P_p

S_v

σ_v

S_3

S_{Hmax}

S_{ij}

$\underline{e}_i$

$\underline{e}_j$

$$dx_i$$

S_{21}

$$S_{31} + \frac{\partial S_{31}}{\partial x_3} dx_3$$

$$S_{11} + \frac{\partial S_{11}}{\partial x_1} dx_1$$

$$S_{33}$$

M

$$E_{load} < E_{unload}$$

ε_{Hmax}

S_{hmin}

σ_r

S_1

$$= P_c$$

σ'_p

○

α

β

γ

$$\tau/\sigma_n \sim \mu$$

$$\tau/\sigma_n \to 0$$

~

ε_{tect}

τ/σ_n

2

P_W

σ_∞

$\Delta\sigma$

$$P_W - P_p = 0$$

σ_{rr}

$\sigma_{\theta\theta}$

$\sigma_{r\theta}$

σ_1

σ_3

$$P_W < P_{W\,shear}$$

UCS

$$P_W > P_b$$

~ 3

$$(x_1, x_2, x_3)$$

θ

T_S

$$P_W = 35$$

$S_v =$

$$S_{Hmax} =$$

$$S_{hmin} =$$

$P_p =$

$$T < T_0$$

d

w_{BO}

$$S_v > S_{Hmax} = S_{hmin}$$

P

$$\sqrt{t_{si}}$$

x_f/r_w

$$x_f > h_f$$

$$x_f < h_f$$

h_f

l_f

l_l

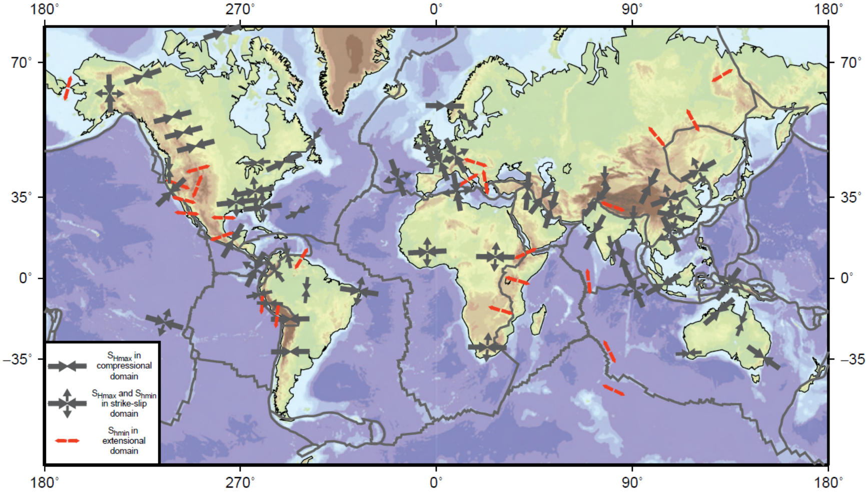
l_w

$$= h_f$$

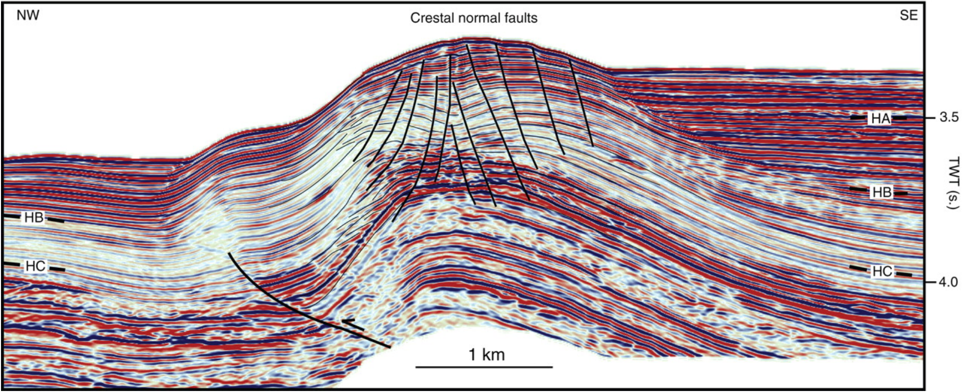
$$p_{net} > 150psi$$

$$l_f =$$

< 10


180°
270°
0°
90°
180°
70°
35°
0°
−35°
S_{Hmax} in compressional domain
S_{Hmax} and S_{hmin} in strike-slip domain
S_{hmin} in extensional domain

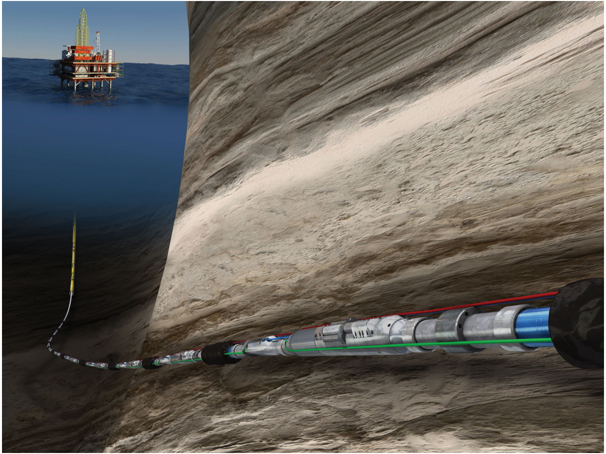
[Zoback 2010]



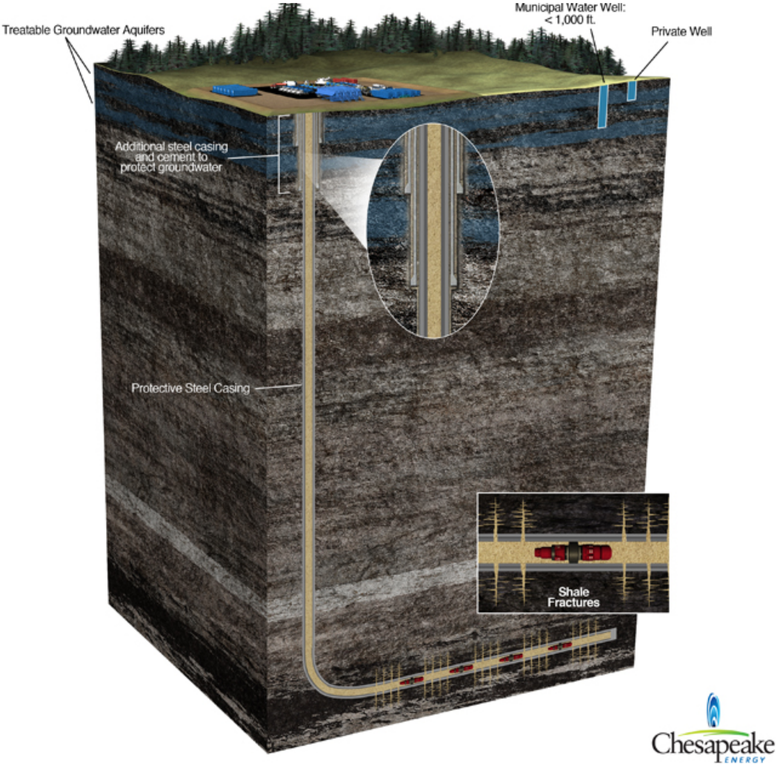
http://geosphere.gsapubs.org/content/5/2/59/F6.large.jpg

~1850

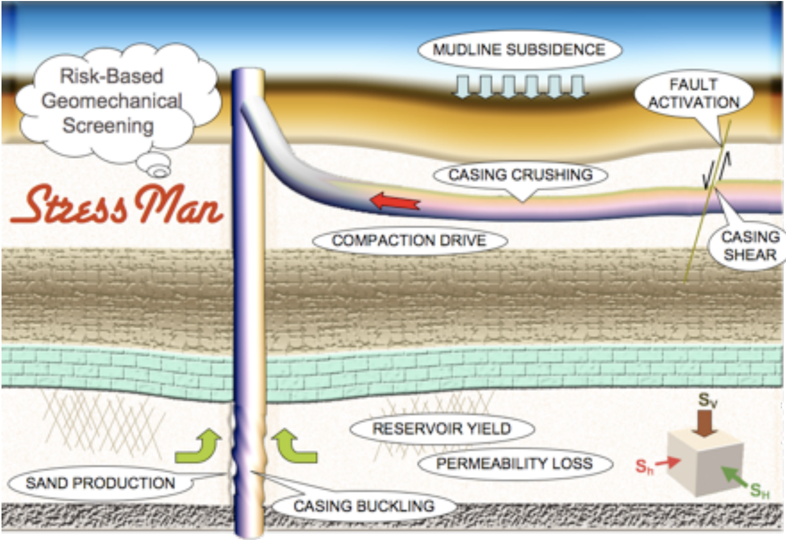
~2010



http://www.drillingcontractor.org



Treatable Groundwater Aquifers
Municipal Water Well:
< 1,000 ft.
Private Well
Additional steel casing and cement to protect groundwater
Protective Steel Casing
Shale Fractures



http://www.higgs-palmer.com

$\underline{F}_1$

$$\underline{F}_2$$

$\mathbb{R}^3$

$$\underline{F}_1 = \underline{F}_2$$

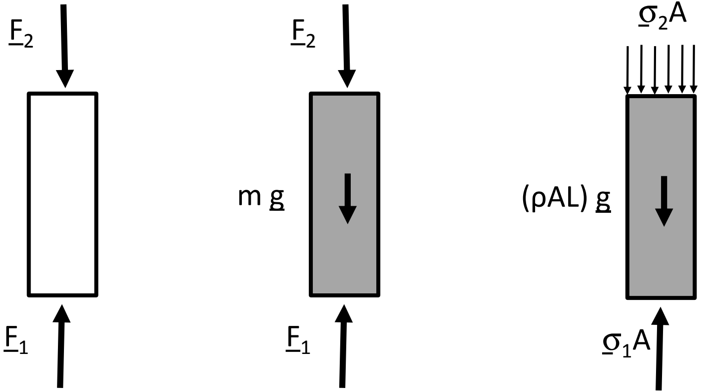
$$\underline{F}_1 = \underline{F}_2 + m\underline{g}$$

$m\underline{g}$

F

A

$$\underline{\sigma} = \frac{F}{A}$$



$\Sigma F_z = + F_1 - F_2 = 0$ $\quad$ $\Sigma F_z = + F_1 - m\,g - F_2$ $\quad$ $\Sigma F_z = +\sigma_1 A - (\rho A L)g - \sigma_2 A$

$$\Delta x \Delta y \Delta z$$

$$\sum \underline{F} = 0$$

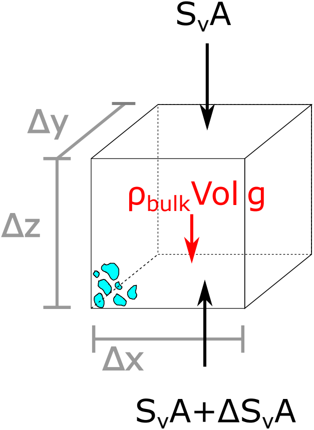
$\underline{g}$

$$\sum F_z = 0$$

$$S_v A + \rho_{bulk} Volg = S_v A + \Delta S_v A$$

$$\rho_{bulk}\,\Delta x \Delta y \Delta z\; g = \Delta S_v \Delta x \Delta y$$

$$\frac{\Delta S_v}{\Delta z} = \rho_{bulk}\, g$$



S_vA
Δy
Δz
ρ_{bulk}Vol g
Δx
$S_vA+\Delta S_vA$

$$\int_0^{S_v(z)} dS_v = \int_0^z \rho_{bulk}(z) g \, dz$$

$$\rho_{bulk}(z)g$$

$$S_v(z = 0) = 0$$

$$\rho_{bulk} = cst$$

z

$$S_v(z) = \rho_{bulk} g \, z.$$

$$\rho_{SiO_2}$$

ρ_w

$$\rho_{bulk}$$

ϕ

$$\rho_{bulk} = (1 - \phi)\rho_{SiO_2} + \phi\rho_w$$

$$\rho_{SiO_2} = 2650$$

3

$$\rho_w = 1000$$

$$\rho_{bulk} = (1 - 0.2) \times 2650$$

$^3 + (0.2) \times 1000$

$^{3} = 2320$

3.

$$\rho_{bulk}\ g = 2320$$

$^3 \times 9.8$

$^2 = 22.7 \times 10^3$

$$= 22.7$$

2

6

g

$$\rho_{bulk} g = 144.8$$

$^3 = 144.8$

$2\ \times$

$$= 144.8$$

$$= 1.01$$

- [redacted]

~ 1

~ 23

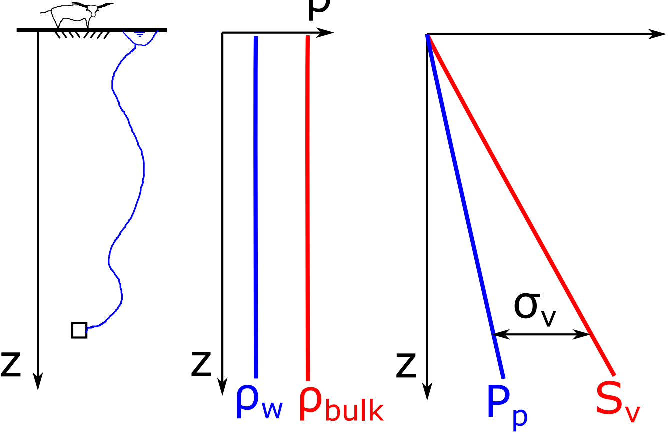
$$\phi = 0.20$$

~ 0.44

~ 10

$$S_w, S_o, S_g$$

$$P_p = \rho_w g\, z$$



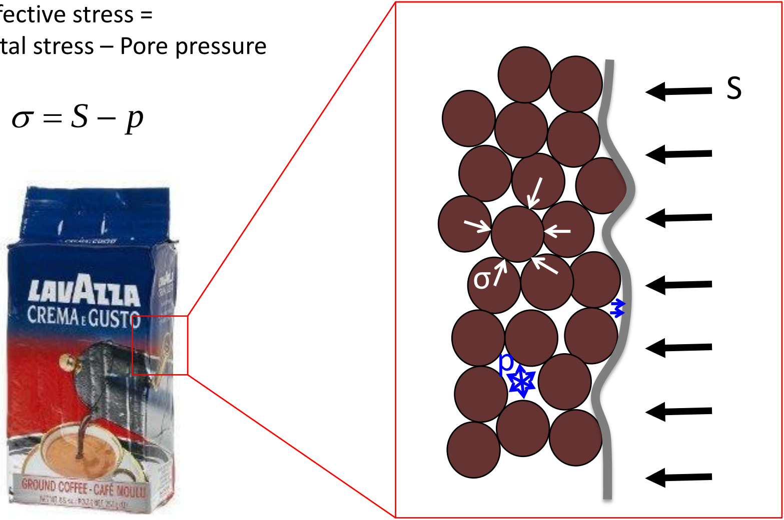
z
ρ
z
ρ_w
ρ_{bulk}
σ_v
z
P_p
S_v

$$\sigma_v = S_v - P_p$$

$$S_v > P_p$$

Effective stress =
Total stress – Pore pressure

$$\sigma = S - p$$



P_w

$$\frac{dP_p}{dz} = \rho_w g = 1040$$

$^{2} = 10.19$

$$\frac{dS_v}{dz} = \rho_{bulk} g = 2350$$

$^2 = 23.03$

$$P_p = \frac{dP_p}{dz} z = 10.19$$

$\times 1.219$

$$= 12.43$$

$$= 1801$$

$$\frac{dS_v}{dz}z = 23.03$$

$$= 28.07$$

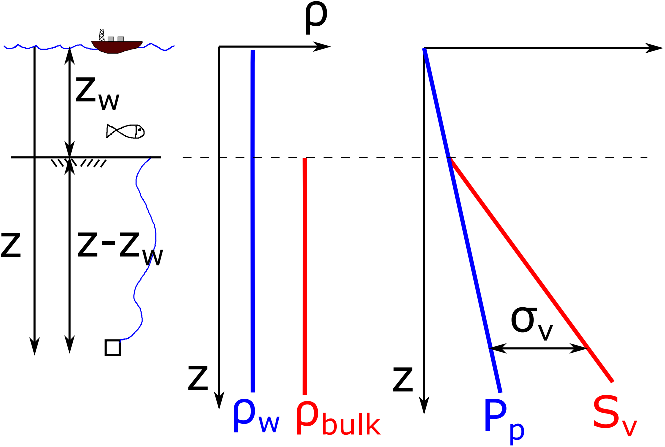
$$= 4070$$

z_w

$$z > z_w$$

$$S_v = \rho_w g\, z_w + \rho_{bulk} g (z - z_w)$$

$$z \geq z_w$$



ρ
z_w
z
$z-z_w$
z
ρ_w
ρ_{bulk}
σ_v
z
P_p
S_v

$$z = z_w$$

$$\sigma_v = S_v - P_p = \rho_{bulk} g (z - z_w)$$

$$P_p = \rho_w g z_w + \frac{dP_p}{dz}(z - z_w) = 0.44$$

$\times 2000$

$+0.44$

$$\times(9000 - 2000)$$

$$= 3960$$

$$S_v = \rho_w g z_w + \frac{dS_v}{dz}(z - z_w) = 0.44$$

$+1.00$

$$= 7880$$

,

$$\sigma_v = S_v - P_p = 7880$$

-3960

$$= 3920$$

$$P_p = 27.31$$

$$S_v = 54.34$$

$$\sigma_v = 27.03$$

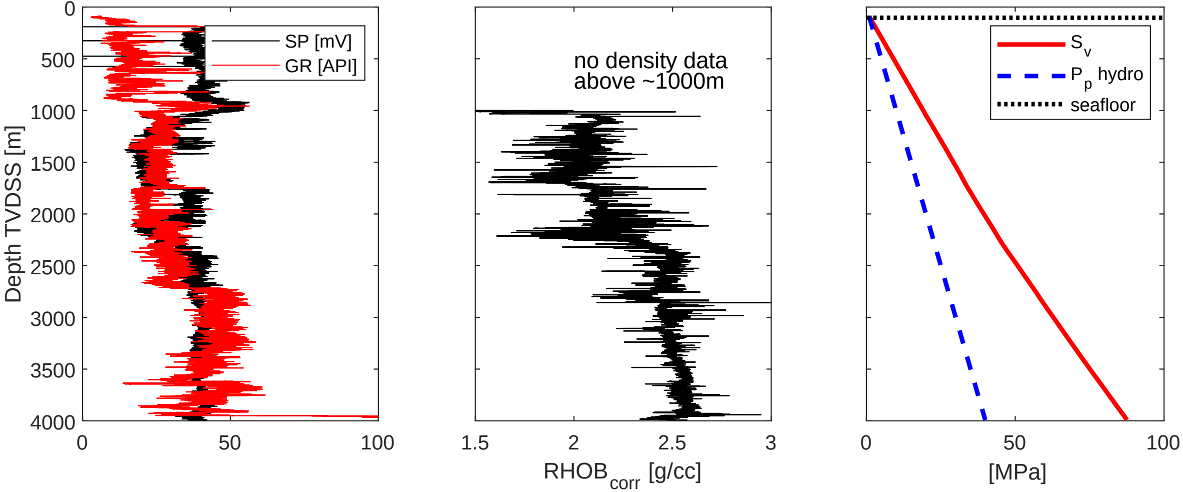
$$S_v(z) = \int_0^z \rho_{bulk}(z) g \, dz$$

$\rho_{bulk}(z)$

$$S_v(z_i) = \sum_{1}^{i} \rho_{bulk}(z_i) g \Delta z_i$$

i

z_i

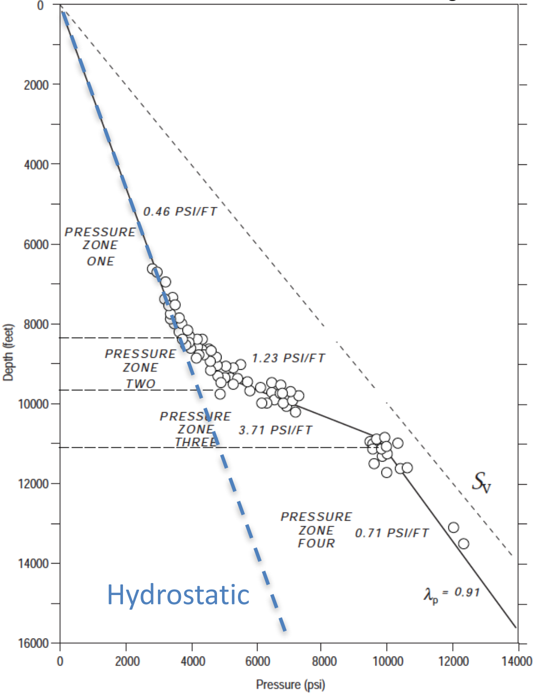


SP [mV]
GR [API]
Depth TVDSS [m]
0
500
1000
1500
2000
2500
3000
3500
4000
0
50
100
no density data
above ~1000m
1.5
2
2.5
3
RHOB$_{corr}$ [g/cc]
S_v
P_p hydro
seafloor
0
50
100
[MPa]

$z \sim 8,500$

$z \sim 11,000$

Monte Cristo field Onshore - Zoback 2013 – Figure 2.2



λ_p

$$\lambda_p(z) = \frac{P_p(z)}{S_v(z)}$$

$$\lambda_p \leq 1$$

$$\lambda_p \sim 0.44$$

$$\lambda_p \neq 0.44$$

$$\lambda_p \to 1$$

$$P_p > P_p^{\text{hydrostatic}}$$

$$\Delta P_p$$

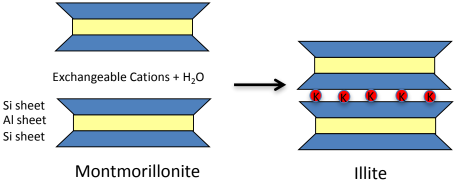
h

$$\rho_{brine} - \rho_{hydrocarbon}$$

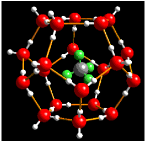
$$\Delta P = (\rho_{brine} - \rho_{hydrocarbon})gh$$

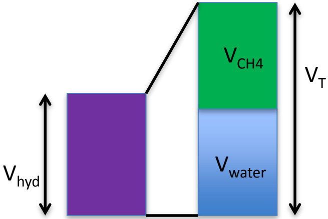
- **Aquathermal pressurization**
  - $\Delta T \rightarrow \Delta P$

- **Dehydration reactions**
  - $\Delta V \rightarrow \Delta P$
  - Montmorillonite to Illite (frees water)



- **Hydrocarbon generation**
  - $\Delta V \rightarrow \Delta P$





V_T/V_{hyd} ~ 1.3 to 6 at cst. P

W

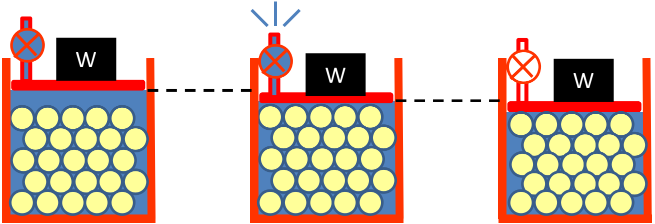
$$\Delta P = W/A$$

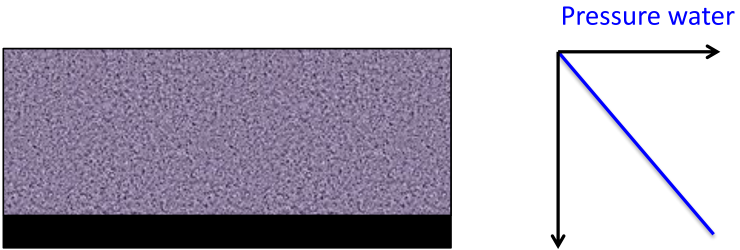
$$\sigma_v = W/A$$

$$\Delta P = 0$$

- **<u>Disequilibrium compaction</u> (Underconsolidation)**
  - ΔS → ΔP (Vertical)

- **Tectonic compression**
  - ΔS → ΔP (Horizontal)




Pressure water

Δz

$$D_h = \frac{Mk}{\mu}$$

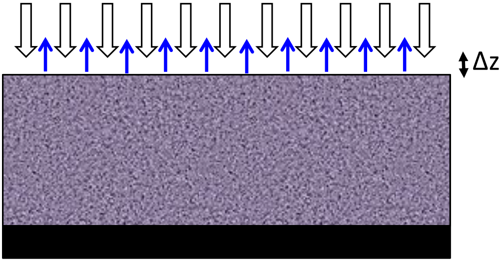
$$C_{bp}$$

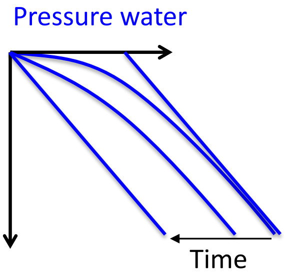
k

μ

$$\frac{\partial P_p}{\partial t} = D_h \frac{d^2 P_p}{dz^2}$$

Rate of sedimentation (loading) and rate of fluid "escape"





T_{ch}

$\sim 2/3$

$$T_{ch} = \frac{L^2}{D_h}$$

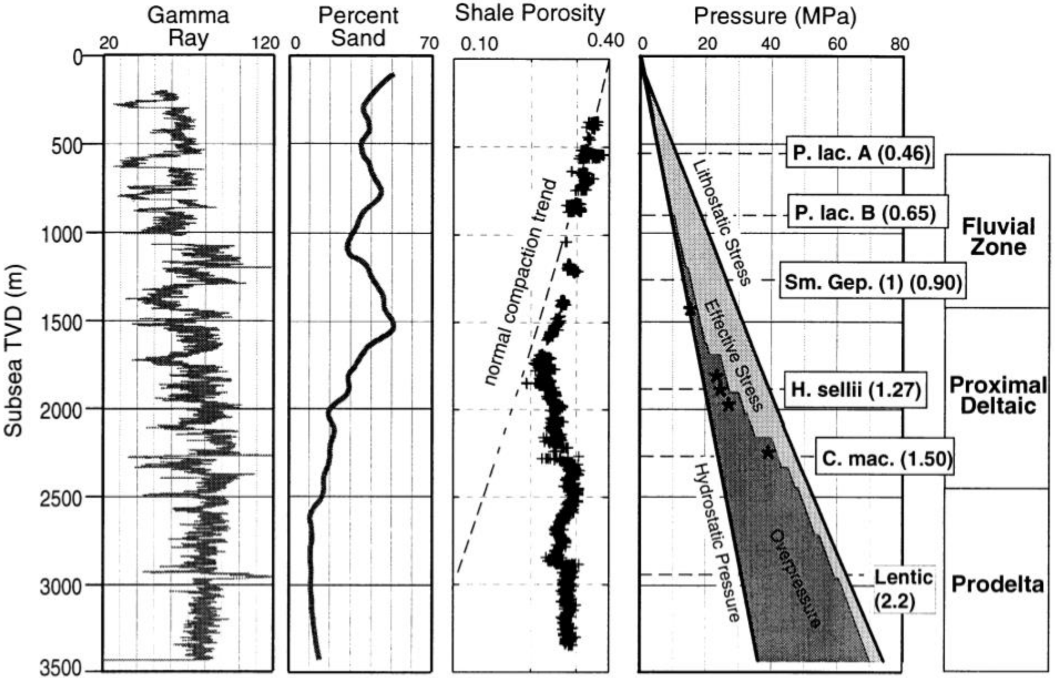
L

$k =$

$M =$

$T_{ch} \sim$

$$\phi = \phi_o \exp(-\beta \sigma_v)$$



Off-shore Louisiana – Gordon and Flemings (1998) Basin Research

$$\phi = 0.298$$

$$\beta = 3.2 \times 10^{-2}$$

$$-1$$

$$\phi_0 = 0.38$$

$$S_v = 10$$

$\times 0.5$

$+22$

$\times 1.5$

$$= 38$$

$$\sigma_v = -\frac{\ln(\phi/\phi_o)}{\beta} = 7.6$$

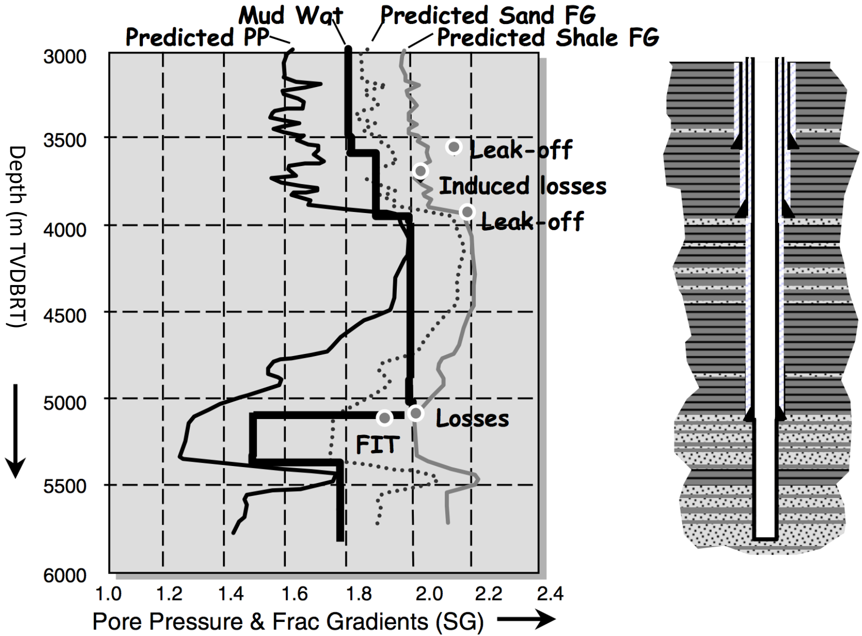
$$S_v = \sigma_v + P_p$$

$$P_p = S_v - \sigma_v = 38$$

-7.6

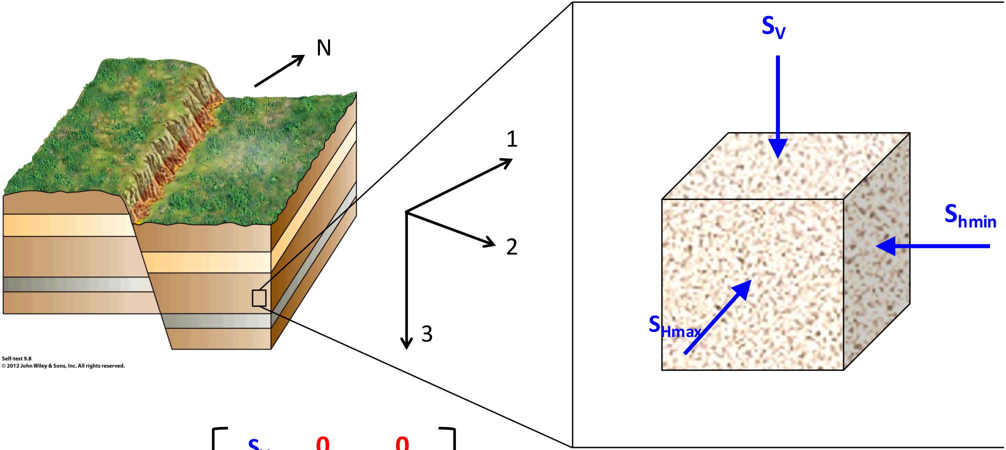
$$= 30.40$$

$$\lambda_p = \frac{P_p}{S_v} = \frac{30.40 \text{ MPa}}{38 \text{ MPa}} = 0.8. \blacksquare$$

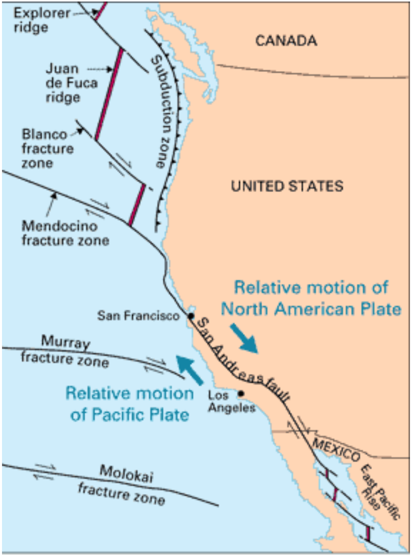


[Caspian Sea, Alberty and McLean – SPE67740]

$$S_{Hmax} \geq S_{hmin}$$

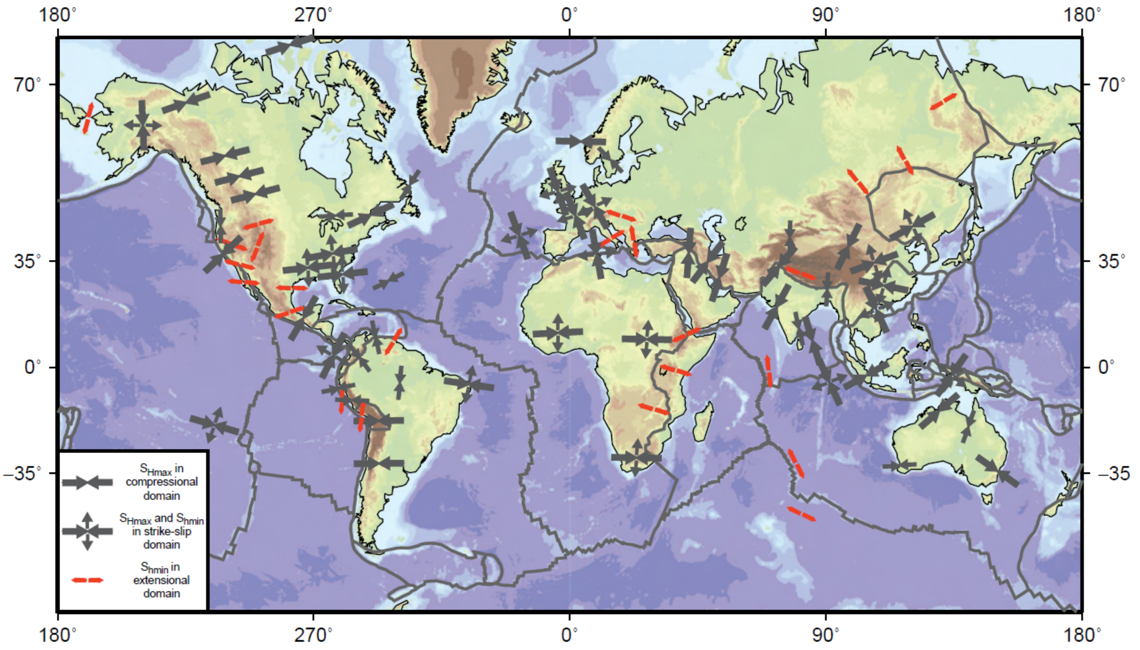


$$\underline{\underline{S}} = \begin{bmatrix} S_V & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_{hmin} \end{bmatrix}$$

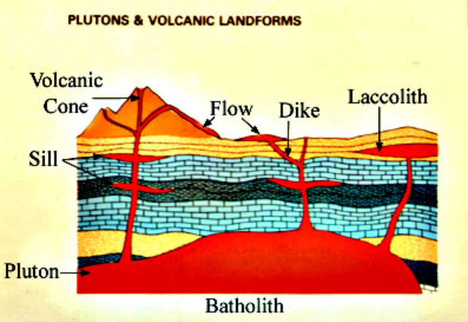


http://pubs.usgs.gov/gip/dynamic/understanding.html#anchor5798673

http://en.wikipedia.org/wiki/File:Aerial-SanAndreas-CarrizoPlain.jpg


180°
270°
0°
90°
180°
70°
35°
0°
–35°
70°
35°
0°
–35
S_{Hmax} in compressional domain
S_{Hmax} and S_{hmin} in strike-slip domain
S_{hmin} in extensional domain

