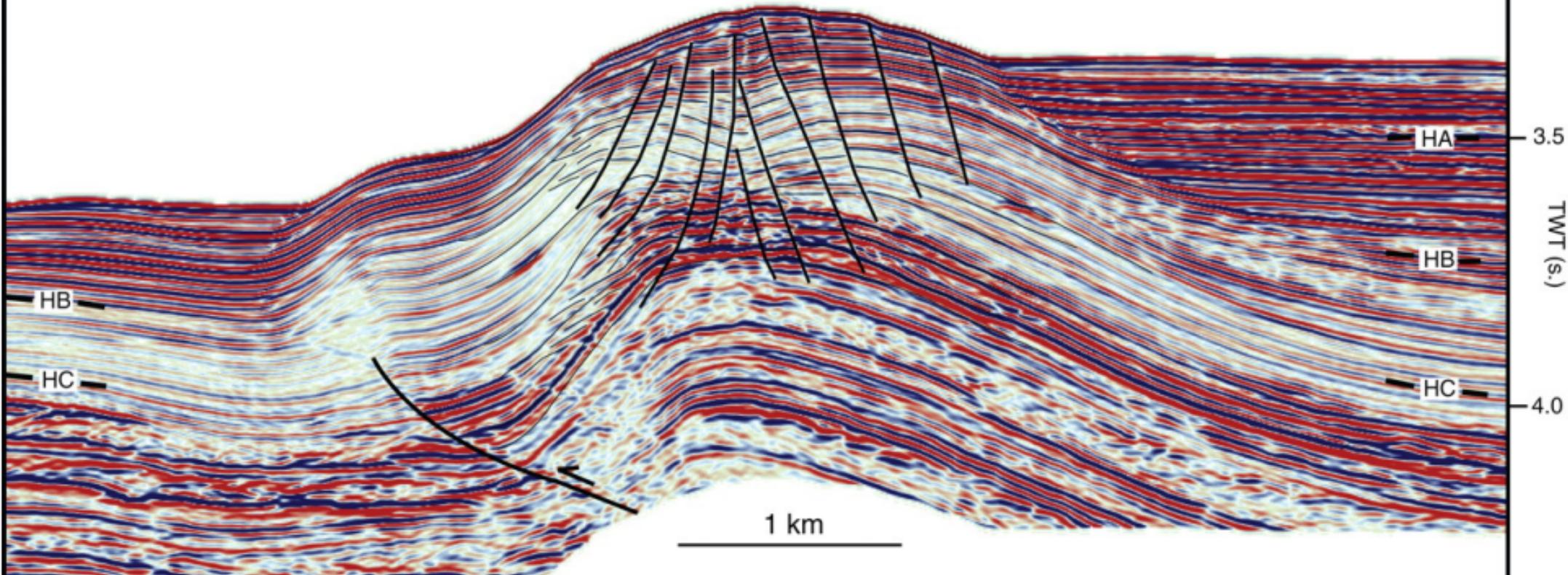


NW

Crestal normal faults

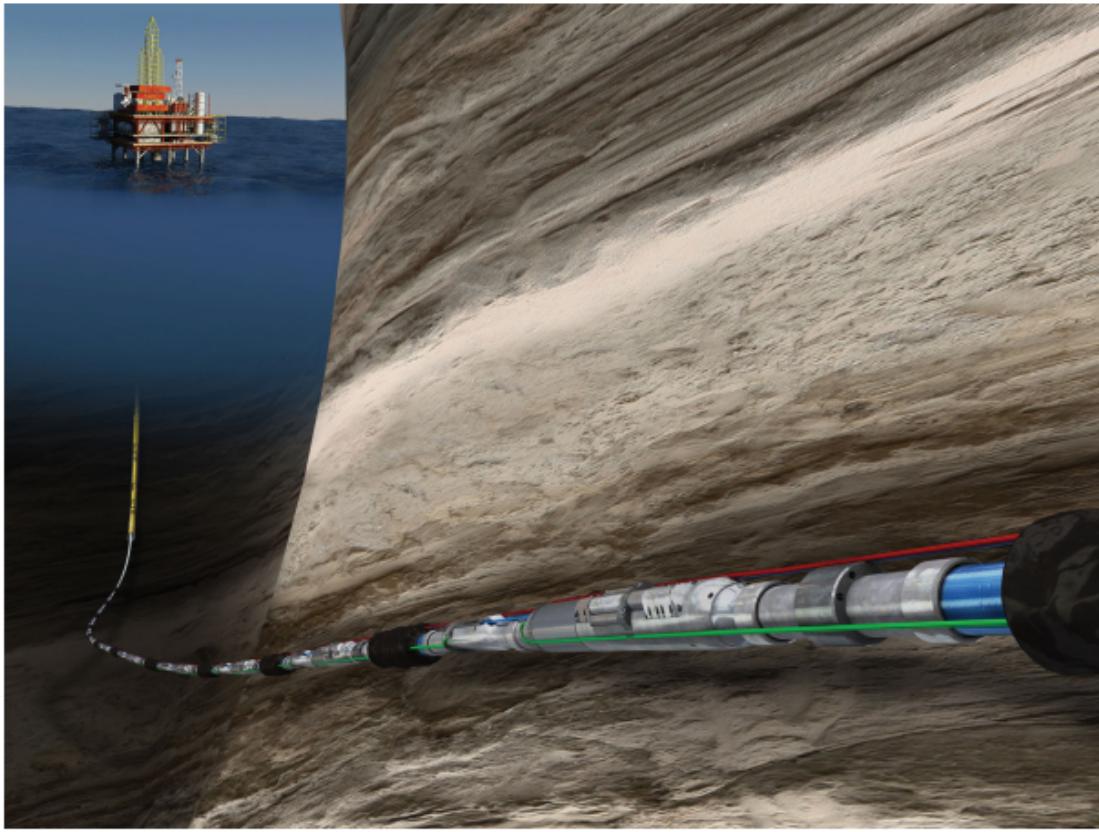
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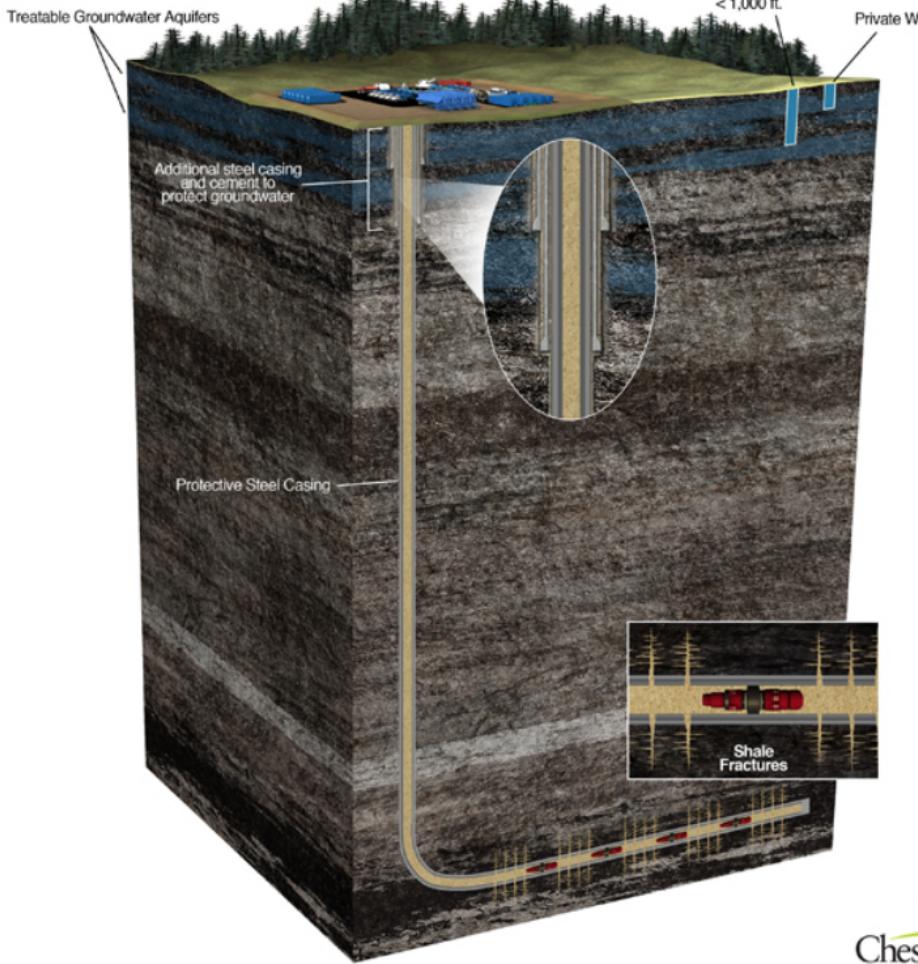


~1850



~2010





Risk-Based
Geomechanical
Screening

Stress Man

MUDLINE SUBSIDENCE



FAULT ACTIVATION

CASING CRUSHING

COMPACTION DRIVE

CASING
SHEAR

SAND PRODUCTION



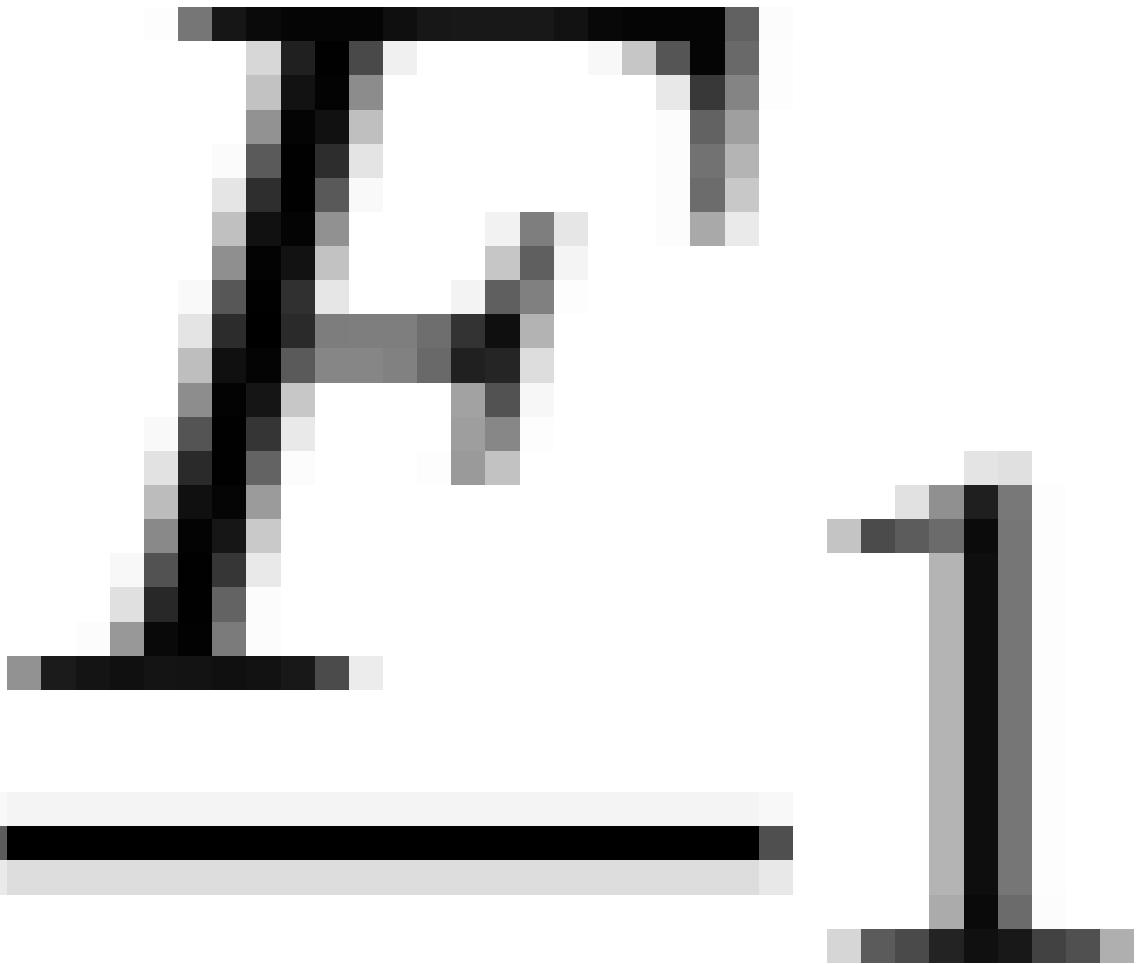
RESERVOIR YIELD

PERMEABILITY LOSS

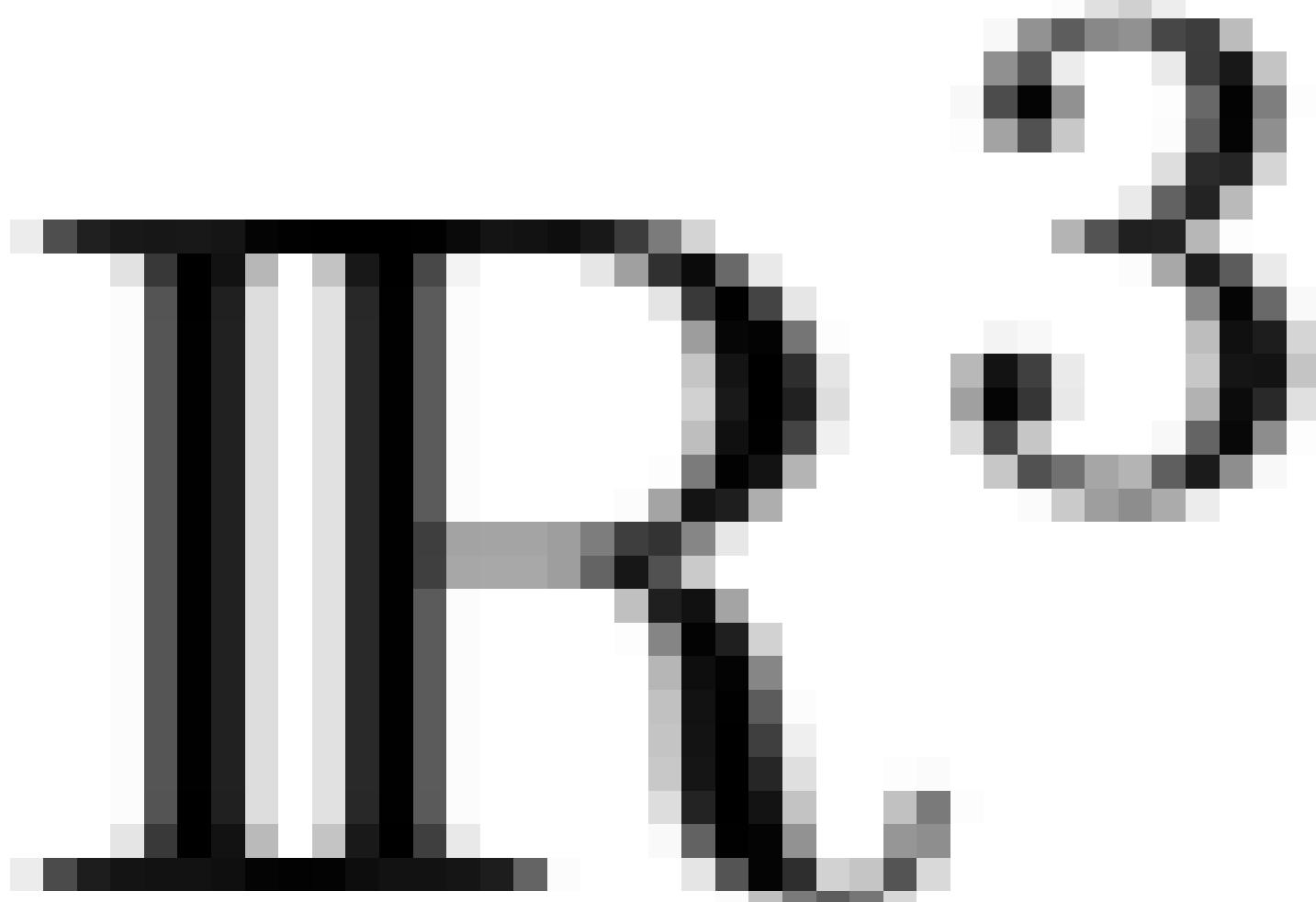
CASING BUCKLING

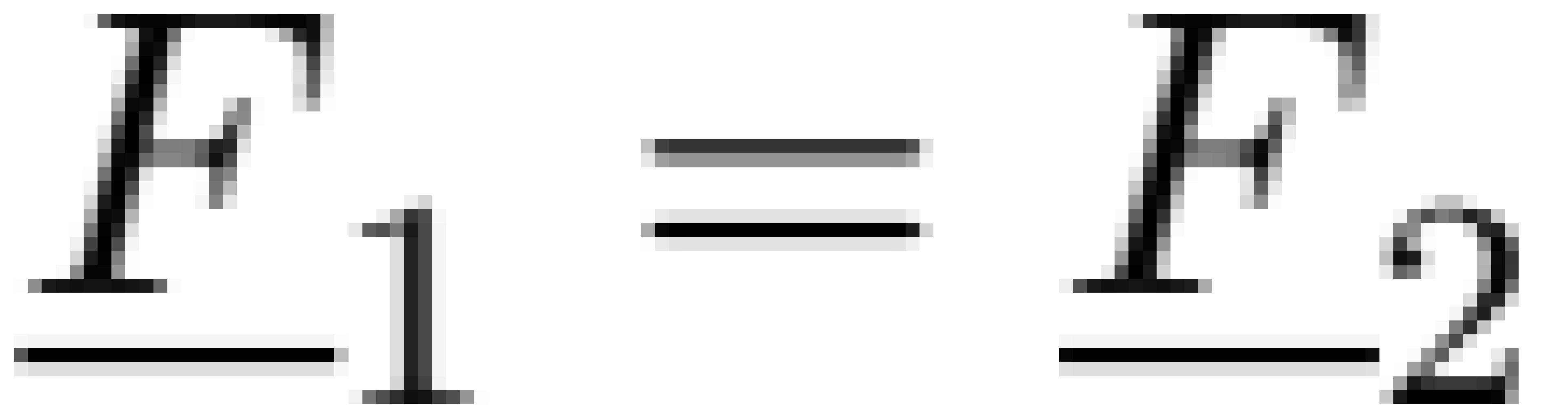
S_V



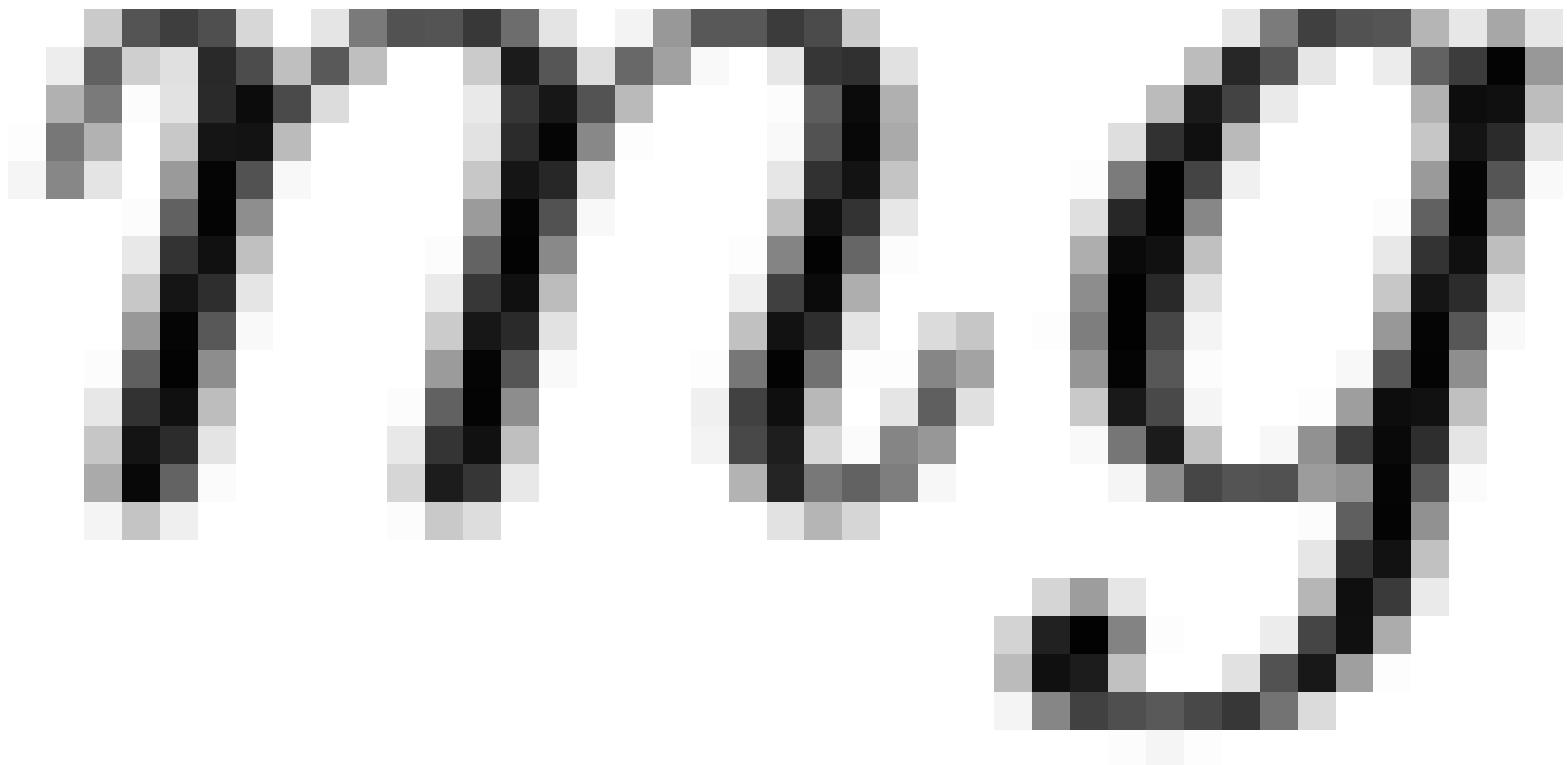


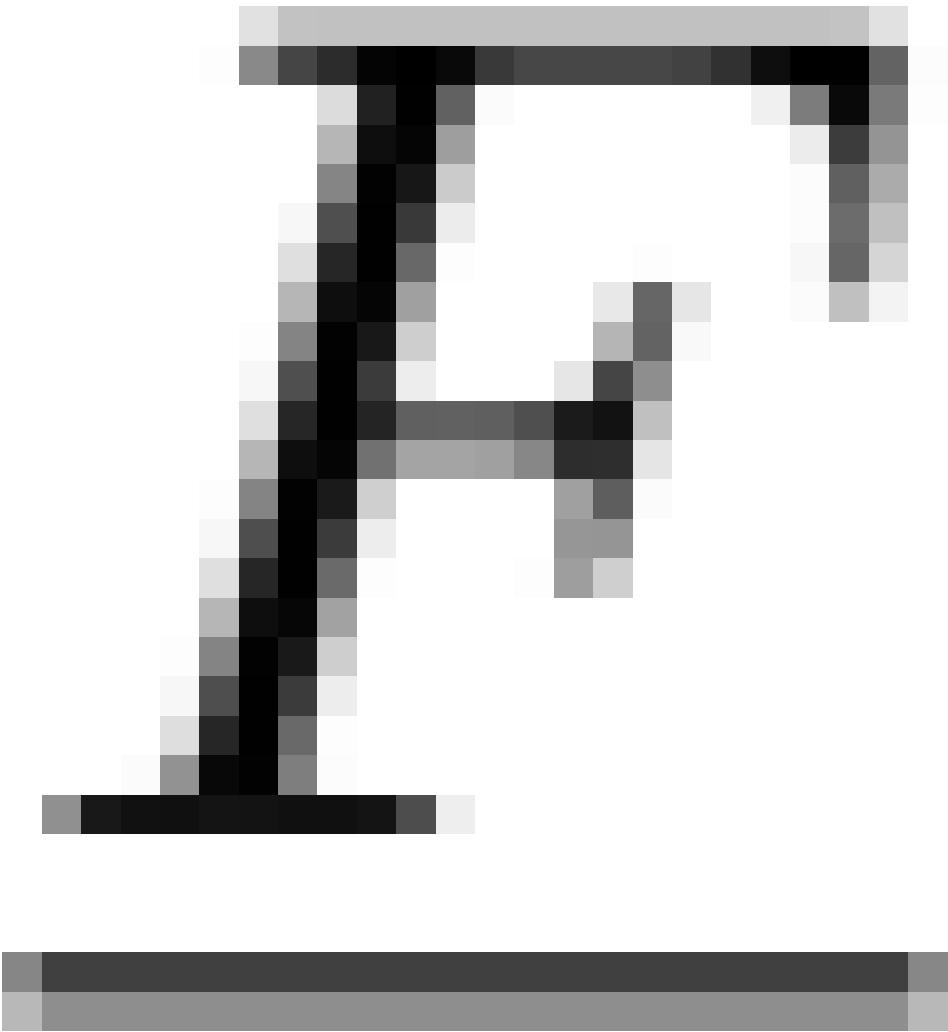


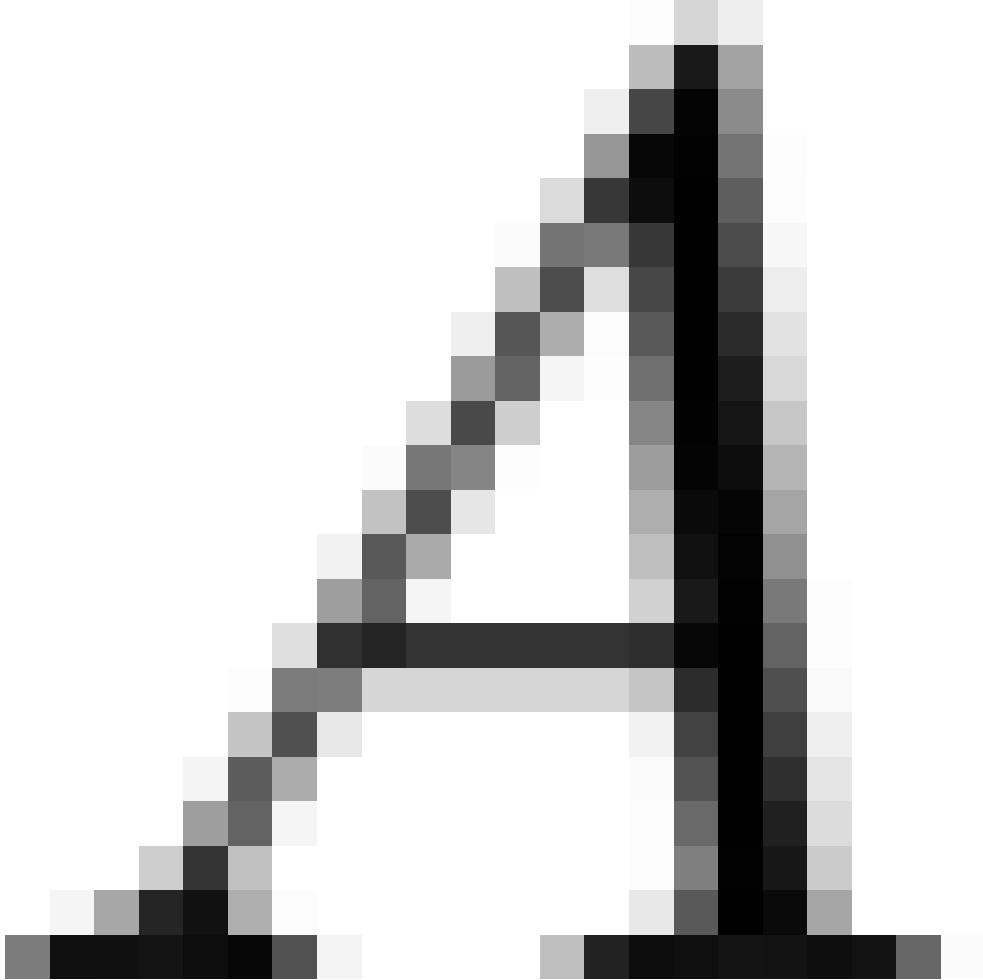












P

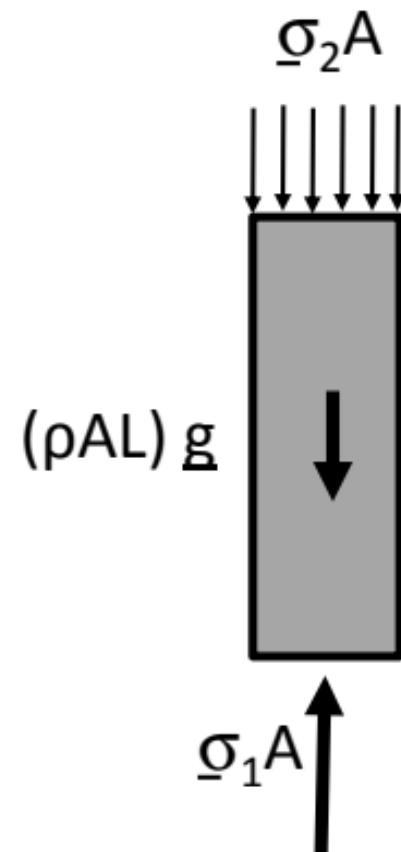
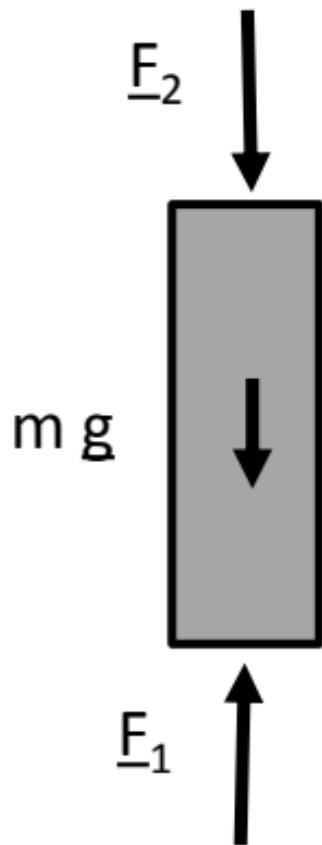
R

S

A

T

C

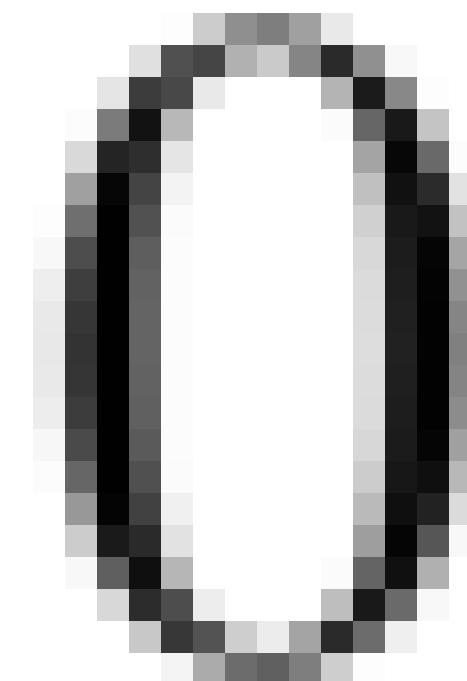
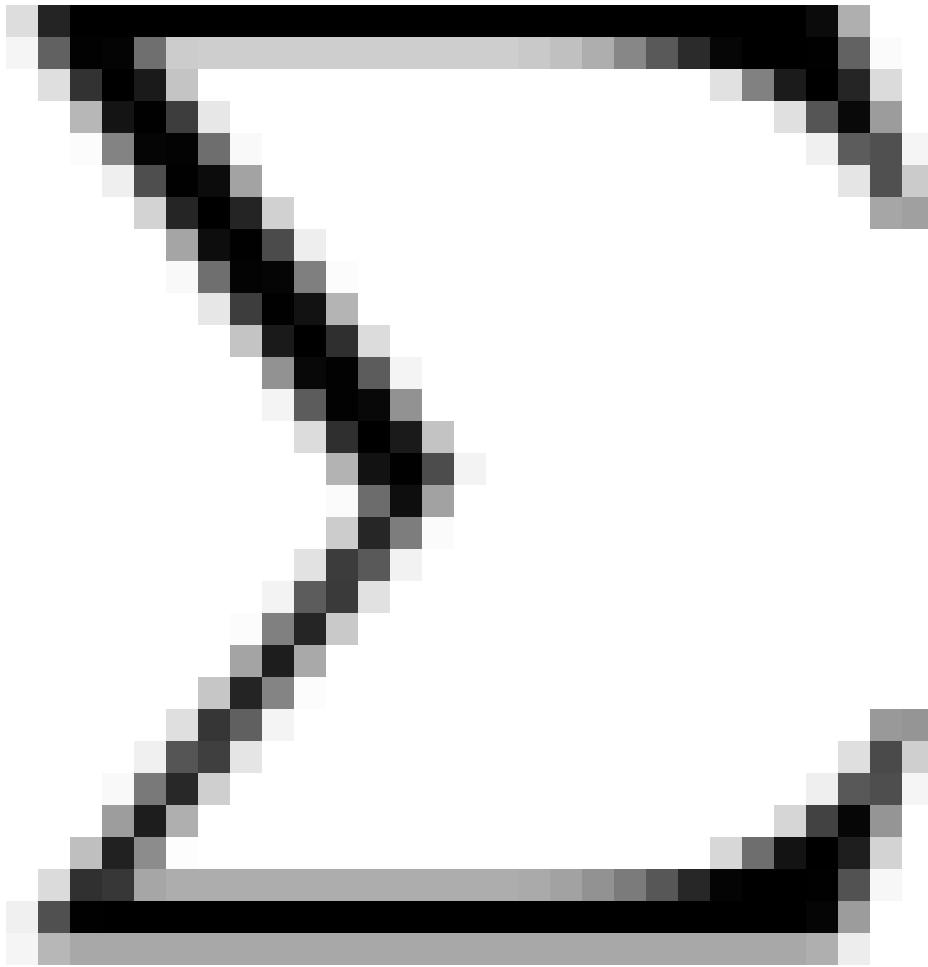


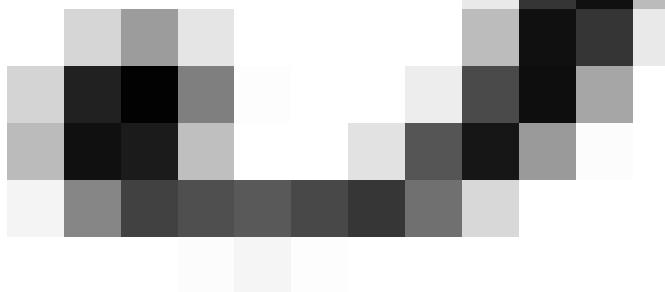
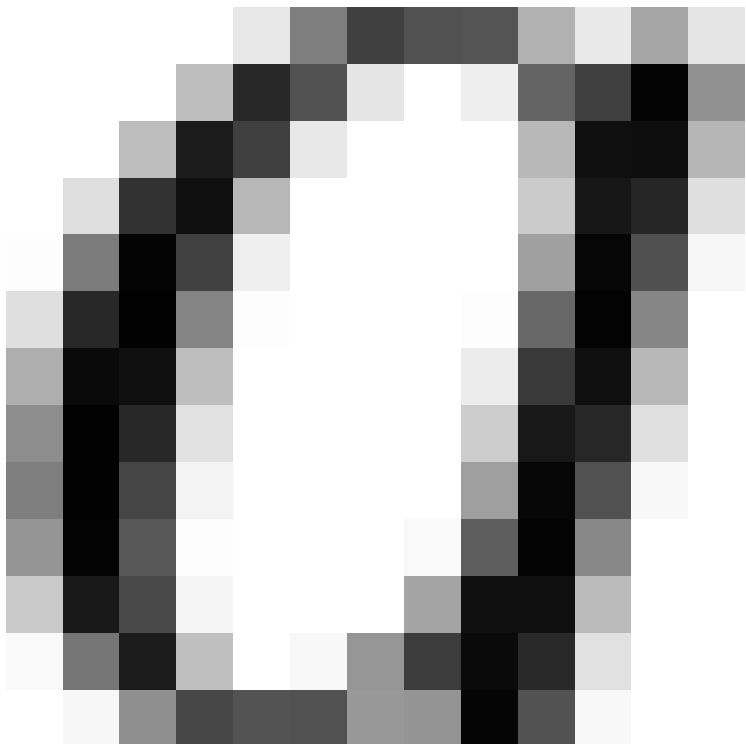
$$\Sigma F_z = +F_1 - F_2 = 0$$

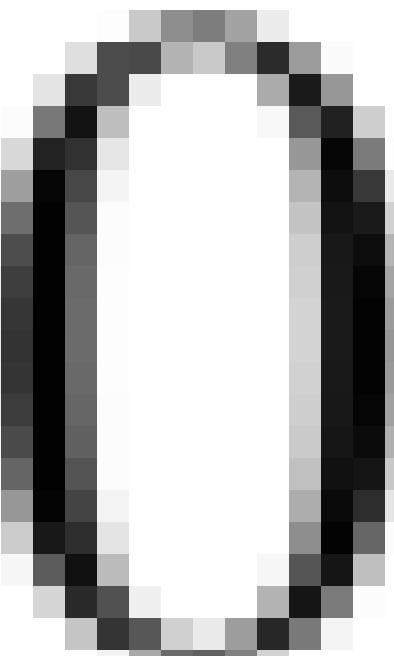
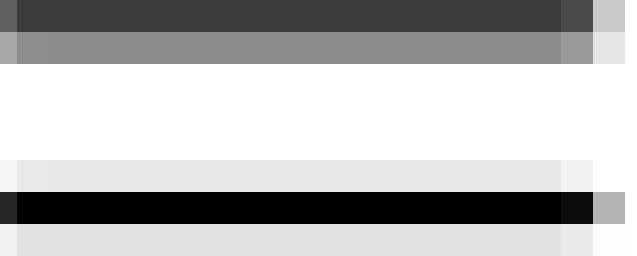
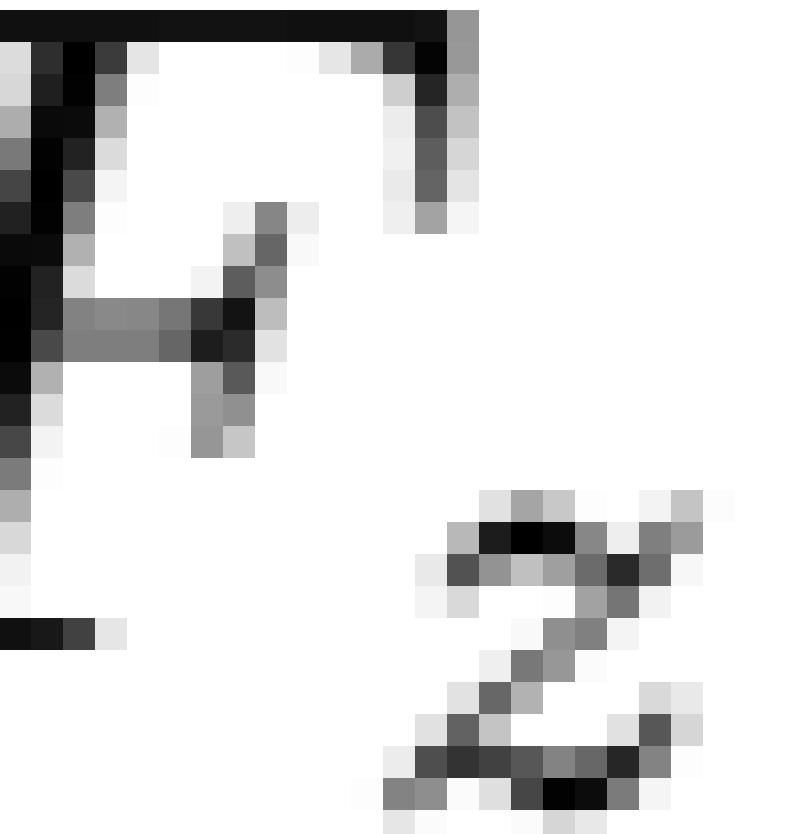
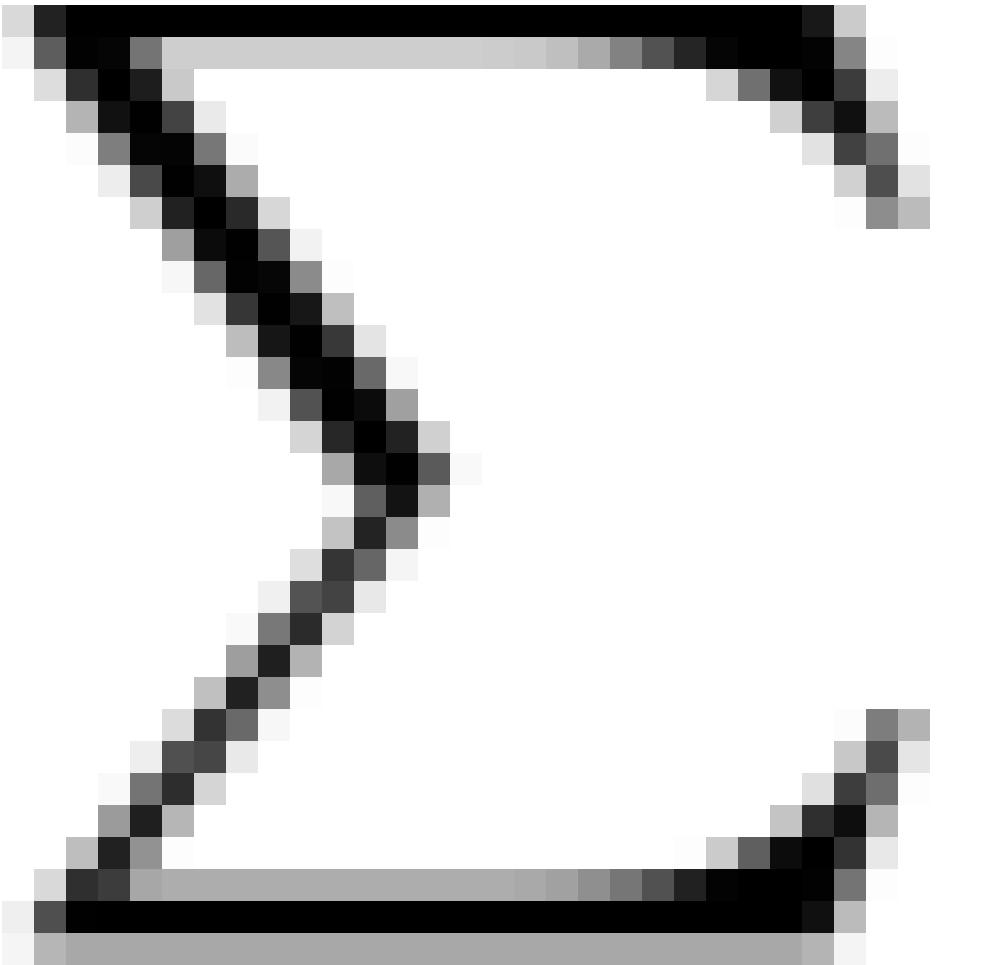
$$\Sigma F_z = +F_1 - m g - F_2$$

$$\Sigma F_z = +\sigma_1 A - (\rho A L)g - \sigma_2 A$$











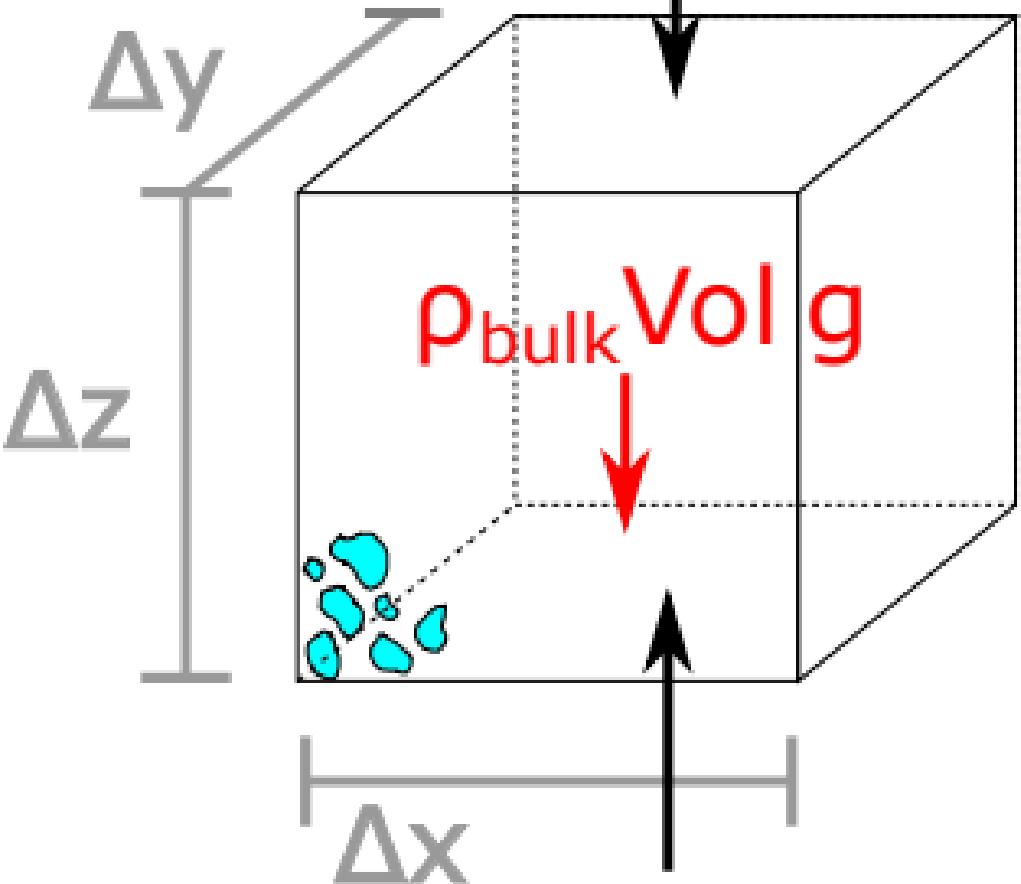


△ S₂

— △ z

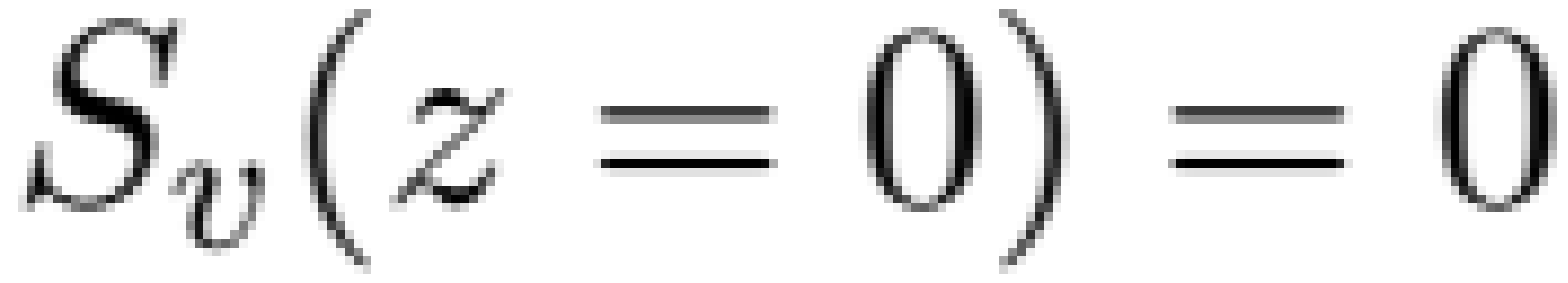
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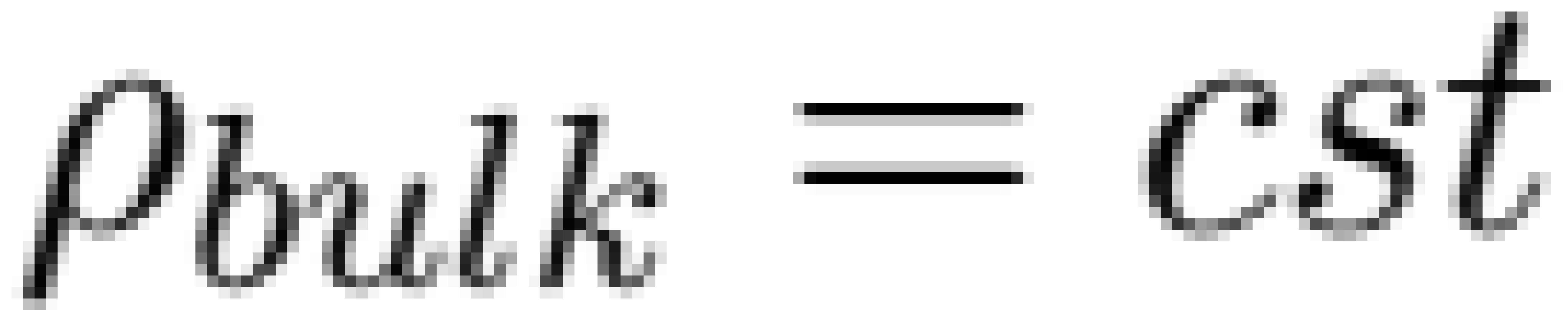
Observe 9

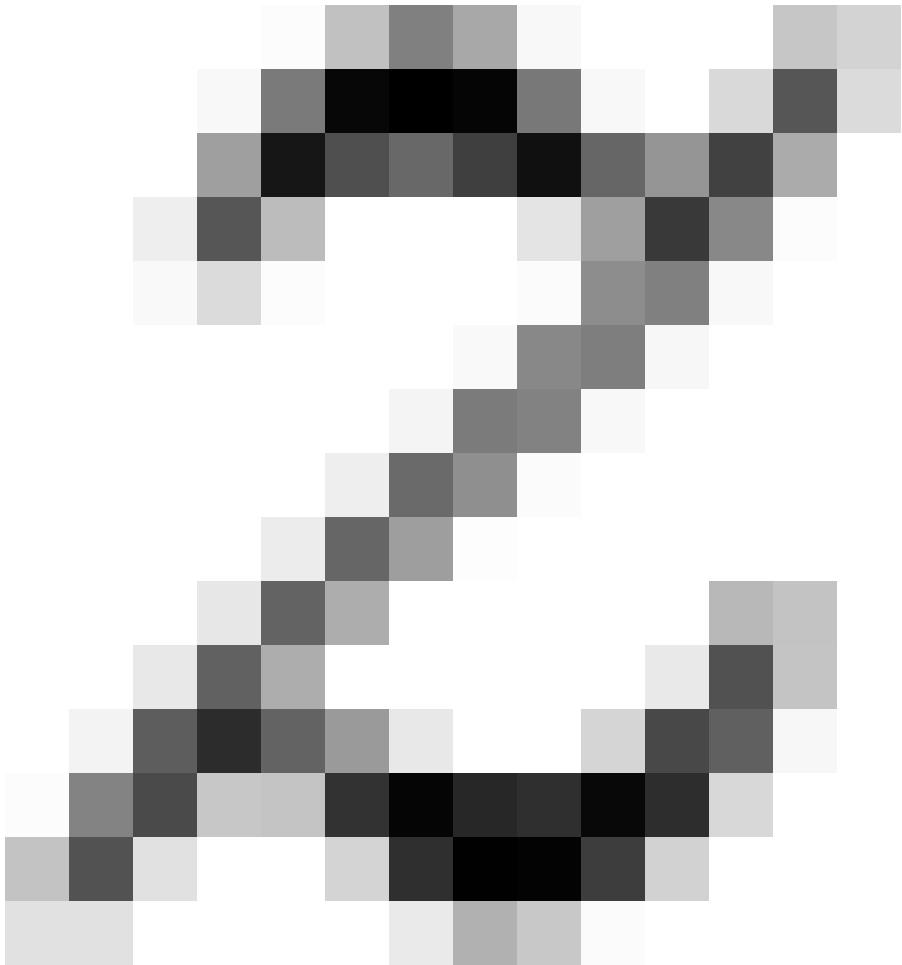
$S_v A$  $S_v A + \Delta S_v A$

$$\int_0^{S_w(z)} ds_w = \int_0^z \rho_{bulk}(z) g dz$$







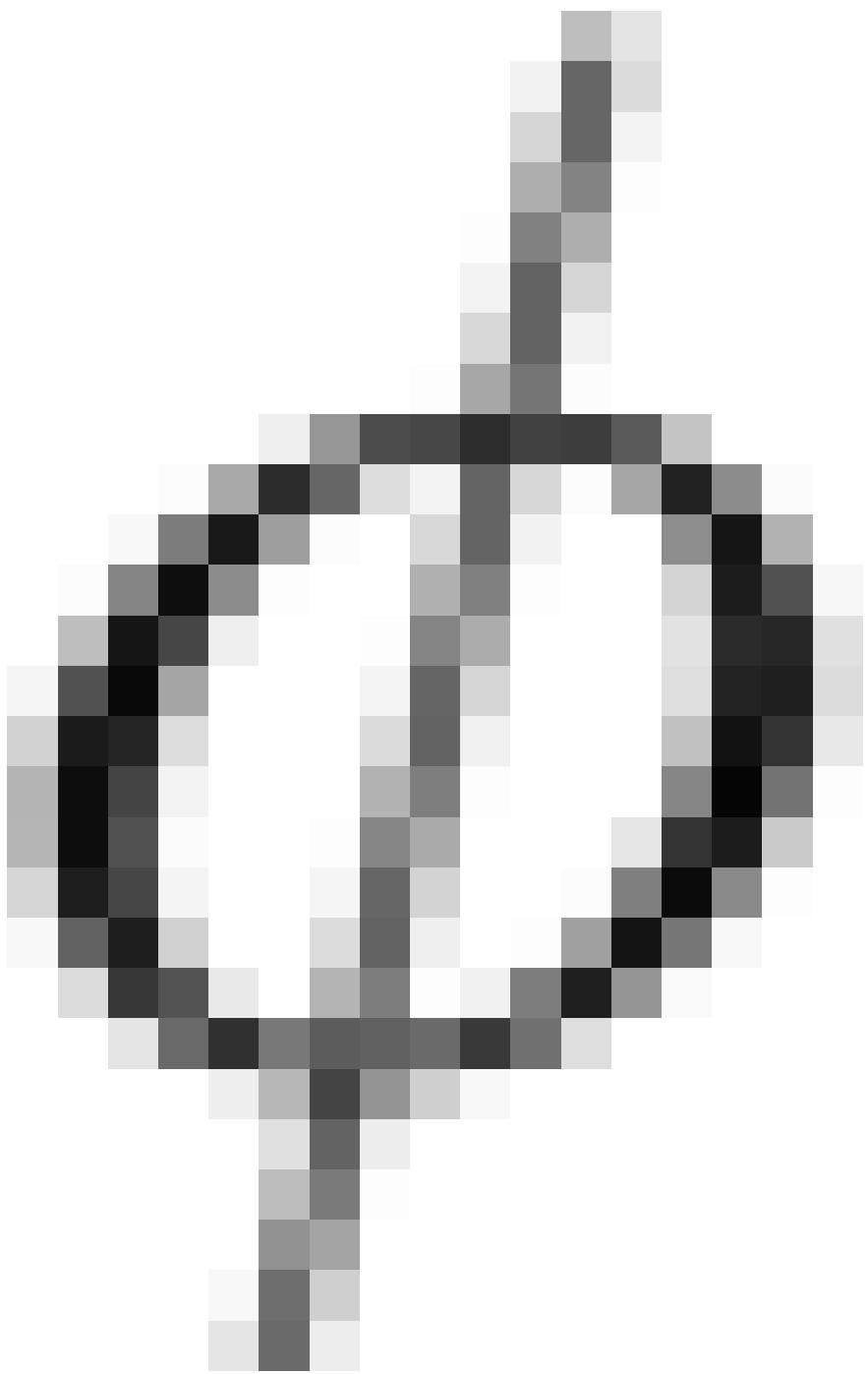


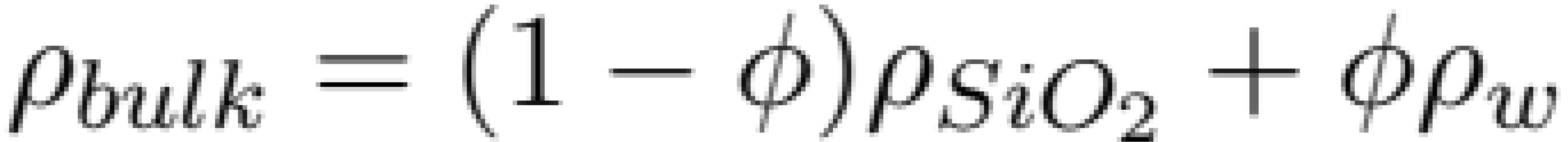




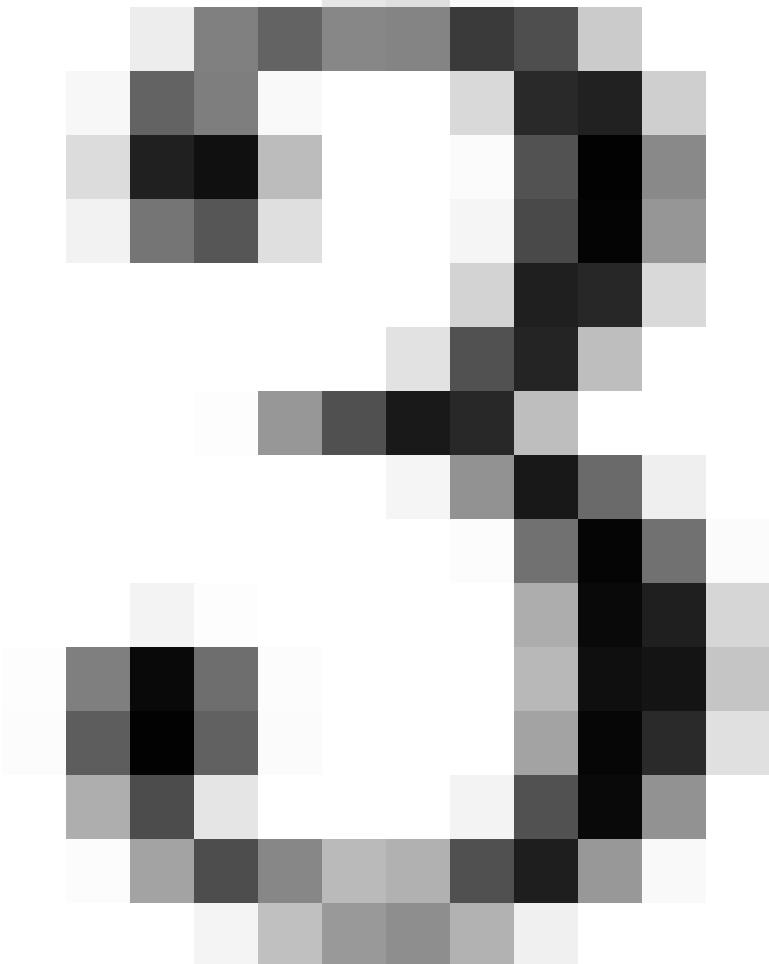


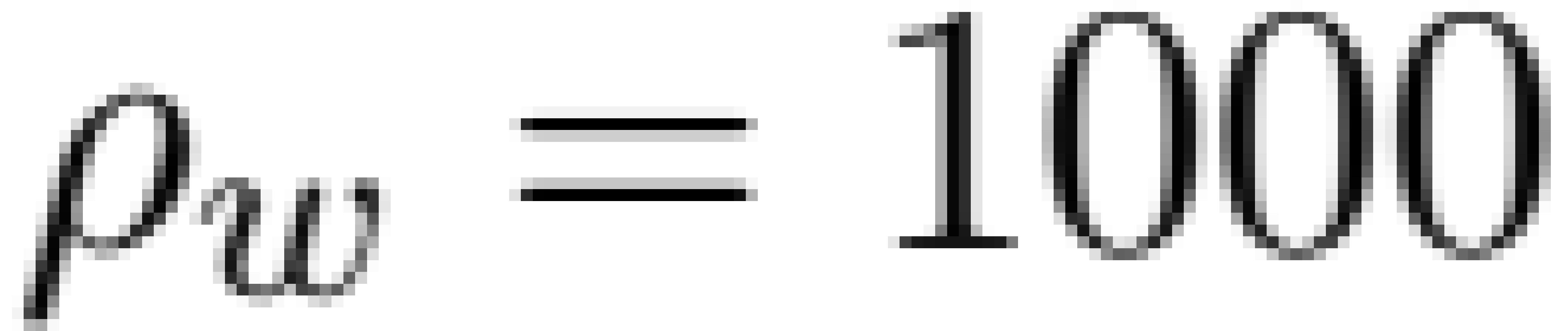


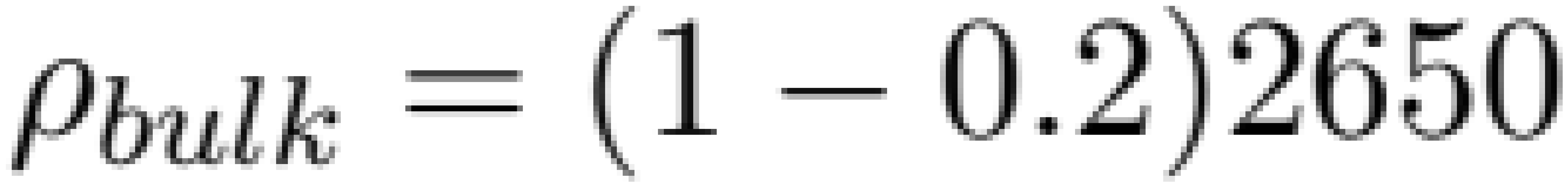


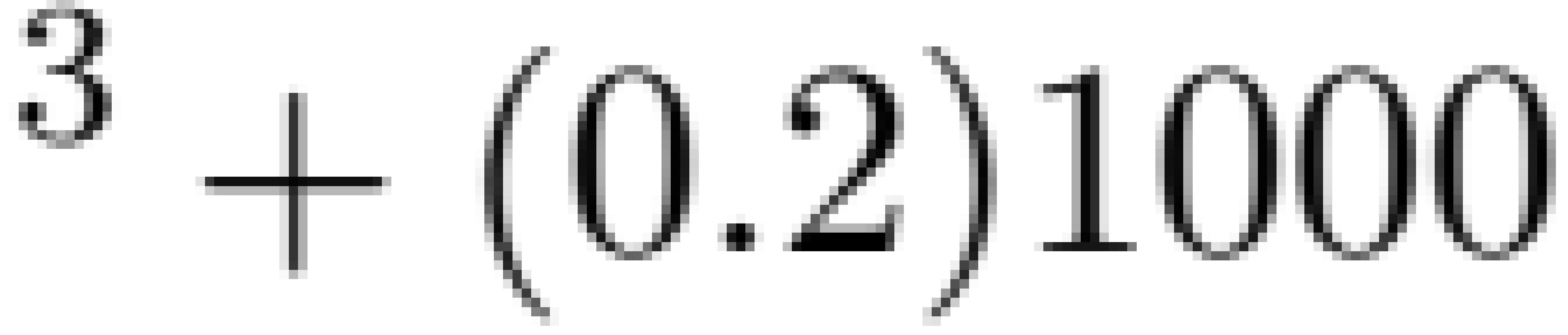




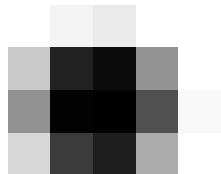
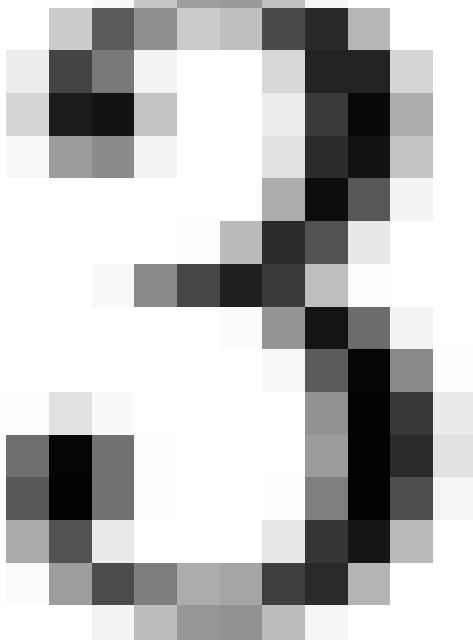


