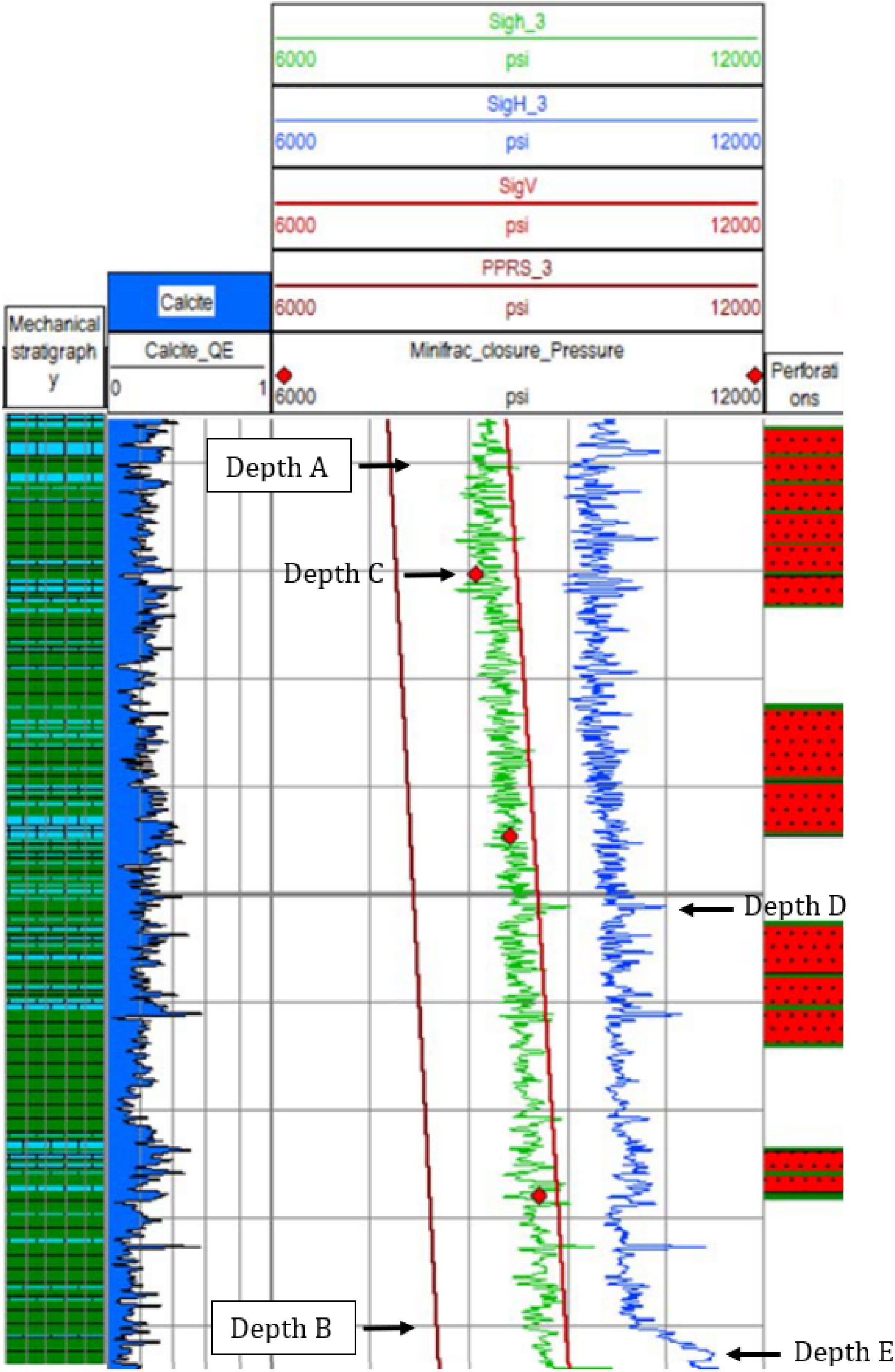
$$P_{mud} = P_W$$

$$S_{Hmax}$$



Sigh_3
6000
psi
12000
SigH_3
6000
psi
12000
SigV
6000
psi
12000
PPRS_3
6000
psi
12000
Calcite
Mechanical stratigraphy
Calcite_QE
0
1
Minifrac_closure_Pressure
6000
psi
12000
Perforations
Depth A
Depth C
Depth D
Depth B
Depth E