[Zoback 2010]



http://www.indiana.edu/~geol105/1425chap5.htm
http://geophysics.ou.edu/geol1114

$$S_v > S_{hmin} = S_3$$

S_2

$$S_v > S_{Hmax} > S_{hmin}$$

$$S_1 = S_v$$

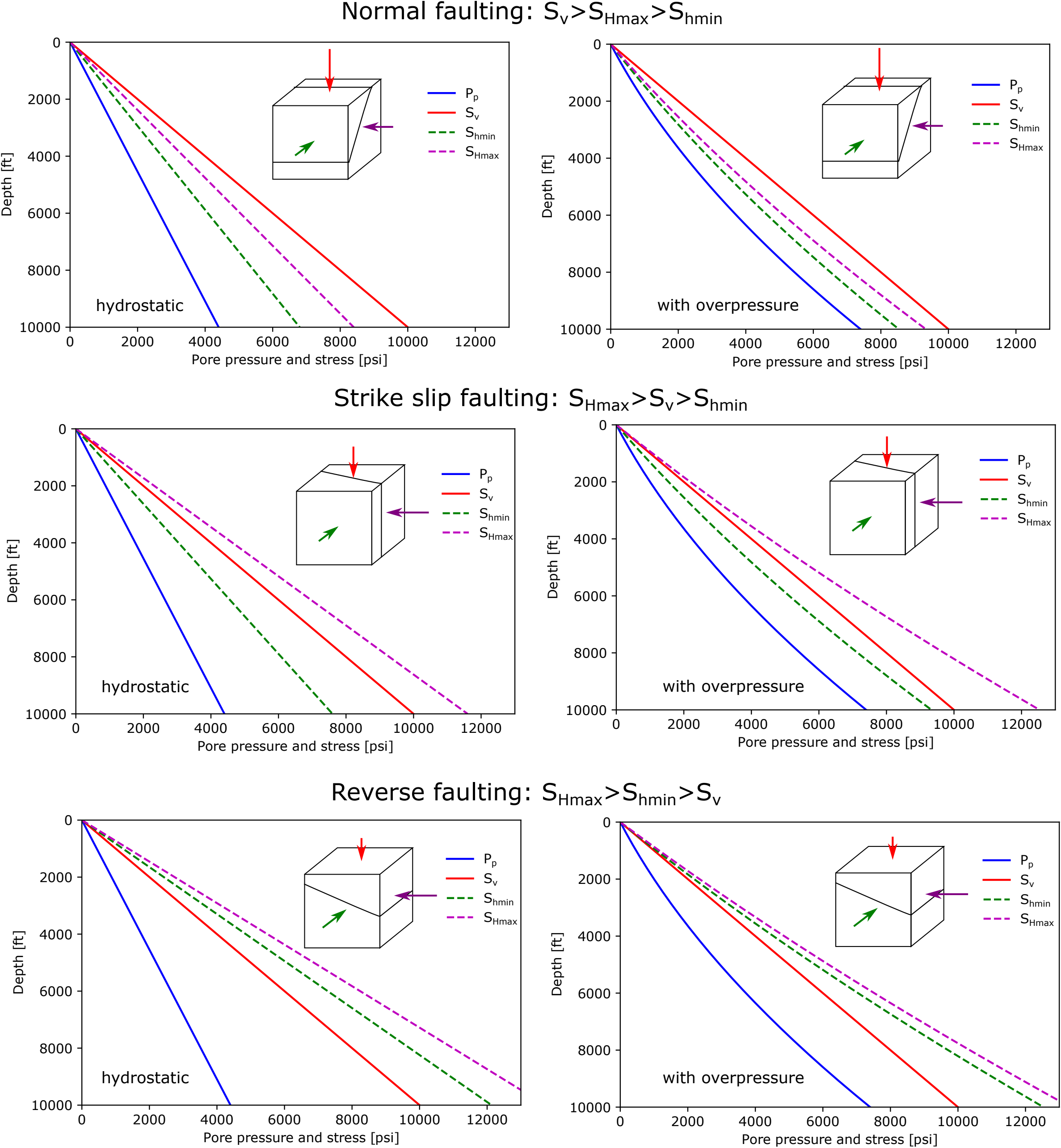
$$S_2 = S_{Hmax}$$

$$S_3 = S_{hmin}$$

~ 60

$$\sigma_1 = S_1 - P_p$$

$$\sigma_3 = S_3 - P_p$$



Normal faulting: $S_v > S_{Hmax} > S_{hmin}$
Strike slip faulting: $S_{Hmax} > S_v > S_{hmin}$
Reverse faulting: $S_{Hmax} > S_{hmin} > S_v$
Depth [ft]
Pore pressure and stress [psi]
P_p
S_v
S_{hmin}
S_{Hmax}
hydrostatic
with overpressure
0
2000
4000
6000
8000
10000
12000

$$S_{Hmax} > S_v > S_{hmin}$$

$$S_1 = S_{Hmax}$$

$$S_2 = S_v$$

$$S_{Hmax} > S_{hmin} > S_v$$

$$S_2 = S_{hmin}$$

$$S_3 = S_v$$

$$\rho_w g z$$

$$S_3 = S_{22}$$

$$S_3 = S_{11}$$

$$S_3 = S_{33}$$

$$S_{hmin} < S_{Hmax} < S_v$$

$$\beta = 3 \times 10^{-2}$$

[

]

3]

[—]

$\underline{e}_1$

$\underline{e}_2$

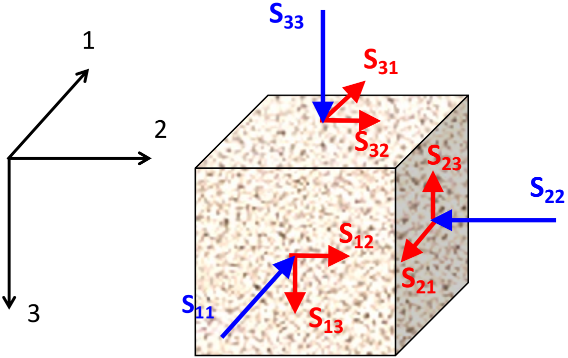
$\underline{e}_3$

T

$\underline{v}$

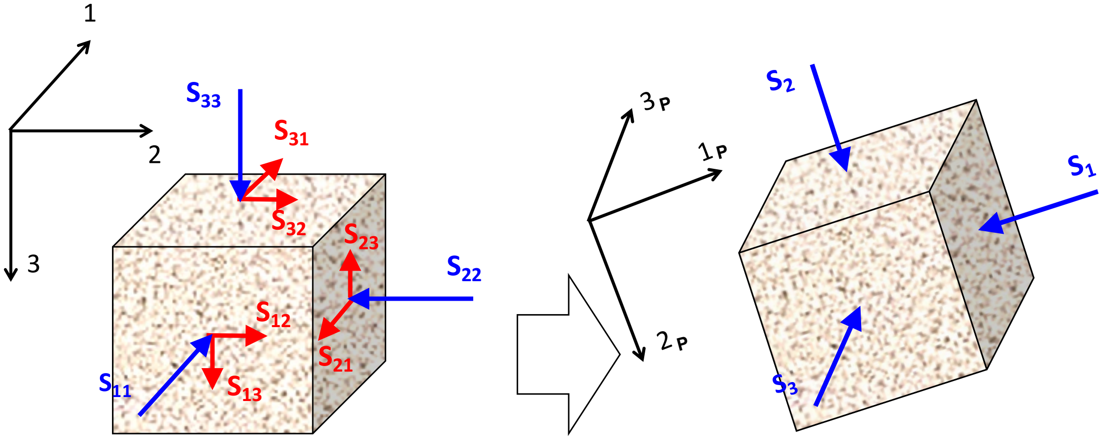
$$S_{ij} = S_{ji}$$

$$i \neq j$$



$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$S_1 \geq S_2 \geq S_3$$



$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad \underline{\underline{S_P}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$\underline{a}$

$$m\underline{a} = 0$$

$$\rho V b_1$$

ρ

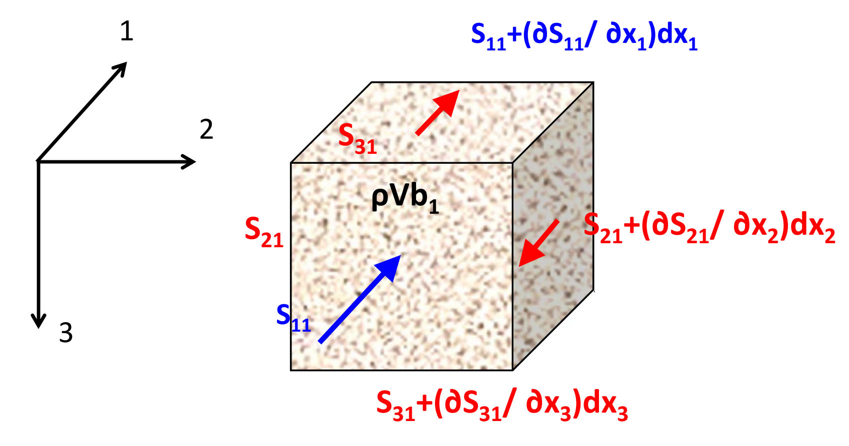
V

b_1

$$
\begin{aligned}
\sum F_1 &= 0 \\
\sum F_1 &= +S_{11}dx_2dx_3 - \left[S_{11} + (\tfrac{\partial S_{11}}{\partial x_1})dx_1\right]dx_2dx_3 \\
&\quad +S_{21}dx_1dx_3 - \left[S_{21} + (\tfrac{\partial S_{21}}{\partial x_2})dx_2\right]dx_1dx_3 \\
&\quad +S_{31}dx_1dx_2 - \left[S_{31} + (\tfrac{\partial S_{31}}{\partial x_3})dx_3\right]dx_1dx_2 \\
&\quad -\rho(dx_1dx_2dx_3)b_1 = 0
\end{aligned}
$$

$$(dx_1 dx_2 dx_3)$$

$$\frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} - \rho b_1 = 0$$



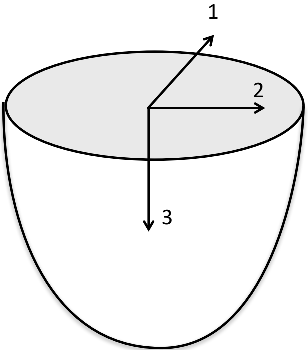
1
2
3
$S_{11}+(\partial S_{11}/ \partial x_1)dx_1$
S_{31}
$\rho V b_1$
S_{21}
$S_{21}+(\partial S_{21}/ \partial x_2)dx_2$
S_{11}
$S_{31}+(\partial S_{31}/ \partial x_3)dx_3$

$$
\left\{
\begin{array}{rcl}
\dfrac{\partial S_{11}}{\partial x_1} + \dfrac{\partial S_{21}}{\partial x_2} + \dfrac{\partial S_{31}}{\partial x_3} - \rho b_1 & = & 0 \\
\dfrac{\partial S_{12}}{\partial x_1} + \dfrac{\partial S_{22}}{\partial x_2} + \dfrac{\partial S_{32}}{\partial x_3} - \rho b_2 & = & 0 \\
\dfrac{\partial S_{13}}{\partial x_1} + \dfrac{\partial S_{23}}{\partial x_2} + \dfrac{\partial S_{33}}{\partial x_3} - \rho b_3 & = & 0
\end{array}
\right.
$$

$$b_3 = g$$

$$\partial()/\partial x_1 = \partial()/\partial x_2 = 0$$

$$S_{33}(x_3) = \int_0^{x_3} \rho(x_3) g \, dx_3$$



$$\begin{cases} \cancel{\dfrac{\partial S_{11}}{\partial x_1}} + \cancel{\dfrac{\partial S_{21}}{\partial x_2}} + \dfrac{\partial S_{31}}{\partial x_3} + \cancel{\rho b_1} = \cancel{\dfrac{\partial^2 (\rho u_1)}{\partial t^2}} \\ \cancel{\dfrac{\partial S_{12}}{\partial x_1}} + \cancel{\dfrac{\partial S_{22}}{\partial x_2}} + \dfrac{\partial S_{32}}{\partial x_3} + \cancel{\rho b_2} = \cancel{\dfrac{\partial^2 (\rho u_2)}{\partial t^2}} \\ \cancel{\dfrac{\partial S_{13}}{\partial x_1}} + \cancel{\dfrac{\partial S_{23}}{\partial x_2}} + \dfrac{\partial S_{33}}{\partial x_3} + \rho b_3 = \cancel{\dfrac{\partial^2 (\rho u_3)}{\partial t^2}} \end{cases}$$

$$\frac{\partial S_{33}}{\partial x_3} - \rho(x_3) g = 0$$

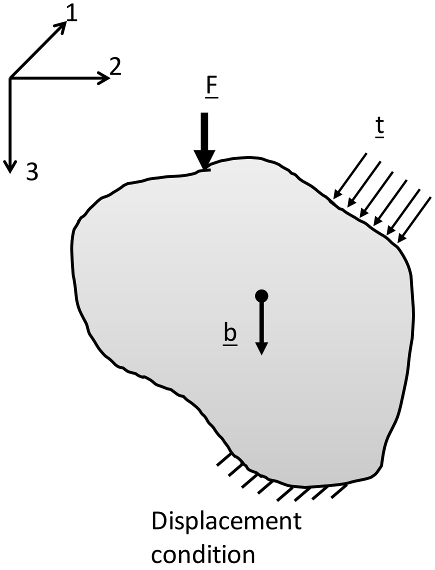
$$S_{33} = \int_0^{x_3} \rho(x_3) g \; dx_3$$

S_{11}

S_{22}

t

$\underline{b}$



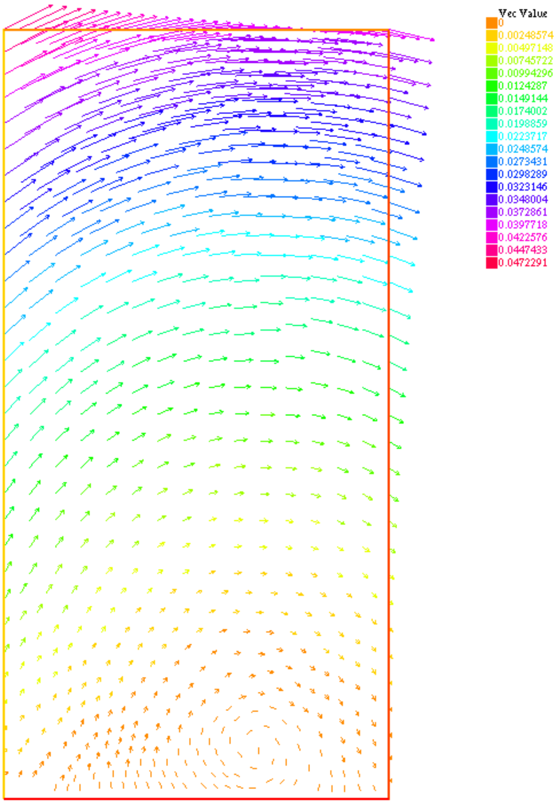
$$
\begin{cases}
\dfrac{\partial S_{11}}{\partial x_1} + \dfrac{\partial S_{21}}{\partial x_2} + \dfrac{\partial S_{31}}{\partial x_3} + \rho b_1 = \dfrac{\partial^2 (\rho u_1)}{\partial t^2} \\[2ex]
\dfrac{\partial S_{12}}{\partial x_1} + \dfrac{\partial S_{22}}{\partial x_2} + \dfrac{\partial S_{32}}{\partial x_3} + \rho b_2 = \dfrac{\partial^2 (\rho u_2)}{\partial t^2} \\[2ex]
\dfrac{\partial S_{13}}{\partial x_1} + \dfrac{\partial S_{23}}{\partial x_2} + \dfrac{\partial S_{33}}{\partial x_3} + \rho b_3 = \dfrac{\partial^2 (\rho u_3)}{\partial t^2}
\end{cases}
$$

And respect the boundary conditions:

- Displacement
- Boundary stresses
- Boundary Forces
- Body Forces

How do we relate stresses to displacements?

- Displacements → Strains (**Kinematic equations**)
- Strains → Stresses (**Constitutive equations**)



Vec Value
0
0.00248574
0.00497148
0.00745722
0.00994296
0.0124287
0.0149144
0.0174002
0.0198859
0.0223717
0.0248574
0.0273431
0.0298289
0.0323146
0.0348004
0.0372861
0.0397718
0.0422576
0.0447433
0.0472291

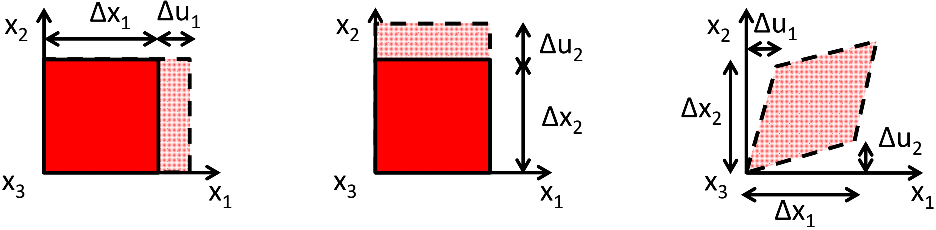
$$\varepsilon_{11} = \frac{\Delta u_1}{\Delta x_1}$$

$$\varepsilon_{22} = \frac{\Delta u_2}{\Delta x_2}$$

$$\pi - [\pi - \arctan(\Delta u_1/\Delta x_2) + \arctan(\Delta u_2/\Delta x_1)]$$

$$\arctan(x/y) \sim x/y$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$



$$\varepsilon_{11} \approx \frac{\Delta u_1}{\Delta x_1} \qquad \varepsilon_{22} \approx \frac{\Delta u_2}{\Delta x_2} \qquad \varepsilon_{12} \approx \frac{1}{2}\left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1}\right)$$

x_i

$$\varepsilon_{vol} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

$$\Delta x_1 \Delta x_2 \Delta x_3$$

ΔV

V_0

$$\varepsilon_{vol} = \frac{\Delta V}{V_0}$$

$$\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3$$

$$(\mathrm{d}x_1 + \mathrm{d}u_1)(\mathrm{d}x_2 + \mathrm{d}u_2)(\mathrm{d}x_3 + \mathrm{d}u_3)$$

$$\varepsilon_{vol} = \frac{[(\mathrm{d}x_1 + \mathrm{d}u_1)(\mathrm{d}x_2 + \mathrm{d}u_2)(\mathrm{d}x_3 + \mathrm{d}u_3) - (\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3)]}{(\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3)}$$

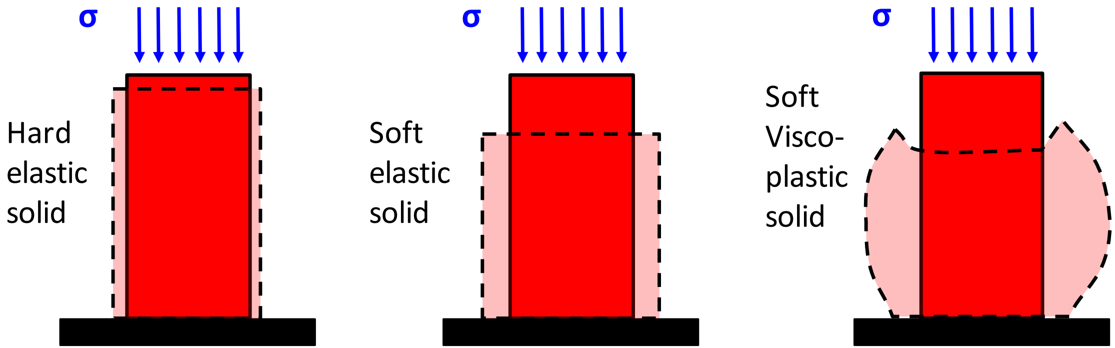
$du_i du_j$

$$du_1 du_2 du_3$$

$$\mathrm{d}u_i$$

$$\mathrm{d}u_i << \mathrm{d}x_j$$

$$\varepsilon_{vol} \sim \frac{(\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}u_3 + \mathrm{d}x_1\mathrm{d}x_3\mathrm{d}u_2 + \mathrm{d}x_2\mathrm{d}x_3\mathrm{d}u_1)}{(\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3)} = \frac{\mathrm{d}u_1}{\mathrm{d}x_1} + \frac{\mathrm{d}u_2}{\mathrm{d}x_2} + \frac{\mathrm{d}u_3}{\mathrm{d}x_3} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \quad \blacksquare$$


σ
Hard
elastic
solid
σ
Soft
elastic
solid
σ
Soft
Visco-
plastic
solid

F

Δx

$$F = k\Delta x$$

E

$$\frac{F}{A} = E\frac{\Delta x}{L}$$

σ

$\mathcal{E}$

$$\sigma = E\varepsilon$$

$\underline{\underline{\sigma}}$

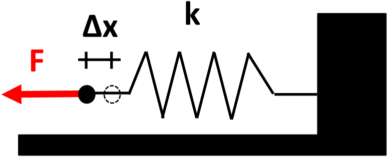
E

$\underline{\underline{C}}$

$$\underline{\underline{\sigma}} = \underline{\underline{C}}\,\underline{\underline{\varepsilon}}$$

Hooke's law

$$F = k\Delta x$$

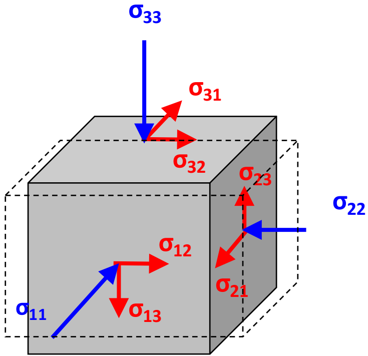


1-D (stress-strain) Hooke's law

$$\sigma = E\varepsilon$$

Generalized Hooke's law

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}}\,\underline{\underline{\varepsilon}}$$



σ_{33}

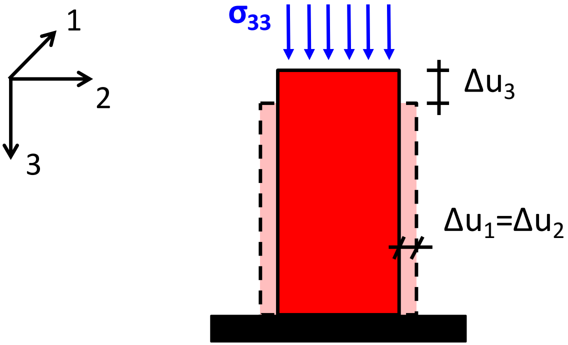
$$\varepsilon_{33}$$

$$E = \frac{\sigma_{33}}{\varepsilon_{33}}$$

ε_{11}

$$\varepsilon_{22}$$

$$\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}}$$



$$E = \frac{\sigma_{33}}{\varepsilon_{33}}$$ Young's Modulus

$$\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}}$$ Poisson's ratio (nu)

σ_a

ε_a

$$\sigma_a = 3000$$

$$= 3000\frac{1}{145}$$

$$= 20.7$$

$\varepsilon_a(1$

$$) = \frac{\sigma_a}{E} = \frac{20.7\ \text{MPa}}{1000\ \text{MPa}} = 0.0207 = 2.07\%$$

$\varepsilon_a(10$

$$) = \frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{10000 \text{ MPa}} = 0.00207 = 0.207\%$$

$\varepsilon_a(50$

$$) = \frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{50000 \text{ MPa}} = 0.00041 = 0.041\%$$

$$
\left\{
\begin{array}{rcl}
\varepsilon_{11} & = & +\dfrac{1}{E}\,\sigma_{11} - \dfrac{\nu}{E}\,\sigma_{22} - \dfrac{\nu}{E}\,\sigma_{33} \\
\varepsilon_{22} & = & -\dfrac{\nu}{E}\,\sigma_{11} + \dfrac{1}{E}\,\sigma_{22} - \dfrac{\nu}{E}\,\sigma_{33} \\
\varepsilon_{33} & = & -\dfrac{\nu}{E}\,\sigma_{11} - \dfrac{\nu}{E}\,\sigma_{22} + \dfrac{1}{E}\,\sigma_{33}
\end{array}
\right.
$$

$$\varepsilon_{ij}$$

$$G = E/[2(1+\nu)]$$

$$\left\{\begin{array}{lcl} 2\varepsilon_{12} & = & \dfrac{1}{G}\,\sigma_{12} \\ 2\varepsilon_{13} & = & \dfrac{1}{G}\,\sigma_{13} \\ 2\varepsilon_{23} & = & \dfrac{1}{G}\,\sigma_{23} \end{array}\right.$$

$$\underline{\underline{D}}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\,\underline{\underline{\sigma}}$$

$$6 \times 1$$

$$6 \times 6$$

$$
\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} +\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & +\frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & +\frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}
$$

$$\underline{\underline{\sigma}} = [0, 0, \sigma_{33}, 0, 0, 0]^T$$

$$\underline{\underline{D}}\ \underline{\underline{\sigma}}$$

$$\underline{\underline{\varepsilon}} = \left[-\frac{\nu}{E}\,\sigma_{33}, -\frac{\nu}{E}\,\sigma_{33}, \frac{1}{E}\,\sigma_{33}, 0, 0, 0 \right]^T$$

ν

$$\underline{\underline{C}} = \underline{\underline{D}}^{-1}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}$$

λ

$$\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{11} + \nu\varepsilon_{22} + \nu\varepsilon_{33}\right]$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + \frac{\nu E}{(1+\nu)(1-2\nu)} \left(-1 + \frac{1-\nu}{\nu} \right) \varepsilon_{11}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$2\mu = \frac{\nu E}{(1+\nu)(1-2\nu)}\left(-1+\frac{1-\nu}{\nu}\right) = \frac{E}{(1+\nu)}$$

$$\mu = G$$

$$
\left\{
\begin{array}{lcl}
\sigma_{11} & = & (\lambda + 2\mu)\,\varepsilon_{11} + \lambda\,\varepsilon_{22} + \lambda\,\varepsilon_{33} \\
\sigma_{22} & = & \lambda\,\varepsilon_{11} + (\lambda + 2\mu)\,\varepsilon_{22} + \lambda\,\varepsilon_{33} \\
\sigma_{33} & = & \lambda\,\varepsilon_{11} + \lambda\,\varepsilon_{22} + (\lambda + 2\mu)\,\varepsilon_{33} \\
\sigma_{23} & = & 2\mu\,\varepsilon_{23} \\
\sigma_{13} & = & 2\mu\,\varepsilon_{13} \\
\sigma_{12} & = & 2\mu\,\varepsilon_{12}
\end{array}
\right. .
$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix} \quad \blacksquare$$

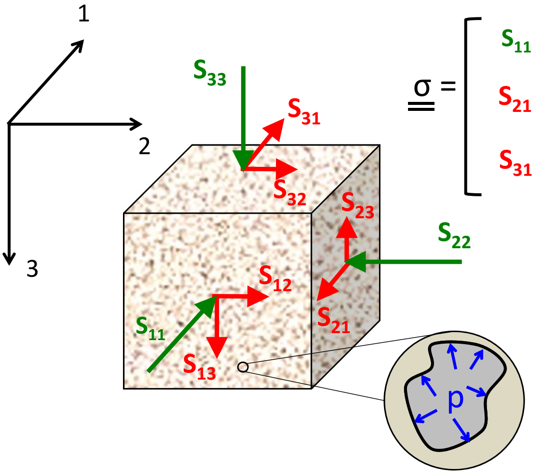
K

G

	(E,ν)	(K,G)	
$G =$	$\dfrac{E}{2(1+\nu)}$		<u>Shear modulus</u> (also noted as μ, S-wave)
$M =$	$\dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}$	$K+\dfrac{4}{3}G$	<u>Constrained modulus</u> (uniaxial compaction, P-wave)
$\lambda =$	$\dfrac{\nu E}{(1+\nu)(1-2\nu)}$		<u>Lamé first parameter</u> (volumetric strain component)
$K =$	$\dfrac{E}{3(1-2\nu)}$		<u>Bulk modulus</u> (relates volumetric strain and isotropic stress)

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\,\underline{\underline{S}}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\left(\underline{\underline{S}} - P_p \underline{\underline{I}}\right) = \underline{\underline{D}}\,\underline{\underline{\sigma}}$$



$$\underline{\underline{\sigma}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} - \begin{bmatrix} P_p & 0 & 0 \\ 0 & P_p & 0 \\ 0 & 0 & P_p \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} S_{11}-P_p & S_{12} & S_{13} \\ S_{21} & S_{22}-P_p & S_{23} \\ S_{31} & S_{32} & S_{33}-P_p \end{bmatrix}$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}} \left(\underline{\underline{S}} - \alpha P_p \underline{\underline{I}} \right)$$

$$\alpha \sim 1$$

$\alpha \sim 0.5$

$$S_v = S_{33}$$

$$\sigma_v(z) = S_v(z) - P_p$$

$$\varepsilon_{11} = \varepsilon_{22}$$

$$\underline{\underline{\varepsilon}} = [0, 0, \varepsilon_{33}, 0, 0, 0]^T$$

$$\underline{\underline{\sigma}} = \underline{\underline{D}}\,\underline{\underline{\varepsilon}}$$

$$\left\{ \begin{array}{l} \sigma_{11} = \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \end{array} \right.$$

σ_{11}

σ_{22}

$$\sigma_{11} = \sigma_{22} = \frac{\nu}{1-\nu}\sigma_{33}$$

$$\sigma_h = \frac{\nu}{1-\nu}\sigma_v$$

$\nu \sim 0.25$

$$\nu/(1-\nu) \sim 1/3$$

$$\nu \to 0.5$$

$\nu/(1-\nu) \to 1$

$\nu \sim 0.5$

$$S_{11} = \sigma_{11} + P_p$$

$$S_{22} = \sigma_{22} + P_p$$

$$\varepsilon_{11} \neq \varepsilon_{22} \neq 0$$

$$\underline{\underline{\sigma}} = \underline{\underline{C}}\,\underline{\underline{\varepsilon}}$$

$$\underline{\underline{\varepsilon}} = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 0, 0, 0]^T$$

$$
\begin{cases}
\sigma_{11} = \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\
\sigma_{22} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\
\sigma_{33} = \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \dfrac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33}
\end{cases}
$$

$$\left\{\begin{array}{l}\sigma_{11}=\dfrac{\nu}{1-\nu}\sigma_{33}+\dfrac{E}{1-\nu^{2}}\varepsilon_{11}+\dfrac{\nu E}{1-\nu^{2}}\varepsilon_{22}\\ \sigma_{22}=\dfrac{\nu}{1-\nu}\sigma_{33}+\dfrac{\nu E}{1-\nu^{2}}\varepsilon_{11}+\dfrac{E}{1-\nu^{2}}\varepsilon_{22}\end{array}\right.$$

ε_{hmin}

$$\begin{cases} \sigma_{Hmax} = \dfrac{\nu}{1-\nu}\sigma_v + E'\varepsilon_{Hmax} + \nu E'\varepsilon_{hmin} \\ \sigma_{hmin} = \dfrac{\nu}{1-\nu}\sigma_v + \nu E'\varepsilon_{Hmax} + E'\varepsilon_{hmin} \end{cases}$$

$$E' = \frac{E}{1-\nu^2}$$

σ_{hmin}

σ_{Hmax}

$$\mathrm{d}S_v/\mathrm{d}z = 23.8$$

$$\lambda_p = 0.7$$

$$\nu = 0.22$$

$$\varepsilon_{hmin} = 0$$

$$\varepsilon_{Hmax} = 0.0002$$

$$S_v = 23.8\frac{\text{MPa}}{\text{km}} \times \frac{1\,\frac{\text{psi}}{\text{ft}}}{23\,\frac{\text{MPa}}{\text{km}}} \times 7950\ \text{ft} = 8227\text{psi}$$

$$P_p = \lambda_p S_v = 0.7 \times 8227$$

$$\mathrm{psi} = 5759\ \mathrm{psi}$$

$$\sigma_v = S_v - P_p = 8227$$

$$\text{psi} - 5759 \text{ psi} = 2468 \text{ psi}$$

$$E' = \frac{E}{1 - \nu^2} = \frac{5 \times 10^6 \text{ psi}}{1 - 0.22^2} = 5.25 \times 10^6 \text{ psi}$$

$$\left\{\begin{array}{l}\sigma_{Hmax}=\frac{\nu}{1-\nu}\sigma_v+E'\varepsilon_{Hmax}+\nu E'\varepsilon_{hmin}=\frac{0.22}{1-0.22}2468\text{ psi}+5.25\times10^6\text{ psi}\times0.0002=1745\text{ psi}\\ \sigma_{hmin}=\frac{\nu}{1-\nu}\sigma_v+\nu E'\varepsilon_{Hmax}+E'\varepsilon_{hmin}=\frac{0.22}{1-0.22}2468\text{ psi}+0.22\times5.25\times10^6\text{ psi}\times0.0002=927\text{ psi}\end{array}\right.$$

$$\begin{cases} S_{Hmax} = \sigma_{Hmax} + P_p = 1745 \text{ psi} + 5759 \text{ psi} = 7504 \text{ psi} \\ S_{hmin} = \sigma_{hmin} + P_p = 927 \text{ psi} + 5759 \text{ psi} = 6686 \text{ psi} \end{cases} \quad \blacksquare$$

C_{pp}

$$\frac{\partial P_p}{\partial t} = \frac{k}{\mu C_t} \frac{\partial^2 P_p}{\partial x^2}$$

$$C_t = C_g S_g + C_w S_w + C_o S_o + C_{pp}$$

$$(C_g S_g + C_w S_w + C_o S_o)$$

V_p

$$C_{pp} = \frac{1}{V_p} \frac{\mathrm{d}V_p}{\mathrm{d}P_p}\bigg|_{S_v,\varepsilon_h}$$

ε_h

$$\mathrm{d}V_p$$

$$\mathrm{d}V_b$$

$$C_{pp} = \frac{1}{\frac{V_p}{V_b}} \left(\frac{1}{V_b} \left. \frac{\mathrm{d}V_b}{\mathrm{d}P_p} \right|_{S_v, \varepsilon_h} \right)$$

$$\phi = V_p / V_b$$

$$\varepsilon_{vol} = \mathrm{d}V_b / V_b$$

$$C_{pp} = \frac{C_{bp}}{\phi}$$

$$C_{bp} \sim M^{-1}$$

$$M = (1 - \nu)E/[(1 + \nu)(1 - 2\nu)]$$

$$C_{pp} = \frac{(1+\nu)(1-2\nu)}{(1-\nu)E\phi}$$

$\times 10^{-6}$

$$^{-1} = \mu$$

$\sim 2\ \mu$

$$\sim 30\ \mu$$

$$E = 10$$

$\nu = 0.20$

$$10^6$$

$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.20)10 \text{ GPa}}{(1+0.20)(1-2\times 0.20)} = 11.11 \text{ GPa} = 1.6\times 10^6 \text{ psi}$$

$$C_{pp} = \frac{1}{M\phi} = \frac{1}{1.6 \times 10^6 \text{psi} \times 0.20} = 3.1\ [10^6 \text{psi}]^{-1} = 3.1\ \mu\text{sip}\ \blacksquare$$

$\underline{u}$

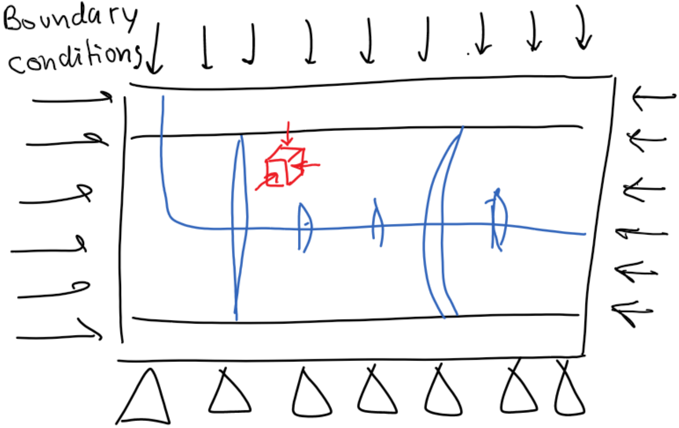
$$(\lambda + G)\nabla(\nabla \cdot \underline{u}) + G\nabla^2\underline{u} + \rho\underline{b} = 0$$

$$\lambda = (\nu E)/[(1+\nu)(1-2\nu)]$$

$$\nabla()$$

$$\nabla \cdot ()$$

$$\nabla^2()$$



1. Equilibrium

$$\frac{\partial \sigma_{ij}}{\partial x_i} + \rho g = 0$$

2. Kinematic

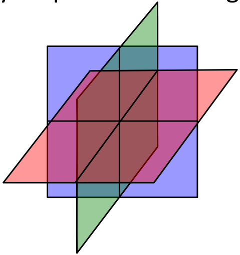
$$\underline{\underline{\varepsilon}} \leftrightarrow \underline{u}$$

