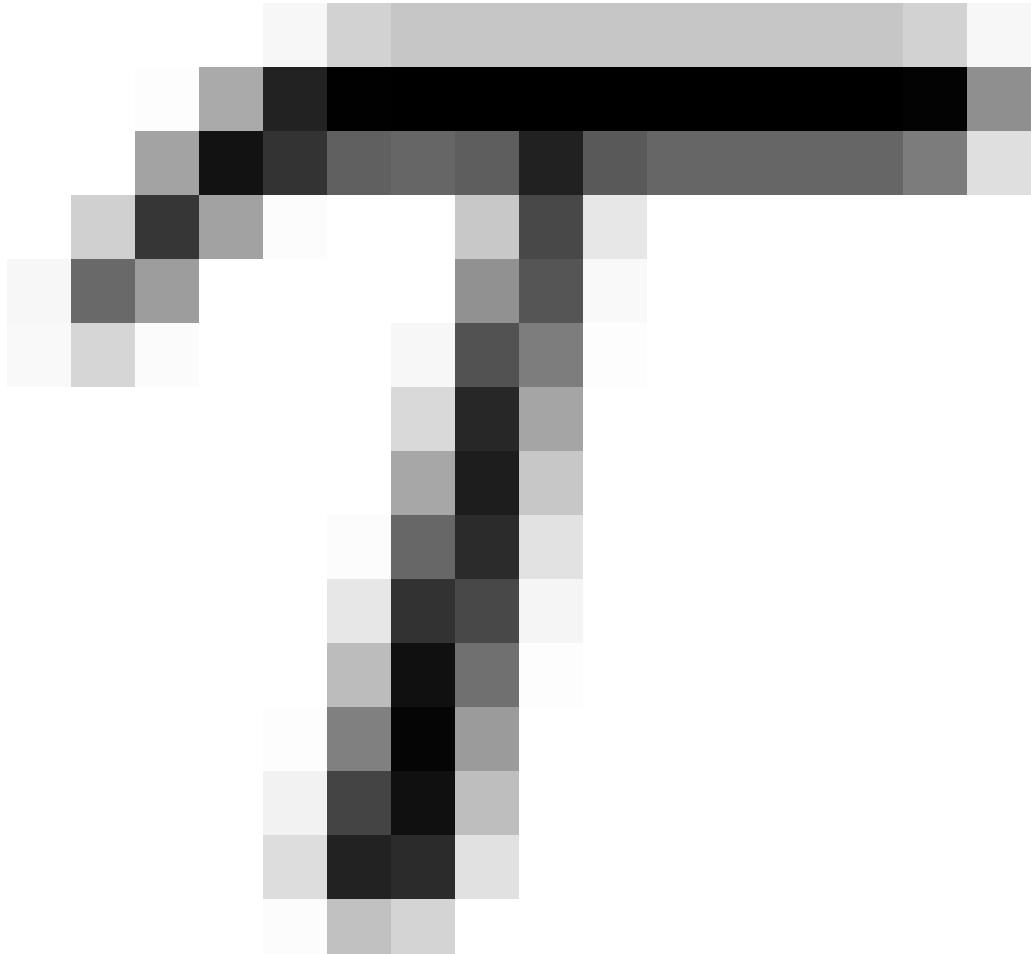


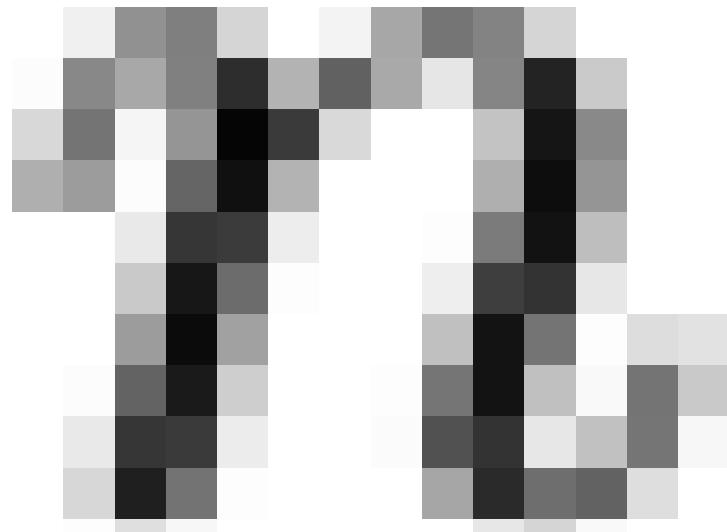
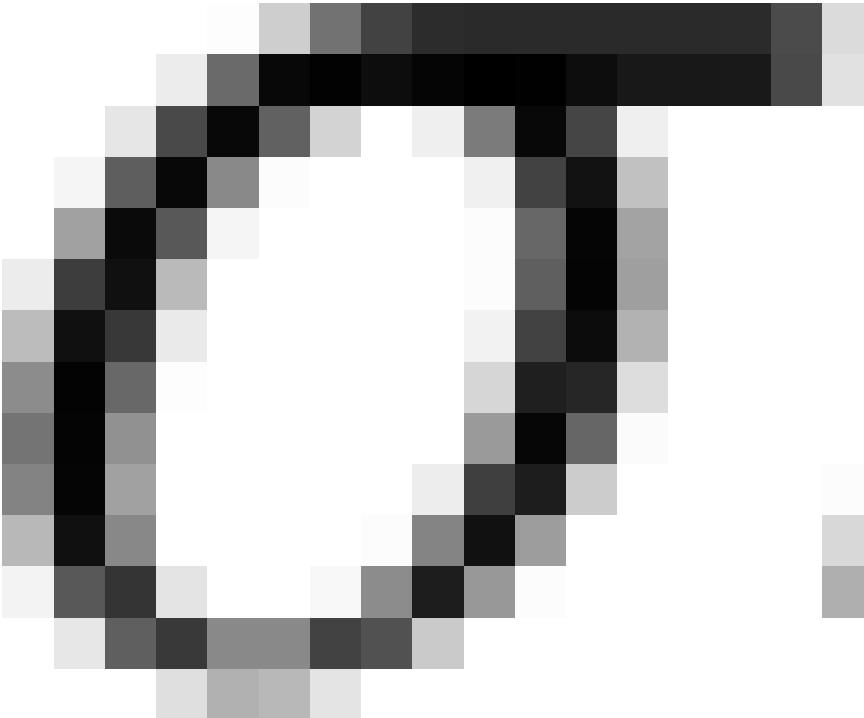




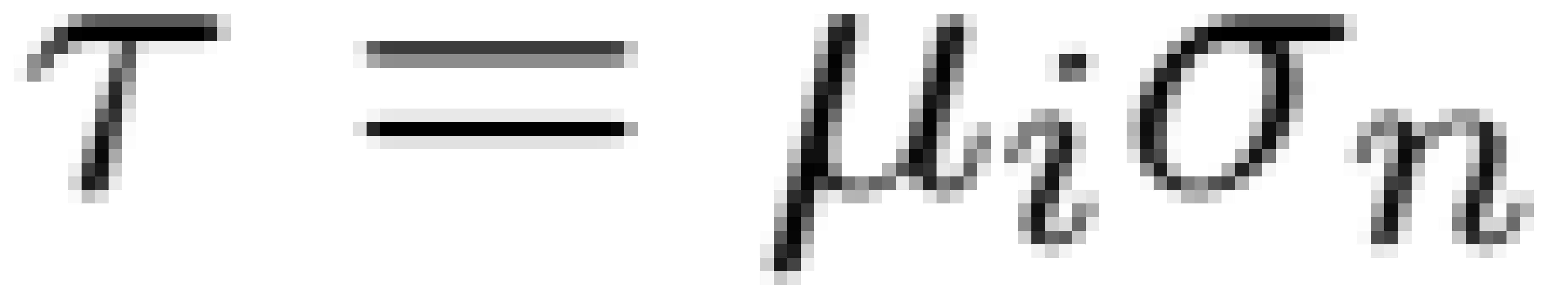


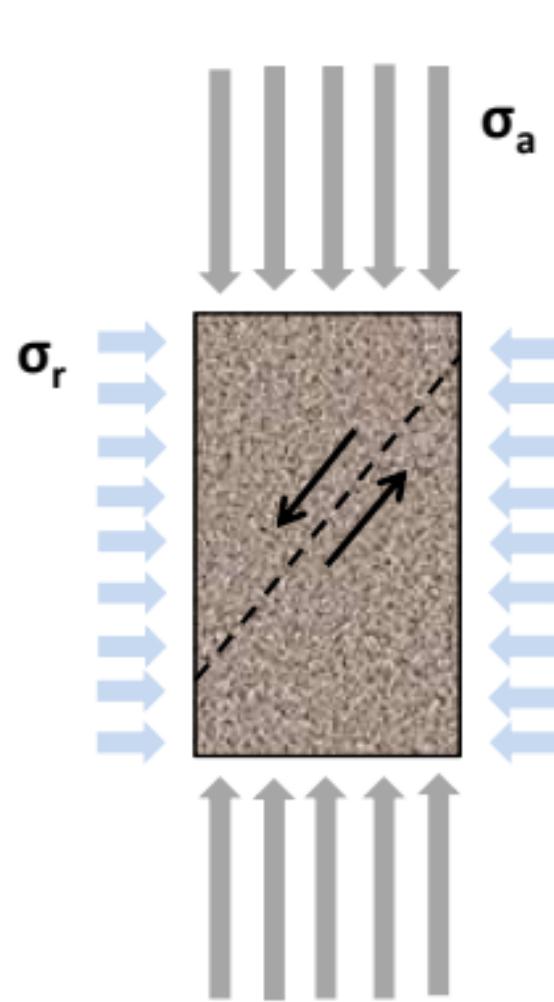




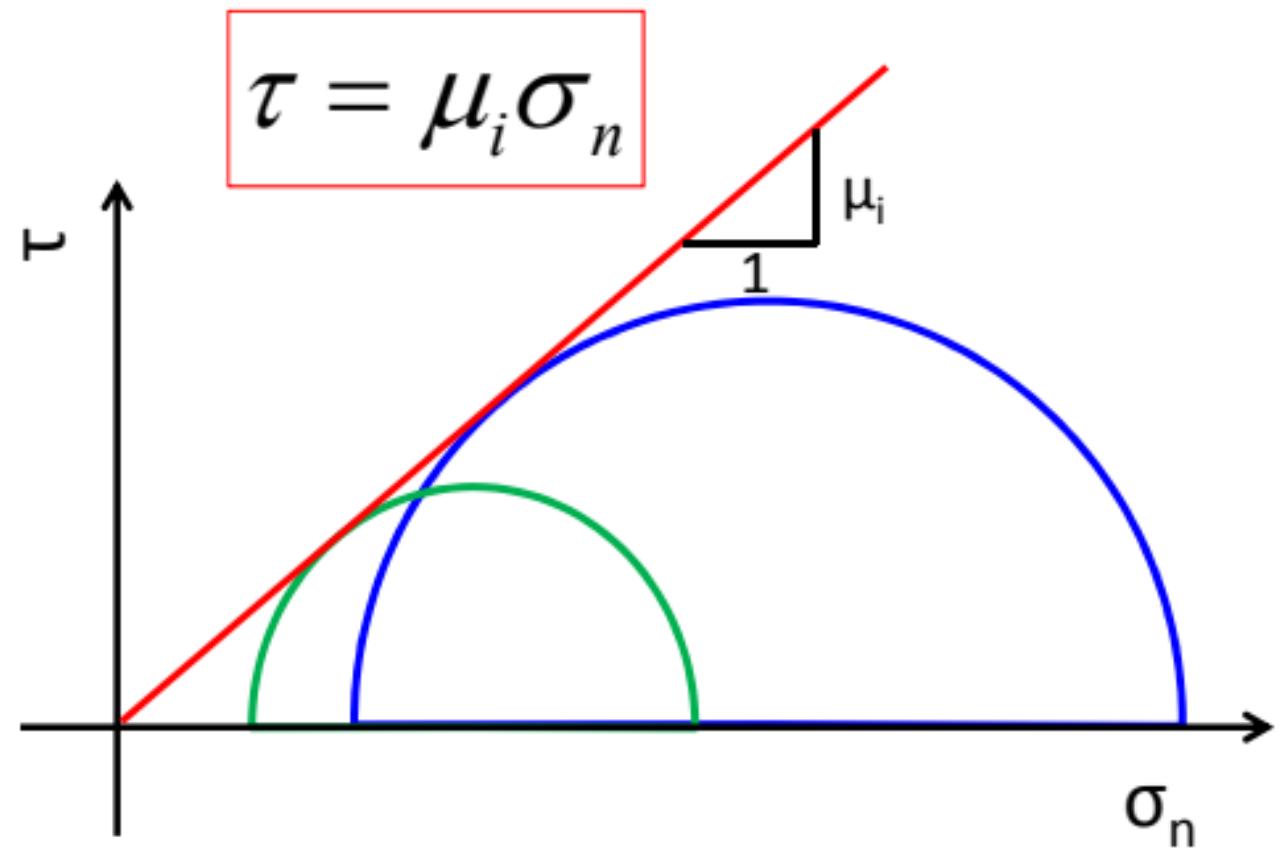


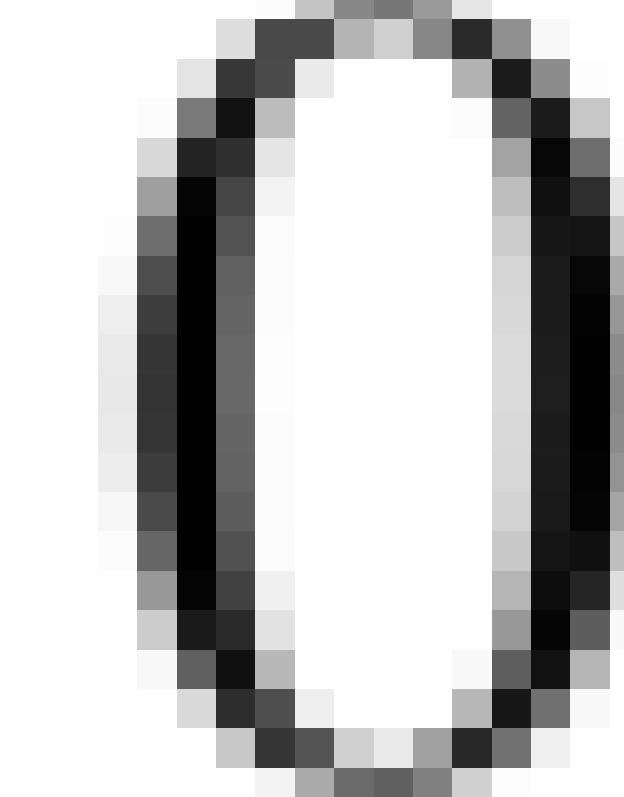
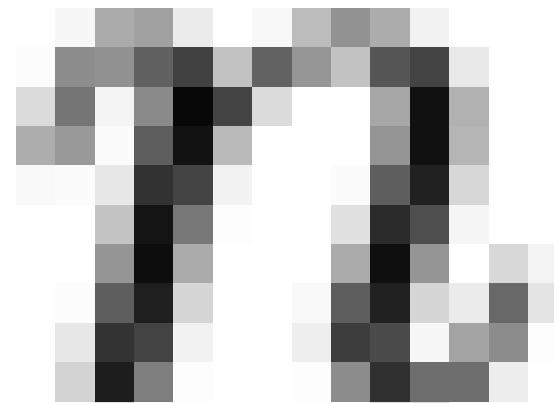
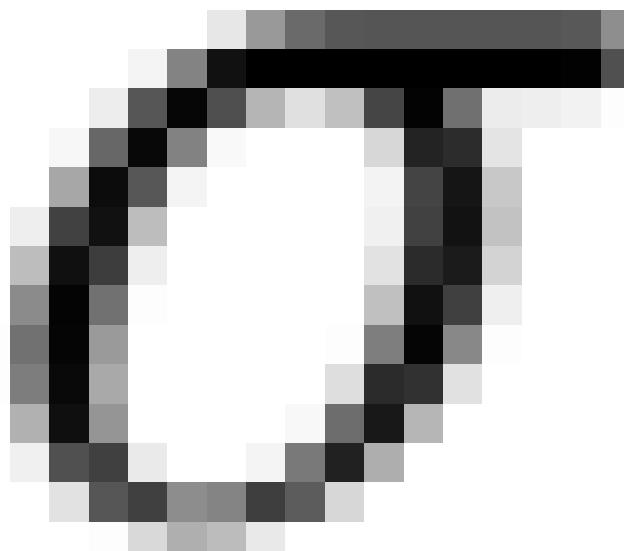


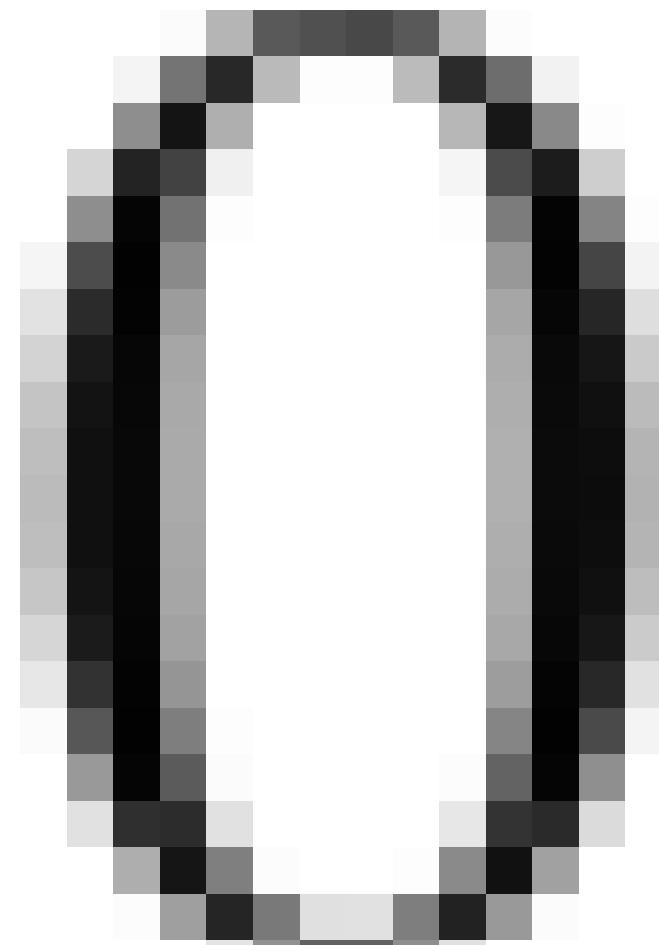
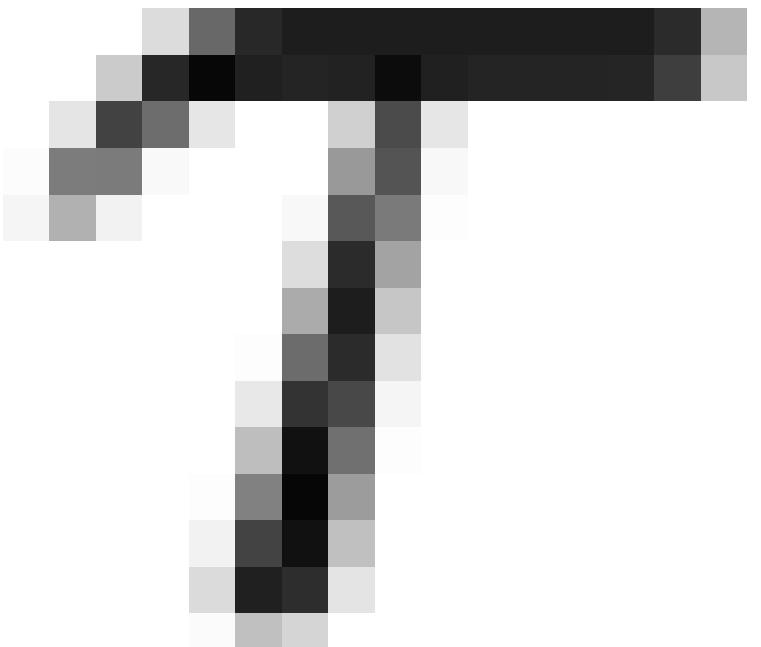


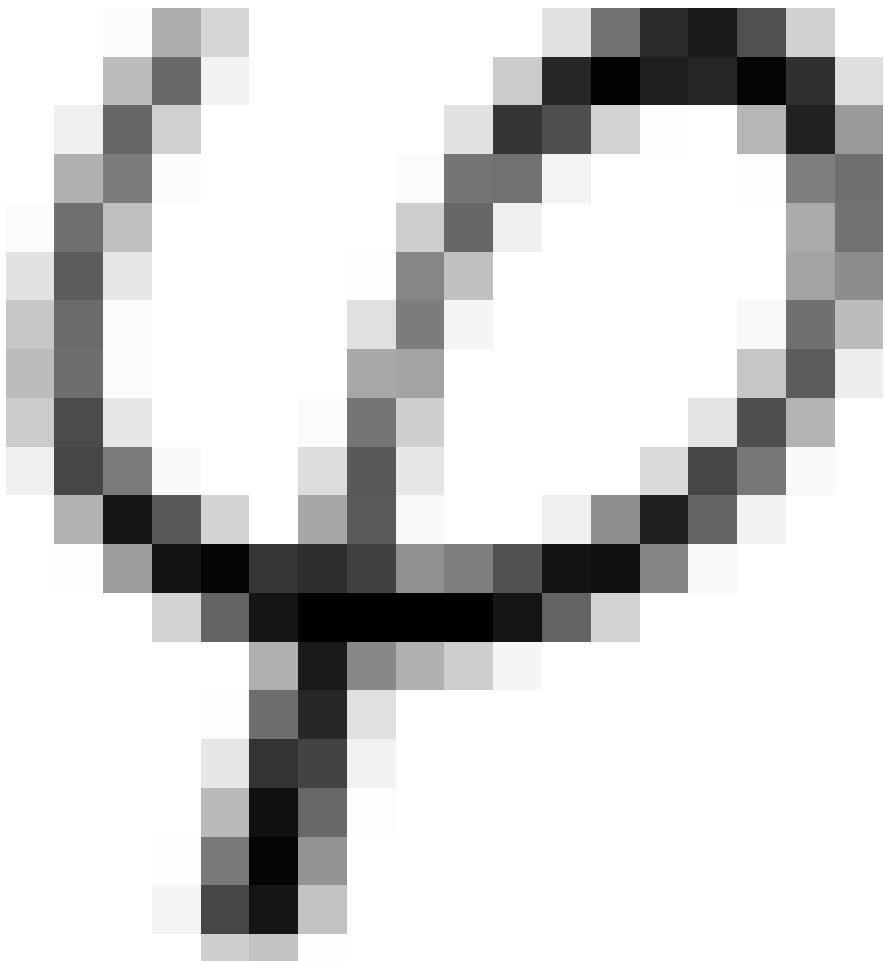


Unconsolidated Sand

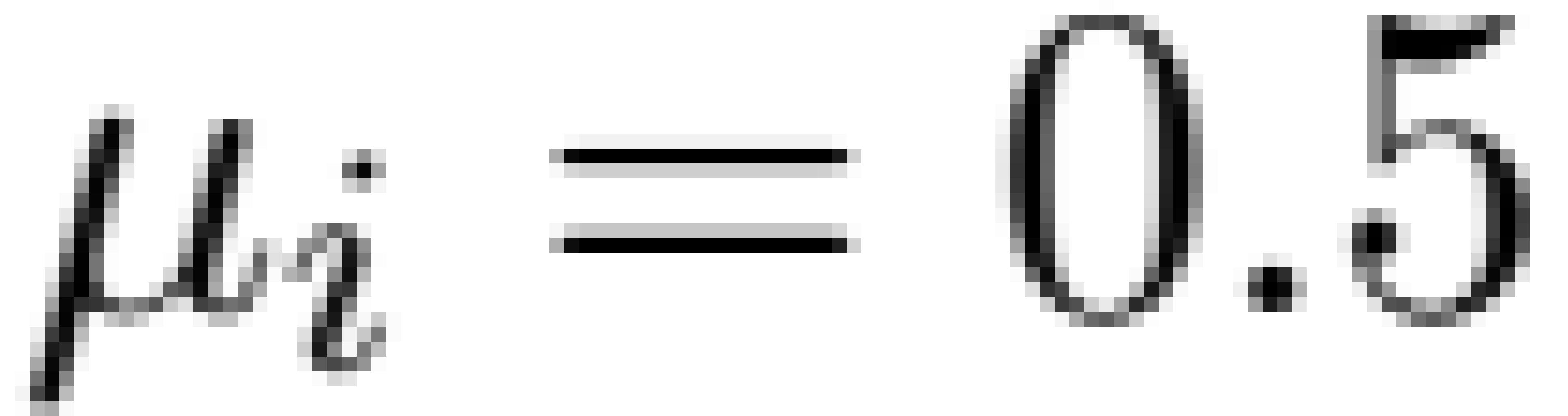


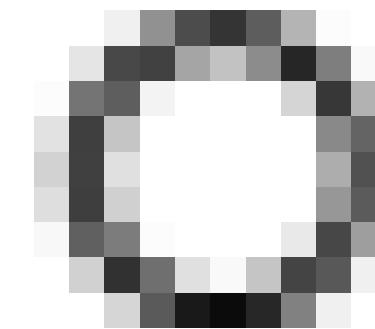
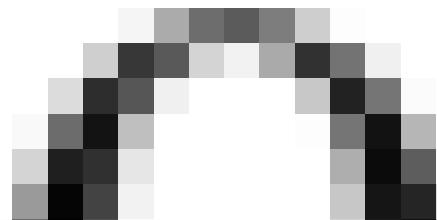
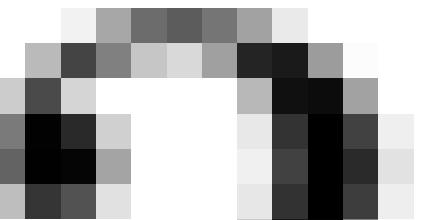
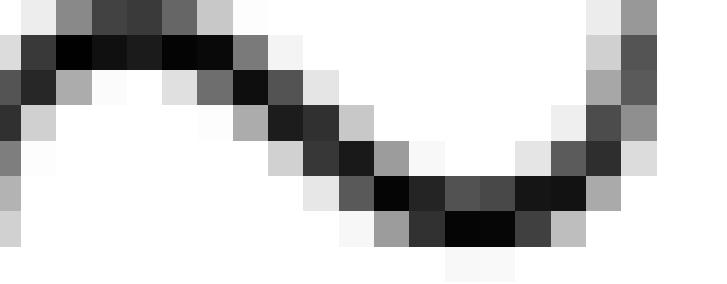
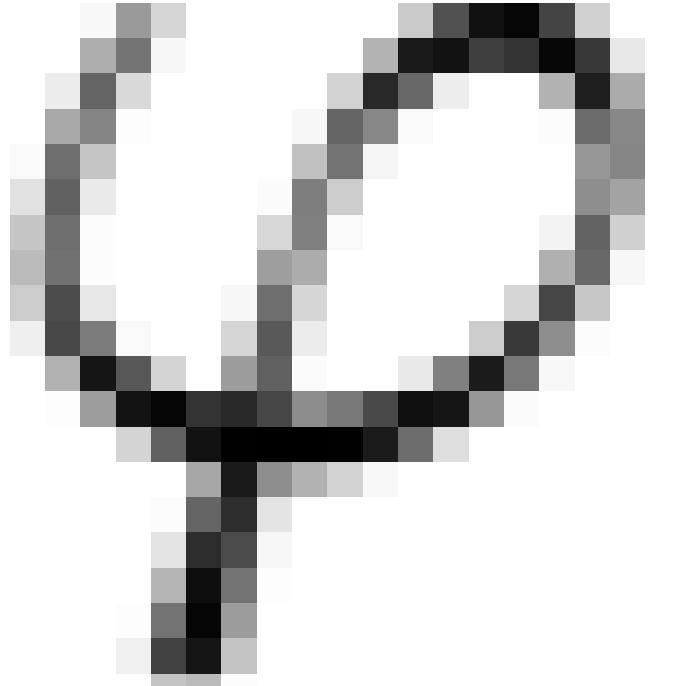


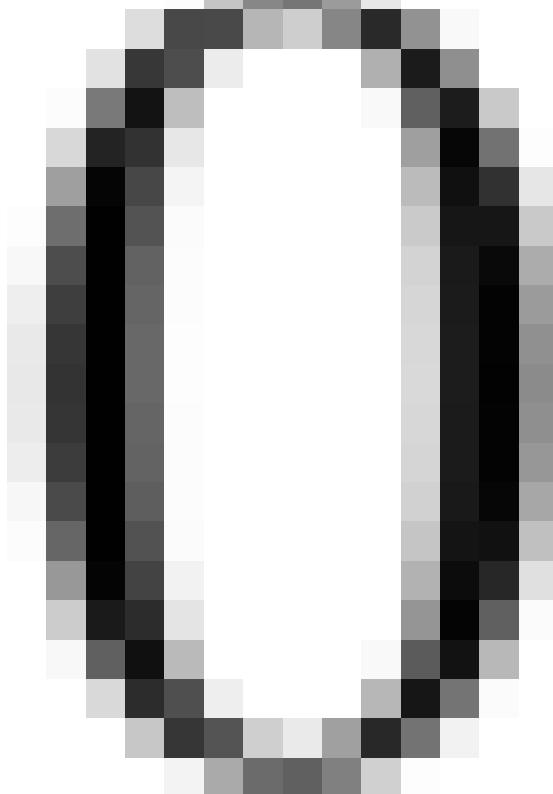
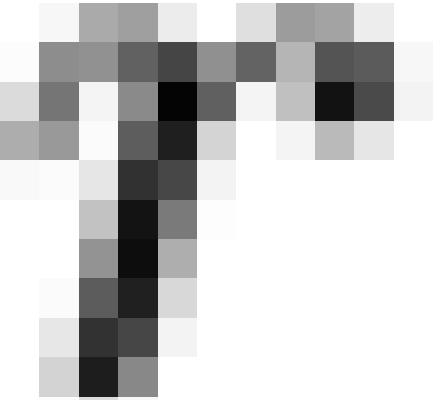
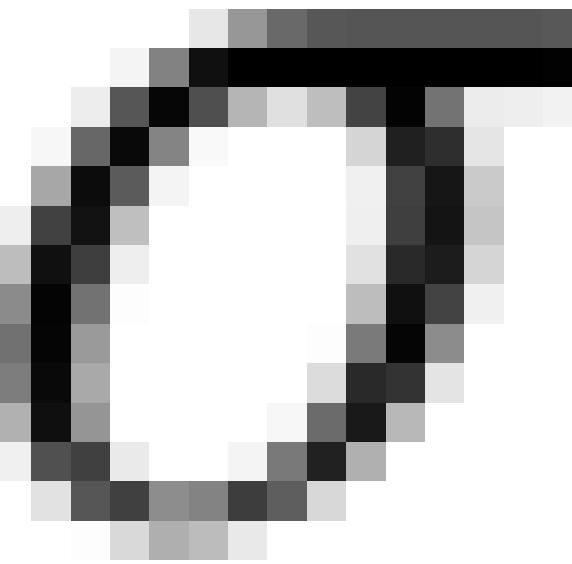




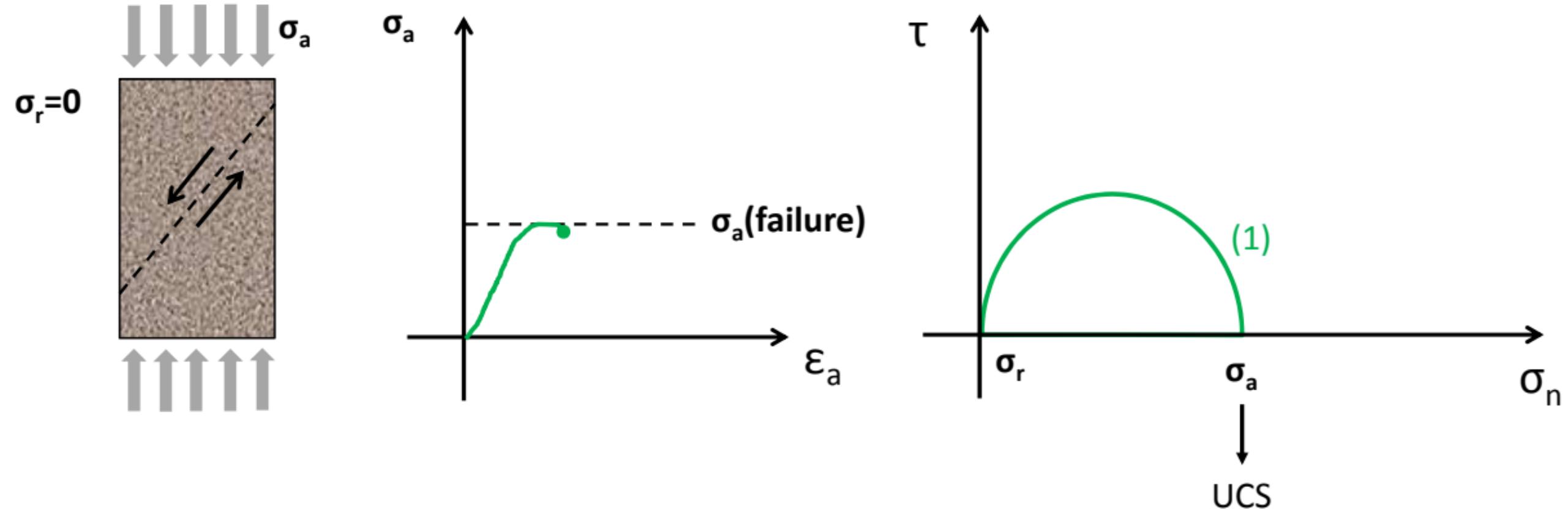


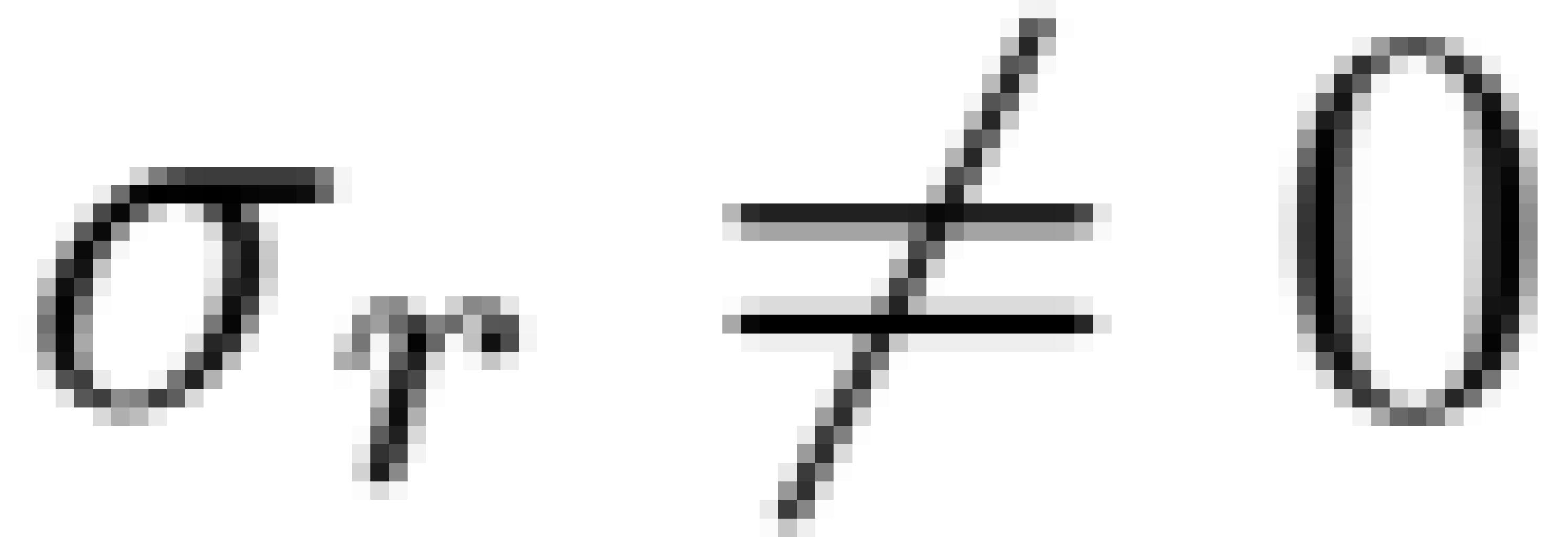


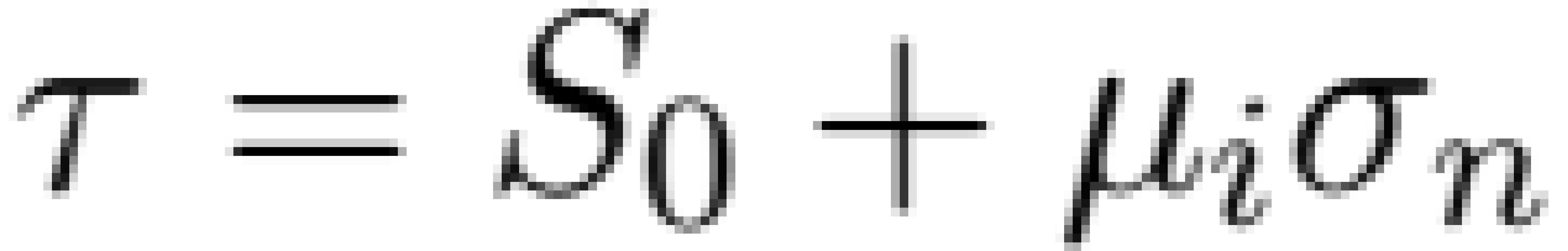




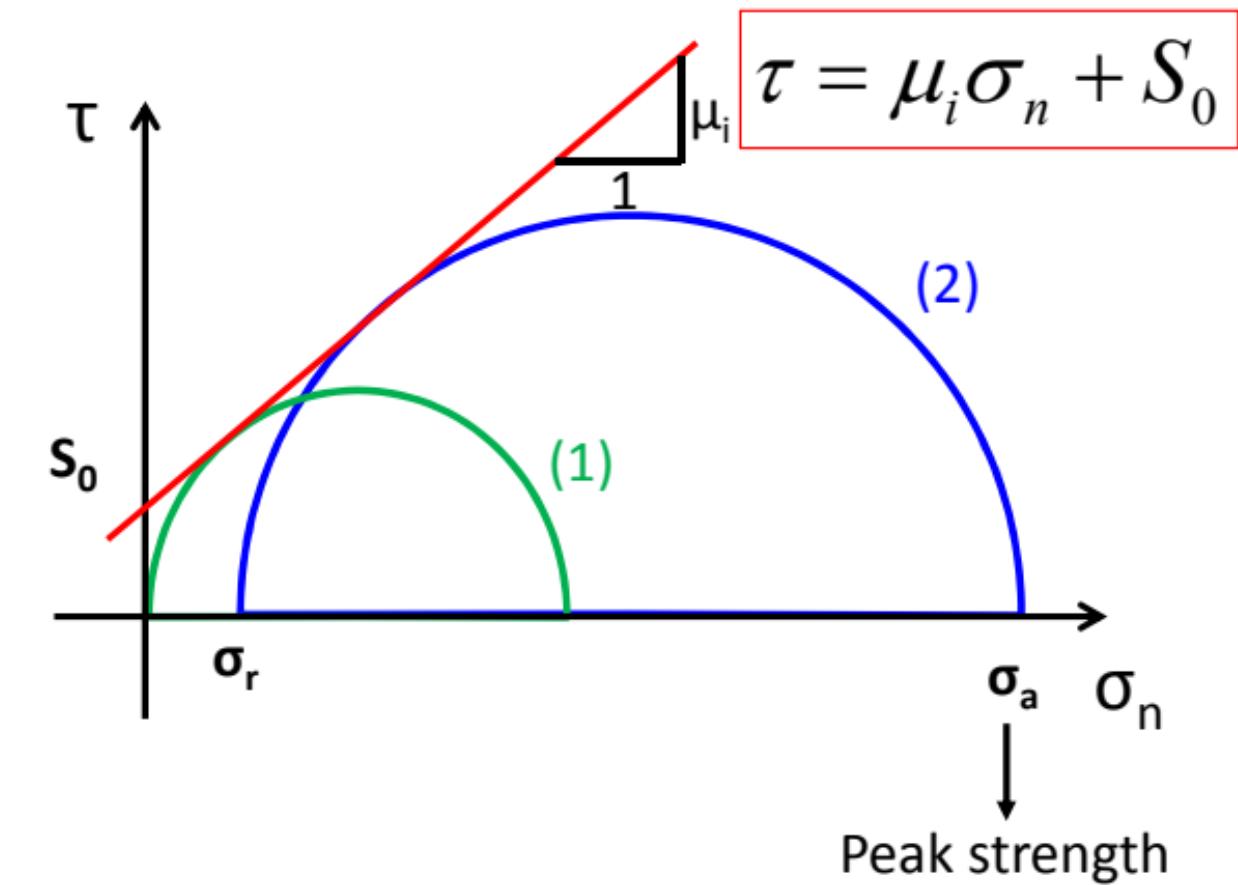
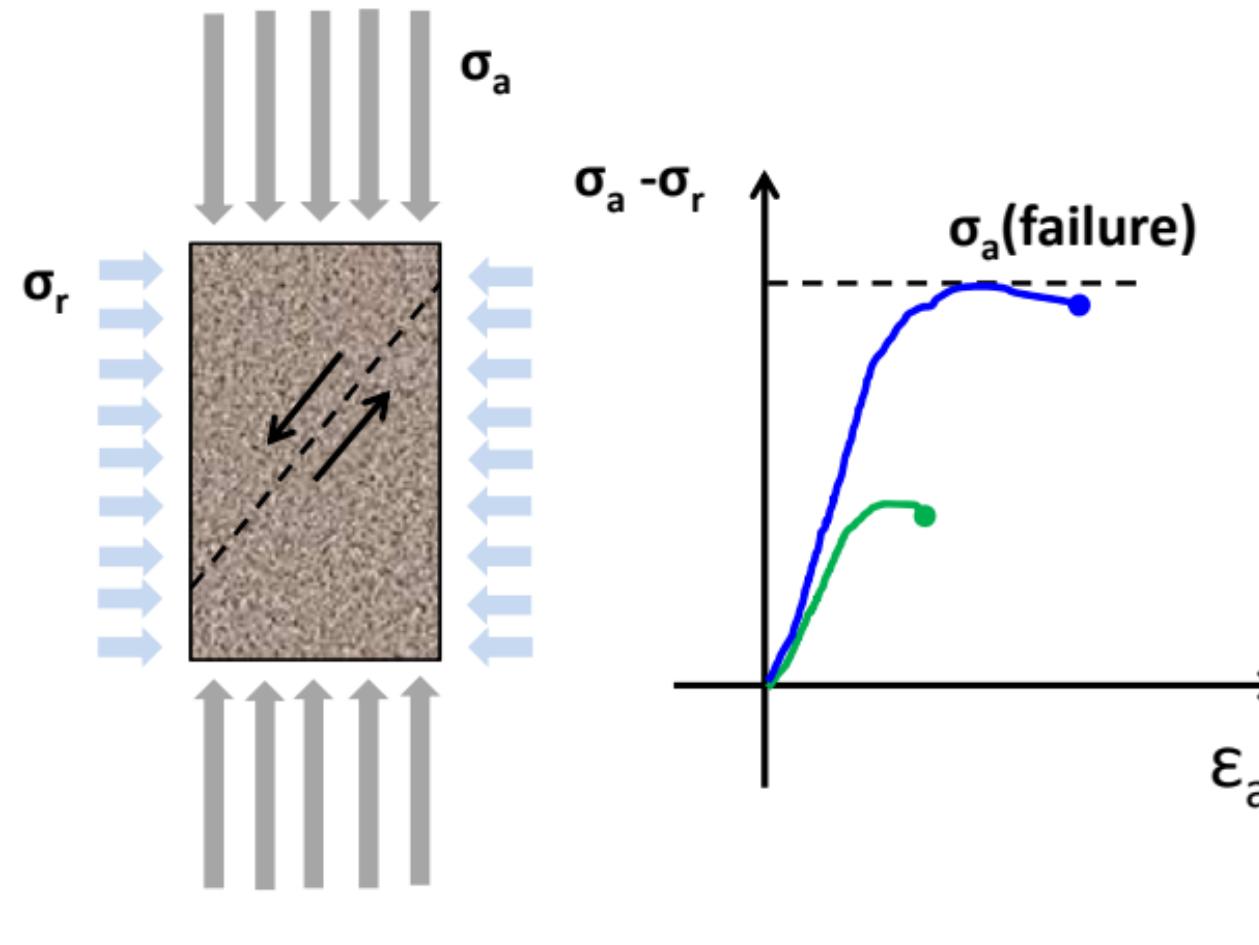
## (1) Unconfined Loading

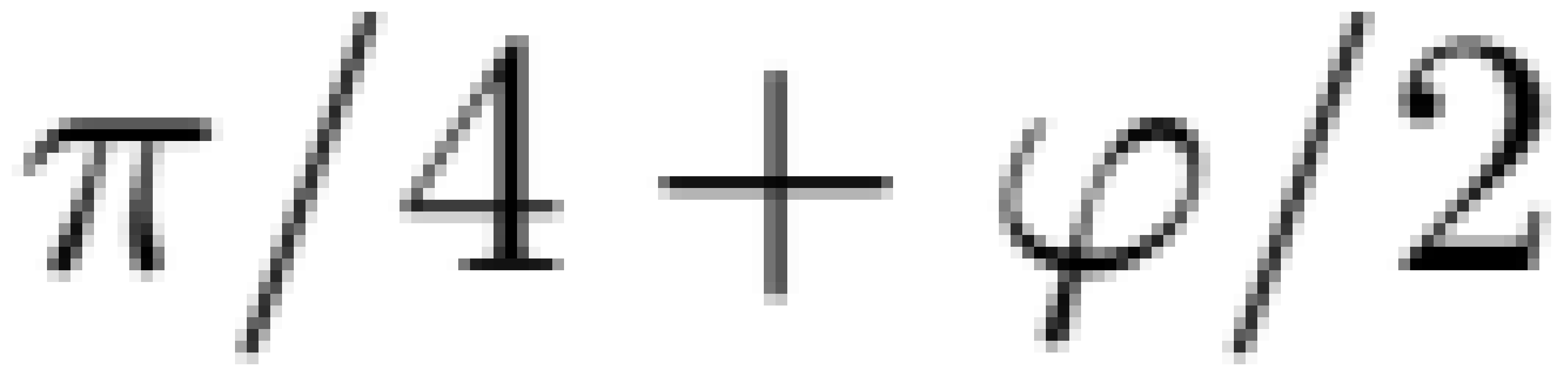






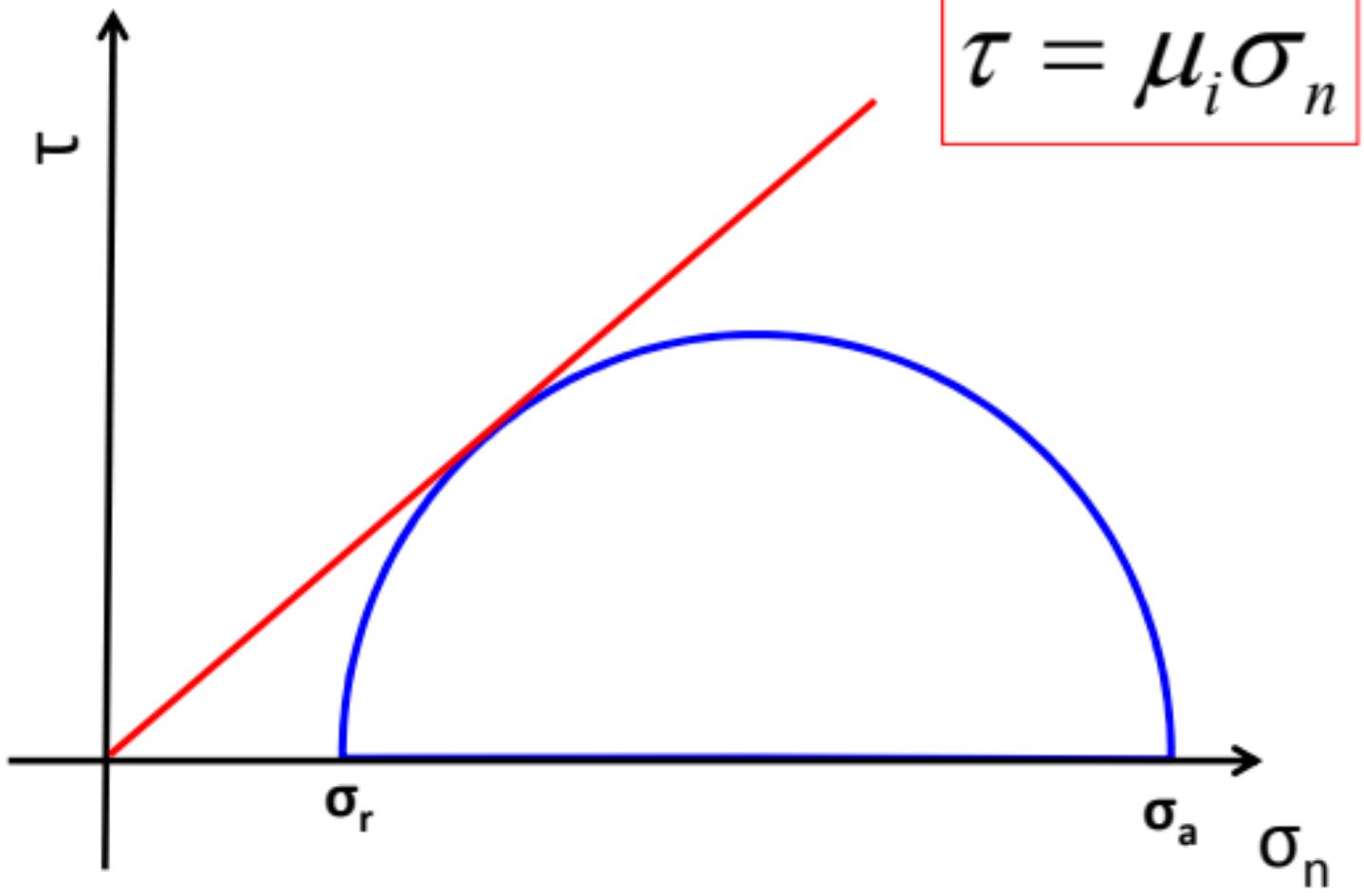
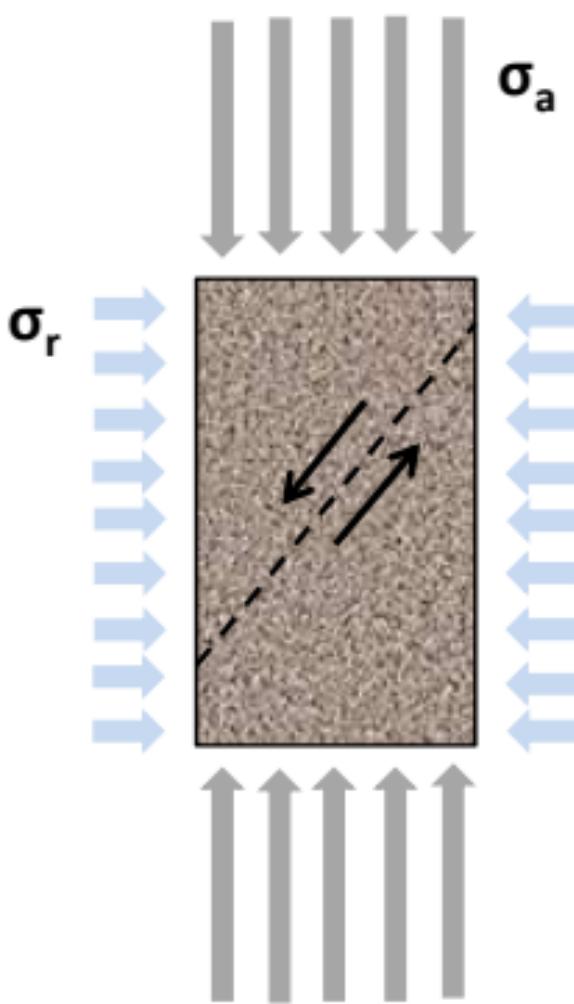
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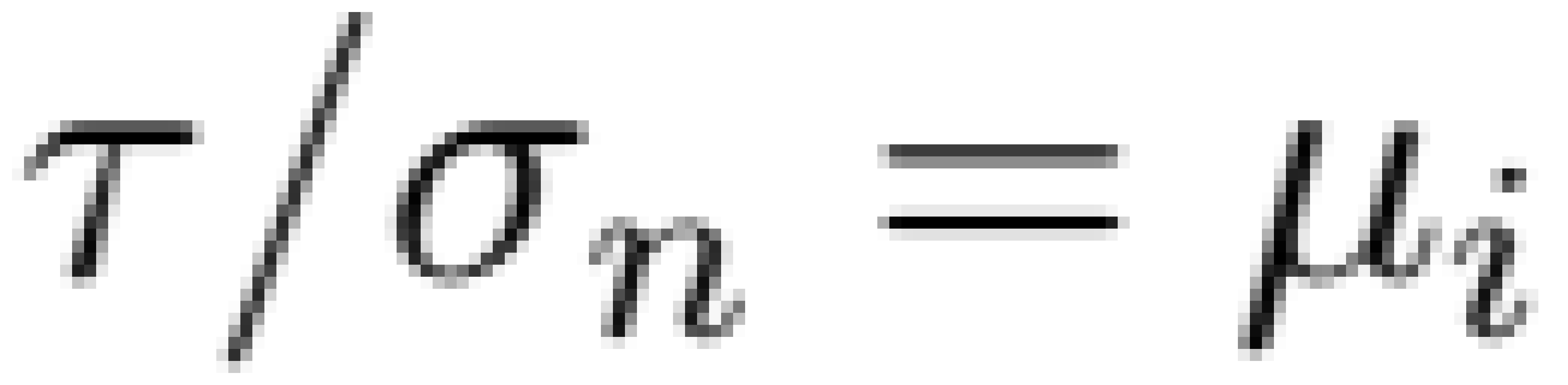


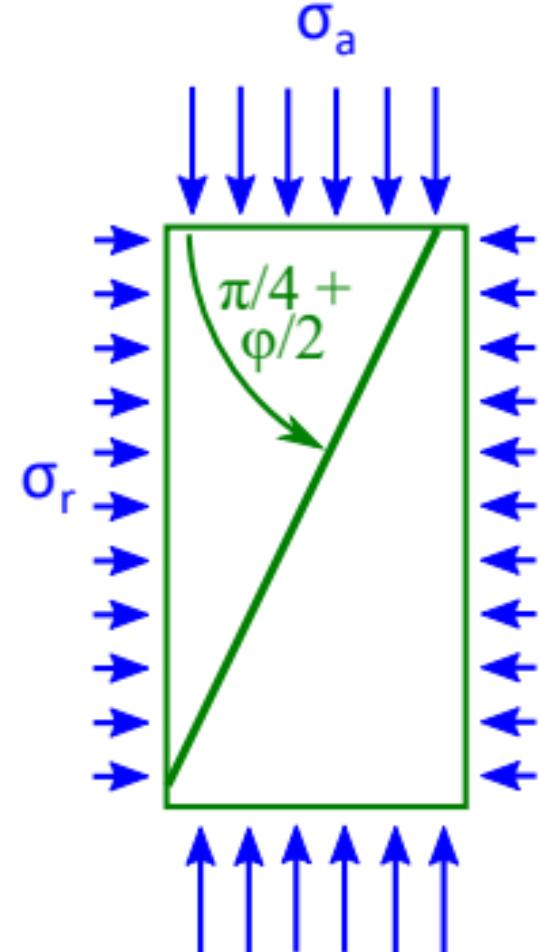
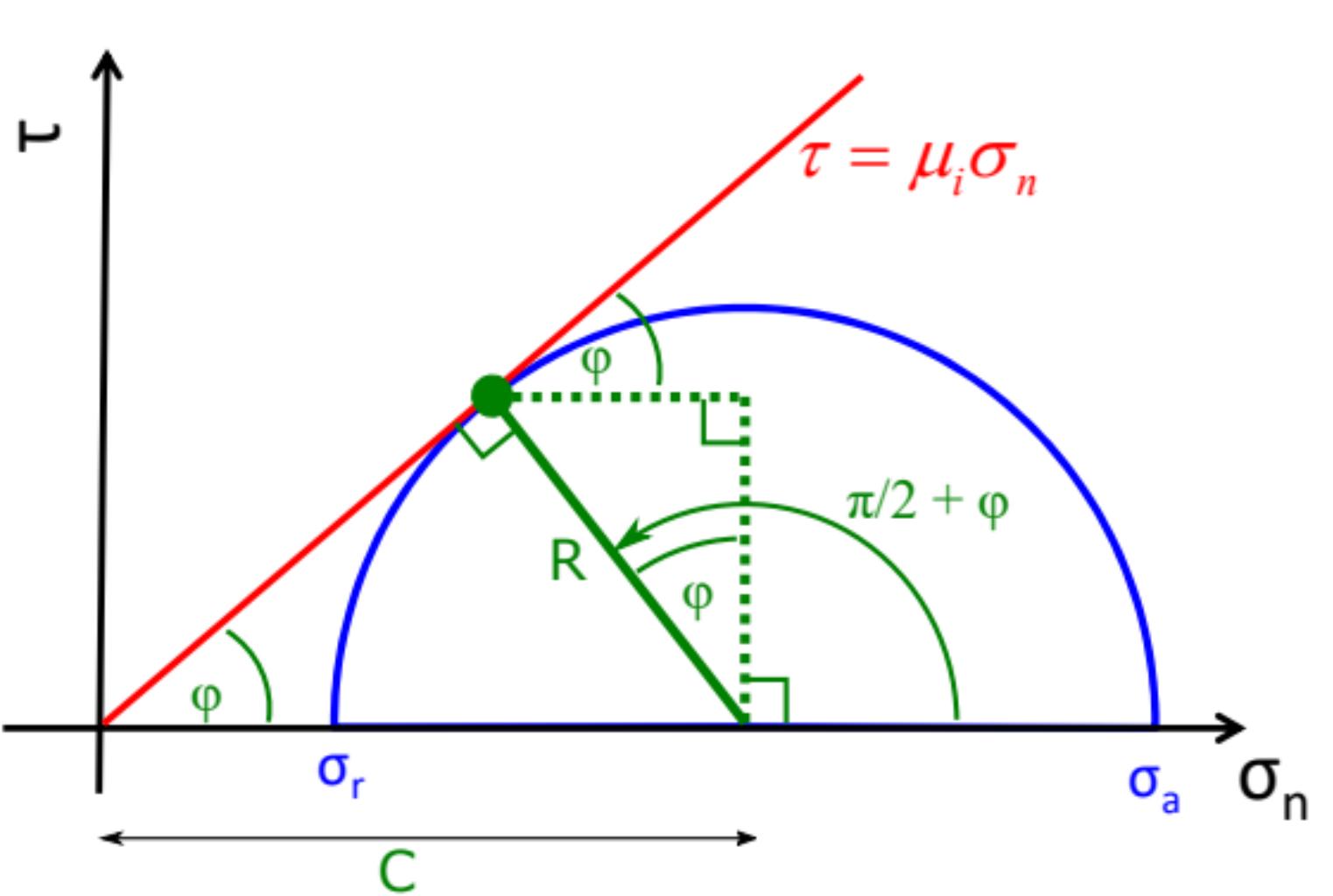








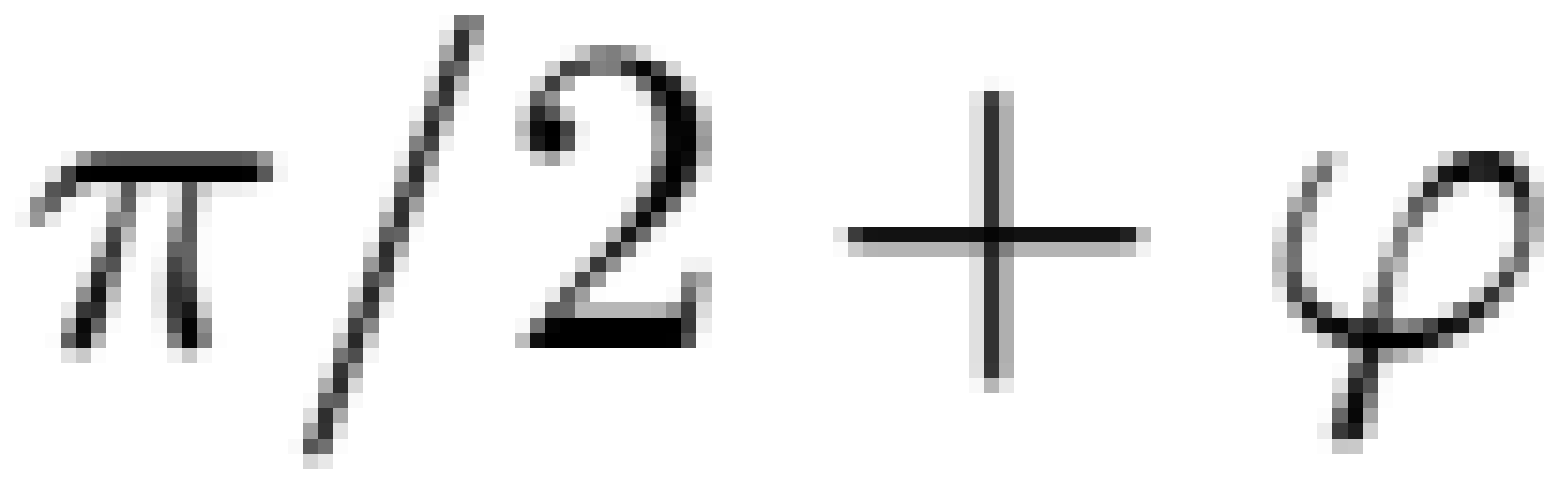


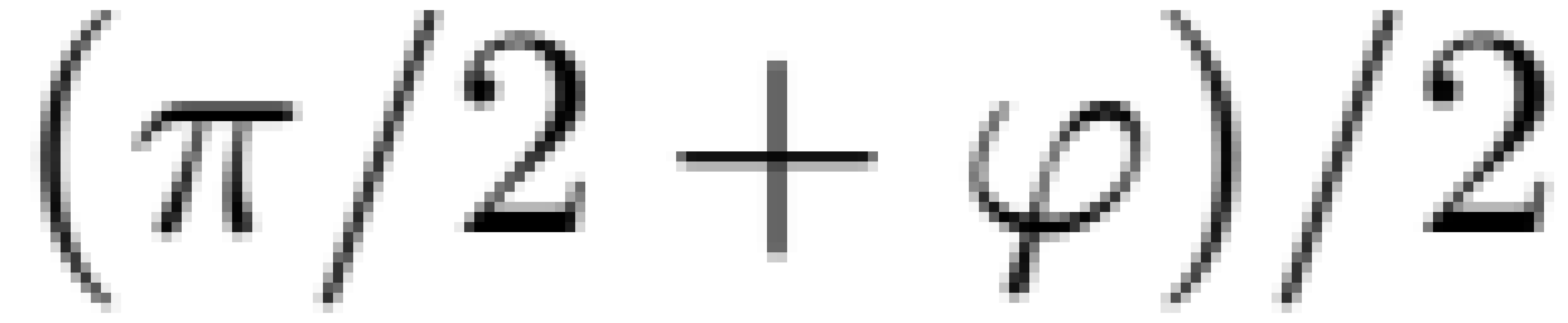


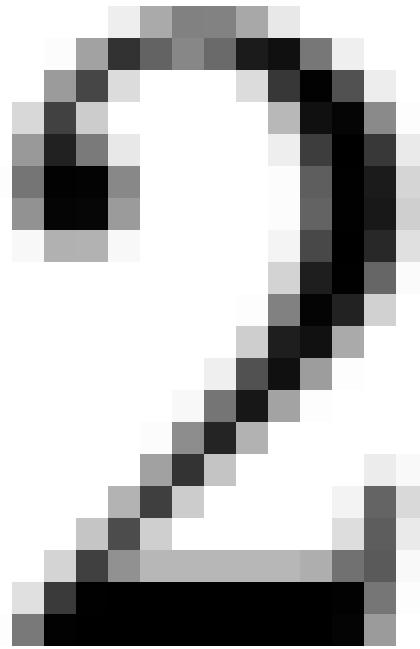
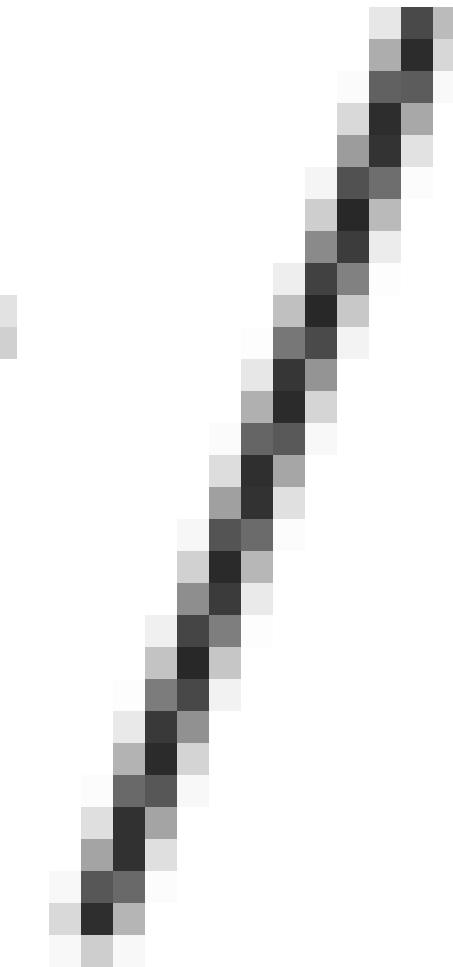
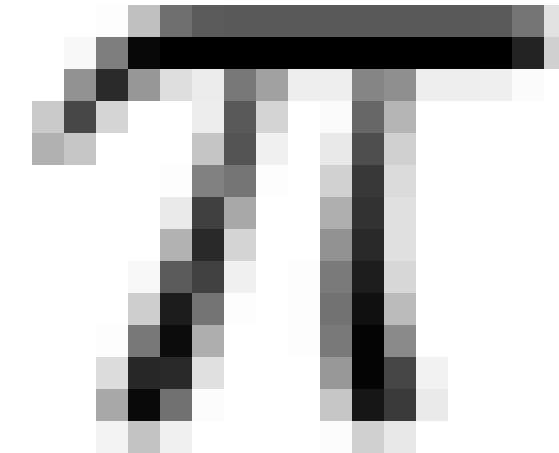
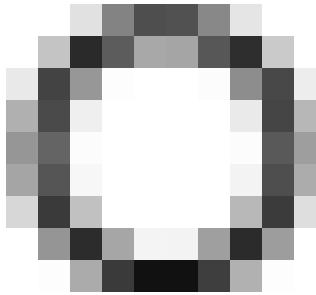


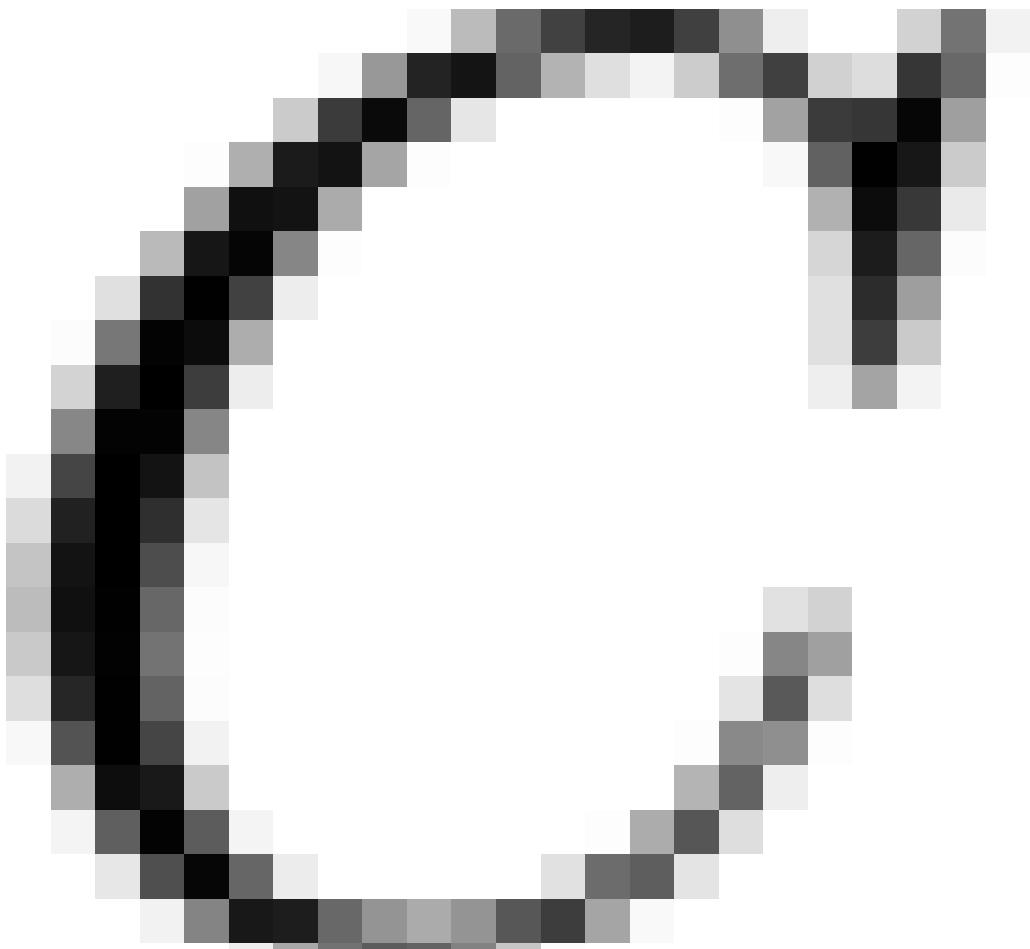






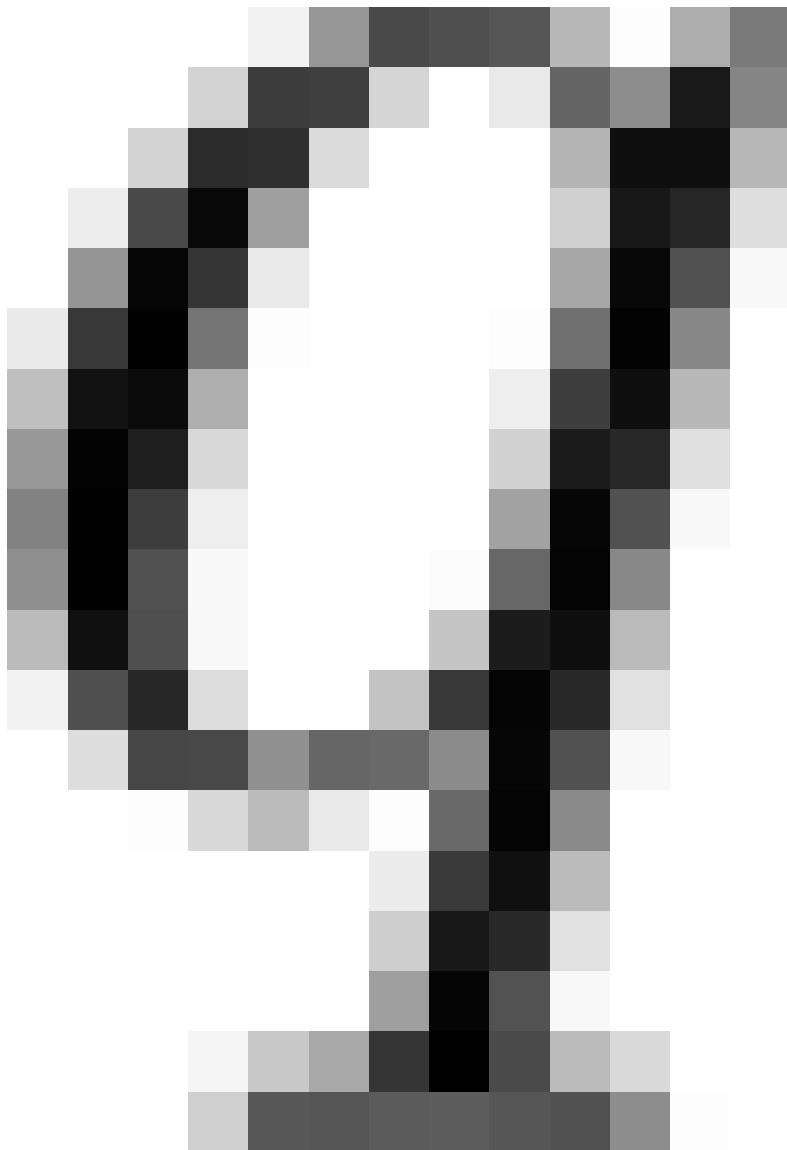


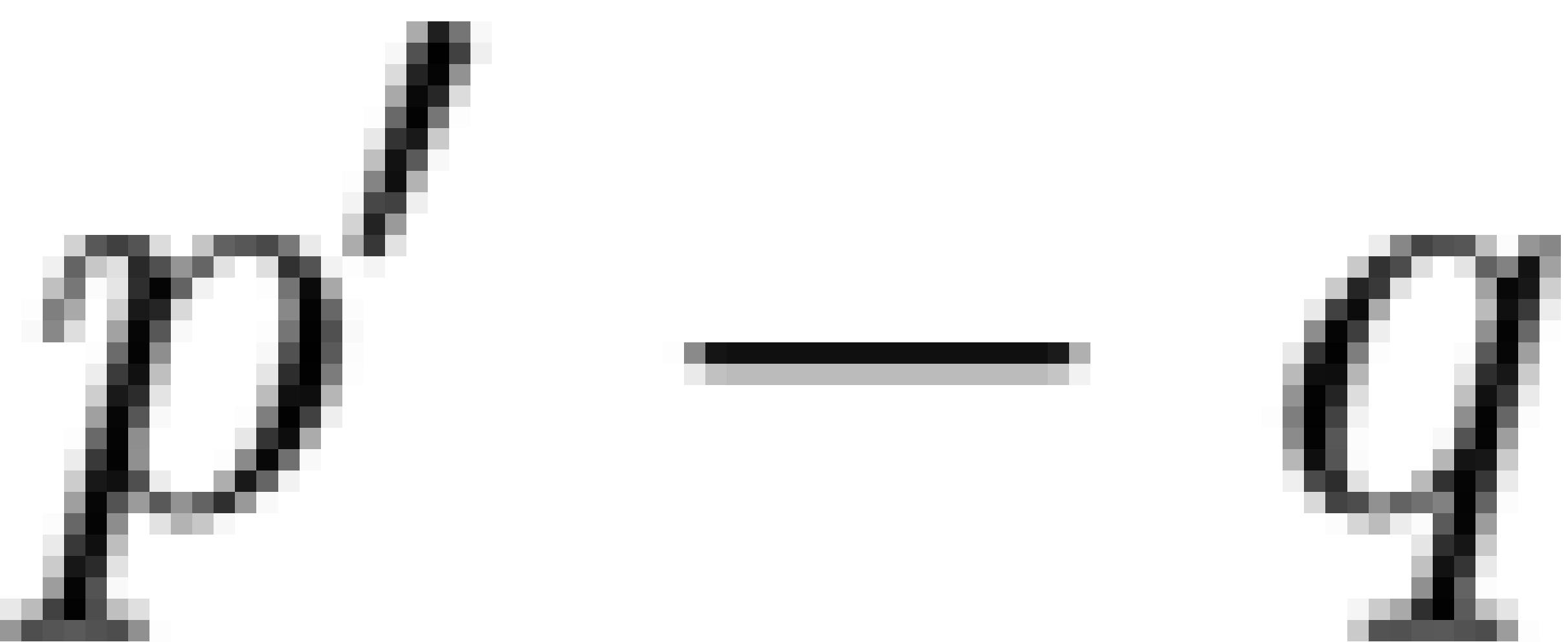


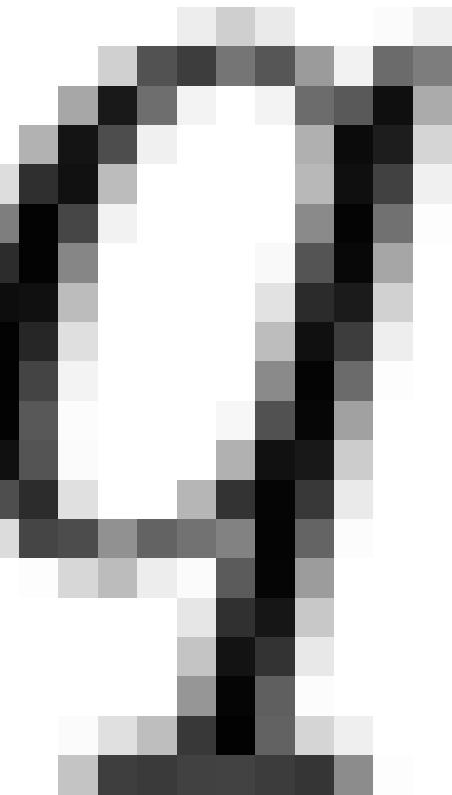
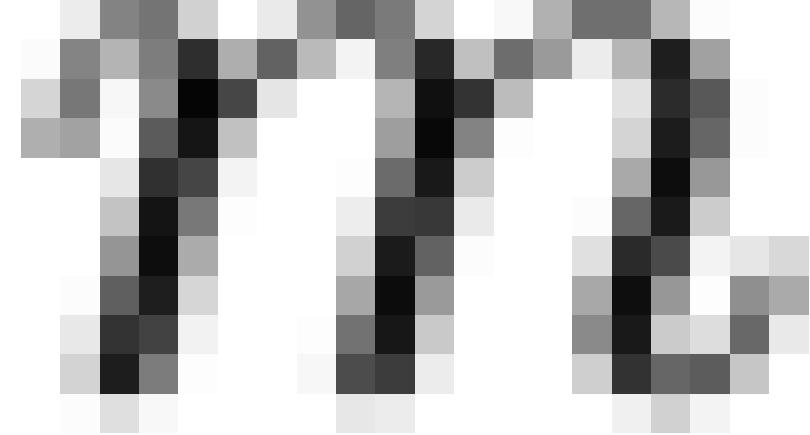
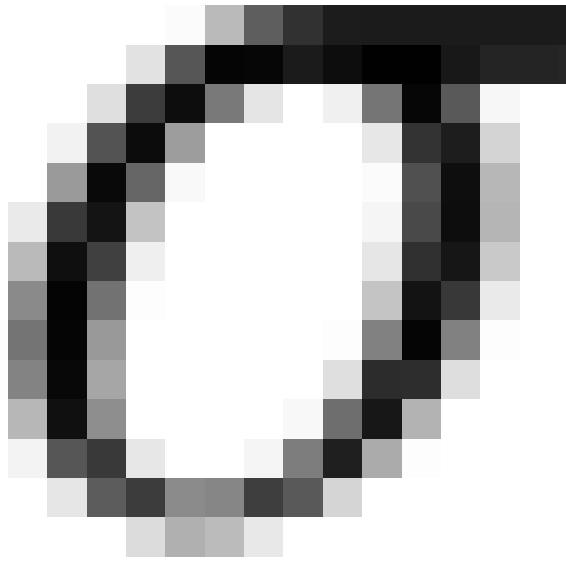


$$\begin{aligned} \sigma_a &= C + R \\ \sigma_r &= C - R \end{aligned}$$
$$\begin{aligned} C + \sin \varphi &= 1 + \sin \varphi \\ C - \sin \varphi &= 1 - \sin \varphi \end{aligned}$$



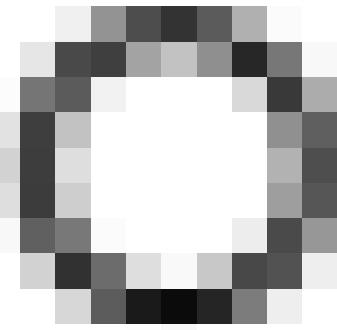
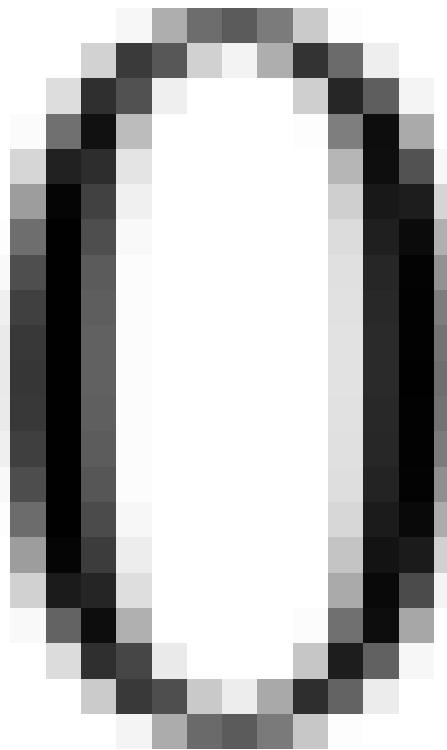
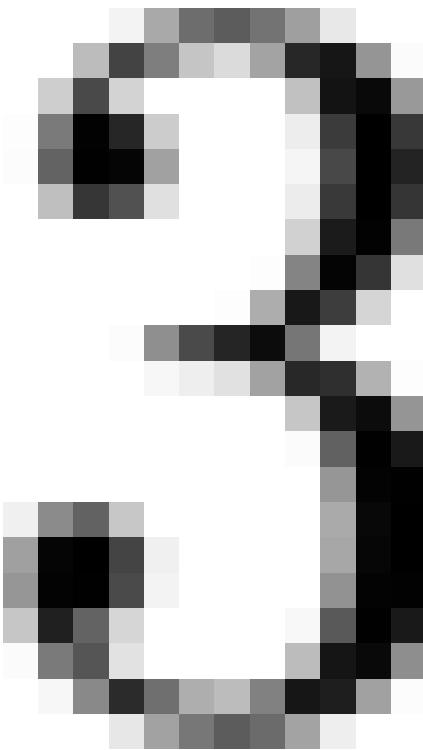
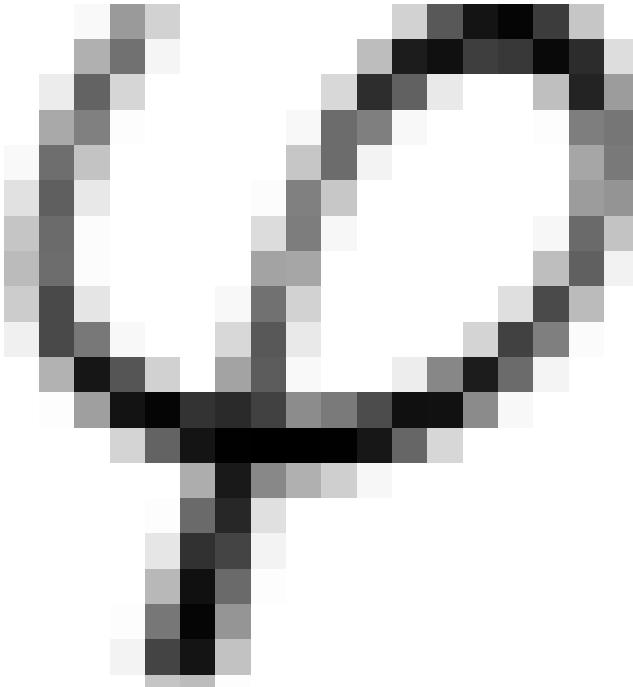


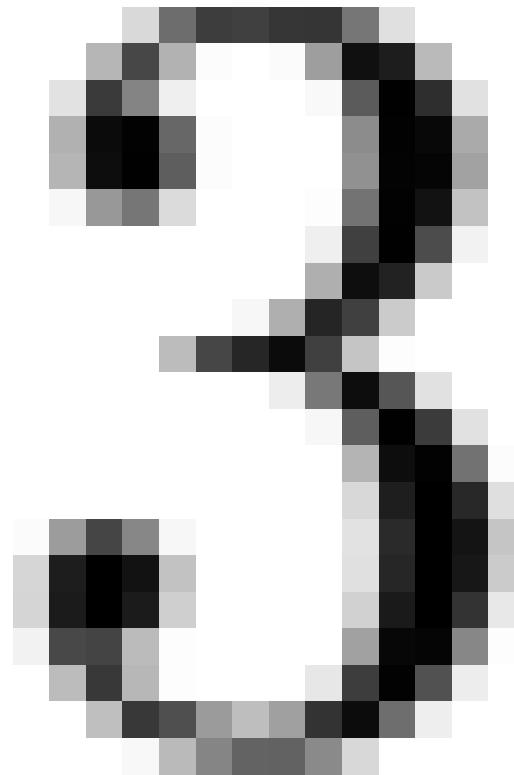
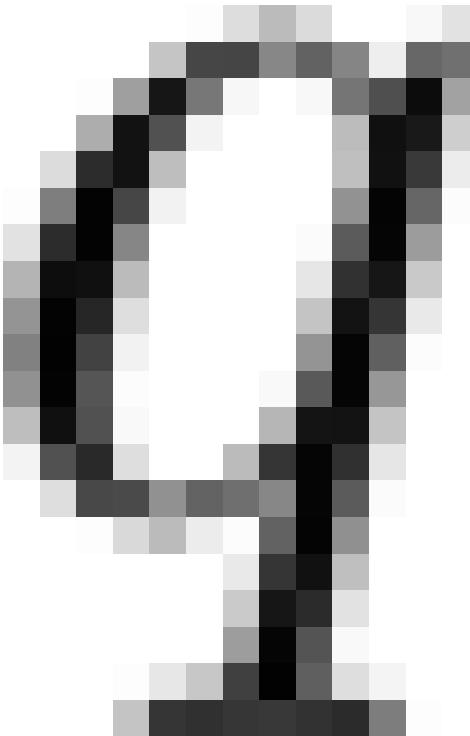




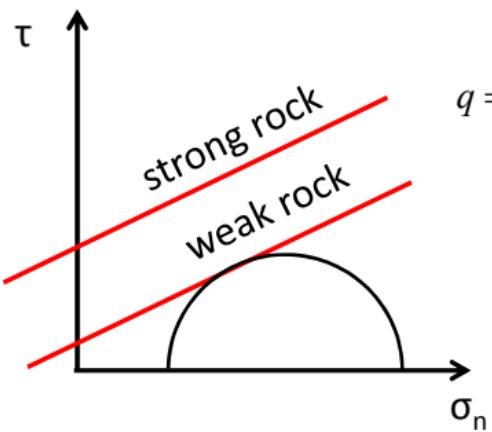
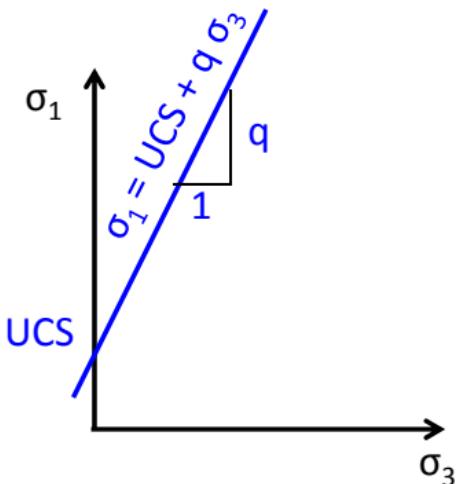
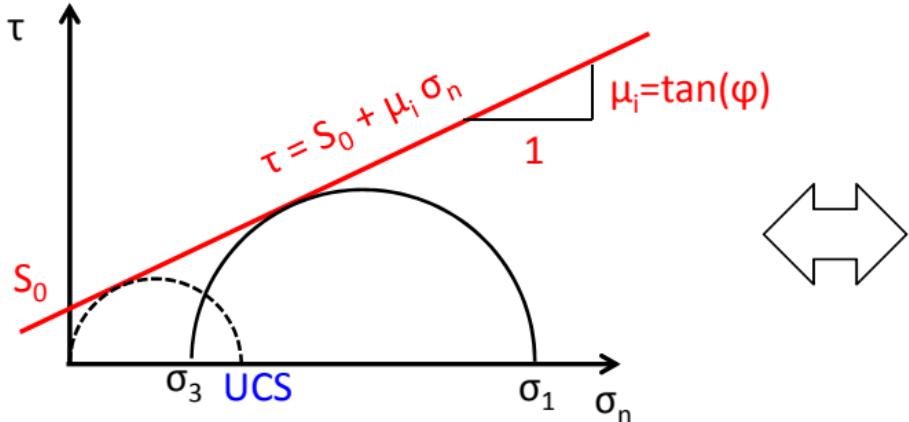
$\varphi$  sin  $\varphi$

$\varphi$  sin  $\varphi$



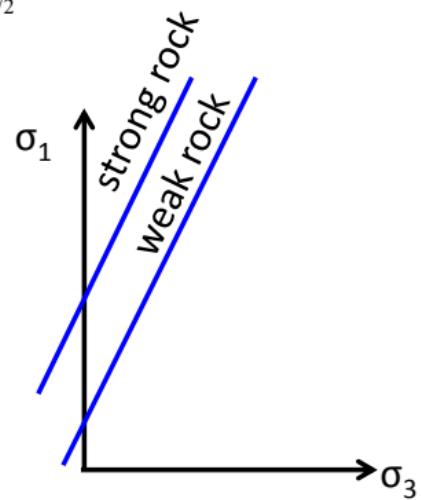


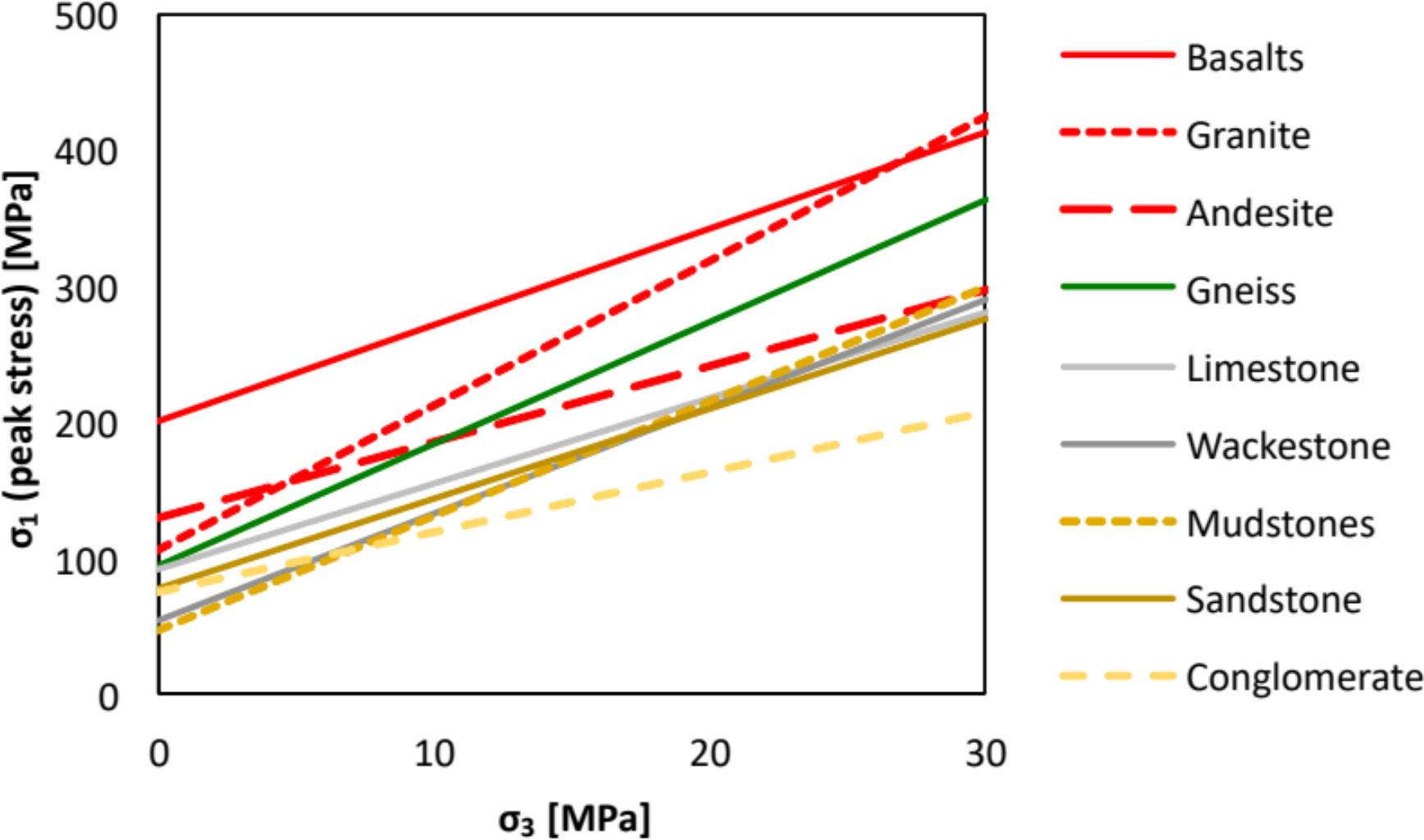
$$U_0 S = 2S_0 \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} = 2S_0 \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi}}$$



$$UCS = 2S_0 \left( \sqrt{\mu_i^2 + 1} + \mu_i \right) = 2S_0 \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2}$$

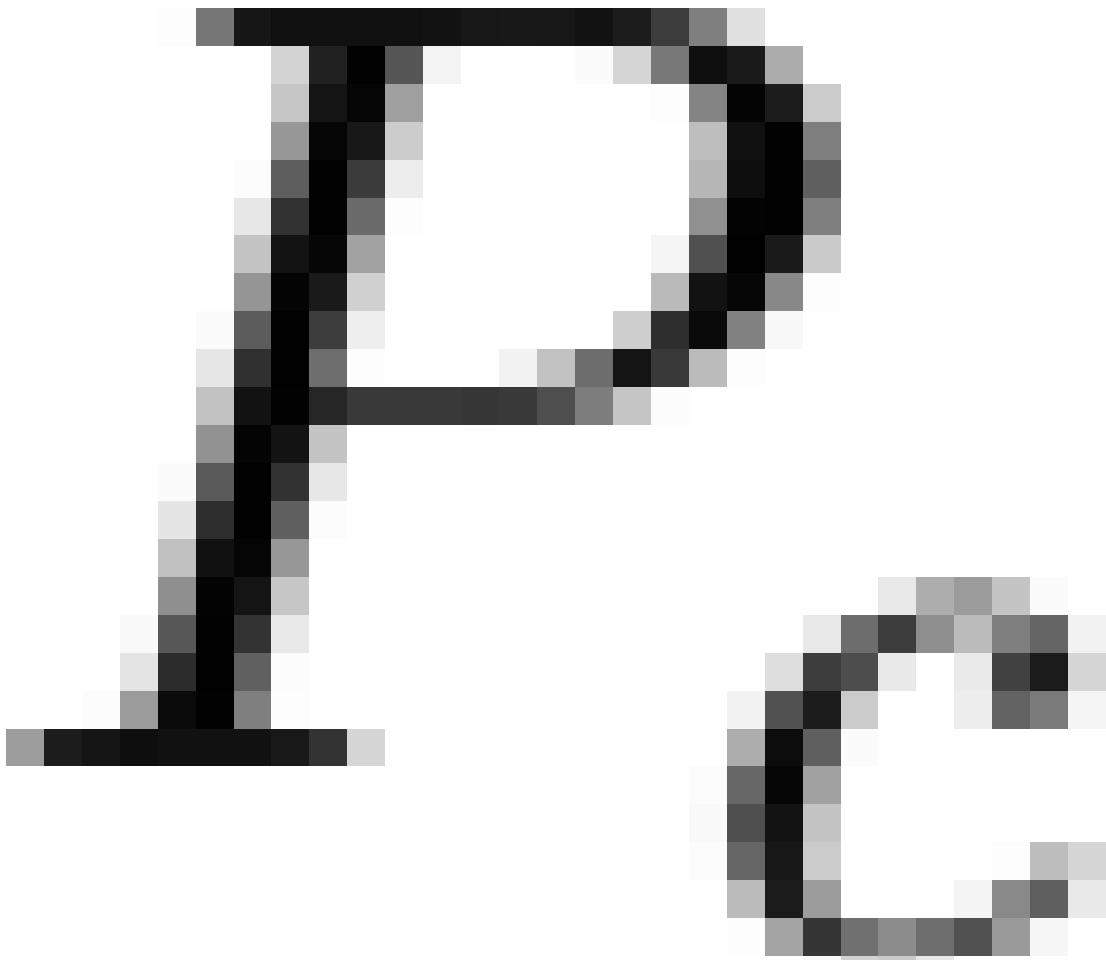
$$q = \left( \sqrt{\mu_i^2 + 1} + \mu_i \right)^2 = \frac{1 + \sin \varphi}{1 - \sin \varphi}$$

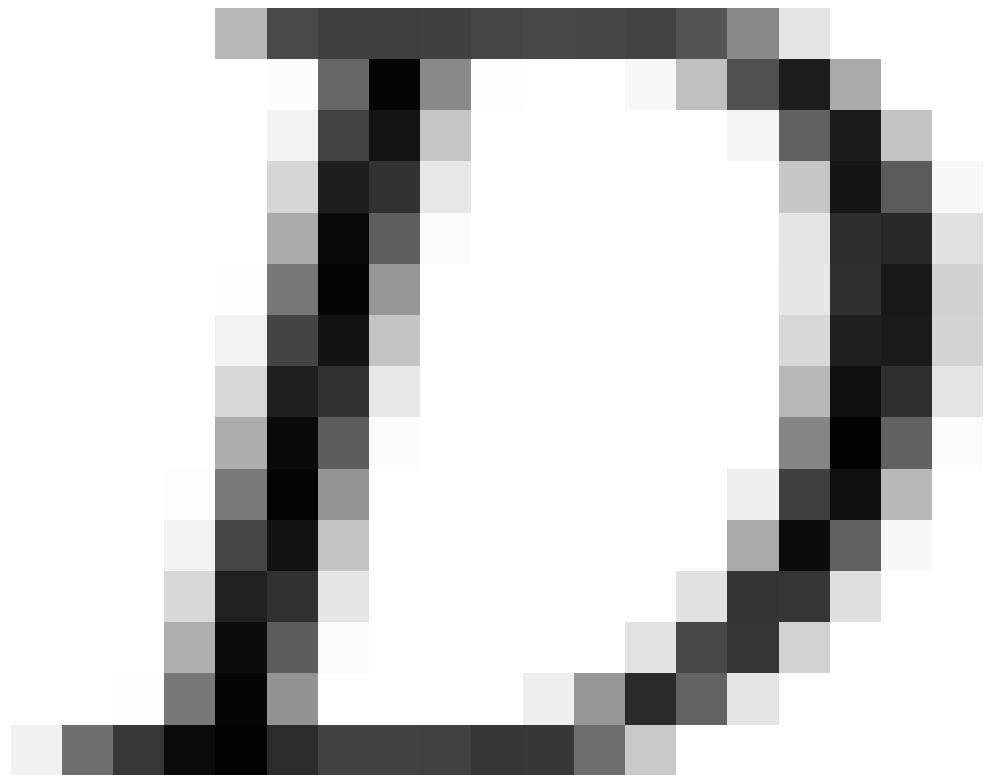
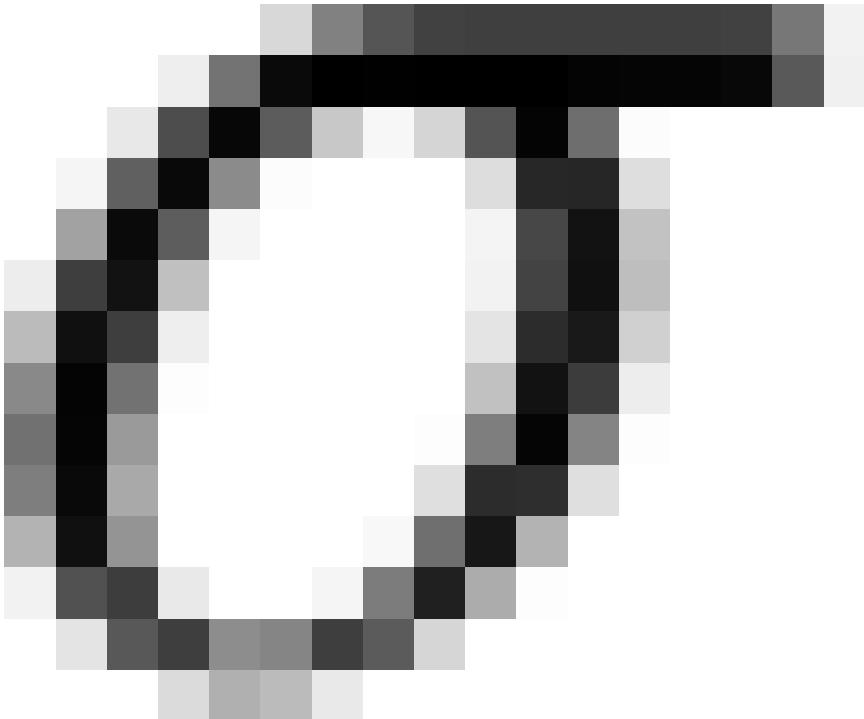




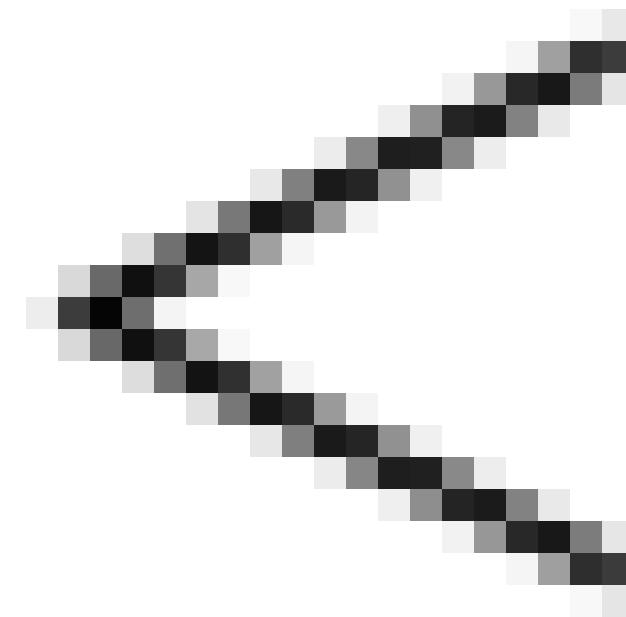












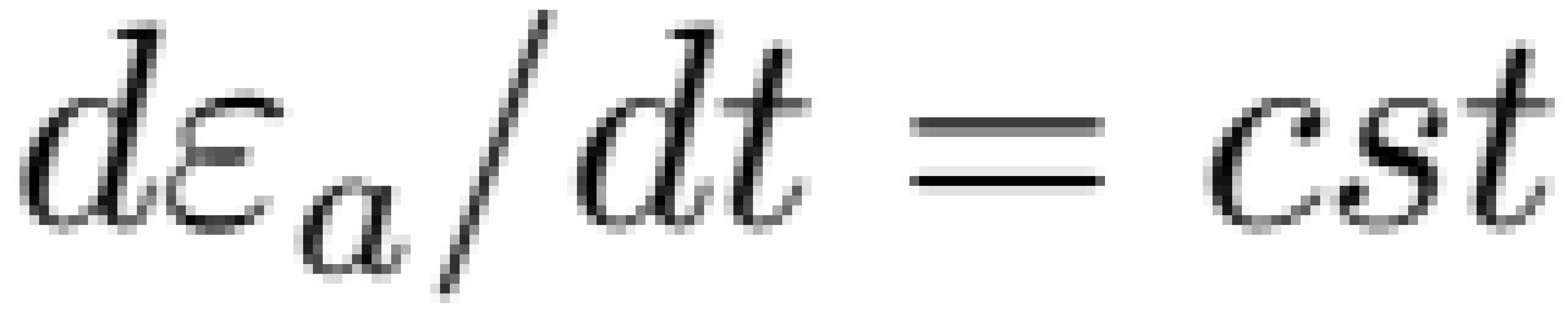










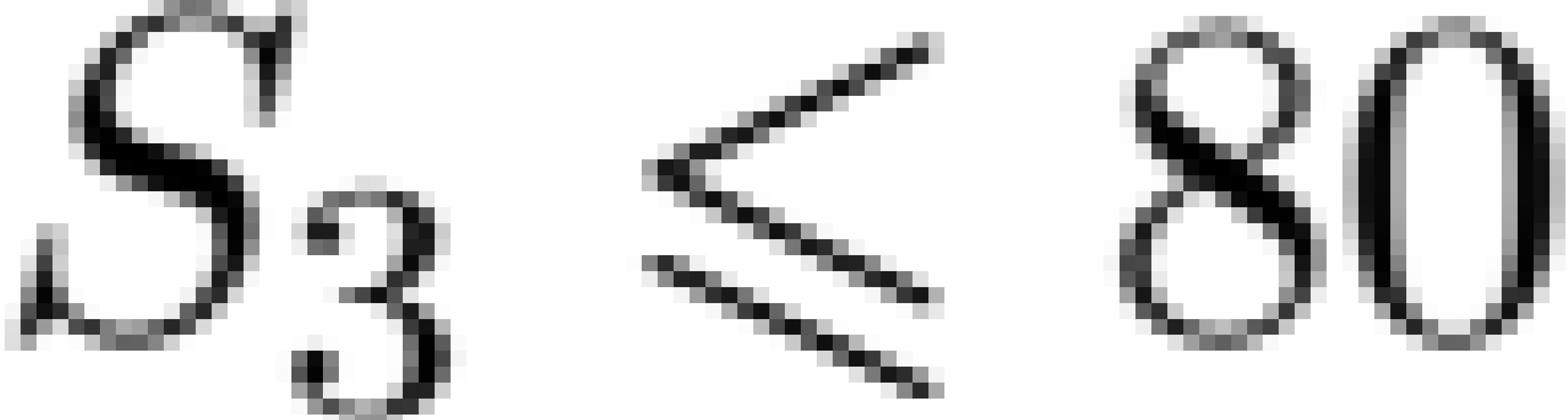


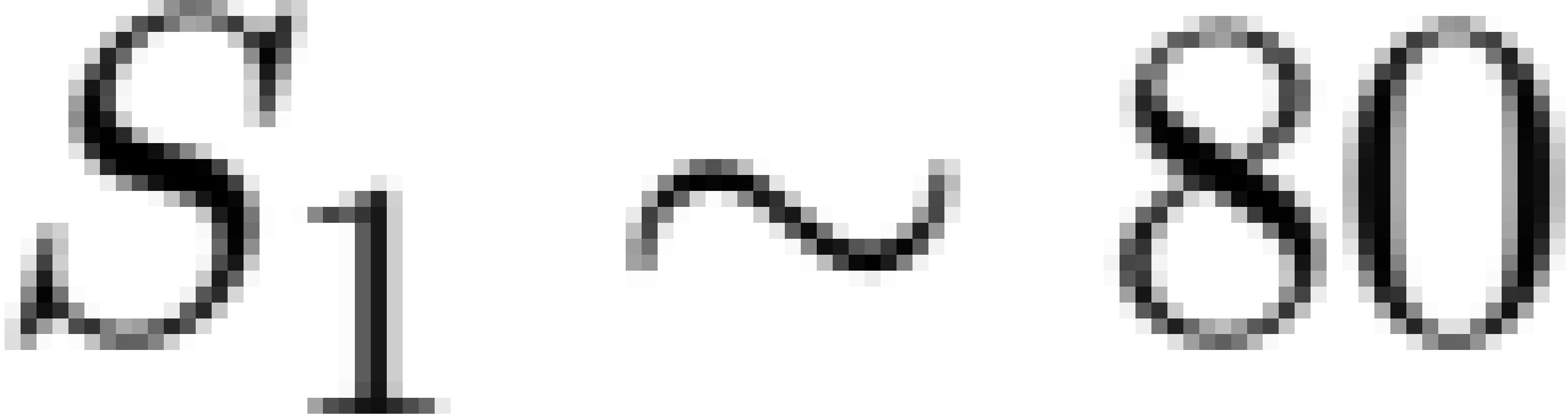


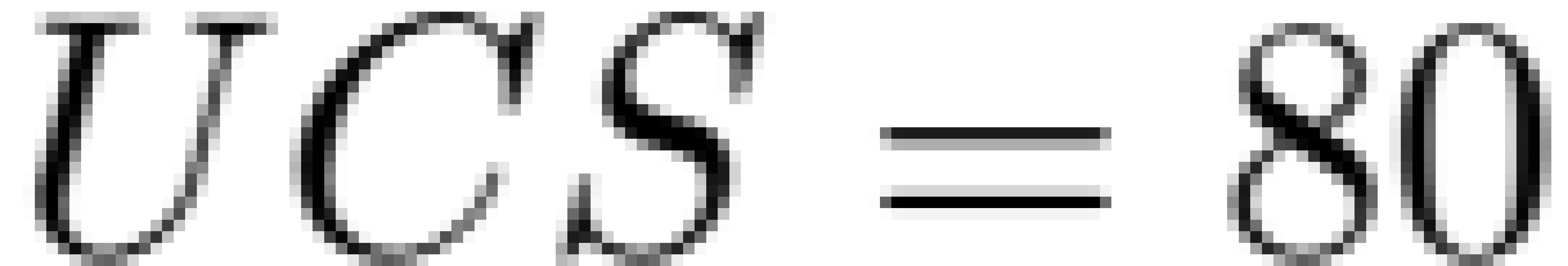


$$\varphi = \arctan$$

$$\left( \frac{q-1}{2\sqrt{q}} \right)$$









$q$

$=$

$\sigma_1$

$=$

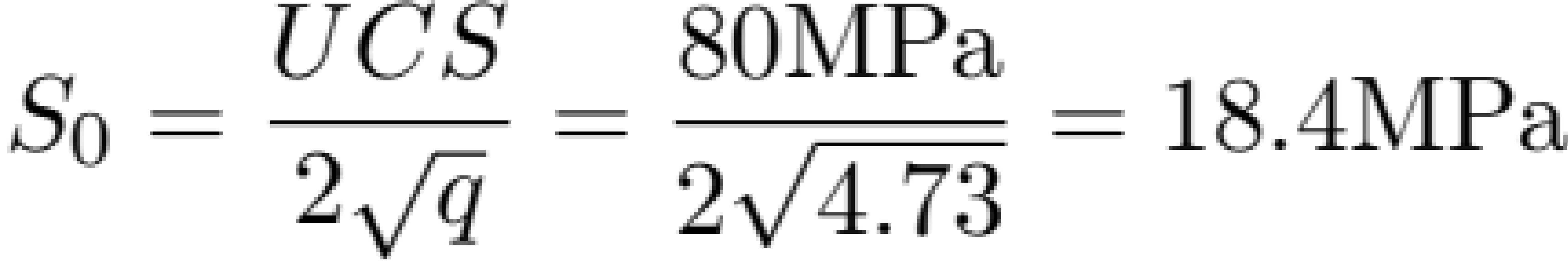
$\sigma_3$

$110 \text{ MPa}$

$520 \text{ MPa}$

$4.73$

$\text{MPa}$

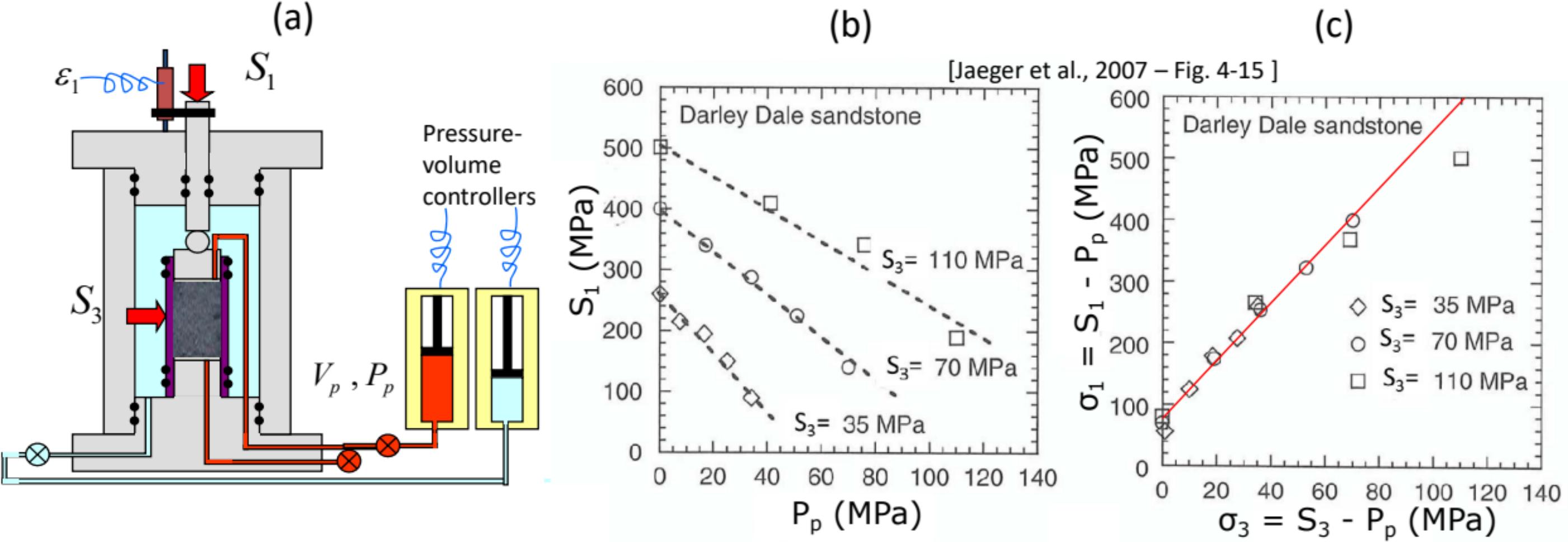


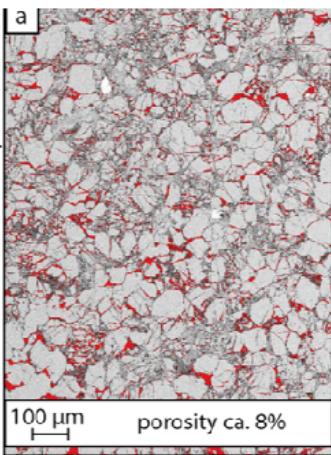
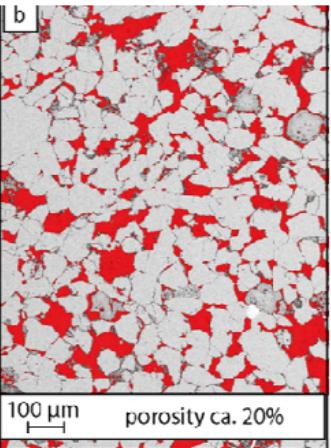
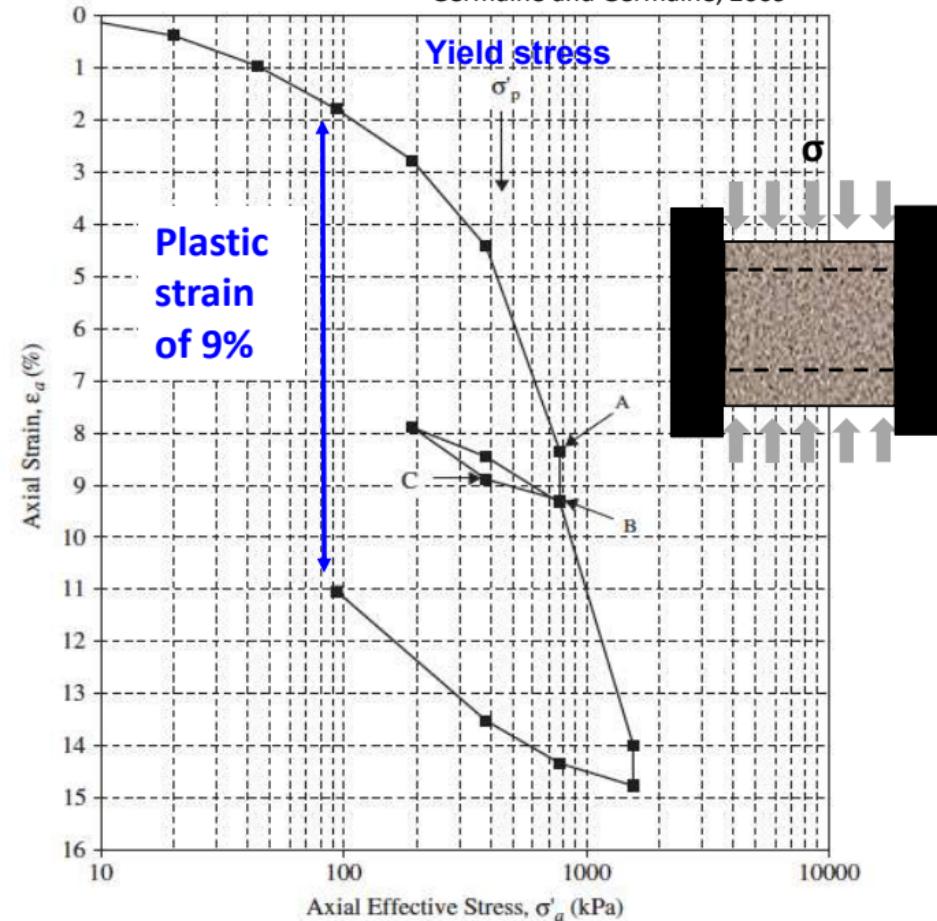
$$\varphi = \arctan \left( \frac{q-1}{2\sqrt{q}} \right) = \arctan \left( \frac{4.73 - 1}{2\sqrt{4.73}} \right) = 45.1^\circ$$

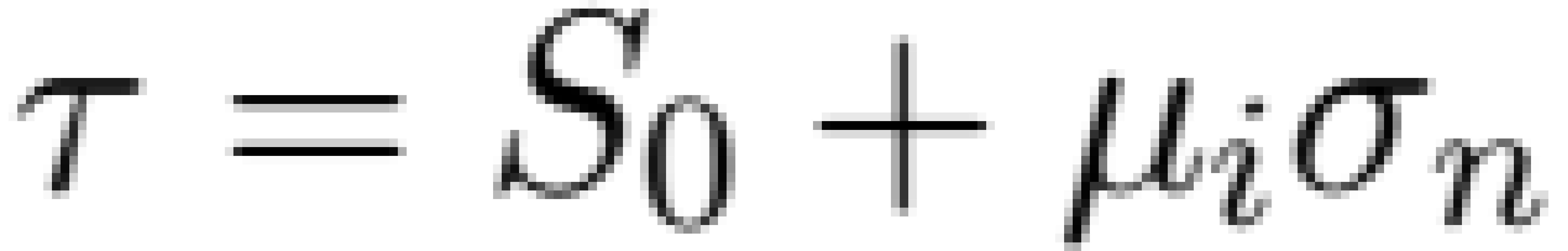


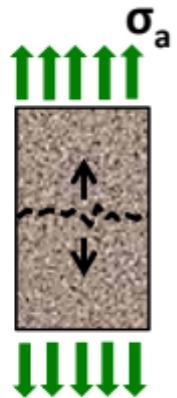








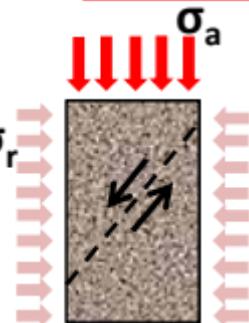


$\tau$ 

$$\sigma_n = T_s$$

 $T_s$ 

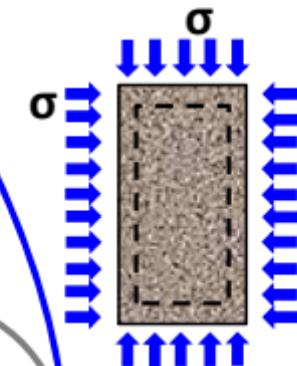
$$\boxed{\tau = \mu_i \sigma_n + S_0}$$



1

 $\mu_i$ 

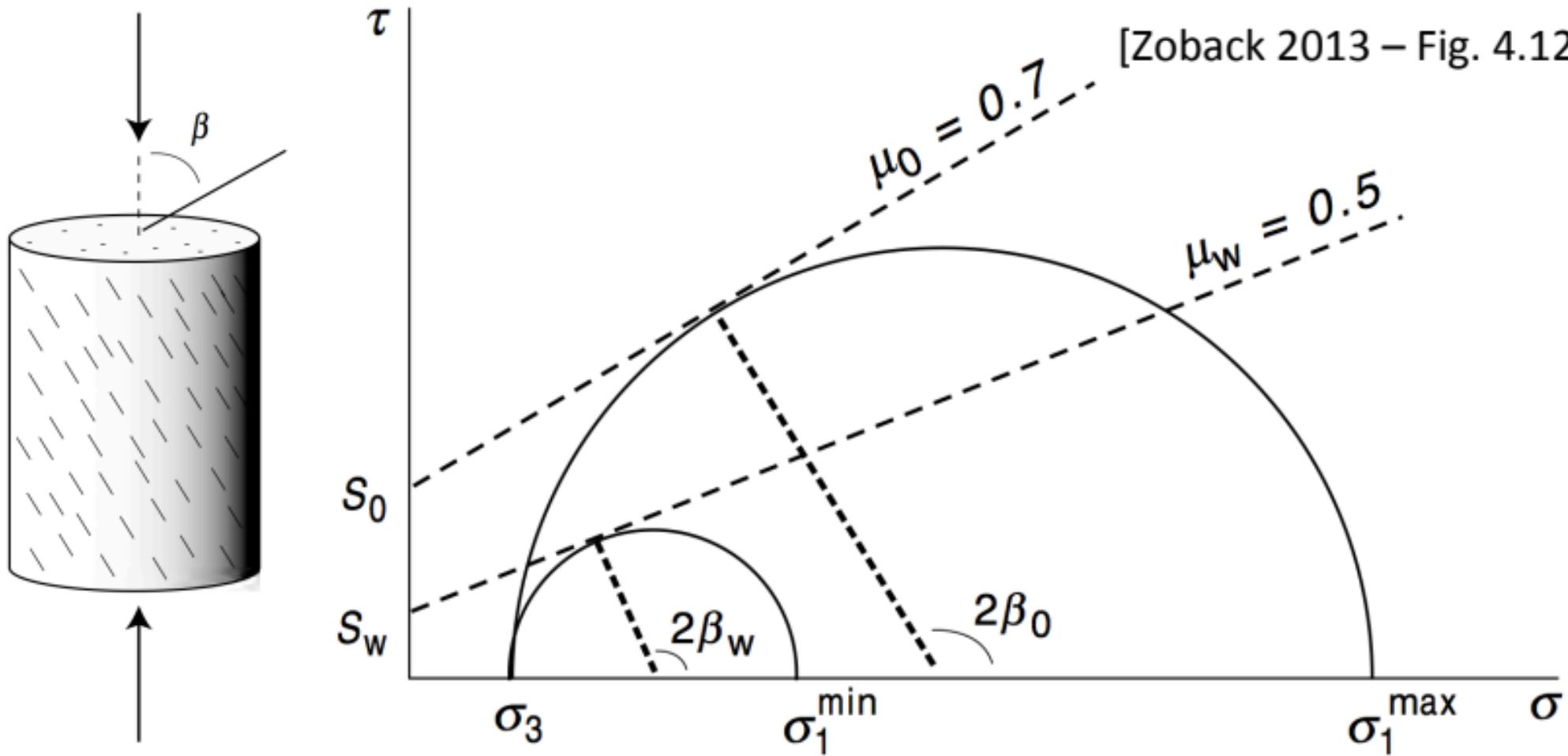
$$\boxed{\mathcal{E}_V^p = \mathcal{E}_{Vcrit}^p}$$

 $\sigma_n$





[Zoback 2013 – Fig. 4.12]

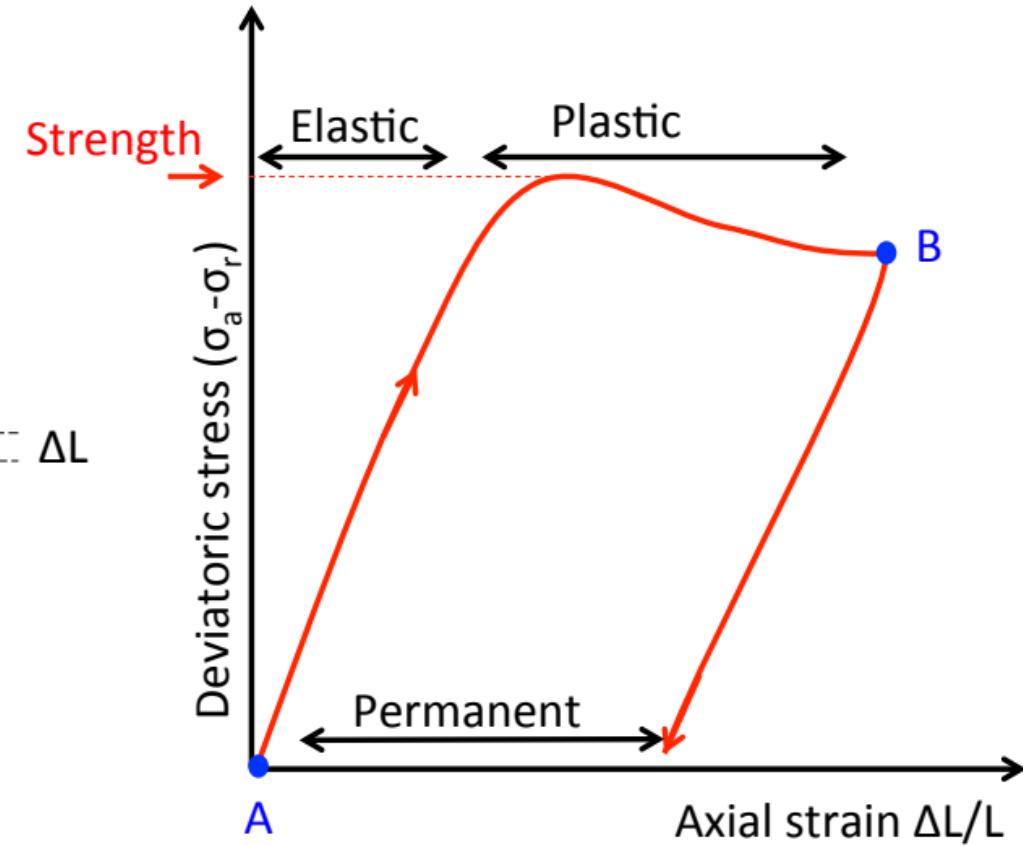
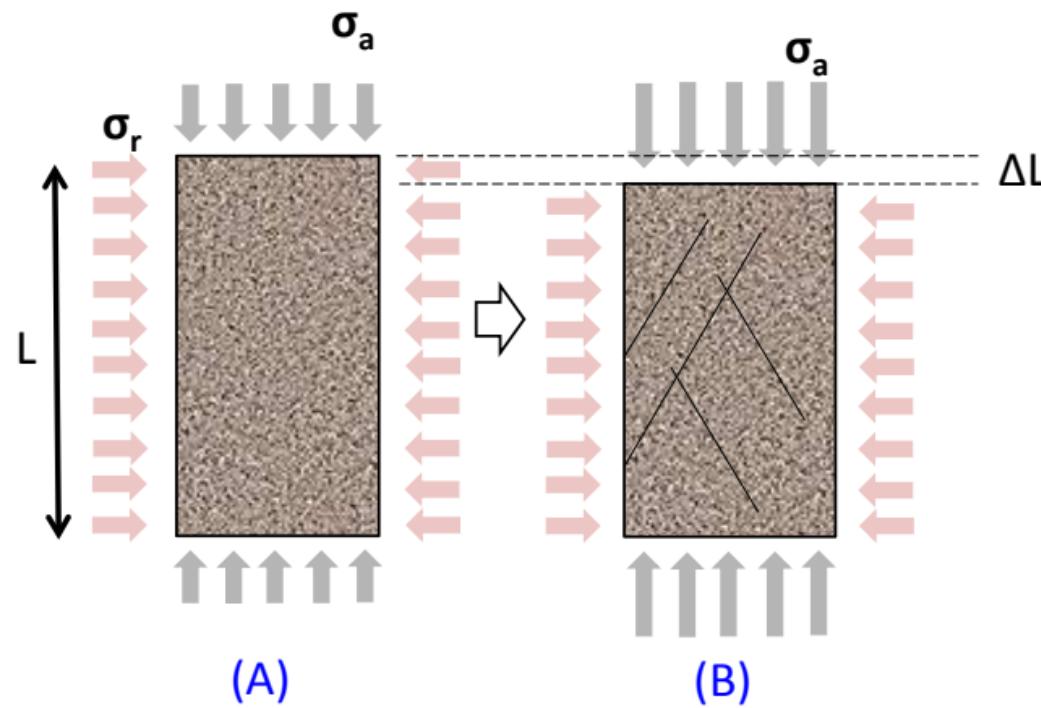


# Rock deformation

Relation strain V.S. stress

Elastic (Young modulus)

Plastic (~Viscosity)



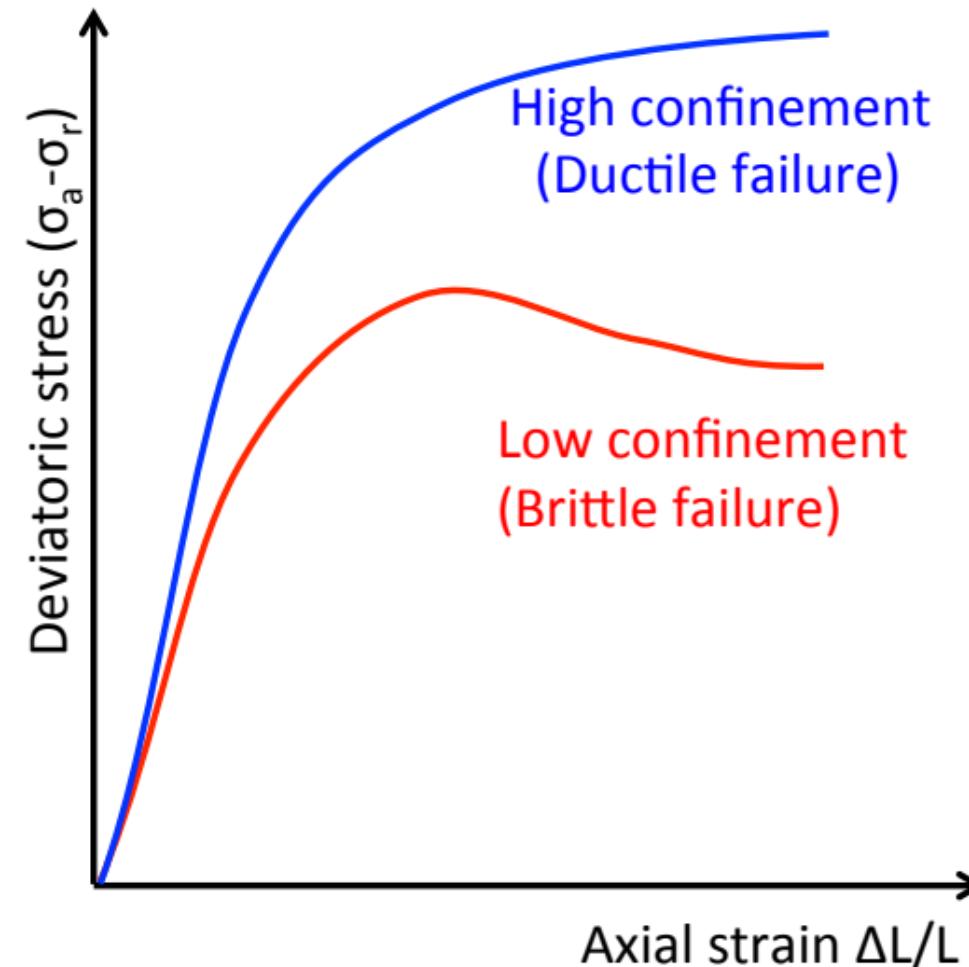
a. This is an undeformed cylinder of rock.



b. This cylinder was subjected to high confining pressure (uniform in all directions) and, at the same time, compression from above. It deformed in a ductile manner, becoming shorter and fatter.



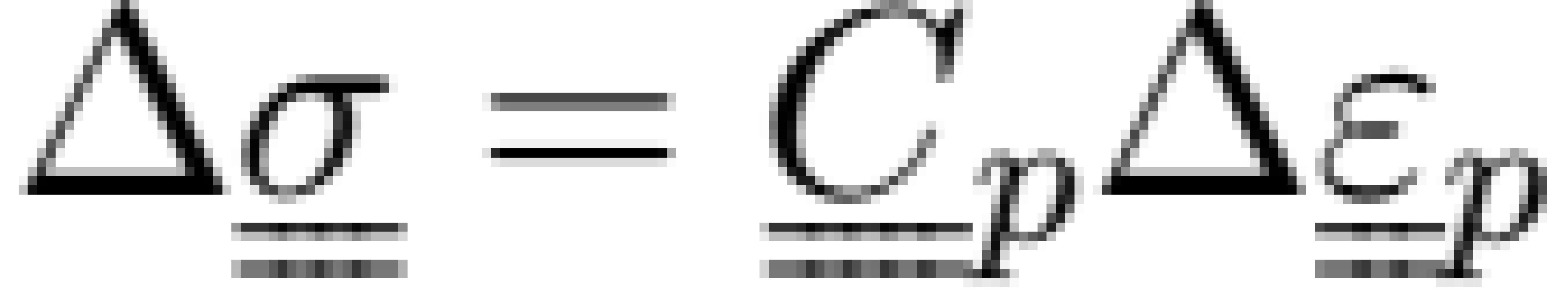
c. An identical cylinder was subjected to the same amount of compression from above, but this time with a lower confining pressure. It deformed in a brittle manner, with many large fractures.



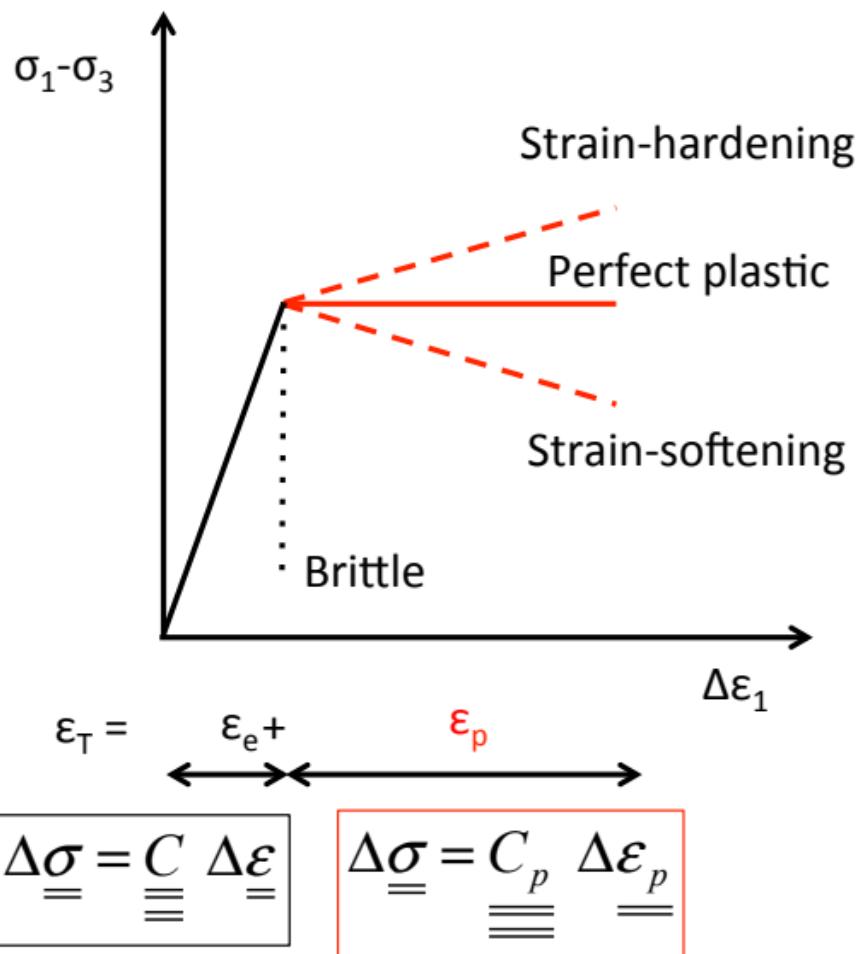
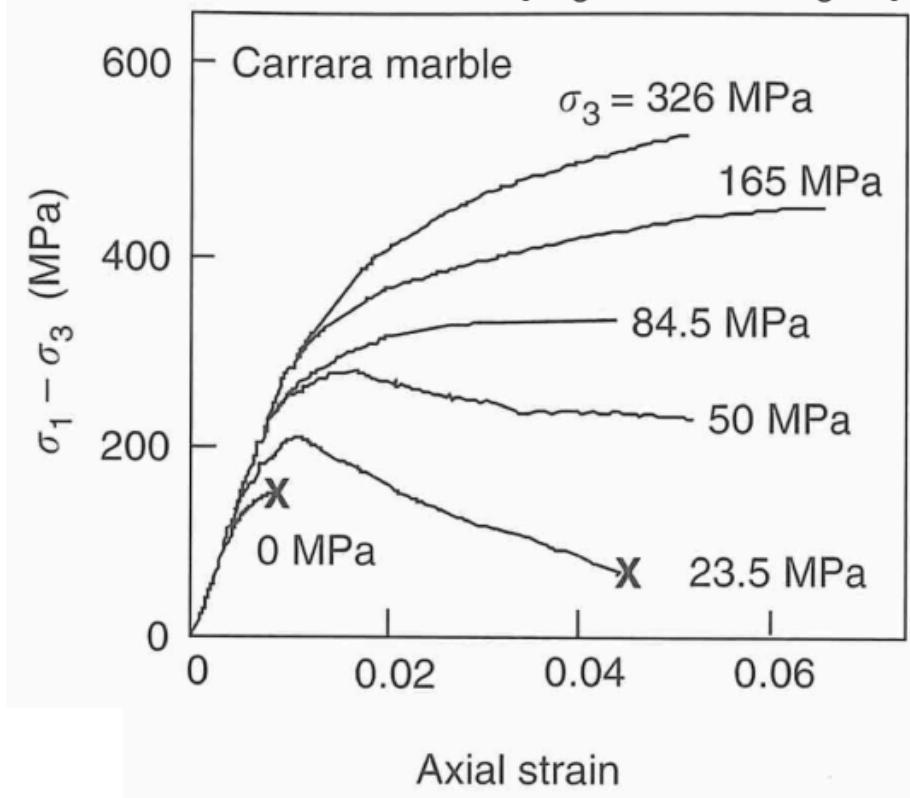




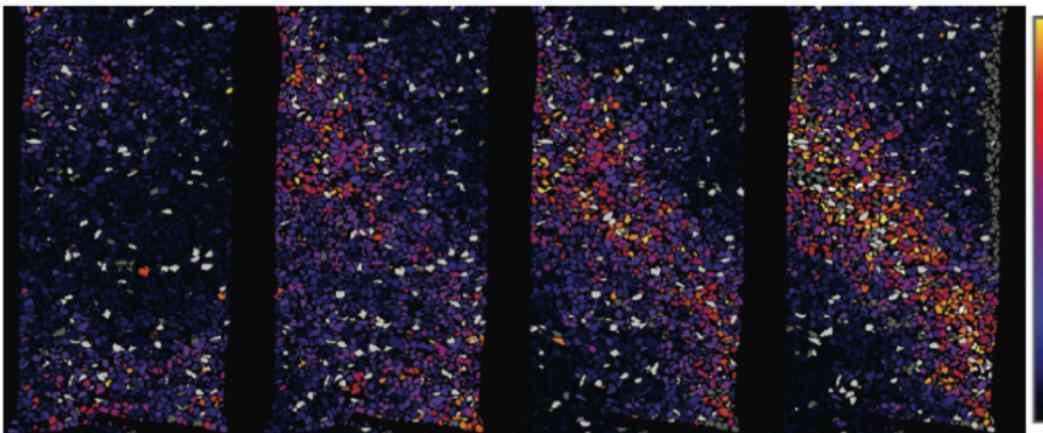
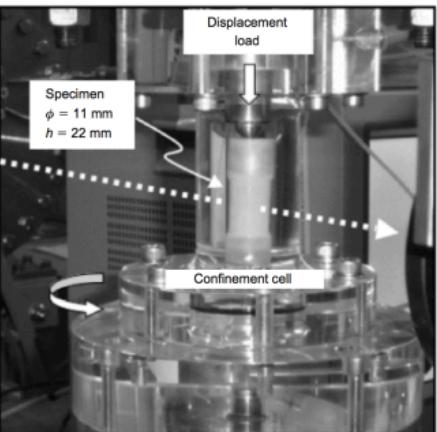




[Jaeger et al. 2007 – Fig. 4.5]



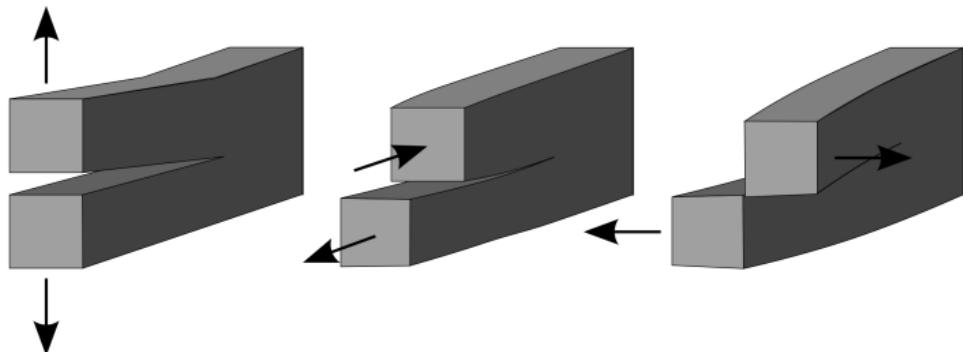
**(a) Uncemented or poorly cemented rock →**



grain friction, dilation, grain crushing/rotation

**(b) Cemented rock →**

Propagation of microfractures, grain friction/crushing

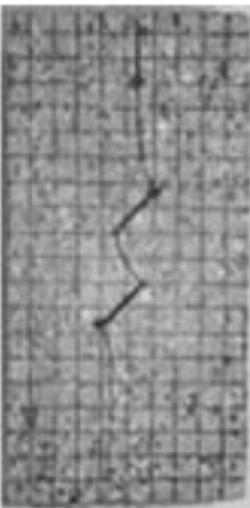
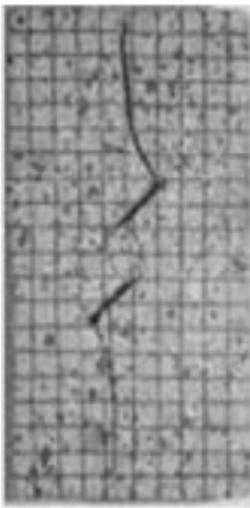
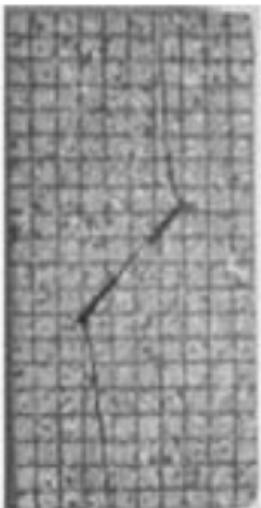
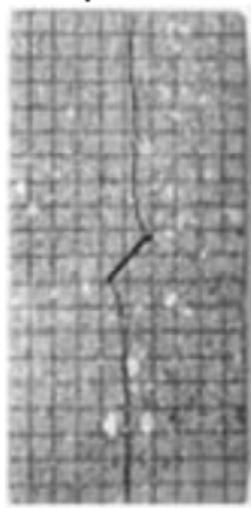


Stress intensification at  
the tip of fractures

Propagation starts at  
fracture tips

# Napolitan Tuffo

[Hall et al. 2006 – Pure Appl. Geophys.]



