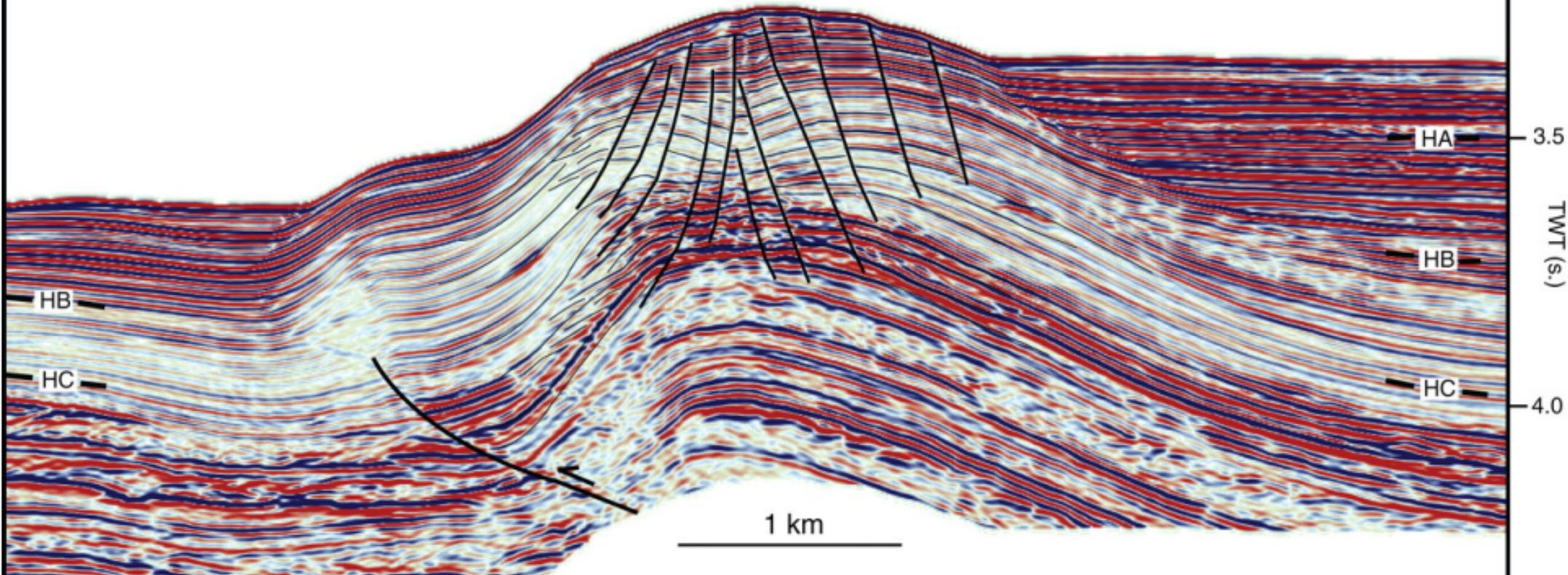
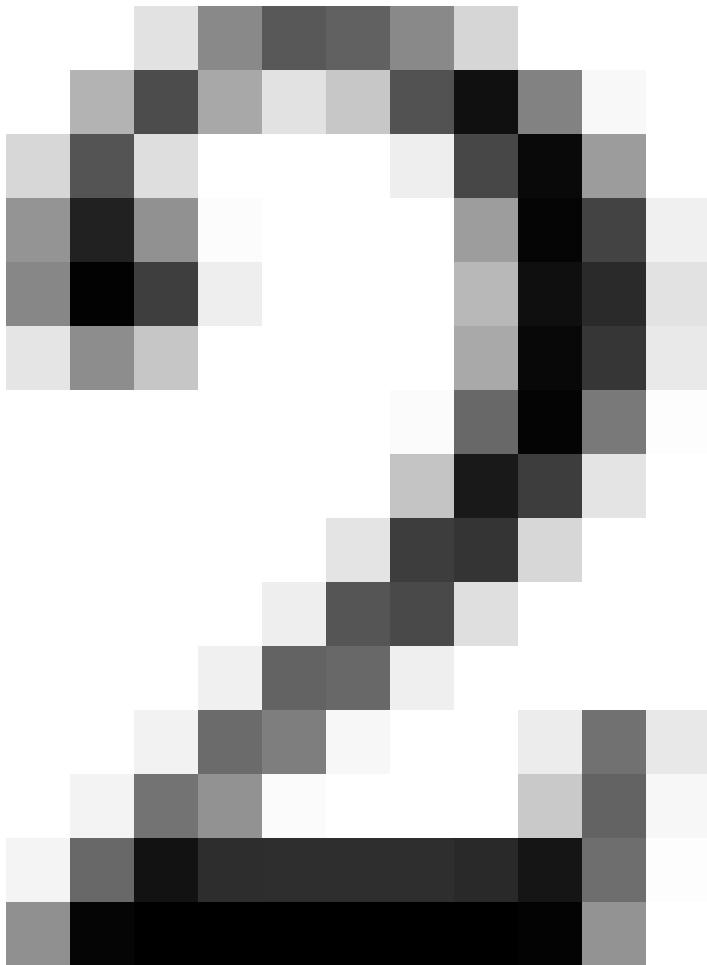


NW

Crestal normal faults

SE

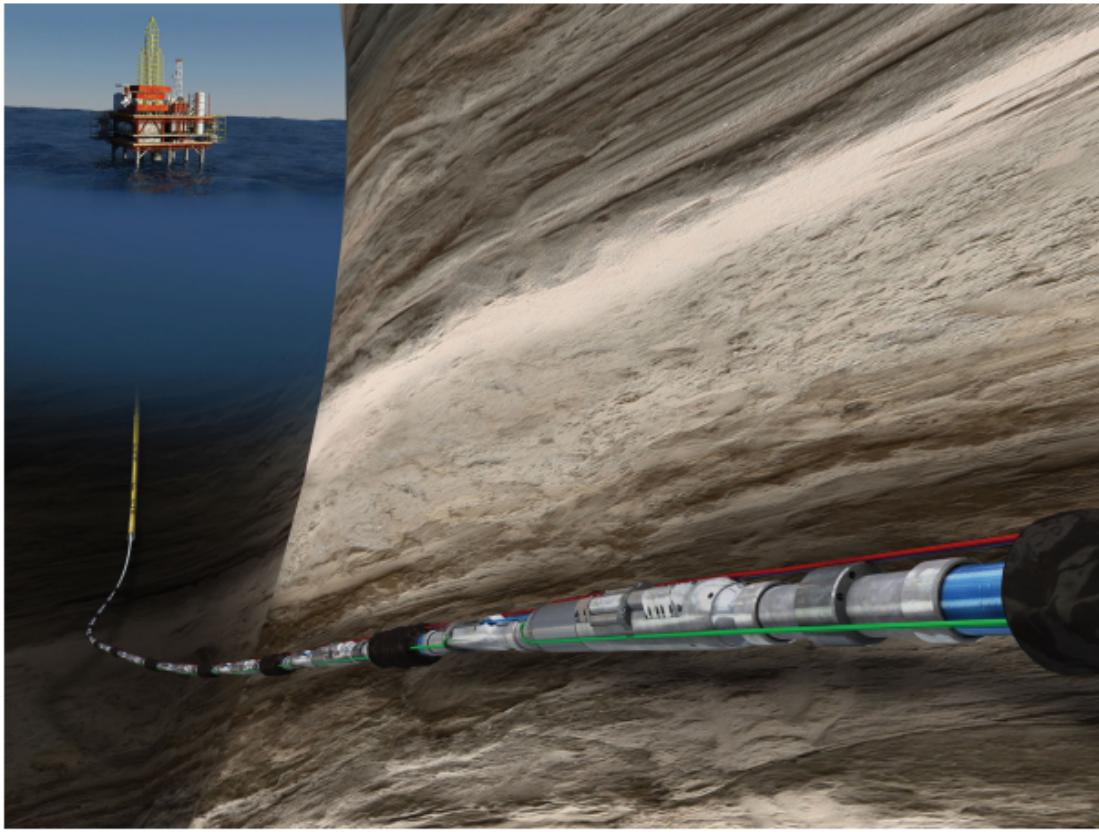


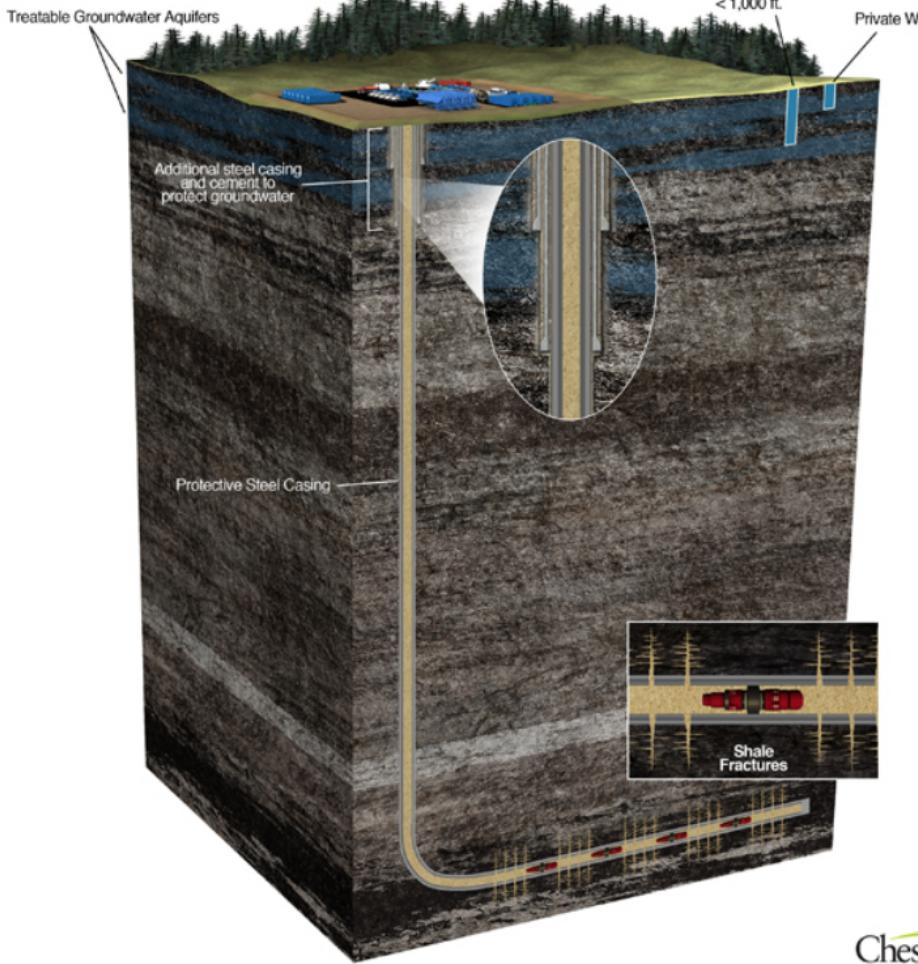


~1850



~2010





Risk-Based  
Geomechanical  
Screening

# Stress Man

MUDLINE SUBSIDENCE



FAULT ACTIVATION

CASING CRUSHING

COMPACTION DRIVE

CASING  
SHEAR

SAND PRODUCTION



RESERVOIR YIELD

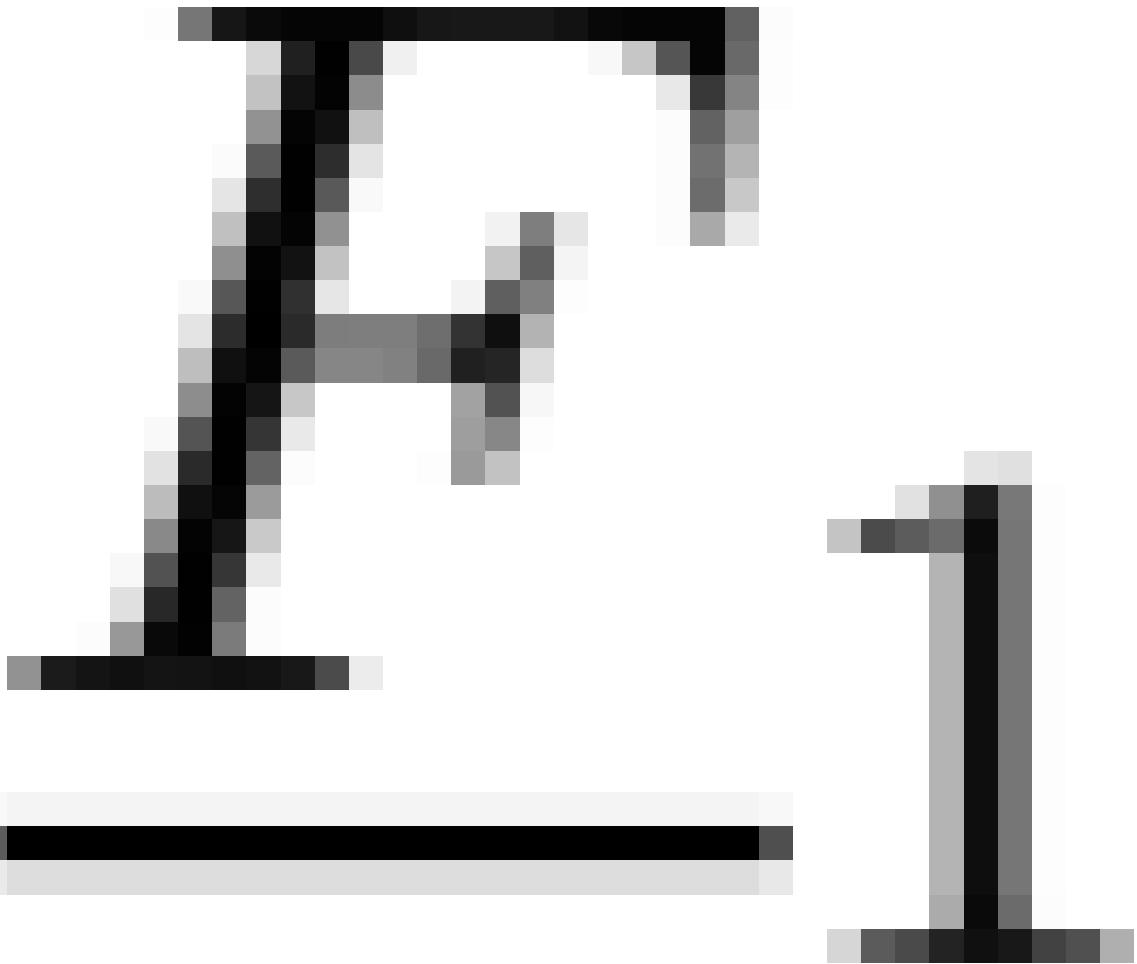
PERMEABILITY LOSS

CASING BUCKLING

$S_V$

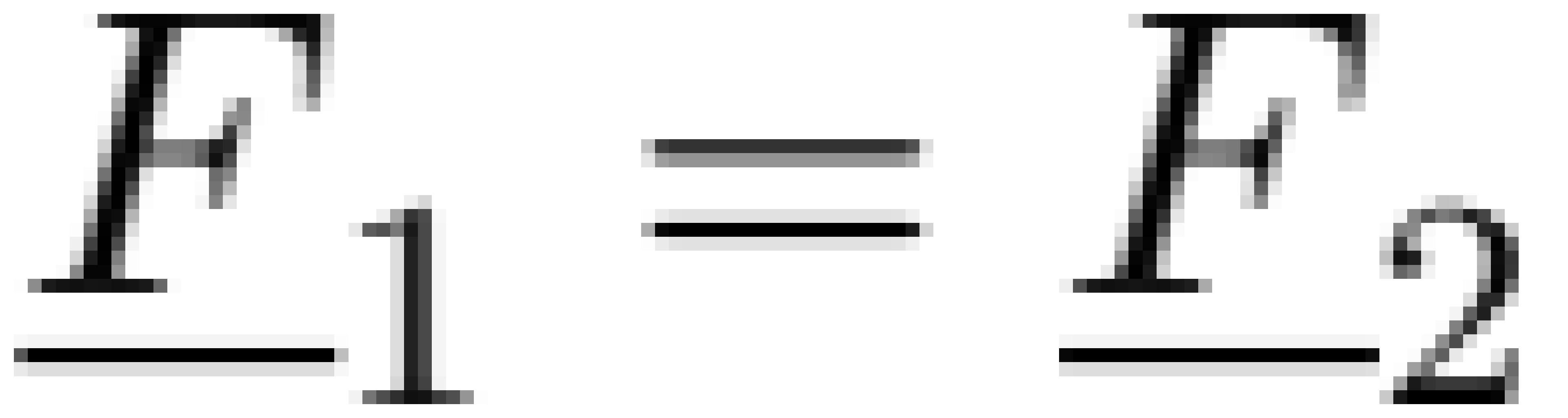




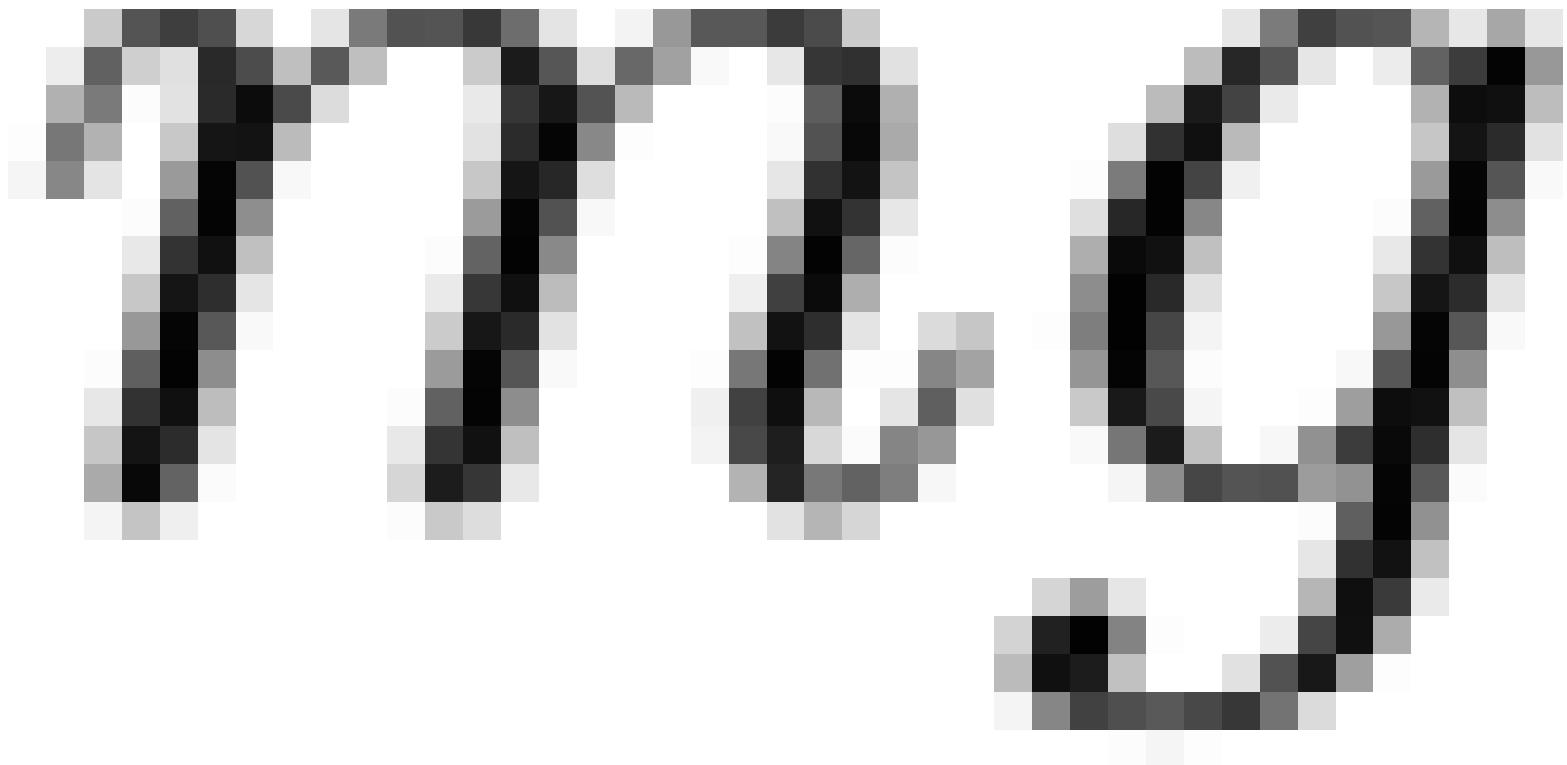


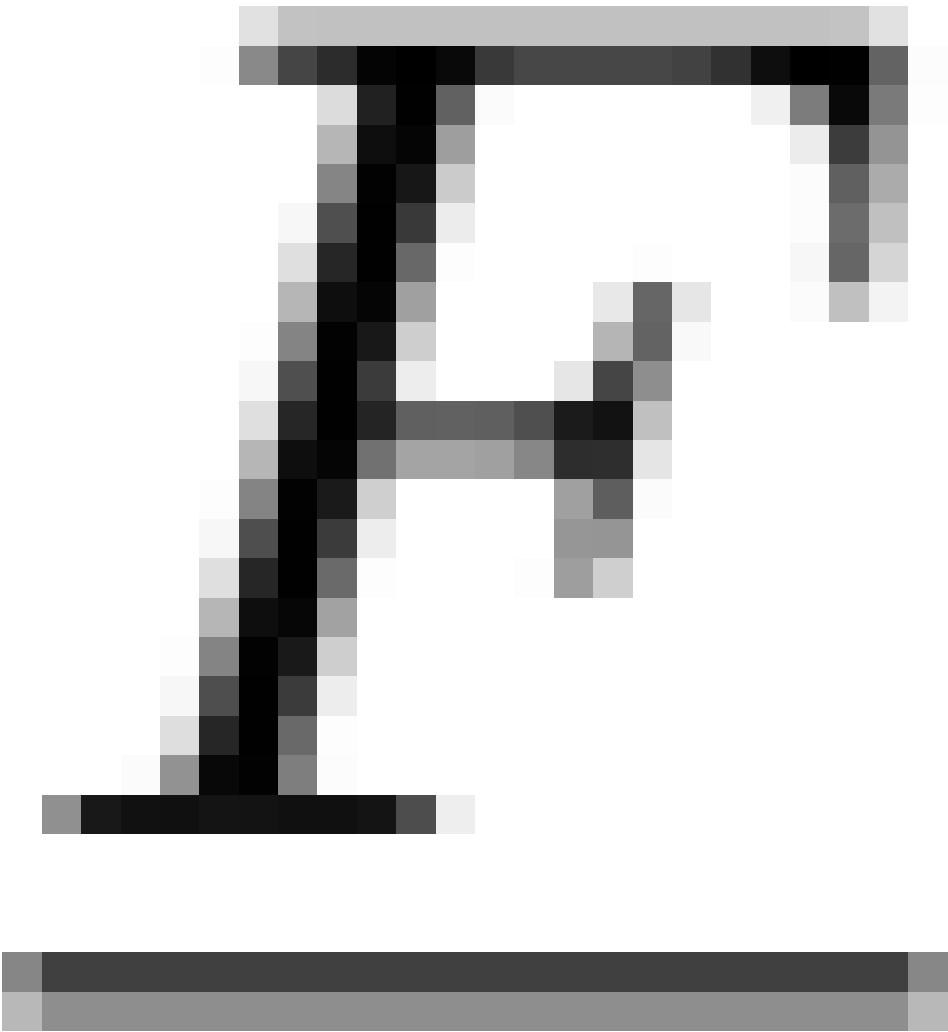


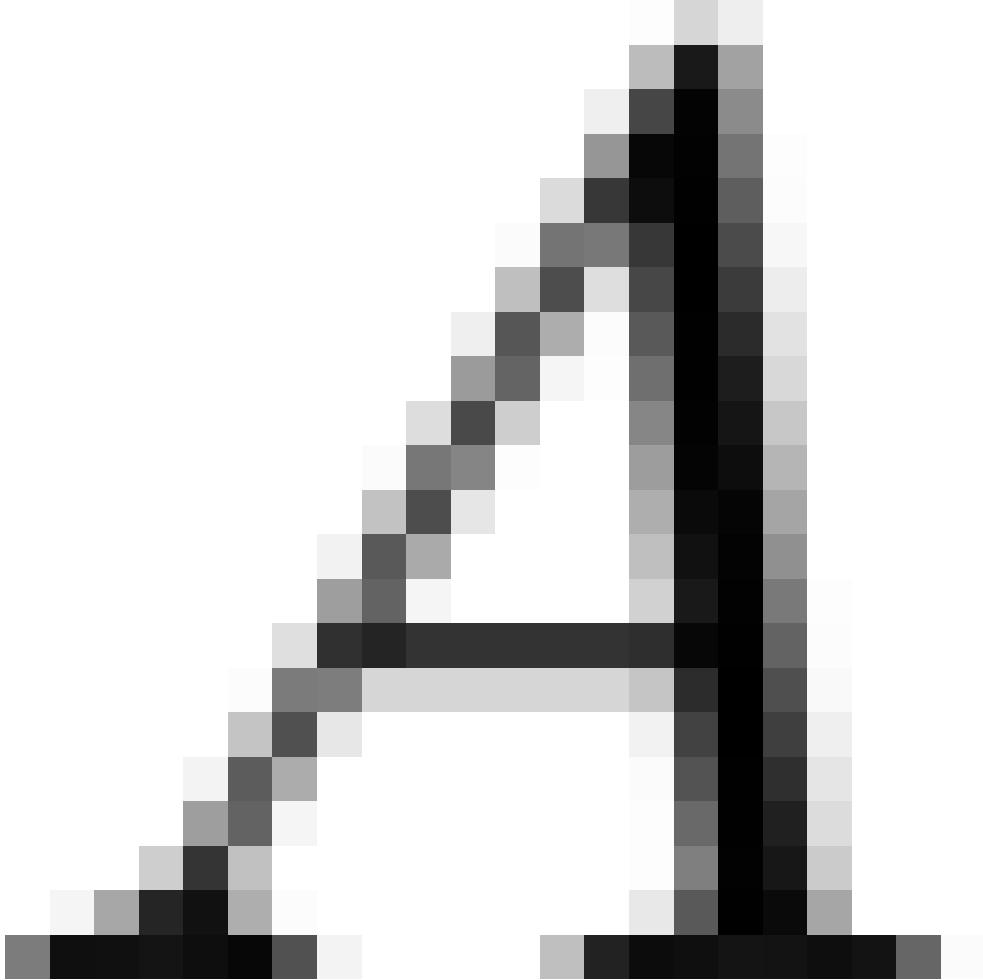












E

S

S

A

S

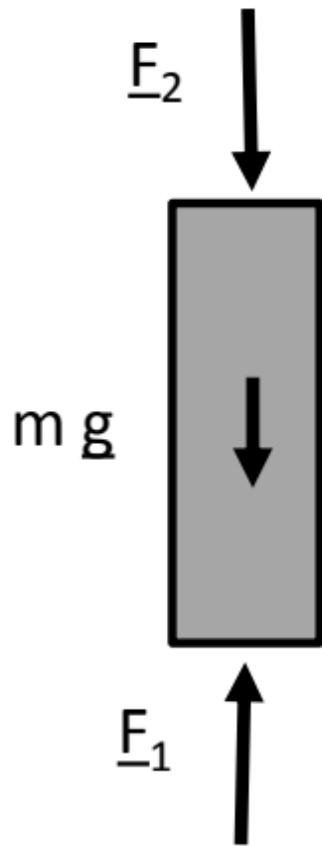
S

C

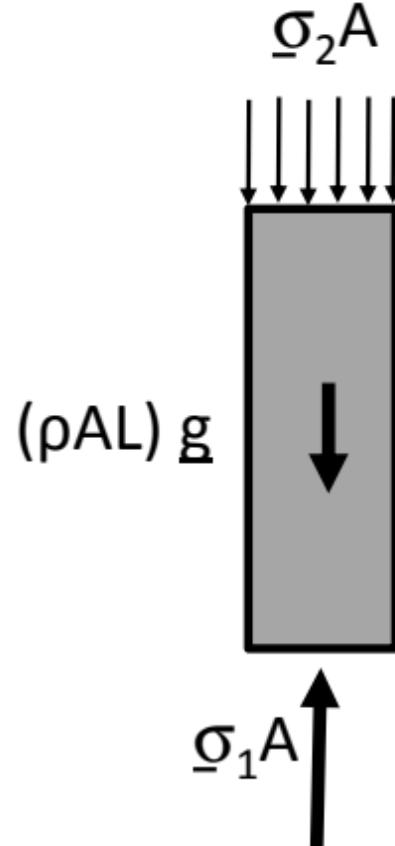
S



$$\Sigma F_z = +F_1 - F_2 = 0$$

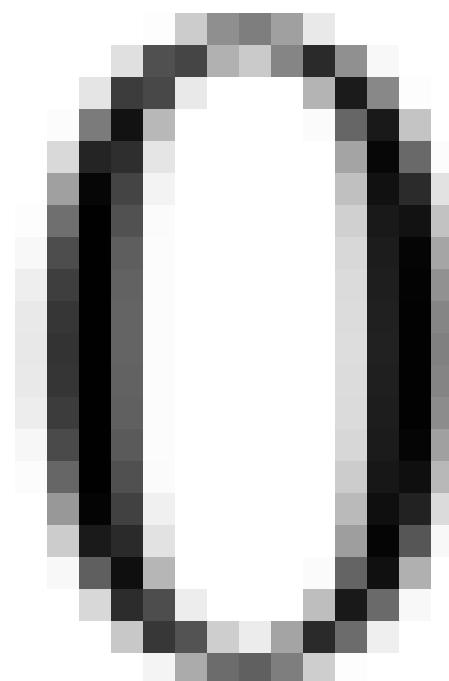
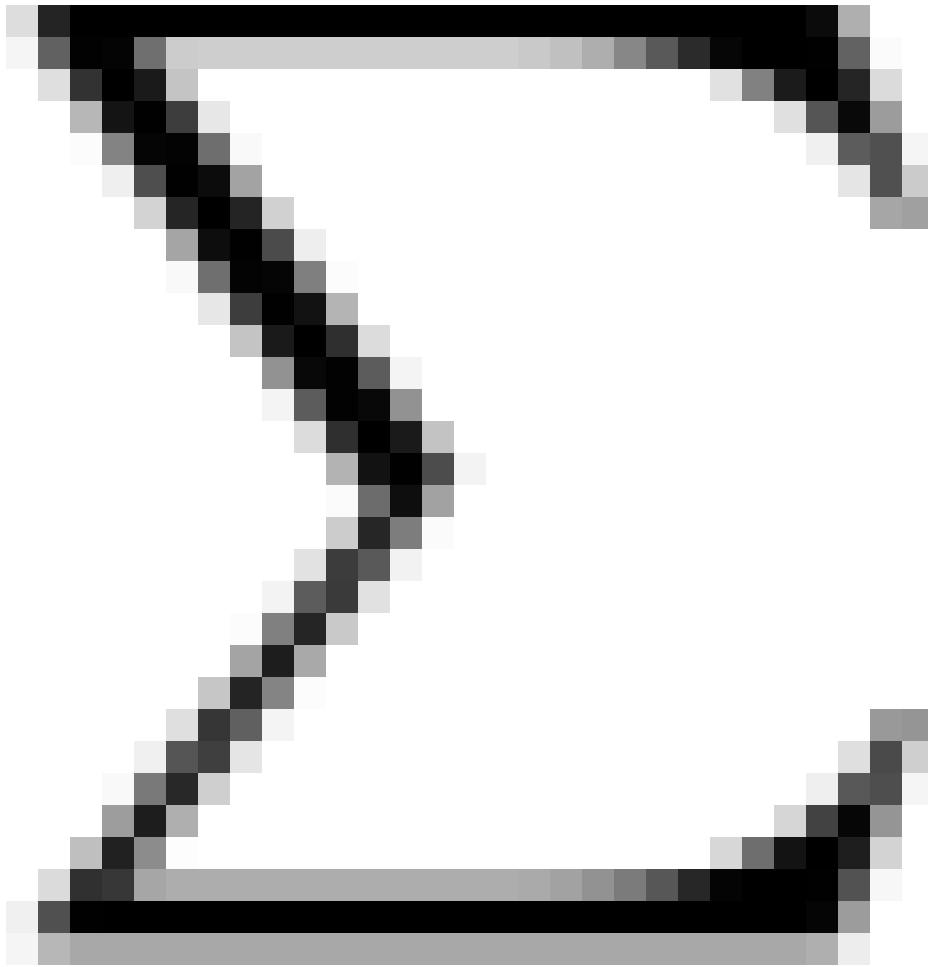


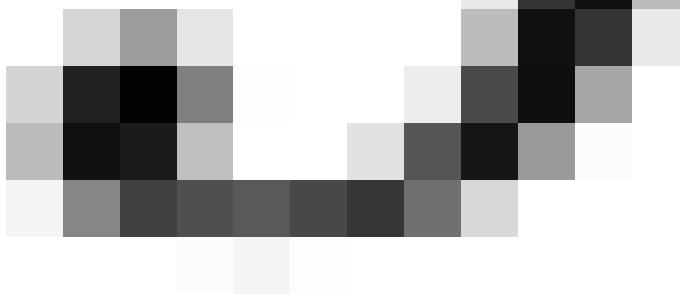
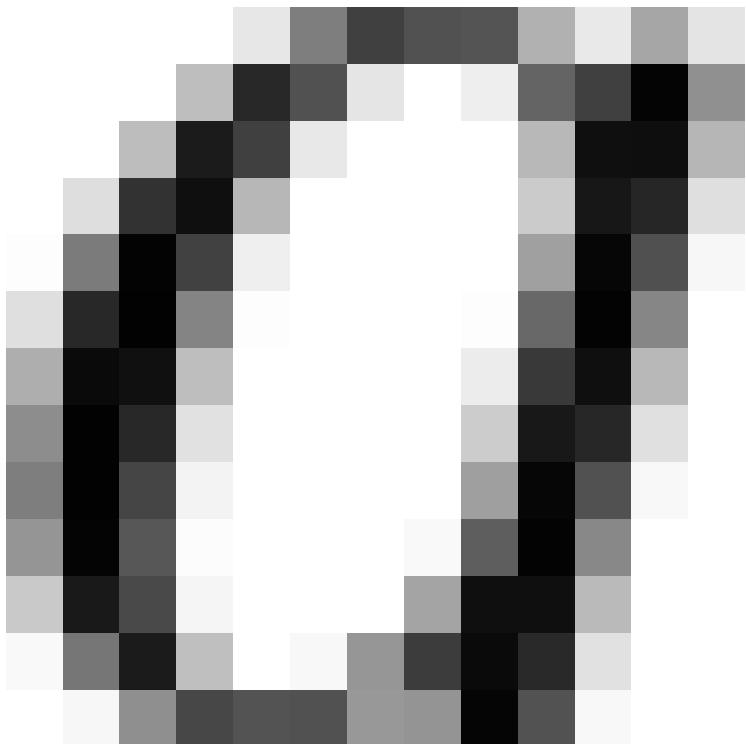
$$\Sigma F_z = +F_1 - m g - F_2$$

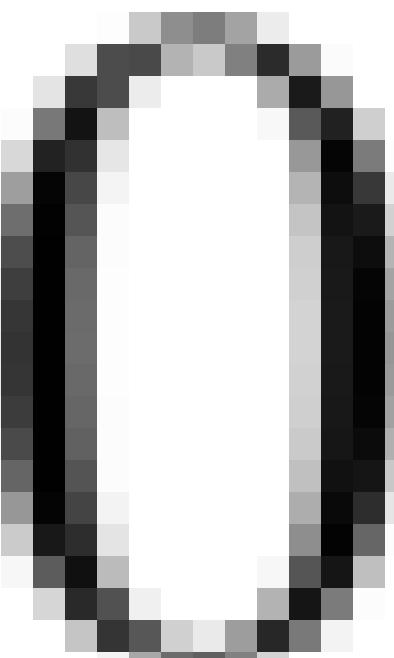
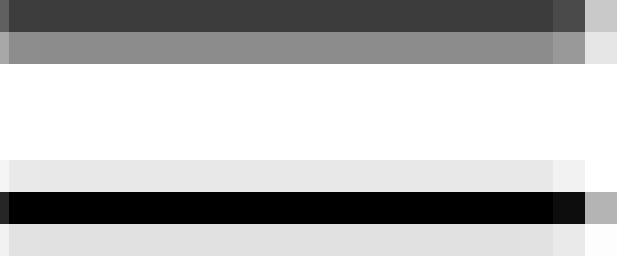
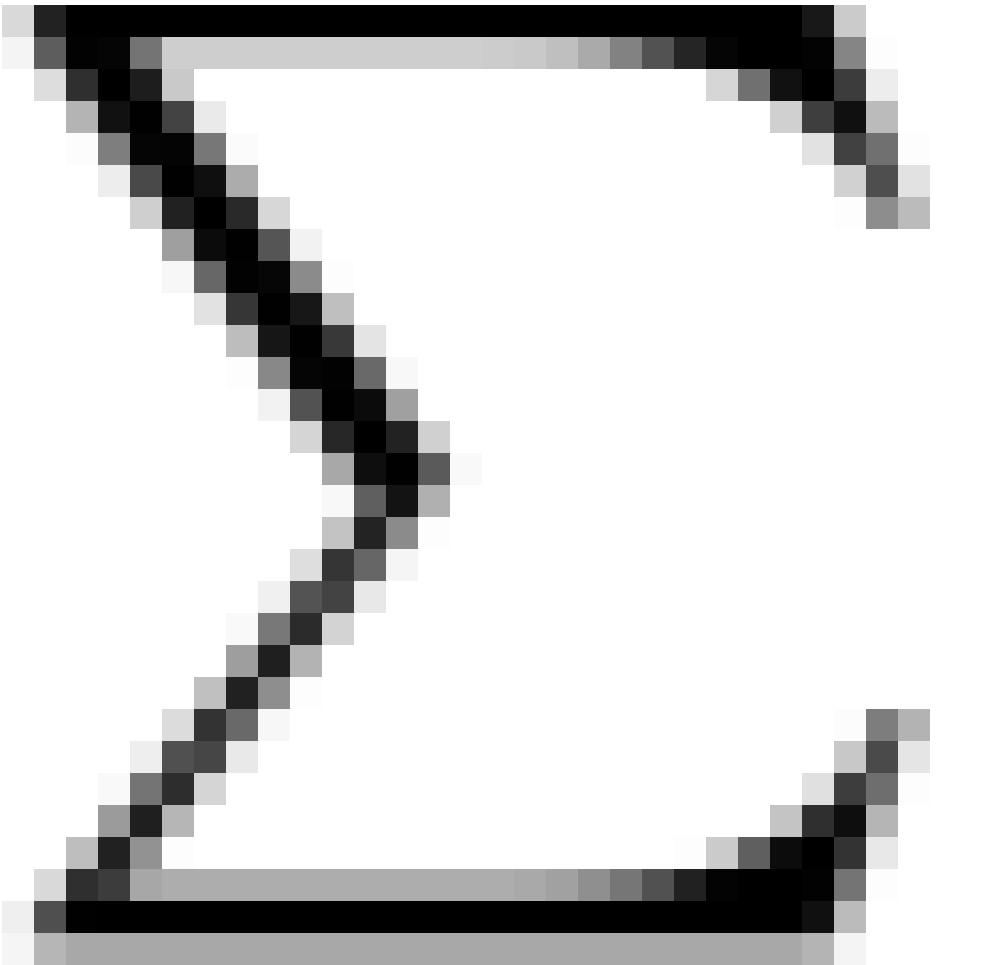


$$\Sigma F_z = +\sigma_1 A - (\rho A L)g - \sigma_2 A$$











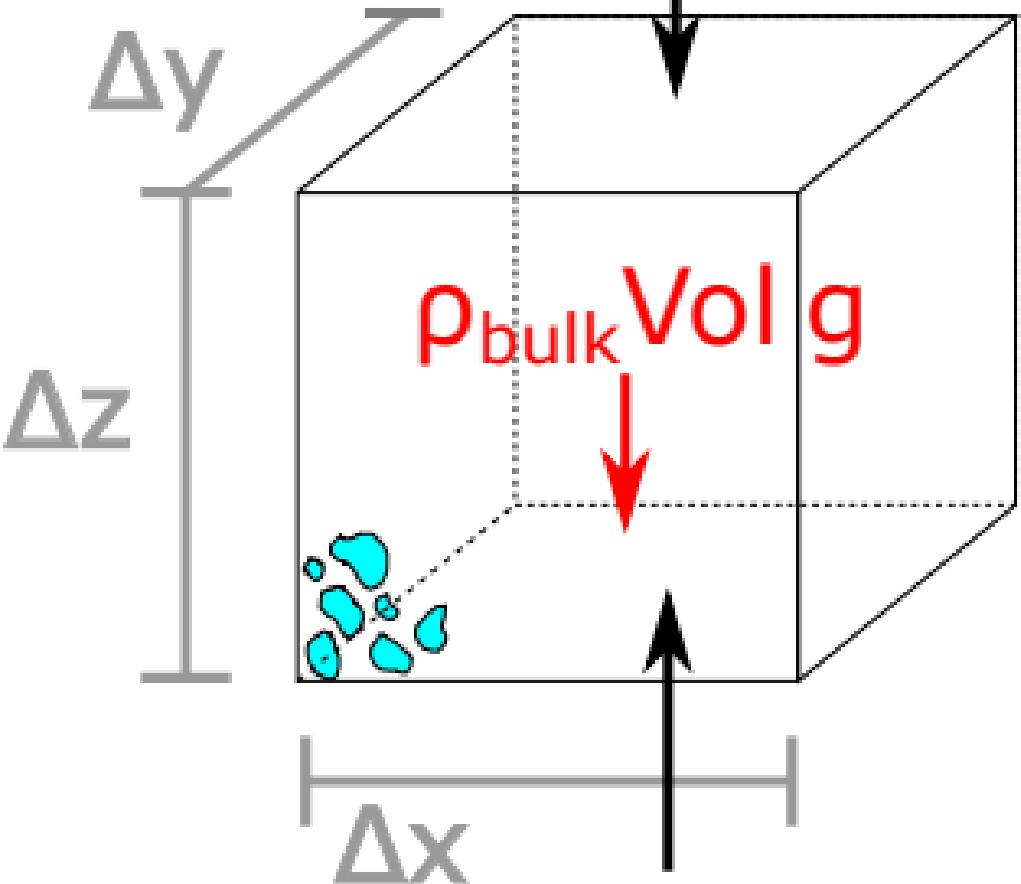


△ S<sub>2</sub>

— △ z

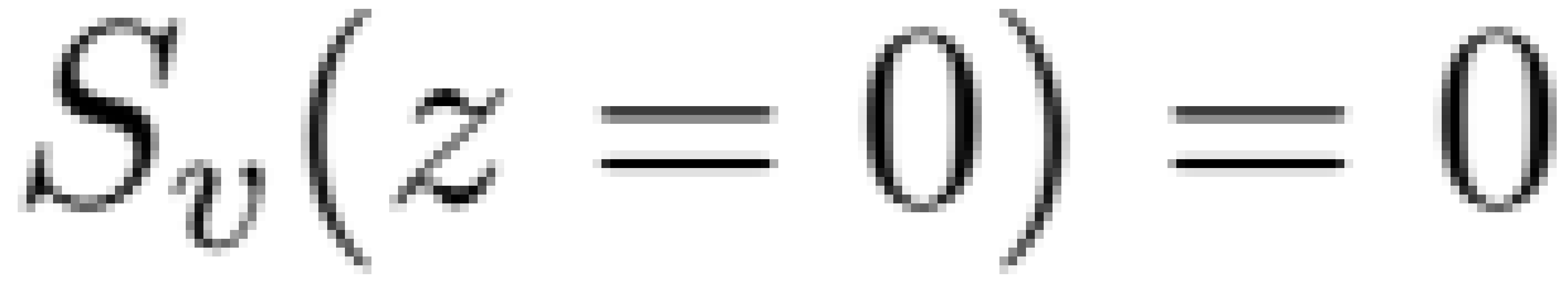
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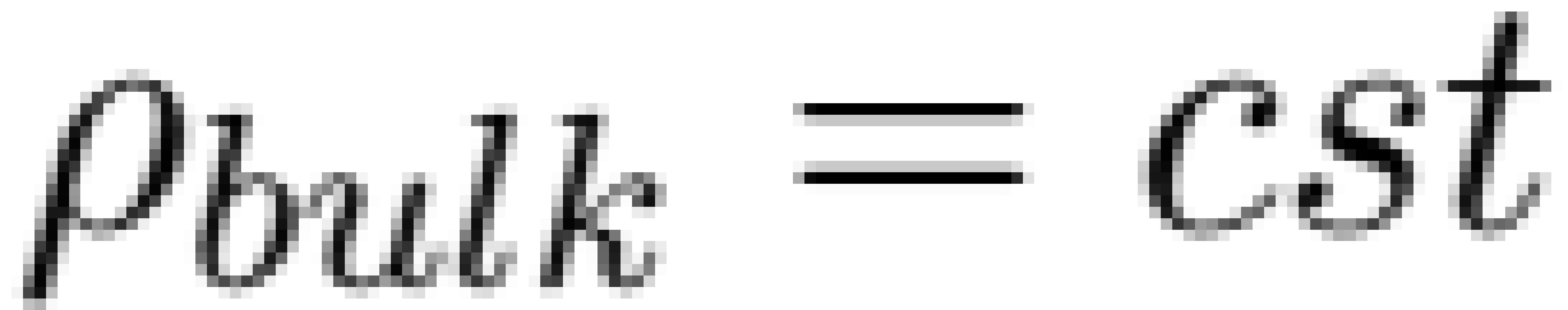
Observe 9

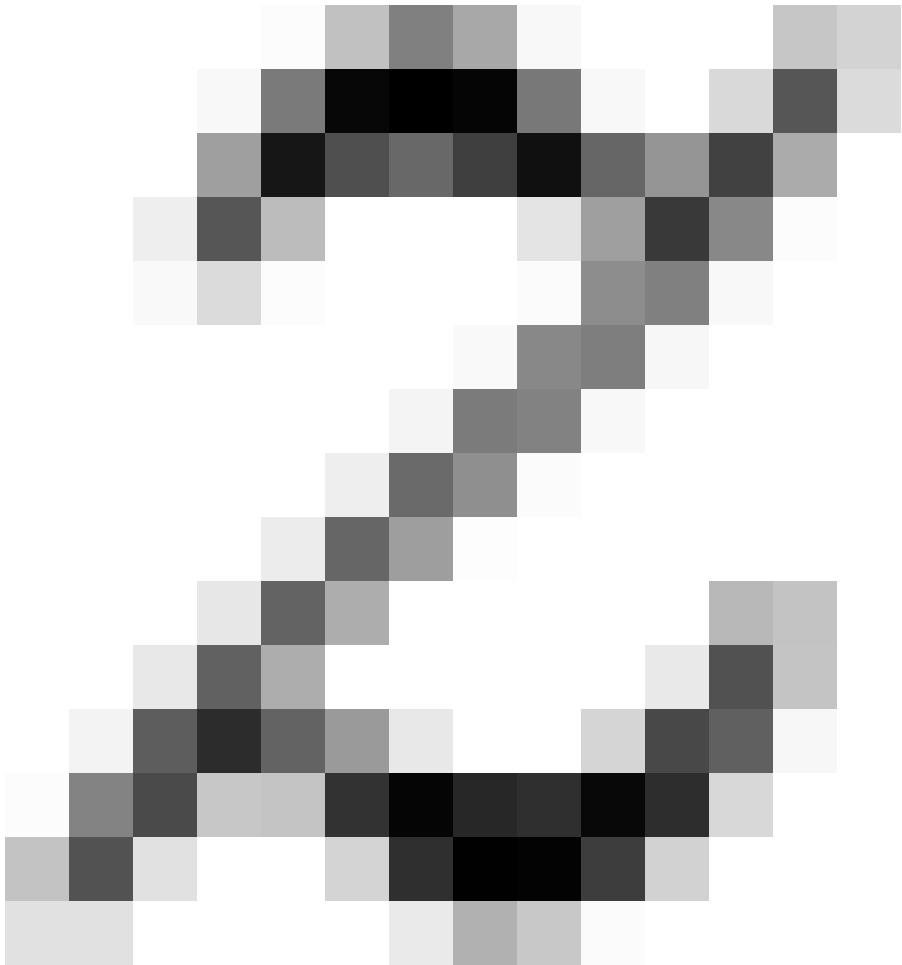
$S_v A$  $S_v A + \Delta S_v A$

$$\int_0^{S_w(z)} ds_w = \int_0^z \rho_{bulk}(z) g dz$$







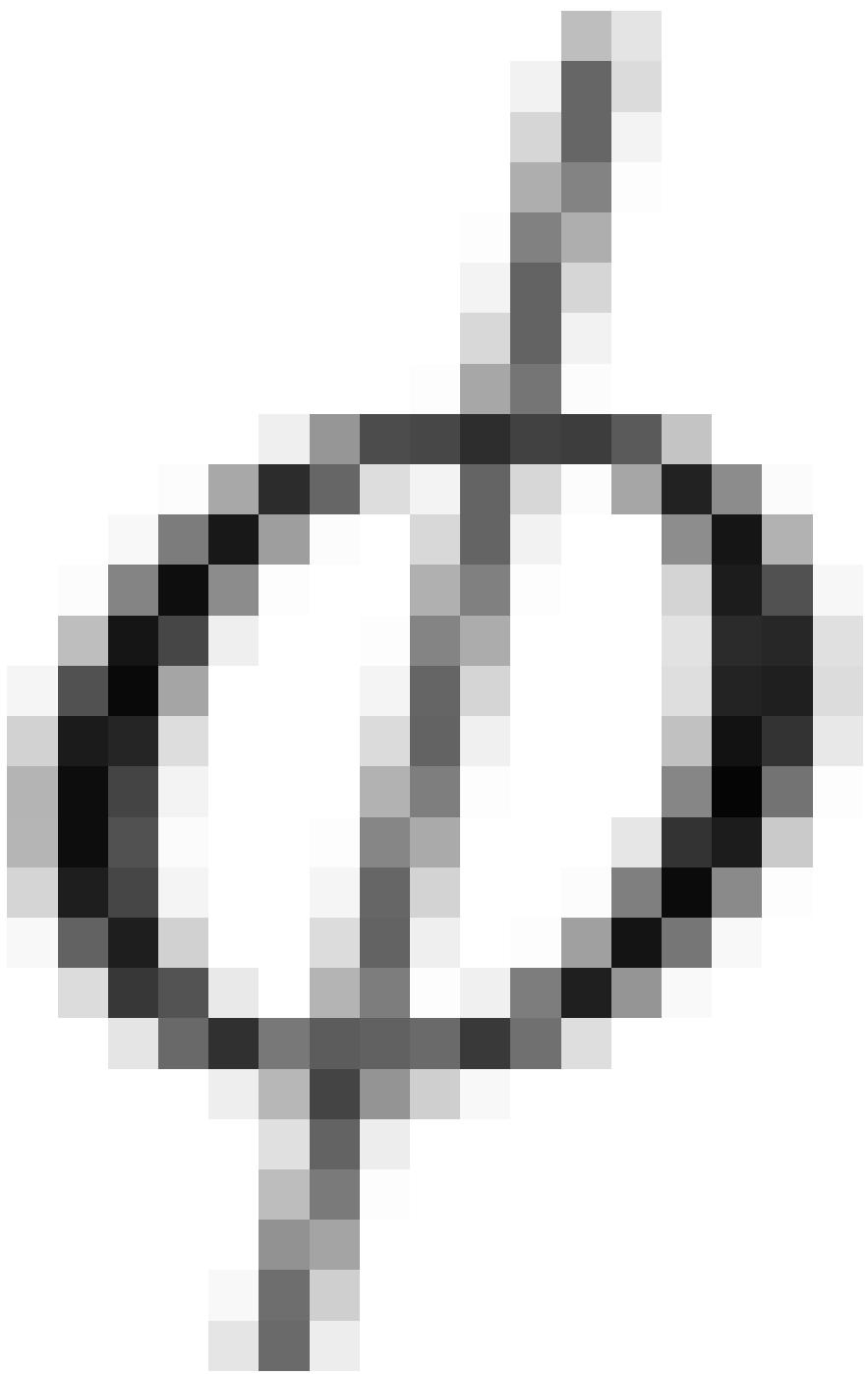


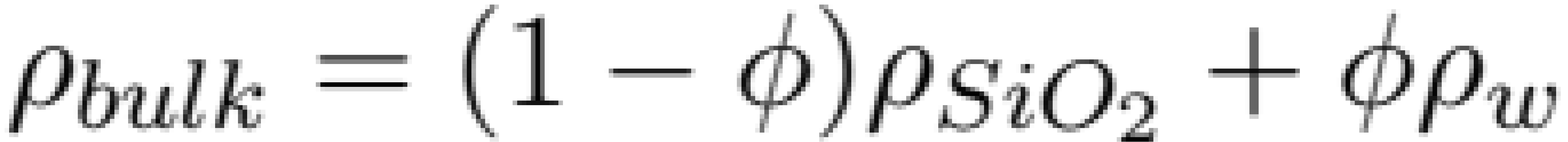


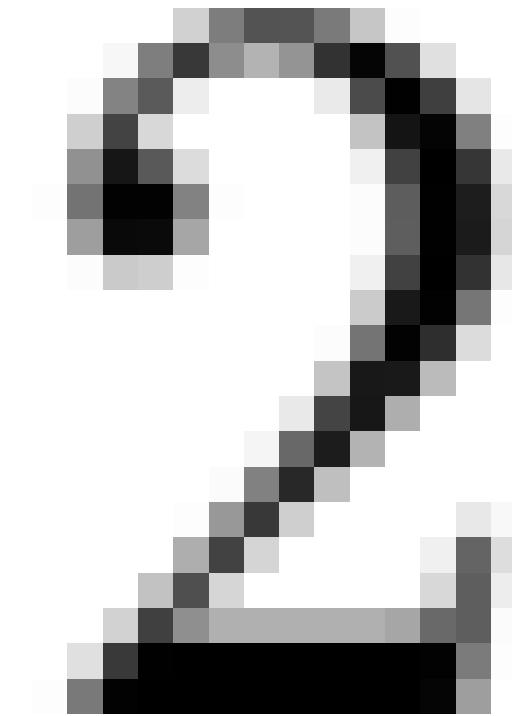
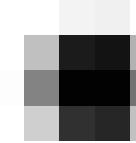
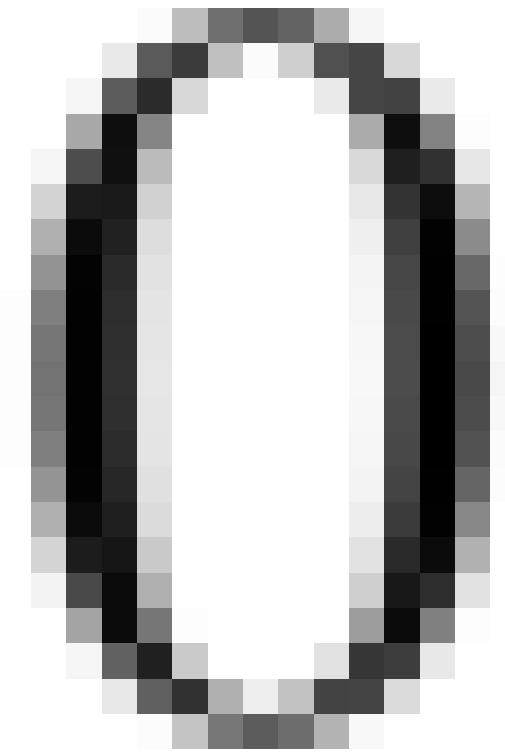
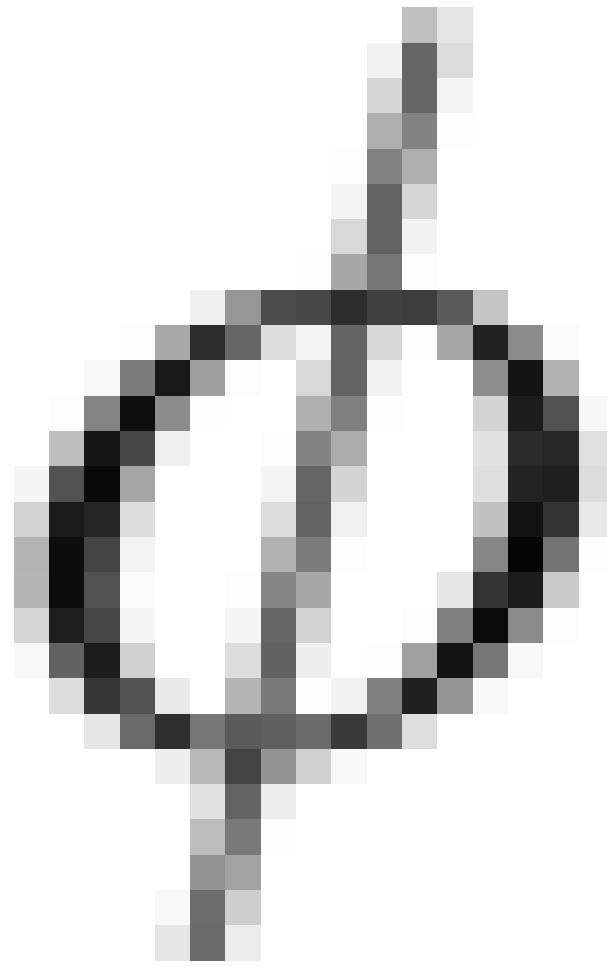




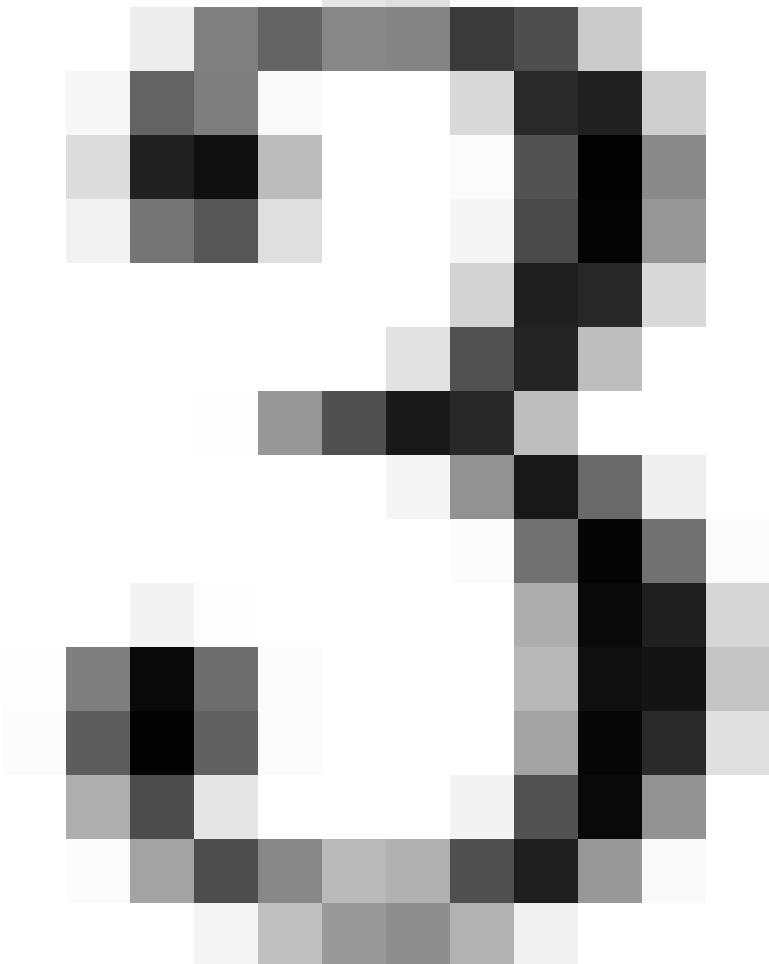


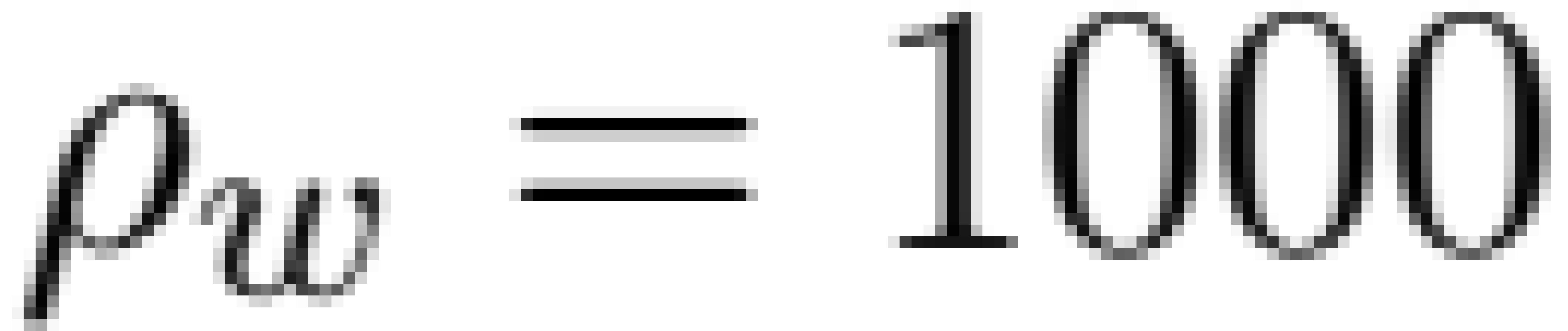


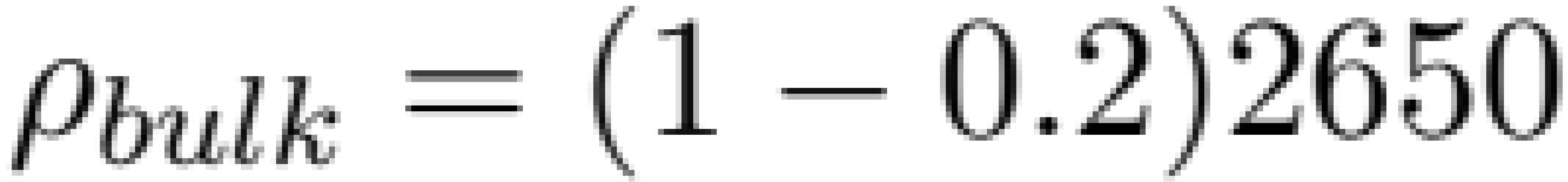


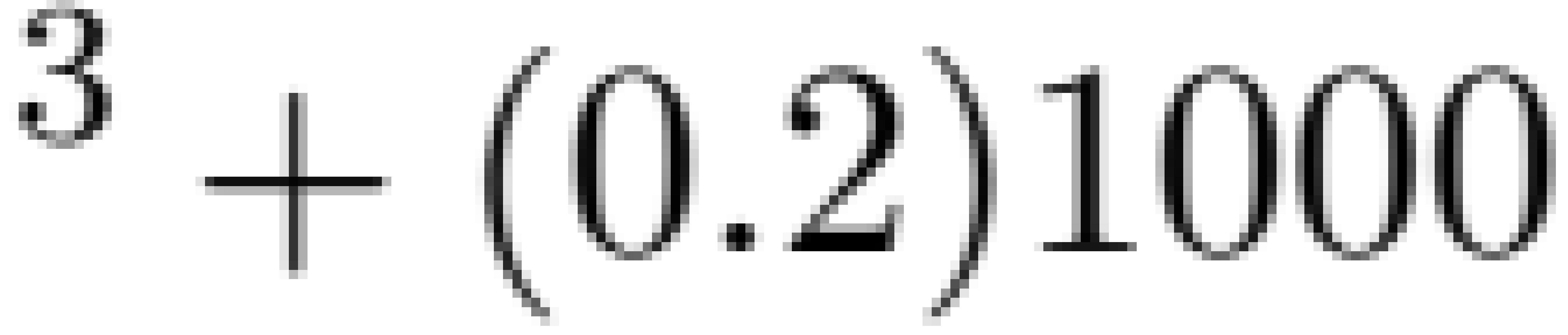




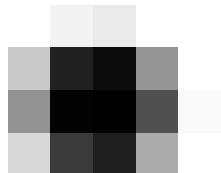
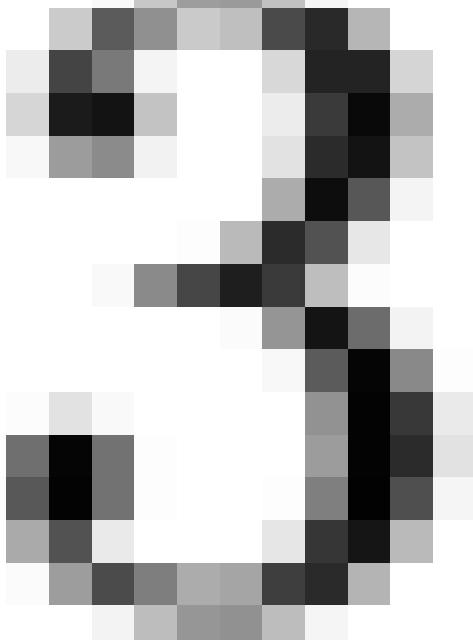


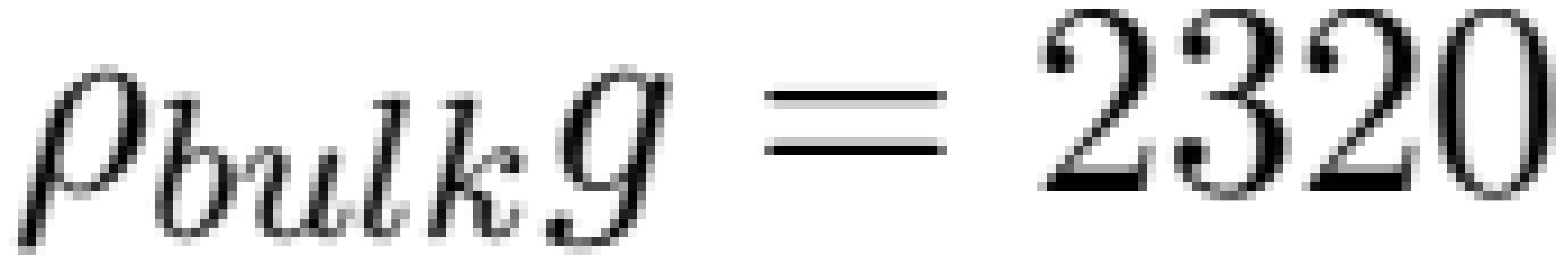


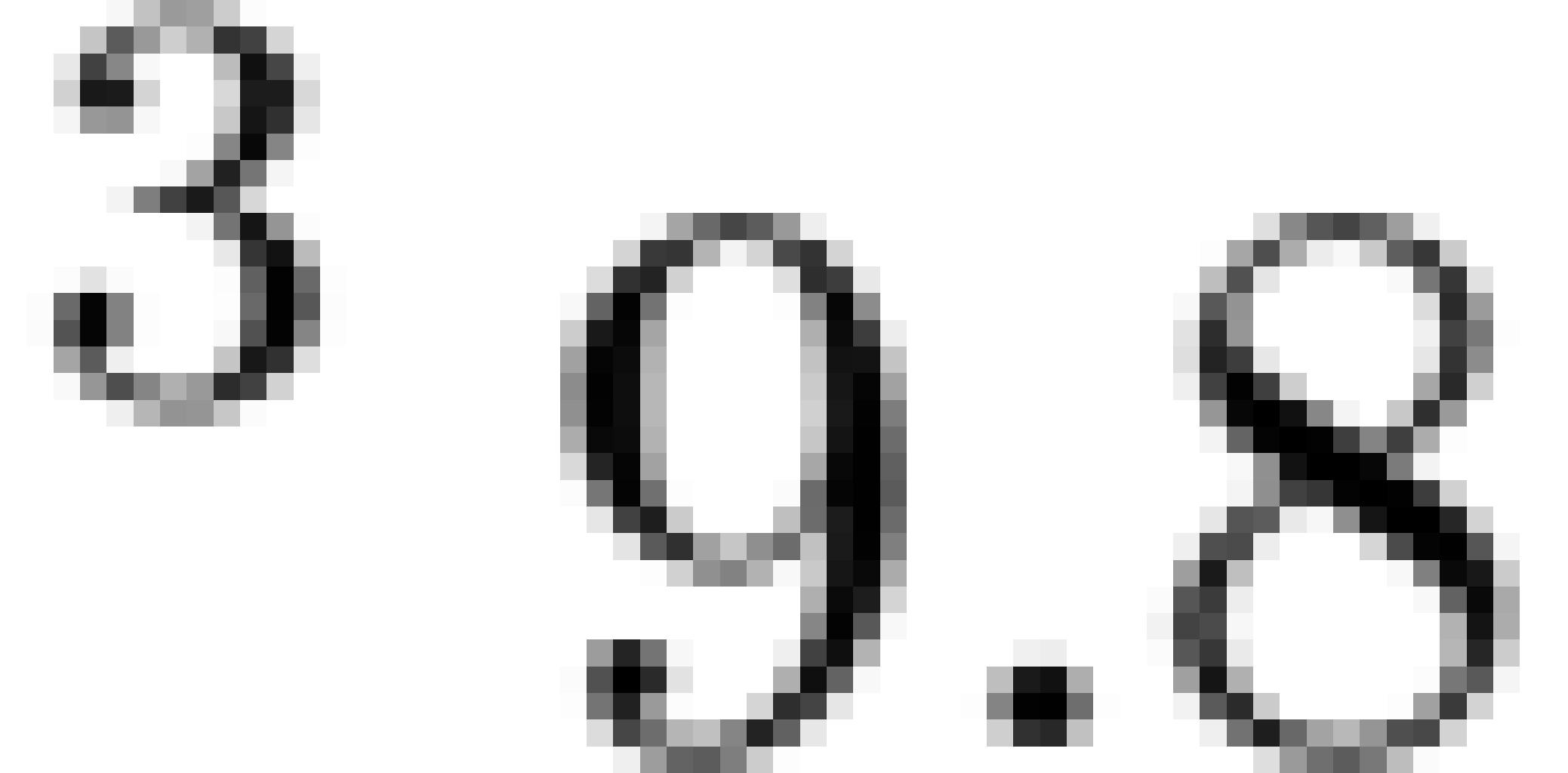




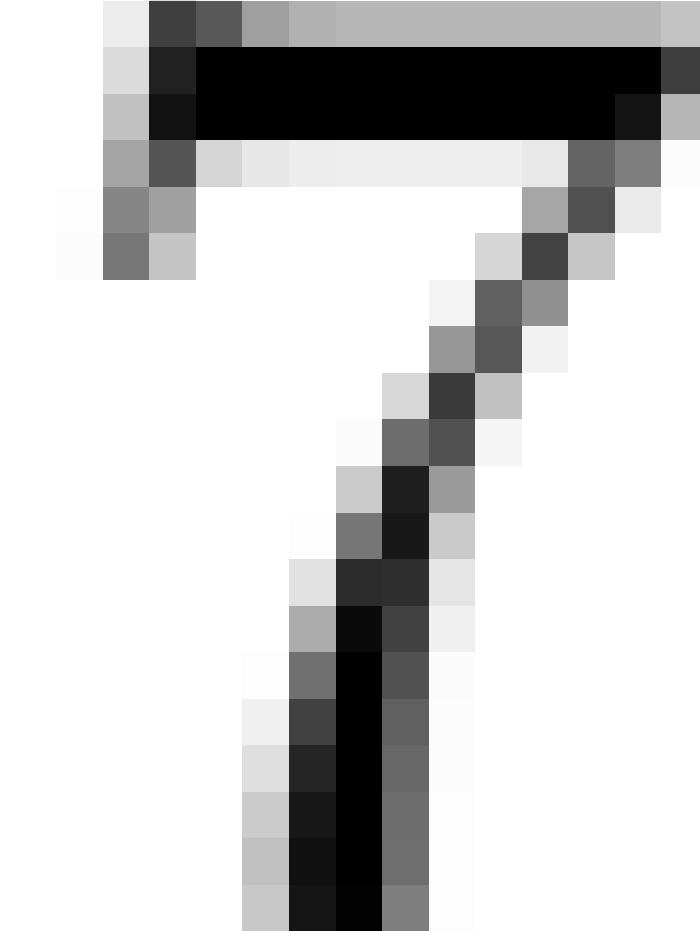
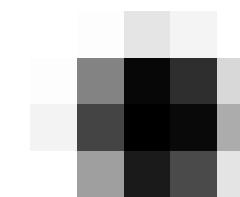
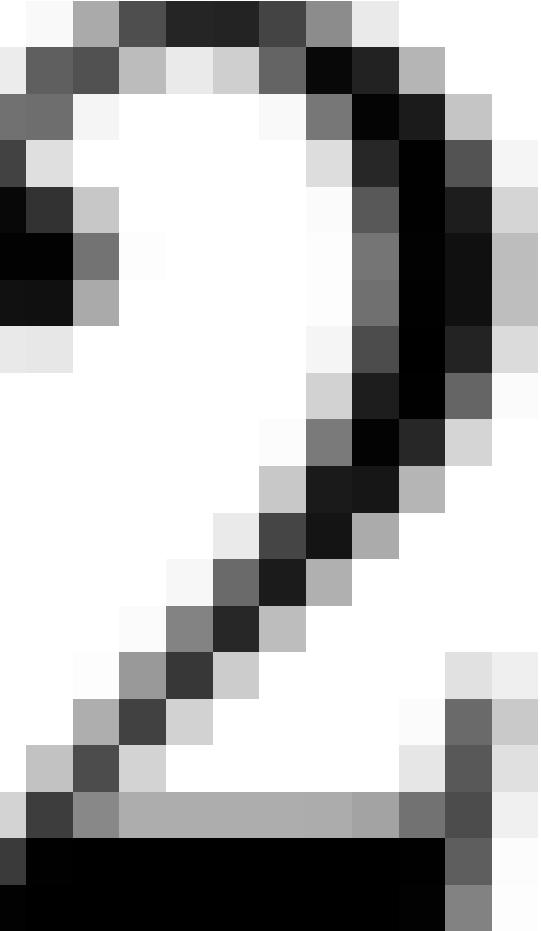
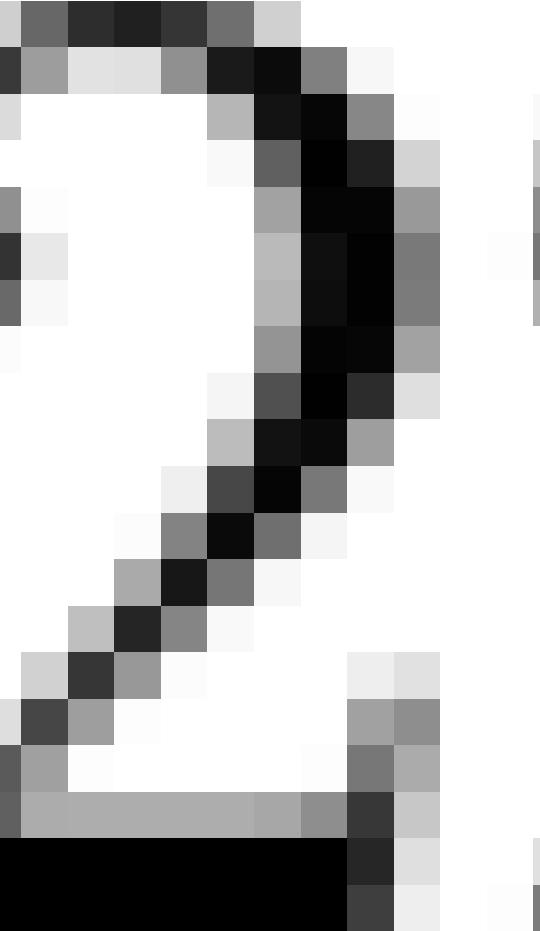


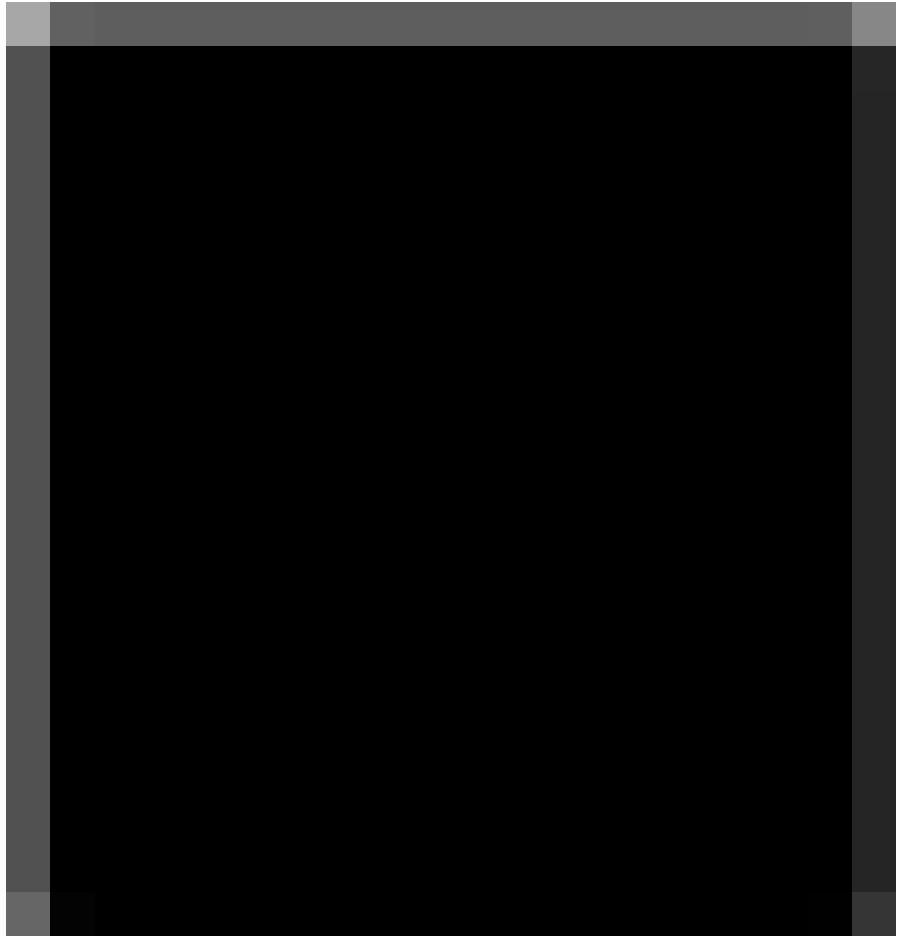
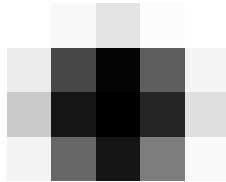


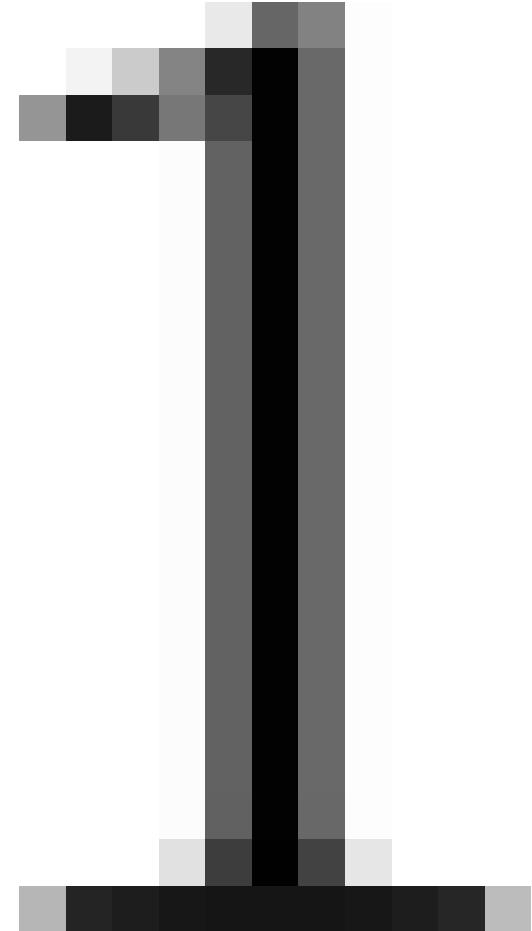
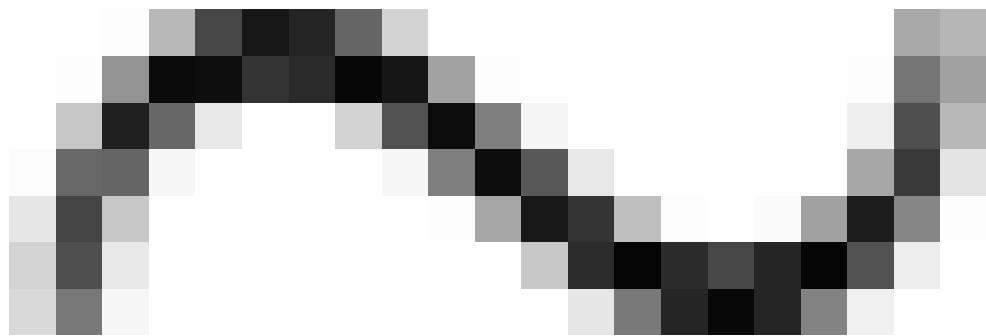


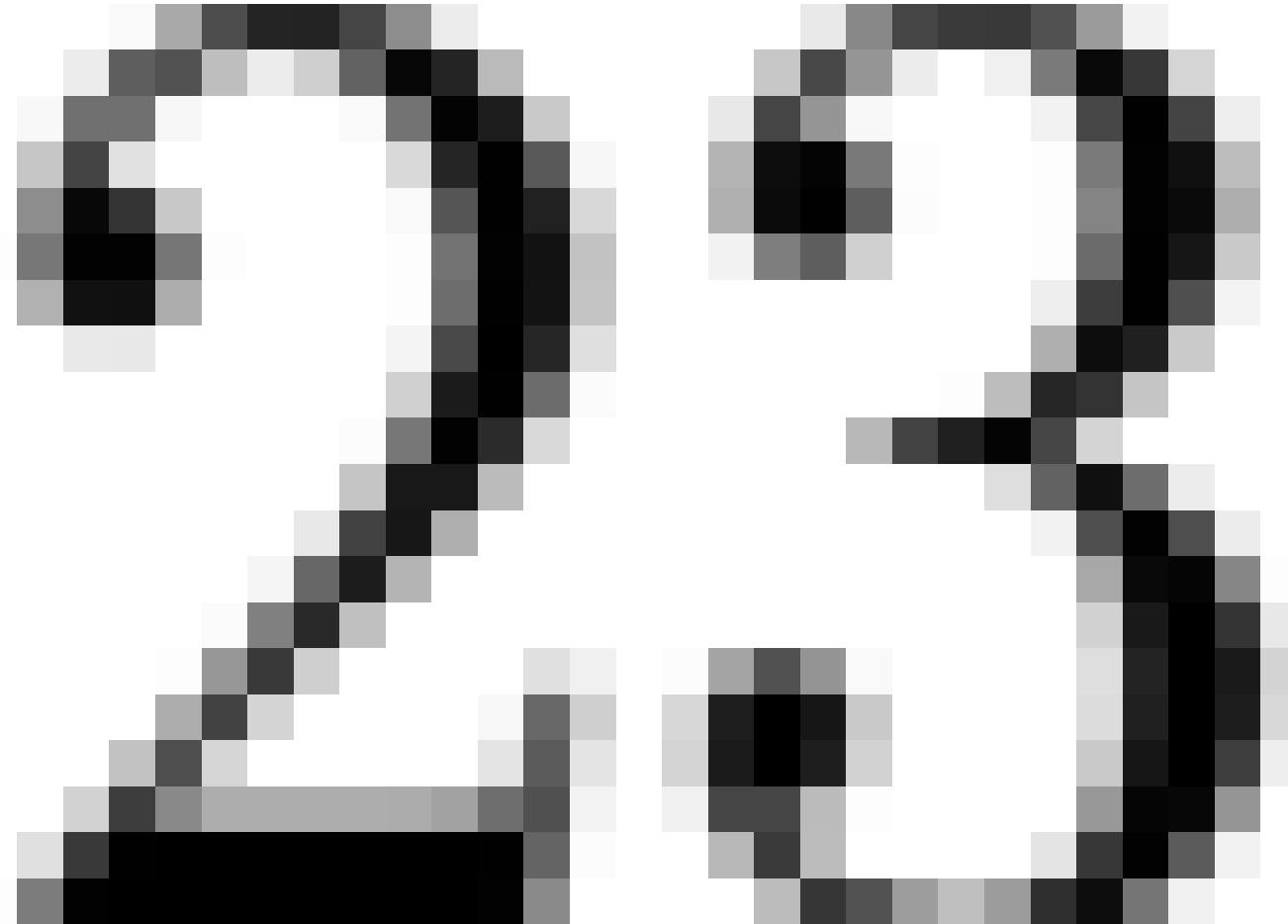
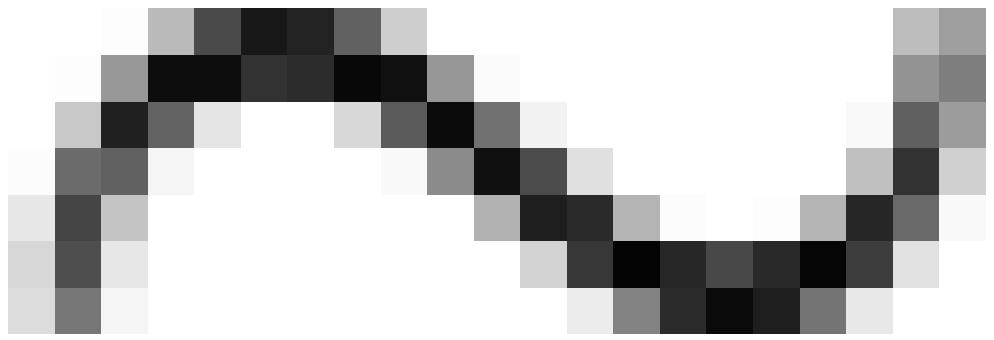


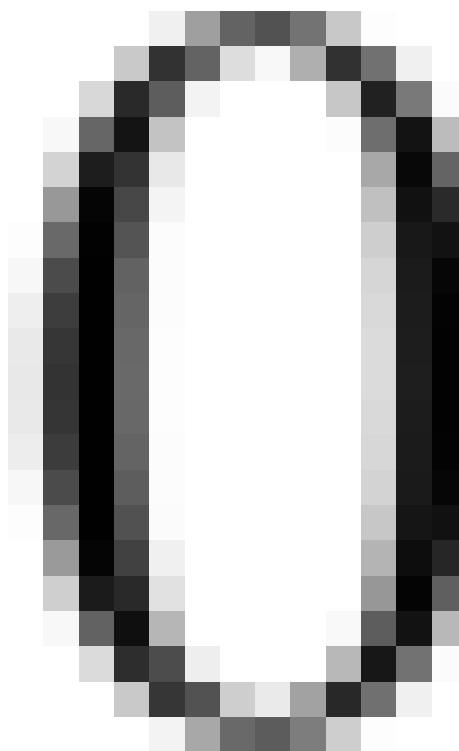
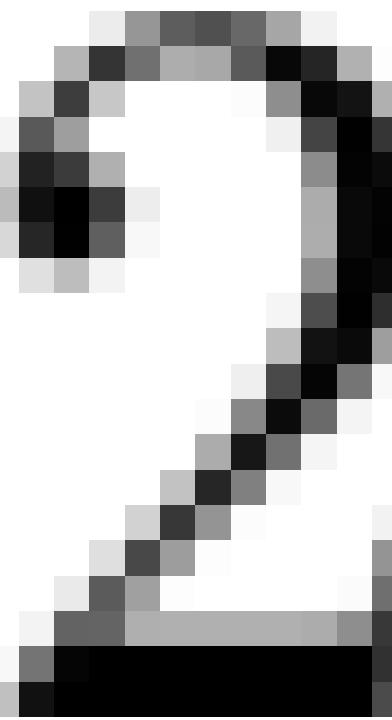
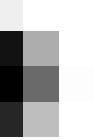
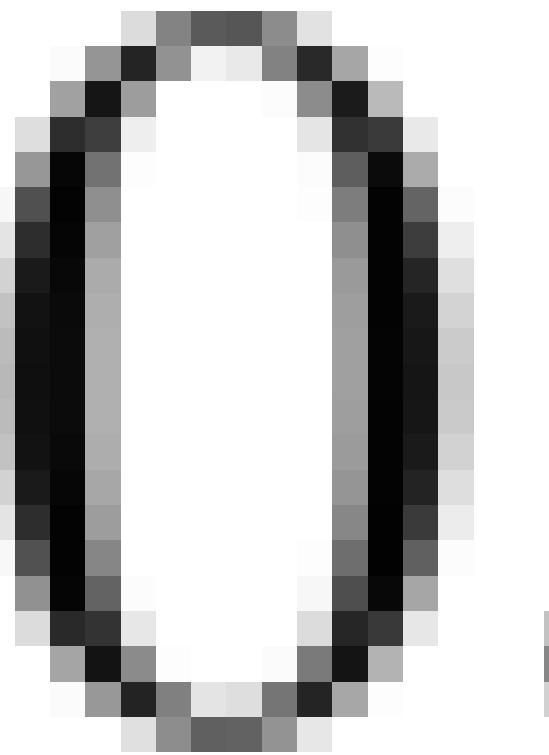
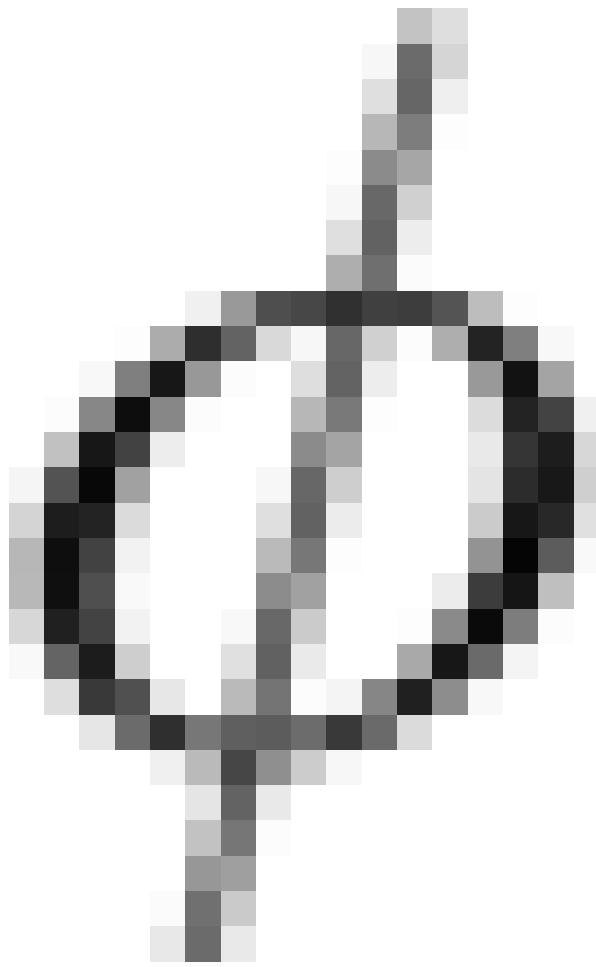


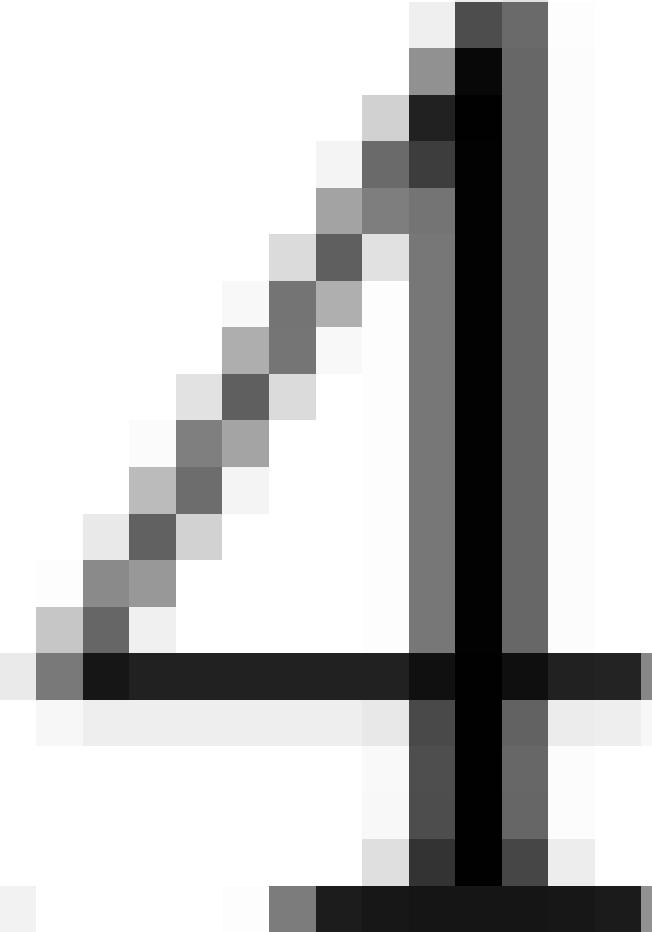
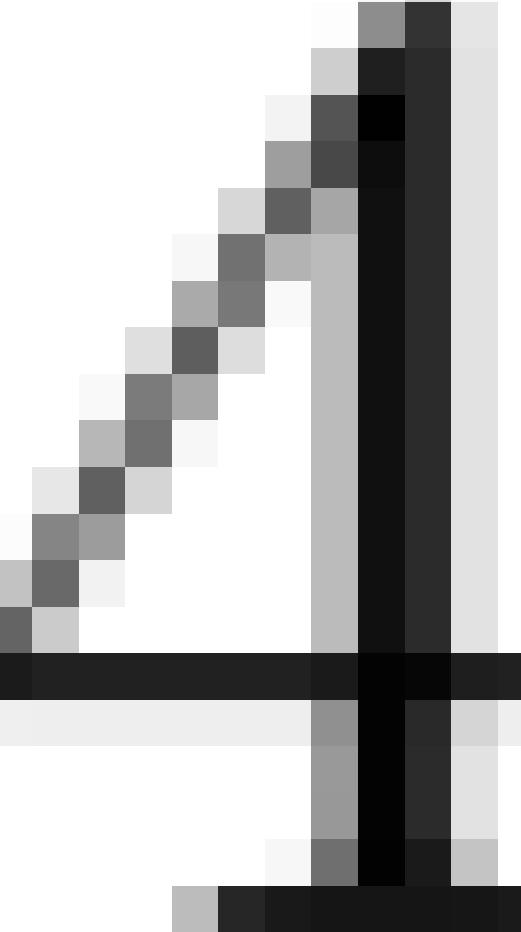
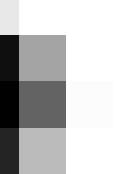
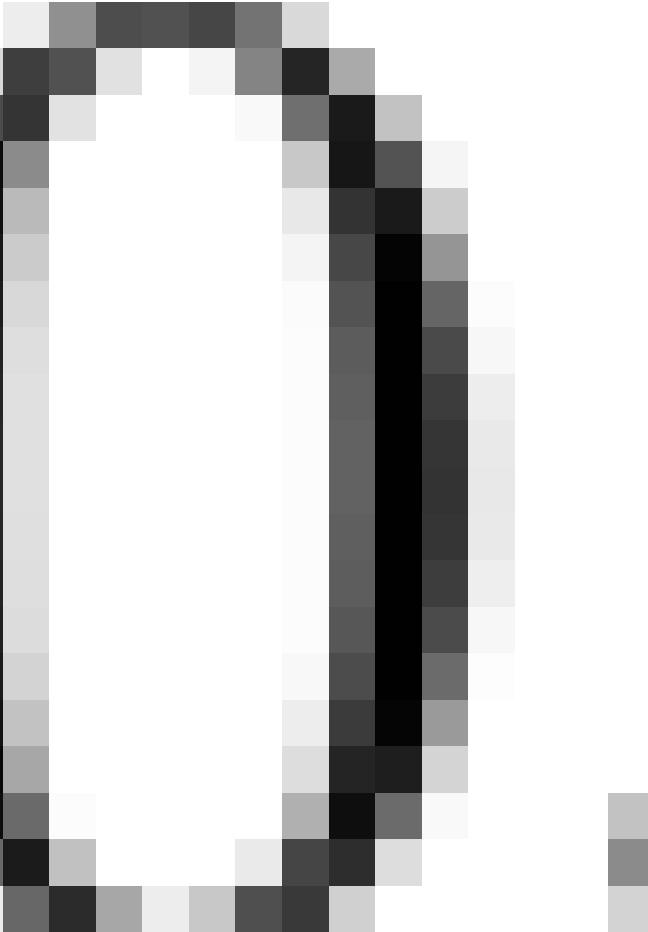
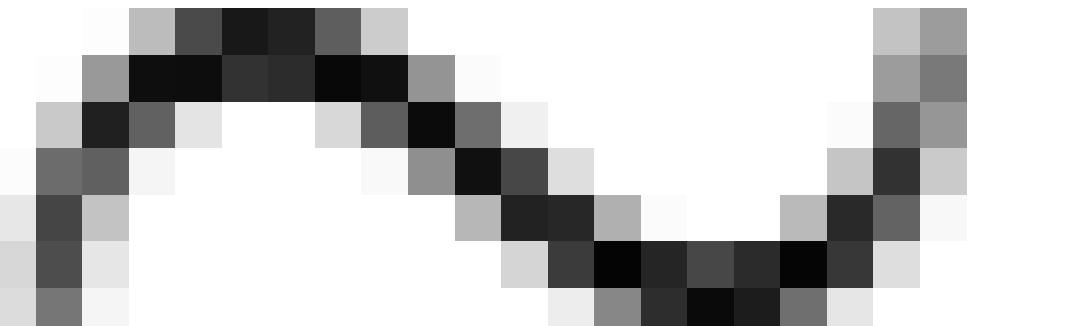


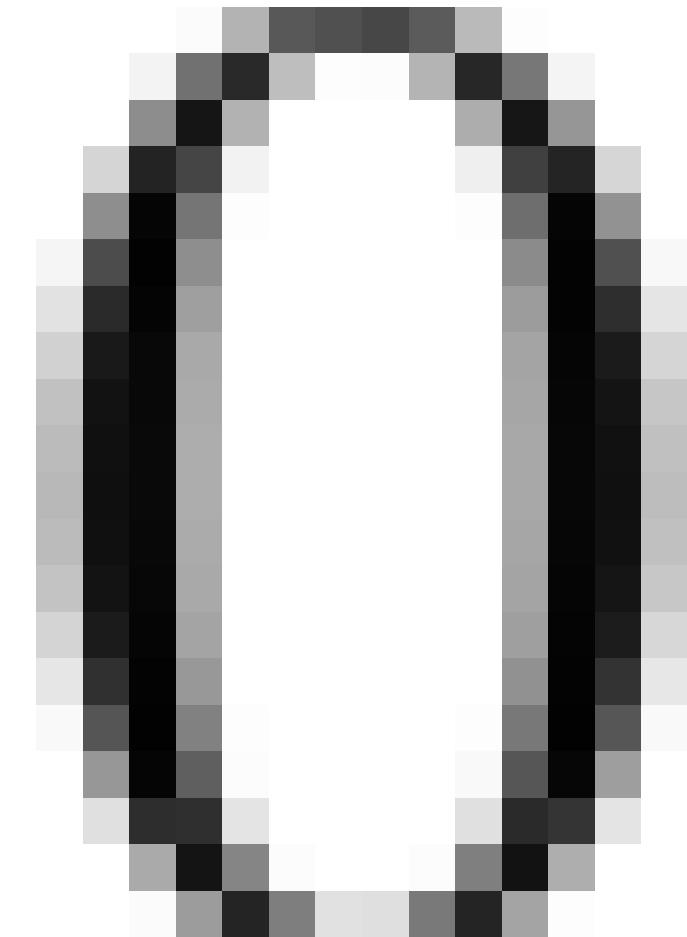
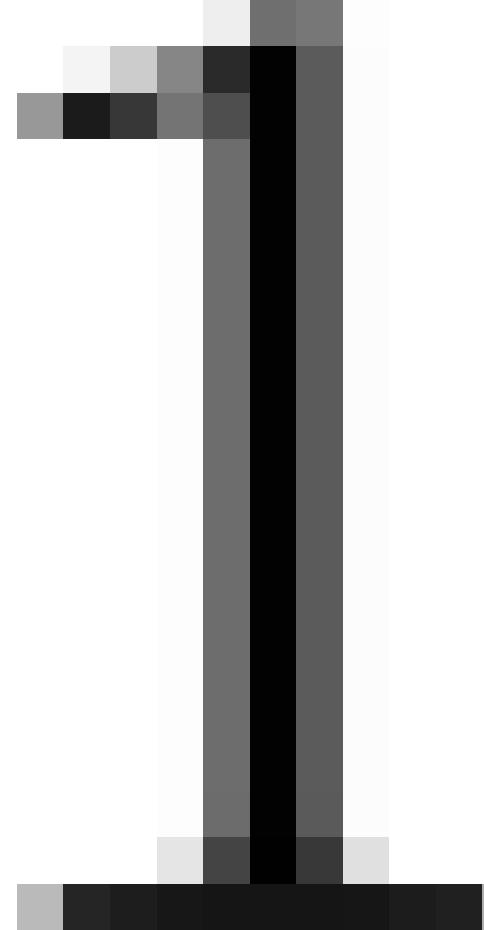
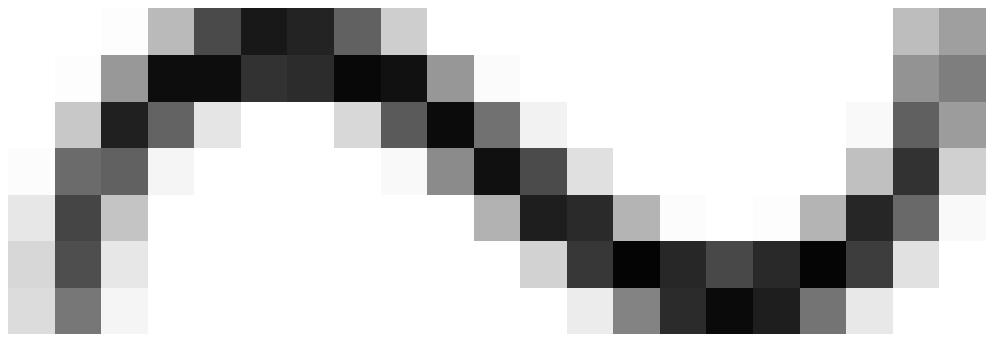




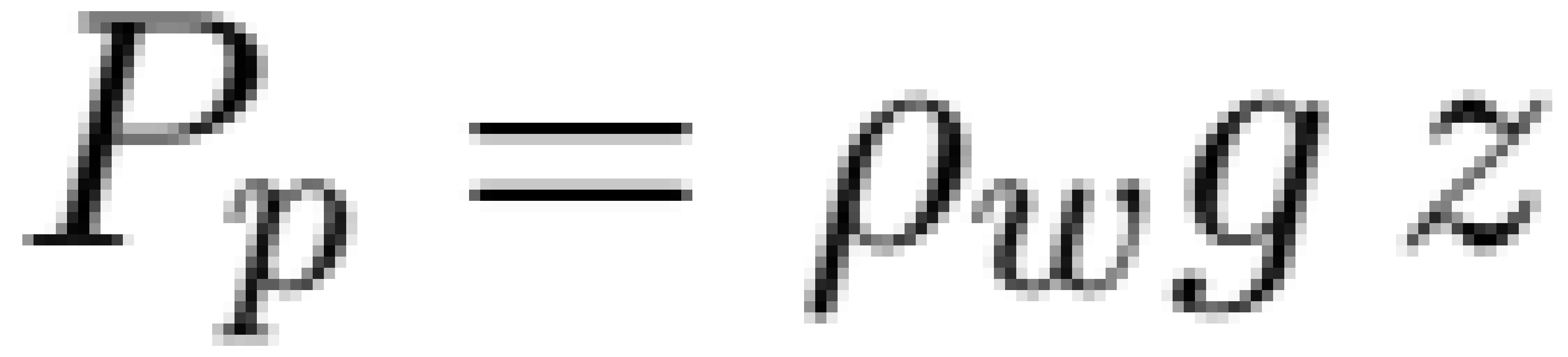




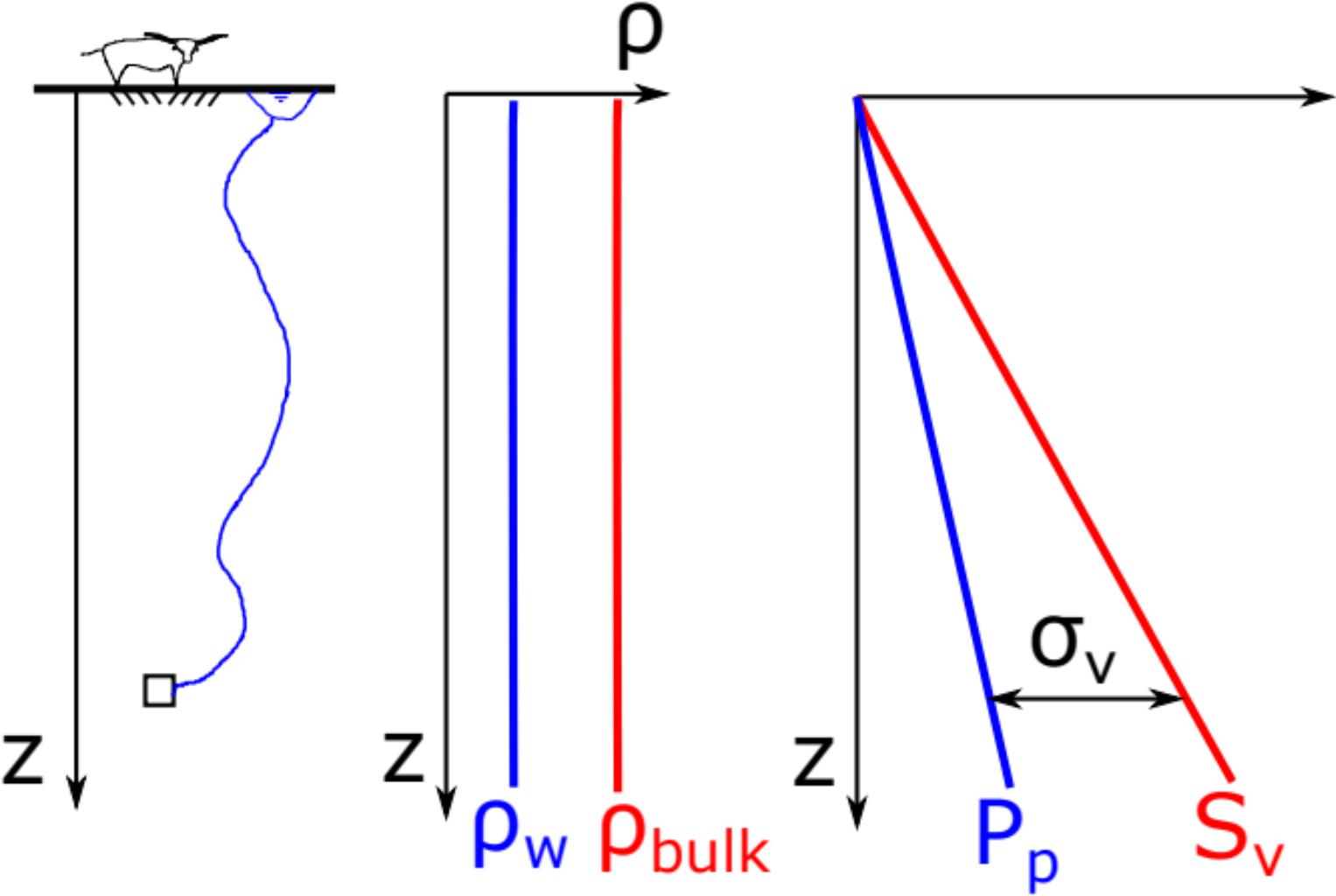










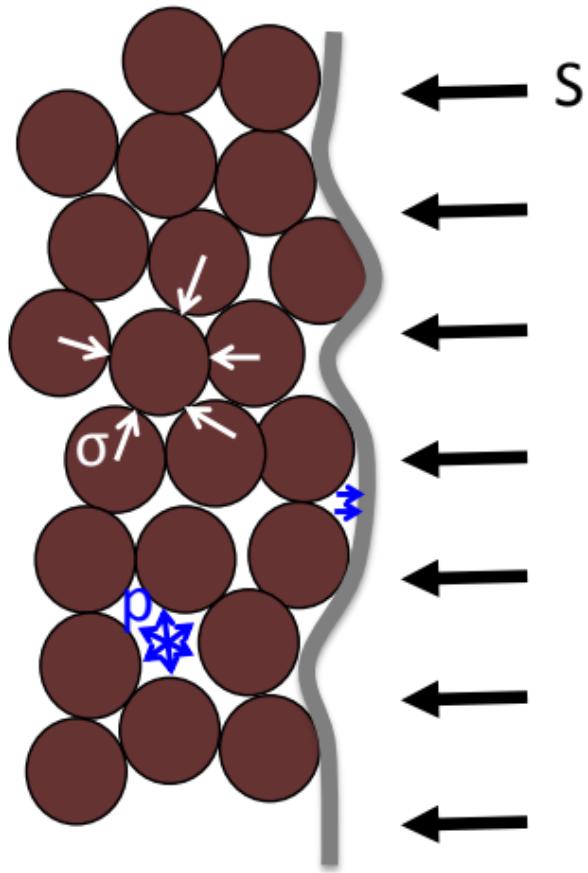




Effective stress =

Total stress – Pore pressure

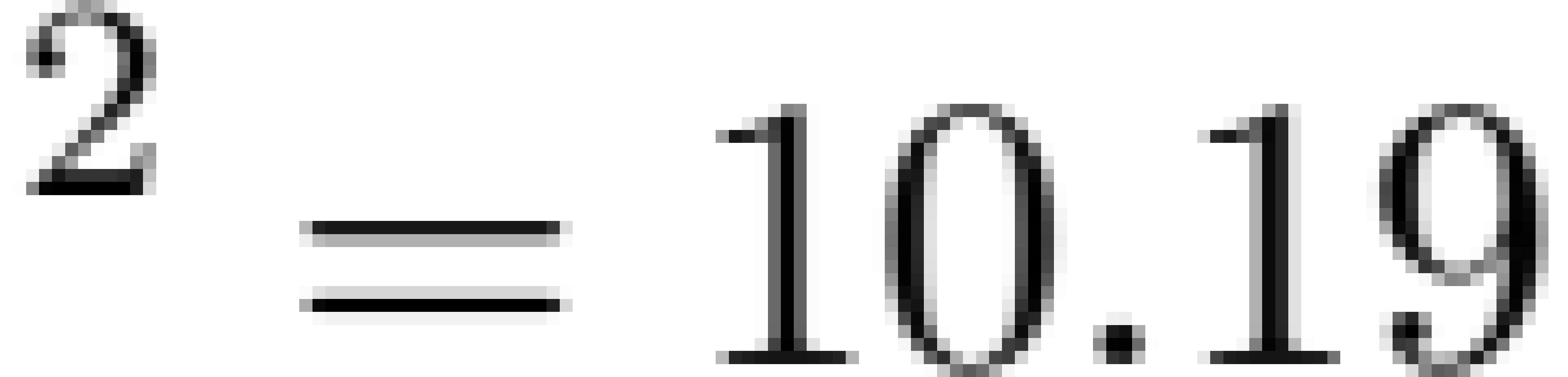
$$\sigma = S - p$$





$dP$   $P$   $\equiv$   $\rho_{w,9}$   $\equiv$   $1040$

$dz$



dsu = Paulk 2350

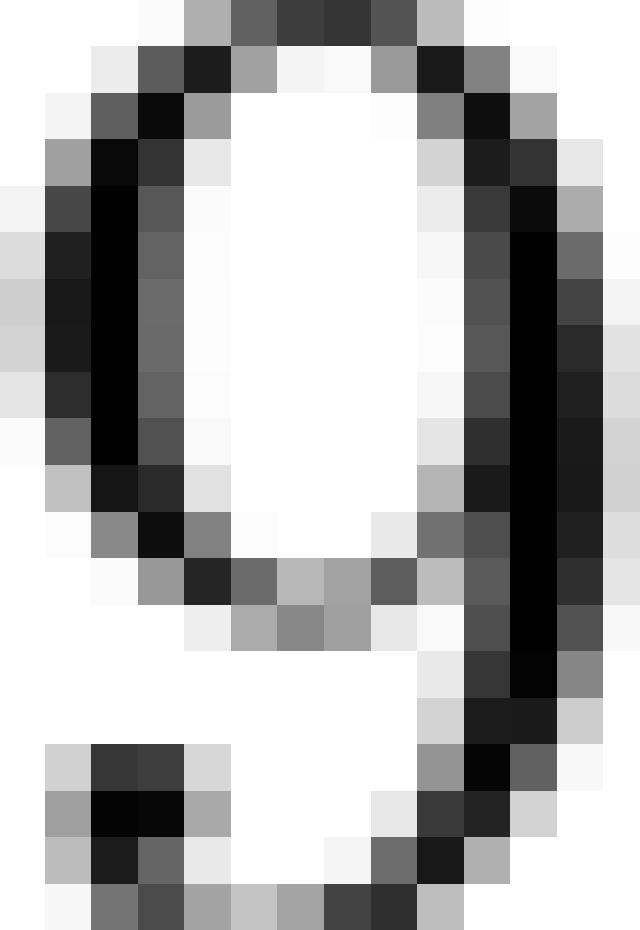
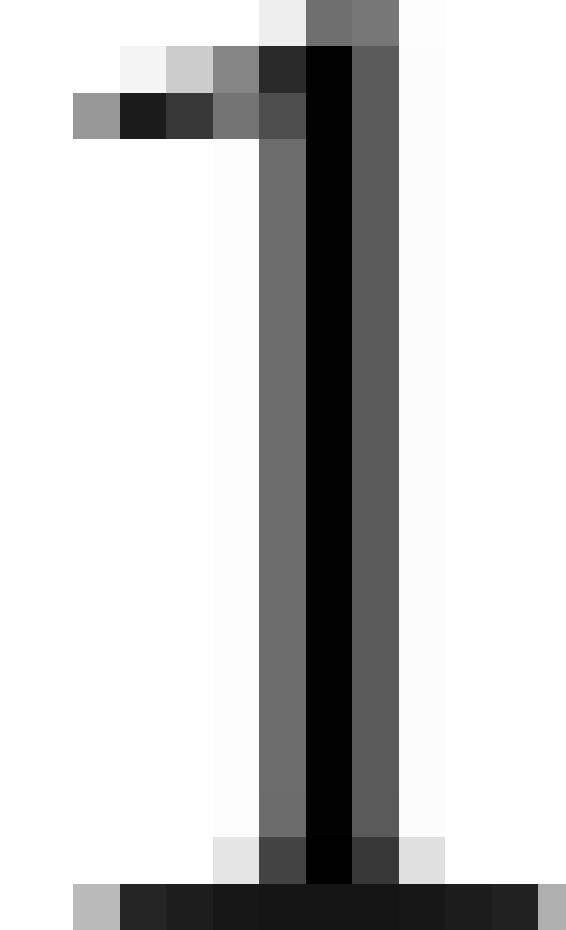
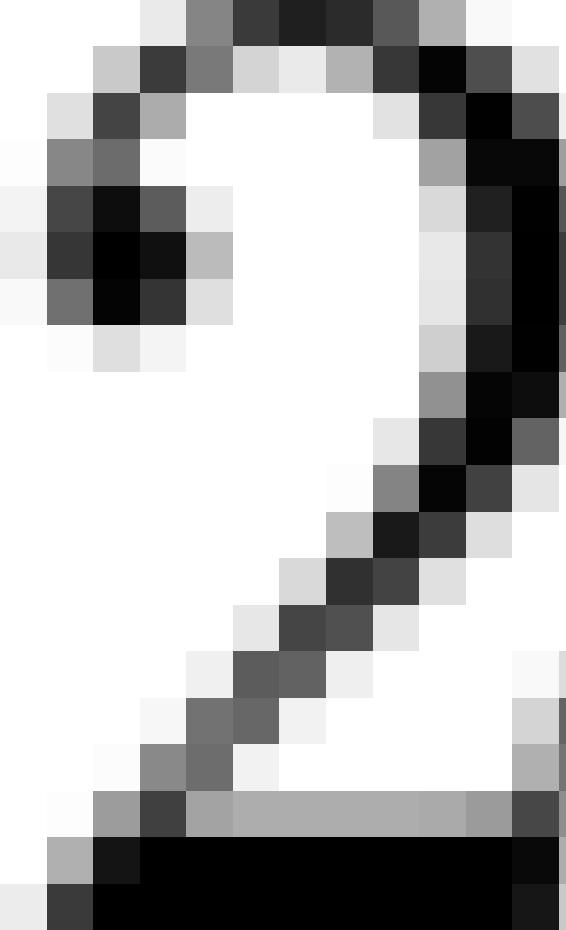
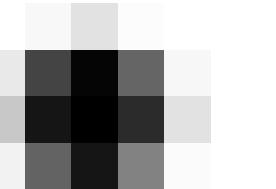
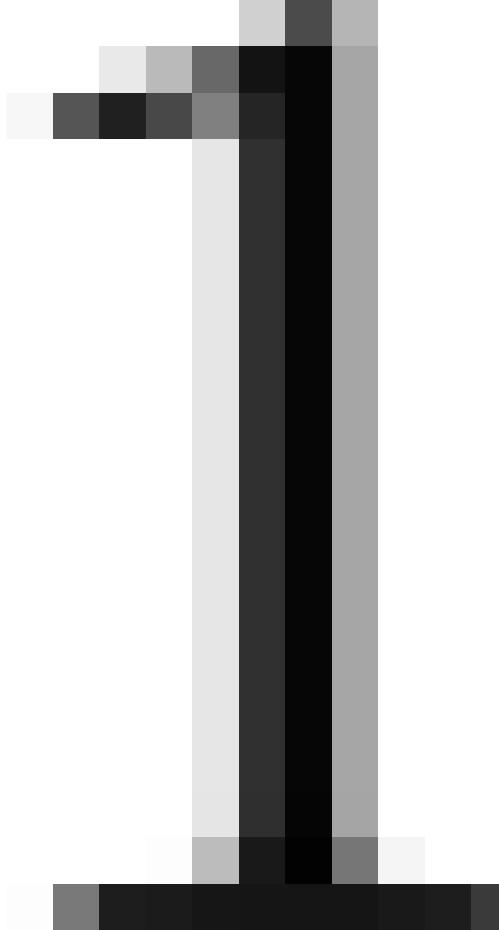


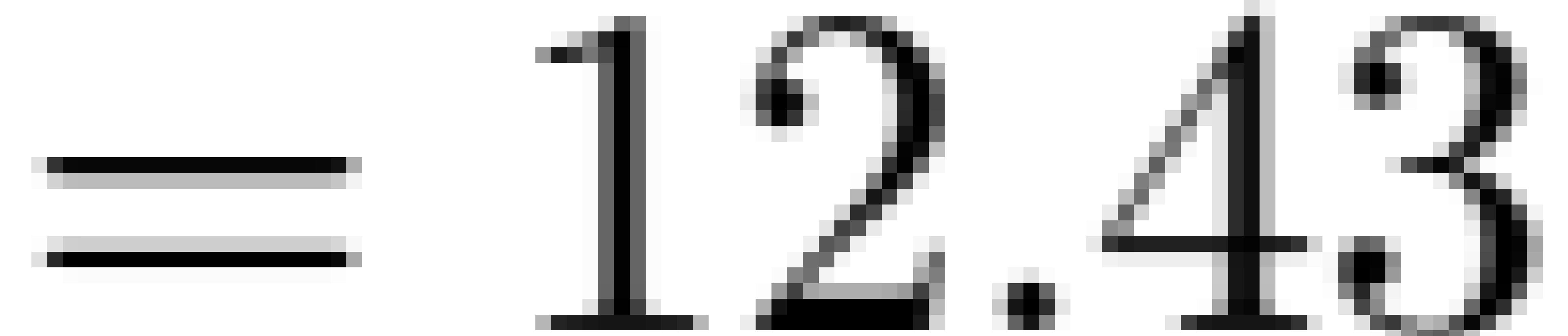
P  
P

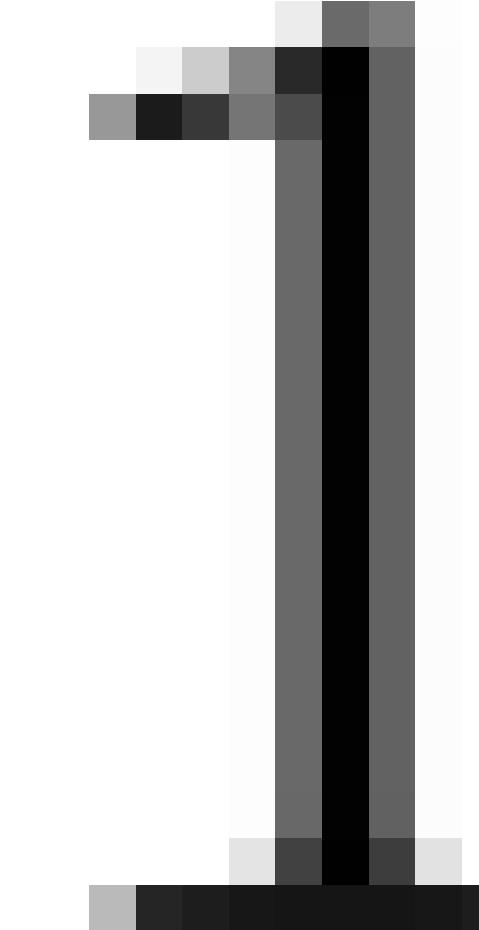
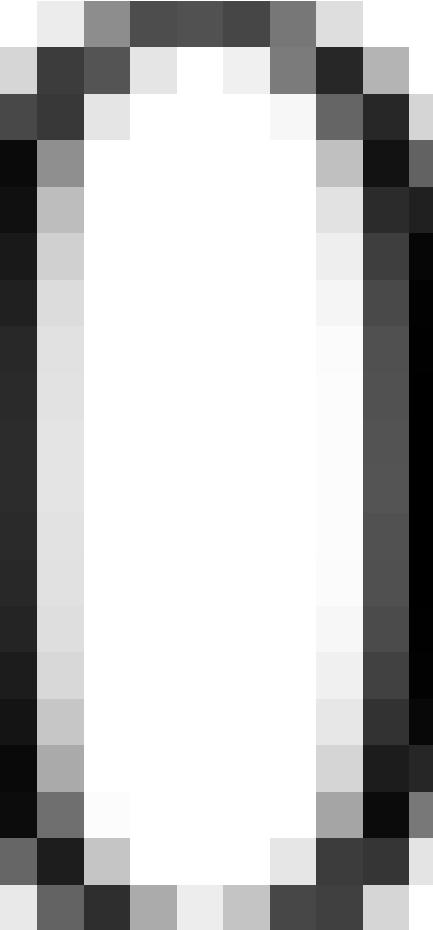
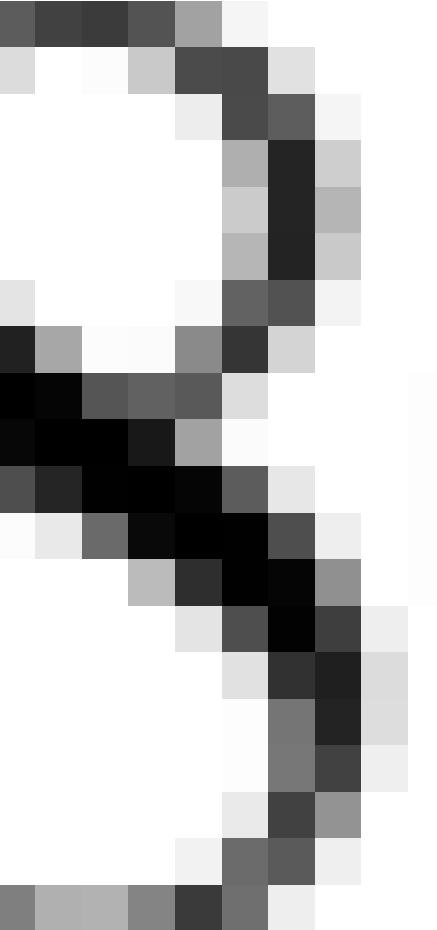
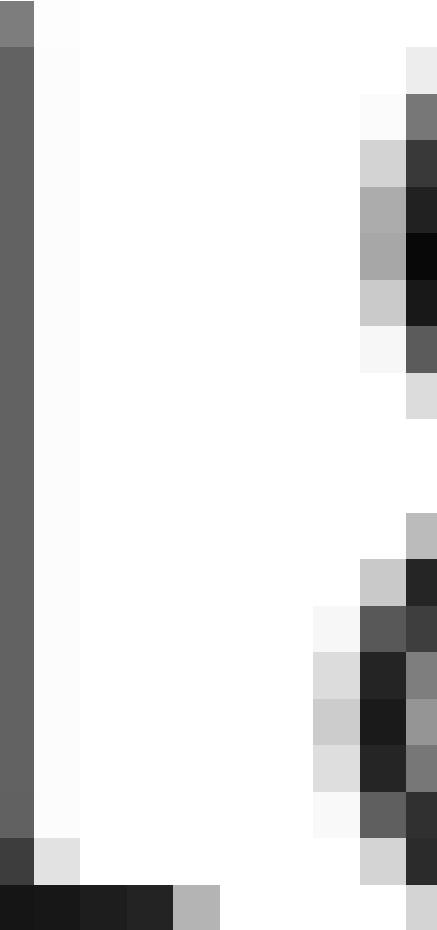
dP  
dP  
dZ  
dZ

—  
—

10.  
19







*d<sub>5</sub>*

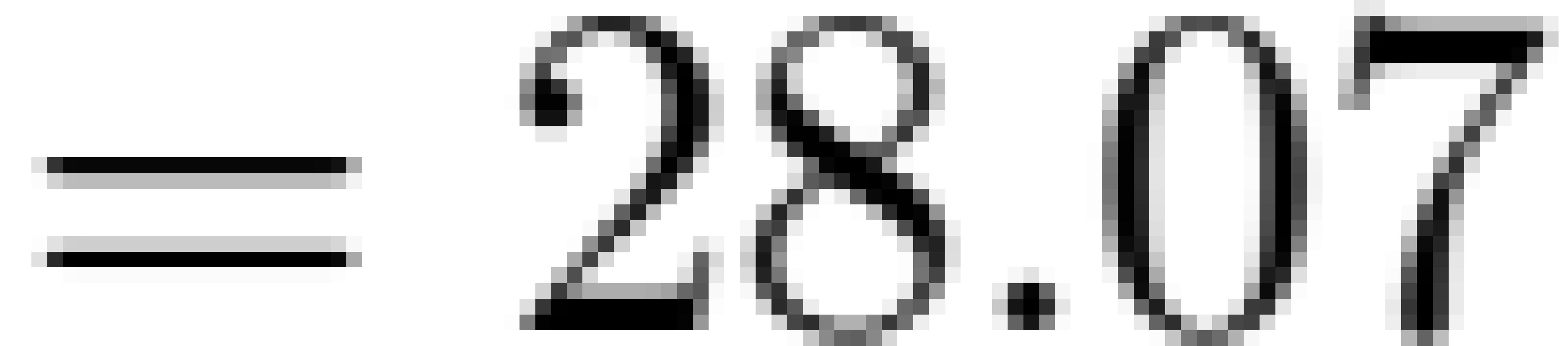
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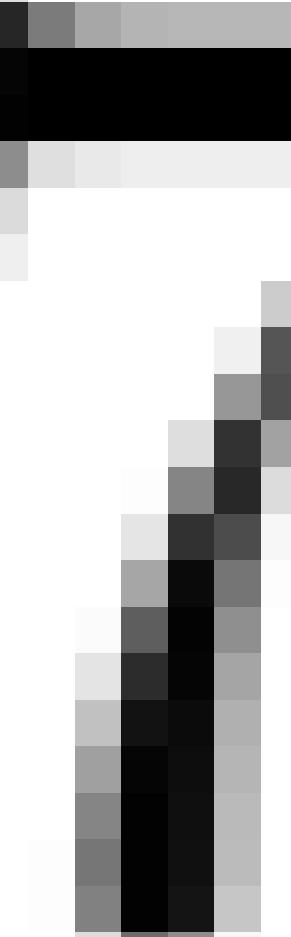
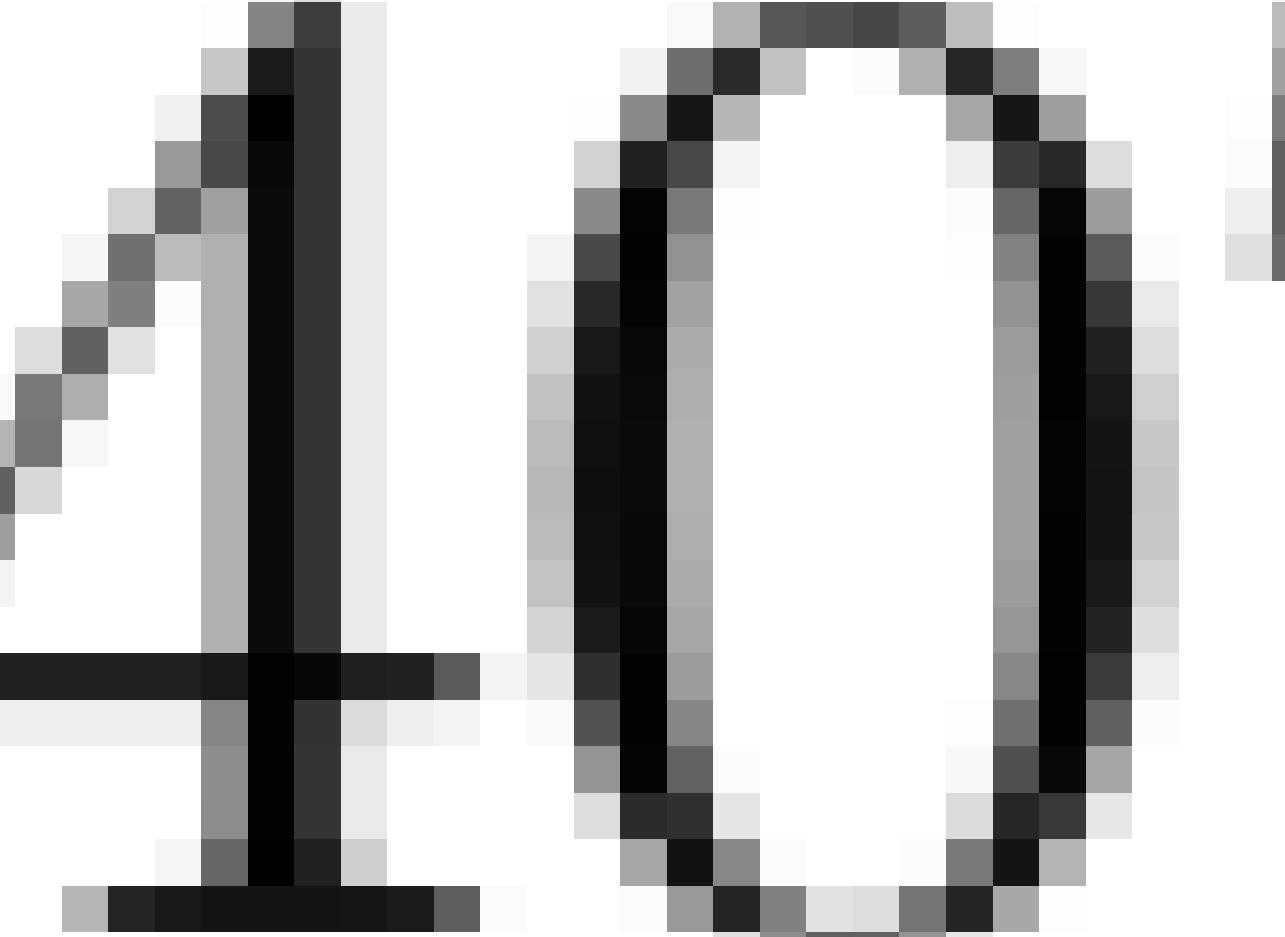
*d<sub>7</sub>*

*z*

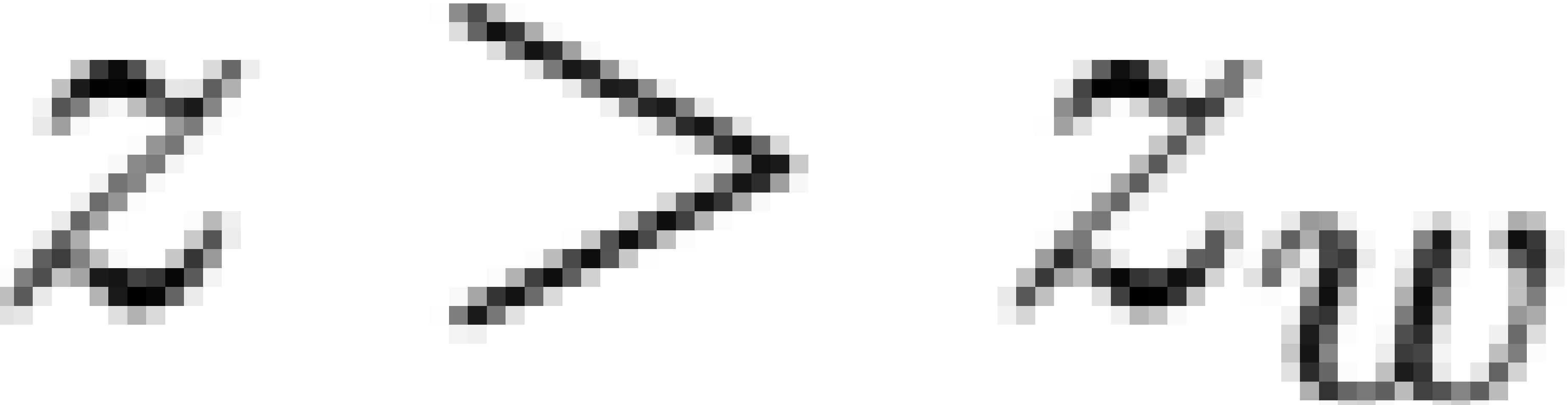
*z*

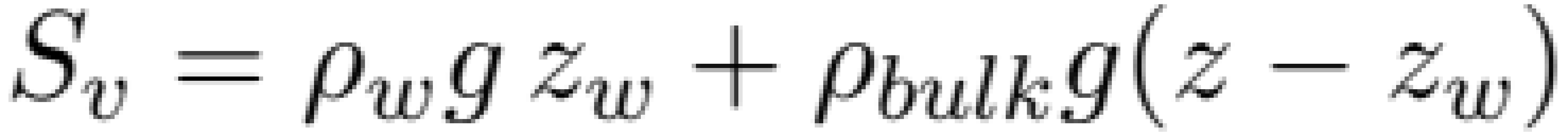
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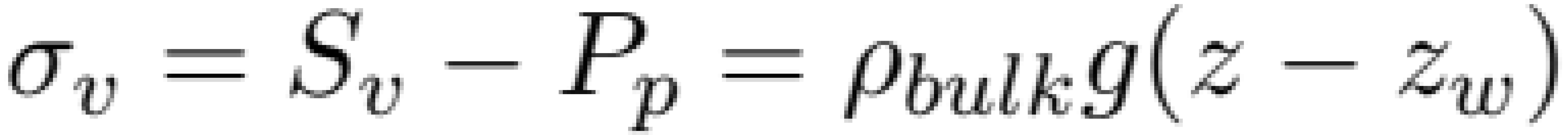


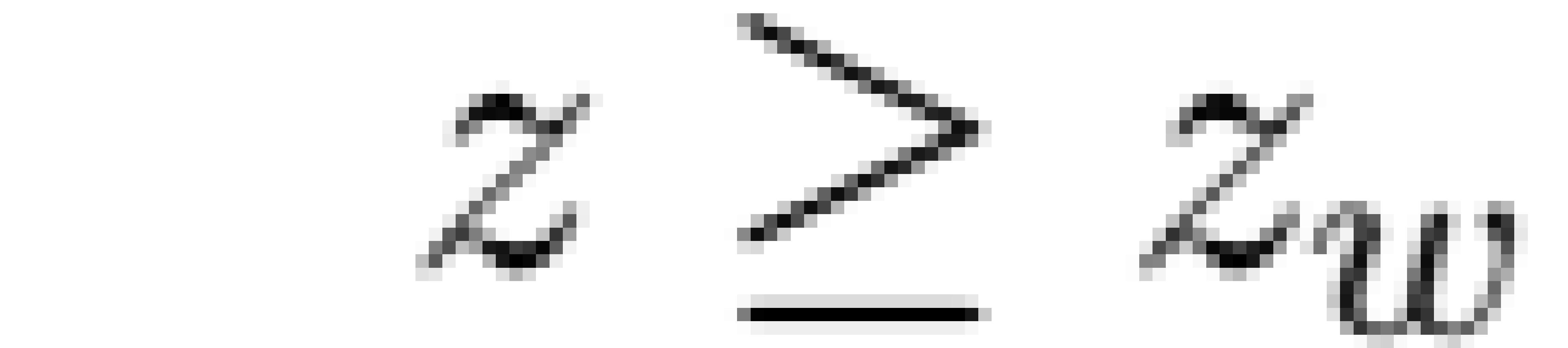


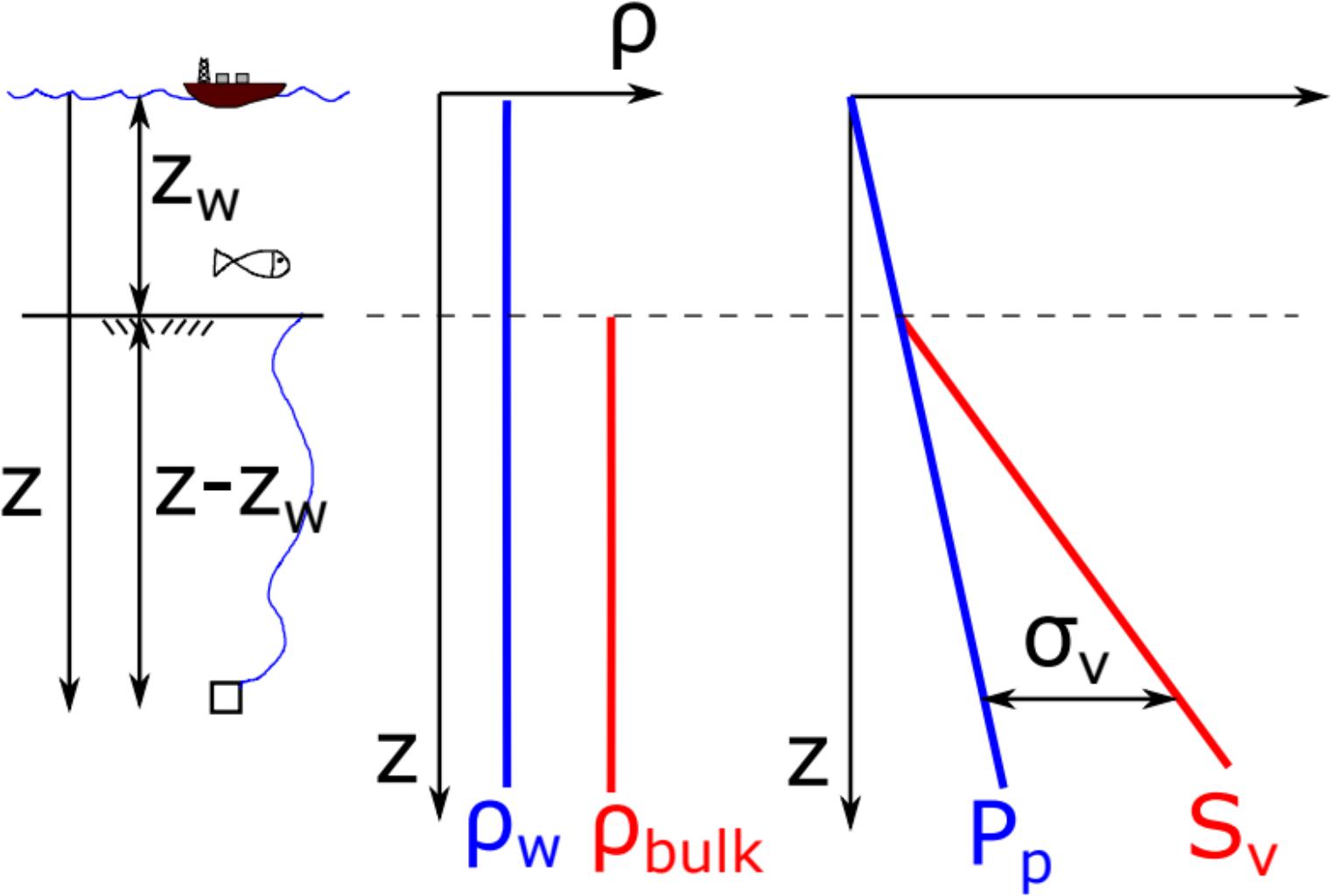




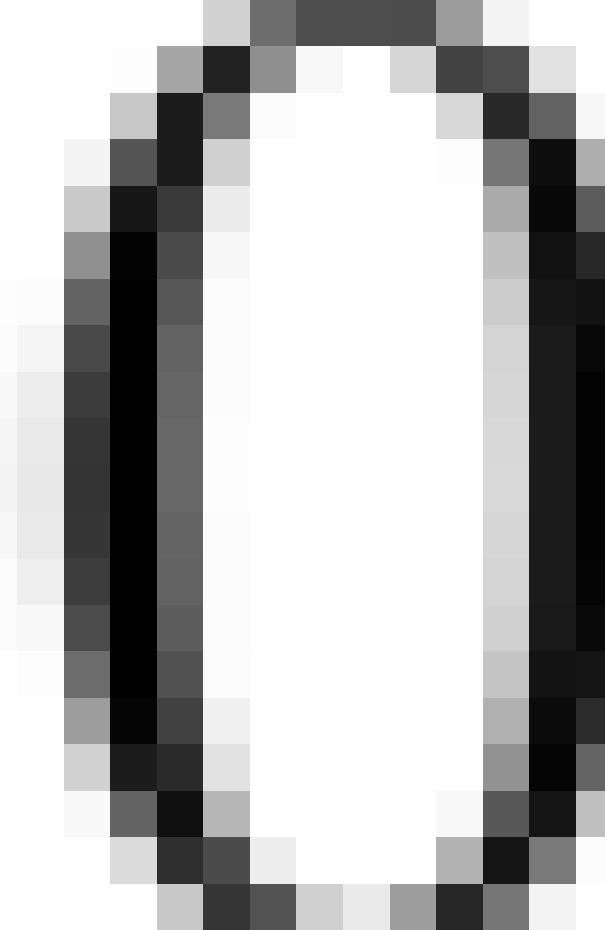
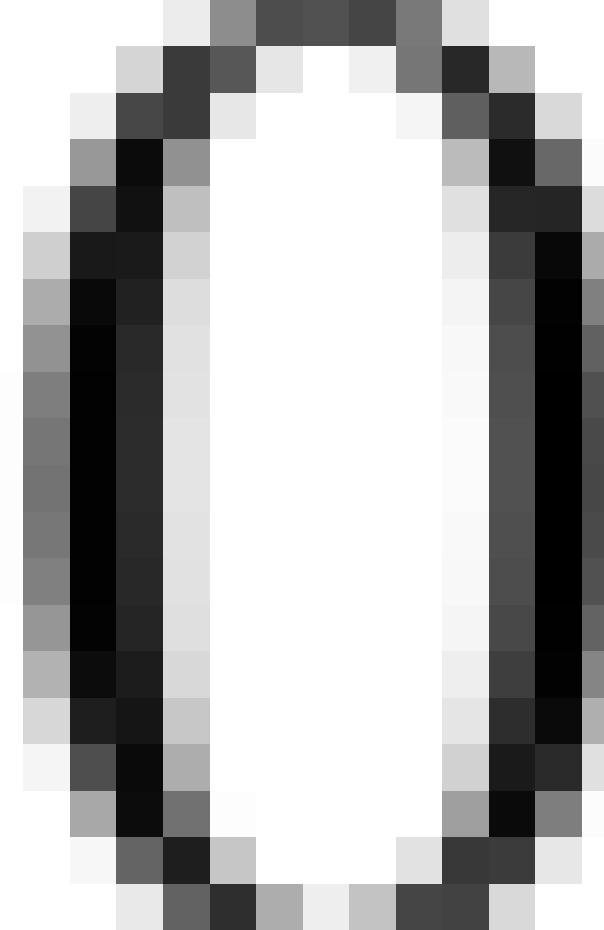
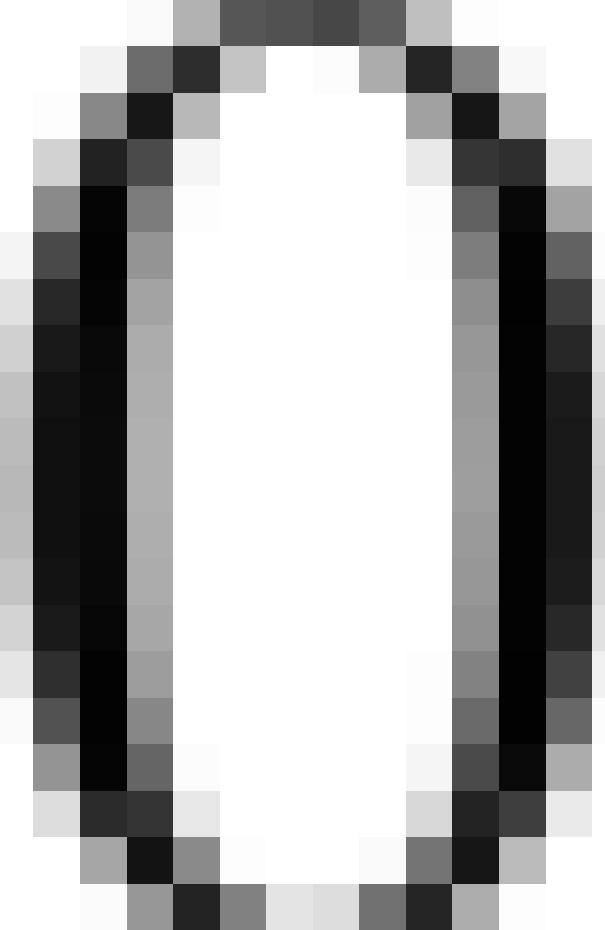
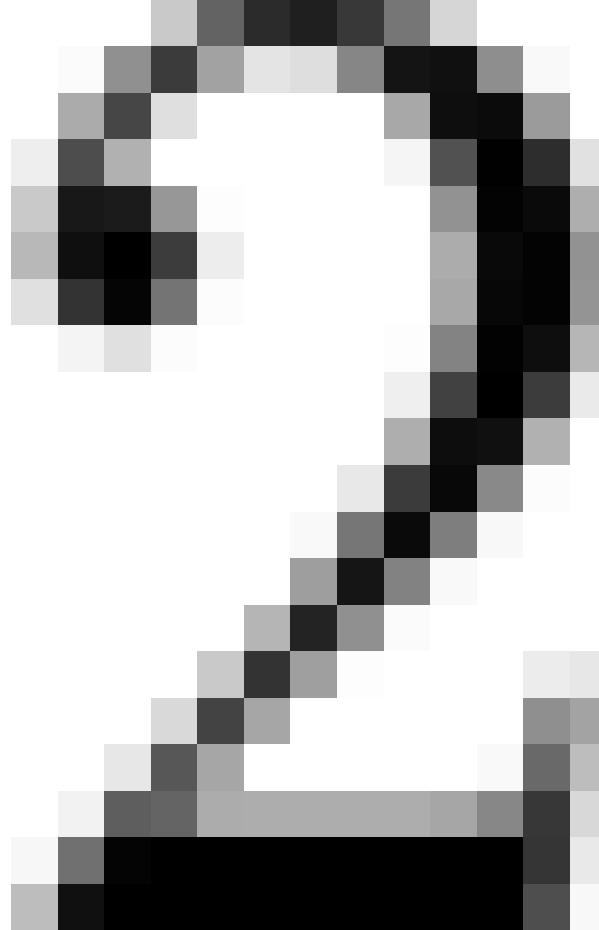