3. Constitutive

$$\underline{\underline{\sigma}} \leftrightarrow \underline{\underline{\varepsilon}}$$

Orthorhombic symmetry

(symmetry respect to 3 orthogonal planes)

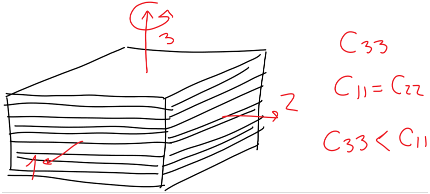


$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & 0 & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{55} & 0 \\ & & & 0 & 0 & C_{66} \end{bmatrix}$$

9 independent parameters

Transverse Isotropy

(symmetry respect to 1 axis)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & 0 & \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{44} & 0 \\ & & & 0 & 0 & C_{66} \end{bmatrix}$$

5 independent parameters ($C_{12}=C_{11}-2C_{66}$)

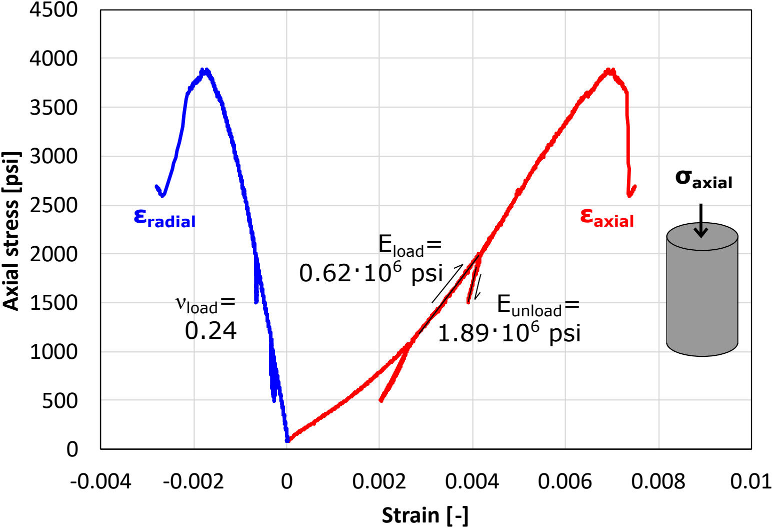
$$E_h > E_v$$

$$\varepsilon \gtrsim 0.001$$

E_{load}

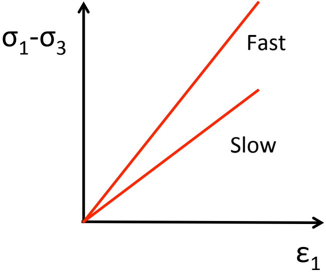
$$E_{unload}$$

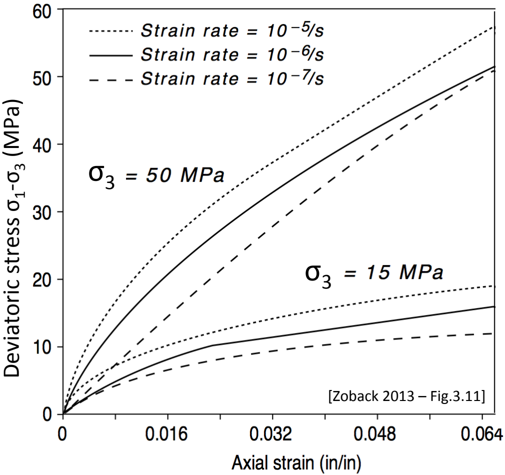
$$E_{unload} \sim E_{unload}$$



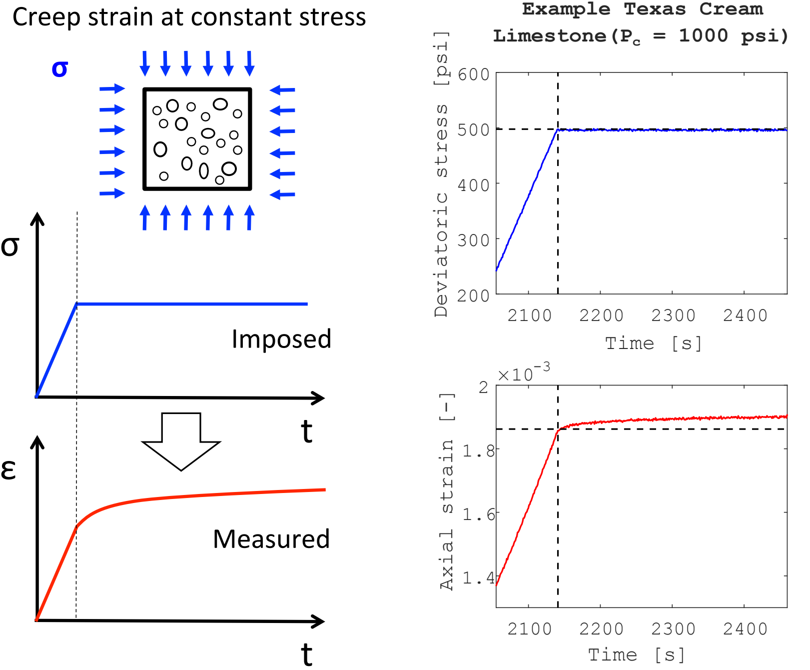
Axial stress [psi]
Strain [-]
4500
4000
3500
3000
2500
2000
1500
1000
500
0
-0.004
-0.002
0
0.002
0.004
0.006
0.008
0.01
ε_{radial}
ε_{axial}
σ_{axial}
E_{load}=
$0.62 \cdot 10^6$ psi
E_{unload}=
$1.89 \cdot 10^6$ psi
ν_{load}=
0.24

Strain rate hardening: The faster the loading, the stiffer the material

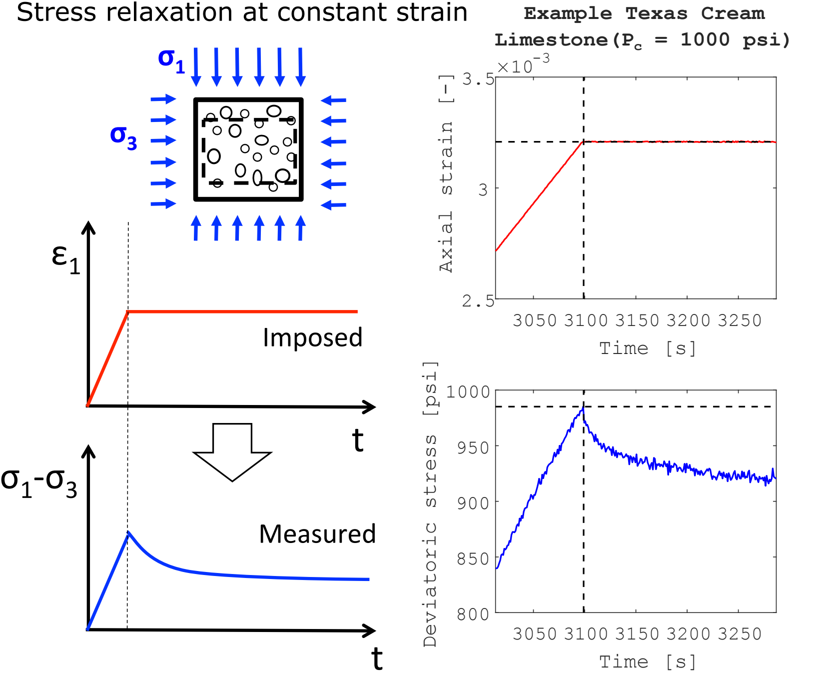




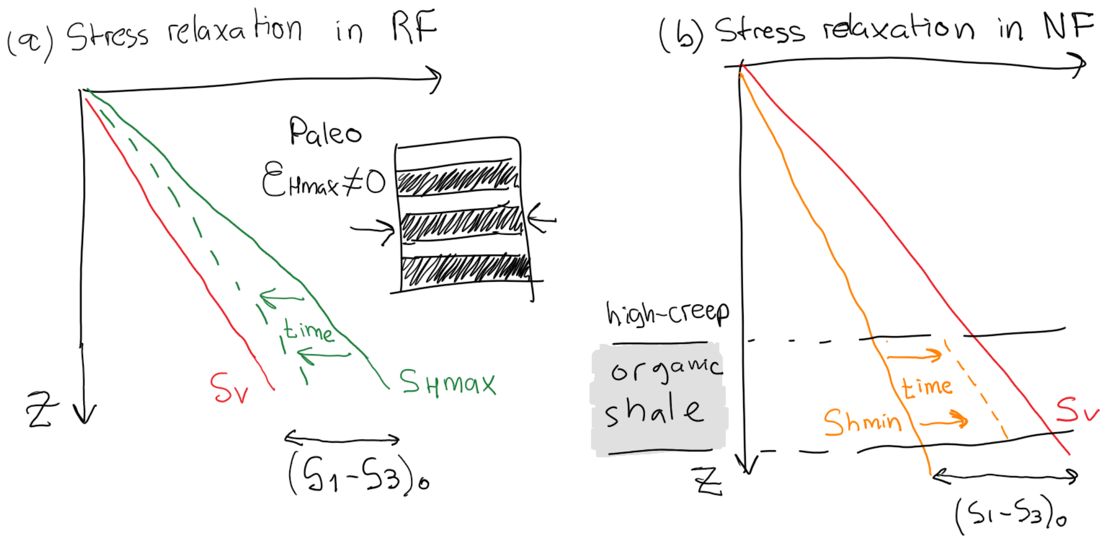
[Zoback 2013 – Fig.3.11]



Creep strain at constant stress
σ
σ
Imposed
t
ε
Measured
t
Example Texas Cream
Limestone (P_c = 1000 psi)
Deviatoric stress [psi]
600
500
400
300
200
2100
2200
2300
2400
Time [s]
×10^-3
2
1.8
1.6
1.4
Axial strain [-]
2100
2200
2300
2400
Time [s]



Stress relaxation at constant strain
σ1
σ3
ε1
Imposed
t
σ1-σ3
Measured
t
Example Texas Cream
Limestone(Pc = 1000 psi)
×10-3
3.5
3
2.5
Axial strain [-]
3050 3100 3150 3200 3250
Time [s]
1000
950
900
850
800
Deviatoric stress [psi]
3050 3100 3150 3200 3250
Time [s]



(a) Stress relaxation in RF
Paleo
$\varepsilon_{Hmax} \neq 0$
time
S_v
S_{Hmax}
Z
$(S_1-S_3)_0$
(b) Stress relaxation in NF
high-creep
organic
shale
time
S_{hmin}
S_v
Z
$(S_1-S_3)_0$

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}\,\underline{\underline{\sigma}}$$

$$\underline{\underline{\sigma}} = \underline{\underline{S}} - \alpha P_p \underline{\underline{I}}$$

$$\underline{\underline{I}}$$

$$\alpha = 1 - \frac{K_{drained}}{K_{unj}}$$

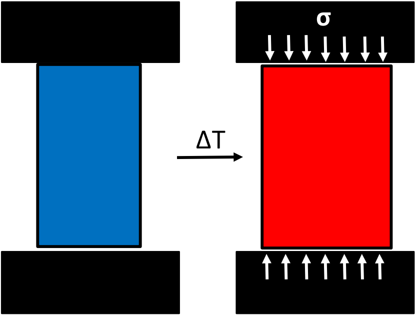
$$K_{drained}$$

K_{unj}

$$K_{unj} = K_m$$

$$\underline{\underline{\alpha}}$$

$$\alpha \neq 1$$


σ
ΔT

α_L

p

$$\alpha_L = \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}T}\bigg|_p$$

ΔT

$$
\left\{
\begin{array}{lll}
\sigma_{11} & = & (\lambda + 2\mu)\,\varepsilon_{11} + \lambda\,\varepsilon_{22} + \lambda\,\varepsilon_{33} + 3K\alpha_L\Delta T \\
\sigma_{22} & = & \lambda\,\varepsilon_{11} + (\lambda + 2\mu)\,\varepsilon_{22} + \lambda\,\varepsilon_{33} + 3K\alpha_L\Delta T \\
\sigma_{33} & = & \lambda\,\varepsilon_{11} + \lambda\,\varepsilon_{22} + (\lambda + 2\mu)\,\varepsilon_{33} + 3K\alpha_L\Delta T \\
\sigma_{23} & = & 2\mu\,\varepsilon_{23} \\
\sigma_{13} & = & 2\mu\,\varepsilon_{13} \\
\sigma_{12} & = & 2\mu\,\varepsilon_{12}
\end{array}
\right.
$$

$$\sigma_{11} = \sigma_{22} = 0$$

$$\varepsilon_{11} = \varepsilon_{22} \neq 0$$

$$\varepsilon_{33} = 0$$

$$\Delta T \neq 0$$

$$\left\{ \begin{array}{rcl} 0 & = & (\lambda + 2\mu)\,\varepsilon_{11} + \lambda\,\varepsilon_{11} + 3K\alpha_L \Delta T \\ \sigma_{33} & = & \lambda\,\varepsilon_{11} + \lambda\,\varepsilon_{11} + 3K\alpha_L \Delta T \end{array} \right.$$

$$\sigma_{33} = \left(\frac{6 - \mu/K}{3 + \mu/K} \right) K \alpha_L \Delta T \ \blacksquare$$

$$D = 1.00$$

$$L = 2.01$$

ΔL

ΔD

$$\underline{\sigma} = \underline{\underline{C}}\,\underline{\varepsilon}$$

σ_{iso}

$$\varepsilon_{vol} = \frac{3(1-2\nu)}{E}\sigma_{iso}$$

ε_{vol}

$$E = 4 \times 10^6$$

$$\nu = 0.2$$

$\sigma_{iso} =$

×

$$\sigma_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\varepsilon_{11}$$

3×3

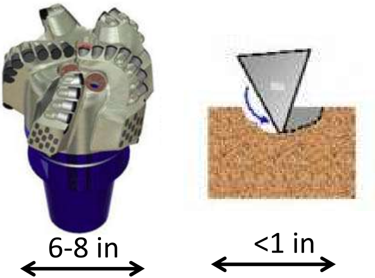
$\phi = 0.35$

(a) Uncemented sand

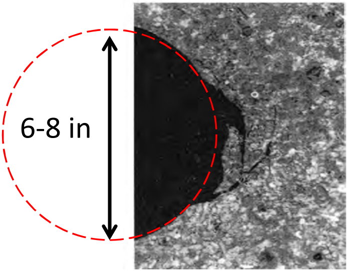
(b) Cemented sandstone

(c) Vuggy carbonate

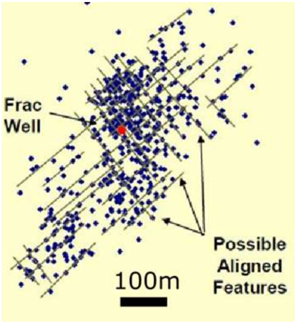
Rock cutting at the drill bit



Wellbore breakout

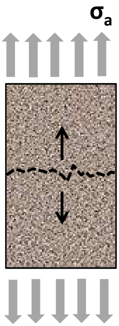


Shale hydraulic fracture

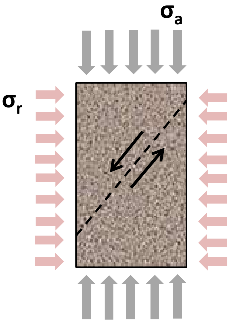


Reservoir depletion

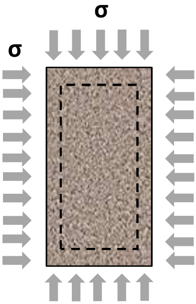
Images: Schlumberger/Terratek, Zoback 2013, Warpinski 2008, doe.gov



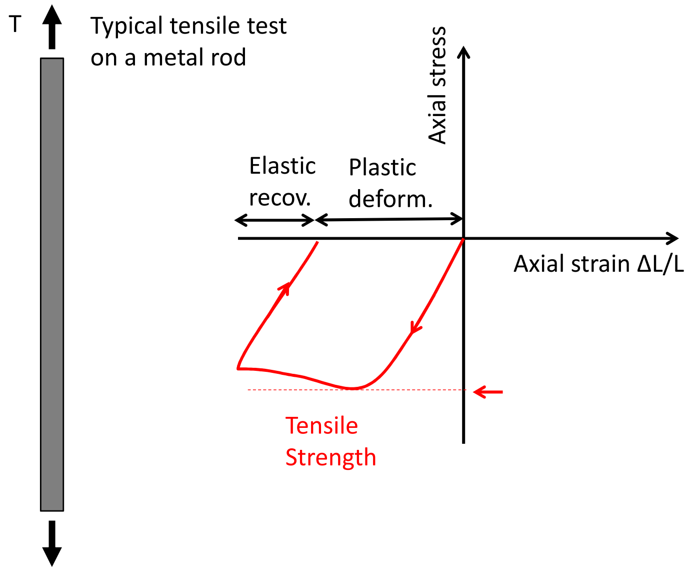
Tension
(bond breakage)
Ex: drilling-induced tens. fracs



Shear
(friction failure)
Ex: fault, breakout



Compression
(pore collapse)
Ex: reservoir compaction


T
Typical tensile test
on a metal rod
Axial stress
Elastic
recov.
Plastic
deform.
Axial strain ΔL/L
Tensile
Strength

Tensile test on a rock rod

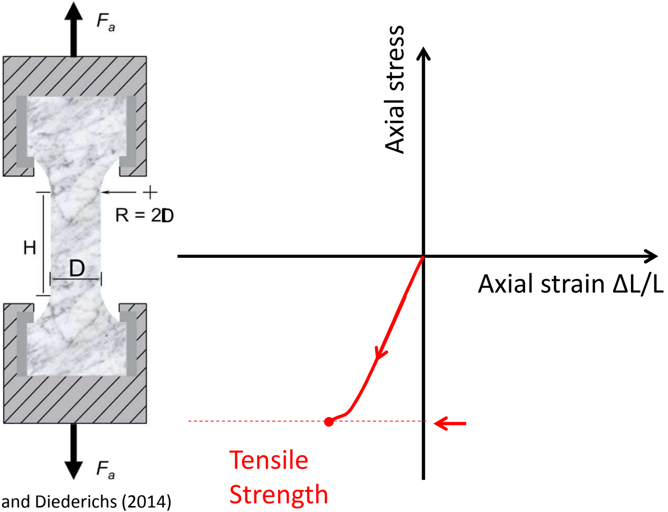
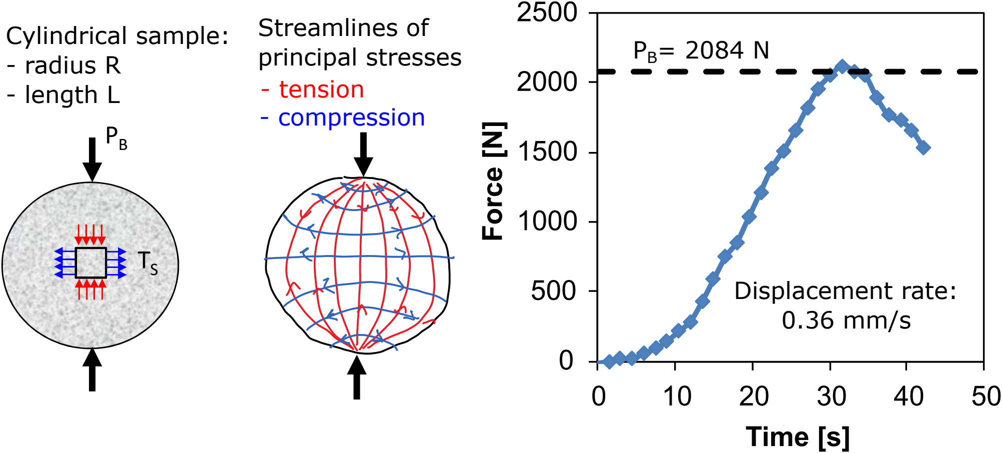


Figure direct tension: Perras and Diederichs (2014)

$$T_S = \frac{P_B}{\pi L R}$$

P_B

R



Cylindrical sample:
- radius R
- length L
P_B
T_S
Streamlines of principal stresses
- tension
- compression
P_B= 2084 N
Displacement rate: 0.36 mm/s
Force [N]
Time [s]
0
500
1000
1500
2000
2500
0
10
20
30
40
50

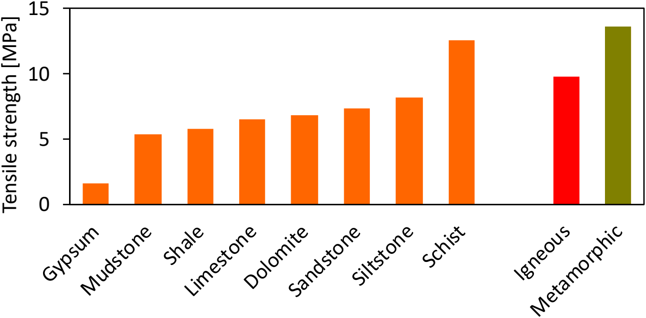
$$R = \frac{1}{2}$$

$$= 0.0127$$

$$L = 1$$

$$= 0.0254$$

$$T_S = \frac{2084 \text{ N}}{\pi(0.0254 \text{ m})(0.0127 \text{ m})} = 2.06 \times 10^6 \text{ Pa} = 2.06 \text{ MPa} \quad \blacksquare$$



Tensile strength [MPa]
15
10
5
0
Gypsum
Mudstone
Shale
Limestone
Dolomite
Sandstone
Siltstone
Schist
Igneous
Metamorphic

S_0

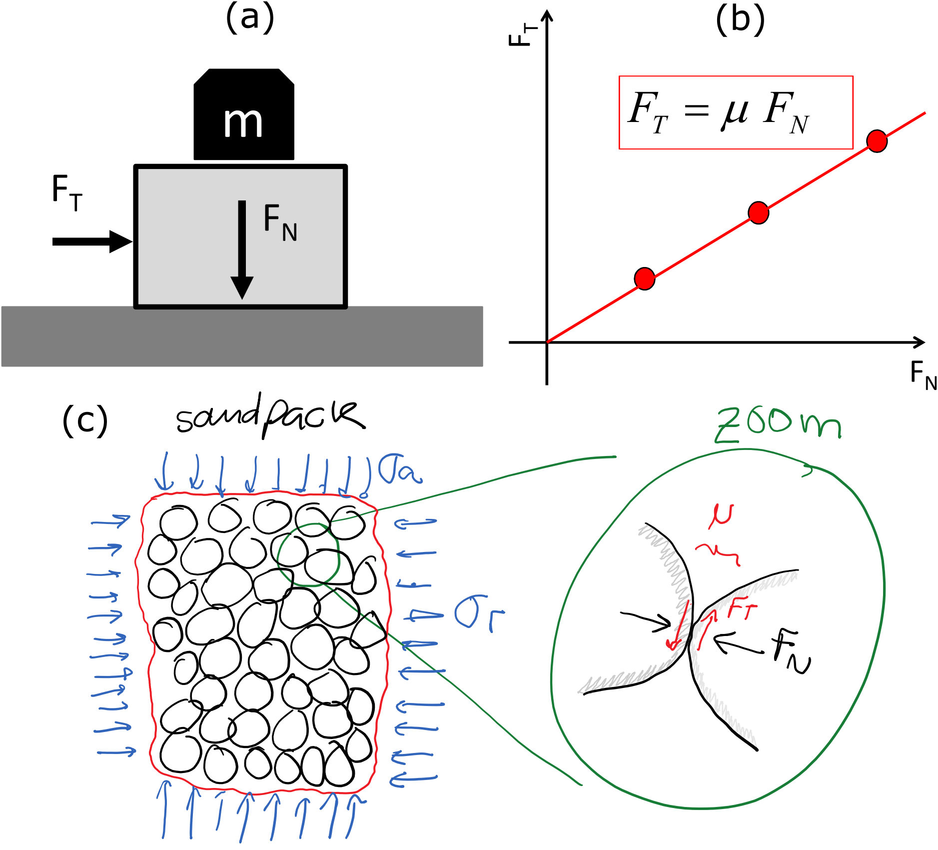
$$F_T$$

F_N

$$F_T = \mu F_N$$

$$F_N = 0$$

$$F_T = 0$$

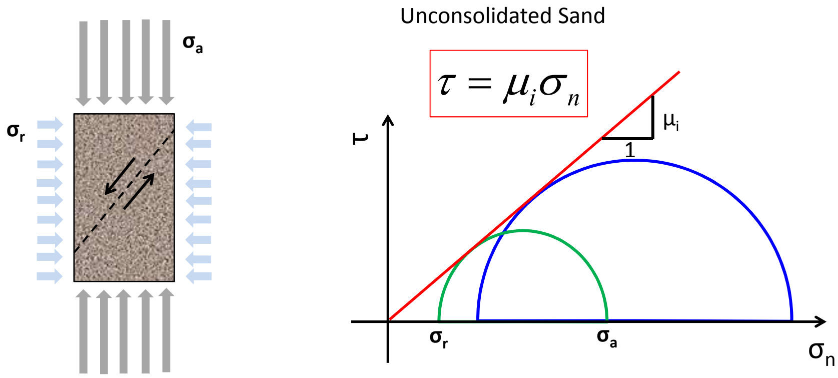

(a)
m
F_T
F_N
(b)
F_T
$F_T = \mu\ F_N$
F_N
(c)
sandpack
σ_a
σ_r
zoom
μ
F_T
F_N

τ

σ_n

μ_i

$$\tau = \mu_i \sigma_n$$


Unconsolidated Sand
$\tau = \mu_i \sigma_n$
σ_a
σ_r
τ
μ_i
1
σ_r
σ_a
σ_n

$$\sigma_n = 0$$

$$\tau = 0$$

φ

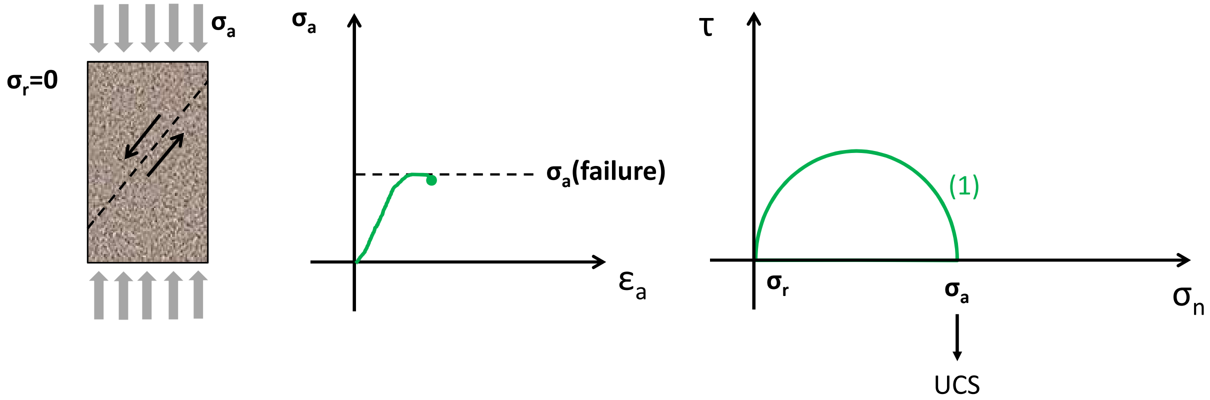
$$\tan(\varphi) = \mu_i$$

$$\mu_i = 0.5$$

$$\varphi \sim 30^\circ$$

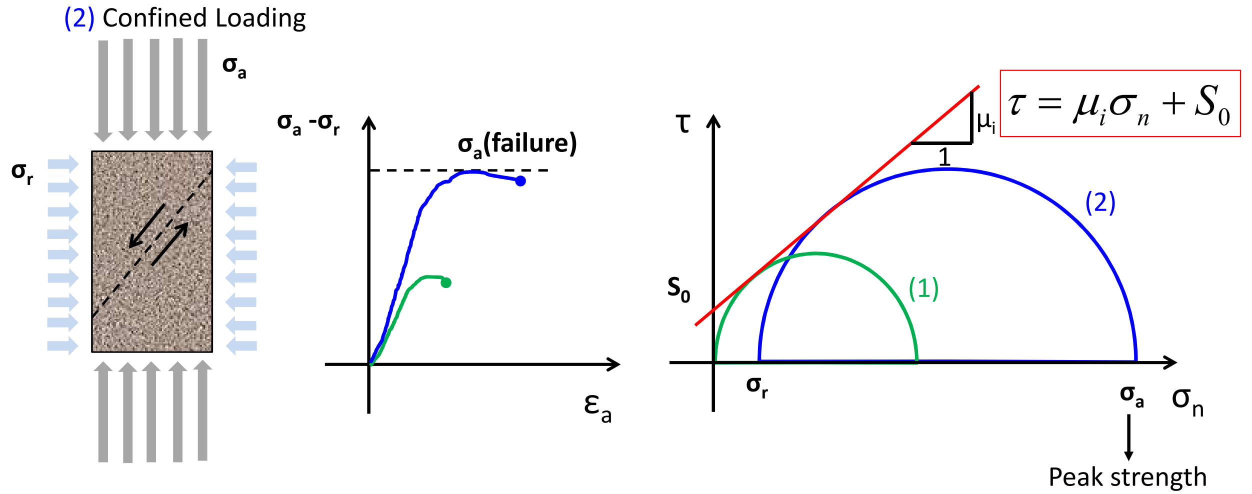
$$\sigma_r = 0$$

(1) Unconfined Loading



$$\sigma_r \neq 0$$

$$\tau = S_0 + \mu_i \sigma_n$$

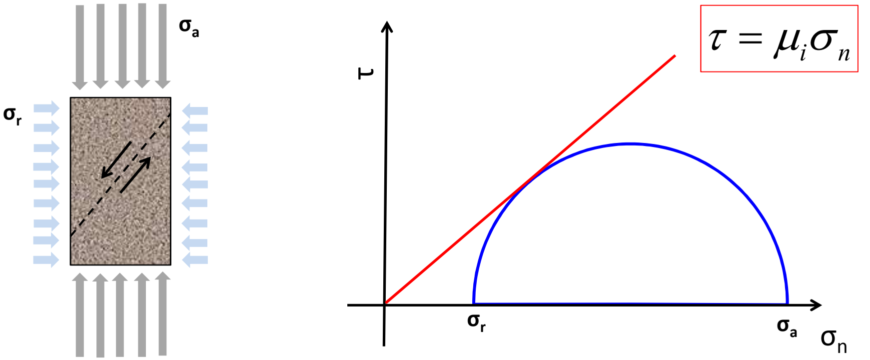


(2) Confined Loading
σ_a
σ_r
$\sigma_a - \sigma_r$
σ_a(failure)
ε_a
τ
μ_i
1
$\tau = \mu_i \sigma_n + S_0$
(2)
(1)
S_0
σ_r
σ_a
σ_n
Peak strength

$$\pi/4 + \varphi/2$$

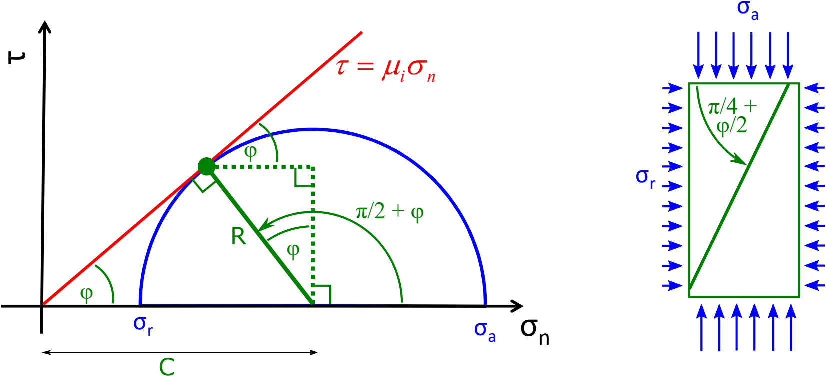
$$\pi/4 + \varphi/2 = 60^\circ$$

σ_a/σ_r



σ_a
σ_r
τ
$\tau = \mu_i \sigma_n$
σ_r
σ_a
σ_n

$$\tau/\sigma_n = \mu_i$$



τ
$\tau = \mu_i \sigma_n$
φ
π/2 + φ
R
φ
φ
σr
σa
σn
C
σa
π/4 + φ/2
σr

$(\sigma_a, 0)$

$(\sigma_r, 0)$

(σ_n, τ)

$$\pi/2 + \varphi$$

$$(\pi/2 + \varphi)/2$$

$^\circ = \pi/2$

C

$$\frac{\sigma_a}{\sigma_r} = \frac{C+R}{C-R} = \frac{C + C\sin\varphi}{C - C\sin\varphi} = \frac{1+\sin\varphi}{1-\sin\varphi} = \quad \blacksquare$$

$$\sigma_1 = UCS + q\,\sigma_3$$

q

$$p' - q$$

$$\sigma_m - q$$

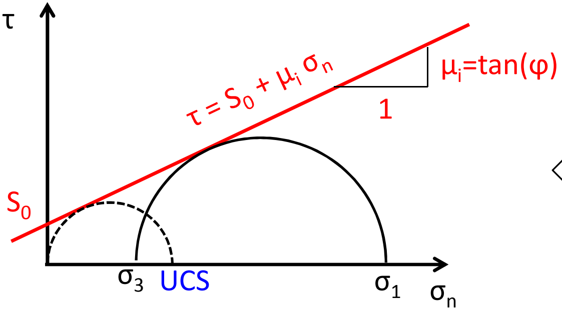
$$q = \frac{1 + \sin \varphi}{1 - \sin \varphi}$$

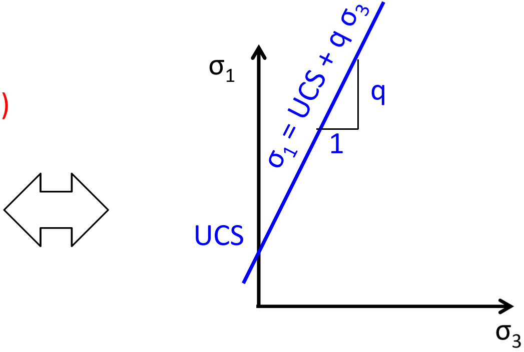
$\varphi\ 30^\circ$

$$q = 3$$

$$\mu_i = \frac{q-1}{2\sqrt{q}}$$

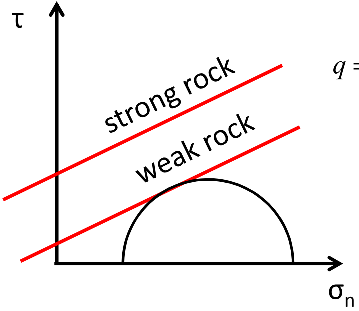
$$UCS = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} = 2S_0 \sqrt{q}$$