p

q

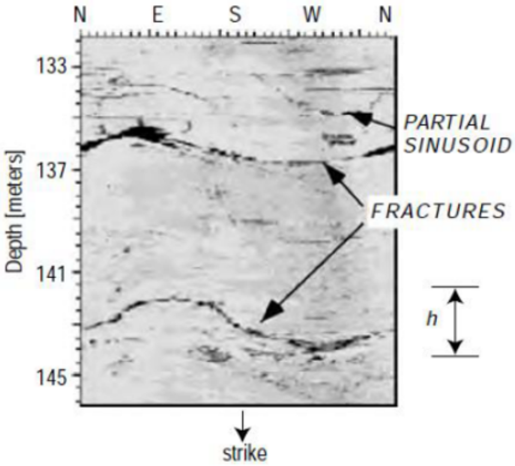
I_1

J_2

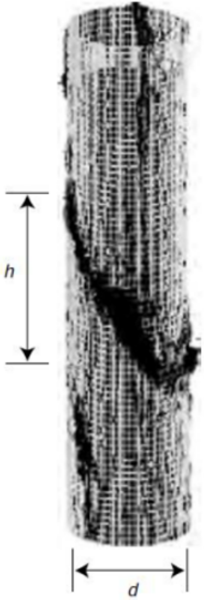
τ

σ_n

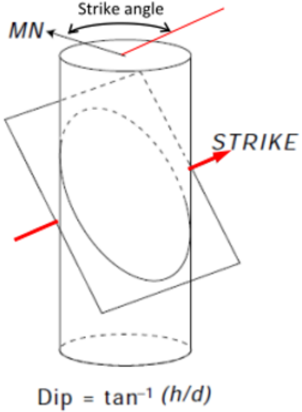
Un-wrapped image
(ultrasonic)



3D-representation



Interpretation



(Zoback 2013, RM, Ch 5.3)

○

(σ_n, τ)