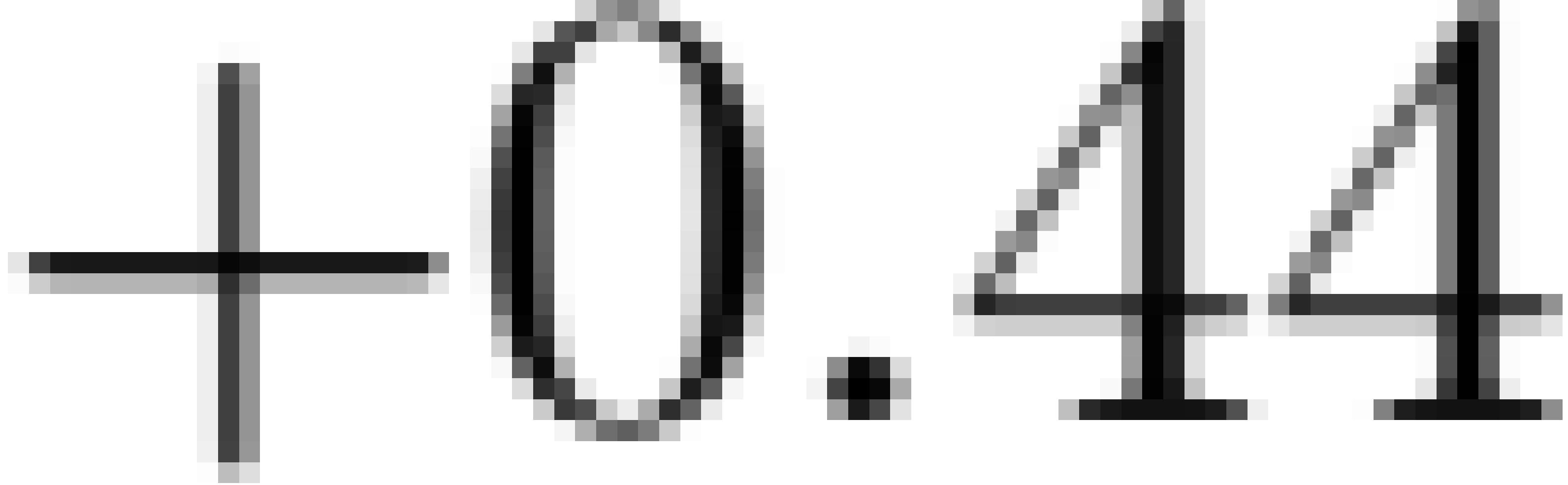


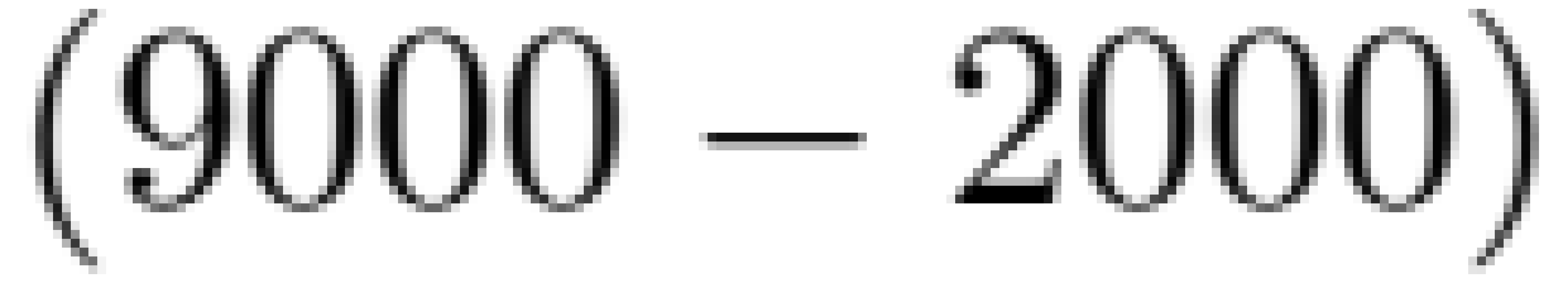


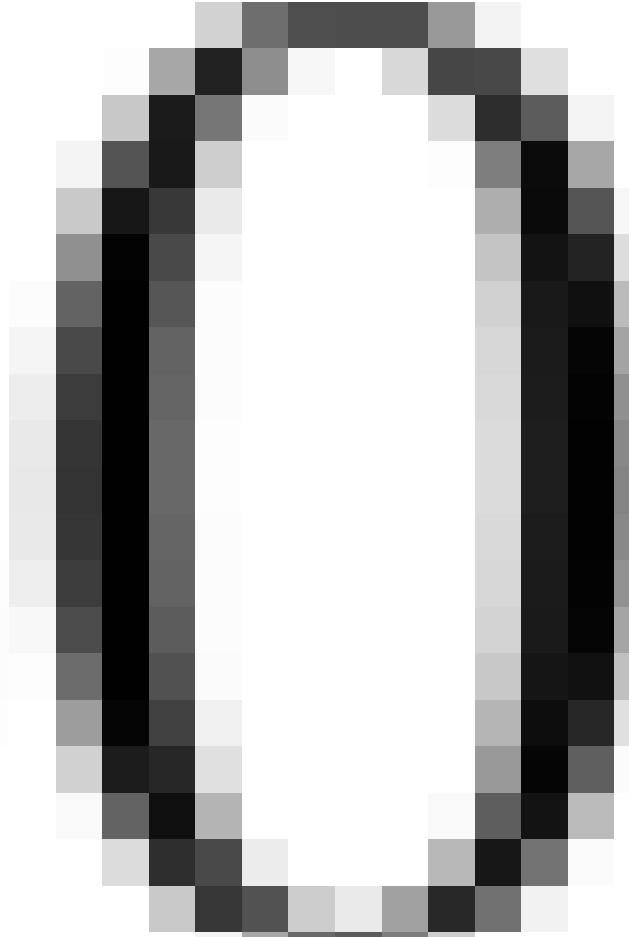
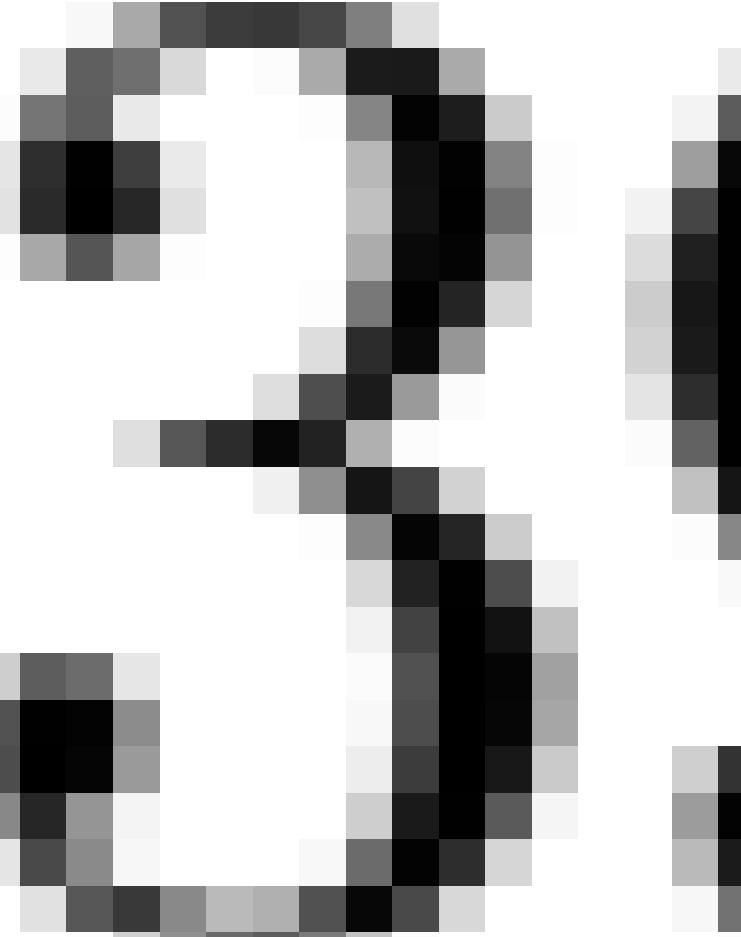


$$P_p = \rho w g z_w + \frac{dP}{dz} (z - z_w) = 0.44$$

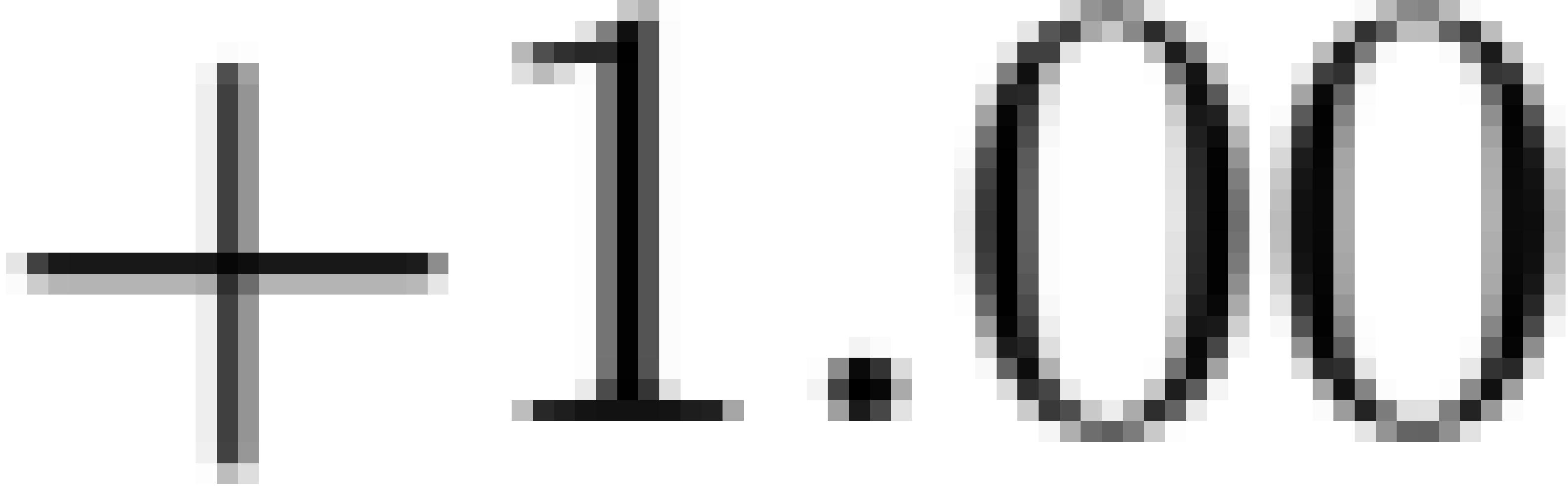


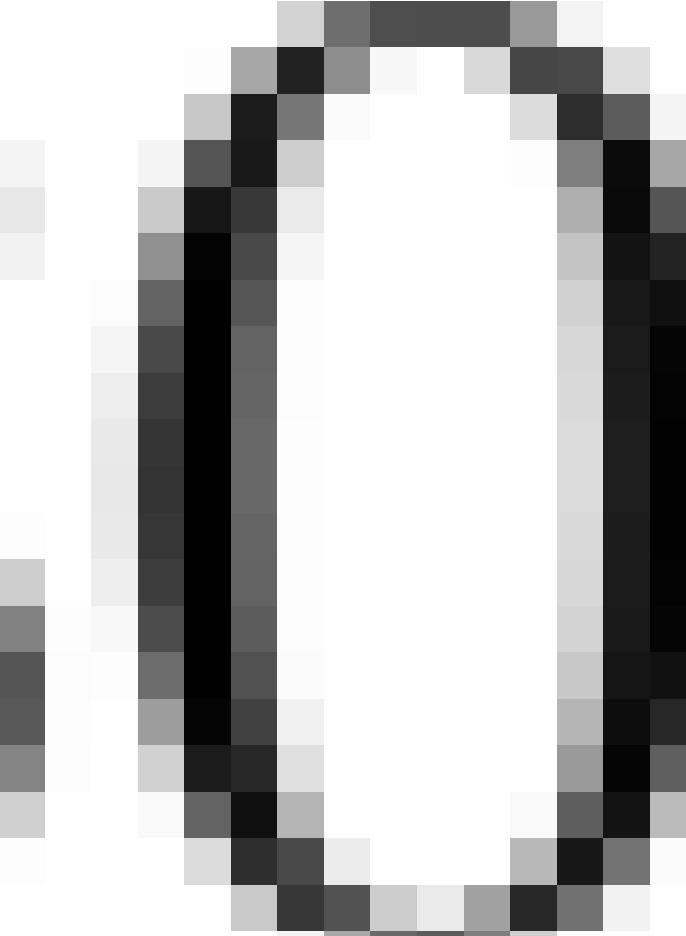
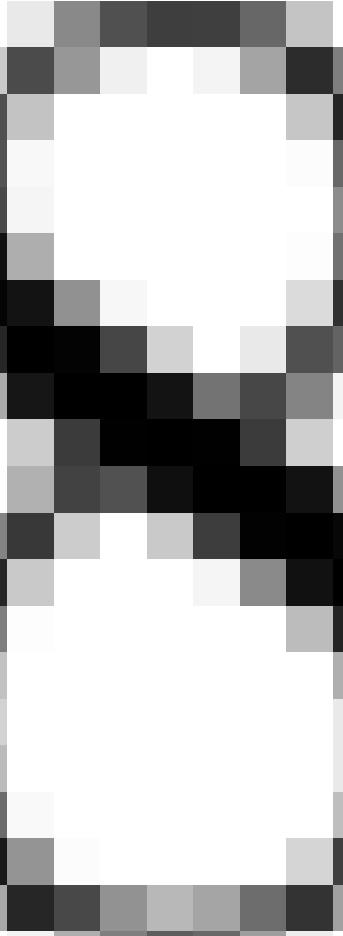
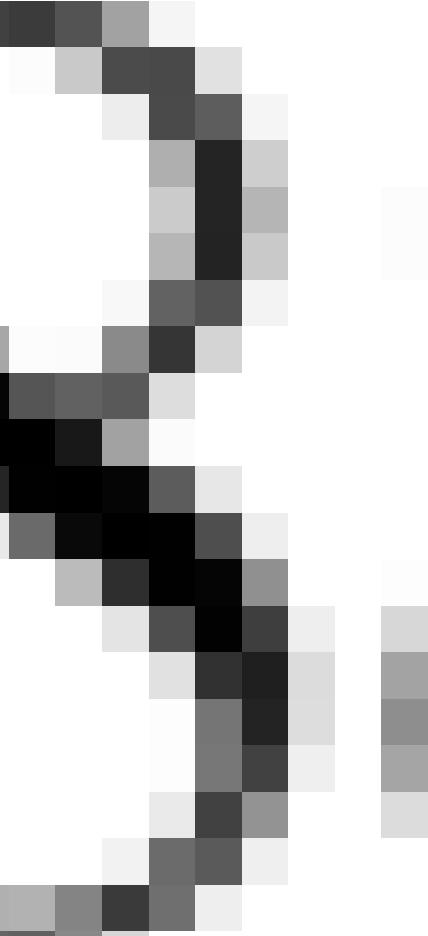
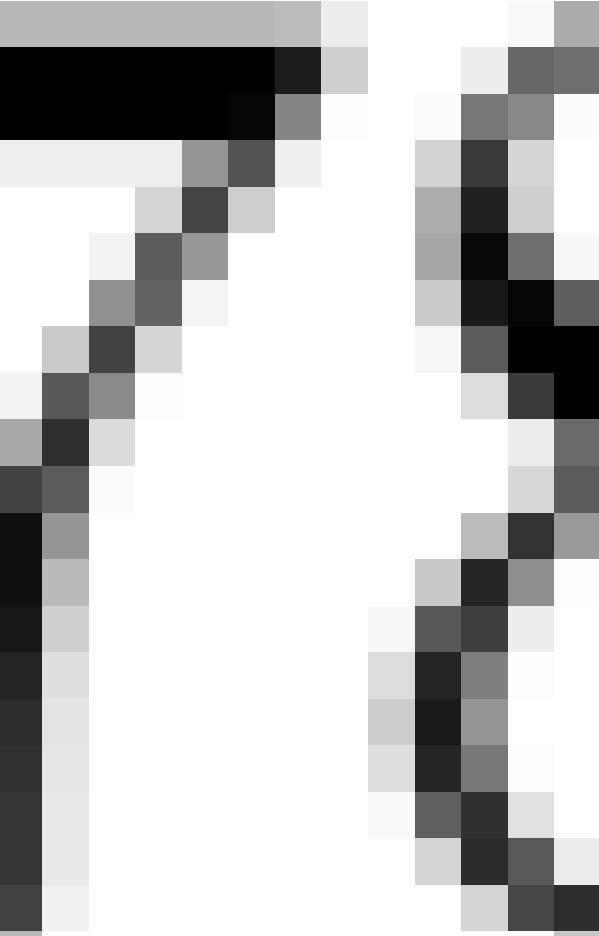


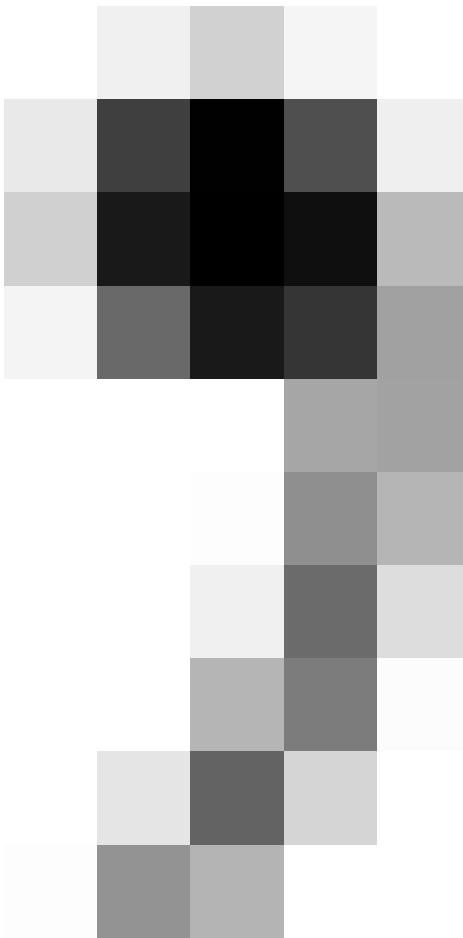


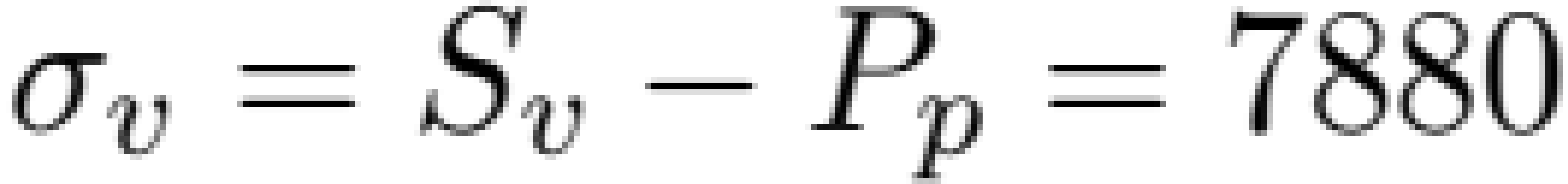


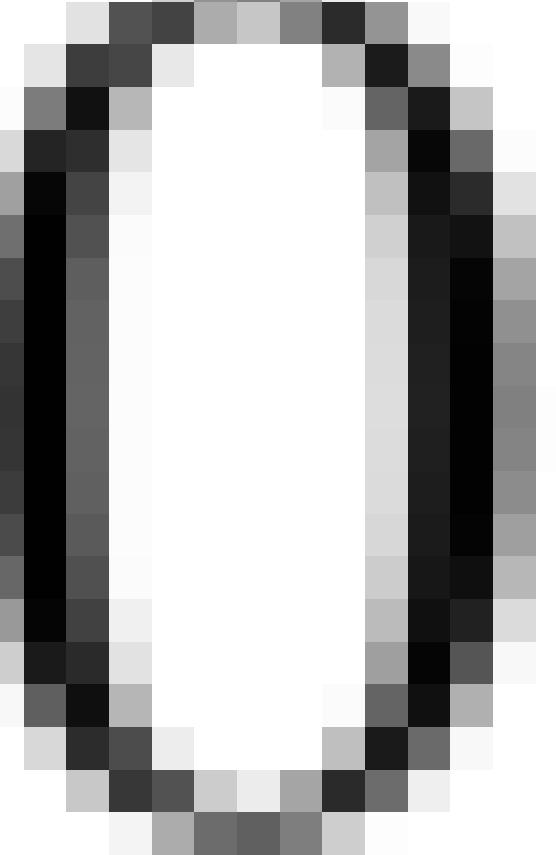
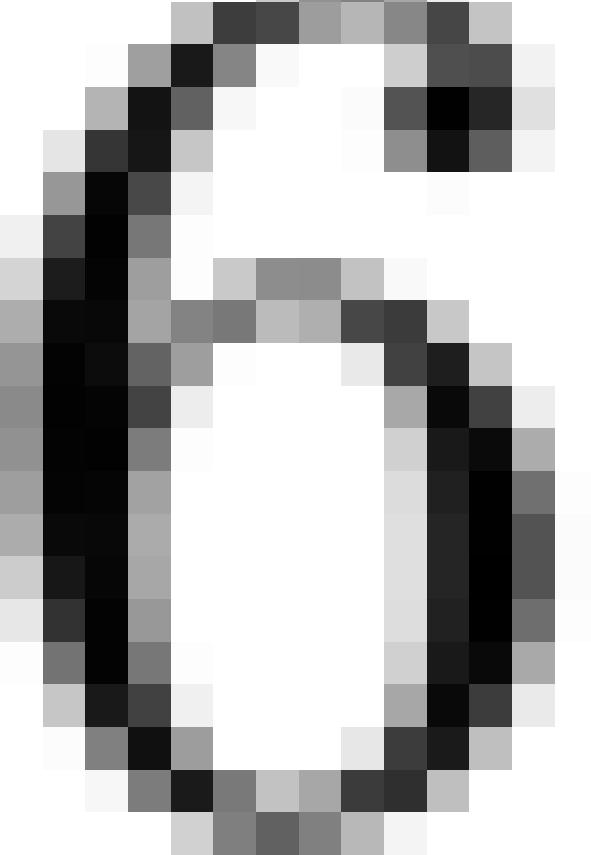
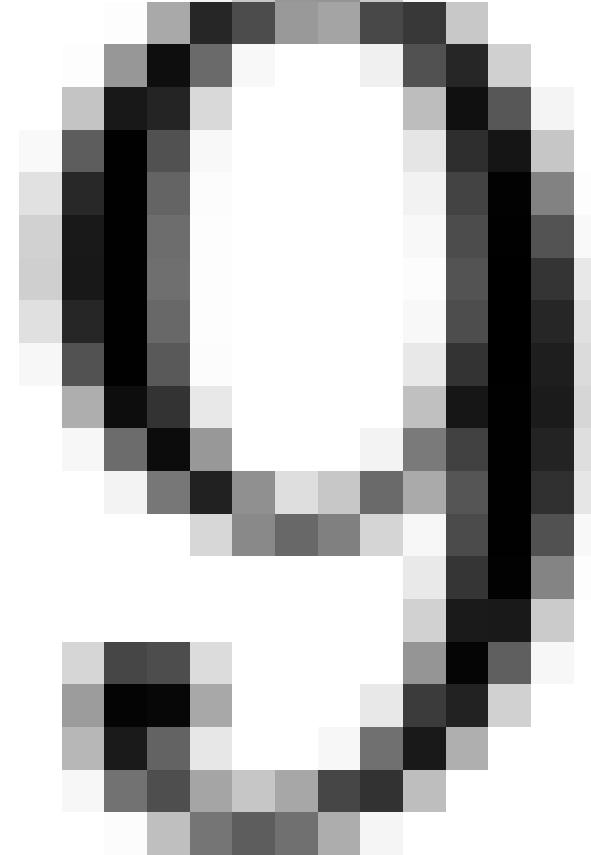
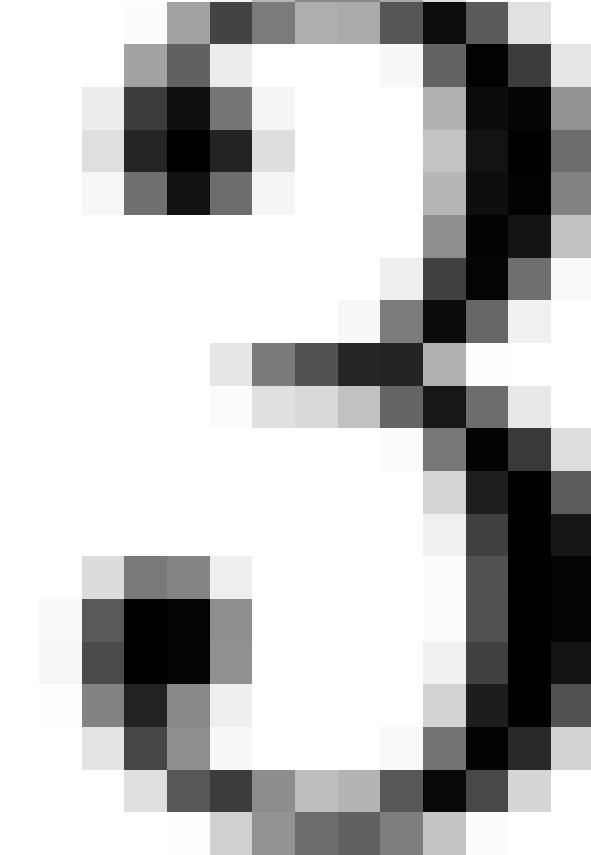
$$S_u = \rho_{u,g} z_u + \frac{dS_u}{dz} (z - z_u) = 0.44$$

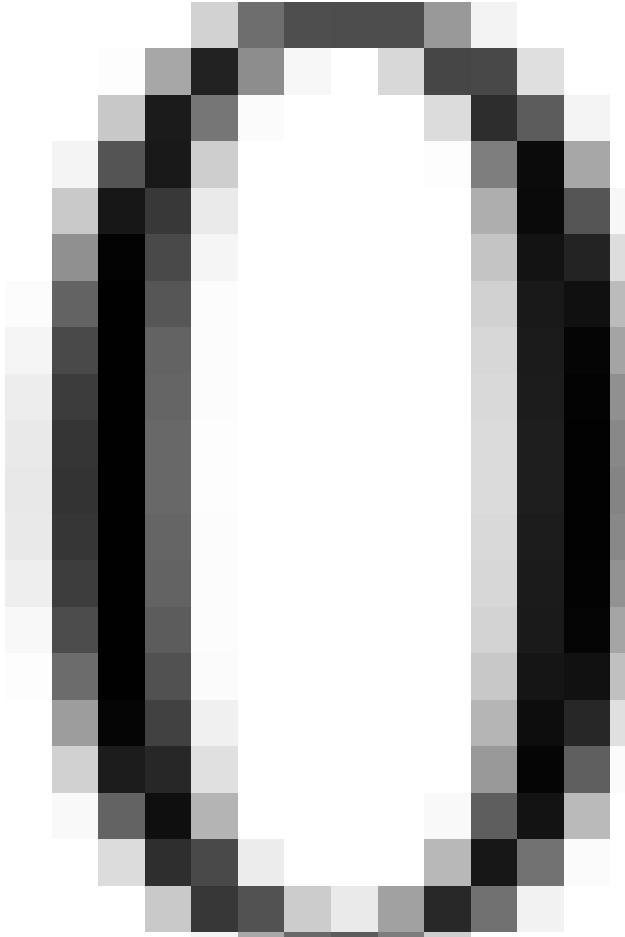
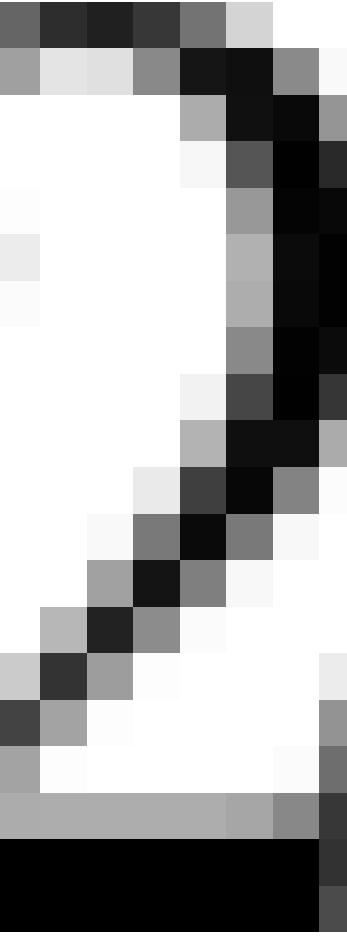
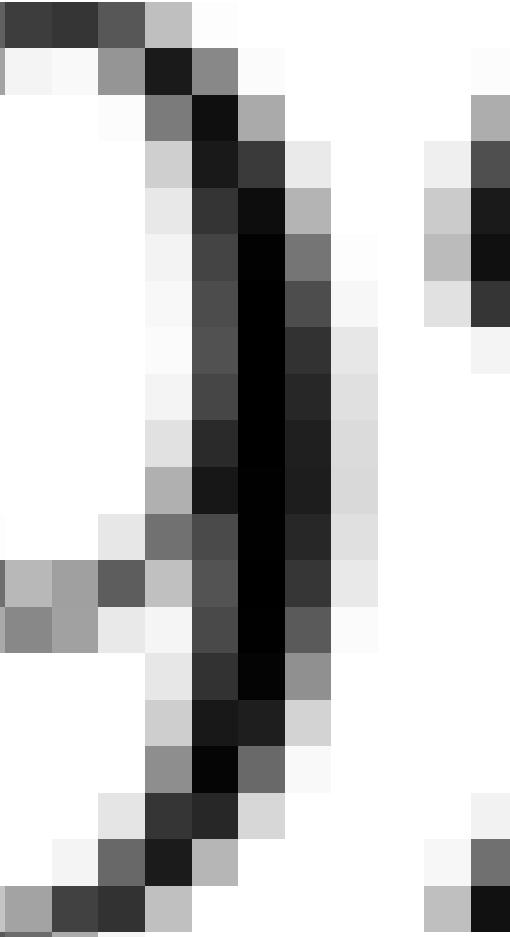
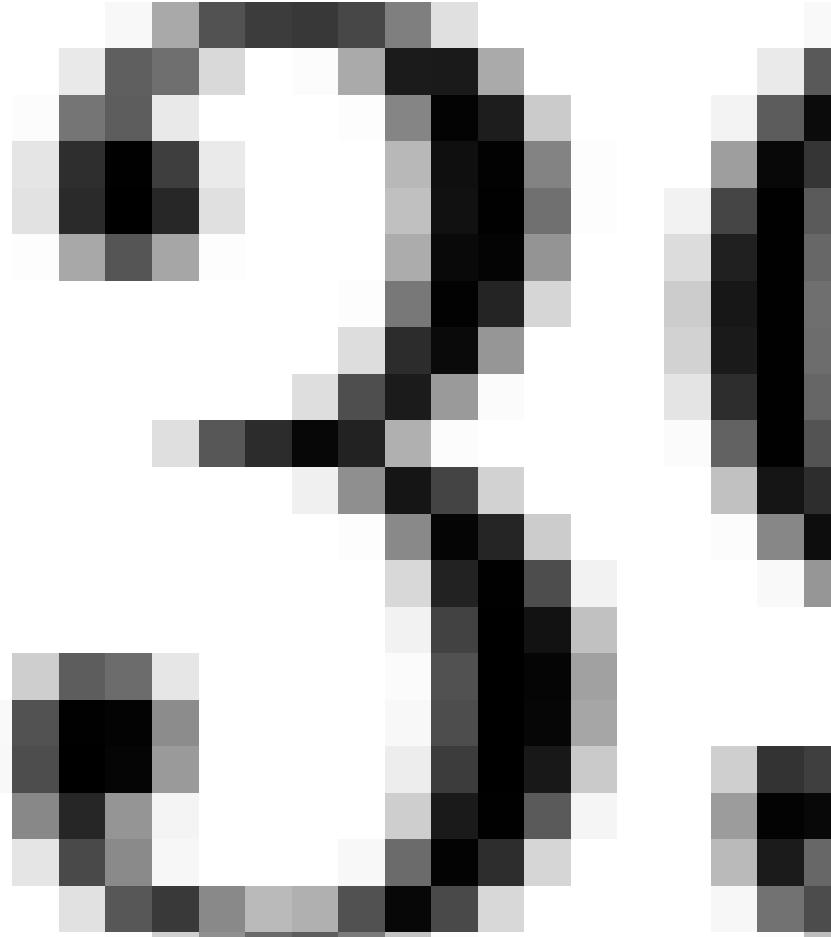


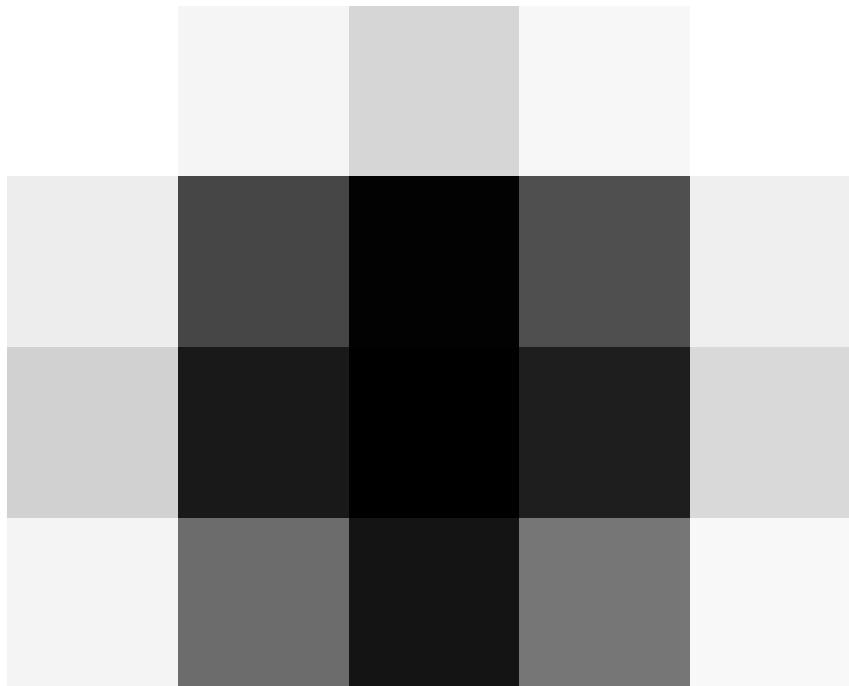


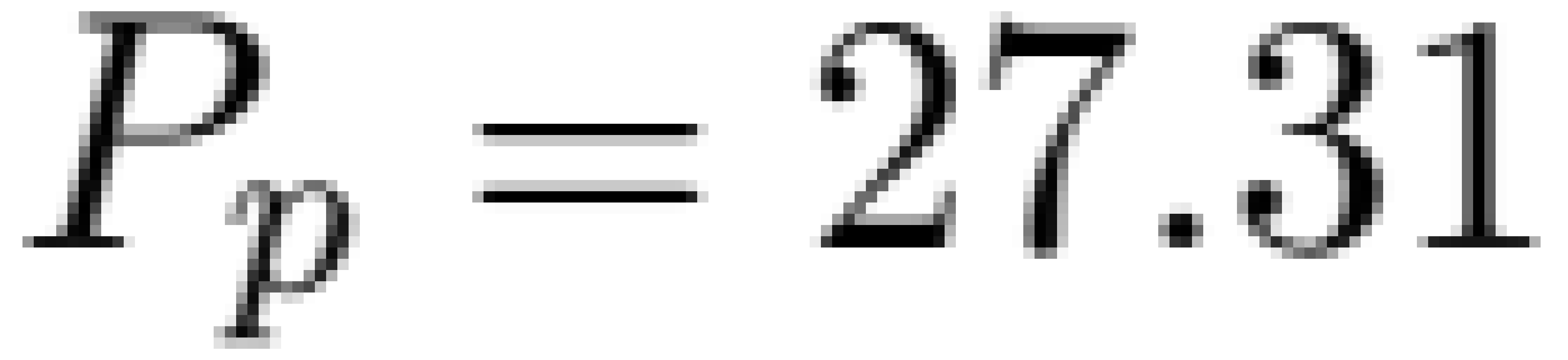






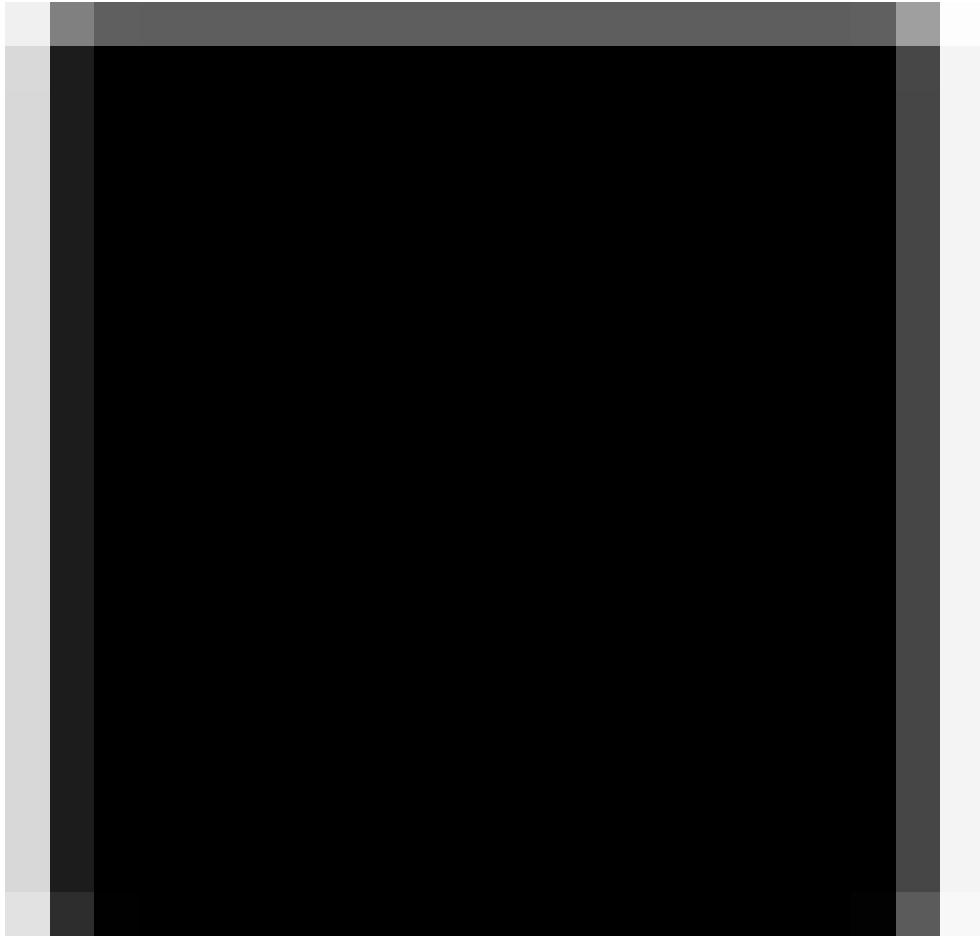








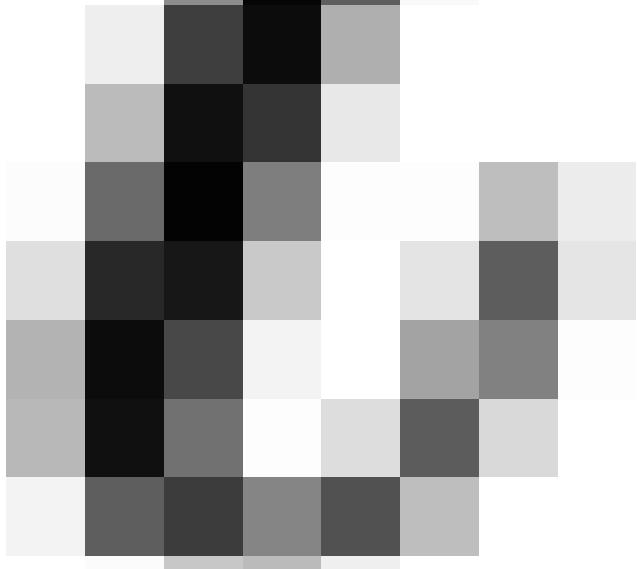
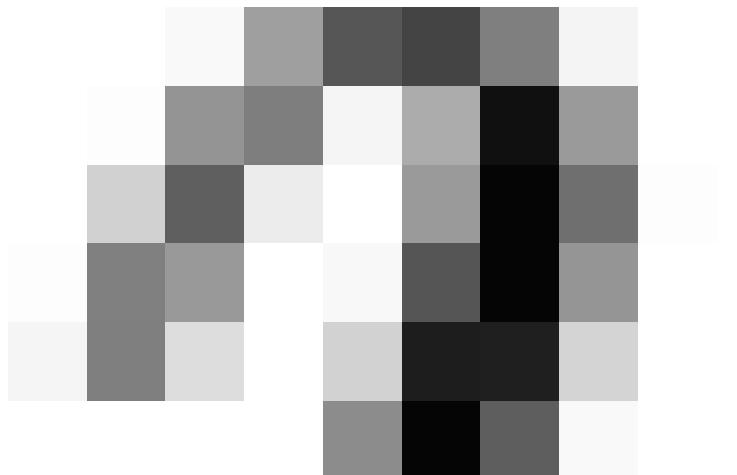
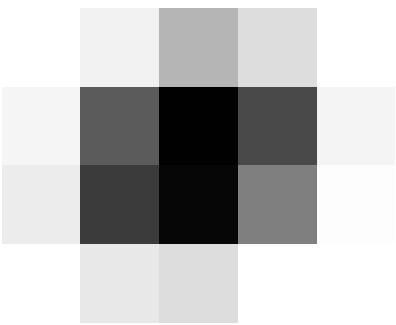


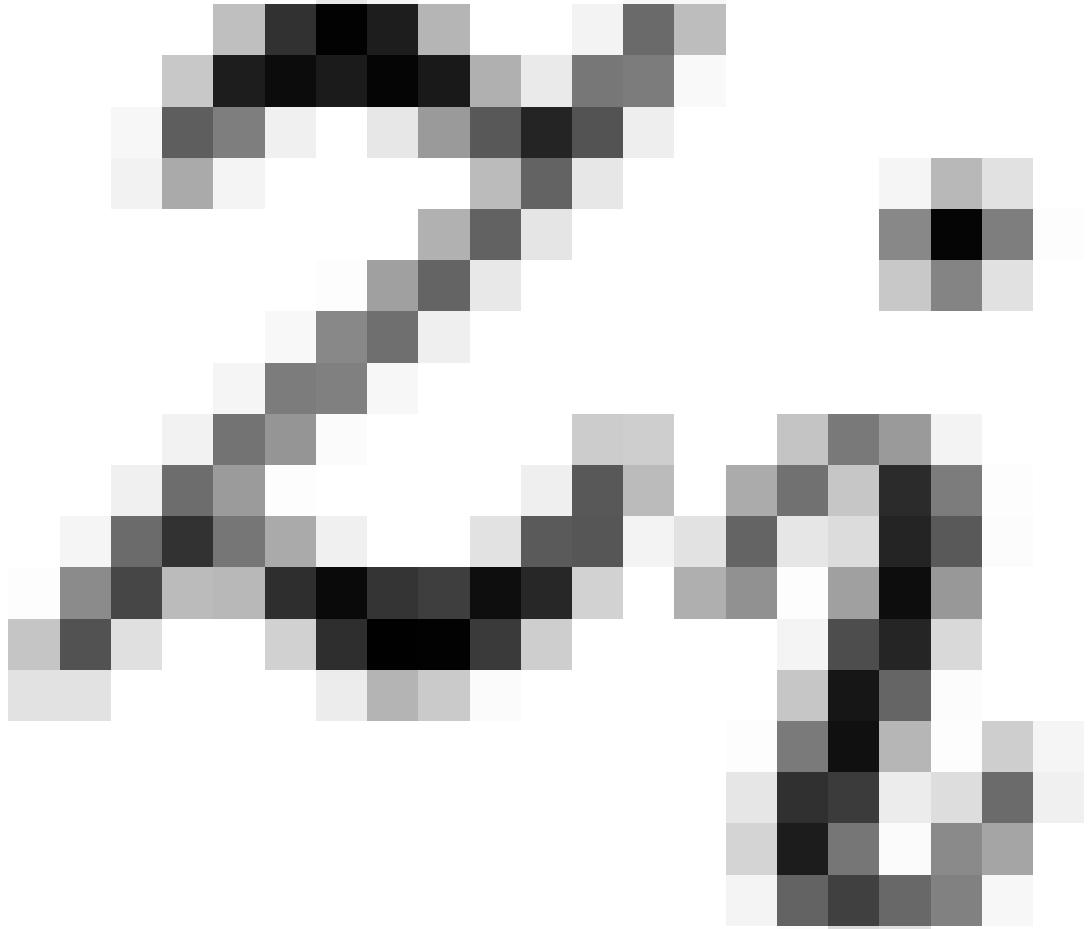


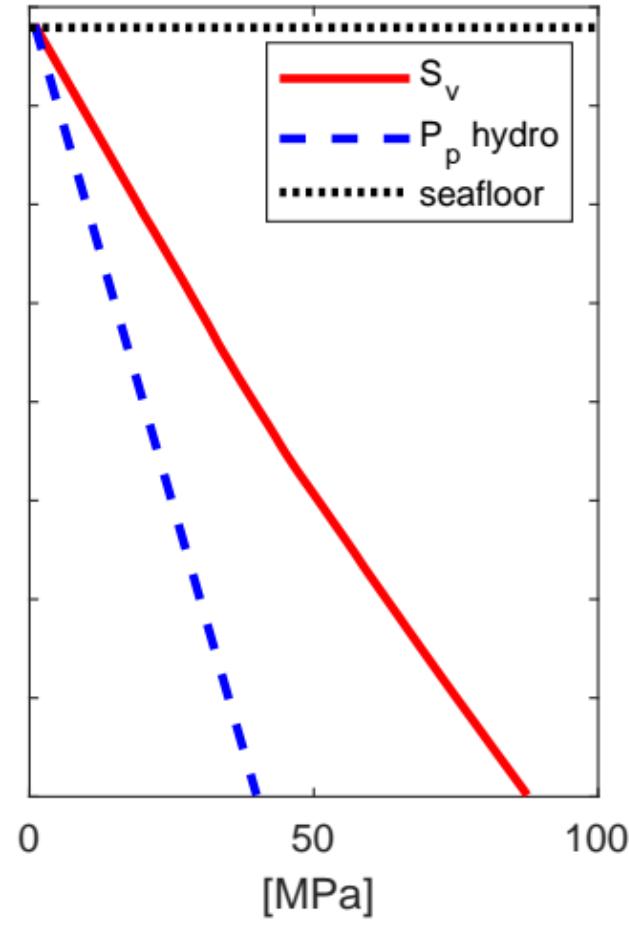
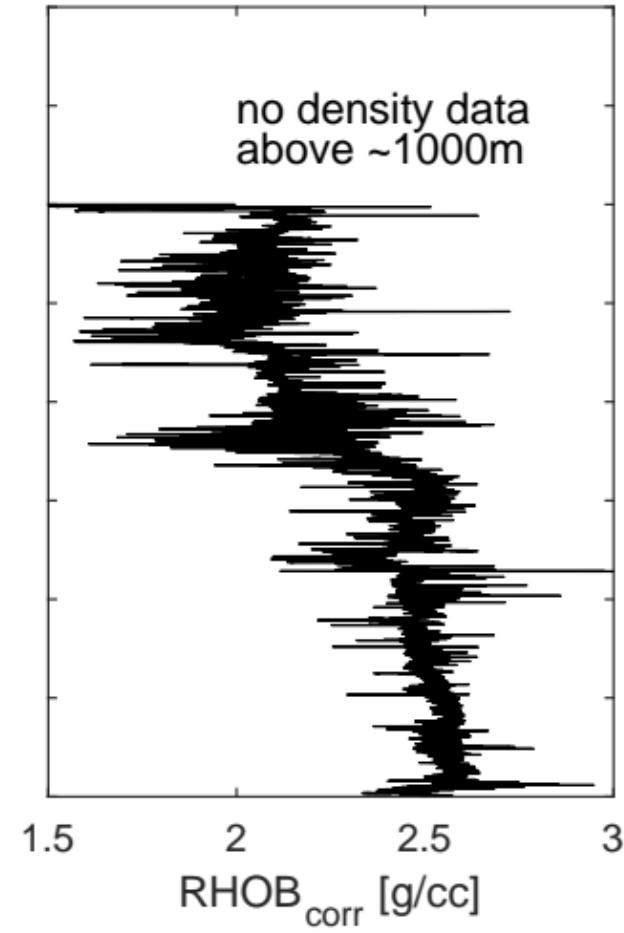
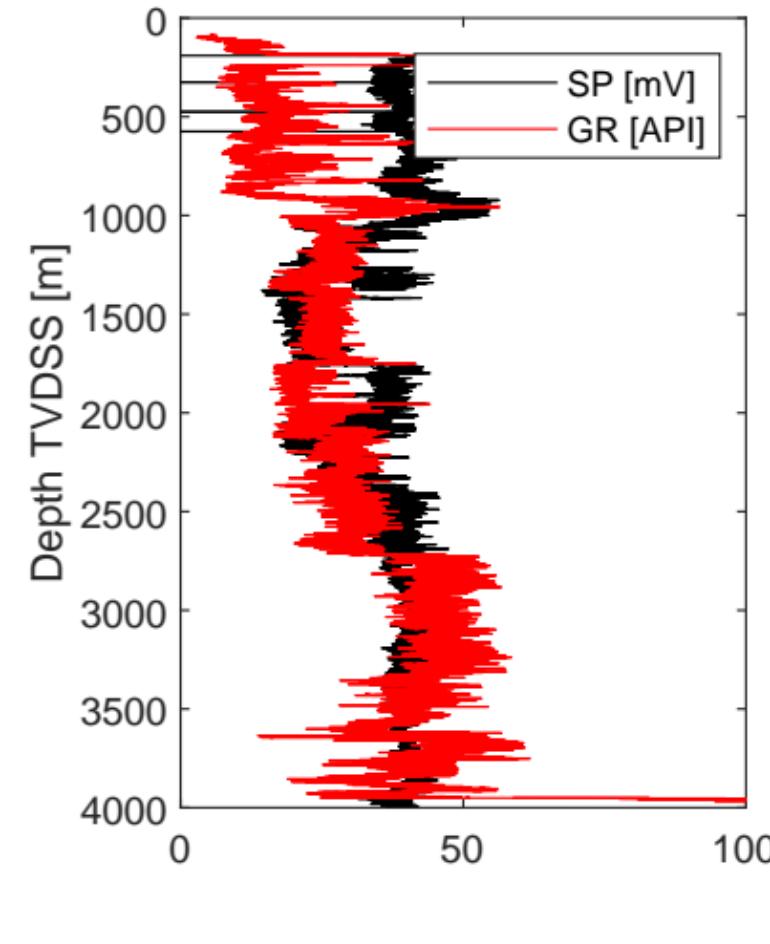
$$S_U(z) = \int_0^z \rho_{bulk}(z') dz'$$

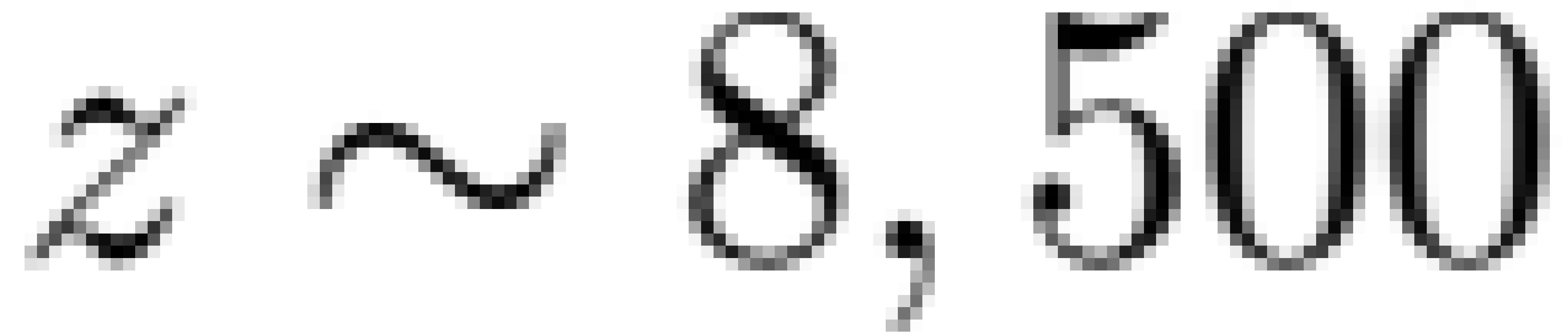


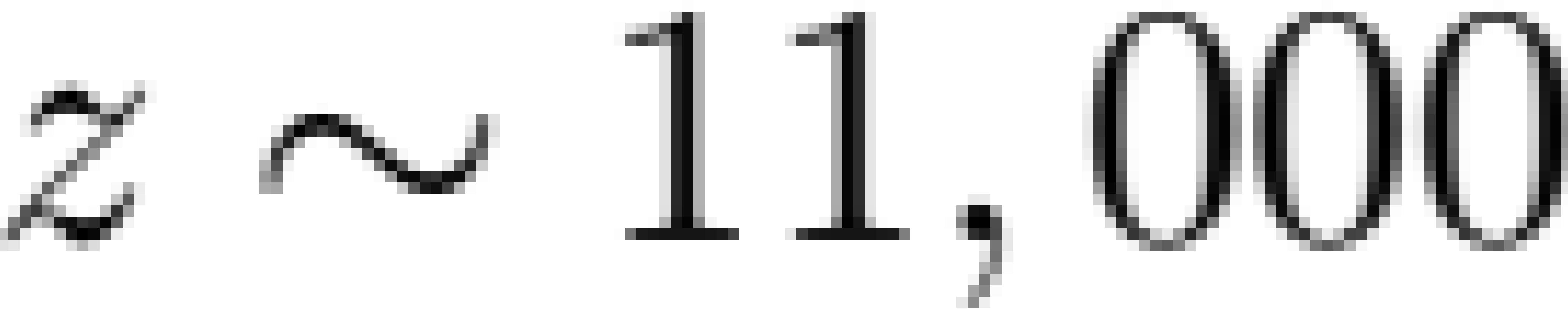
$$S_v(z_i) = \sum_{j=1}^i \rho_{bulk}(z_i) g\Delta z_i$$



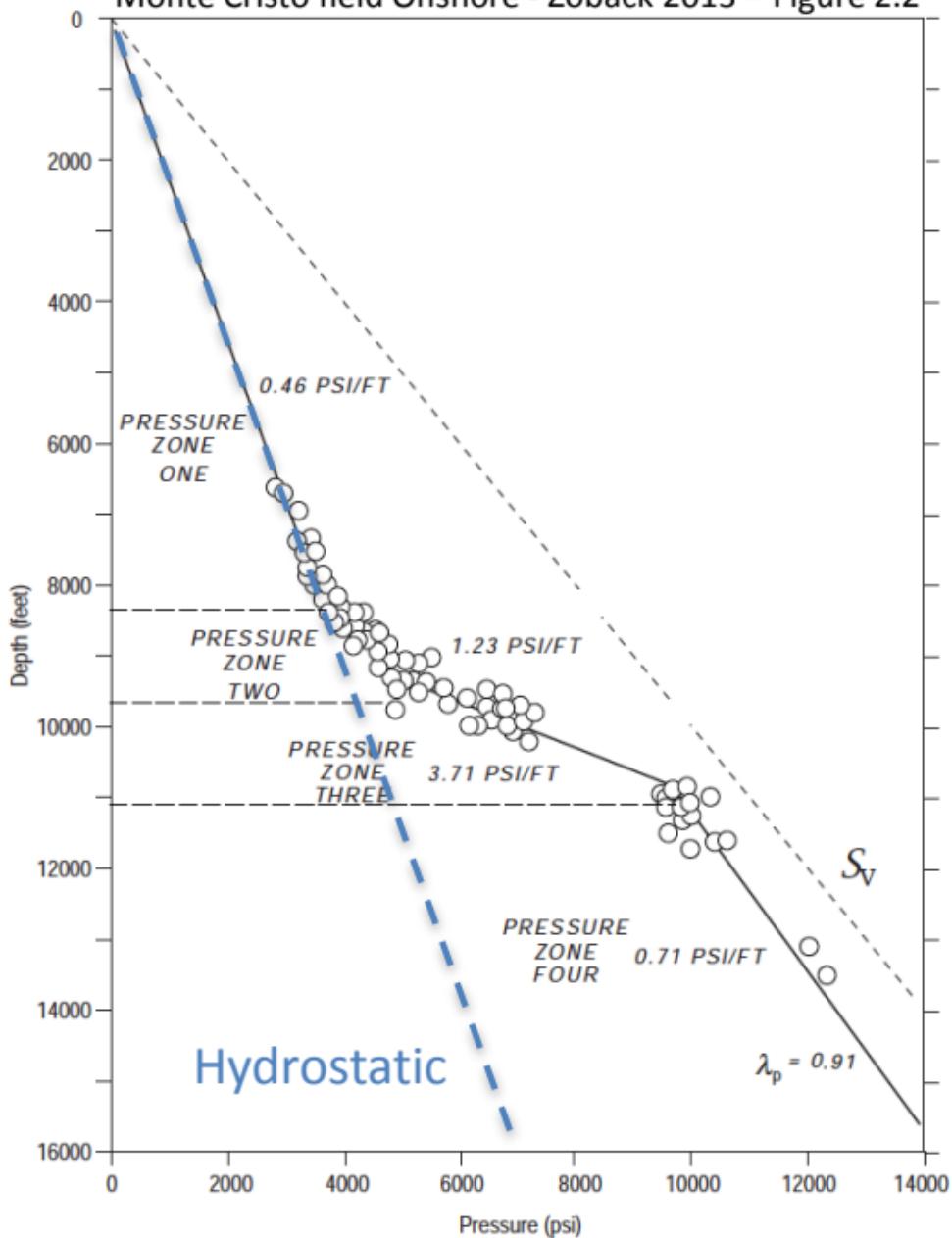








# Monte Cristo field Onshore - Zoback 2013 – Figure 2.2



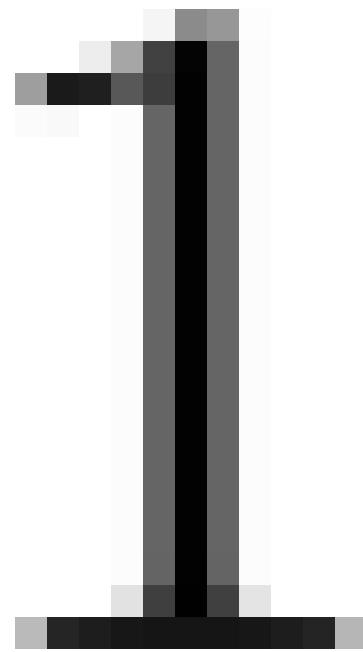
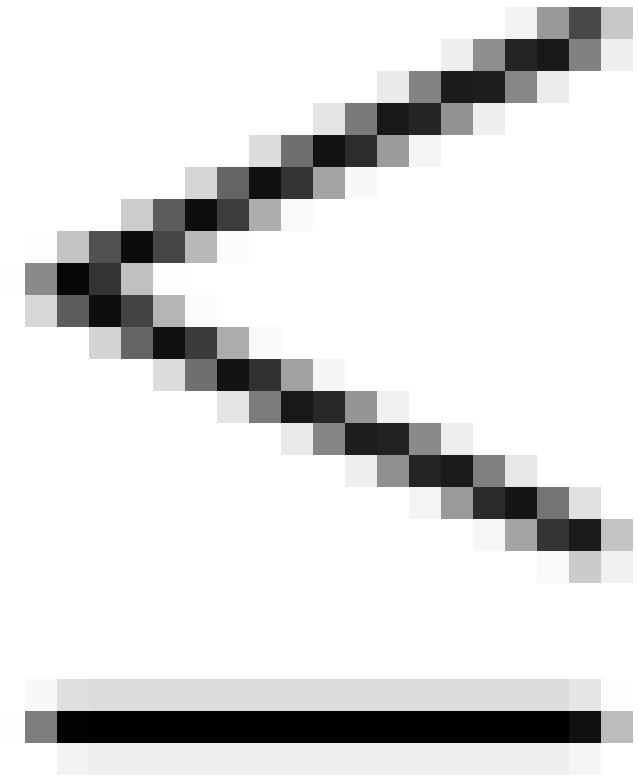


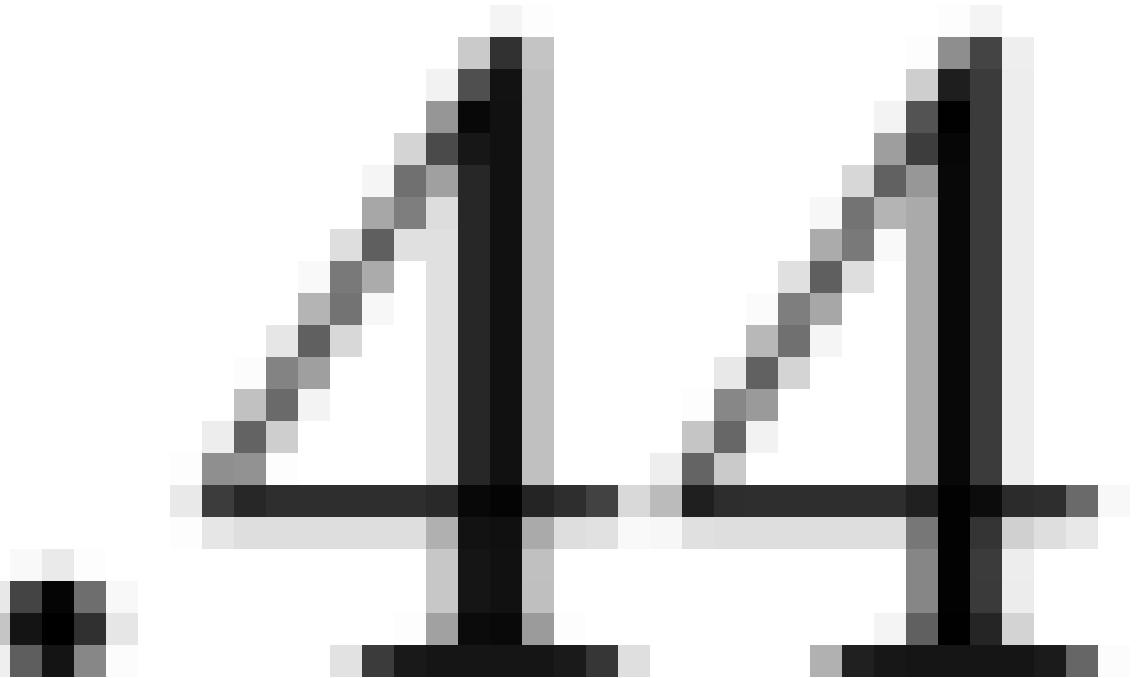
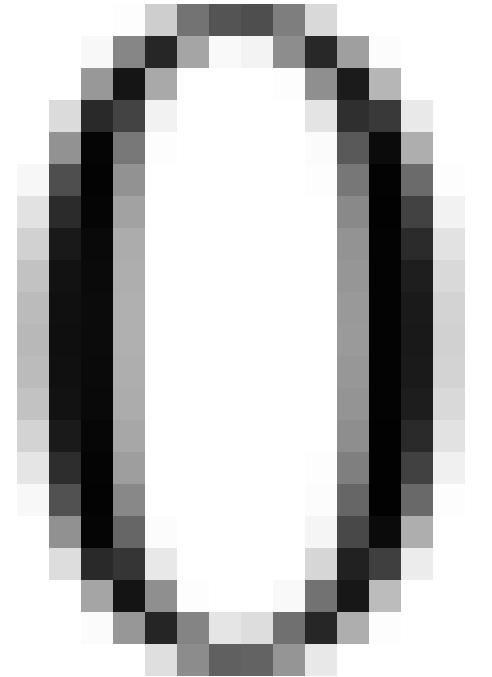
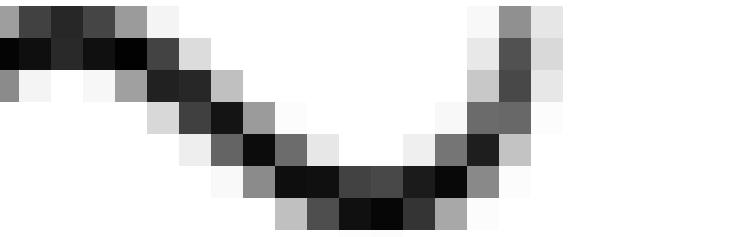
$\lambda_p(z)$

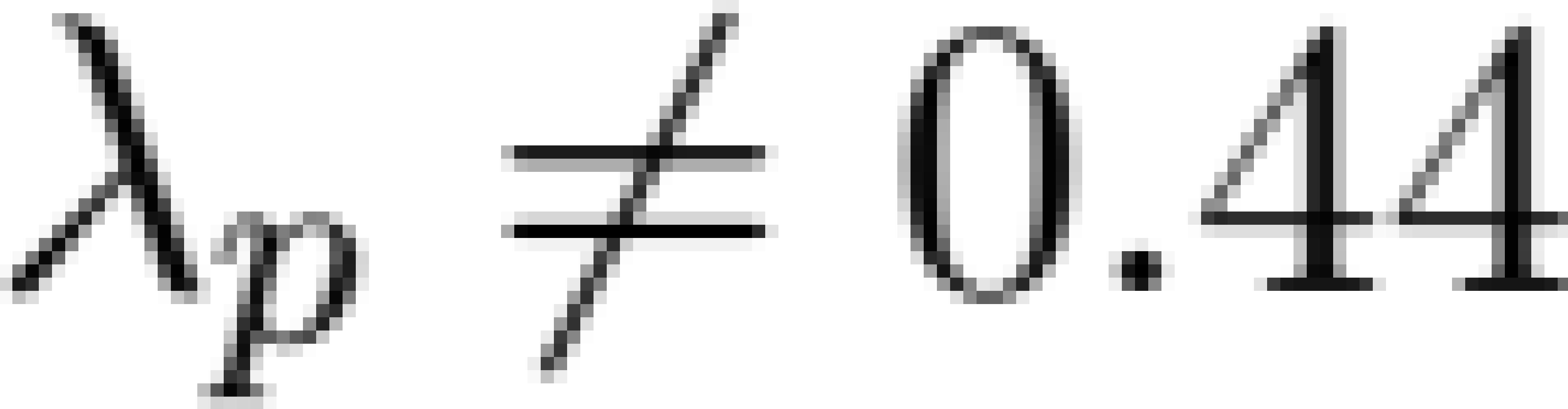


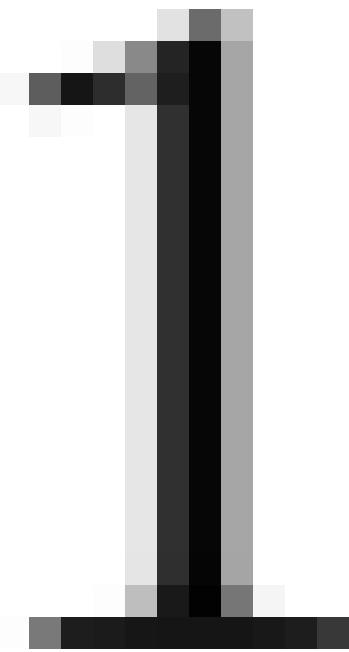
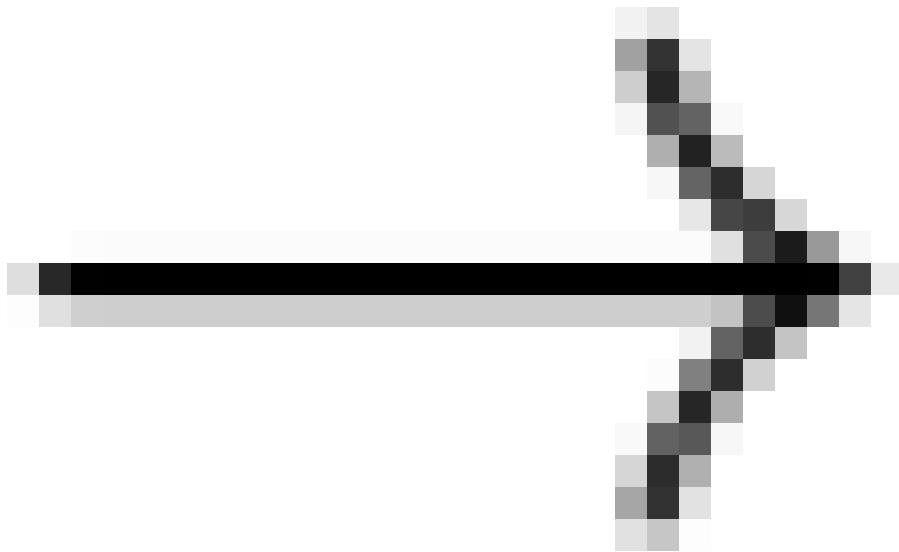
$P(z)$

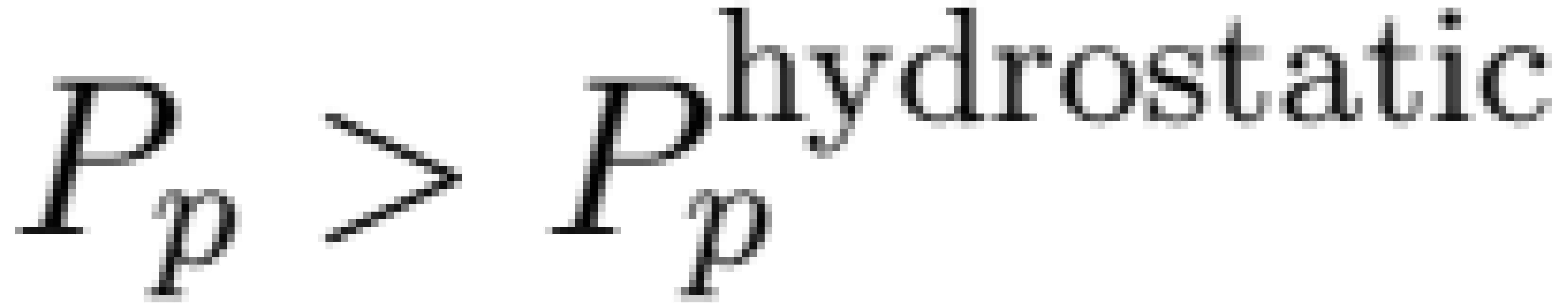
$S_T(z)$



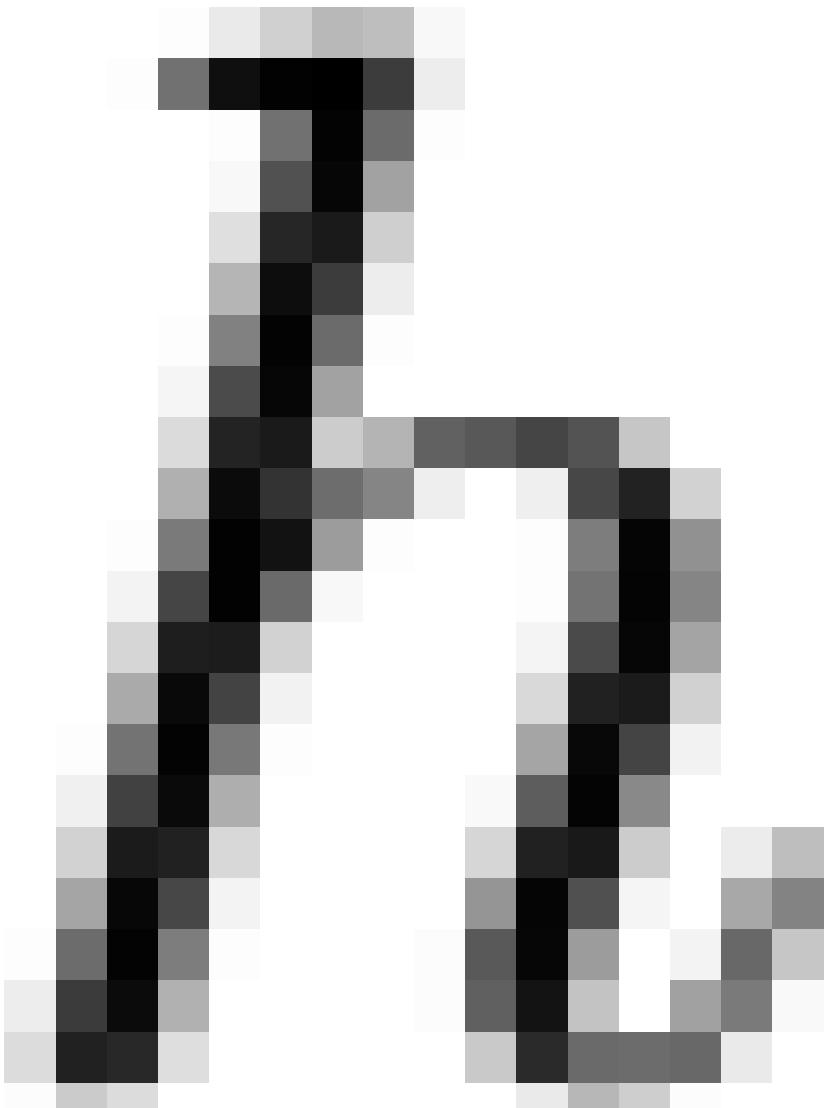














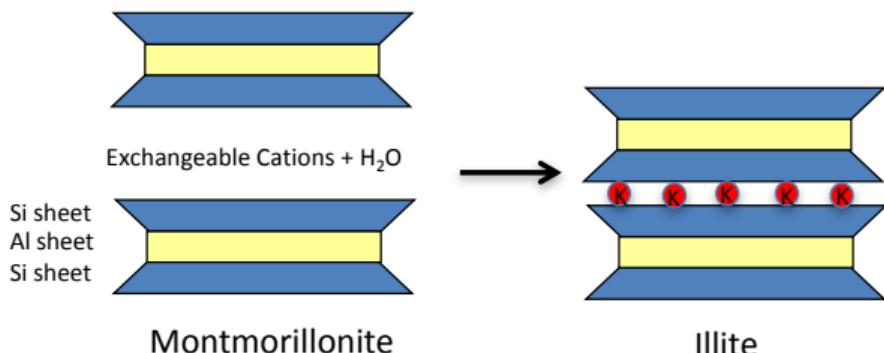


- **Aquathermal pressurization**

- $\Delta T \rightarrow \Delta P$

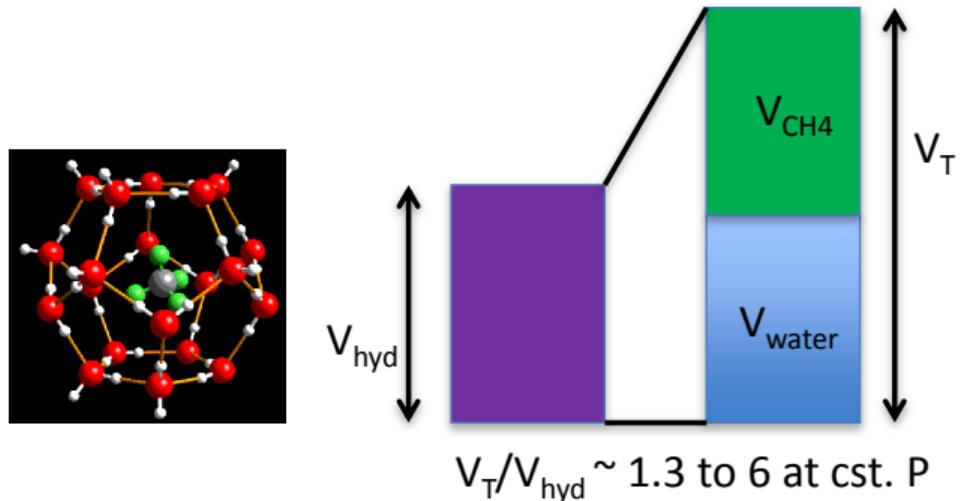
- **Dehydration reactions**

- $\Delta V \rightarrow \Delta P$
  - Montmorillonite to Illite (frees water)

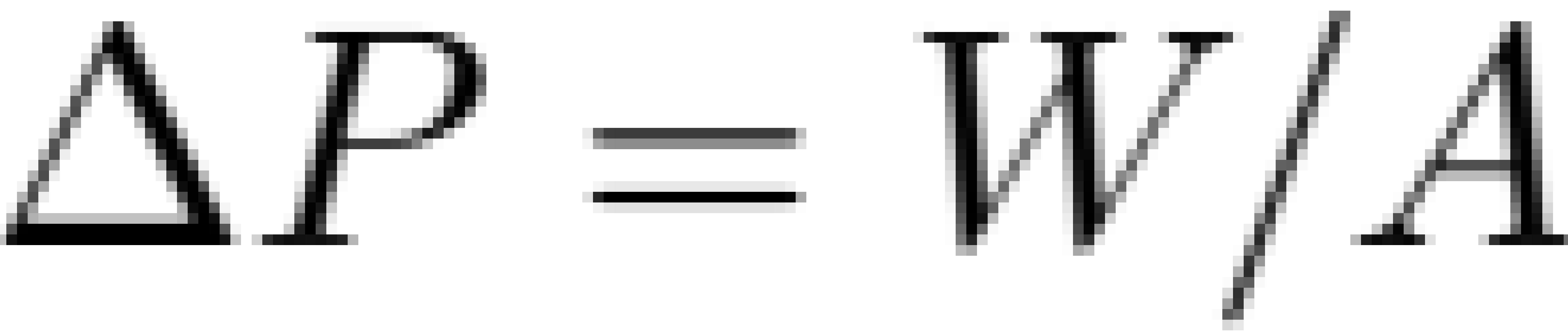


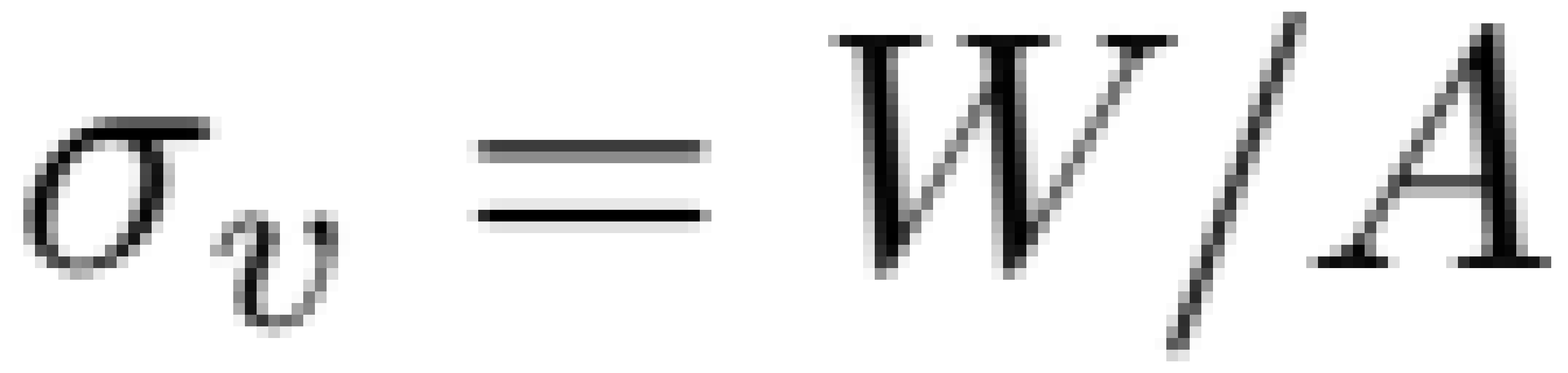
- **Hydrocarbon generation**

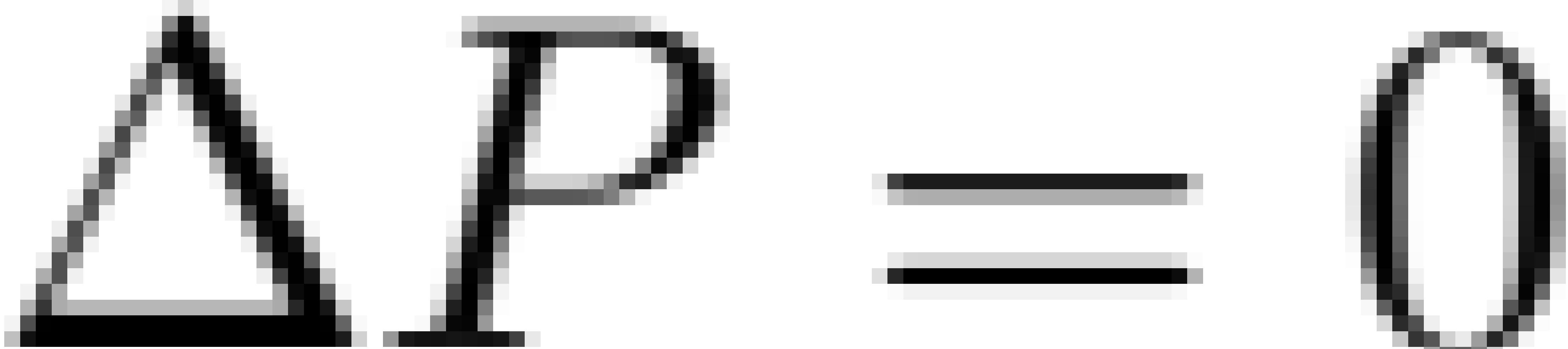
- $\Delta V \rightarrow \Delta P$



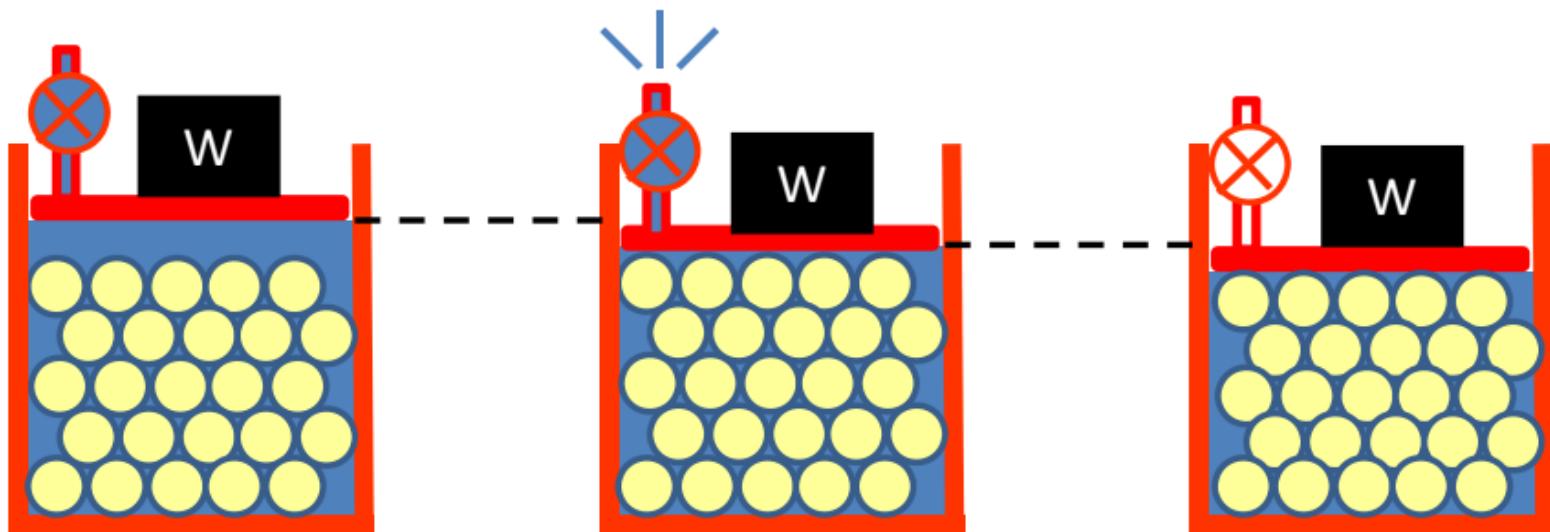




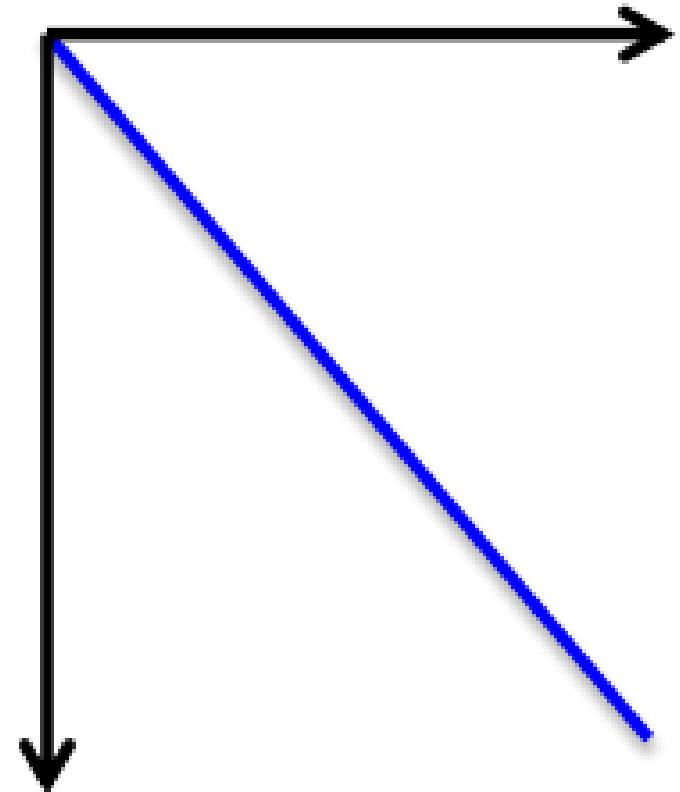
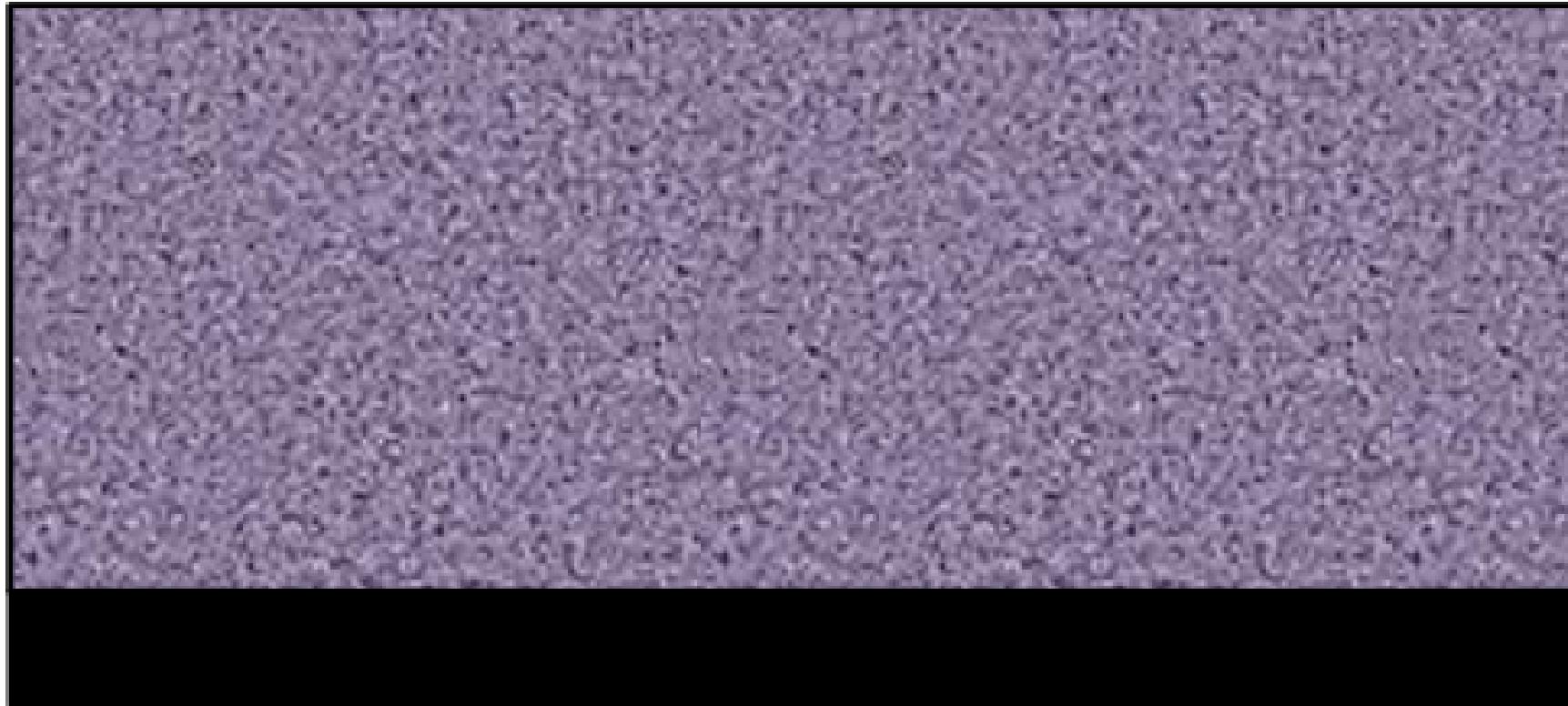


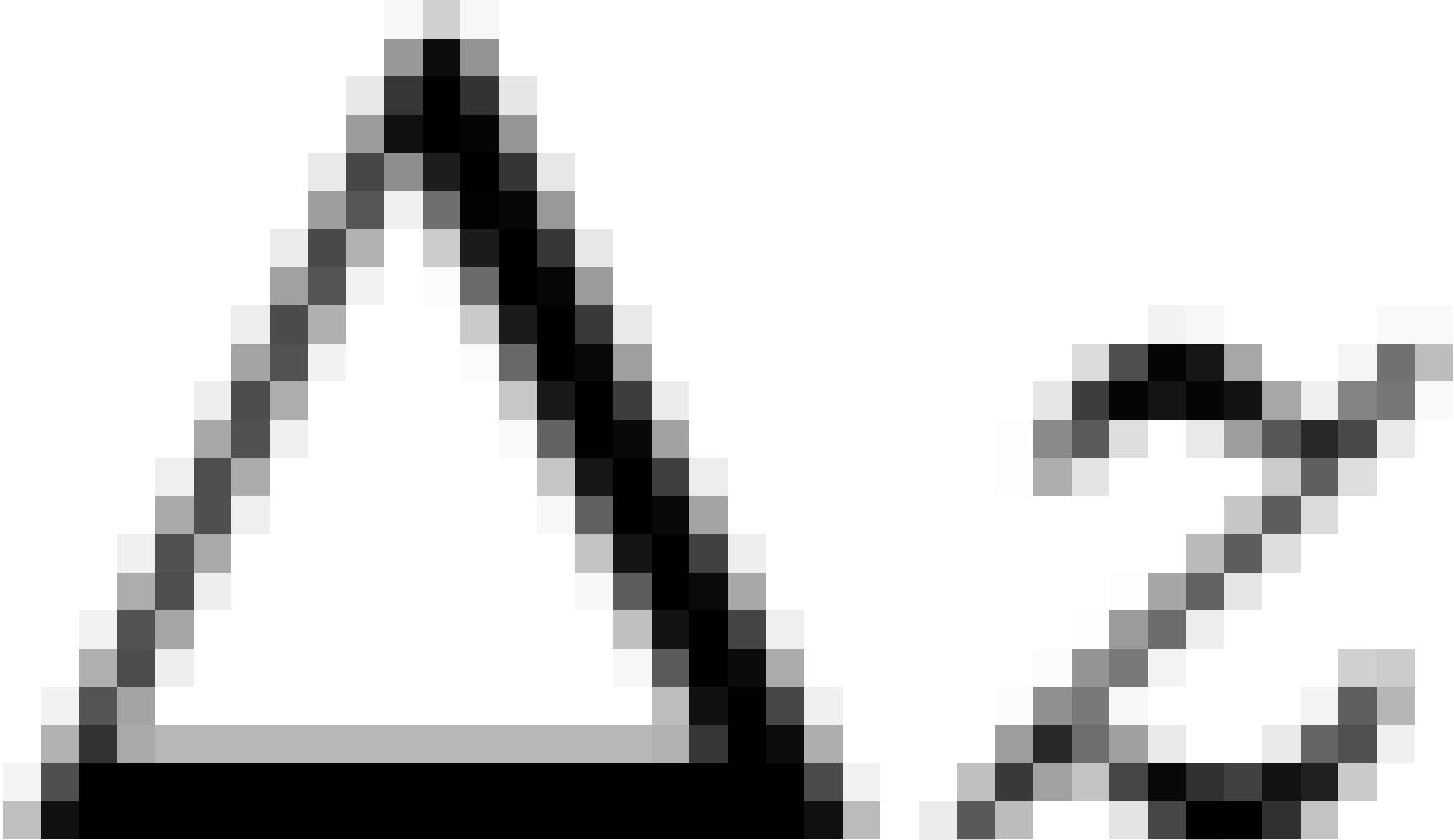


- **Disequilibrium compaction (Underconsolidation)**
  - $\Delta S \rightarrow \Delta P$  (Vertical)
- **Tectonic compression**
  - $\Delta S \rightarrow \Delta P$  (Horizontal)



Pressure water



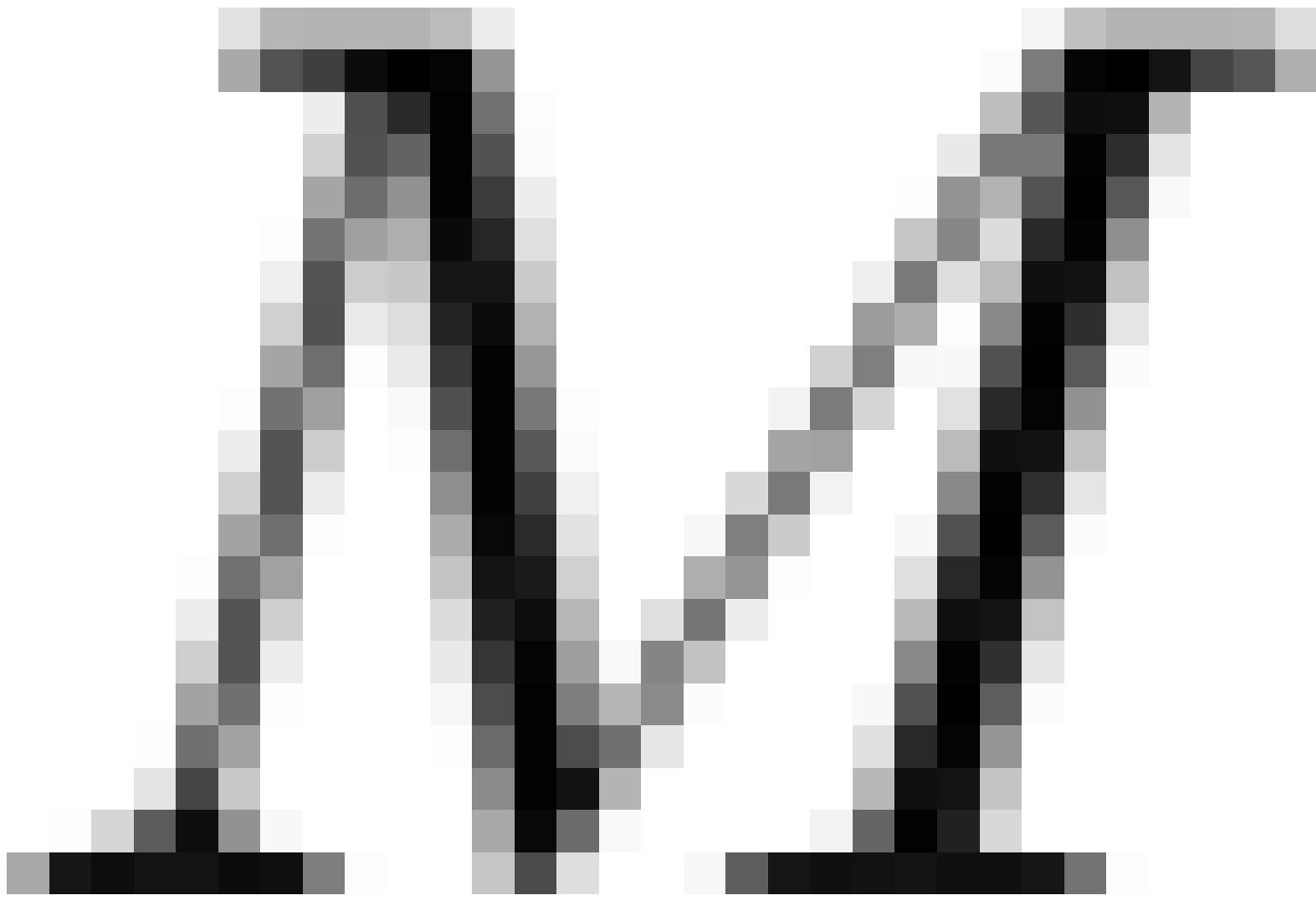


Dh

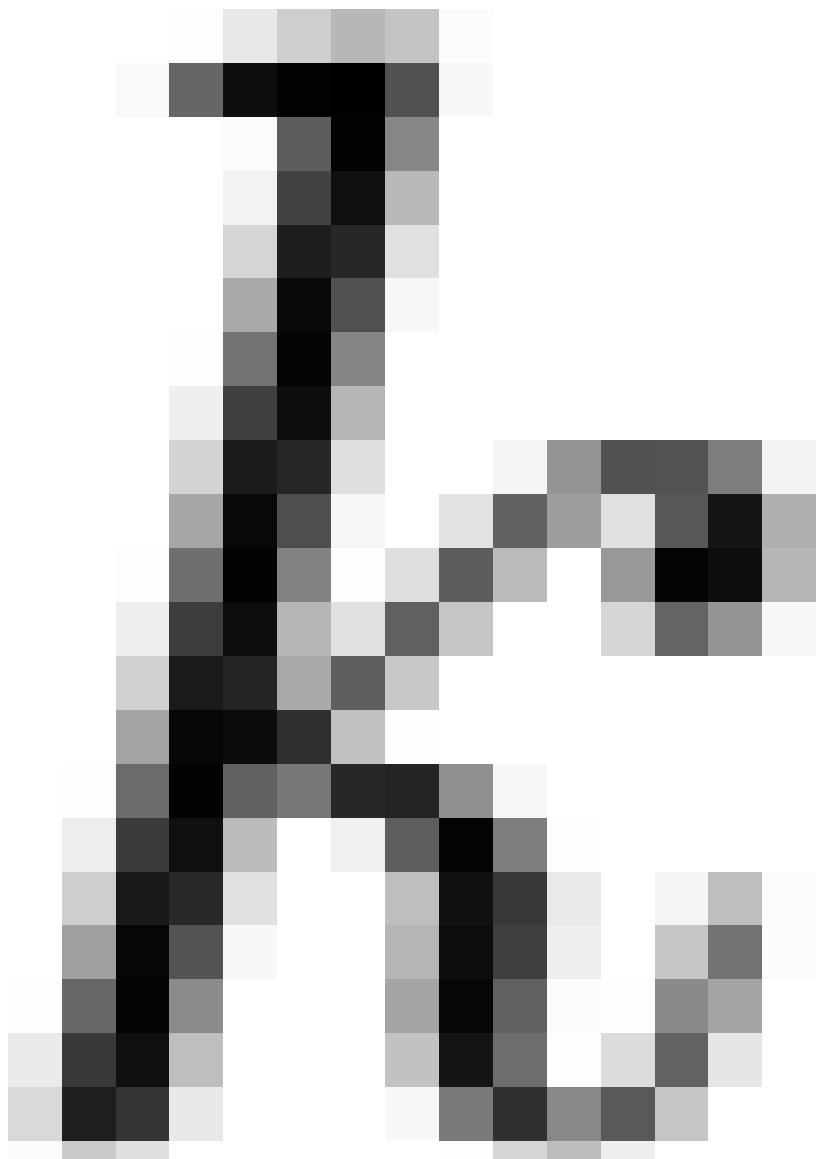
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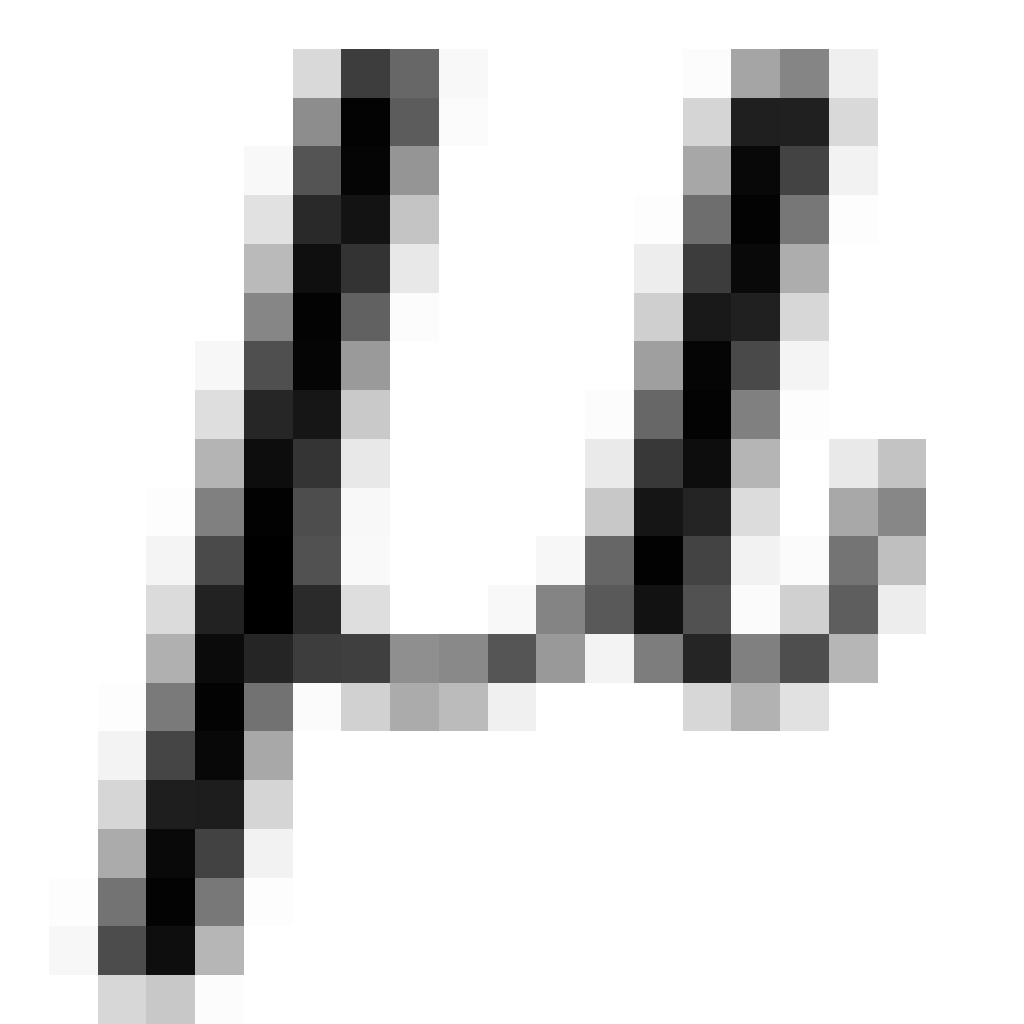
MK

ll









$\alpha p$

$\beta$

$\delta t$

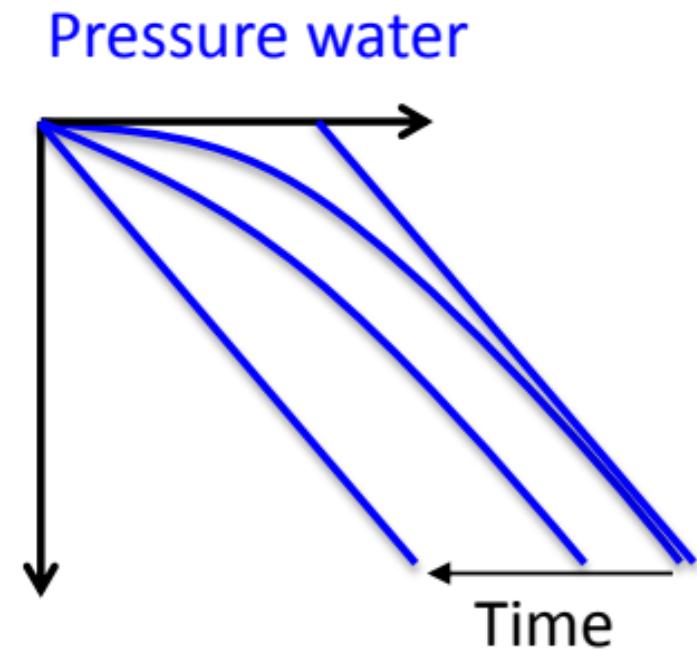
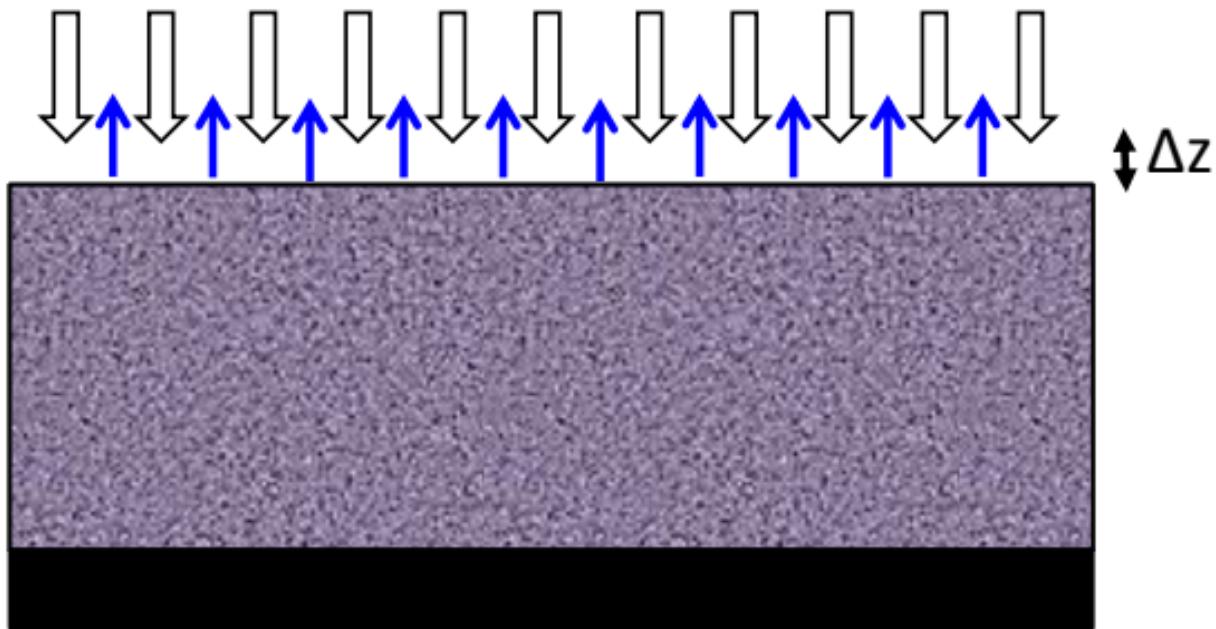
$=$

$D_h$

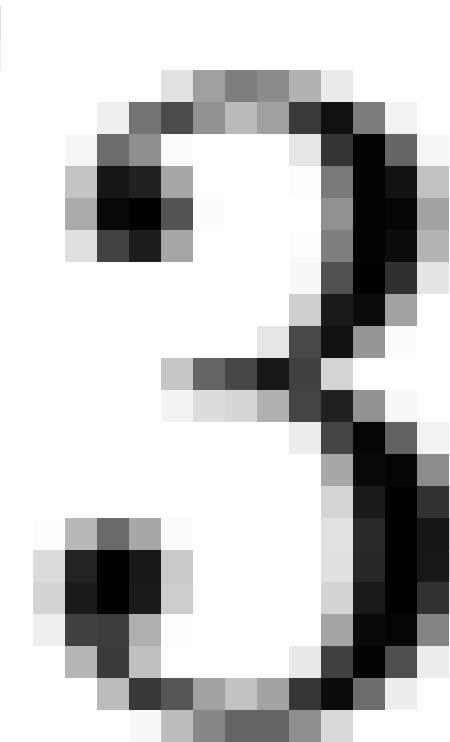
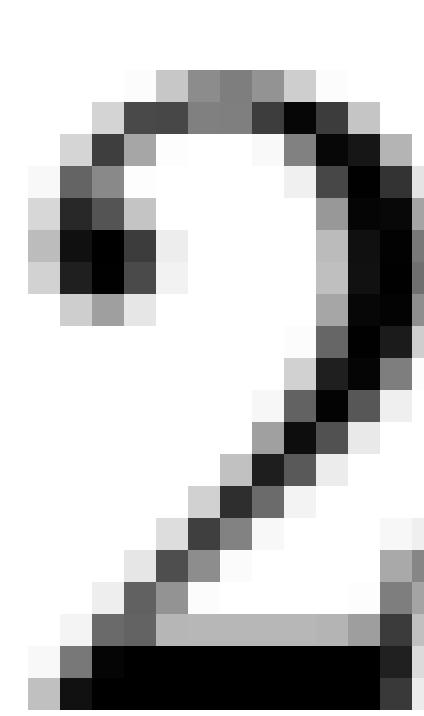
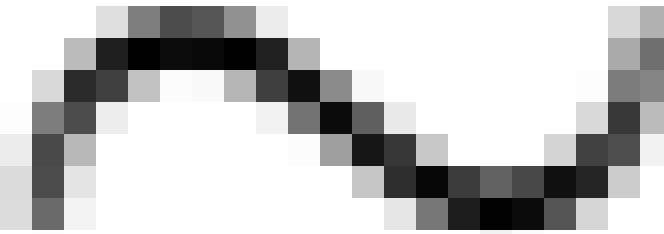
$dz^2$

$d^2 p$

## Rate of sedimentation (loading) and rate of fluid “escape”





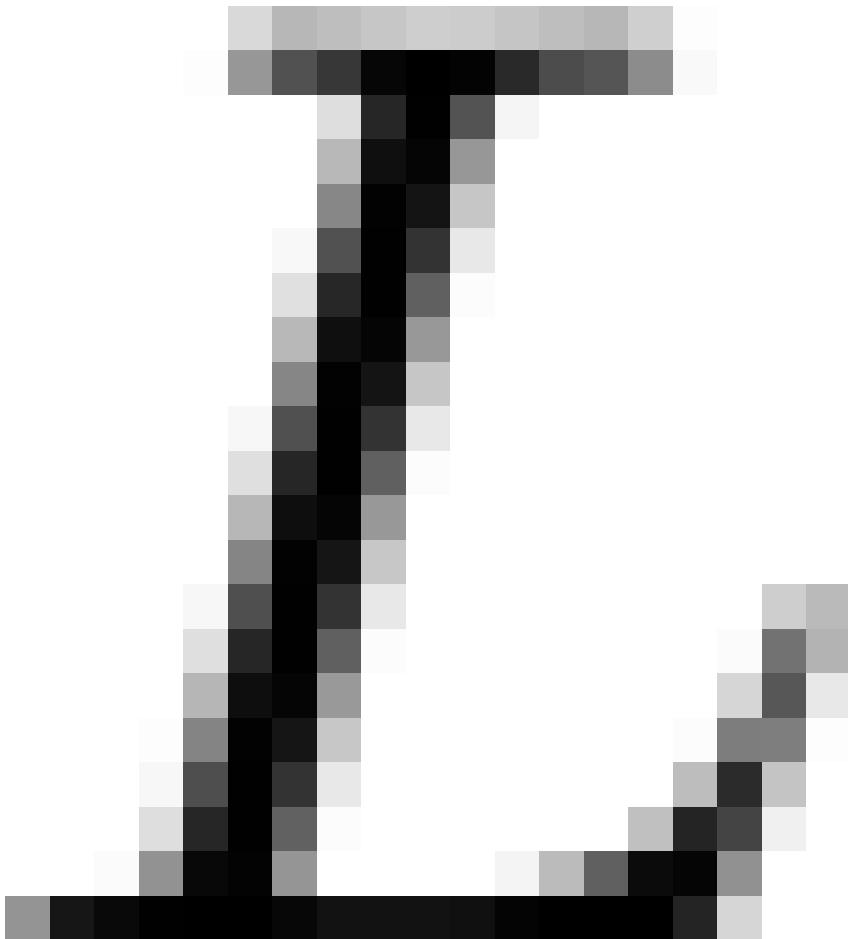


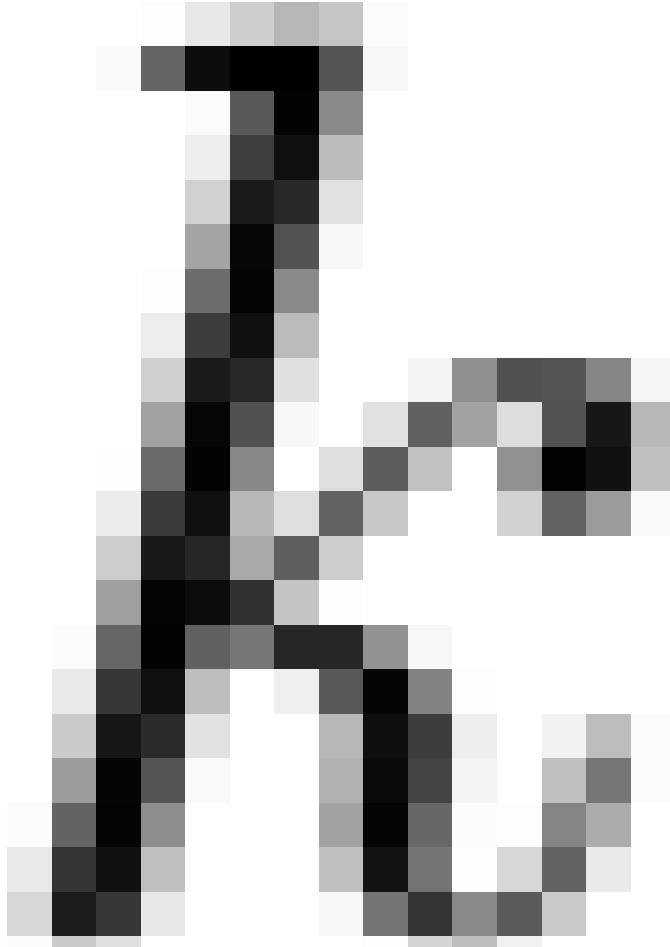
$\Gamma_{ch}$

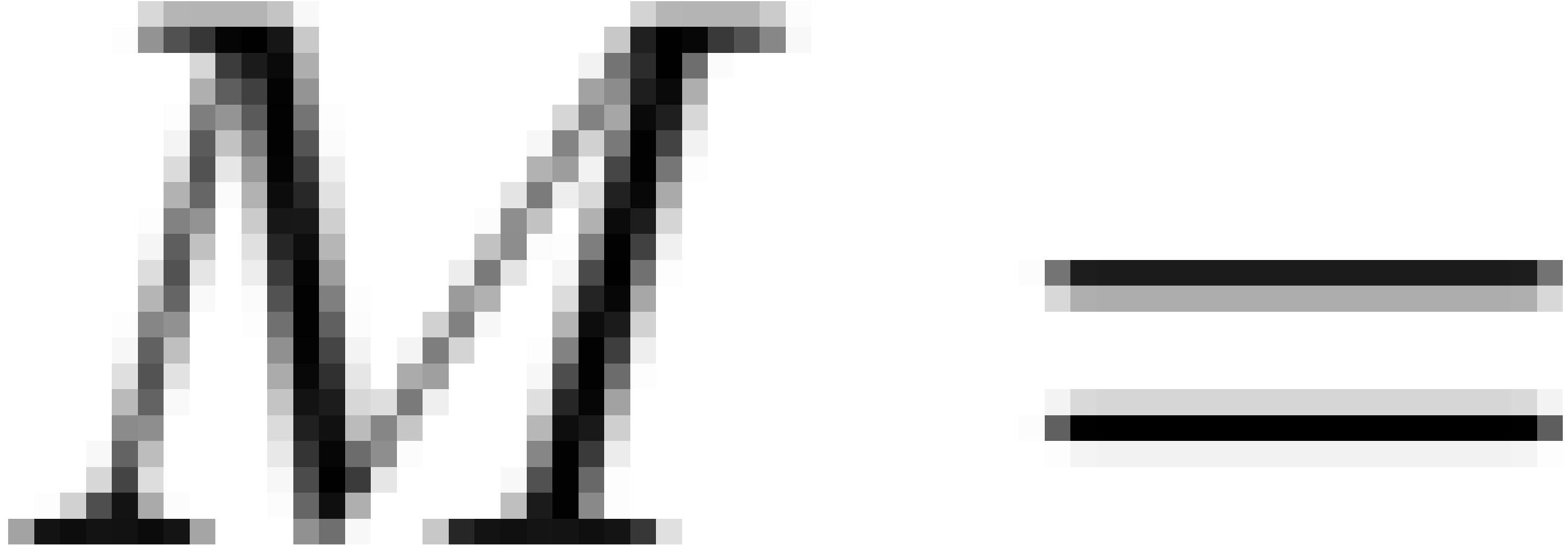


$D_h$

$\Gamma^2$

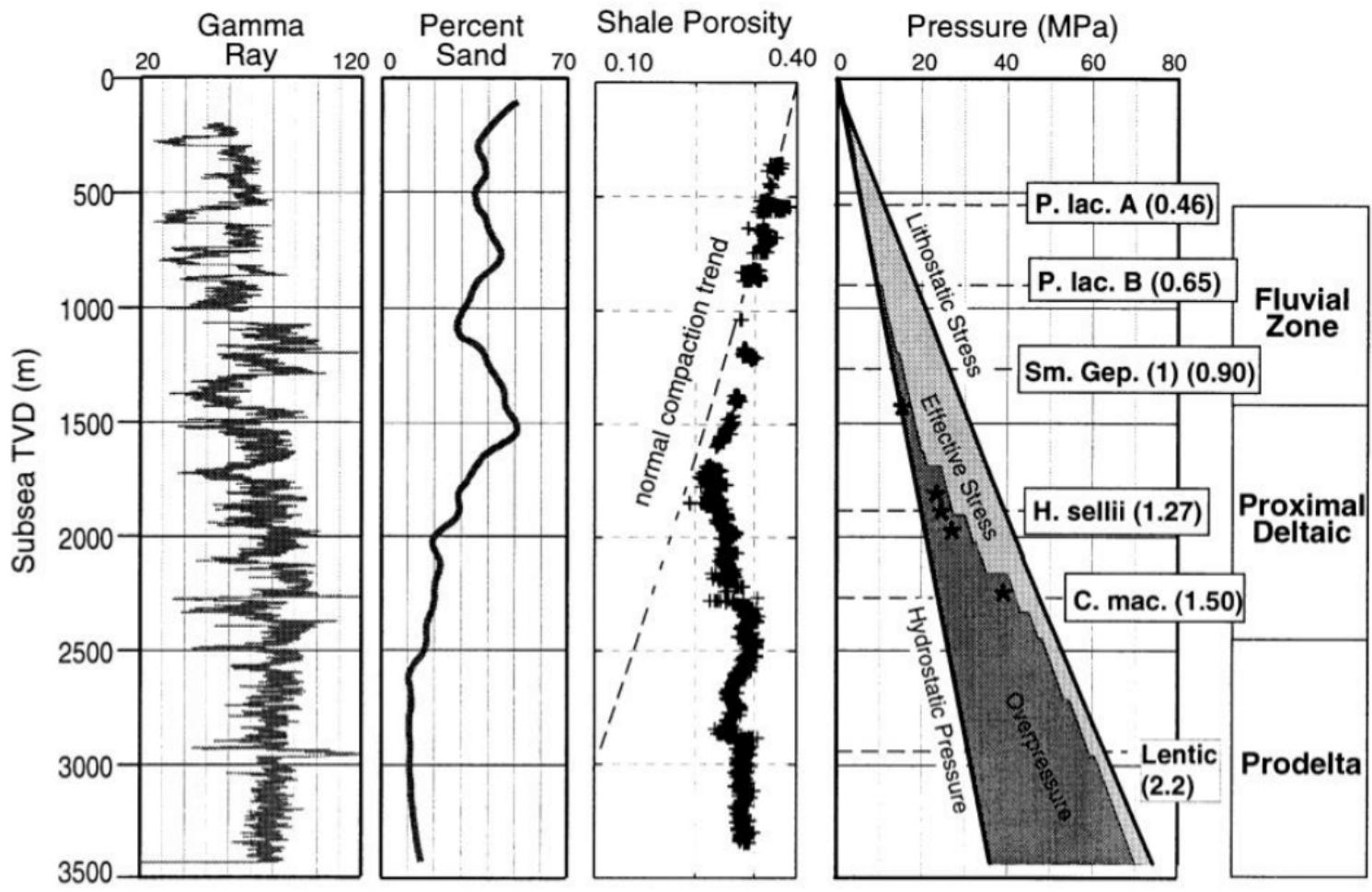


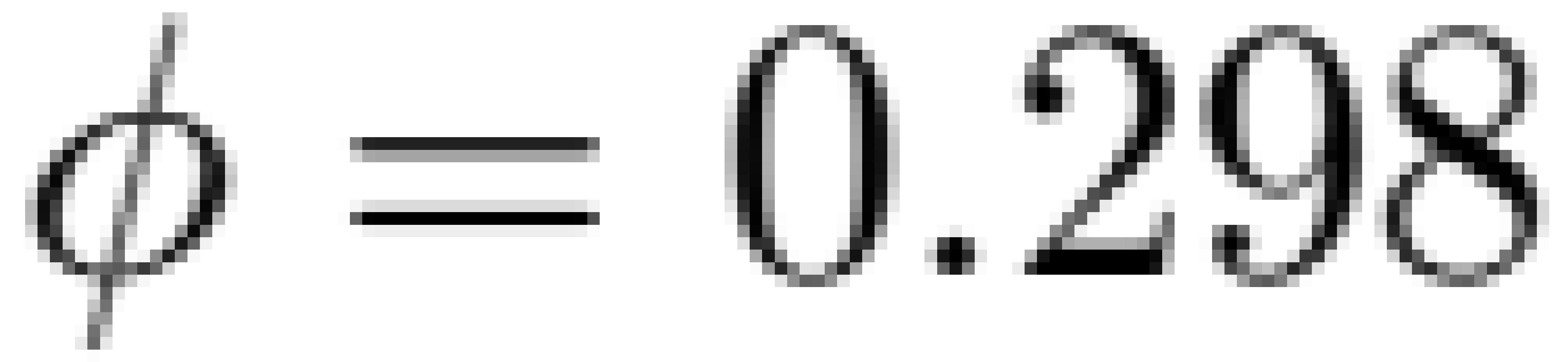


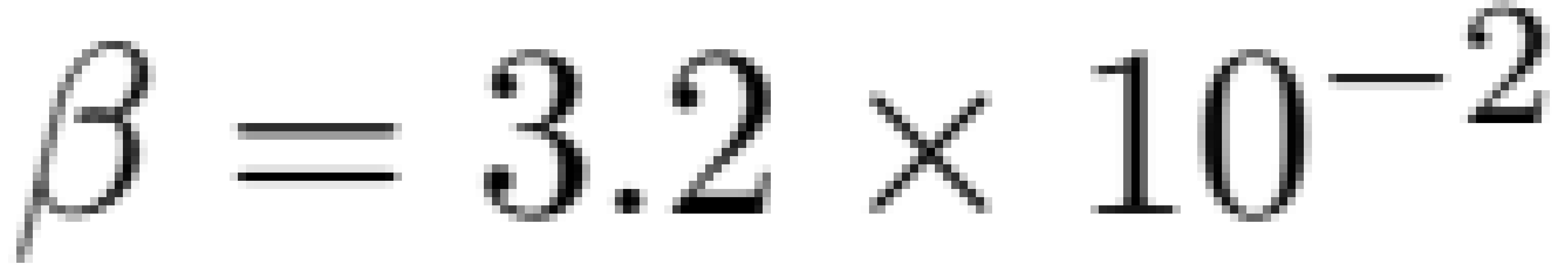


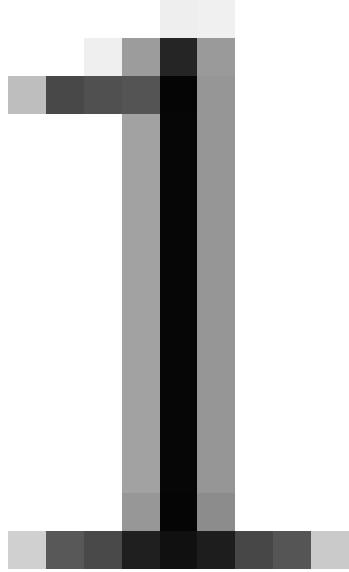


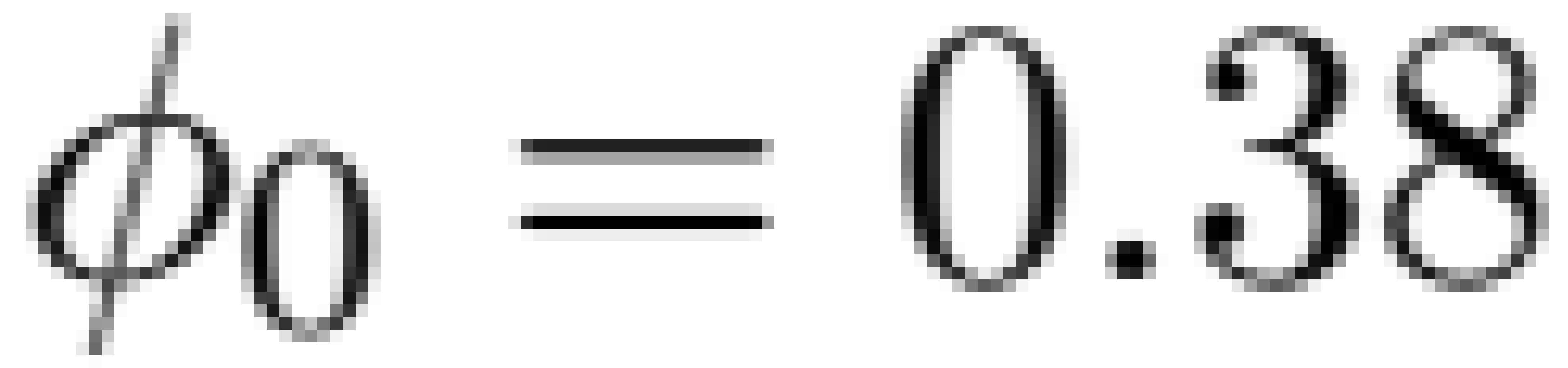






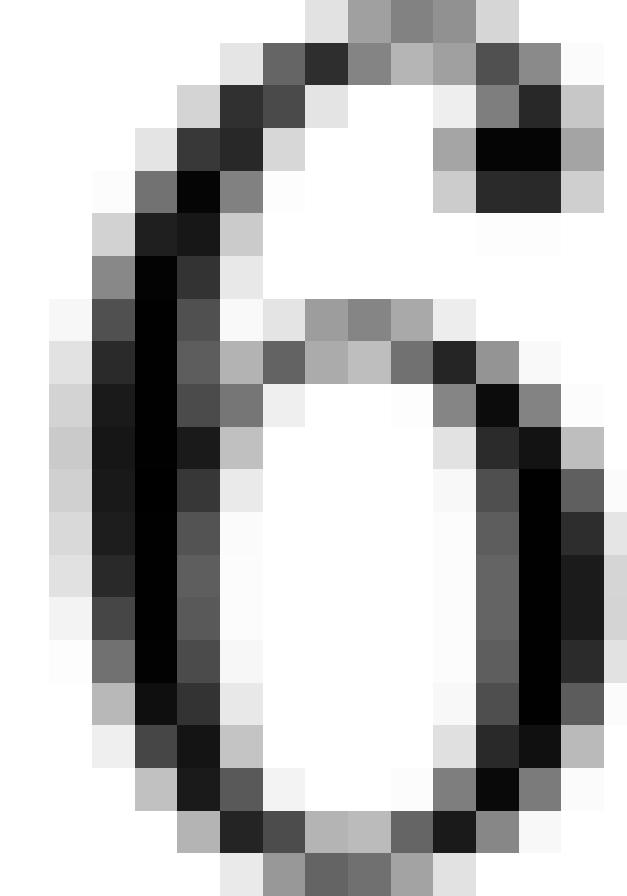
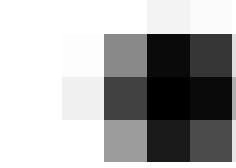
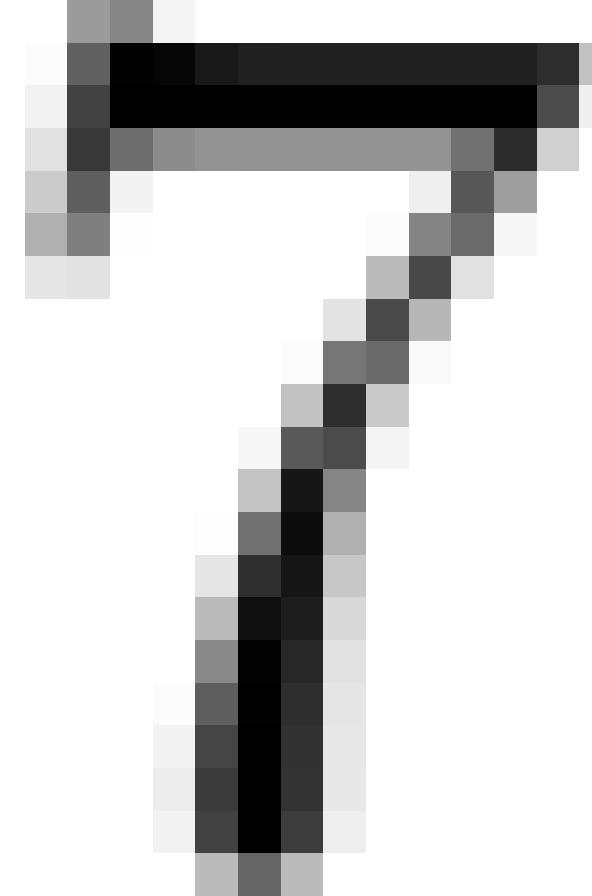


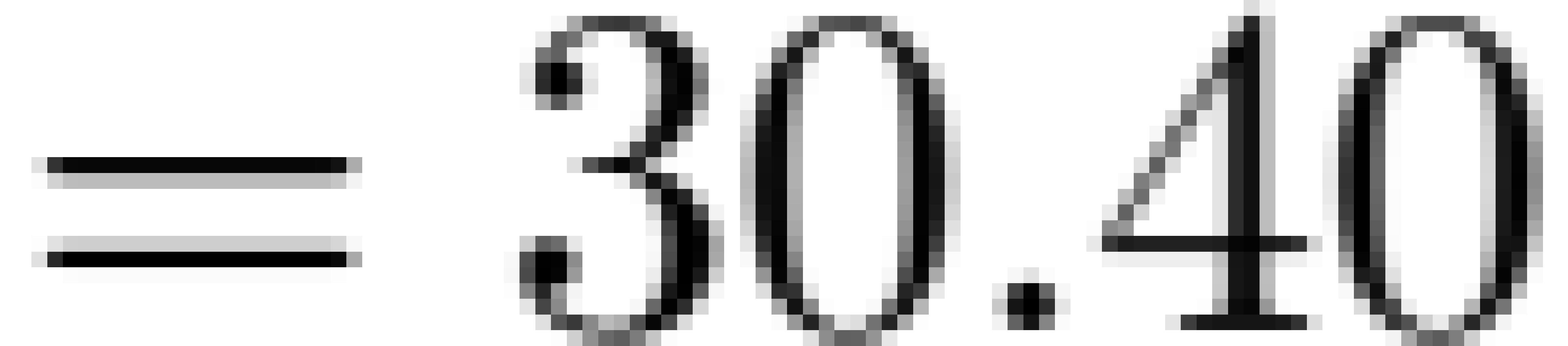




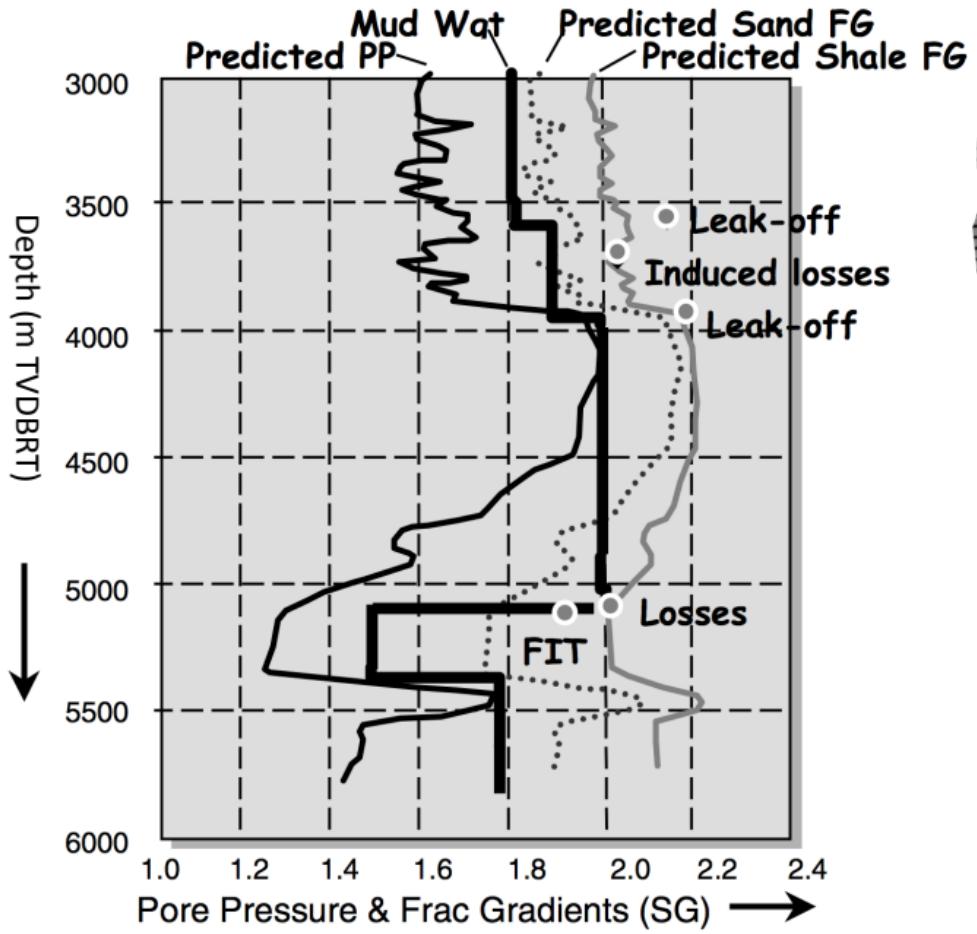


$$P_p = S_v + \frac{\ln \left( \frac{\phi}{\phi_0} \right)}{\beta} = 38$$





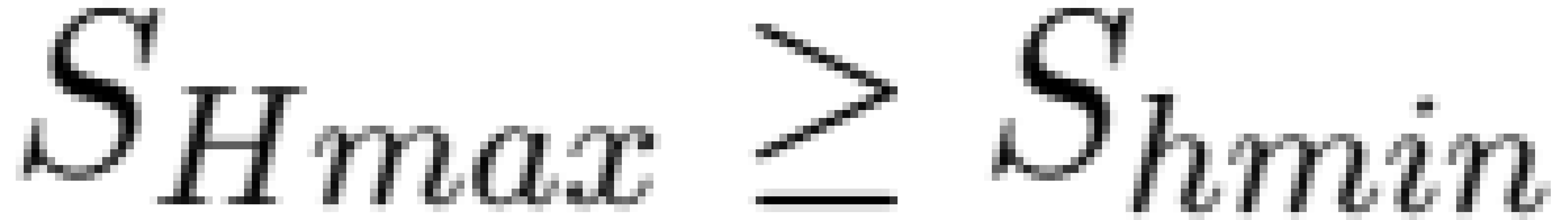
$P$  = 30.40 MPa  
 $P$  = 0.8 MPa  
 $S_2$  = 38 MPa