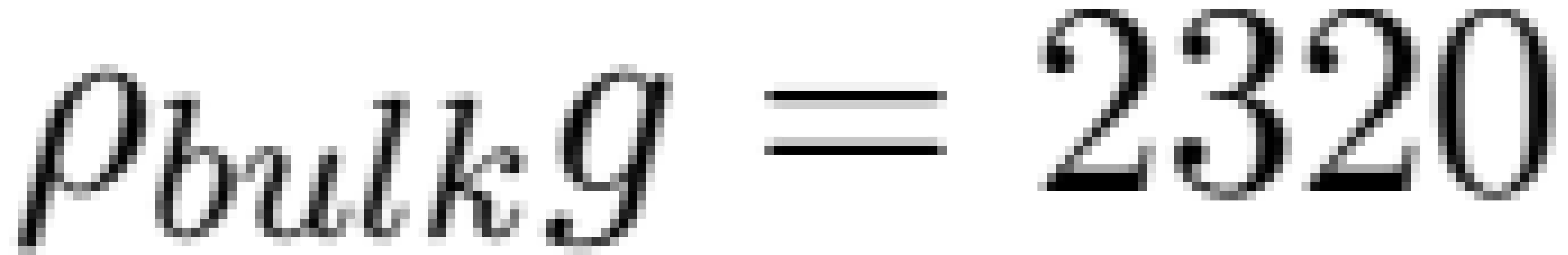




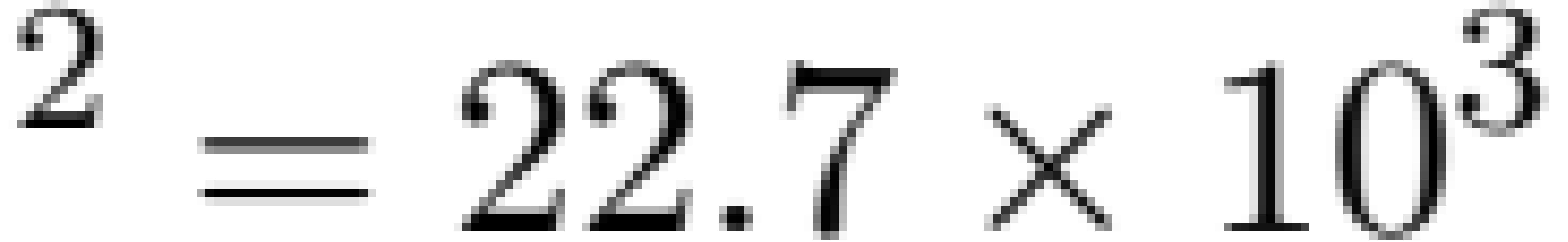


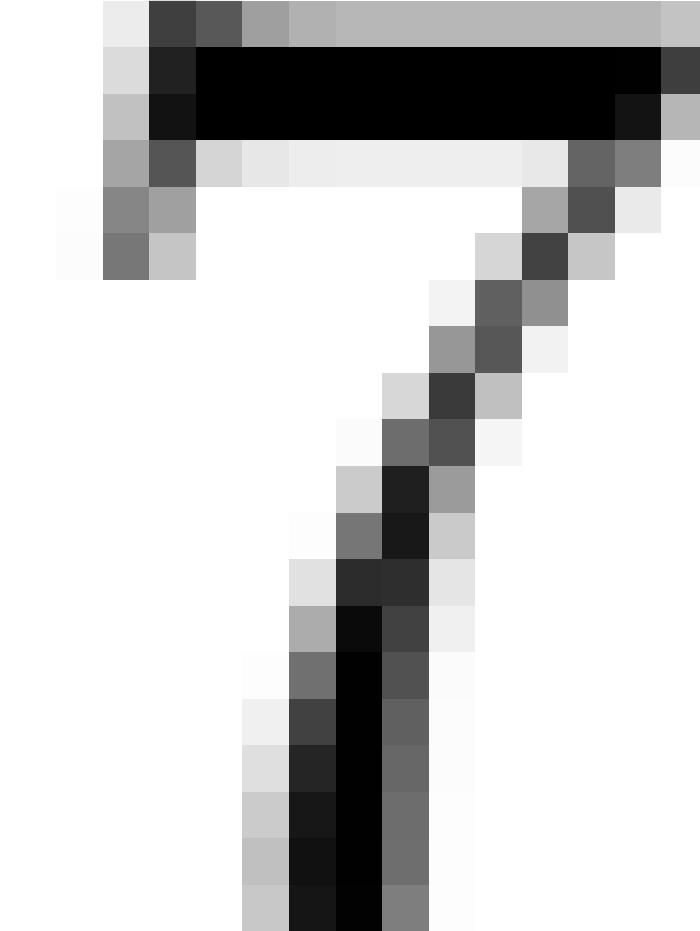
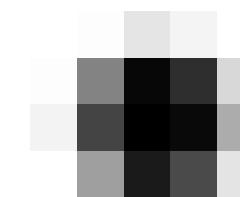
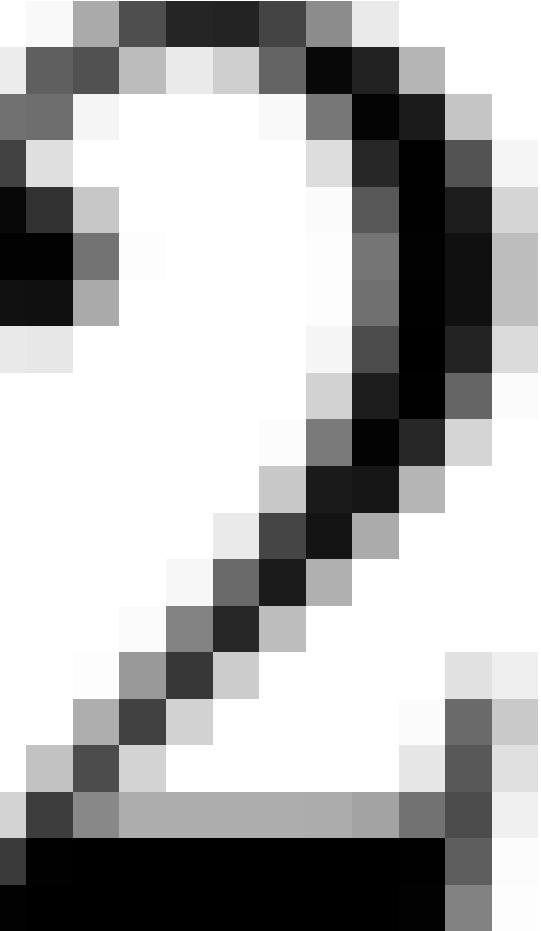
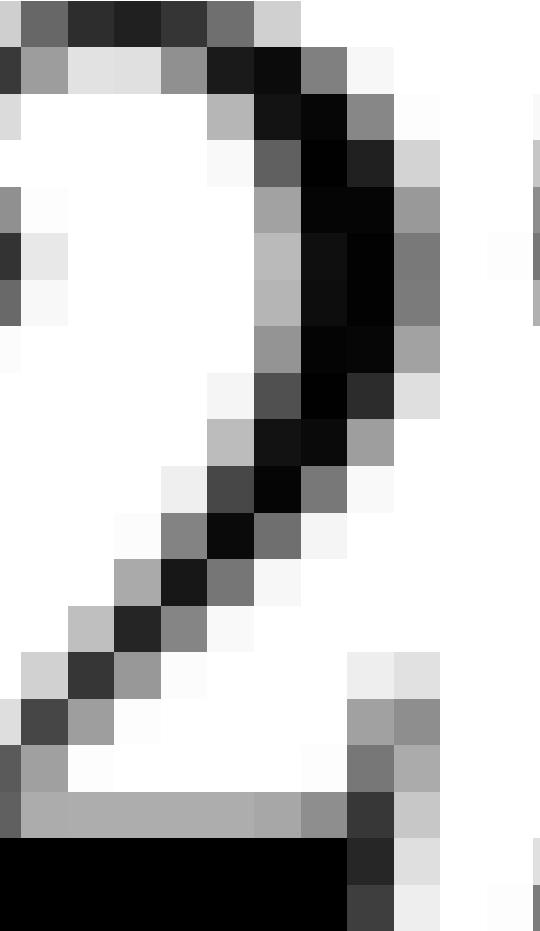


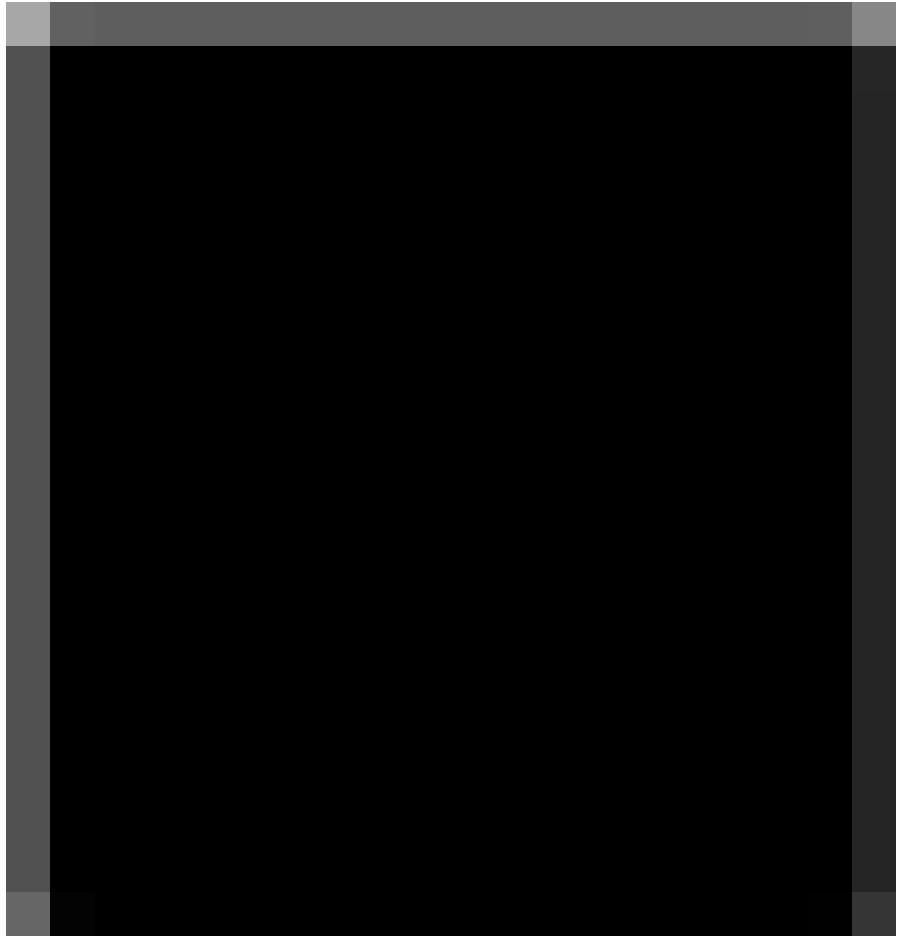
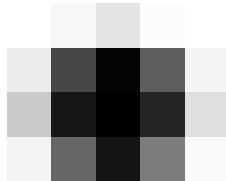


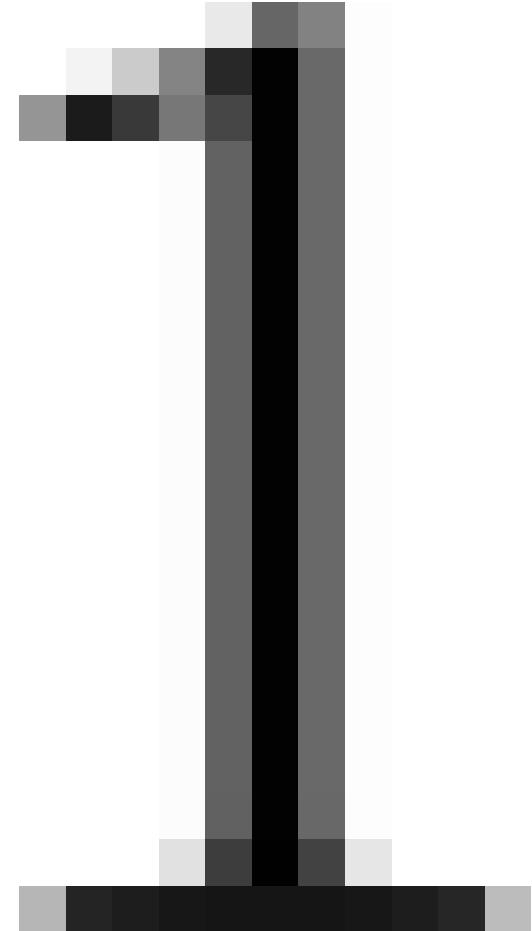
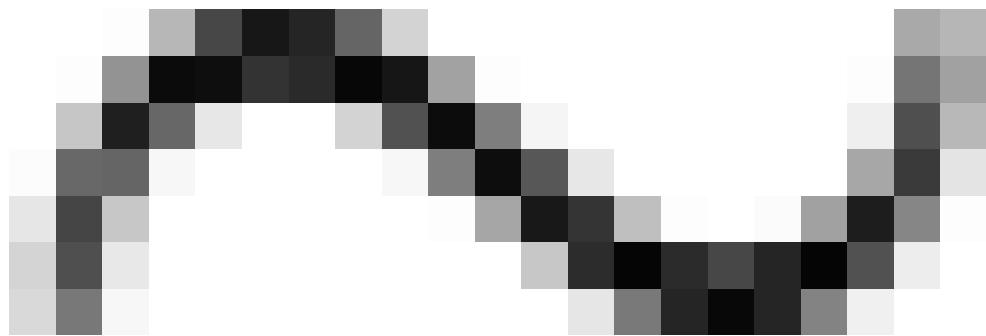


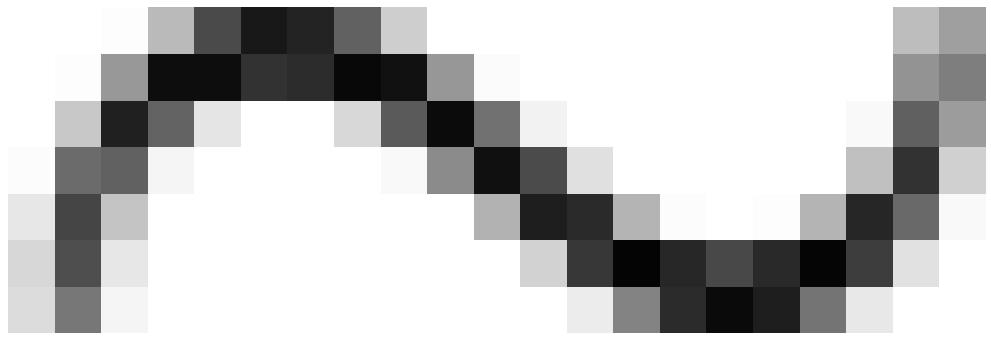


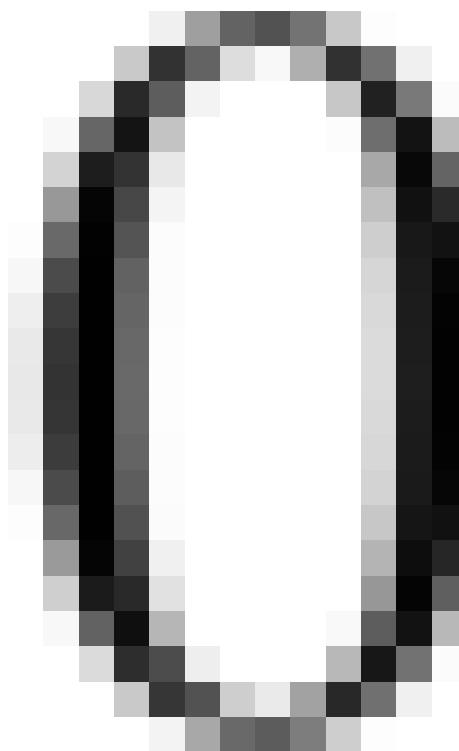
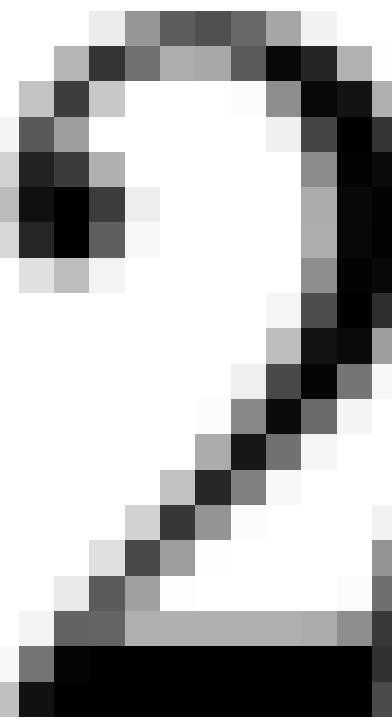
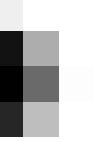
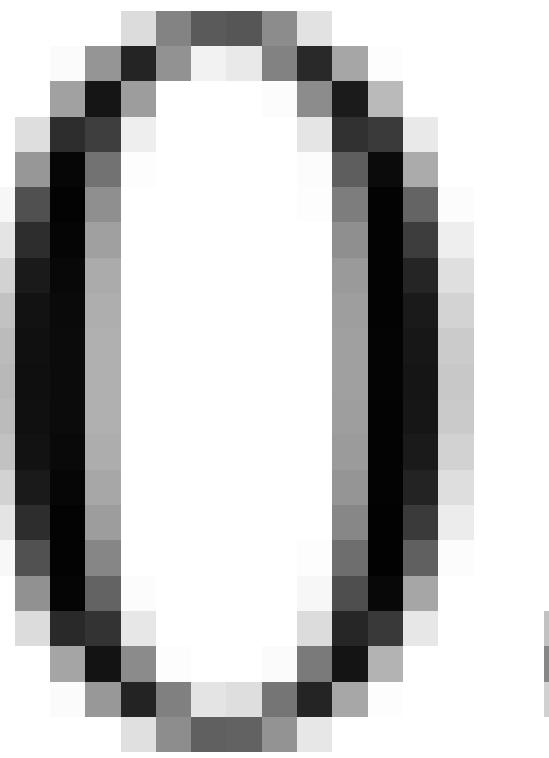
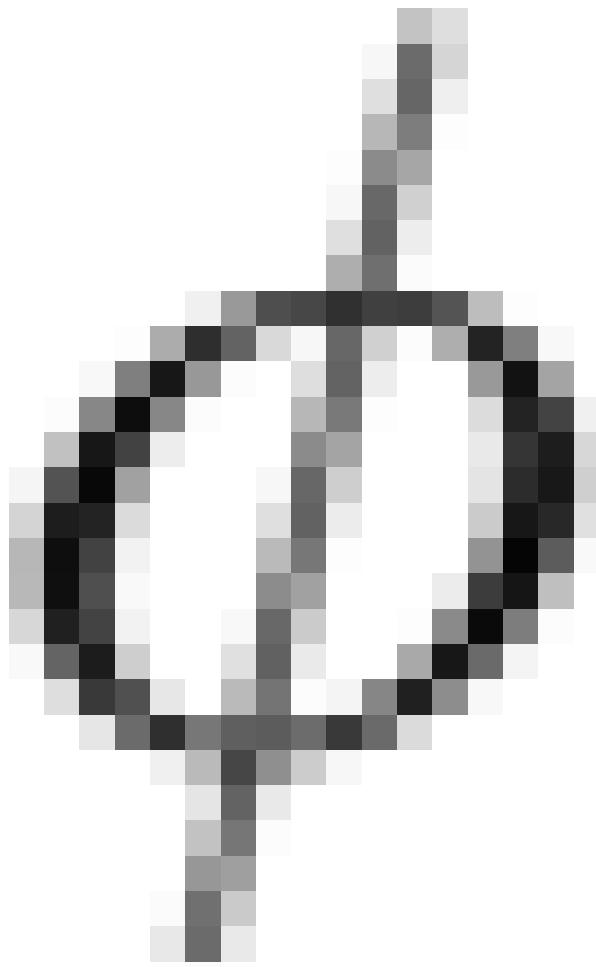


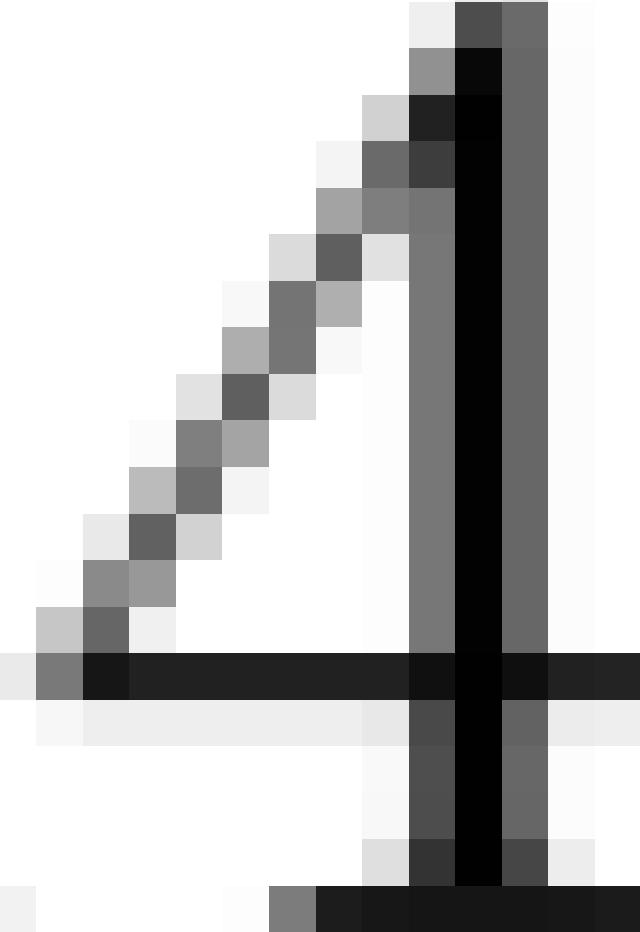
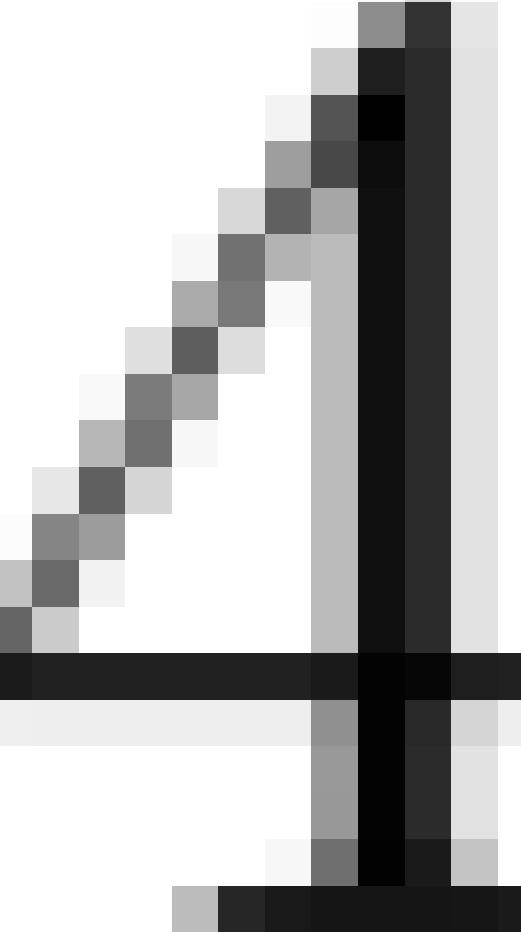
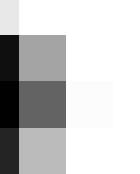
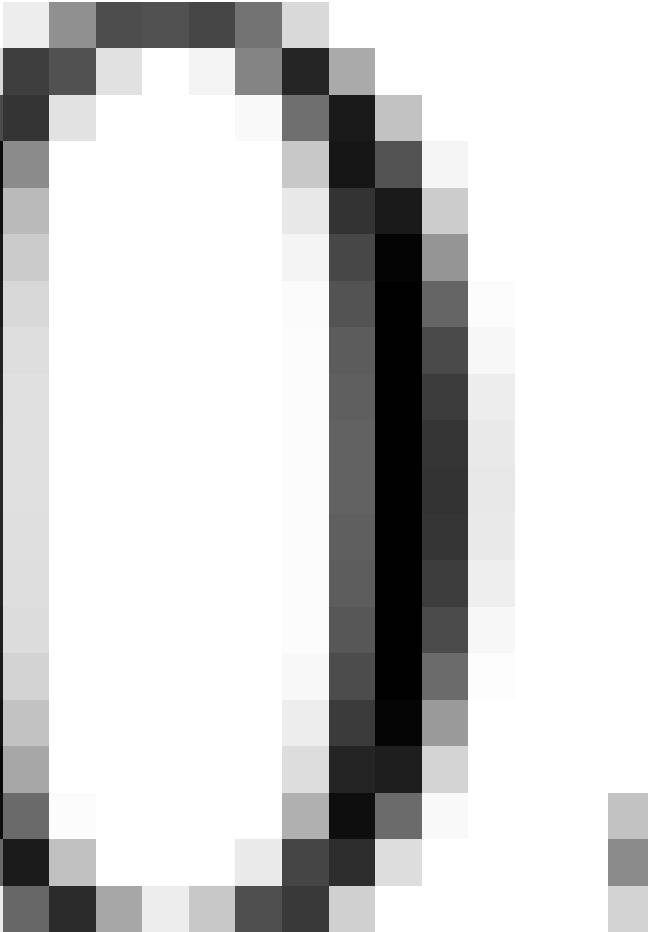
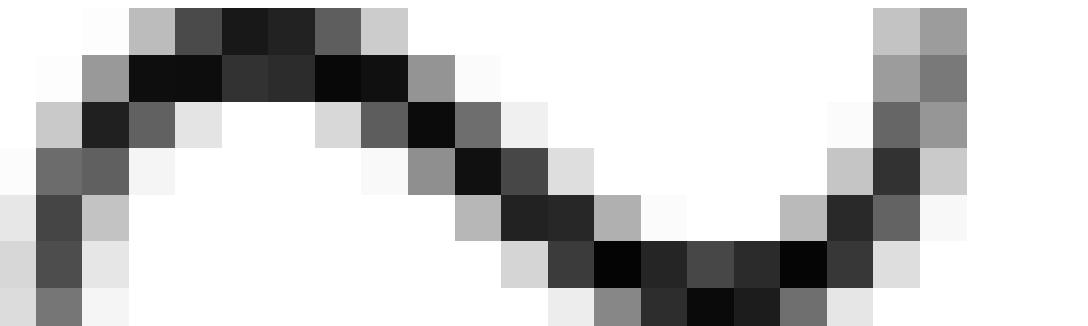


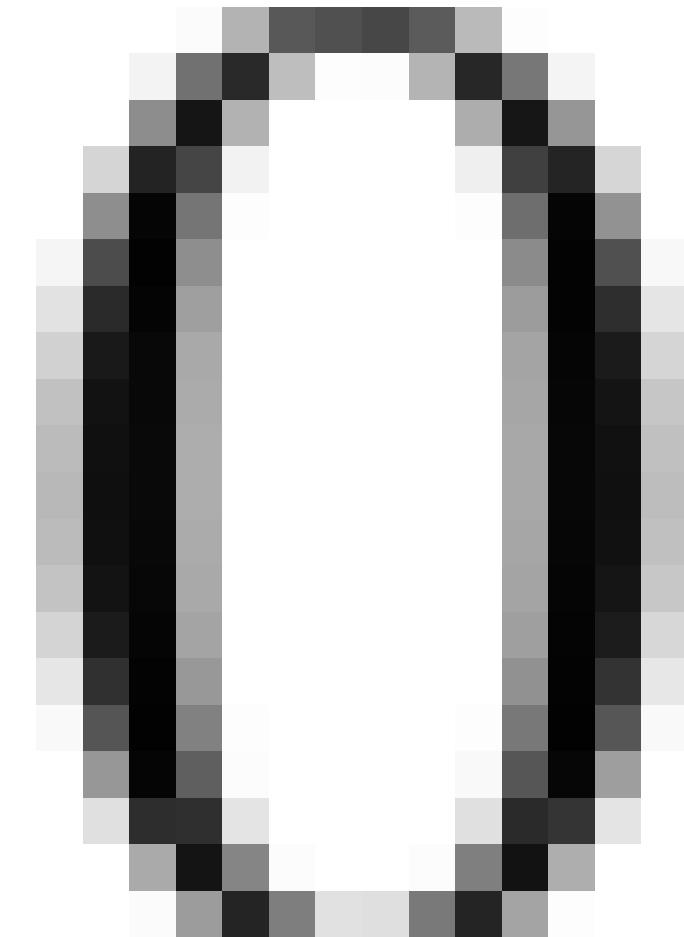
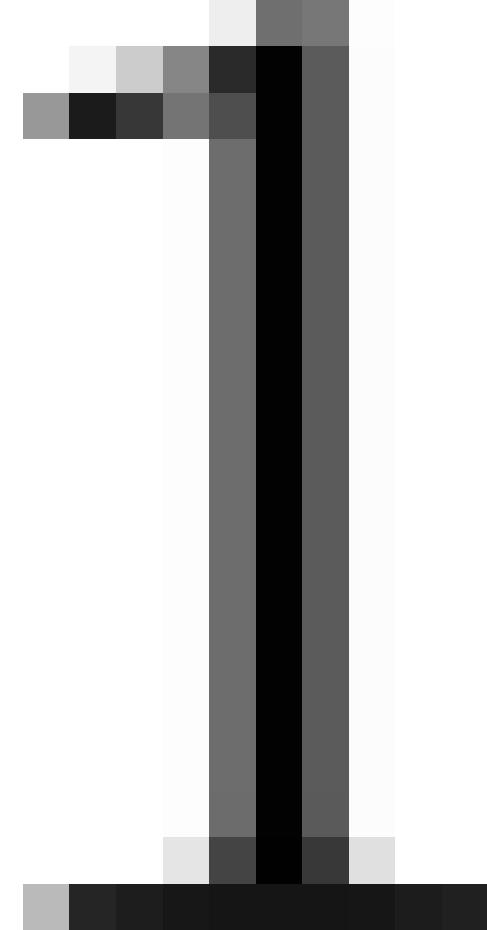
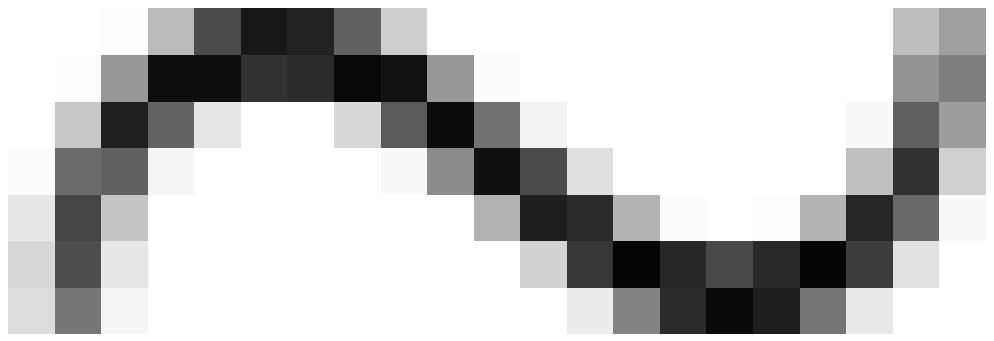




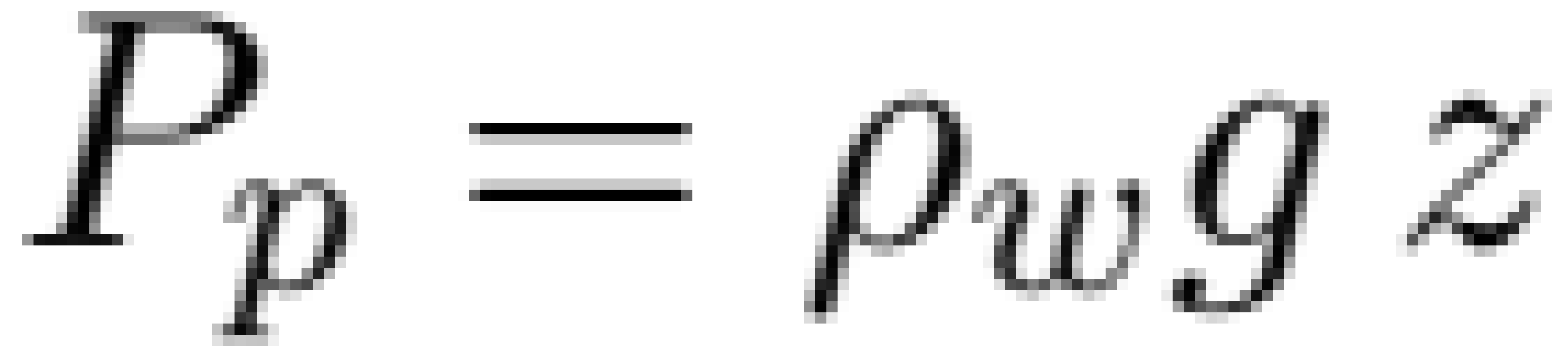


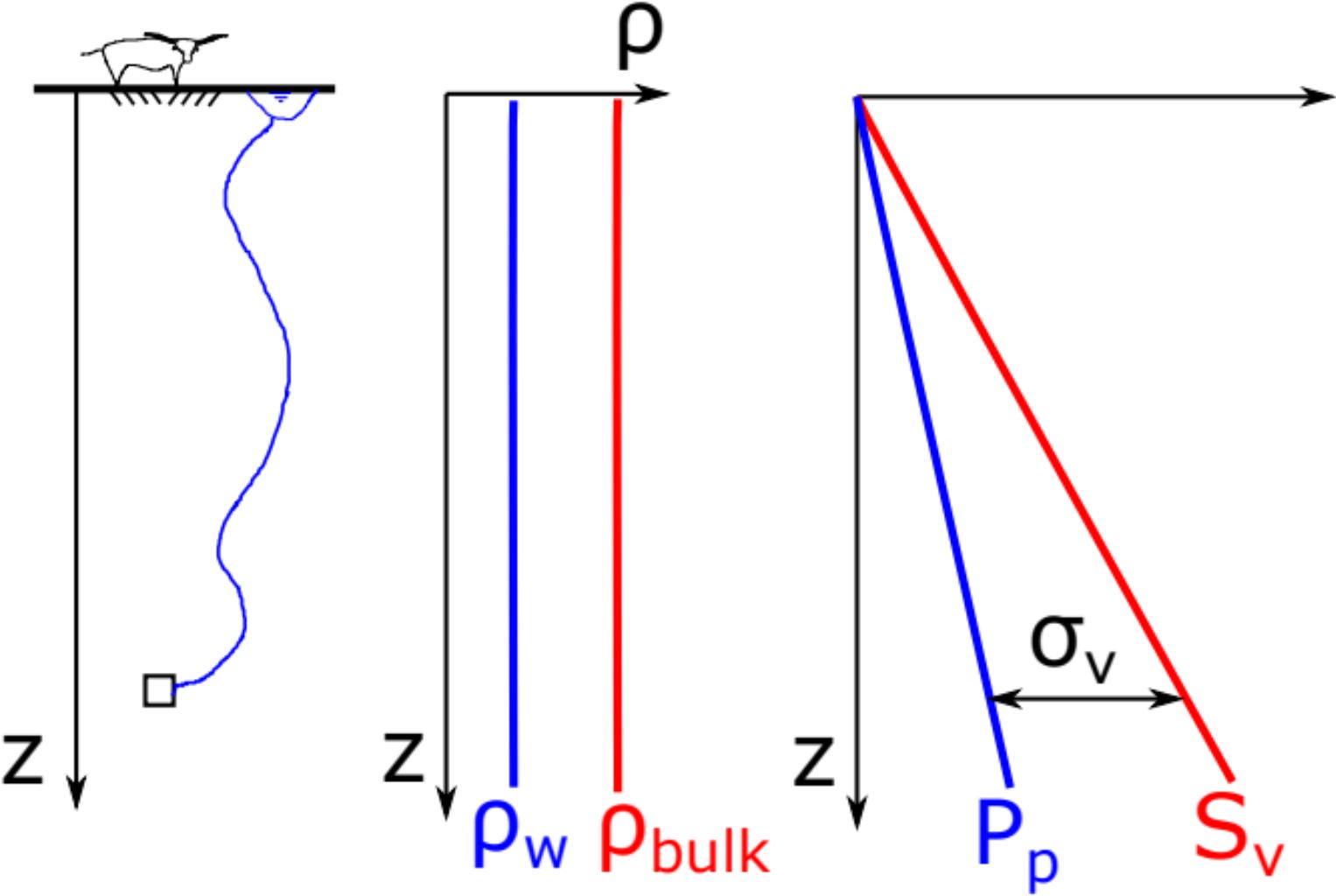




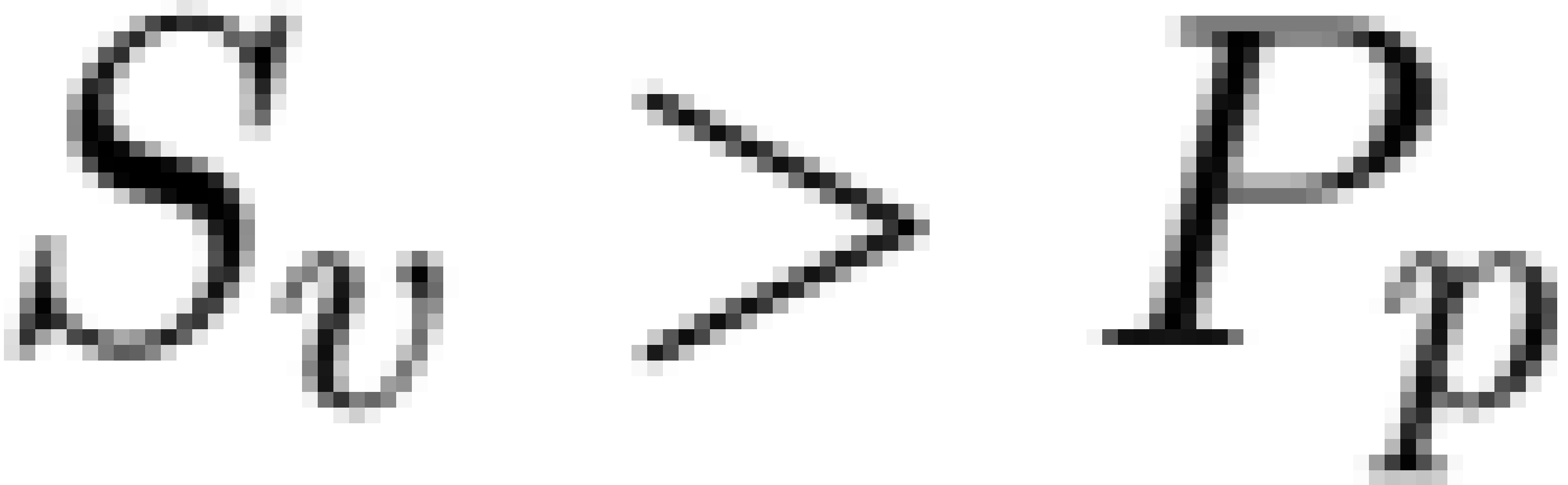








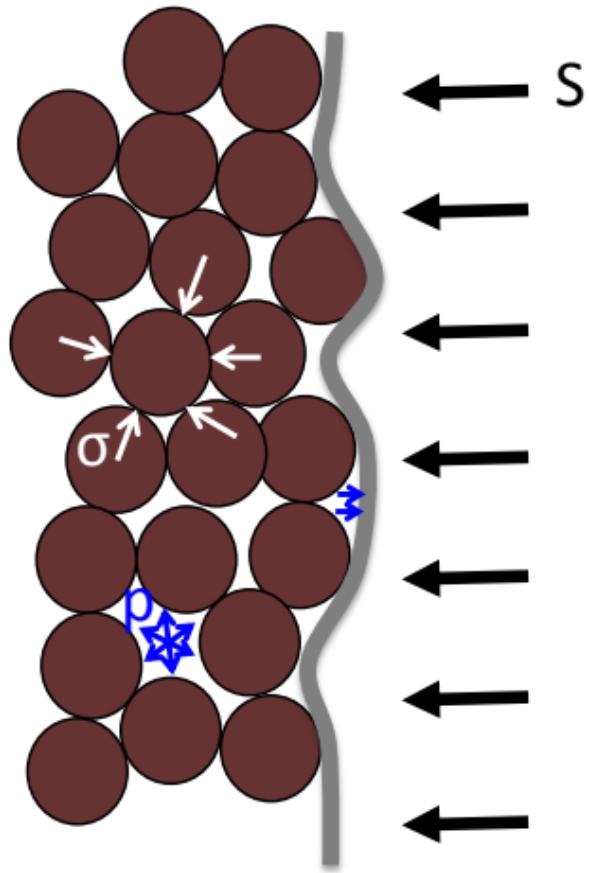




Effective stress =

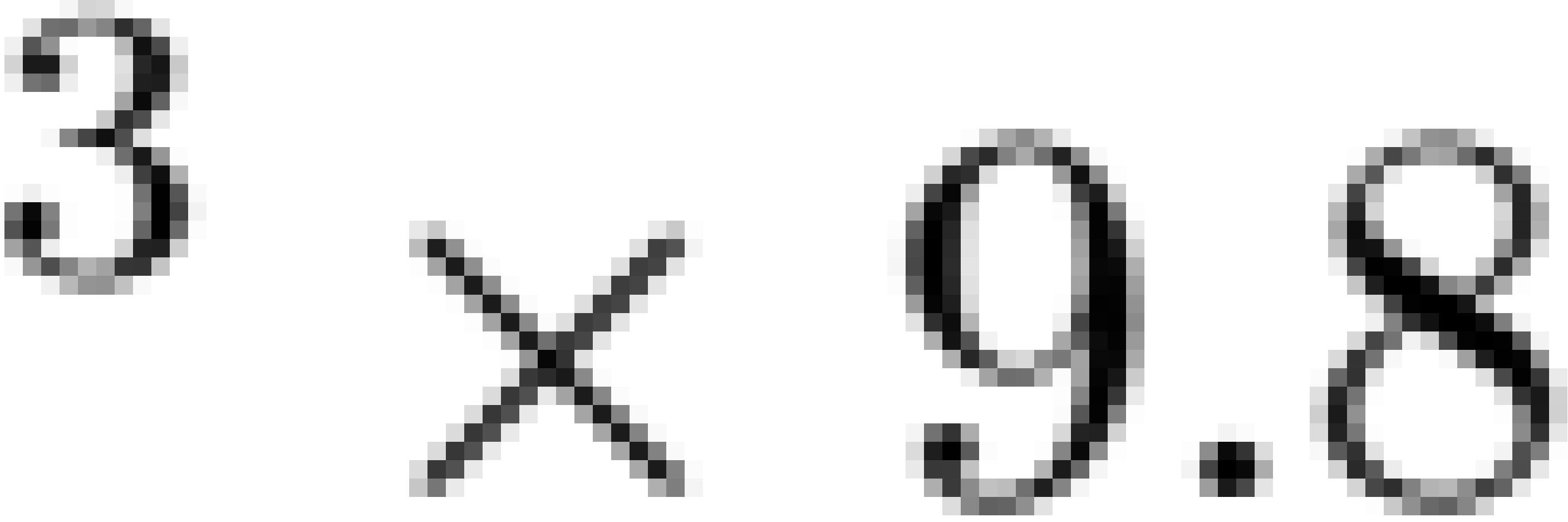
Total stress – Pore pressure

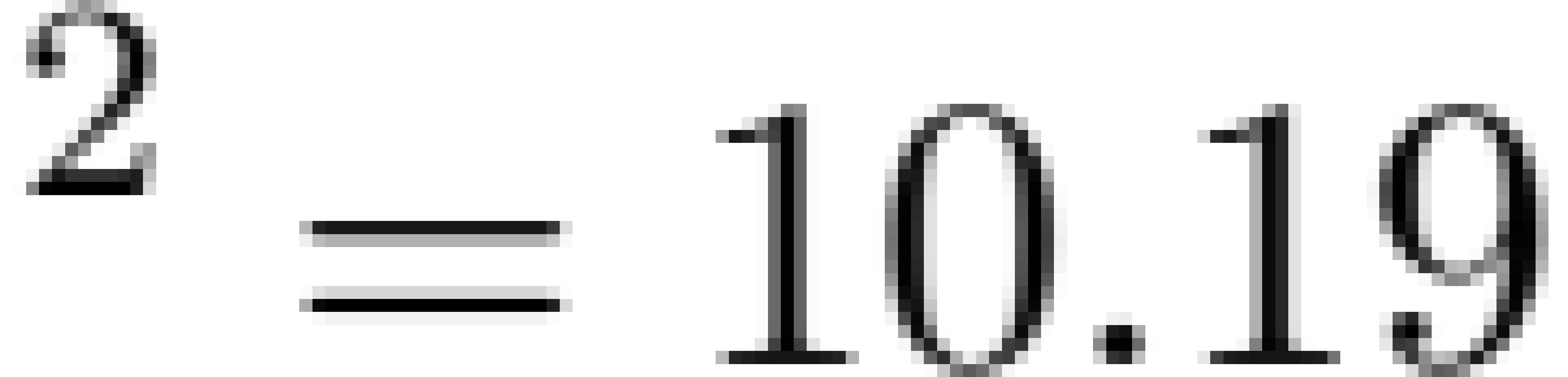
$$\sigma = S - p$$





dP P $\rho_{w,9}$ 1040
 dz





dsu = Paulk 2350

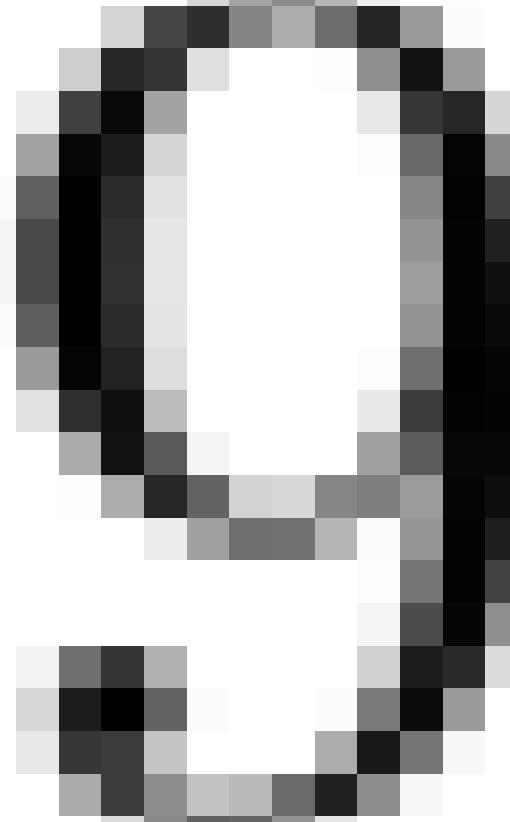
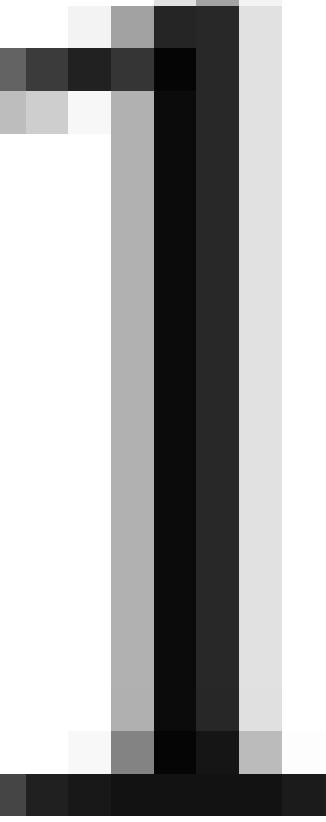
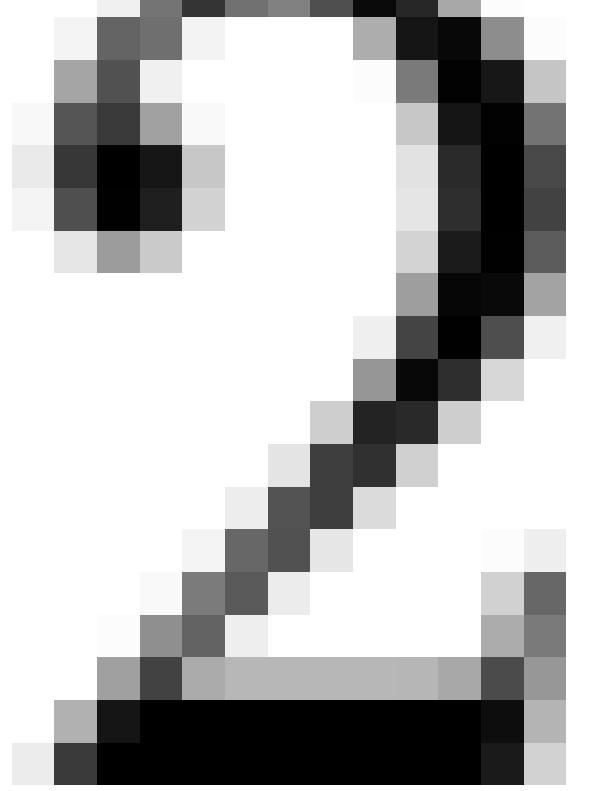
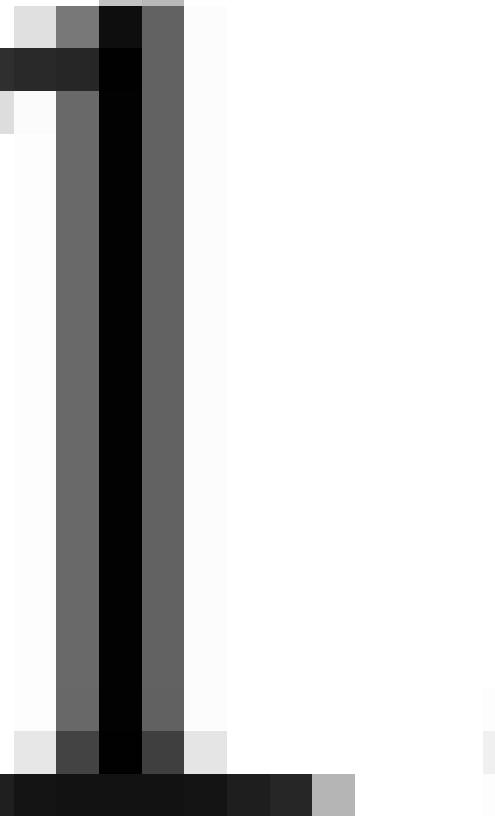
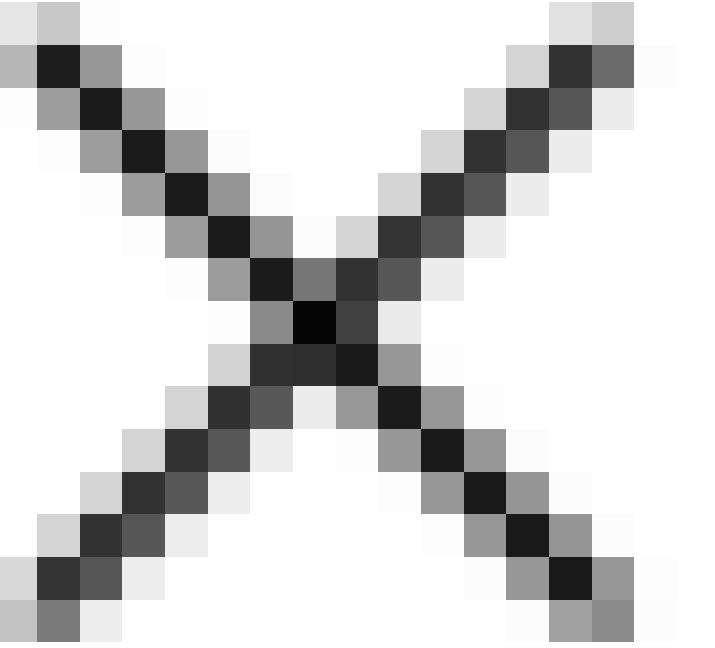


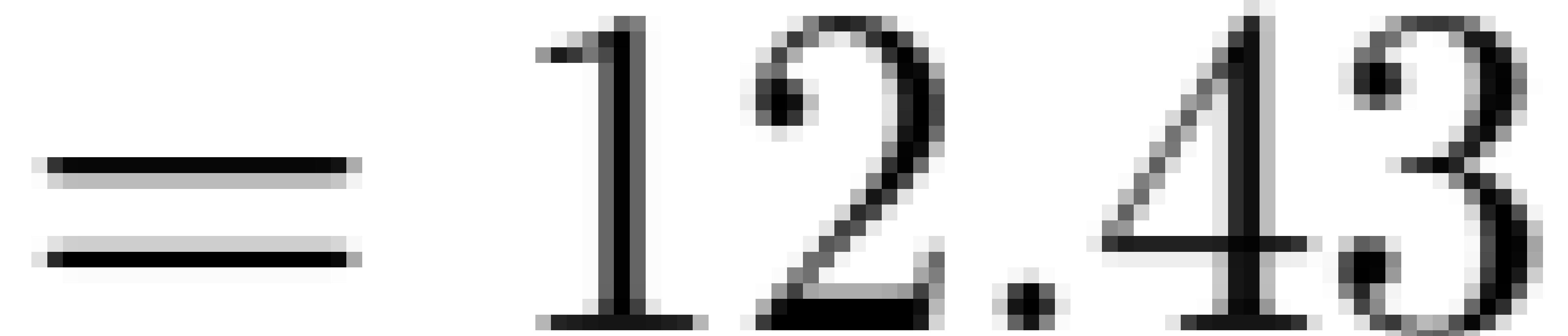
P
P

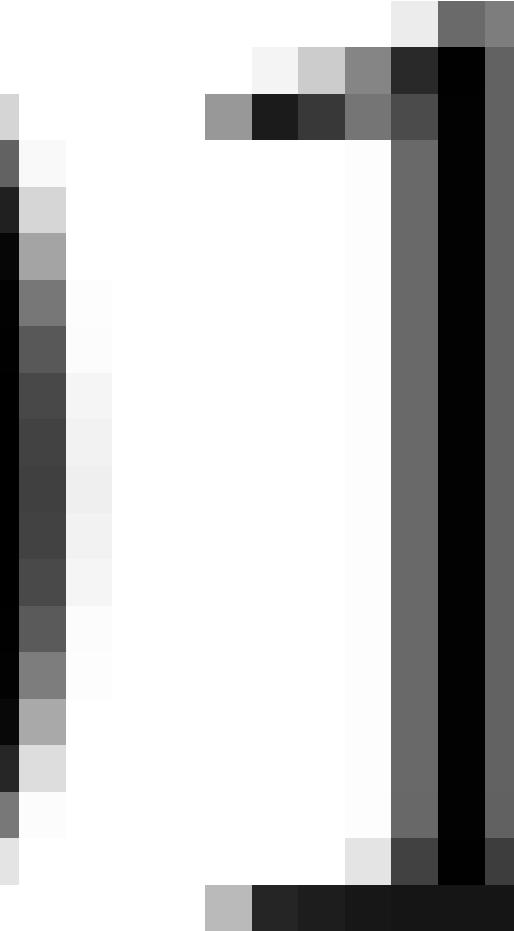
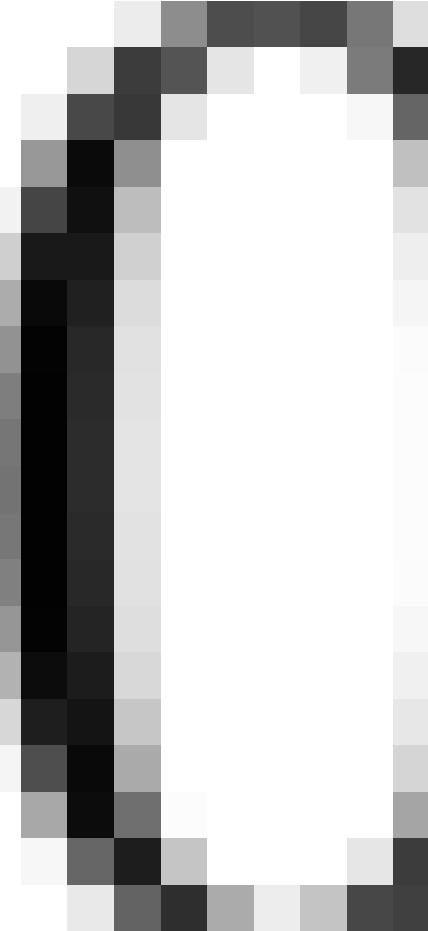
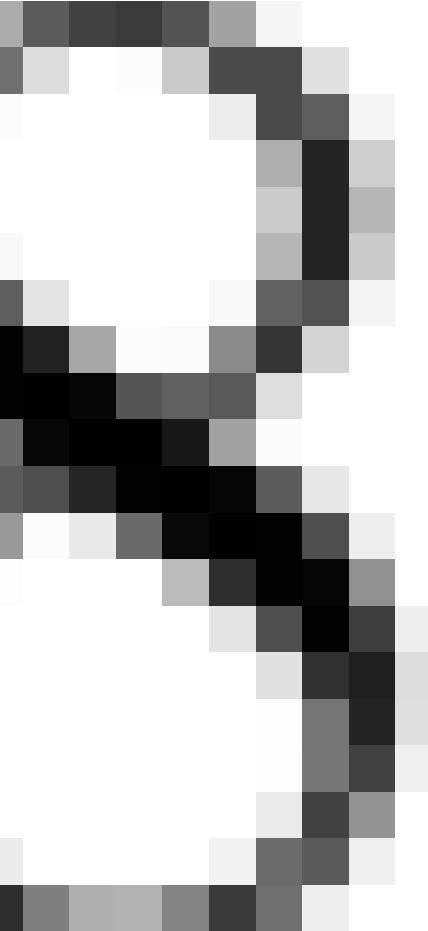
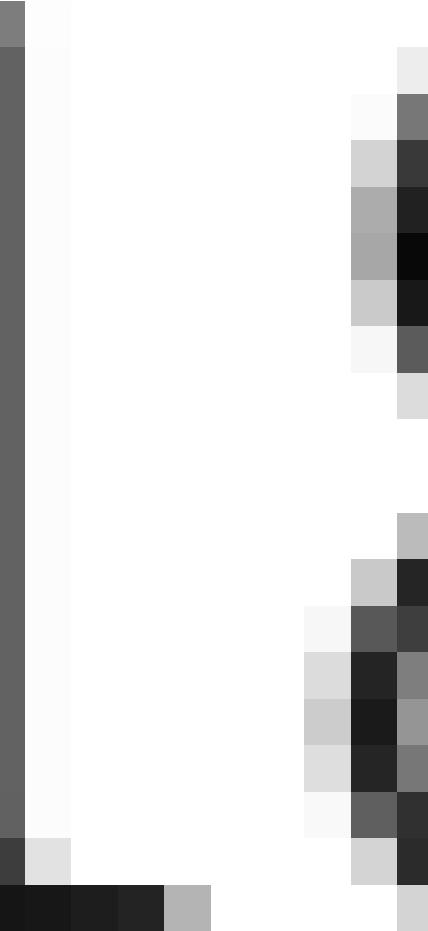
dP
dP
dZ
dZ

—
—

10.
19







d₅

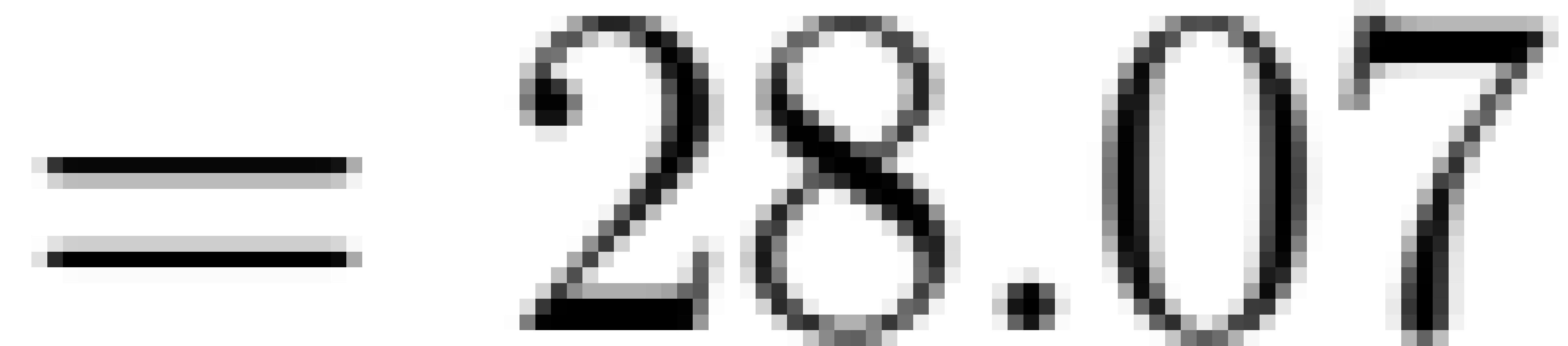


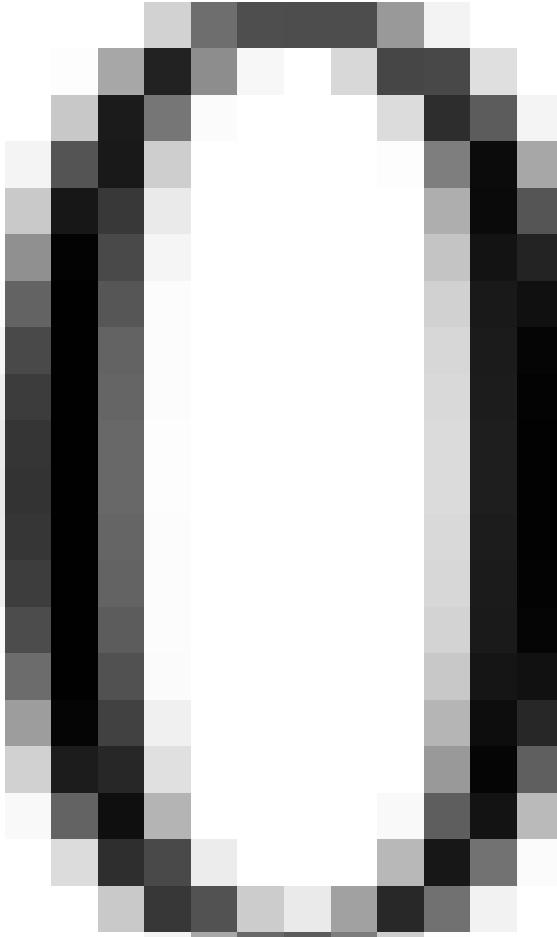
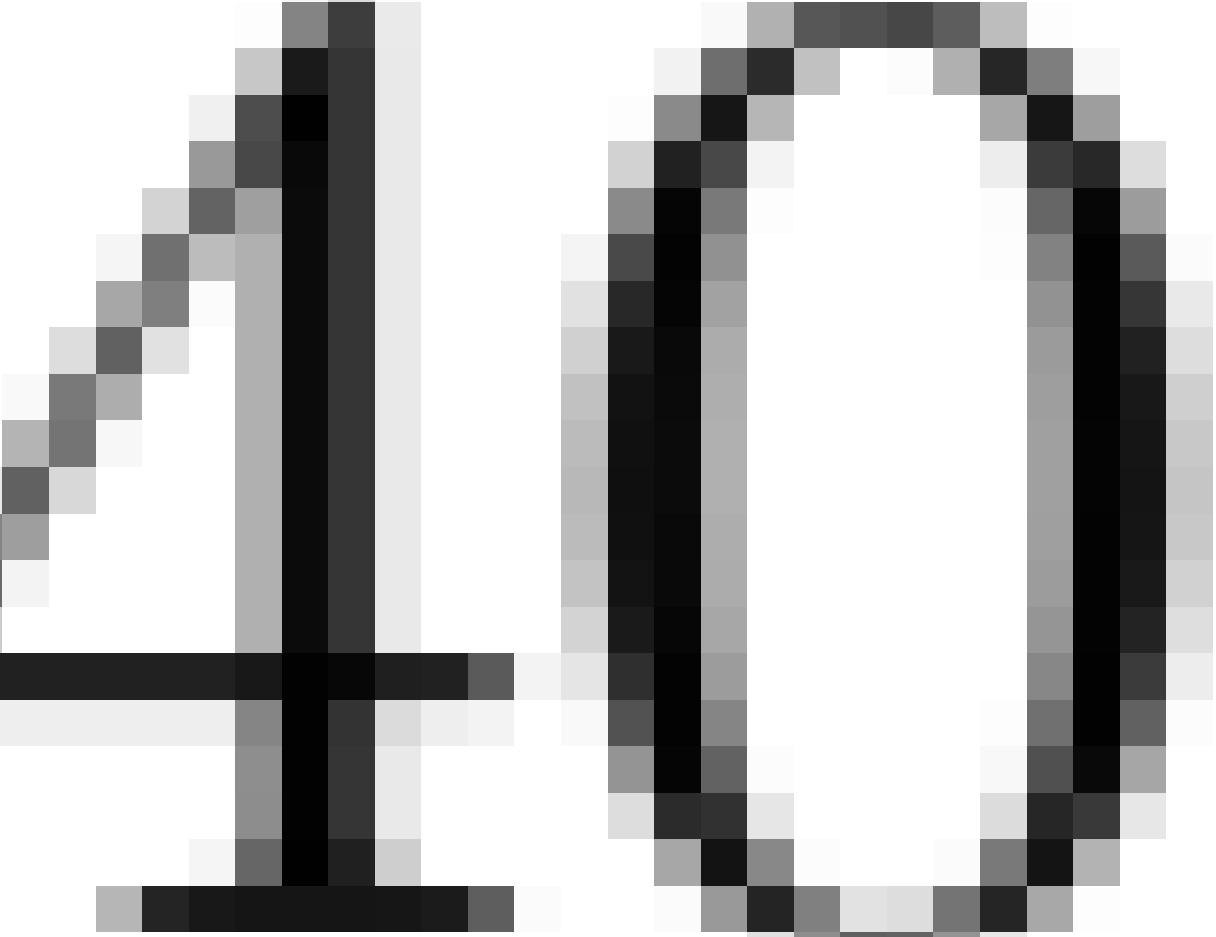
d₂



