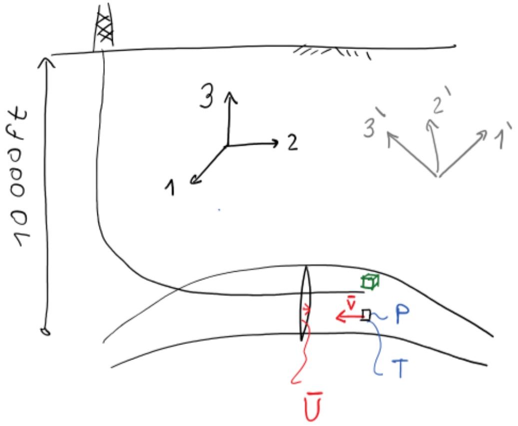
p'

q

$$P_{mud} = P_W$$

S_{Hmax}



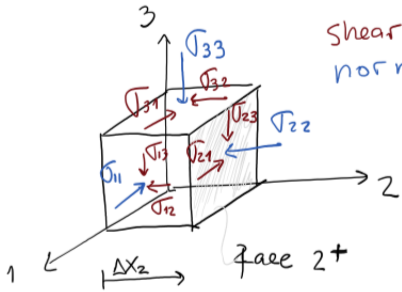
scalar : P , T

vector : $\bar{v} = [0, -0.1, 0]\ \frac{m}{day}$

$\bar{U} = [0, 1, 0]$ cm

(2nd order) tensor :

stress $\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$



shear stresses

normal stresses

$\sigma_{ij} = \sigma_{ji}$

$\sigma_{\underbrace{i}_{\text{face}}\ \underbrace{j}_{\text{direction}}}$

symmetric

$\sigma_{ij} \in \mathbb{R}$

$\to$ eigenvalues $\in \mathbb{R}$

$$\underline{\underline{S}} = \begin{bmatrix} 7000 & 0 & 0 \\ 0 & 6500 & 0 \\ 0 & 0 & 10000 \end{bmatrix} psi$$

in coord system 1,2,3

principal stresses

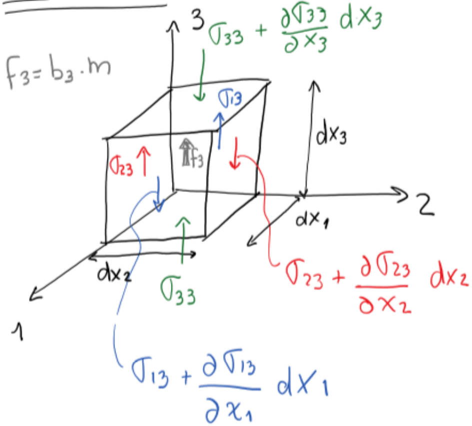
$\updownarrow$

$\perp \leftarrow$ principal directions

$\rightarrow \underline{\underline{\sigma}}^P = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$

$\sigma_1 \neq \sigma_2 \neq \sigma_3$

$\sigma_1 \geq \sigma_2 \geq \sigma_3$



$$Vol = dx_1\, dx_2\, dx_3$$

$$\sum F_3 = m \cancel{a_3}^{\,0} = 0$$

$$\cancel{\sigma_{33}}\, dx_1 dx_2 - \left(\cancel{\sigma_{33}} + \frac{\partial \sigma_{33}}{\partial x_3} dx_3\right) dx_1 dx_2 +$$

$$\cancel{\sigma_{23}}\, dx_1 dx_3 - \left(\cancel{\sigma_{23}} + \frac{\partial \sigma_{23}}{\partial x_2} dx_2\right) dx_1 dx_3 +$$

$$\cancel{\sigma_{13}}\, dx_2 dx_3 - \left(\cancel{\sigma_{13}} + \frac{\partial \sigma_{13}}{\partial x_1} dx_1\right) dx_2 dx_3 +$$

$$F_3 = 0$$

$$\frac{\partial \sigma_{33}}{\partial x_3} dx_1 dx_2 dx_3 + \frac{\partial \sigma_{23}}{\partial x_2} dx_1 dx_2 dx_3 +$$

$$\frac{\partial \sigma_{13}}{\partial x_1} dx_1 dx_2 dx_3 + b_3 m = 0$$

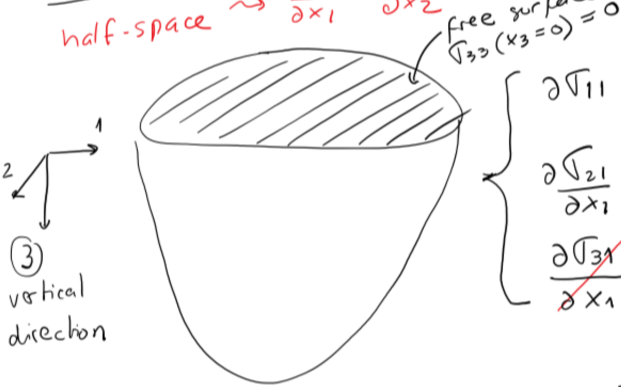
$$\underline{\underline{\sum F_3}} \longrightarrow \frac{\partial \sigma_{33}}{\partial x_3} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_1} + b_3 \frac{m}{Vol} = 0$$

Cauchy's equilibrium equations

- $$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i \rho = 0$$
- $$\nabla \cdot \underline{\underline{\sigma}} + \underline{b}\, \rho = 0$$

$$\begin{cases} \dfrac{\partial \sigma_{11}}{\partial x_1} + \dfrac{\partial \sigma_{12}}{\partial x_2} + \dfrac{\partial \sigma_{13}}{\partial x_3} + b_1 \rho = 0 \\ \dfrac{\partial \sigma_{21}}{\partial x_1} + \dfrac{\partial \sigma_{22}}{\partial x_2} + \dfrac{\partial \sigma_{23}}{\partial x_3} + b_2 \rho = 0 \\ \dfrac{\partial \sigma_{31}}{\partial x_1} + \dfrac{\partial \sigma_{32}}{\partial x_2} + \dfrac{\partial \sigma_{33}}{\partial x_3} + \underbrace{b_3}_{\substack{\text{gravity in 3} \\ b_3 = -g}} \underbrace{\rho}_{\frac{m}{Vol}} = 0 \end{cases}$$

half-space $\rightsquigarrow \frac{\partial}{\partial x_1} = \frac{\partial}{\partial x_2} = 0$