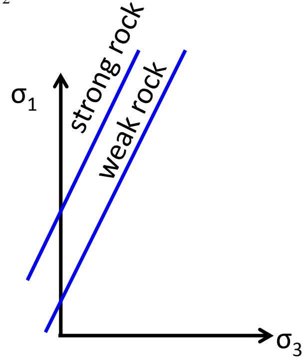


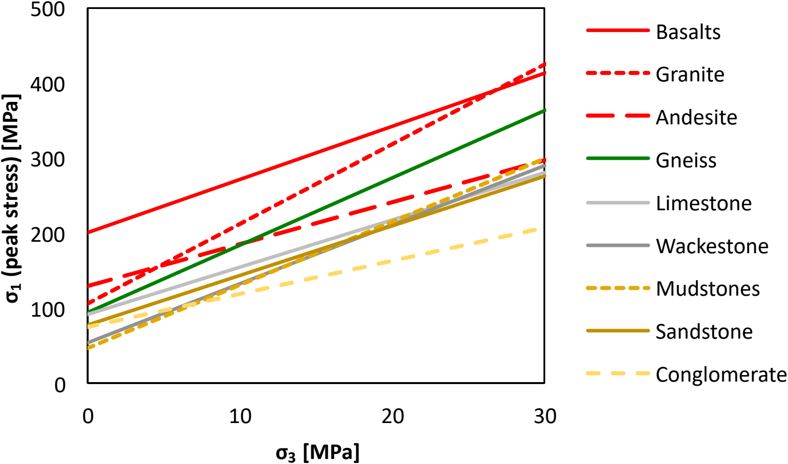


$$UCS = 2S_0\left(\sqrt{\mu_i^2+1}+\mu_i\right) = 2S_0\left(\frac{1+\sin\varphi}{1-\sin\varphi}\right)^{1/2}$$

$$q = \left(\sqrt{\mu_i^2+1}+\mu_i\right)^2 = \frac{1+\sin\varphi}{1-\sin\varphi}$$







0
100
200
300
400
500
σ1 (peak stress) [MPa]
0
10
20
30
σ3 [MPa]
Basalts
Granite
Andesite
Gneiss
Limestone
Wackestone
Mudstones
Sandstone
Conglomerate

$$(S_1, S_2, S_3)$$

$(\sigma_1, \sigma_2, \sigma_3)$

P_c

σ_D

$$P_c = cst$$

$$P_p < P_c$$

$$P_p = cst$$

$$\sigma_D = S_1 - S_3$$

$$S_3 = P_c$$

$$S_2 = S_3$$

$$S_1 - S_3 = \sigma_1 - \sigma_3$$

$$d\varepsilon_a/dt = cst$$

$$\sigma_1 = (S_1 - P_p)$$

$$\sigma_3 = (S_3 - P_p)$$

$$\varphi = \arctan\left(\frac{q-1}{2\sqrt{q}}\right)$$

$$S_3 \leqslant 80$$

$$S_1 \sim 80$$

$$UCS = 80$$

$$q = \frac{\Delta\sigma_1}{\Delta\sigma_3} = \frac{520\ \text{MPa}}{110\ \text{MPa}} = 4.73$$

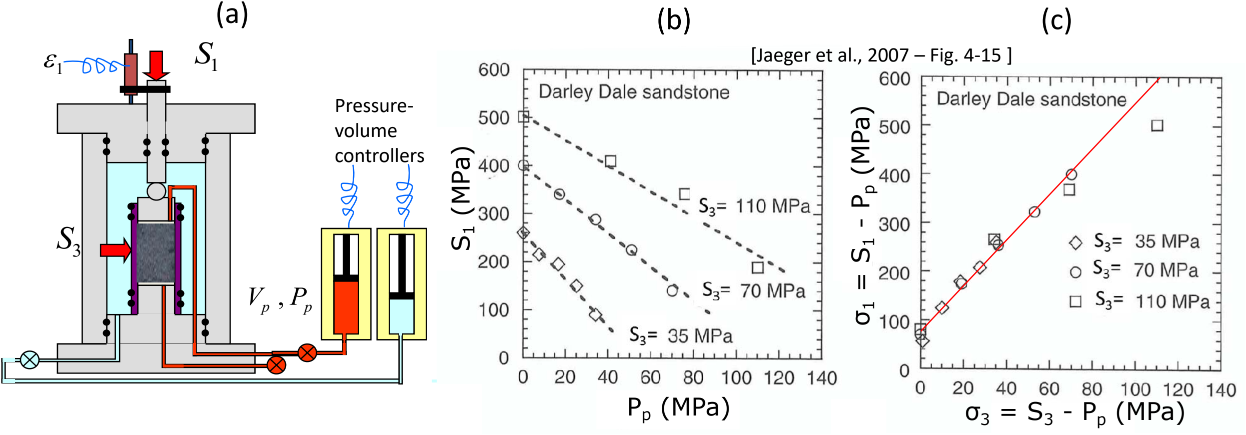
$$S_0 = \frac{UCS}{2\sqrt{q}} = \frac{80\mathrm{MPa}}{2\sqrt{4.73}} = 18.4\mathrm{MPa}$$

$$\varphi = \arctan\left(\frac{q-1}{2\sqrt{q}}\right) = \arctan\left(\frac{4.73-1}{2\sqrt{4.73}}\right) = 40.6^\circ$$

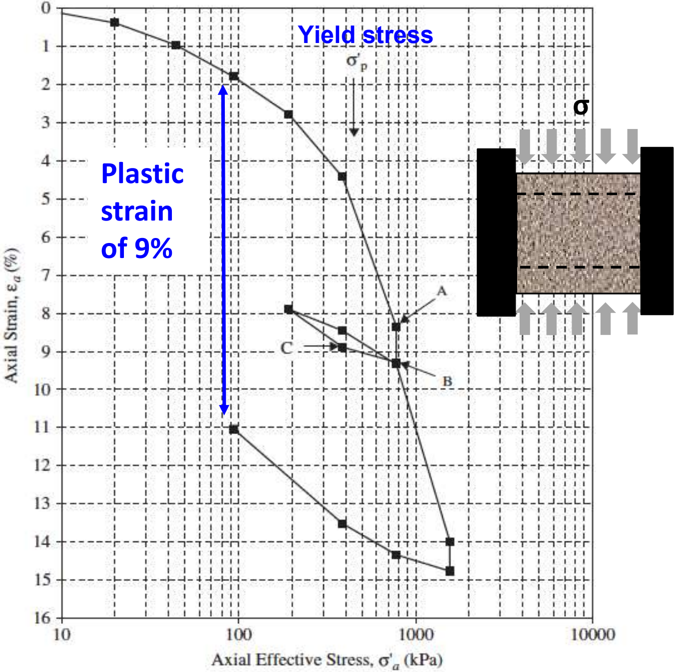
$$\mu_i = \tan(\varphi) = \tan(40.6^\circ) = 0.86 \quad \blacksquare$$

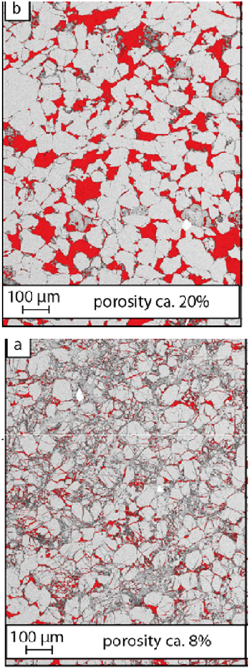
$$\varepsilon_v = \varepsilon_a + 2\varepsilon_r$$

$$d\varepsilon_v/dt < 0$$


(a)
ε_1
S_1
S_3
Pressure-volume controllers
V_p , P_p
(b)
(c)
[Jaeger et al., 2007 – Fig. 4-15]
Darley Dale sandstone
S_1 (MPa)
P_p (MPa)
S_3= 110 MPa
S_3= 70 MPa
S_3= 35 MPa
Darley Dale sandstone
$\sigma_1 = S_1 - P_p$ (MPa)
$\sigma_3 = S_3 - P_p$ (MPa)
S_3= 35 MPa
S_3= 70 MPa
S_3= 110 MPa

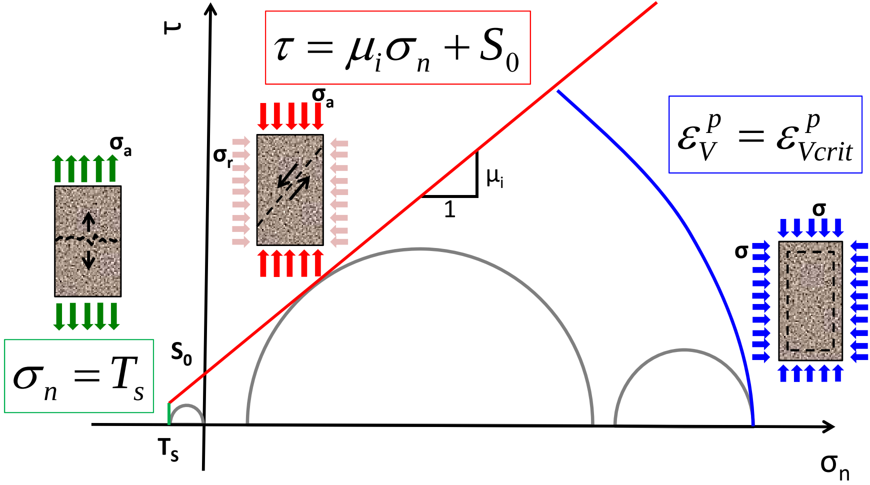
Germaine and Germaine, 2009



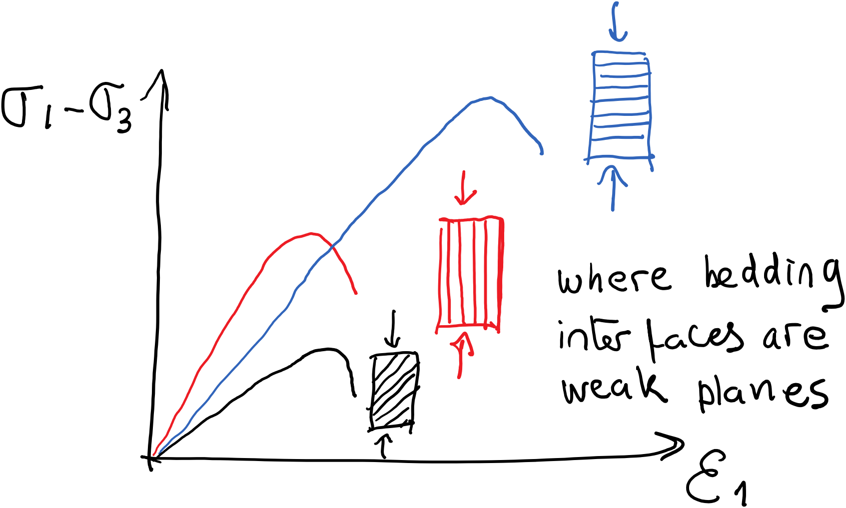


www.memin.de/results-tp-2.html

$$\tau = S_0 + \mu_i \sigma_n$$



τ
$\tau = \mu_i \sigma_n + S_0$
σ_a
σ_r
μ_i
1
$\varepsilon_V^p = \varepsilon_{Vcrit}^p$
σ
σ
σ_a
S_0
$\sigma_n = T_s$
T_s
σ_n

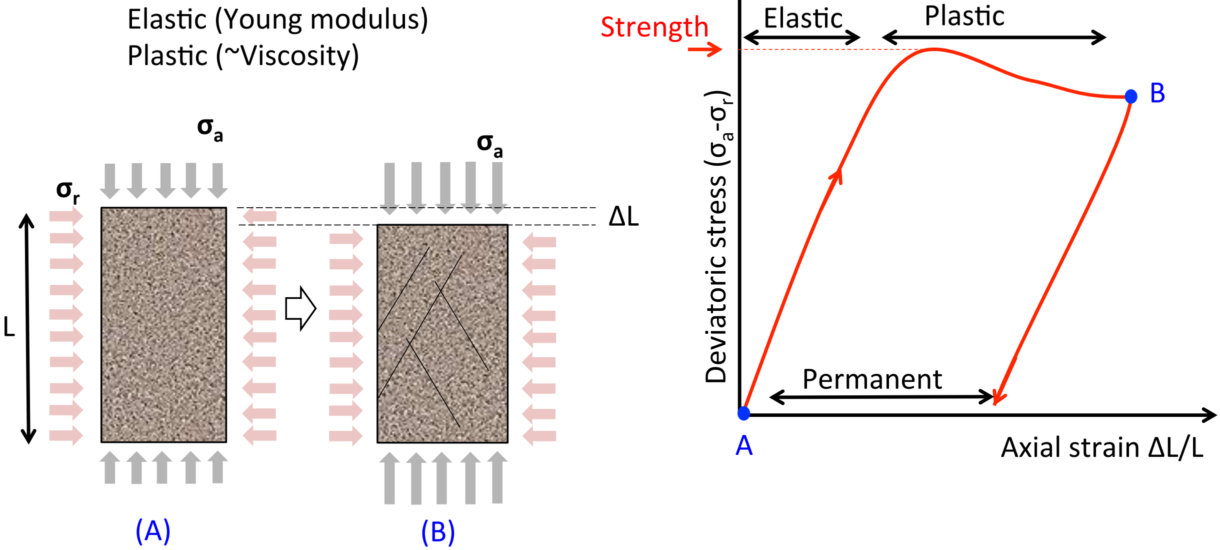

$\sigma_1 - \sigma_3$
ε_1
where bedding interfaces are weak planes

Rock deformation

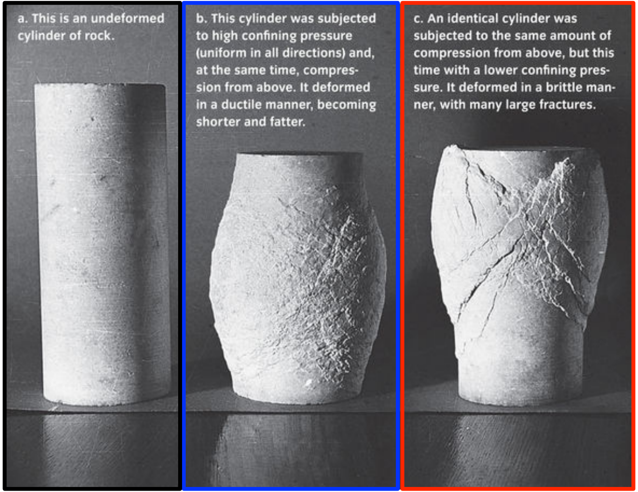
Relation strain V.S. stress

Elastic (Young modulus)

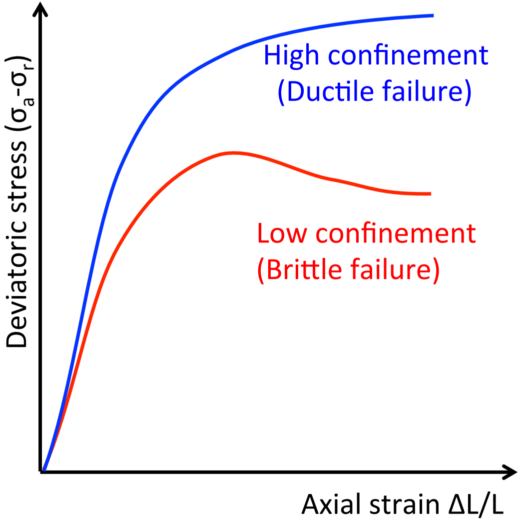
Plastic (~Viscosity)



E/ν



Visualizing Geology, WileyPlus



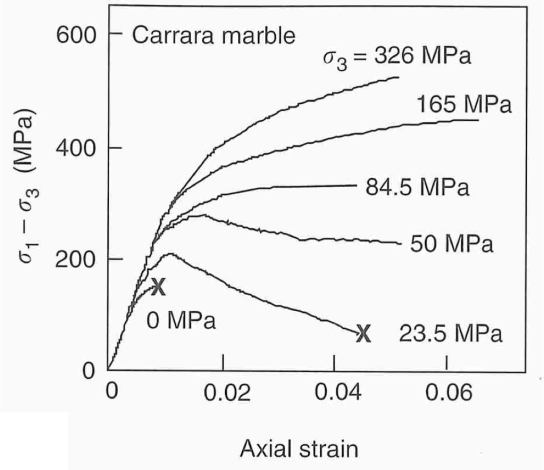
$$\sigma_d = (\sigma_1 - \sigma_3)$$

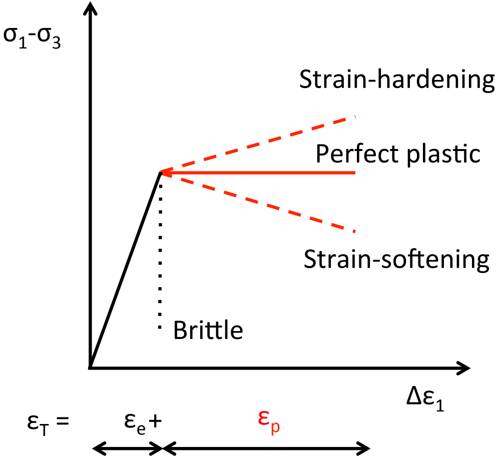
$\underline{\mathcal{E}}p$

$$\underline{\underline{C}}_p$$

$$\Delta\underline{\underline{\sigma}} = \underline{\underline{C}}_p \Delta\underline{\underline{\varepsilon}}_p$$

[Jaeger et al. 2007 – Fig. 4.5]

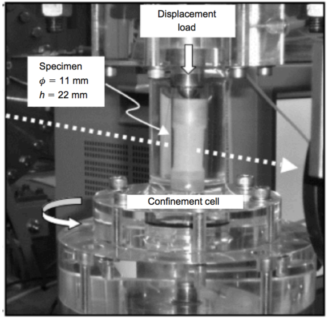


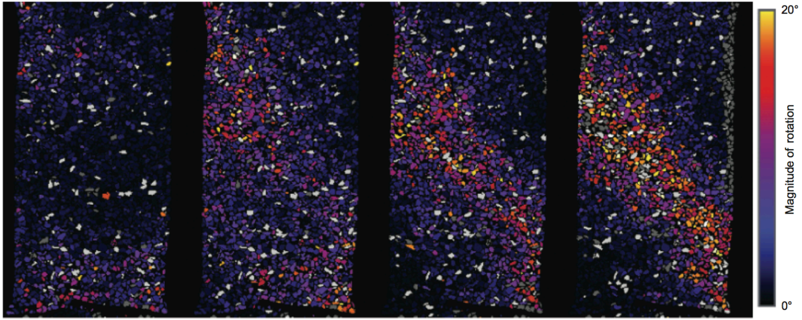


$$\boxed{\Delta \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \; \Delta \underline{\underline{\varepsilon}}}$$

$$\boxed{\Delta \underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C_p}}}} \; \Delta \underline{\underline{\varepsilon_p}}}$$

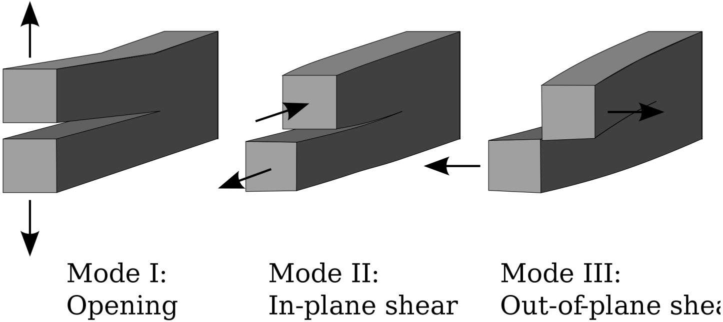
(a) Uncemented or poorly cemented rock → grain friction, dilation, grain crushing/rotation





Laboratoire 3SR - Navier

(b) Cemented rock → Propagation of microfractures, grain friction/crushing

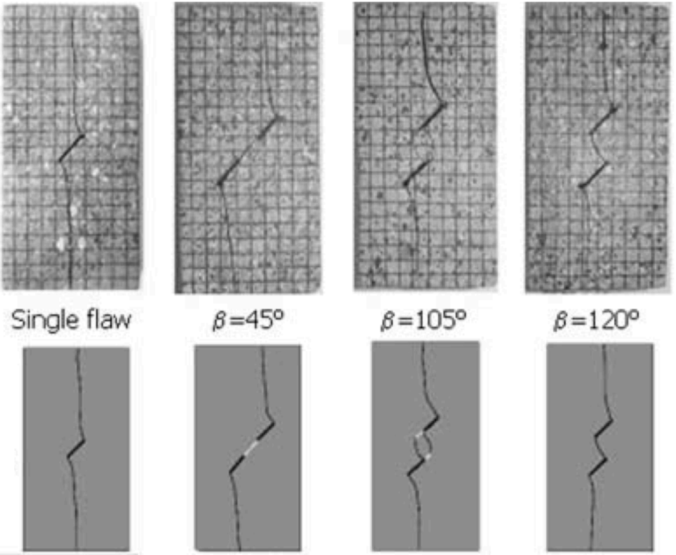


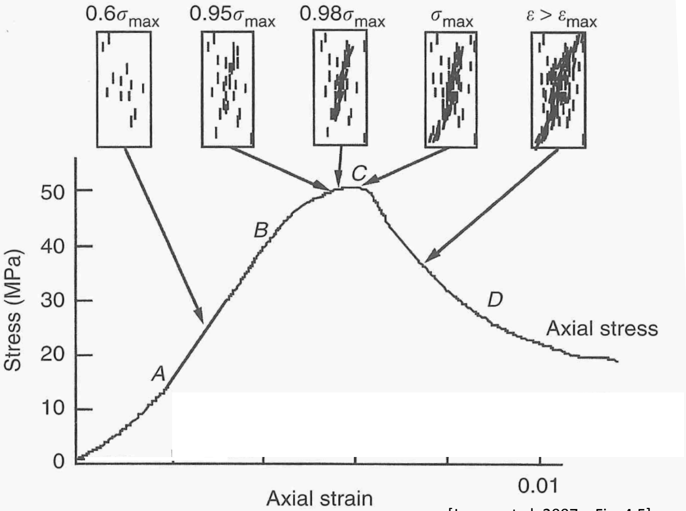
Stress intensification at the tip of fractures

Propagation starts at fracture tips

Napolitan Tuffo

[Hall et al. 2006 – Pure Appl. Geophys.]





[Jaeger et al. 2007 – Fig. 4.5]

$S_1 - S_3$

$$\varphi = f(q)$$

Sig_D

E_x

E_y

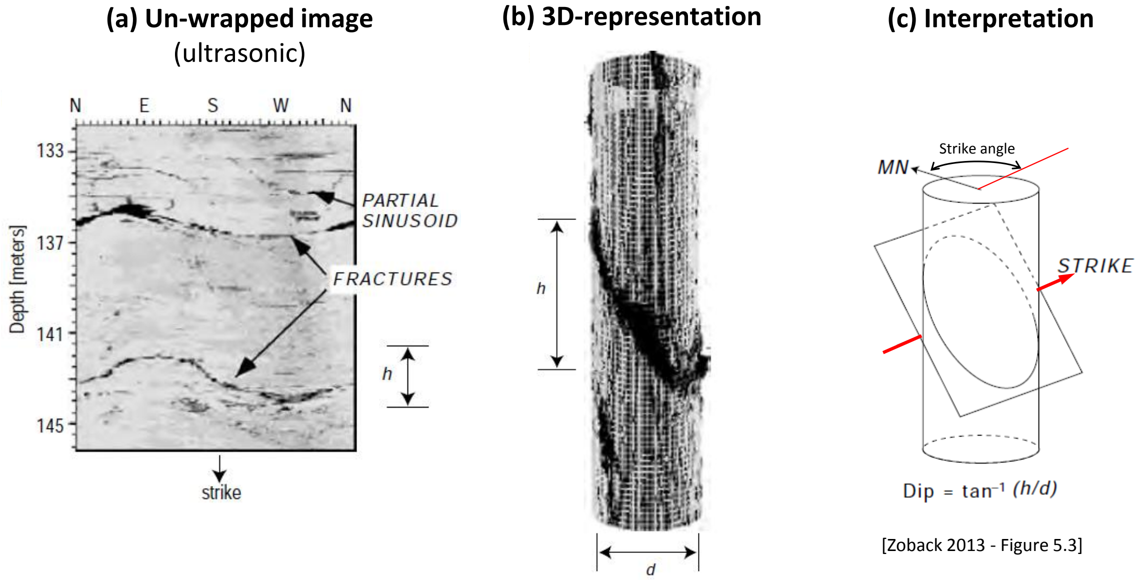
$$q = 5.3$$

$$S_1 = S_3 \to S_1 - S_3 = 0$$

$$S_o$$

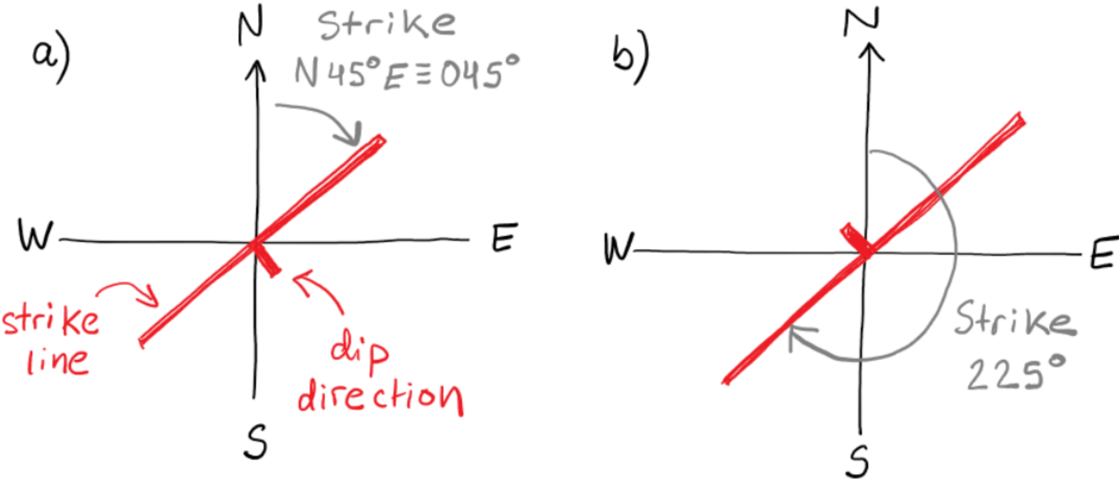
Outcrop of a normal faults on the footwall of the Moab Fault in front of the entry of Arches National Park, Utah (Photo: DNE. https://bit.ly/2UDwiEt)

Outcrop of a normal fault in Split Mountain gorge
http://geology.csupomona.edu/janourse/TectonicsFieldTrips.htm



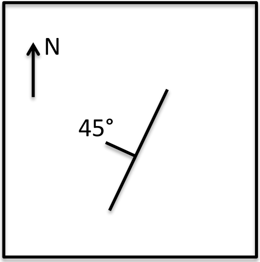
[Zoback 2013 - Figure 5.3]

dip

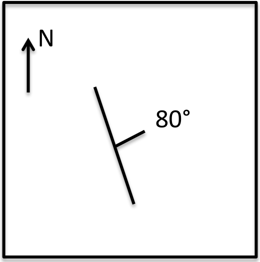

a)
N
Strike
N45°E ≡ 045°
W
E
S
strike line
dip direction
b)
N
W
E
S
Strike 225°

Geologic map

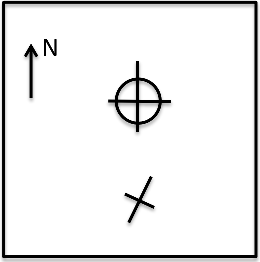
Figure from Prof. Prodanovic



a)
N30°E, 45°NW
or
030°, 45°NW



b)
N10°W, 80°NE
or
350°, 80°NE



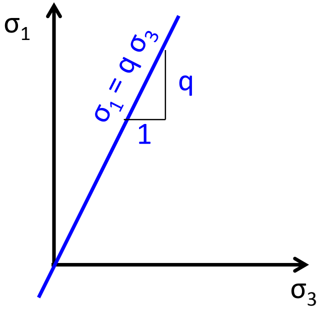
c)
Symbols for
horizontal plane
and vertical plane

$$\tau = \mu\, \sigma_n$$

$$\sigma_1 = q\,\sigma_3$$

$$q = \left(\sqrt{\mu^2 + 1} + \mu\right)^2 = \frac{1 + \sin\ \varphi}{1 - \sin\ \varphi}$$

$\sigma_1/\sigma_3 \sim$



σ_1
$\sigma_1 = q\,\sigma_3$
q
1
σ_3

$$\pi/4 + \phi/2$$

$\sim 30^{\circ}$

$\sim 60^\circ$

$45°$

$67.5°$

50°

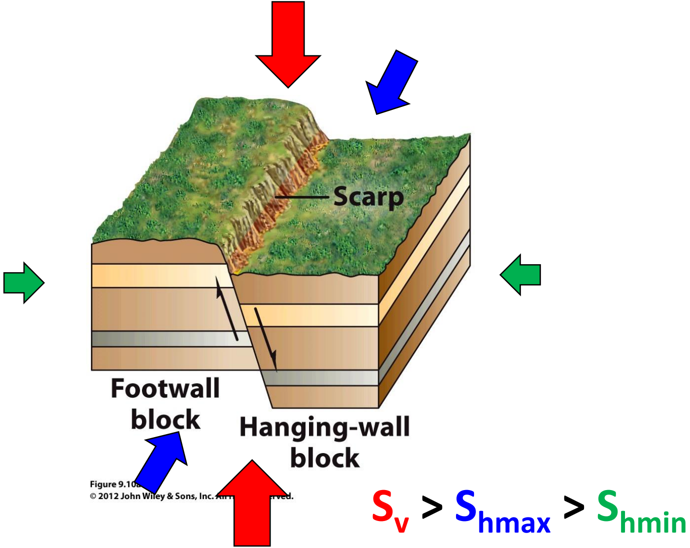
70°

$$\sigma_1/\sigma_3 > q$$

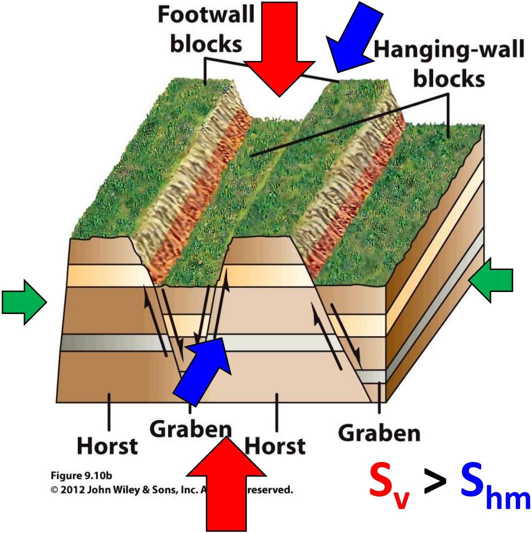
$\sigma_1/\sigma_3 < q$

$$\begin{cases} S_1 = S_v \\ S_2 = S_{Hmax} \\ S_3 = S_{hmin} \end{cases}$$

$$S_1 > S_2 > S_3$$


Scarp
Footwall block
Hanging-wall block
Figure 9.10a
© 2012 John Wiley & Sons, Inc.
$S_v > S_{hmax} > S_{hmin}$

$$S_1 \neq S_2 \neq S_3$$



$$S_v > S_{hmax} > S_{hmin}$$

$$\begin{cases} S_1 = S_{Hmax} \\ S_2 = S_{hmin} \\ S_3 = S_v \end{cases}$$

$$(\pi/4 + \varphi/2)$$

S_v

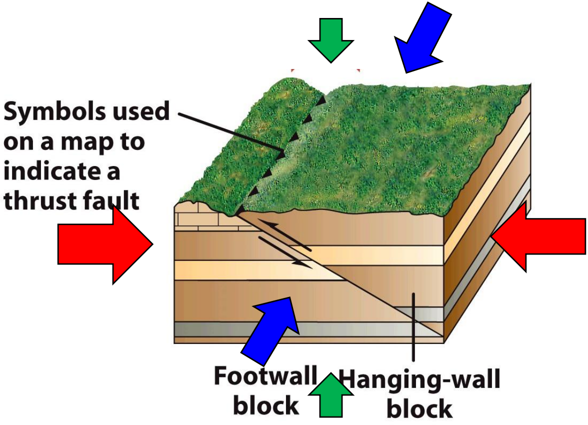


Figure 9.10d

$S_{Hmax} > S_{hmin} > S_v$

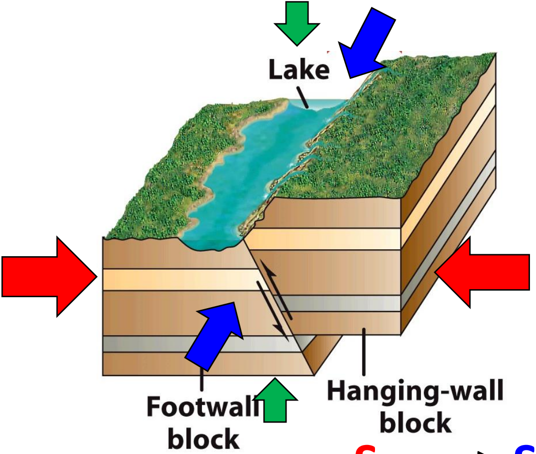


Figure 9.10c

$$S_{Hmax} > S_{hmin} > S_v$$

$$\begin{cases} S_1 = S_{Hmax} \\ S_2 = S_v \\ S_3 = S_{hmin} \end{cases}$$

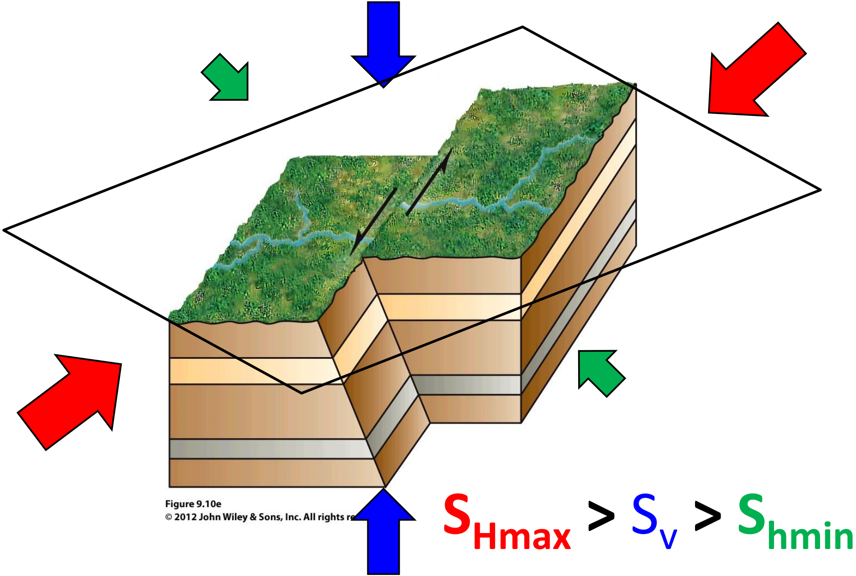
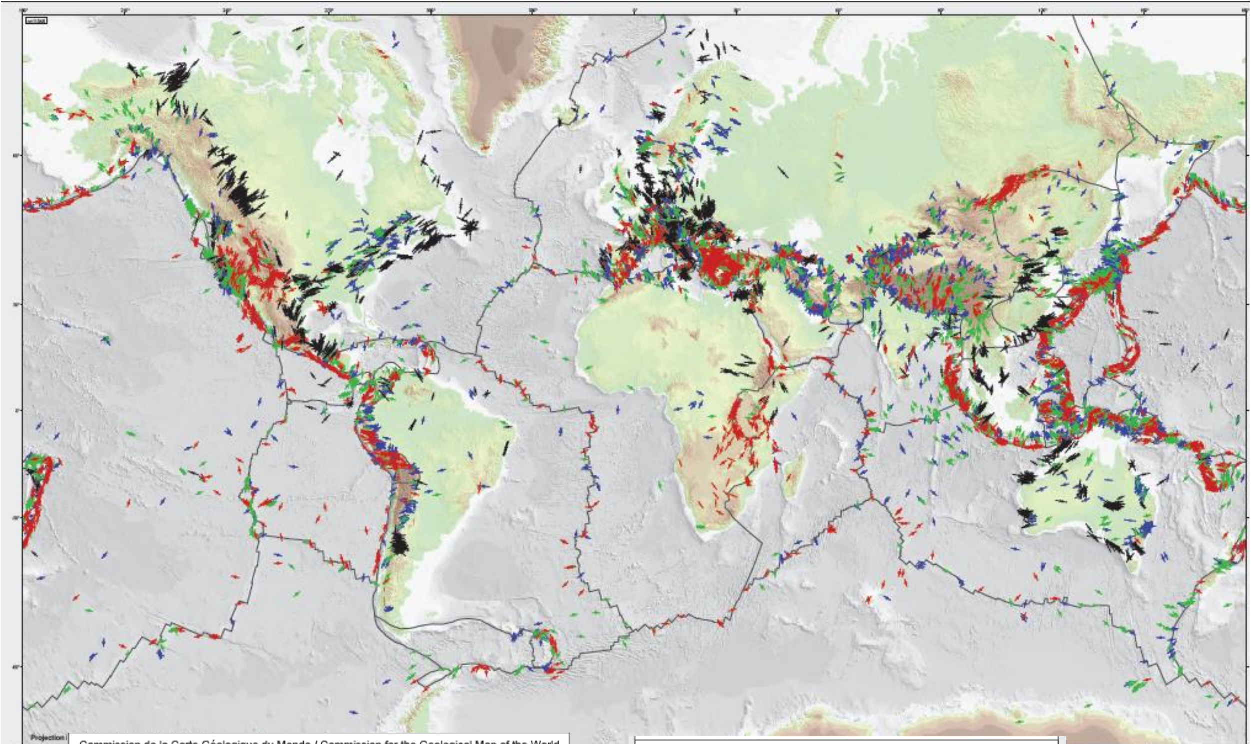