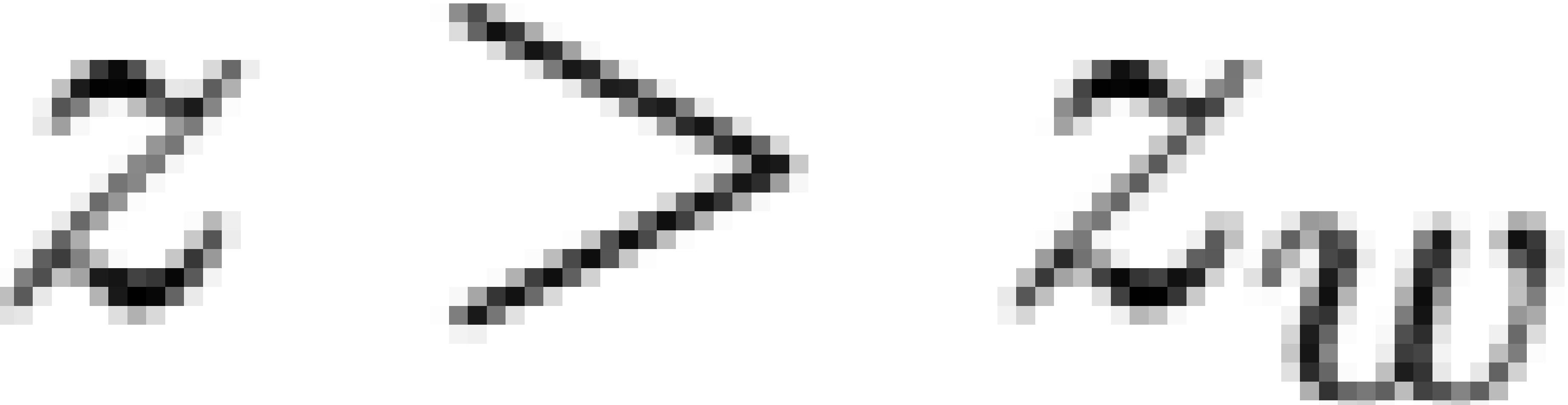
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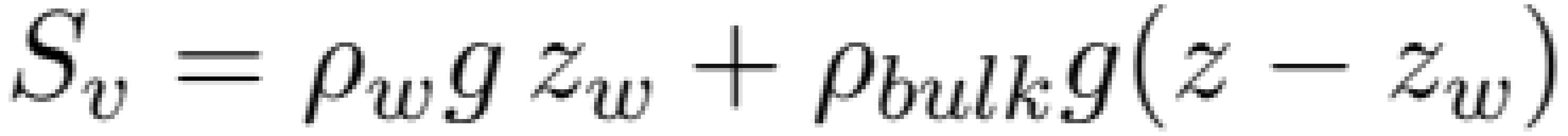
d₂

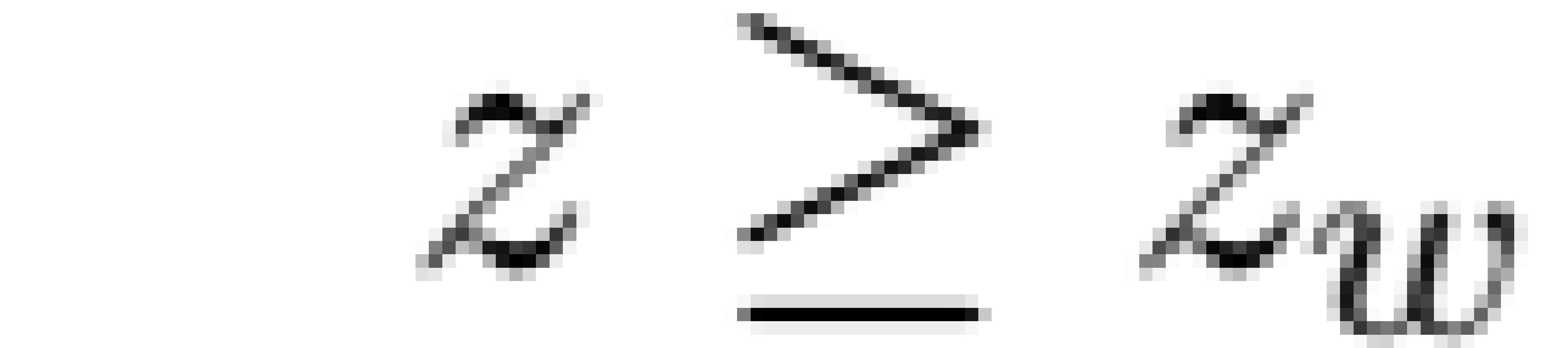


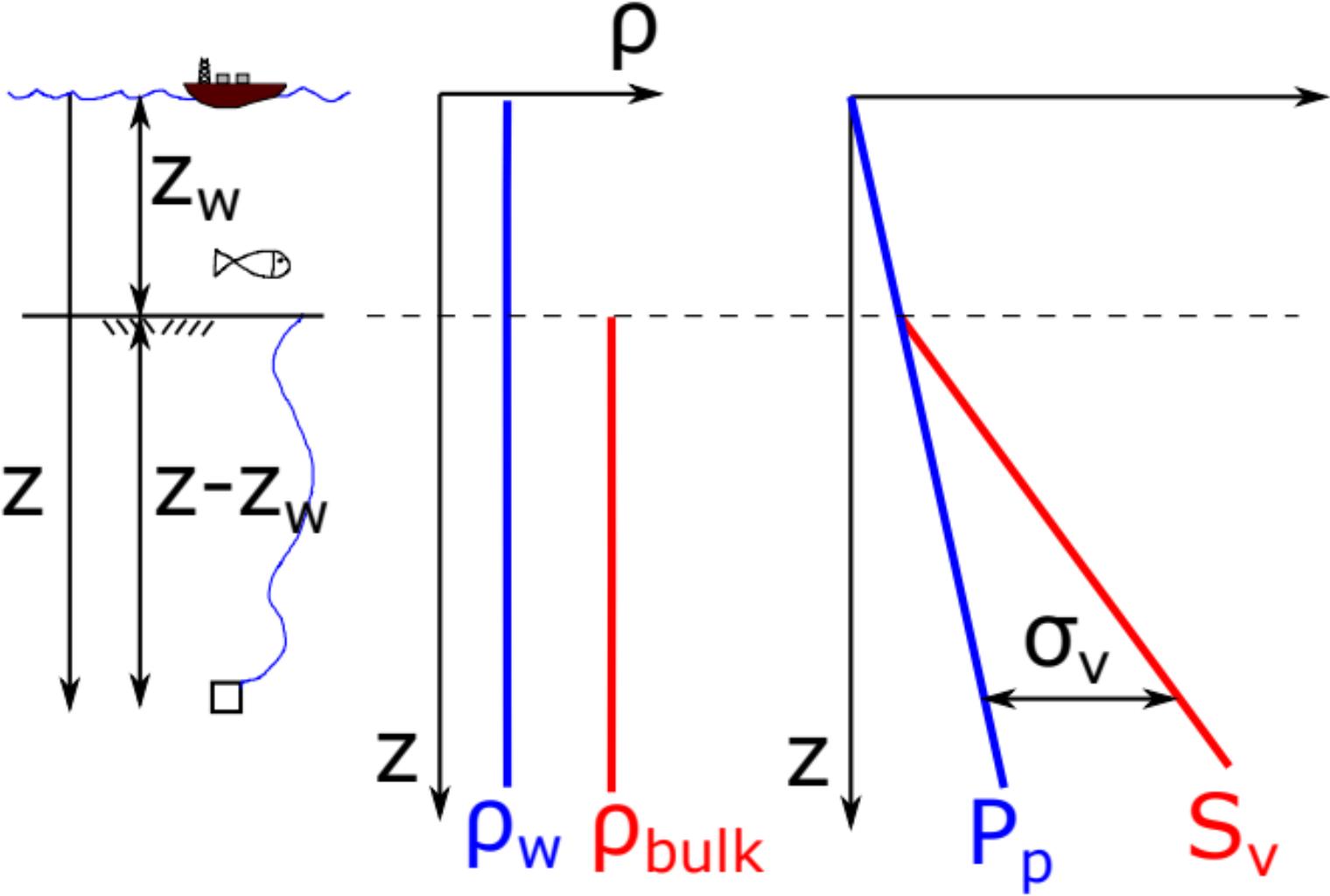




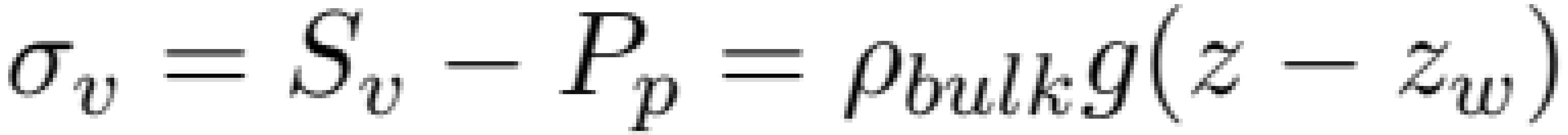




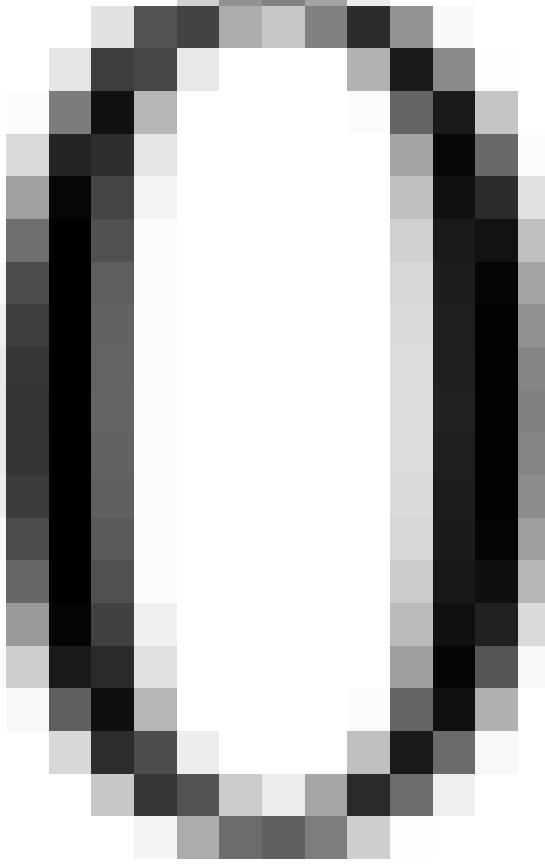
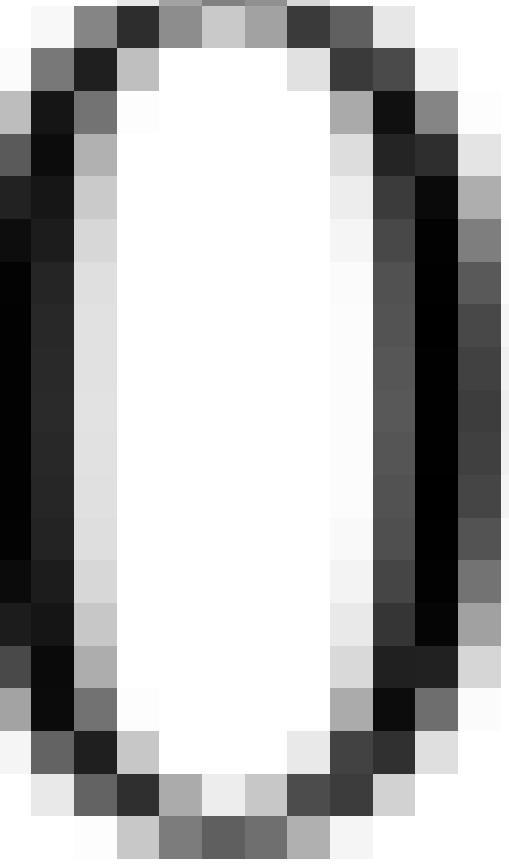
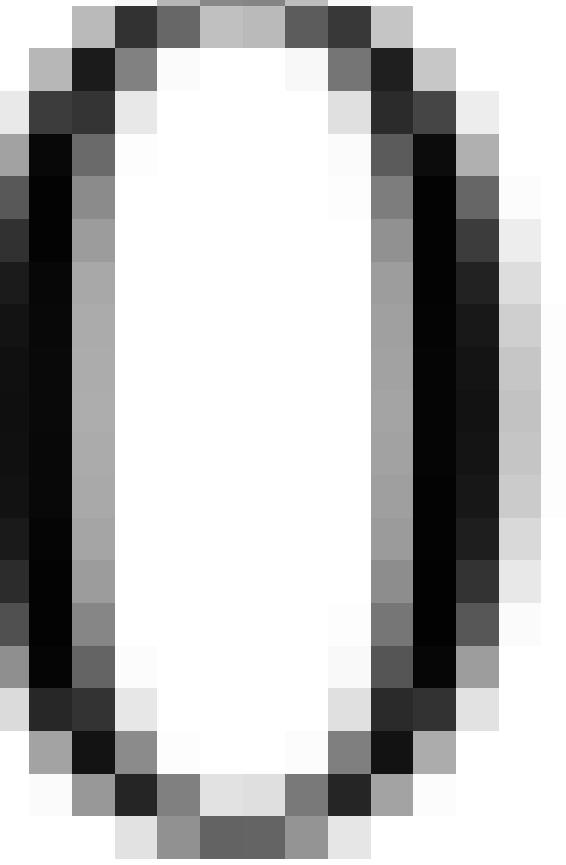
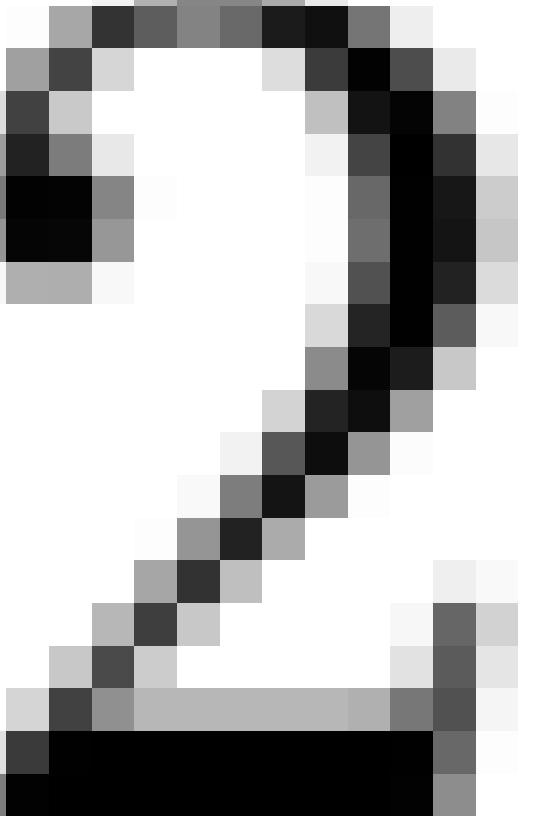
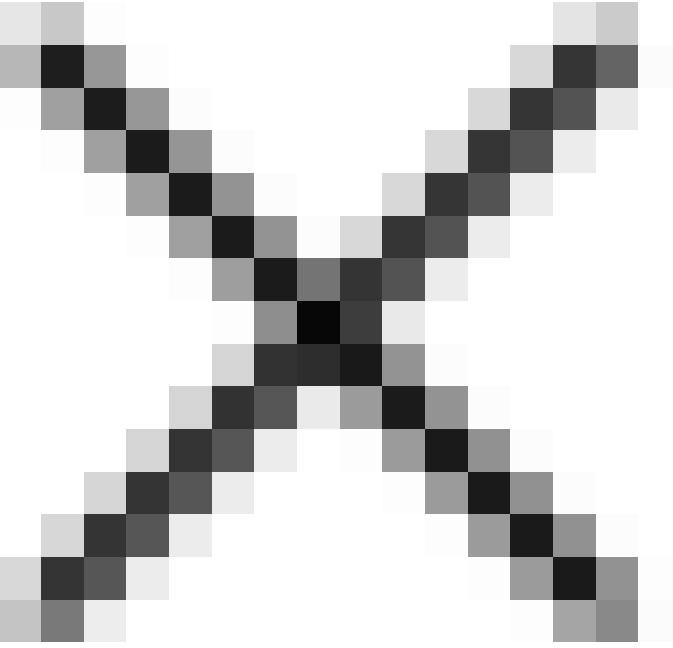


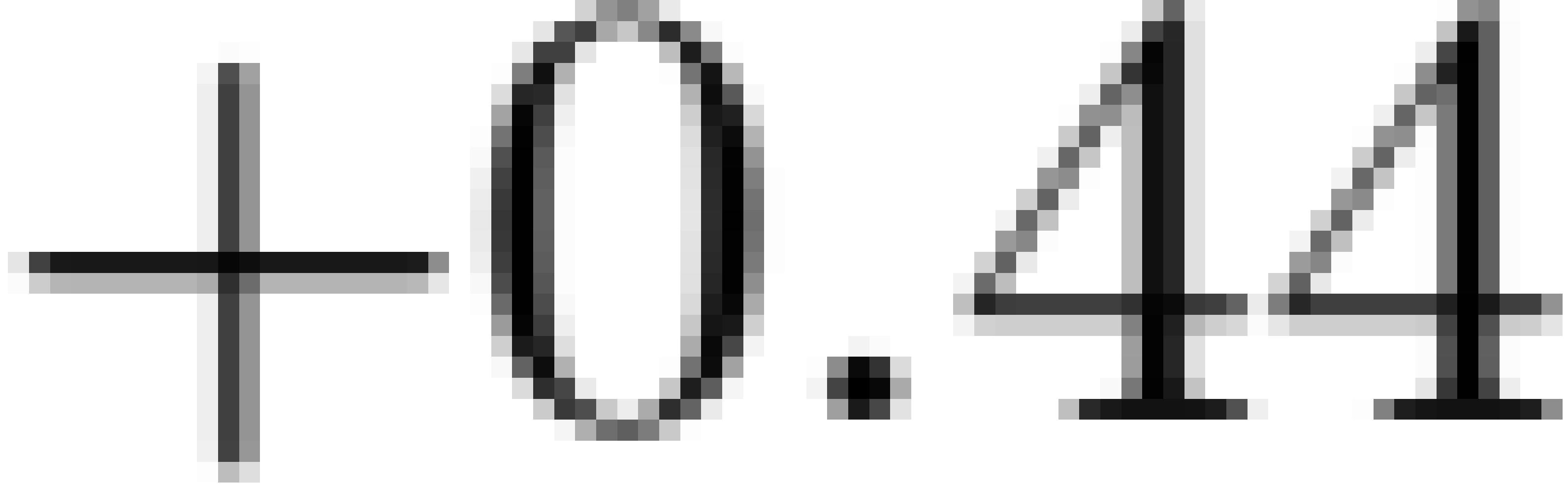


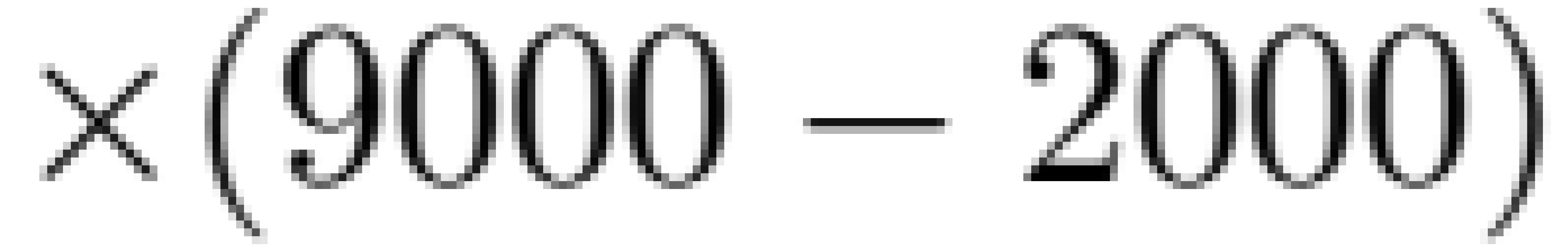


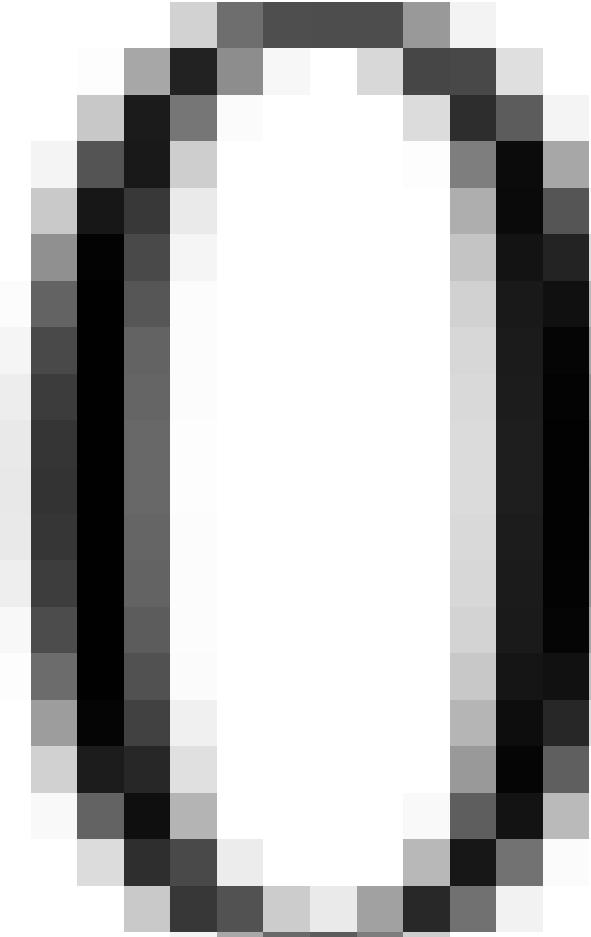
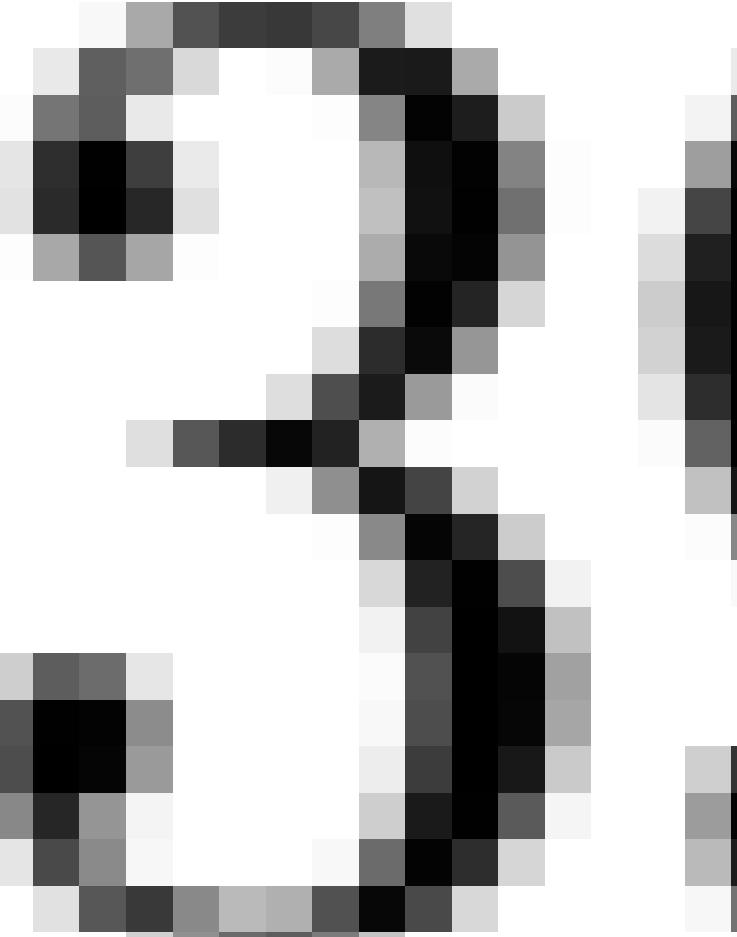


$$P_p = \rho w g z_w + \frac{dP}{dz} (z - z_w) = 0.44$$

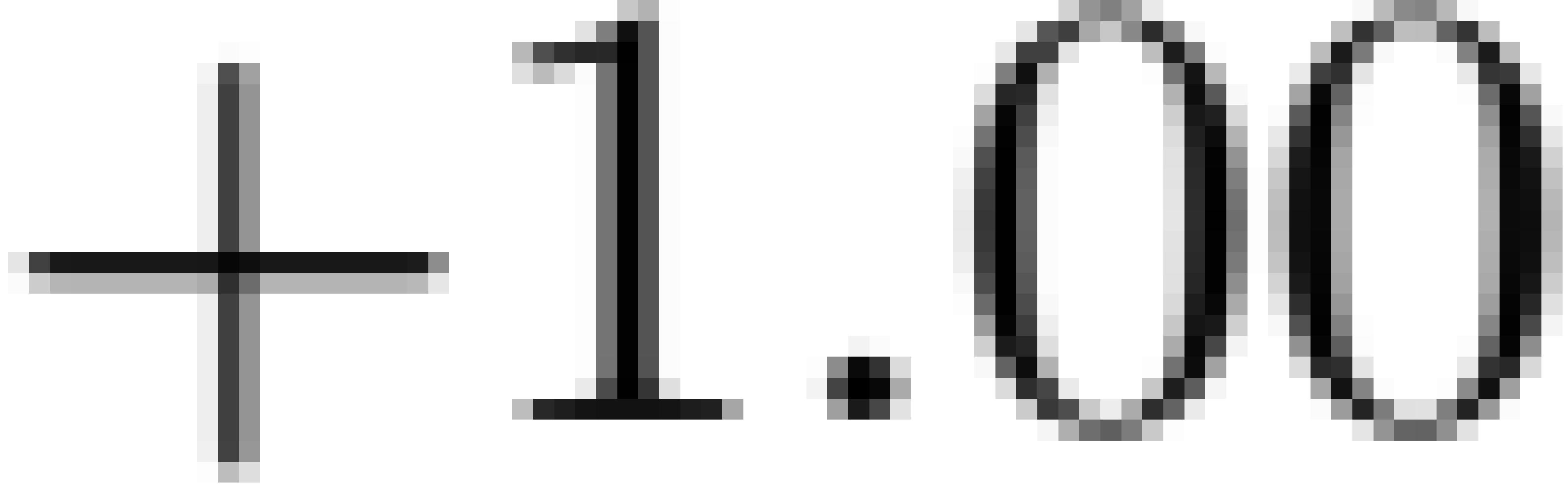


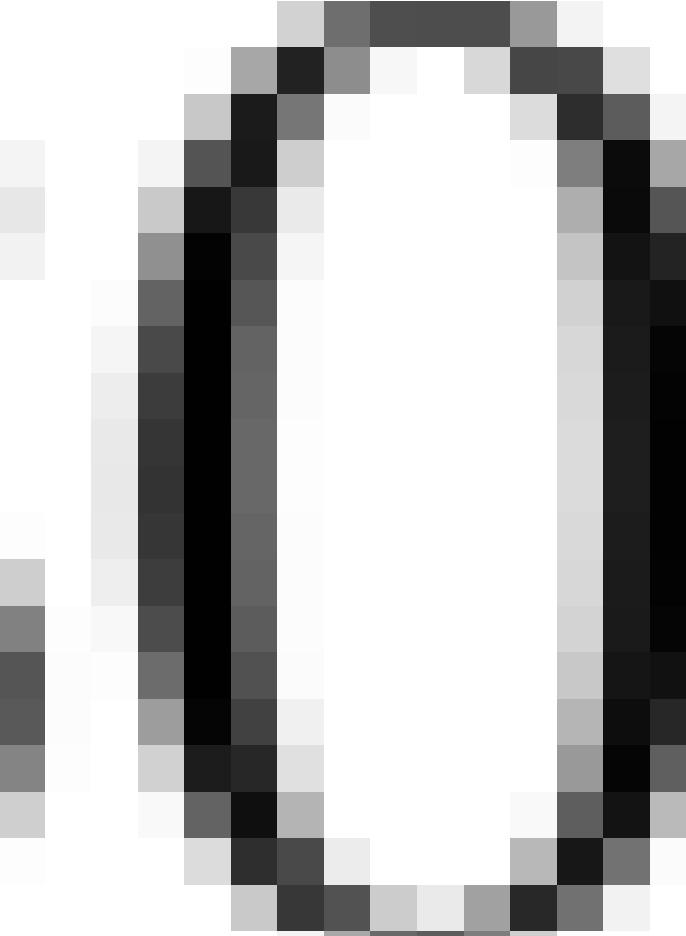
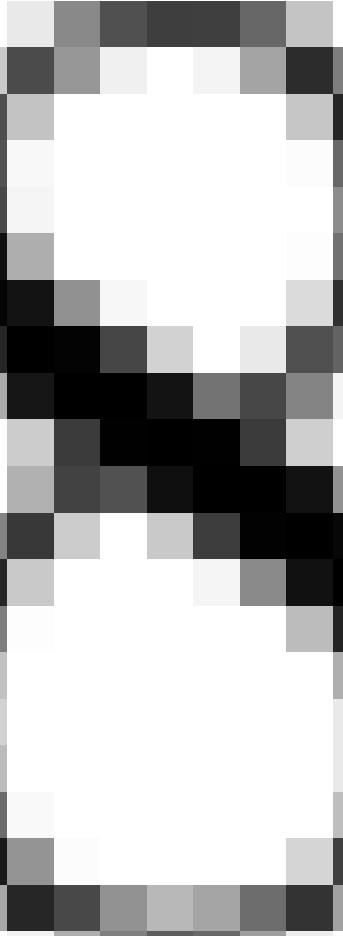
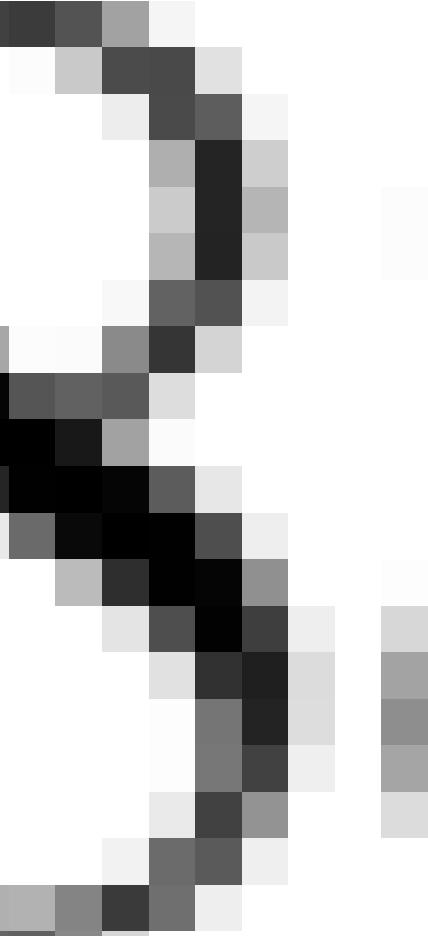
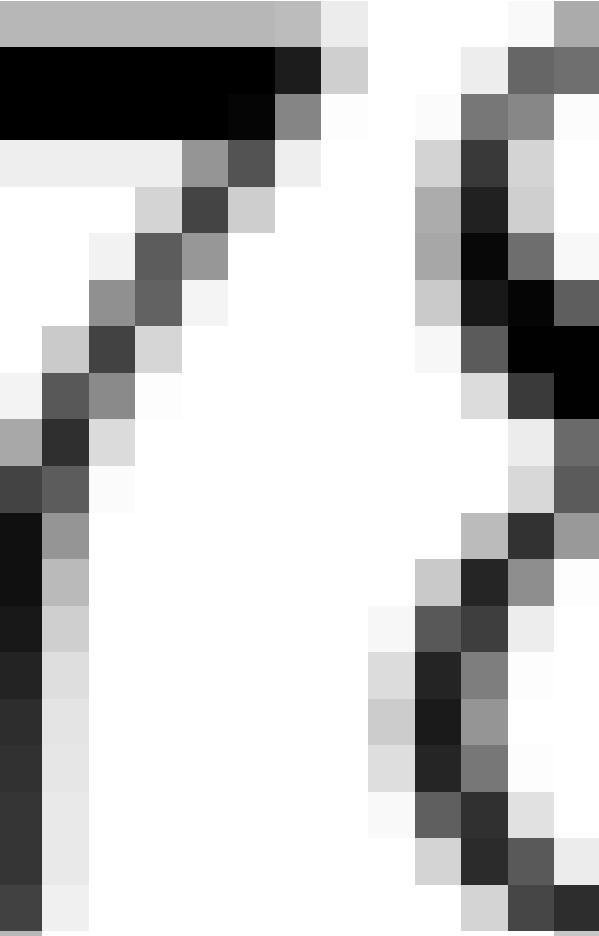


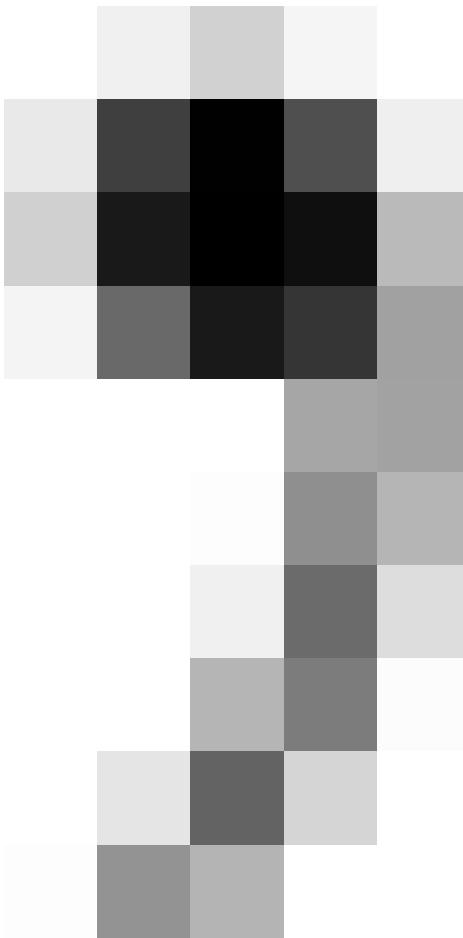


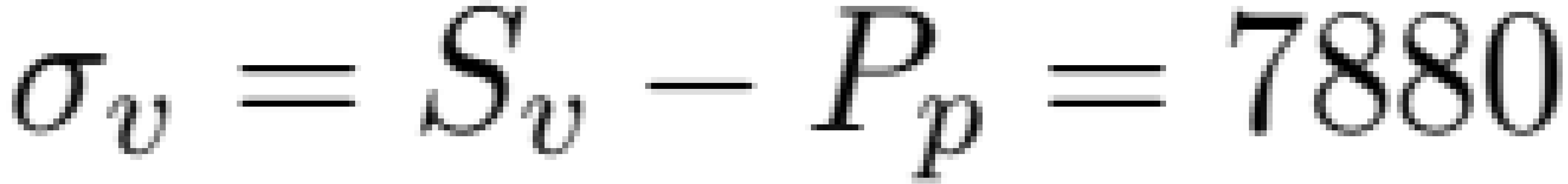


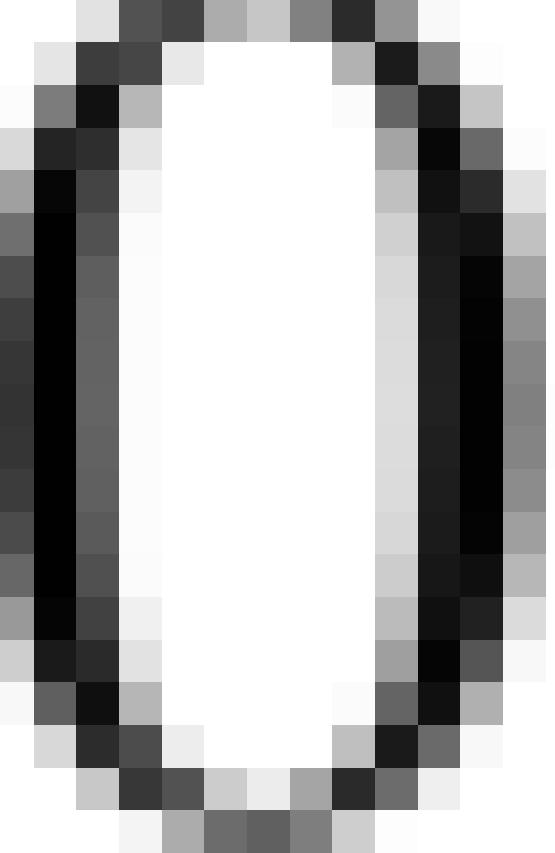
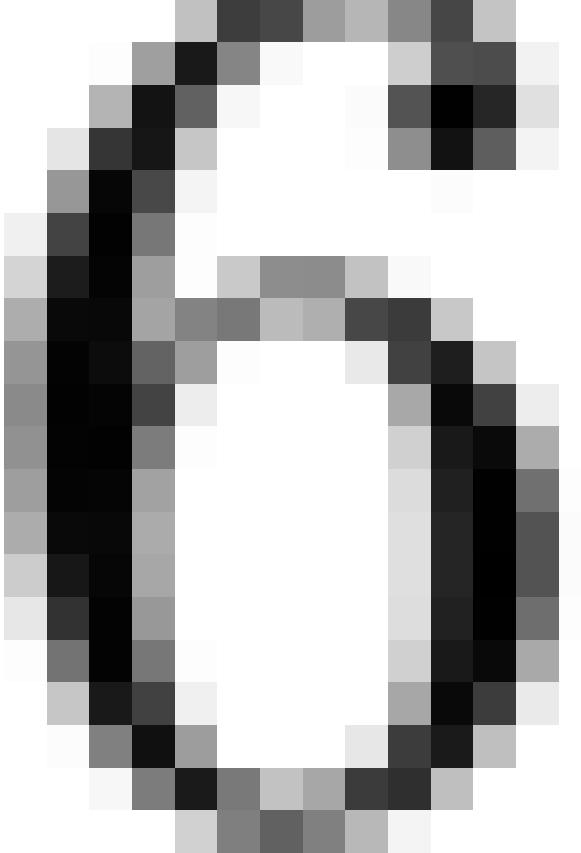
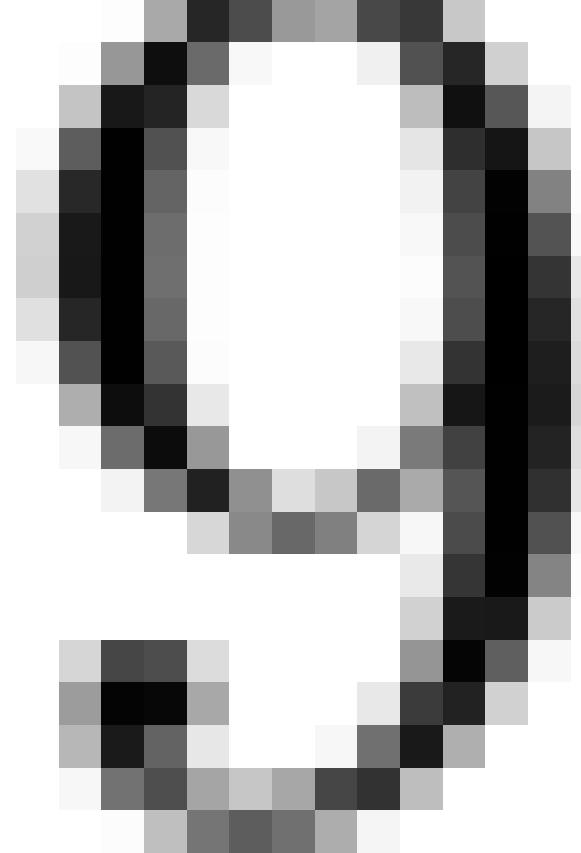
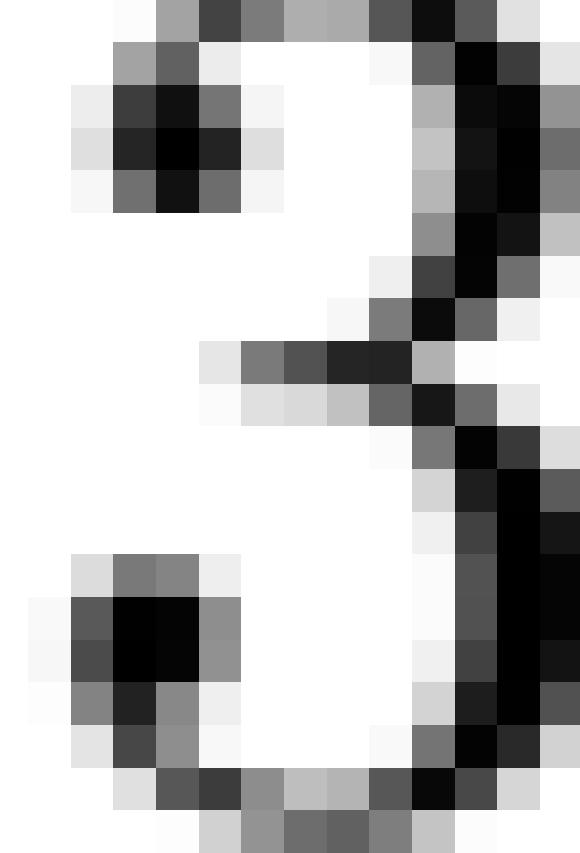
$$S_u = \rho_{u,g} z_u + \frac{dS_u}{dz} (z - z_u) = 0.44$$

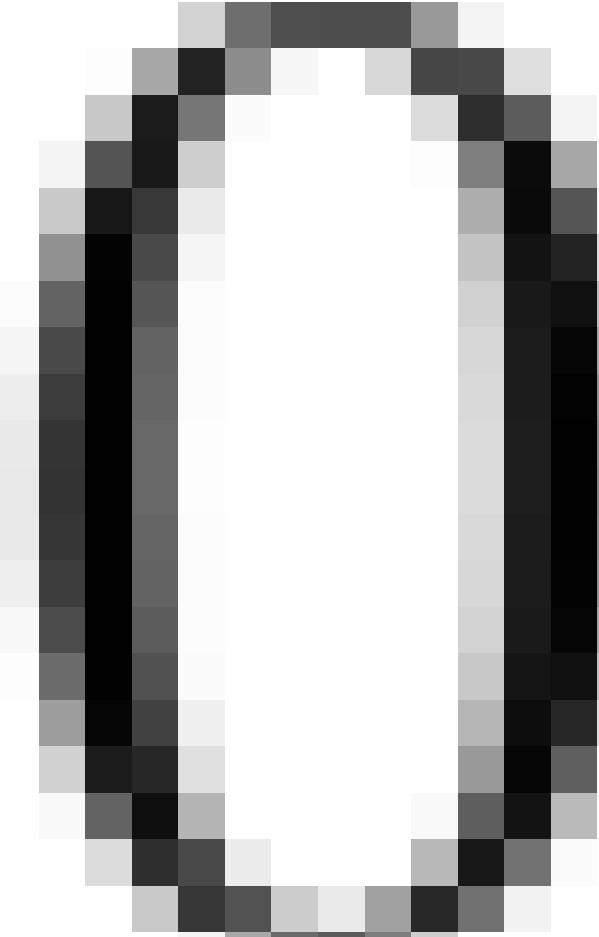
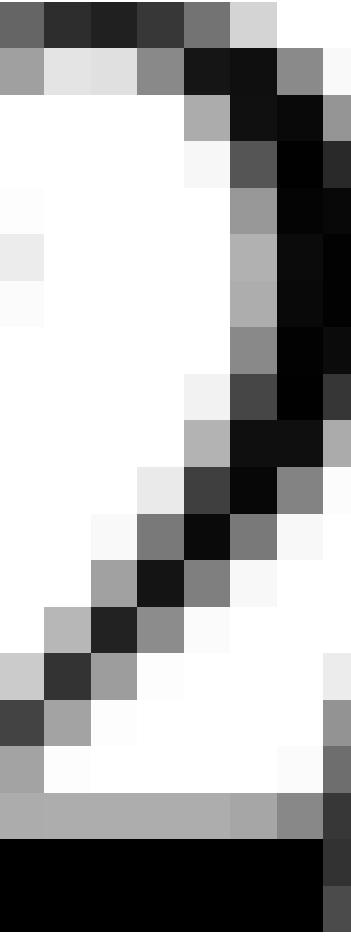
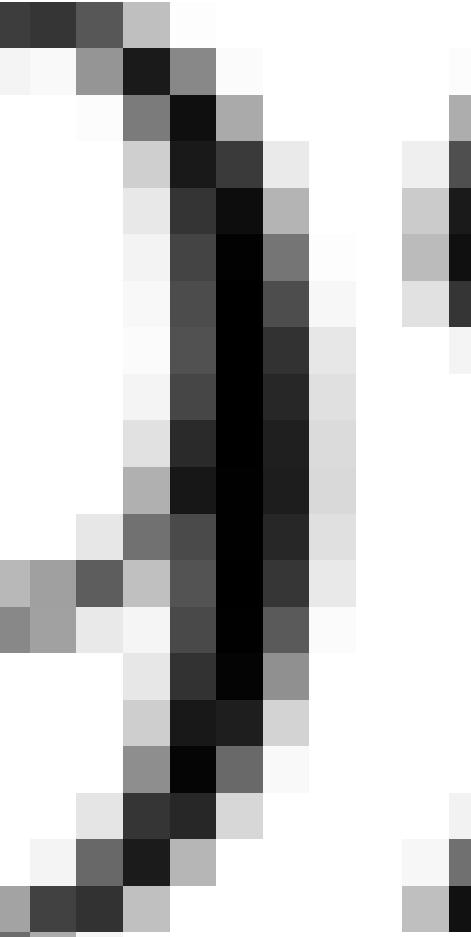
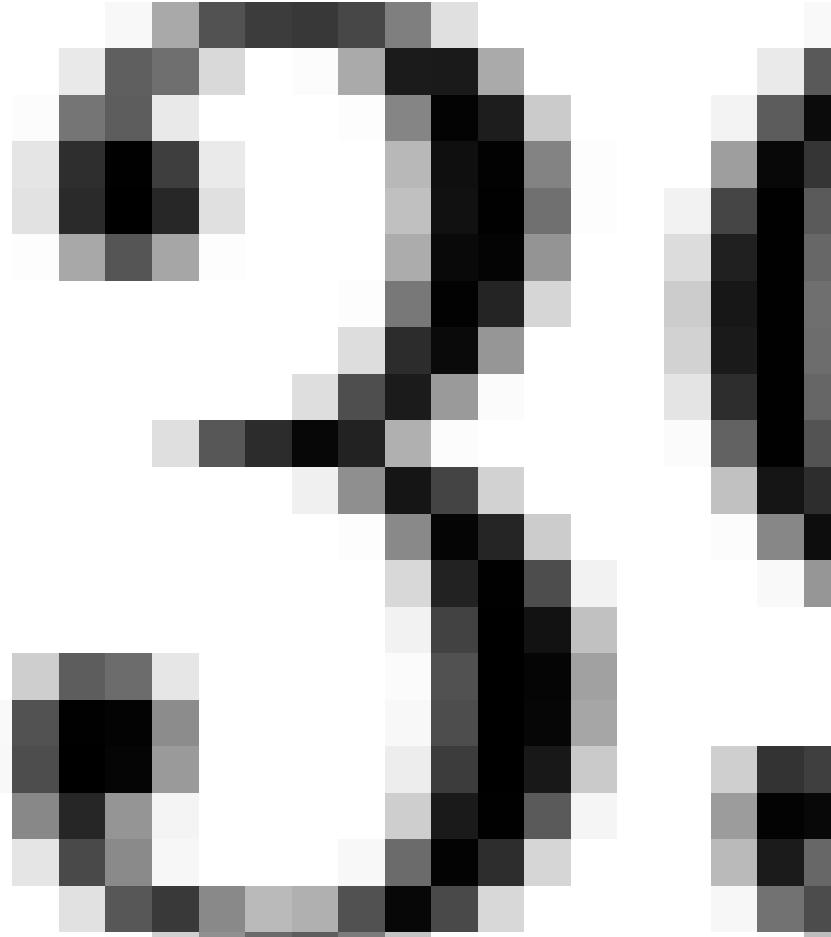


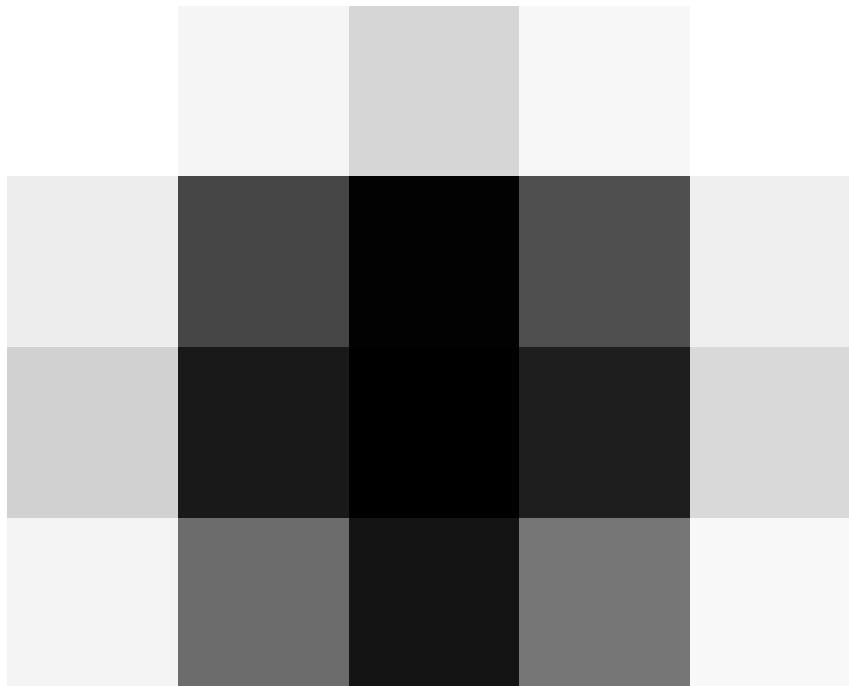


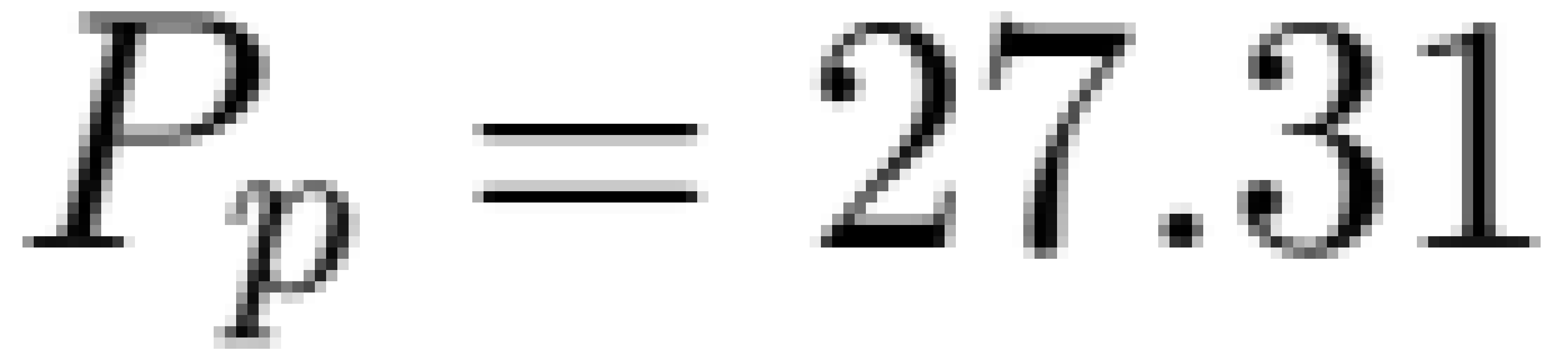






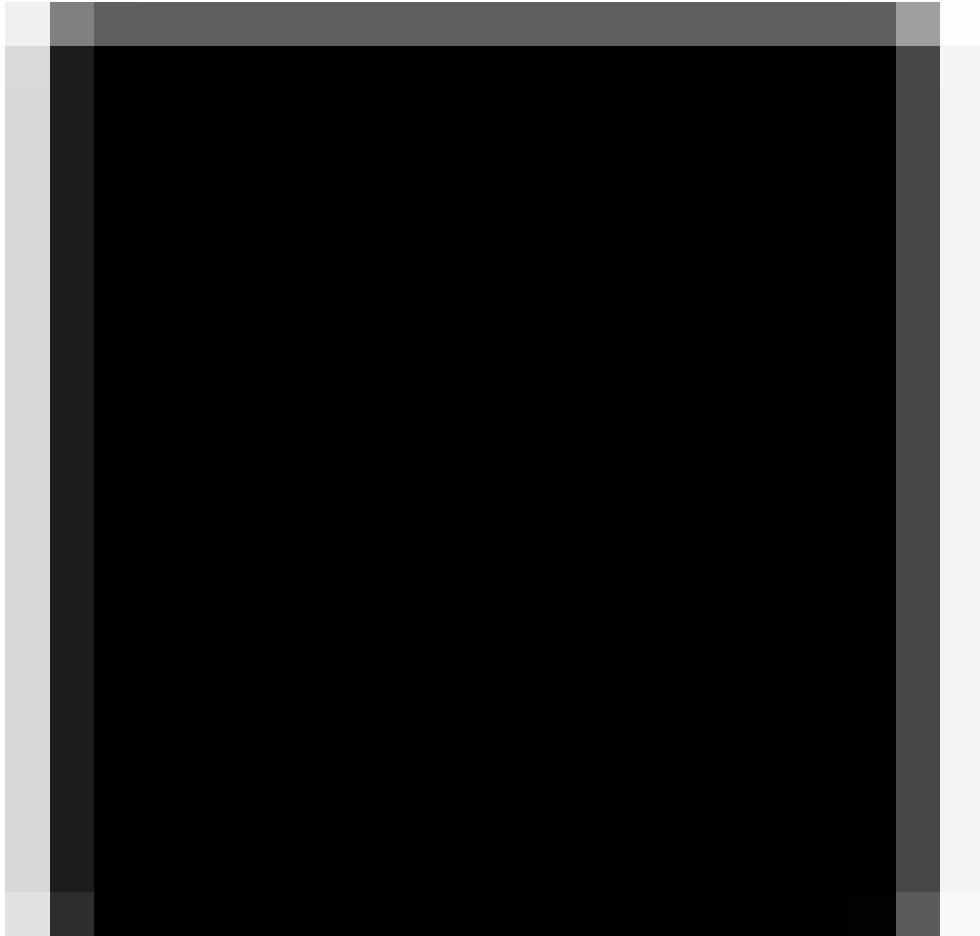








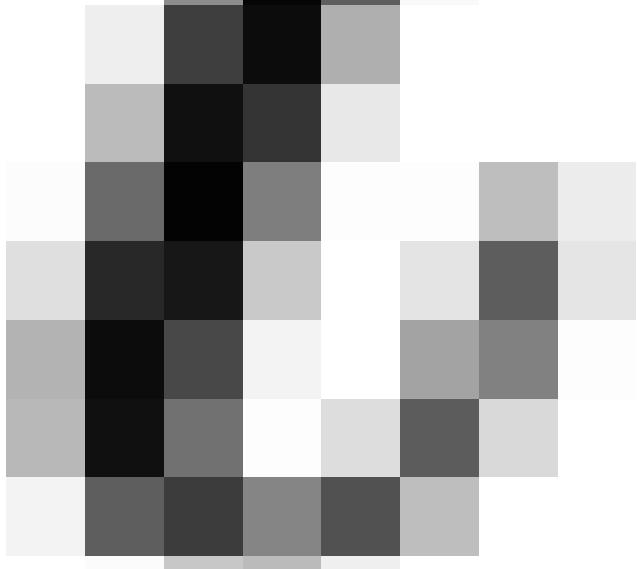
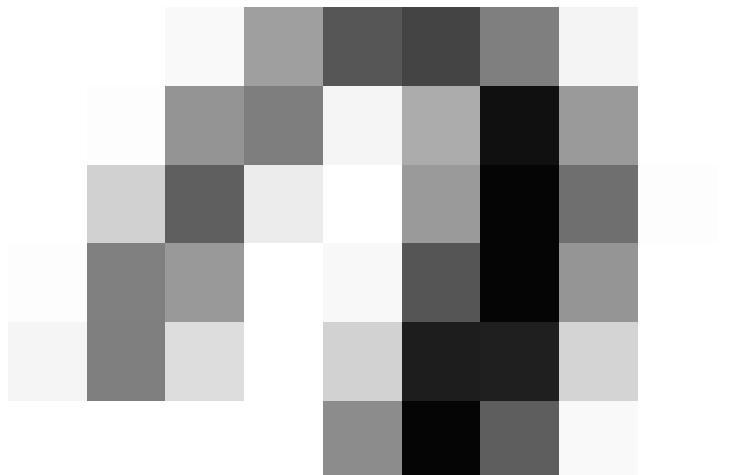
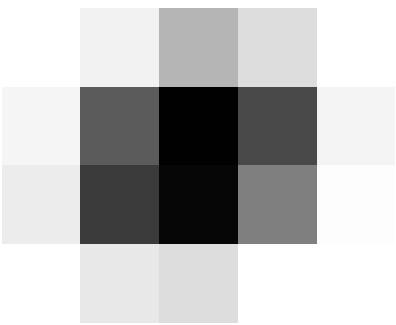


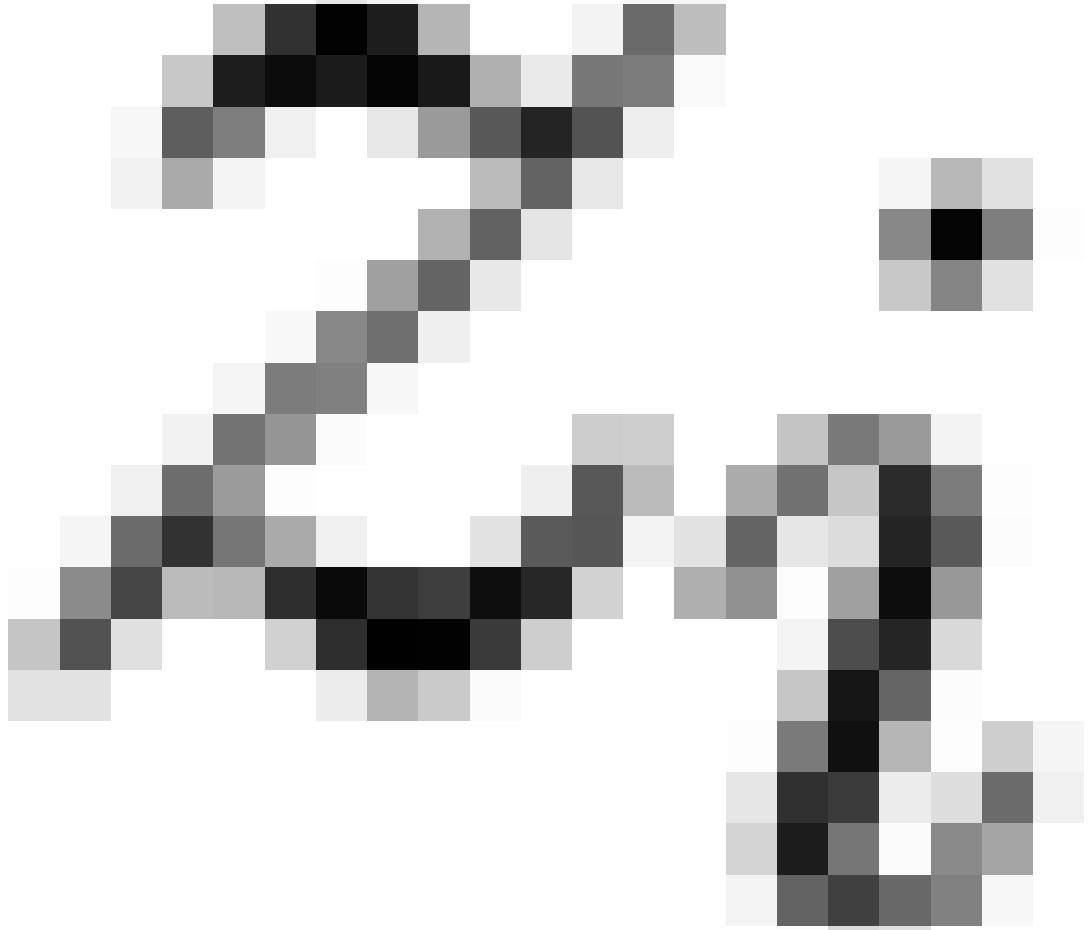


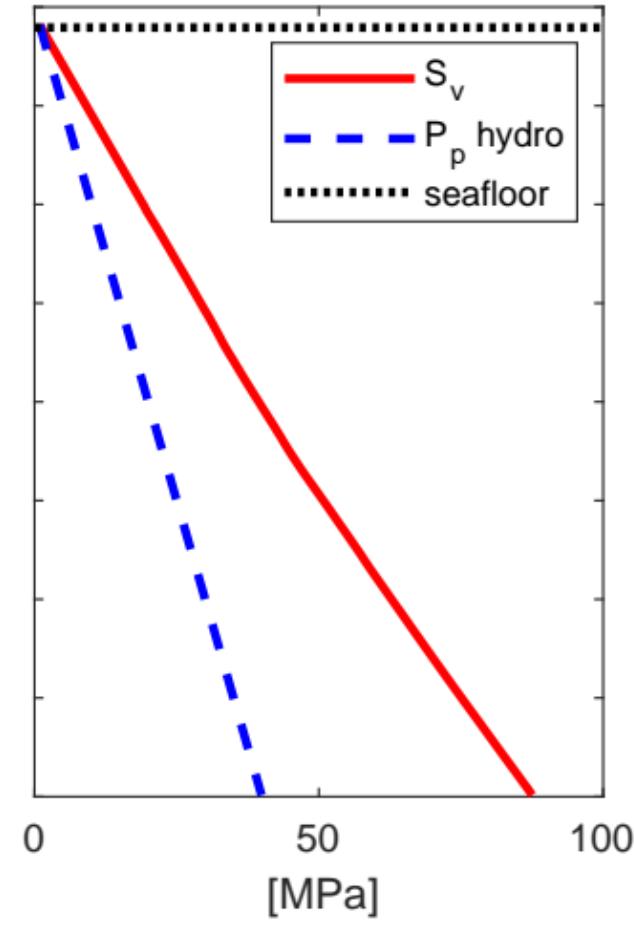
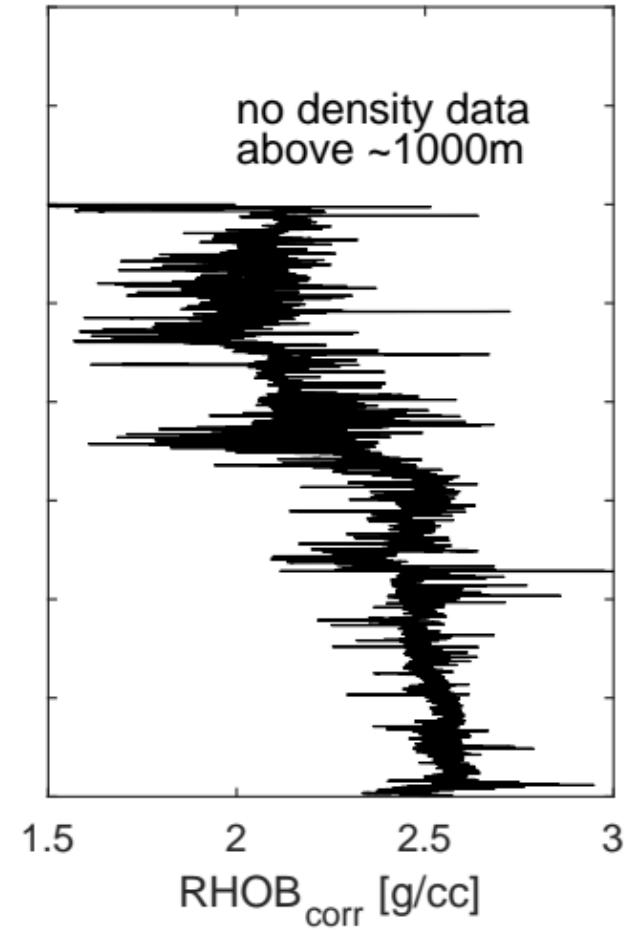
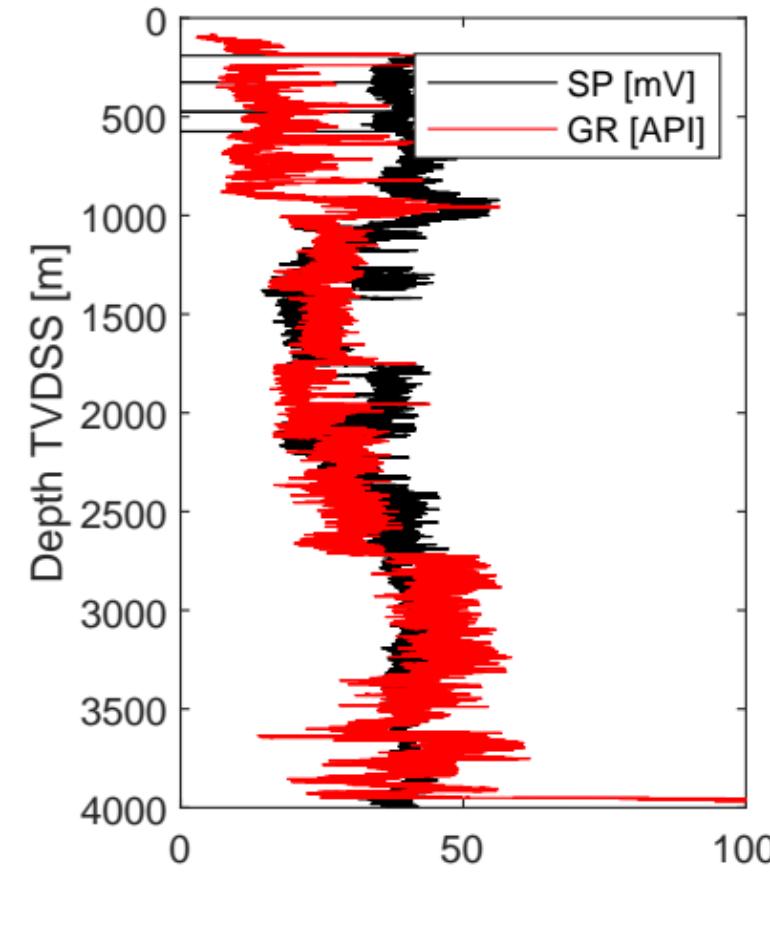
$$S_U(z) = \int_0^z \rho_{bulk}(z') dz'$$