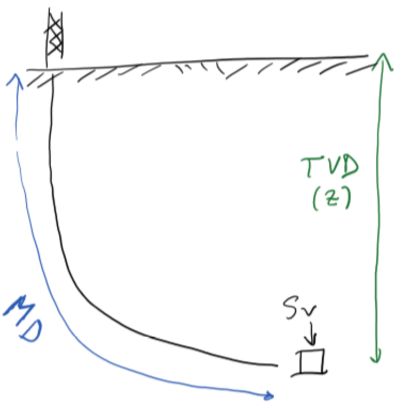


$$\left\{\begin{aligned}
&\partial\sigma_{11} \quad \partial x_2 + \frac{\partial \sigma_{13}}{\partial x_3} + b_1 \rho = 0\\
&\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + b_2 \rho = 0\\
&\cancelto{0}{\frac{\partial \sigma_{31}}{\partial x_1}} + \cancelto{0}{\frac{\partial \sigma_{32}}{\partial x_2}} + \boxed{\frac{\partial \sigma_{33}}{\partial x_3}} + b_3 \rho = 0
\end{aligned}\right.$$

$$\frac{\partial \sigma_{33}}{\partial x_3} = g \cdot \rho$$

$$\int_{\sigma_{33}(x_3=0)}^{\sigma_{33}(x_3)} d\sigma_{33} = \int_{x_3=0}^{x_3} g \cdot \rho \cdot dx_3$$

$$\sigma_{33}(x_3) = \int_0^{x_3} g \cdot \rho(x_3)\, dx_3$$



vertical depth

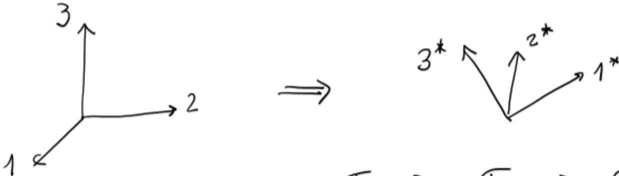
$$\boxed{S_v(z) = \int_0^{z} g \cdot \rho_{bulk}(z)\, dz}$$

$\rho_{bulk}(z) = \rho_{bulk}$ $\qquad S_v(z) = \rho_{bulk} \cdot g \cdot z$

$$\frac{dS_v}{dz} = \underset{\sim}{\rho_{bulk}} \cdot g \left\{\begin{array}{l} \underline{23\ MPa/km} \\ \\ \underline{1\ psi/ft} \end{array}\right.$$

$$2300\ \frac{kg}{m^3} \cdot 9.8\ \frac{m}{s^2}$$

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \Longrightarrow \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$



$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

$$\sigma_1 \perp \sigma_2 \perp \sigma_3$$

σ_v is a principal stress:

↳ $\sigma_v \perp \sigma_{Hmax} \perp \sigma_{hmin}$

(*)

$$\underline{\underline{\sigma}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} - \begin{bmatrix} P_p & 0 & 0 \\ 0 & P_p & 0 \\ 0 & 0 & P_p \end{bmatrix}$$

NORMAL FAULTING

$\sigma_v \geq \sigma_{Hmax} \geq \sigma_{hmin}$

$$\begin{bmatrix} \sigma_v & 0 & 0 \\ 0 & \sigma_{Hmax} & 0 \\ 0 & 0 & \sigma_{hmin} \end{bmatrix}$$

STRIKE-SLIP FAULTING

$\sigma_{Hmax} \geq \sigma_v \geq \sigma_{hmin}$

$$\begin{bmatrix} \sigma_{Hmax} & 0 & 0 \\ 0 & \sigma_v & 0 \\ 0 & 0 & \sigma_{hmin} \end{bmatrix}$$

REVERSE F.

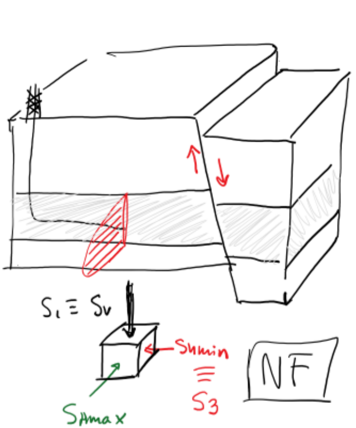
$\sigma_{Hmax} \geq \sigma_{hmin} \geq \sigma_v$

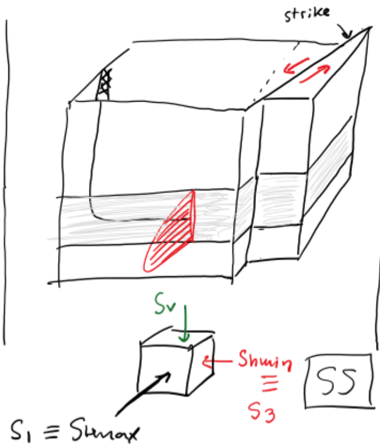
$$\begin{bmatrix} \sigma_{Hmax} & 0 & 0 \\ 0 & \sigma_{hmin} & 0 \\ 0 & 0 & \sigma_v \end{bmatrix}$$

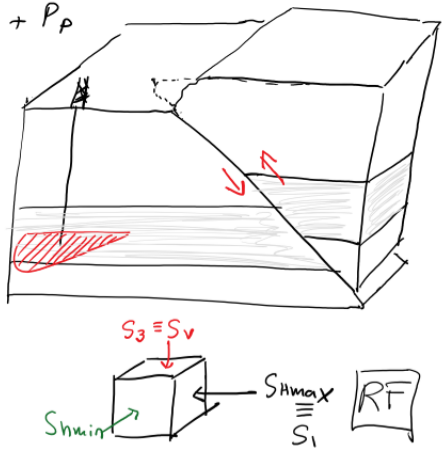
Effective stresses : $\underline{\underline{\sigma}}$; Total stresses $\underline{\underline{S}} = \underline{\underline{\sigma}} + P_p \underline{\underline{I}}$ (*)

$$\begin{bmatrix} S_v & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_{hmin} \end{bmatrix} \qquad \begin{bmatrix} S_{Hmax} & 0 & 0 \\ 0 & S_v & 0 \\ 0 & 0 & S_{hmin} \end{bmatrix} \qquad \begin{bmatrix} S_{Hmax} & 0 & 0 \\ 0 & S_{hmin} & 0 \\ 0 & 0 & S_v \end{bmatrix}$$

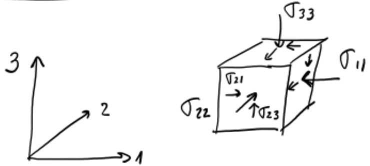
$S_v = \sigma_v + P_p$; $S_{Hmax} = \sigma_{Hmax} + P_p$; $S_{hmin} = \sigma_{hmin} + P_p$

