P_p

S_v

σ_v

S_3

S_{ij}

$\underline{e}_i$

$\underline{e}_j$

$$dx_i$$

S_{21}

$$S_{31} + \frac{\partial S_{31}}{\partial x_3} dx_3$$

$$S_{11} + \frac{\partial S_{11}}{\partial x_1} dx_1$$

S_{33}

M

$$E_{load} < E_{unload}$$

σ_r

σ'_p

○

α

β

γ

S_V

$$(S_n - P_p)/S_v$$

~

τ/σ_n

P_W

σ_{∞}

$$\Delta\sigma$$

$$P_W - P_p = 0$$

σ_{rr}

$\sigma_{\theta\theta}$

$\sigma_{r\theta}$

σ_1

σ_3

~ 3

$$\Delta P = P_W - P_p$$

T_S

$$P_W = 35$$

$S_v =$

$S_{Hmax} =$

$S_{hmin} =$

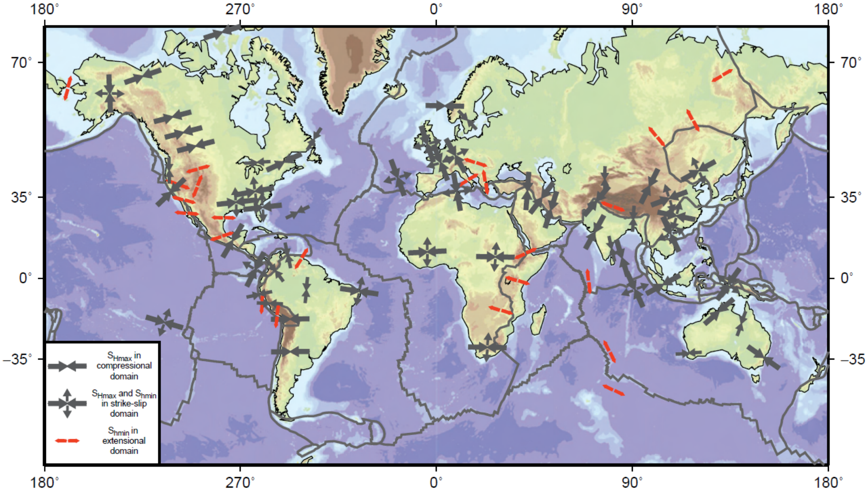
$P_p =$

$$w_{BO}$$

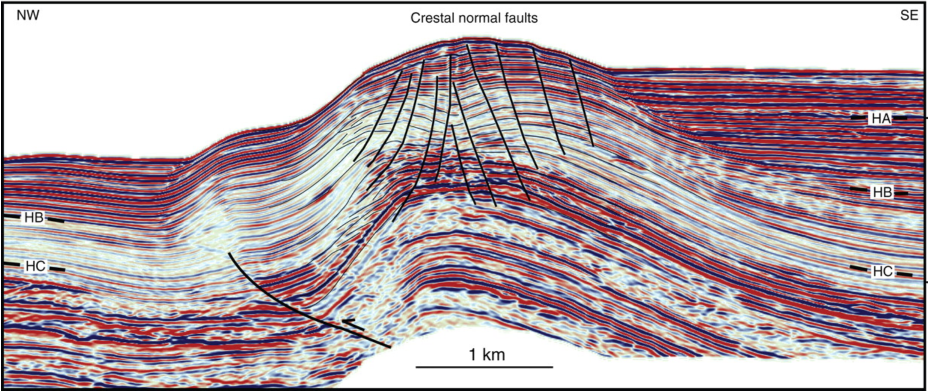
$$S_v > S_{Hmax} = S_{hmin}$$

S_{hmin}

< 10


180°
270°
0°
90°
180°
70°
35°
0°
−35°
70°
35°
0°
−35
S_{Hmax} in compressional domain
S_{Hmax} and S_{hmin} in strike-slip domain
S_{hmin} in extensional domain

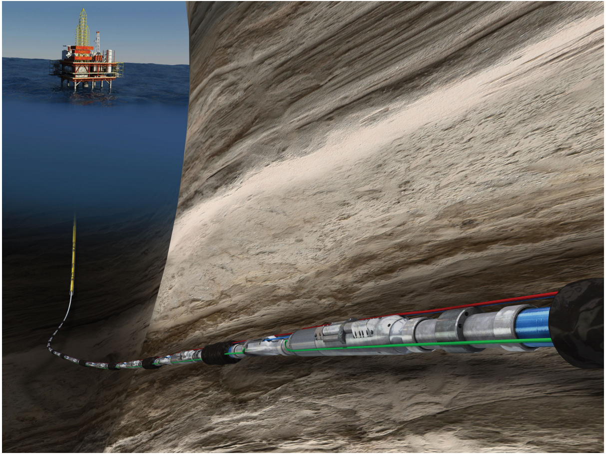
[Zoback 2010]



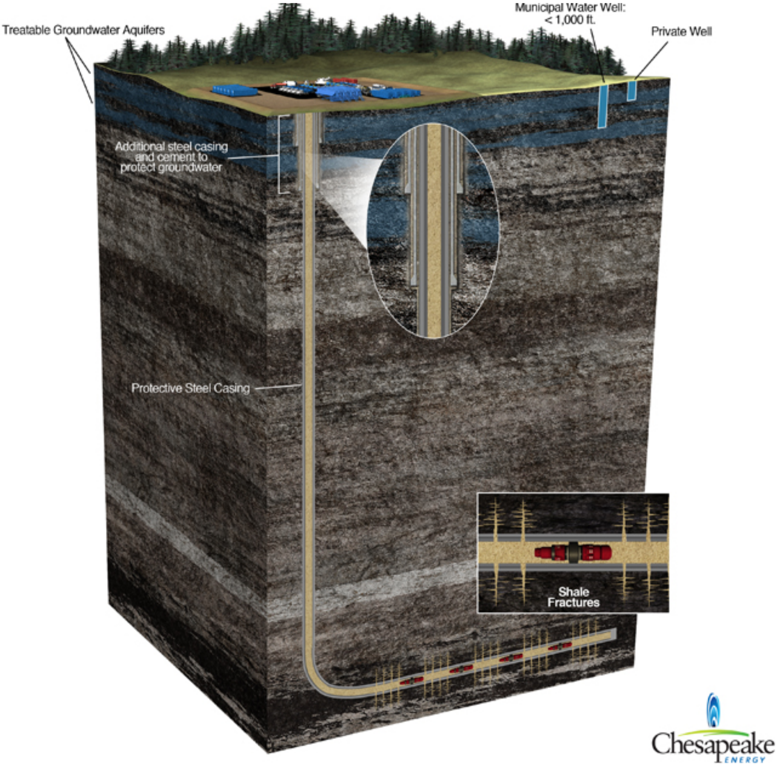
http://geosphere.gsapubs.org/content/5/2/59/F6.large.jpg

~1850

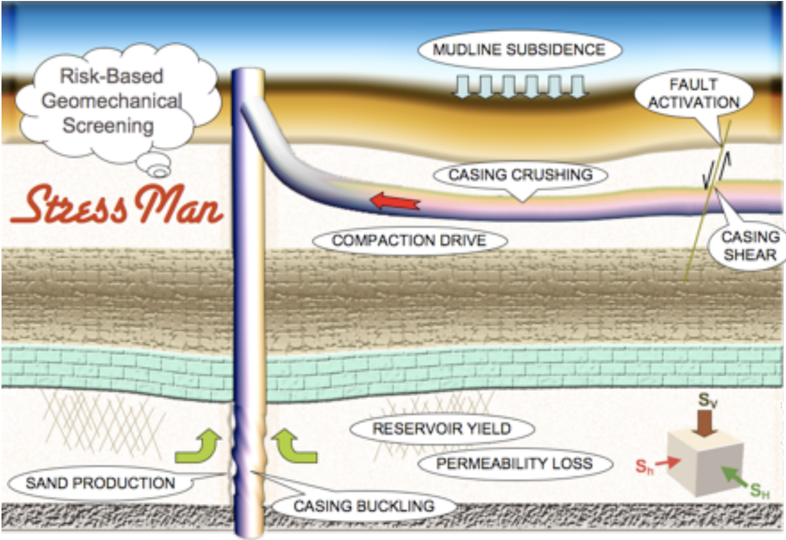
~2010



http://www.drillingcontractor.org



Treatable Groundwater Aquifers
Municipal Water Well:
< 1,000 ft.
Private Well
Additional steel casing and cement to protect groundwater
Protective Steel Casing
Shale Fractures
Chesapeake
ENERGY



http://www.higgs-palmer.com

$$\underline{F}_1$$

$$\underline{F}_2$$

$$\mathbb{R}^3$$

$$\underline{F}_1 = \underline{F}_2$$

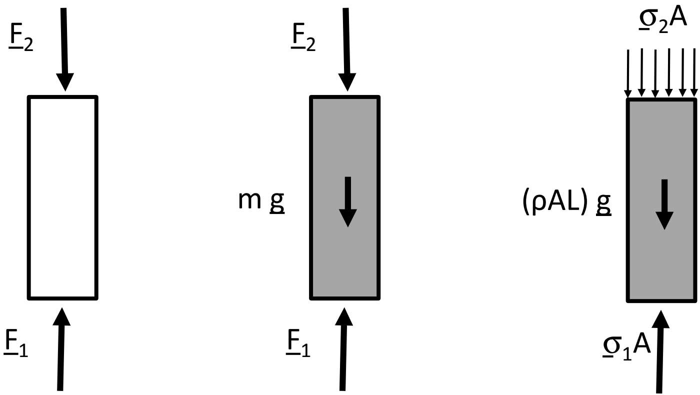
$$\underline{F}_1 = \underline{F}_2 + m\underline{g}$$

$$m\underline{g}$$

$\overline{F}$

A

$$\underline{\sigma} = \frac{F}{A}$$



$\Sigma F_z = + F_1 - F_2 = 0$ $\Sigma F_z = + F_1 - m\,g - F_2$ $\Sigma F_z = +\sigma_1 A - (\rho A L) g - \sigma_2 A$

$$\Delta x \Delta y \Delta x$$

$$\sum \underline{F} = 0$$

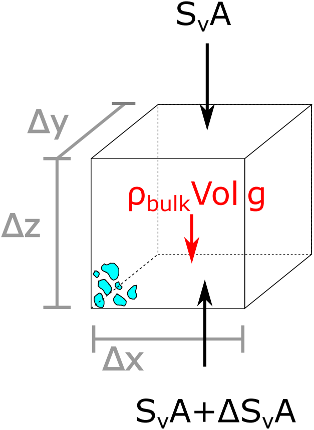
$\underline{g}$

$$\sum F_z = 0$$

$$S_v A + \rho_{bulk} Vol g = S_v A + \Delta S_v A$$

$$\rho_{bulk}\ \Delta x \Delta y \Delta z\ g = \Delta S_v \Delta x \Delta y$$

$$\frac{\Delta S_v}{\Delta z} = \rho_{bulk}\, g$$



S_vA
Δy
Δz
ρ_{bulk}Vol g
Δx
$S_vA+\Delta S_vA$

$$\int_0^{S_v(z)} dS_v = \int_0^z \rho_{bulk}(z) g \, dz$$

$$\rho_{bulk}(z)g$$

$$S_v(z = 0) = 0$$

$$\rho_{bulk} = cst$$

$\mathcal{Z}$

$$S_v(z) = \rho_{bulk} g \, z.$$

ρ_{SiO_2}

ρ_w

ρ_{bulk}

ϕ

$$\rho_{bulk} = (1 - \phi)\rho_{SiO_2} + \phi\rho_w$$

$$\rho_{SiO_2} = 2650$$

3

$$\rho_w = 1000$$

$$\rho_{bulk} = (1 - 0.2)2650$$

$^{3} + (0.2)1000$

$^3 = 2320$

3.

$$\rho_{bulk} g = 2320$$

[3] 9.8

$^{2} = 22.7 \times 10^{3}$

$$= 22.7$$

- [redacted]

~ 1

~ 23

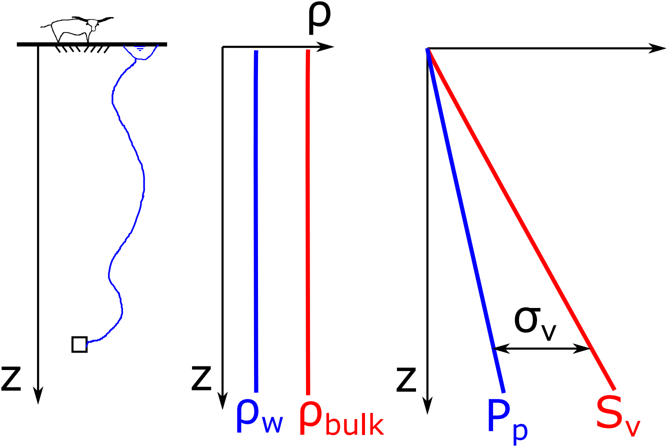
$$\phi = 0.20$$

~ 0.44

~ 10

S_w, S_o, S_g

$$P_p = \rho_w g \, z$$



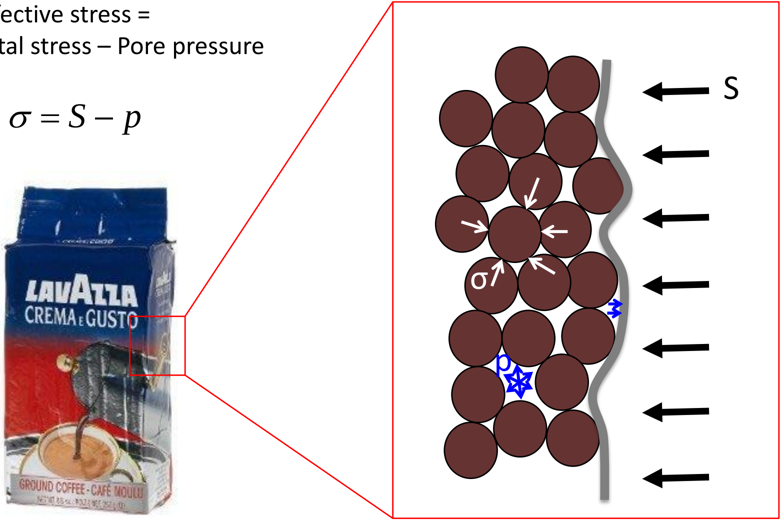
z
ρ
z
ρ_w
ρ_{bulk}
σ_v
z
P_p
S_v

$$\sigma_v = S_v - P_p$$

$$S_v > P_p$$

Effective stress =
Total stress – Pore pressure

$$\sigma = S - p$$



P_w

$$\frac{dP_p}{dz} = \rho_w g = 1040$$

$^{3} \times 9.8$

$^2 = 10.19$

$$\frac{dS_v}{dz} = \rho_{bulk} g = 2350$$

$^2 = 23.03$

$$P_p = \frac{dP_p}{dz} z = 10.19$$

$\times 1.219$

$$= 12.43$$

$$= 1801$$

$$\frac{dS_v}{dz}z = 23.03$$

$$= 28.07$$

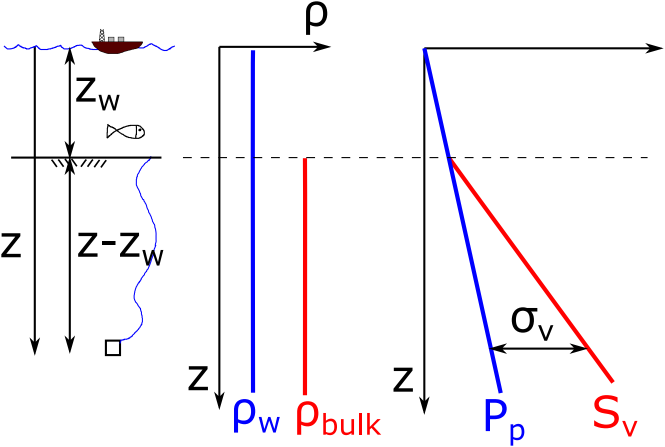
$$= 4070$$

z_w

$z > z_w$

$$S_v = \rho_w g\, z_w + \rho_{bulk} g (z - z_w)$$

$$z \geq z_w$$



Z_w
Z
Z-Z_w
ρ
Z
ρ_w
ρ_bulk
σ_v
Z
P_p
S_v

$$z = z_w$$

$$\sigma_v = S_v - P_p = \rho_{bulk} g(z - z_w)$$

$$P_p = \rho_w g z_w + \frac{dP_p}{dz}(z - z_w) = 0.44$$

$\times 2000$

$+0.44$

$$\times(9000 - 2000)$$

$$= 3960$$

$$S_v = \rho_w g z_w + \frac{dS_v}{dz}(z - z_w) = 0.44$$

$+1.00$

$$= 7880$$

,

$$\sigma_v = S_v - P_p = 7880$$

$$-3960$$

$$= 3920$$

$$P_p = 27.31$$

$$S_v = 54.34$$

$$\sigma_v = 27.03$$

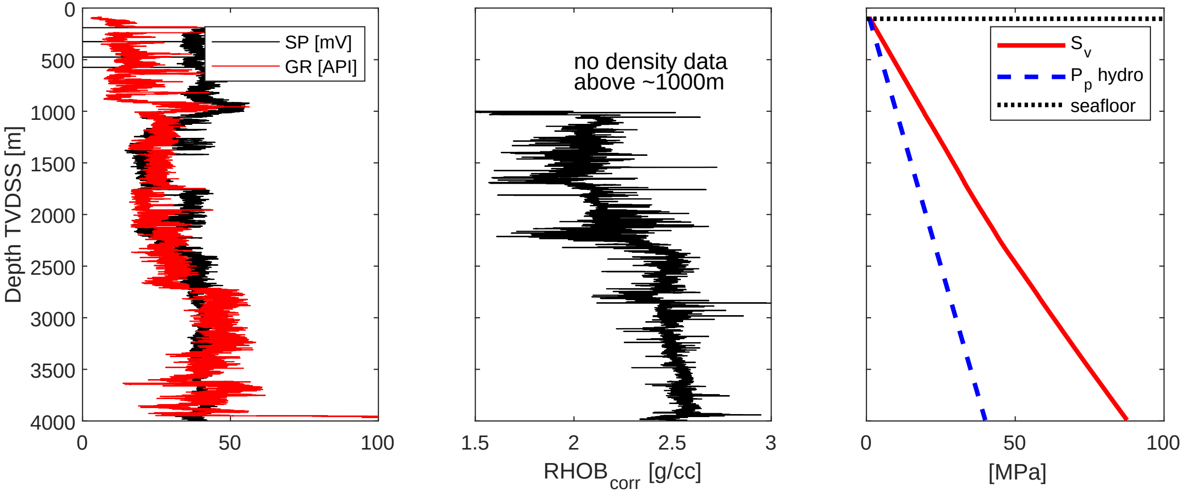
$$S_v(z) = \int_0^z \rho_{bulk}(z) g \, dz$$

$$\rho_{bulk}(z)$$

$$S_v(z_i) = \sum_{1}^{i} \rho_{bulk}(z_i) g \Delta z_i$$

i

z_i

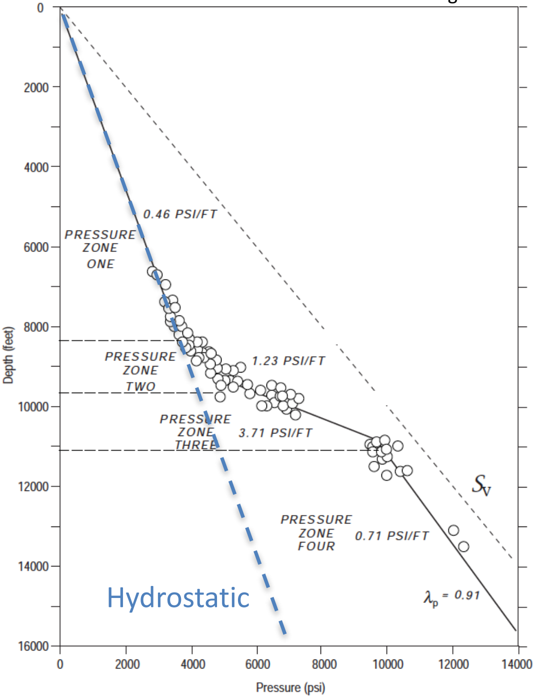


SP [mV]
GR [API]
Depth TVDSS [m]
0
500
1000
1500
2000
2500
3000
3500
4000
0
50
100
no density data
above ~1000m
1.5
2
2.5
3
RHOB corr [g/cc]
S v
P p hydro
seafloor
0
50
100
[MPa]

$z \sim 8,500$

$z \sim 11,000$

Monte Cristo field Onshore - Zoback 2013 – Figure 2.2



λ_p

$$\lambda_p(z) = \frac{P_p(z)}{S_v(z)}$$

$\lambda_p \leq 1$

$$\lambda_p \sim 0.44$$

$$\lambda_p \neq 0.44$$

$$\lambda_p \to 1$$

$$P_p > P_p^{\text{hydrostatic}}$$

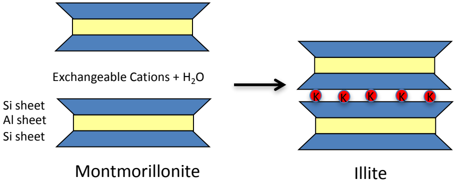
$$\Delta P_p$$

h

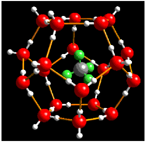
$$\rho_{brine} - \rho_{hydrocarbon}$$

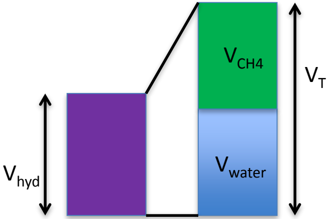
$$\Delta P = (\rho_{brine} - \rho_{hydrocarbon})gh$$

- **Aquathermal pressurization**
  - $\Delta T \rightarrow \Delta P$
- **Dehydration reactions**
  - $\Delta V \rightarrow \Delta P$
  - Montmorillonite to Illite (frees water)



- **Hydrocarbon generation**
  - $\Delta V \rightarrow \Delta P$





V_T/V_{hyd} ~ 1.3 to 6 at cst. P

W

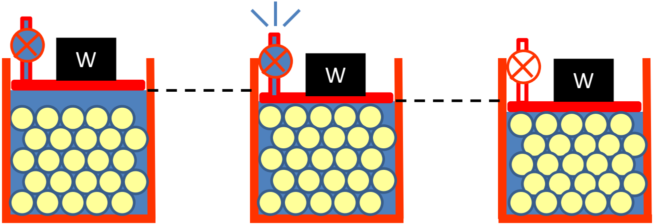
$$\Delta P = W/A$$

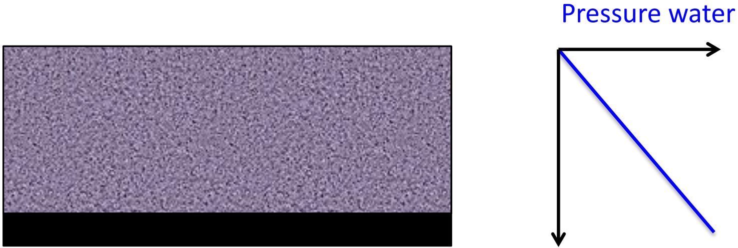
$$\sigma_v = W/A$$

$$\Delta P = 0$$

- <u>Disequilibrium compaction</u> **(Underconsolidation)**
    - o ΔS → ΔP (Vertical)

- **Tectonic compression**
    - o ΔS → ΔP (Horizontal)




Pressure water

$$\Delta z$$

$$D_h = \frac{Mk}{\mu}$$

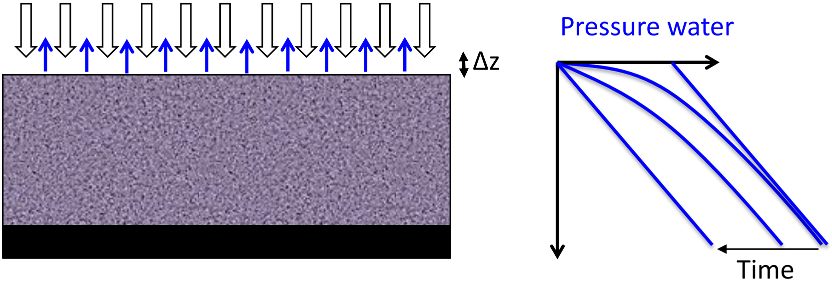
C_{bp}

k

$$\mu$$

$$\frac{\partial P_p}{\partial t} = D_h \frac{d^2 P_p}{dz^2}$$

Rate of sedimentation (loading) and rate of fluid “escape”



T_{ch}

$\sim 2/3$

$$T_{ch} = \frac{L^2}{D_h}$$

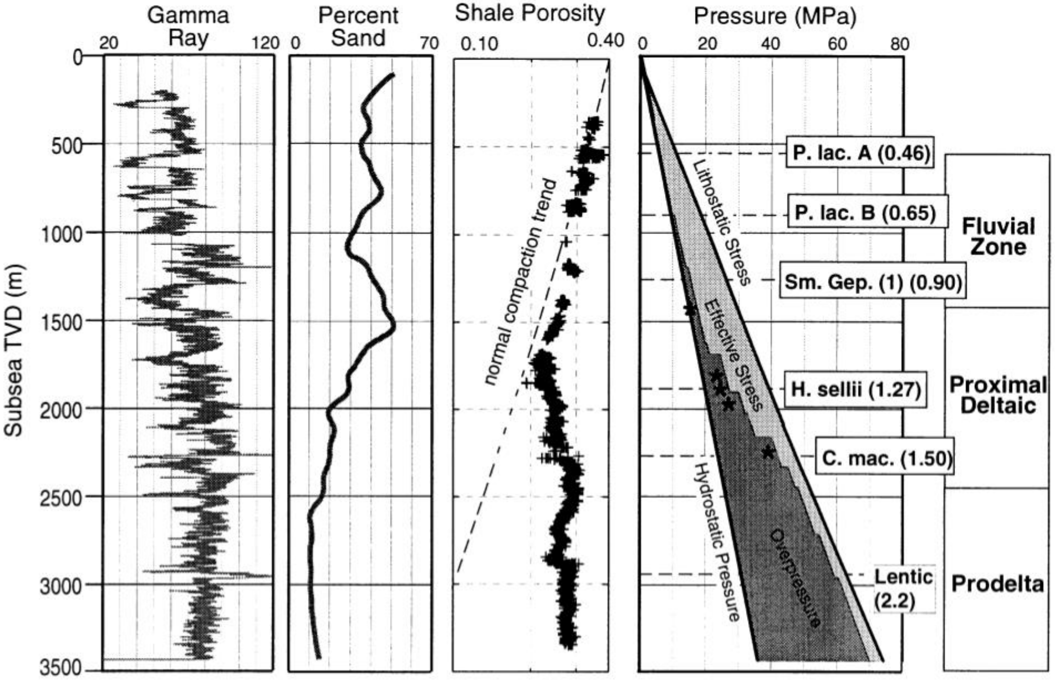
L

$k =$

$M =$

$T_{ch} \sim$

$$\phi = \phi_o \exp(-\beta\sigma_v)$$



Off-shore Louisiana – Gordon and Flemings (1998) Basin Research

$$\phi = 0.298$$

$$\beta = 3.2 \times 10^{-2}$$

-1

$$\phi_0 = 0.38$$

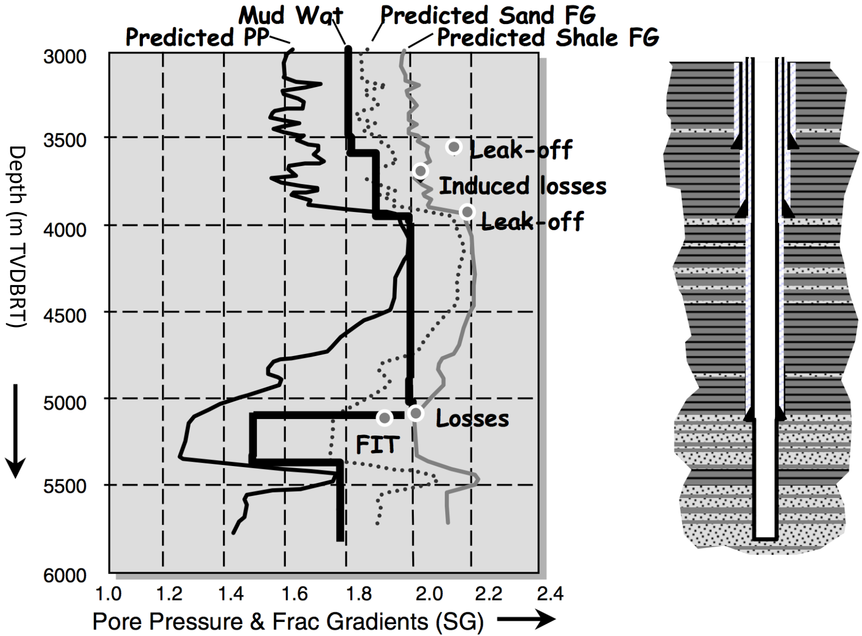
×

$$P_p = S_v + \frac{\ln(\phi/\phi_o)}{\beta} = 38$$

-7.6

$$= 30.40$$

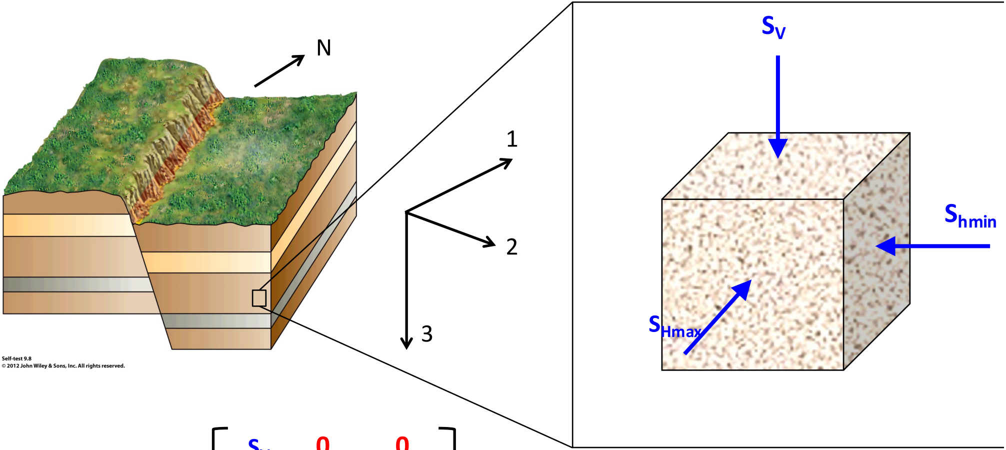
$$\lambda_p = \frac{P_p}{S_v} = \frac{30.40 \text{ MPa}}{38 \text{ MPa}} = 0.8. \blacksquare$$



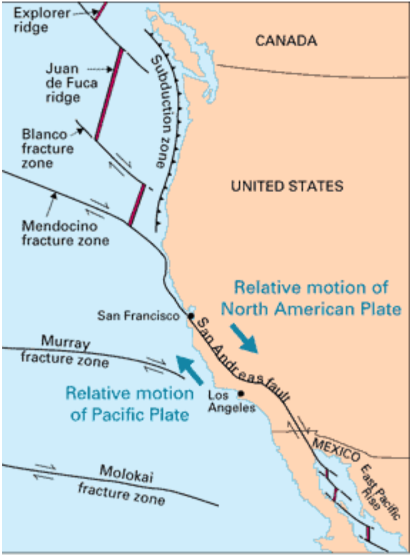
[Caspian Sea, Alberty and McLean – SPE67740]

S_{Hmax}

$$S_{Hmax} \geq S_{hmin}$$

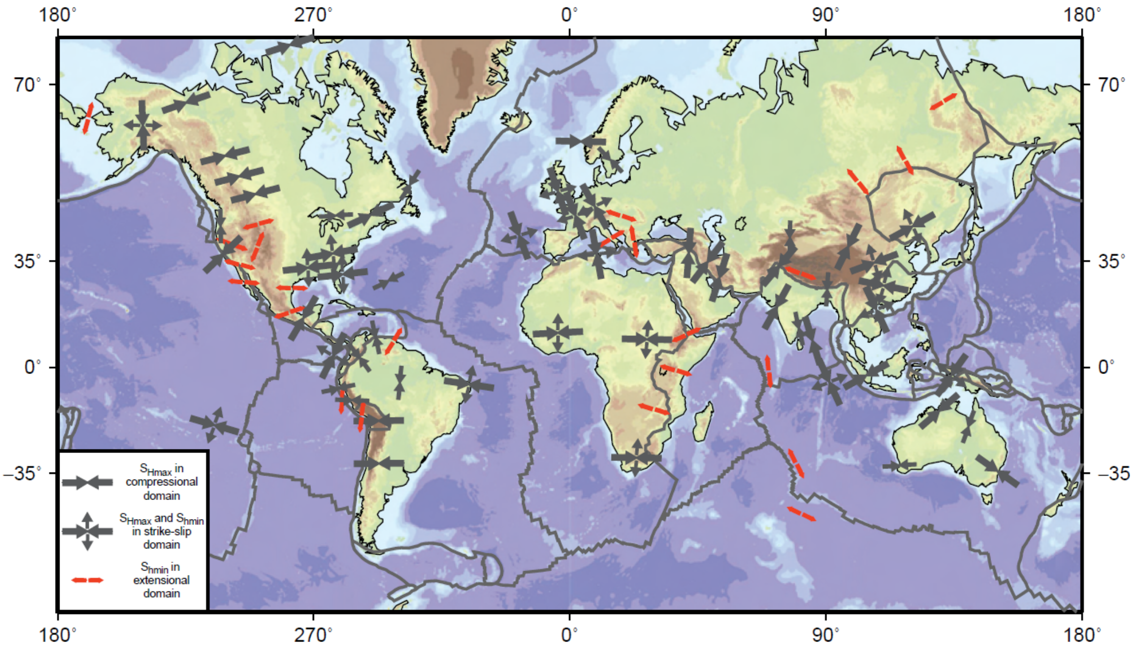


$$\underline{\underline{S}} = \begin{bmatrix} S_V & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_{hmin} \end{bmatrix}$$



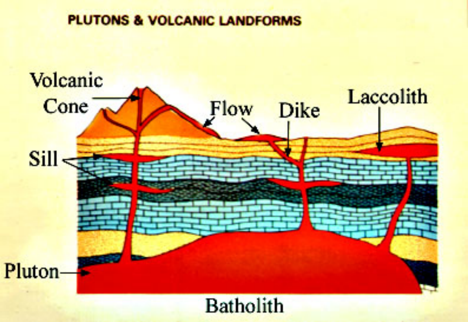
http://pubs.usgs.gov/gip/dynamic/understanding.html#anchor5798673

http://en.wikipedia.org/wiki/File:Aerial-SanAndreas-CarrizoPlain.jpg



180°
270°
0°
90°
180°
70°
35°
0°
–35°
70°
35°
0°
–35
S_{Hmax} in compressional domain
S_{Hmax} and S_{hmin} in strike-slip domain
S_{hmin} in extensional domain