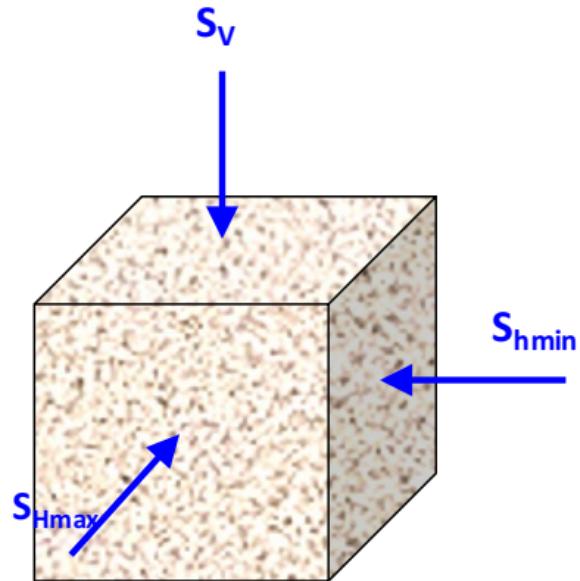
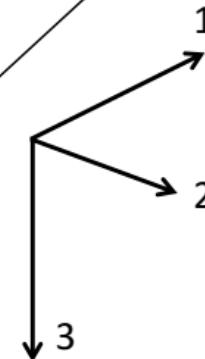
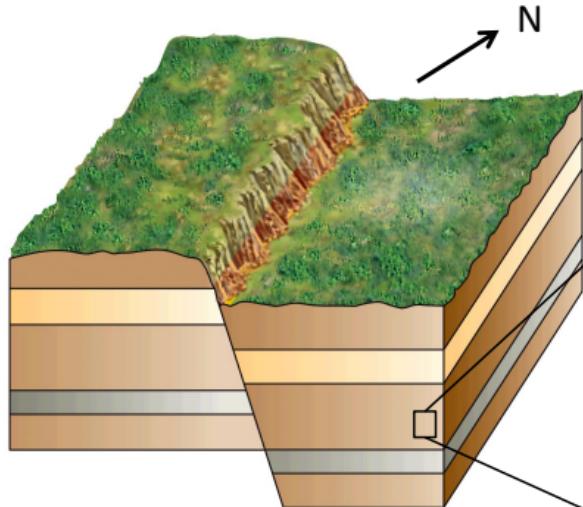


[Caspian Sea, Alberta and McLean – SPE67740]

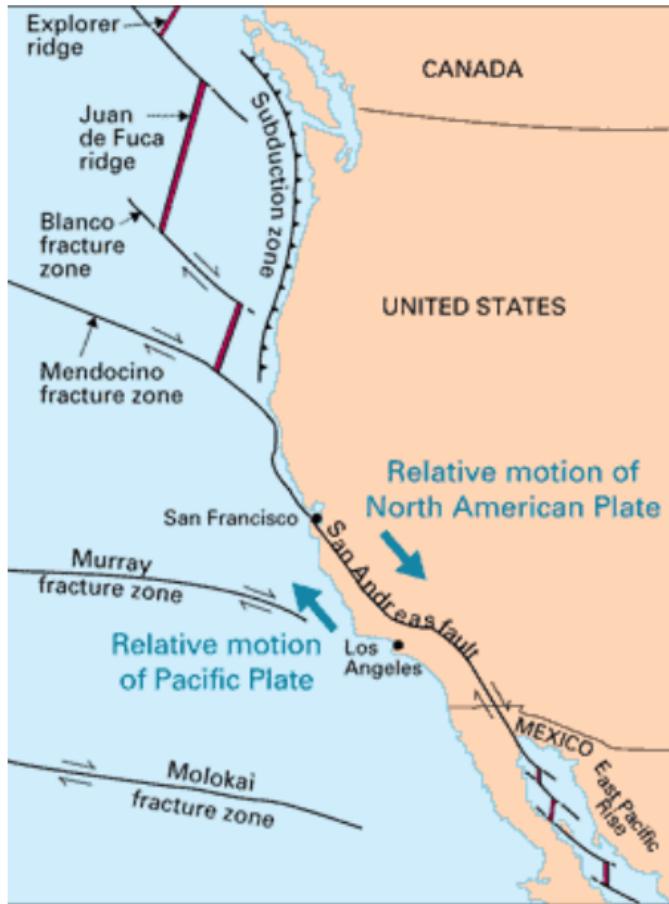






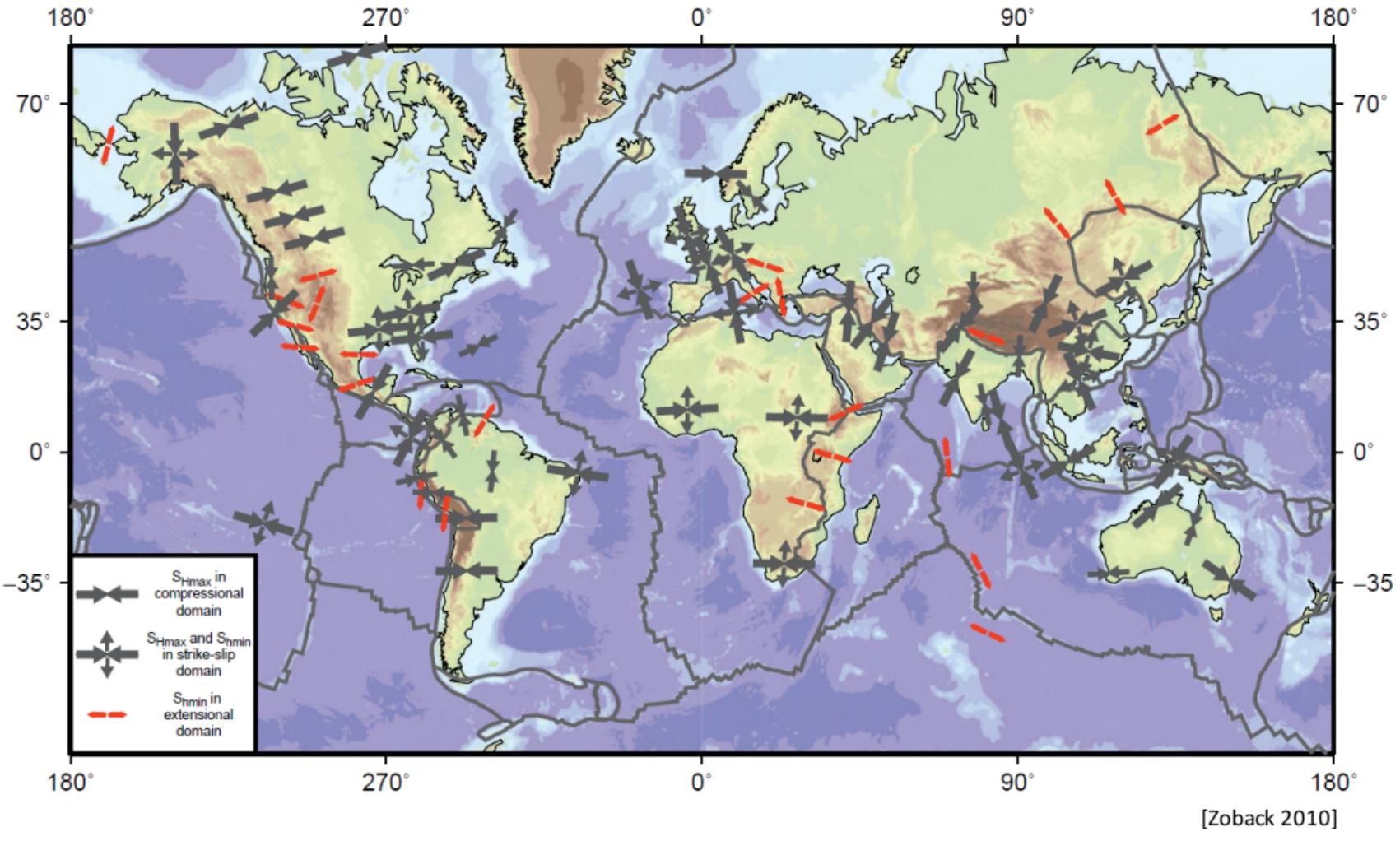


$$S = \begin{bmatrix} S_V & 0 & 0 \\ 0 & S_{H\max} & 0 \\ 0 & 0 & S_{H\min} \end{bmatrix}$$

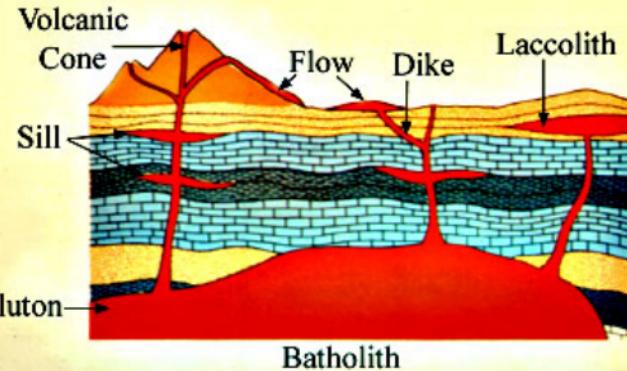


<http://pubs.usgs.gov/gip/dynamic/understanding.html#anchor5798673>

<http://en.wikipedia.org/wiki/File:Aerial-SanAndreas-CarrizoPlain.jpg>



## PLUTONS & VOLCANIC LANDFORMS





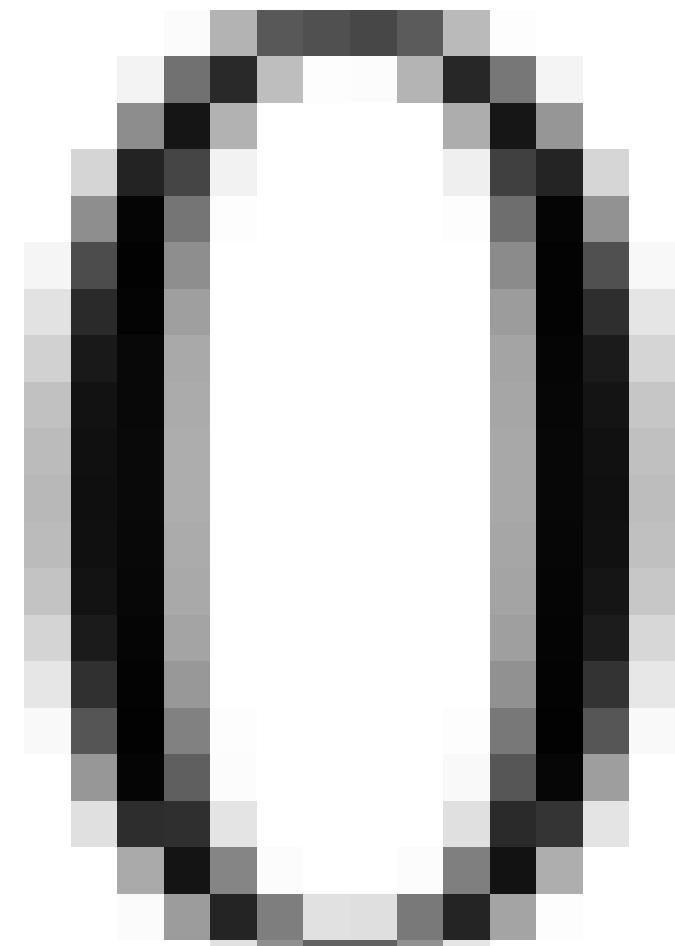
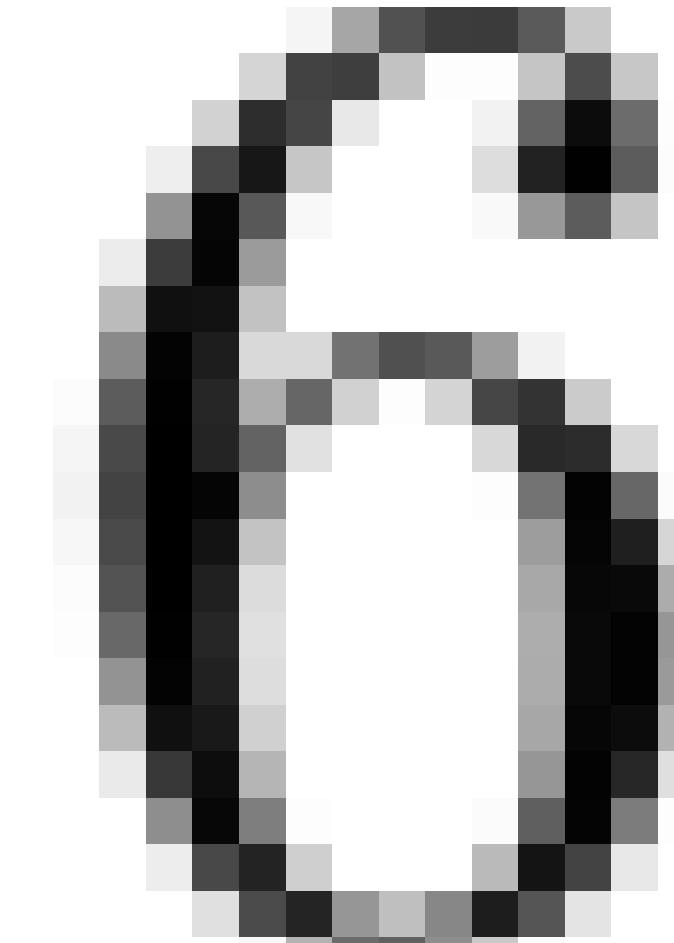
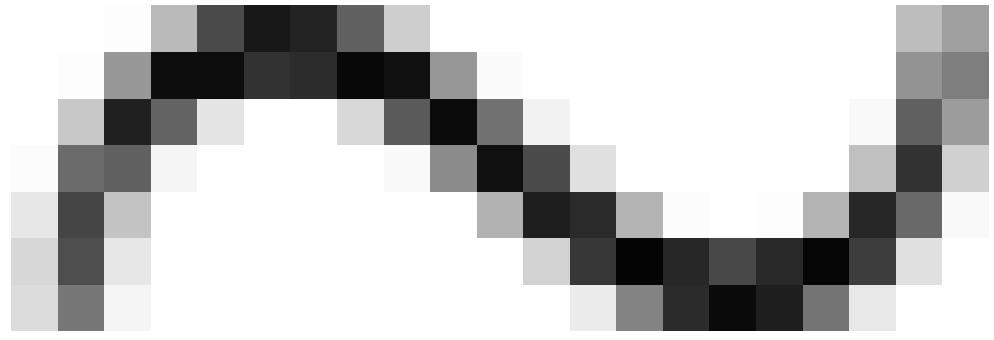








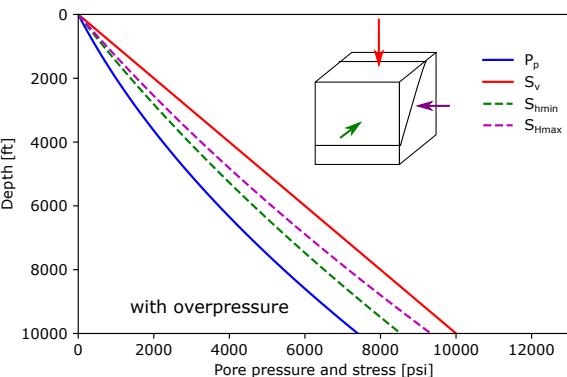
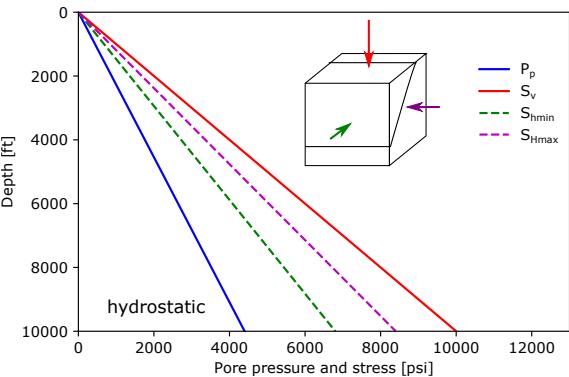




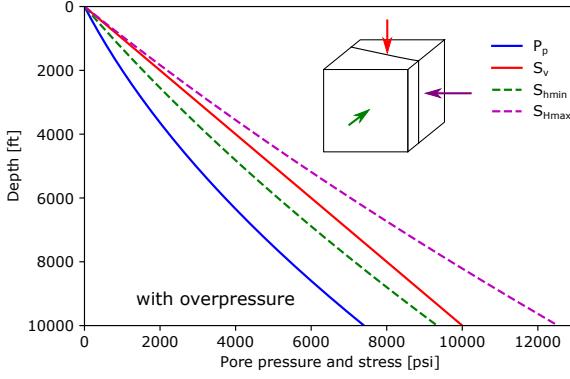
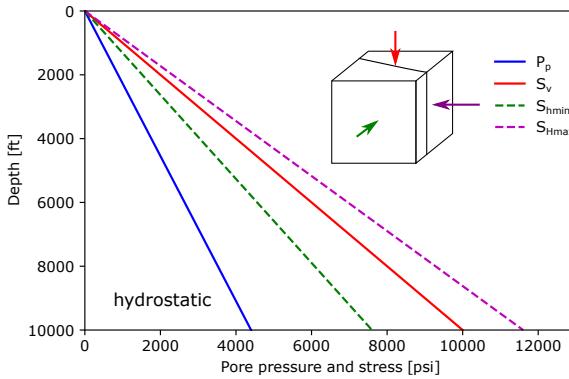




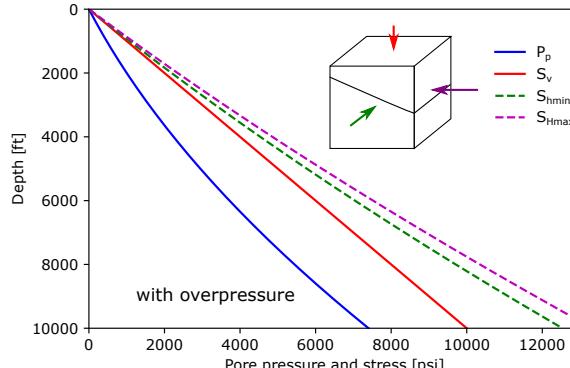
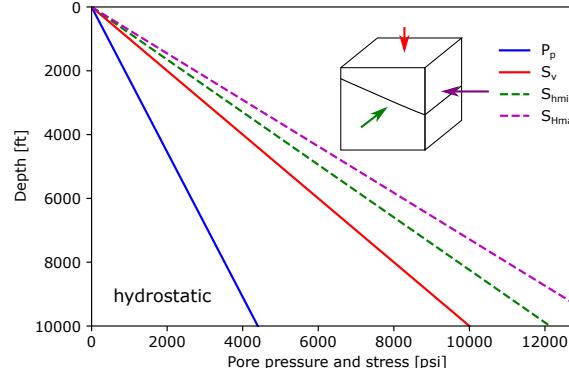
### Normal faulting: $S_v > S_{H\max} > S_{h\min}$

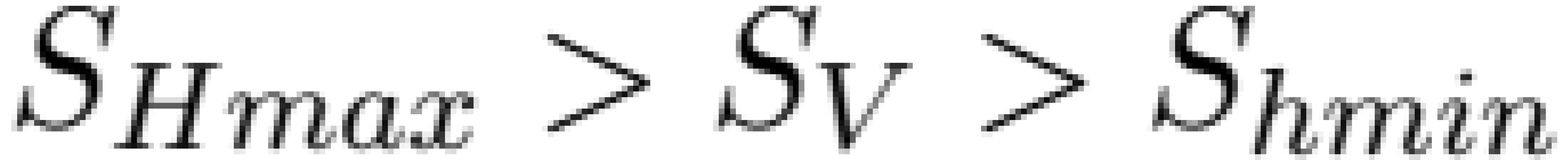


### Strike slip faulting: $S_{H\max} > S_v > S_{h\min}$



### Reverse faulting: $S_{H\max} > S_{h\min} > S_v$





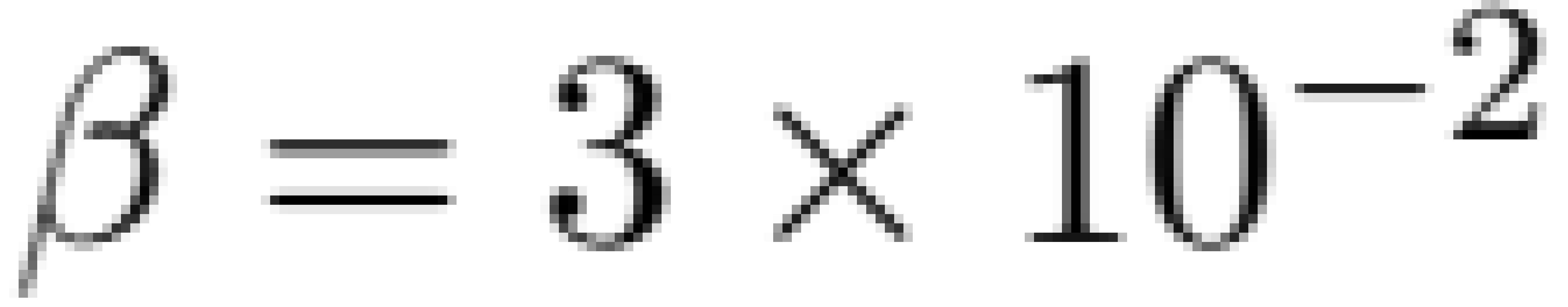


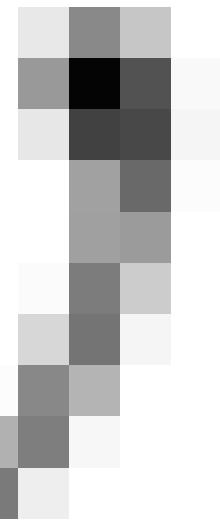
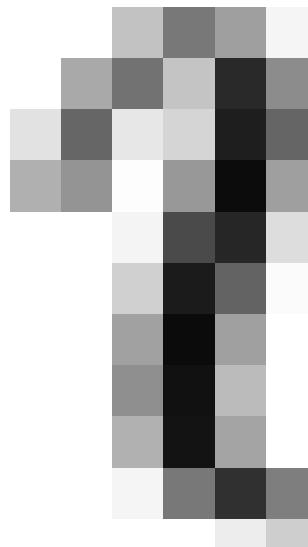
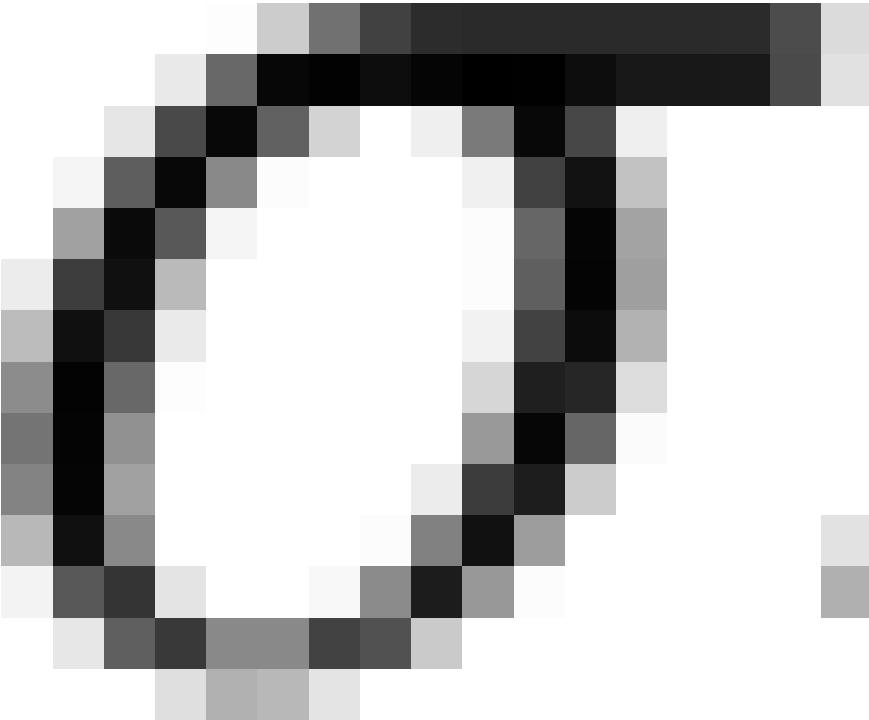


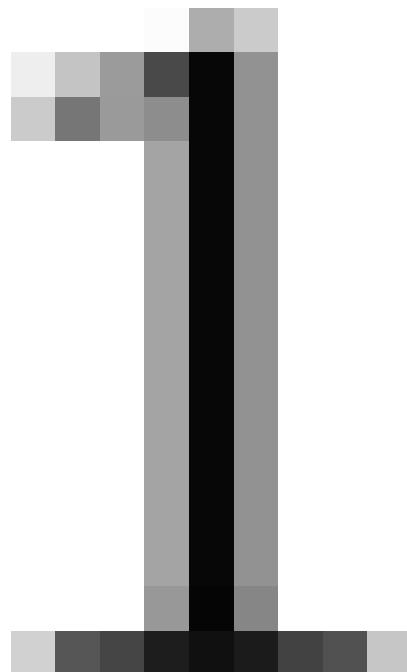
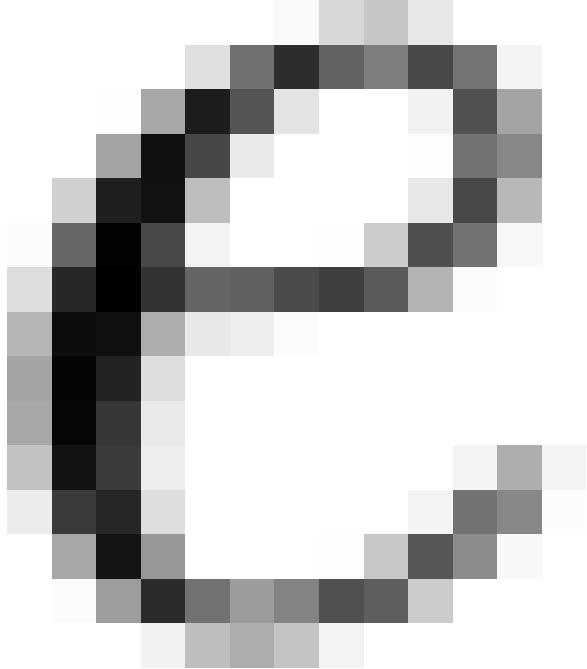


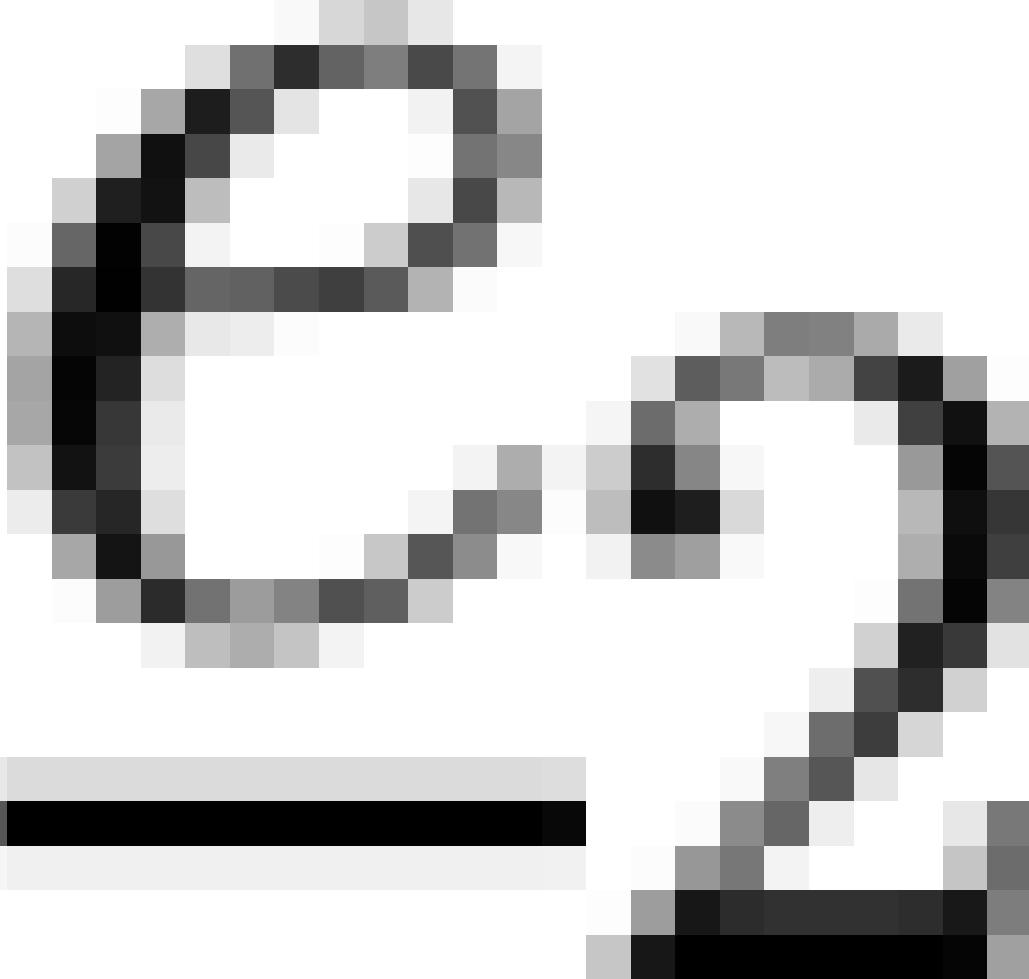


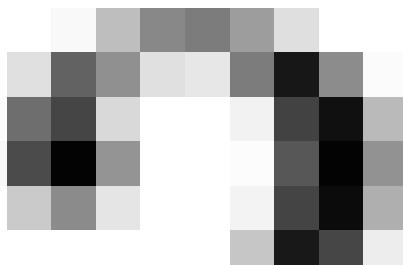
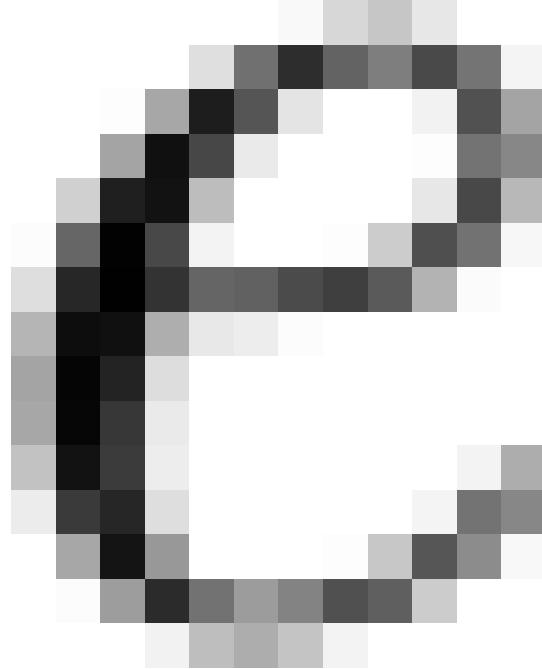


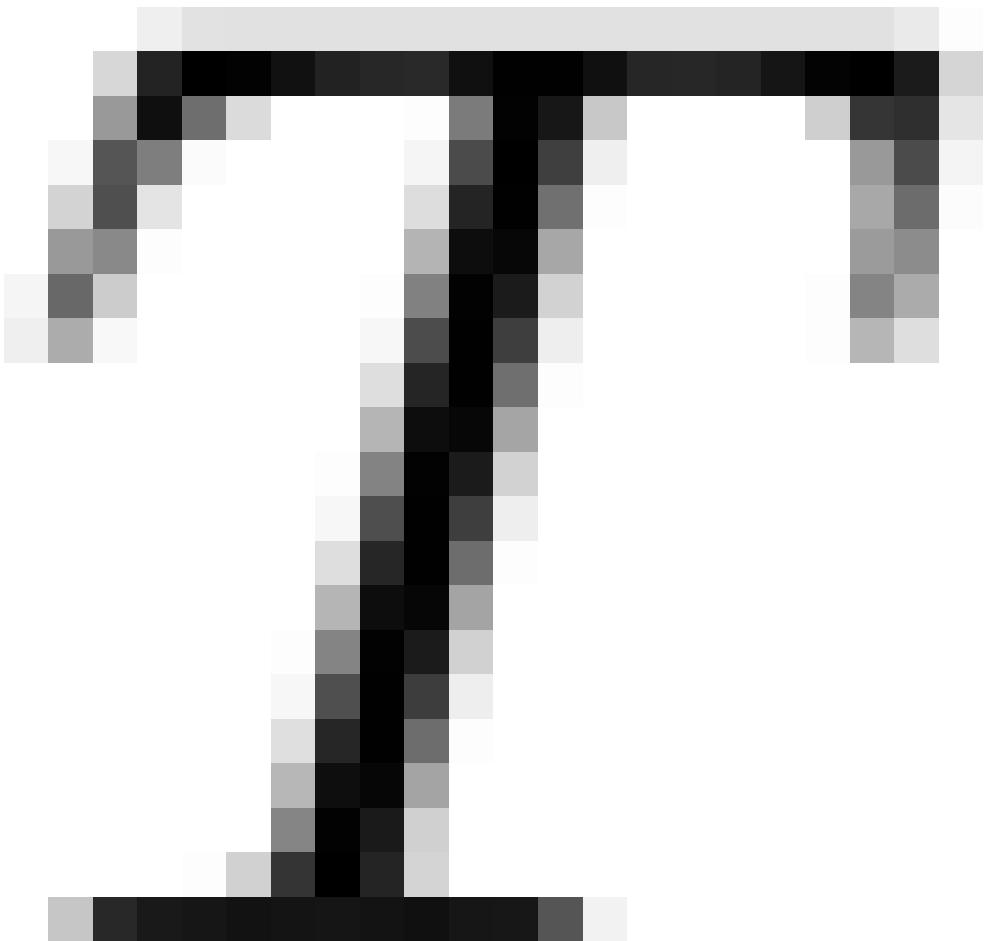




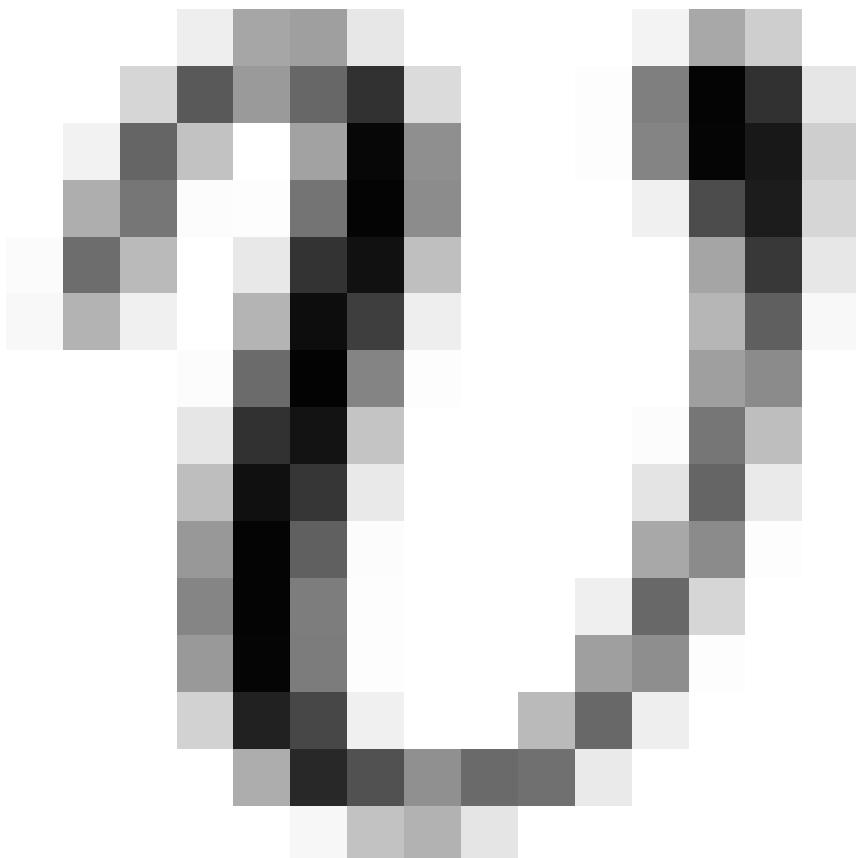




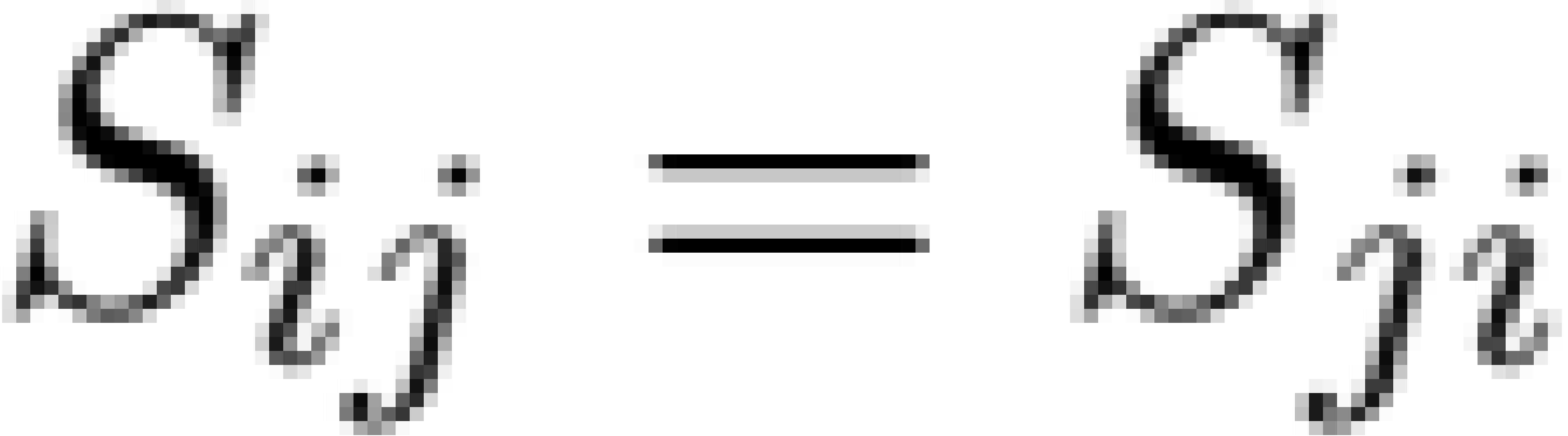


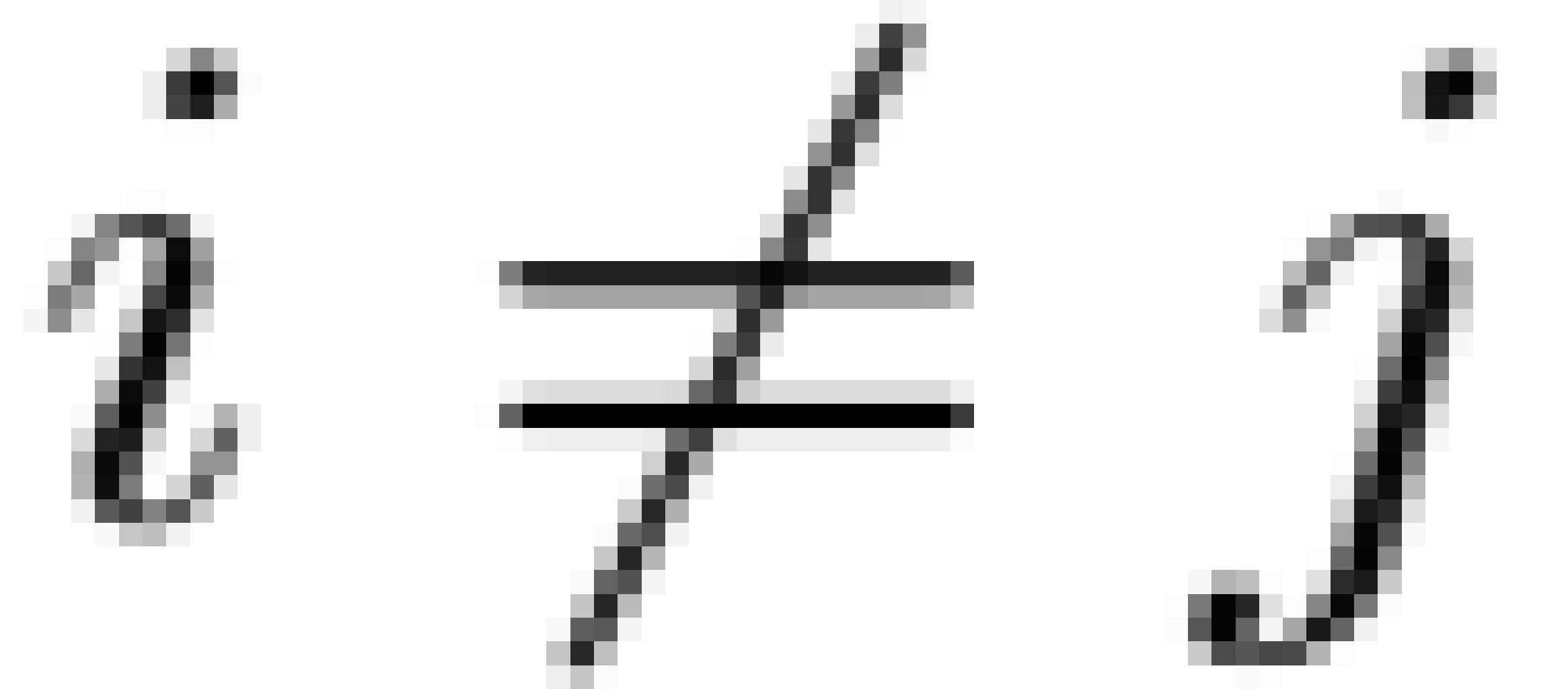


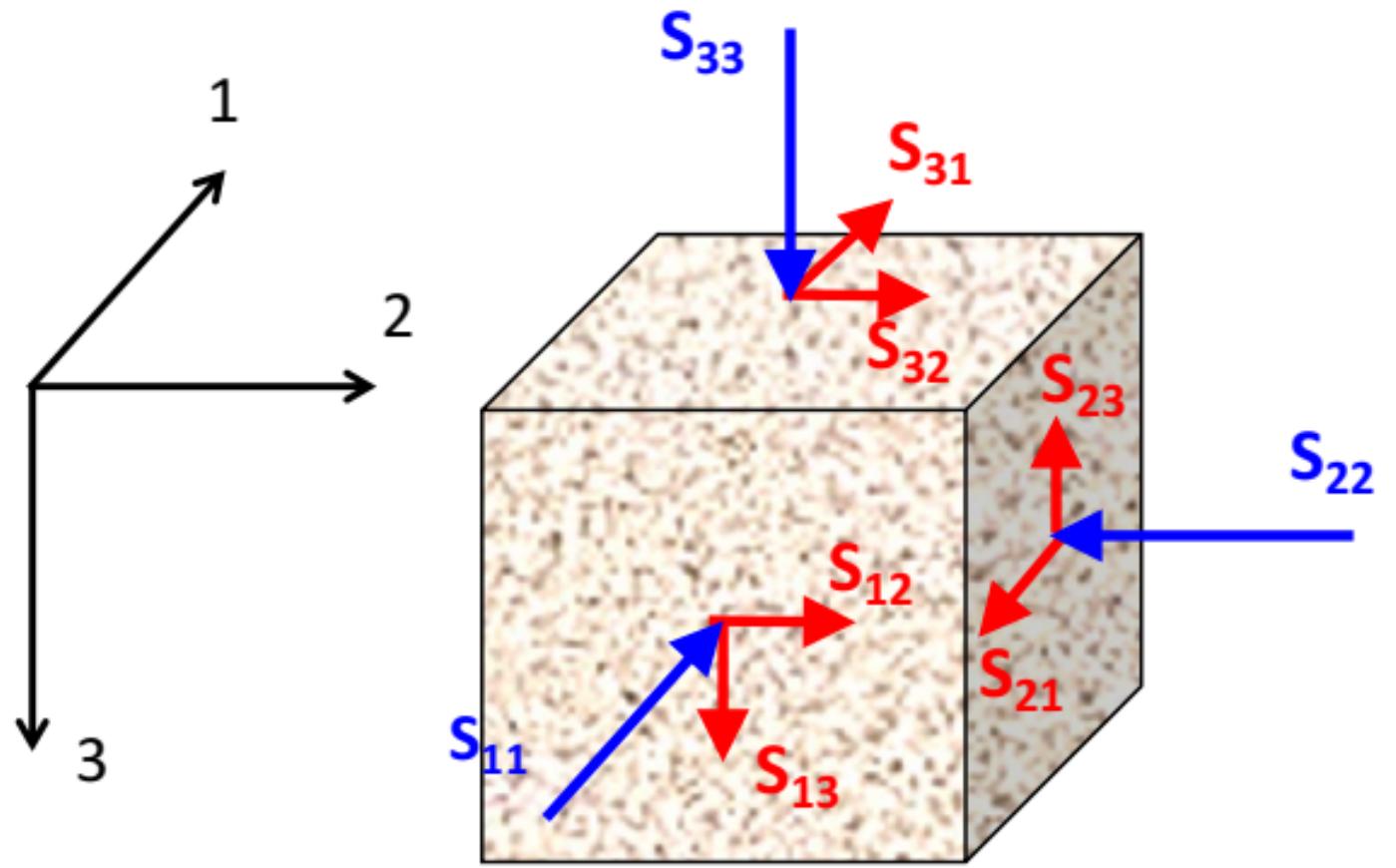






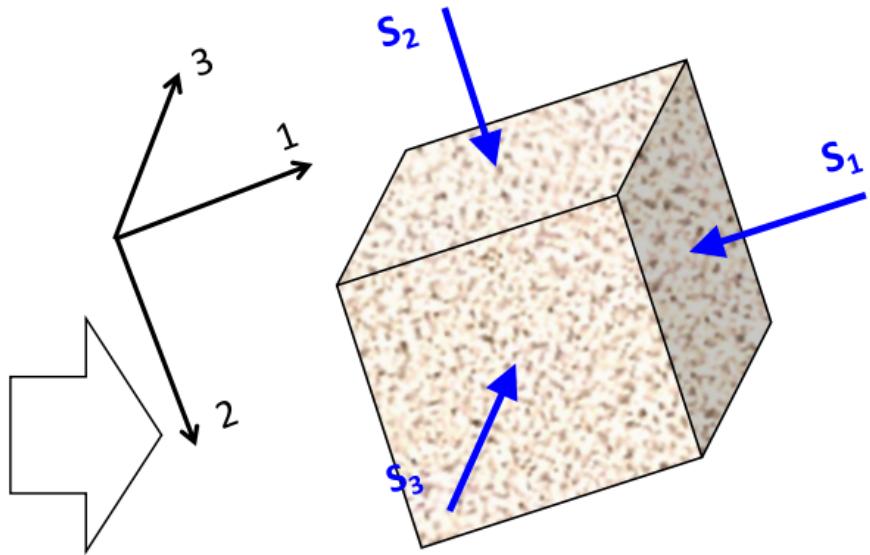
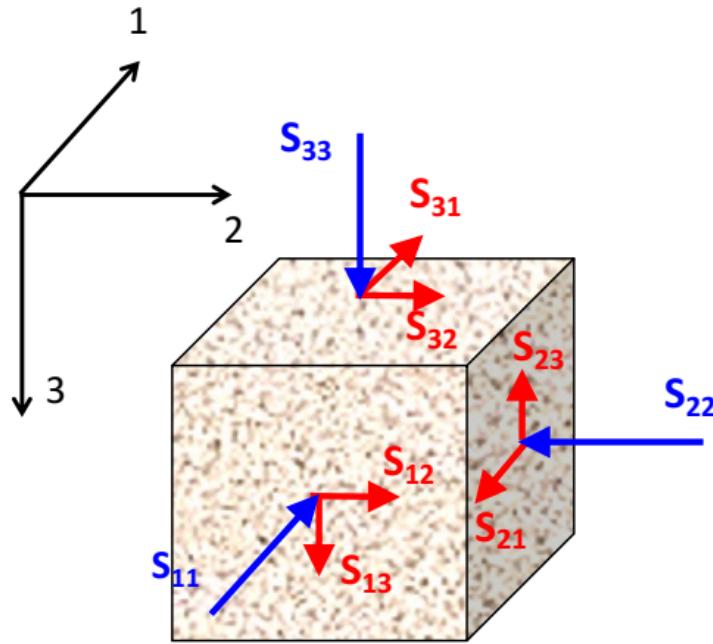






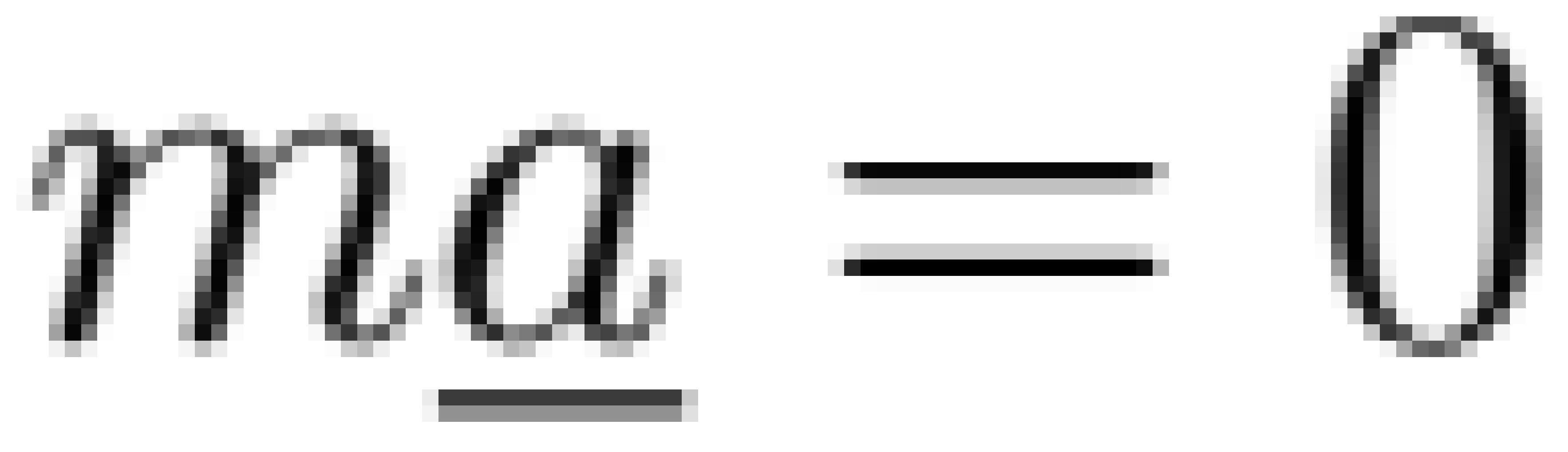
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

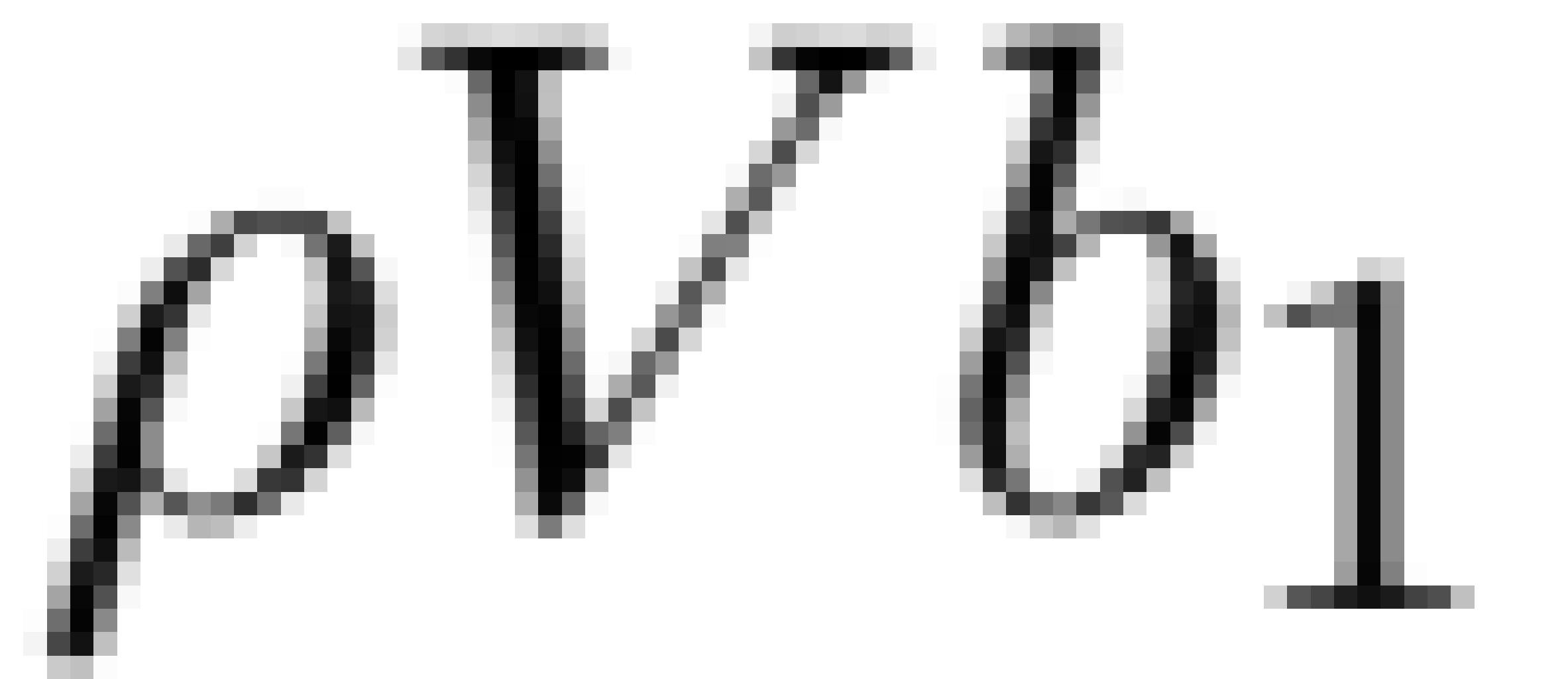


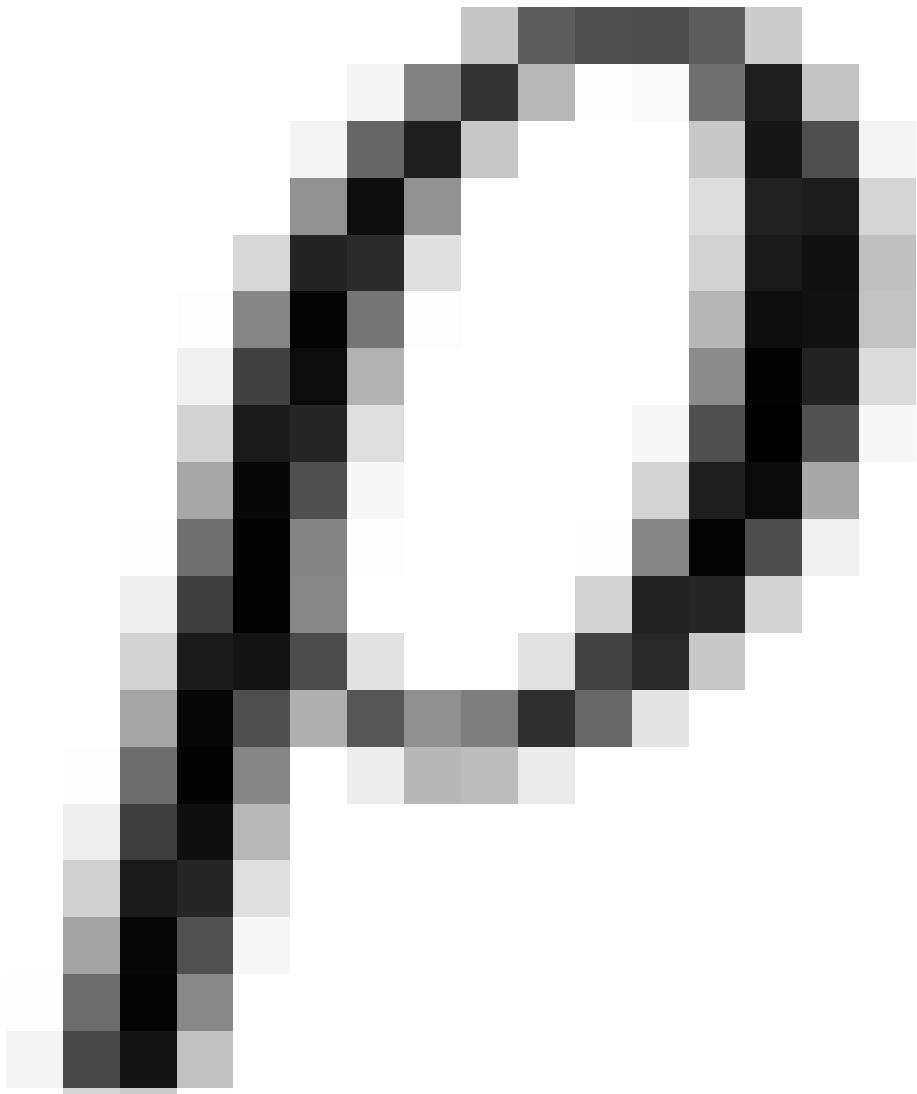


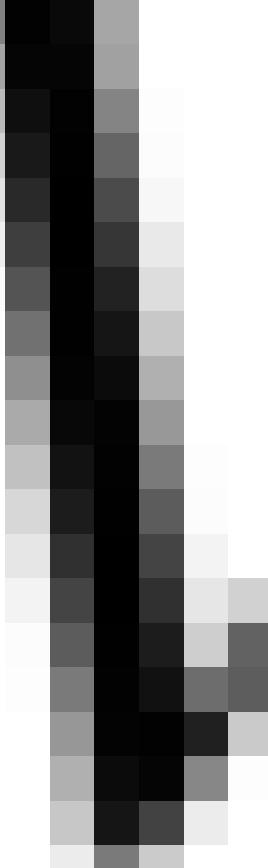
$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

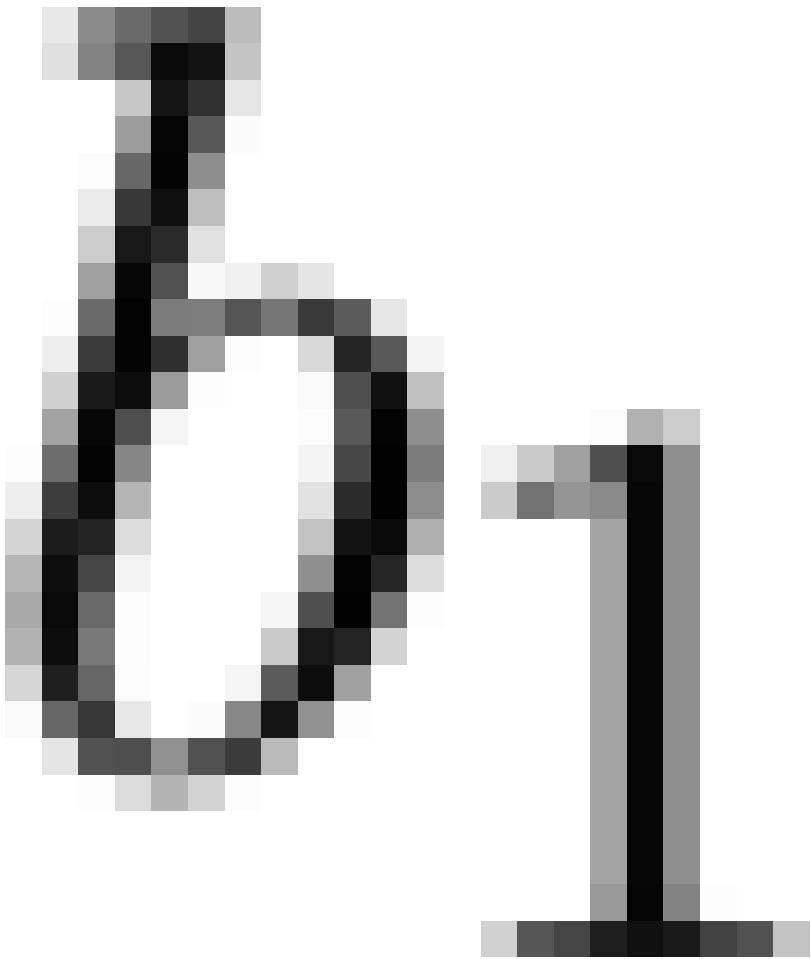
$$\underline{\underline{S}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$







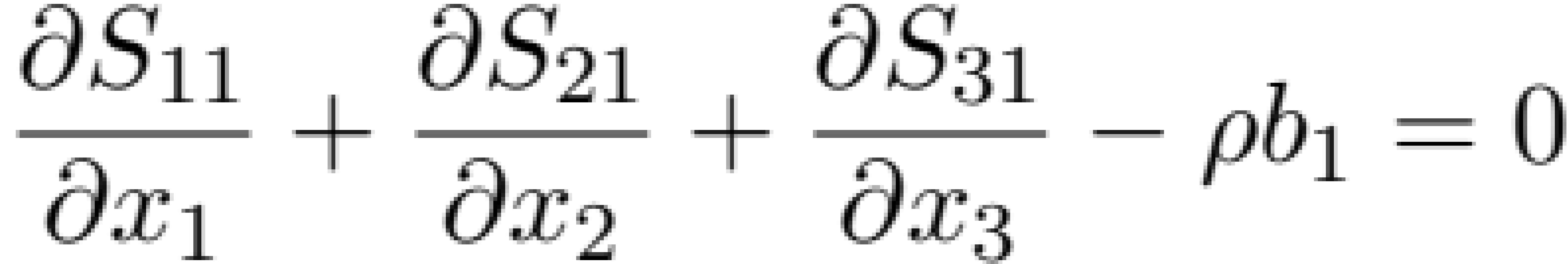


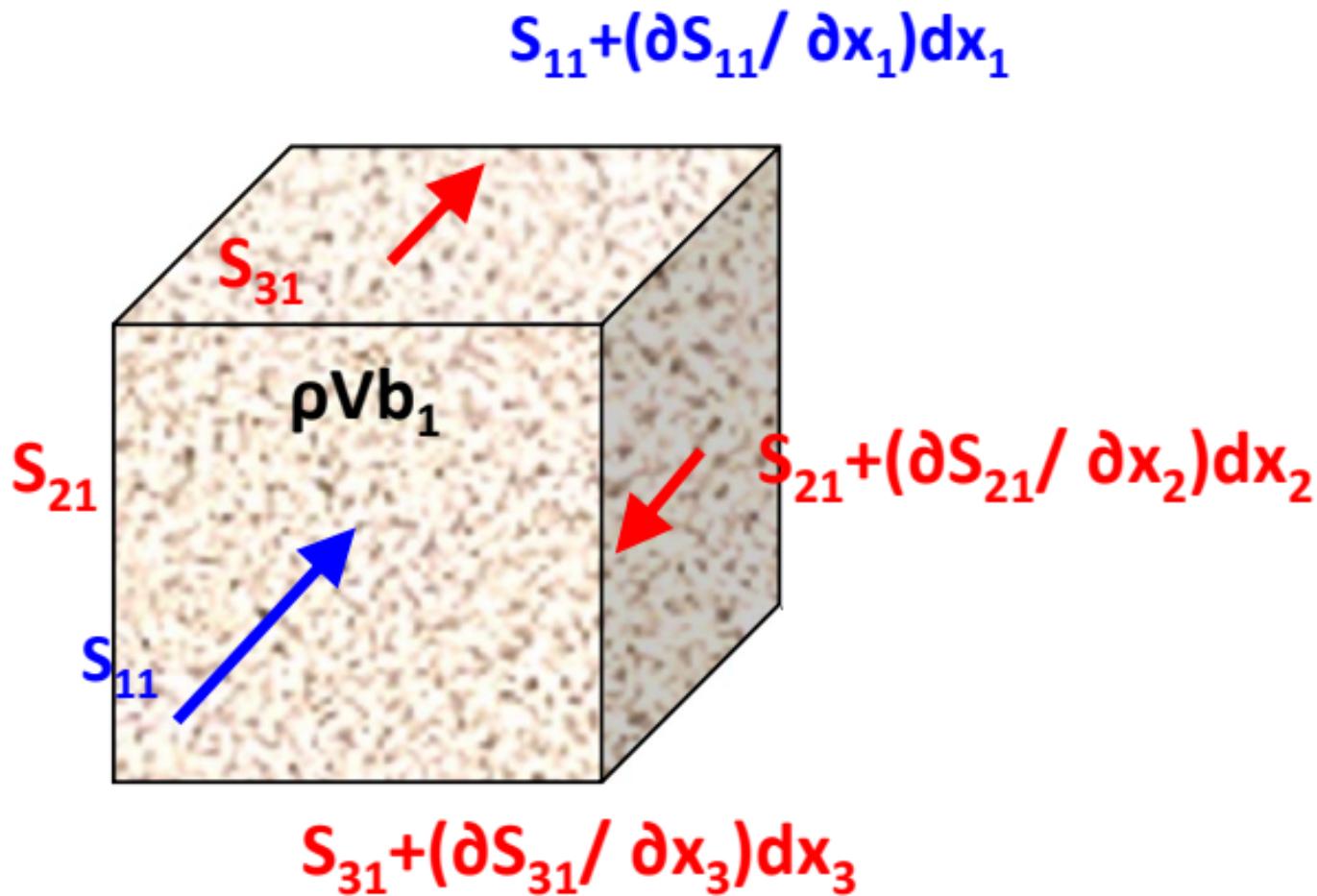
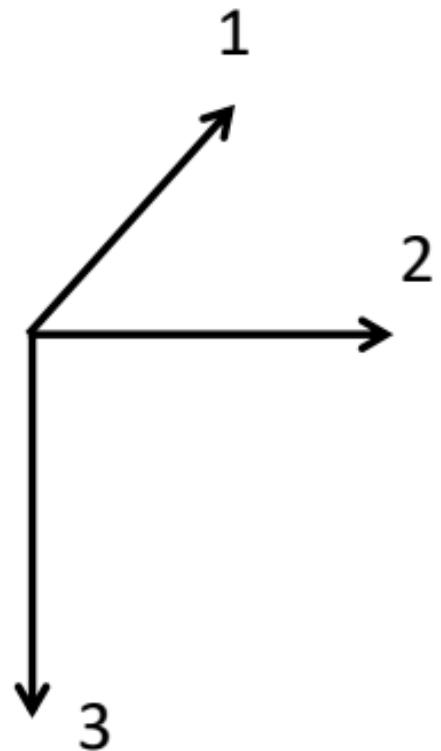


$$\sum F_1 = 0$$

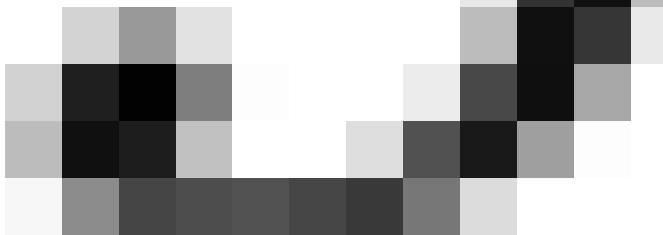
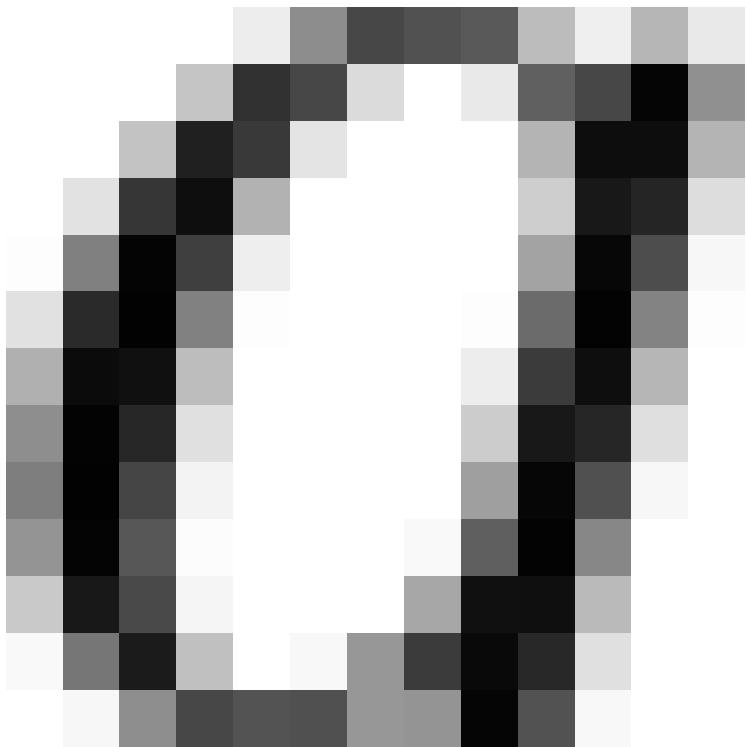
$$\begin{aligned}\sum F_1 &= +S_{11}dx_2dx_3 - \left[ S_{11} + \left( \frac{\partial S_{11}}{\partial x_1} \right) dx_1 \right] dx_2dx_3 \\ &\quad + S_{21}dx_1dx_3 - \left[ S_{21} + \left( \frac{\partial S_{21}}{\partial x_2} \right) dx_2 \right] dx_1dx_3 \\ &\quad + S_{31}dx_1dx_2 - \left[ S_{31} + \left( \frac{\partial S_{31}}{\partial x_3} \right) dx_3 \right] dx_1dx_2 \\ &\quad - \rho(dx_1dx_2dx_3)b_1 = 0\end{aligned}$$

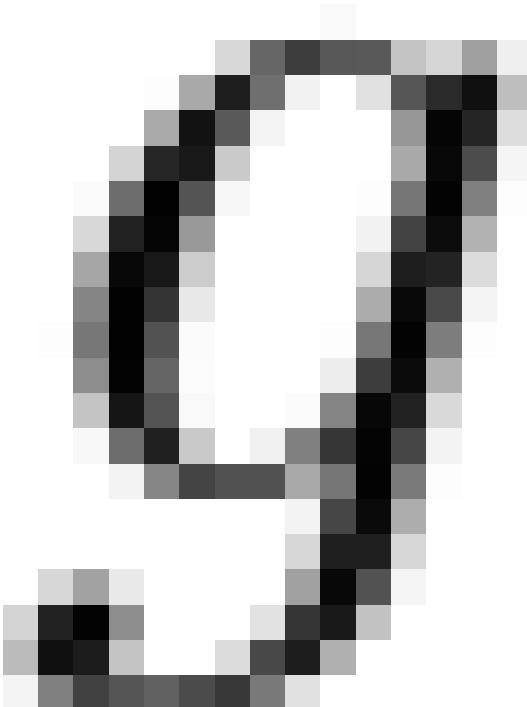
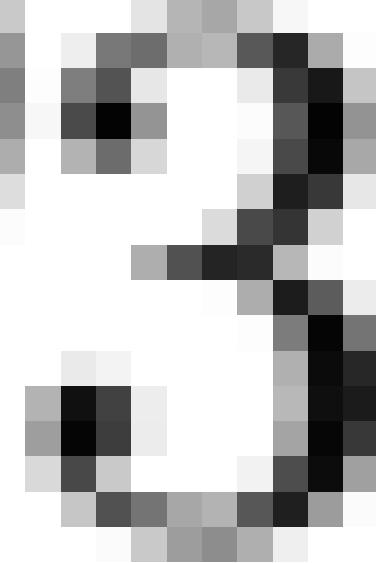
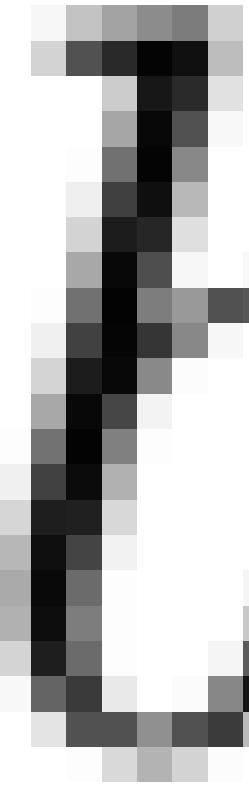






$$\left\{ \begin{array}{l} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} - \rho b_1 = 0 \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} - \rho b_2 = 0 \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} - \rho b_3 = 0 \end{array} \right.$$

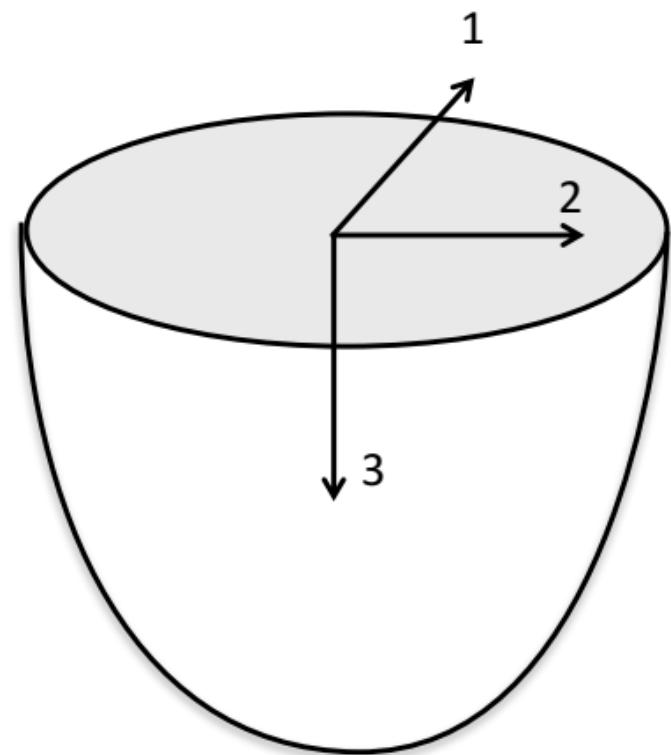








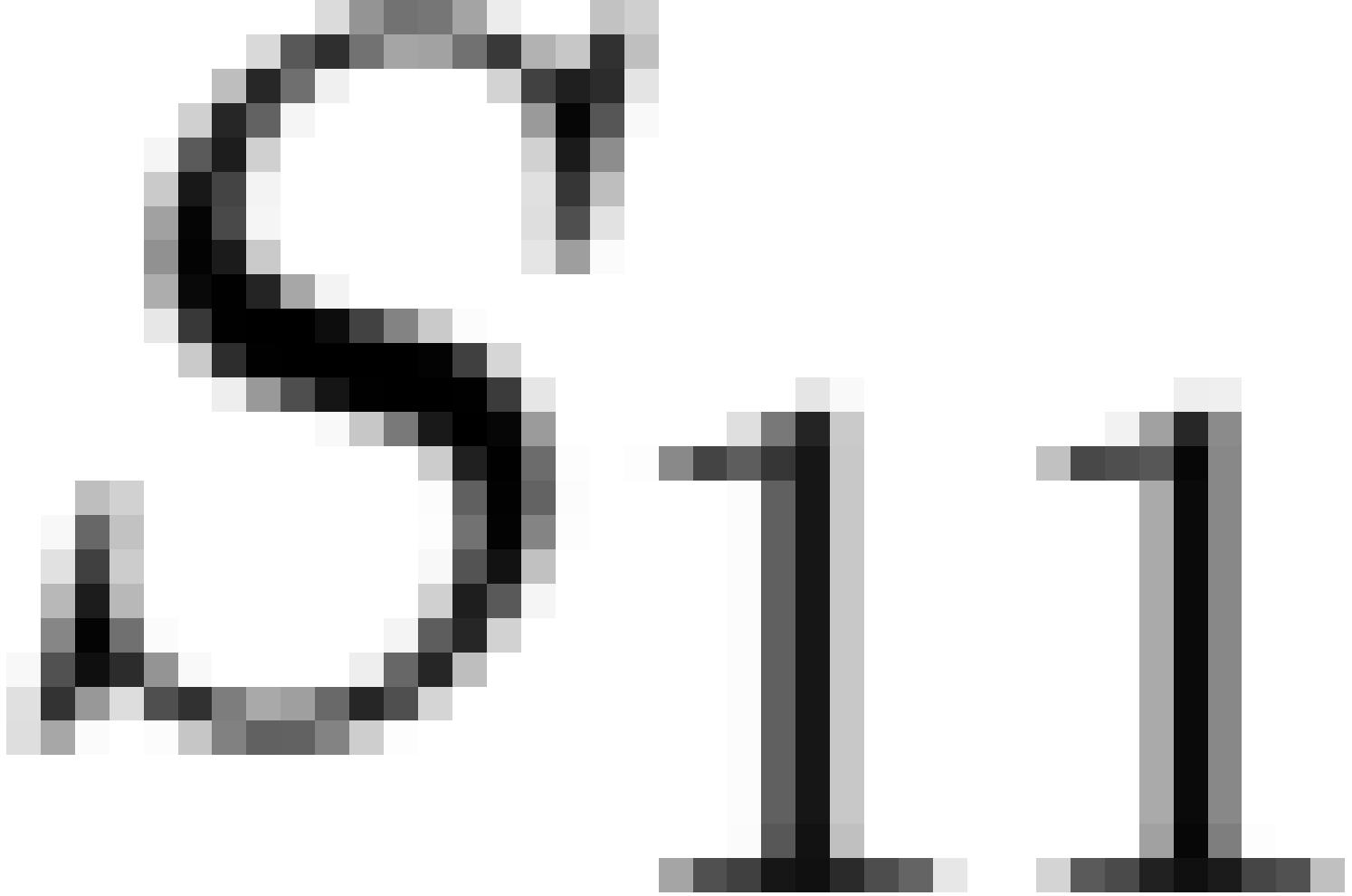
$$s_{33}(c_3) = \rho(c_3) \circ d_{c_3}$$



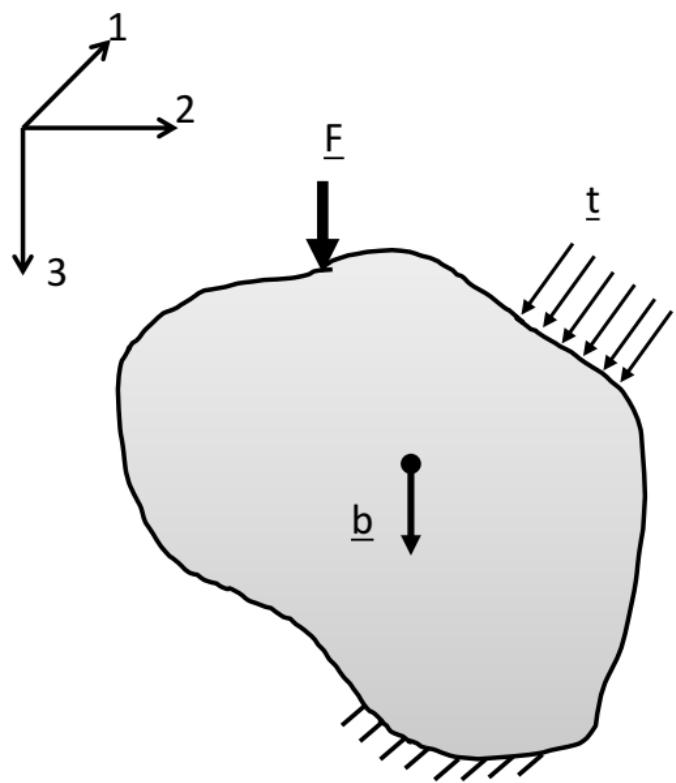
$$\left\{ \begin{array}{l} \cancel{\frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2}} \\ \cancel{\frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2}} \\ \cancel{\frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2}} \end{array} \right.$$

$$\frac{\partial S_{33}}{\partial x_3} - \rho(x_3)g = 0$$

$$S_{33} = \int_0^{x_3} \rho(x_3)g \, dx_3$$







Displacement condition

$$\begin{cases} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2} \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2} \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2} \end{cases}$$

And respect the boundary conditions:

- Displacement
- Boundary stresses
- Boundary Forces
- Body Forces

How do we relate stresses to displacements?

- Displacements → Strains (**Kinematic equations**)
- Strains → Stresses (**Constitutive equations**)



$\epsilon_{11}$



$\alpha_1$

$\alpha_1$

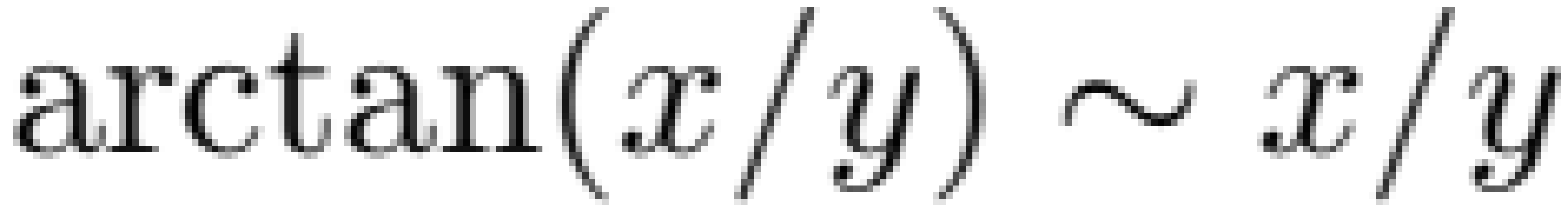
$\epsilon_{22}$

$=$

$\alpha_2$

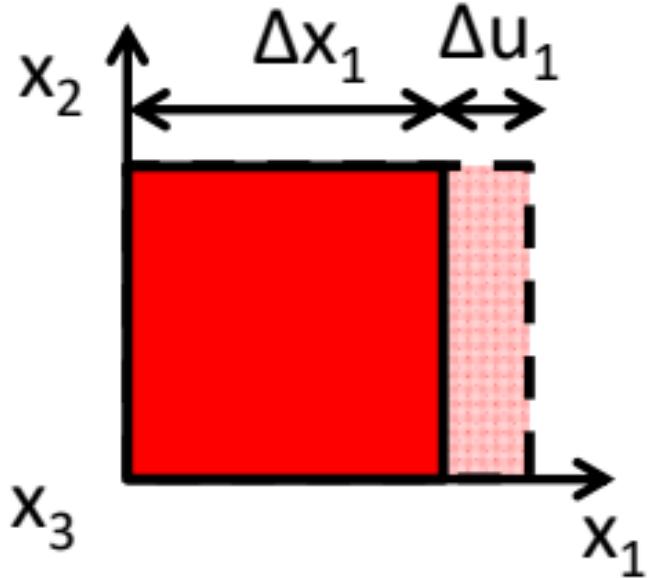
$\alpha_2$

actuator1(0) + actuator2(0))

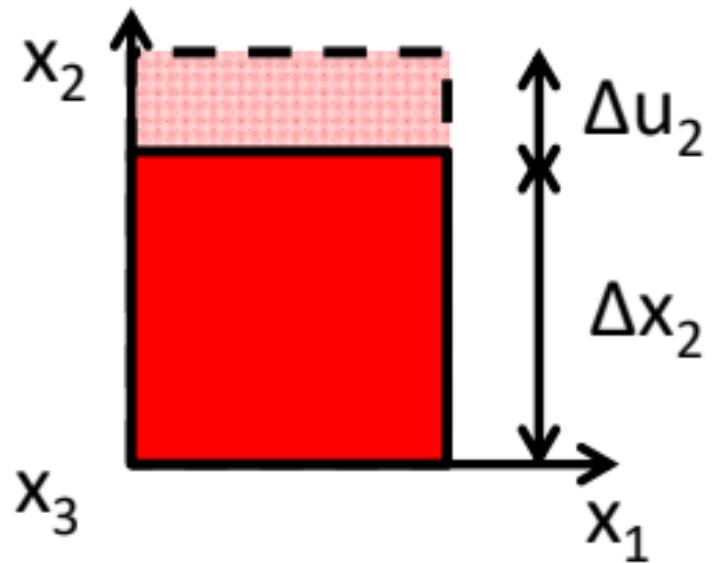


$\epsilon_{12} =$  $-\frac{1}{2}$ 

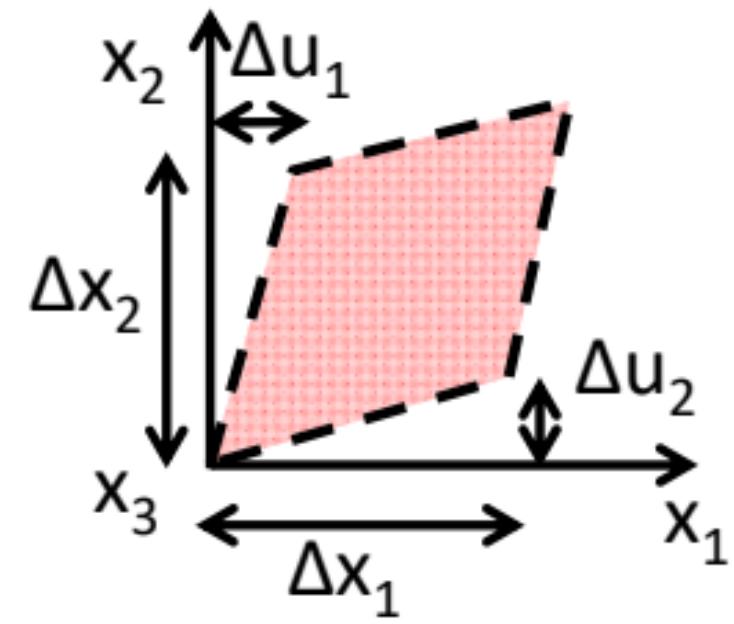
$$\frac{1}{2} \left( u_1 - u_2 + c_1 - c_2 \right)$$



$$\varepsilon_{11} \approx \frac{\Delta u_1}{\Delta x_1}$$



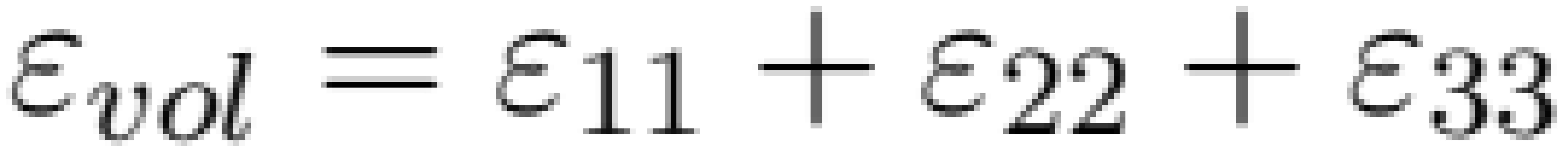
$$\varepsilon_{22} \approx \frac{\Delta u_2}{\Delta x_2}$$



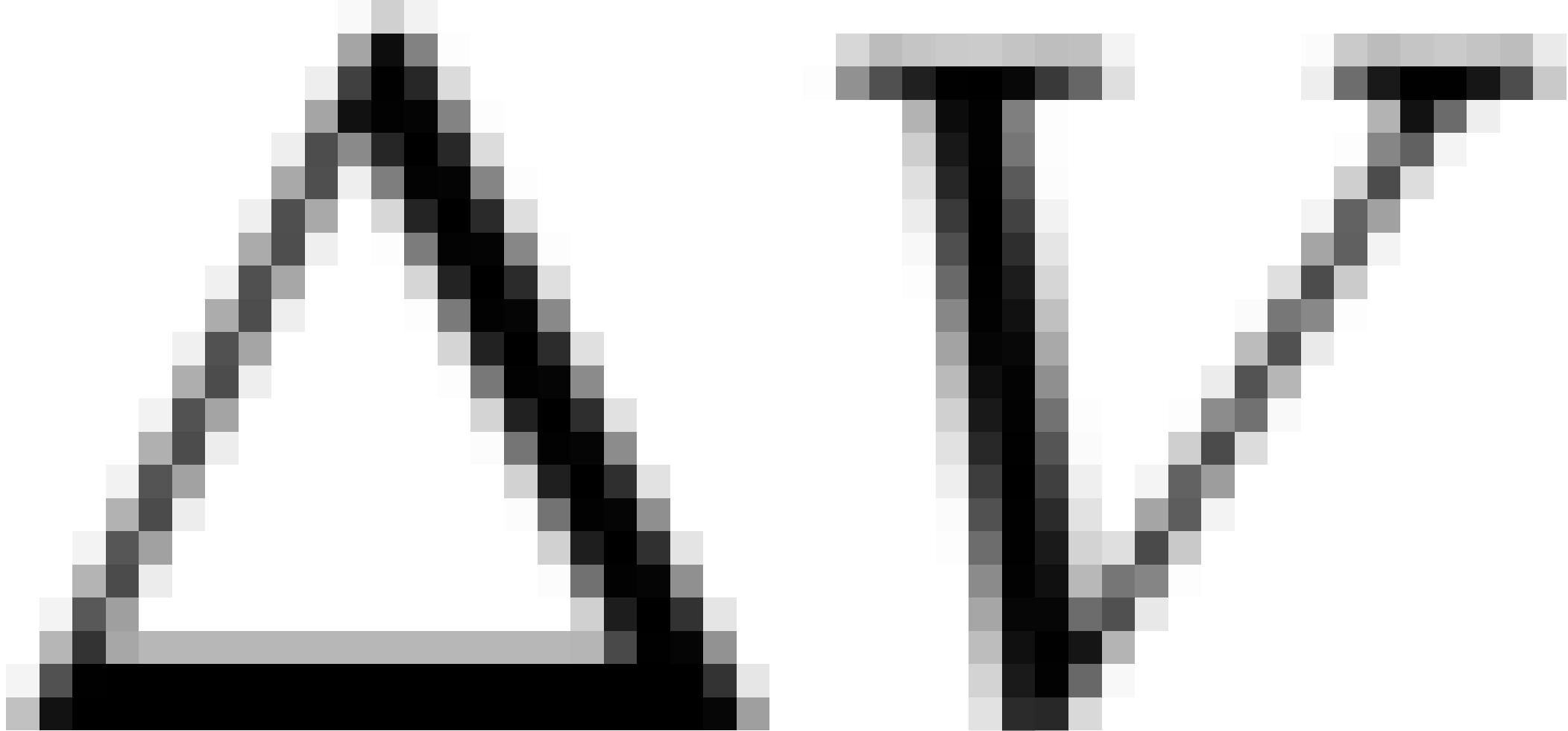
$$\varepsilon_{12} \approx \frac{1}{2} \left( \frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1} \right)$$

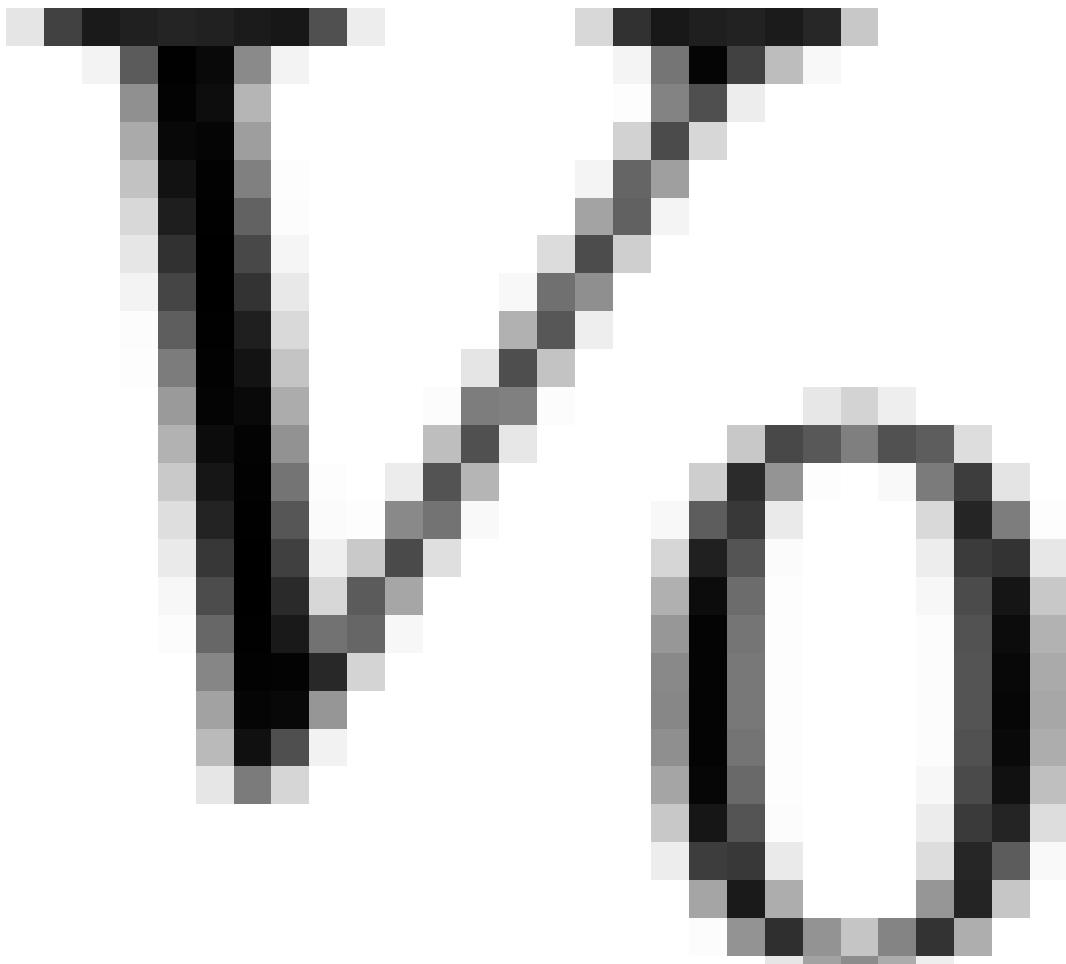




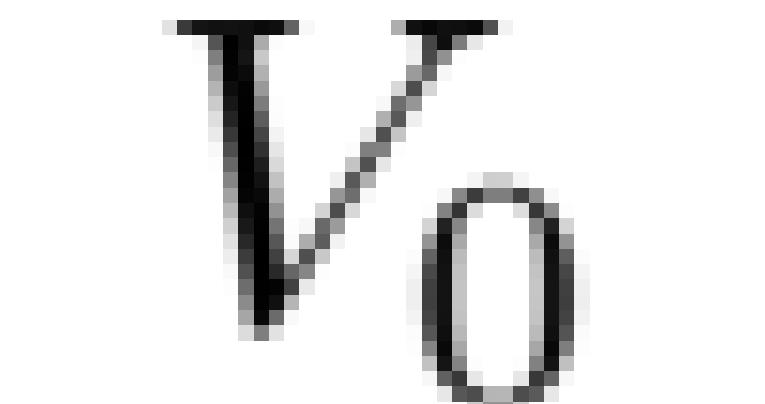
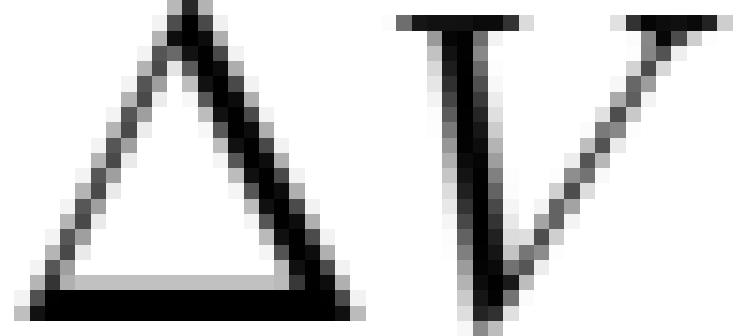
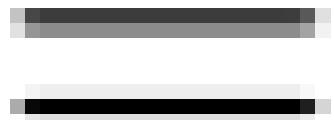




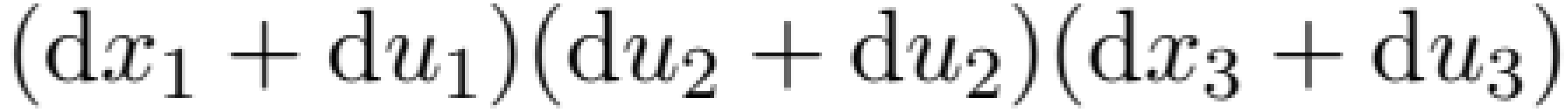




cool



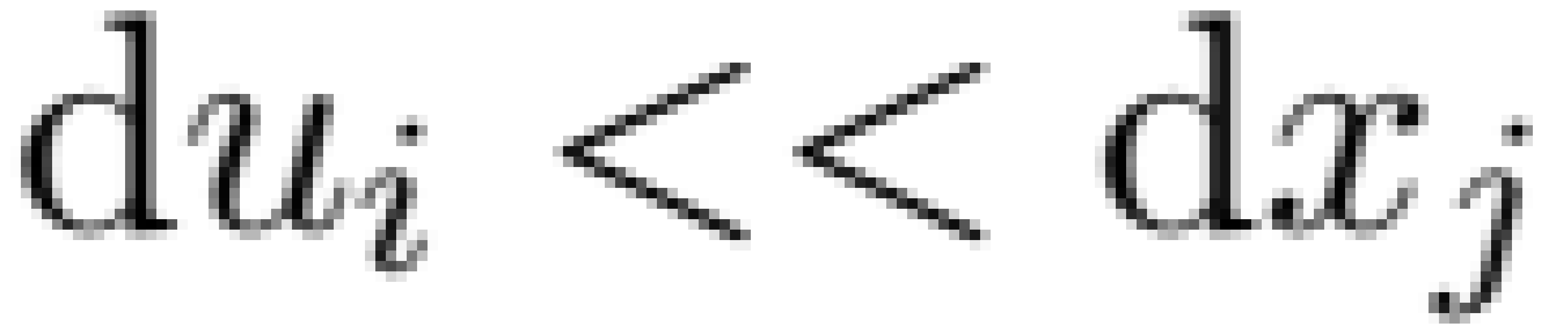




$$\epsilon_{vol} = \frac{[(dx_1 + du_1)(dx_2 + du_3) - (dx_1 dx_2 dx_3)]}{(dx_1 dx_2 dx_3)}$$

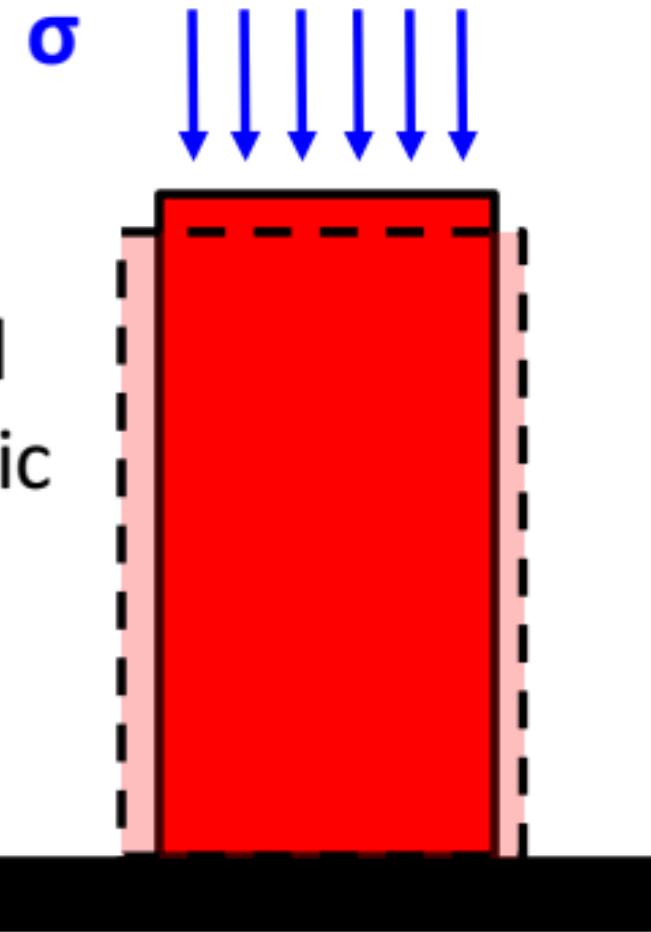




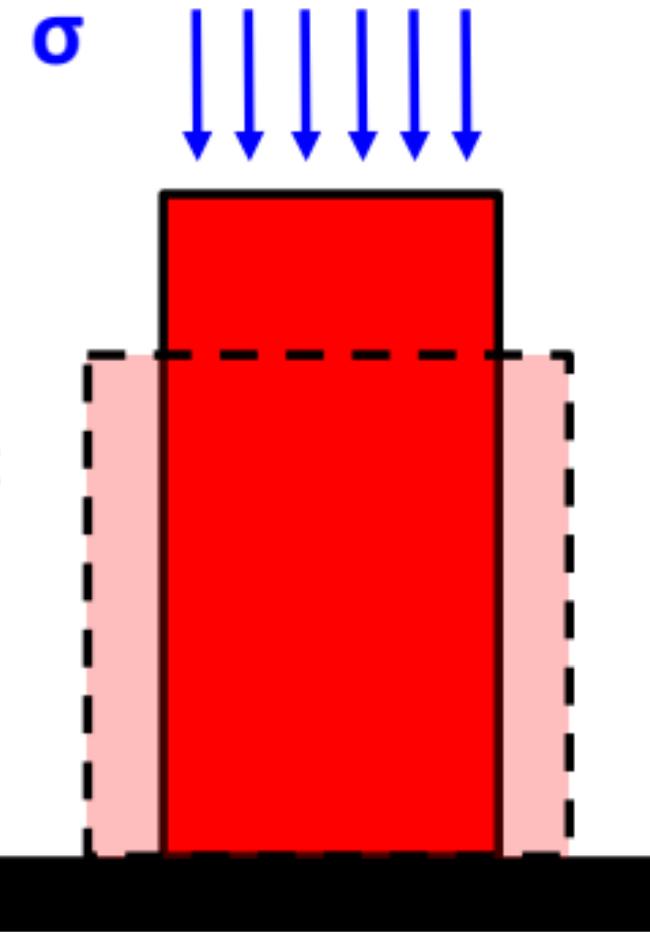


$$\text{vol}^2 \frac{(dx_1 dx_2 du_3 + dx_1 dx_3 du_2 + dx_2 dx_3 du_1)}{(dx_1 dx_2 dx_3)} = \frac{du_1}{\epsilon_{11}} + \frac{du_2}{\epsilon_{22}} + \frac{du_3}{\epsilon_{33}}$$

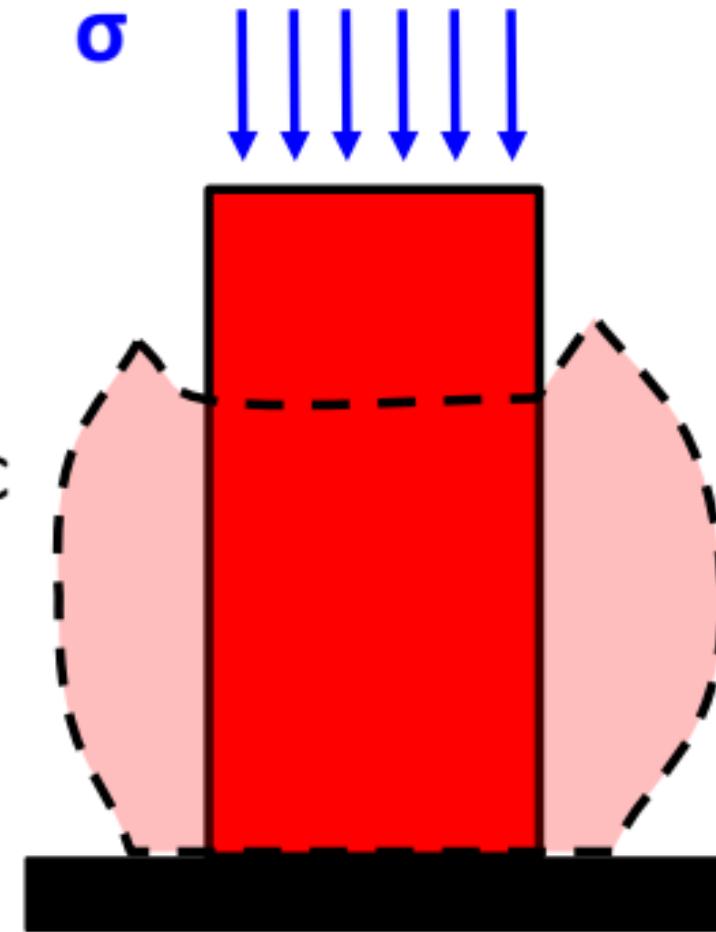
Hard  
elastic  
solid

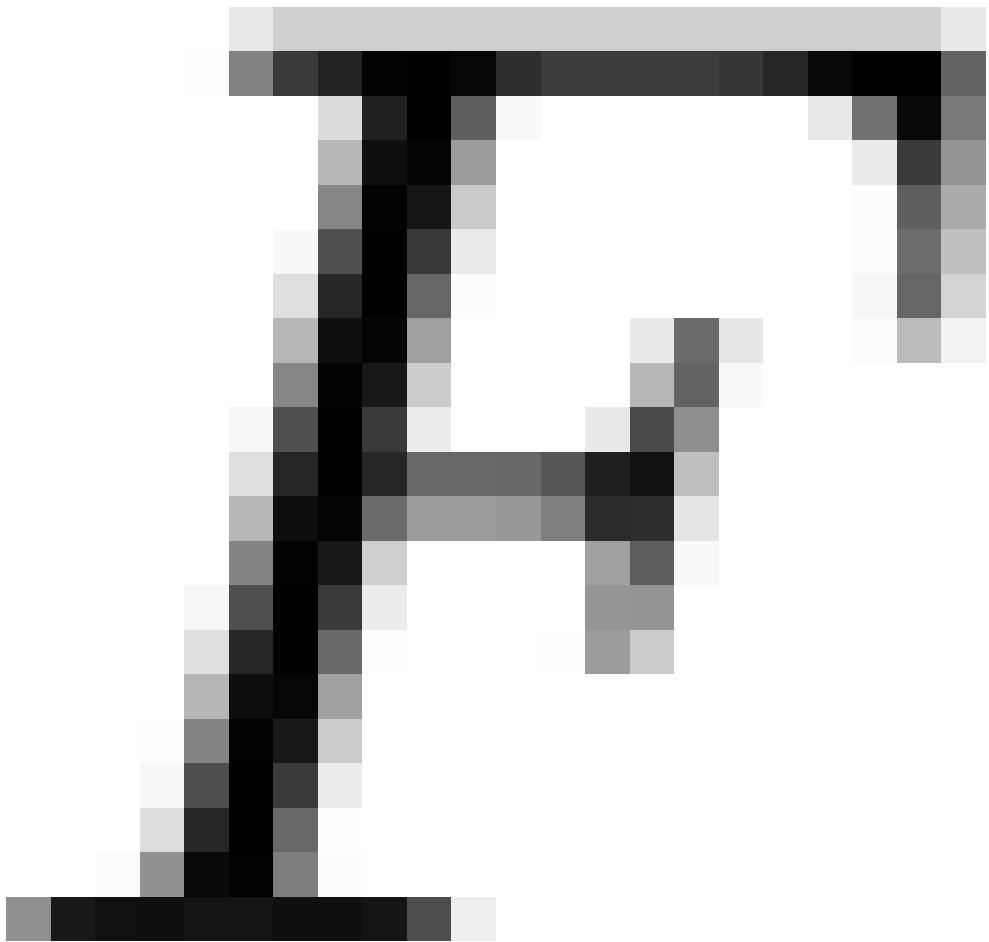


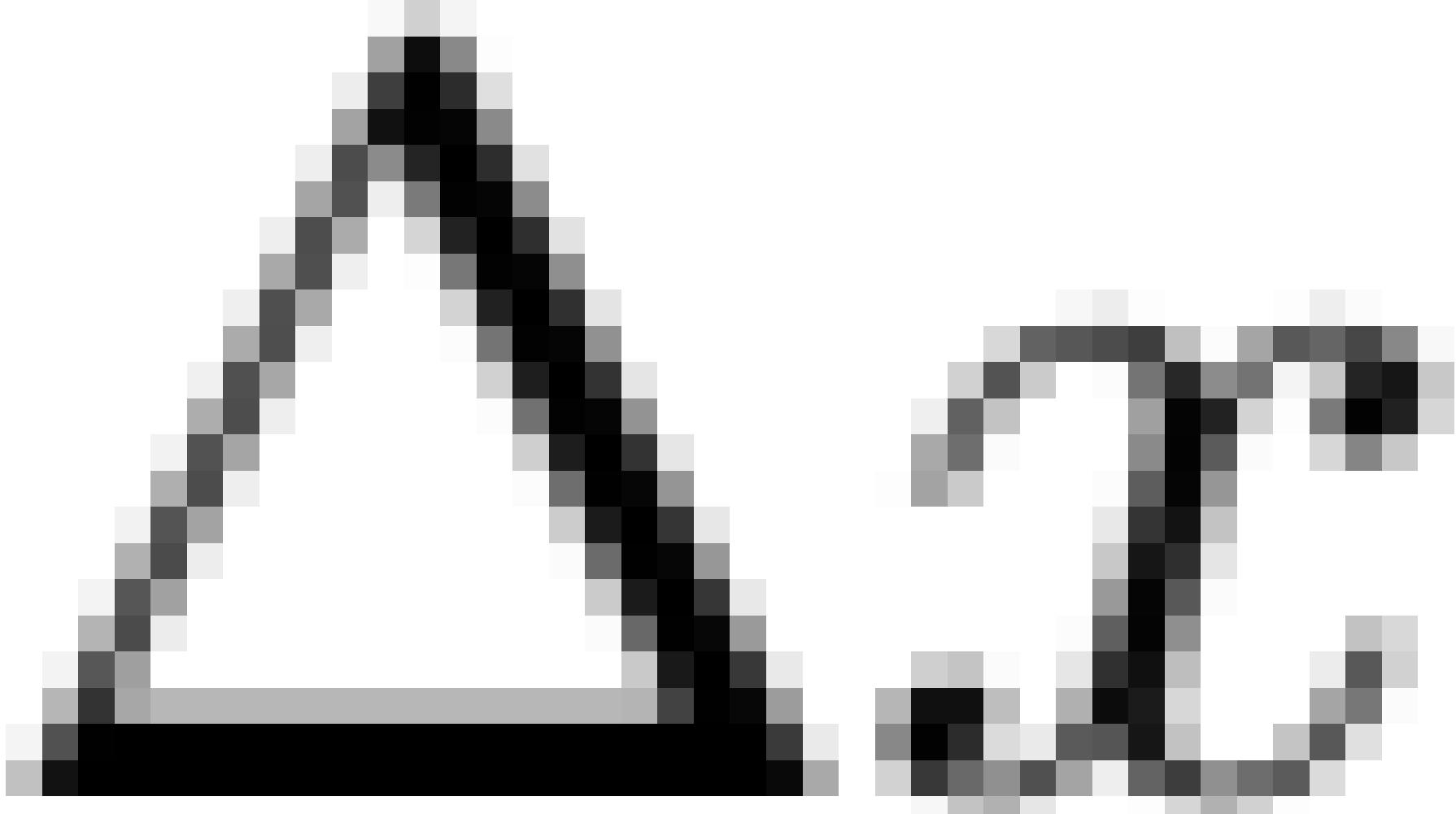
Soft  
elastic  
solid

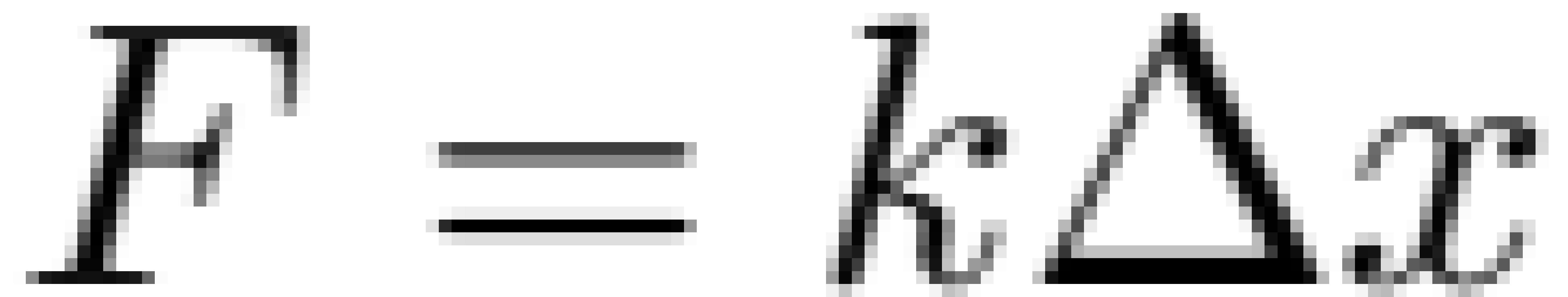


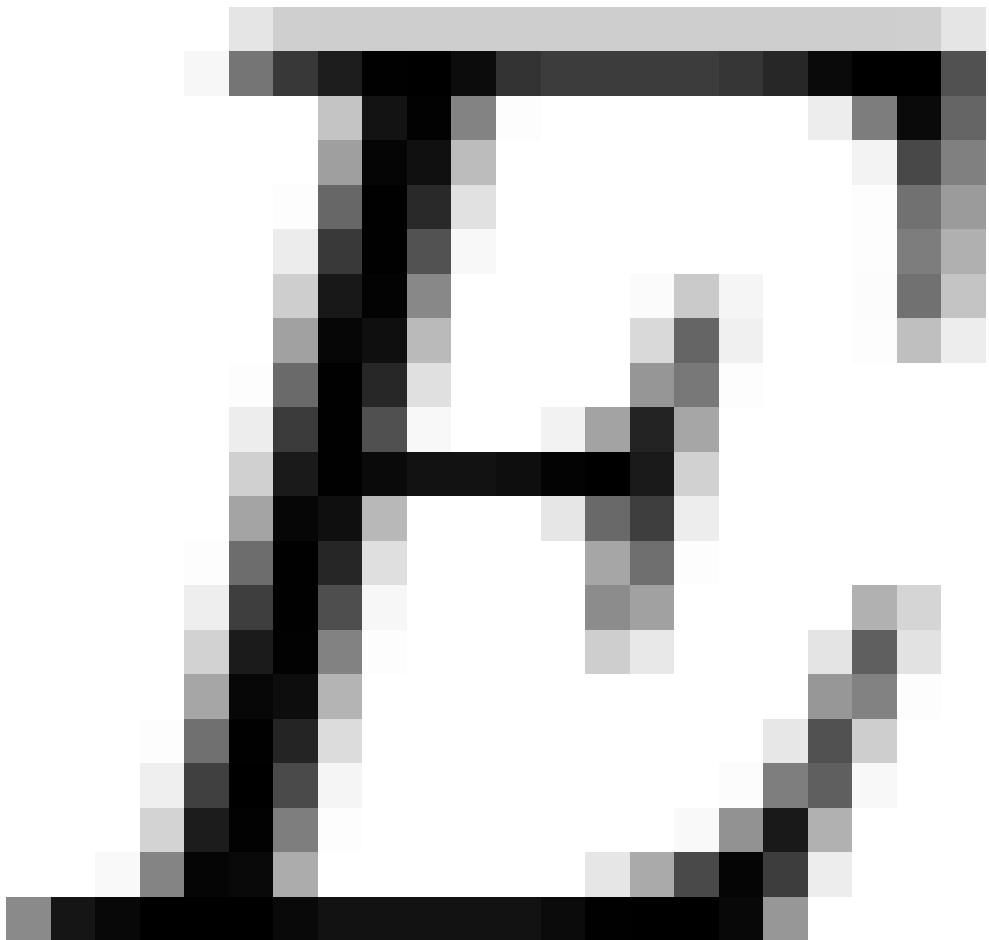
Soft  
Visco-  
plastic  
solid











A

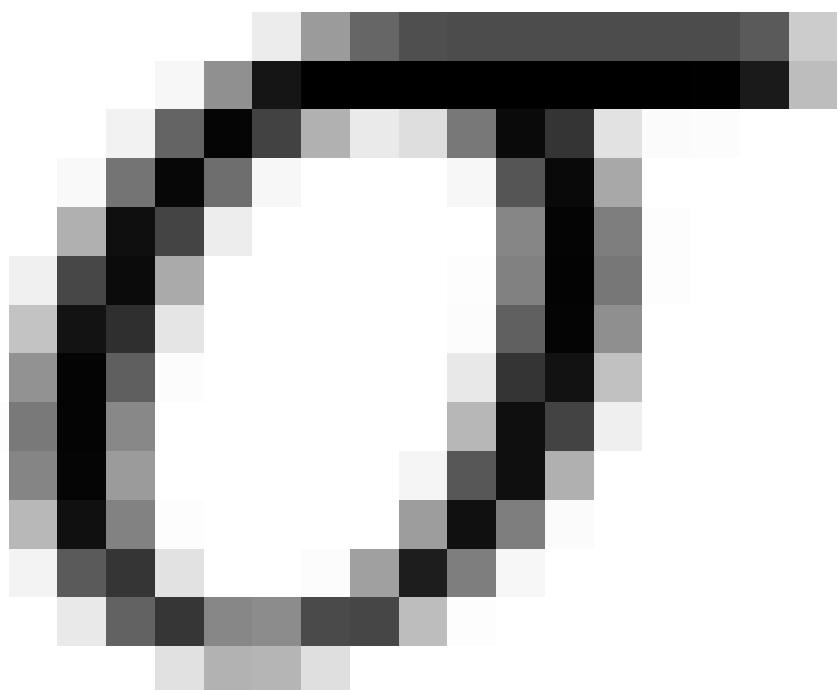
A

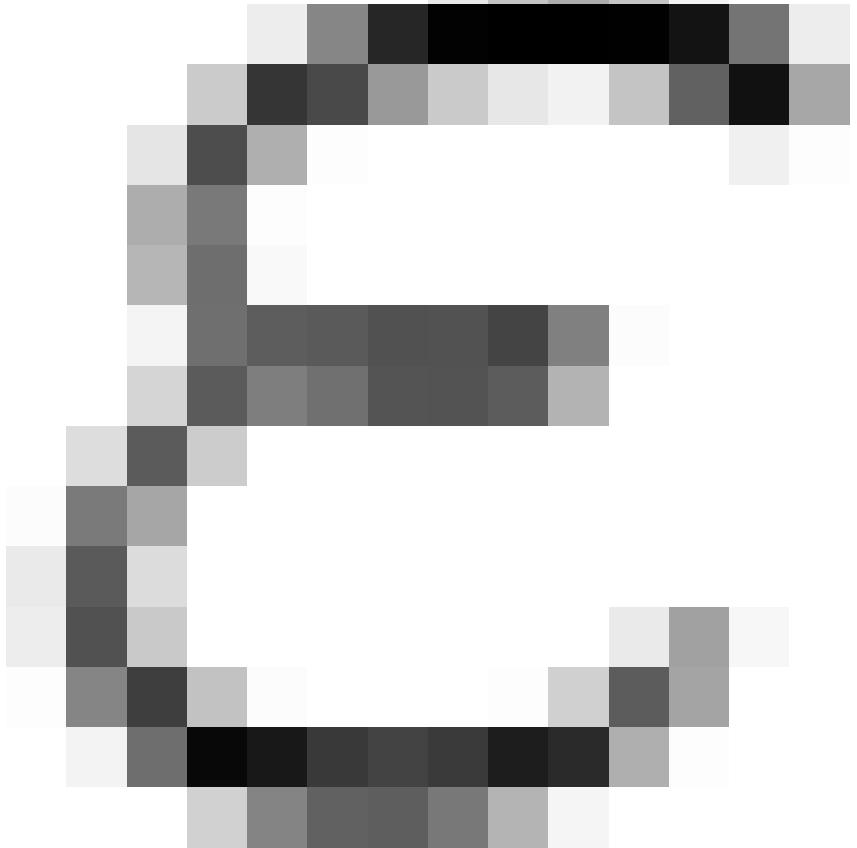
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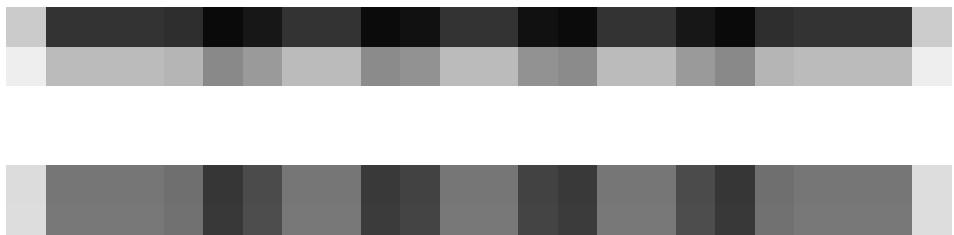
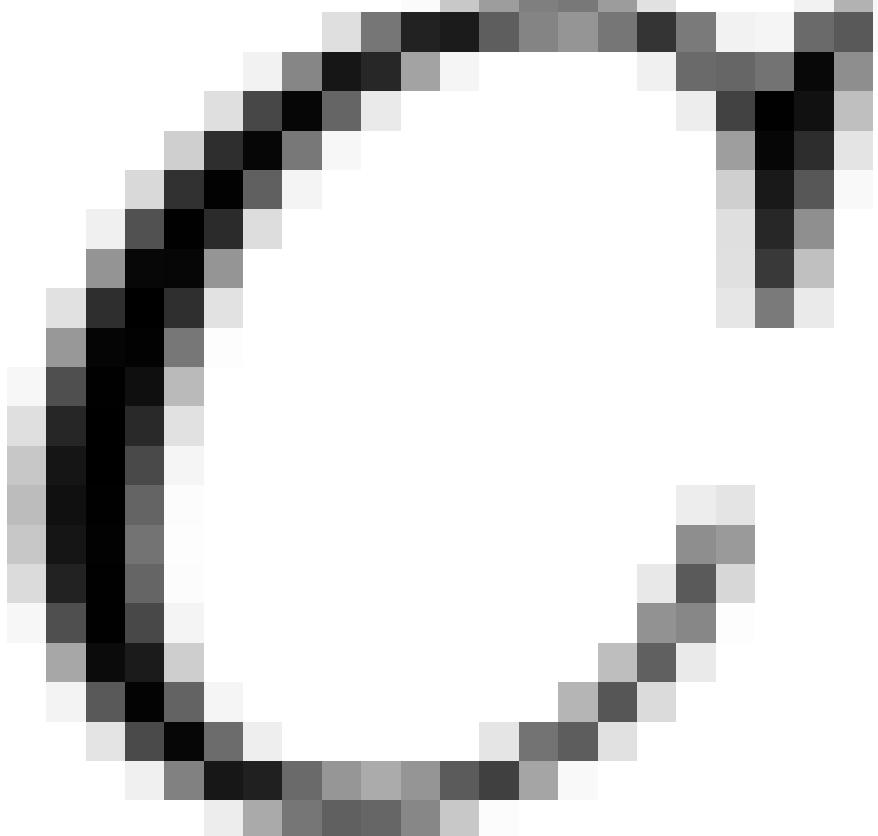
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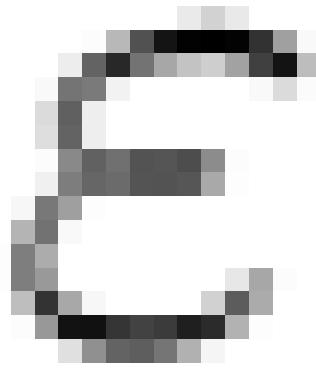
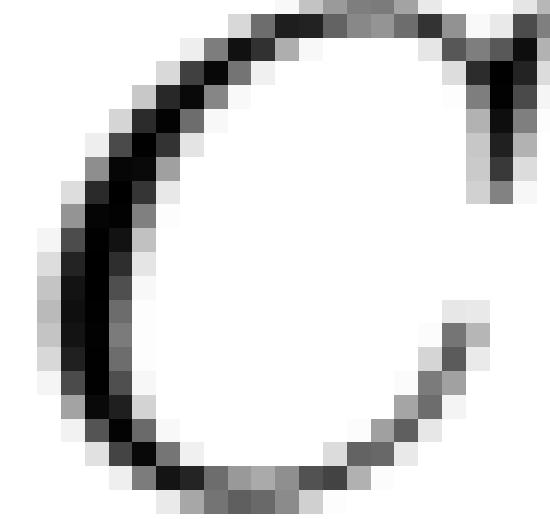
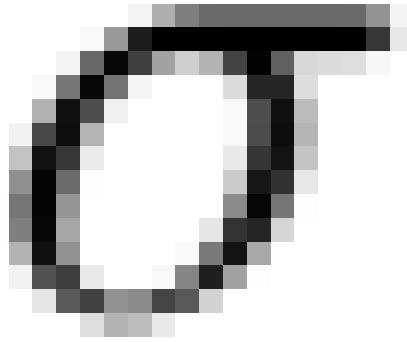
C

T



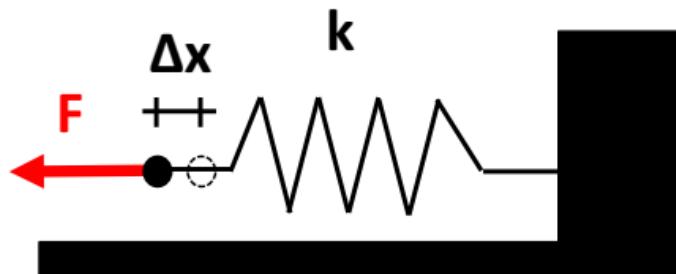






Hooke's law

$$F = k\Delta x$$

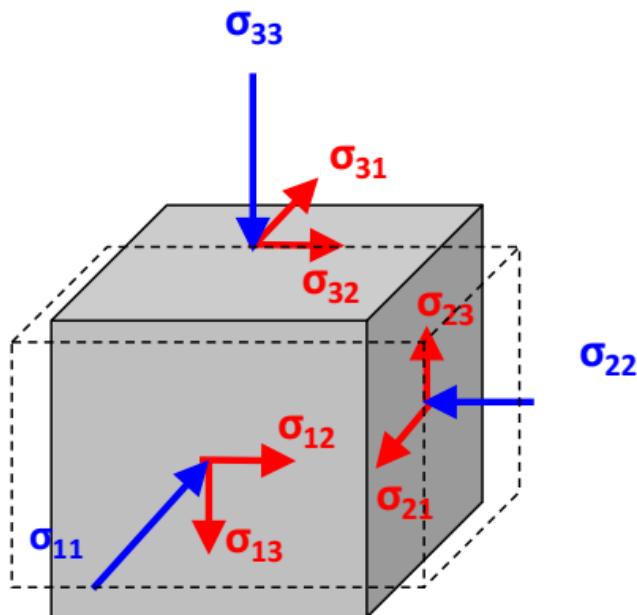


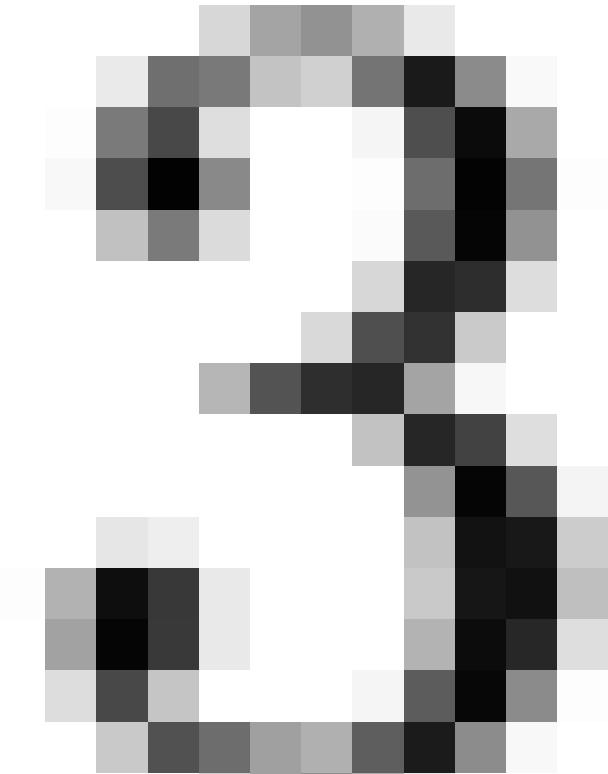
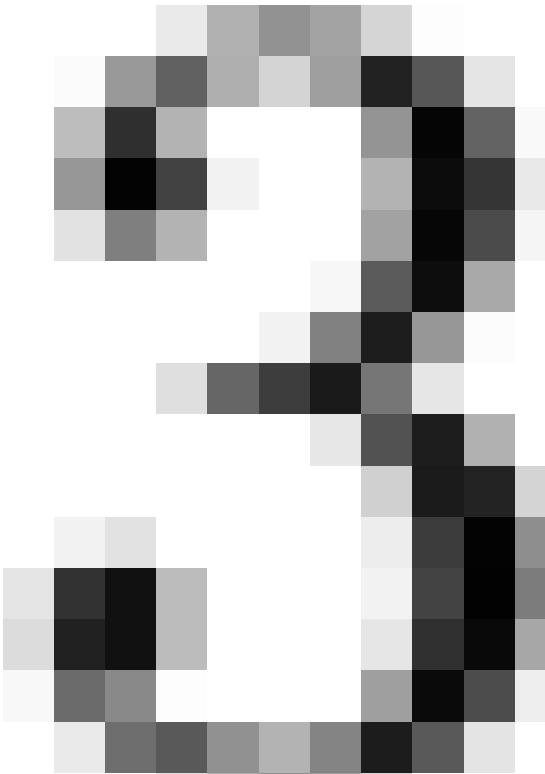
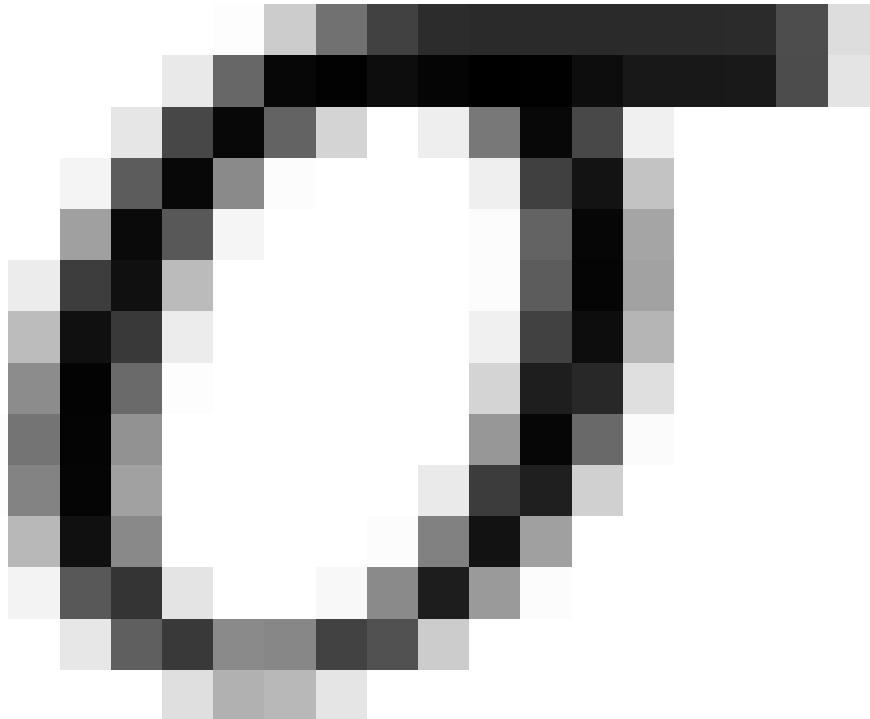
1-D (stress-strain) Hooke's law

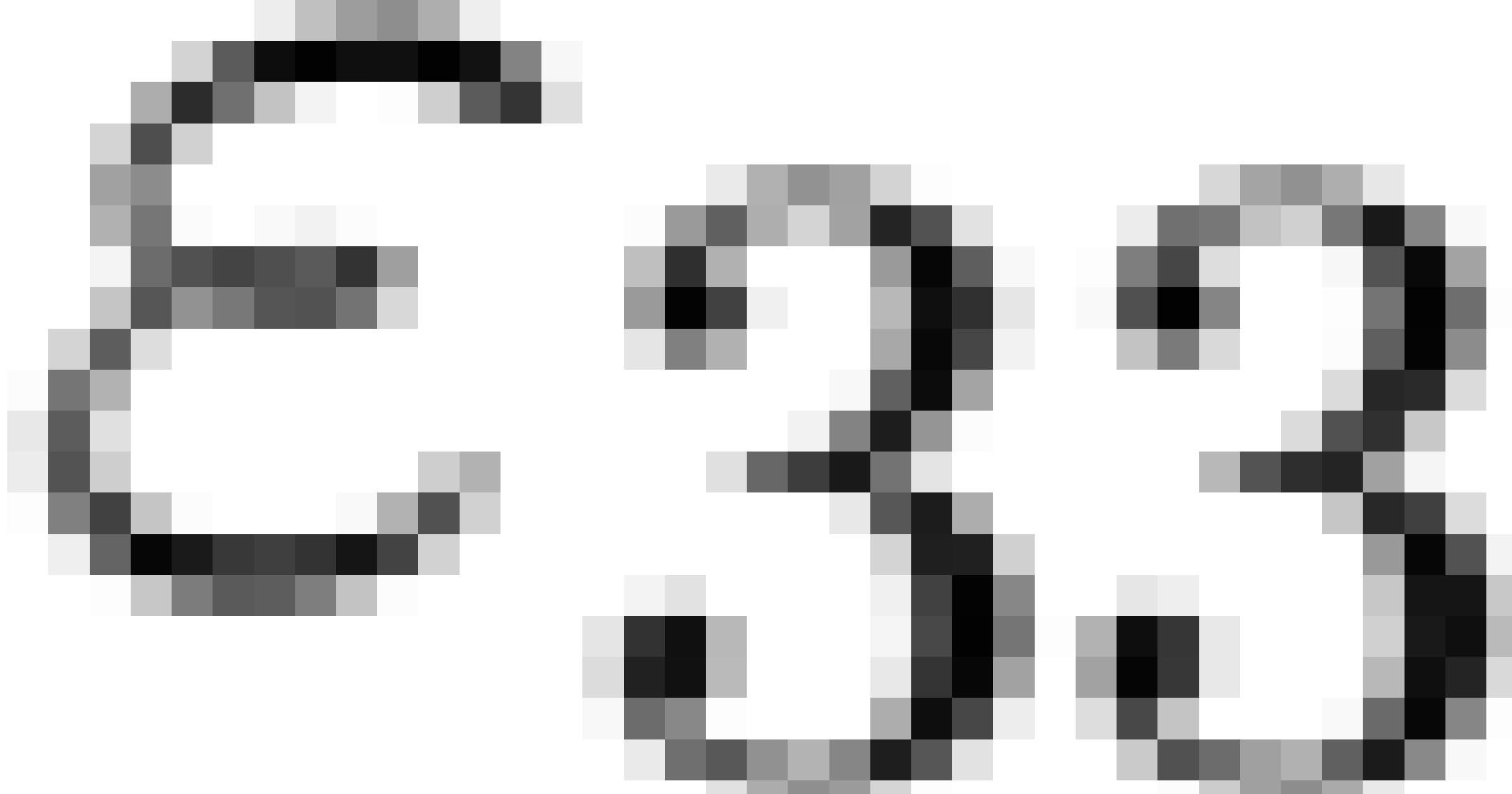
$$\sigma = E\varepsilon$$

Generalized Hooke's law

$$\underline{\sigma} = \underline{C}\underline{\varepsilon}$$







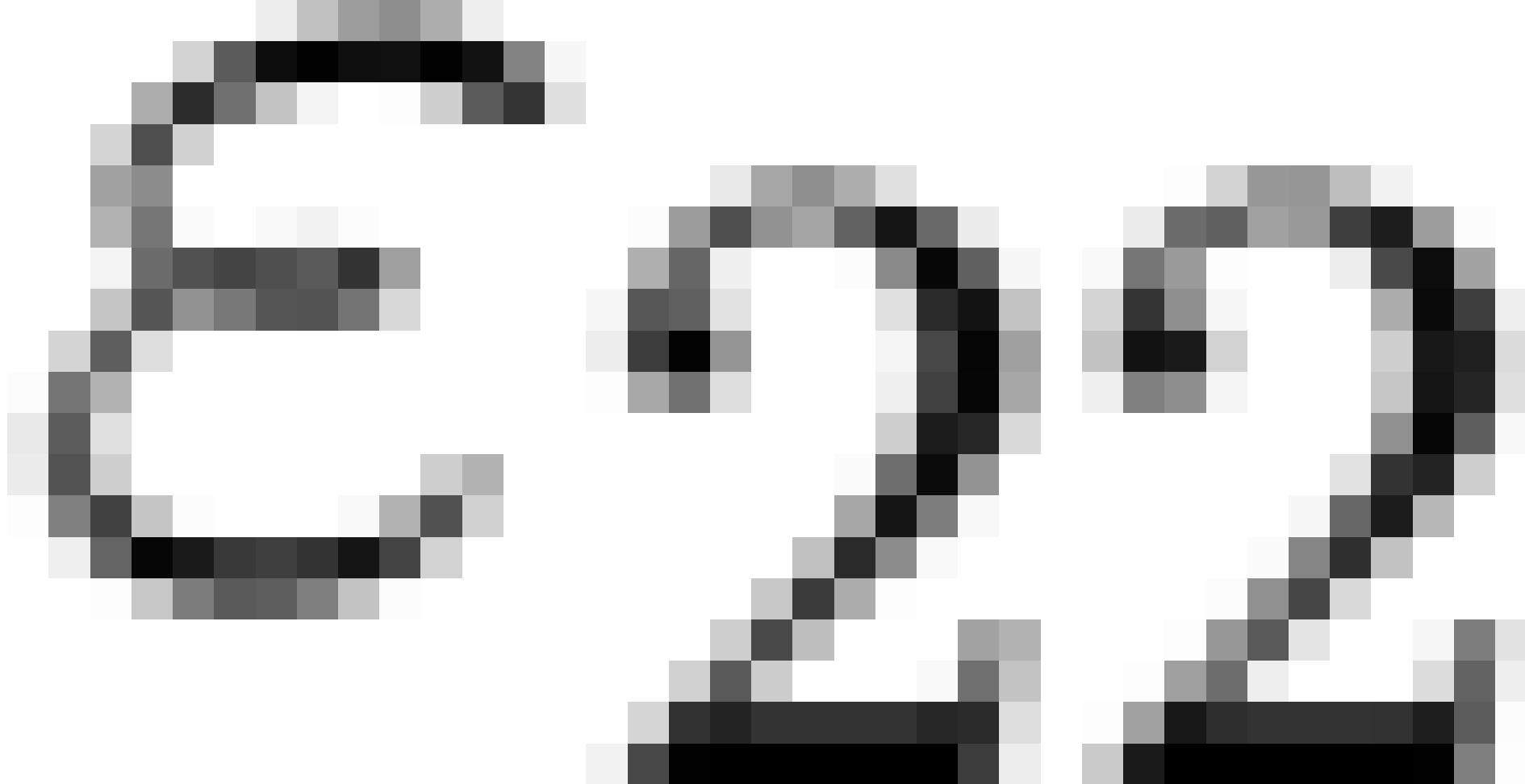
E

—  
—

σ33

σ33





611

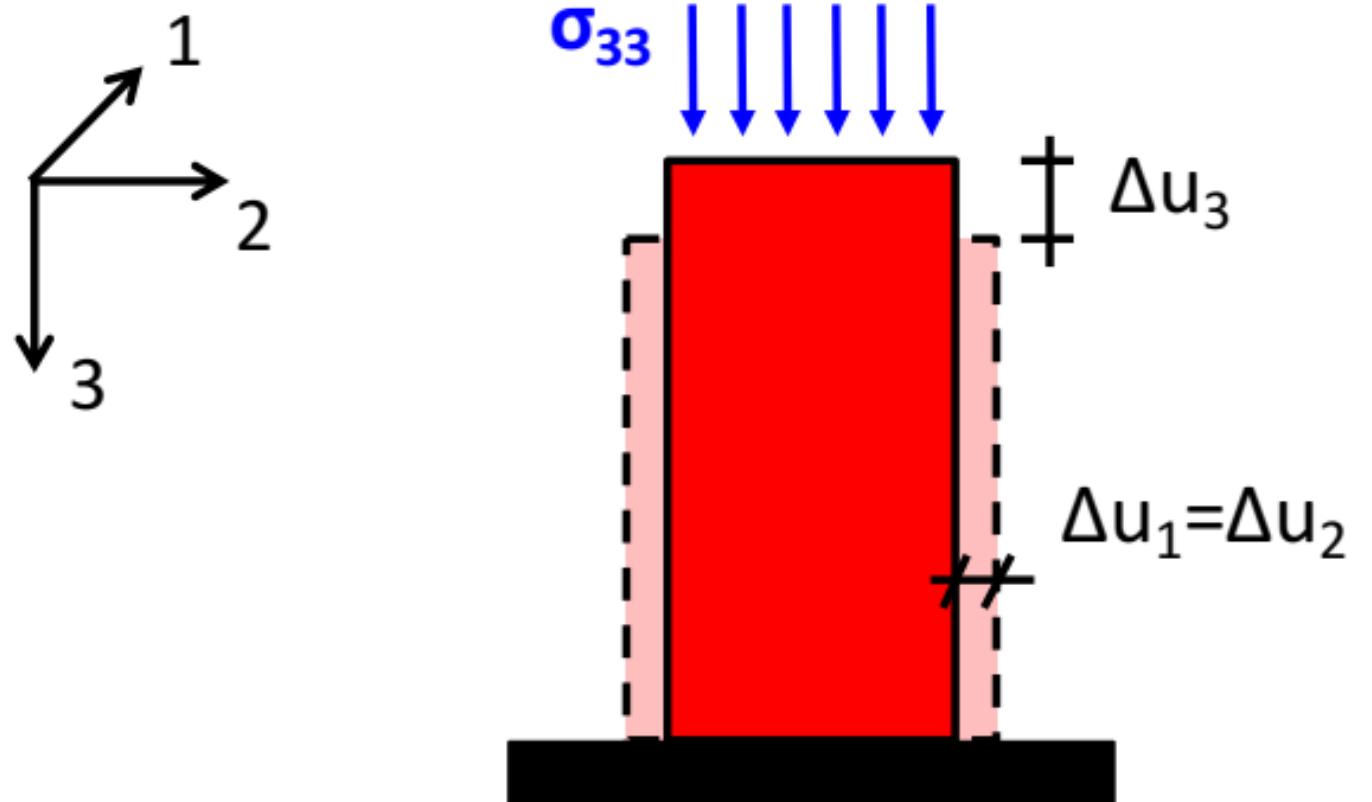
W

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633

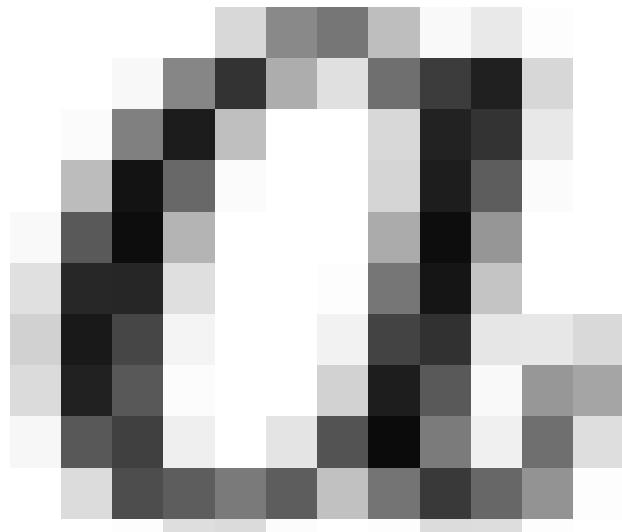
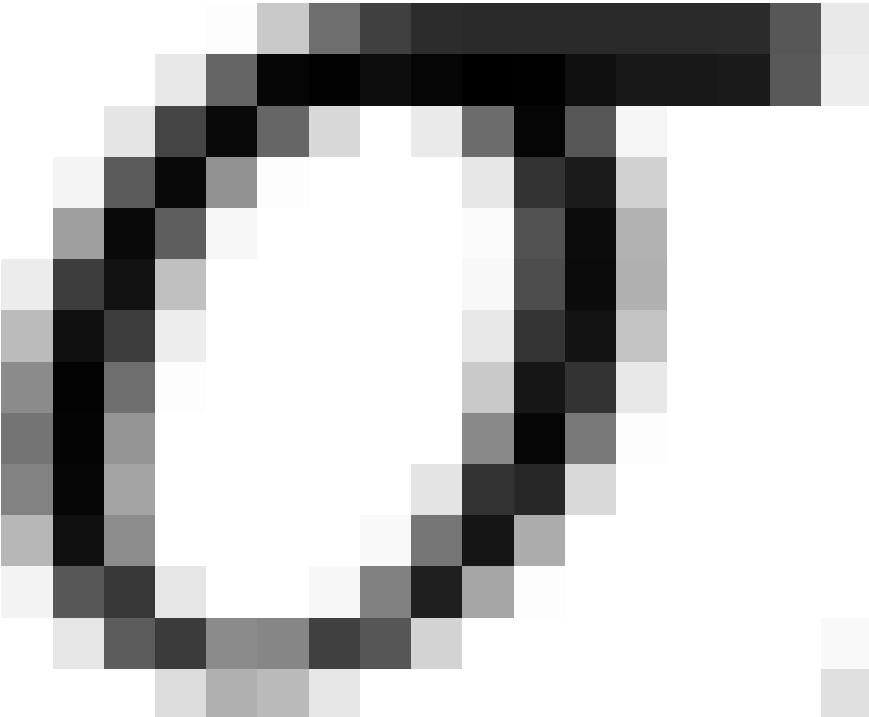