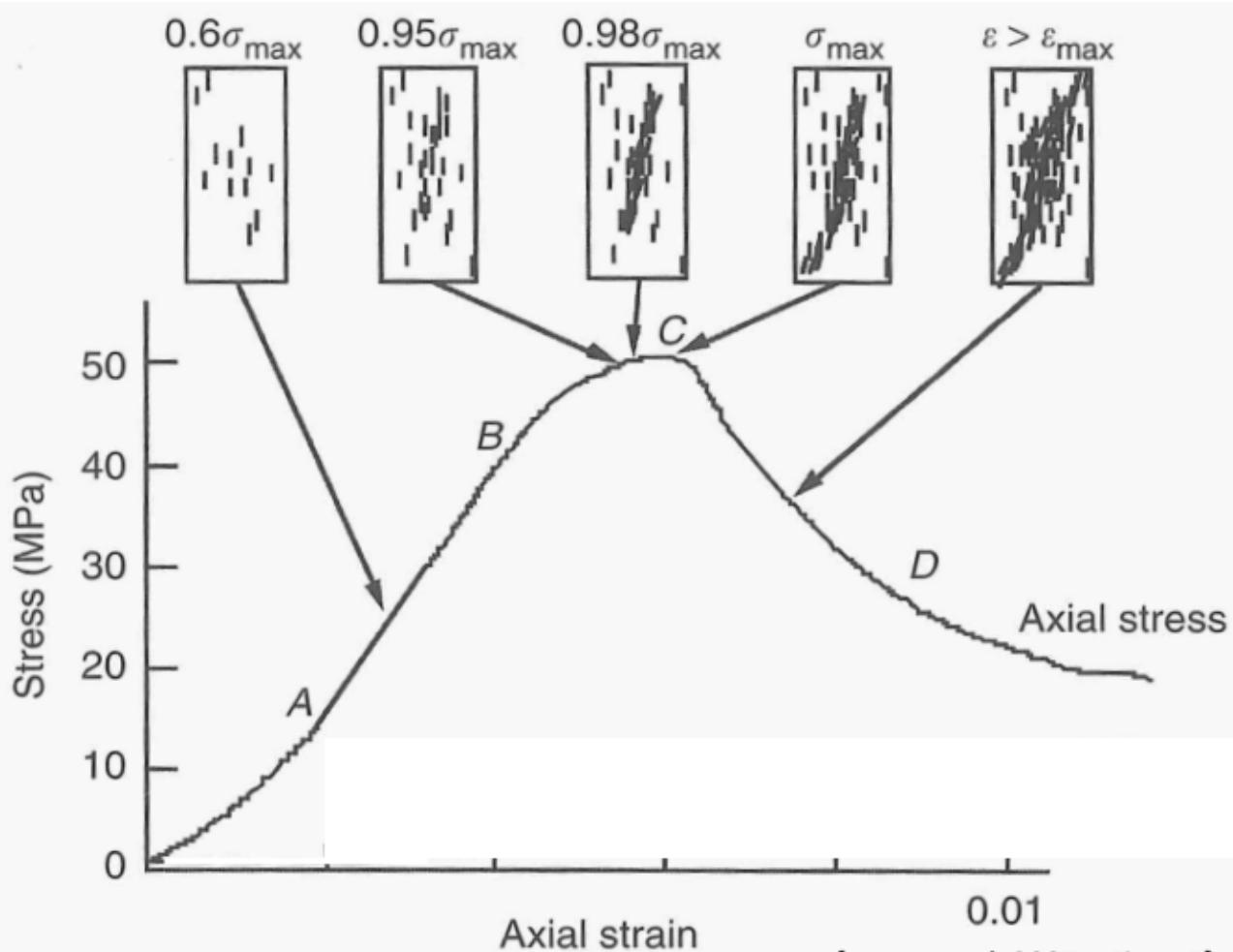
Single flaw

$\beta = 45^\circ$

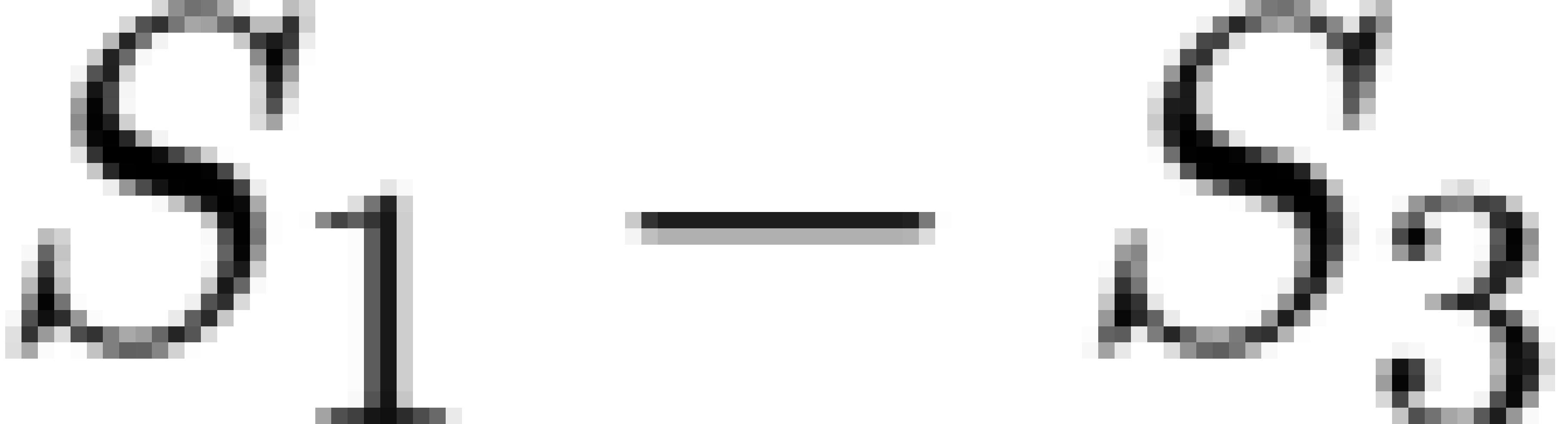
$\beta = 105^\circ$

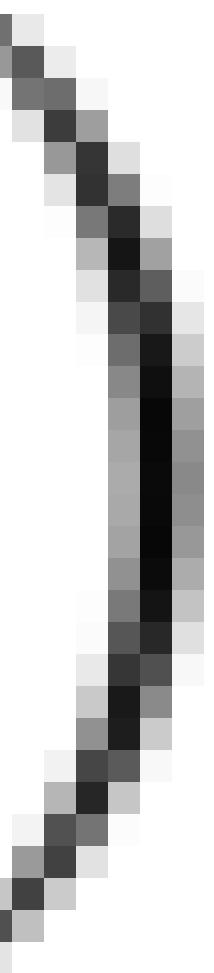
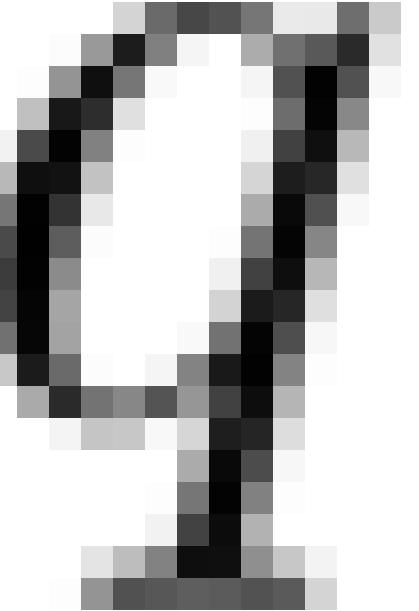
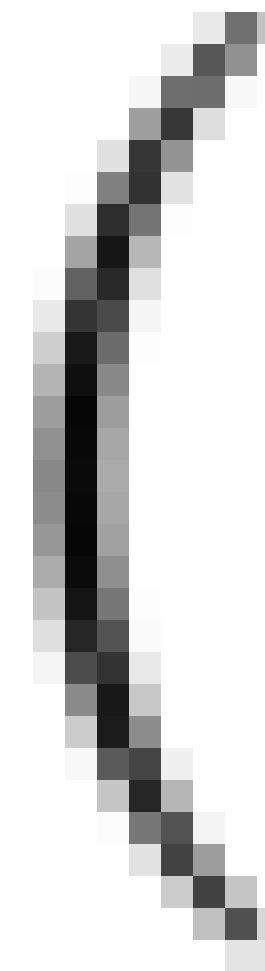
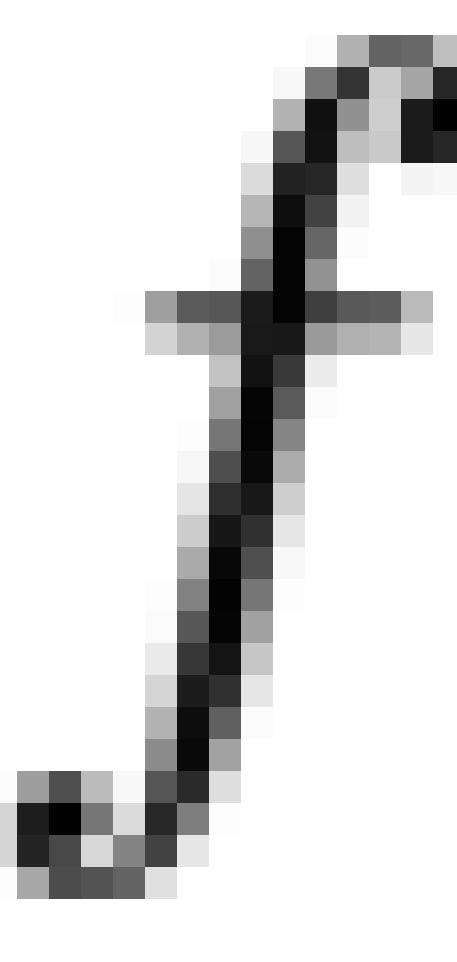
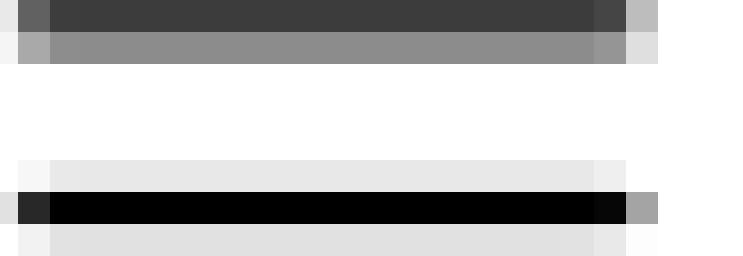
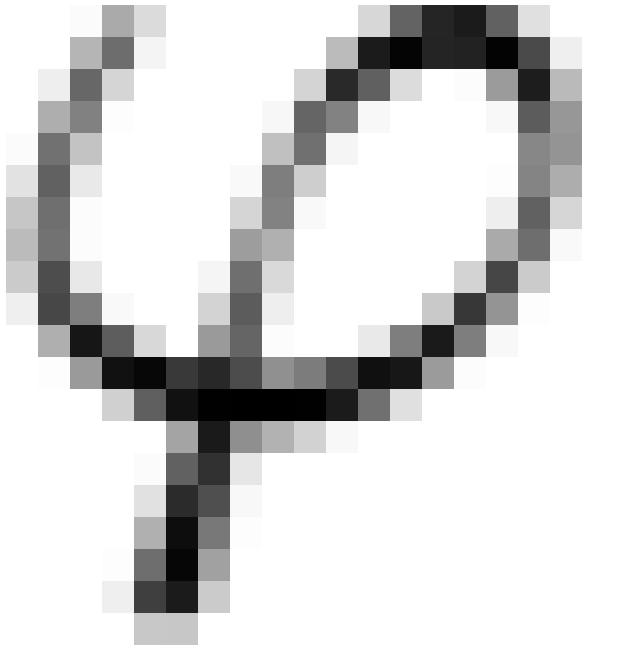
$\beta = 120^\circ$





[Jaeger et al. 2007 – Fig. 4.5]

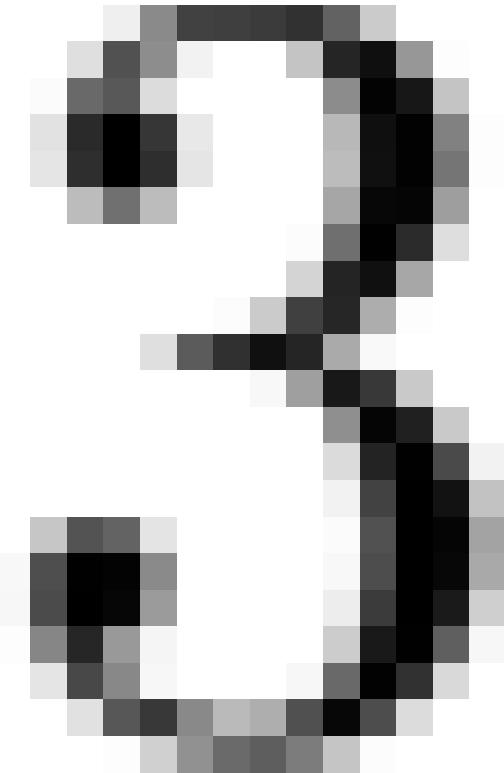
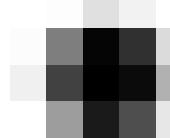
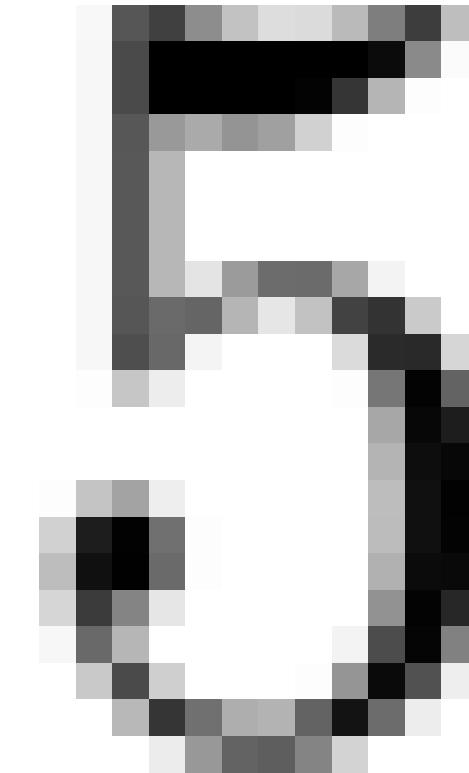
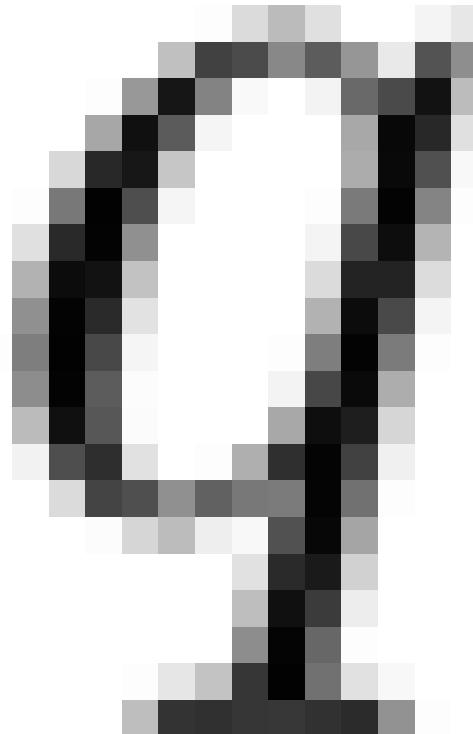












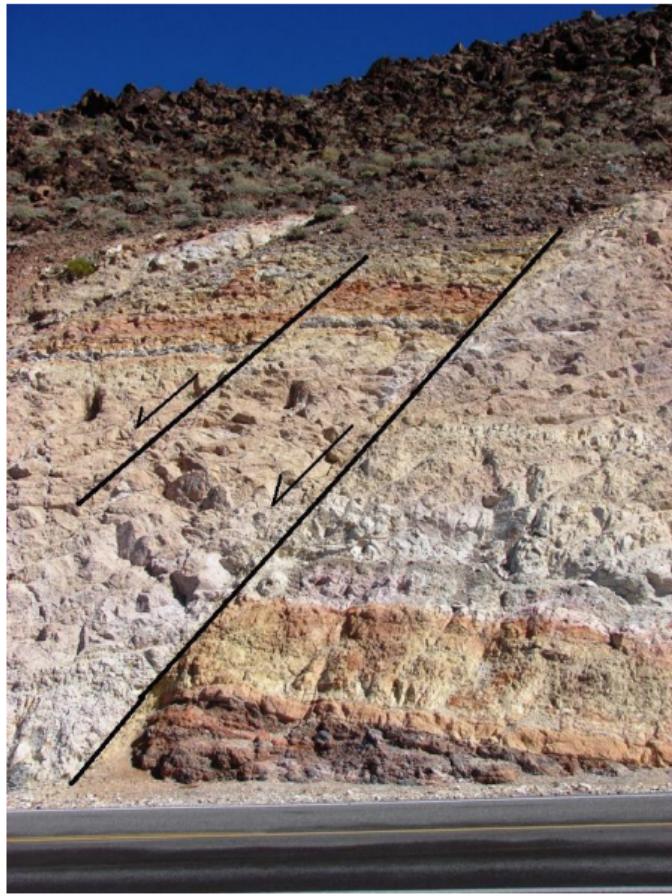






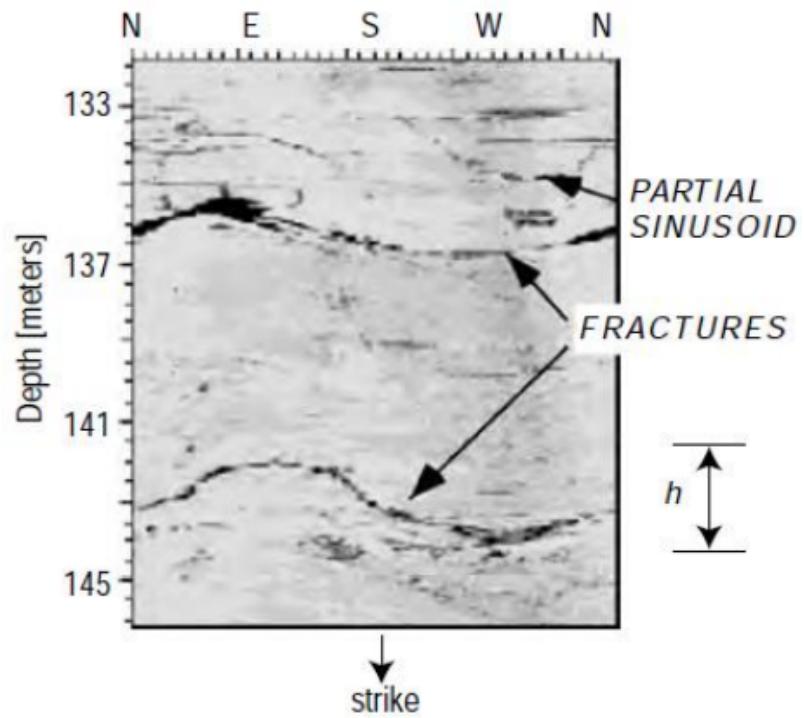
Outcrop of a normal fault in Split Mountain gorge

<http://geology.csupomona.edu/janourse/TectonicsFieldTrips.htm>

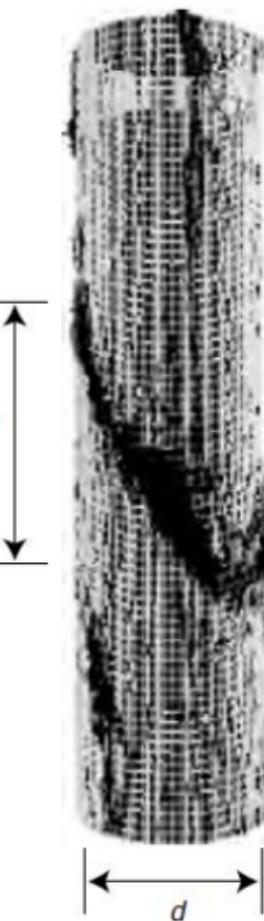


<http://geotripperimages.com/images/DSC03438%20Charlie%20Brown%20b.jpg>

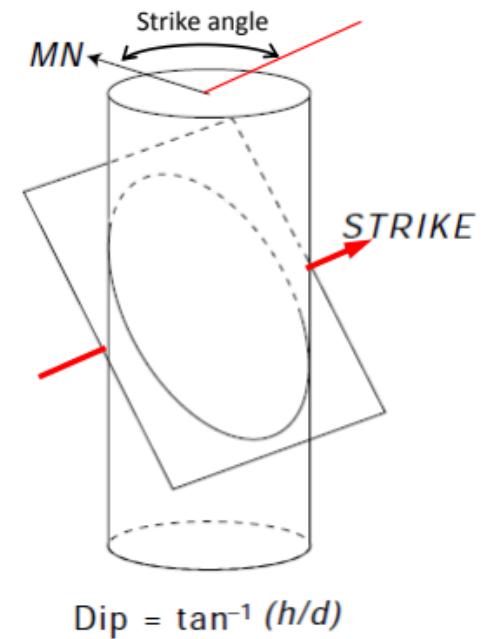
**(a) Un-wrapped image  
(ultrasonic)**



**(b) 3D-representation**



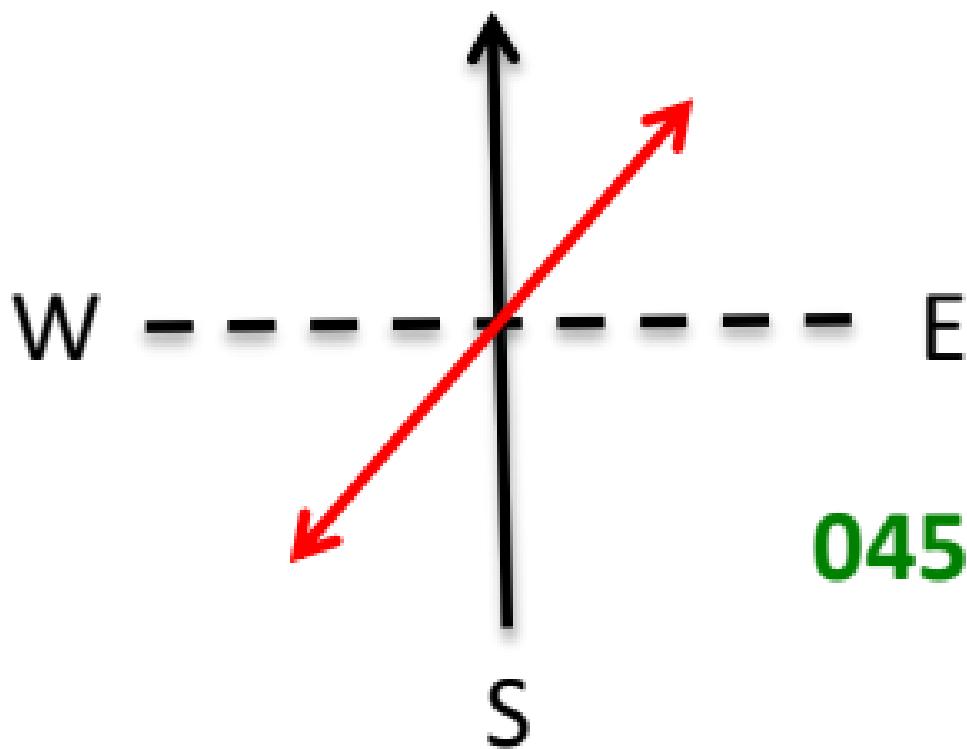
**(c) Interpretation**



[Zoback 2013 - Figure 5.3]



**N45°E==S45°W**



**045° == 225°**

# Geologic map

Figure from Prof. Prodanovic



a)

$N30^\circ E$ ,  $45^\circ NW$

or

$030^\circ$ ,  $45^\circ NW$



b)

$N10^\circ W$ ,  $80^\circ NE$

or

$350^\circ$ ,  $80^\circ NE$



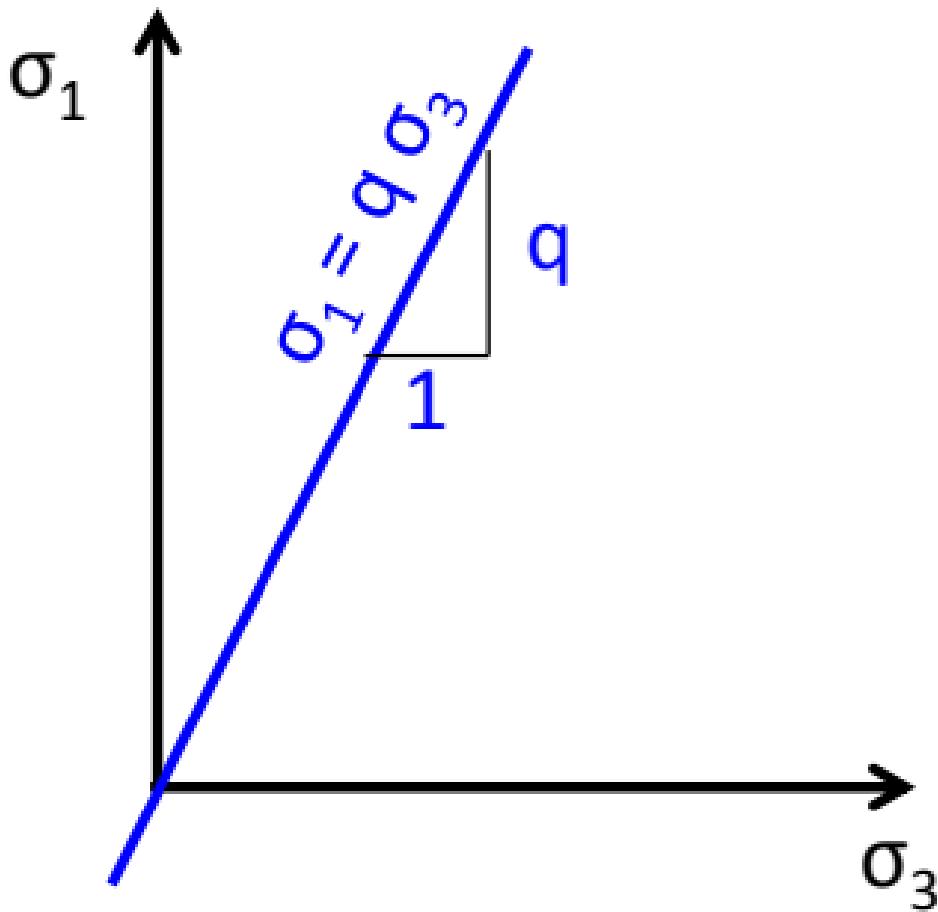
c)

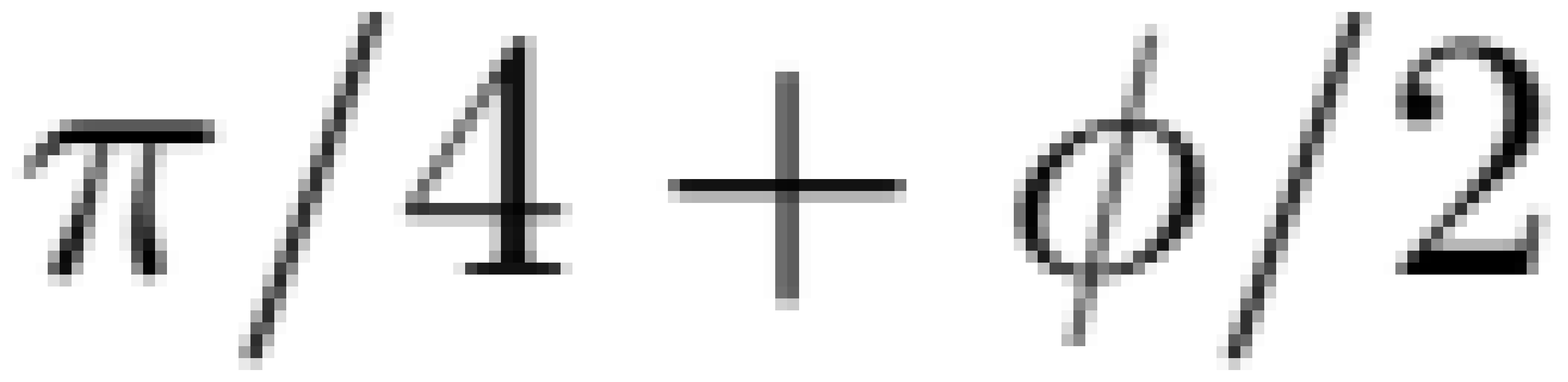
Symbols for  
horizontal plane  
and vertical plane

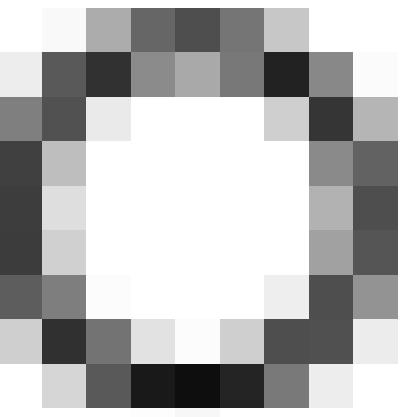
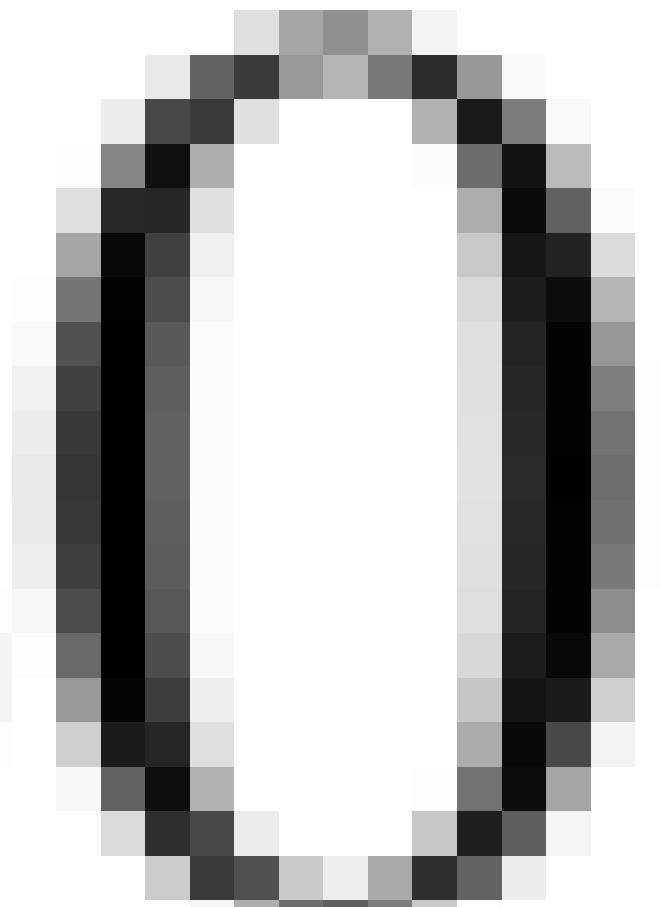
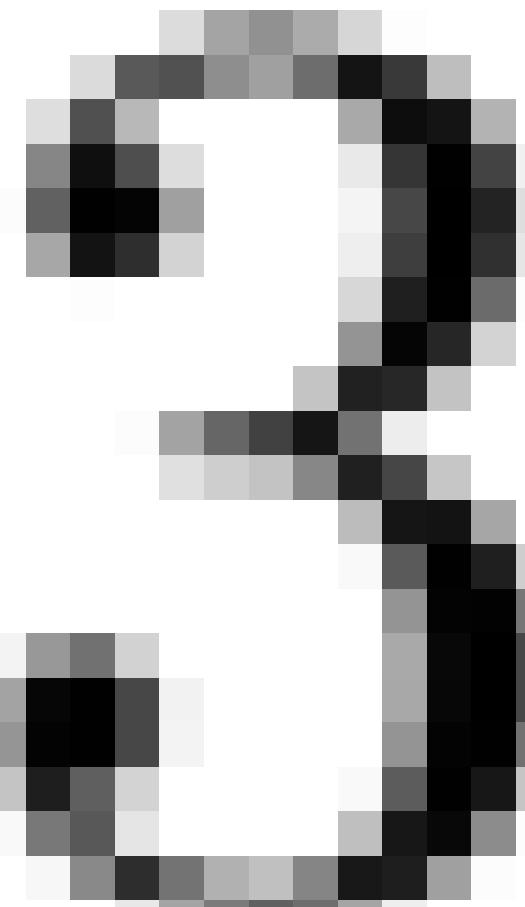
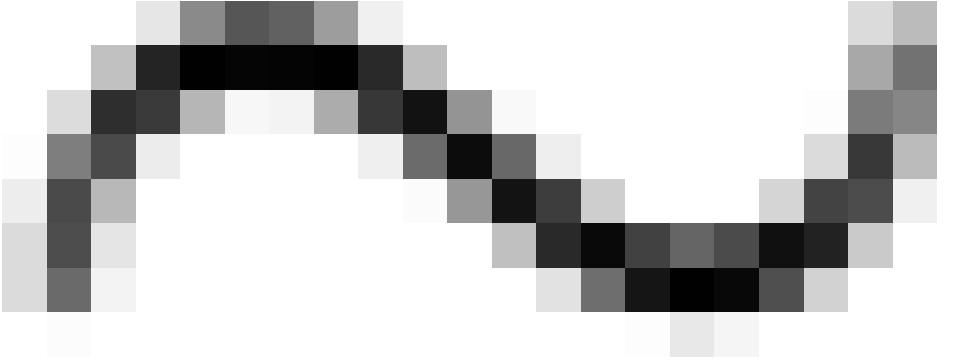


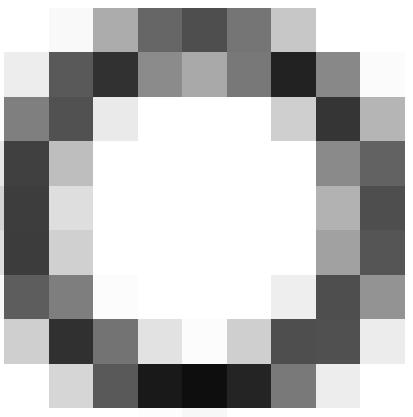
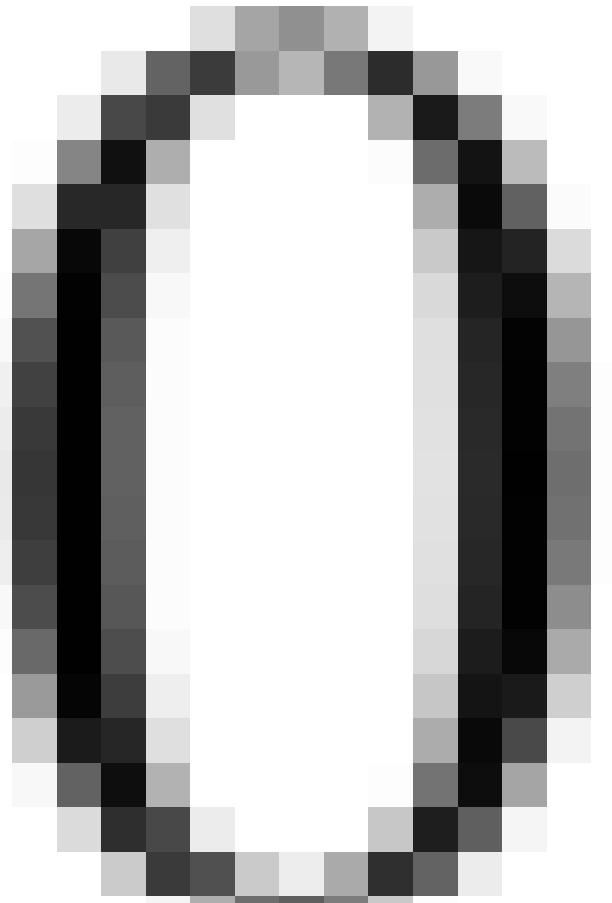
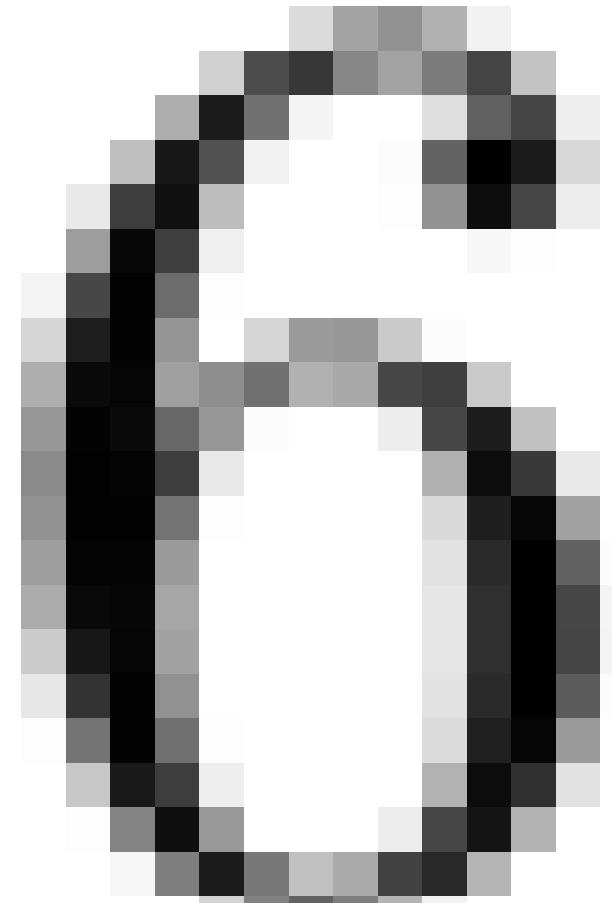
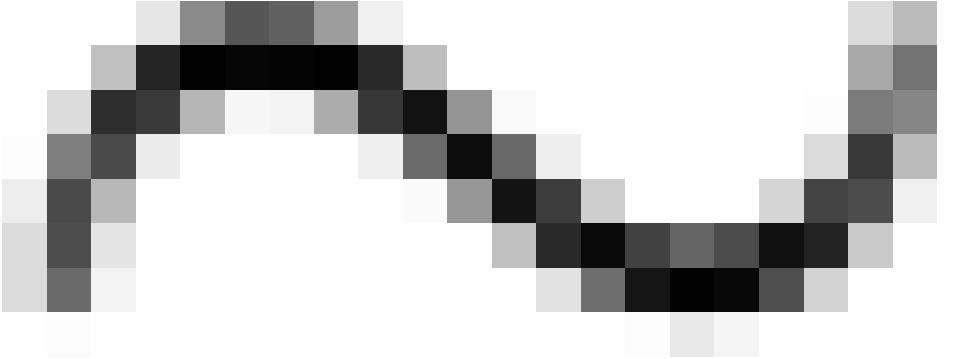
$$\begin{aligned} q &= \sin^2 \varphi + 1 \\ &= 1 - \sin^2 \varphi \end{aligned}$$

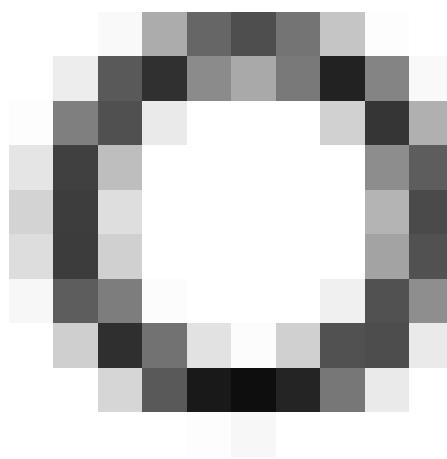
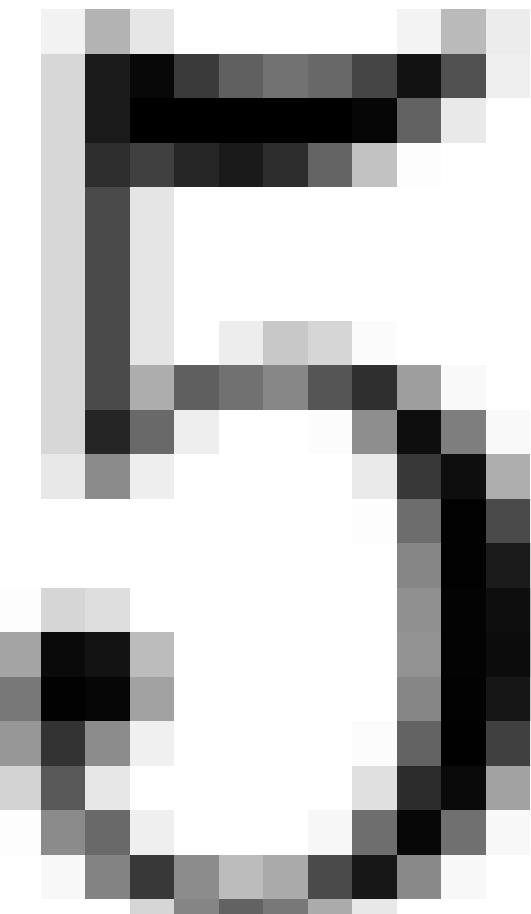
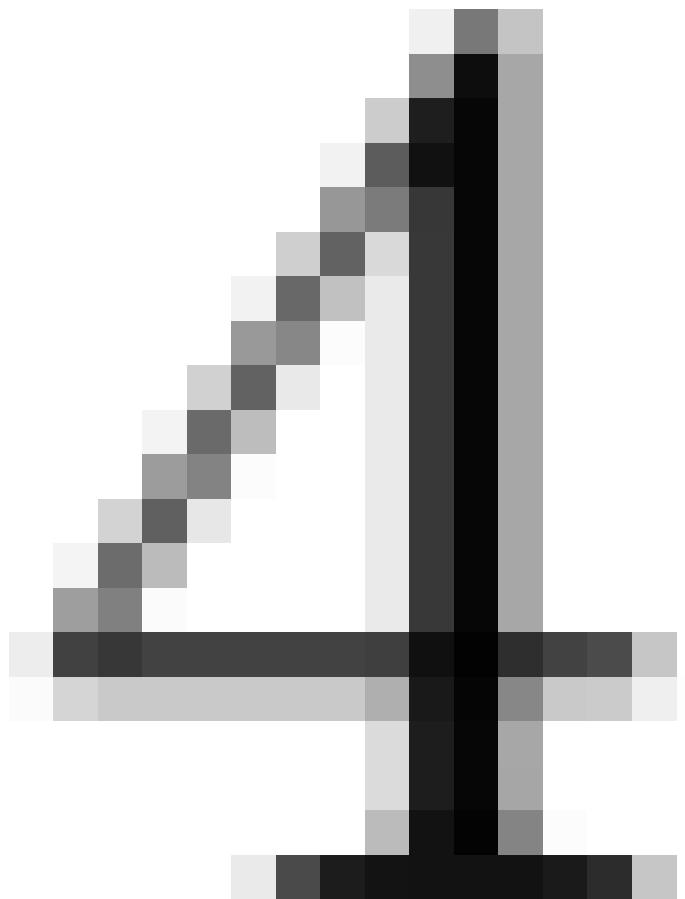


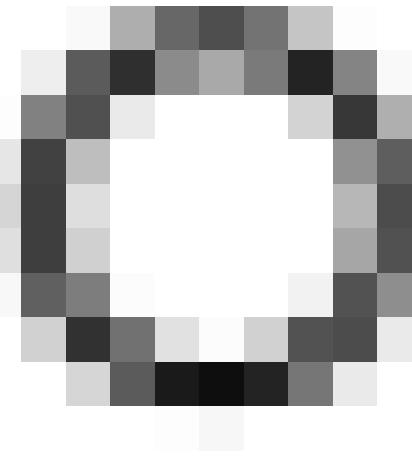
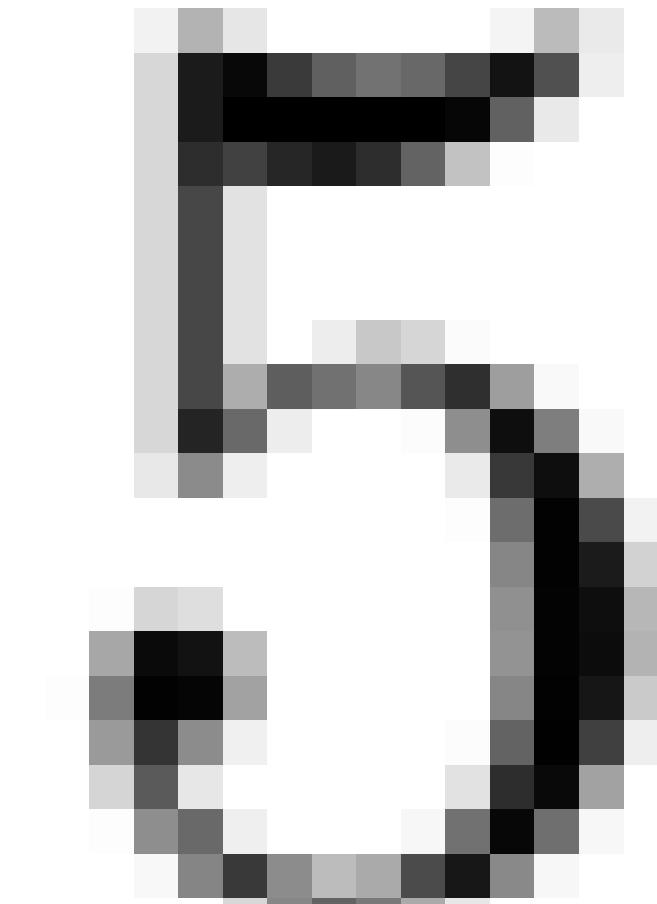
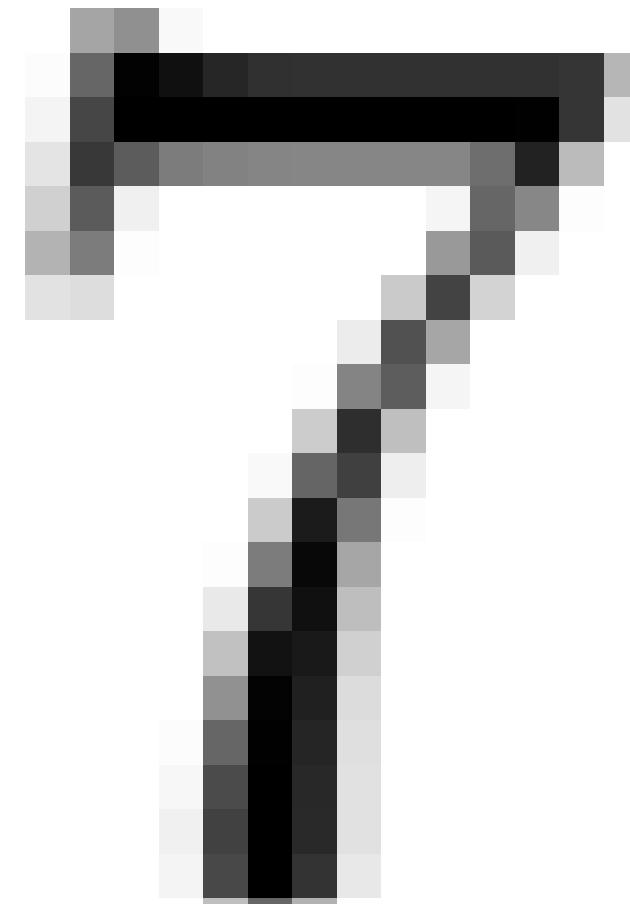
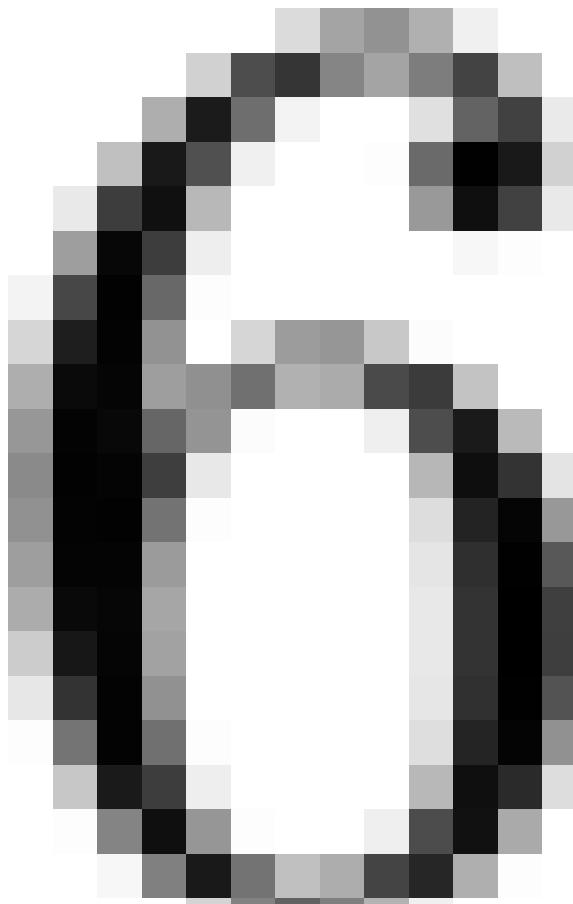


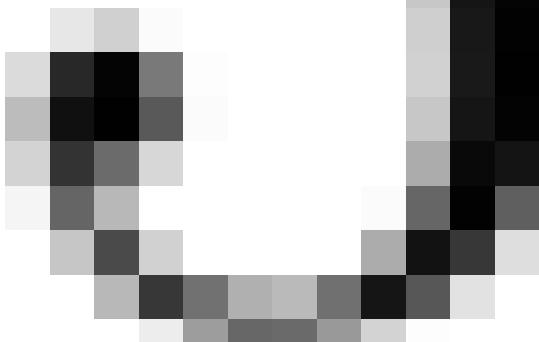
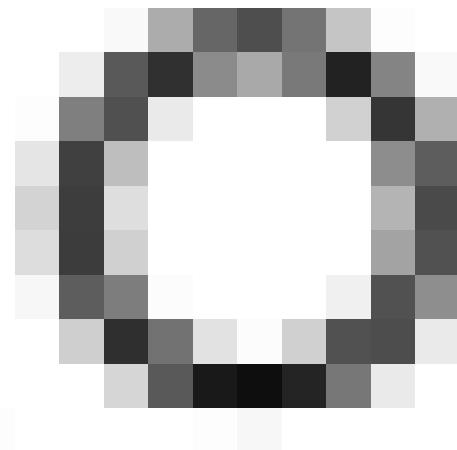
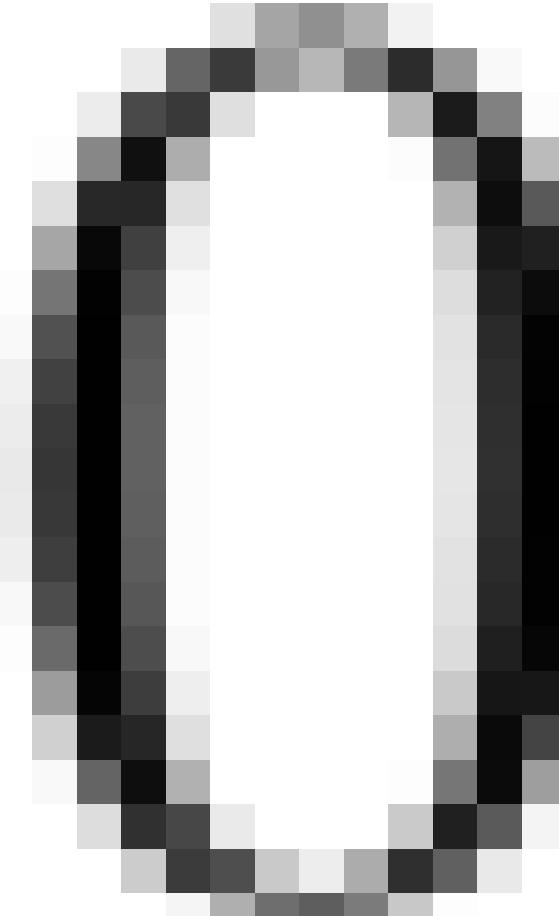
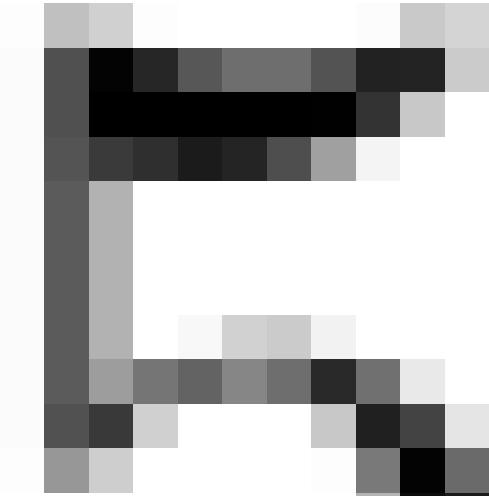


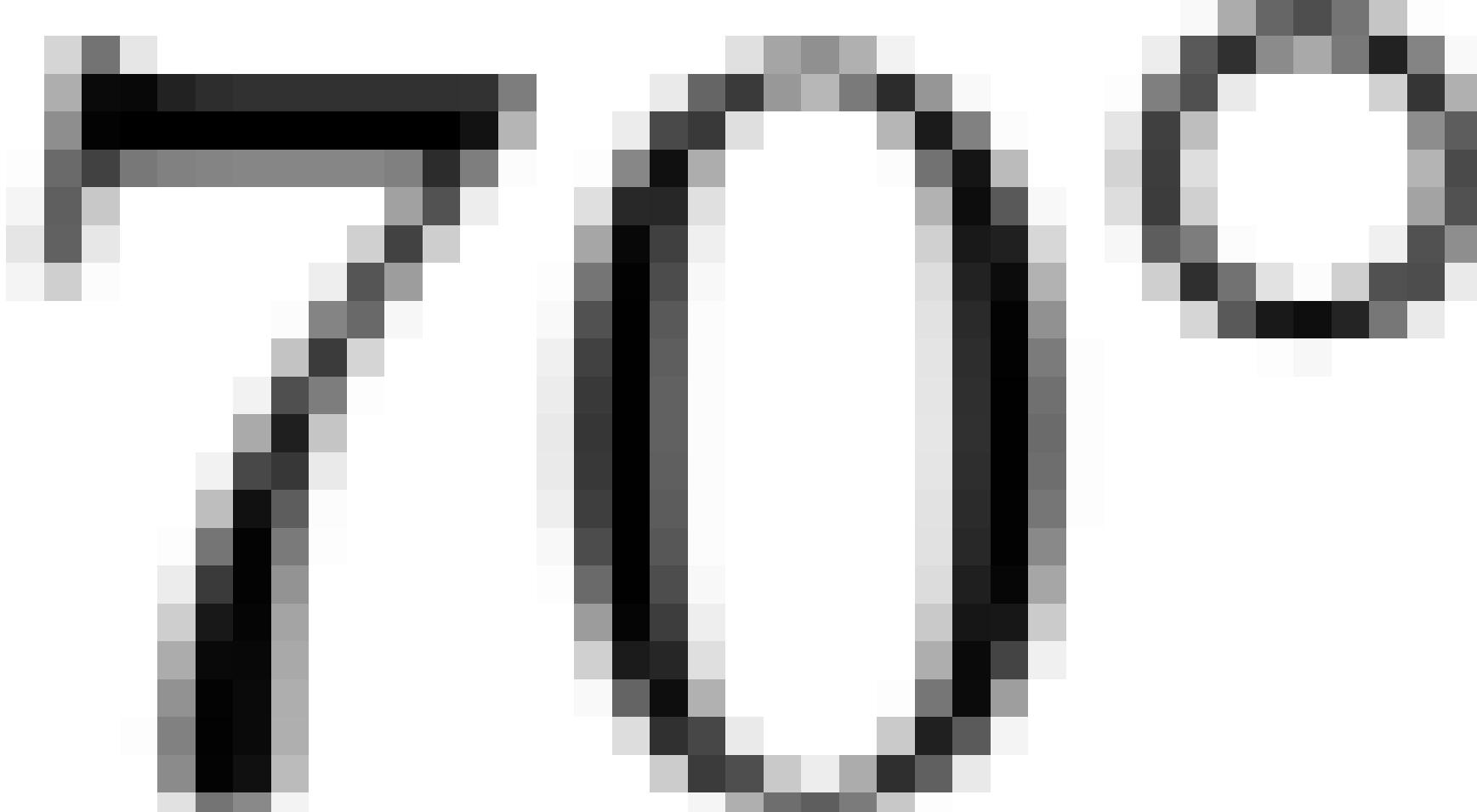


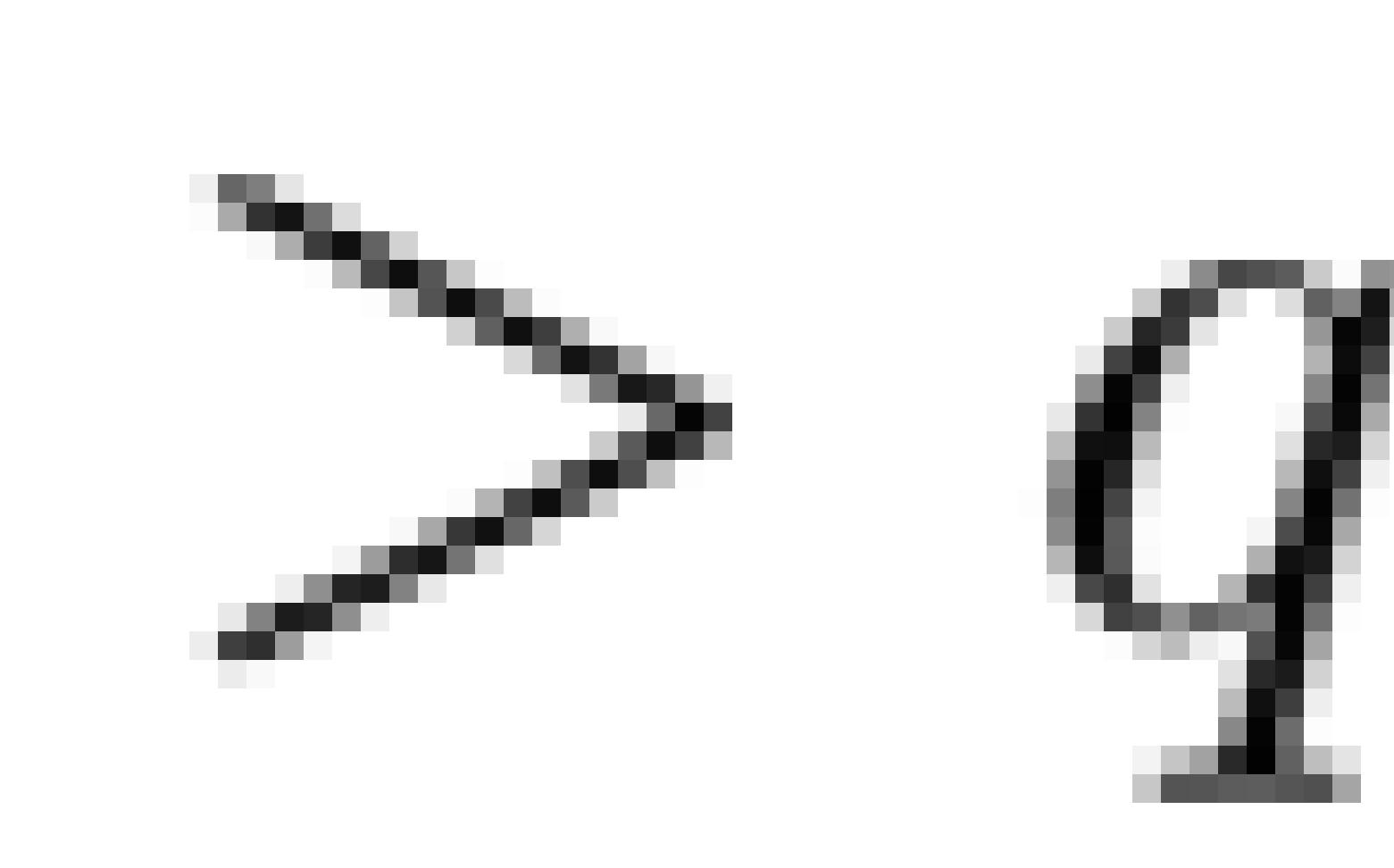














$$S_1 = s_v$$

$$S_2 = s_{H\max}$$

$$S_3 = s_{hmin}$$



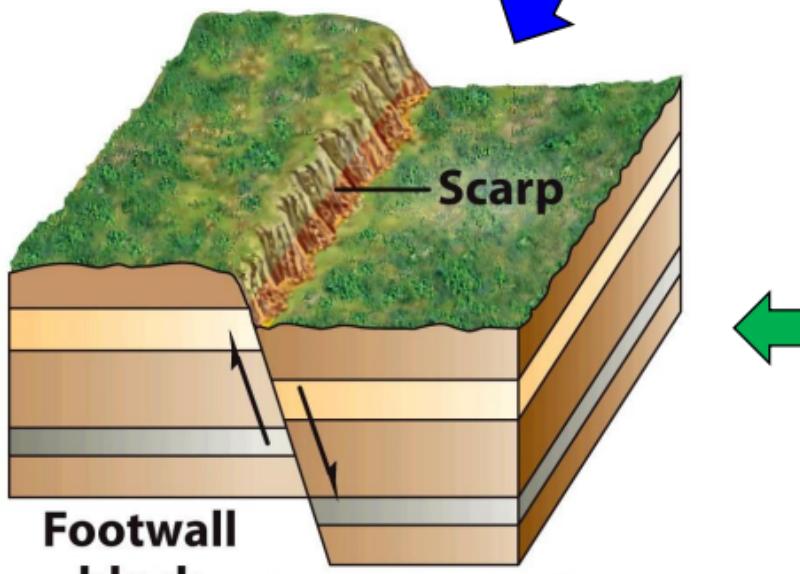


Figure 9.10b  
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$$S_v > S_{h\max} > S_{h\min}$$



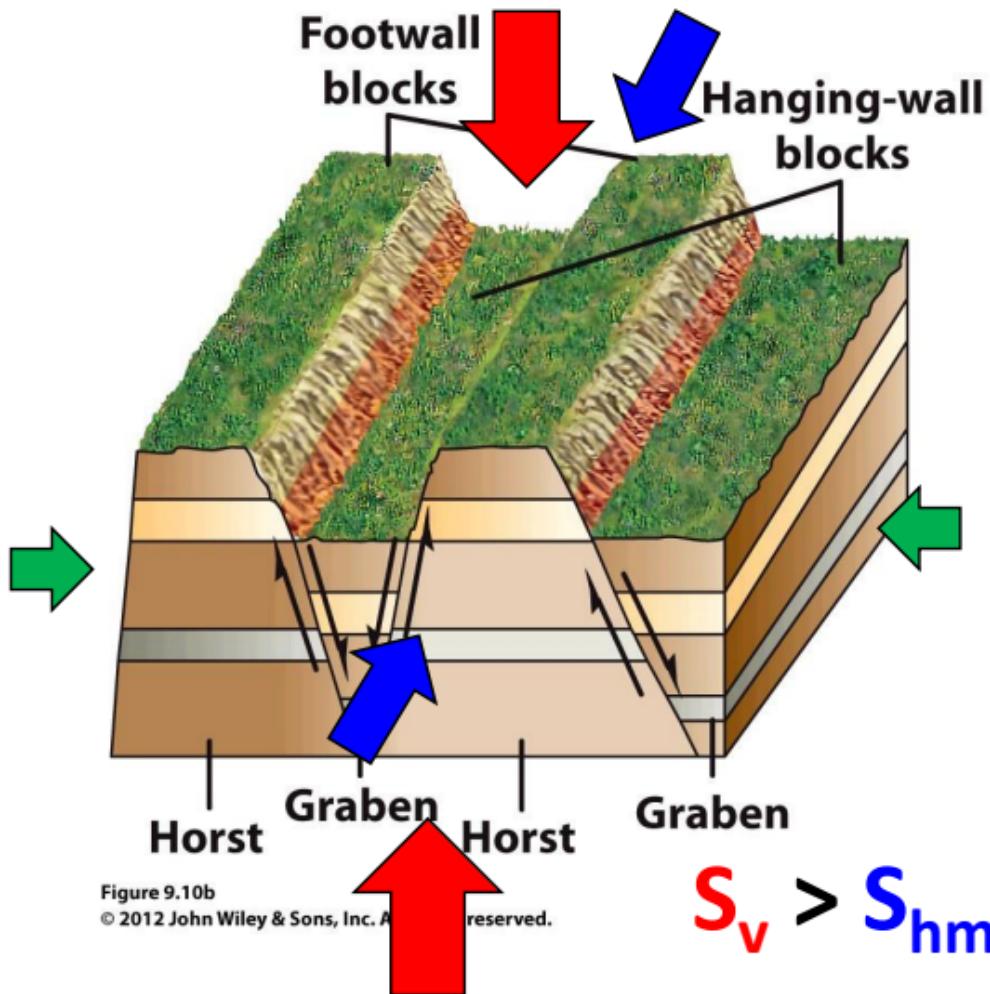


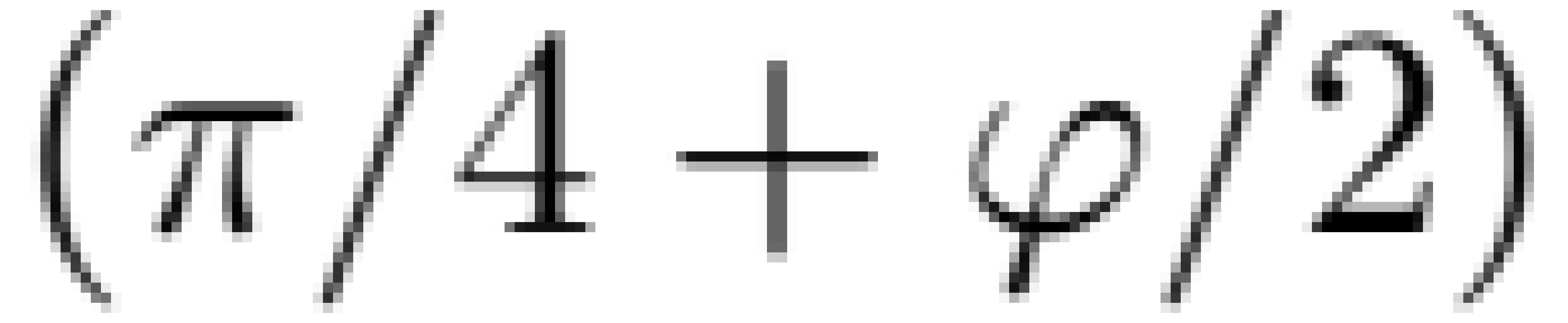
Figure 9.10b  
© 2012 John Wiley & Sons, Inc. All rights reserved.

$$S_v > S_{h\max} > S_{h\min}$$

$$S_1 = S_{H\max}$$

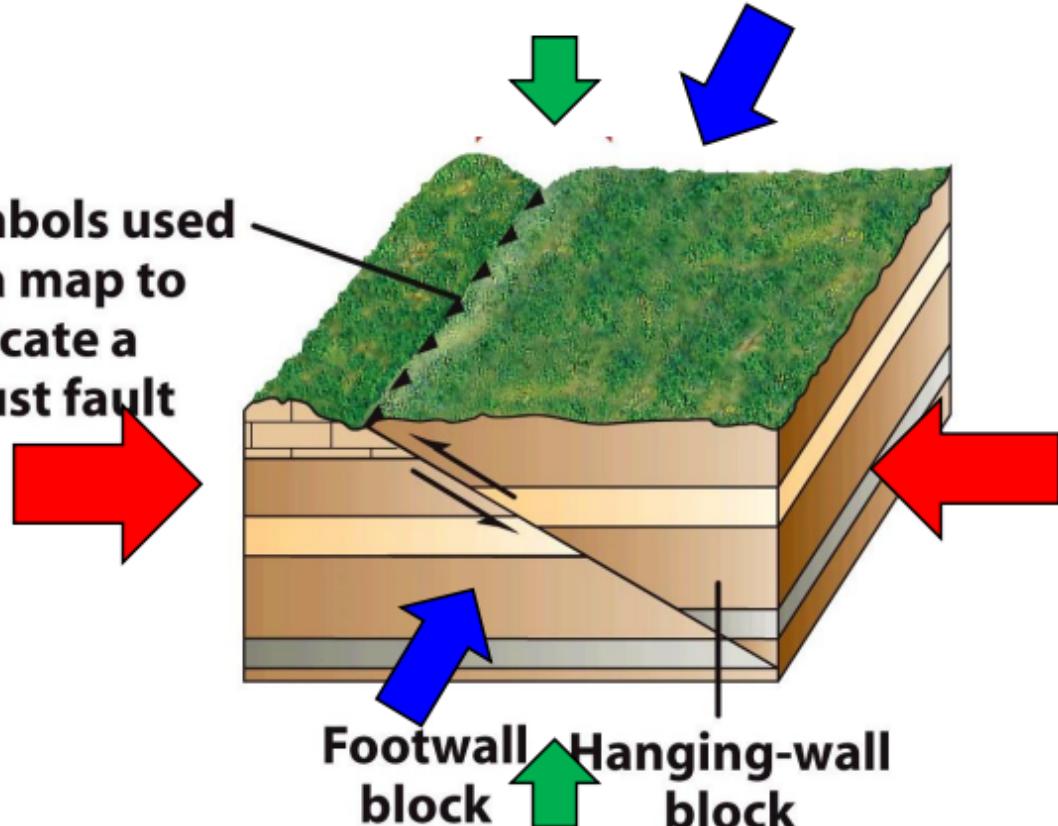
$$S_2 = S_{h\min}$$

$$S_3 = S_v$$





**Symbols used  
on a map to  
indicate a  
thrust fault**



**Footwall  
block**      **Hanging-wall  
block**

Figure 9.10d  
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$$S_{H\max} > S_{h\min} > S_v$$

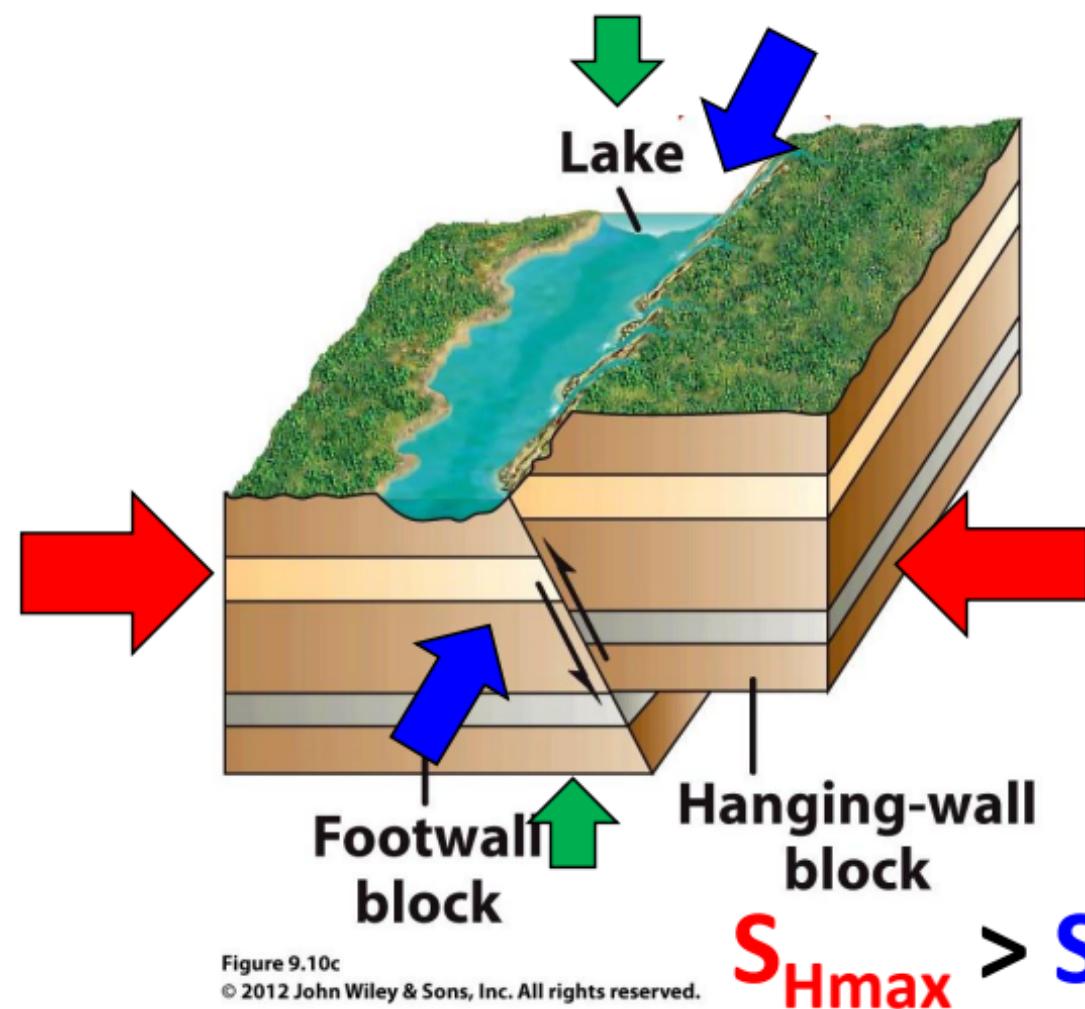


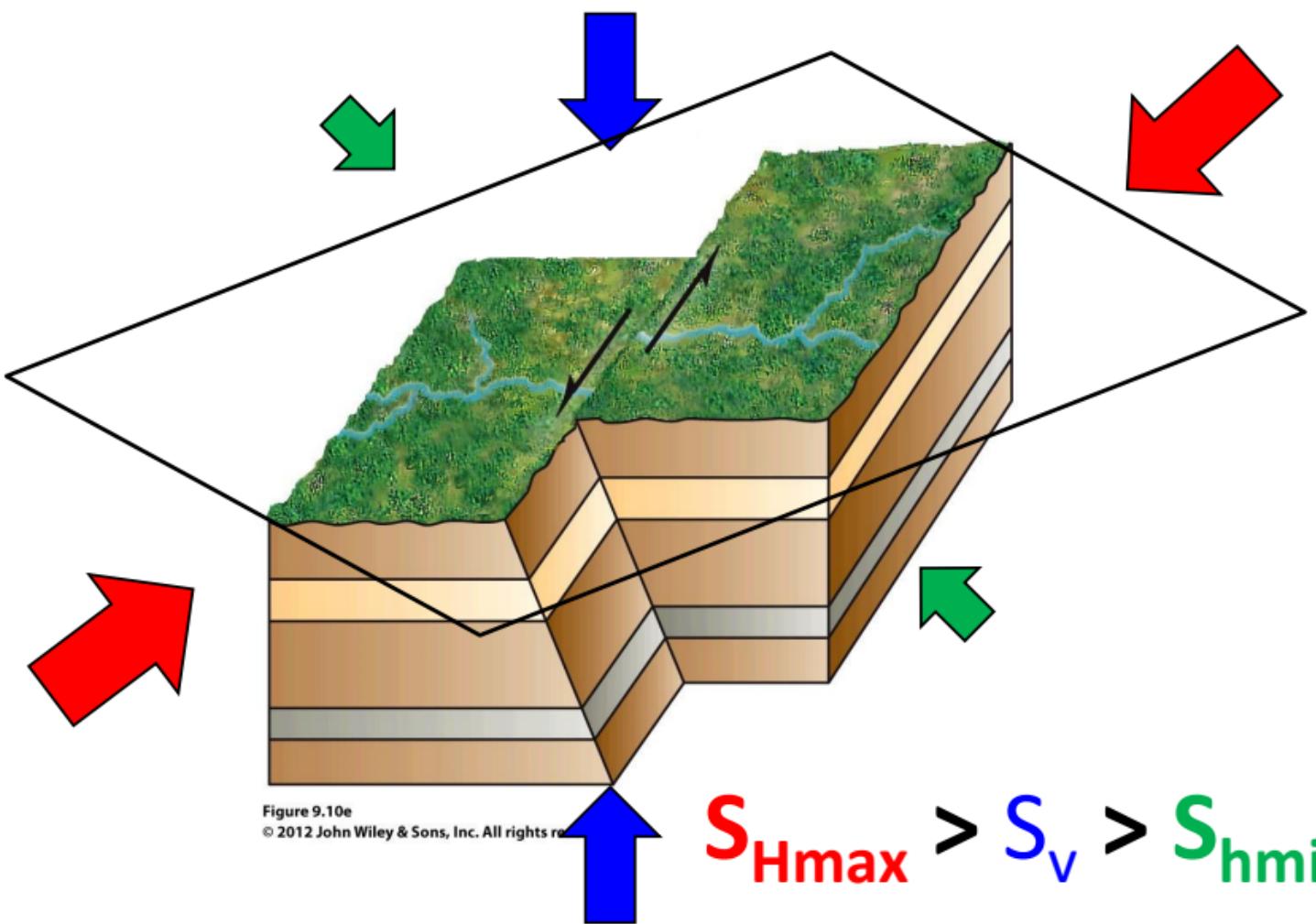
Figure 9.10c

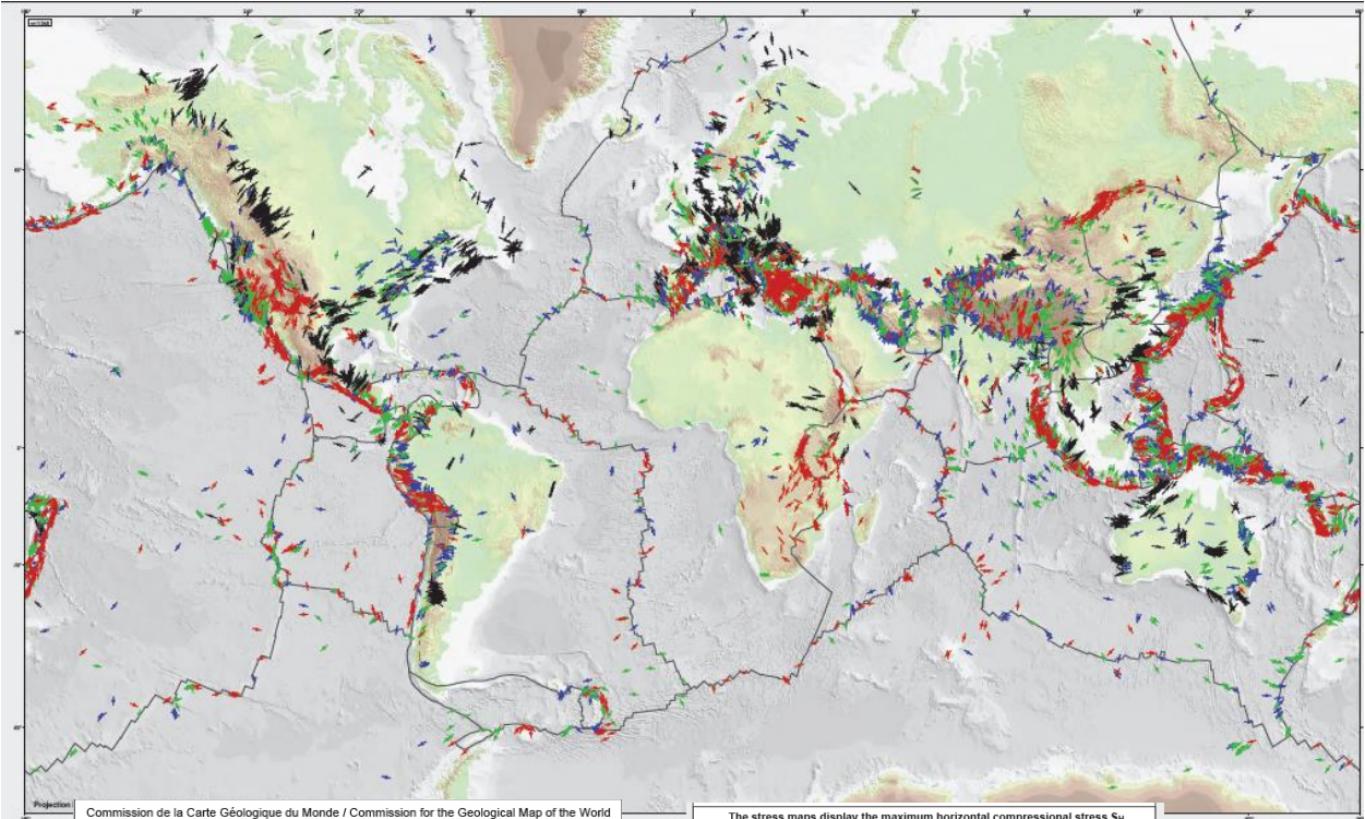
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$$S_1 = S_{H\max}$$

$$S_2 = S_v$$

$$S_3 = S_{hmin}$$





Commission de la Carte Géologique du Monde / Commission for the Geological Map of the World



# WORLD STRESS MAP



2009 - 2<sup>nd</sup> edition, based on the WSM database release 2008

Helmholtz Centre Potsdam - GFZ German Research Centre for Geosciences

## Authors

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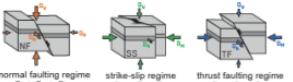
The stress maps display the maximum horizontal compressional stress  $S_H$

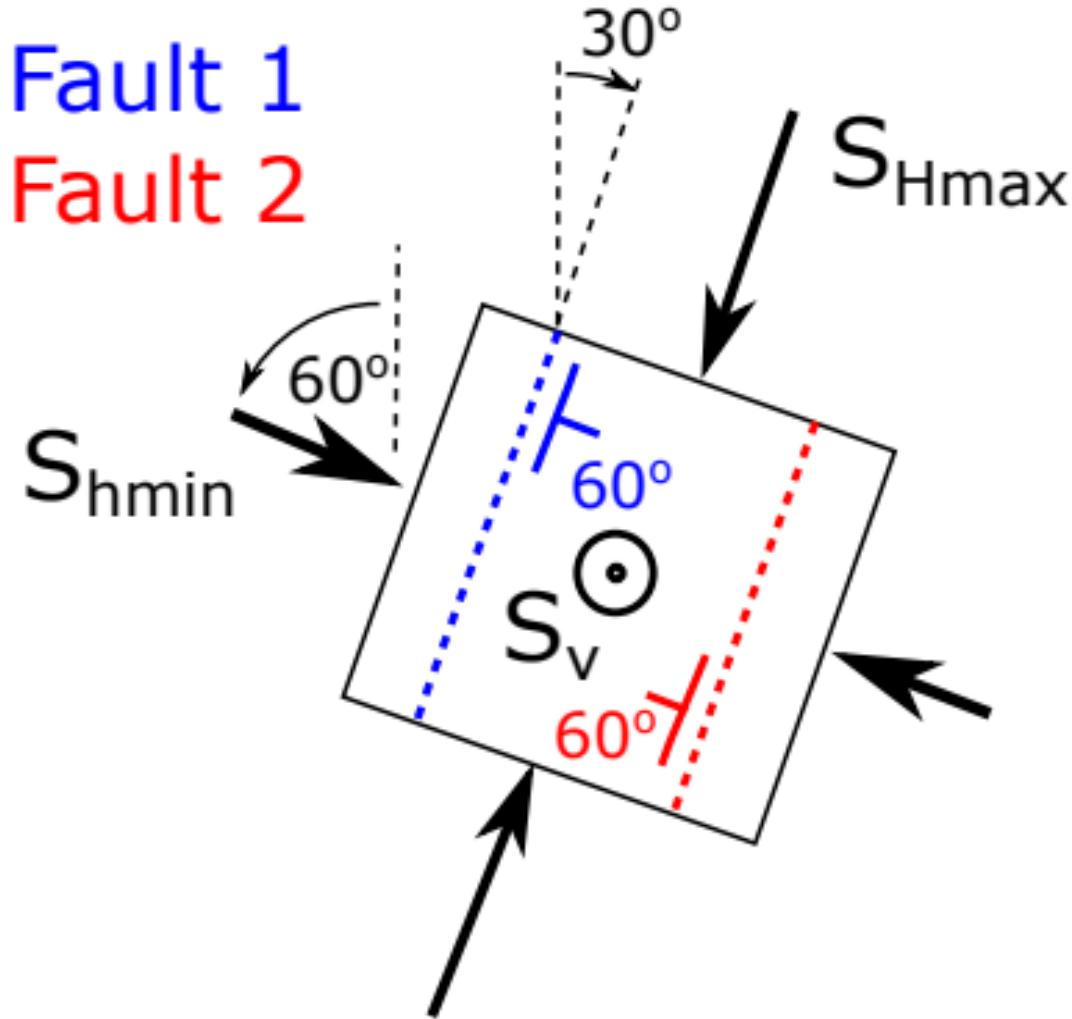
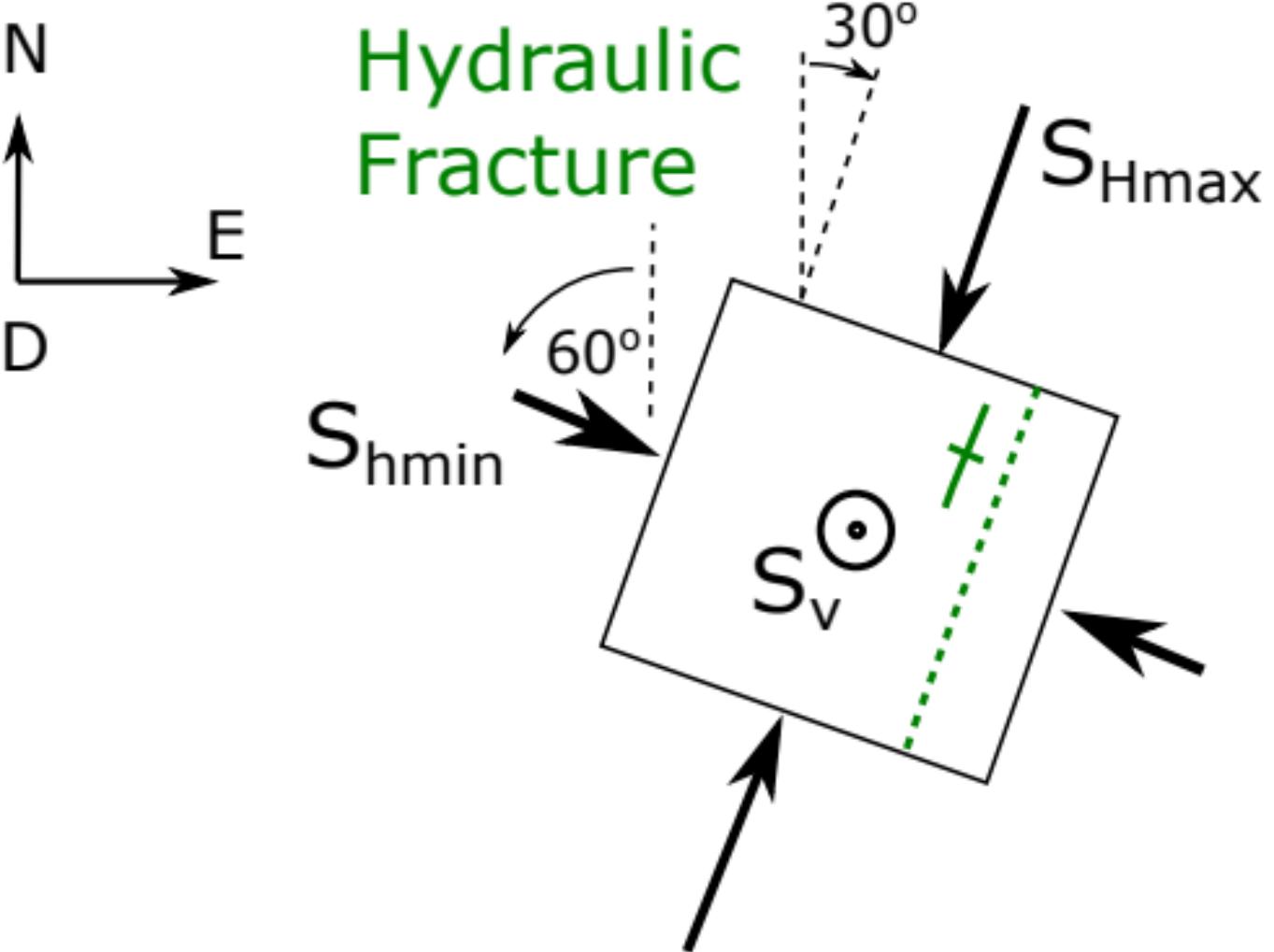
Method	Quality
focal mechanism	$S_H$ is within $\pm 15^\circ$
breakouts	$S_H$ is within $\pm 20^\circ$
drl. induced frac.	$S_H$ is within $\pm 25^\circ$
overcoring	
hydro. fractures	
geol. indicators	

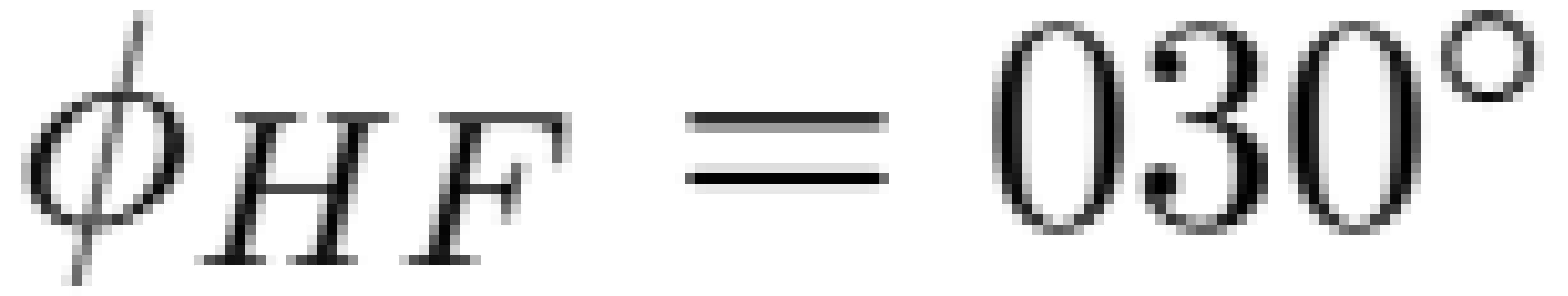
Data depth range

0-40 km

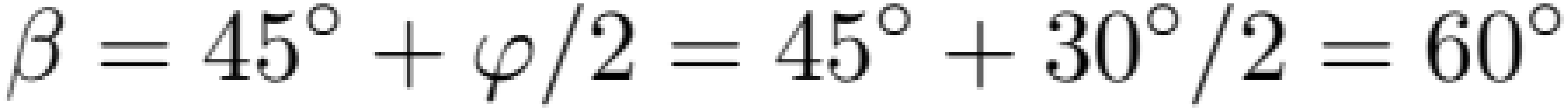
Stress Regime
Normal faulting
Strike-slip faulting
Thrust faulting
Unknown regime



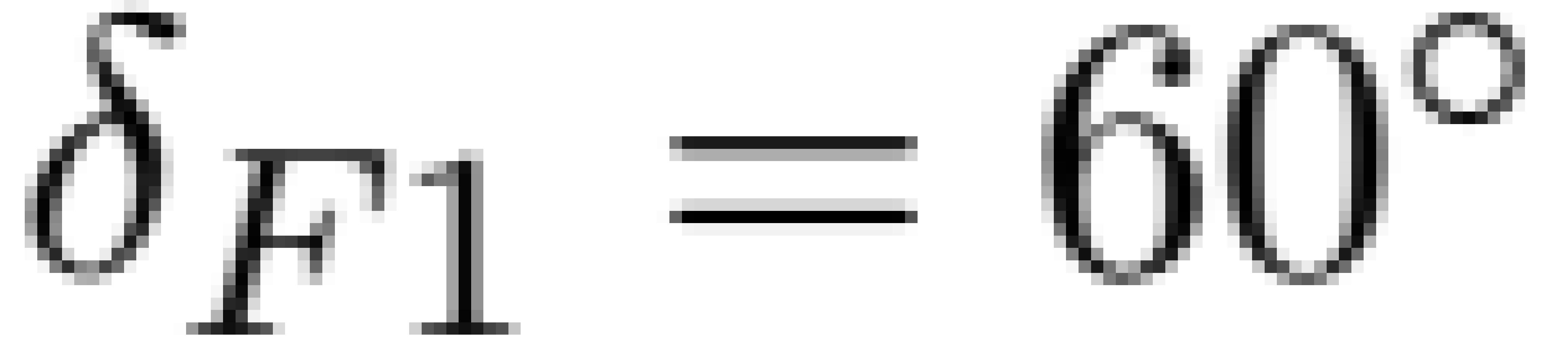


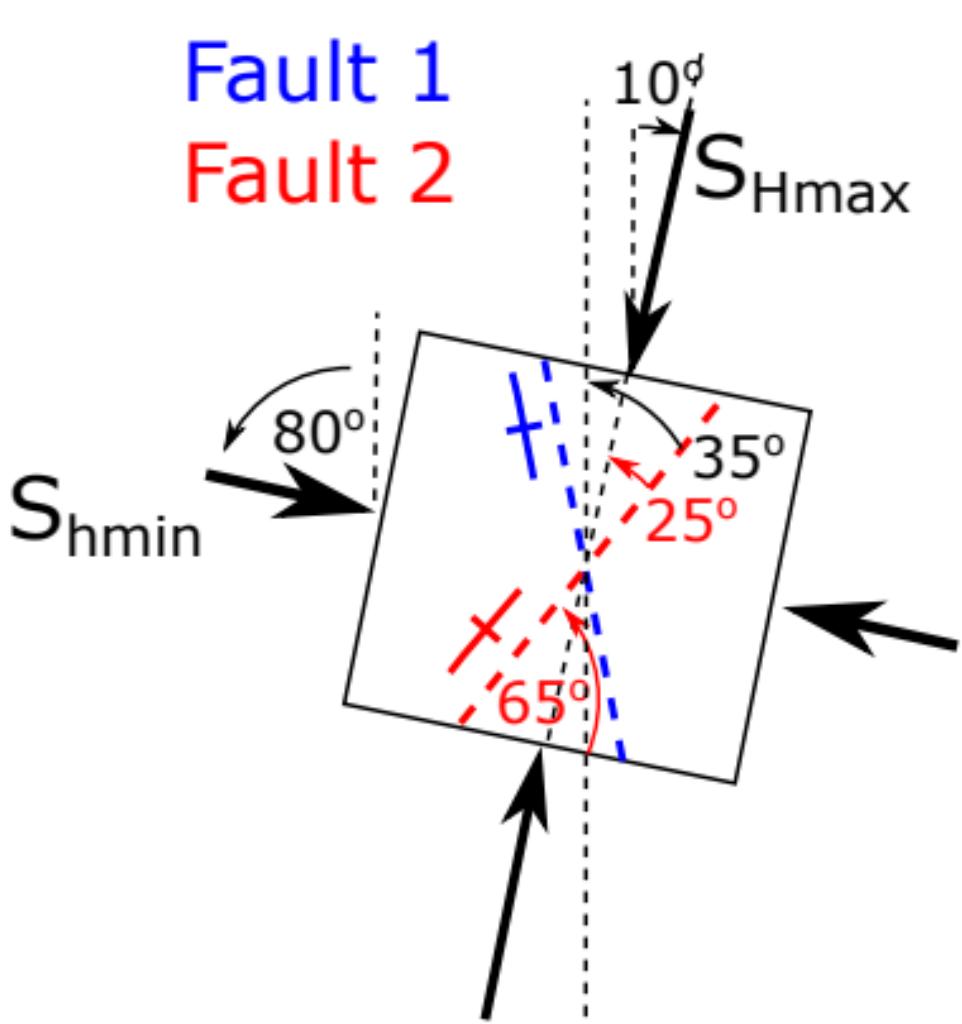
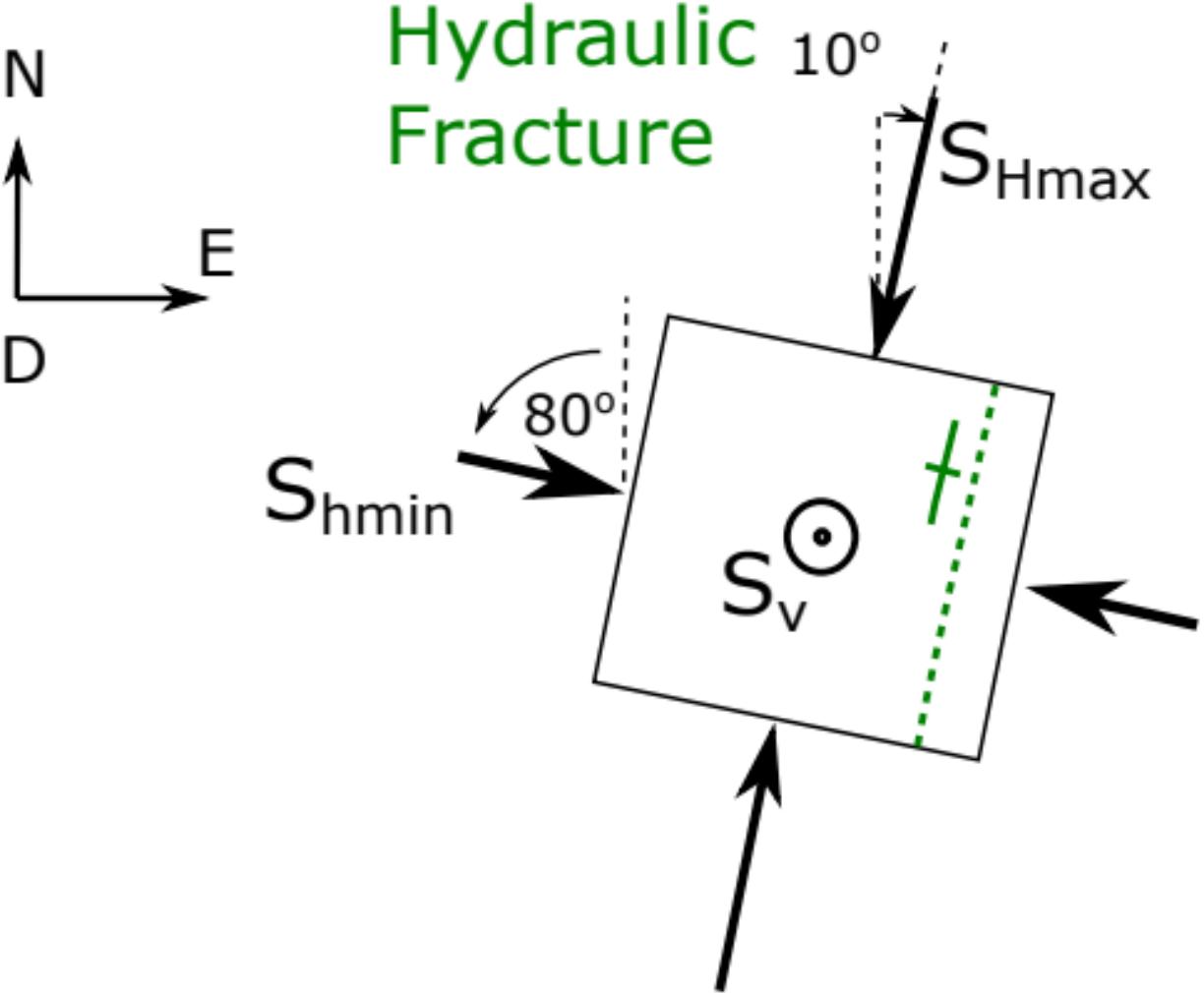


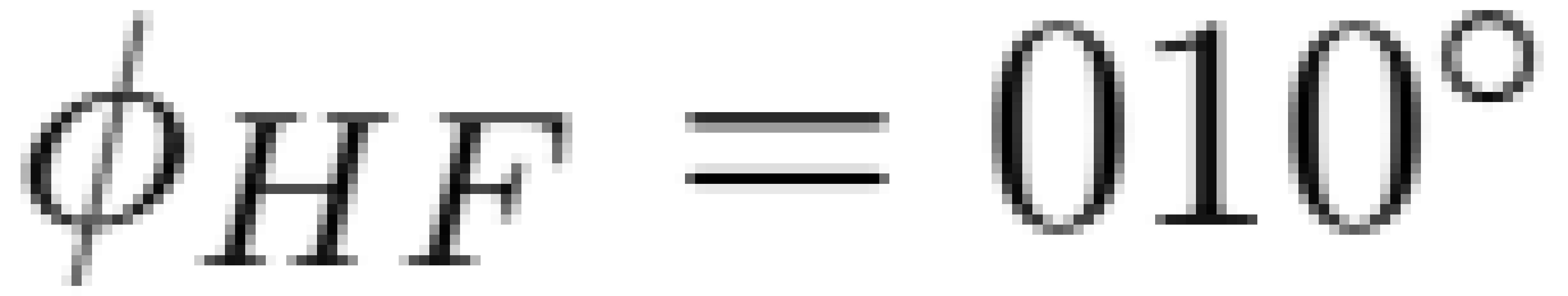


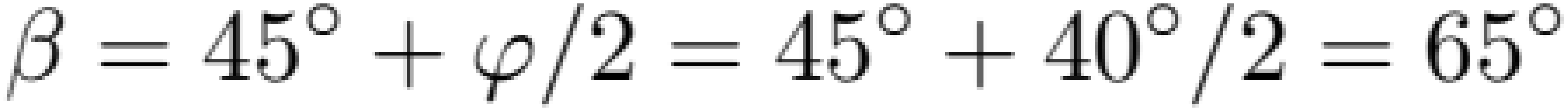


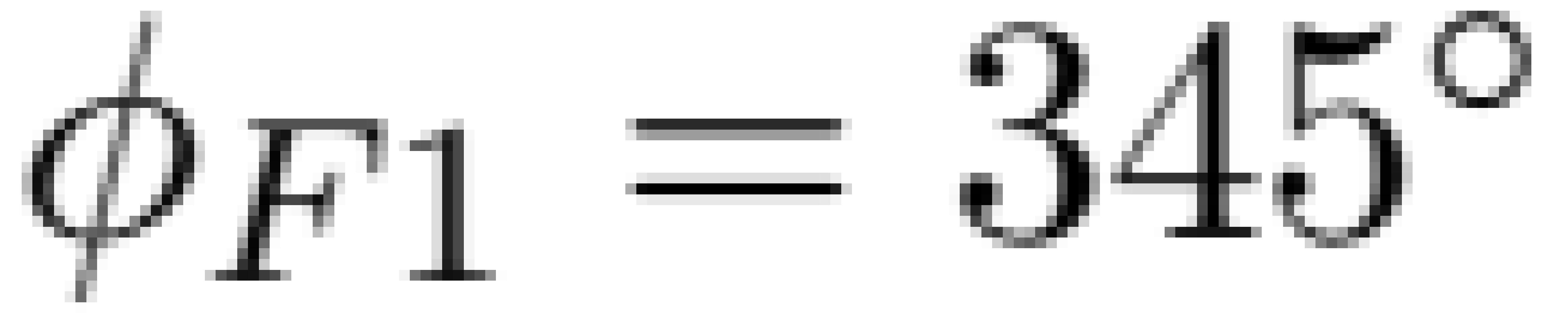


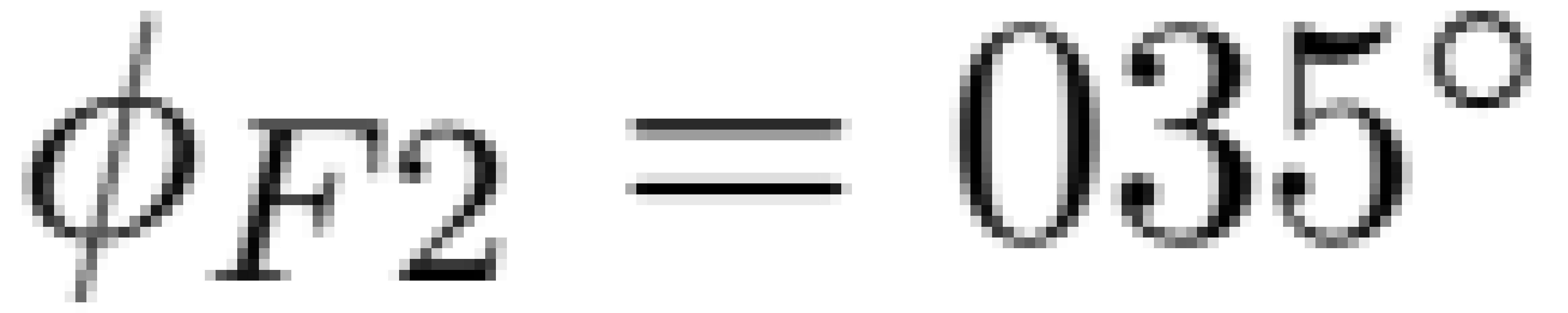




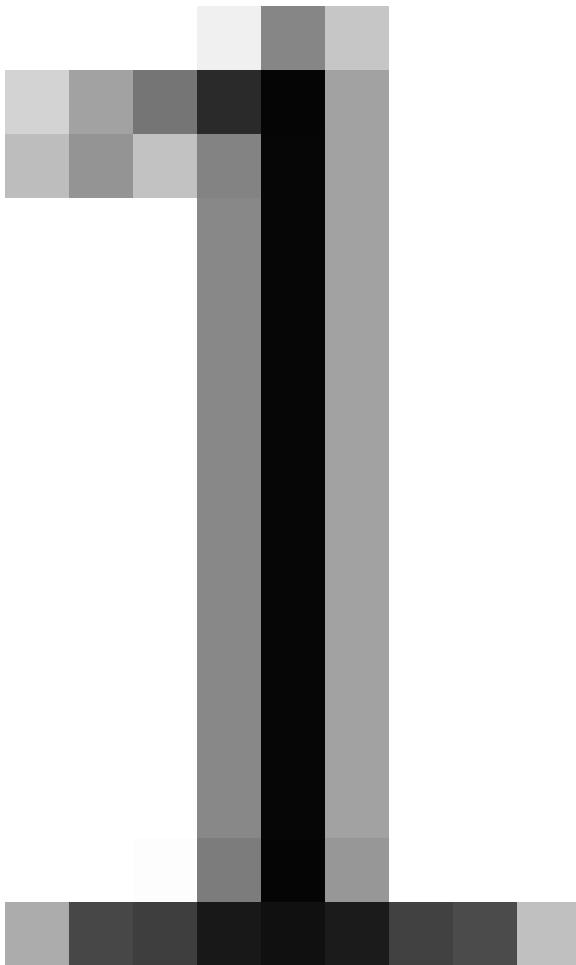




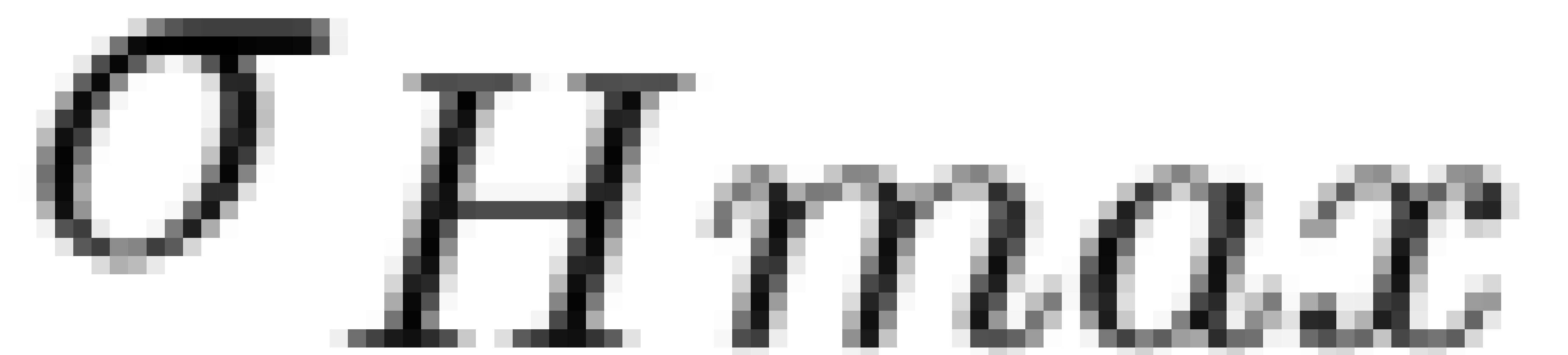




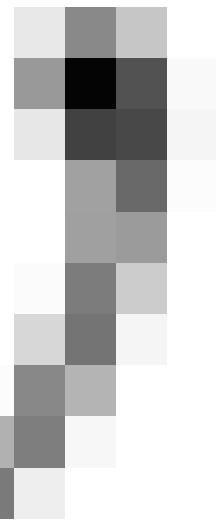
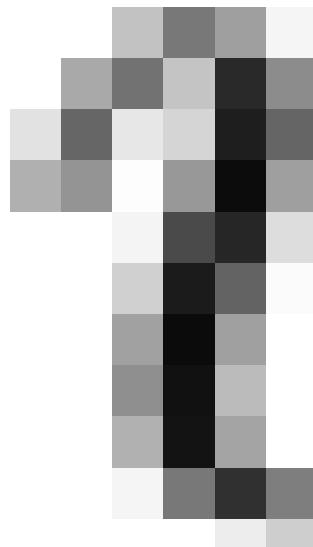
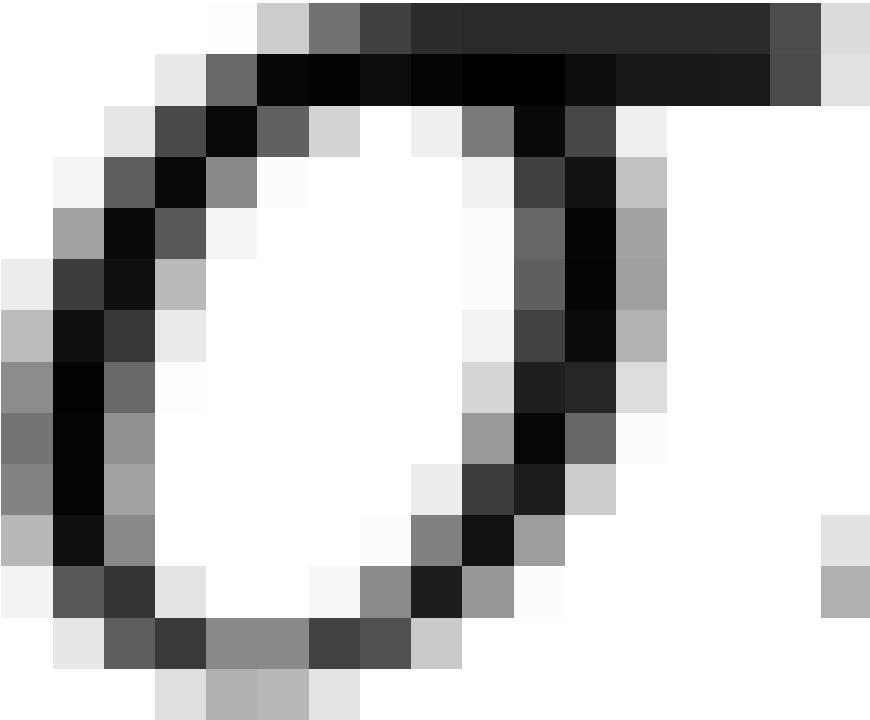




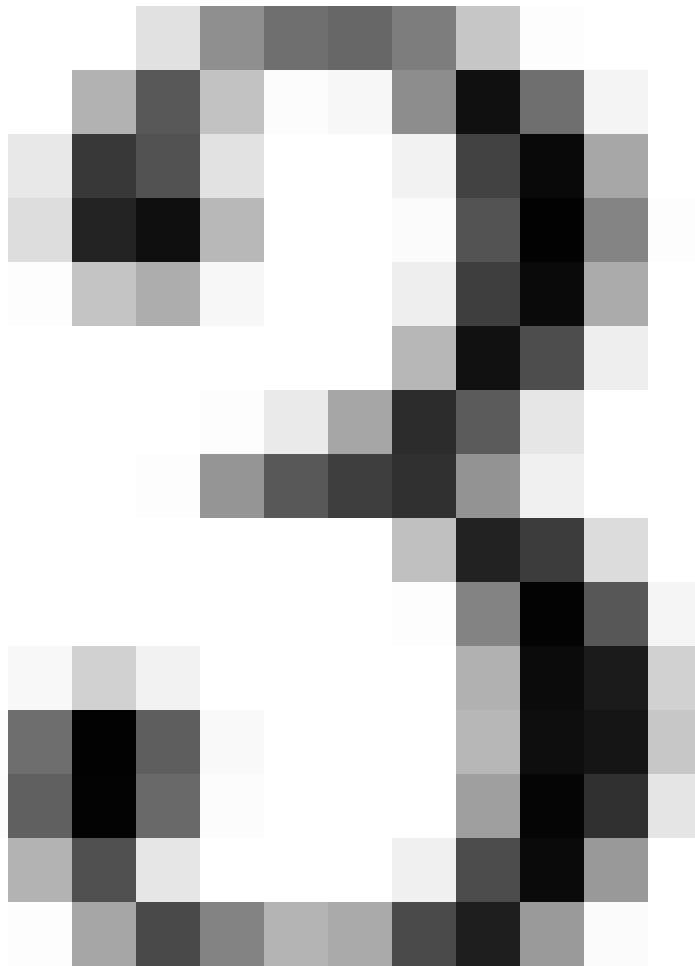




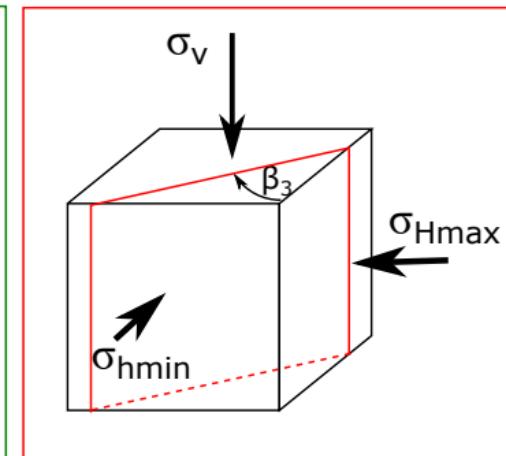
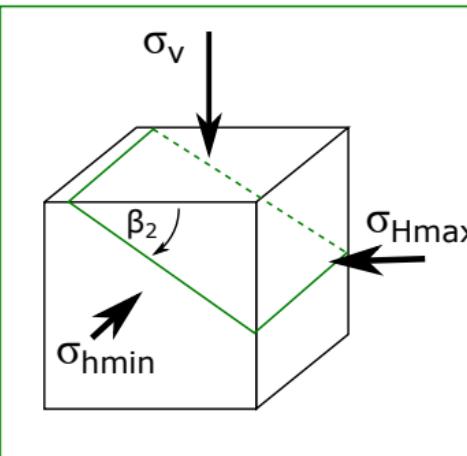
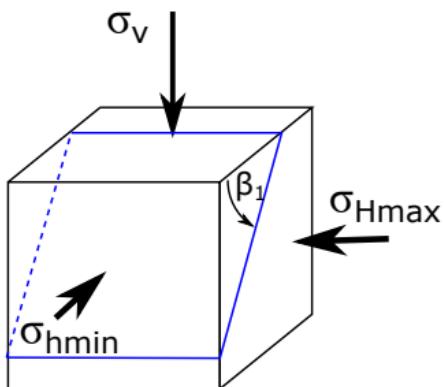
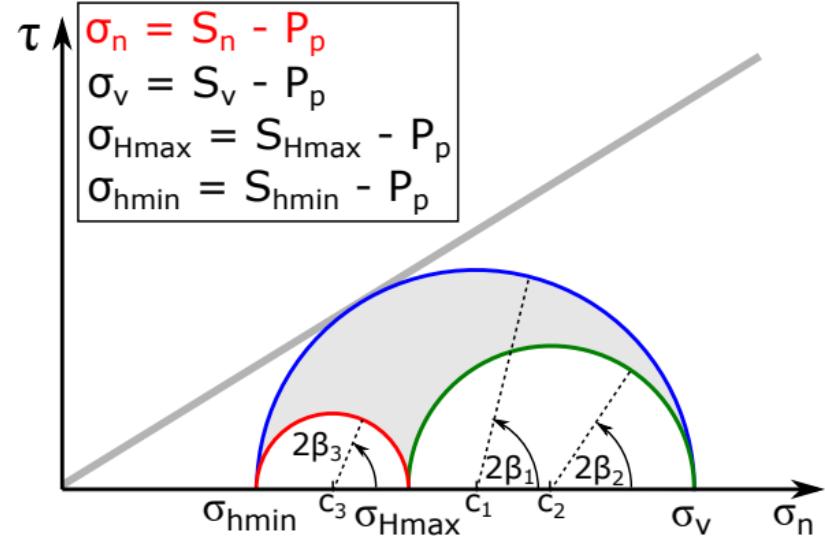
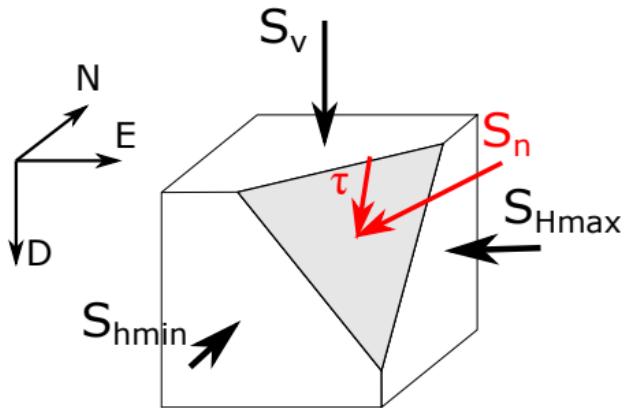


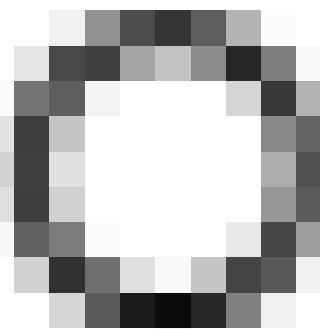
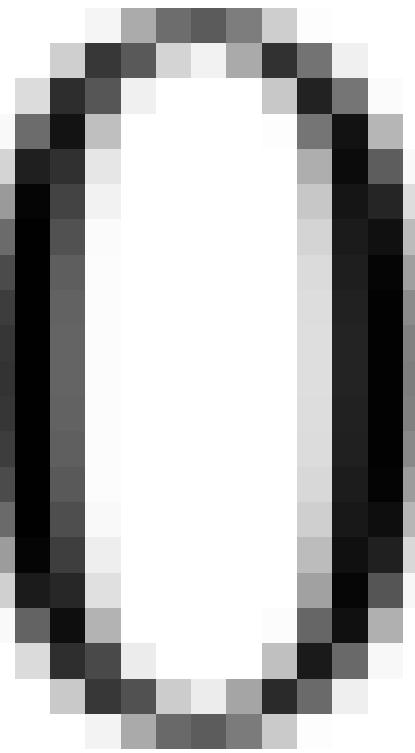
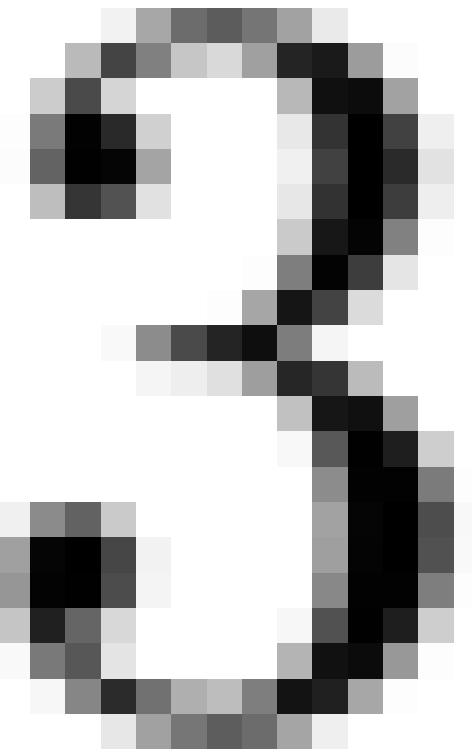
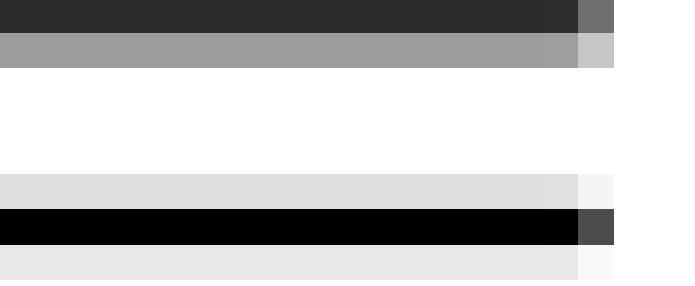
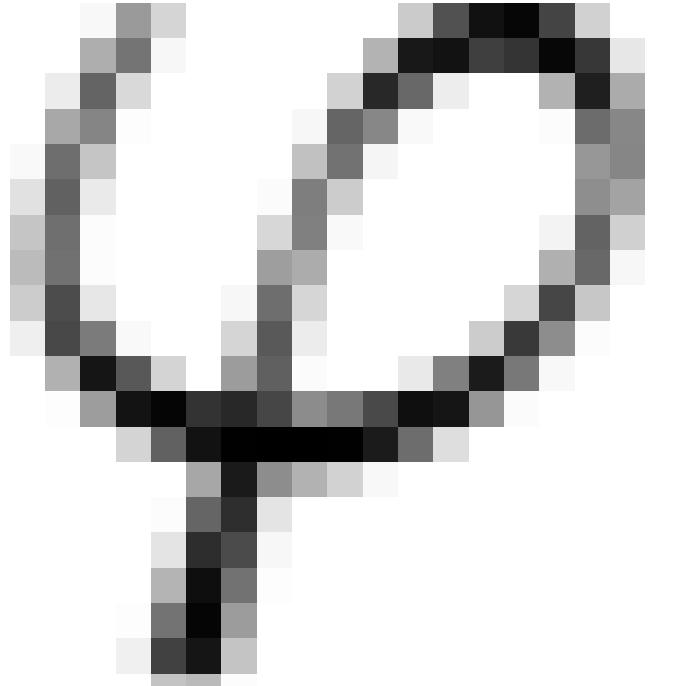


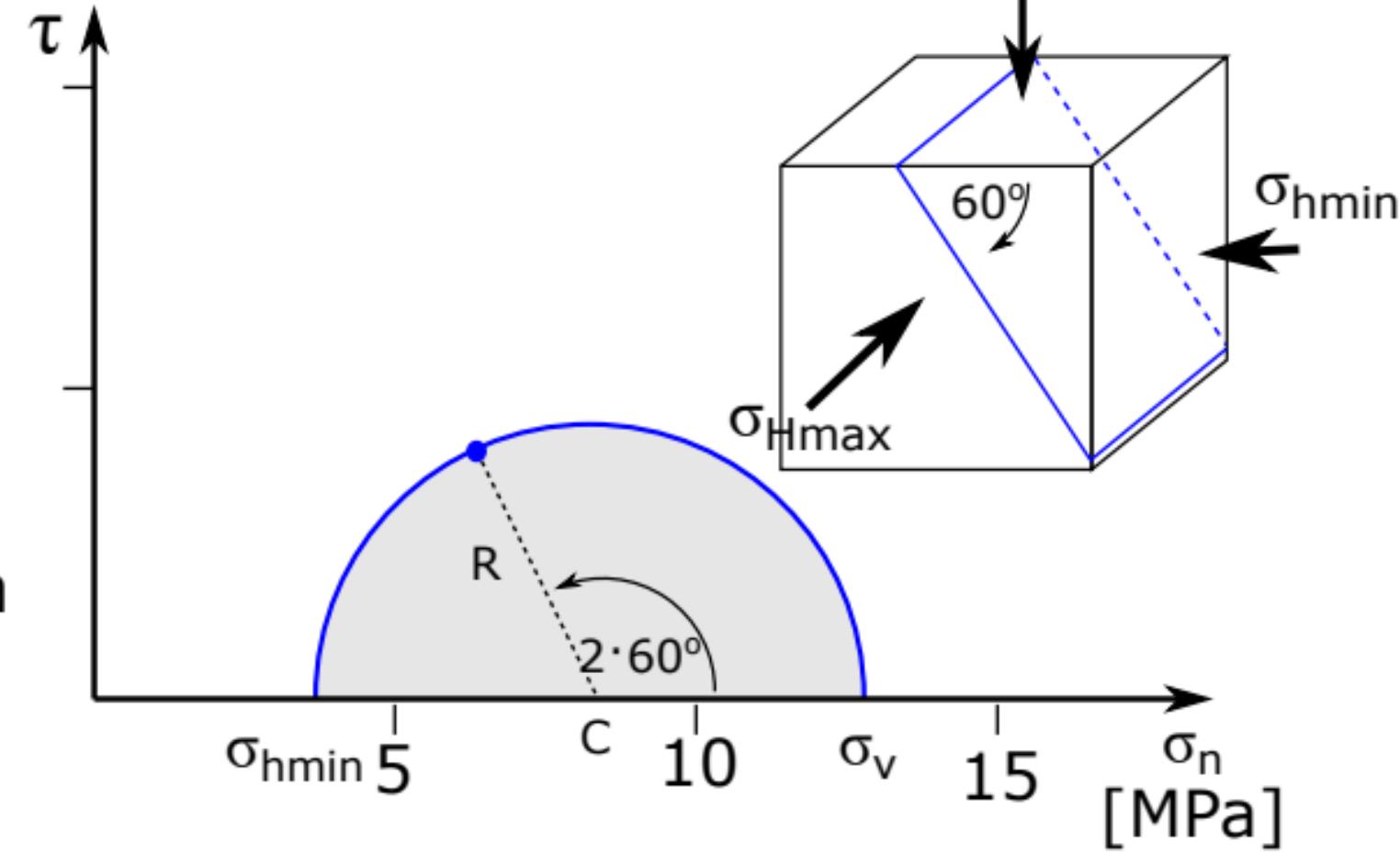
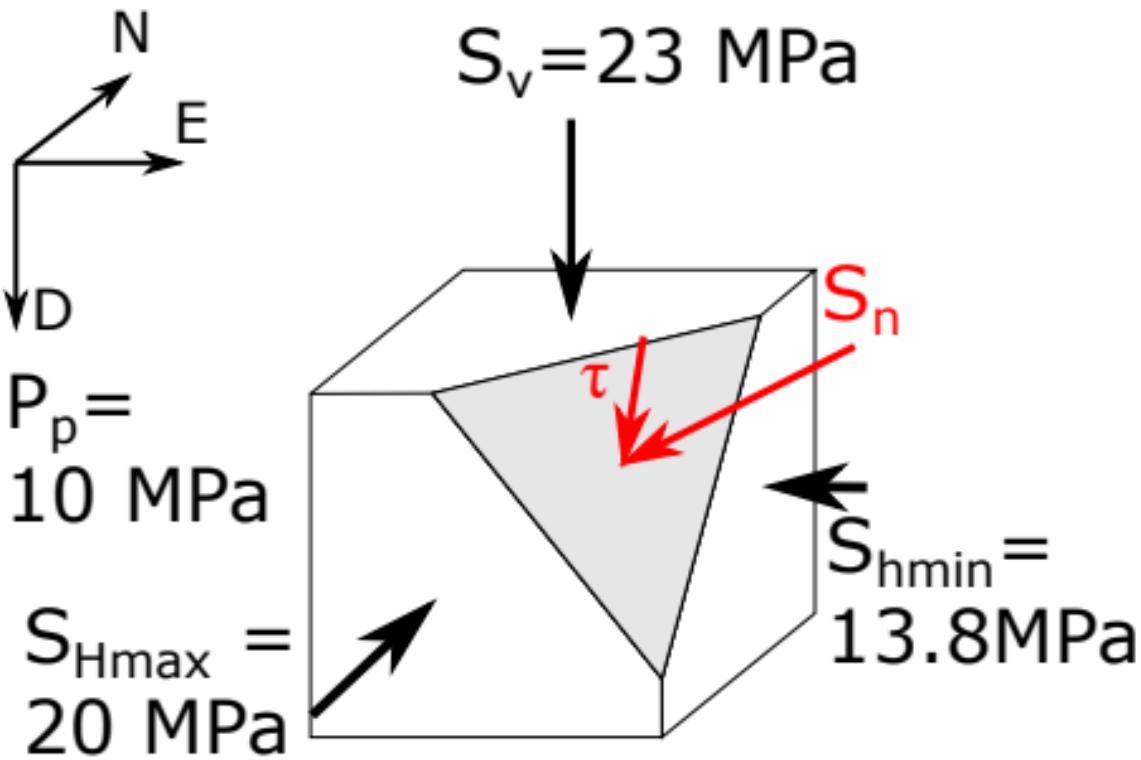


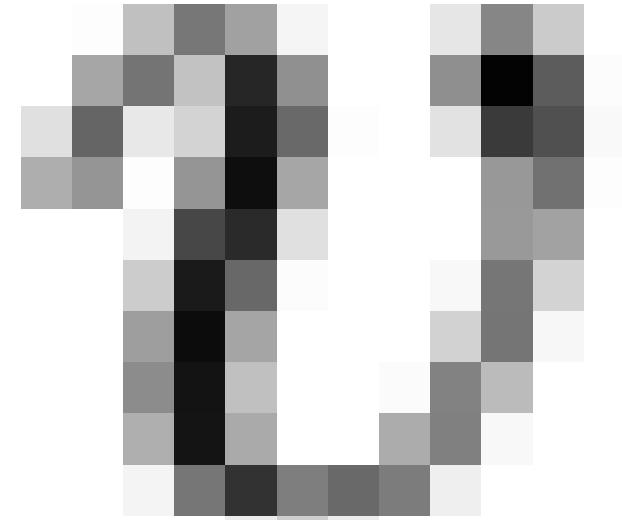
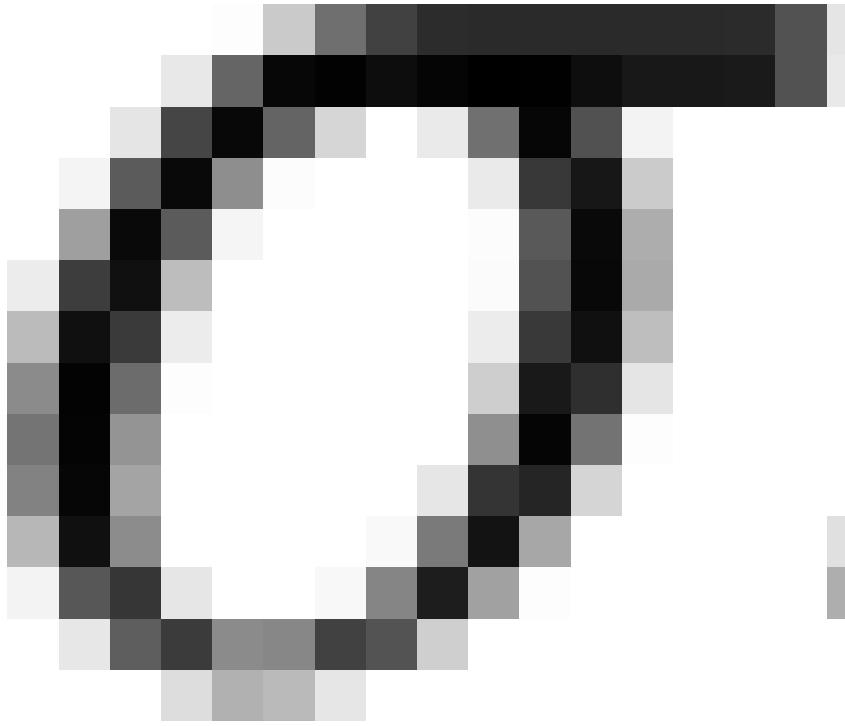




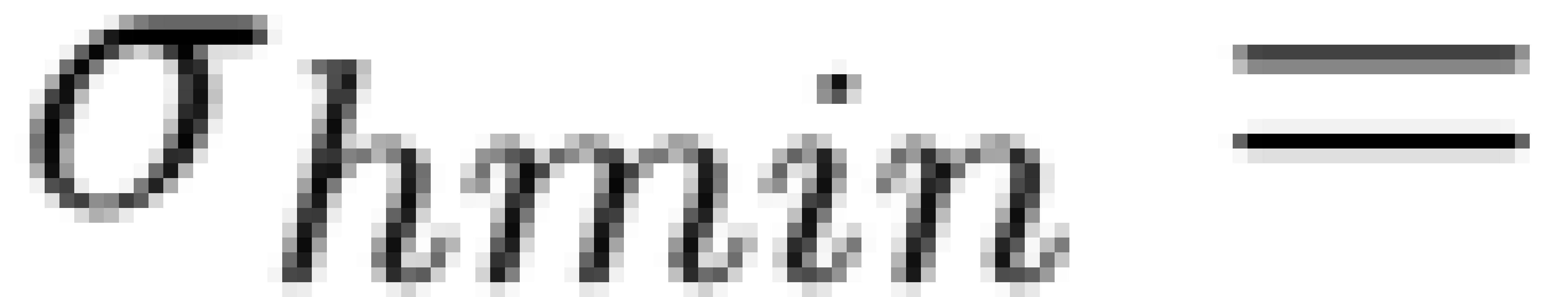








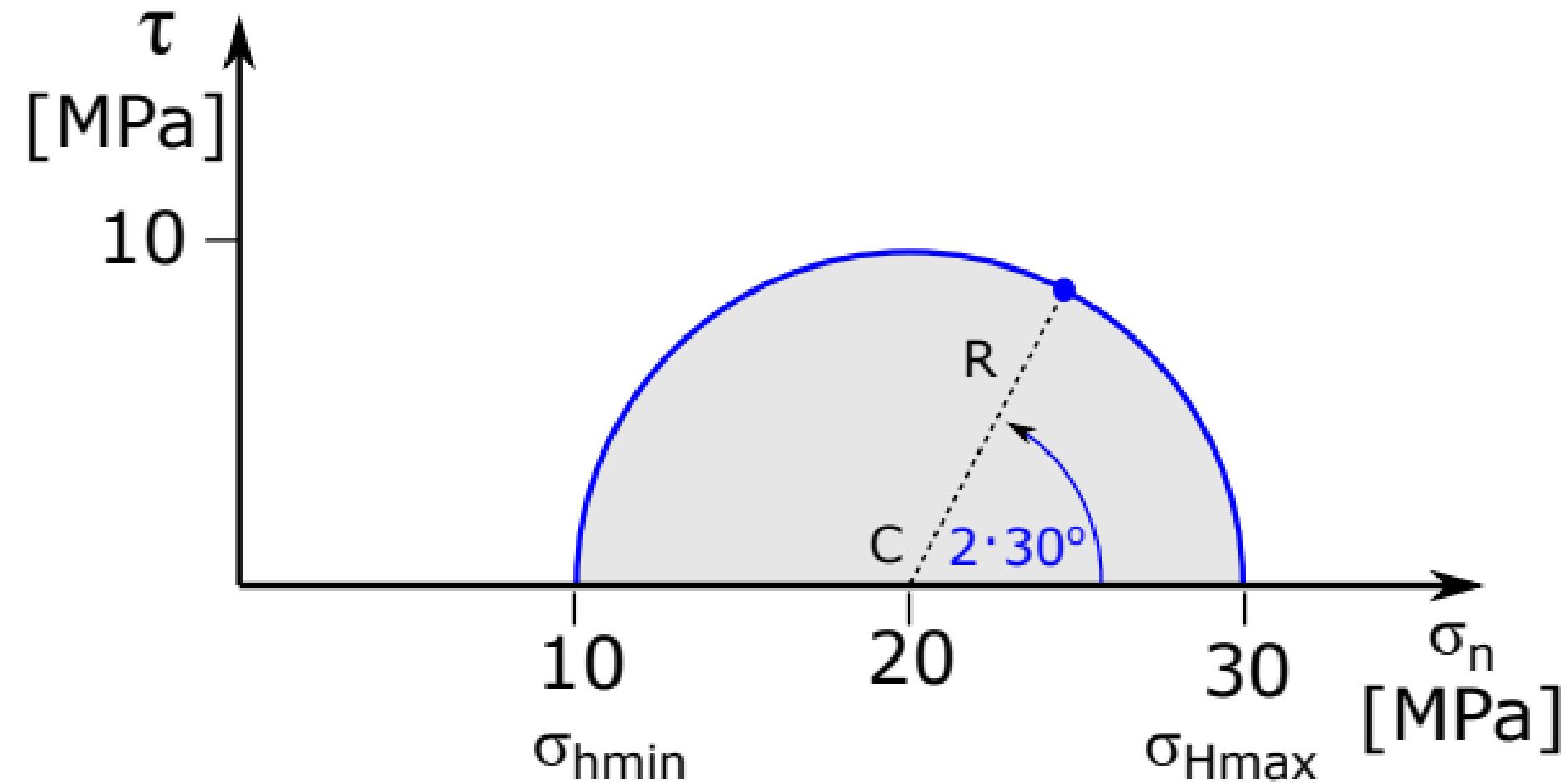
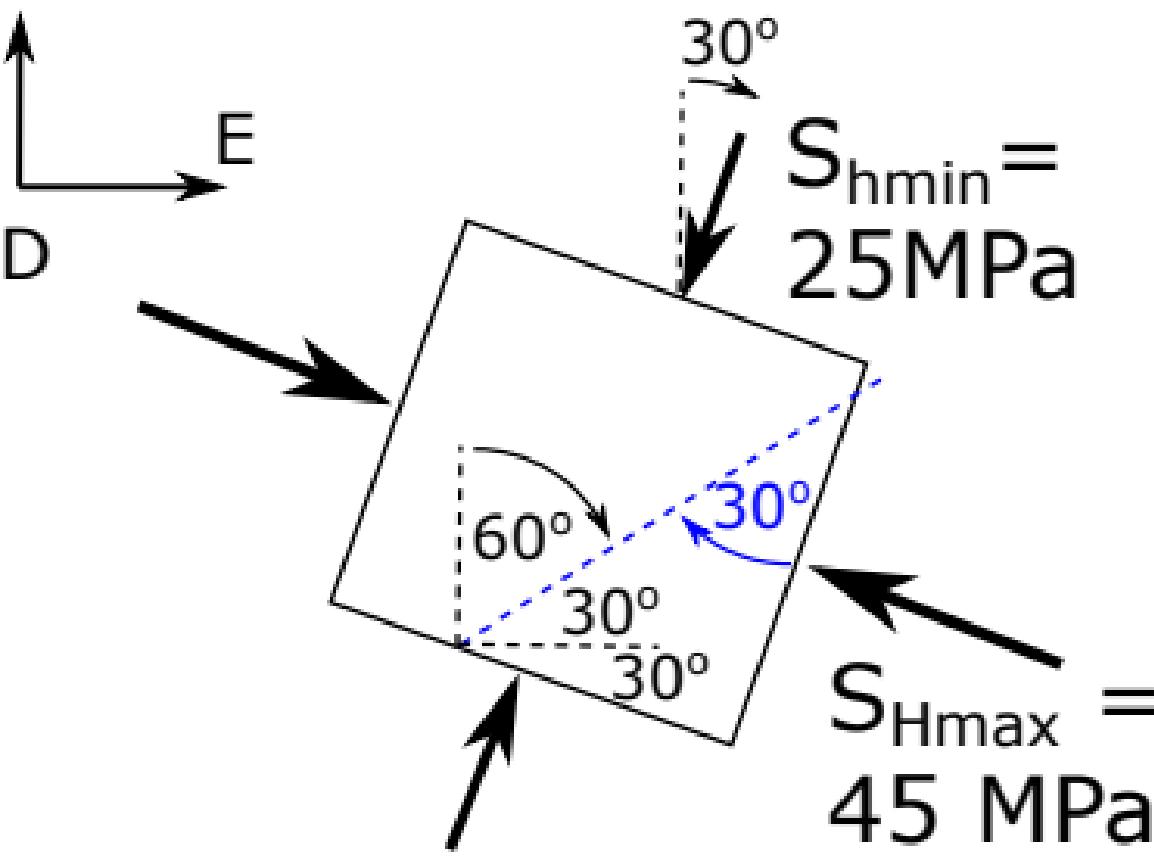




$$\sigma_n = \frac{(13 \text{ MPa} + 3.8 \text{ MPa})}{2} + \frac{(13 \text{ MPa} - 3.8 \text{ MPa}) \cos(2 \cdot 60^\circ)}{2} = 6.1 \text{ MPa}$$

$$\tau = \frac{(13 \text{ MPa} - 3.8 \text{ MPa})}{2} \sin(2 \cdot 60^\circ) = 4.0 \text{ MPa}$$

N  $P_p = 15 \text{ MPa}$   $S_v = 30 \text{ MPa}$



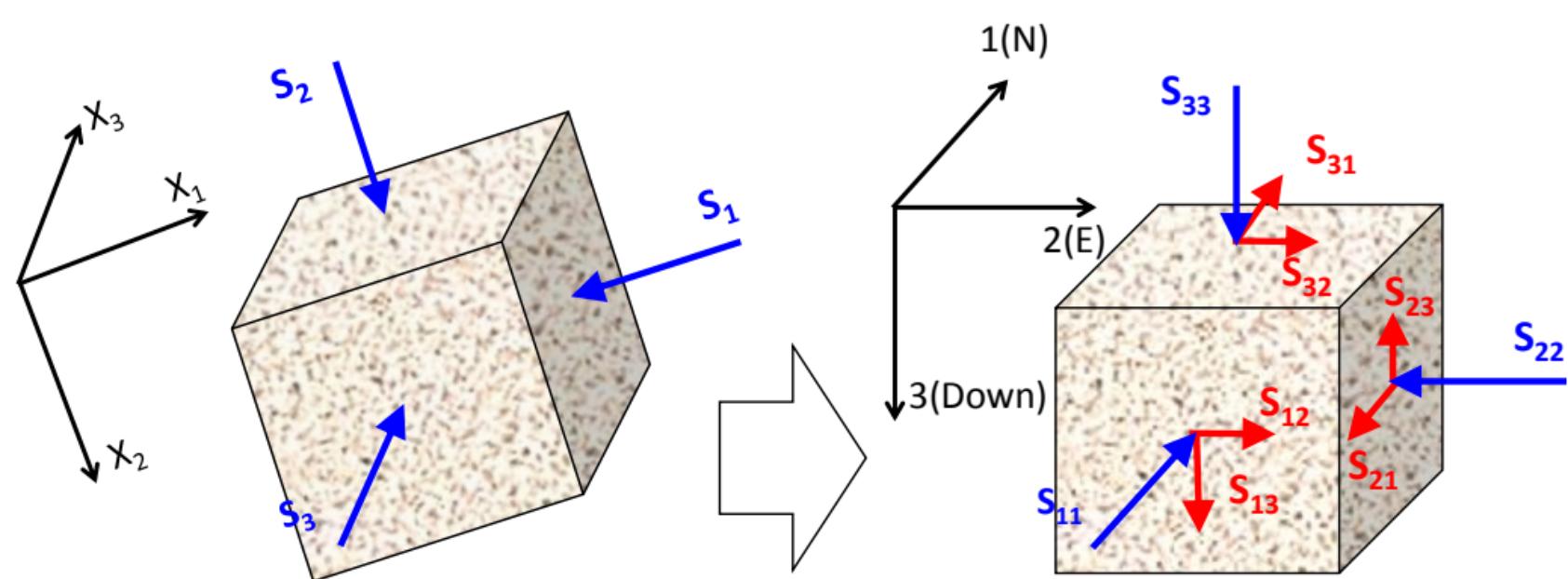
$$\sigma_n = \frac{(30 \text{ MPa} + 10 \text{ MPa})}{2} + \frac{(30 \text{ MPa} - 10 \text{ MPa}) \cos(2 \cdot 30^\circ)}{2} = 25 \text{ MPa}$$

$$\tau = \frac{(30 \text{ MPa} - 10 \text{ MPa})}{2} \sin(2 \cdot 30^\circ) = 8.7 \text{ MPa}$$





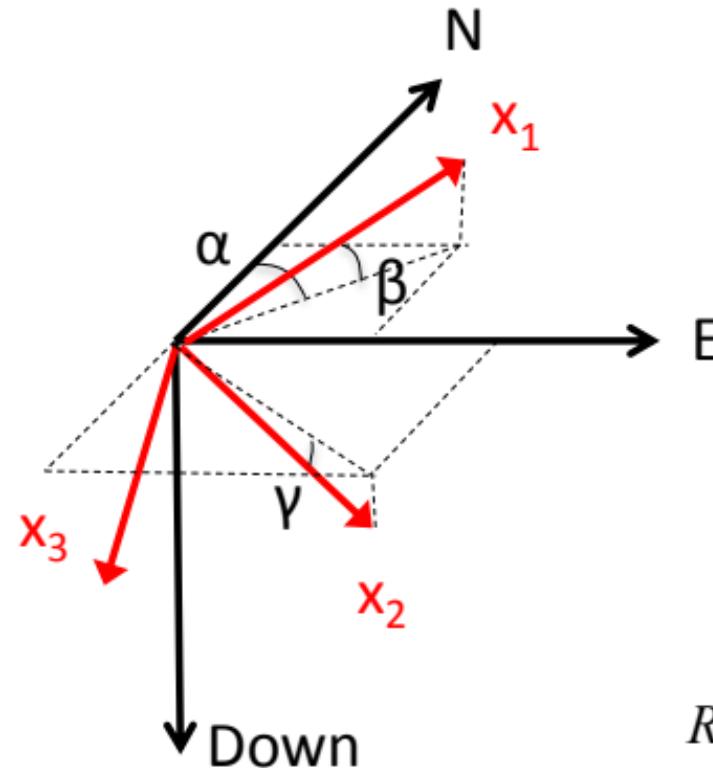




$$S_p = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$S_g = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$





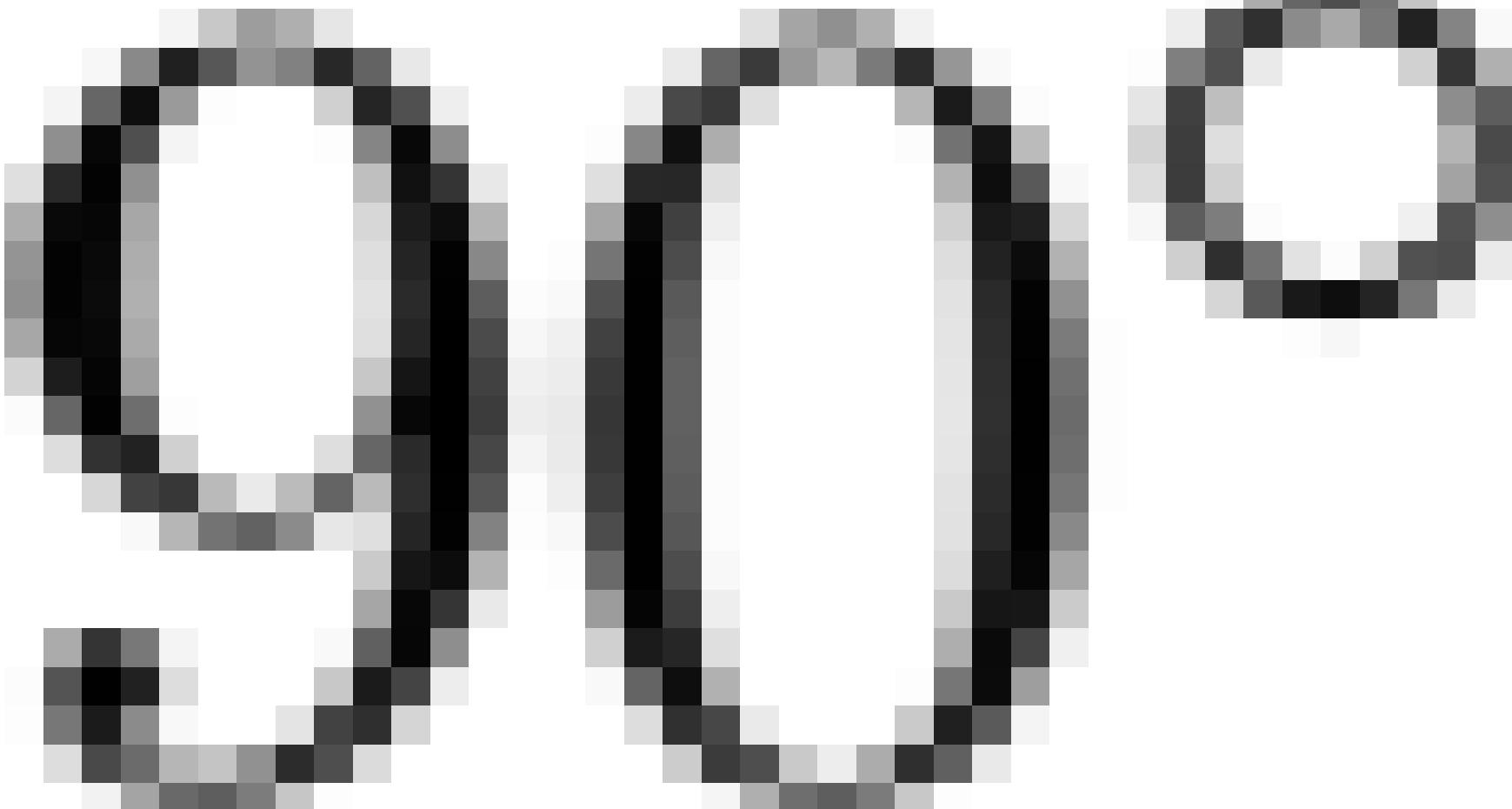
$$\underline{\underline{S}}' = \underline{\underline{A}} \underline{\underline{S}} \underline{\underline{A}}^T$$

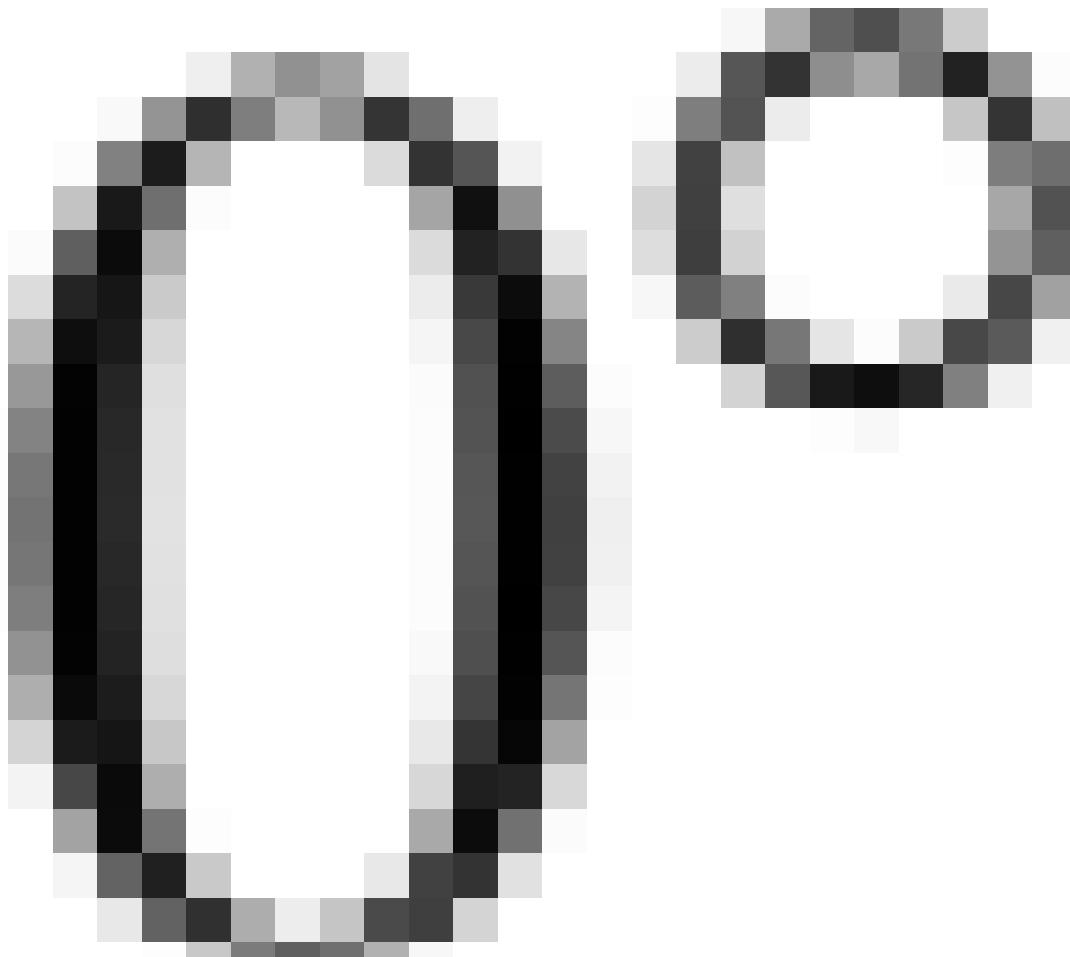
$$\underline{\underline{A}} = \begin{bmatrix} \underline{e}'_1 \cdot \underline{e}_1 & \underline{e}'_1 \cdot \underline{e}_2 & \underline{e}'_1 \cdot \underline{e}_3 \\ \underline{e}'_2 \cdot \underline{e}_1 & \underline{e}'_2 \cdot \underline{e}_2 & \underline{e}'_2 \cdot \underline{e}_3 \\ \underline{e}'_3 \cdot \underline{e}_1 & \underline{e}'_3 \cdot \underline{e}_2 & \underline{e}'_3 \cdot \underline{e}_3 \end{bmatrix}$$

where  $\underline{\underline{A}}$  is the transformation matrix from the old base  $\underline{e}_i$  to base  $\underline{e}'_i$  and the components are the projection of the elements of the new base on the old base.