[Zoback 2010]



http://www.indiana.edu/~geol105/1425chap5.htm
http://geophysics.ou.edu/geol1114

$$S_v > S_{hmin} = S_3$$

S_1

S_2

$$S_v > S_{Hmax} > S_{hmin}$$

$$S_1 = S_v$$

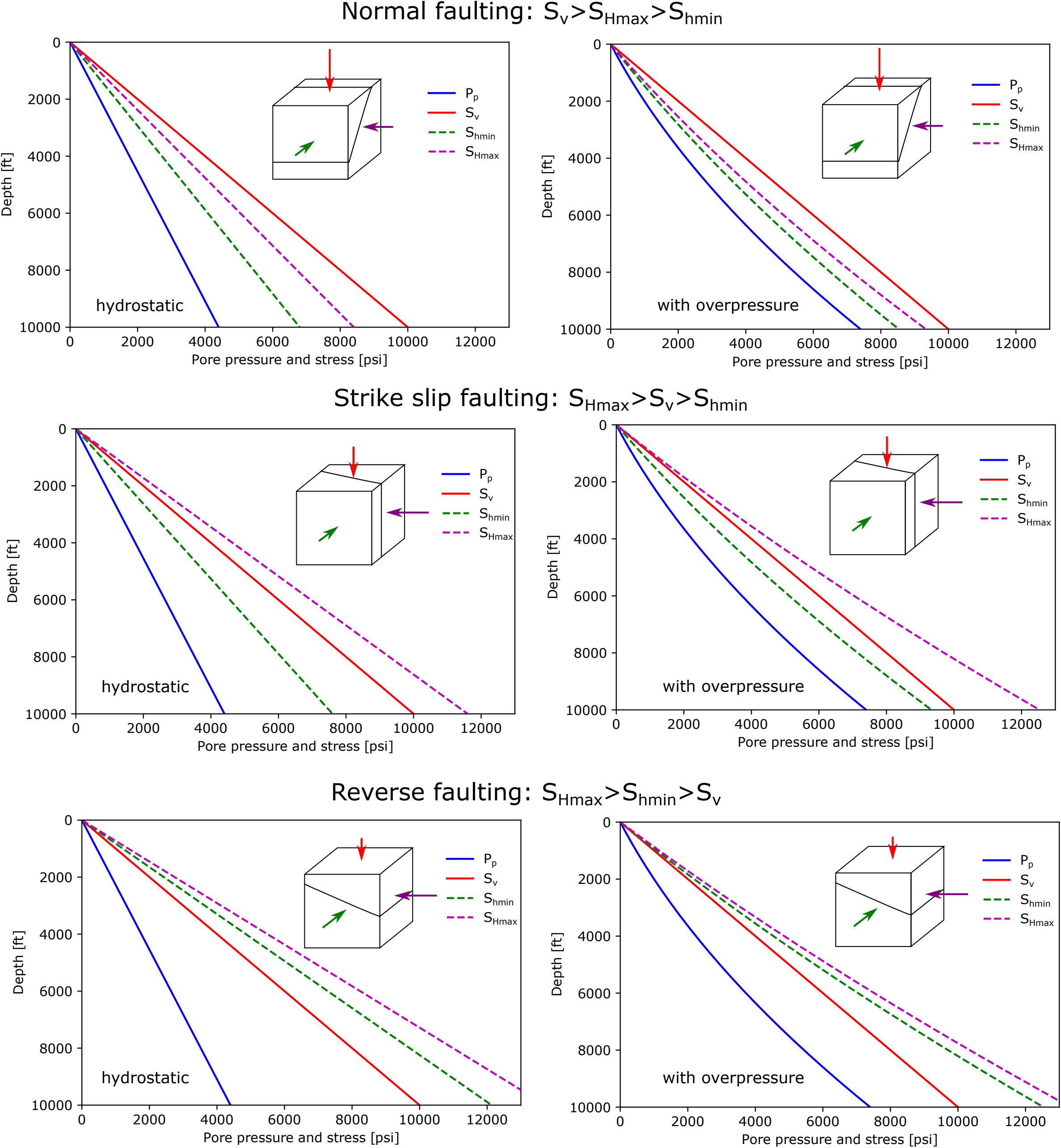
$$S_2 = S_{Hmax}$$

$$S_3 = S_{hmin}$$

~ 60

$$\sigma_1 = S_1 - P_p$$

$$\sigma_3 = S_3 - P_p$$



Normal faulting: $S_v > S_{Hmax} > S_{hmin}$
Strike slip faulting: $S_{Hmax} > S_v > S_{hmin}$
Reverse faulting: $S_{Hmax} > S_{hmin} > S_v$
Depth [ft]
Pore pressure and stress [psi]
P_p
S_v
S_{hmin}
S_{Hmax}
hydrostatic
with overpressure
0
2000
4000
6000
8000
10000
12000

$$S_{Hmax} > S_v > S_{hmin}$$

$$S_1 = S_{Hmax}$$

$$S_2 = S_v$$

$$S_{Hmax} > S_{hmin} > S_v$$

$$S_2 = S_{hmin}$$

$$S_3 = S_v$$

$$\rho_w g z$$

$$S_3 = S_{22}$$

$$S_3 = S_{11}$$

$$S_3 = S_{33}$$

$$S_{hmin} < S_{Hmax} < S_v$$

$$\beta = 3 \times 10^{-2}$$

]

3]

[—]

$\underline{e}_1$

$\underline{e}_2$

$$\underline{e}_3$$

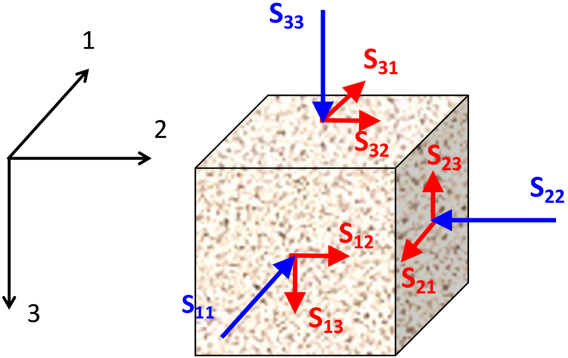
T

$$(x_1, x_2, x_3)$$

$\underline{v}$

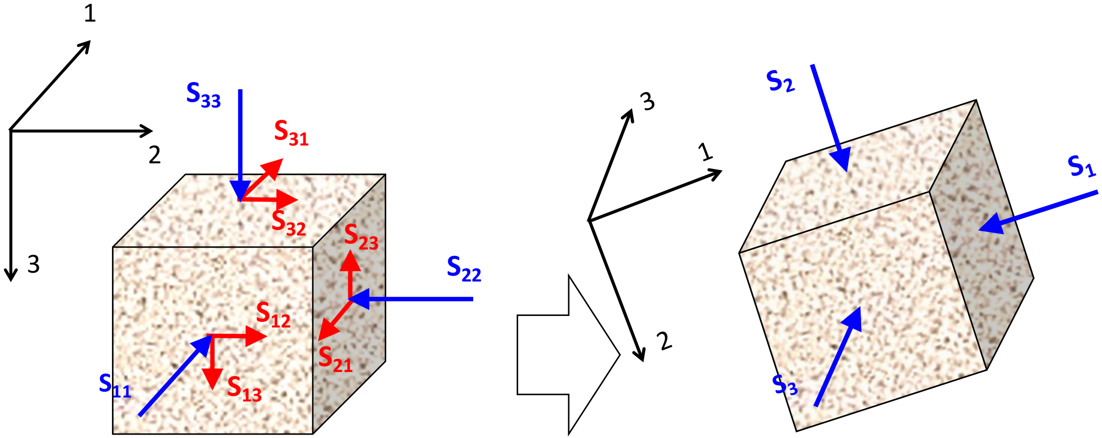
$$S_{ij} = S_{ji}$$

$$i \neq j$$



$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$S_1 \geq S_2 \geq S_3$$



$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$\underline{\underline{S}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$\underline{a}$

$$m\underline{a} = 0$$

$$\rho V b_1$$

ρ

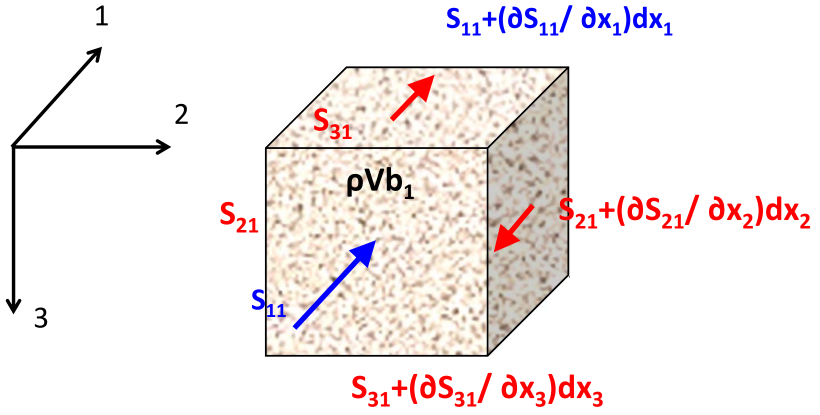
V

b_1

$$
\begin{aligned}
\sum F_1 &= 0 \\
\sum F_1 &= +S_{11}dx_2dx_3 - \left[S_{11} + (\tfrac{\partial S_{11}}{\partial x_1})dx_1\right]dx_2dx_3 \\
&\quad +S_{21}dx_1dx_3 - \left[S_{21} + (\tfrac{\partial S_{21}}{\partial x_2})dx_2\right]dx_1dx_3 \\
&\quad +S_{31}dx_1dx_2 - \left[S_{31} + (\tfrac{\partial S_{31}}{\partial x_3})dx_3\right]dx_1dx_2 \\
&\quad -\rho(dx_1dx_2dx_3)b_1 = 0
\end{aligned}
$$

$$(dx_1 dx_2 dx_3)$$

$$\frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} - \rho b_1 = 0$$



1
2
3
$S_{11}+(\partial S_{11}/ \partial x_1)dx_1$
S_{31}
$\rho V b_1$
S_{21}
$S_{21}+(\partial S_{21}/ \partial x_2)dx_2$
S_{11}
$S_{31}+(\partial S_{31}/ \partial x_3)dx_3$

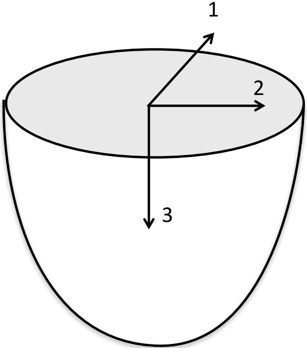
$$\left\{\begin{array}{rcl}\dfrac{\partial S_{11}}{\partial x_1}+\dfrac{\partial S_{21}}{\partial x_2}+\dfrac{\partial S_{31}}{\partial x_3}-\rho b_1 & = & 0\\ \dfrac{\partial S_{12}}{\partial x_1}+\dfrac{\partial S_{22}}{\partial x_2}+\dfrac{\partial S_{33}}{\partial x_3}-\rho b_2 & = & 0\\ \dfrac{\partial S_{13}}{\partial x_1}+\dfrac{\partial S_{23}}{\partial x_2}+\dfrac{\partial S_{33}}{\partial x_3}-\rho b_3 & = & 0\end{array}\right.$$

g

$$b_3 = g$$

$$\partial()/\partial x_1 = \partial()/\partial x_2 = 0$$

$$S_{33}(x_3) = \int_0^{x_3} \rho(x_3) g \, dx_3$$



$$\begin{cases} \cancel{\dfrac{\partial S_{11}}{\partial x_1}} + \cancel{\dfrac{\partial S_{21}}{\partial x_2}} + \dfrac{\partial S_{31}}{\partial x_3} + \cancel{\rho b_1} = \cancel{\dfrac{\partial^2 (\rho u_1)}{\partial t^2}} \\ \cancel{\dfrac{\partial S_{12}}{\partial x_1}} + \cancel{\dfrac{\partial S_{22}}{\partial x_2}} + \dfrac{\partial S_{32}}{\partial x_3} + \cancel{\rho b_2} = \cancel{\dfrac{\partial^2 (\rho u_2)}{\partial t^2}} \\ \cancel{\dfrac{\partial S_{13}}{\partial x_1}} + \cancel{\dfrac{\partial S_{23}}{\partial x_2}} + \dfrac{\partial S_{33}}{\partial x_3} + \rho b_3 = \cancel{\dfrac{\partial^2 (\rho u_3)}{\partial t^2}} \end{cases}$$

$$\frac{\partial S_{33}}{\partial x_3} - \rho(x_3) g = 0$$

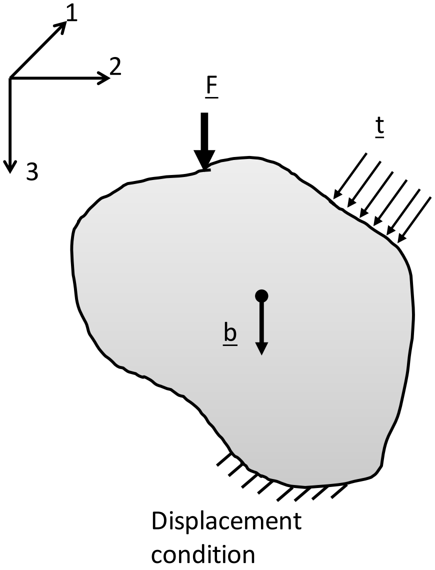
$$S_{33} = \int_0^{x_3} \rho(x_3) g \; dx_3$$

S_{11}

S_{22}

t

$\underline{b}$



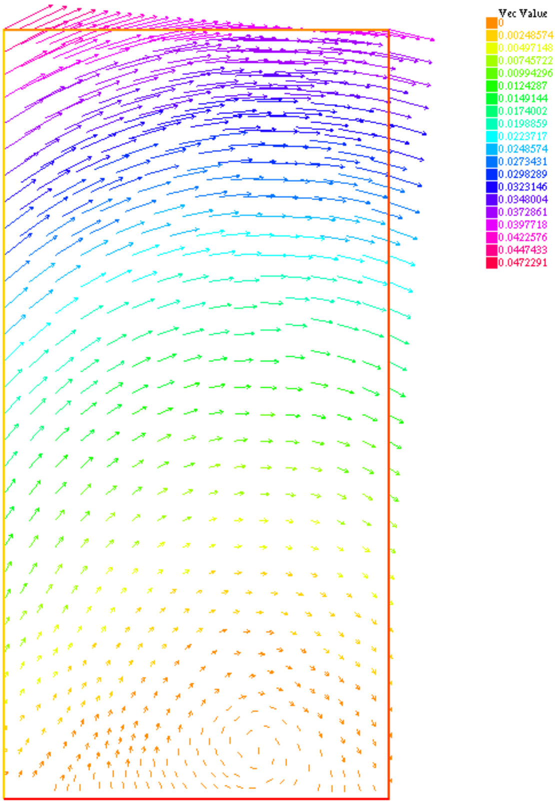
$$\begin{cases} \dfrac{\partial S_{11}}{\partial x_1} + \dfrac{\partial S_{21}}{\partial x_2} + \dfrac{\partial S_{31}}{\partial x_3} + \rho b_1 = \dfrac{\partial^2 (\rho u_1)}{\partial t^2} \\ \dfrac{\partial S_{12}}{\partial x_1} + \dfrac{\partial S_{22}}{\partial x_2} + \dfrac{\partial S_{32}}{\partial x_3} + \rho b_2 = \dfrac{\partial^2 (\rho u_2)}{\partial t^2} \\ \dfrac{\partial S_{13}}{\partial x_1} + \dfrac{\partial S_{23}}{\partial x_2} + \dfrac{\partial S_{33}}{\partial x_3} + \rho b_3 = \dfrac{\partial^2 (\rho u_3)}{\partial t^2} \end{cases}$$

And respect the boundary conditions:

- Displacement
- Boundary stresses
- Boundary Forces
- Body Forces

How do we relate stresses to displacements?

- Displacements → Strains (**Kinematic equations**)
- Strains → Stresses (**Constitutive equations**)


Vec Value
0
0.00248574
0.00497148
0.00745722
0.00994296
0.0124287
0.0149144
0.0174002
0.0198859
0.0223717
0.0248574
0.0273431
0.0298289
0.0323146
0.0348004
0.0372861
0.0397718
0.0422576
0.0447433
0.0472291

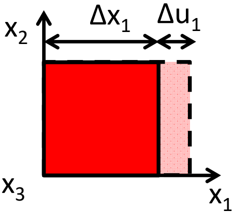
$$\varepsilon_{11} = \frac{\Delta u_1}{\Delta x_1}$$

$$\varepsilon_{22} = \frac{\Delta u_2}{\Delta x_2}$$

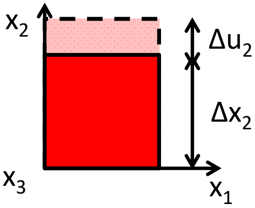
$$\pi - [\pi - \arctan(\Delta u_1/\Delta x_2) + \arctan(\Delta u_2/\Delta x_1)]$$

$$\arctan(x/y) \sim x/y$$

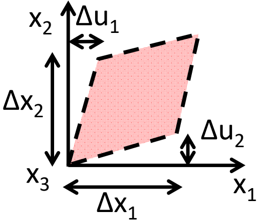
$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$



$$\varepsilon_{11} \approx \frac{\Delta u_1}{\Delta x_1}$$



$$\varepsilon_{22} \approx \frac{\Delta u_2}{\Delta x_2}$$



$$\varepsilon_{12} \approx \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$

x_i

$$\varepsilon_{vol} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

$$\Delta x_1 \Delta x_2 \Delta x_3$$

ΔV

V_0

$$\varepsilon_{vol} = \frac{\Delta V}{V_0}$$

$$\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3$$

$$(\mathrm{d}x_1 + \mathrm{d}u_1)(\mathrm{d}u_2 + \mathrm{d}u_2)(\mathrm{d}x_3 + \mathrm{d}u_3)$$

$$\varepsilon_{vol} = \frac{[(\mathrm{d}x_1 + \mathrm{d}u_1)(\mathrm{d}x_2 + \mathrm{d}u_2)(\mathrm{d}x_3 + \mathrm{d}u_3) - (\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3)]}{(\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3)}$$

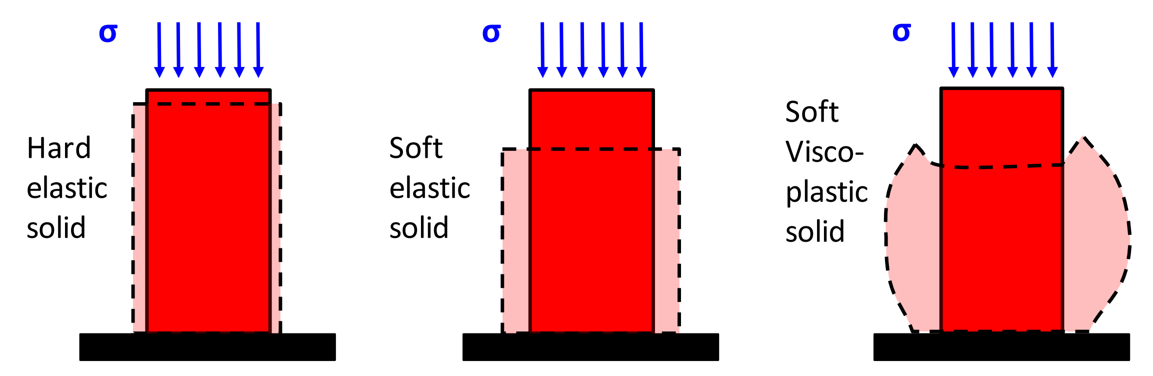
$$\mathrm{d}u_i \mathrm{d}u_j$$

$$\mathrm{d}u_1\mathrm{d}u_2\mathrm{d}u_3$$

$$\mathrm{d}u_i$$

$$\mathrm{d}u_i << \mathrm{d}x_j$$

$$\varepsilon_{vol} \sim \frac{(\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}u_3 + \mathrm{d}x_1\mathrm{d}x_3\mathrm{d}u_2 + \mathrm{d}x_2\mathrm{d}x_3\mathrm{d}u_1)}{(\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3)} = \frac{\mathrm{d}u_1}{\mathrm{d}x_1} + \frac{\mathrm{d}u_2}{\mathrm{d}x_2} + \frac{\mathrm{d}u_3}{\mathrm{d}x_3} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \quad \blacksquare$$


σ
Hard
elastic
solid
σ
Soft
elastic
solid
σ
Soft
Visco-
plastic
solid

F

$$\Delta x$$

$$F = k\Delta x$$

E

$$\frac{F}{A} = E\frac{\Delta x}{L}$$

σ

$\mathcal{E}$

$$\sigma = E\varepsilon$$

$$\underline{\underline{\sigma}}$$

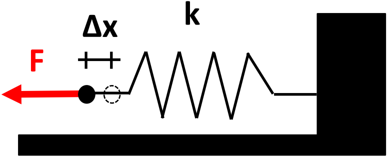
$\underline{\underline{\mathcal{E}}}$

$$\underline{\underline{C}}$$

$$\underline{\underline{\sigma}} = \underline{\underline{C}}\,\underline{\underline{\varepsilon}}$$

Hooke's law

$$F = k\Delta x$$

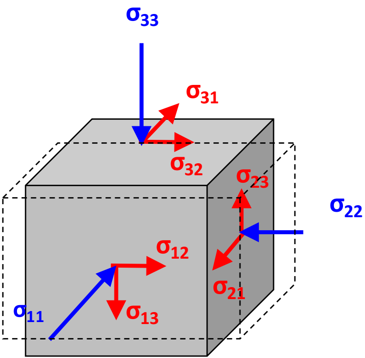


1-D (stress-strain) Hooke's law

$$\sigma = E\varepsilon$$

Generalized Hooke's law

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\,\underline{\underline{\varepsilon}}$$



σ_{33}

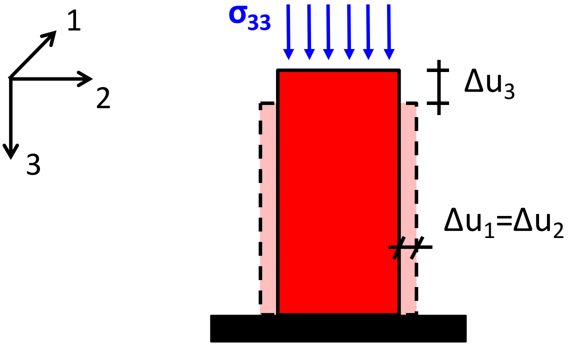
ε_{33}

$$E = \frac{\sigma_{33}}{\varepsilon_{33}}$$

$$\varepsilon_{11}$$

$$\varepsilon_{22}$$

$$\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}}$$



$$E = \frac{\sigma_{33}}{\varepsilon_{33}}$$ Young's Modulus

$$\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}}$$ Poisson's ratio (nu)

σ_a

ε_a

$$\sigma_a = 3000$$

$$= 3000\frac{1}{145}$$

$$= 20.7$$

$\varepsilon_a(1$

$$) = \frac{\sigma_a}{E} = \frac{20.7\text{ MPa}}{1000\text{ MPa}} = 0.0207 = 2.07\%$$

$\varepsilon_a(10$

$$) = \frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{10000 \text{ MPa}} = 0.00207 = 0.207\%$$

$\varepsilon_a(50$

$$) = \frac{\sigma_a}{E} = \frac{20.7\ \mathrm{MPa}}{50000\ \mathrm{MPa}} = 0.00041 = 0.041\%$$

$$\left\{\begin{array}{lcl}\varepsilon_{11} & = & +\dfrac{1}{E}\,\sigma_{11}-\dfrac{\nu}{E}\,\sigma_{22}-\dfrac{\nu}{E}\,\sigma_{33}\\ \varepsilon_{22} & = & -\dfrac{\nu}{E}\,\sigma_{11}+\dfrac{1}{E}\,\sigma_{22}-\dfrac{\nu}{E}\,\sigma_{33}\\ \varepsilon_{33} & = & -\dfrac{\nu}{E}\,\sigma_{11}-\dfrac{\nu}{E}\,\sigma_{22}+\dfrac{1}{E}\,\sigma_{33}\end{array}\right.$$

ε_{ij}

$$G = E/[2(1 + \nu)]$$

$$\begin{cases} 2\varepsilon_{12} &= \dfrac{1}{G}\,\sigma_{12} \\ 2\varepsilon_{13} &= \dfrac{1}{G}\,\sigma_{13} \\ 2\varepsilon_{23} &= \dfrac{1}{G}\,\sigma_{23} \end{cases}$$

$$\underline{\underline{D}}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\,\underline{\underline{\sigma}}$$

6×1

$$6 \times 6$$

$$
\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}
=
\begin{bmatrix}
+\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\
-\frac{\nu}{E} & +\frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\
-\frac{\nu}{E} & -\frac{\nu}{E} & +\frac{1}{E} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E}
\end{bmatrix}
\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}
$$

$$\underline{\underline{\sigma}} = [0, 0, \sigma_{33}, 0, 0, 0]^T$$

$$\underline{\underline{D}}\ \underline{\underline{\sigma}}$$

$$\underline{\underline{\varepsilon}} = \left[-\frac{\nu}{E}\,\sigma_{33}, -\frac{\nu}{E}\,\sigma_{33}, \frac{1}{E}\,\sigma_{33}, 0, 0, 0 \right]^T$$

ν

$$\underline{\underline{C}} = \underline{\underline{D}}^{-1}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$

λ

$$\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{11} + \nu\varepsilon_{22} + \nu\varepsilon_{33}\right]$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(-1 + \frac{1-\nu}{\nu}\right)\varepsilon_{11}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$2\mu = \frac{\nu E}{(1+\nu)(1-2\nu)}\left(-1 + \frac{1-\nu}{\nu}\right) = \frac{E}{(1+\nu)}$$

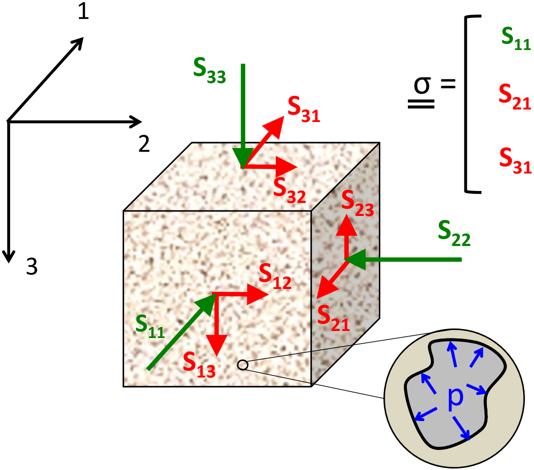
$$\mu = G$$

$$\left\{\begin{array}{lcl}\sigma_{11} & = & (\lambda+2\mu)\,\varepsilon_{11}+\lambda\,\varepsilon_{22}+\lambda\,\varepsilon_{33}\\ \sigma_{22} & = & \lambda\,\varepsilon_{11}+(\lambda+2\mu)\,\varepsilon_{22}+\lambda\,\varepsilon_{33}\\ \sigma_{33} & = & \lambda\,\varepsilon_{11}+\lambda\,\varepsilon_{22}+(\lambda+2\mu)\,\varepsilon_{33}\\ \sigma_{12} & = & 2\mu\,\varepsilon_{12}\\ \sigma_{13} & = & 2\mu\,\varepsilon_{13}\\ \sigma_{23} & = & 2\mu\,\varepsilon_{23}\end{array}\right. .$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} \quad \blacksquare$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\;\underline{\underline{S}}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\left(\underline{\underline{S}} - P_p \underline{\underline{I}}\right) = \underline{\underline{D}}\,\underline{\underline{\sigma}}$$



$$\underline{\underline{\sigma}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} - \begin{bmatrix} P_p & 0 & 0 \\ 0 & P_p & 0 \\ 0 & 0 & P_p \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} S_{11}-P_p & S_{12} & S_{13} \\ S_{21} & S_{22}-P_p & S_{23} \\ S_{31} & S_{32} & S_{33}-P_p \end{bmatrix}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{D}}}} \left(\underline{\underline{S}} - \alpha P_p \underline{\underline{I}} \right)$$

$\alpha \sim 1$

$\alpha \sim 0.5$

$$S_v = S_{33}$$

$$\sigma_v(z) = S_v(z) - P_p$$

$$\varepsilon_{11} = \varepsilon_{22}$$

$$\underline{\underline{\varepsilon}} = [0, 0, \varepsilon_{33}, 0, 0, 0]^T$$

$$\underline{\underline{\sigma}} = \underline{\underline{D}}\,\underline{\varepsilon}$$

$$\begin{cases} \sigma_{11} = \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \end{cases}$$

σ_{11}

σ_{22}

$$\sigma_{11} = \sigma_{22} = \frac{\nu}{1-\nu}\sigma_{33}$$

$$\sigma_h = \frac{\nu}{1-\nu}\sigma_v$$

$\nu \sim 0.25$

$\nu/(1-\nu) \sim 1/3$

$\nu \to 0.5$

$\nu/(1-\nu) \to 1$

$\nu \sim 0.5$

$$S_{11} = \sigma_{11} + P_p$$

$$S_{33} = \sigma_{33} + P_p$$

$$\varepsilon_{11} \neq \varepsilon_{22} \neq 0$$

$$\underline{\underline{\varepsilon}} = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 0, 0, 0]^T$$

$$\begin{cases} \sigma_{11} = \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{22} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{33} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \end{cases}$$

$$\left\{\begin{array}{l}\sigma_{11}=\dfrac{\nu}{1-\nu}\sigma_{33}+\dfrac{E}{1-\nu^2}\varepsilon_{11}+\dfrac{\nu E}{1-\nu^2}\varepsilon_{22}\\ \sigma_{22}=\dfrac{\nu}{1-\nu}\sigma_{33}+\dfrac{\nu E}{1-\nu^2}\varepsilon_{11}+\dfrac{E}{1-\nu^2}\varepsilon_{22}\end{array}\right.$$

ε_{Hmax}

ε_{hmin}

$$
\begin{cases}
\sigma_{Hmax} = \dfrac{\nu}{1-\nu}\sigma_v + E'\varepsilon_{Hmax} + \nu E'\varepsilon_{hmin} \\
\sigma_{hmin} = \dfrac{\nu}{1-\nu}\sigma_v + \nu E'\varepsilon_{Hmax} + E'\varepsilon_{hmin}
\end{cases}
$$

$$E' = \frac{E}{1-\nu^2}$$

$$S_{Hmax} = \sigma_{Hmax} + P_p$$

$$S_{hmin} = \sigma_{hmin} + P_p$$

$$\sigma_v = S_v - P_p$$

K

G

	(E,ν)	(K,G)
$G=$	$\dfrac{E}{2(1+\nu)}$	
$M=$	$\dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}$	$K+\dfrac{4}{3}G$
$\lambda=$	$\dfrac{\nu E}{(1+\nu)(1-2\nu)}$	
$K=$	$\dfrac{E}{3(1-2\nu)}$	

Shear modulus
(also noted as μ, S-wave)

Constrained modulus
(uniaxial compaction, P-wave)

Lamé first parameter
(volumetric strain component)

Bulk modulus
(relates volumetric strain and isotropic stress)

$$\mathrm{d}S_v/\mathrm{d}z = 23.8$$

$$\lambda_p = 0.7$$

$$E = 5$$

$$\nu = 0.22$$

$$\varepsilon_{hmin} = 0$$

$$\varepsilon_{Hmax} = 0.0002$$

$$S_v = 23.8\frac{\text{MPa}}{\text{km}} \times \frac{1\frac{\text{psi}}{\text{ft}}}{\frac{\text{MPa}}{\text{km}}} \times 7950 \text{ ft} = 8227\text{psi}$$

$$P_p = \lambda_p S_v = 0.7 \times 8227$$

$$\text{psi} = 5759 \text{ psi}$$

$$\sigma_v = S_v - P_p = 8227$$

$$\text{psi} - 5759\ \text{psi} = 2468\ \text{psi}$$

$$E' = \frac{E}{1 - \nu^2} = \frac{5 \times 10^6 \text{ psi}}{1 - 0.22^2} = 5.25 \times 10^6 \text{ psi}$$

$$\begin{cases} \sigma_{Hmax} = \frac{\nu}{1-\nu}\sigma_v + E'\varepsilon_{Hmax} + \nu E'\varepsilon_{hmin} = \frac{0.22}{1-0.22}2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} \times 0.0002 = 1745 \text{ psi} \\ \sigma_{hmin} = \frac{\nu}{1-\nu}\sigma_v + \nu E'\varepsilon_{Hmax} + E'\varepsilon_{hmin} = \frac{0.22}{1-0.22}2468 \text{ psi} + 0.22 \times 5.25 \times 10^6 \text{ psi} \times 0.0002 = 927 \text{ psi} \end{cases}$$

$$\begin{cases} S_{Hmax} = \sigma_{Hmax} + P_p = 1745 \text{ psi} + 5759 \text{ psi} = 7504 \text{ psi} \\ S_{hmin} = \sigma_{hmin} + P_p = 927 \text{ psi} + 5759 \text{ psi} = 6686 \text{ psi} \end{cases} \qquad \blacksquare$$

C_{pp}

V_p

$$C_{pp} = \frac{1}{V_p} \frac{\mathrm{d}V_p}{\mathrm{d}P_p}\bigg|_{S_v,\varepsilon_h}$$

ε_h

$$C_{pp} = \frac{C_{bp}}{\phi}$$

$$C_{bp} \sim 1/M$$

$$M = (1 - \nu)E/[(1 + \nu)(1 - 2\nu)]$$

$$C_{pp} = \frac{(1+\nu)(1-2\nu)}{(1-\nu)E\phi}$$

$$E = 10$$

$$\nu = 0.20$$

10^6

$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.20)10 \text{ GPa}}{(1+0.20)(1-2\times 0.20)} = 11.11 \text{ GPa} = 1.6\times 10^6 \text{ psi}$$

$$C_{pp} = \frac{1}{M\phi} = \frac{1}{1.6 \times 10^6 psi \times 0.20} = 3.1\ [10^6$$

$]^{-1} = 3.1\,\mu$

$\underline{u}$

$$(\lambda + G)\nabla(\nabla \cdot \underline{u}) + G\nabla^2 \underline{u} + \rho \underline{b} = 0$$