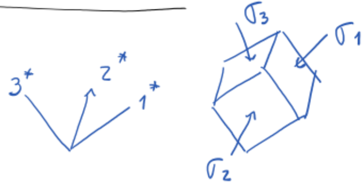




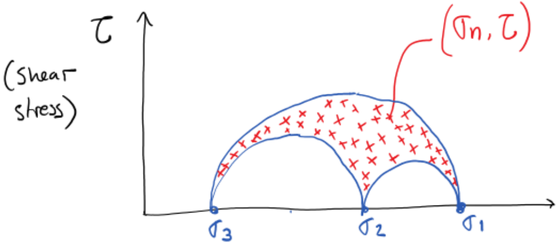


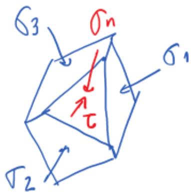
Stress Invariants and graphical representation

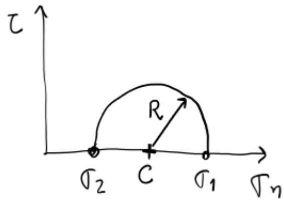




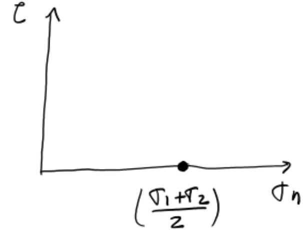
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \Longrightarrow \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$





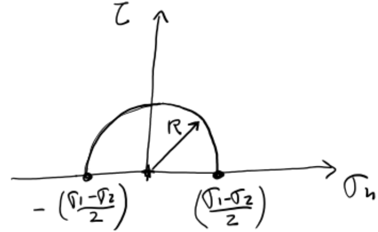


$=$



isotropic

$+$



deviatoric
(pure shear)

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}}_{\text{isotropic}} + \underbrace{\begin{bmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{bmatrix}}_{\text{deviatoric}}$$

$$\sigma_m = \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3}$$

$$\boxed{\underline{\underline{\sigma}} = \sigma_m \underline{\underline{I}} + \underline{\underline{S}}_d}$$

Invariants (do not change wrt coord system)

$$\Rightarrow I_1(\underline{\underline{\sigma}}) = \sigma_{11} + \sigma_{22} + \sigma_{33} = \sigma_1 + \sigma_2 + \sigma_3 \quad \Rightarrow \sigma_m = \frac{I_1(\underline{\underline{\sigma}})}{3}$$

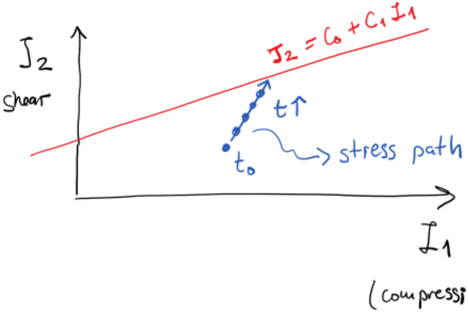
$$I_2(\underline{\underline{\sigma}}) = \sigma_{11}\sigma_{22} + \sigma_{11}\cdot\sigma_{33} + \sigma_{22}\sigma_{33} - \sigma_{12}^2 - \sigma_{13}^2 - \sigma_{23}^2$$

$$I_3(\underline{\underline{\sigma}}) = \det(\underline{\underline{\sigma}}) = \sigma_1\cdot\sigma_2\cdot\sigma_3 \leftarrow$$

$$J_1(\underline{\underline{s}}_d) = 0$$

$$\Rightarrow J_2(\underline{\underline{s}}_d) = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2\right]$$

$$J_3(\underline{\underline{s}}_d) = (\sigma_1-\sigma_m)\cdot(\sigma_2-\sigma_m)\cdot(\sigma_3-\sigma_m)$$



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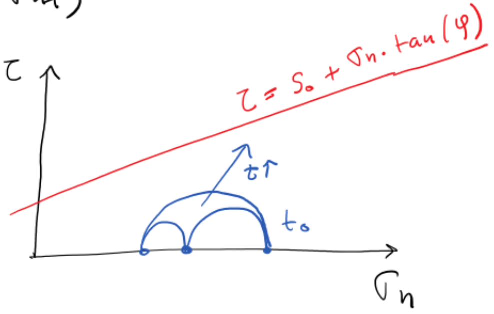
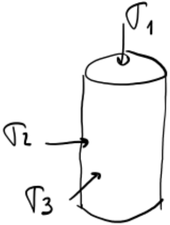


soft sediments (soil mechanics)

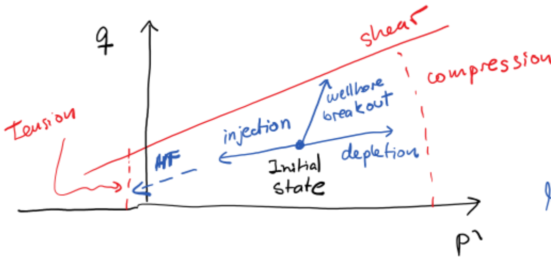
$$\begin{cases} p' = \underbrace{\sigma_m}_{\text{effective}} = I_1(\underline{\sigma})/3 \\ q = \sqrt{3\,J_2} \end{cases}$$



$\sigma_2 = \sigma_3$

$\Longrightarrow$ Axisymmetric stress conditions

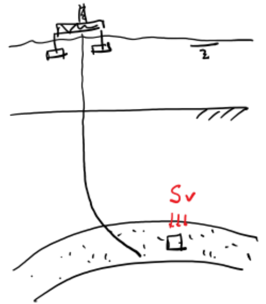
$$\begin{cases} p' = \dfrac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \dfrac{\sigma_1 + 2\sigma_3}{3} \\ q = \sqrt{\cancel{3}\left\{\dfrac{1}{\cancel{6}_{\cancel{3}}}\left[(\sigma_1-\sigma_3)^2 + \cancel{(\sigma_1-\sigma_3)^2}\right]\right\}} = \sigma_1 - \sigma_3 \end{cases}$$