τ/σ_n

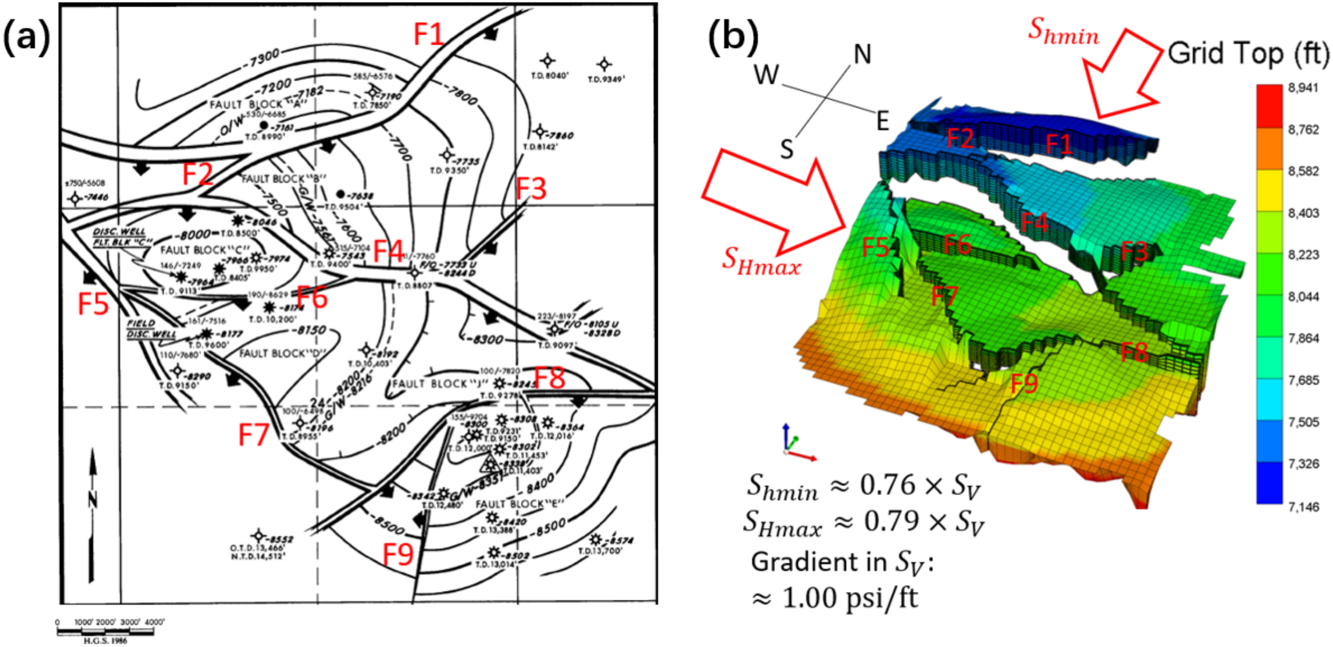
$$\mu = 0.5 \pm 0.1$$

$$60^\circ \pm 10^\circ$$

$$80^\circ \pm 5^\circ$$

$$10^\circ \pm 10^\circ$$

$$60^\circ \pm 5^\circ$$



(a)
F1
F2
F3
F4
F5
F6
F7
F8
F9
FAULT BLOCK "A"
FAULT BLOCK "B"
FAULT BLOCK "C"
FAULT BLOCK "D"
FAULT BLOCK "J"
FAULT BLOCK "E"
DISC. WELL FLT. BLK "C"
FIELD DISC. WELL
N
H.G.S. 1986
(b)
W
N
E
S
S_{hmin}
S_{Hmax}
Grid Top (ft)
8,941
8,762
8,582
8,403
8,223
8,044
7,864
7,685
7,505
7,326
7,146
$S_{hmin} \approx 0.76 \times S_V$
$S_{Hmax} \approx 0.79 \times S_V$
Gradient in S_V:
≈ 1.00 psi/ft