- Old system: N-E-D (Right-handed) Geographical system
- New system: 1-2-3 (Right-handed) Principal stress system

$$R_{PG} = \begin{bmatrix} \cos \alpha \cos b & \sin \alpha \cos b & -\sin b \\ \cos \alpha \sin b \sin g - \sin \alpha \cos g & \sin \alpha \sin b \sin g + \cos \alpha \cos g & \cos b \sin g \\ \cos \alpha \sin b \cos g + \sin \alpha \sin g & \sin \alpha \sin b \cos g - \cos \alpha \sin g & \cos b \cos g \end{bmatrix}$$







S

G

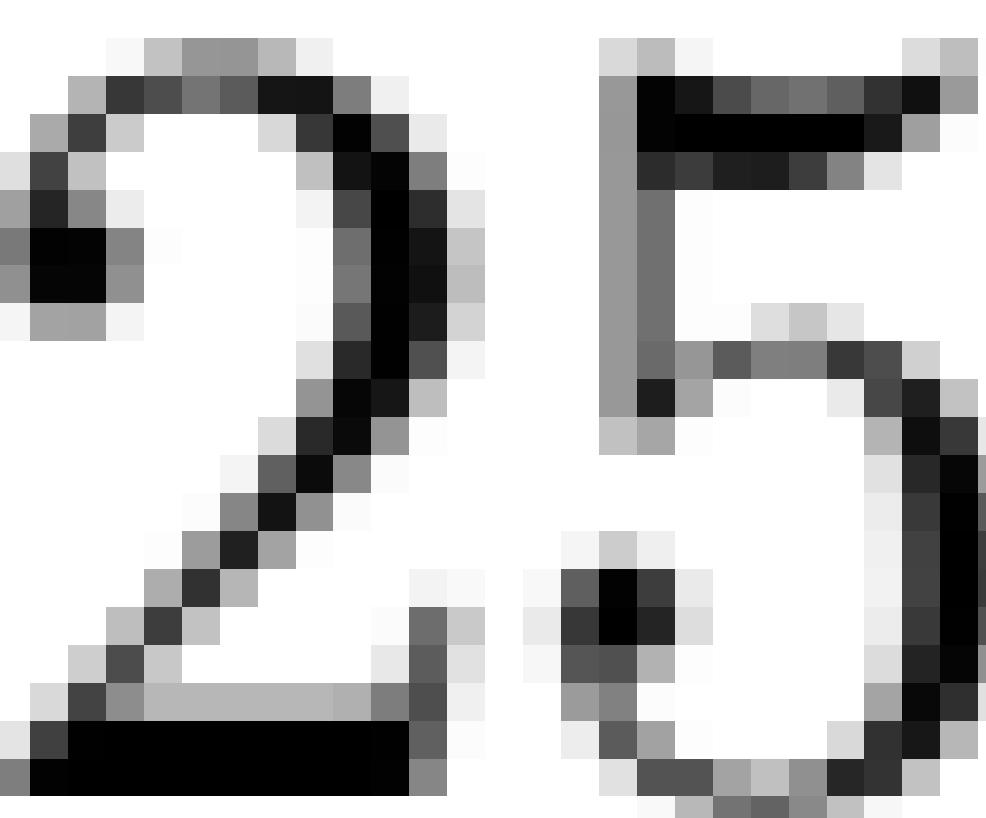
S

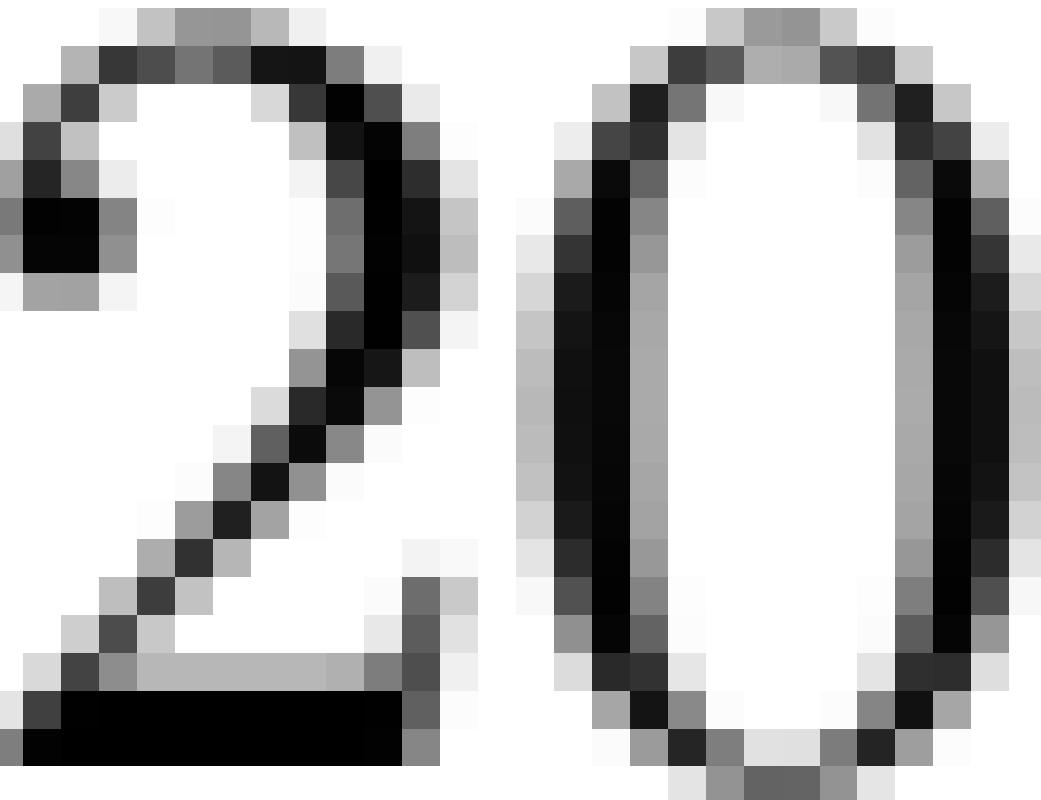
SP

S

SP

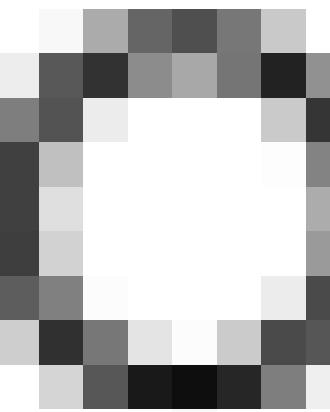
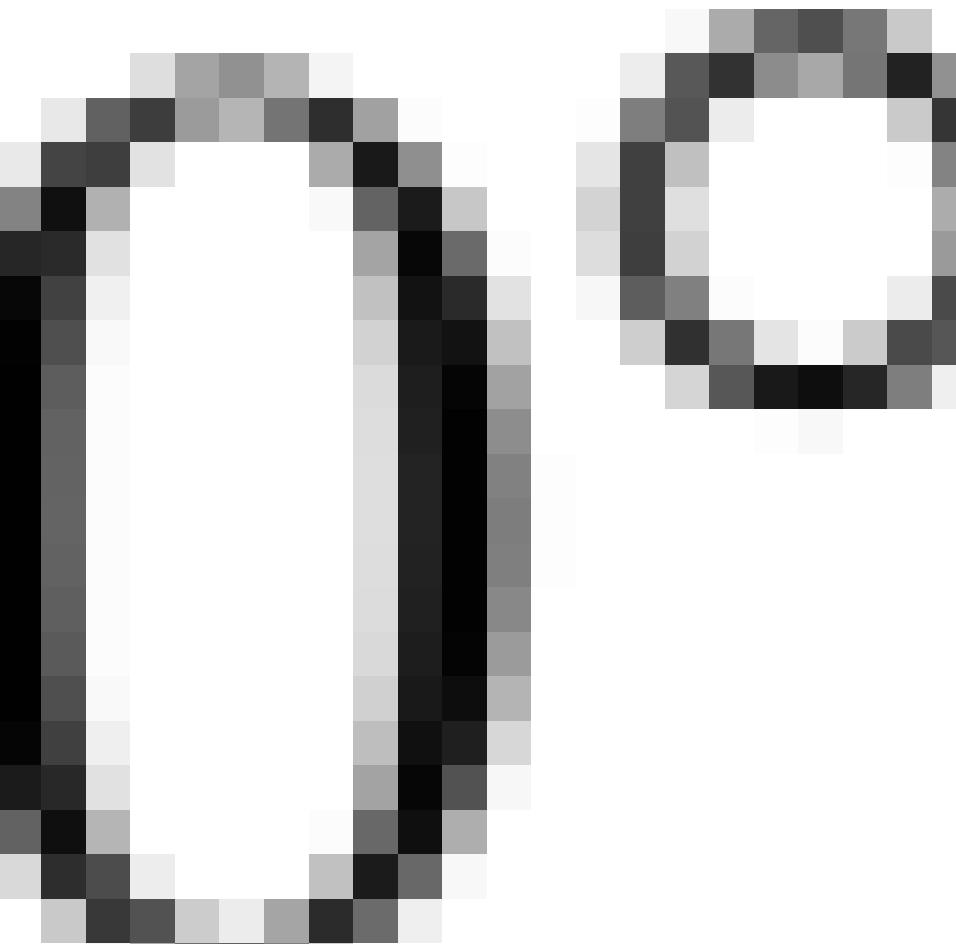
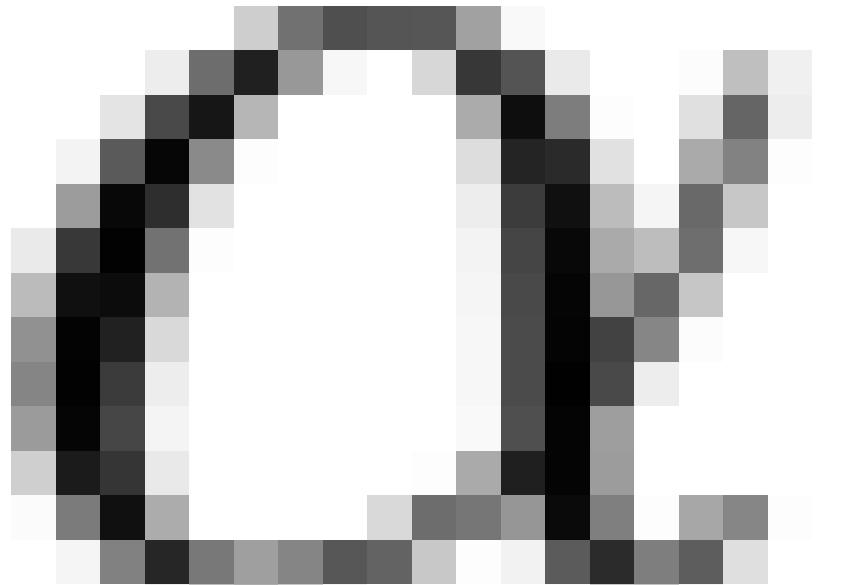


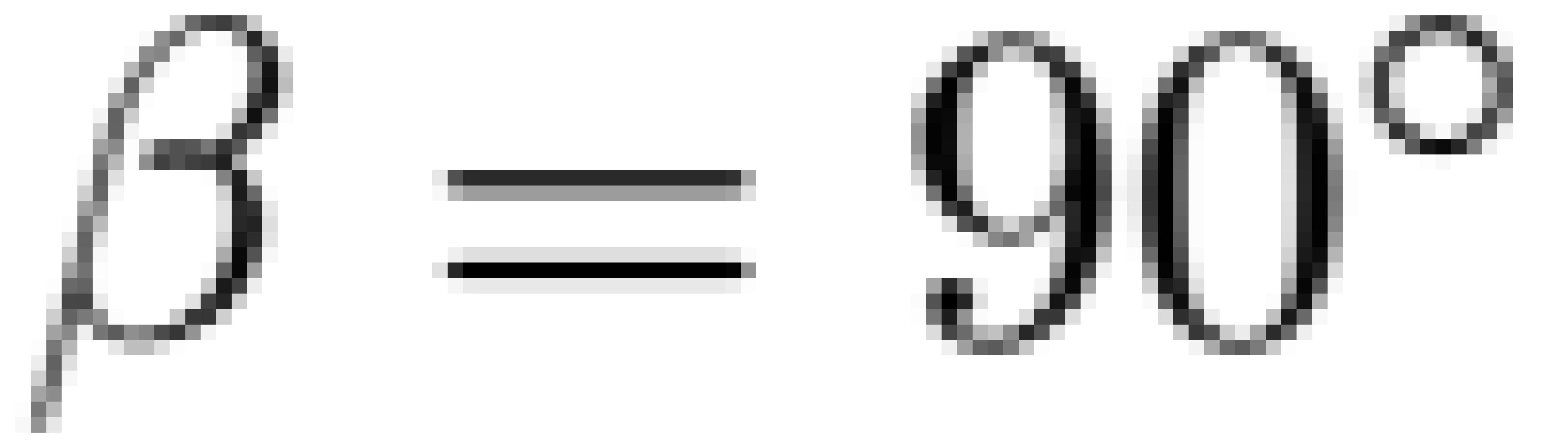




$$\underline{S}_P =$$

$$\begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$



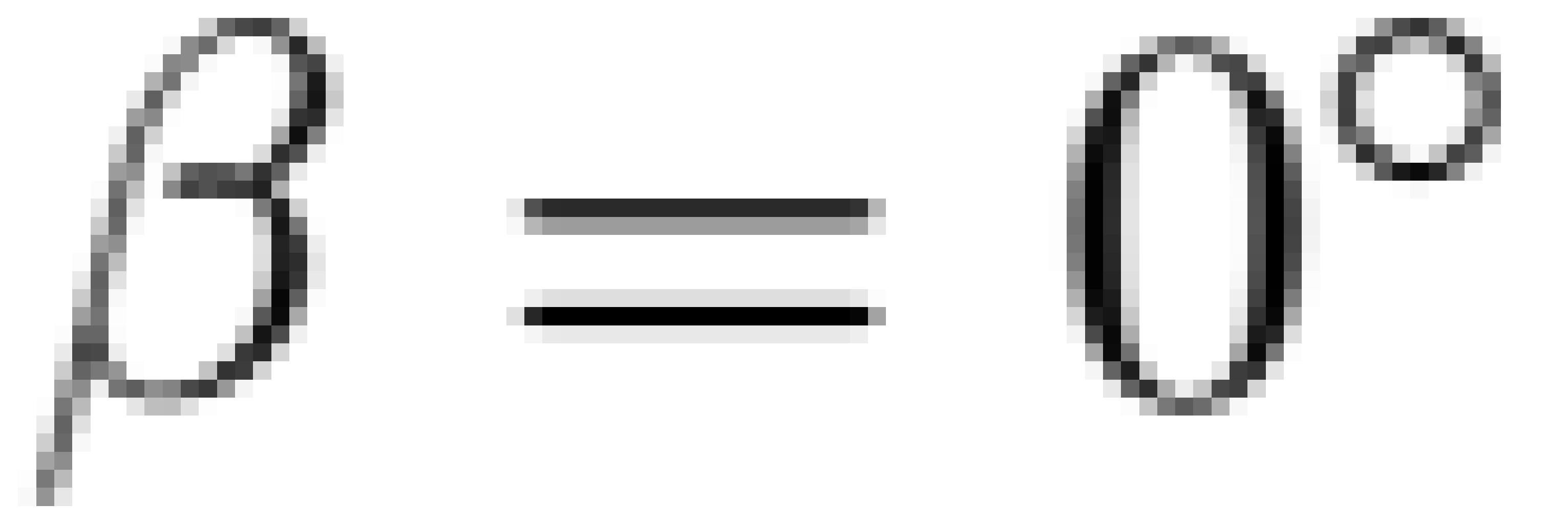


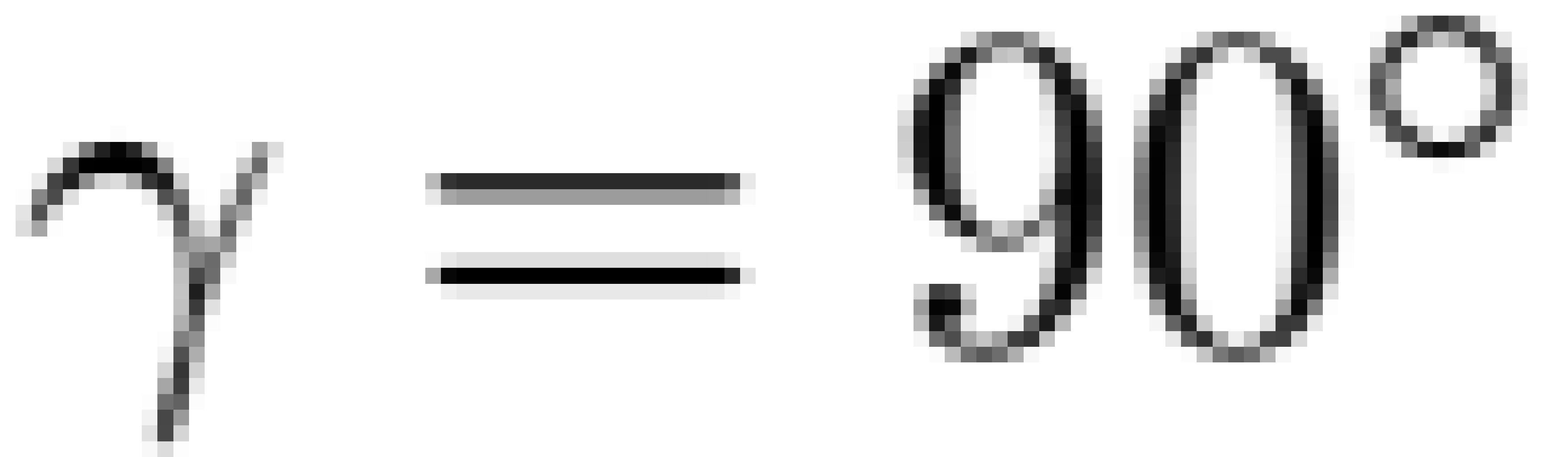


$$R_{PG} =$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 10 & 5 \\ 1 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 1 & 0 & 30 \end{bmatrix}$$





$$R_{PG} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 10 \\ 0 & -1 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & -1 & 25 \end{bmatrix}$$

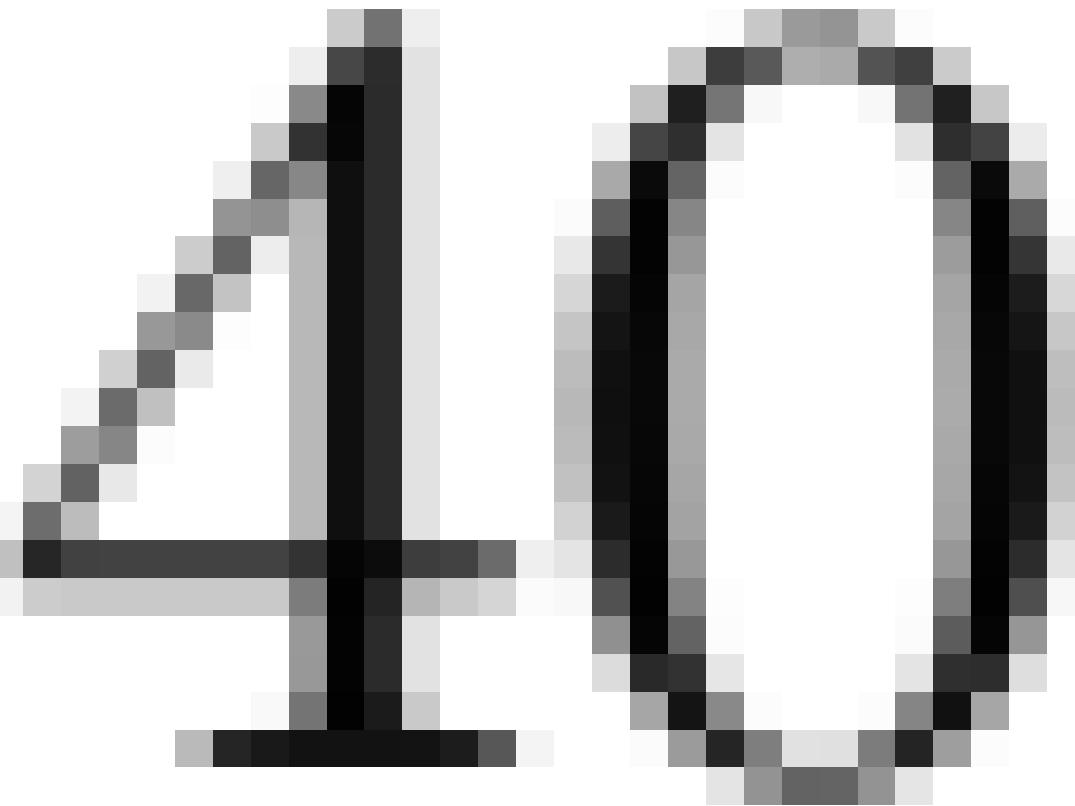


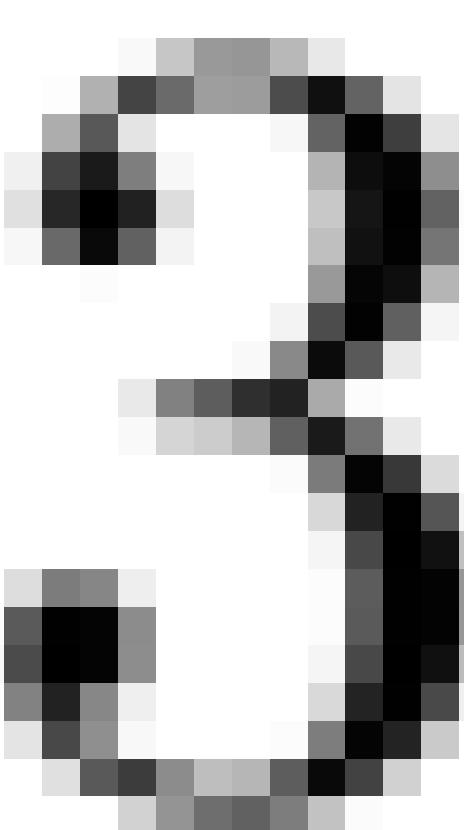
$$R_{PG} =$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 10 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

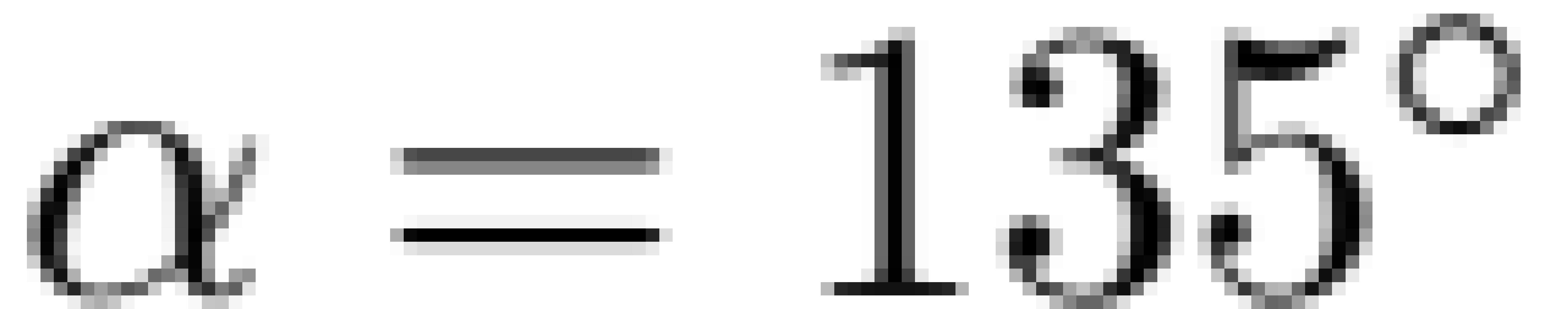






$$\underline{S}_P =$$

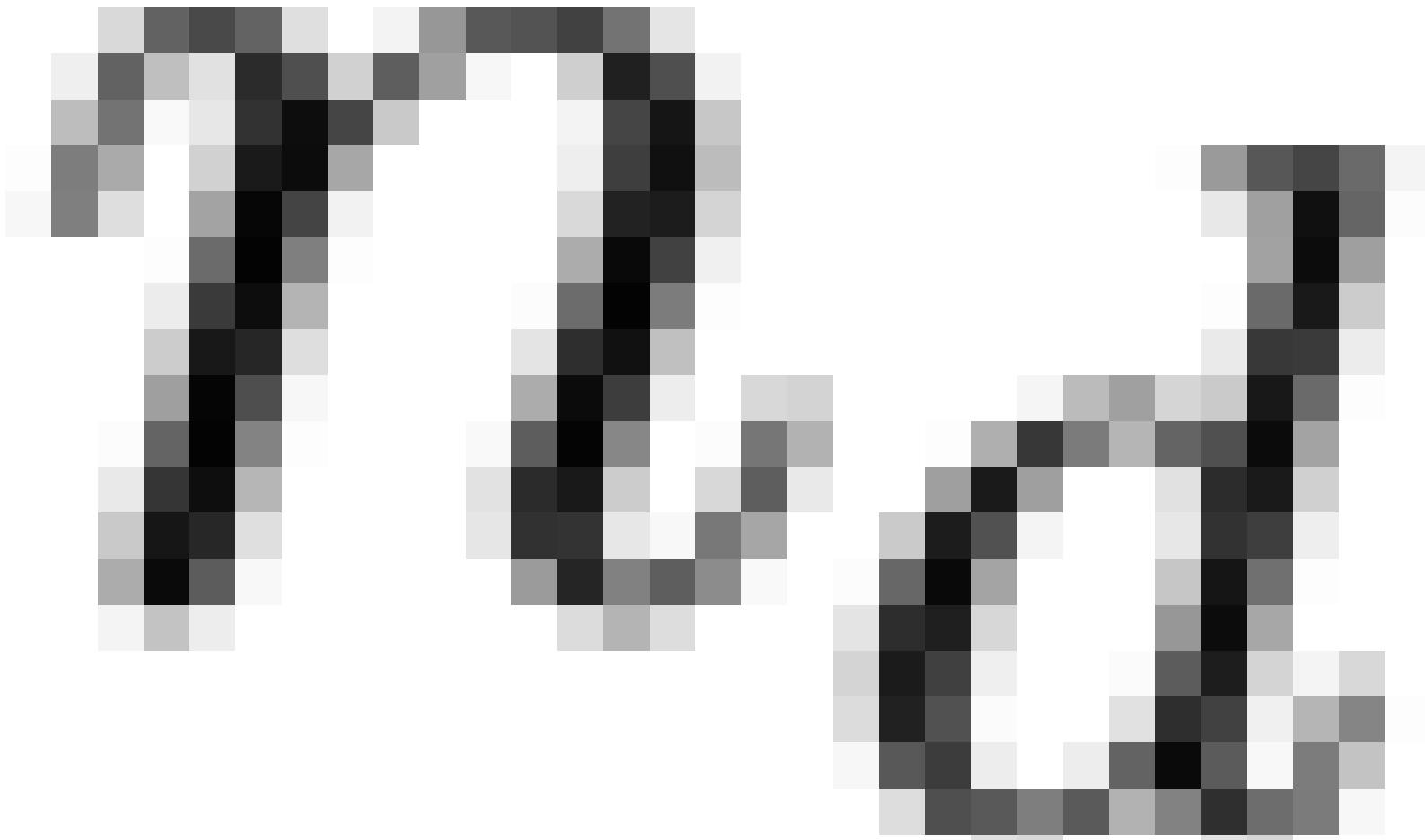
$$\begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

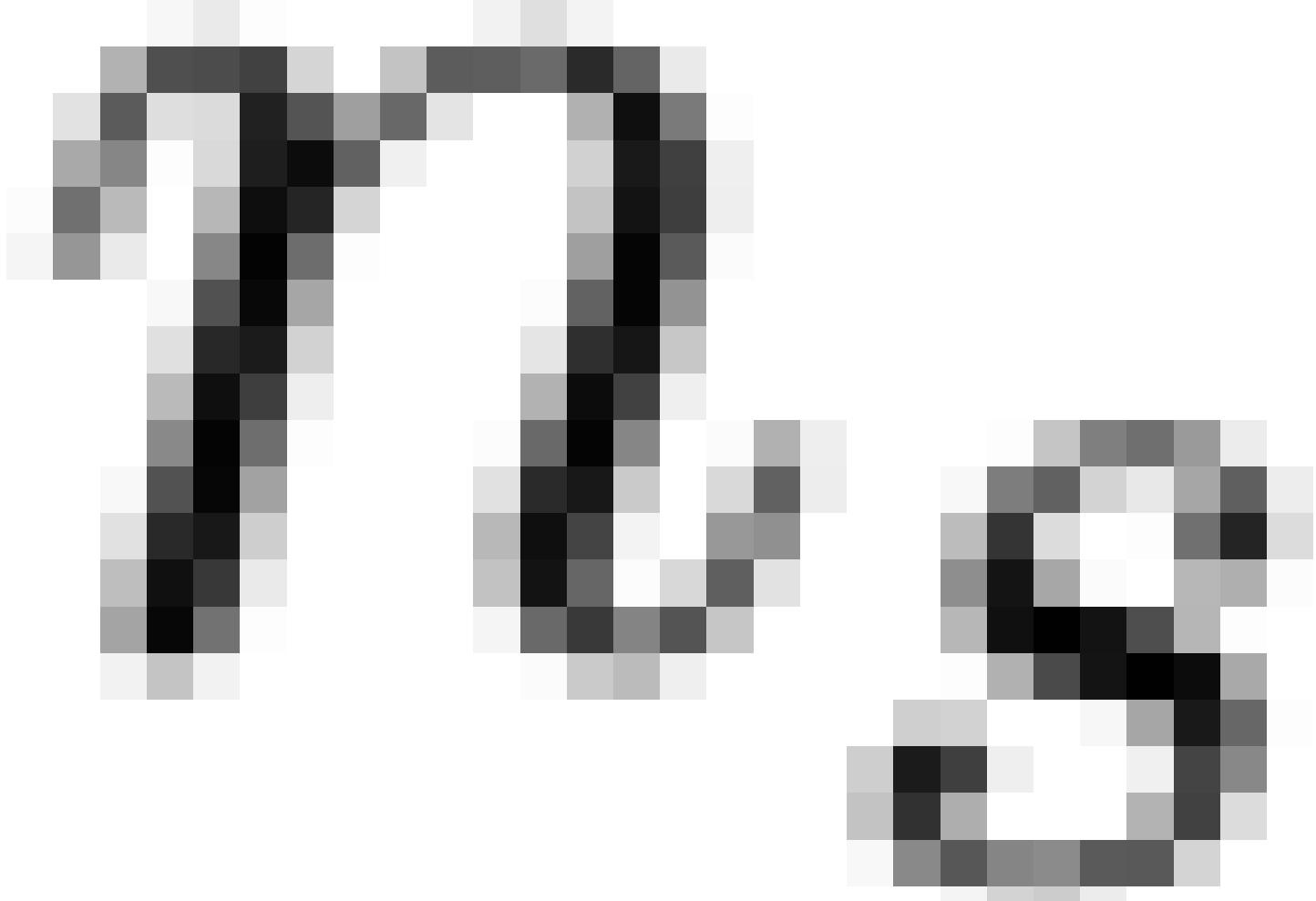


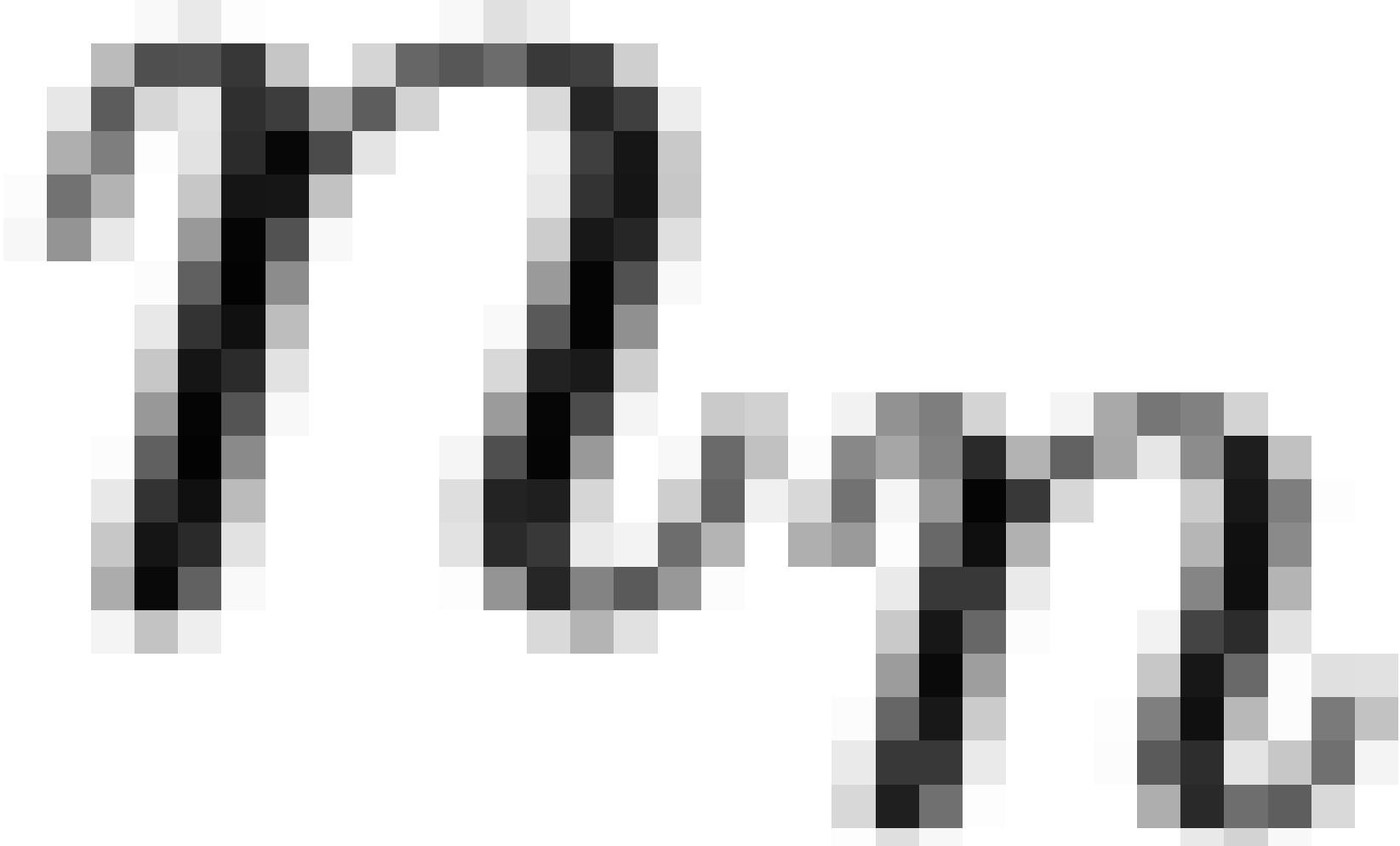
$$R_{PG} =$$

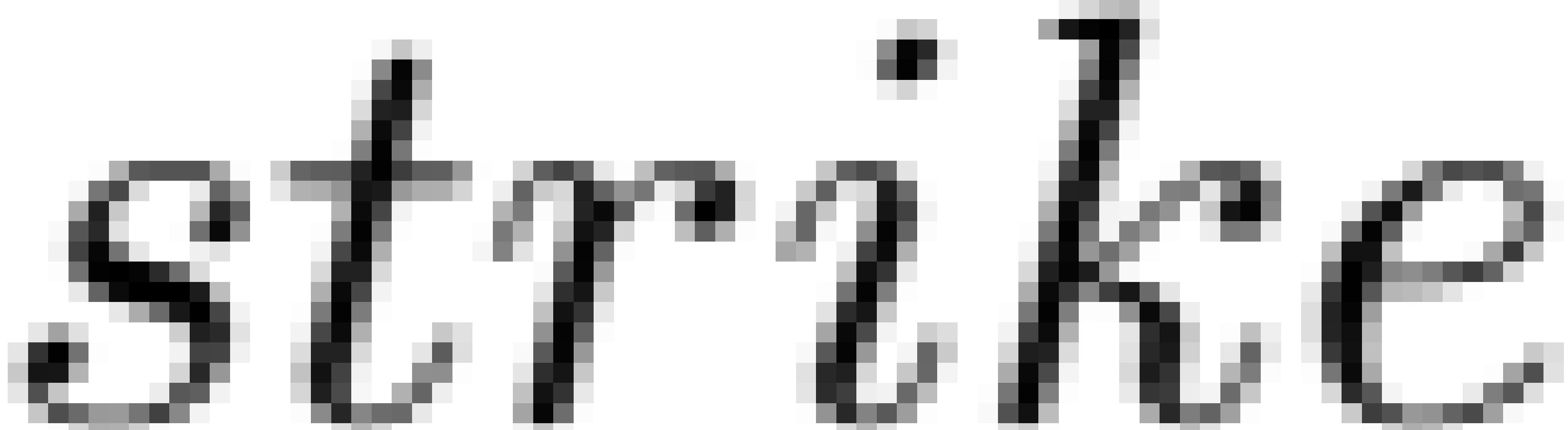
$$\begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}^T \begin{bmatrix} 60 & 0 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -0.707 & 47.5 & -12.5 \\ 0 & 40 & 0 \\ -12.5 & 47.5 & 0 \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 & 0 \\ 0 & 0 & 35 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$



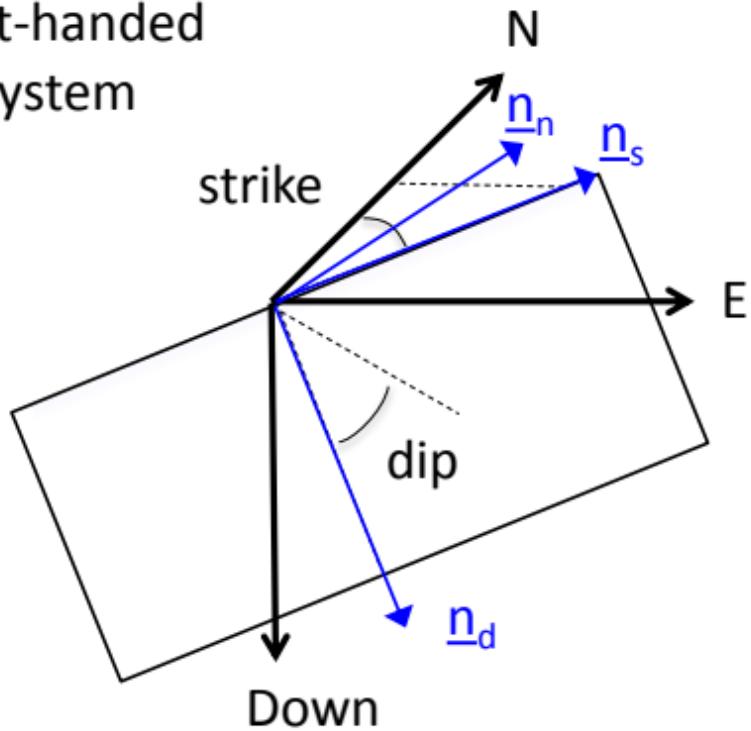






d-s-n

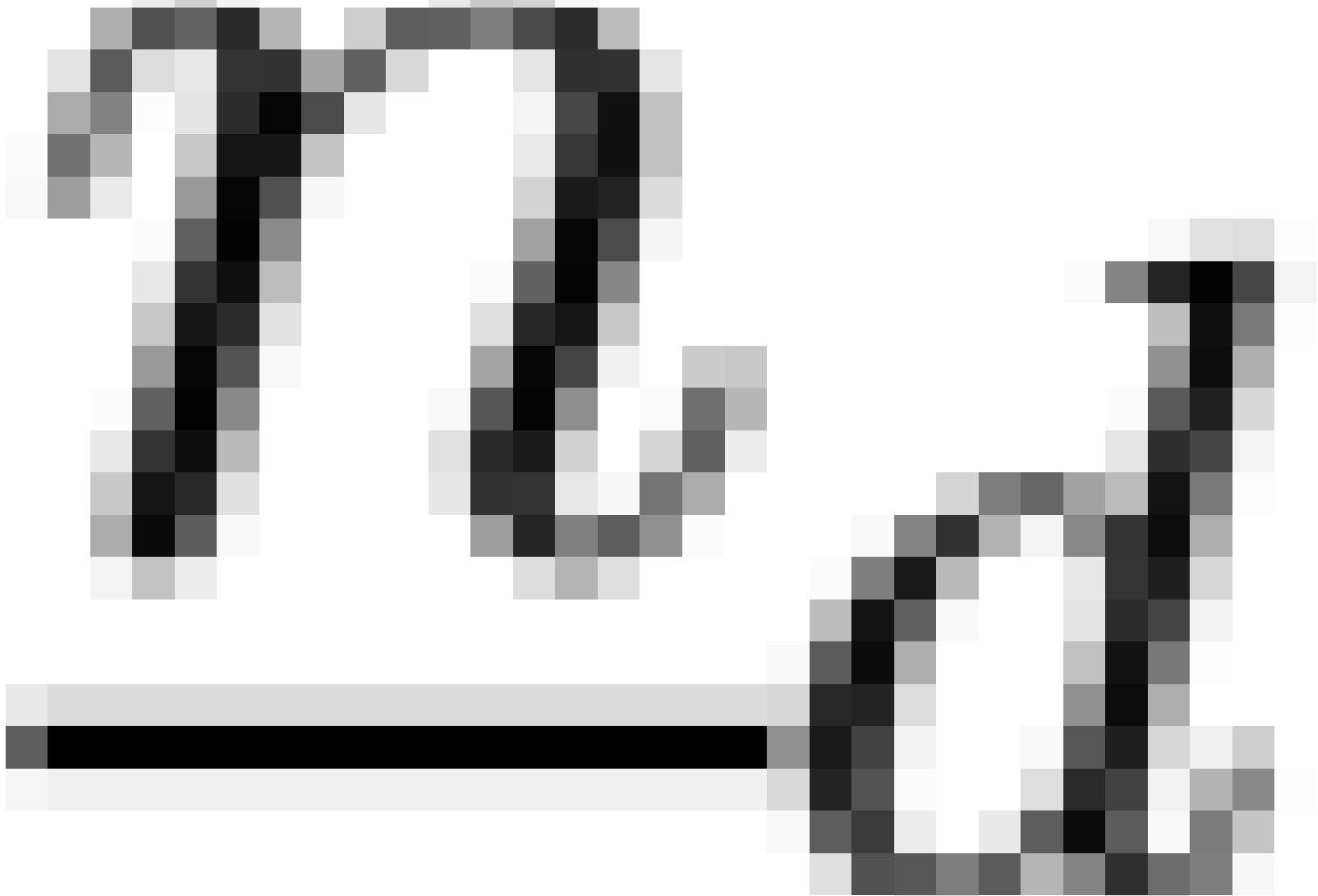
Right-handed  
system



$$\underline{n}_n = \begin{bmatrix} -\sin(strike)\sin(dip) \\ \cos(strike)\sin(dip) \\ -\cos(dip) \end{bmatrix}$$

$$\underline{n}_s = \begin{bmatrix} \cos(strike) \\ \sin(strike) \\ 0 \end{bmatrix}$$

$$\underline{n}_d = \begin{bmatrix} -\sin(strike)\cos(dip) \\ \cos(strike)\cos(dip) \\ \sin(dip) \end{bmatrix}$$

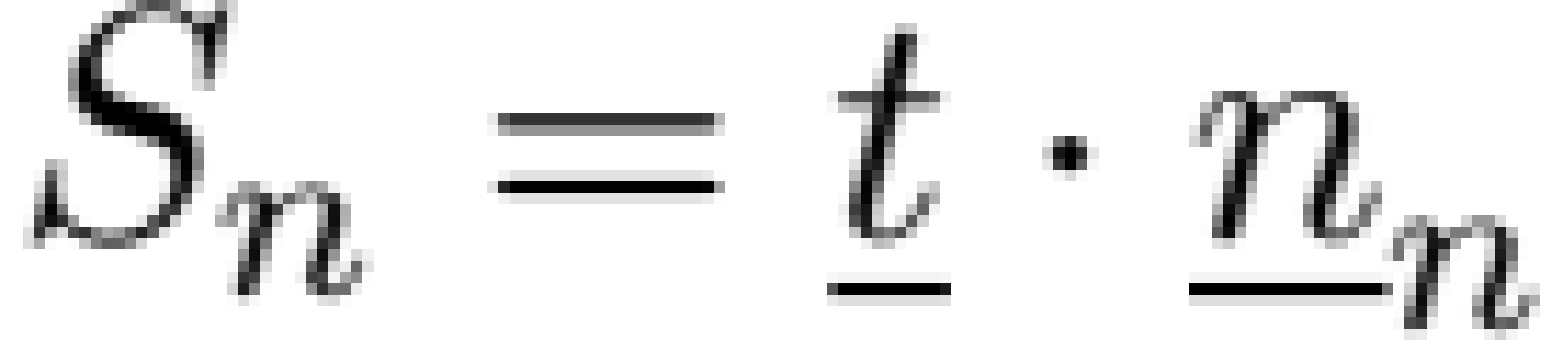








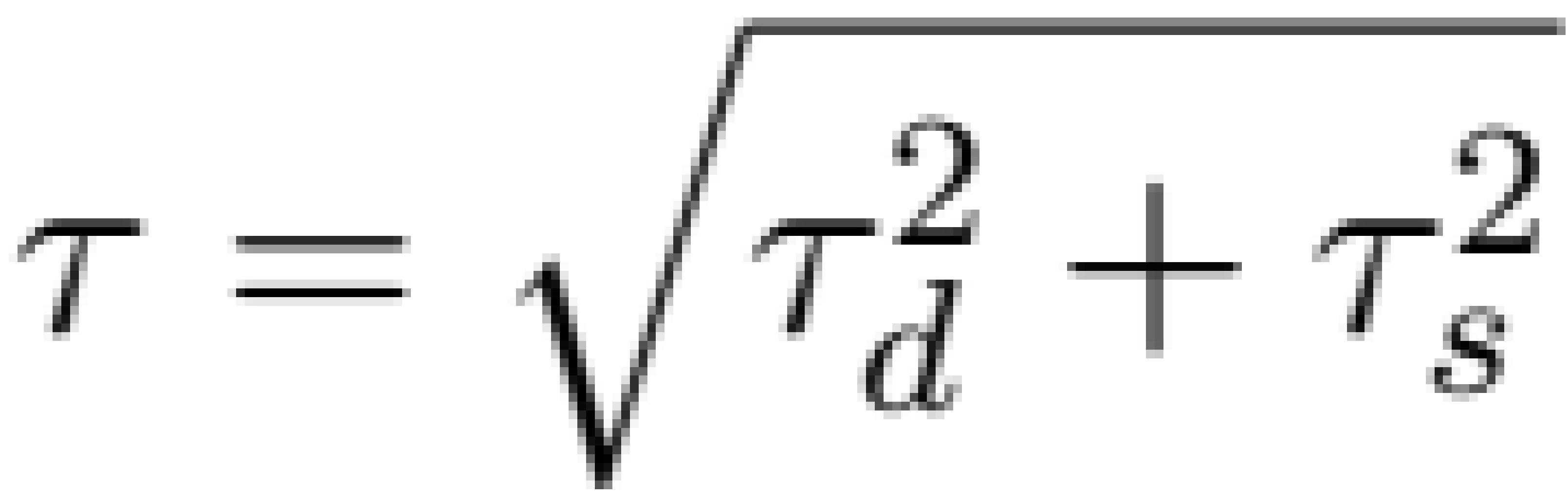


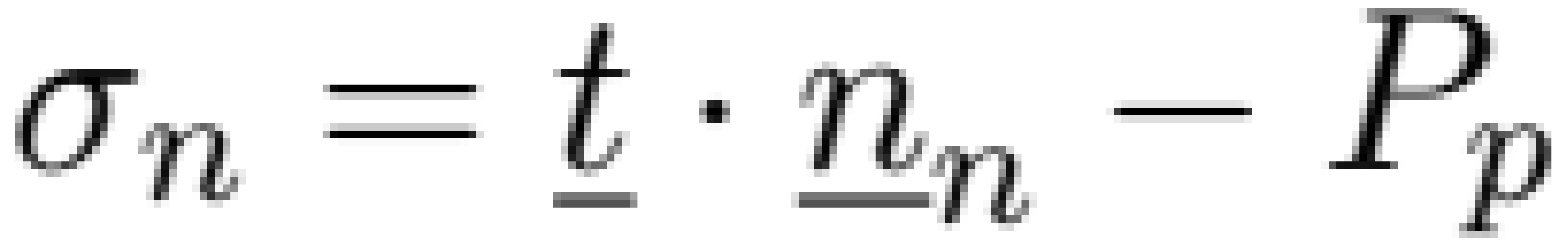


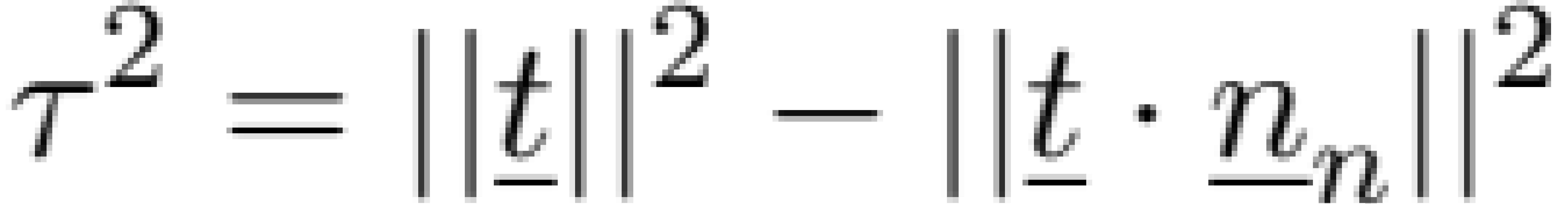


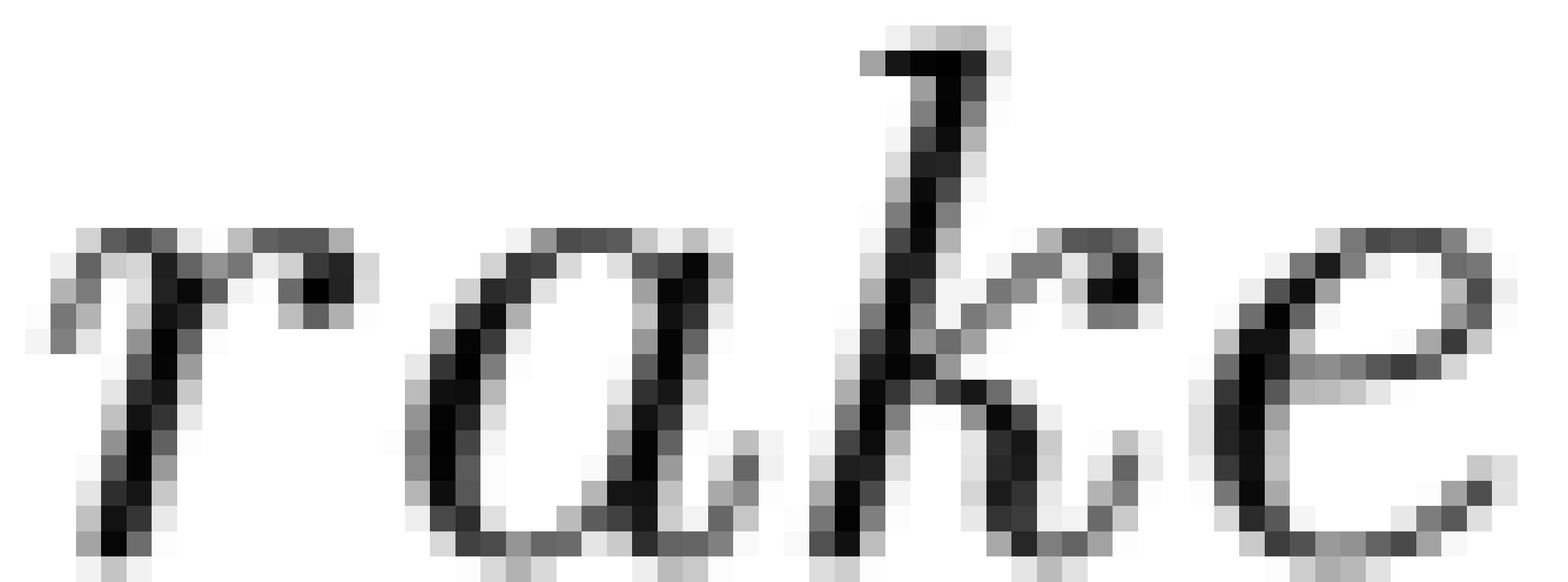
$$\tau_d = t \cdot \frac{r_d}{n_d}$$

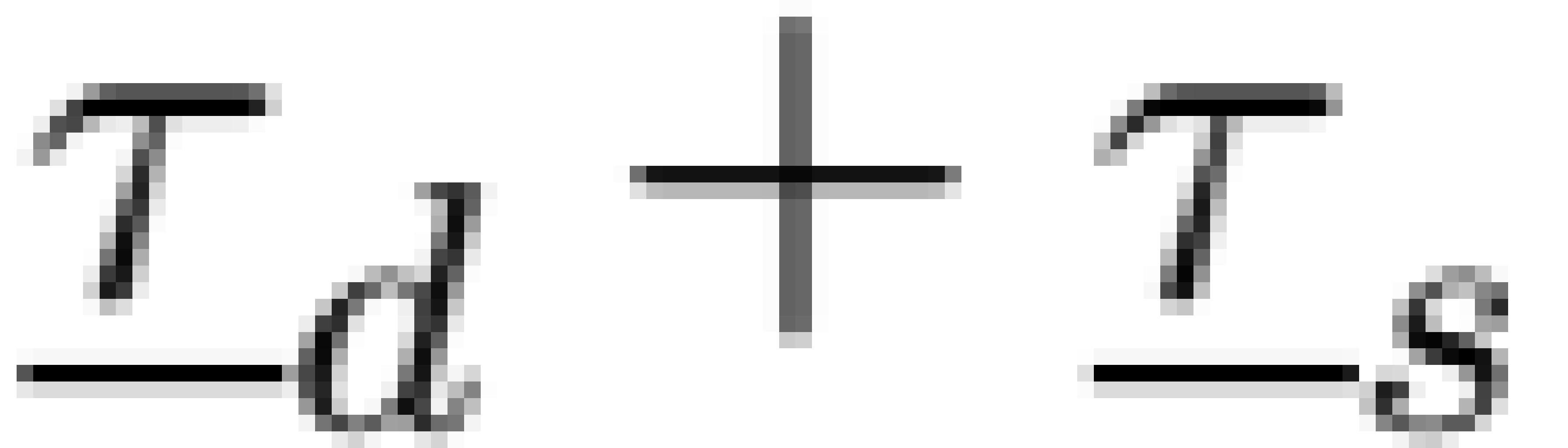
$$\tau_s = t \cdot \frac{r_s}{n_s}$$





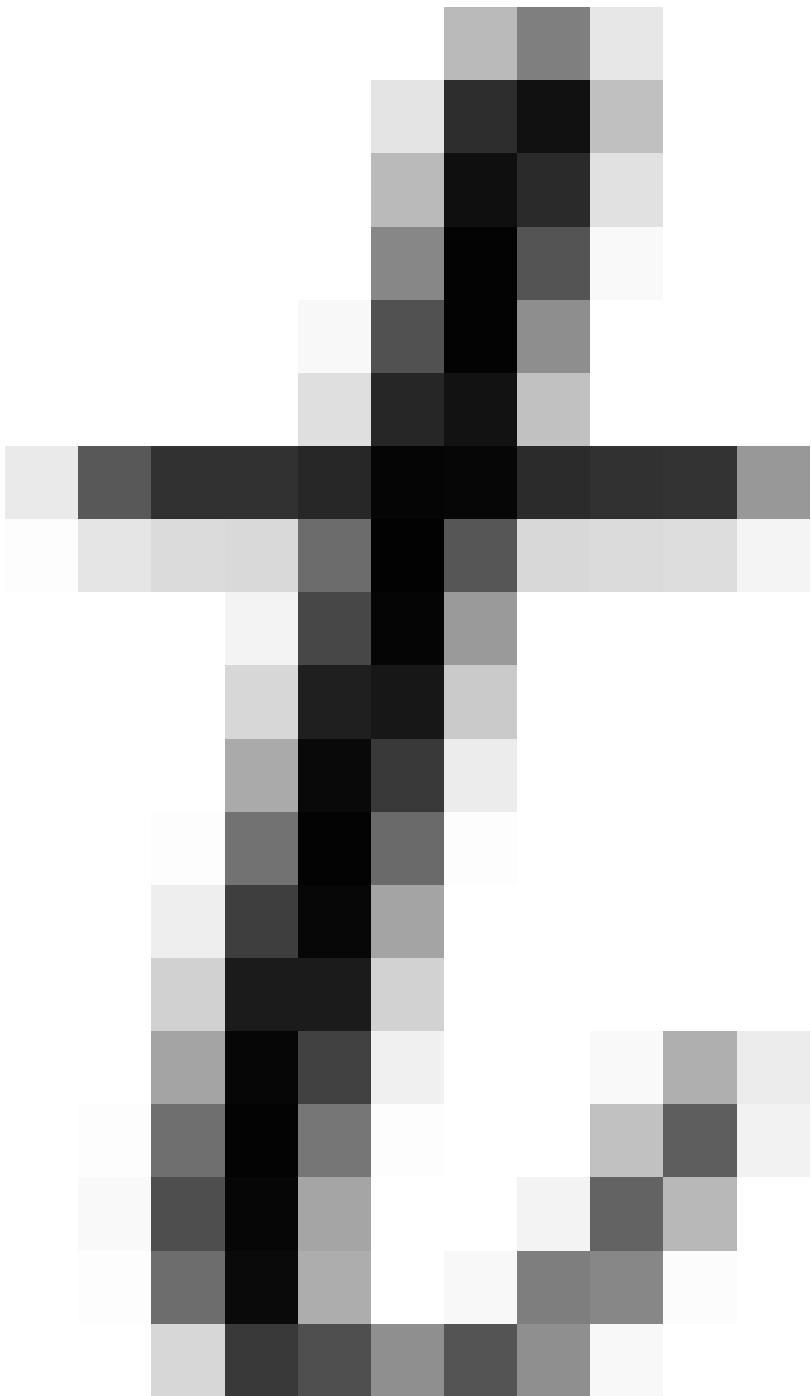


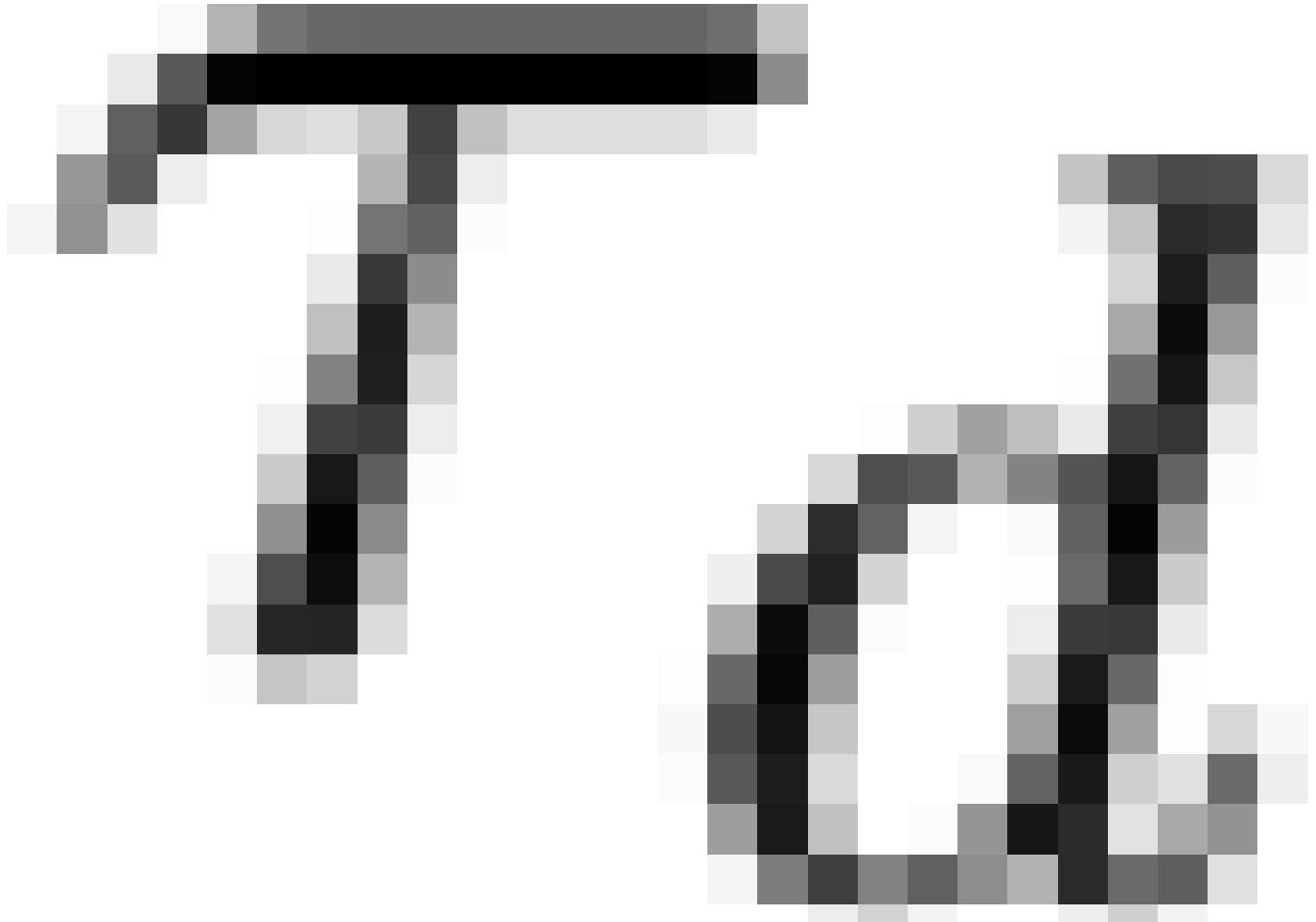


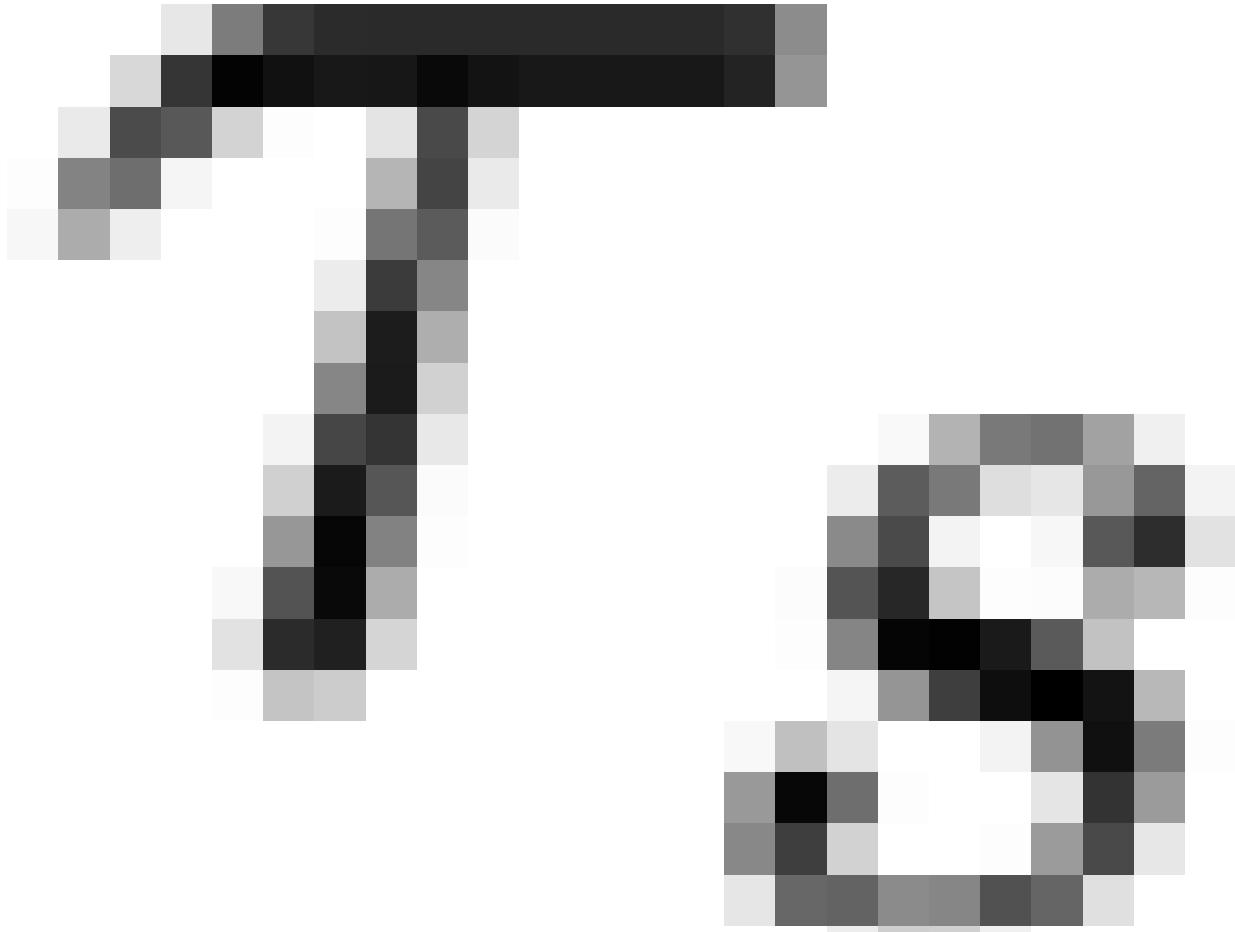


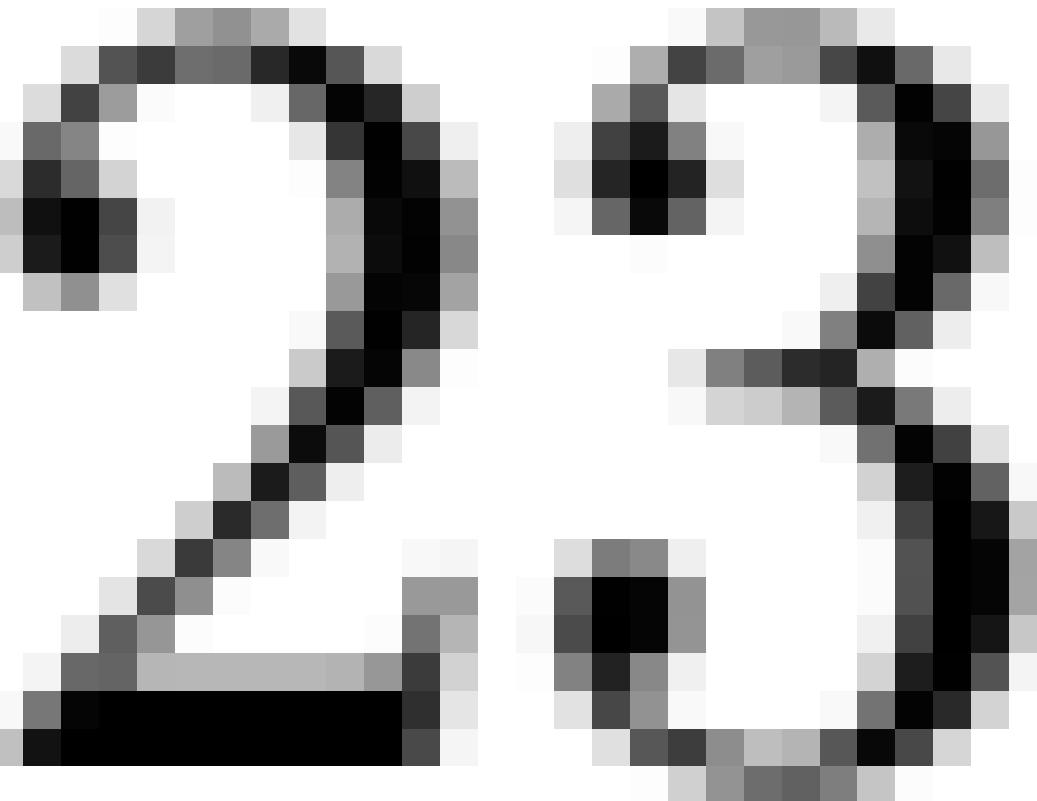
rake = arctan

$$\frac{\tau_d}{\tau_s}$$

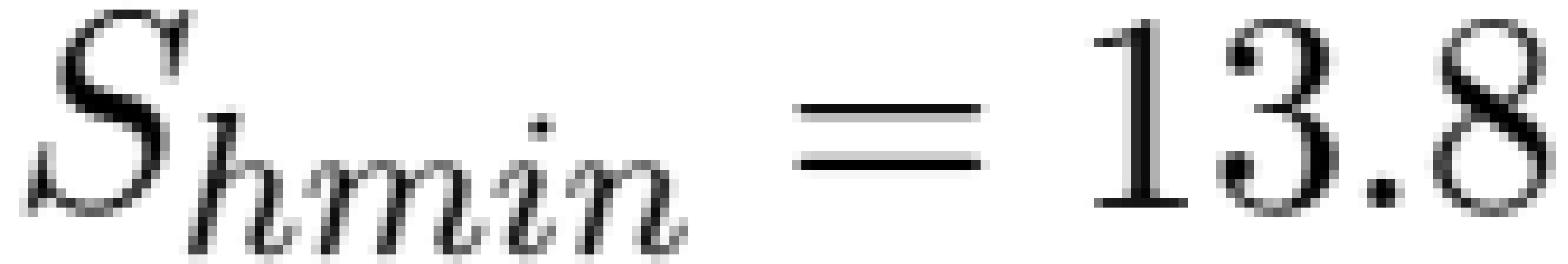


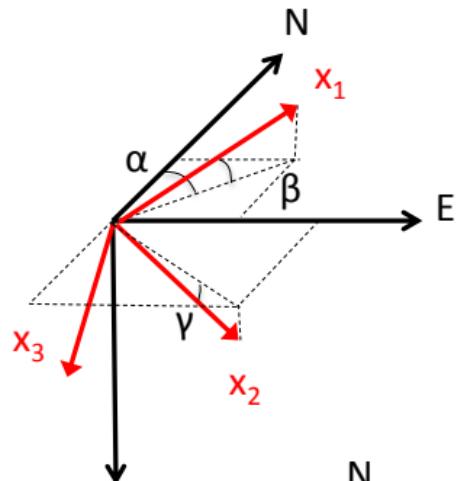










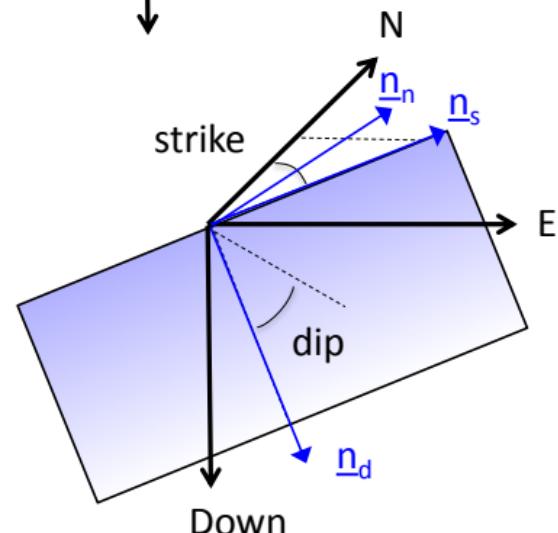


$$\underline{\underline{S}}_P = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 13.8 \end{bmatrix}$$

$a = 90^\circ$  Azimuth of  $S_{h\min}$   
 $b = 90^\circ$   $S_1 = S_V$   
 $g = 0^\circ$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 13.8 & 0 \\ 0 & 0 & 23 \end{bmatrix}$$



Fault geometry

Strike =  $000^\circ$   
 Dip =  $60^\circ$

$$\underline{n}_n = \begin{bmatrix} 0 \\ 0.867 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [0, 11.95, -11.50] \text{ MPa}$$

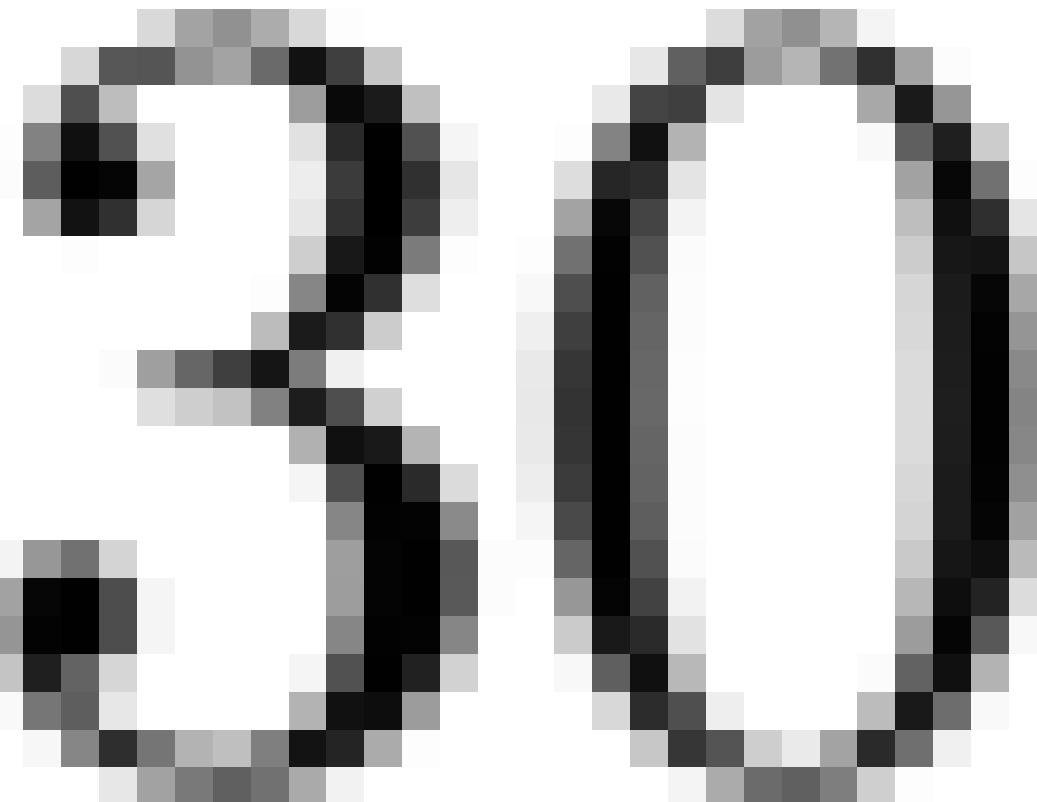
$$S_n = 16.1 \text{ MPa}$$

$$t_d = -3.98 \text{ MPa}$$

$$t_s = 0 \text{ MPa}$$

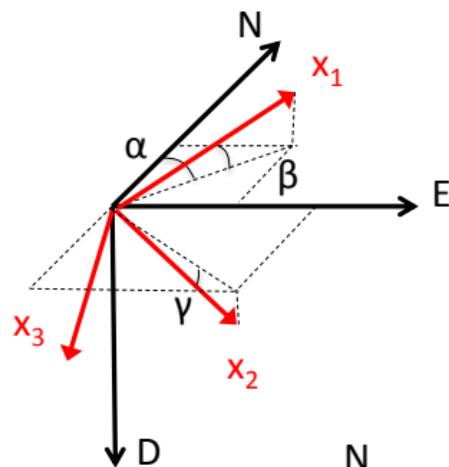
$$\text{rake} = 90^\circ$$

Actual stresses opposite to d-s-n



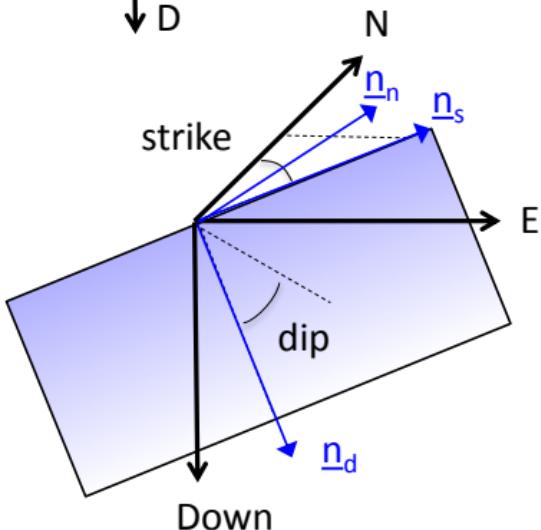






$$\underline{\underline{S}}_P = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix} \quad \begin{array}{ll} a = 120^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 90^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \\ 0.866 & 0.5 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$



Fault geometry

Strike = 060°

Dip = 90°

$$\underline{n}_n = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$\underline{t} = [-30.31, 27.5, 0] \text{ MPa}$$

$$S_n = 40 \text{ MPa}$$

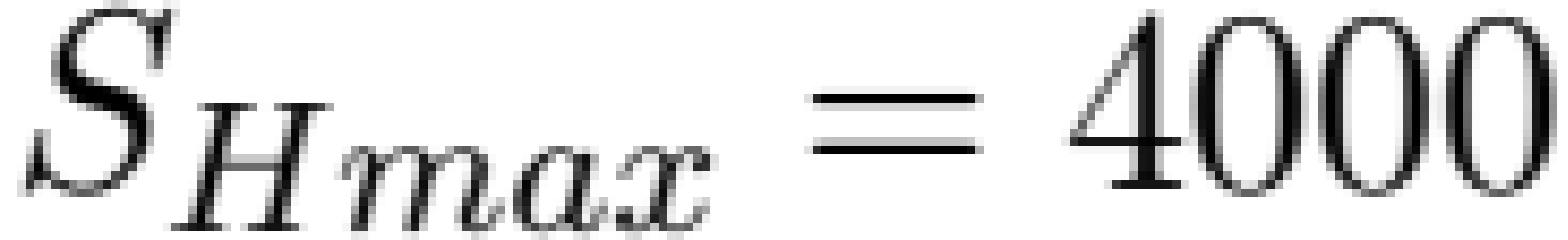
$$t_d = 0 \text{ MPa}$$

$$t_s = 8.66 \text{ MPa}$$

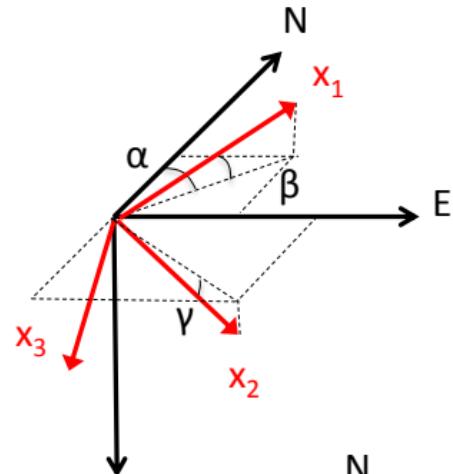
$$rake = 0^\circ$$

Actual stresses opposite to d-s-n



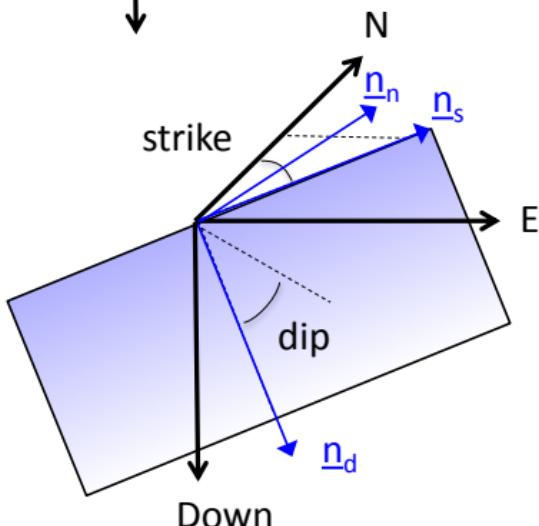






$$\underline{\underline{S}}_P = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \quad \begin{array}{ll} a = 90^\circ & \text{Azimuth of } S_{h\min} \\ b = 90^\circ & \\ g = 0^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$



Fault geometry

Strike =  $045^\circ$

Dip =  $60^\circ$

$$\underline{n}_n = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [-2450, 1840, -2500] \text{ psi}$$

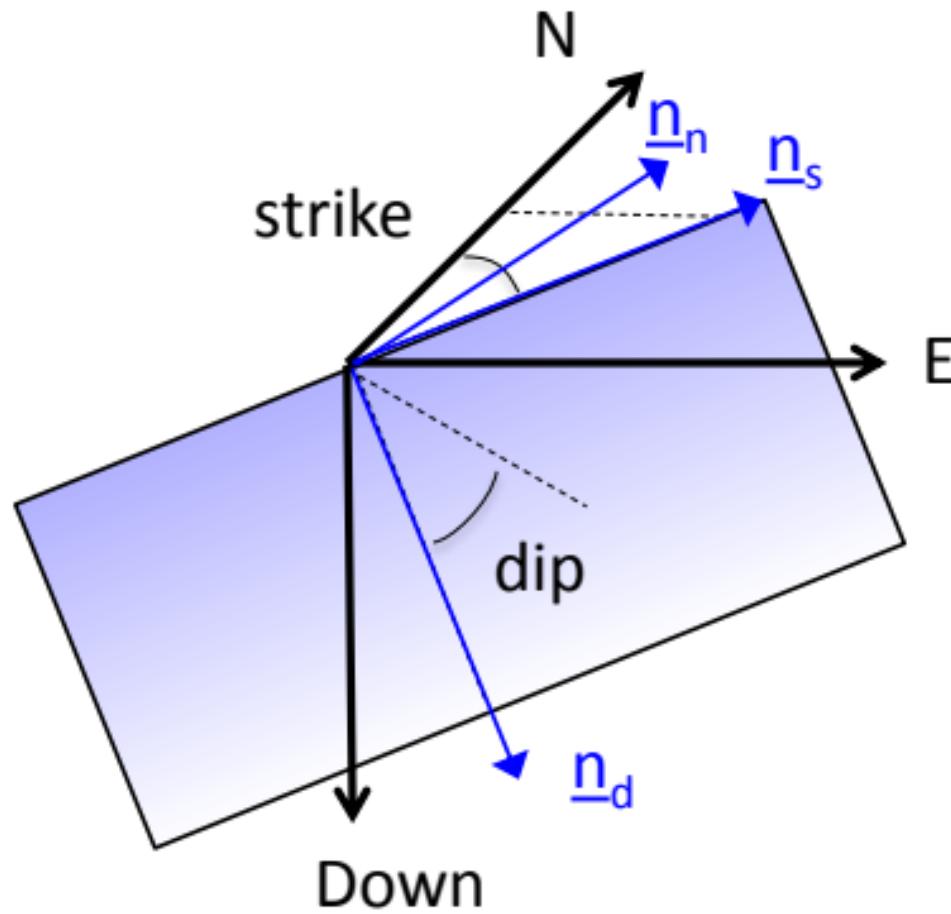
$$S_n = 3870 \text{ psi}$$

$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

$$\text{rake} = 56.3^\circ$$

Actual stresses opposite to d-s-n



Fault geometry

Strike =  $225^\circ$

Dip =  $60^\circ$

$$\underline{n}_n = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [2450, -1840, -2500] \text{ psi}$$

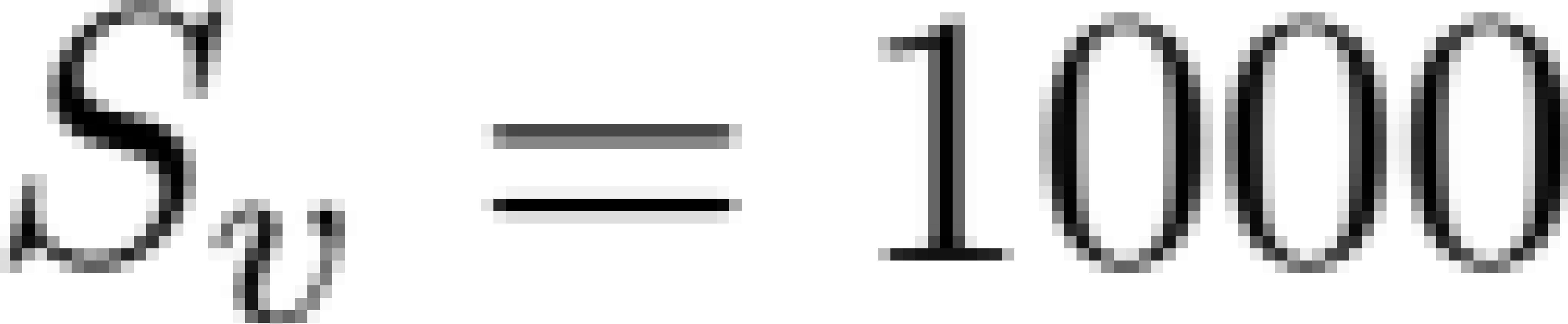
$$S_n = 3870 \text{ psi}$$

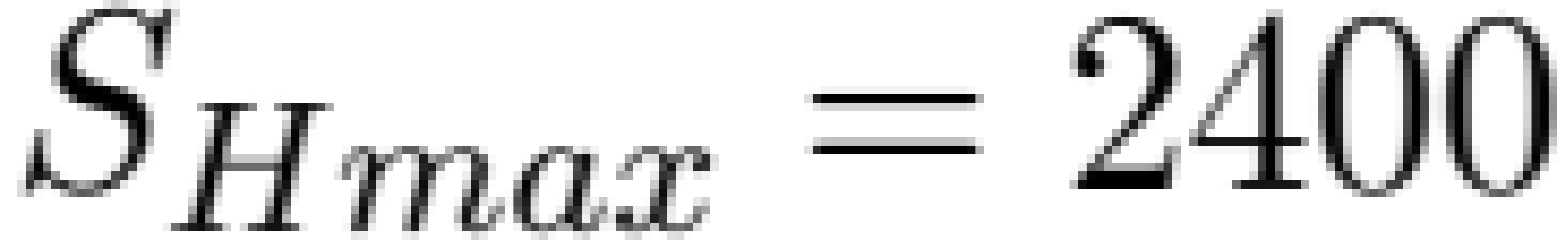
$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

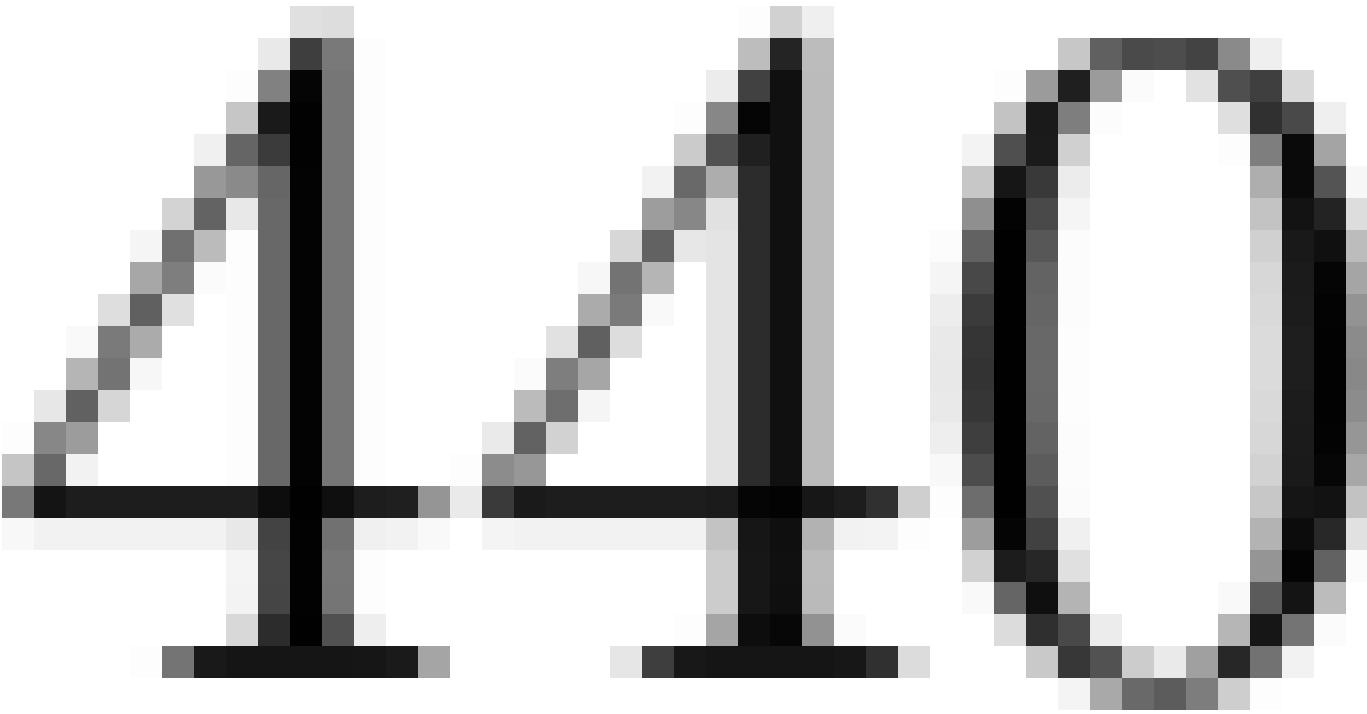
$$\text{rake} = 56.3^\circ$$

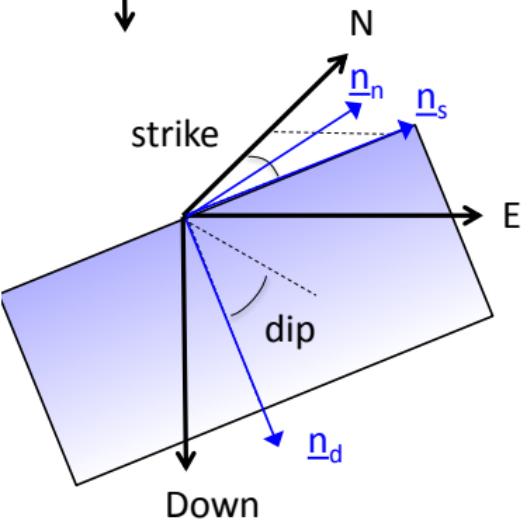
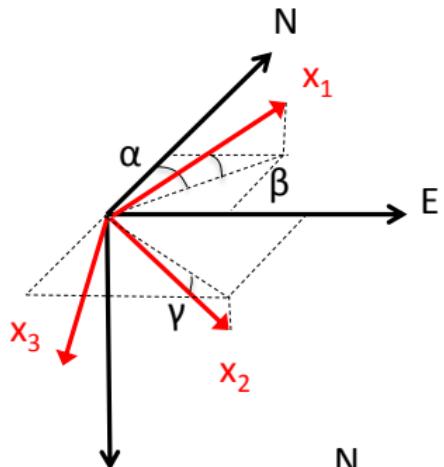
Actual stresses opposite to d-s-n











$$\underline{\underline{S}}_P = \begin{bmatrix} 2400 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \quad \begin{array}{ll} \alpha = 150^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 0^\circ & S_3 = S_V \end{array}$$

$$P_w = 440 \text{ psi}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.866 & 0.5 & 0 \\ -0.5 & -0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

Fault geometry

Strike =  $120^\circ$

Dip =  $70^\circ$

$$\underline{n}_n = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix}$$

$$\underline{t} = [-1465, -281, -342] \text{ psi}$$

$$S_n = 1441 \text{ psi}$$

$$t_d = 160 \text{ psi}$$

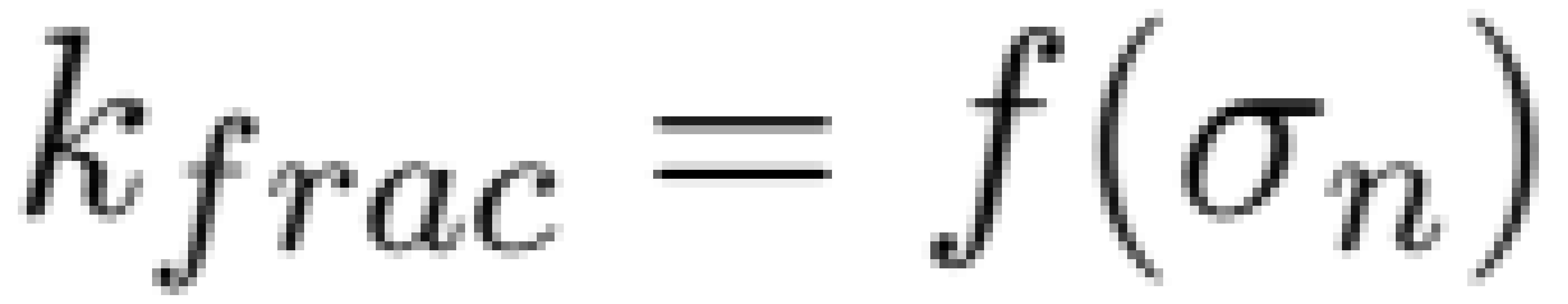
$$t_s = 488 \text{ psi}$$

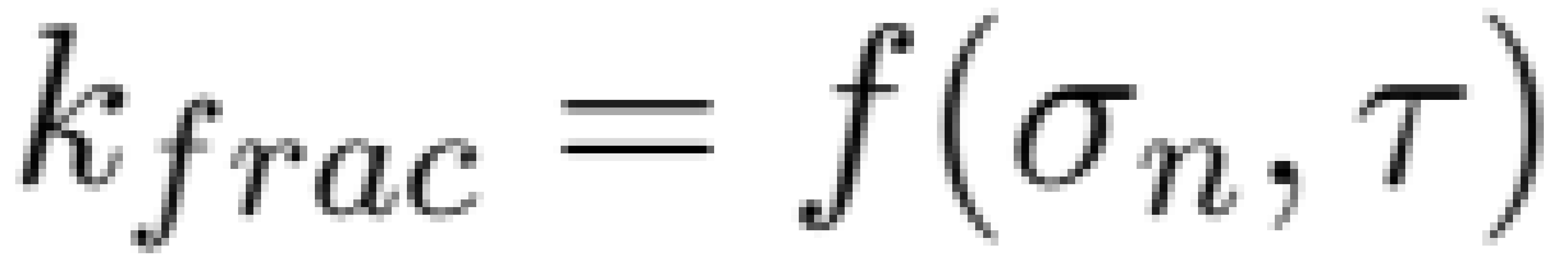
$$\text{rake} = 18.21^\circ$$

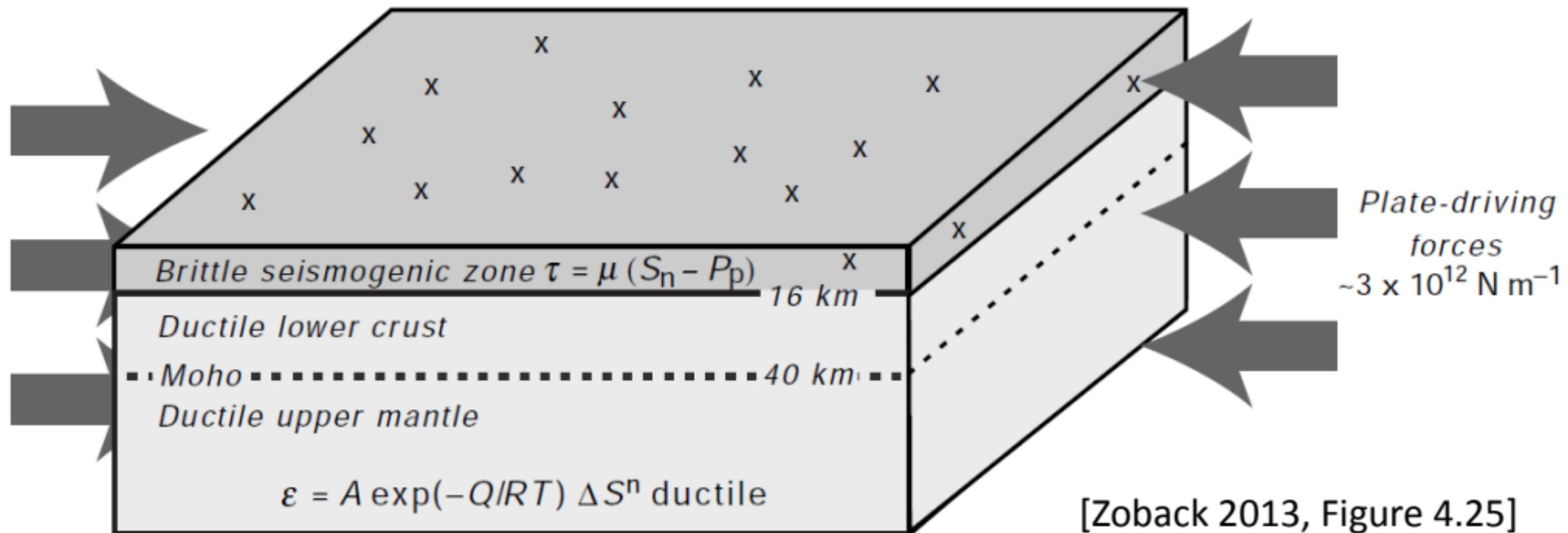
$$t / S_n = 0.51$$

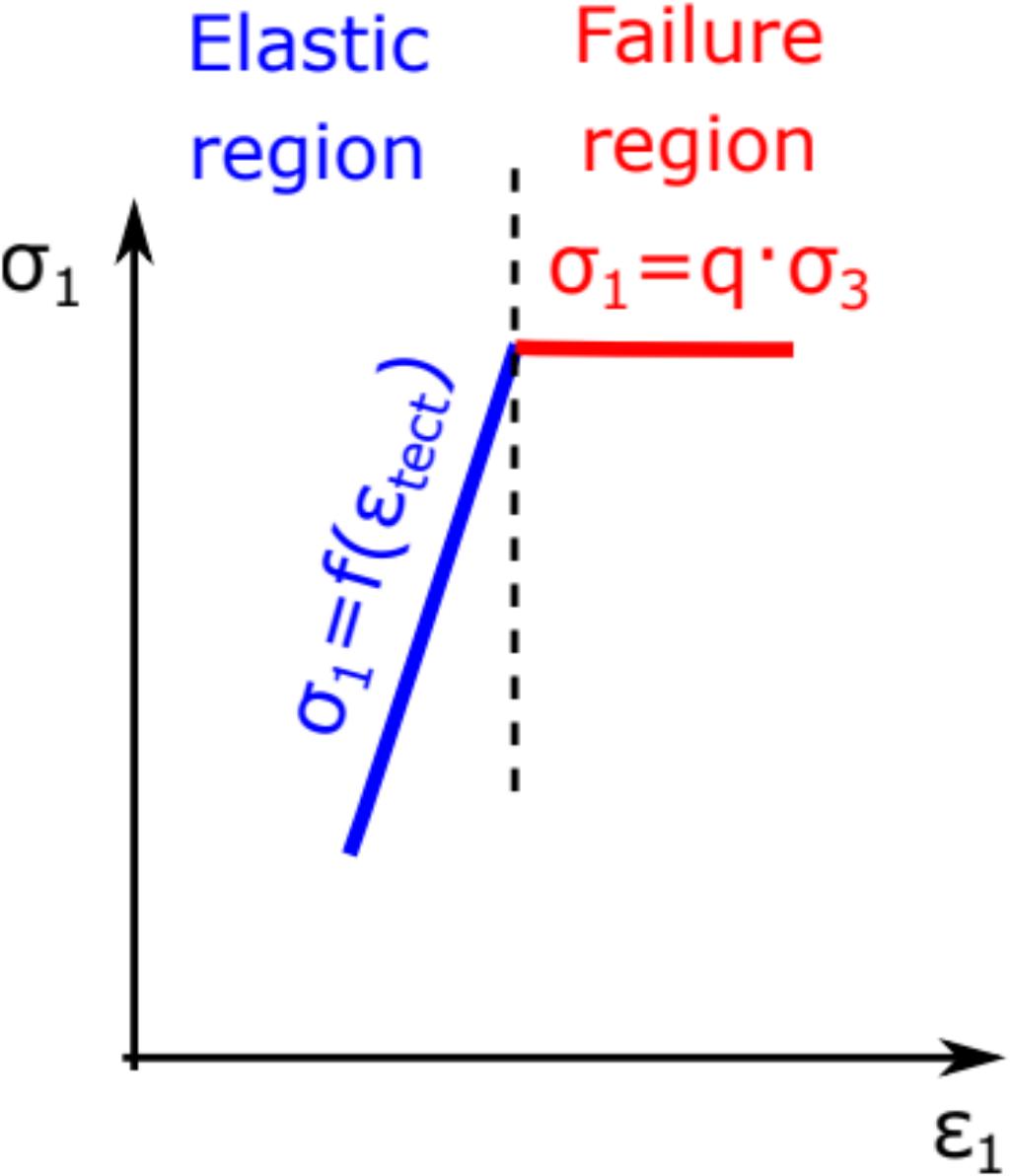
Actual stresses opposite to d-s-n





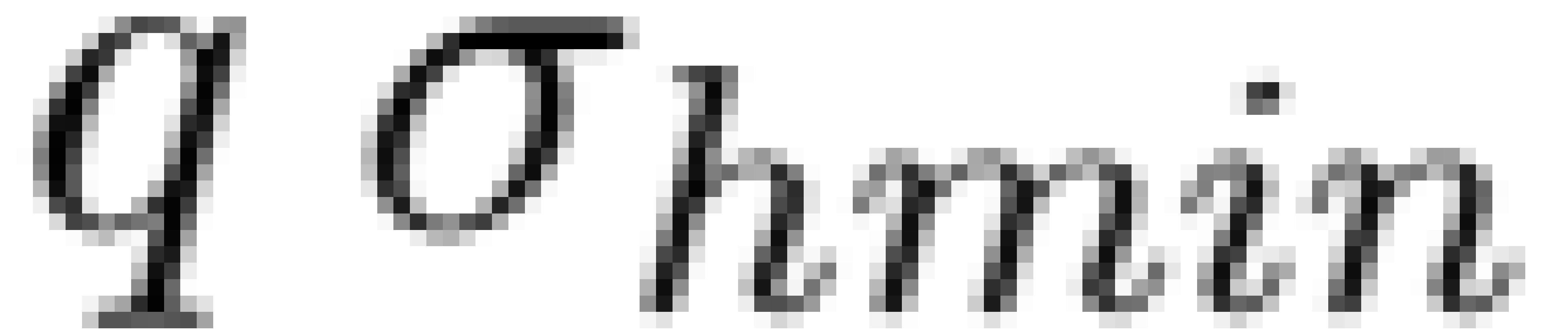


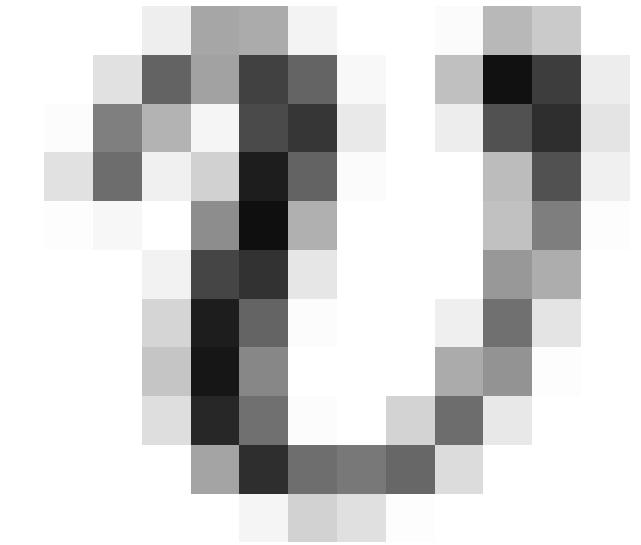
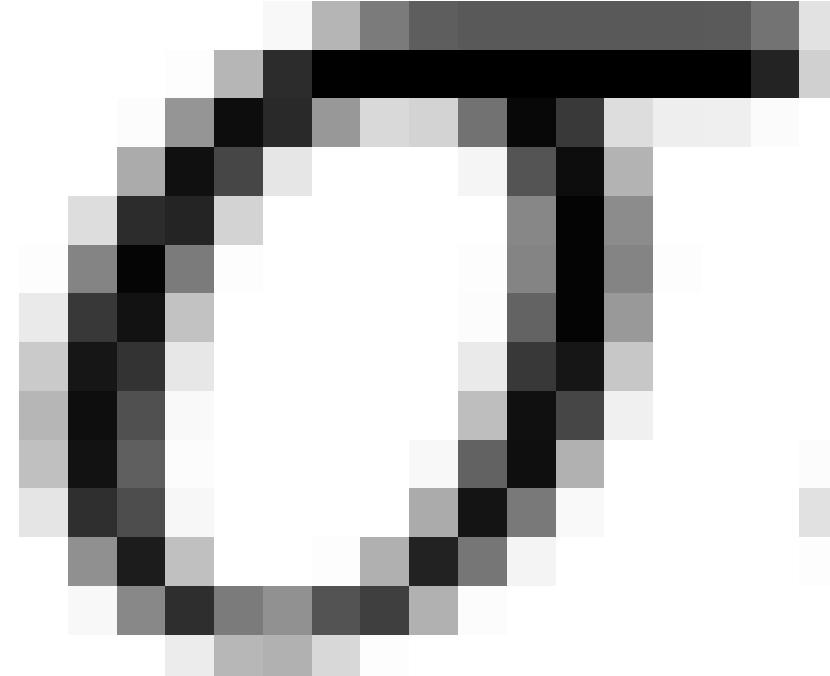
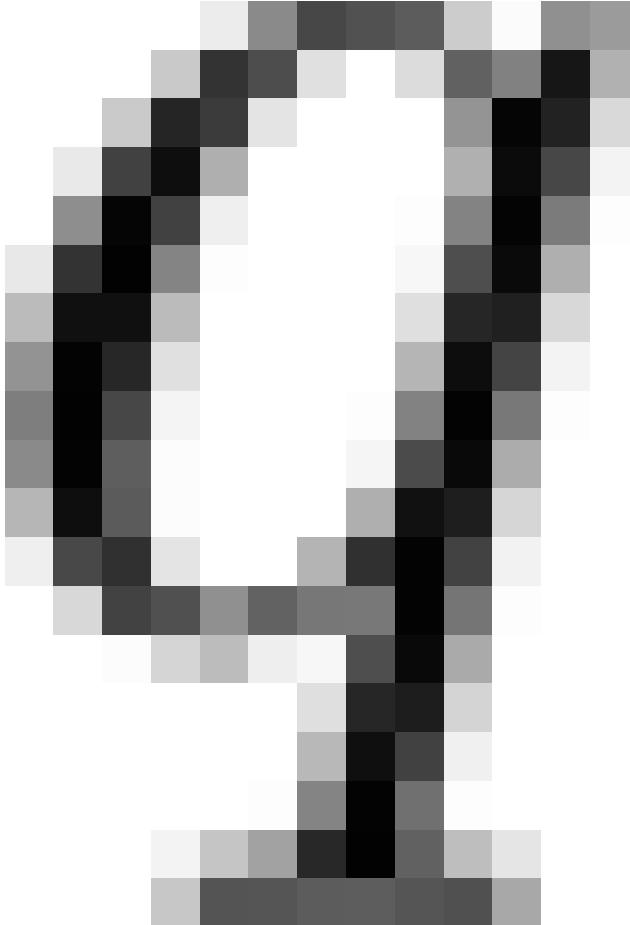


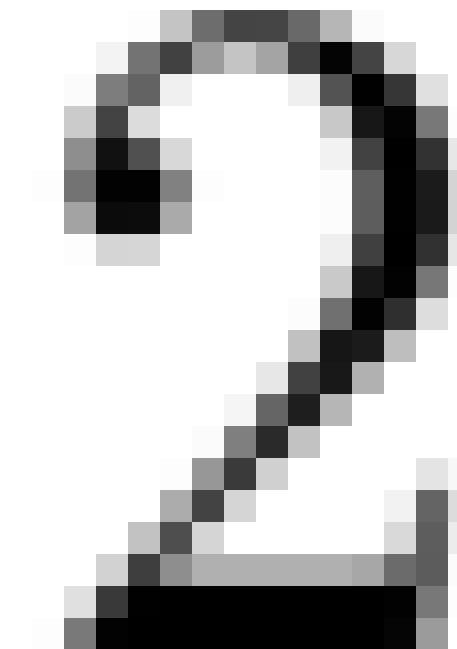
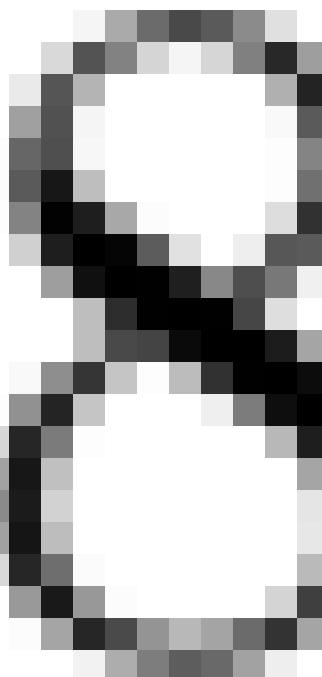
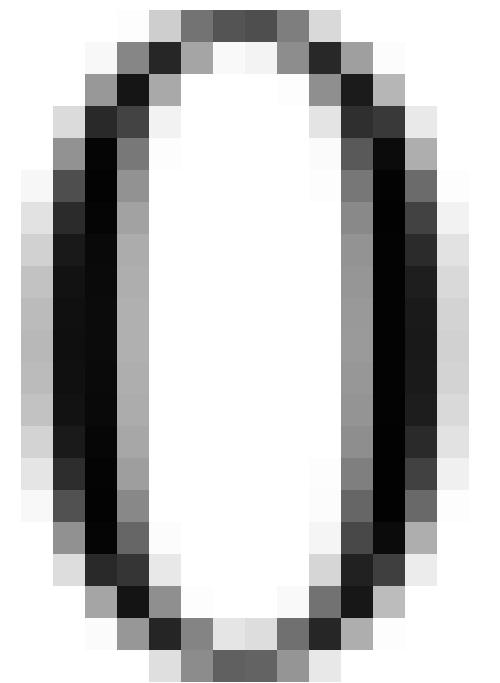




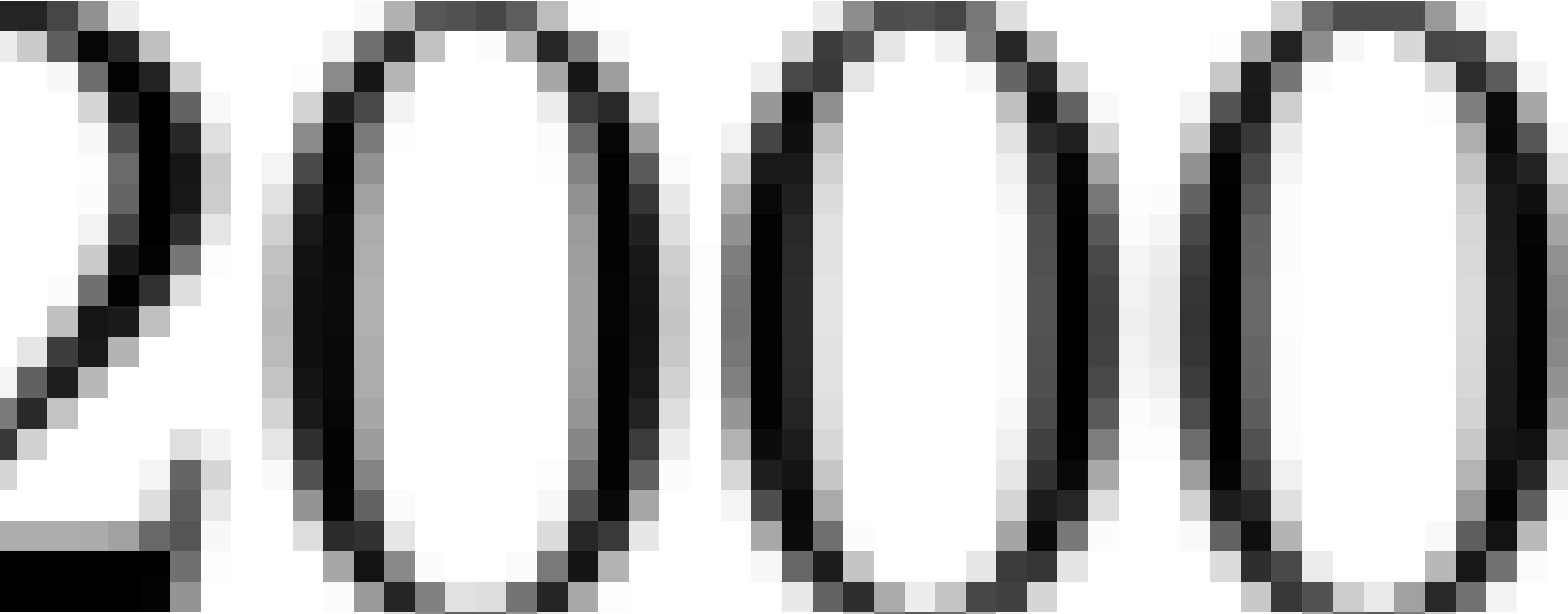


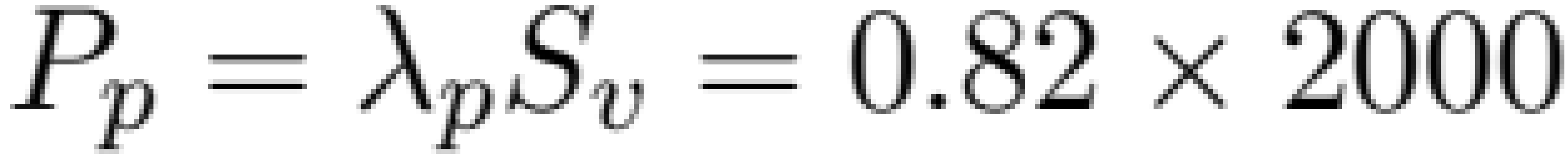


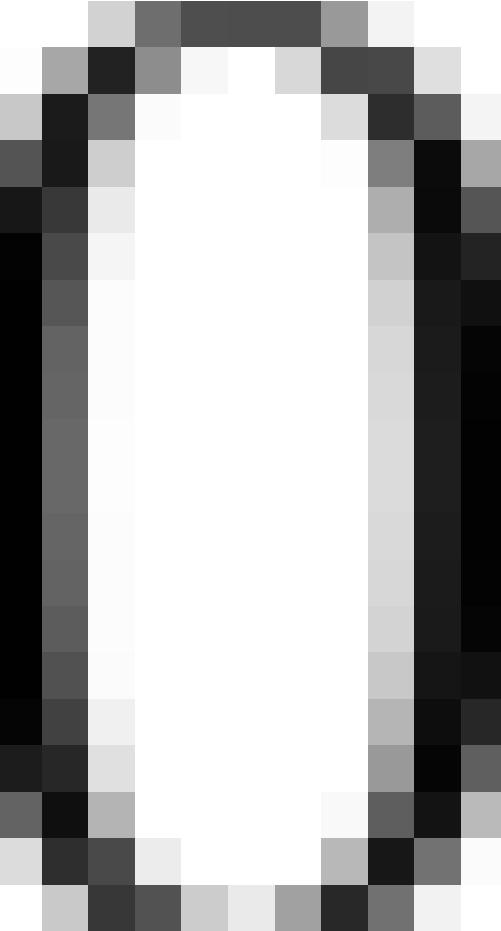
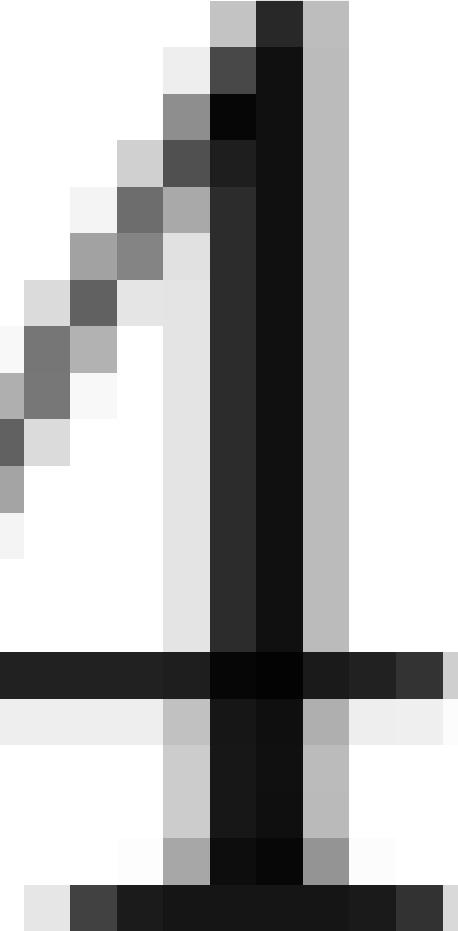
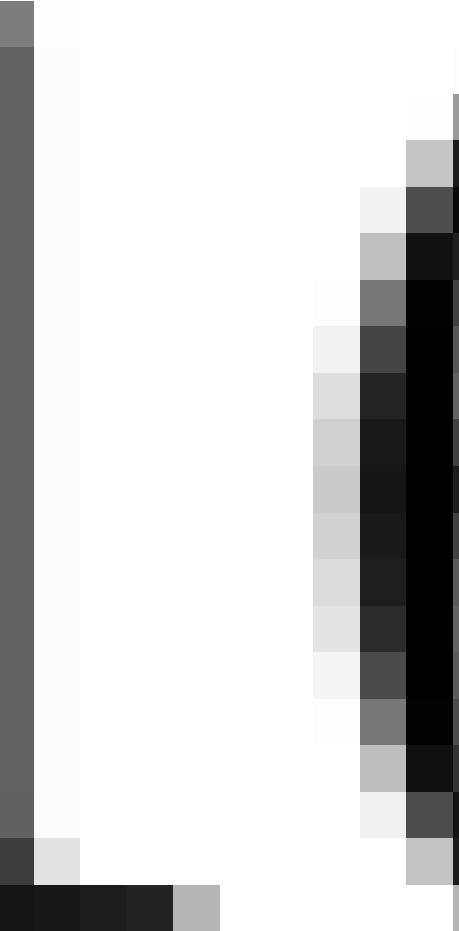


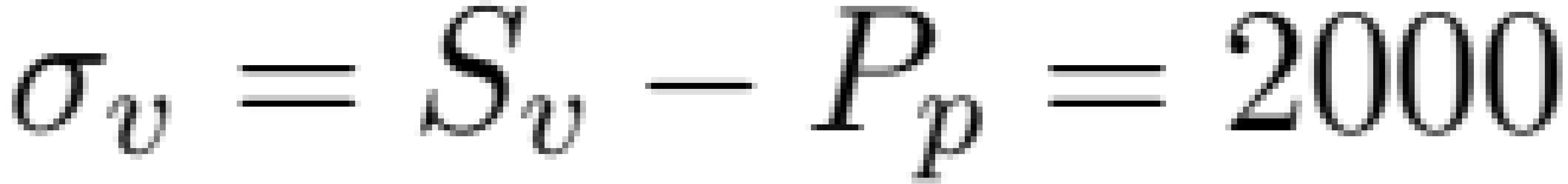


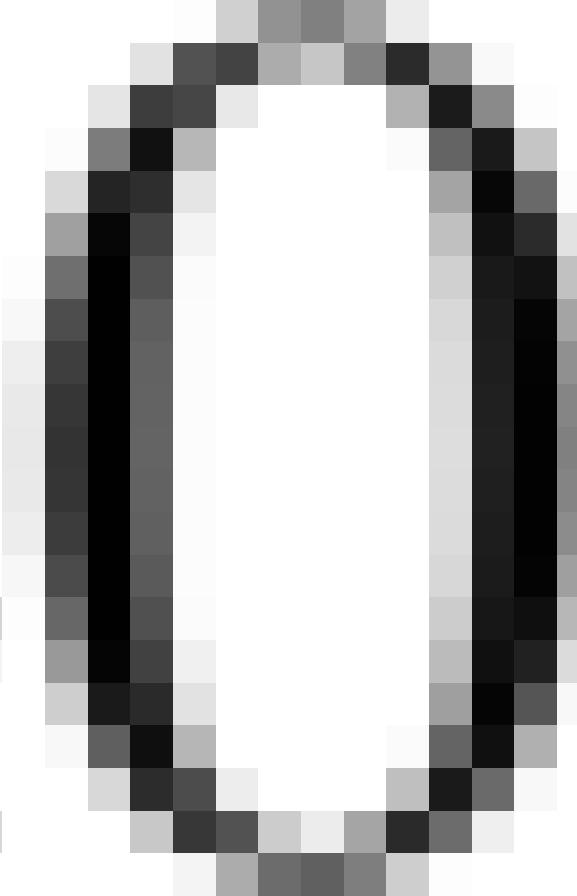
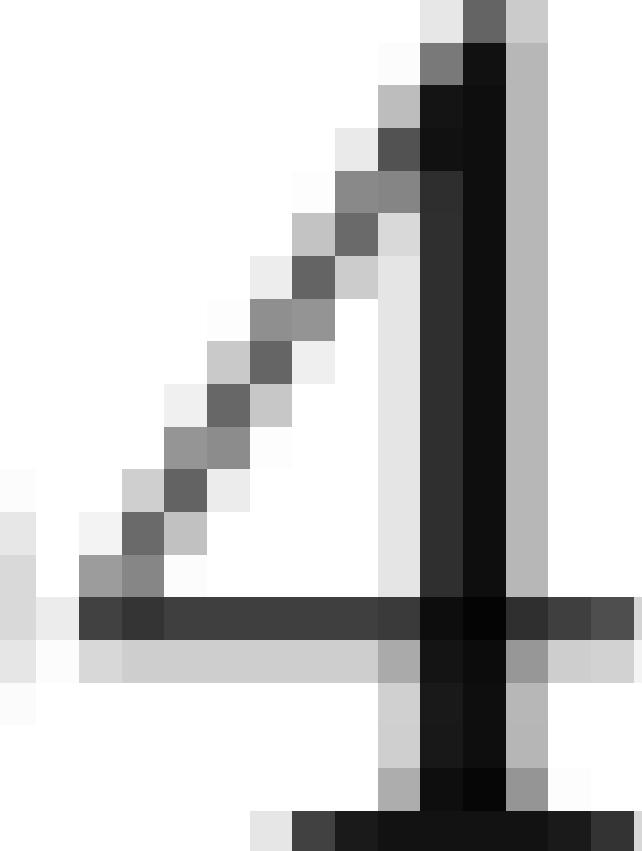
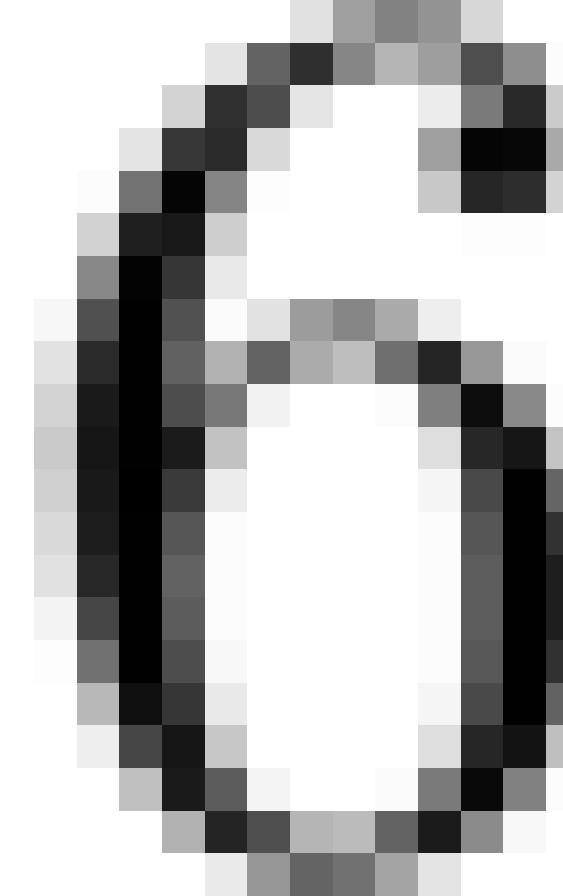
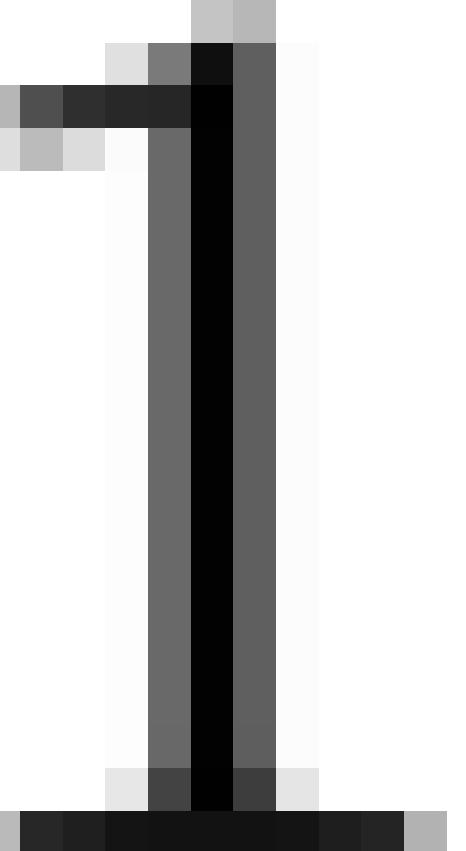


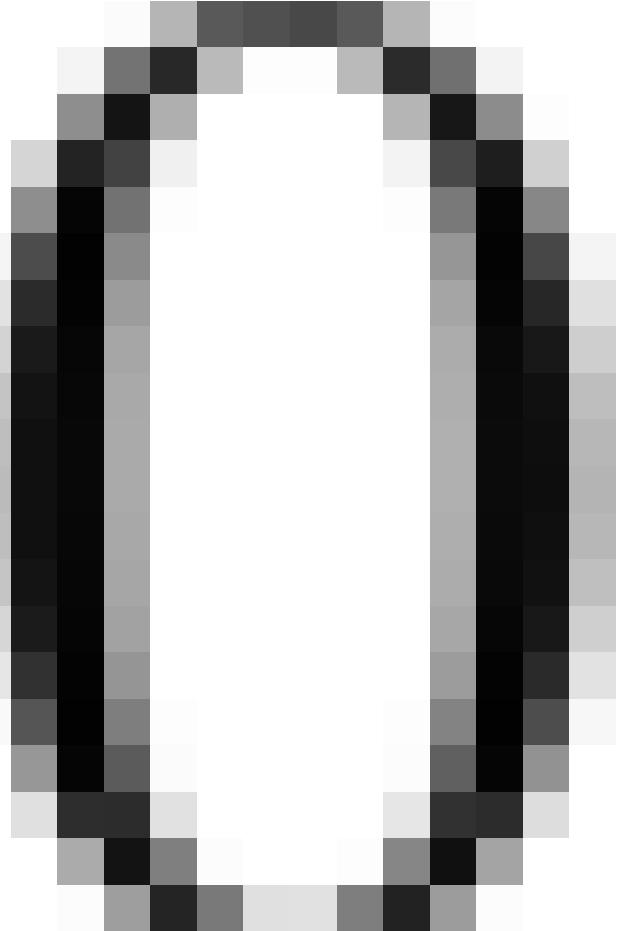
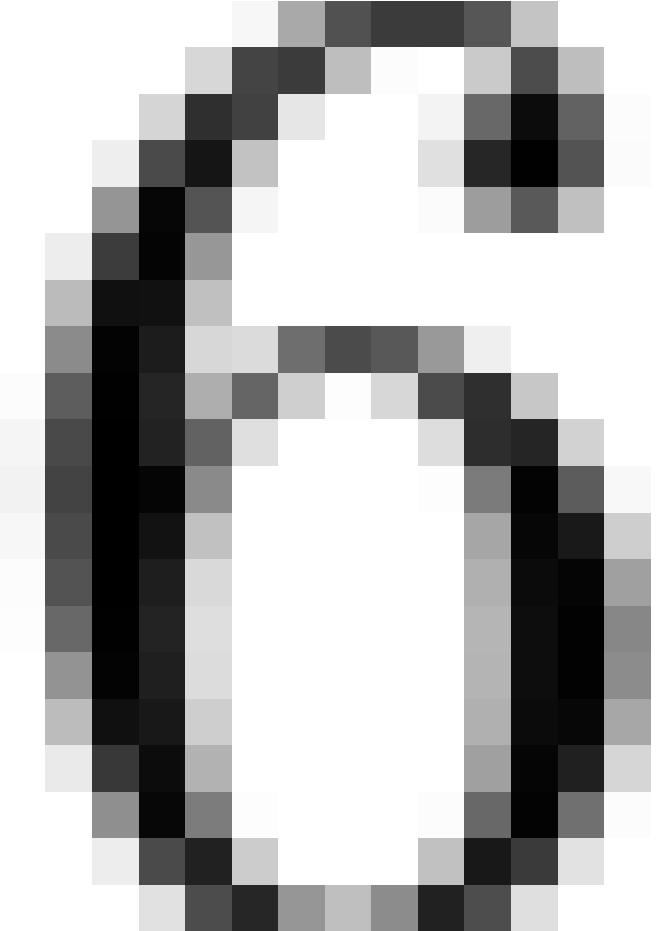
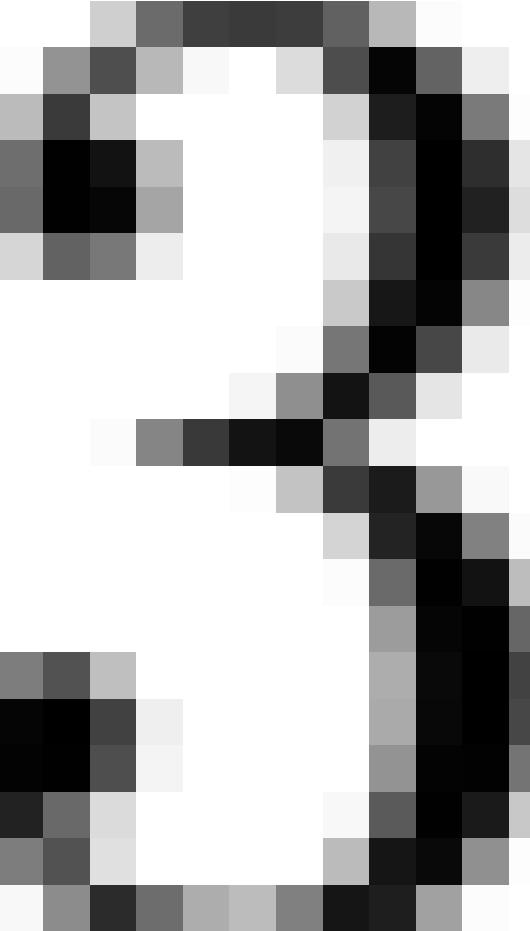








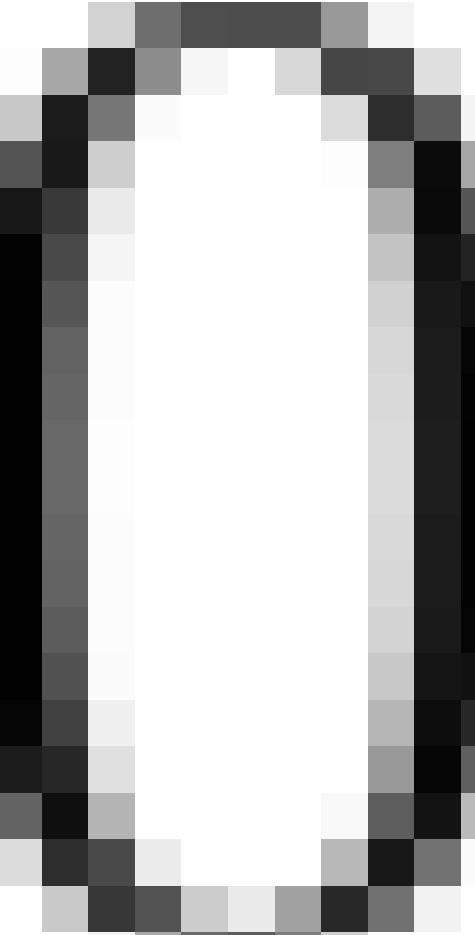
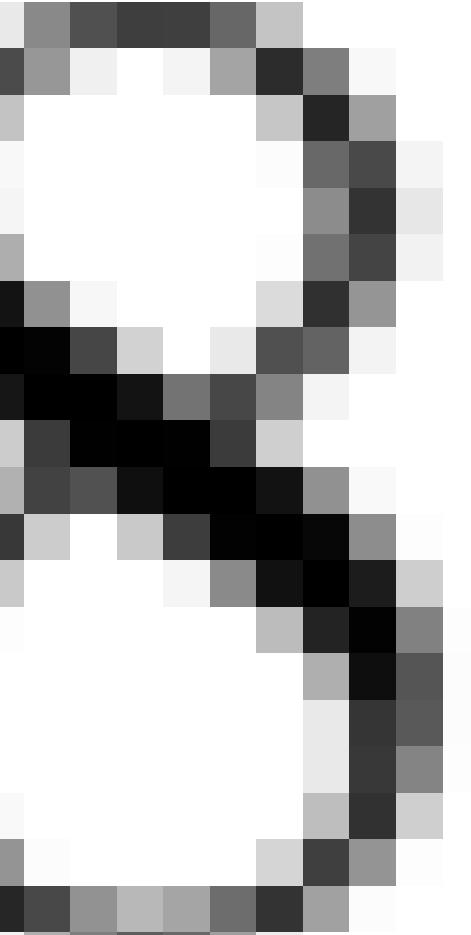
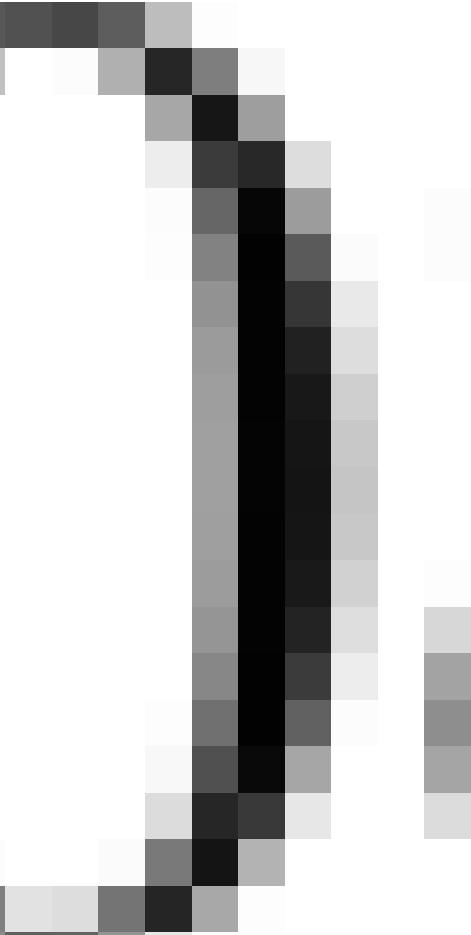
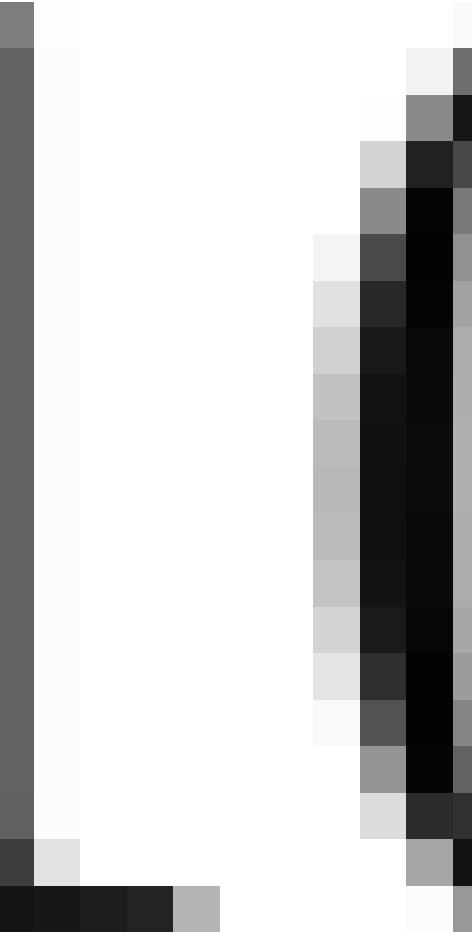


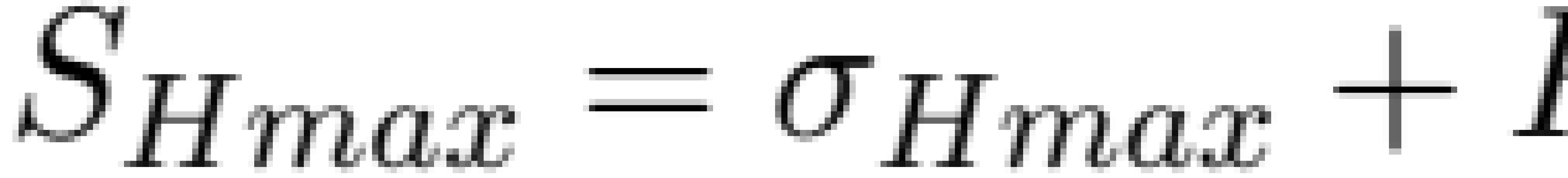


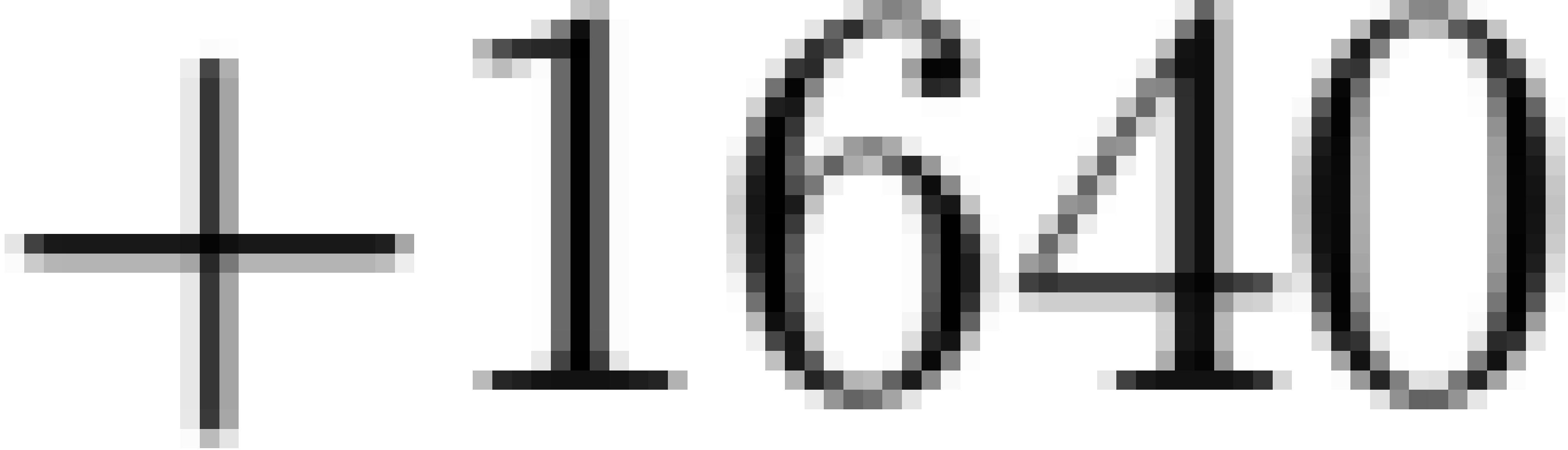
$$\sigma_{H\text{max}} =$$

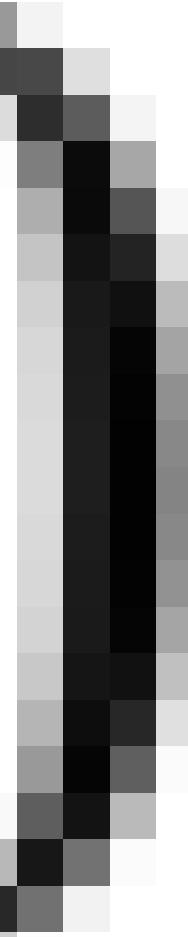
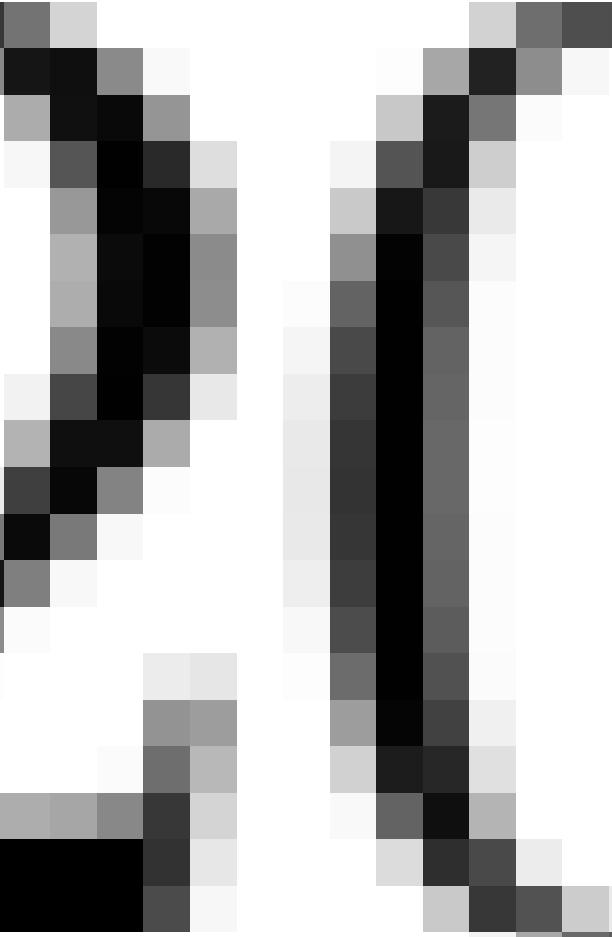
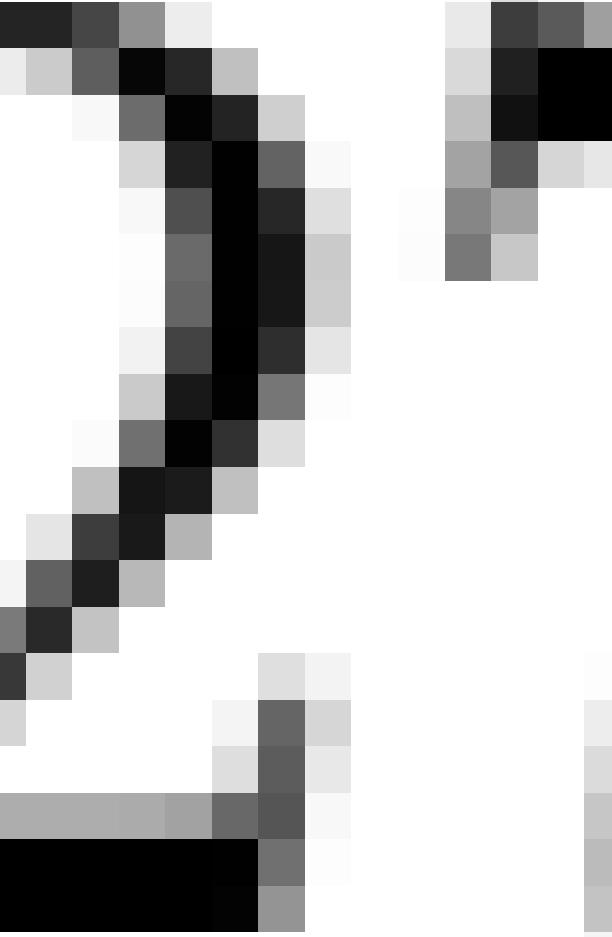
$$90^\circ =$$

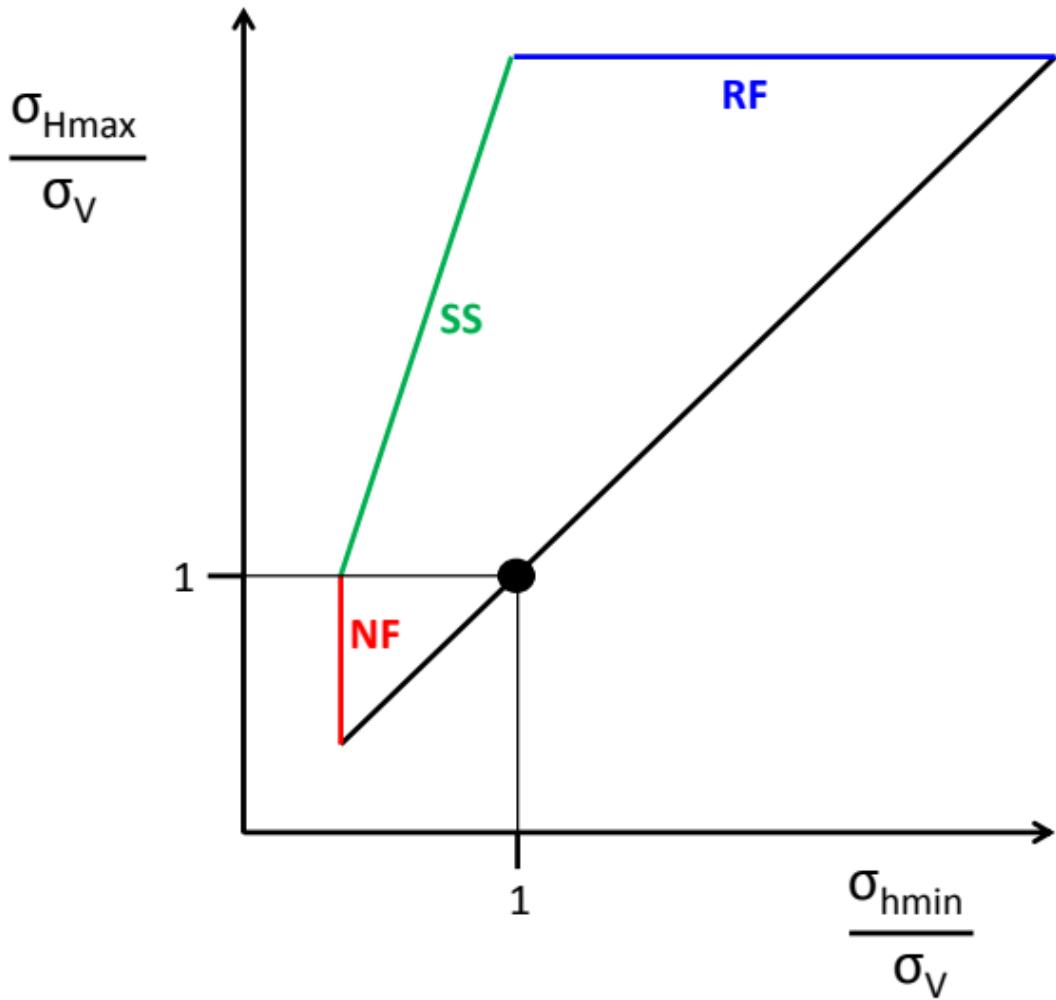
$$\frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} 360$$



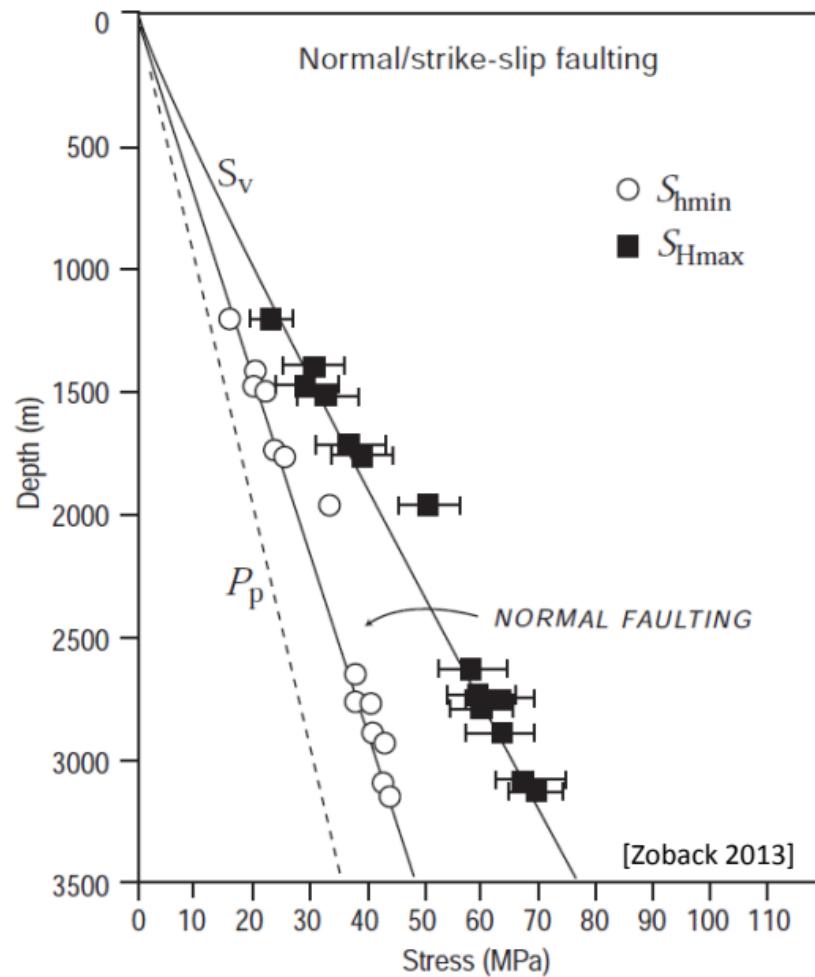
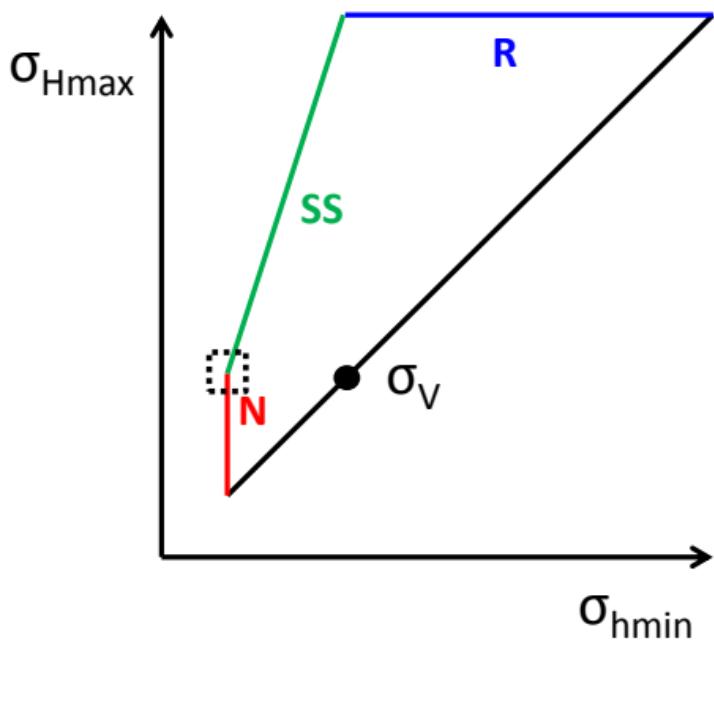