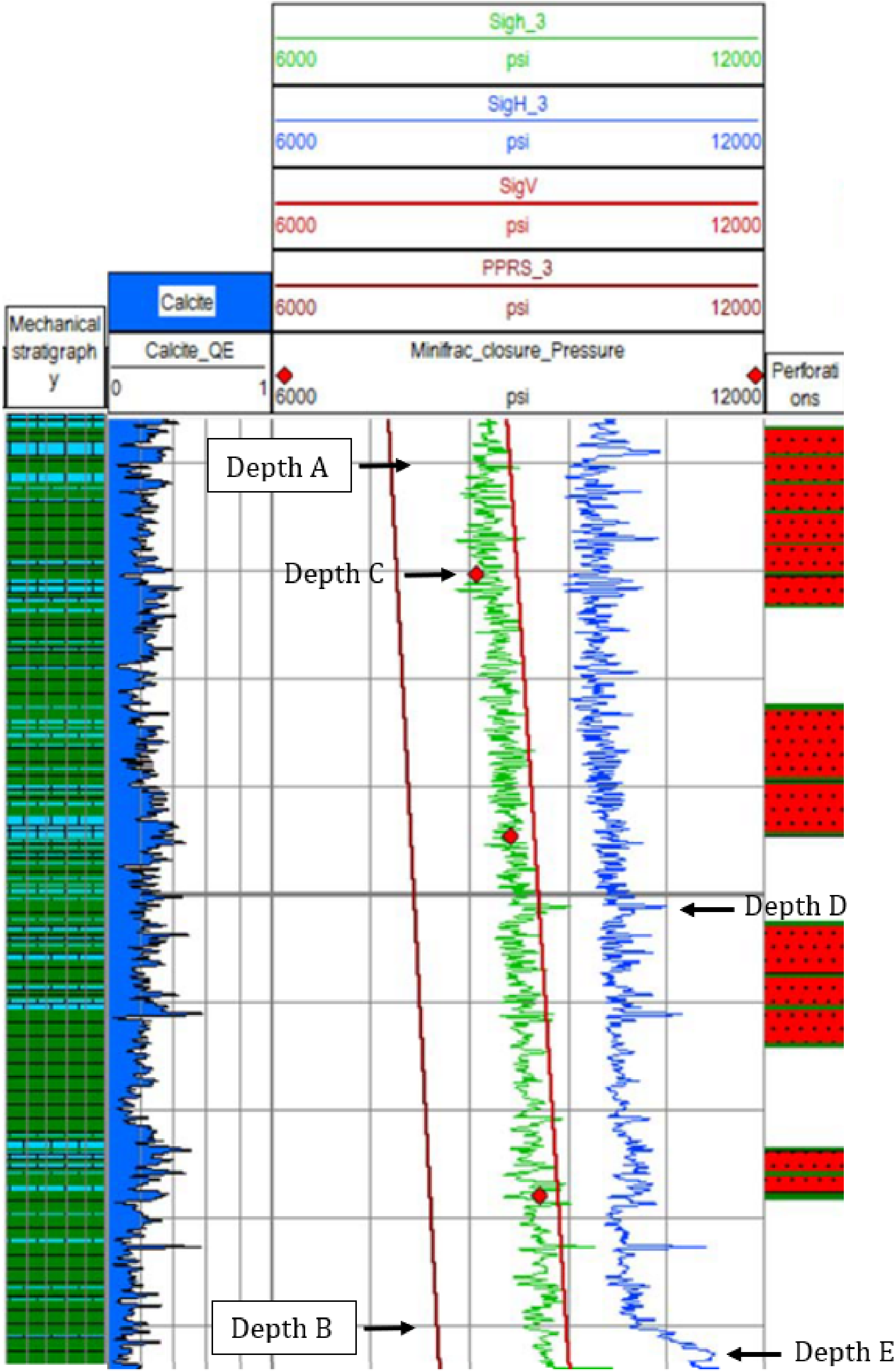


stress path

$\Downarrow$

track state of stress as a function of time





Sigh_3
6000 psi 12000
SigH_3
6000 psi 12000
SigV
6000 psi 12000
PPRS_3
6000 psi 12000
Minifrac_closure_Pressure
6000 psi 12000
Mechanical stratigraphy
Calcite
Calcite_QE
0 1
Perforations
Depth A
Depth C
Depth D
Depth B
Depth E

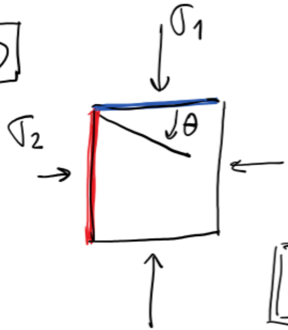
p

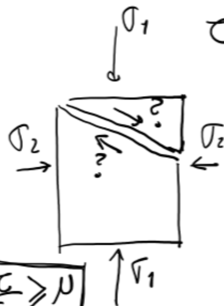
I_1

$$J_2$$

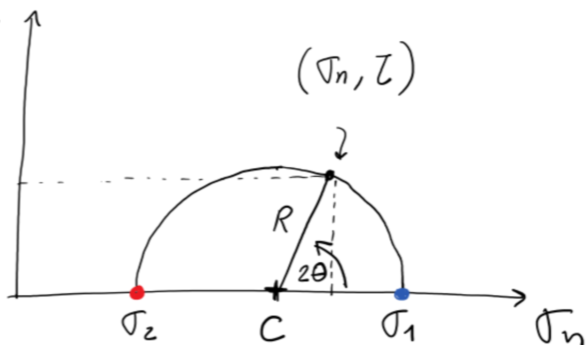
Stress projection on a plane

$\boxed{2D}$

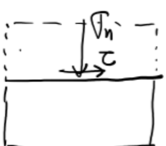




$$\boxed{\frac{\tau}{\sigma_n} \geq \mu}$$



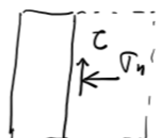
$\theta = 0$



$\underline{\sigma_n = \sigma_1 \; ; \; \tau = 0}$

σ_n, τ

$\theta = 90^\circ$



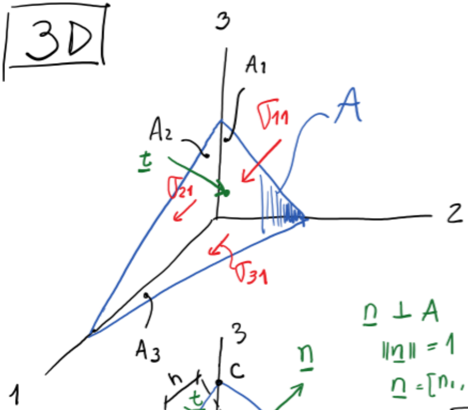
$\sigma_n = \sigma_2 \; ; \; \tau = 0$

$$\begin{cases} \sigma_n = C + R \cdot \cos 2\theta \\ \tau = R \cdot \sin 2\theta \end{cases}$$

$$C = (\sigma_1 + \sigma_2)/2$$

$$R = (\sigma_1 - \sigma_2)/2$$

$\boxed{3D}$



$$\Sigma F_1 = 0$$

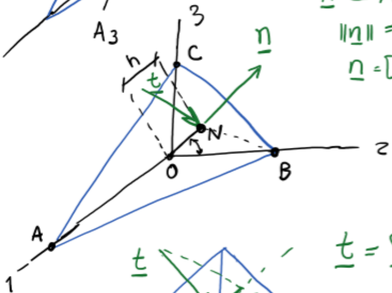
① $\sigma_{11} A_1 + \sigma_{21} A_2 + \sigma_{31} A_3 = t_1 A$