S_{hmin}

$$\sigma_n/\sigma_v$$

τ / σ_v

$P_p/$

$$z = 0.44$$

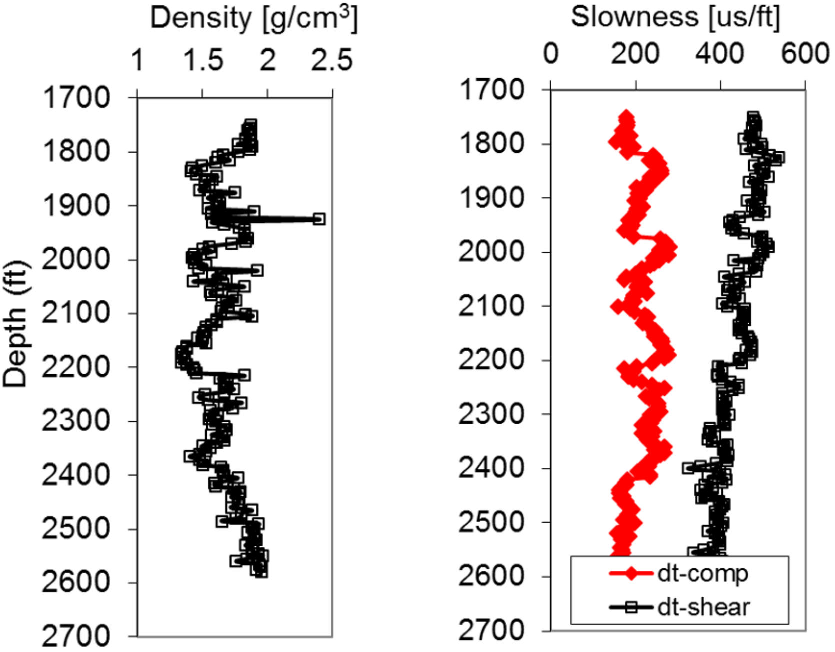
$\mu = 0.6$

$$E_{static} = 0.65 \cdot E_{dynamic}$$

$$E'_{static} = E_{static}/(1 - \nu^2)$$

$$\varepsilon_{Hmax} = 0.0015$$

$$\varepsilon_{hmin} = 0$$



Density [g/cm3]
1
1.5
2
2.5
1700
1800
1900
2000
2100
2200
2300
2400
2500
2600
2700
Depth (ft)
Slowness [us/ft]
0
200
400
600
1700
1800
1900
2000
2100
2200
2300
2400
2500
2600
2700
dt-comp
dt-shear

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} +\dfrac{1}{E_h} & -\dfrac{\nu_h}{E_h} & -\dfrac{\nu_v}{E_v} & 0 & 0 & 0 \\ -\dfrac{\nu_h}{E_h} & +\dfrac{1}{E_h} & -\dfrac{\nu_v}{E_v} & 0 & 0 & 0 \\ -\dfrac{\nu_v}{E_v} & -\dfrac{\nu_v}{E_v} & +\dfrac{1}{E_v} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{G_v} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{G_v} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{G_h} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}$$

$$G_h = \frac{E_h}{2(1+\nu_h)}$$

G_v

$$E_h = \frac{(C_{11} - C_{12})\left[C_{33}(C_{11} + C_{12}) - 2\, C_{13}^2\right]}{C_{11}C_{33} - C_{13}^2}$$

$$E_v = C_{33} - \frac{2\, C_{13}^2}{C_{11} + C_{12}}$$

$$\nu_h = \frac{C_{12}C_{33} - C_{13}^2}{C_{11}C_{33} - C_{13}^2}$$

$$\nu_v = \frac{C_{13}}{C_{11} + C_{12}}$$

$$G_v = C_{44}$$

$$G_h = C_{66} = \frac{C_{11} - C_{12}}{2}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}$$

$$C_{11} = \left[\frac{1}{(1-\nu_h)E_v - 2\nu_v^2 E_h}\right]\left(\frac{E_h E_v - \nu_v^2 E_h^2}{1+\nu_h}\right)$$

$$C_{33} = \left[\frac{1}{(1-\nu_h)E_v - 2\nu_v^2 E_h} \right] (E_v^2 - \nu_h E_v^2)$$

$$C_{12} = \left[\frac{1}{(1-\nu_h)E_v - 2\nu_v^2 E_h}\right]\left(\frac{\nu_v^2 E_h^2 + \nu_h E_h E_v}{1+\nu_h}\right)$$