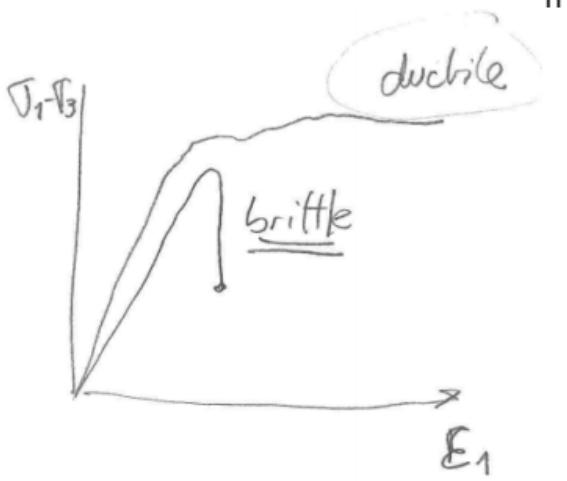
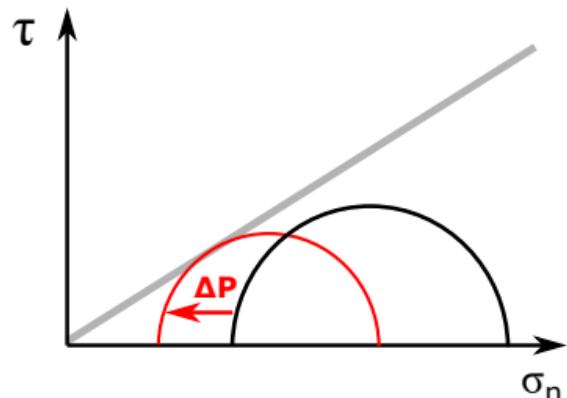
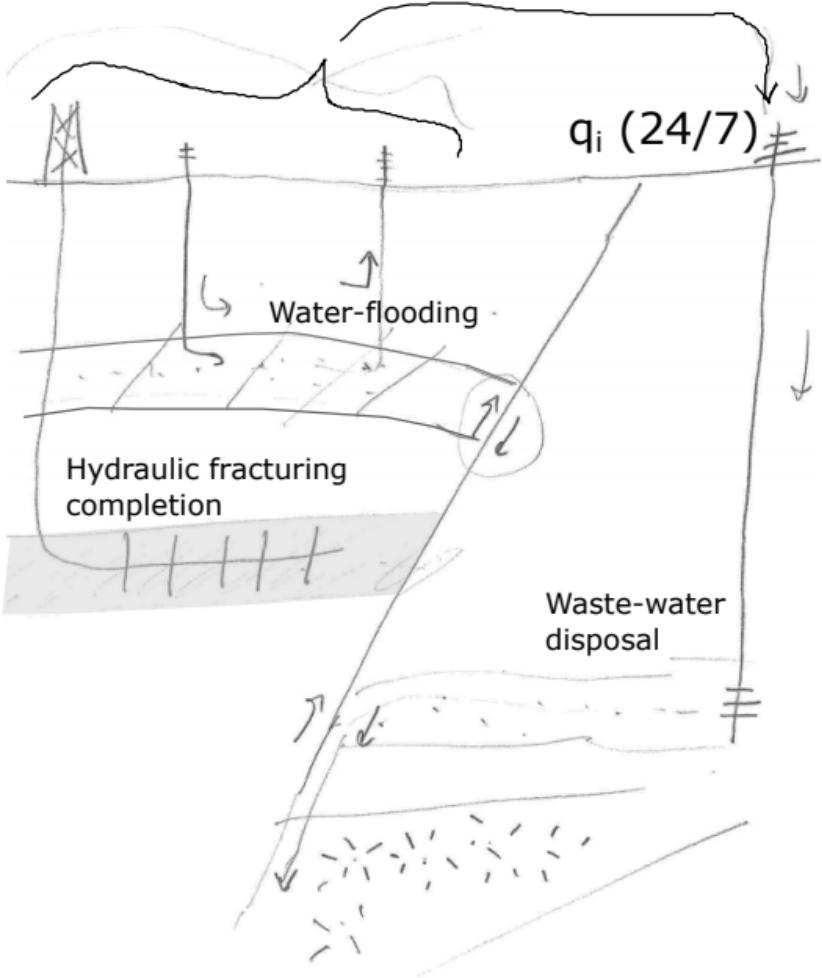






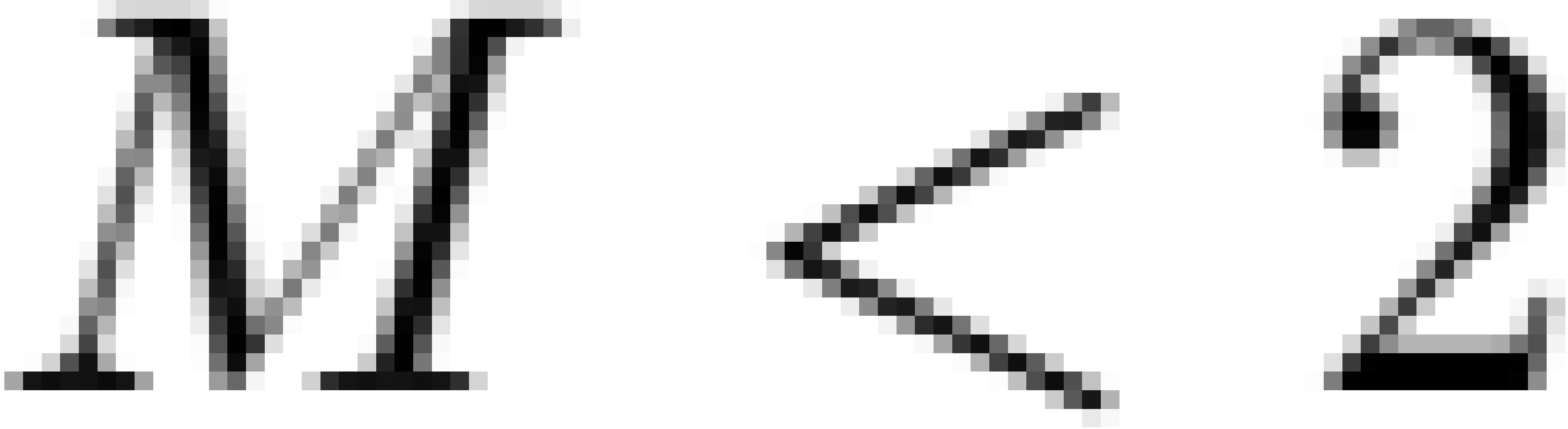
$$\sigma_{h\min} < \sigma_{H\max} \approx \sigma_v$$

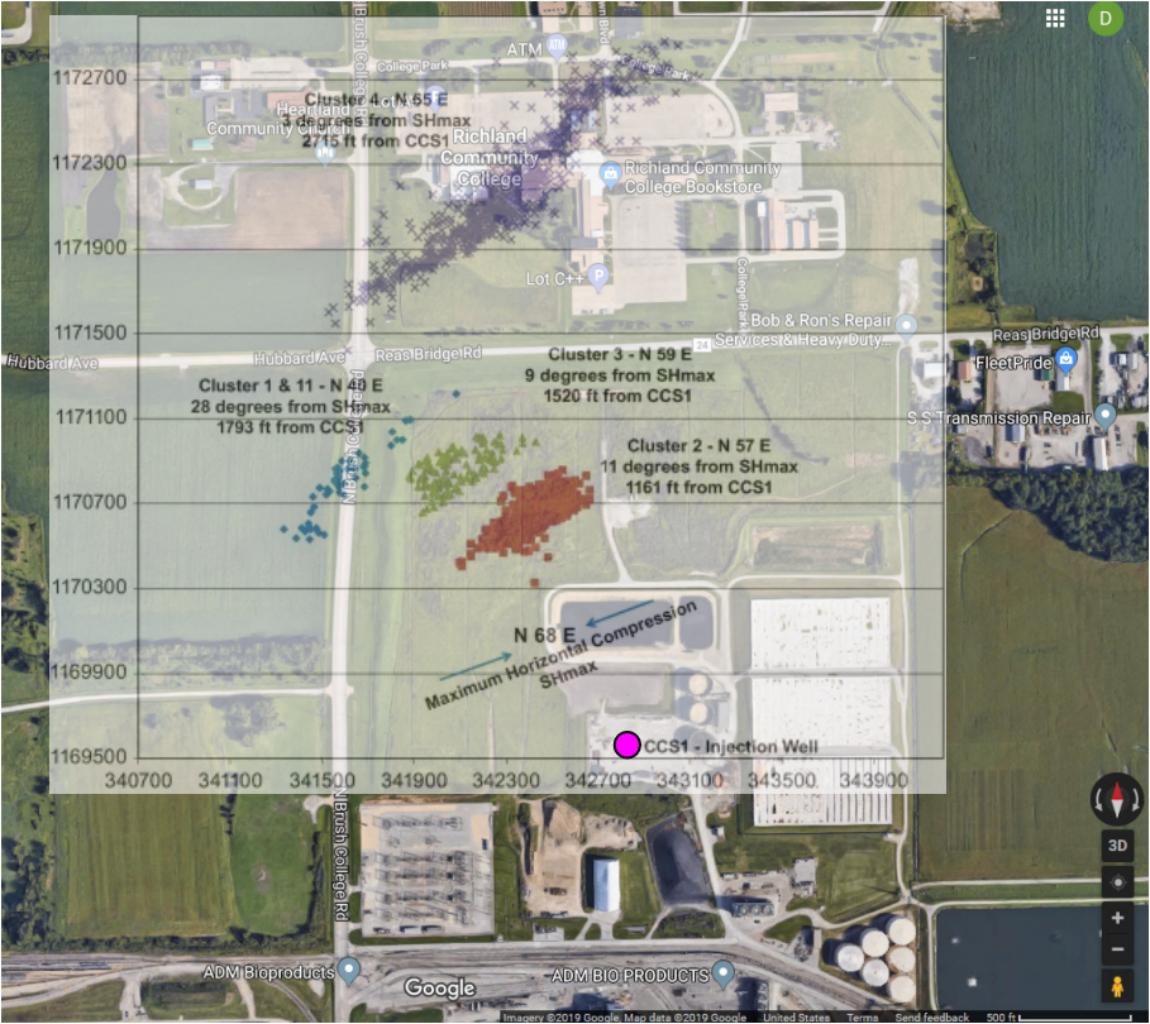




$$\Delta\sigma_a = -\Delta P_p$$

$$\Delta\sigma_{\text{Hermann}} \leq -\Delta P_p$$

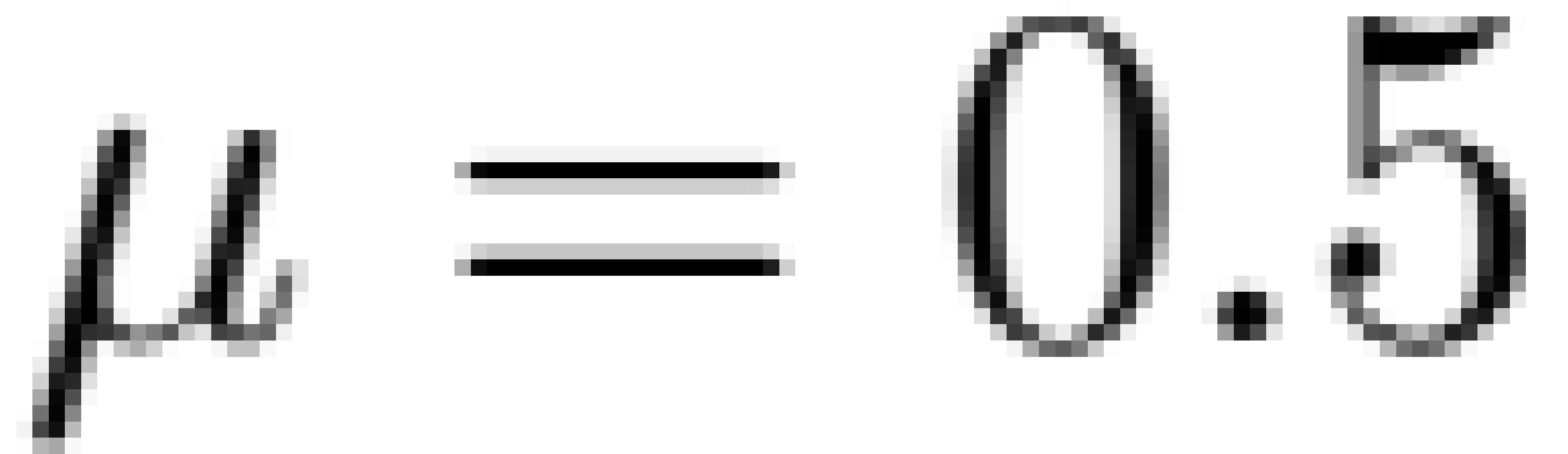


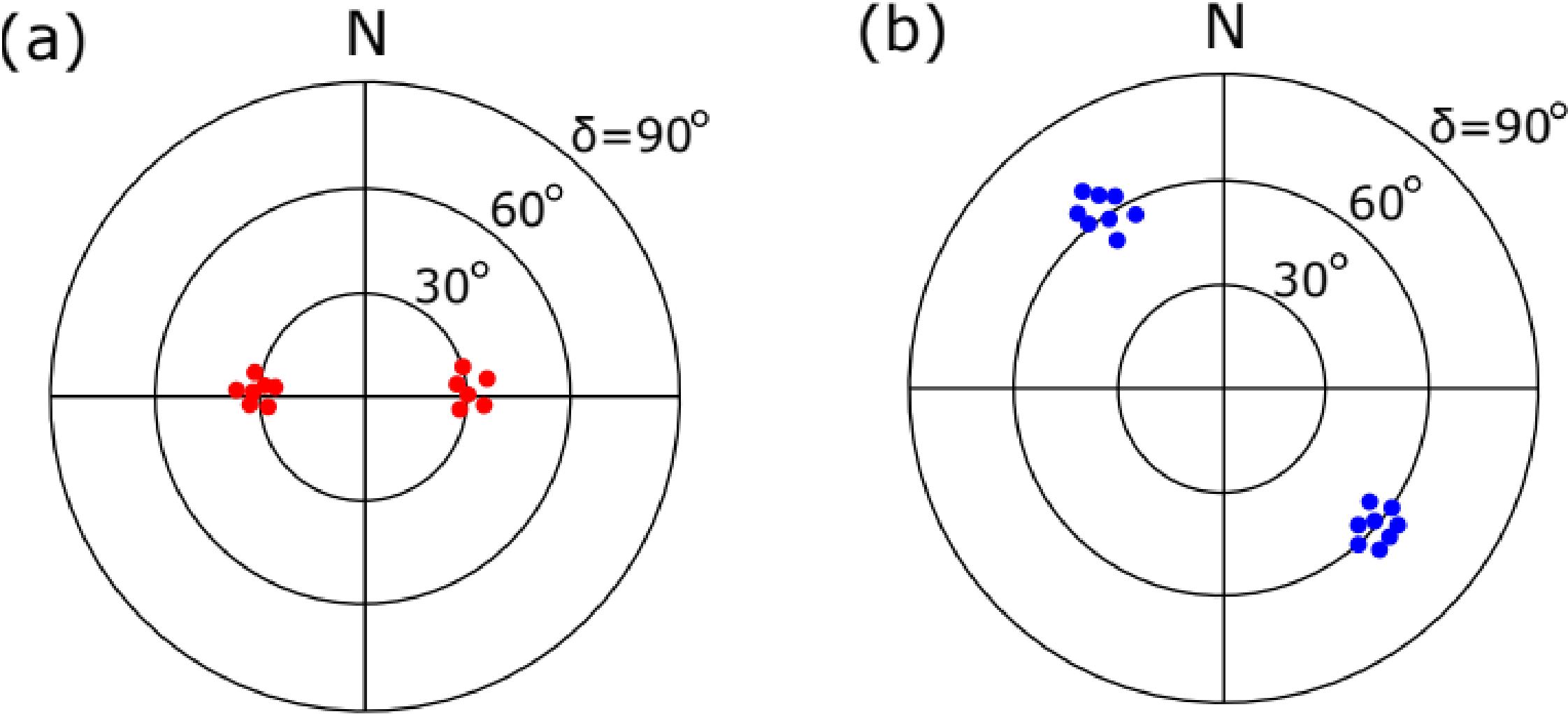




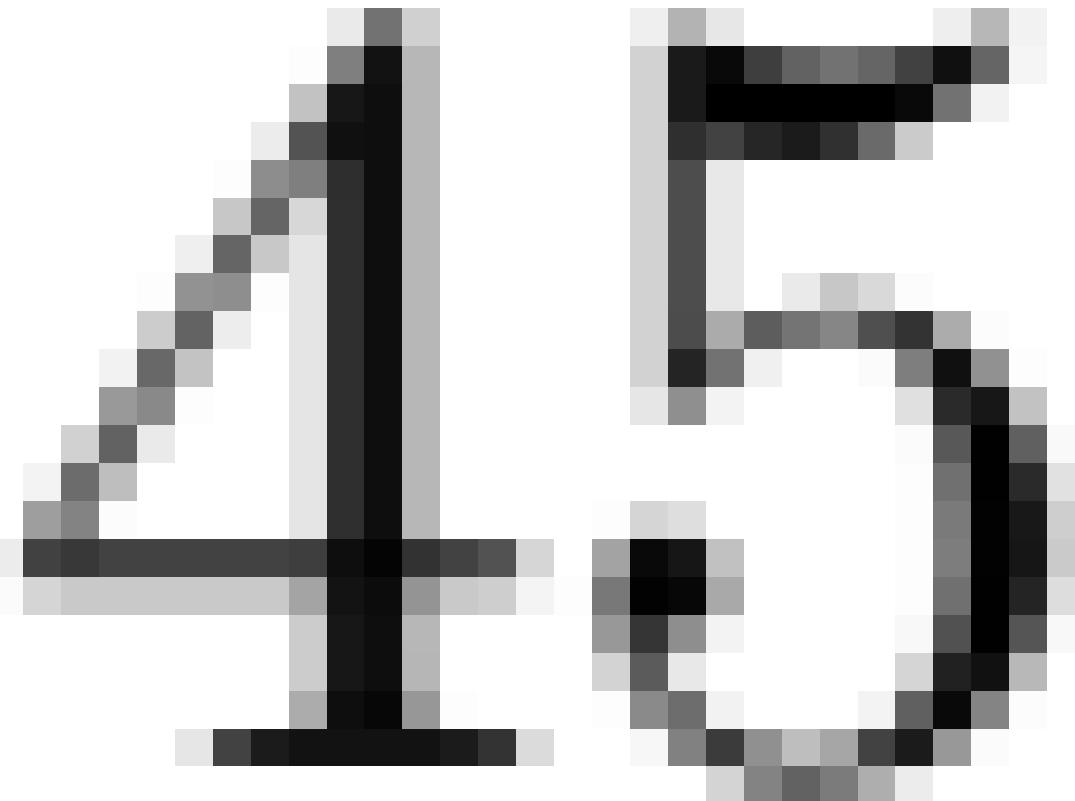


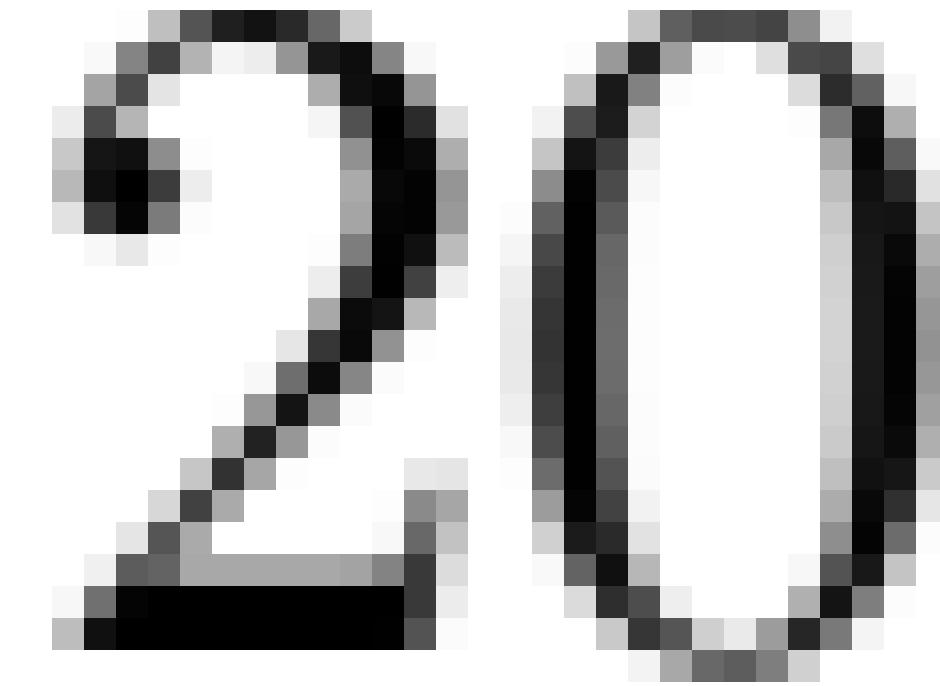


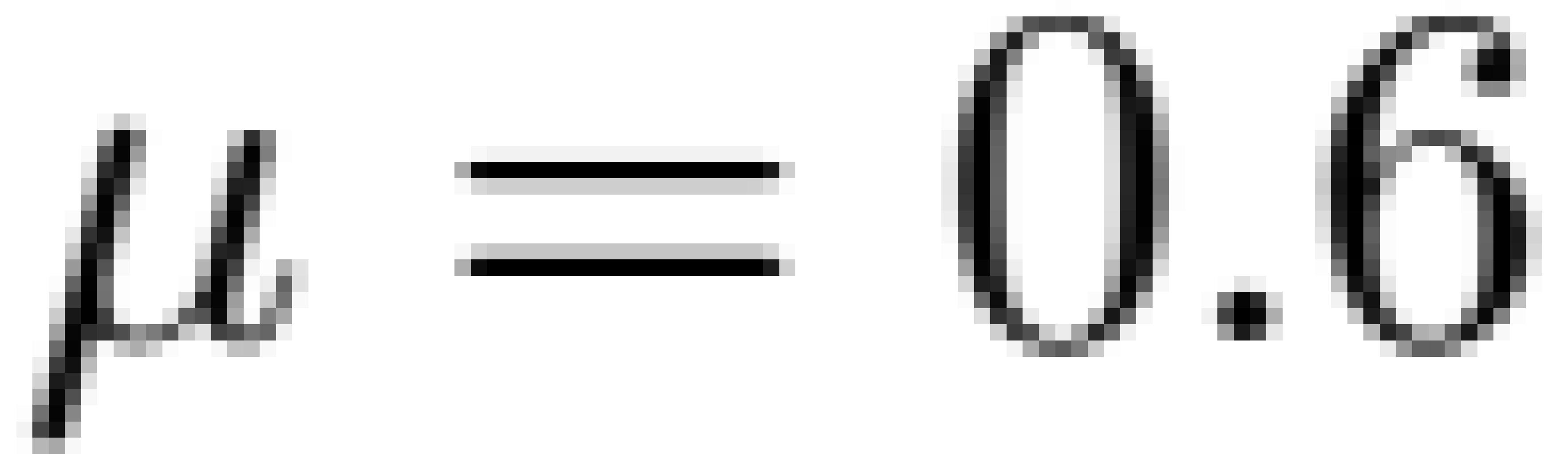






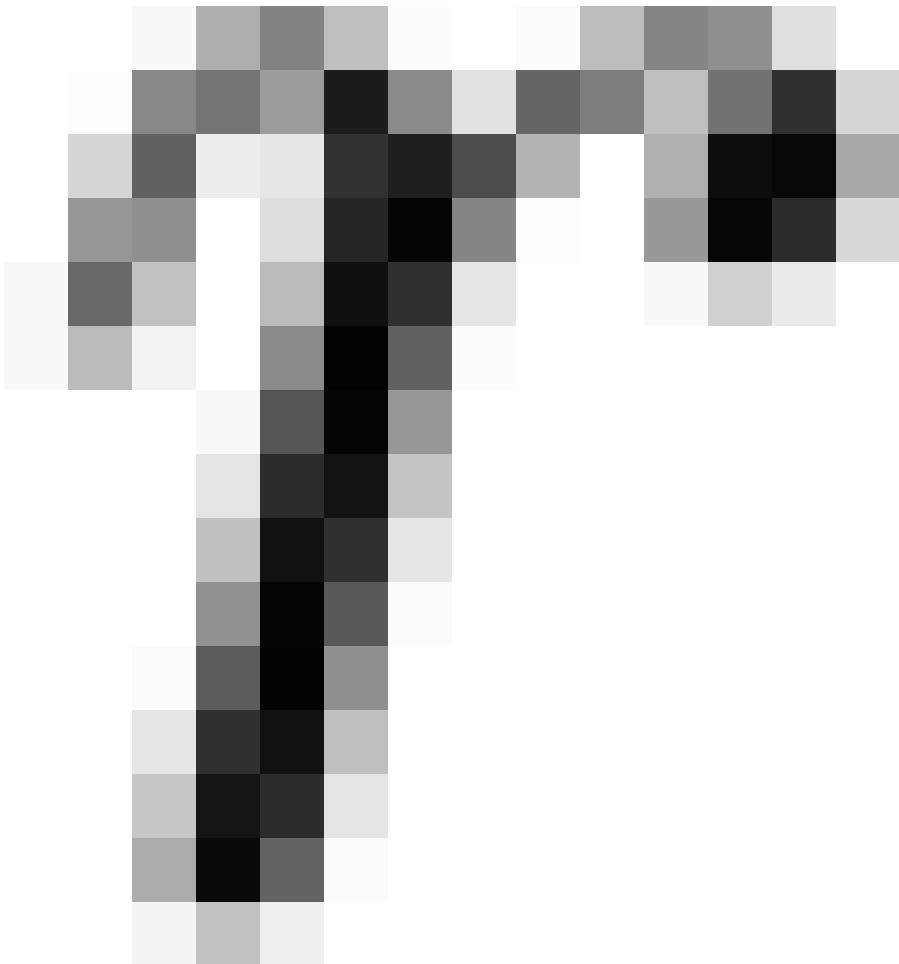


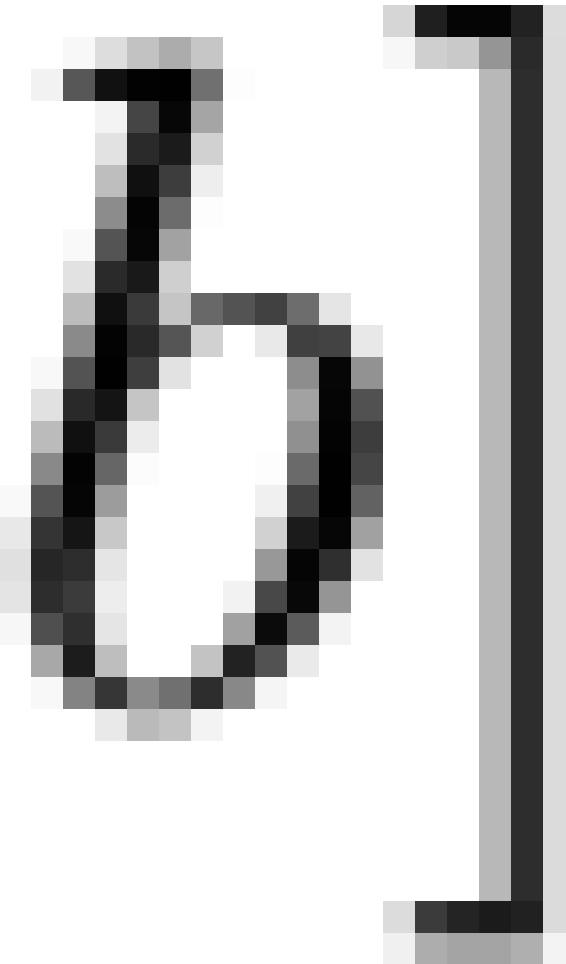
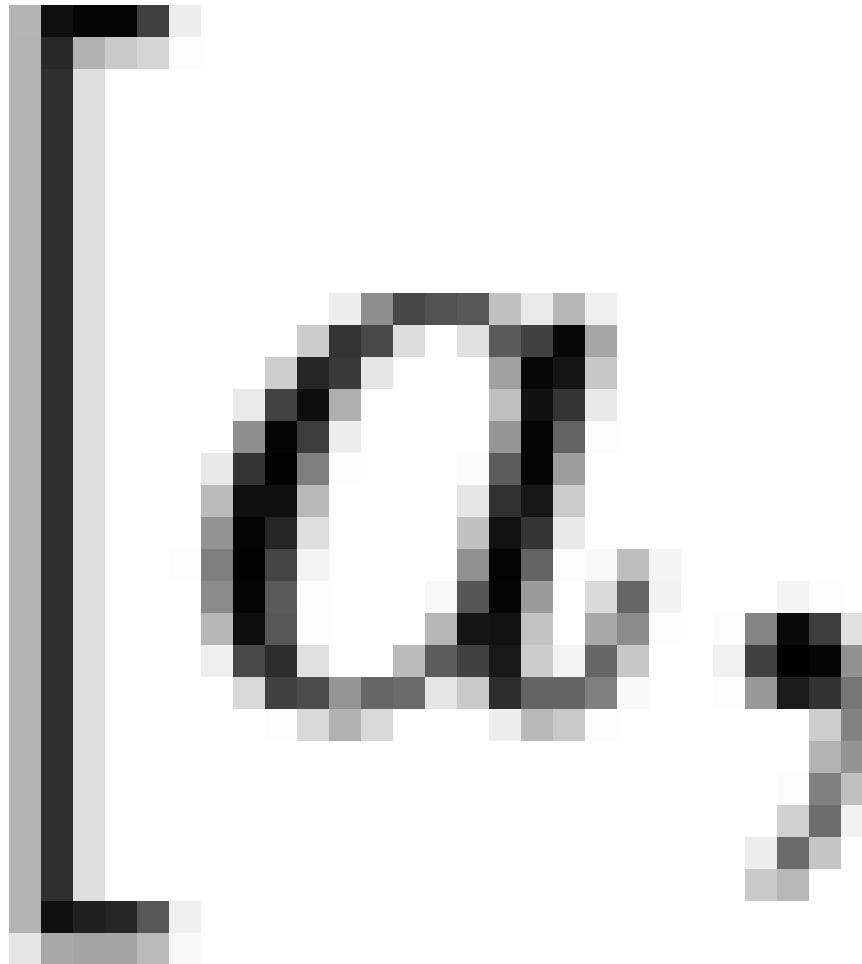


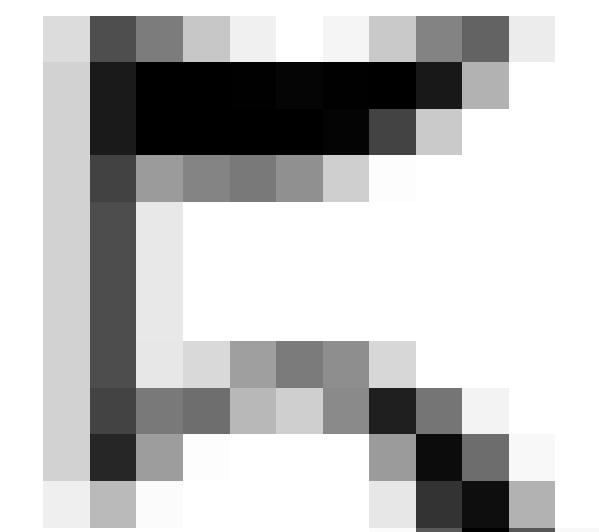
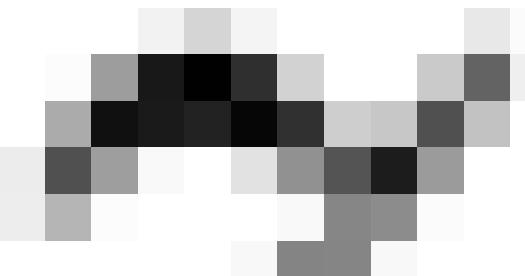
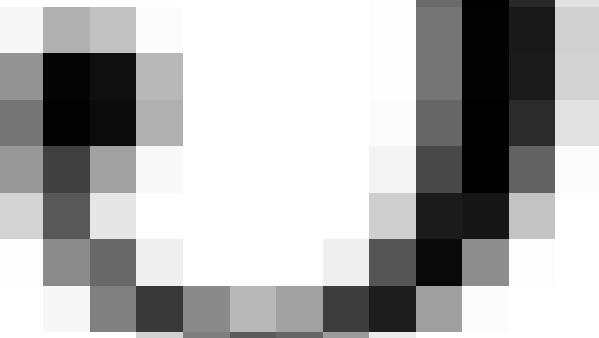
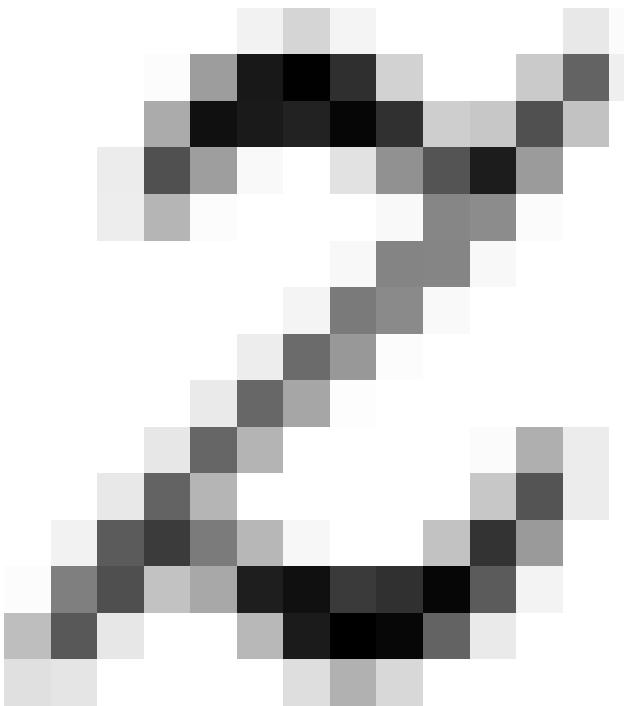






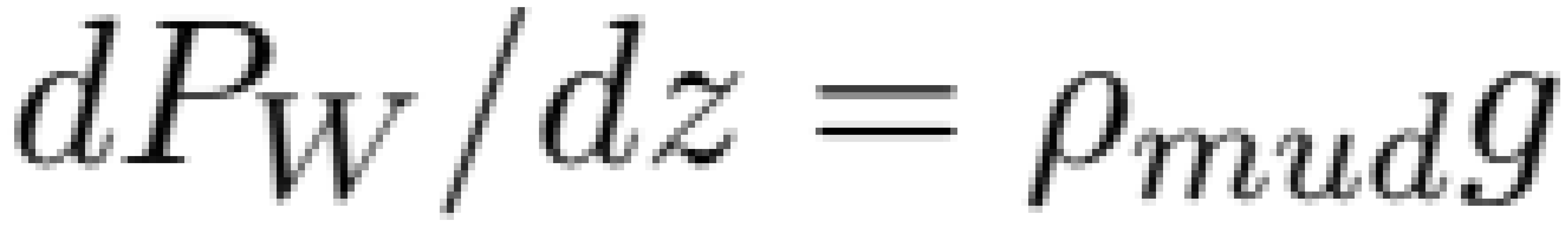


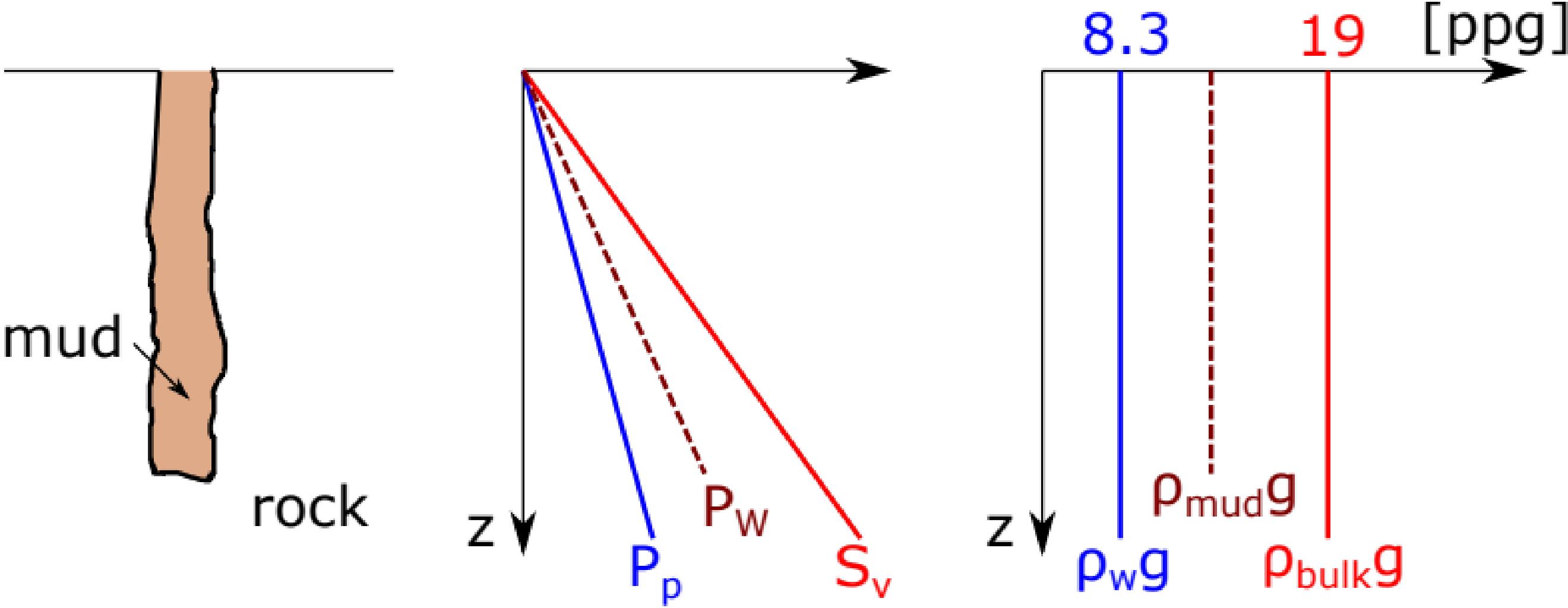


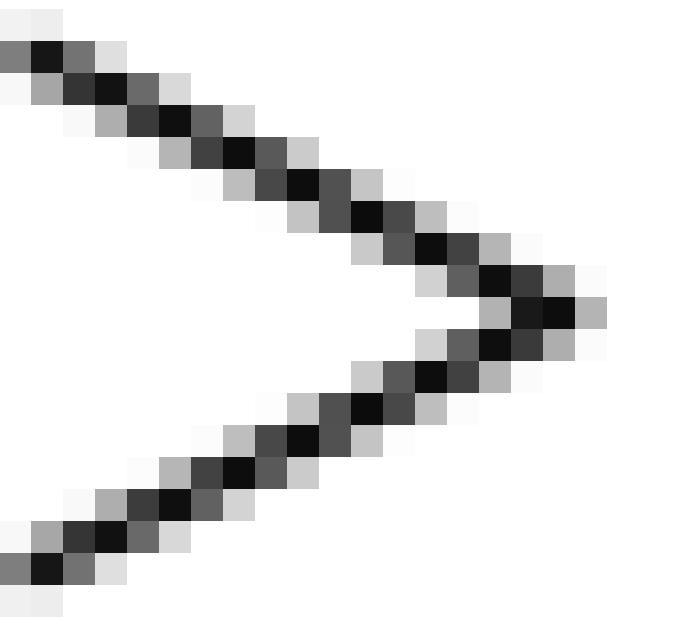




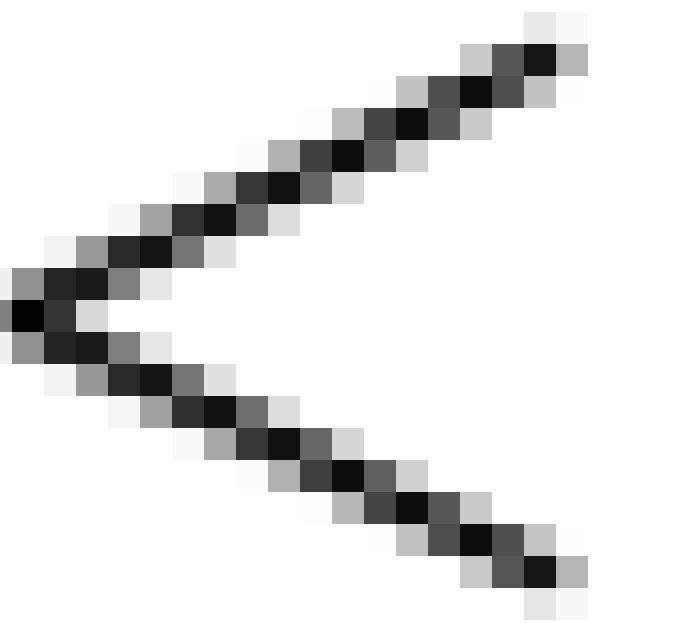


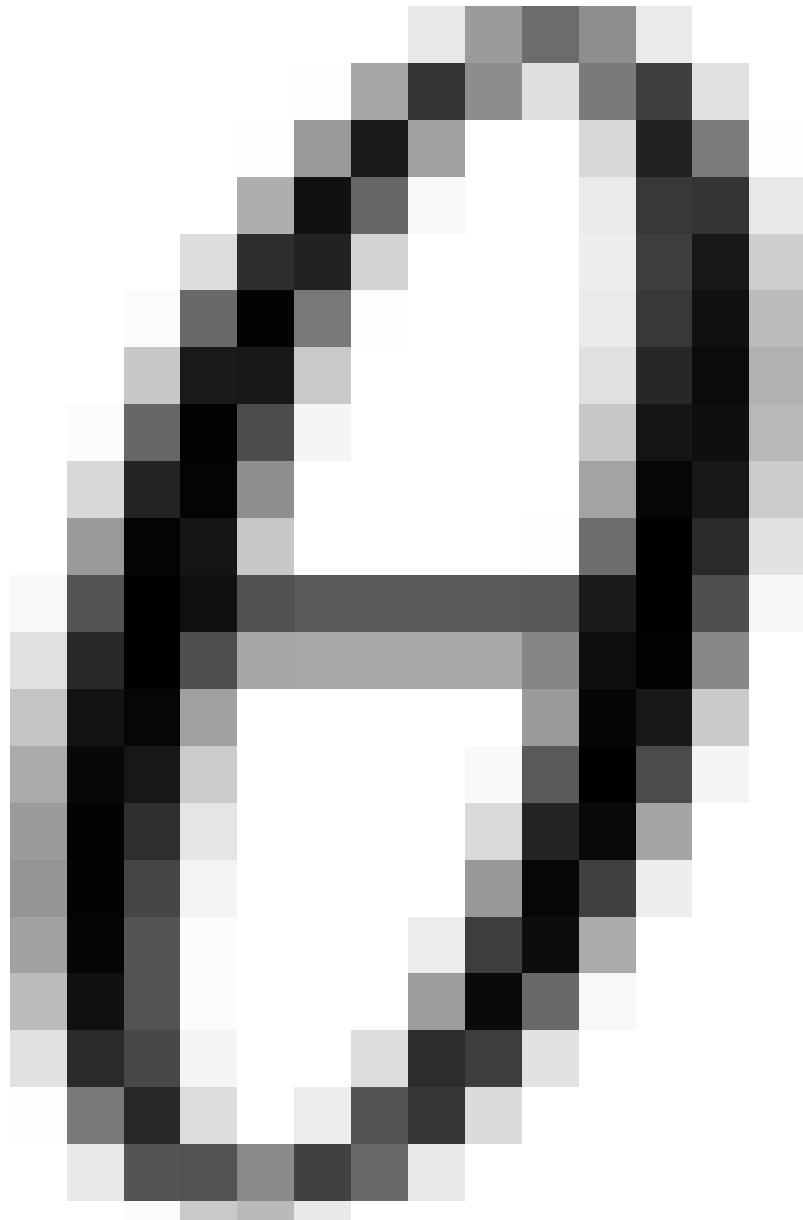


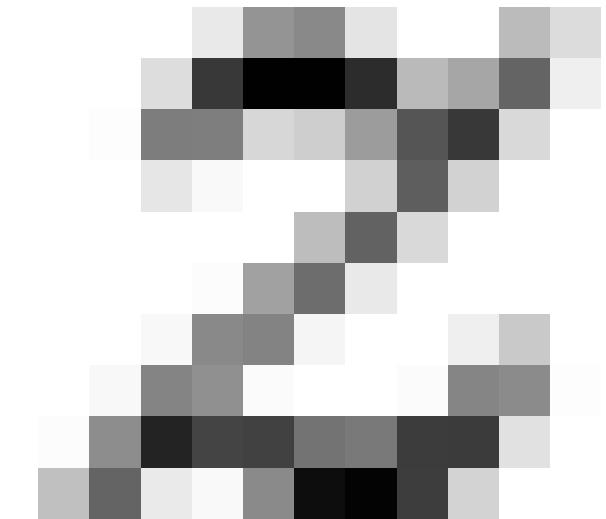
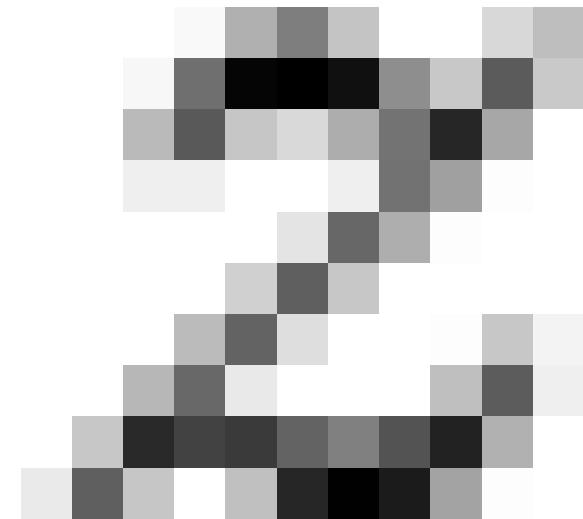
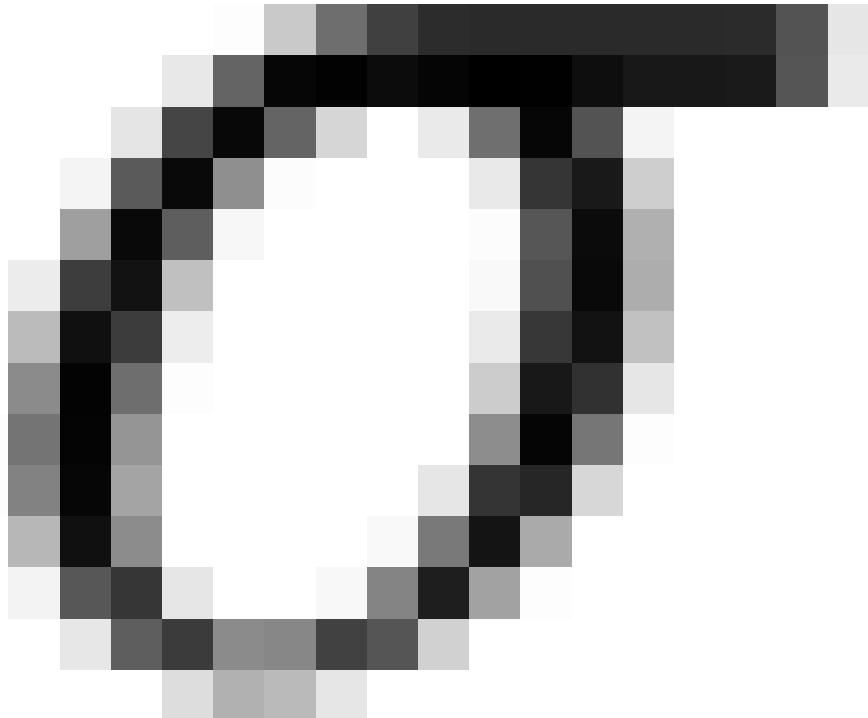


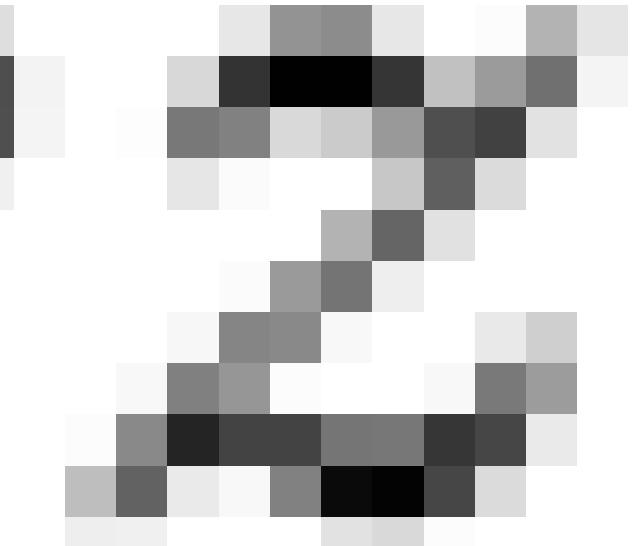
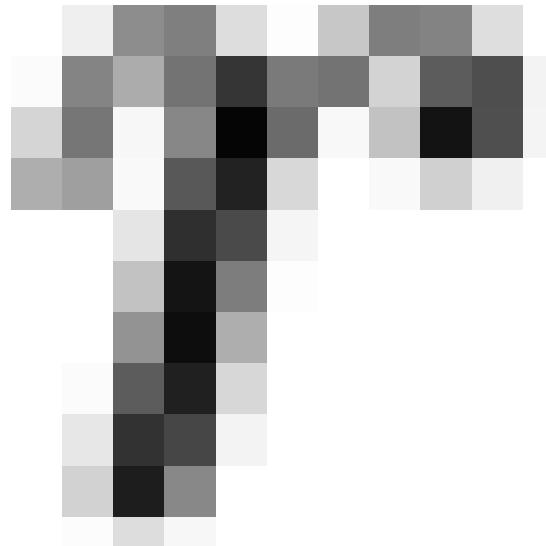
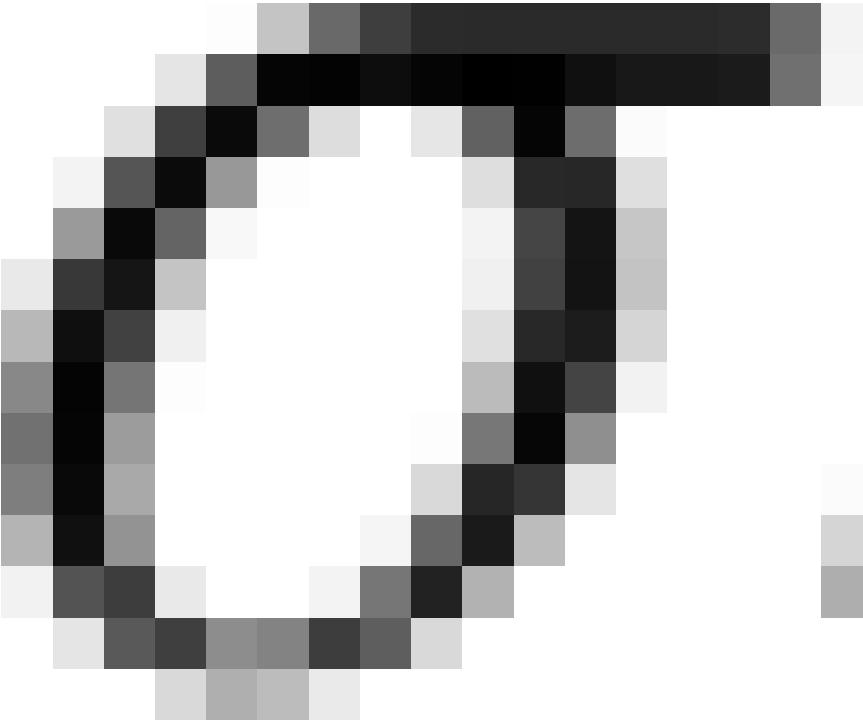


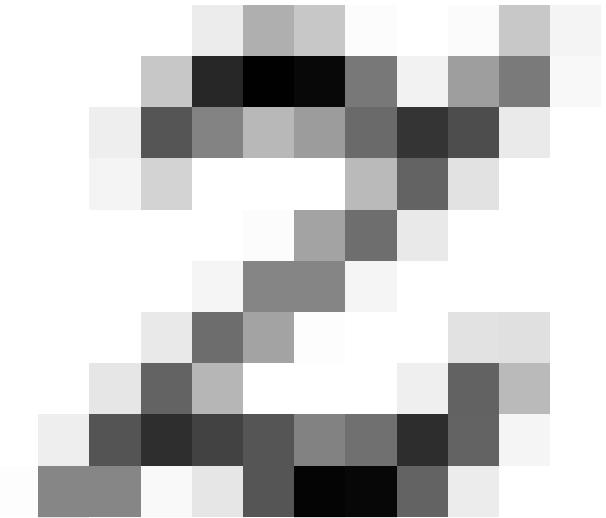
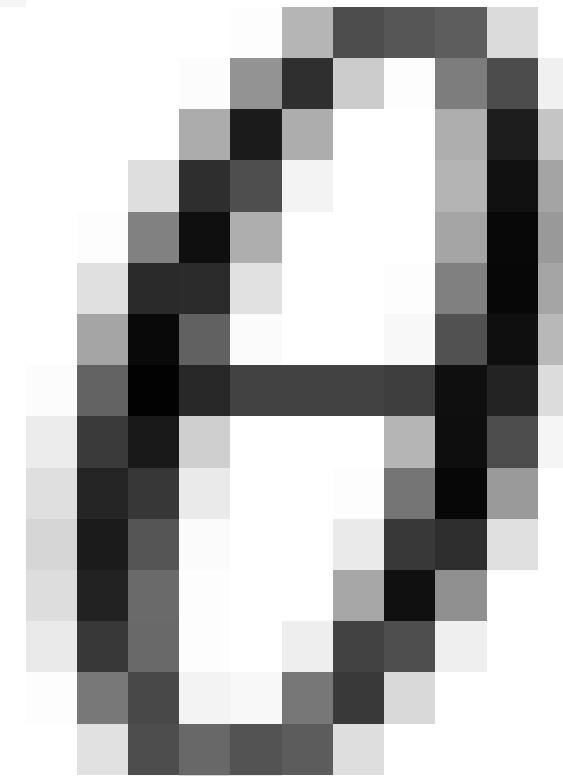
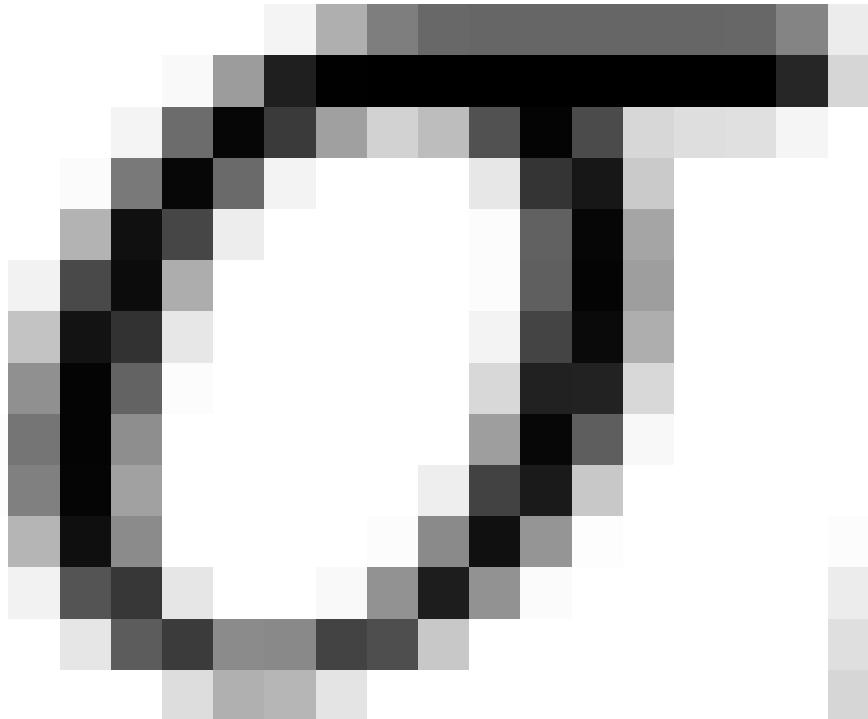




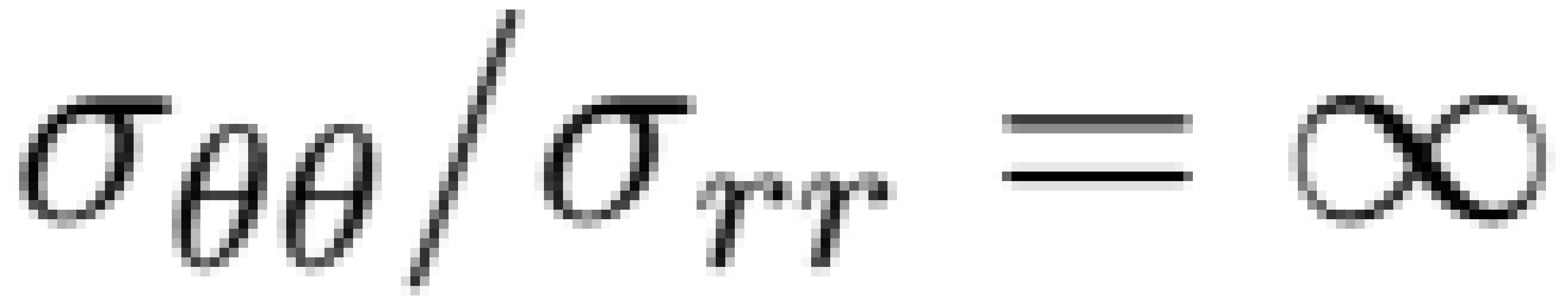


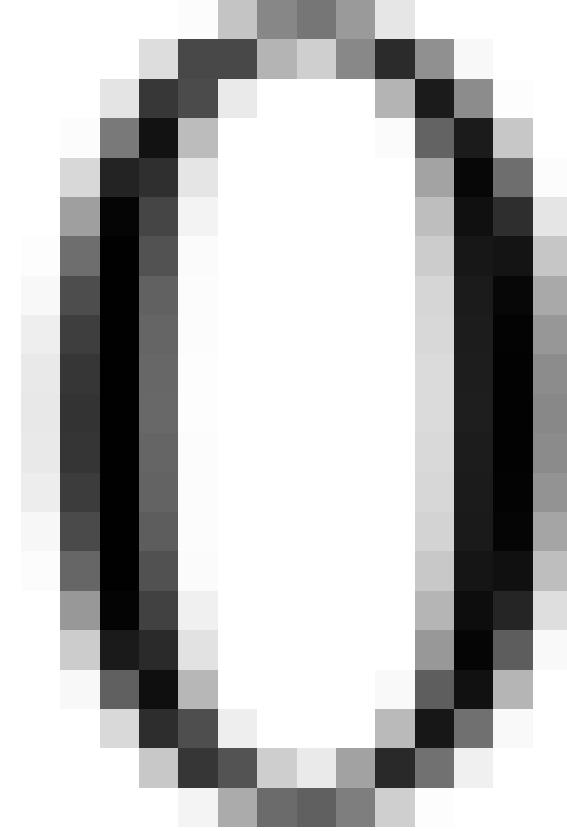
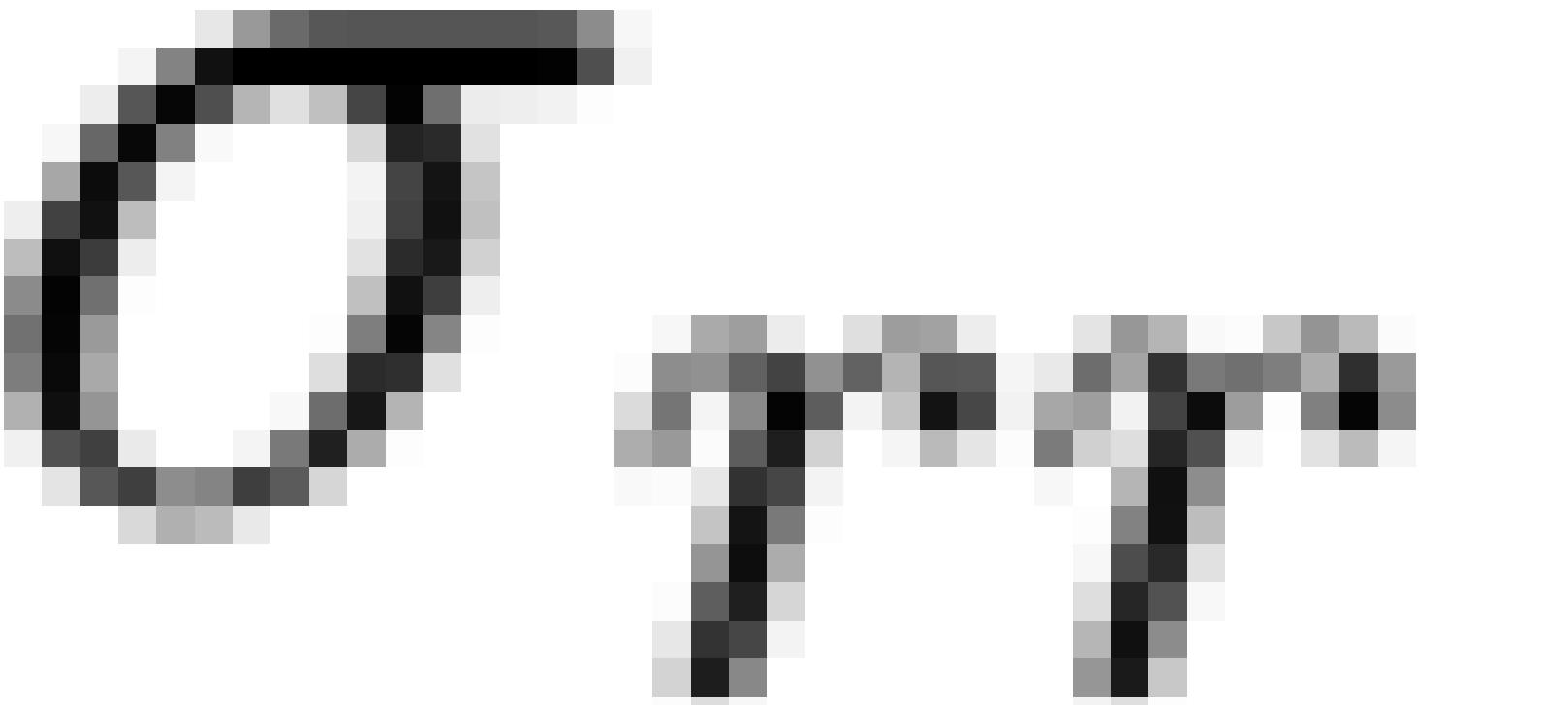




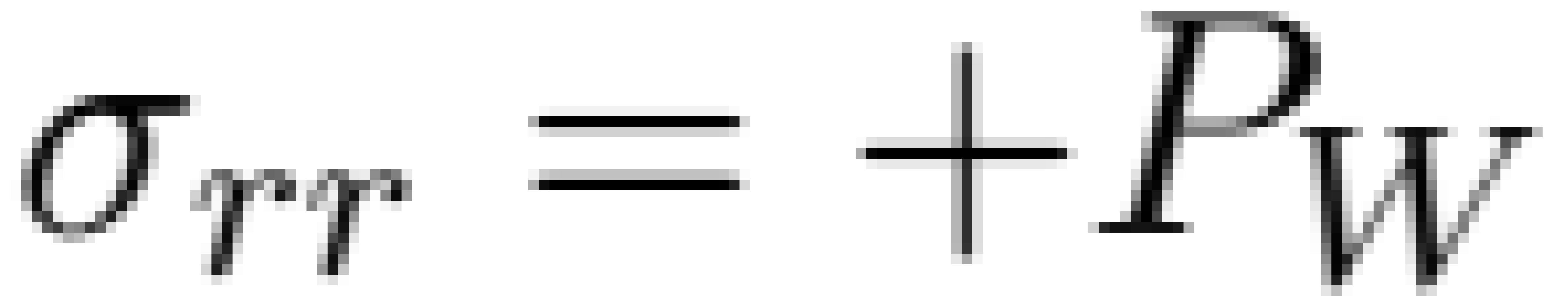


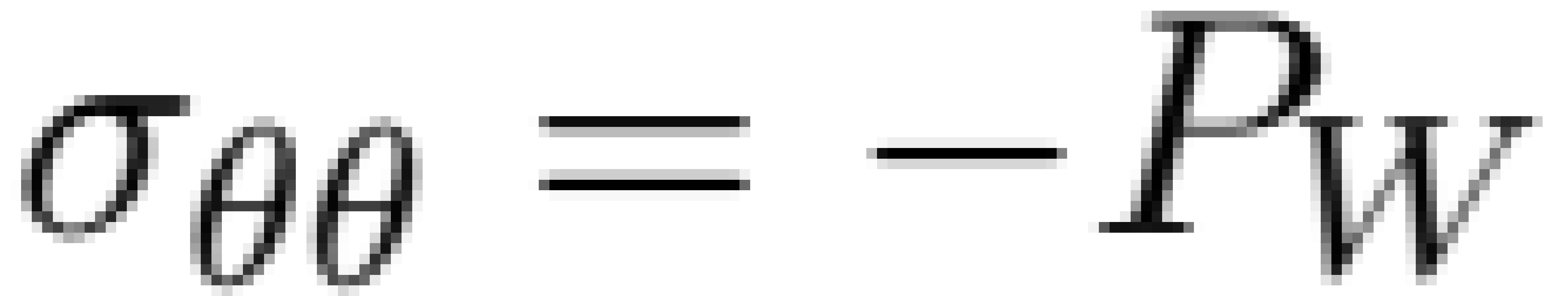


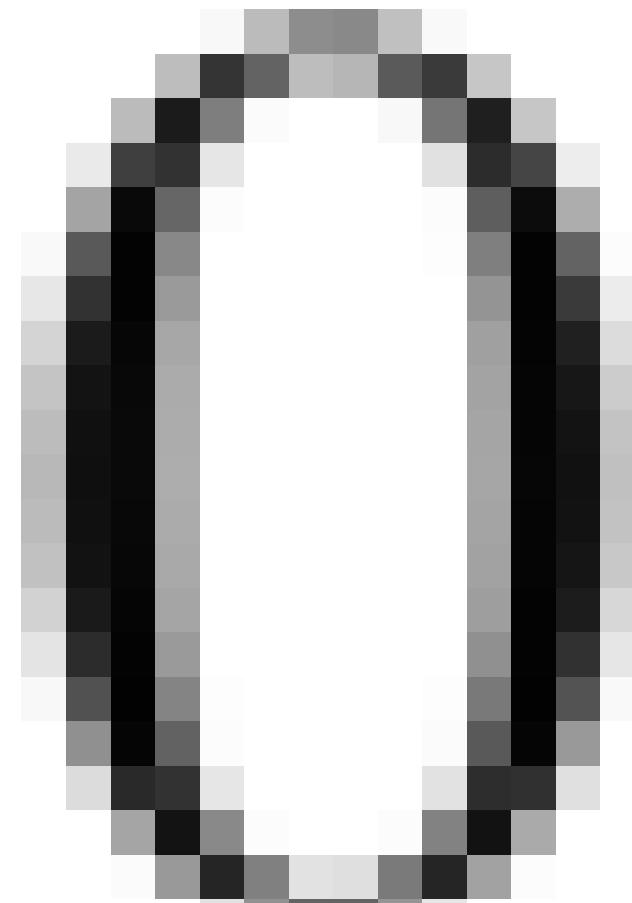
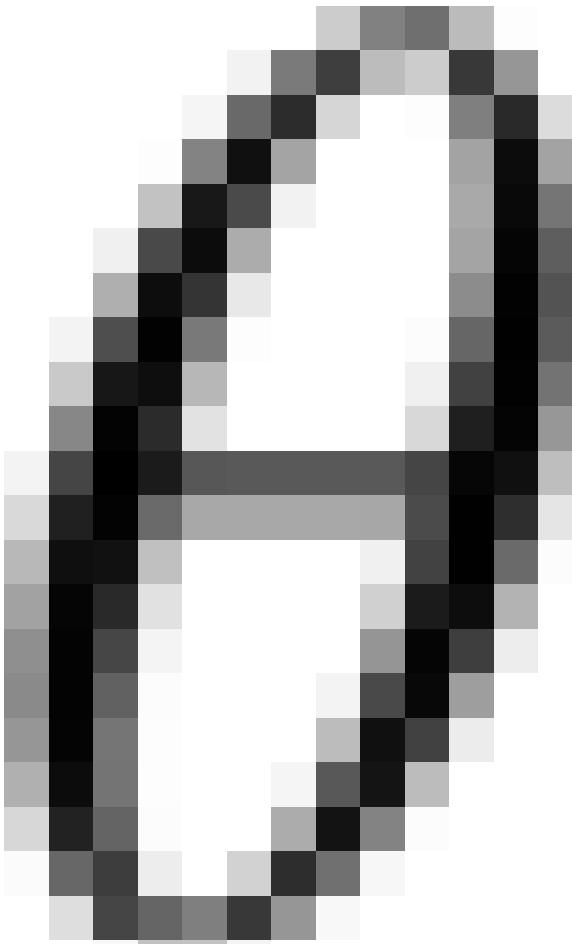


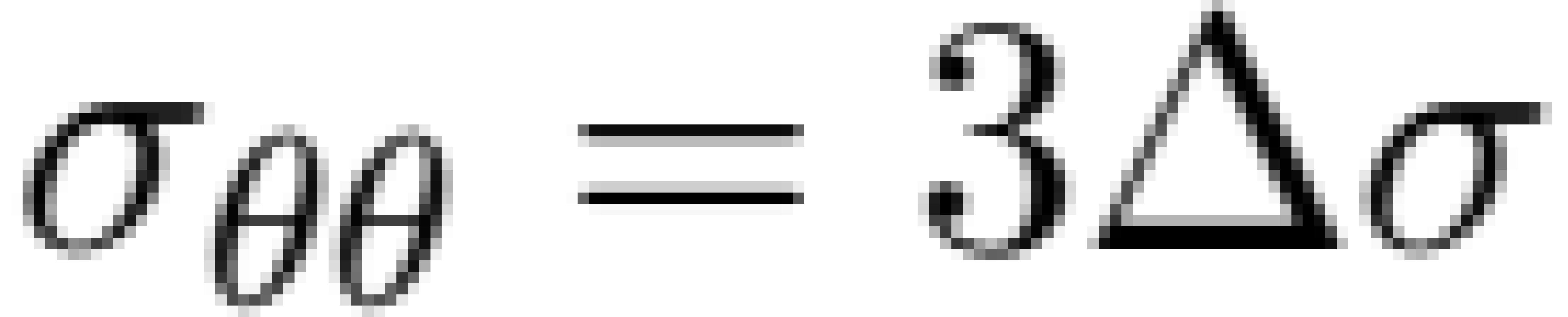


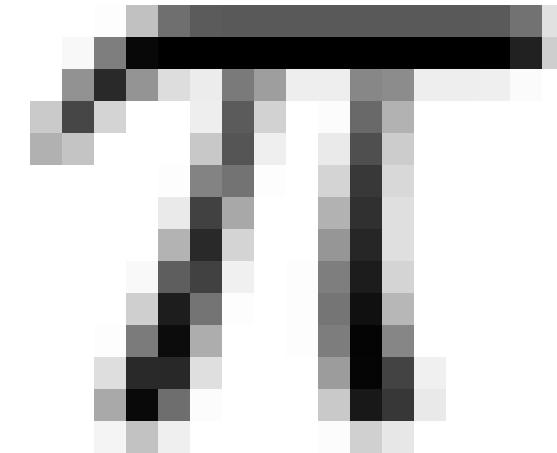
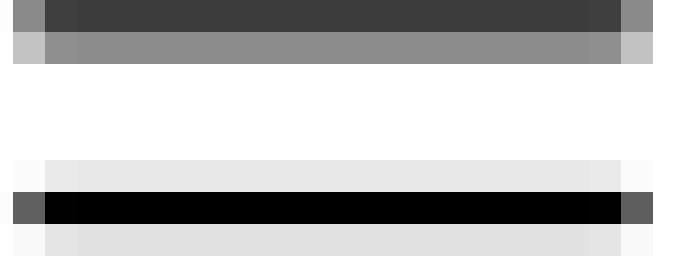
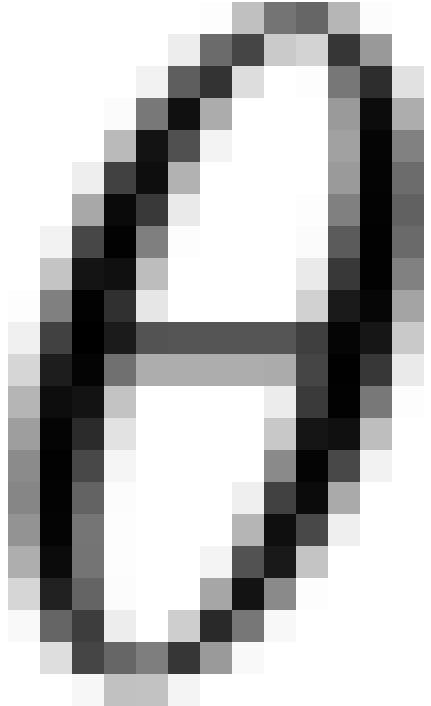


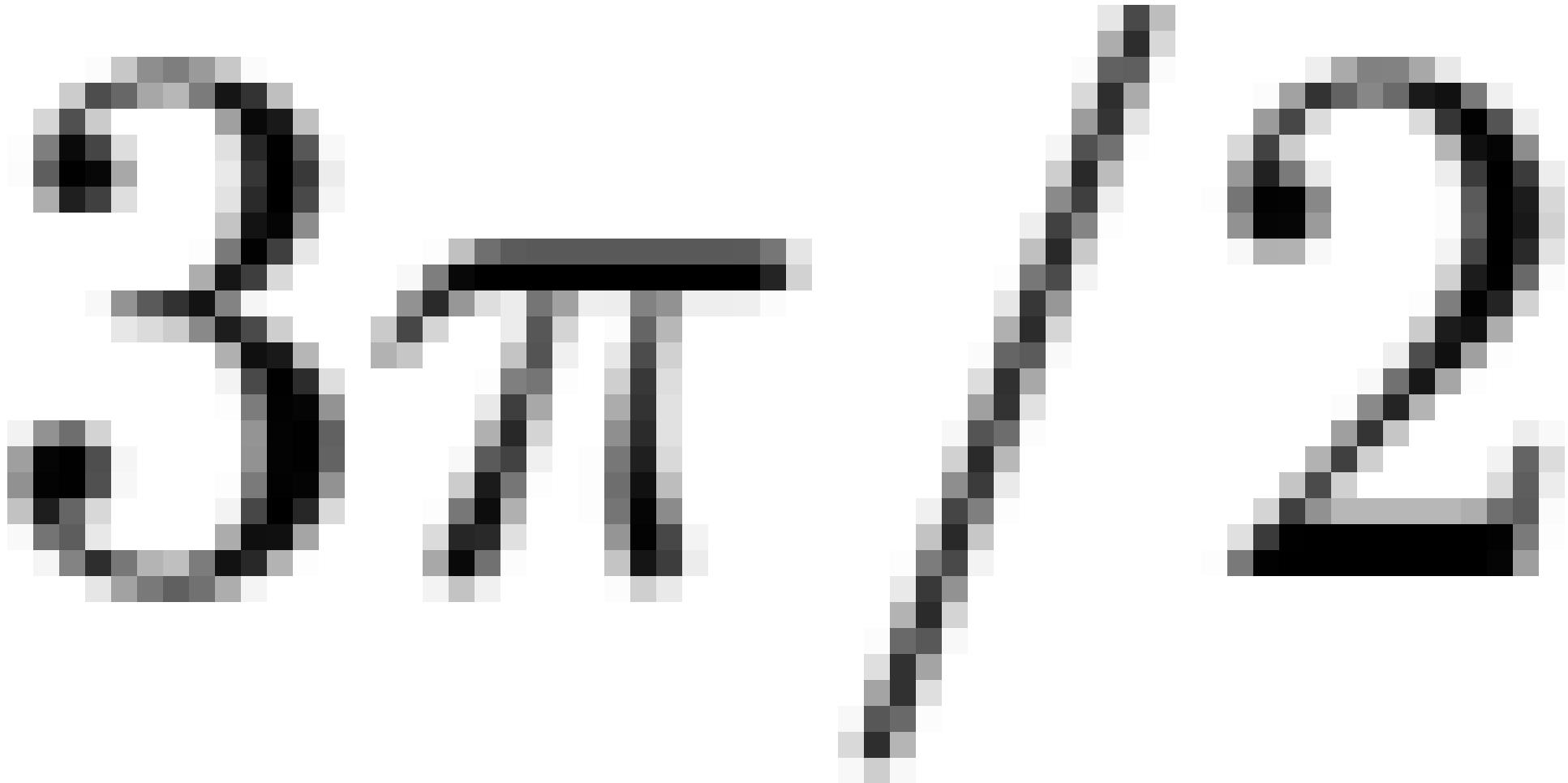


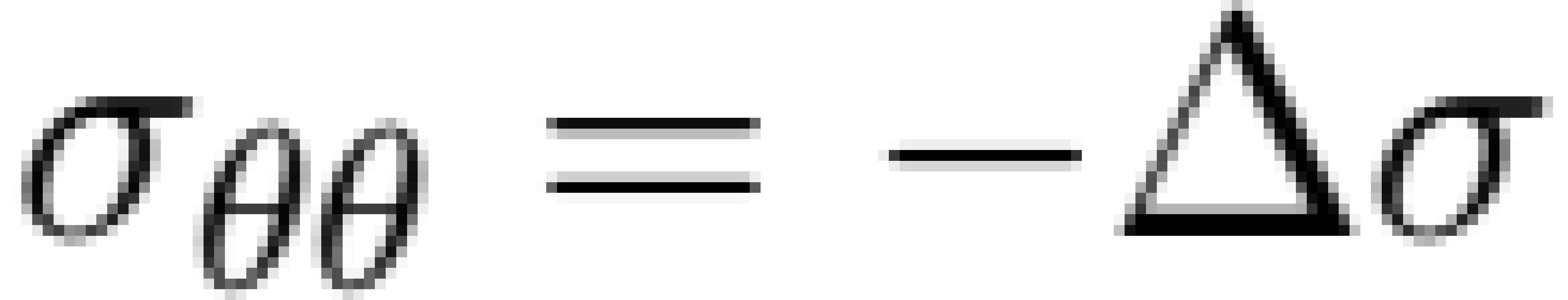


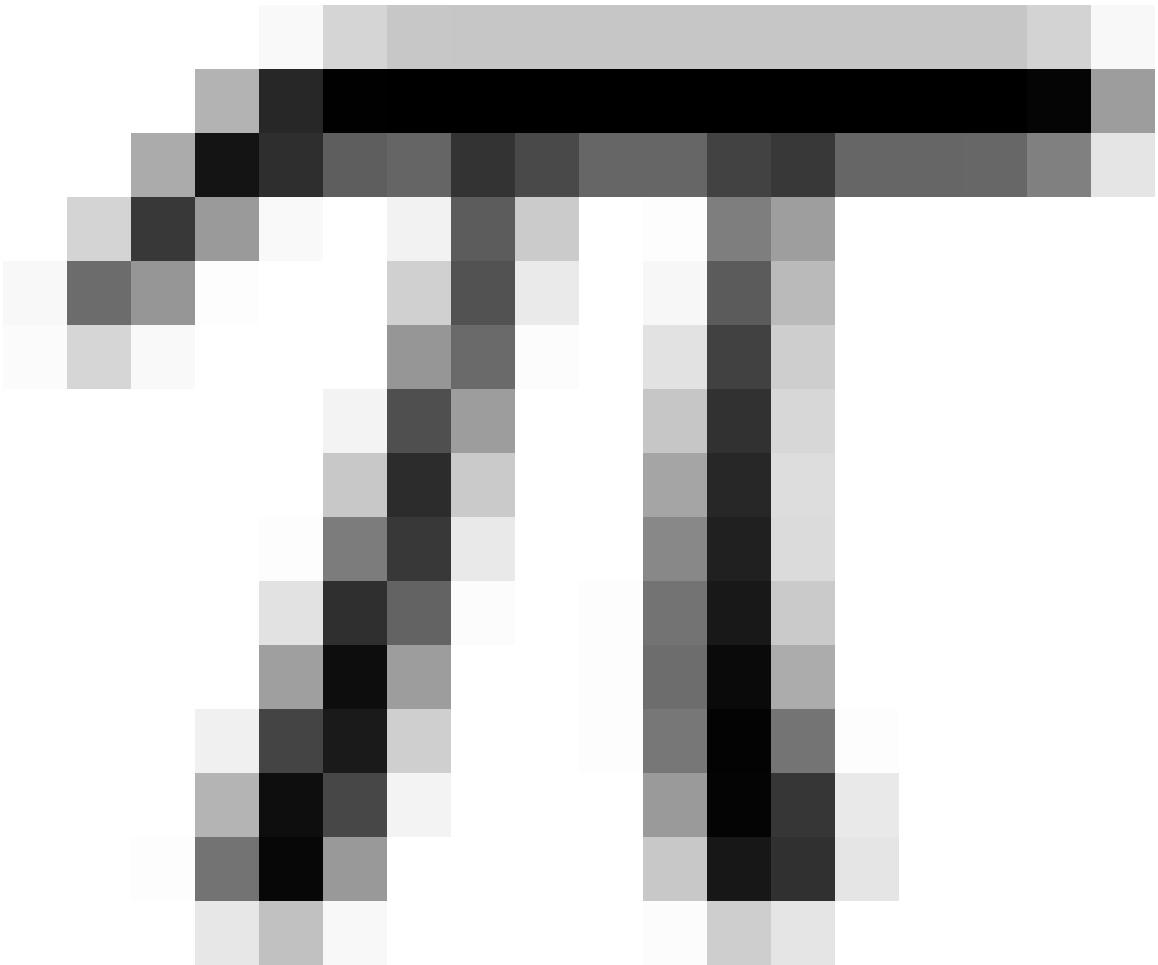




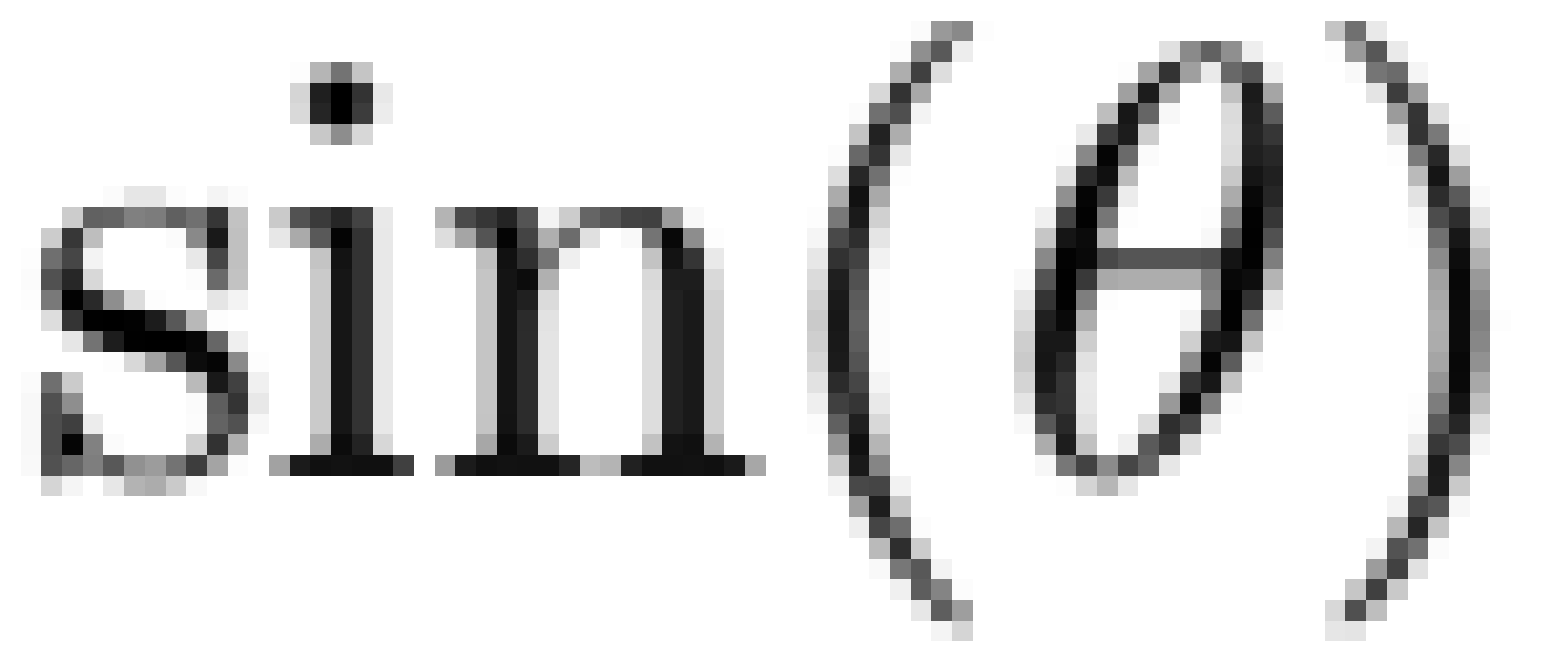




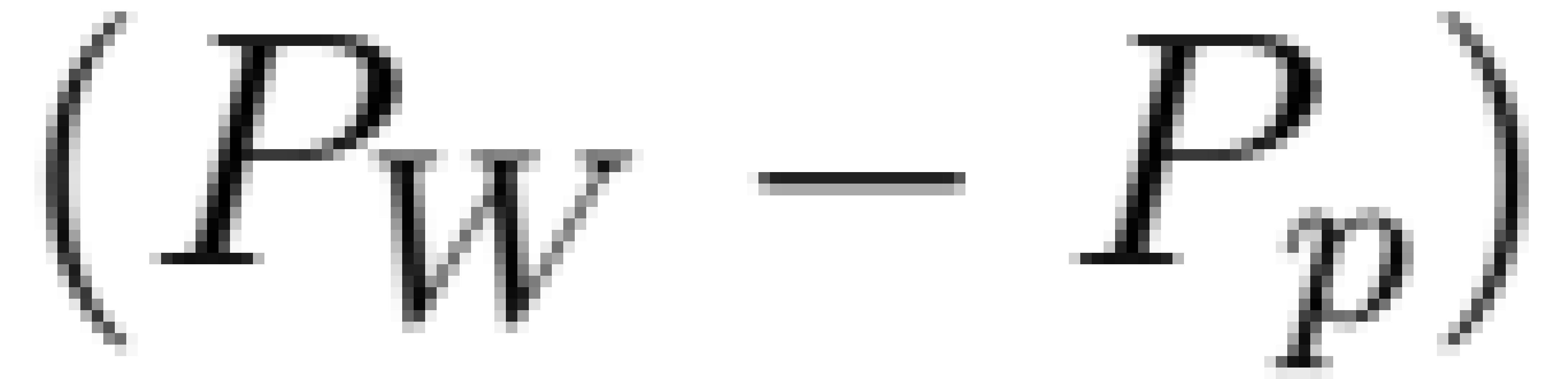


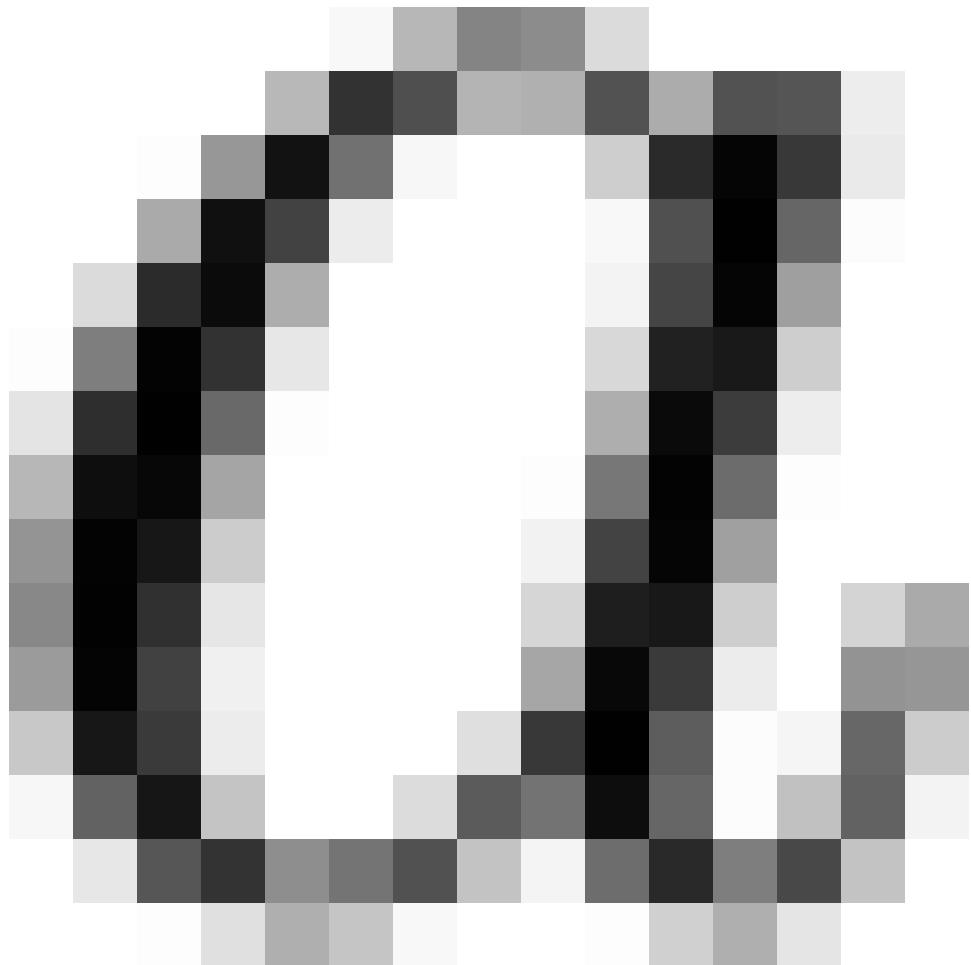




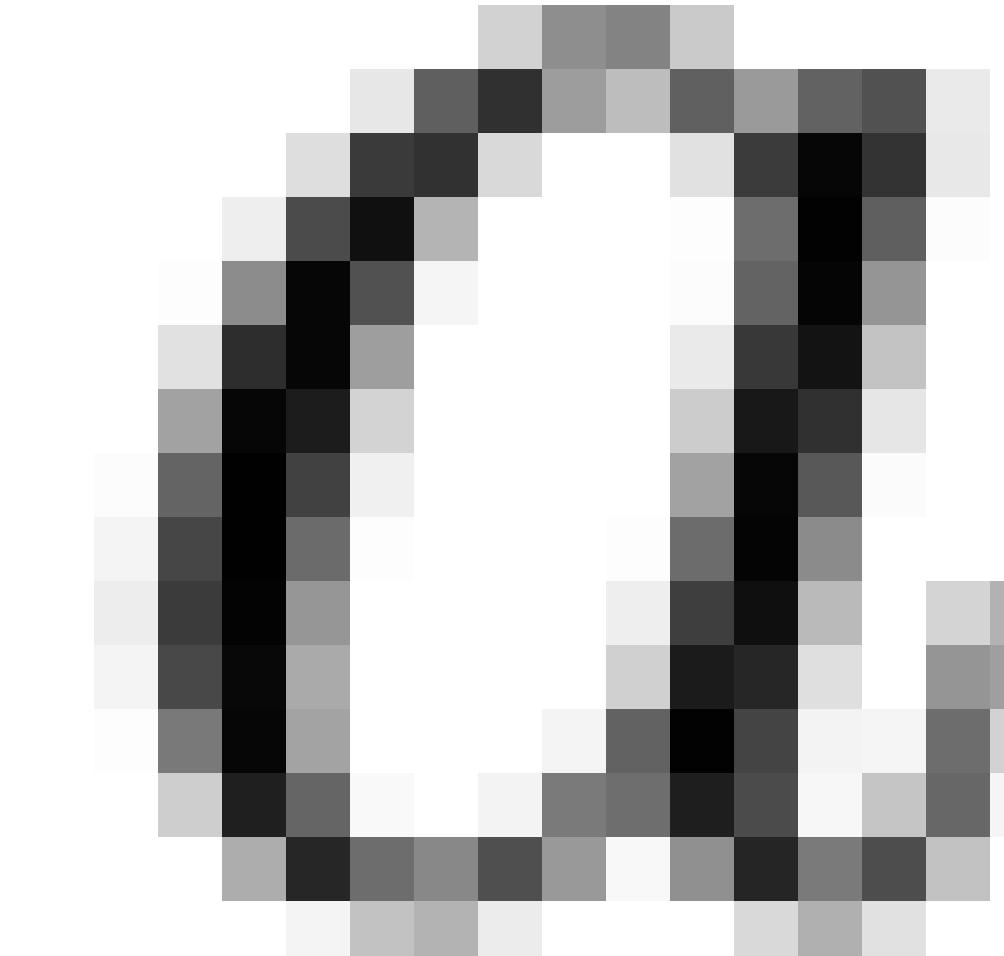
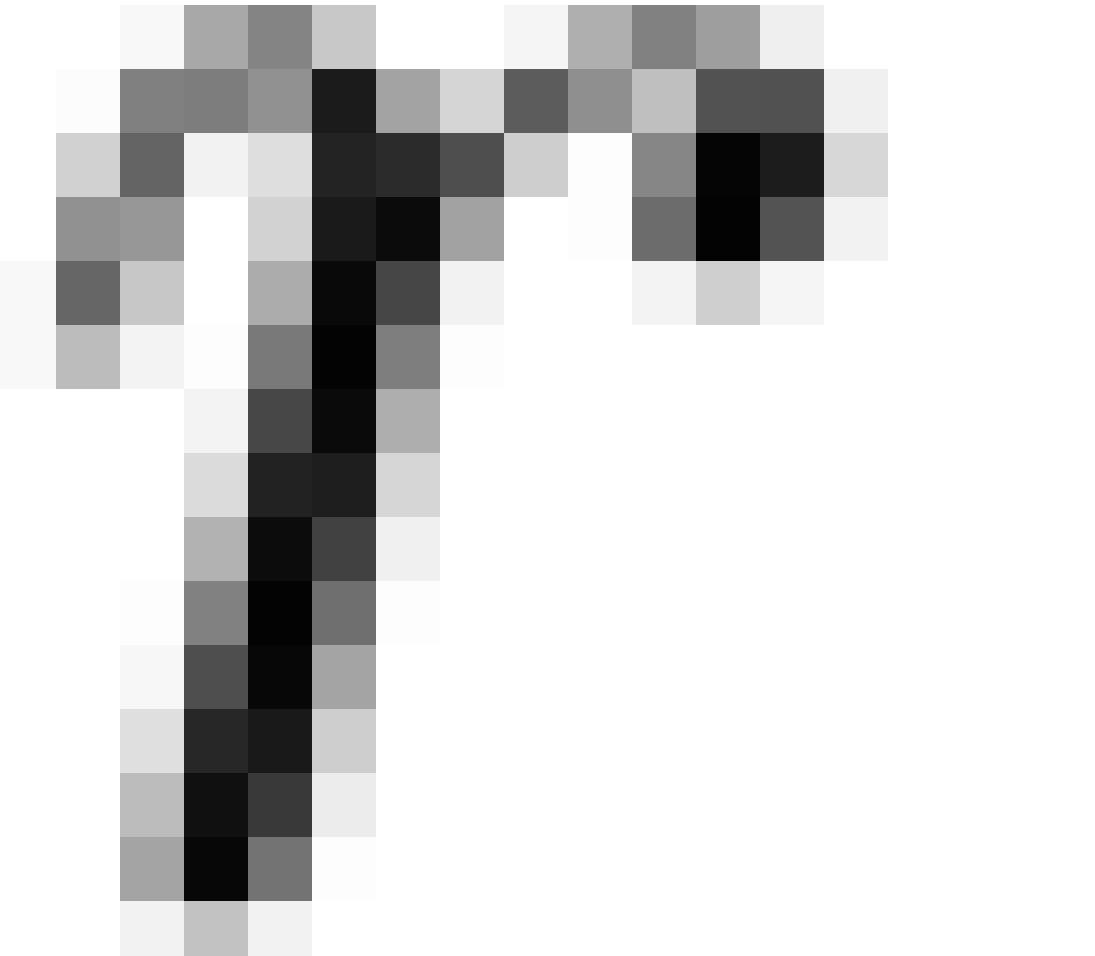






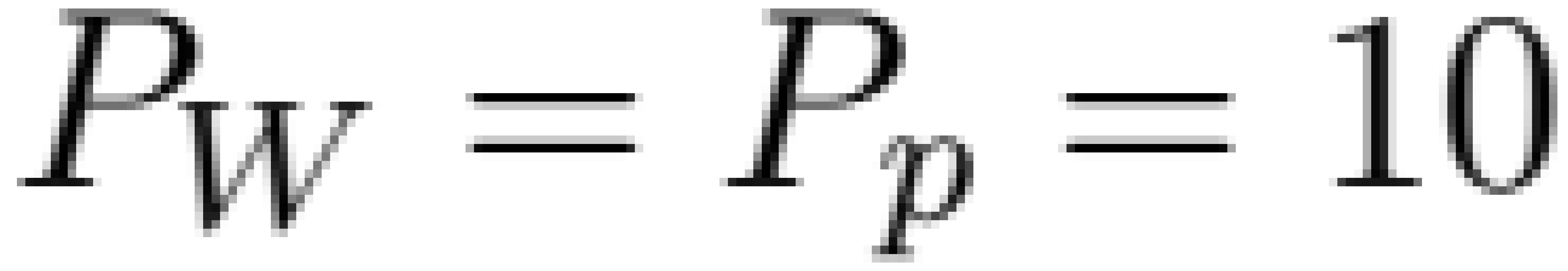


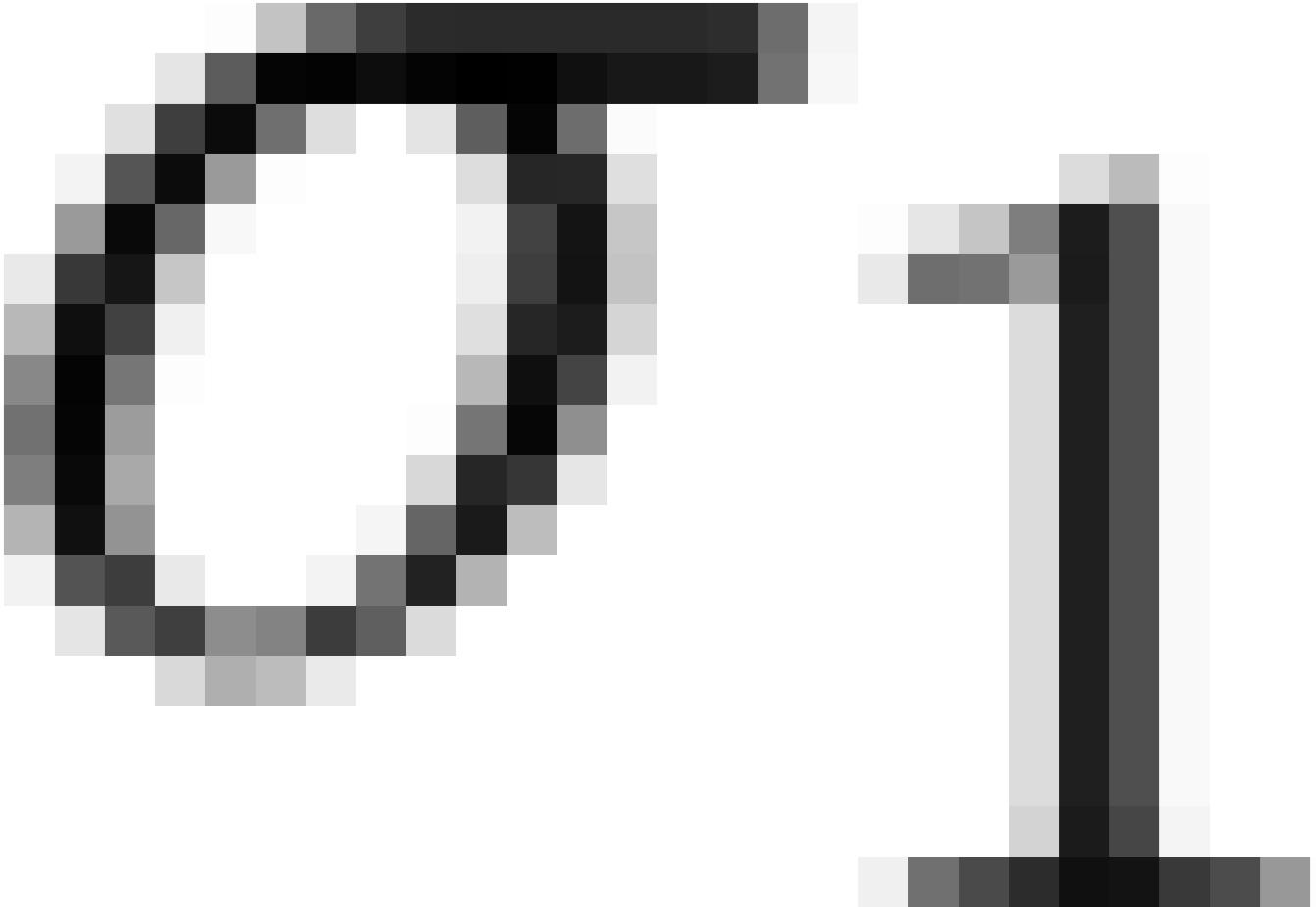
$$\left\{ \begin{array}{lcl} \sigma_{rr} & = & (P_W - P_p) \left( \frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left( 1 - \frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left( 1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{\theta\theta} & = & -(P_W - P_p) \left( \frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left( 1 + \frac{a^2}{r^2} \right) - \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left( 1 + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{r\theta} & = & \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left( 1 + 2 \frac{a^2}{r^2} - 3 \frac{a^4}{r^4} \right) \sin(2\theta) \\ \\ \sigma_{zz} & = & \sigma_v - 2\nu (\sigma_{Hmax} - \sigma_{hmin}) \left( \frac{a^2}{r^2} \right) \cos(2\theta) \end{array} \right.$$



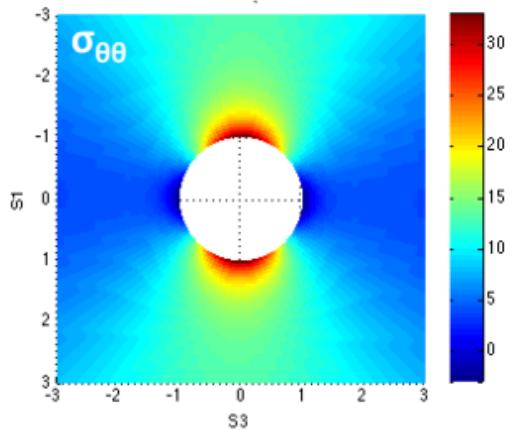
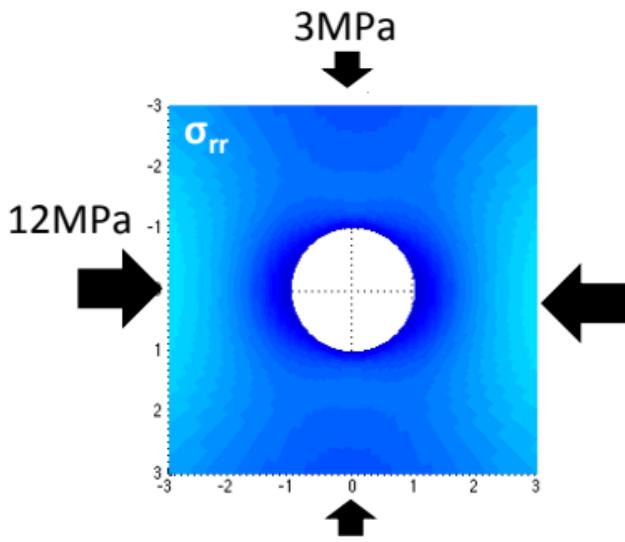




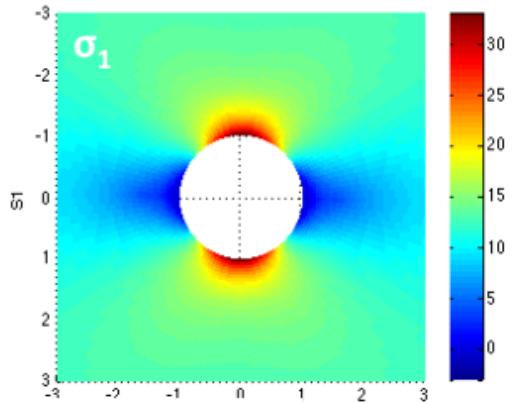
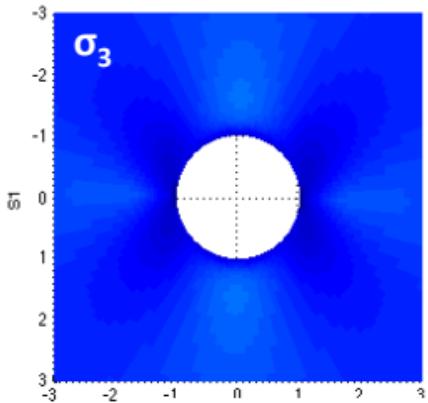


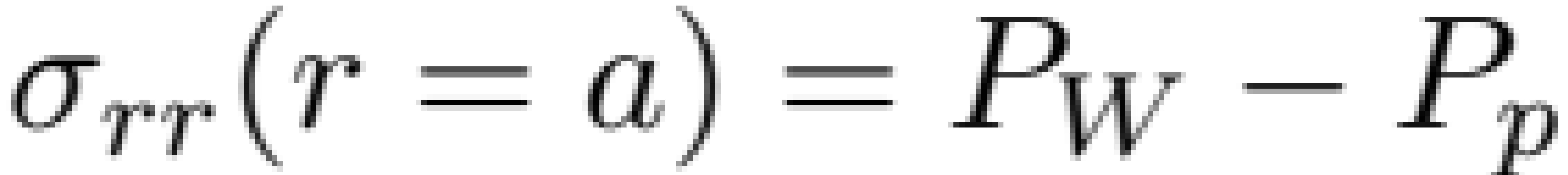


Stresses in cylindrical coordinates



Principal Stresses





$$\sigma_{\text{H}\alpha\alpha} = \sigma_{\text{H}\alpha\alpha}^{\text{max}} + \sigma_{\text{H}\alpha\alpha}^{\text{min}} \left( \frac{P_{\text{pump}}}{P_{\text{pump}} + P_{\text{H}\alpha\alpha}} \right)^2 \left( \frac{\sin(\theta)}{\sin(\theta) + \cos(\theta)} \right)^2$$

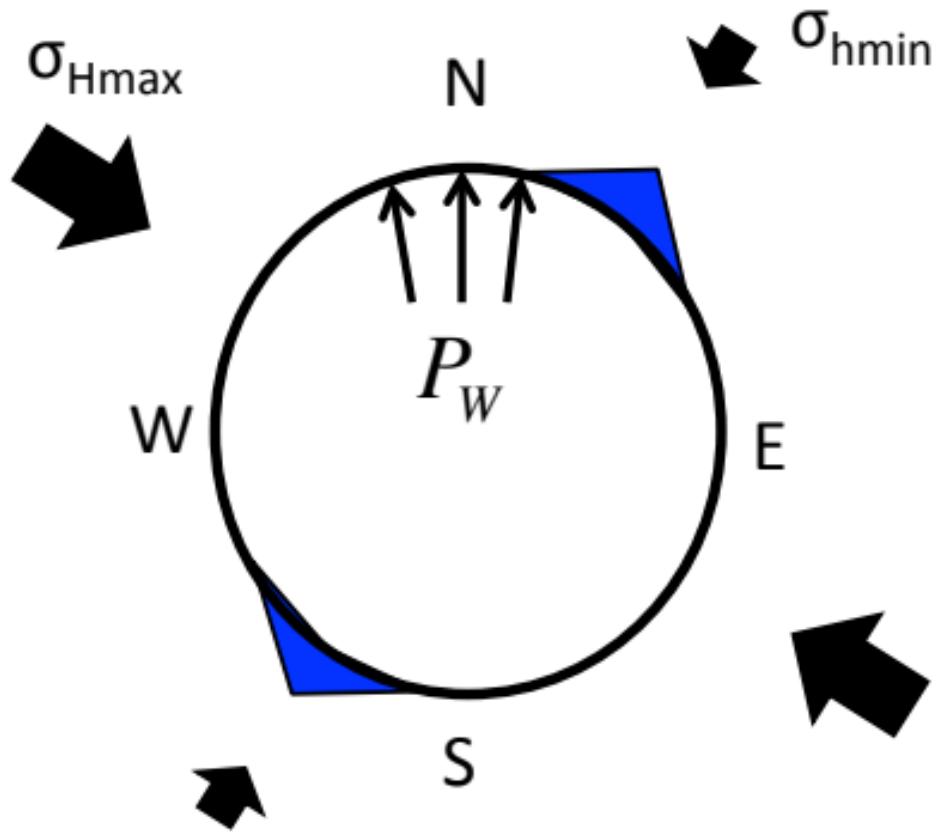
$$\begin{aligned} \sigma_{\theta\theta}(r=a, \theta=0) &= (P_W - P_p) + 3\sigma_{h\min} \\ \sigma_{\theta\theta}(r=a, \theta=\pi/2) &= -(P_W - P_p) + 3\sigma_{h\max} \end{aligned}$$







$$\left\{ \begin{array}{l} \sigma_1 = \sigma_{\theta\theta} = - (P_W - P_p) + 3 \sigma_{H\max} - \sigma_{hmin} \\ \sigma_3 = \sigma_{rr} = (P_W - P_p) \end{array} \right.$$



Stress anisotropy

$$P_{W\text{shear}} = P_p + \frac{3\sigma_{H\max} - \sigma_{h\min} - UCS}{1+q}$$

Pore pressure in the formation

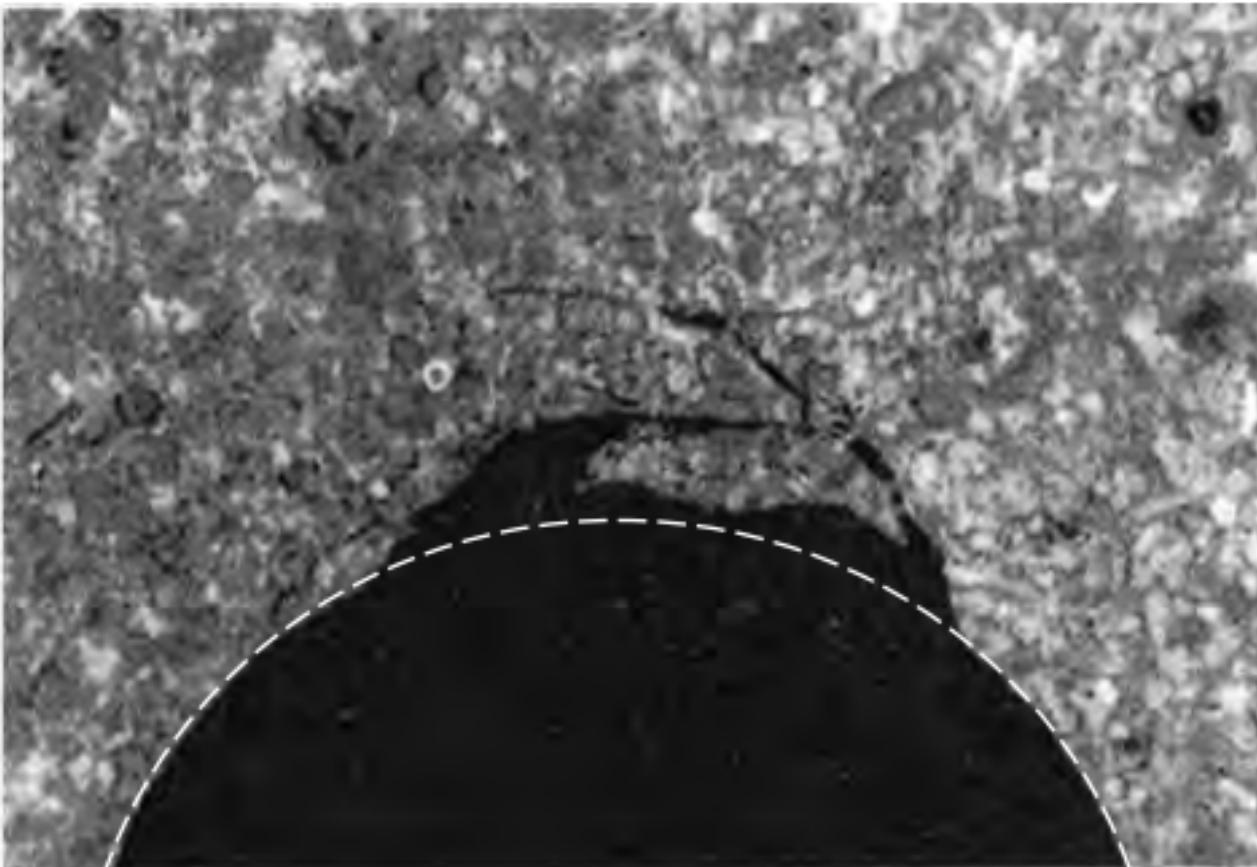
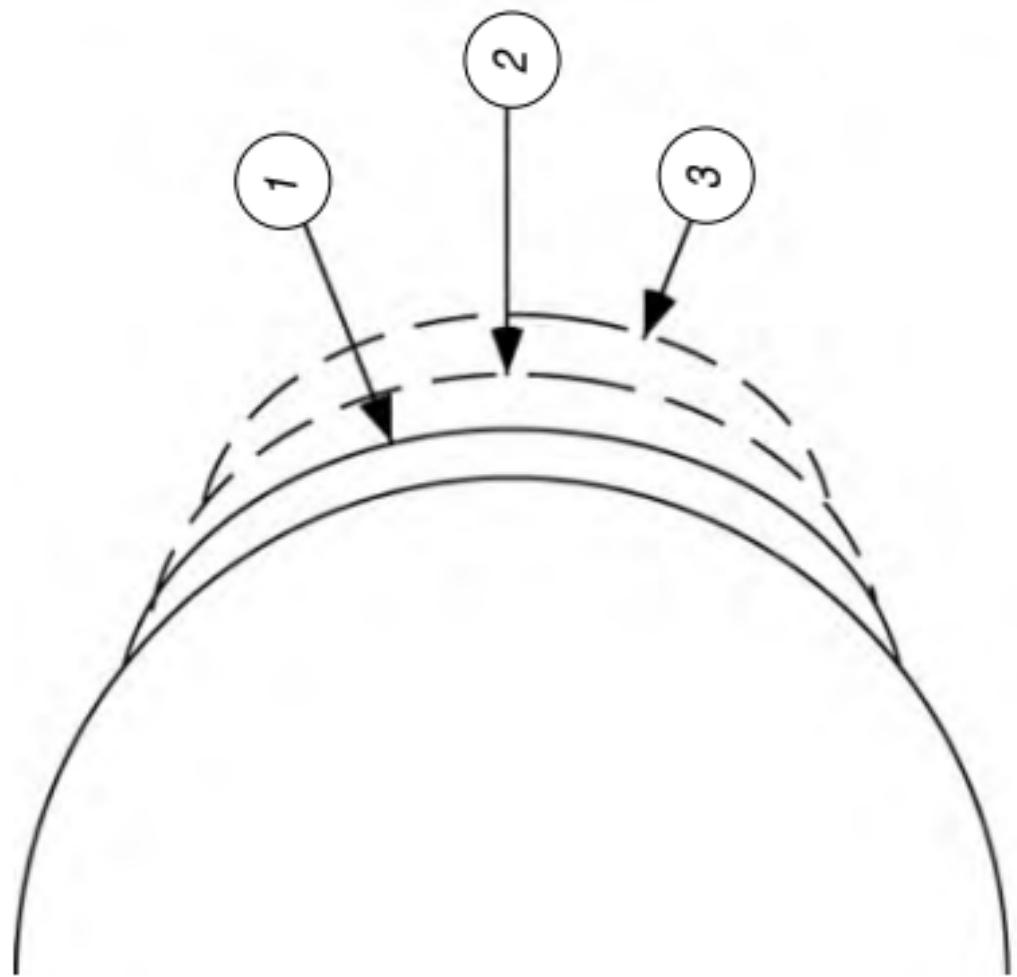
Shear strength

$P_W \leq P_{W\text{shear}}$  leads to breakouts



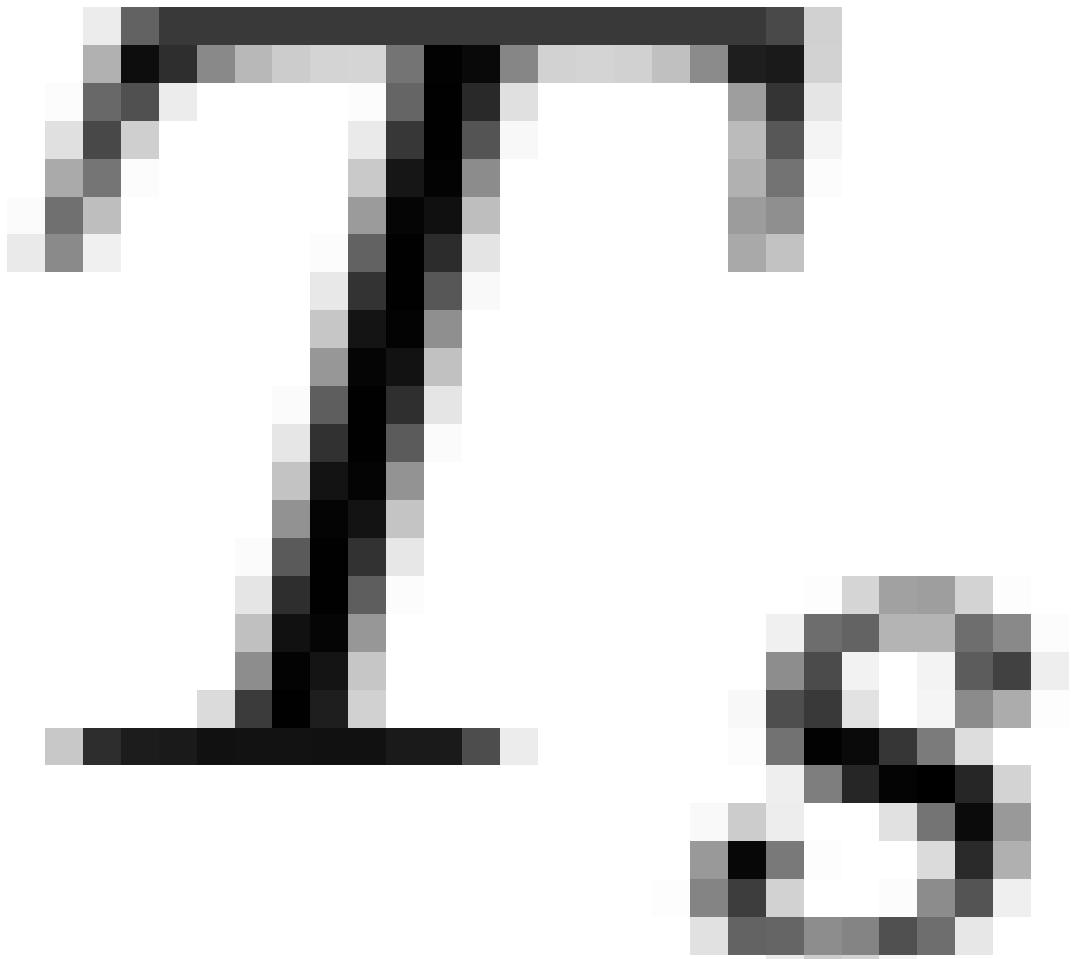
$\left[ \frac{1}{\sqrt{2}}(P_+ + P_-) \right] = \frac{1}{\sqrt{2}}(S_x + iS_y)$

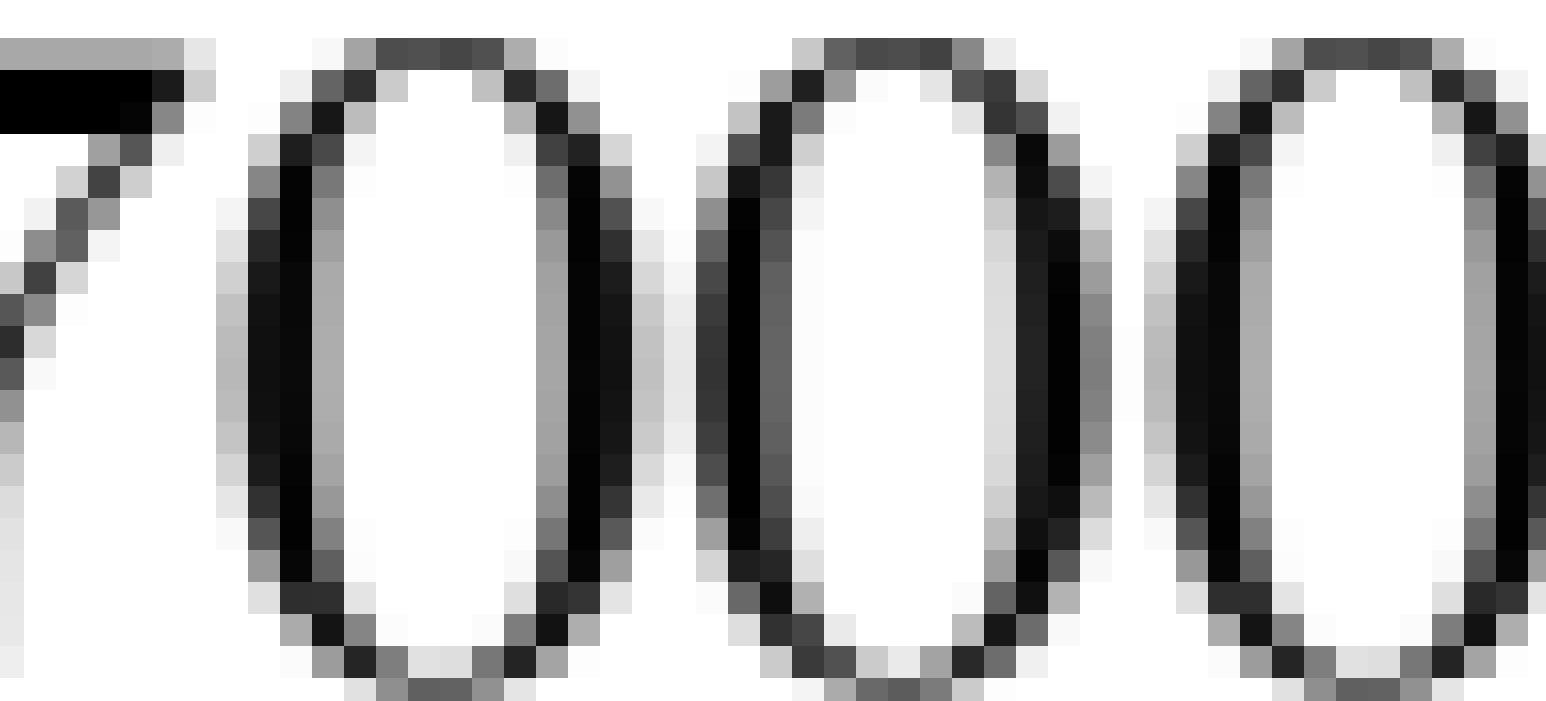
$$P_{W\text{ shear}} = \frac{3\sigma_H \max - \sigma_{hmin} - U_{CS}}{1 + q}$$

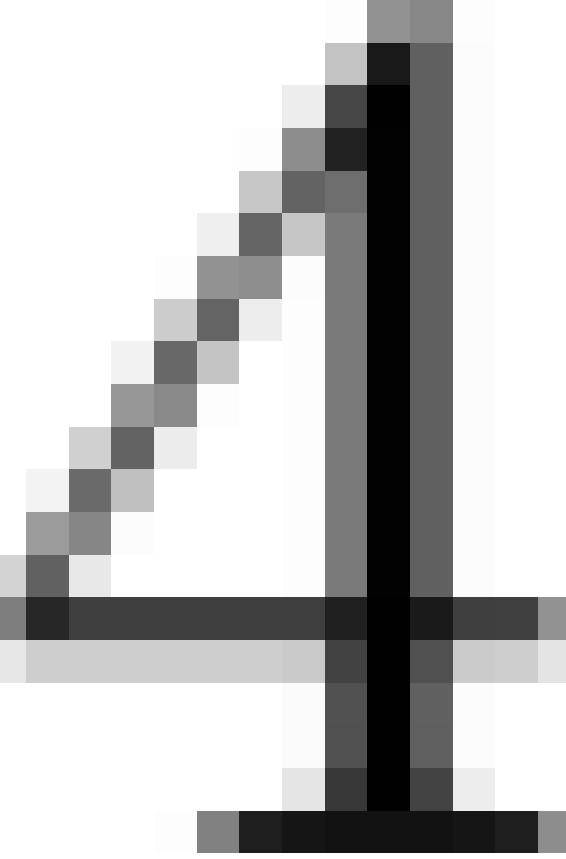
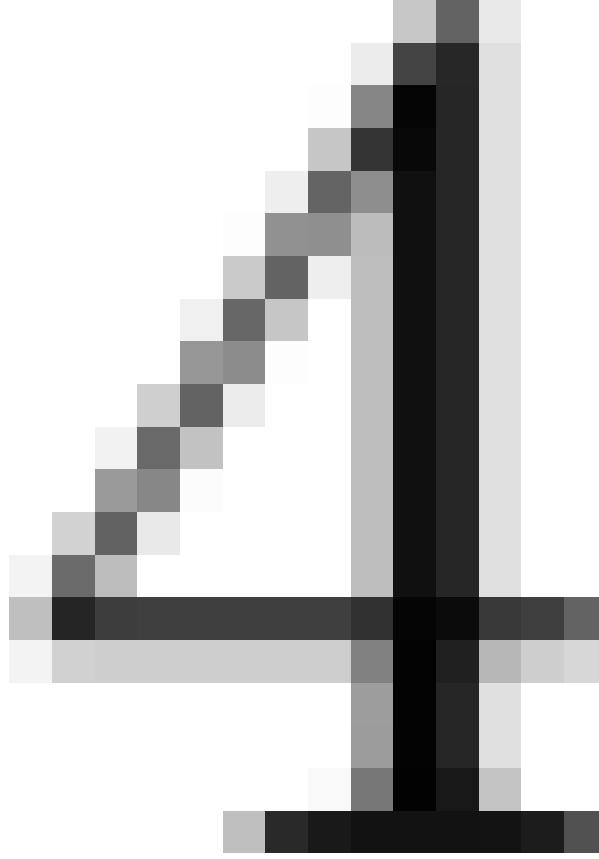
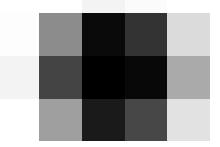
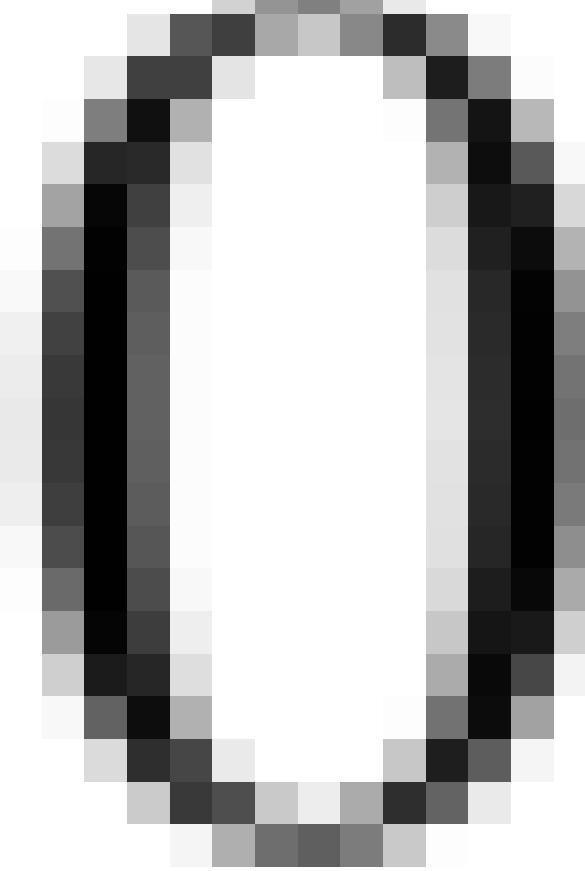
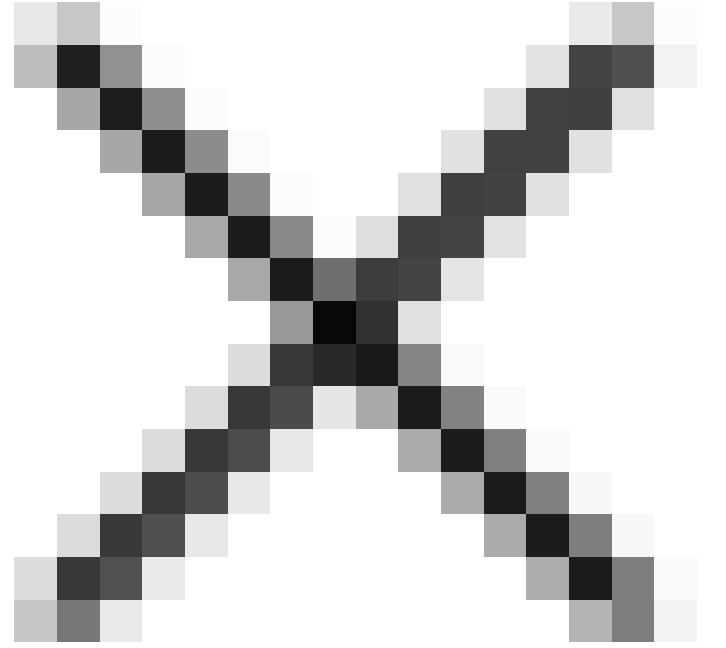


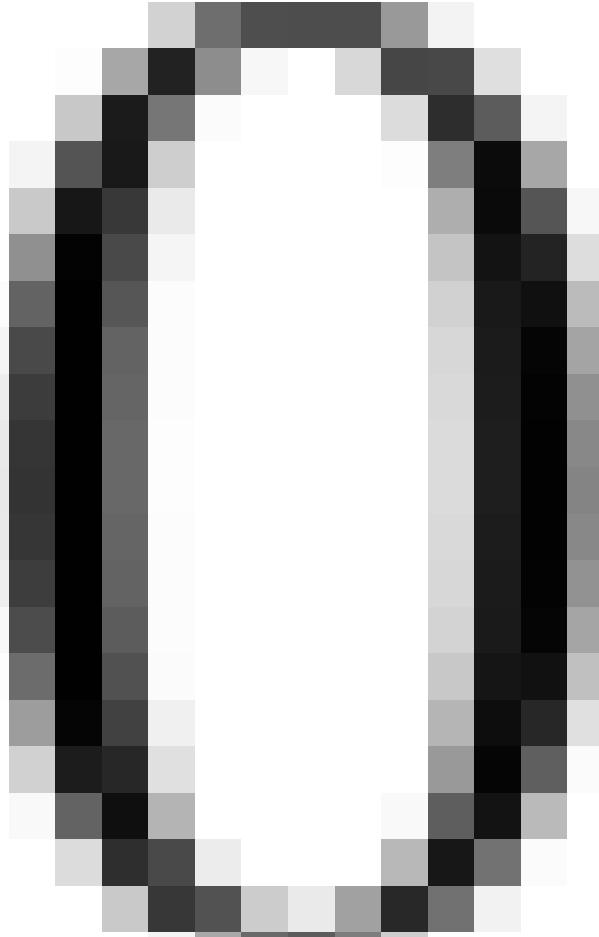
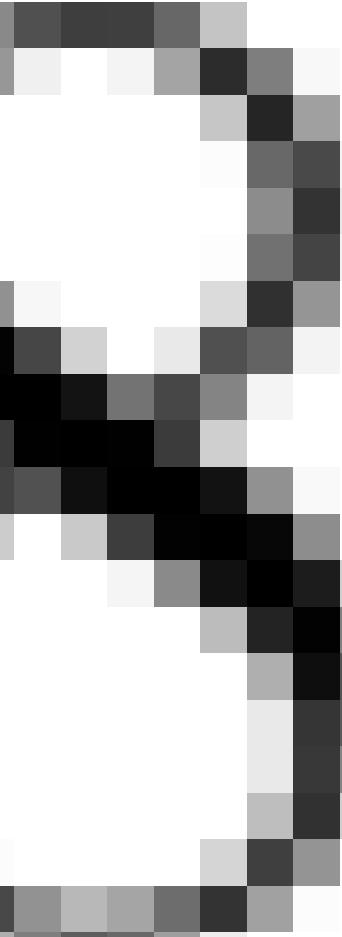
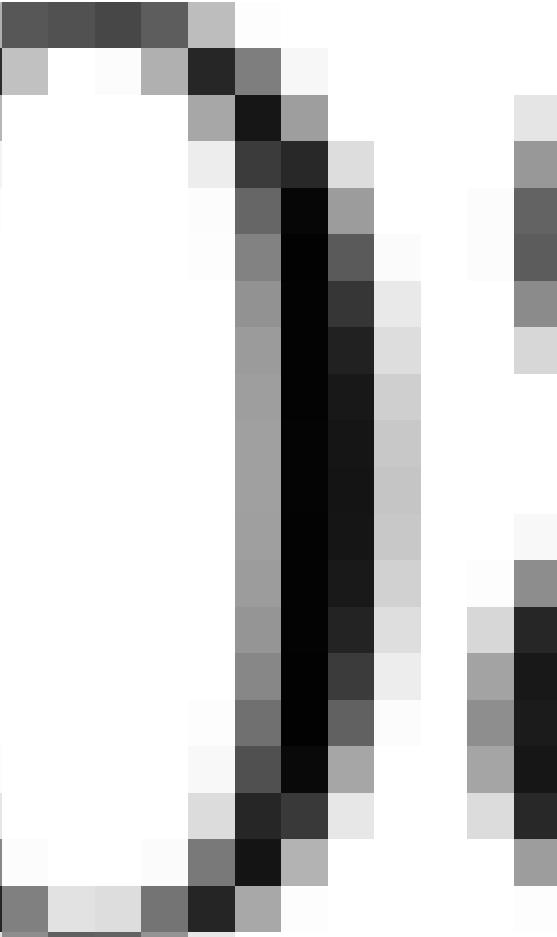
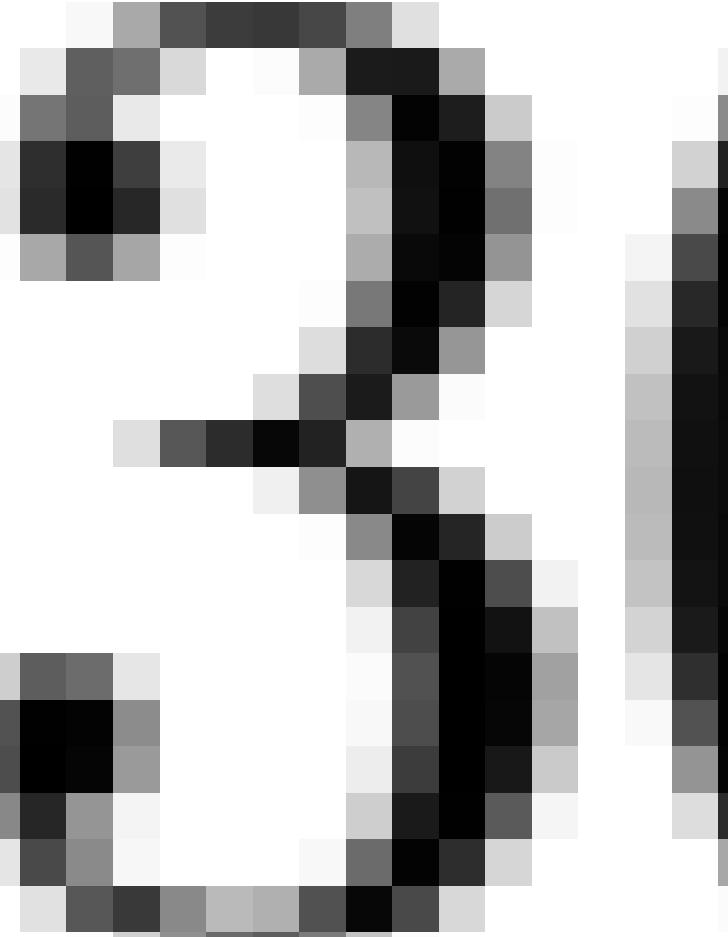




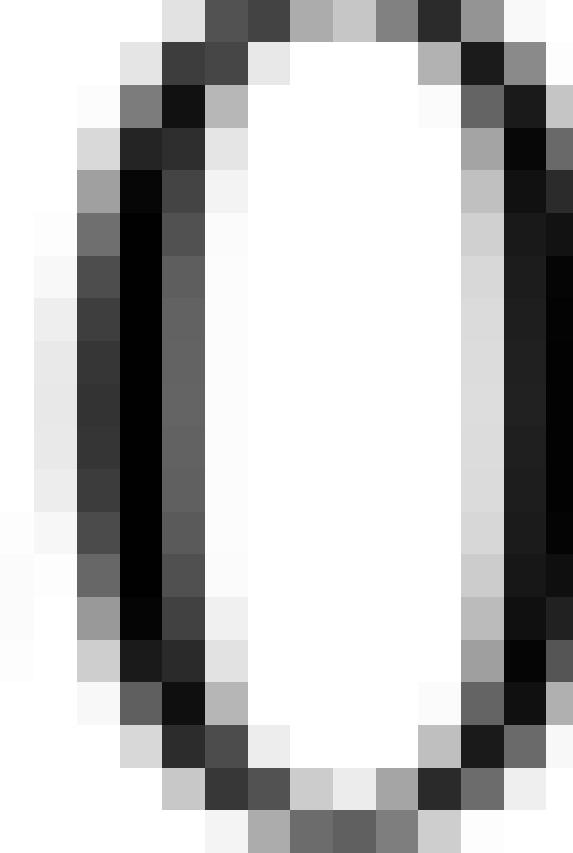
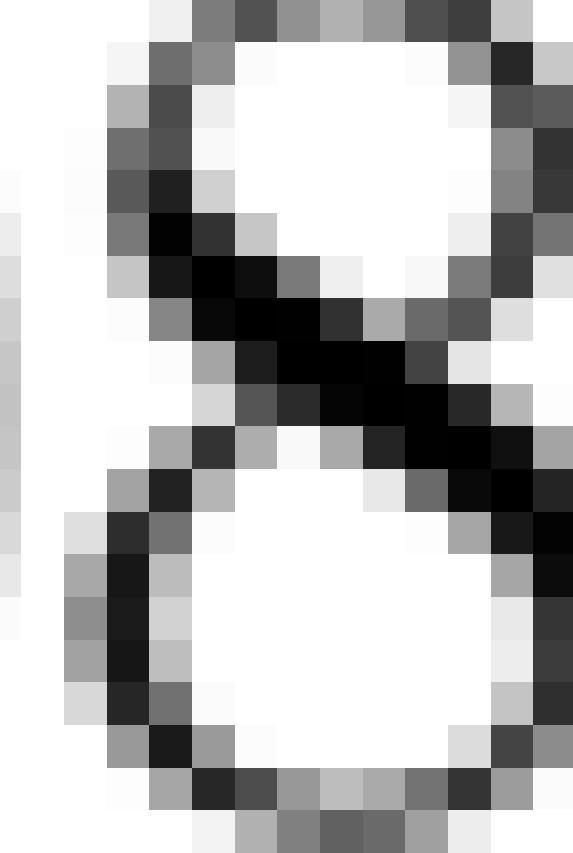
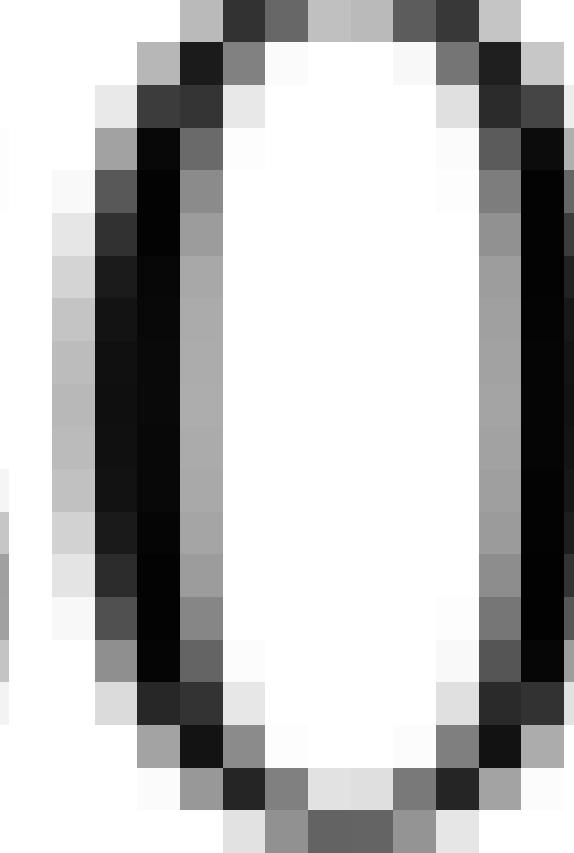
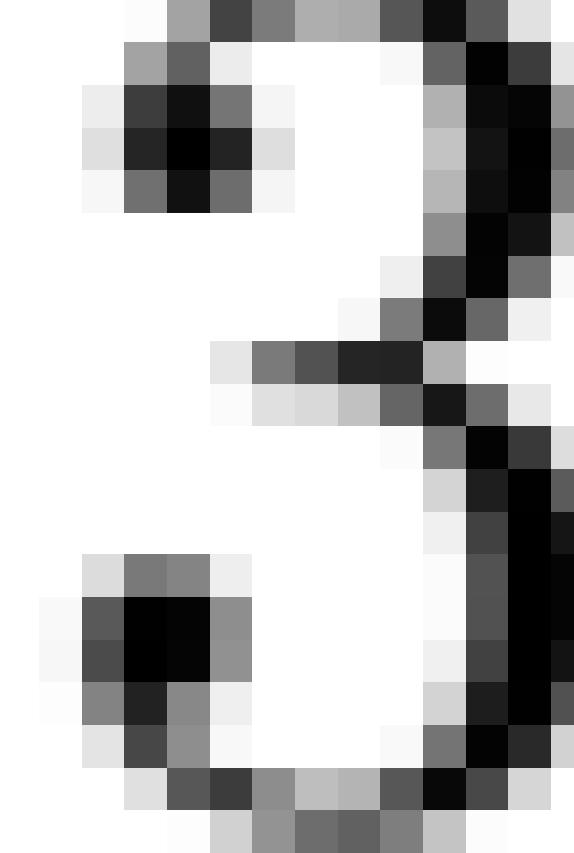


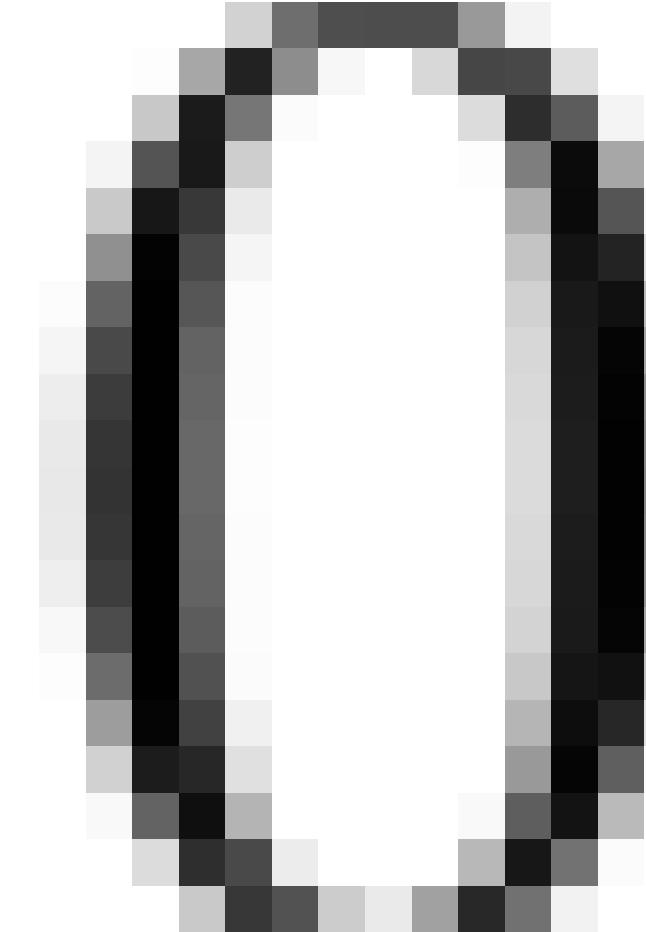
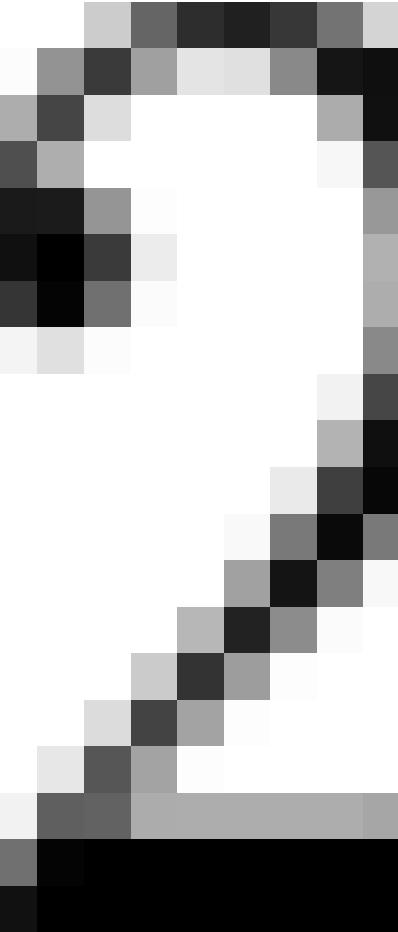
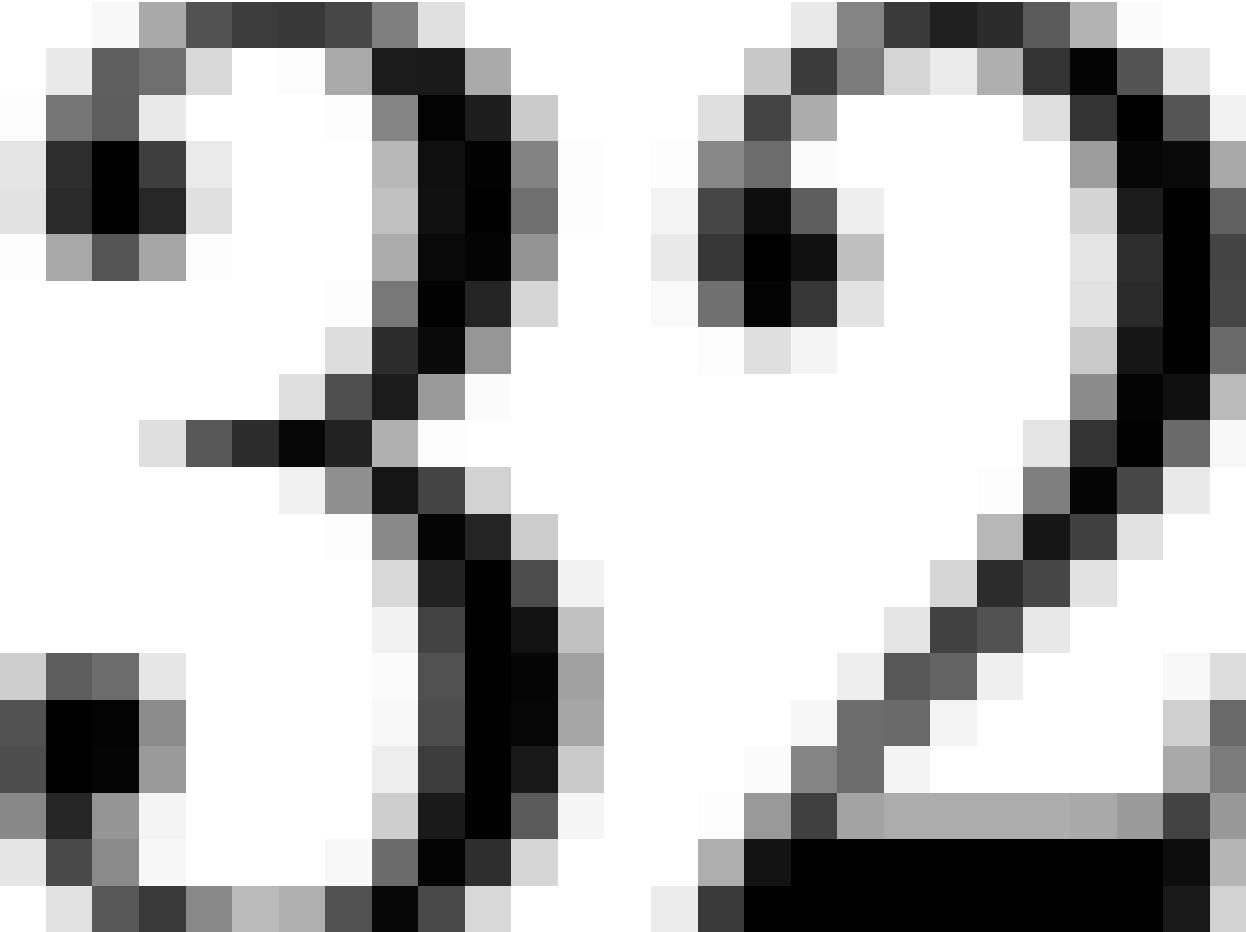