② Vol $= \frac{1}{3} A \cdot h$

$$= \frac{1}{3} A_1 \cdot \overline{OA}$$

$$\cancel{\tfrac{1}{3}} A h = \cancel{\tfrac{1}{3}} A_1 \overline{OA}$$

$$A_1 = \frac{h}{\overline{OA}} A$$



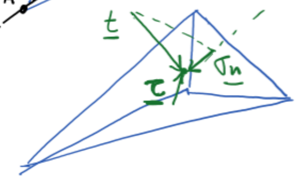
$\underline{n} \perp A$

$\|\underline{n}\| = 1$

$\underline{n} = [n_1, n_2, n_3]$

$$\begin{cases} A_1 = \cos A\hat{O}N \cdot A = n_1 A \\ A_2 = \cos B\hat{O}N \cdot A = n_2 A \\ A_3 = \underbrace{\cos C\hat{O}N}_{\text{cosine directors}} \cdot A = n_3 A \end{cases}$$

$$\boxed{A_i = n_i A}$$



$\underline{t} = \underline{\sigma_n} + \underline{\tau}$

$\rightarrow$ ① + ② $\sigma_{11} n_1 \cancel{A} + \underset{=\sigma_{12}}{\sigma_{21}} n_2 \cancel{A} + \underset{=\sigma_{13}}{\sigma_{31}} n_3 \cancel{A} = t_1 \cancel{A}$

$\Downarrow$ $\sigma_{21} = \sigma_{12}$ (angular momentum equil.)

$$\rightarrow \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

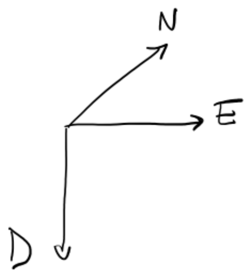
row × column

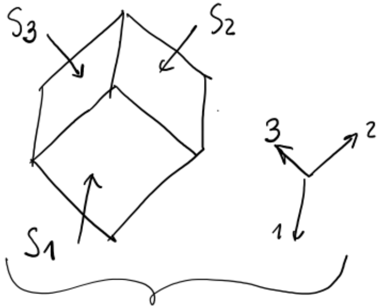
$$\underline{t} = \underline{\sigma} \cdot \underline{n}$$

$$\begin{cases} \sigma_n = \underline{t} \cdot \underline{n} \quad \} \text{ proyection of } \underline{\sigma_n} \text{ on } \underline{n} \\ \tau = \sqrt{\|\underline{t}\|^2 - \|\sigma_n\|^2} \quad \} \Longleftarrow \|\underline{t}\|^2 = \|\sigma_n\|^2 + \|\tau\|^2 \end{cases}$$

Geographical coordinate system

N - E - D





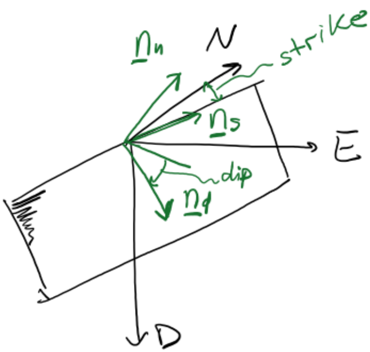
Principal stresses and direction

$$\underline{\underline{S}}_P = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} S_{NN} & S_{NE} & S_{ND} \\ S_{EN} & S_{EE} & S_{ED} \\ S_{DN} & S_{DE} & S_{DD} \end{bmatrix}$$

$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \, \underline{\underline{S}}_P \, R_{PG}}$$

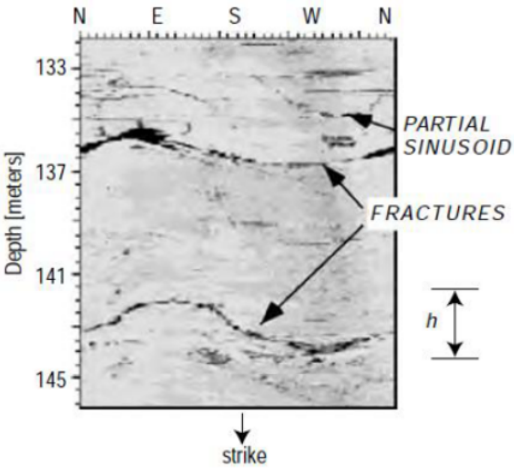
$$R_{PG} = \underbrace{\begin{bmatrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{bmatrix}}_{f(\alpha, \beta, \gamma)}$$



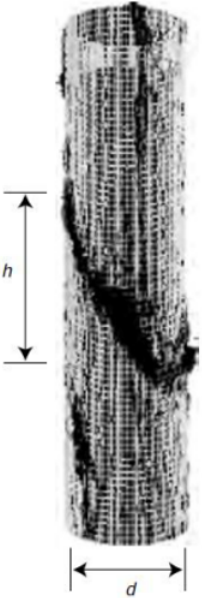
$\mathcal{T}$

σ_n

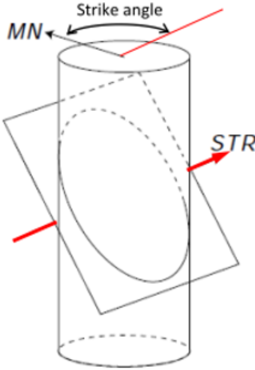
Un-wrapped image
(ultrasonic)



3D-representation



Interpretation



Dip = $\tan^{-1}(h/d)$

(Zoback 2013, RM, Ch 5.3)

○

(σ_n, τ)