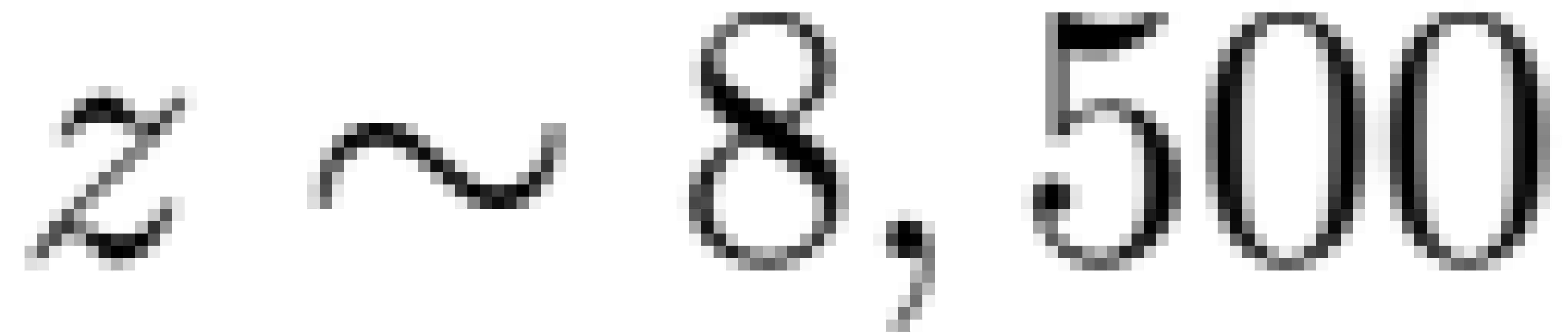


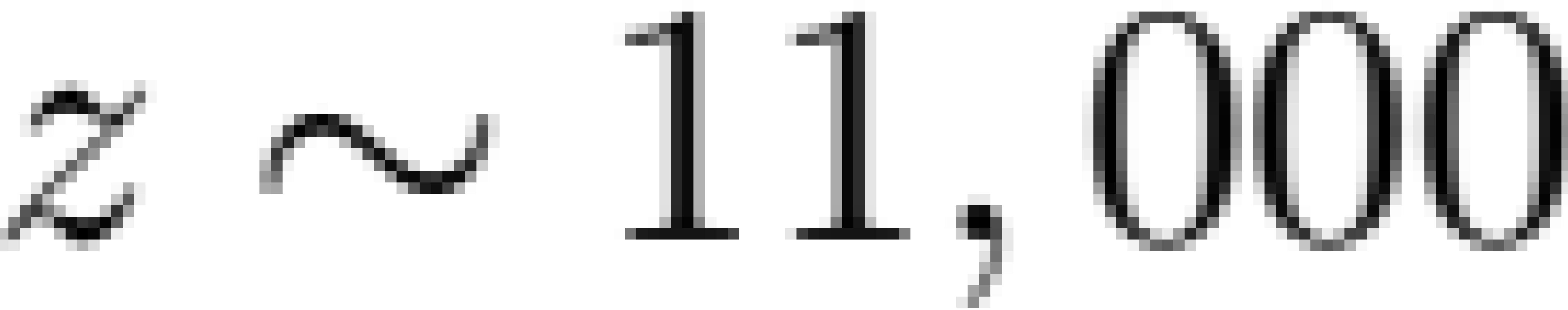
$$S_v(z_i) = \sum_{j=1}^i \rho_{bulk}(z_i) g \Delta z_i$$



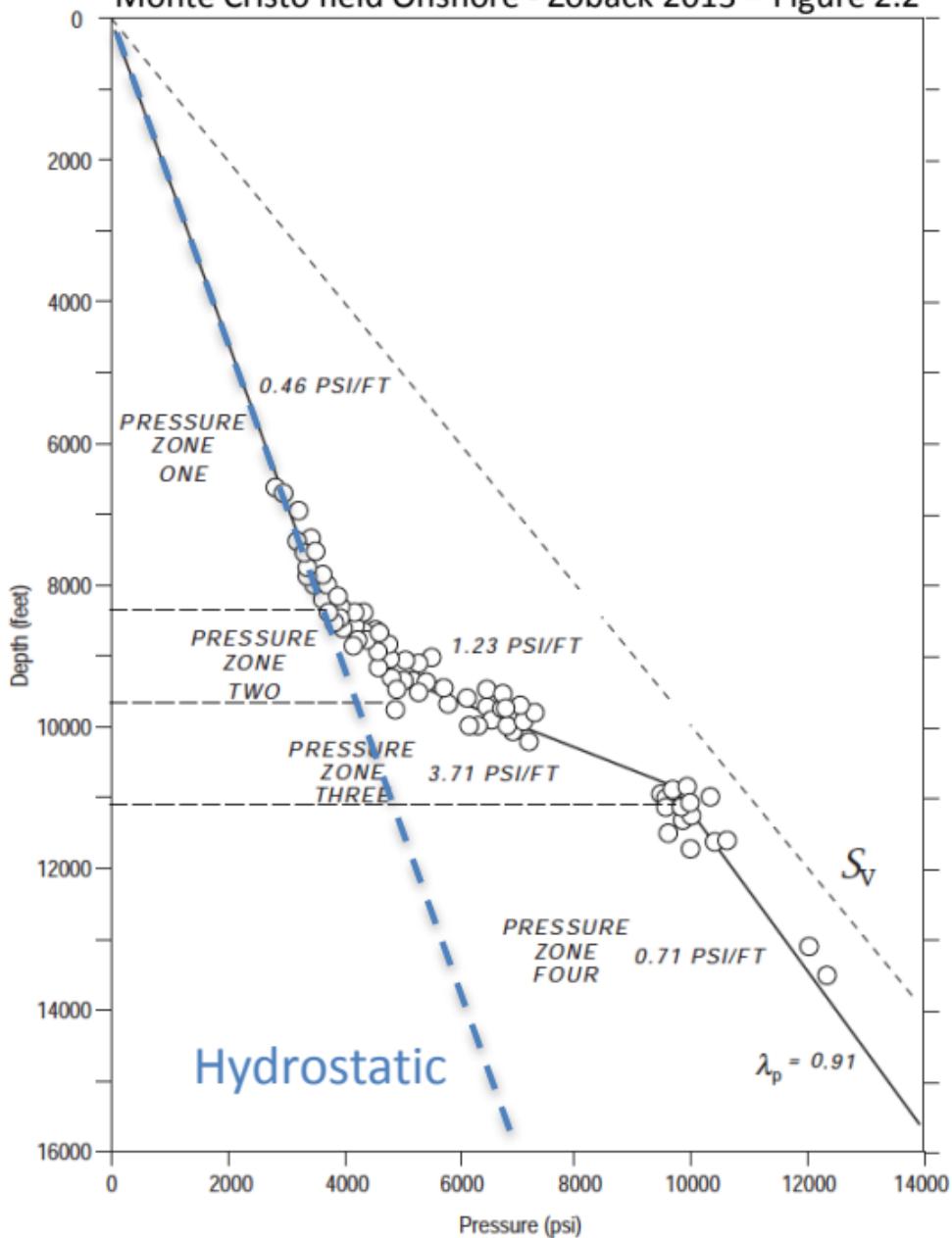








Monte Cristo field Onshore - Zoback 2013 – Figure 2.2

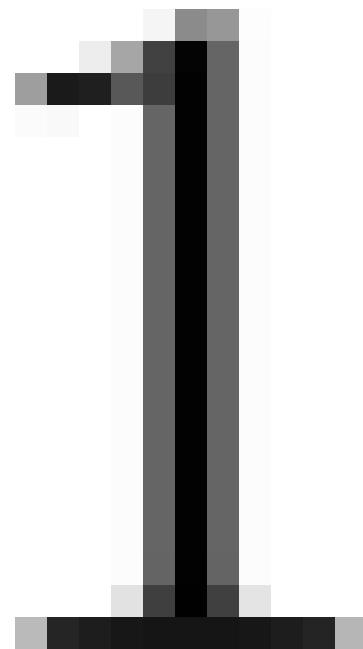
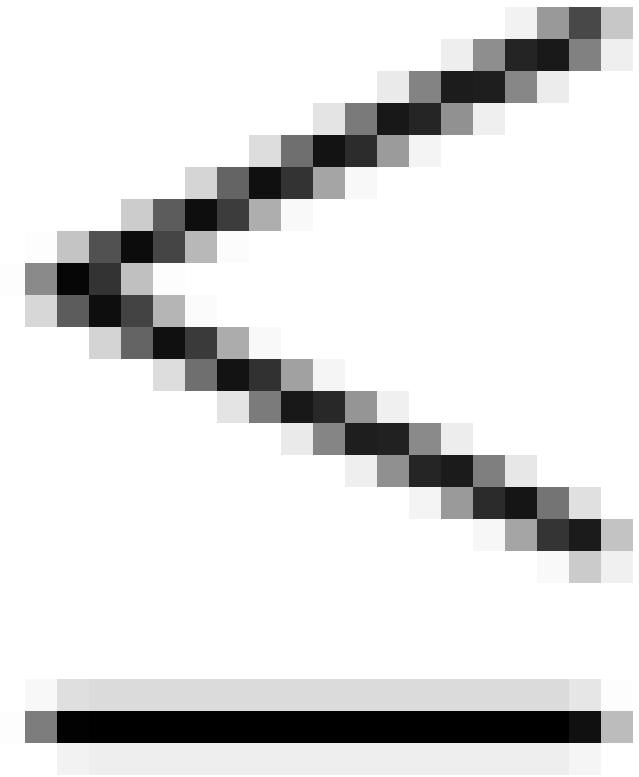


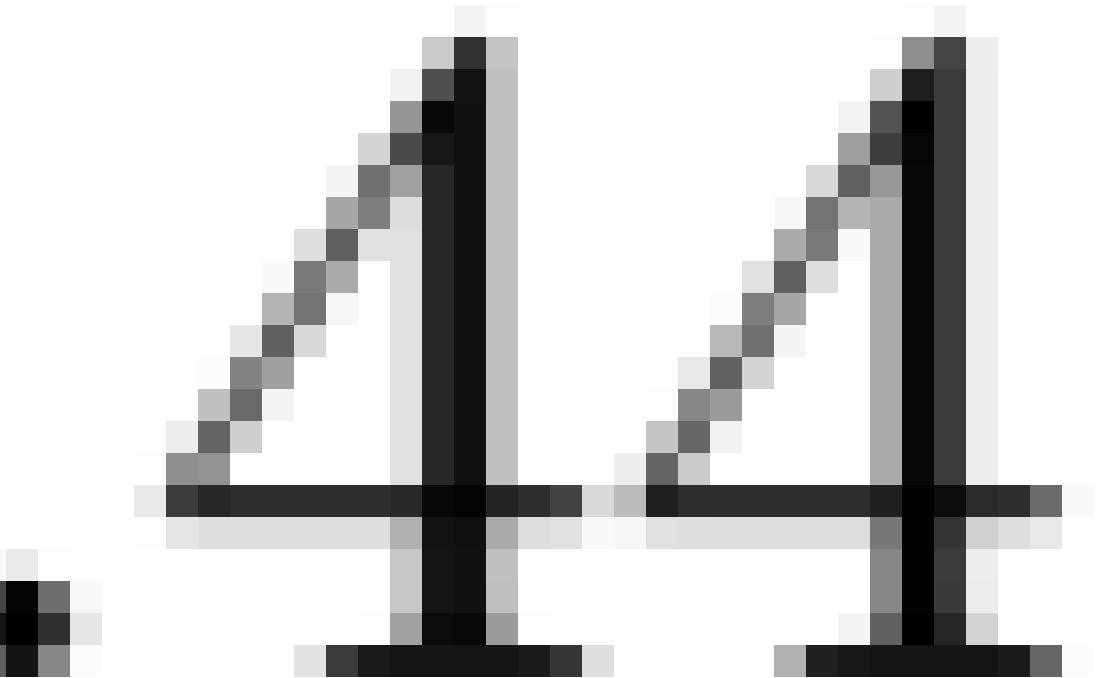
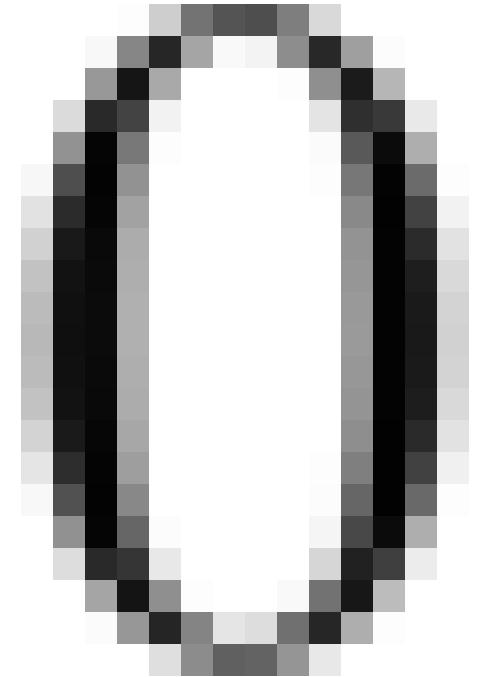
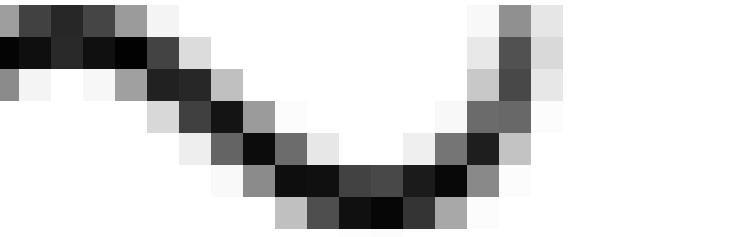


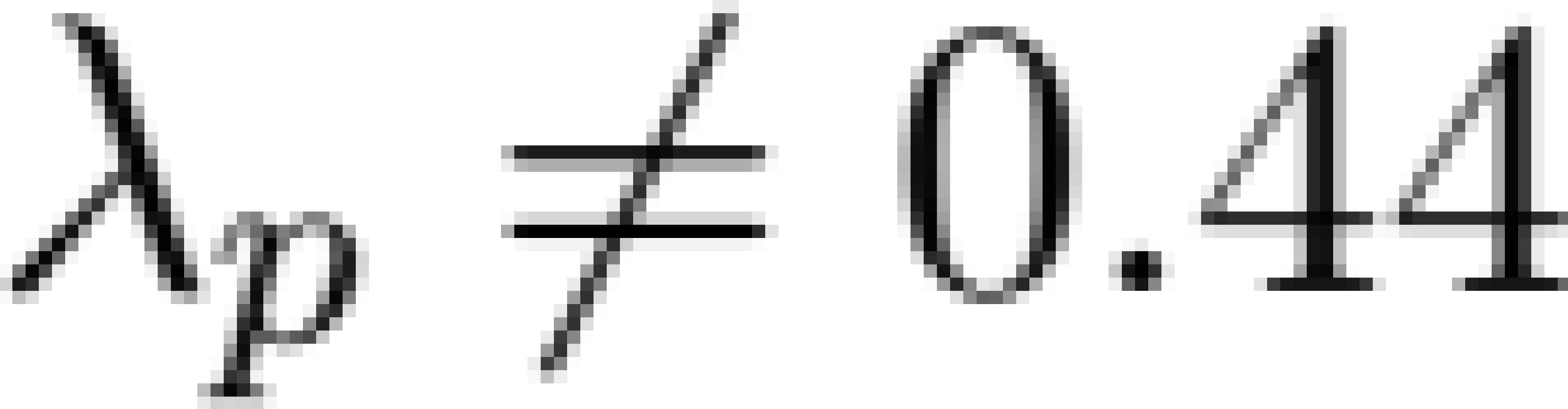
$p(z)$

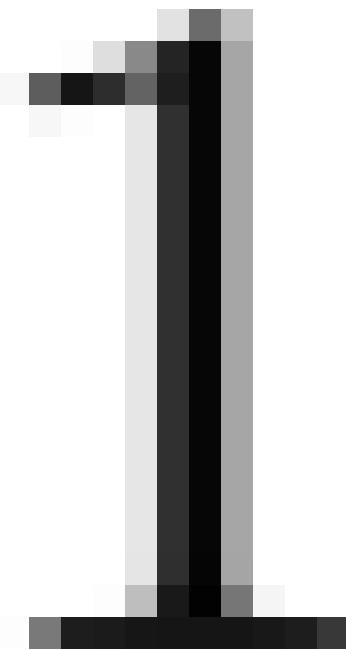
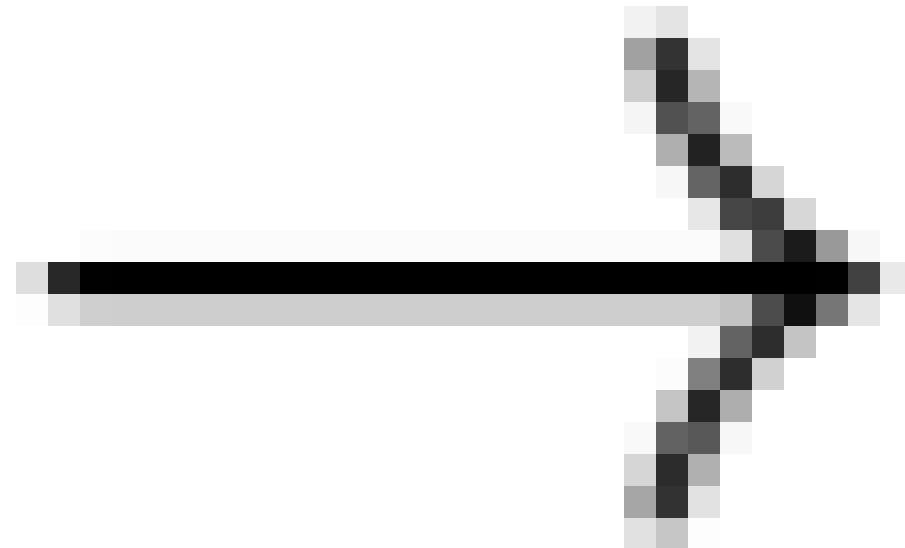


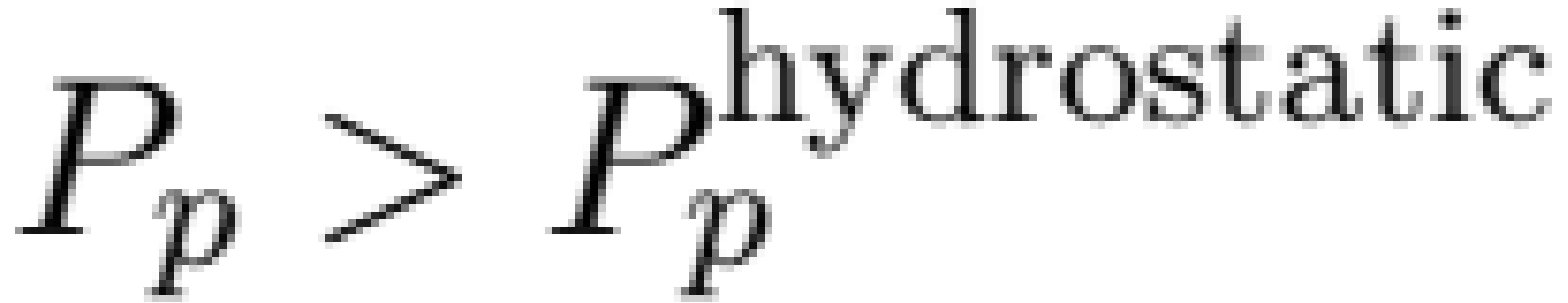
$P_p(z)$
 $S_q(z)$



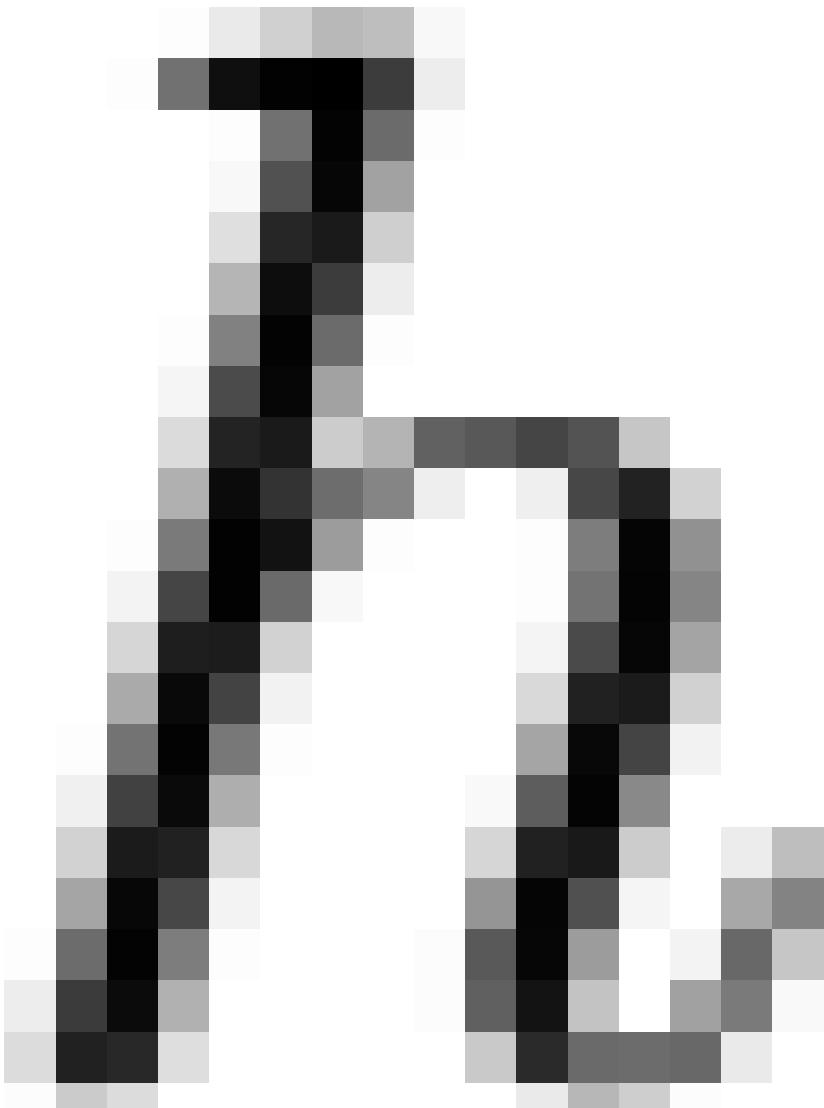














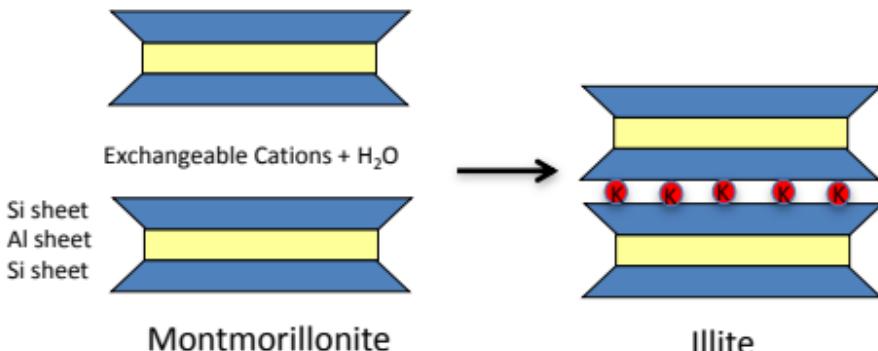


- **Aquathermal pressurization**

- $\Delta T \rightarrow \Delta P$

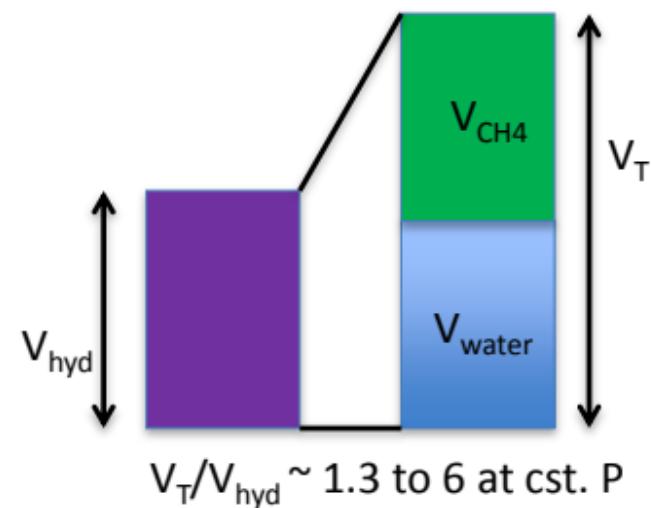
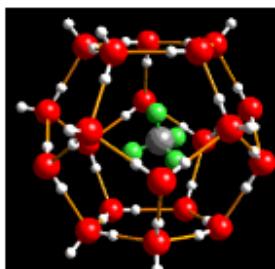
- **Dehydration reactions**

- $\Delta V \rightarrow \Delta P$
 - Montmorillonite to Illite (frees water)

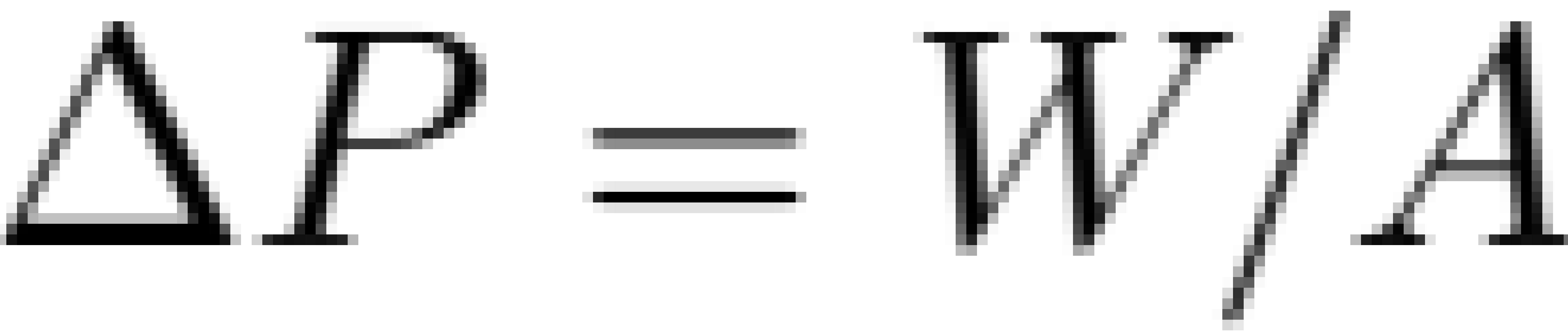


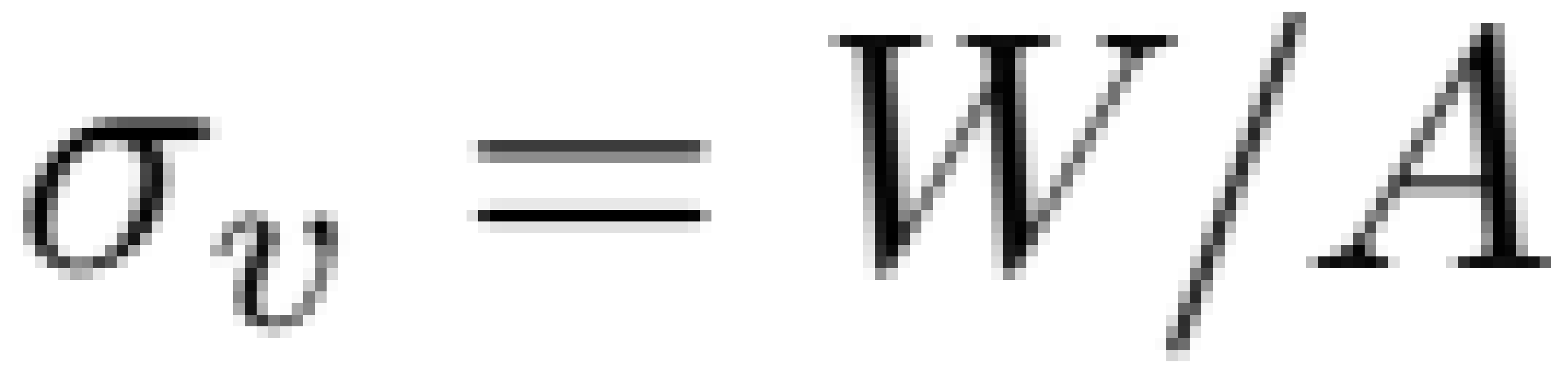
- **Hydrocarbon generation**

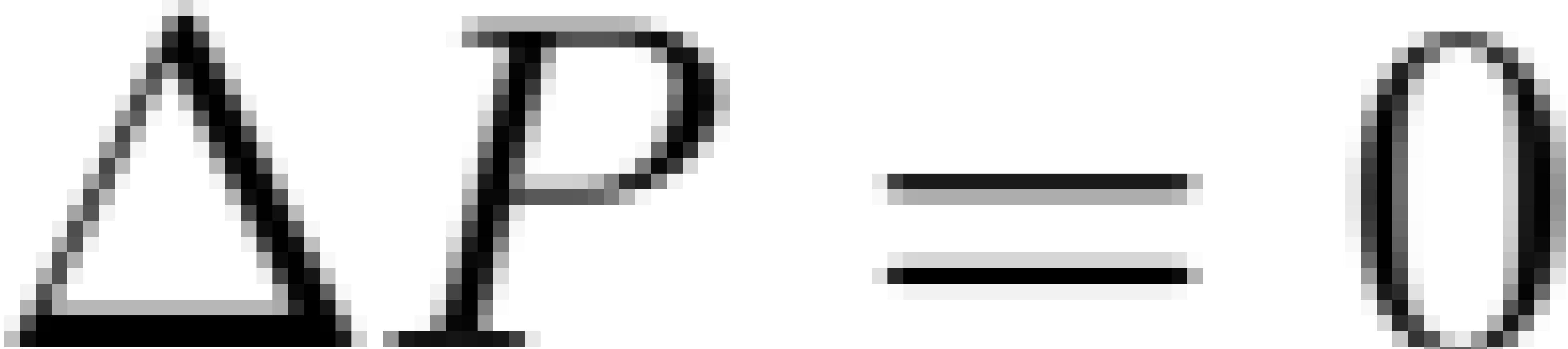
- $\Delta V \rightarrow \Delta P$



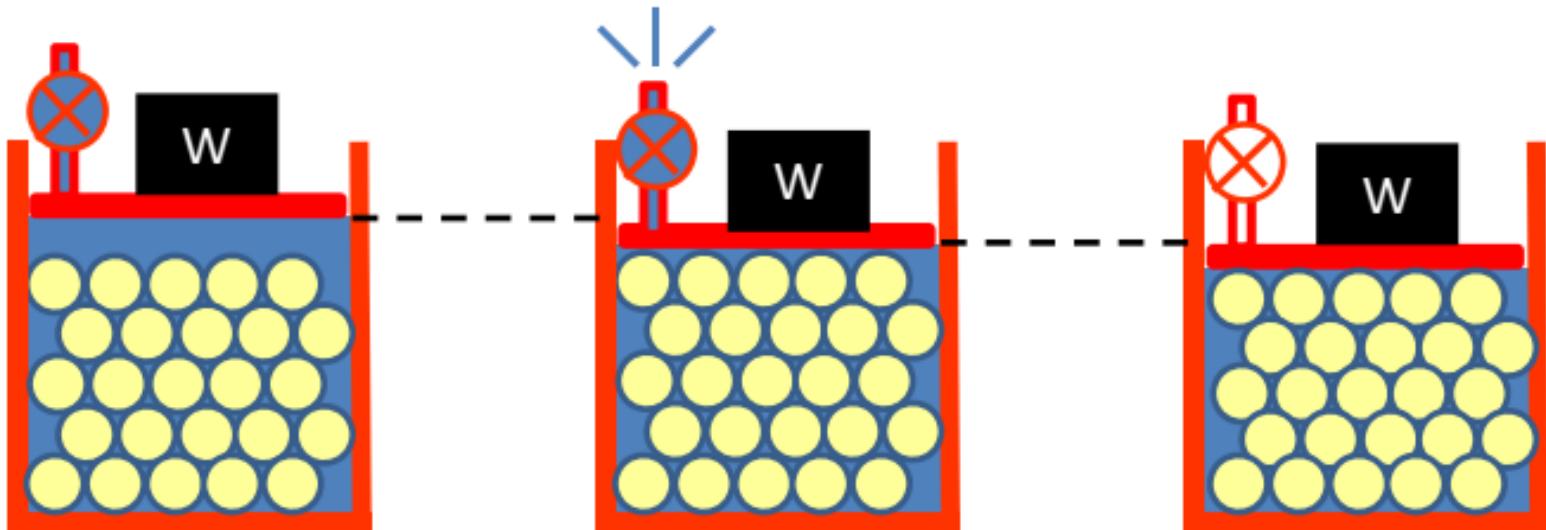




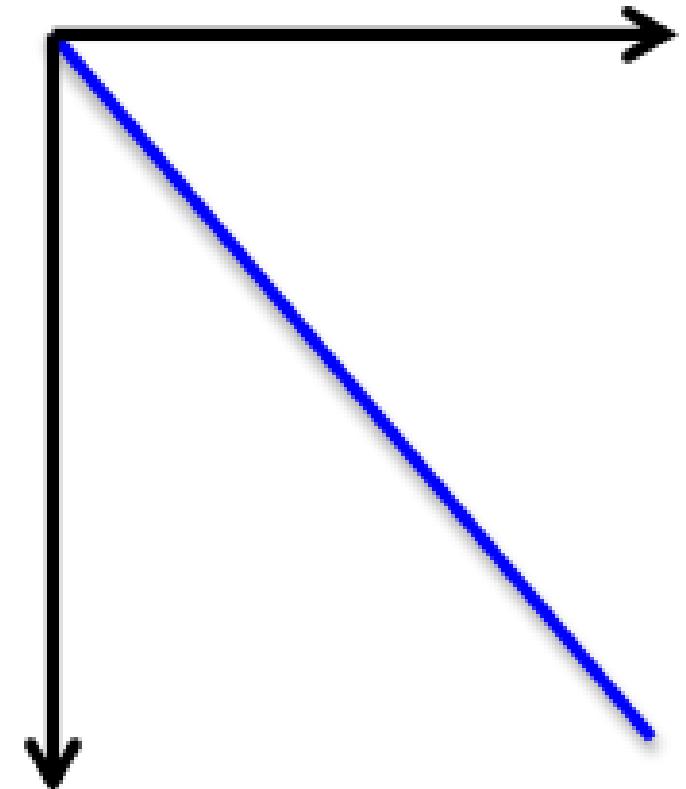
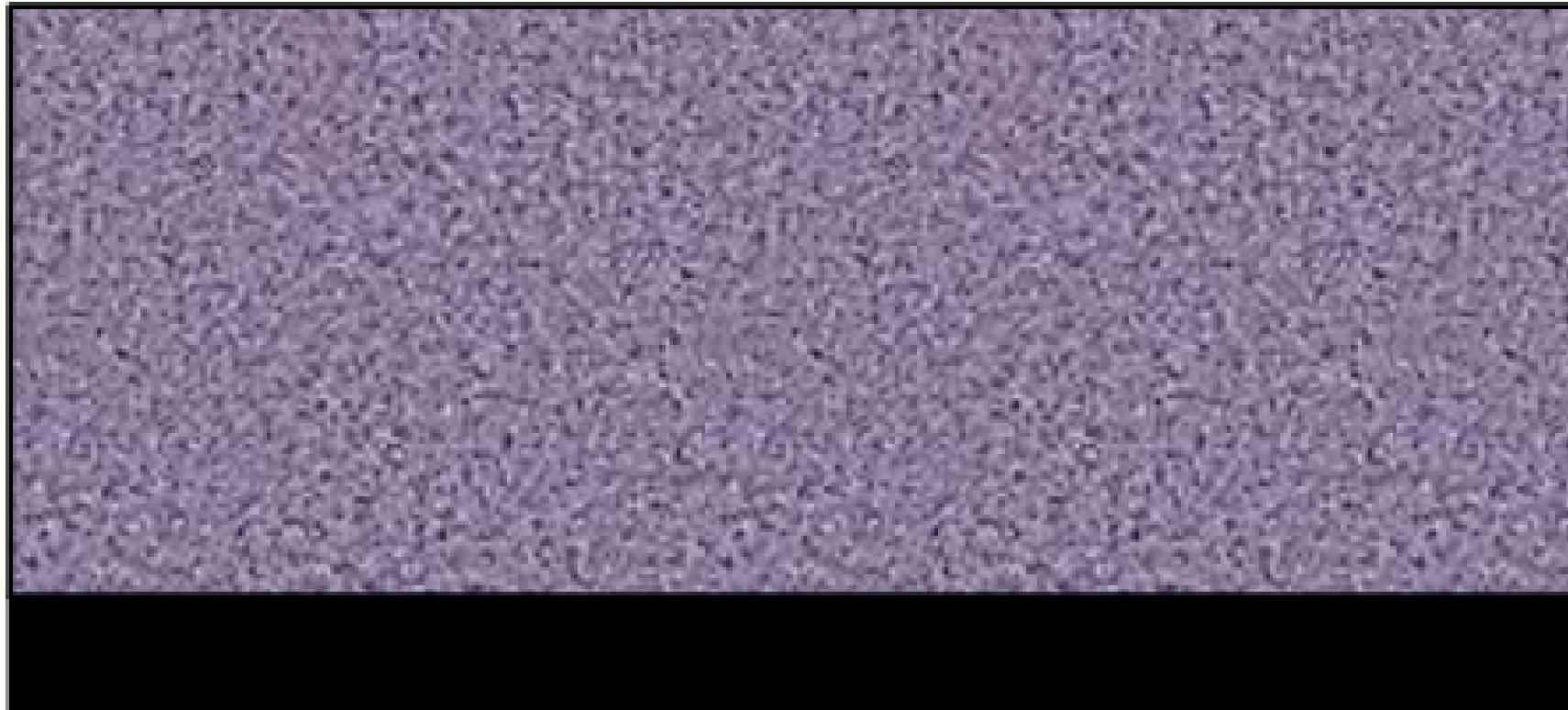


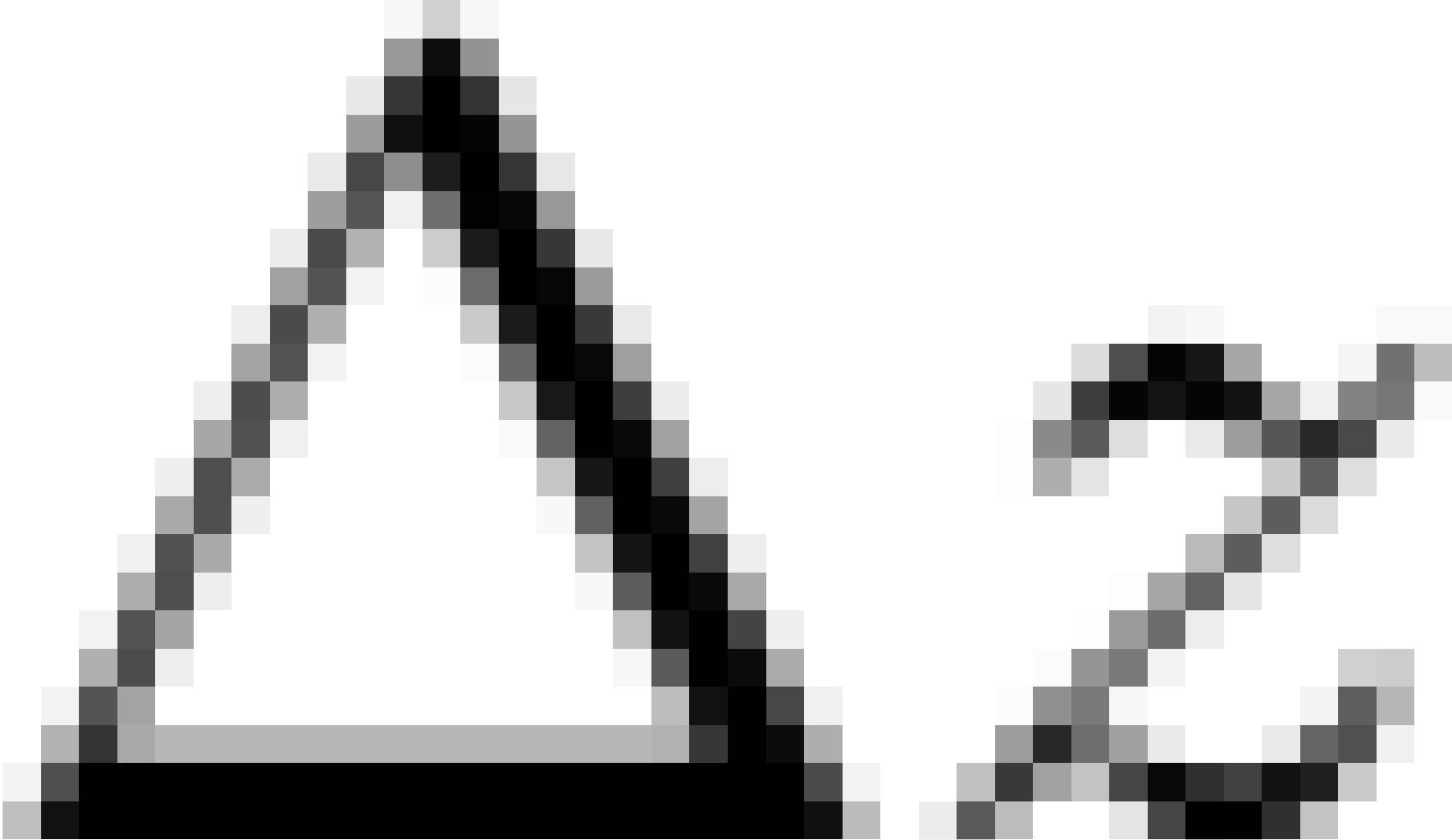


- **Disequilibrium compaction (Underconsolidation)**
 - $\Delta S \rightarrow \Delta P$ (Vertical)
- **Tectonic compression**
 - $\Delta S \rightarrow \Delta P$ (Horizontal)



Pressure water





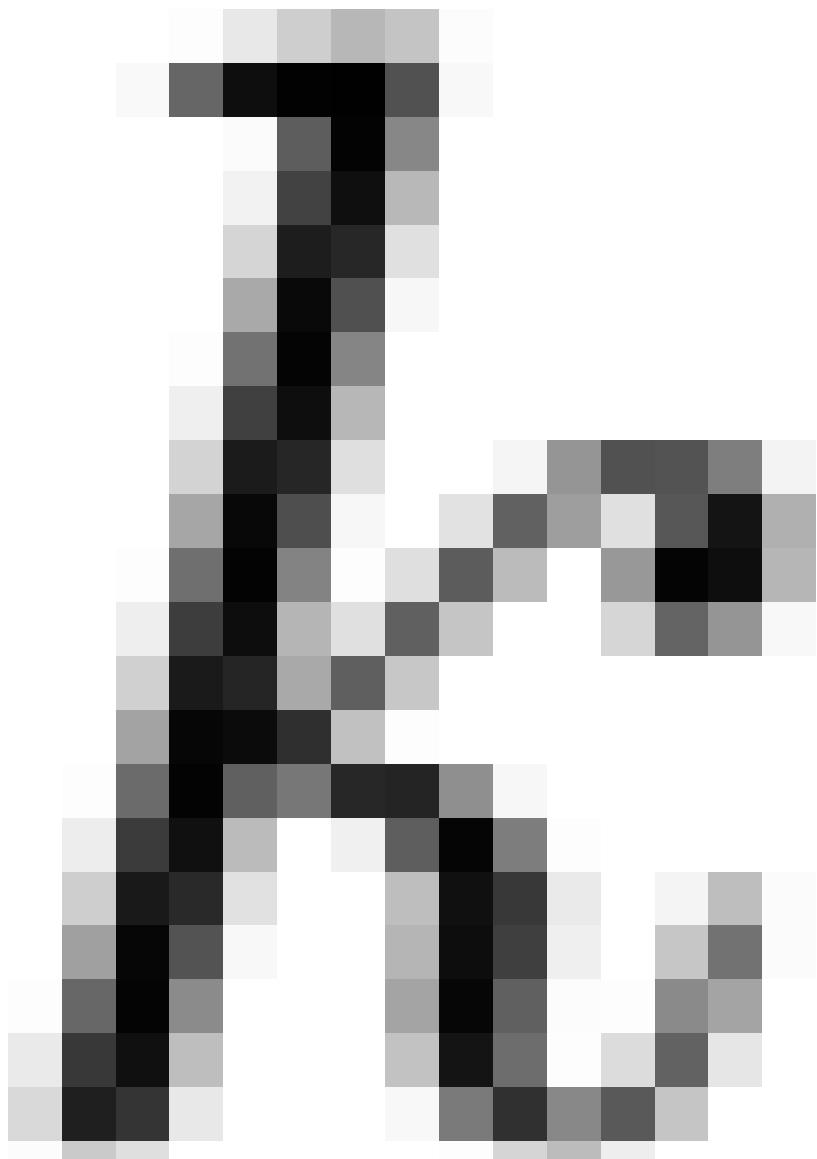
Dh

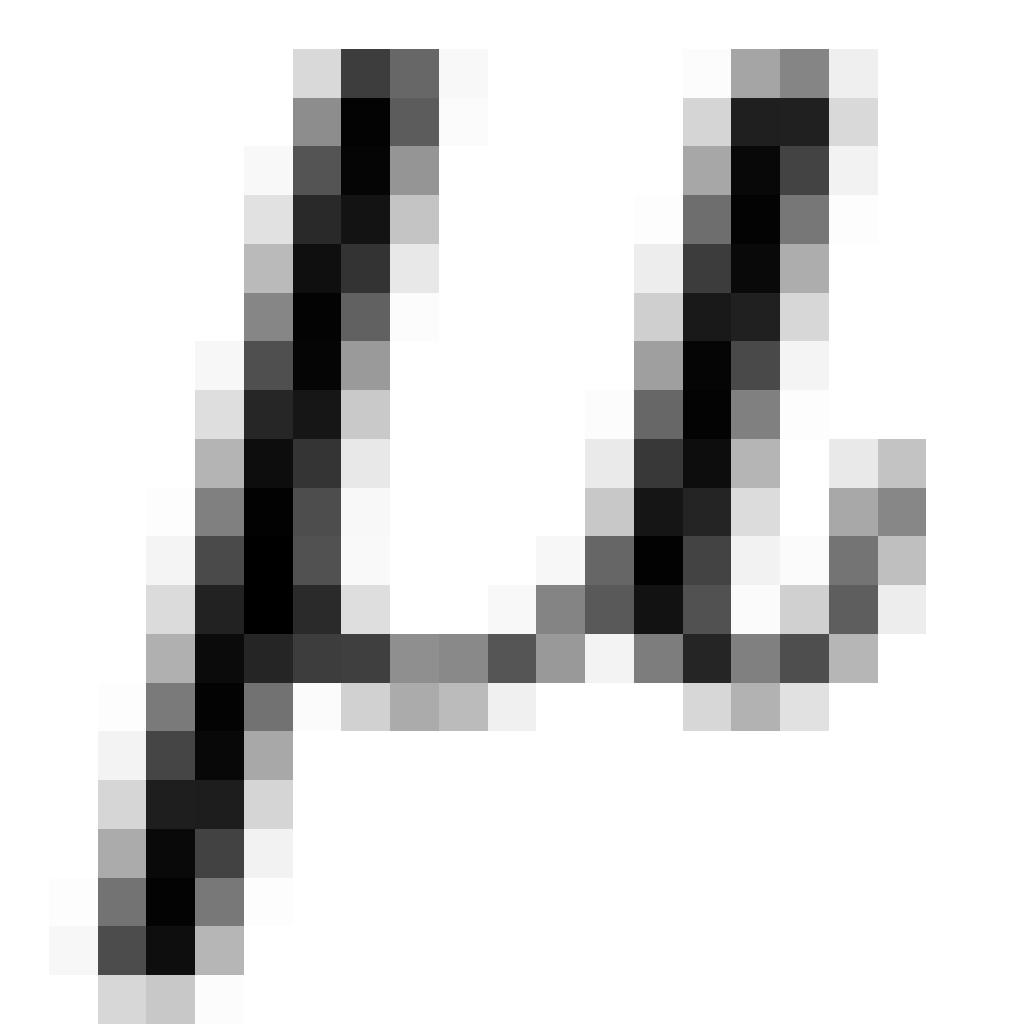
=

MK

ll







αp



αt

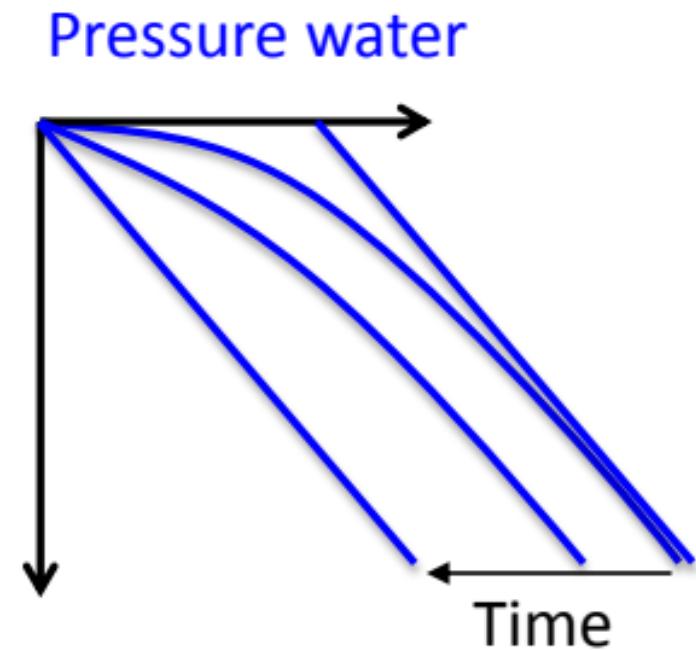
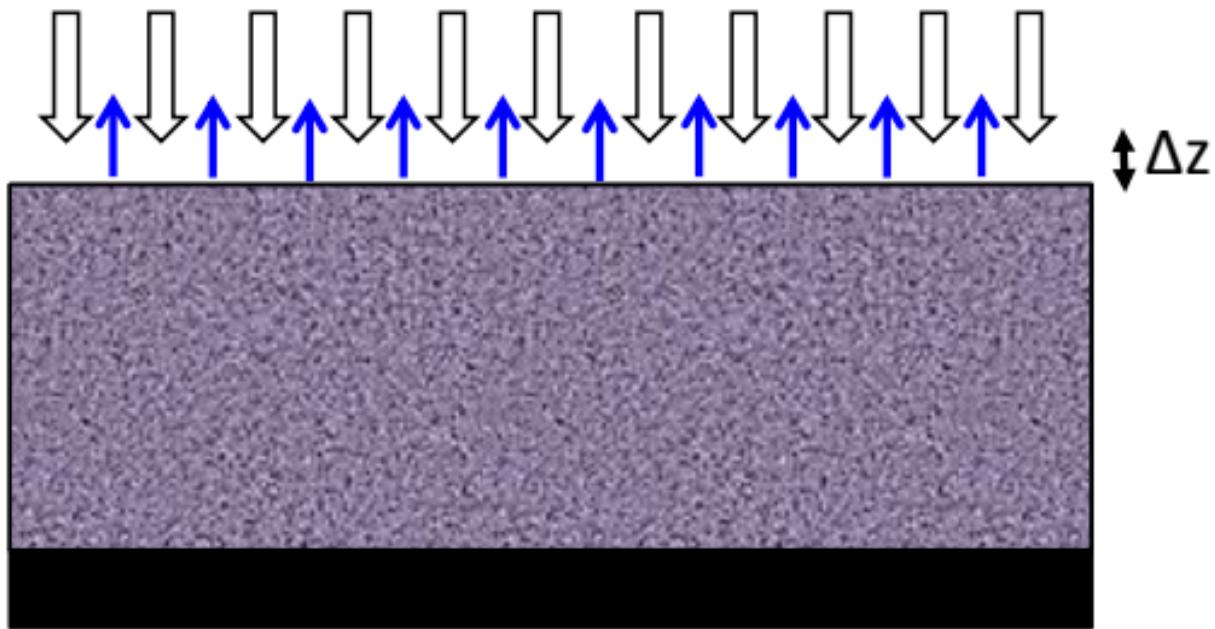


D_b

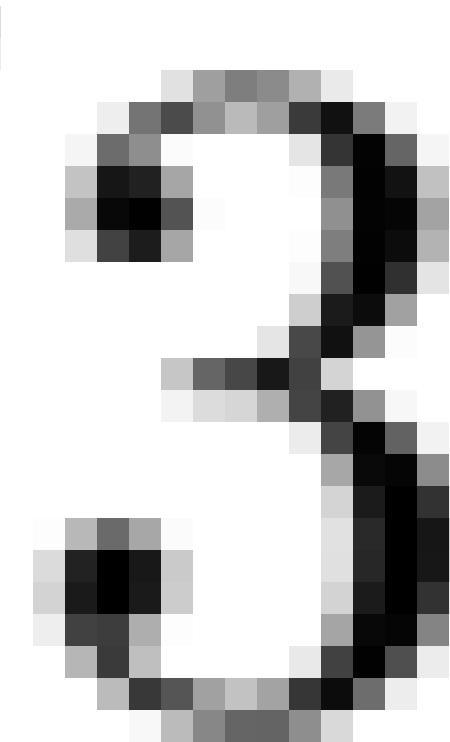
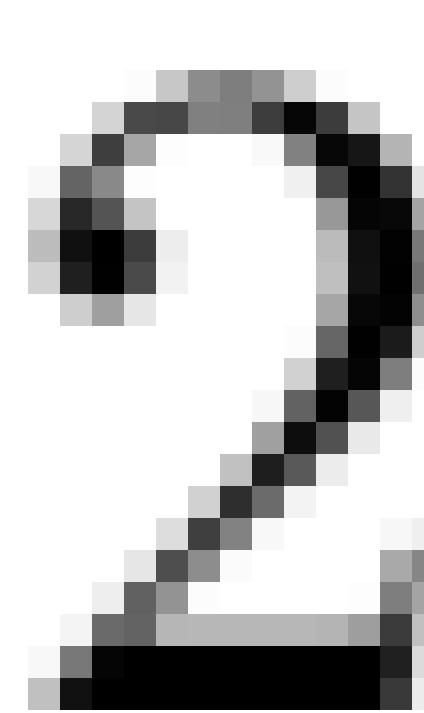
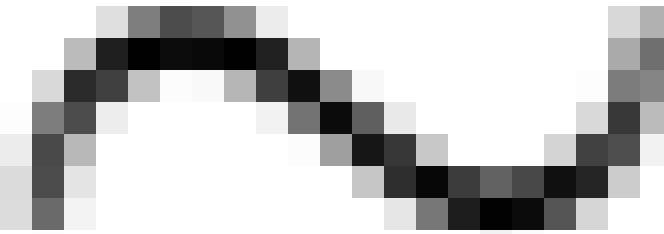
$d^2 p$

dz^2

Rate of sedimentation (loading) and rate of fluid “escape”





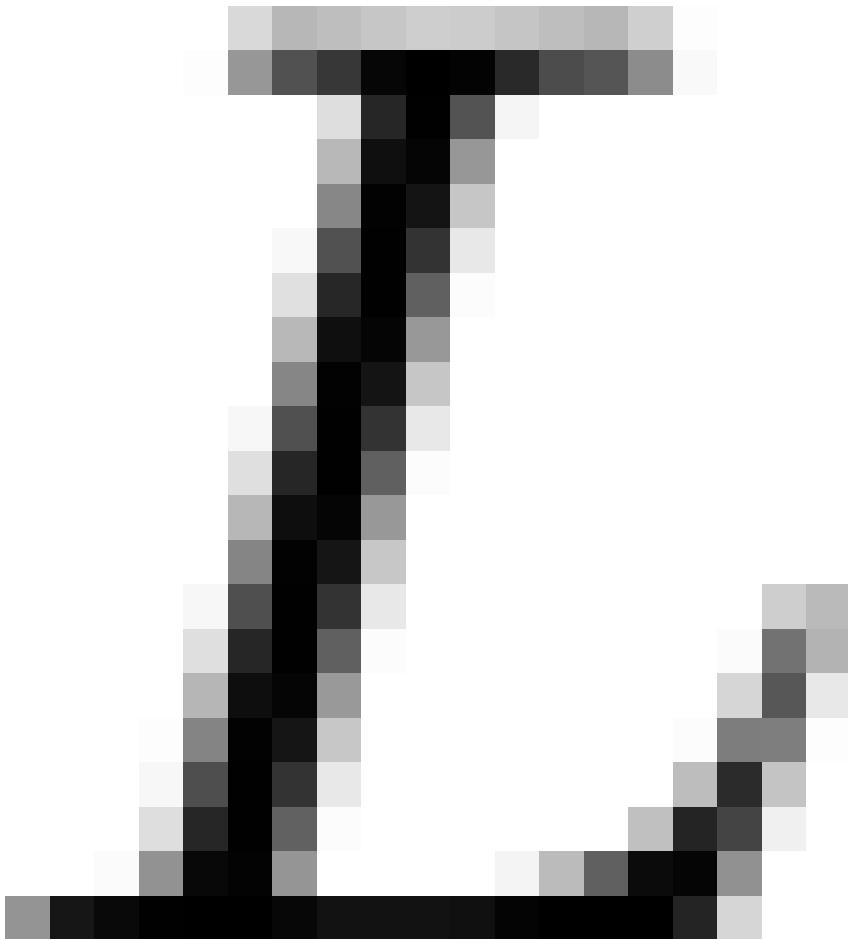


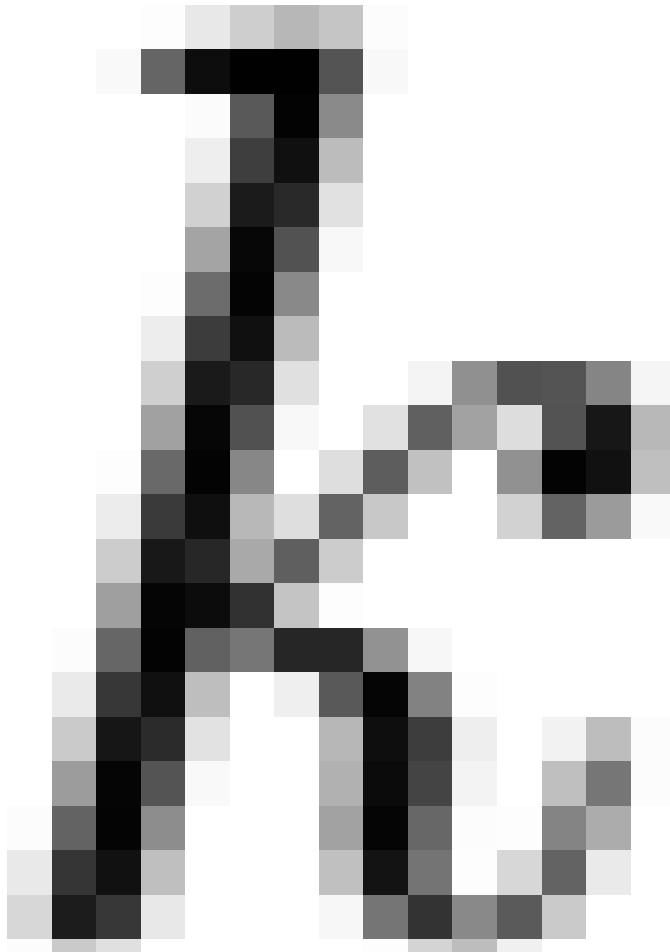
Γ_{ch}

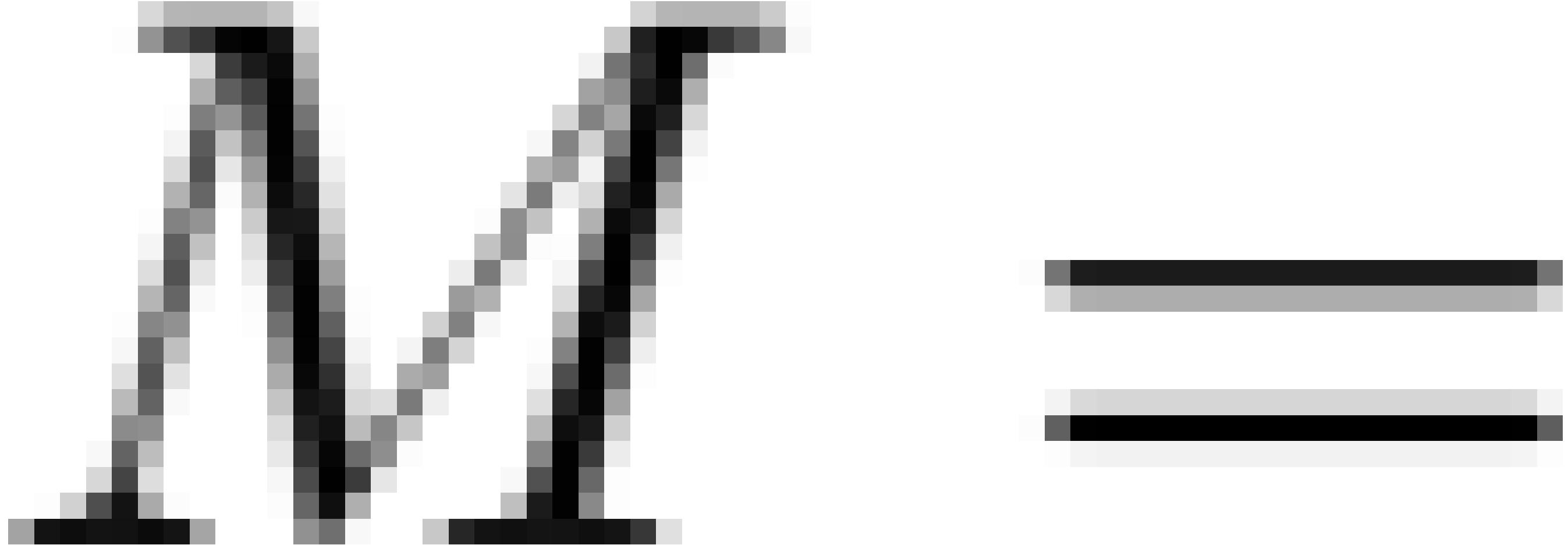


D_h

Γ^2

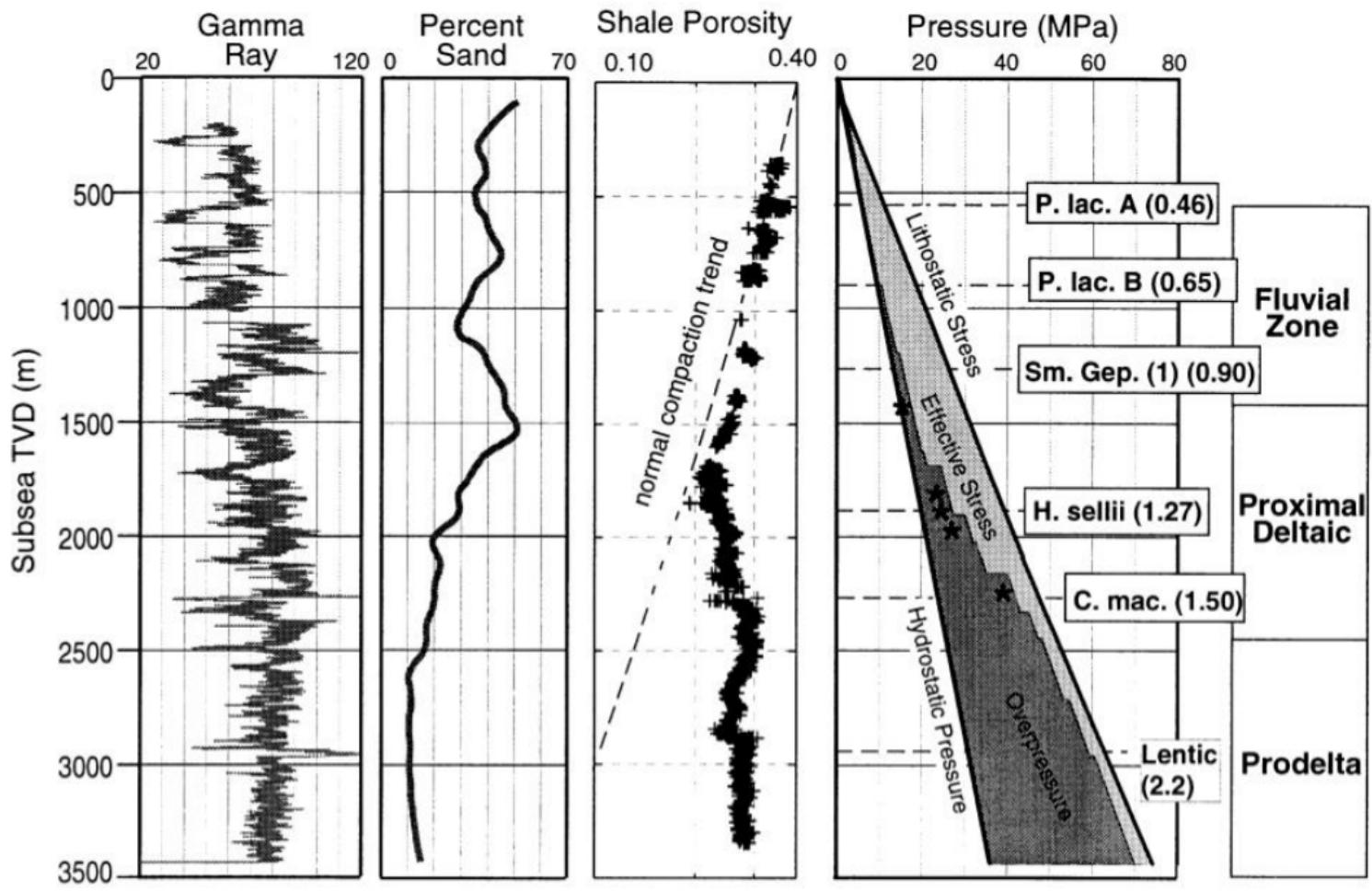




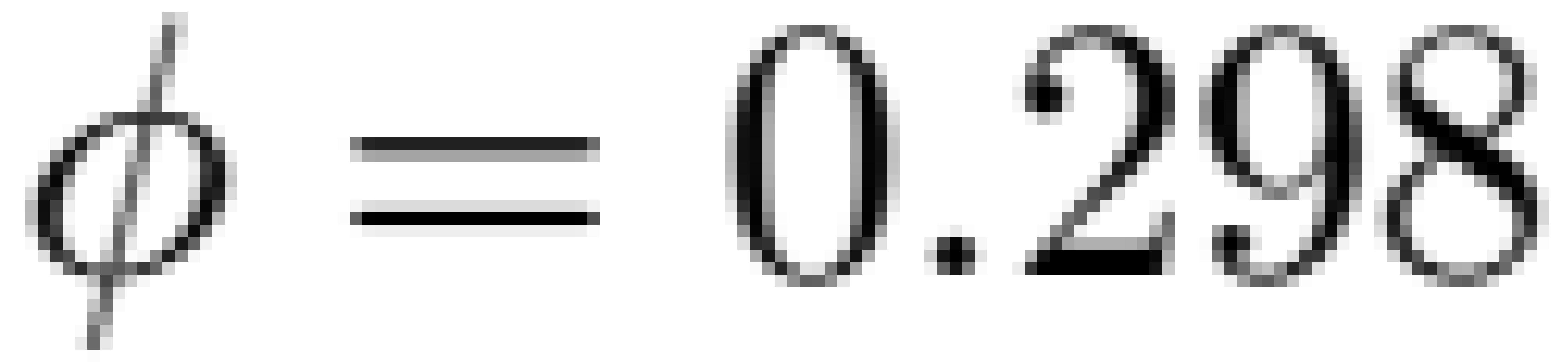


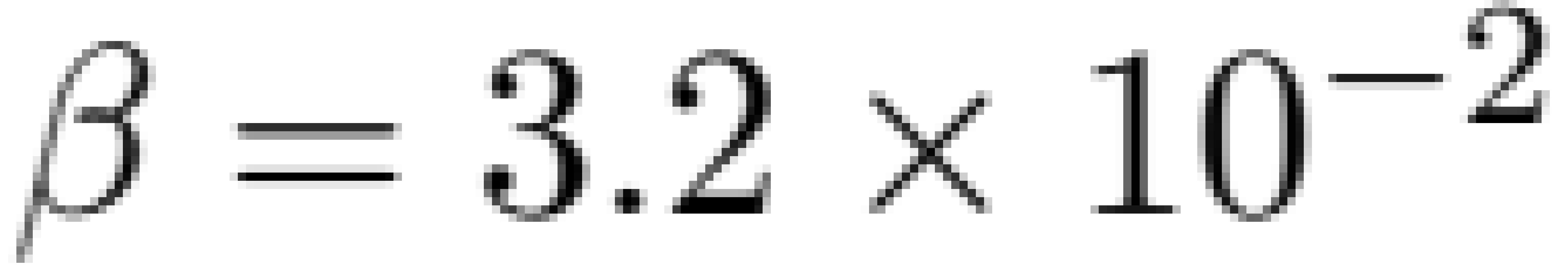


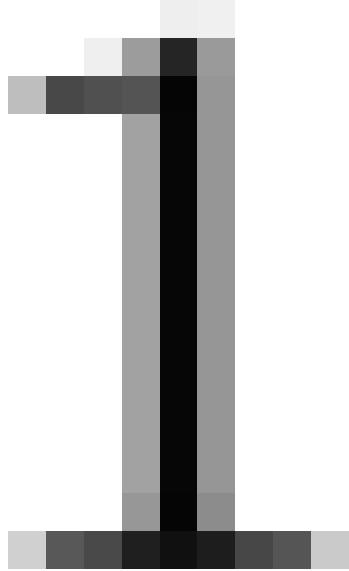


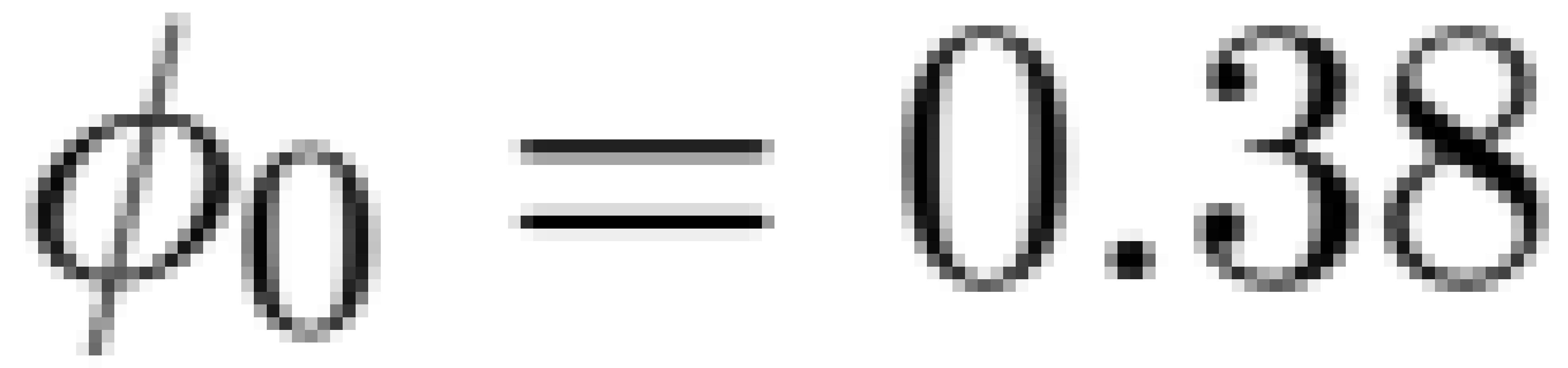


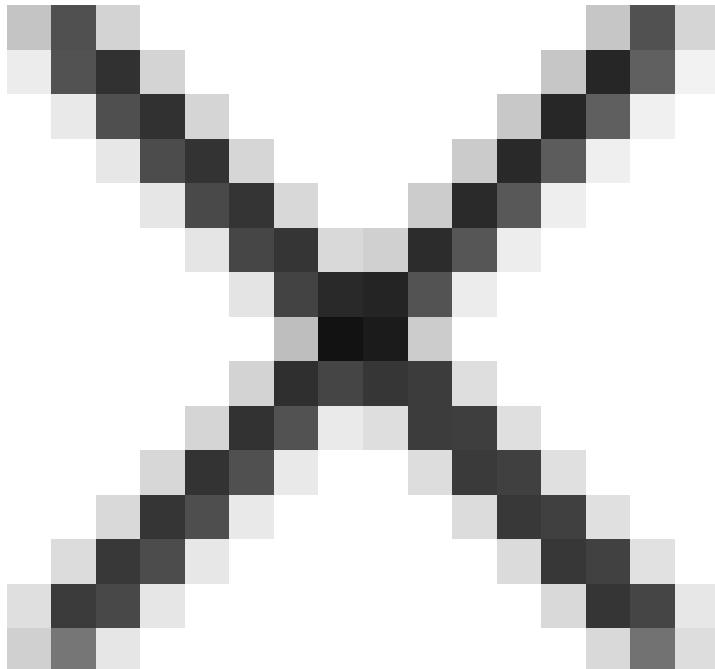
Off-shore Louisiana – Gordon and Flemings (1998) Basin Research