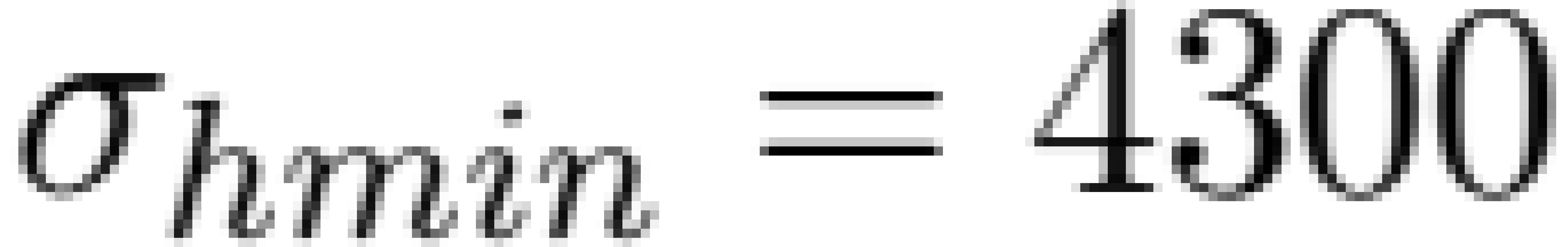


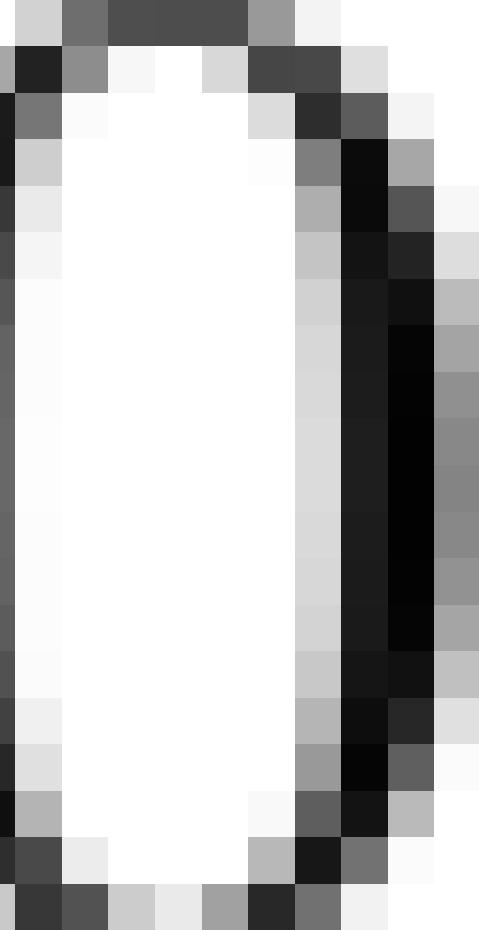
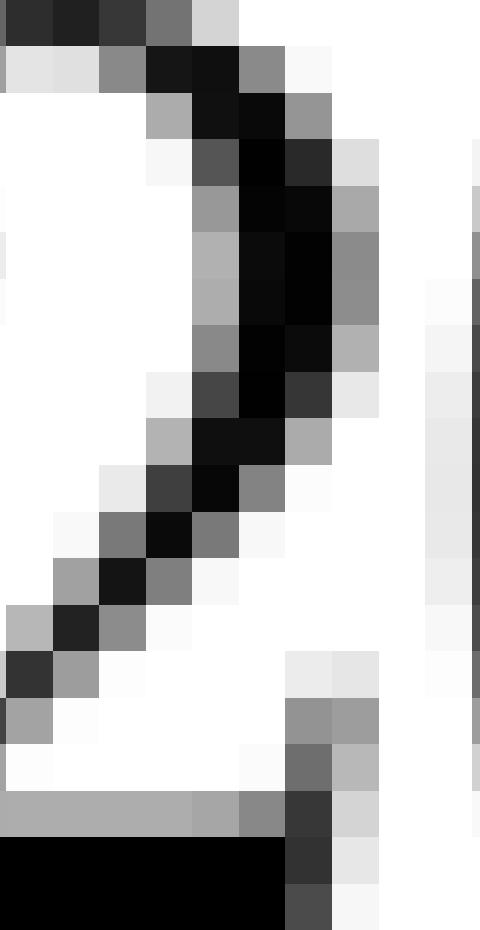
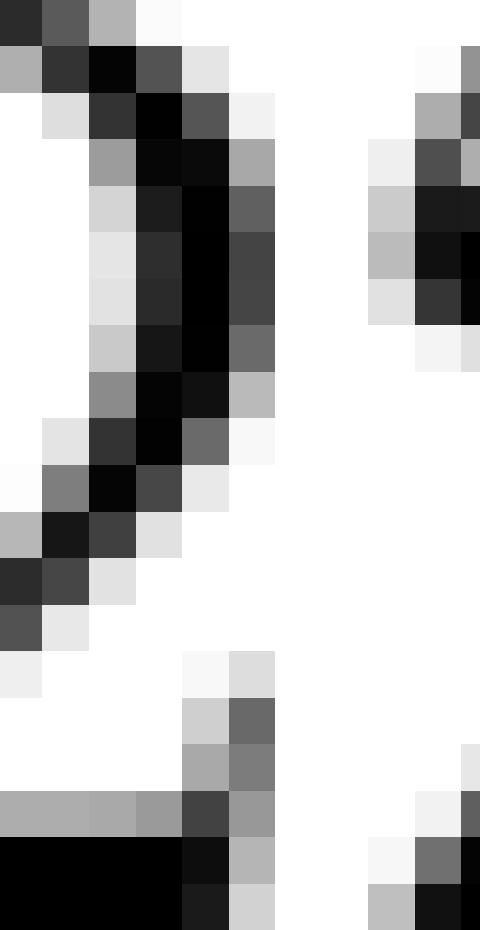
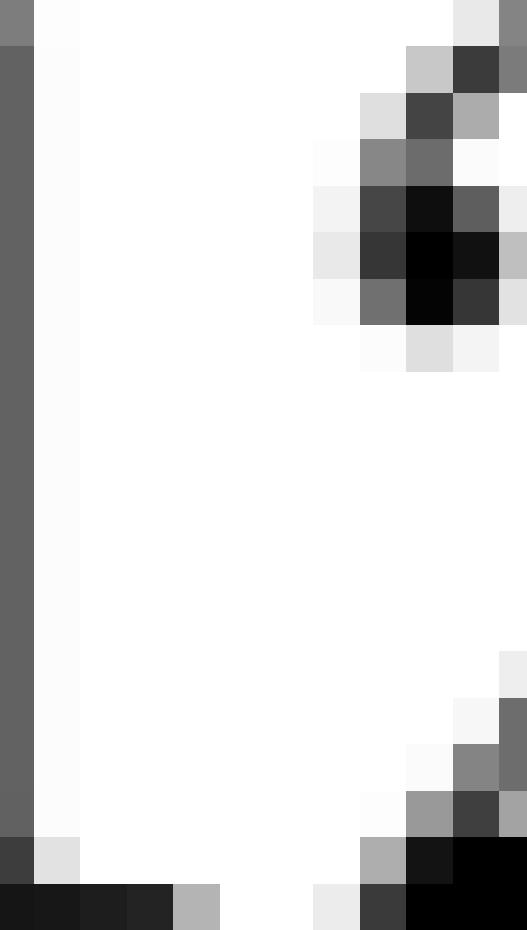














9 2

1

$$1 + \sin 30.96^\circ$$

$$- \sin 30.96^\circ$$

—

3.12



$$\frac{3 \times 3220 \text{ psi} - 1220 \text{ psi}}{1 + 3.12} = 4279 \text{ psi}$$

4279 psi

7000 ft

X

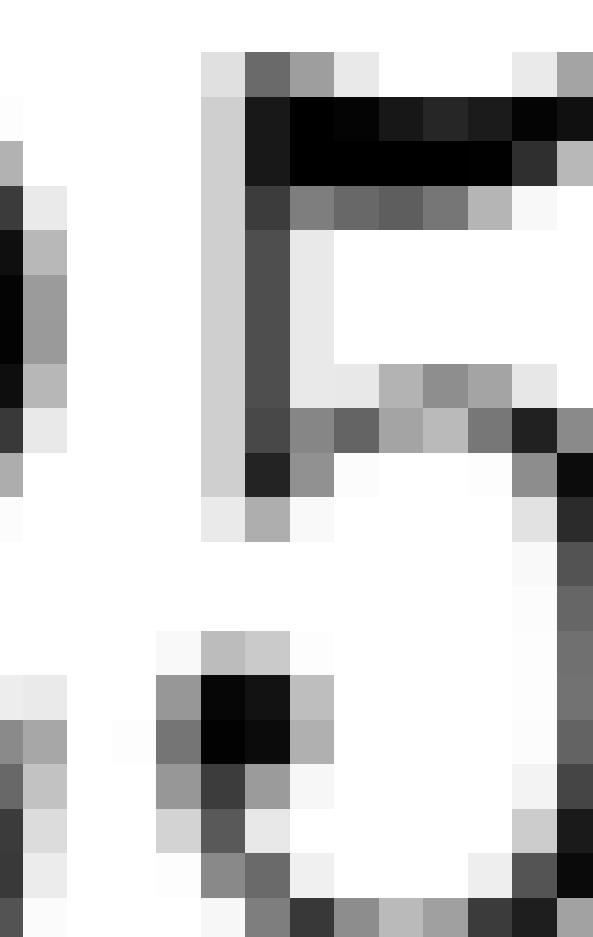
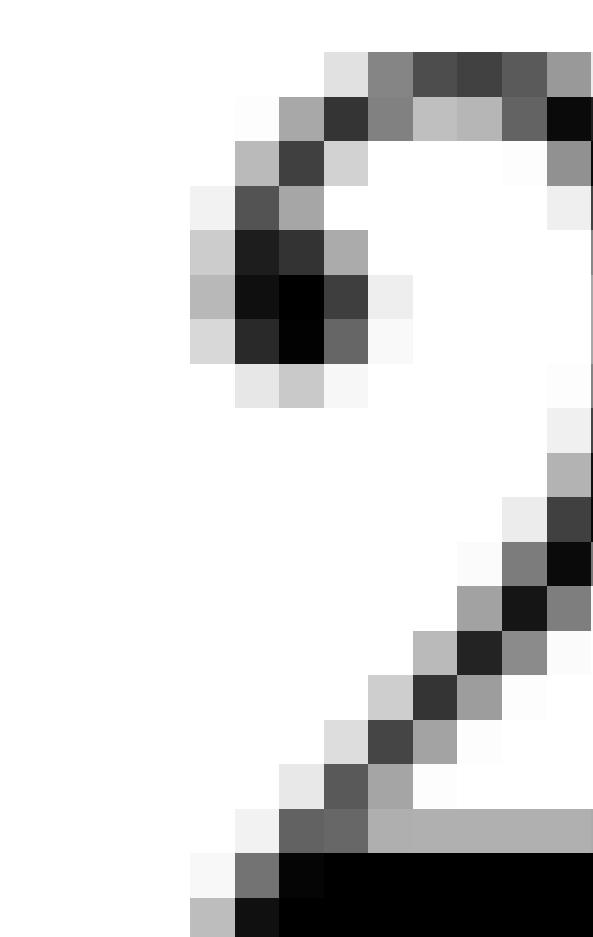
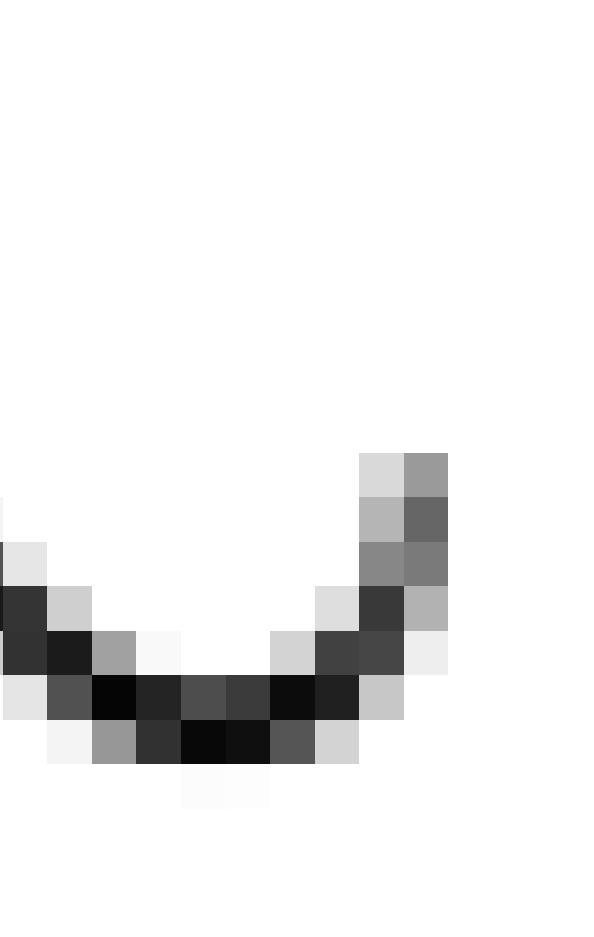
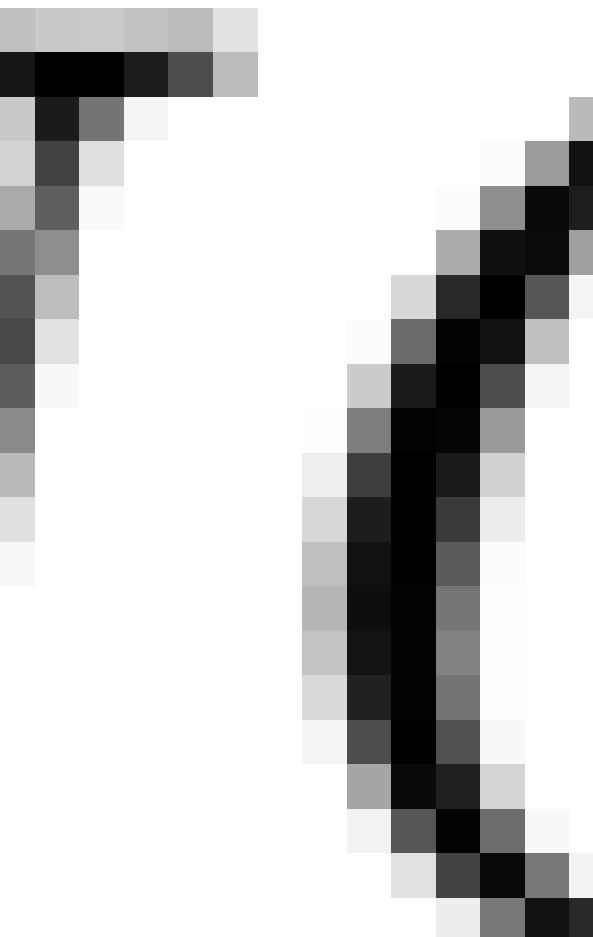
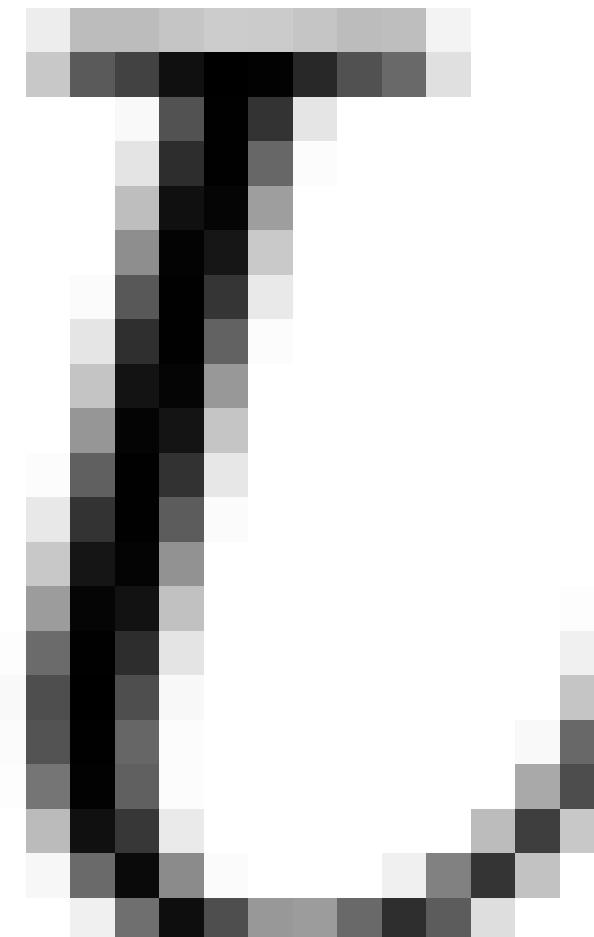
8.3 ppg

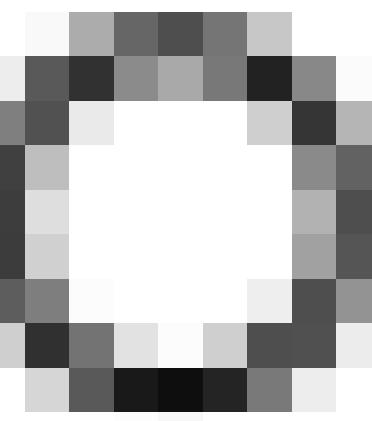
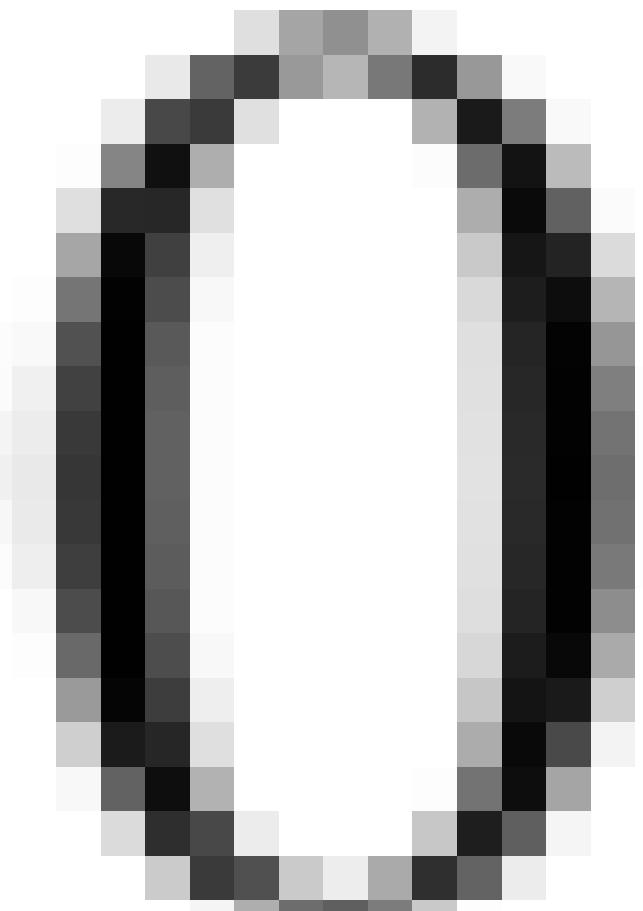
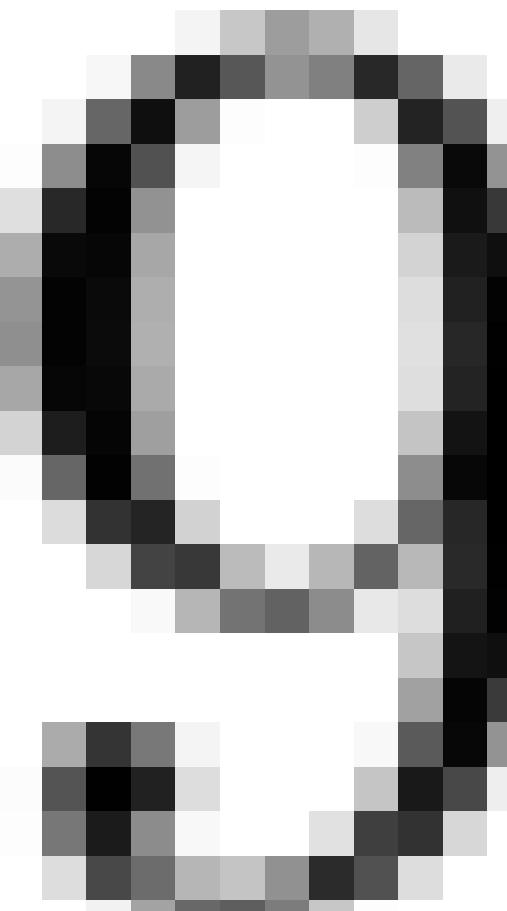
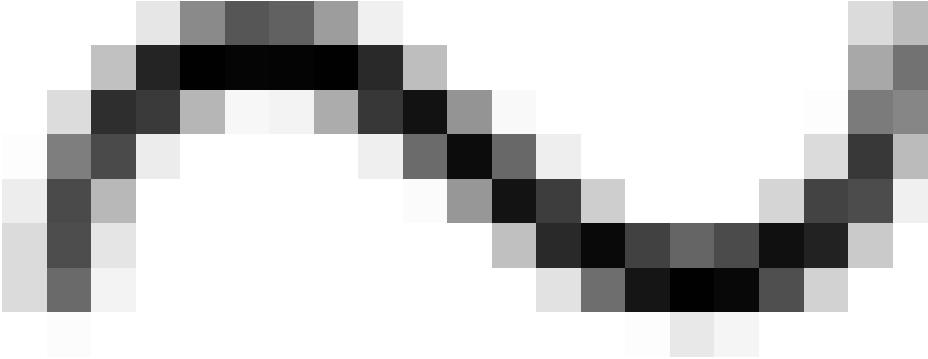
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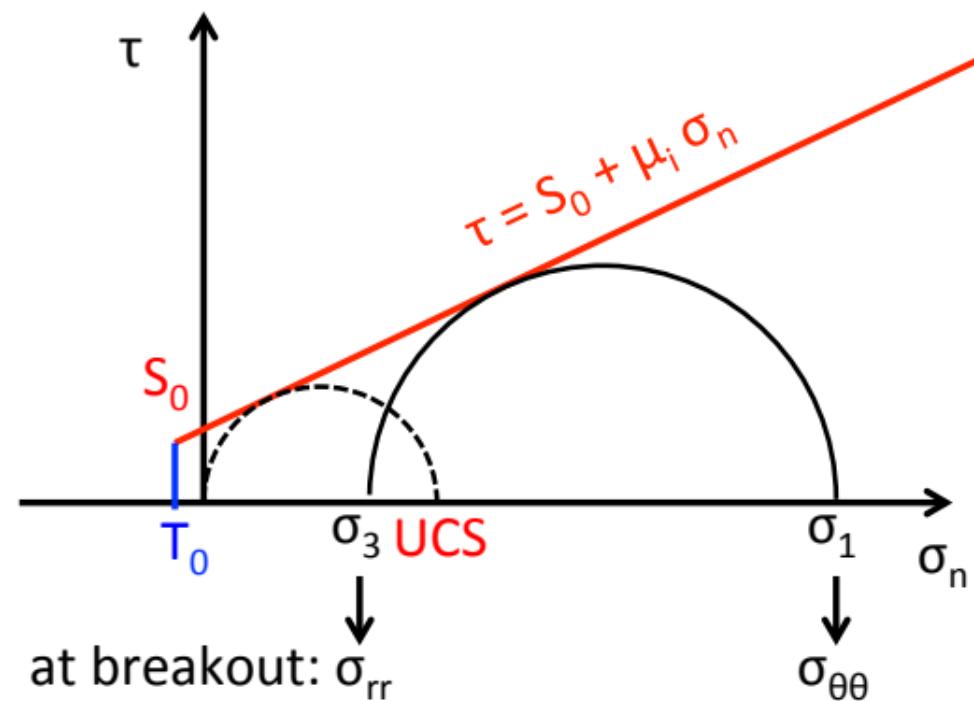
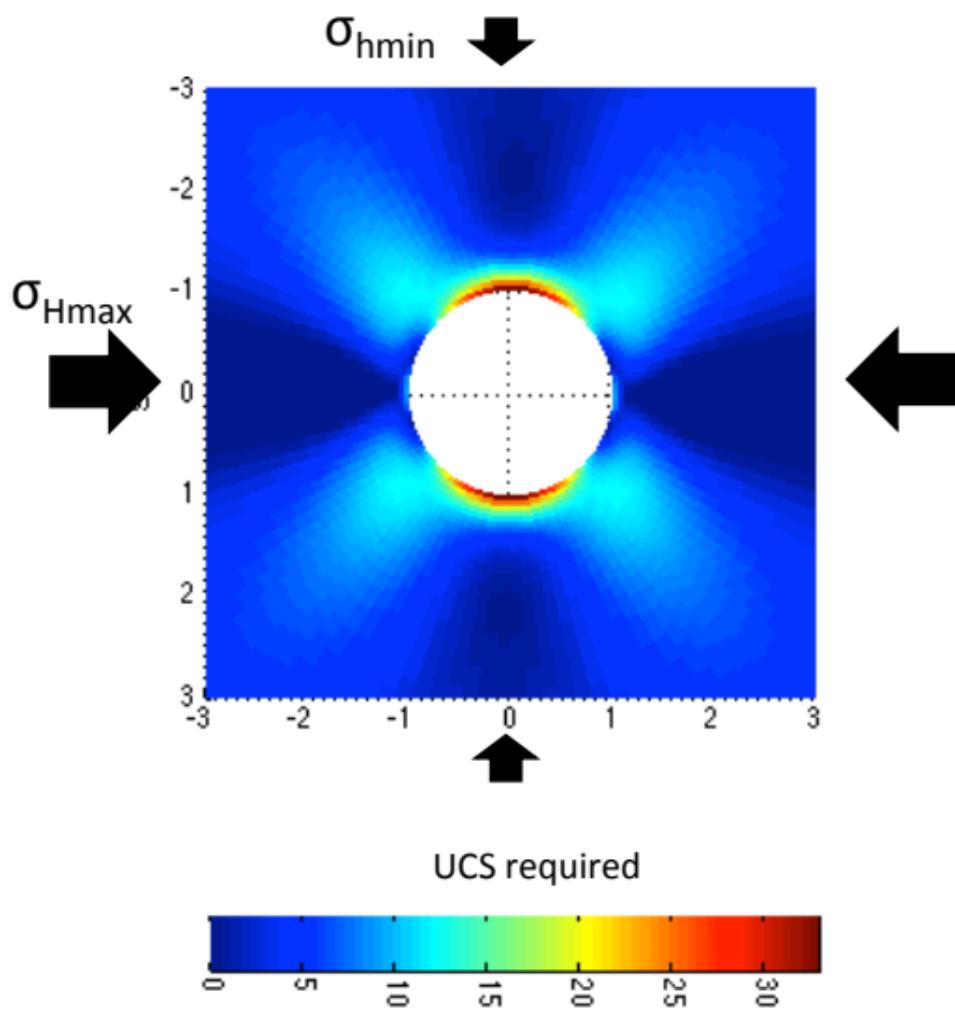
11.57 ppg

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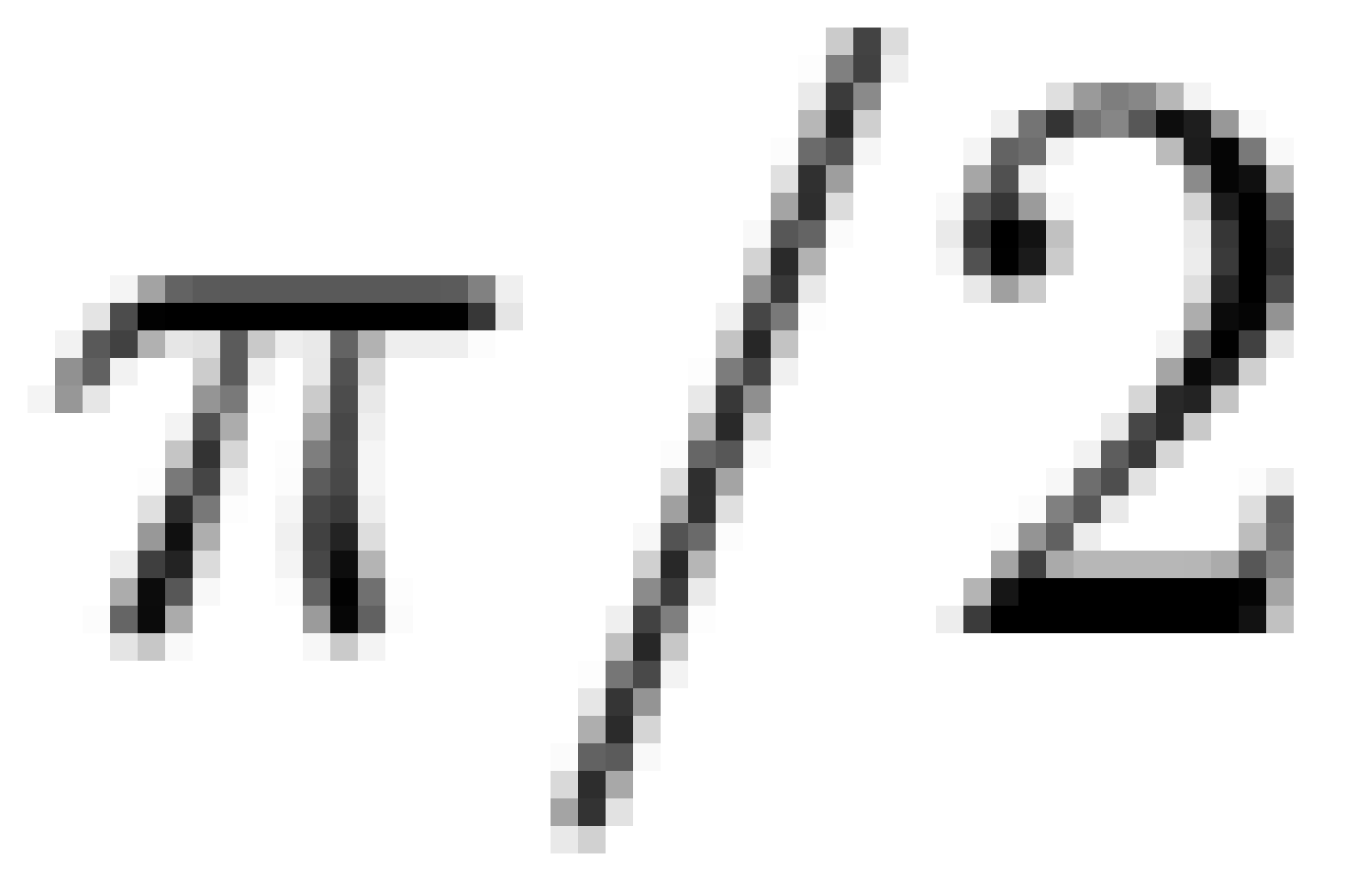
0.44 psi/ft

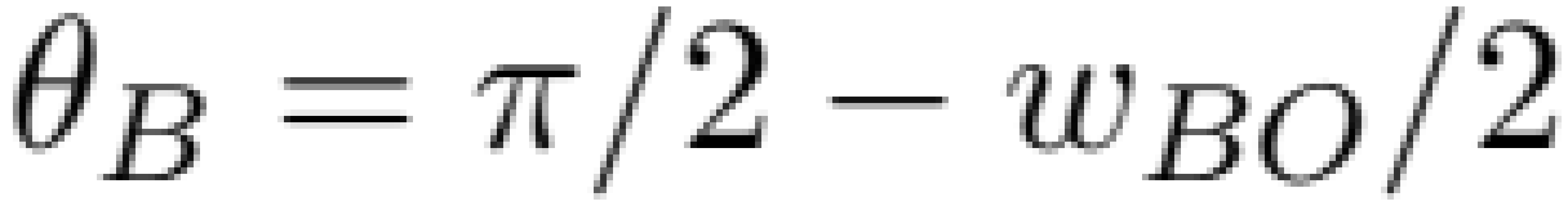




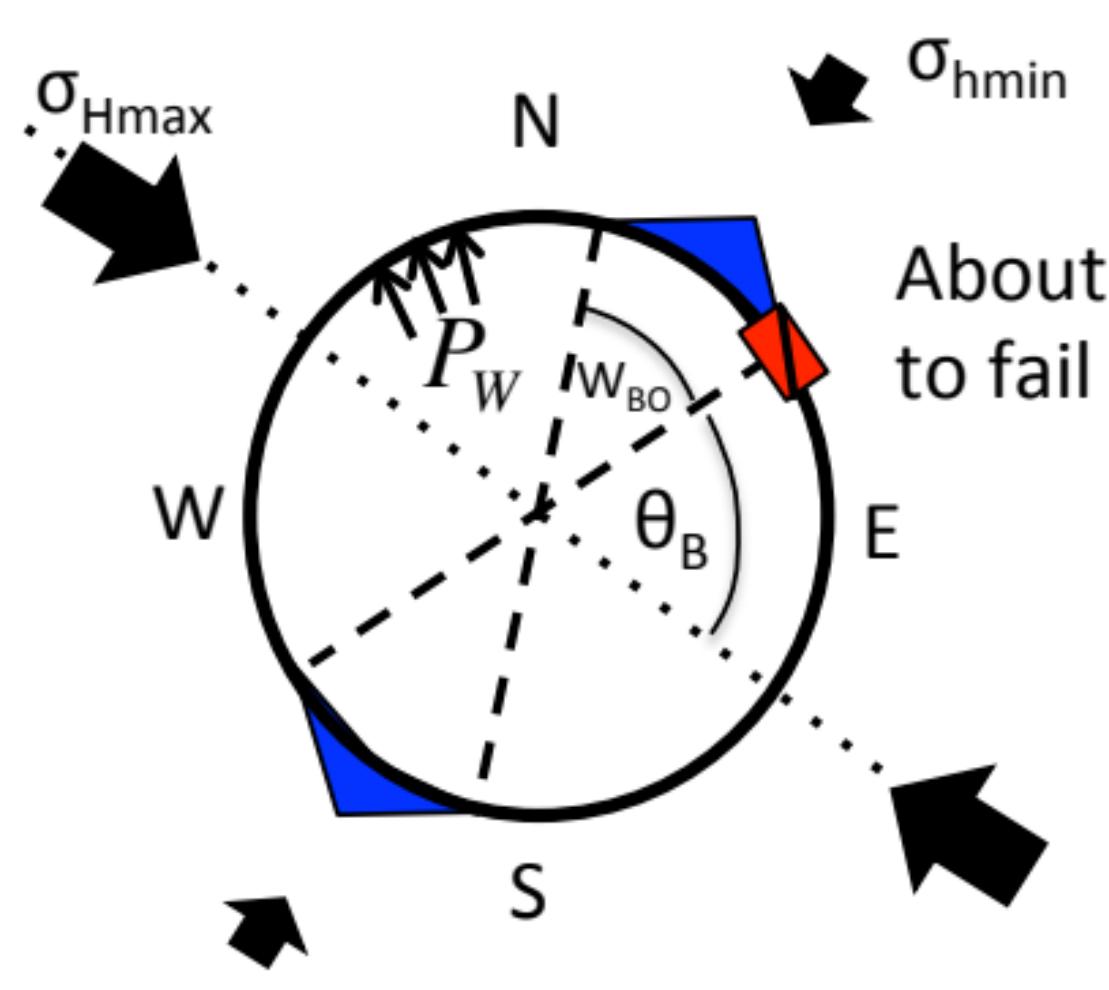








$$\begin{aligned} \sigma_{\theta\theta} &= (P_W - P_p) + 2(\sigma_{H\max} - \sigma_{h\min}) \cos(2\theta_B) \\ \sigma_{rr} &= + (P_W - P_p) \end{aligned}$$

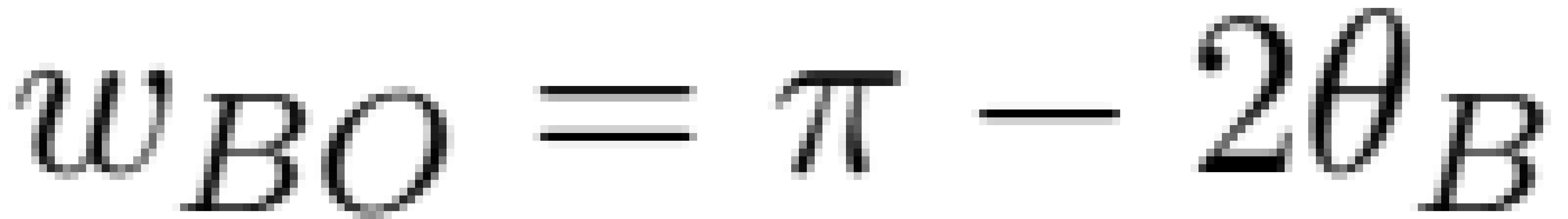


About  
to fail



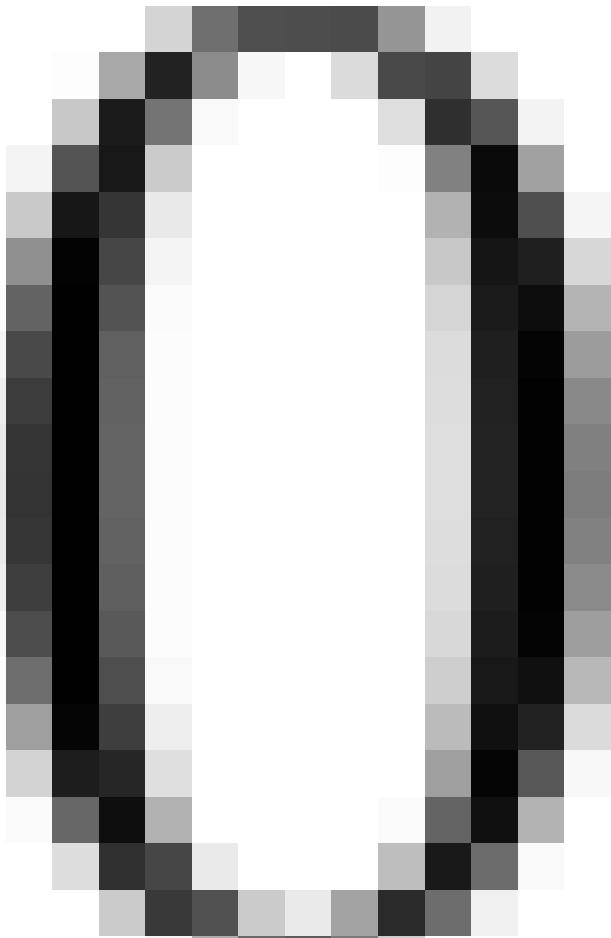
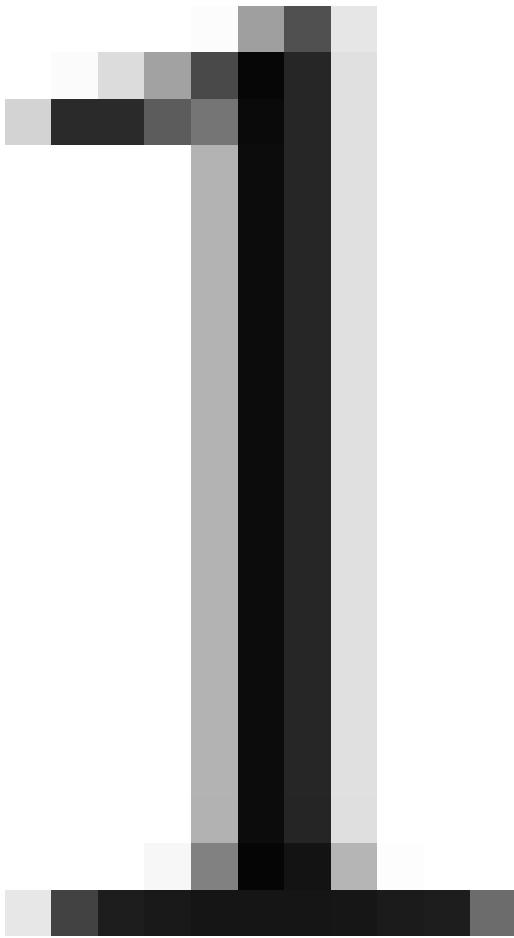
$$S_{\text{H}} = \frac{1}{2} \left[ \sigma_{W_{\text{max}}}^2 + \sigma_{H_{\text{max}}}^2 \right] \cos(2\theta) + \sigma_{H_{\text{max}}}^2 \sin(2\theta) \cos(\phi_{H_{\text{max}}}) + \sigma_{W_{\text{max}}}^2 \sin(2\theta) \sin(\phi_{H_{\text{max}}})$$

$$2\theta_B = \arccos \left[ \frac{\sigma_{Hmax} - \sigma_{hmin} - (1+q)(P_W - P_p)}{2(\sigma_{Hmax} - \sigma_{hmin})} \right]$$





$$P_{WBO} = P + \frac{(\sigma_{Hmax} - \sigma_{hmin}) \cos(\pi - w_{BO}) - Ucs}{1 + q}$$

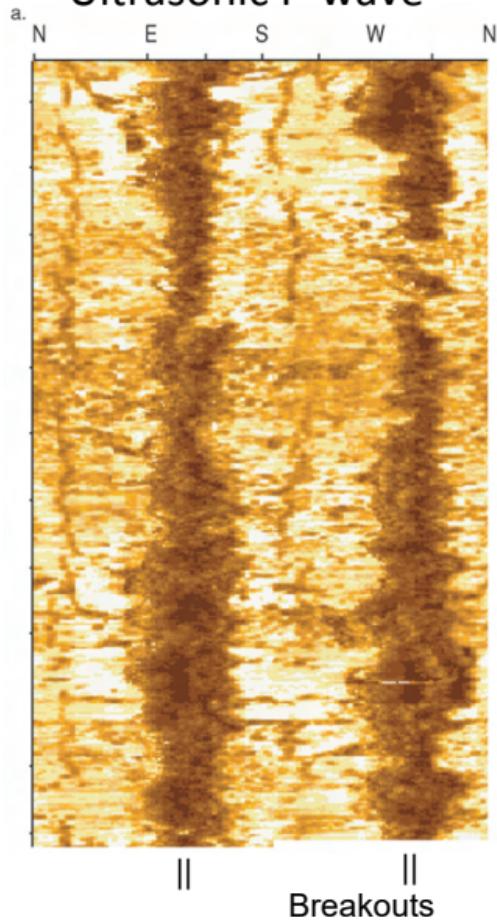


$$0.44 \text{ psi/ft} \times 8.3 \text{ PPG}$$

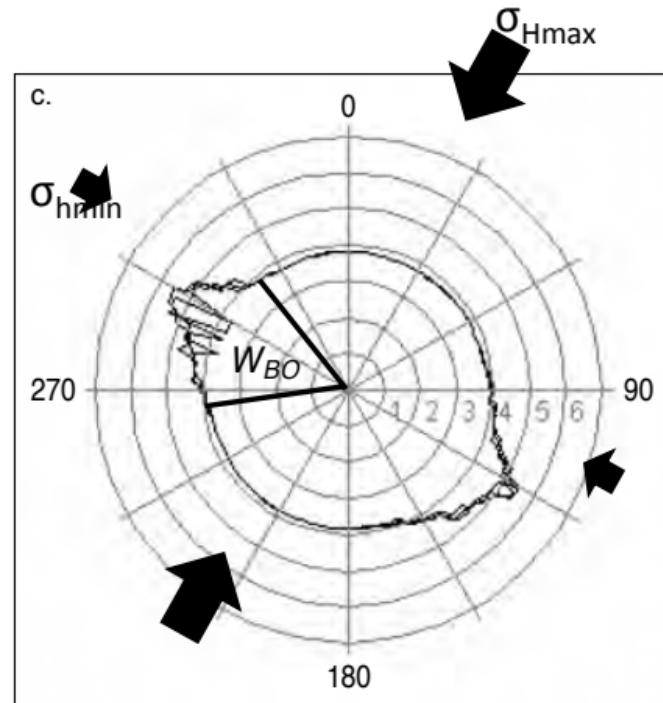
$$\times 7000 \text{ ft} = 3710 \text{ psi}$$

$$w_{BO}^{\circ} = 180^{\circ} - \arccos \frac{[3220 \text{ psi} + 1220 \text{ psi} - (1 + 3.12)(3710 \text{ psi} - 3080 \text{ psi})]}{2(3220 \text{ psi} - 1220 \text{ psi})} = 66^{\circ}$$

## Ultrasonic P-wave

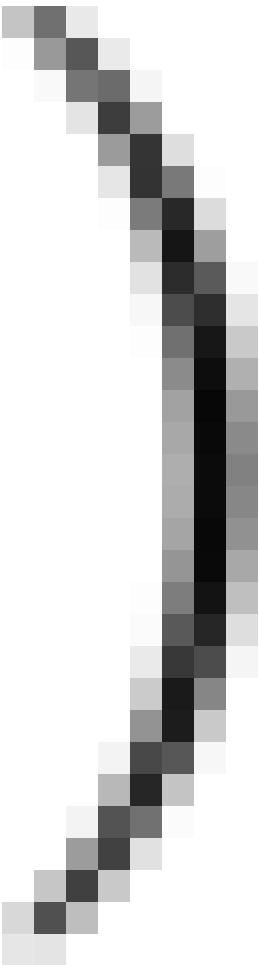
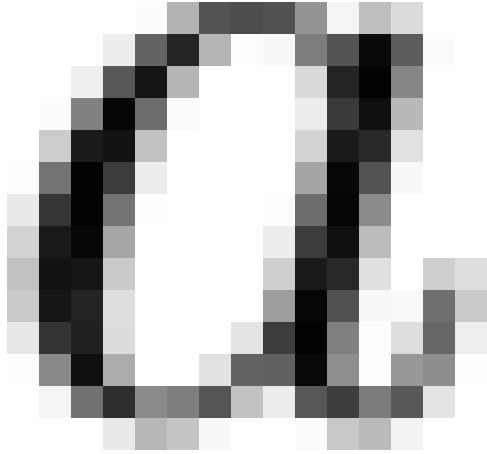
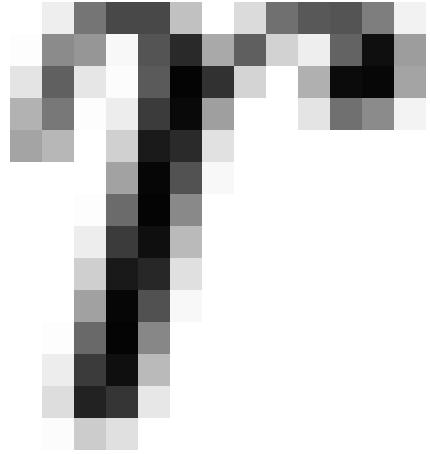
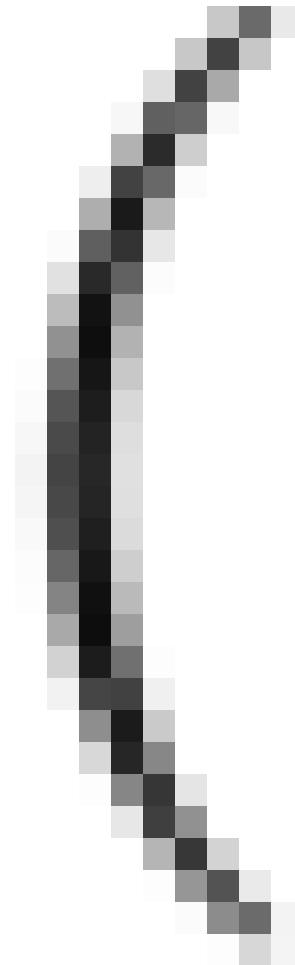


## Electrical resistivity



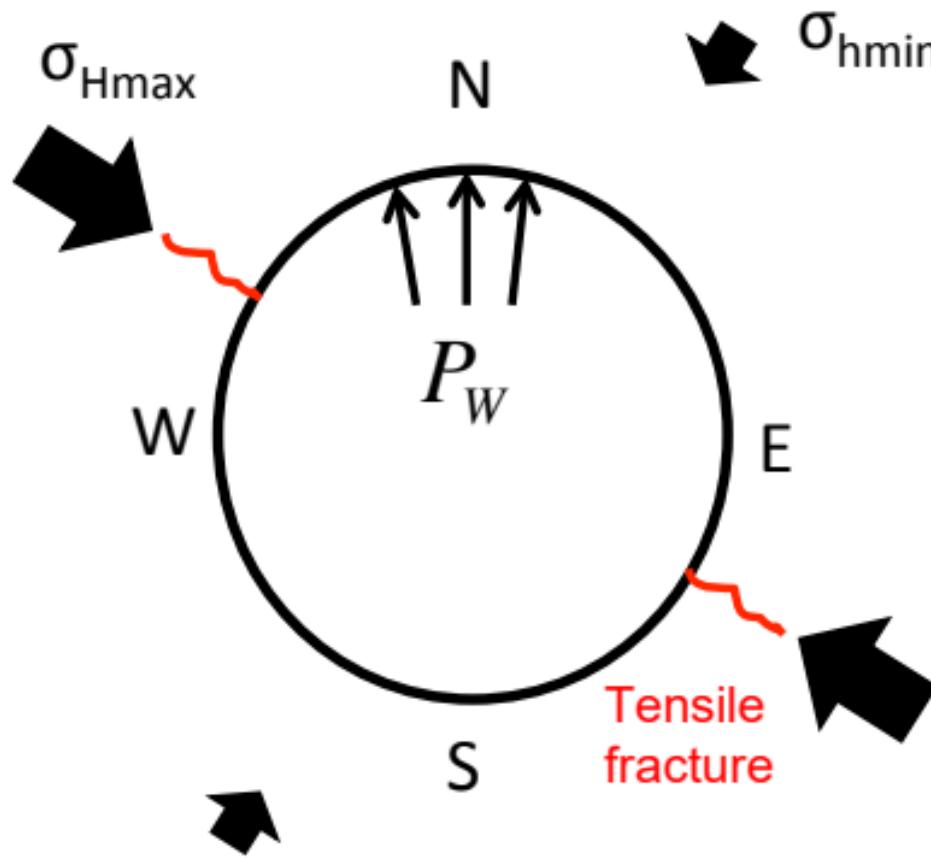
[Zoback 2013 - Figure 6.4]

$$S_{H\max} = \frac{P}{P} + \frac{UCS + (1+q)(P_W - P_p) - \sigma_{hmin}[1 + 2\cos(\pi - w_{BO})]}{1 - 2\cos(\pi - w_{BO})}$$



σθαντικός + σούχος + στ



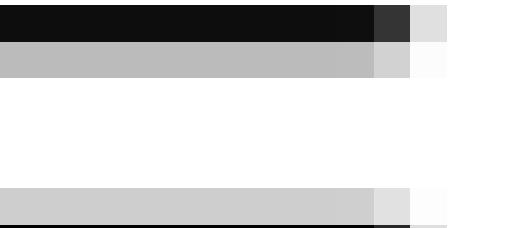
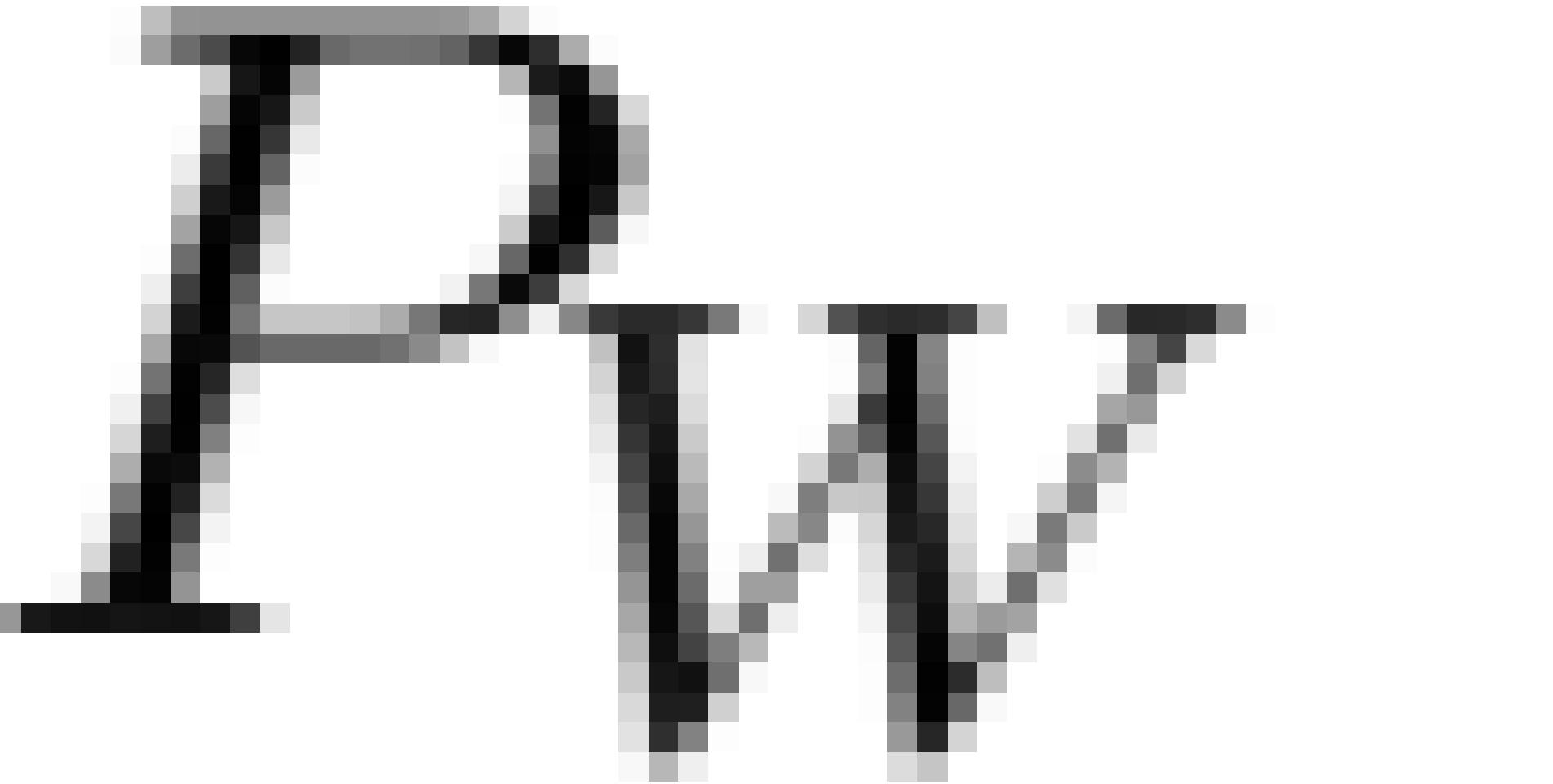


$$P_b = P_p + 3\sigma_{h\min} - \sigma_{H\max} + T_s + \sigma^{\Delta T}$$

Pore pressure  
in the formation

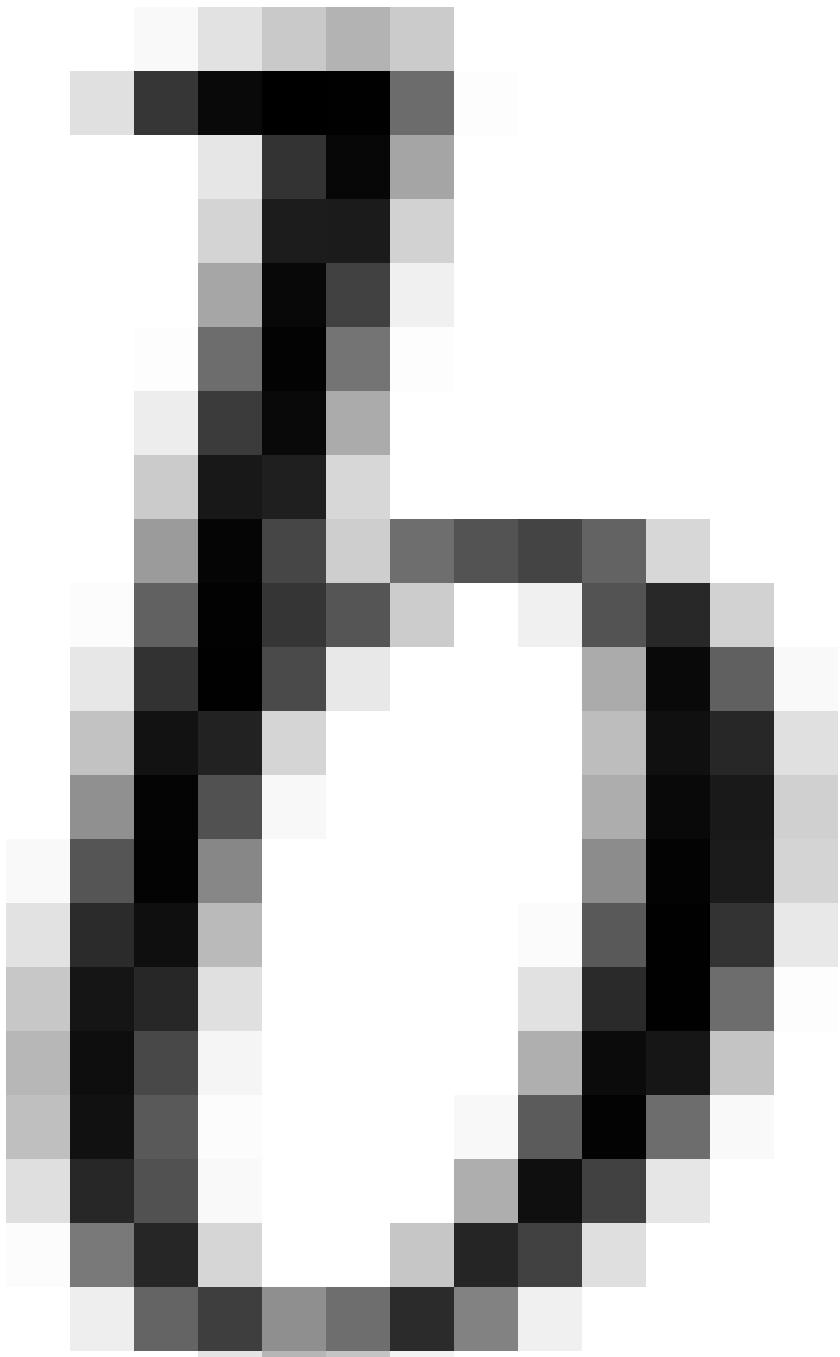
Stress anisotropy

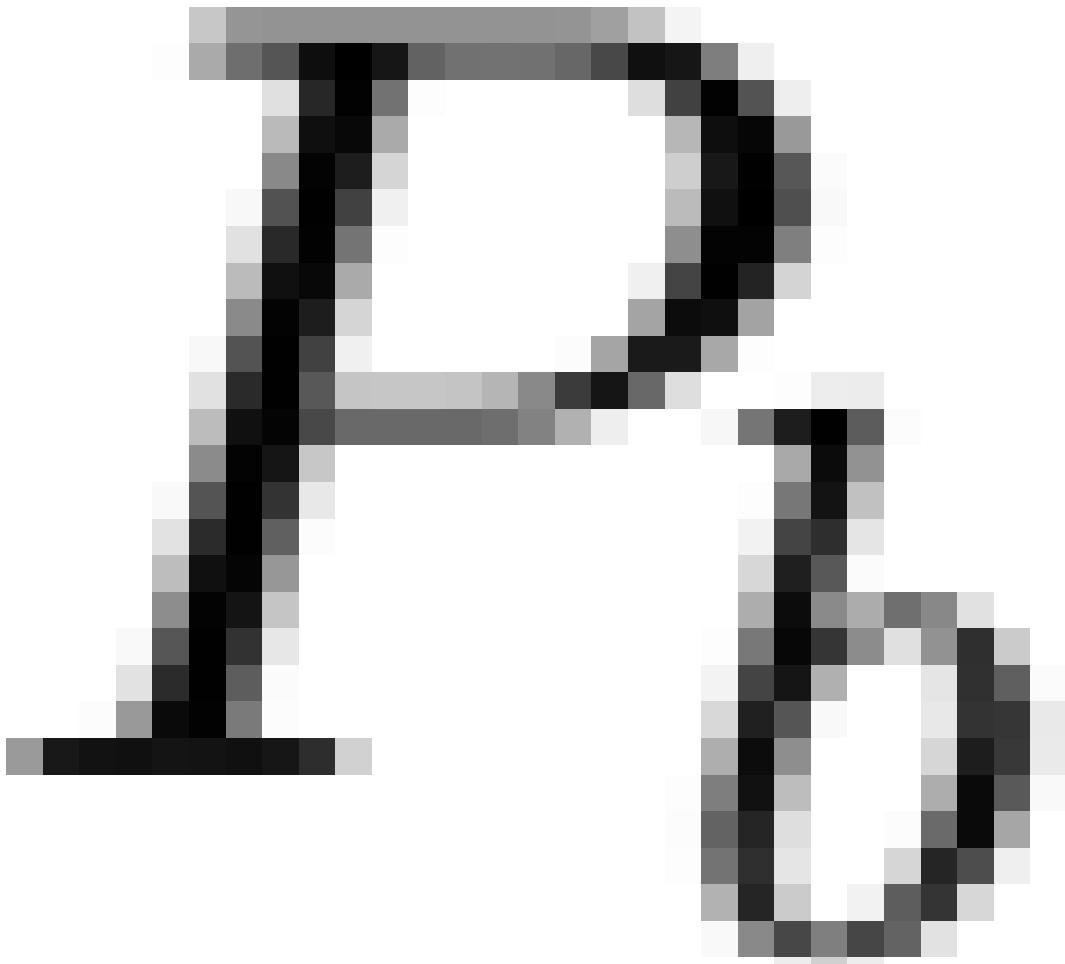
Tensile strength  
Cooling stress

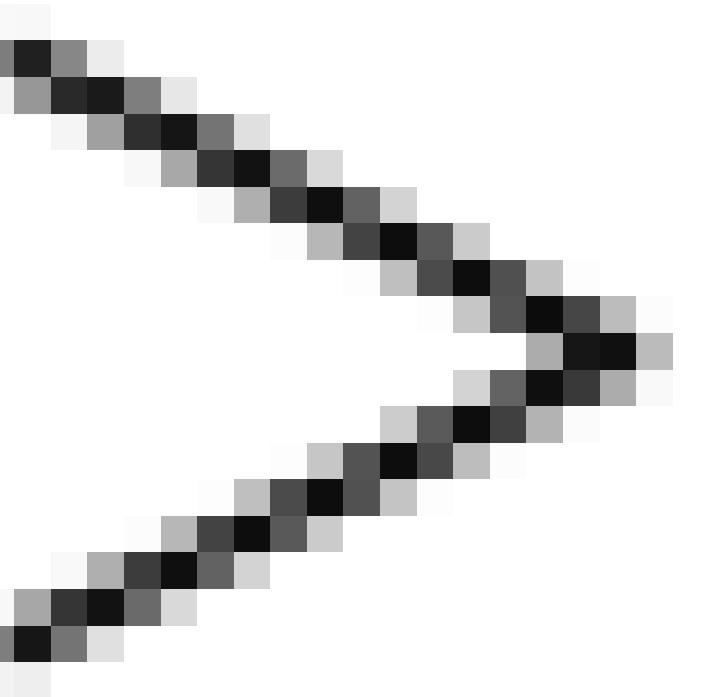
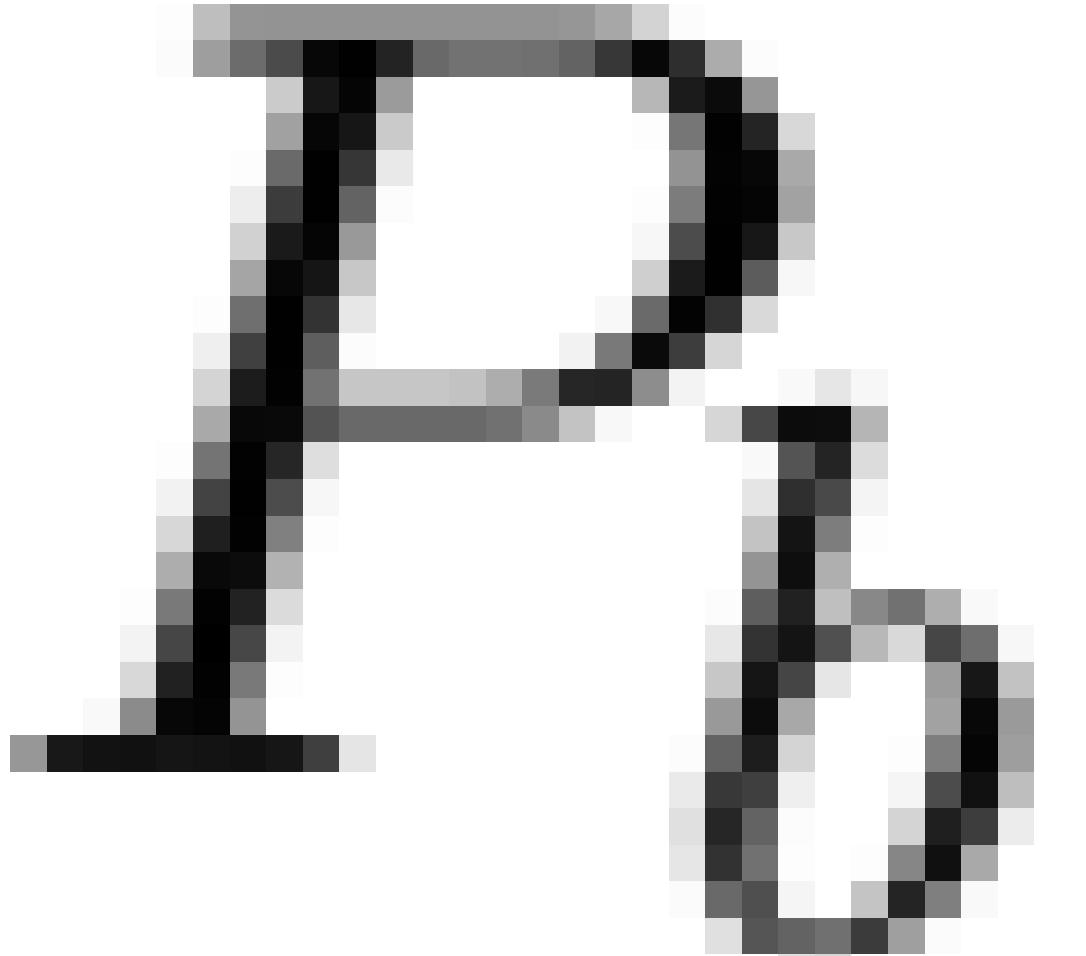


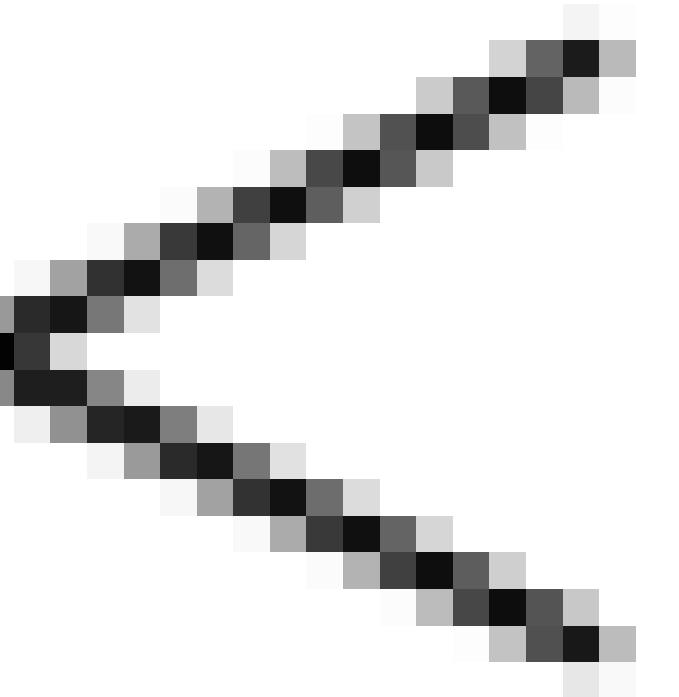
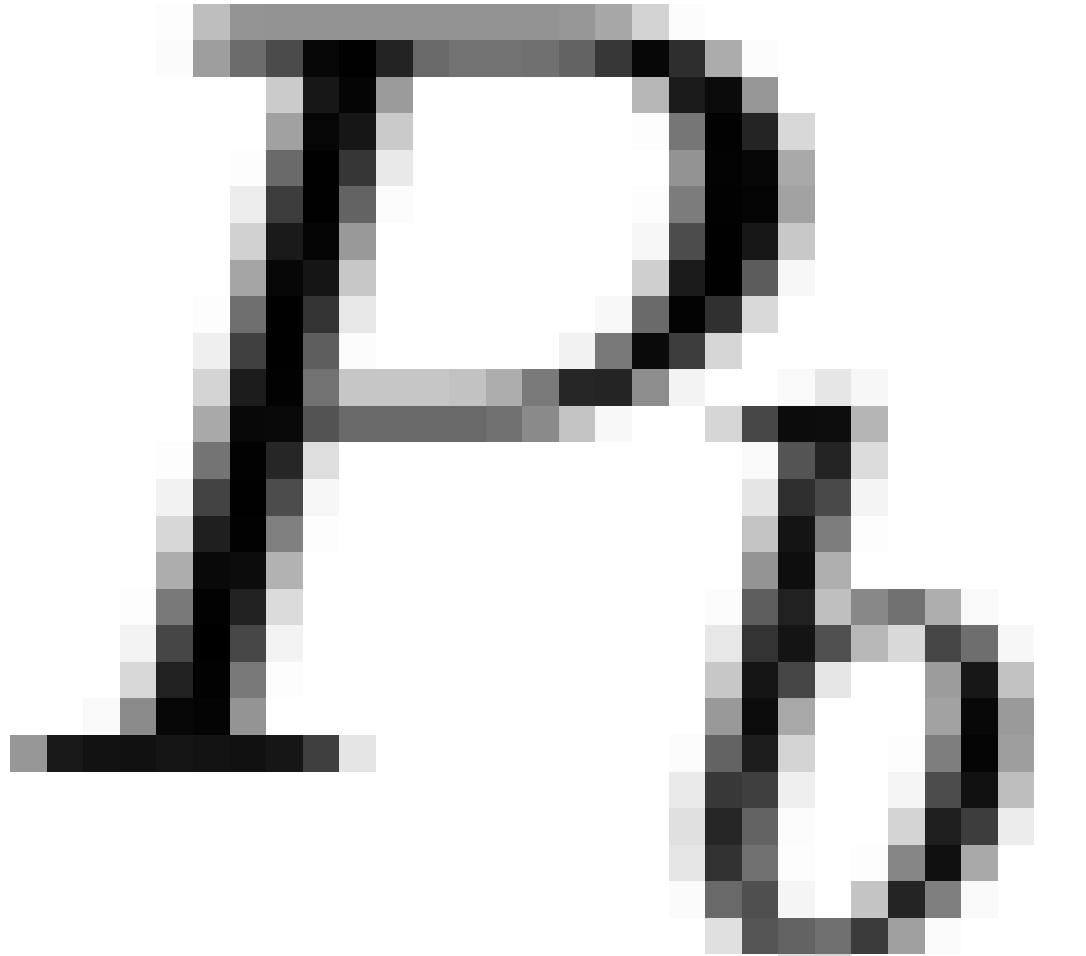
$\sigma_{\text{Hannac}} + \sigma_{\text{Hannac}}^2$

Bob Smith + 30 hours

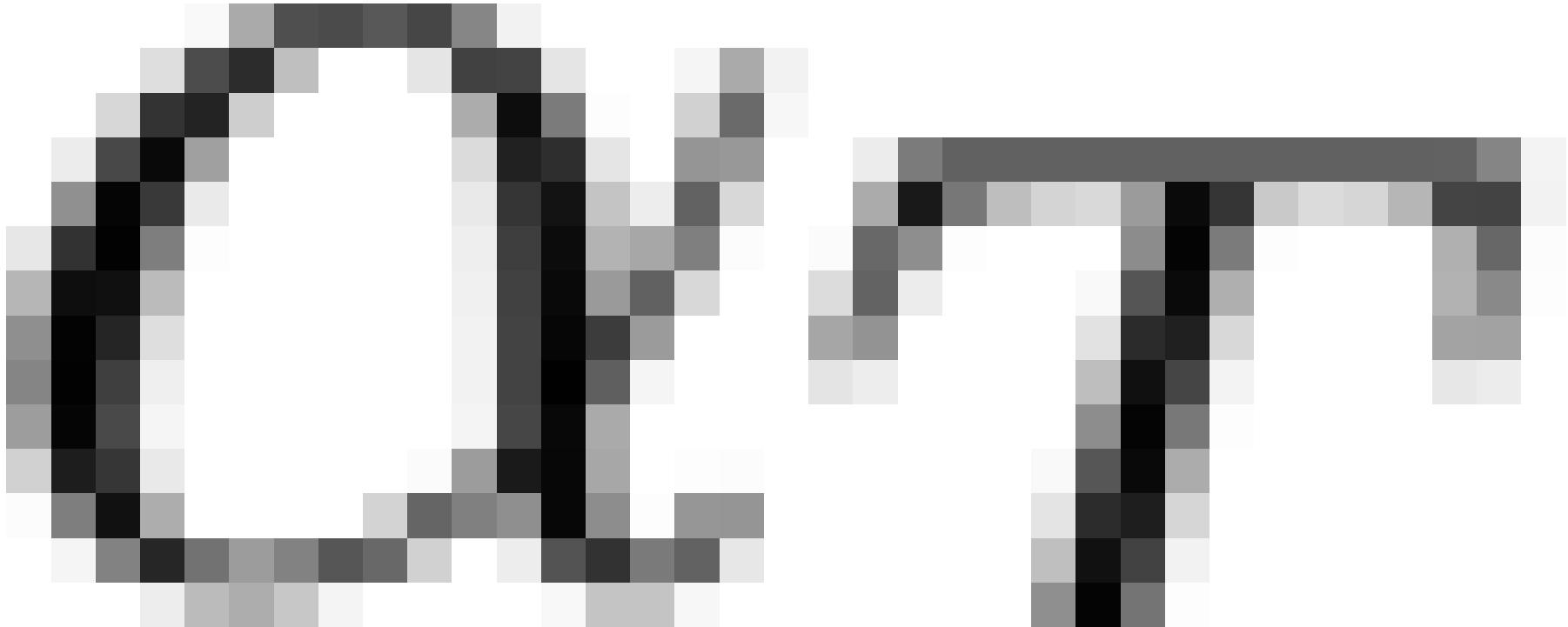


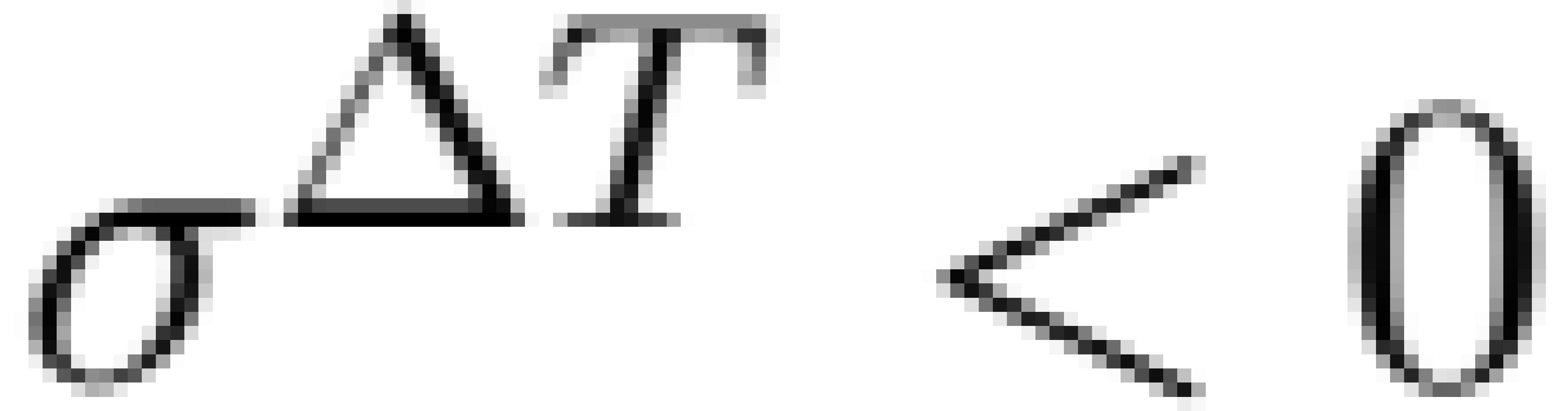


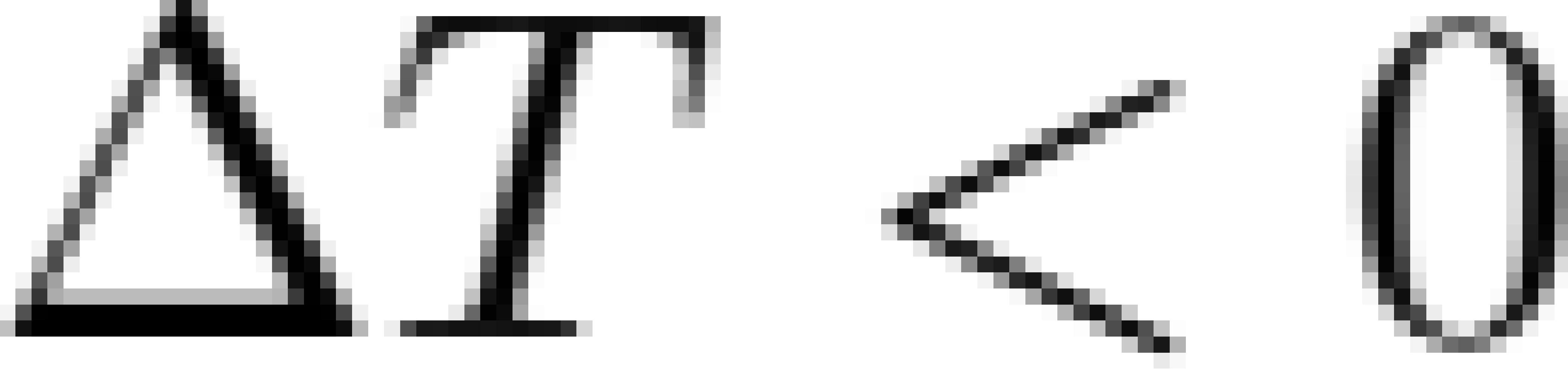


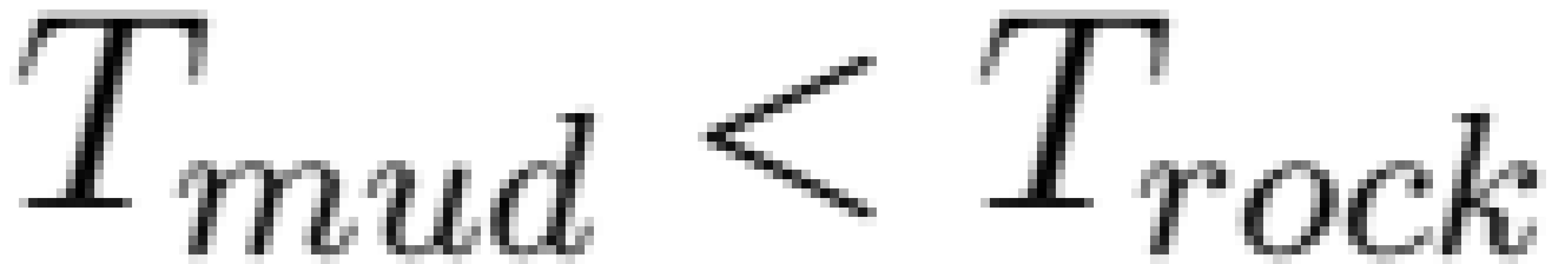


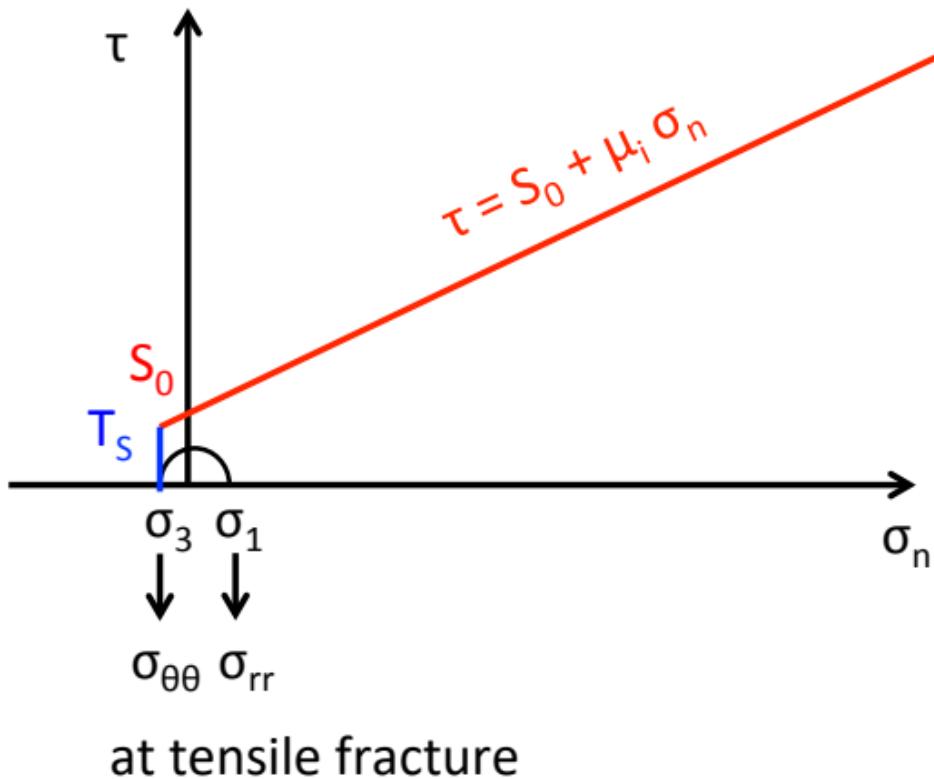
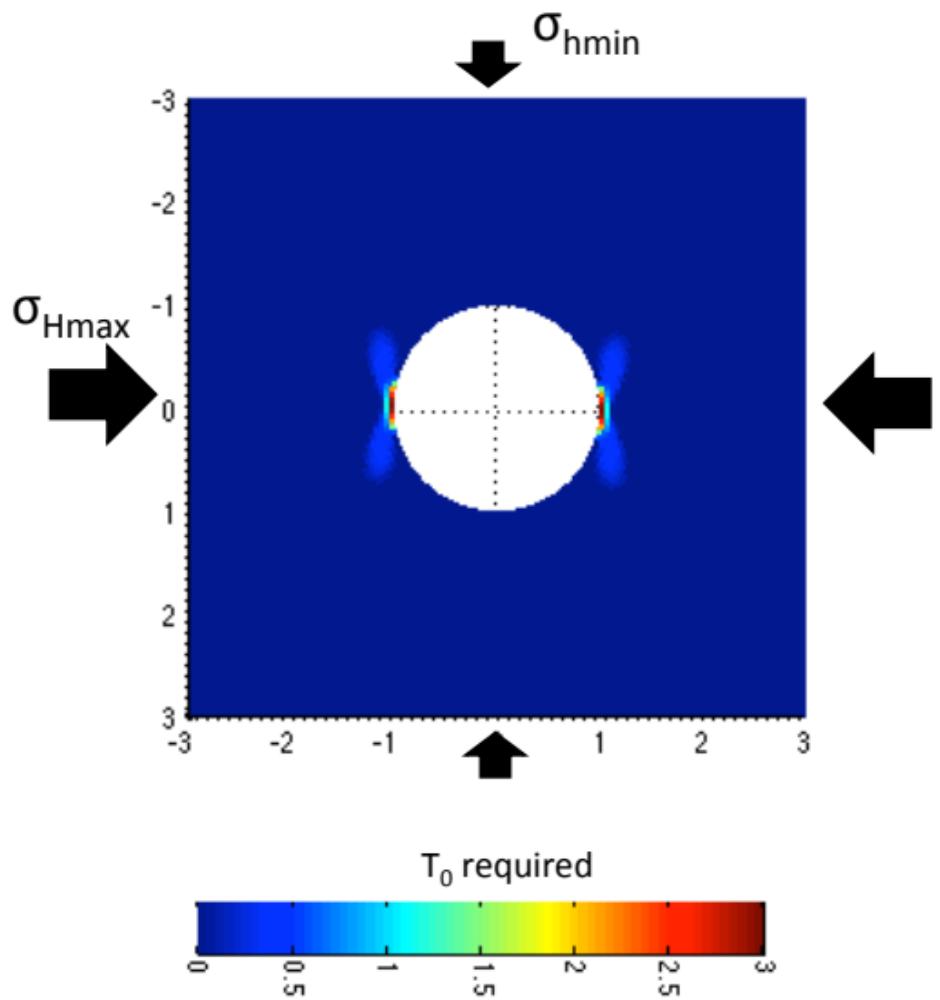


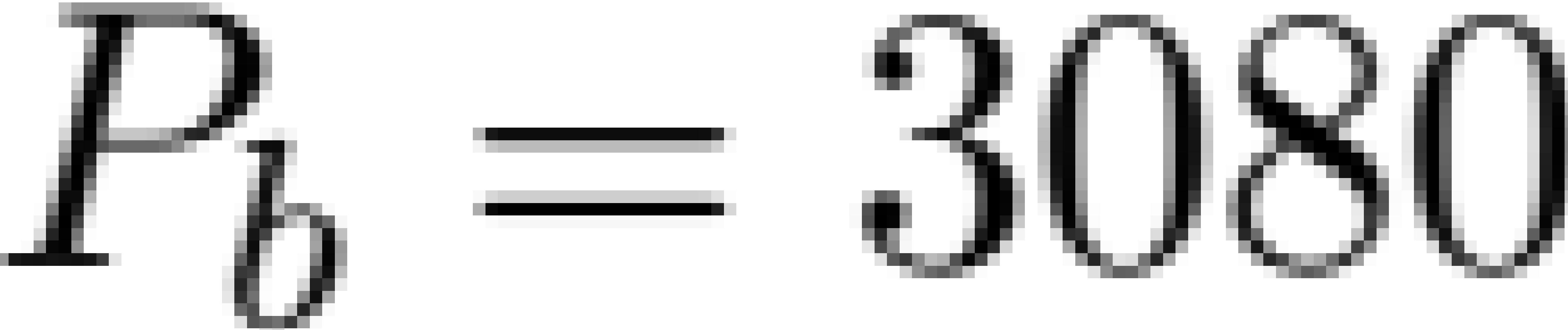


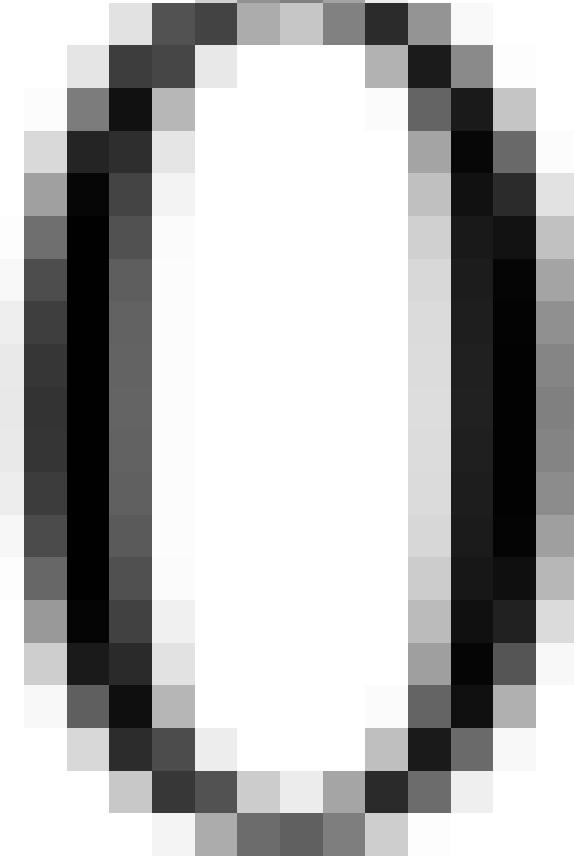
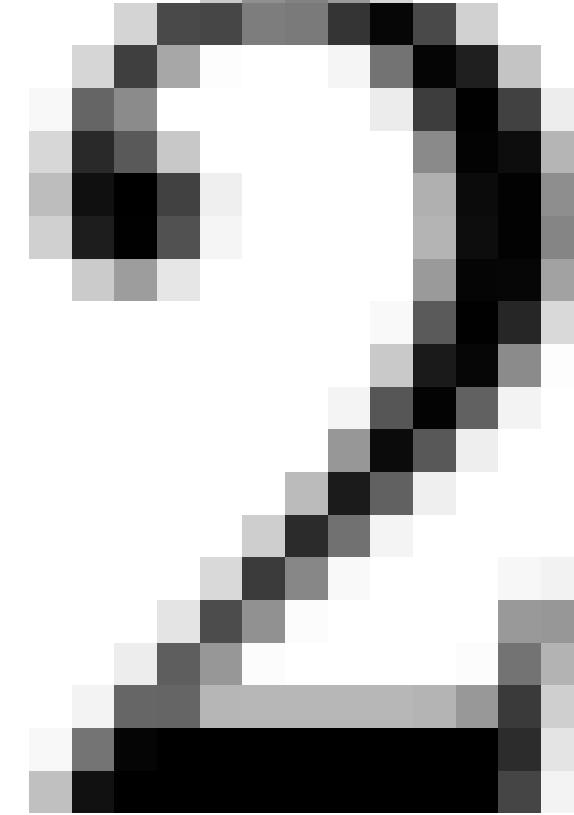
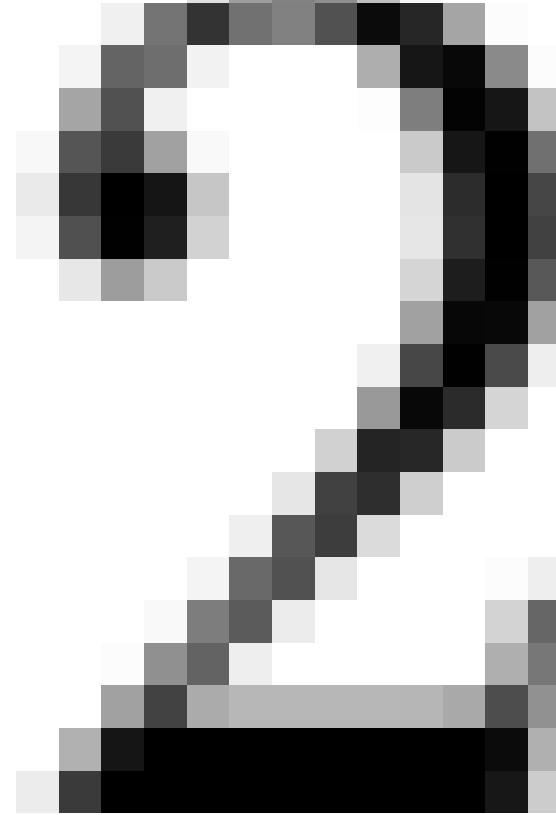
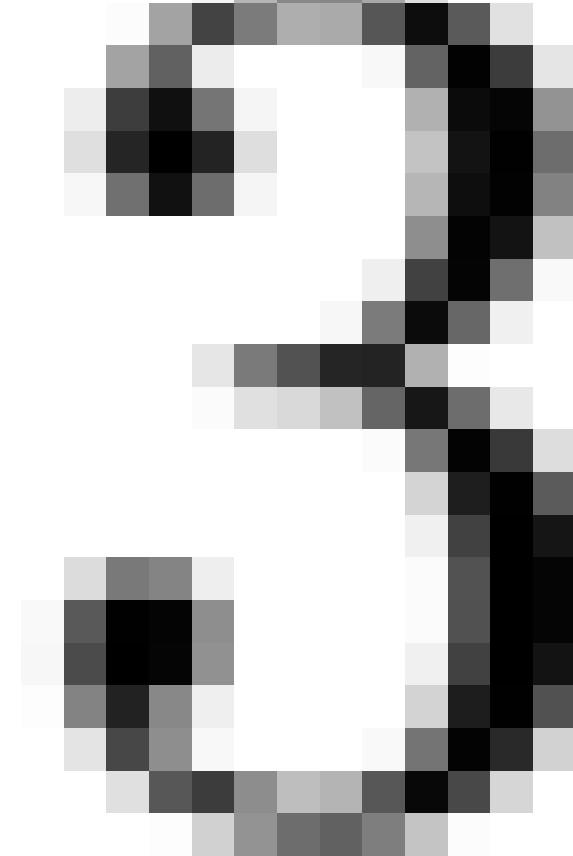


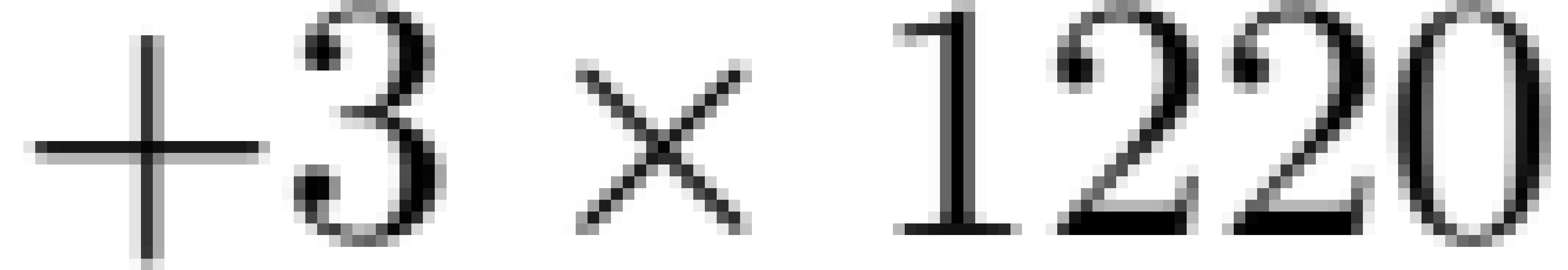


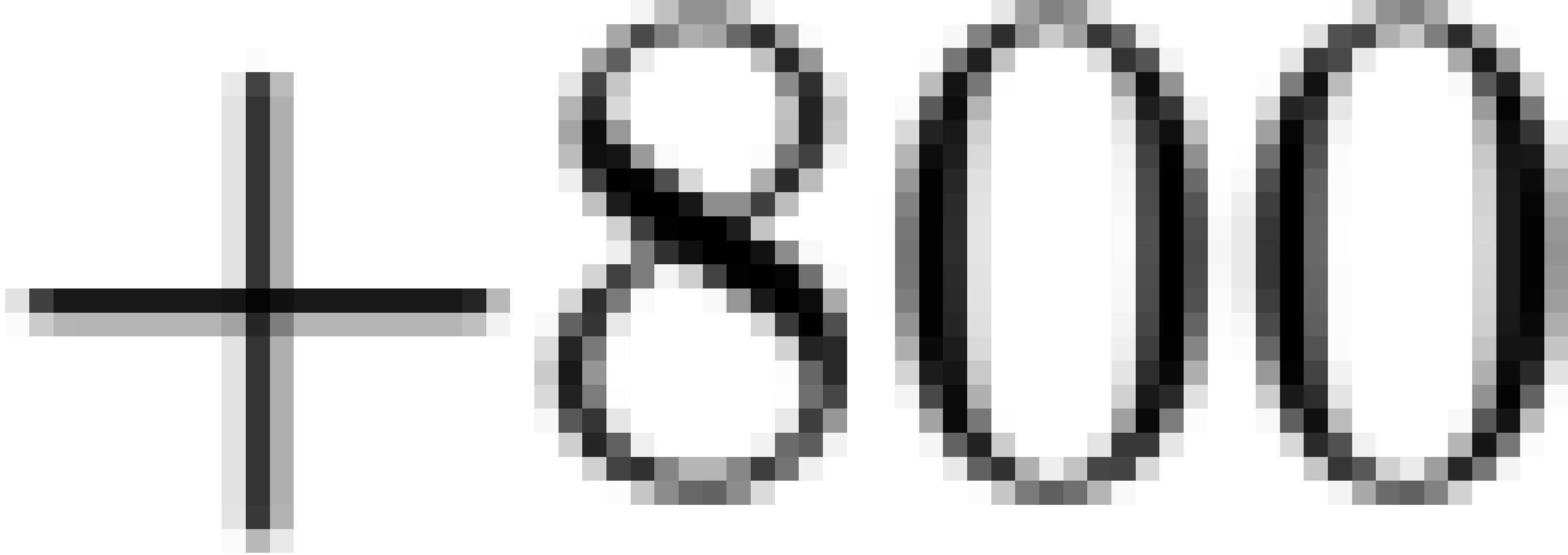


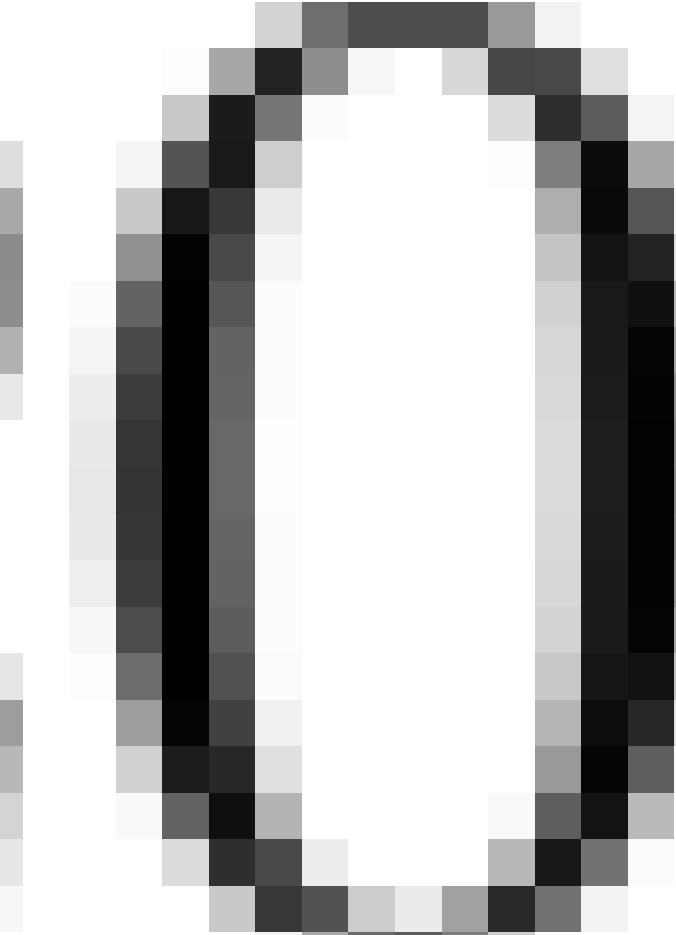
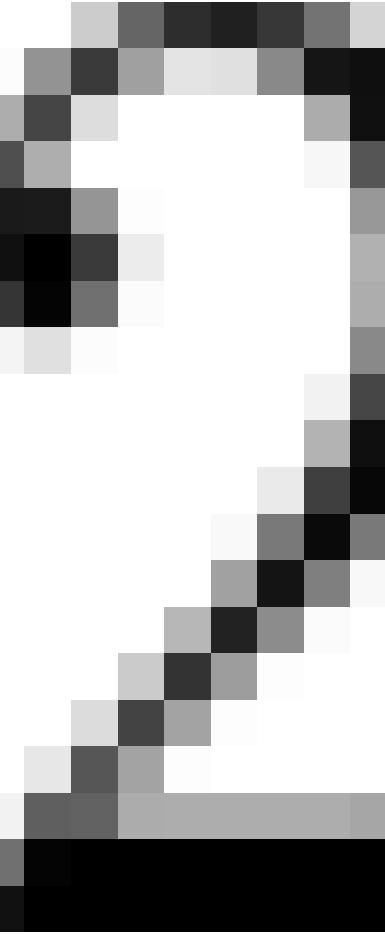
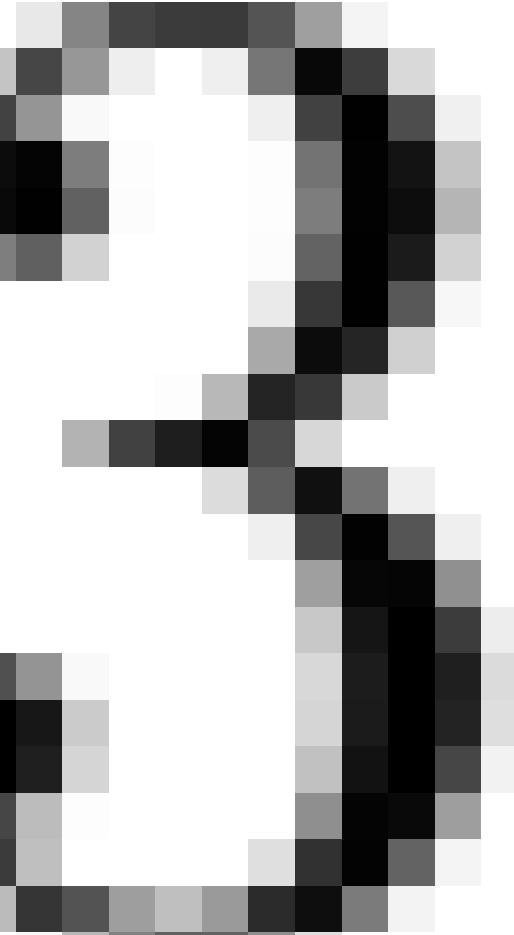
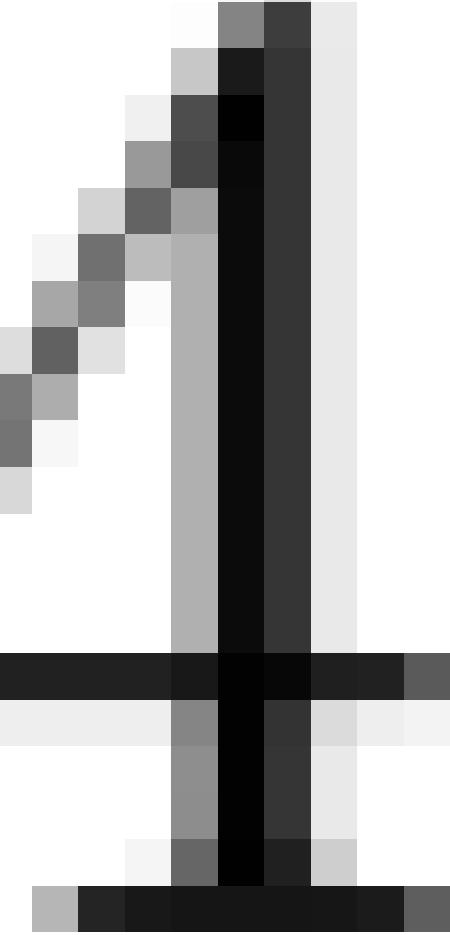












4320 psi

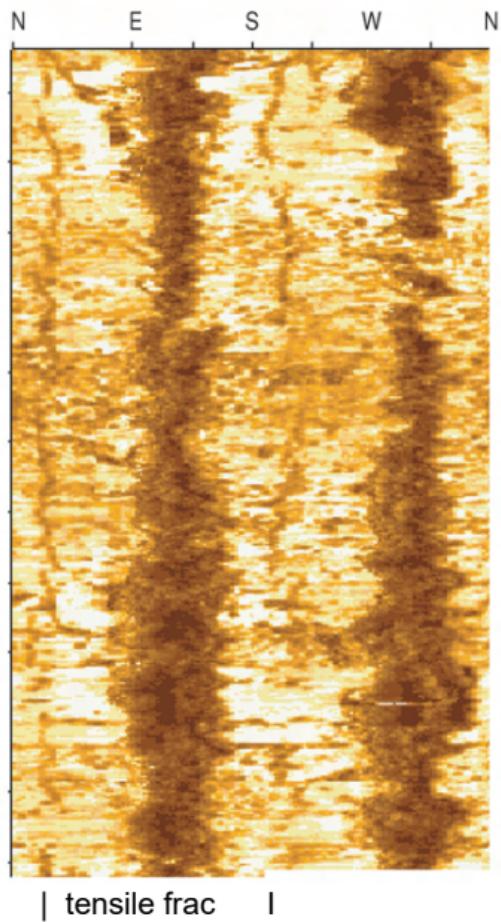
7000 ft

8.33 ppg

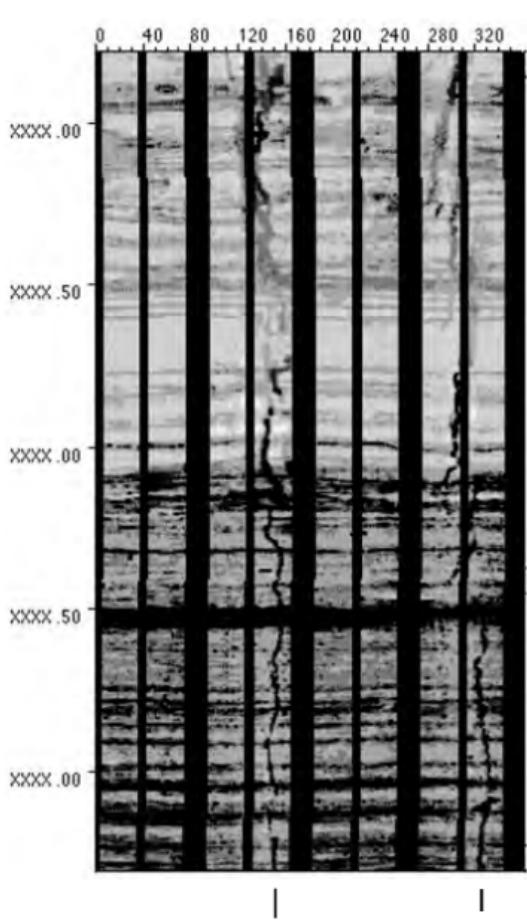
0.44 psi/ft

11.68 ppg

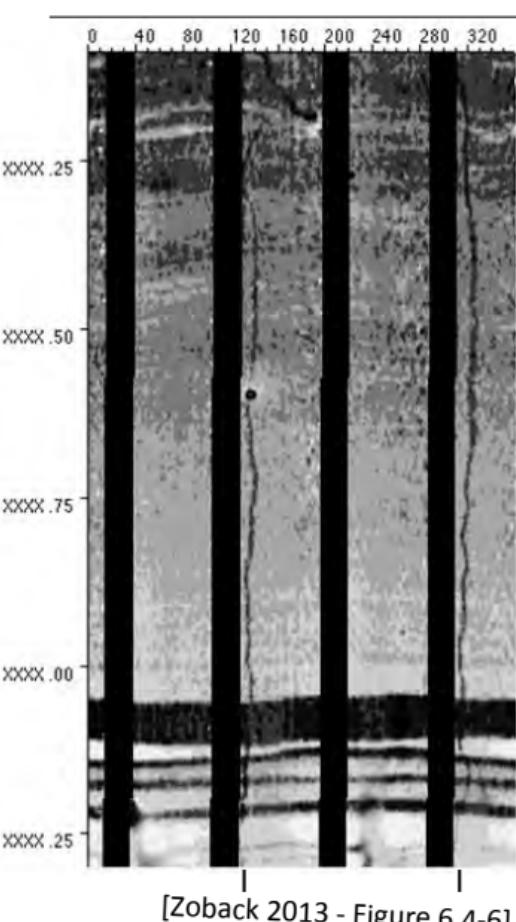
### Ultrasonic P-wave

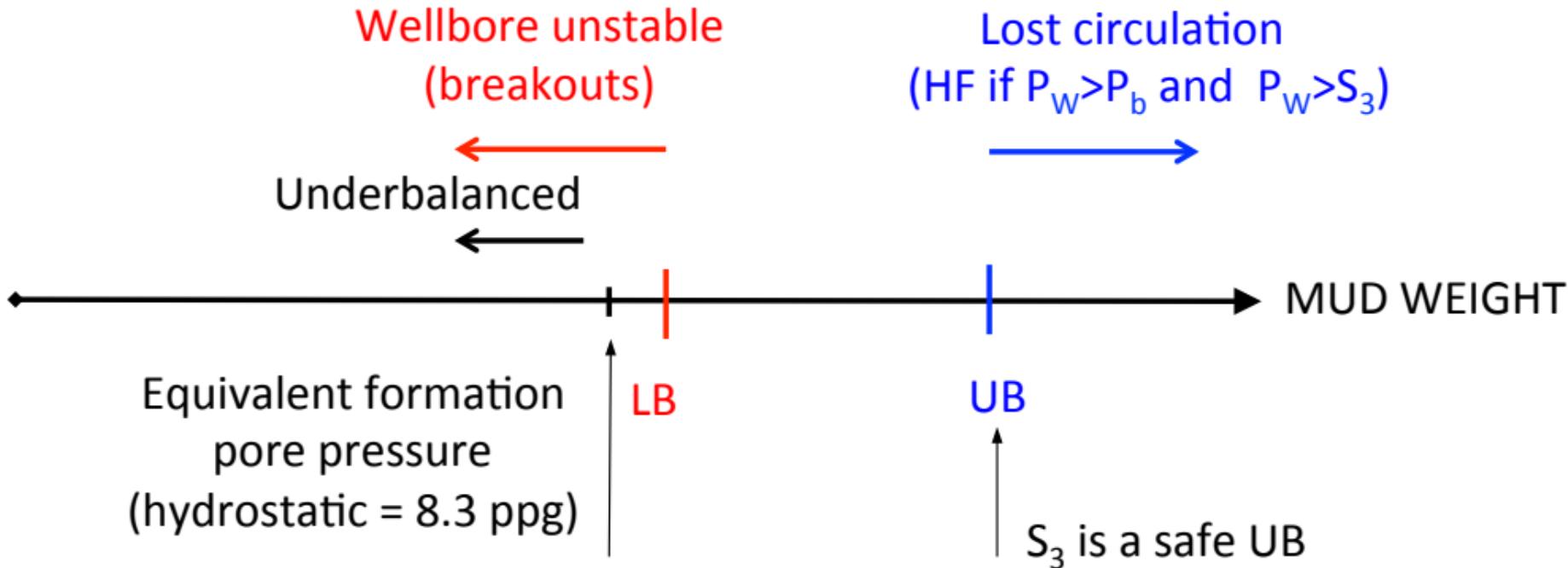


### Electrical resistivity



### Electrical resistivity



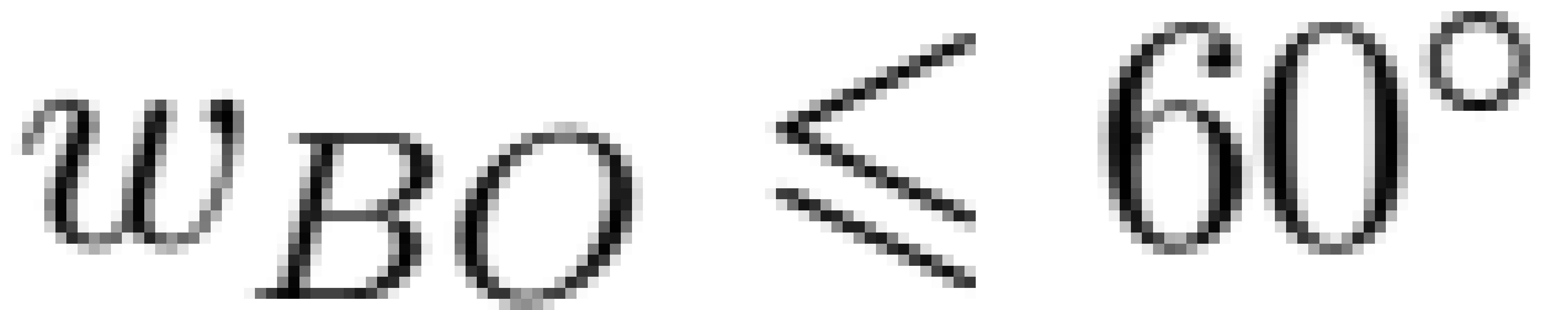


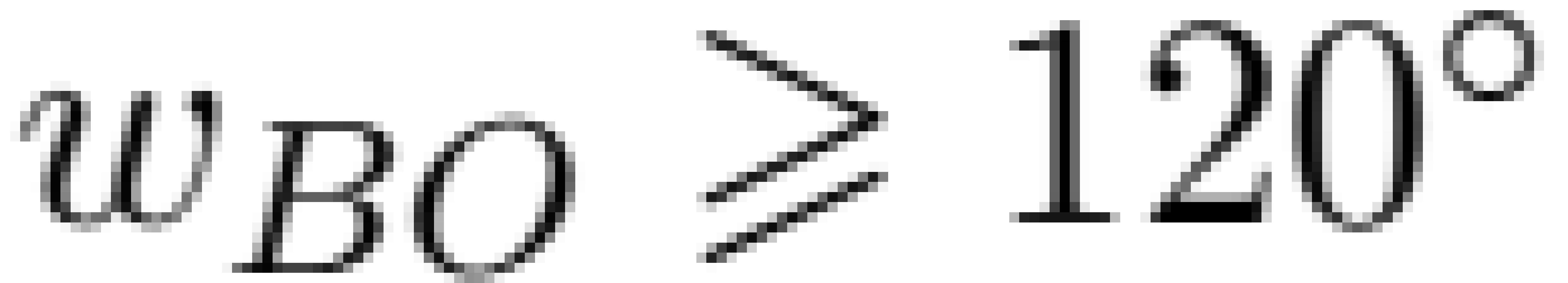
### LIGHT MUDS

- May promote water production
- Compromise wellbore stability

### HEAVY MUDS

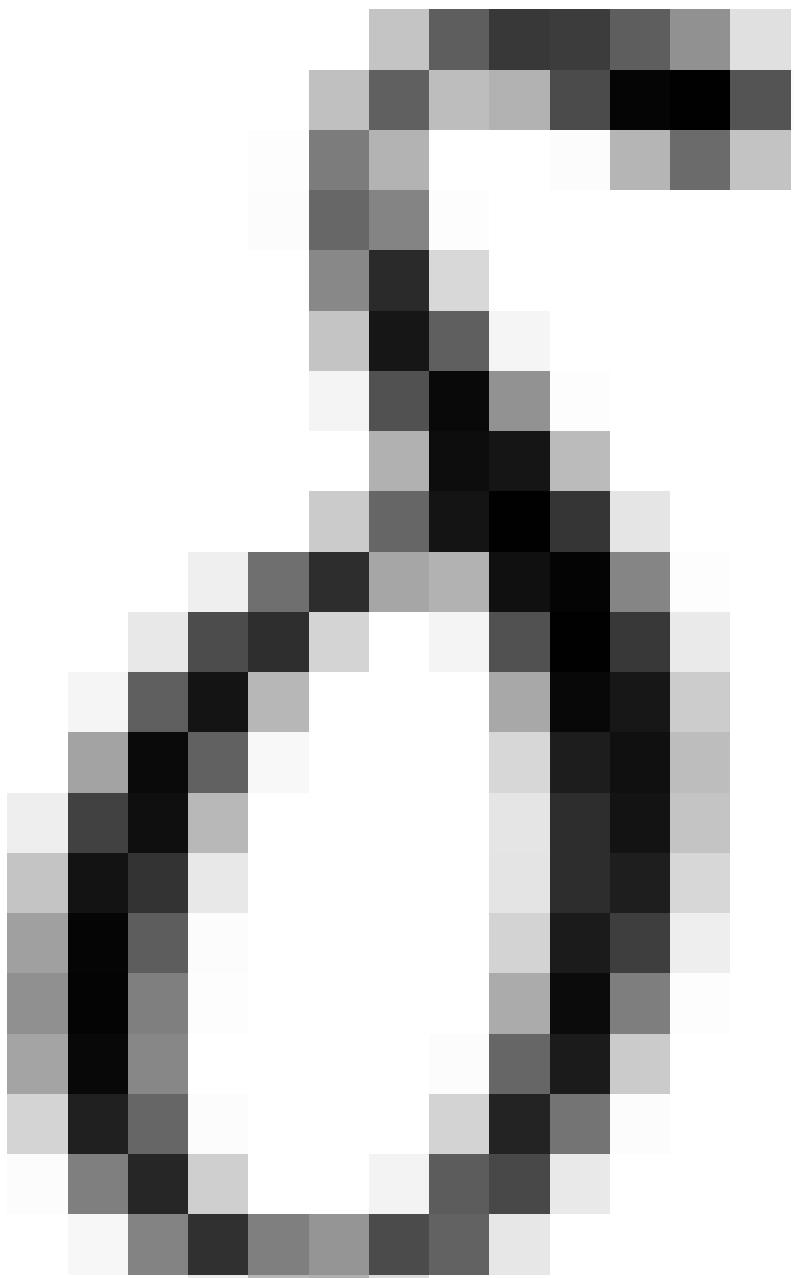
- Damage formation ( $k$ ) by mud infiltration
- Promote mud losses in permeable strata
- Low ROP because of stronger rock



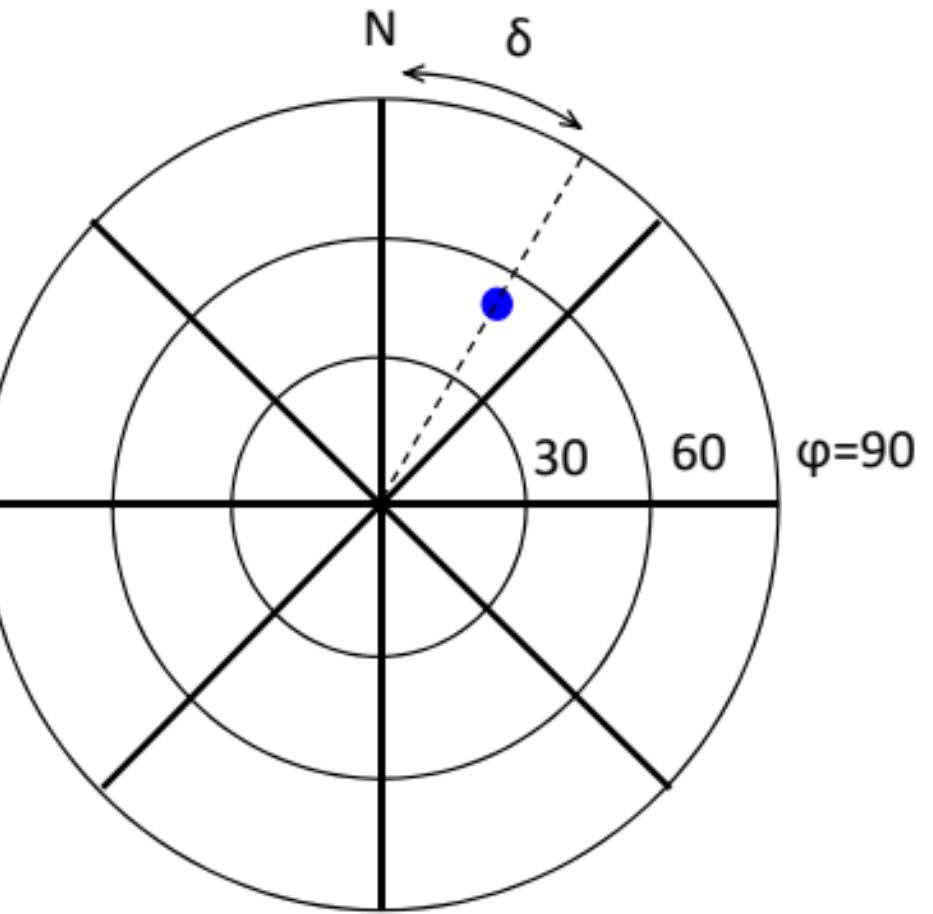
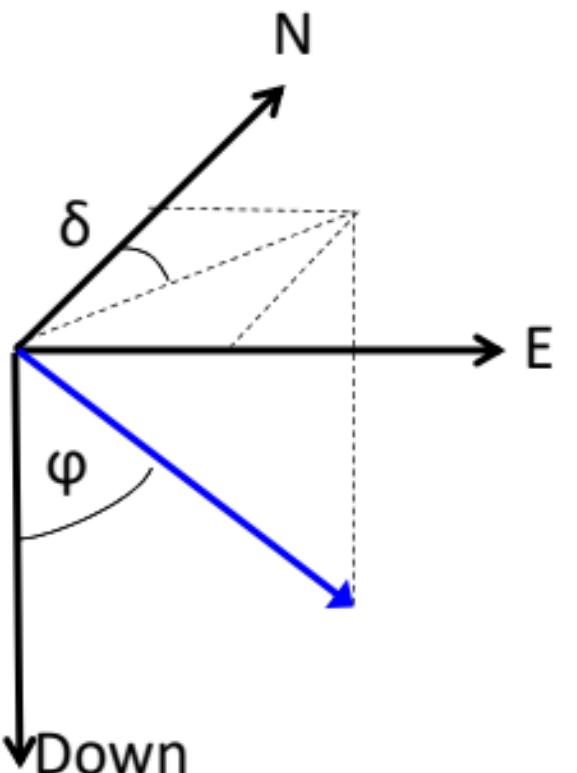




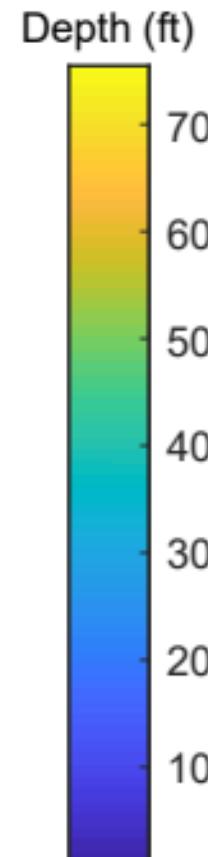
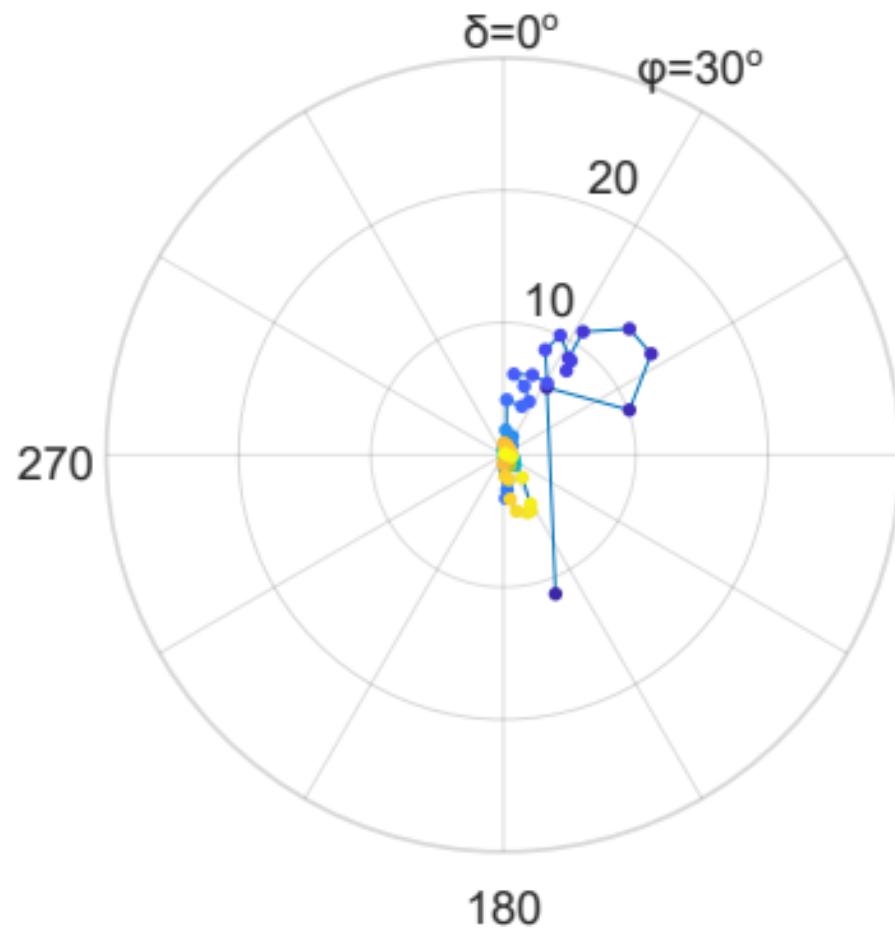


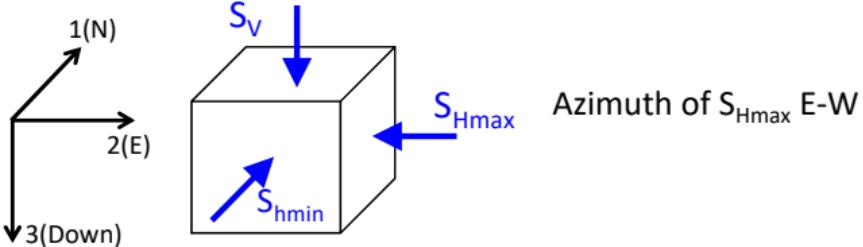


(a) Definitions

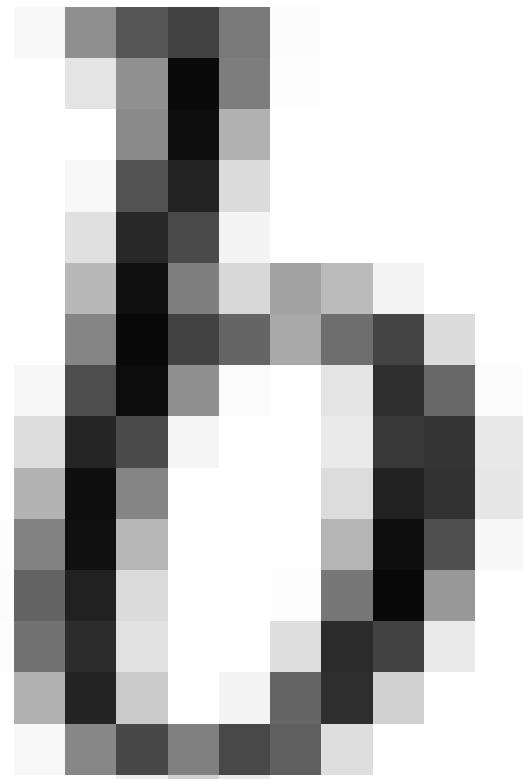
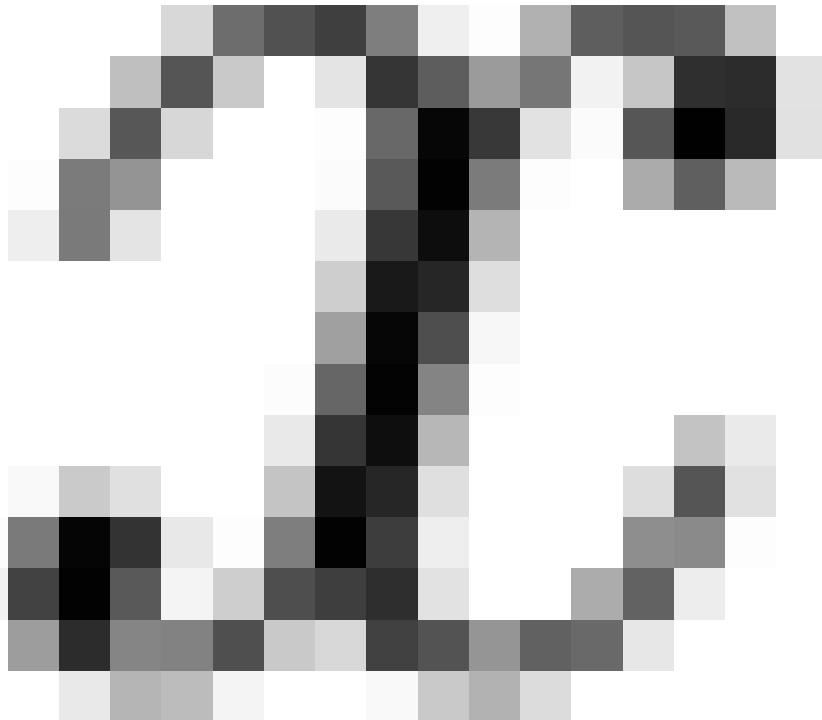


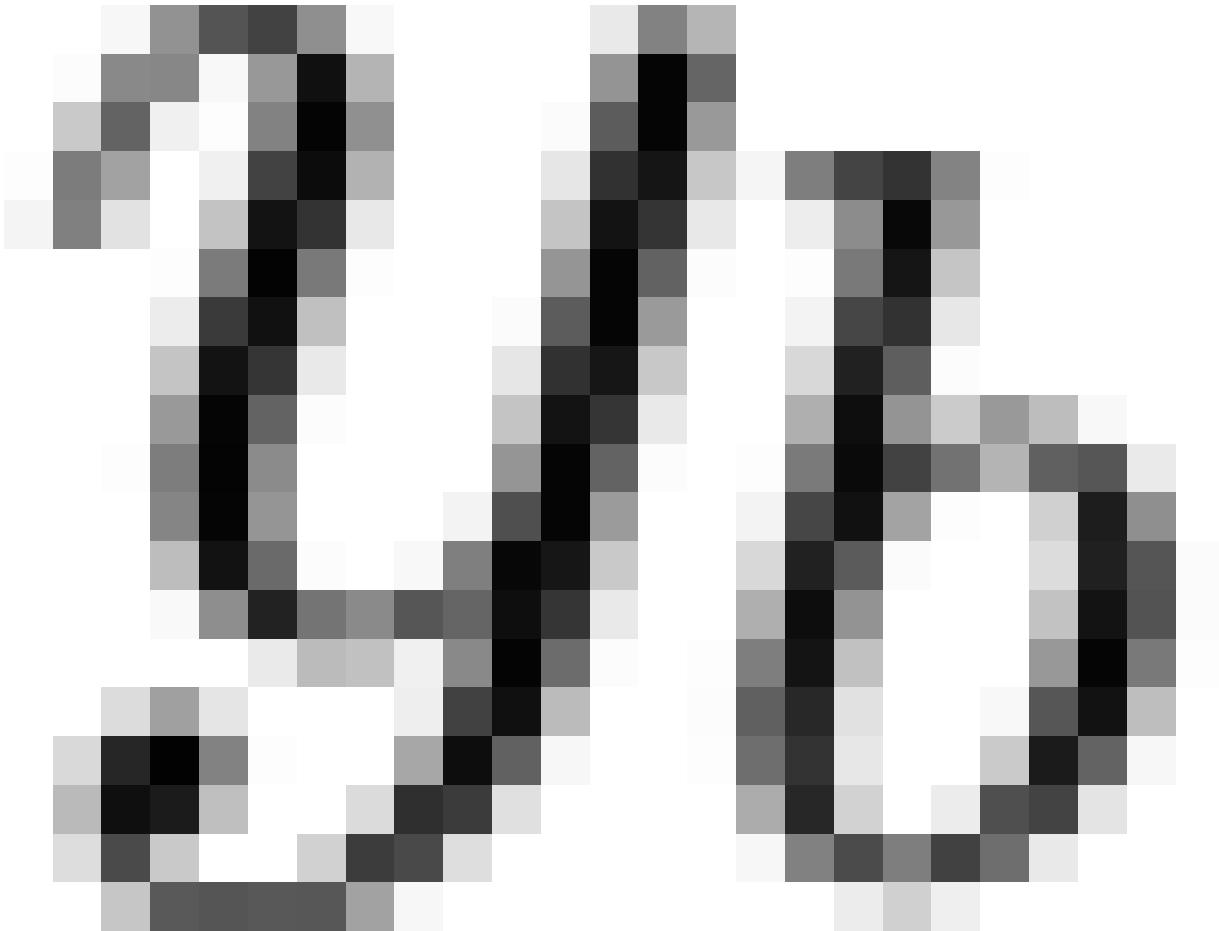
(b) Example

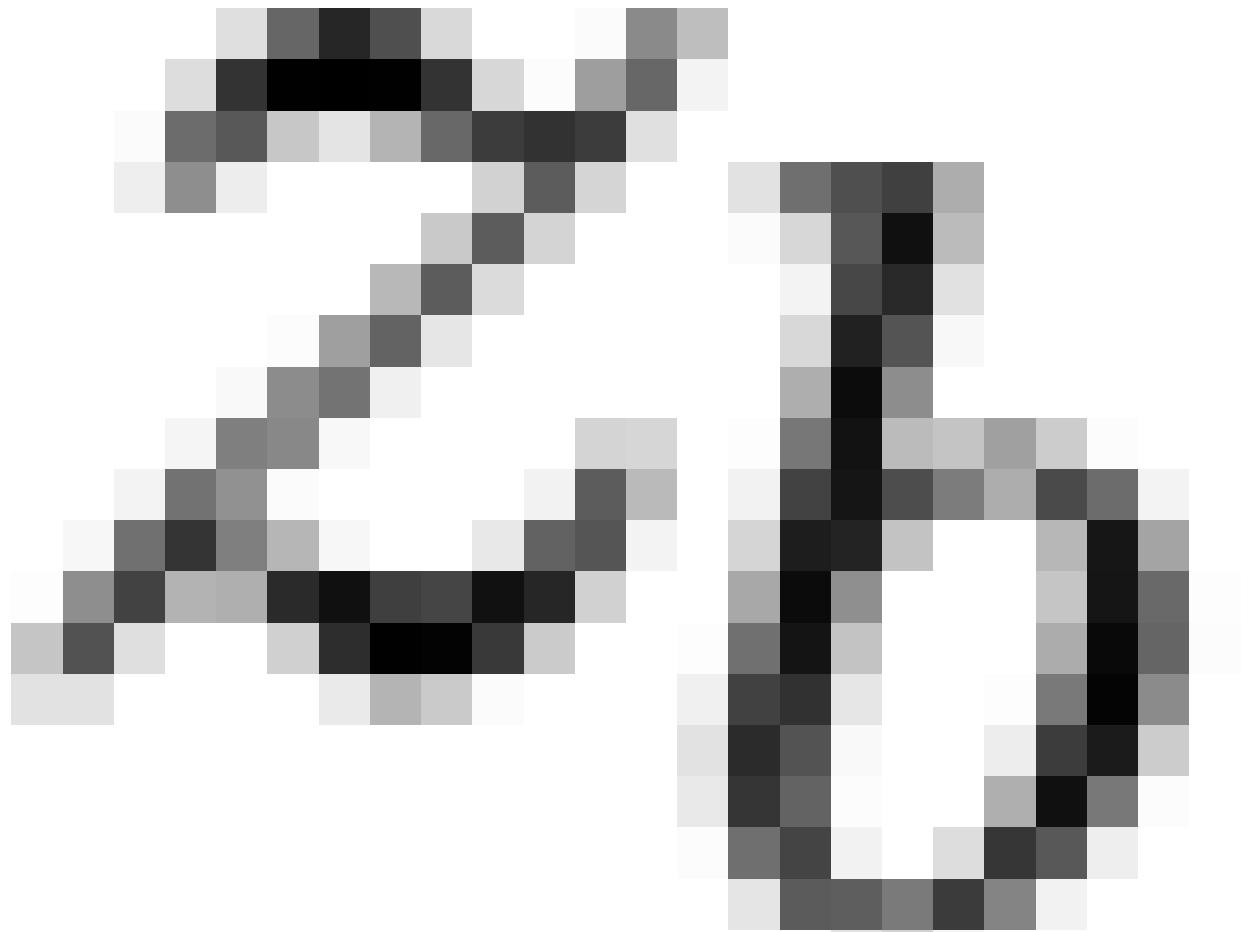


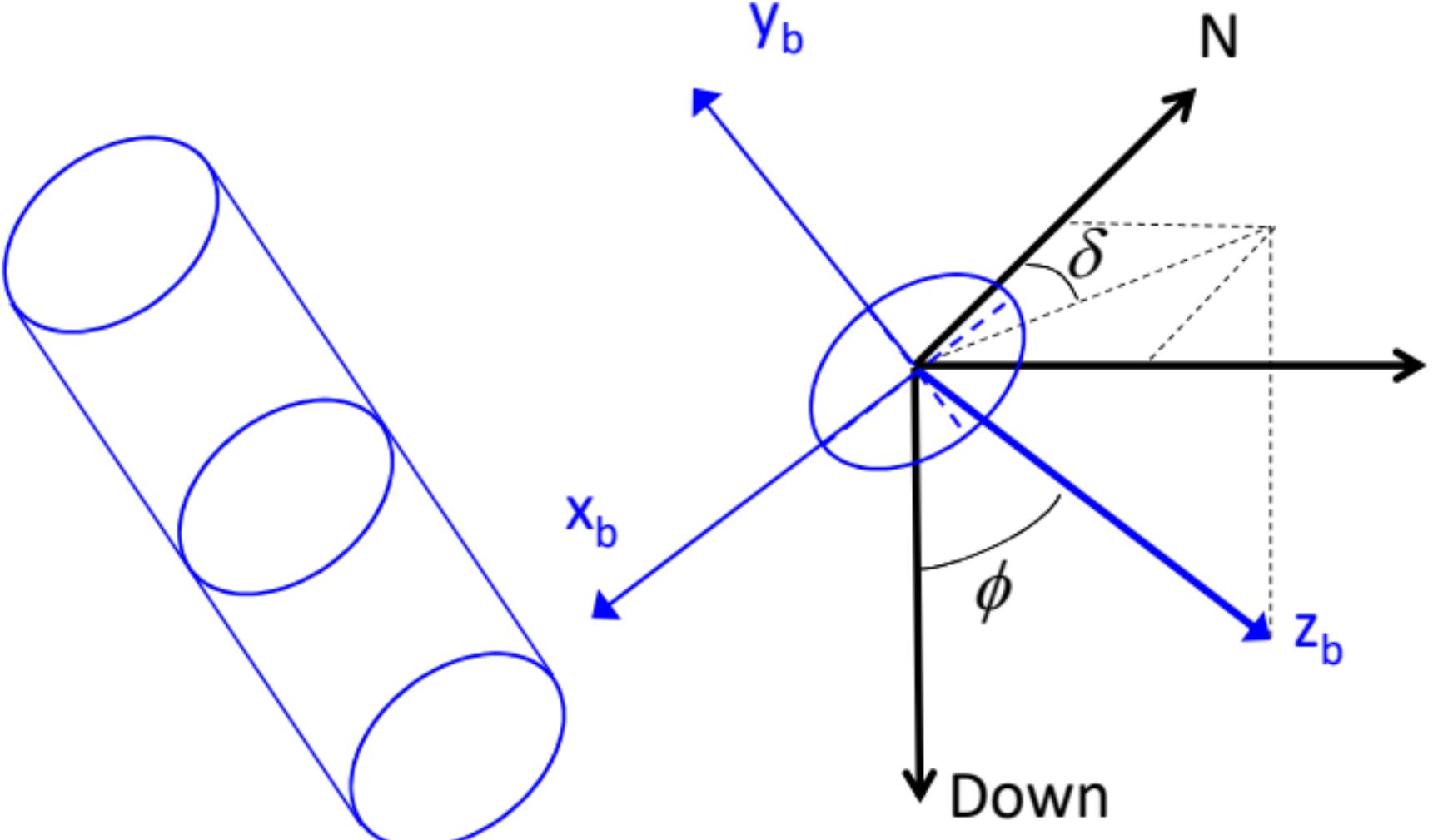


Stress Environment	NF	SS	RF
Plane with highest stress anisotropy and location of breakouts and tensile fracs	<p>3D stress element for Normal Faulting (NF) showing the vertical stress <math>S_V</math> acting downwards, the minimum horizontal stress <math>S_{h\text{min}}</math> acting to the right, and the maximum horizontal stress <math>S_{H\text{max}}</math> acting to the left. A red dot marks the top face.</p>	<p>3D stress element for Strike-Slip (SS) showing the vertical stress <math>S_V</math> acting downwards, the minimum horizontal stress <math>S_{h\text{min}}</math> acting to the right, and the maximum horizontal stress <math>S_{H\text{max}}</math> acting to the left. A red dot marks the top face.</p>	<p>3D stress element for Reverse Fault (RF) showing the vertical stress <math>S_V</math> acting downwards, the minimum horizontal stress <math>S_{h\text{min}}</math> acting to the right, and the maximum horizontal stress <math>S_{H\text{max}}</math> acting to the left. A red dot marks the top face.</p>
Narrower drilling window in a stereonet projection	<p>N</p>	<p>N</p>	<p>N</p>

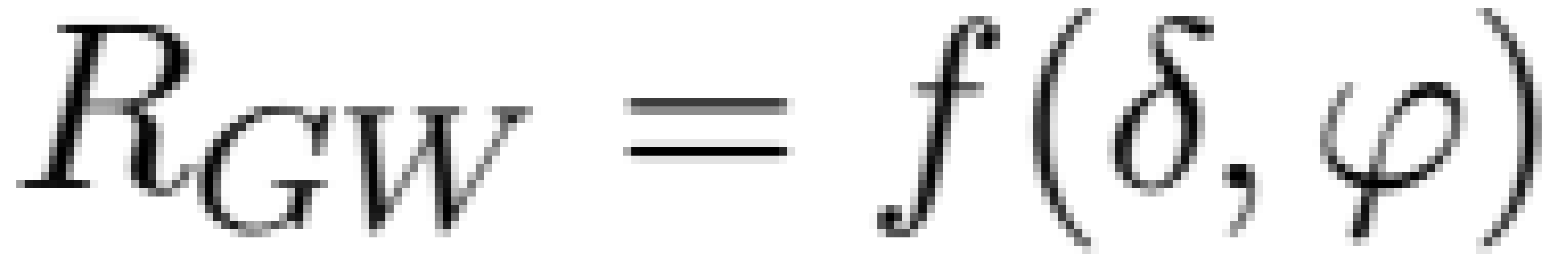








$$R_{GW} = \begin{bmatrix} -\cos\delta\cos\phi & -\sin\delta\cos\phi & \sin\phi \\ \sin\delta & -\cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix}$$









$$\underline{S}_W =$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$





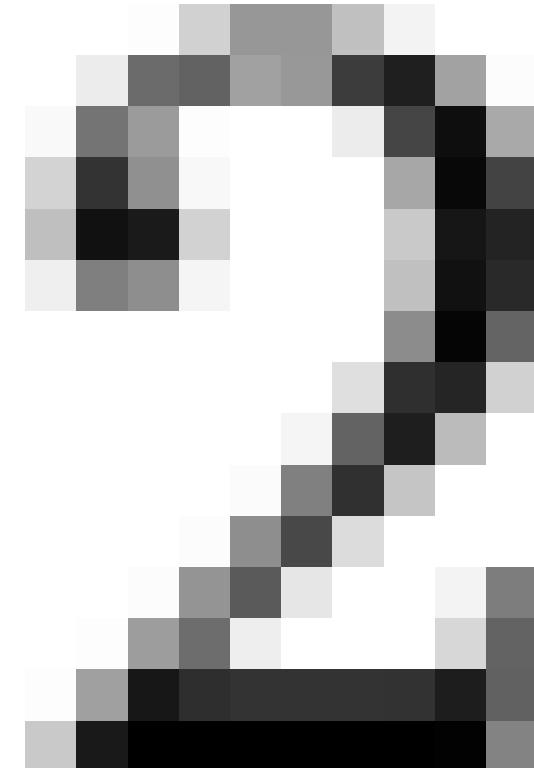
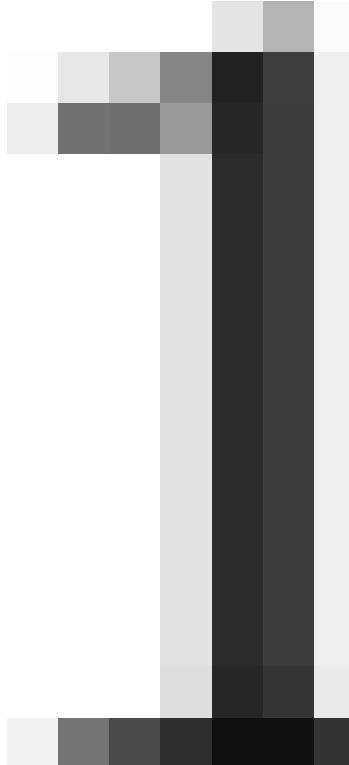
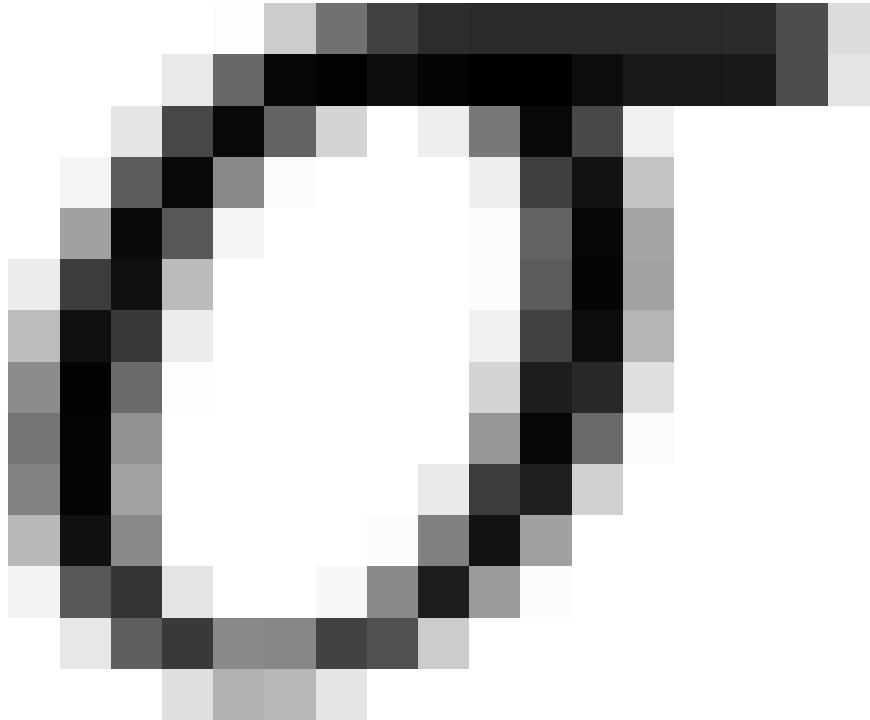


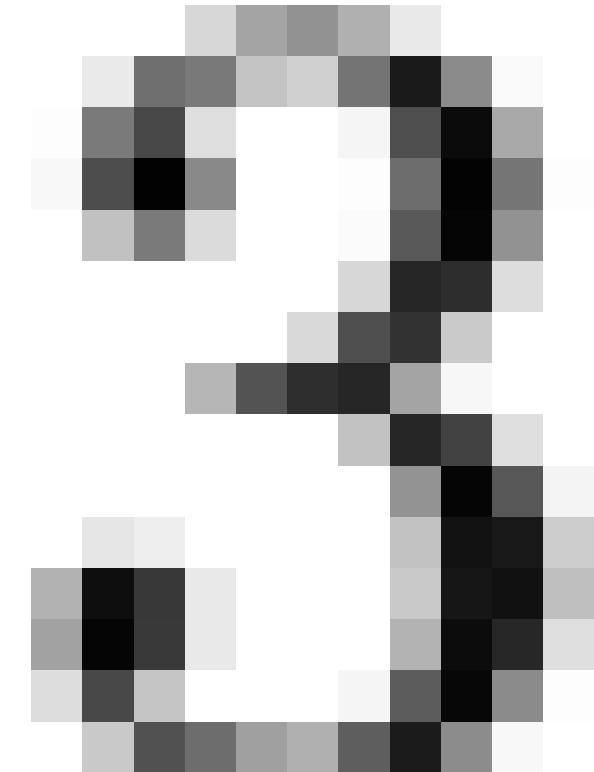
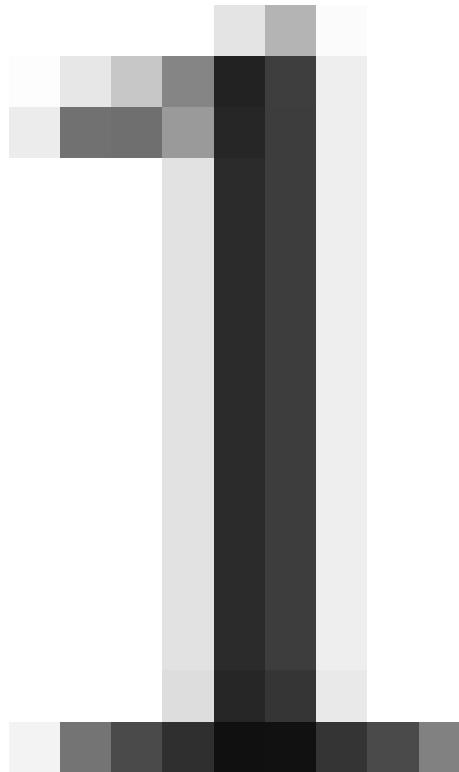
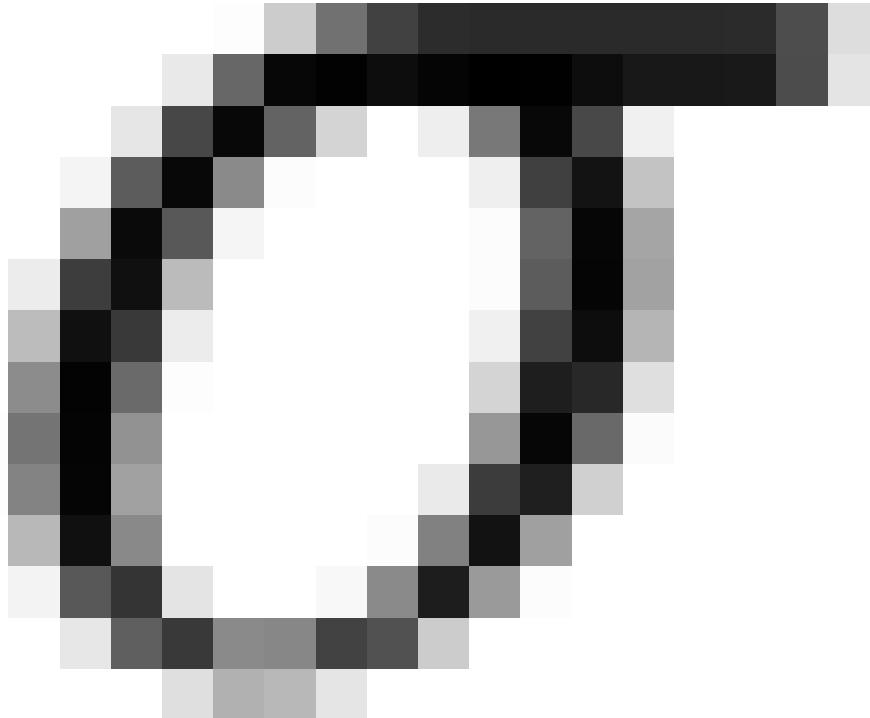