τ / σ_n

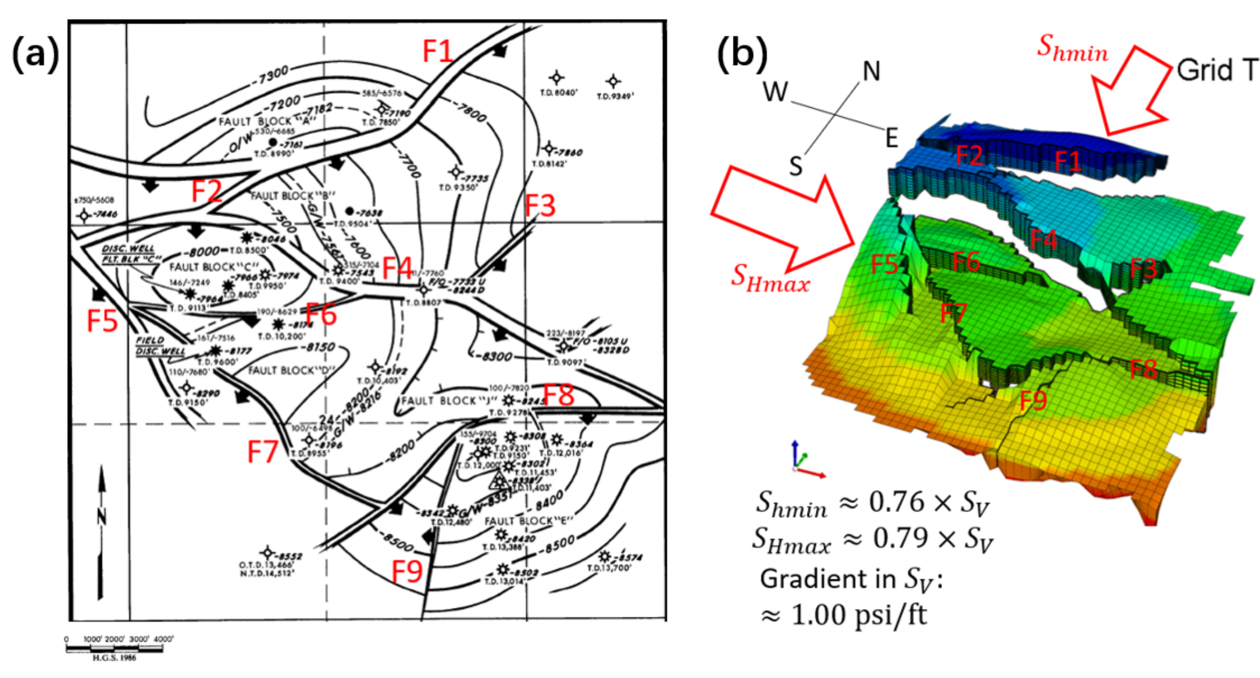
$$\mu = 0.5 \pm 0.1$$

$$60^\circ \pm 10^\circ$$

$$80^\circ \pm 5^\circ$$

$$10^\circ \pm 10^\circ$$

$$60^\circ \pm 5^\circ$$



(a)
F1
F2
F3
F4
F5
F6
F7
F8
F9
FAULT BLOCK "A"
FAULT BLOCK "B"
FAULT BLOCK "C"
FAULT BLOCK "D"
FAULT BLOCK "E"
FAULT BLOCK "J"
DISC. WELL FLT. BLK "C"
FIELD DISC. WELL
-7300
-7200
-7800
-7700
-7500
-7600
-8000
-8150
-8300
-8200
-8400
-8500
N
0 1000' 2000' 3000' 4000'
H.G.S. 1986
(b)
W
N
E
S
S_{hmin}
Grid T
S_{Hmax}
F1
F2
F3
F4
F5
F6
F7
F8
F9
$S_{hmin} \approx 0.76 \times S_V$
$S_{Hmax} \approx 0.79 \times S_V$
Gradient in S_V:
≈ 1.00 psi/ft

S_{hmin}

$$\sigma_n/\sigma_v$$

τ/σ_v

$P_p/$

$$z = 0.44$$

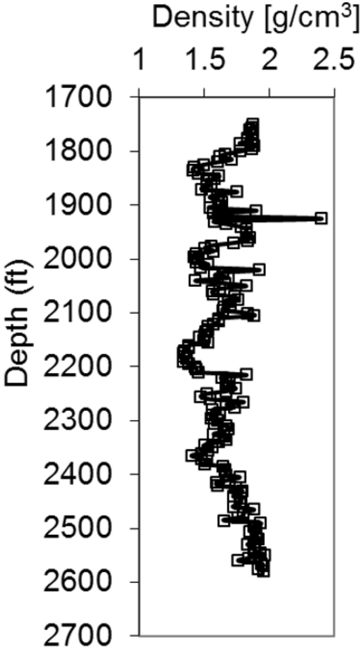
$$\mu = 0.6$$

$$E_{static} = 0.65 \cdot E_{dynamic}$$

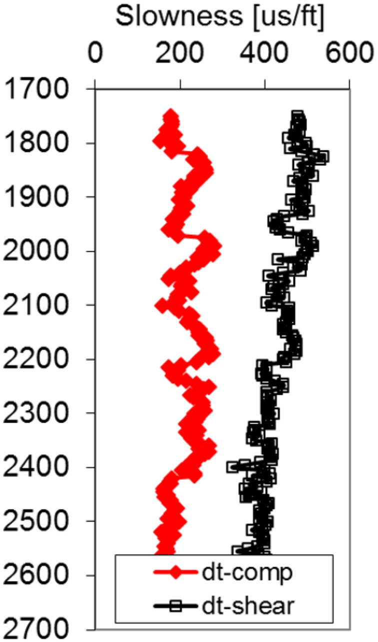
$$E'_{static} = E_{static}/(1 - \nu^2)$$

$$\varepsilon_{Hmax} = 0.0015$$

$$\varepsilon_{hmin} = 0$$



Density [g/cm3]
1
1.5
2
2.5
1700
1800
1900
2000
2100
2200
2300
2400
2500
2600
2700
Depth (ft)



Slowness [us/ft]
0
200
400
600
1700
1800
1900
2000
2100
2200
2300
2400
2500
2600
2700
dt-comp
dt-shear

$$
\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} +\dfrac{1}{E_h} & -\dfrac{\nu_h}{E_h} & -\dfrac{\nu_v}{E_v} & 0 & 0 & 0 \\ -\dfrac{\nu_h}{E_h} & +\dfrac{1}{E_h} & -\dfrac{\nu_v}{E_v} & 0 & 0 & 0 \\ -\dfrac{\nu_v}{E_v} & -\dfrac{\nu_v}{E_v} & +\dfrac{1}{E_v} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{G_v} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{G_v} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{G_h} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}
$$

$$G_h = \frac{E_h}{2(1+\nu_h)}$$

G_v

$$E_h = \frac{(C_{11} - C_{12})\left[C_{33}(C_{11} + C_{12}) - 2\, C_{13}^2\right]}{C_{11}C_{33} - C_{13}^2}$$

$$E_v = C_{33} - \frac{2\,C_{13}^2}{C_{11} + C_{12}}$$

$$\nu_h = \frac{C_{12}C_{33} - C_{13}^2}{C_{11}C_{33} - C_{13}^2}$$

$$\nu_v = \frac{C_{13}}{C_{11} + C_{12}}$$

$$G_v = C_{44}$$

$$G_h = C_{66} = \frac{C_{11} - C_{12}}{2}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}$$