Figure 9.10e
© 2012 John Wiley & Sons, Inc. All rights re
$S_{Hmax} > S_V > S_{hmin}$



Commission de la Carte Géologique du Monde / Commission for the Geological Map of the World

WORLD STRESS MAP

2009 - 2nd edition, based on the WSM database release 2008

Helmholtz Centre Potsdam - GFZ German Research Centre for Geosciences

Authors

Oliver Heidbach, Mark Tingay, Andreas Barth, John Reinecker, Daniel Kurfeß, and Birgit Müller

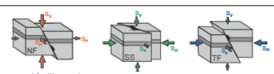
The stress maps display the maximum horizontal compressional stress S_H

Method
- focal mechanism
- breakouts
- drill. induced frac.
- overcoring
- hydro. fractures
- geol. indicators

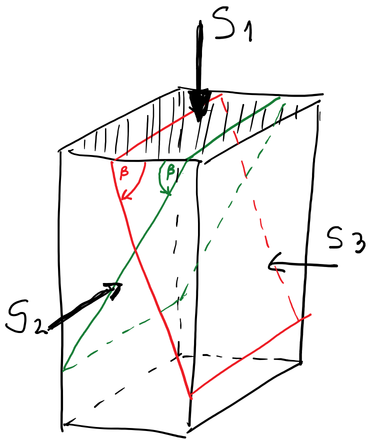
Data depth range
0-40 km

Quality
- A — S_H is within ±15°
- B — S_H is within ±20°
- C — S_H is within ±25°

Stress Regime
- Normal faulting
- Strike-slip faulting
- Thrust faulting
- Unknown regime

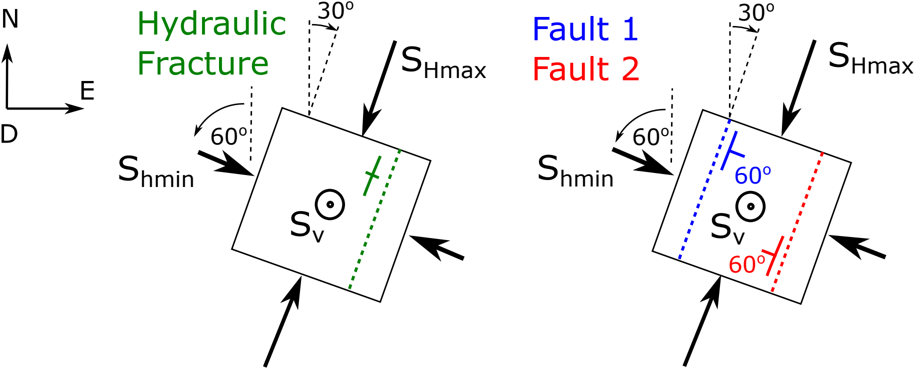


normal faulting regime $S_V > S_H > S_h$ — strike-slip regime $S_H > S_V > S_h$ — thrust faulting regime $S_H > S_h > S_V$



$$\beta = 45^\circ + \varphi/2$$

$$\beta = 45^\circ + \varphi/2$$



N
E
D
Hydraulic
Fracture
30°
60°
S_{Hmax}
S_{hmin}
S_v
Fault 1
Fault 2
30°
60°
S_{Hmax}
S_{hmin}
60°
S_v
60°

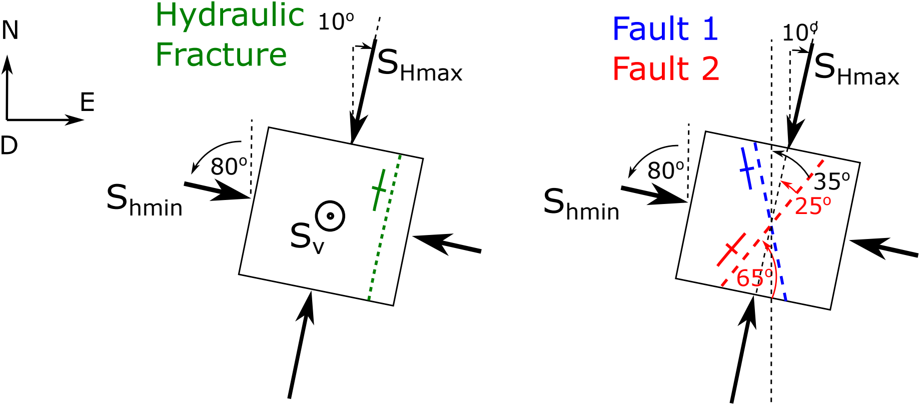
$$\phi_{HF} = 030^\circ$$

$$\delta_{HF} = 90^\circ$$

$$\beta = 45^\circ + \varphi/2 = 45^\circ + 30^\circ/2 = 60^\circ$$

$$\phi_{F1} = \phi_{F2} = 030^\circ$$

$$\delta_{F1} = 60^\circ$$



N
E
D
Hydraulic
Fracture
10°
S_{Hmax}
80°
S_{hmin}
S_v
Fault 1
Fault 2
10°
S_{Hmax}
80°
S_{hmin}
35°
25°
65°

$$\phi_{HF} = 010^\circ$$

$$\beta = 45^\circ + \varphi/2 = 45^\circ + 40^\circ/2 = 65^\circ$$

$$\phi_{F1} = 345^\circ$$

$$\phi_{F2} = 035^\circ$$

$$\delta_{F1} = \delta_{F2} = 90^\circ$$

1

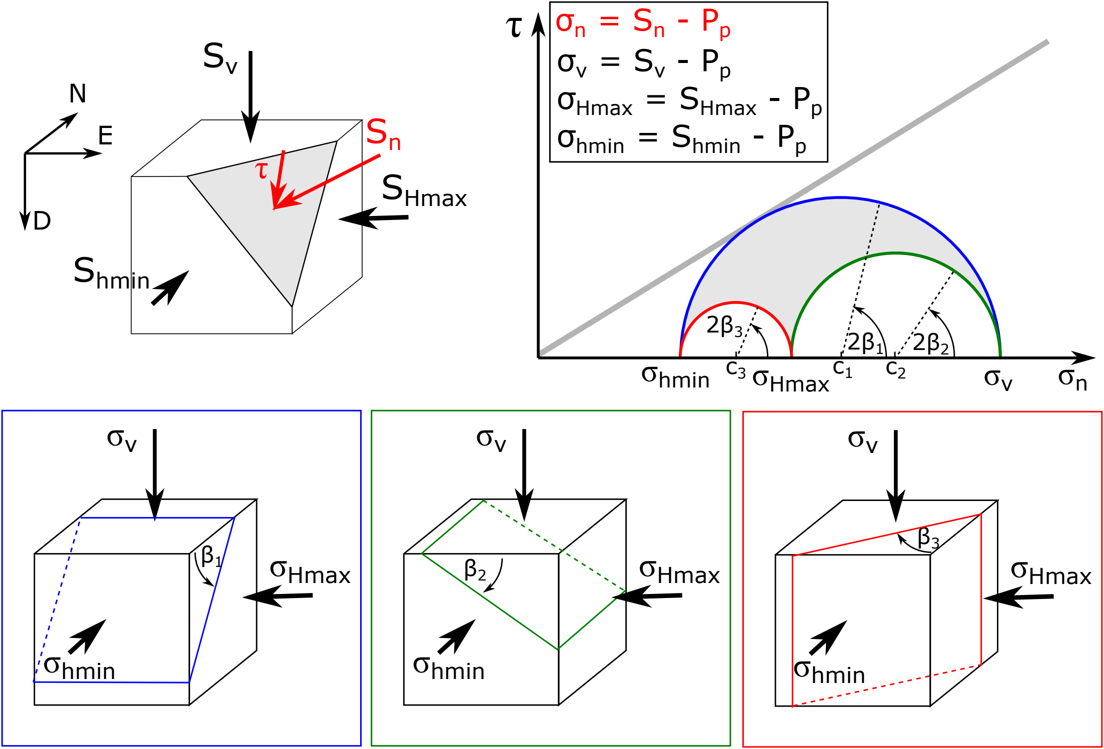
β_1

σ_v

β_2

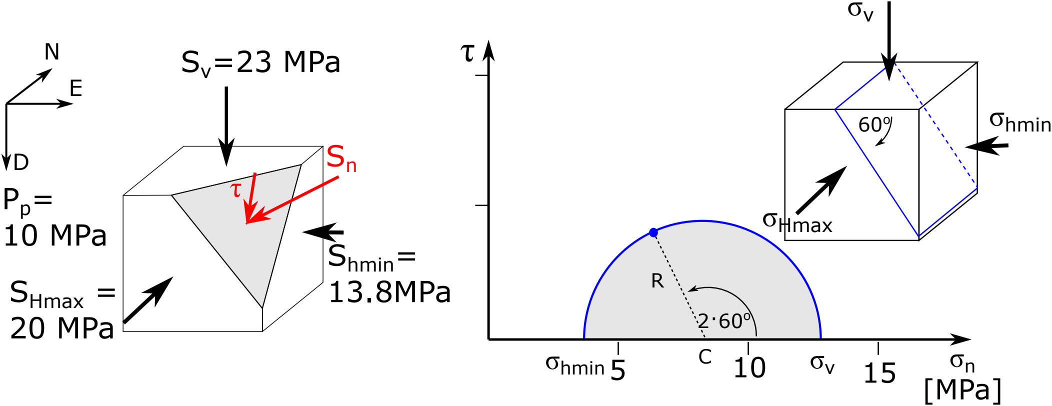
3

β_3



S_v
N
E
D
S_n
τ
S_{Hmax}
S_{hmin}
τ
$\sigma_n = S_n - P_p$
$\sigma_v = S_v - P_p$
$\sigma_{Hmax} = S_{Hmax} - P_p$
$\sigma_{hmin} = S_{hmin} - P_p$
$2\beta_3$
$2\beta_1$
$2\beta_2$
σ_{hmin}
c_3
σ_{Hmax}
c_1
c_2
σ_v
σ_n
σ_v
β_1
σ_{Hmax}
σ_{hmin}
σ_v
β_2
σ_{Hmax}
σ_{hmin}
σ_v
β_3
σ_{Hmax}
σ_{hmin}

$$\varphi = 30^\circ$$



N
E
D
S_v=23 MPa
S_n
τ
P_p=
10 MPa
S_{Hmax} =
20 MPa
S_{hmin}=
13.8MPa
τ
σ_v
σ_{hmin}
60º
σ_{Hmax}
R
2·60º
σ_{hmin} 5
C
10
σ_v
15
σ_n
[MPa]

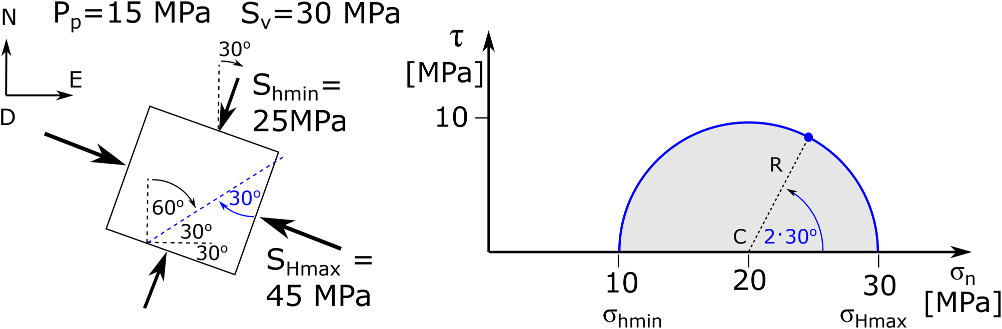
$\sigma_v =$

$\sigma_{Hmax} =$

$$\sigma_{hmin} =$$

$$\sigma_n = \left(\frac{13\ \text{MPa} + 3.8\ \text{MPa}}{2}\right) + \left(\frac{13\ \text{MPa} - 3.8\ \text{MPa}}{2}\right)\cos(2 \cdot 60^\circ) = 6.1\ \text{MPa}$$

$$\tau = \left(\frac{13\ \text{MPa} - 3.8\ \text{MPa}}{2} \right) \sin(2 \cdot 60^\circ) = 4.0\ \text{MPa}\ \blacksquare$$



N
E
D
P_p=15 MPa
S_v=30 MPa
30º
S_{hmin}=
25MPa
60º
30º
30º
30º
S_{Hmax} =
45 MPa
τ
[MPa]
10
R
C
2·30º
10
σ_{hmin}
20
30
σ_{Hmax}
σ_n
[MPa]

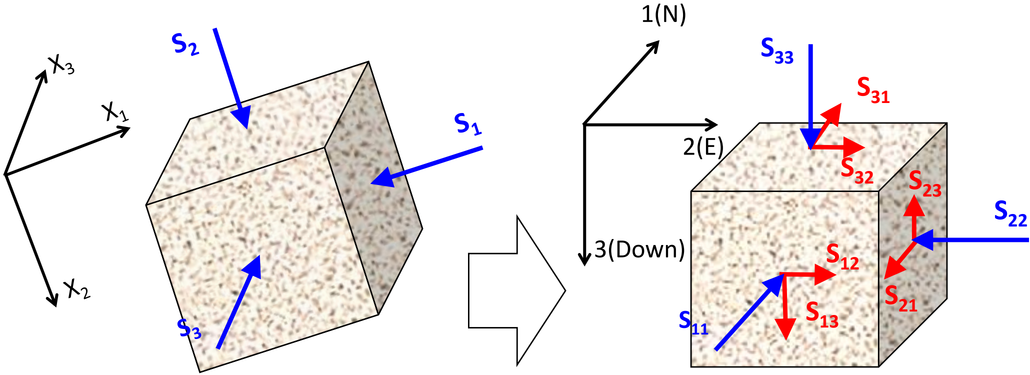
$$\sigma_n = \left(\frac{30\ \text{MPa} + 10\ \text{MPa}}{2}\right) + \left(\frac{30\ \text{MPa} - 10\ \text{MPa}}{2}\right)\cos(2 \cdot 30^\circ) = 25\ \text{MPa}$$

$$\tau = \left(\frac{30 \text{ MPa} - 10 \text{ MPa}}{2} \right) \sin(2 \cdot 30^\circ) = 8.7 \text{ MPa} \quad \blacksquare$$

$$(strike, dip)$$

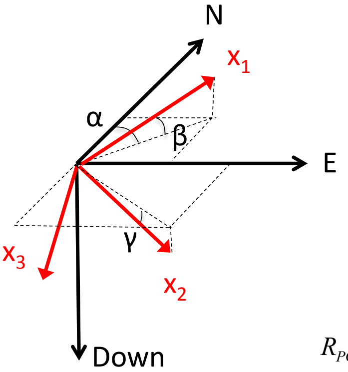
$\underline{\underline{S}}_P$

$$\underline{\underline{S}}_G$$



$$\underline{\underline{S}}_P = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix} \qquad \underline{\underline{S}}_G = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$R_{PG}$$



$$\underline{\underline{S}}' = A\underline{\underline{S}}A^T \qquad A = \begin{bmatrix} \underline{e}'_1 \cdot \underline{e}_1 & \underline{e}'_1 \cdot \underline{e}_2 & \underline{e}'_1 \cdot \underline{e}_3 \\ \underline{e}'_2 \cdot \underline{e}_1 & \underline{e}'_2 \cdot \underline{e}_2 & \underline{e}'_2 \cdot \underline{e}_3 \\ \underline{e}'_3 \cdot \underline{e}_1 & \underline{e}'_3 \cdot \underline{e}_2 & \underline{e}'_3 \cdot \underline{e}_3 \end{bmatrix}$$

where $\underline{\underline{A}}$ is the transformation matrix from the old base $\underline{e}_i$ to base $\underline{e}'_i$ and the components are the projection of the elements of the new base on the old base.

- Old system: N-E-D (Right-handed) Geographical system
- New system: 1-2-3 (Right-handed) Principal stress system

$$R_{PG} = \begin{bmatrix} \cos a \cos b & \sin a \cos b & -\sin b \\ \cos a \sin b \sin g - \sin a \cos g & \sin a \sin b \sin g + \cos a \cos g & \cos b \sin g \\ \cos a \sin b \cos g + \sin a \sin g & \sin a \sin b \cos g - \cos a \sin g & \cos b \cos g \end{bmatrix}$$

$$\mathrm{R}_{PG} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & -\sin\beta \\ \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \cos\beta\sin\gamma \\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

90°

$0°$

$$\underline{\underline{S}}_P = R_{PG}\underline{\underline{S}}_G R_{PG}^T$$

$$\underline{\underline{S}}_G = R_{PG}^T \underline{\underline{S}}_P R_{PG}$$

$$S_1 = 30$$

$$S_2 = 25$$

$$S_3 = 20$$

$$\underline{\underline{\mathrm{S}}}_P = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$\alpha = 0^\circ$

$$\beta = 90^\circ$$

$$\gamma = 0^{\circ}$$

$$\mathrm{R}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{\mathrm{S}}}_G = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 30 \end{bmatrix} \quad \blacksquare$$

$$\beta = 0^\circ$$

$$\gamma = 90^\circ$$

$$\mathrm{R}_{PG} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\underline{\underline{\mathrm{S}}}_G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 25 \end{bmatrix} \quad \blacksquare$$

$$\alpha = 90^\circ$$

$$\mathrm{R}_{PG} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{\mathrm{S}}}_G = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix} \quad \blacksquare$$

$$S_1 = 60$$

$$S_2 = 40$$

$$S_3 = 35$$

$$\underline{\underline{S}}_P = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

$$\alpha = 135^\circ$$

$$\mathrm{R}_{PG} = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$

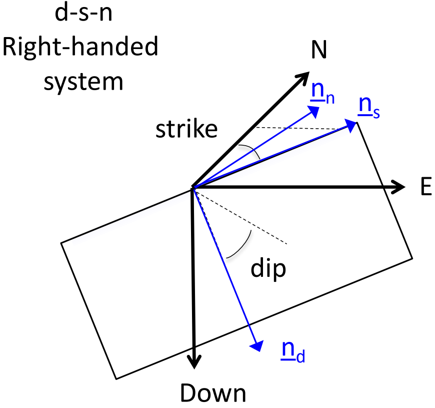
$$\underline{\underline{\mathrm{S}}}_G = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}^T \begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix} \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix} = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix} \blacksquare$$

n_d

n_s

n_n

$strike$



$$\underline{n}_n = \begin{bmatrix} -\sin(strike)\sin(dip) \\ \cos(strike)\sin(dip) \\ -\cos(dip) \end{bmatrix}$$

$$\underline{n}_s = \begin{bmatrix} \cos(strike) \\ \sin(strike) \\ 0 \end{bmatrix}$$

$$\underline{n}_d = \begin{bmatrix} -\sin(strike)\cos(dip) \\ \cos(strike)\cos(dip) \\ \sin(dip) \end{bmatrix}$$

$\underline{n}_d$

$$\underline{n}_s$$

$\underline{n}_n$

$$\underline{t} = \underline{\underline{S}}_G \underline{n}_n$$

S_n

$$S_n = \underline{t} \cdot \underline{n}_n$$

$$\sigma_n = S_n - P_p$$

$$\begin{cases} \tau_d = \underline{t} \cdot \underline{n}_d \\ \tau_s = \underline{t} \cdot \underline{n}_s \end{cases}$$

$$\tau = \sqrt{\tau_d^2 + \tau_s^2}$$

$$\sigma_n = \underline{t} \cdot \underline{n}_n - P_p$$

$$\tau^2 = ||\underline{t}||^2 - ||\underline{t} \cdot \underline{n}_n||^2$$

$rake$

$$\underline{\tau}_d + \underline{\tau}_s$$

$$rake = \arctan\left(\frac{\tau_d}{\tau_s}\right)$$

t

τ_d

τ_s

$$S_v = 23$$

$$S_{Hmax} = 15$$

$$S_{hmin} = 13.8$$

$$\underline{\underline{\mathrm{S}}}_P = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 13.8 \end{bmatrix}$$

$$\mathrm{R}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\underline{\mathrm{S}}}_G = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 13.8 & 0 \\ 0 & 0 & 23 \end{bmatrix}$$

$$\underline{\mathrm{n}}_n = \begin{bmatrix} 0 \\ 0.867 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [0, 11.95, -11.50]$$

$$S_n = 16.1$$

$$\tau_d = -3.98$$

$$\tau_s = 0$$

○ ■

$$S_v = 30$$

$$S_{Hmax} = 45$$

$$S_{hmin} = 25$$

$$\underline{\underline{\mathrm{S}}}_P = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix} \ \mathrm{MPa}$$

$$\alpha = 120^\circ$$

$$\mathrm{R}_{PG} = \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \\ 0.866 & 0.5 & 0 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix} \text{ MPa}$$

$$\underline{\mathrm{n}}_n = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$\underline{t} = [-30.31, 27.5, 0]$$

$$S_n = 40$$

$$\tau_d = 0$$

$$\tau_s = 8.66$$

$$S_v = 5000$$

$$S_{Hmax} = 4000$$

$$S_{hmin} = 3000$$

$$\underline{\underline{\mathrm{S}}}_P = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \text{ psi}$$

$$\underline{\underline{\mathrm{S}}}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix} \text{ psi}$$

$$\underline{\mathrm{n}}_n = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [-2450, 1840, -2500]$$

$$S_n = 3870$$

$$\tau_d = -649$$

$$\tau_s = -433$$

$$\underline{\mathrm{n}}_n = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [2450, -1840, -2500]$$

$$S_v = 1000$$

$$S_{Hmax} = 2400$$

$$S_{hmin} = 1200$$

$$P_p = 440$$

$$\underline{\underline{\mathrm{S}}}_P = \begin{bmatrix} 2400 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \text{ psi}$$

$$\alpha = 150^\circ$$

$$\mathrm{R}_{PG} = \begin{bmatrix} -0.866 & 0.5 & 0 \\ -0.5 & -0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \text{ psi}$$

$$\underline{\mathrm{n}}_n = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix}$$

$$\underline{t} = [-1465, -281, -342]$$

$$S_n = 1441$$

$$\sigma_n = 1001$$

$$\tau_d = 160$$

$$\tau_s = 488$$

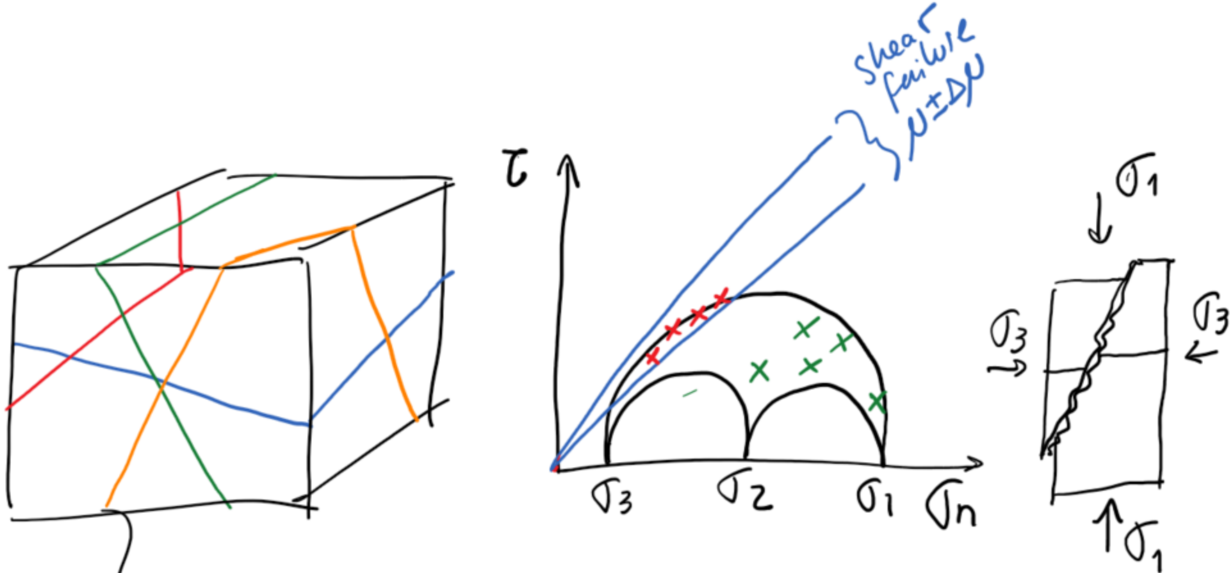
$$\tau = 514$$

$\tau/\sigma_n = 0.51$ ■

$\mu \sim$

$$k_{frac} = f(\sigma_n)$$

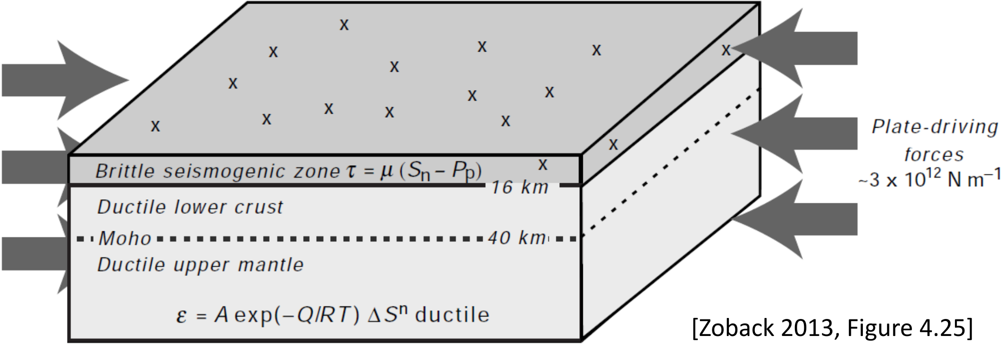
$$k_{frac} = f(\sigma_n, \tau)$$



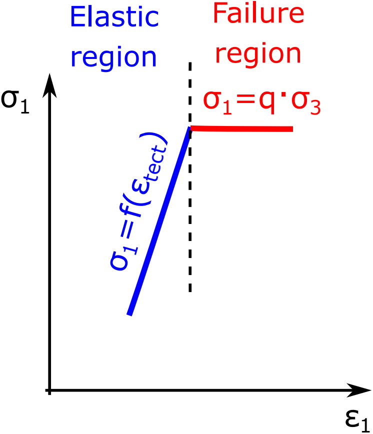
low matrix K

with fractures

$$\uparrow \frac{\tau}{\sigma_n} \Rightarrow \uparrow K_{frac}$$



[Zoback 2013, Figure 4.25]



Elastic region
Failure region
σ1
σ1=q·σ3
σ1=f(εtect)
ε1

$$q = [1 + \sin(\varphi)]/[1 - \sin(\varphi)]$$

σ_v/q

$q\ \sigma_{hmin}$

$q\ \sigma_v$

$$\lambda_p = 0.82$$

$$S_v = 1$$

$$= 2000$$

$$P_p = \lambda_p S_v = 0.82 \times 2000$$

$$= 1640$$

$$\sigma_v = S_v - P_p = 2000$$

-1640

$$= 360$$

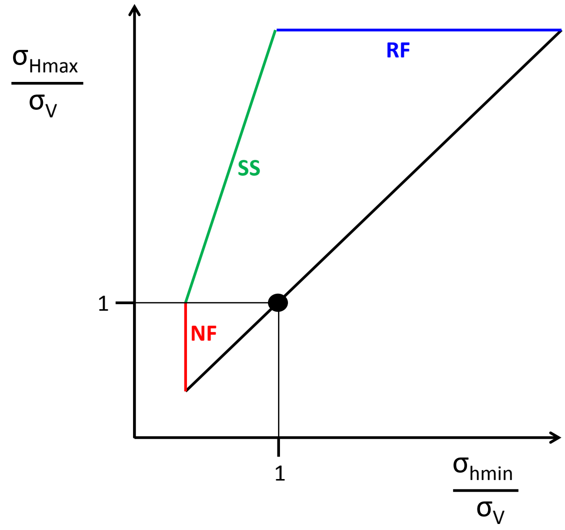
$$\sigma_{Hmax} = q\ \sigma_v = \frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} 360$$

$$= 1080$$

$$S_{Hmax} = \sigma_{Hmax} + P_p = 1080$$

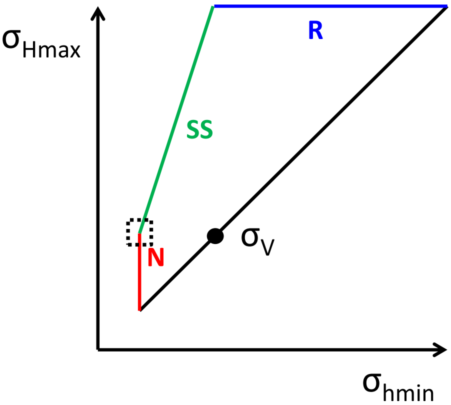
$+1640$

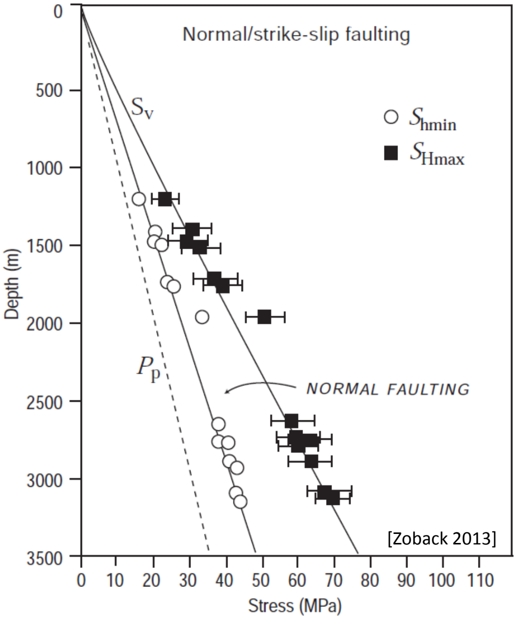
$$= 2720$$

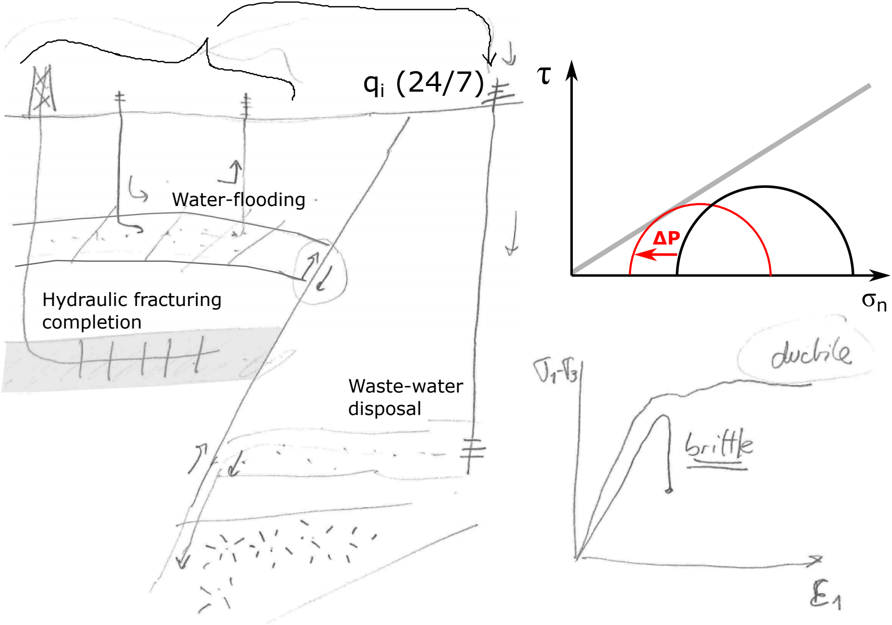


$\frac{\sigma_{Hmax}}{\sigma_V}$
RF
SS
1
NF
1
$\frac{\sigma_{hmin}}{\sigma_V}$

$$\sigma_{hmin} < \sigma_{Hmax} \approx \sigma_V$$



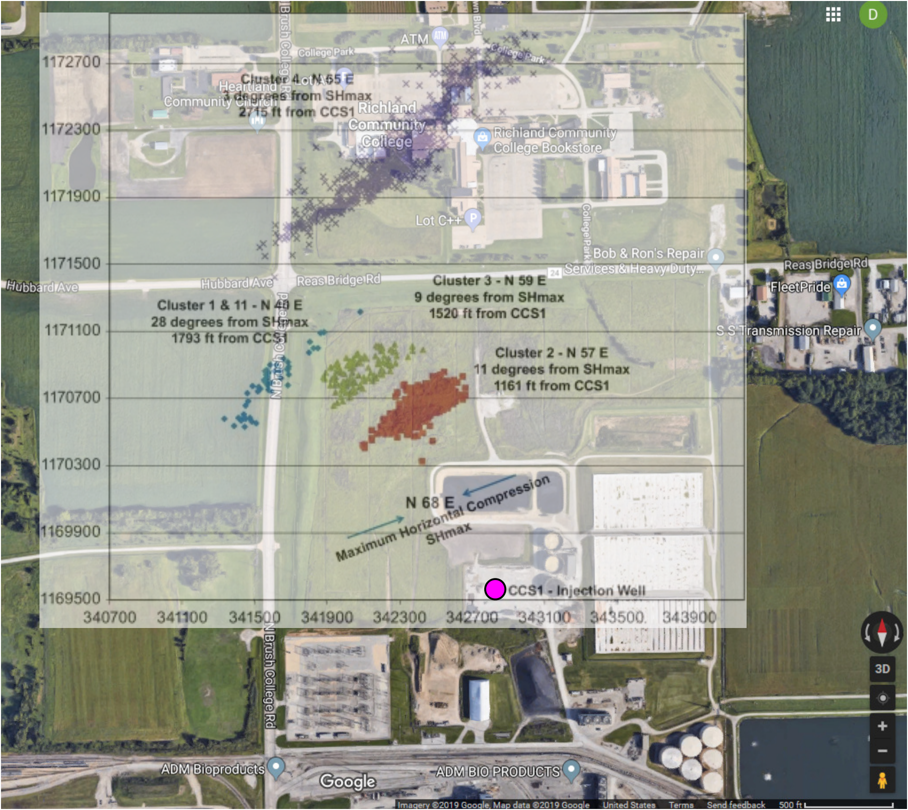




q_i (24/7)
Water-flooding
Hydraulic fracturing completion
Waste-water disposal
τ
ΔP
σ_n
$\sigma_1-\sigma_3$
ductile
brittle
ε_1

$$\begin{cases} \Delta\sigma_v = -\Delta P_p \\ \Delta\sigma_{hmin} \leq -\Delta P_p \end{cases}$$