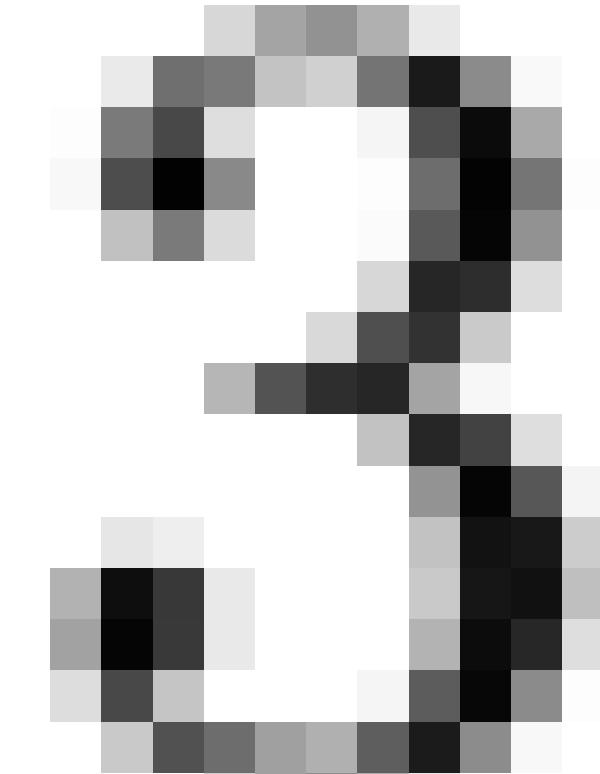
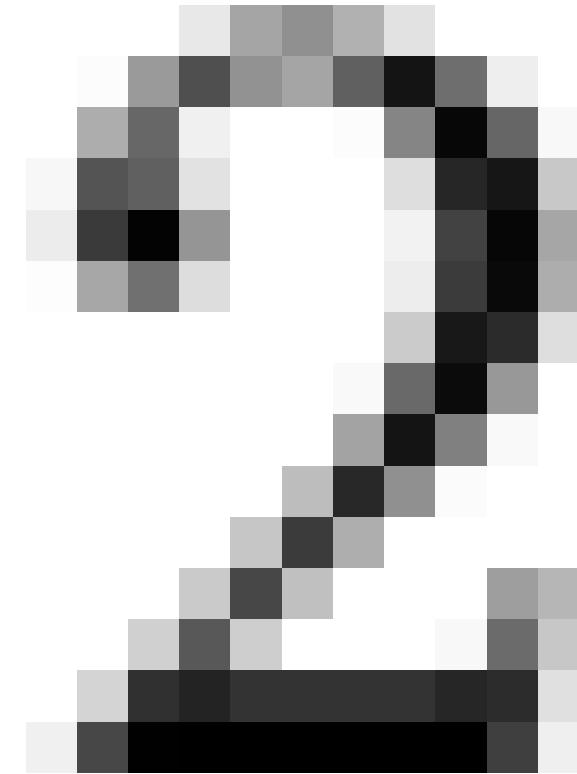
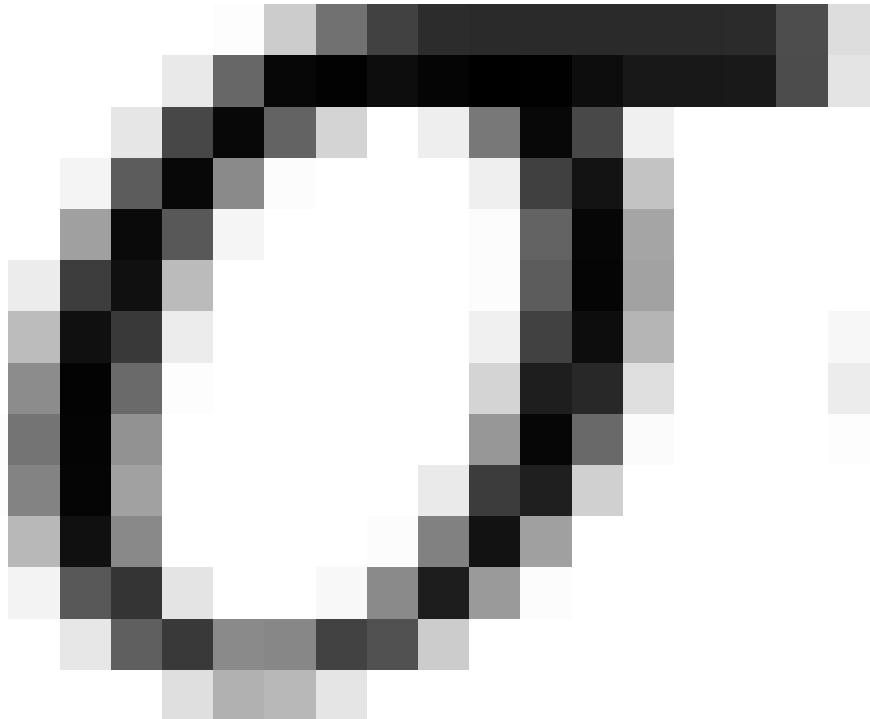


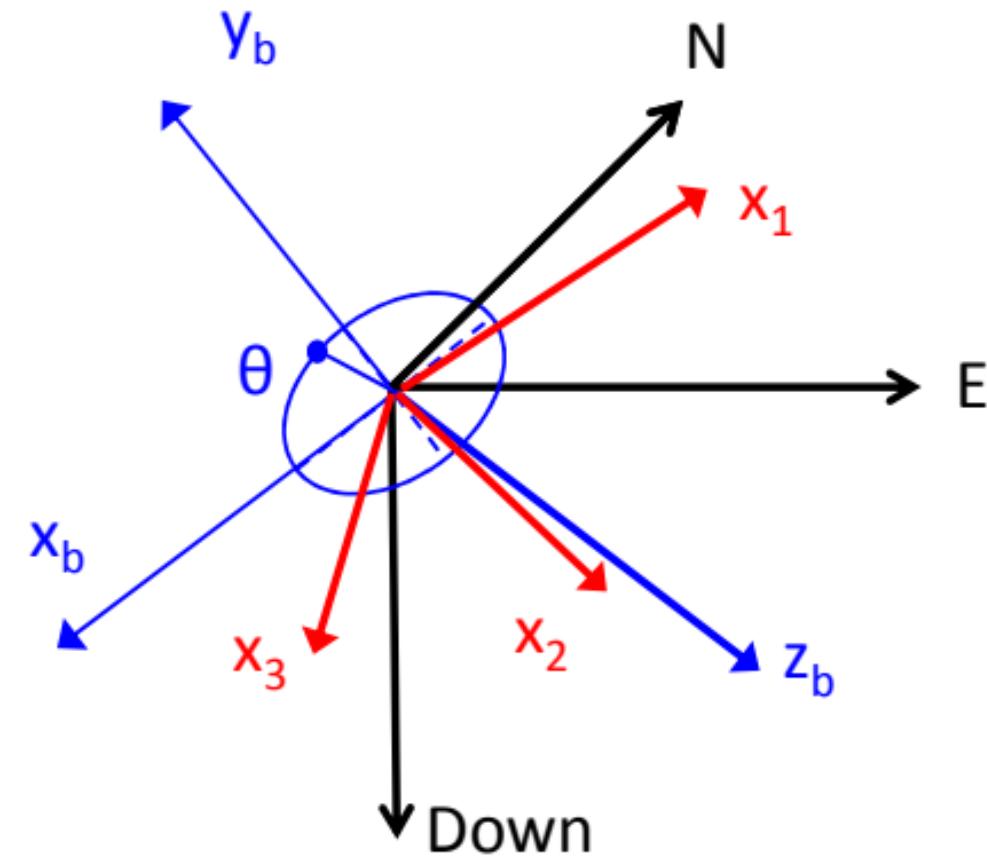








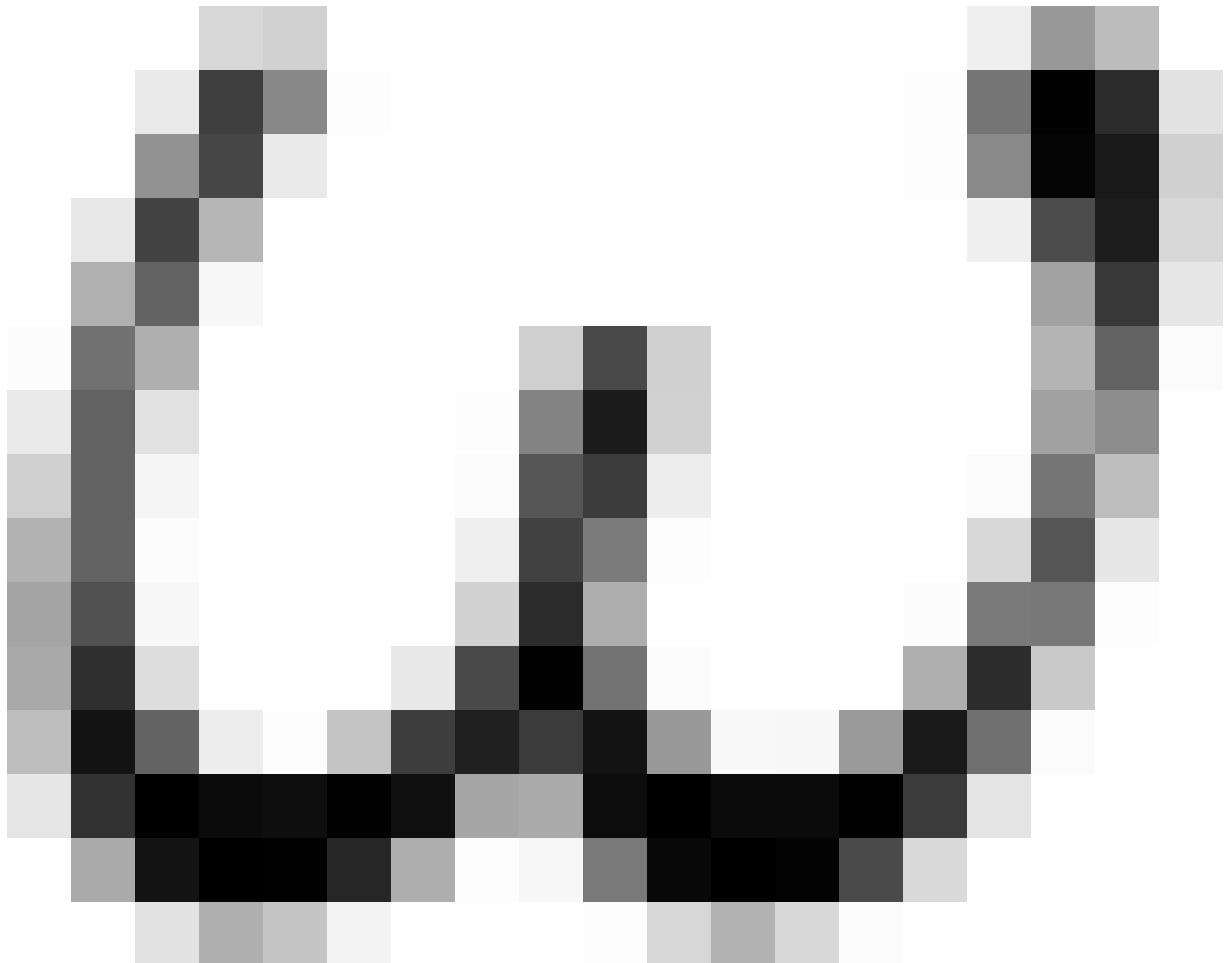


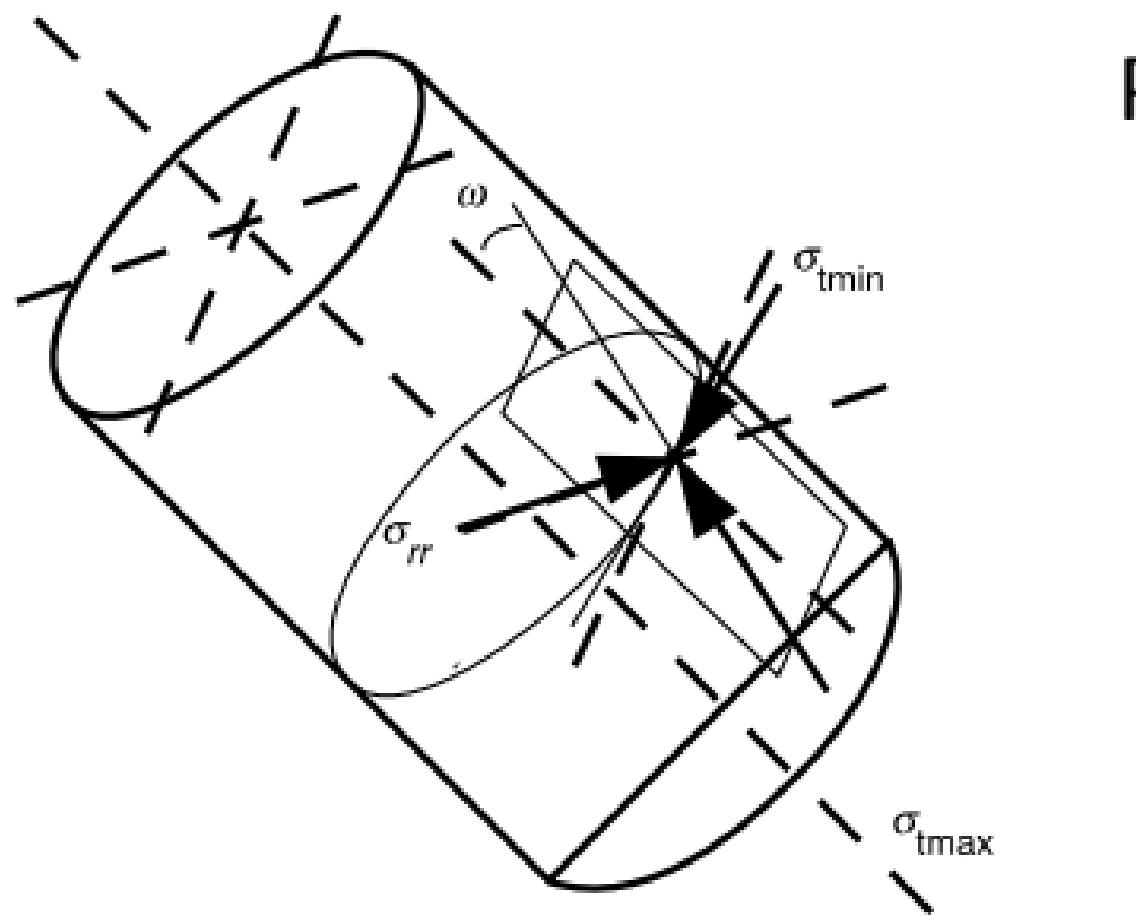


Stresses at the wellbore wall (Kirsch[PP]+Kirsch[S])

$$\begin{cases} \sigma_{rr} = \Delta P \\ \sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\sigma_{12}\sin 2\theta - \Delta P \\ \tau_{\theta z} = 2(\sigma_{23}\cos\theta - \sigma_{13}\sin\theta) \\ \sigma_{zz} = \sigma_{33} - 2\nu(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\nu\sigma_{12}\sin 2\theta \end{cases}$$

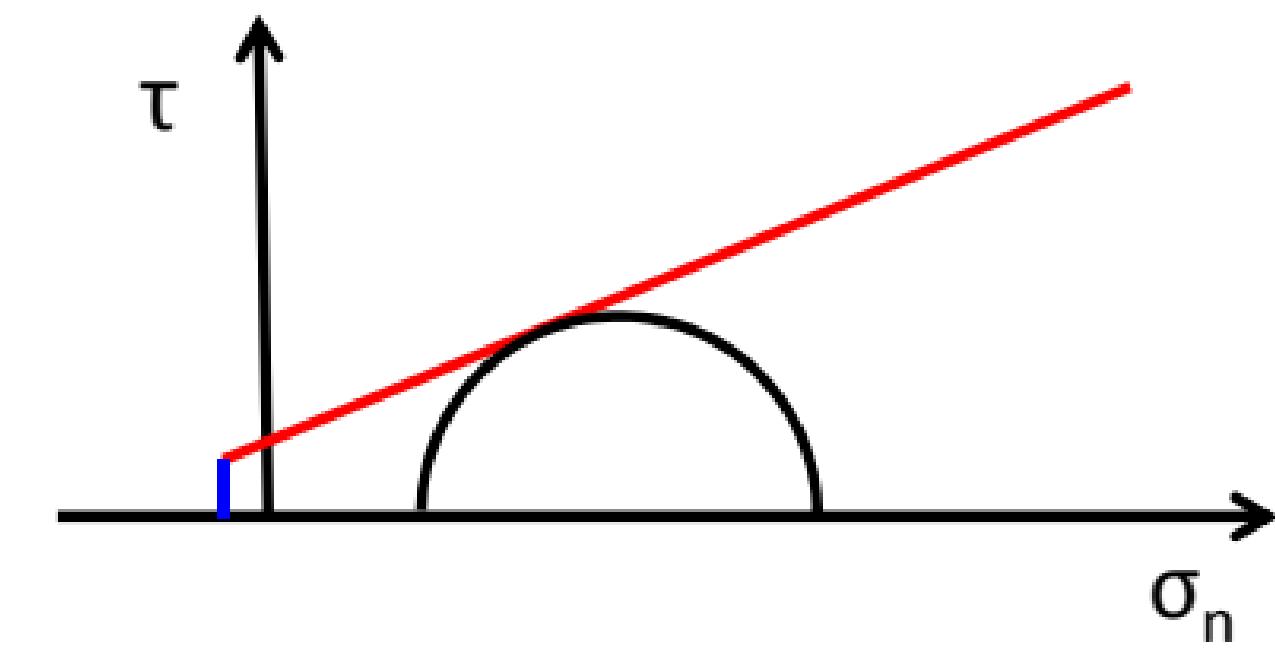




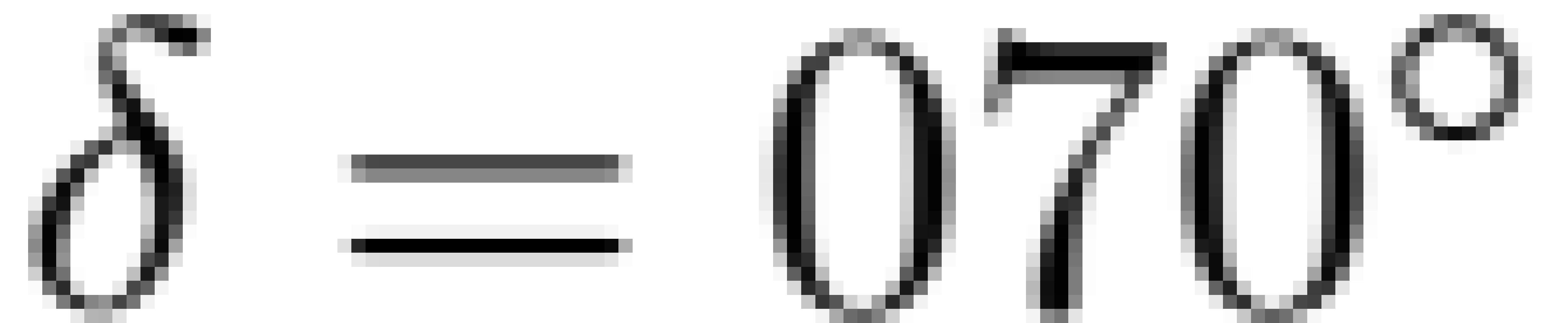


## Principal stresses at the wellbore wall

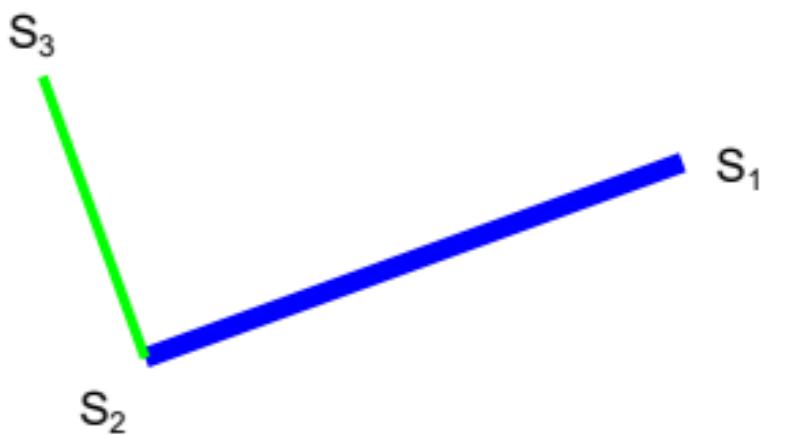
$$\left\{ \begin{array}{l} \sigma_{t\max} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} + \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \\ \sigma_{t\min} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} - \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \end{array} \right.$$



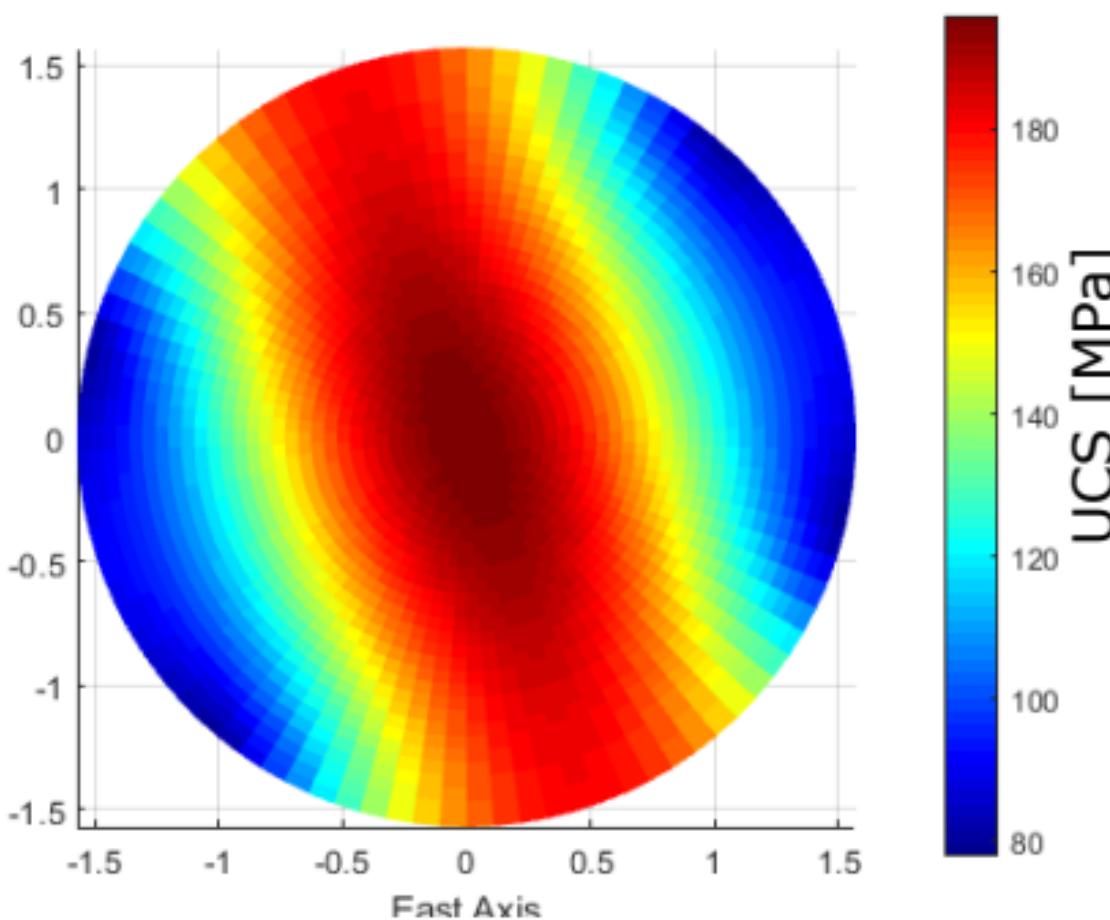




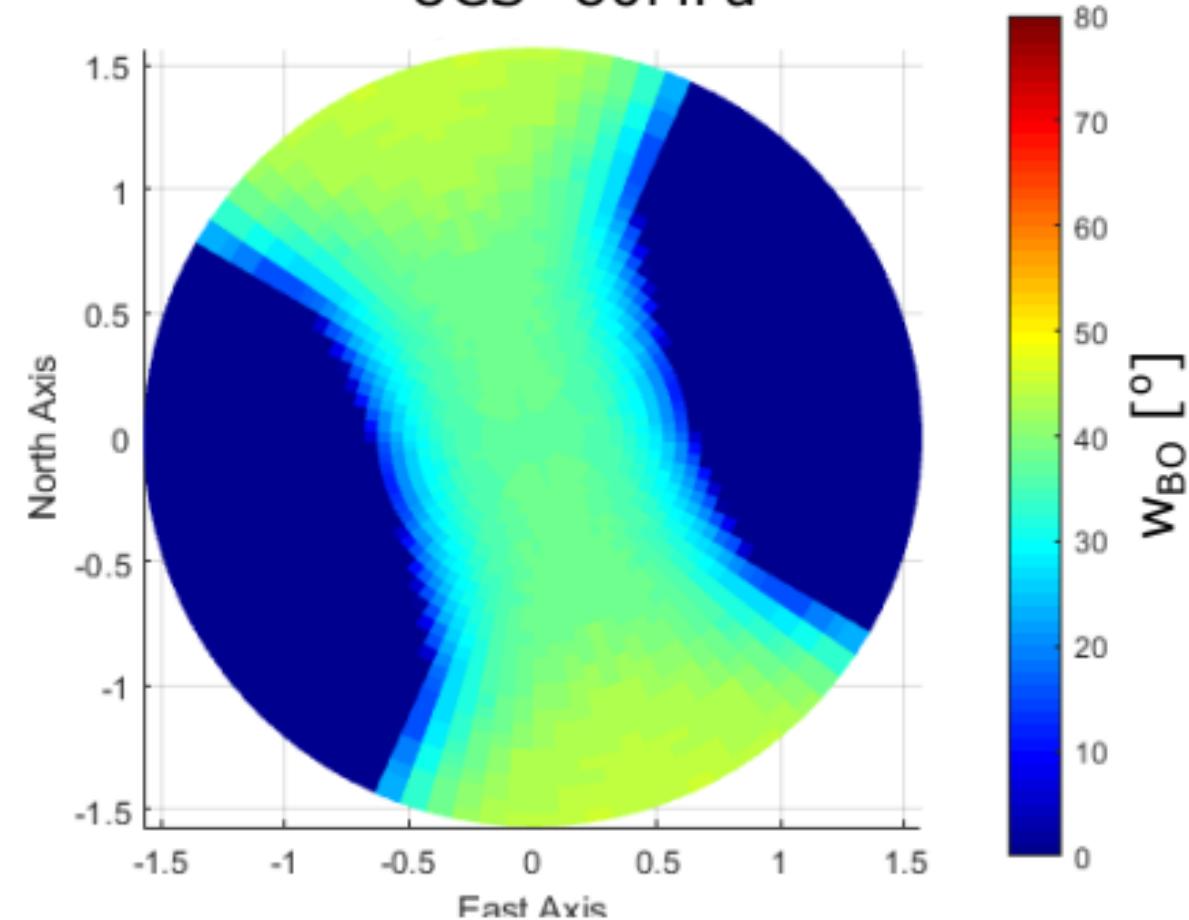
## Geographical principal stresses

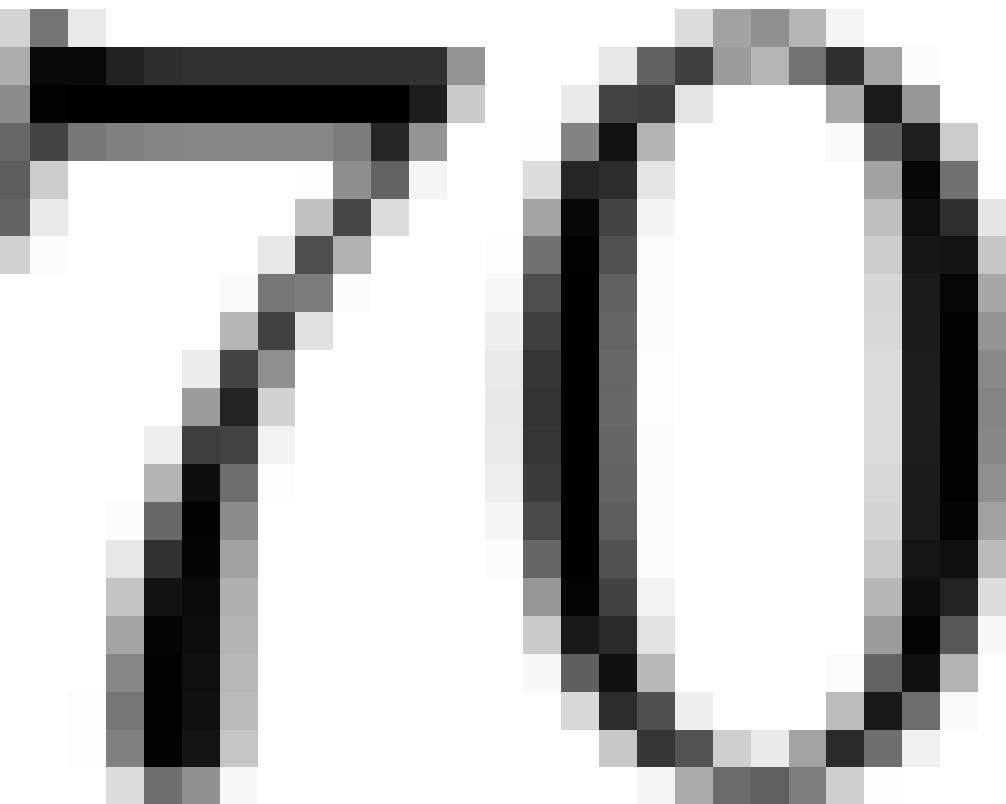


## Required UCS ( $P_w = P_p$ )



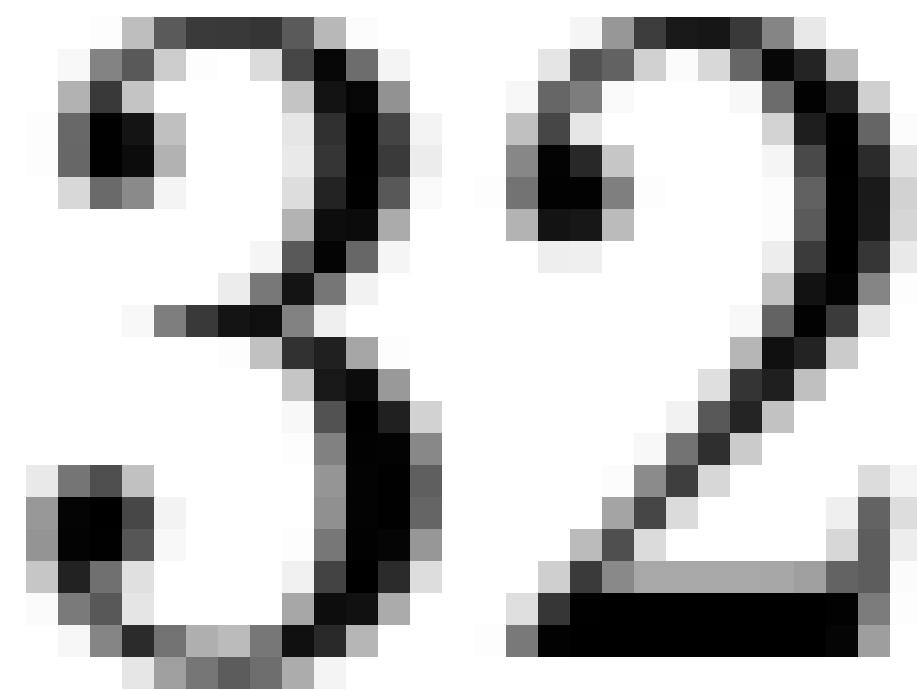
## Breakout angle by Lade - $P_w = 45\text{MPa}$ UCS=80MPa

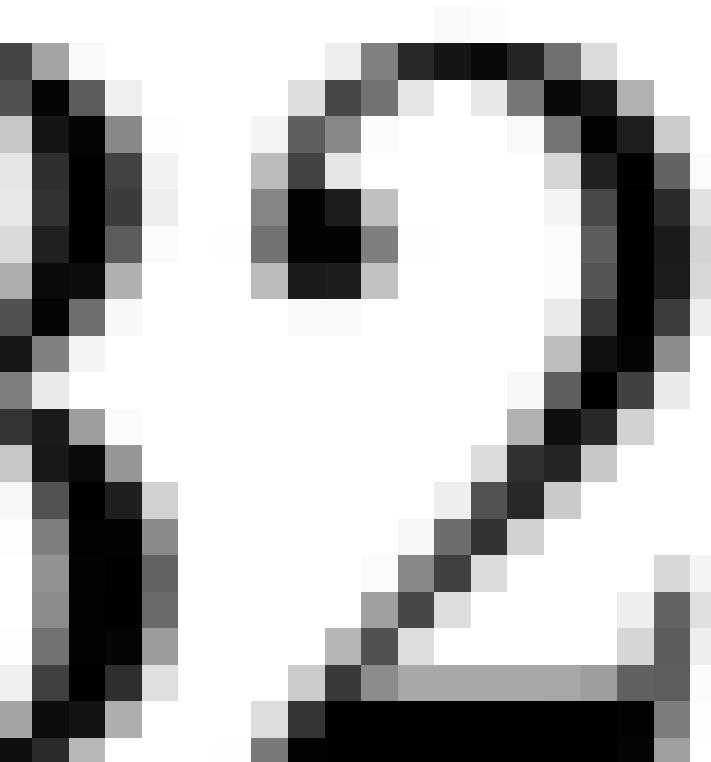
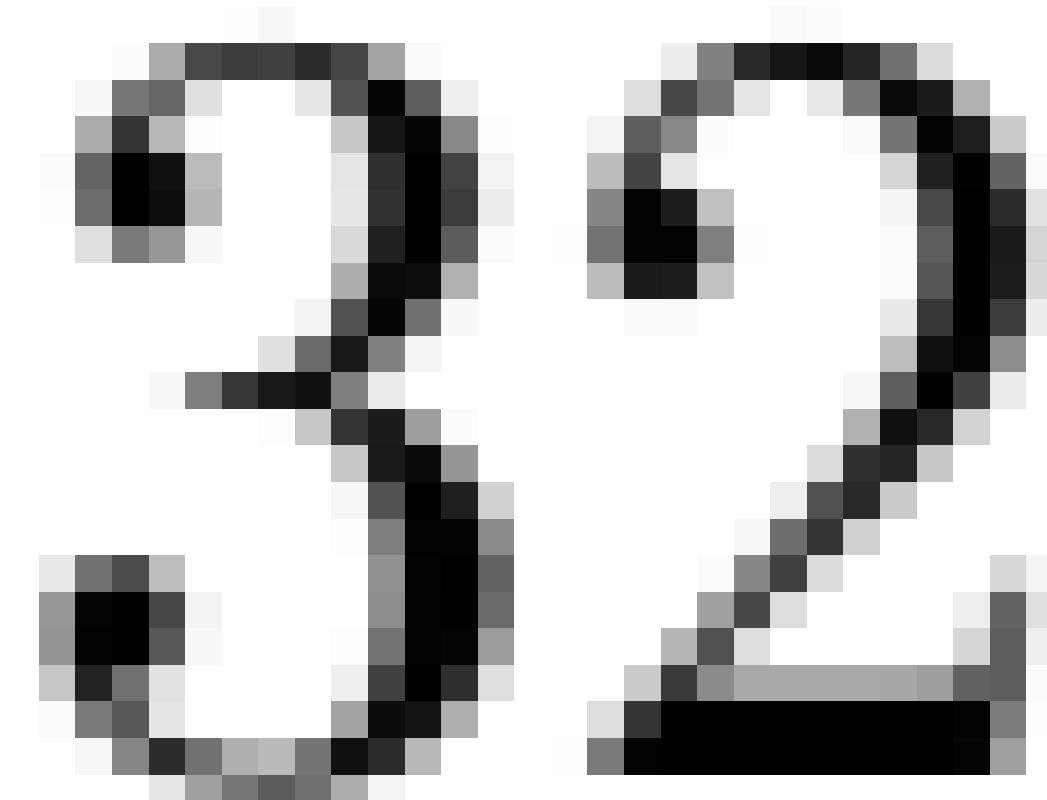


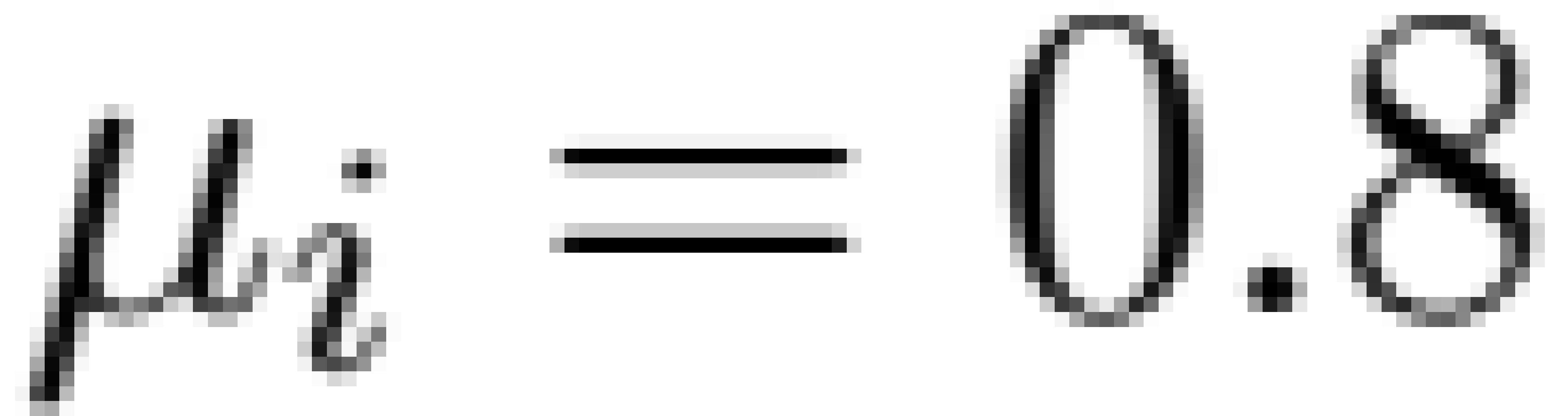




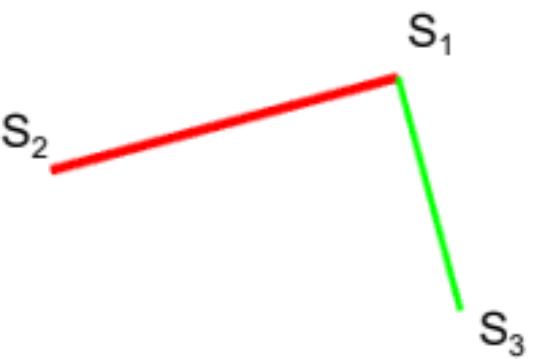




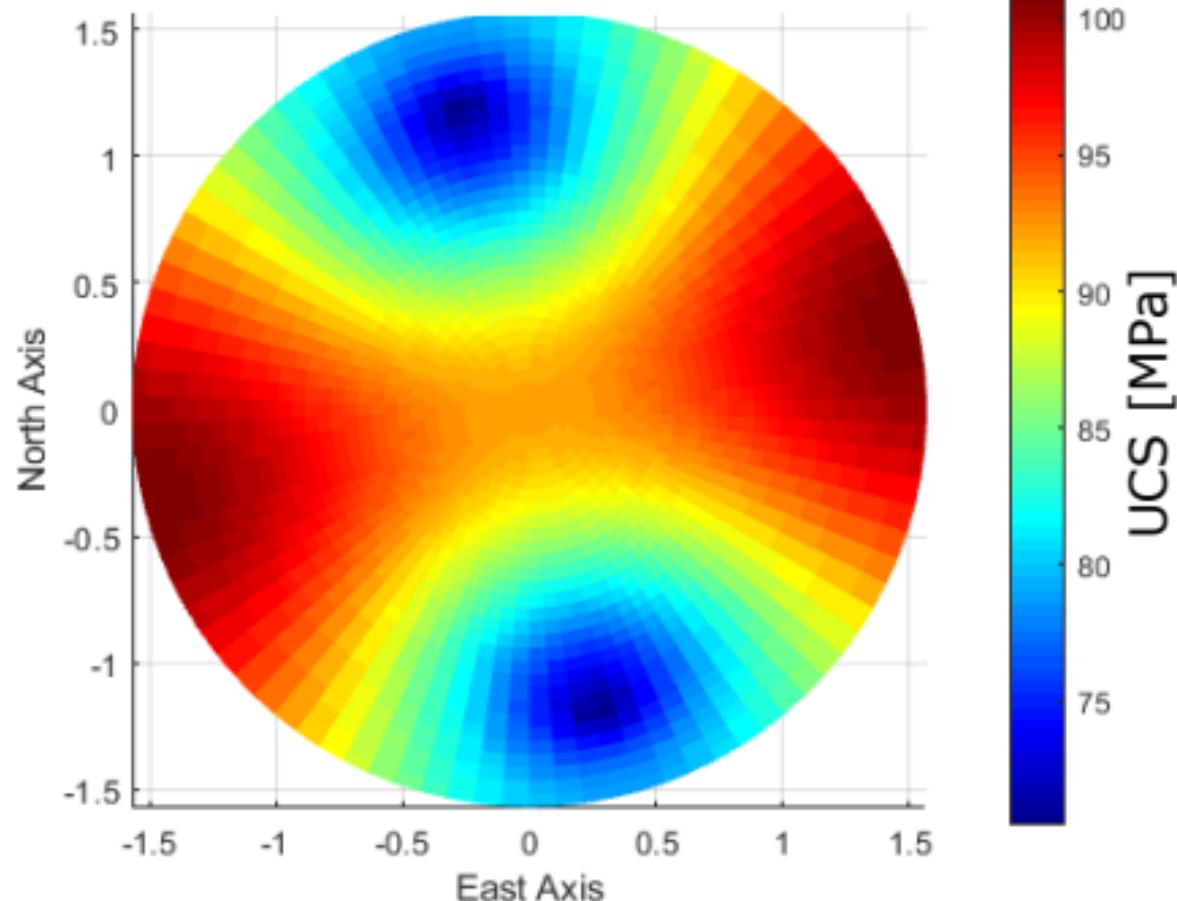




## Principal stresses



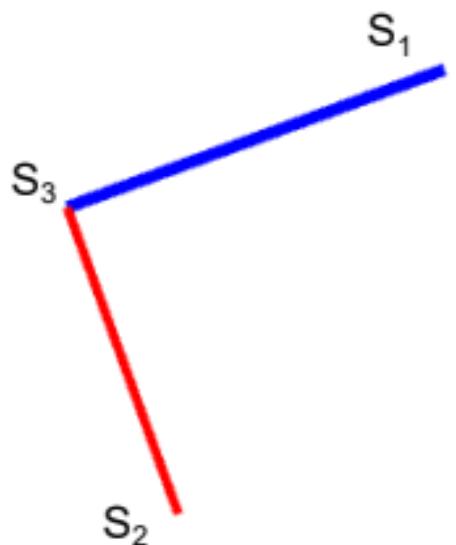
## Required UCS ( $P_w = P_p$ )



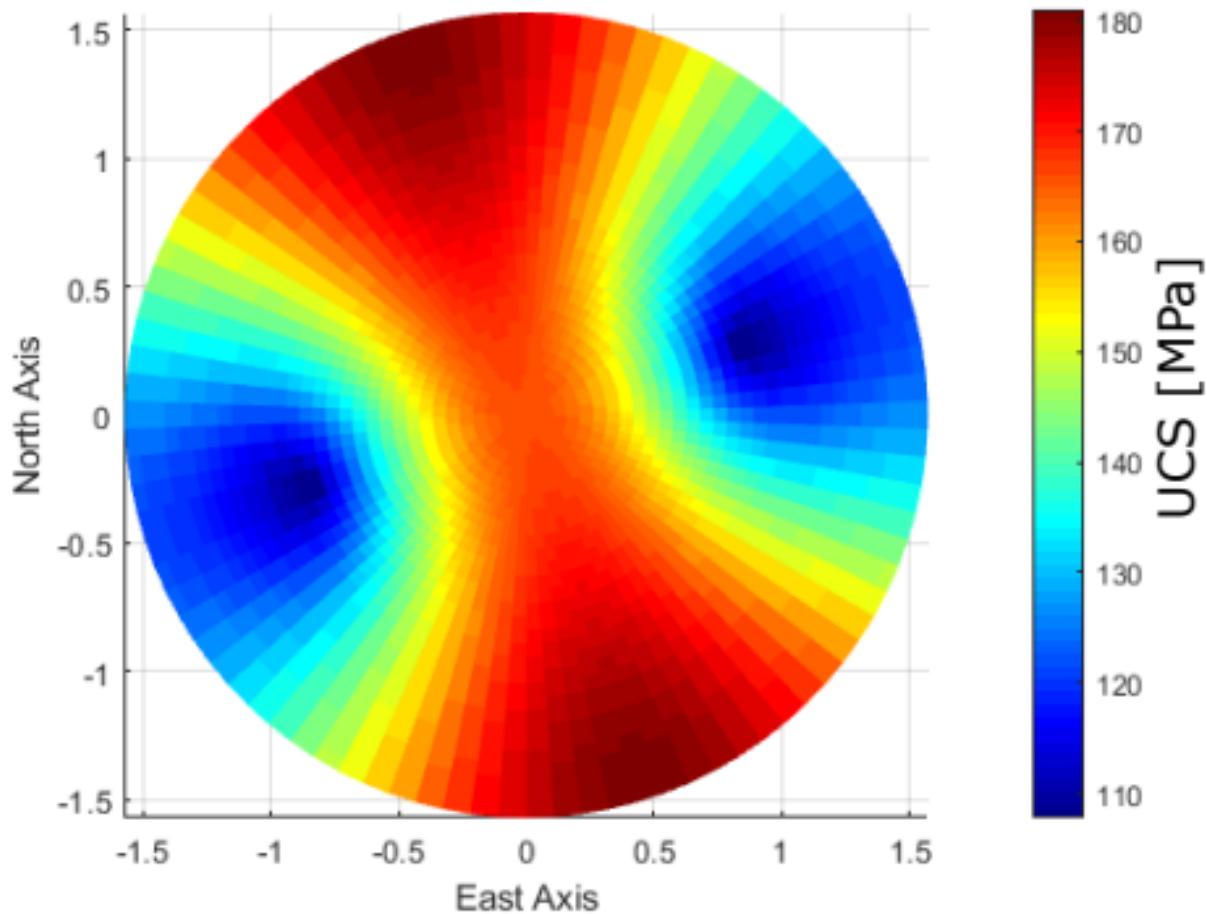


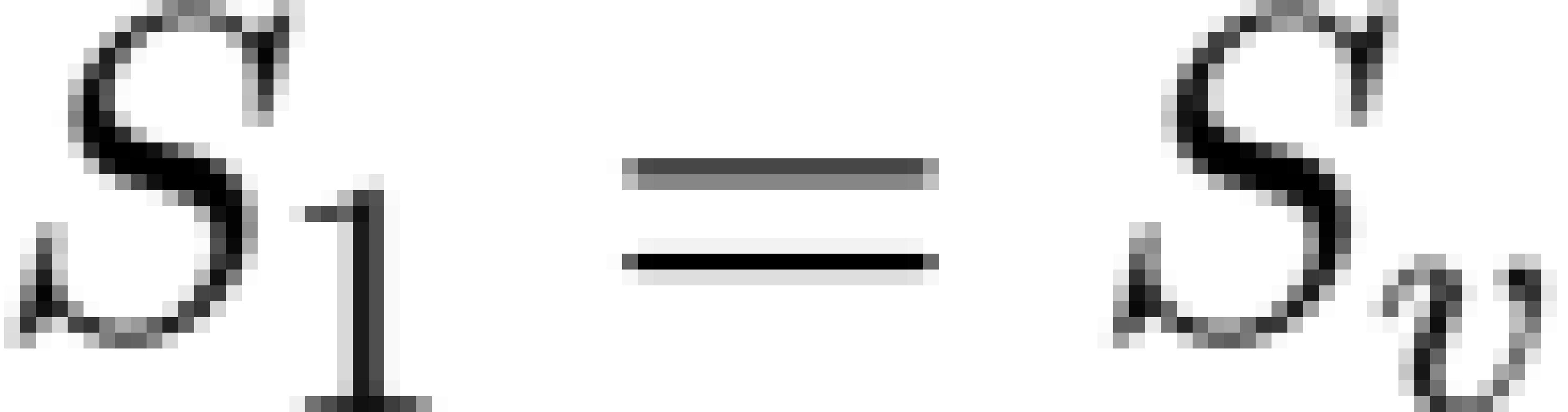


# Principal stresses

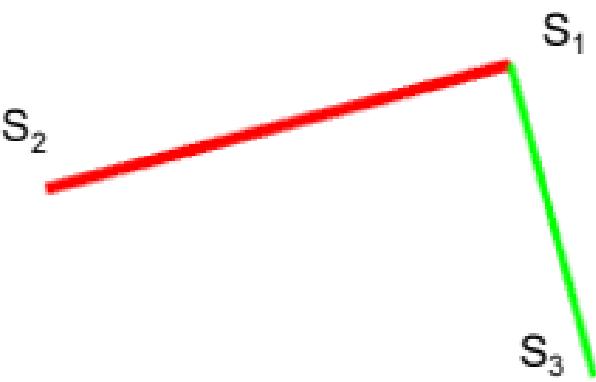


# Required UCS ( $P_w = P_p$ )

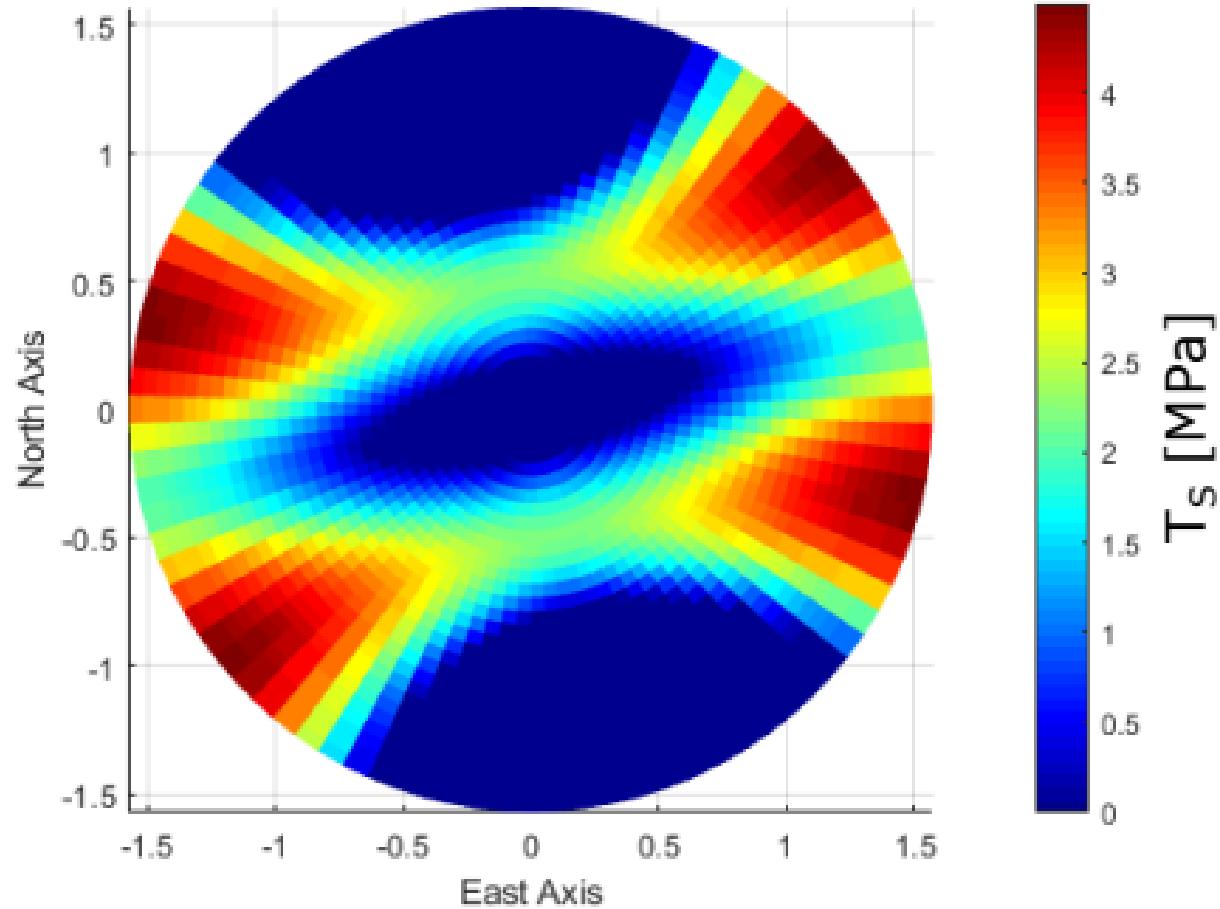




# Principal stresses



# Required $T_s$ ( $P_w=35\text{MPa}$ )





$\delta\Omega$

$\Delta$

$\delta\Omega$

$\Delta$

$\Delta\Omega$

$\Delta\Omega$

$\Delta\Omega$









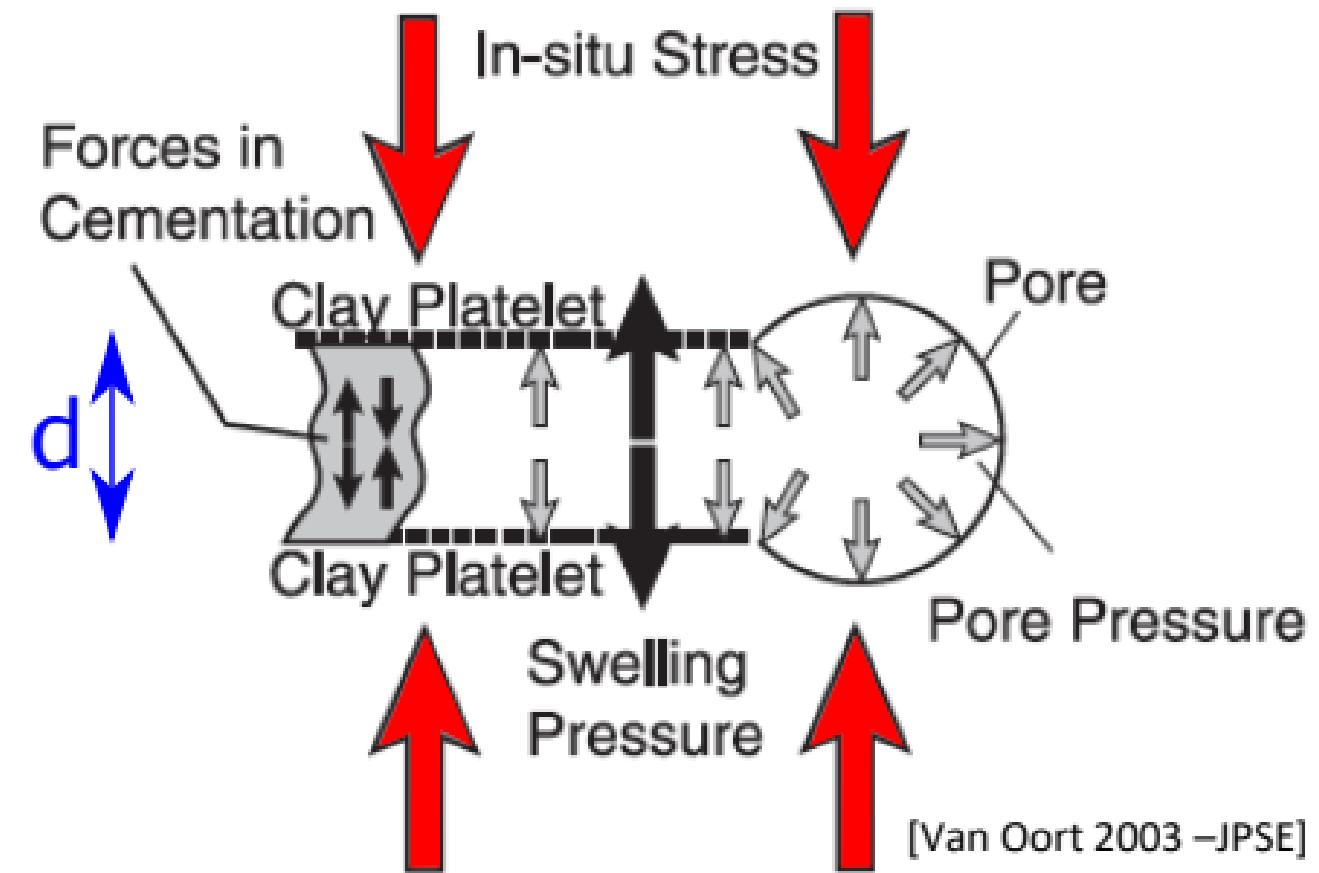
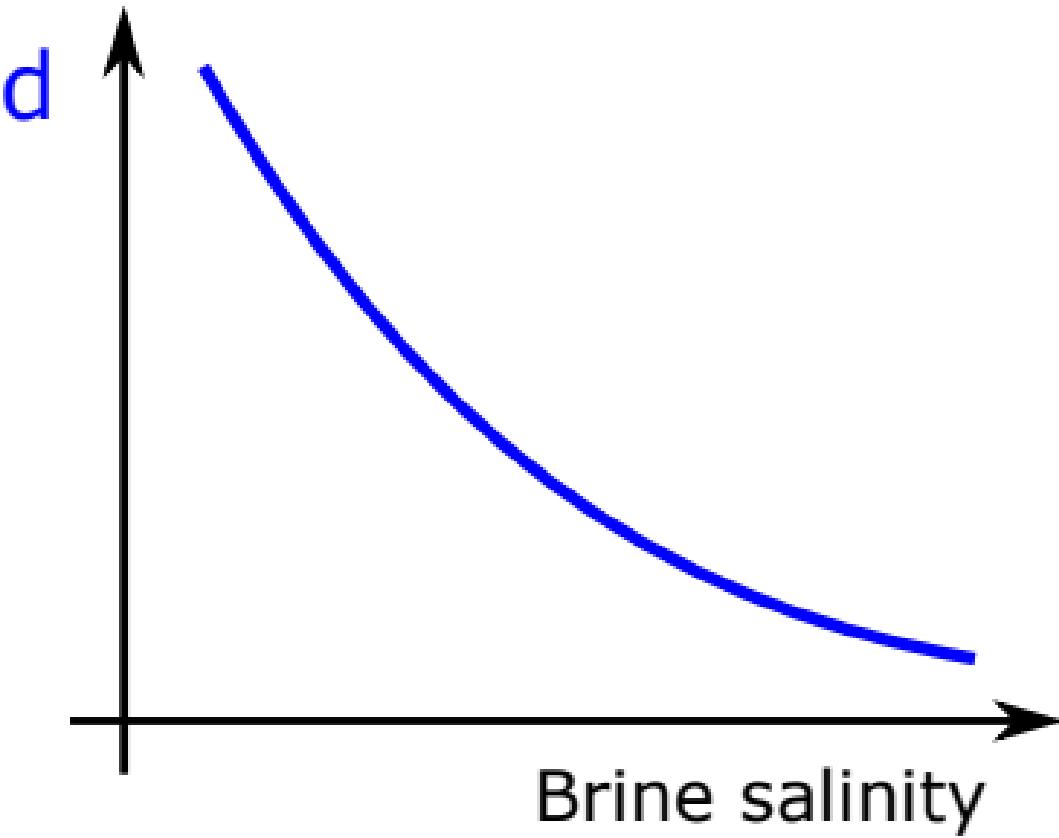


$\Delta$   $\sigma_{\theta\theta}$

$=$

$\alpha \Gamma B \Delta T$

$1 - \sqrt{ }$



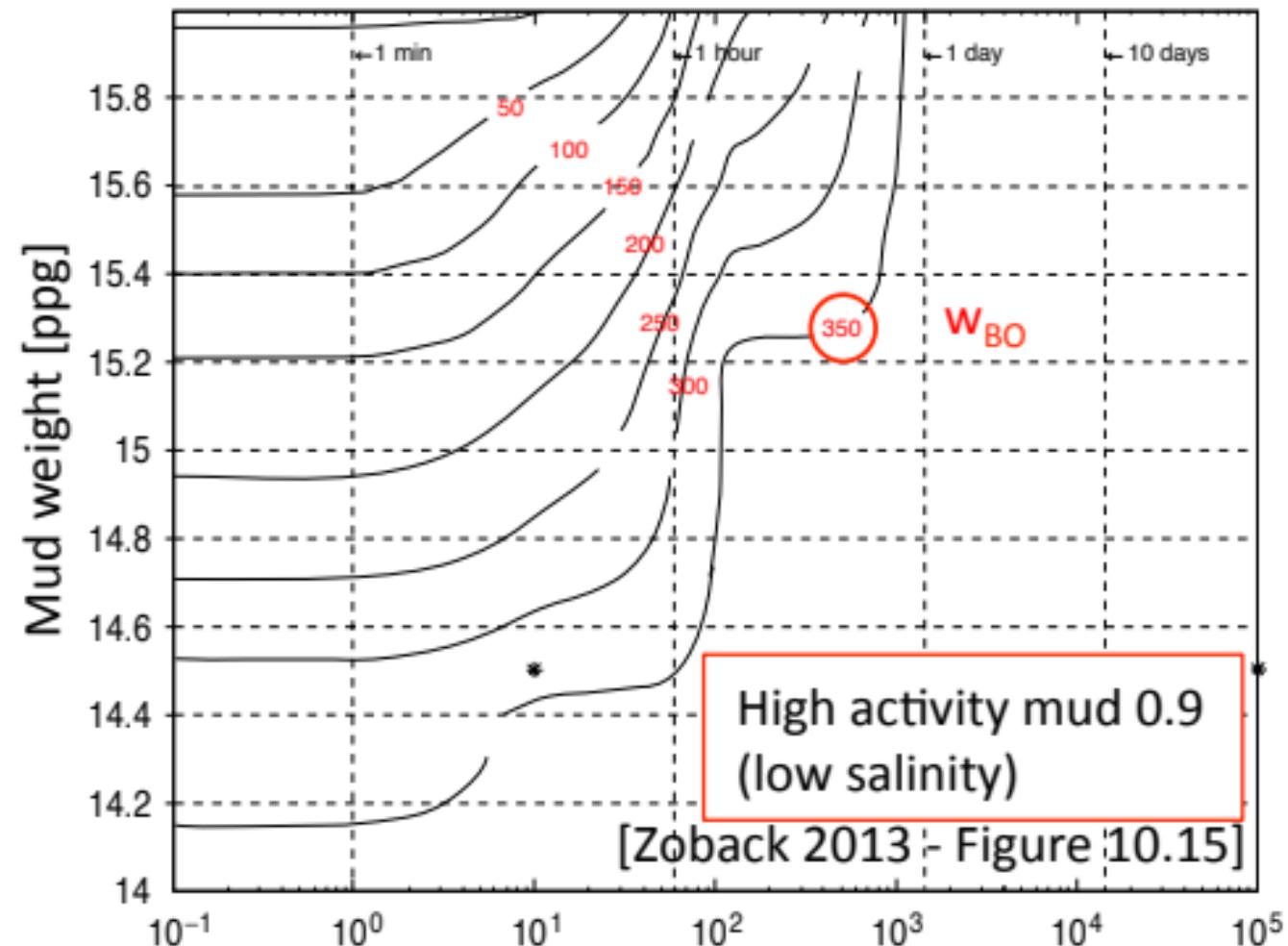
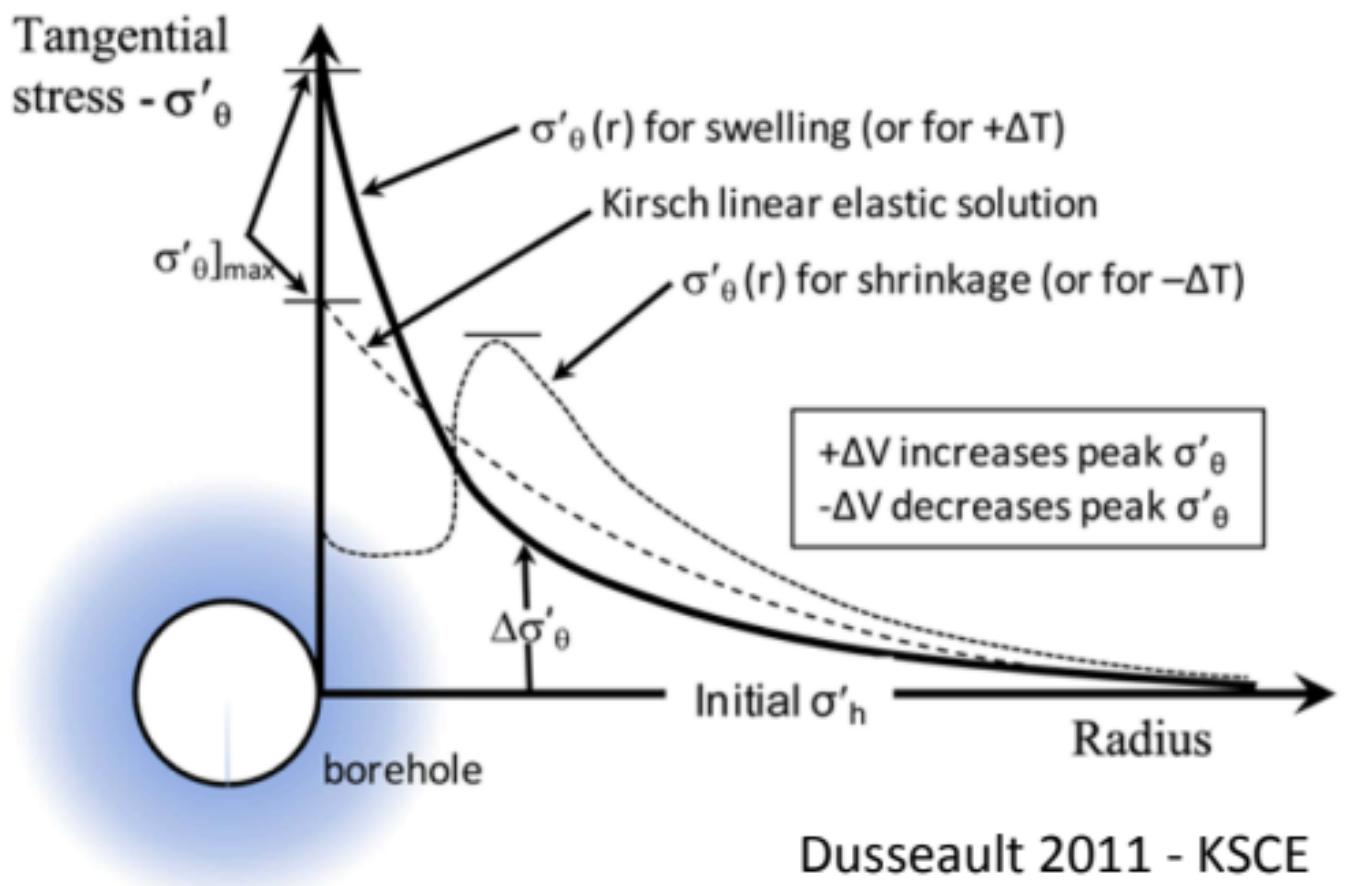
# Norway shale



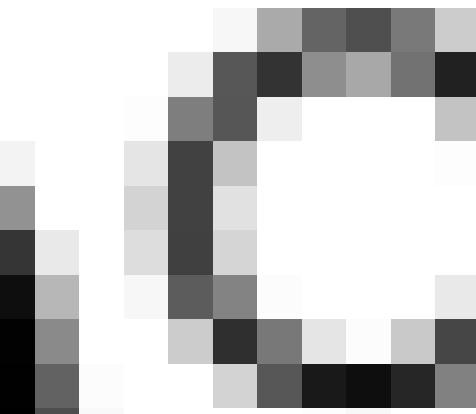
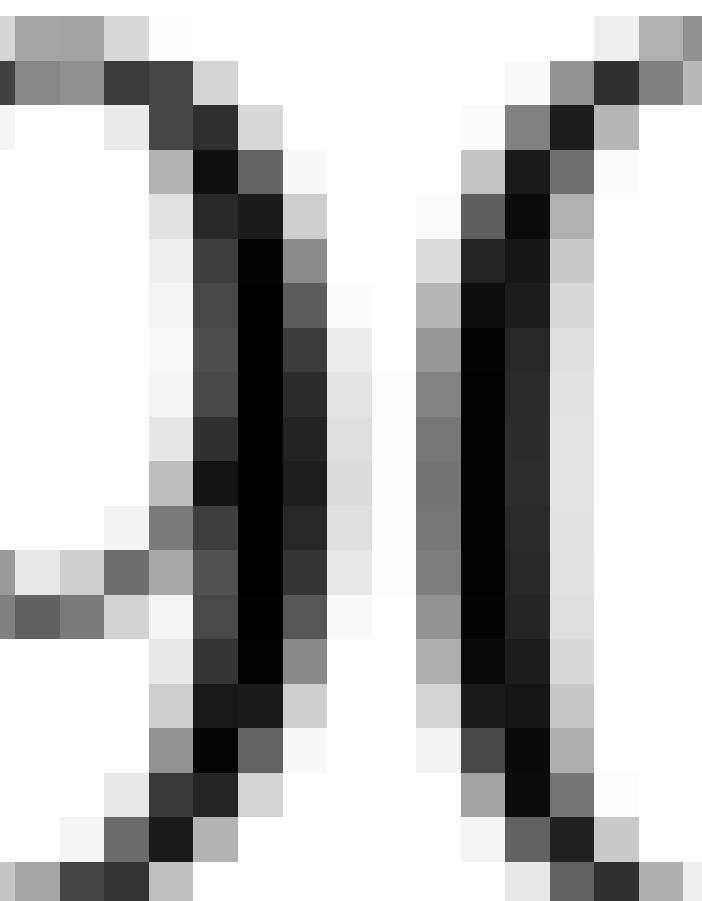
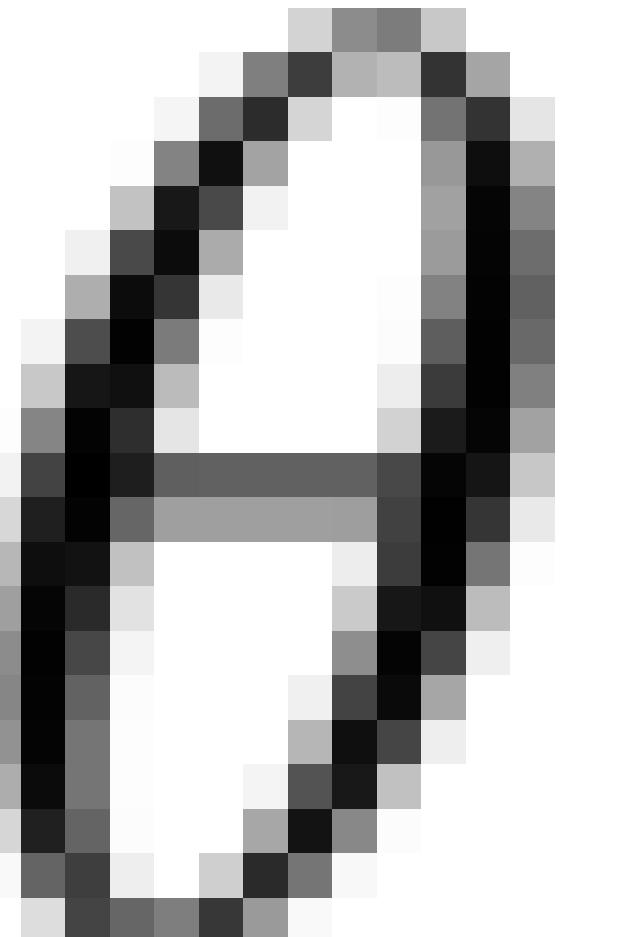
# After 2hs exposition

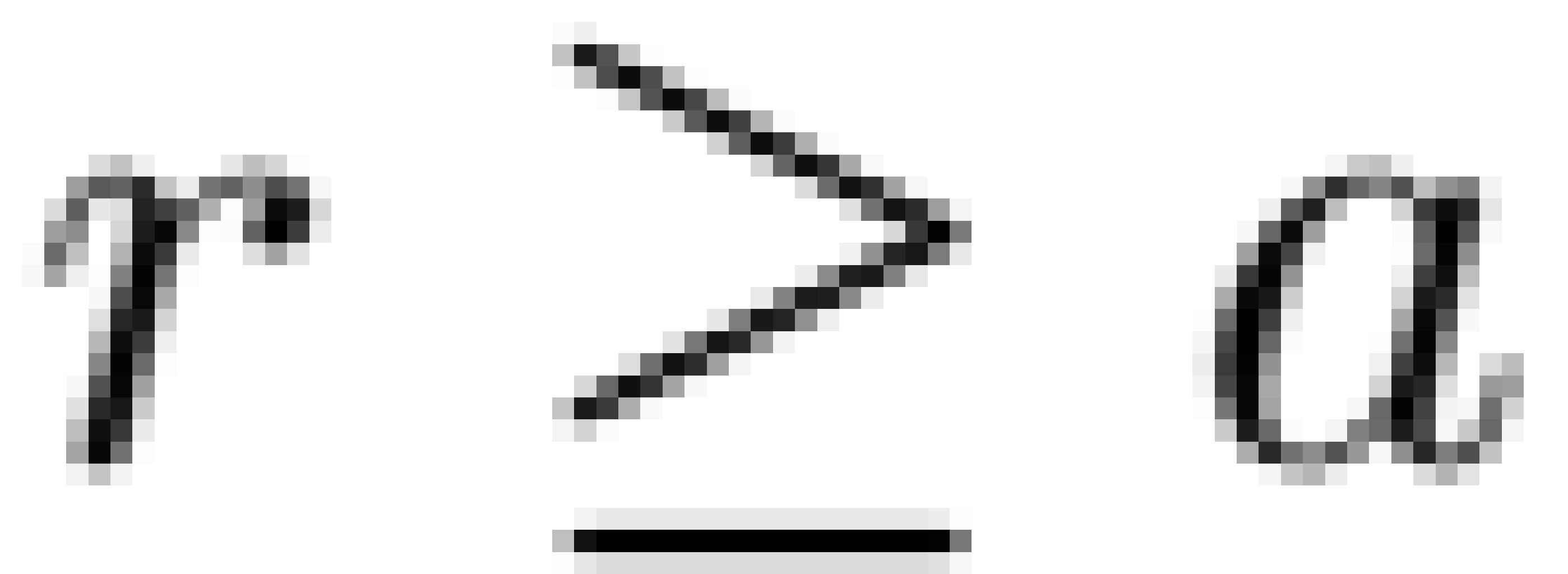


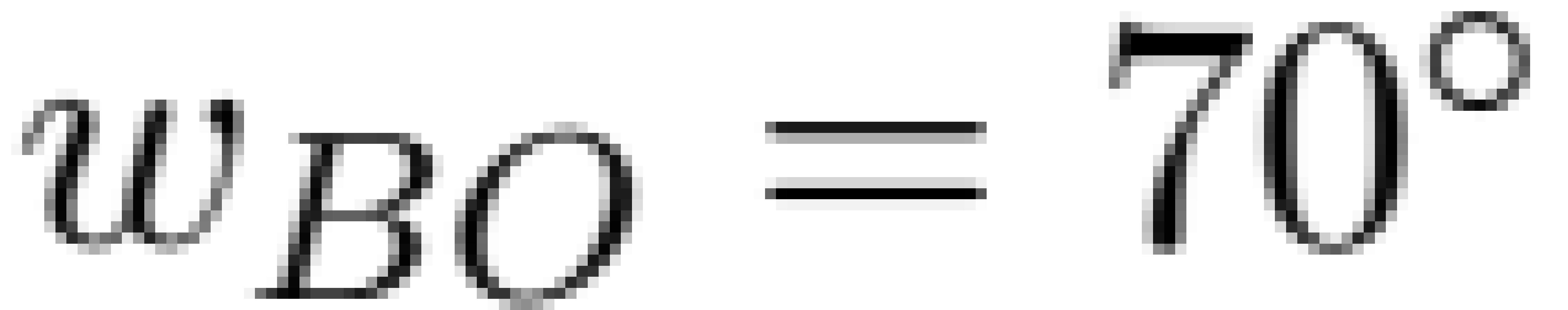
CPGE – M. Chenevert



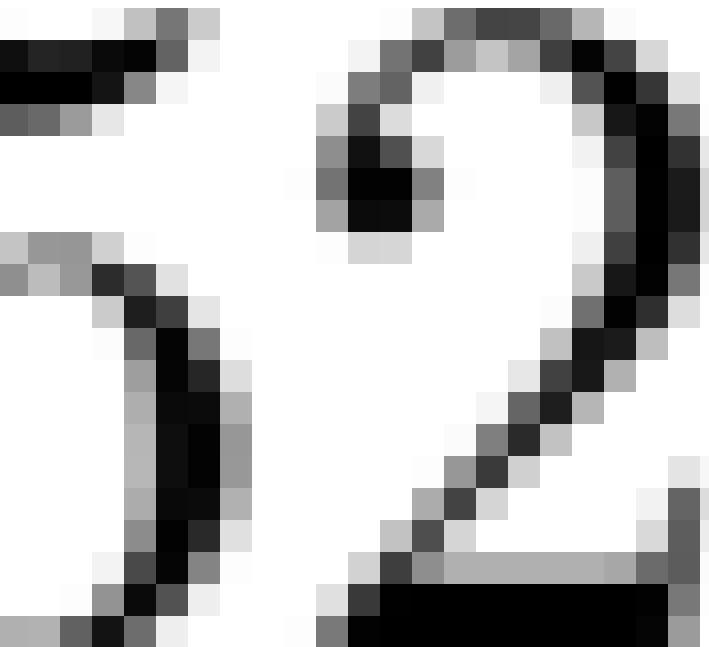
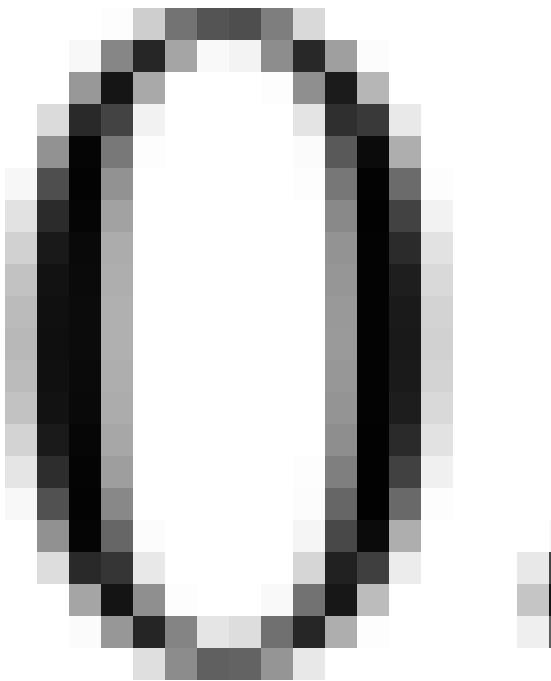


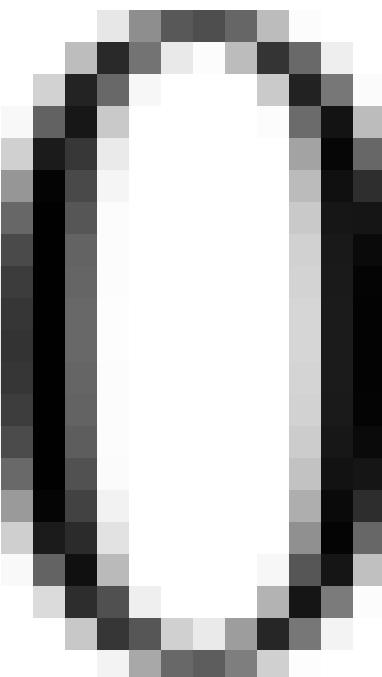
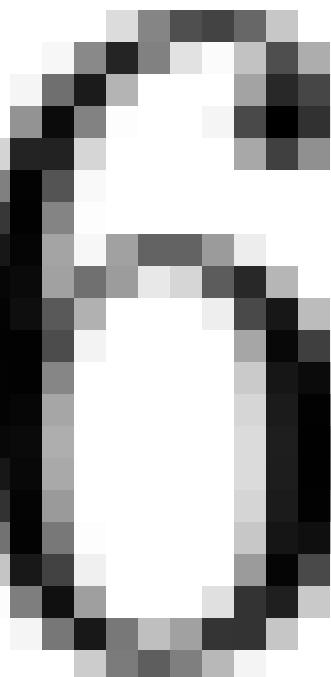
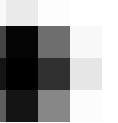
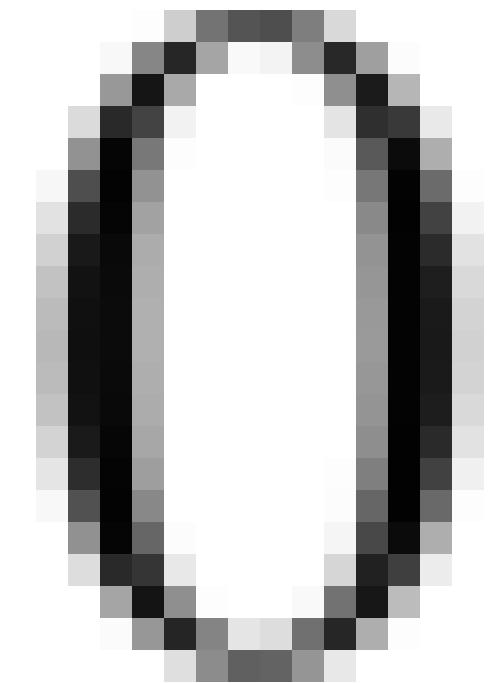


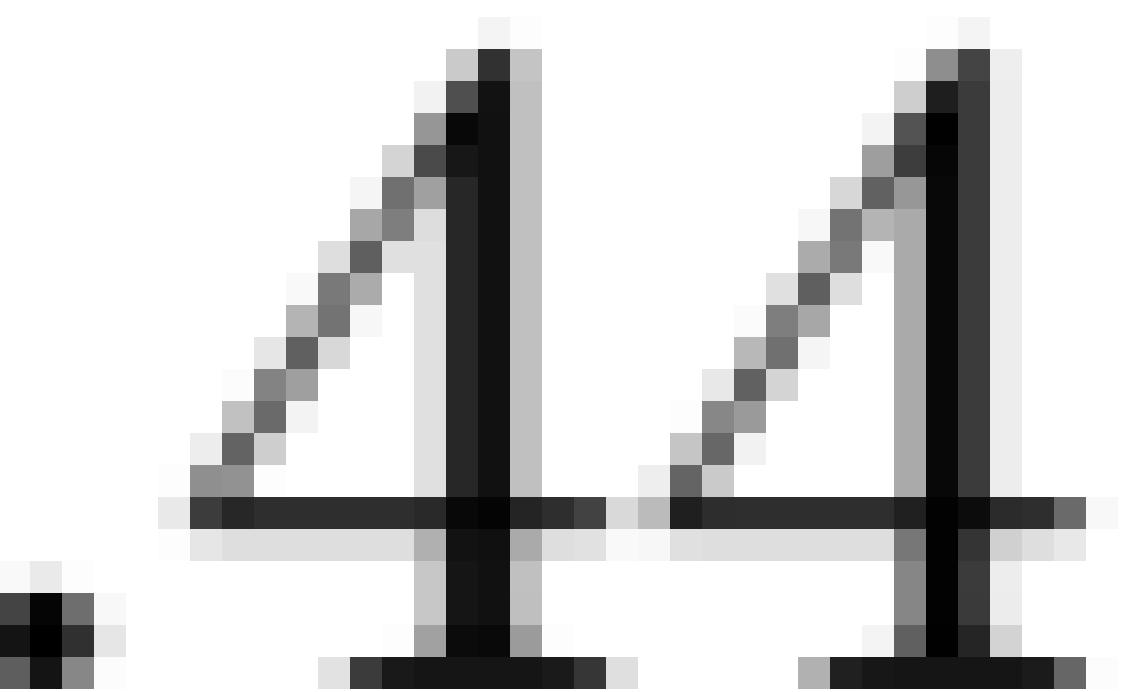
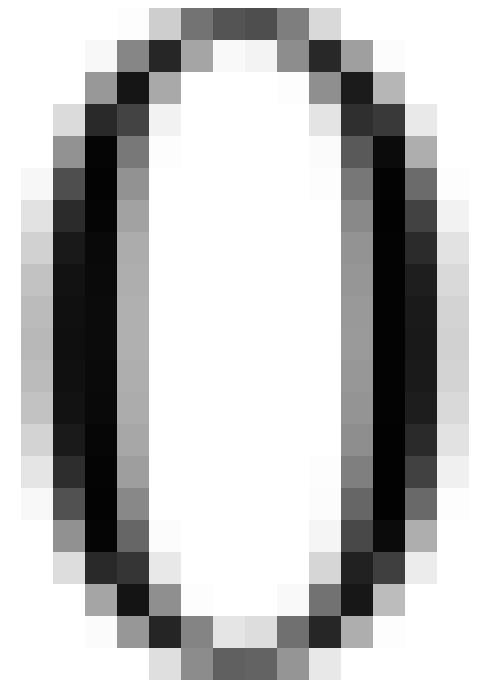


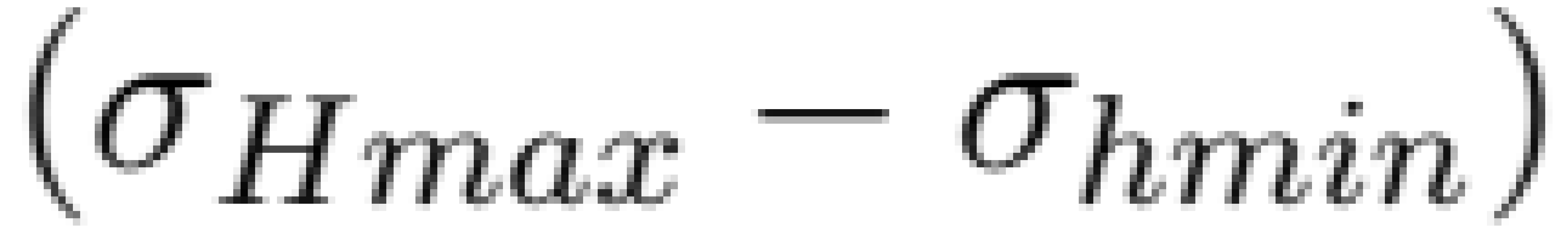


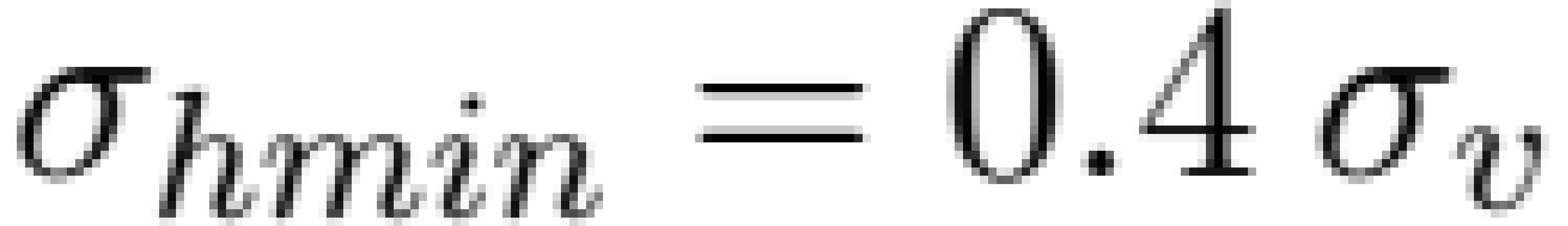


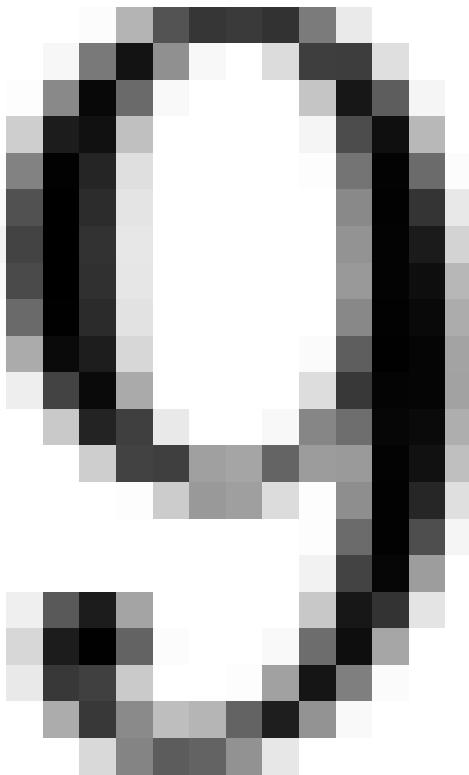
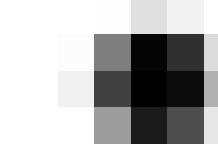
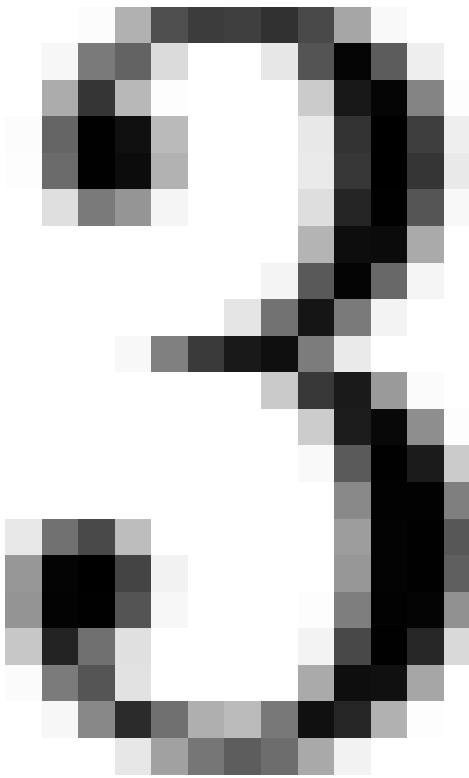
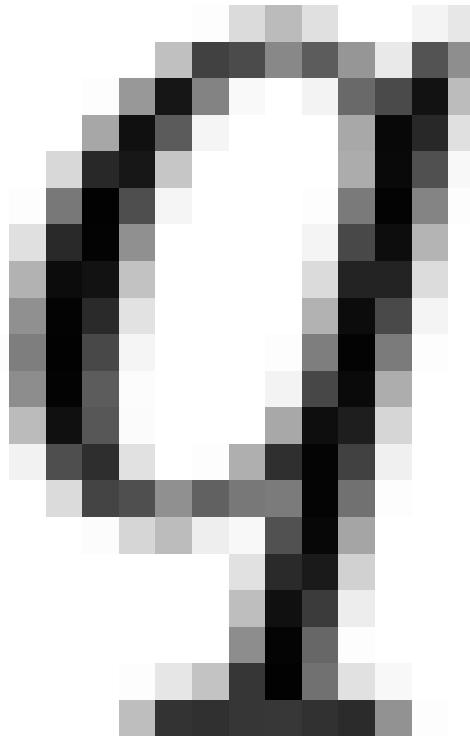


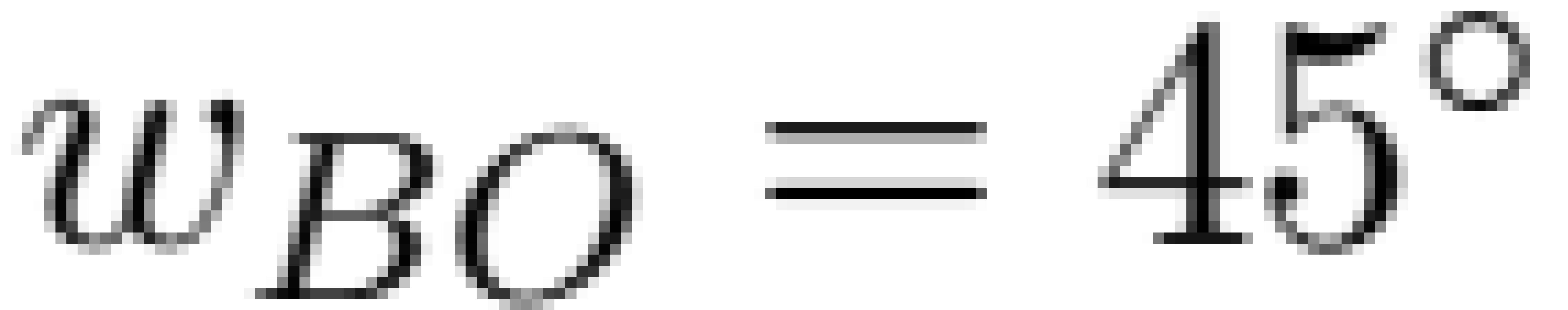








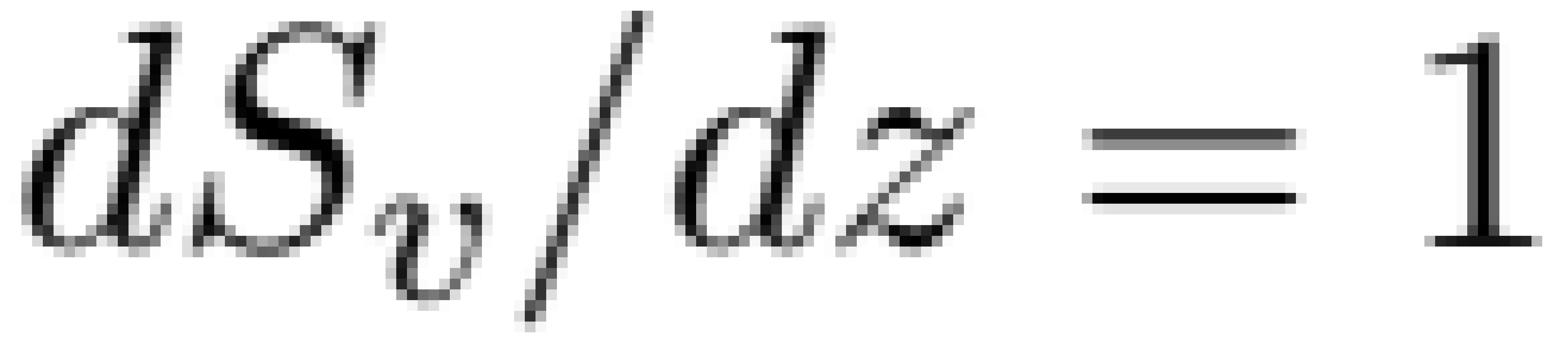




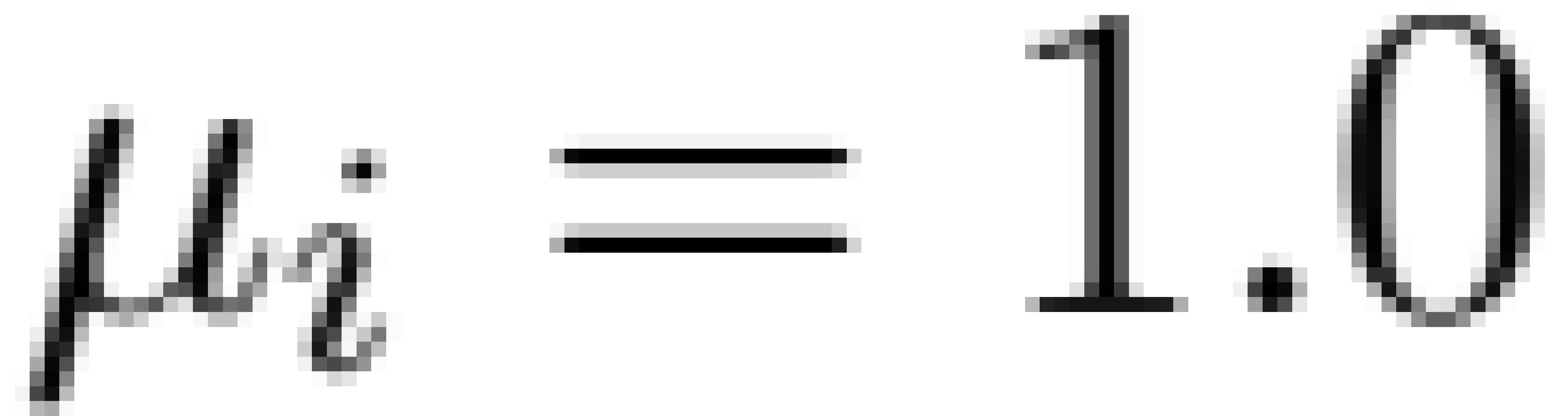




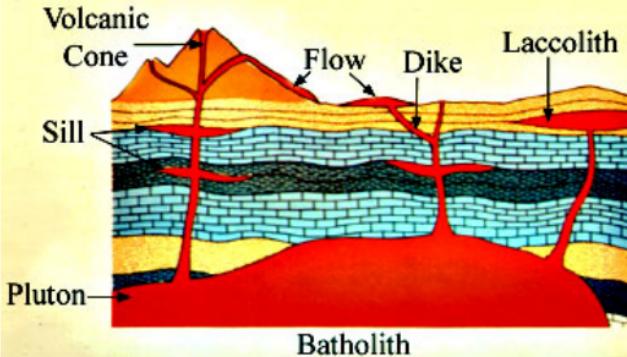


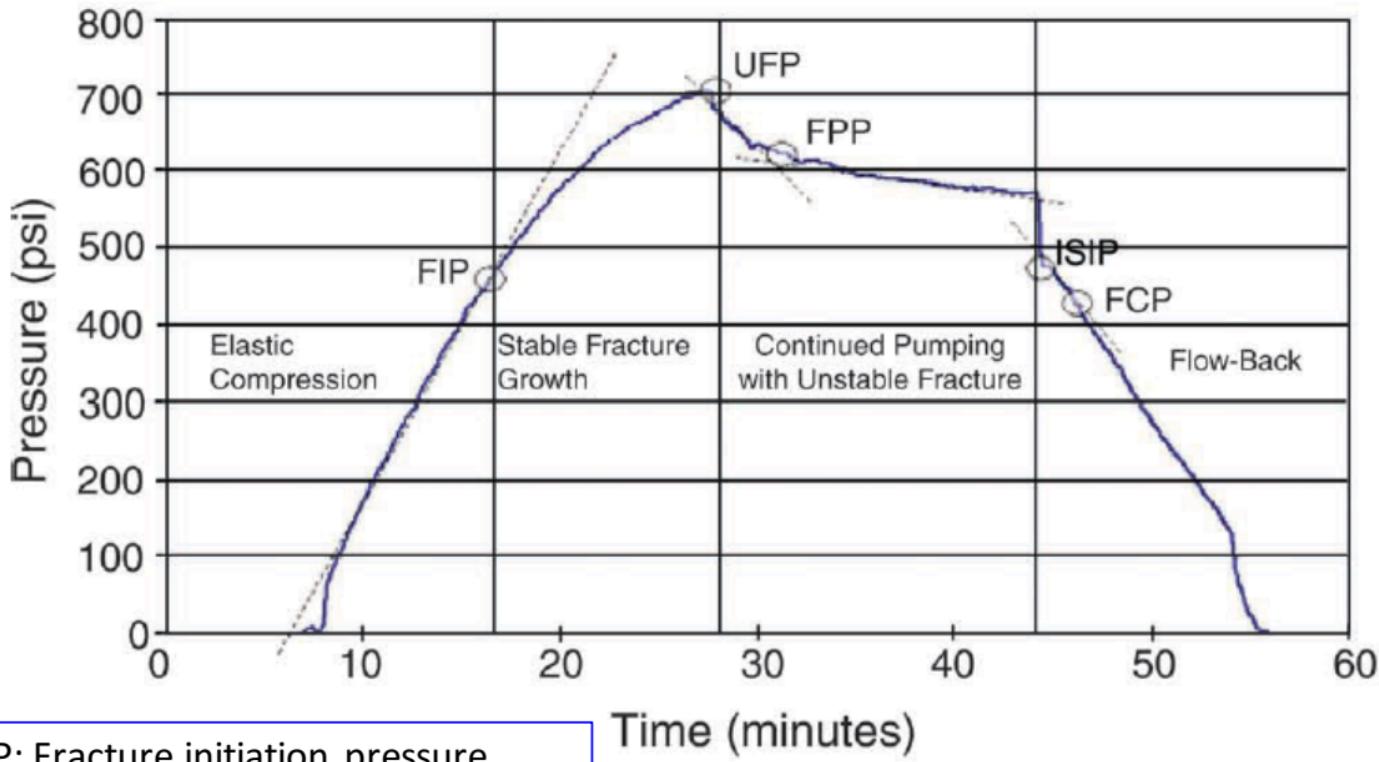






## PLUTONS & VOLCANIC LANDFORMS





FIP: Fracture initiation pressure

UFP: Unstable fracture pressure

FPP: Fracture propagation pressure

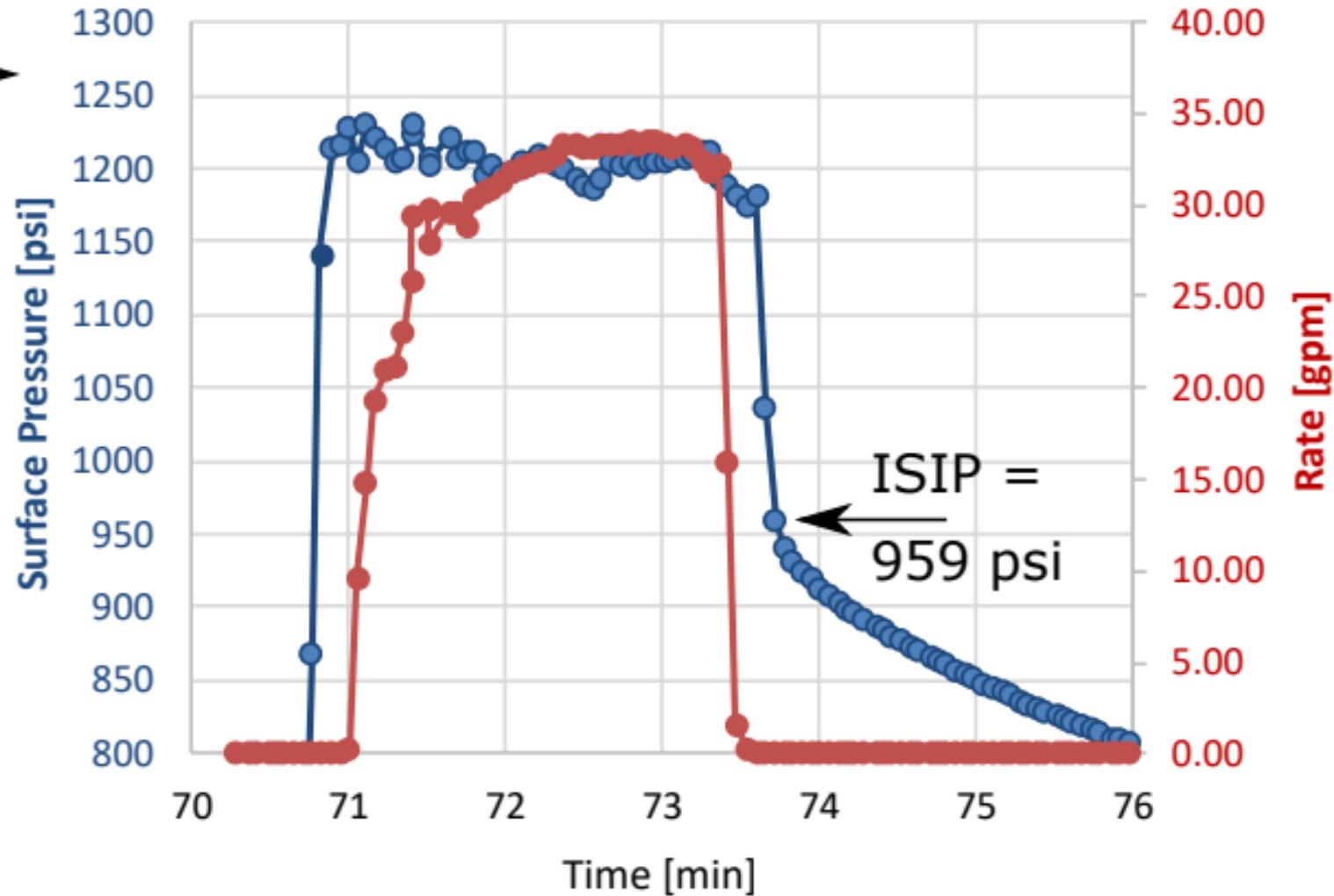
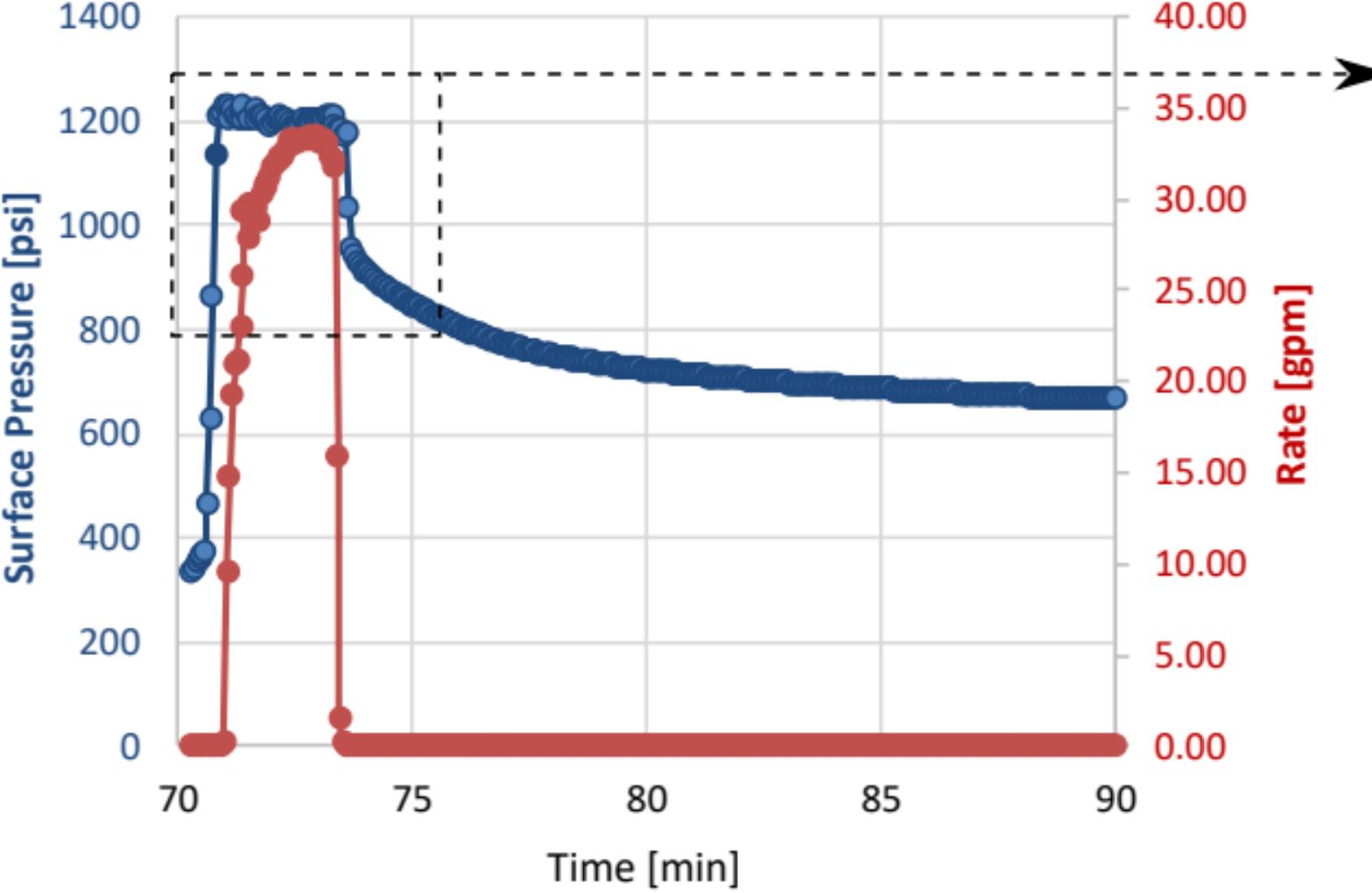
ISIP: Instantaneous shut-in pressure

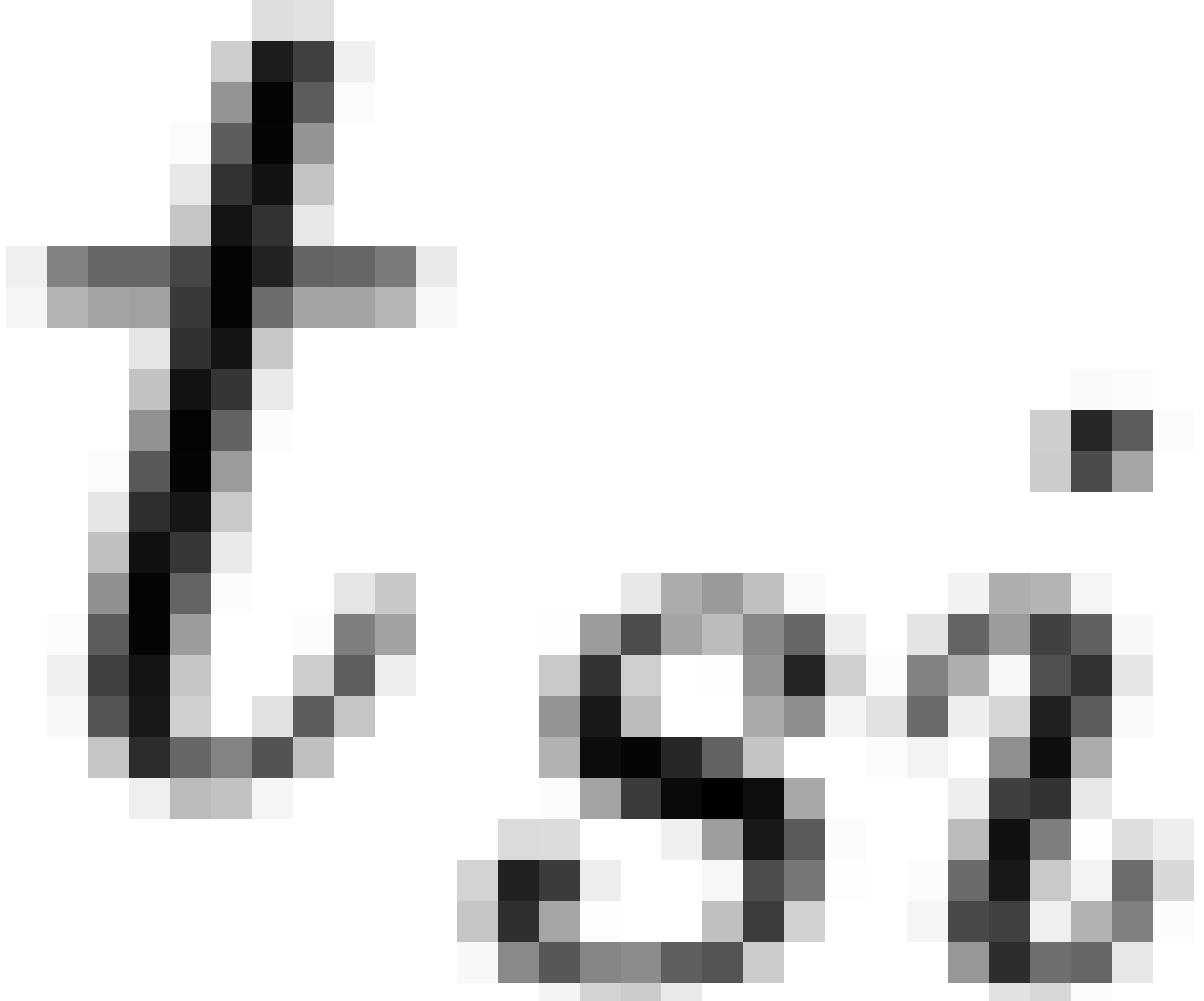
FCP: Fracture closure pressure

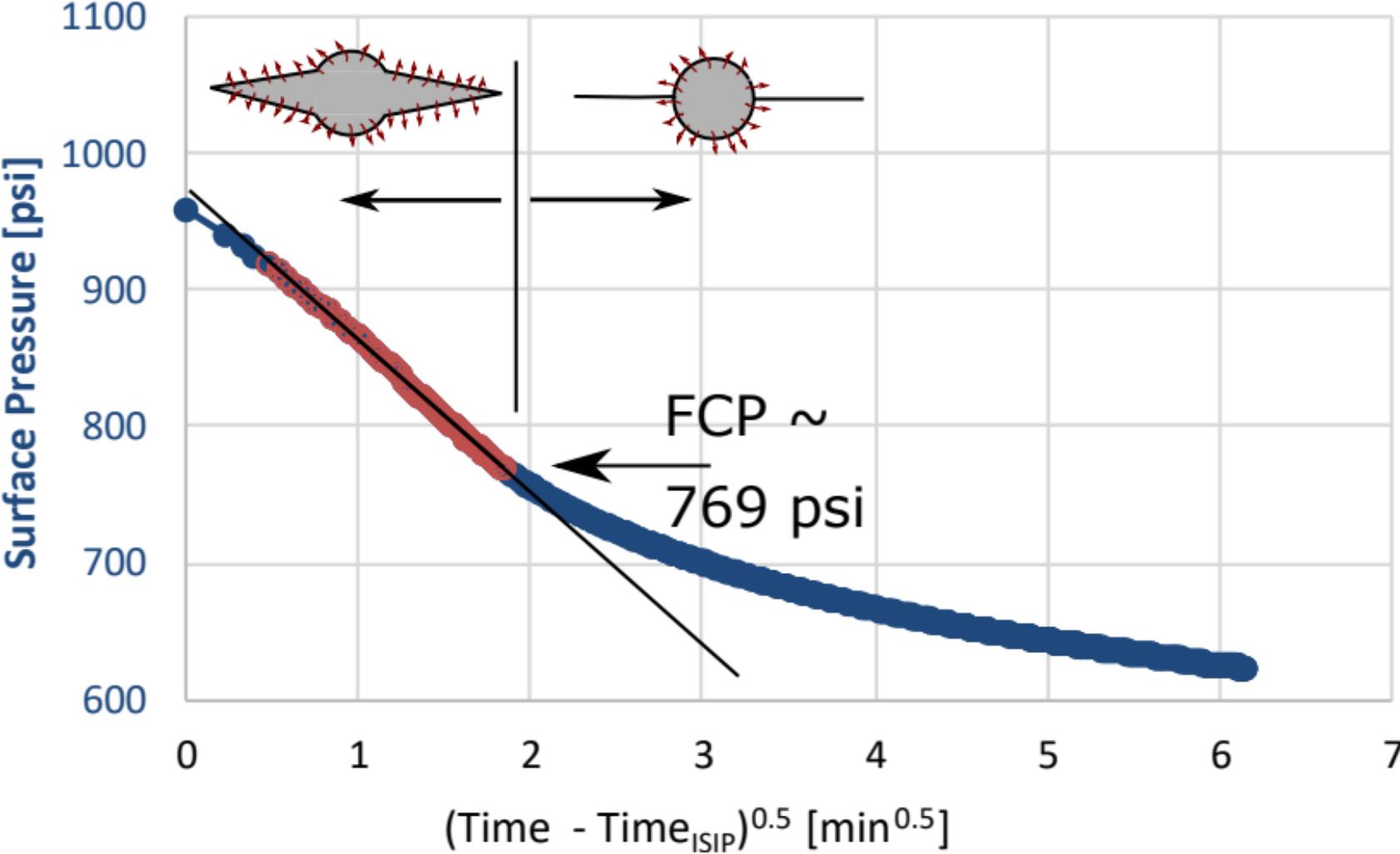
Time (minutes)

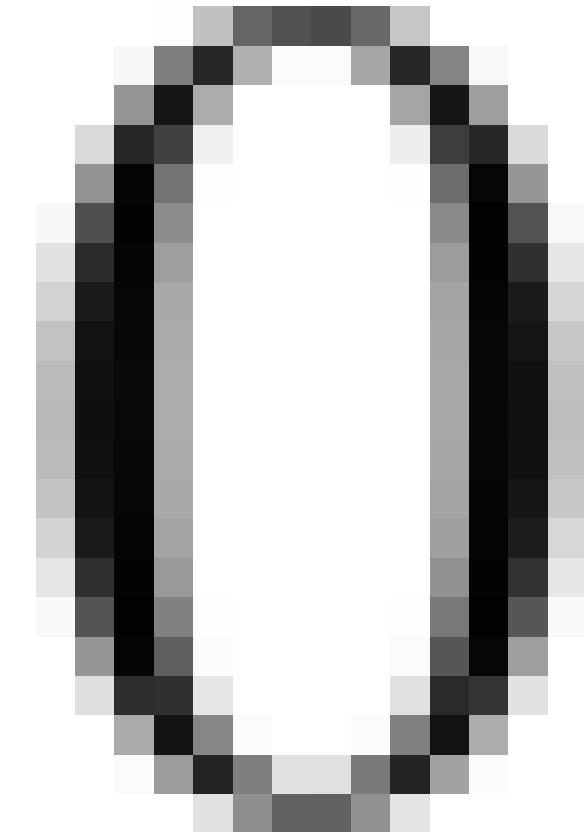
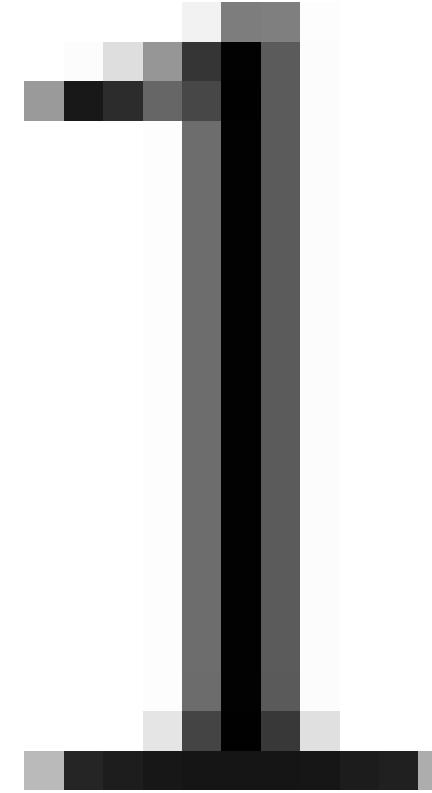
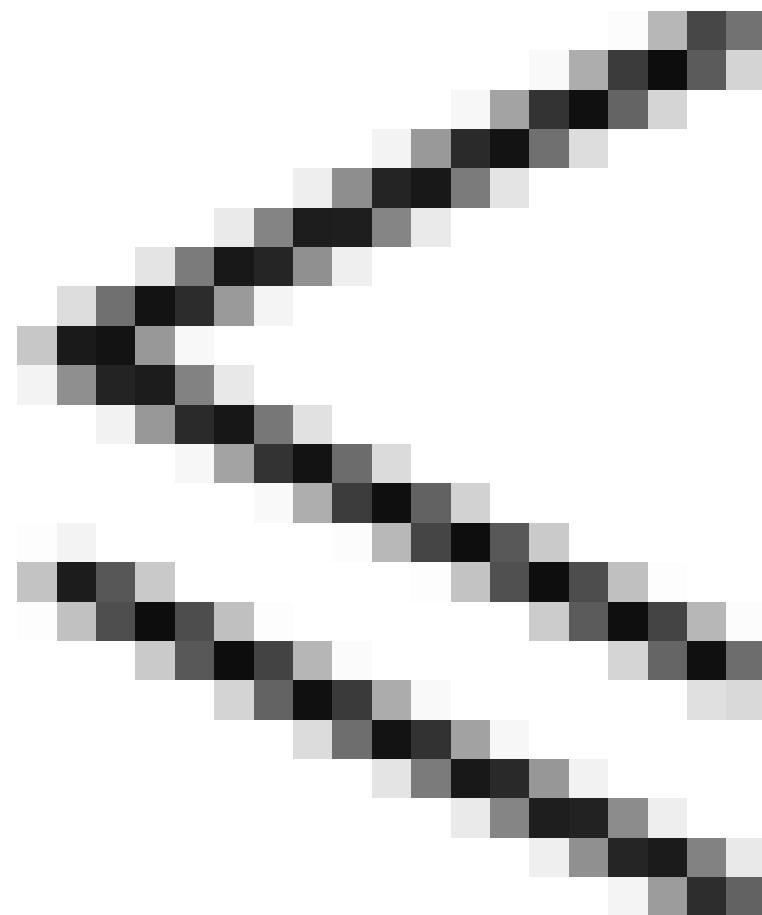
$$\Delta V = q \Delta t$$

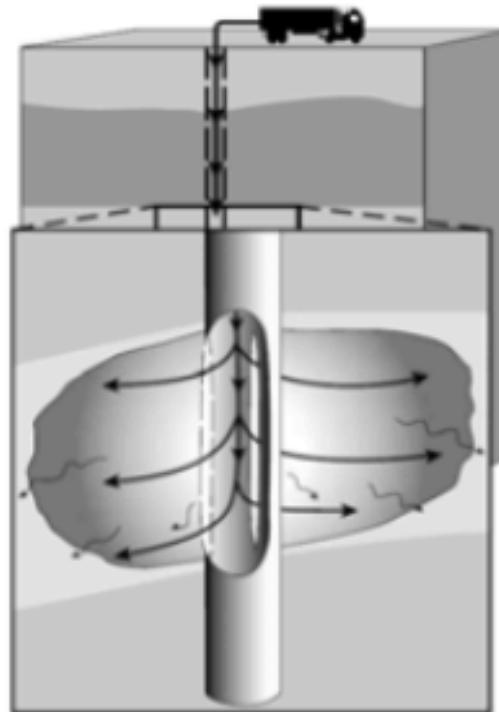
[van Oort and Vargo, 2008]



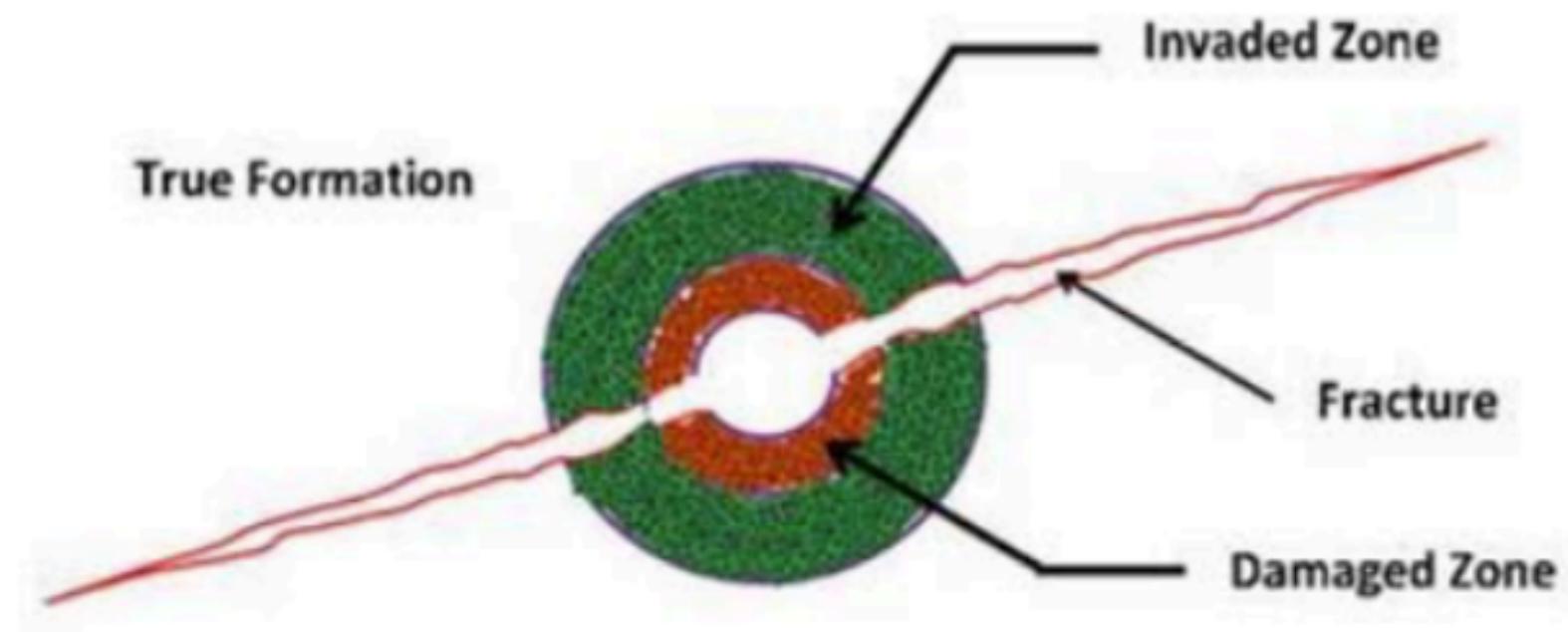








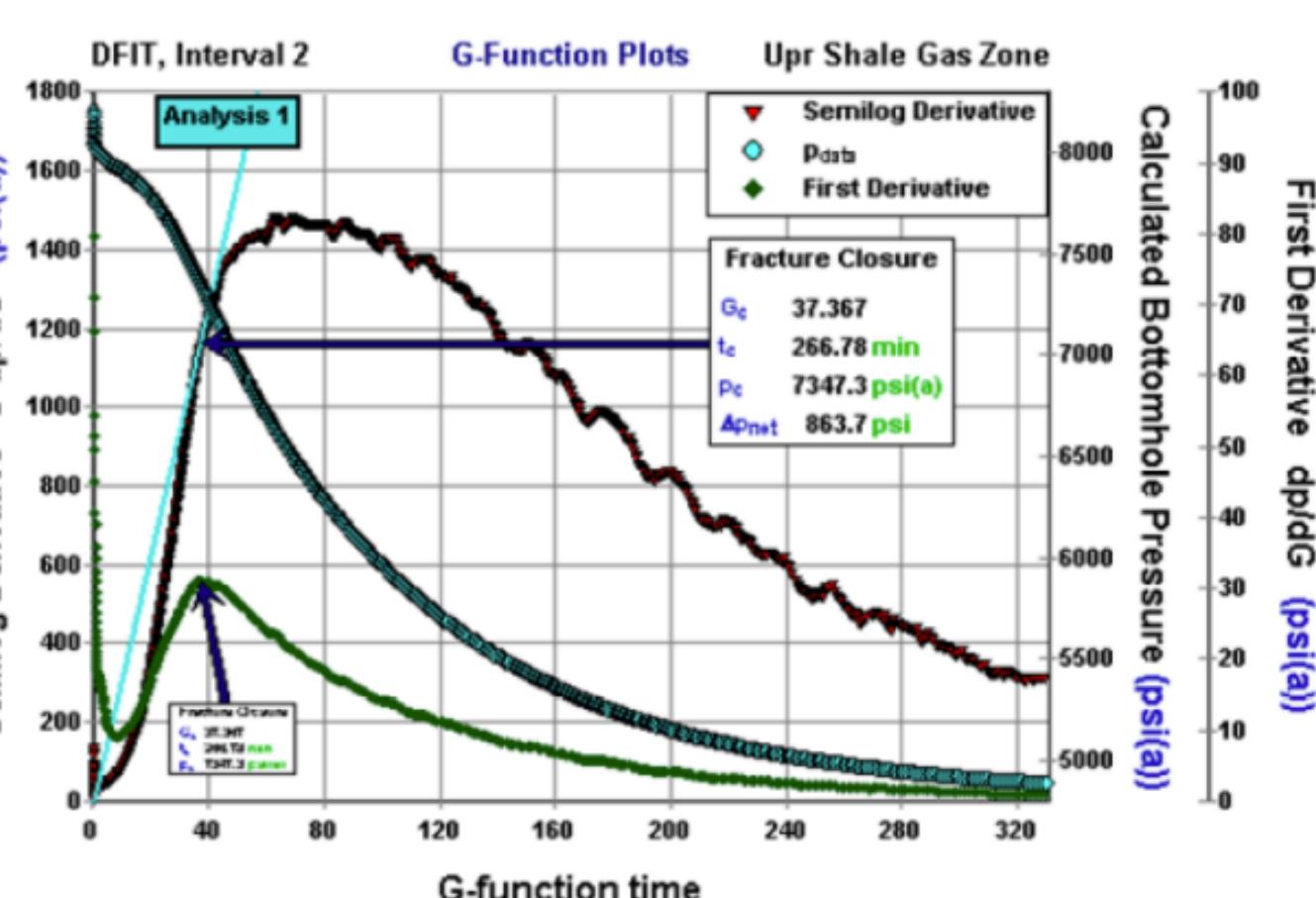
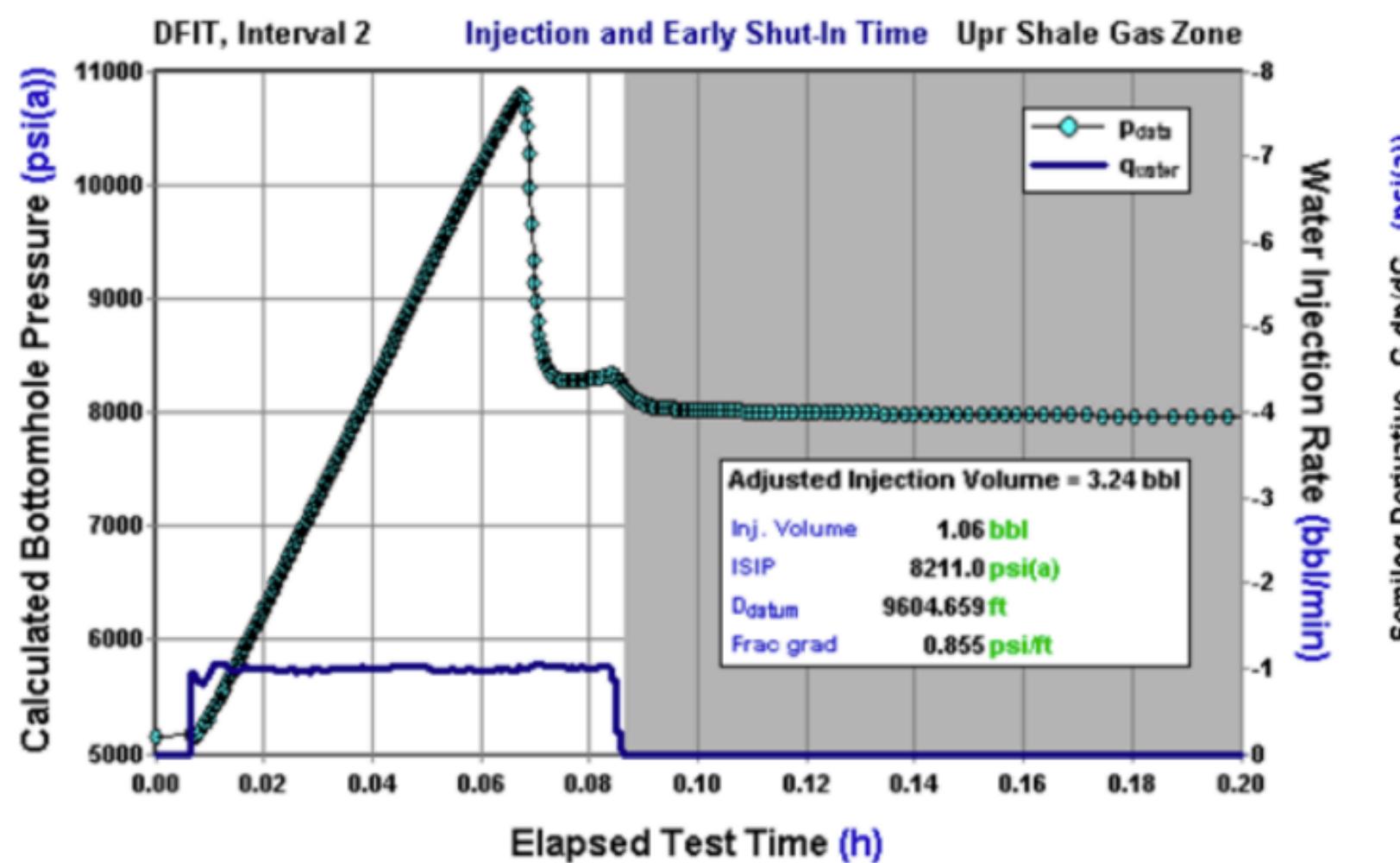
**Side View**



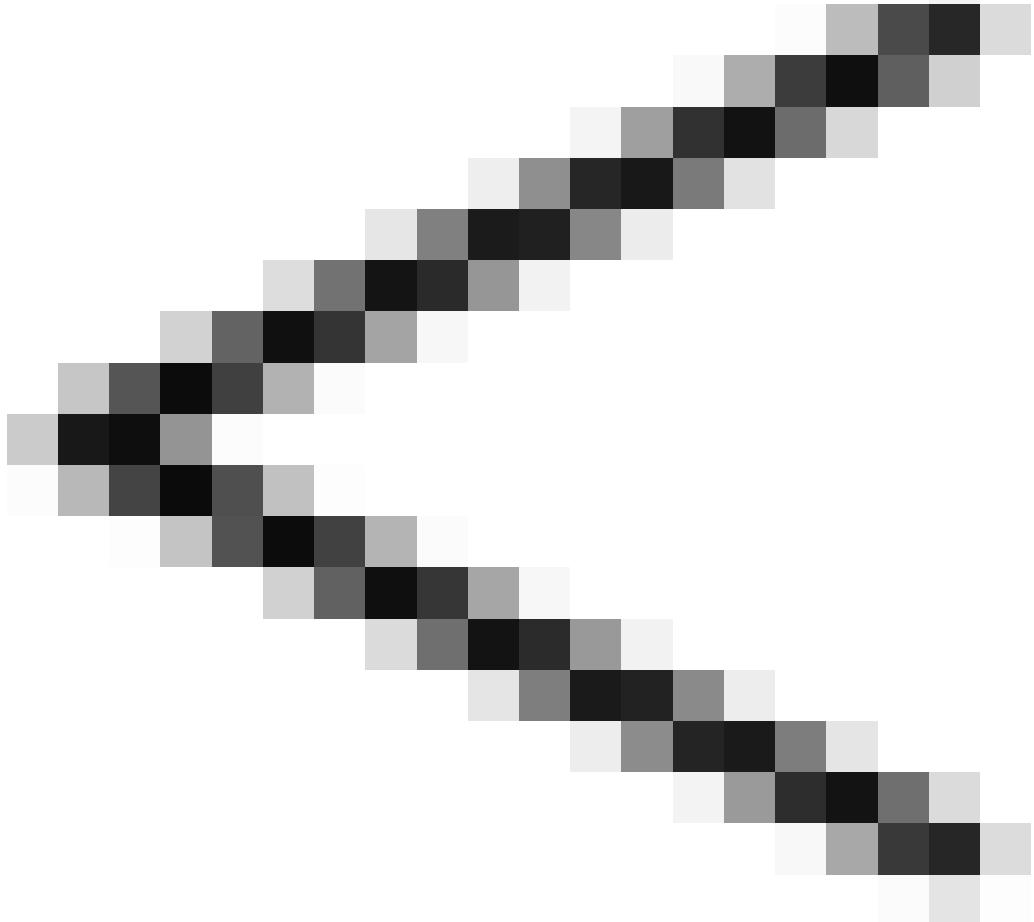
**Plan View**

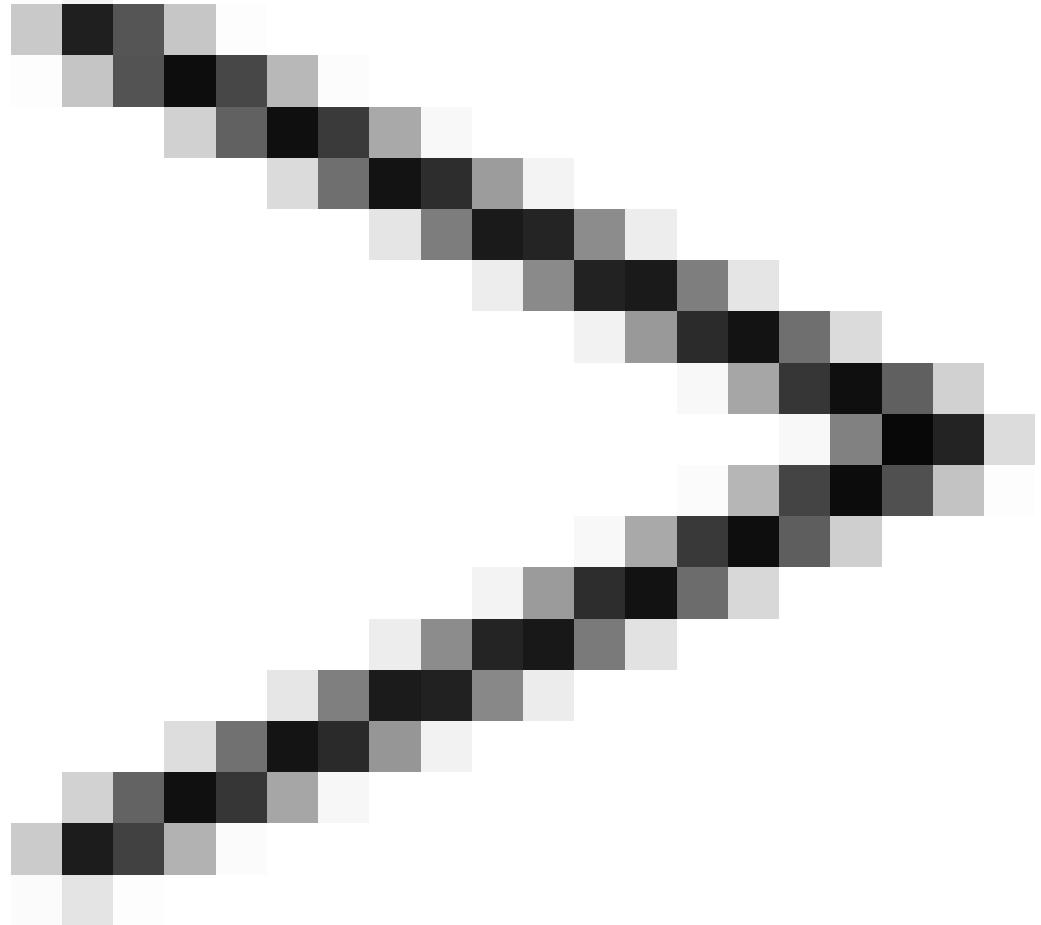
Fekete, 2011

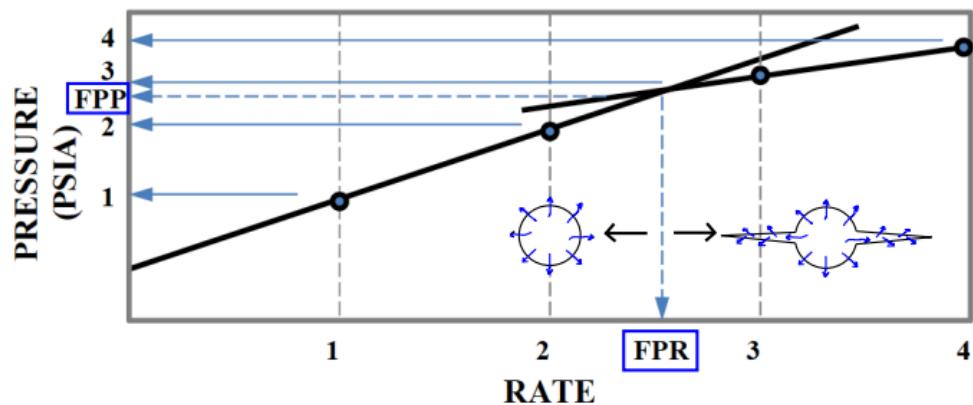
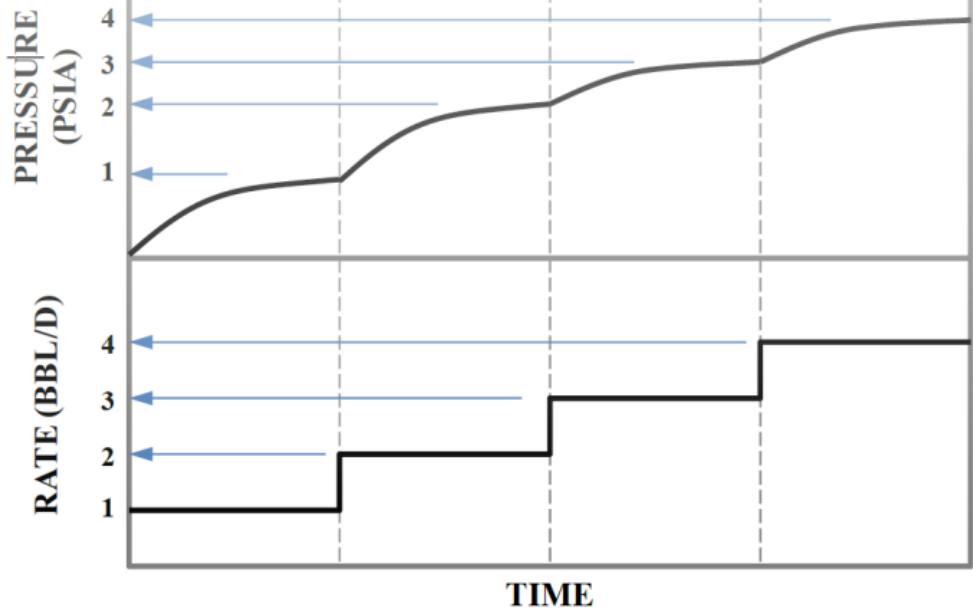
[SPE 163863]

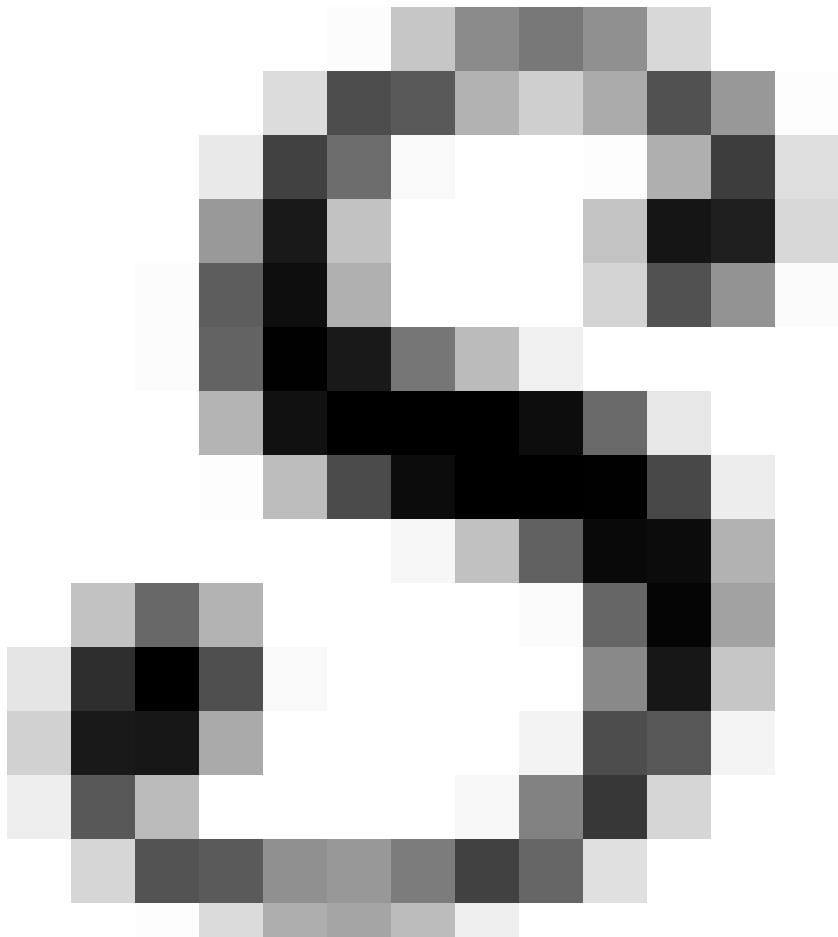


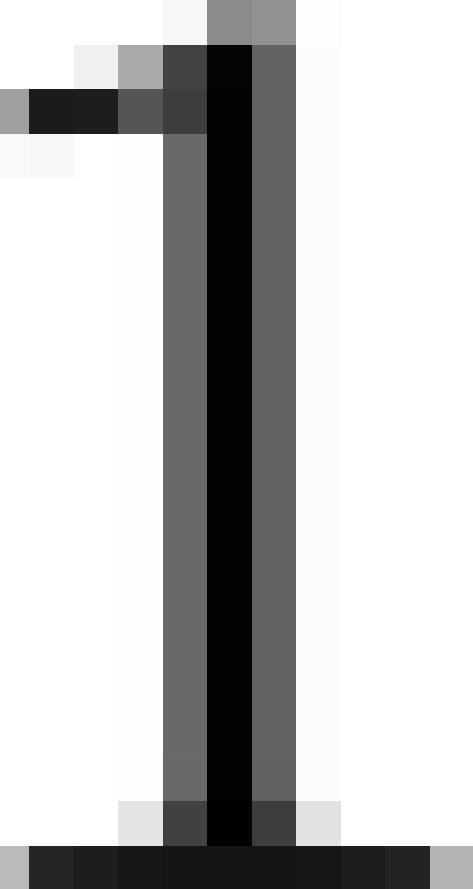
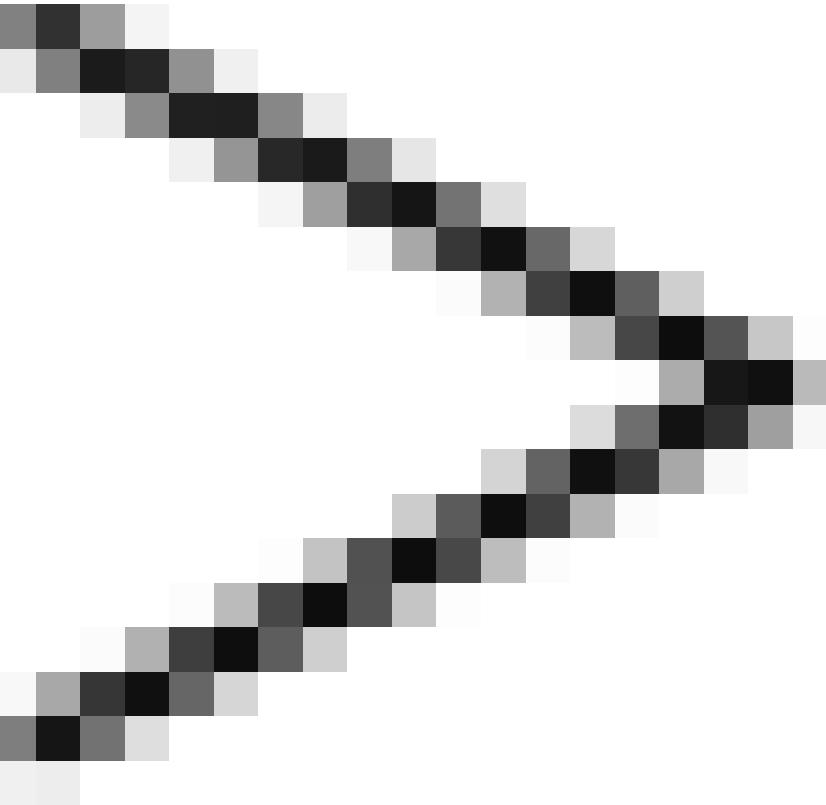
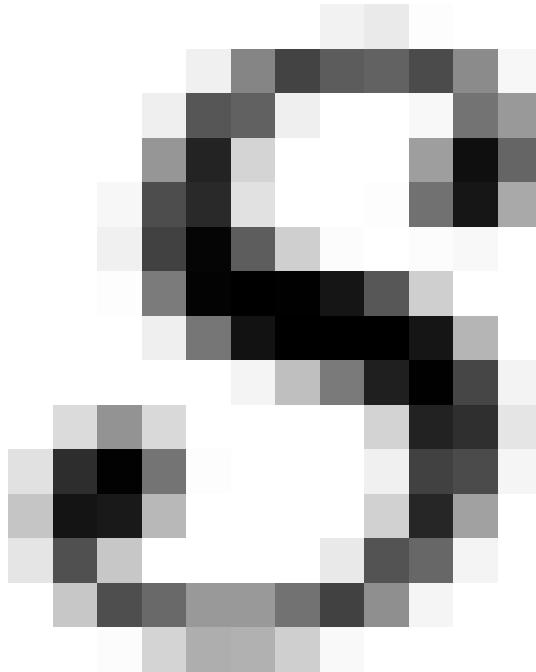
[SPE 163863]