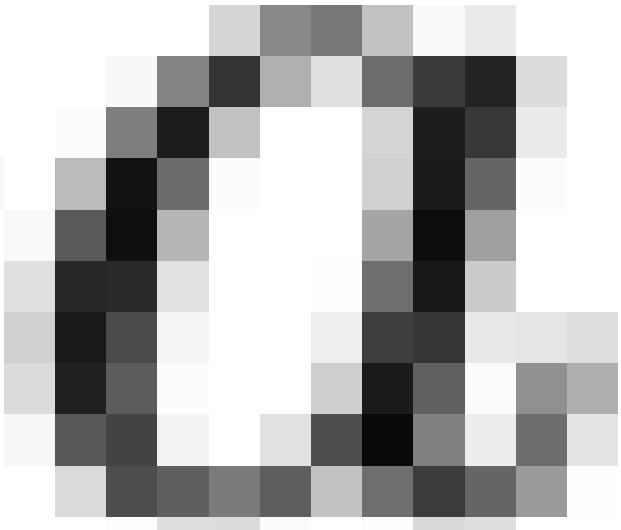
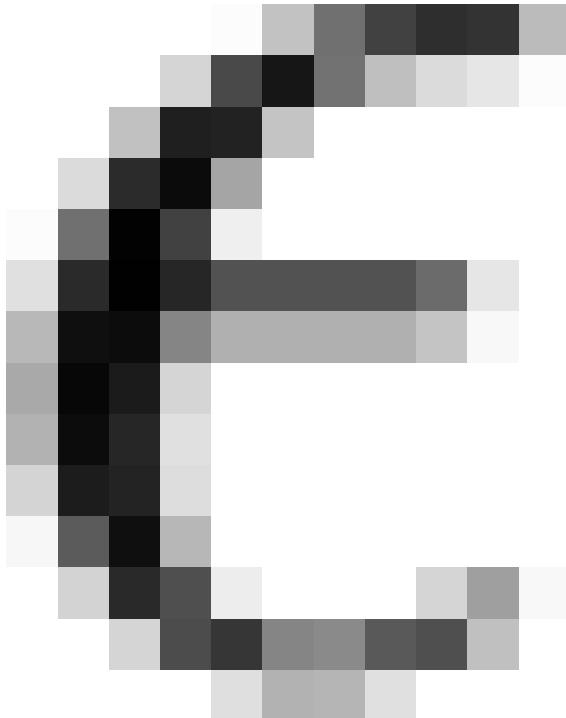
$$E = \frac{\sigma_{33}}{\epsilon_{33}}$$

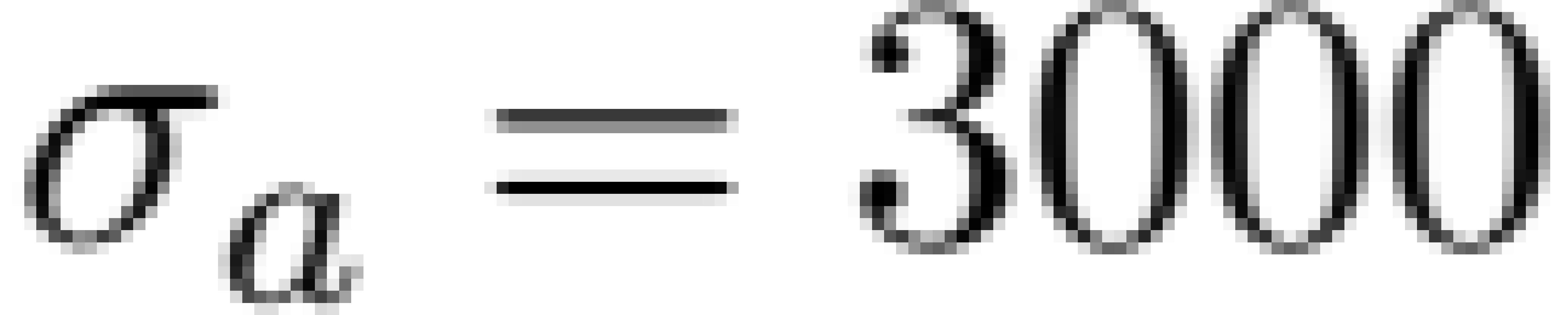
Young's Modulus

$$\nu = -\frac{\epsilon_{11}}{\epsilon_{33}}$$

Poisson's ratio (nu)







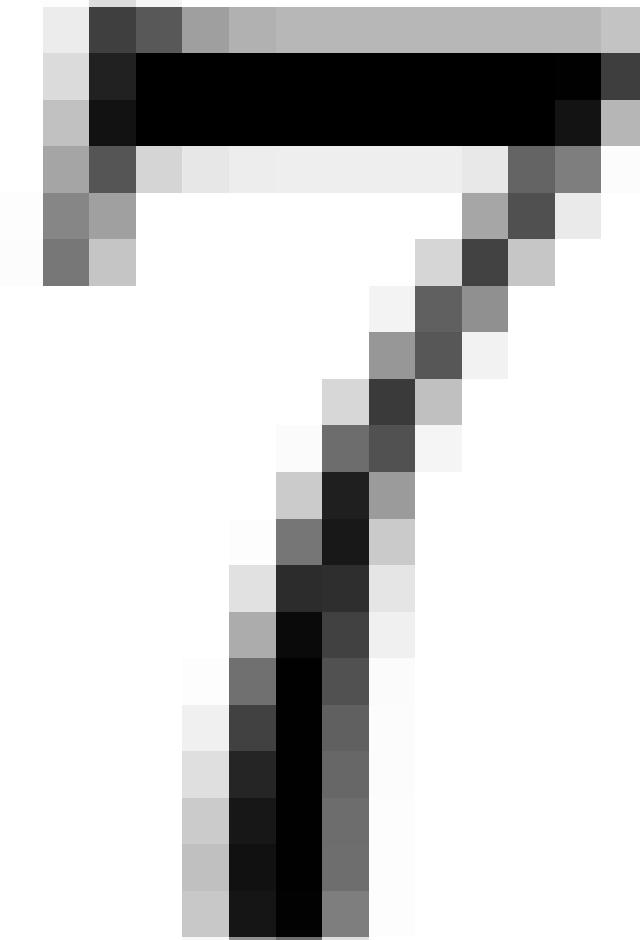
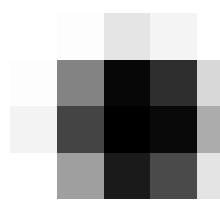
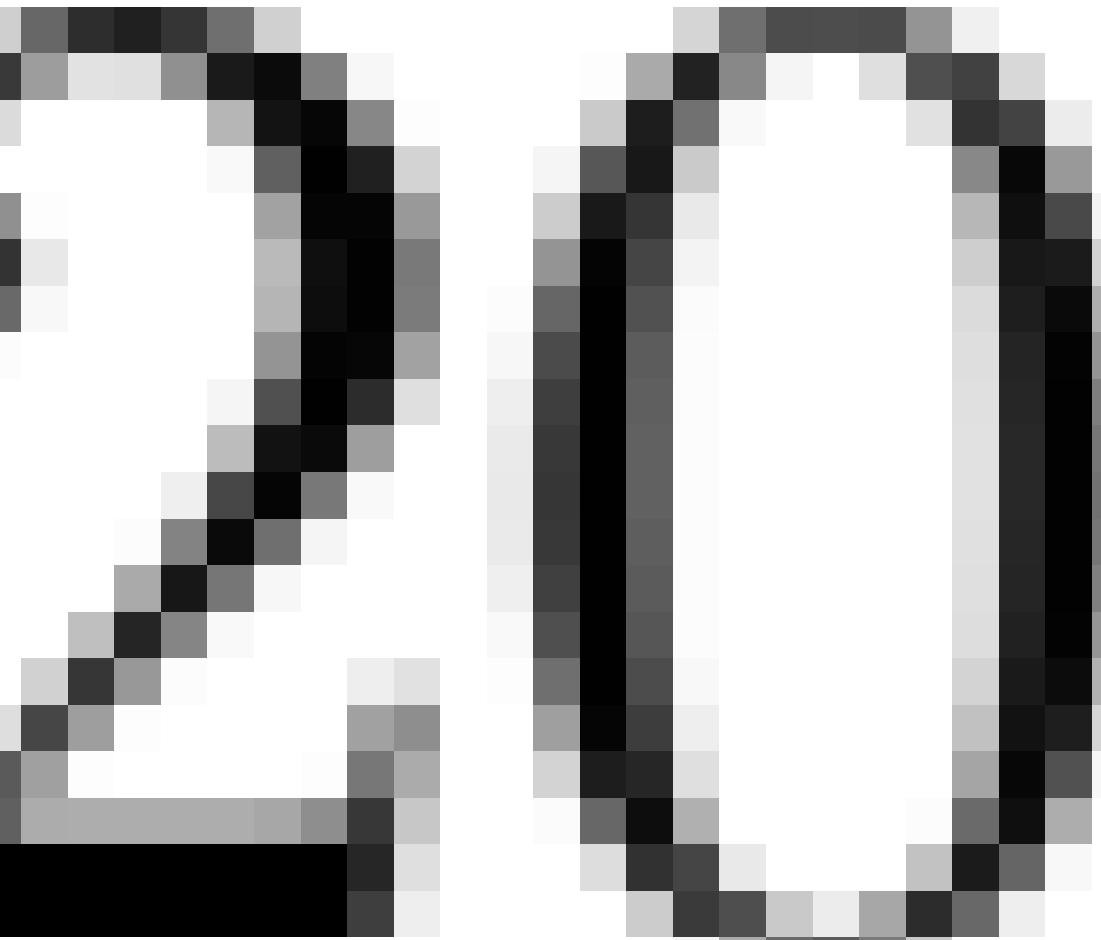
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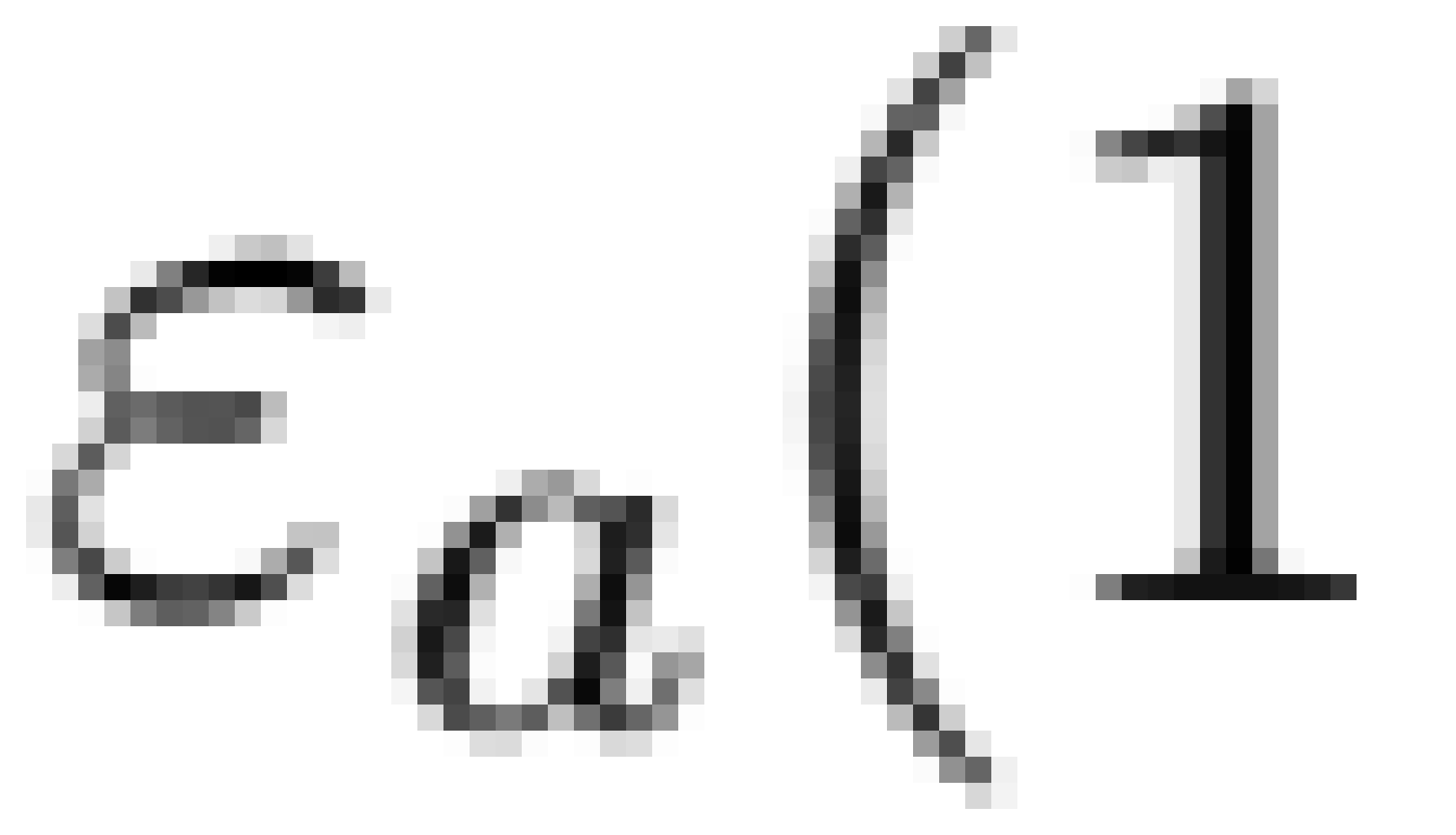
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145

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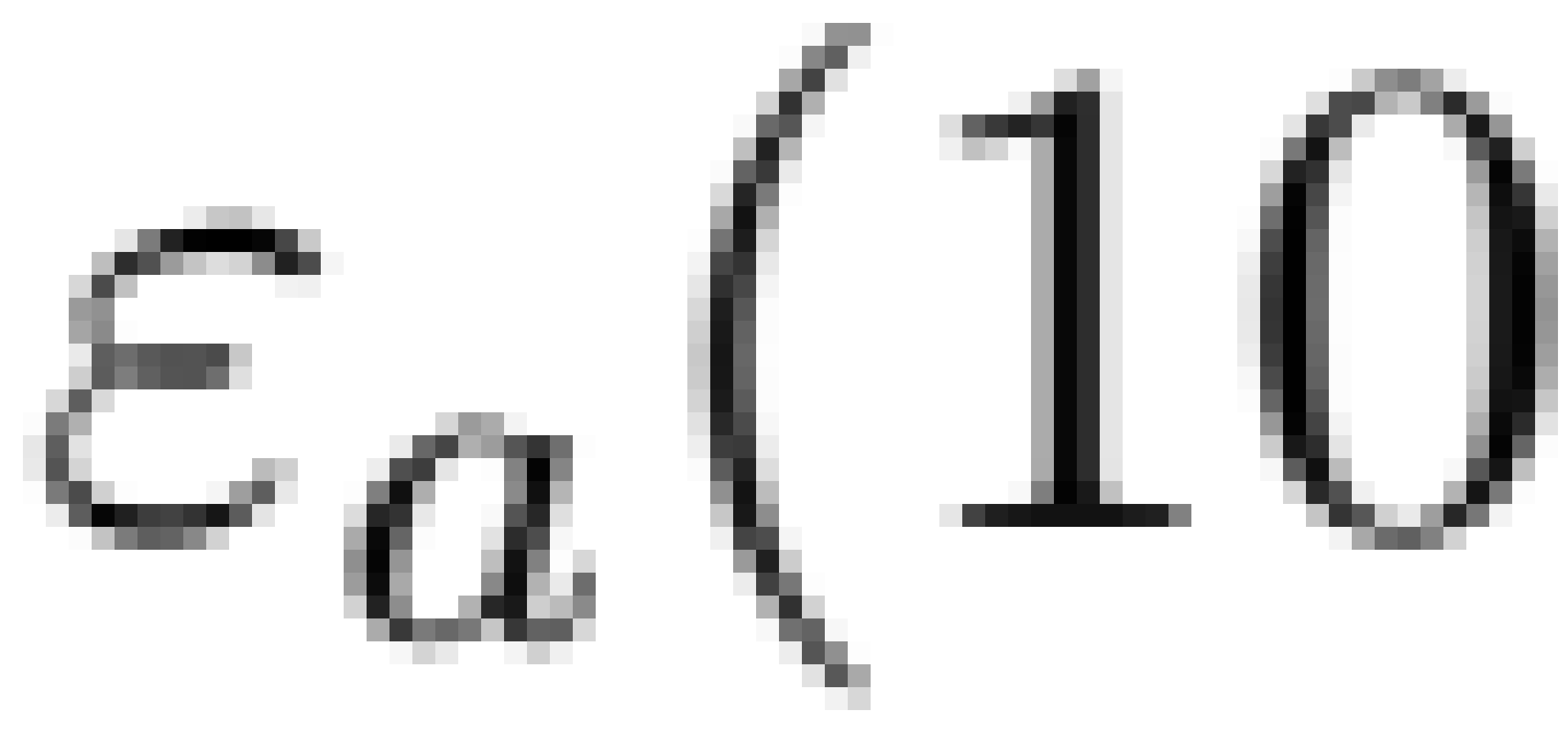
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$$\frac{\sigma_a}{E} = \frac{20.7}{100} = 0.207$$

$$0.207 \times 100 \text{ MPa} = 20.7 \text{ MPa}$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{100 \times 10^9 \text{ MPa}} = 0.207 \times 10^{-7}$$

$$= 0.00207 \times 10^{-7} = 0.00207 \%$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{50000 \text{ MPa}} = 0.00041$$

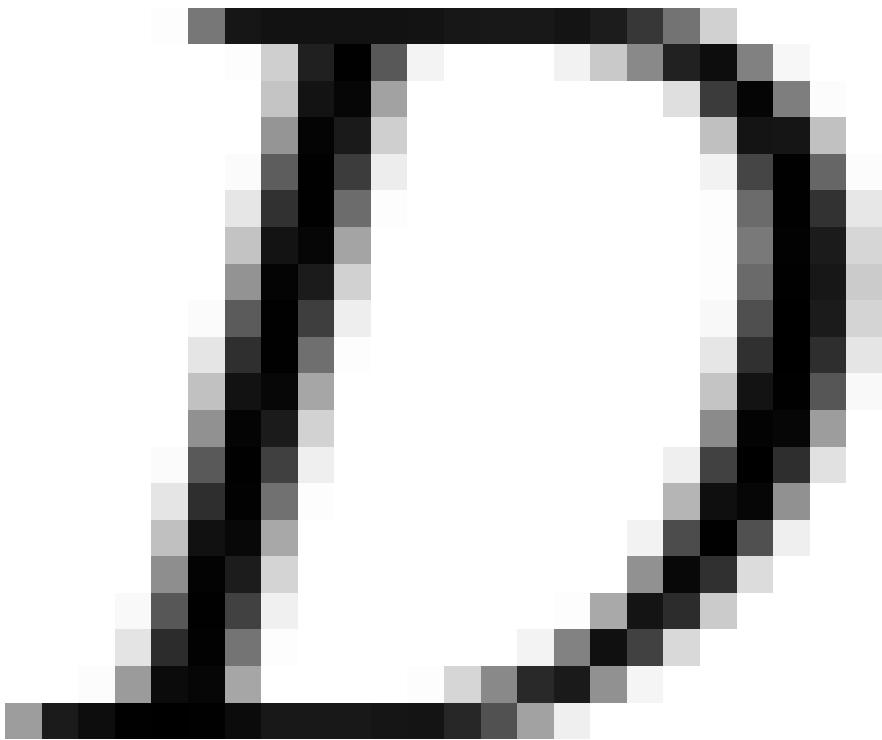
$$= 0.041\%$$

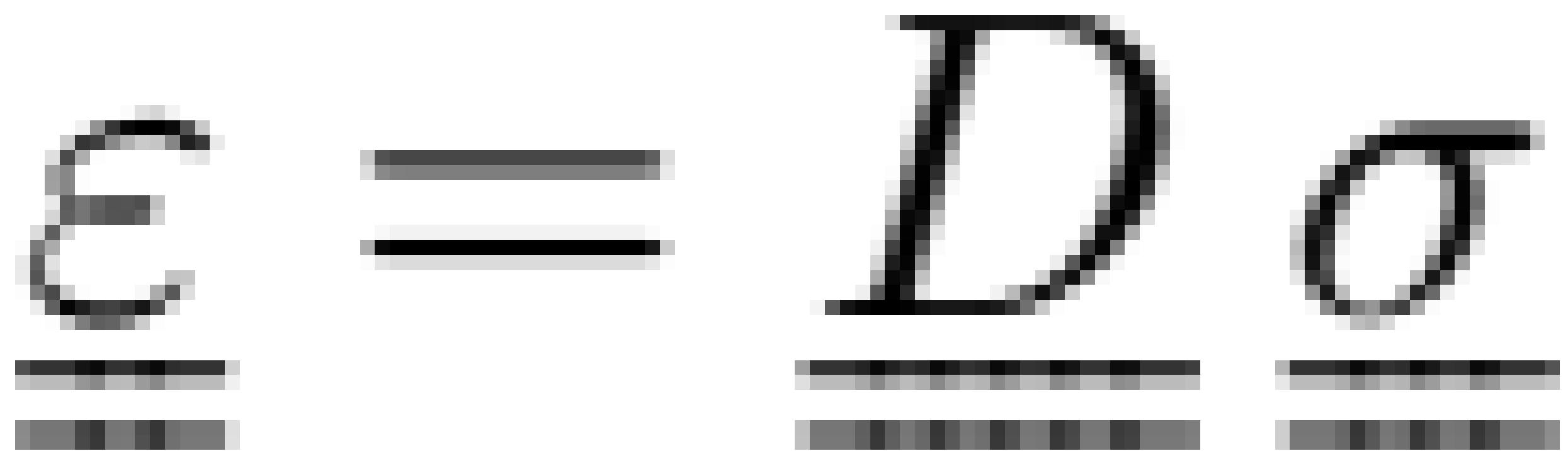
$$\left\{ \begin{array}{lcl} \epsilon_{11} & = & +\frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ & & \nu \quad \quad \quad 1 \quad \quad \quad \nu \\ \epsilon_{22} & = & -\frac{1}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{1}{E}\sigma_{33} \\ & & \nu \quad \quad \quad \nu \quad \quad \quad 1 \\ \epsilon_{33} & = & -\frac{1}{E}\sigma_{11} - \frac{1}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{array} \right.$$

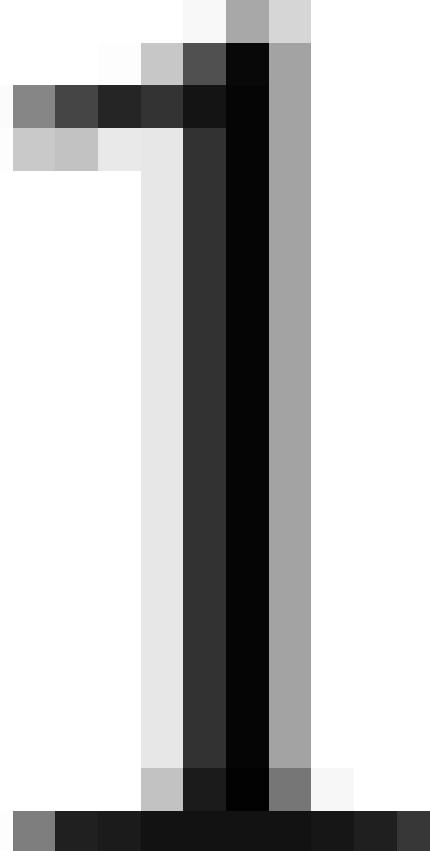
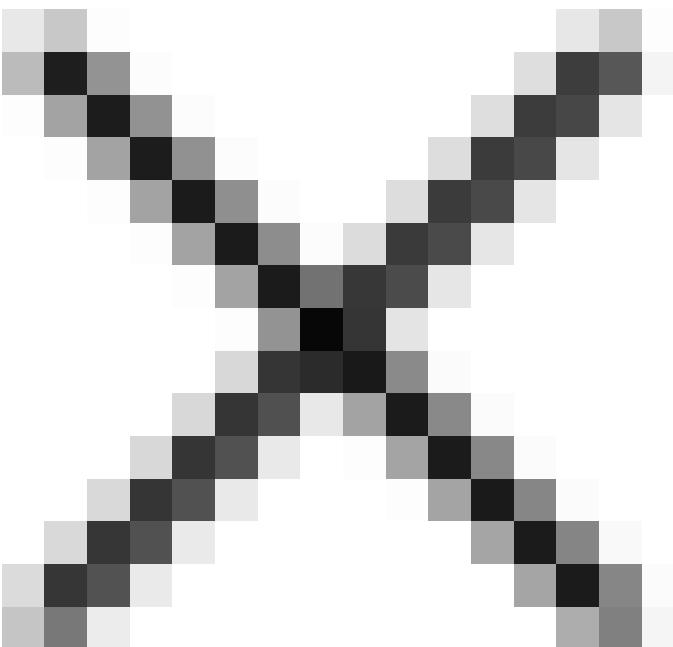
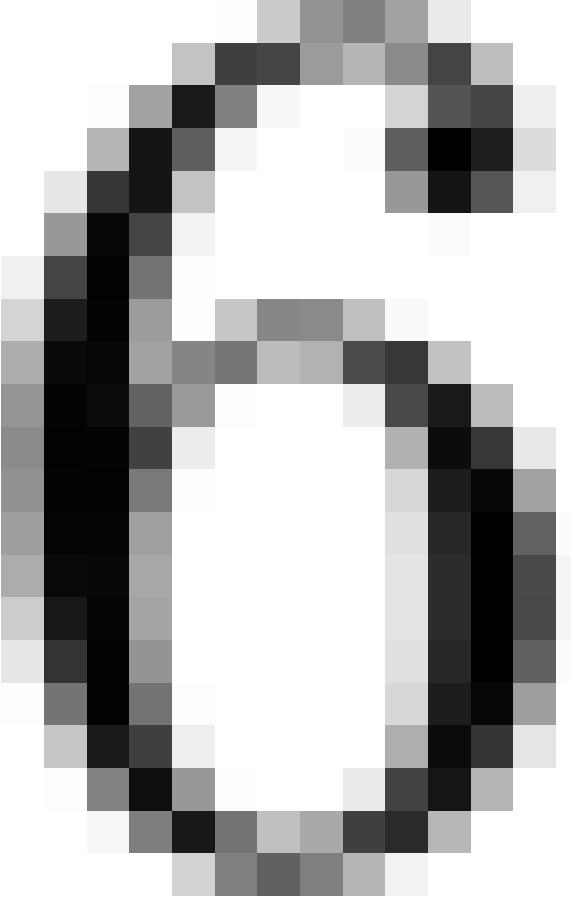


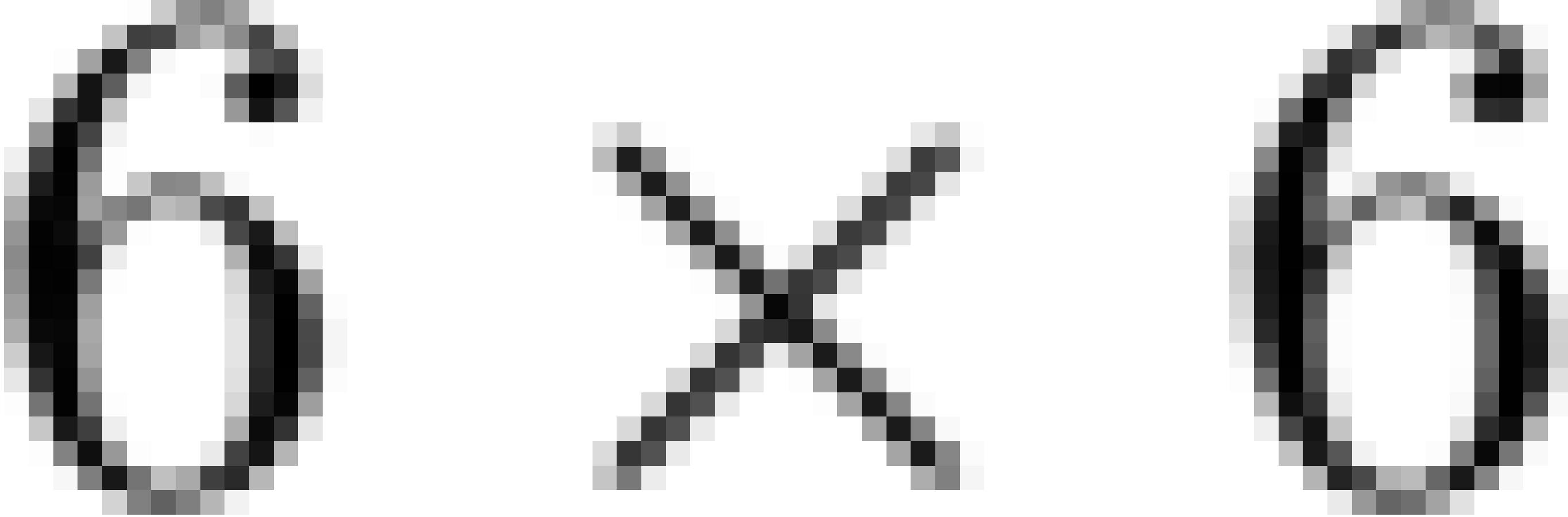


$$\left\{ \begin{array}{l} 2\epsilon_{12} = (1/G) \sigma_{12} \\ 2\epsilon_{13} = (1/G) \sigma_{13} \\ 2\epsilon_{23} = (1/G) \sigma_{23} \end{array} \right.$$



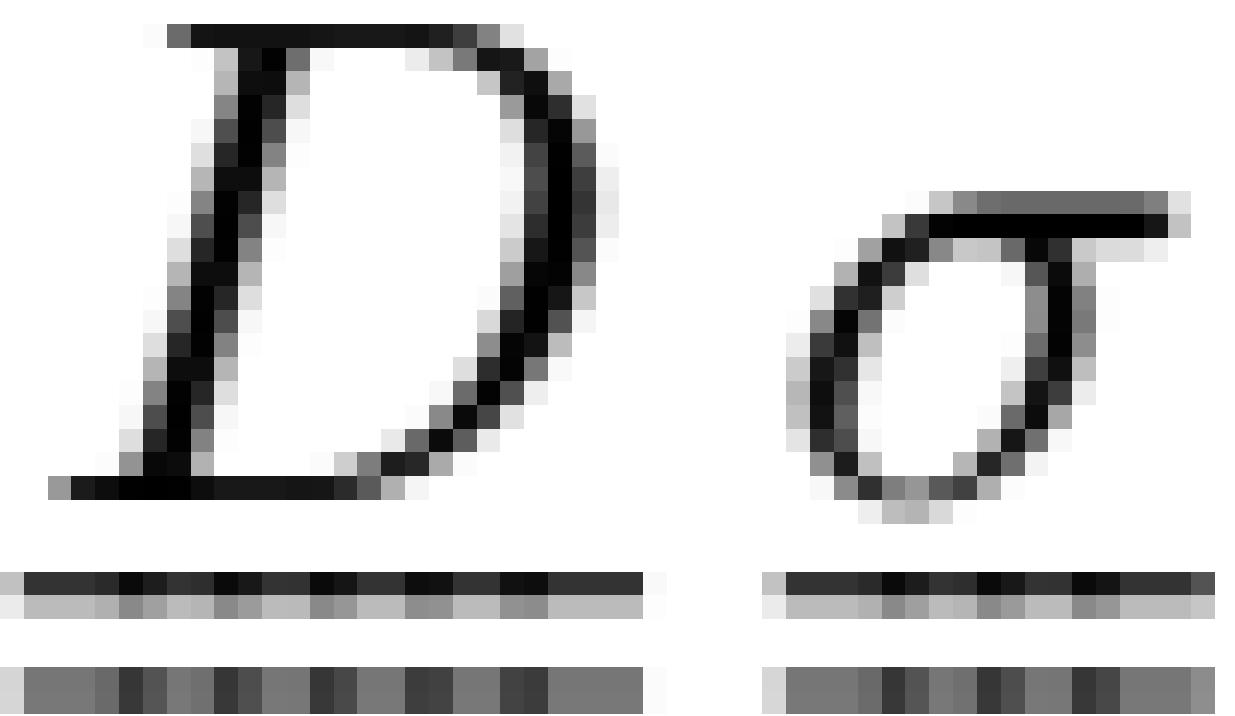




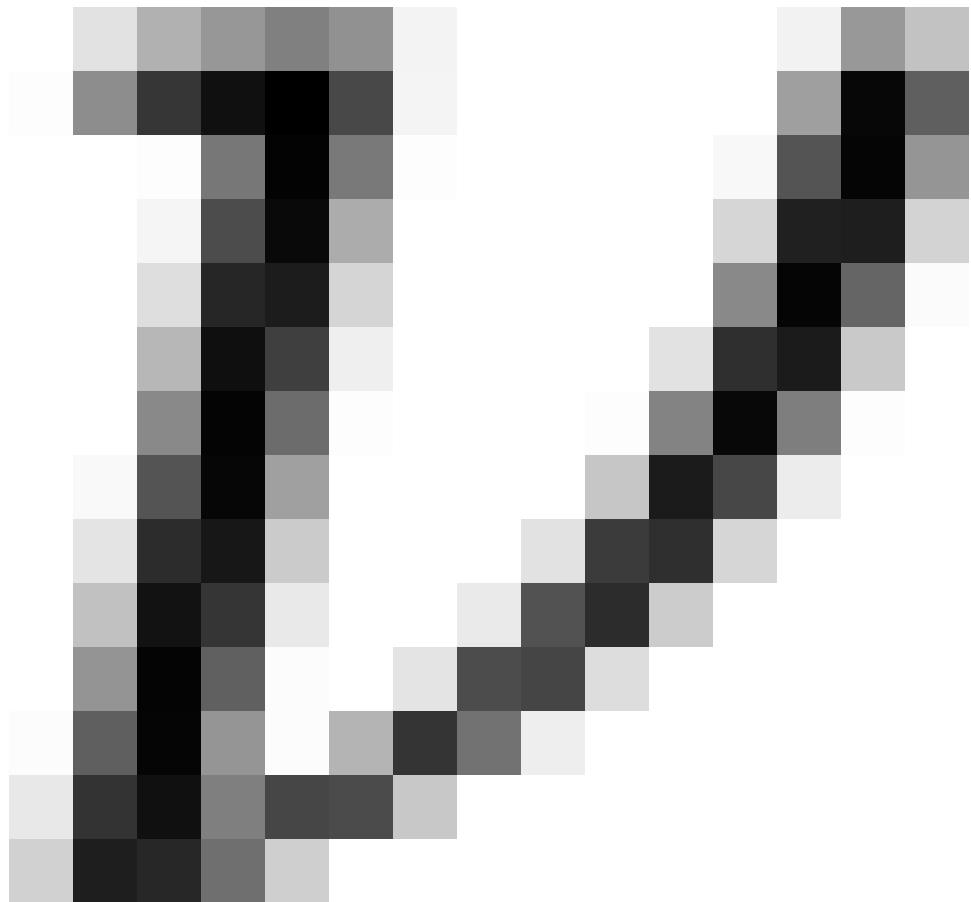


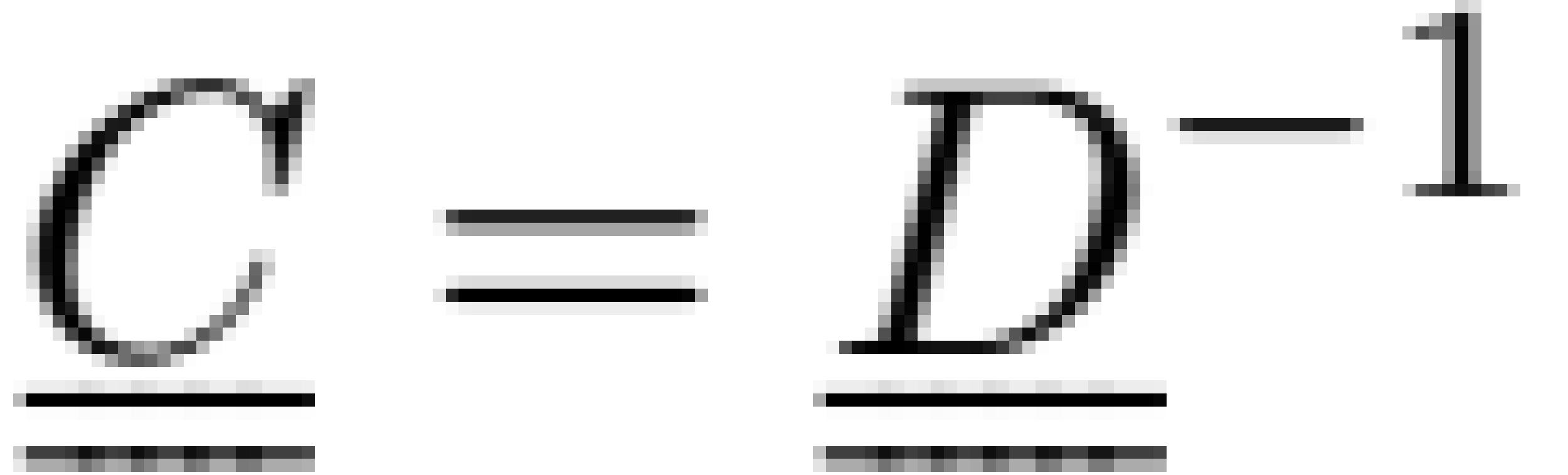
$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} = \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & 1 & \nu & 0 & 0 & 0 \\ -\frac{\nu}{E} & +\frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & \nu & 1 & 0 & 0 & 0 \\ -\frac{1}{E} & -\frac{\nu}{E} & +\frac{1}{E} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}$$



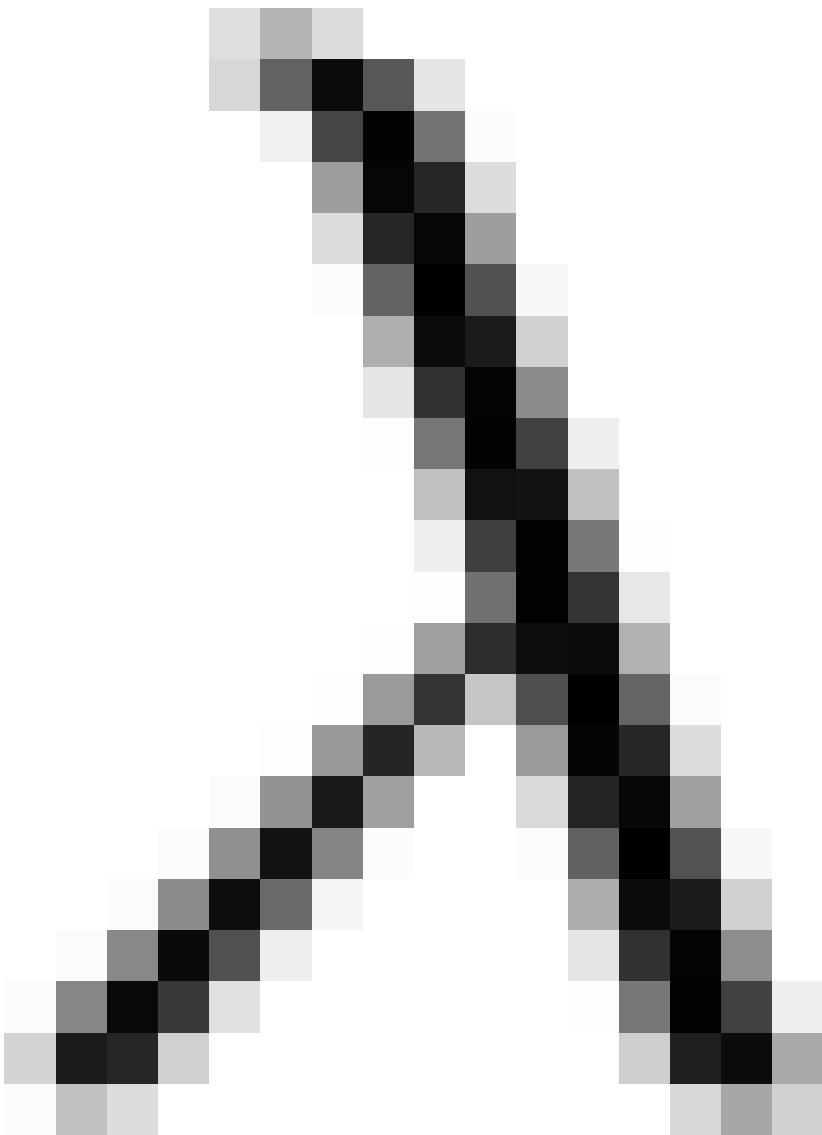


$$\underline{\underline{\epsilon}} = \begin{bmatrix} -\frac{v}{E}\sigma_{33}, -\frac{v}{E}\sigma_{33}, \frac{1}{E}\sigma_{33}, 0, 0, 0 \end{bmatrix}^T$$





$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$



$$\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)} [(1-\nu)e_{11} + \nu e_{22} + \nu e_{33}]$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + \frac{(1-2\nu)}{(1+\nu)(1-2\nu)} \epsilon_{11}$$

$\lambda$



$(1 + \nu)$

$(1 - \nu)$



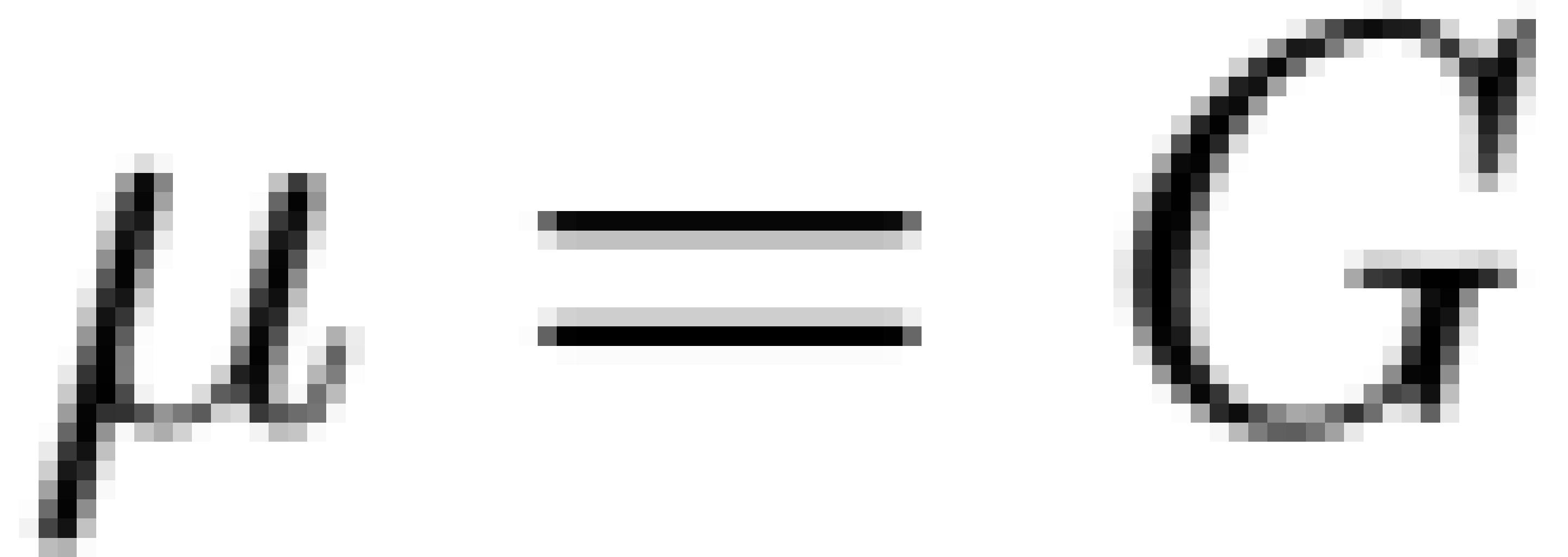
$(2\nu)$

$\nu E$

$$2\mu =$$

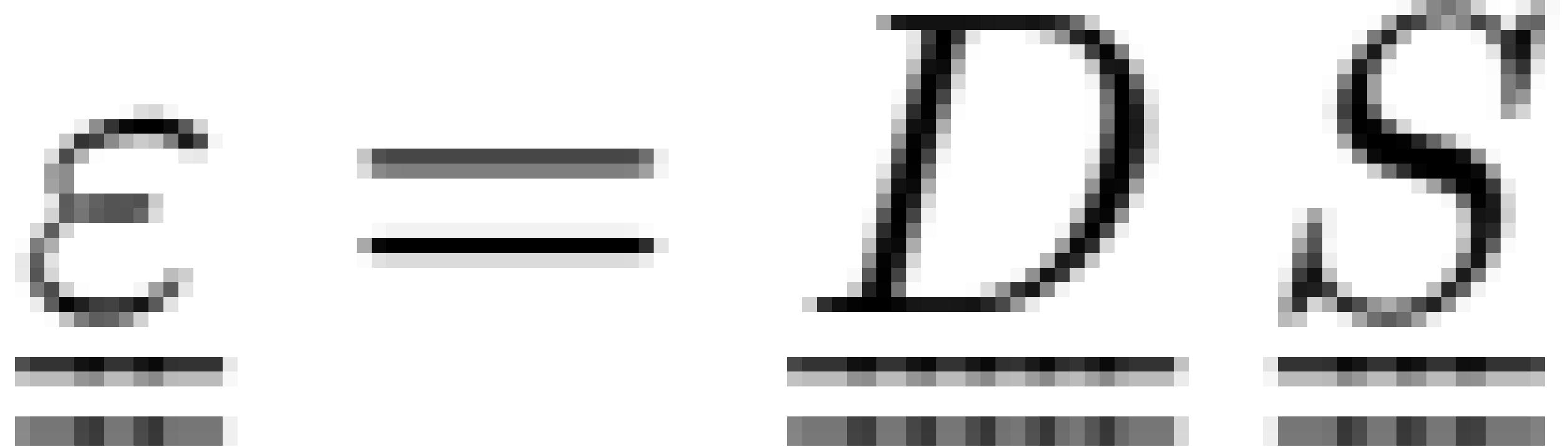
$$\frac{\nu E}{(1+\nu)(1-2\nu)} =$$

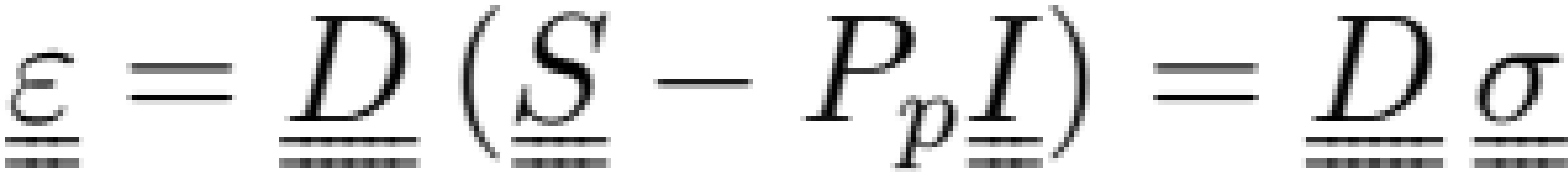
$$\frac{(1-2\nu)E}{\nu} = \frac{E}{(1+\nu)}$$

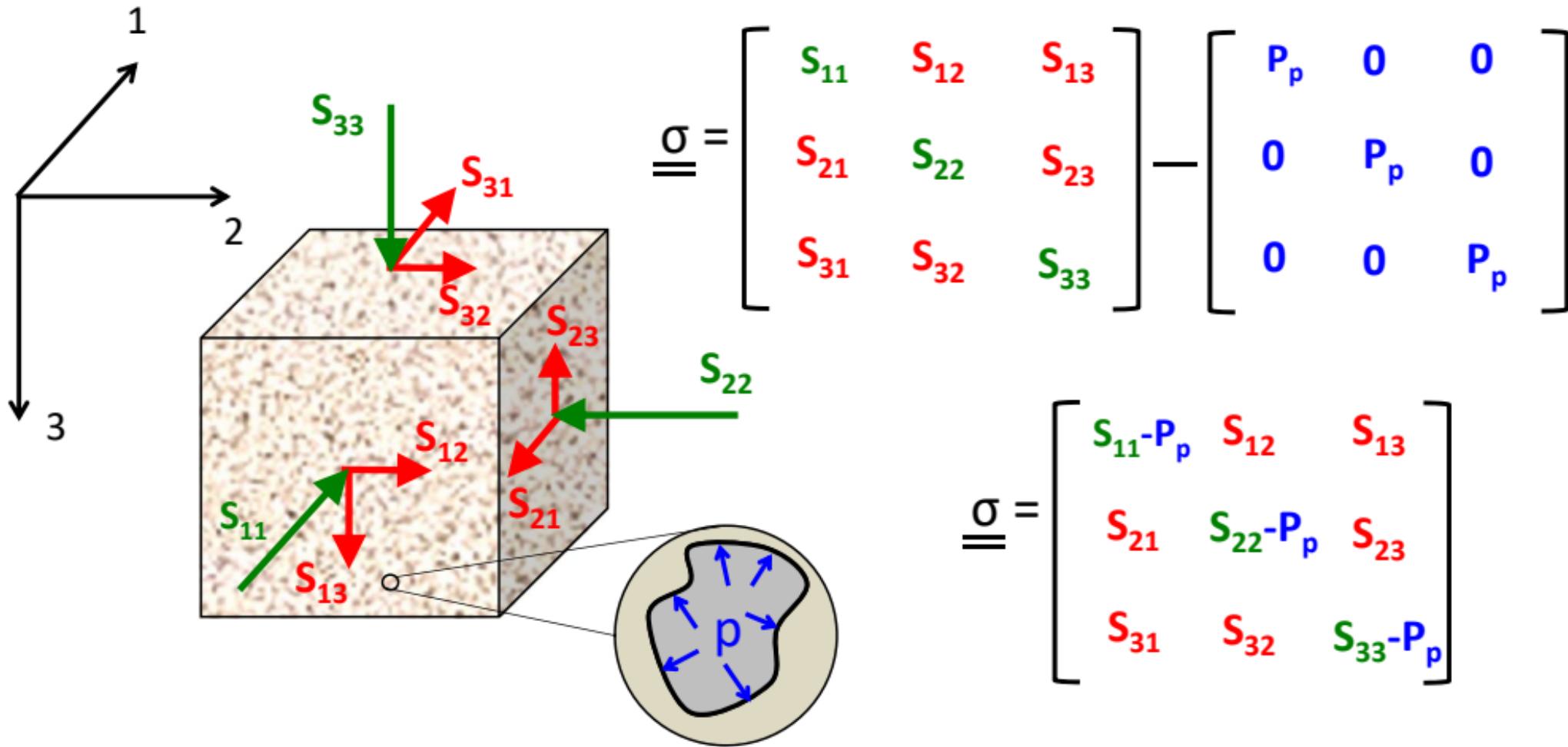


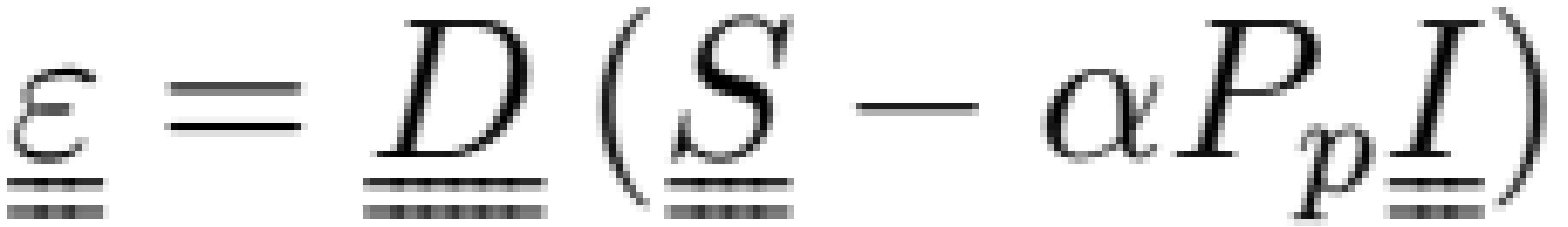
$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right. .$$

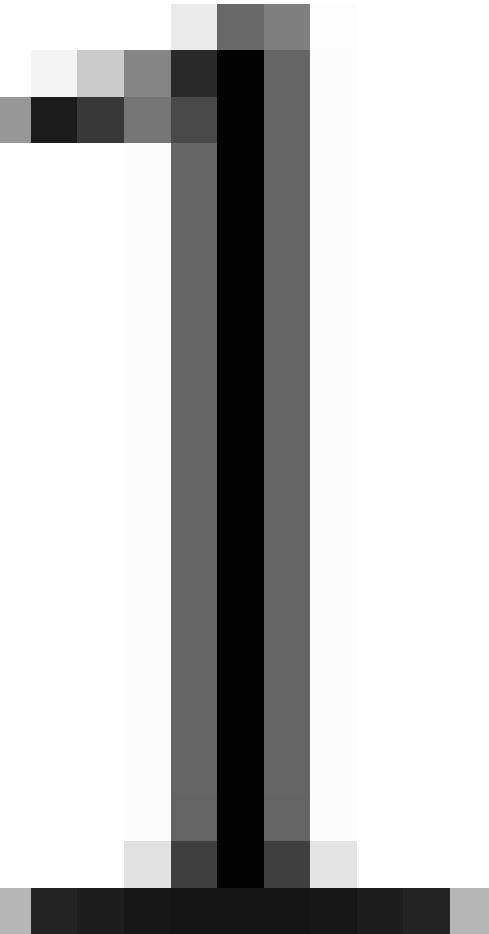
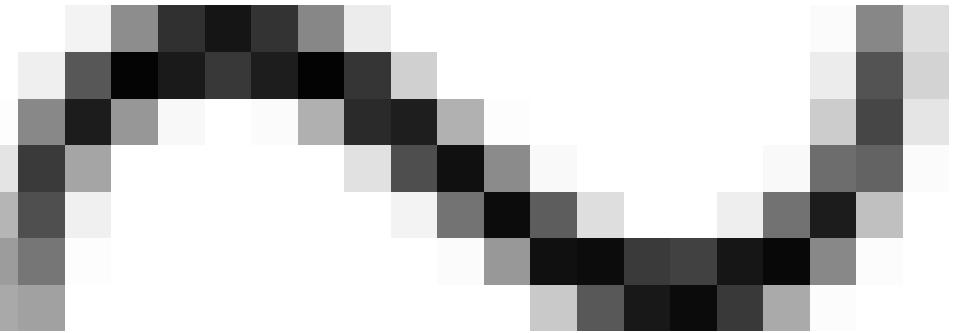
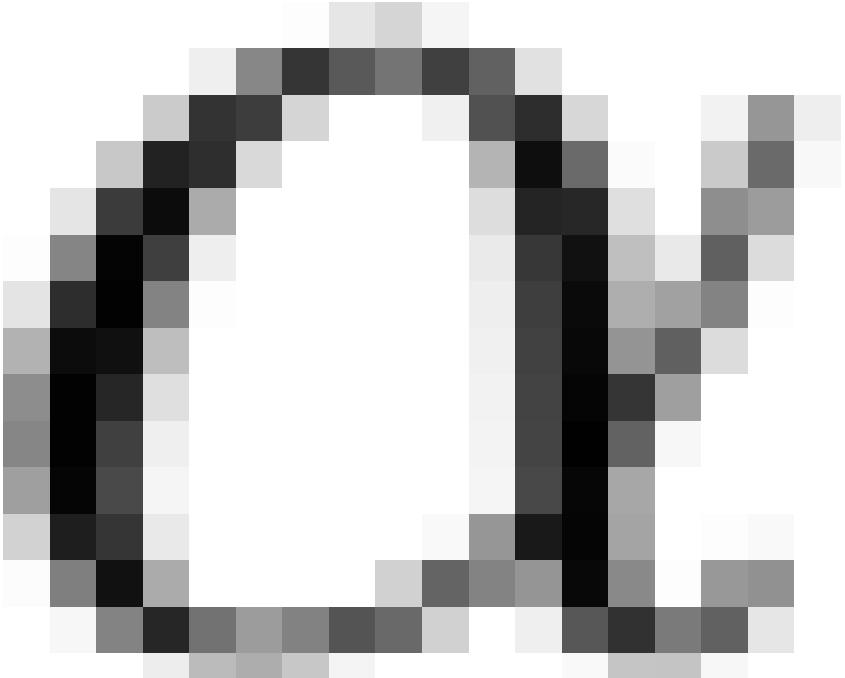
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$









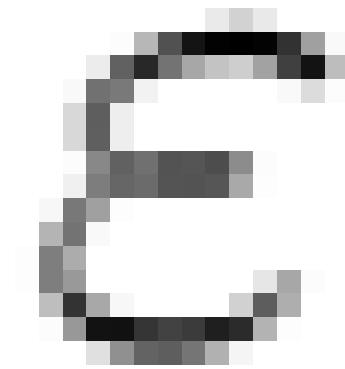
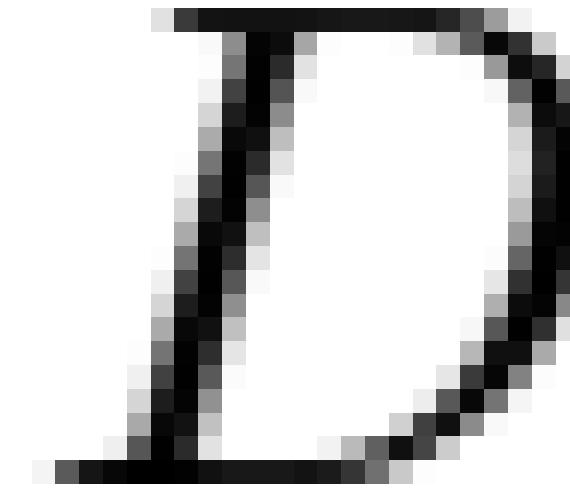
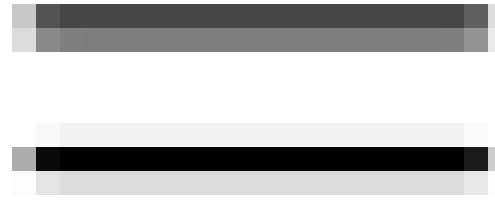
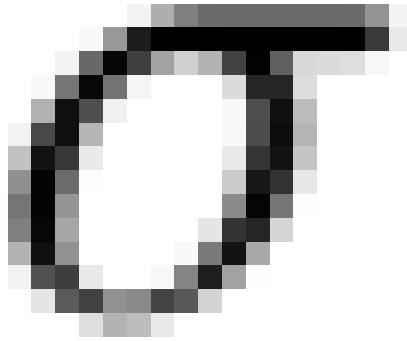














$$\left\{ \begin{array}{l} \sigma_{11} = \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)} \epsilon_{33} \\ \sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \epsilon_{33} \end{array} \right.$$

$\sigma_{11}$ 

$$= \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

 $\sigma_{22}$ 

$$= \begin{array}{c} \text{---} \\ | \end{array}$$

 $\sigma_{33}$  $\nu$  $\nu$

$\sigma_b$

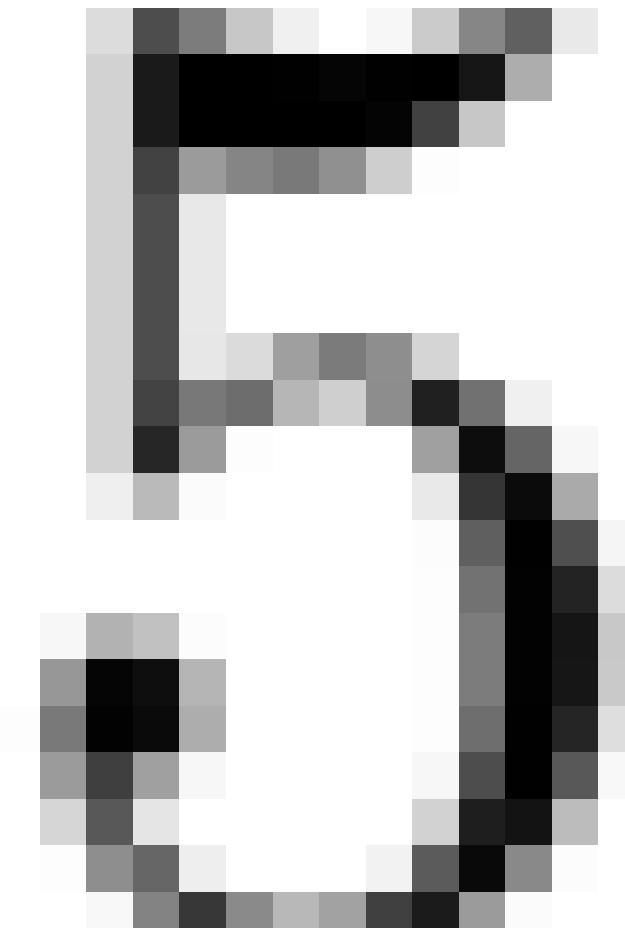
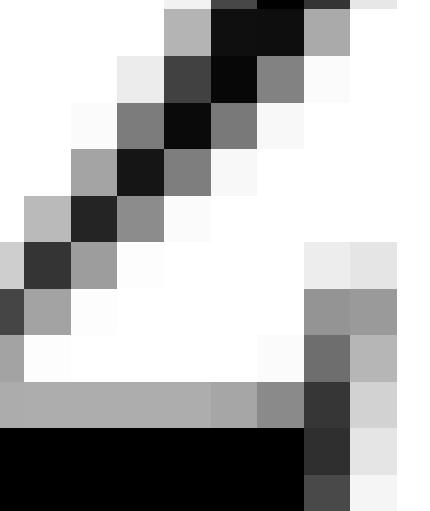
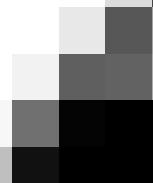
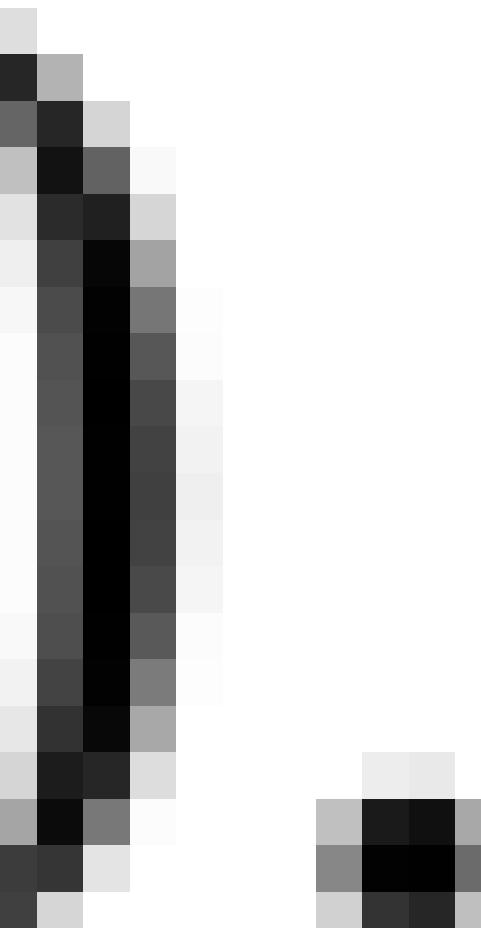
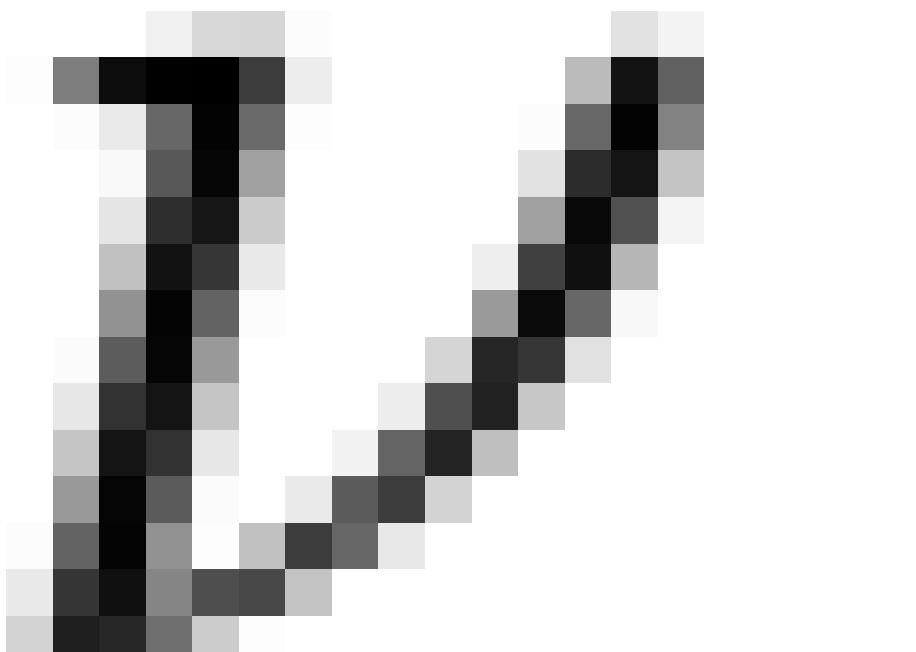
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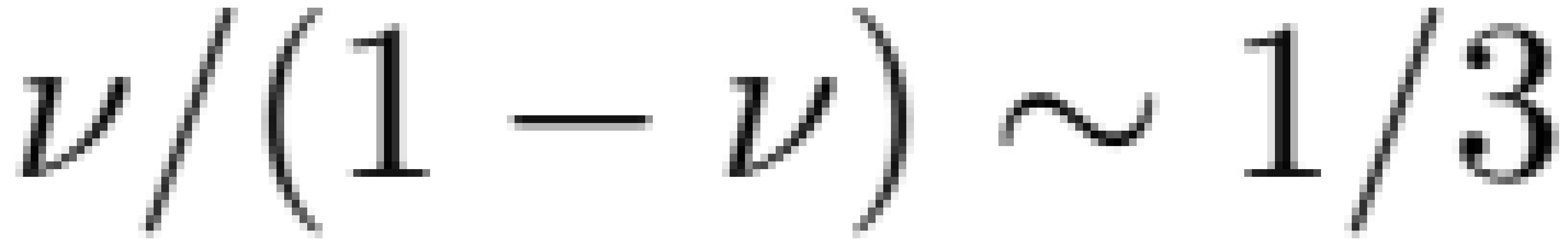
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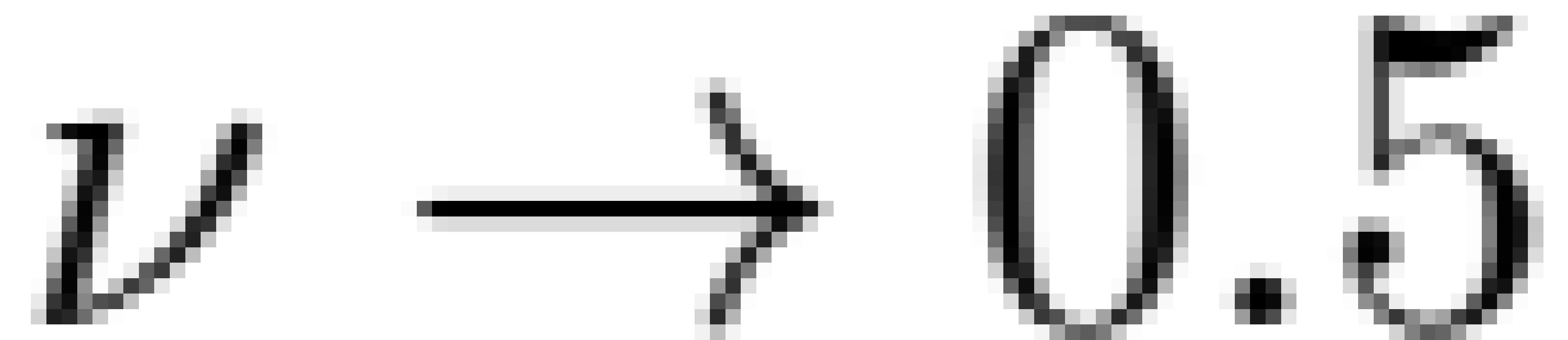
V

V

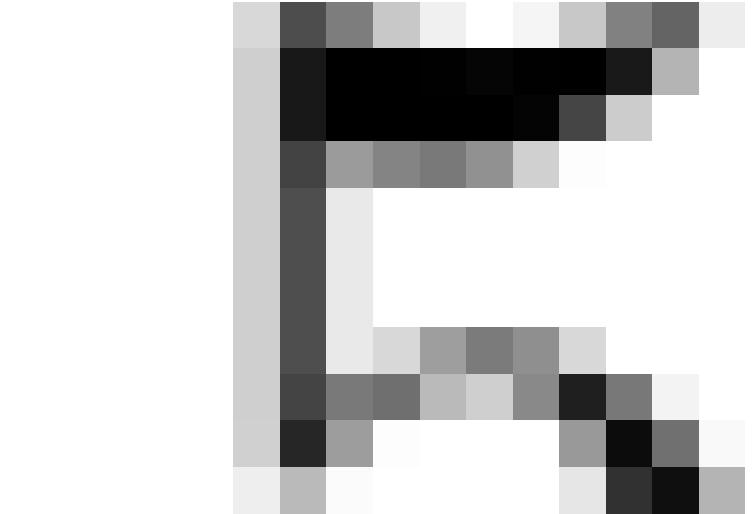
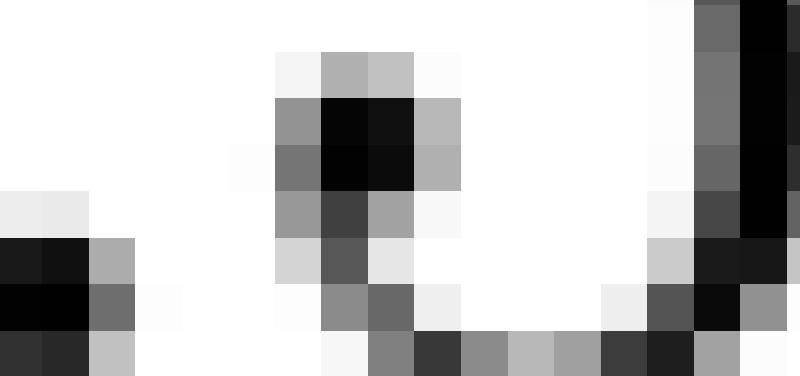
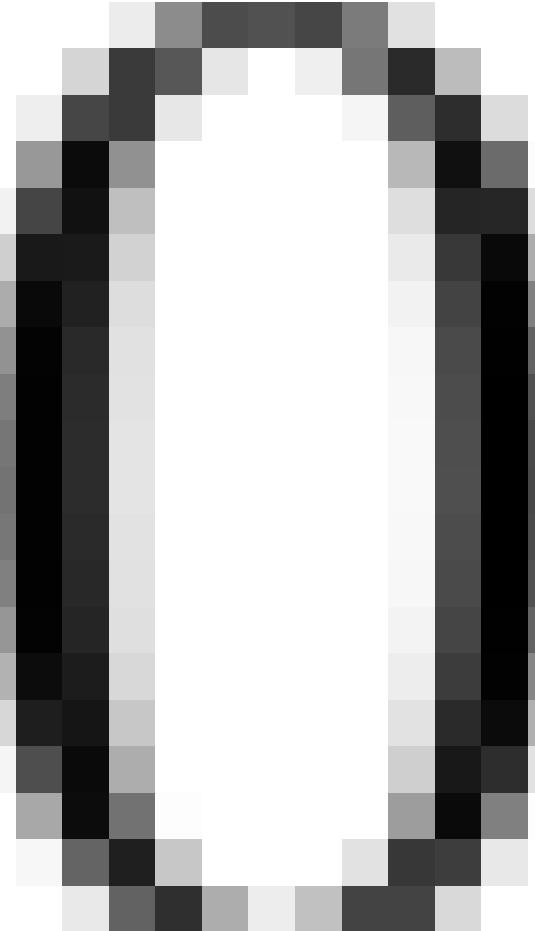
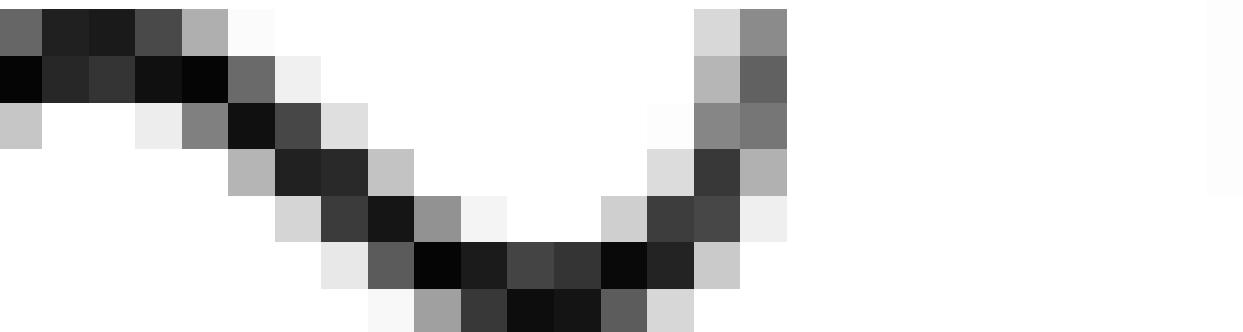
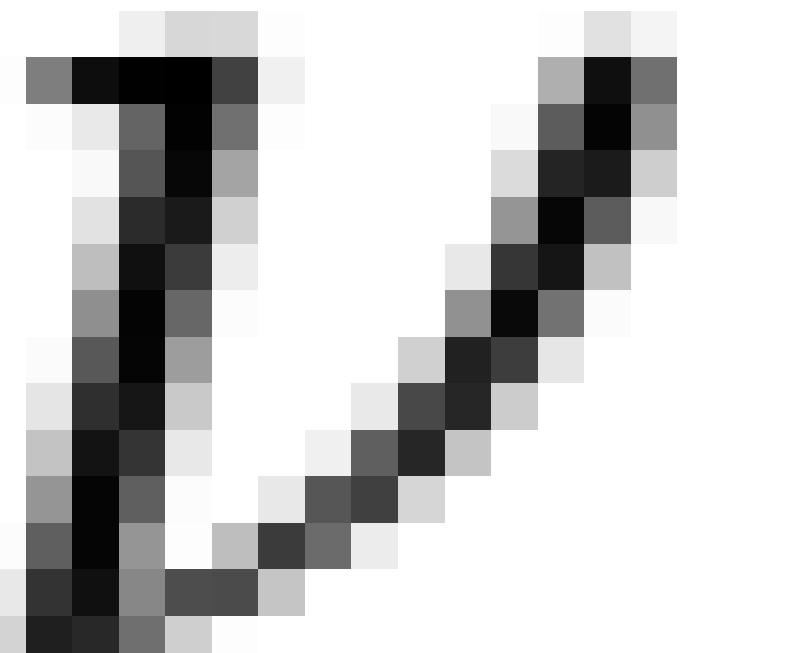
$\sigma_w$







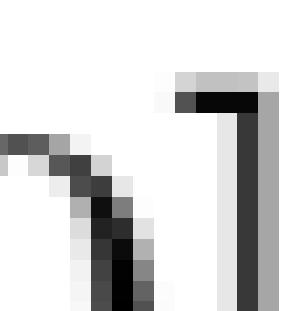
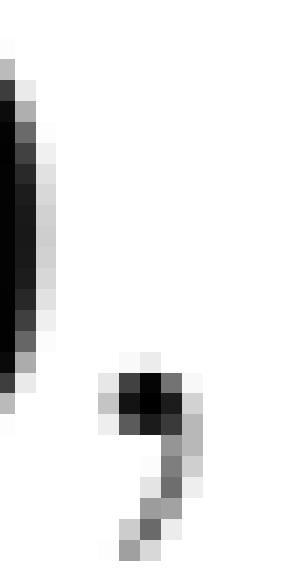
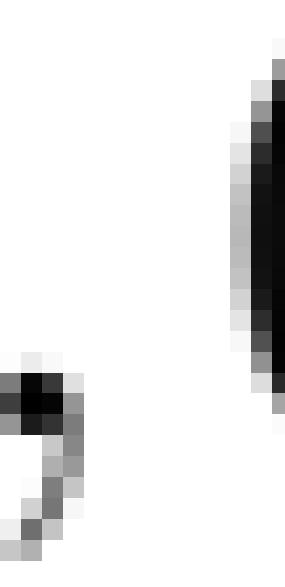
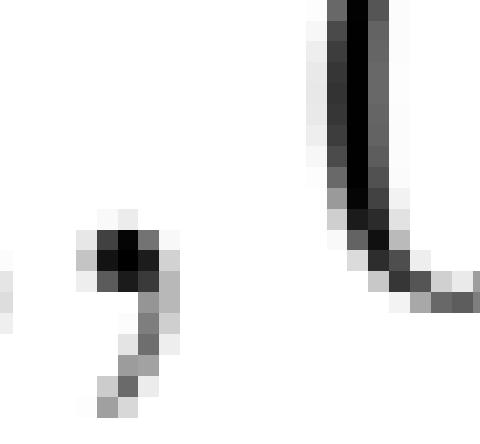
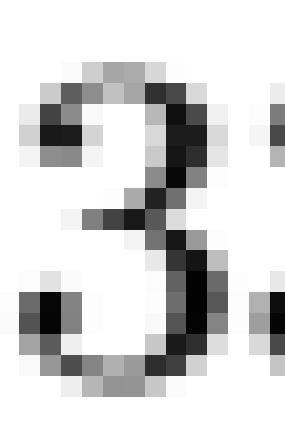
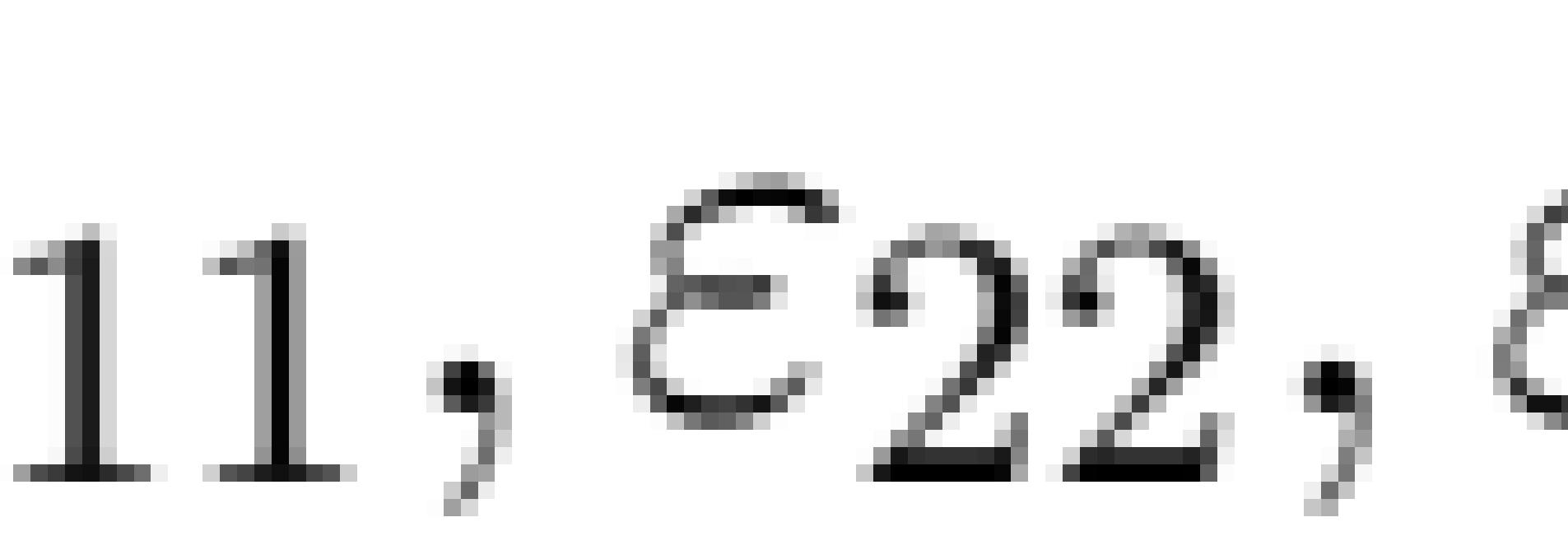
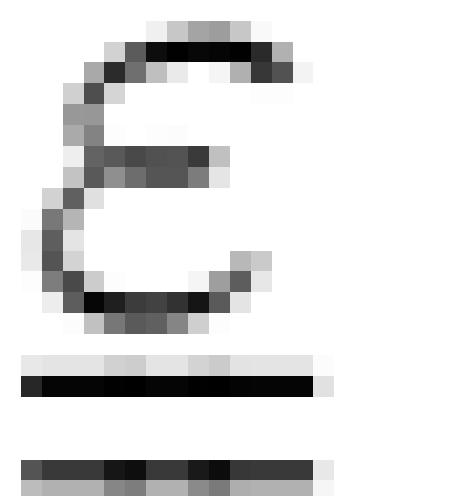




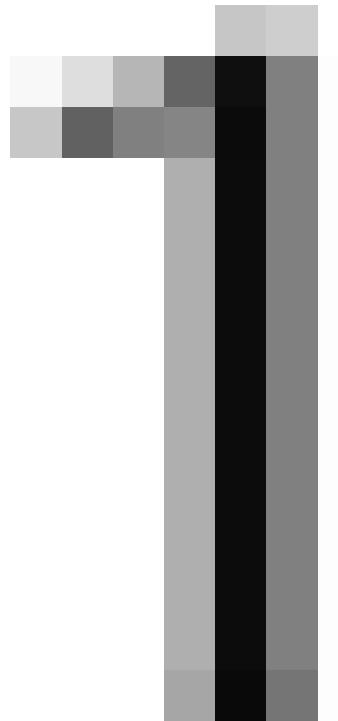
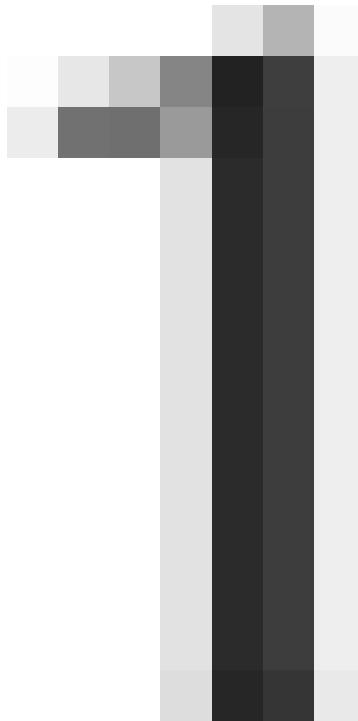
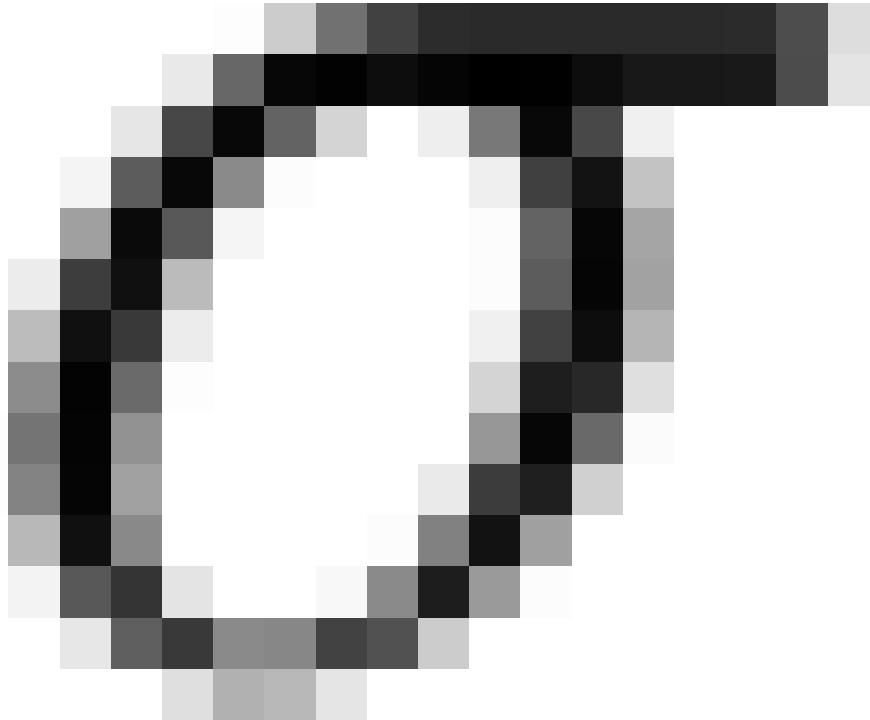




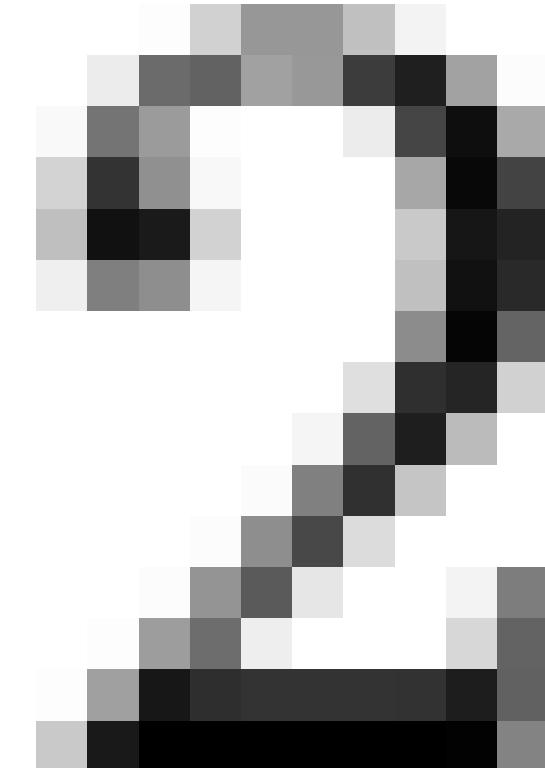
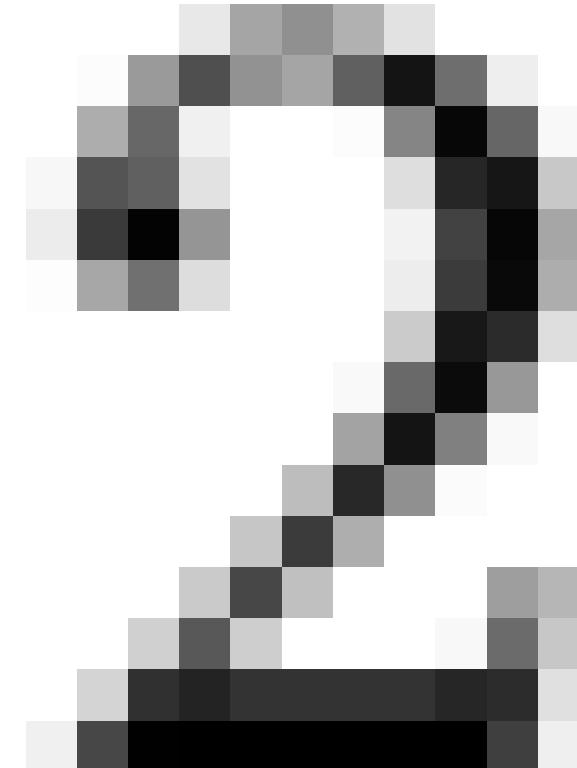
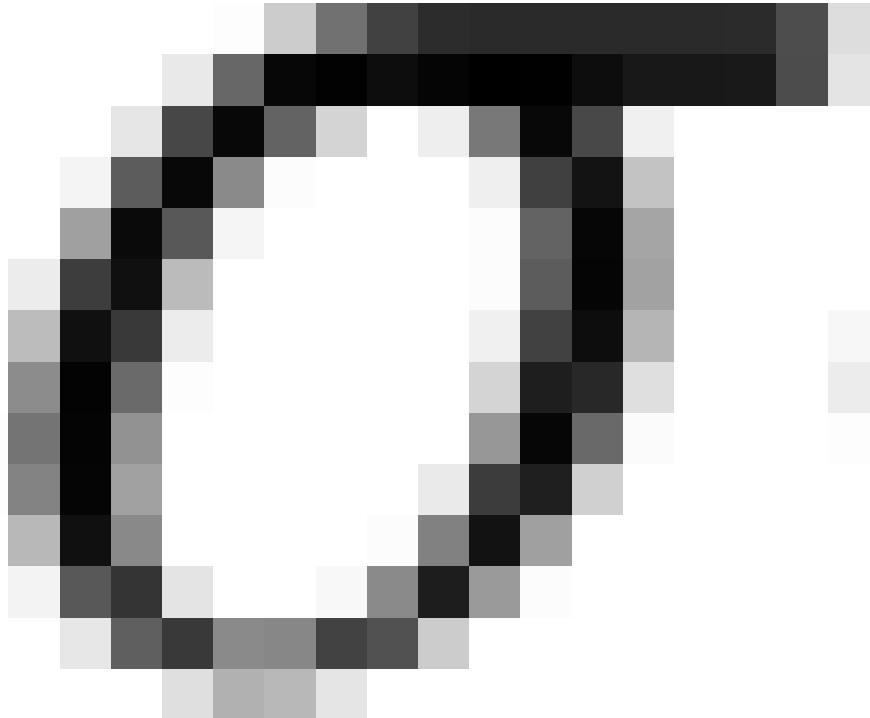




$$\left\{ \begin{array}{l} \sigma_{11} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{33} \\ \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{11} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{33} \\ \sigma_{33} = \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\epsilon_{22} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon_{33} \end{array} \right.$$

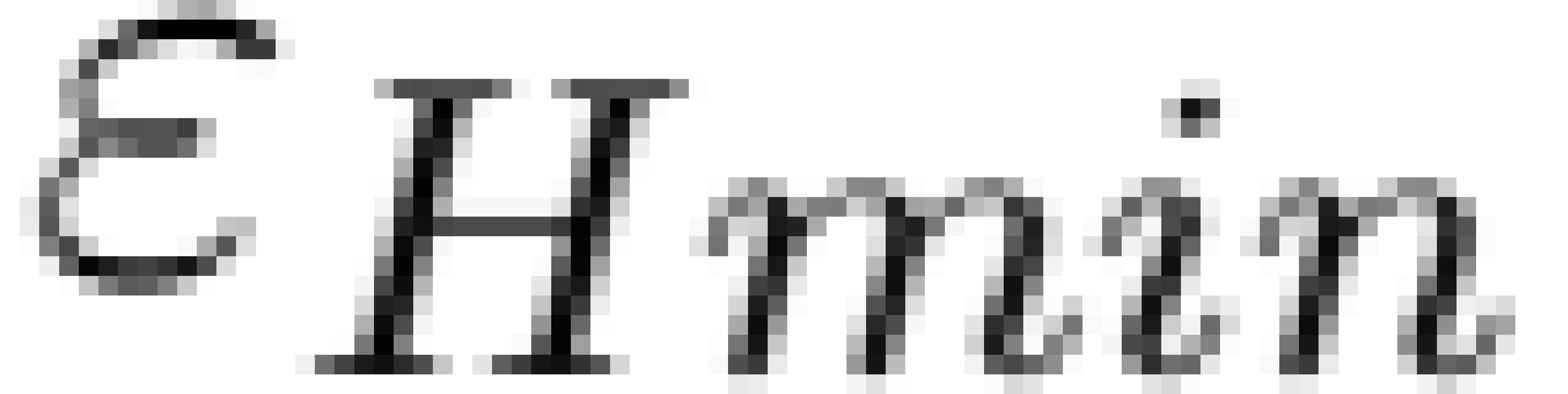


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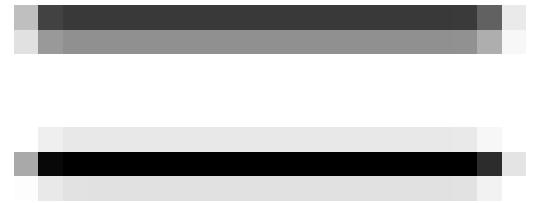
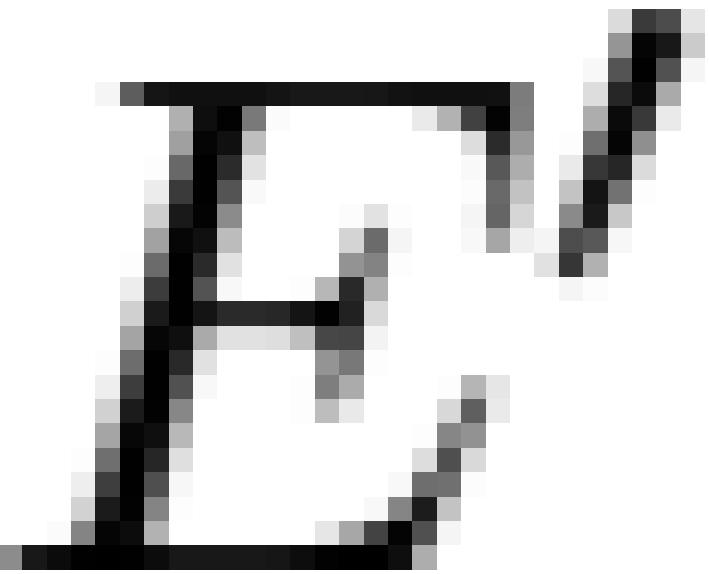


$$\left\{ \begin{array}{l} \sigma_{11} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{E}{1-\nu^2} \epsilon_{11} + \frac{\nu E}{1-\nu^2} \epsilon_{22} \\ \sigma_{22} = \frac{\nu}{1-\nu} \sigma_{33} \frac{\nu E}{1-\nu^2} \epsilon_{11} + \frac{E}{1-\nu^2} \epsilon_{22} \end{array} \right.$$

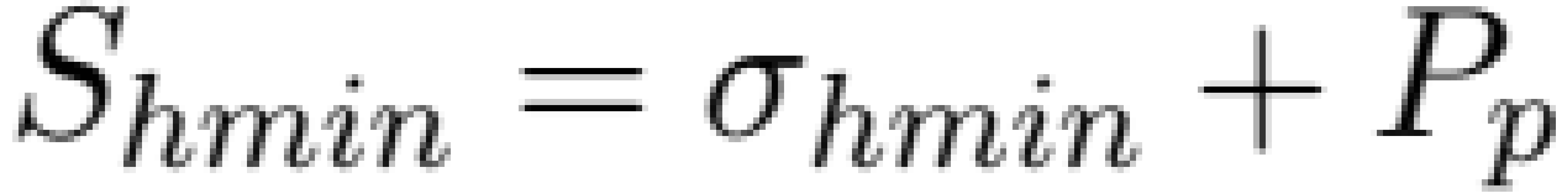




$$\left\{ \begin{array}{l} \sigma_{H\max} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{H\max} + \nu E' \epsilon_{hmin} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{H\max} + \nu E' \epsilon_{hmin} \end{array} \right.$$

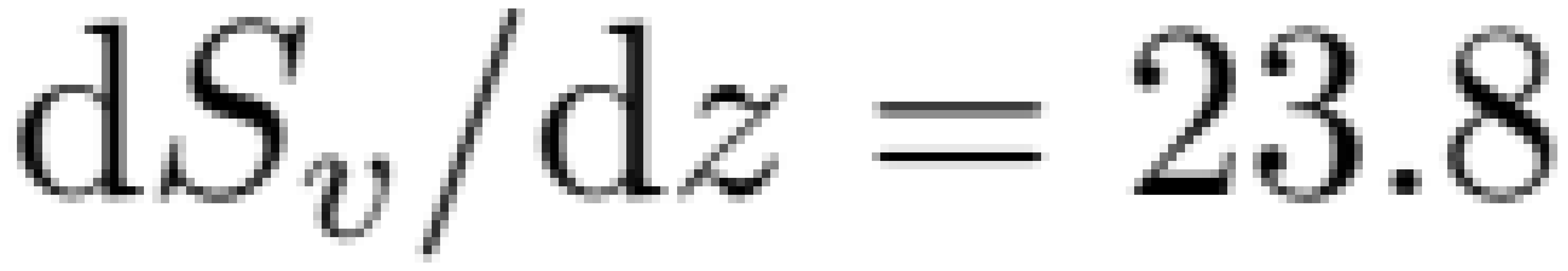


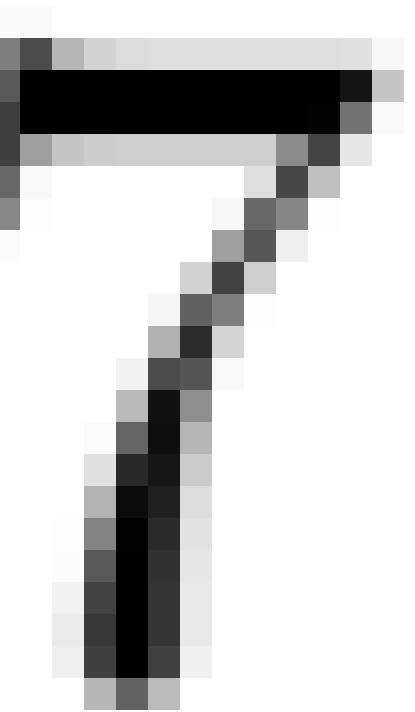
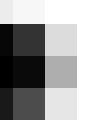
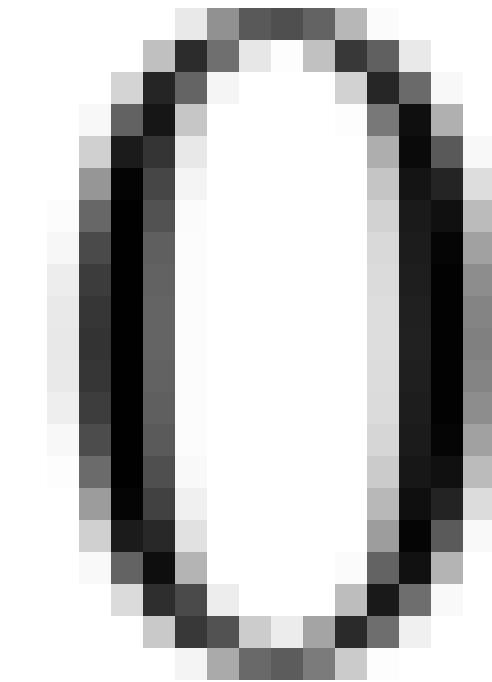


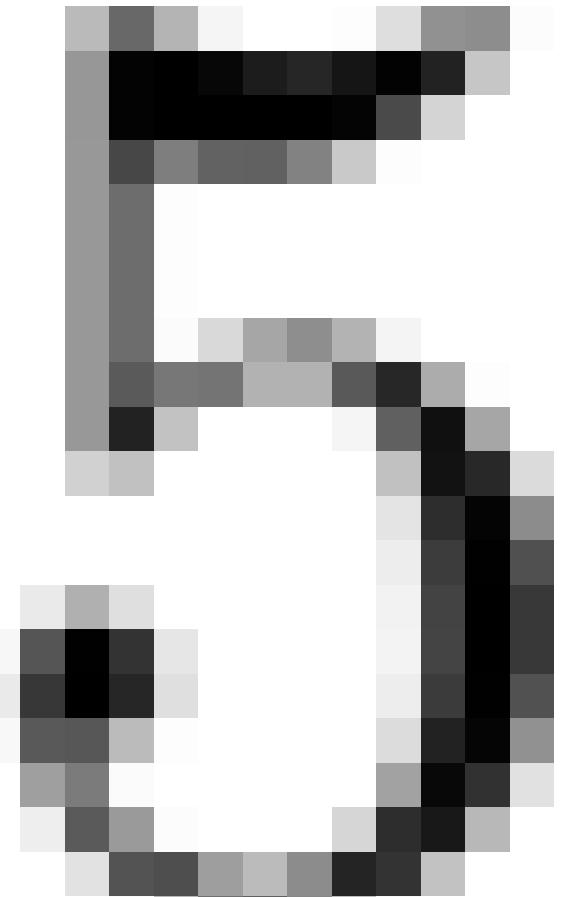
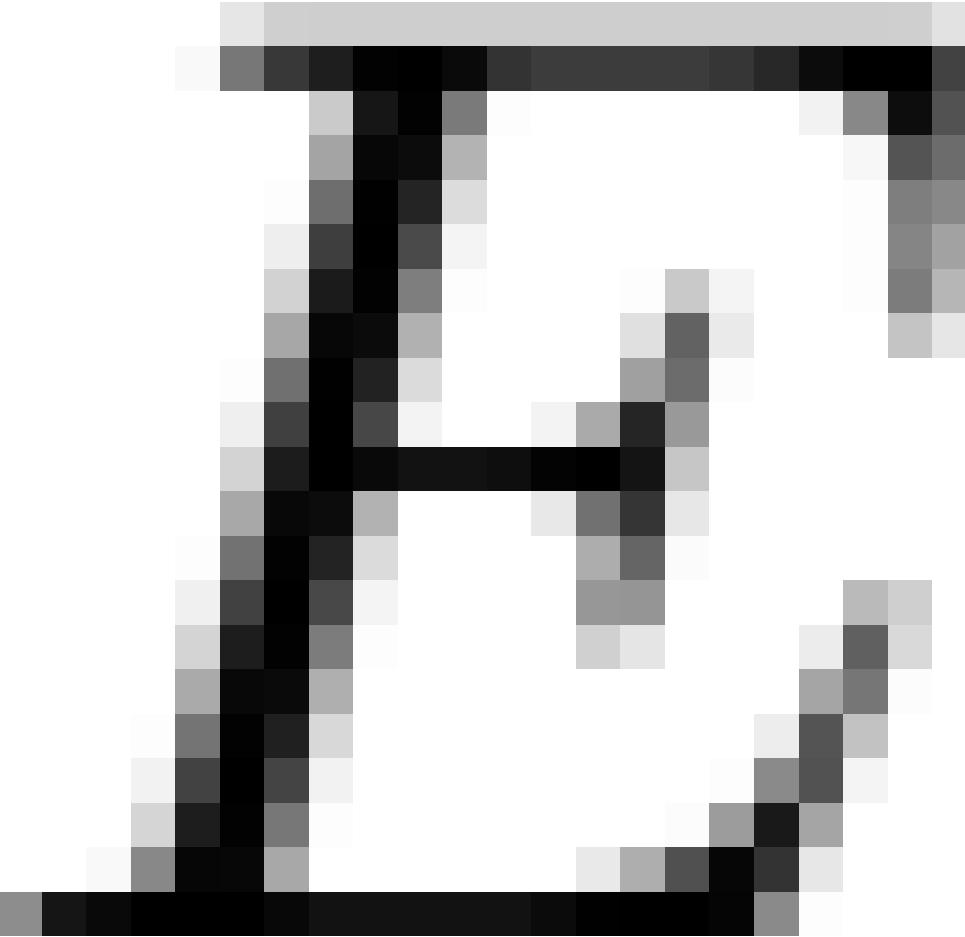


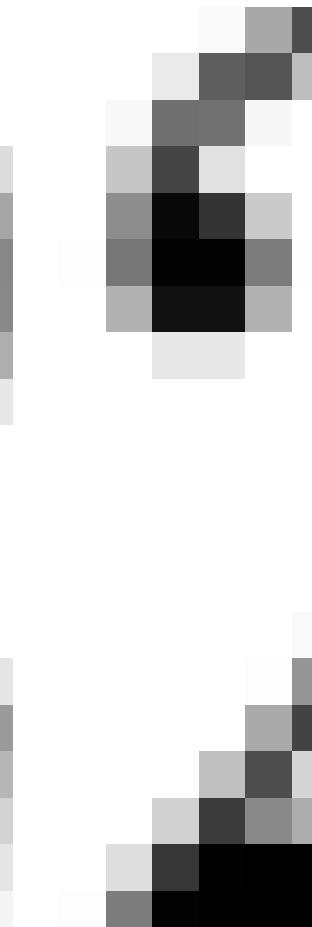
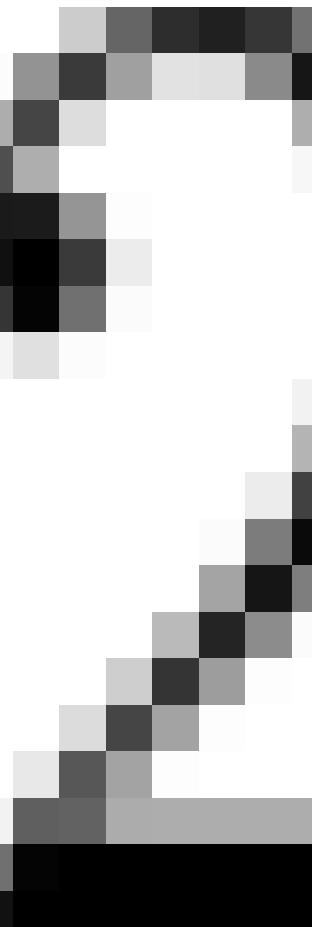
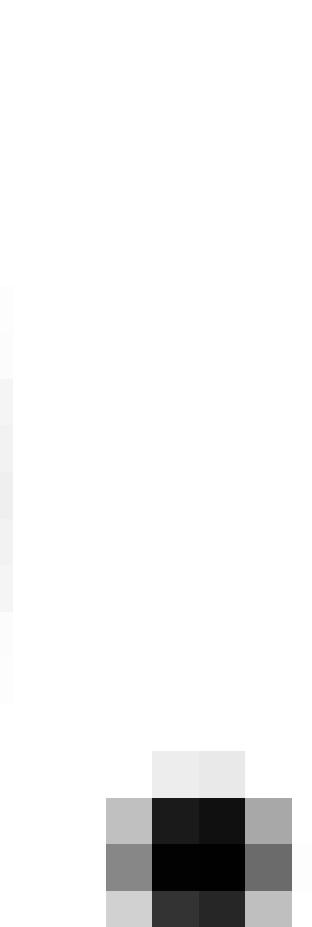
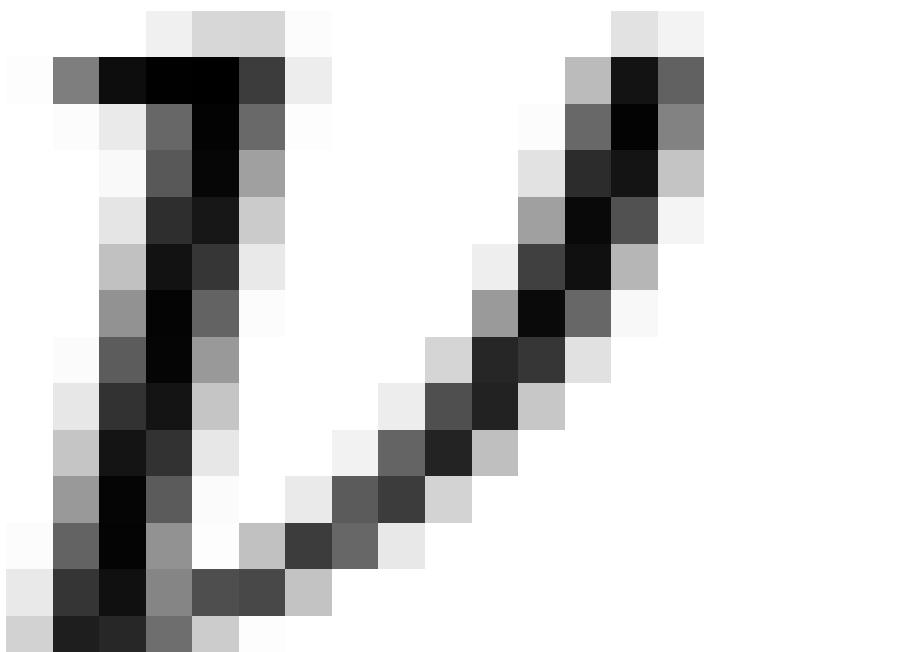


|             | $(E, v)$                     | $(K, G)$  |
|-------------|------------------------------|---|
| $G =$       | $\frac{E}{2(1+v)}$           | <u>Shear modulus</u><br>(also noted as $\mu$ , S-wave)                  |
| $M =$       | $\frac{(1-v)E}{(1+v)(1-2v)}$ | <u>Constrained modulus</u><br>(uniaxial compaction, P-wave)             |
| $\lambda =$ | $\frac{vE}{(1+v)(1-2v)}$     | <u>Lamé first parameter</u><br>(volumetric strain component)            |
| $K =$       | $\frac{E}{3(1-2v)}$          | <u>Bulk modulus</u><br>(relates volumetric strain and isotropic stress) |





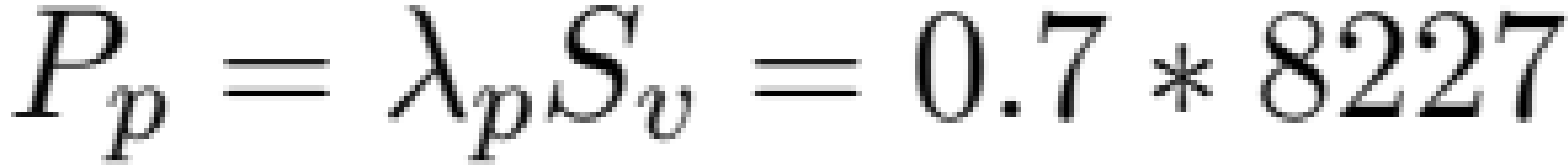




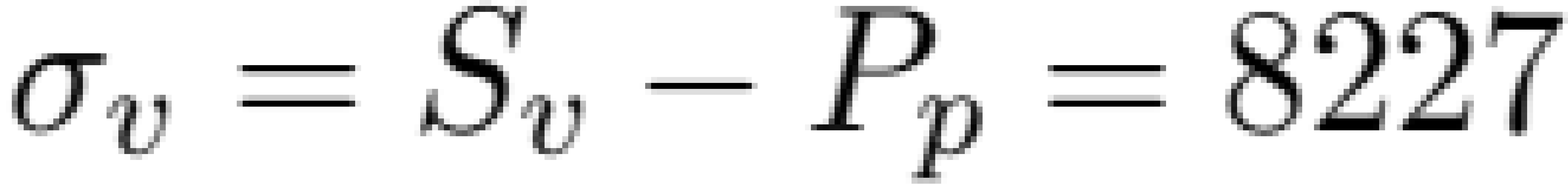




$$S_v = 23.8 \frac{\text{MPa}}{\text{km}} \cdot \frac{1 \frac{\text{psi}}{\text{ft}}}{\frac{\text{MPa}}{\text{km}}} \cdot 7950 \text{ ft} = 8227 \text{ psi}$$









$$\frac{E'}{1 - \nu^2} = \frac{E}{1 - 0.22^2} = \frac{5 \times 10^6 \text{ psi}}{5.25 \times 10^6 \text{ psi}}$$

$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} * 0.0002 = 1745 \text{ psi} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{hmin} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 0.22 * 5.25 \times 10^6 \text{ psi} * 0.0002 = 927 \text{ psi} \end{array} \right.$$

$$S_{H\max} = \sigma_{H\max} + P_p = 1745 \text{ psi} + 5759 \text{ psi} = 7504 \text{ psi}$$

$$S_{h\min} = \sigma_{h\min} + P_p = 927 \text{ psi} + 5759 \text{ psi} = 6686 \text{ psi}$$

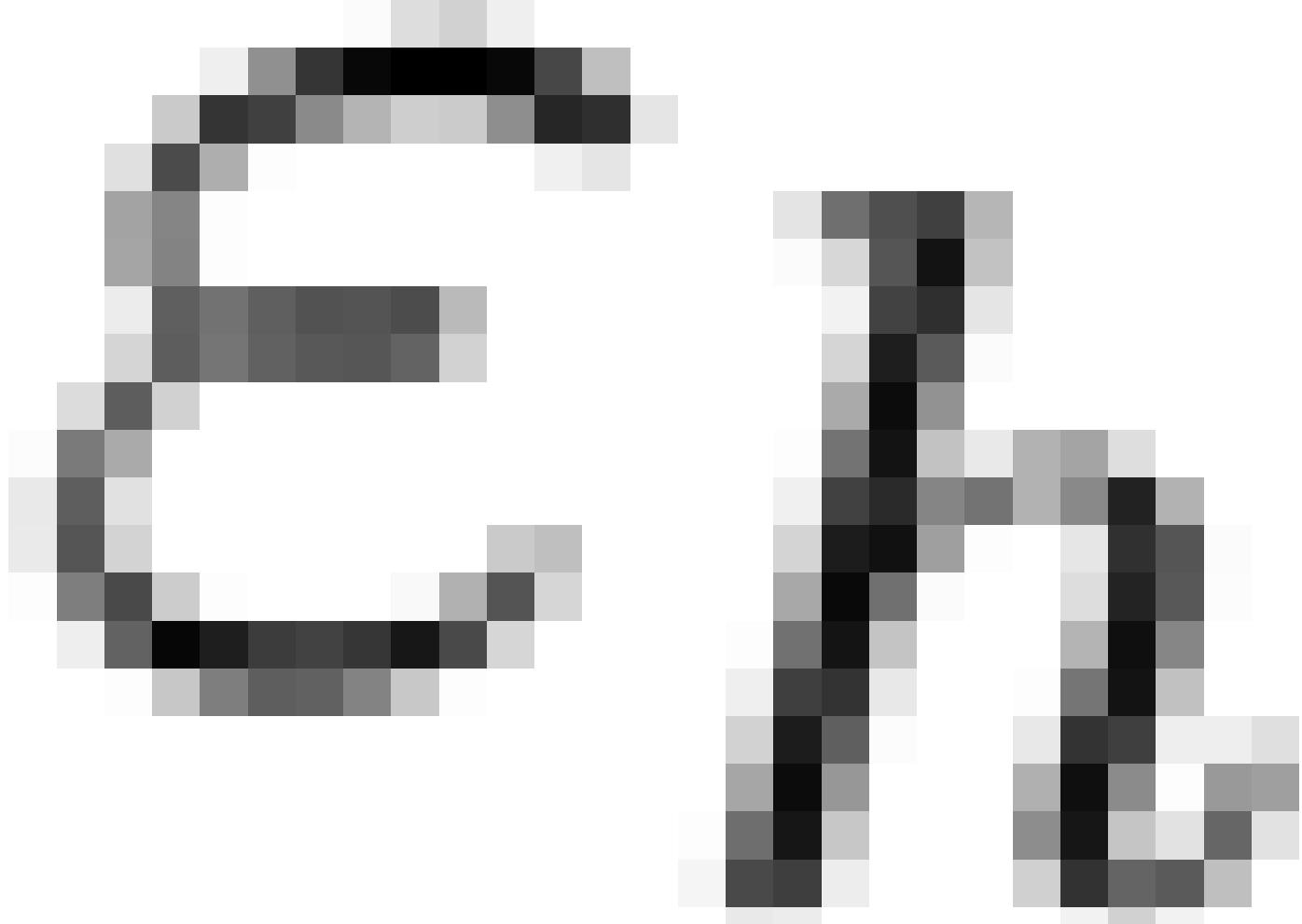




$$C_{pp} =$$

$$\frac{1}{V_p} \frac{dV_p}{dP_p}$$

$S_u, \epsilon_h$



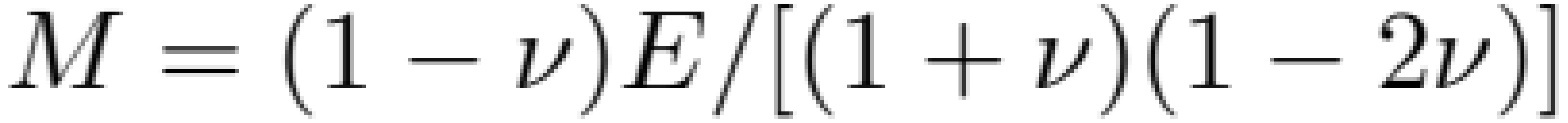
$\text{Op}$

$=$

$\text{Op}$

$\phi$



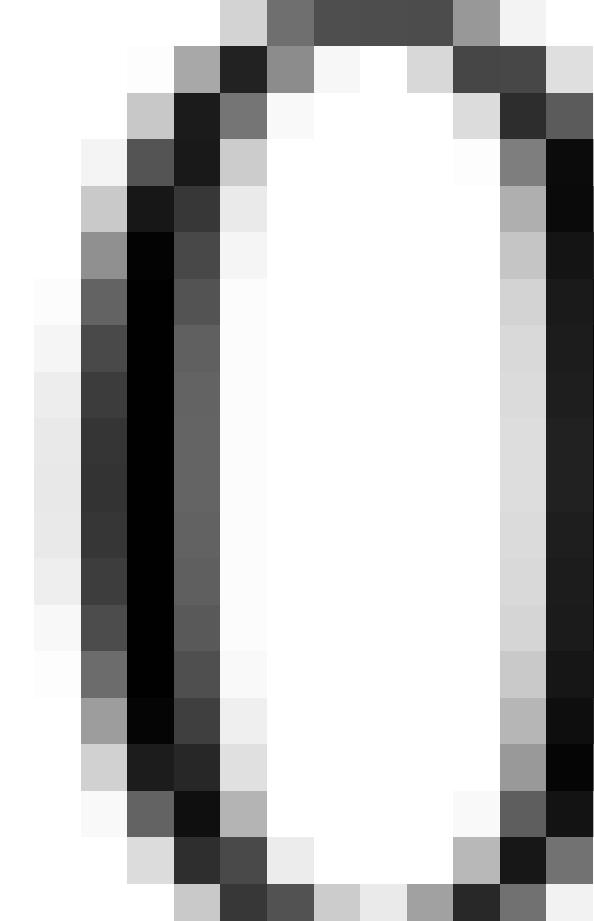
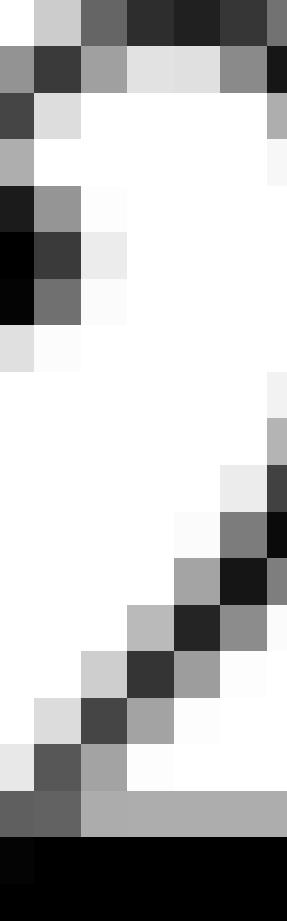
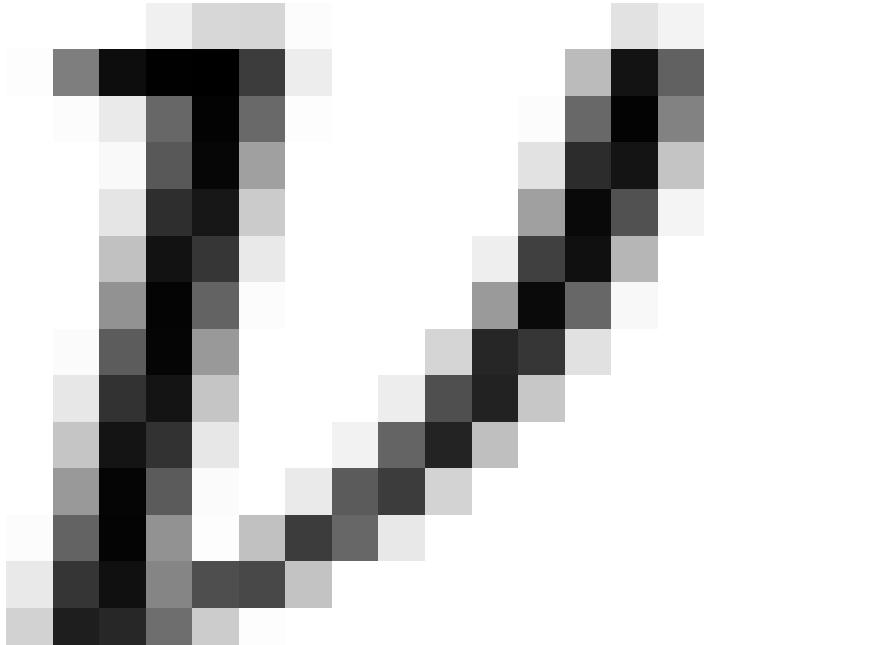


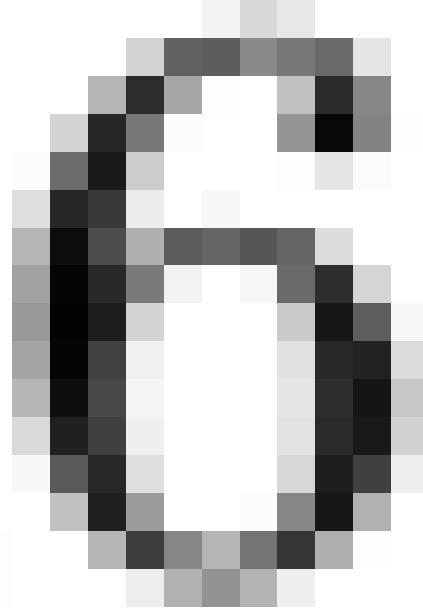
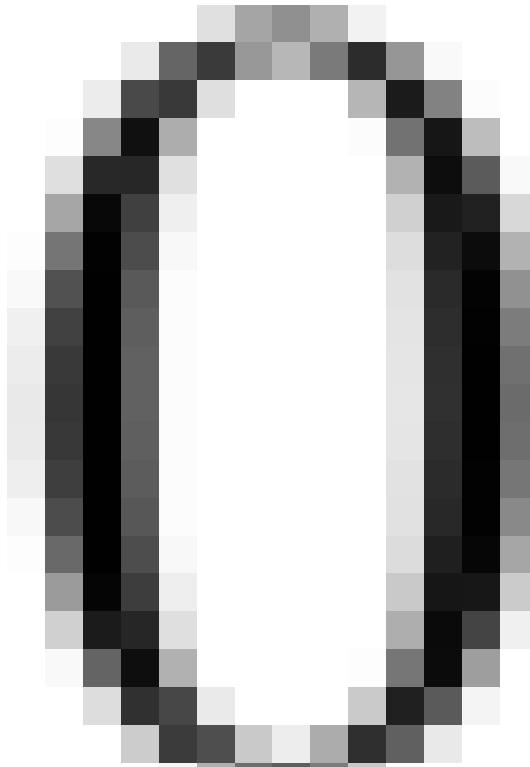
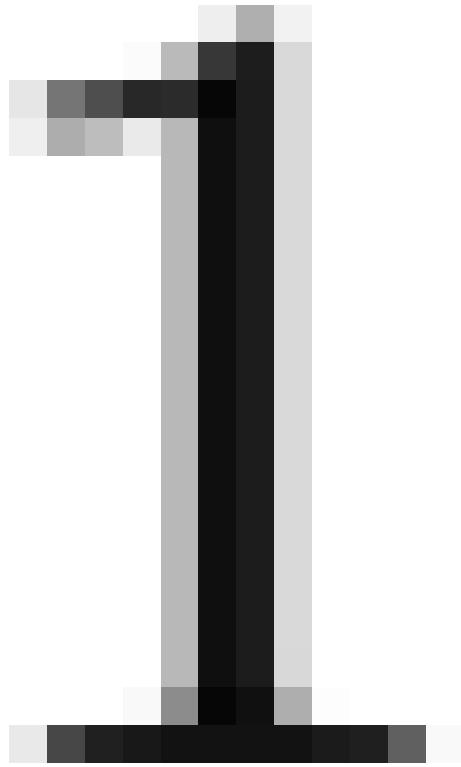
$\langle pp \rangle$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\frac{(1 + \nu)(1 - \nu)}{2\nu} E^\phi$$

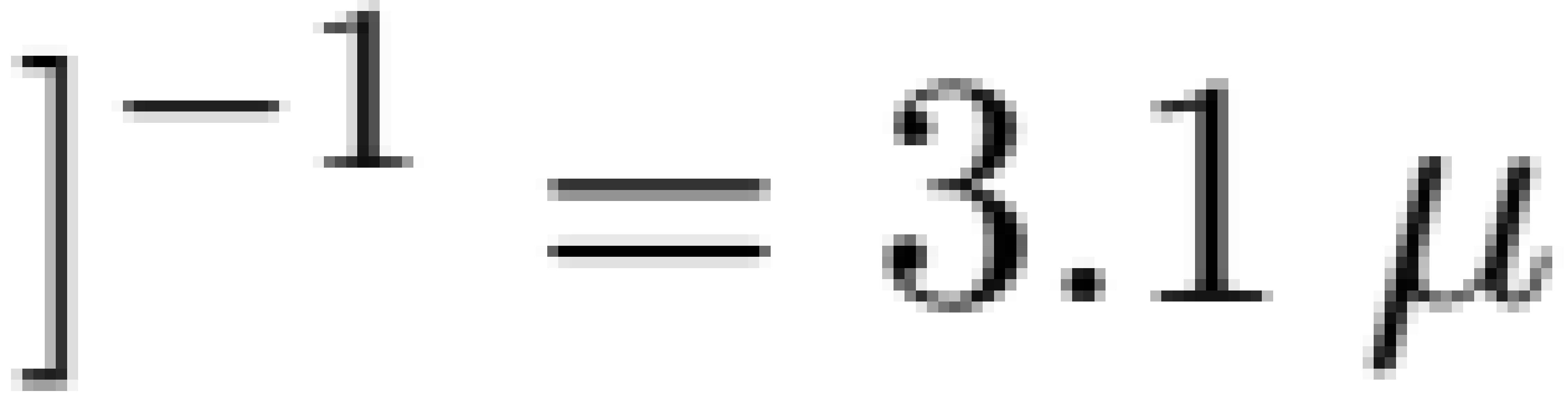


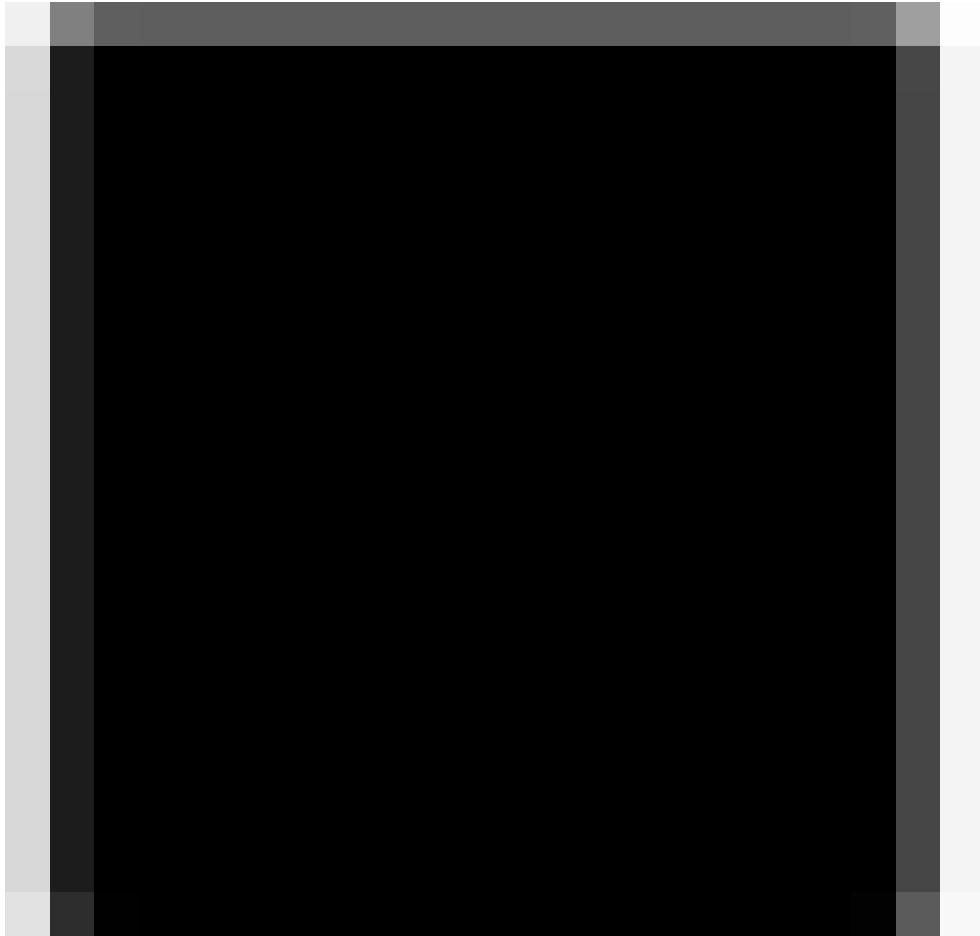


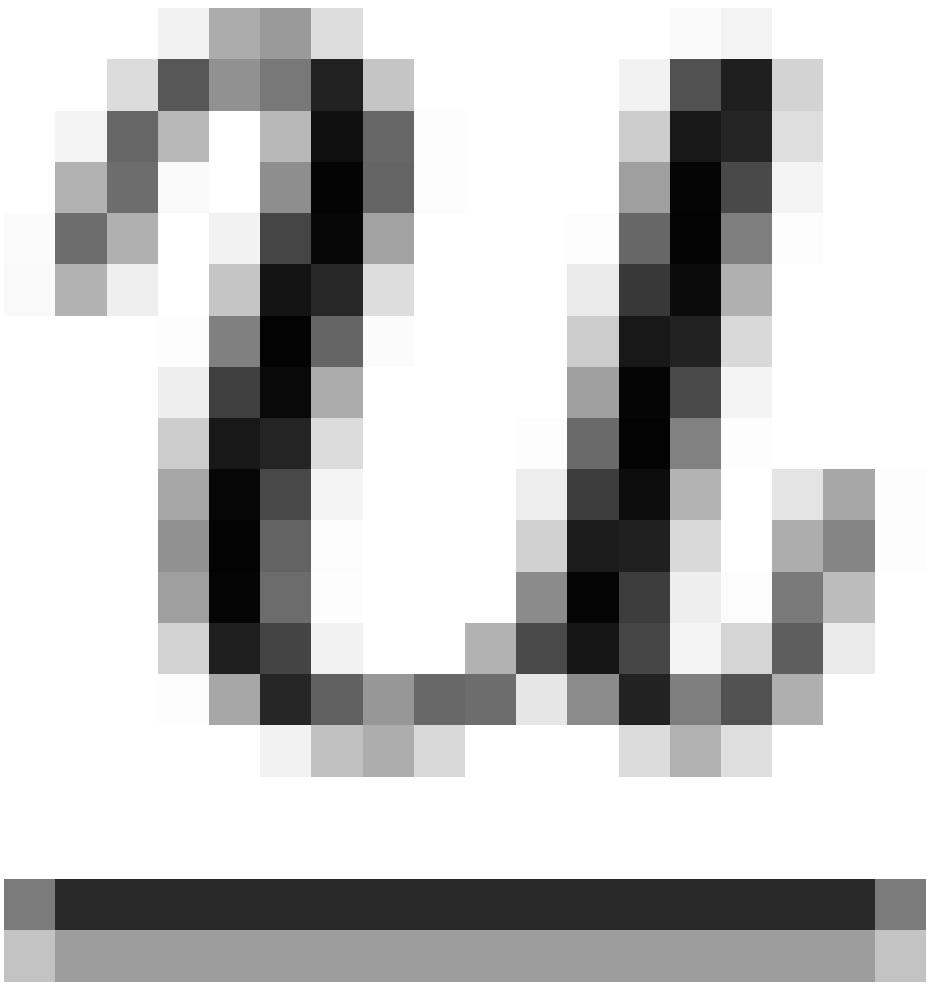


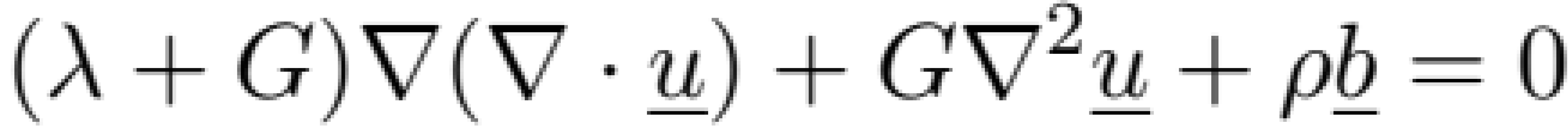
$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.20)10 \text{ GPa}}{1.6 \times 10^6 \text{ psi}} = \frac{11.11 \text{ GPa}}{(1+0.20)(1-2 \times 0.20)}$$

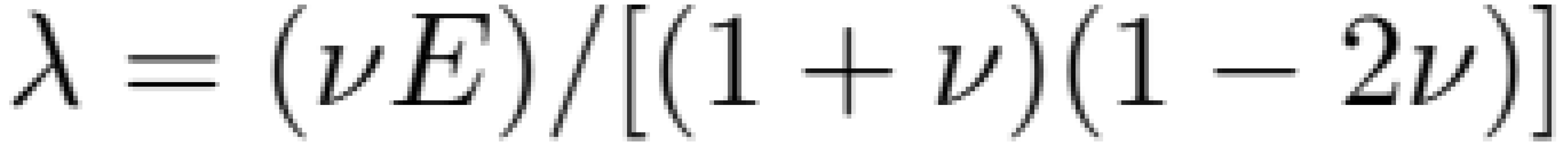
$$\frac{C_{pp}}{M_\phi} = \frac{1}{3.1 \times 10^6 \text{ psi} \times 0.20} = \frac{1}{1.6 \times 10^6}$$

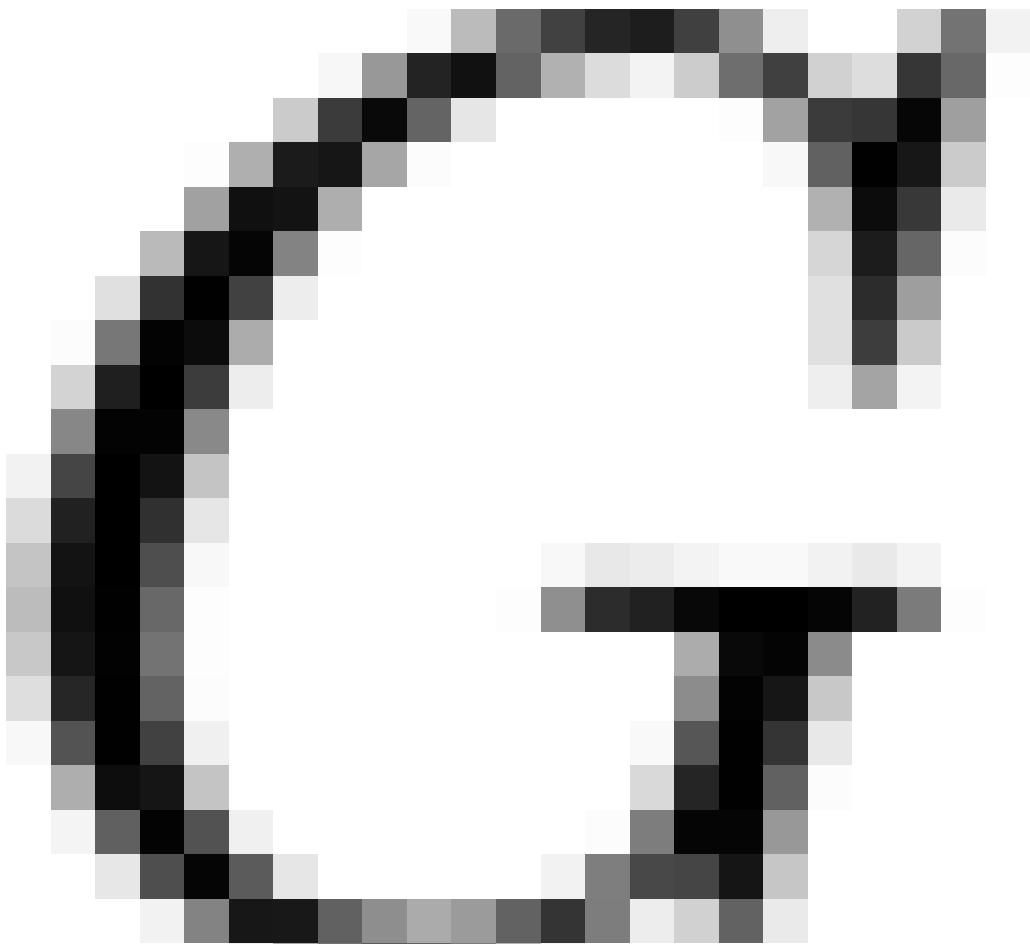


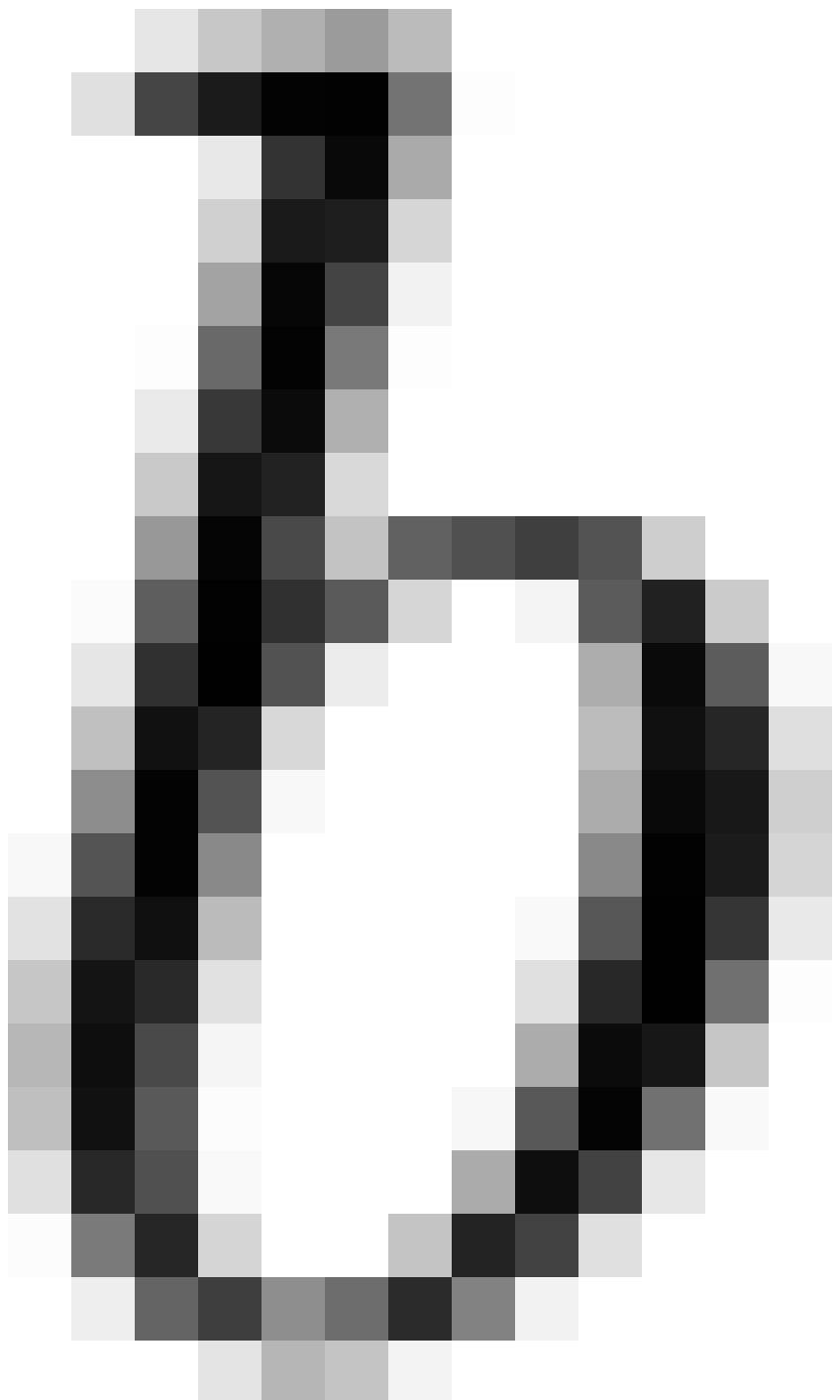




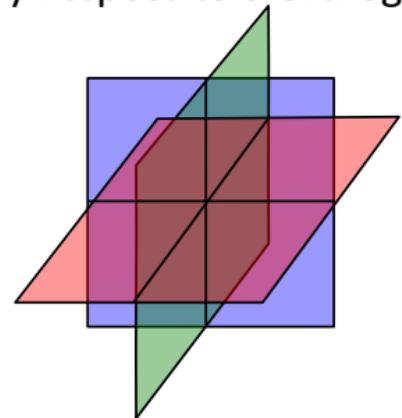








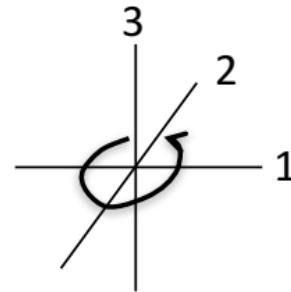
Orthorhombic symmetry  
(symmetry respect to 3 orthogonal planes)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & 0 \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{55} & 0 \\ & 0 & 0 & 0 & & C_{66} \end{bmatrix}$$