$$C_{13} = \left[\frac{1}{(1-\nu_h)E_v - 2\nu_v^2 E_h}\right](\nu_v E_h E_v)$$

$$C_{66} = \frac{C_{11} - C_{12}}{2} = G_h = \frac{E_h}{2(1 + \nu_h)}$$

C_{44}

$$\sigma_{xx}$$

σ_{yy}

E

ν

UCS

T_s

σ_{rr}

$\sigma_{\theta\theta}$

$$\sigma_{r\theta}$$

x

y

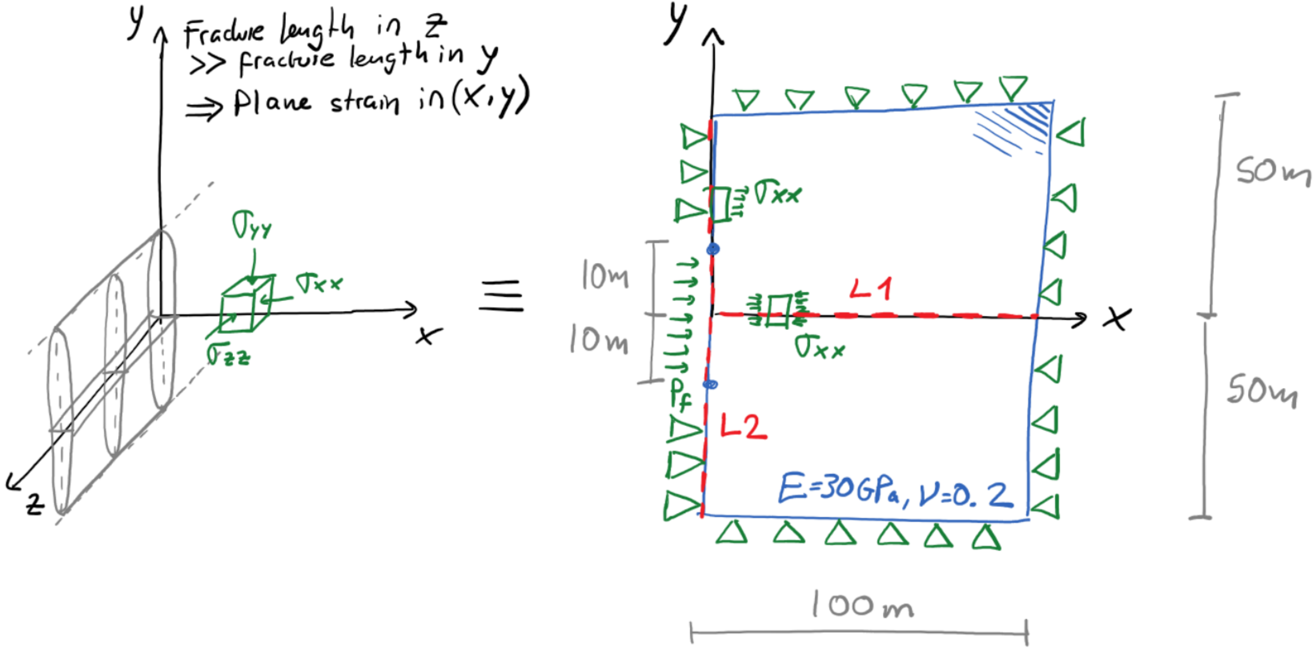
(r, θ)

r

σ_{xy}

c

$(x, y) =$



Fracture length in z
>> fracture length in y
⇒ Plane strain in (x, y)
σ_{yy}
σ_{xx}
σ_{zz}
x
y
z
≡
σ_{xx}
10m
10m
P_f
L1
L2
σ_{xx}
E=30GPa, ν=0.2
50m
50m
100m