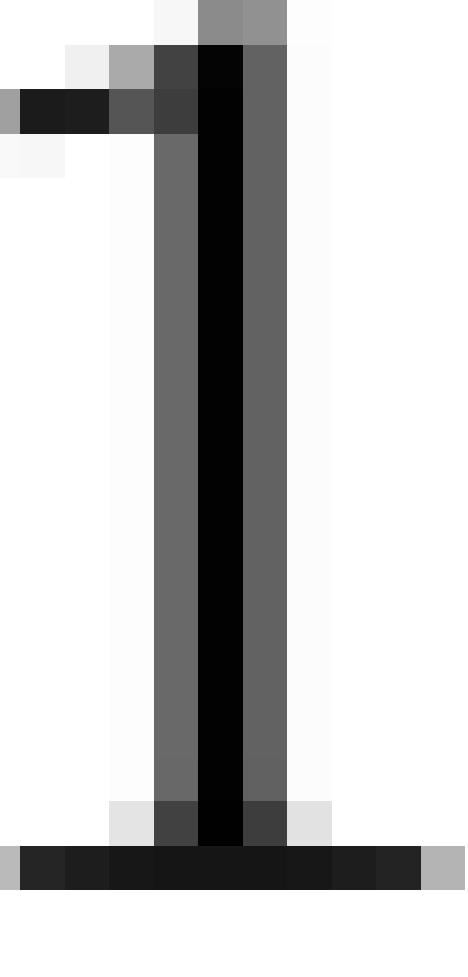
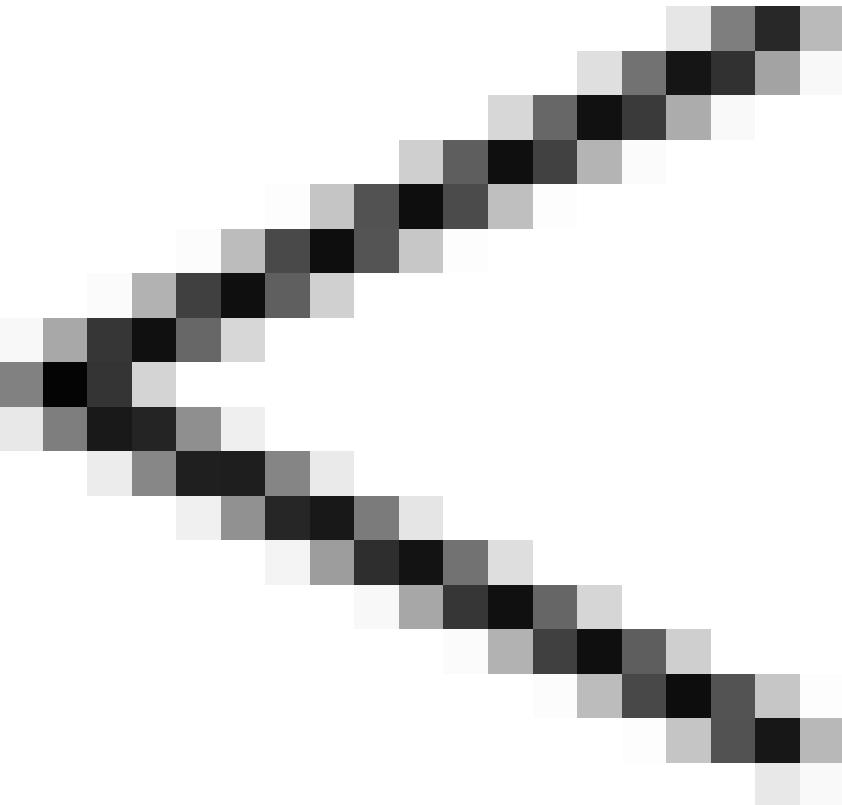
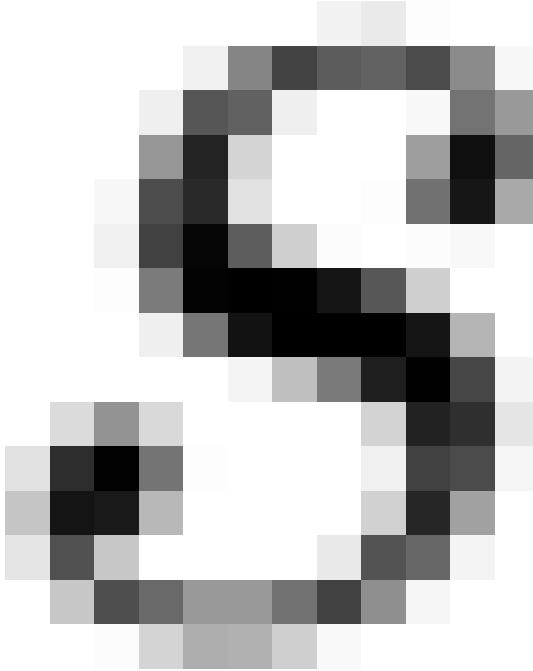










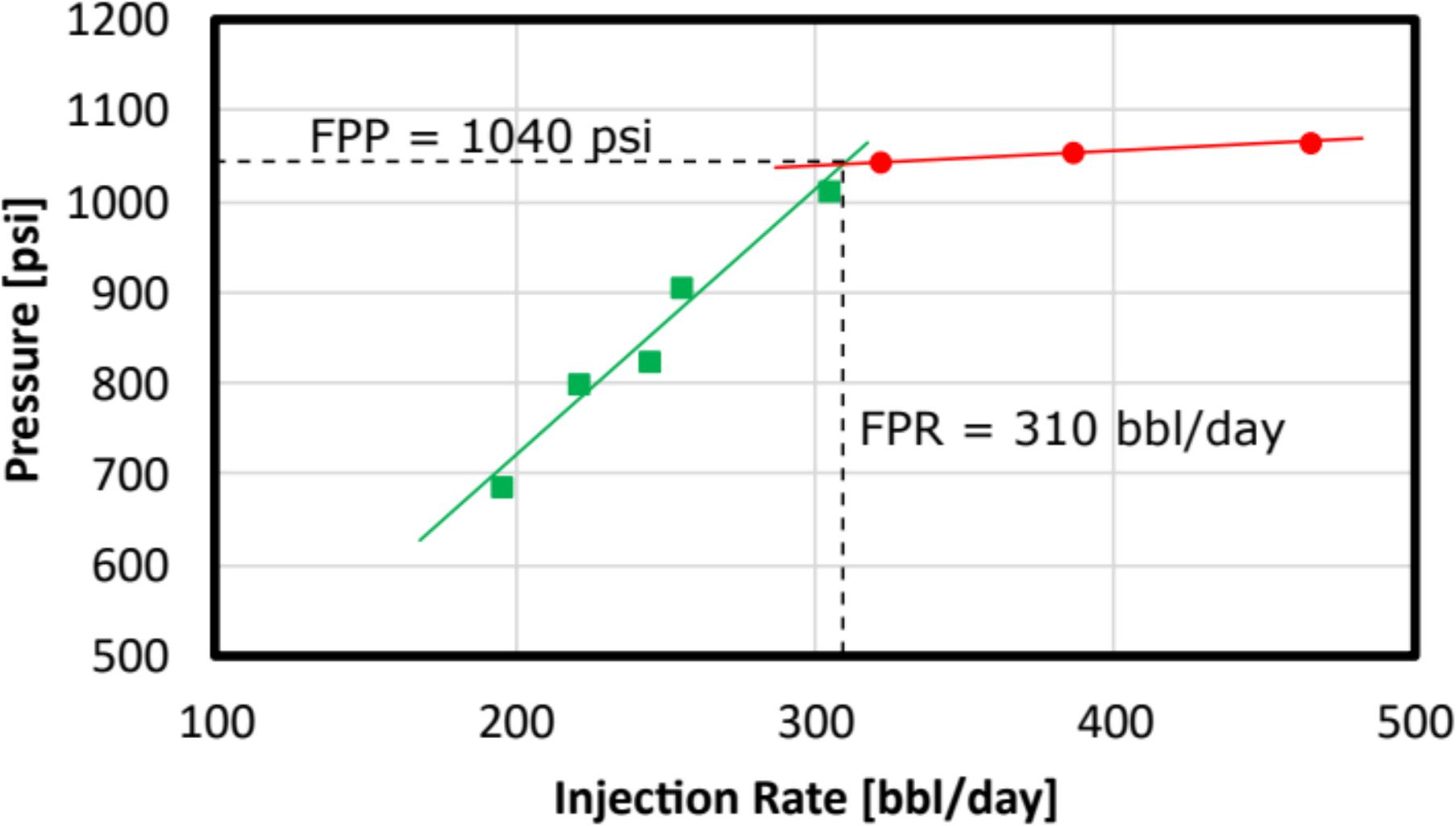


$q$

$$2\pi\hbar k$$

$\mu$

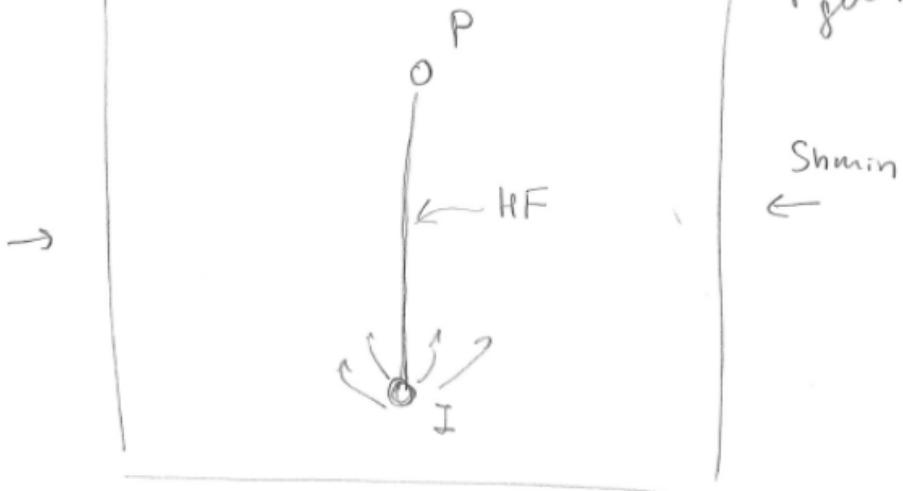
$$\frac{P_e - P_w}{\ln\left(\frac{\tau_e}{\tau_w}\right) + s}$$



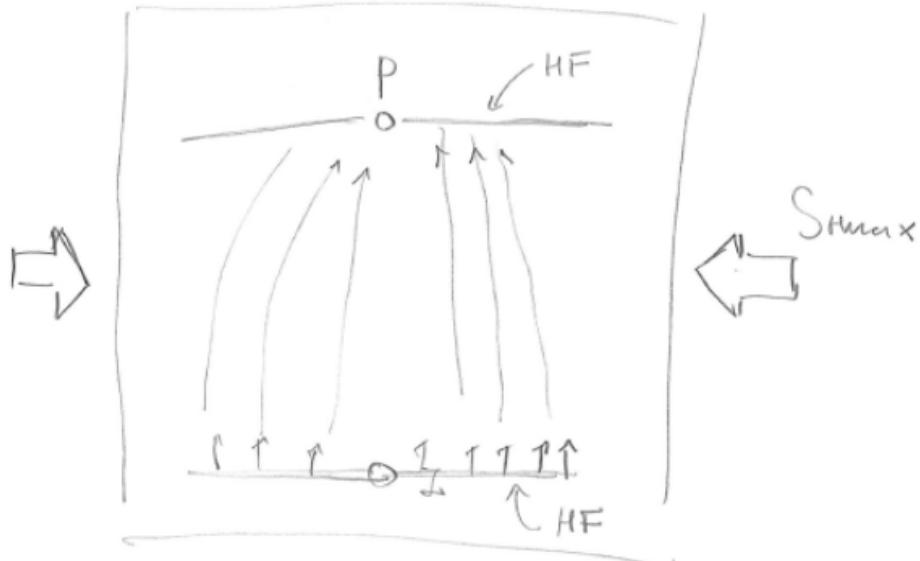
(NF)

$\downarrow S_{\text{max}}$

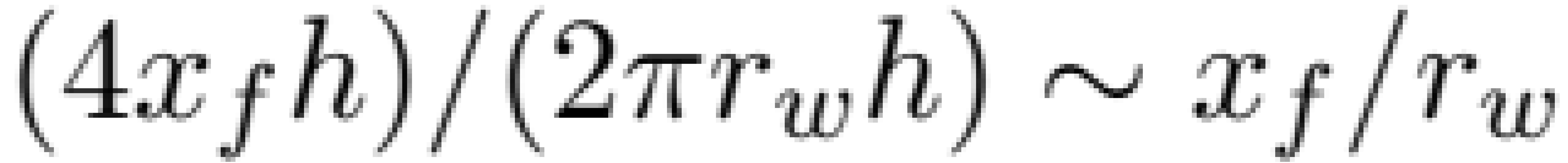
Not good



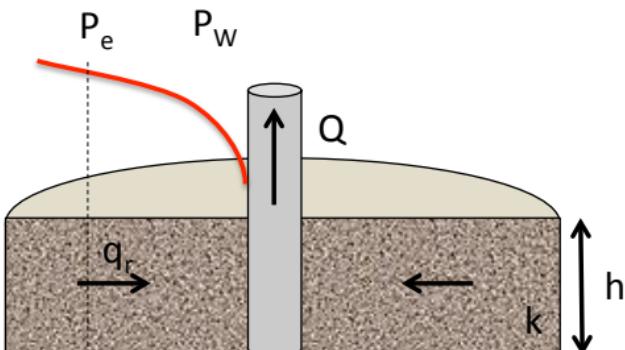
$\downarrow S_{\text{min}}$







## Intact

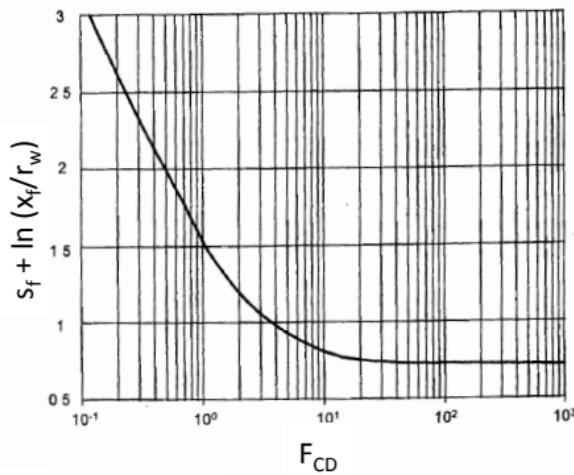
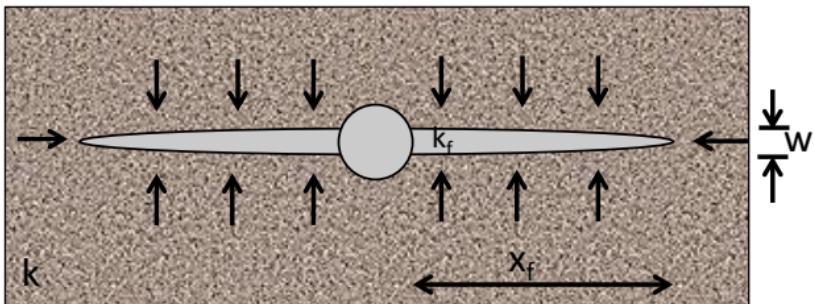


$$Q = -\frac{2\pi kh}{\mu} \frac{(P_e - P_w)}{p_D + s}$$

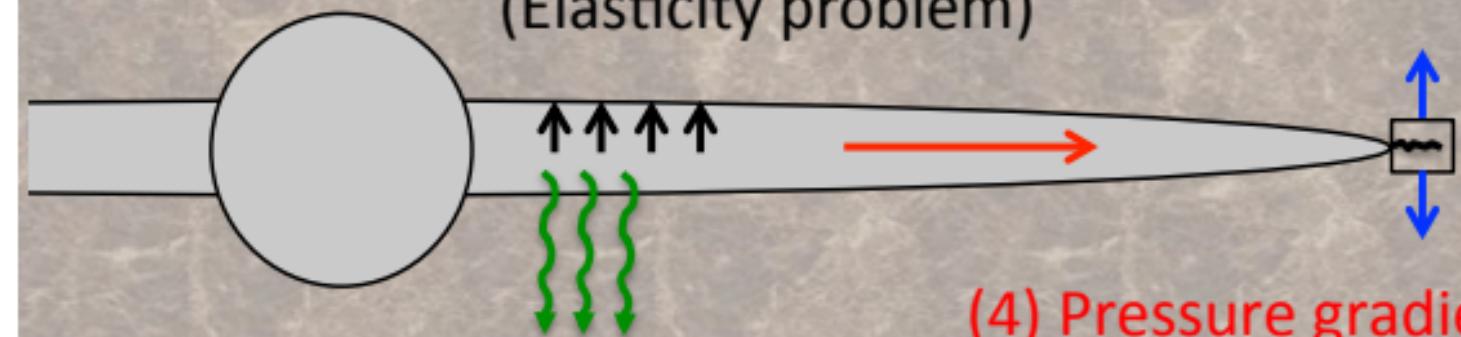
$p_D$ : Dimensionless pressure  
 $= \ln(r_e/r_w)$  if steady state

$s$ : skin factor  
 $> 1 \rightarrow$  damage  
 $< 1 \rightarrow$  stimulation

## Fractured



$w, x_f, k_f \rightarrow F_{CD} = (k_f w)/(k x_f)$   
 $x_f, r_w, F_{CD} \rightarrow s_f$  from above plot



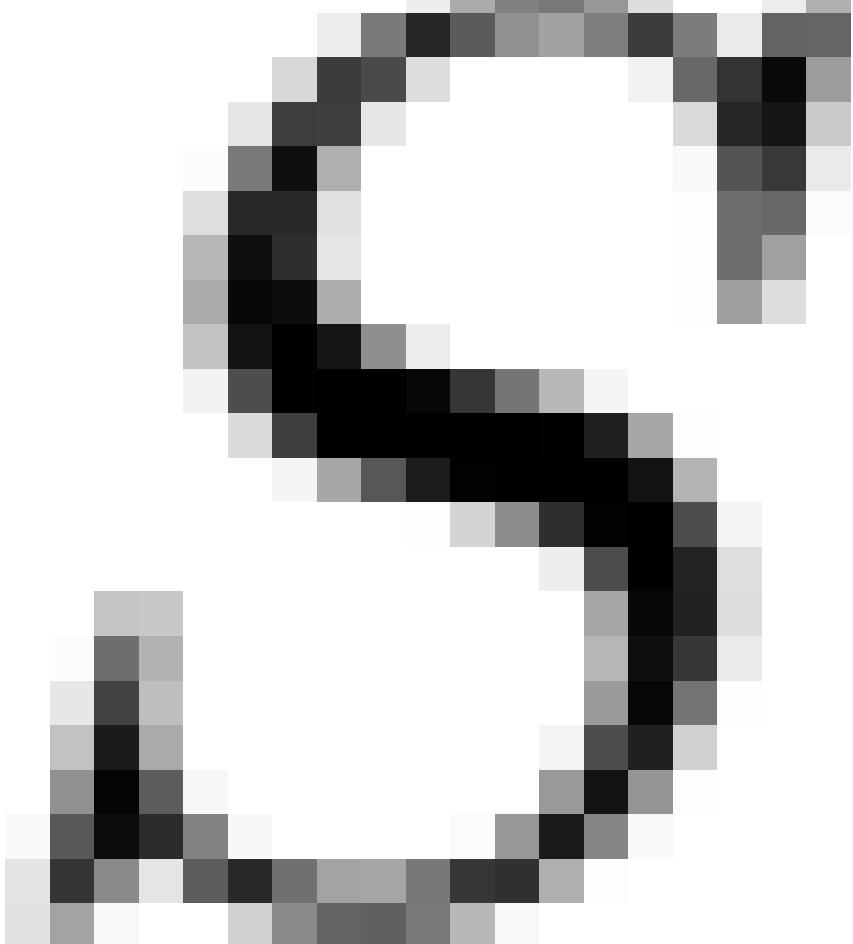
(1) Pressure on the fracture deforms adjacent rock  
(Elasticity problem)

(2) Fracture propagates if the “stress intensity factor” is higher than what the rock can resist “rock toughness”  
(Fracture mechanics problem)

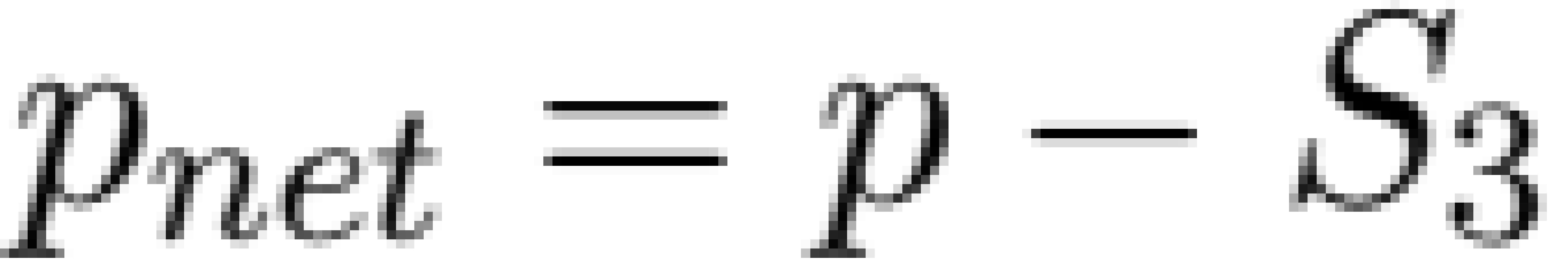
(3) Fracturing fluids can leak off to the formation  
(Mud-cake design)

(4) Pressure gradient leads to flow of fracturing fluid through the fracture  
(Lubrication problem)

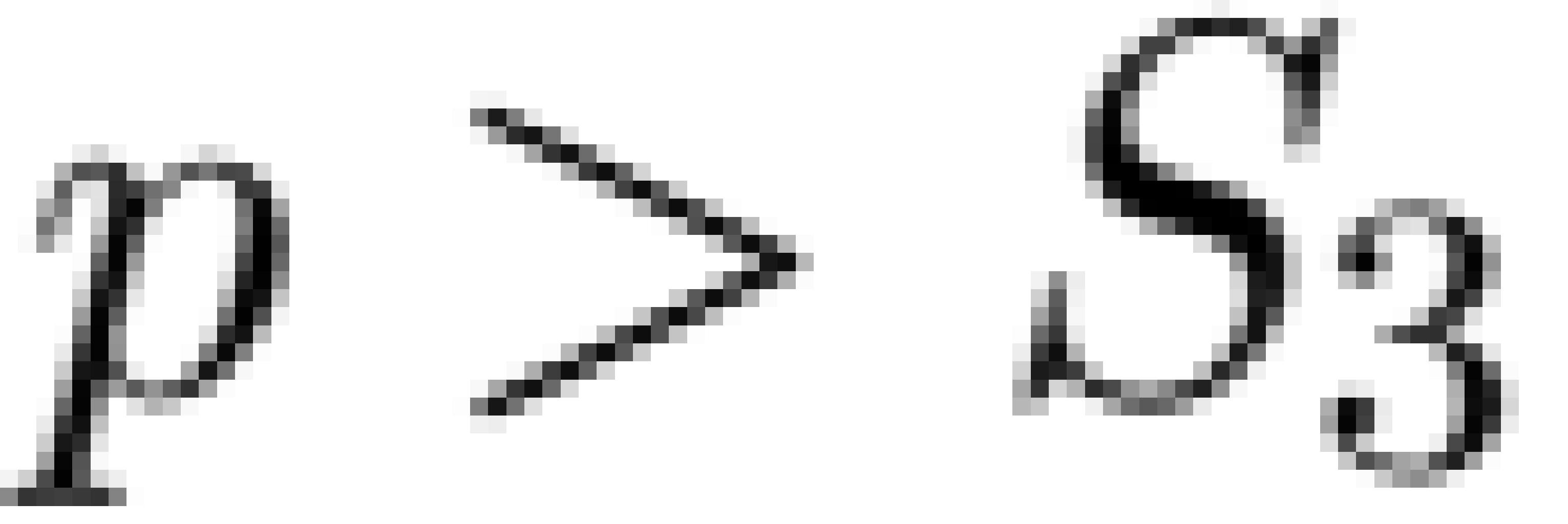
Net pressure  
 $p_{\text{net}} = p - S_3$

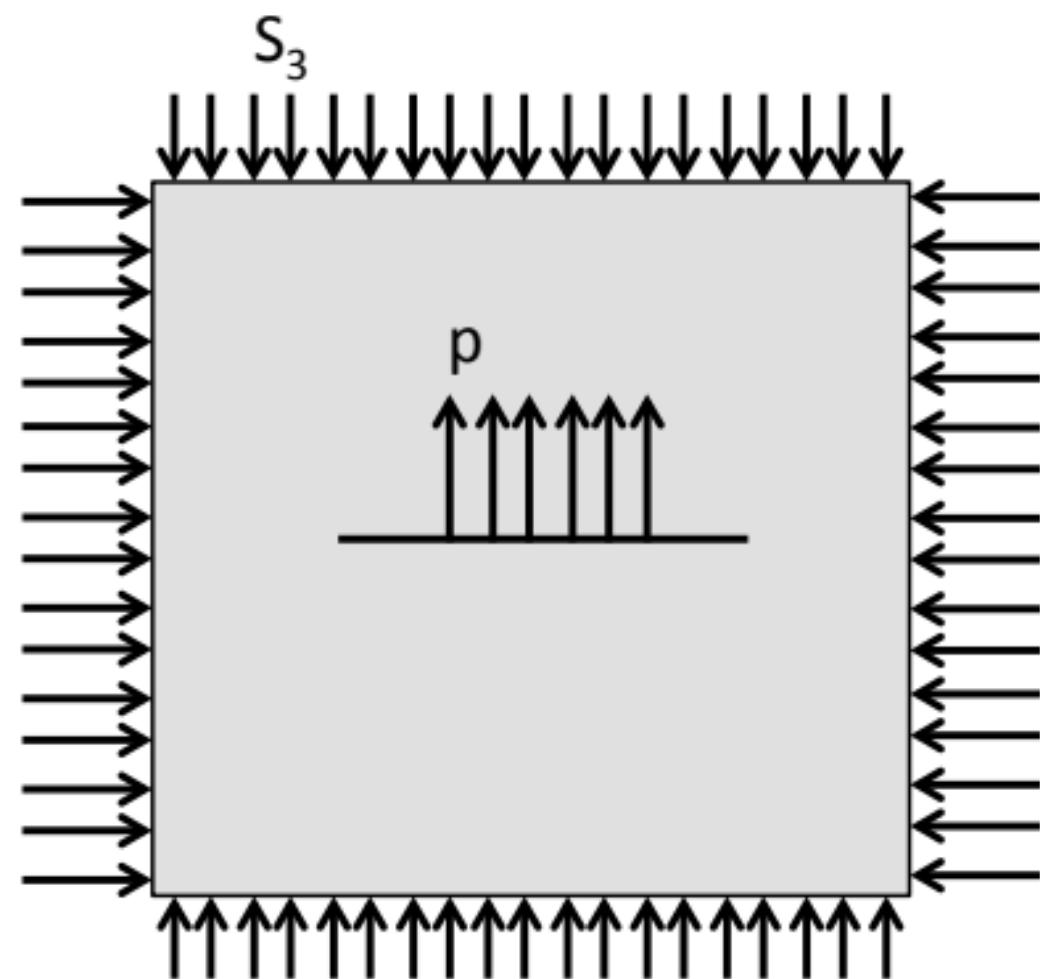




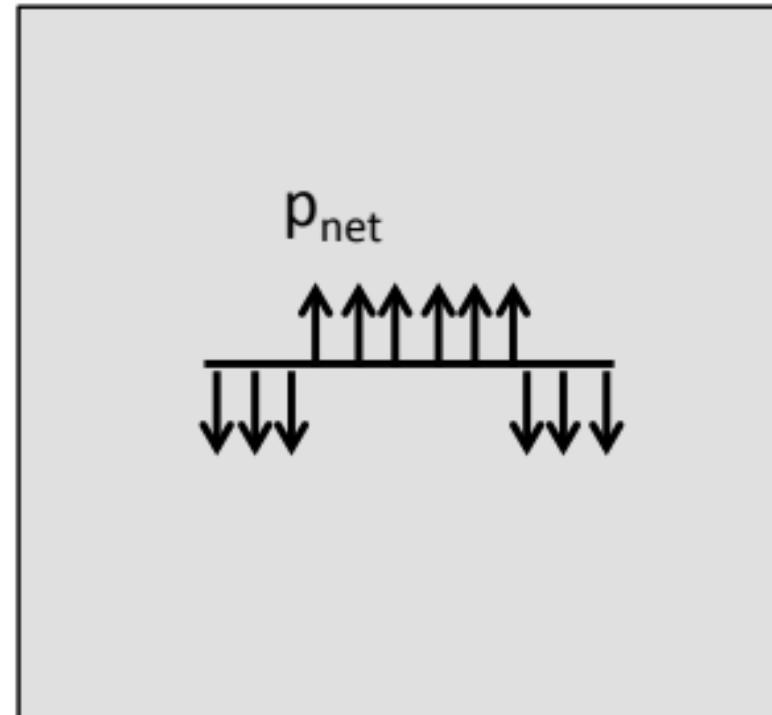




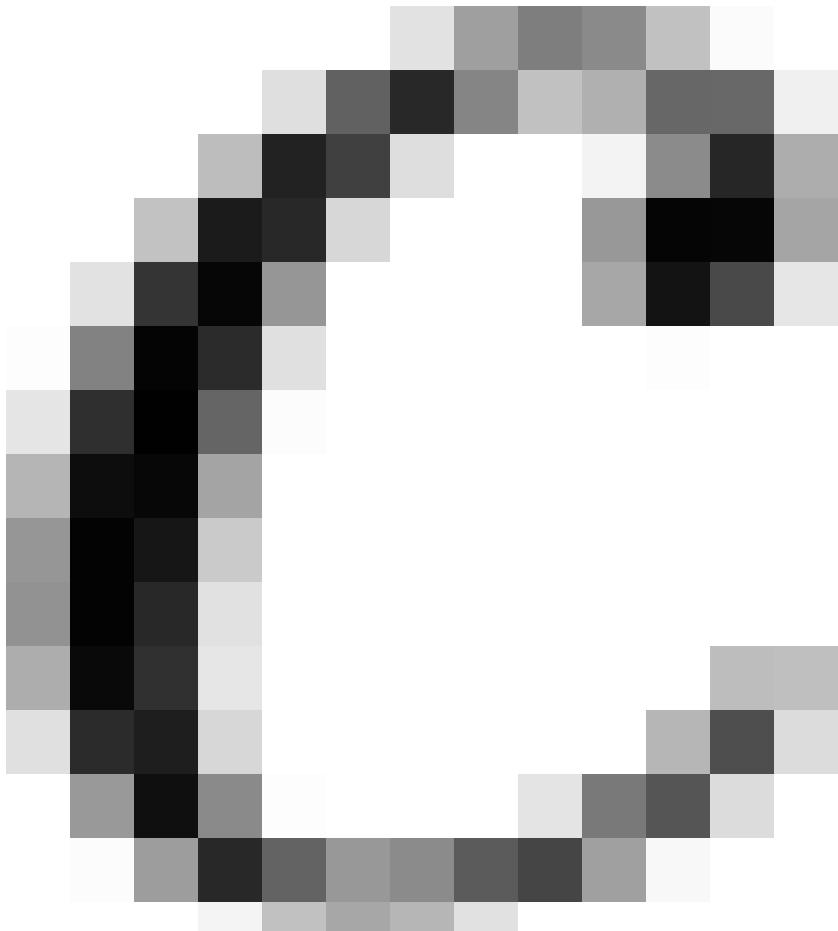




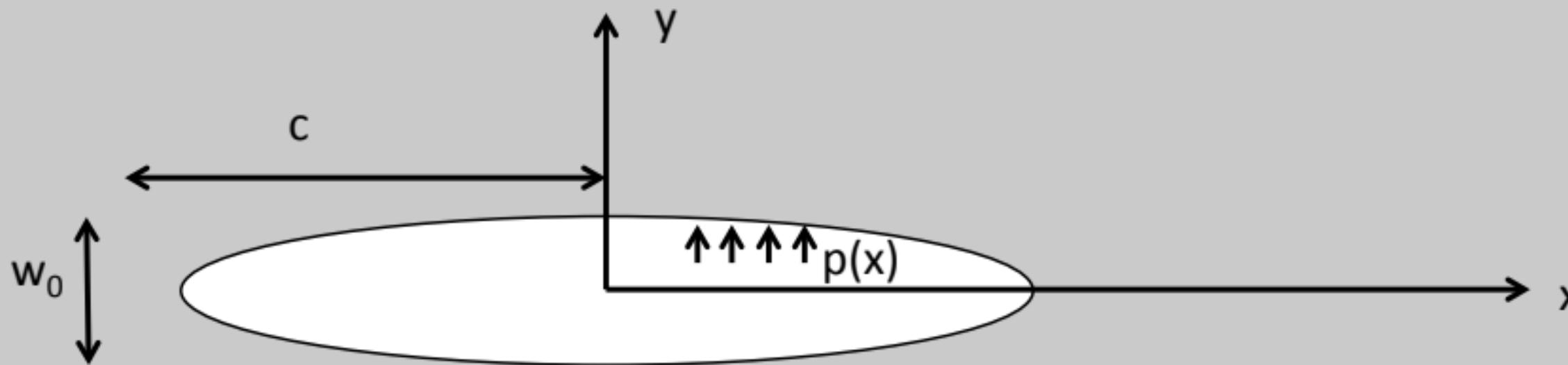
≡







$$\begin{cases} \sigma_{yy} = p(x) & \text{for } 0 \leq x \leq c \\ u_y = 0 & \text{for } x > c \end{cases}$$

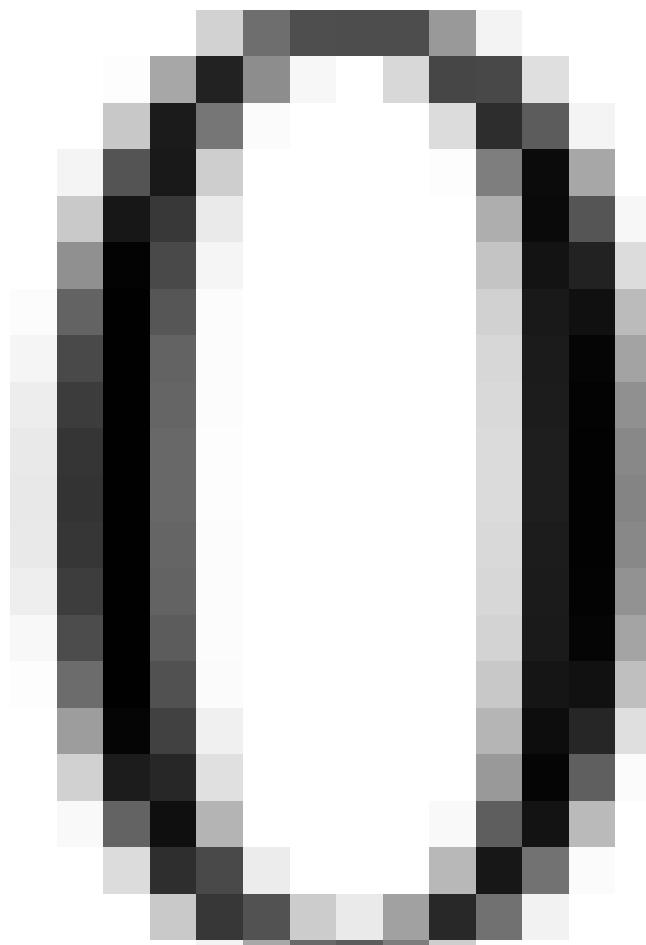
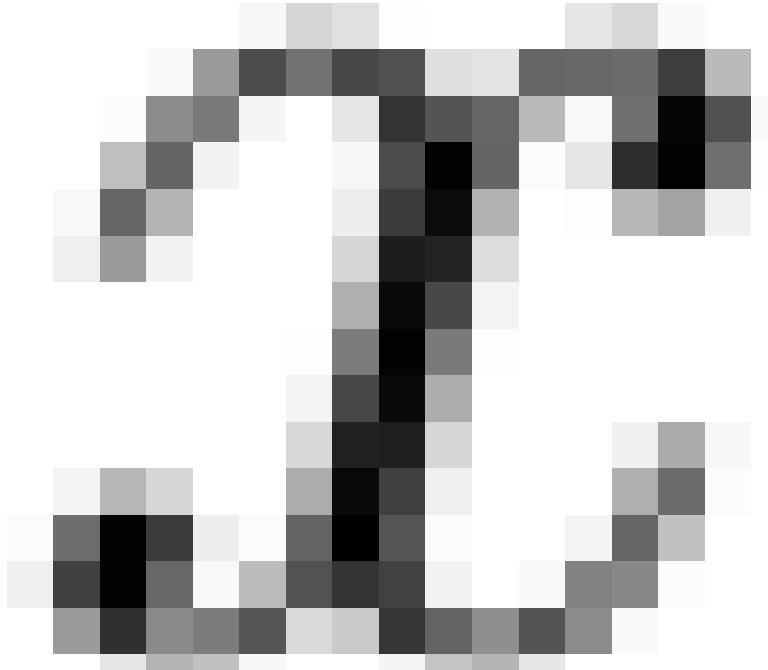


Linear elastic and homogeneous solid  
 $(E, v)$   
 $E' = E / (1 - v^2)$



$$\begin{cases} u_y(x, 0) = \frac{2p_o}{E'} \sqrt{c^2 - x^2} & \text{for } 0 \leq x \leq c \\ \sigma_{yy}(x, 0) = -p_o \frac{x}{\sqrt{x^2 - c^2 - 1}} & \text{for } x > c \end{cases}$$









ω

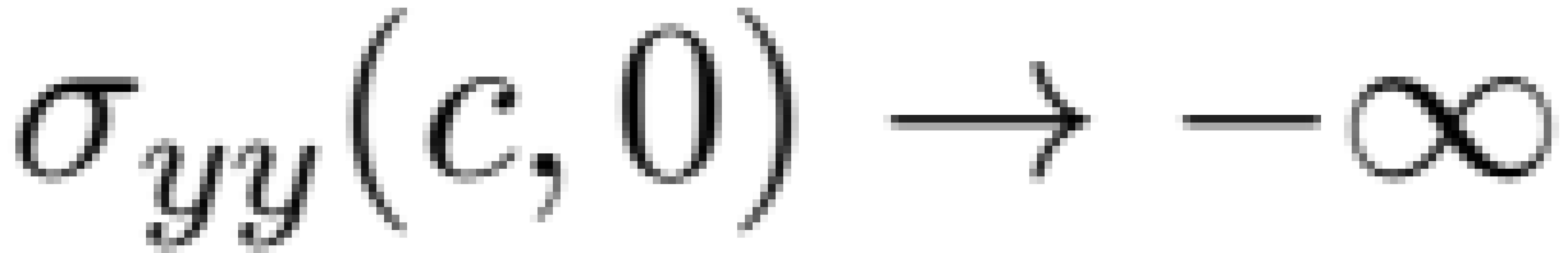


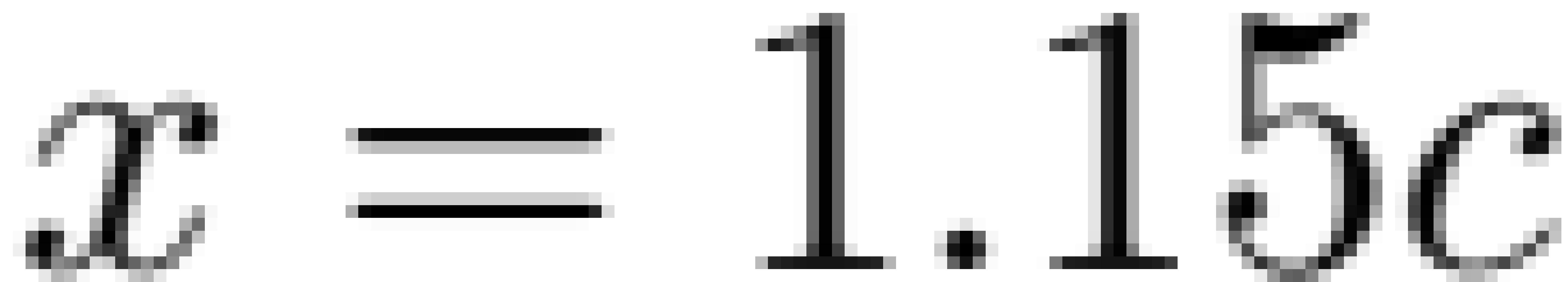
4P0C



EY

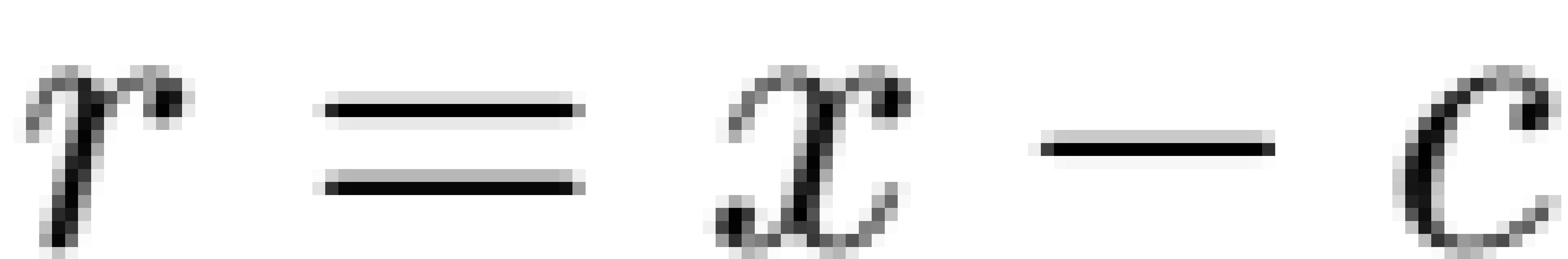


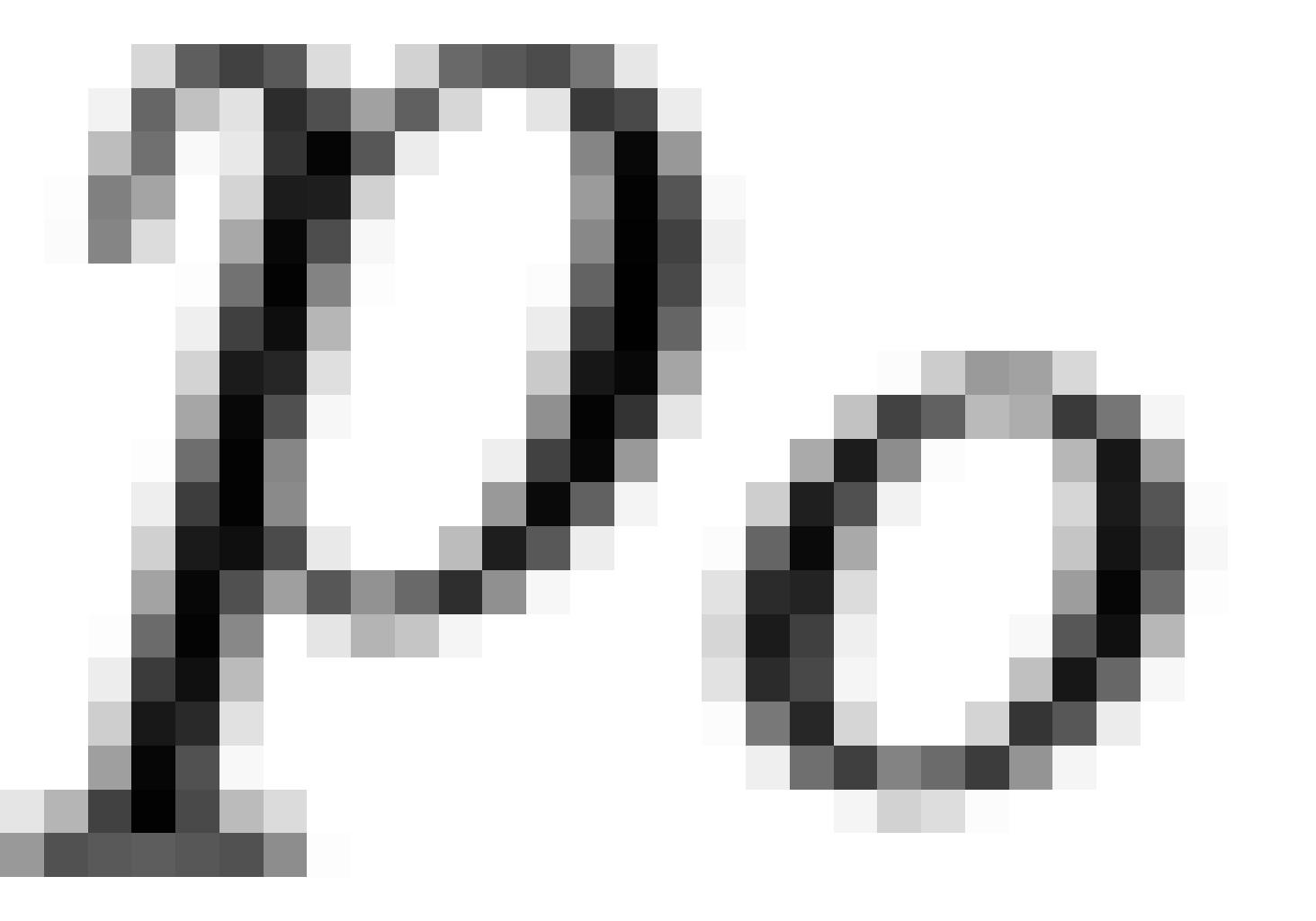


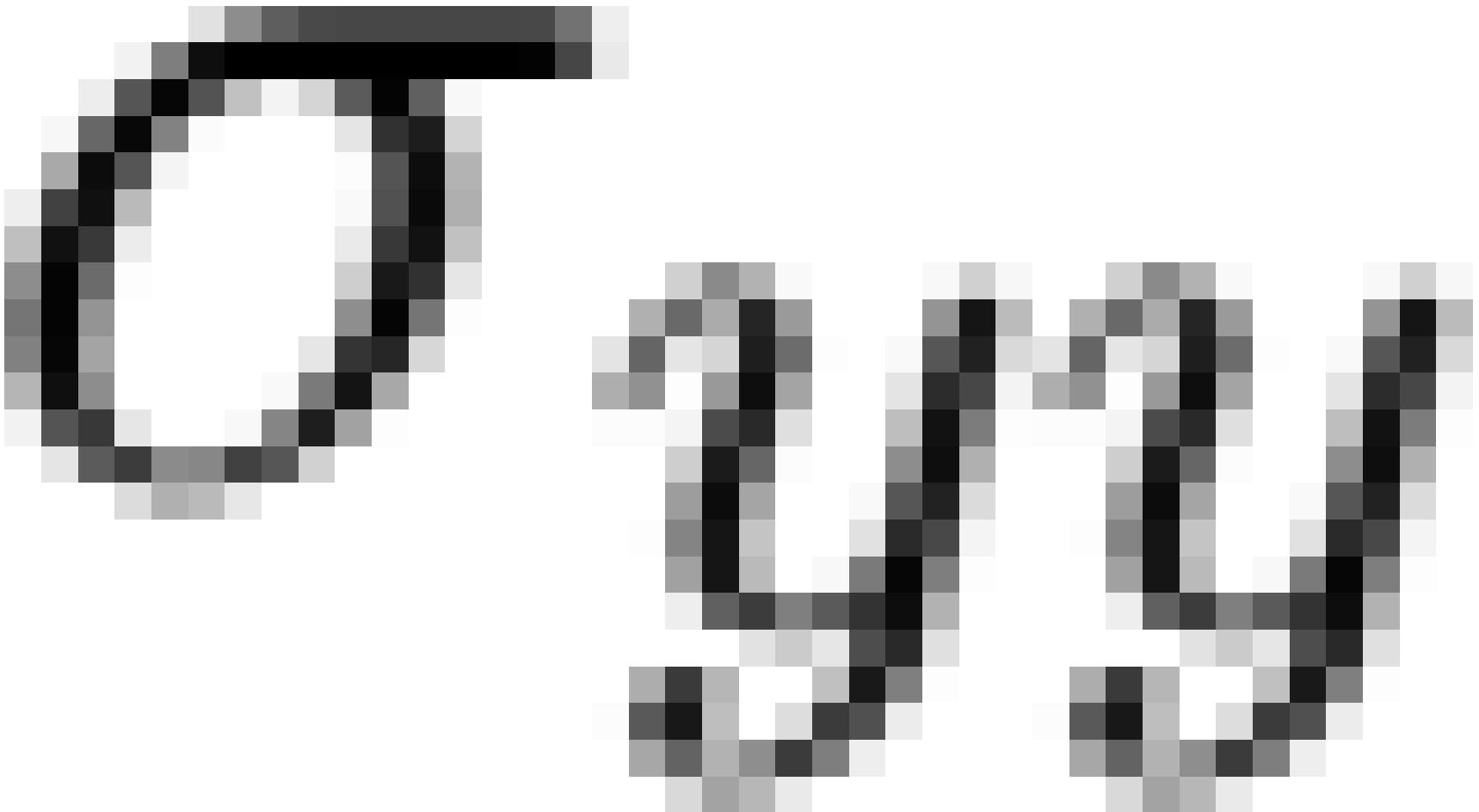




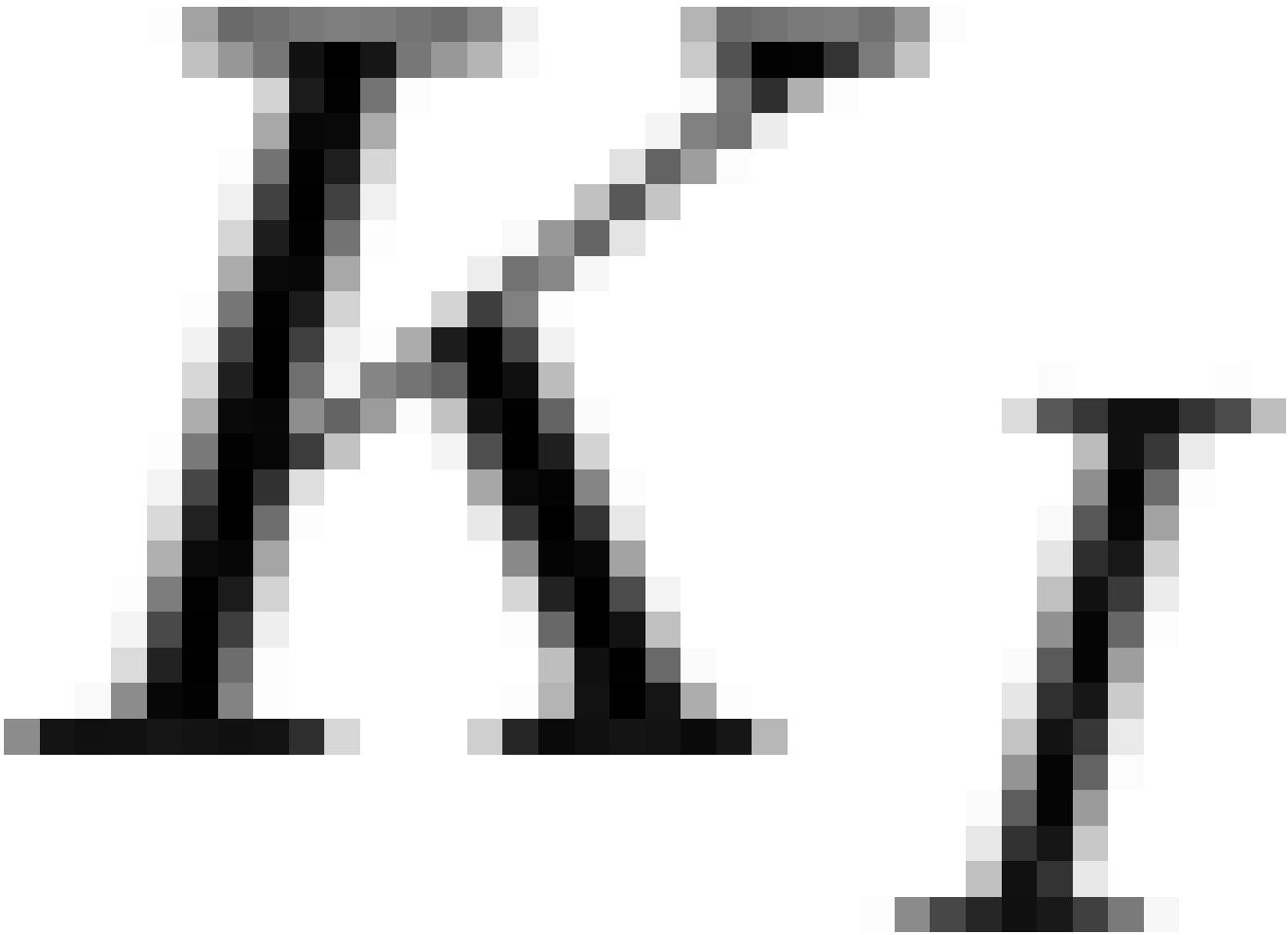
$$\kappa_I = \lim_{r \rightarrow 0^+} \left[ (2\pi r)^{1/2} \sigma_{uv}(c + r, y=0) \right]$$

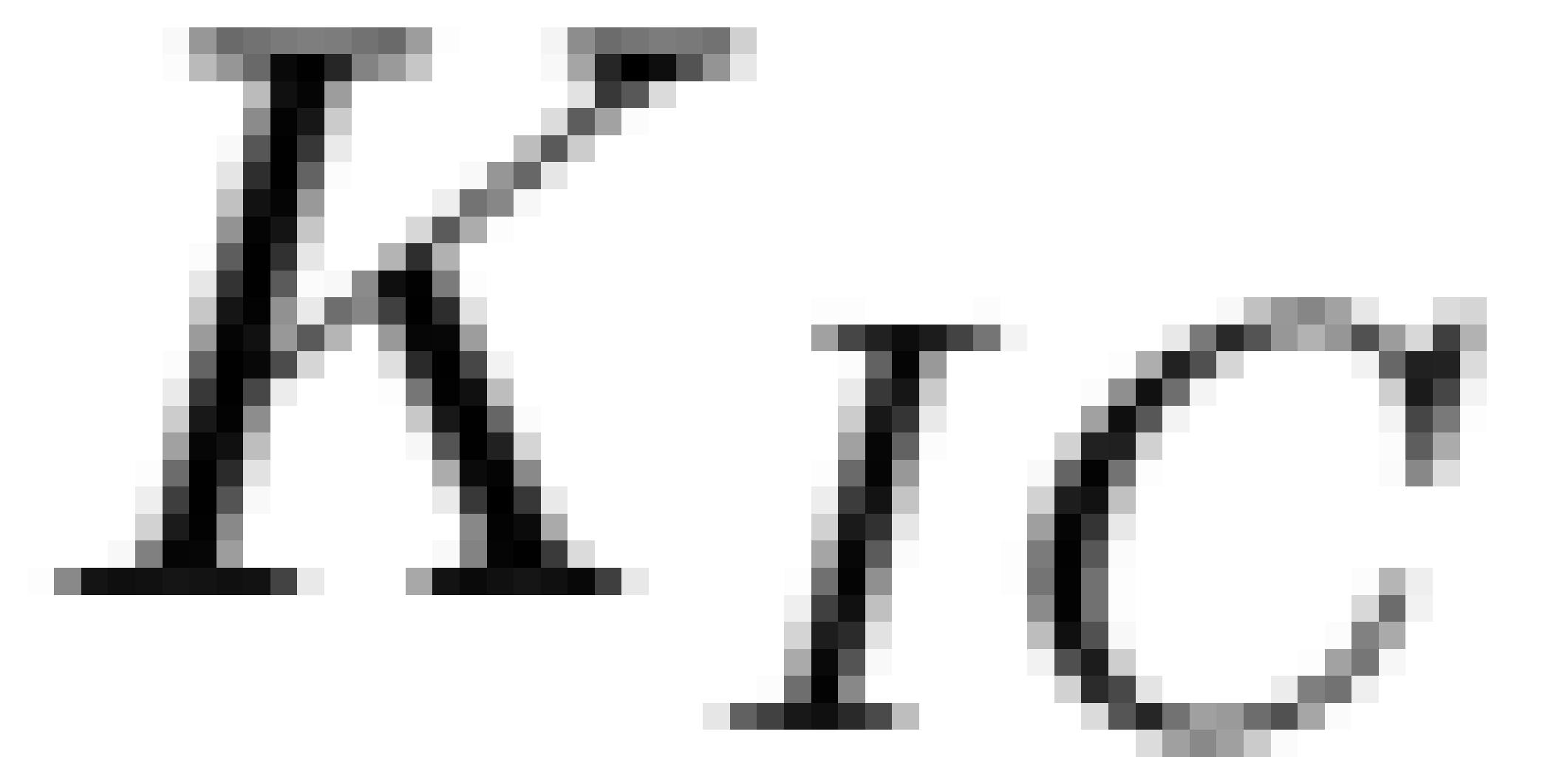


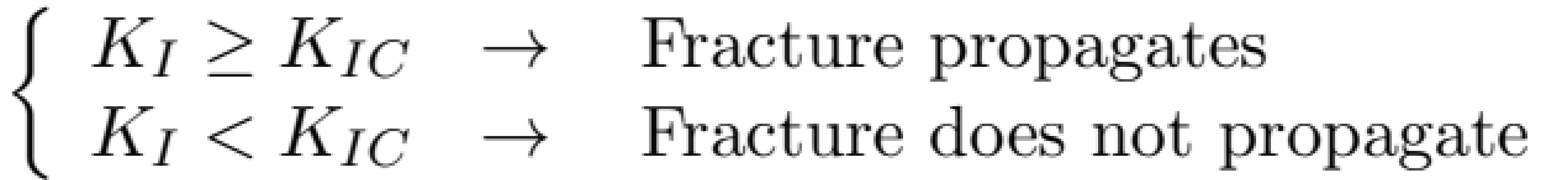


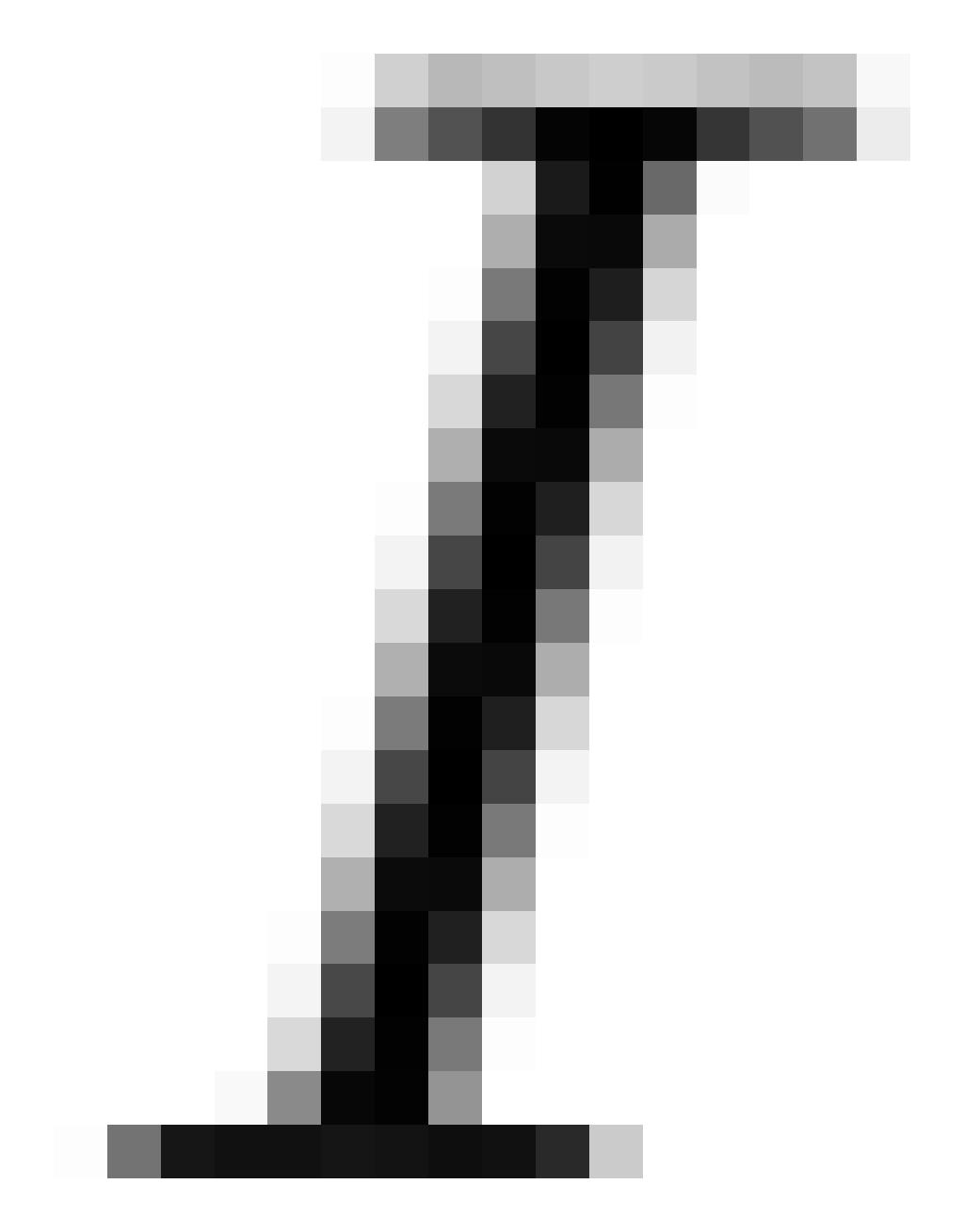






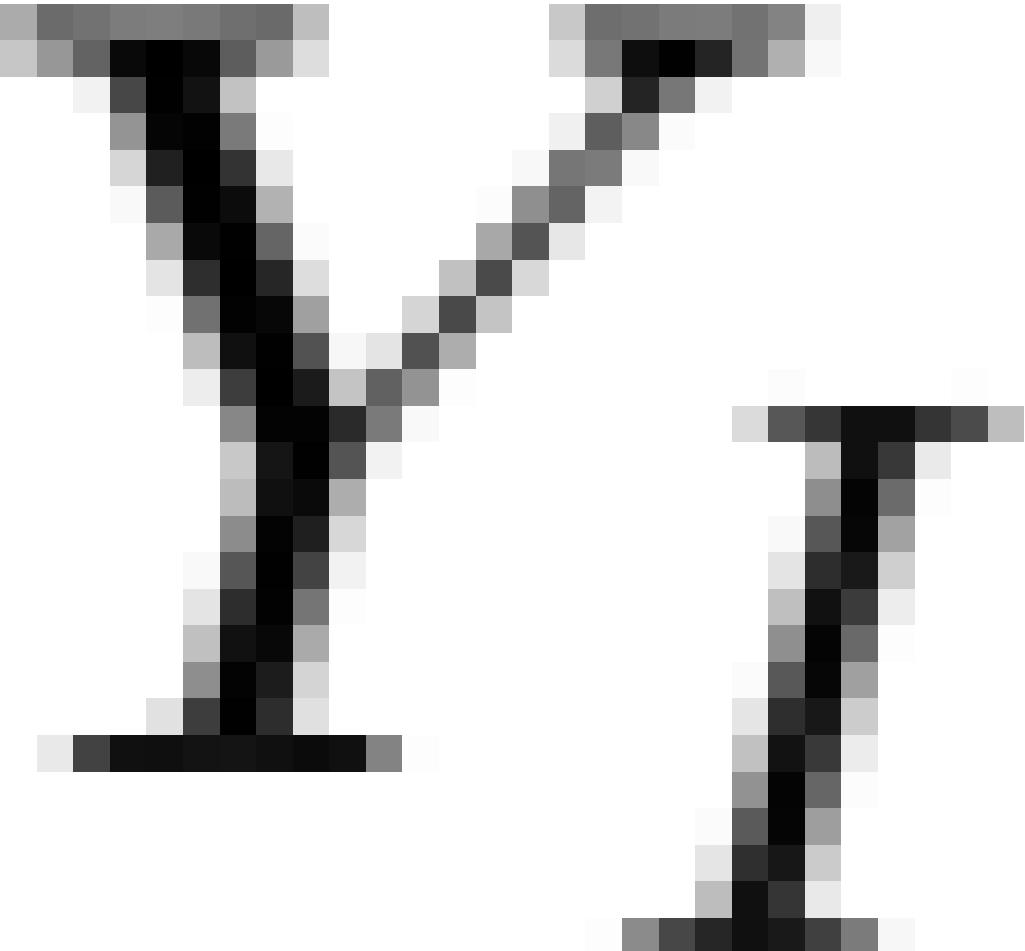


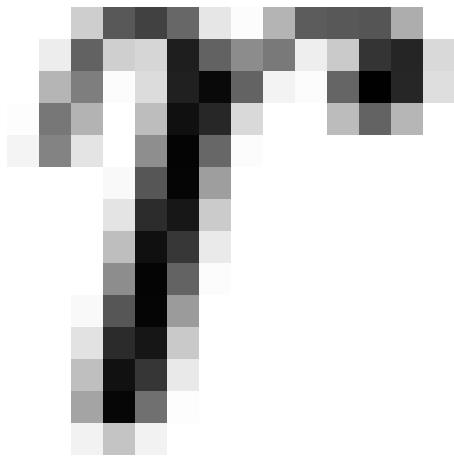
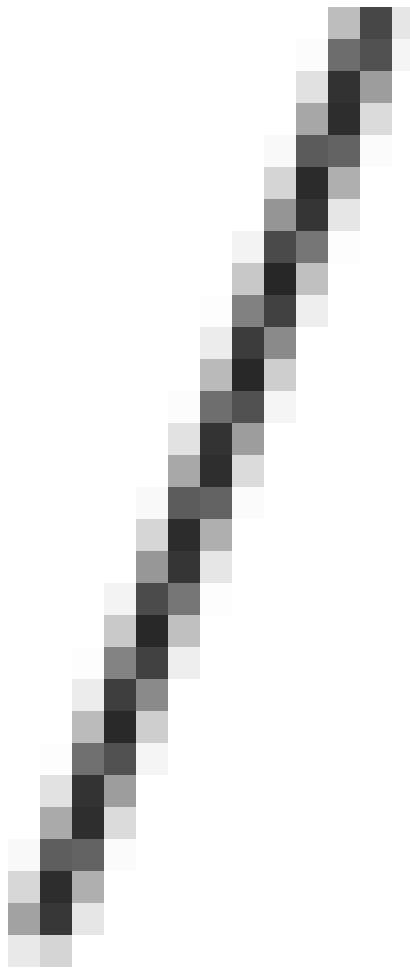
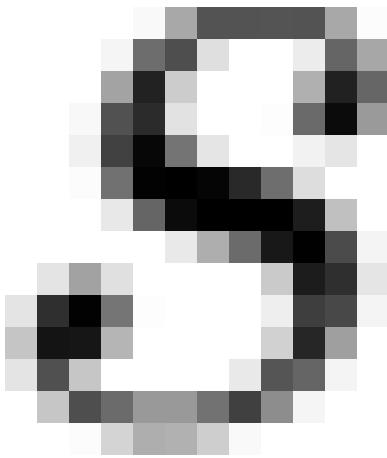


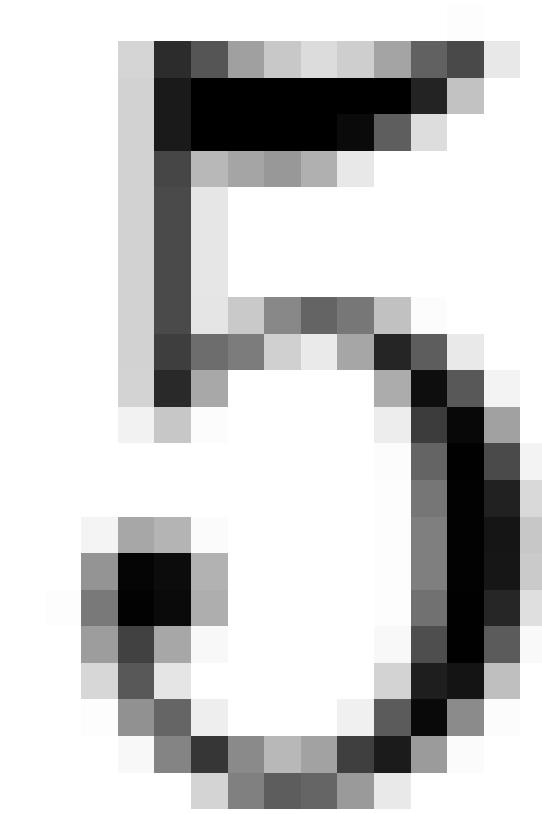
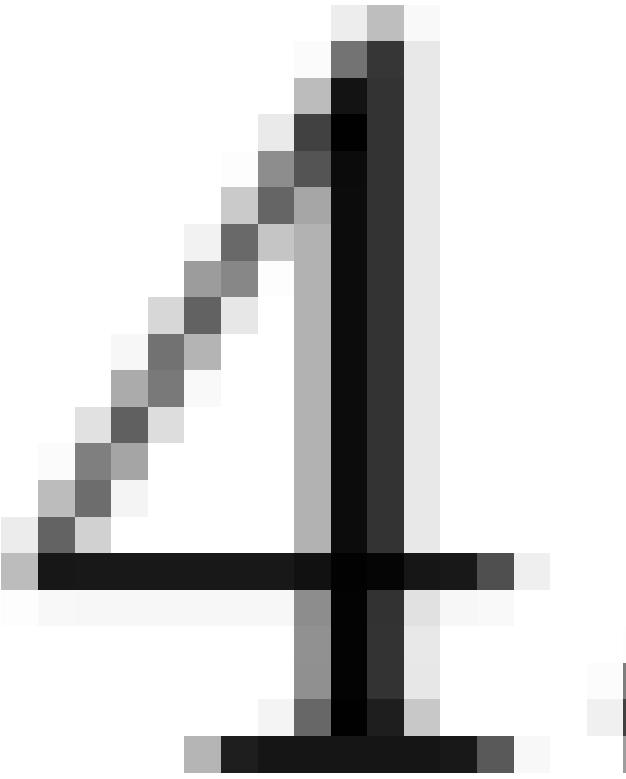
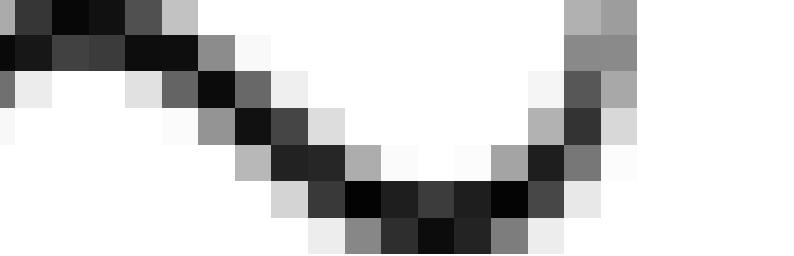
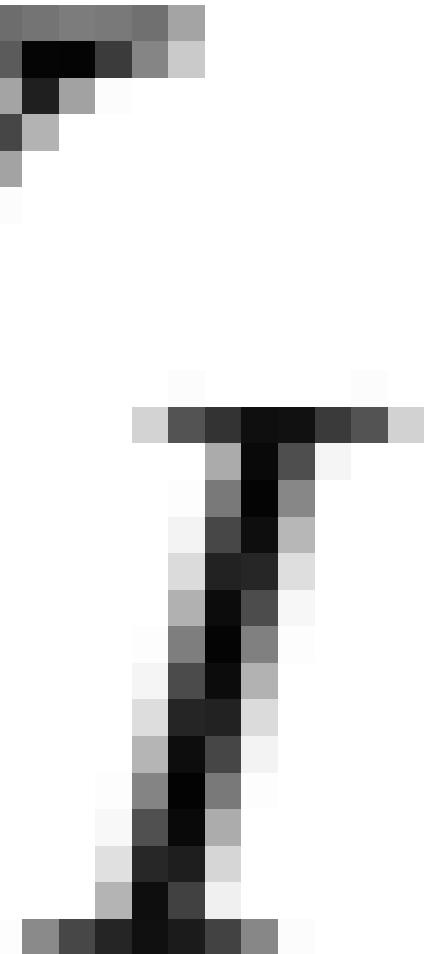
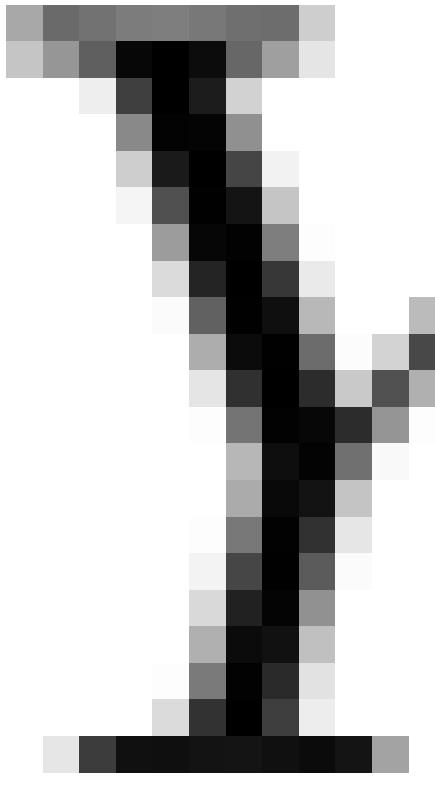


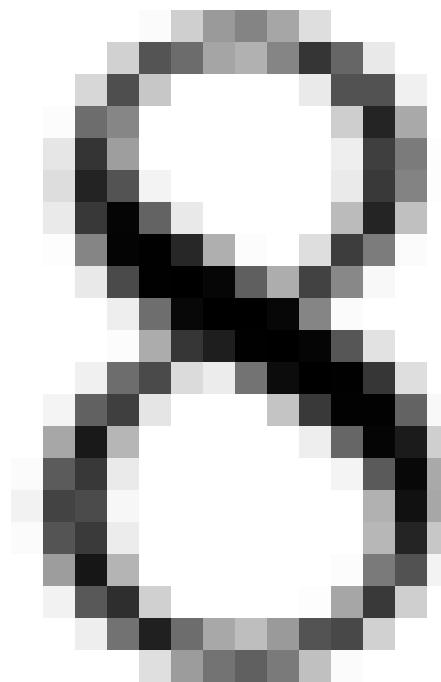
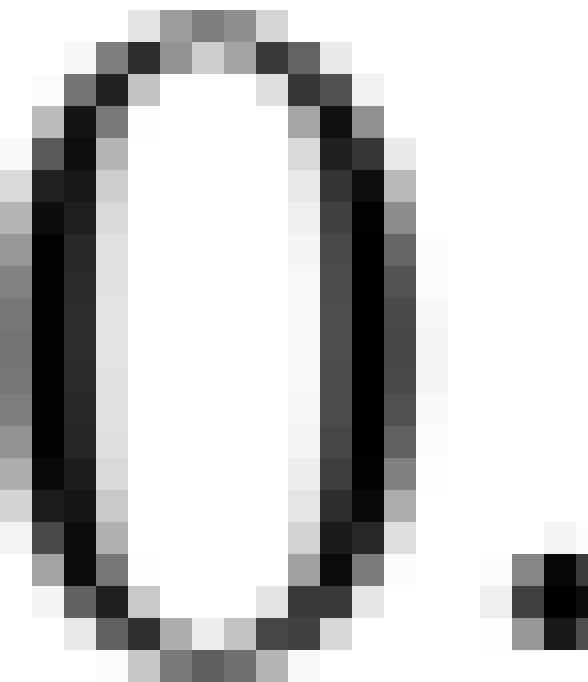
$$K_{IC} = \frac{P_{max}(\pi_0)^{1/2}}{2\pi L} M$$



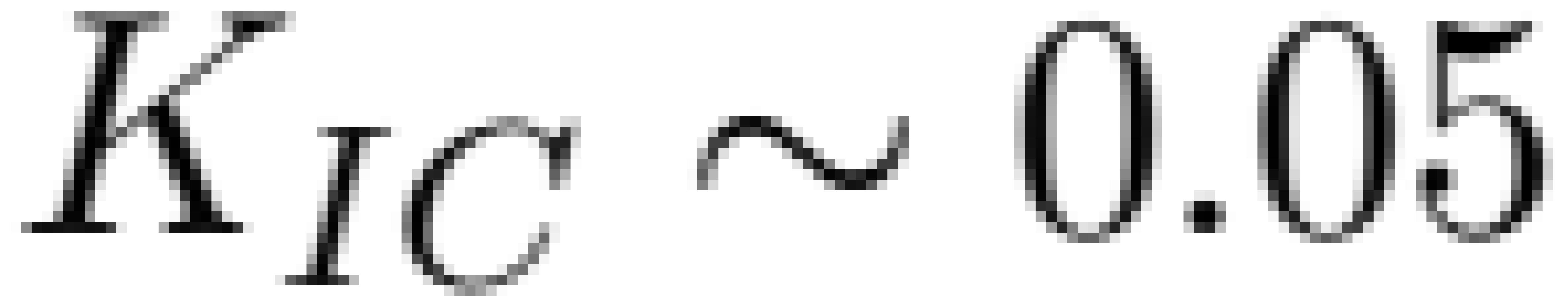


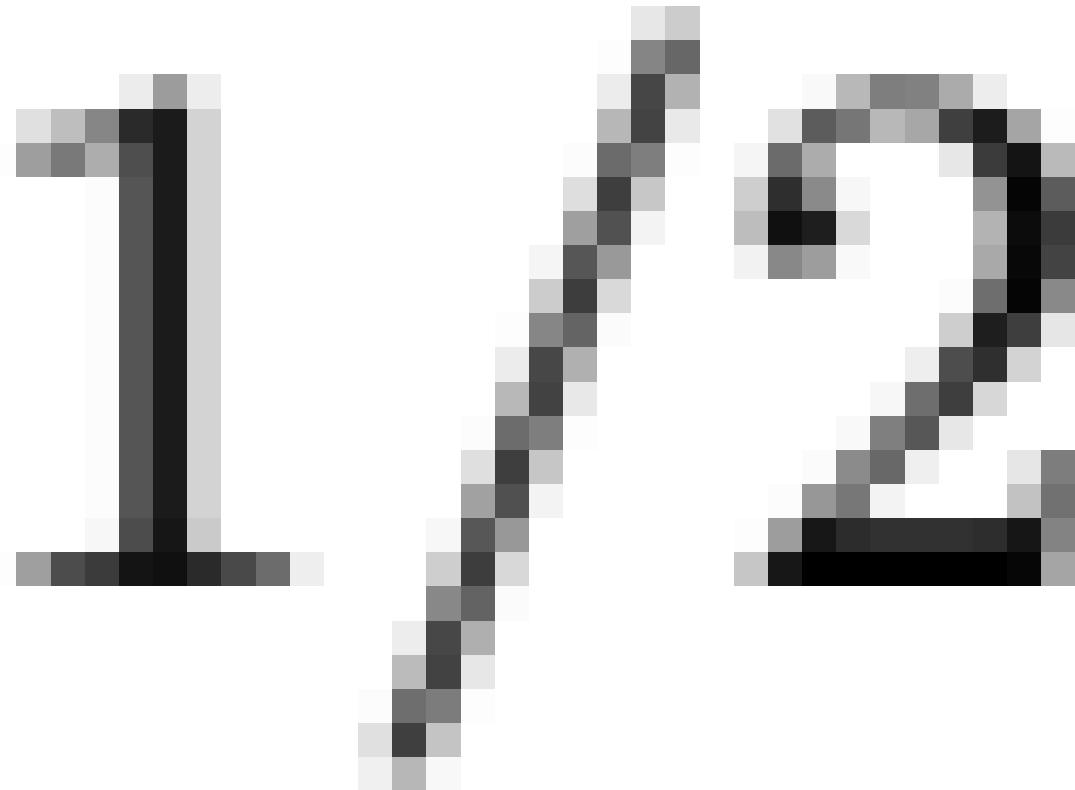




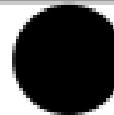
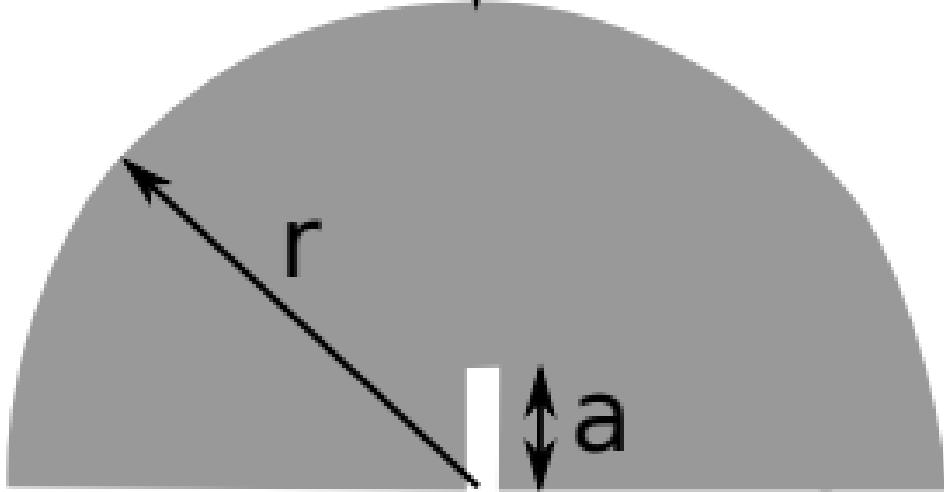




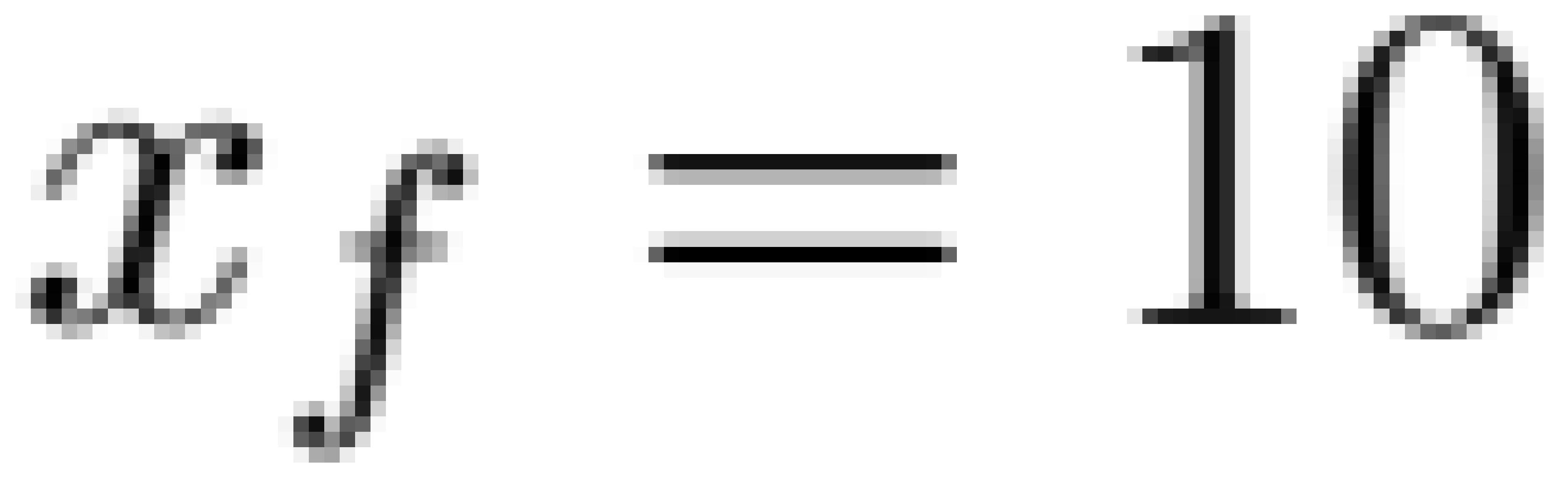


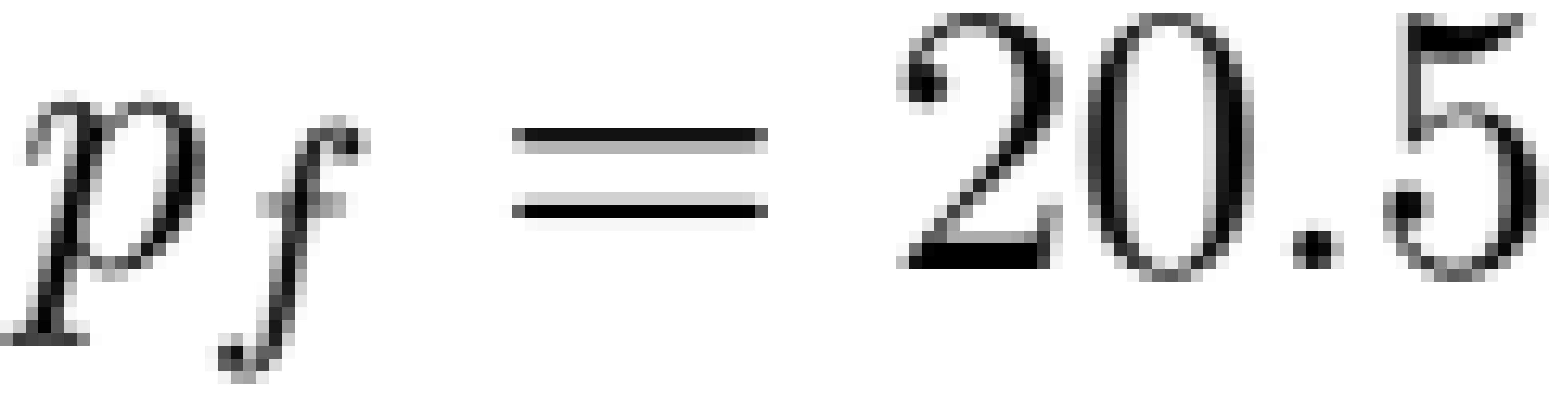


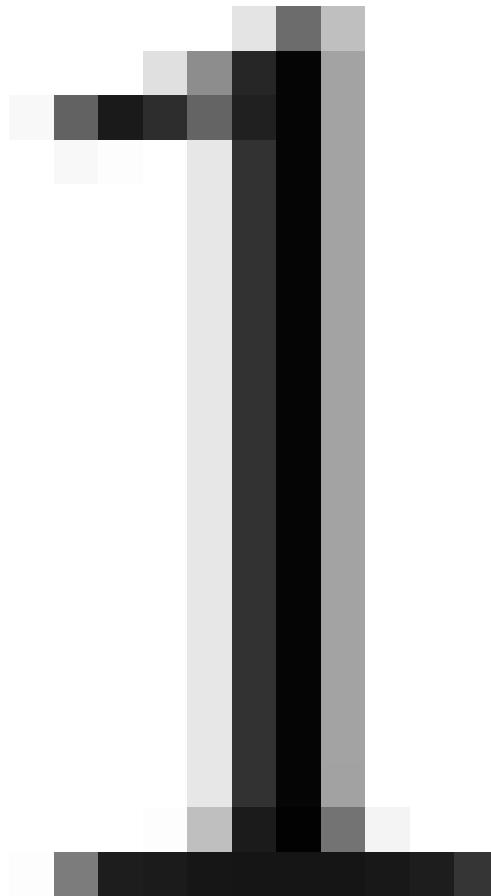
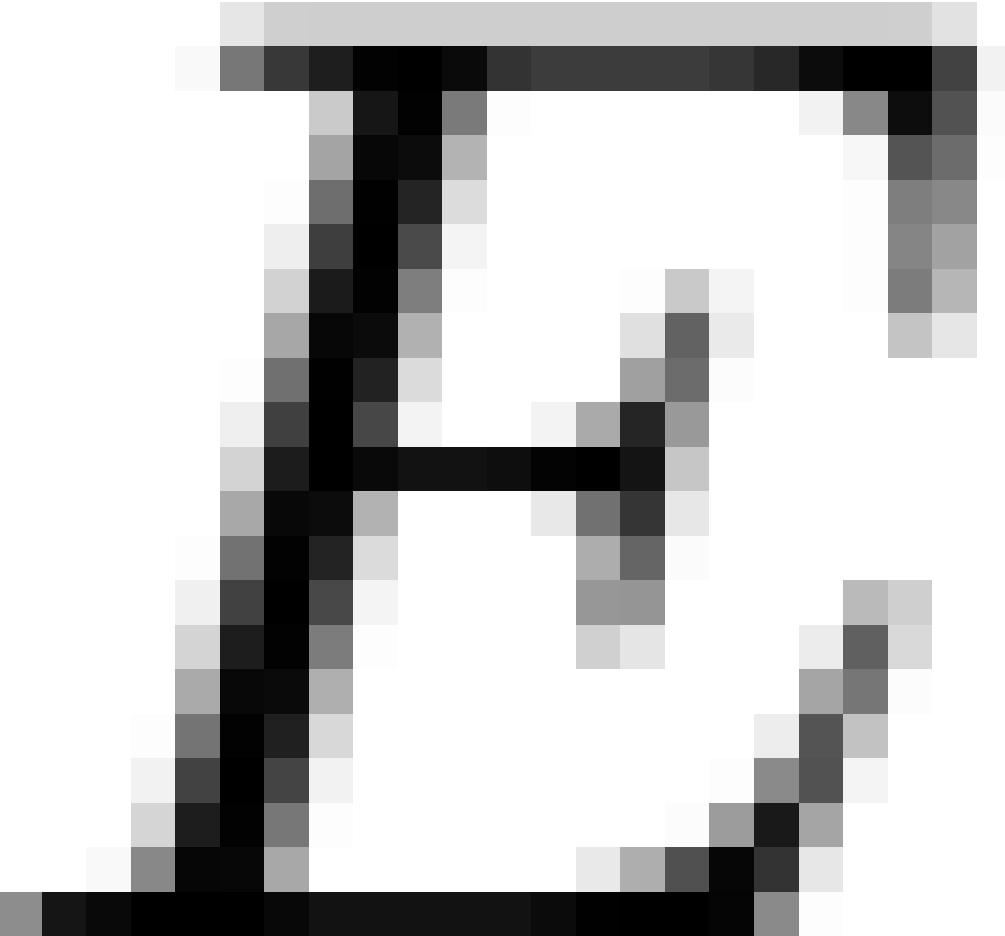
$P$

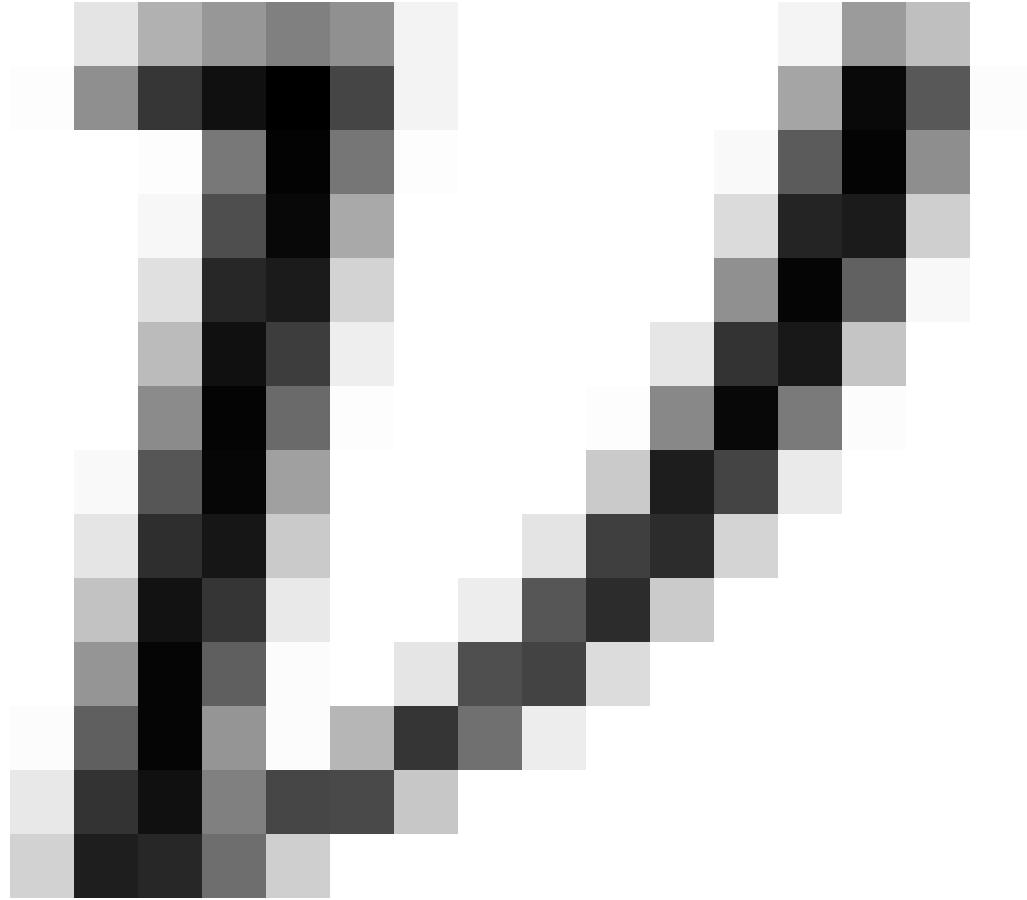


$2s$

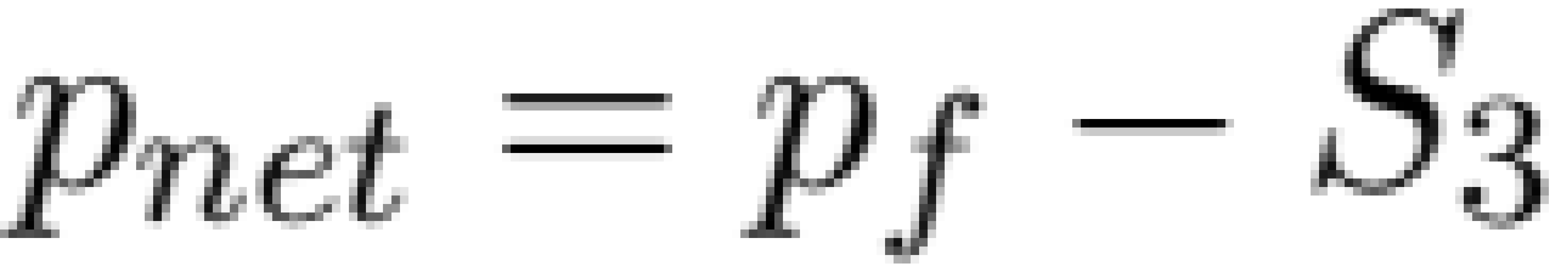






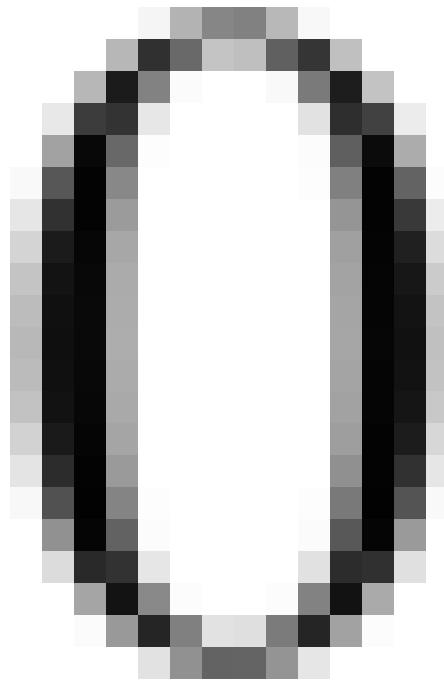
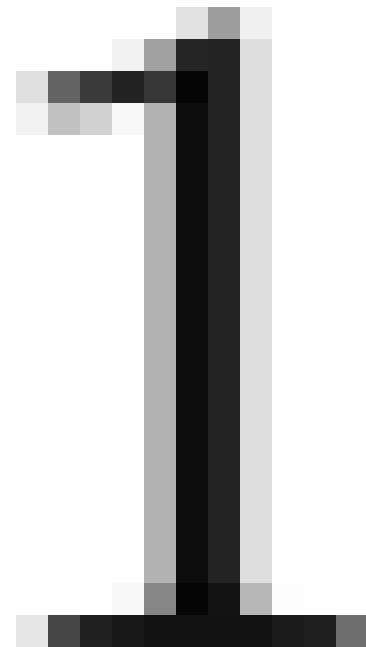
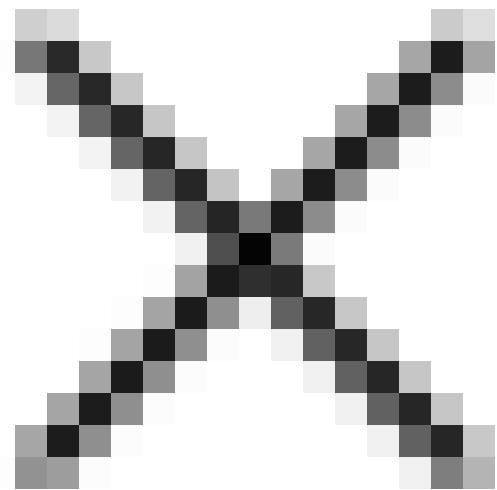
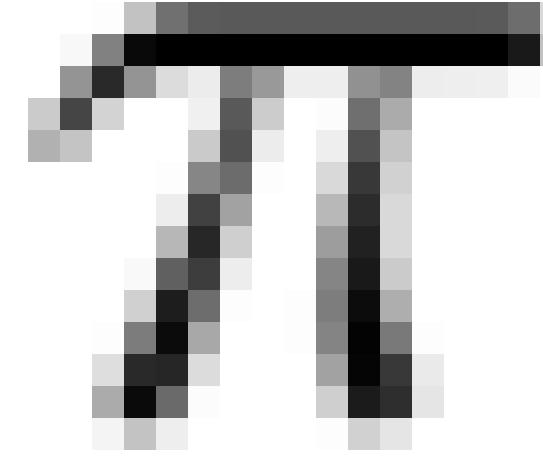
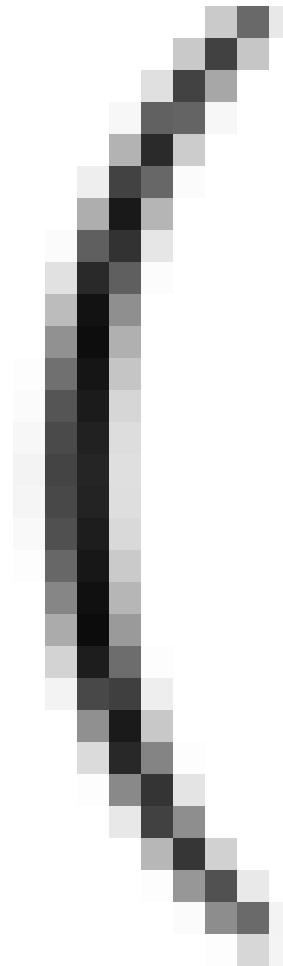


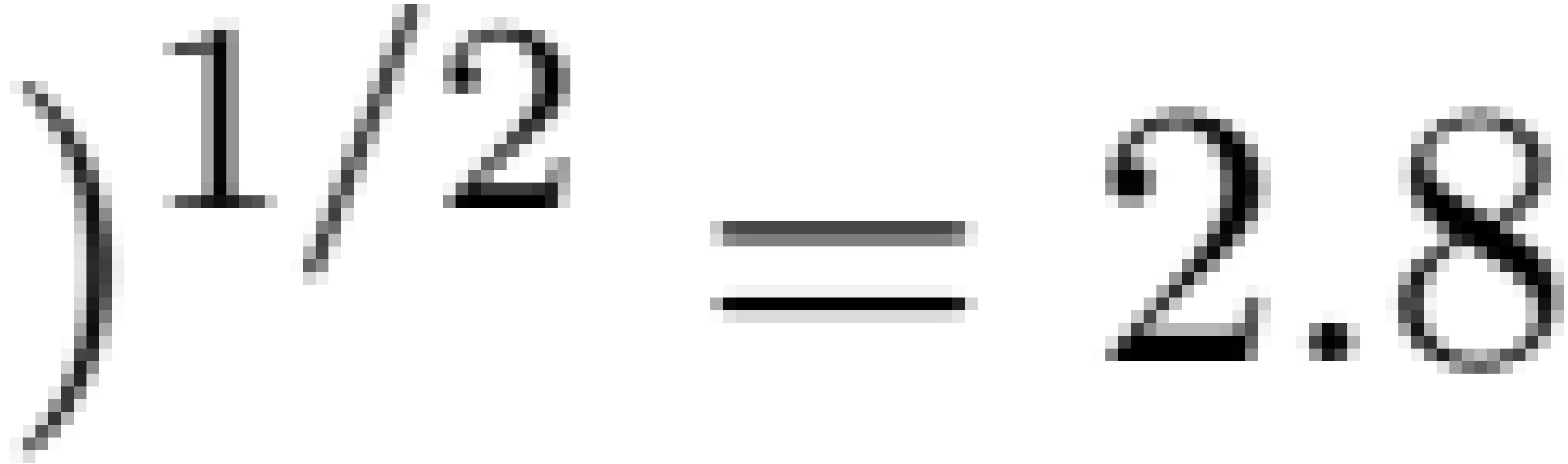
$$\frac{E}{1 - \nu^2} = \frac{1 \text{ GPa}}{1 - 0.25^2} = 1.07 \text{ GPa} = 1.07 \times 10^7 \text{ MPa}$$

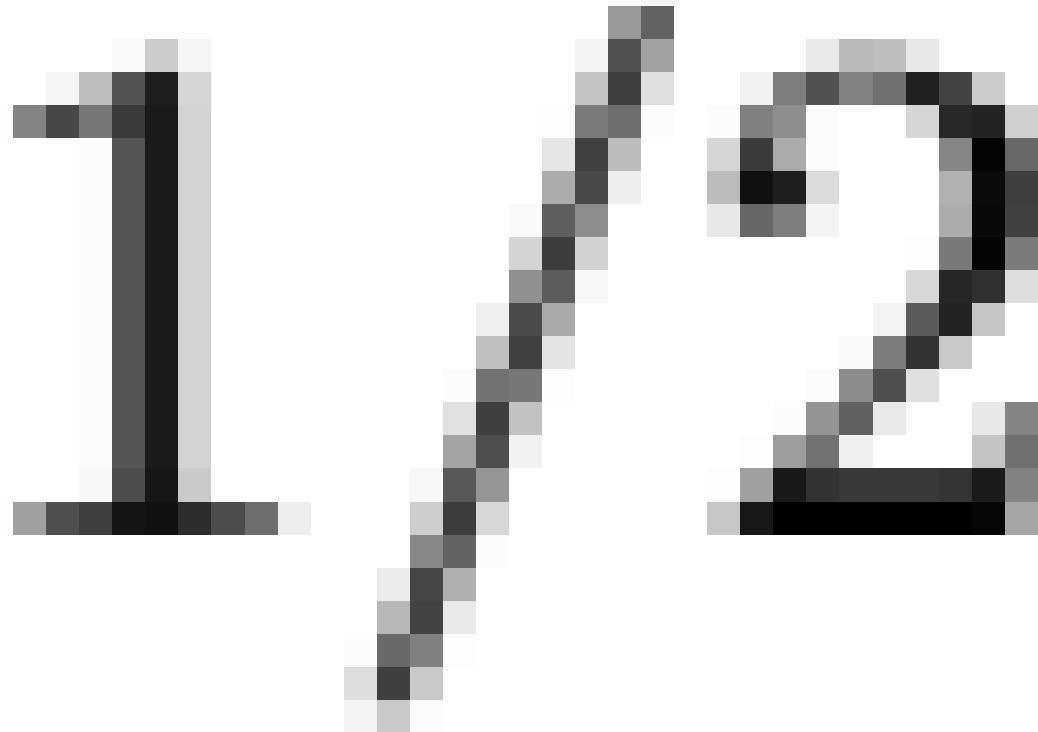


$$\omega_0 = \frac{4\rho_0 c}{E'} = \frac{4 \times 0.5 \text{ MPa} \times 10 \text{ m}}{1070 \text{ MPa}} = 0.019 \text{ m} = 19 \text{ mm}$$



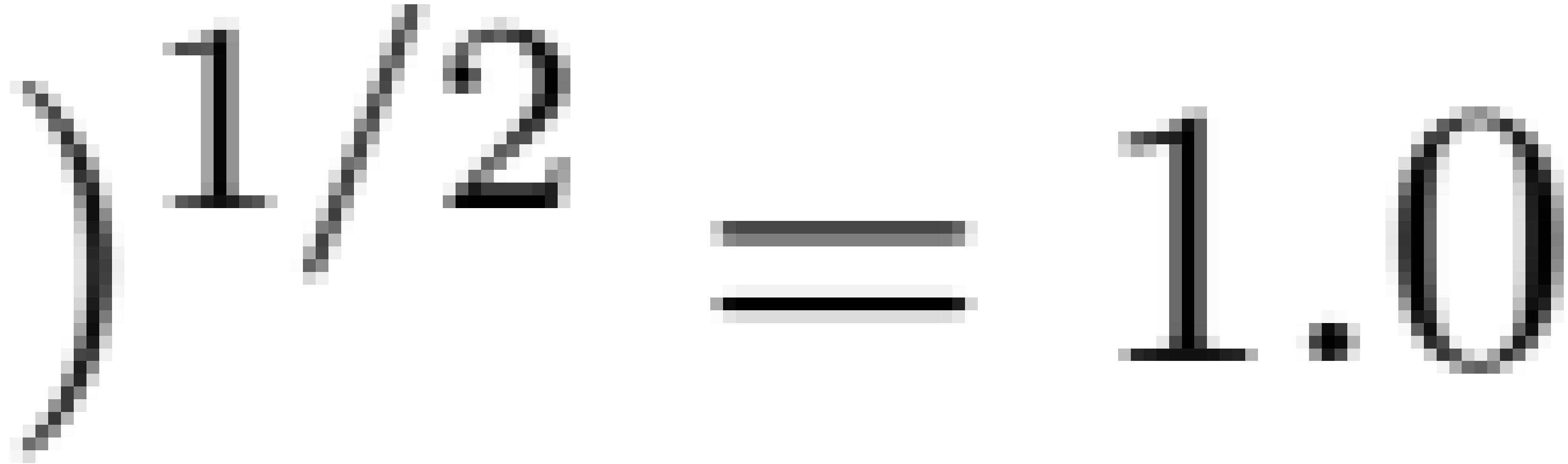


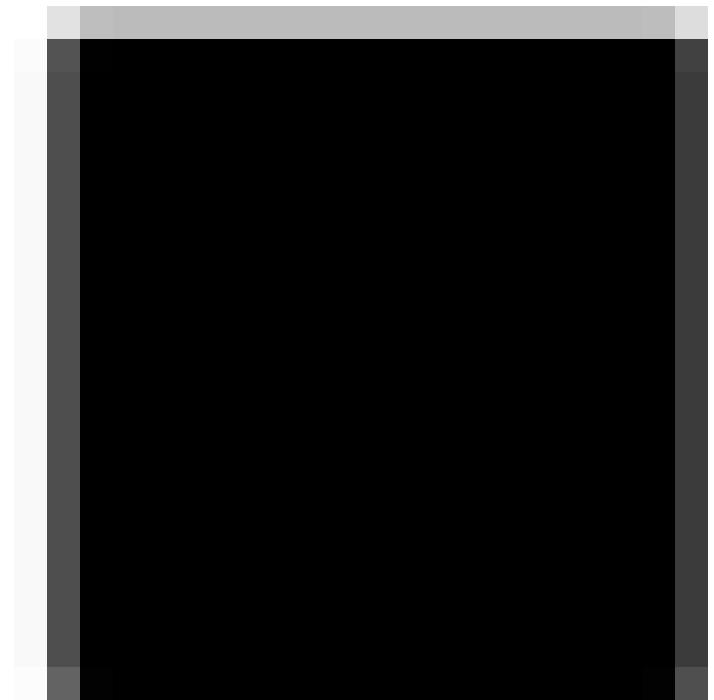
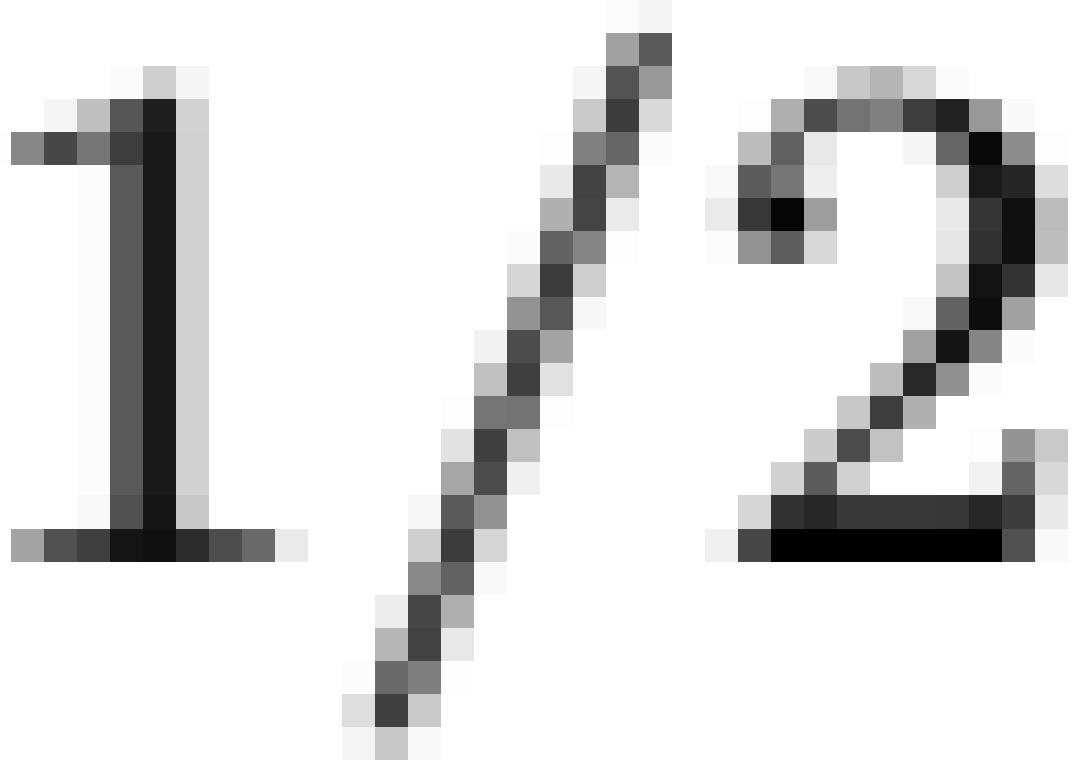


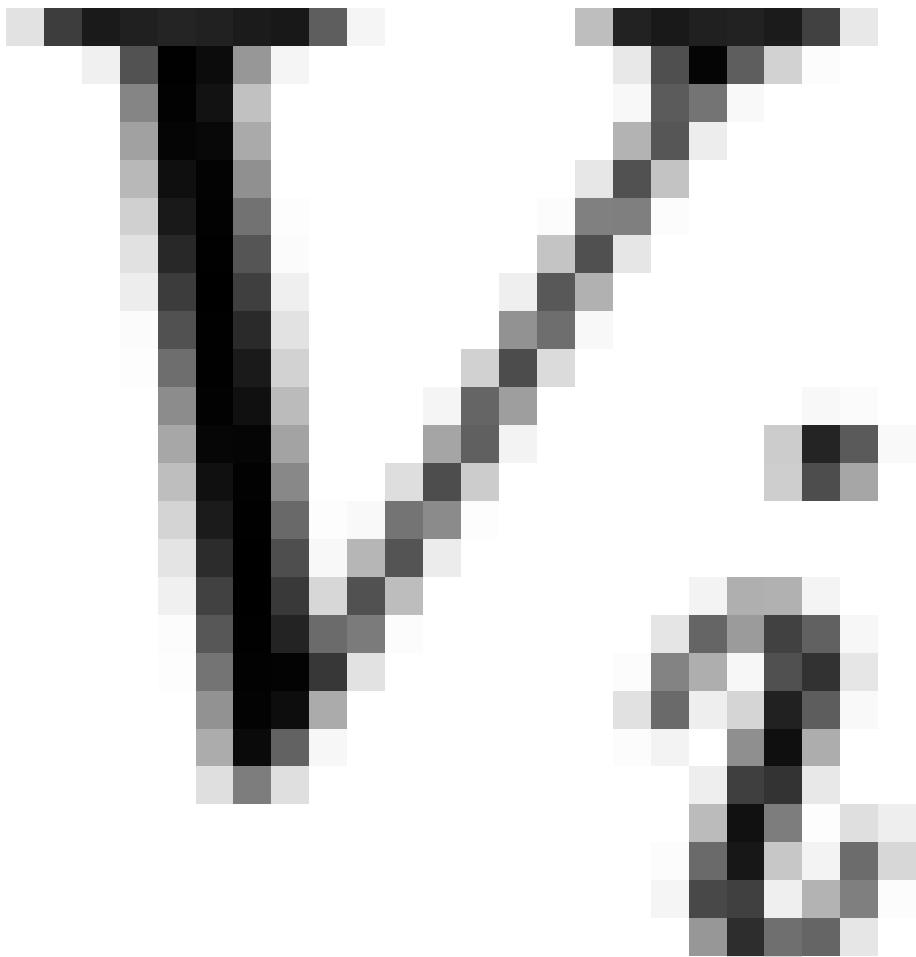


$$w_0 = \frac{2\rho_0 c}{E'} = \frac{2 \times 0.5 \text{ MPa} \times 10 \text{ m}}{1070 \text{ MPa}} = 0.0095 \text{ m} = 9.5 \text{ mm}$$

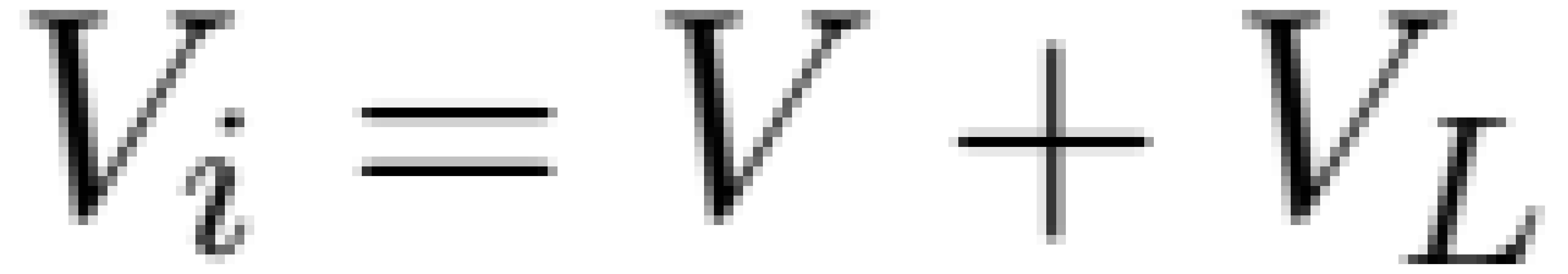
$$K_I = \left(1 - \frac{2}{\pi}\right)^{1/2} = \rho_0(\pi c)^{1/2} = 0.5$$

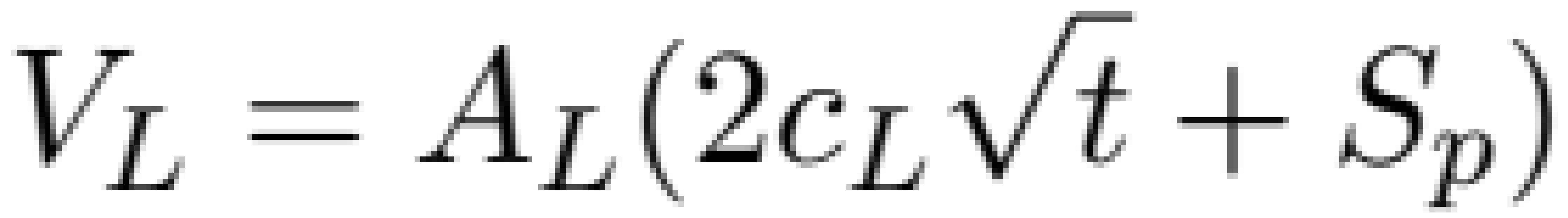


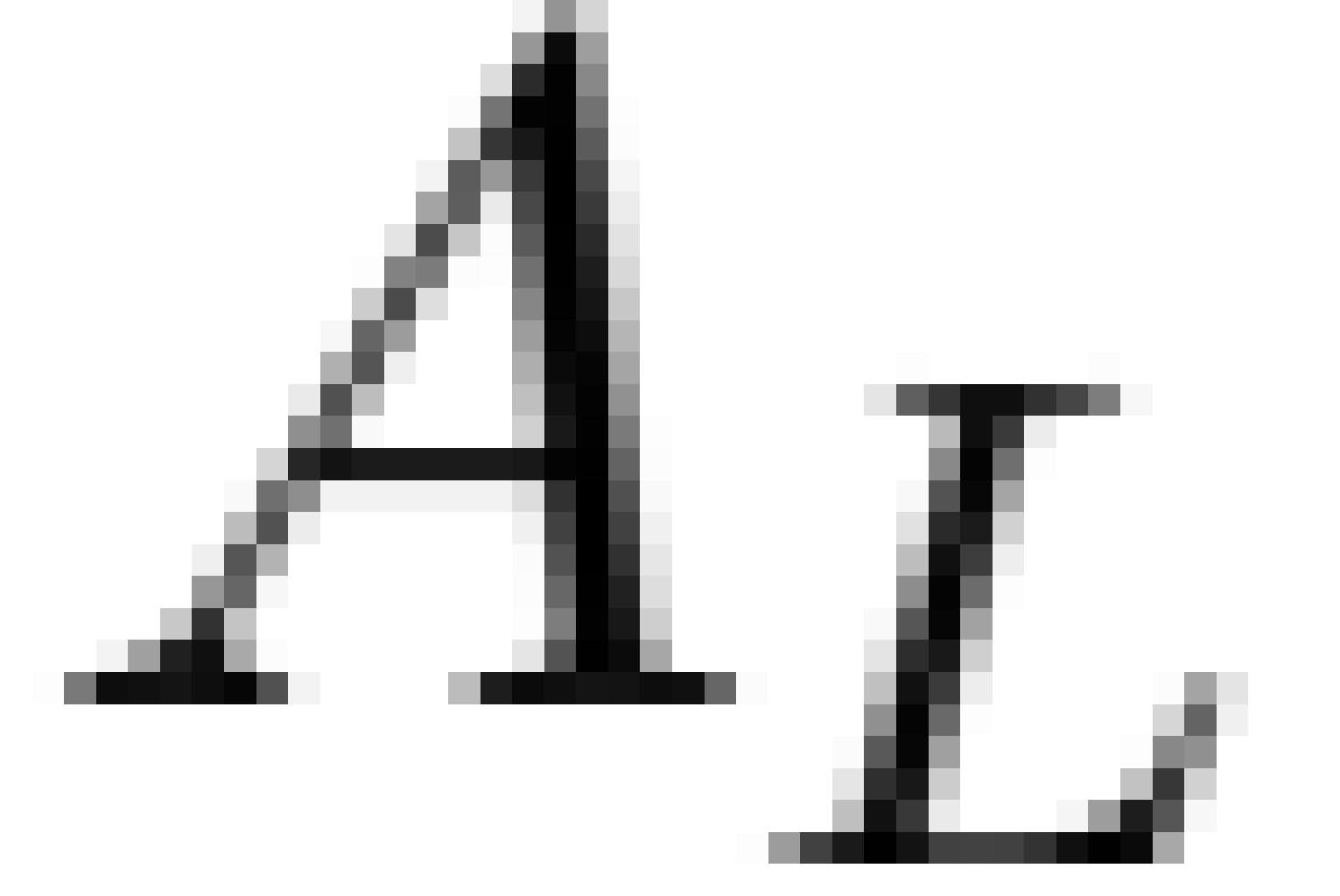
















W

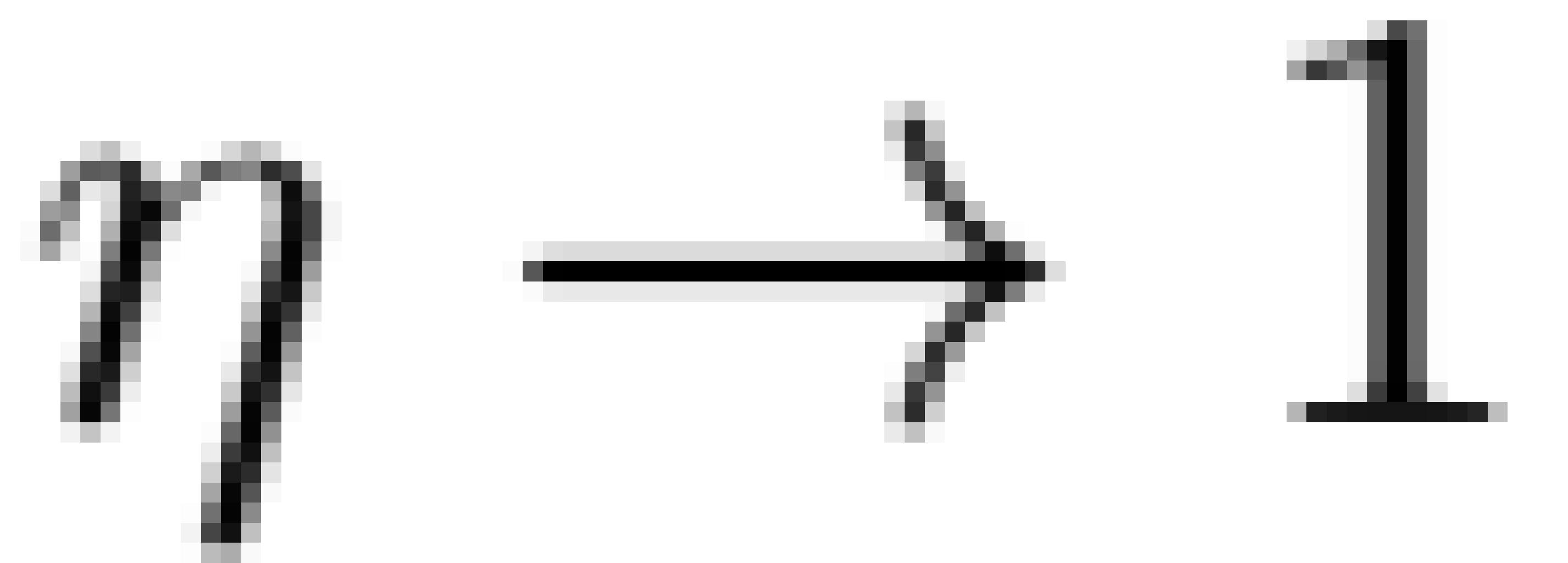
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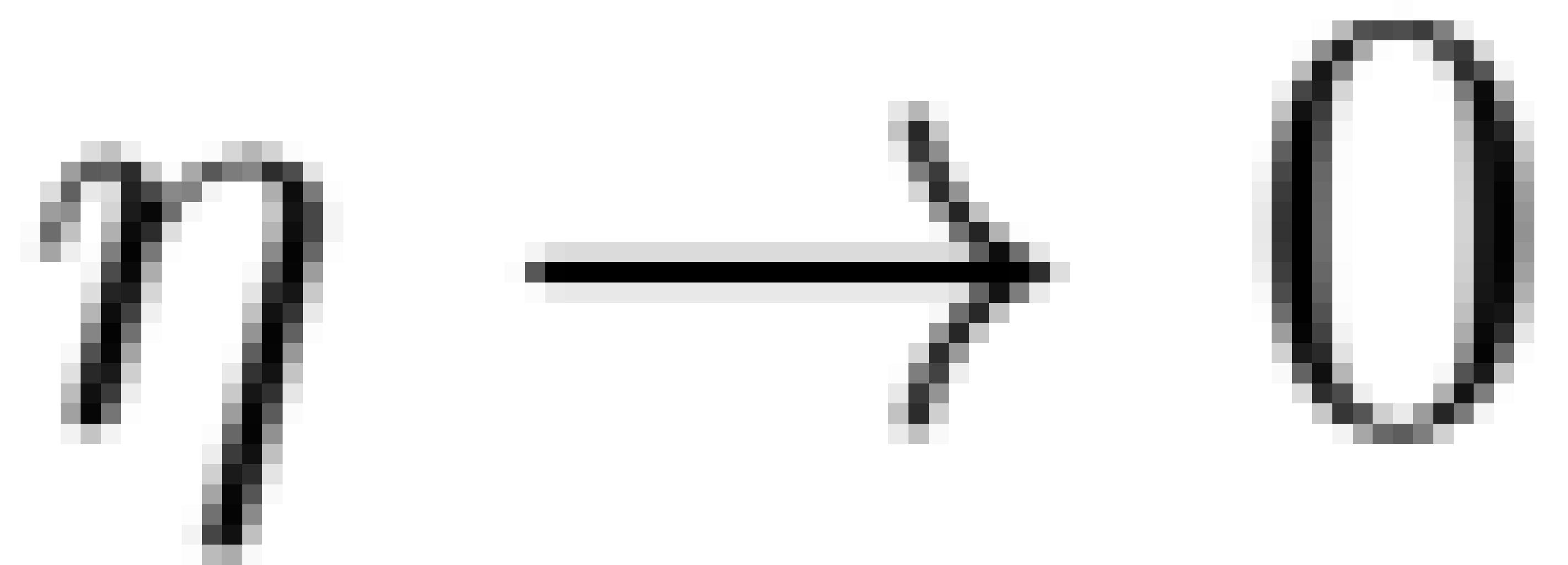
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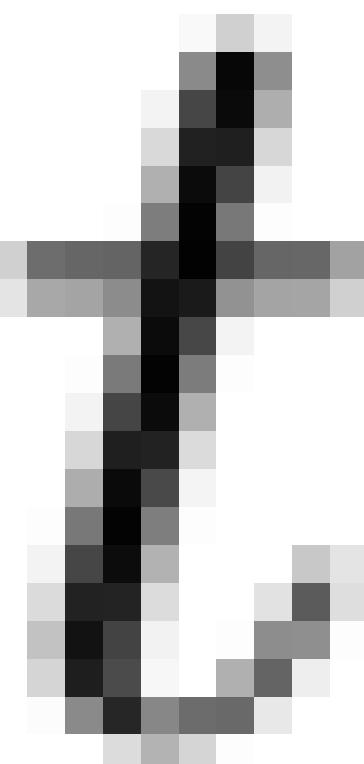
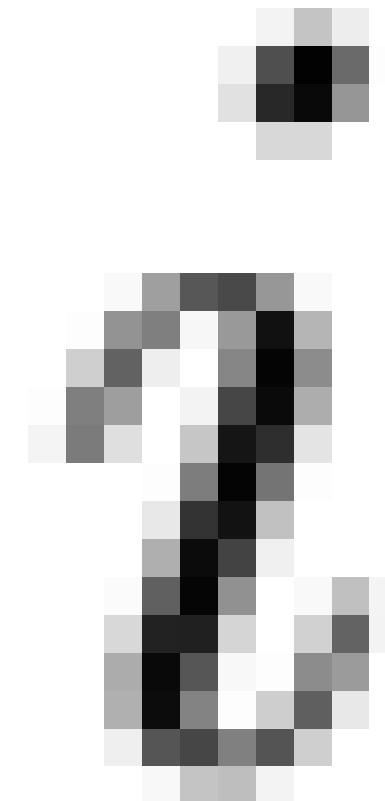
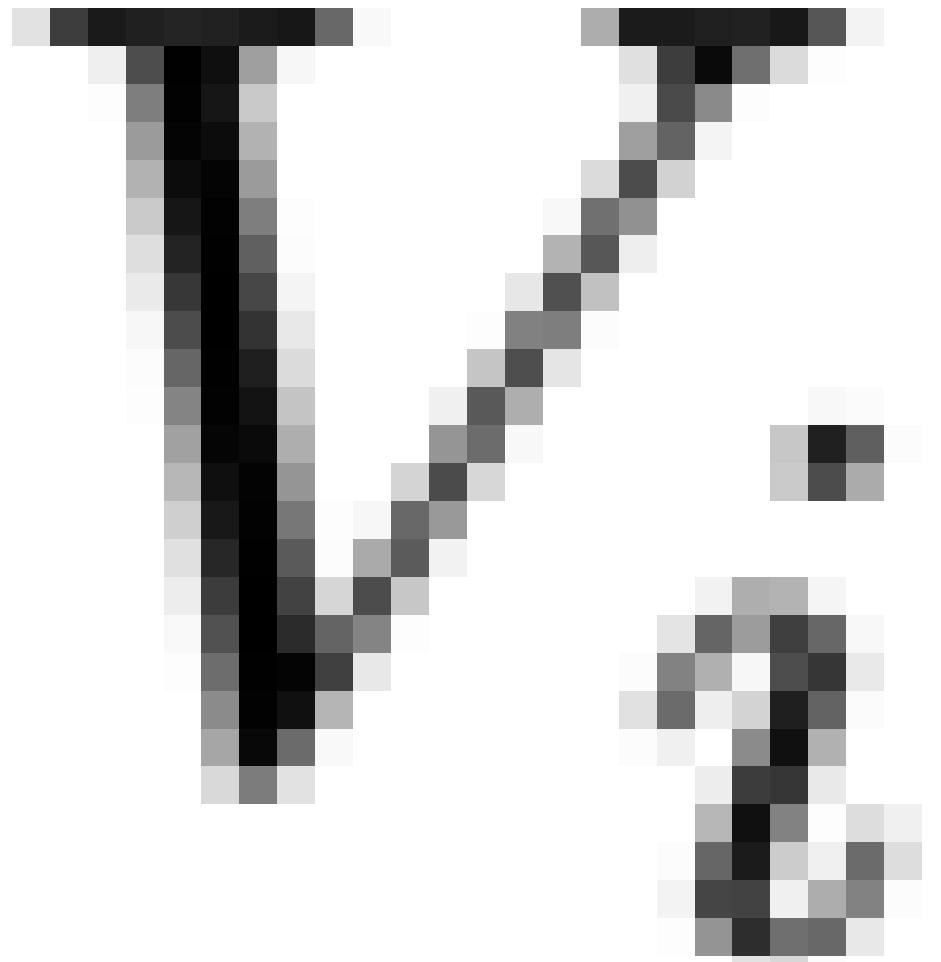
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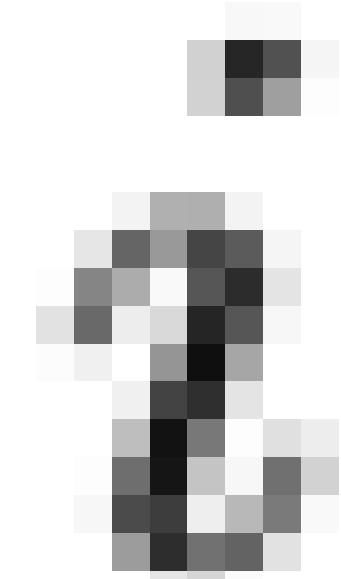
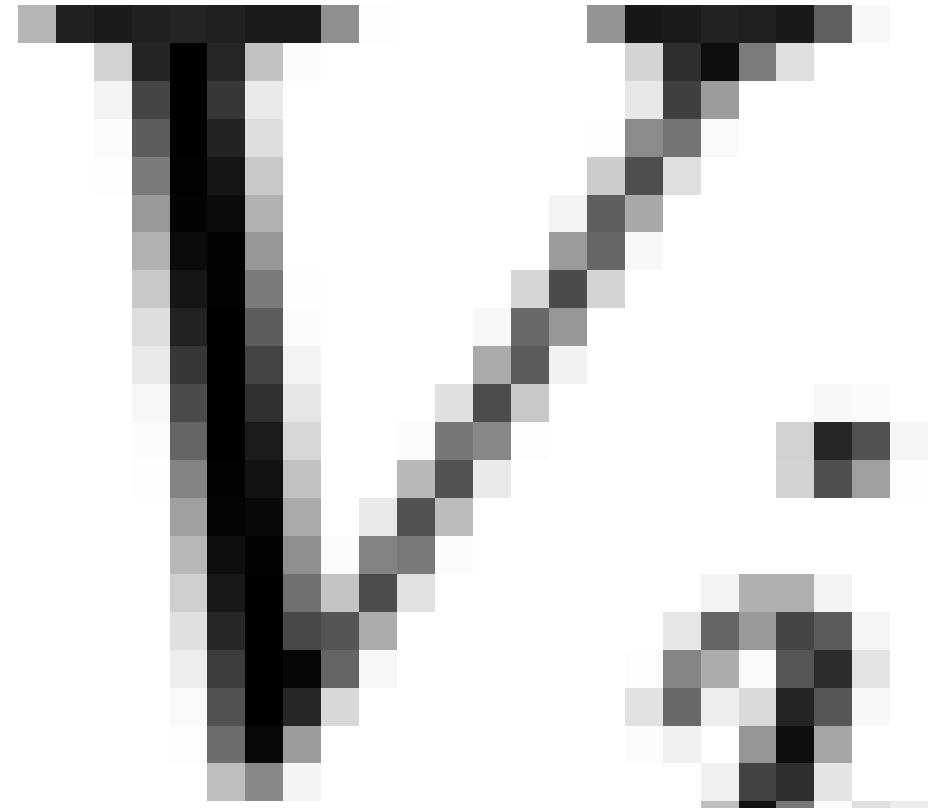
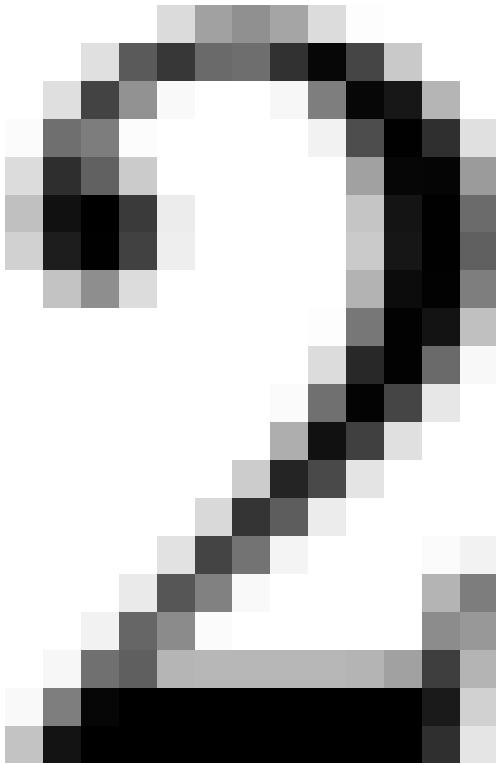
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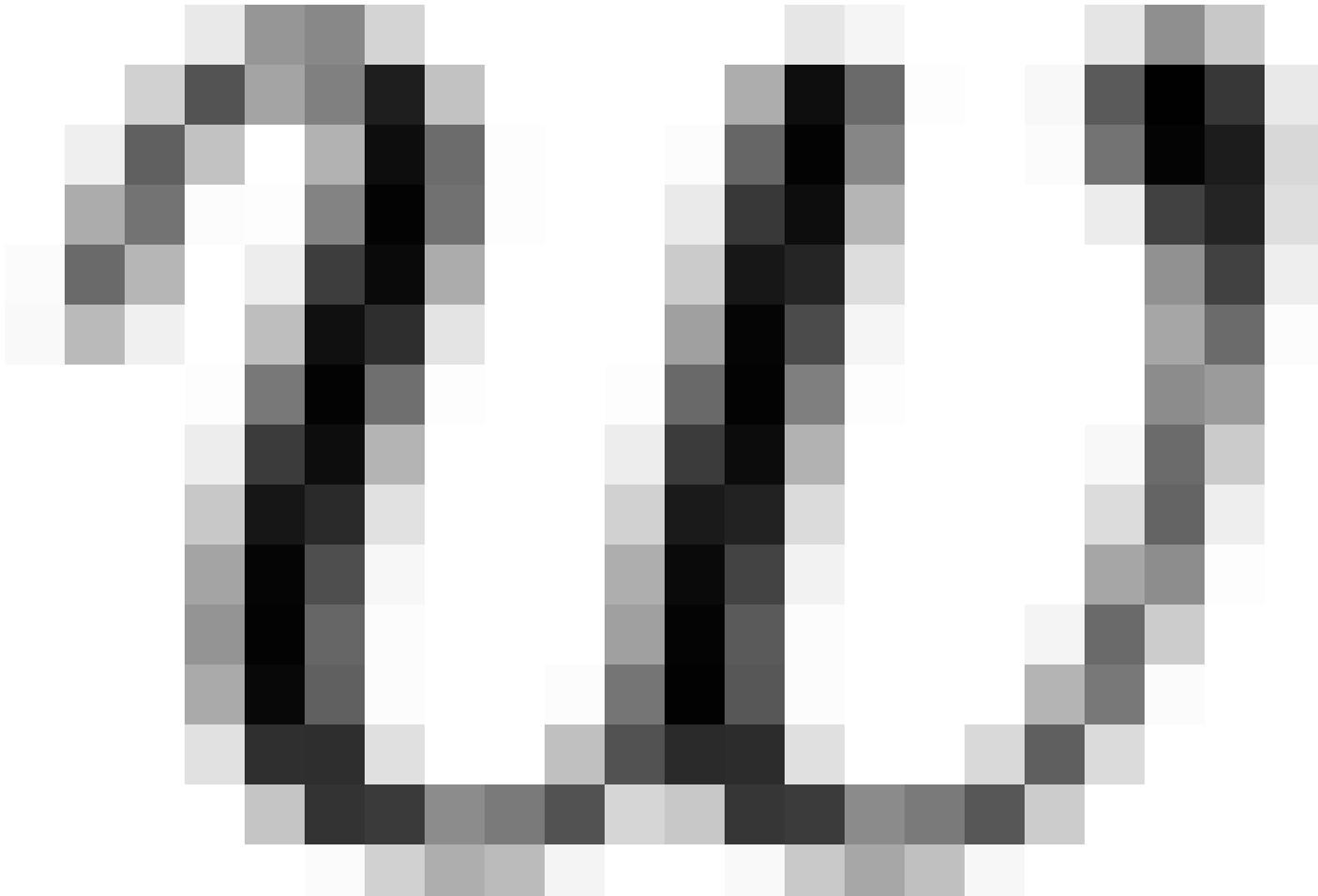
$q$

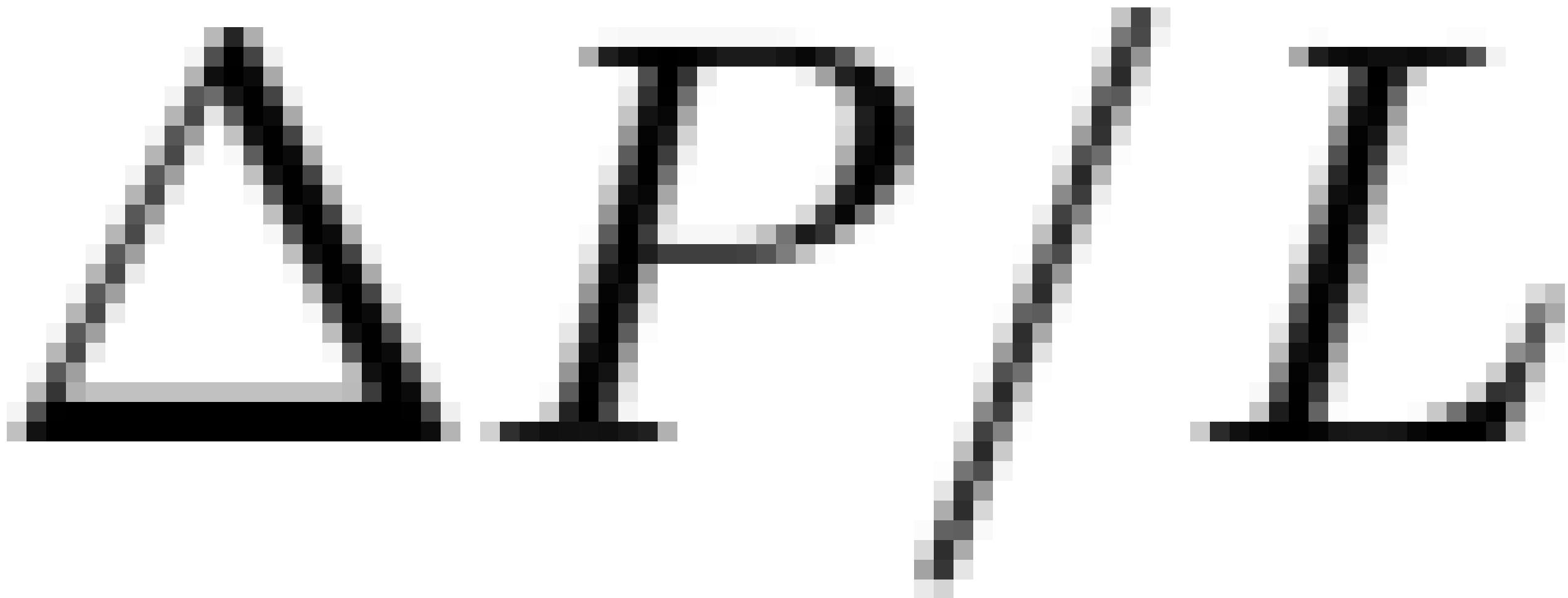
$\omega^3 h_f$

$P$

$12\mu L$

$L$

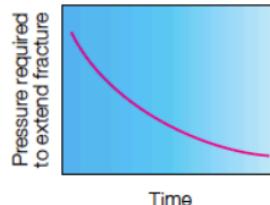
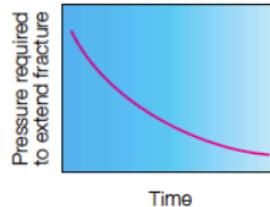
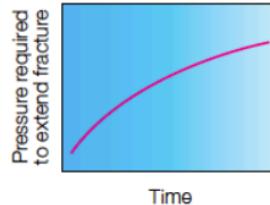
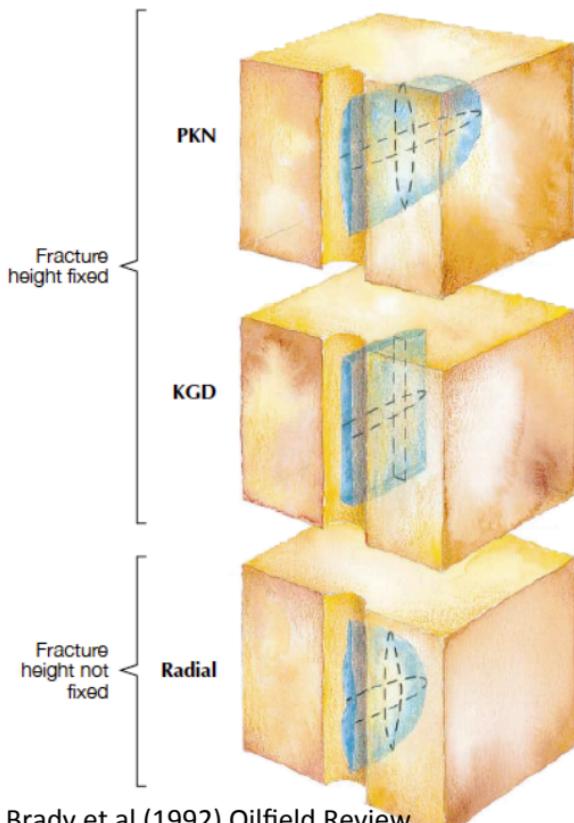








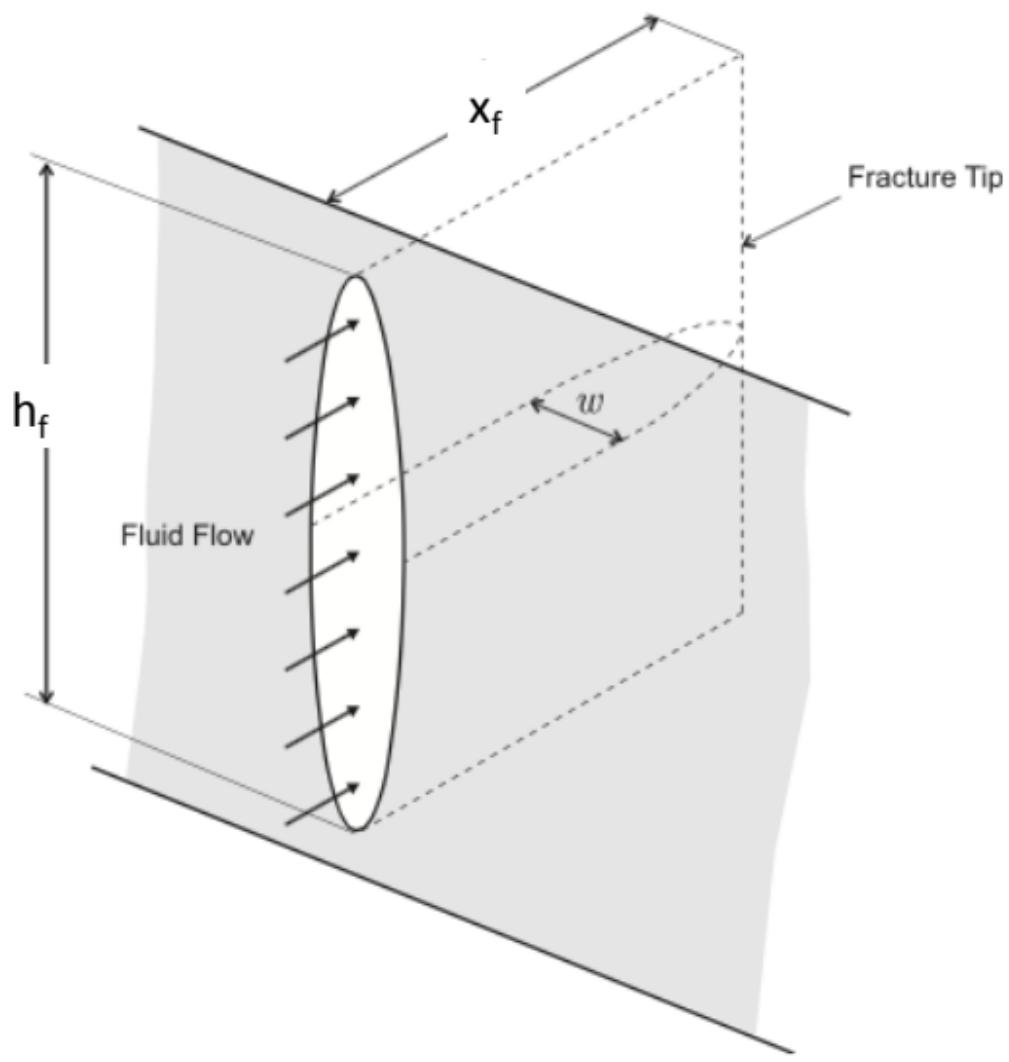
## 2D Fracture Models



- Elliptical cross section
- Width  $\propto$  height
- Width < KGD; length > KGD
- More appropriate when fracture length > height

- Rectangular cross section
- Width  $\propto$  length
- More appropriate when fracture length < height

- Appropriate when fracture length = height



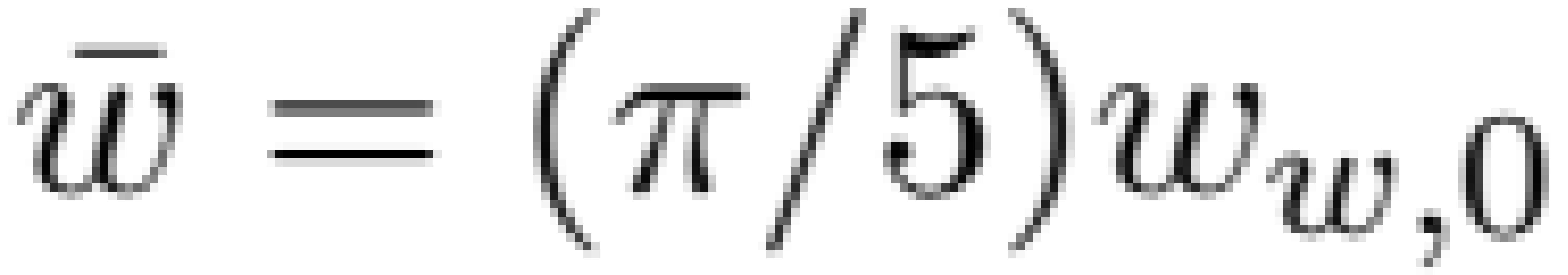
$$x_f = \left( \frac{625}{512\pi^3} \right)^{1/5} \left( \frac{i^3 E'}{\mu h_f^4} \right)^{1/5} t^{4/5} = 0.524 t^{4/5}$$

$$w_{w,0} = \left( \frac{2560}{\pi^2} \right)^{1/5} \left( \frac{i^2 \mu}{E' h_f} \right)^{1/5} t^{1/5} = 3.040 \left( \frac{i^2 \mu}{E' h_f} \right)^{1/5} t^{1/5}$$