$$C_{11} = \left[\frac{1}{(1-\nu_h)E_v - 2\nu_v^2 E_h}\right]\left(\frac{E_h E_v - \nu_v^2 E_h^2}{1+\nu_h}\right)$$

$$C_{33} = \left[\frac{1}{(1-\nu_h)E_v - 2\nu_v^2 E_h}\right](E_v^2 - \nu_h E_v^2)$$

$$C_{12} = \left[\frac{1}{(1-\nu_h)E_v - 2\nu_v^2 E_h}\right]\left(\frac{\nu_v^2 E_h^2 + \nu_h E_h E_v}{1+\nu_h}\right)$$

$$C_{13} = \left[\frac{1}{(1 - \nu_h)E_v - 2\nu_v^2 E_h} \right] (\nu_v E_h E_v)$$

$$C_{66} = \frac{C_{11} - C_{12}}{2} = G_h = \frac{E_h}{2(1 + \nu_h)}$$

C_{44}

σ_{xx}

$$\sigma_{yy}$$

E

ν

UCS

$$T_s$$

$$\sigma_{rr}$$

$\sigma_{\theta\theta}$

$\sigma_{r\theta}$

x

y

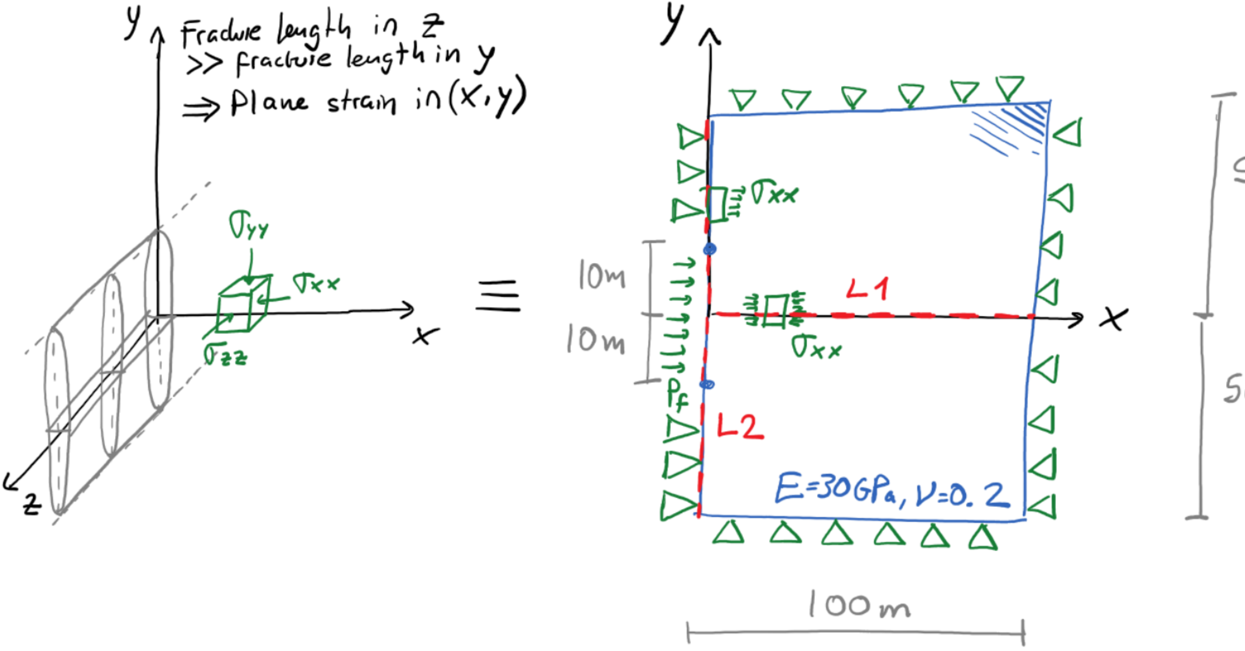
(r, θ)

r

σ_{xy}

c

$$(x, y) =$$


Fracture length in z
>> fracture length in y
⇒ Plane strain in (x,y)
y
x
z
σyy
σxx
σzz
≡
y
x
10m
10m
Pf
σxx
σxx
L1
L2
E=30GPa, ν=0.2
100m
5
5

$y =$

P_c

P_p

$$\varepsilon_{vol}$$

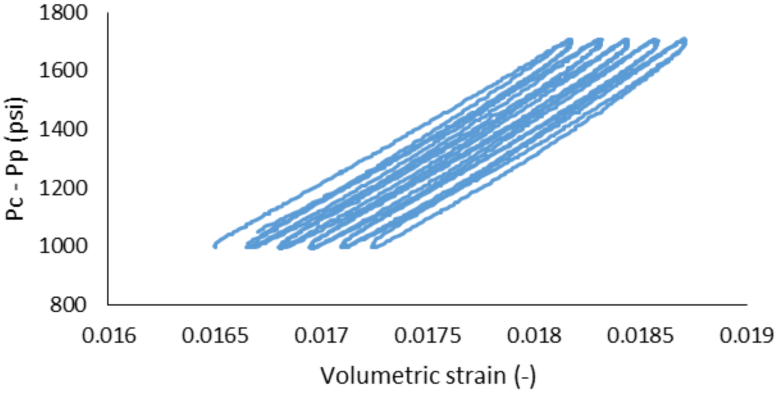
σ_{mean}

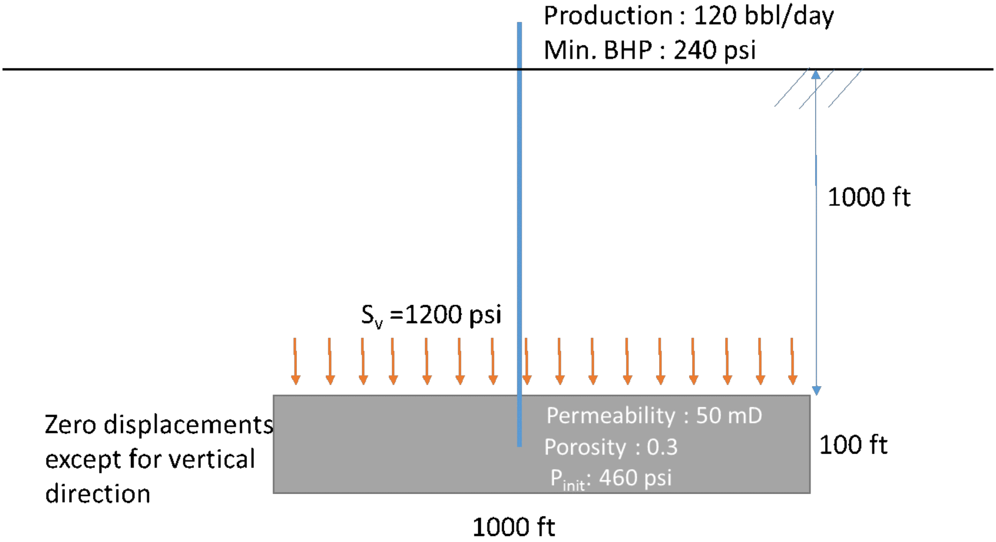
α

T_{ch}

$$\alpha_h$$

α_v


1800
1600
1400
1200
1000
800
Pc - Pp (psi)
0.016
0.0165
0.017
0.0175
0.018
0.0185
0.019
Volumetric strain (-)