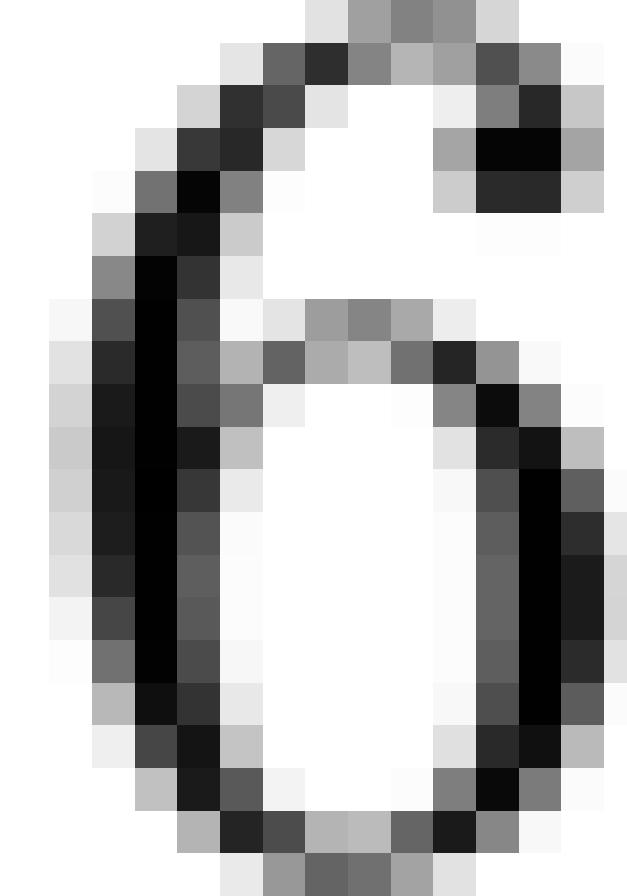
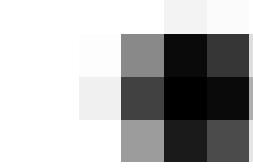
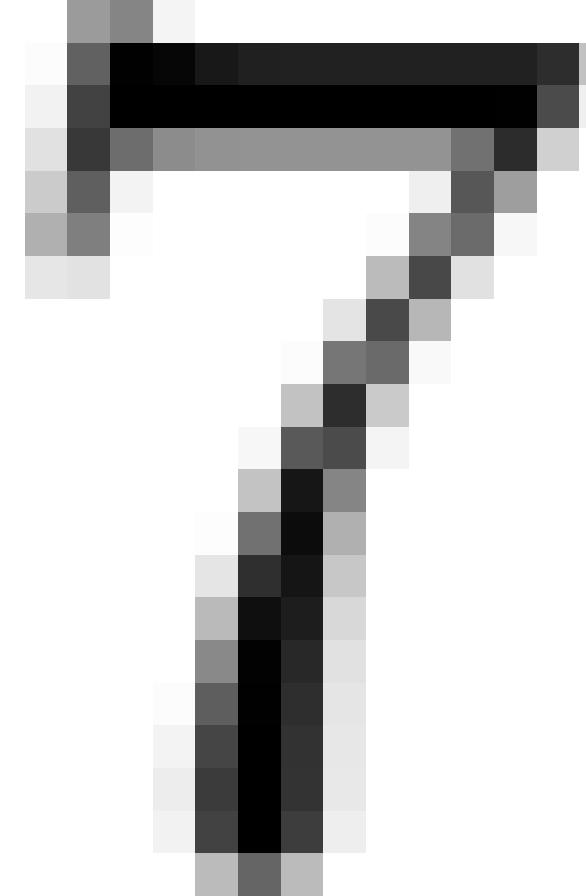


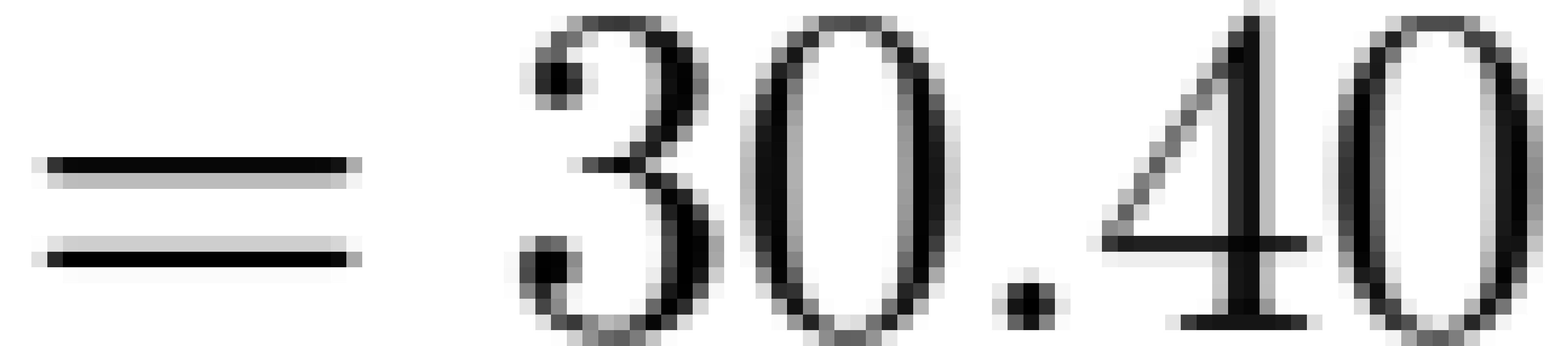




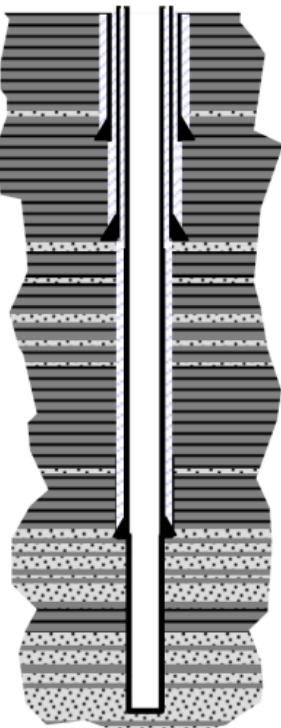
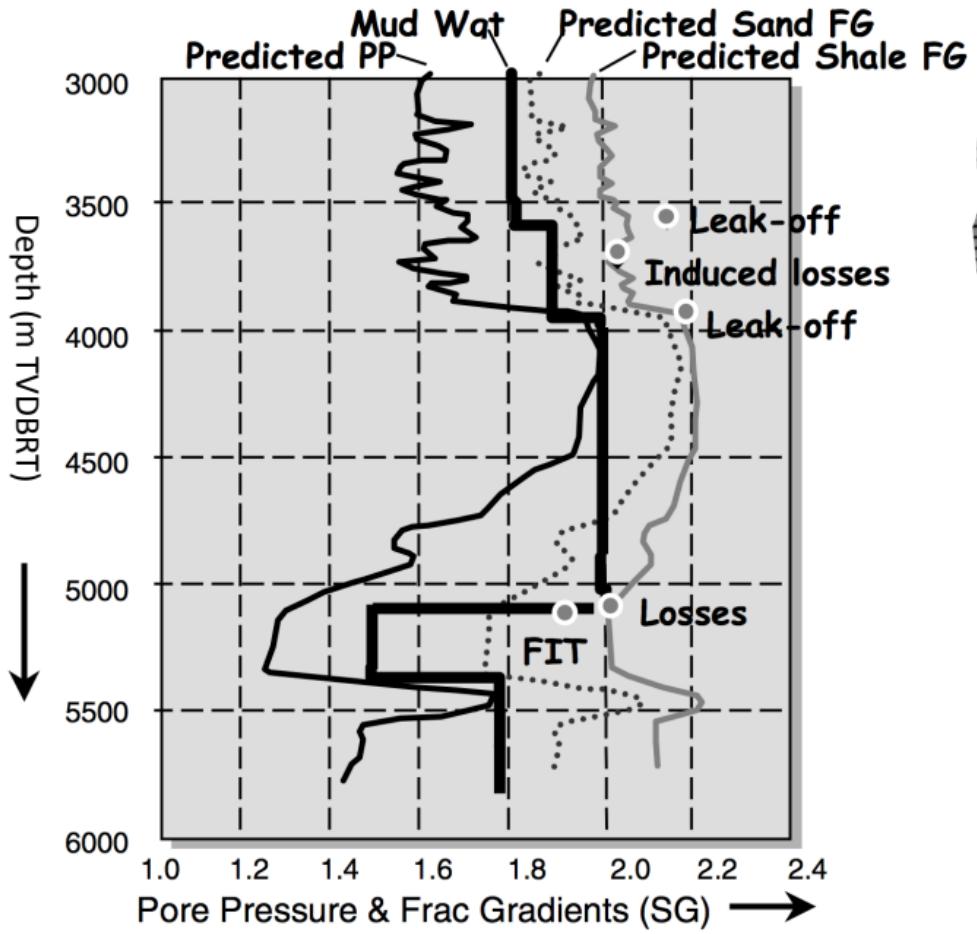


$$P_p = S_p + \ln(\phi/\phi_0) = 38$$

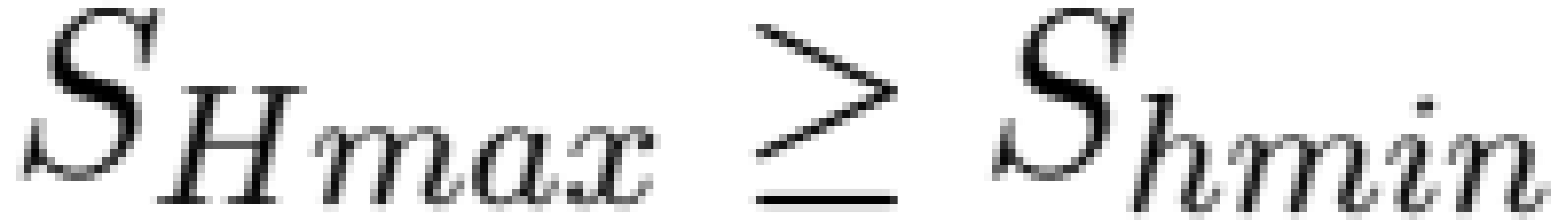


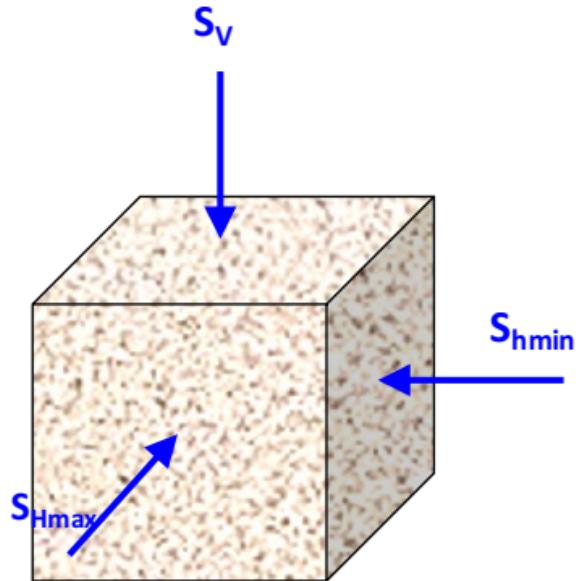
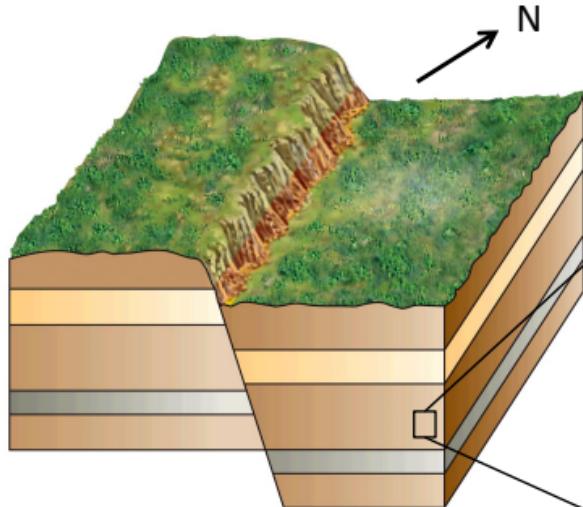


P = 30.40 MPa
 S_2 = 0.8.
38 MPa



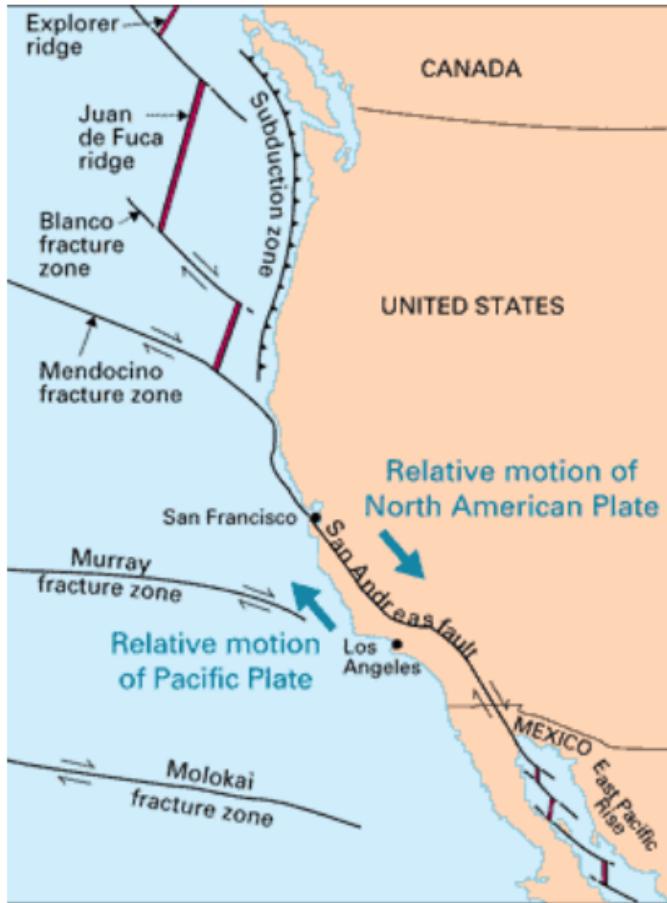
[Caspian Sea, Alberta and McLean – SPE67740]





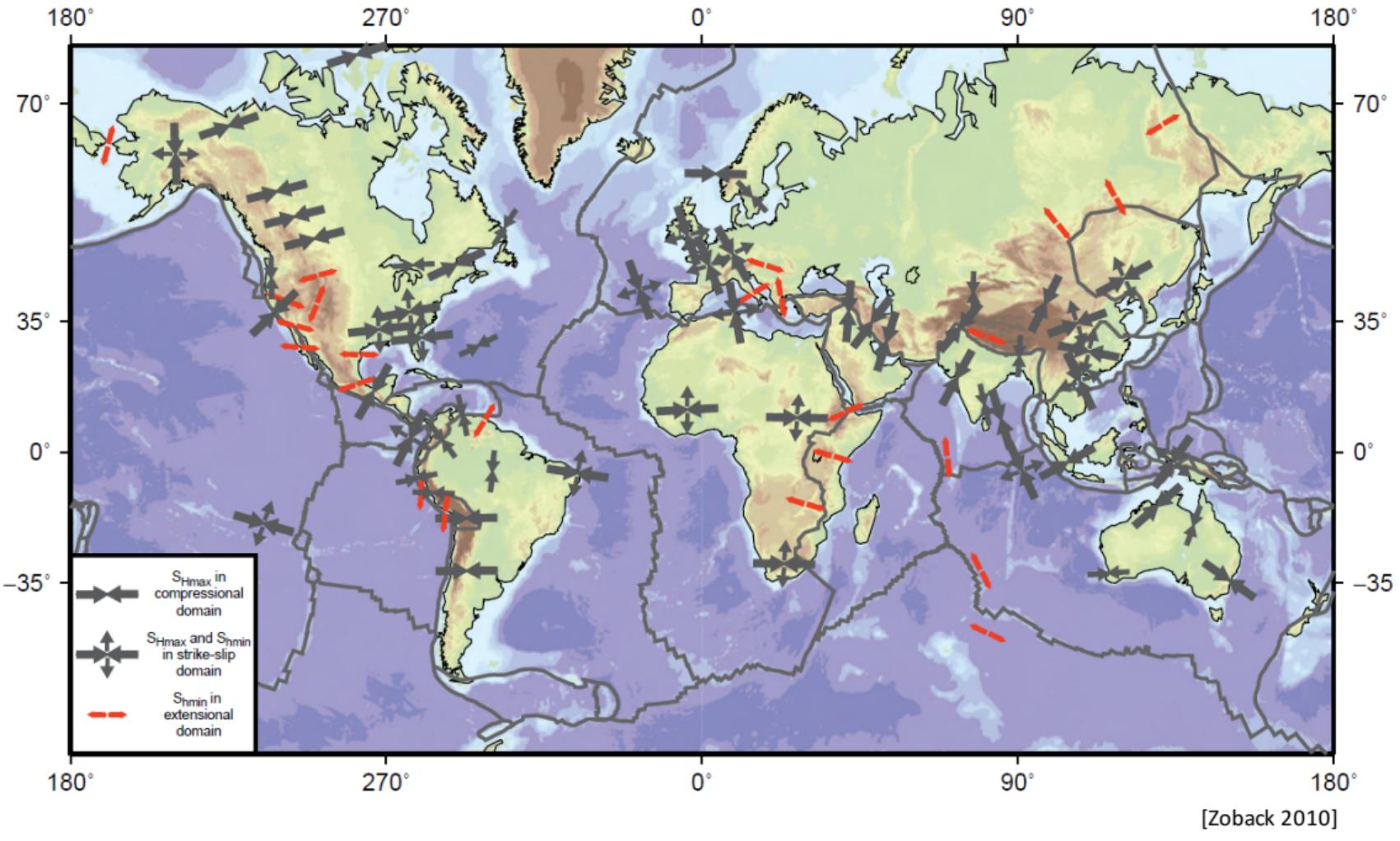
Self-test 9.8
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$$\underline{\underline{S}} = \begin{bmatrix} S_V & 0 & 0 \\ 0 & S_{H\max} & 0 \\ 0 & 0 & S_{h\min} \end{bmatrix}$$

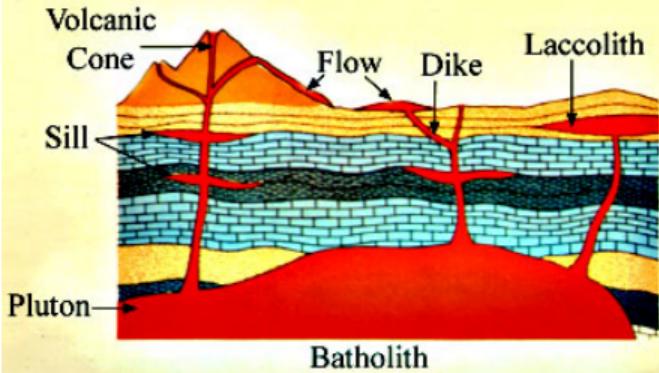


<http://pubs.usgs.gov/gip/dynamic/understanding.html#anchor5798673>

<http://en.wikipedia.org/wiki/File:Aerial-SanAndreas-CarrizoPlain.jpg>



PLUTONS & VOLCANIC LANDFORMS



<http://www.indiana.edu/~geol105/1425chap5.htm>
<http://geophysics.ou.edu/geol1114>



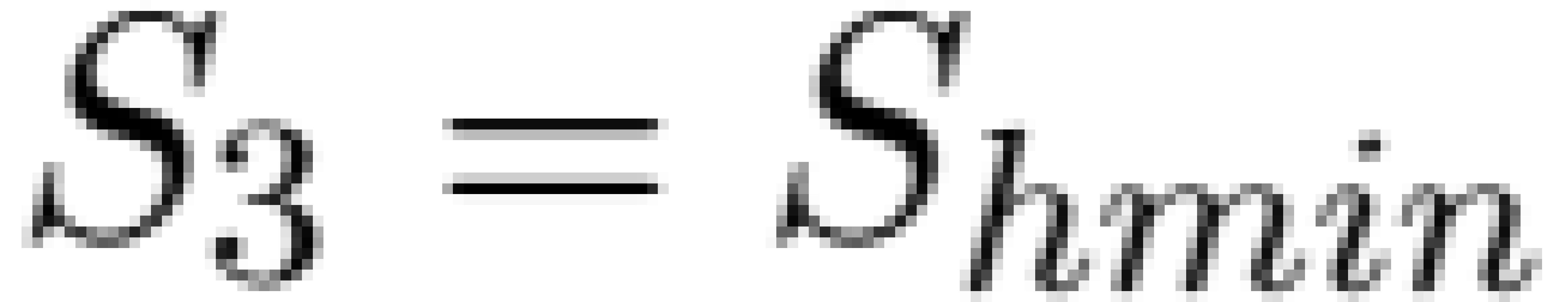


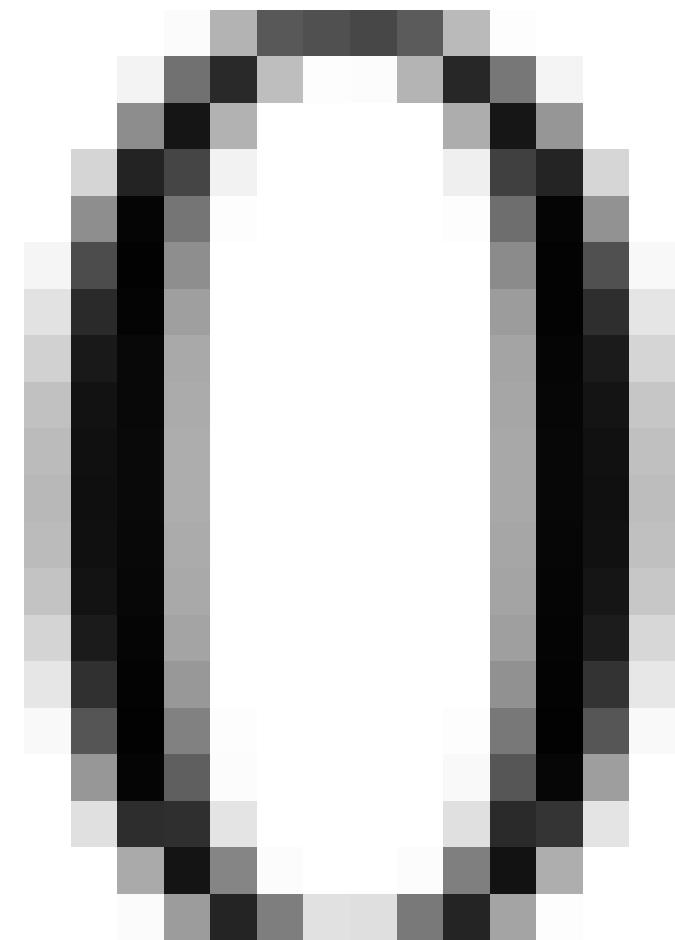
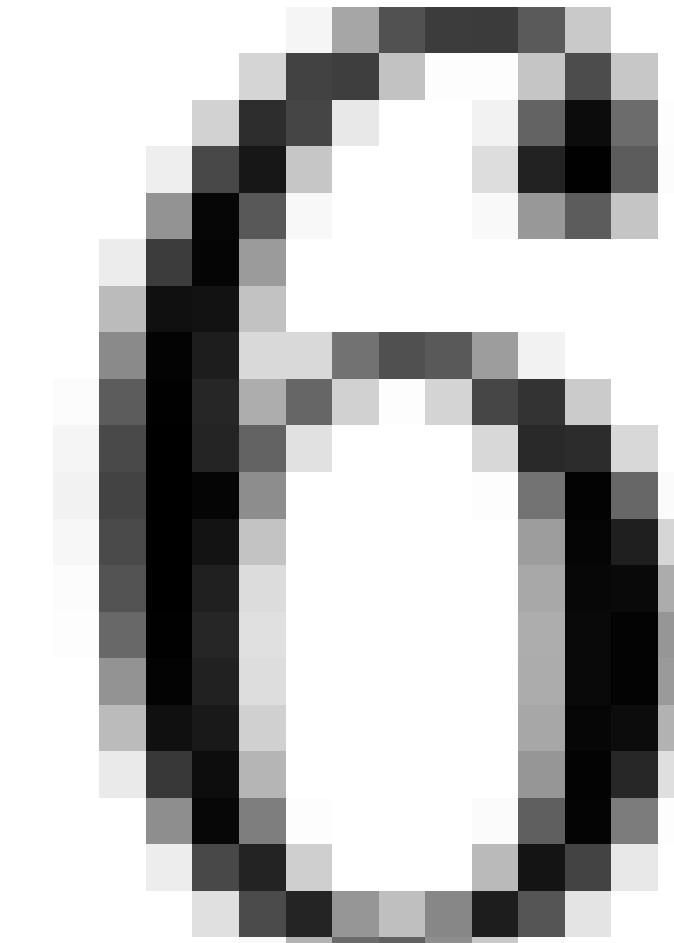
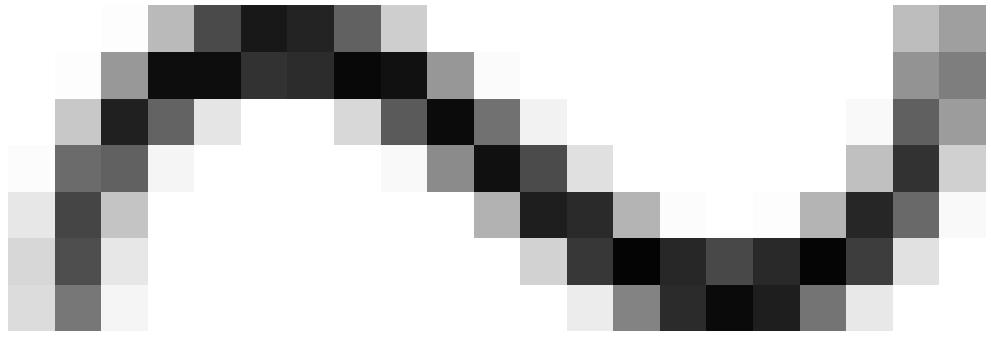








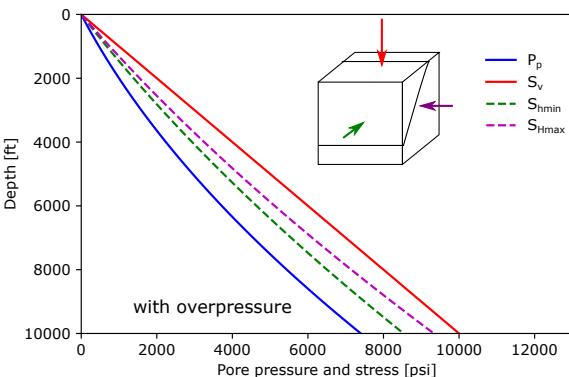
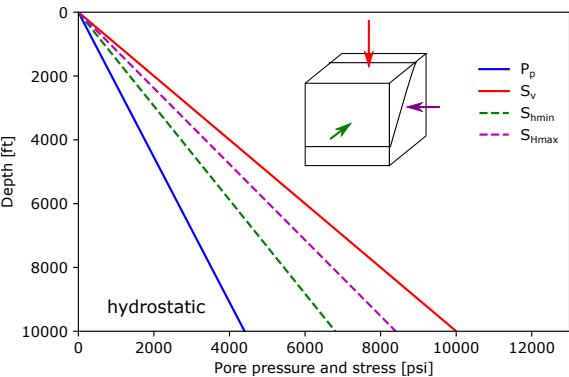




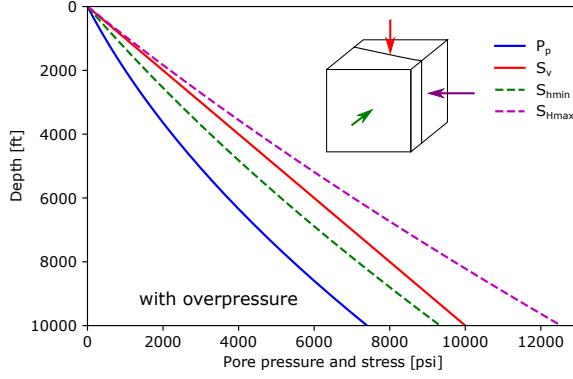
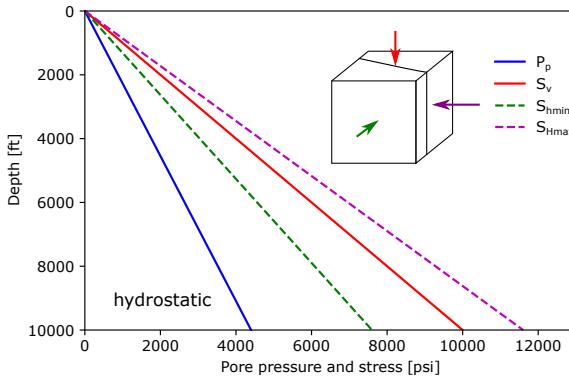




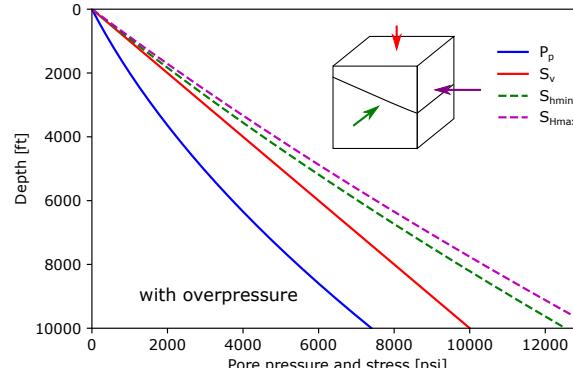
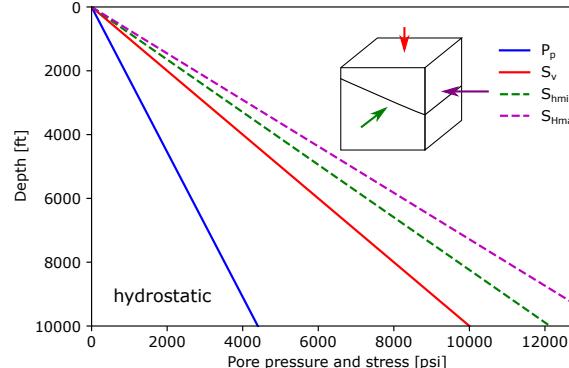
Normal faulting: $S_v > S_{H\max} > S_{h\min}$

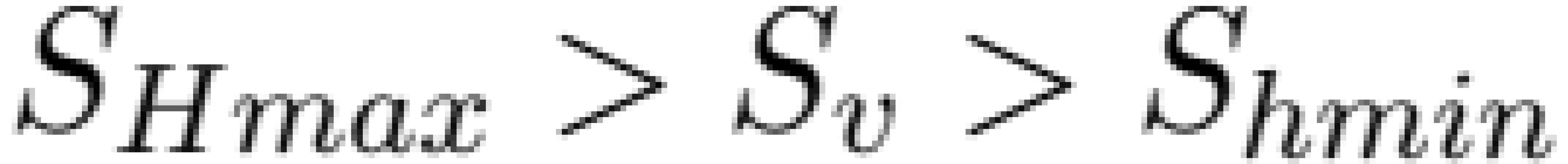


Strike slip faulting: $S_{H\max} > S_v > S_{h\min}$



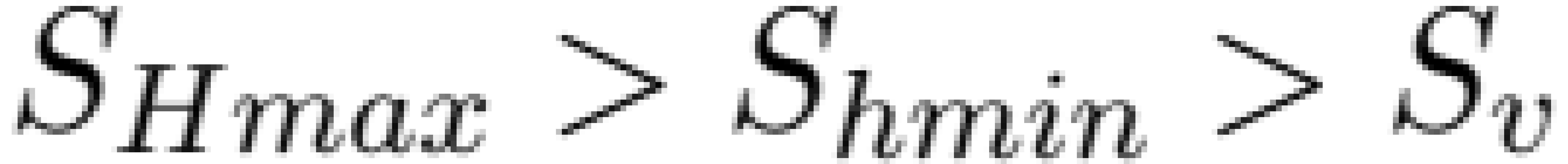
Reverse faulting: $S_{H\max} > S_{h\min} > S_v$

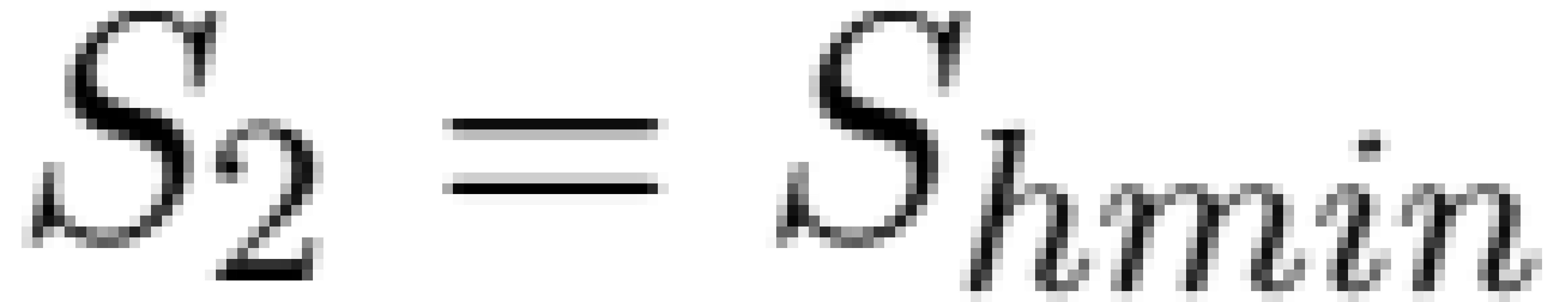




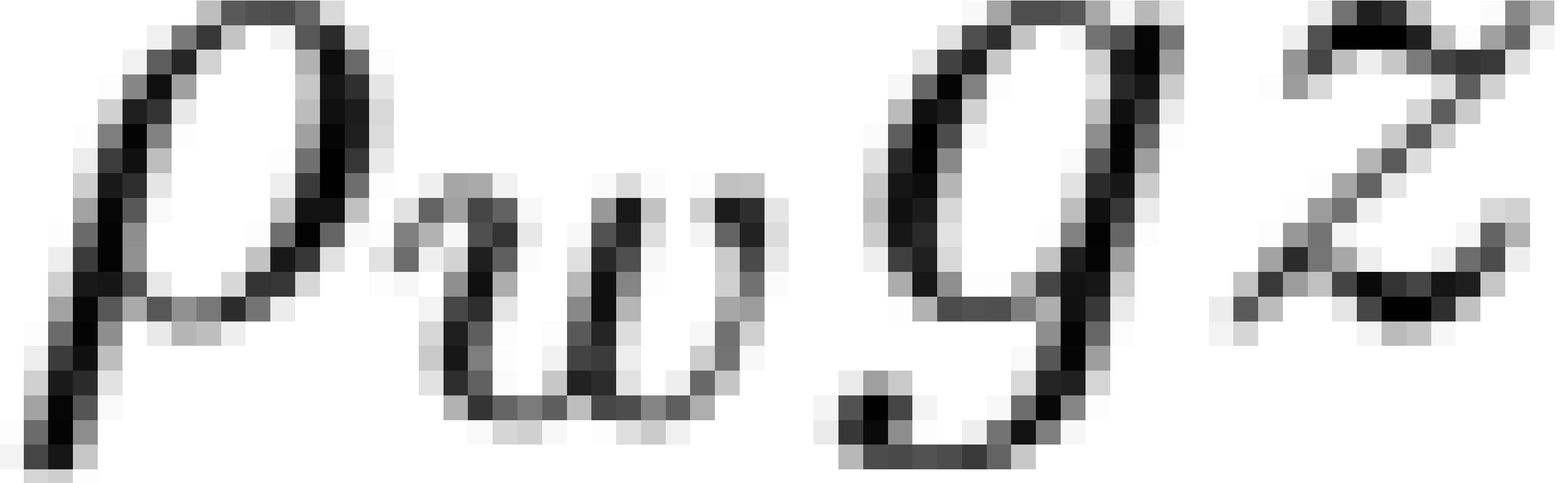










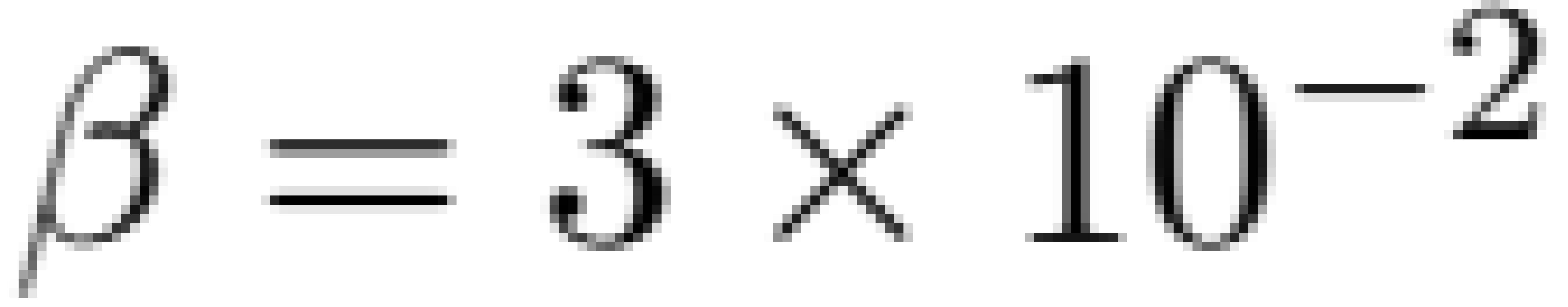


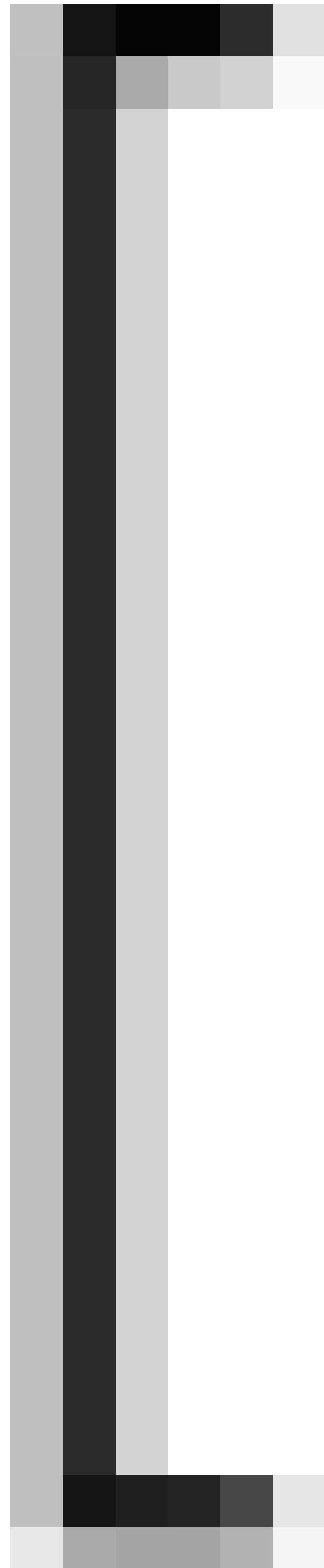


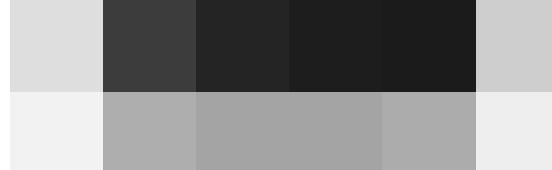


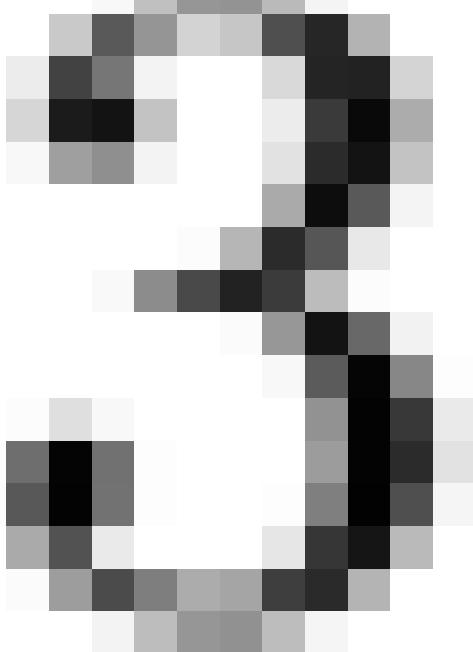


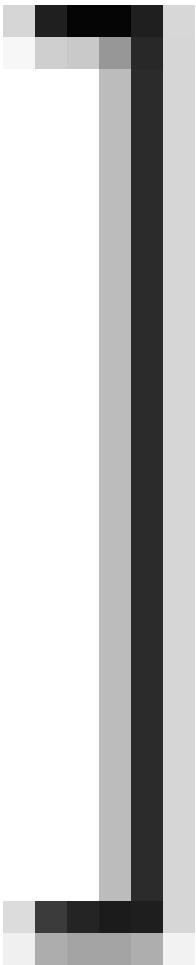
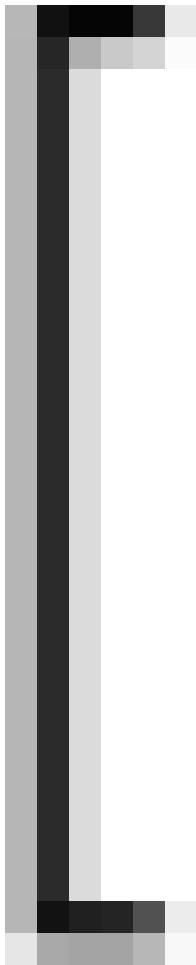


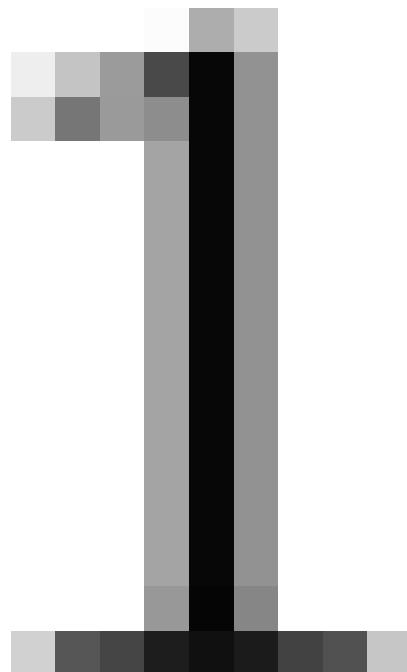
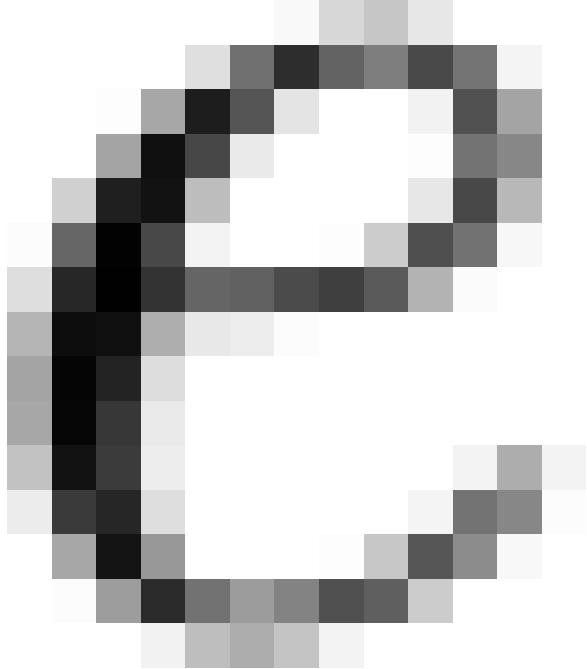


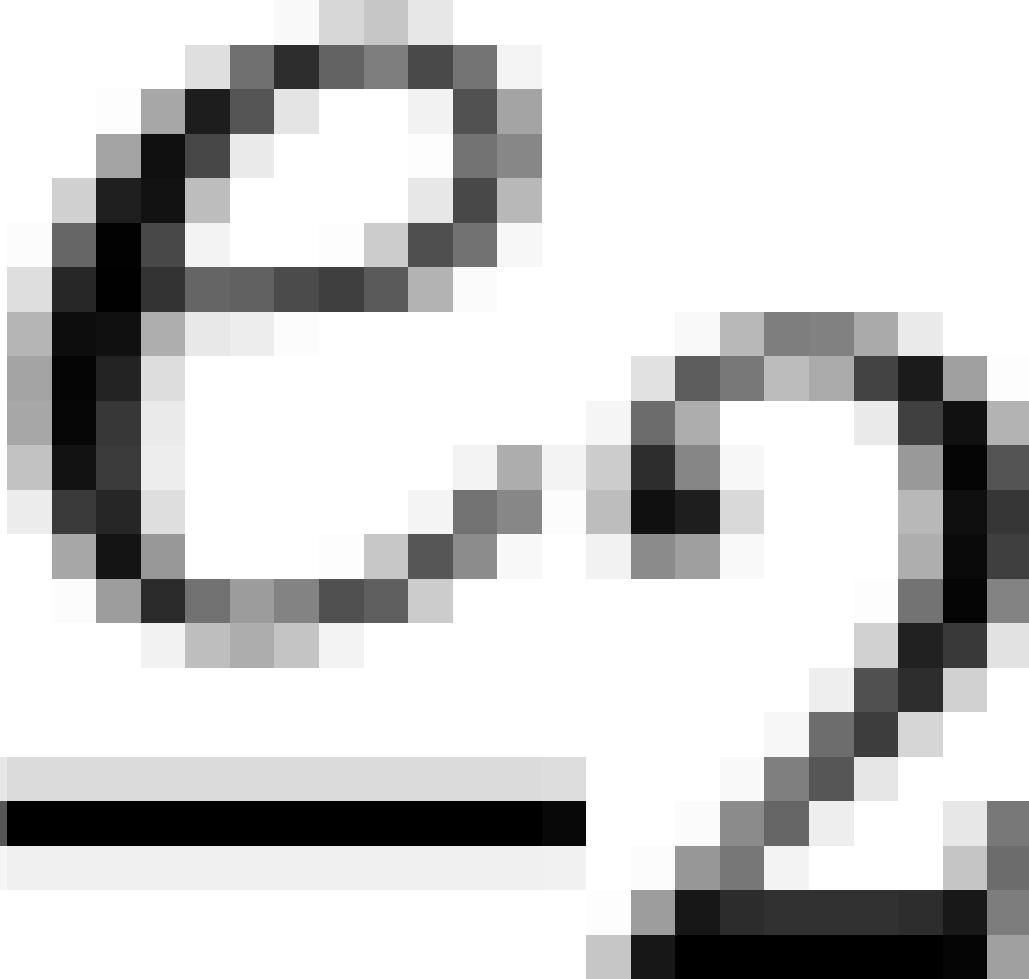


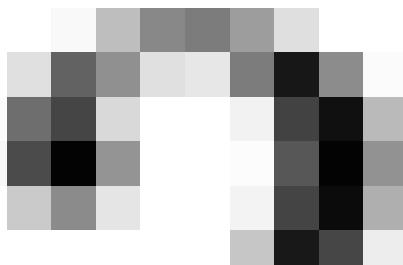
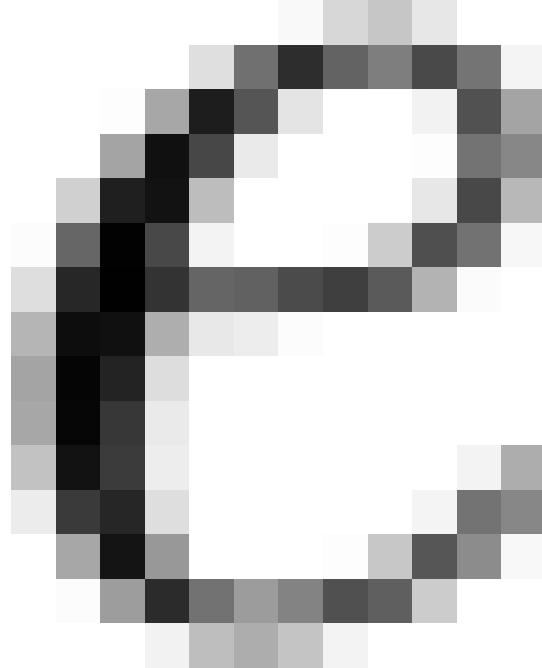


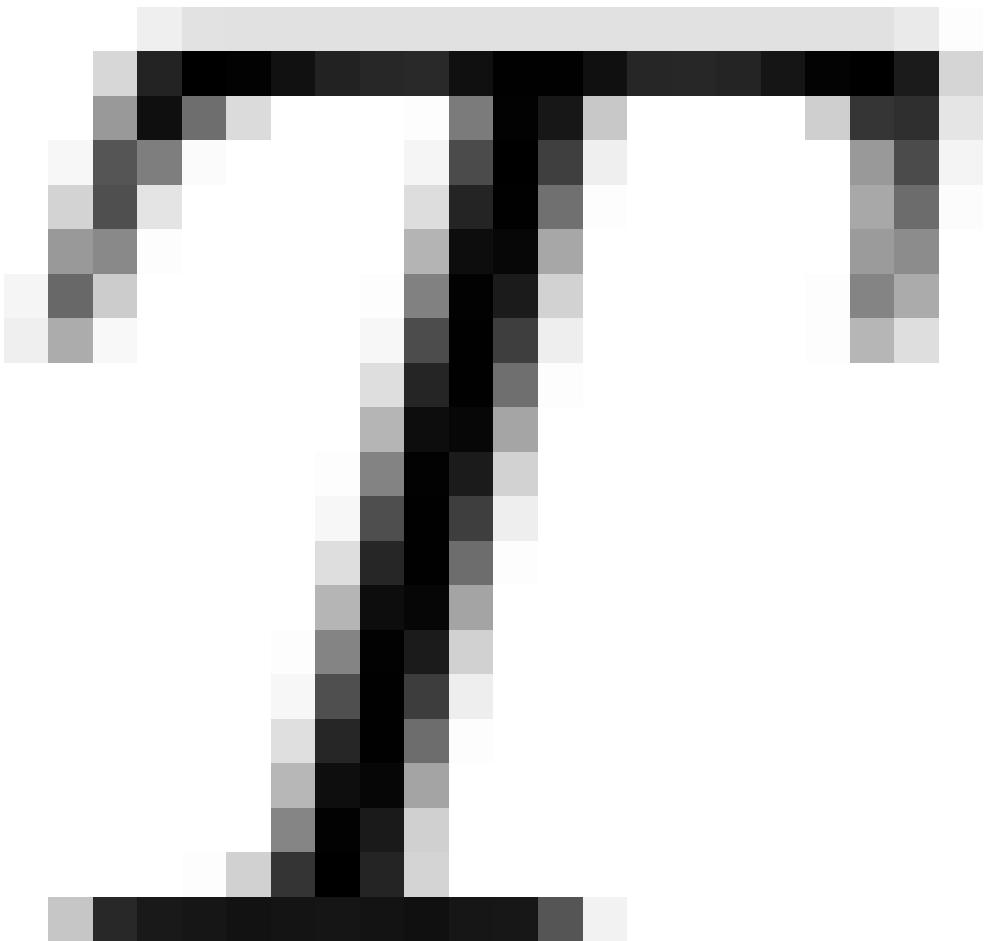


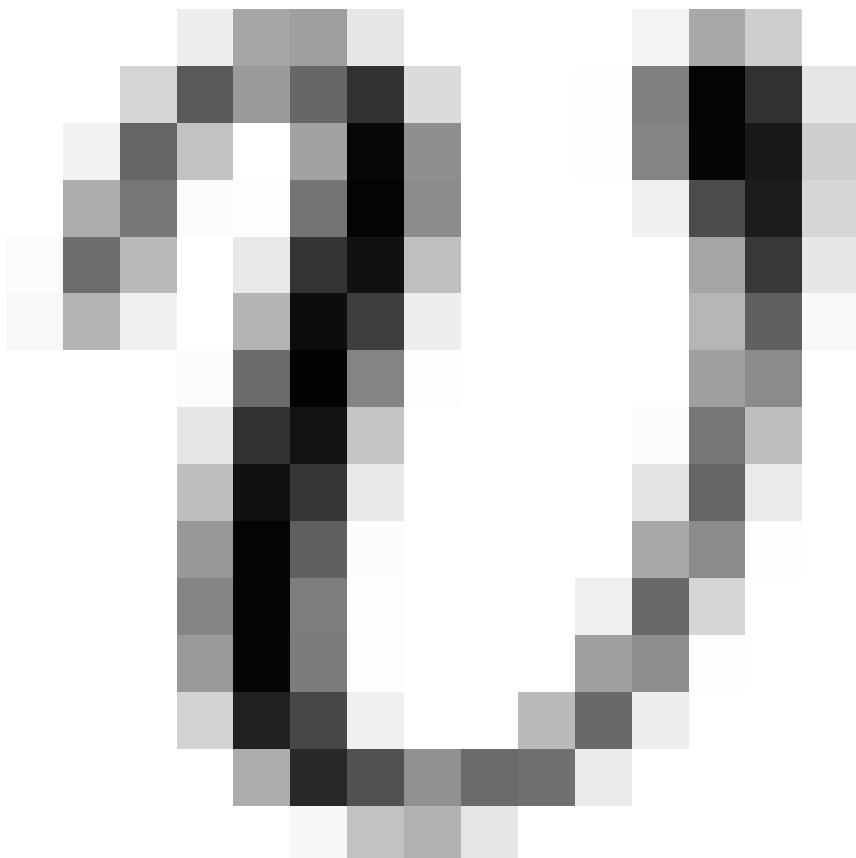


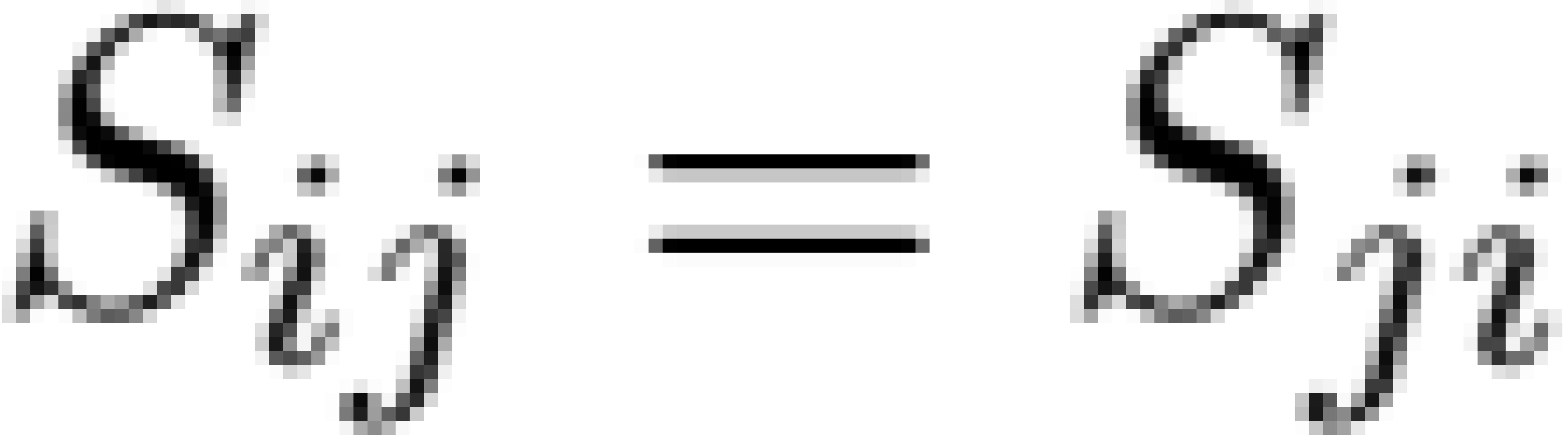


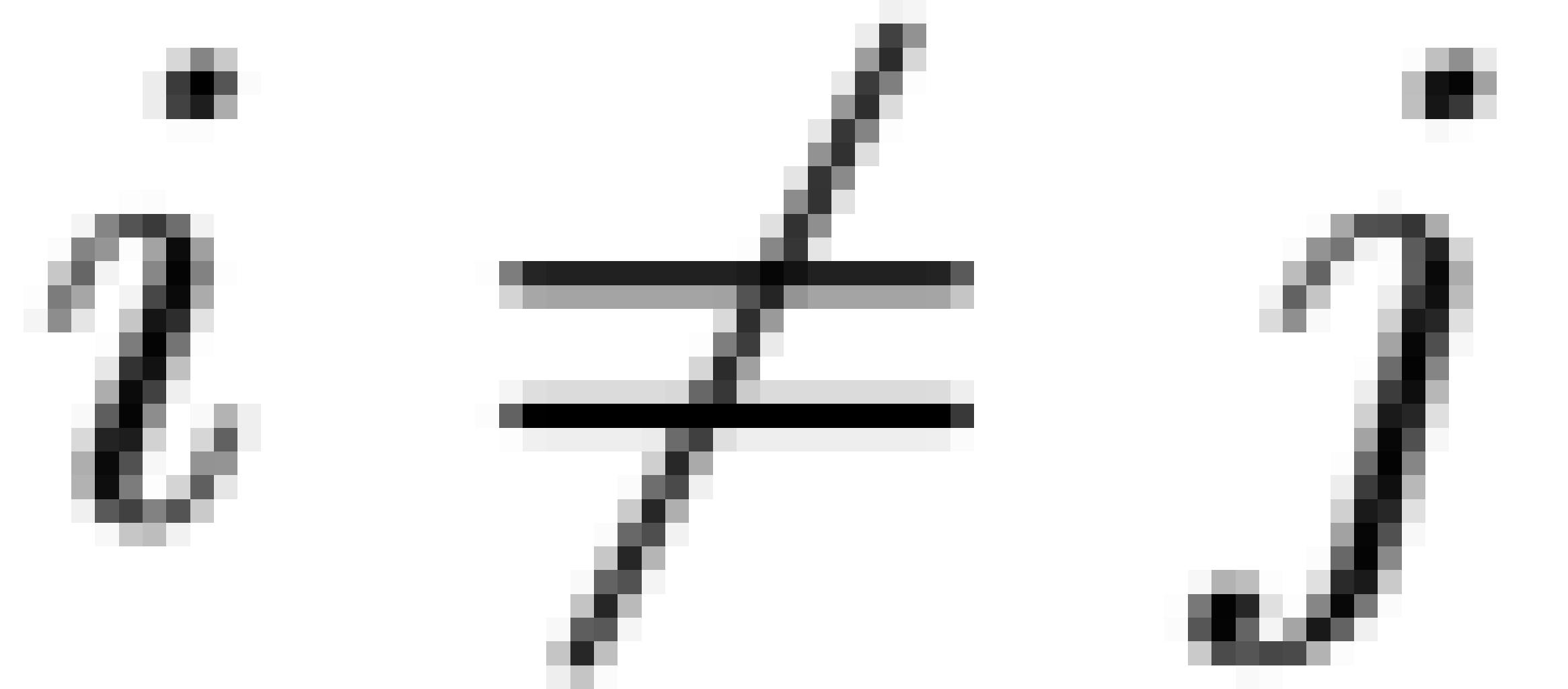


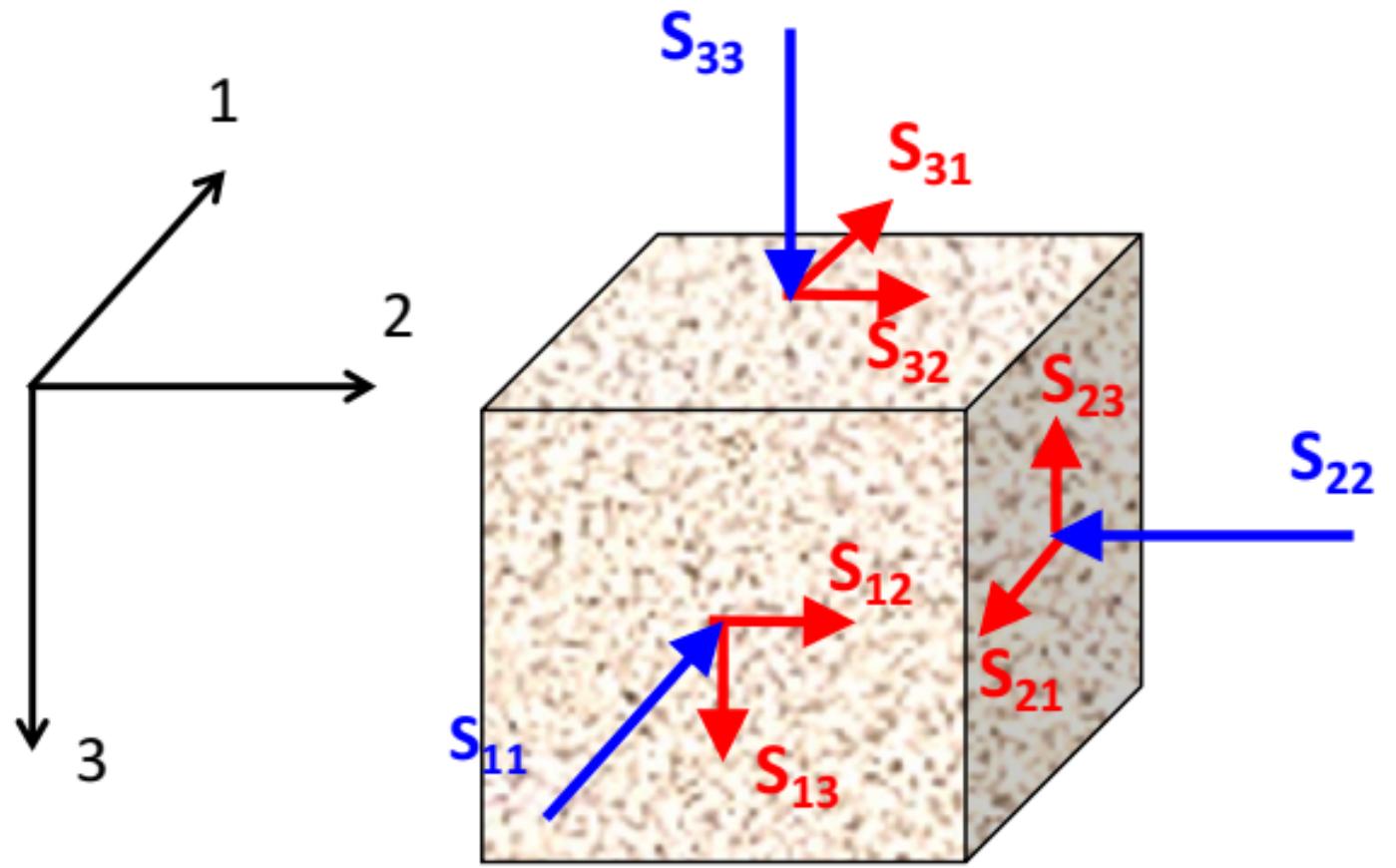






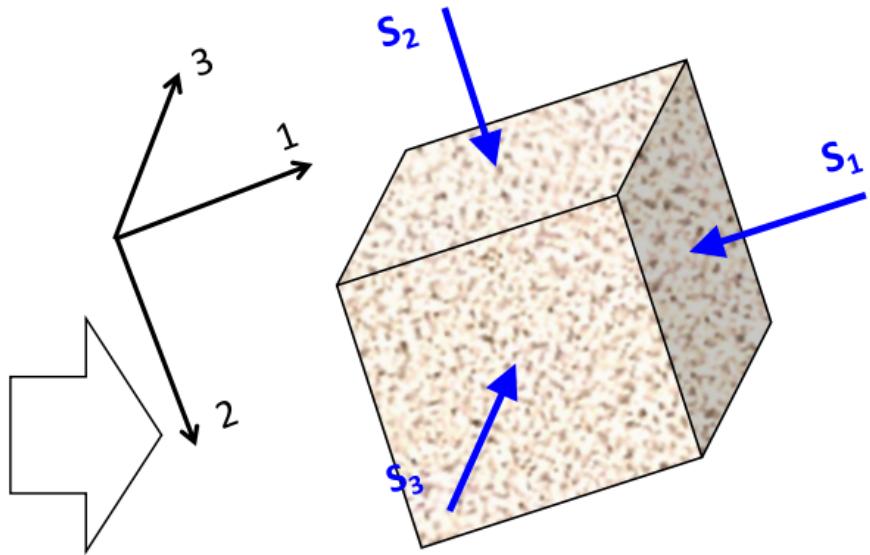
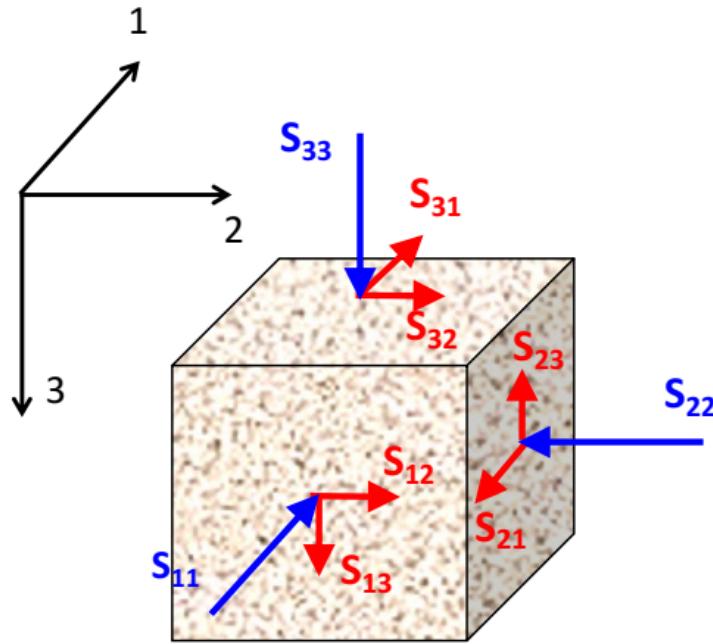