$M < 2$



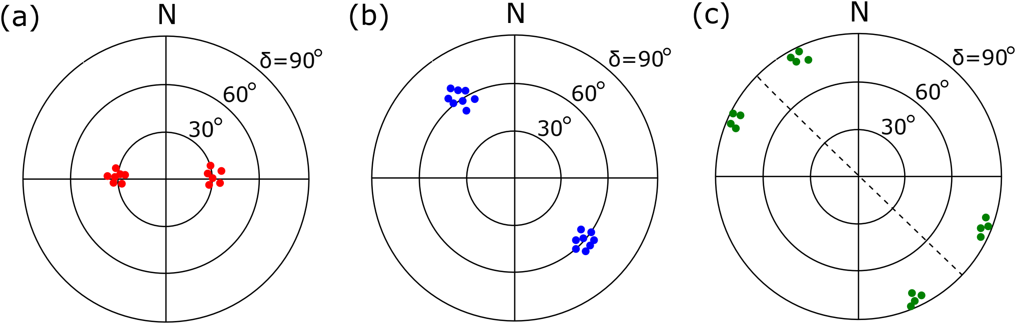
Cluster 4 - N 65 E
3 degrees from SHmax
2715 ft from CCS1
Cluster 3 - N 59 E
9 degrees from SHmax
1520 ft from CCS1
Cluster 1 & 11 - N 40 E
28 degrees from SHmax
1793 ft from CCS1
Cluster 2 - N 57 E
11 degrees from SHmax
1161 ft from CCS1
N 68 E
Maximum Horizontal Compression
SHmax
CCS1 - Injection Well
1172700
1172300
1171900
1171500
1171100
1170700
1170300
1169900
1169500
340700
341100
341500
341900
342300
342700
343100
343500
343900
ATM
College Park
Heartland Community Church
Richland Community College
Richland Community College Bookstore
Lot C++
Bob & Ron's Repair Services & Heavy Duty...
Hubbard Ave
Reas Bridge Rd
FleetPride
S S Transmission Repair
N Brush College Rd
ADM Bioproducts
ADM BIO PRODUCTS
Google
Imagery ©2019 Google, Map data ©2019 Google
United States
Terms
Send feedback
500 ft

$$S_v < S_{hmin} < S_{Hmax}$$

$\mu =$

$$S_{hmin} < S_v < S_{Hmax}$$

$\mu = 0.5$


(a)
N
δ=90°
60°
30°
(b)
N
δ=90°
60°
30°
(c)
N
δ=90°
60°
30°

$$S_{Hmax} = 60$$

$$S_v = 45$$

$$P_p = 20$$

$$\mu = 0.6$$

$$(\underline{\underline{S}}_P, \alpha, \beta, \gamma)$$

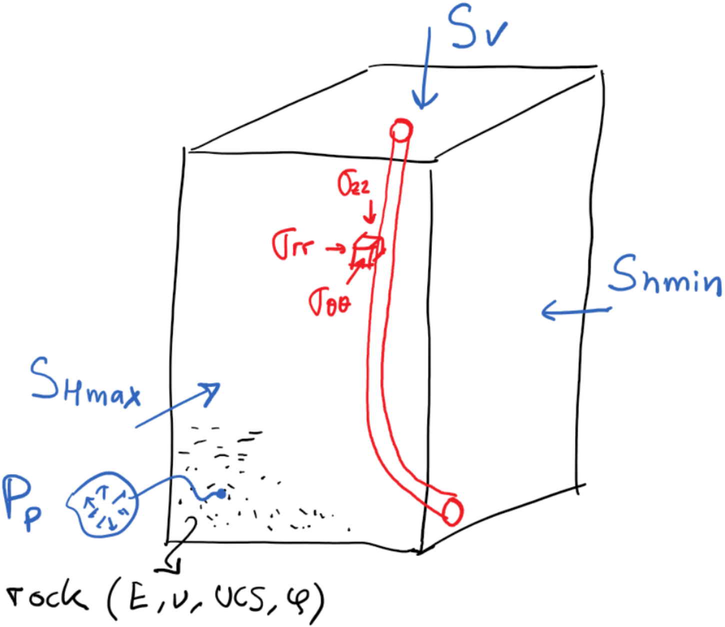
$$r = a + (b - a) * rand(N, 1)$$

r

$$[a, b]$$

$$z = 5$$

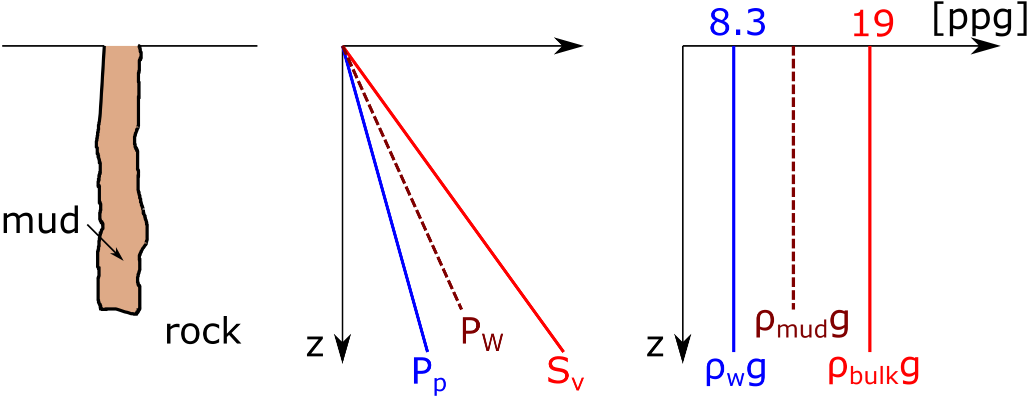
$$\lambda_p =$$



Sv
σzz
σrr
σθθ
Shmin
SHmax
Pp
rock (E, ν, UCS, φ)

$$P_W = \rho_{mud}\, g\, z$$

$$dP_W/dz = \rho_{mud} g$$



mud
rock
z
P_p
P_W
S_v
8.3
19
[ppg]
z
$\rho_{mud}g$
$\rho_w g$
$\rho_{bulk}g$

$$P_W > P_p$$

$$P_W - P_p$$

$$P_W < P_p$$

σ_{zz}

σ_{rz}

$\sigma_{\theta z}$

$$\sigma_{\theta\theta}/\sigma_{\infty} = 2$$

$$\sigma_{\theta\theta}/\sigma_{rr} = \infty$$

$$\sigma_{rr} = 0$$

r^2

$$\sigma_{rr} = +P_W$$

$$\sigma_{\theta\theta} = -P_W$$

$$\theta = 0$$

$$\sigma_{\theta\theta} = 3\Delta\sigma$$

$\theta = \pi/2$

$3\pi/2$

$$\sigma_{\theta\theta} = -\Delta\sigma$$

π

$$\sigma_{\theta\theta}/\sigma_{\infty} = 3$$

$$\sin(\theta)$$

$$\cos(\theta)$$

$$(P_W - P_p)$$

a

$$
\left\{
\begin{array}{lcl}
\sigma_{rr} & = & (P_W - P_p)\left(\frac{a^2}{r^2}\right) + \frac{\sigma_{Hmax}+\sigma_{hmin}}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma_{Hmax}-\sigma_{hmin}}{2}\left(1 - 4\frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right)\cos(2\theta) \\
\sigma_{\theta\theta} & = & -(P_W - P_p)\left(\frac{a^2}{r^2}\right) + \frac{\sigma_{Hmax}+\sigma_{hmin}}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma_{Hmax}-\sigma_{hmin}}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos(2\theta) \\
\sigma_{r\theta} & = & \frac{\sigma_{Hmax}-\sigma_{hmin}}{2}\left(1 + 2\frac{a^2}{r^2} - 3\frac{a^4}{r^4}\right)\sin(2\theta) \\
\sigma_{zz} & = & \sigma_v - 2\nu\left(\sigma_{Hmax} - \sigma_{hmin}\right)\left(\frac{a^2}{r^2}\right)\cos(2\theta)
\end{array}
\right.
$$

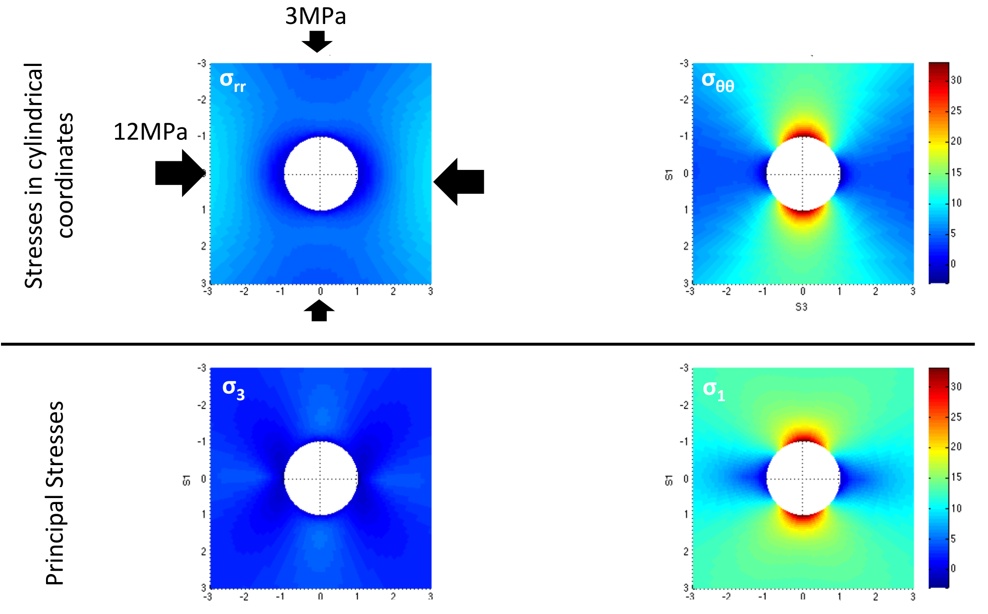
$$r = a$$

$$S_{Hmax} = 22$$

$$S_{hmin} = 13$$

$$P_W = P_p = 10$$

σ_1



Stresses in cylindrical coordinates
3MPa
12MPa
σrr
σθθ
S1
S3
Principal Stresses
σ3
σ1
S1
-3 -2 -1 0 1 2 3
0 5 10 15 20 25 30

$$\sigma_{rr}(r = a) = P_W - P_p$$

$$\sigma_{\theta\theta}(r=a) = -(P_W - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(2\theta)$$

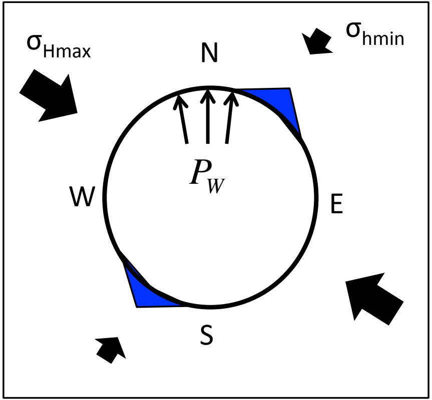
$$\begin{cases} \sigma_{\theta\theta}(r=a,\theta=0) & = & -(P_W - P_p) - \sigma_{Hmax} + 3\,\sigma_{hmin} \\ \sigma_{\theta\theta}(r=a,\theta=\pi/2) & = & -(P_W - P_p) + 3\,\sigma_{Hmax} - \sigma_{hmin} \end{cases}$$

$$\sigma_{r\theta} = 0$$

$$\sigma_{zz}(r = a) = \sigma_v - 2\nu(\sigma_{Hmax} - \sigma_{hmin}) \cos(2\theta)$$

σ_1/σ_3

$$\begin{cases} \sigma_1 = \sigma_{\theta\theta} & = & -(P_W - P_p) + 3\,\sigma_{Hmax} - \sigma_{hmin} \\ \sigma_3 = \sigma_{rr} & = & (P_W - P_p) \end{cases}$$



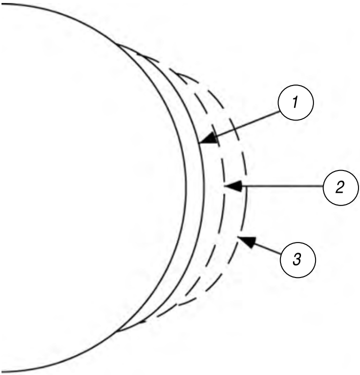
$$P_{Wshear} = \underbrace{P_p}_{\text{Pore pressure in the formation}} + \frac{\overbrace{3\sigma_{H\max} - \sigma_{h\min}}^{\text{Stress anisotropy}} \underbrace{- UCS}_{\text{Shear strength}}}{1+q}$$

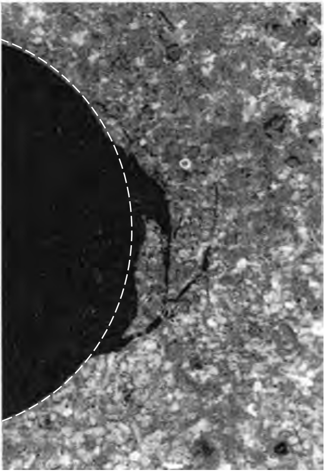
$P_W \leq P_{Wshear}$ leads to breakouts

$P_{W\,shear}$

$$[-(P_W - P_p) + 3\,\sigma_{Hmax} - \sigma_{hmin}] = UCS + q\,[P_W - P_p]$$

$$P_{Wshear} = P_p + \frac{3\sigma_{Hmax} - \sigma_{hmin} - UCS}{1+q}$$


1
2
3



$UCS =$

$\mu_i =$

T_s

$$P_p = 7000$$

$\times 0.44$

$$= 3080$$

$$\sigma_{Hmax} = 6300$$

$$-3080$$

$$= 3220$$

$$\sigma_{hmin} = 4300$$

$$= 1220$$

$$\arctan(0.6) = 30.96^\circ$$

$$q = \frac{1 + \sin 30.96^\circ}{1 - \sin 30.96^\circ} = 3.12$$

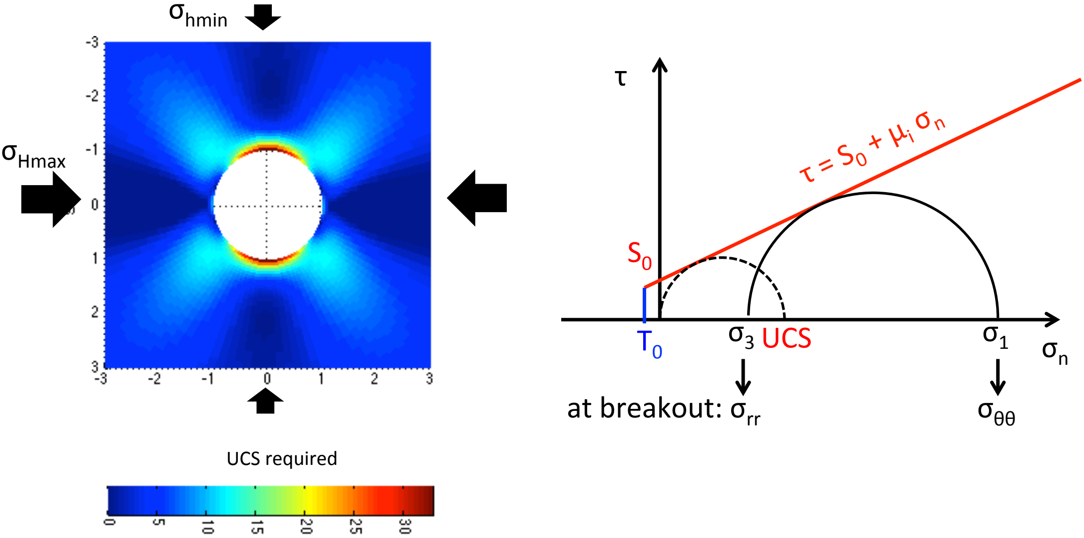
$$P_{W\,shear} = 3080$$

$$+\frac{3 \times 3220 \text{ psi} - 1220 \text{ psi} - 3500 \text{ psi}}{1 + 3.12} = 4279 \text{ psi}$$

$$\frac{4279 \text{ psi}}{7000 \text{ ft}} \times \frac{8.3 \text{ ppg}}{0.44 \text{ psi/ft}} = 11.57 \text{ ppg} \quad \blacksquare$$

$$UCS \sim 25$$

$$\sim 90^\circ$$



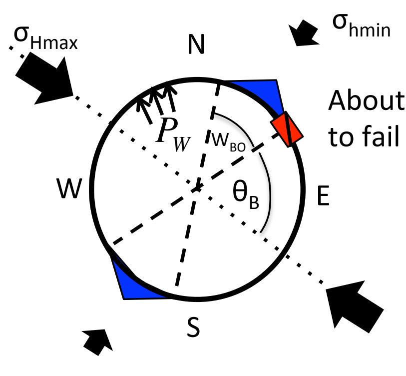
σhmin
σHmax
UCS required
0
5
10
15
20
25
30
τ
τ = S0 + μi σn
S0
T0
σ3
UCS
σ1
σn
at breakout: σrr
σθθ

$(w_{BO}/2)$

$\pi/2$

$$\theta_B = \pi/2 - w_{BO}/2$$

$$\begin{cases} \sigma_{\theta\theta} & = & -(P_W - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(2\theta_B) \\ \sigma_{rr} & = & +(P_W - P_p) \end{cases}$$



σ_{Hmax}
σ_{hmin}
N
About to fail
P_W
W_{BO}
W
θ_B
E
S

$$\sigma_1 = UCS + q\,\sigma_3$$

$$[-(P_W - P_p) + (\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(2\theta_B)] = UCS + q(P_W - P_p)$$

$$2\theta_B = \arccos\left[\frac{\sigma_{Hmax} + \sigma_{hmin} - UCS - (1+q)(P_W - P_p)}{2(\sigma_{Hmax} - \sigma_{hmin})}\right]$$

$$w_{BO} = \pi - 2\theta_B$$

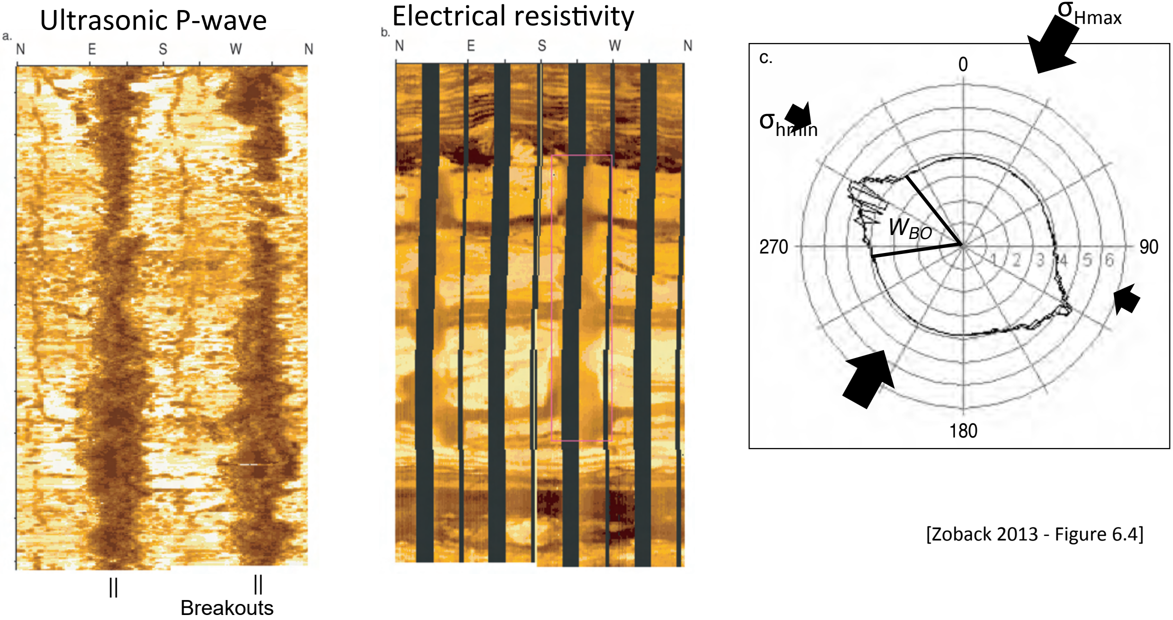
$$w_{BO} \gtrsim 60^\circ$$

$$P_{WBO} = P_p + \frac{(\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(\pi - w_{BO}) - UCS}{1 + q}$$

10

$$\times \frac{0.44 \text{ psi/ft}}{8.3 \text{ ppg}} \times 7000 \text{ ft} = 3710 \text{ psi}$$

$$w_{BO} = 180^\circ - \arccos\left[\frac{3220\ \text{psi} + 1220\ \text{psi} - 3500\ \text{psi} - (1 + 3.12)(3710\ \text{psi} - 3080\ \text{psi})}{2\,(3220\ \text{psi} - 1220\ \text{psi})}\right] = 66^\circ \quad \blacksquare$$



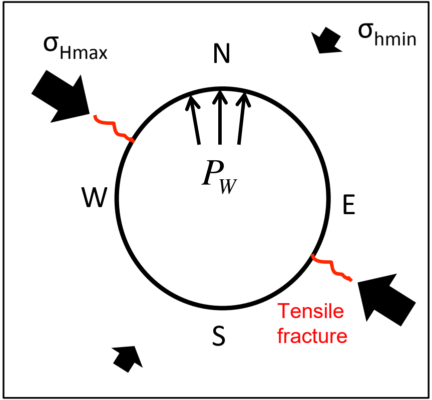
[Zoback 2013 - Figure 6.4]

$$S_{Hmax} = P_p + \frac{UCS + (1+q)(P_W - P_p) - \sigma_{hmin}\left[1 + 2\cos(\pi - w_{BO})\right]}{1 - 2\cos(\pi - w_{BO})}$$

$(r = a)$

$$\sigma_{\theta\theta} = -(P_W - P_p) - \sigma_{Hmax} + 3\sigma_{hmin} + \sigma^{\Delta T}$$

$\sigma^{\Delta T}$



$$\boxed{P_b = \underbrace{P_p}_{\text{Pore pressure in the formation}} + \underbrace{3\sigma_{hmin} - \sigma_{H\max}}_{\text{Stress anisotropy}} + \underbrace{T_s}_{\text{Tensile strength}} + \underbrace{\sigma^{\Delta T}}_{\text{Cooling stress}}}$$

$$P_W = P_b$$

$$-T_s = -(P_b - P_p) - \sigma_{Hmax} + 3\sigma_{hmin} + \sigma^{\Delta T}$$

$$P_b = P_p - \sigma_{Hmax} + 3\sigma_{hmin} + T_s + \sigma^{\Delta T}$$

b

P_b

$P_b > S_3$

$$P_b < S_3$$

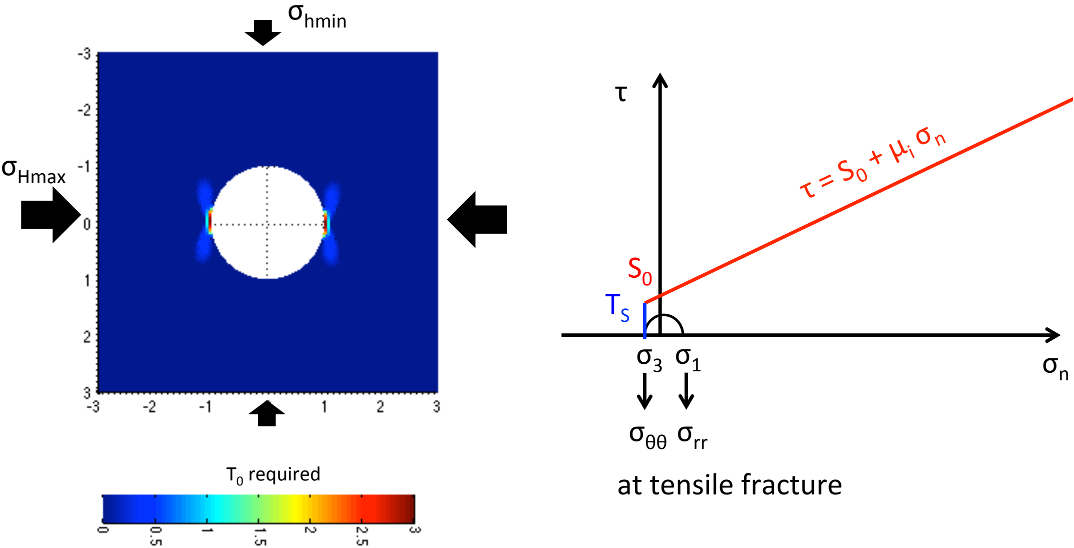
$$\sigma^{\Delta T} = [\alpha_T E/(1-\nu)]\Delta T$$

α_T

$$\sigma^{\Delta T} < 0$$

$$\Delta T < 0$$

$$T_{mud} < T_{rock}$$



σ_{hmin}
σ_{Hmax}
-3
-2
-1
0
1
2
3
-3
-2
-1
1
2
3
T_0 required
0
0.5
1
1.5
2
2.5
3
τ
$\tau = S_0 + \mu_i \sigma_n$
S_0
T_S
σ_3
σ_1
σ_n
$\sigma_{\theta\theta}$
σ_{rr}
at tensile fracture

$$P_b = 3080$$

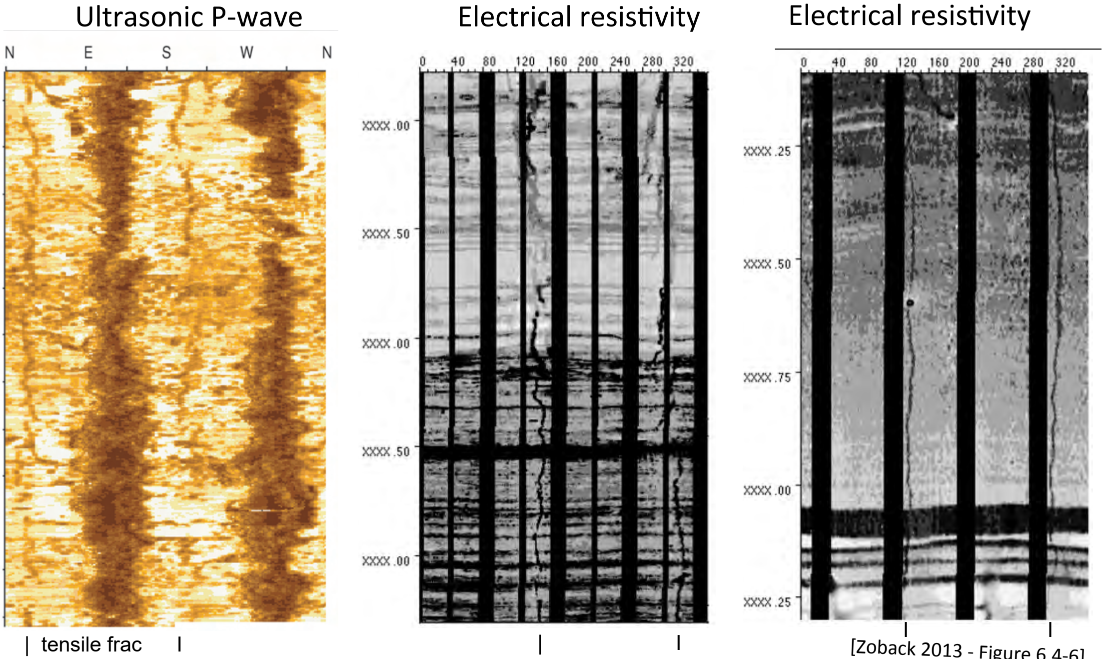
-3220

$+3 \times 1220$

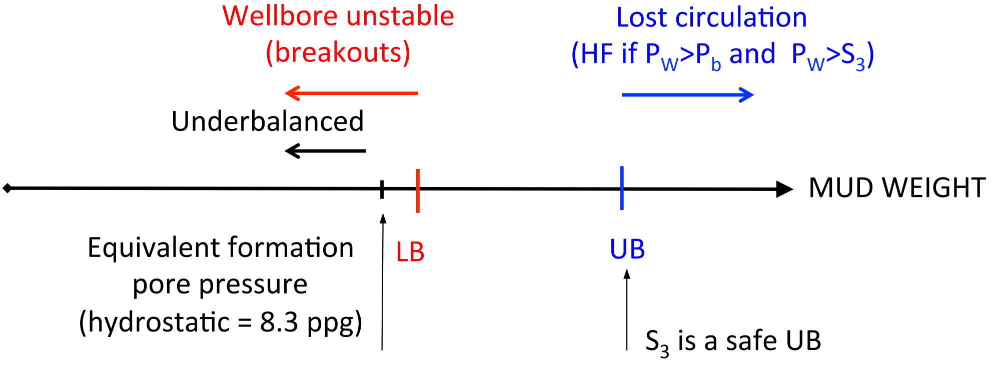
$+800$

$$= 4320$$

$$\frac{4320 \text{ psi}}{7000 \text{ ft}} \times \frac{8.33 \text{ ppg}}{0.44 \text{ psi/ft}} = 11.68 \text{ ppg} \blacksquare$$



[Zoback 2013 - Figure 6.4-6]



LIGHT MUDS
- May promote water production
- Compromise wellbore stability

HEAVY MUDS
- Damage formation (*k*) by mud infiltration
- Promote mud losses in permeable strata
- Low ROP because of stronger rock

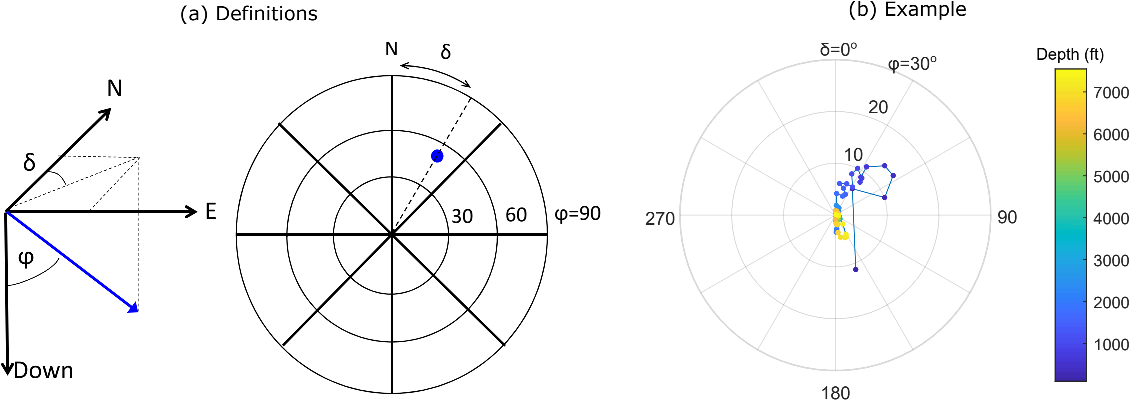
$$w_{BO} \leqslant 60^\circ$$

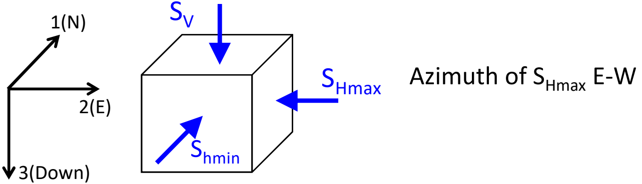
$$w_{BO} \geqslant 120^\circ$$

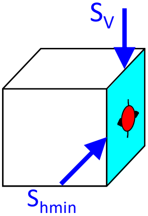
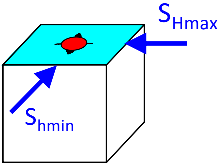
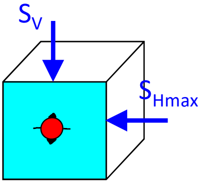
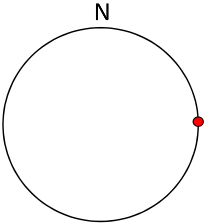
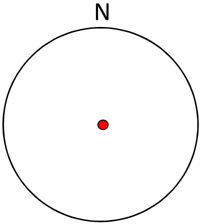
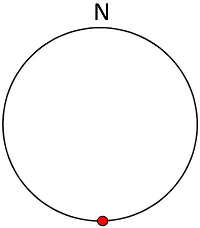
$$T_s = 0$$

$$\Delta P_W / \Delta z$$

δ


(a) Definitions
N
δ
E
φ
Down
N
δ
30
60
φ=90
(b) Example
δ=0°
φ=30°
20
10
270
90
180
Depth (ft)
7000
6000
5000
4000
3000
2000
1000

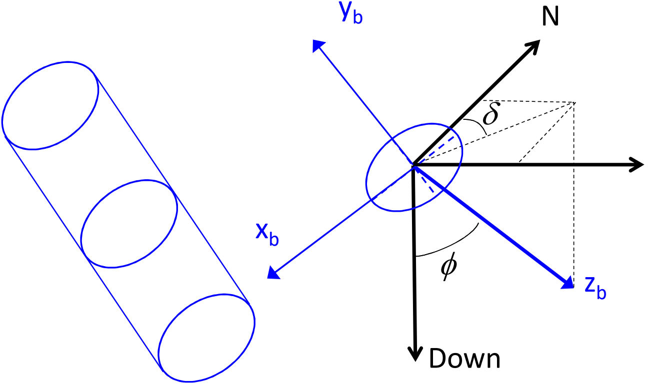


Stress Environment	NF	SS	RF
Plane with highest stress anisotropy and location of breakouts and tensile fracs			
Narrower drilling window in a stereonet projection			

x_b

y_b

z_b



$$R_{GW} = \begin{bmatrix} -\cos\delta\cos\phi & -\sin\delta\cos\phi & \sin\phi \\ \sin\delta & -\cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix}$$

$$R_{GW} = f(\delta, \varphi)$$

$$\underline{\underline{S}}_W = R_{GW} \underline{\underline{S}}_G R_{GW}^T$$

$$\underline{\underline{S}}_W = R_{GW} R_{PG}^T \underline{\underline{S}}_P R_{PG} R_{GW}^T$$

$$\underline{\underline{S}}_W$$

$$\underline{\underline{\mathrm{S}}}_W = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$(\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \sigma_{\theta z})$$

$$\sigma_{11} = S_{11} - P_p$$

$$\sigma_{22} = S_{22} - P_p$$

$$\sigma_{33} = S_{33} - P_p$$

$$\sigma_{12} = S_{12}$$

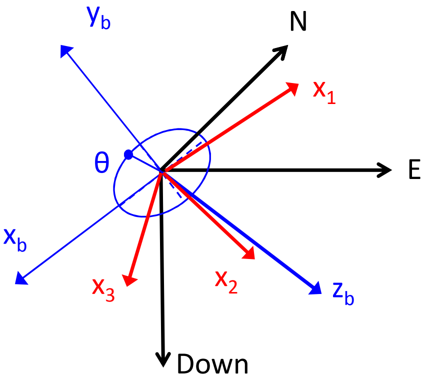
$$\sigma_{13} = S_{13}$$

$$\sigma_{23} = S_{23}$$

σ_{12}

σ_{13}

σ_{23}



Stresses at the wellbore wall (Kirsch[PP]+Kirsch[S])