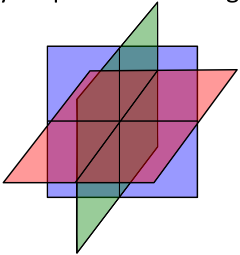
$$\lambda = (\nu E)/[(1+\nu)(1-2\nu)]$$

$\nabla()$

$$\nabla \cdot ()$$

$$\nabla^2()$$

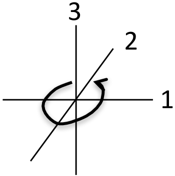
Orthorhombic symmetry
(symmetry respect to 3 orthogonal planes)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & 0 & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{55} & 0 \\ & & & 0 & 0 & C_{66} \end{bmatrix}$$

9 independent parameters

Transverse Isotropy
(symmetry respect to 1 axis)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & 0 & \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{44} & 0 \\ & & & 0 & 0 & C_{66} \end{bmatrix}$$

5 independent parameters ($C_{12}=C_{11}-2C_{66}$)

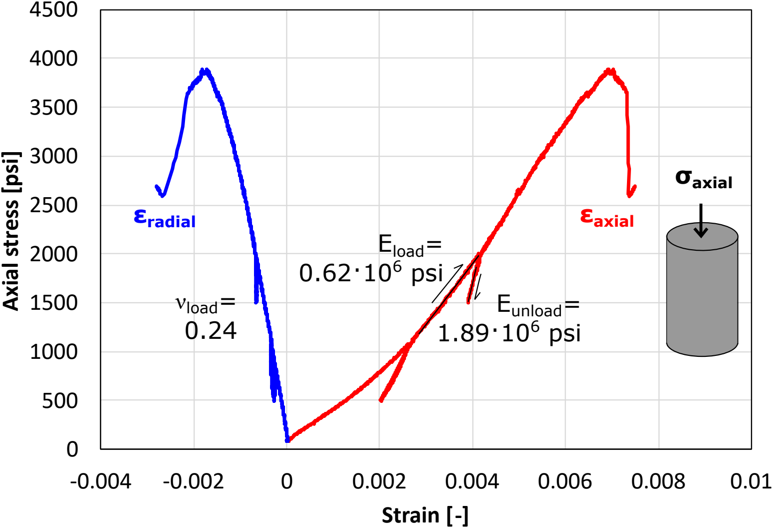
$$E_h > E_v$$

$$\varepsilon \gtrsim 0.001$$

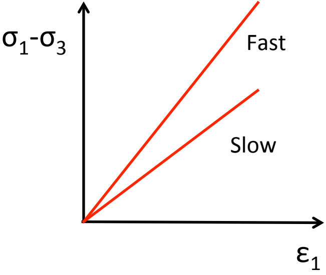
$$E_{load}$$

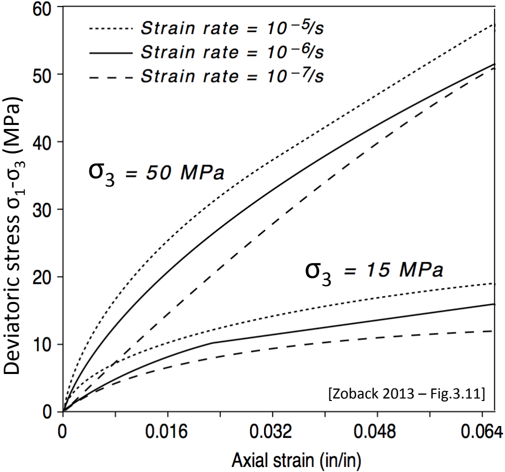
E_{unload}

$$E_{unload} \sim E_{unload}$$

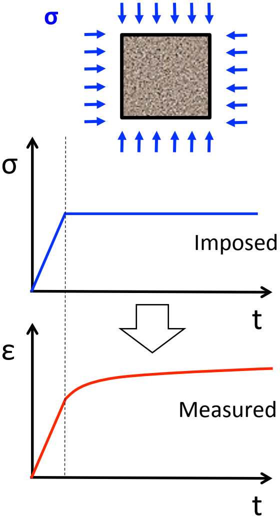

4500
4000
3500
3000
2500
2000
1500
1000
500
0
Axial stress [psi]
-0.004
-0.002
0
0.002
0.004
0.006
0.008
0.01
Strain [-]
ε_{radial}
ε_{axial}
ν_{load}= 0.24
E_{load}= $0.62 \cdot 10^6$ psi
E_{unload}= $1.89 \cdot 10^6$ psi
σ_{axial}

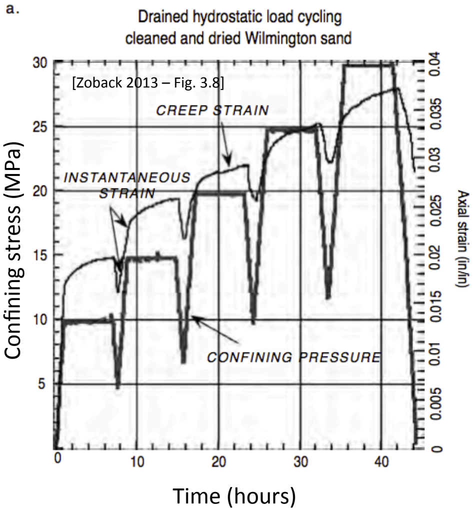
Strain rate hardening: The faster the loading, the stiffer the material



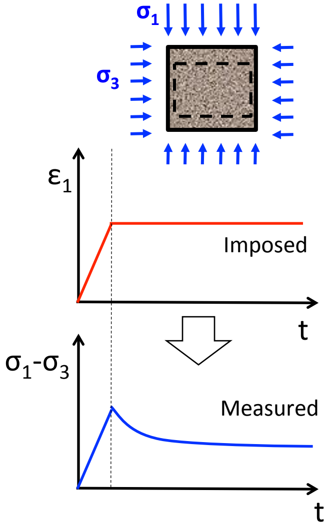


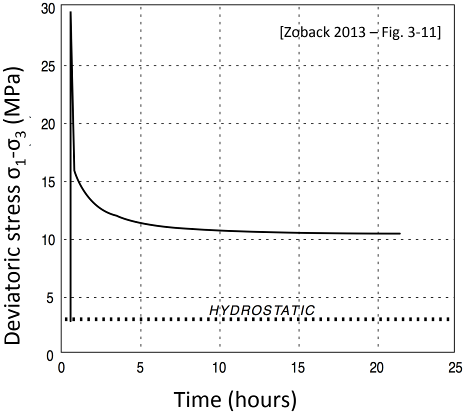
Creep strain at constant stress





(2) Stress relaxation at constant strain





$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\, \underline{\underline{\sigma}}$$

$$\underline{\underline{\sigma}} = \underline{\underline{S}} - \alpha P_p \underline{\underline{I}}$$

$\underline{\underline{I}}$

$$\alpha = 1 - \frac{K_{drained}}{K_{unj}}$$

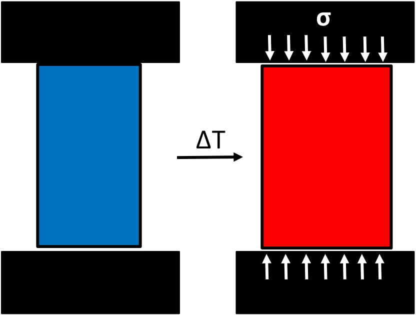
$$K_{drained}$$

K_{unj}

$$K_{unj} = K_m$$

$$\underline{\underline{\alpha}}$$

$\alpha \neq 1$


σ
ΔT

α_L

p

$$\alpha_L = \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}T}\bigg|_p$$

$$\Delta T$$

$$
\left\{
\begin{array}{rcl}
\sigma_{11} & = & (\lambda + 2\mu)\,\varepsilon_{11} + \lambda\,\varepsilon_{22} + \lambda\,\varepsilon_{33} + 3K\alpha_L \Delta T \\
\sigma_{22} & = & \lambda\,\varepsilon_{11} + (\lambda + 2\mu)\,\varepsilon_{22} + \lambda\,\varepsilon_{33} + 3K\alpha_L \Delta T \\
\sigma_{33} & = & \lambda\,\varepsilon_{11} + \lambda\,\varepsilon_{22} + (\lambda + 2\mu)\,\varepsilon_{33} + 3K\alpha_L \Delta T \\
\sigma_{12} & = & 2\mu\,\varepsilon_{12} \\
\sigma_{13} & = & 2\mu\,\varepsilon_{13} \\
\sigma_{23} & = & 2\mu\,\varepsilon_{23}
\end{array}
\right.
$$

$$\sigma_{11} = \sigma_{22} = 0$$

$$\varepsilon_{11} = \varepsilon_{22} \neq 0$$

$$\varepsilon_{33} = 0$$

$$\Delta T \neq 0$$

$$\left\{ \begin{array}{rcl} 0 & = & (\lambda + 2\mu)\,\varepsilon_{11} + \lambda\,\varepsilon_{11} + 3K\alpha_L\Delta T \\ \sigma_{33} & = & \lambda\,\varepsilon_{11} + \lambda\,\varepsilon_{11} + 3K\alpha_L\Delta T \end{array} \right.$$

$$\sigma_{33} = \left(\frac{6 - \mu/K}{3 + \mu/K} \right) K \alpha_L \Delta T \quad \blacksquare$$

$$D = 1.00$$

$$L = 2.01$$

ΔL

$$\Delta D$$

$$\underline{\sigma} = \underline{\underline{C}}\,\underline{\varepsilon}$$

σ_{iso}

$$\varepsilon_{vol} = \frac{3(1-2\nu)}{E}\sigma_{iso}$$

ε_{vol}

$$E = 4$$

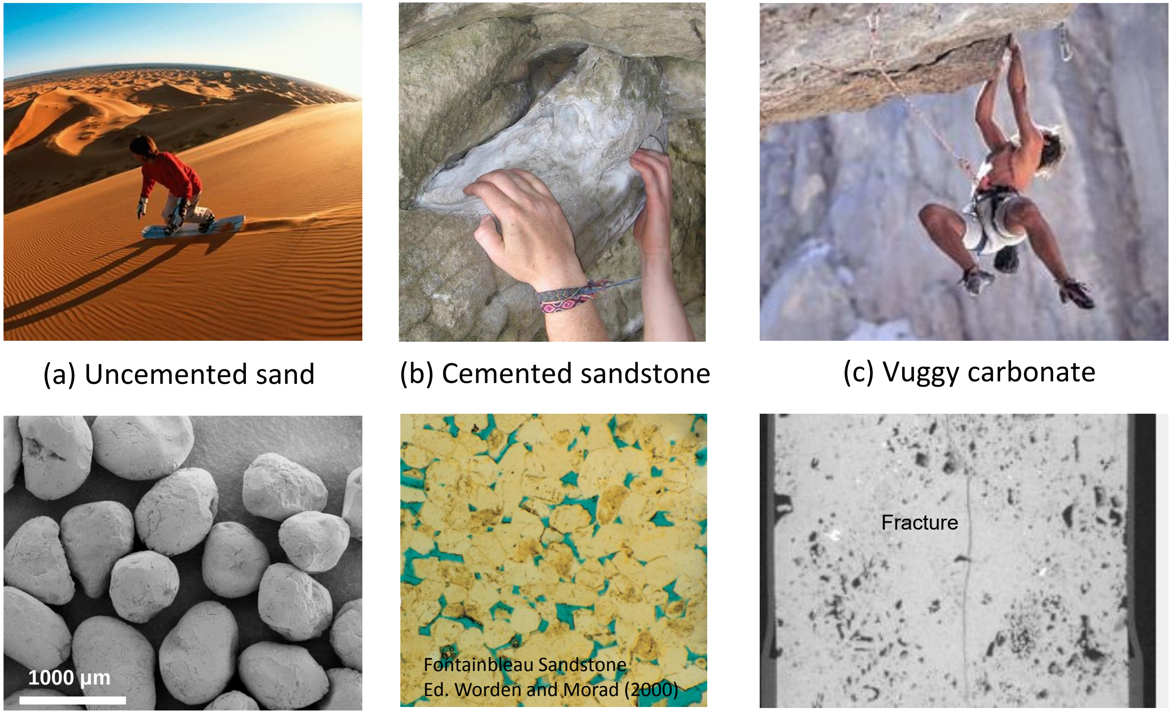
$\nu = 0.2$

$\sigma_{iso} =$

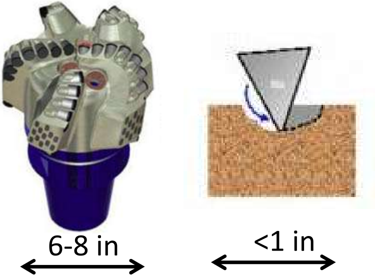
$$\sigma_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\varepsilon_{11}$$

3×3

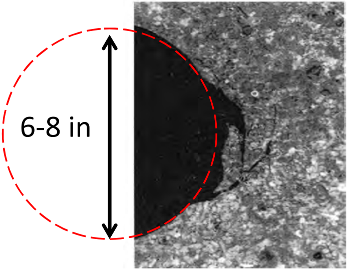
$$\phi = 0.35$$


(a) Uncemented sand
(b) Cemented sandstone
(c) Vuggy carbonate
1000 μm
Fontainbleau Sandstone
Ed. Worden and Morad (2000)
Fracture

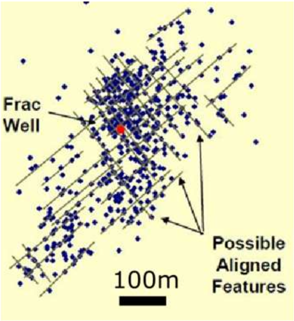
Rock cutting at the drill bit



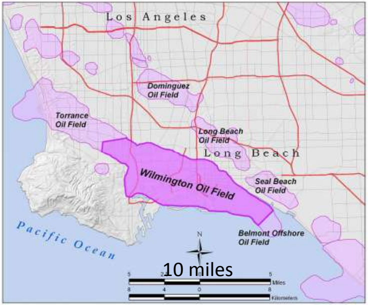
Wellbore breakout



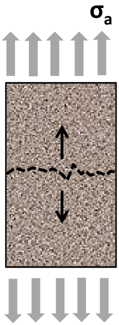
Shale hydraulic fracture



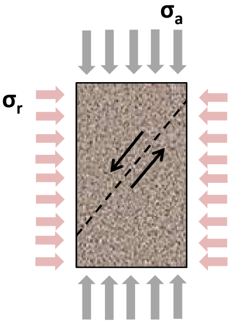
Reservoir depletion



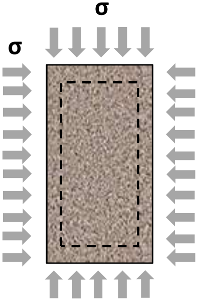
Images: Schlumberger/Terratek, Zoback 2013, Warpinski 2008, doe.gov



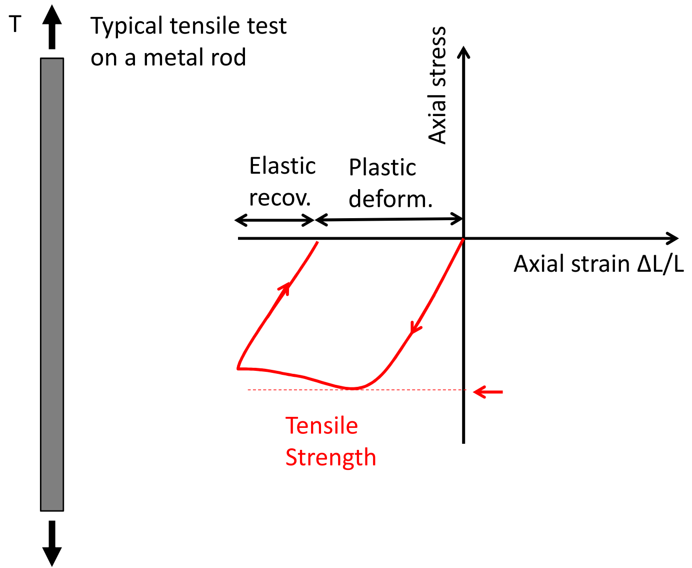
Tension
(bond breakage)
Ex: drilling-induced tens. fracs



Shear
(friction failure)
Ex: fault, breakout



Compression
(pore collapse)
Ex: reservoir compaction


T
Typical tensile test
on a metal rod
Axial stress
Elastic
recov.
Plastic
deform.
Axial strain ΔL/L
Tensile
Strength

Tensile test on a rock rod

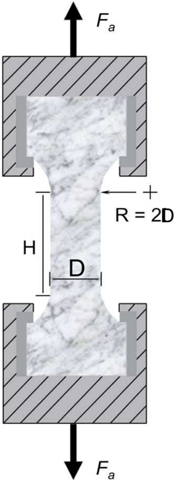
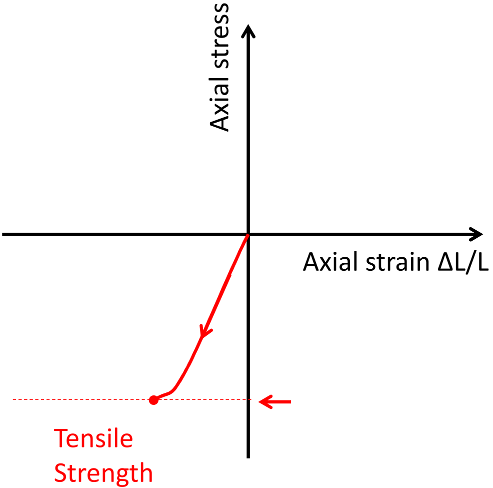


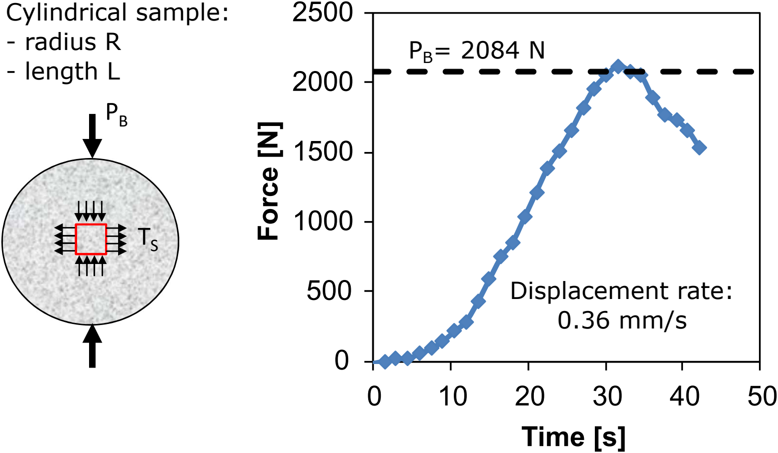
Figure direct tension: Perras and Diederichs (2014)



$$T_S = \frac{P_B}{\pi L R}$$

P_B

R



Cylindrical sample:
- radius R
- length L
P_B
T_S
P_B= 2084 N
Displacement rate:
0.36 mm/s
Force [N]
Time [s]
2500
2000
1500
1000
500
0
0
10
20
30
40
50

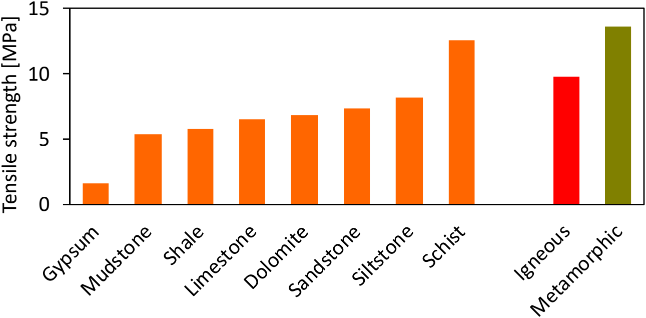
$$R = \frac{1}{2}$$

$$= 0.0127$$

$$L = 1$$

$$= 0.0254$$

$$T_S = \frac{2084 \text{ N}}{\pi(0.0254 \text{ m})(0.0127 \text{ m})} = 2.06 \times 10^6 \text{ Pa} = 2.06 \text{ MPa} \quad \blacksquare$$



Tensile strength [MPa]
15
10
5
0
Gypsum
Mudstone
Shale
Limestone
Dolomite
Sandstone
Siltstone
Schist
Igneous
Metamorphic

S_0

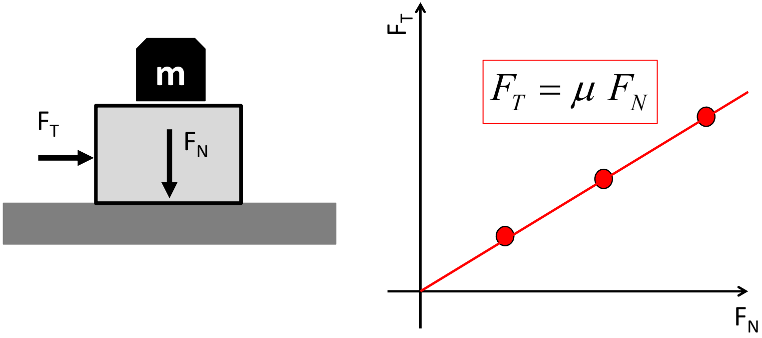
F_T

$$F_N$$

$$F_T = \mu F_N$$

$$F_N = 0$$

$$F_T = 0$$

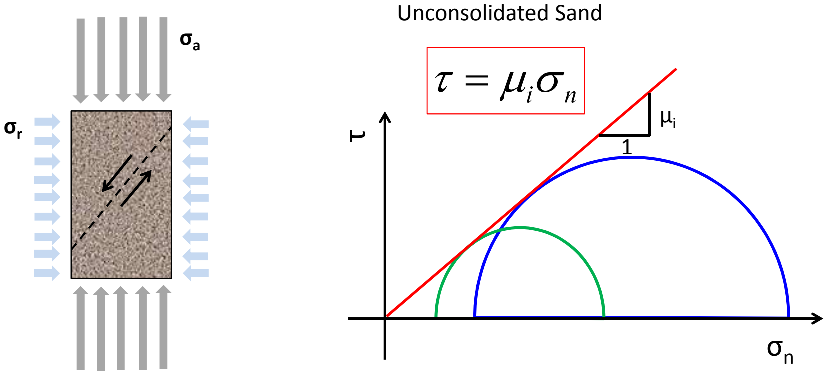

m
F_T
F_N
$F_T = \mu F_N$
F_T
F_N

τ

σ_n

μ_i

$$\tau = \mu_i \sigma_n$$



Unconsolidated Sand
σa
σr
τ = μiσn
μi
1
τ
σn

$$\sigma_n = 0$$

$$\tau = 0$$

φ

$$\tan(\varphi) = \mu_i$$

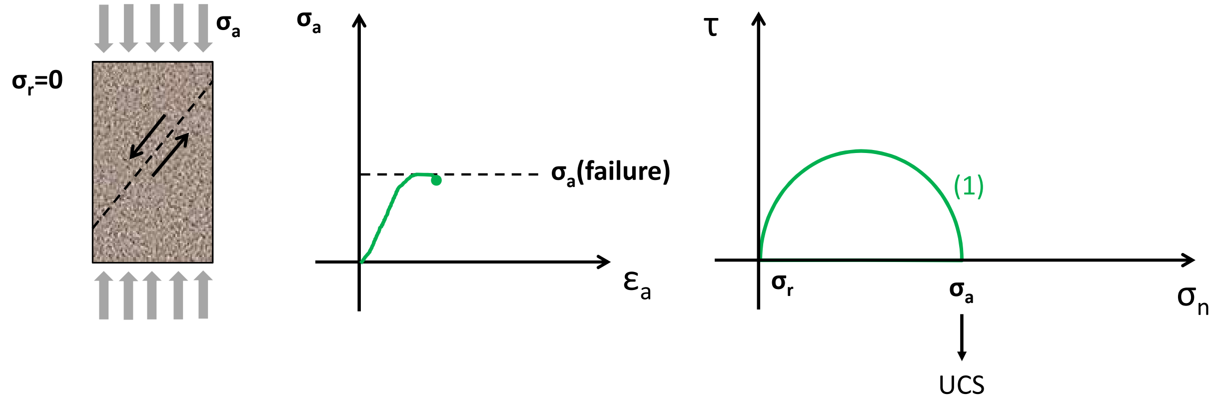
$\mu_i = 0.5$

$\varphi \sim 30^\circ$

$$\sigma_r = 0$$

UCS

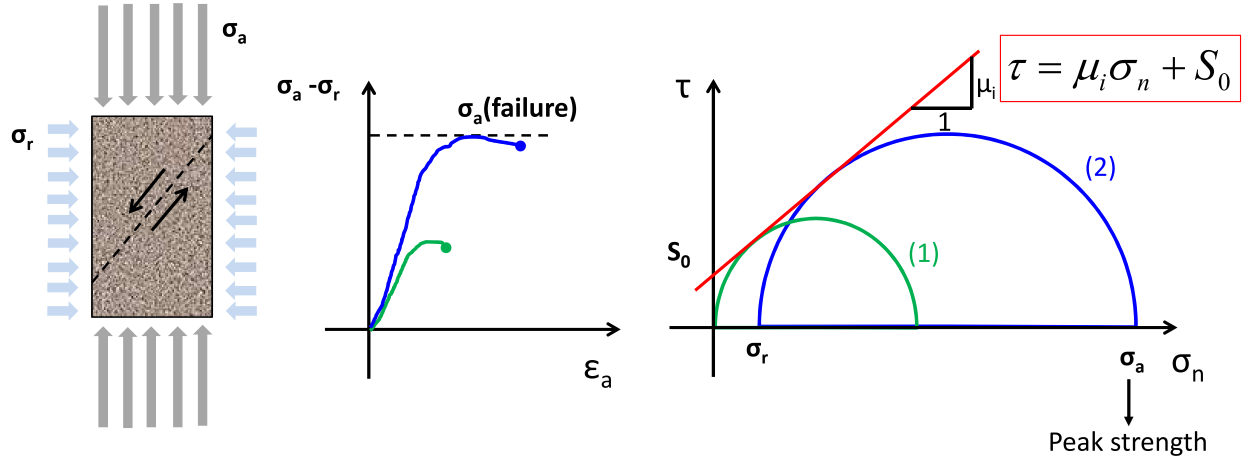
(1) Unconfined Loading



$\sigma_r \neq 0$

$$\tau = S_0 + \mu_i \sigma_n$$

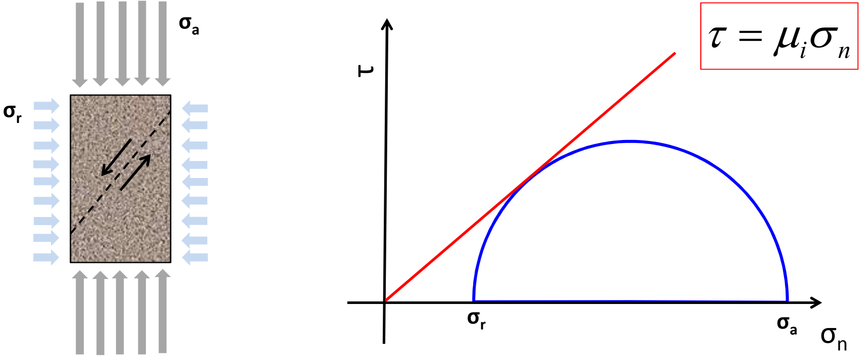
(2) Confined Loading



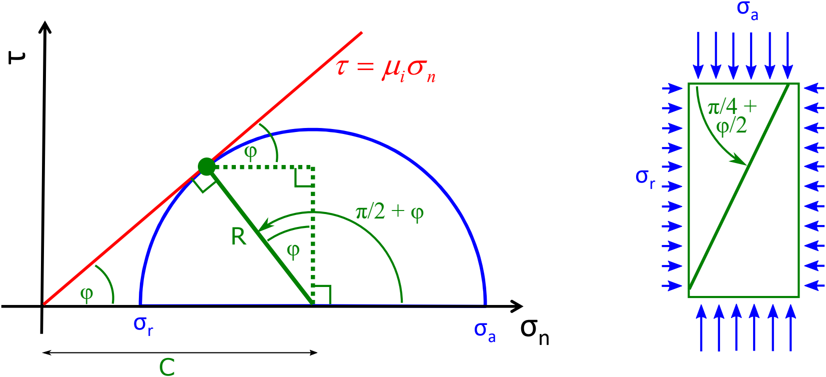
$$\pi/4 + \varphi/2$$

$$\pi/4 + \varphi/2 = 60^\circ$$

σ_a/σ_r


σ_a
σ_r
τ
$\tau = \mu_i \sigma_n$
σ_r
σ_a
σ_n

$$\tau/\sigma_n = \mu_i$$



τ
$\tau = \mu_i \sigma_n$
φ
φ
φ
π/2 + φ
R
σr
σa
σn
C
σa
σr
π/4 + φ/2

$$(\sigma_a, 0)$$

$$(\sigma_r, 0)$$

$$(\sigma_n, \tau)$$

$$\pi/2+\varphi$$

$$(\pi/2+\varphi)/2$$

$^{\circ} = \pi/2$

C

$$\frac{\sigma_a}{\sigma_r} = \frac{C+R}{C-R} = \frac{C + C\sin\varphi}{C - C\sin\varphi} = \frac{1+\sin\varphi}{1-\sin\varphi} =$$ [redacted]

$$\sigma_1 = UCS + q\,\sigma_3$$

q

$$p' - q$$

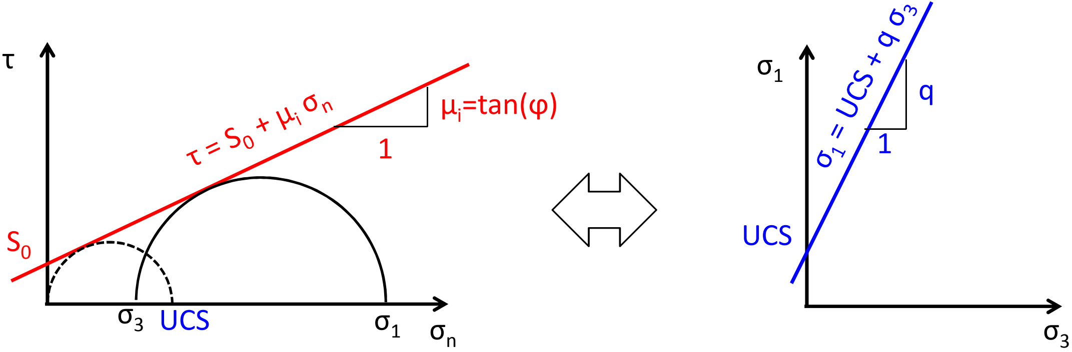
$$\sigma_m - q$$

$$q = \frac{1 + \sin\varphi}{1 - \sin\varphi}$$

$\varphi\ 30^\circ$

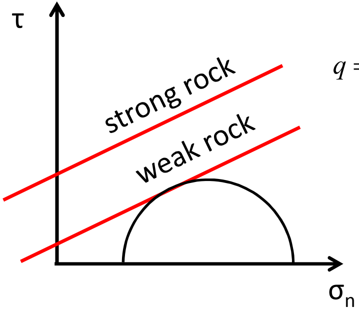
$$q = 3$$

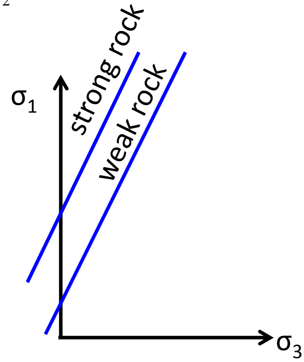
$$UCS = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} = 2S_0 \sqrt{q}$$

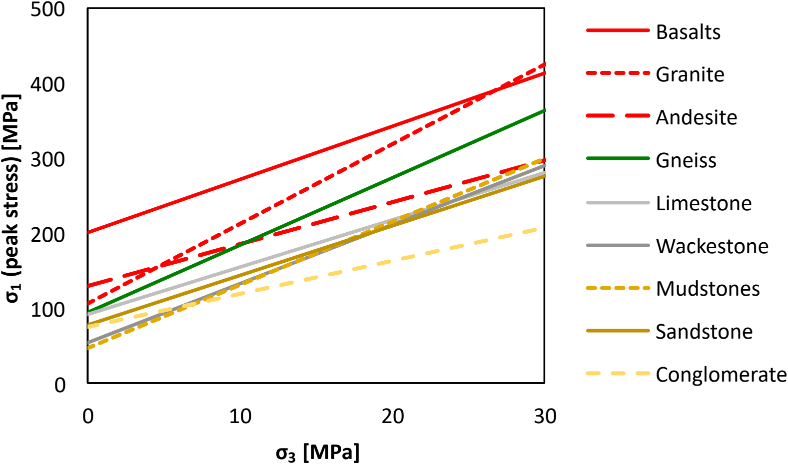


$$UCS = 2S_0\left(\sqrt{\mu_i^2+1}+\mu_i\right) = 2S_0\left(\frac{1+\sin\varphi}{1-\sin\varphi}\right)^{1/2}$$

$$q = \left(\sqrt{\mu_i^2+1}+\mu_i\right)^2 = \frac{1+\sin\varphi}{1-\sin\varphi}$$






500
400
300
200
100
0
σ1 (peak stress) [MPa]
0
10
20
30
σ3 [MPa]
Basalts
Granite
Andesite
Gneiss
Limestone
Wackestone
Mudstones
Sandstone
Conglomerate

$$(S_1, S_2, S_3)$$

$$(\sigma_1, \sigma_2, \sigma_3)$$

P_c

σ_D

$$P_c = cst$$

$$P_p < P_c$$

$$P_p = cst$$

$$\sigma_D = S_1 - S_3$$

$$S_3 = P_c$$

$$S_2 = S_3$$

$$S_1 - S_3 = \sigma_1 - \sigma_3$$

$$d\varepsilon_a/dt = cst$$

P

$$\sigma_1 = (S_1 - P_p)$$

$$\sigma_3 = (S_3 - P_p)$$

$$\varphi = \arctan\left(\frac{q-1}{2\sqrt{q}}\right)$$

$$S_3 \leqslant 80$$

$$S_1 \sim 80$$

$$UCS = 80$$

$$q = \frac{\Delta\sigma_1}{\Delta\sigma_3} = \frac{520\ \text{MPa}}{110\ \text{MPa}} = 4.73$$