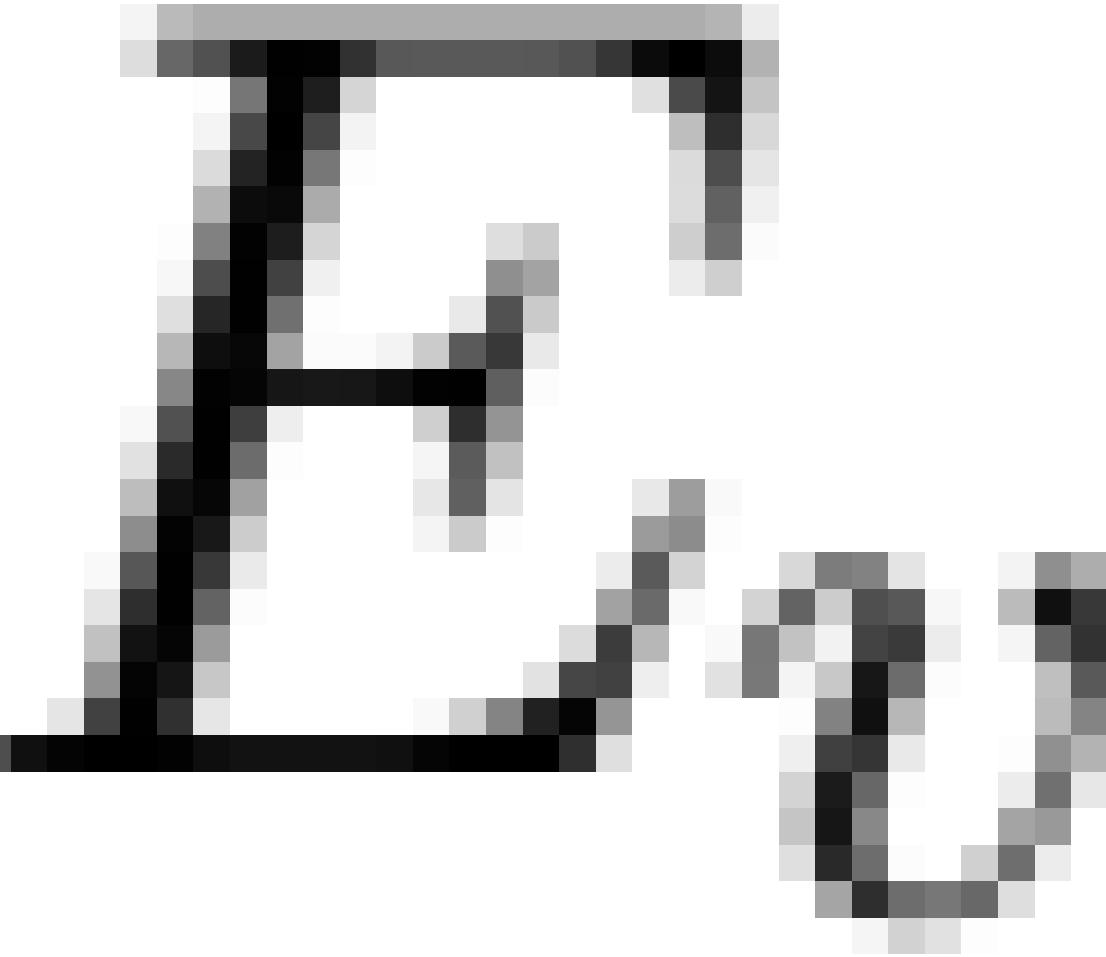
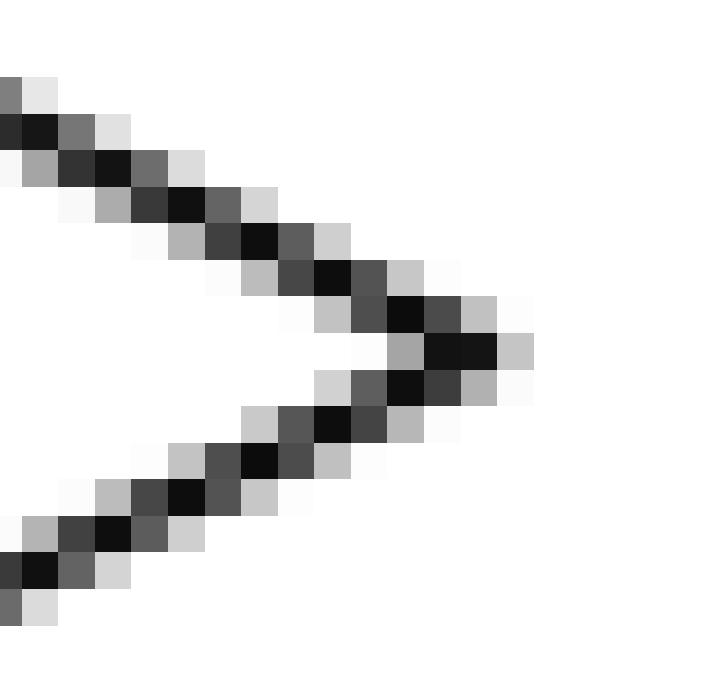
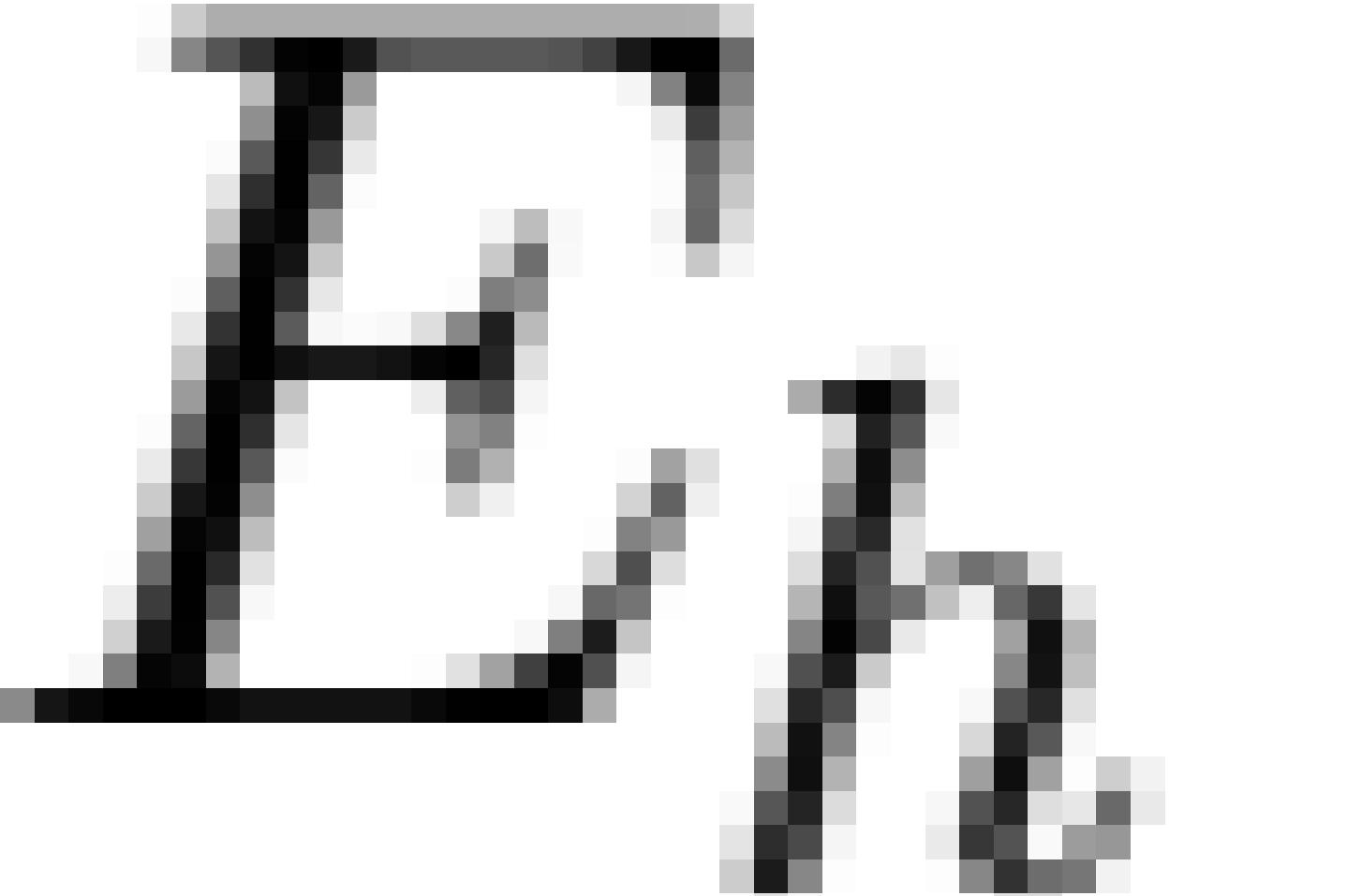
9 independent parameters

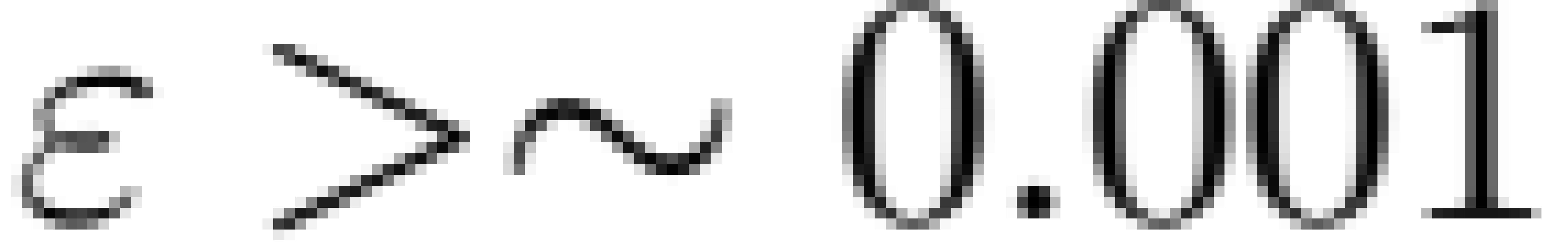
Transverse Isotropy  
(symmetry respect to 1 axis)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & & 0 \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{44} & 0 \\ & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

5 independent parameters ( $C_{12}=C_{11}-2C_{66}$ )



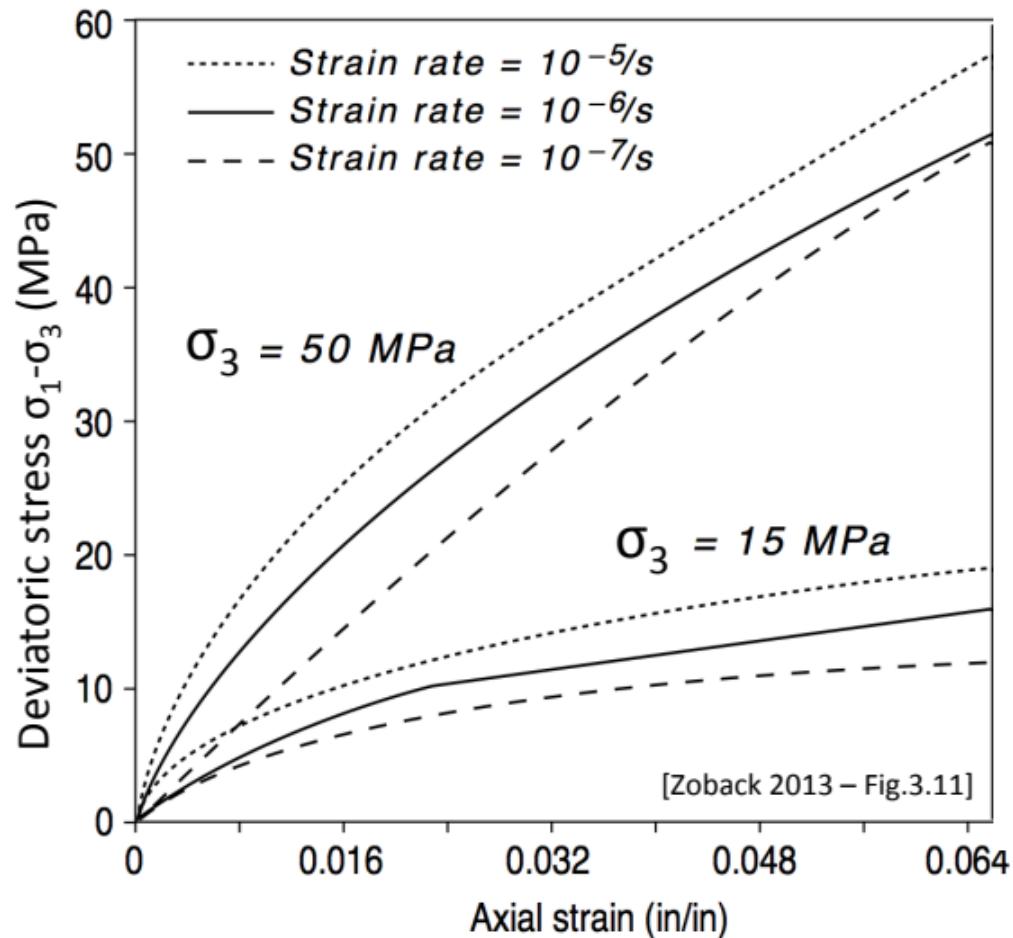
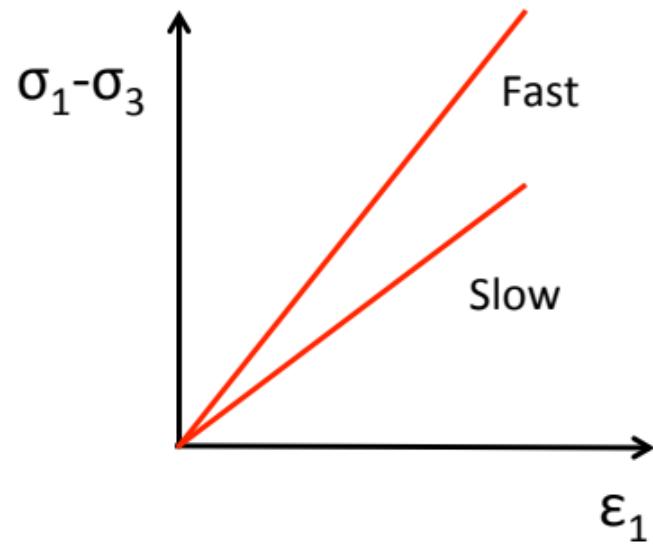




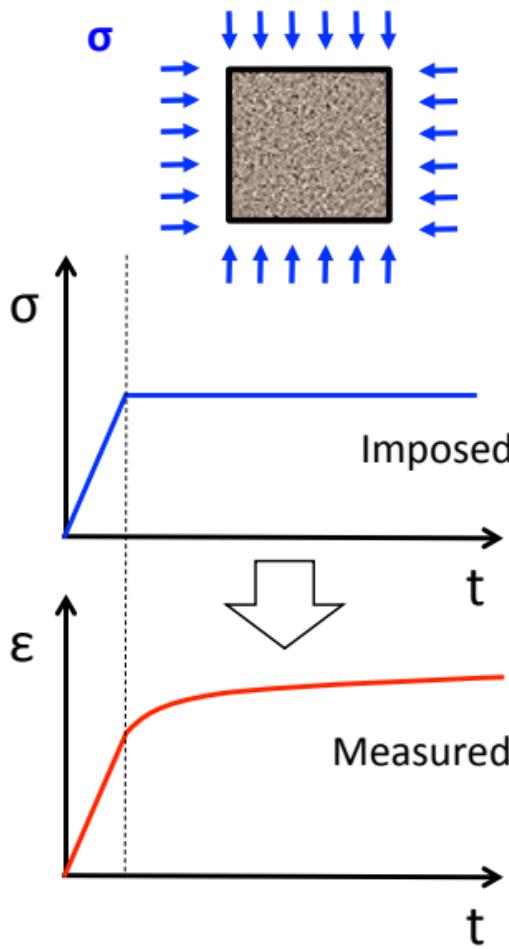




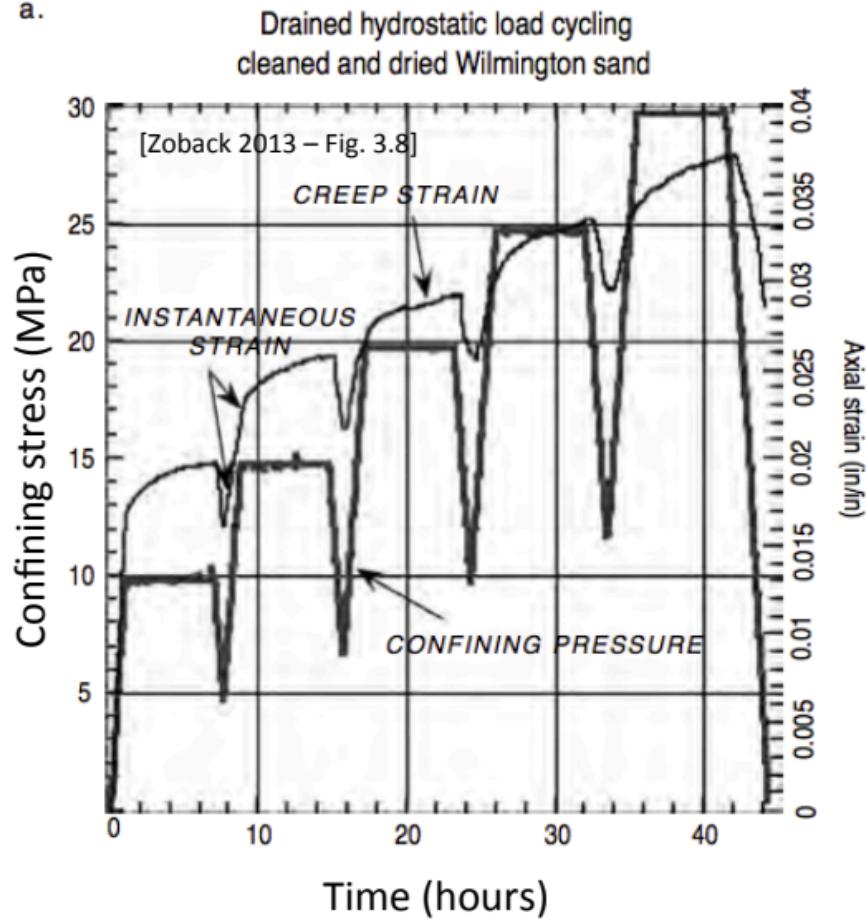
Strain rate hardening: The faster the loading, the stiffer the material



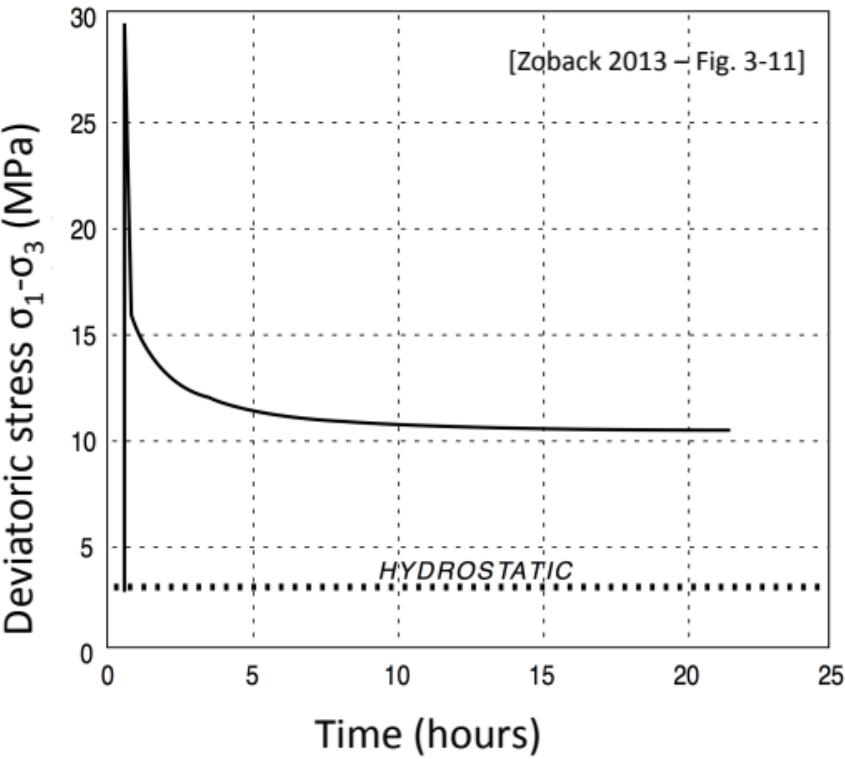
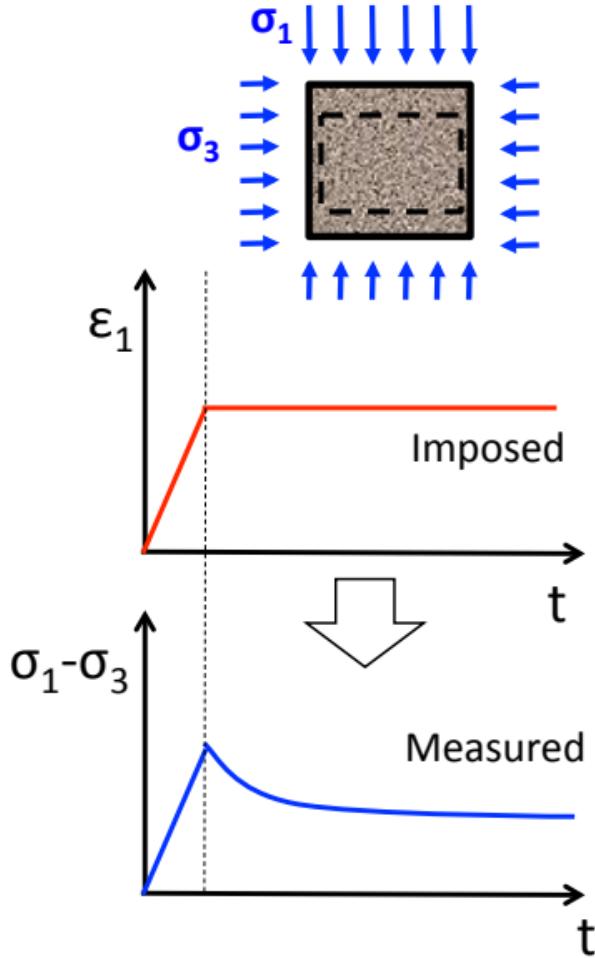
## Creep strain at constant stress

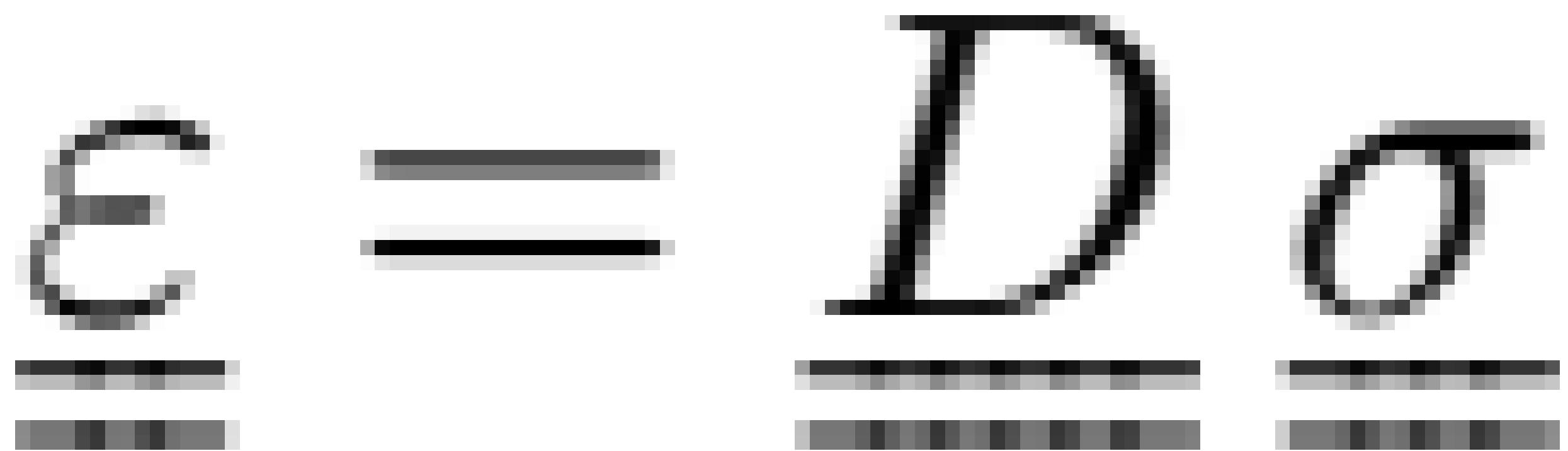


a.

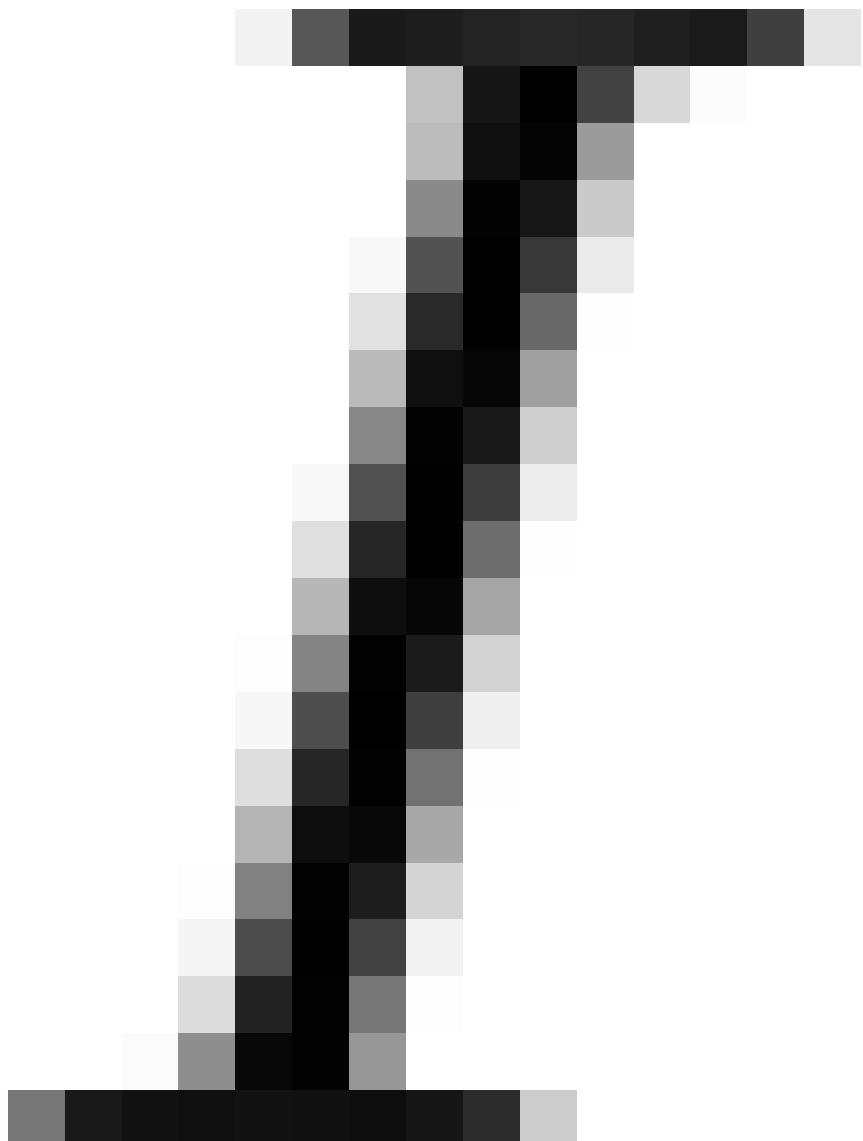


## (2) Stress relaxation at constant strain









$\alpha$

=

1

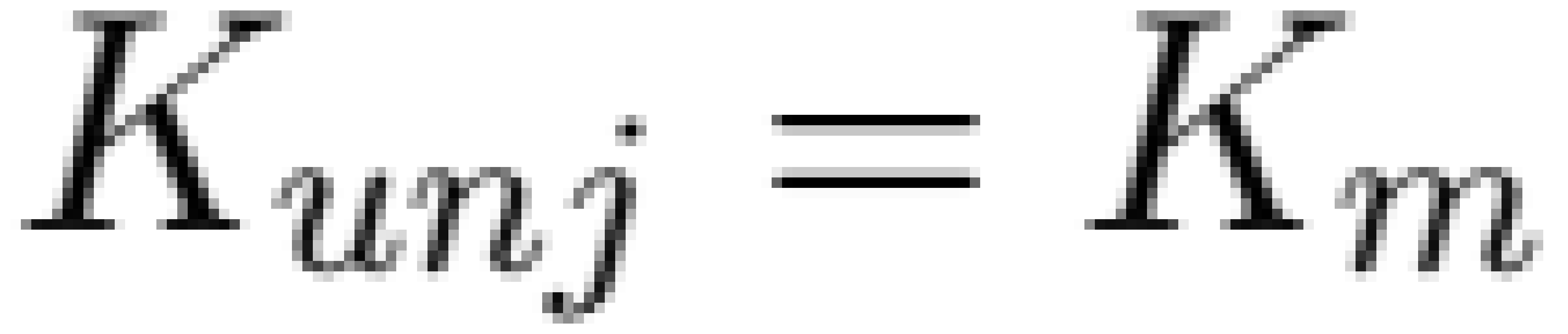
-

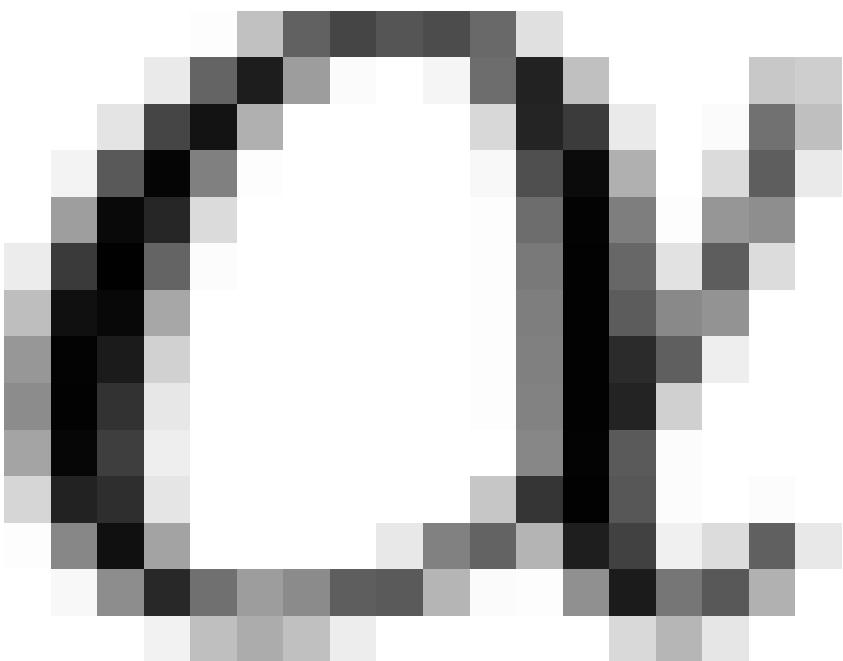
$K_{drained}$

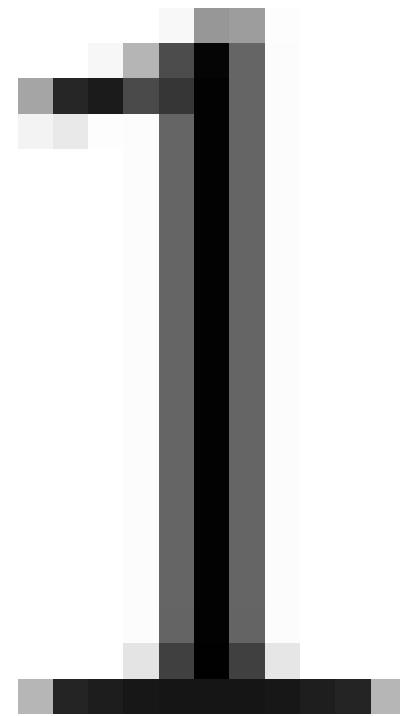
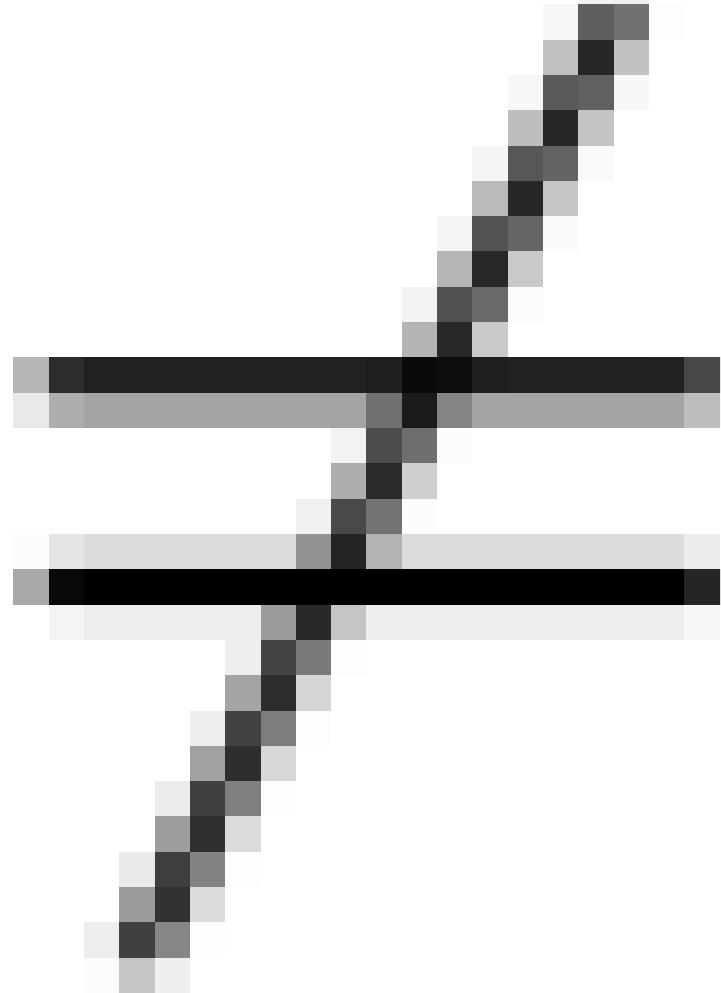
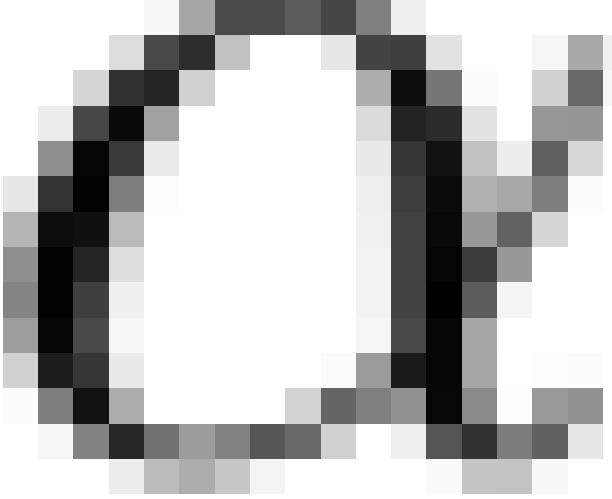
$K_{unq}$ .

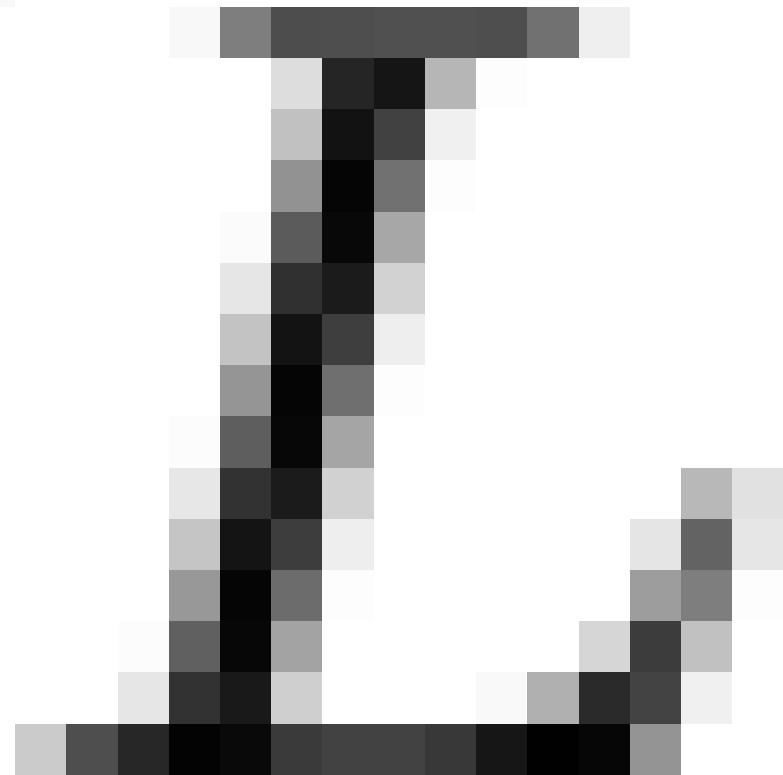
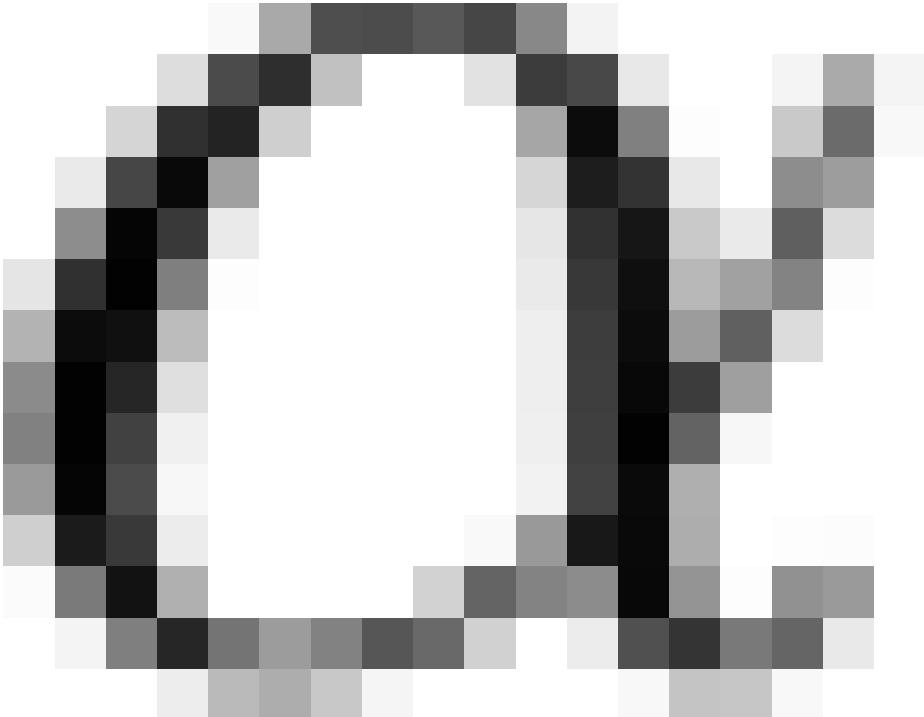


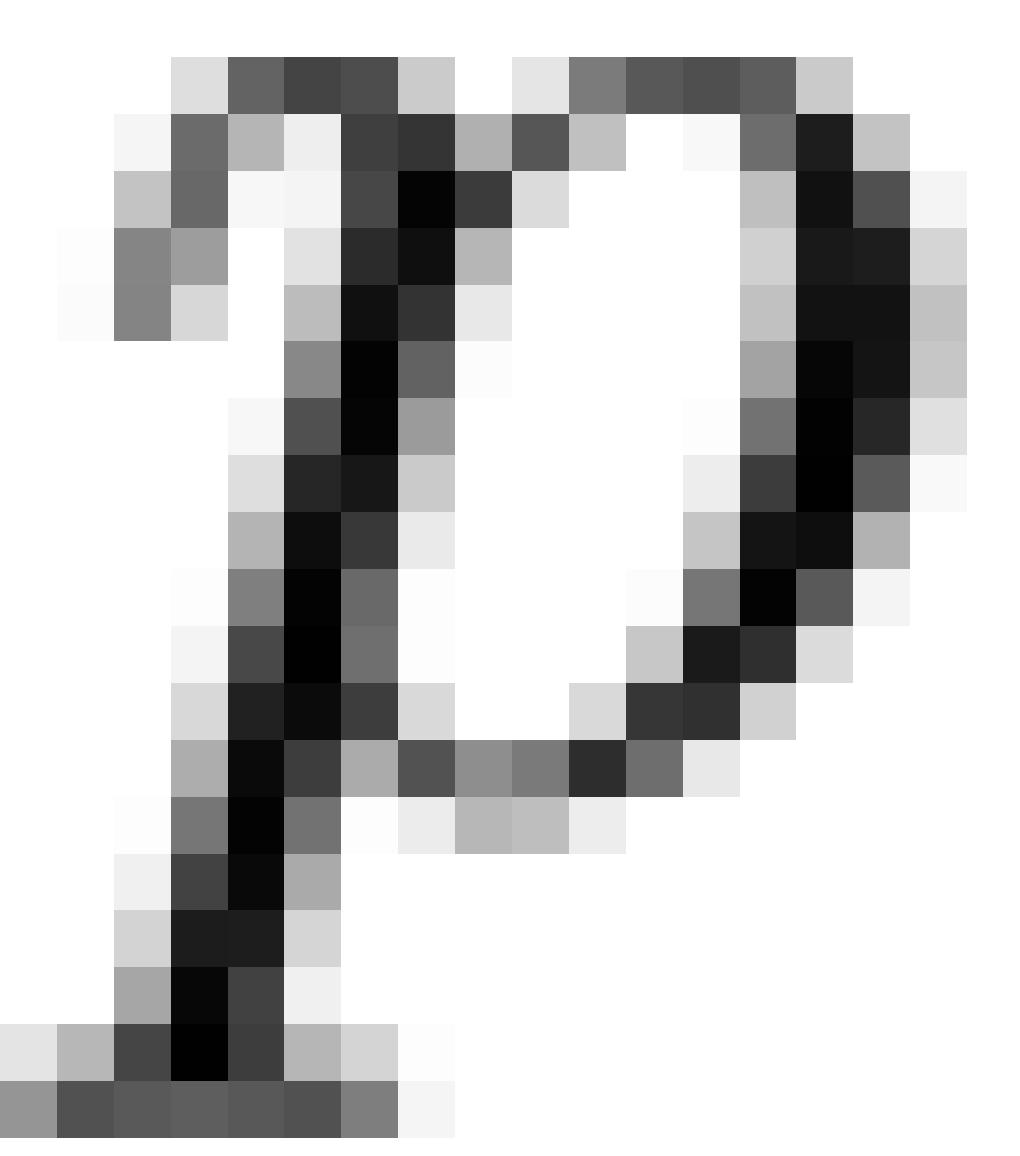




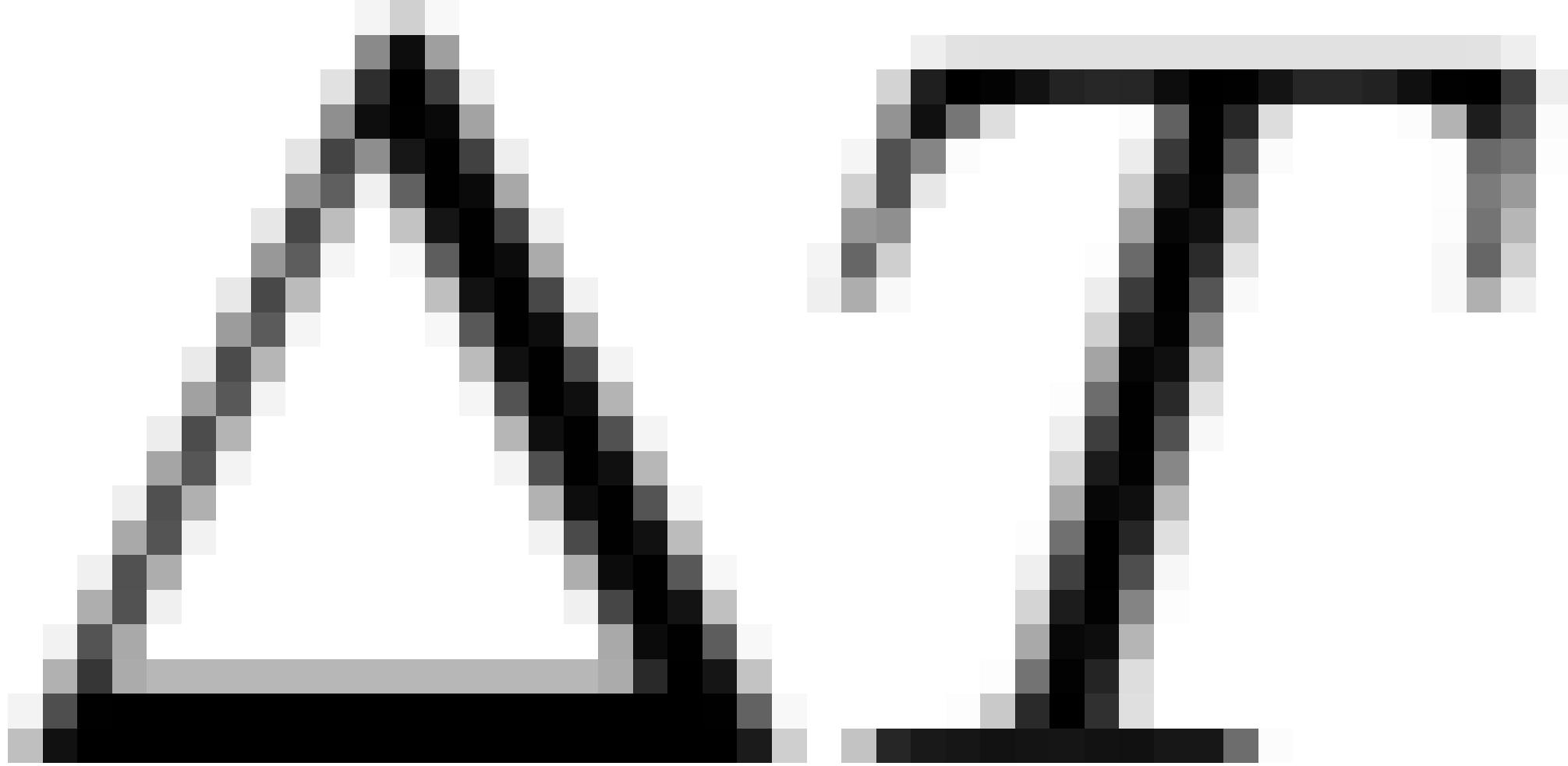




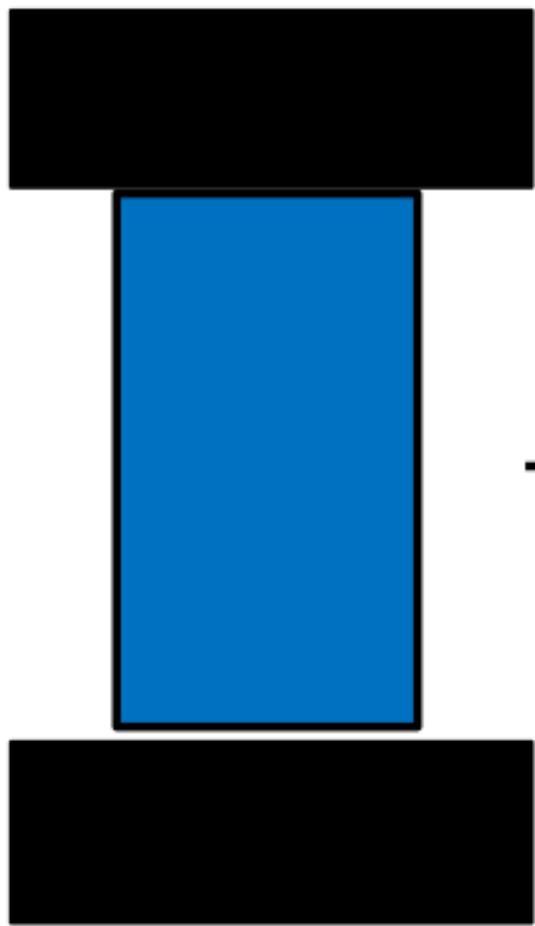




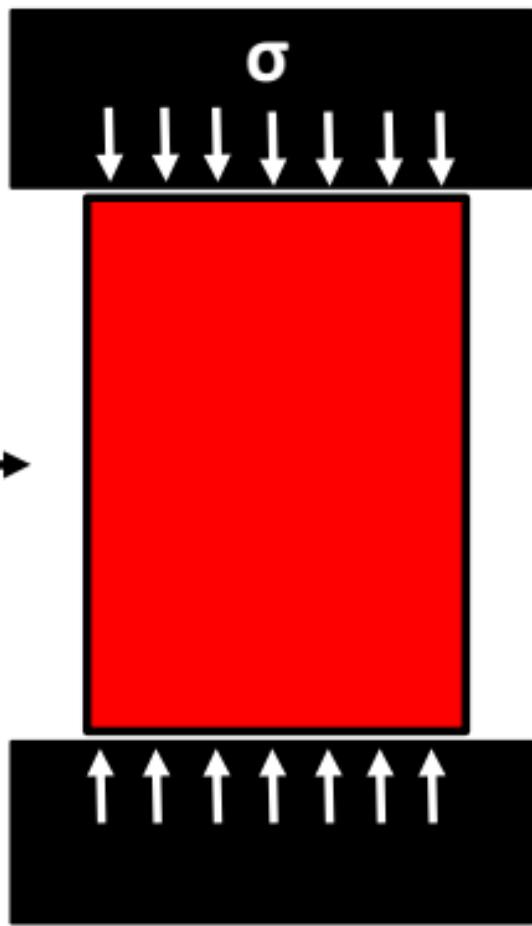
$$\alpha_L = \frac{1}{LDT} \frac{dL}{dp}$$



$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right.$$

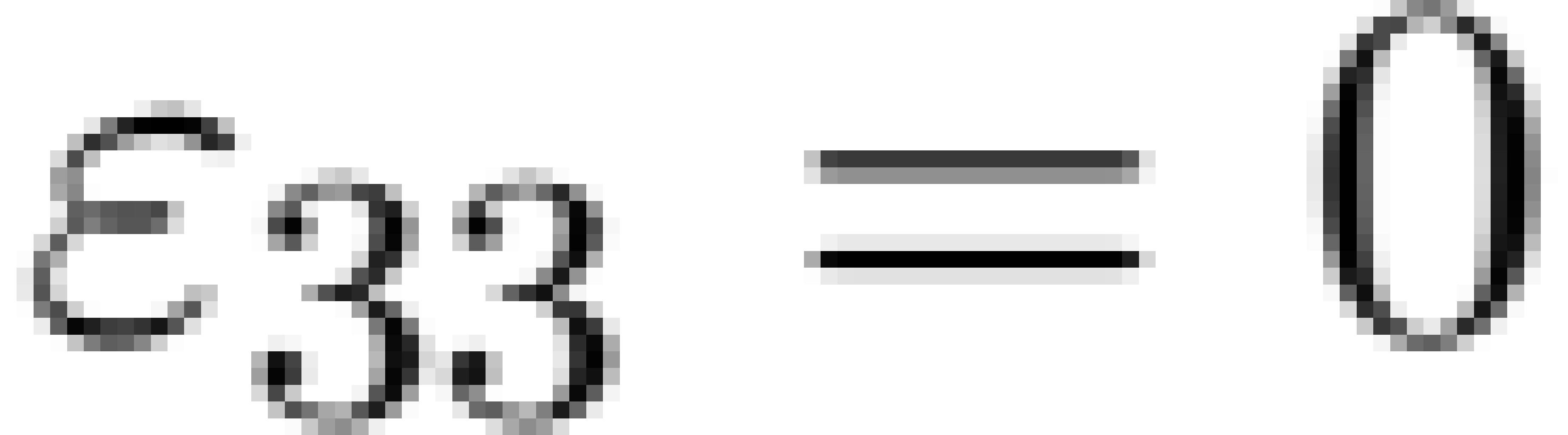


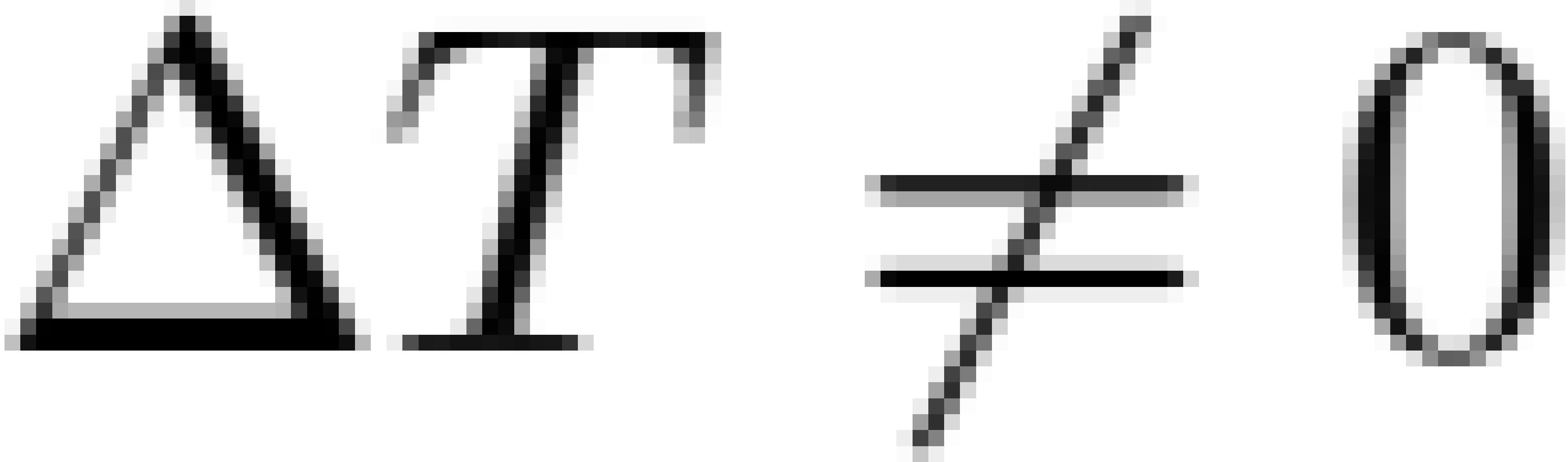
$\Delta T$  →











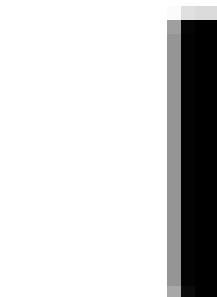
$$\begin{aligned} \sigma_{11}^0 &= (\lambda + 2\mu) \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \\ \sigma_{33} &= \lambda \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \end{aligned}$$

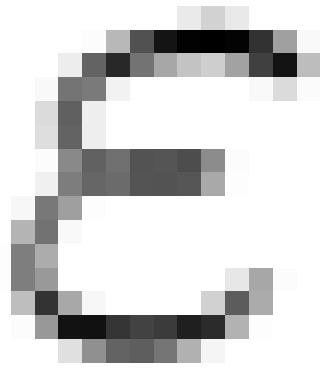
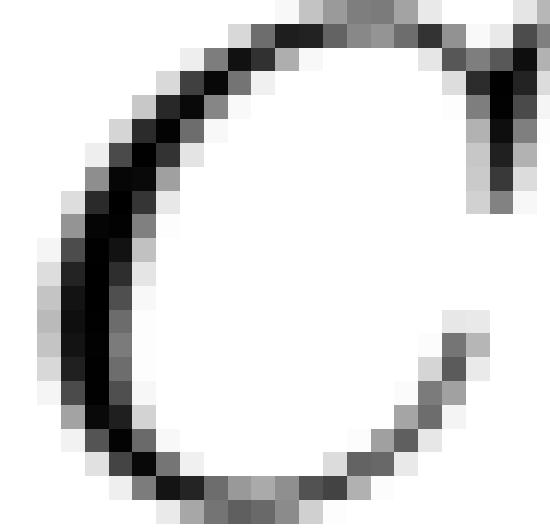
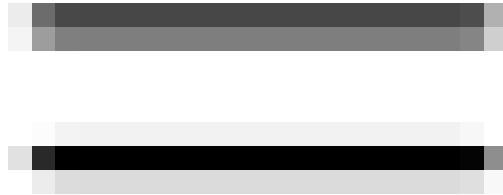
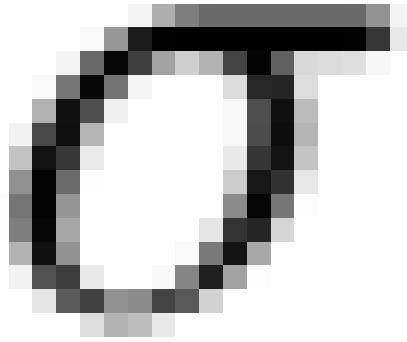
$\sigma_{33}$

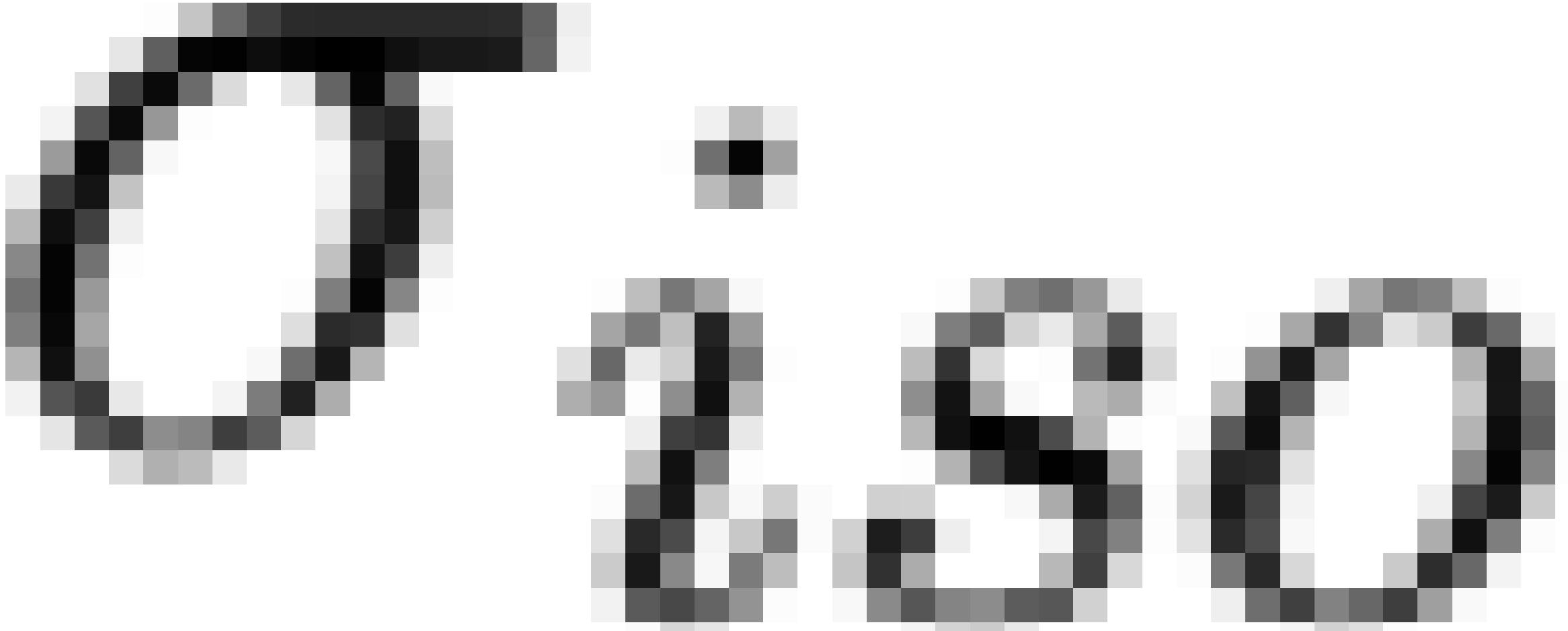
$=$

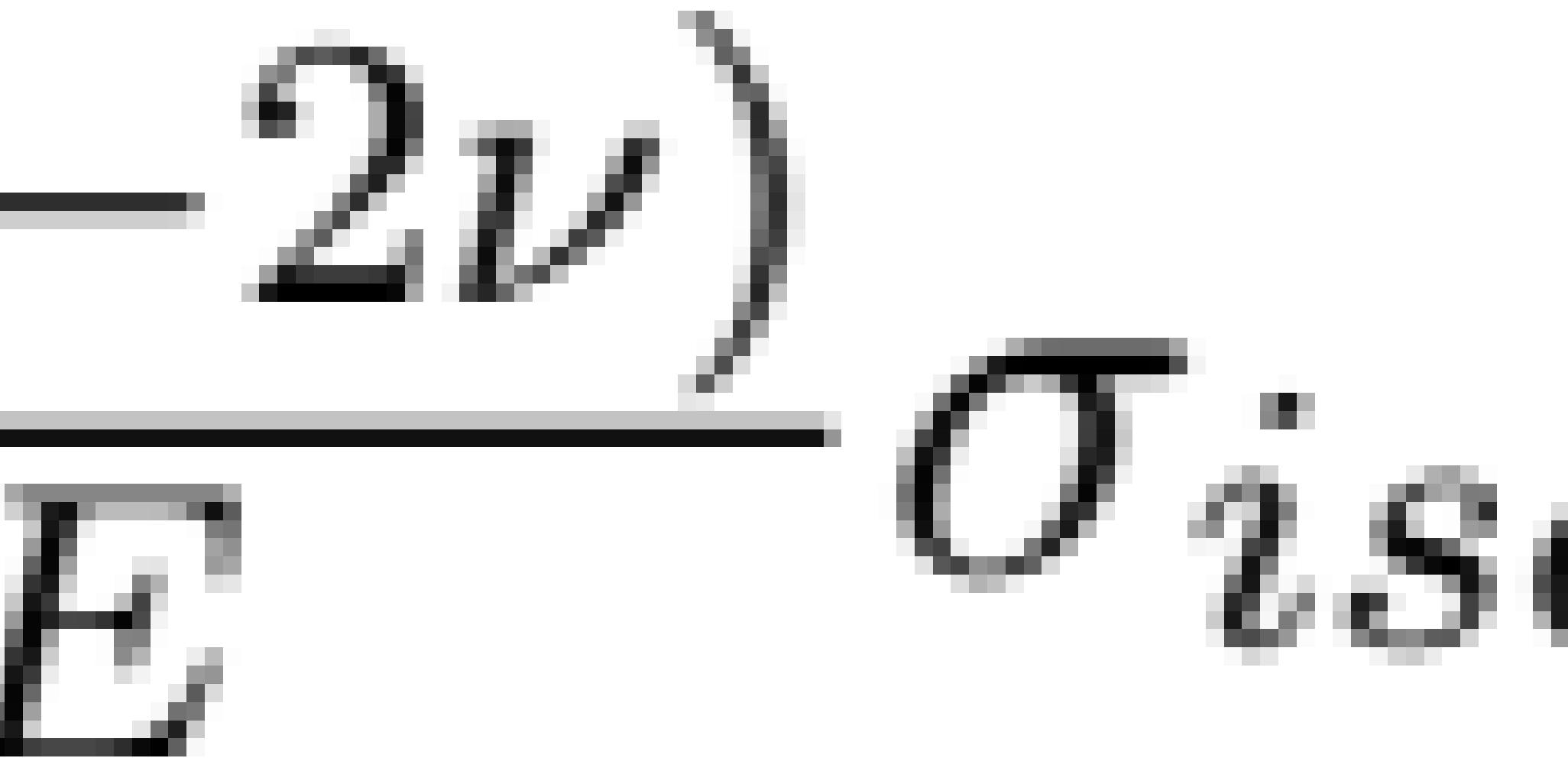
$$\frac{6}{3} \mu K$$

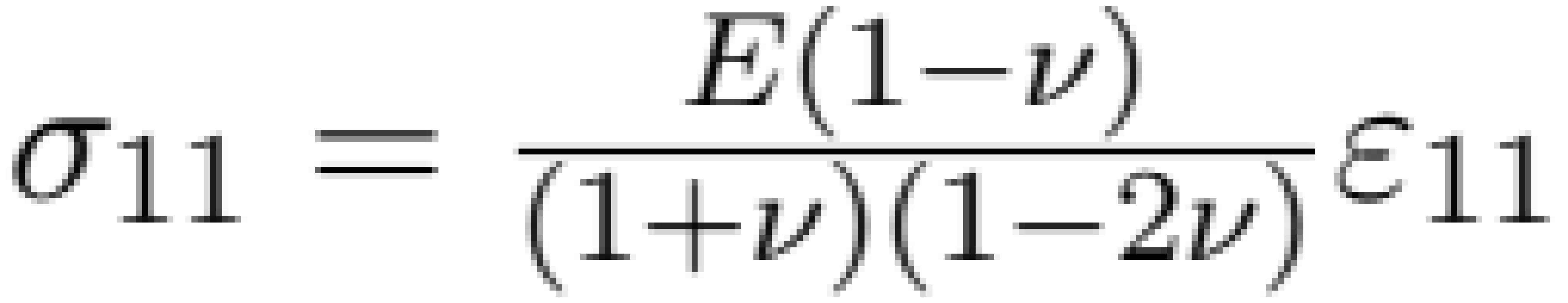
$K \alpha L \Delta T$













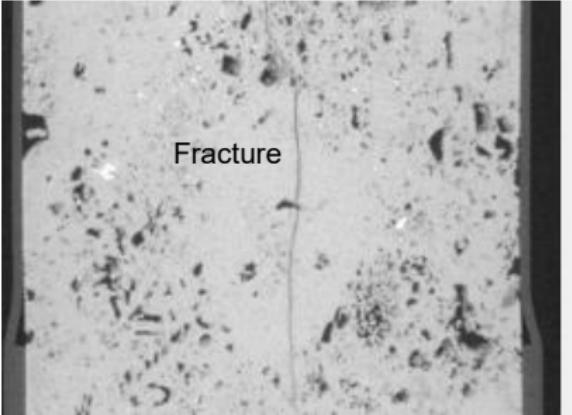
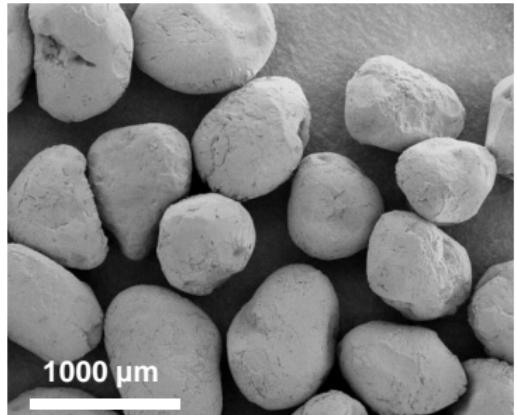
(a) Uncemented sand



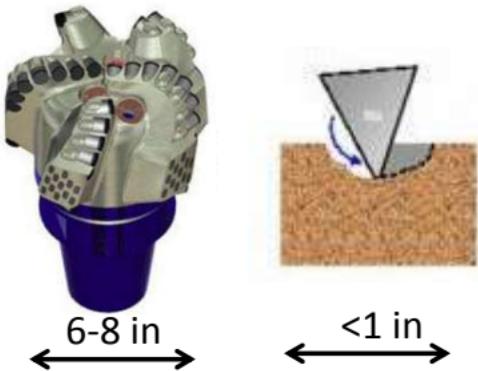
(b) Cemented sandstone



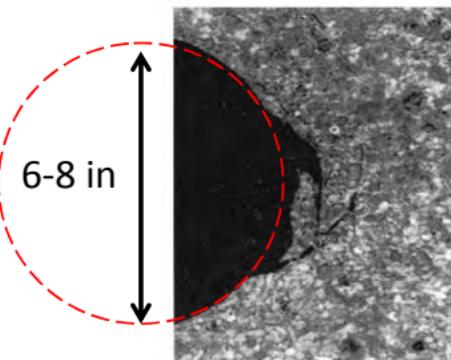
(c) Vuggy carbonate



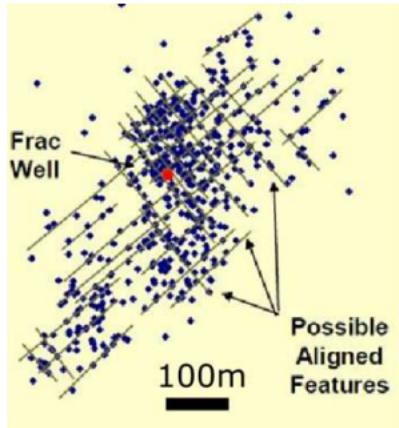
## Rock cutting at the drill bit



## Wellbore breakout



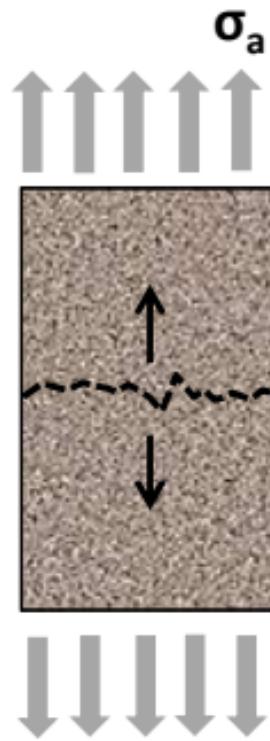
## Shale hydraulic fracture



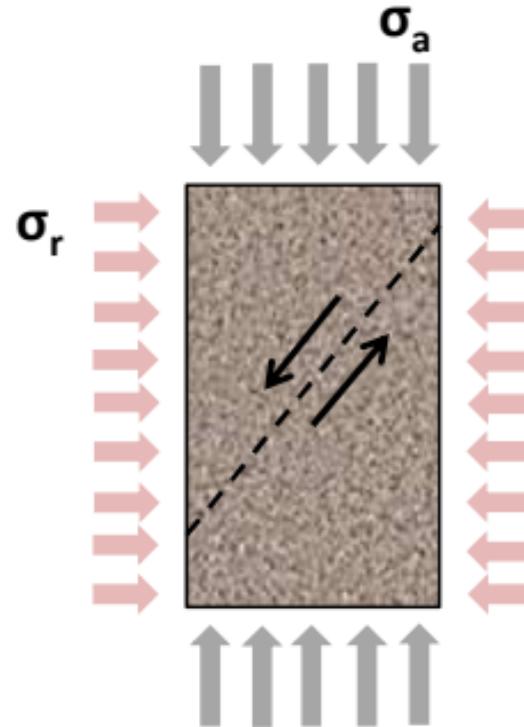
## Reservoir depletion



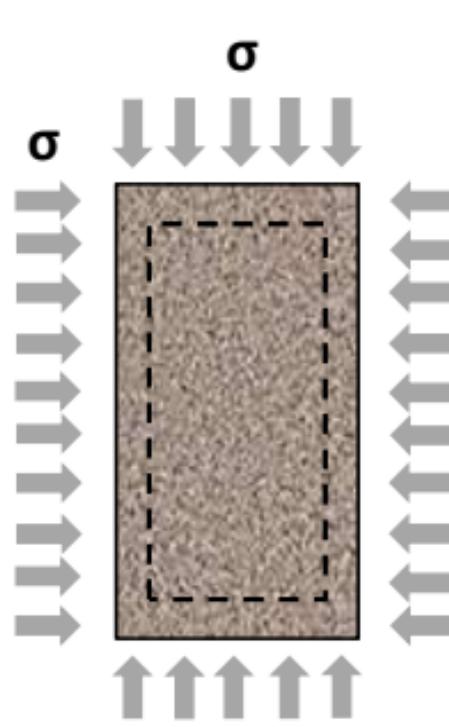
Images: Schlumberger/Terratek, Zoback 2013, Warpinski 2008, doe.gov



Tension  
(bond breakage)  
Ex: drilling-induced tens. fracs



Shear  
(friction failure)  
Ex: fault, breakout

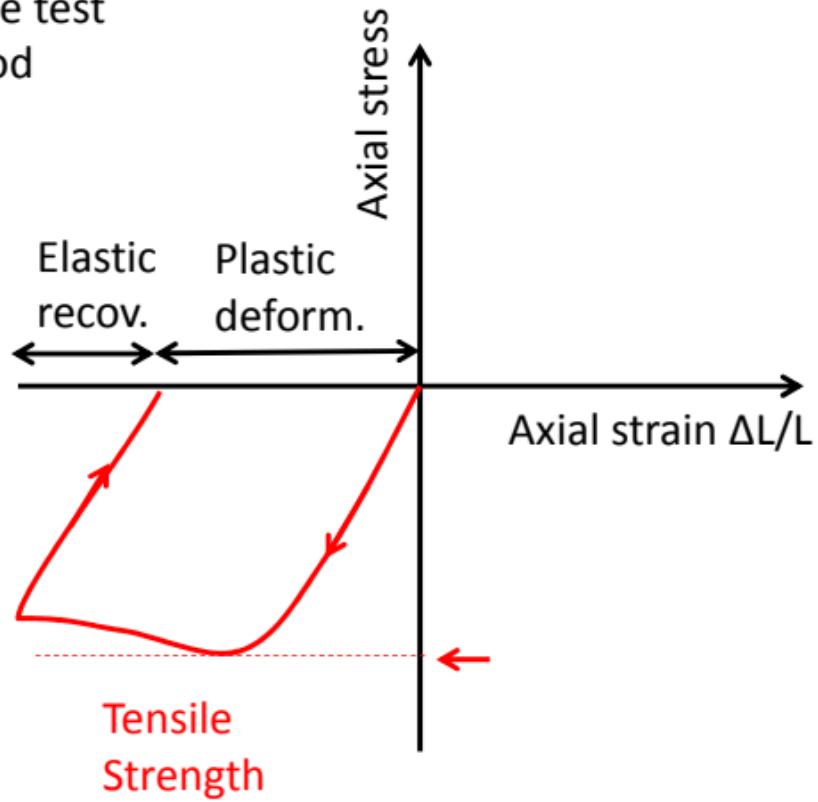


Compression  
(pore collapse)  
Ex: reservoir compaction

T



Typical tensile test  
on a metal rod



Tensile test on a rock rod

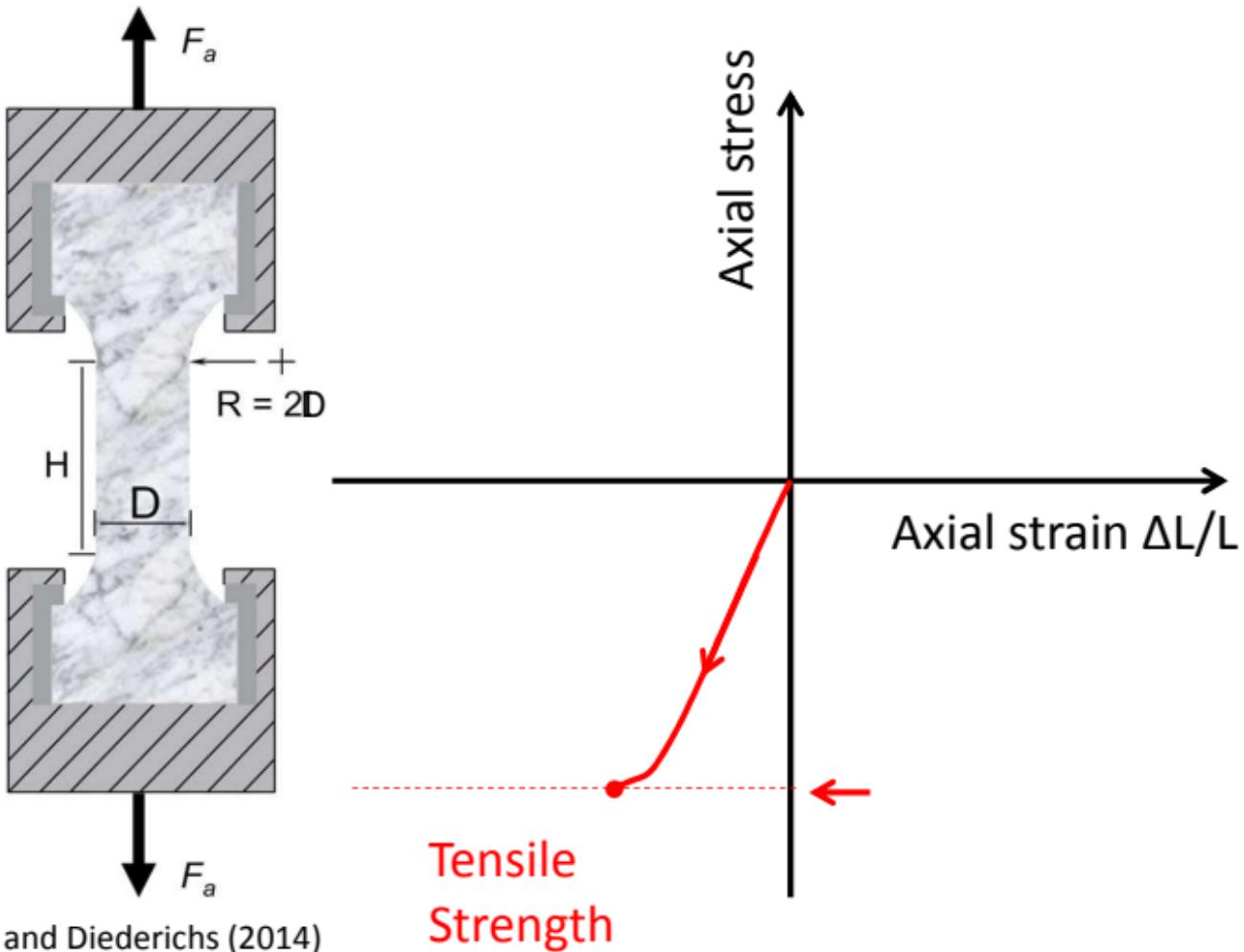
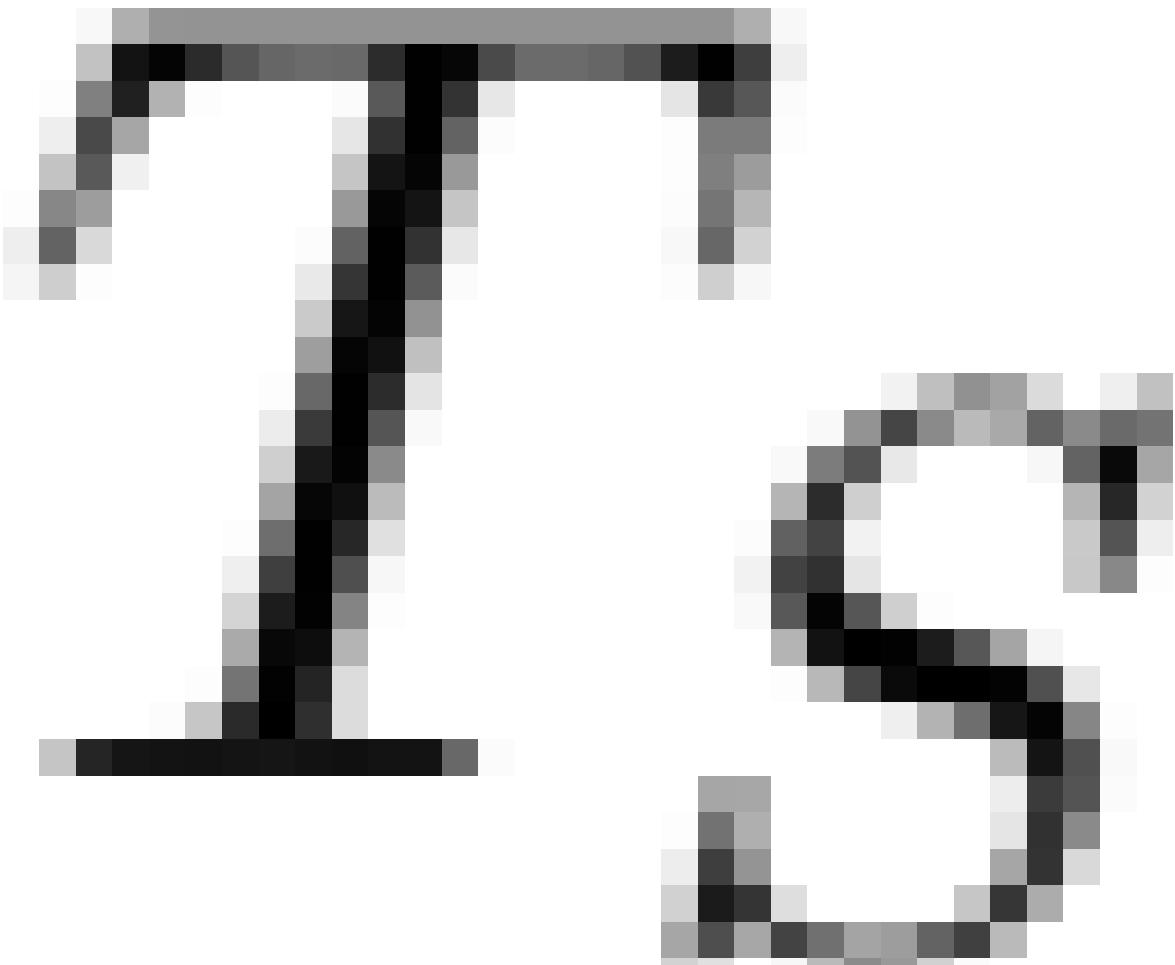


Figure direct tension: Perras and Diederichs (2014)



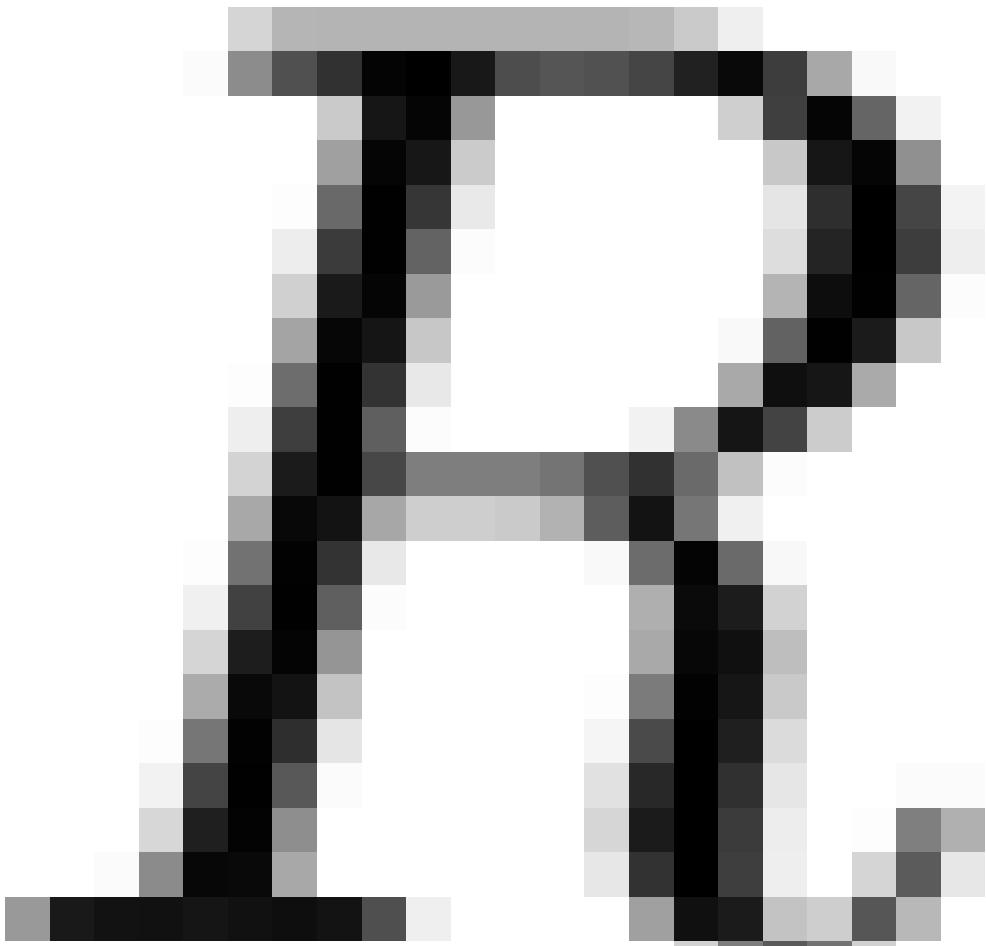
IS

—  
—

πLR

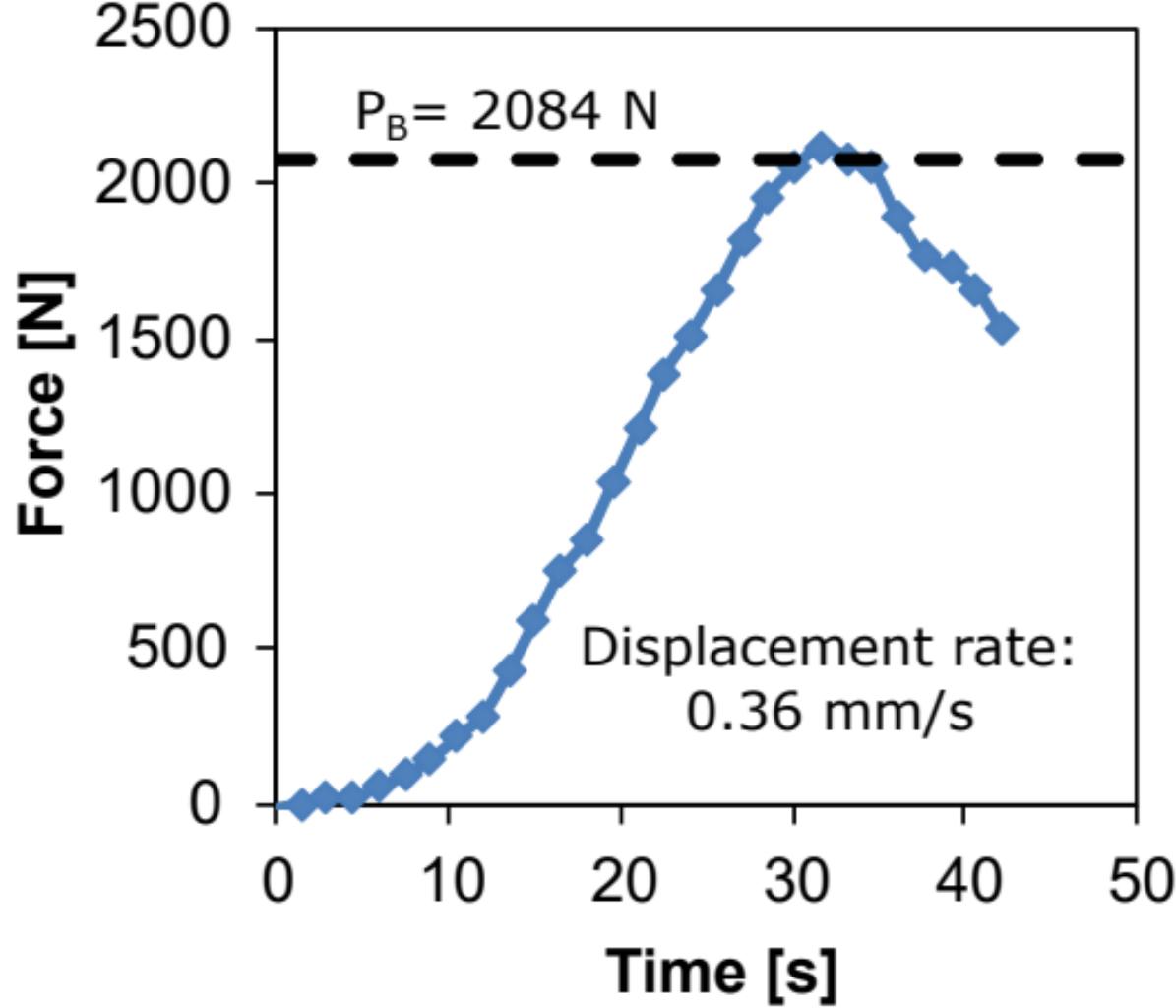
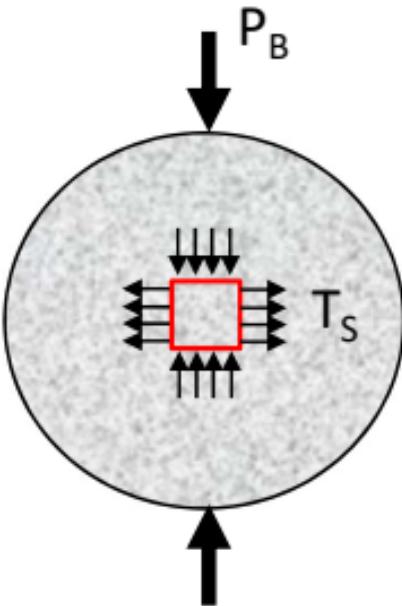
PB





Cylindrical sample:

- radius R
- length L

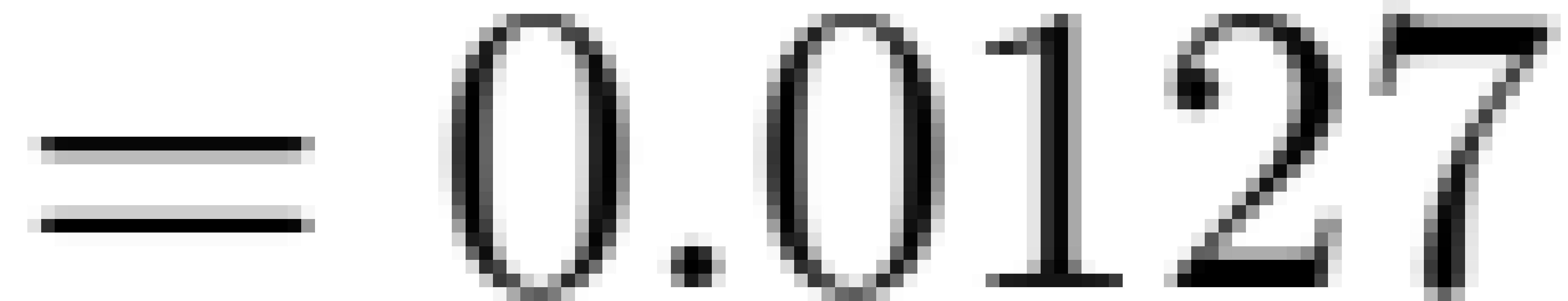


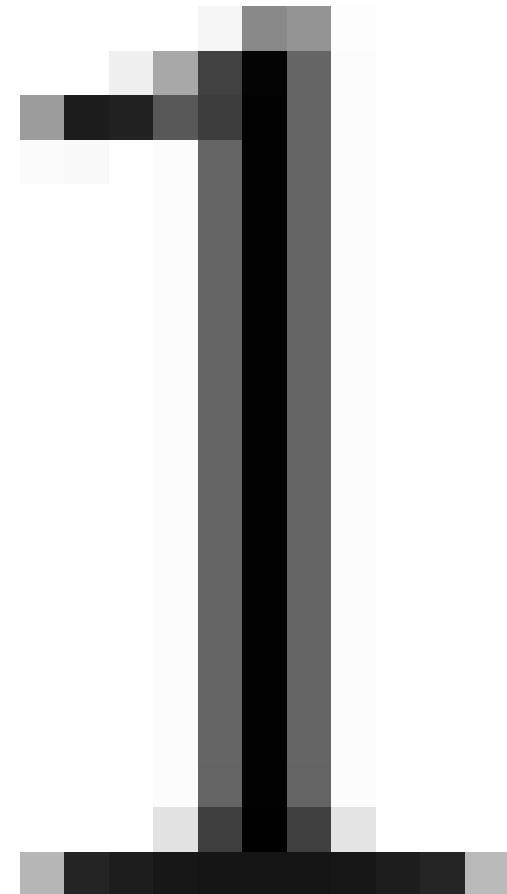
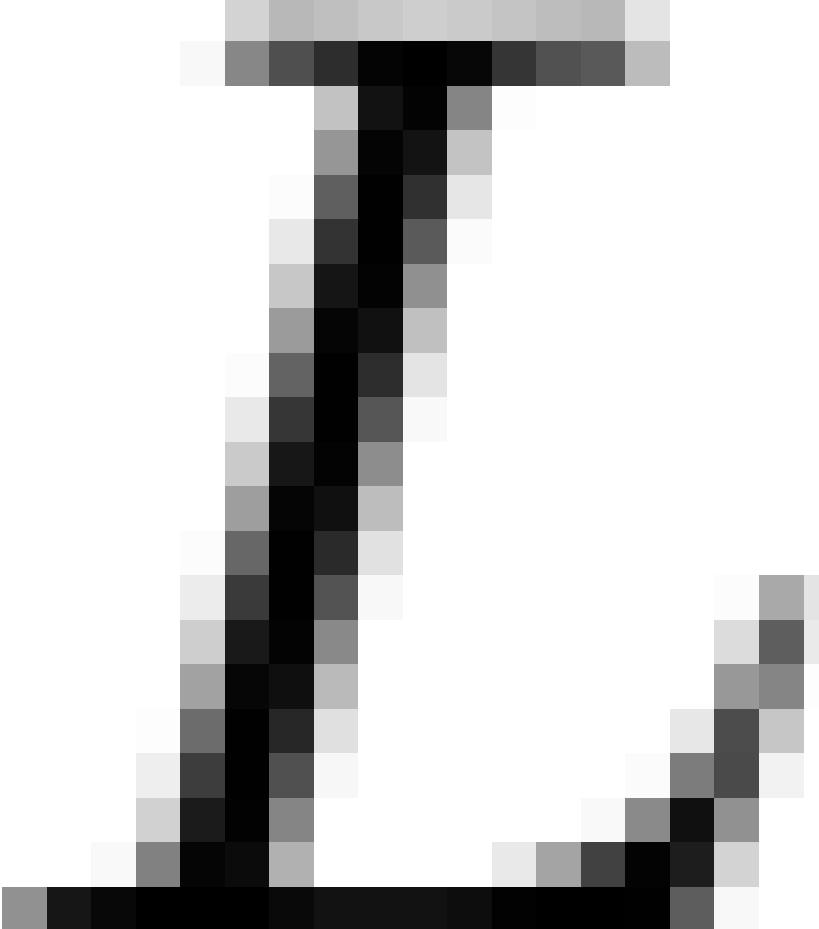
R

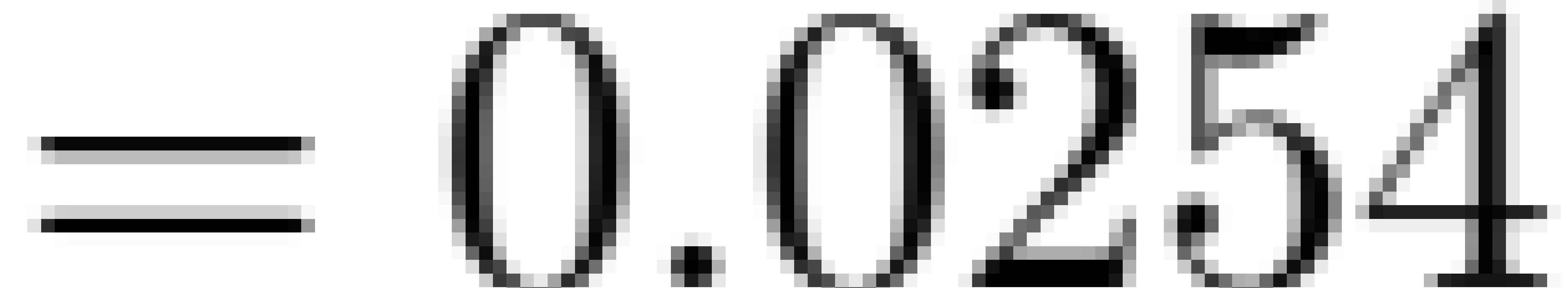
1  
2

1  
2

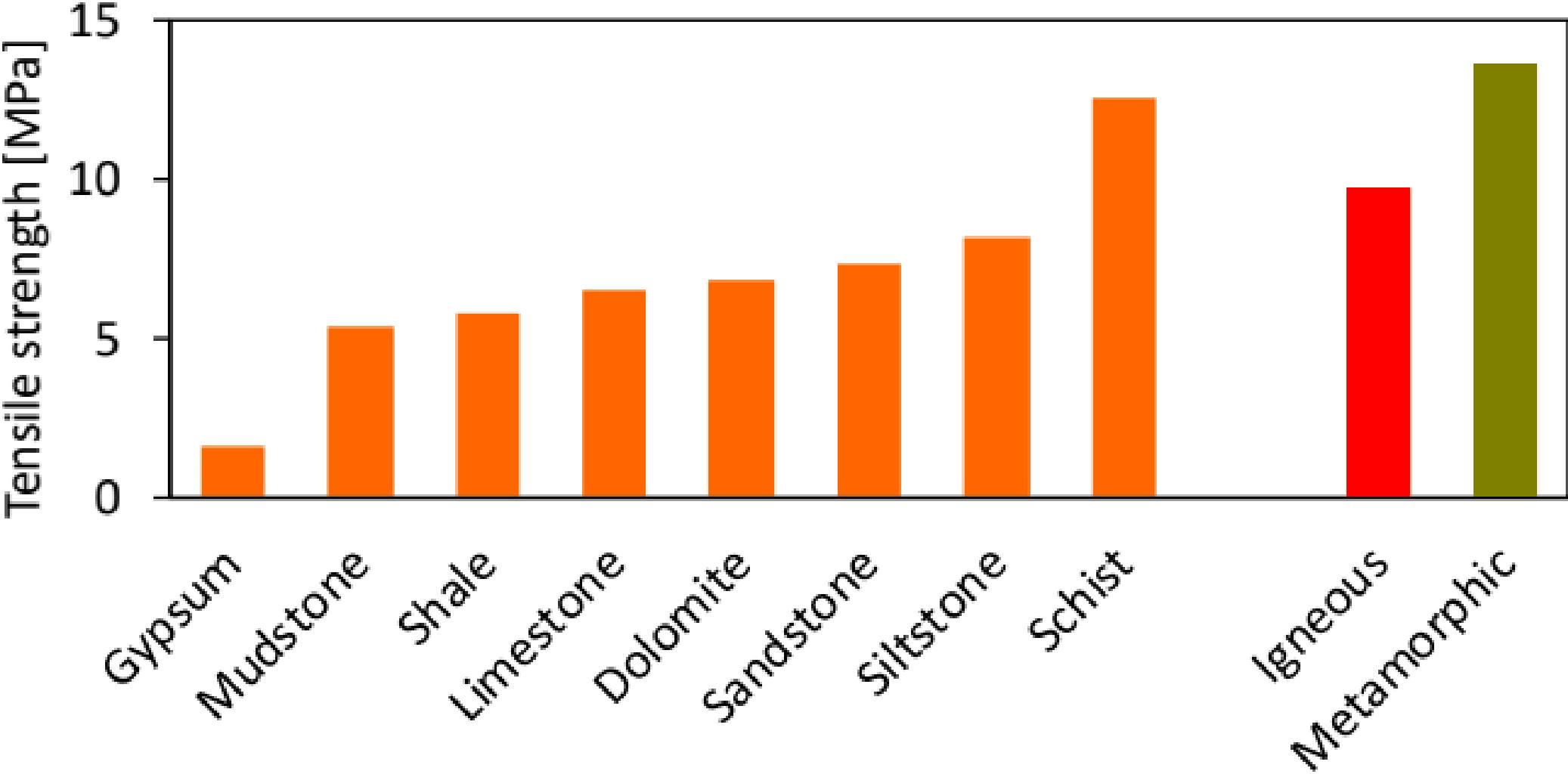
1  
2

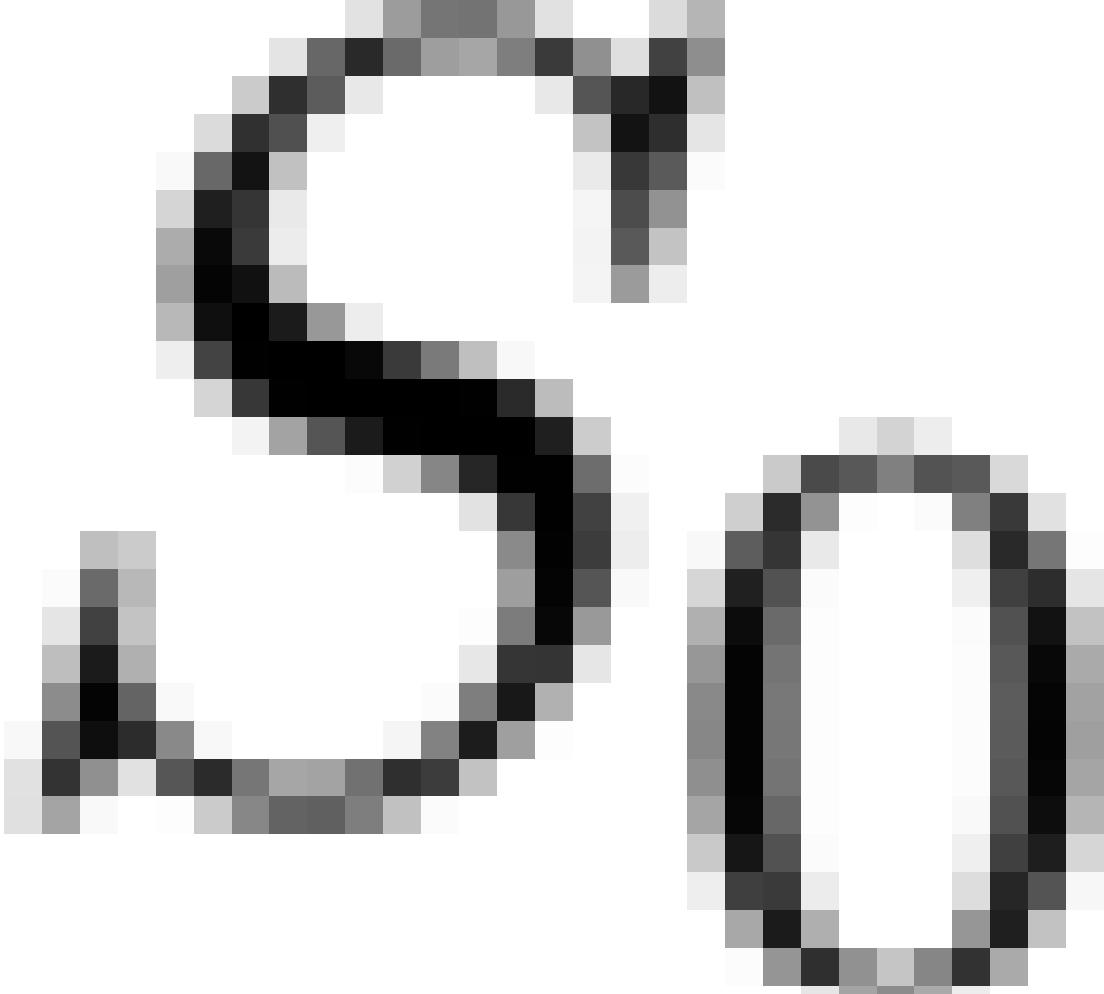


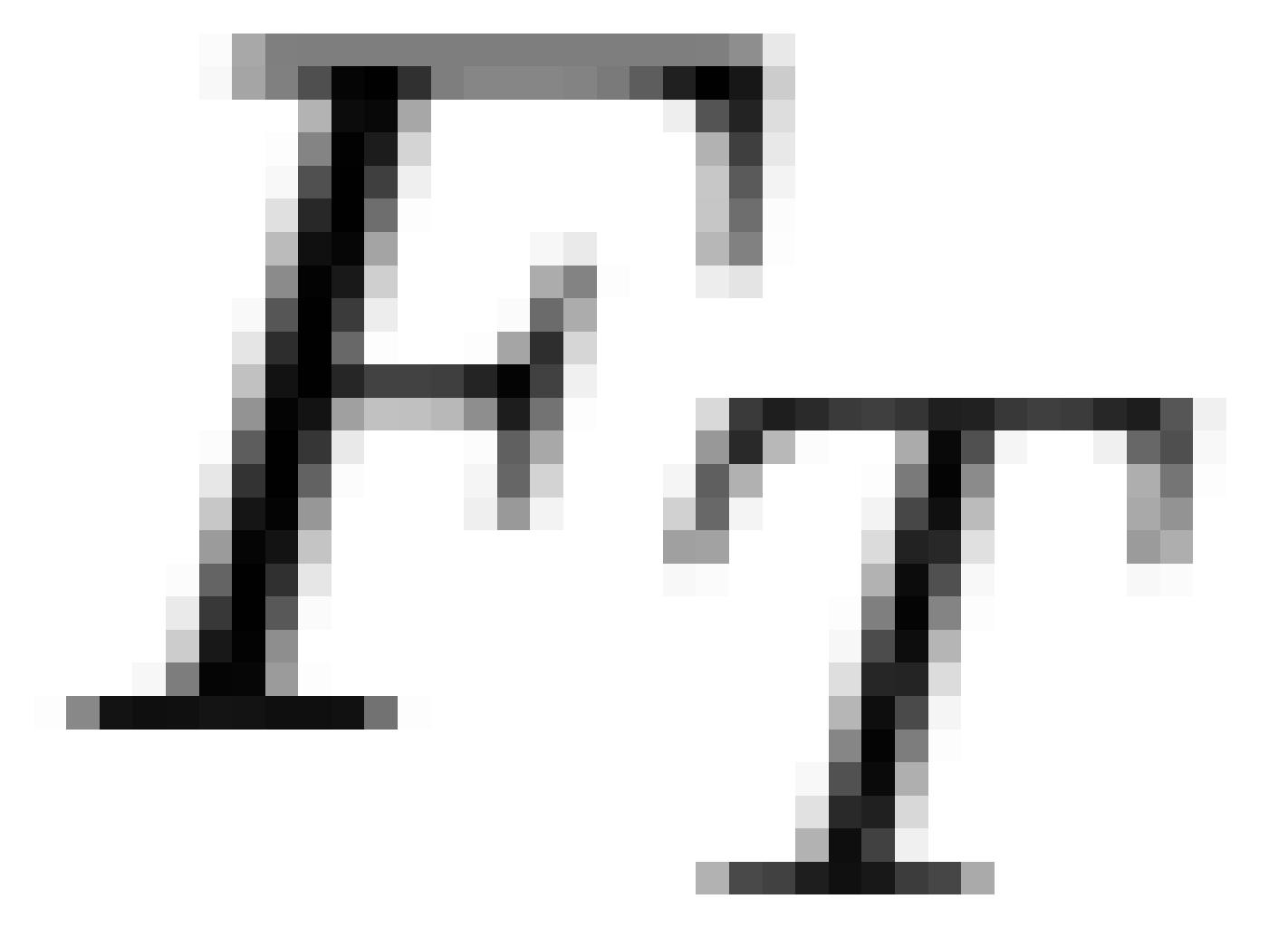




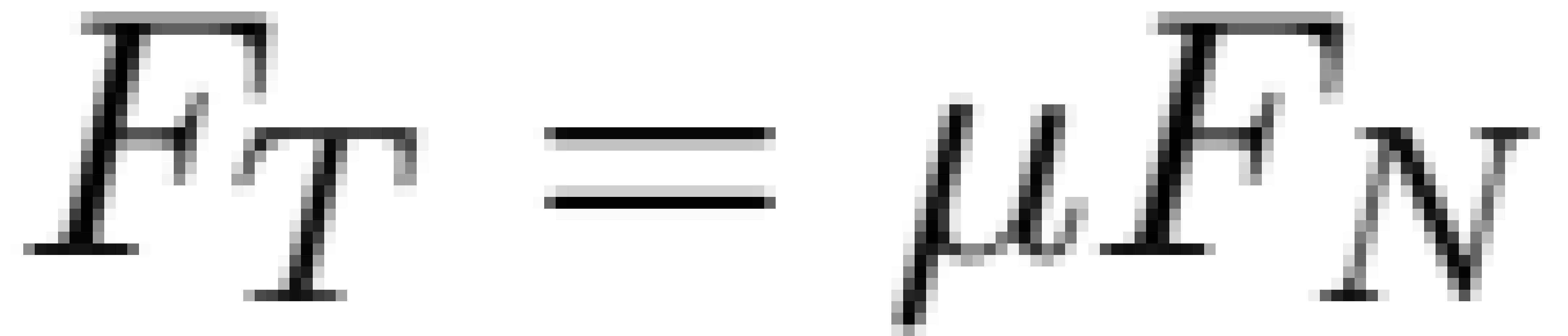
$$\frac{2084 \text{ N}}{\pi (0.0254 \text{ m})(0.0127 \text{ m})} = 2.06 \times 10^6 \text{ Pa} = 2.06 \text{ MPa}$$

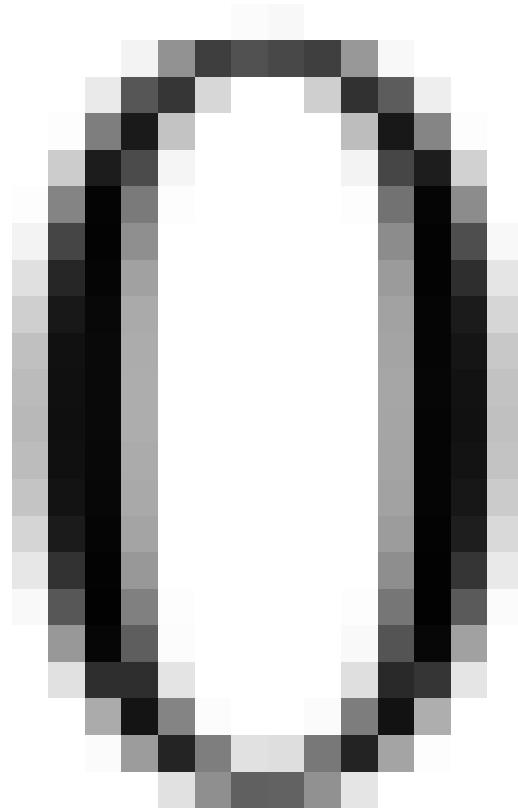


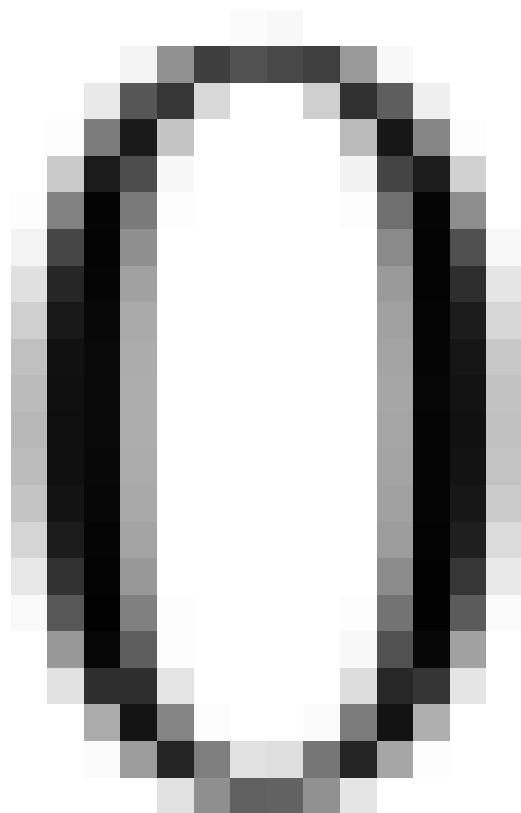
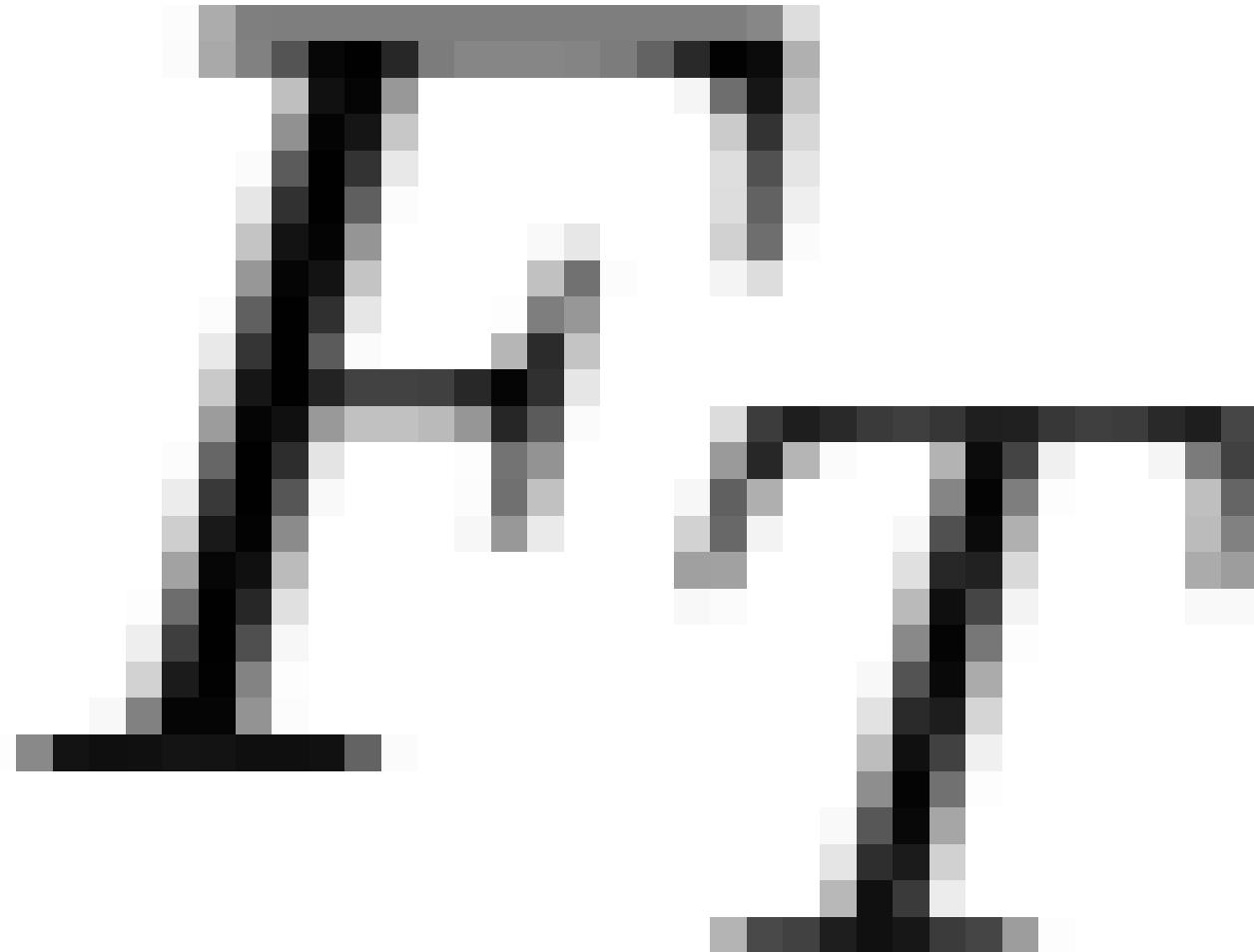


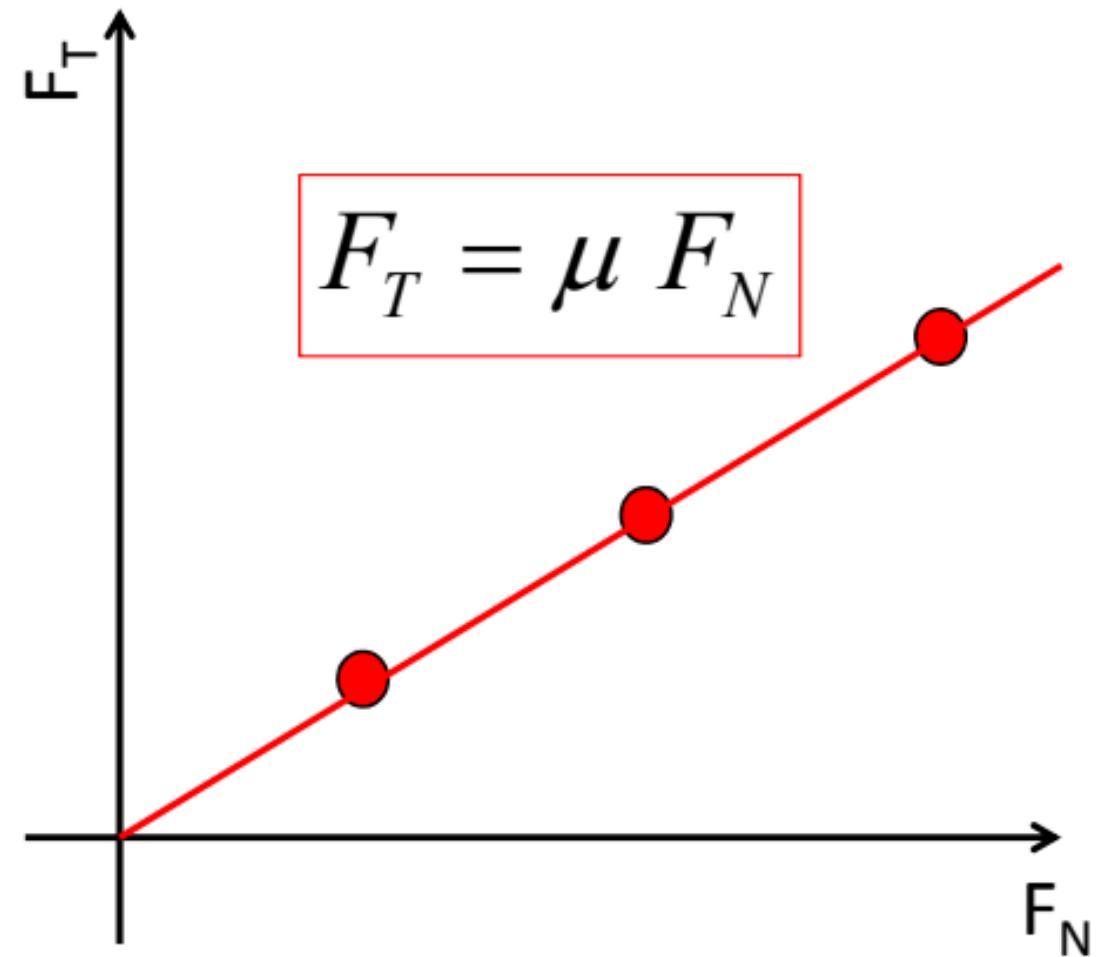
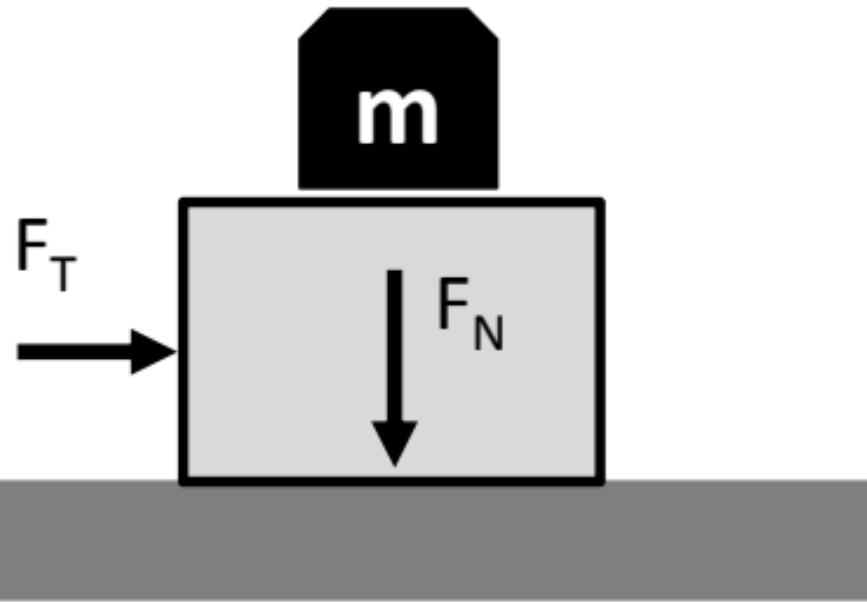


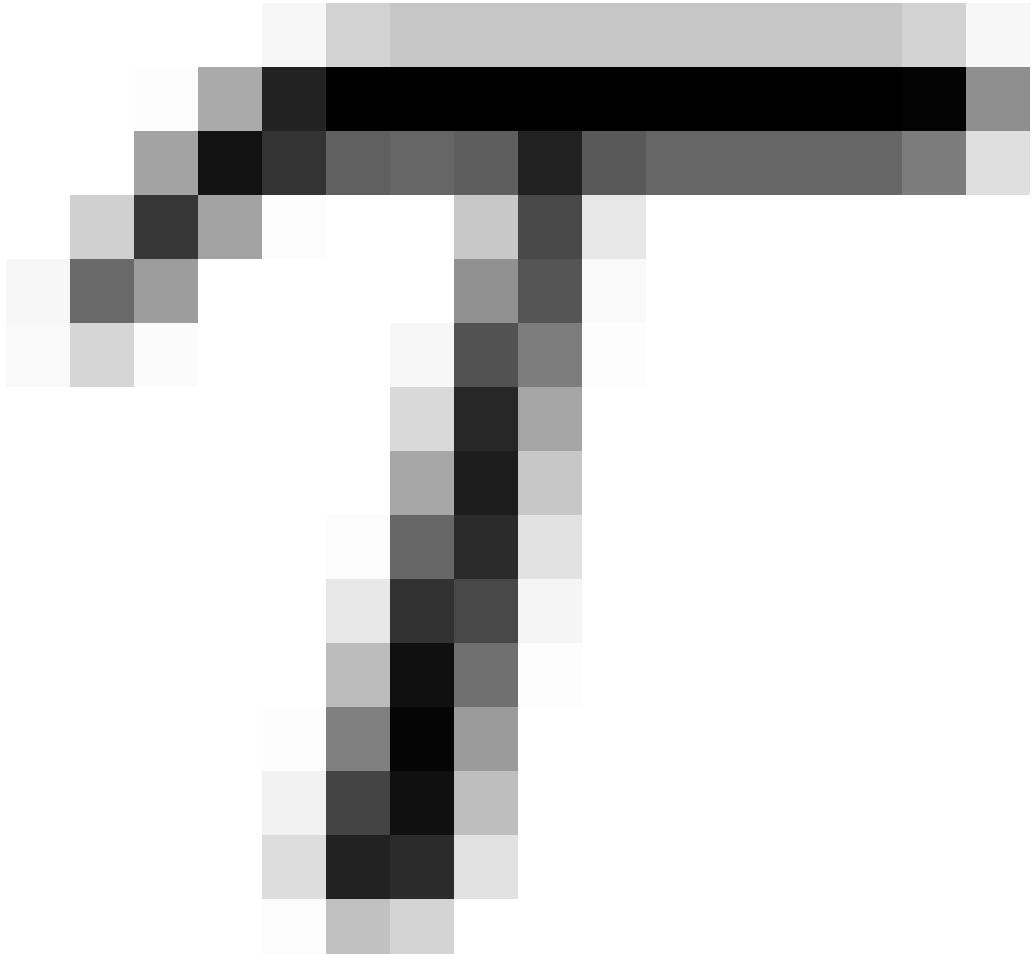


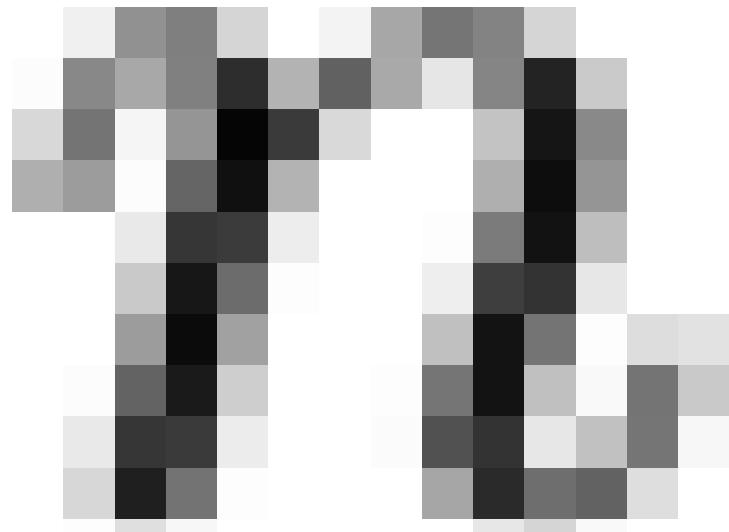
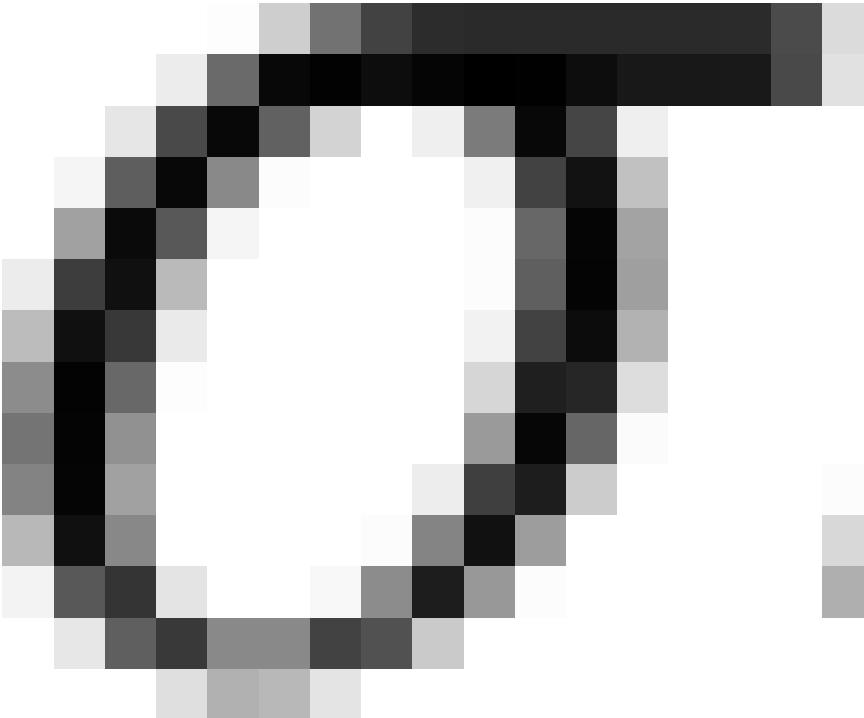


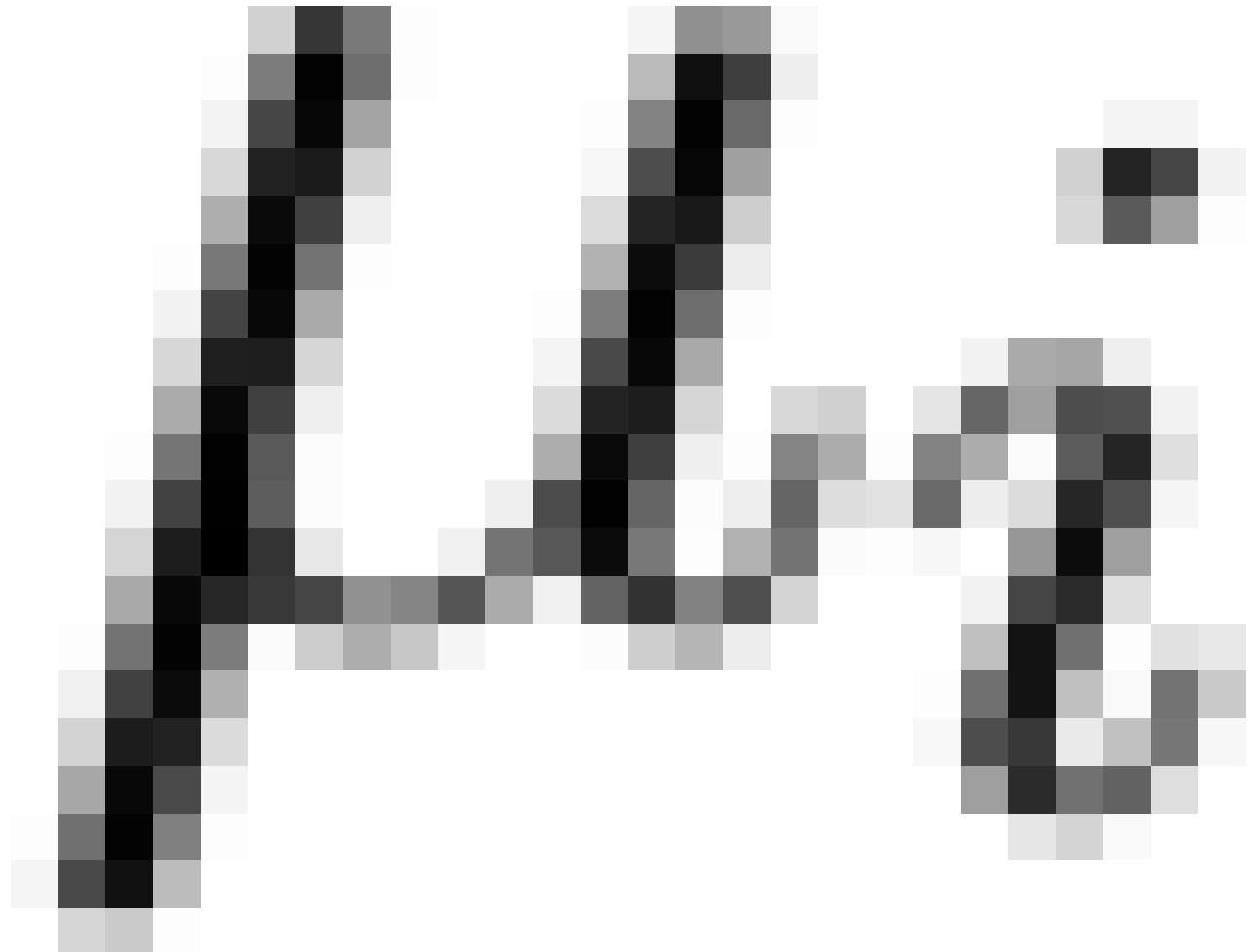


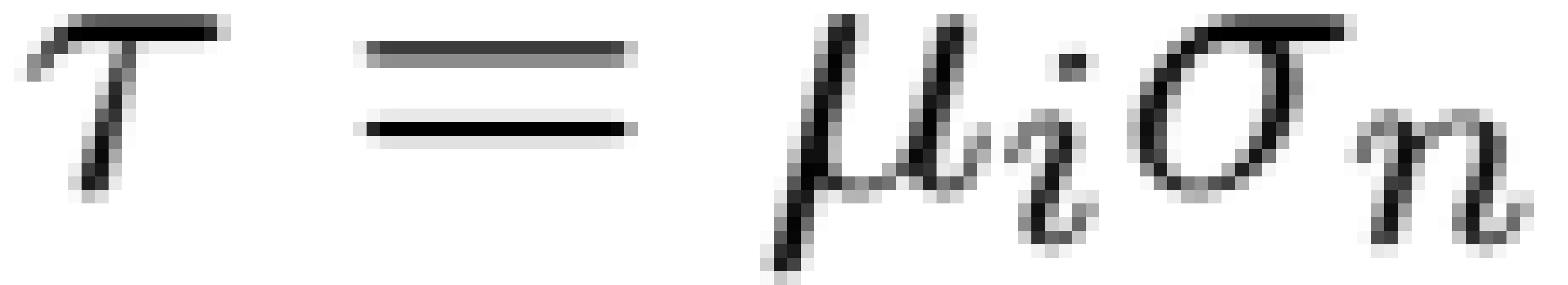


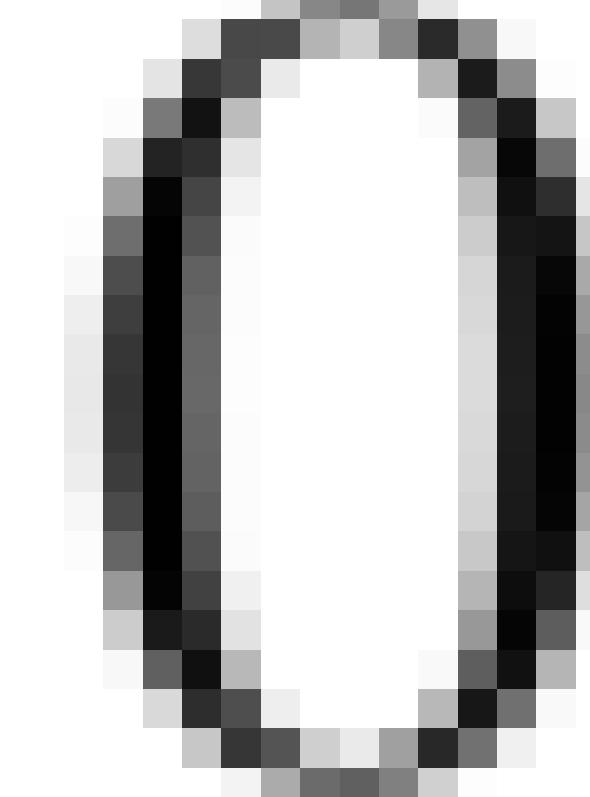
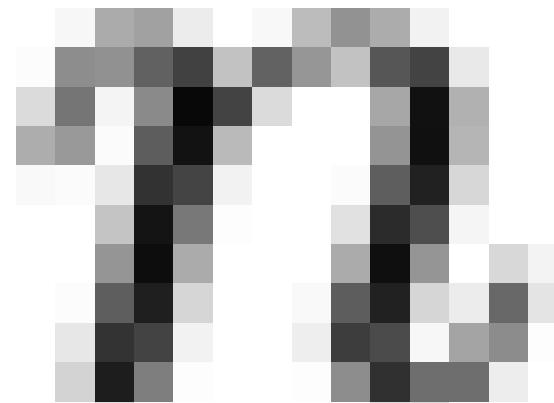
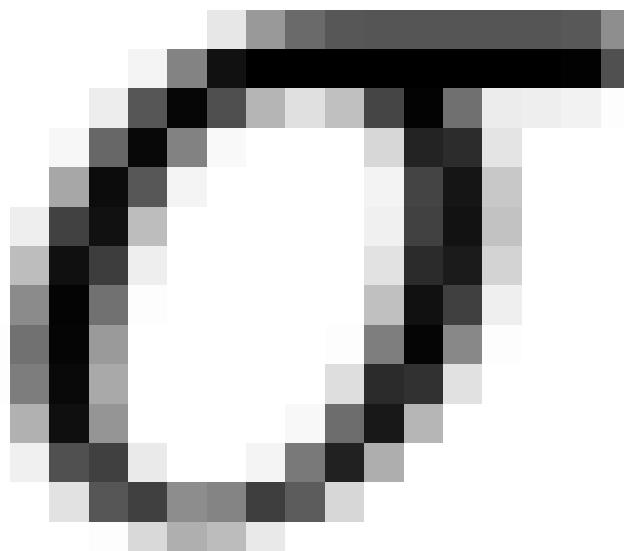


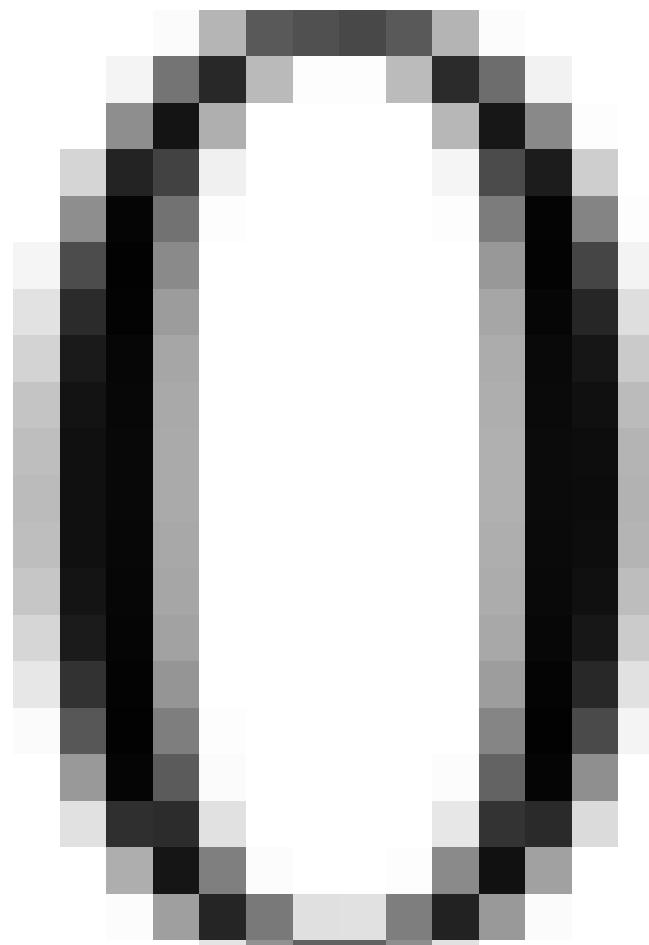
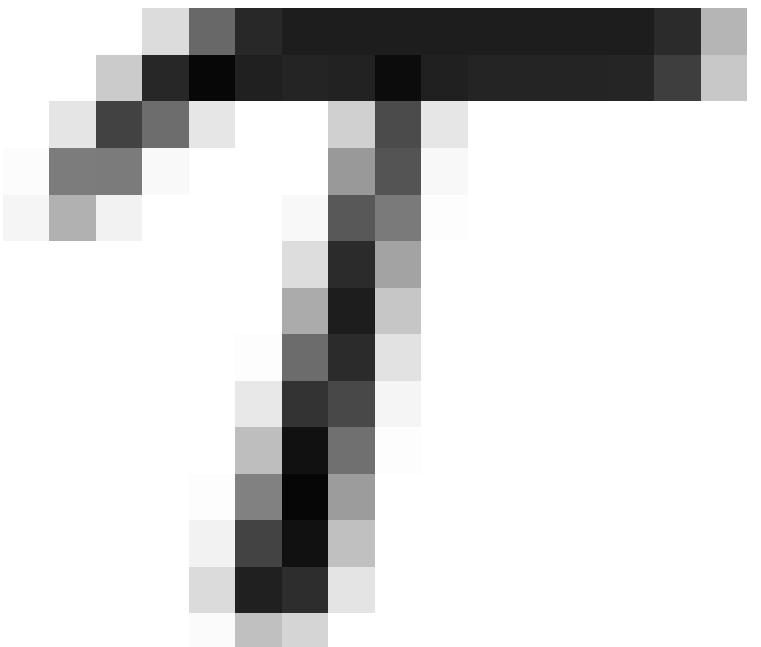