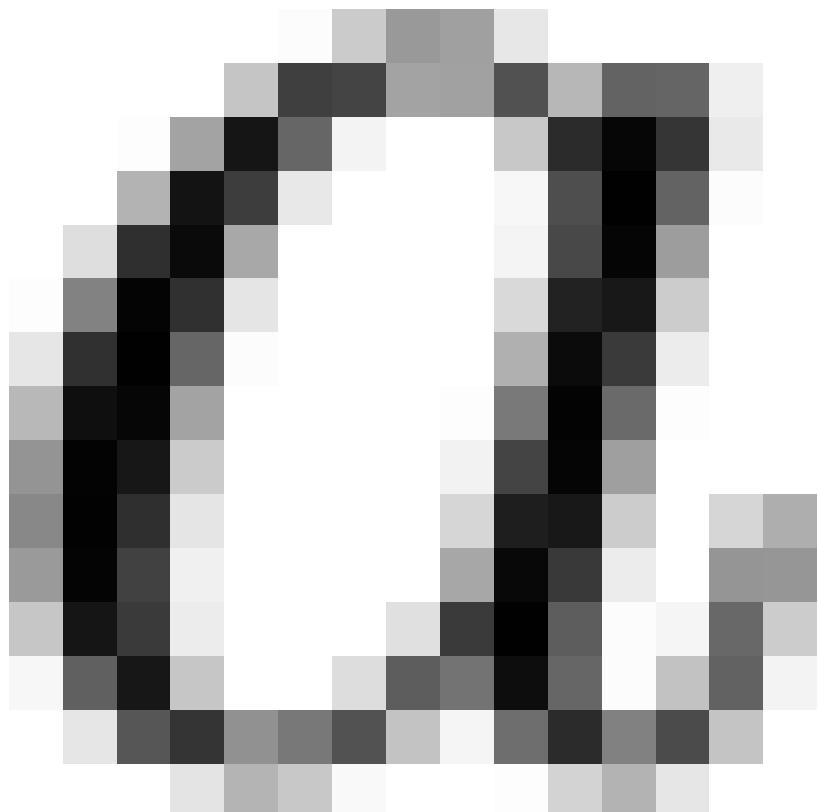
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

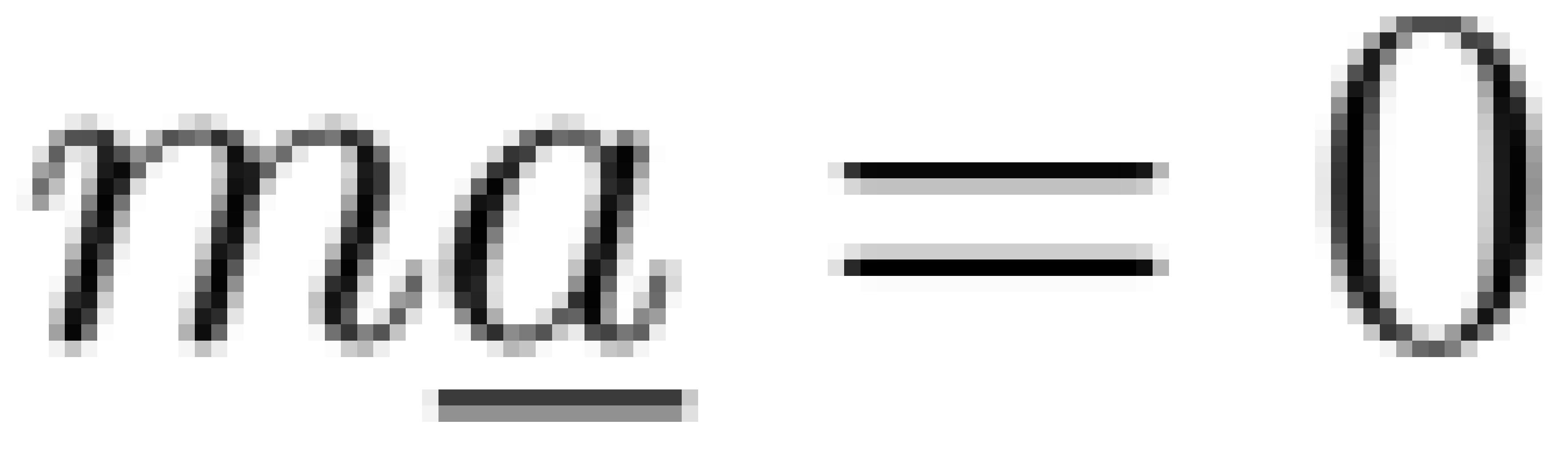




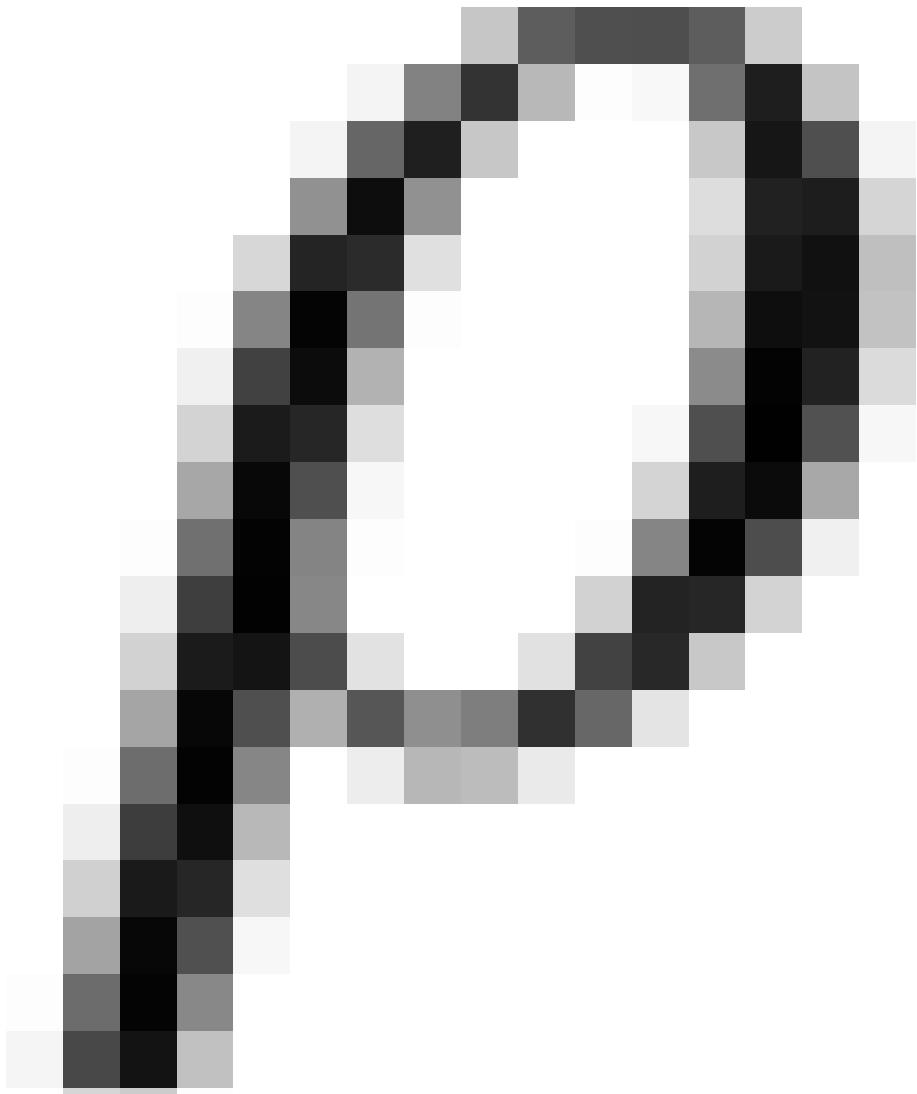
$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

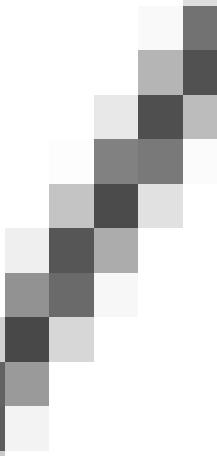
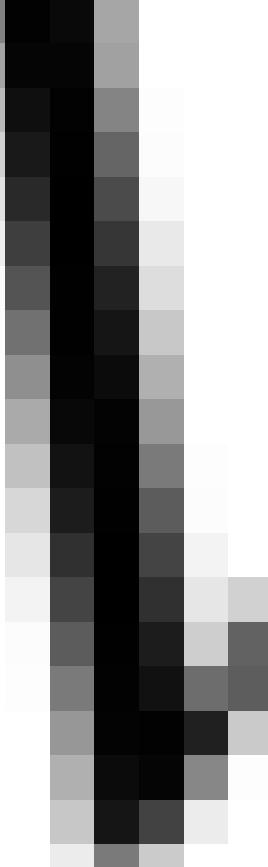
$$\underline{\underline{S}} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

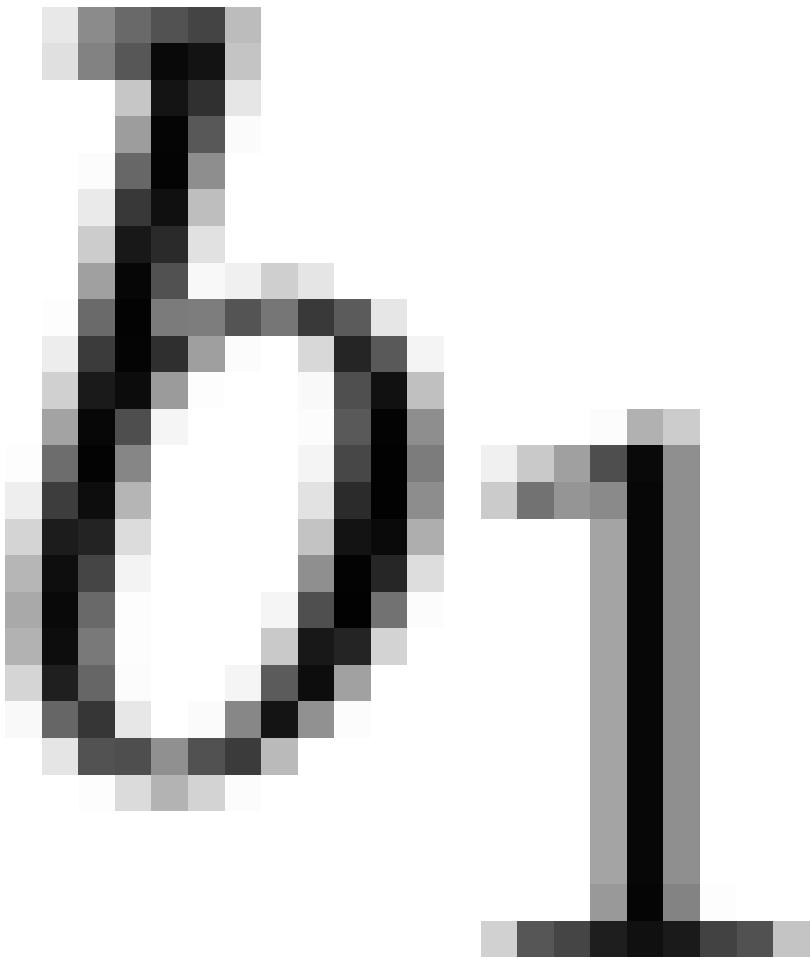








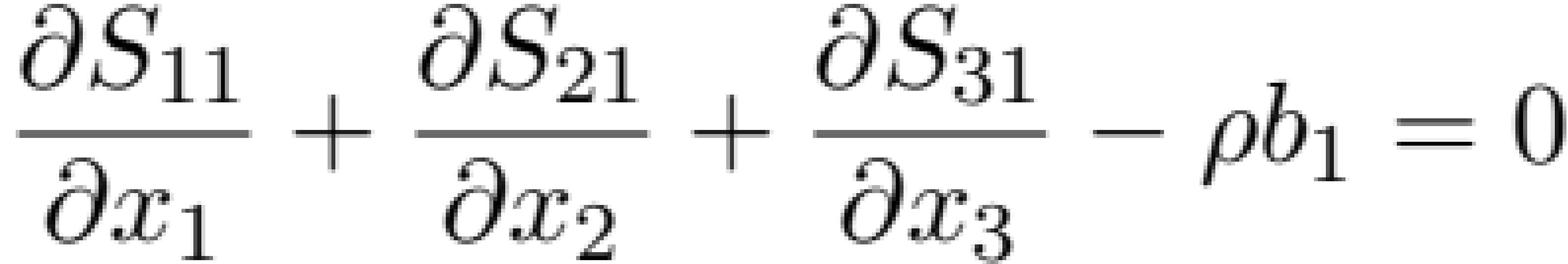


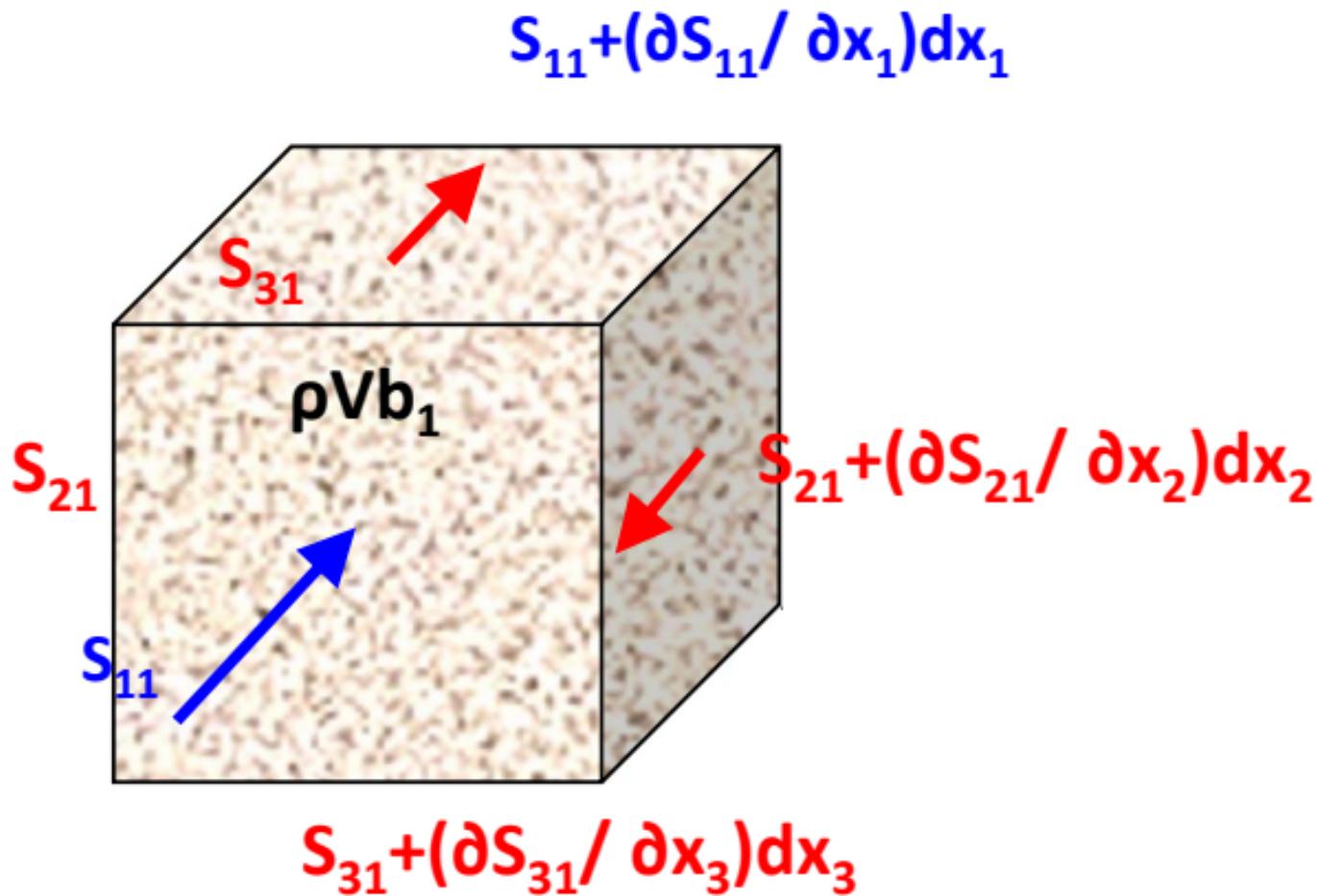
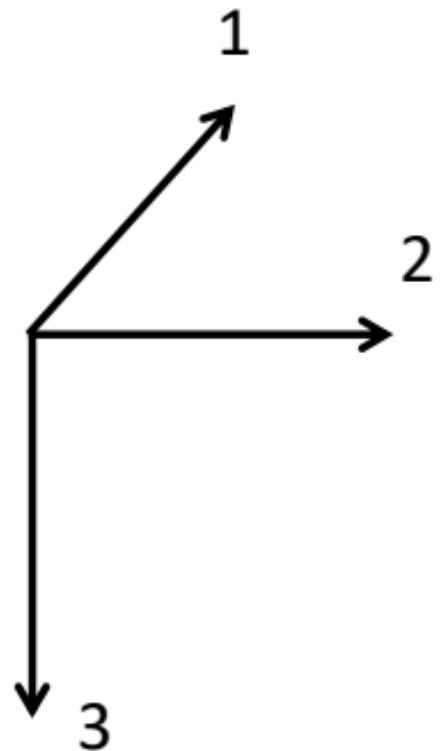


$$\sum F_1 = 0$$

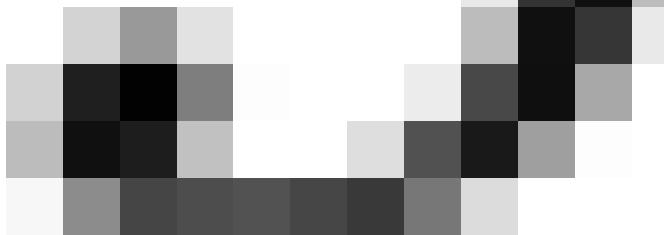
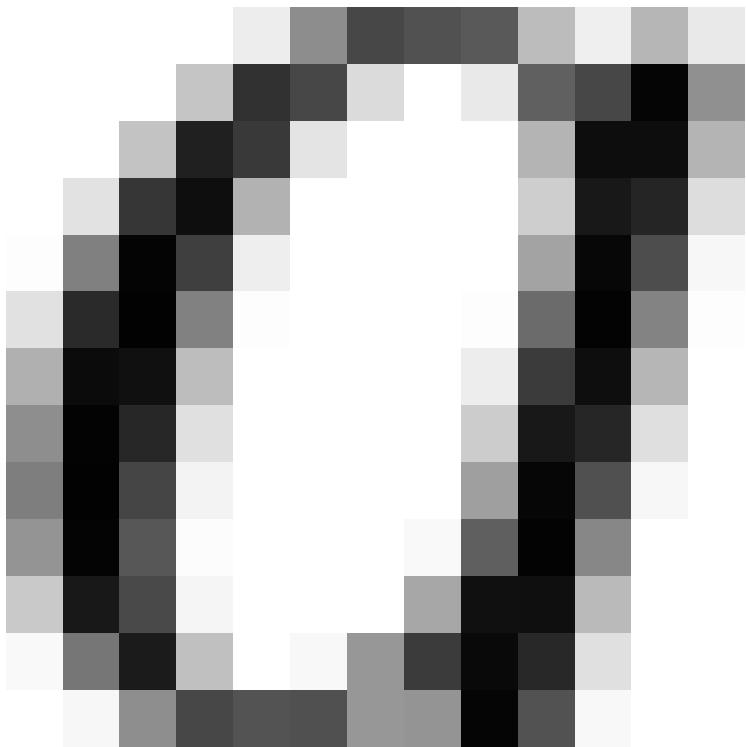
$$\begin{aligned}\sum F_1 &= +S_{11}dx_2dx_3 - \left[S_{11} + \left(\frac{\partial S_{11}}{\partial x_1} \right) dx_1 \right] dx_2dx_3 \\ &\quad + S_{21}dx_1dx_3 - \left[S_{21} + \left(\frac{\partial S_{21}}{\partial x_2} \right) dx_2 \right] dx_1dx_3 \\ &\quad + S_{31}dx_1dx_2 - \left[S_{31} + \left(\frac{\partial S_{31}}{\partial x_3} \right) dx_3 \right] dx_1dx_2 \\ &\quad - \rho(dx_1dx_2dx_3)b_1 = 0\end{aligned}$$

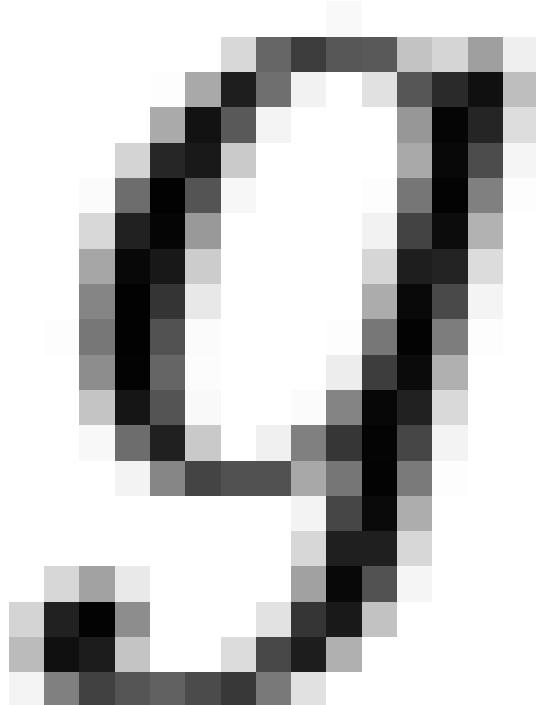






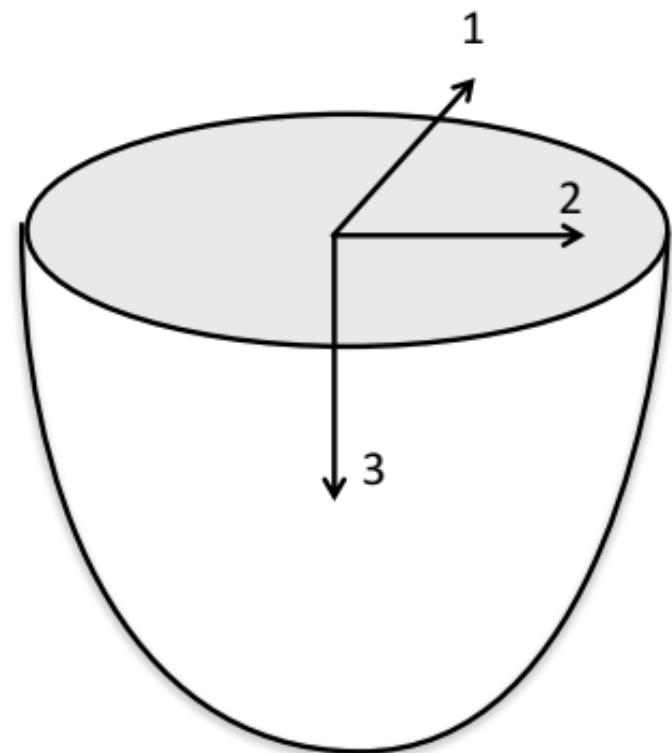
$$\left\{ \begin{array}{l} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} - \rho b_1 = 0 \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} - \rho b_2 = 0 \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} - \rho b_3 = 0 \end{array} \right.$$







$$s_{33}(c_3) = \rho(c_3) \circ d_{c_3}$$



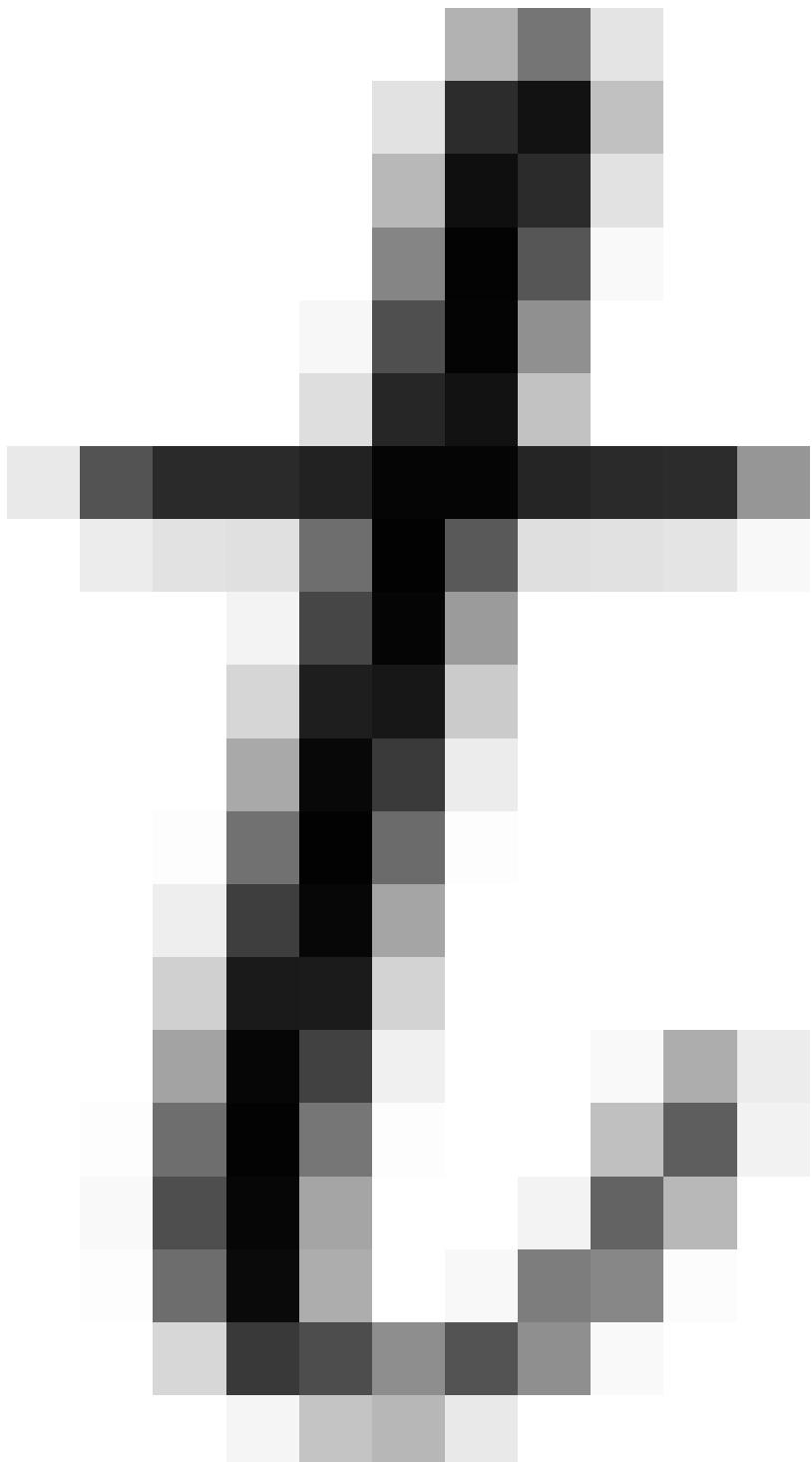
$$\left\{ \begin{array}{l} \cancel{\frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2}} \\ \cancel{\frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2}} \\ \cancel{\frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2}} \end{array} \right.$$

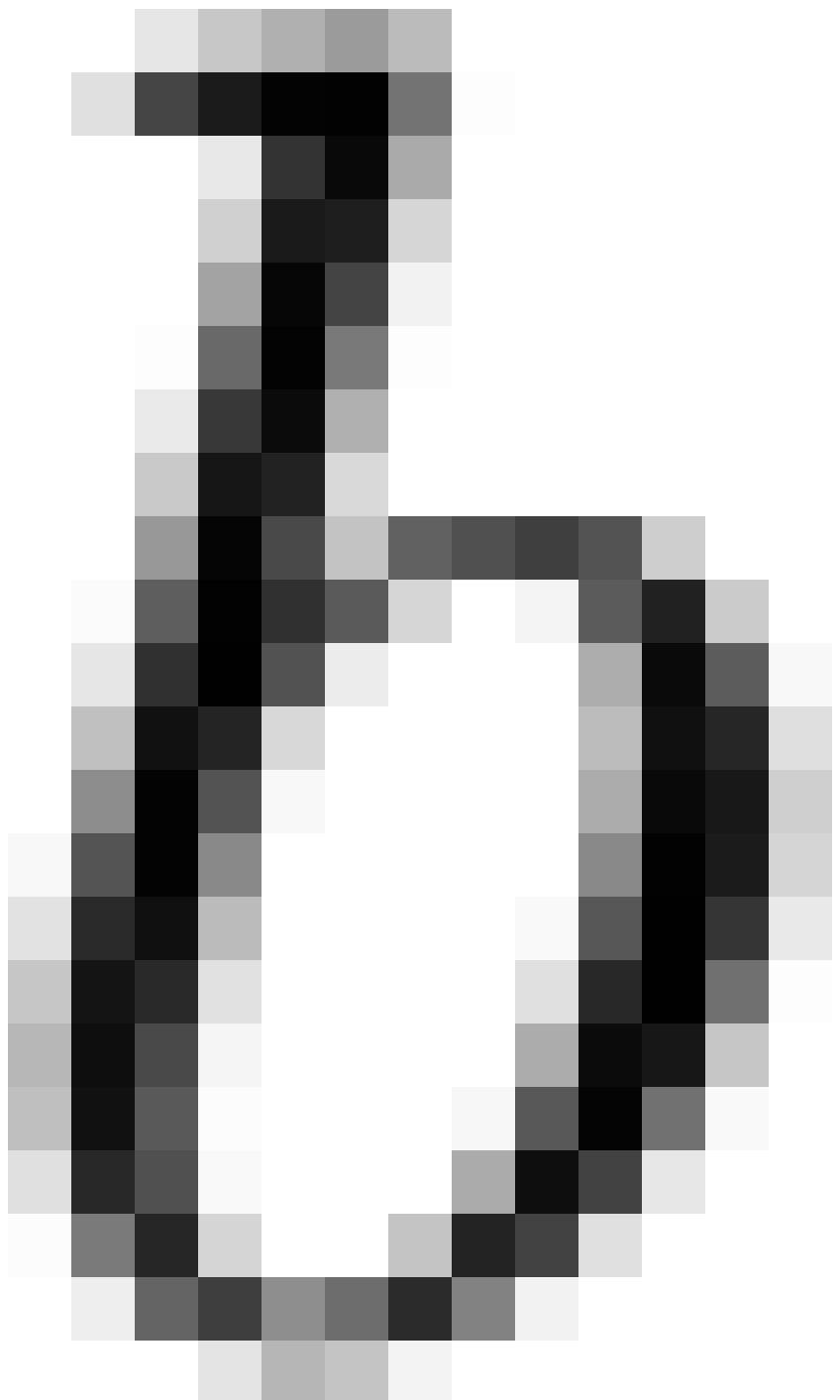
$$\frac{\partial S_{33}}{\partial x_3} - \rho(x_3)g = 0$$

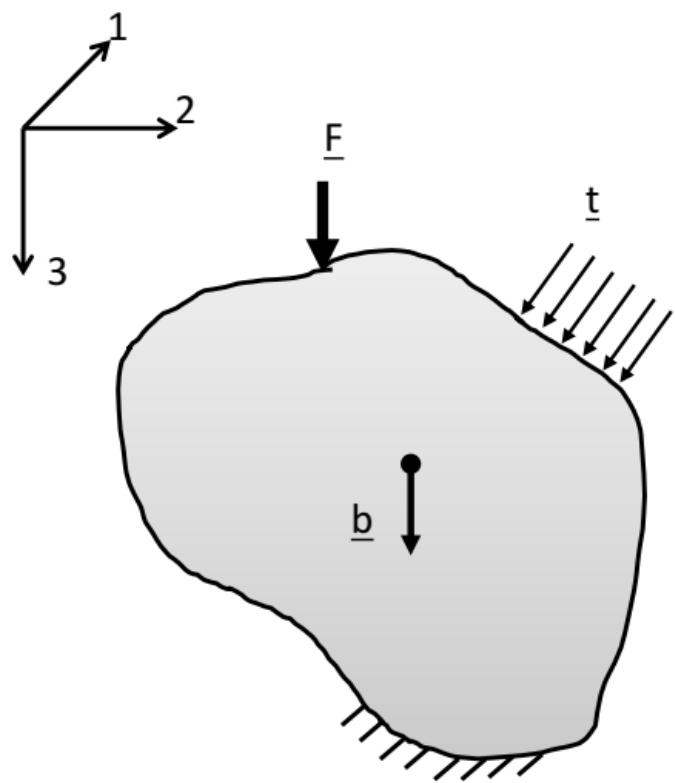
$$S_{33} = \int_0^{x_3} \rho(x_3)g \, dx_3$$











Displacement condition

$$\begin{cases} \frac{\partial S_{11}}{\partial x_1} + \frac{\partial S_{21}}{\partial x_2} + \frac{\partial S_{31}}{\partial x_3} + \rho b_1 = \frac{\partial^2 (\rho u_1)}{\partial t^2} \\ \frac{\partial S_{12}}{\partial x_1} + \frac{\partial S_{22}}{\partial x_2} + \frac{\partial S_{32}}{\partial x_3} + \rho b_2 = \frac{\partial^2 (\rho u_2)}{\partial t^2} \\ \frac{\partial S_{13}}{\partial x_1} + \frac{\partial S_{23}}{\partial x_2} + \frac{\partial S_{33}}{\partial x_3} + \rho b_3 = \frac{\partial^2 (\rho u_3)}{\partial t^2} \end{cases}$$

And respect the boundary conditions:

- Displacement
- Boundary stresses
- Boundary Forces
- Body Forces

How do we relate stresses to displacements?

- Displacements → Strains (**Kinematic equations**)
- Strains → Stresses (**Constitutive equations**)



ϵ_{11}



α_1

α_1

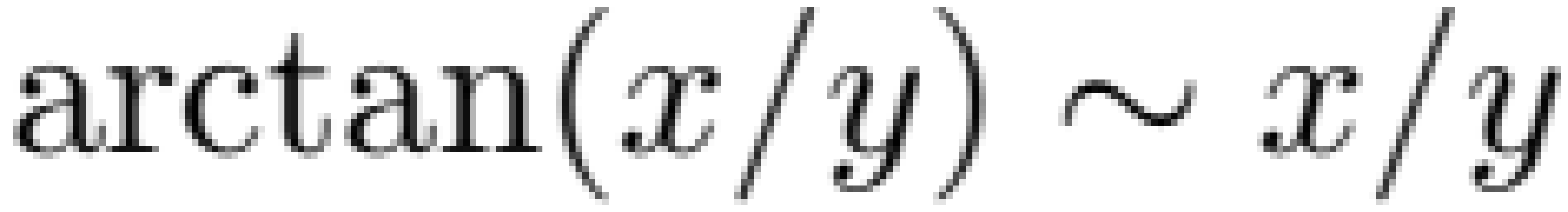
ϵ_{22}

$=$

α_2

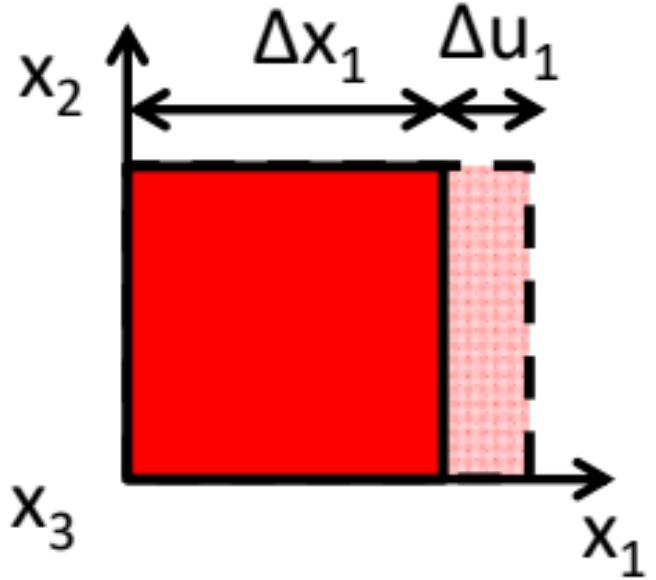
α_2

acc10(ut1) + acc10(ut2) + acc10(ut3)

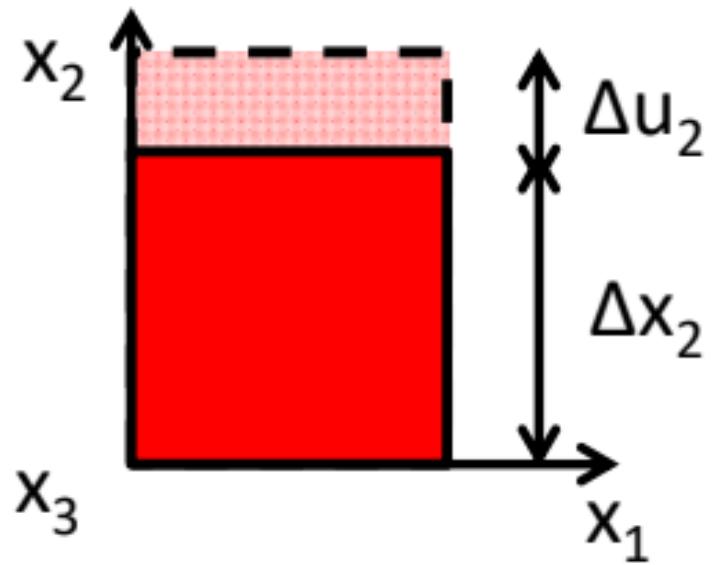


$\epsilon_{12} =$ $\frac{1}{2}$

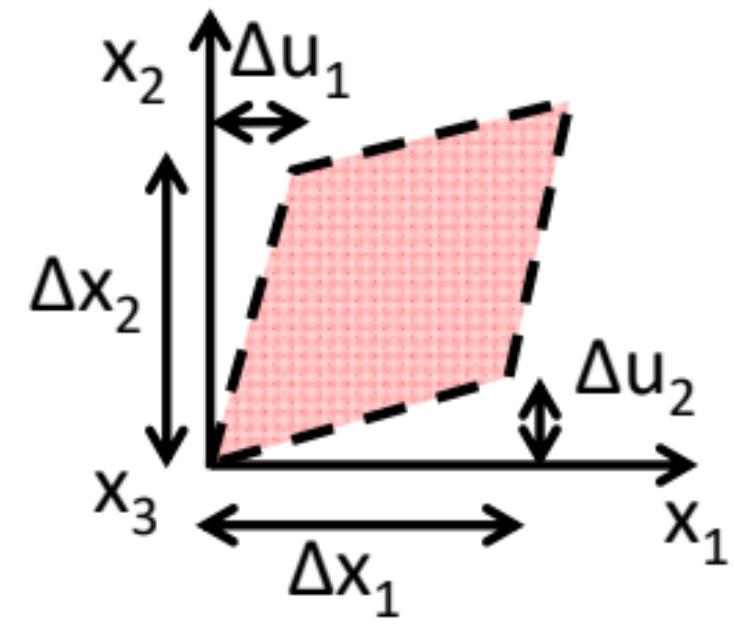
$$\frac{1}{2} \left(u_1 - u_2 + c_1 - c_2 \right)$$



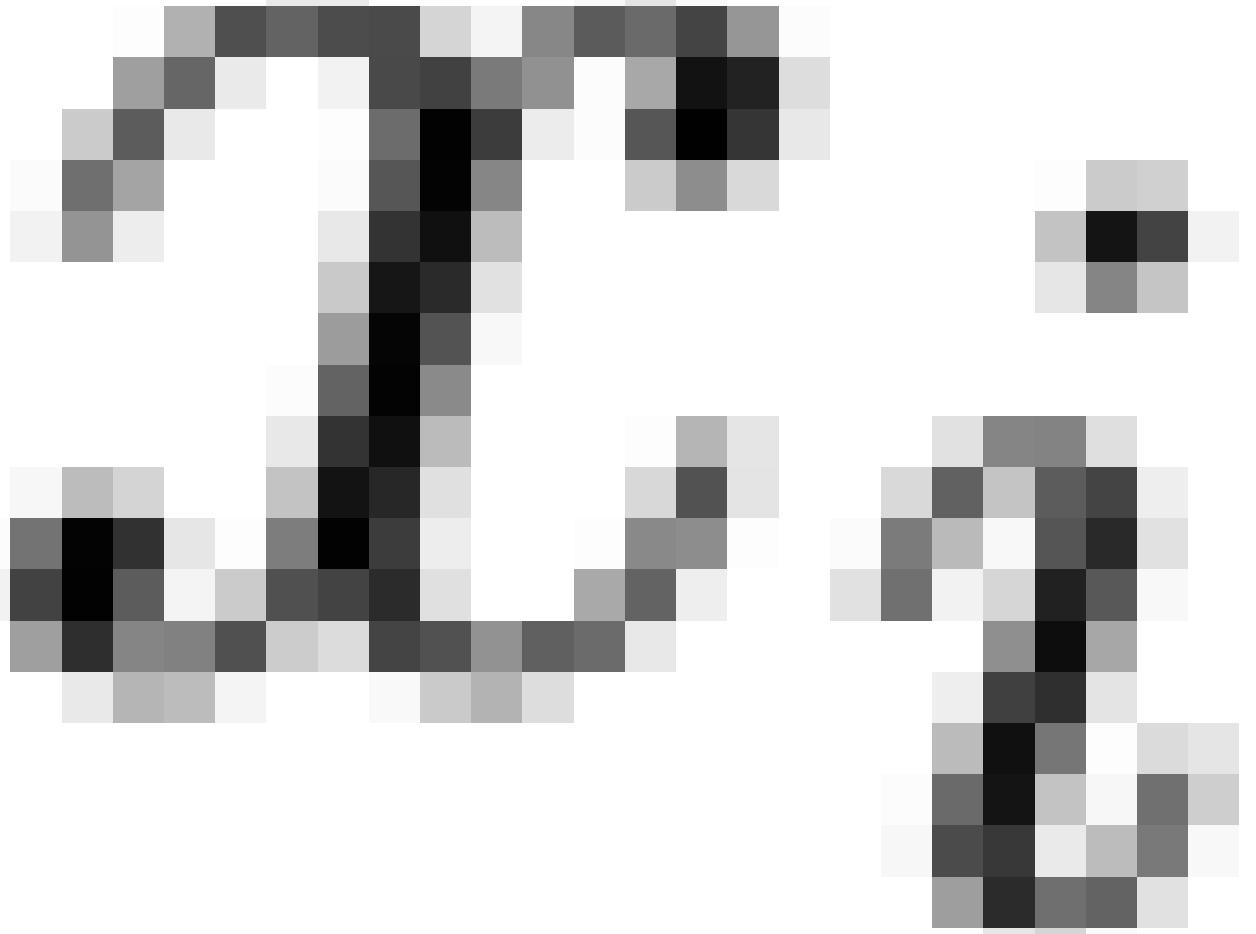
$$\varepsilon_{11} \approx \frac{\Delta u_1}{\Delta x_1}$$

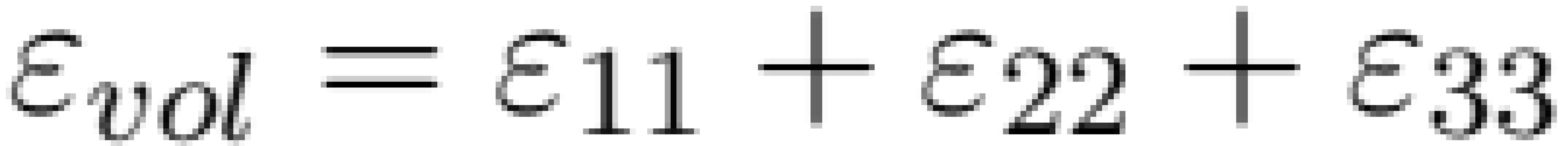


$$\varepsilon_{22} \approx \frac{\Delta u_2}{\Delta x_2}$$

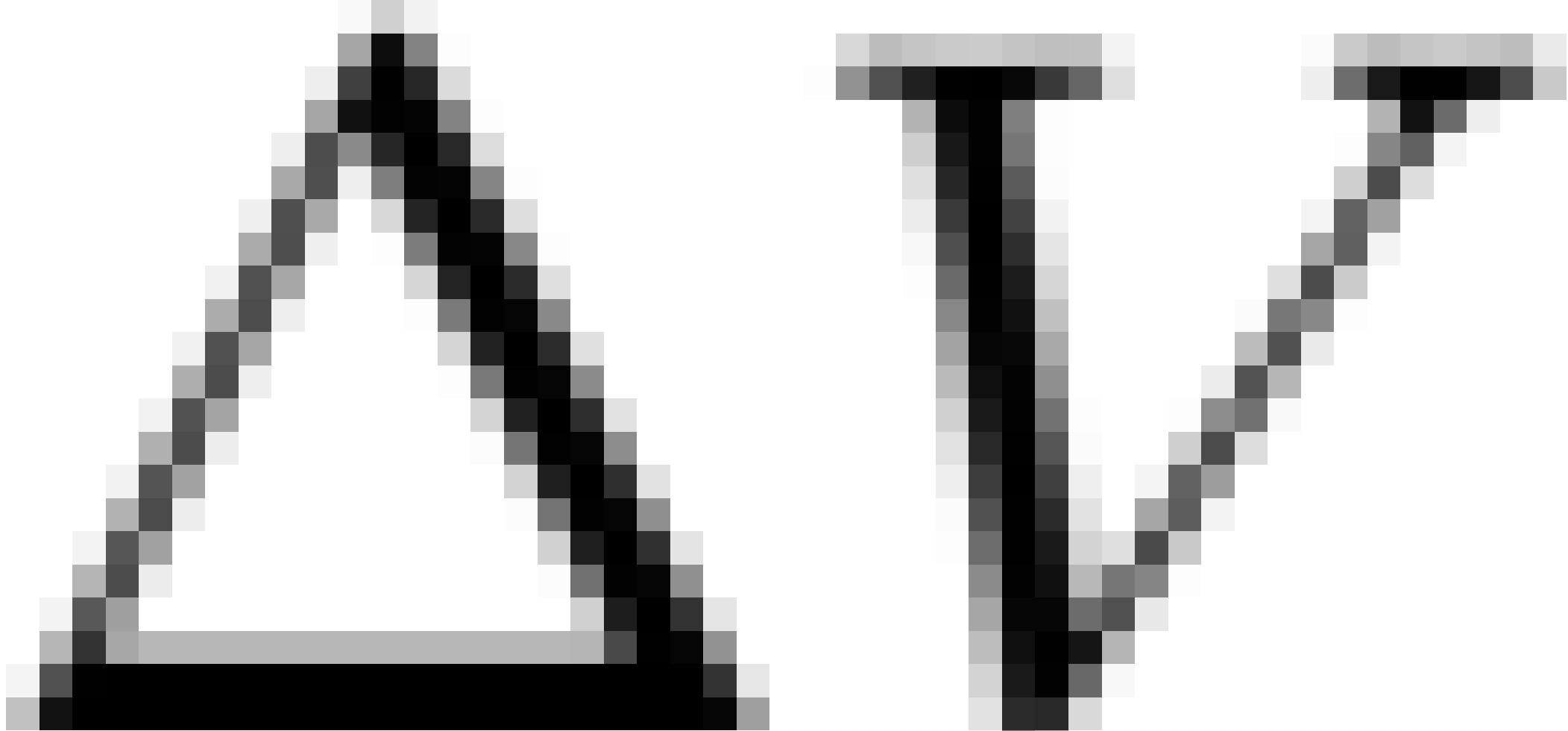


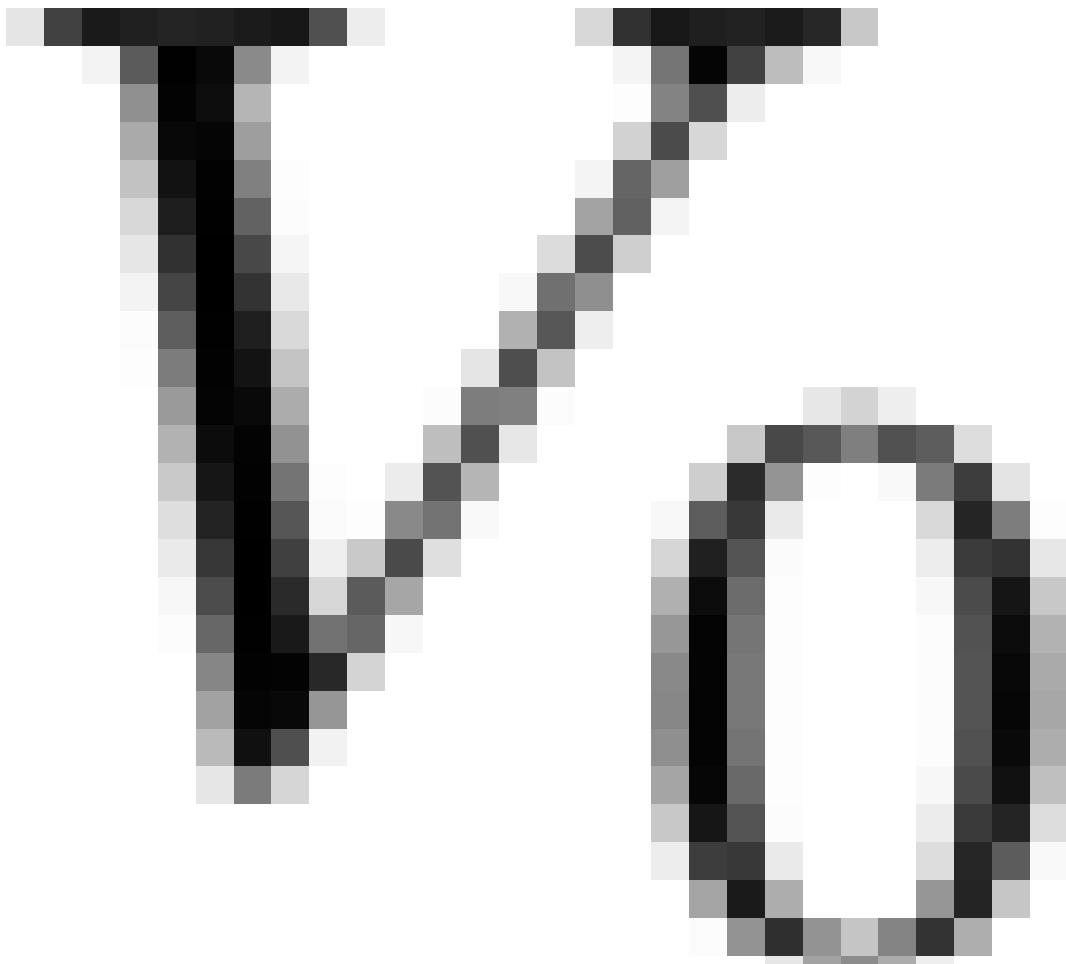
$$\varepsilon_{12} \approx \frac{1}{2} \left(\frac{\Delta u_1}{\Delta x_2} + \frac{\Delta u_2}{\Delta x_1} \right)$$



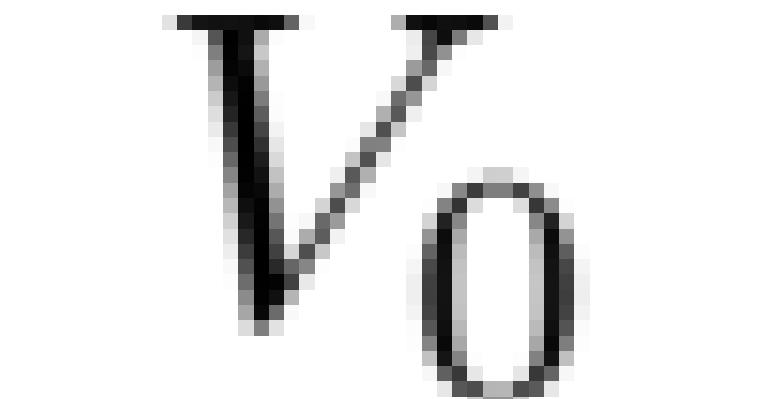
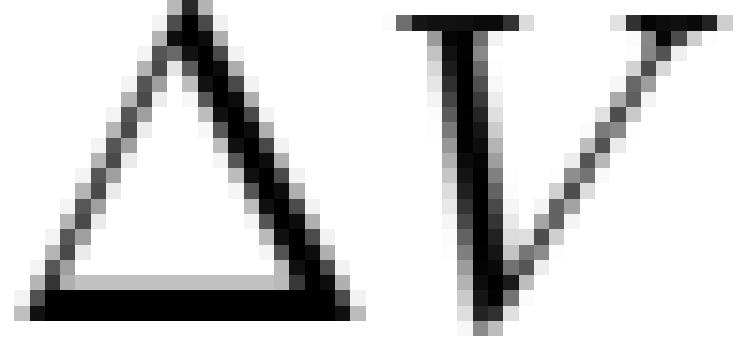




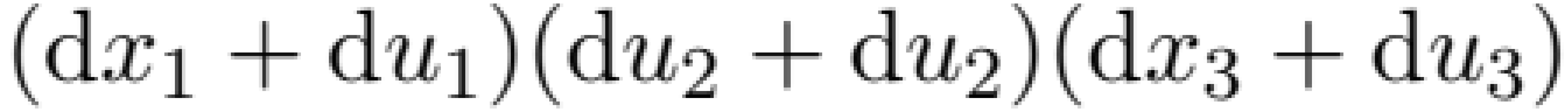




cool





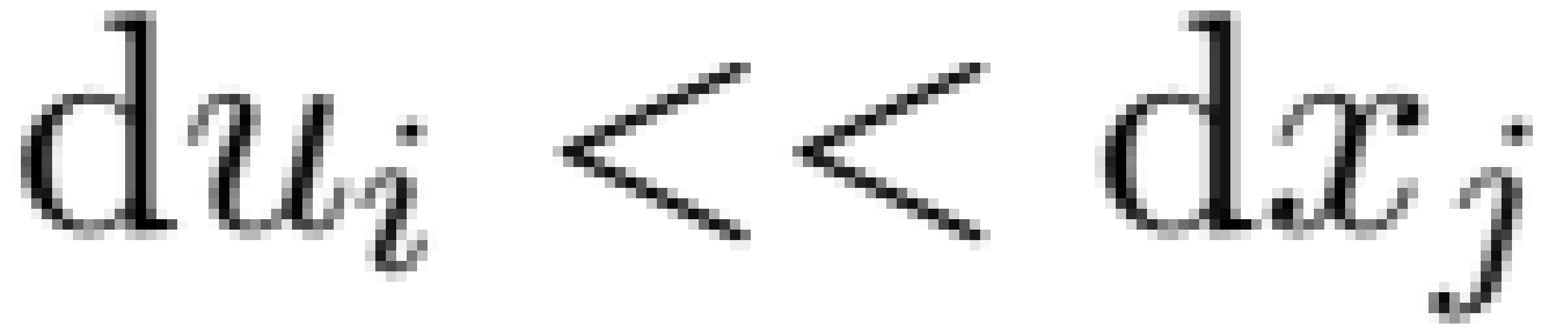


$$\epsilon_{vol} = \frac{[(dx_1 + du_1)(dx_2 + du_3) - (dx_1 dx_2 dx_3)]}{(dx_1 dx_2 dx_3)}$$



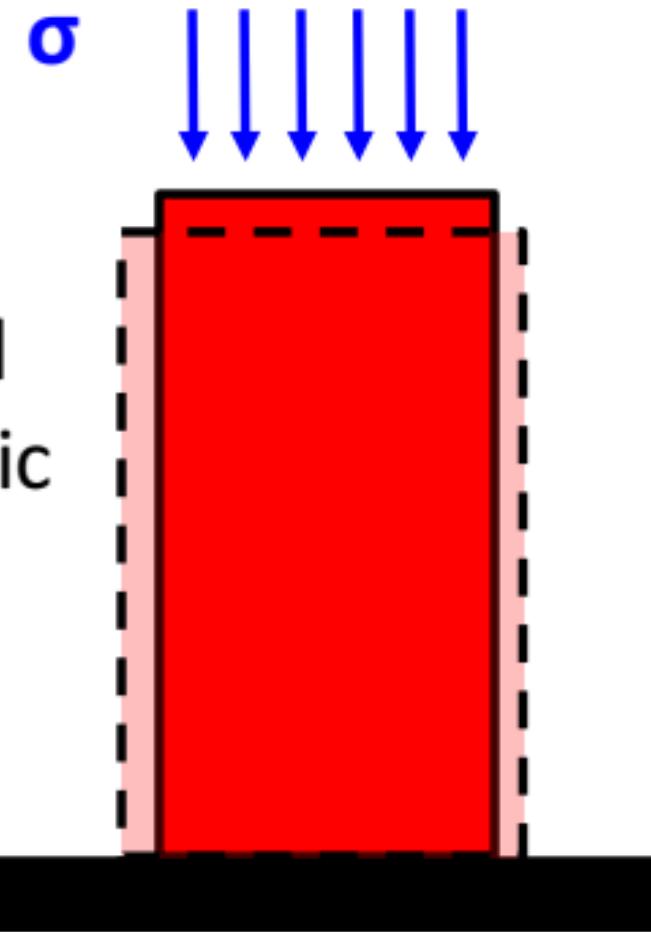




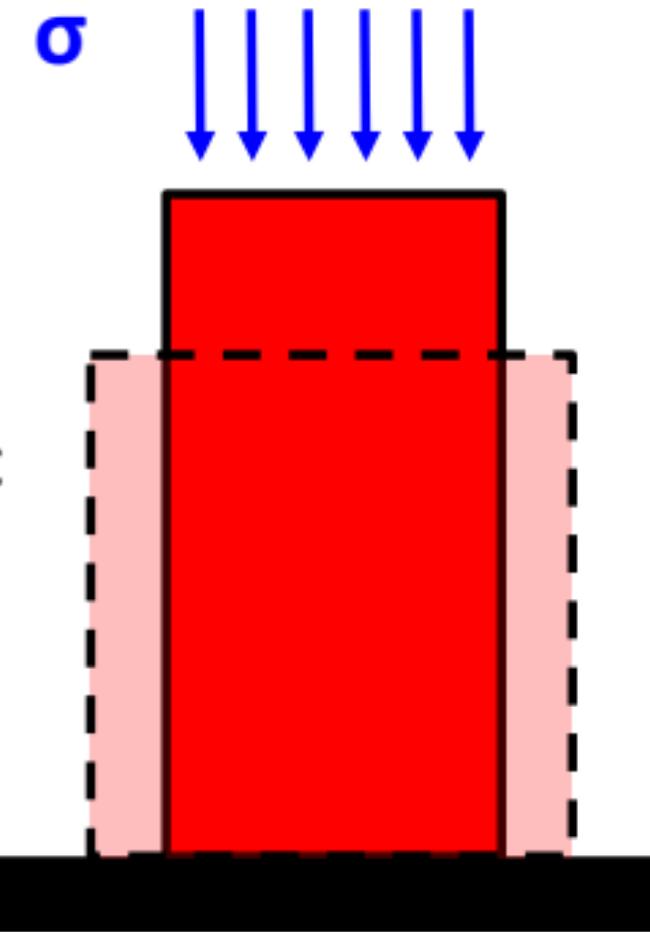


$$\text{vol}^2 \frac{(dx_1 dx_2 du_3 + dx_1 dx_3 du_2 + dx_2 dx_3 du_1)}{(dx_1 dx_2 dx_3)} = \frac{du_1}{\epsilon_{11}} + \frac{du_2}{\epsilon_{22}} + \frac{du_3}{\epsilon_{33}}$$