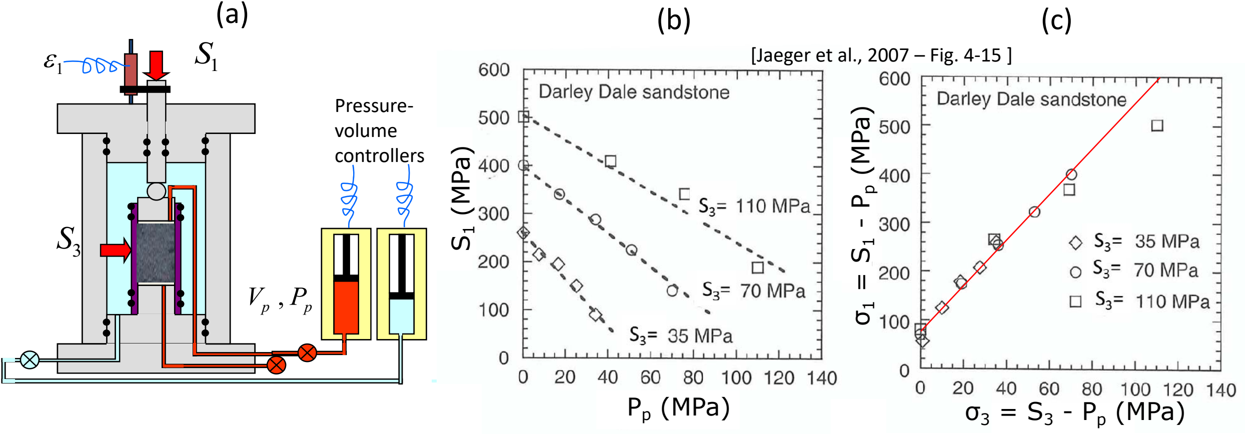
$$S_0 = \frac{UCS}{2\sqrt{q}} = \frac{80\text{MPa}}{2\sqrt{4.73}} = 18.4\text{MPa}$$

$$\varphi = \arctan\left(\frac{q-1}{2\sqrt{q}}\right) = \arctan\left(\frac{4.73-1}{2\sqrt{4.73}}\right) = 45.1^\circ \quad \blacksquare$$

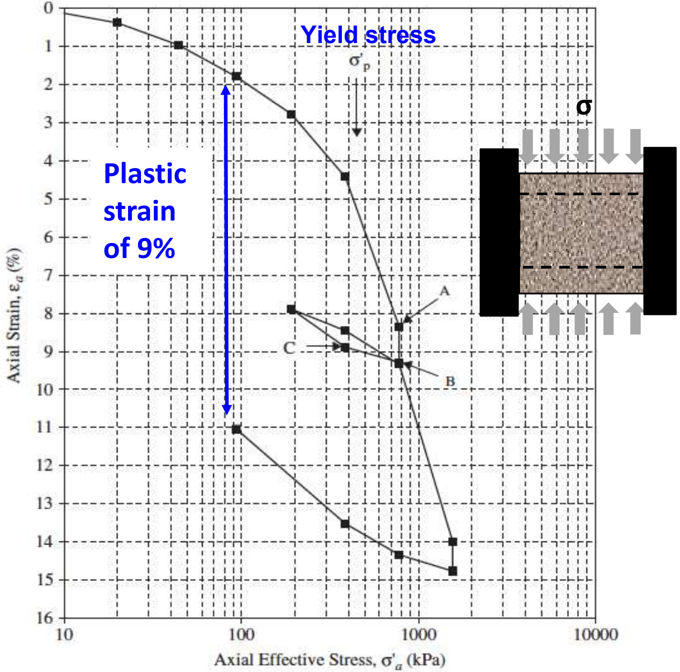
$$\mu_i = \tan(\varphi) = \tan(45.1^\circ) = 0.86 \quad \blacksquare$$

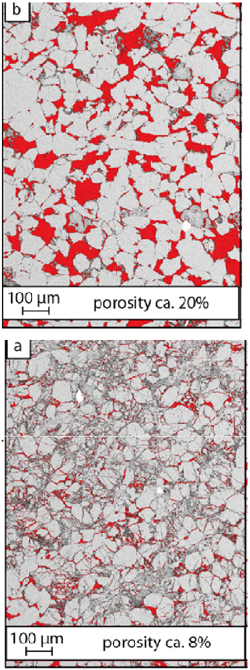
$$\varepsilon_v = \varepsilon_a + 2\varepsilon_r$$

$$d\varepsilon_v/dt < 0$$


(a)
(b)
(c)
[Jaeger et al., 2007 – Fig. 4-15]
ε1
S1
S3
Pressure-volume controllers
Vp , Pp
Darley Dale sandstone
S1 (MPa)
Pp (MPa)
S3= 110 MPa
S3= 70 MPa
S3= 35 MPa
σ1 = S1 - Pp (MPa)
σ3 = S3 - Pp (MPa)

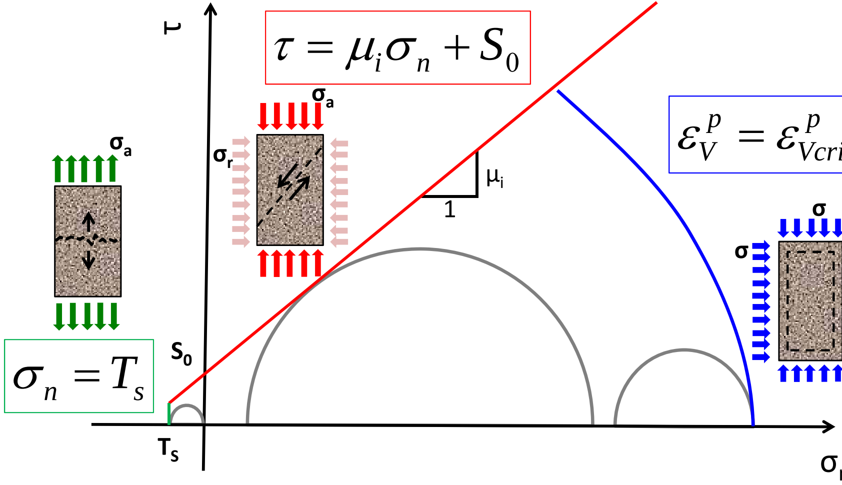
Germaine and Germaine, 2009





www.memin.de/results-tp-2.html

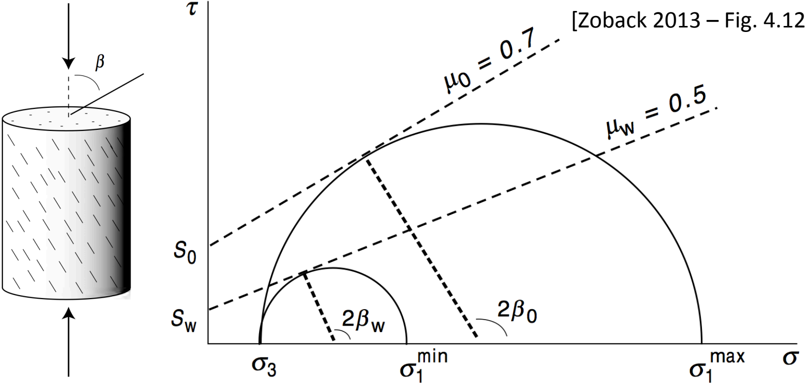
$$\tau = S_0 + \mu_i \sigma_n$$



$\tau = \mu_i \sigma_n + S_0$
$\varepsilon_V^p = \varepsilon_{Vcrit}^p$
$\sigma_n = T_s$
τ
σ_a
σ_r
σ_a
μ_i
1
σ
σ
S_0
T_S
σ_n

$$\beta_w$$

$$S_w + \mu_w \sigma_n$$



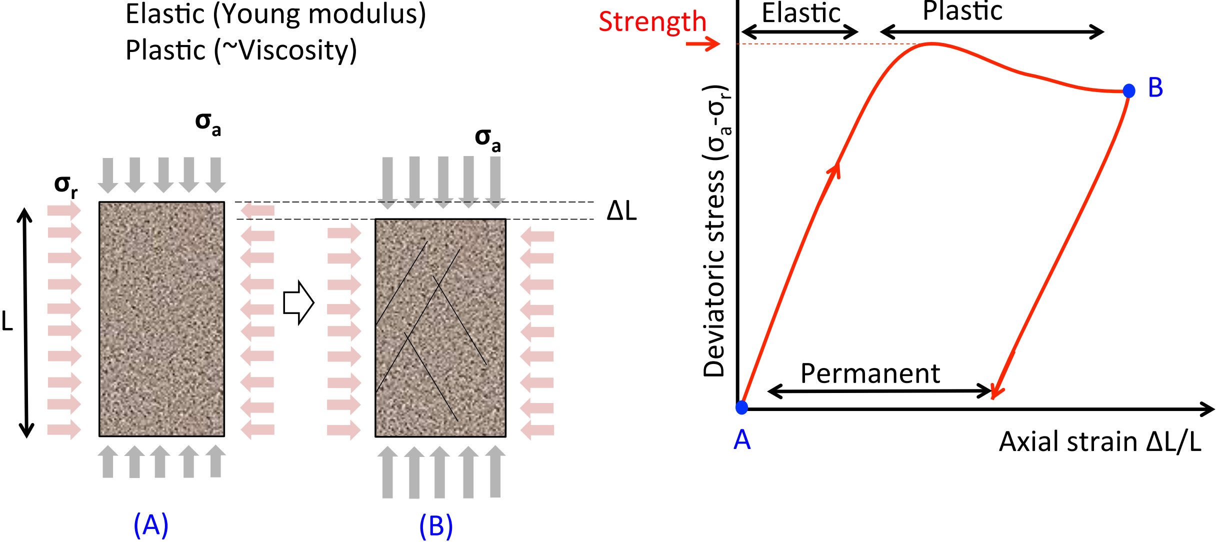
[Zoback 2013 – Fig. 4.12]

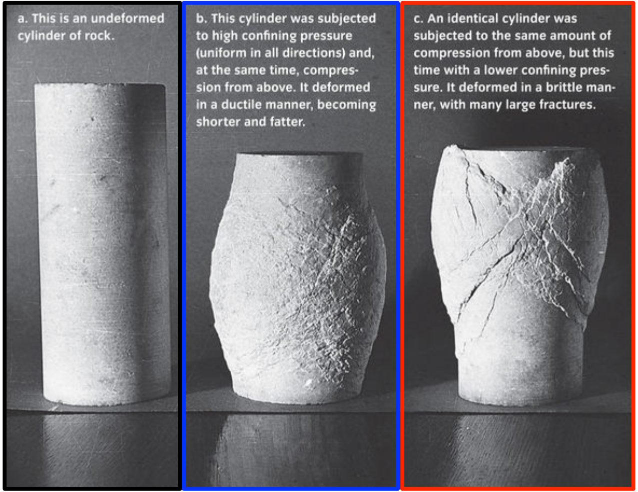
Rock deformation

Relation strain V.S. stress

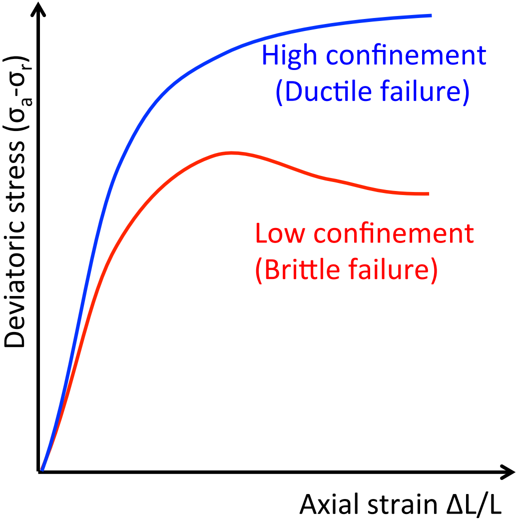
Elastic (Young modulus)

Plastic (~Viscosity)





Visualizing Geology, WileyPlus



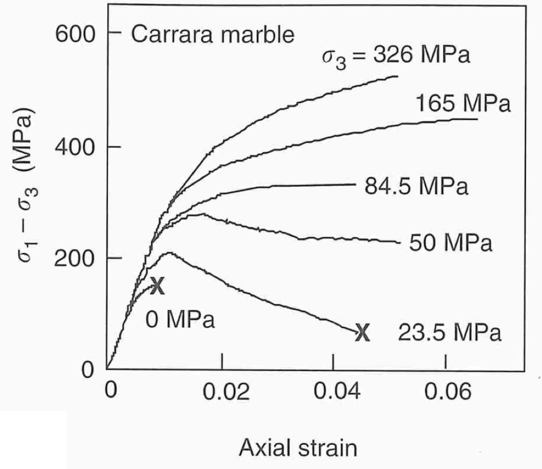
$$\sigma_d = (\sigma_1 - \sigma_3)$$

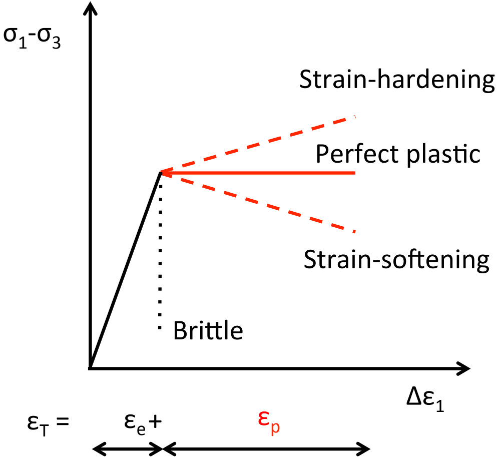
$\underline{\mathcal{E}}p$

$$\underline{\underline{C}}_p$$

$$\Delta\underline{\underline{\sigma}} = \underline{\underline{C}}_p \Delta\underline{\underline{\varepsilon}}_p$$

[Jaeger et al. 2007 – Fig. 4.5]

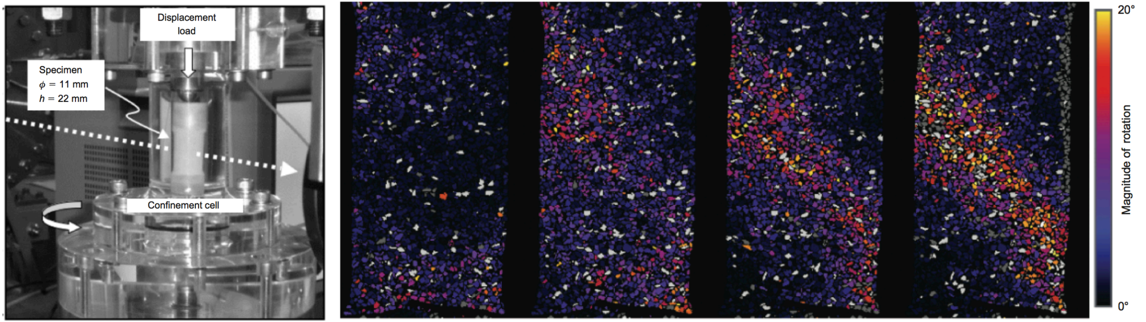




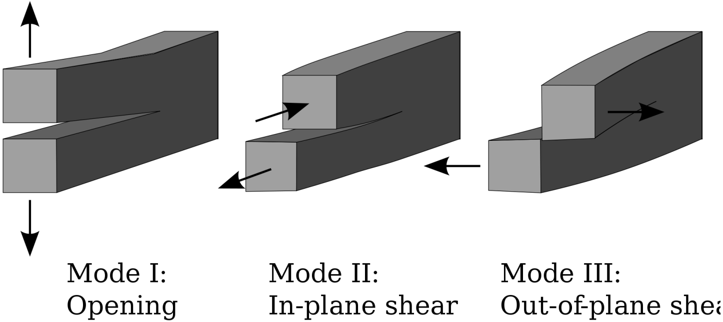
$$\Delta \underline{\underline{\sigma}} = \underline{\underline{\underline{C}}} \; \Delta \underline{\underline{\varepsilon}}$$

$$\Delta \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}_p \; \Delta \underline{\underline{\varepsilon}}_p$$

(a) Uncemented or poorly cemented rock → grain friction, dilation, grain crushing/rotation



(b) Cemented rock → Propagation of microfractures, grain friction/crushing

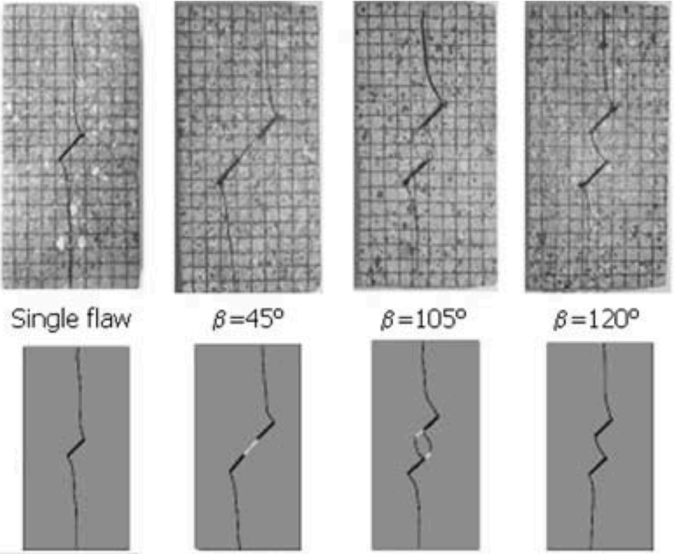


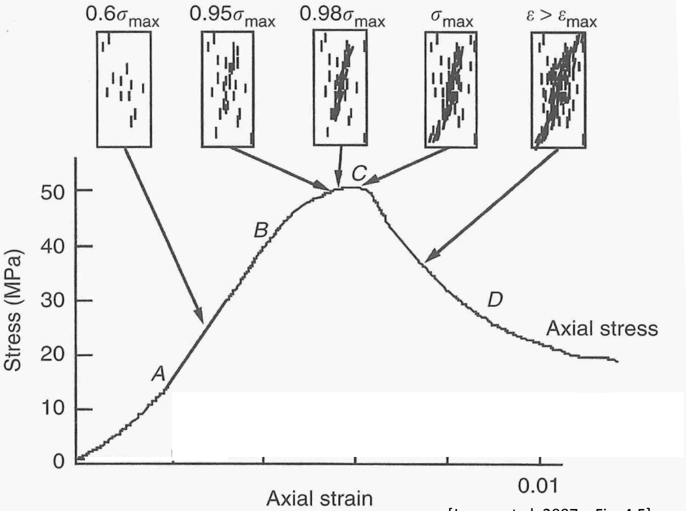
Stress intensification at the tip of fractures

Propagation starts at fracture tips

Napolitan Tuffo

[Hall et al. 2006 – Pure Appl. Geophys.]





[Jaeger et al. 2007 – Fig. 4.5]

$$S_1 - S_3$$

$$\varphi = f(q)$$

Sig_D

$$E_x$$

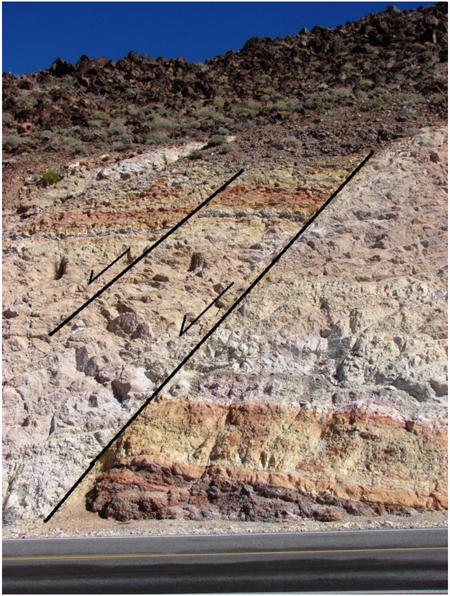
E_y

$$q = 5.3$$

$$S_1 = S_3 \to S_1 - S_3 = 0$$

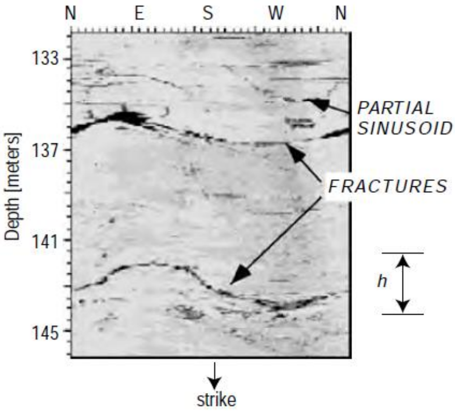
S_o

Outcrop of a normal fault in Split Mountain gorge
http://geology.csupomona.edu/janourse/TectonicsFieldTrips.htm

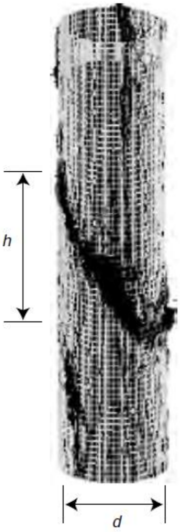


http://geotripperimages.com/images/DSC03438%20Charlie%20Brown%20b.jpg

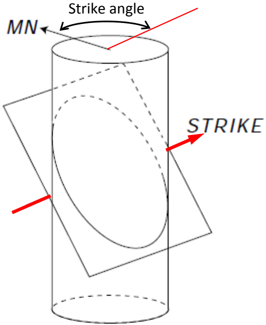
(a) Un-wrapped image (ultrasonic)



(b) 3D-representation



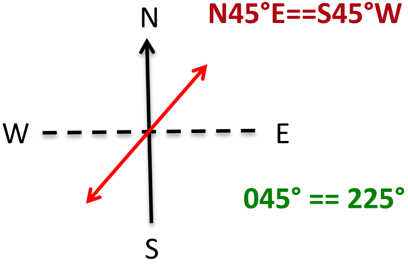
(c) Interpretation



Dip = $\tan^{-1}(h/d)$

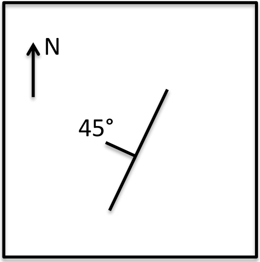
[Zoback 2013 - Figure 5.3]

dip

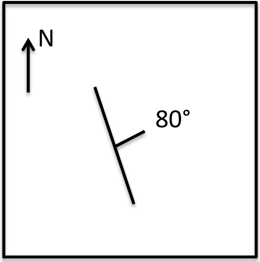

N45°E==S45°W
N
W
E
S
045° == 225°

Geologic map

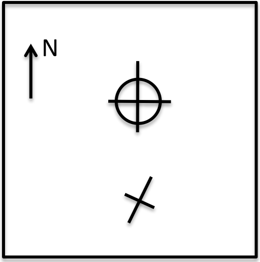
Figure from Prof. Prodanovic



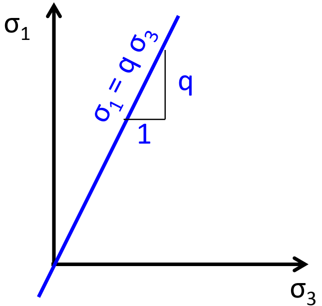
a)
N30°E, 45°NW
or
030°, 45°NW



b)
N10°W, 80°NE
or
350°, 80°NE



c)
Symbols for
horizontal plane
and vertical plane



σ_1
$\sigma_1 = q\,\sigma_3$
q
1
σ_3

$$\sigma_1 = q\, \sigma_3$$

$$q = \left(\sqrt{\mu^2 + 1} + \mu\right)^2 = \frac{1 + \sin\ \varphi}{1 - \sin\ \varphi}$$

$\sigma_1/\sigma_3 <$

$\pi/4 + \phi/2$

$\sim 30^{\circ}$

$\sim 60^\circ$

45°

$67.5°$

50°

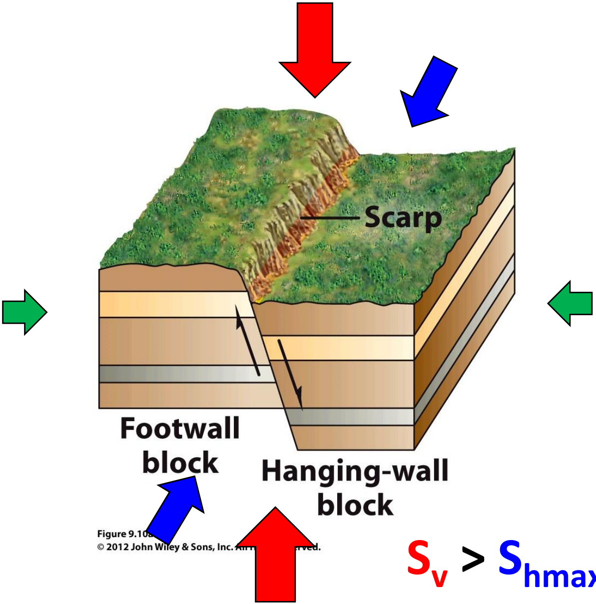
70°

$\sigma_1/\sigma_3 > q$

$\sigma_1/\sigma_3 < q$

$$\begin{cases} S_1 = S_v \\ S_2 = S_{Hmax} \\ S_3 = S_{hmin} \end{cases}$$

$$S_1 > S_2 > S_3$$



$$S_v > S_{hmax} > S_{hmin}$$

$$S_1 \neq S_2 \neq S_3$$

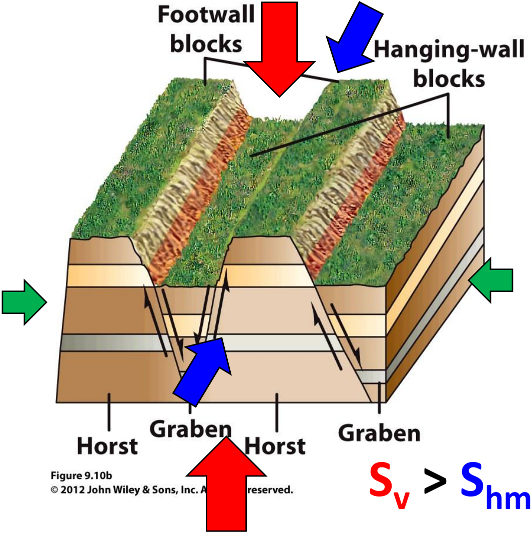


Figure 9.10b

$$S_v > S_{hmax} > S_{hmin}$$

$$\begin{cases} S_1 = S_{Hmax} \\ S_2 = S_{hmin} \\ S_3 = S_v \end{cases}$$

$(\pi/4 + \varphi/2)$

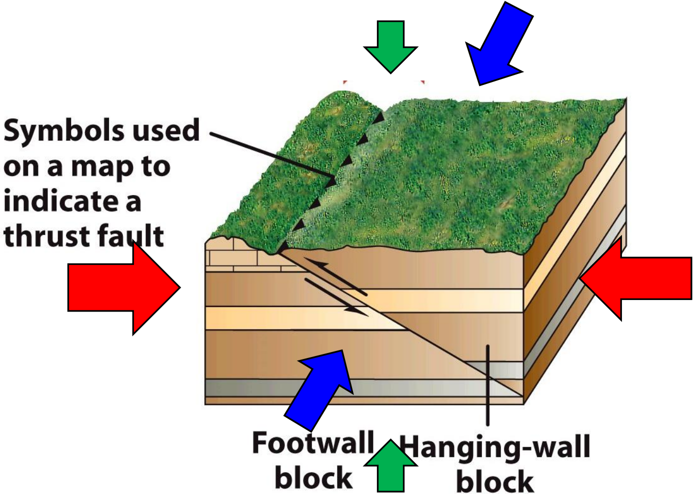
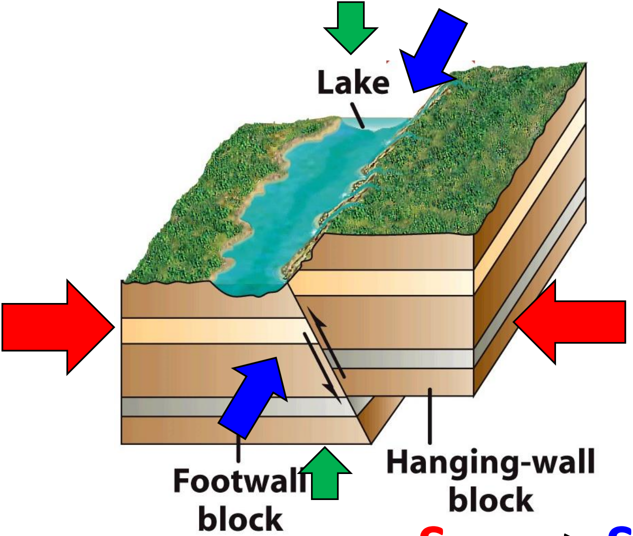


Figure 9.10d

$S_{Hmax} > S_{hmin} > S_v$



$$S_{Hmax} > S_{hmin} > S_v$$

Figure 9.10c

$$\begin{cases} S_1 = S_{Hmax} \\ S_2 = S_v \\ S_3 = S_{hmin} \end{cases}$$

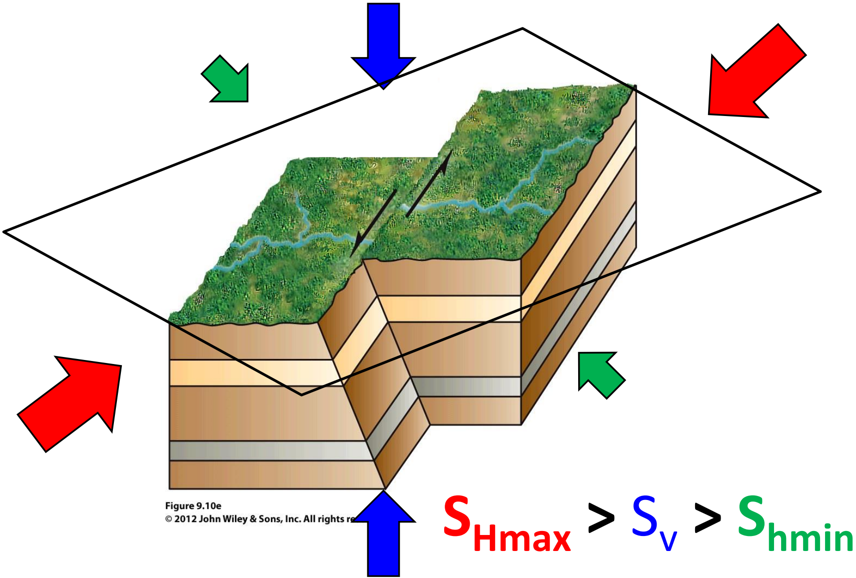
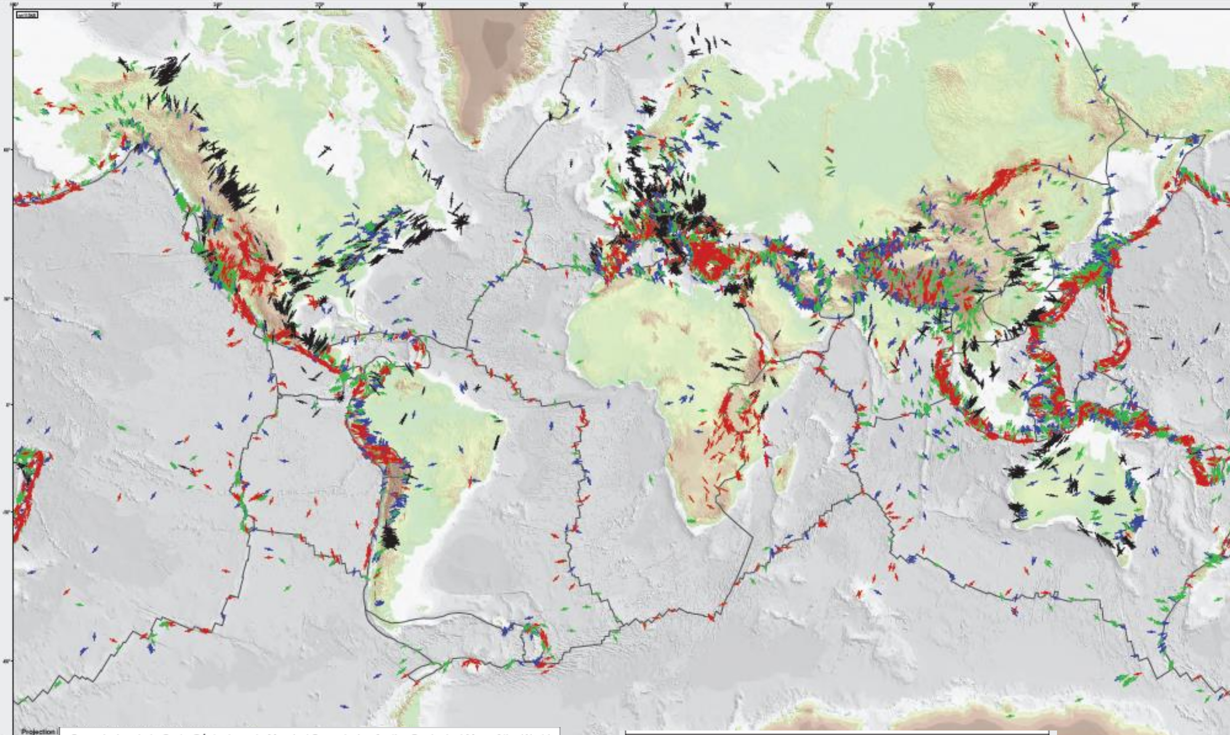


Figure 9.10e
© 2012 John Wiley & Sons, Inc. All rights re
$S_{Hmax} > S_V > S_{hmin}$

S_v



Commission de la Carte Géologique du Monde / Commission for the Geological Map of the World

WORLD STRESS MAP

2009 - 2nd edition, based on the WSM database release 2008
Helmholtz Centre Potsdam - GFZ German Research Centre for Geosciences

Authors
Oliver Heidbach, Mark Tingay, Andreas Barth, John Reinecker, Daniel Kurfeß, and Birgit Müller

The stress maps display the maximum horizontal compressional stress S_H

Method

- focal mechanism
- breakouts
- drill. induced frac.
- overcoring
- hydro. fractures
- geol. indicators

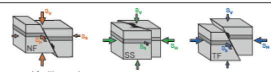
Data depth range
0-40 km

Quality

A	——	S_H is within ±15°
B	——	S_H is within ±20°
C	—	S_H is within ±25°

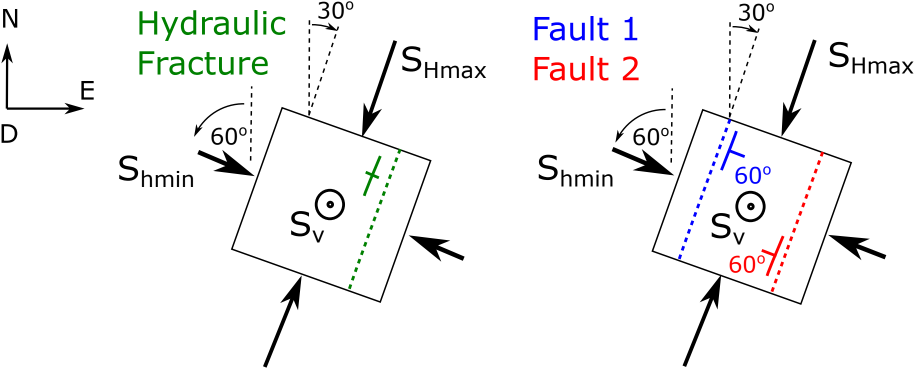
Stress Regime

- Normal faulting
- Strike-slip faulting
- Thrust faulting
- Unknown regime



normal faulting regime $S_V > S_H > S_h$ | strike-slip regime $S_H > S_V > S_h$ | thrust faulting regime $S_H > S_h > S_V$