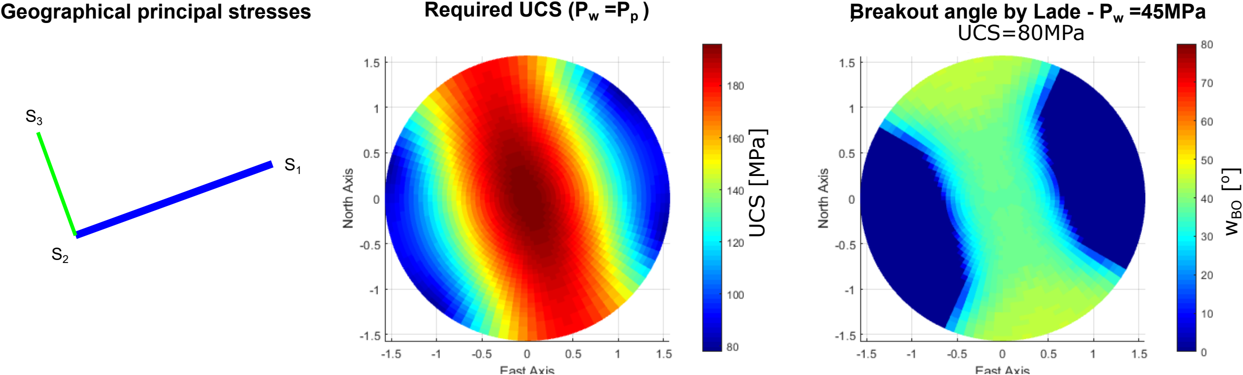
$$
\begin{cases}
\sigma_{rr} = \Delta P \\
\sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2\left(\sigma_{11} - \sigma_{22}\right)\cos 2\theta - 4\sigma_{12}\sin 2\theta - \Delta P \\
\tau_{\theta z} = 2\left(\sigma_{23}\cos\theta - \sigma_{13}\sin\theta\right) \\
\sigma_{zz} = \sigma_{33} - 2\nu\left(\sigma_{11} - \sigma_{22}\right)\cos 2\theta - 4\nu\sigma_{12}\sin 2\theta
\end{cases}
$$

$$(\sigma_{tmax}, \sigma_{rr}, \sigma_{tmin})$$

ω

$$S_2 = S_v$$

$$\delta = 070^\circ$$



Geographical principal stresses
S3
S1
S2
Required UCS (Pw =Pp)
North Axis
East Axis
UCS [MPa]
180
160
140
120
100
80
Breakout angle by Lade - Pw =45MPa
UCS=80MPa
North Axis
East Axis
wBO [o]
80
70
60
50
40
30
20
10
0

$$S_v = 70$$

$$S_{Hmax} = 67$$

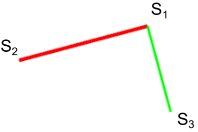
$$S_{hmin} = 45$$

$$P_p = 32$$

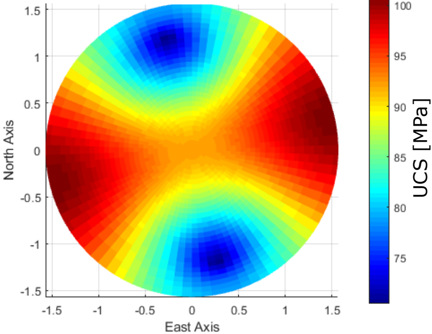
$$P_W = 32$$

$$\mu_i = 0.8$$

Principal stresses

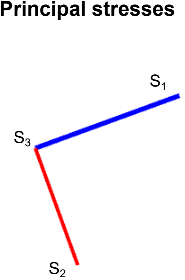


Required UCS ($P_w = P_p$)

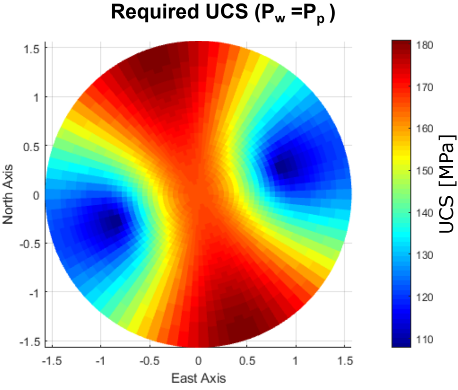


$$S_{Hmax} = 105$$

$$S_{hmin} = 85$$



Principal stresses
S_1
S_3
S_2

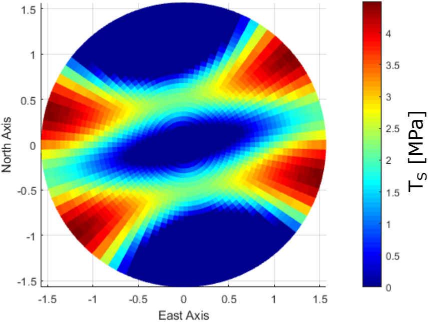


Required UCS ($P_w = P_p$)
North Axis
East Axis
UCS [MPa]
1.5
1
0.5
0
-0.5
-1
-1.5
110
120
130
140
150
160
170
180

$$S_1 = S_v$$

Principal stresses

Required T_S (P_W=35MPa)



$$S_{hmin} = 55$$

$$\frac{\partial T}{\partial t} = D_T \nabla^2 T$$

$$D_T = k_T/(\rho c_p)$$

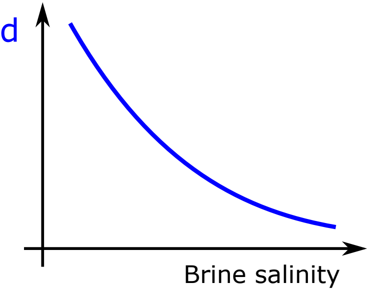
k_T

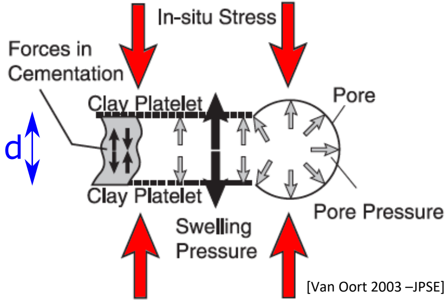
c_p

$$\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)$$

$\Delta\sigma_{\theta\theta}$

$$\Delta\sigma_{\theta\theta} = \frac{\alpha_L E \Delta T}{1 - \nu}$$

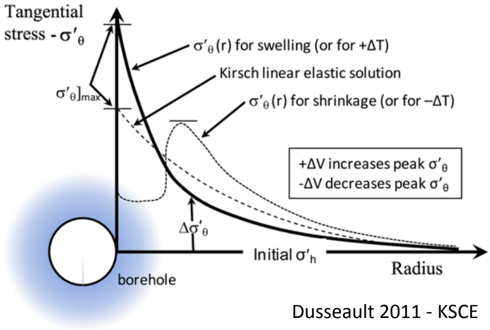

d
Brine salinity


In-situ Stress
Forces in Cementation
Clay Platelet
Pore
d
Clay Platelet
Pore Pressure
Swelling Pressure
[Van Oort 2003 –JPSE]

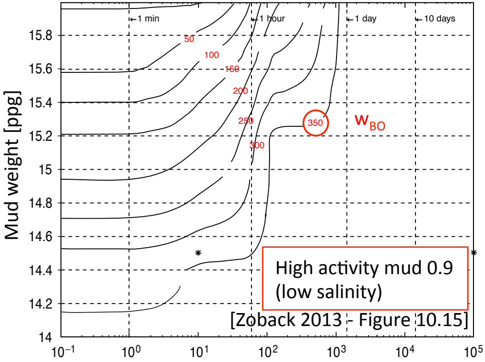
Norway shale

After 2hs exposition

CPGE – M. Chenevert



Dusseault 2011 - KSCE



[Zoback 2013 - Figure 10.15]

$$\sigma_{rr} = P_W - P_p$$

$\theta = 90^\circ$

$$r \geq a$$

$$w_{BO} = 70^\circ$$

$$w_{BO} \sim 0^\circ$$

$$\lambda_p = 0.52$$

$$\lambda_p = 0.60$$

$$\lambda_p = 0.44$$

$$(\sigma_{Hmax} - \sigma_{hmin})$$

$$dS_v/dz = 23$$

$$\sigma_{hmin} = 0.4\,\sigma_v$$

$q = 3.9$

$$w_{BO} = 45^\circ$$

$$\sigma_{Hmax} = 0.6\,\sigma_v$$

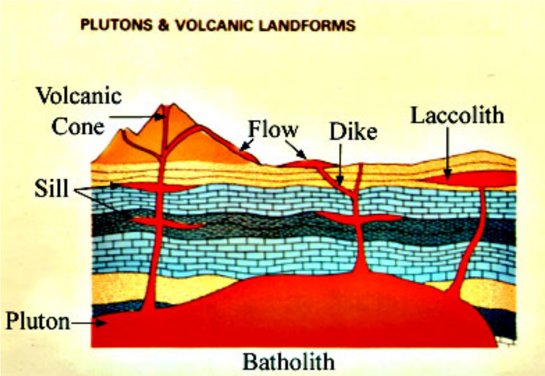
$$\sigma_{Hmax} = 0.8\,\sigma_v$$

$$\sigma_{Hmax} = 1.0\,\sigma_v$$

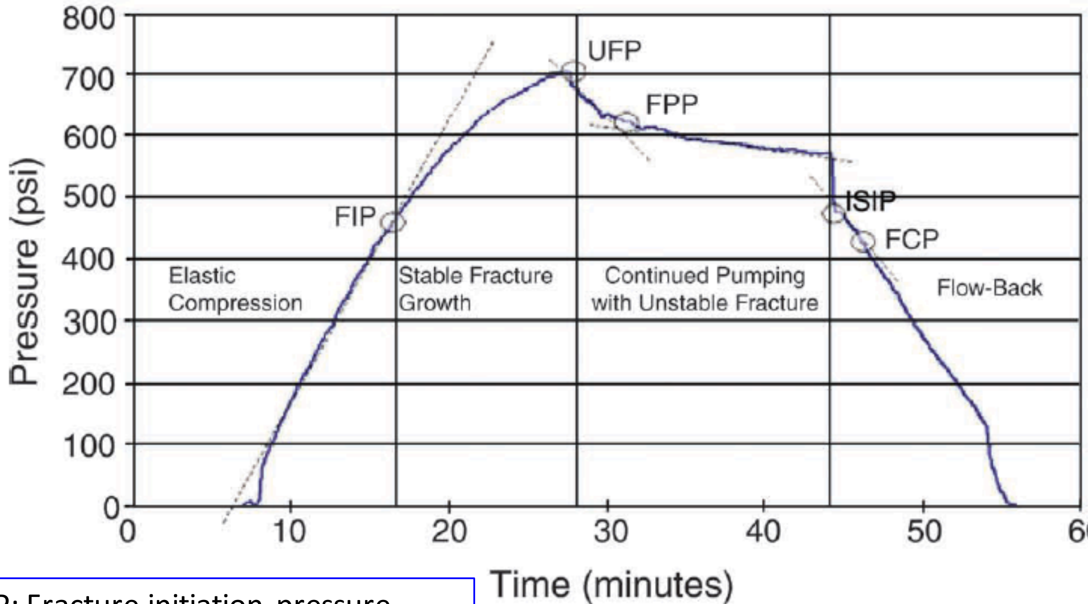
$$dS_v/dz = 1$$

$$\lambda_p = 0.6$$

$\mu_i = 1.0$



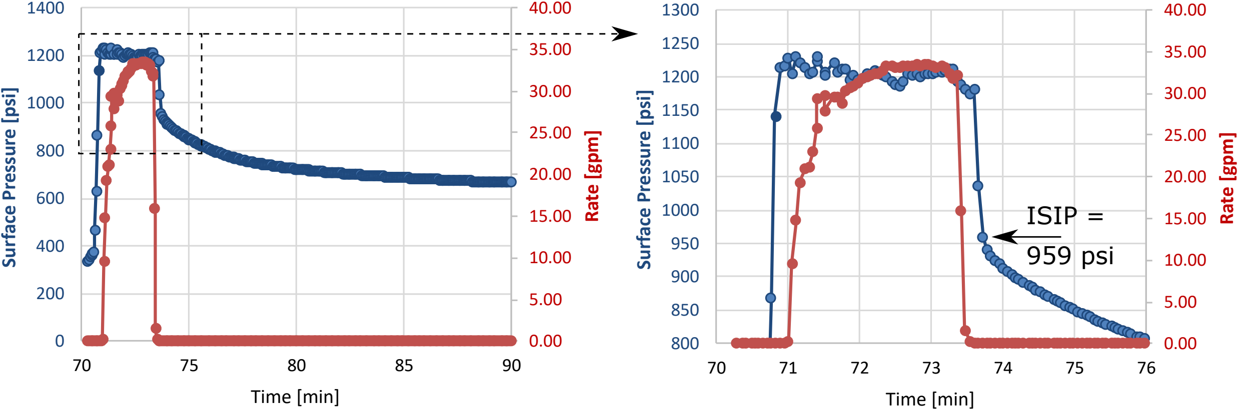
http://www.indiana.edu/~geol105/1425chap5.htm
http://geophysics.ou.edu/geol1114



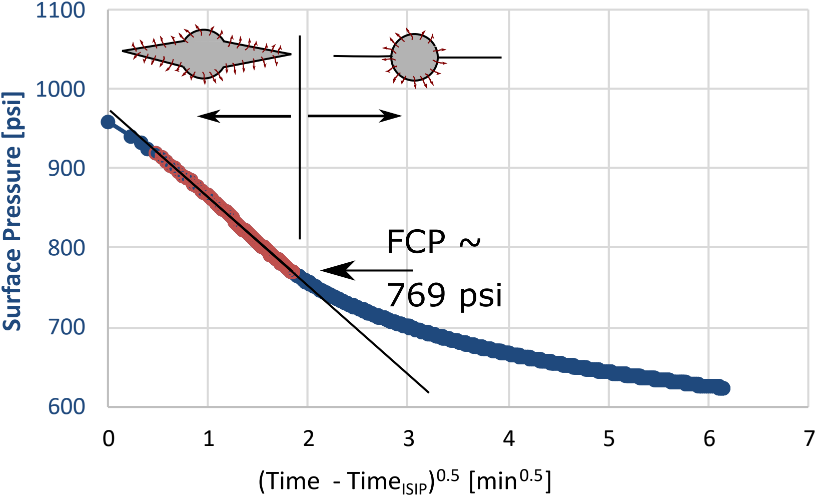
$\Delta V = q\ \Delta t$

FIP: Fracture initiation pressure
UFP: Unstable fracture pressure
FPP: Fracture propagation pressure
ISIP: Instantaneous shut-in pressure
FCP: Fracture closure pressure

[van Oort and Vargo, 2008]

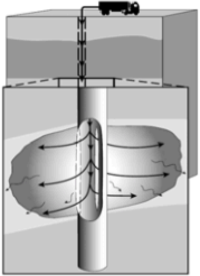

Surface Pressure [psi]
Rate [gpm]
Time [min]
1400
1200
1000
800
600
400
200
0
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
70
75
80
85
90
Surface Pressure [psi]
Rate [gpm]
Time [min]
1300
1250
1200
1150
1100
1050
1000
950
900
850
800
70
71
72
73
74
75
76
ISIP =
959 psi

t_{si}

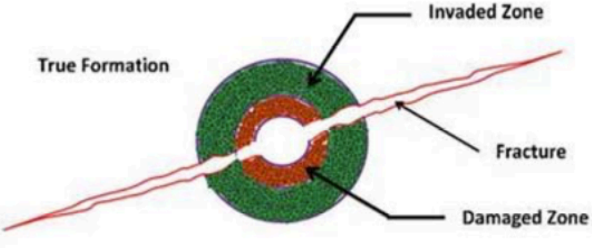


1100
1000
900
800
700
600
Surface Pressure [psi]
FCP ~
769 psi
0
1
2
3
4
5
6
7
(Time - Time_ISIP)^0.5 [min^0.5]

$\leqslant 10$



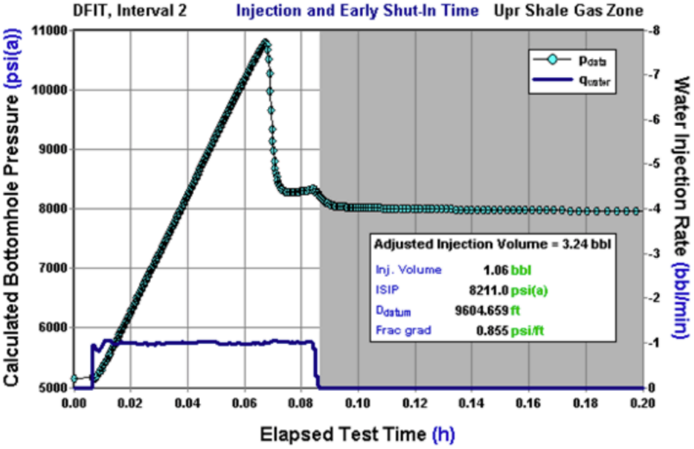
Side View



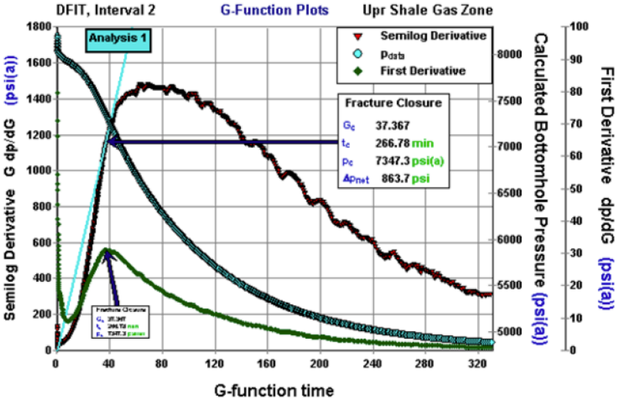
Plan View

Fekete, 2011

[SPE 163863]



DFIT, Interval 2
Injection and Early Shut-In Time
Upr Shale Gas Zone
Calculated Bottomhole Pressure (psi(a))
Water Injection Rate (bbl/min)
Elapsed Test Time (h)
pdata
qwater
Adjusted Injection Volume = 3.24 bbl
Inj. Volume 1.06 bbl
ISIP 8211.0 psi(a)
Ddatum 9604.659 ft
Frac grad 0.855 psi/ft



DFIT, Interval 2
G-Function Plots
Upr Shale Gas Zone
Analysis 1
Semilog Derivative
pdata
First Derivative
Fracture Closure
Gc 37.367
tc 266.78 min
pc 7347.3 psi(a)
Δpnet 863.7 psi
Semilog Derivative G dp/dG (psi(a))
Calculated Bottomhole Pressure (psi(a))
First Derivative dp/dG (psi(a))
G-function time

[SPE 163863]

<

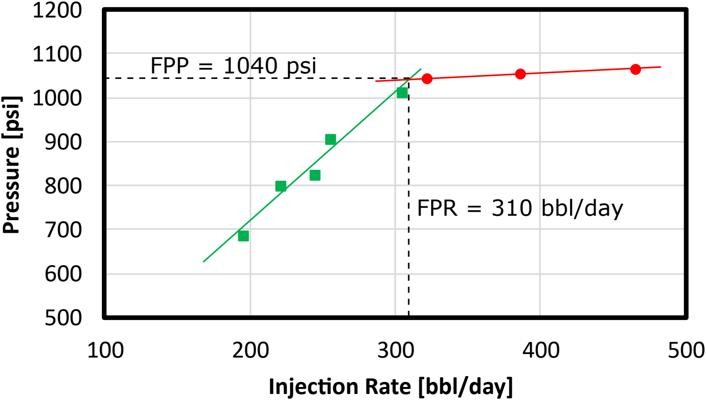
>

S

$s > 1$

$s < 1$

$$q = \frac{2\pi hk}{\mu} \frac{P_e - P_w}{\ln(\frac{r_e}{r_w}) + s}$$

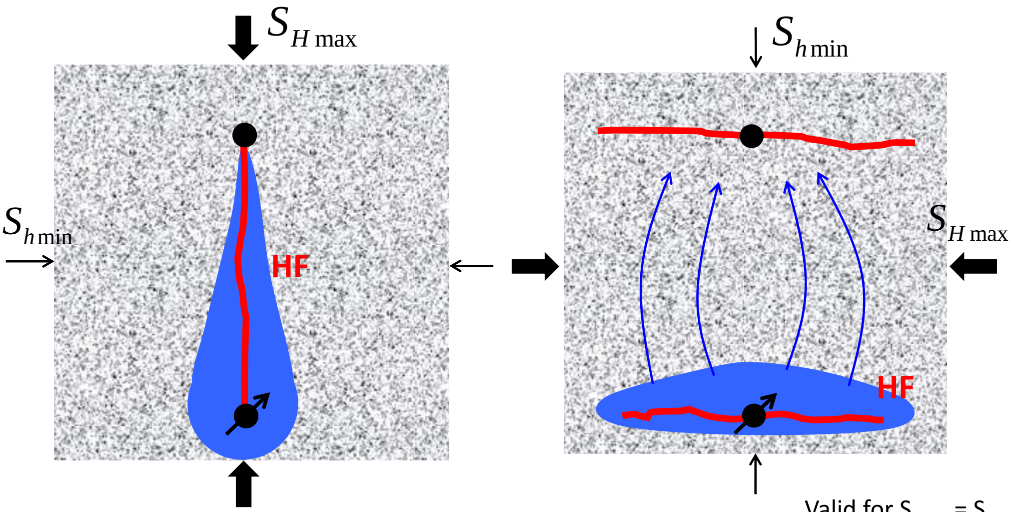


FPP = 1040 psi
FPR = 310 bbl/day
Pressure [psi]
Injection Rate [bbl/day]
1200
1100
1000
900
800
700
600
500
100
200
300
400
500

Fracture Influence on Flooding

Fractures modify :

1) Sweep Efficiency
2) Injectivity
3) Oil Productivity

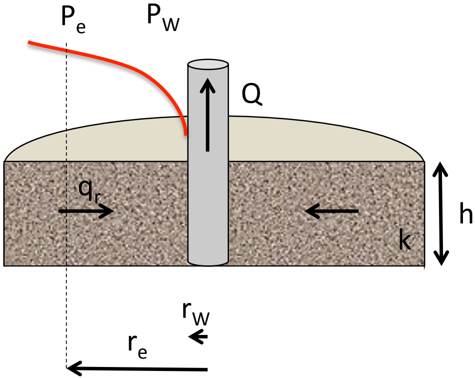


Valid for $S_{hmin} = S_3$

x_f

$$(4x_f h)/(2\pi r_w h) \sim x_f/r_w$$

Intact



$$Q = -\frac{2\pi kh}{\mu}\frac{(P_e - P_W)}{p_D + s}$$

p_D: Dimensionless pressure

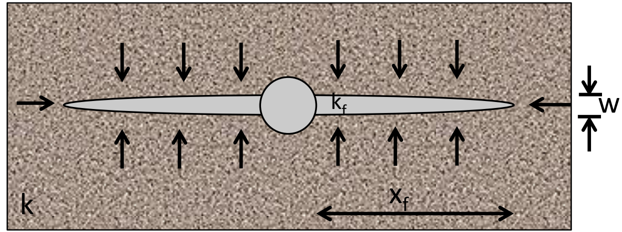
$= \ln(r_e/r_W)$ if steady state

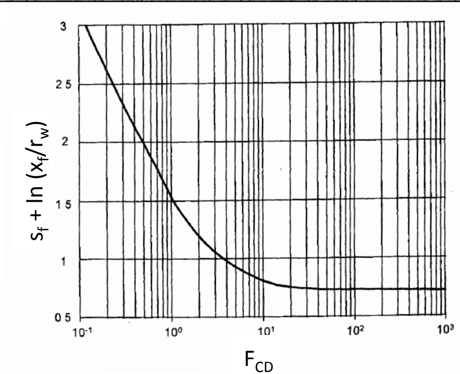
s: skin factor

$> 1 \rightarrow$ damage

$< 1 \rightarrow$ stimulation

Fractured

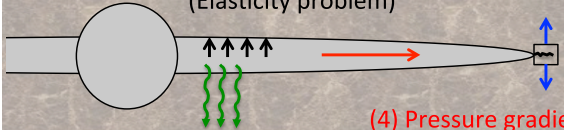




$w, x_f, k_f \rightarrow F_{CD} = (k_f w)/(k\, x_f)$

$x_f, r_W, F_{CD} \rightarrow s_f$ from above plot

(1) Pressure on the fracture deforms adjacent rock (Elasticity problem)



(2) Fracture propagates if the "stress intensity factor" is higher than what the rock can resist "rock toughness" (Fracture mechanics problem)

(3) Fracturing fluids can leak off to the formation (Mud-cake design)

(4) Pressure gradient leads to flow of fracturing fluid through the fracture (Lubrication problem)

Net pressure

$p_{net} = p - S_3$

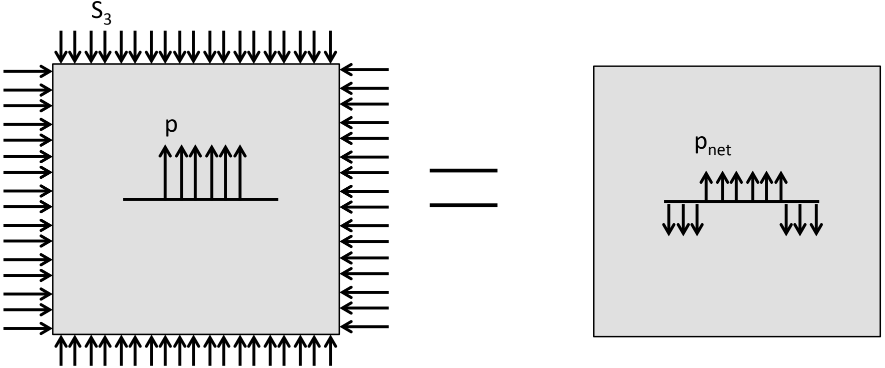
$\underline{\underline{S}}$

p_{net}

$$p_{net} = p - S_3$$

$$p_{net} > 0$$

$p > S_3$

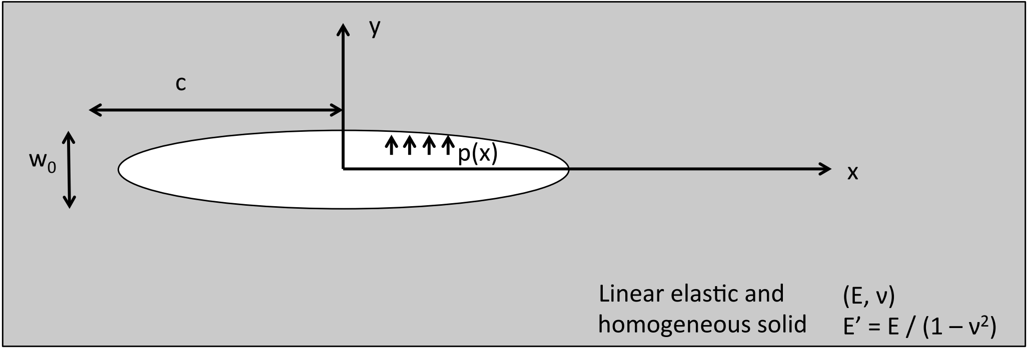


S_3
p
=
p_{net}

$$p_{net} = p_o$$

c

$$\begin{cases} \sigma_{yy} &= p(x) = p_o \quad \text{for} \quad 0 \le x \le c \\ u_y &= 0 \qquad\qquad\;\; \text{for} \quad x > c \end{cases}$$


y
c
w_0
p(x)
x
Linear elastic and homogeneous solid
(E, ν)
$E' = E / (1 - \nu^2)$

$$(x, y = 0)$$

$$
\begin{cases}
u_y(x,0) &= \dfrac{2p_o}{E'}\sqrt{c^2 - x^2} \qquad \text{for} \quad 0 \le x \le c \\
\sigma_{yy}(x,0) &= -p_o\left(\dfrac{x}{\sqrt{x^2 - c^2}} - 1\right) \quad \text{for} \quad x > c
\end{cases}
$$

$$E' = E/(1 - \nu^2)$$

$$x = 0$$

$$x = c$$

$$u_y(0,0) = 2p_0c/E'$$

$$w_o = \frac{4p_0c}{E'}$$

$$u_y$$

$$\sigma_{yy}(c,0) \rightarrow -\infty$$

$$x = 1.15c$$

$$\sigma_{yy}(1.15c, 0) \sim -p_o$$

$$K_I = \lim_{r \to 0^+} \left[(2\pi r)^{1/2} \sigma_{yy}(x = c + r, y = 0) \right]$$

$$r = x - c$$

p_o

σ_{yy}

$$K_I = p_o(\pi c)^{1/2}$$

$$K_I$$

K_{IC}

$$
\begin{cases} K_I \geq K_{IC} & \rightarrow \quad \text{Fracture propagates} \\ K_I < K_{IC} & \rightarrow \quad \text{Fracture does not propagate} \end{cases}
$$

I

$$K_{IC} = \frac{P_{max}(\pi a)^{1/2}}{2rL} Y_I$$

P_{max}

Y_I

s/r

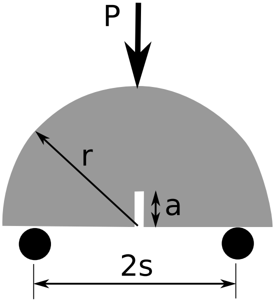
$$Y_I \sim 4.5$$

$$s/r = 0.8$$

$$0.1 < a/r < 0.2$$

$$K_{IC} \sim 0.05$$

$1/2$


P
r
a
2s

$$x_f = 10$$

$$p_f = 20.5$$

$$E = 1$$

$\nu =$

$$E' = \frac{E}{1 - \nu^2} = \frac{1 \text{ GPa}}{1 - 0.25^2} = 1.07 \text{ GPa} = 1070 \text{ MPa}$$

$$p_{net} = p_f - S_3$$

$$w_o = \frac{4p_0 c}{E'} = \frac{4 \times 0.5 \text{ MPa} \times 10 \text{ m}}{1070 \text{ MPa}} = 0.019 \text{ m} = 19 \text{ mm}$$

$$K_I = p_o(\pi c)^{1/2} = 0.5$$

$(\pi \times 10$

$)^{1/2} = 2.8$

$1/2$

$$w_o = \frac{2p_0c}{E'} = \frac{2 \times 0.5\ \text{MPa} \times 10\ \text{m}}{1070\ \text{MPa}} = 0.0095\ \text{m} = 9.5\ \text{mm}$$

$$K_I = \left(1 - \frac{2}{\pi}\right) p_o (\pi c)^{1/2} = \left(1 - \frac{2}{\pi}\right) 0.5$$

$)^{1/2} = 1.0$

$1/2 \quad \blacksquare$

V_i

V_L

$$V_i = V + V_L$$

$$V_L = A_L(2c_L\sqrt{t} + S_p)$$

A_L

c_L

$$S_p$$

$$\eta = \frac{V}{V_i}$$

$\eta \to 1$

$\eta \to 0$

$$V_i = i\,t$$

$$2V_i$$

$$q = \frac{w^3 h_f}{12\mu} \frac{\Delta P}{L}$$

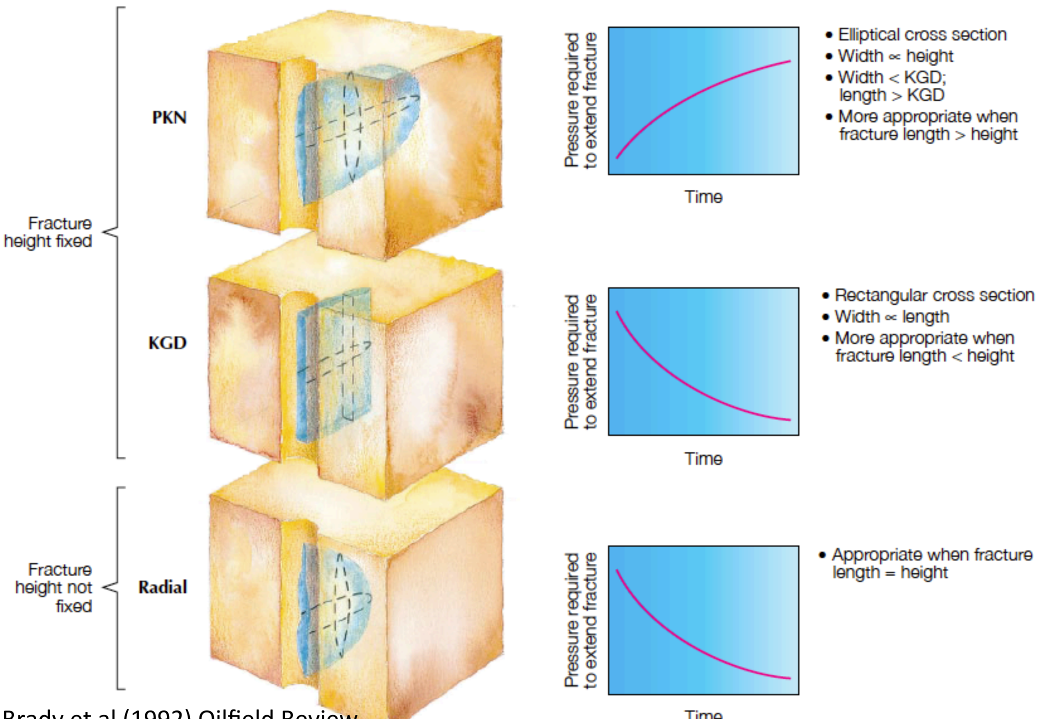
w

$\Delta P/L$

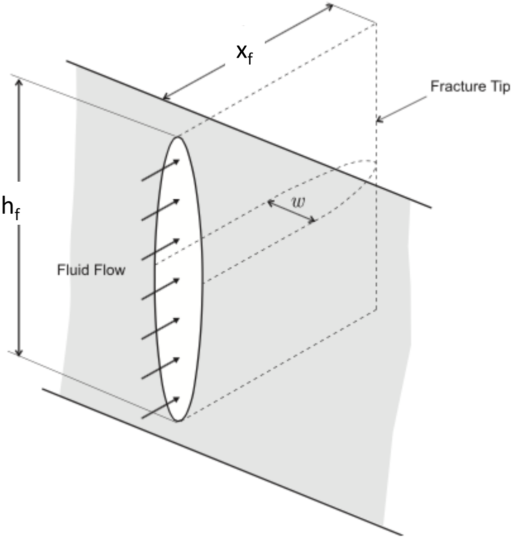
$w(x)$

$\Delta P(x)/dx$

2D Fracture Models



Brady et al (1992) Oilfield Review



x_f
Fracture Tip
h_f
w
Fluid Flow

$$x_f = \left(\frac{625}{512\pi^3}\right)^{1/5} \left(\frac{i^3 E'}{\mu\, h_f^4}\right)^{1/5} t^{4/5} = 0.524 \left(\frac{i^3 E'}{\mu\, h_f^4}\right)^{1/5} t^{4/5}$$

$$w_{w,0} = \left(\frac{2560}{\pi^2}\right)^{1/5} \left(\frac{i^2 \mu}{E' \, h_f}\right)^{1/5} t^{1/5} = 3.040 \, \left(\frac{i^2 \mu}{E' \, h_f}\right)^{1/5} t^{1/5}$$

$$p_{net,w} = \left(\frac{80}{\pi^2}\right)^{1/4} \left(\frac{E'^4 i^2 \mu}{h_f^6}\right)^{1/5} t^{1/5} = 1.520 \left(\frac{E'^4 i^2 \mu}{h_f^6}\right)^{1/5} t^{1/5}$$

$$t^{1/5}$$

$$\bar{w} = (\pi/5) w_{w,0}$$

$$V_{frac} = (\pi/5)w_{w,0}h_f x_f$$

$$w_{w,0}$$

$p_{net,w}$

$$h_f = 100$$

$$E' = 8.9 \times 10^6$$

$$i = 25$$

$$h_f = 30.5$$

$$E' = 61.4 \times 10^9$$

•

$$i = 0.066$$

$$x_f = 0.524 \left[\frac{(0.066 \text{ m}^3/\text{s})^3 \times (61.4 \times 10^9 \text{ Pa})}{0.001 \text{ Pa s} \times (30.5 \text{ m})^4} \right]^{1/5} (1800 \text{ s})^{4/5} = 1536.2 \text{ m}$$

$$w_{w,0} = 3.040 \left[\frac{(0.066 \text{ m}^3/\text{s})^2 \times (0.001 \text{ Pa s})}{(61.4 \times 10^9 \text{ Pa}) \times (30.5 \text{ m})} \right]^{1/5} (1800 \text{ s})^{1/5} = 4.05 \times 10^{-3} m$$