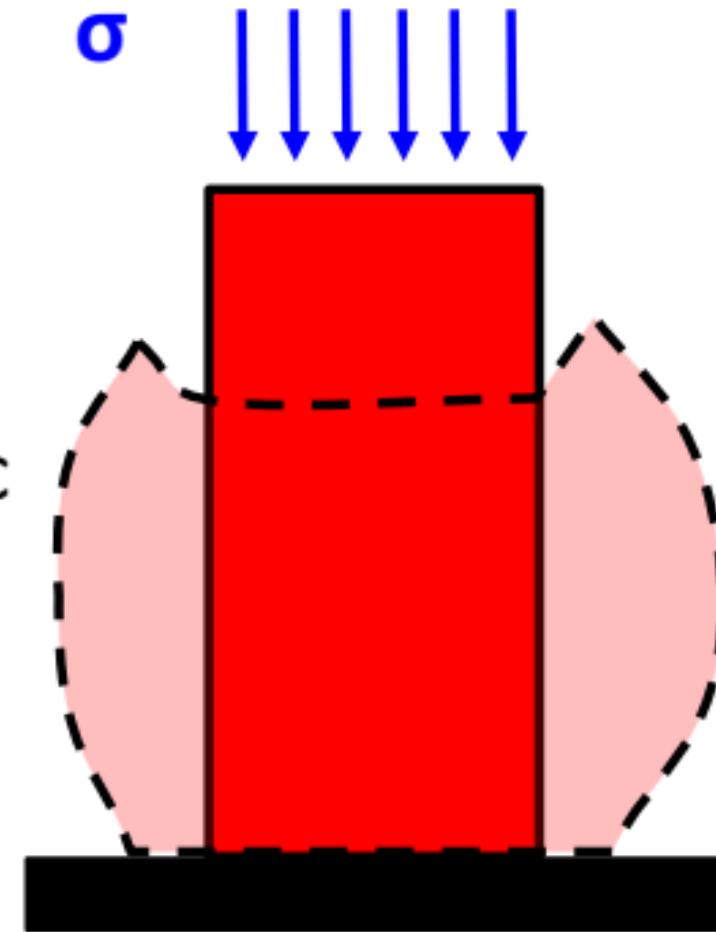
Hard
elastic
solid

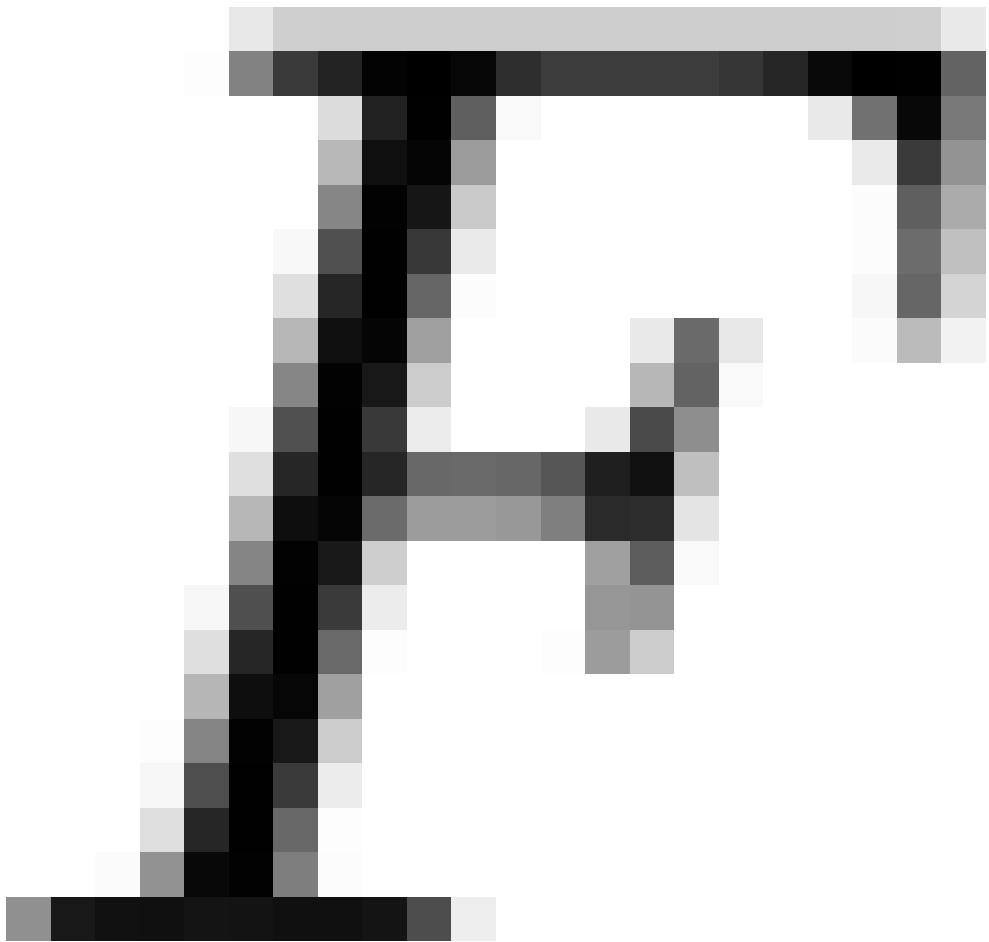


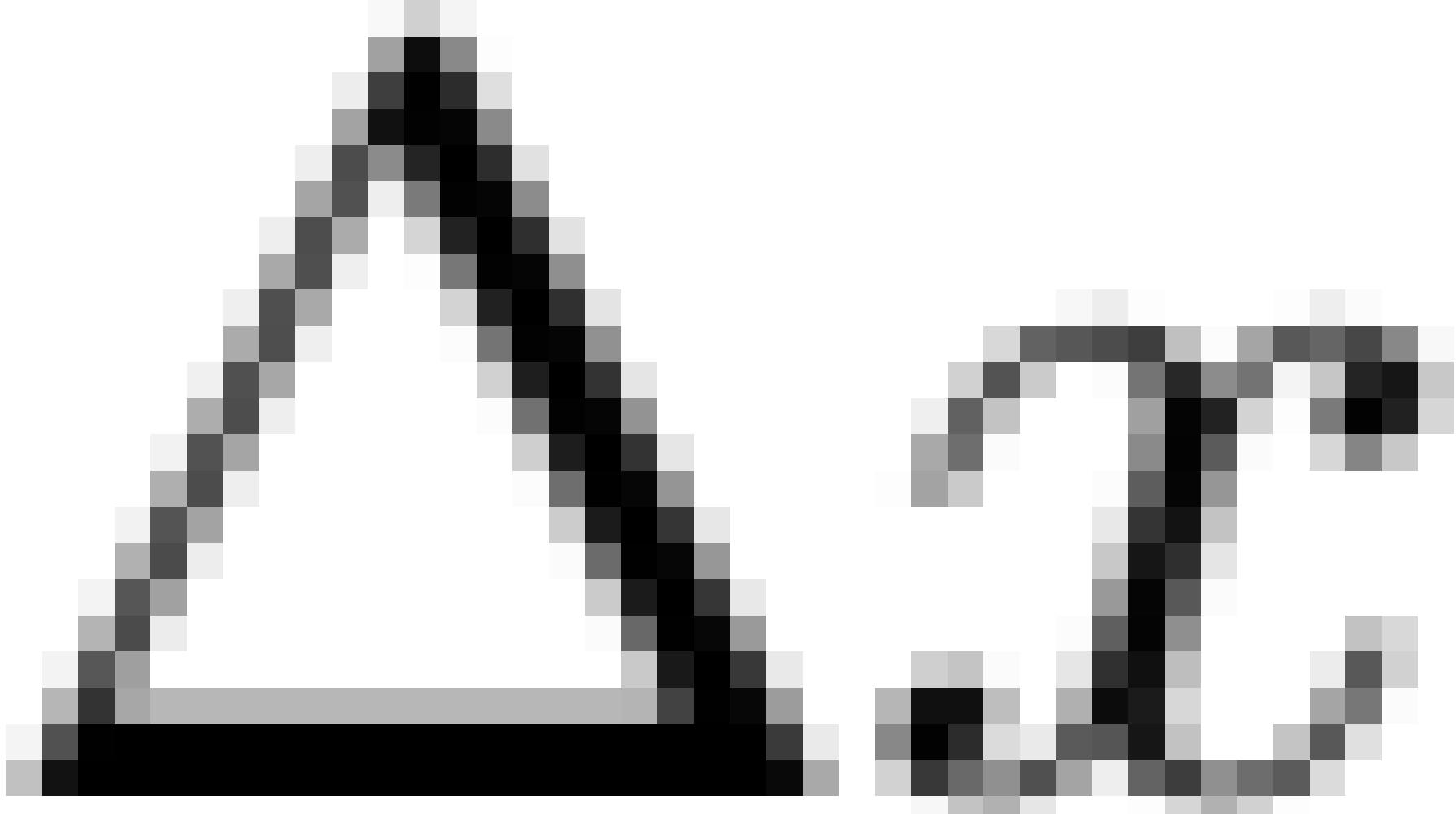
Soft
elastic
solid

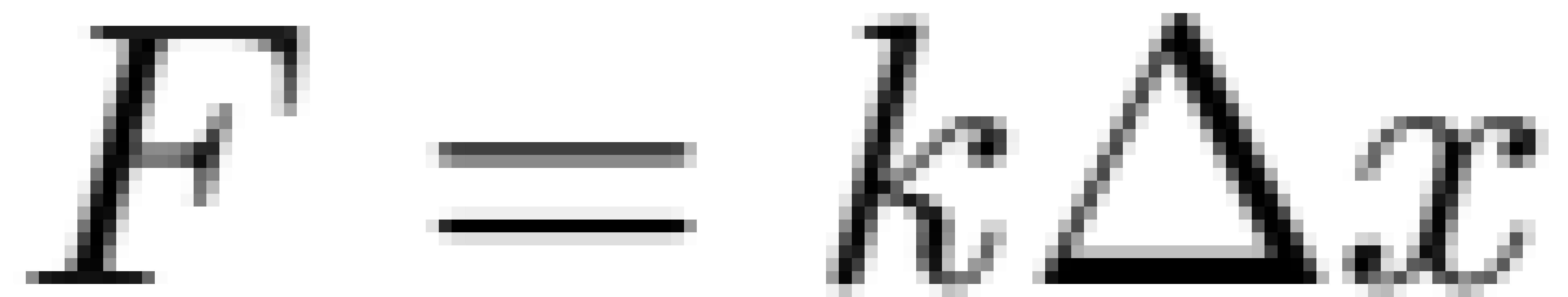


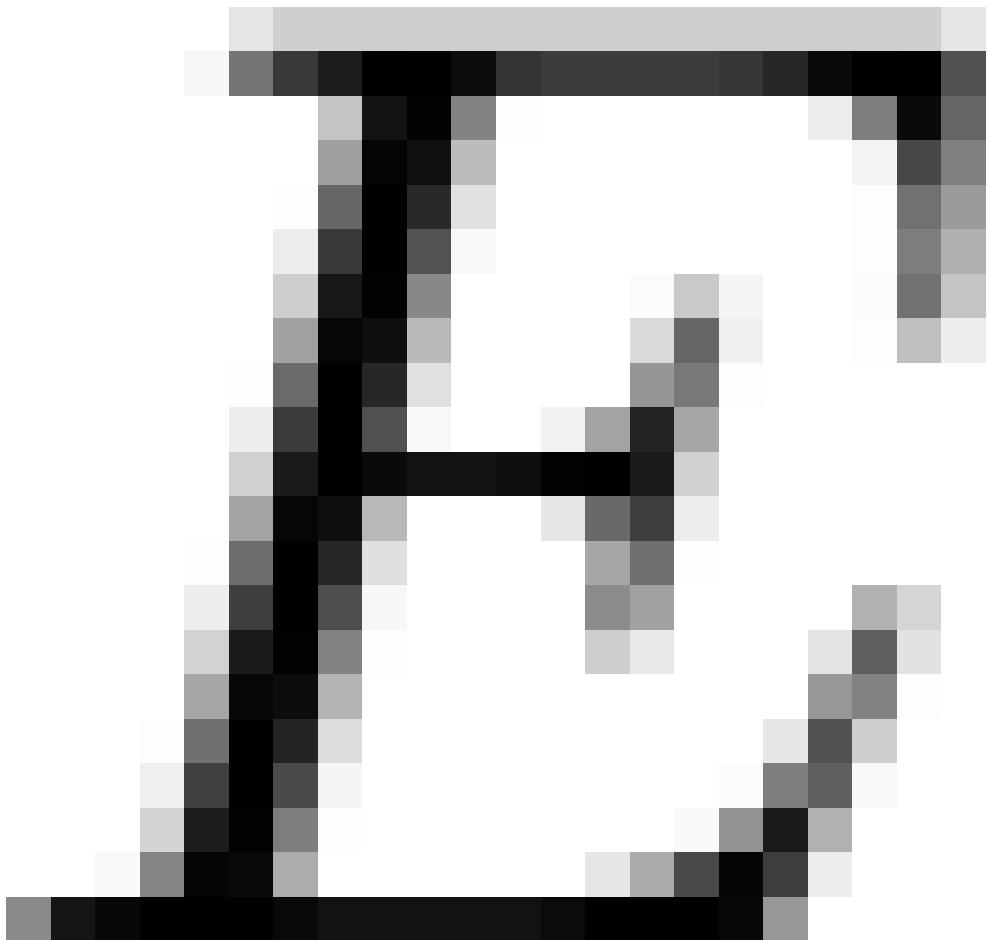
Soft
Visco-
plastic
solid











A

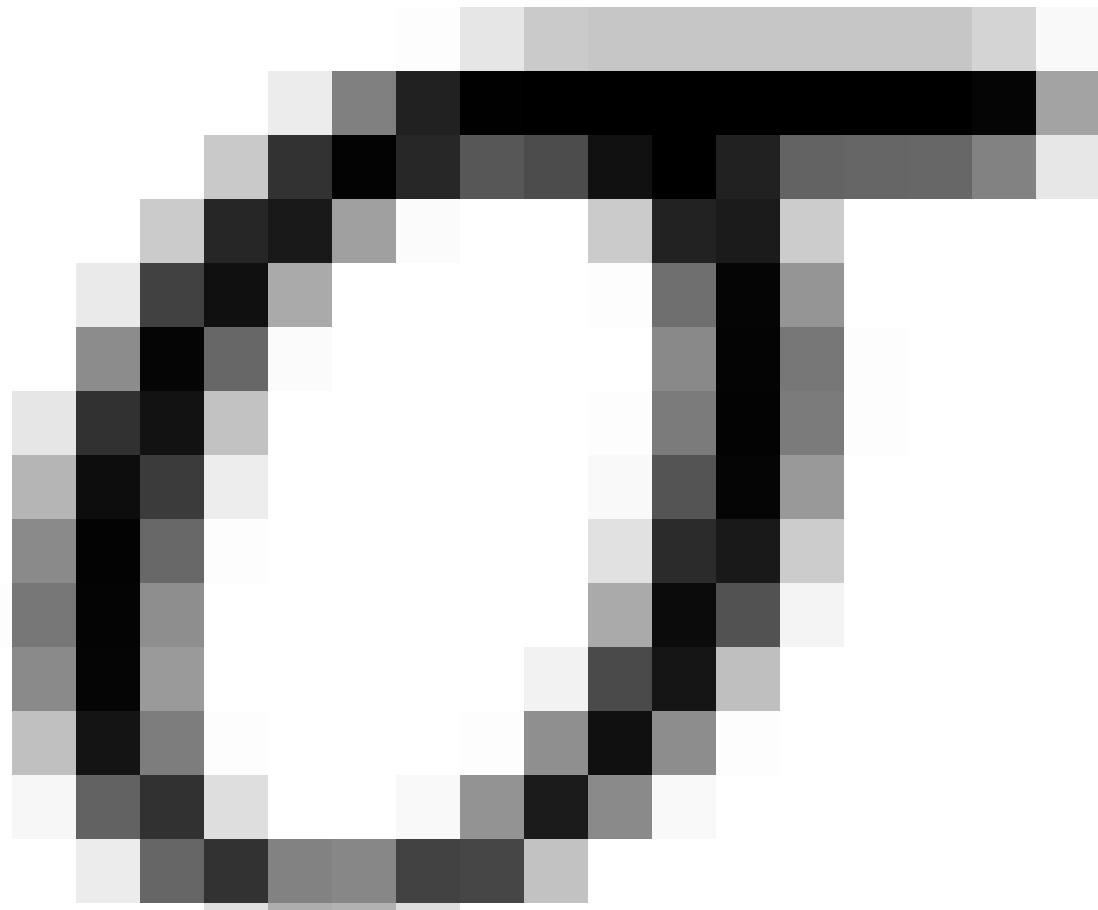
A

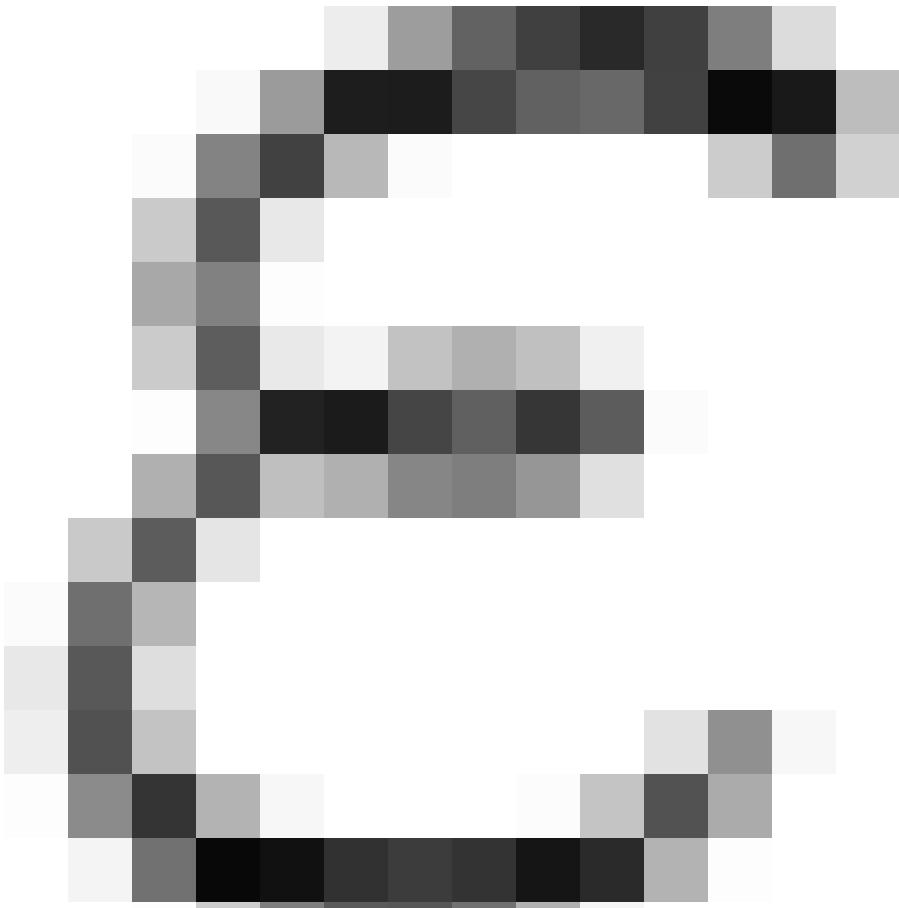
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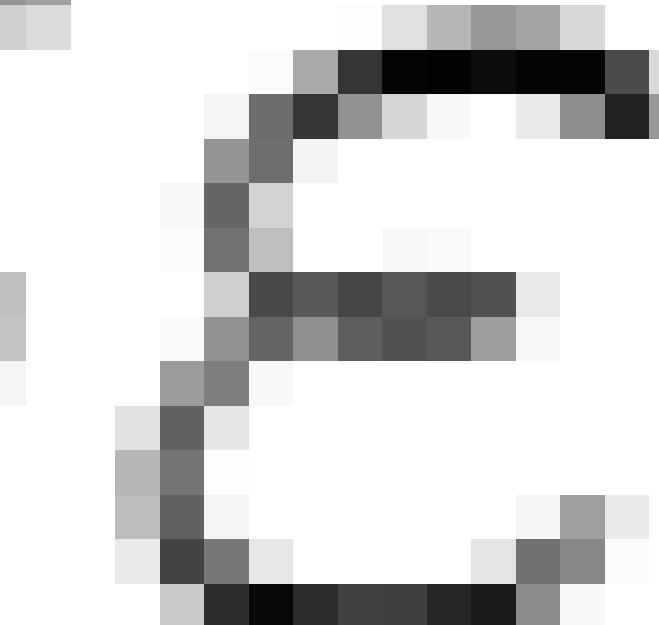
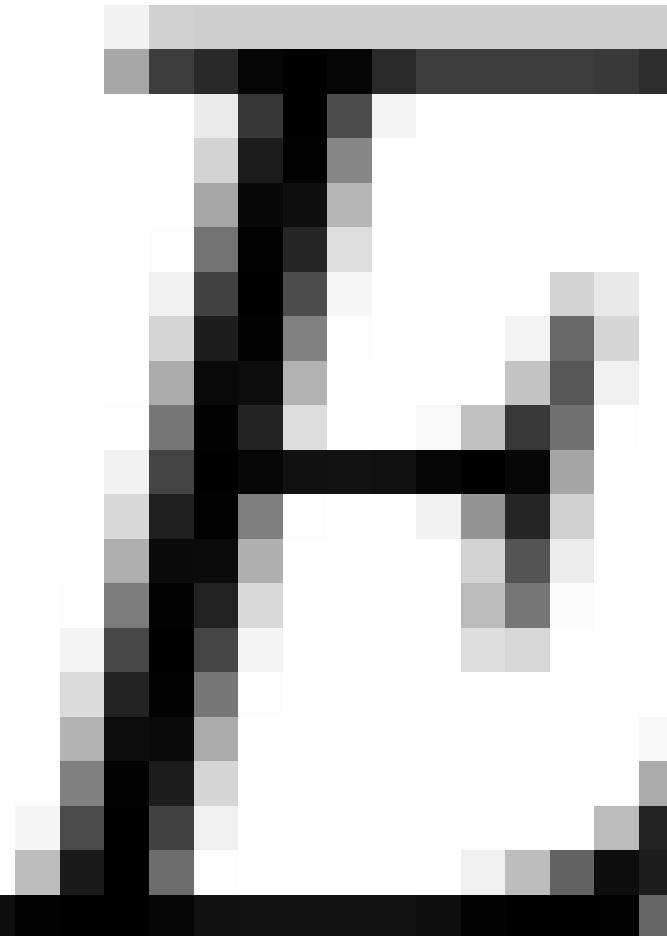
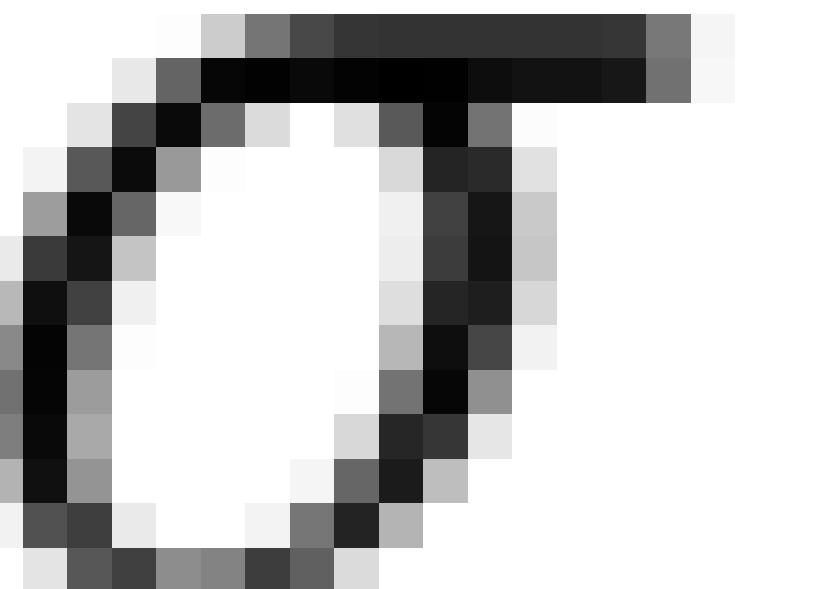
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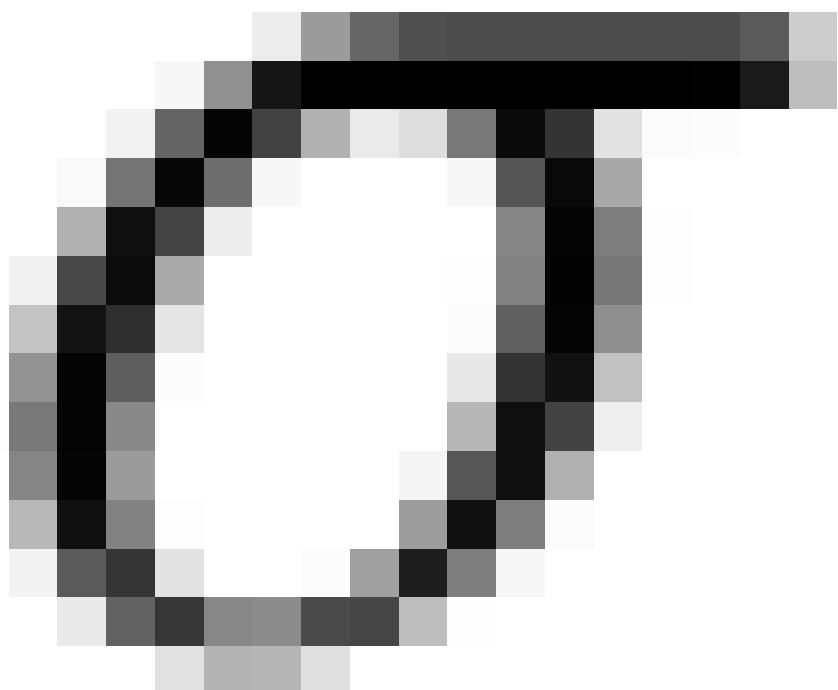
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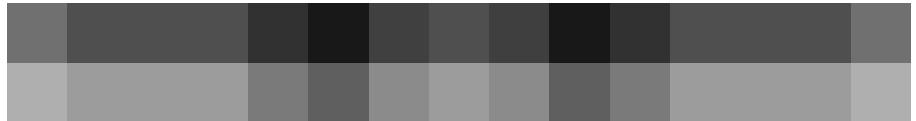
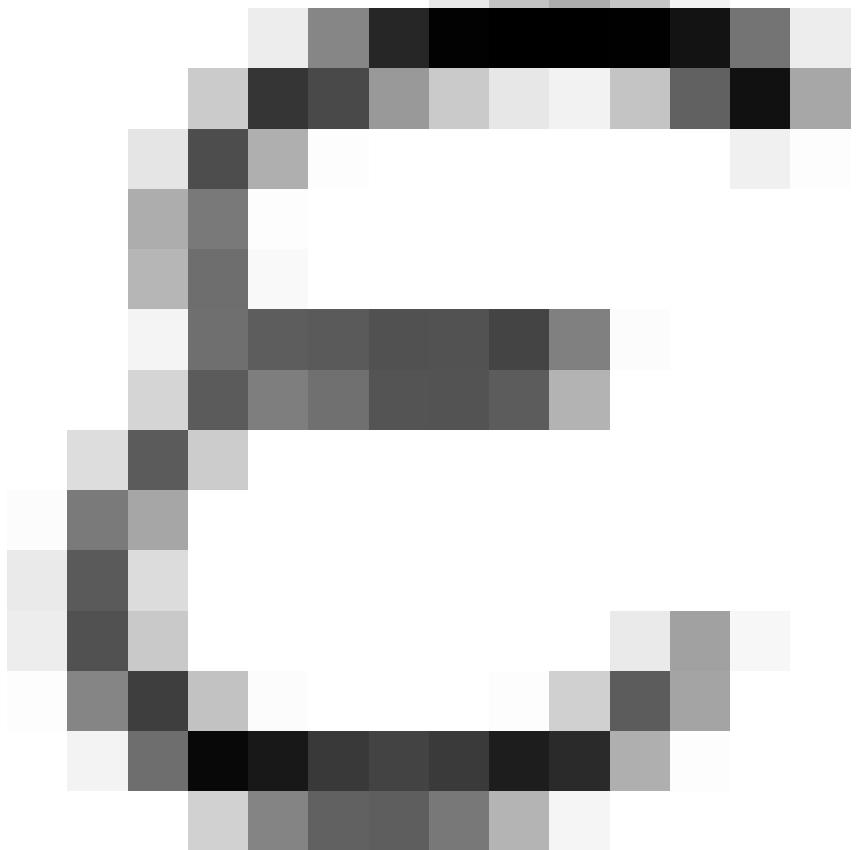
T

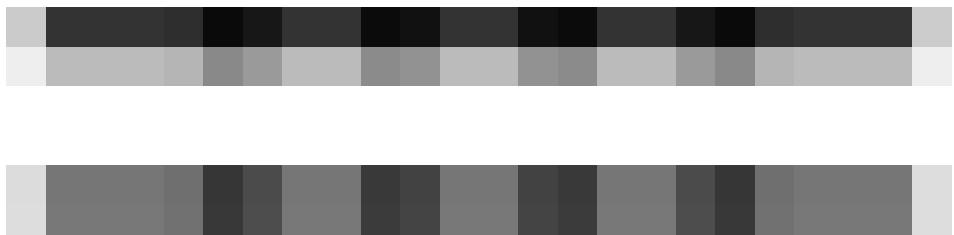
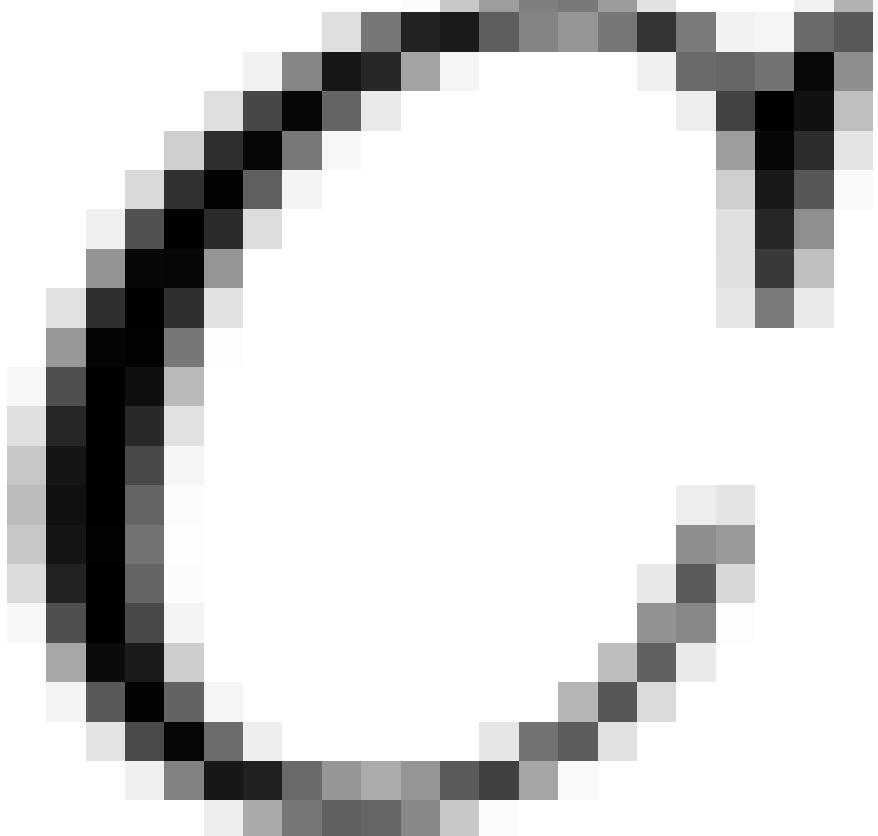


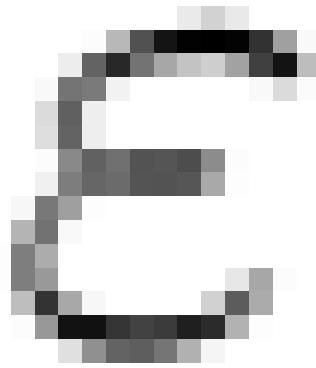
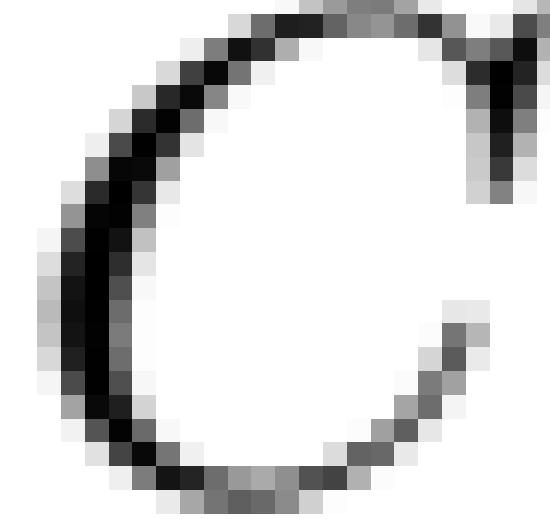
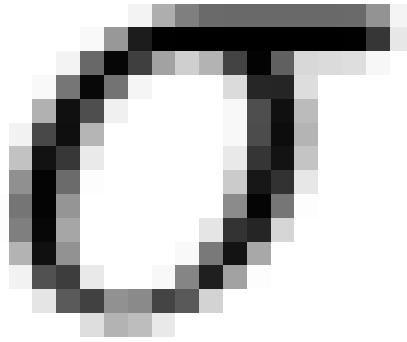






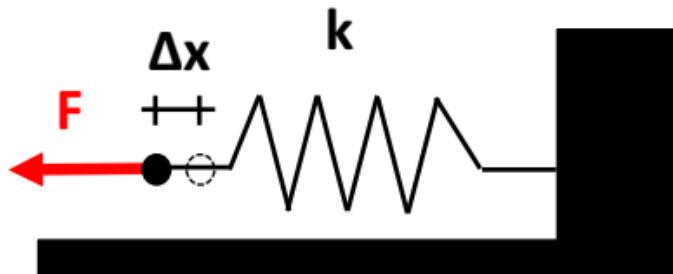






Hooke's law

$$F = k\Delta x$$

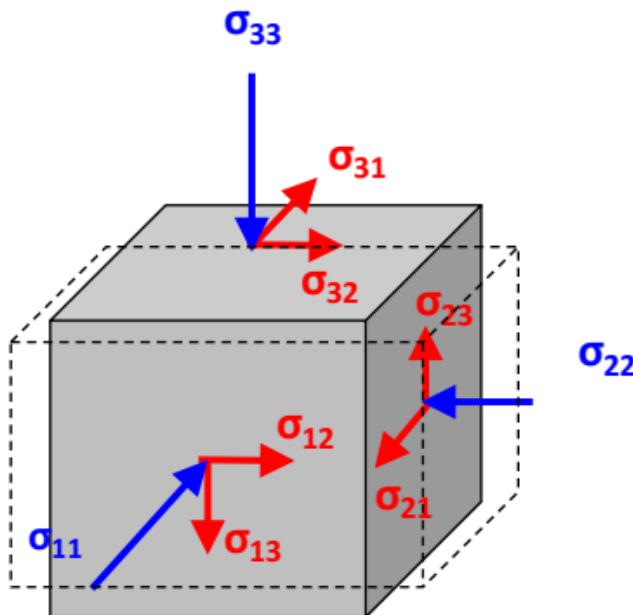


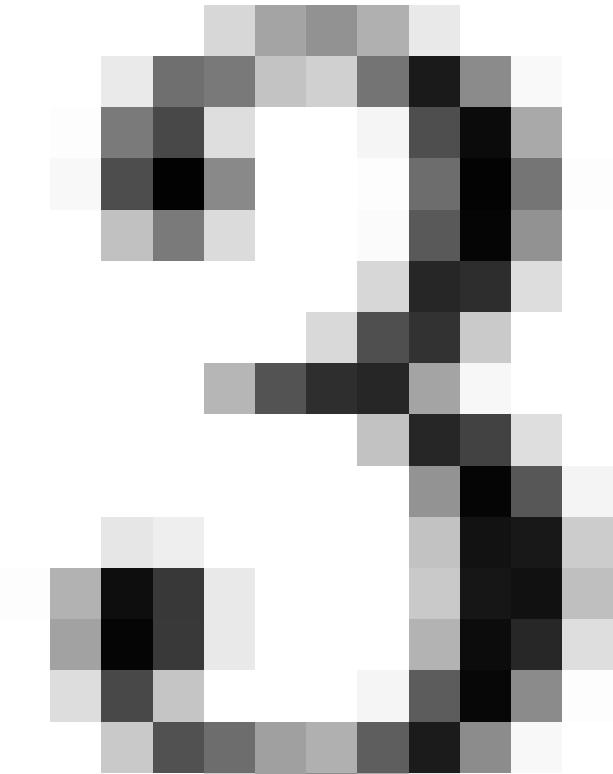
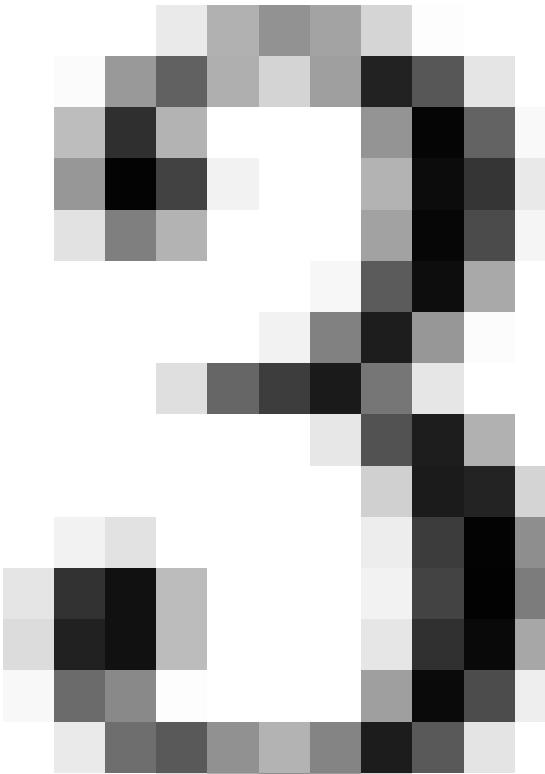
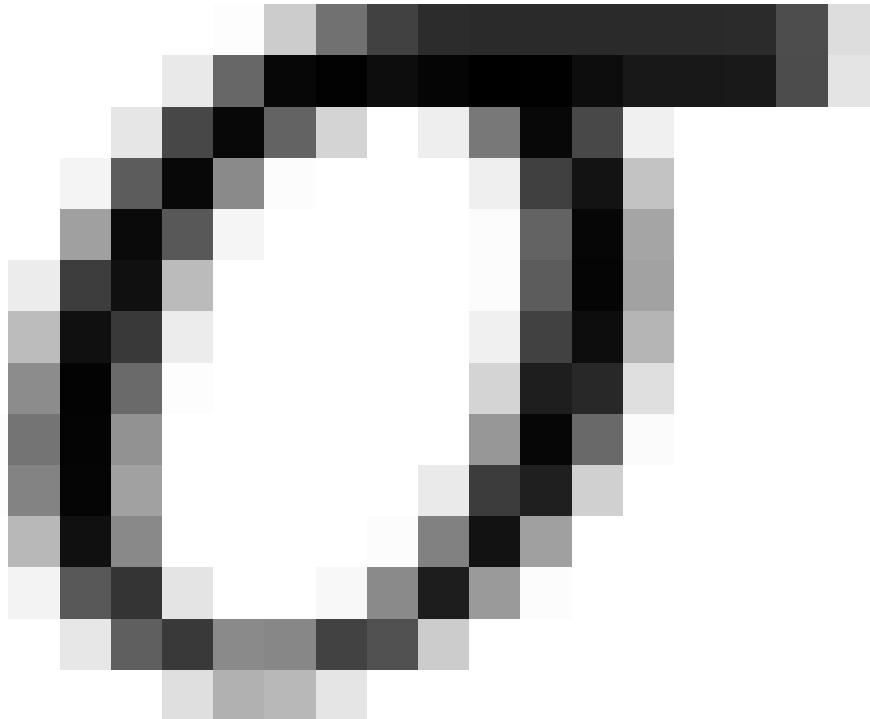
1-D (stress-strain) Hooke's law

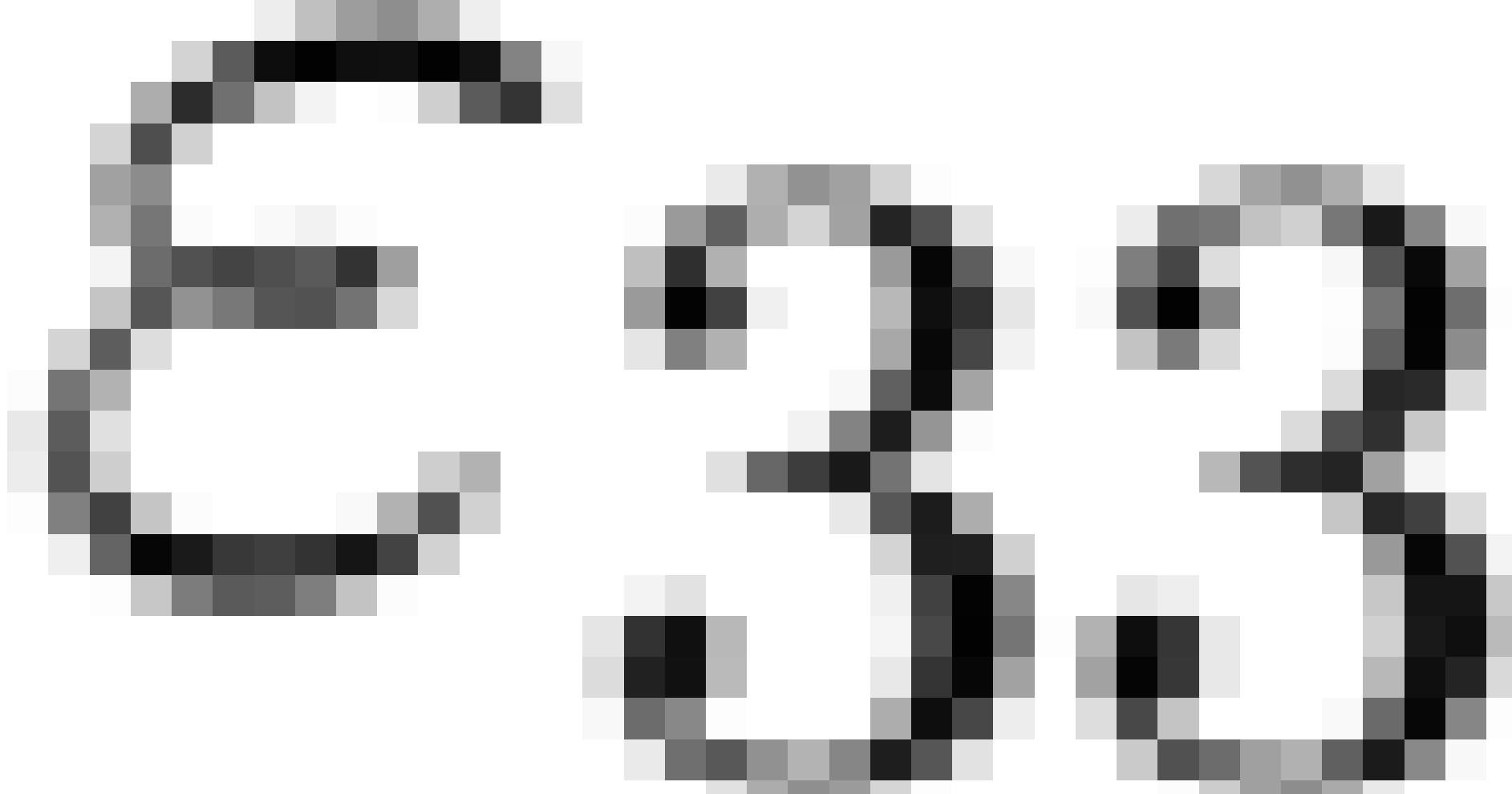
$$\sigma = E\varepsilon$$

Generalized Hooke's law

$$\underline{\sigma} = \underline{C}\underline{\varepsilon}$$







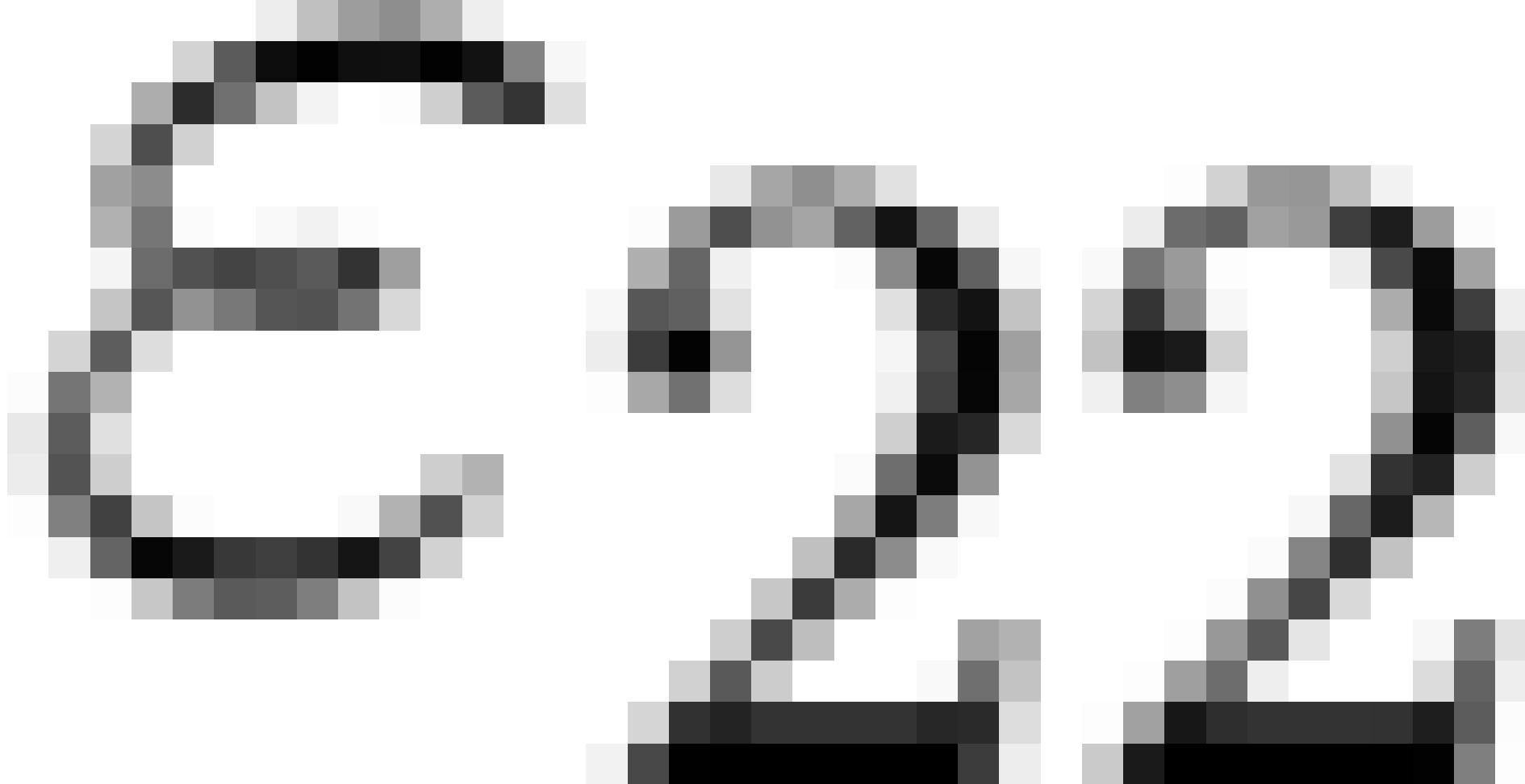
E

—
—

σ33

c33





611

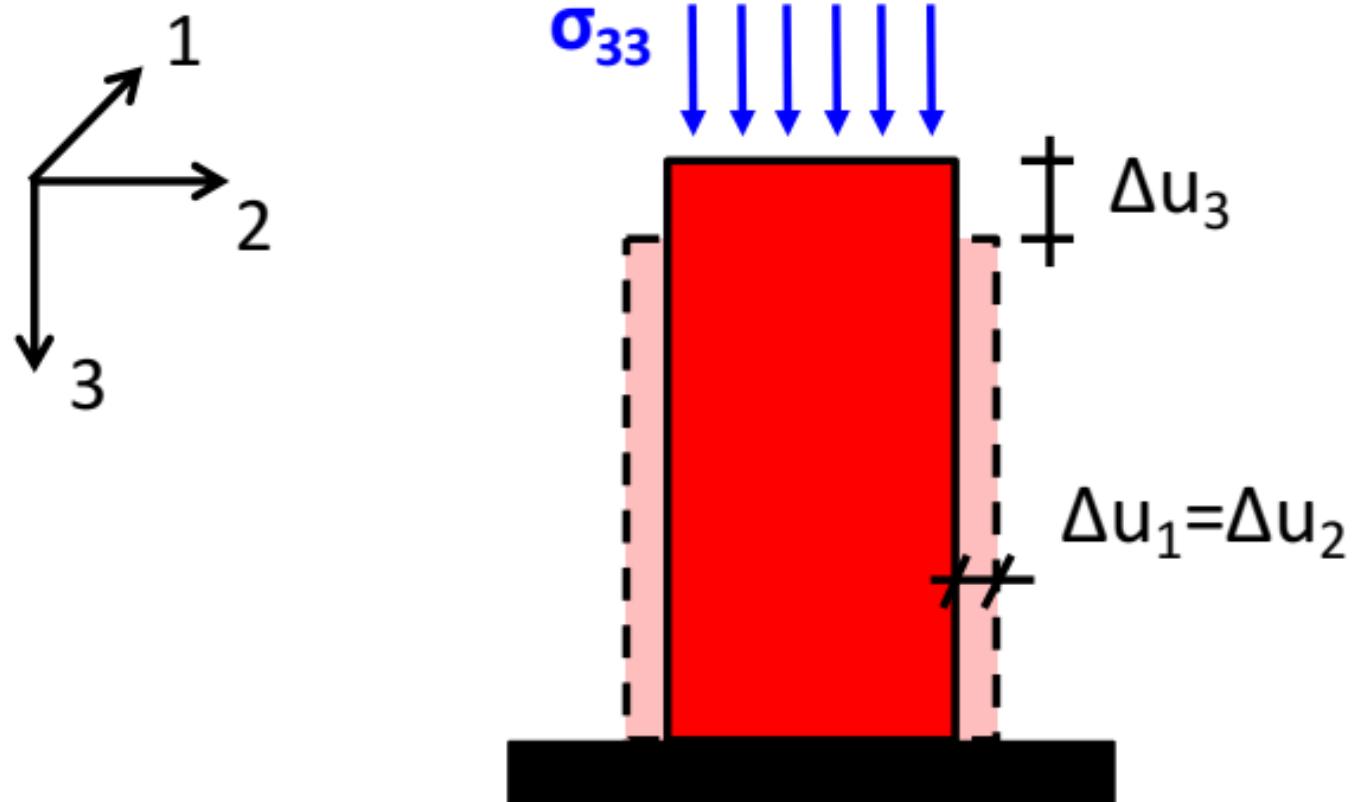
W

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633

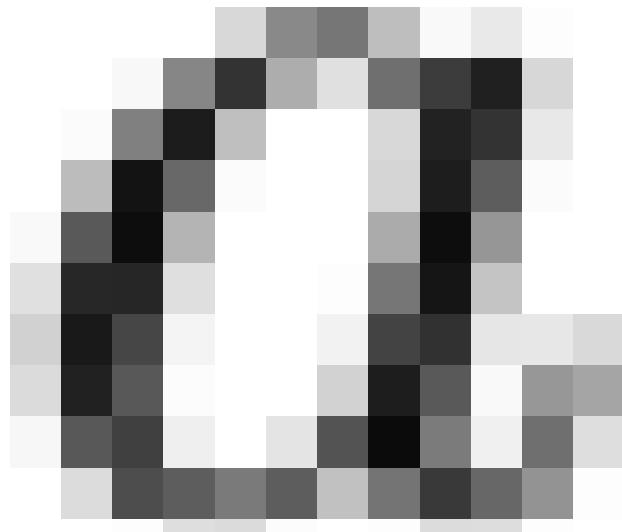
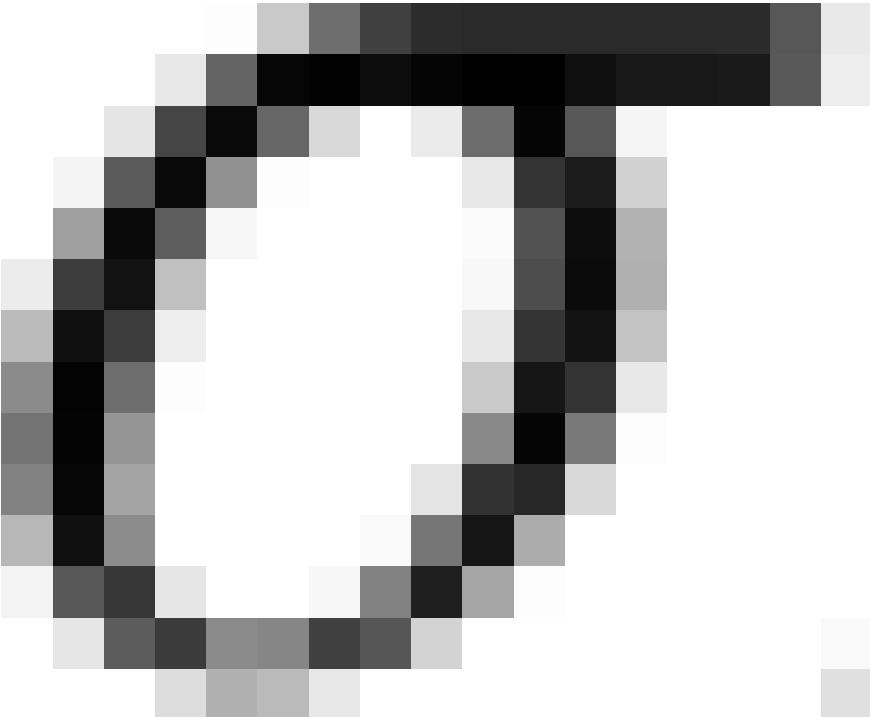


$$E = \frac{\sigma_{33}}{\epsilon_{33}}$$

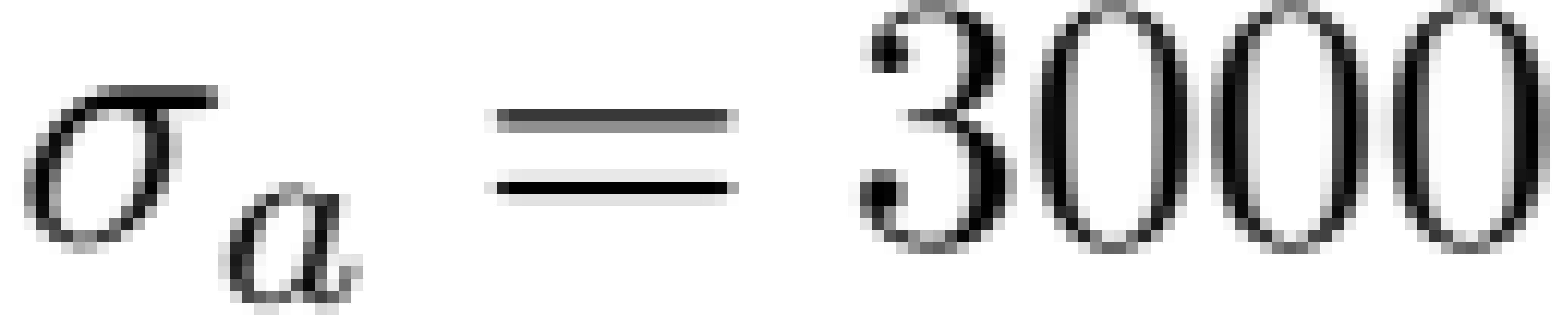
Young's Modulus

$$\nu = -\frac{\epsilon_{11}}{\epsilon_{33}}$$

Poisson's ratio (nu)