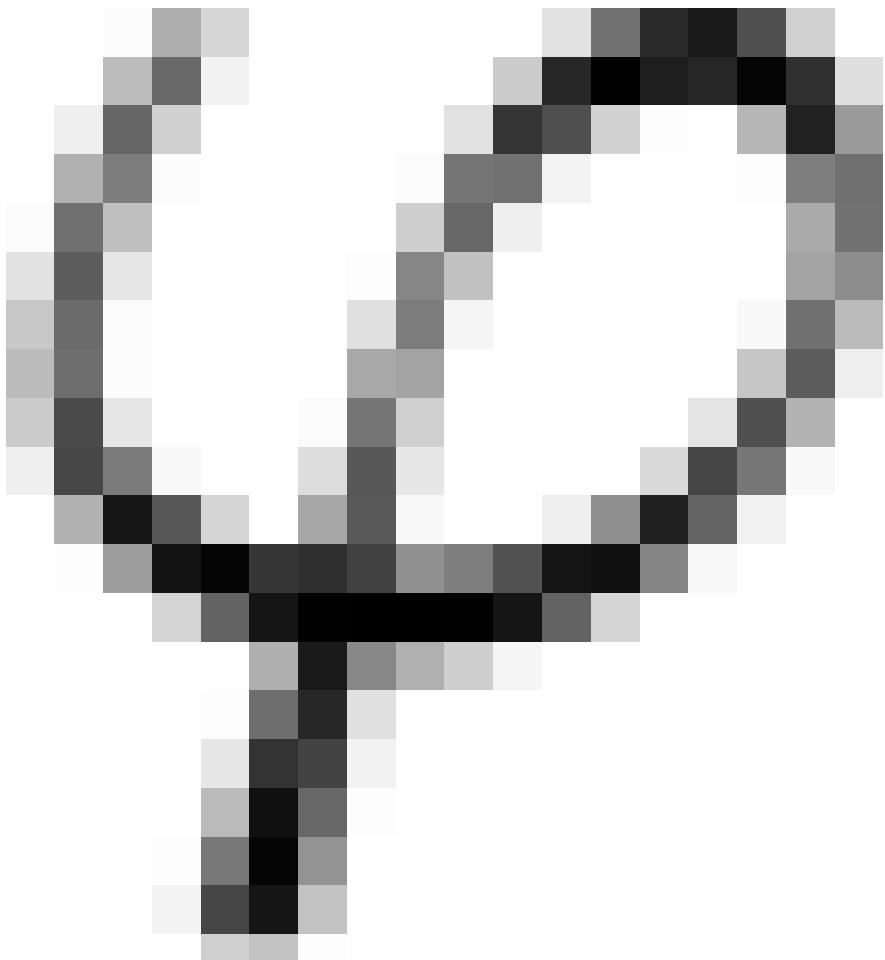




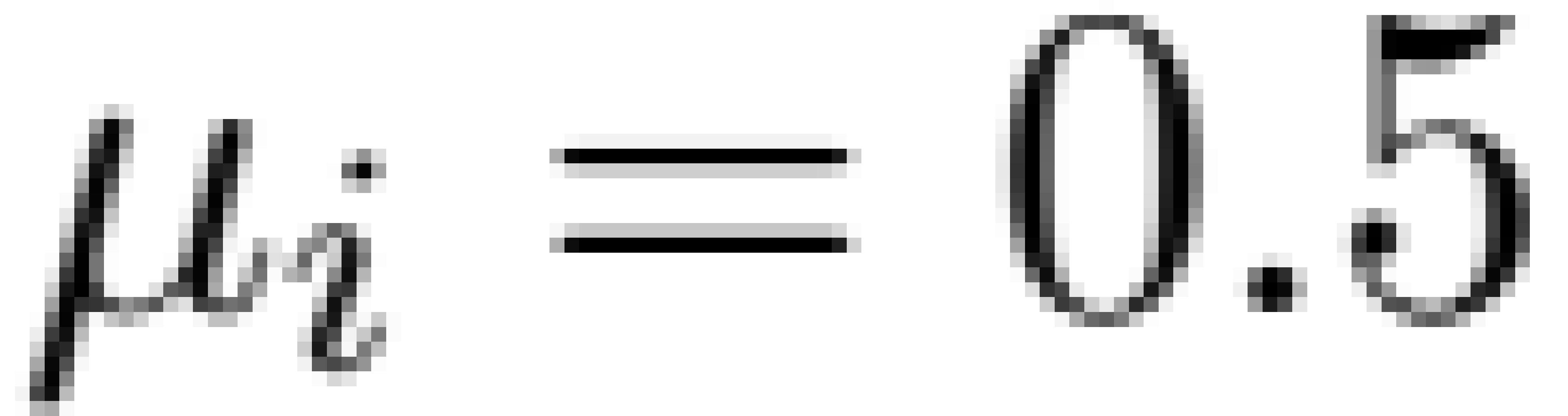


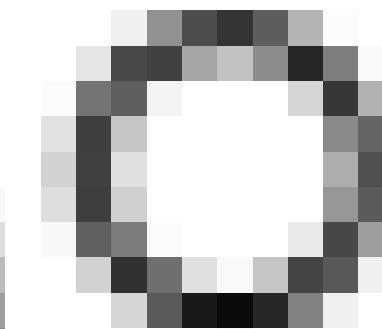
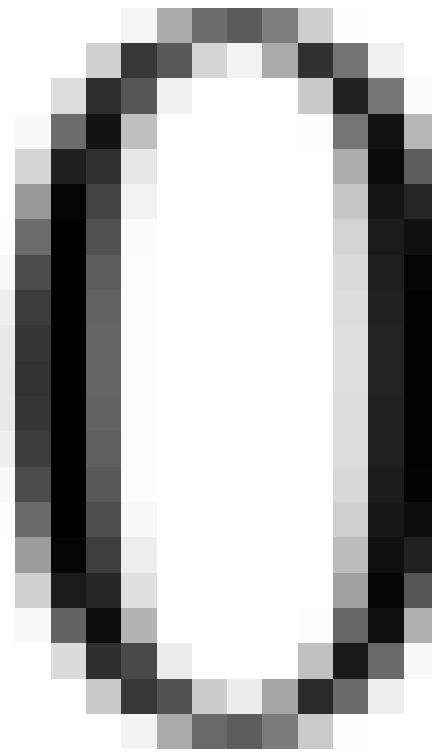
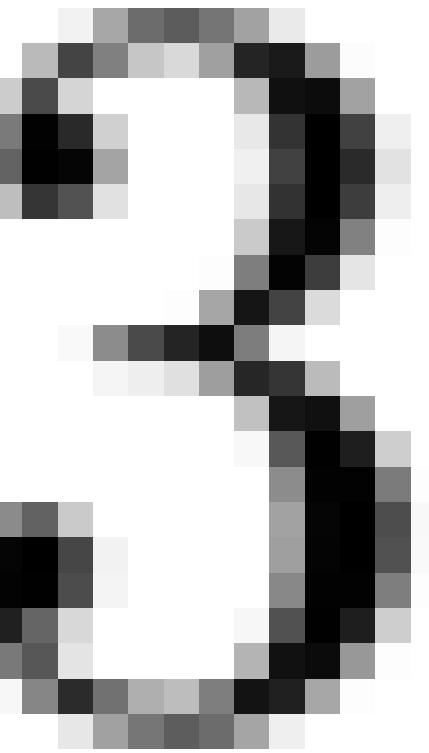
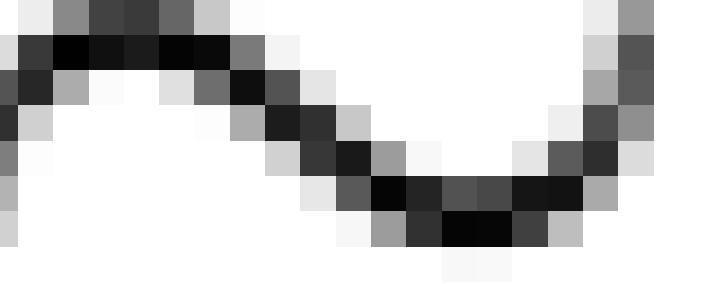
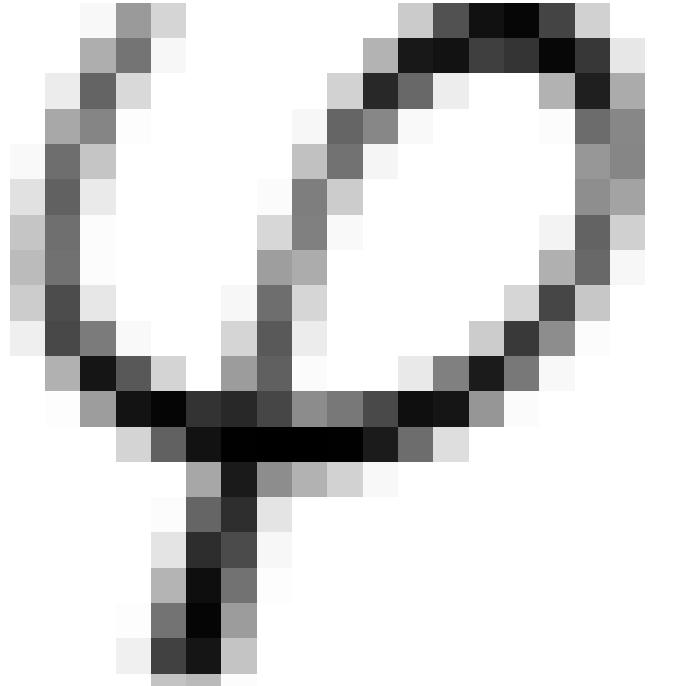


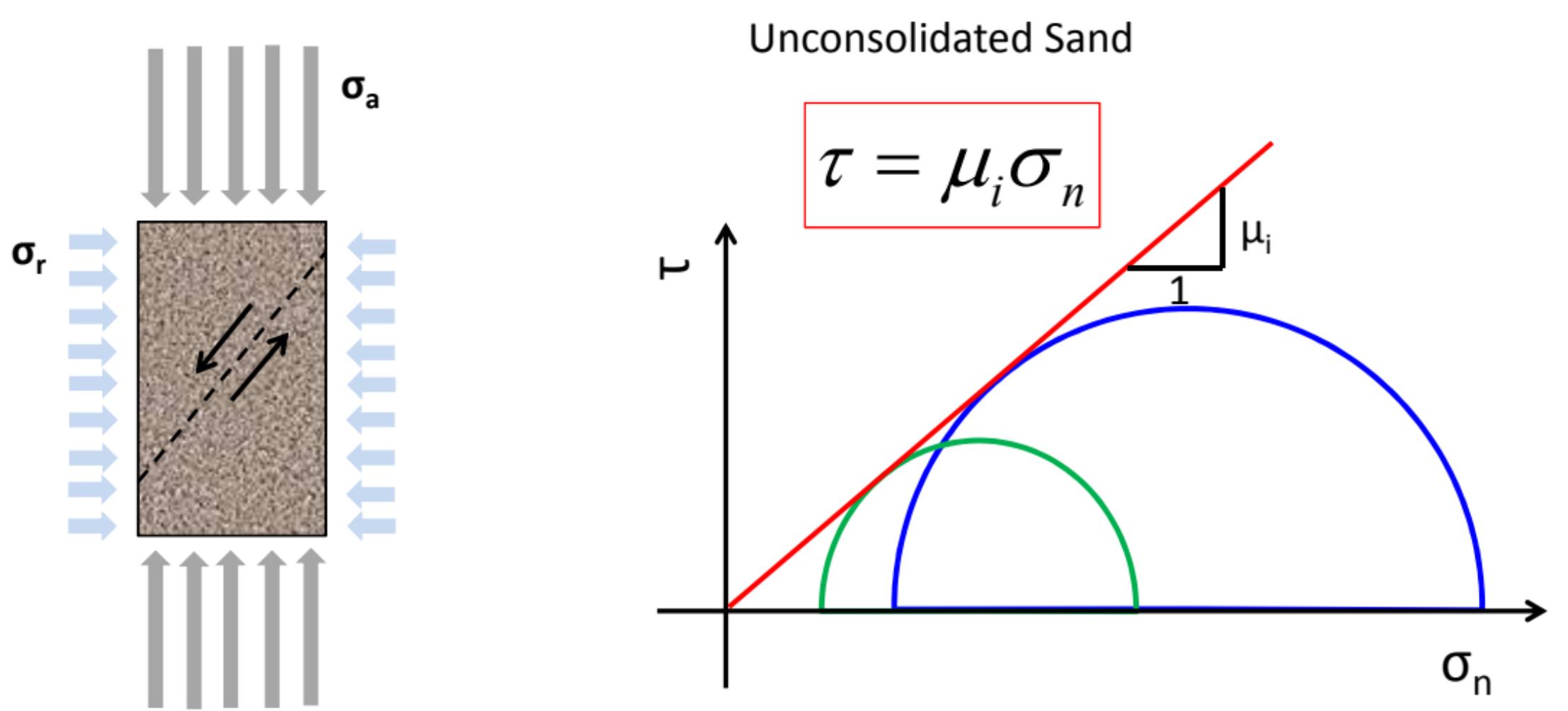


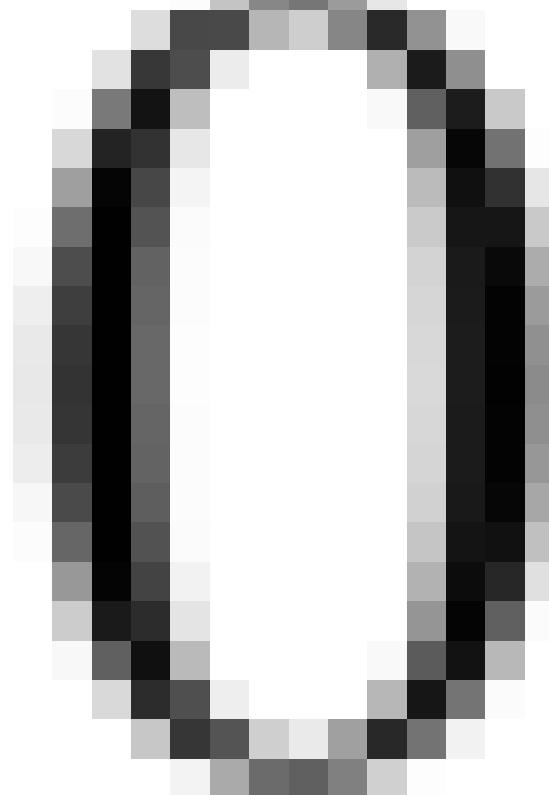
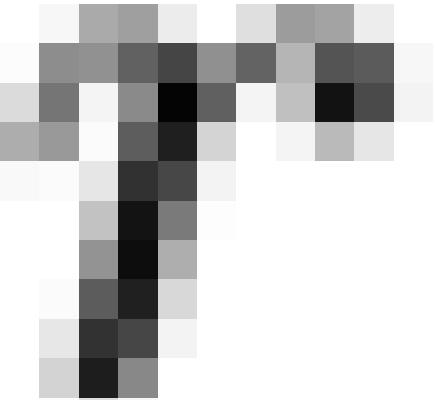
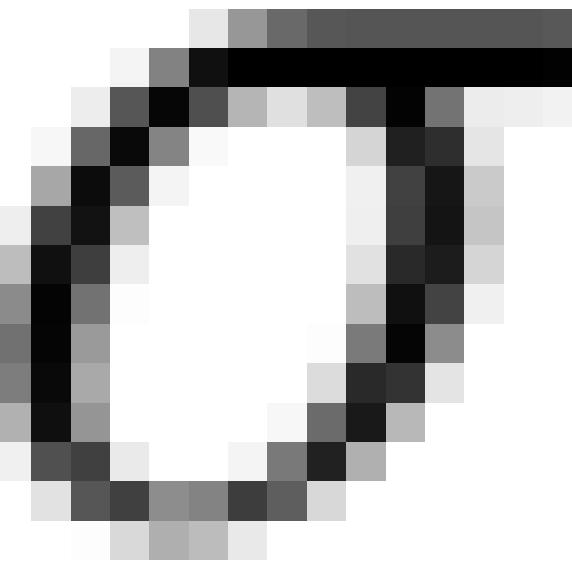


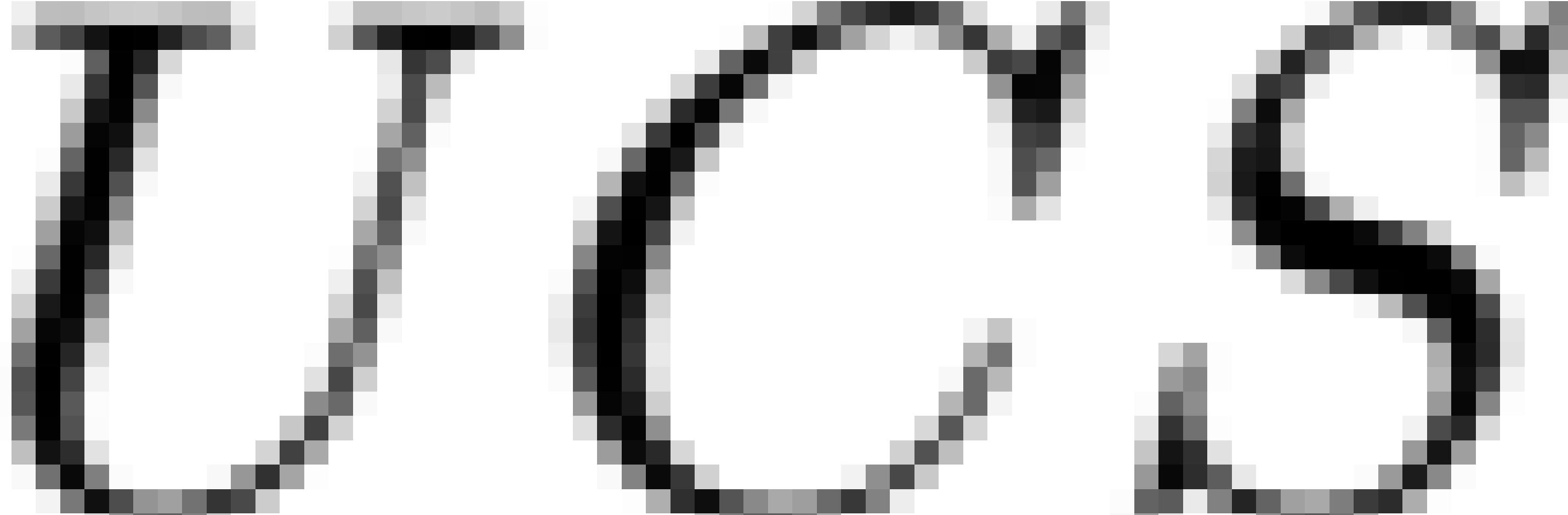




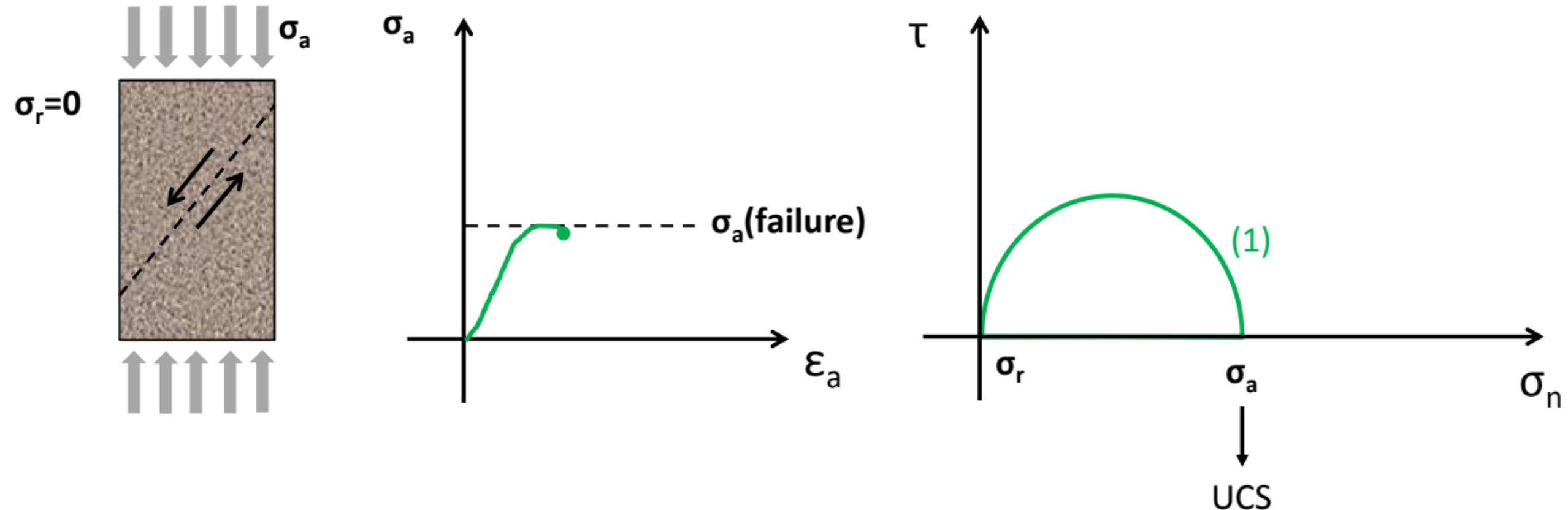


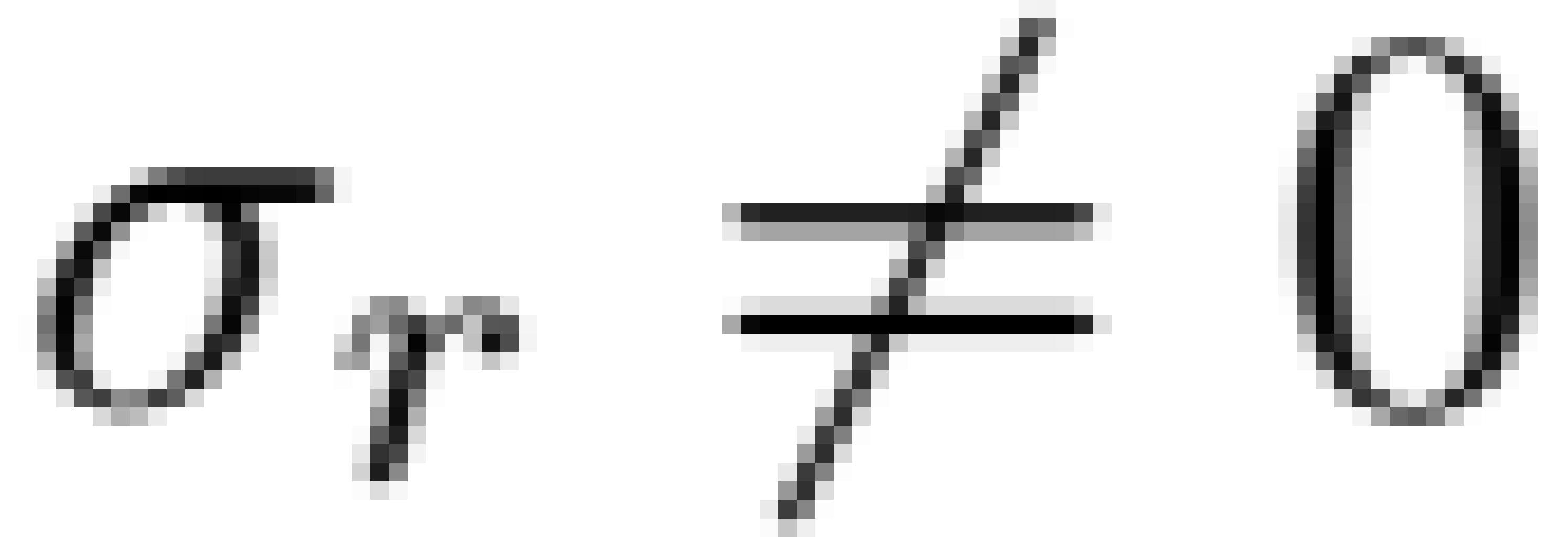


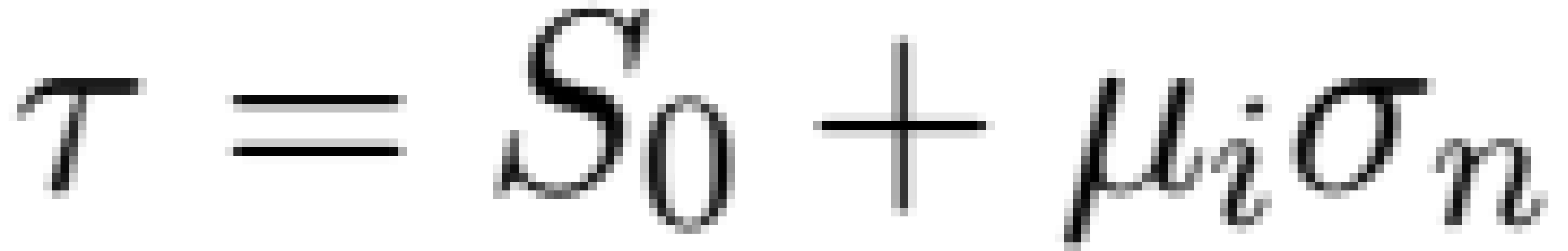




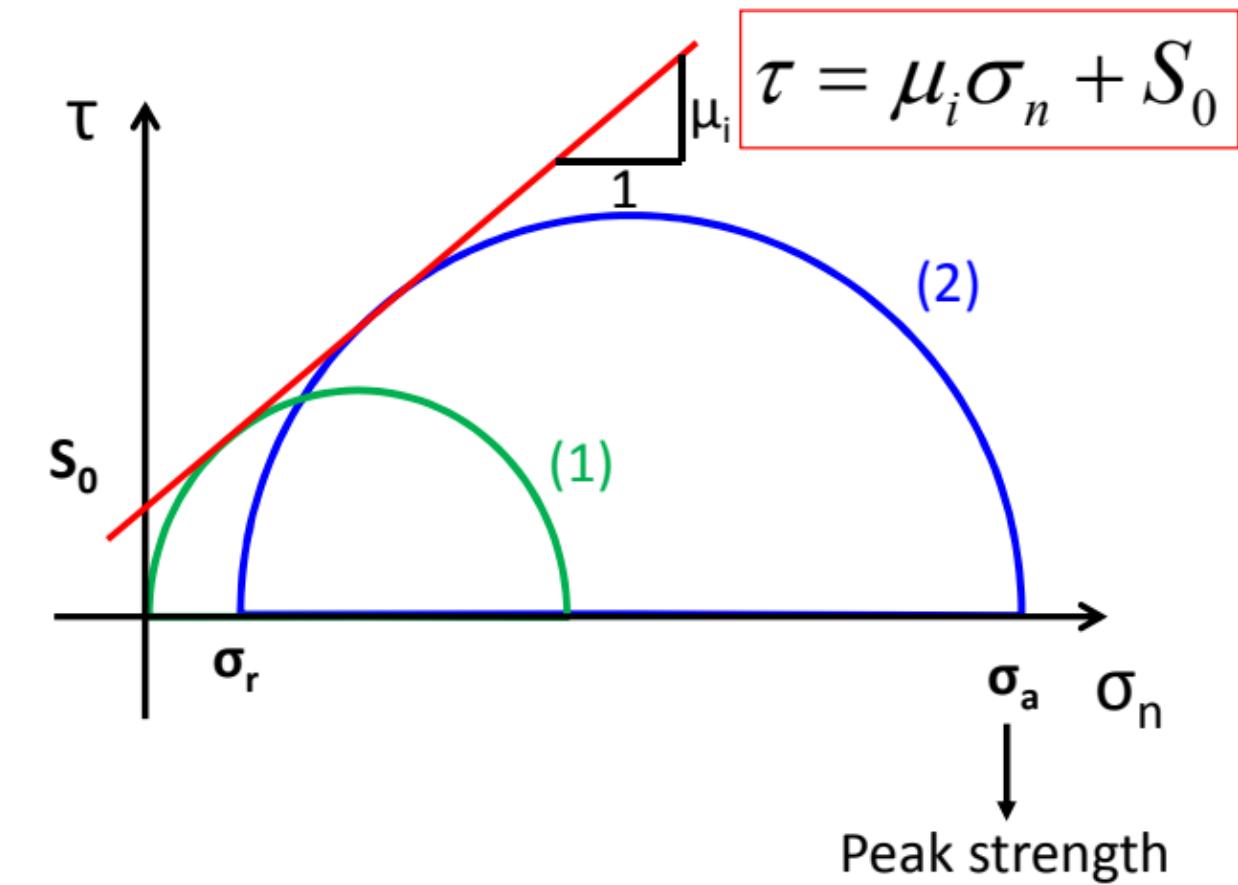
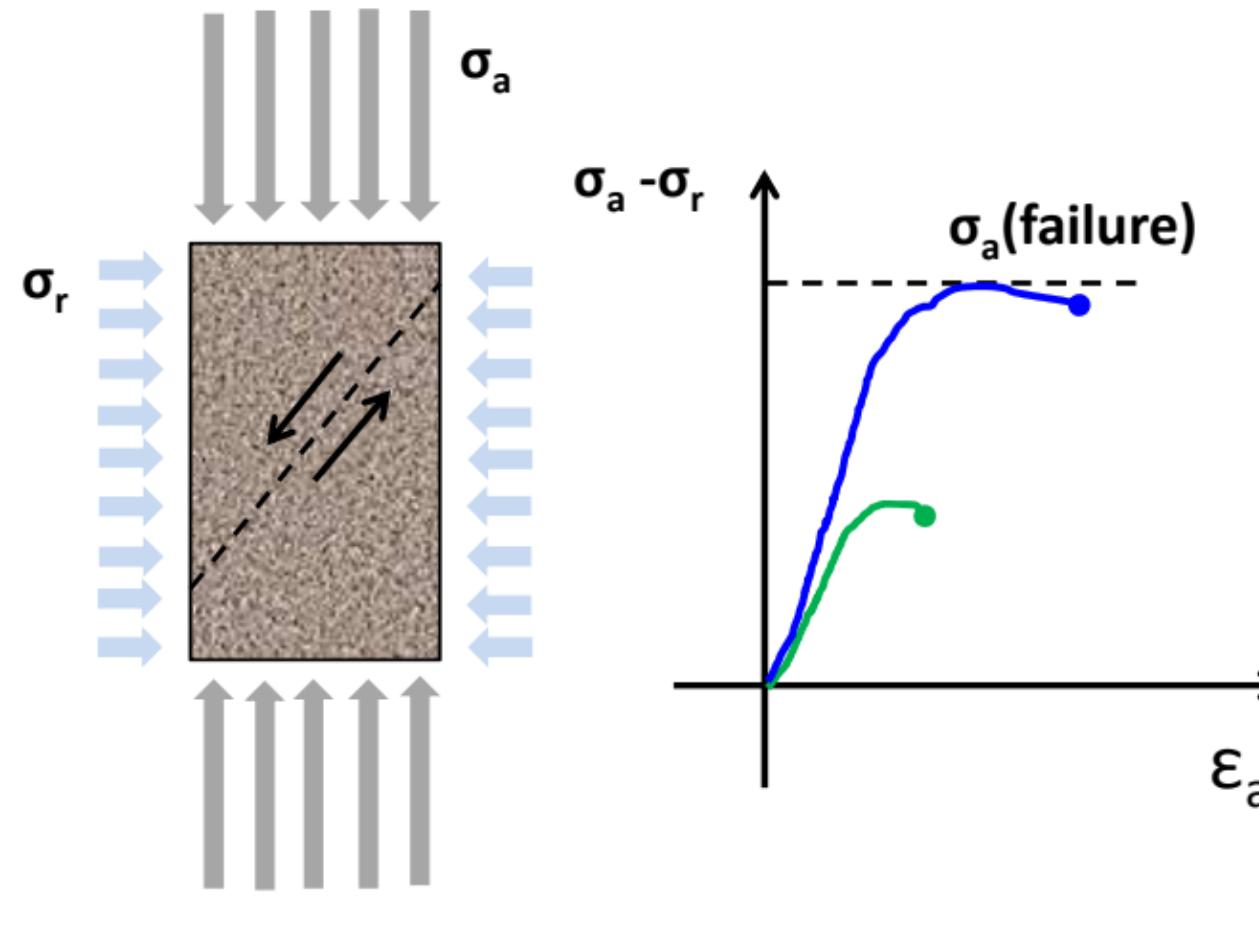
## (1) Unconfined Loading

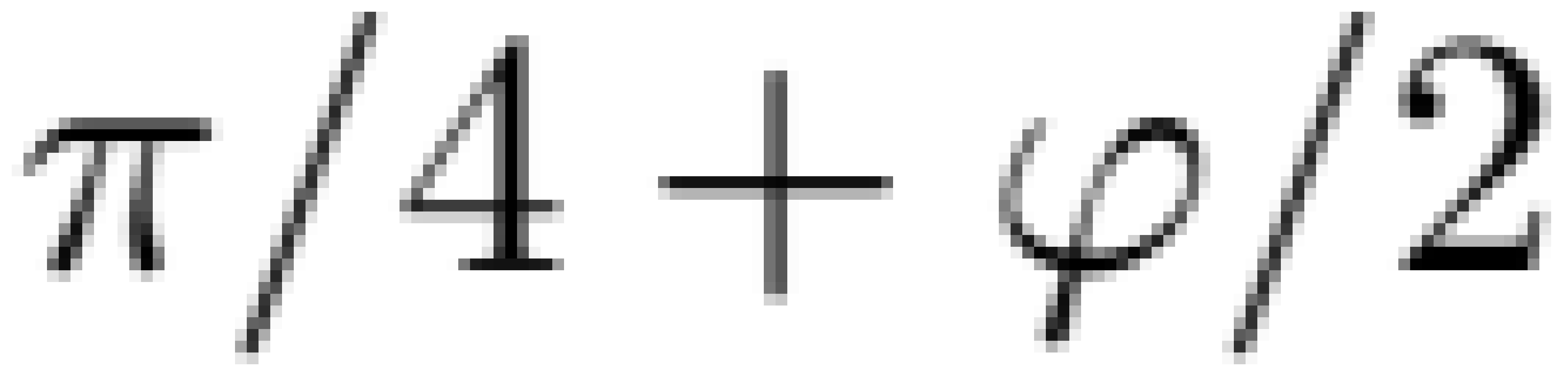






## (2) Confined Loading

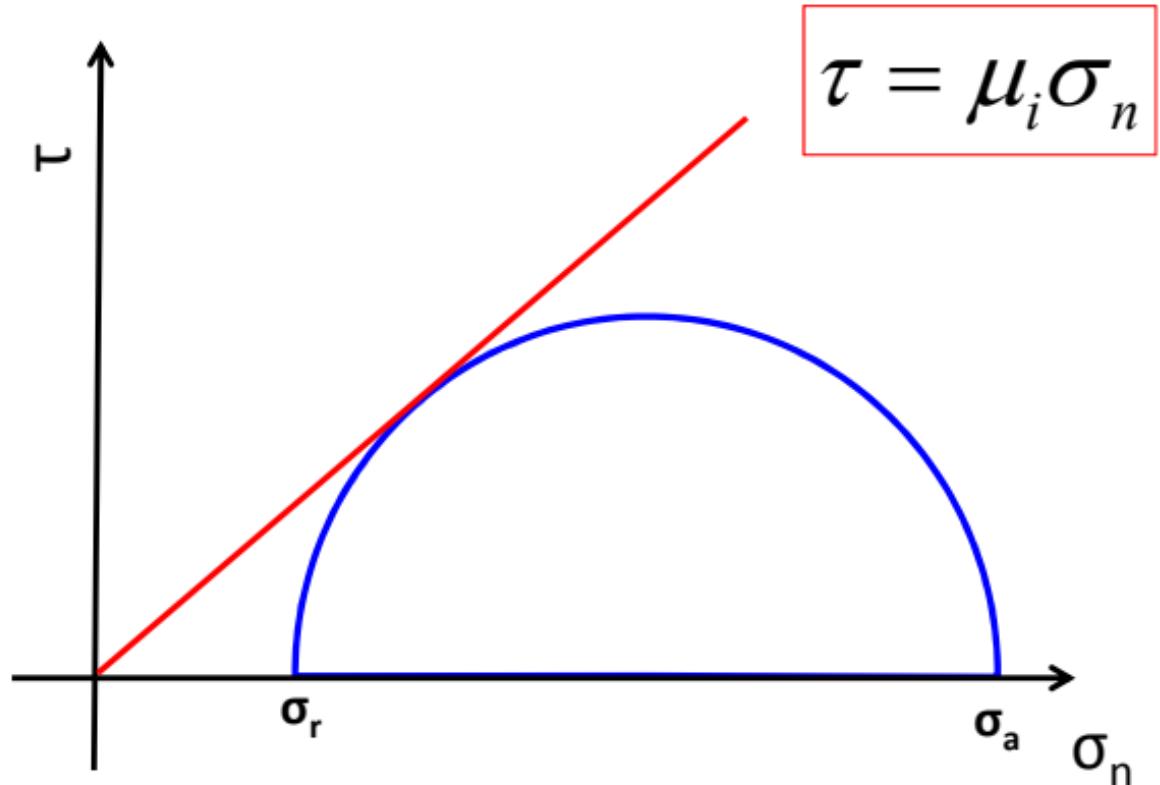
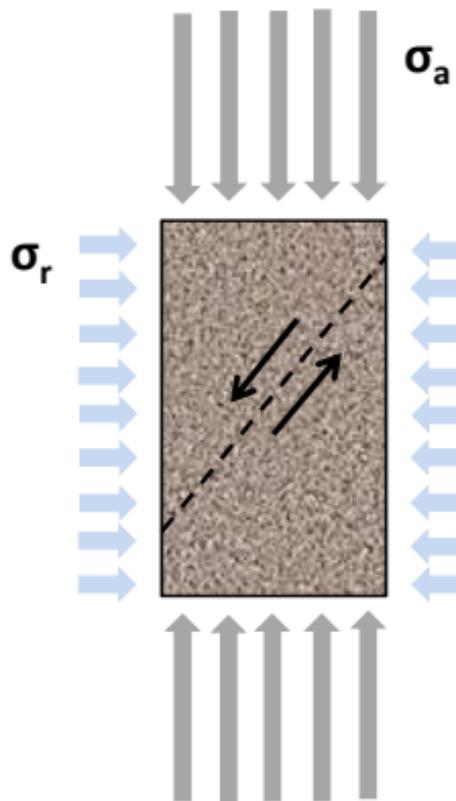




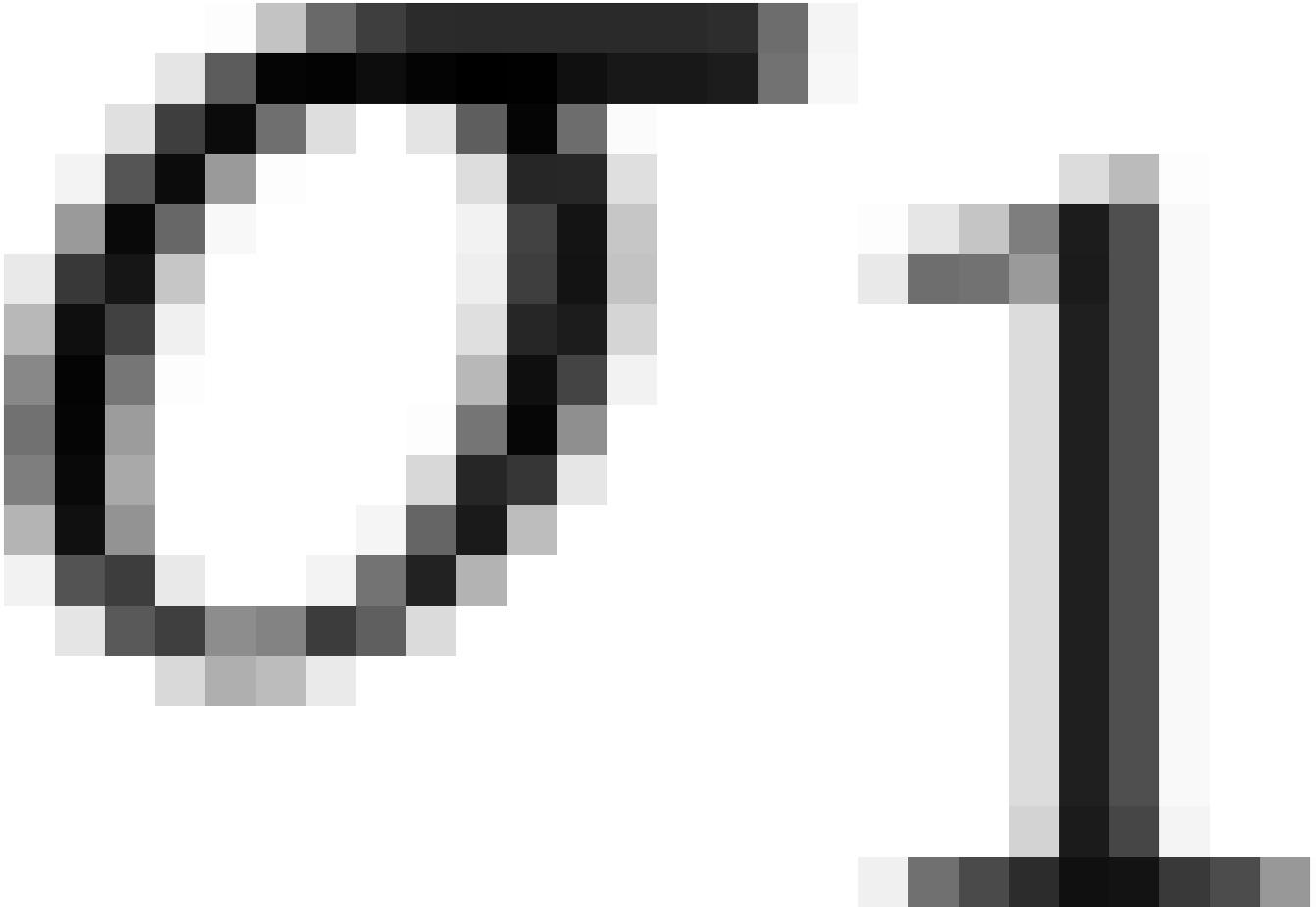


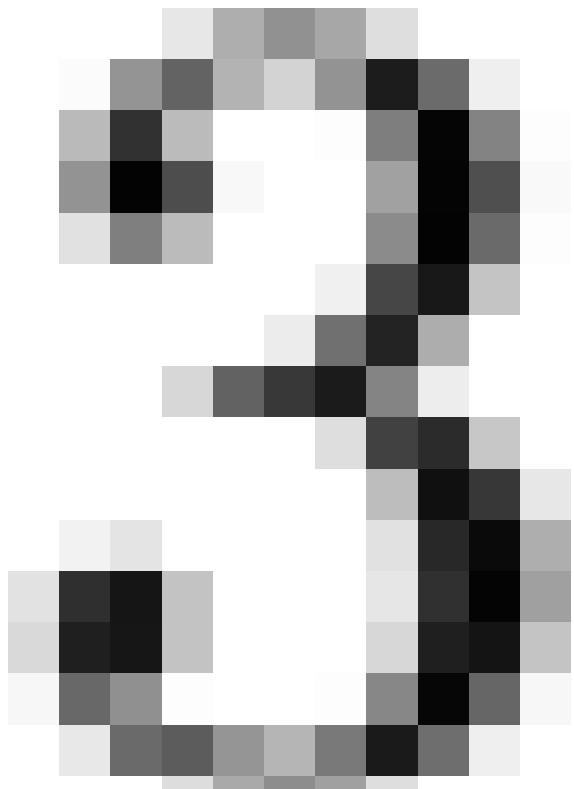
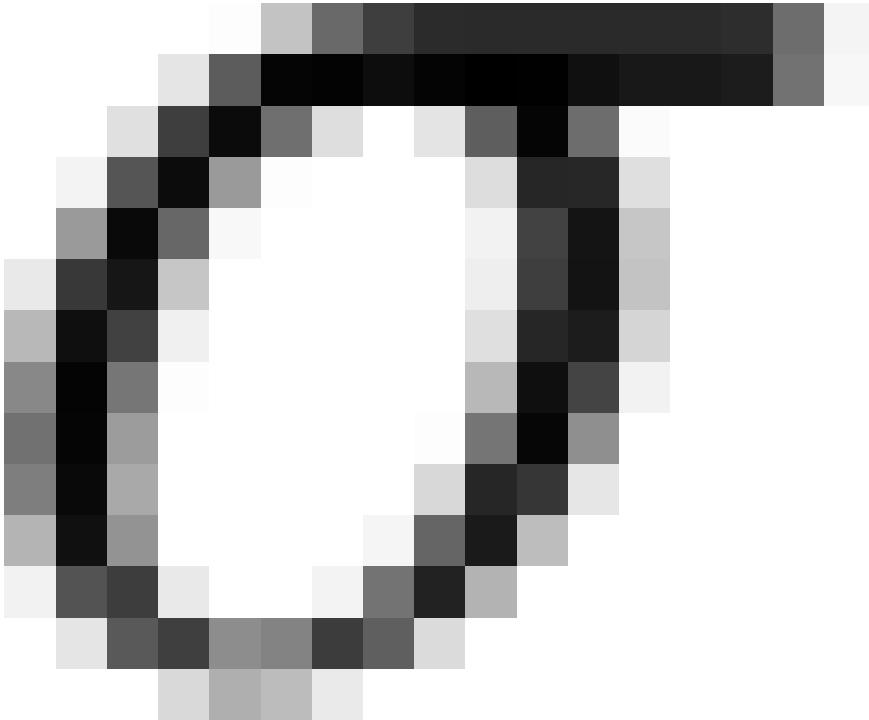
Which is point in the Mohr circle with maximum  $\tau/\sigma_n$ ?

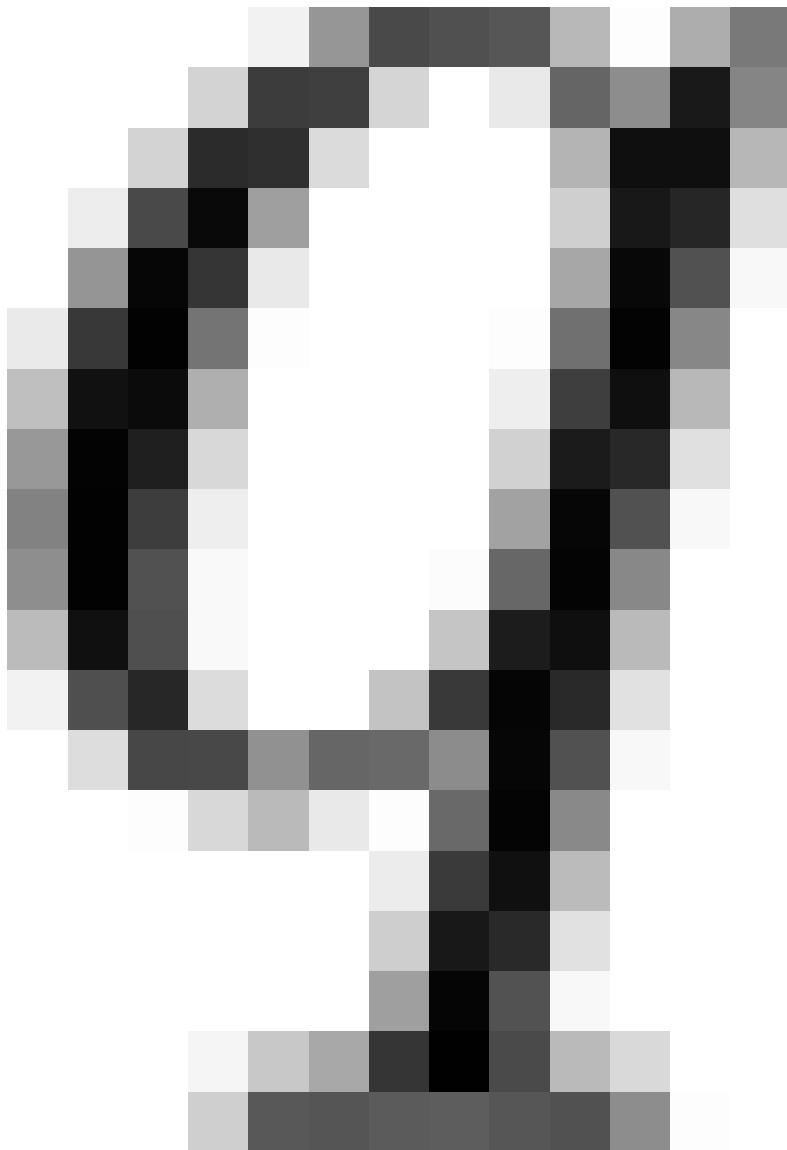
What is the angle of the failure plane?



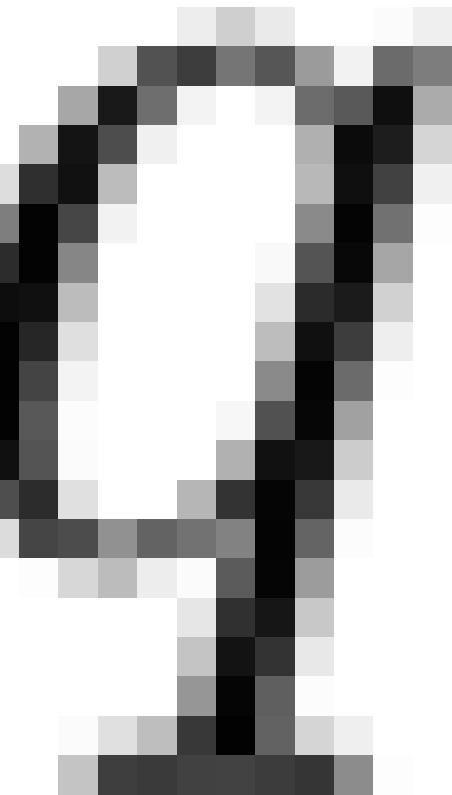
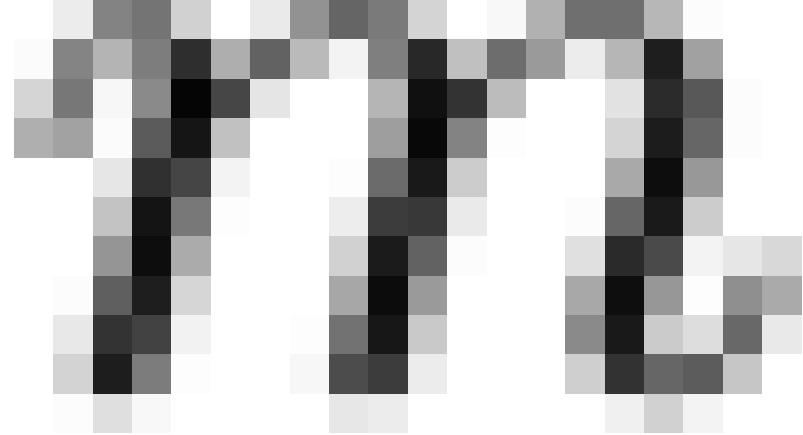
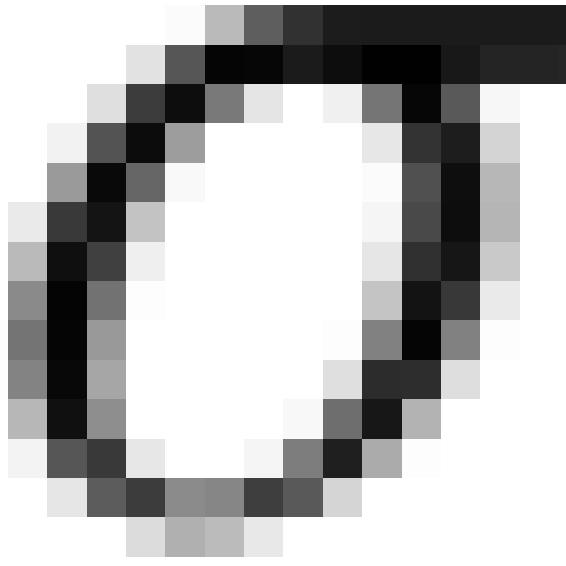






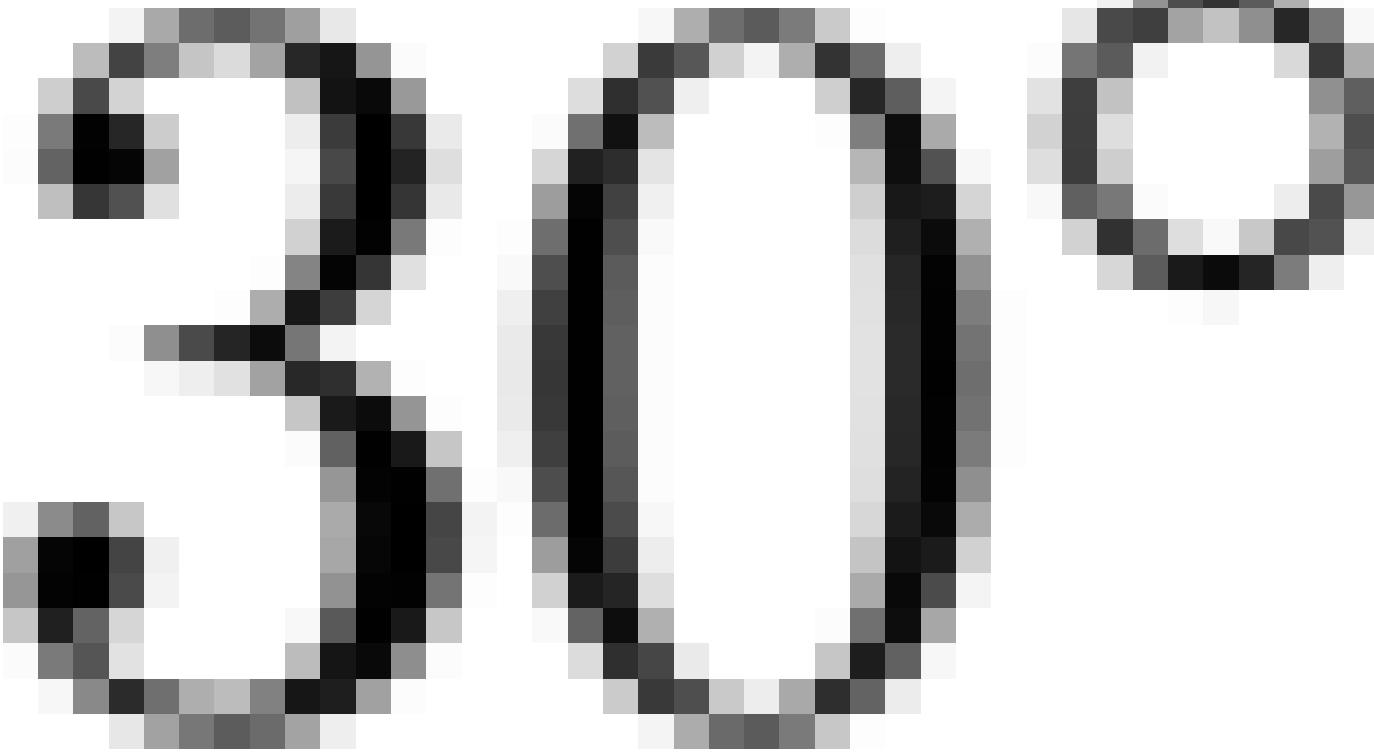
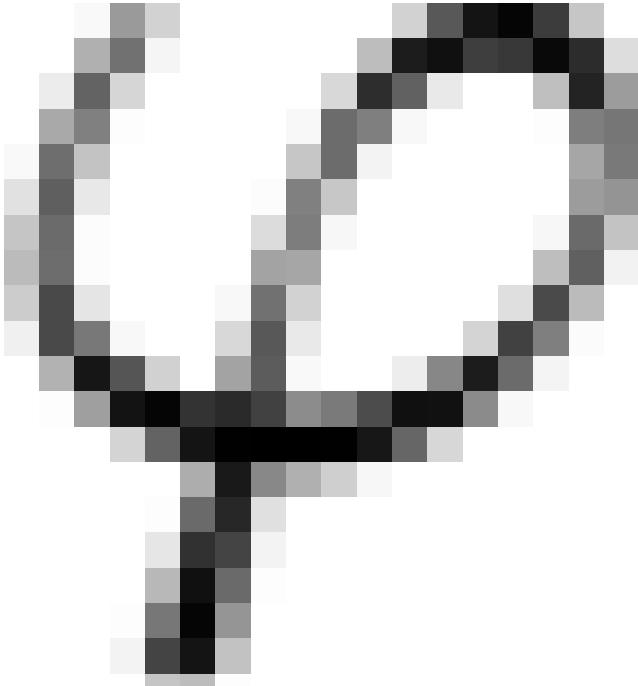


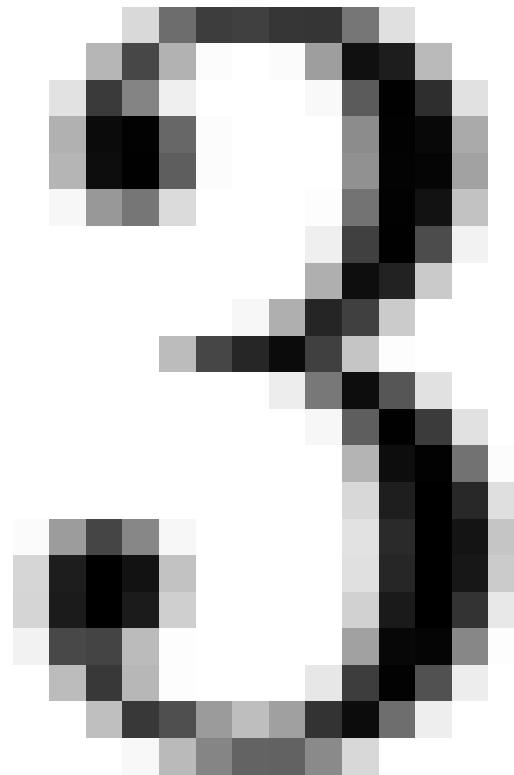
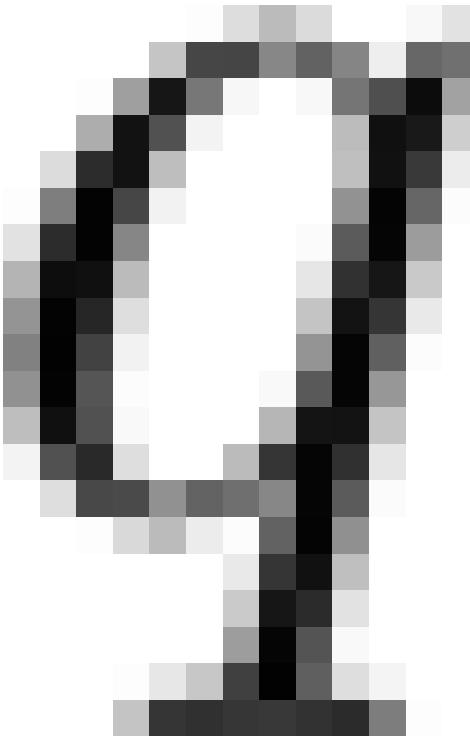




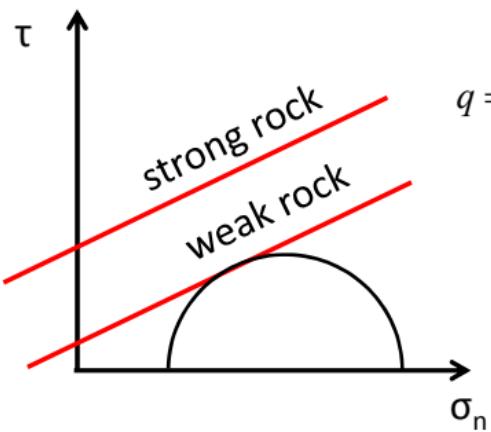
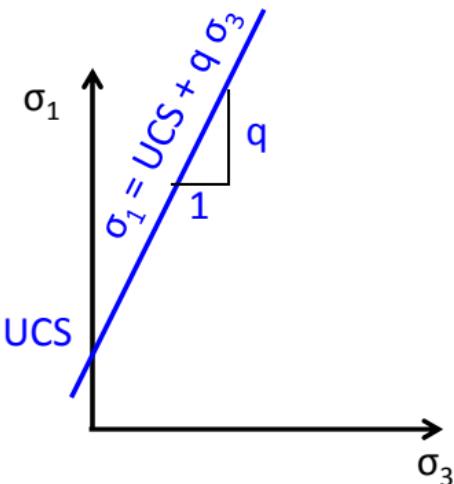
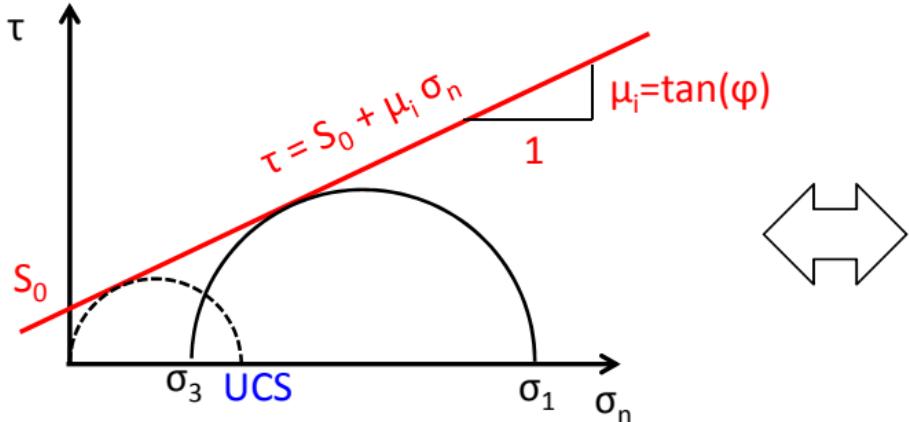
$\varphi$  sin  $\varphi$

$\varphi$  sin  $\varphi$



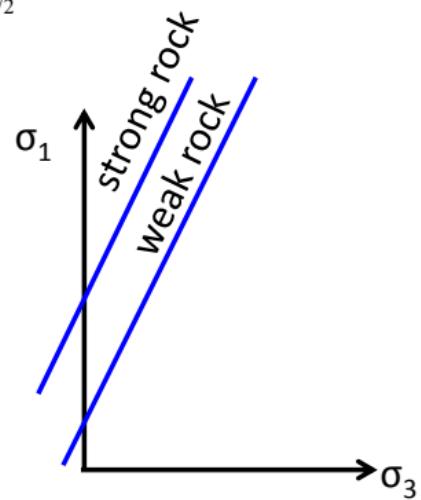


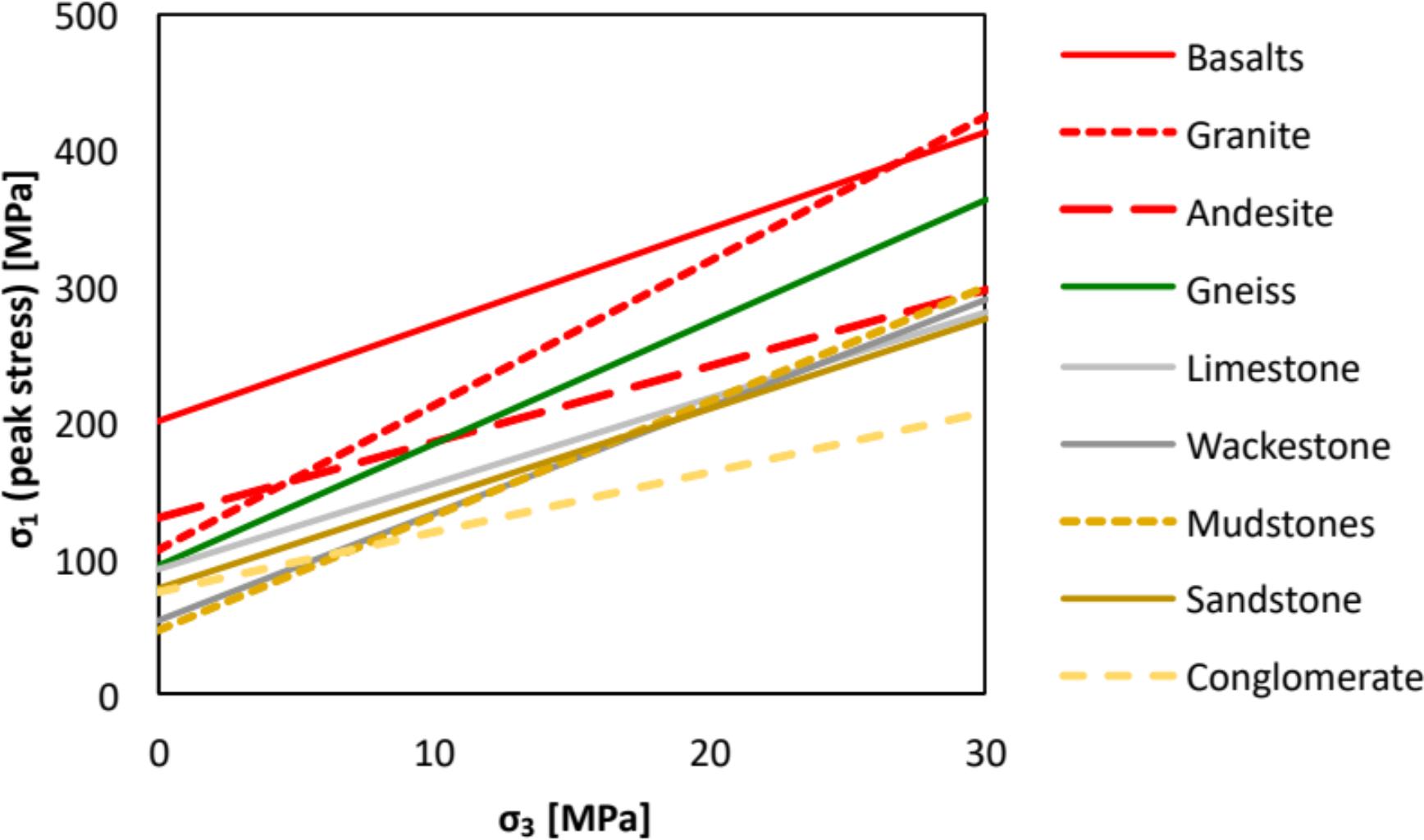
$$U_0 S = 2S_0 \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} = 2S_0 \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi}}$$



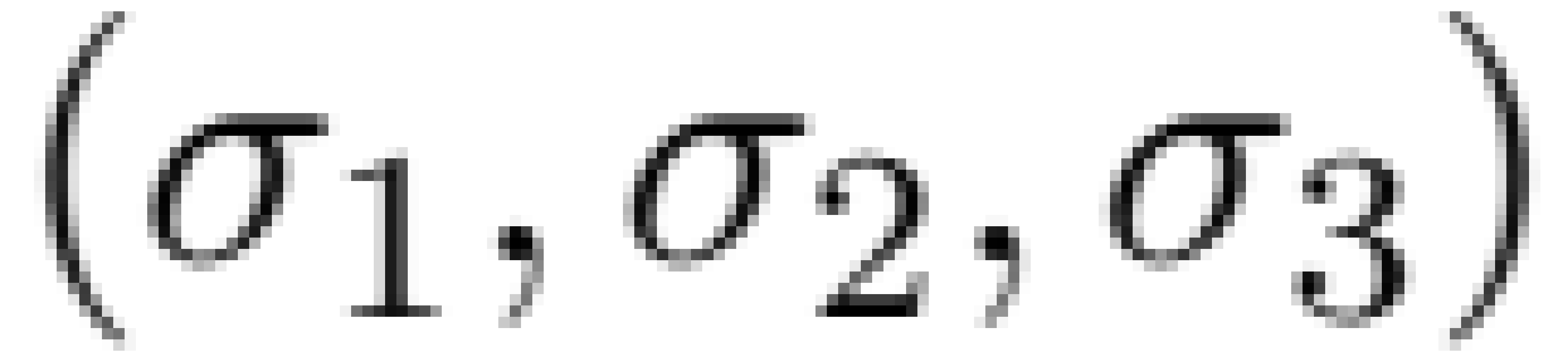
$$UCS = 2S_0 \left( \sqrt{\mu_i^2 + 1} + \mu_i \right) = 2S_0 \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2}$$

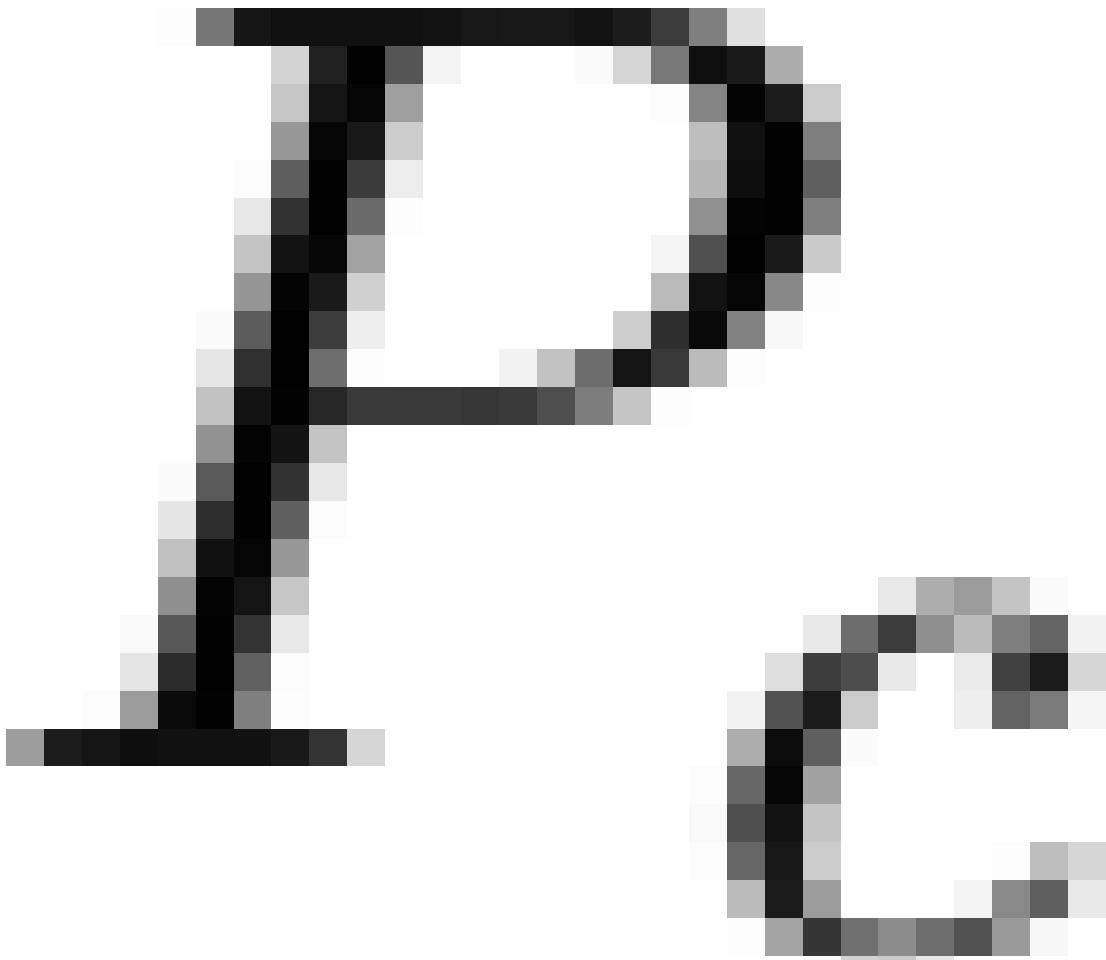
$$q = \left( \sqrt{\mu_i^2 + 1} + \mu_i \right)^2 = \frac{1 + \sin \varphi}{1 - \sin \varphi}$$

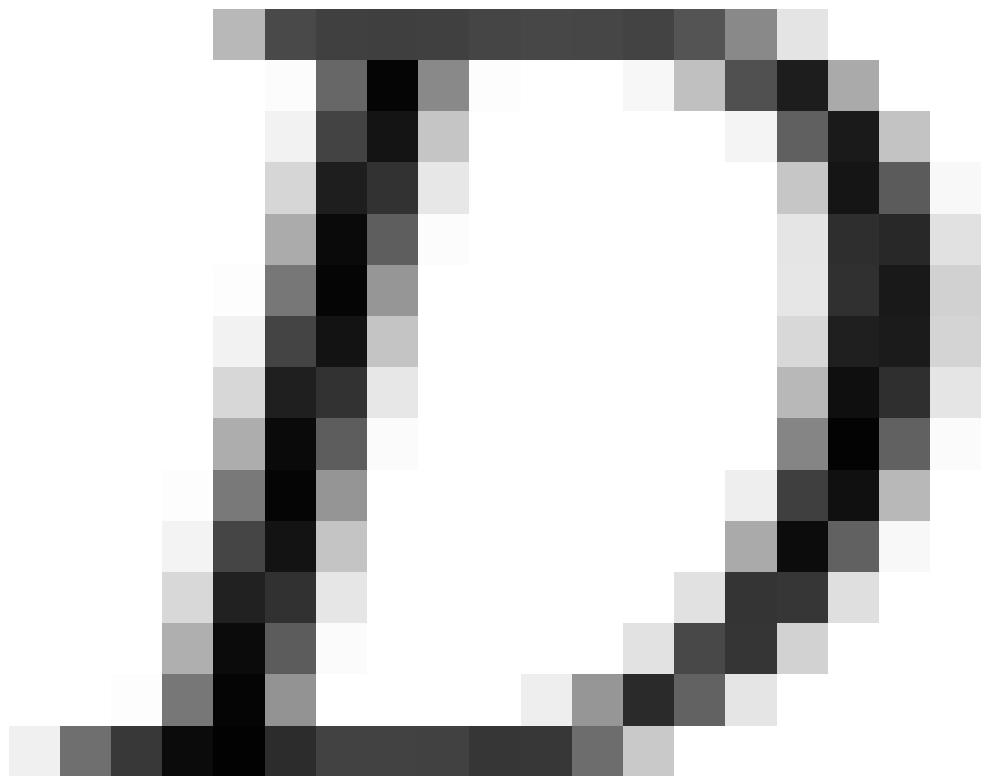
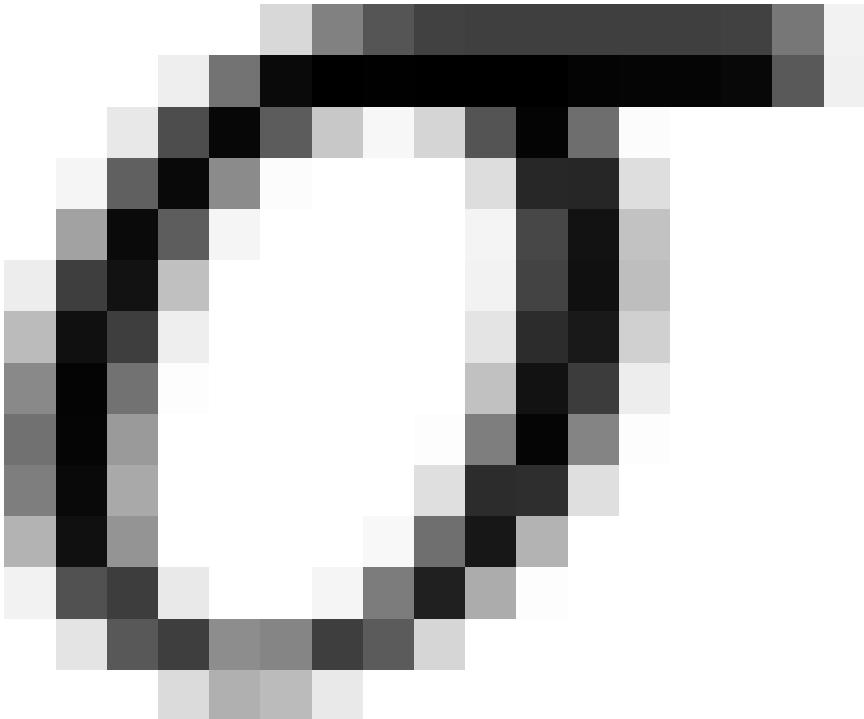


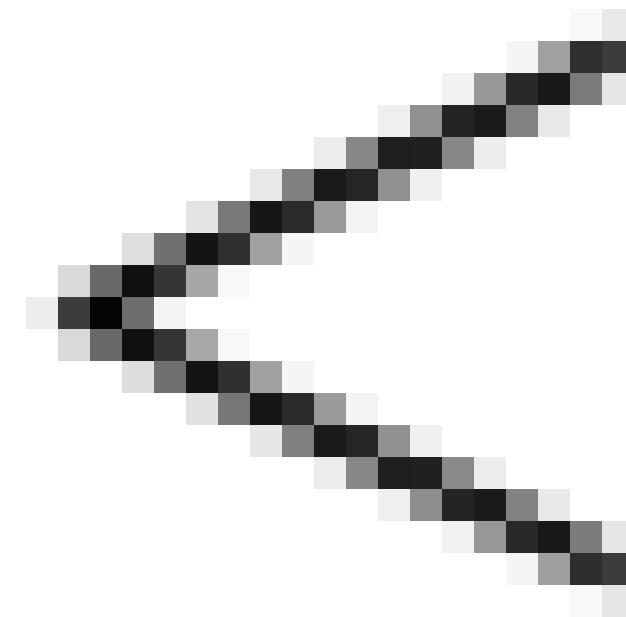
















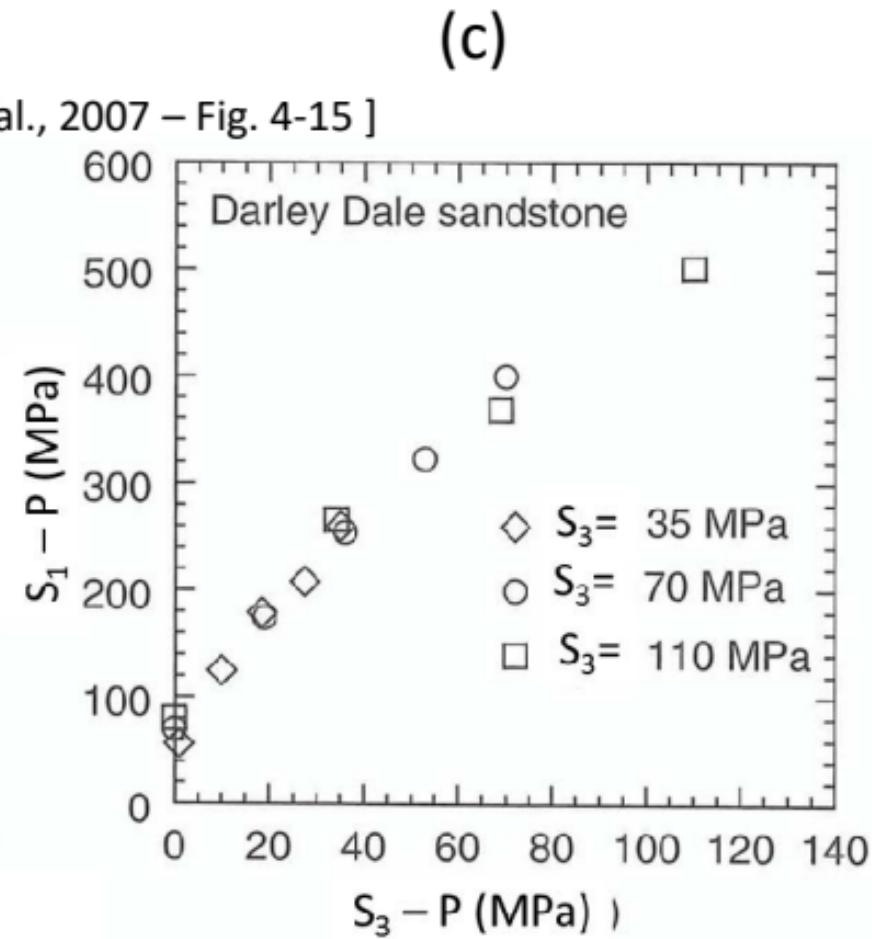
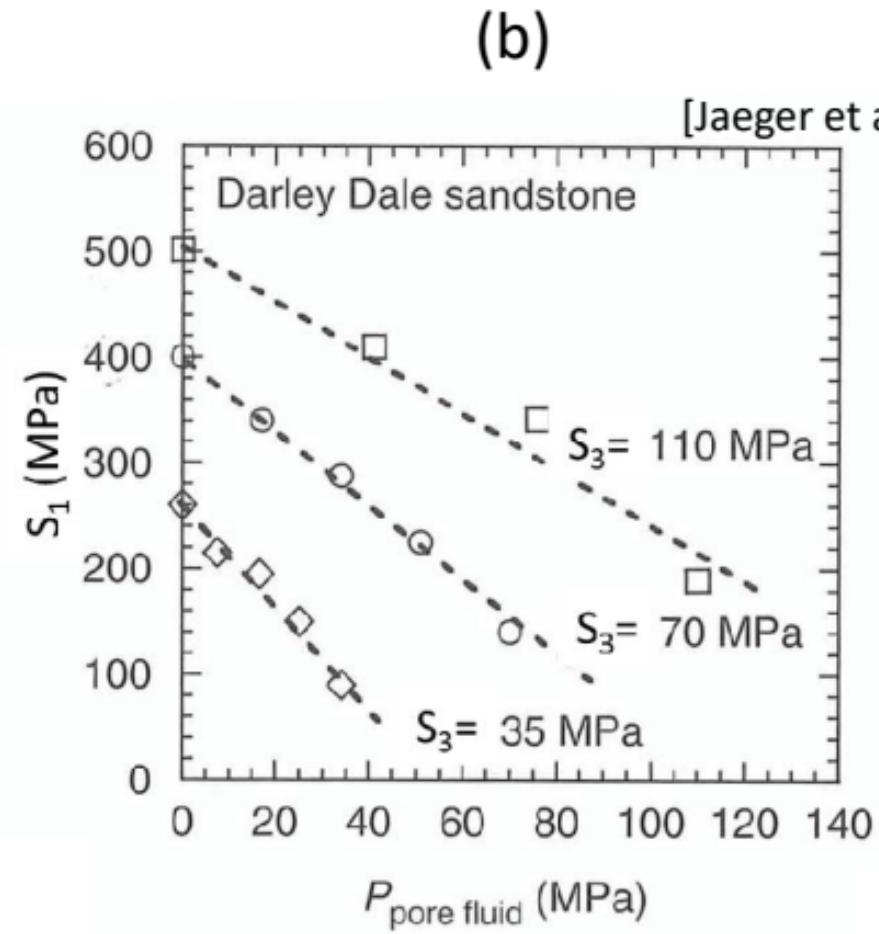
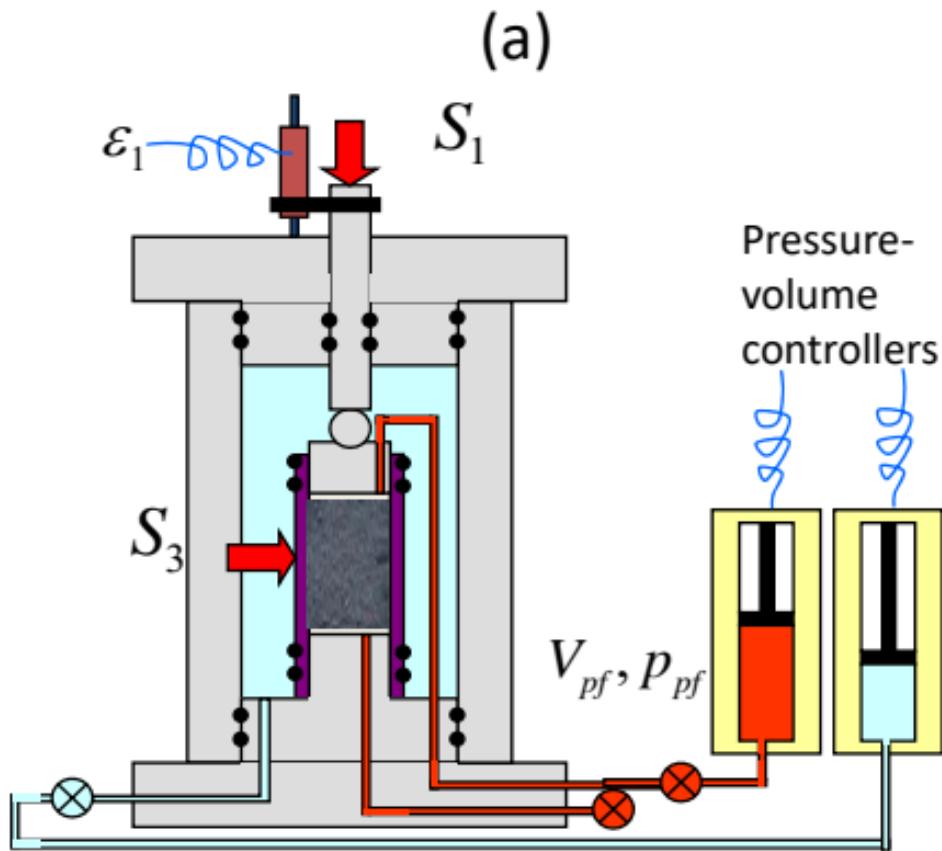


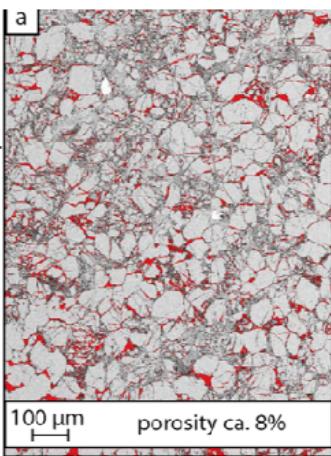
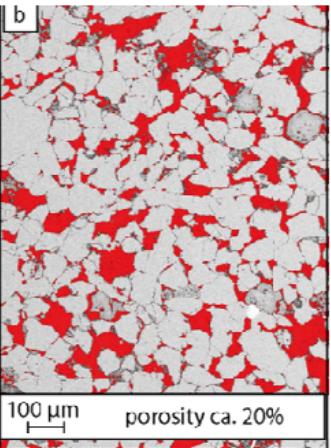
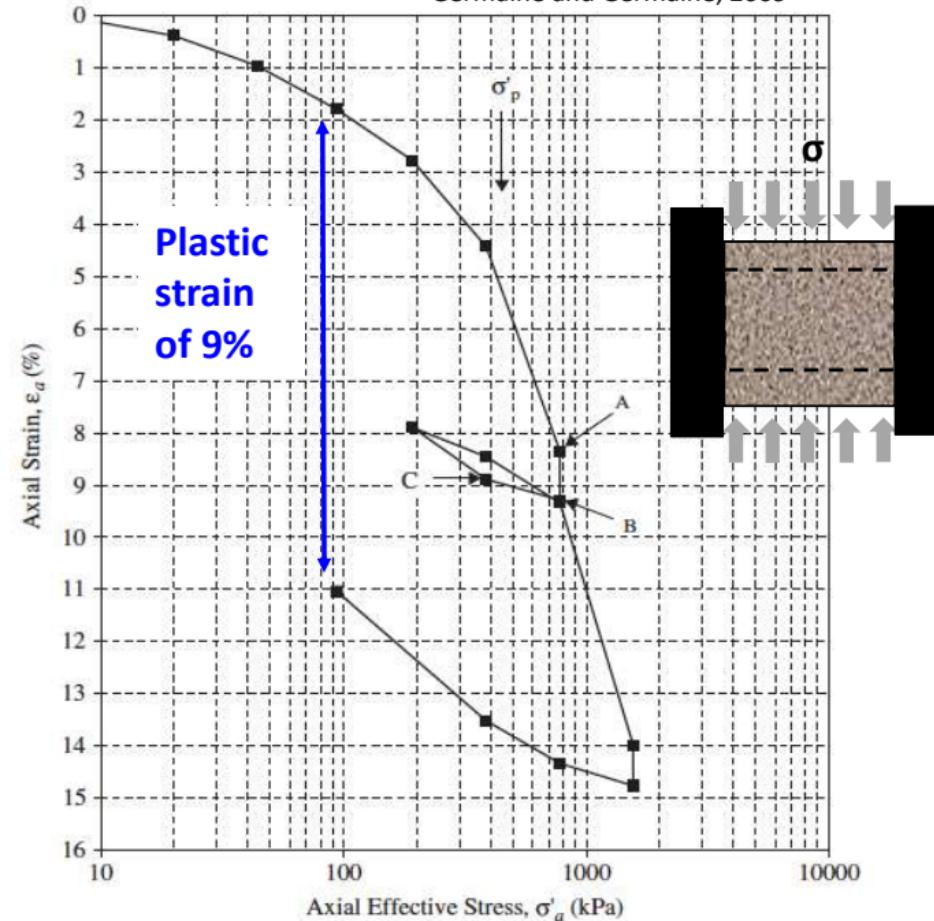




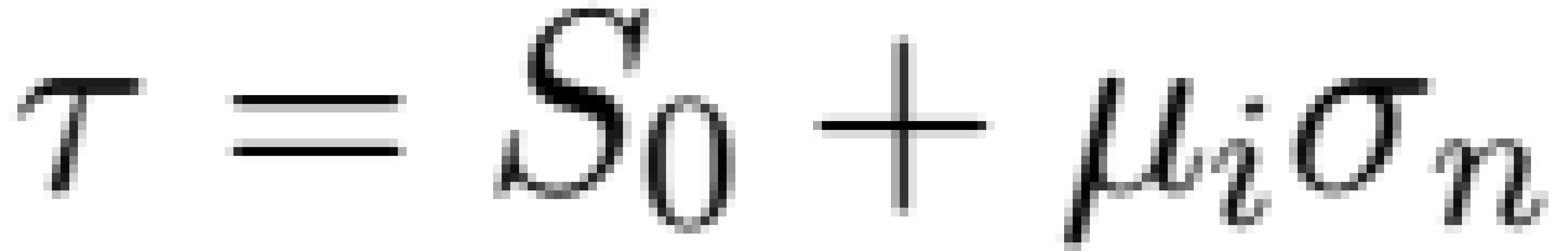


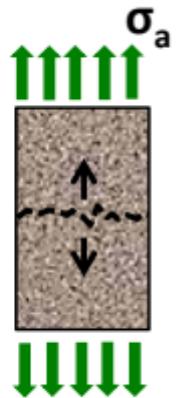








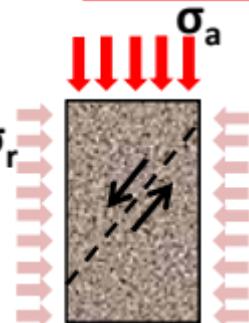


$\tau$ 

$$\sigma_n = T_s$$

 $T_s$ 

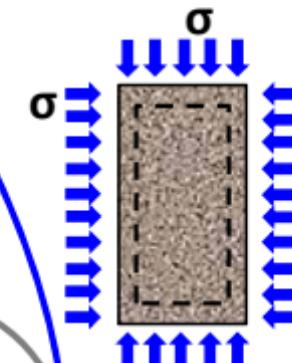
$$\boxed{\tau = \mu_i \sigma_n + S_0}$$



1

 $\mu_i$ 

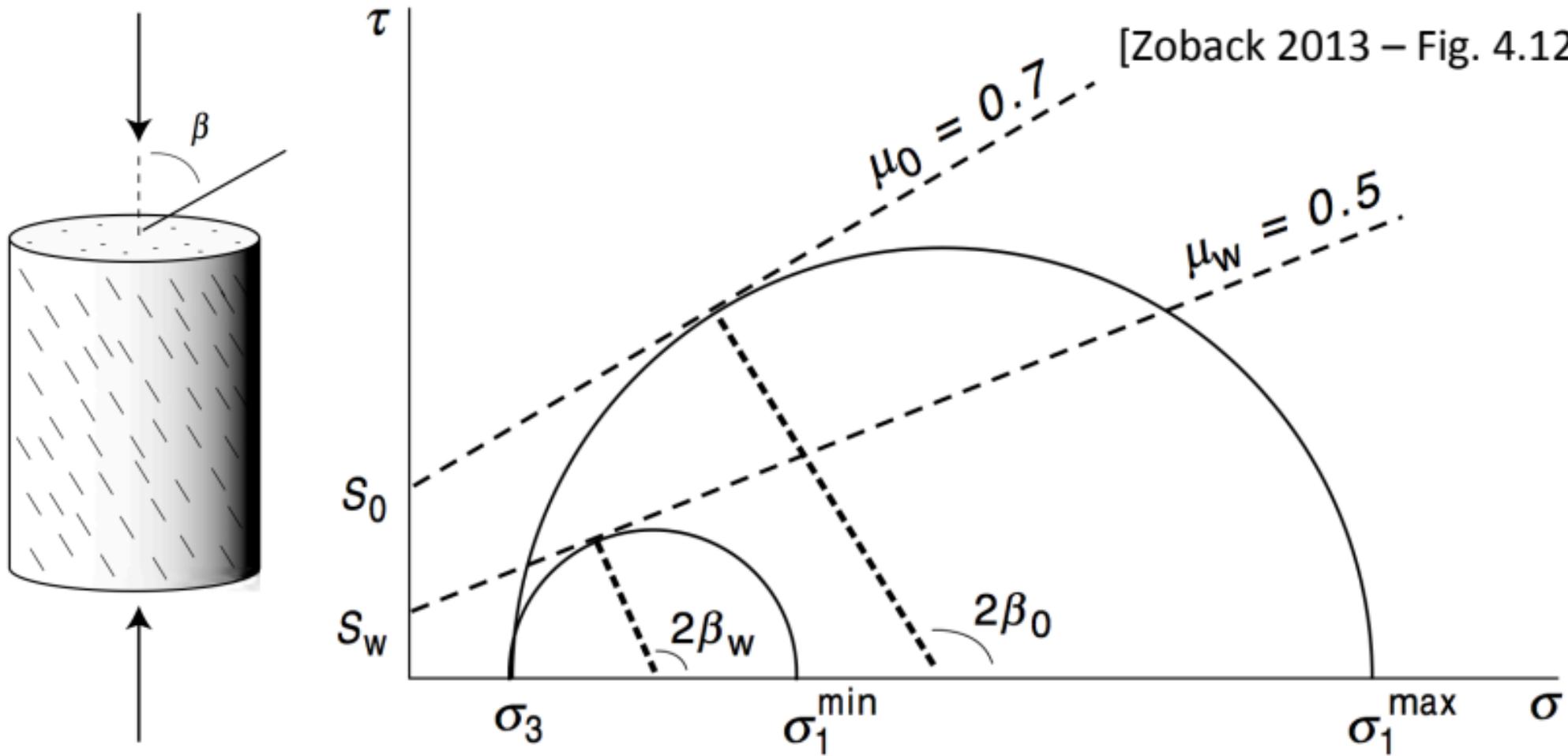
$$\boxed{\mathcal{E}_V^p = \mathcal{E}_{Vcrit}^p}$$

 $\sigma_n$





[Zoback 2013 – Fig. 4.12]

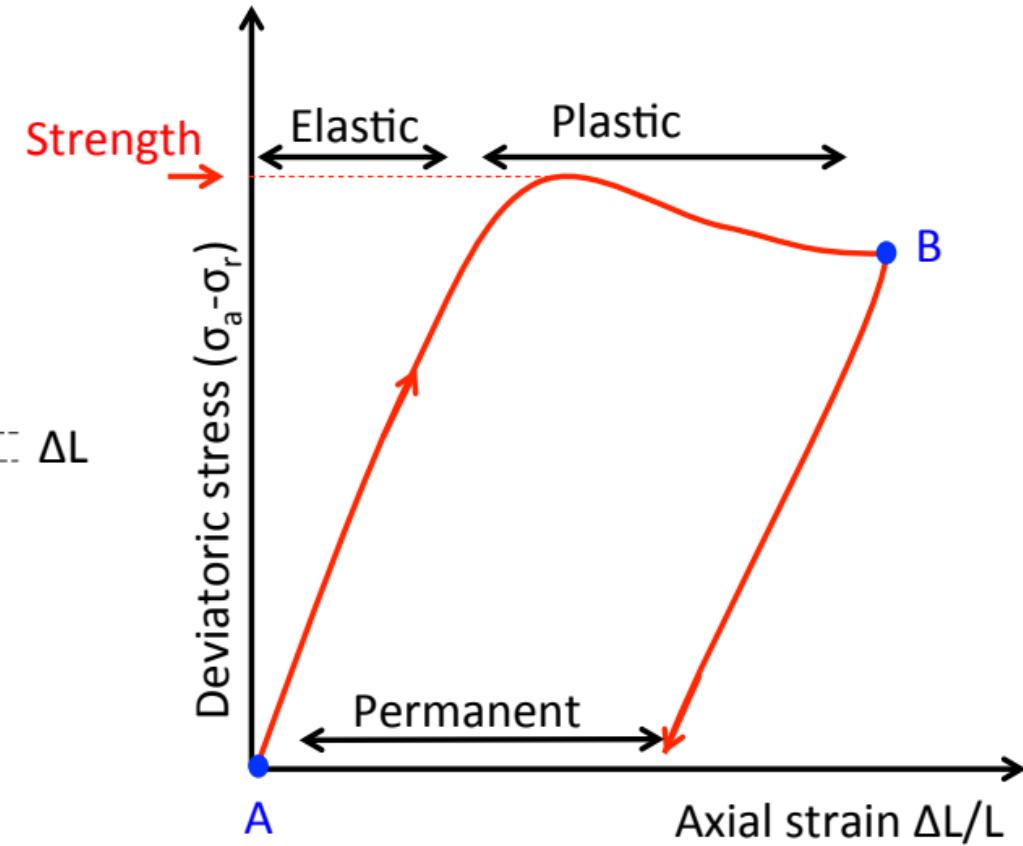
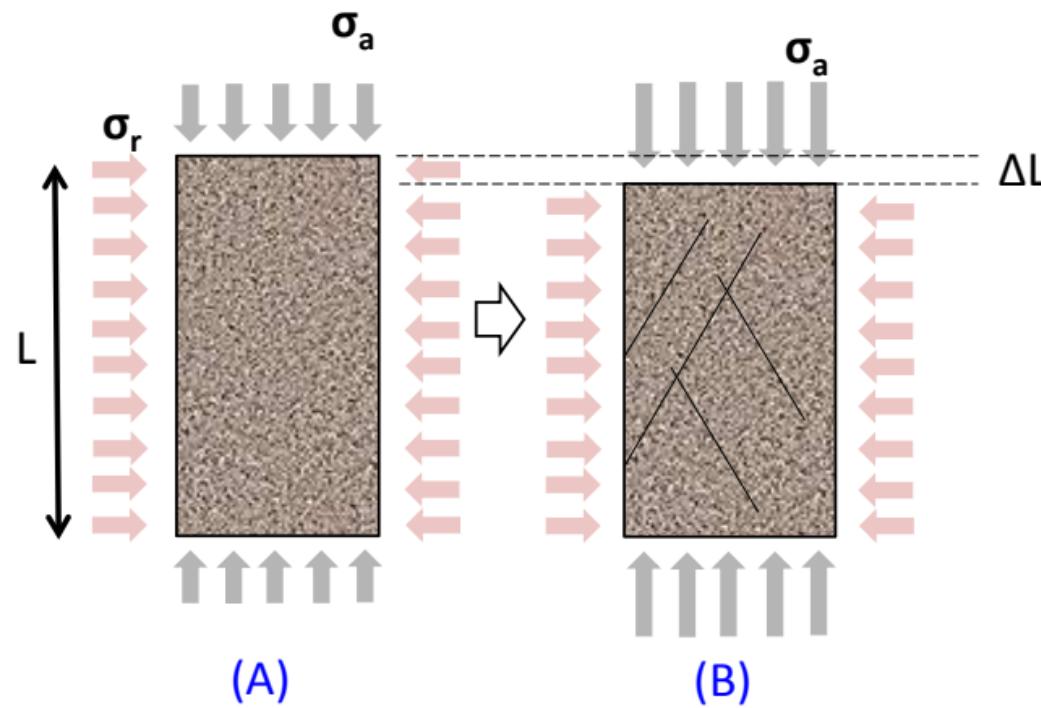


# Rock deformation

Relation strain V.S. stress

Elastic (Young modulus)

Plastic (~Viscosity)



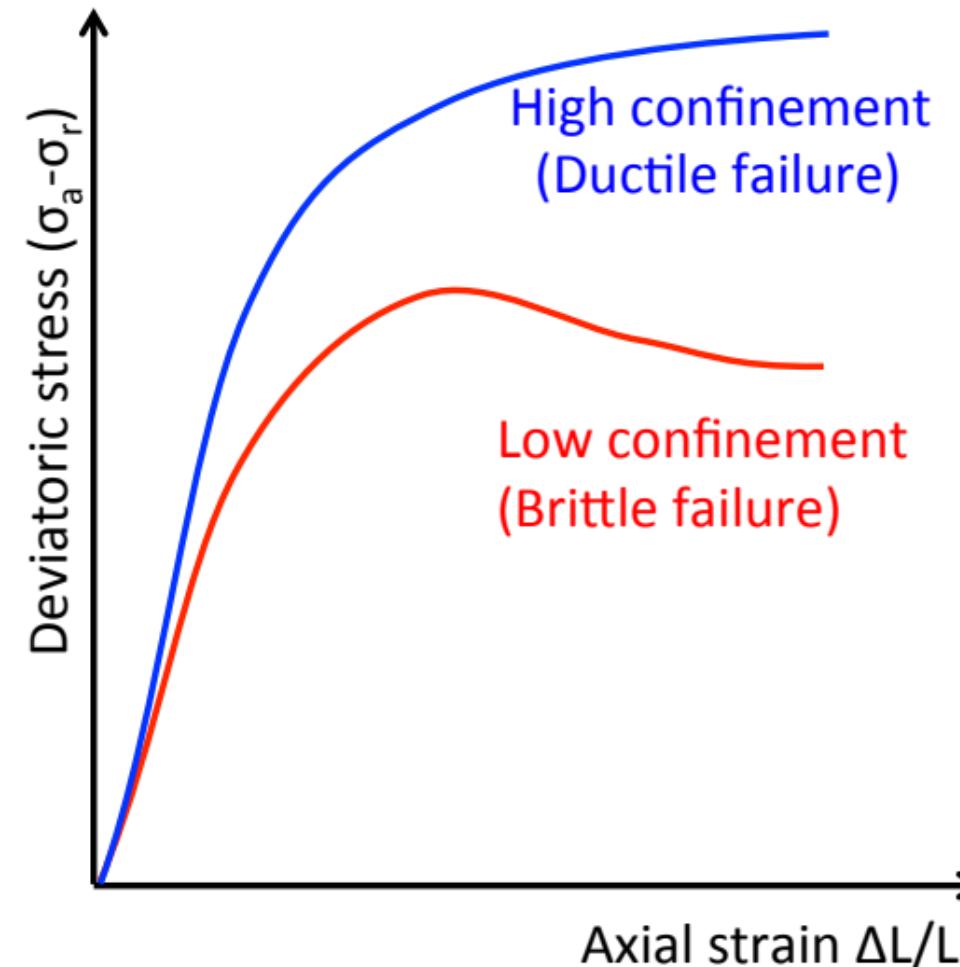
a. This is an undeformed cylinder of rock.



b. This cylinder was subjected to high confining pressure (uniform in all directions) and, at the same time, compression from above. It deformed in a ductile manner, becoming shorter and fatter.



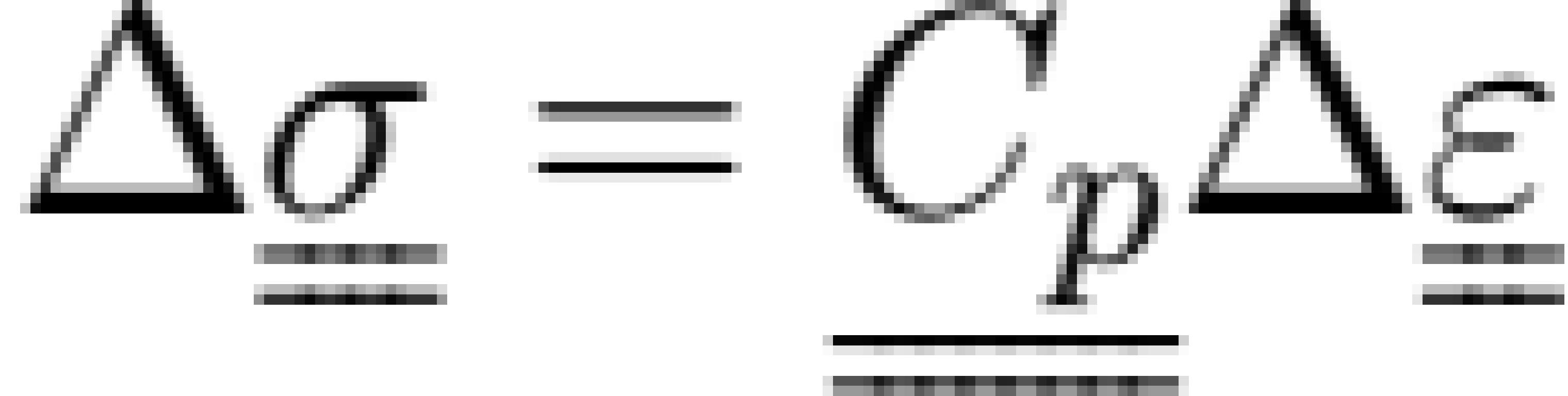
c. An identical cylinder was subjected to the same amount of compression from above, but this time with a lower confining pressure. It deformed in a brittle manner, with many large fractures.



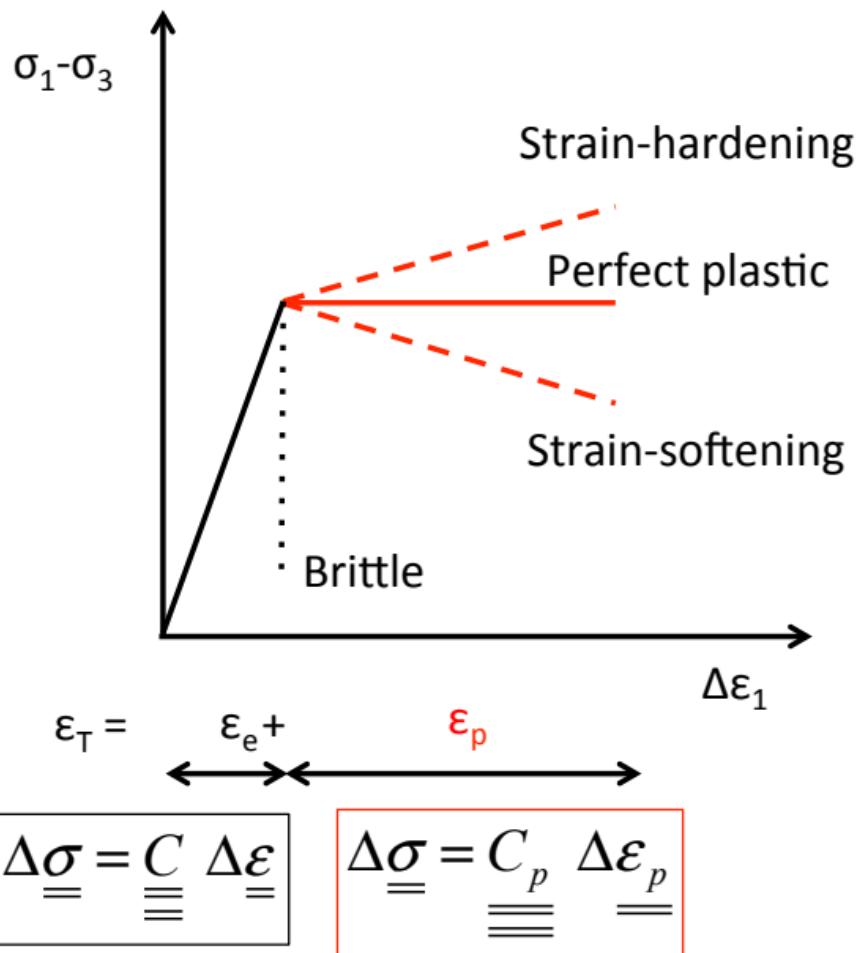
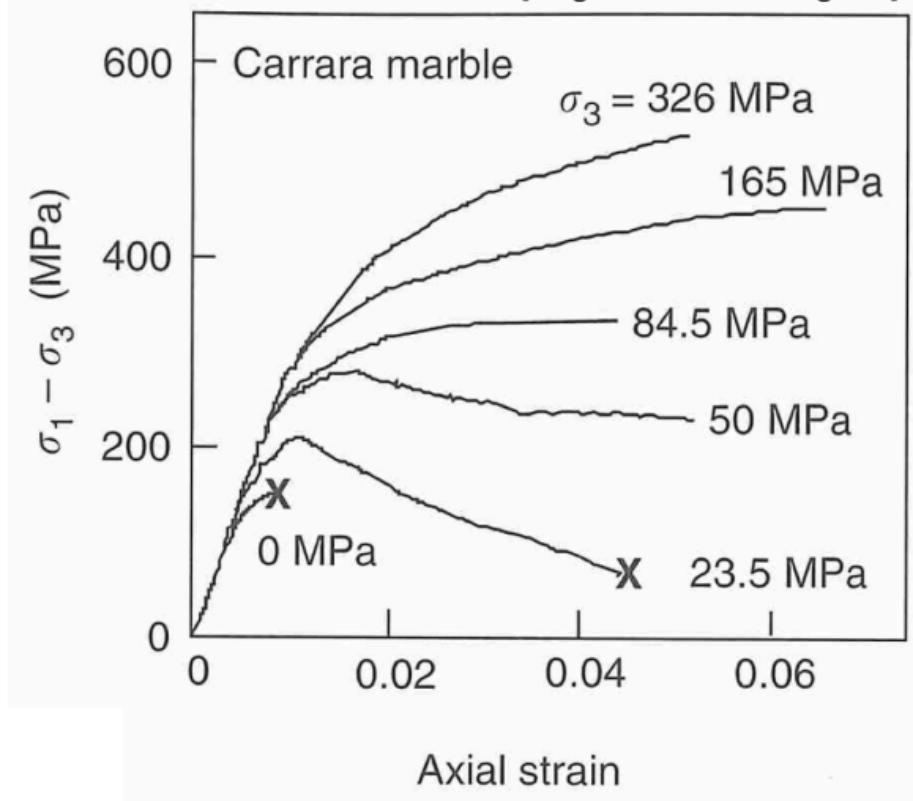




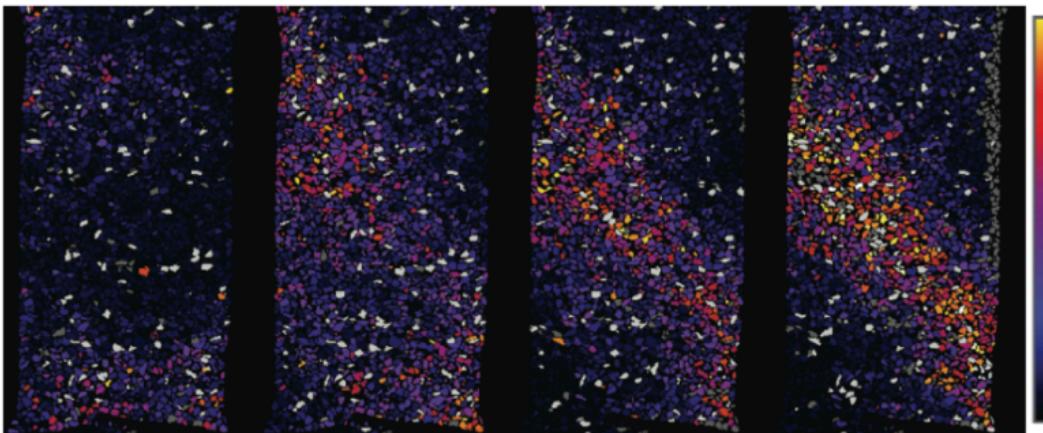
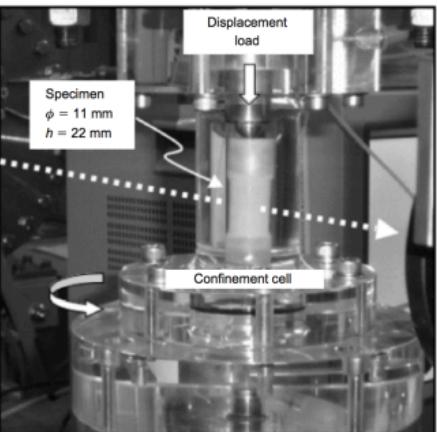




[Jaeger et al. 2007 – Fig. 4.5]



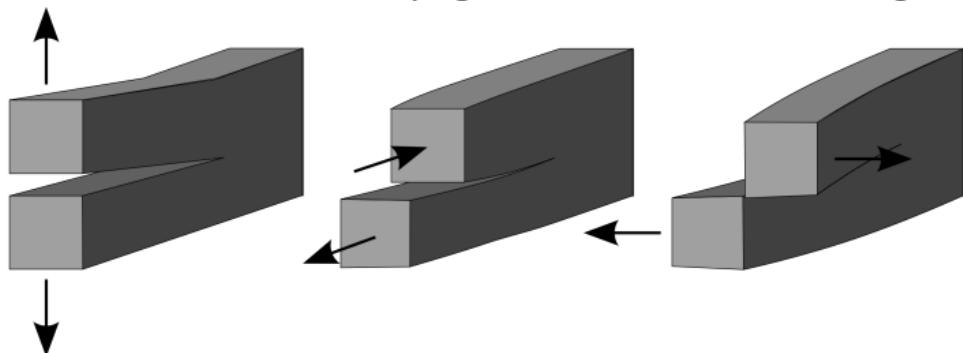
**(a) Uncemented or poorly cemented rock →**



grain friction, dilation, grain crushing/rotation

**(b) Cemented rock →**

Propagation of microfractures, grain friction/crushing

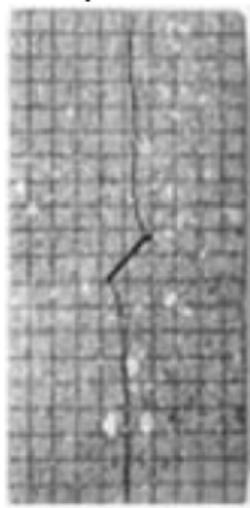


Stress intensification at  
the tip of fractures

Propagation starts at  
fracture tips

# Napolitan Tuffo

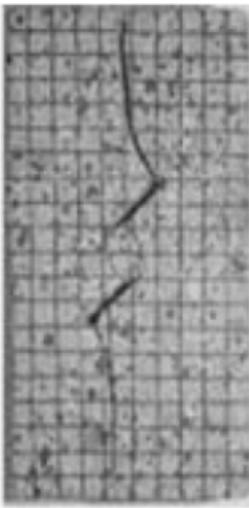
[Hall et al. 2006 – Pure Appl. Geophys.]



Single flaw



$\beta=45^\circ$

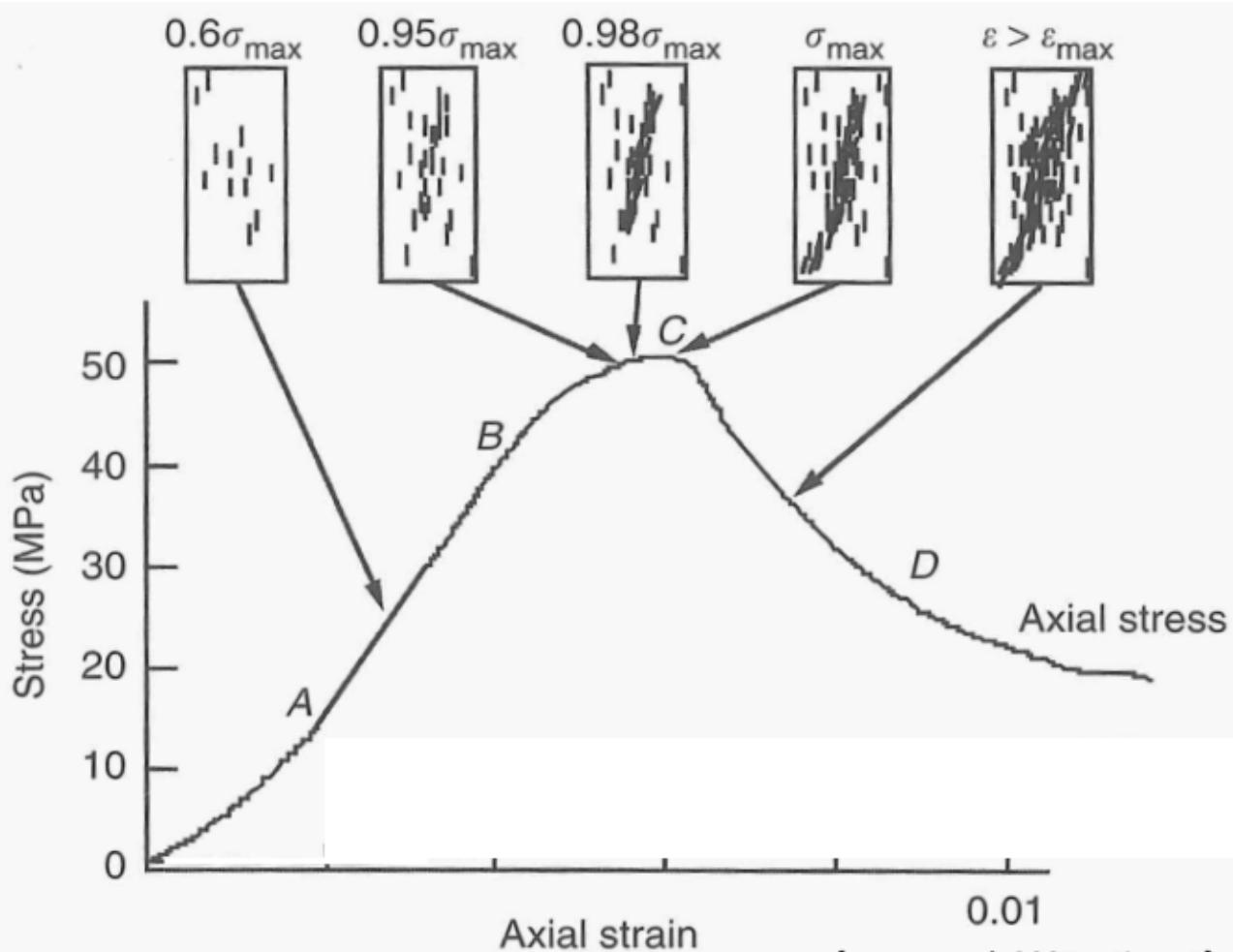


$\beta=105^\circ$



$\beta=120^\circ$



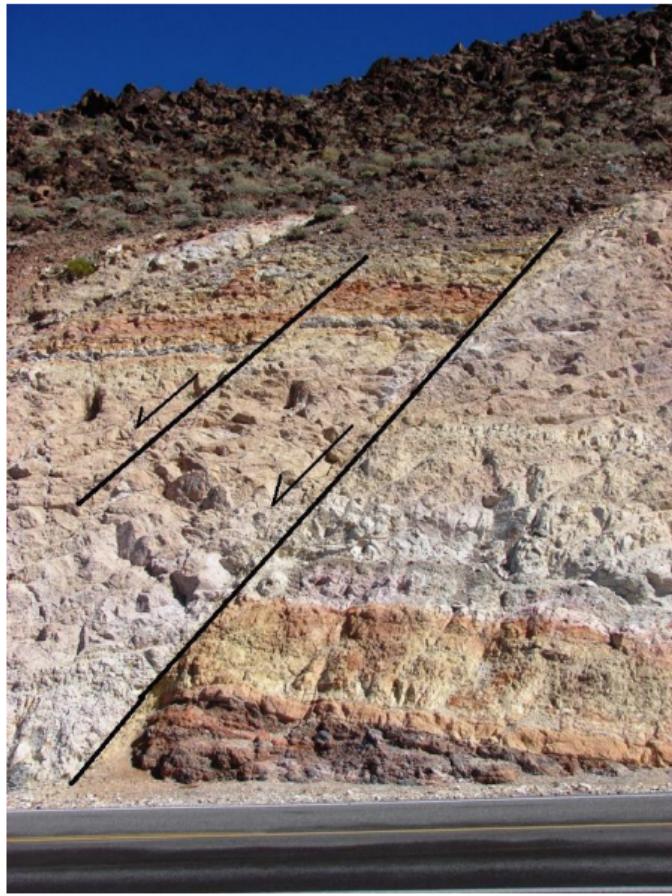


[Jaeger et al. 2007 – Fig. 4.5]



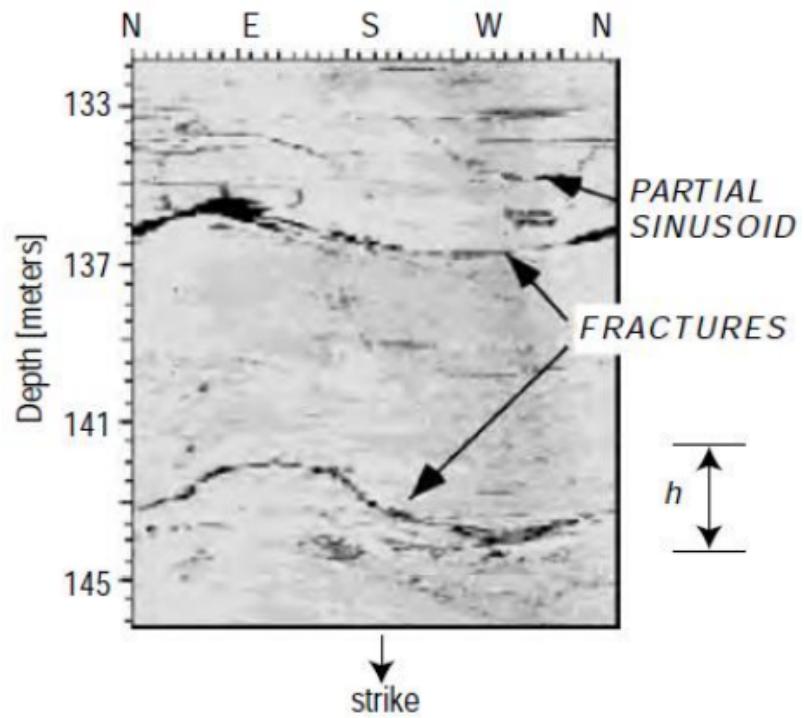
Outcrop of a normal fault in Split Mountain gorge

<http://geology.csupomona.edu/janourse/TectonicsFieldTrips.htm>

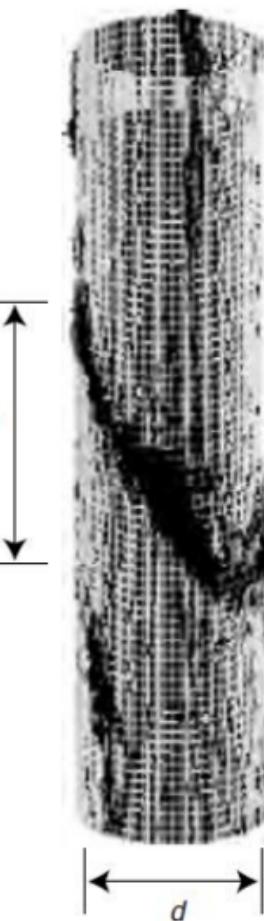


<http://geotripperimages.com/images/DSC03438%20Charlie%20Brown%20b.jpg>

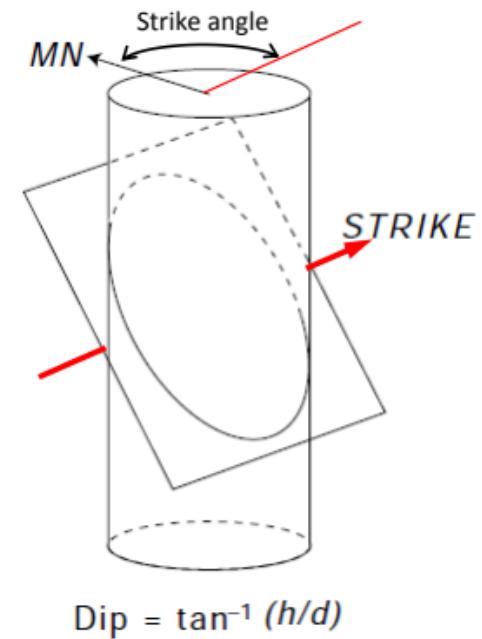
**(a) Un-wrapped image  
(ultrasonic)**



**(b) 3D-representation**



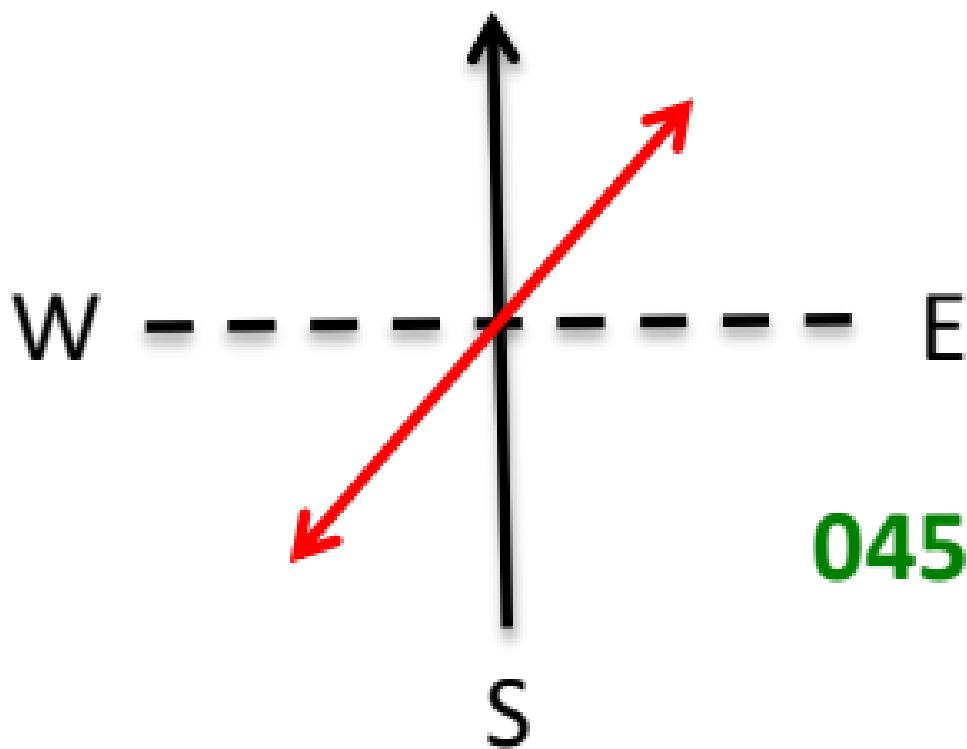
**(c) Interpretation**



[Zoback 2013 - Figure 5.3]



**N45°E==S45°W**



**045° == 225°**

# Geologic map

Figure from Prof. Prodanovic

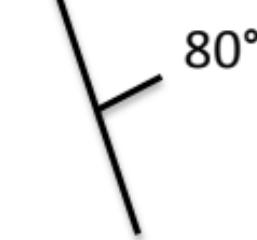


a)

$N30^\circ E$ ,  $45^\circ NW$

or

$030^\circ$ ,  $45^\circ NW$



b)

$N10^\circ W$ ,  $80^\circ NE$

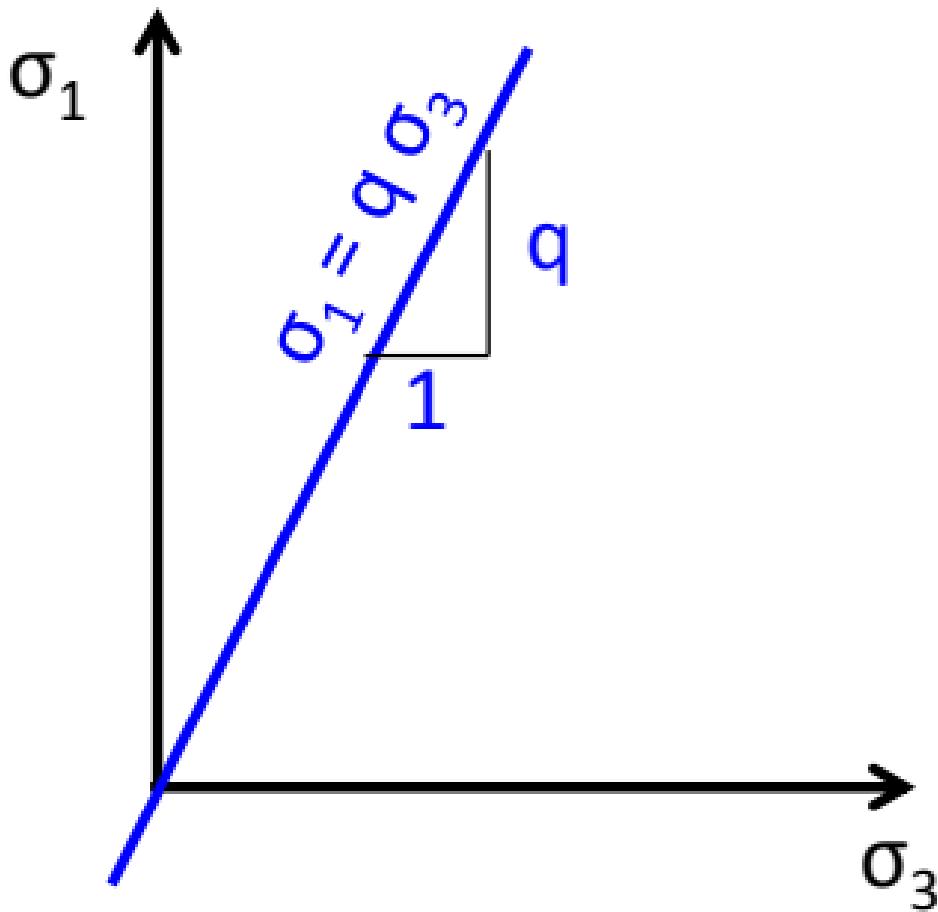
or

$350^\circ$ ,  $80^\circ NE$



c)

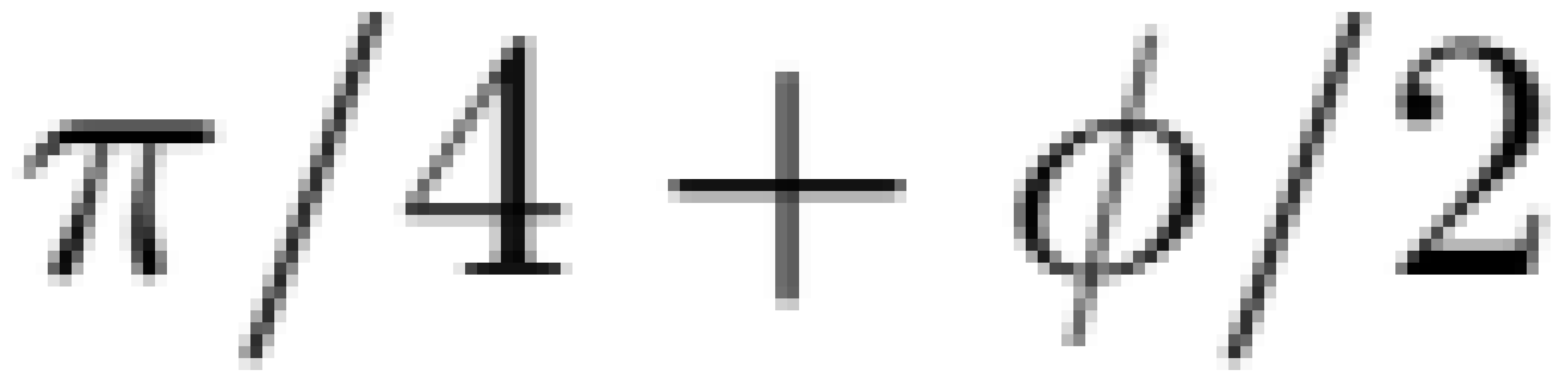
Symbols for  
horizontal plane  
and vertical plane

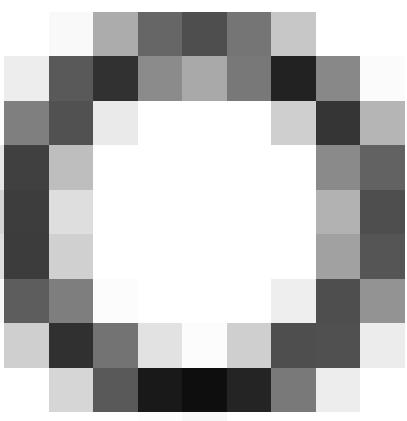
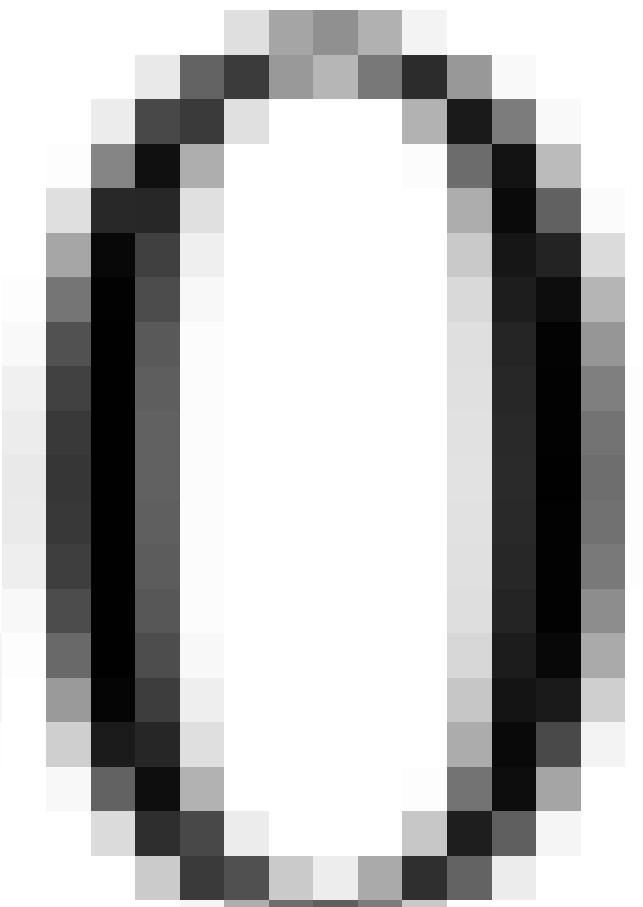
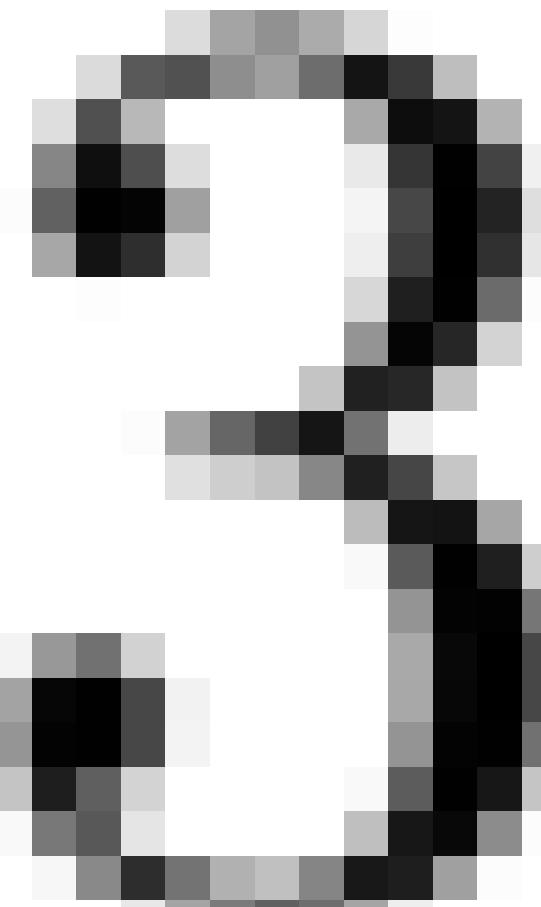
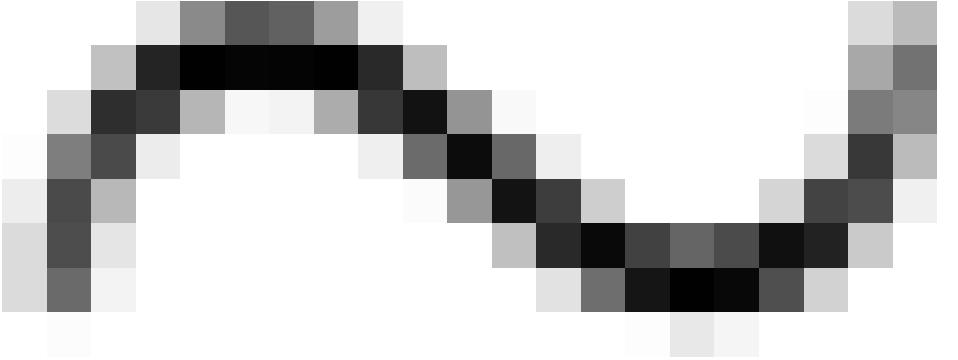


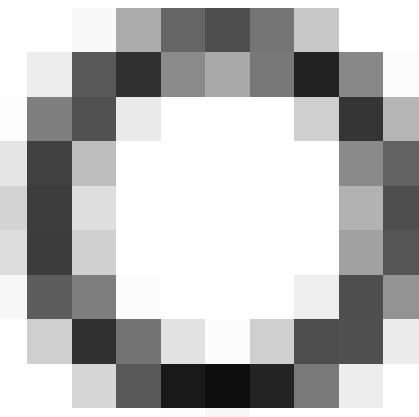
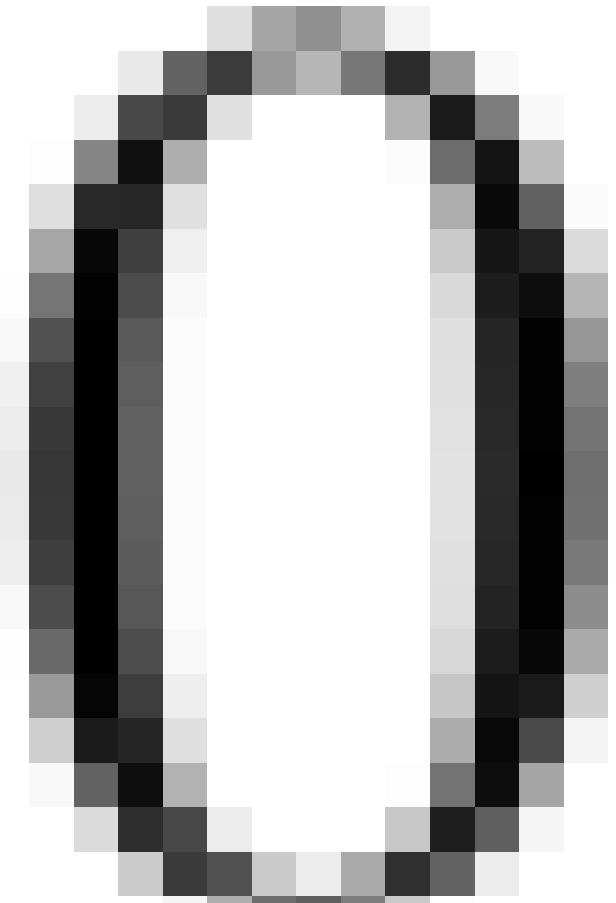
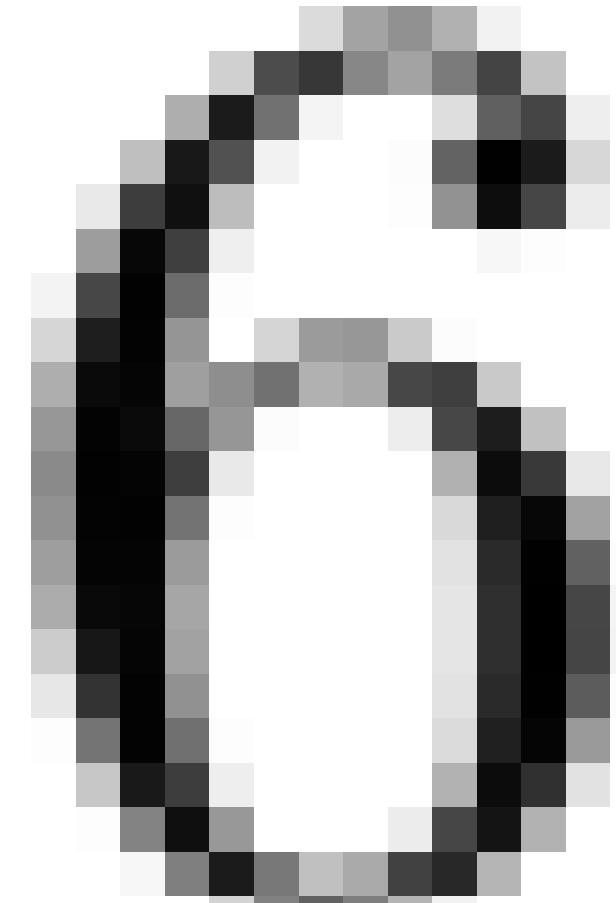
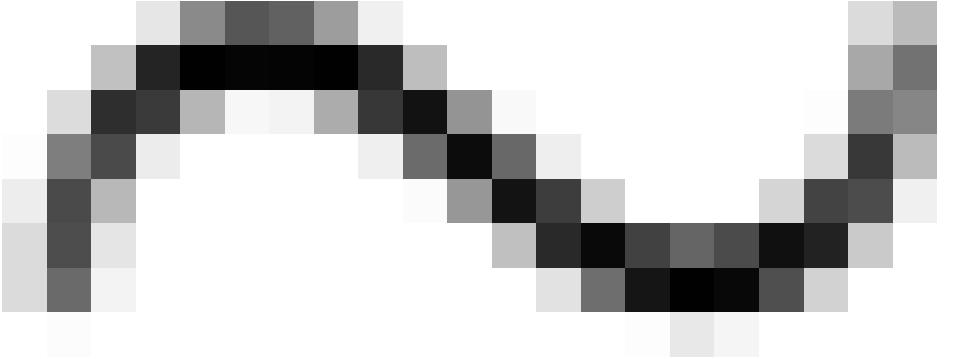


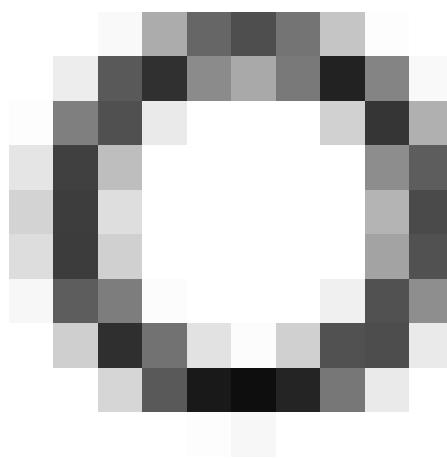
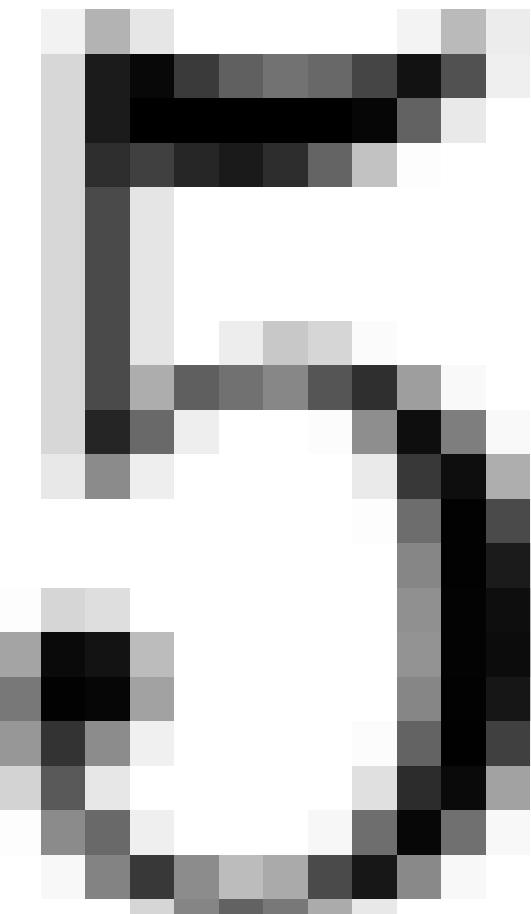
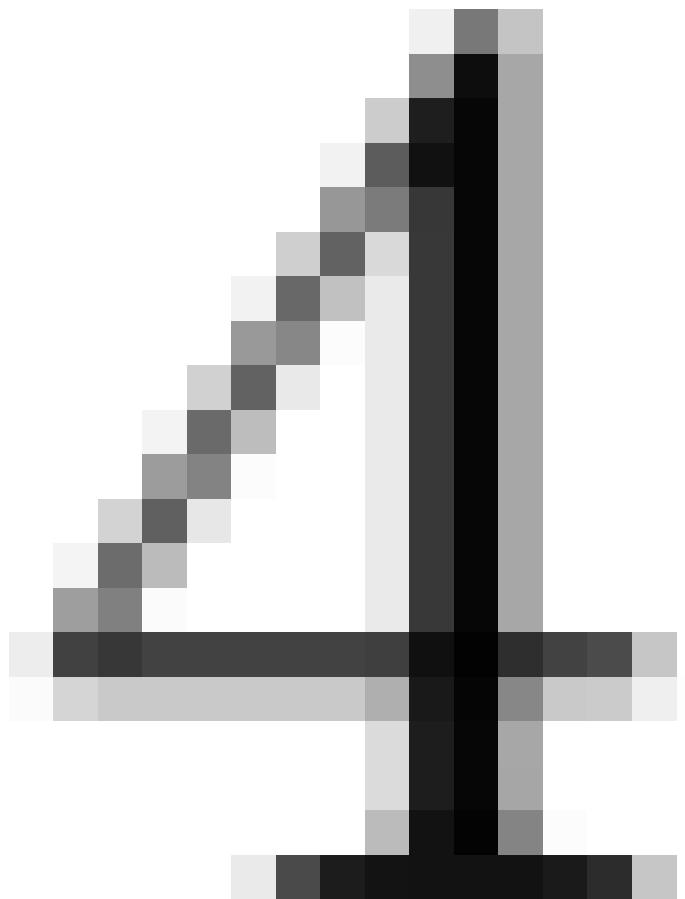
$$\begin{aligned} q &= \sin^2 \varphi + \frac{1}{\sin^2 \varphi} \\ &= \frac{1}{\sin^2 \varphi} + \frac{1}{\sin^2 \varphi} \end{aligned}$$