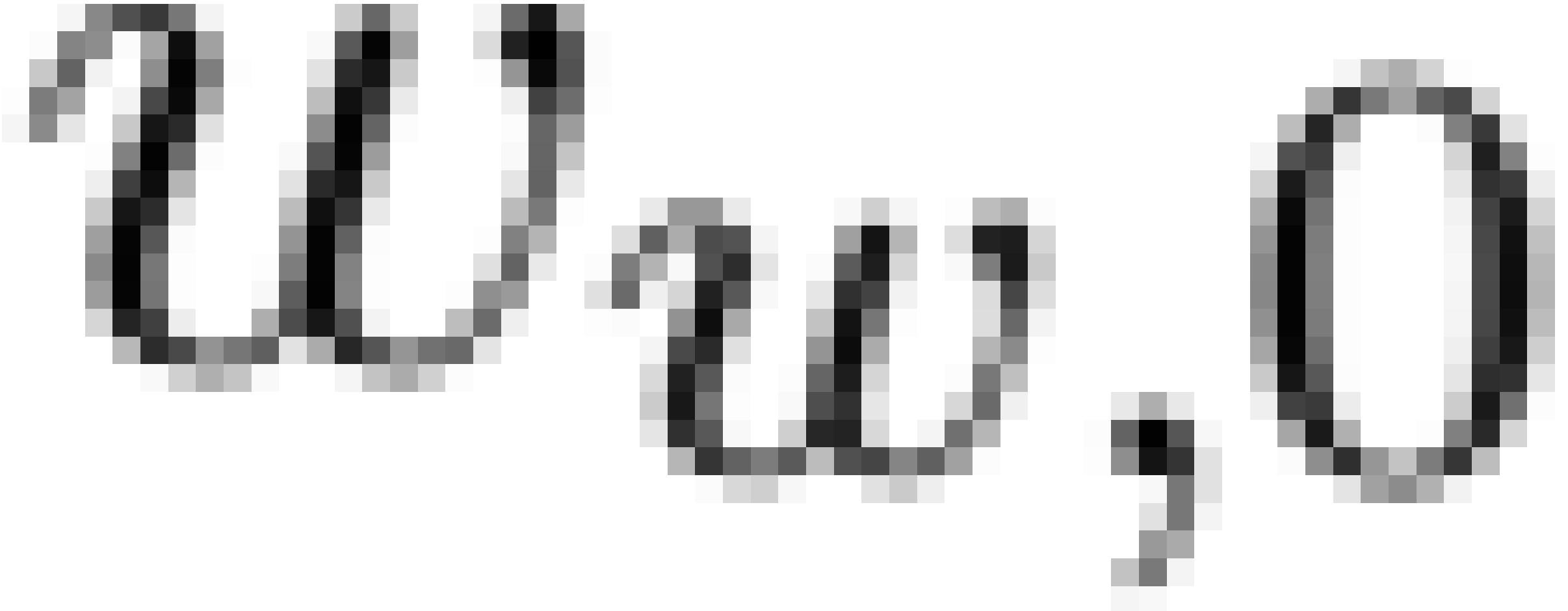
$$p_{net,w} = \left( \frac{80}{\pi^2} \right)^{1/4} \left( \frac{E'^4 i^2 \mu}{h_f^6} \right)^{1/5} t^{1/5} = 1.520 \left( \frac{E'^4 i^2 \mu}{h_f^6} \right)^{1/5} t^{1/5}$$



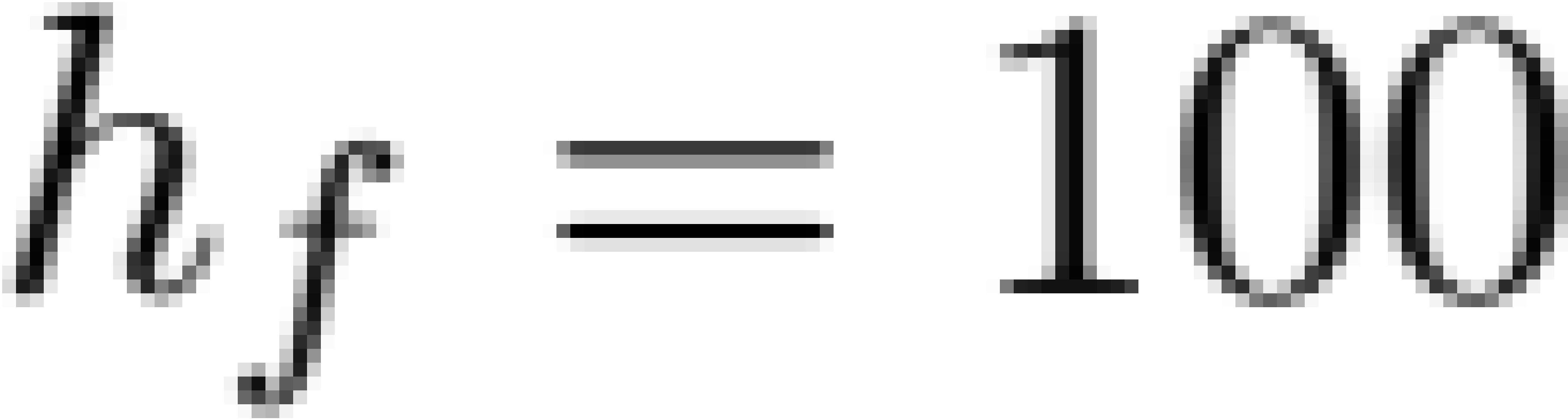






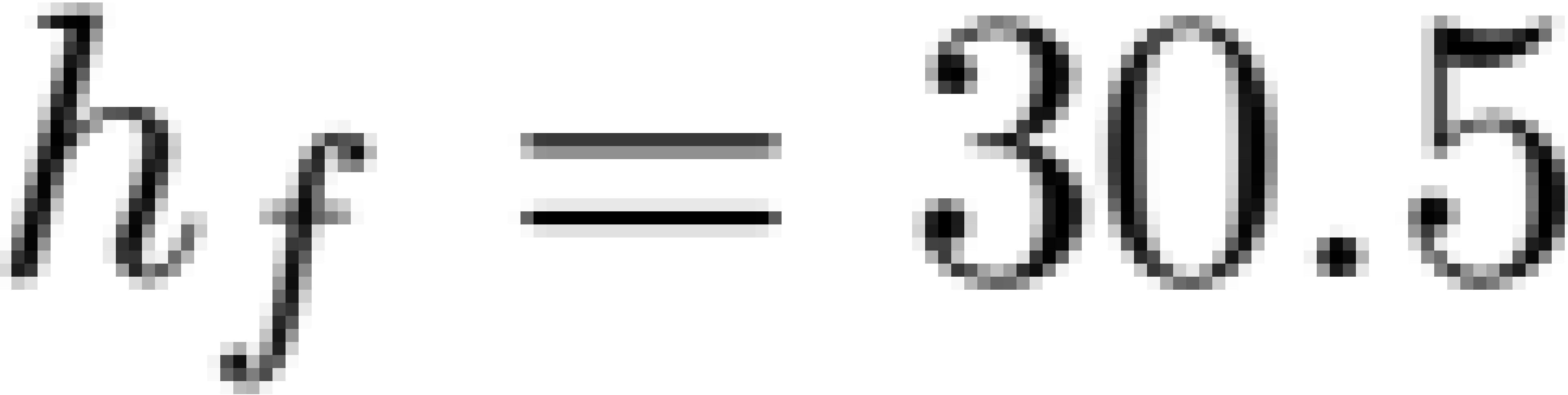


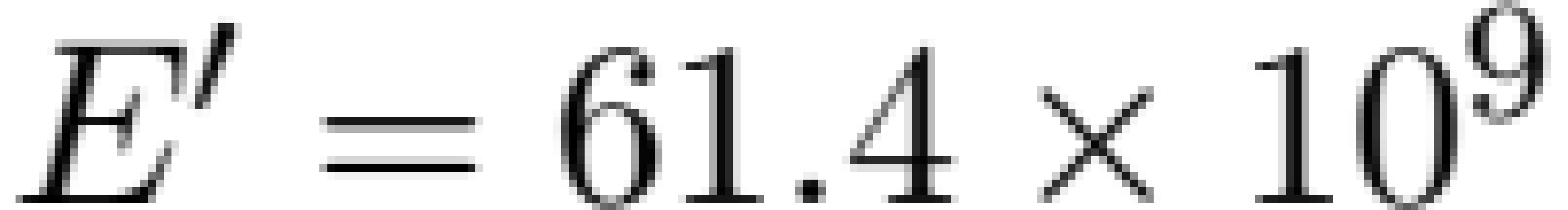


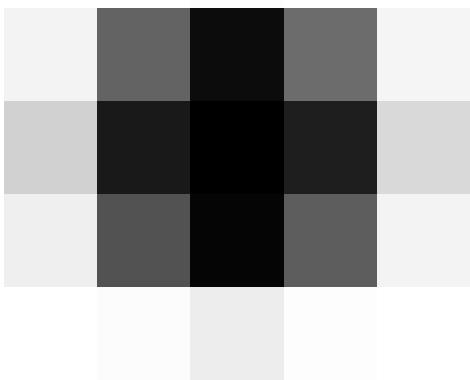


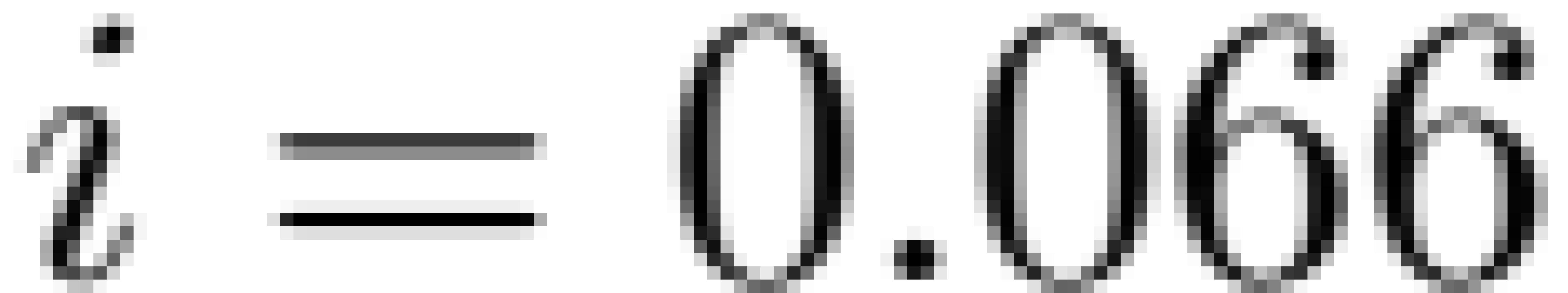












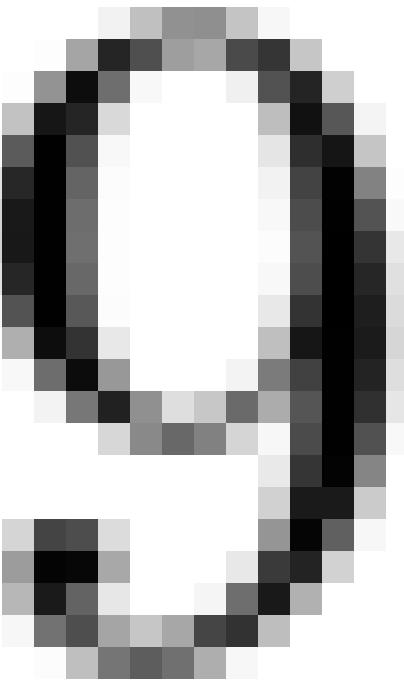
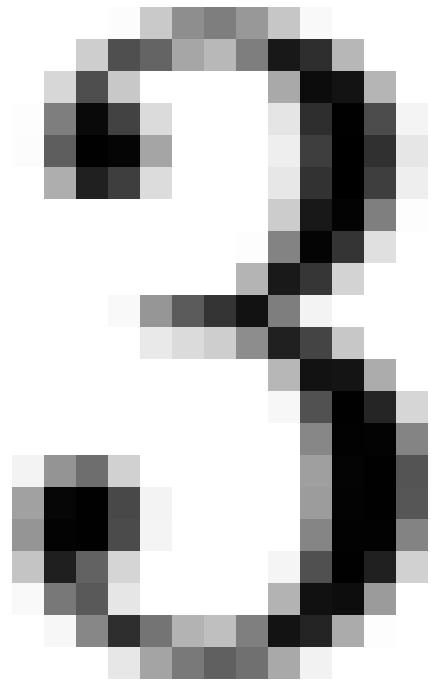
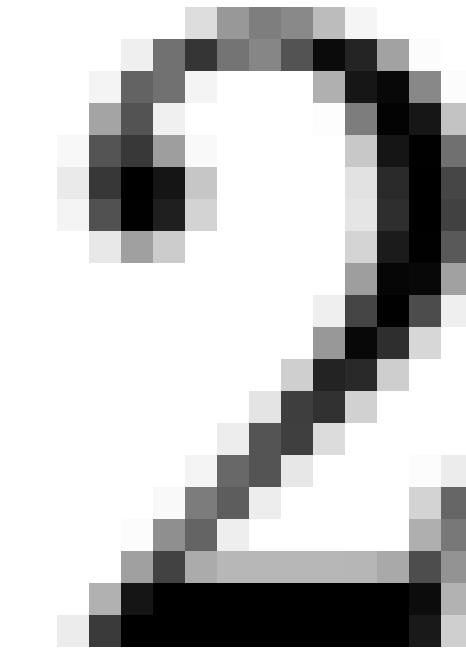
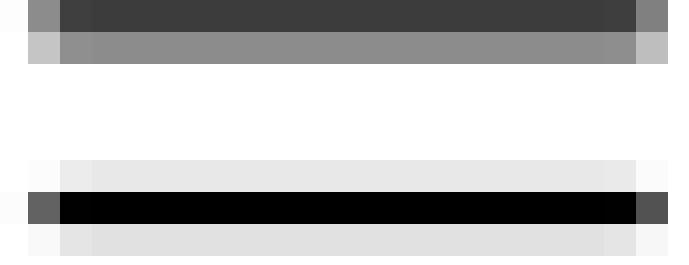
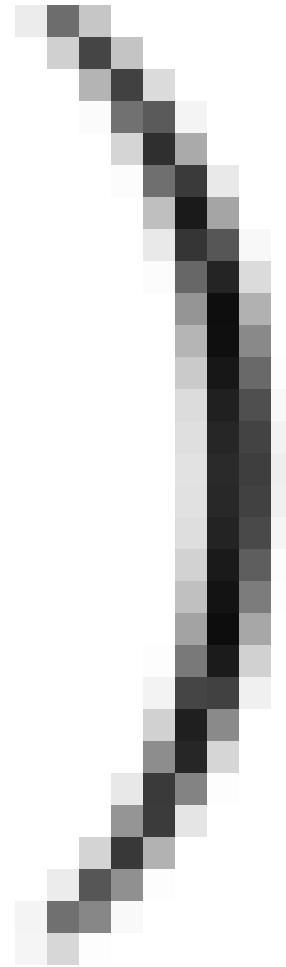
$$x_f = 0.524 \left[ \frac{(0.066 \text{ m}^3/\text{s})^3 \times (61.4 \times 10^9 \text{ Pa})}{0.001 \text{ Pa s} \times (30.5 \text{ m})^4} \right]^{1/5} = 1536.2 \text{ m}$$

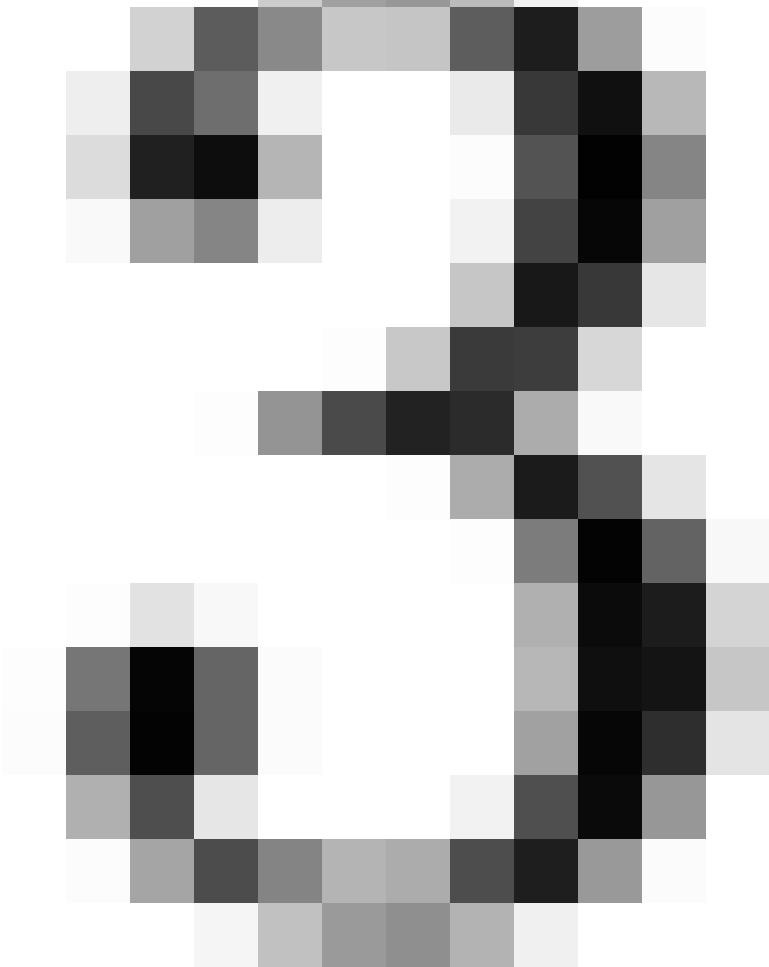
$$w_{w,0} = 3.040 \left[ \frac{(0.066 \text{ m}^3/\text{s})^2 \times (0.001 \text{ Pa s})}{(61.4 \times 10^9 \text{ Pa}) \times (30.5 \text{ m})} \right]^{1/5} = 4.05 \times 10^{-3} \text{ m} = 4.05 \text{ mm}$$

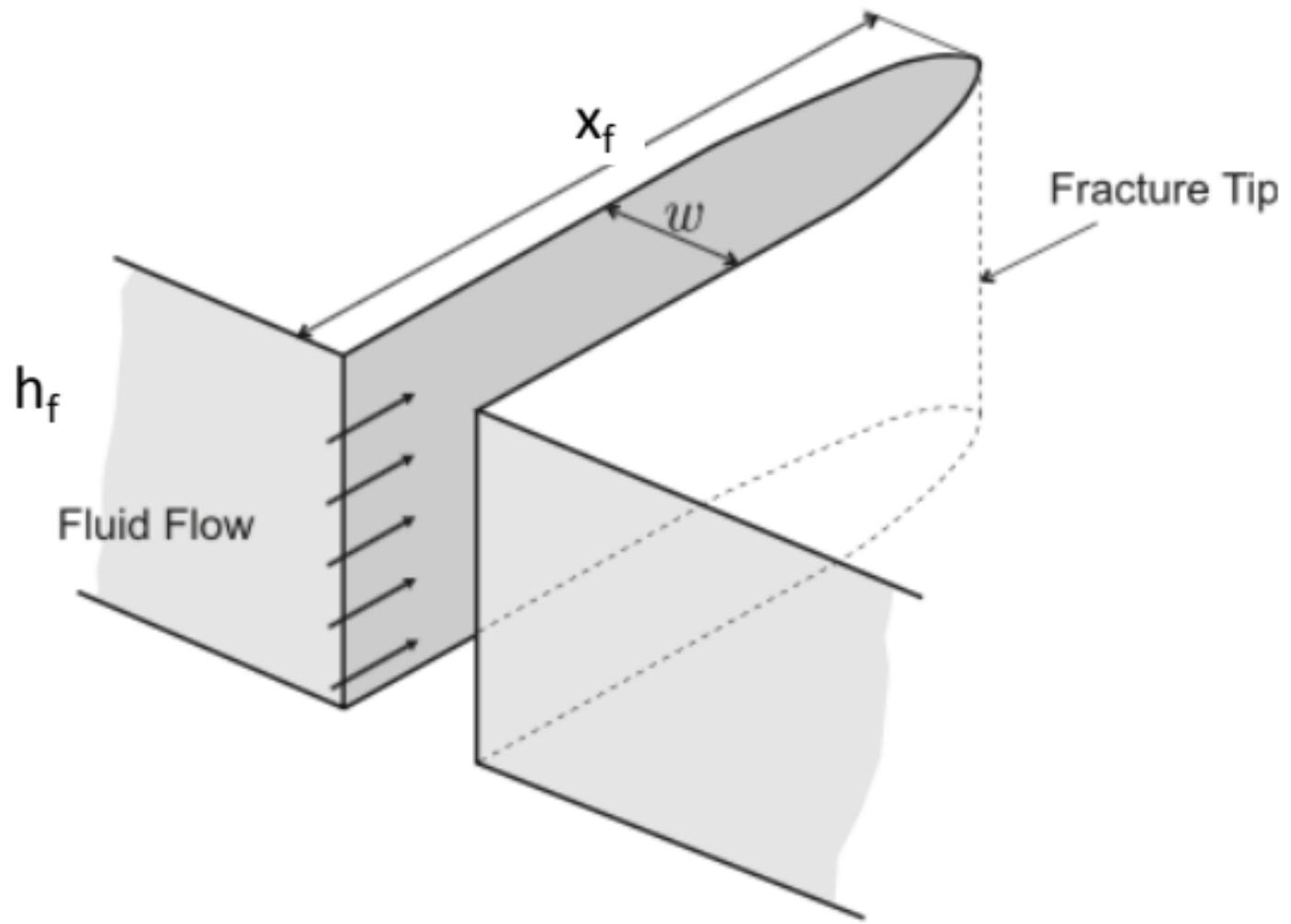
$$p_{net,w} = 1.520 \left[ \frac{(61.4 \times 10^9 \text{ Pa})^4 \times (0.066 \text{ m}^3/\text{s})^2 \times (0.001 \text{ Pa s})}{(1800 \text{ s})^{1/5} \times (30.5 \text{ m})^6} \right]^{1/5} = 4.1 \times 10^6 \text{ Pa}$$

$$2.13 \times 10^{-3} \pi$$





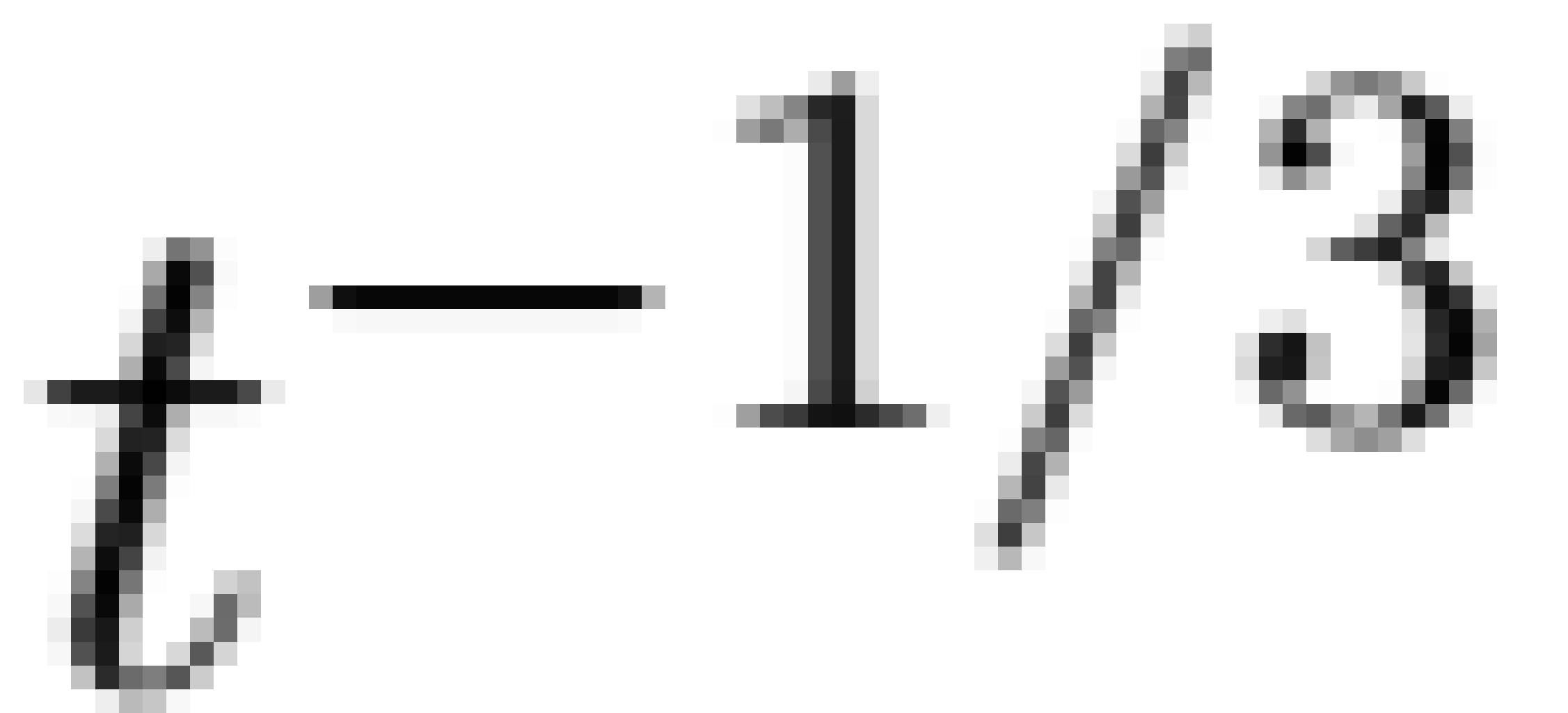


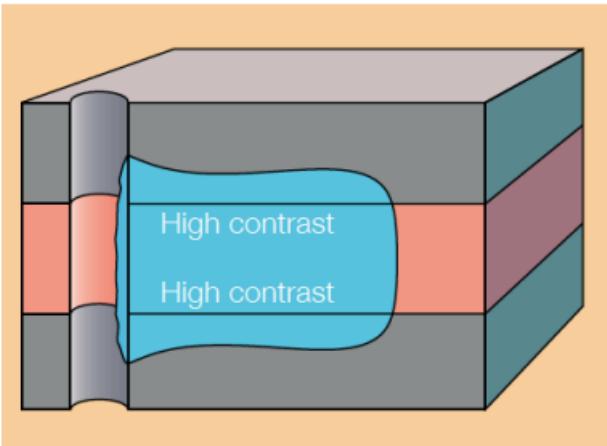
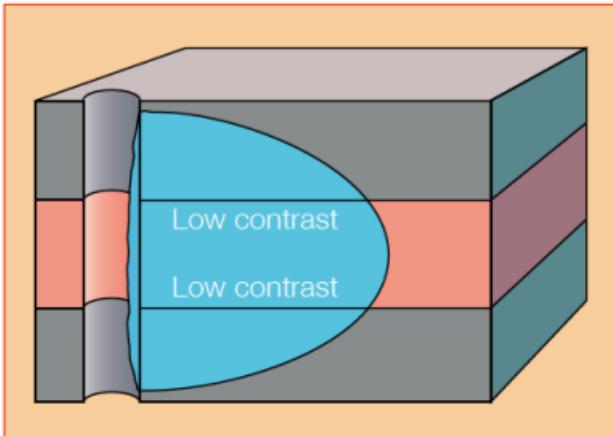


$$x_f = \left( \frac{16}{21\pi^3} \right)^{1/6} \left( \frac{i^3 E'}{\mu h_f^3} \right)^{1/5} t^{2/3} = 0.539 t^{2/3}$$

$$w_{w,0} = \left( \frac{5376}{\pi^3} \right)^{1/6} \left( \frac{\dot{x}^3 \mu}{E' h_f^3} \right)^{1/6} t^{1/3} = 2.360 t^{1/3}$$

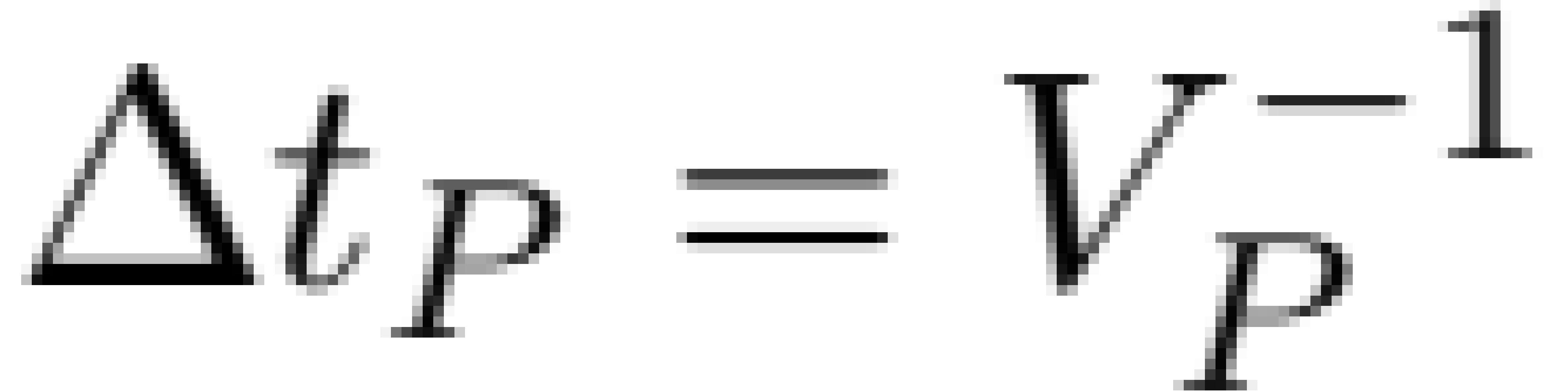
$$p_{net,w} = \left(\frac{21}{16}\right)^{1/3} (E^2 \mu)^{1/3} t^{-1/3} = 1.090 (E^2 \mu)^{1/3} t^{-1/3}$$

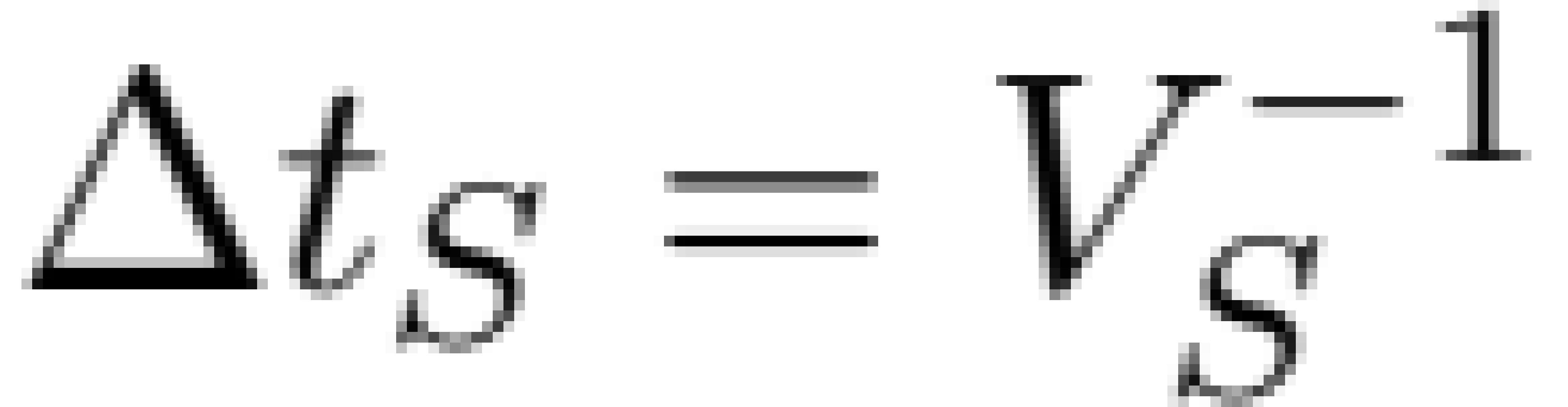










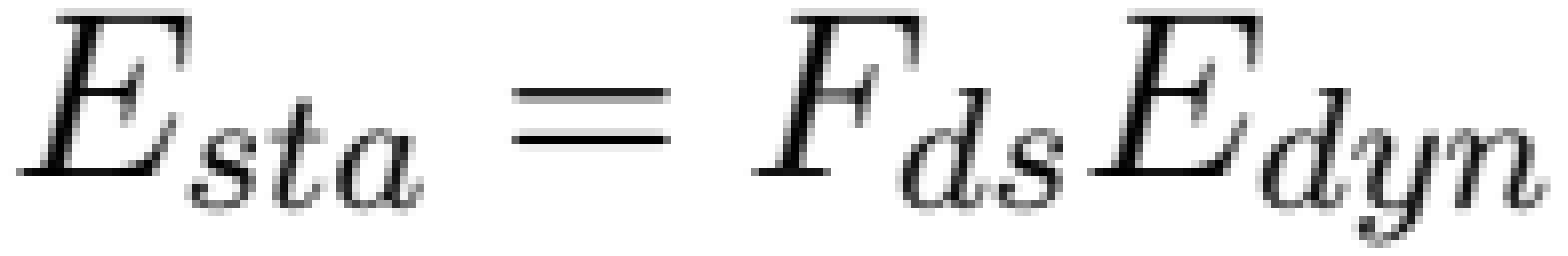


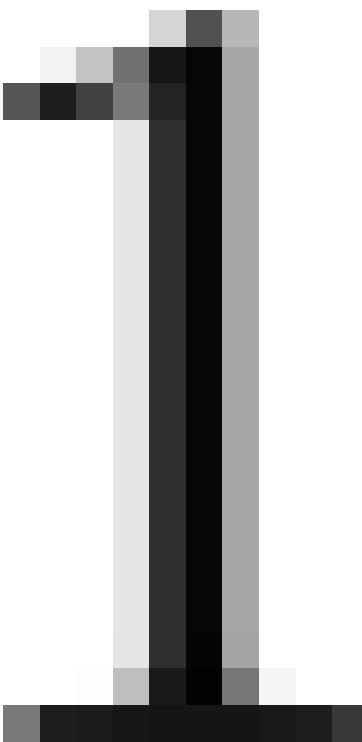
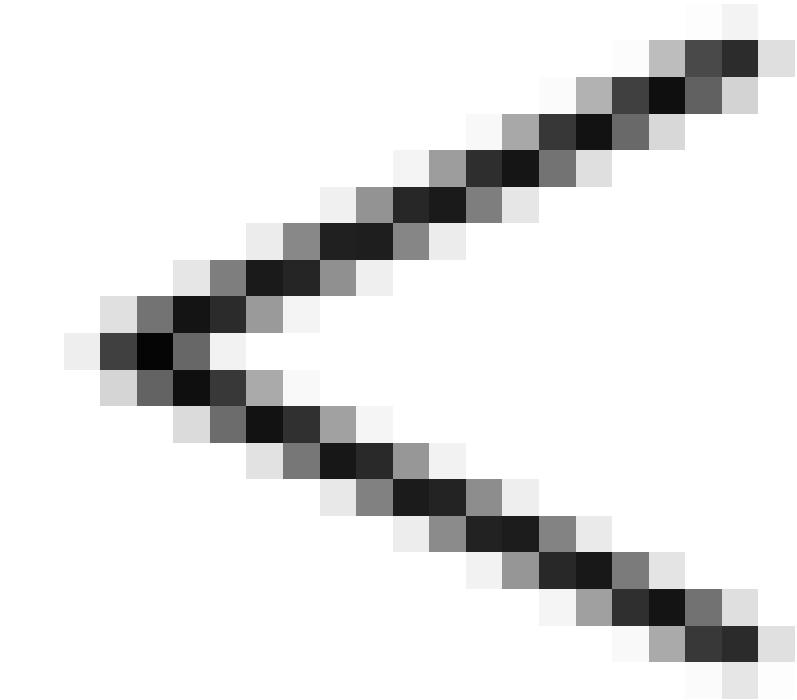
$$E_{\text{edge}} = \rho_{\text{bulk}} V_s^2 \left( \frac{3V_p^2}{V_s^2} - \frac{4V_s^2}{V_p^2} \right)$$

$$v_{dyn} = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)}$$



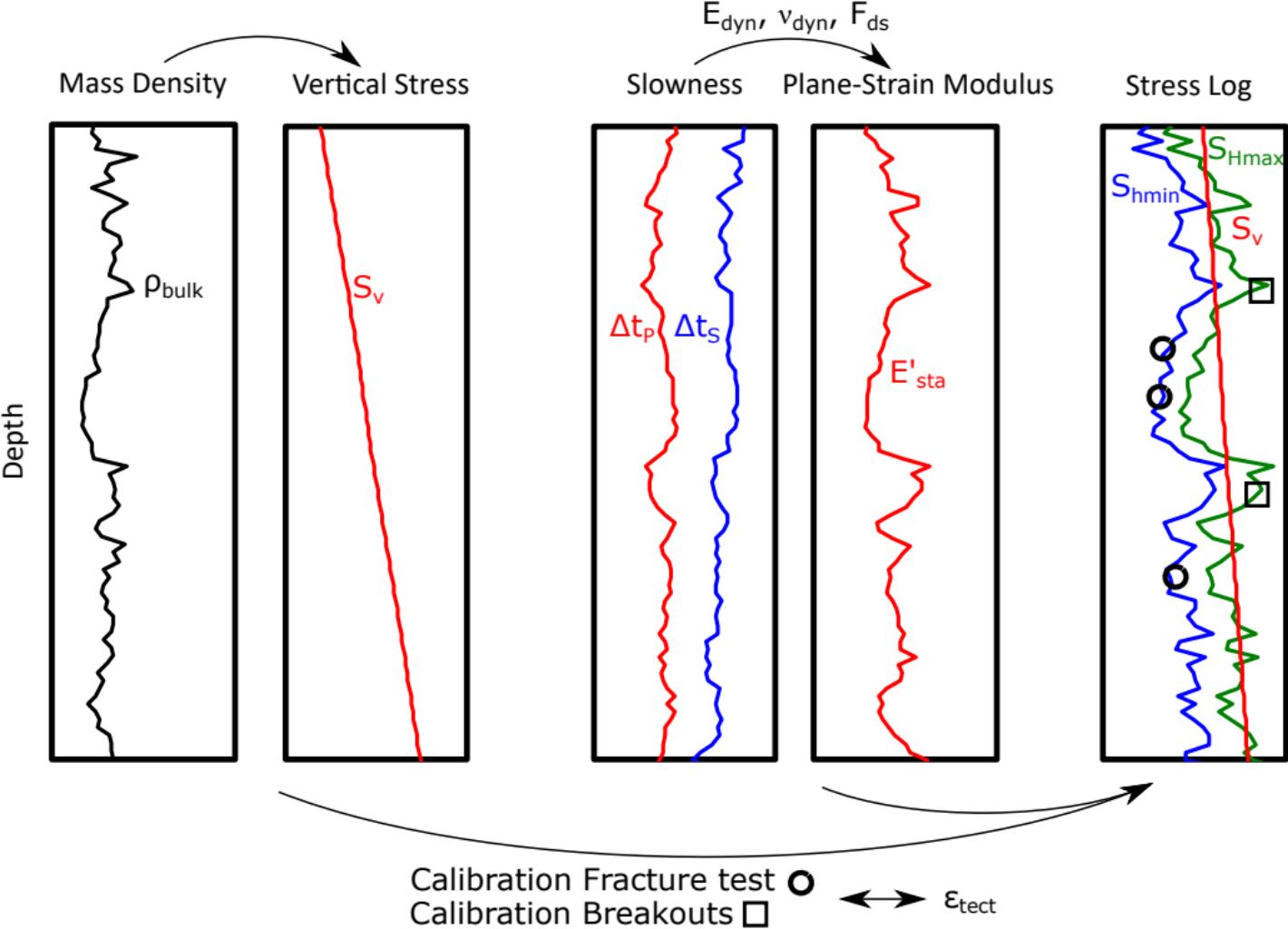


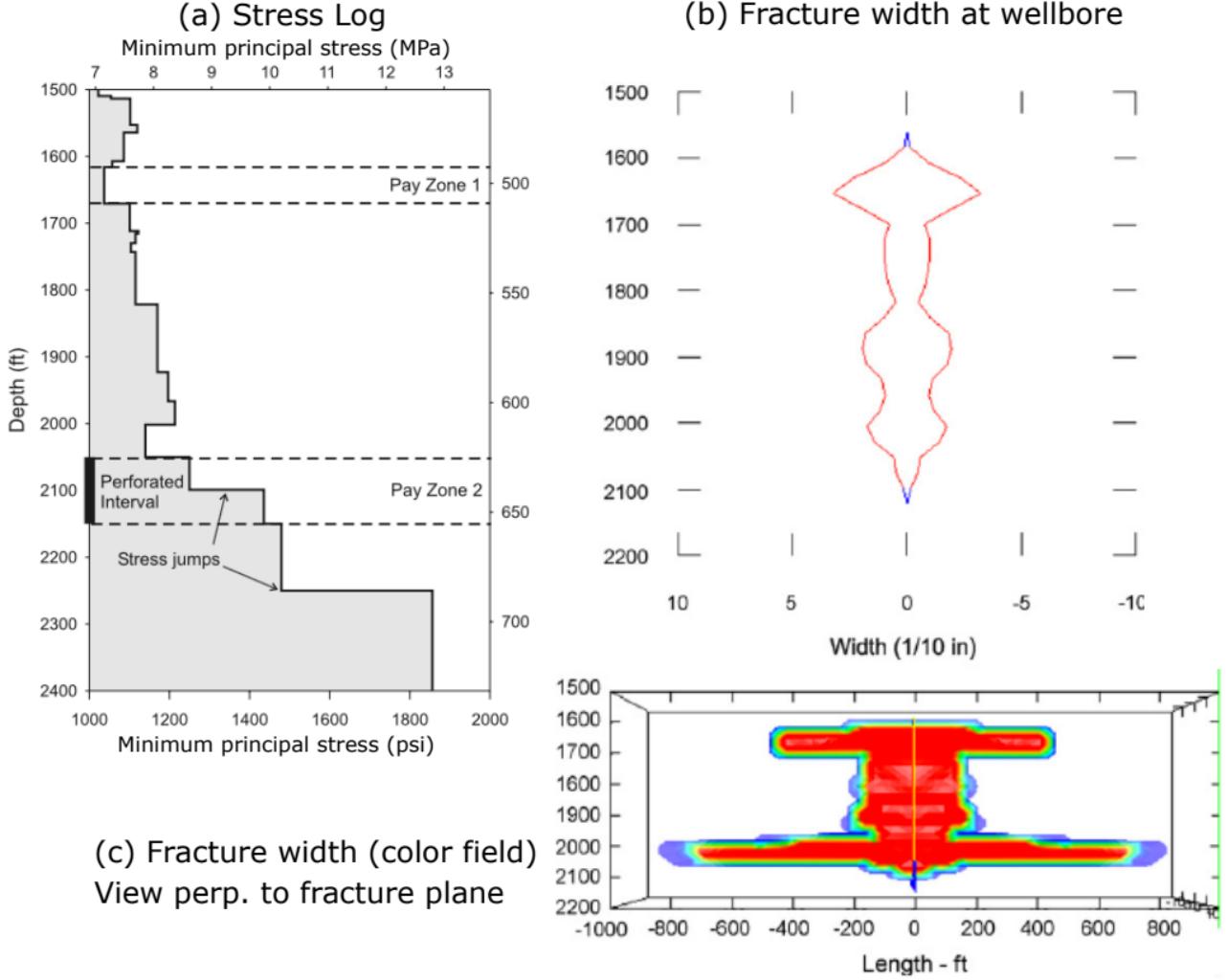


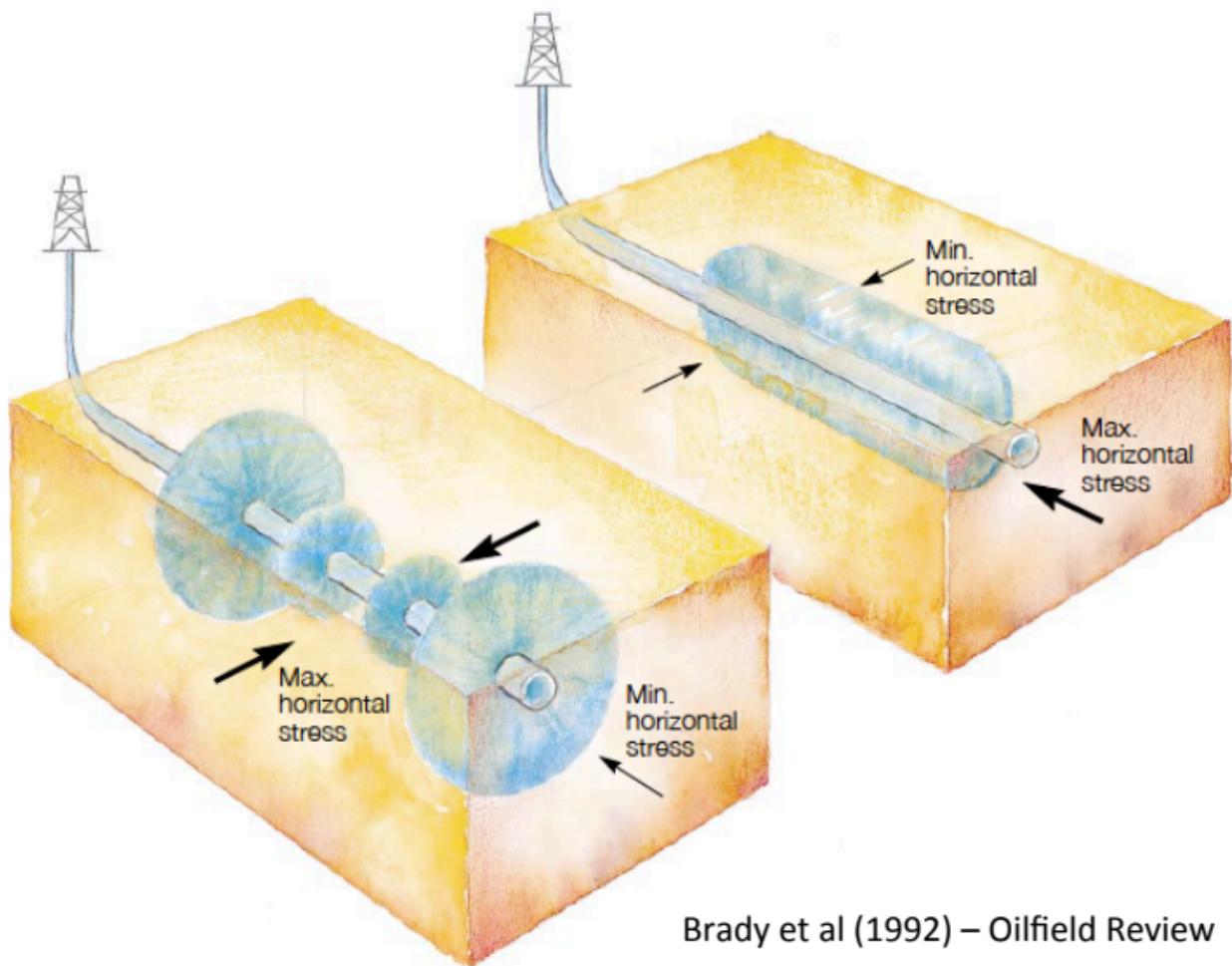






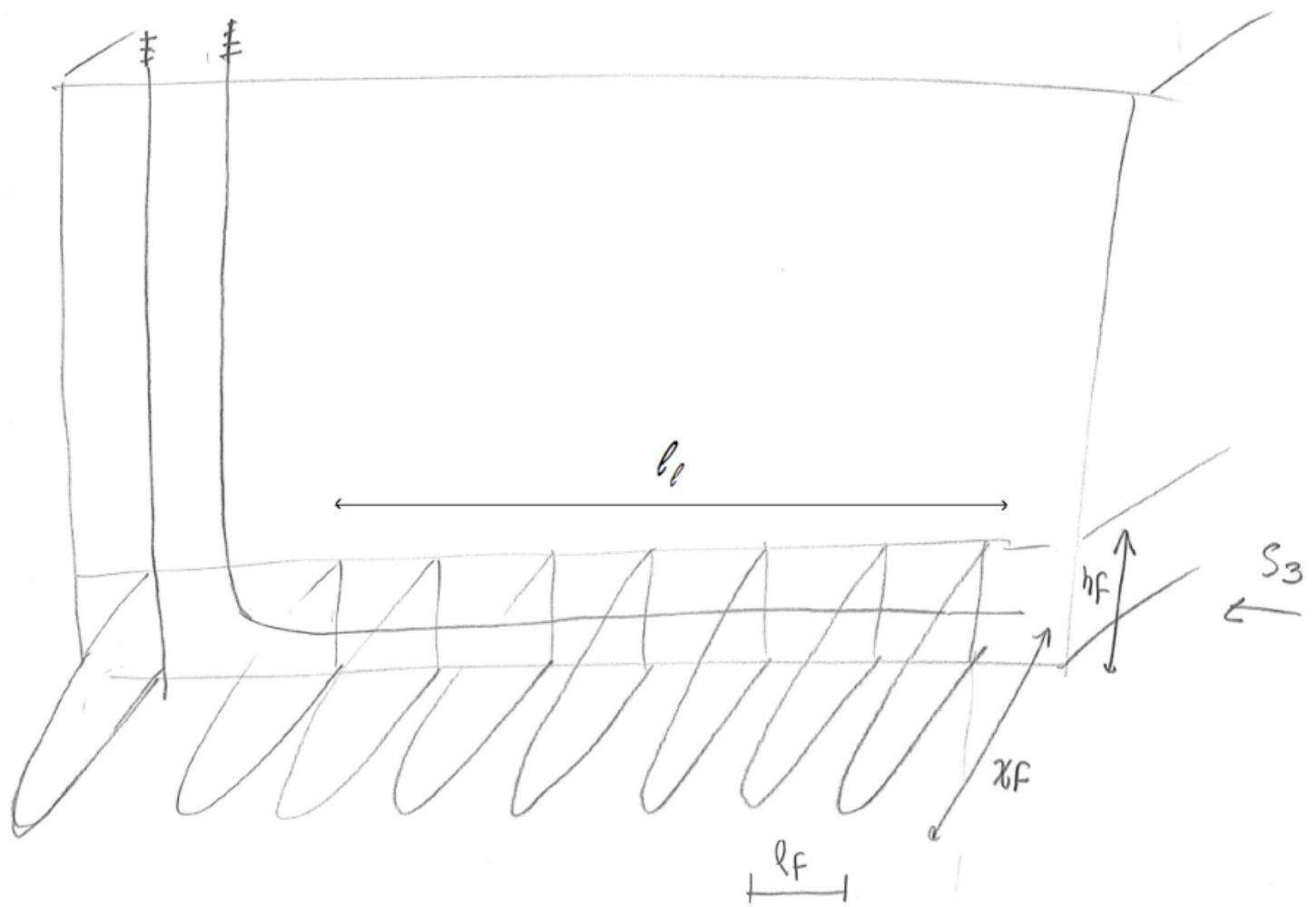






Brady et al (1992) – Oilfield Review





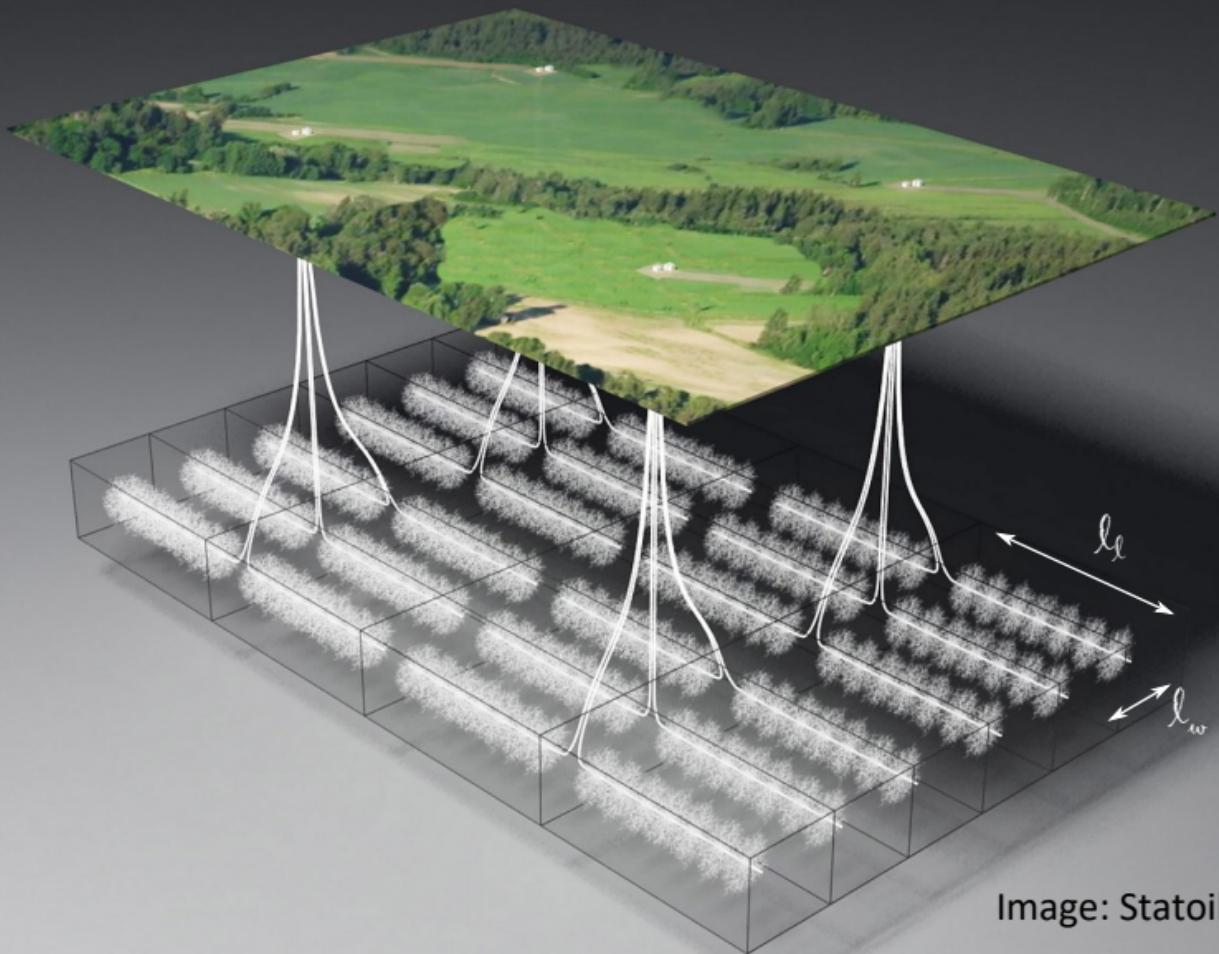
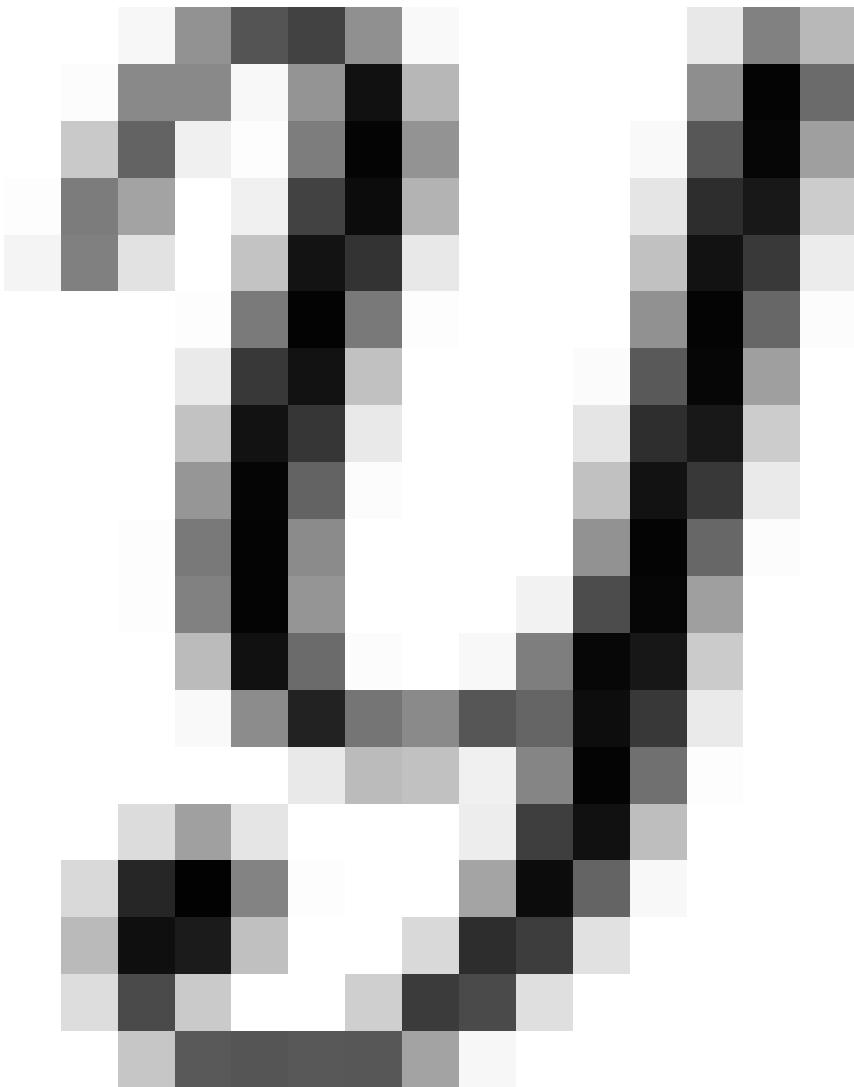
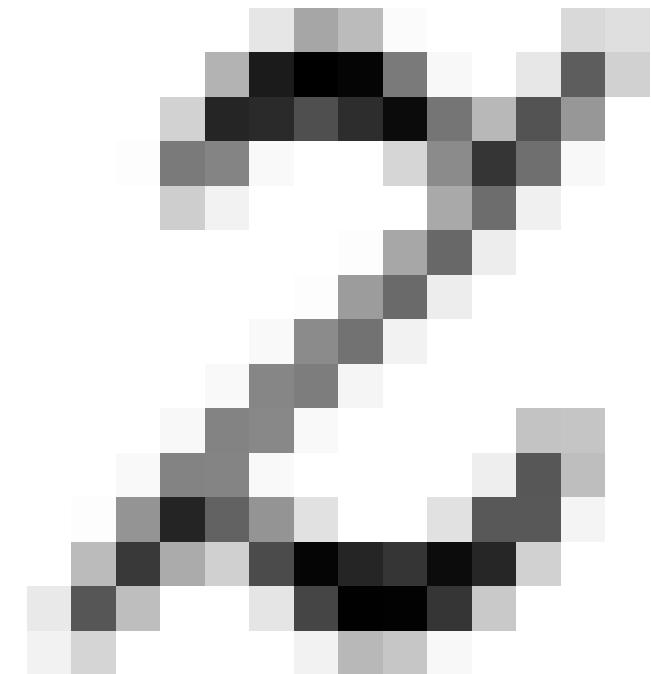
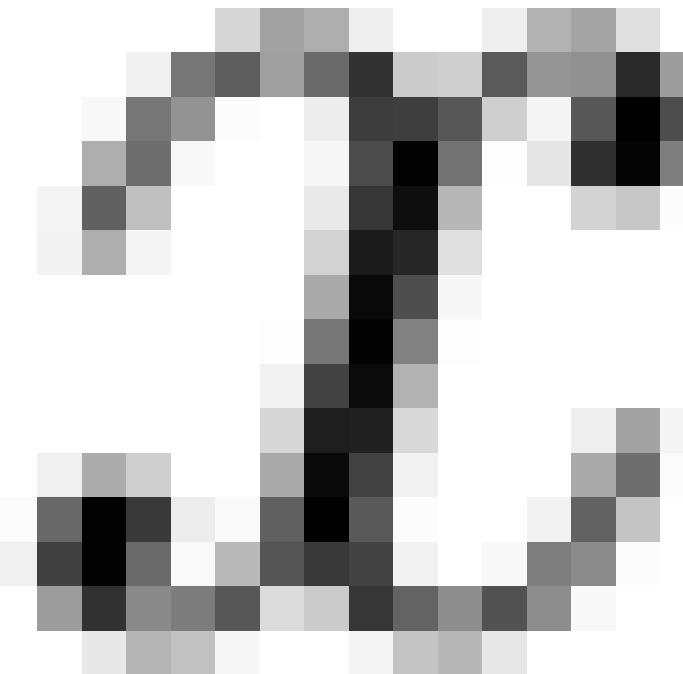


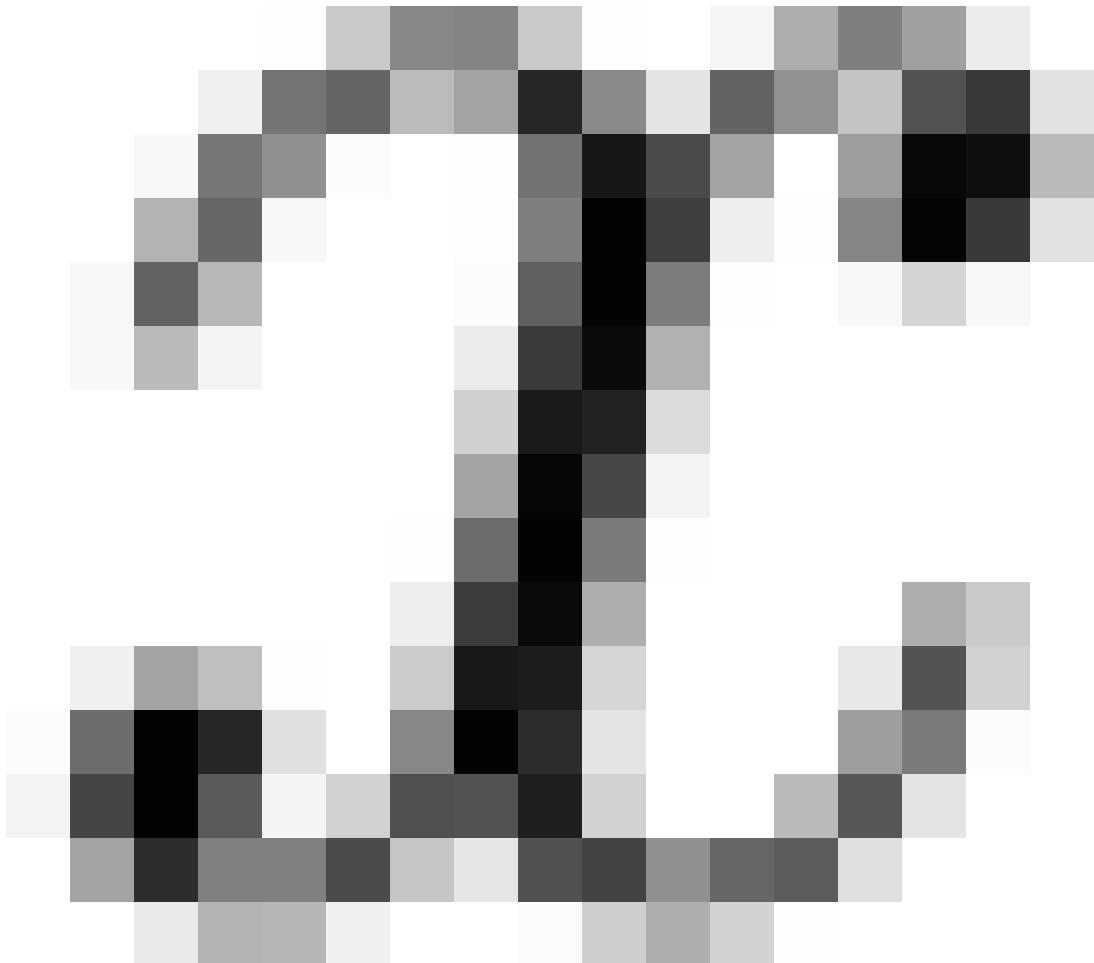
Image: Statoil



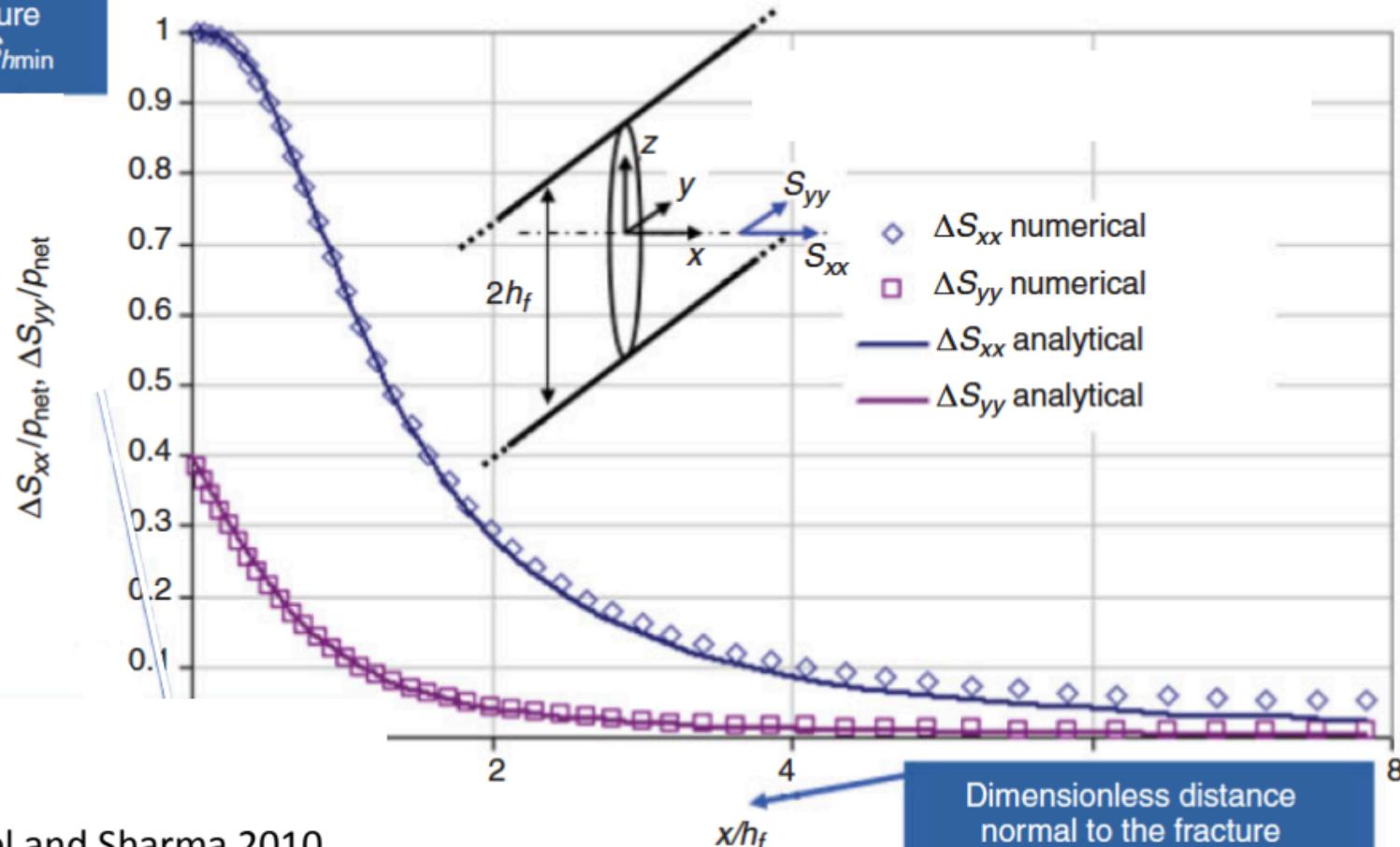






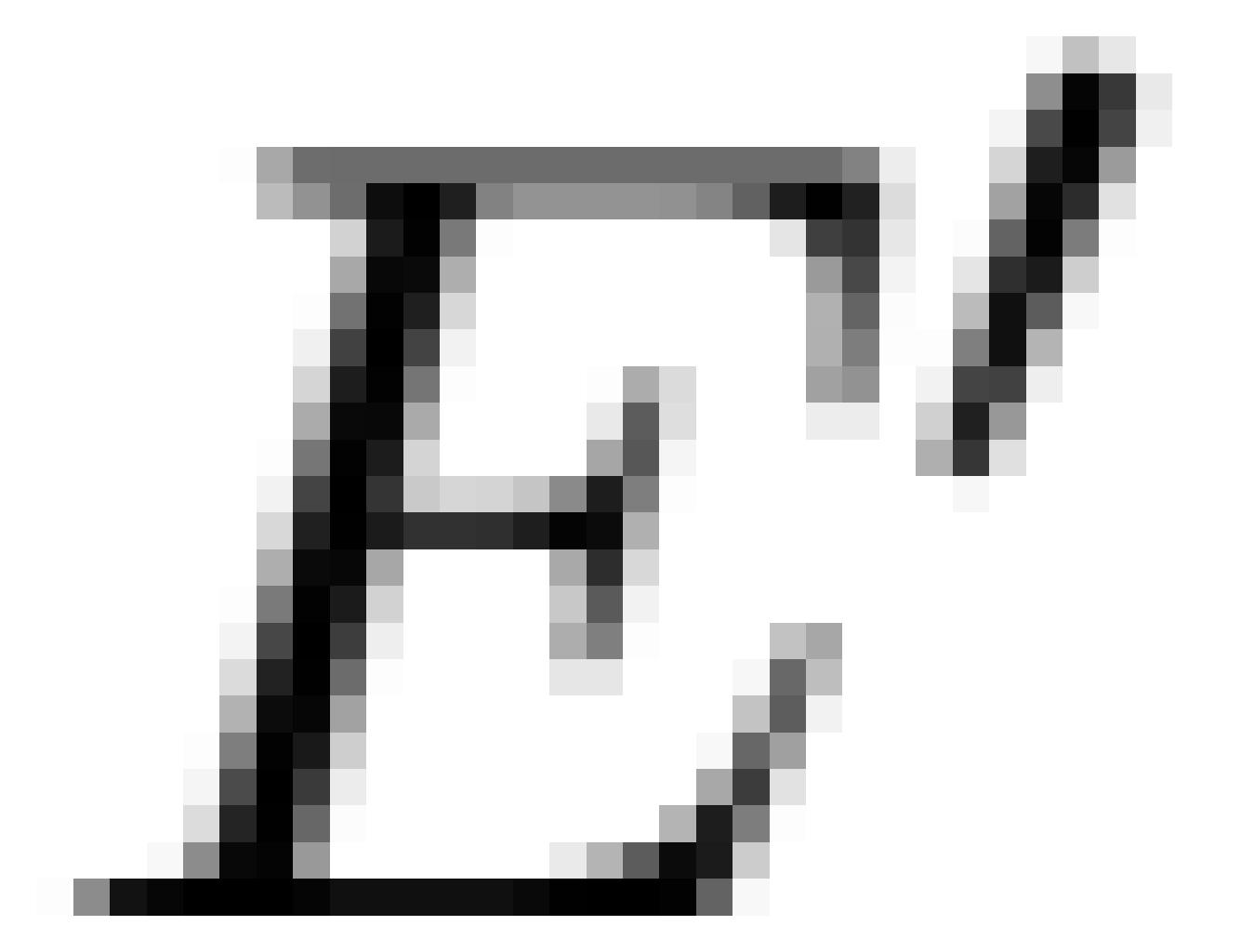


Net extension pressure  
 $= p_f - S_{h\min}$



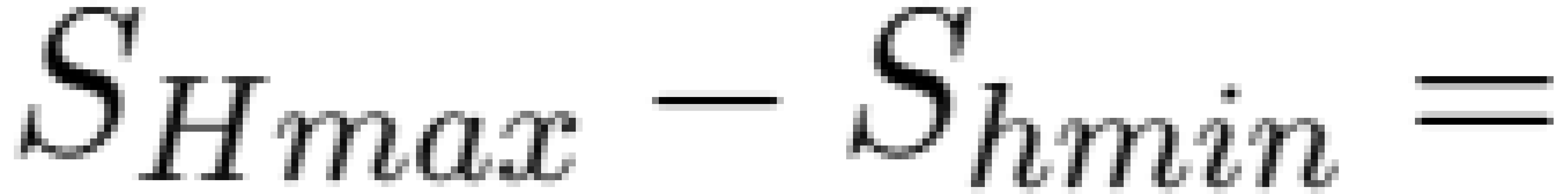
$\ell_f \propto$

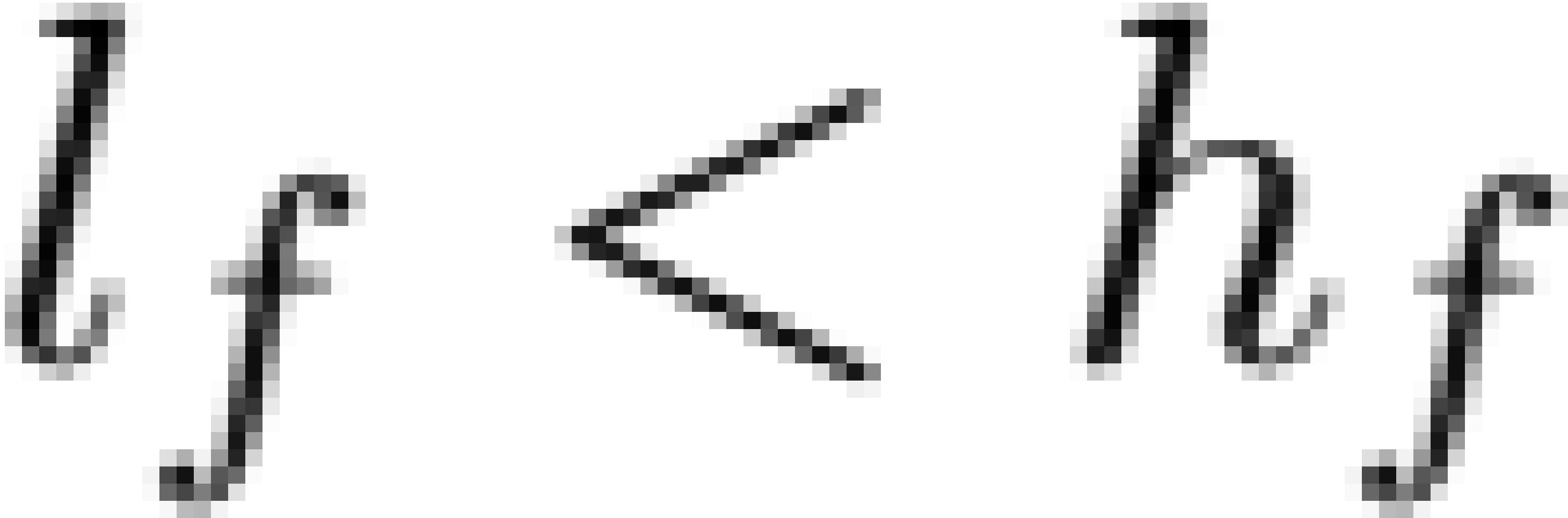
Preeti f  
—  
 $s_2 -$  —  $s_3$



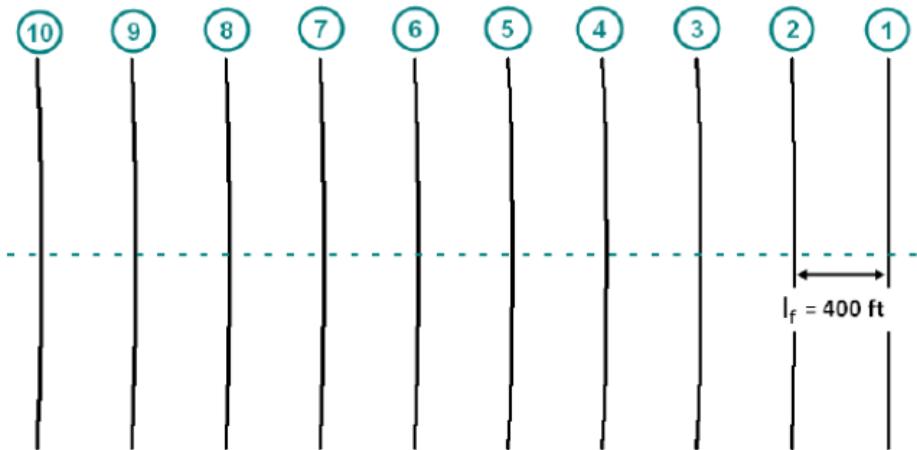




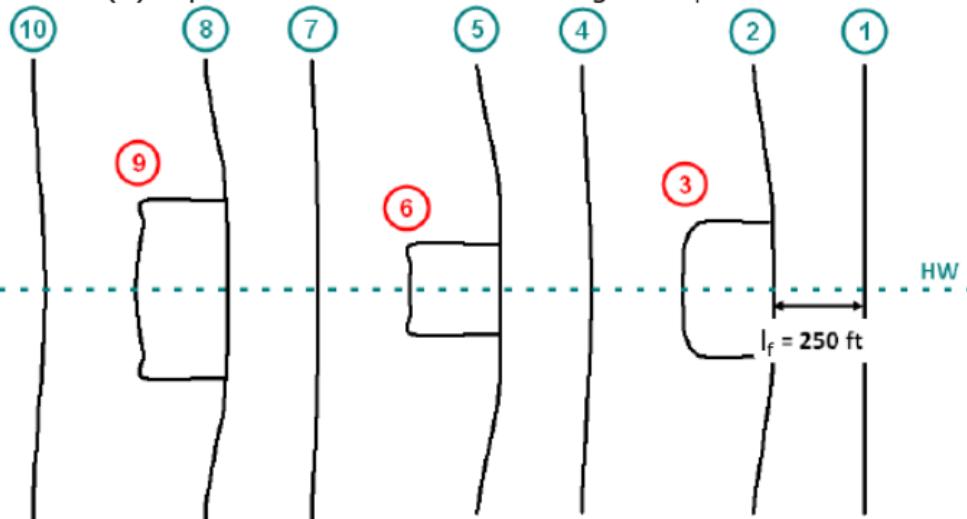


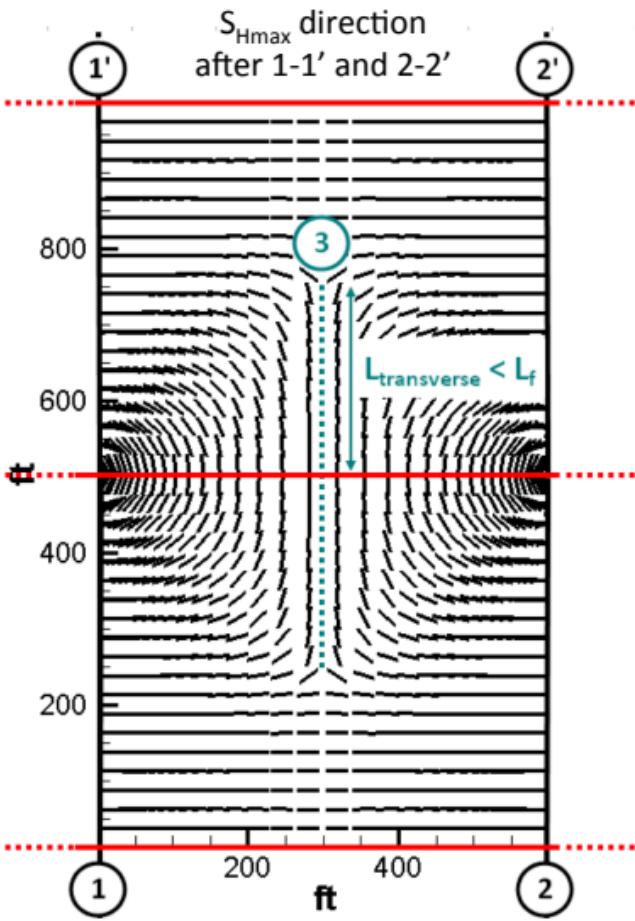
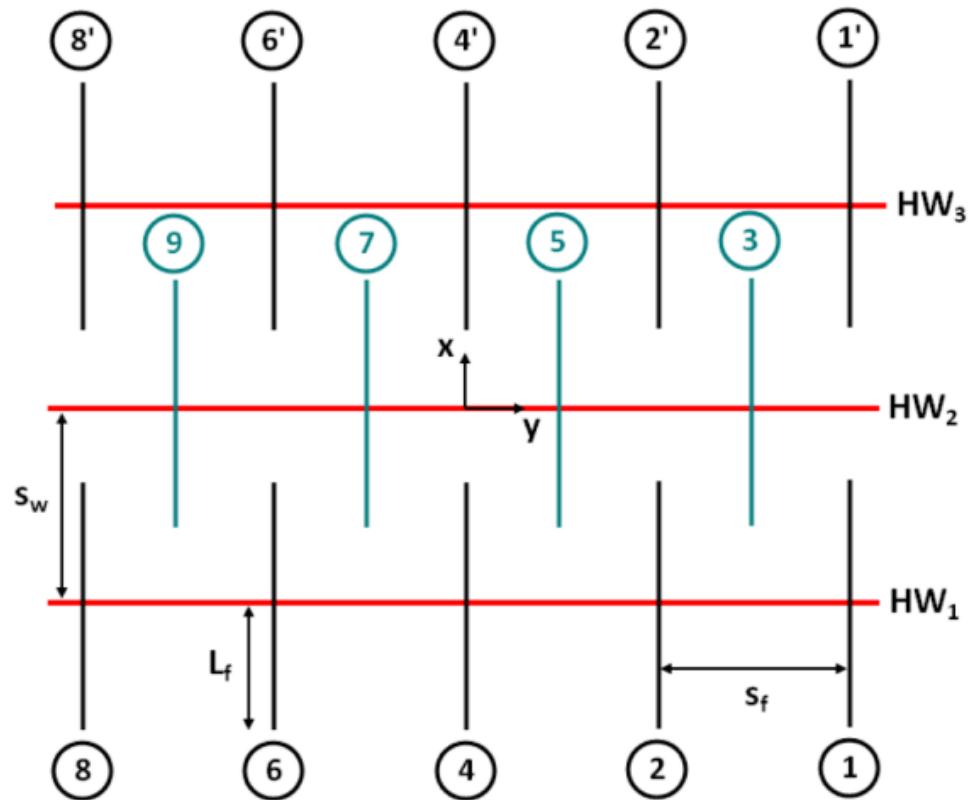


(a) Top view: consecutive fracturing with  $l_f = 400\text{ft}$

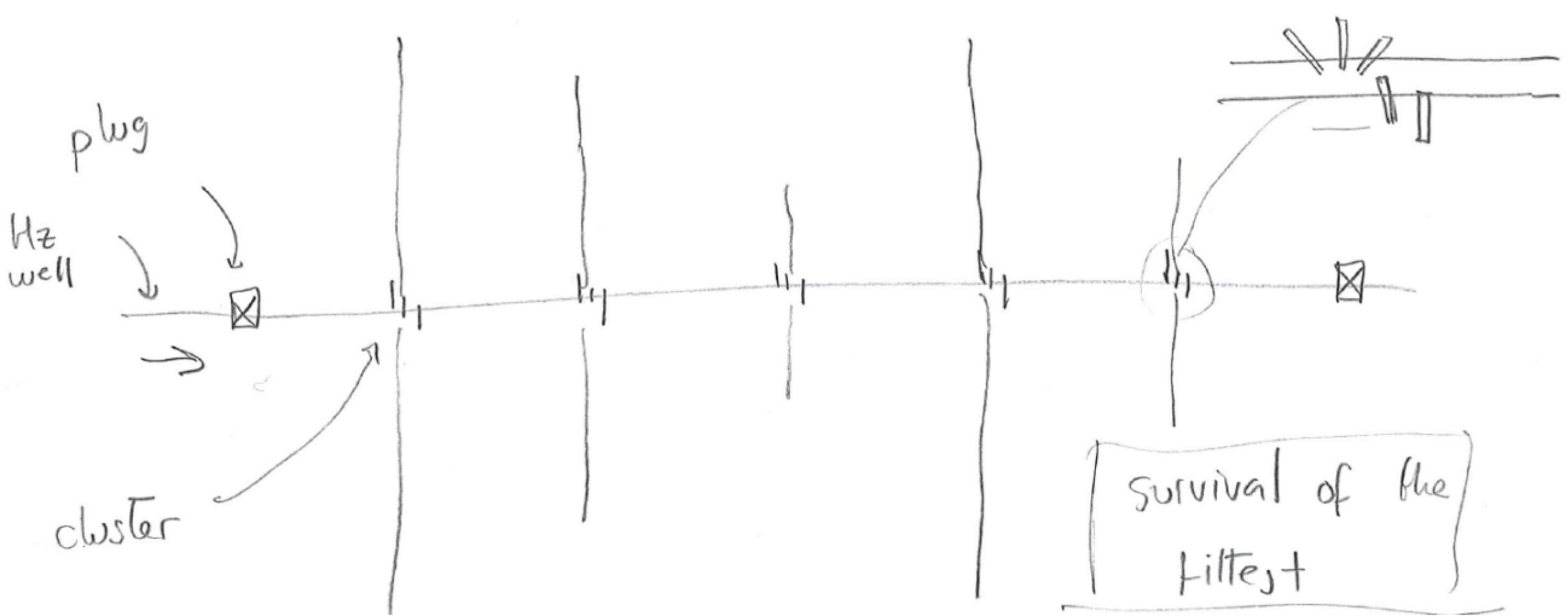


(b) Top view: consecutive fracturing with  $l_f = 250\text{ft}$

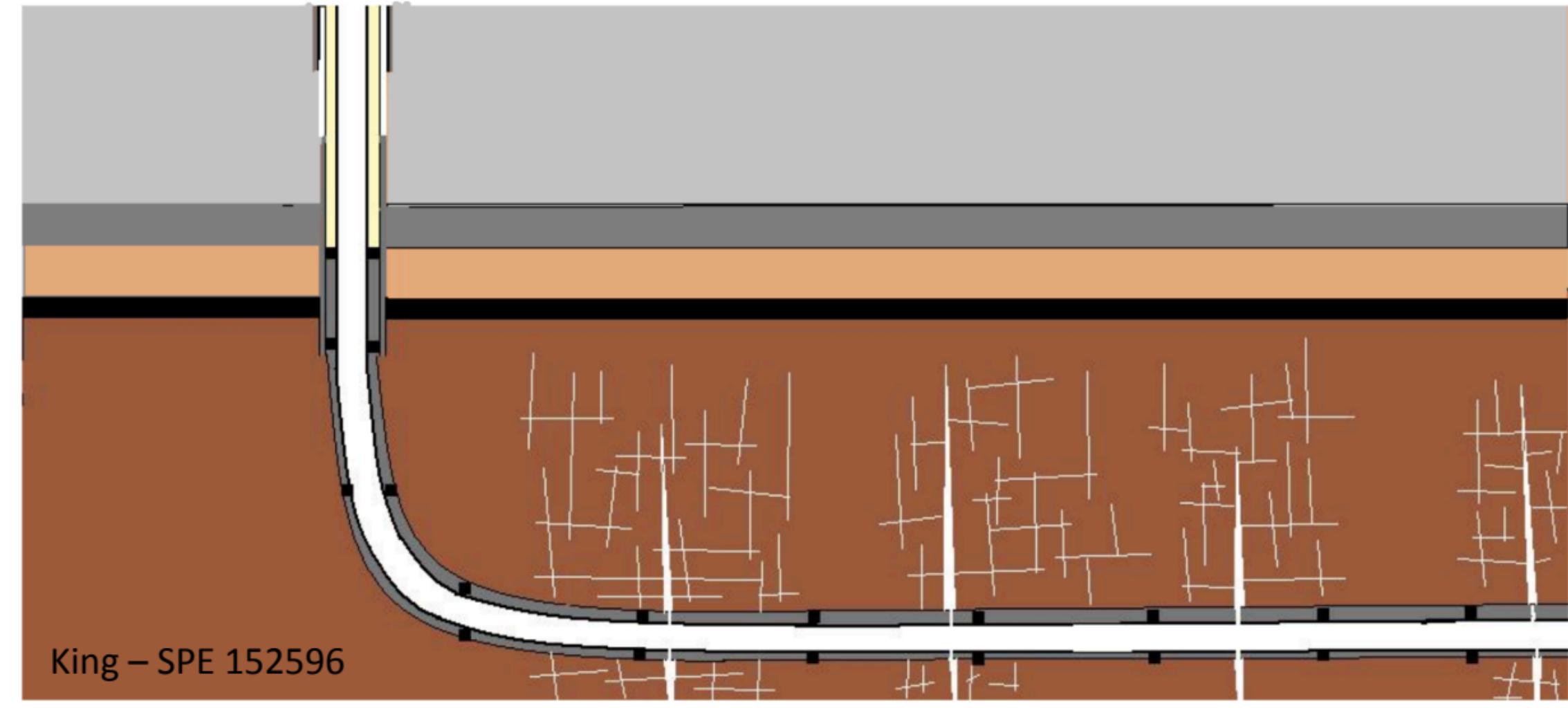


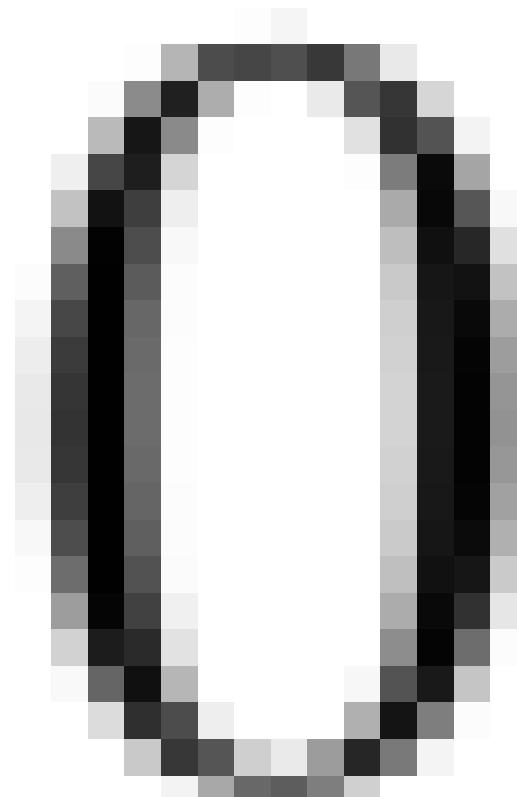
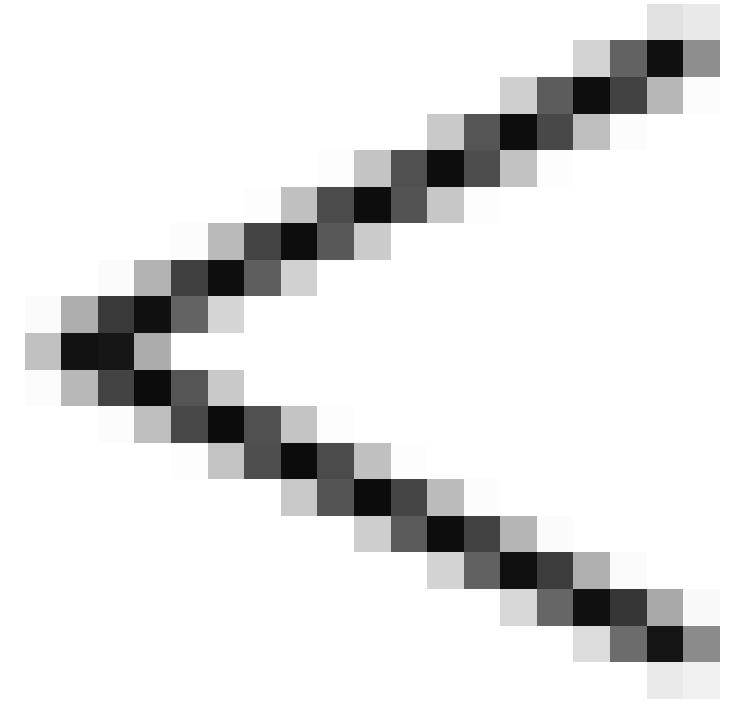






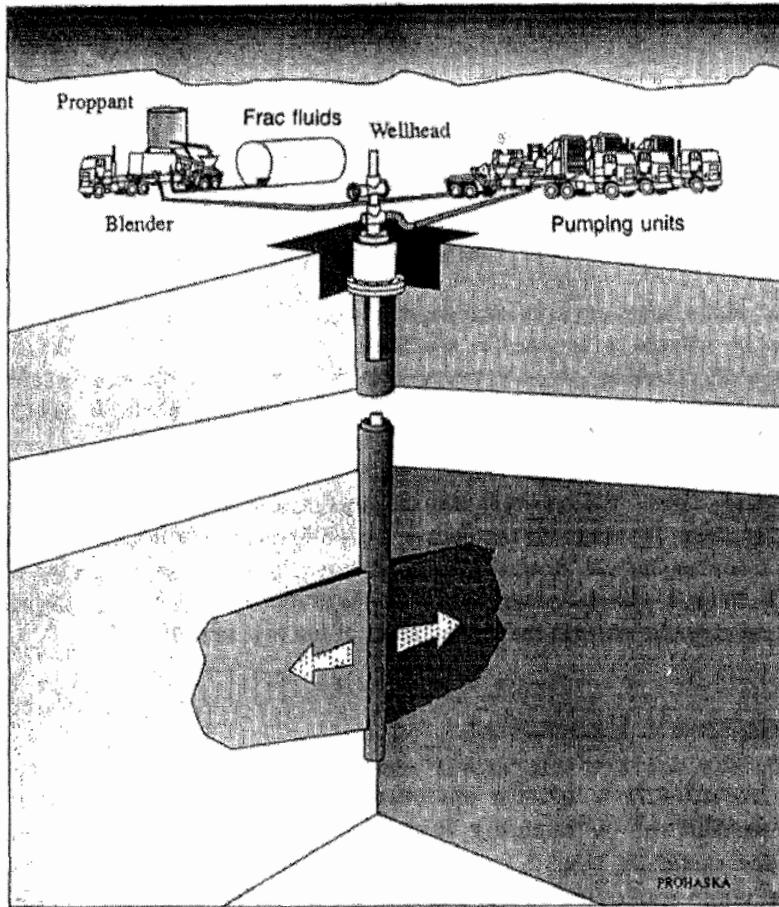
King – SPE 152596





... / Fig

$EVR = RH - SRV(1 - \phi)$

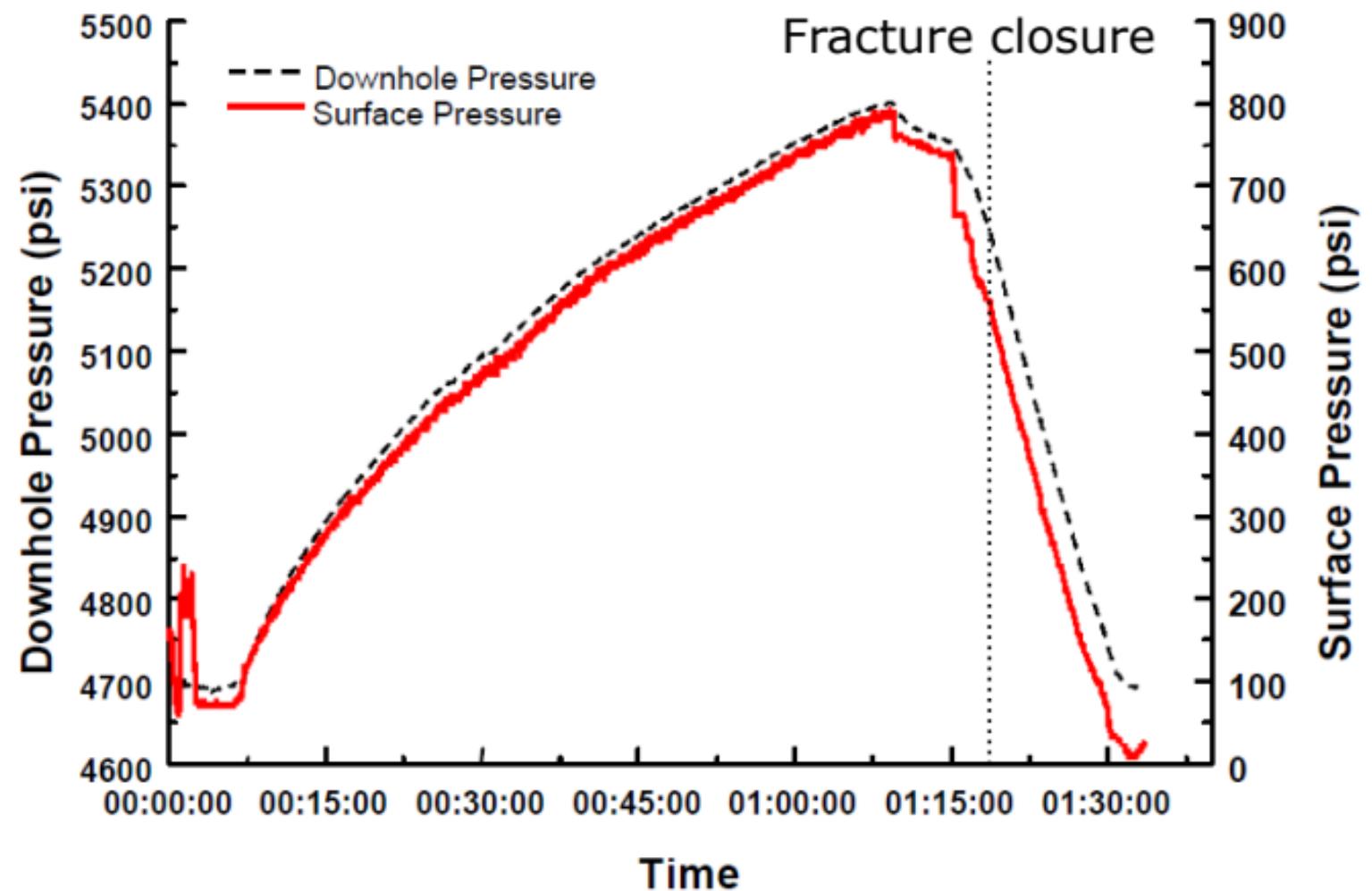
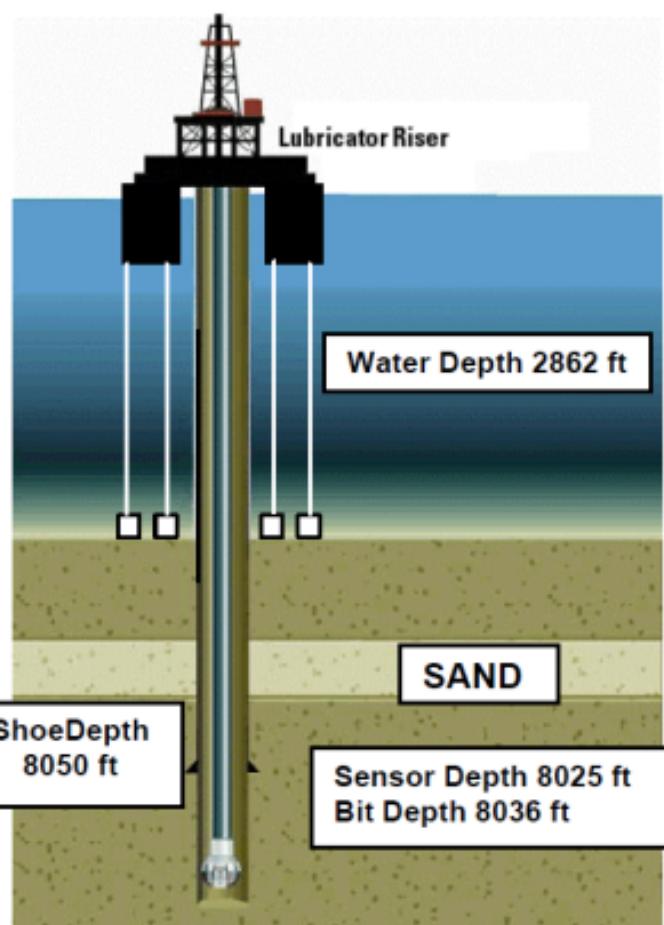


From  
Valko and Economides, 1996

Surf Press [Tbg] (psi) Slurry Flow Rate (bpm)  
Proppant Conc (ppg)





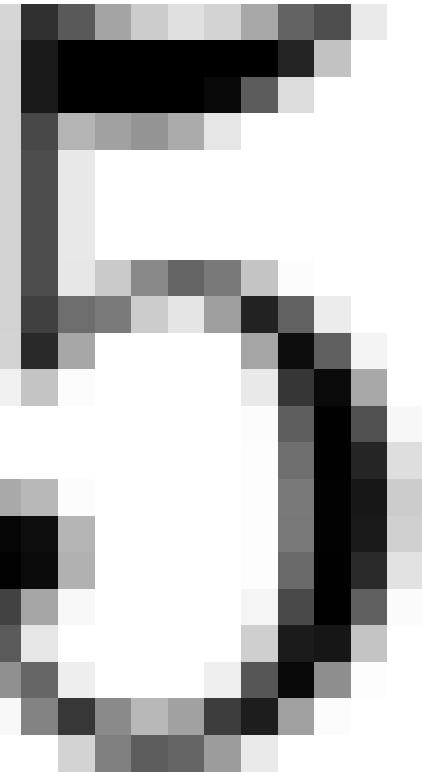
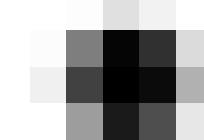
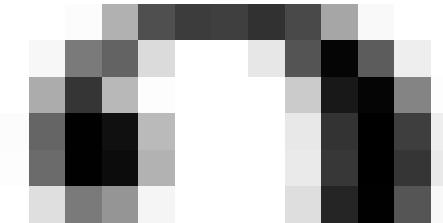
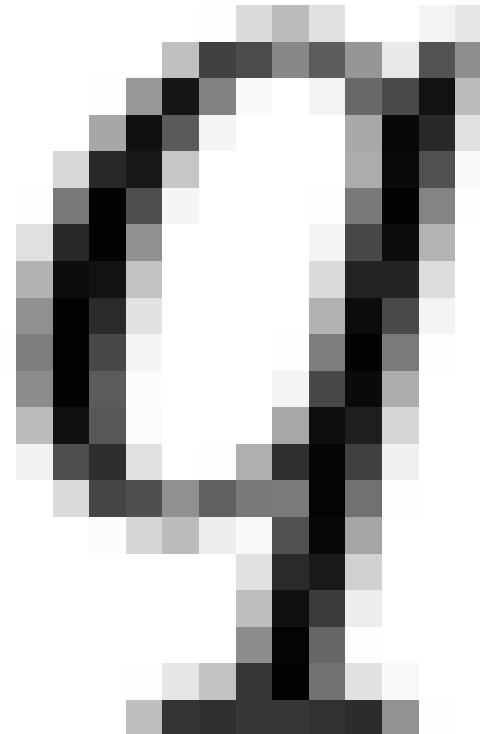


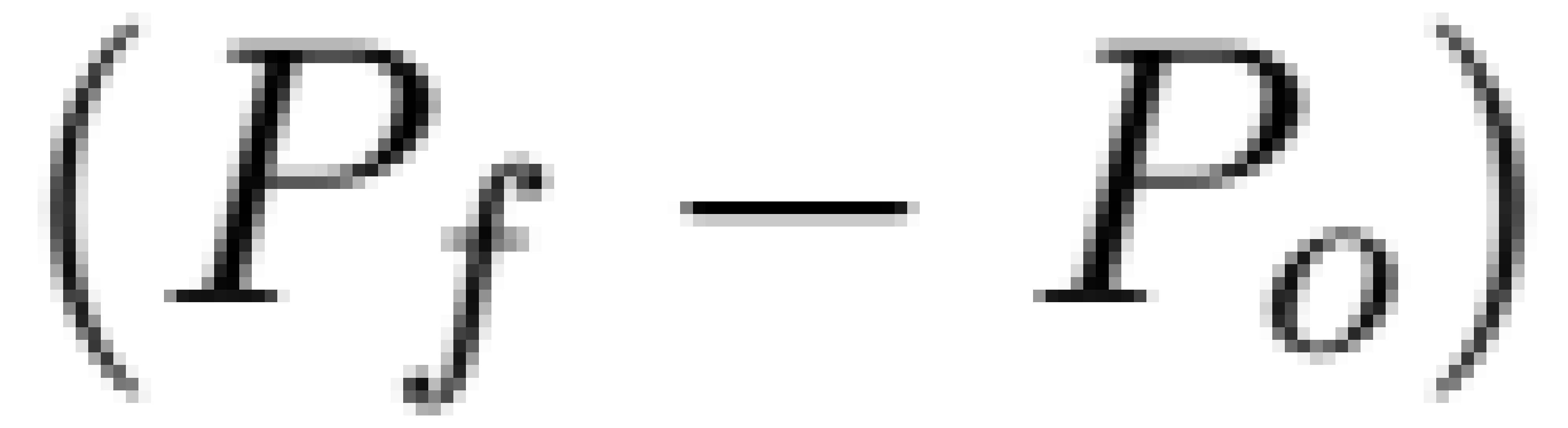
Step #	Test rate (bbl/min)	max. rate)	Test rate (% of								
			Time (min)	0	5	10	15	20	25	30	
1	0.2	5	Pressure (psi)	0	99	105	108	109	110	110	
			Time (min)	0	5	10	15	20	25	30	
2	0.4	10	Pressure (psi)	88	187	204	215	219	220	220	
			Time (min)	0	5	10	15	20	25	30	
3	0.8	20	Pressure (psi)	209	358	424	431	438	439	440	
			Time (min)	0	5	10	15	20	25	30	
4	1.6	40	Pressure (psi)	418	770	869	871	875	878	882	
			Time (min)	0	5	10	15	20	25	30	
5	2.4	60	Pressure (psi)	825	1089	1133	1199	1265	1298	1321	
			Time (min)	0	5	10	15	20	25	30	
6	3.2	80	Pressure (psi)	1210	1375	1459	1507	1529	1535	1540	
			Time (min)	0	5	10	15	20	25	30	
7	4	100	Pressure (psi)	1485	1595	1650	1683	1727	1749	1760	
			Time (min)	0	5	10	15	20	25	30	

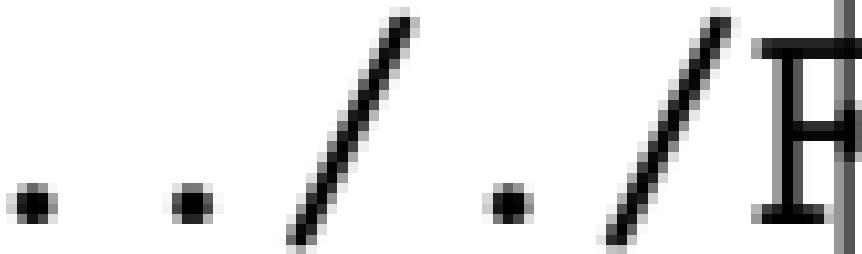














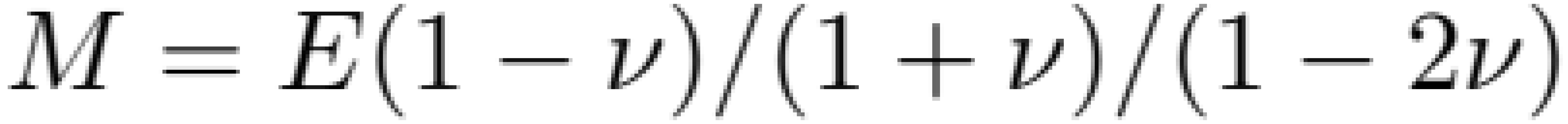


• . / E

$$\begin{bmatrix} S_{11} - \alpha P_p \\ S_{22} - \alpha P_p \\ S_{33} - \alpha P_p \\ S_{12} \\ S_{13} \\ S_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varepsilon_{33} \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma_{33} =$$

$$\frac{S_{33} - \alpha P_p}{E(1-\nu)}$$
$$\frac{(1+\nu)(1-2\nu)}{(1+\nu)(1-2\nu)}$$



AC33

E

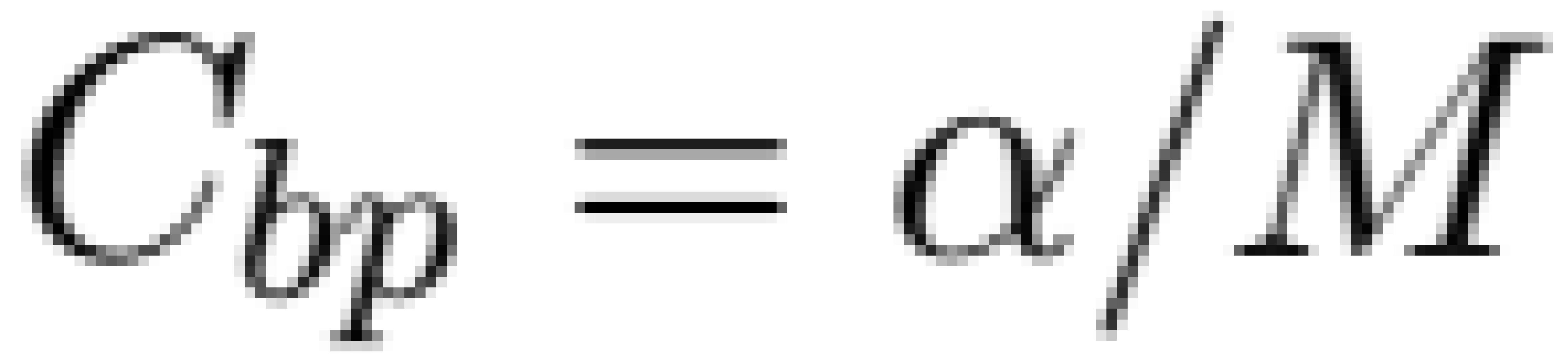
M

O

AP





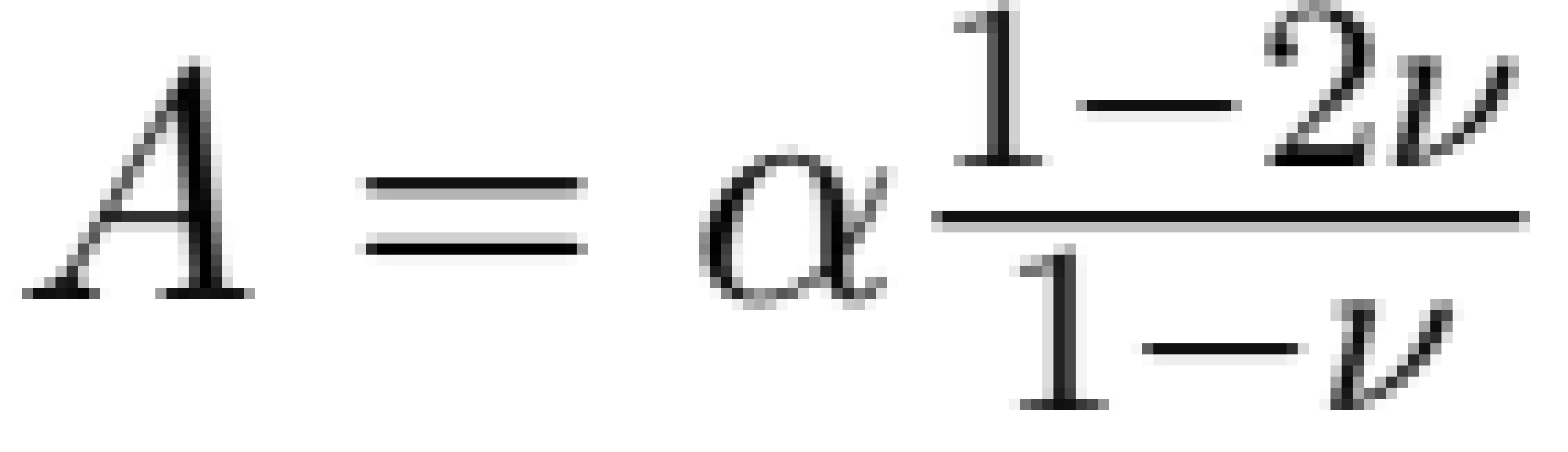


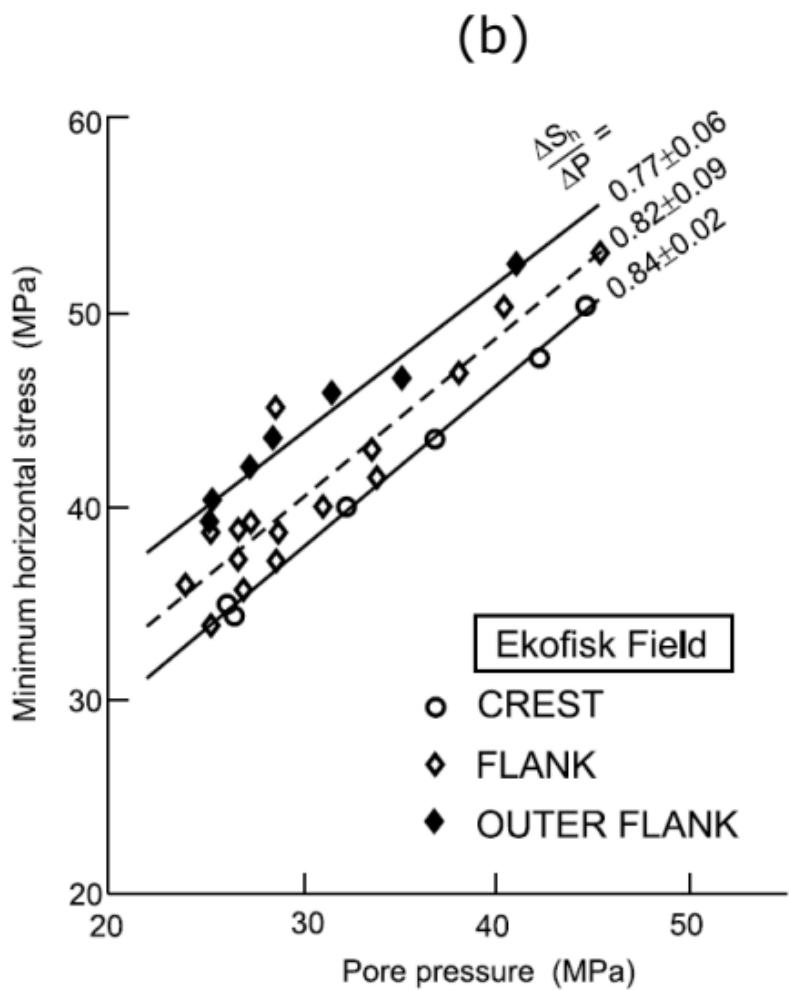
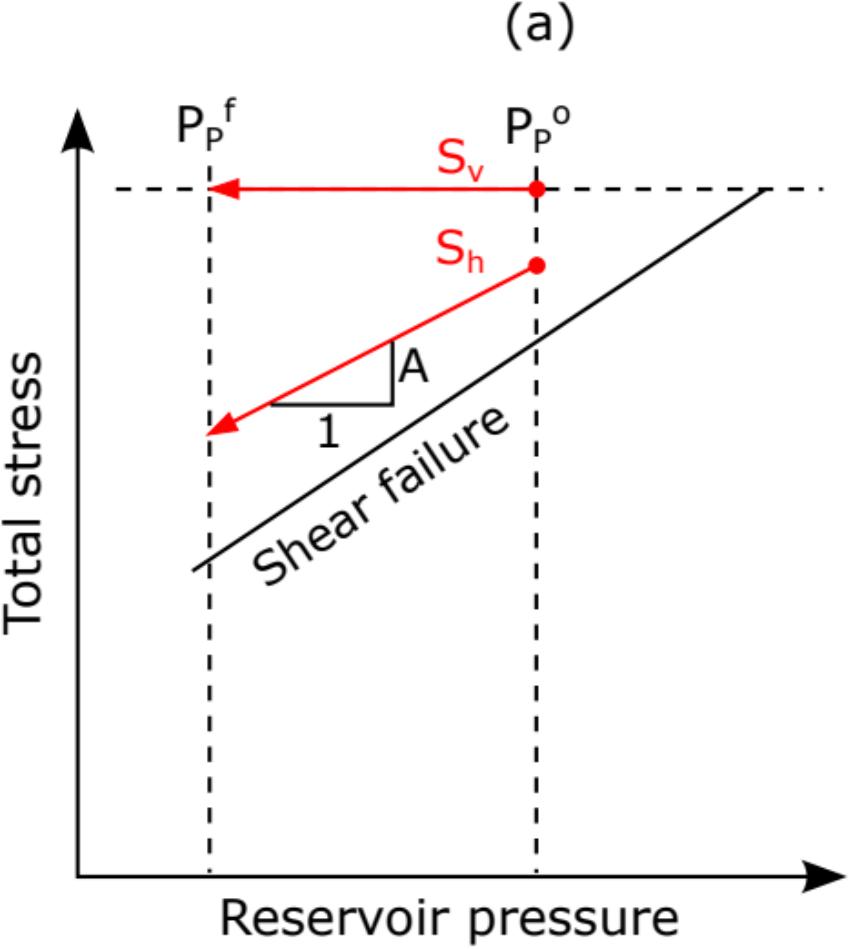
$\Delta S_{11}$

$= \alpha$

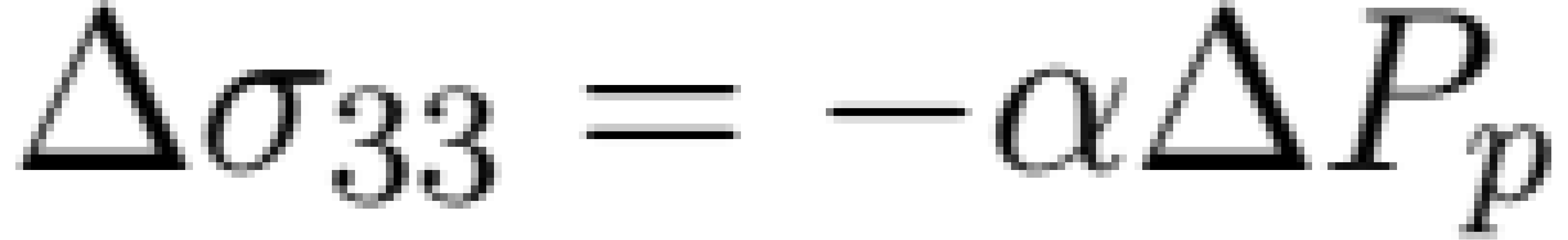
$\Delta V = \frac{1}{2} \Delta P$

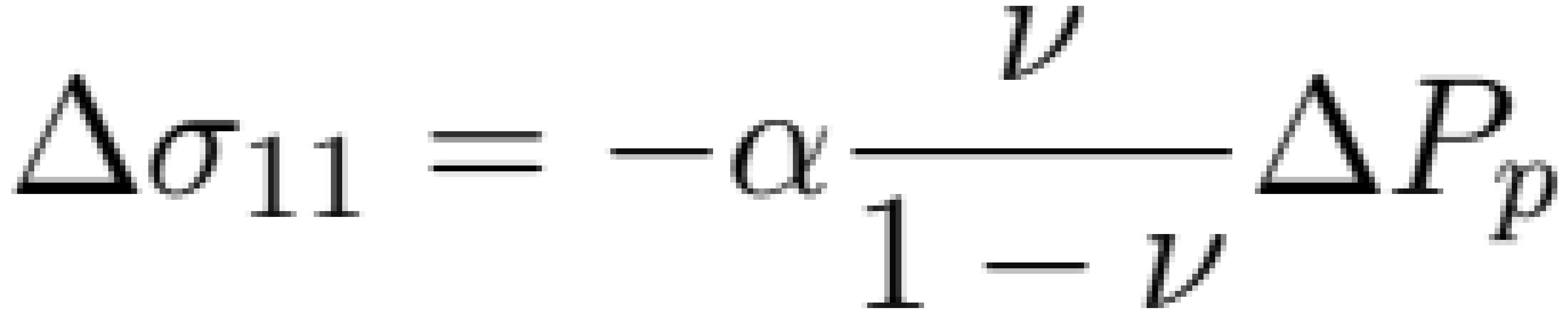






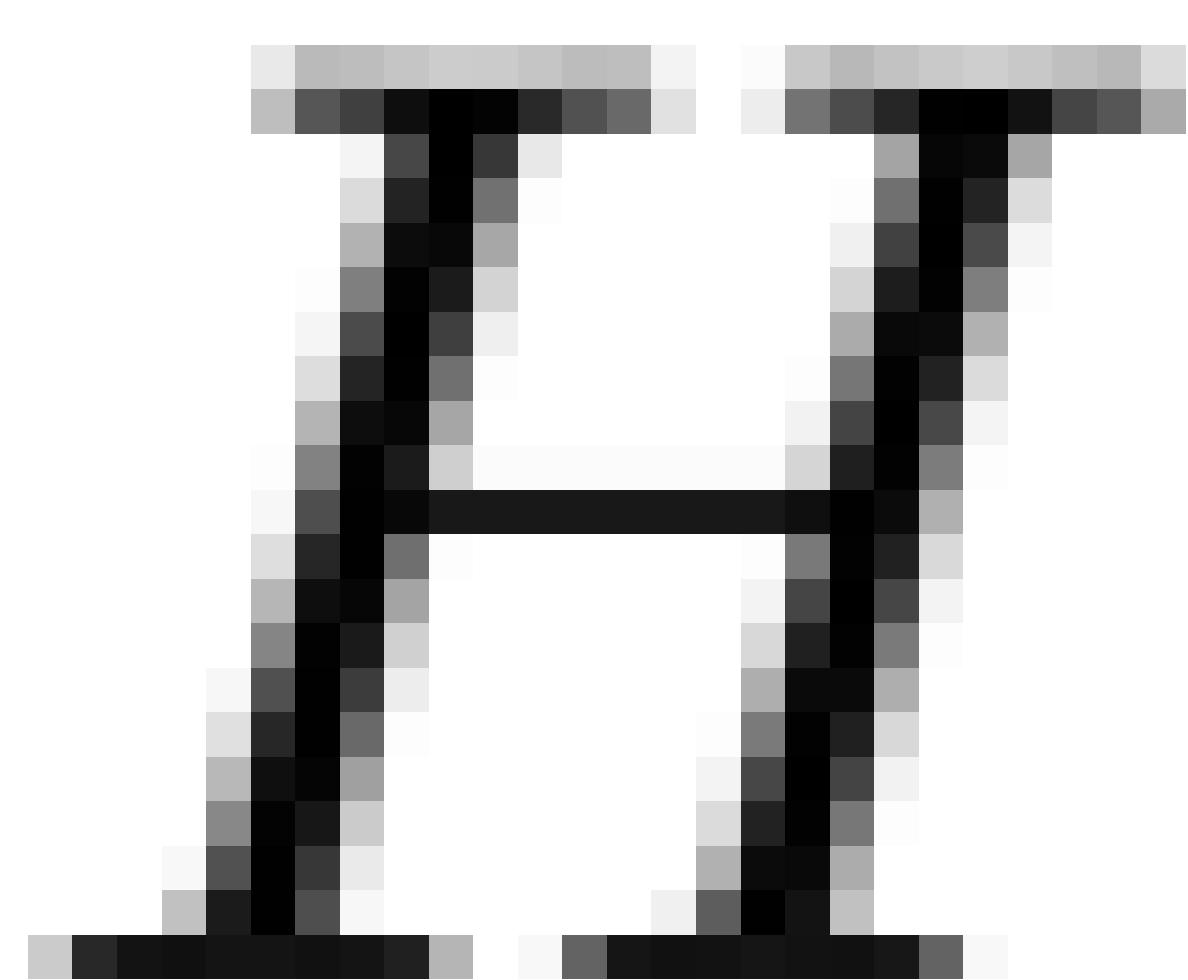
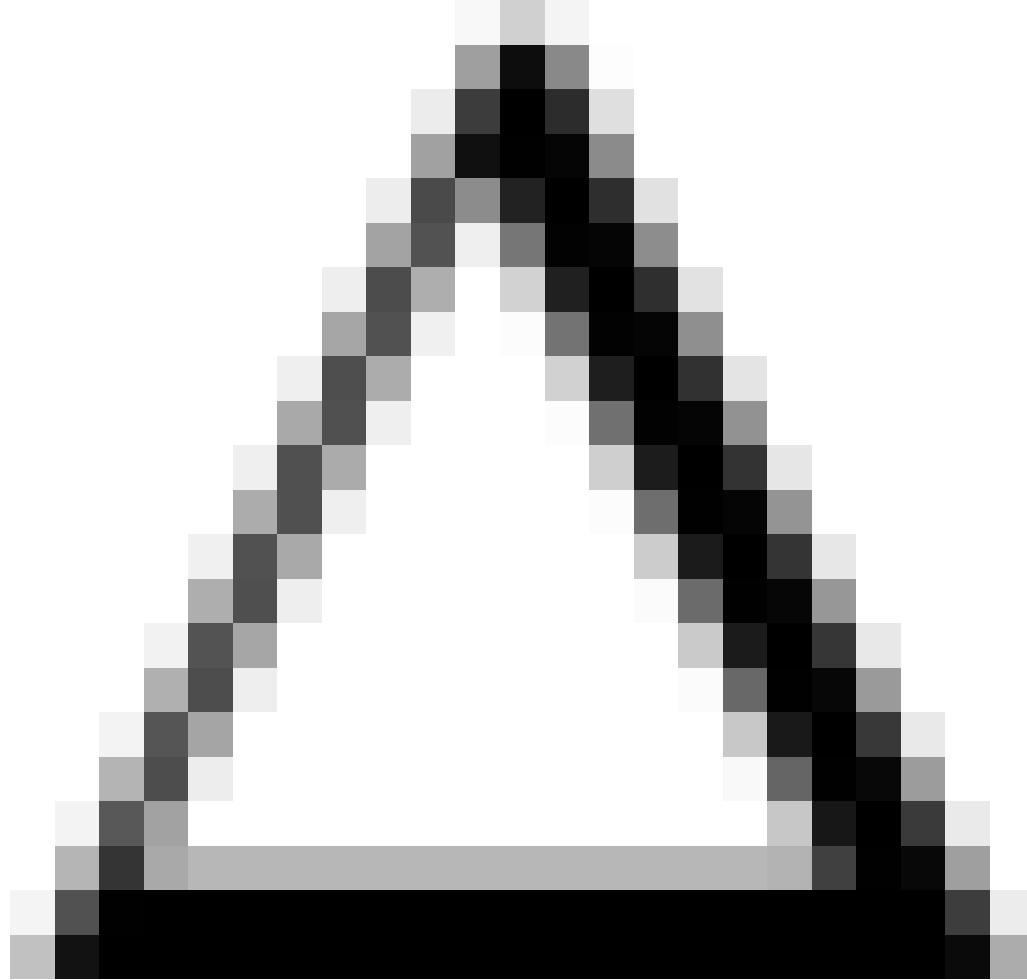






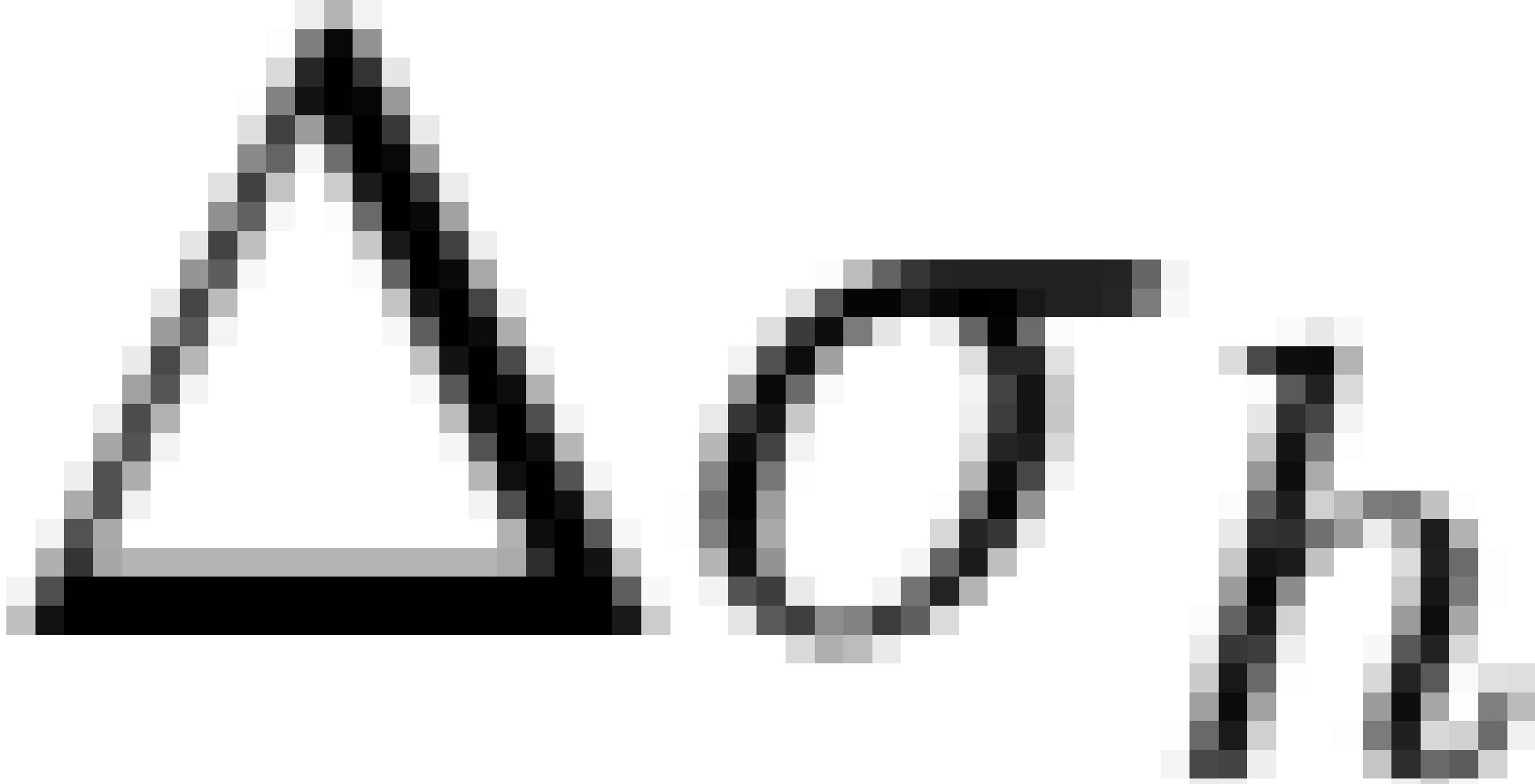
• . / E

• . / E

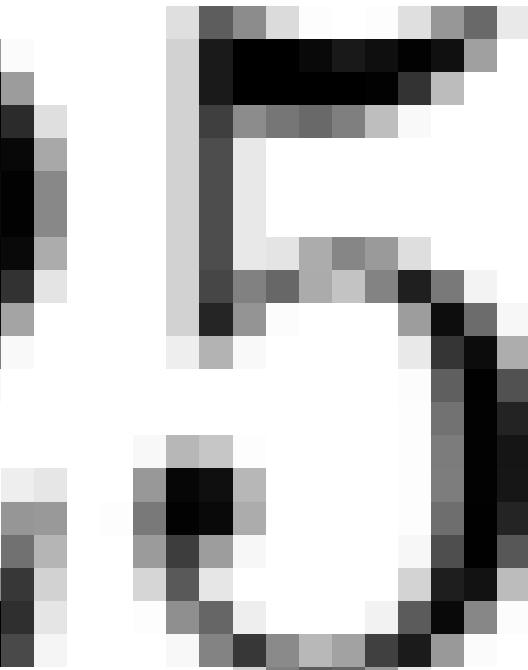
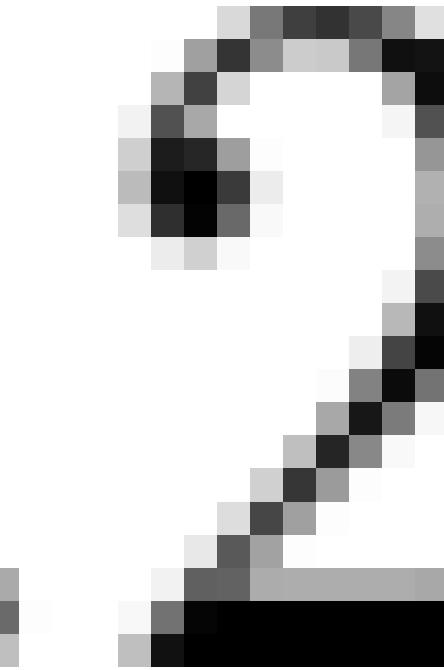
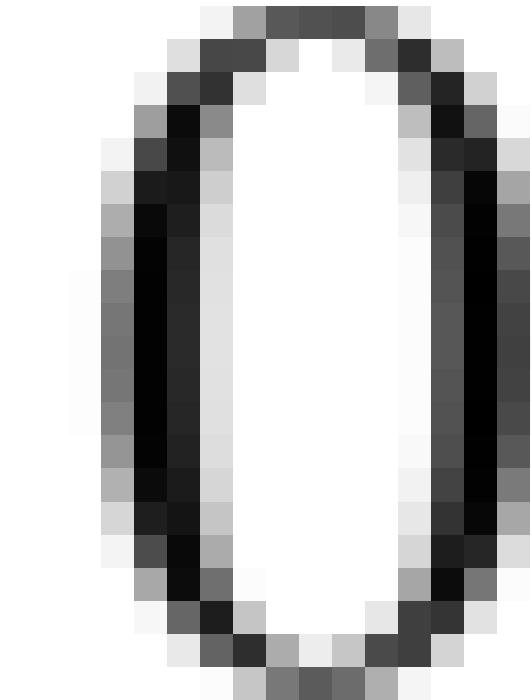
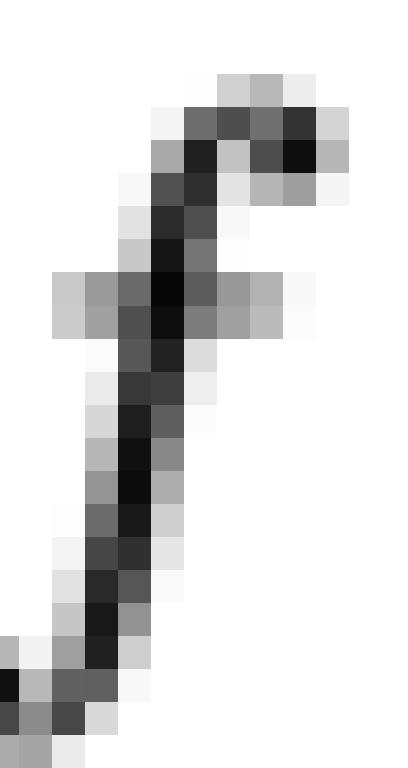
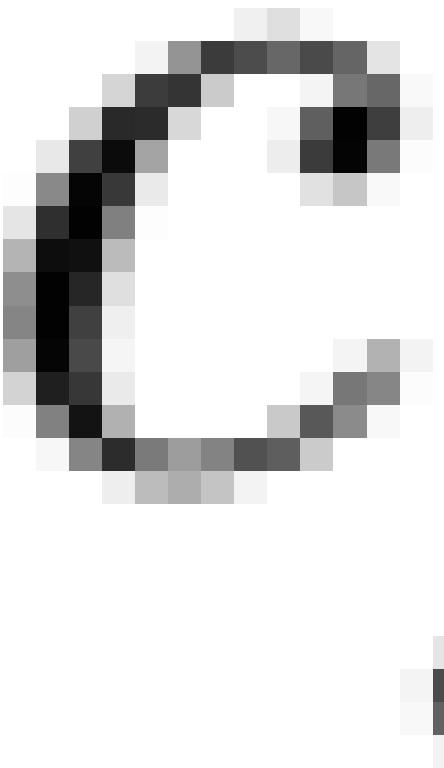


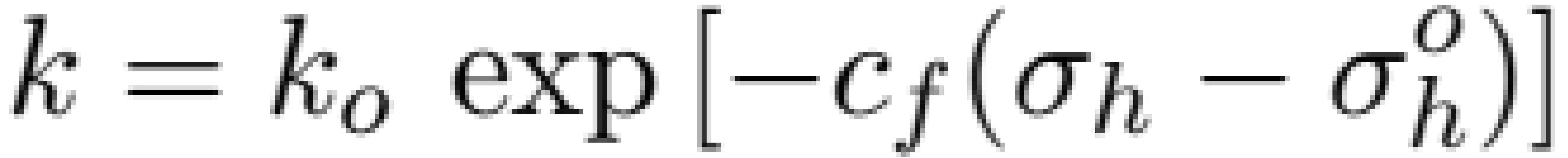










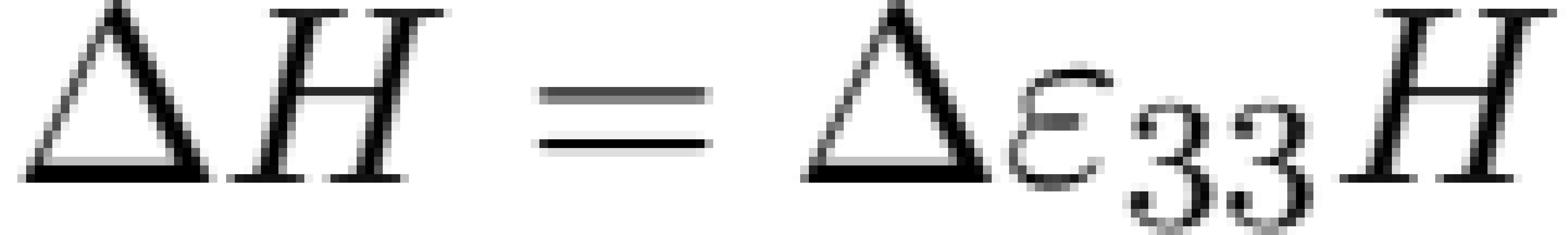


$$M_2 =$$

$$E \left( \frac{1}{1 + v} \right) \left( \frac{1 - v}{1 - 2v} \right)$$

$$=$$

$$14.8$$



633

25

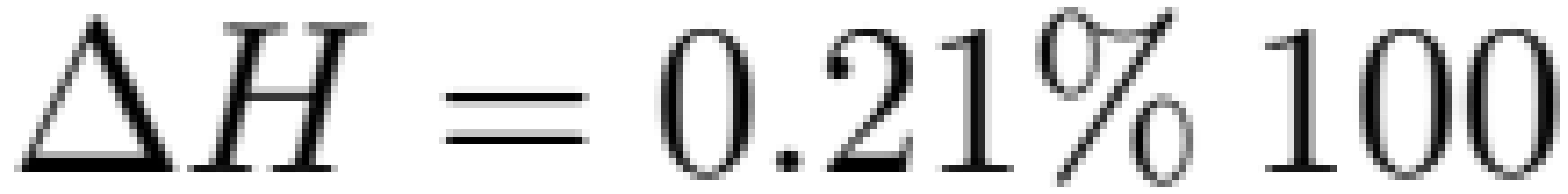
-75

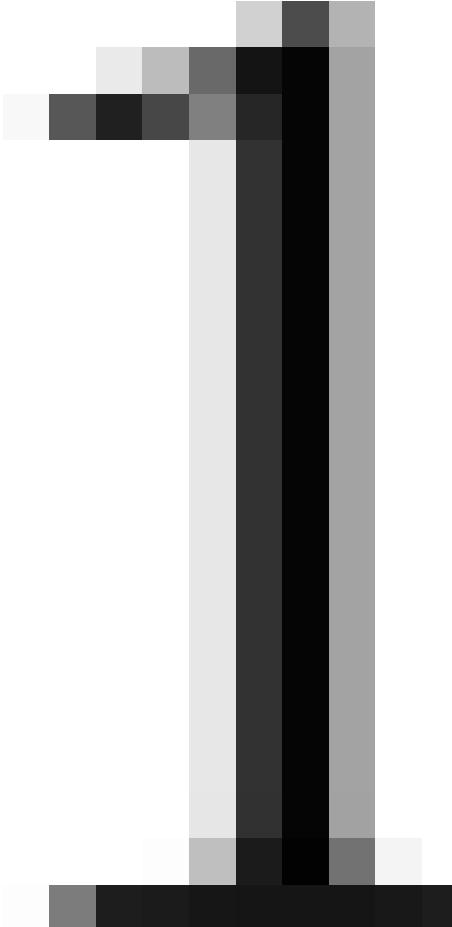
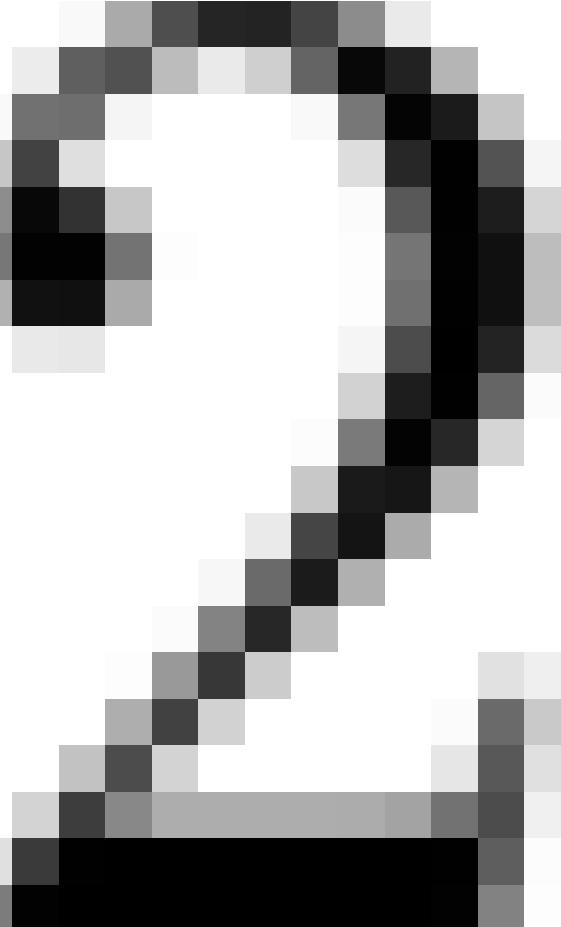
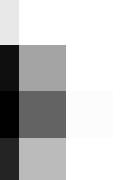
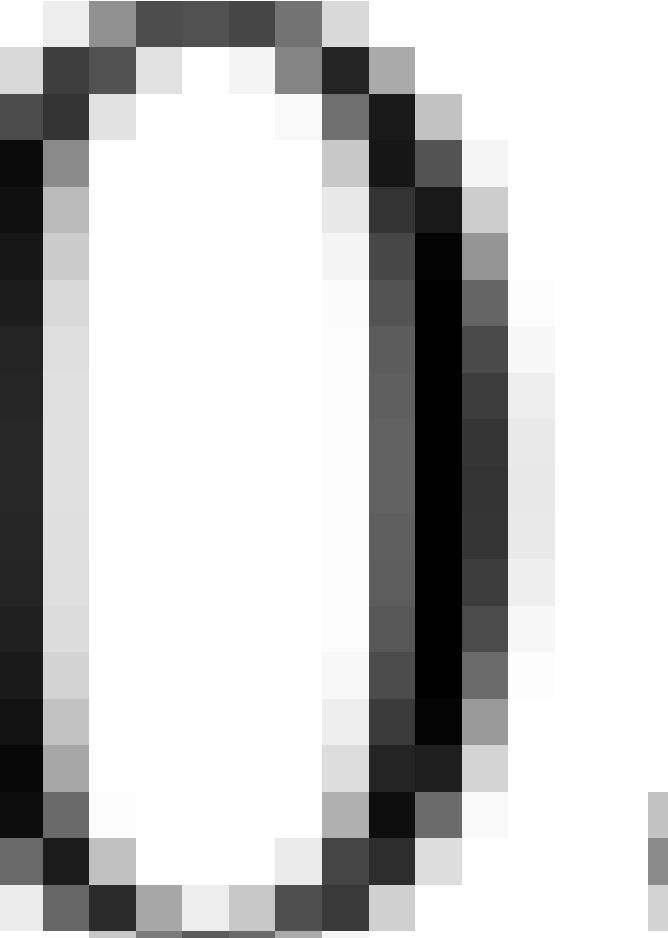
0

.21

0.06

0  
D  
P  
M



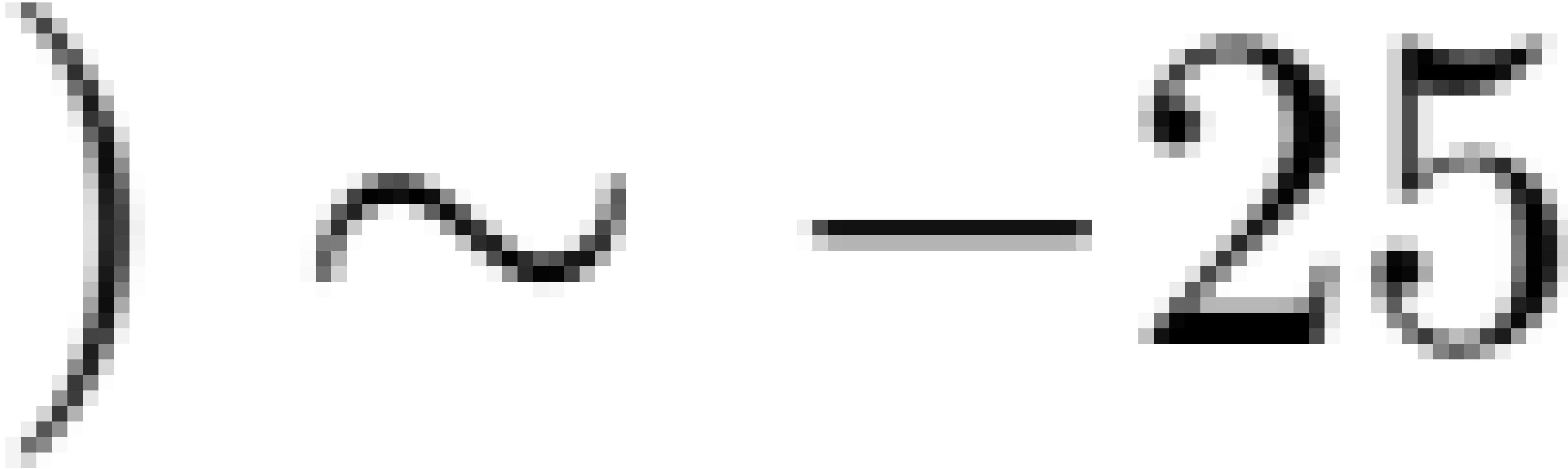


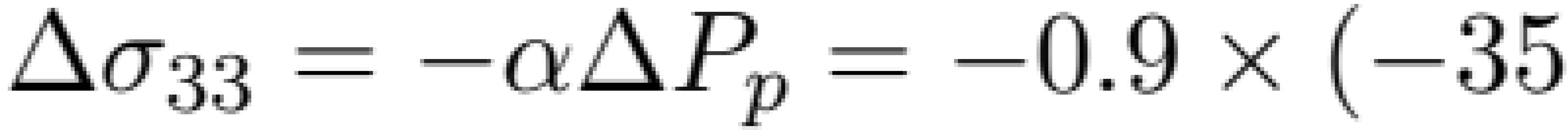
$$C_{bp} = \frac{\alpha}{M} = 0.06 \times 10^{-6} \frac{1}{\text{MPa}} = 0.42 \times 10^{-9} \frac{1}{\text{psi}}$$

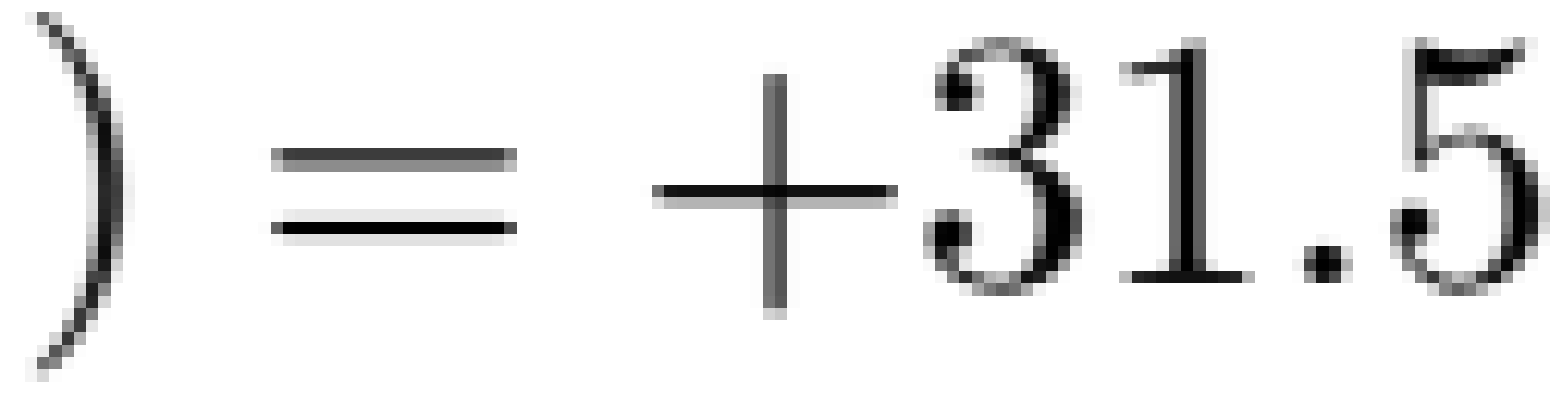
$$C_{pp} = \frac{C_{bp}}{\phi} = 2.0 \times 10^{-6} \frac{1}{\text{psi}}$$

$\Delta S_h \geq 0$

$$\frac{1}{1-v} - \frac{2v}{1+v}$$







$\sigma_{11} = \alpha_1$   $P = 6.45$

$V$

$\nu$

$P_p = 1$

$$\frac{k}{k_0} = \exp[-c_f(\sigma_h - \sigma_h^0)] = \exp[-0.25 \text{ MPa}^{-1} (+6.45 \text{ MPa})]$$