Production : 120 bbl/day
Min. BHP : 240 psi
1000 ft
S_v =1200 psi
Permeability : 50 mD
Porosity : 0.3
P_{init}: 460 psi
100 ft
Zero displacements except for vertical direction
1000 ft

$$\alpha \frac{(1-2\nu)}{(1-\nu)}$$

(p', q)

$$
\begin{cases}
\nabla \underline{\underline{\sigma}} + \underline{f} = \underline{0} & \text{Equilibrium} \\[2ex]
\underline{\underline{\varepsilon}} = \dfrac{1}{2}(\nabla \underline{u} + \nabla \underline{u}^T) & \text{Kinematic} \\[1ex]
\underline{\underline{\sigma}} = \underline{\underline{C}} \cdot \underline{\underline{\varepsilon}} + 3\alpha_L K \theta \underline{\underline{I}}; \;\; \theta = T - T_0 & \text{Constitutive} \\[2ex]
\dfrac{\partial \theta}{\partial t} = \dfrac{k_T}{\rho c_v} \nabla^2 \theta + \dfrac{3\beta K T_0}{\rho c_v} \dfrac{\partial \varepsilon_{vol}}{\partial t} & \text{Diffusivity}
\end{cases}
$$

$$
\begin{cases}
\dfrac{\partial p}{\partial t} - \dfrac{k}{\mu} M \nabla^2 p = -\alpha M \dfrac{\partial \varepsilon_{vol}}{\partial t} + \beta_e M \dfrac{\partial T}{\partial t} & \text{Pore pressure diffusivity} \\
\dfrac{\partial T}{\partial t} - \kappa_T \nabla^2 T = -\dfrac{\alpha_d}{m_d} \dfrac{\partial \varepsilon_{vol}}{\partial t} + \dfrac{\beta_e}{m_d} \dfrac{\partial p}{\partial t} & \text{Temperature diffusivity}
\end{cases}
$$

$$\beta_e = \alpha\beta_d + \phi(\beta_f - \beta_s)$$

$$\beta_f > \beta_s$$

$$\beta_d = \beta_s;$$

K

$$\alpha_d = K\ \beta_d$$

$$m_d = \frac{c_d}{T_0}$$

$$\underline{\underline{\sigma}} = \underline{\underline{C}} \cdot \underline{\underline{\varepsilon}} + 3\alpha_L K\theta \underline{\underline{I}}; \quad \theta = T - T_0$$

$$\varepsilon_{11} = \varepsilon_{22} = 0$$

$\sigma_{33} =$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \varepsilon_{33} \end{bmatrix} + \begin{bmatrix} 3\alpha_L K\theta \\ 3\alpha_L K\theta \\ 3\alpha_L K\theta \end{bmatrix}$$

$$\sigma_{11}$$

σ_{33}

$$
\begin{cases}
\sigma_{11} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} + 3\alpha_L K\theta \\
\sigma_{33} = \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} + 3\alpha_L K\theta
\end{cases}
$$

ε_{33}

$$\sigma_{11} = \left(\frac{\nu}{1-\nu} \right) \sigma_{33} + \left(\frac{1-2\nu}{1-\nu} \right) 3\alpha_L K\theta$$

$$K = E/[3(1 - 2\nu)]$$

$$\sigma_{11} = \left(\frac{\nu}{1-\nu} \right) \sigma_{33} + \frac{\alpha_L E}{1-\nu} \theta$$

$$\frac{\partial \sigma_{11}}{\partial \theta} = +\frac{\alpha_L E}{1-\nu}$$

$$\frac{\partial \sigma_{11}}{\partial \theta} = 0.1 \frac{\mathrm{MPa}}{{}^{\circ}\mathrm{C}}$$

$$\alpha_L = 10^{-5}$$

$1/^\circ$

$$E = 10$$

$\nu = 0.2$

S_1

S_3

$$P_{pore\ fluid}$$

$$\sigma_1 = UCS + q\sigma_3$$

S_0

φ

$$(J_2)^{1/2}$$

$$P_W = P_p$$

w_{BO}

$$0^\circ < w_{BO} < 60^\circ$$

$$w_{BO} > 60^\circ$$

$$P_W$$

$$dP_p/dz$$

$$dS_v/dz$$

$$\phi = \phi_0 \exp\left(-\beta \sigma_v\right)$$

$$e = e_0 - C_c \ln\left(\frac{\sigma_v}{1\ \mathrm{MPa}}\right)$$

λ_p

S_v

$$dq = 3dp'$$

G

$$p'_o$$

$$\phi_{CS} = 24^{\circ}$$

λ

κ

e_o

ε_q

e

$$dq = 0.9\, dp'$$

$$p' = 400$$

$$d\varepsilon^e_{p'} = \frac{\kappa}{\upsilon}\frac{dp'}{p'};\ d\varepsilon^e_q = \frac{dq}{3G}$$

$$\begin{bmatrix} d\varepsilon_{p'}^{p} \\ d\varepsilon_q^{p} \end{bmatrix} = \frac{\lambda-\kappa}{\upsilon p'(M^2+\eta^2)} \begin{bmatrix} M^2-\eta^2 & 2\eta \\ 2\eta & \frac{4\eta^2}{M^2-\eta^2} \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix}$$

$$v = 1 + e$$

$$\eta = q/p'$$

$$de = -\upsilon d\varepsilon_p$$

$$dp'_o = d\varepsilon^p_{p'} \frac{\upsilon}{\lambda - \kappa} p'_o$$

$$\Delta S_{yy} = \nu(\Delta S_{xx} + \Delta S_{zz})$$

h_f

$$E' = 8.9 \times 10^6$$

x_f

$$w_{w,0}$$

p_n

p_n

-6

$$p' - q$$

$$10^{-6}$$

$$E_{sta}/E_{dyn}$$