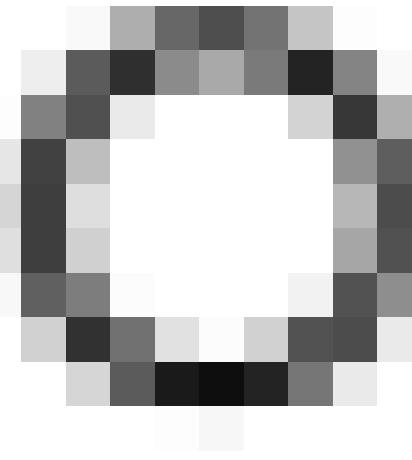
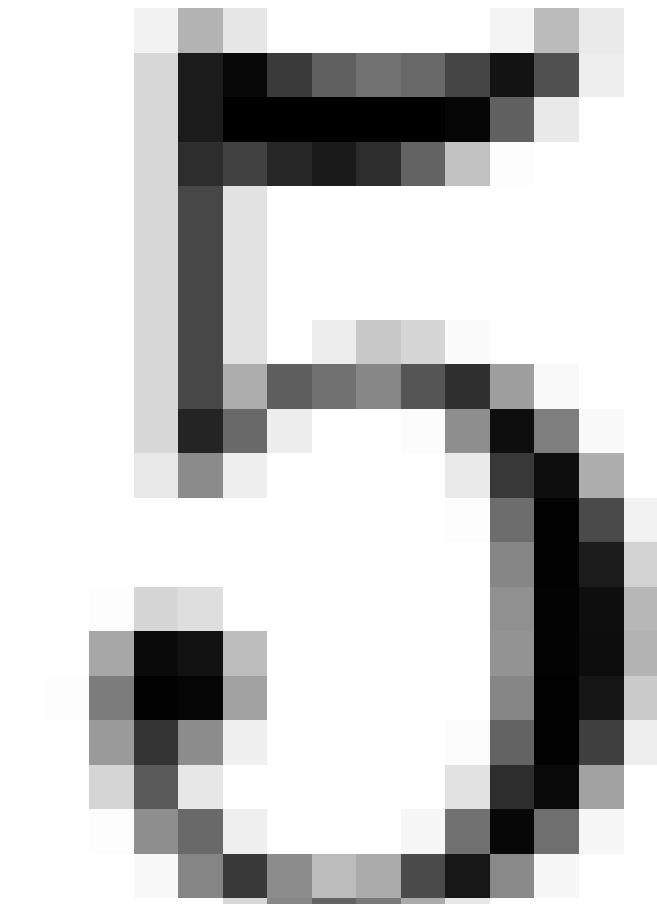
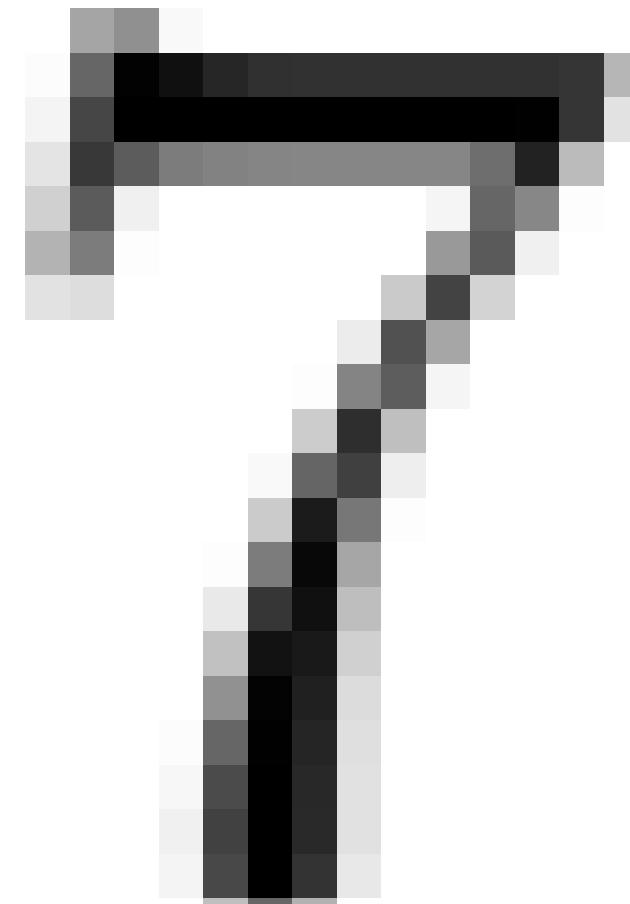
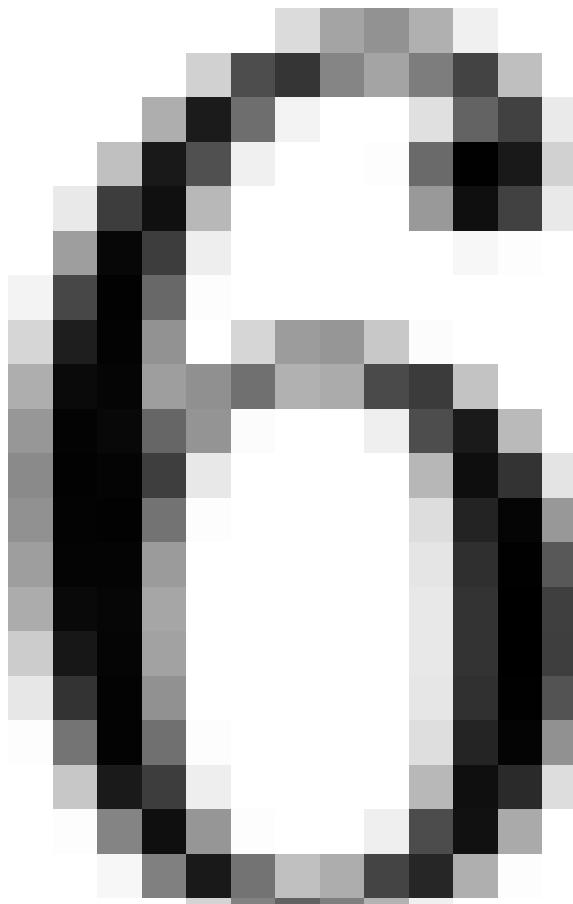


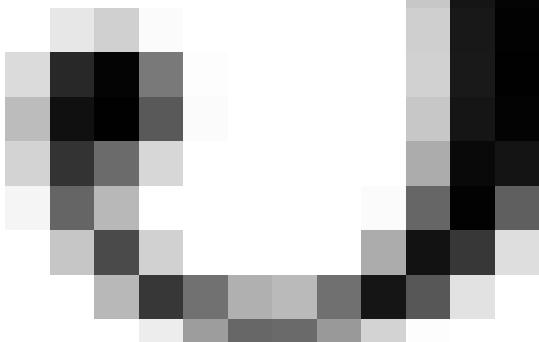
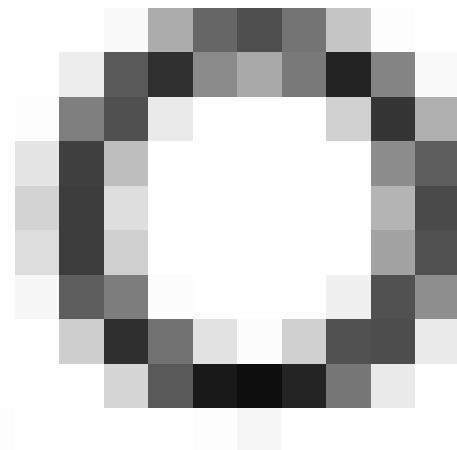
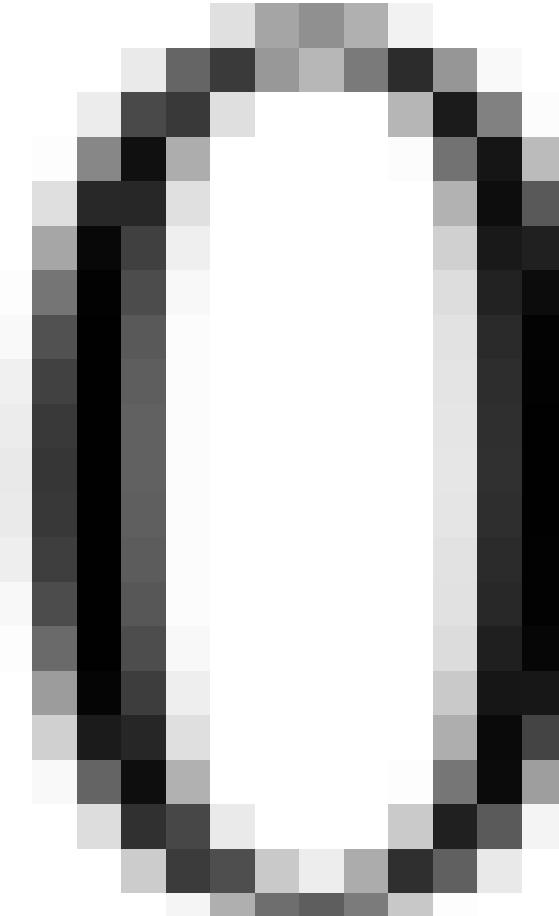
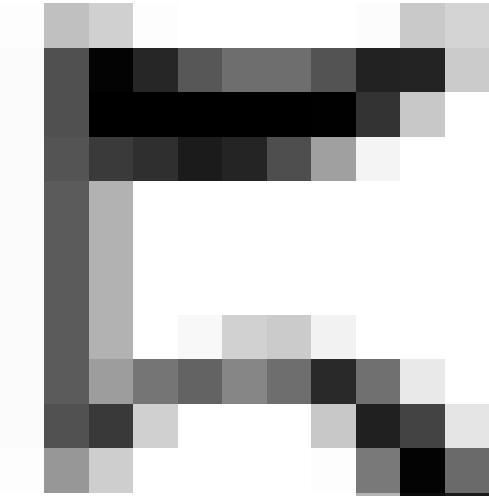


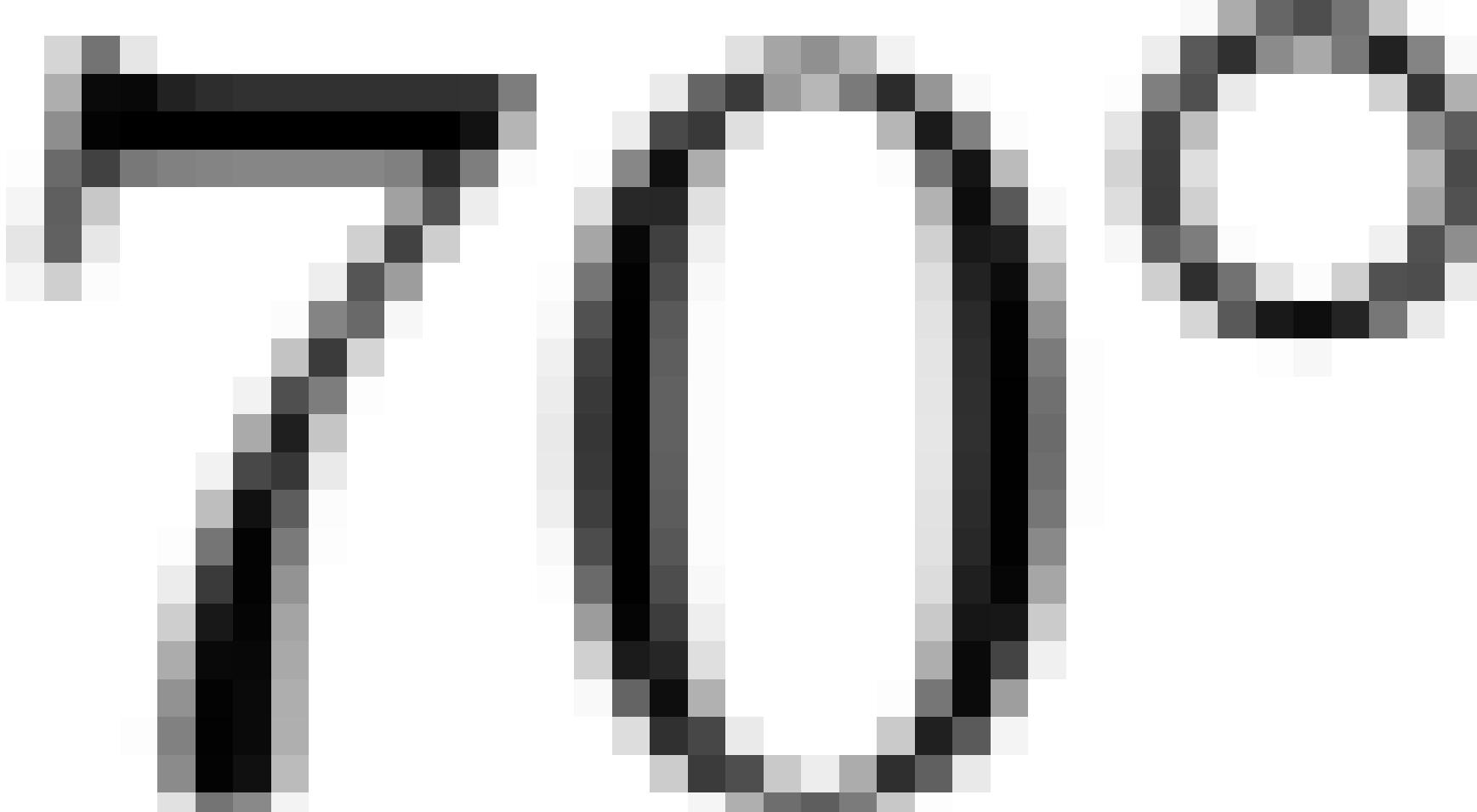


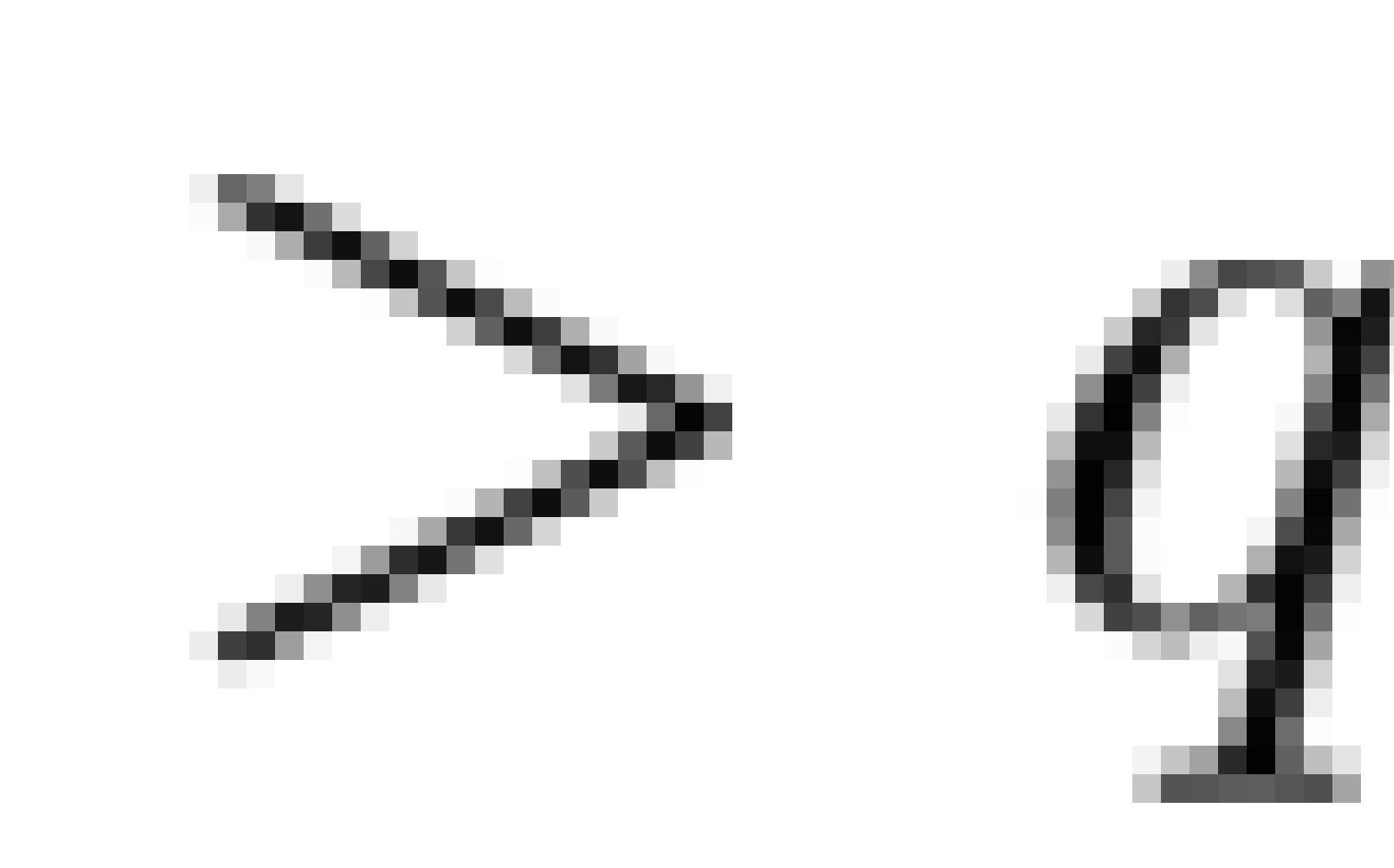














$$S_1 = s_v$$

$$S_2 = s_{H\max}$$

$$S_3 = s_{h\min}$$



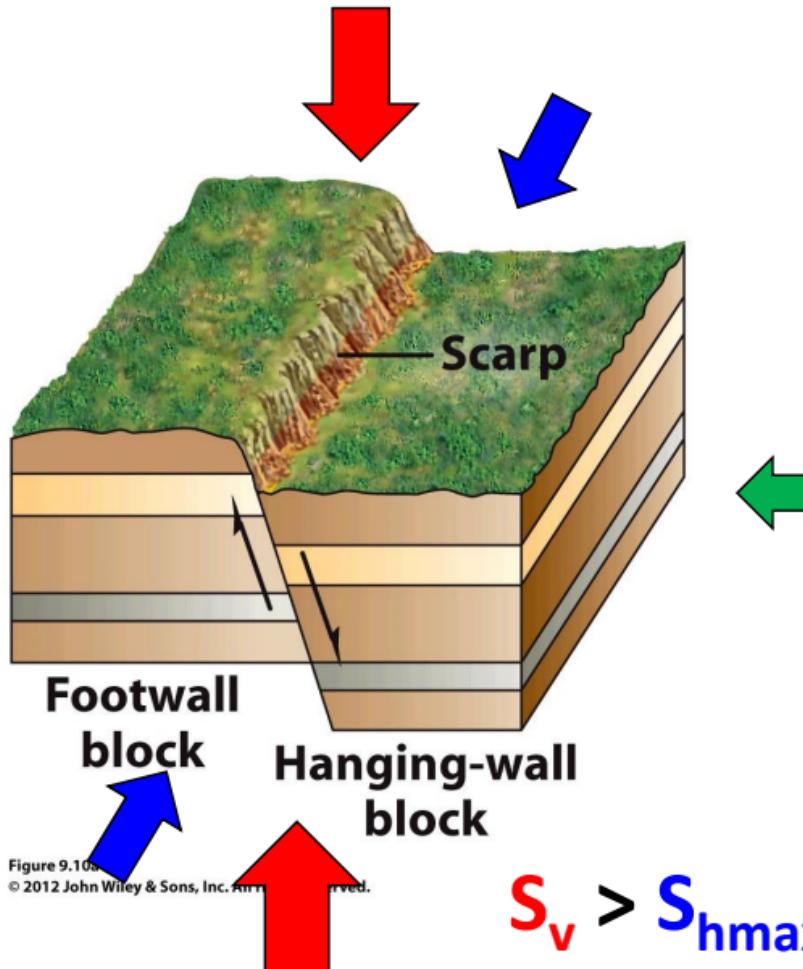


Figure 9.10b  
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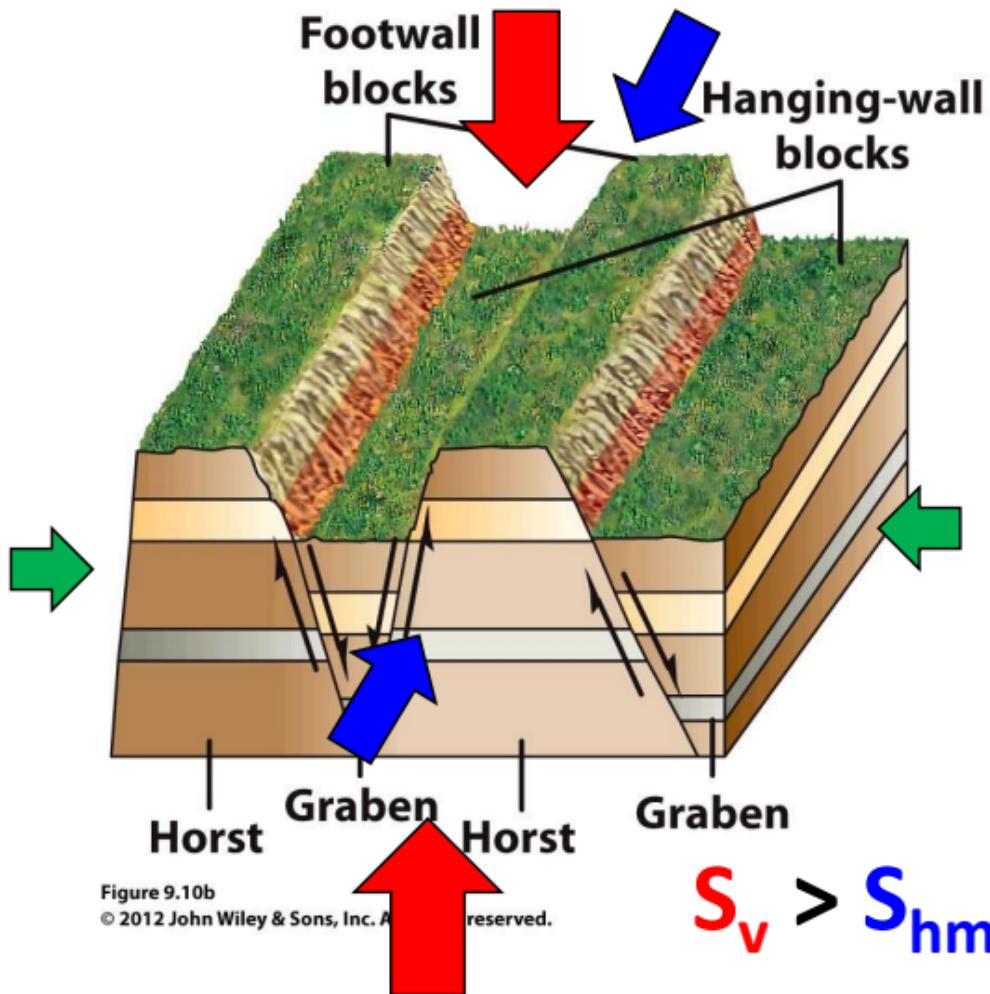


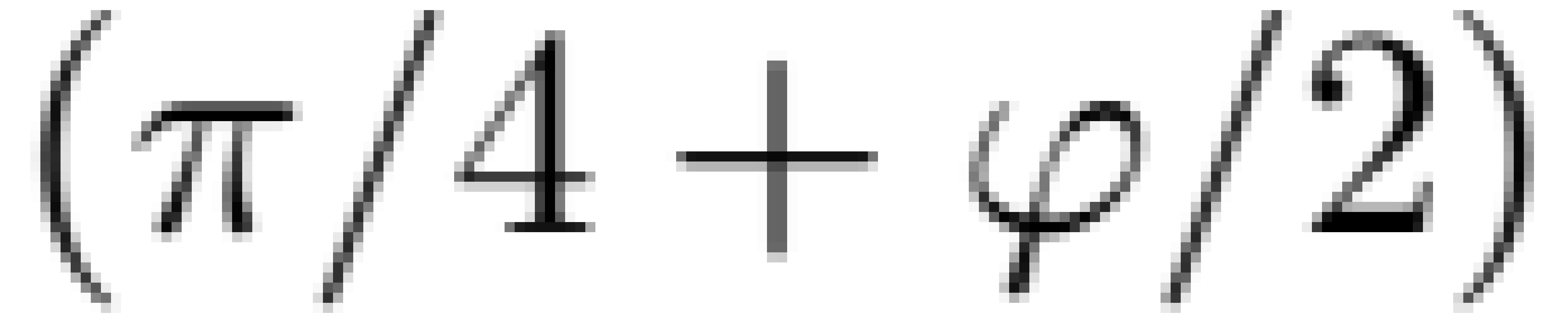
Figure 9.10b  
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$$S_v > S_{h\max} > S_{h\min}$$

$$S_1 = S_{H\max}$$

$$S_2 = S_{h\min}$$

$$S_3 = S_v$$



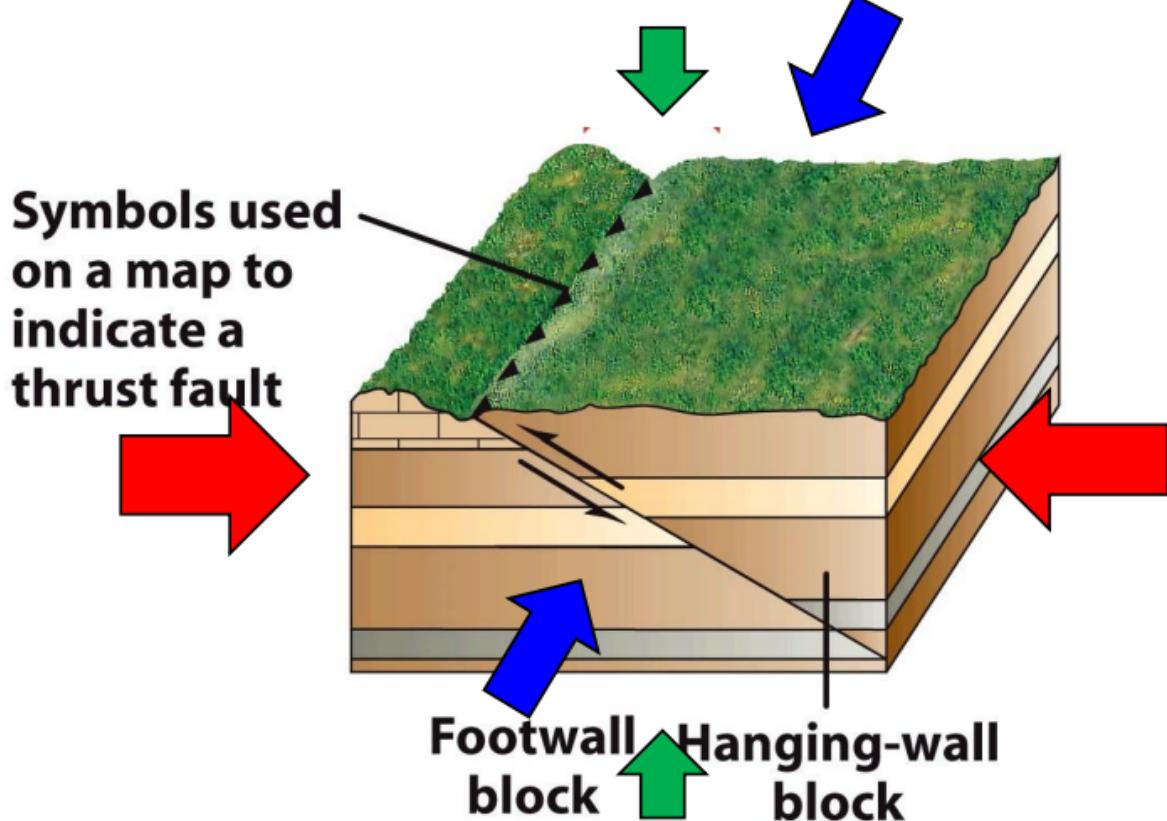


Figure 9.10d  
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$$S_{H\max} > S_{h\min} > S_v$$

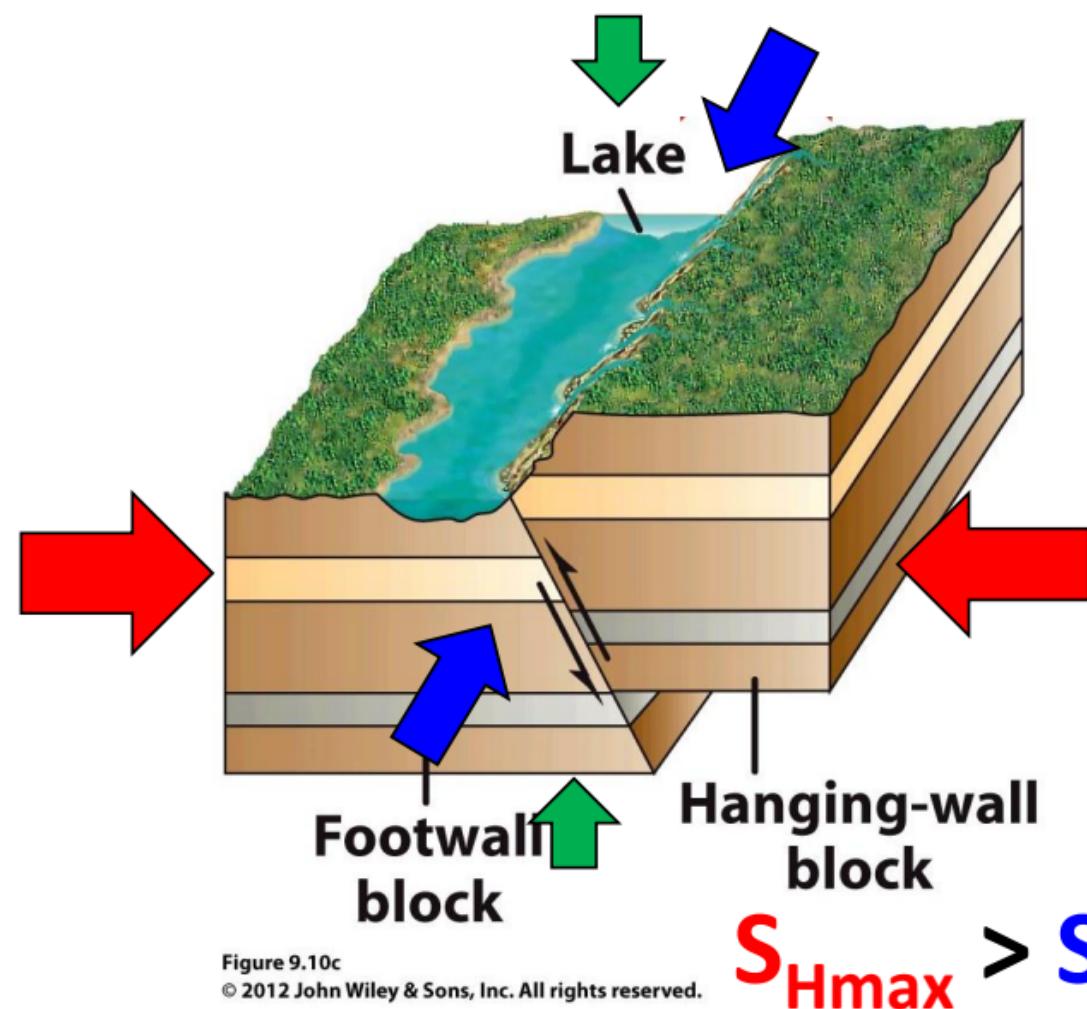


Figure 9.10c

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$$S_1 = S_{H\max}$$

$$S_2 = S_v$$

$$S_3 = S_{hmin}$$

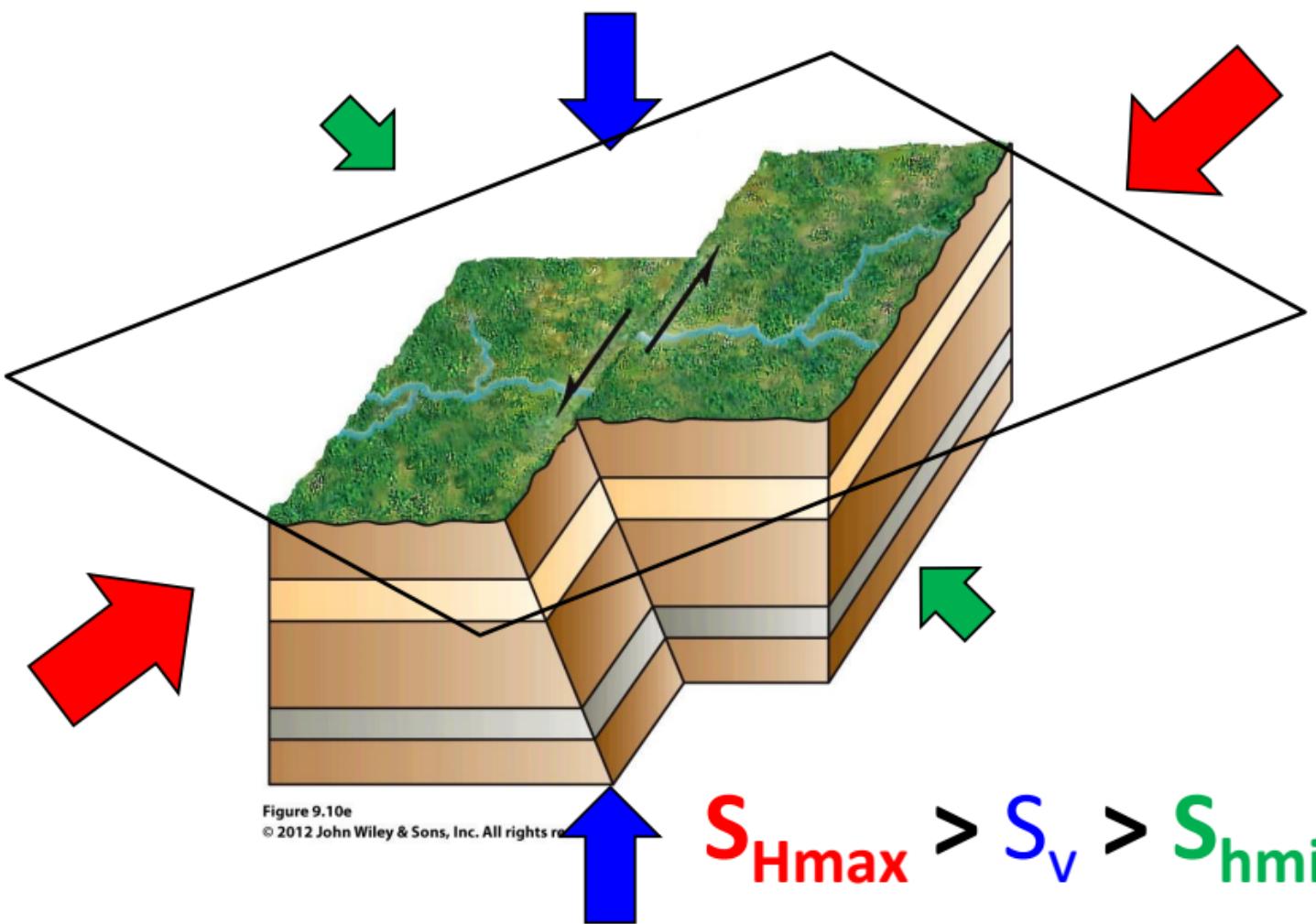
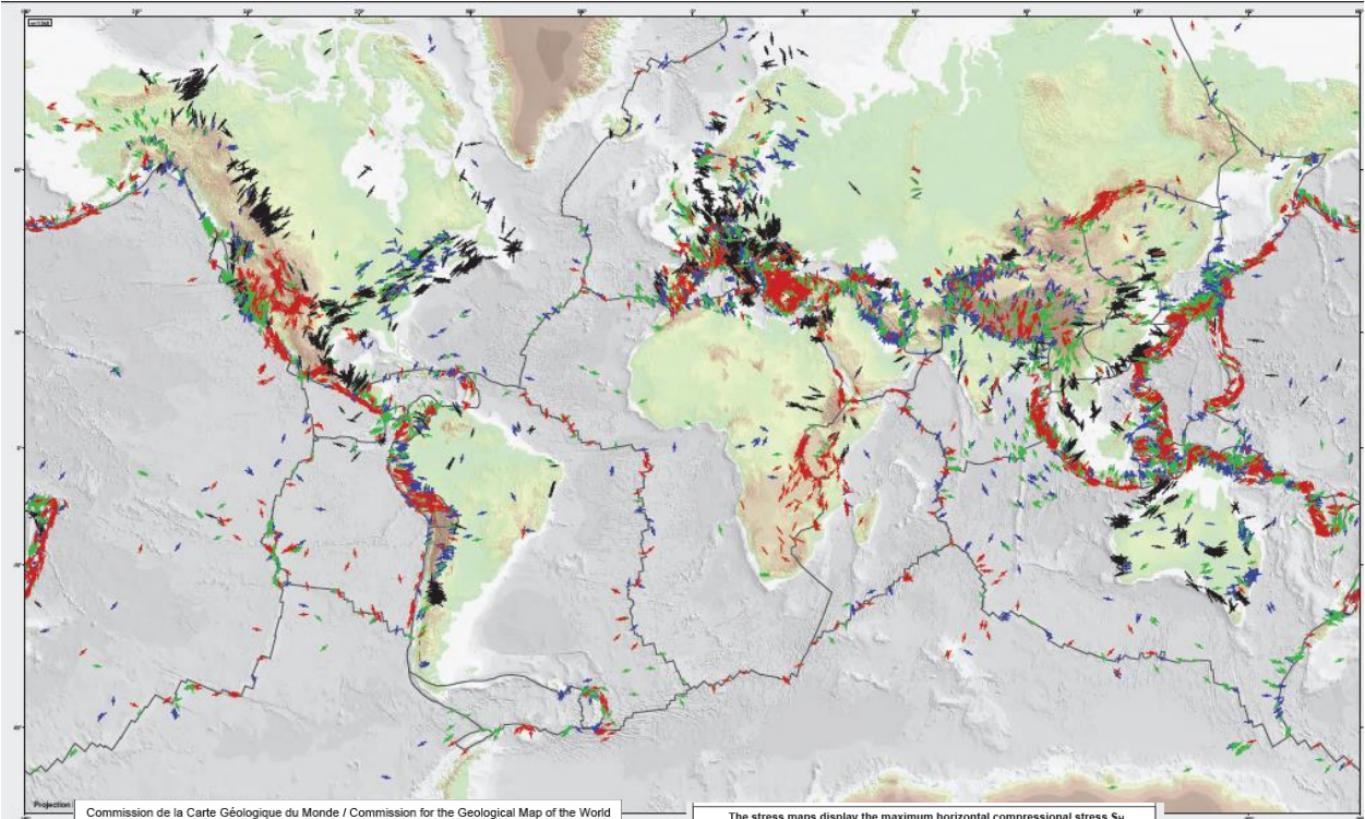


Figure 9.10e

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Commission de la Carte Géologique du Monde / Commission for the Geological Map of the World



# WORLD STRESS MAP



2009 - 2<sup>nd</sup> edition, based on the WSM database release 2008

Helmholtz Centre Potsdam - GFZ German Research Centre for Geosciences

## Authors

Oliver Heidbach, Mark Tingay, Andreas Barth, John Reinecker, Daniel Kurfürst, and Birgit Müller



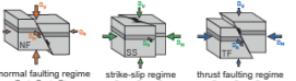
The stress maps display the maximum horizontal compressional stress  $S_H$

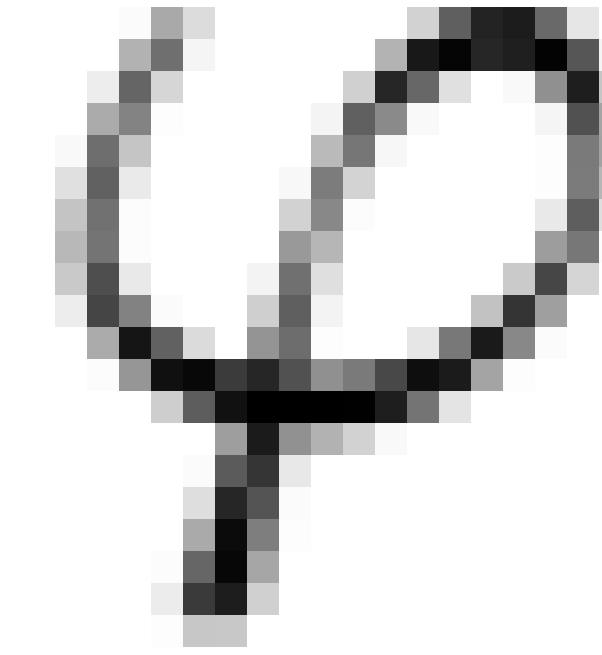
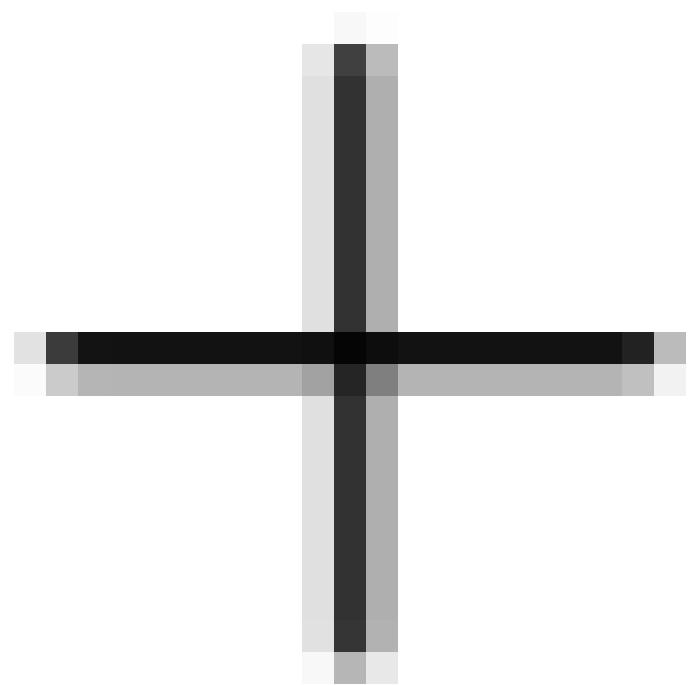
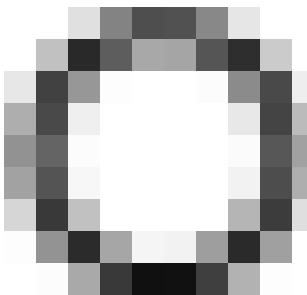
| Method             | Quality                        |
|--------------------|--------------------------------|
| focal mechanism    | $S_H$ is within $\pm 15^\circ$ |
| breakouts          | $S_H$ is within $\pm 20^\circ$ |
| drl. induced frac. | $S_H$ is within $\pm 25^\circ$ |
| overcoring         |                                |
| hydro. fractures   |                                |
| geol. indicators   |                                |

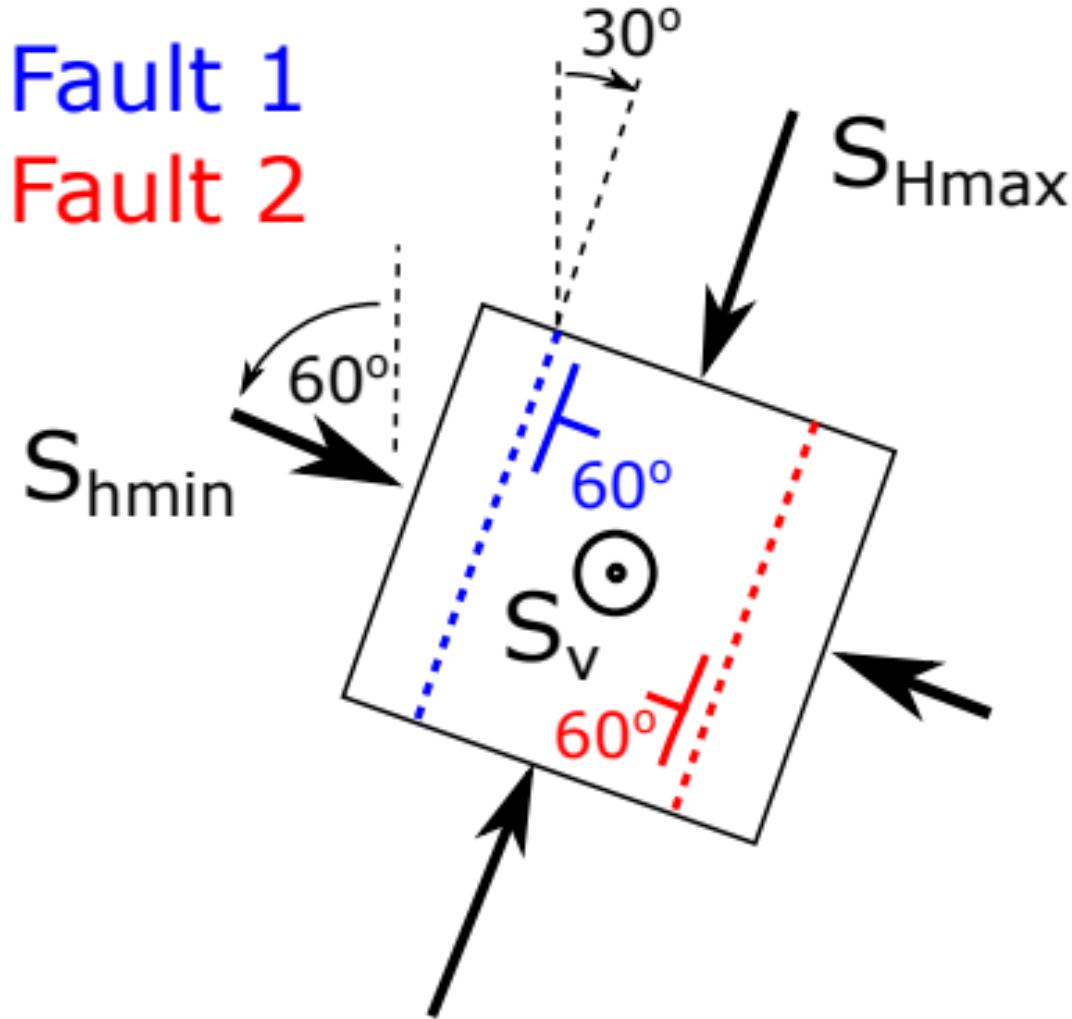
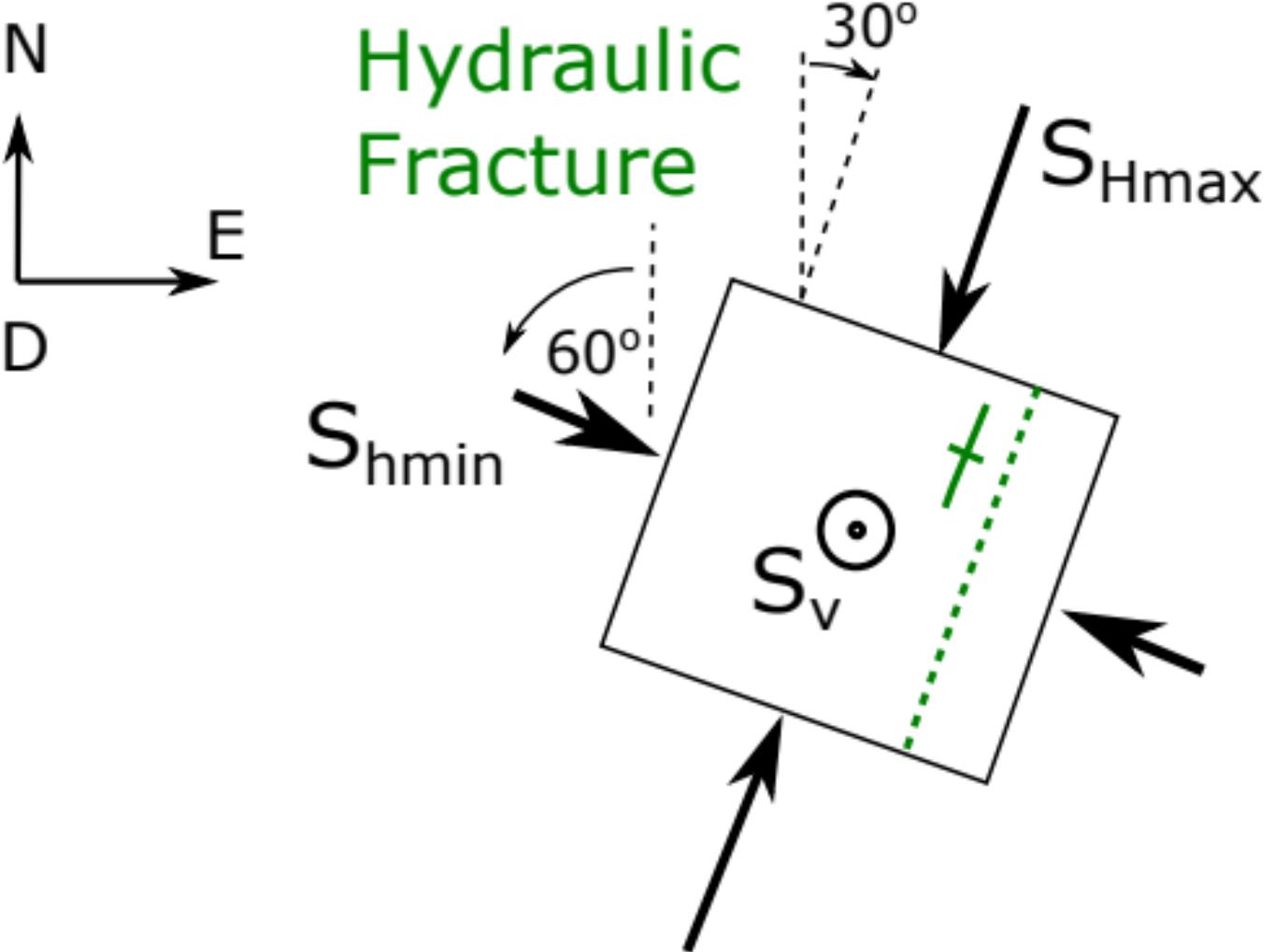
Data depth range

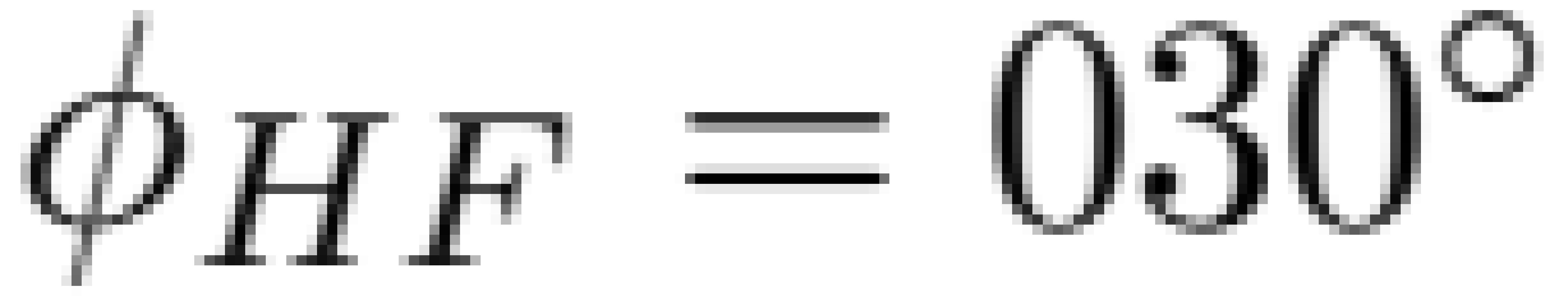
0-40 km

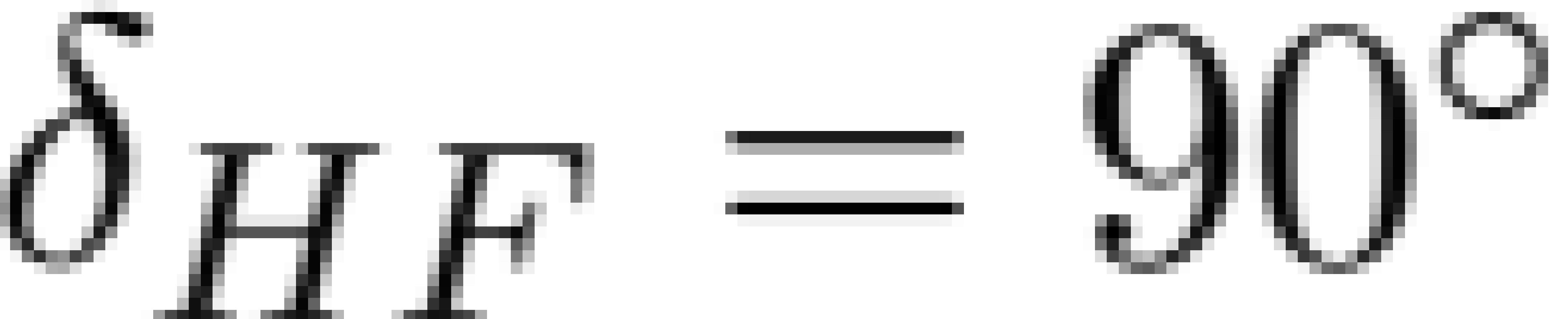
| Stress Regime        |
|----------------------|
| Normal faulting      |
| Strike-slip faulting |
| Thrust faulting      |
| Unknown regime       |

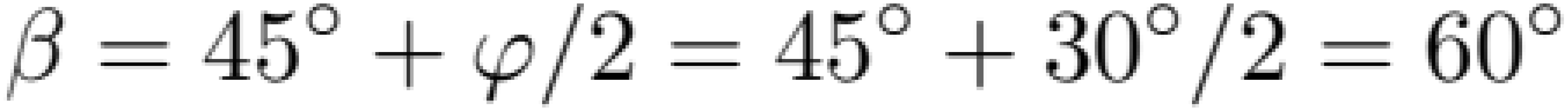




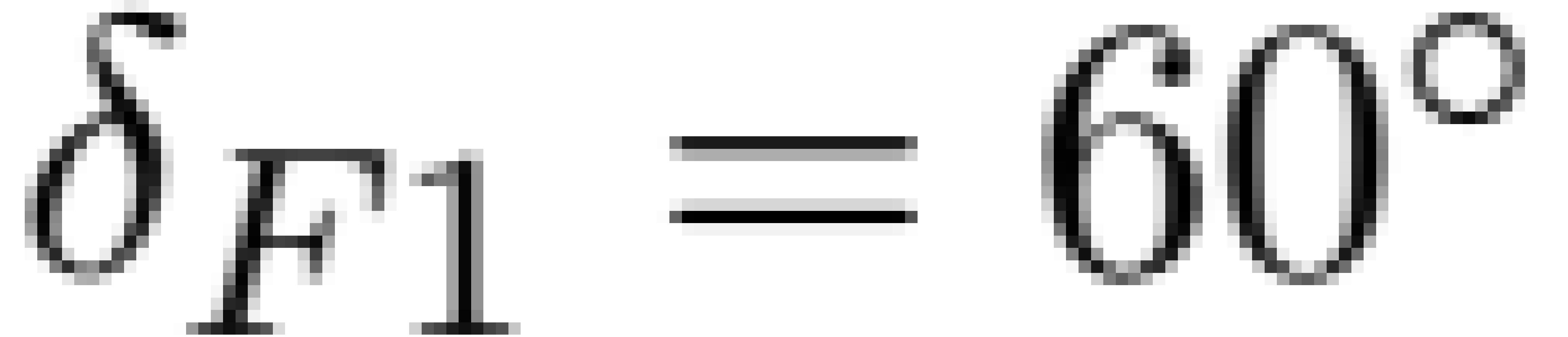


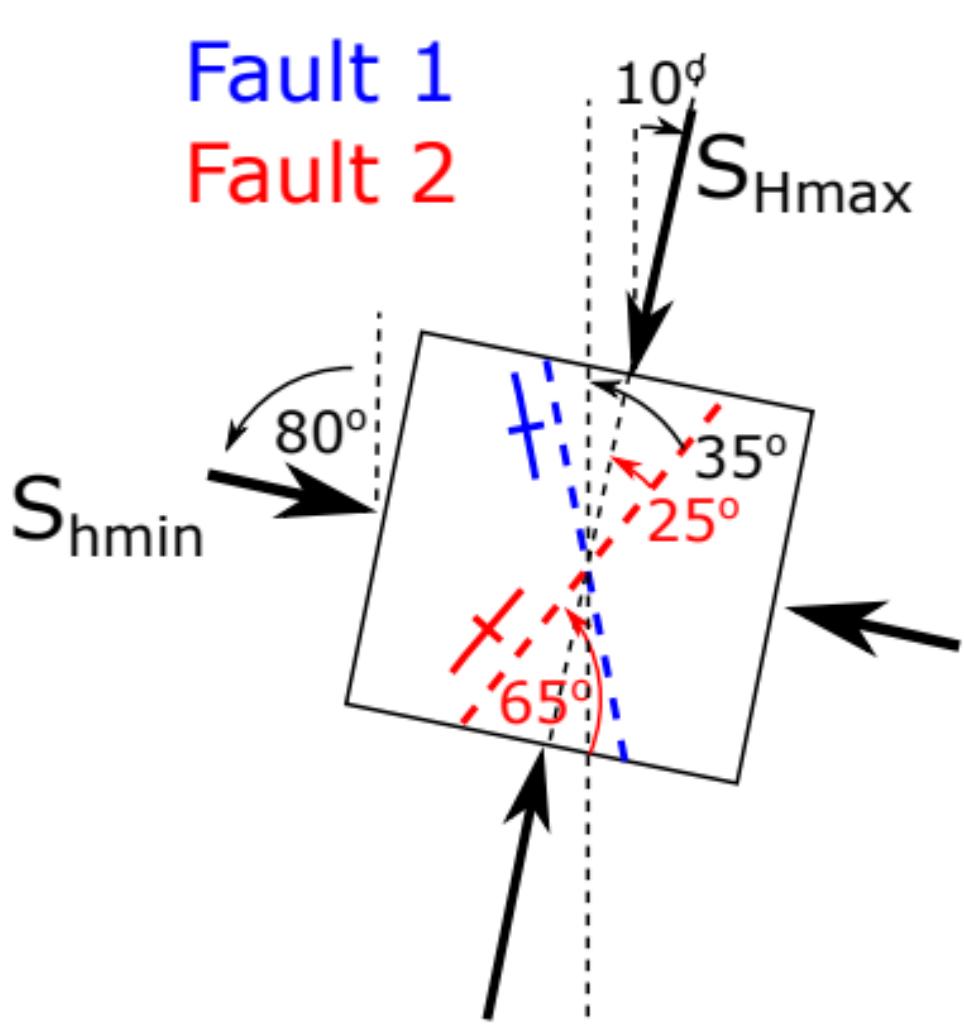
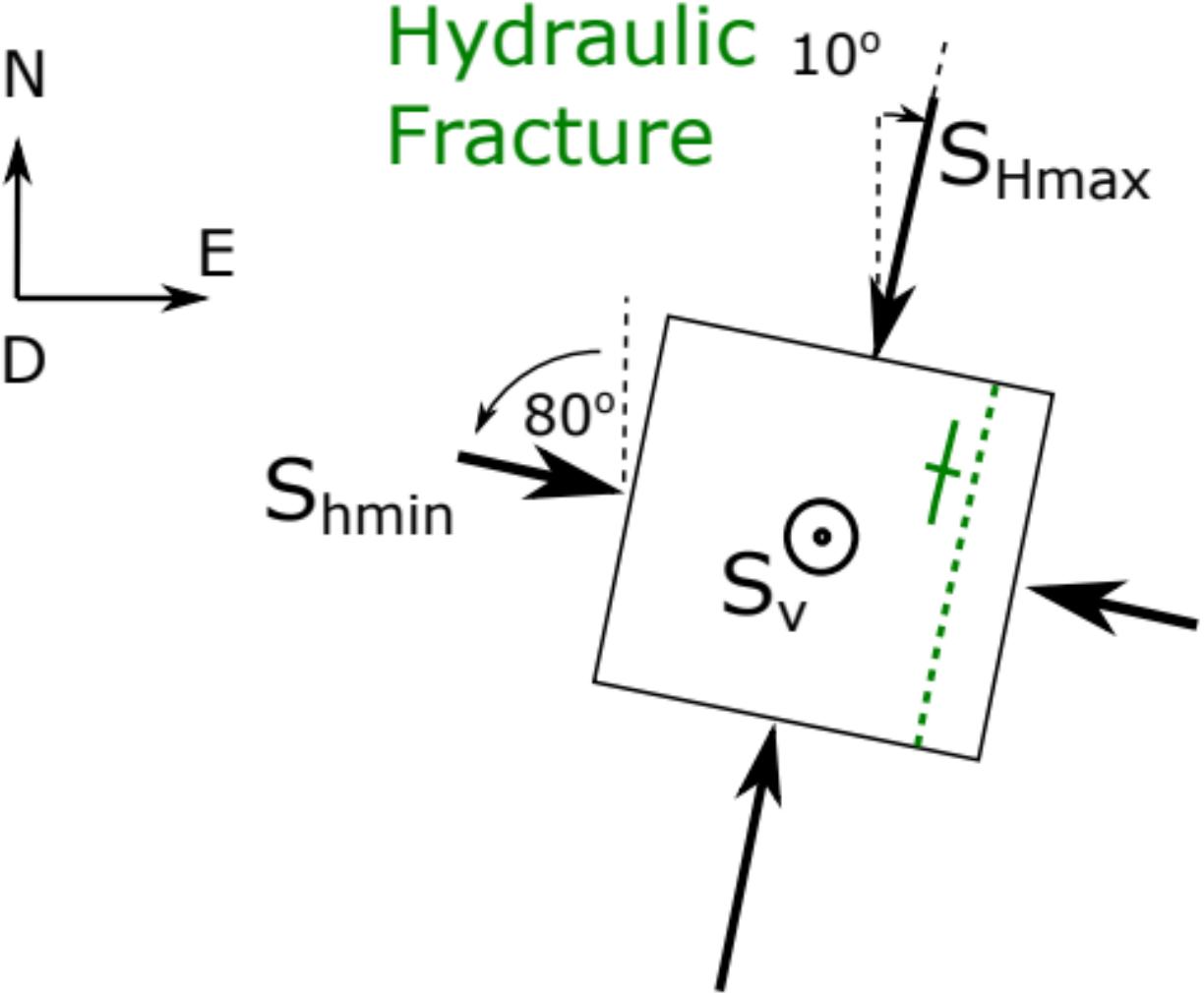


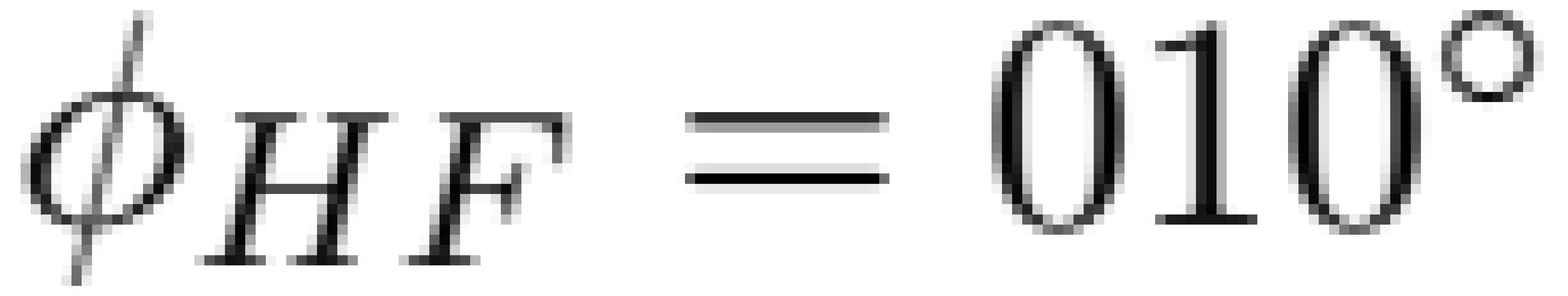


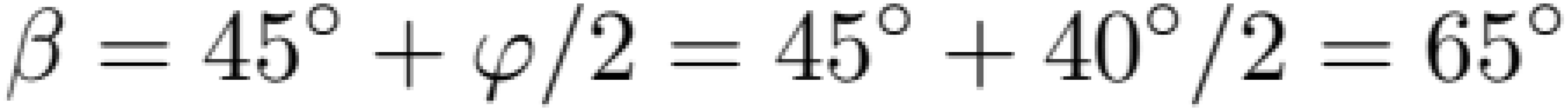




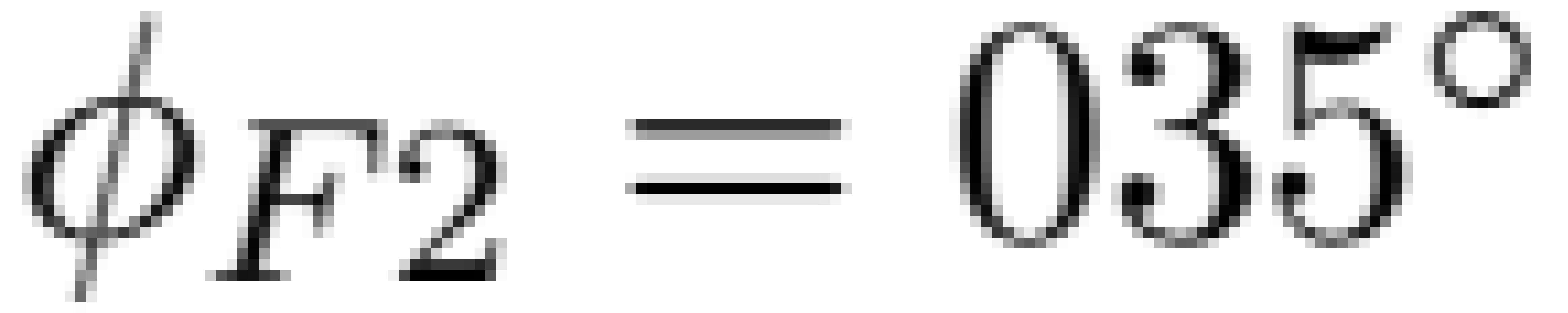






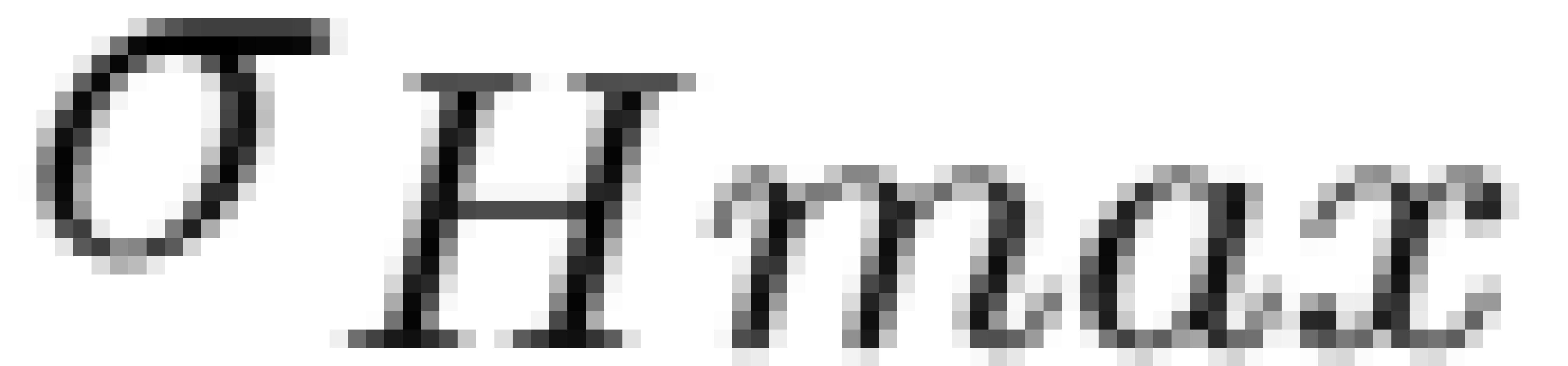




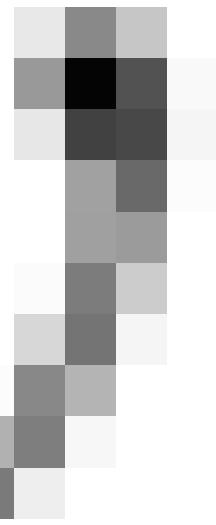
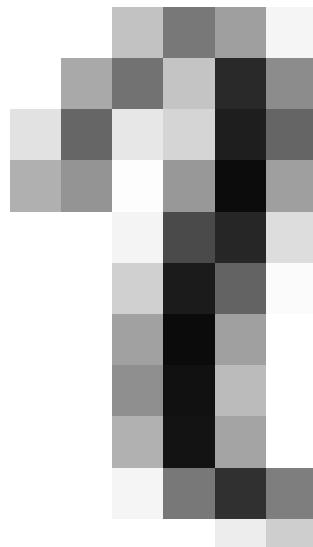
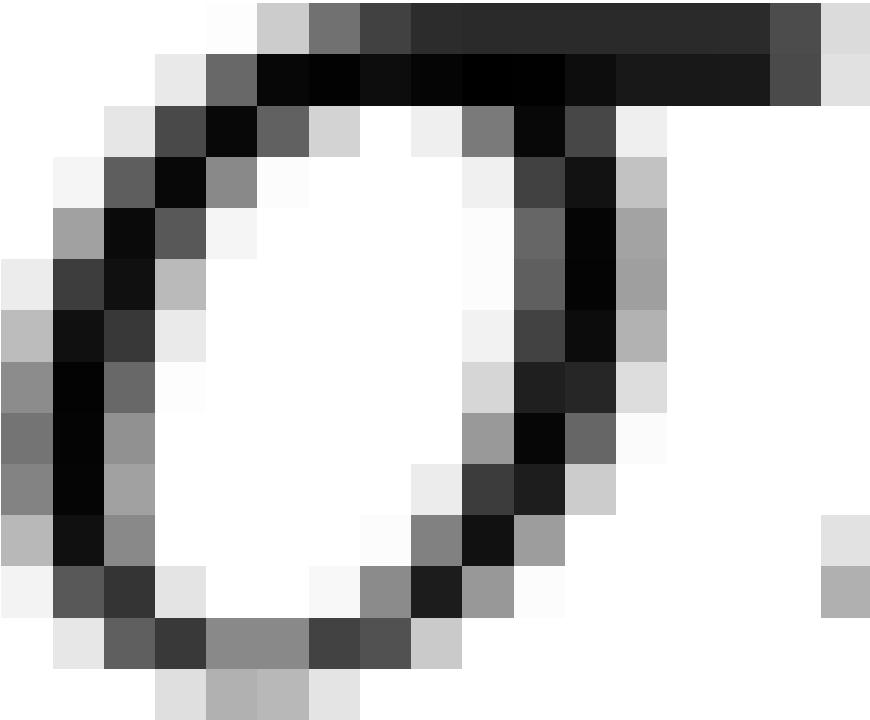






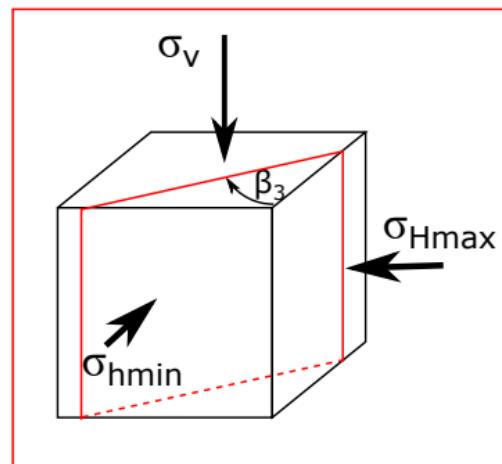
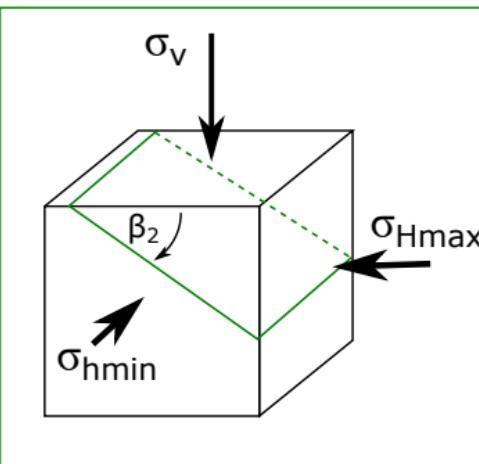
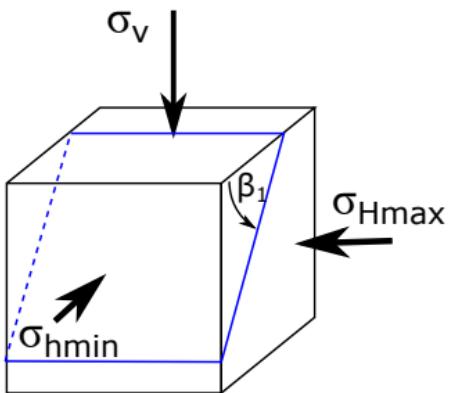
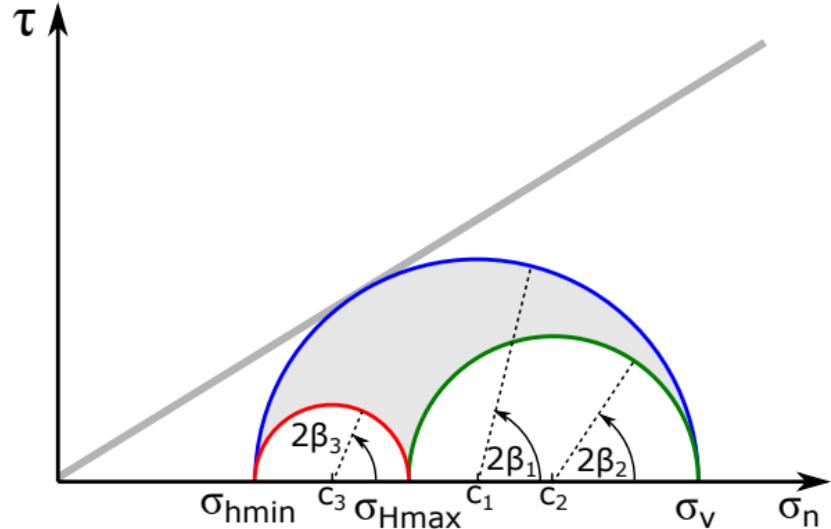
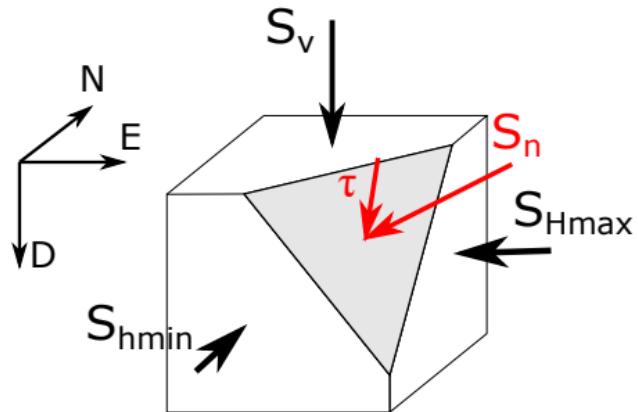


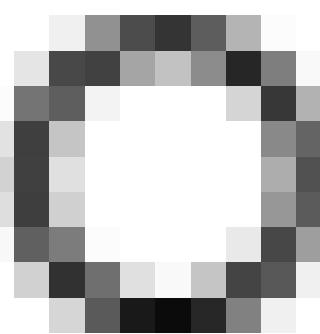
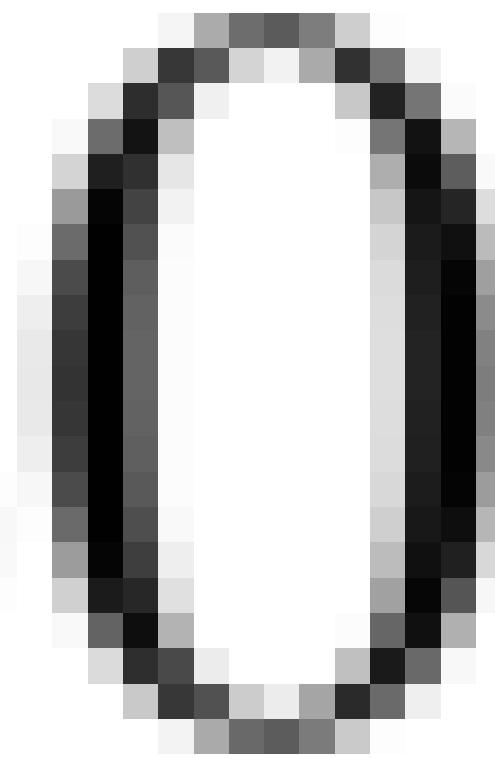
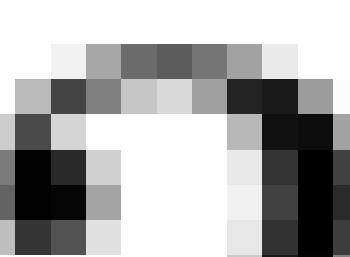
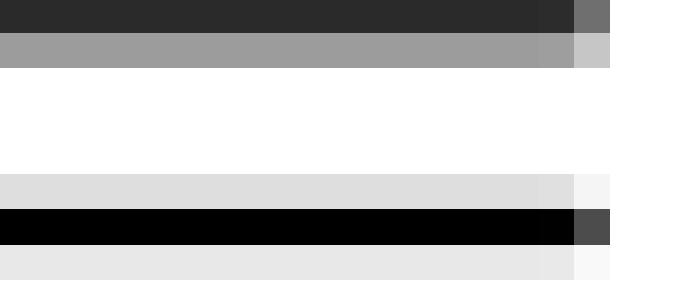
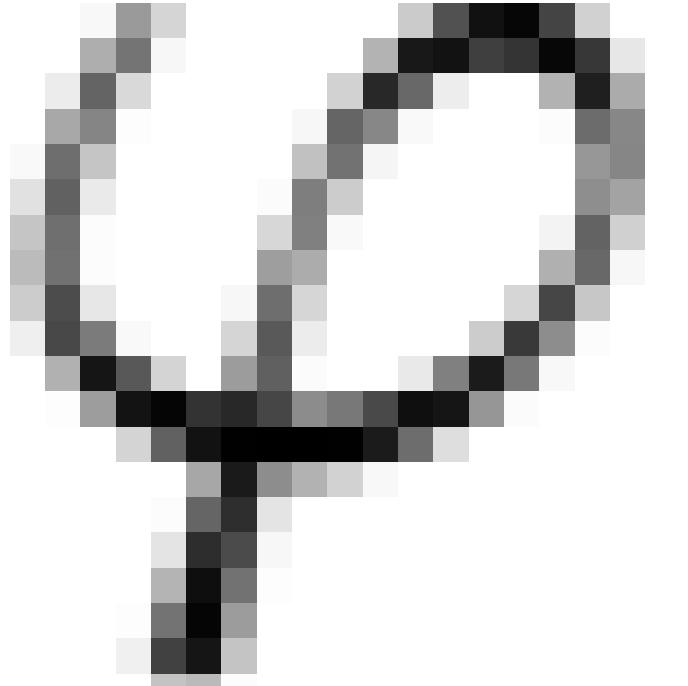




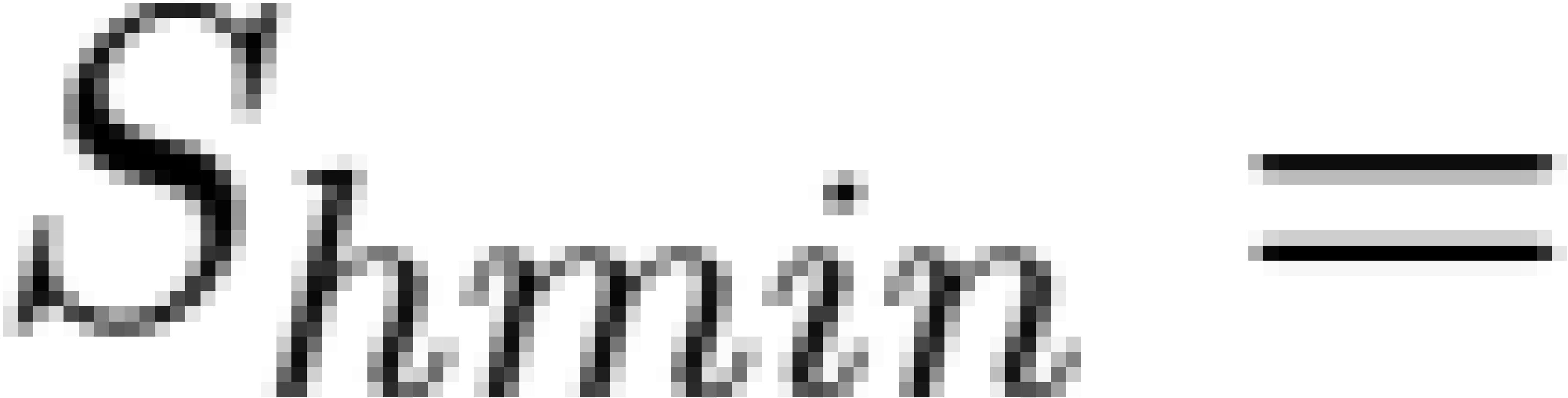




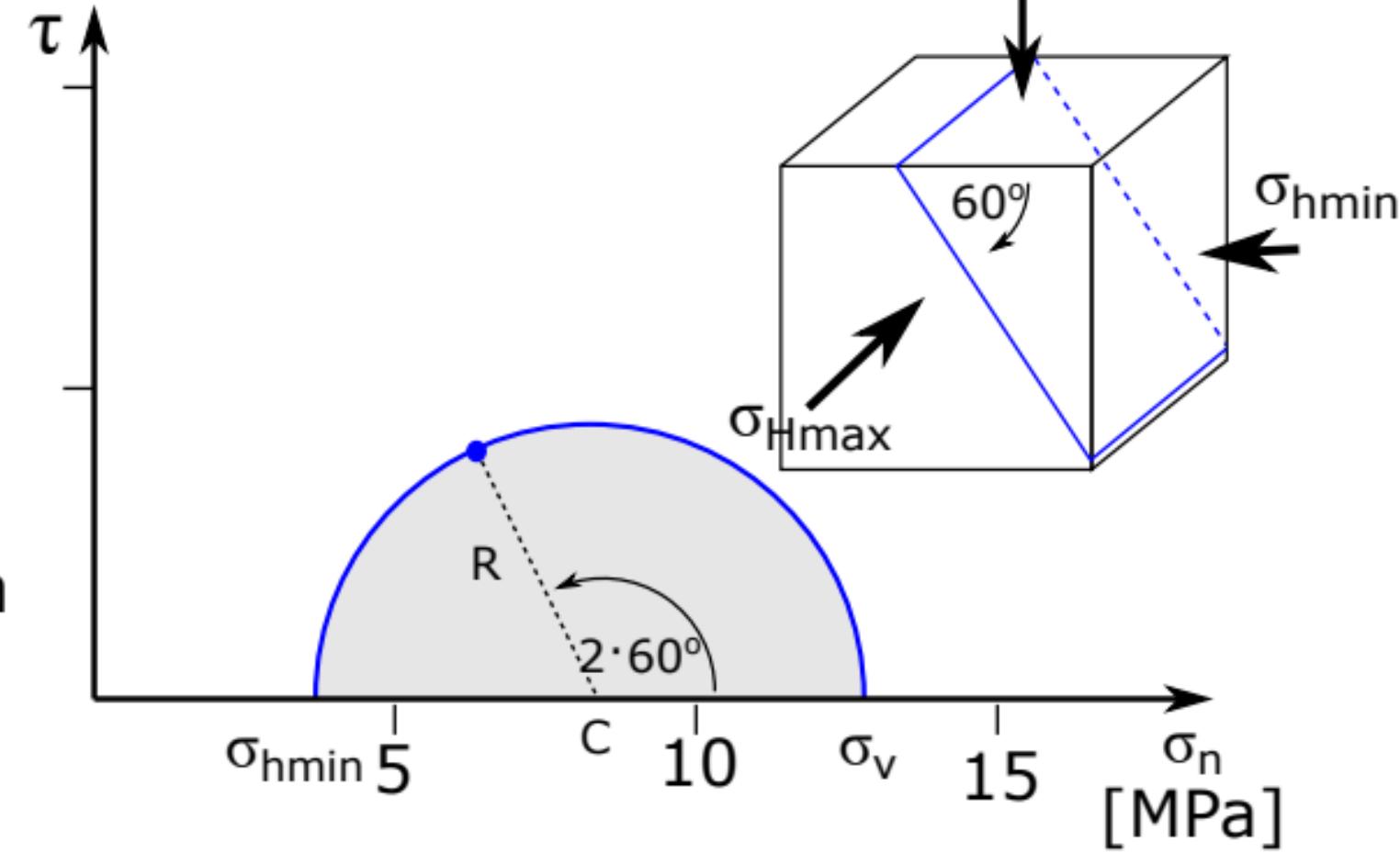
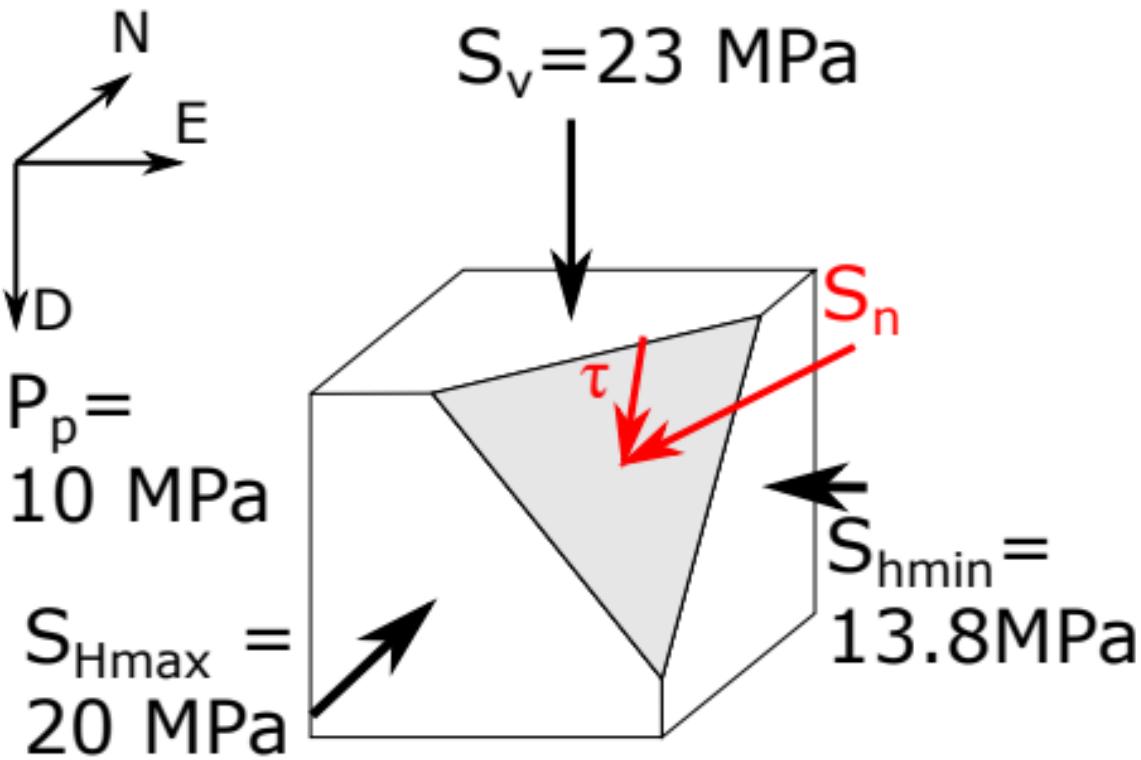


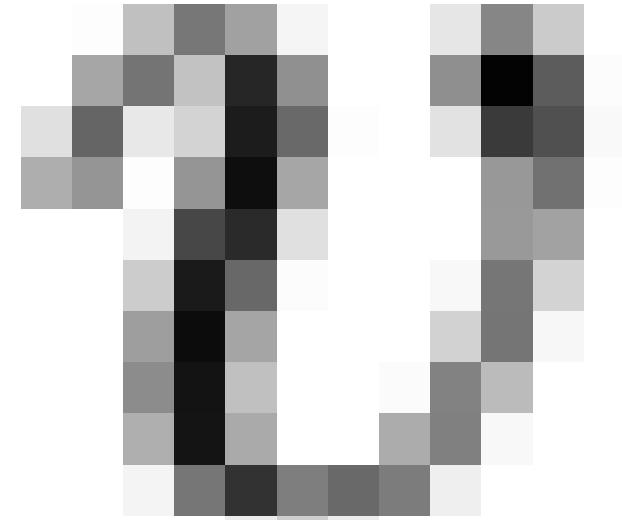
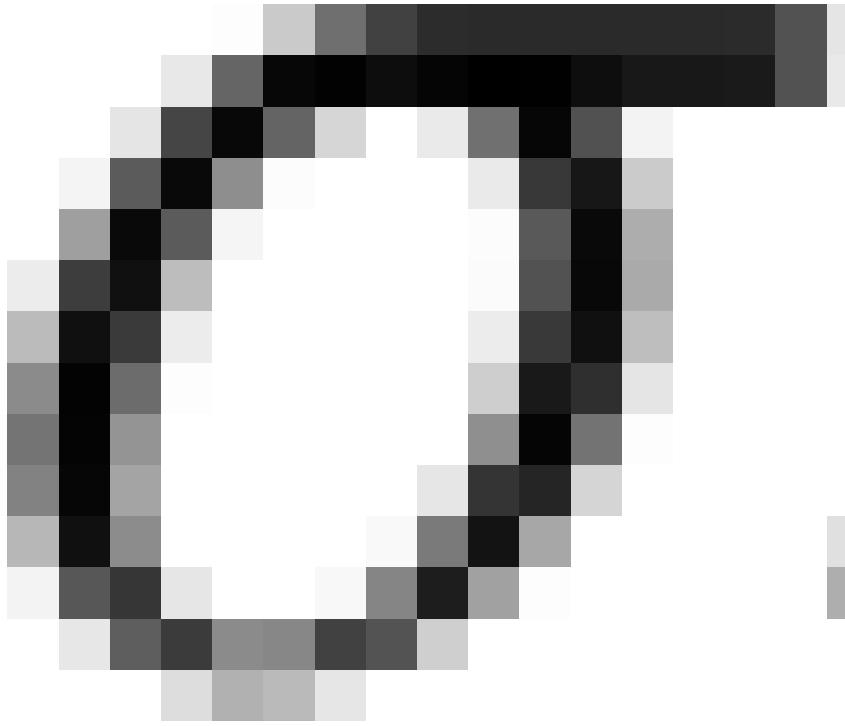




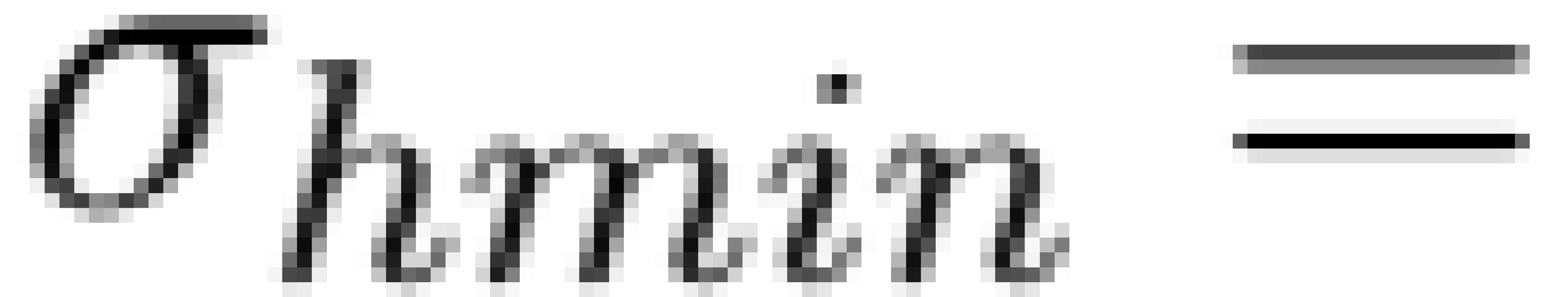








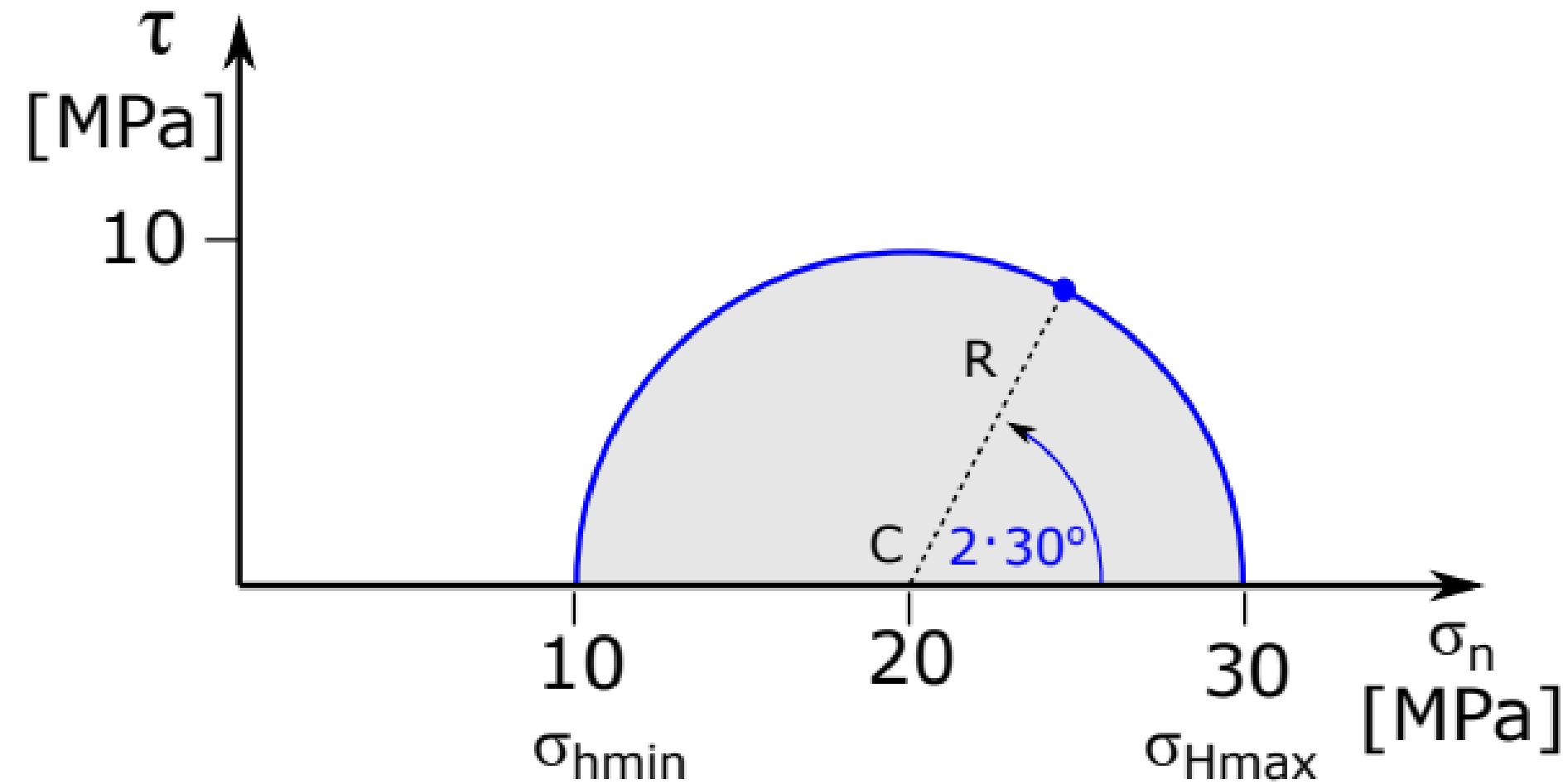
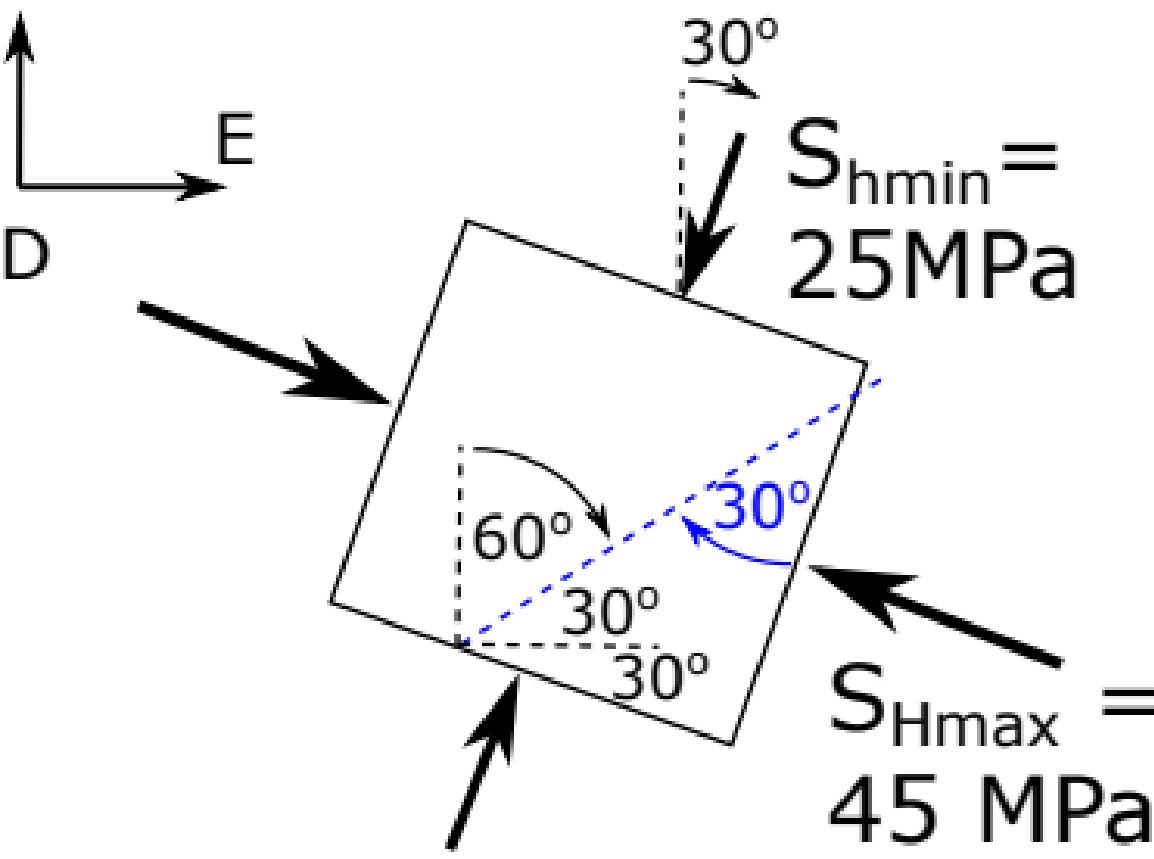




$$\sigma_n = \frac{(13 \text{ MPa} + 3.8 \text{ MPa})}{2} + \frac{(13 \text{ MPa} - 3.8 \text{ MPa}) \cos(2 \cdot 60^\circ)}{2} = 6.1 \text{ MPa}$$

$$\tau = \frac{(13 \text{ MPa} - 3.8 \text{ MPa})}{2} \sin(2 \cdot 60^\circ) = 4.0 \text{ MPa}$$

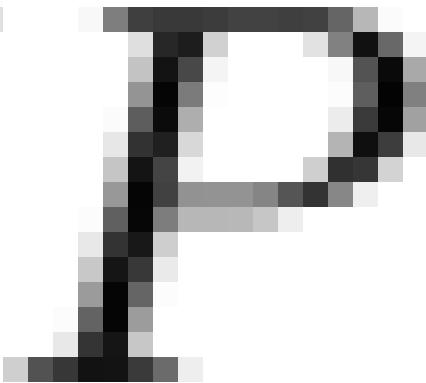
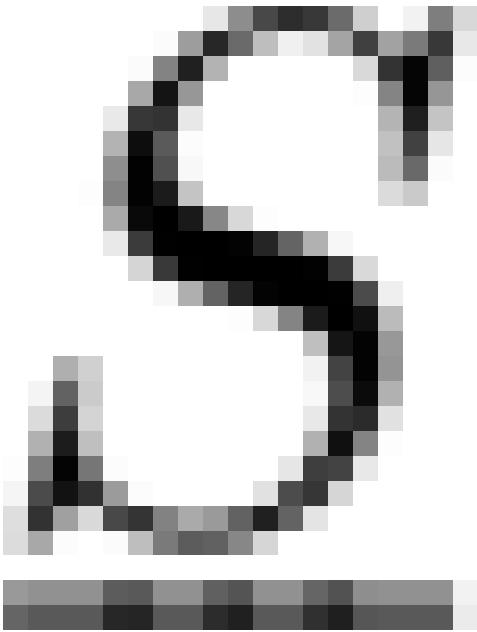
N  $P_p = 15 \text{ MPa}$   $S_v = 30 \text{ MPa}$

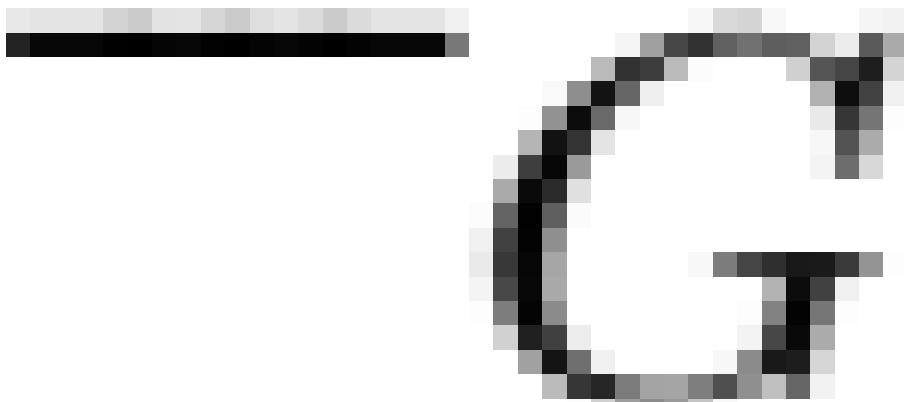
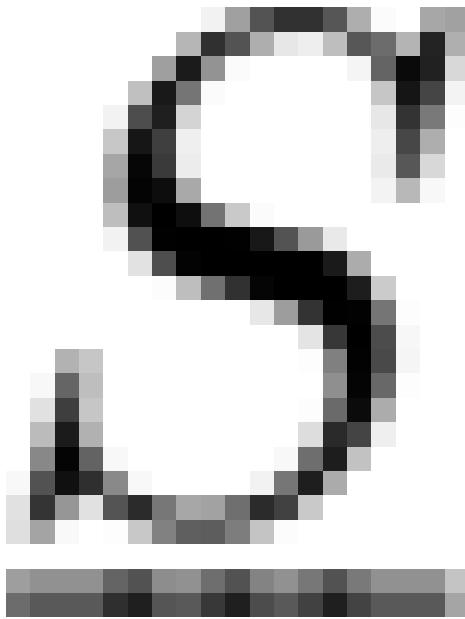


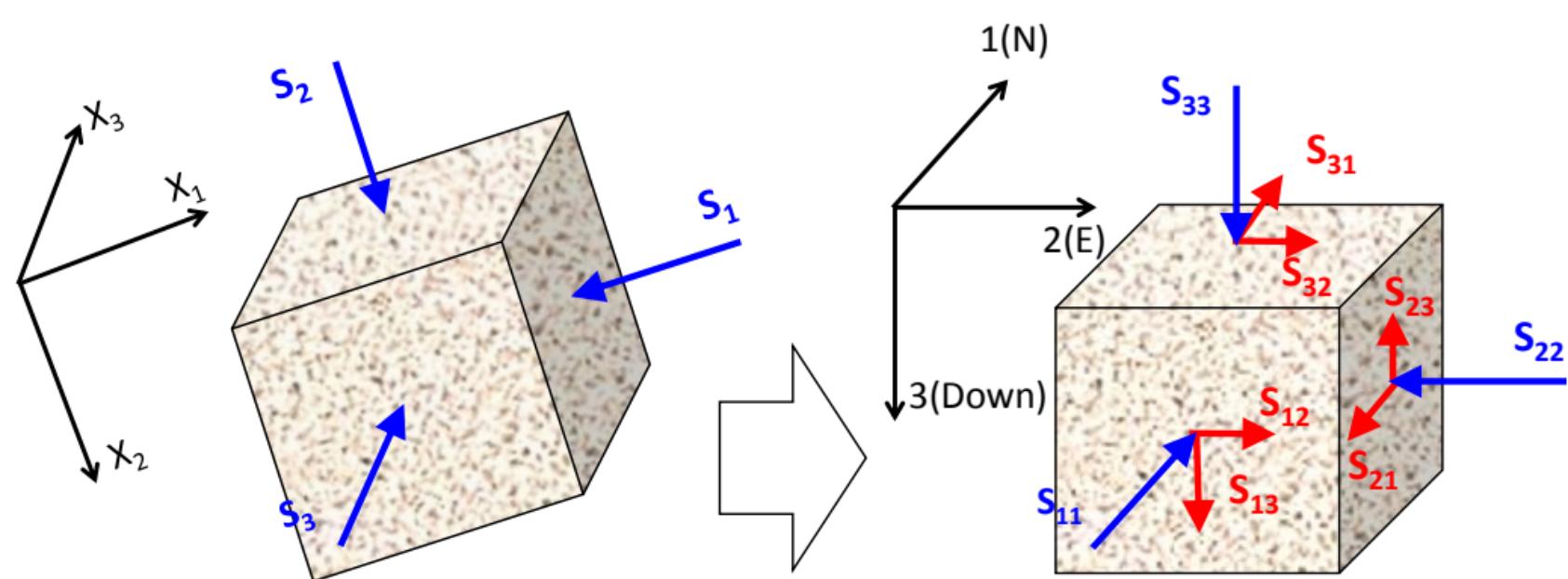
$$\sigma_n = \frac{(30 \text{ MPa} + 10 \text{ MPa})}{2} + \frac{(30 \text{ MPa} - 10 \text{ MPa}) \cos(2 \cdot 30^\circ)}{2} = 25 \text{ MPa}$$

$$\tau = \frac{(30 \text{ MPa} - 10 \text{ MPa})}{2} \sin(2 \cdot 30^\circ) = 8.7 \text{ MPa}$$



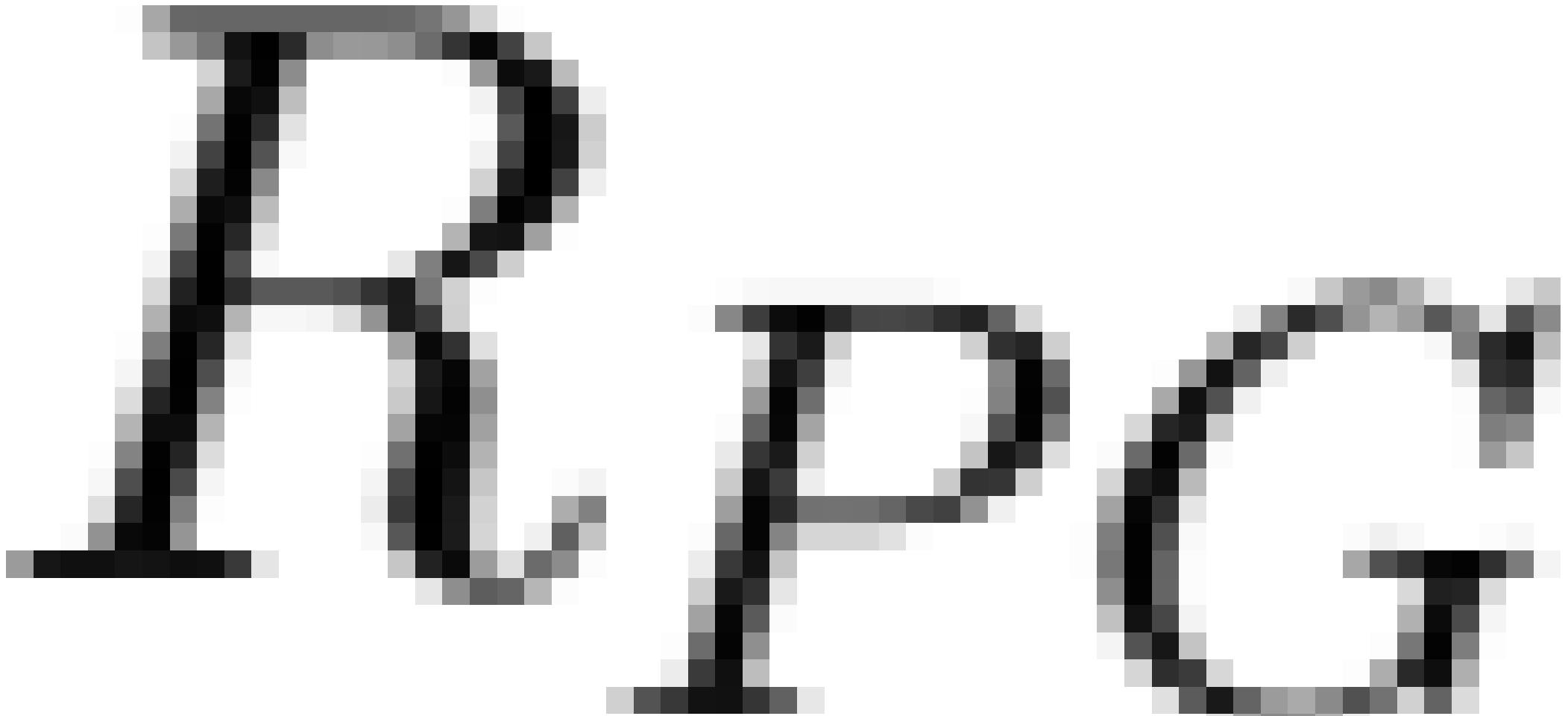


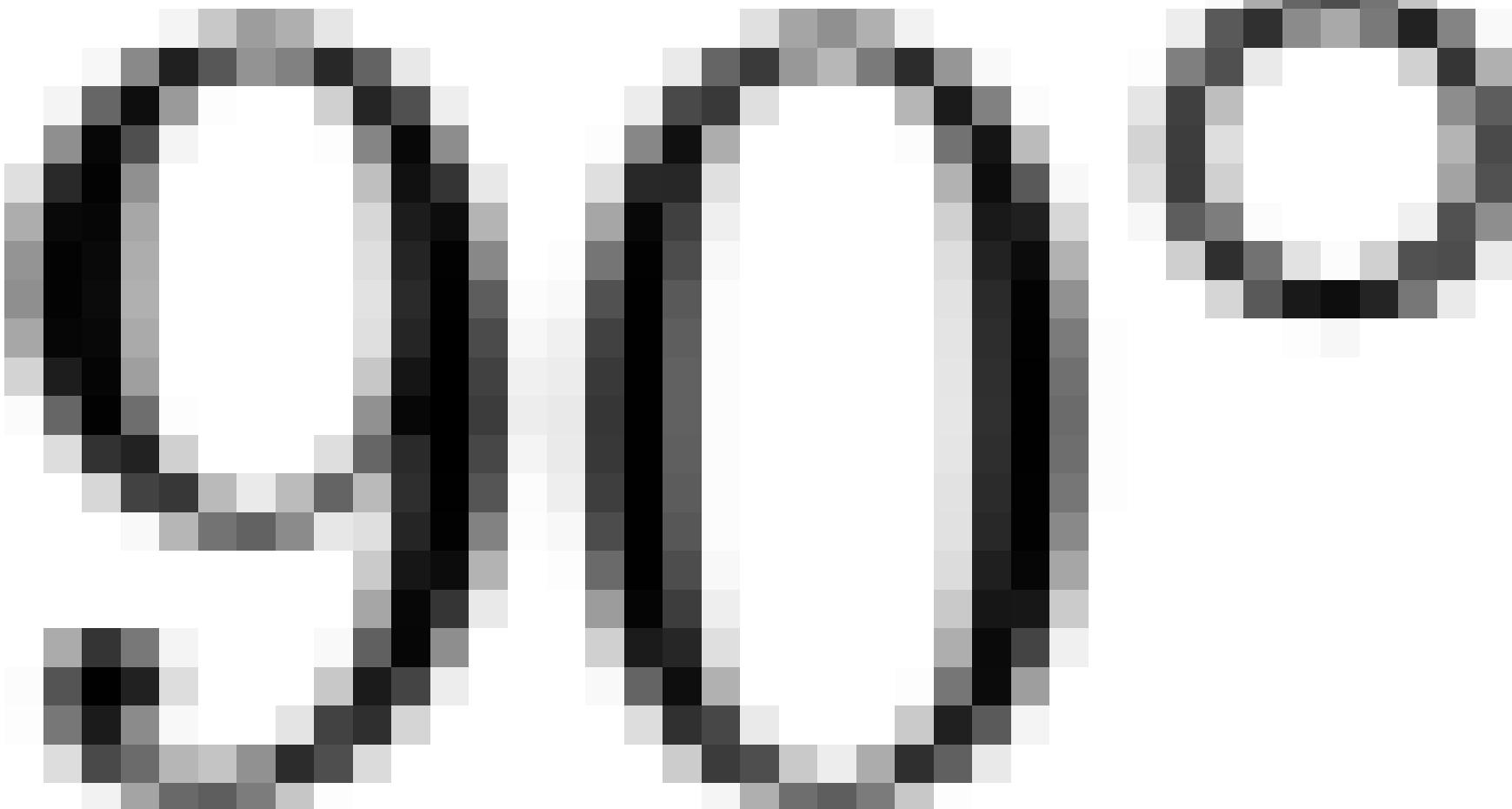


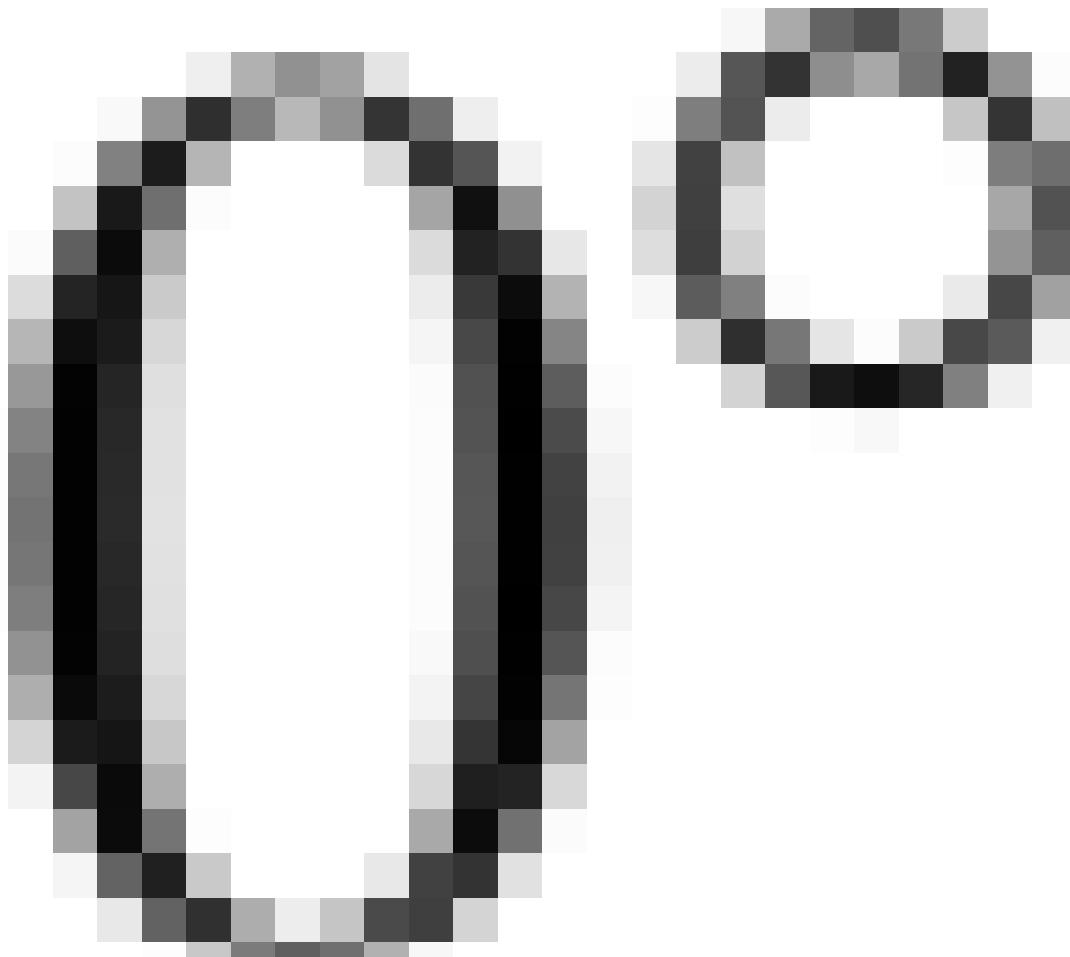


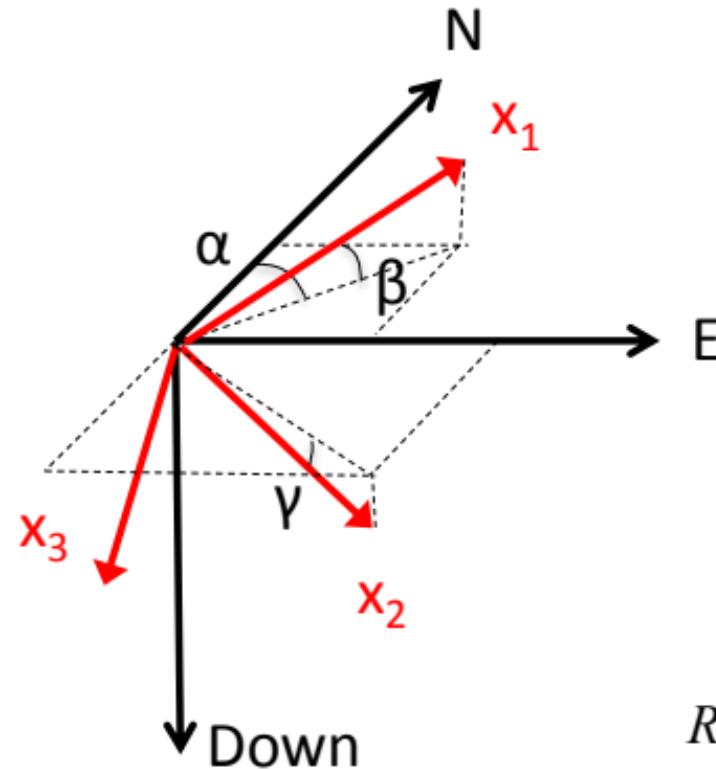
$$S_P = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$S_G = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$









$$\underline{\underline{S}}' = \underline{\underline{A}} \underline{\underline{S}} \underline{\underline{A}}^T$$

$$\underline{\underline{A}} = \begin{bmatrix} \underline{e}'_1 \cdot \underline{e}_1 & \underline{e}'_1 \cdot \underline{e}_2 & \underline{e}'_1 \cdot \underline{e}_3 \\ \underline{e}'_2 \cdot \underline{e}_1 & \underline{e}'_2 \cdot \underline{e}_2 & \underline{e}'_2 \cdot \underline{e}_3 \\ \underline{e}'_3 \cdot \underline{e}_1 & \underline{e}'_3 \cdot \underline{e}_2 & \underline{e}'_3 \cdot \underline{e}_3 \end{bmatrix}$$

where  $\underline{\underline{A}}$  is the transformation matrix from the old base  $\underline{e}_i$  to base  $\underline{e}'_i$  and the components are the projection of the elements of the new base on the old base.

- Old system: N-E-D (Right-handed) Geographical system
- New system: 1-2-3 (Right-handed) Principal stress system

$$R_{PG} = \begin{bmatrix} \cos \alpha \cos b & \sin \alpha \cos b & -\sin b \\ \cos \alpha \sin b \sin g - \sin \alpha \cos g & \sin \alpha \sin b \sin g + \cos \alpha \cos g & \cos b \sin g \\ \cos \alpha \sin b \cos g + \sin \alpha \sin g & \sin \alpha \sin b \cos g - \cos \alpha \sin g & \cos b \cos g \end{bmatrix}$$

2

P

E

MPG

G

PG

Q

Q

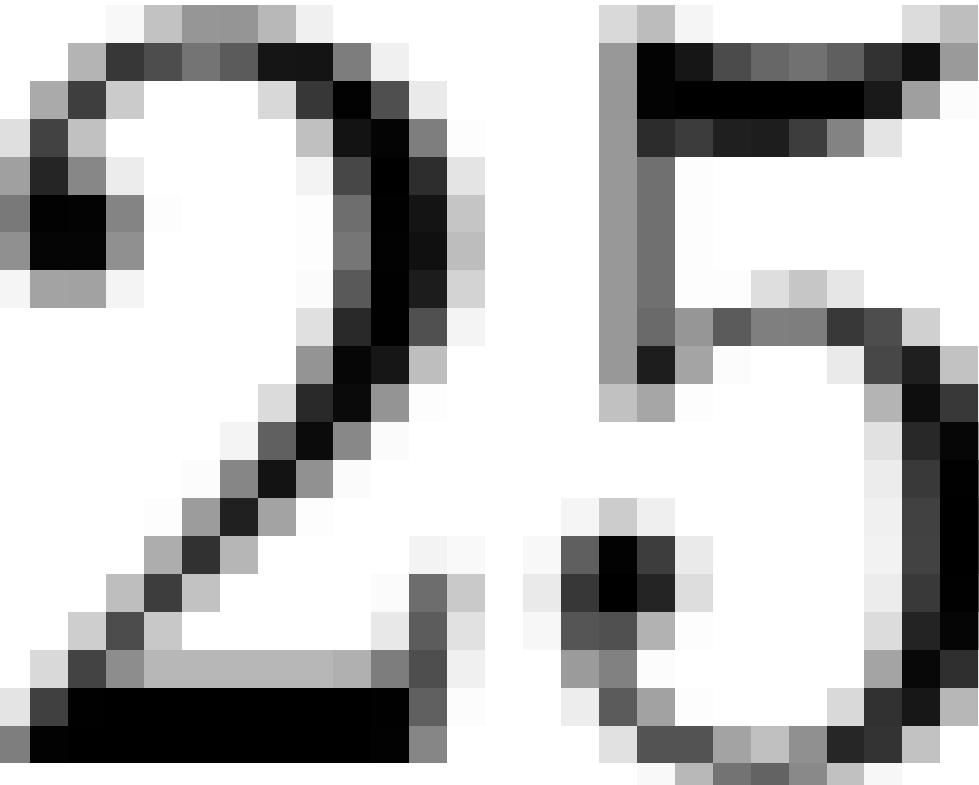
S

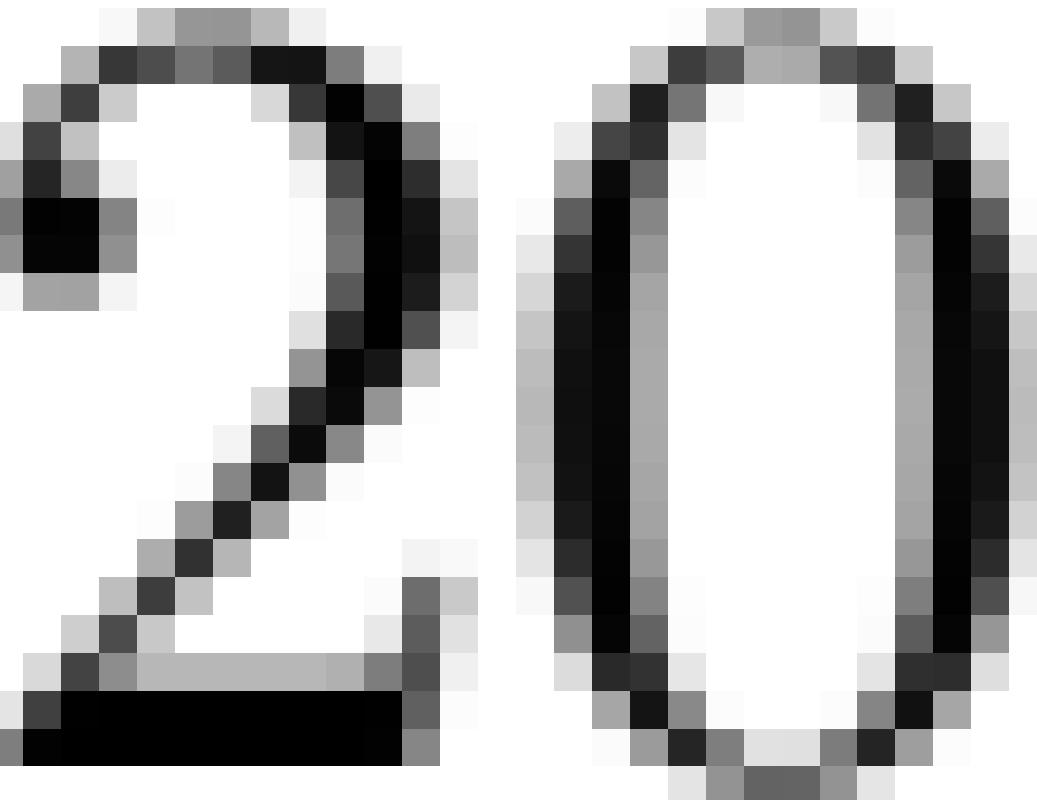
Q P Q S

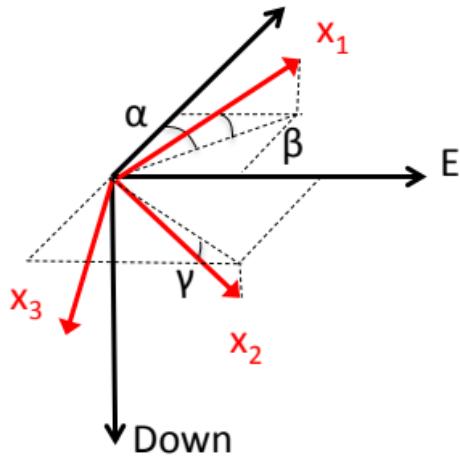
P

Q P Q









$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$a = 0^\circ$$

$$b = 90^\circ$$

$$g = 0^\circ$$

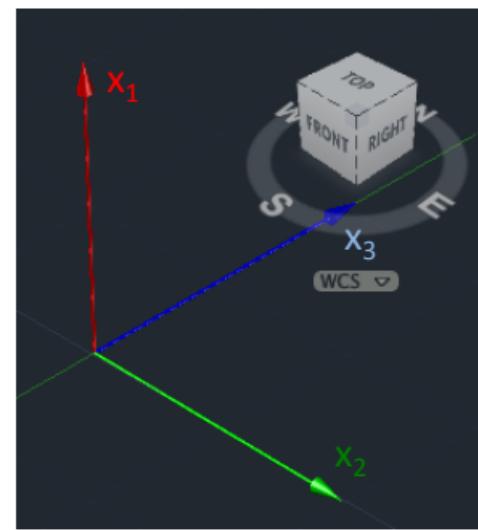
Azimuth of  $S_{h\min}$

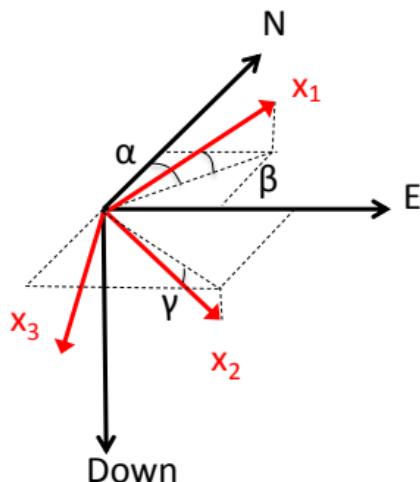
$$S_1 = S_V$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

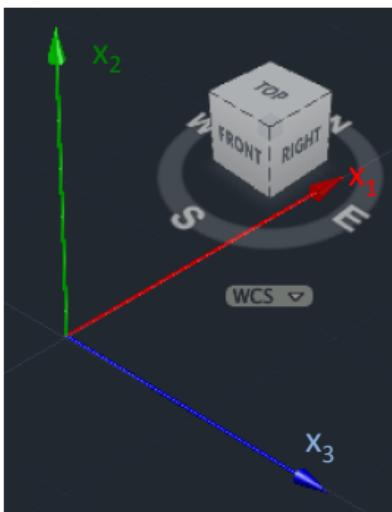
$$\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$





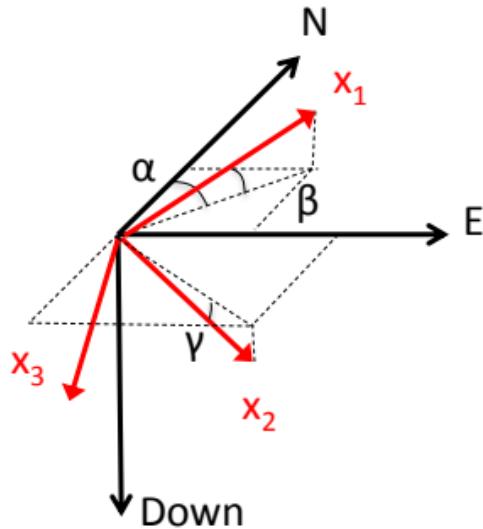
$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \quad \begin{array}{ll} a = 0^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & S_1 = S_{H\max} \\ g = 90^\circ & S_2 = S_V \end{array}$$



$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

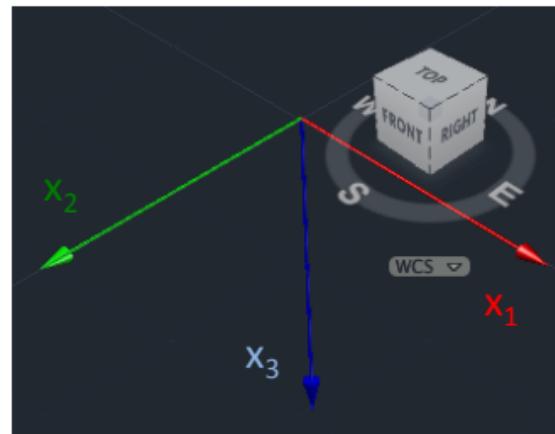
$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$



$$\underline{\underline{S}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$a = 90^\circ$  Azimuth of  $S_{H\max}$   
 $b = 0^\circ$   
 $g = 0^\circ$   
 $S_1 = S_{H\max}$   
 $S_2 = S_{h\min}$

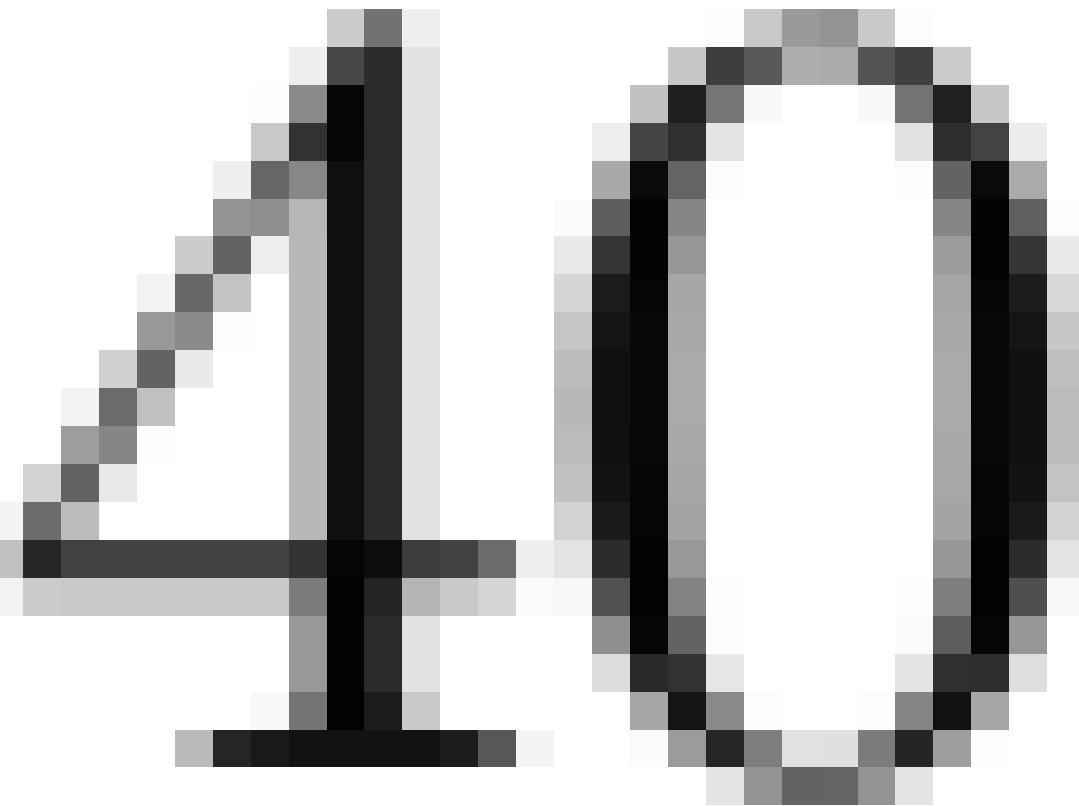


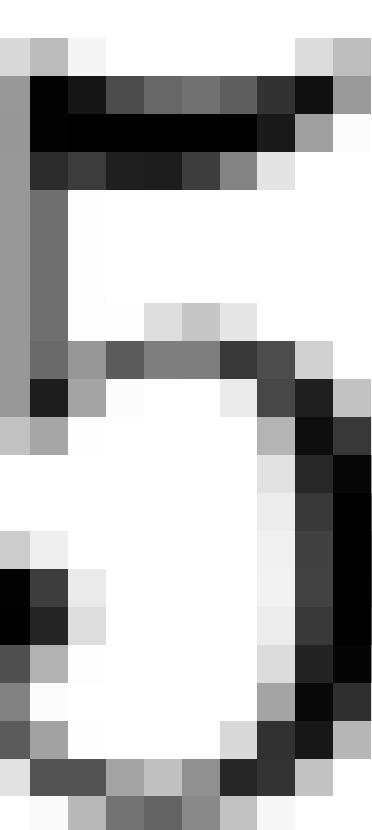
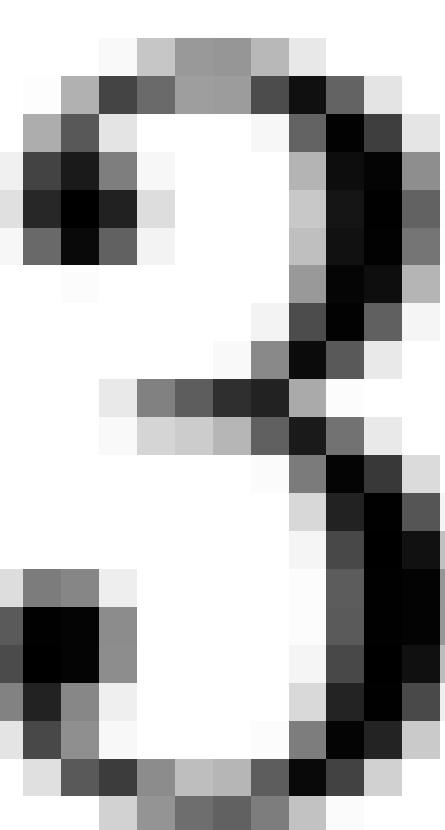
$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

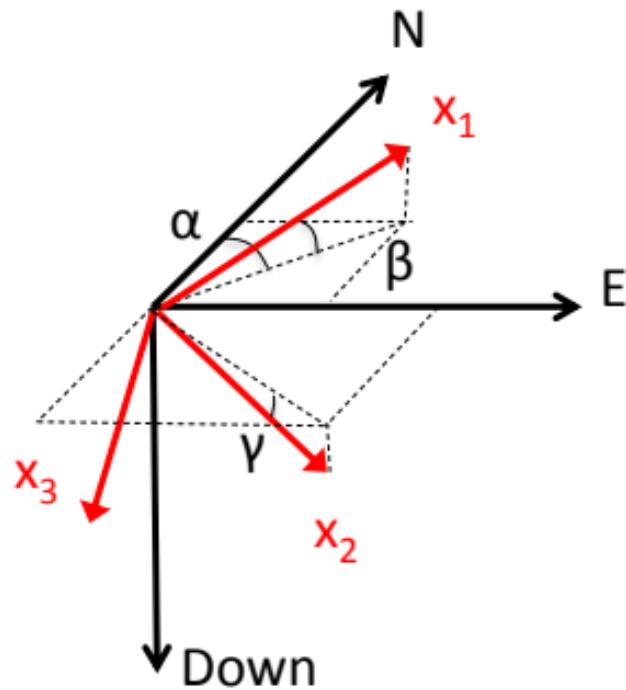
$$\boxed{\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$







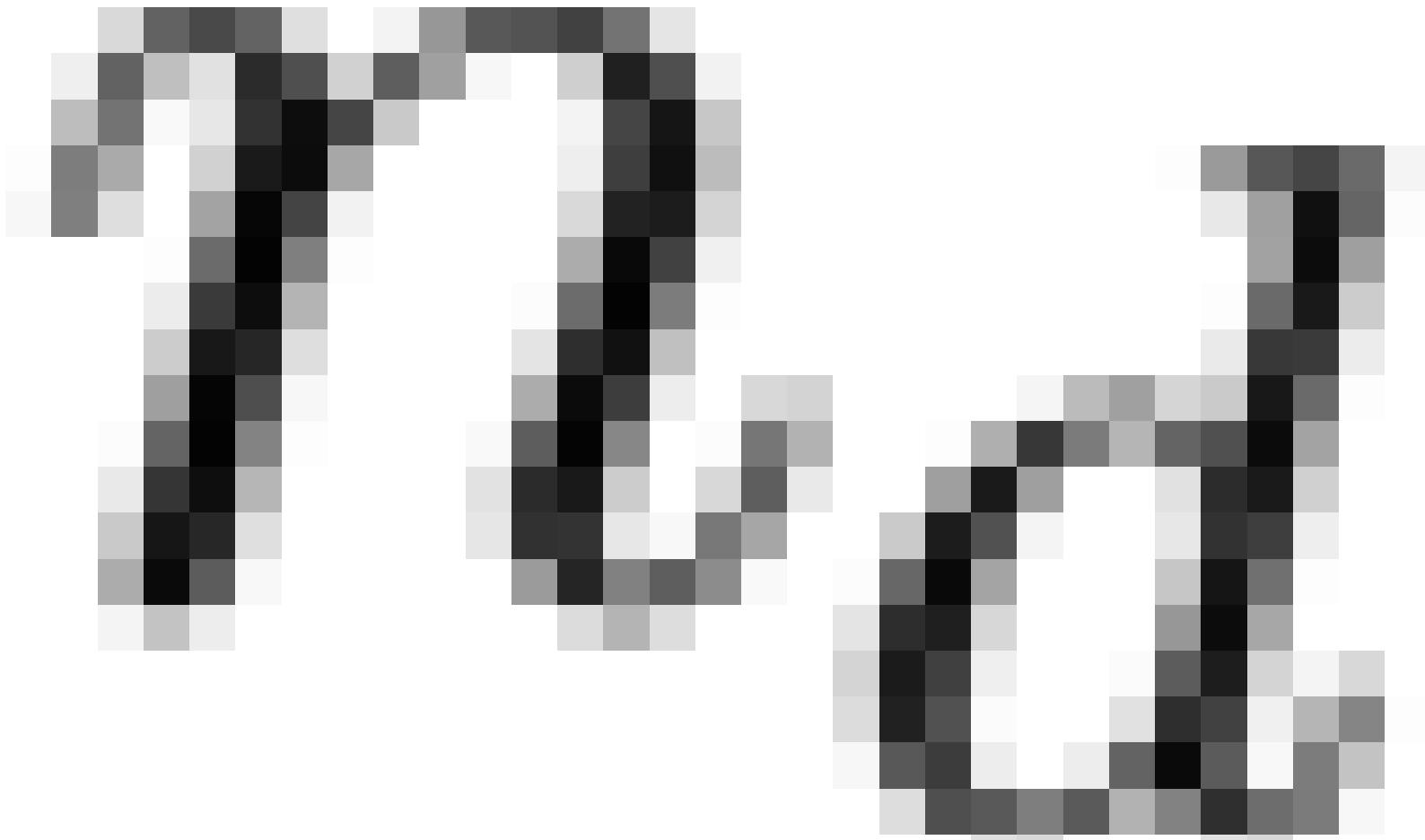


$$\underline{\underline{S}}_P = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix} \quad \begin{array}{ll} a = 135^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & S_1 = S_{H\max} \\ g = 90^\circ & S_2 = S_V \end{array}$$