$y =$

P_c

P_p

ε_{vol}

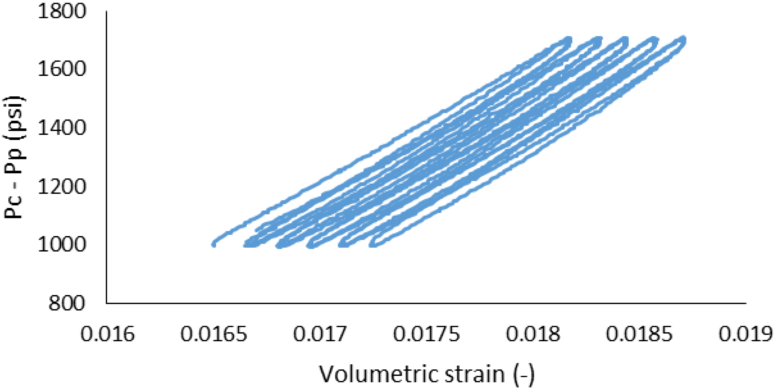
σ_{mean}

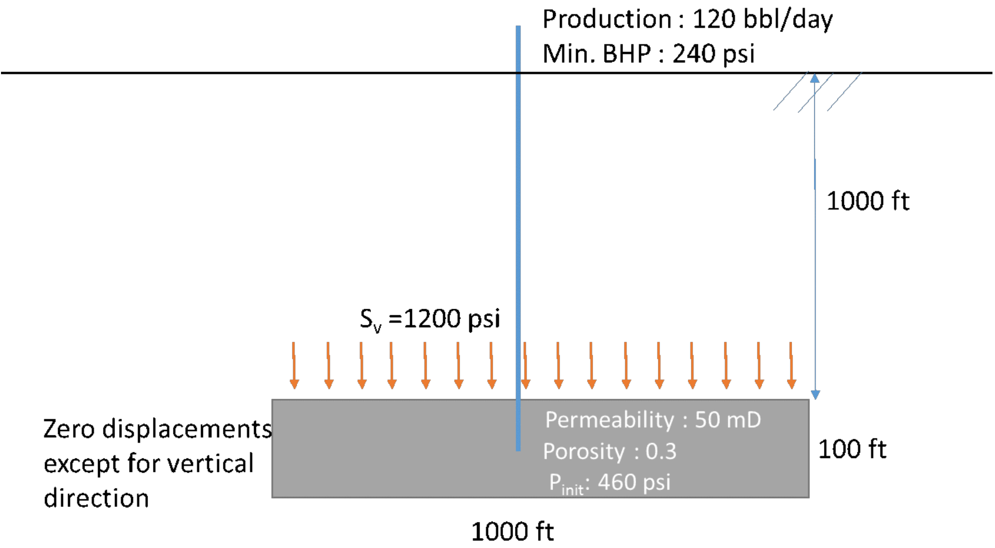
α

$$T_{ch}$$

α_h

$$\alpha_v$$


1800
1600
1400
1200
1000
800
Pc - Pp (psi)
0.016
0.0165
0.017
0.0175
0.018
0.0185
0.019
Volumetric strain (-)


Production : 120 bbl/day
Min. BHP : 240 psi
1000 ft
S_v =1200 psi
Zero displacements except for vertical direction
Permeability : 50 mD
Porosity : 0.3
P_{init}: 460 psi
100 ft
1000 ft

$$\alpha\frac{(1-2\nu)}{(1-\nu)}$$

(p', q)

$$
\begin{cases}
\nabla \underline{\underline{\sigma}} + \underline{f} = \underline{0} & \text{Equilibrium} \\
\underline{\underline{\varepsilon}} = \dfrac{1}{2}(\nabla \underline{u} + \nabla \underline{u}^T) & \text{Kinematic} \\
\underline{\underline{\sigma}} = \underline{\underline{C}} \cdot \underline{\underline{\varepsilon}} + 3\alpha_L K \theta \underline{\underline{I}}; \;\; \theta = T - T_0 & \text{Constitutive} \\
\dfrac{\partial \theta}{\partial t} = \dfrac{k_T}{\rho c_v} \nabla^2 \theta + \dfrac{3\beta K T_0}{\rho c_v} \dfrac{\partial \varepsilon_{vol}}{\partial t} & \text{Diffusivity}
\end{cases}
$$

$$\begin{cases} \dfrac{\partial p}{\partial t} - \dfrac{k}{\mu} M \nabla^2 p = -\alpha M \dfrac{\partial \varepsilon_{vol}}{\partial t} + \beta_e M \dfrac{\partial T}{\partial t} & \text{Pore pressure diffusivity} \\ \dfrac{\partial T}{\partial t} - \kappa_T \nabla^2 T = -\dfrac{\alpha_d}{m_d} \dfrac{\partial \varepsilon_{vol}}{\partial t} + \dfrac{\beta_e}{m_d} \dfrac{\partial p}{\partial t} & \text{Temperature diffusivity} \end{cases}$$