$^\circ + \varphi/2$



N
E
D
Hydraulic
Fracture
30°
60°
S_{Hmax}
S_{hmin}
S_v
Fault 1
Fault 2
30°
60°
S_{Hmax}
S_{hmin}
60°
60°
S_v

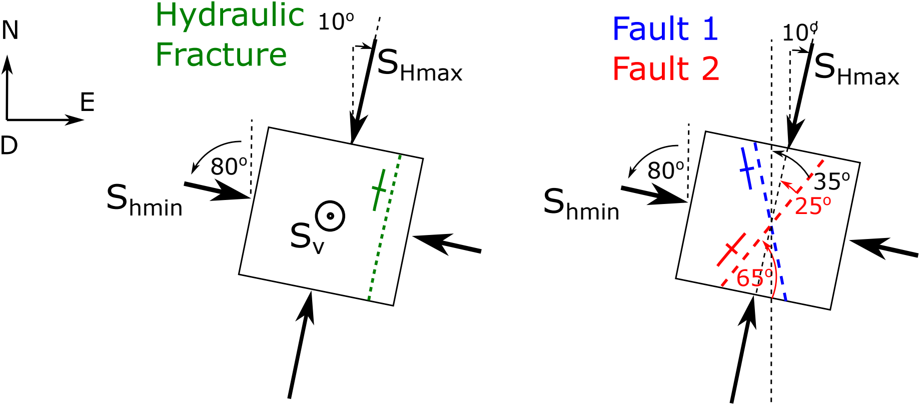
$$\phi_{HF} = 030^\circ$$

$$\delta_{HF} = 90^\circ$$

$$\beta = 45^\circ + \varphi/2 = 45^\circ + 30^\circ/2 = 60^\circ$$

$$\phi_{F1} = \phi_{F2} = 030^\circ$$

$$\delta_{F1} = 60^\circ$$



N
E
D
Hydraulic Fracture
10°
S_{Hmax}
80°
S_{hmin}
S_v
Fault 1
Fault 2
10°
S_{Hmax}
80°
S_{hmin}
35°
25°
65°

$$\phi_{HF} = 010^\circ$$

$$\beta = 45^\circ + \varphi/2 = 45^\circ + 40^\circ/2 = 65^\circ$$

$$\phi_{F1} = 345^\circ$$

$$\phi_{F2} = 035^{\circ}$$

$$\delta_{F1} = \delta_{F2} = 90^\circ$$

σ_{hmin}

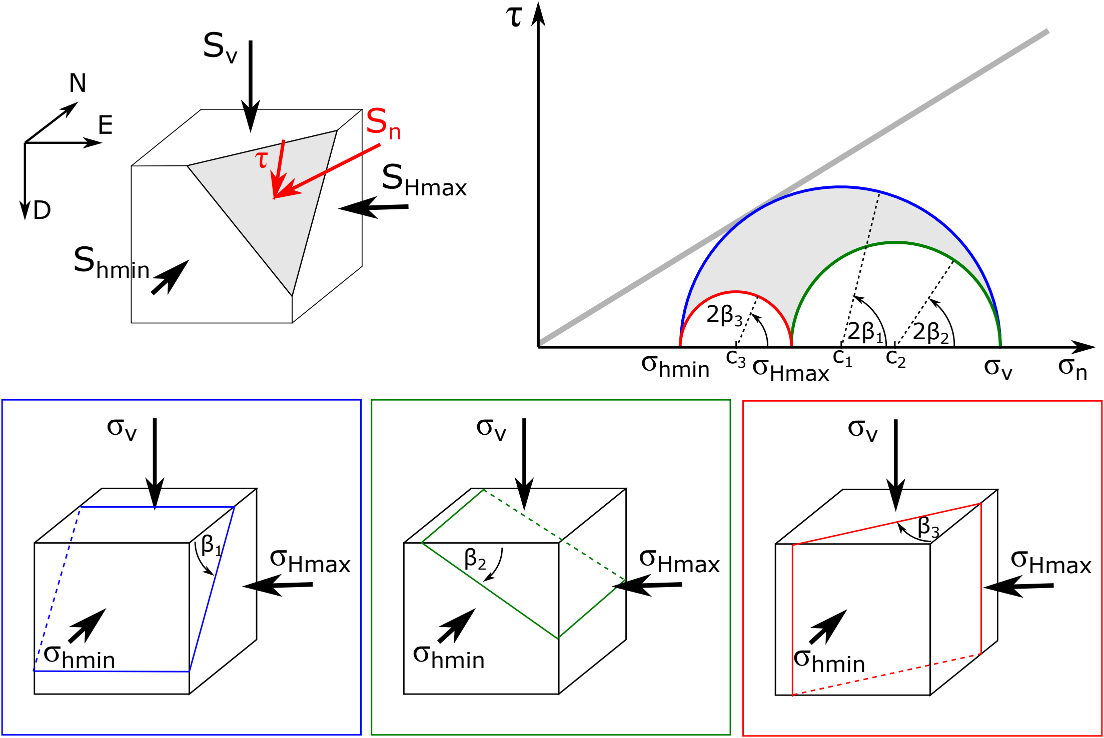
σ_{Hmax}

$$\beta_1$$

$$\sigma_v$$

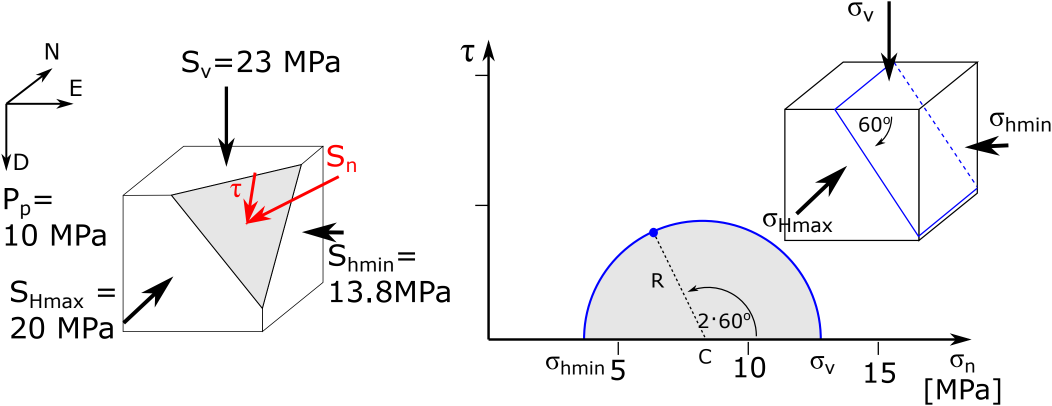
β_2

β_3



S_v
N
E
D
S_n
τ
S_Hmax
S_hmin
τ
2β3
2β1
2β2
σhmin
c3
σHmax
c1
c2
σv
σn
σv
σHmax
σhmin
β1
σv
β2
σHmax
σhmin
σv
β3
σHmax
σhmin

$$\varphi = 30^\circ$$



N
E
D
S_v=23 MPa
S_n
τ
P_p= 10 MPa
S_{hmin}= 13.8MPa
S_{Hmax} = 20 MPa
τ
σ_v
60º
σ_{hmin}
σ_{Hmax}
R
2·60º
σ_{hmin} 5
C
10
σ_v
15
σ_n [MPa]

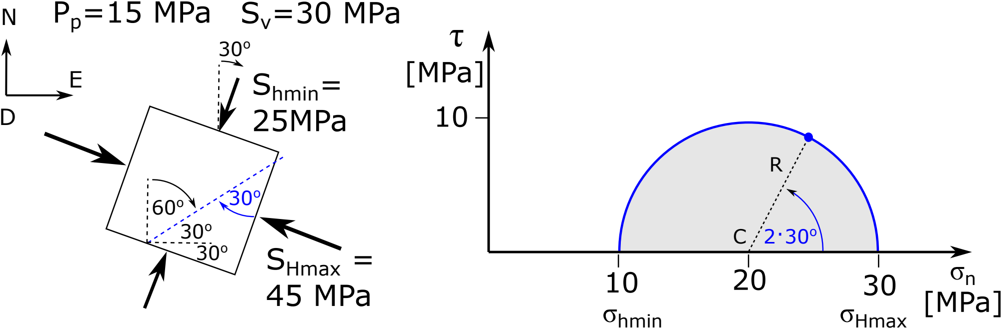
$\sigma_v =$

$$\sigma_{Hmax} =$$

$\sigma_{hmin} =$

$$\sigma_n = \left(\frac{13 \text{ MPa} + 3.8 \text{ MPa}}{2} \right) + \left(\frac{13 \text{ MPa} - 3.8 \text{ MPa}}{2} \right) \cos(2 \cdot 60^\circ) = 6.1 \text{ MPa}$$

$$\tau = \left(\frac{13 \text{ MPa} - 3.8 \text{ MPa}}{2} \right) \sin(2 \cdot 60^\circ) = 4.0 \text{ MPa} \ \blacksquare$$



N
E
D
P_p=15 MPa
S_v=30 MPa
30°
S_hmin=
25MPa
60°
30°
30°
30°
S_Hmax =
45 MPa
τ
[MPa]
10
R
C
2·30°
10
σ_hmin
20
30
σ_Hmax
σ_n
[MPa]

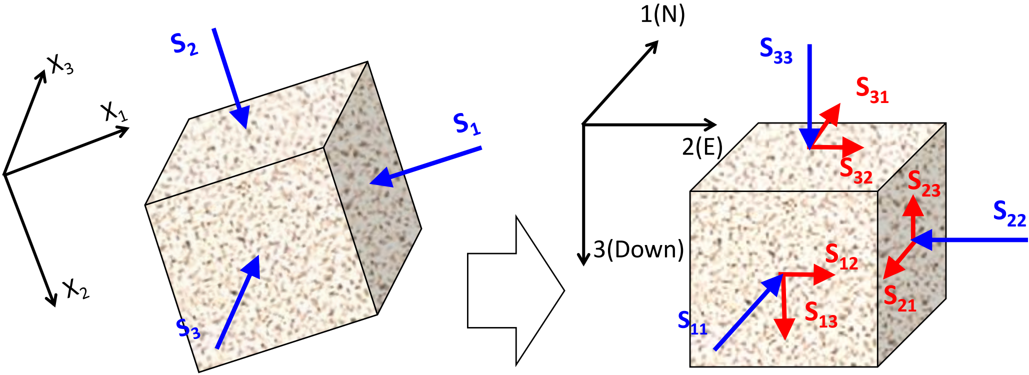
$$\sigma_n = \left(\frac{30\ \text{MPa} + 10\ \text{MPa}}{2}\right) + \left(\frac{30\ \text{MPa} - 10\ \text{MPa}}{2}\right)\cos(2 \cdot 30^\circ) = 25\ \text{MPa}$$

$$\tau = \left(\frac{30 \text{ MPa} - 10 \text{ MPa}}{2} \right) \sin(2 \cdot 30^\circ) = 8.7 \text{ MPa} \quad \blacksquare$$

$$(strike, dip)$$

$$\underline{\underline{S}}_P$$

$$\underline{\underline{S}}_G$$

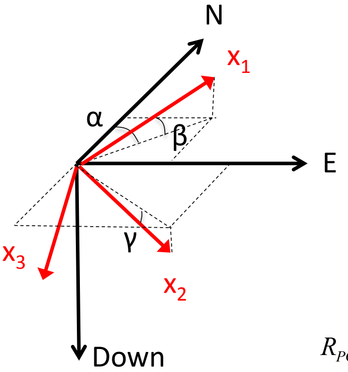


$$\underline{\underline{S_P}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix} \qquad \underline{\underline{S_G}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

R_{PG}

90°

$0°$



$$\underline{\underline{S}}' = A\underline{\underline{S}}A^T \qquad A = \begin{bmatrix} \underline{e}'_1 \cdot \underline{e}_1 & \underline{e}'_1 \cdot \underline{e}_2 & \underline{e}'_1 \cdot \underline{e}_3 \\ \underline{e}'_2 \cdot \underline{e}_1 & \underline{e}'_2 \cdot \underline{e}_2 & \underline{e}'_2 \cdot \underline{e}_3 \\ \underline{e}'_3 \cdot \underline{e}_1 & \underline{e}'_3 \cdot \underline{e}_2 & \underline{e}'_3 \cdot \underline{e}_3 \end{bmatrix}$$

where $\underline{\underline{A}}$ is the transformation matrix from the old base $\underline{e}_i$ to base $\underline{e}'_i$ and the components are the projection of the elements of the new base on the old base.

- Old system: N-E-D (Right-handed) Geographical system
- New system: 1-2-3 (Right-handed) Principal stress system

$$R_{PG} = \begin{bmatrix} \cos a \cos b & \sin a \cos b & -\sin b \\ \cos a \sin b \sin g - \sin a \cos g & \sin a \sin b \sin g + \cos a \cos g & \cos b \sin g \\ \cos a \sin b \cos g + \sin a \sin g & \sin a \sin b \cos g - \cos a \sin g & \cos b \cos g \end{bmatrix}$$

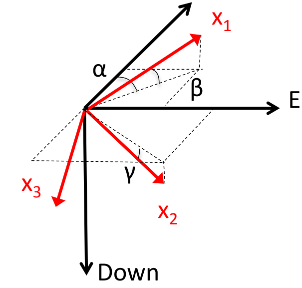
$$\underline{\underline{S}}_P = R_{PG} \underline{\underline{S}}_G R_{PG}^T$$

$$\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}$$

$$S_1 = 30$$

$$S_2 = 25$$

$$S_3 = 20$$



$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

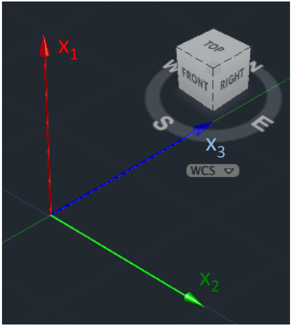
$a = 0^\circ$

$b = 90^\circ$

$g = 0^\circ$

Azimuth of S_{hmin}

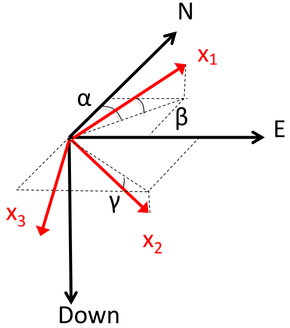
$S_1 = S_V$



$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$

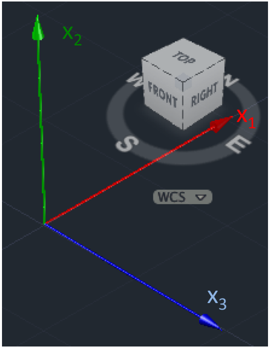


$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$a = 0^\circ$ Azimuth of S_{Hmax}

$b = 0^\circ$ $S_1 = S_{Hmax}$

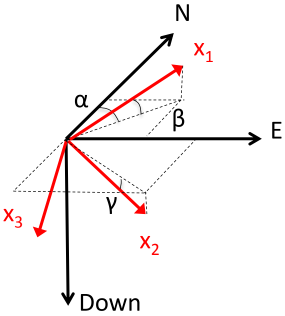
$g = 90^\circ$ $S_2 = S_V$



$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

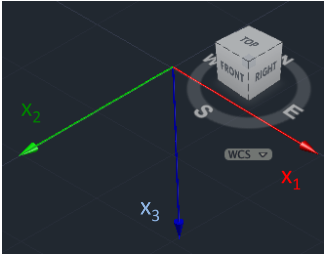


$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$a = 90^\circ$ Azimuth of S_{Hmax}

$b = 0^\circ$ S_1=S_{Hmax}

$g = 0^\circ$ S_2=S_{hmin}



$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

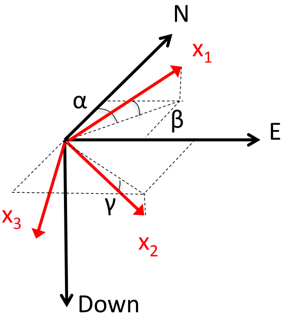
$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$S_1 = 60$$

$$S_2 = 40$$

$$S_3 = 35$$



$$\underline{\underline{S}}_P = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

$a = 135^\circ$ Azimuth of S_{Hmax}

$b = 0^\circ$ $S_1 = S_{Hmax}$

$g = 90^\circ$ $S_2 = S_V$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$

$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

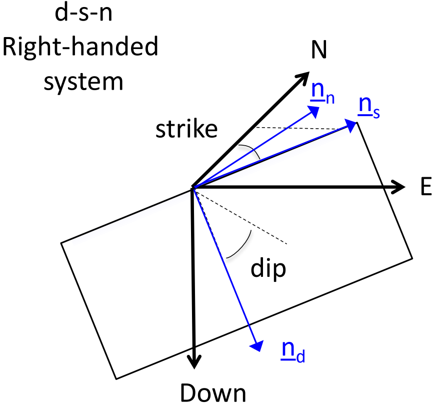
$$\underline{\underline{S}}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

n_d

n_s

$$n_n$$

$strike$



$$\underline{n}_n = \begin{bmatrix} -\sin(strike)\sin(dip) \\ \cos(strike)\sin(dip) \\ -\cos(dip) \end{bmatrix}$$

$$\underline{n}_s = \begin{bmatrix} \cos(strike) \\ \sin(strike) \\ 0 \end{bmatrix}$$

$$\underline{n}_d = \begin{bmatrix} -\sin(strike)\cos(dip) \\ \cos(strike)\cos(dip) \\ \sin(dip) \end{bmatrix}$$

$\underline{n}_d$

$\underline{n}_s$

$$\underline{n}_n$$

$$\underline{t} = \underline{\underline{S}}_G \underline{n}_n$$

S_n

$$S_n = \underline{t} \cdot \underline{n}_n$$

$$\sigma_n = S_n - P_p$$

$$\begin{cases} \tau_d = \underline{t} \cdot \underline{n}_d \\ \tau_s = \underline{t} \cdot \underline{n}_s \end{cases}$$

$$\tau = \sqrt{\tau_d^2 + \tau_s^2}$$

$$\sigma_n = \underline{t} \cdot \underline{n}_n - P_p$$

$$\tau^2 = ||\underline{t}||^2 - ||\underline{t} \cdot \underline{n}_n||^2$$

$rake$

$$\underline{\tau}_d + \underline{\tau}_s$$

$$rake = \arctan\left(\frac{\tau_d}{\tau_s}\right)$$

t

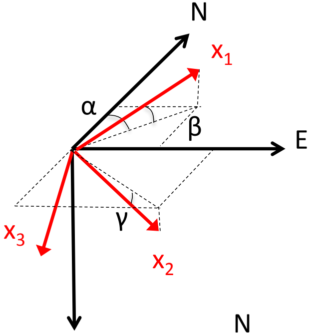
τ_d

τ_s

$$S_v = 23$$

$$S_{Hmax} = 15$$

$$S_{hmin} = 13.8$$



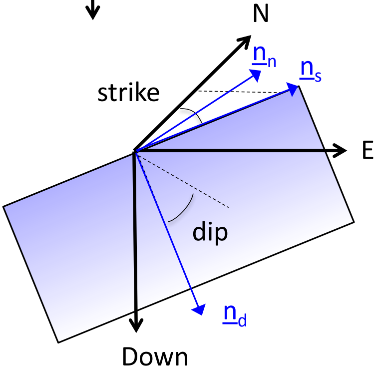
$$\underline{\underline{S}}_P = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 13.8 \end{bmatrix}$$

$a = 90^\circ$ Azimuth of S_{hmin}

$b = 90^\circ$ $S_1 = S_V$

$g = 0^\circ$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \underline{\underline{S}}_G = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 13.8 & 0 \\ 0 & 0 & 23 \end{bmatrix}$$



Fault geometry

Strike = 000°

Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0 \\ 0.867 \\ -0.5 \end{bmatrix}$$

$\underline{t} = [0, 11.95, -11.50]\text{MPa}$

$S_n = 16.1\text{MPa}$

$t_d = -3.98\text{MPa}$

$t_s = 0\text{MPa}$

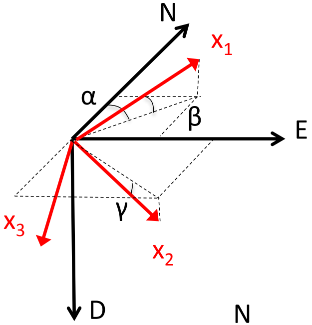
$\text{rake} = 90^\circ$

Actual stresses opposite to d-s-n

$$S_v = 30$$

$$S_{Hmax} = 45$$

$$S_{hmin} = 25$$



$$\underline{\underline{S}}_P = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

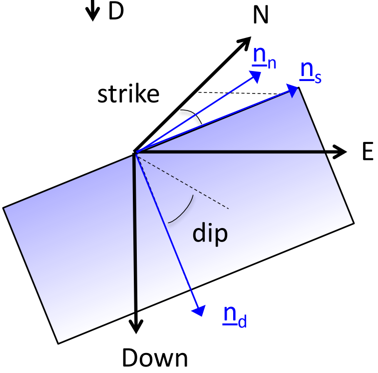
$a = 120^\circ$ Azimuth of S_{Hmax}

$b = 0^\circ$

$g = 90^\circ$

$S_1 = S_V$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \\ 0.866 & 0.5 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$



Fault geometry

Strike = 060°

Dip = 90°

$$\underline{n}_n = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$\underline{t} = [-30.31, 27.5, 0]\mathrm{MPa}$$

$$S_n = 40\mathrm{MPa}$$

$$t_d = 0\mathrm{MPa}$$

$$t_s = 8.66\mathrm{MPa}$$

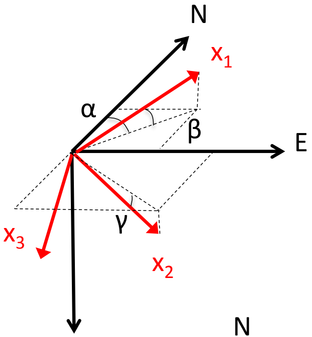
$$rake = 0^\circ$$

Actual stresses opposite to d-s-n

$$S_v = 5000$$

$$S_{Hmax} = 4000$$

$$S_{hmin} = 3000$$



$$\underline{\underline{S}}_P = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix}$$

$a = 90^\circ$

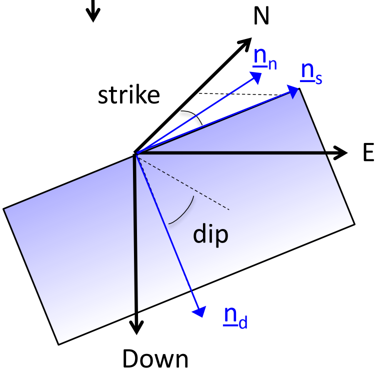
$b = 90^\circ$

$g = 0^\circ$

Azimuth of S_{hmin}

$S_1=S_V$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \underline{\underline{S}}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$



Fault geometry

Strike = 045°
Dip = 60°

$$\underline{n}_n = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix}$$

$\underline{t} = [-2450, 1840, -2500]\text{psi}$

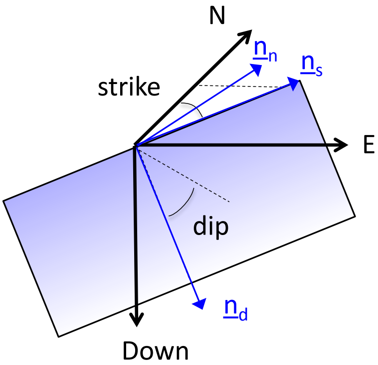
$S_n = 3870\text{psi}$

$t_d = -649\text{psi}$

$t_s = -433\text{psi}$

$\text{rake}=56.3^\circ$

Actual stresses opposite to d-s-n



Fault geometry

Strike = 225°
Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [2450, -1840, -2500]\text{psi}$$
$$S_n = 3870\text{psi}$$
$$t_d = -649\text{psi}$$
$$t_s = -433\text{psi}$$
$$\text{rake} = 56.3^\circ$$

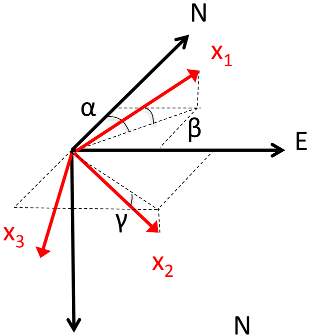
Actual stresses opposite to d-s-n

$$S_v = 1000$$

$$S_{Hmax} = 2400$$

$$S_{hmin} = 1200$$

$$P_p = 440$$



$$\underline{\underline{S}}_P = \begin{bmatrix} 2400 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

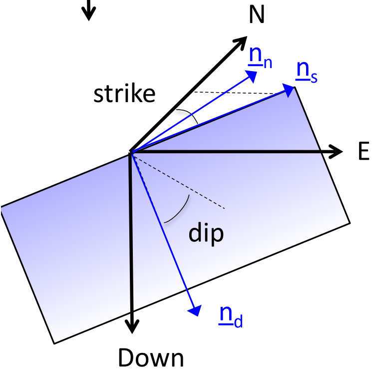
$a = 150^\circ$ Azimuth of S_{Hmax}

$b = 0^\circ$ $S_3 = S_V$

$g = 0^\circ$

P_w = 440 psi

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.866 & 0.5 & 0 \\ -0.5 & -0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \underline{\underline{S}}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$



Fault geometry

Strike = 120°

Dip = 70°

$$\underline{n}_n = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix}$$

$\underline{t} = [-1465, -281, -342]$psi

$S_n = 1441$psi

$t_d = 160$psi

$t_s = 488$psi

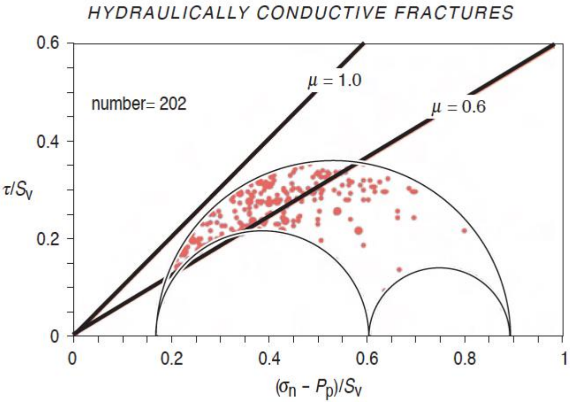
rake $= 18.21^\circ$

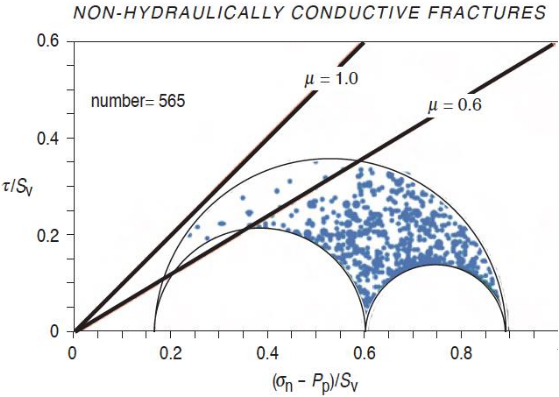
$t / s_n = 0.51$

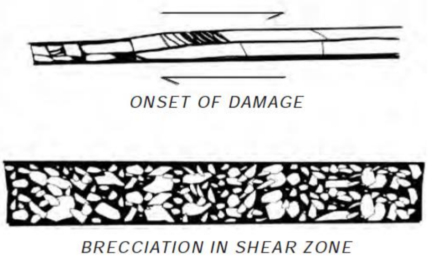
Actual stresses opposite to d-s-n

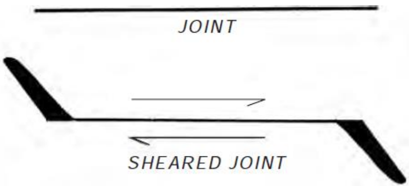
$\mu \sim$

Zoback 2013 - Figure 11.1-5.2

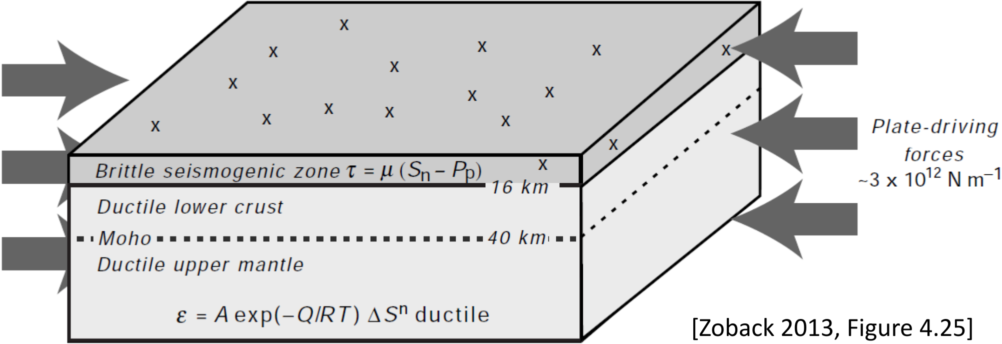








+ Mineral precipitation
+ Mineralization to clays



[Zoback 2013, Figure 4.25]

$$\sigma_1 = q\sigma_3$$

$$q = [1 + \sin(\varphi)]/[1 - \sin(\varphi)]$$

σ_v/q

$q\ \sigma_{hmin}$

$q\ \sigma_v$

$$\phi = 30^\circ$$

$$\lambda_p = 0.82$$

$$S_v = 1$$

$$= 2000$$

$$P_p = \lambda_p S_v = 0.82 \times 2000$$

$$= 1640$$

$$\sigma_v = S_v - P_p = 2000$$

-1640

$$= 360$$

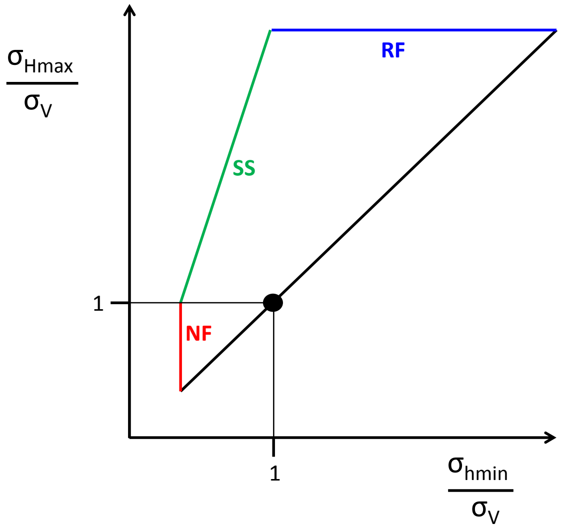
$$\sigma_{Hmax} = q\ \sigma_v = \frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} 360$$

$$= 1080$$

$$S_{Hmax} = \sigma_{Hmax} + P_p = 1080$$

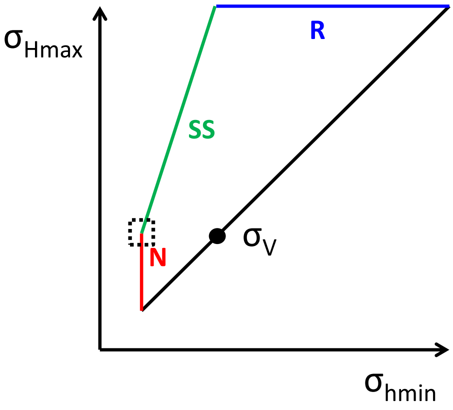
$+1640$

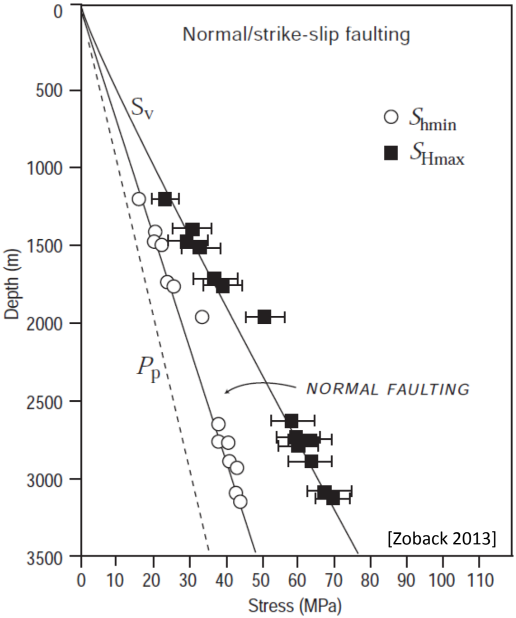
$$= 2720$$

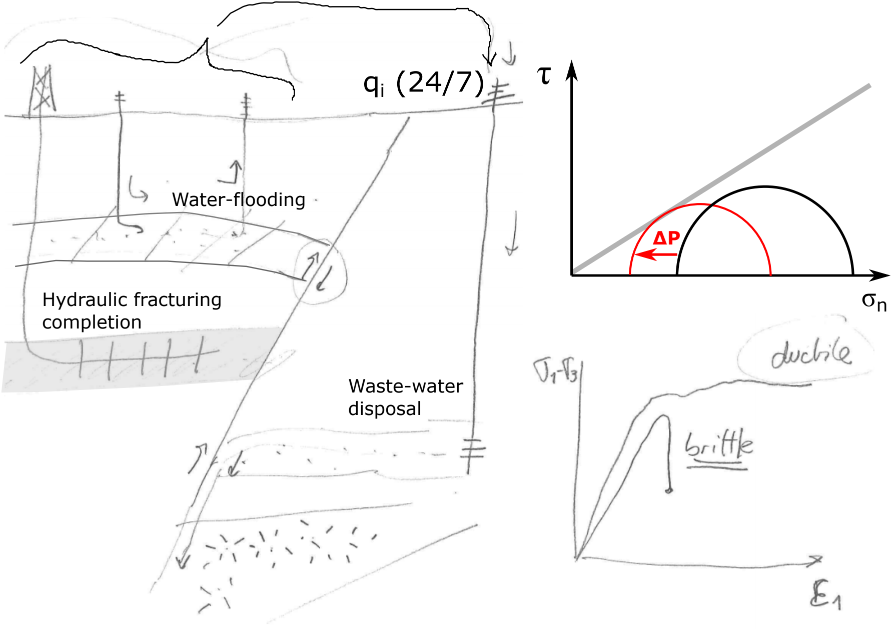


$\frac{\sigma_{Hmax}}{\sigma_V}$
RF
SS
1
NF
1
$\frac{\sigma_{hmin}}{\sigma_V}$

$$\sigma_{hmin} < \sigma_{Hmax} \approx \sigma_V$$







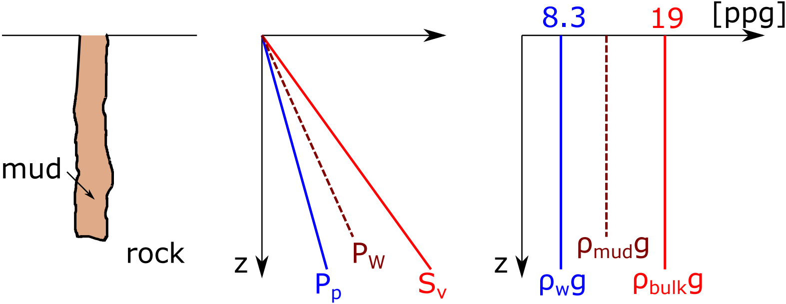
q_i (24/7)
Water-flooding
Hydraulic fracturing completion
Waste-water disposal
τ
ΔP
σ_n
$\sigma_1 - \sigma_3$
ductile
brittle
ε_1

$$\begin{cases} \Delta\sigma_v = -\Delta P_p \\ \Delta\sigma_{hmin} \leq -\Delta P_p \end{cases}$$