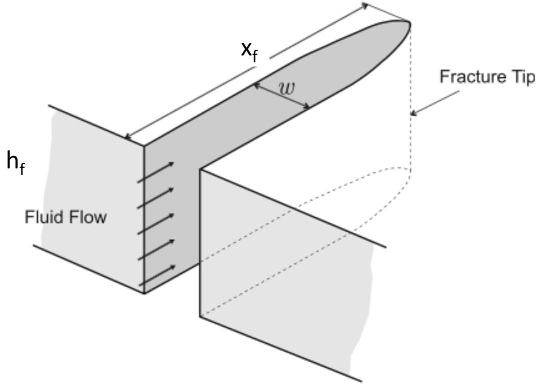
$$p_{net,w} = 1.520 \left[\frac{(61.4 \times 10^9 \text{ Pa})^4 \times (0.066 \text{ m}^3/\text{s})^2 \times (0.001 \text{ Pa s})}{(30.5 \text{ m})^6} \right]^{1/5} (1800 \text{ s})^{1/5} = 4.1 \times 10^6 \text{Pa}$$

$$2V_{frac} = 2\frac{\pi}{5}(4.05 \times 10^{-3}m) \times (30.5$$

$) \times (1536.2$

$) = 239$

3



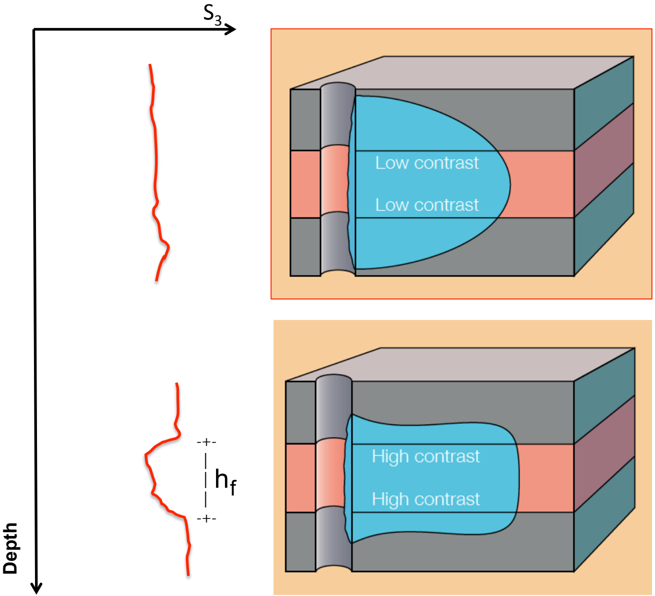
x_f
w
Fracture Tip
h_f
Fluid Flow

$$x_f = \left(\frac{16}{21\pi^3}\right)^{1/6} \left(\frac{i^3 E'}{\mu\, h_f^3}\right)^{1/5} t^{2/3} = 0.539 \left(\frac{i^3 E'}{\mu\, h_f^3}\right)^{1/5} t^{2/3}$$

$$w_{w,0} = \left(\frac{5376}{\pi^3}\right)^{1/6} \left(\frac{i^3 \mu}{E' \, h_f^3}\right)^{1/6} t^{1/3} = 2.360 \, \left(\frac{i^3 \mu}{E' \, h_f^3}\right)^{1/6} t^{1/3}$$

$$p_{net,w} = \left(\frac{21}{16}\right)^{1/3} \left(E'^2\mu\right)^{1/3} t^{-1/3} = 1.090 \ \left(E'^2\mu\right)^{1/3} t^{-1/3}$$

$$t^{-1/3}$$



S_3
Depth
h_f
Low contrast
Low contrast
High contrast
High contrast

$V_P(z)$

$$V_S(z)$$

$$\Delta t_P = V_P^{-1}$$

$$\Delta t_S = V_S^{-1}$$

$$E_{dyn} = \rho_{bulk} V_S^2 \left(\frac{3V_P^2 - 4V_S^2}{V_P^2 - V_S^2} \right)$$

$$\nu_{dyn} = \frac{V_P^2 - 2V_S^2}{2\left(V_P^2 - V_S^2\right)}$$

E_{sta}

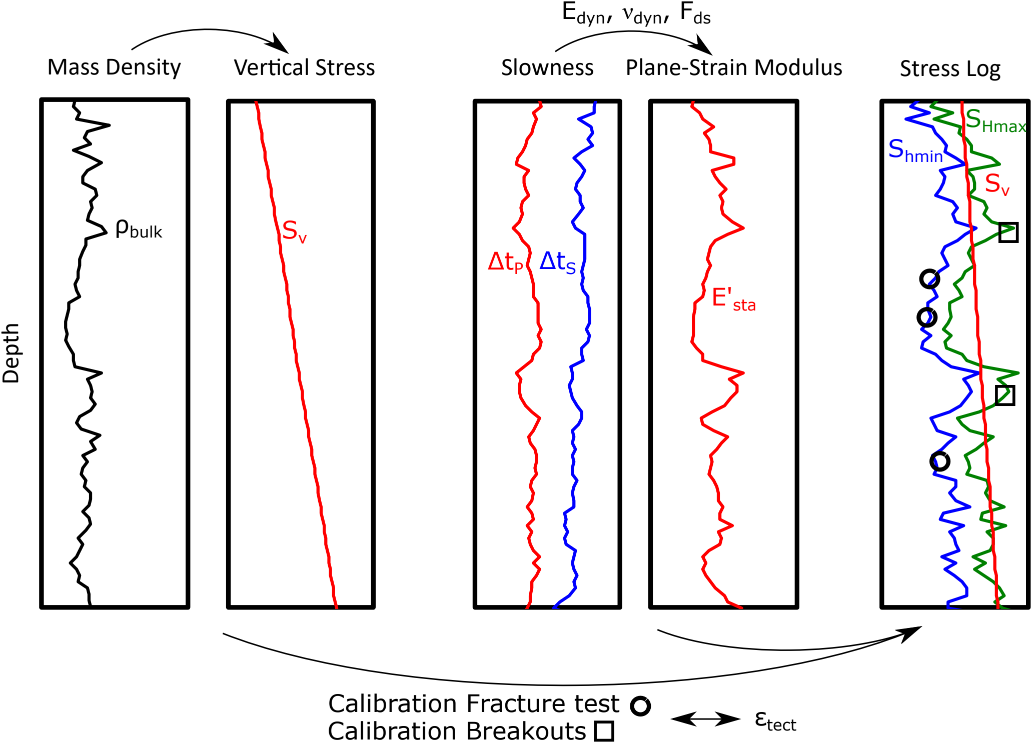
$$E_{dyn}$$

$$E_{sta} = F_{ds} E_{dyn}$$

$$F_{ds} < 1$$

$$\nu_{sta} = \nu_{dyn}$$

$$E'_{sta}$$

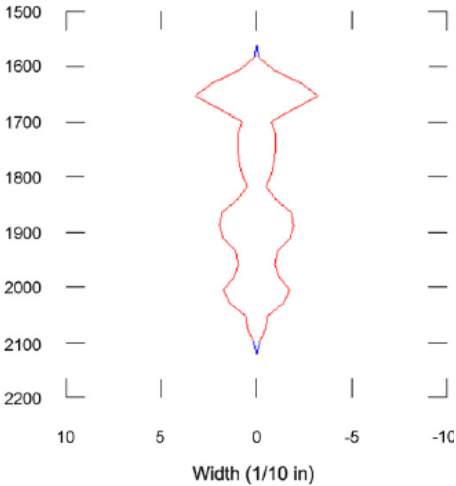


E_{dyn}, ν_{dyn}, F_{ds}
Mass Density
Vertical Stress
Slowness
Plane-Strain Modulus
Stress Log
Depth
ρ_{bulk}
S_v
Δt_P
Δt_S
E'_{sta}
S_{Hmax}
S_{hmin}
S_v
Calibration Fracture test
Calibration Breakouts
ε_{tect}

(a) Stress Log

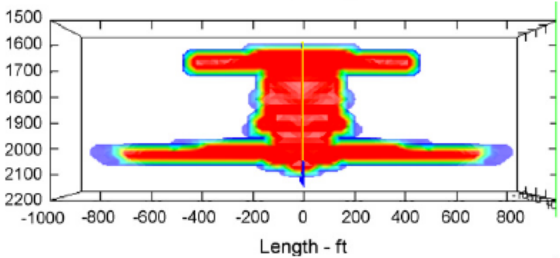
Minimum principal stress (MPa)

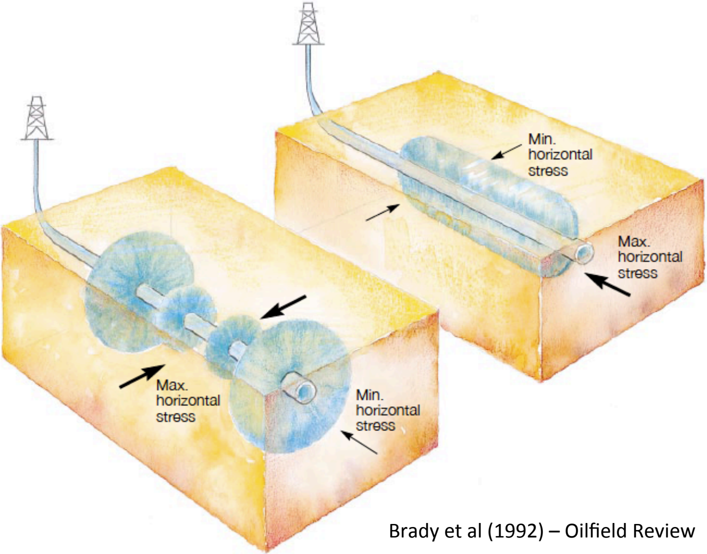
(b) Fracture width at wellbore



Minimum principal stress (psi)

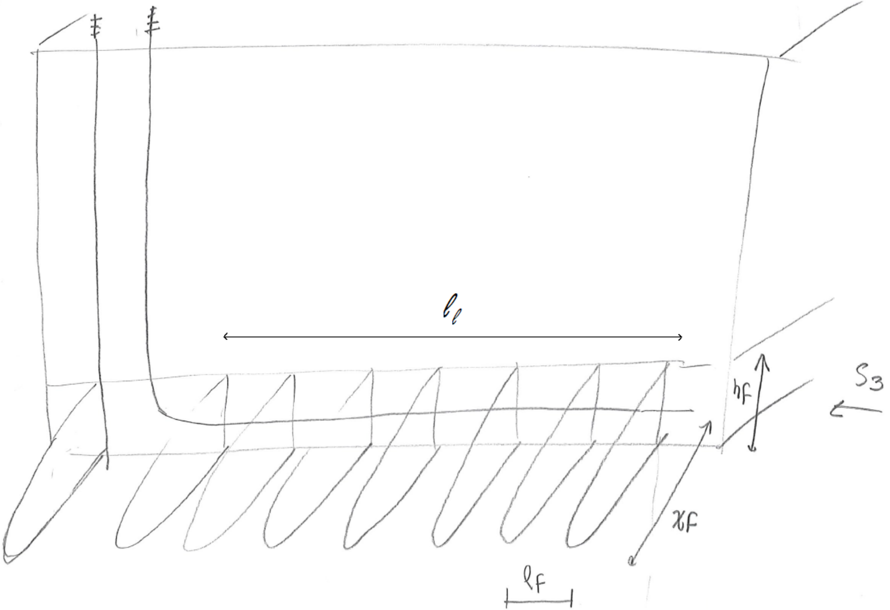
(c) Fracture width (color field)
View perp. to fracture plane





Brady et al (1992) – Oilfield Review

(x_f, h_f, w_0)



ℓ_ℓ
h_F
S_3
x_F
ℓ_F

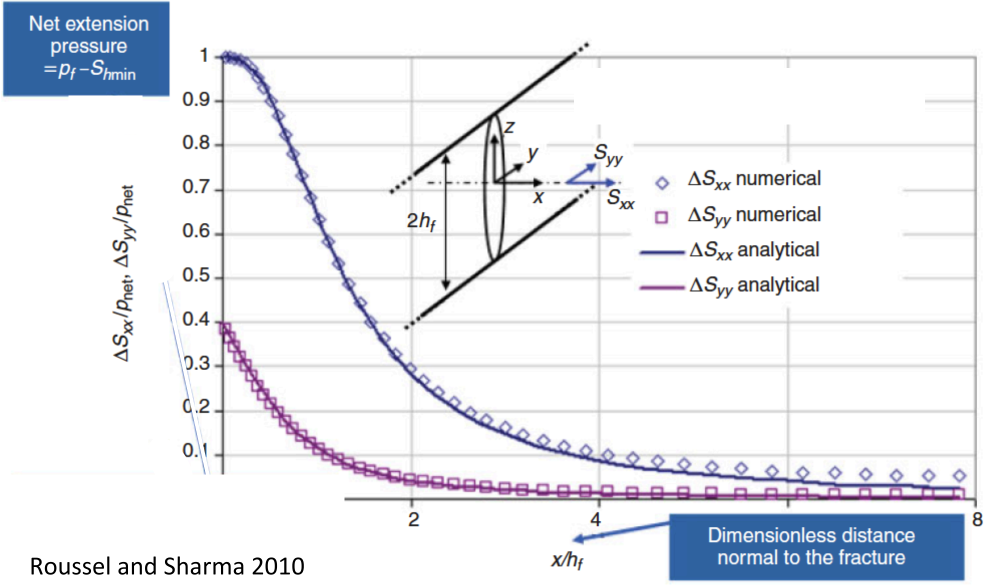
$$S_2 - S_3$$

$$S_{Hmax} - S_{hmin}$$

y

$$x - z$$

x



Roussel and Sharma 2010

$$l_f \propto \frac{p_{net} h_f}{S_2 - S_3}$$

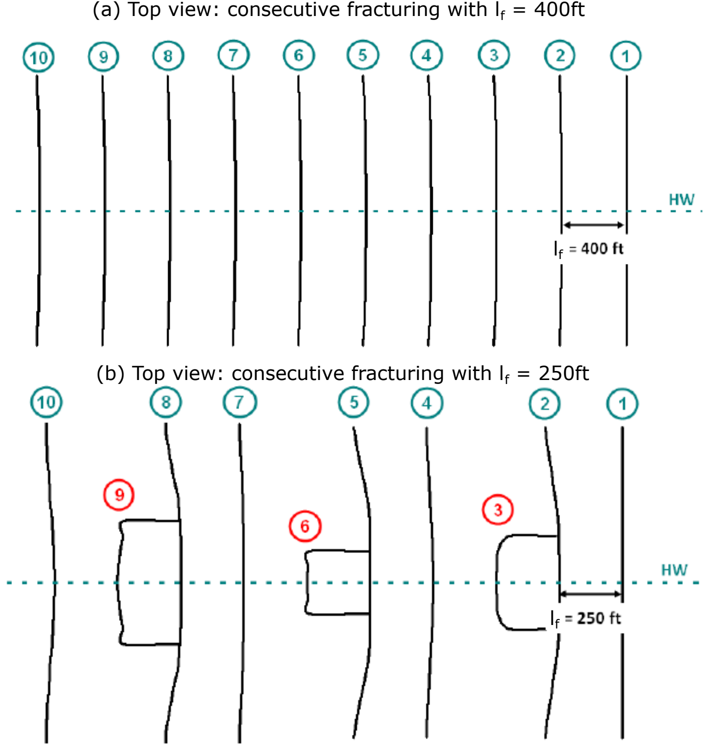
$$E'$$

$$S_2 - S_3 \sim 100$$

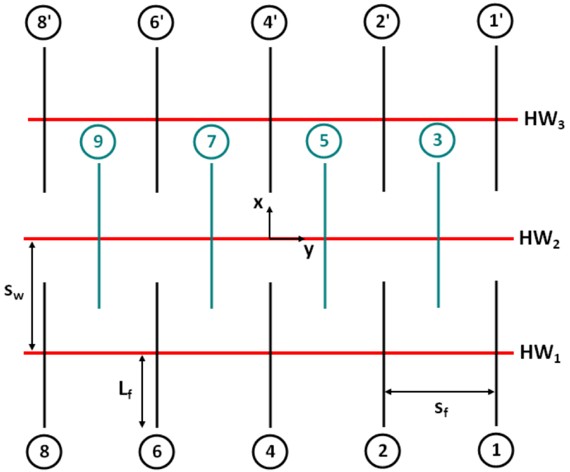
$p_{net} >$

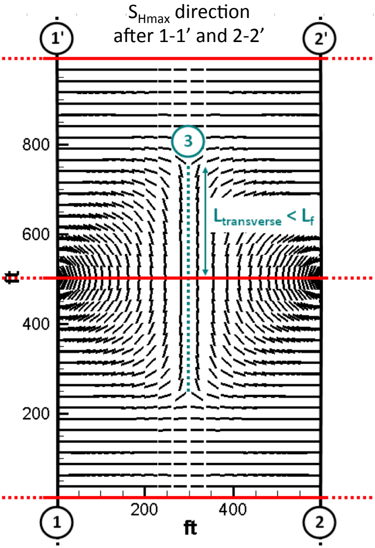
$S_{Hmax} - S_{hmin} =$

$$l_f < h_f$$



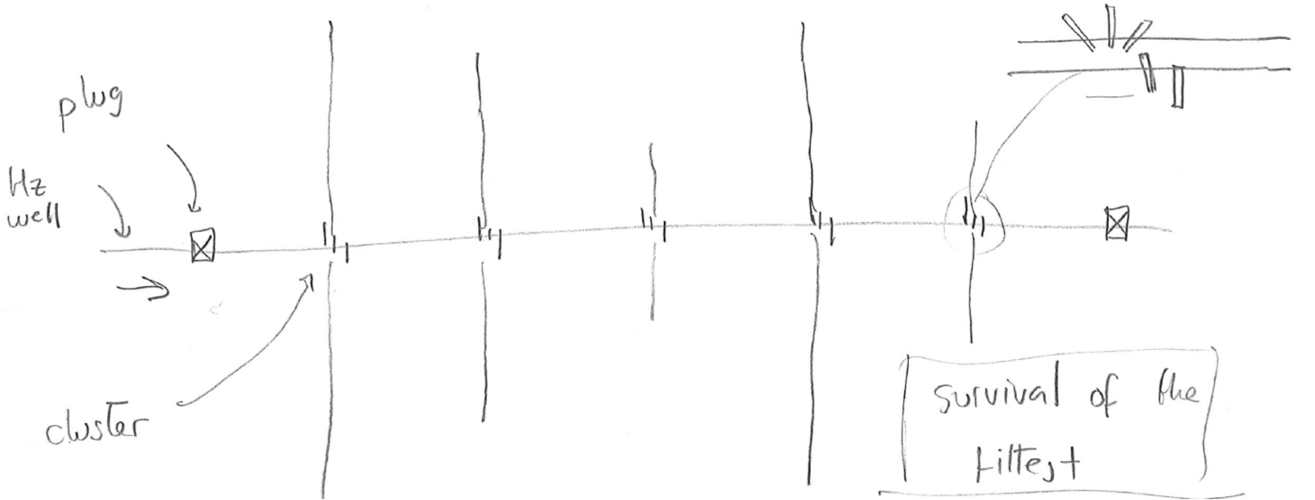
(a) Top view: consecutive fracturing with l_f = 400ft
10
9
8
7
6
5
4
3
2
1
HW
l_f = 400 ft
(b) Top view: consecutive fracturing with l_f = 250ft
10
8
7
5
4
2
1
9
6
3
HW
l_f = 250 ft

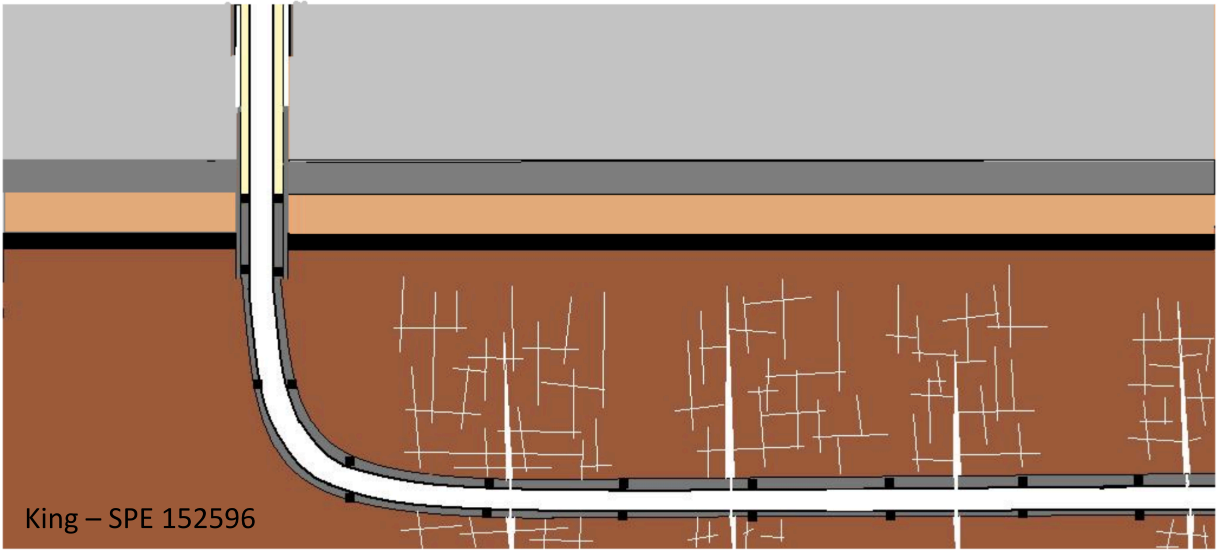




[Roussel and Sharma – SPE 146104]

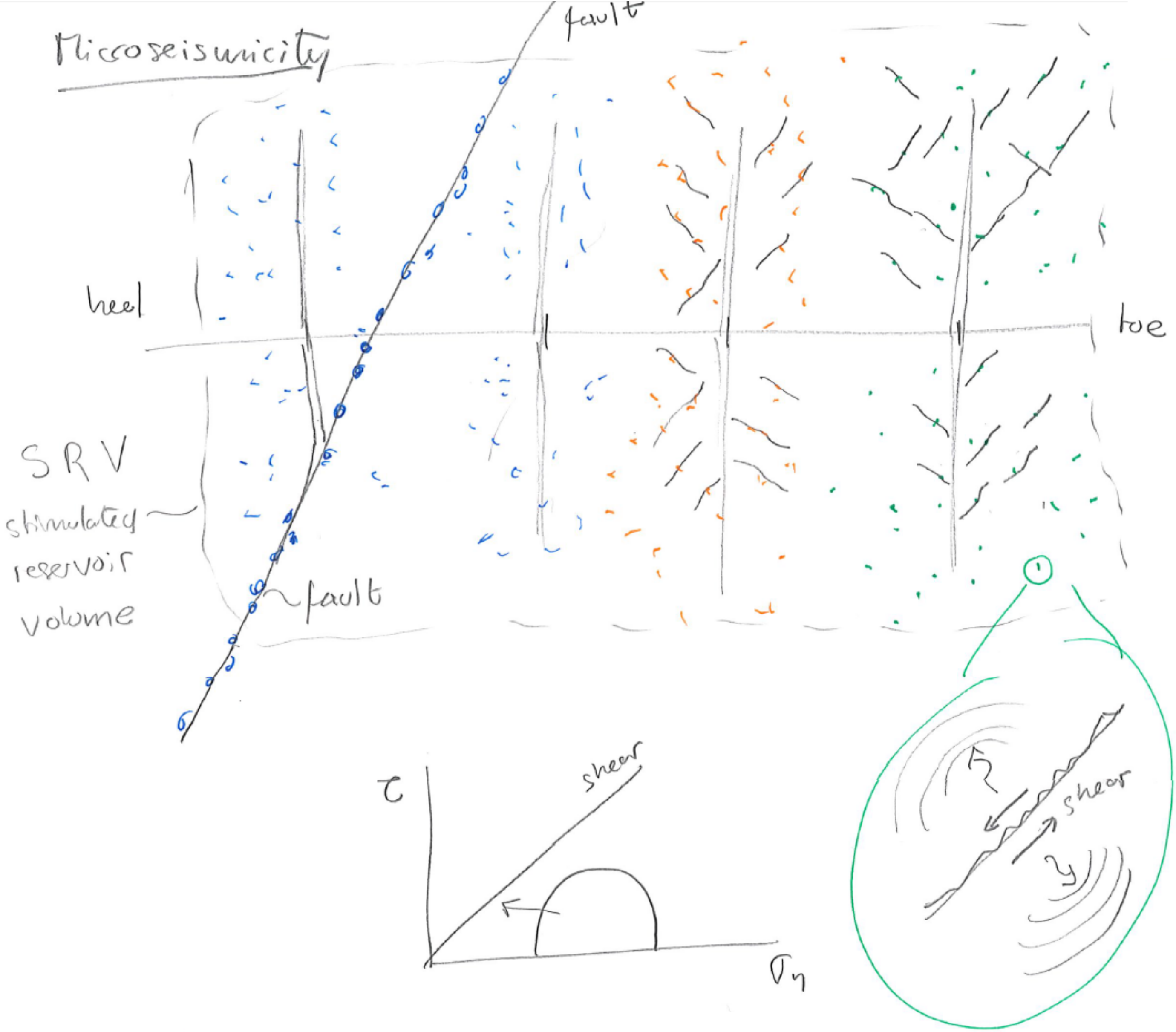
$l_l >$


plug
Hz
well
cluster
survival of the fittest

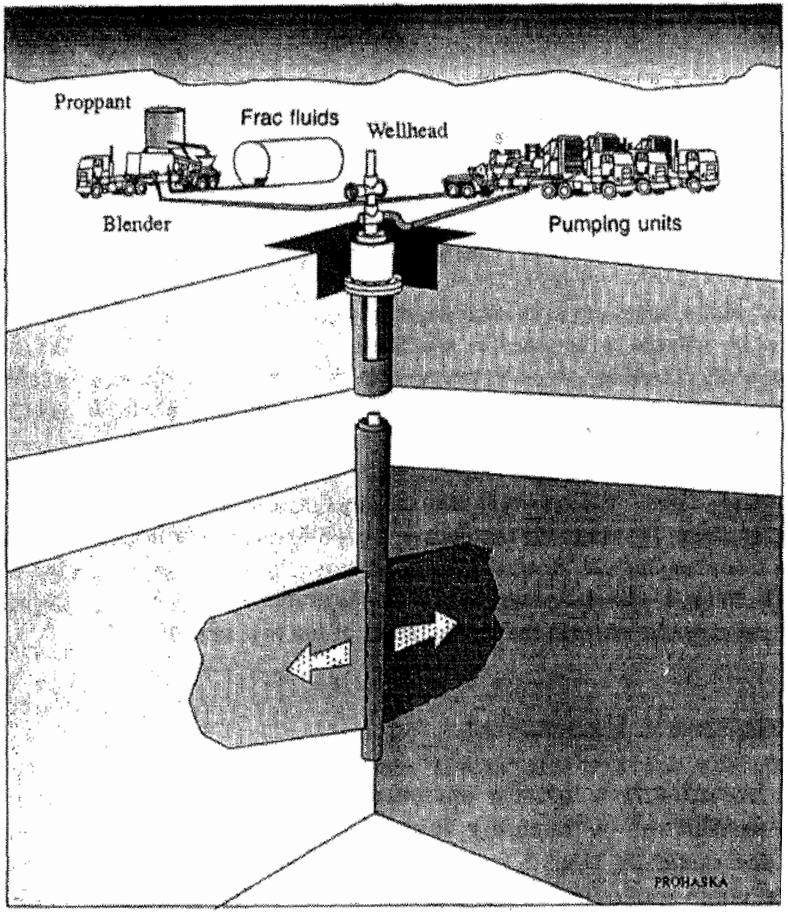

King – SPE 152596

$$M_L < 0$$

Microseismicity



$$EUR = RF \frac{SRV\ \phi(1 - S_w)}{B_{oi}}$$

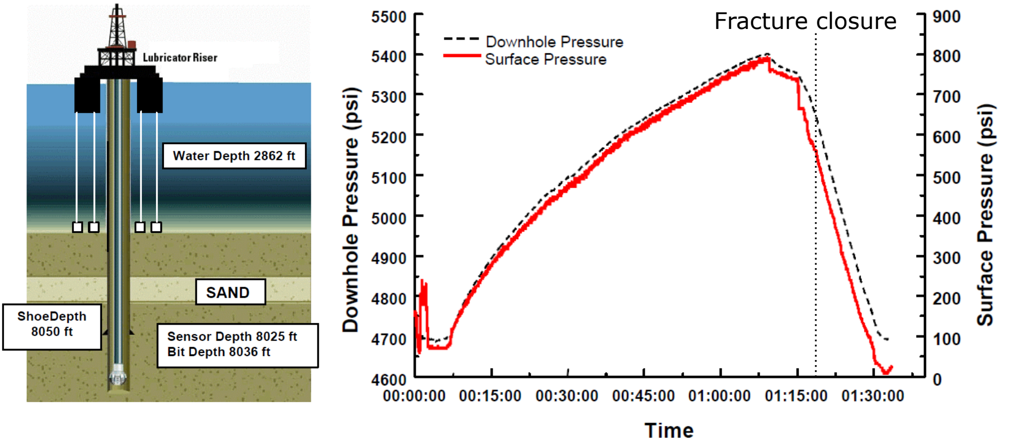


From
Valko and Economides, 1996

SPE-169959

Surf Press [Tbg] (psi)
Slurry Flow Rate (bpm)
Proppant Conc (ppg)
5000
20.00
4000
16.00
3000
12.00
2000
8.000
1000
4.000
0.0
0.0
60.00
48.00
36.00
24.00
12.00
0.0
210.0
226.0
242.0
258.0
274.0
290.0
Time (min)

$$\sigma_v/\sigma_{hmin}$$



Lubricator Riser
Water Depth 2862 ft
SAND
ShoeDepth
8050 ft
Sensor Depth 8025 ft
Bit Depth 8036 ft
Fracture closure
Downhole Pressure
Surface Pressure
Downhole Pressure (psi)
Surface Pressure (psi)
5500
5400
5300
5200
5100
5000
4900
4800
4700
4600
900
800
700
600
500
400
300
200
100
0
00:00:00
00:15:00
00:30:00
00:45:00
01:00:00
01:15:00
01:30:00
Time

Step #	Test rate (bbl/min)	Test rate (% of max. rate)								
1	0.2	5	Time (min)	0	5	10	15	20	25	30
			Pressure (psi)	0	99	105	108	109	110	110
2	0.4	10	Time (min)	0	5	10	15	20	25	30
			Pressure (psi)	88	187	204	215	219	220	220
3	0.8	20	Time (min)	0	5	10	15	20	25	30
			Pressure (psi)	209	358	424	431	438	439	440
4	1.6	40	Time (min)	0	5	10	15	20	25	30
			Pressure (psi)	418	770	869	871	875	878	882
5	2.4	60	Time (min)	0	5	10	15	20	25	30
			Pressure (psi)	825	1089	1133	1199	1265	1298	1321
6	3.2	80	Time (min)	0	5	10	15	20	25	30
			Pressure (psi)	1210	1375	1459	1507	1529	1535	1540
7	4	100	Time (min)	0	5	10	15	20	25	30
			Pressure (psi)	1485	1595	1650	1683	1727	1749	1760

$$E_{static} = 0.65E_{dynamic}$$

$$E'_{static} = E_{static}/(1 - \nu^2)$$

$$\varepsilon_{Hmax} = 0.0015$$

$$q = 3.5$$

$$(P_f - P_o)$$

$$d_{pore}^2$$

w_f^3

$$\begin{bmatrix} S_{11}-\alpha P_p \\ S_{22}-\alpha P_p \\ S_{33}-\alpha P_p \\ S_{12} \\ S_{13} \\ S_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \varepsilon_{33} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\varepsilon_{33} = \frac{S_{33} - \alpha P_p}{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}}$$

$$M = E(1-\nu)/(1+\nu)/(1-2\nu)$$

$$\Delta\varepsilon_{33} = -\frac{\alpha}{M}\Delta P_p$$

$$\Delta P_p < 0$$

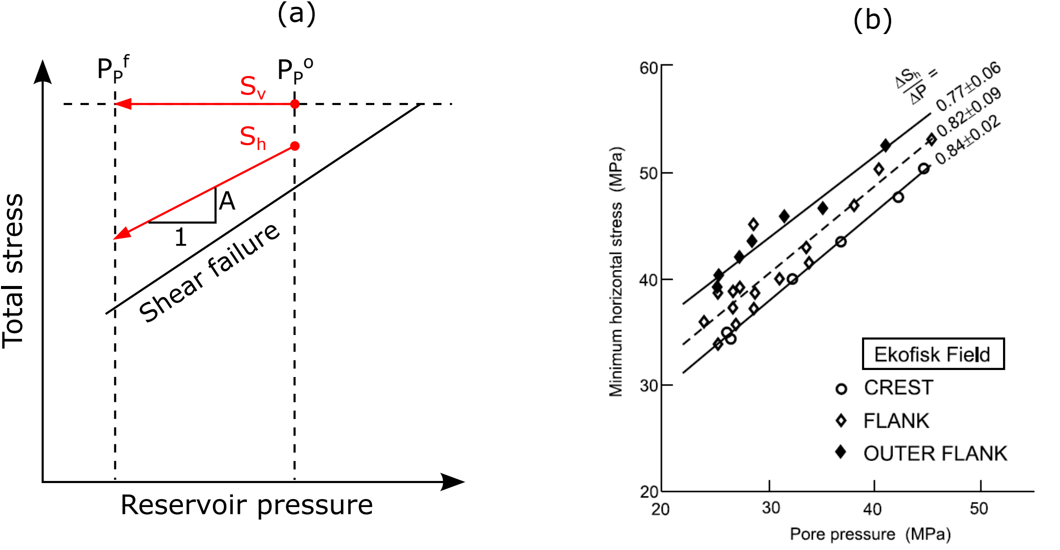
$$\Delta\varepsilon_{33} > 0$$

$$C_{bp} = \alpha / M$$

$$\Delta S_{11} = \alpha \frac{1 - 2\nu}{1 - \nu} \, \Delta P_p$$

$$\Delta S_{11} < 0$$

$$A = \alpha \frac{1-2\nu}{1-\nu}$$


(a)
P_P^f
P_P^o
S_v
S_h
A
1
Shear failure
Total stress
Reservoir pressure
(b)
$\frac{\Delta S_h}{\Delta P}$ =
0.77±0.06
0.82±0.09
0.84±0.02
Ekofisk Field
CREST
FLANK
OUTER FLANK
Minimum horizontal stress (MPa)
Pore pressure (MPa)
20
30
40
50
60

$$\sigma_{33} = S_{33} - \alpha P_p$$

$$\Delta\sigma_{33} = -\alpha\Delta P_p$$

$$\Delta\sigma_{11} = -\alpha\frac{\nu}{1-\nu}\Delta P_p$$

ΔH

$$\Delta S_h$$

$$\Delta\sigma_v$$

$$\Delta\sigma_h$$

k/k_o

$$c_f = 0.25$$

$$k = k_o \exp\left[-c_f(\sigma_h - \sigma_h^o)\right]$$

$$M = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = 14.8$$

$$\Delta H = \Delta \varepsilon_{33} H$$

$$\varepsilon_{33} = -\frac{\alpha}{M}\Delta P_p = 0.21\%$$

$$\Delta H = 0.21\% \; 100$$

$$= 0.21$$

$$C_{bp} = \frac{\alpha}{M} = 0.06 \times 10^{-9} \frac{1}{\mathrm{Pa}} = 0.42 \times 10^{-6} \frac{1}{\mathrm{psi}}$$

$$C_{pp} = \frac{C_{bp}}{\phi} = 2.0 \times 10^{-6} \frac{1}{\text{psi}} = 2.0\ \mu\text{sip}$$

$$\Delta S_h = \alpha \frac{1 - 2\nu}{1 - \nu} \, \Delta P_p = 0.716 \times (-35$$

$) \sim -25$

$$\Delta\sigma_{33} = -\alpha \Delta P_p = -0.9 \times (-35$$

$) = +31.5$

$$\Delta\sigma_{11} = -\alpha\frac{\nu}{1-\nu}\Delta P_p = +6.45$$

$$\frac{k}{k_o} = \exp\left[-c_f(\sigma_h - \sigma_h^o)\right] = \exp\left[-0.25 \text{ MPa}^{-1}(+6.45 \text{ MPa})\right] \sim 0.2 \quad \blacksquare$$