$$\beta_e = \alpha\beta_d + \phi(\beta_f - \beta_s)$$

$$\beta_f > \beta_s$$

$$\beta_d = \beta_s;$$

K

$$\alpha_d = K\ \beta_d$$

$$m_d = \frac{c_d}{T_0}$$

$$\underline{\underline{\sigma}} = \underline{\underline{C}} \cdot \underline{\underline{\varepsilon}} + 3\alpha_L K \theta \underline{\underline{I}}; \quad \theta = T - T_0$$

$$\varepsilon_{11} = \varepsilon_{22} = 0$$

$$\sigma_{33} =$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \varepsilon_{33} \end{bmatrix} + \begin{bmatrix} 3\alpha_L K\theta \\ 3\alpha_L K\theta \\ 3\alpha_L K\theta \end{bmatrix}$$

σ_{11}

σ_{33}

$$\begin{cases} \sigma_{11} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} + 3\alpha_L K\theta \\ \sigma_{33} = \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} + 3\alpha_L K\theta \end{cases}$$

ε_{33}

$$\sigma_{11} = \left(\frac{\nu}{1-\nu}\right)\sigma_{33} + \left(\frac{1-2\nu}{1-\nu}\right)3\alpha_L K\theta$$

$$K = E/[3(1 - 2\nu)]$$

$$\sigma_{11} = \left(\frac{\nu}{1-\nu}\right)\sigma_{33} + \frac{\alpha_L E}{1-\nu}\theta$$

$$\frac{\partial \sigma_{11}}{\partial \theta} = +\frac{\alpha_L E}{1-\nu}$$

$$\frac{\partial \sigma_{11}}{\partial \theta} = 0.1 \frac{\mathrm{MPa}}{^{\circ}\mathrm{C}}$$

$$\alpha_L = 10^{-5}$$

$1/^{\circ}$

$$E = 10$$

$$\nu = 0.2$$

S_1

$$S_3$$

$$P_{pore\ fluid}$$

$$\sigma_1 = UCS + q\sigma_3$$

S_0

φ

$$(J_2)^{1/2}$$

$$P_W = P_p$$

w_{BO}

$$0^\circ < w_{BO} < 60^\circ$$

$$w_{BO} > 60^\circ$$

$$P_W$$

$$dP_p/dz$$

dS_v/dz

$$\phi = \phi_0 \exp\left(-\beta \sigma_v\right)$$

$$e = e_0 - C_c \ln\left(\frac{\sigma_v}{1\ \mathrm{MPa}}\right)$$

λ_p

$$S_v$$

$$dq = 3dp'$$

G

p'_o

$$\phi_{CS} = 24^\circ$$

λ

$\mathcal{K}$

e_o

p'

ε_q

e

$$dq = 0.9\, dp'$$

$$p' = 400$$

$$d\varepsilon_{p'}^{e} = \frac{\kappa}{\upsilon}\frac{dp'}{p'};\; d\varepsilon_{q}^{e} = \frac{dq}{3G}$$

$$\begin{bmatrix} d\varepsilon_{p'}^{p} \\ d\varepsilon_{q}^{p} \end{bmatrix} = \frac{\lambda-\kappa}{\upsilon p'(M^2+\eta^2)} \begin{bmatrix} M^2-\eta^2 & 2\eta \\ 2\eta & \frac{4\eta^2}{M^2-\eta^2} \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix}$$

$$v = 1 + e$$

$$\eta = q/p'$$

$$de = -\upsilon d\varepsilon_p$$

$$dp'_o = d\varepsilon^{p}_{p'} \frac{\upsilon}{\lambda - \kappa} p'_o$$

$$\Delta S_{yy} = \nu(\Delta S_{xx} + \Delta S_{zz})$$

h_f

$$E' = 8.9 \times 10^6$$

x_f

$$w_{w,0}$$

p_n

p_n

-6

$$p' - q$$

10^{-6}

$$E_{sta}/E_{dyn}$$