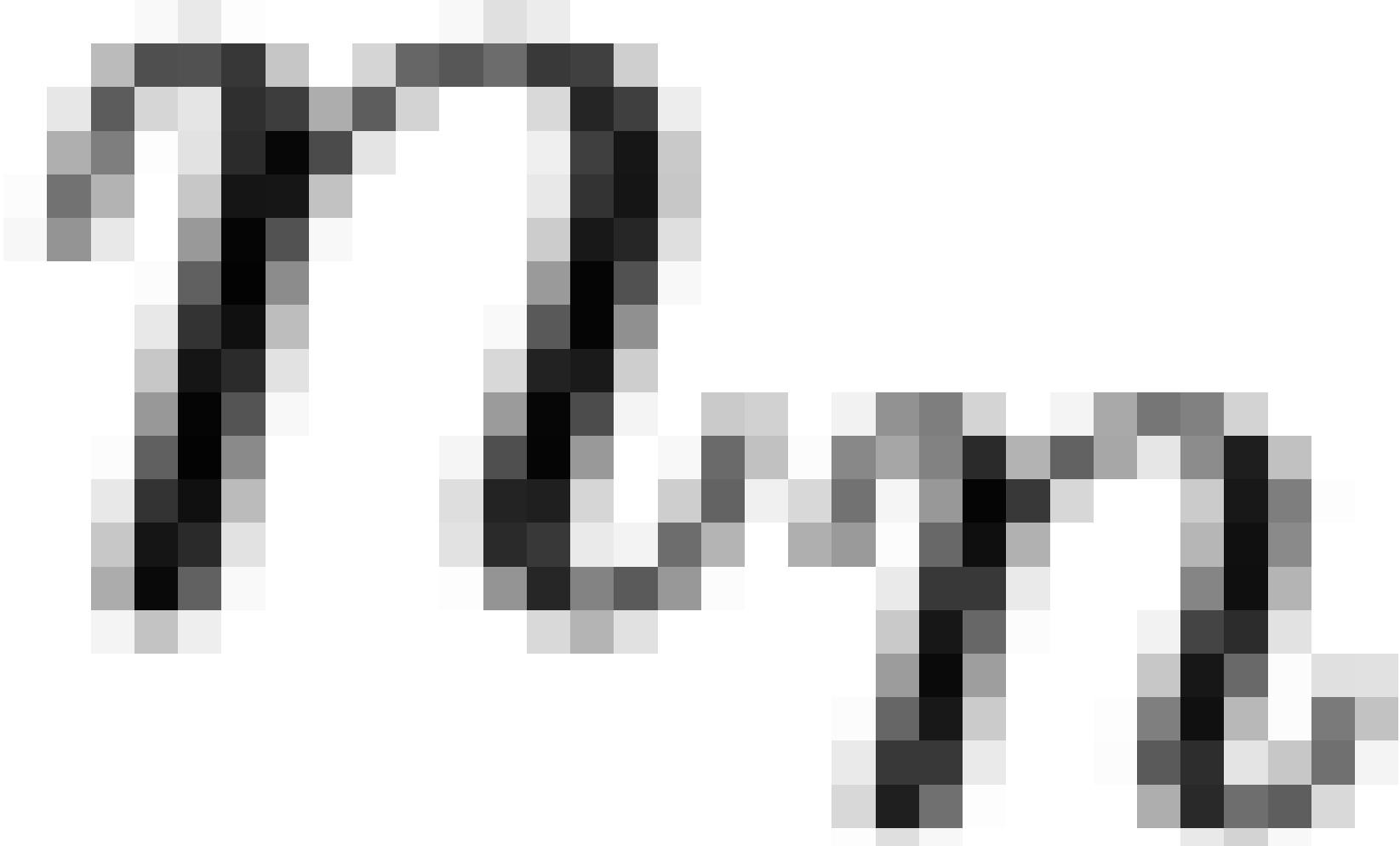
$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$

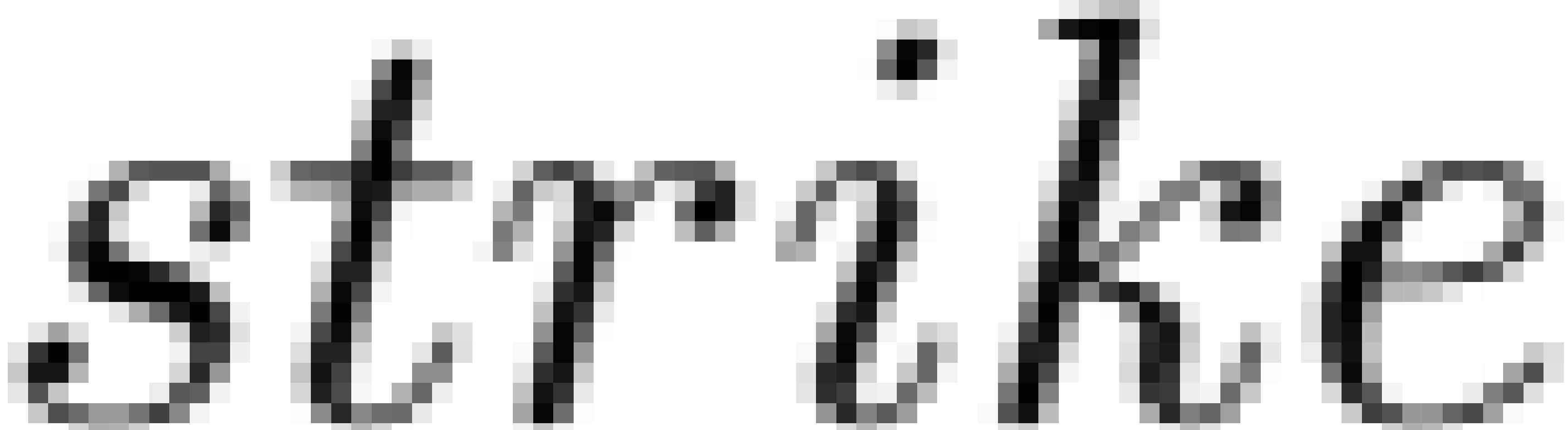
$$\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$



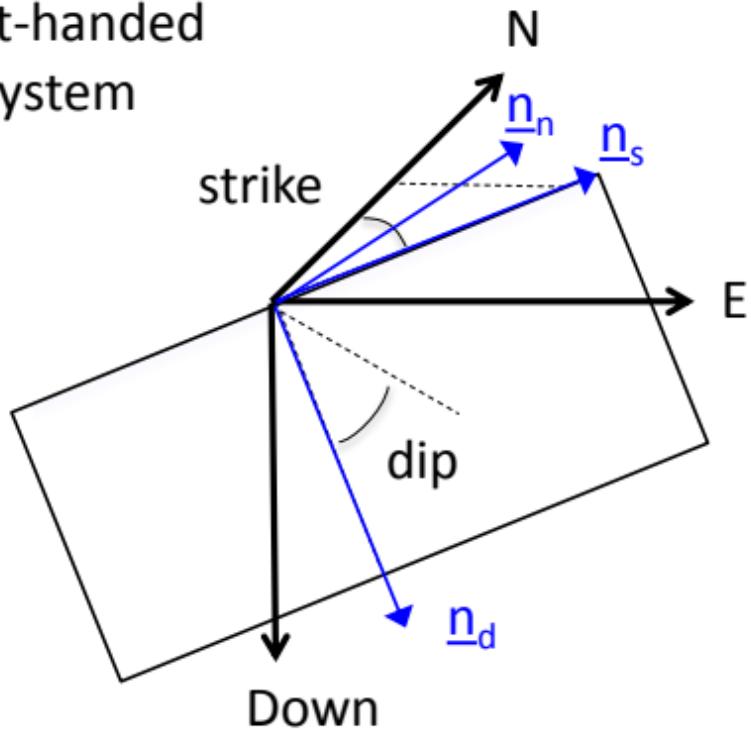






d-s-n

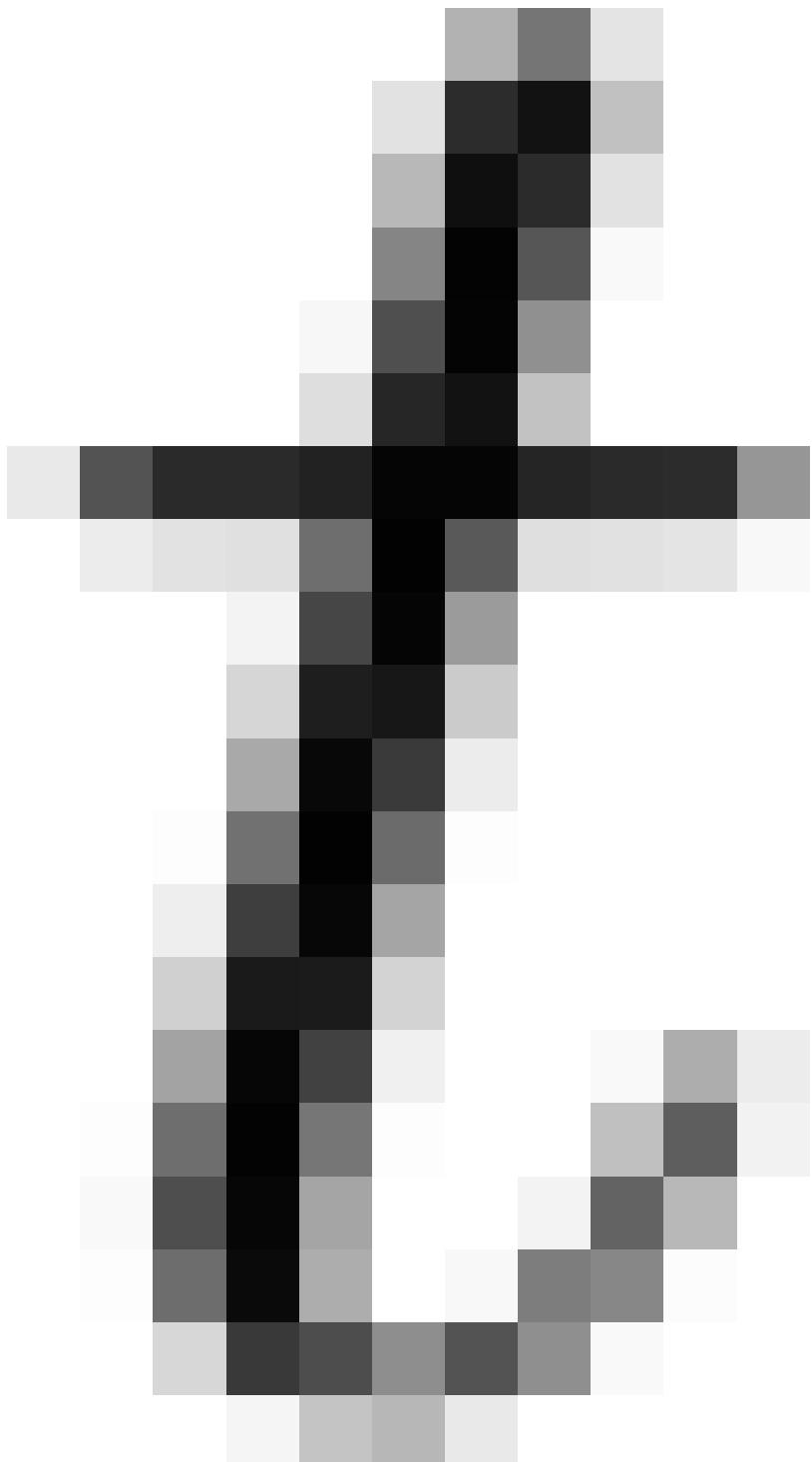
Right-handed  
system

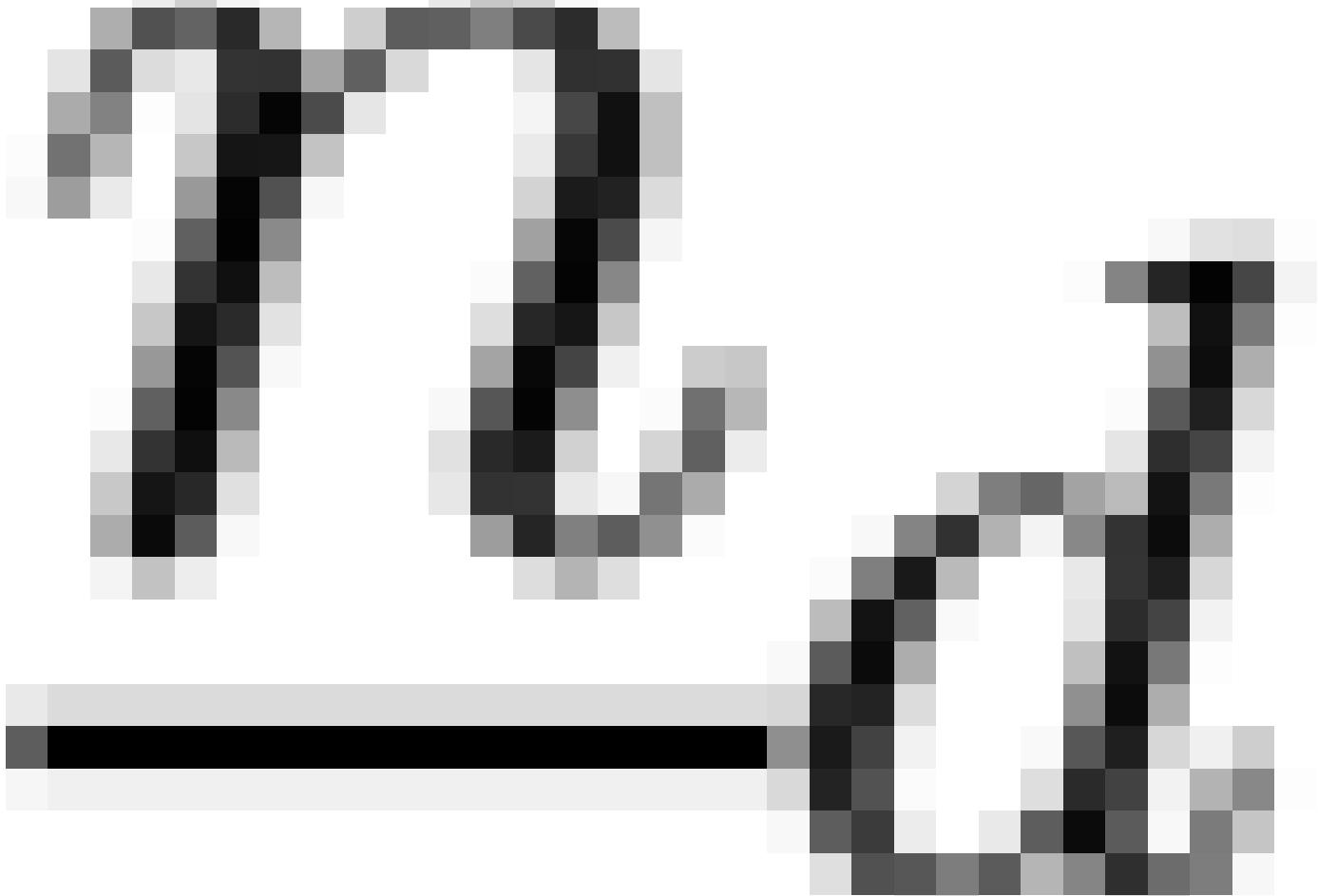


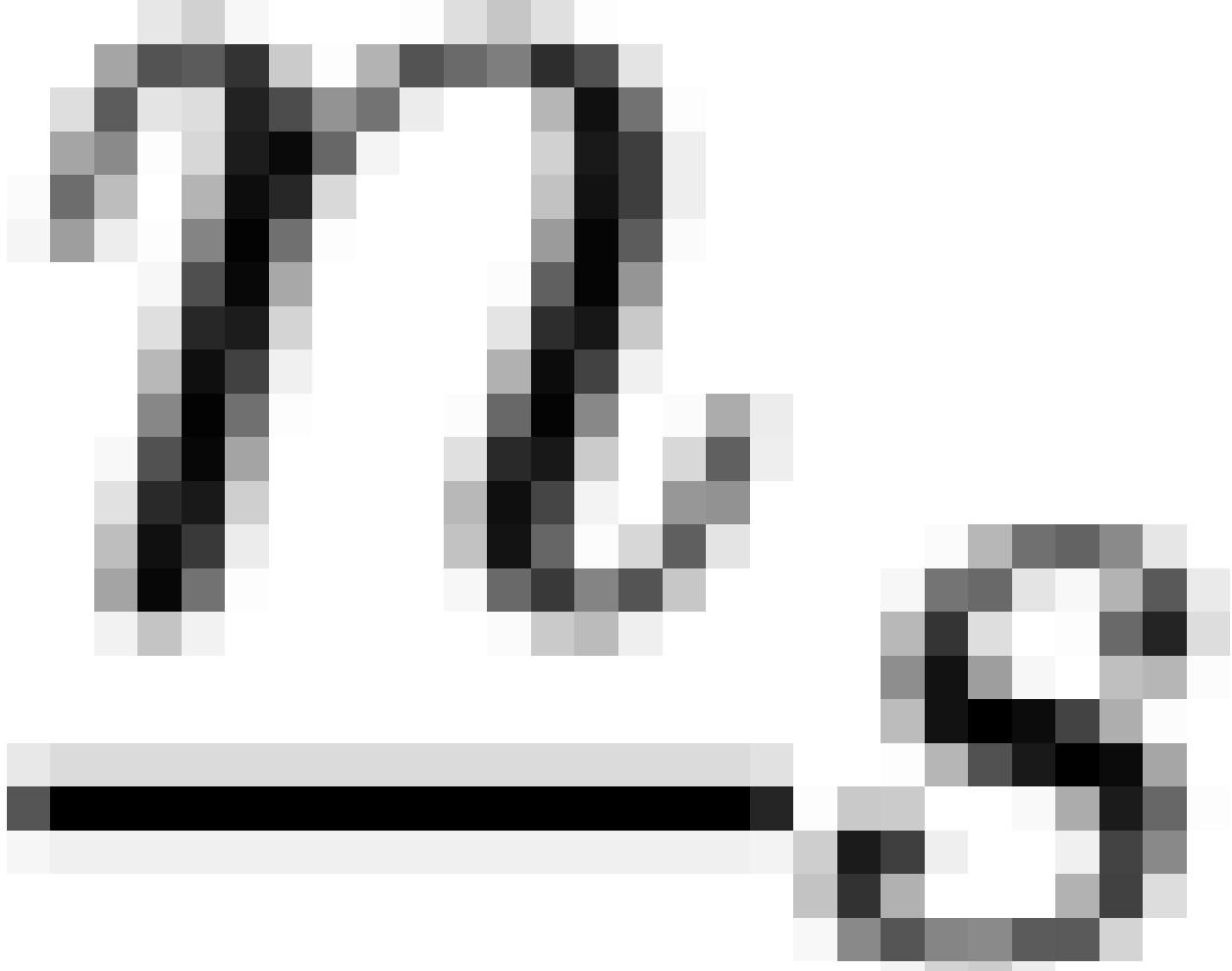
$$\underline{n}_n = \begin{bmatrix} -\sin(strike)\sin(dip) \\ \cos(strike)\sin(dip) \\ -\cos(dip) \end{bmatrix}$$

$$\underline{n}_s = \begin{bmatrix} \cos(strike) \\ \sin(strike) \\ 0 \end{bmatrix}$$

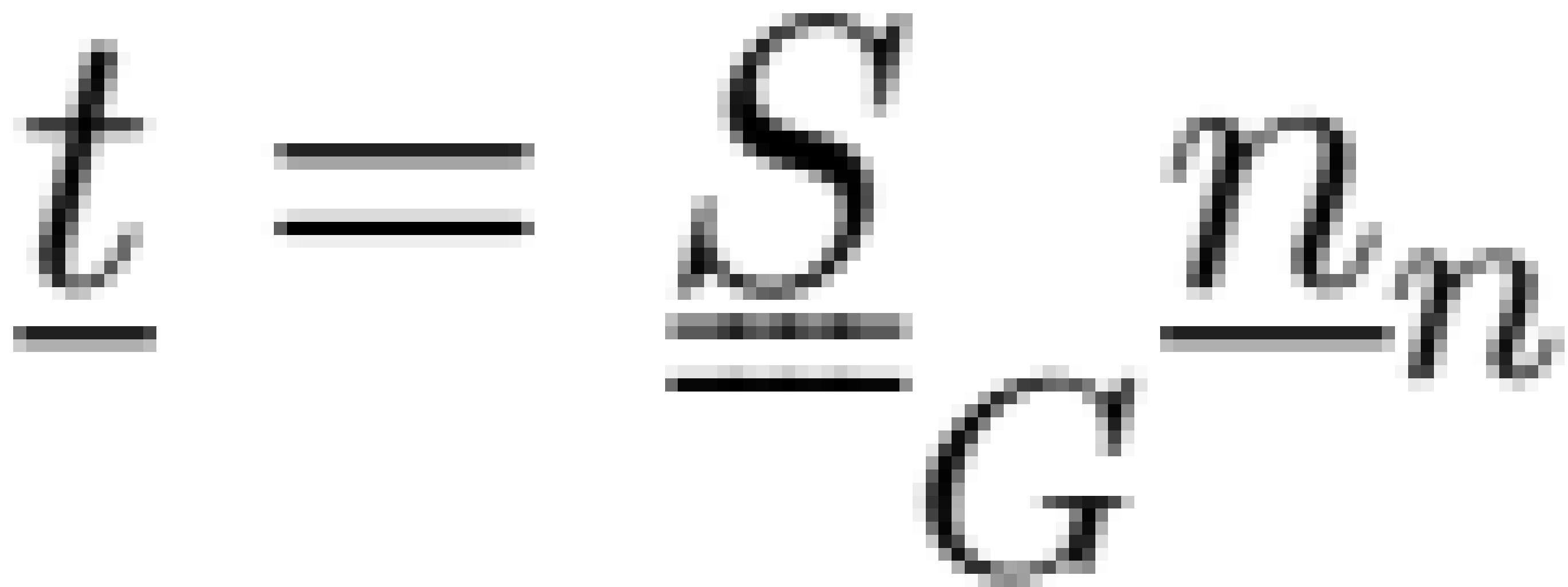
$$\underline{n}_d = \begin{bmatrix} -\sin(strike)\cos(dip) \\ \cos(strike)\cos(dip) \\ \sin(dip) \end{bmatrix}$$



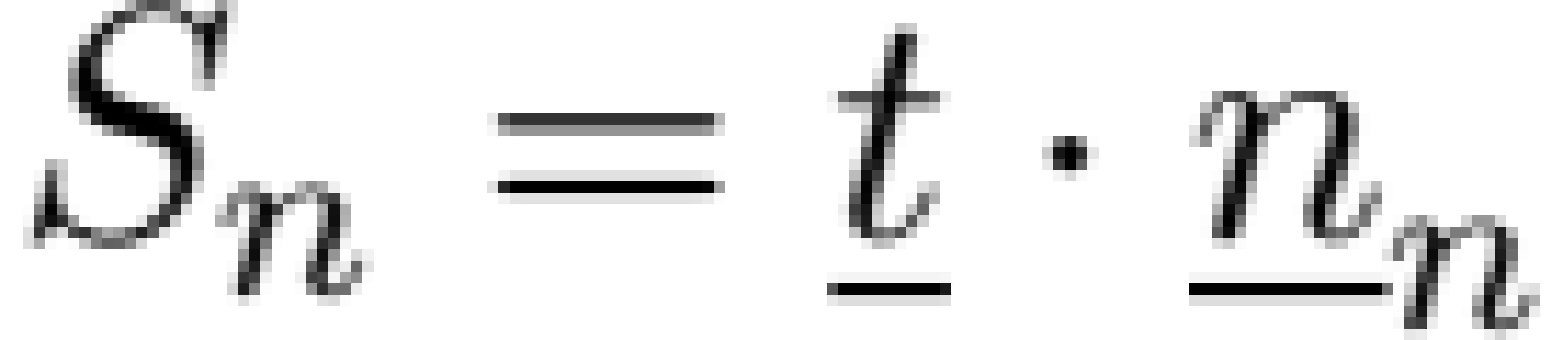








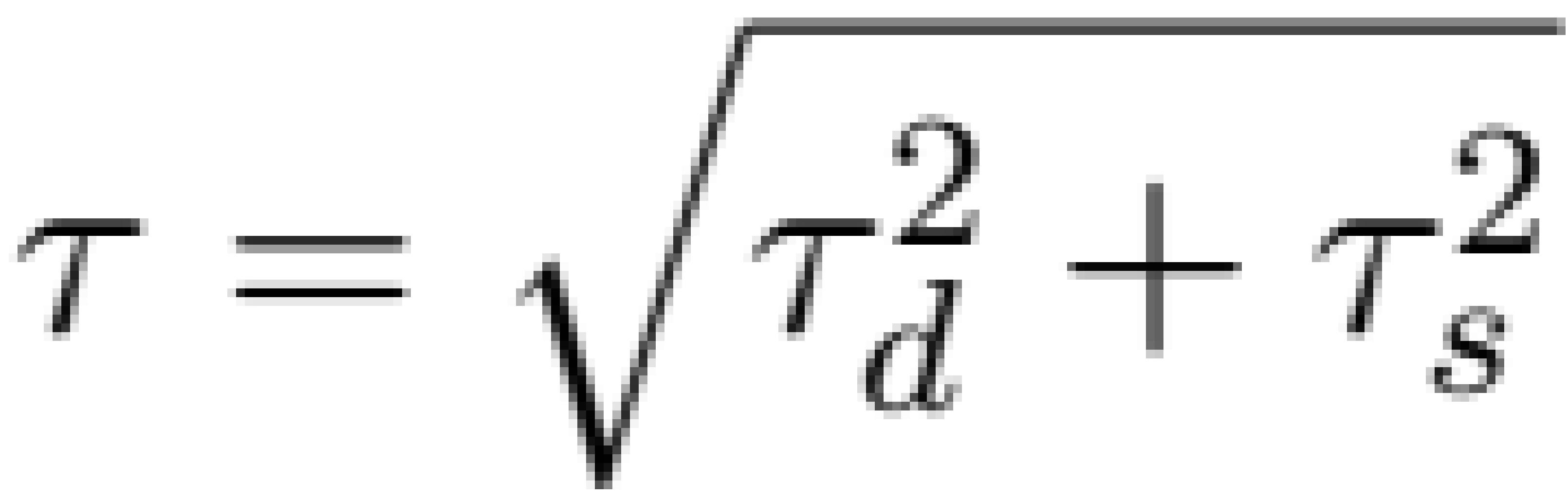


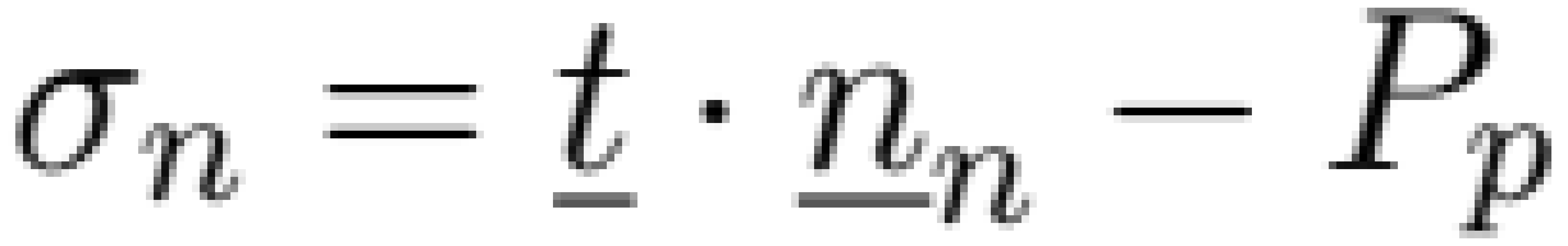


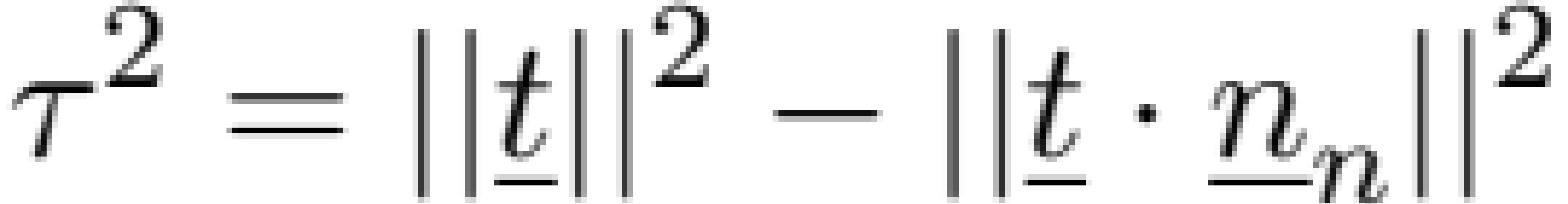


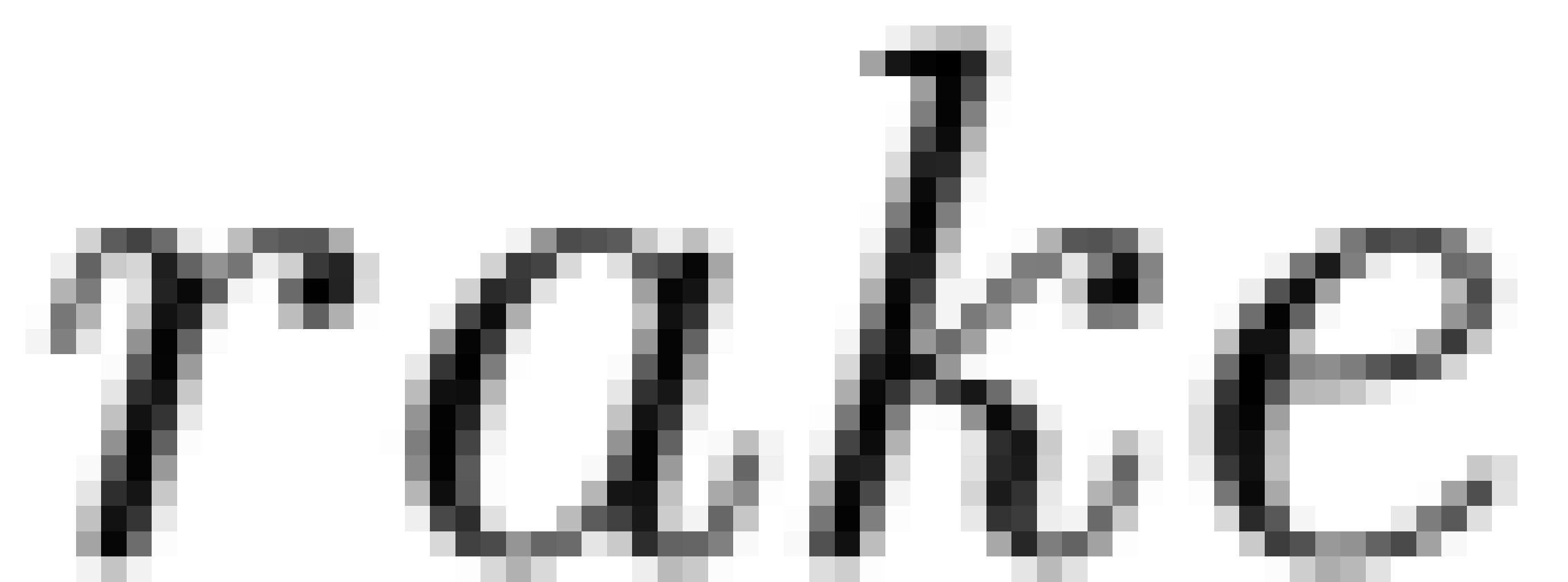
$$\tau_d = t \cdot \frac{r_d}{n_d}$$

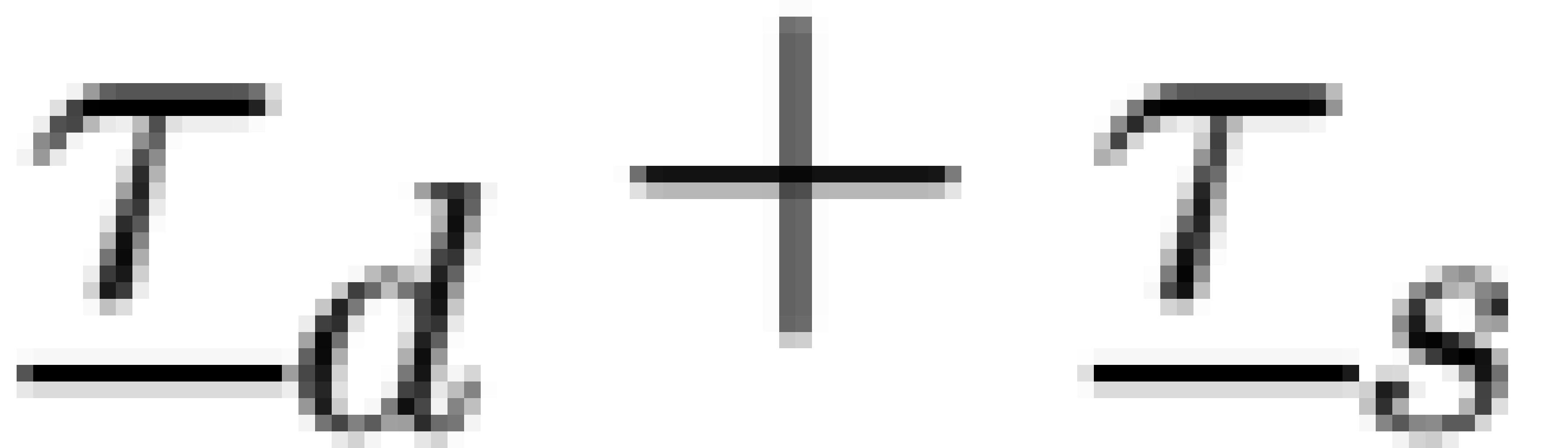
$$\tau_s = t \cdot \frac{r_s}{n_s}$$





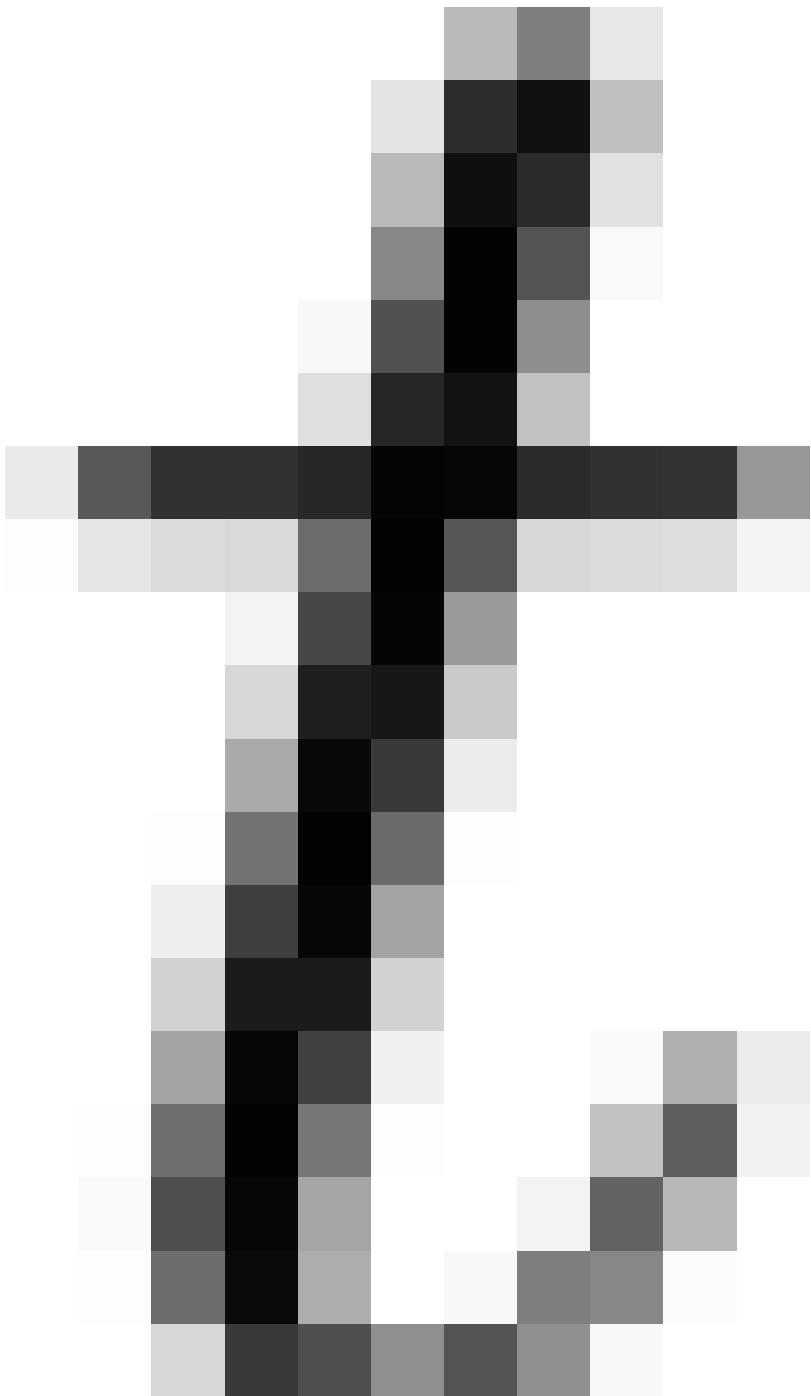


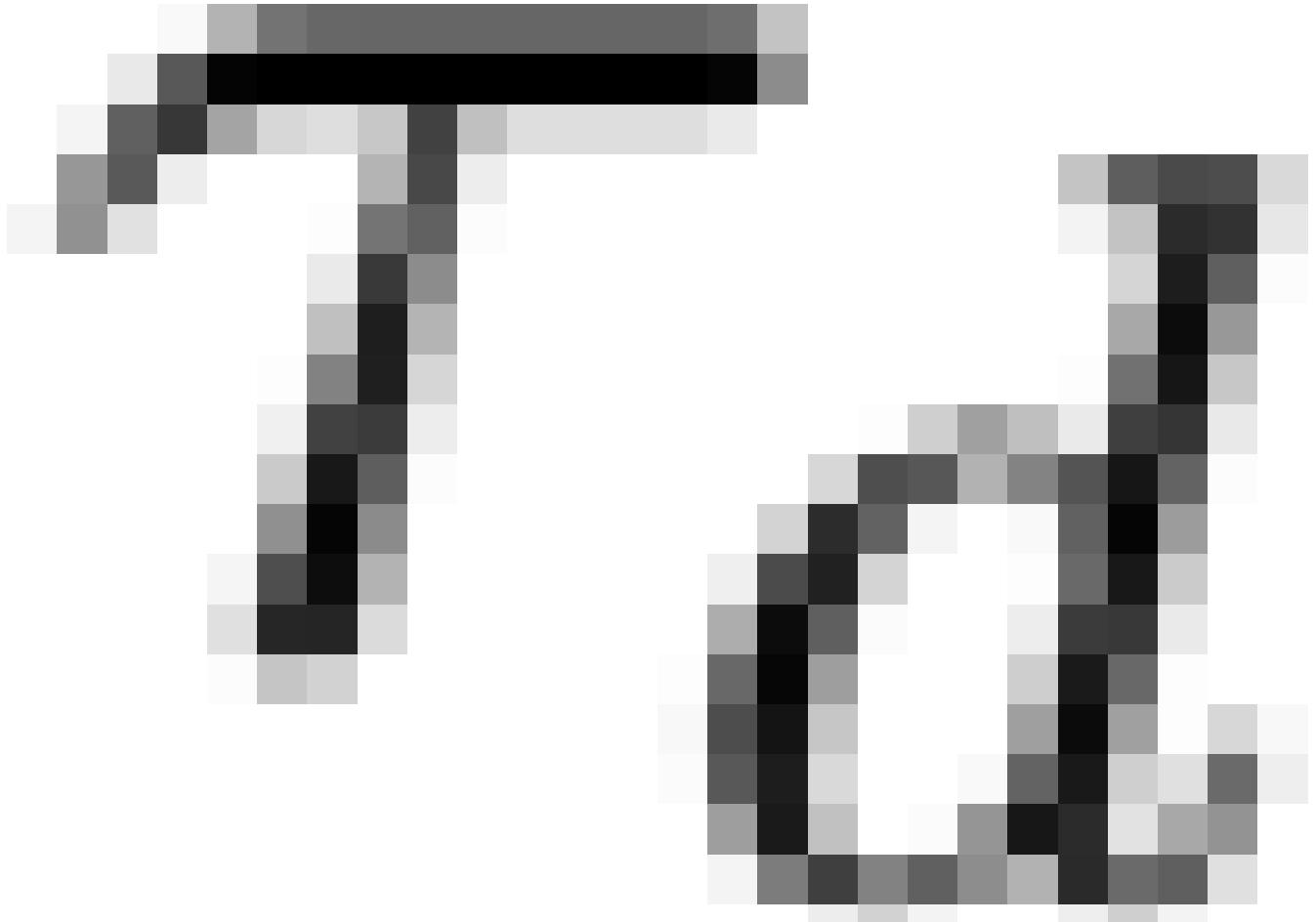


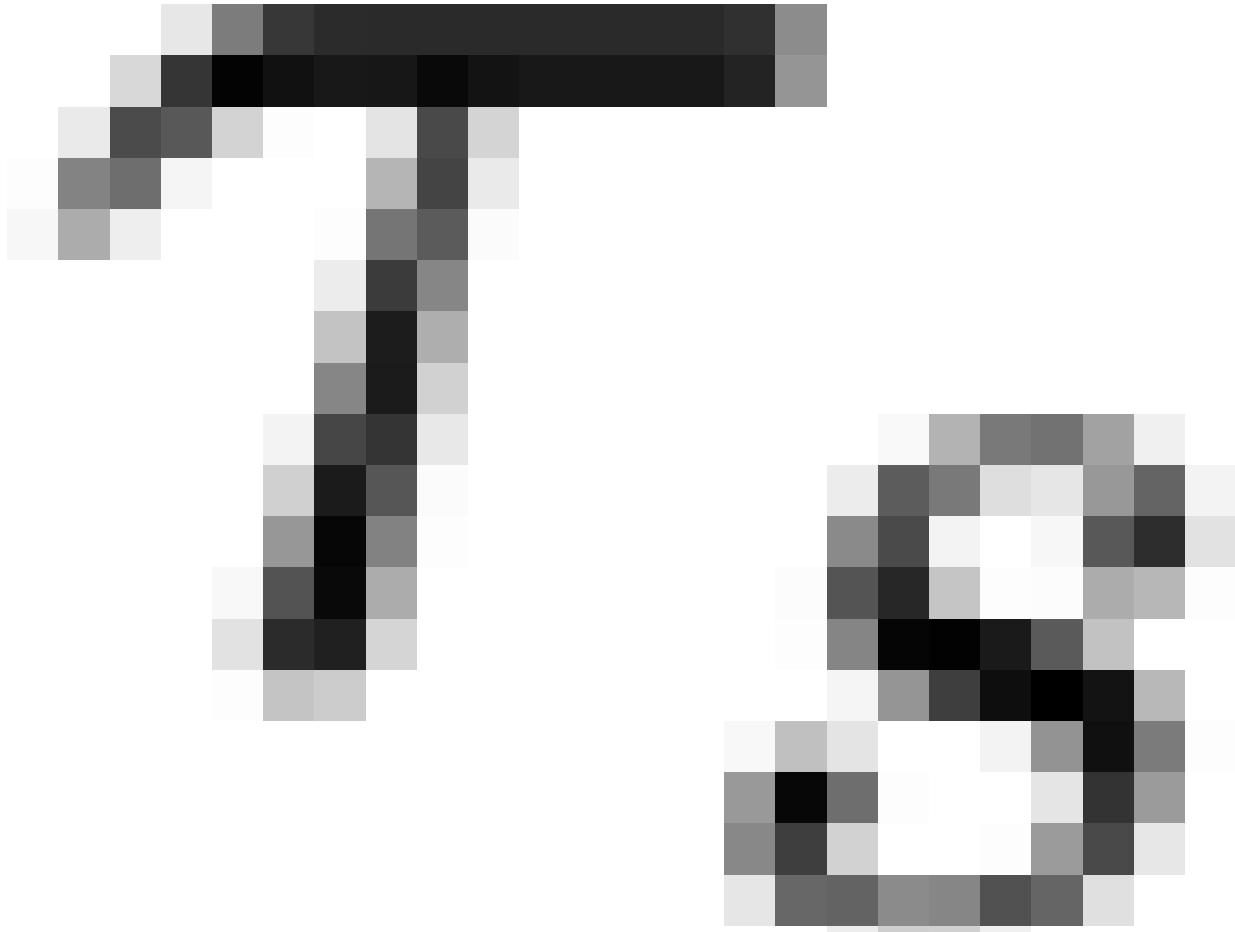


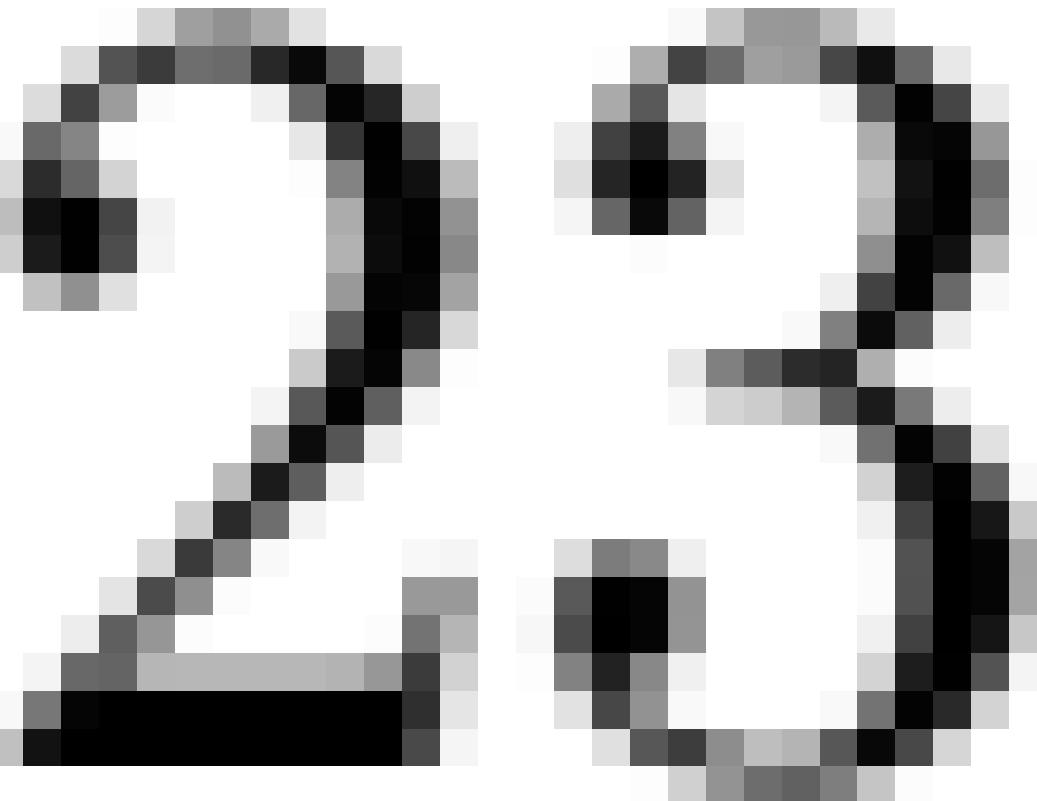
rake = arctan

$$\frac{\tau_d}{\tau_s}$$

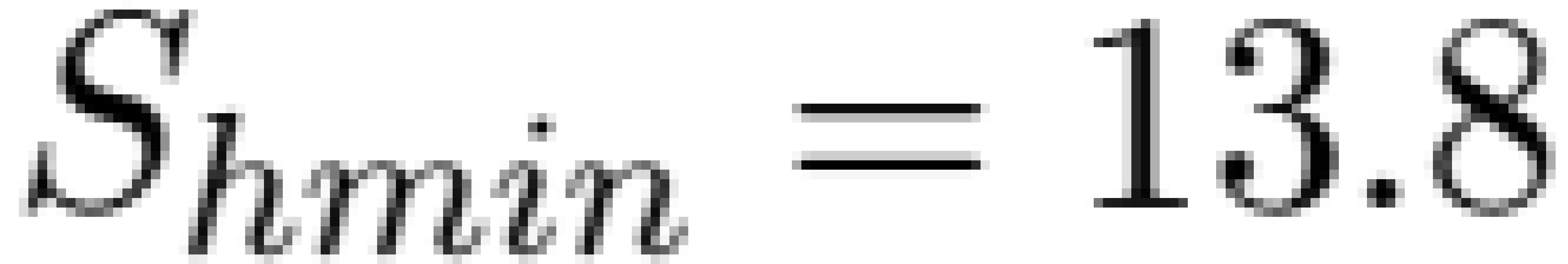


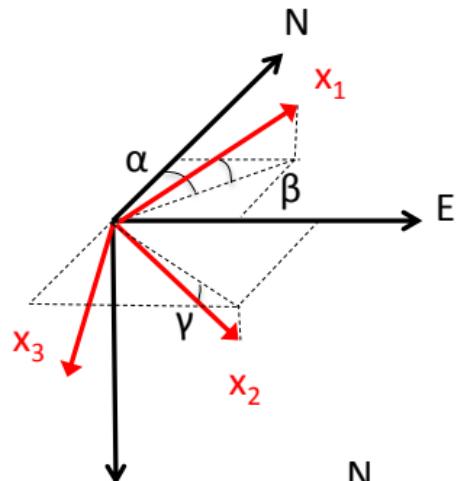










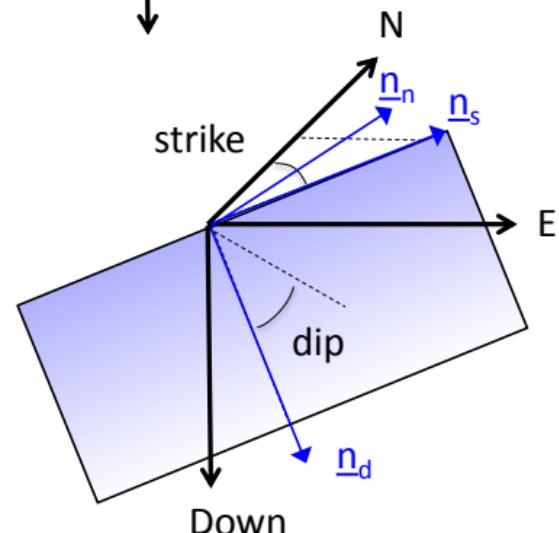


$$\underline{\underline{S}}_P = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 13.8 \end{bmatrix}$$

$a = 90^\circ$  Azimuth of  $S_{h\min}$   
 $b = 90^\circ$   $S_1 = S_V$   
 $g = 0^\circ$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 13.8 & 0 \\ 0 & 0 & 23 \end{bmatrix}$$



Fault geometry

Strike =  $000^\circ$   
 Dip =  $60^\circ$

$$\underline{n}_n = \begin{bmatrix} 0 \\ 0.867 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [0, 11.95, -11.50] \text{ MPa}$$

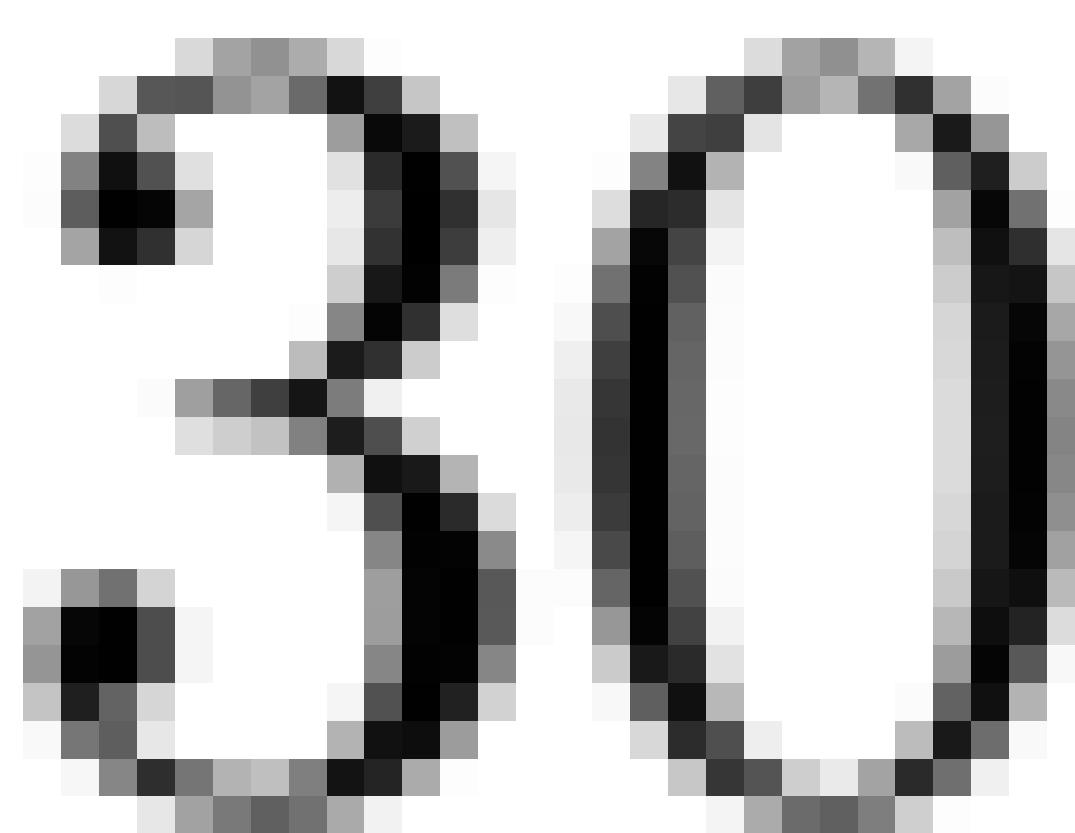
$$S_n = 16.1 \text{ MPa}$$

$$t_d = -3.98 \text{ MPa}$$

$$t_s = 0 \text{ MPa}$$

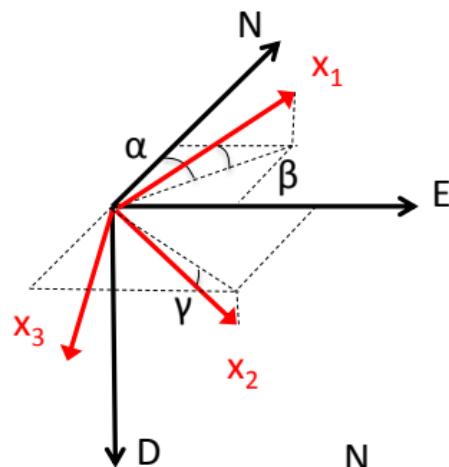
$$\text{rake} = 90^\circ$$

Actual stresses opposite to d-s-n



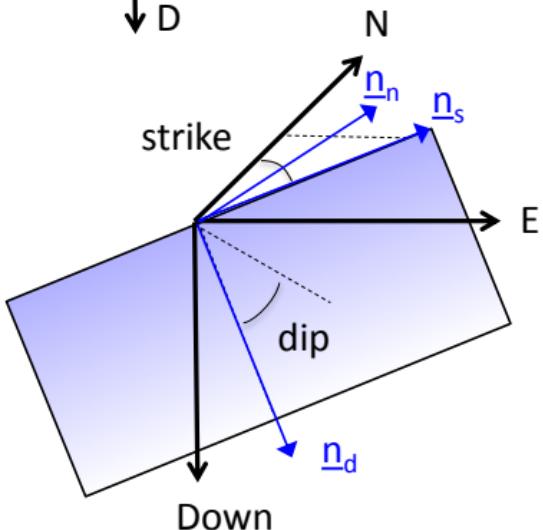






$$\underline{\underline{S}}_P = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix} \quad \begin{array}{ll} a = 120^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 90^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \\ 0.866 & 0.5 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$



Fault geometry

Strike = 060°

Dip = 90°

$$\underline{n}_n = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$\underline{t} = [-30.31, 27.5, 0] \text{ MPa}$$

$$S_n = 40 \text{ MPa}$$

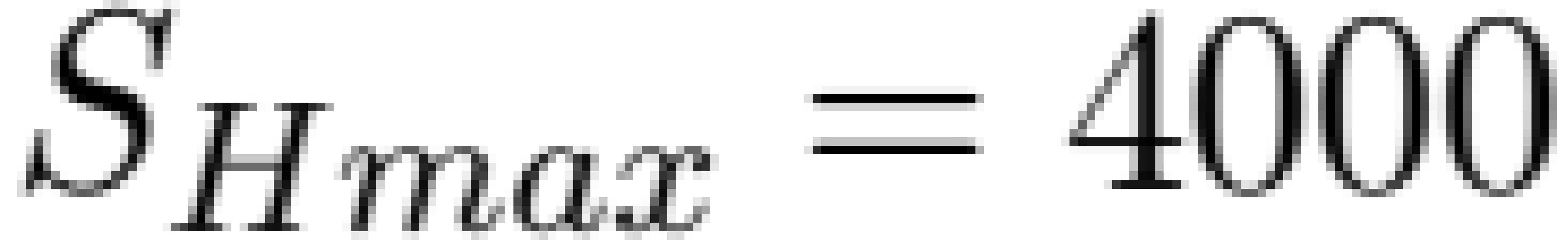
$$t_d = 0 \text{ MPa}$$

$$t_s = 8.66 \text{ MPa}$$

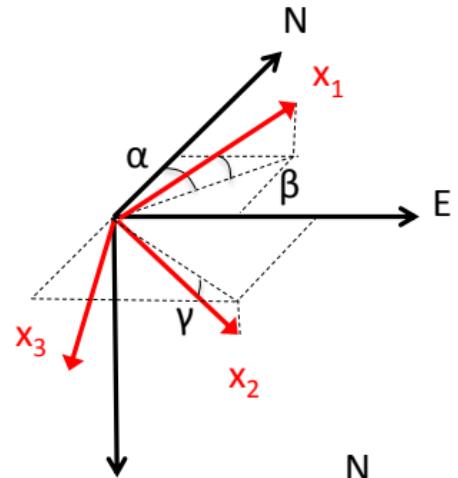
$$rake = 0^\circ$$

Actual stresses opposite to d-s-n



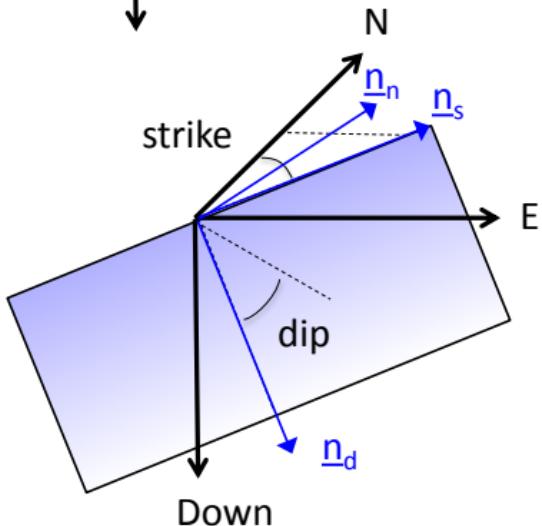






$$\underline{\underline{S}}_P = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \quad \begin{array}{ll} a = 90^\circ & \text{Azimuth of } S_{h\min} \\ b = 90^\circ & \\ g = 0^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$



Fault geometry

Strike = 045°

Dip = 60°

$$n_n = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [-2450, 1840, -2500] \text{ psi}$$

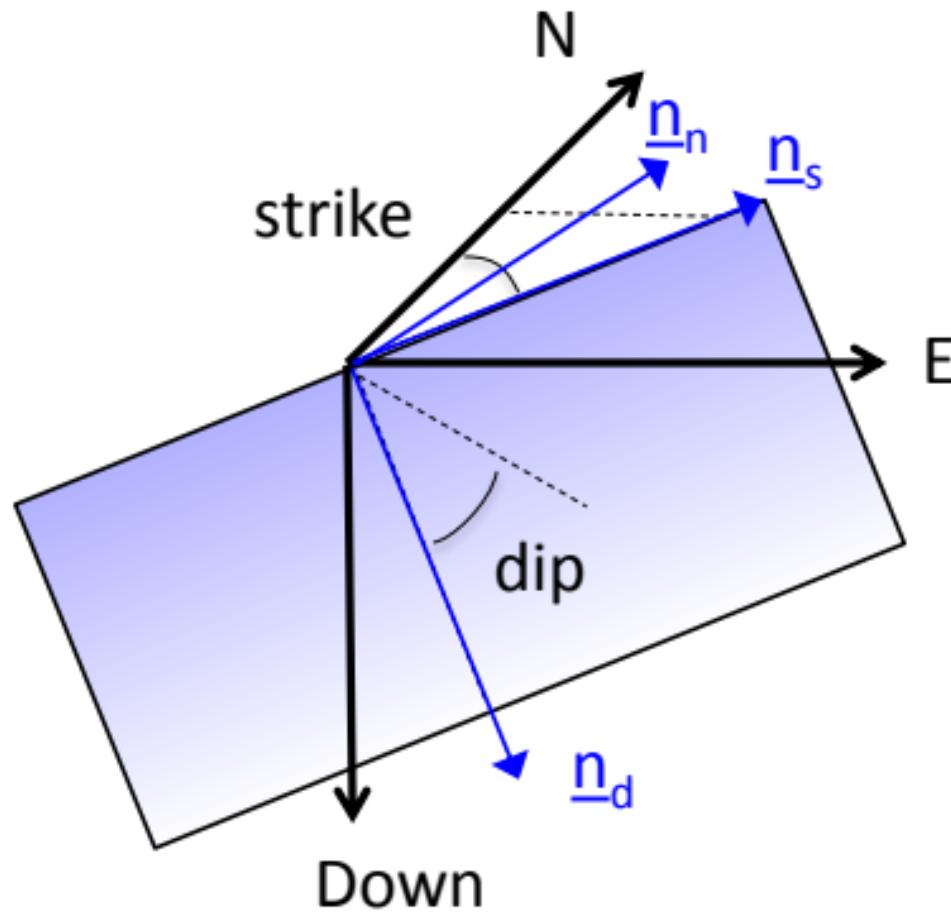
$$S_n = 3870 \text{ psi}$$

$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

$$\text{rake} = 56.3^\circ$$

Actual stresses opposite to d-s-n



Fault geometry

Strike =  $225^\circ$

Dip =  $60^\circ$

$$\underline{n}_n = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix}$$

$$t = [2450, -1840, -2500] \text{ psi}$$

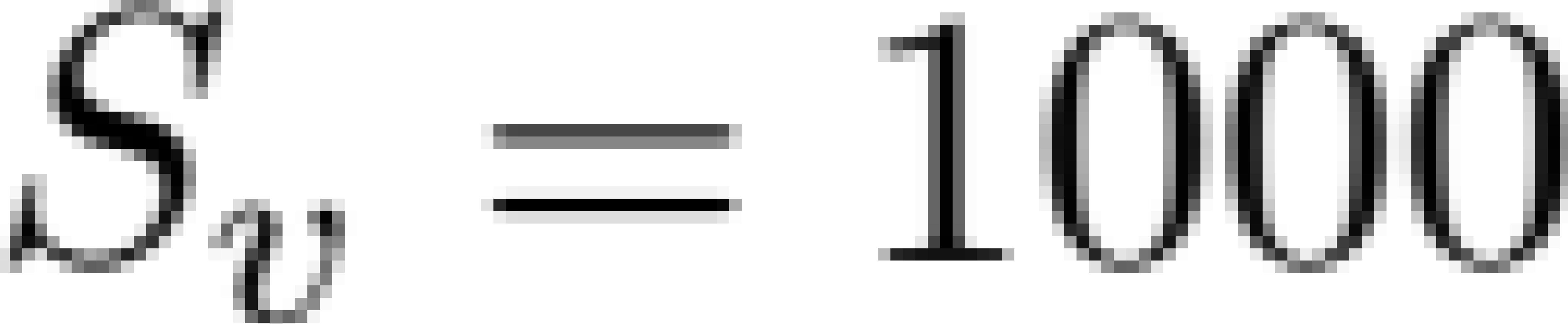
$$S_n = 3870 \text{ psi}$$

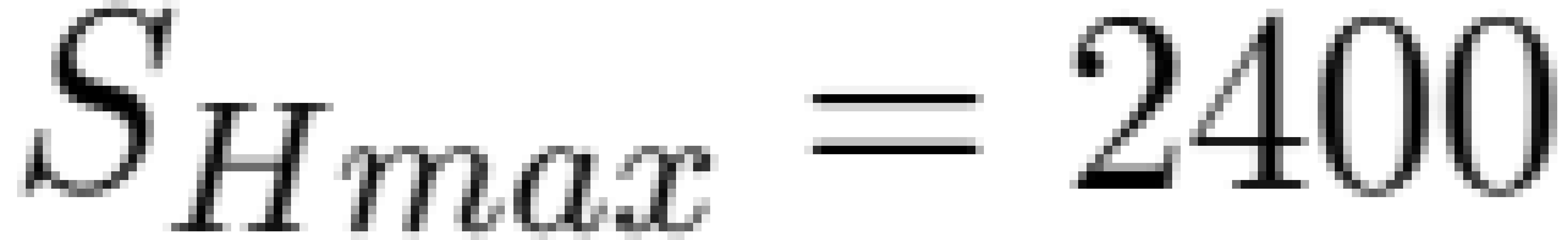
$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

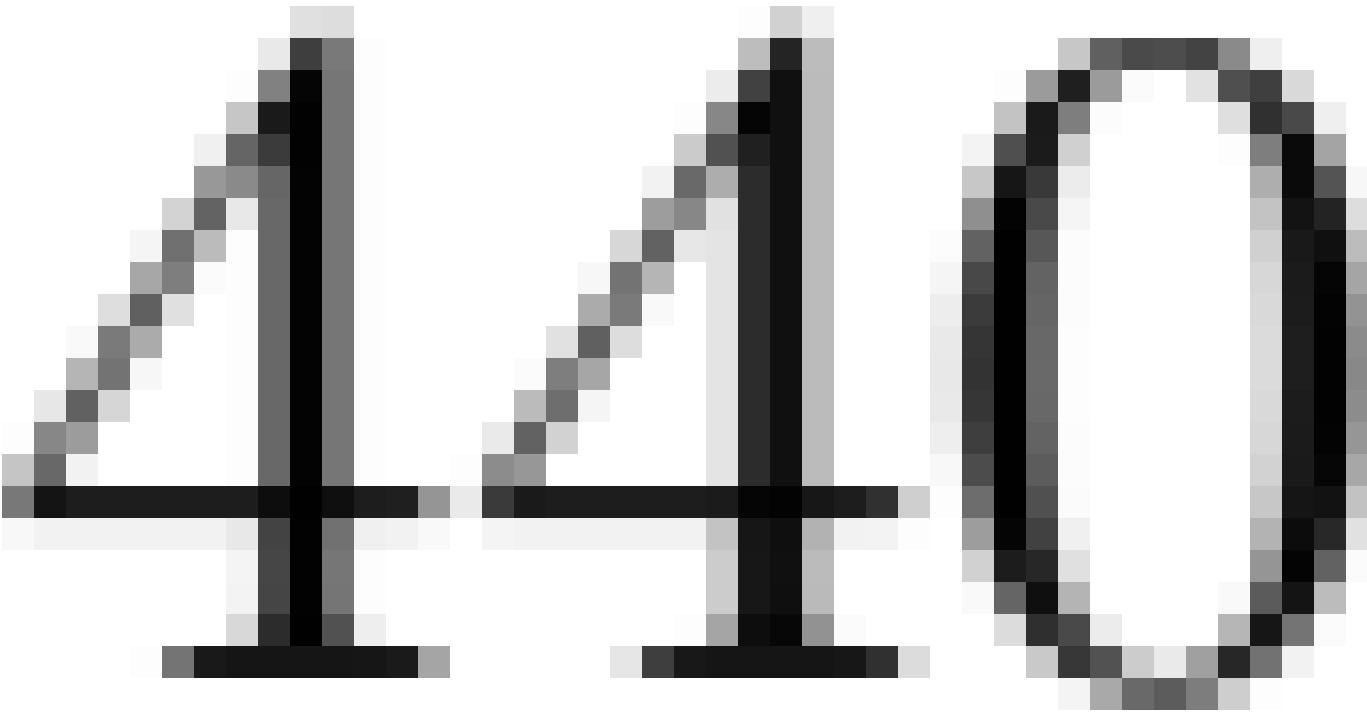
$$\text{rake} = 56.3^\circ$$

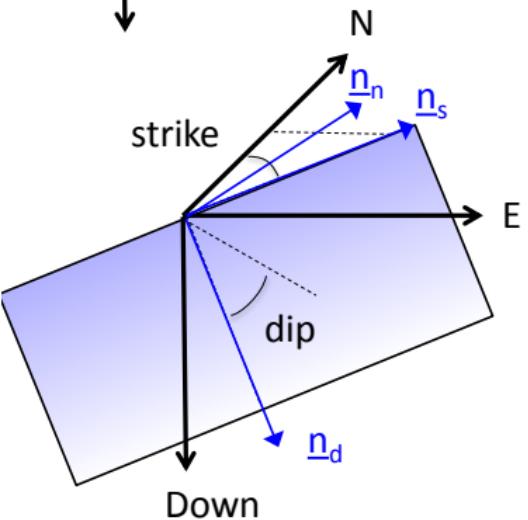
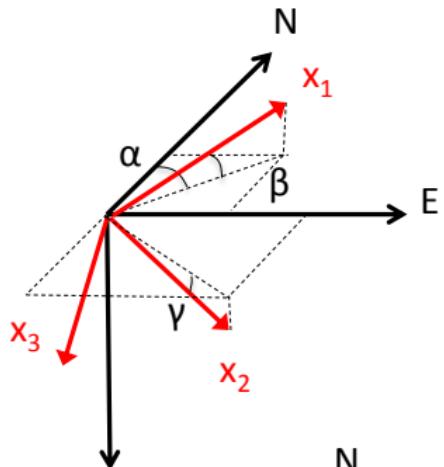
Actual stresses opposite to d-s-n











$$\underline{\underline{S}}_P = \begin{bmatrix} 2400 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \quad \begin{array}{ll} \alpha = 150^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 0^\circ & S_3 = S_V \end{array}$$

$$P_w = 440 \text{ psi}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.866 & 0.5 & 0 \\ -0.5 & -0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

Fault geometry

Strike =  $120^\circ$

Dip =  $70^\circ$

$$\underline{n}_n = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix}$$

$$\underline{t} = [-1465, -281, -342] \text{ psi}$$

$$S_n = 1441 \text{ psi}$$

$$t_d = 160 \text{ psi}$$

$$t_s = 488 \text{ psi}$$

$$\text{rake} = 18.21^\circ$$

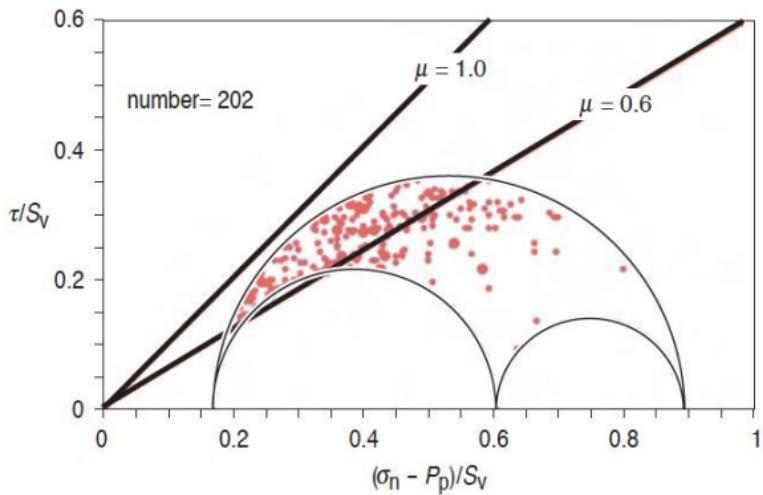
$$t / S_n = 0.51$$

Actual stresses opposite to d-s-n



Zoback 2013 - Figure 11.1-5.2

HYDRAULICALLY CONDUCTIVE FRACTURES

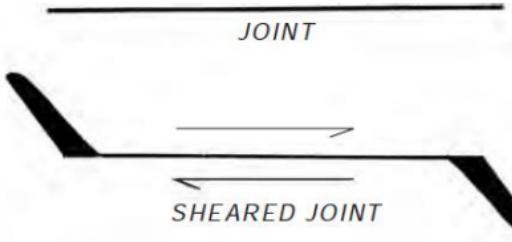
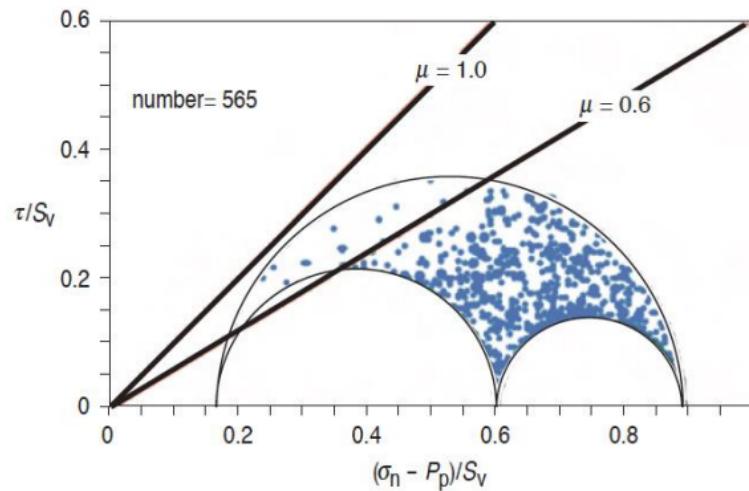


ONSET OF DAMAGE

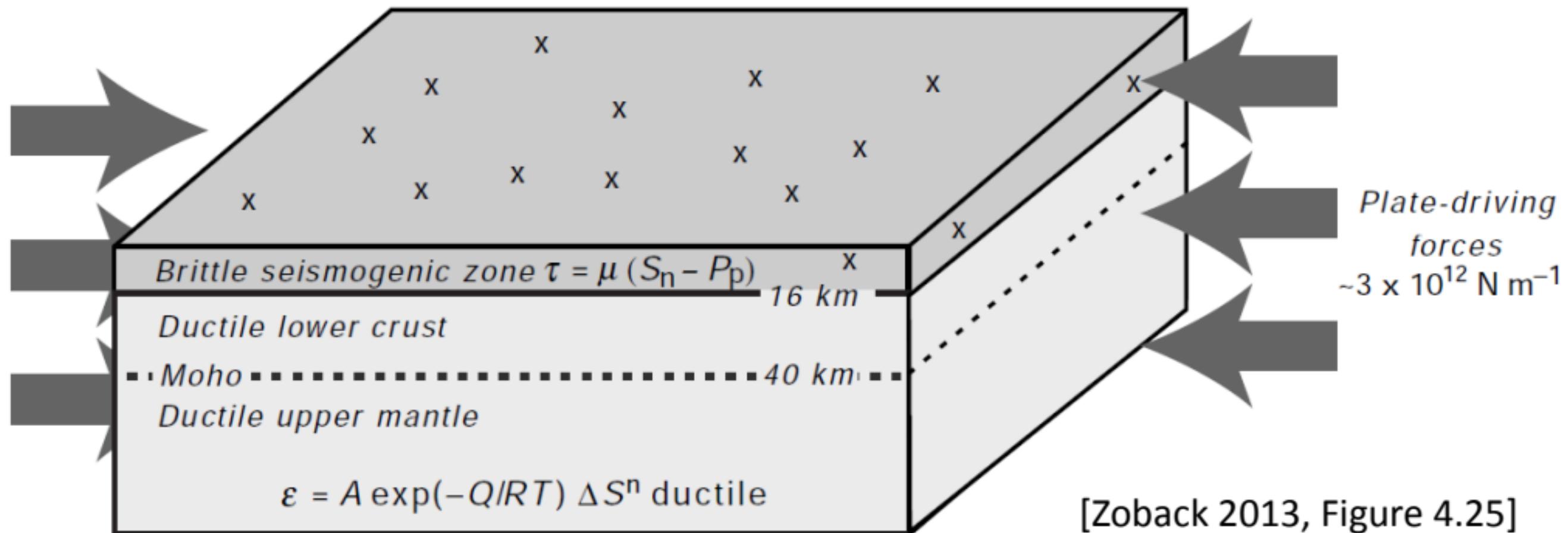


BRECCIAZION IN SHEAR ZONE

NON-HYDRAULICALLY CONDUCTIVE FRACTURES



- + Mineral precipitation
- + Mineralization to clays

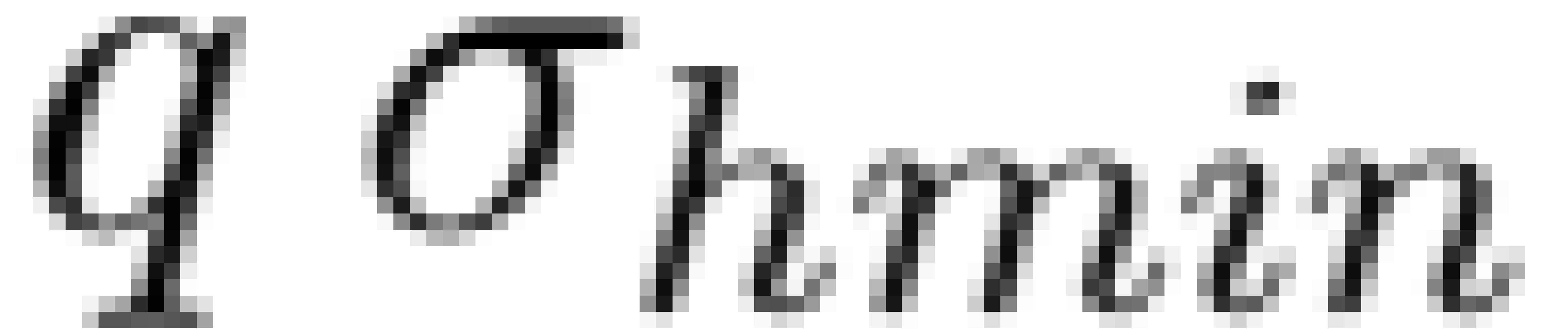


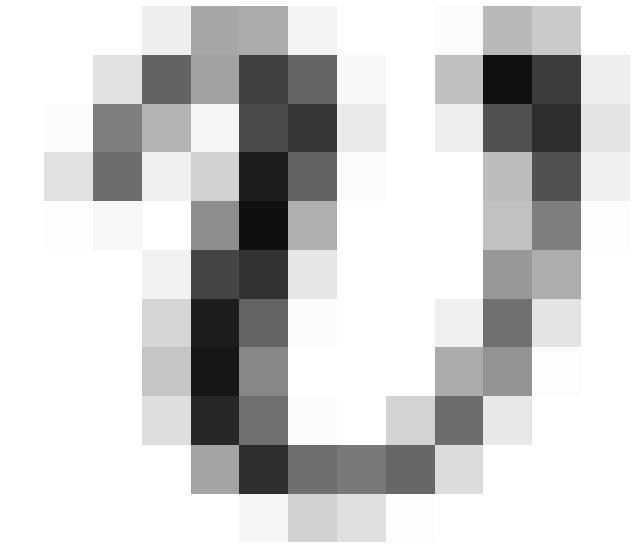
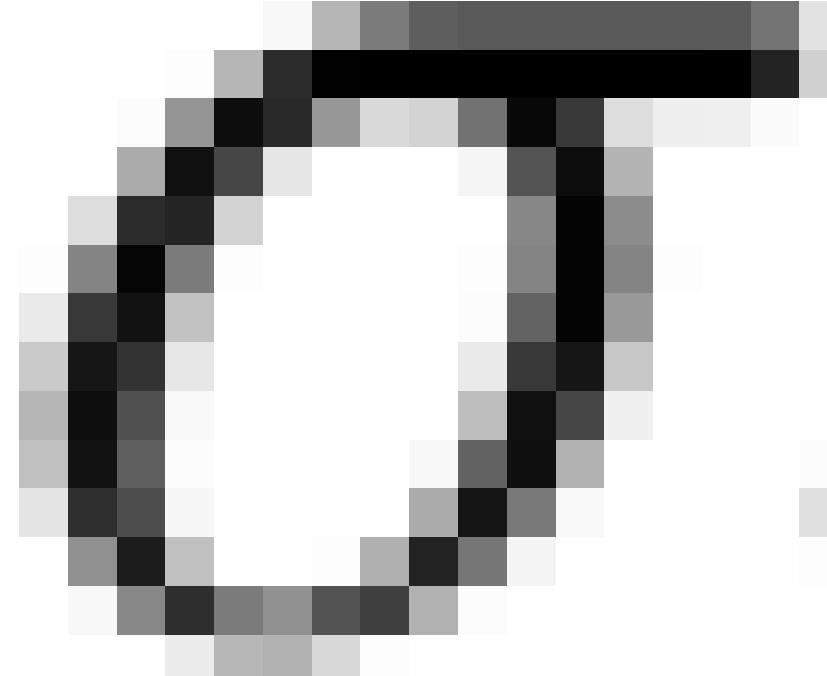
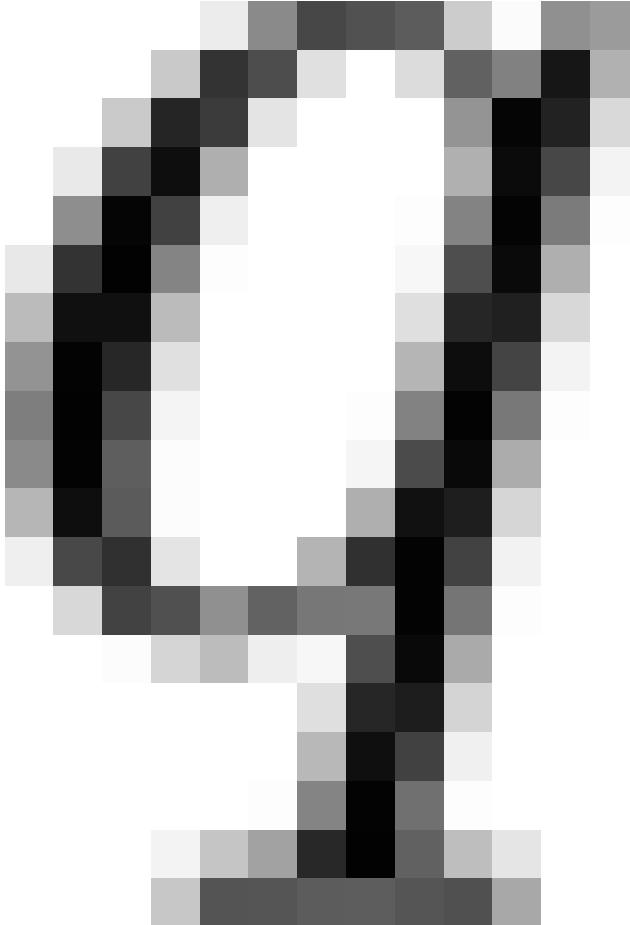
[Zoback 2013, Figure 4.25]

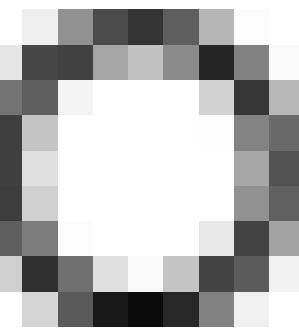
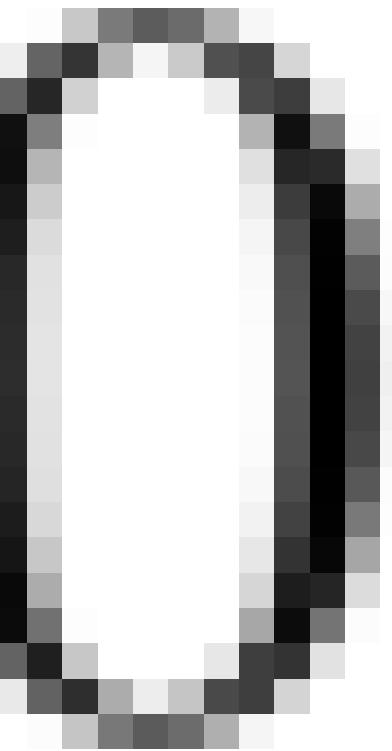
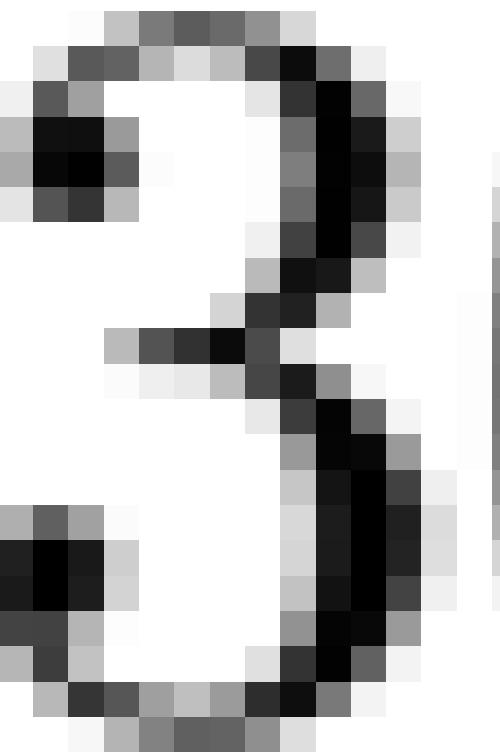
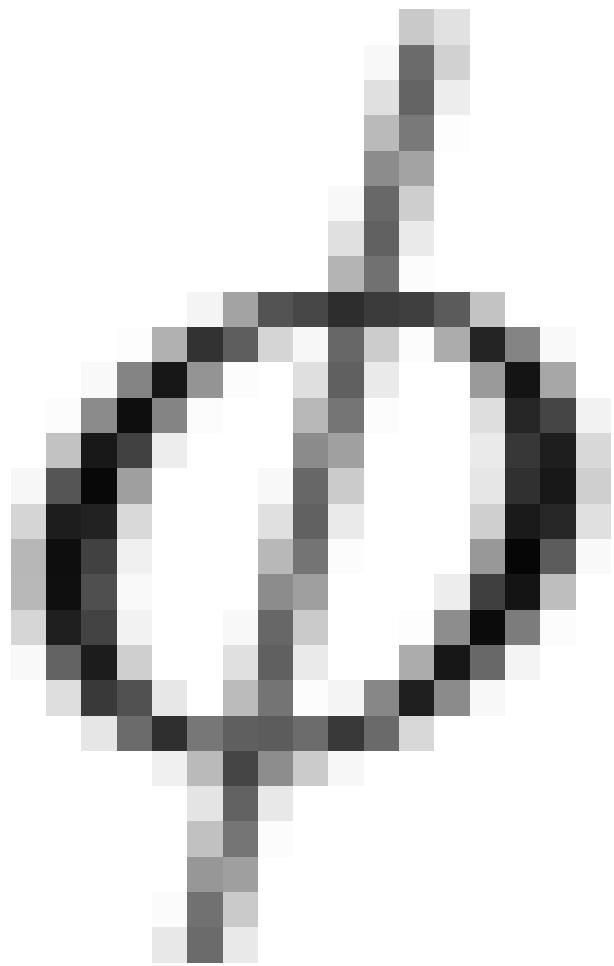


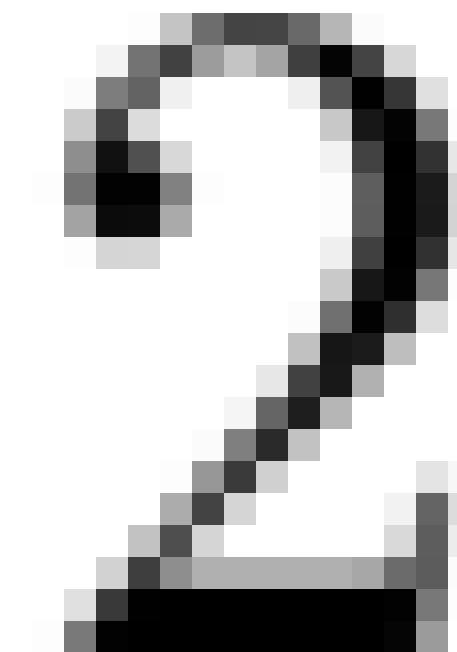
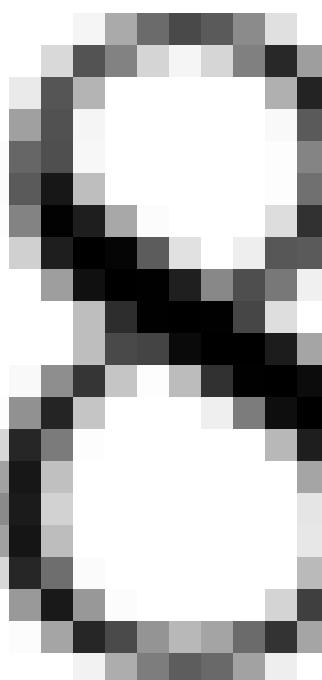
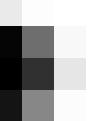
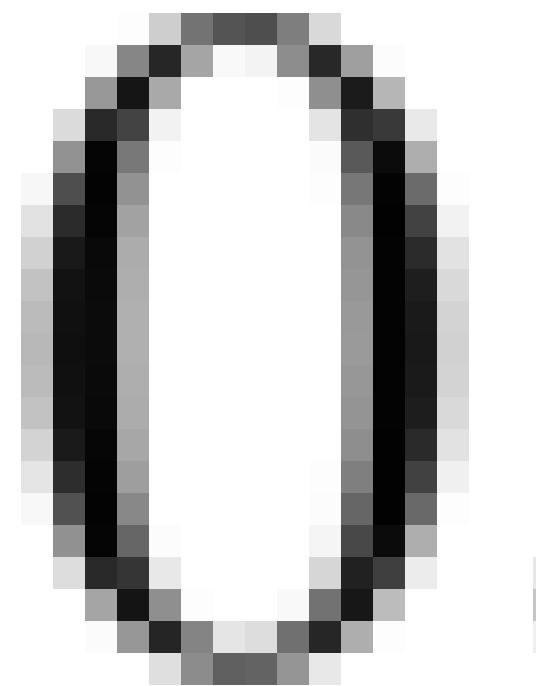




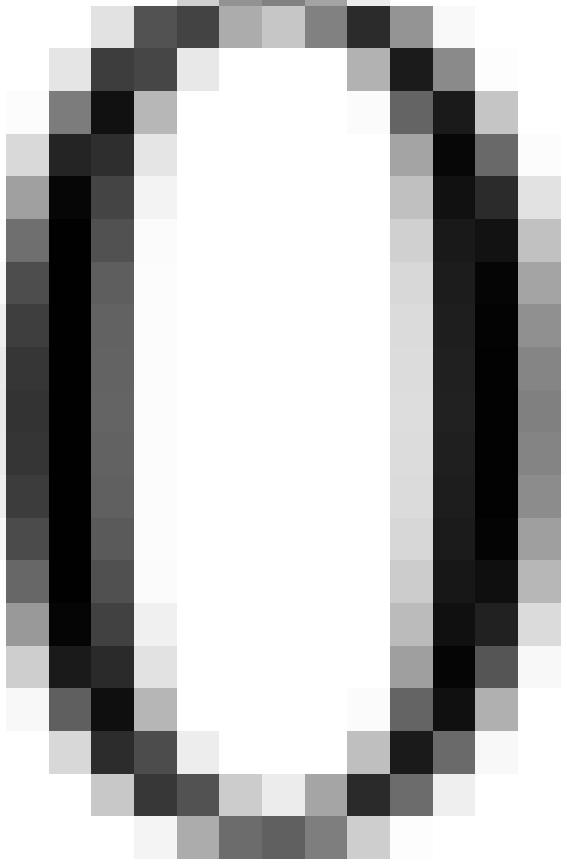
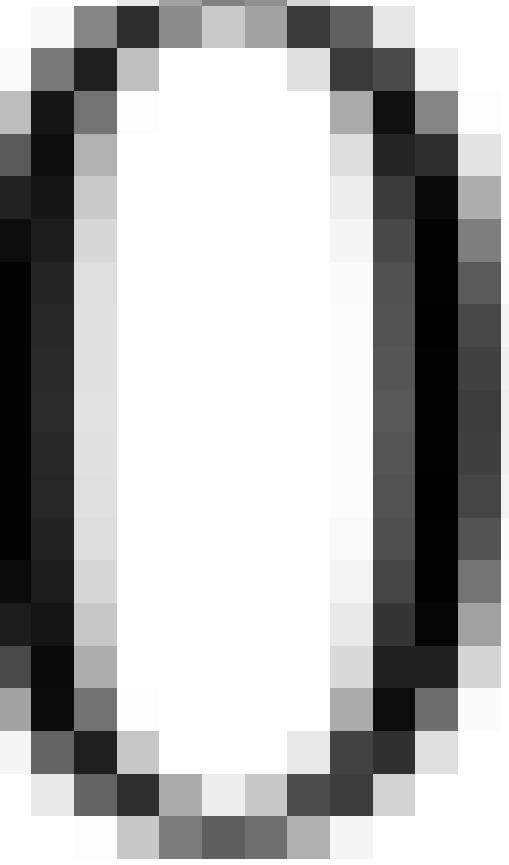
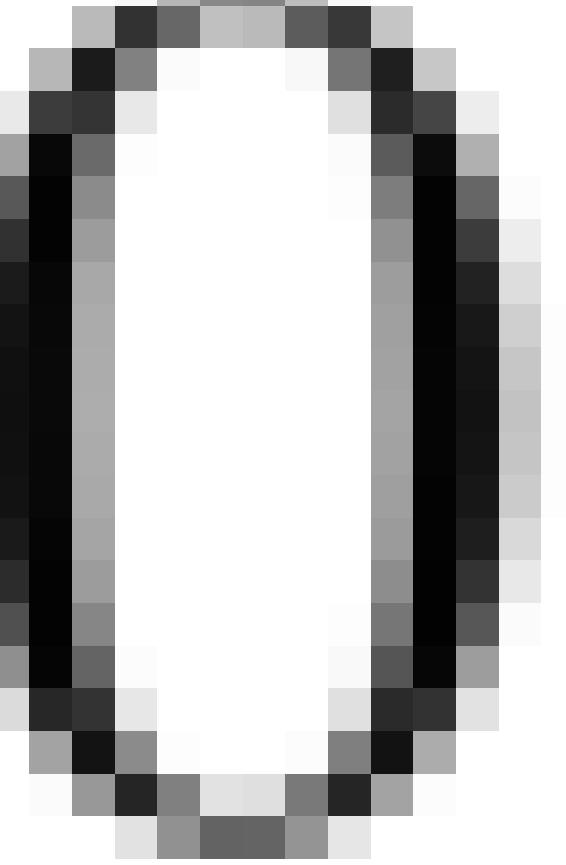
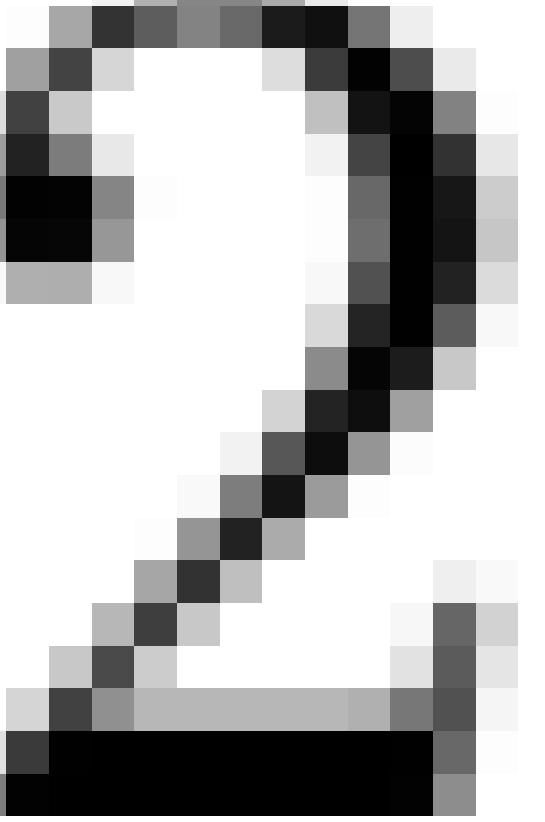
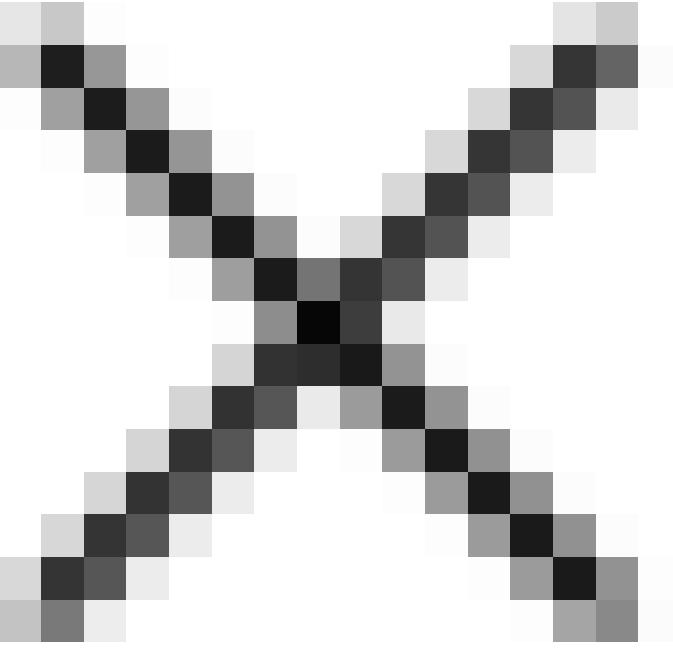


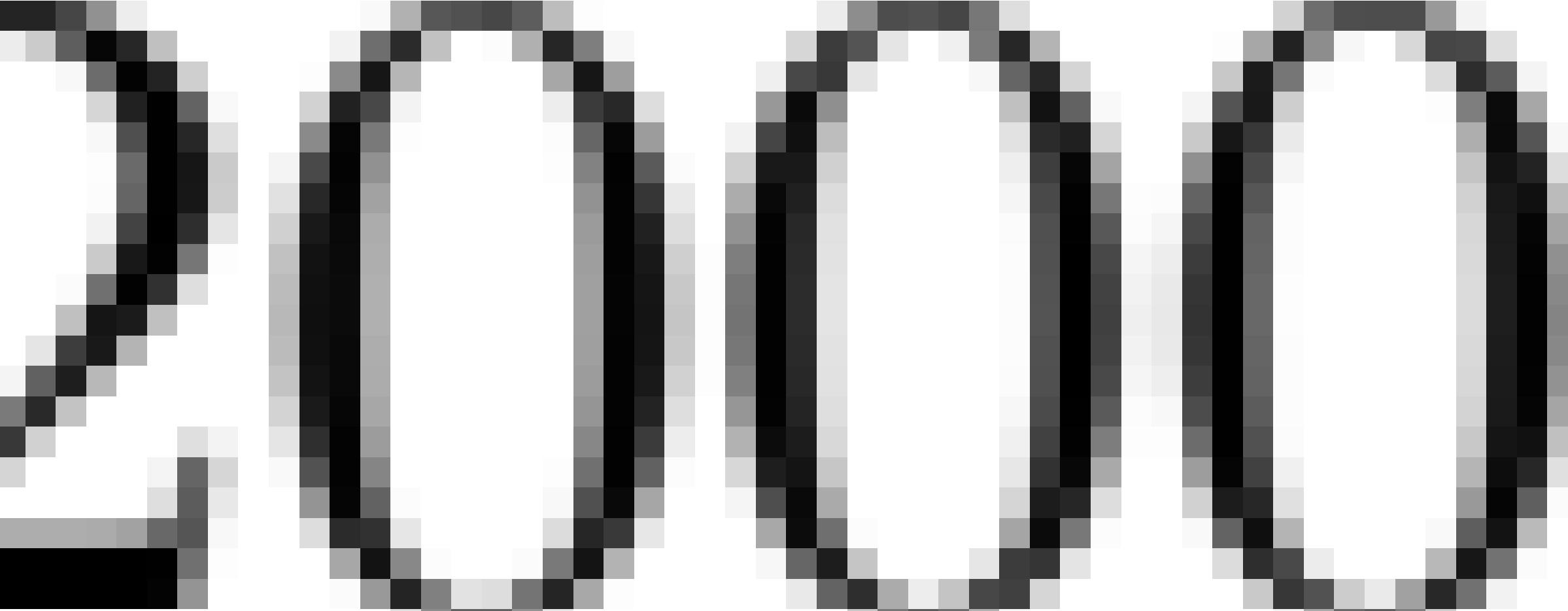




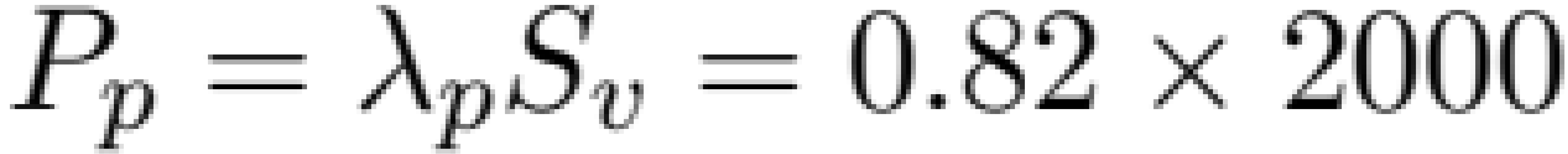


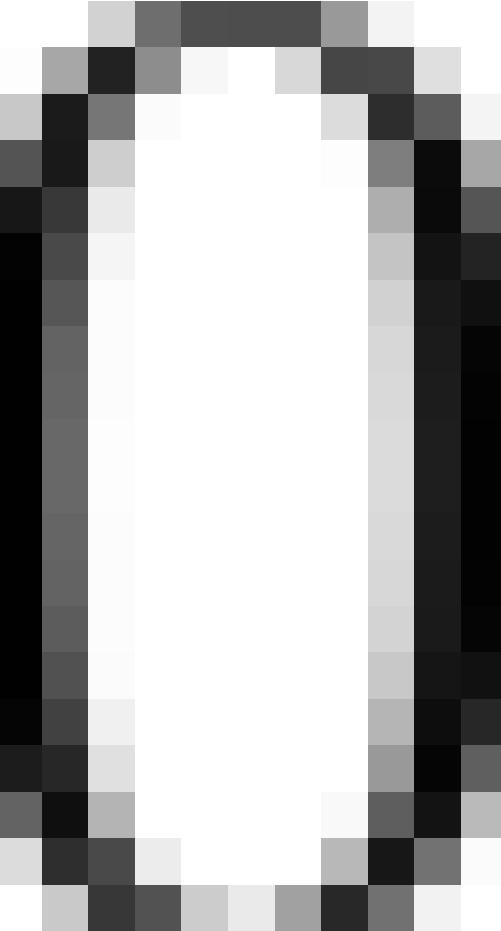
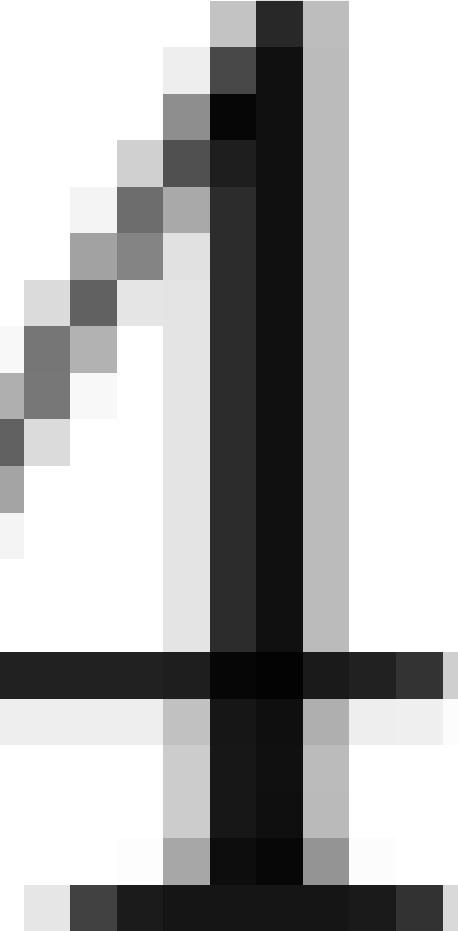
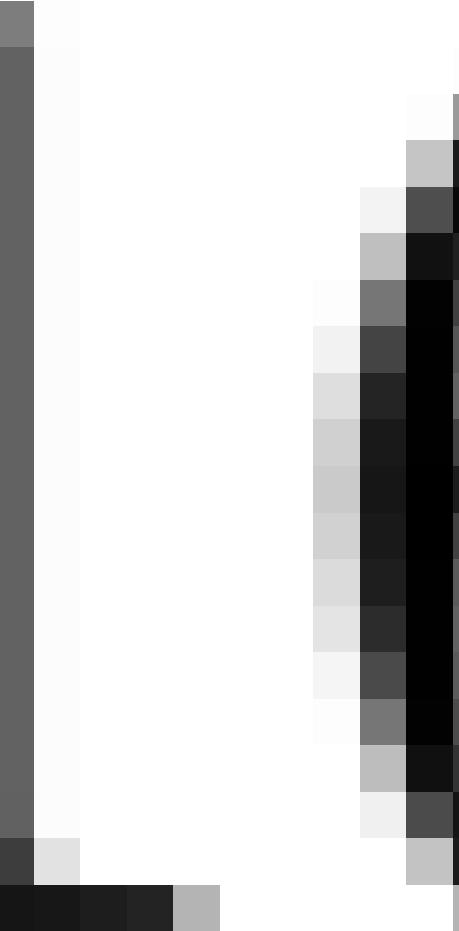


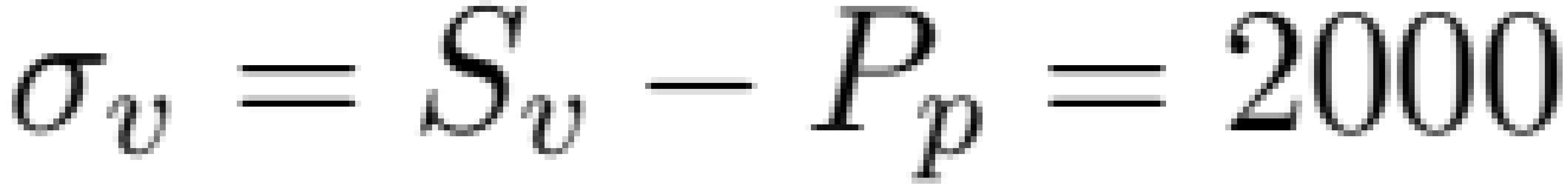


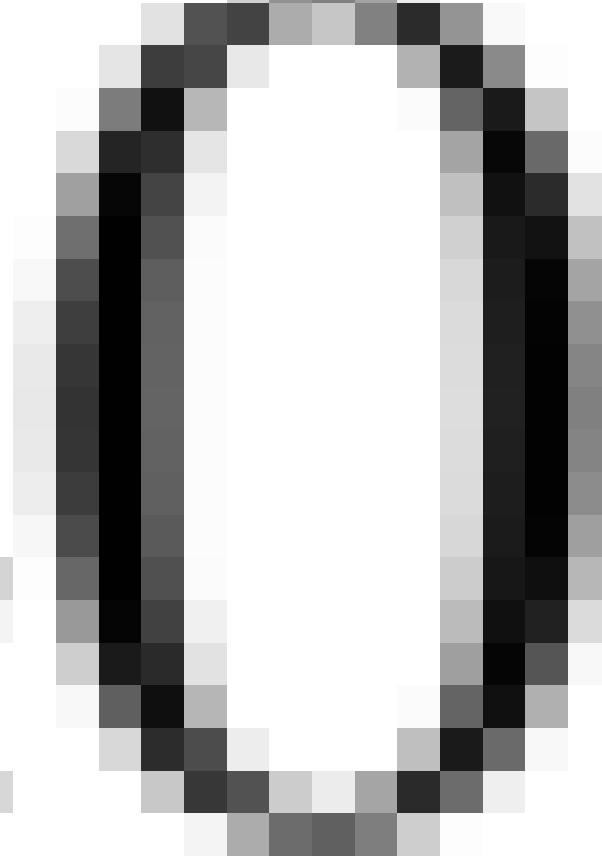
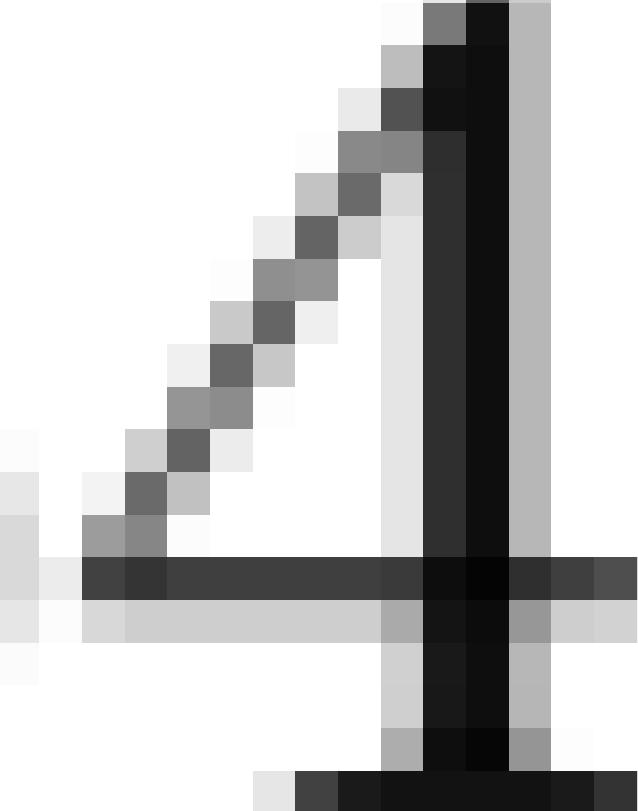
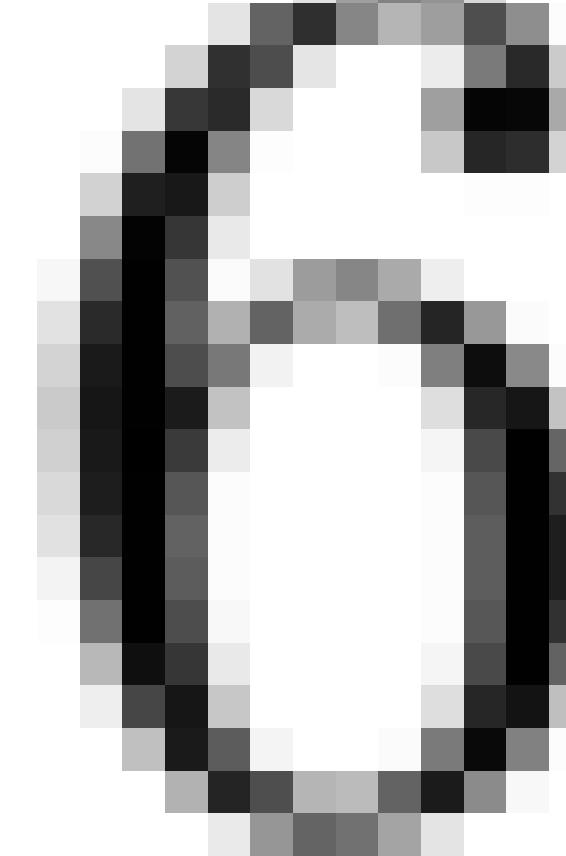
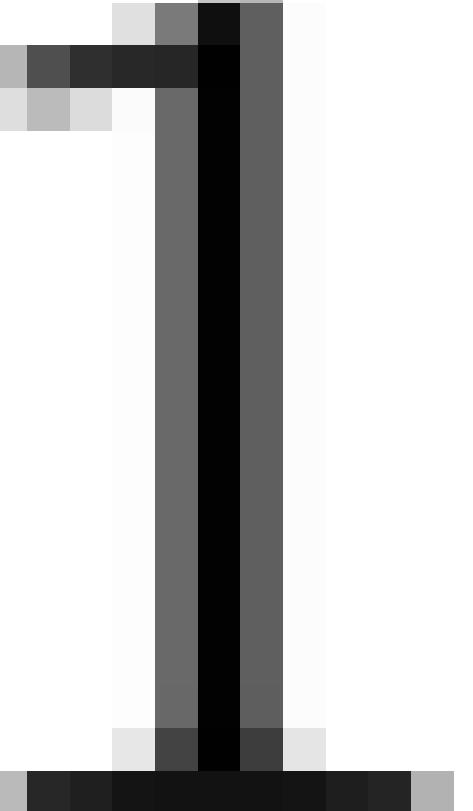


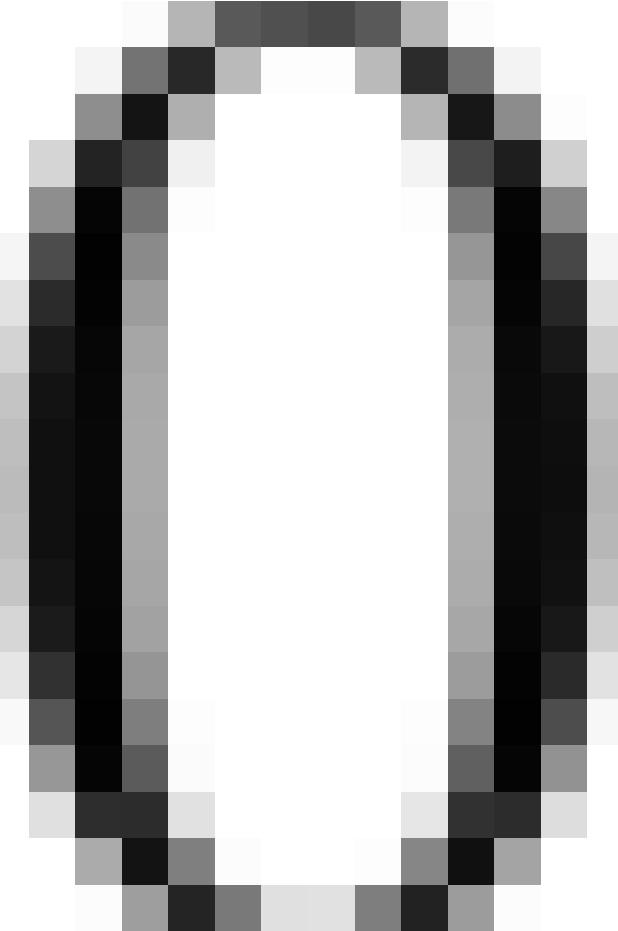
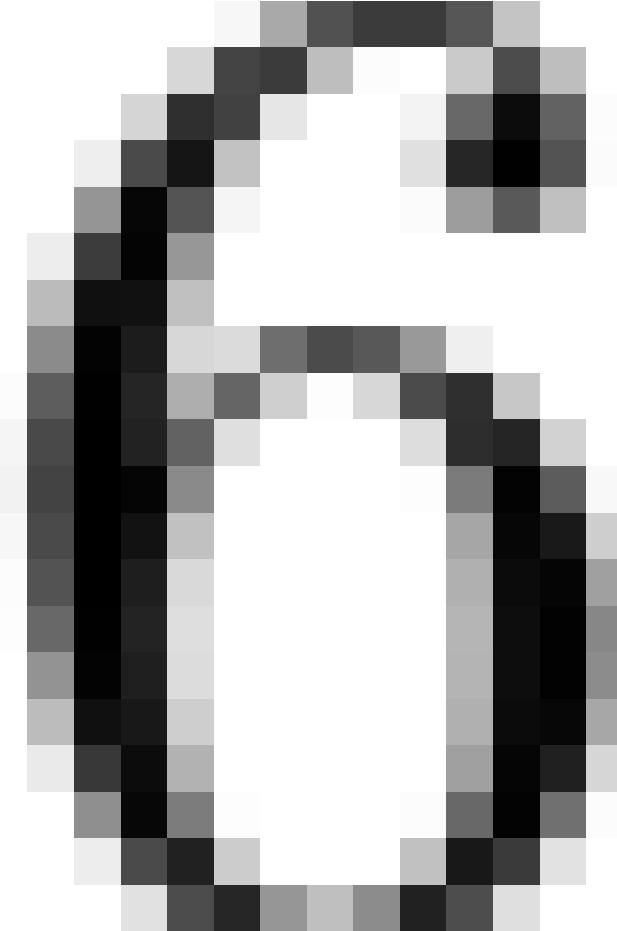
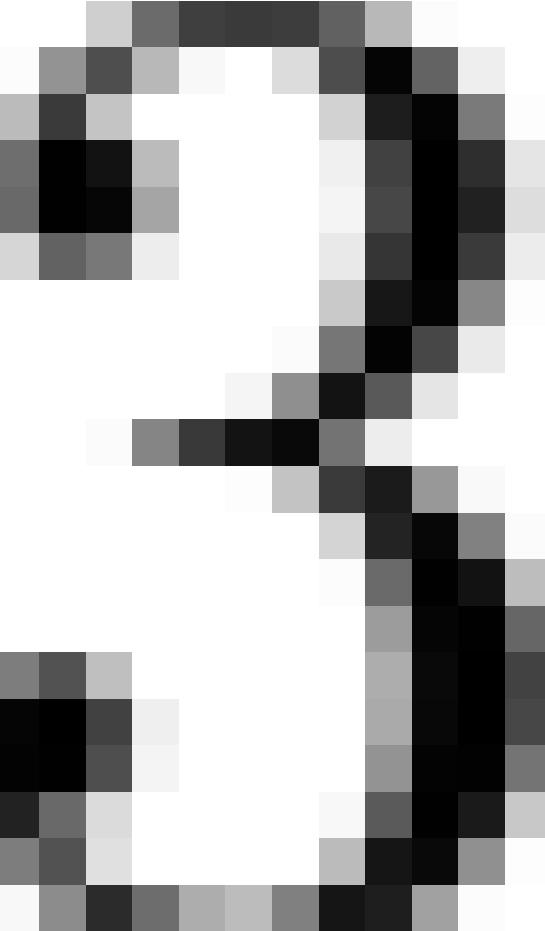








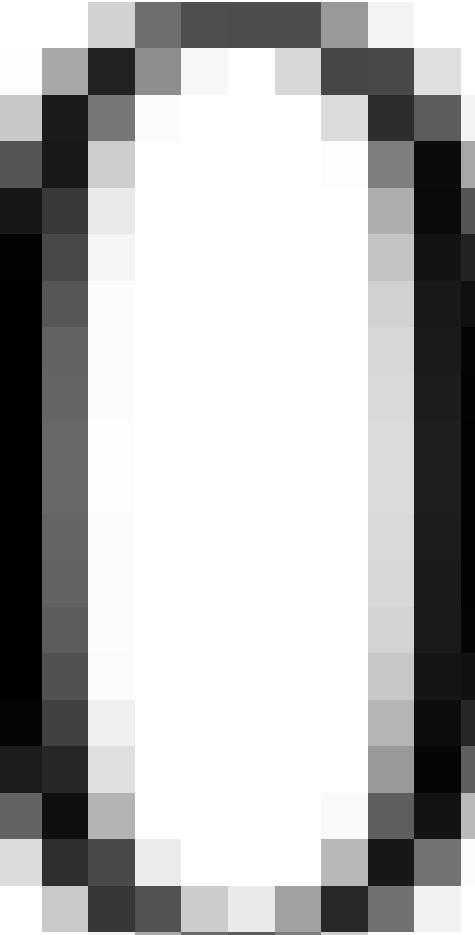
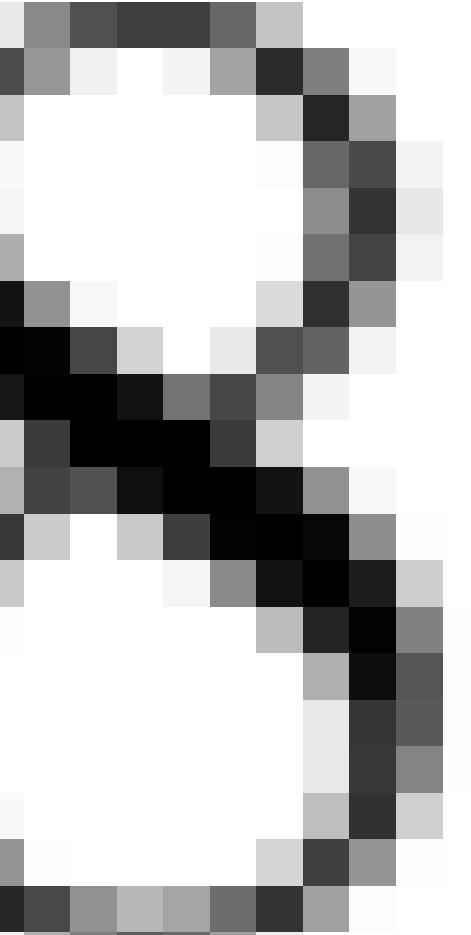
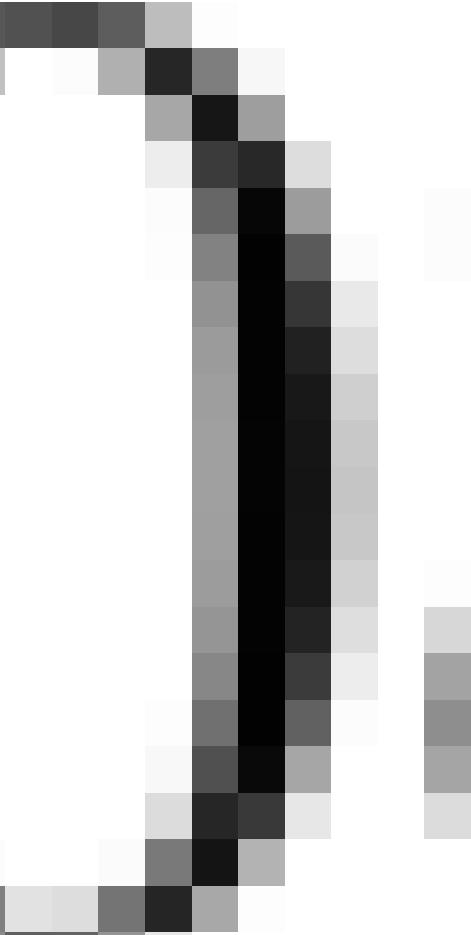
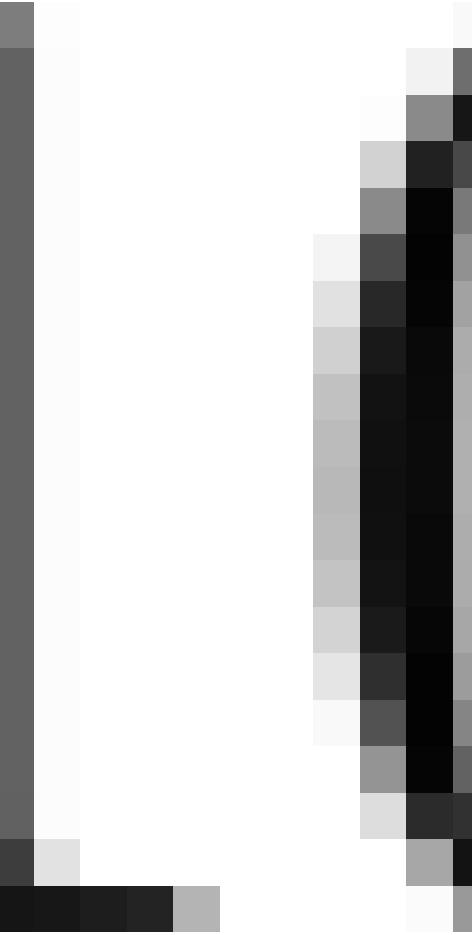


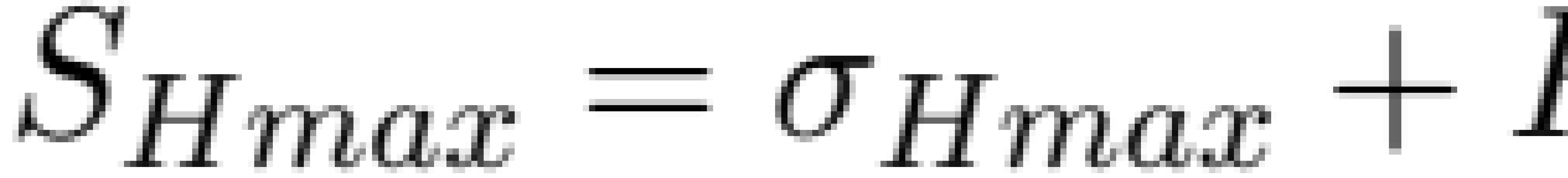


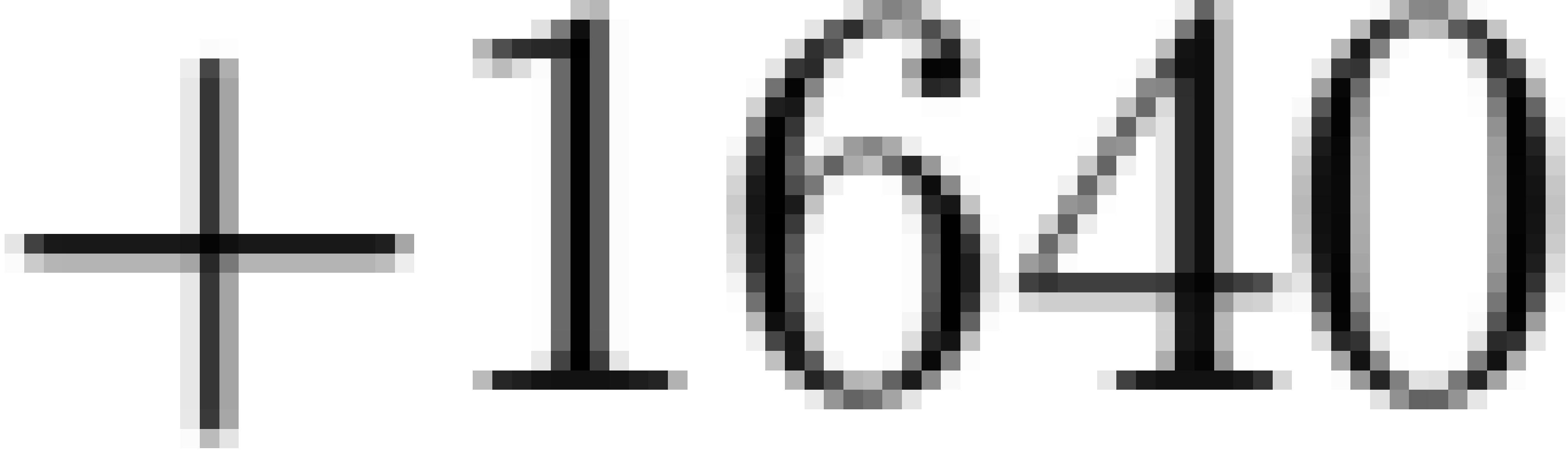
$$\sigma_{H\text{max}} =$$

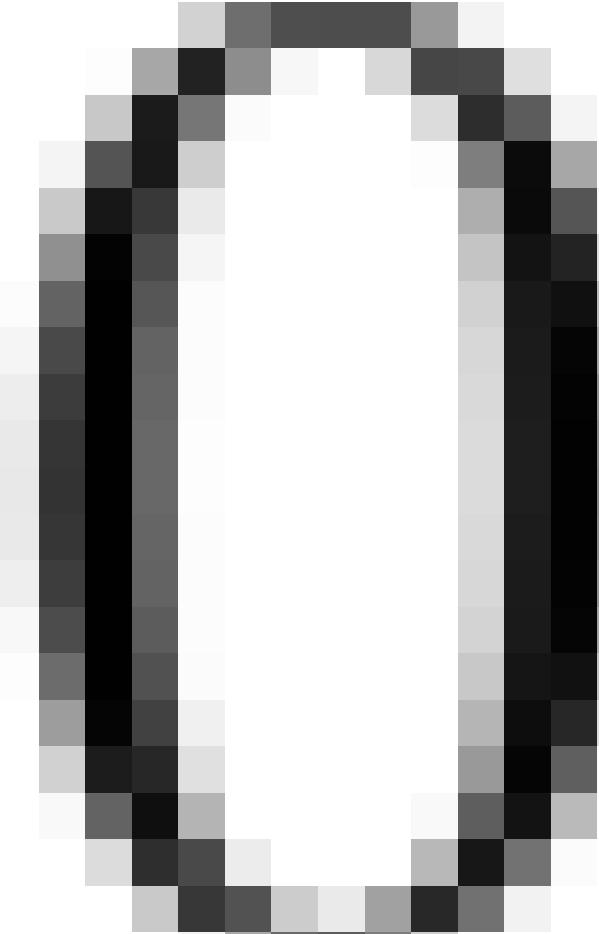
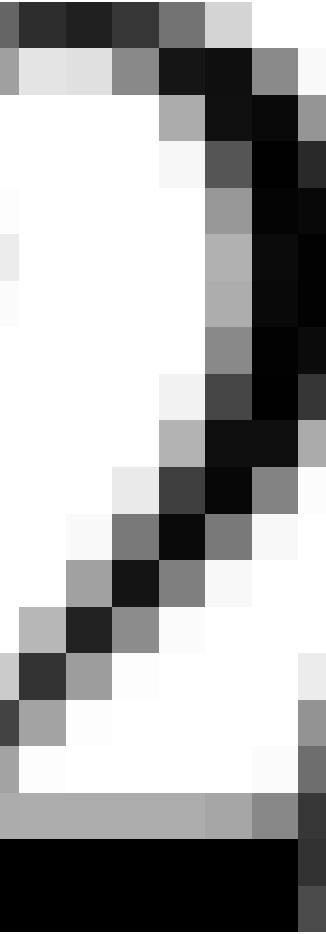
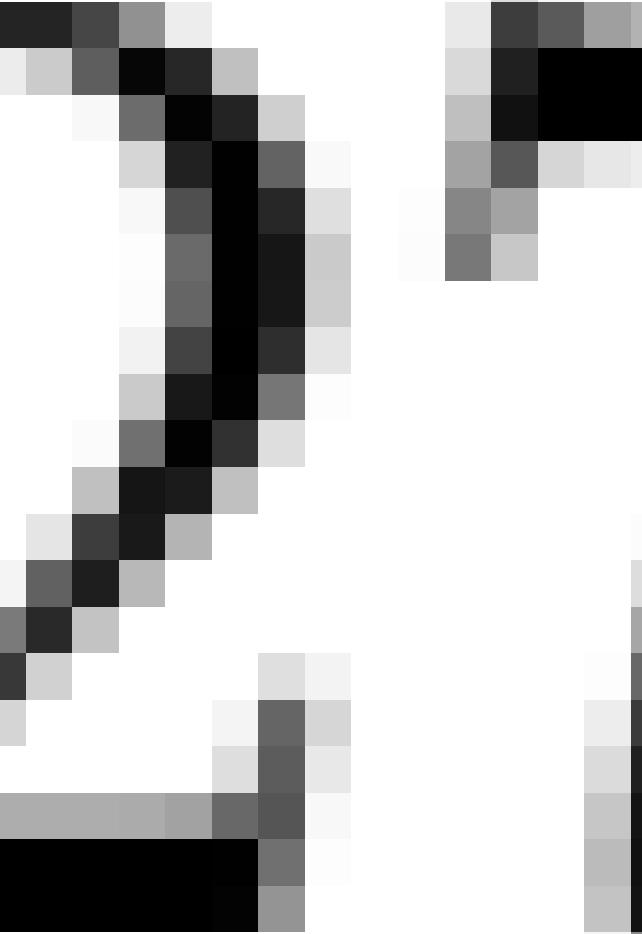
$$90^\circ =$$

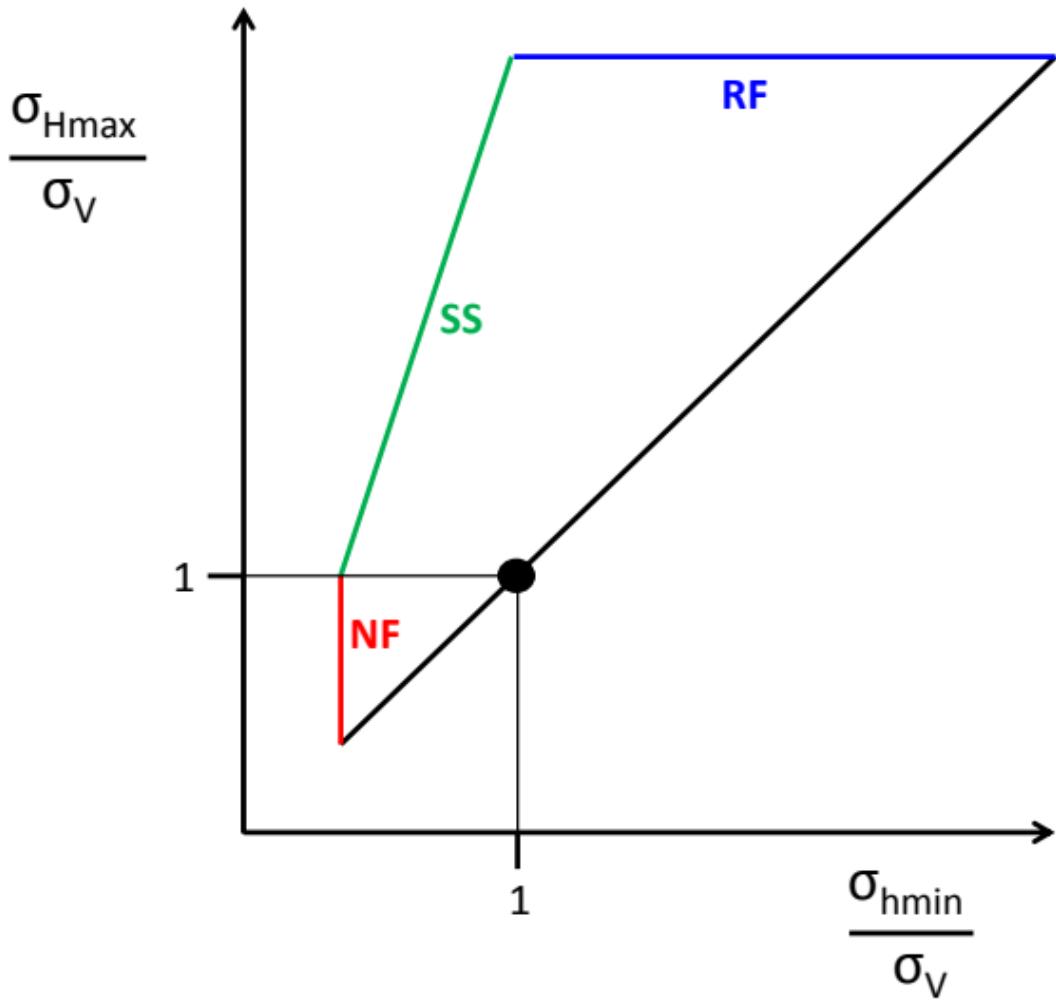
$$\frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} 360$$



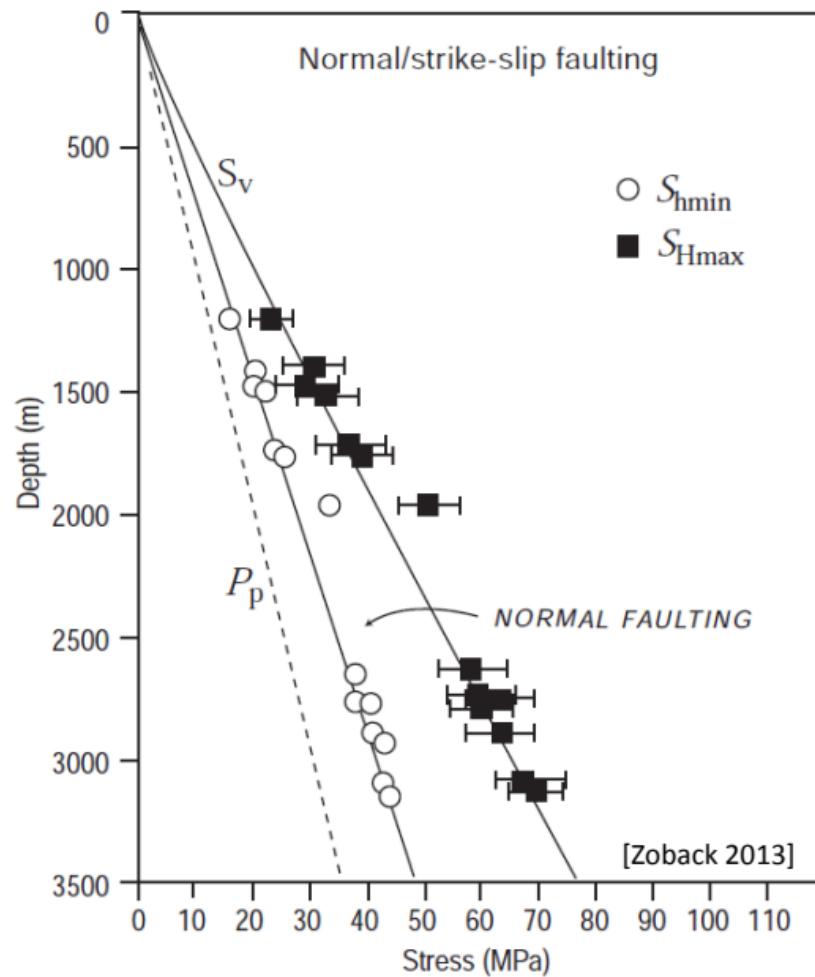
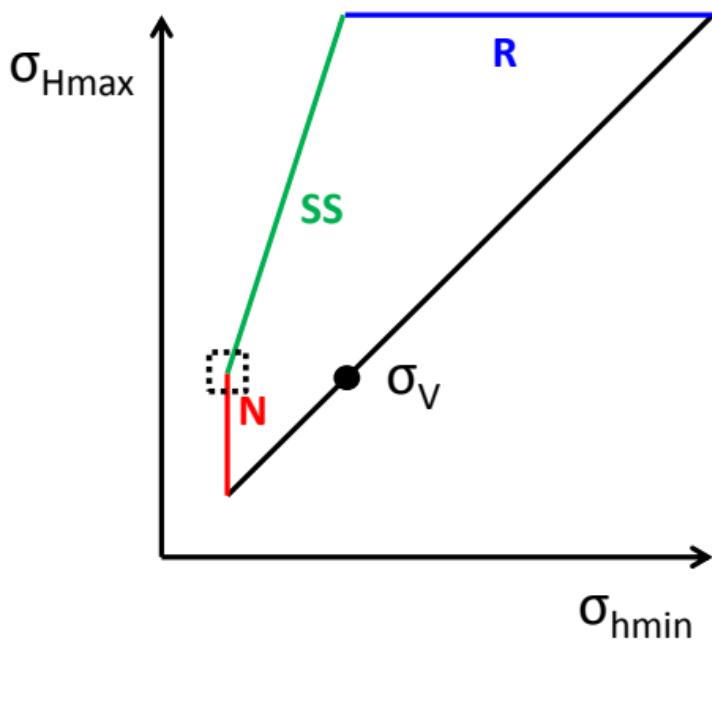


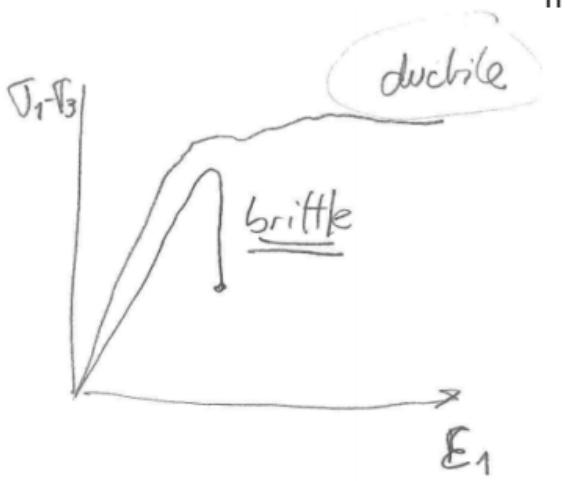
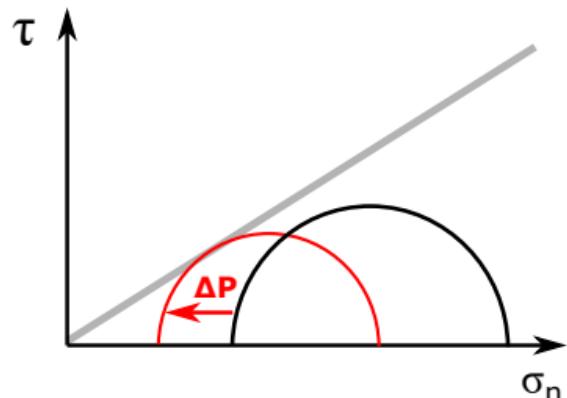
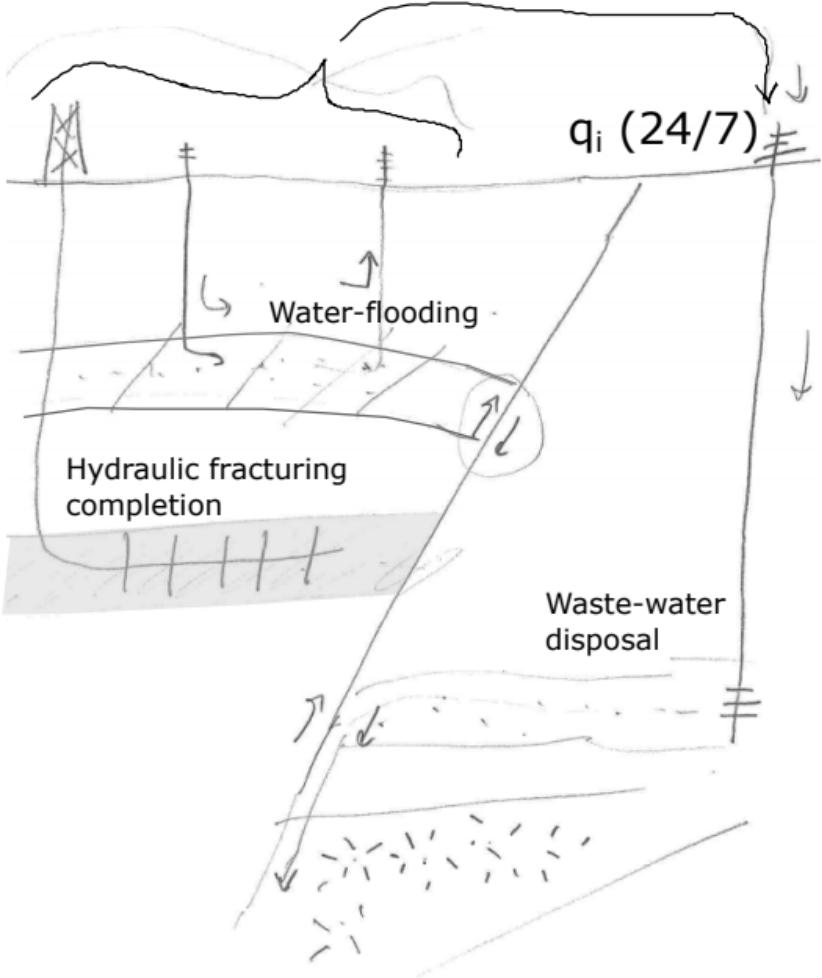






$$\sigma_{h\min} < \sigma_{H\max} \approx \sigma_v$$





$$\Delta\sigma_a = -\Delta P_p$$

$$\Delta\sigma_{\text{Hermann}} \leq -\Delta P_p$$