$\underline{\underline{S}}_G$

$$P_W = \rho_{mud}\, g\, z$$

$$dP_W/dz = \rho_{mud} g$$



mud
rock
z
P_p
P_W
S_v
8.3
19
[ppg]
z
$\rho_w g$
$\rho_{mud} g$
$\rho_{bulk} g$

$$P_W > P_p$$

$$P_W - P_p$$

$$P_W < P_p$$

r

θ

σ_{zz}

σ_{rz}

$\sigma_{\theta z}$

$$\sigma_{\theta\theta}/\sigma_{\infty} = 2$$

$$\sigma_{\theta\theta}/\sigma_{rr} = \infty$$

r^2

$$\sigma_{rr} = +P_W$$

$$\sigma_{rr} = -P_W$$

$\theta = 0$

$$\sigma_{\theta\theta} = 3\Delta\sigma$$

$$\theta = \pi/2$$

$3\pi/2$

$$\sigma_{\theta\theta} = -\Delta\sigma$$

π

$$\sigma_{\theta\theta}/\sigma_{\infty} = 3$$

$\sin$

$\cos$

$$(P_W - P_p)$$

$$
\begin{cases}
\sigma_{rr} & = & (P_W - P_p)\left(\frac{a^2}{r^2}\right) + \frac{\sigma_{Hmax}+\sigma_{hmin}}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_{Hmax}-\sigma_{hmin}}{2}\left(1 - 4\frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right)\cos(2\theta) \\
\sigma_{\theta\theta} & = & -(P_W - P_p)\left(\frac{a^2}{r^2}\right) + \frac{\sigma_{Hmax}+\sigma_{hmin}}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_{Hmax}-\sigma_{hmin}}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos(2\theta) \\
\sigma_{r\theta} & = & \frac{\sigma_{Hmax}-\sigma_{hmin}}{2}\left(1 + 2\frac{a^2}{r^2} - 3\frac{a^4}{r^4}\right)\sin(2\theta) \\
\sigma_{zz} & = & \sigma_v - 2\nu\left(\sigma_{Hmax} - \sigma_{hmin}\right)\left(\frac{a^2}{r^2}\right)\cos(2\theta)
\end{cases}
$$

a

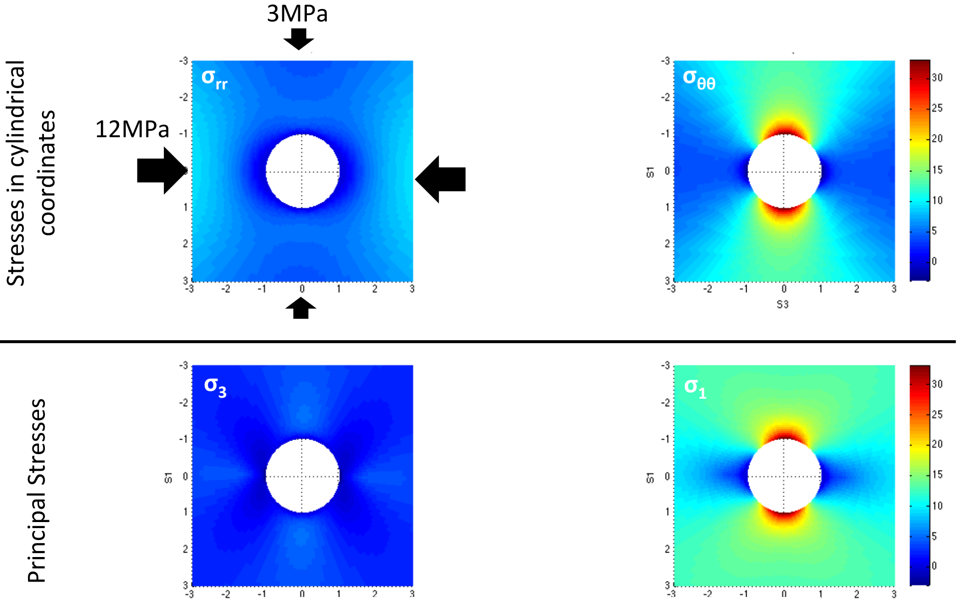
$$r = a$$

$$S_{Hmax} = 22$$

$$S_{hmin} = 13$$

$$P_W = P_p = 10$$

σ_1



3MPa
12MPa
Stresses in cylindrical coordinates
Principal Stresses
σrr
σθθ
σ3
σ1
S1
S3
-3 -2 -1 0 1 2 3
0 5 10 15 20 25 30

$$\sigma_{rr}(r = a) = P_W - P_p$$

$$\sigma_{\theta\theta}(r=a) = -(P_W - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(2\theta)$$

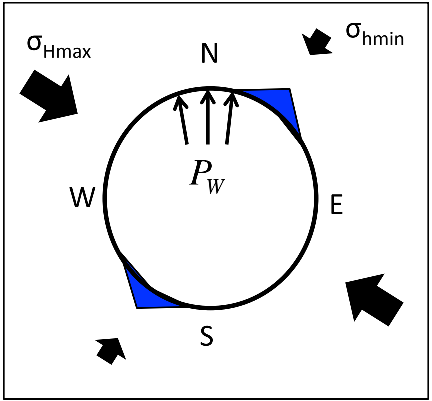
$$\begin{cases} \sigma_{\theta\theta}(r=a,\theta=0) & = & -(P_W - P_p) - \sigma_{Hmax} + 3\sigma_{hmin} \\ \sigma_{\theta\theta}(r=a,\theta=\pi/2) & = & -(P_W - P_p) + 3\sigma_{Hmax} - \sigma_{hmin} \end{cases}$$

$$\sigma_{r\theta} = 0$$

$$\sigma_{zz}(r = a) = \sigma_v - 2\nu(\sigma_{Hmax} - \sigma_{hmin}) \cos(2\theta)$$

σ_1/σ_3

$$\begin{cases} \sigma_1 = \sigma_{\theta\theta} & = & -(P_W - P_p) + 3\sigma_{Hmax} - \sigma_{hmin} \\ \sigma_3 = \sigma_{rr} & = & (P_W - P_p) \end{cases}$$



$$P_{Wshear} = \underbrace{P_p}_{\text{Pore pressure in the formation}} + \frac{\overbrace{3\sigma_{H\max} - \sigma_{h\min}}^{\text{Stress anisotropy}} - UCS}{\underbrace{1+q}_{\text{Shear strength}}}$$

$$P_{Wshear} = \frac{3S_{H\max} - S_{h\min} - UCS + (q-1)P_p}{1+q}$$

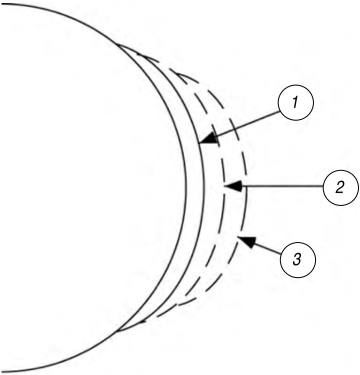
$P_W \leq P_{Wshear}$ leads to breakouts and potential wellbore colapse

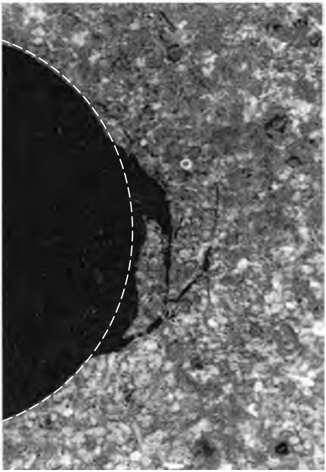
$P_{W\,shear}$

$$[-(P_W - P_p) + 3\sigma_{Hmax} - \sigma_{hmin}] = UCS + q\,[P_W - P_p]$$

$$P_{Wshear} = P_p + \frac{3\sigma_{Hmax} - \sigma_{hmin} - UCS}{1 + q}$$

$$P_W < P_{W\,shear}$$


1
2
3



$UCS =$

$\mu_i =$

T_s

$$P_p = 7000$$

$\times 0.44$

$$= 3080$$

$$\sigma_{Hmax} = 6300$$

-3080

$$= 3220$$

$$\sigma_{hmin} = 4300$$

$$= 1220$$

$$\arctan(0.6) = 30.96^\circ$$

$$q = \frac{1 + \sin 30.96^\circ}{1 - \sin 30.96^\circ} = 3.12$$

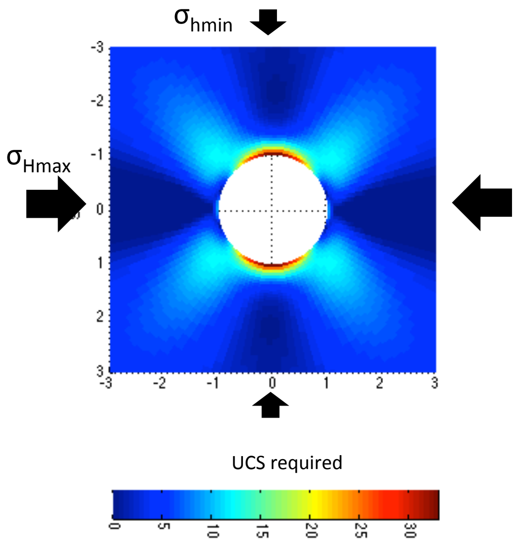
$$P_{W\,shear} = 3080$$

$$+\frac{3 \times 3220 \text{ psi} - 1220 \text{ psi} - 3500 \text{ psi}}{1 + 3.12} = 4279 \text{ psi}$$

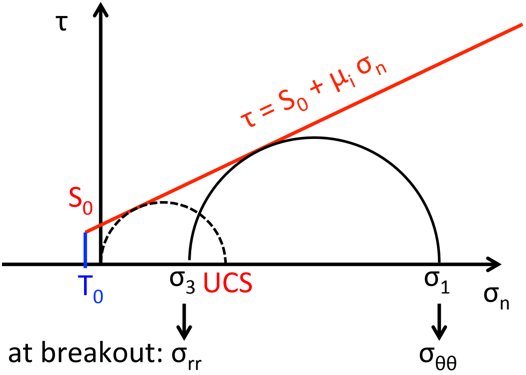
$$\frac{4279 \text{ psi}}{7000 \text{ ft}} \times \frac{8.3 \text{ ppg}}{0.44 \text{ psi/ft}} = 11.57 \text{ ppg} \blacksquare$$

$$UCS \sim 25$$

$\sim 90^\circ$



σ_{hmin}
σ_{Hmax}
-3 -2 -1 0 1 2 3
-3 -2 -1 0 1 2 3
UCS required
0 5 10 15 20 25 30



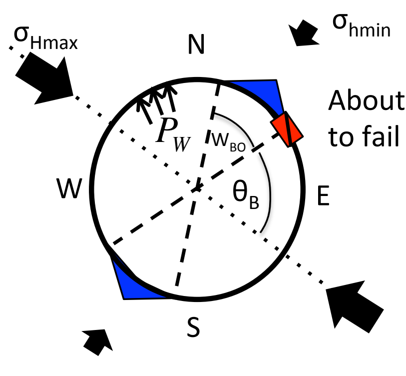
τ
$\tau = S_0 + \mu_i \sigma_n$
S_0
T_0
σ_3 UCS
σ_1
σ_n
at breakout: σ_{rr}
$\sigma_{\theta\theta}$

$(w_{BO}/2)$

$\pi/2$

$$\theta_B = \pi/2 - w_{BO}/2$$

$$\begin{cases} \sigma_{\theta\theta} & = & -(P_W - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(2\theta_B) \\ \sigma_{rr} & = & +(P_W - P_p) \end{cases}$$



σ_{Hmax}
N
σ_{hmin}
About to fail
P_W
w_{BO}
W
θ_B
E
S

$$\sigma_1 = UCS + q\,\sigma_3$$

$$[-(P_W - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(2\theta_B)] = UCS + q(P_W - P_p)$$

$$2\theta_B = \arccos\left[\frac{\sigma_{Hmax} + \sigma_{hmin} - UCS - (1+q)(P_W - P_p)}{2(\sigma_{Hmax} - \sigma_{hmin})}\right]$$

$$w_{BO} = \pi - 2\theta_B$$

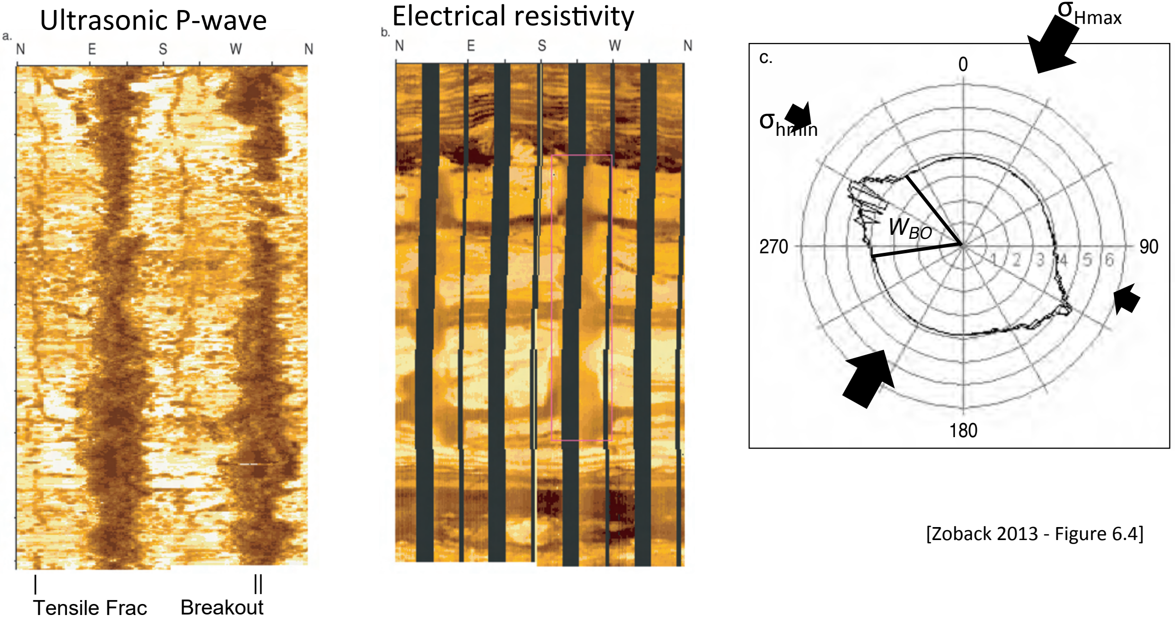
$$\sim w_{BO} > 60^\circ$$

$$P_{WBO} = P_p + \frac{(\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(\pi - w_{BO}) - UCS}{1 + q}$$

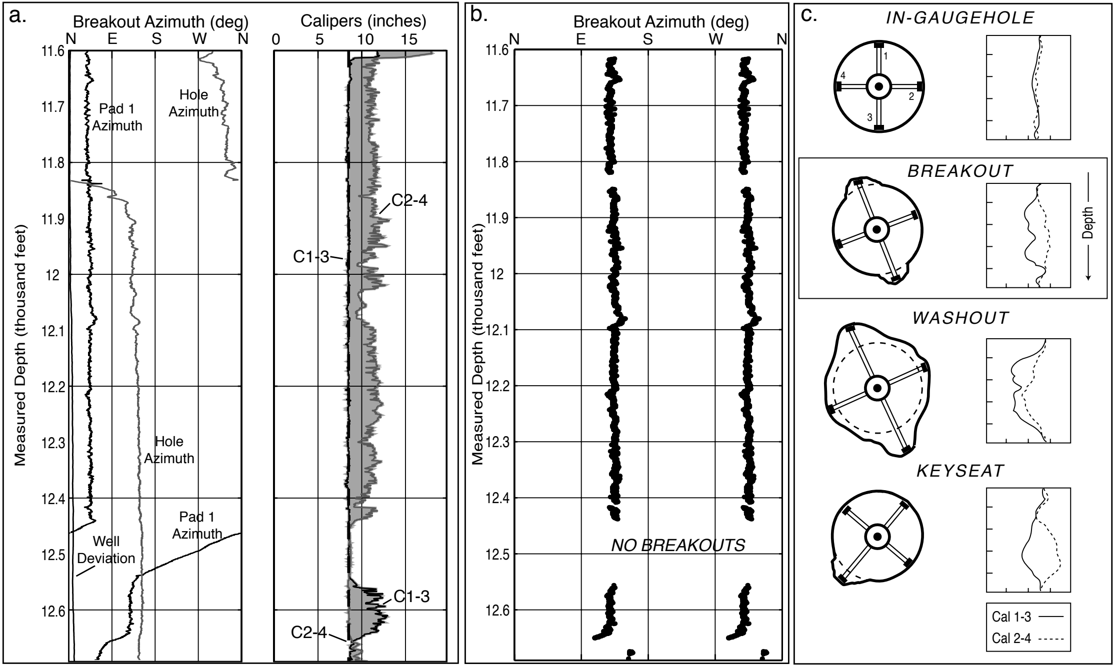
10

$$\times \frac{0.44 \text{ psi/ft}}{8.3 \text{ ppg}} \times 7000 \text{ psi} = 3710 \text{ psi}$$

$$w_{BO} = 180^\circ - \arccos\left[\frac{3220 \text{ psi} + 1220 \text{ psi} - 3500 \text{ psi} - (1 + 3.12)(3710 \text{ psi} - 3080 \text{ psi})}{2(3220 \text{ psi} - 1220 \text{ psi})}\right] = 66^\circ \quad \blacksquare$$



[Zoback 2013 - Figure 6.4]



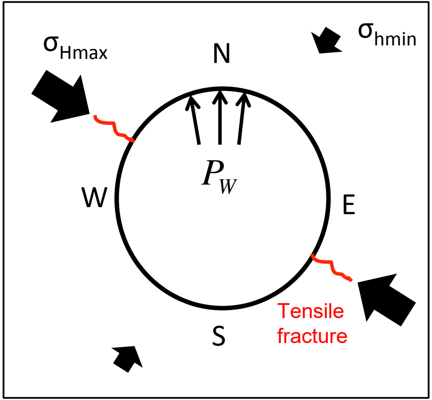
[Zoback 2013 - Figure 6.9]

$$S_{Hmax} = P_p + \frac{UCS + (1+q)(P_W - P_p) - \sigma_{hmin}\left[1 + 2\cos(\pi - w_{BO})\right]}{1 - 2\cos(\pi - w_{BO})}$$

$$(r = a)$$

$$\sigma_{\theta\theta} = -(P_W - P_p) - \sigma_{Hmax} + 3\sigma_{hmin} + \sigma^{\Delta T}$$

$$\sigma^{\Delta T}$$



$$\boxed{P_b = \underbrace{P_p}_{\text{Pore pressure in the formation}} + \underbrace{3\sigma_{h\min} - \sigma_{H\max}}_{\text{Stress anisotropy}} + \underbrace{T_s}_{\text{Tensile strength}} + \underbrace{\sigma^{\Delta T}}_{\text{Cooling stress}}}$$

$$\boxed{P_b = 3S_{h\min} - S_{H\max} - P_p + T_s + \sigma^{\Delta T}}$$

$$P_W = P_b$$

$$-T_s = -(P_b - P_p) - \sigma_{Hmax} + 3\sigma_{hmin} + \sigma^{\Delta T}$$

$$P_b = P_p - \sigma_{Hmax} + 3\sigma_{hmin} + T_s + \sigma^{\Delta T}$$

b

P_b

$$P_b > S_3$$

$$P_W > P_b$$

$$P_b < S_3$$

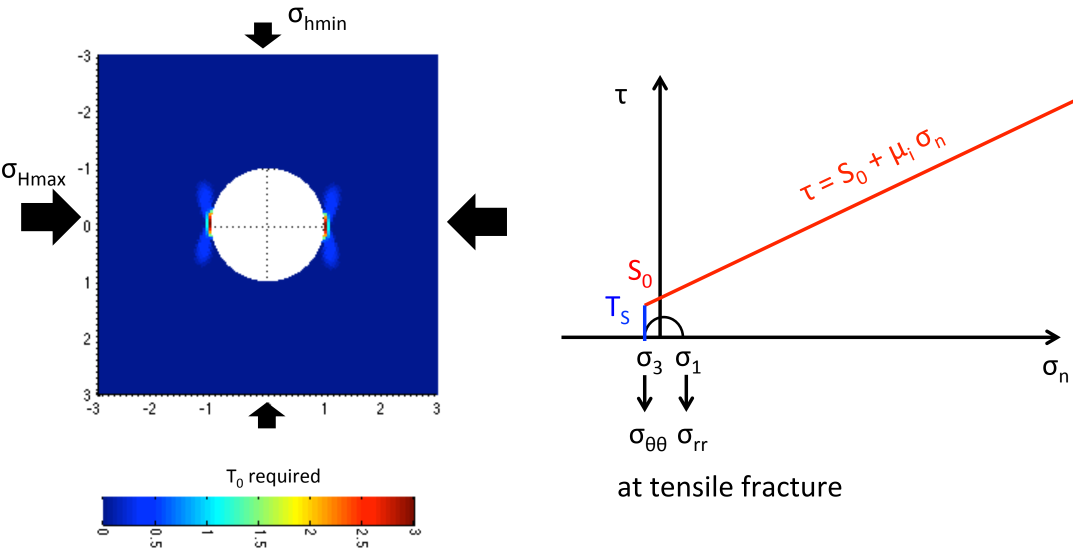
$$\sigma^{\Delta T} = [\alpha_T E/(1-\nu)]\Delta T$$

α_T

$$\sigma^{\Delta T} < 0$$

$$\Delta T < 0$$

$$T_{mud} < T_{rock}$$



σ_{hmin}
σ_{Hmax}
-3
-2
-1
0
1
2
3
T_0 required
0
0.5
1
1.5
2
2.5
3
τ
$\tau = S_0 + \mu_i \sigma_n$
S_0
T_S
σ_3
σ_1
σ_n
$\sigma_{\theta\theta}$
σ_{rr}
at tensile fracture

$$P_b = 3080$$

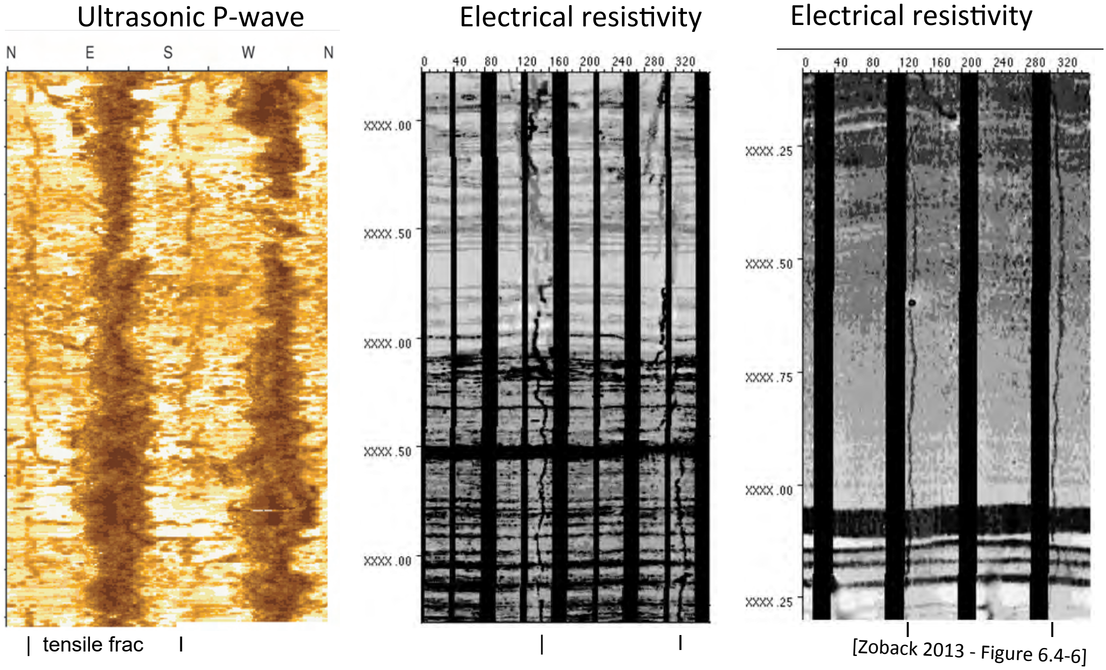
-3220

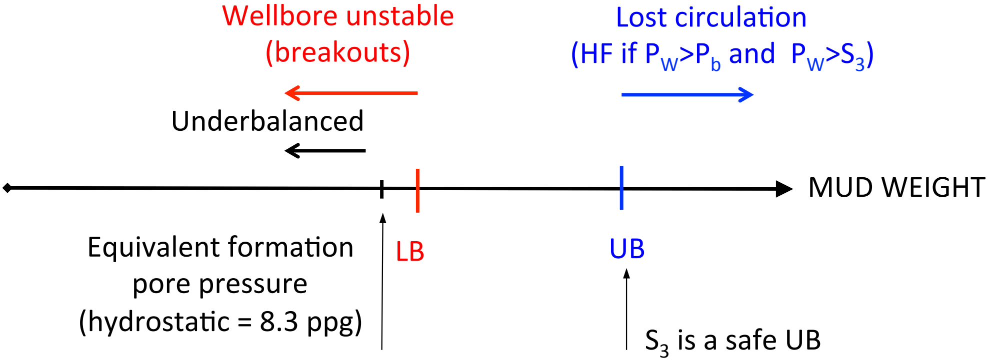
$$+3 \times 1220$$

$+800$

$$= 4320$$

$$\frac{4320 \text{ psi}}{7000 \text{ ft}} \times \frac{8.33 \text{ ppg}}{0.44 \text{ psi/ft}} = 11.68 \text{ ppg} \quad \blacksquare$$


Ultrasonic P-wave
N
E
S
W
N
| tensile frac |
Electrical resistivity
0 40 80 120 160 200 240 280 320
XXXX .00
XXXX .50
XXXX .00
XXXX .50
XXXX .00
Electrical resistivity
0 40 80 120 160 200 240 280 320
XXXX .25
XXXX .50
XXXX .75
XXXX .00
XXXX .25
[Zoback 2013 - Figure 6.4-6]



LIGHT MUDS
- May promote water production
- Compromise wellbore stability

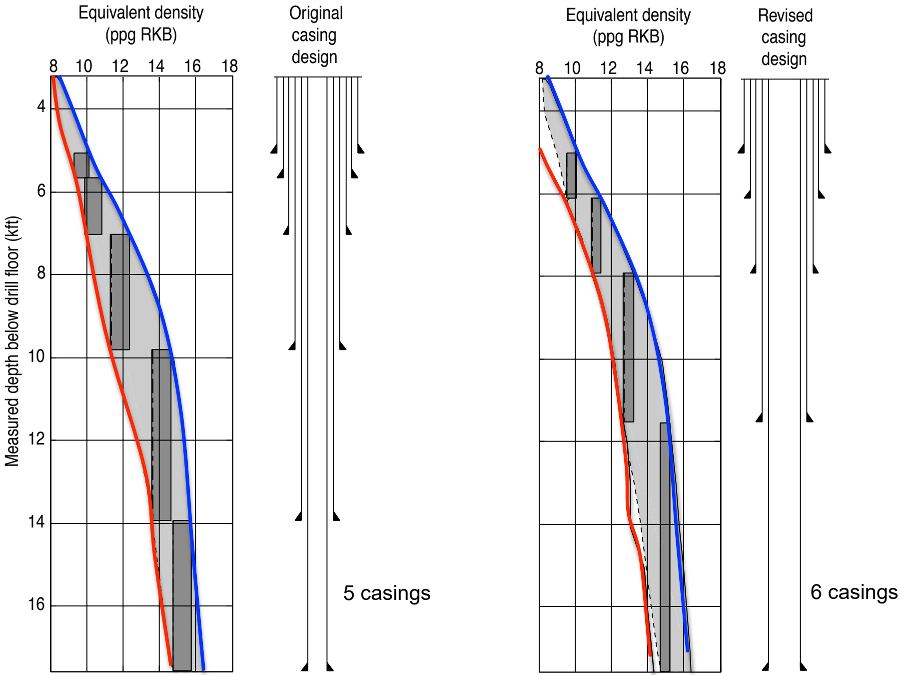
HEAVY MUDS
- Damage formation (*k*) by mud infiltration
- Promote mud losses in permeable strata
- Low ROP because of stronger rock

$$w_{BO} \leqslant 60^\circ$$

$$w_{BO} \geqslant 120^\circ$$

$$T_S = 0MPa$$

$$dP_W/dz$$

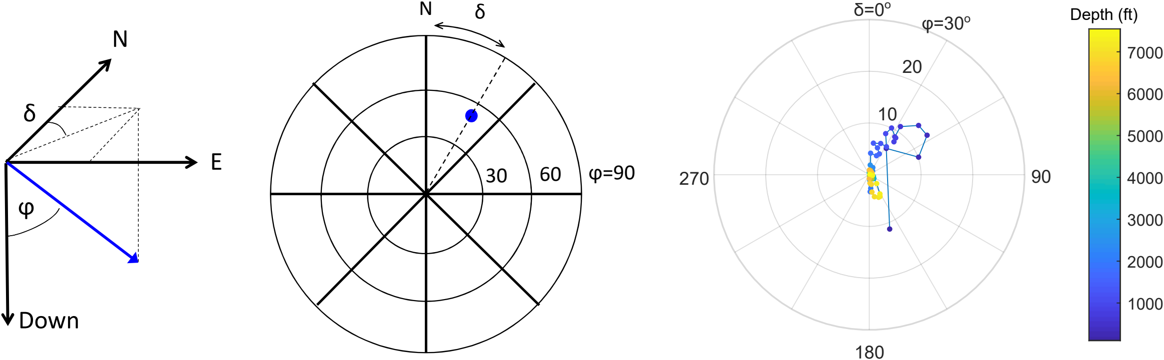


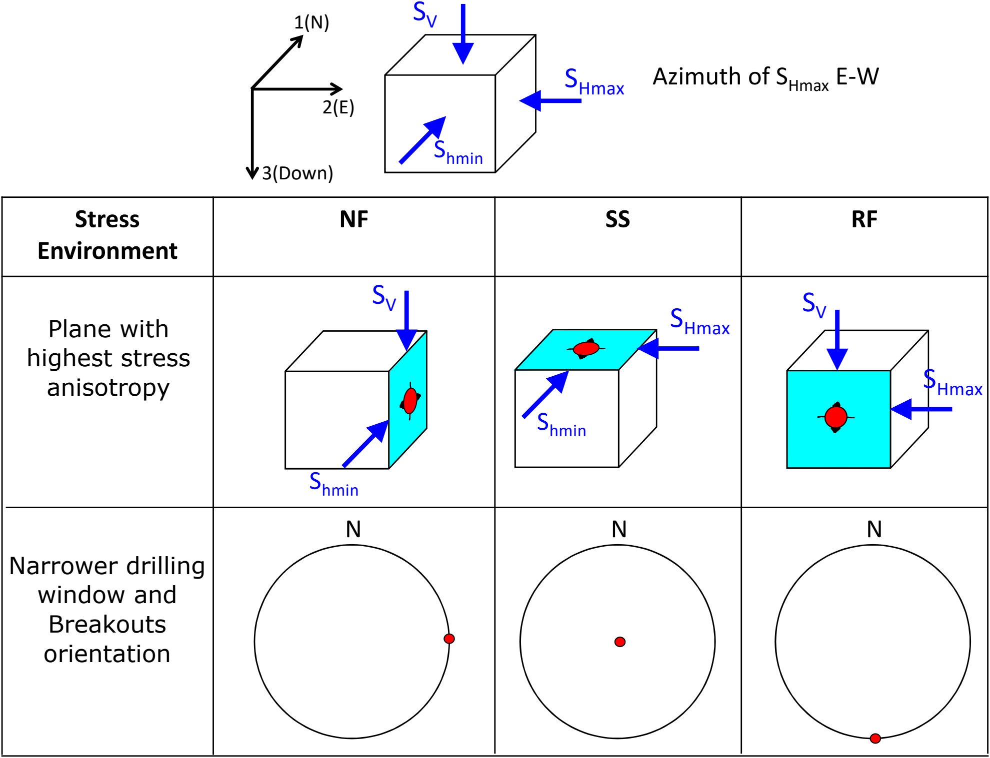
LB: Pore pressure
UB: Frac gradient

LB: Collapse pressure
UB: Frac gradient

[Zoback 2013 – Figure 10.3]

δ


N
δ
E
φ
Down
N
δ
30
60
φ=90
δ=0°
φ=30°
20
10
270
90
180
Depth (ft)
7000
6000
5000
4000
3000
2000
1000

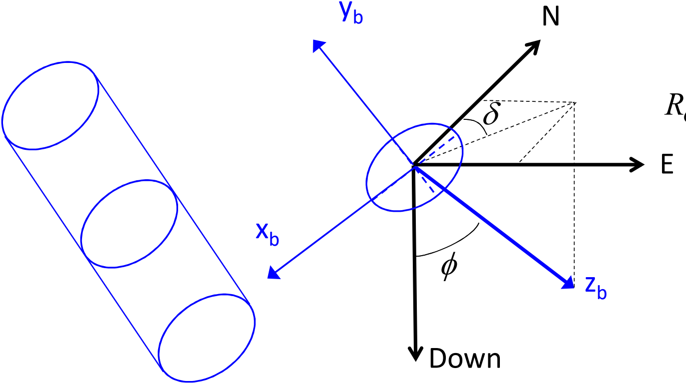


1(N)
2(E)
3(Down)
SV
SHmax
Shmin
Azimuth of SHmax E-W
Stress Environment
NF
SS
RF
Plane with highest stress anisotropy
SV
Shmin
SHmax
Shmin
SV
SHmax
Narrower drilling window and Breakouts orientation
N
N
N

x_b

y_b

z_b



$$R_{GW} = \begin{bmatrix} -\cos\delta\cos\phi & -\sin\delta\cos\phi & \sin\phi \\ \sin\delta & -\cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix}$$

$$R_{GW} = f(\delta, \varphi)$$

$$\underline{\underline{S}}_W = R_{GW} \underline{\underline{S}}_G R_{GW}^T$$

$$\underline{\underline{S}}_W = R_{GW} R_{PG}^T \underline{\underline{S}}_P R_{PG} R_{GW}^T$$

$$\underline{\underline{S}}_P$$

$$\underline{\underline{S}}_W$$

$$\underline{\underline{\mathrm{S}}}_W = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$(\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{\theta z})$$

$$\sigma_{11} = S_{11} - P_p$$

$$\sigma_{22} = S_{22} - P_p$$

$$\sigma_{33} = S_{33} - P_p$$

$$\sigma_{12} = S_{12}$$

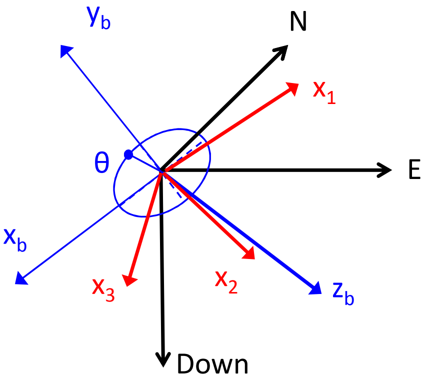
$$\sigma_{13} = S_{13}$$

$$\sigma_{23} = S_{23}$$

σ_{12}

σ_{13}

σ_{23}

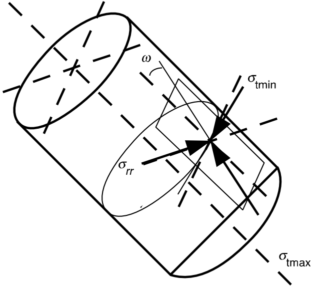


Stresses at the wellbore wall (Kirsch[PP]+Kirsch[S])

$$\begin{cases} \sigma_{rr} = \Delta P \\ \sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\sigma_{12}\sin 2\theta - \Delta P \\ \tau_{\theta z} = 2(\sigma_{23}\cos\theta - \sigma_{13}\sin\theta) \\ \sigma_{zz} = \sigma_{33} - 2\nu(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\nu\sigma_{12}\sin 2\theta \end{cases}$$

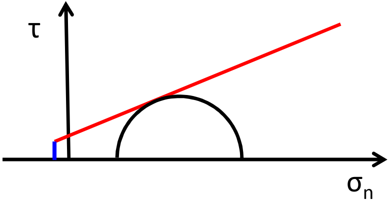
$$(\sigma_{tmax}, \sigma_{rr}, \sigma_{tmin})$$

ω



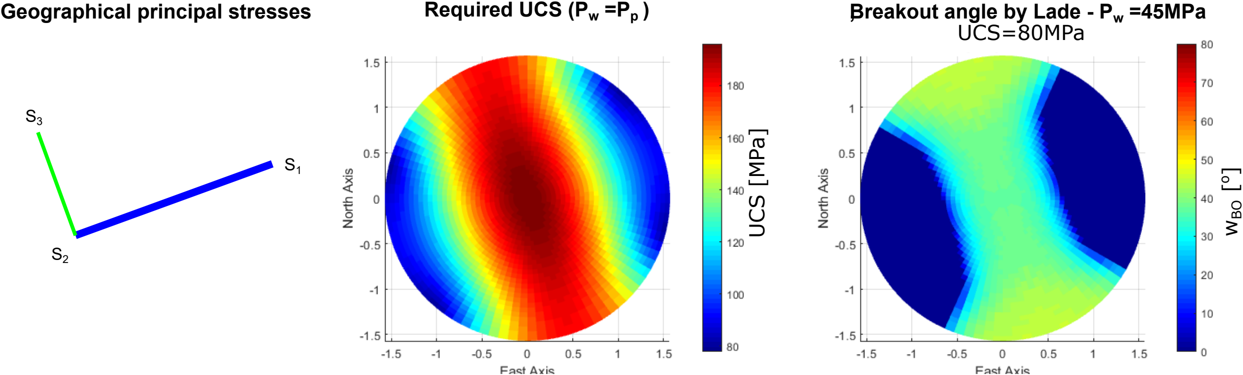
Principal stresses at the wellbore wall

$$\begin{cases} \sigma_{t\max} = \dfrac{\sigma_{zz} + \sigma_{\theta\theta}}{2} + \sqrt{\left(\dfrac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \\ \sigma_{t\min} = \dfrac{\sigma_{zz} + \sigma_{\theta\theta}}{2} - \sqrt{\left(\dfrac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \end{cases}$$



$$S_2 = S_v$$

$$\delta = 070^\circ$$


Geographical principal stresses
S3
S1
S2
Required UCS (Pw =Pp)
North Axis
East Axis
UCS [MPa]
Breakout angle by Lade - Pw =45MPa
UCS=80MPa
wBO [°]

$$S_v = 70$$

$$S_{Hmax} = 67$$

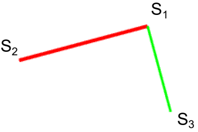
$$S_{hmin} = 45$$

$$P_p = 32$$

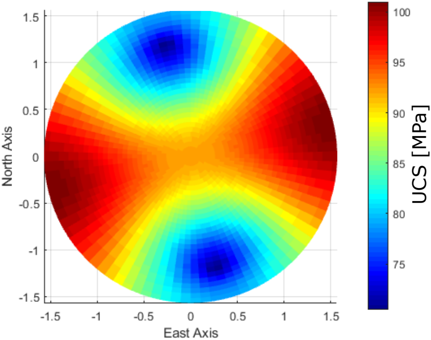
$$P_W = 32$$

$\mu_i = 0.8$

Principal stresses

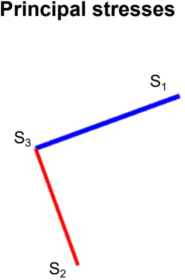


Required UCS ($P_w = P_p$)

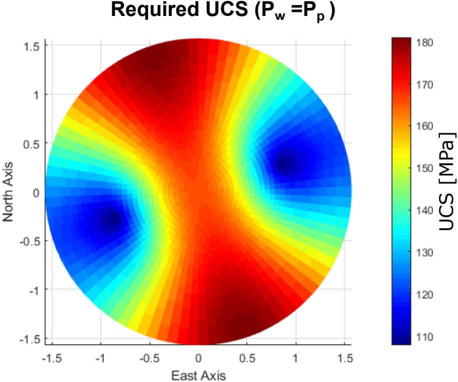


$$S_{Hmax} = 105$$

$$S_{hmin} = 85$$



Principal stresses
S1
S3
S2

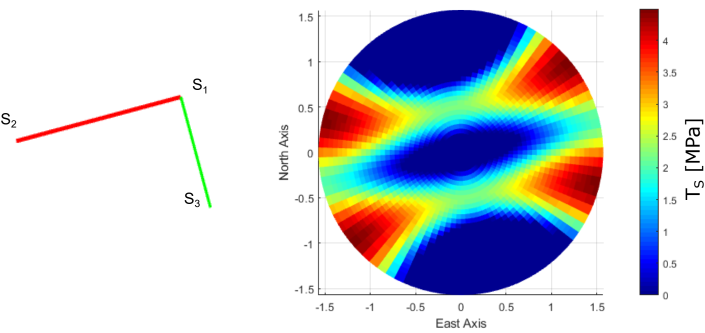


Required UCS ($P_w = P_p$)
North Axis
East Axis
1.5
1
0.5
0
-0.5
-1
-1.5
UCS [MPa]
180
170
160
150
140
130
120
110

$$S_1 = S_v$$

Principal stresses

Required T_S (P_W=35MPa)



$$S_{hmin} = 55$$

$$T_s = 0$$

$$\frac{\partial T}{\partial t} = D_T \nabla^2 T$$

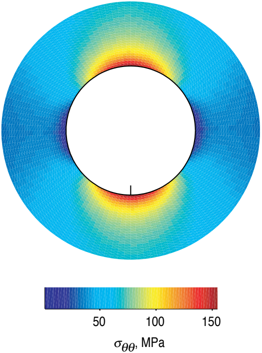
$$D_T = k_T/(\rho c_p)$$

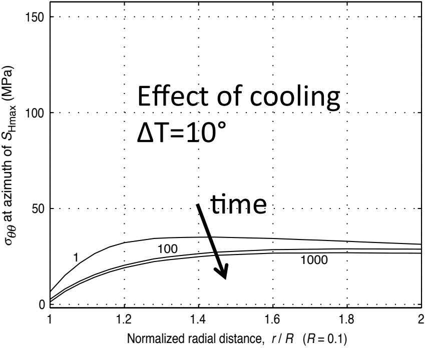
k_T

c_p

$$\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)$$

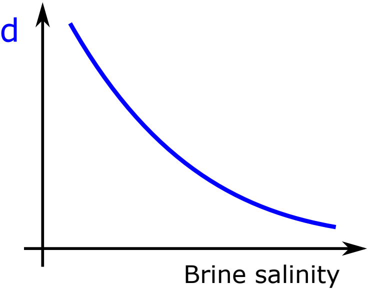
[Zoback 2013 - Figure 6.14]

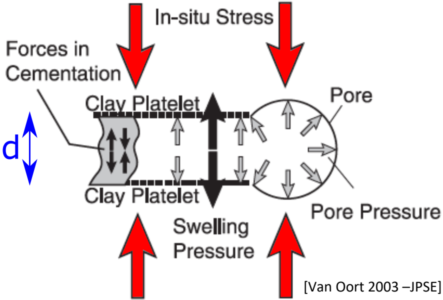




$$\Delta\sigma_{\theta\theta}$$

$$\Delta\sigma_{\theta\theta} = \frac{\alpha_L E \Delta T}{1-\nu}$$



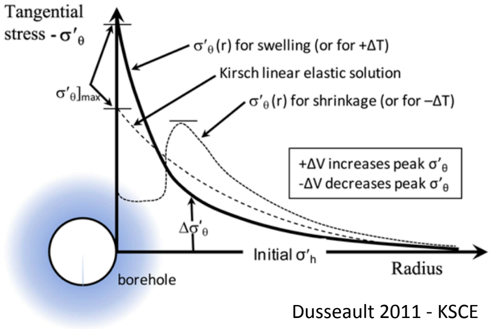


[Van Oort 2003 –JPSE]

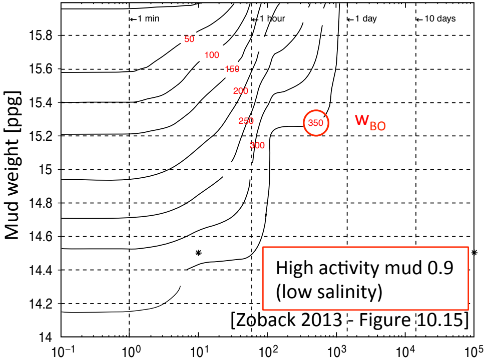
Norway shale

After 2hs exposition

CPGE – M. Chenevert

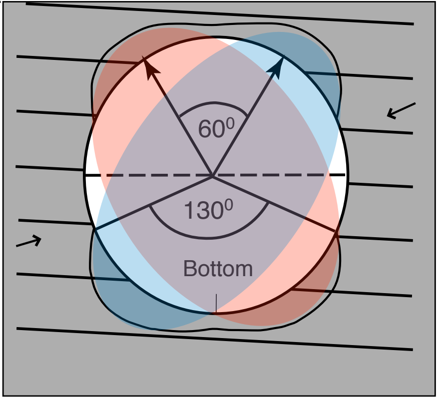


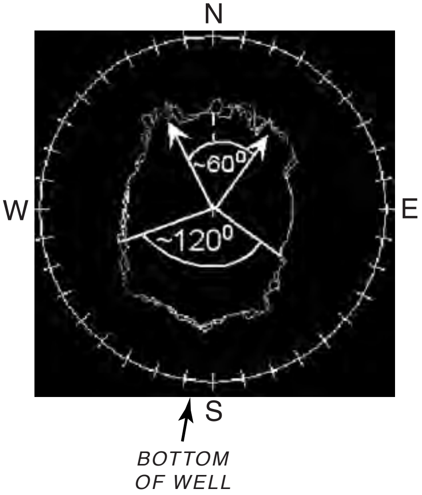
Dusseault 2011 - KSCE



[Zoback 2013 - Figure 10.15]

$$\sigma_{rr} = P_W - P_p$$

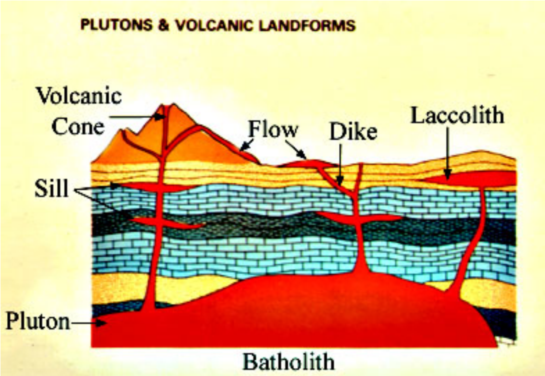




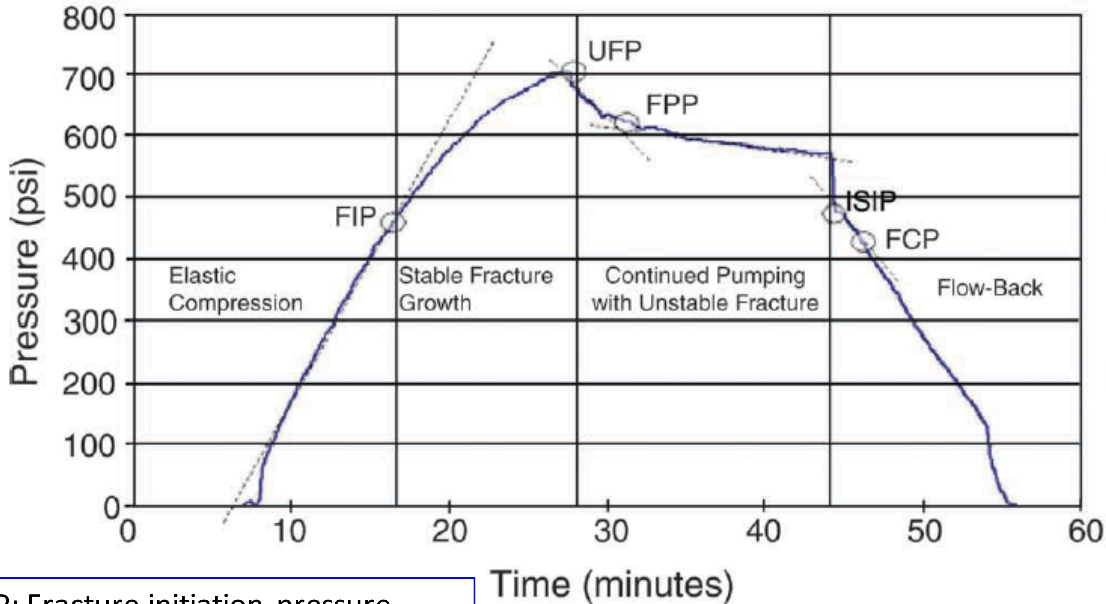
Shear failure intact rock

Shear failure at weak interfaces
(usually weak bedding planes)

[Zoback 2013 - Figure 6.14]



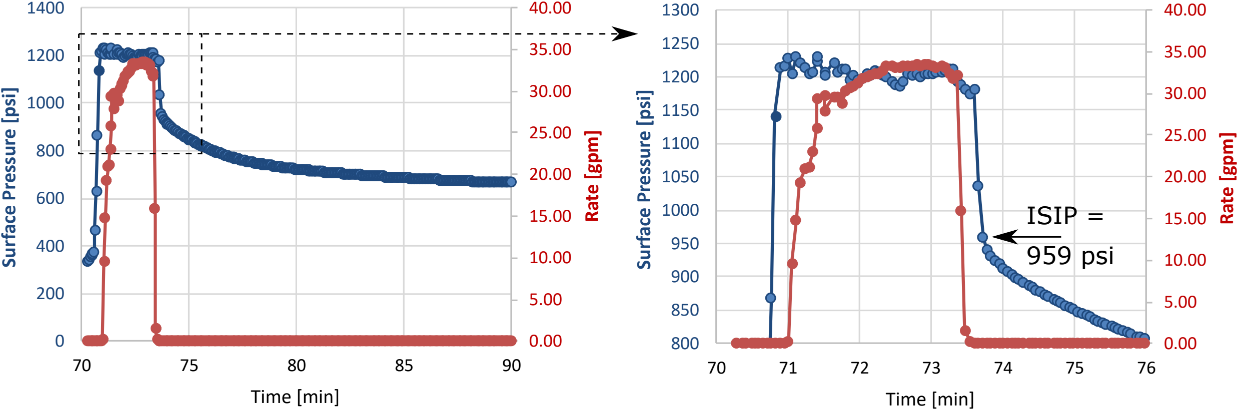
http://www.indiana.edu/~geol105/1425chap5.htm
http://geophysics.ou.edu/geol1114



$\Delta V = q \, \Delta t$

FIP: Fracture initiation pressure
UFP: Unstable fracture pressure
FPP: Fracture propagation pressure
ISIP: Instantaneous shut-in pressure
FCP: Fracture closure pressure

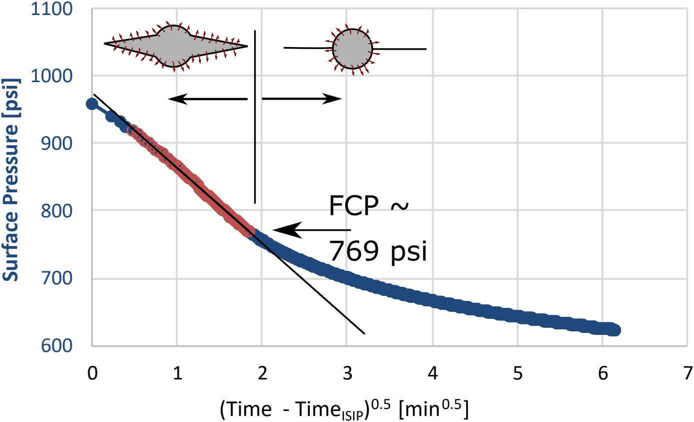
[van Oort and Vargo, 2008]



Surface Pressure [psi]
Rate [gpm]
Time [min]
1400
1200
1000
800
600
400
200
0
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
70
75
80
85
90
1300
1250
1200
1150
1100
1050
1000
950
900
850
800
71
72
73
74
75
76
ISIP =
959 psi

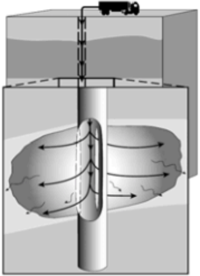
$$\sqrt{t_{si}}$$

t_{si}

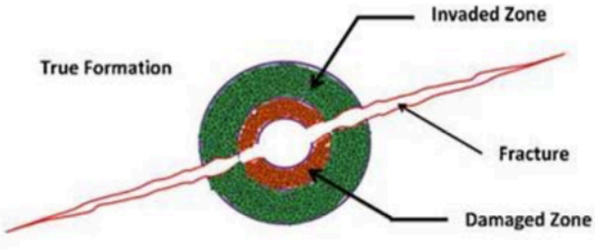


1100
1000
900
800
700
600
Surface Pressure [psi]
FCP ~
769 psi
0
1
2
3
4
5
6
7
(Time - Time$_{ISIP}$)$^{0.5}$ [min$^{0.5}$]

$\leqslant 10$



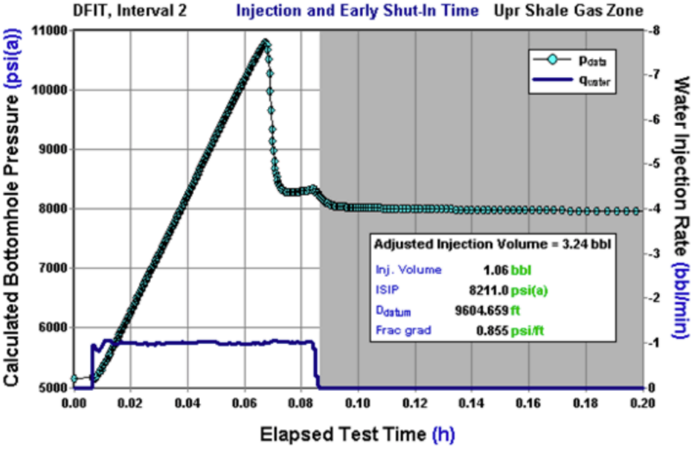
Side View

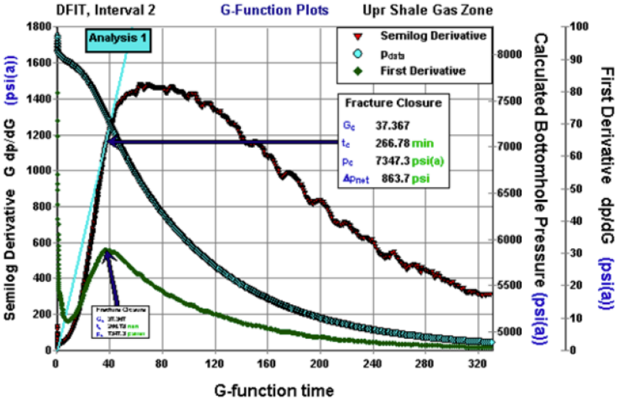


Plan View

Fekete, 2011

[SPE 163863]


DFIT, Interval 2
Injection and Early Shut-In Time
Upr Shale Gas Zone
Calculated Bottomhole Pressure (psi(a))
Water Injection Rate (bbl/min)
Elapsed Test Time (h)
p_{data}
q_{water}
Adjusted Injection Volume = 3.24 bbl
Inj. Volume 1.06 bbl
ISIP 8211.0 psi(a)
D_{datum} 9604.659 ft
Frac grad 0.855 psi/ft


DFIT, Interval 2
G-Function Plots
Upr Shale Gas Zone
Analysis 1
Semilog Derivative
p_{data}
First Derivative
Fracture Closure
G_c 37.367
t_c 266.78 min
p_c 7347.3 psi(a)
Δp_{net} 863.7 psi
Semilog Derivative G dp/dG (psi(a))
Calculated Bottomhole Pressure (psi(a))
First Derivative dp/dG (psi(a))
G-function time

[SPE 163863]

<

>

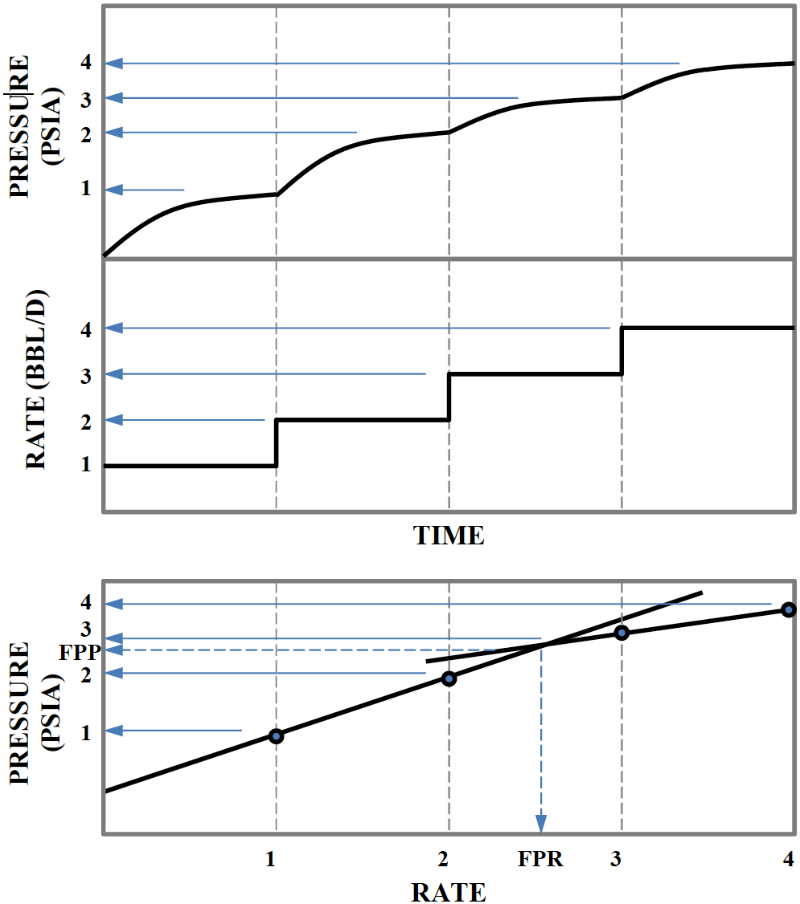


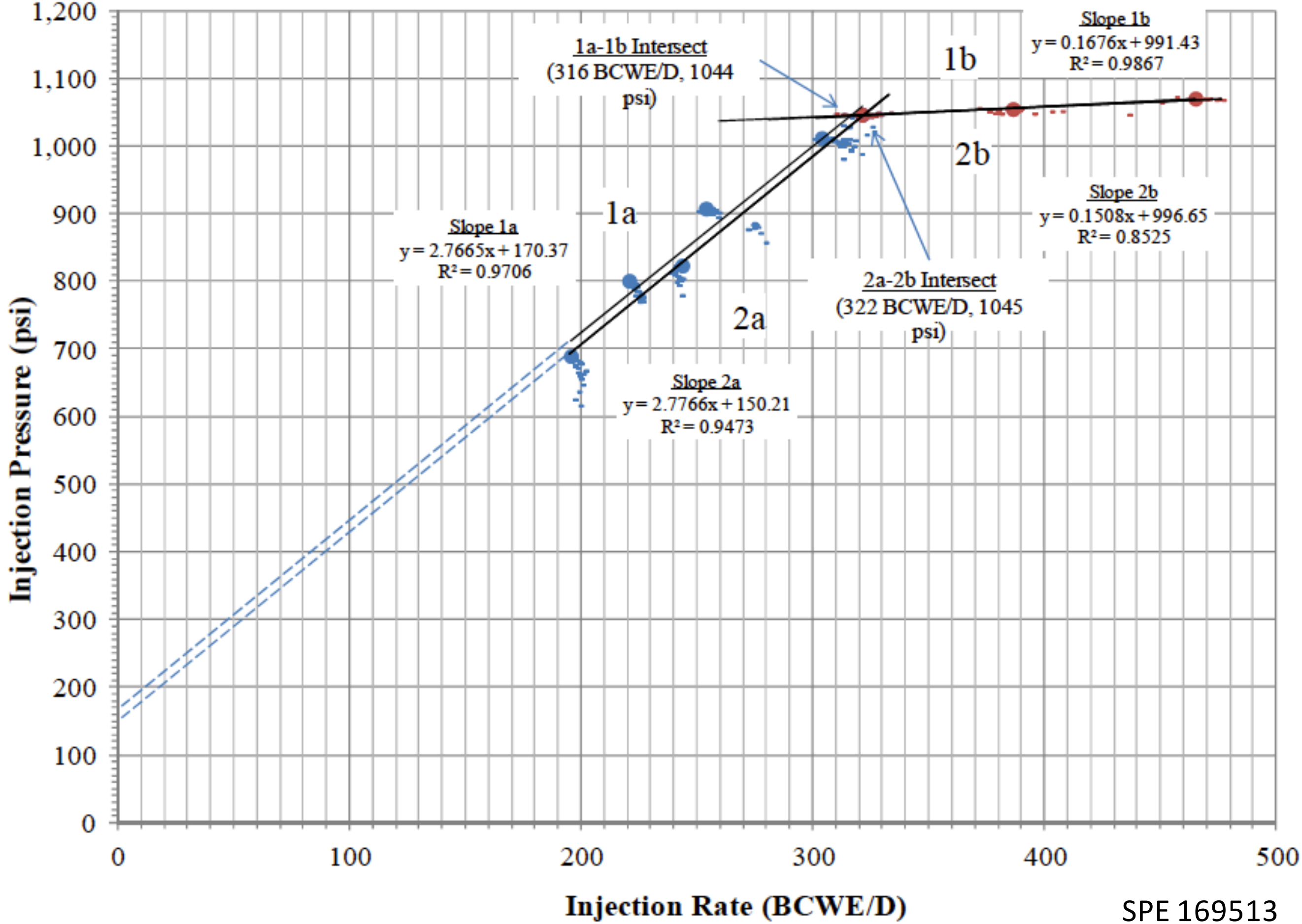
Figure 1. Schematic of a Generalized SRT

S

$s > 1$

$s < 1$

$$q = \frac{2\pi hk}{\mu} \frac{P_e - P_w}{\ln(\frac{r_e}{r_w}) + s}$$

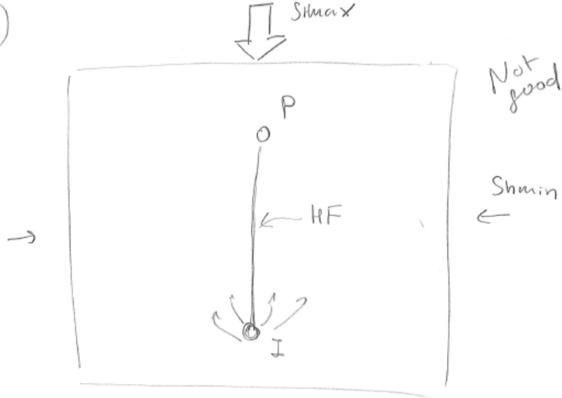


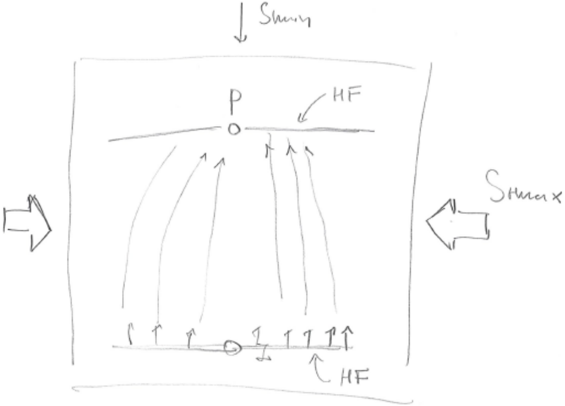
Slope 1b
y = 0.1676x + 991.43
R² = 0.9867
1a-1b Intersect
(316 BCWE/D, 1044 psi)
1b
2b
1a
2a
Slope 2b
y = 0.1508x + 996.65
R² = 0.8525
Slope 1a
y = 2.7665x + 170.37
R² = 0.9706
2a-2b Intersect
(322 BCWE/D, 1045 psi)
Slope 2a
y = 2.7766x + 150.21
R² = 0.9473
Injection Pressure (psi)
Injection Rate (BCWE/D)
0
100
200
300
400
500
600
700
800
900
1,000
1,100
1,200

SPE 169513

NF

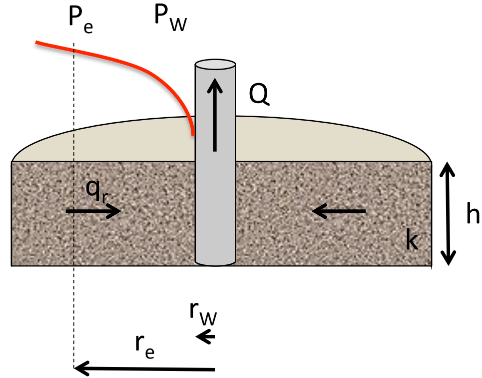
Not good





x_f/r

Intact



$$Q = -\frac{2\pi kh}{\mu}\frac{(P_e - P_W)}{p_D + s}$$

p_D: Dimensionless pressure

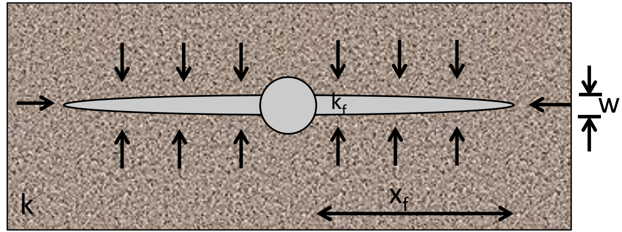
= ln (r_e/r_W) if steady state

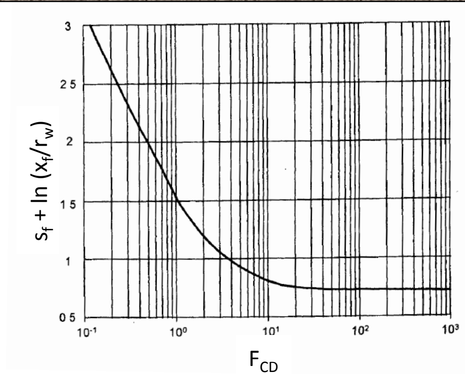
s: skin factor

> 1 → damage

< 1 → stimulation

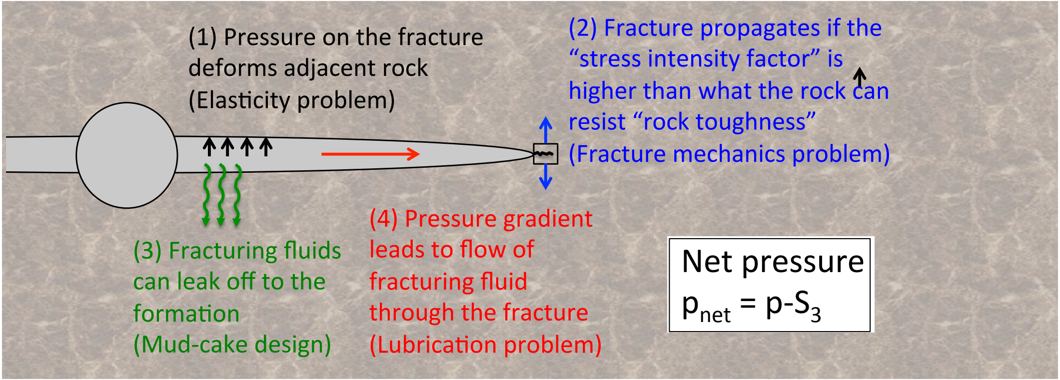
Fractured

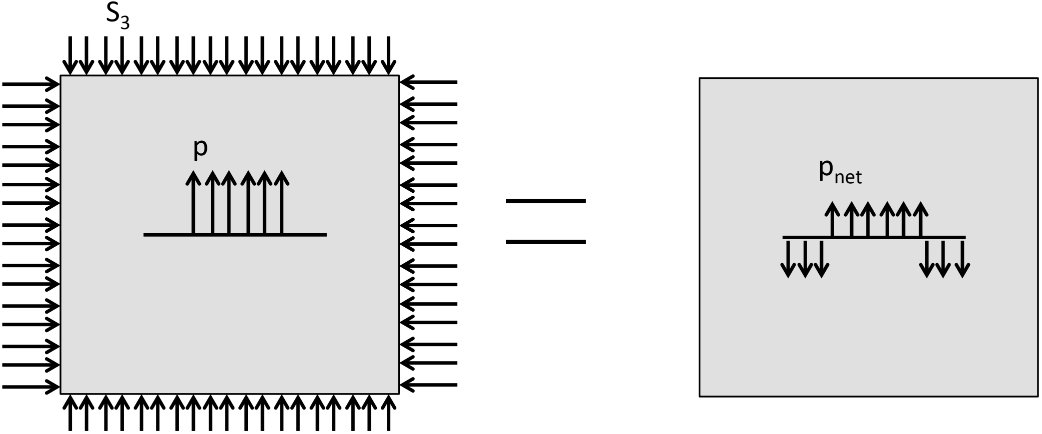




w, x_f, k_f → $F_{CD} = (k_f w)/(k\ x_f)$

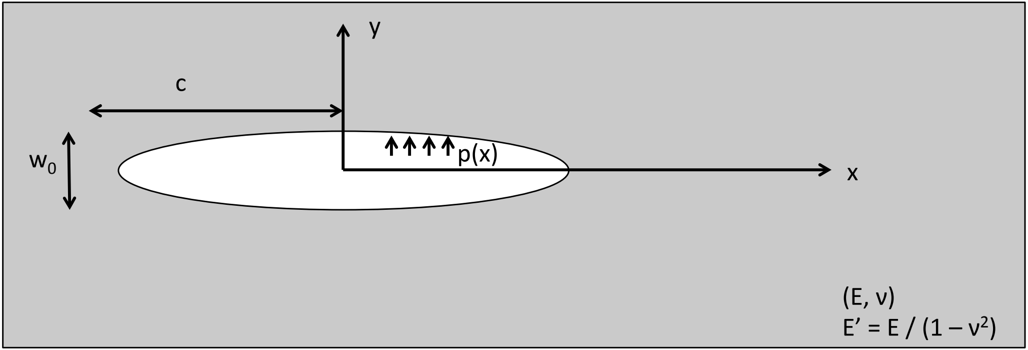
x_f, r_W, F_{CD} → s_f from above plot


(1) Pressure on the fracture deforms adjacent rock (Elasticity problem)
(2) Fracture propagates if the "stress intensity factor" is higher than what the rock can resist "rock toughness" (Fracture mechanics problem)
(3) Fracturing fluids can leak off to the formation (Mud-cake design)
(4) Pressure gradient leads to flow of fracturing fluid through the fracture (Lubrication problem)
Net pressure
$p_{net} = p - S_3$



S3
p
pnet

Griffith crack problem



<u>Boundary conditions</u>

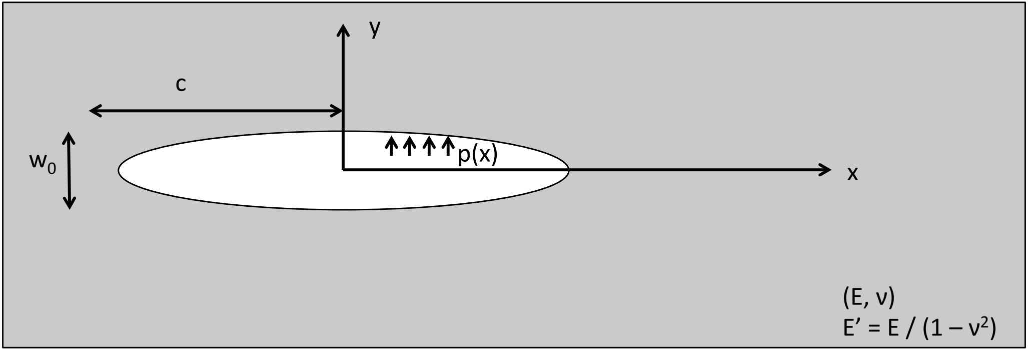
$$\begin{cases} \sigma_{yy}(x,0) = -\,p(x) & \text{for } 0 \le x \le c \\ u_y(x,0) = 0 & \text{for } x > c \end{cases}$$

- Linear elastic and homogeneous solid
- Constant pressure p_0
- No failure

$$\begin{cases} \sigma_{yy}(x,0) = p_0[x/(x^2-c^2)^{1/2} - 1] \\ u_y(x,0) = 2p_0/E' * (c^2-x^2)^{1/2} \end{cases}$$

- σ_{yy}: <u>tension</u> for x>c; $\sigma_{yy}=-p_0$ at 1.15c
- $w_0 = 4 \cdot c \cdot p_0 / E'$

Griffith crack problem



$\sigma_{yy}(x,0) = p_0[x/(x^2-c^2)^{1/2} -1]$

- $\sigma_{yy} \rightarrow \infty$ at x=c!

A more convenient amount to compare is the stress intensity factor

$$K_I = \lim_{r\to 0^+}\left[2\pi\, r^{1/2}\sigma_{yy}\left(x=c+r, y=0\right)\right]$$

Constant pressure crack:

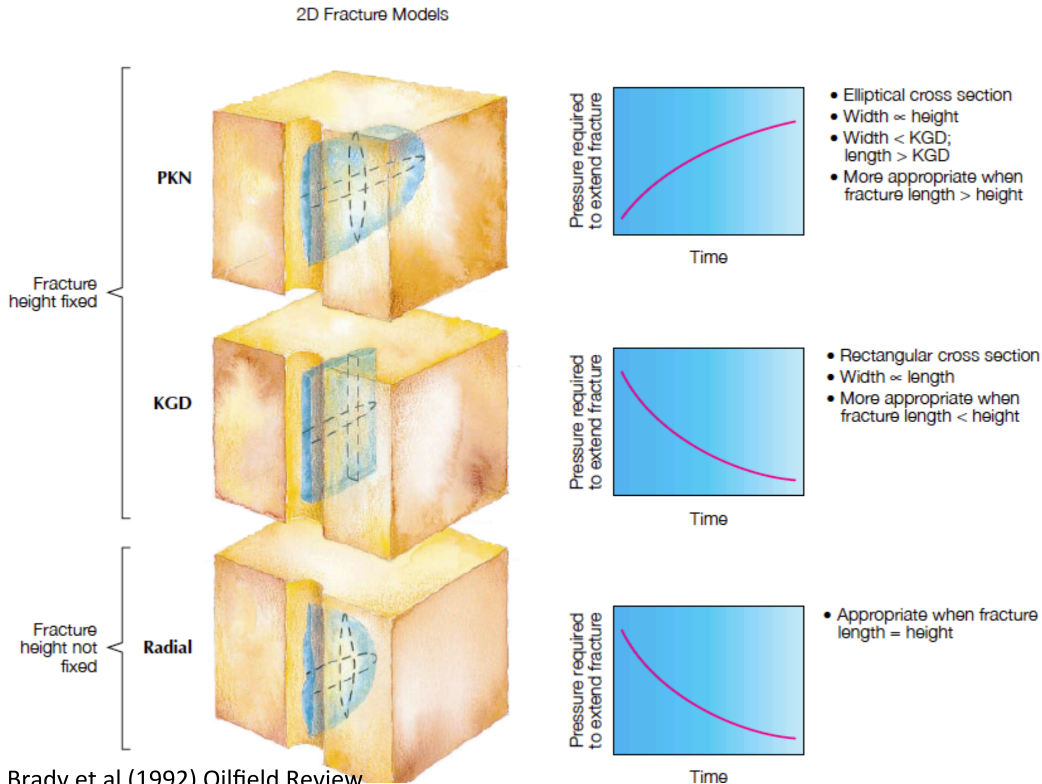
$$K_I = p_0\left(\pi c\right)^{1/2}$$

If $K_I > K_{IC}$ (Fracture Toughness)
→ Fracture propagates

xf

w

h_f



Brady et al (1992) Oilfield Review

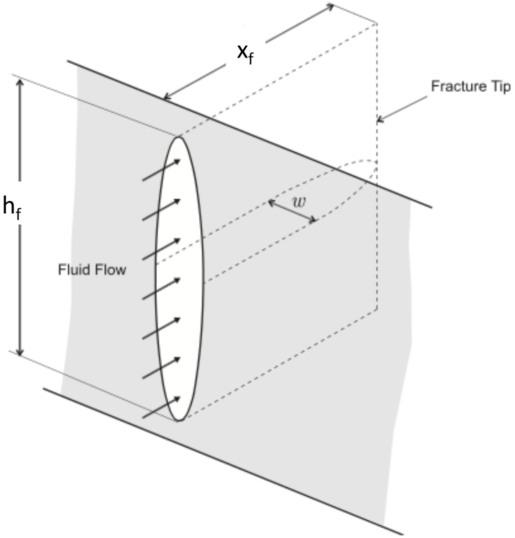


Fig. 1. Schematic showing PKN fracture geometry.

Figure: Adachi et al. 2007

(1) Linear elasticity
(2) Negligible Toughness
(3) No leak-off
(4) Laminar flow, constant injection rate

- Fracture half-length

$$x_f = 0.524\left(\frac{i^3 E'}{\mu\, h_f^4}\right)^{1/5} (t)^{4/5}$$

- Maximum width at the wellbore

$$w_{w,0} = 3.04\left(\frac{i^2 \mu}{E' h_f}\right)^{1/5} (t)^{1/5}$$

- Net Pressure at the wellbore

$$p_{n,w} = 1.52\left(\frac{E'^4\, i^2 \mu}{h_f^6}\right)^{1/5} (t)^{1/5}$$

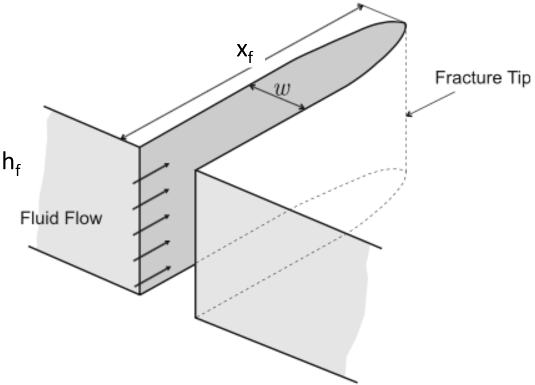


Fig. 2. Schematic showing KGD fracture geometry.

Figure: Adachi et al. 2007

- Fracture half-length

$$x_f = 0.539\left(\frac{i^3 E'}{\mu\, h_f^3}\right)^{1/5} (t)^{2/3}$$

- Width at the wellbore

$$w_w = 2.36\left(\frac{i^3 \mu}{E' h_f^3}\right)^{1/6} (t)^{1/3}$$

- Net Pressure at the wellbore

$$p_{n,w} = 1.09\left(E'^2\, \mu\right)^{1/3} (t)^{-1/3}$$

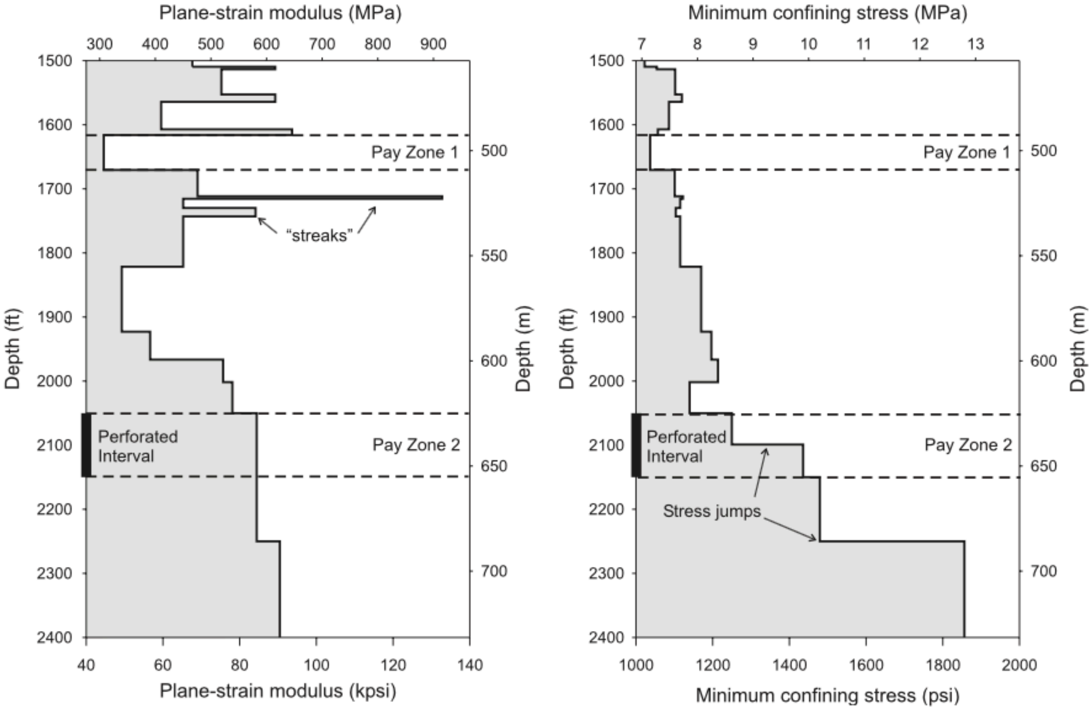
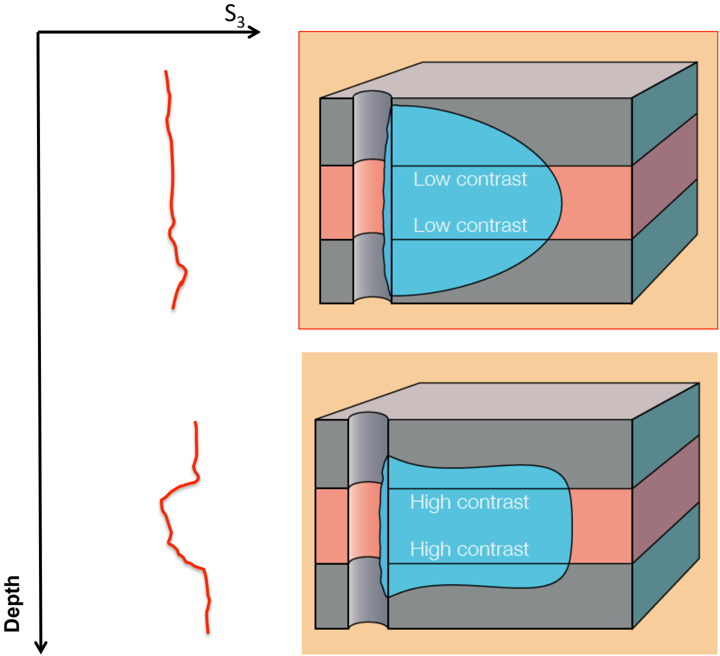
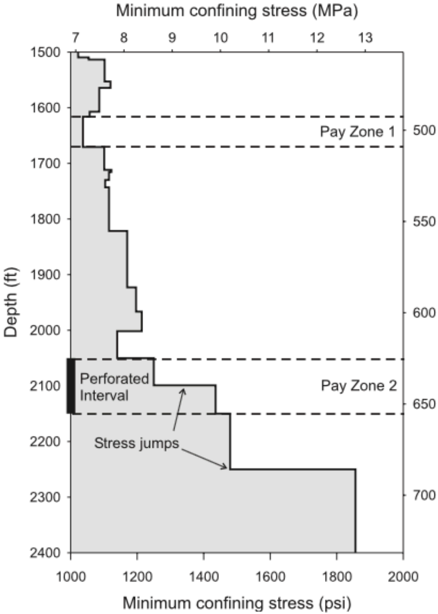
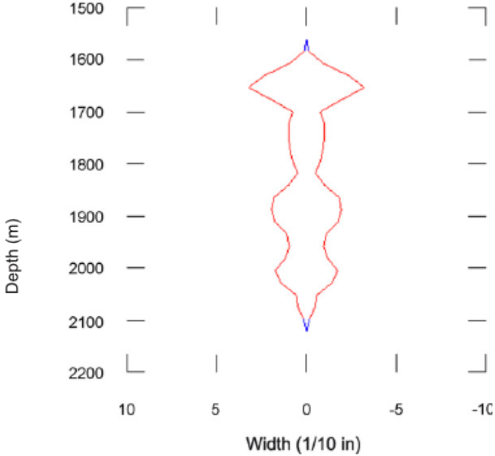


Fig. 9. Profiles of plane-strain modulus (left) and minimum *in situ* confining stress (right) versus depth.

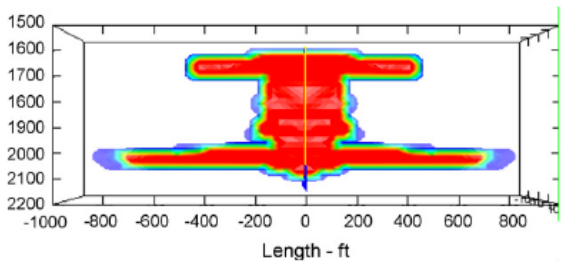


S_3
Depth
Low contrast
Low contrast
High contrast
High contrast



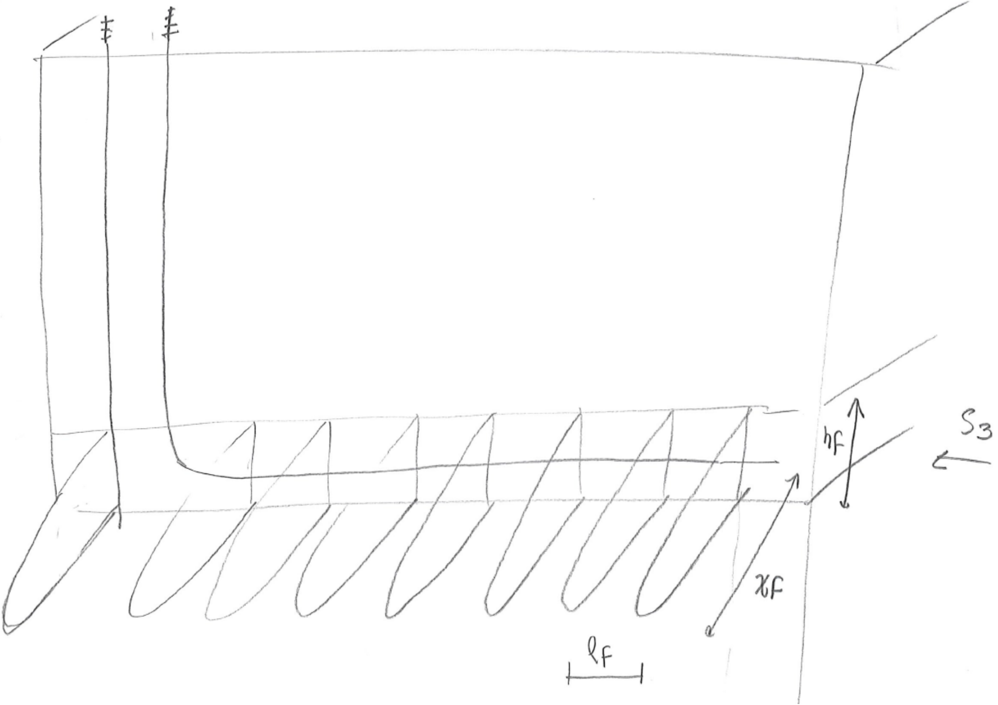


View perpendicular to fracture propagation

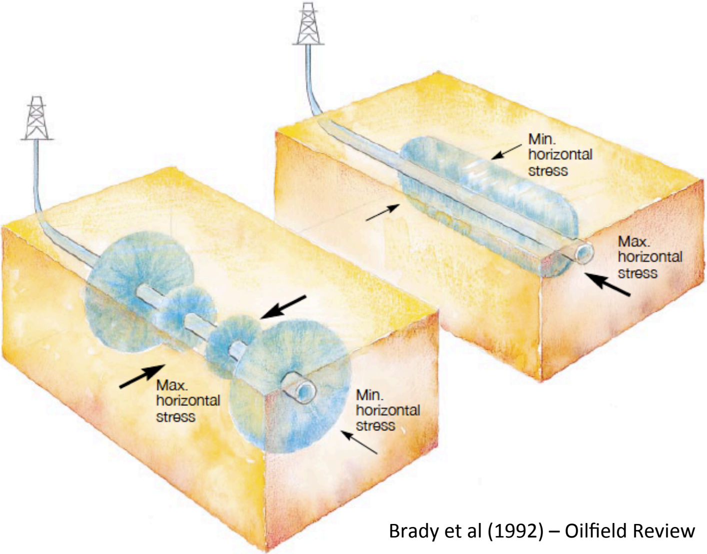


View perpendicular to fracture plane

Figures – Adachi et al. 2007



h_F
S_3
x_F
l_F



Brady et al (1992) – Oilfield Review

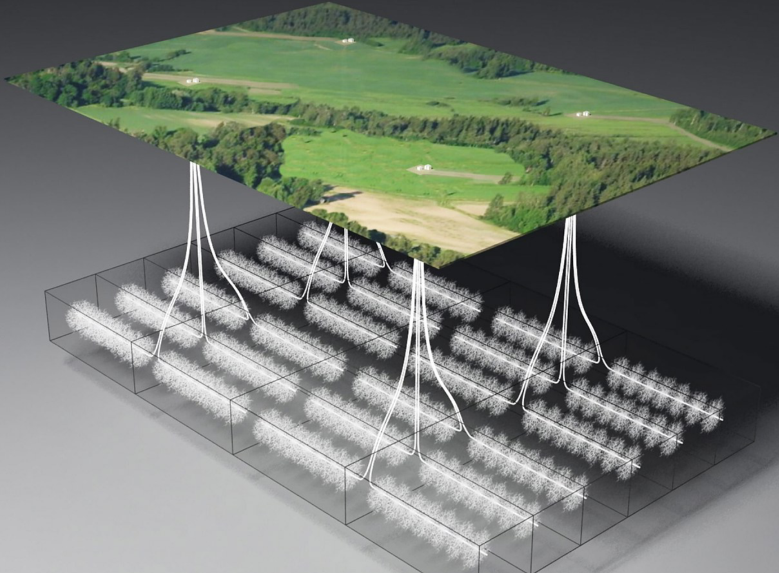
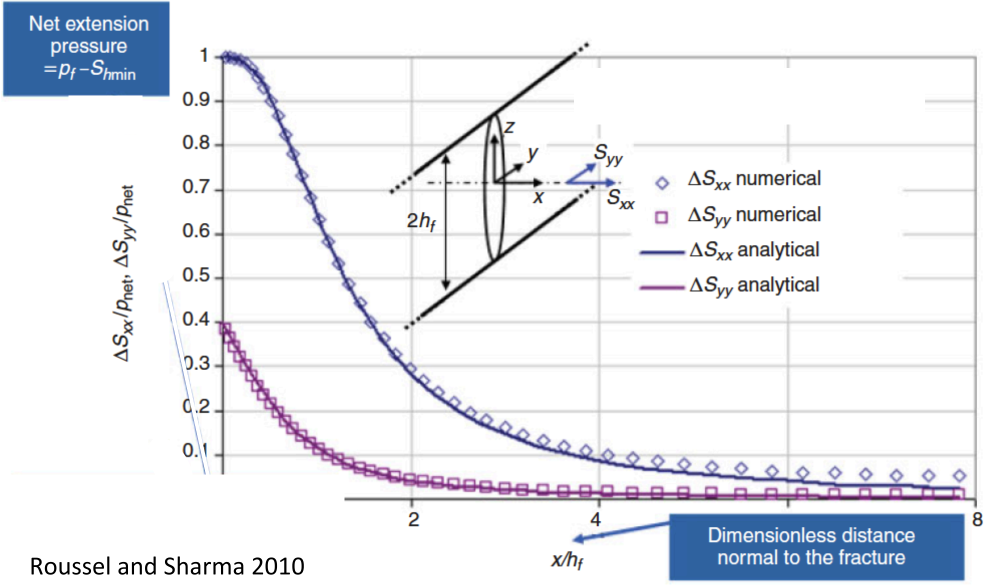


Image: Statoil



Roussel and Sharma 2010

TOP VIEW

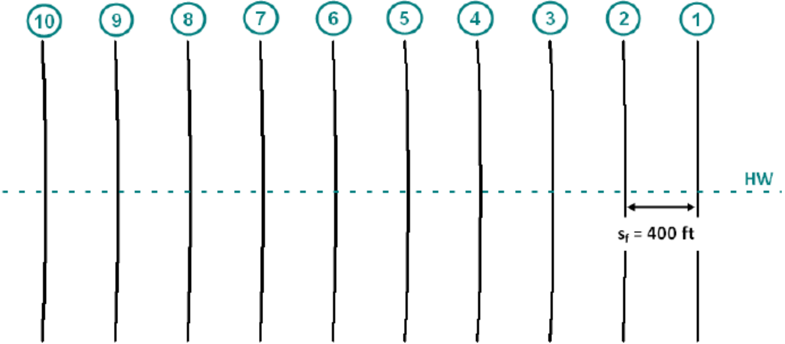


Fig. 7 – Trajectory of multiple consecutive transverse fractures spaced 400 ft apart

Roussel and Sharma 2011

TOP VIEW

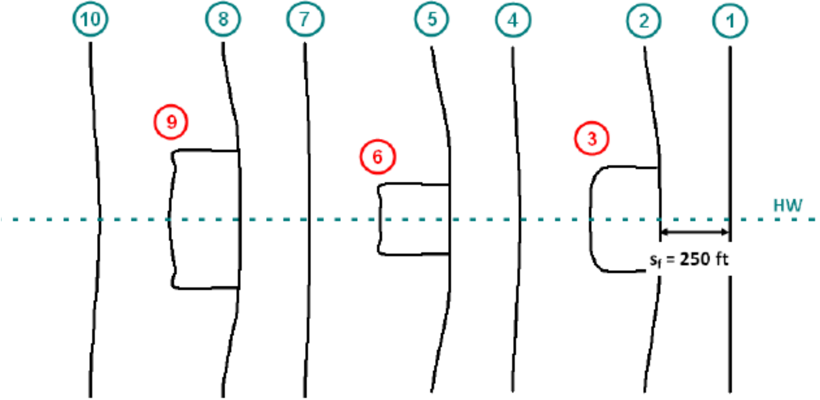
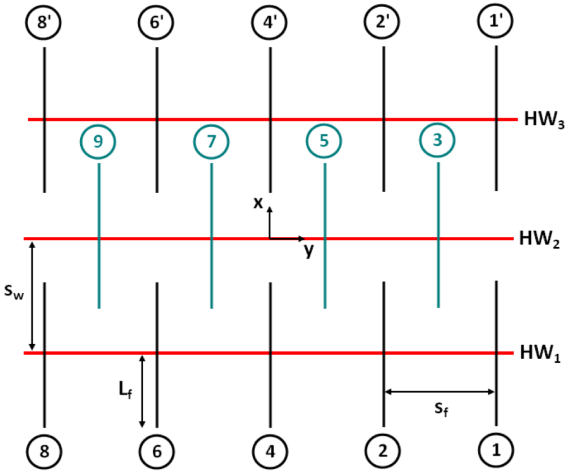
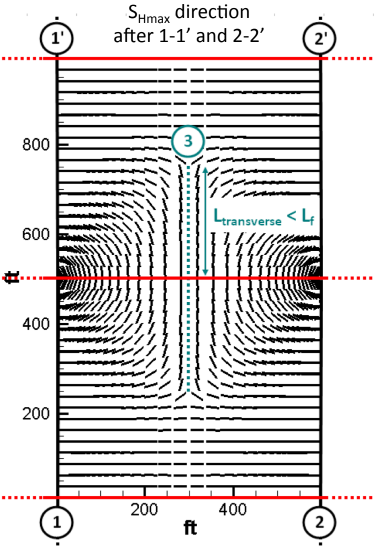


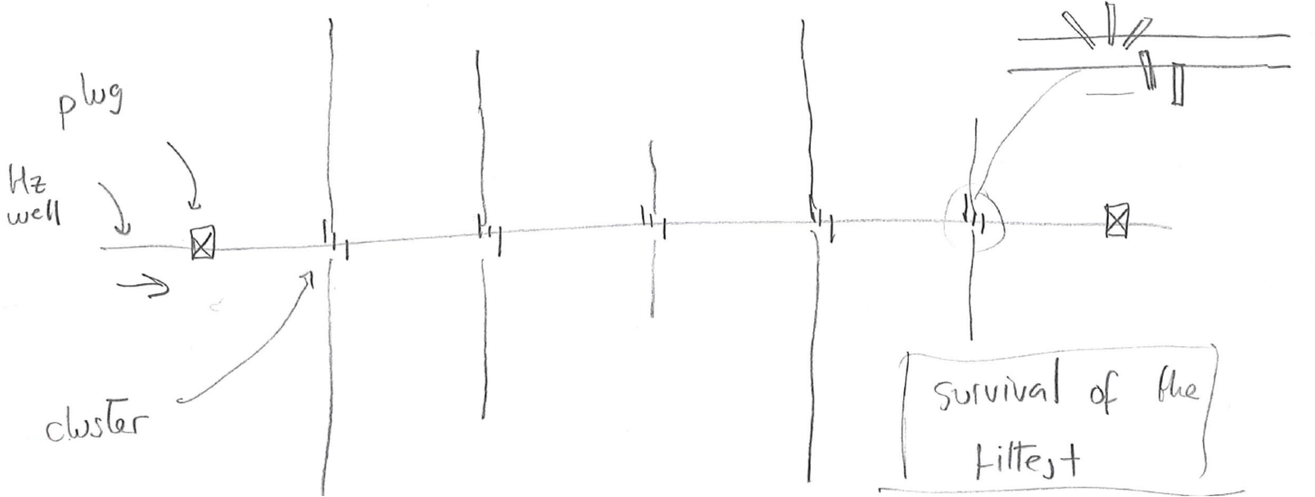
Fig. 10 – Trajectory of multiple consecutive transverse fractures spaced 250 ft apart

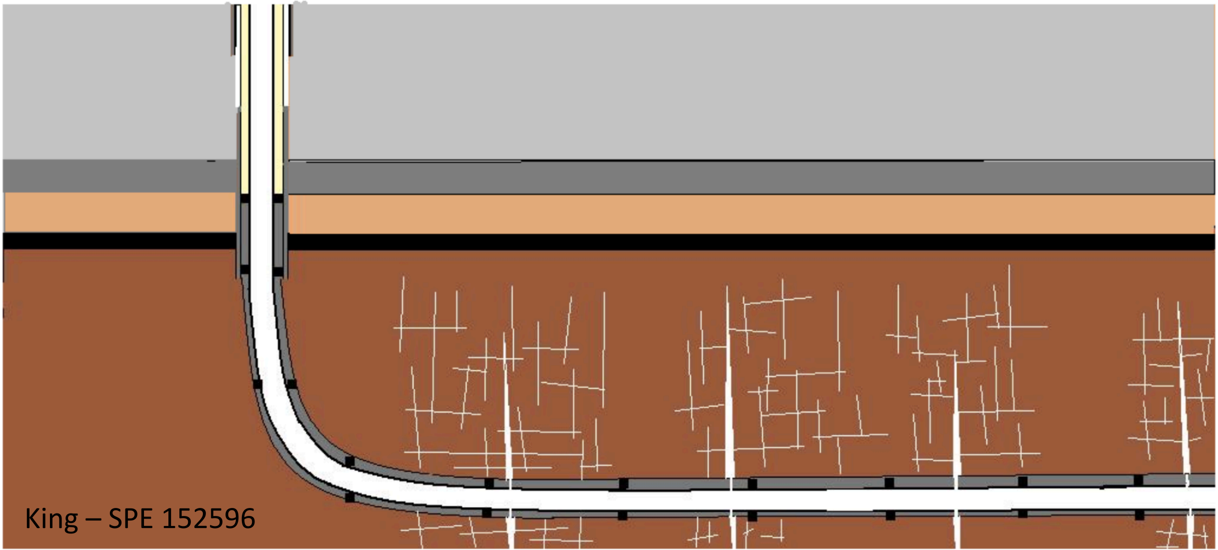
Roussel and Sharma 2011



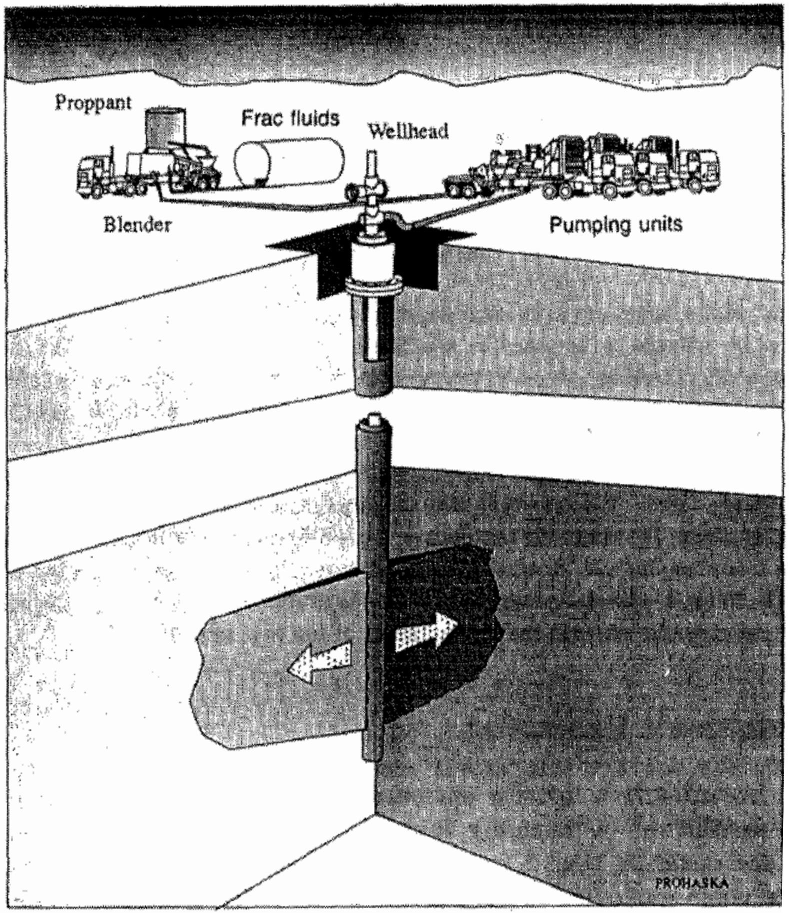


[Roussel and Sharma – SPE 146104]


plug
Hz well
cluster
survival of the fittest


King – SPE 152596

../../Fig



From
Valko and Economides, 1996

Surf Press [Tbg] (psi)
Slurry Flow Rate (bpm)
Proppant Conc (ppg)
SPE-169959
5000
20.00
4000
16.00
3000
12.00
2000
8.000
1000
4.000
0.0
0.0
60.00
48.00
36.00
24.00
12.00
0.0
210.0
226.0
242.0
258.0
274.0
290.0
Time (min)

$$E_{static} = 0.65 E_{dynamic}$$

$$E'_{static} = E_{static}/(1 - \nu^2)$$

$$\varepsilon_{Hmax} = 0.0015$$

$$E'$$

x_f