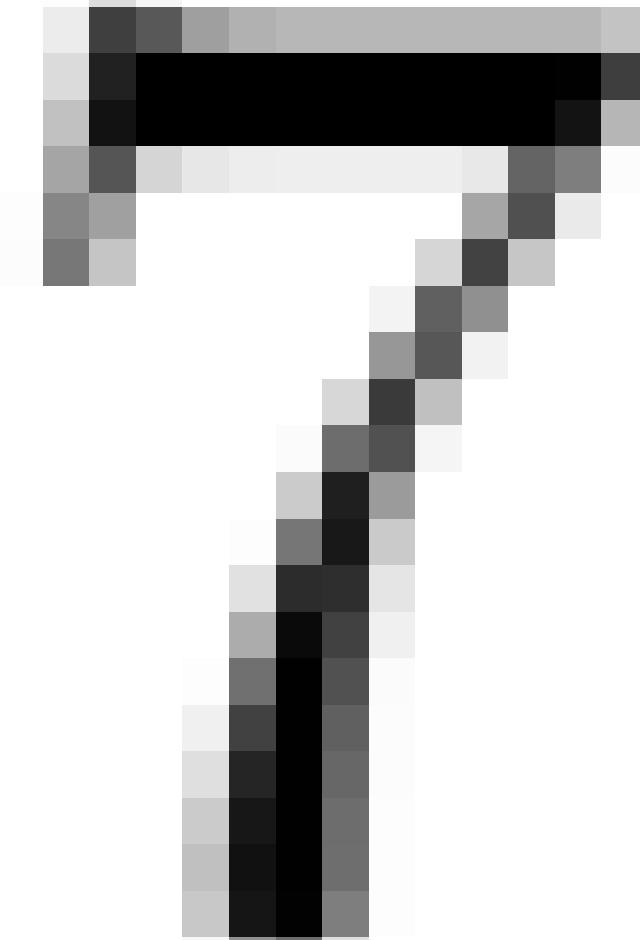
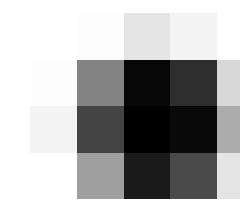
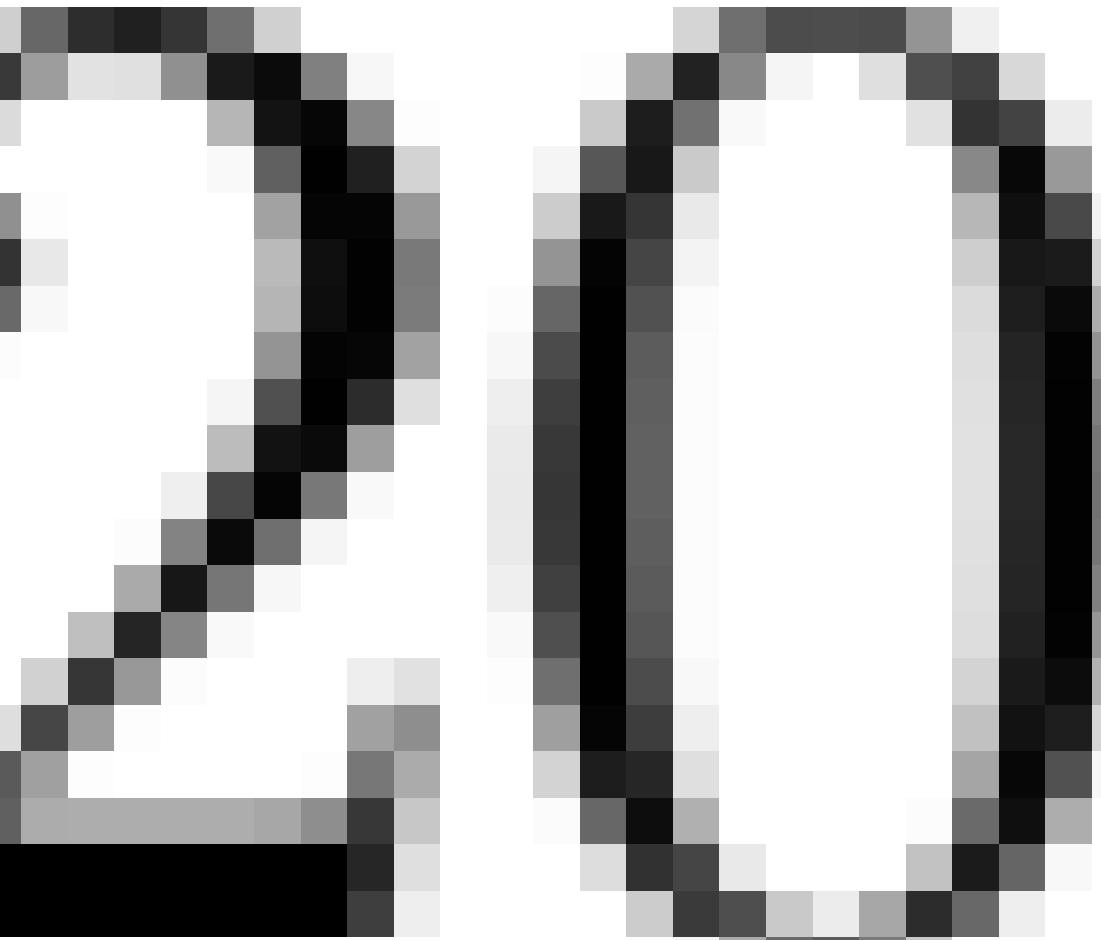
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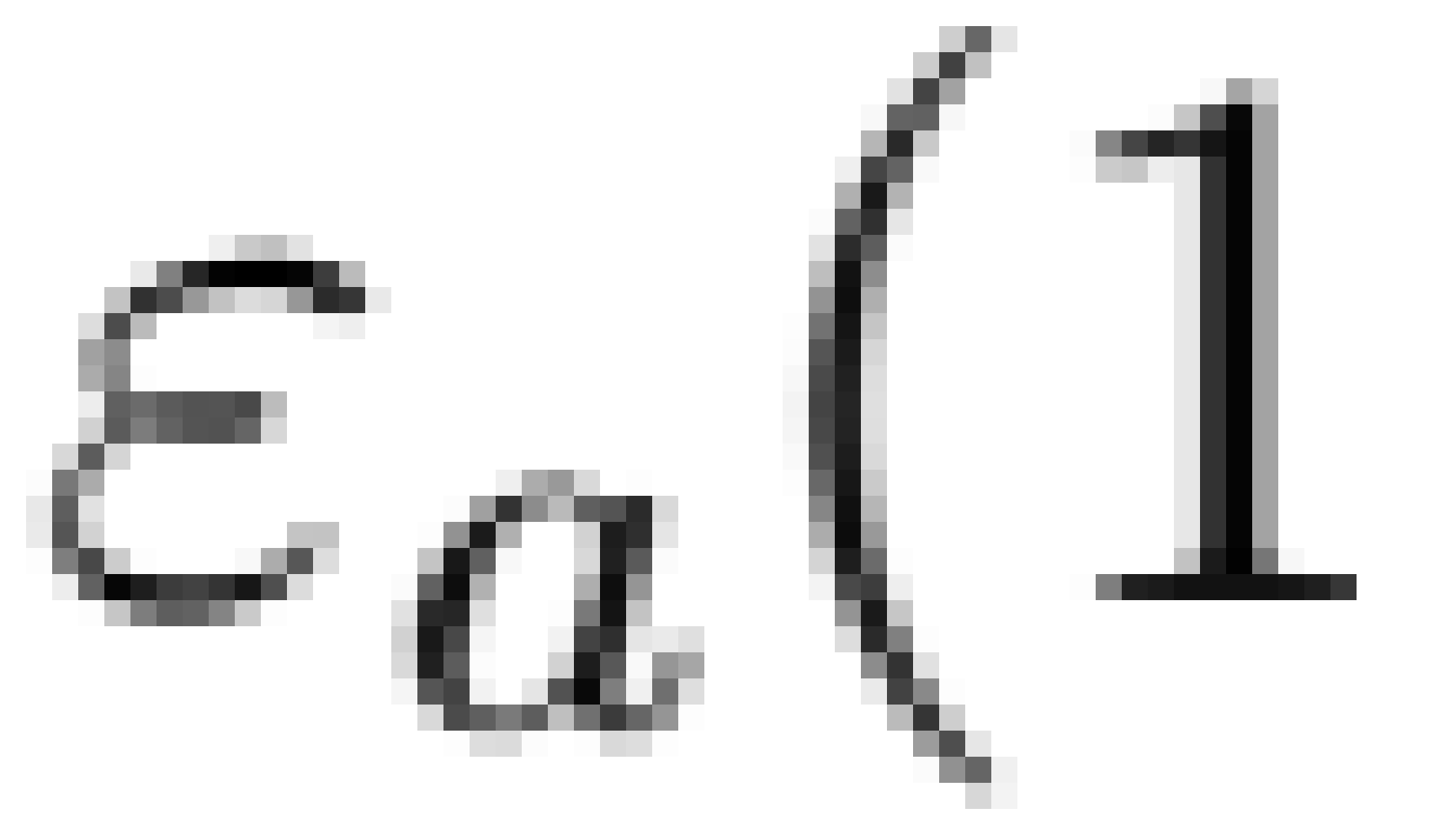
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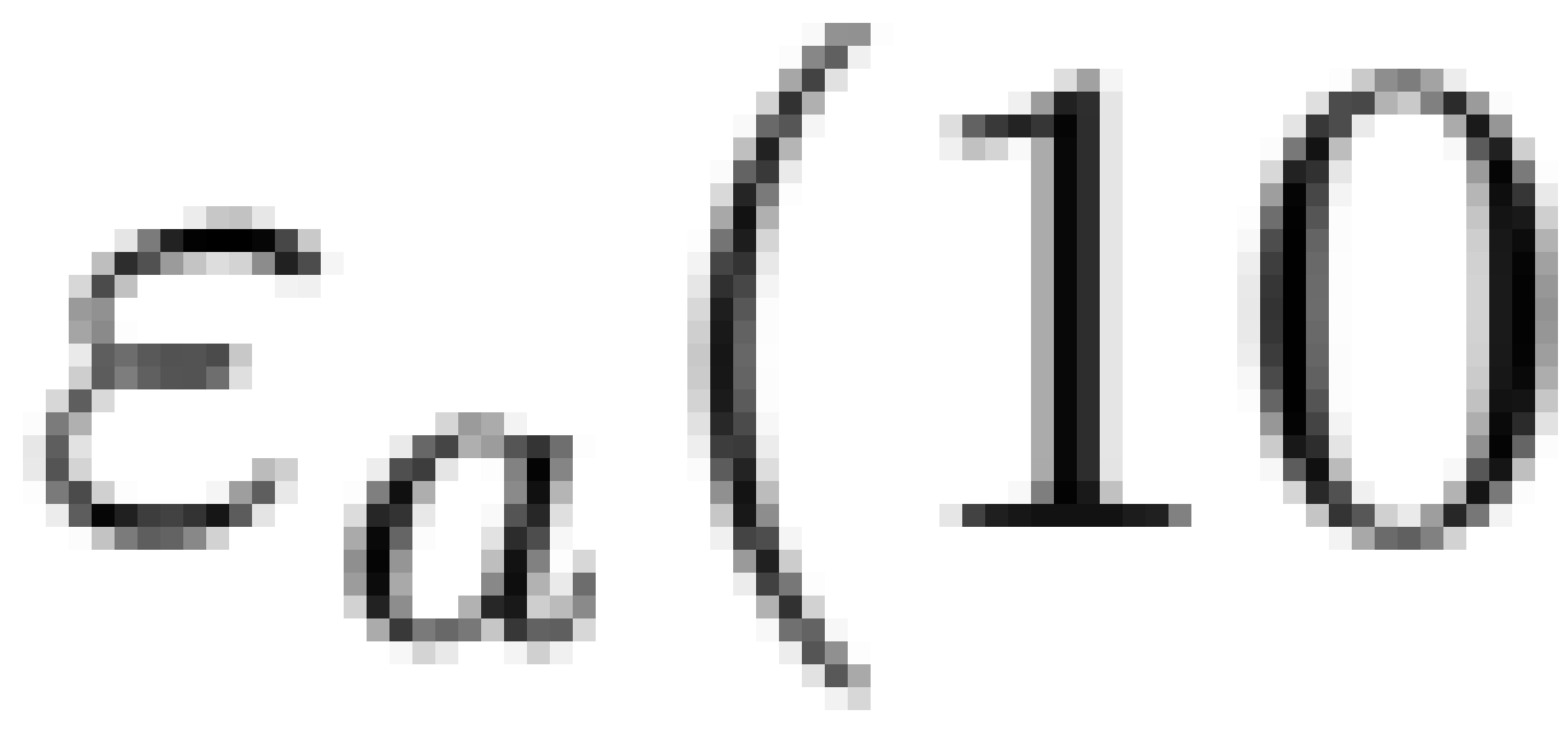
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$$\frac{\sigma_a}{E} = \frac{20.7}{100} = 0.207$$

$$0.207 \times 100 \text{ MPa} = 20.7 \text{ MPa}$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{100 \times 10^9 \text{ MPa}} = 0.207\%$$



$$\frac{\sigma_a}{E} = \frac{20.7 \text{ MPa}}{50000 \text{ MPa}} = 0.00041$$

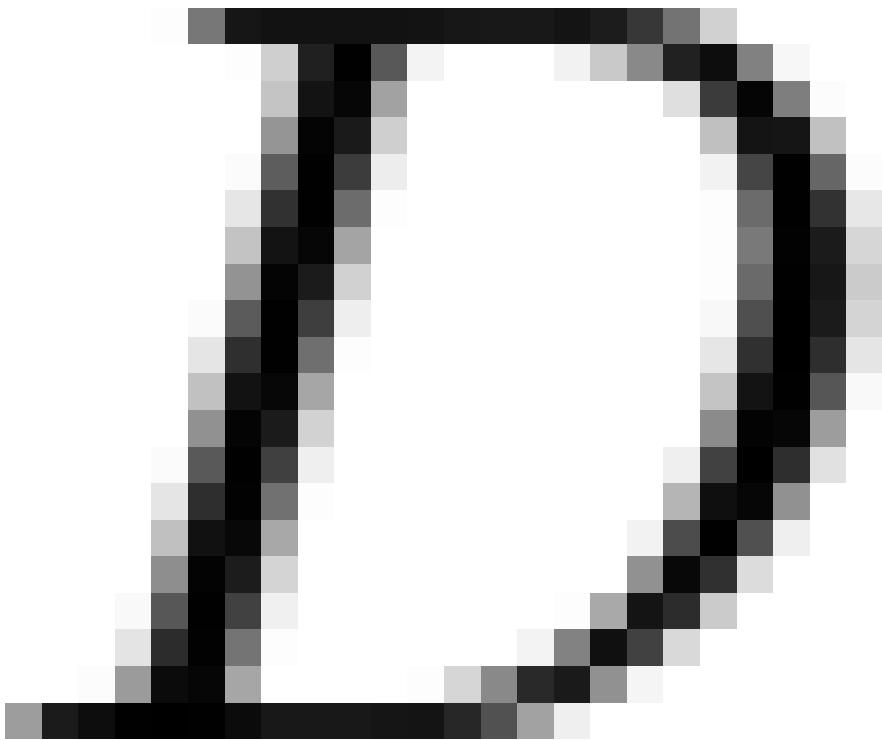
$$= 0.041\%$$

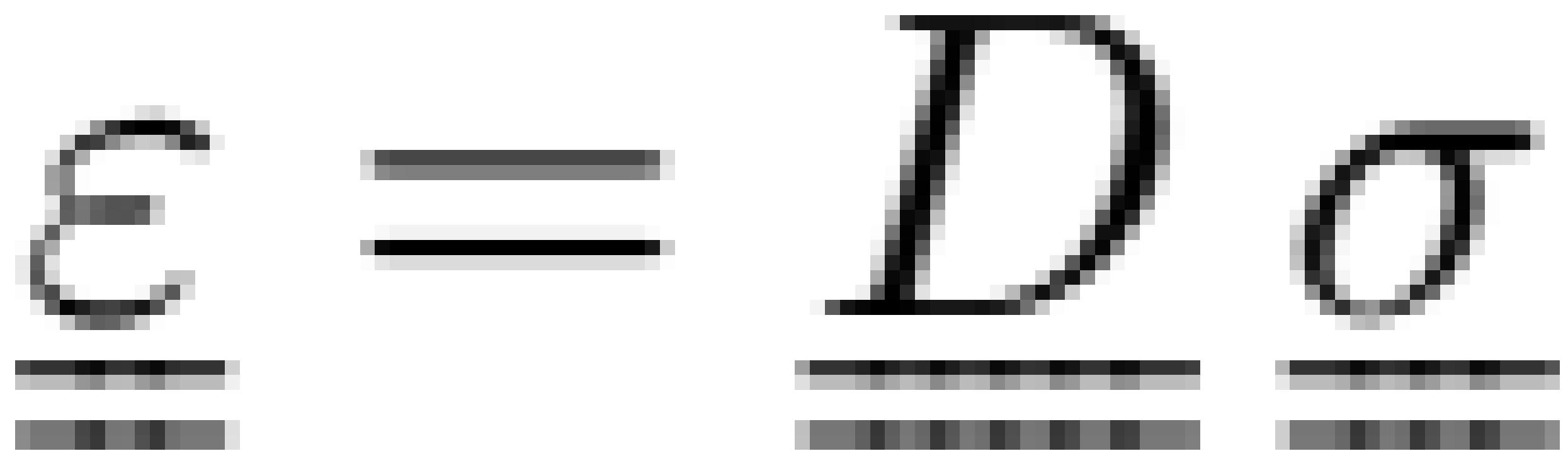
$$\left\{ \begin{array}{lcl} \epsilon_{11} & = & +\frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ & & \nu \quad \quad \quad 1 \quad \quad \quad \nu \\ \epsilon_{22} & = & -\frac{1}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{1}{E}\sigma_{33} \\ & & \nu \quad \quad \quad \nu \quad \quad \quad 1 \\ \epsilon_{33} & = & -\frac{1}{E}\sigma_{11} - \frac{1}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{array} \right.$$

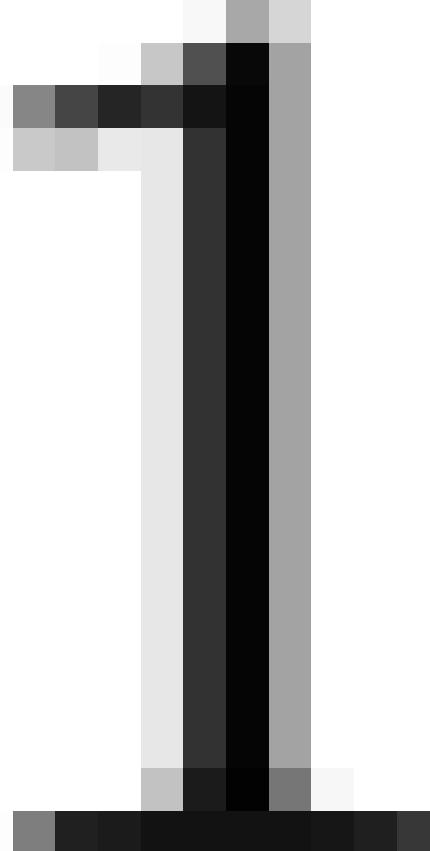
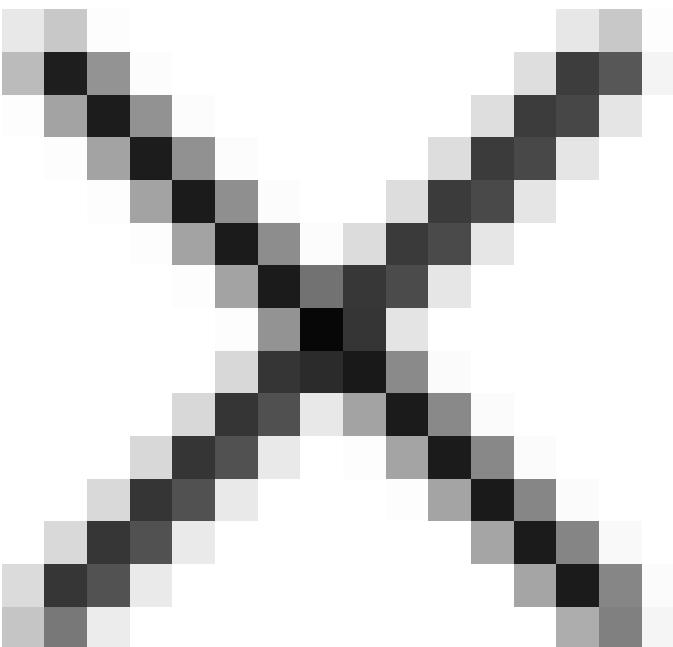
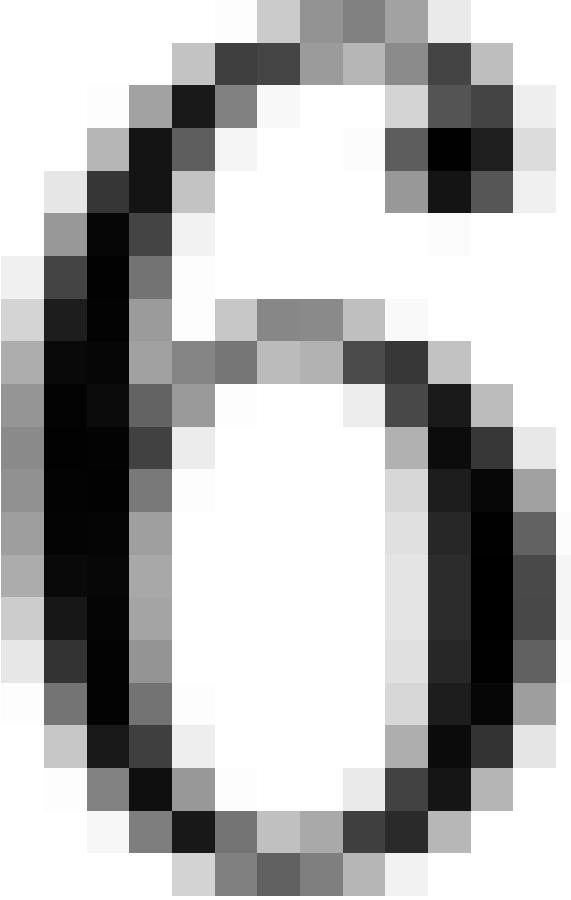


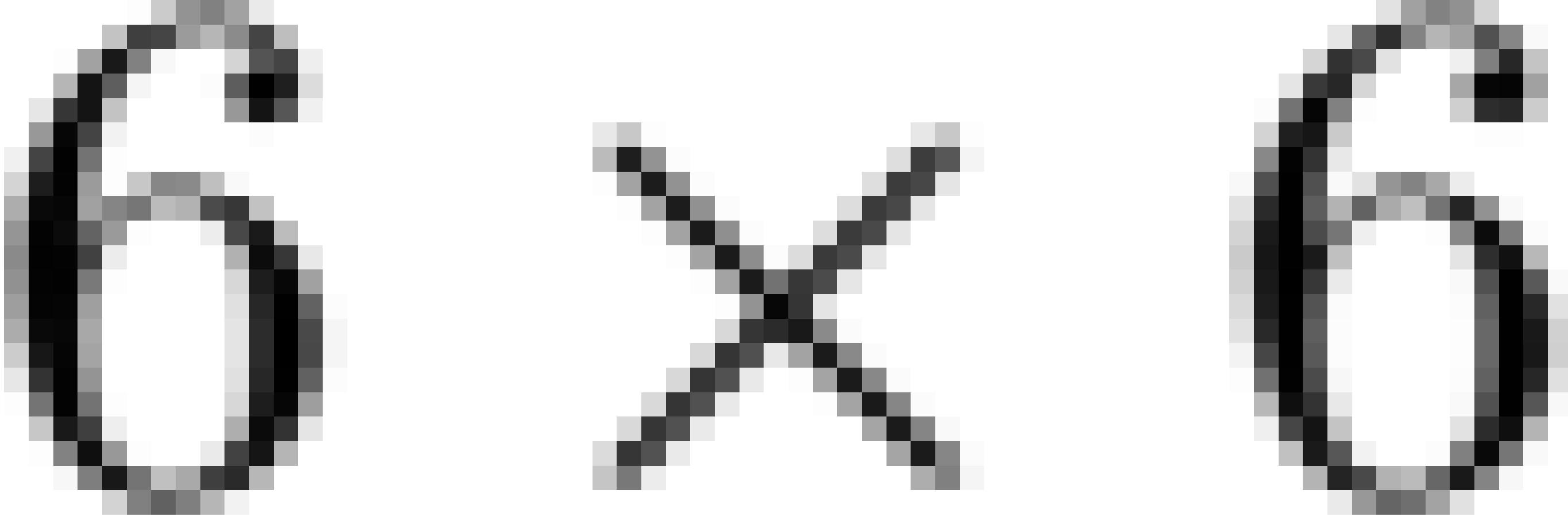


$$\left\{ \begin{array}{l} 2\varepsilon_{12} = \frac{1}{G} \sigma_{12} \\ \\ 2\varepsilon_{13} = \frac{1}{G} \sigma_{13} \\ \\ 2\varepsilon_{23} = \frac{1}{G} \sigma_{23} \end{array} \right.$$



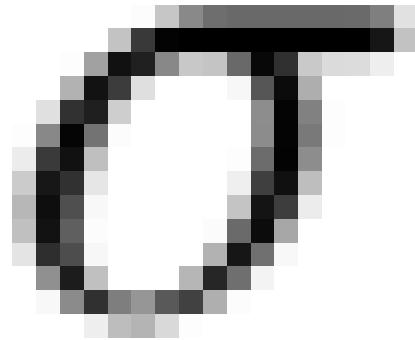
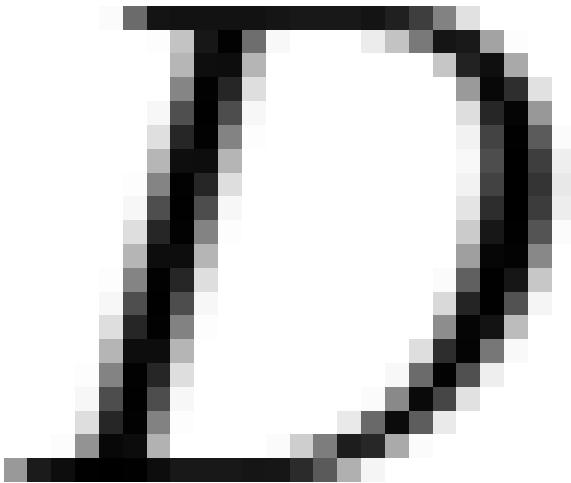




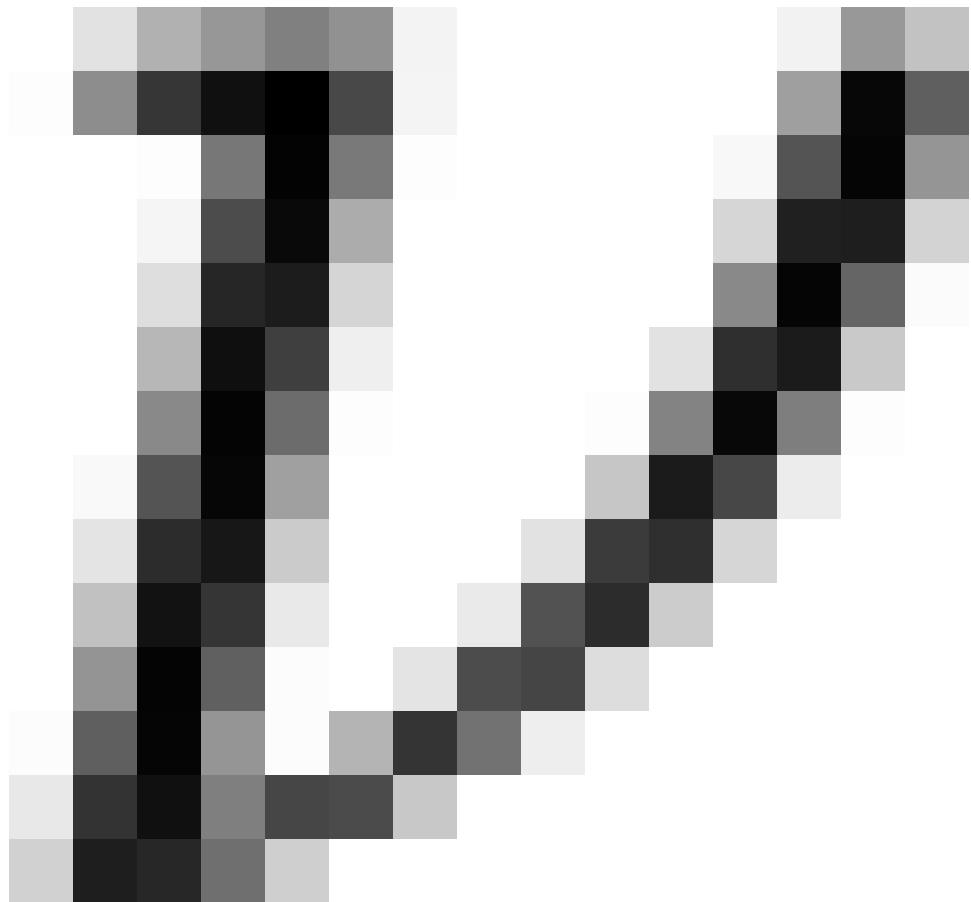


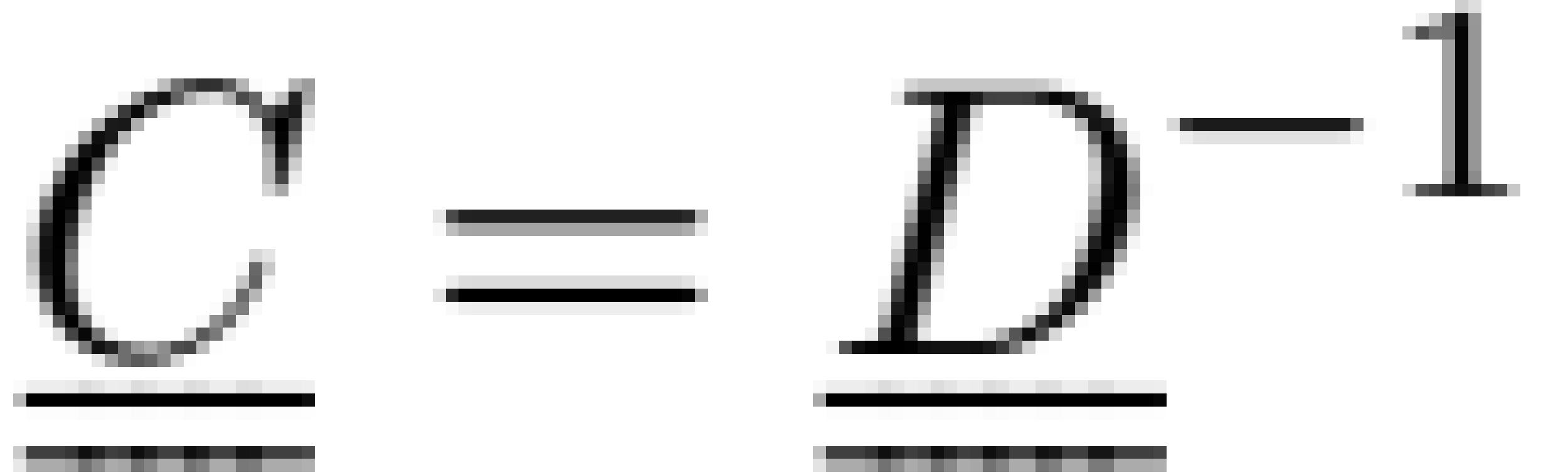
$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix} = \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & 1 & \nu & 0 & 0 & 0 \\ -\frac{\nu}{E} & +\frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ \nu & \nu & 1 & 0 & 0 & 0 \\ -\frac{1}{E} & -\frac{\nu}{E} & +\frac{1}{E} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}$$



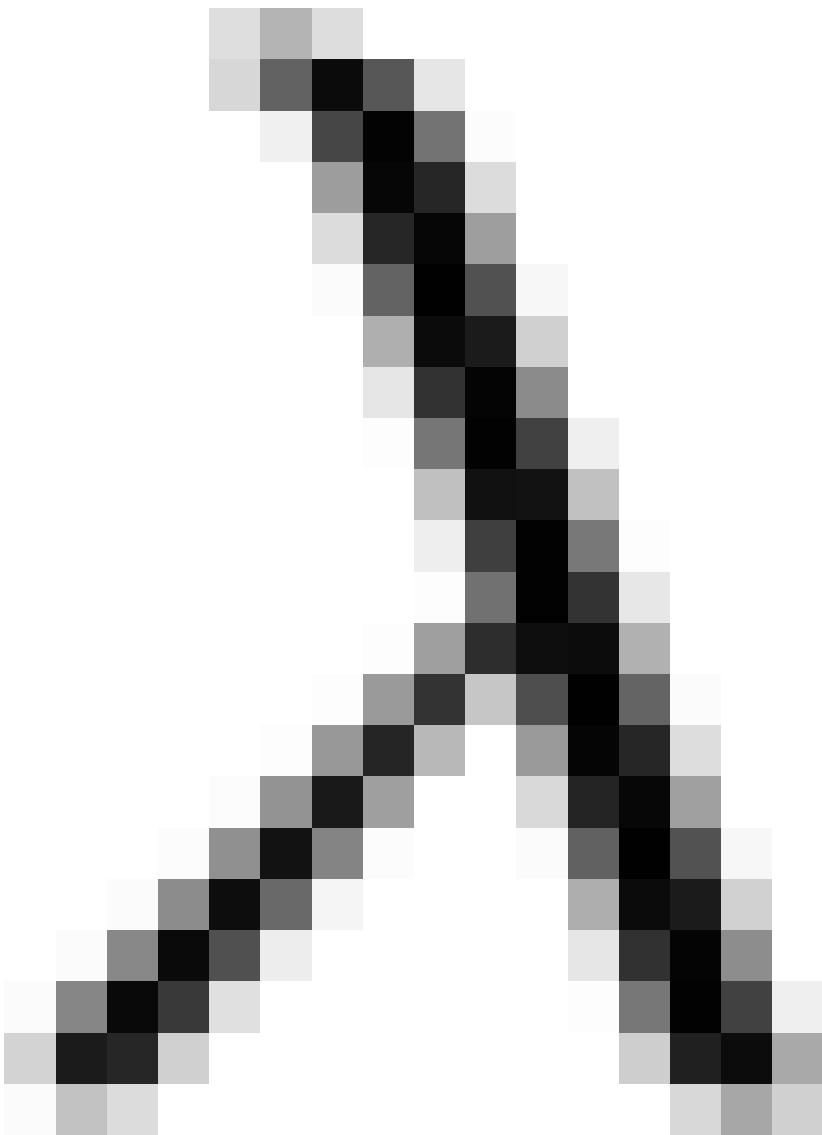


$$\underline{\underline{\epsilon}} = \begin{bmatrix} -\frac{v}{E}\sigma_{33}, -\frac{v}{E}\sigma_{33}, \frac{1}{E}\sigma_{33}, 0, 0, 0 \end{bmatrix}^T$$





$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$



$$\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)} [(1-\nu)e_{11} + \nu e_{22} + \nu e_{33}]$$

$$\sigma_{11} = \frac{\nu E}{(1+\nu)(1-2\nu)} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + \frac{\nu E}{(1+\nu)(1-2\nu)} \left(\frac{1-\nu}{1+\nu} \epsilon_{11} - \frac{1+\nu}{1-\nu} \epsilon_{22} \right)$$

λ



$$(1 + \nu)$$

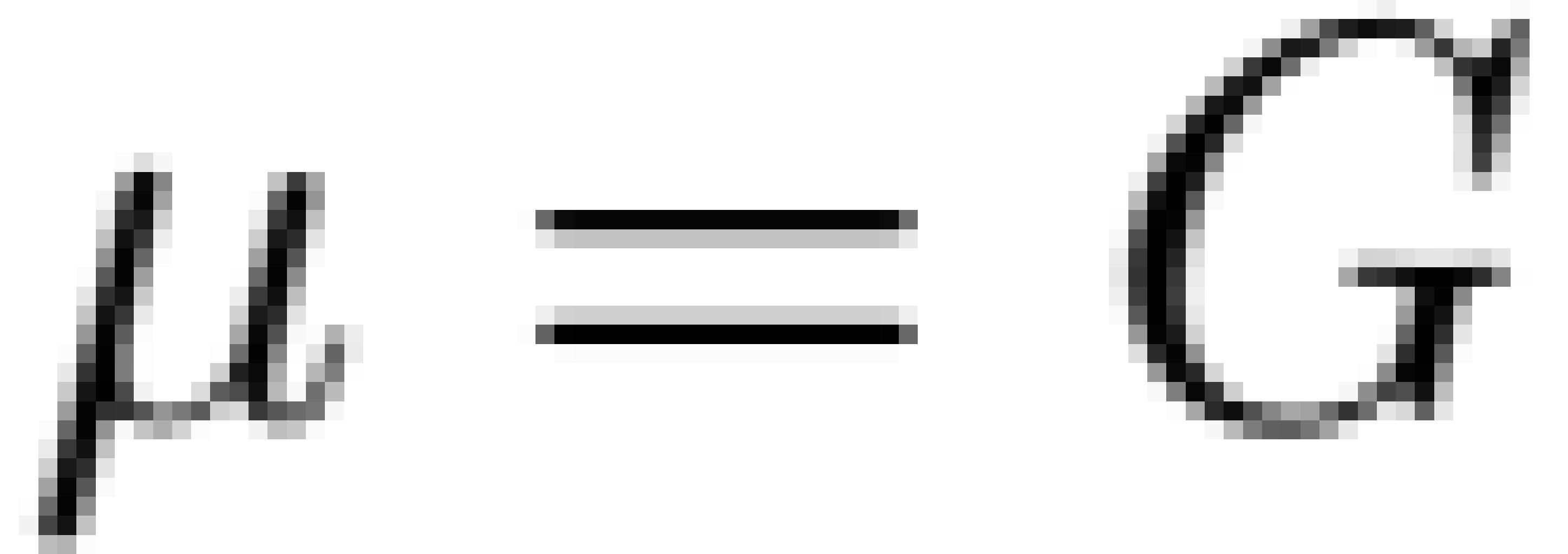
$$(1 - \nu)$$



$$2\nu$$

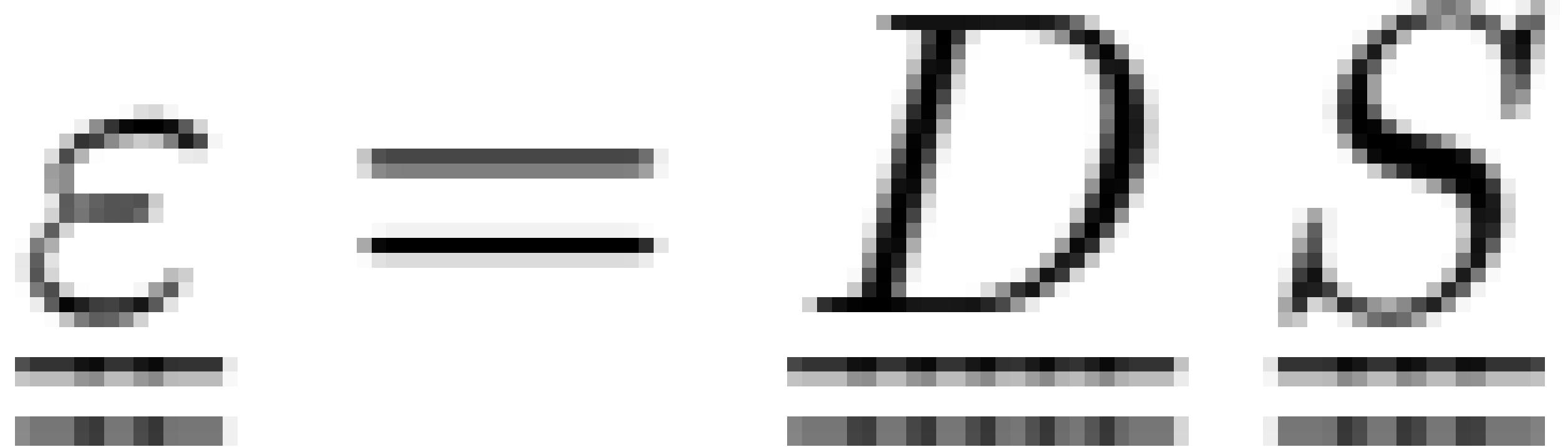
νE

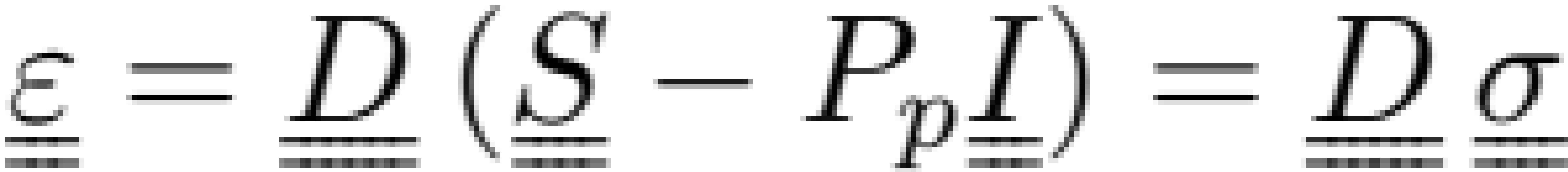
$$2\mu = \frac{\nu E}{(1+\nu)(1-2\nu)} = \frac{-1}{(1+\nu)} + \frac{1-\nu}{(1+\nu)}$$

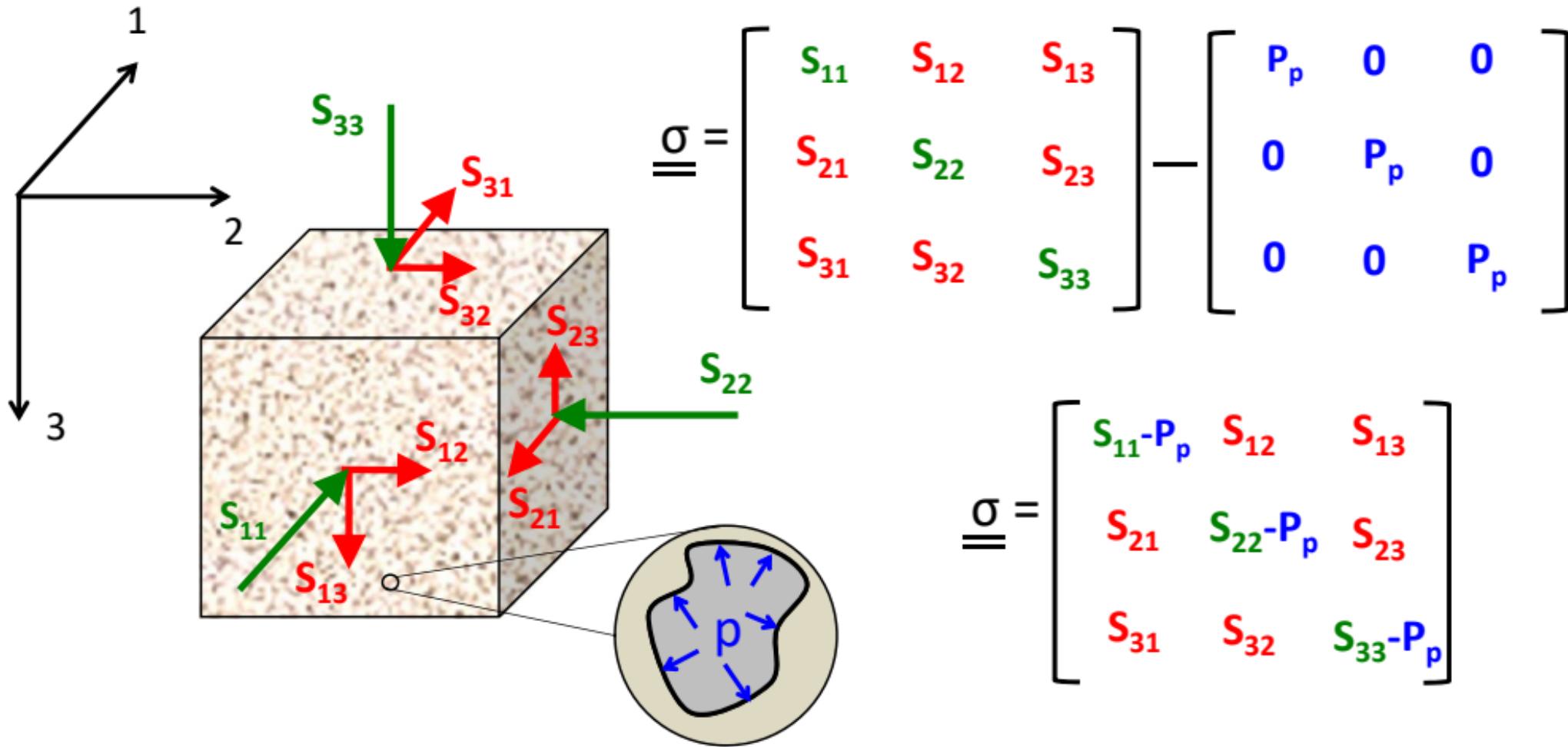


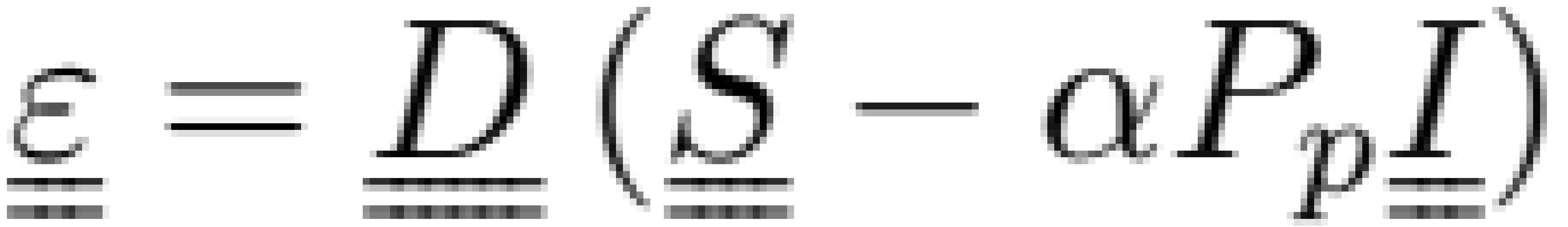
$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right. .$$

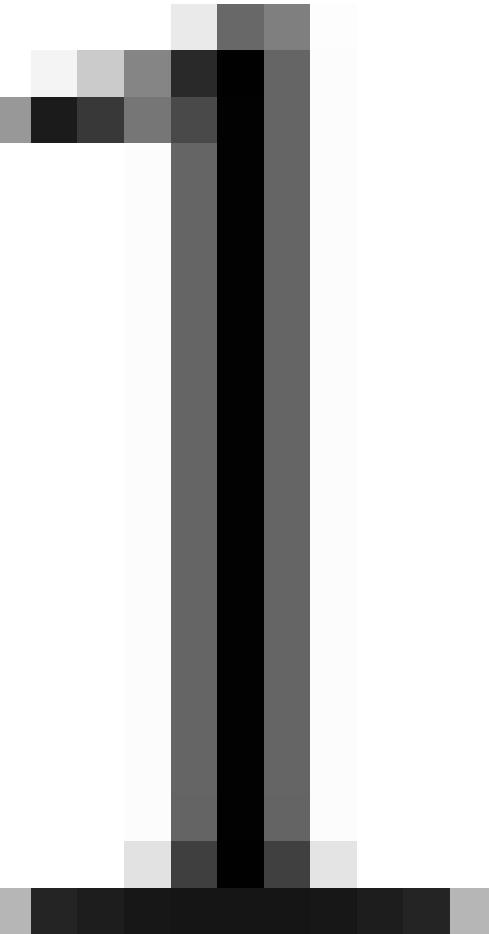
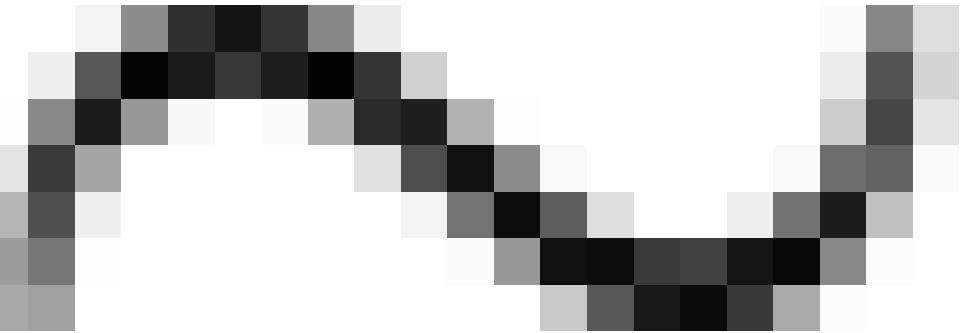
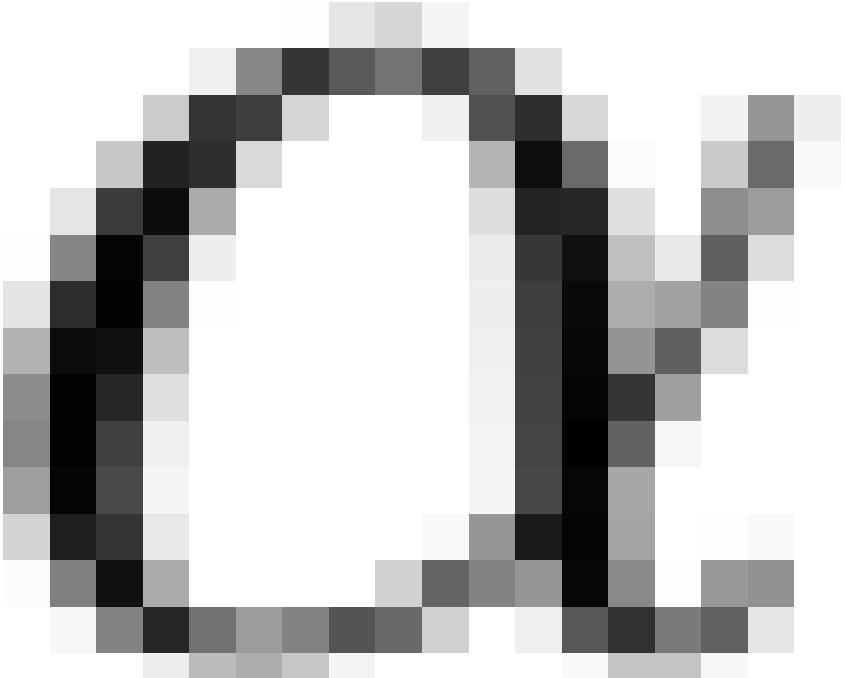
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$







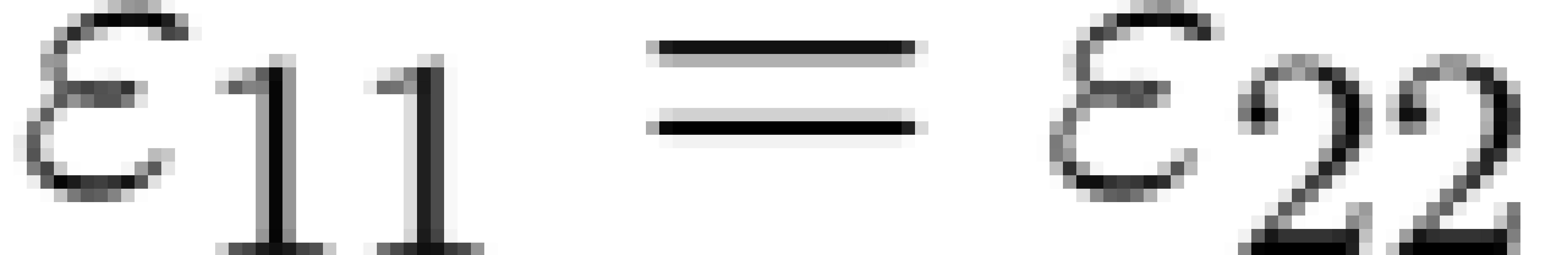




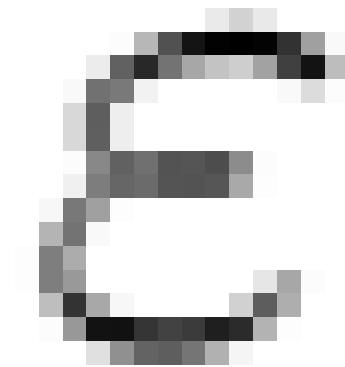
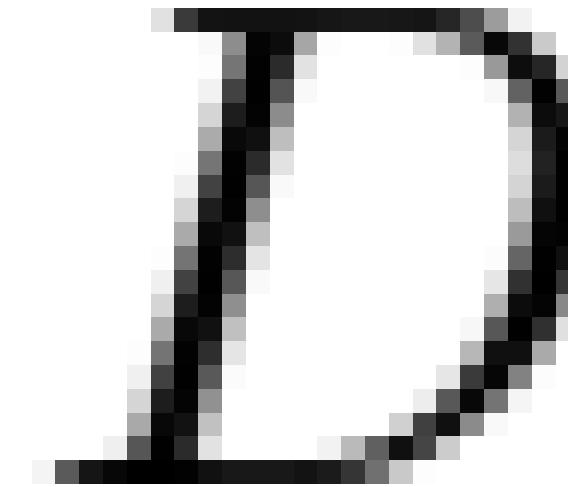
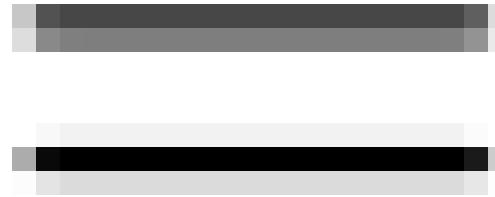
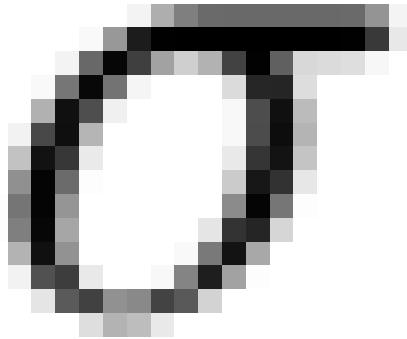




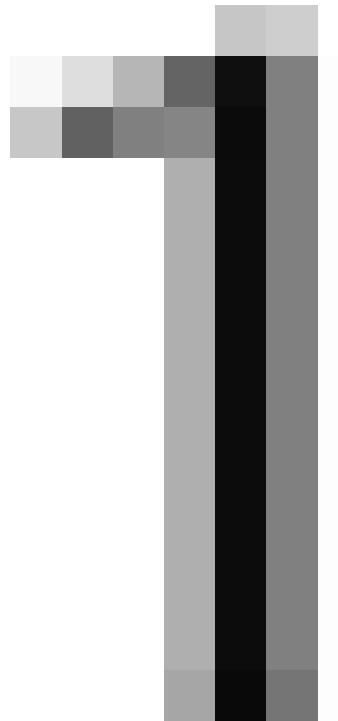
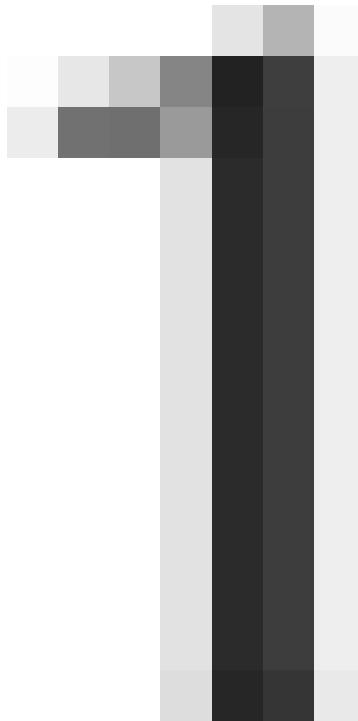
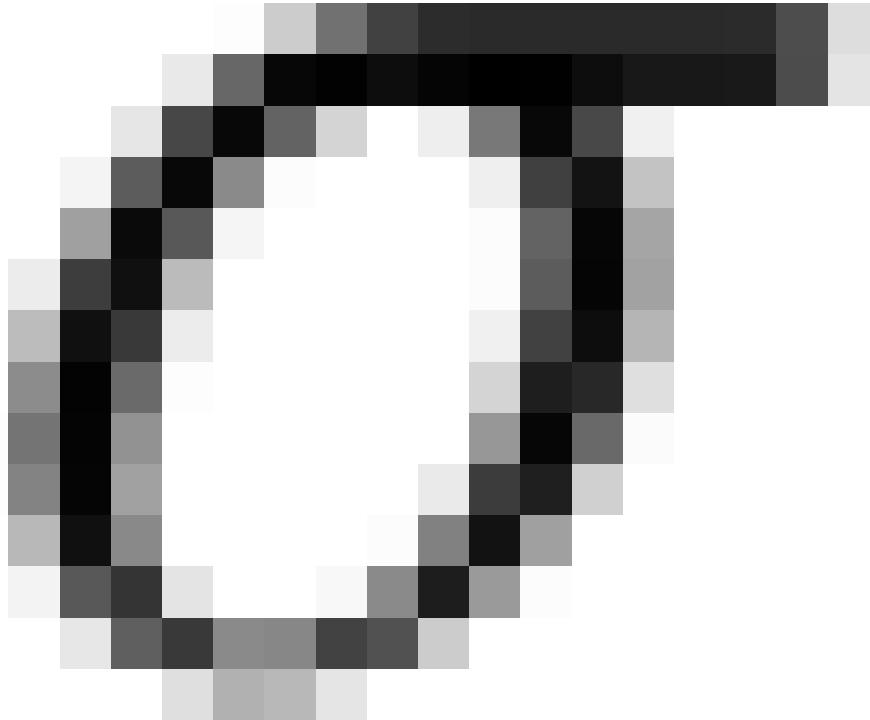


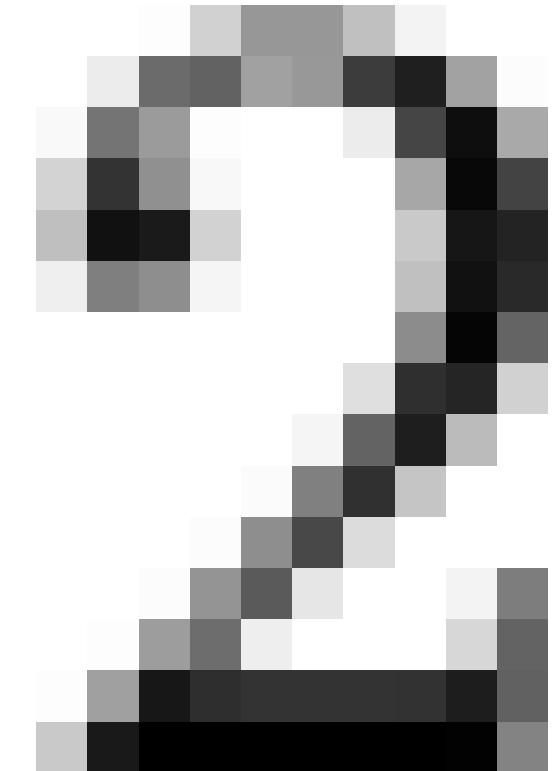
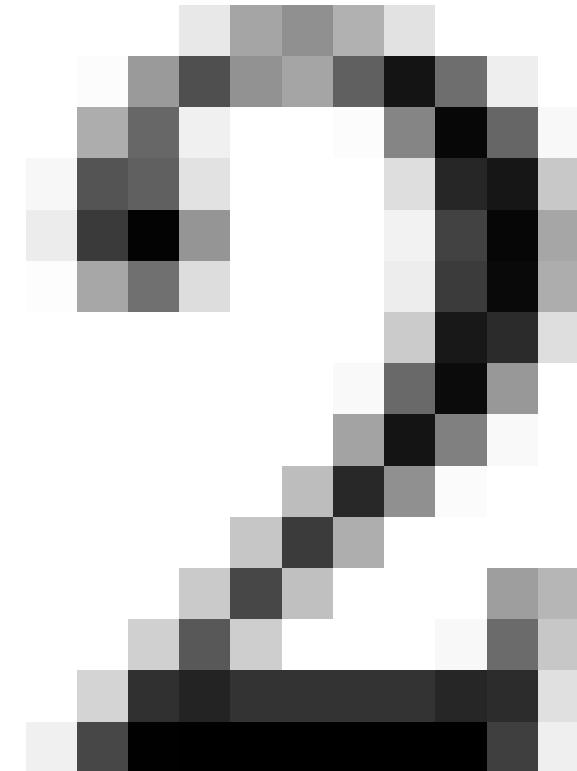
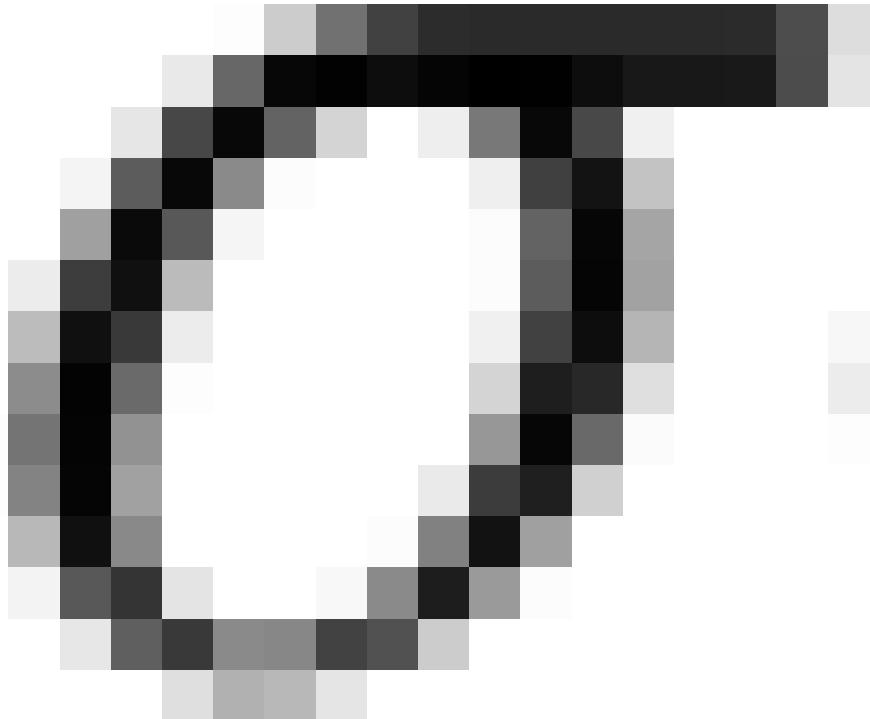






$$\left\{ \begin{array}{l} \sigma_{11} = \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)} \epsilon_{33} \\ \sigma_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \epsilon_{33} \end{array} \right.$$





σ_{11}

$$= \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

 σ_{22}

$$= \begin{array}{c} \text{---} \\ | \end{array}$$

 σ_{33} ν ν

σ_b

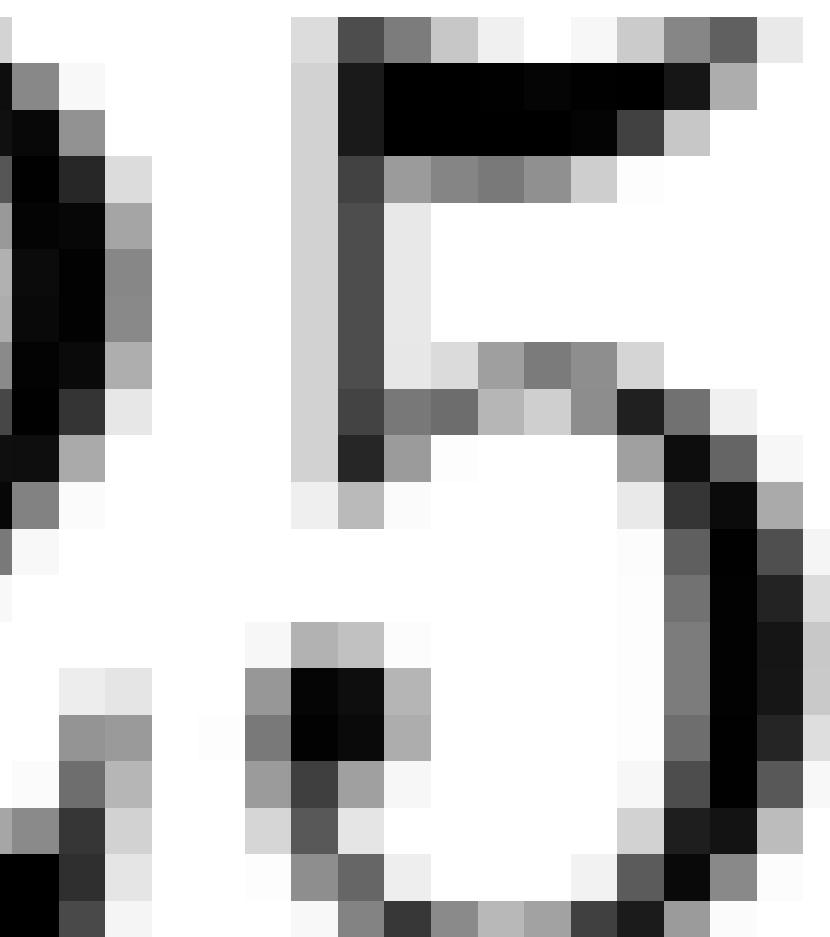
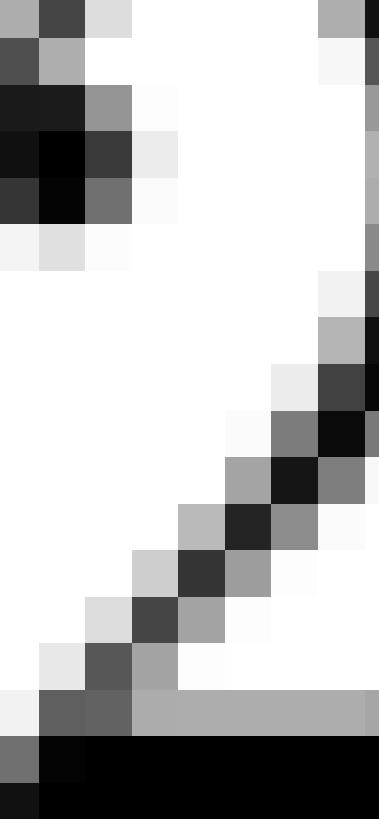
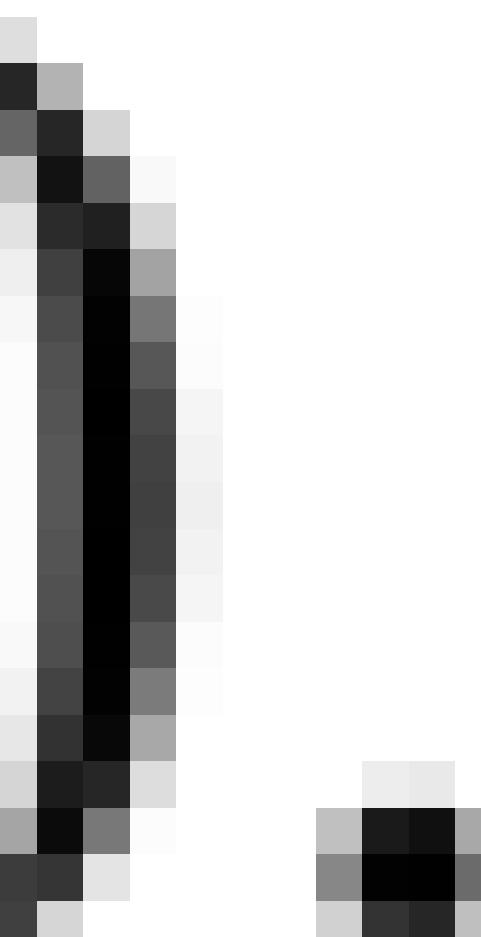
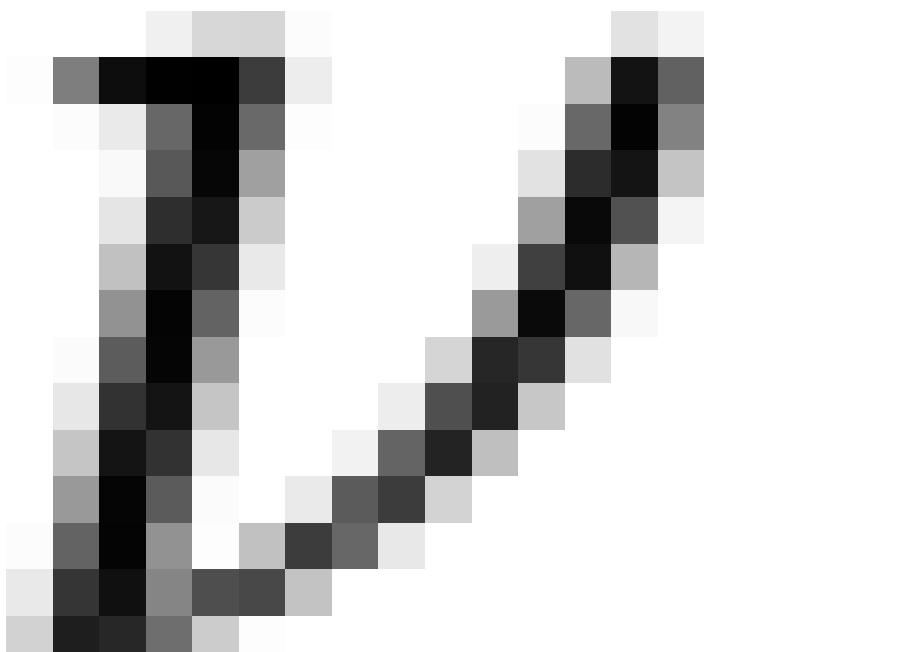
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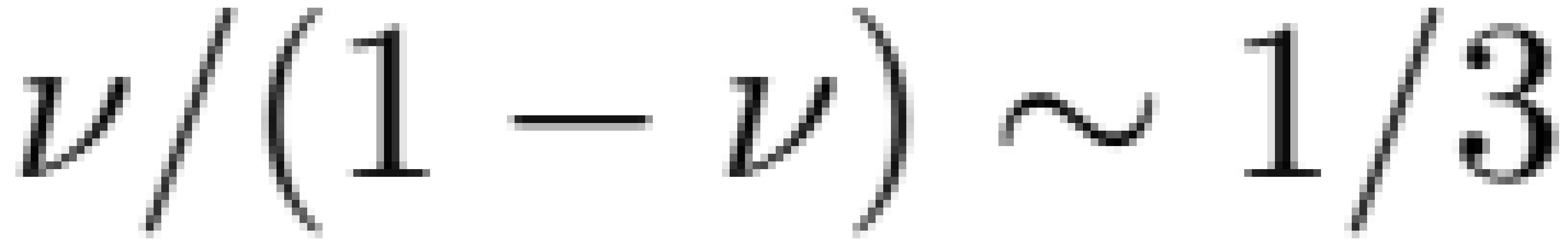
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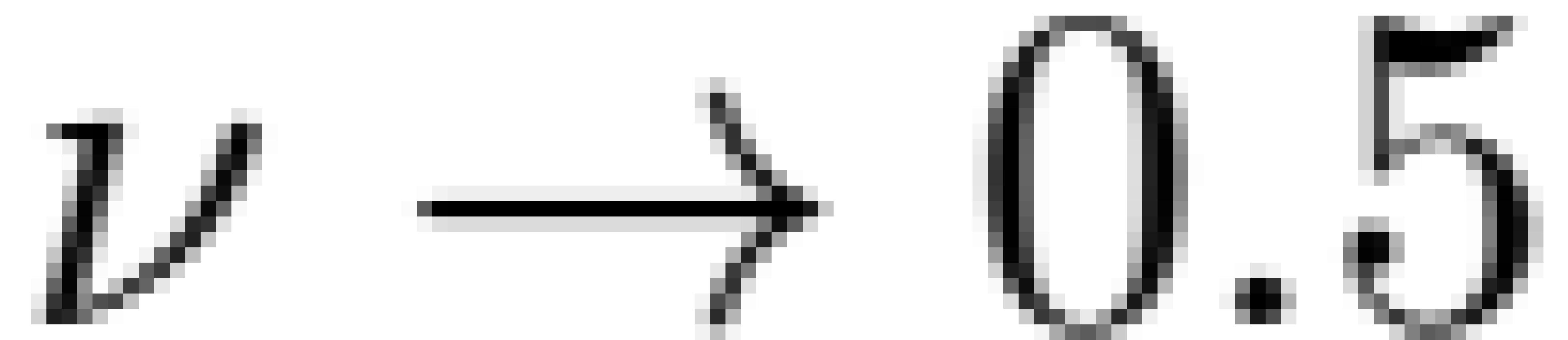
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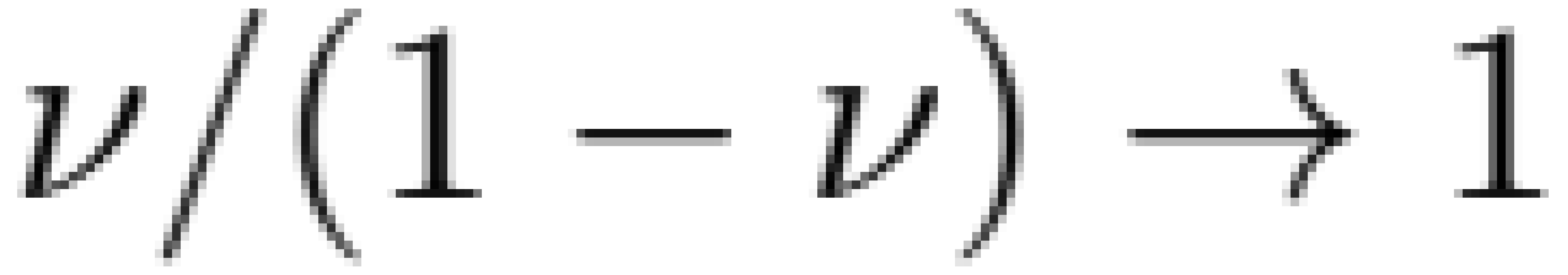
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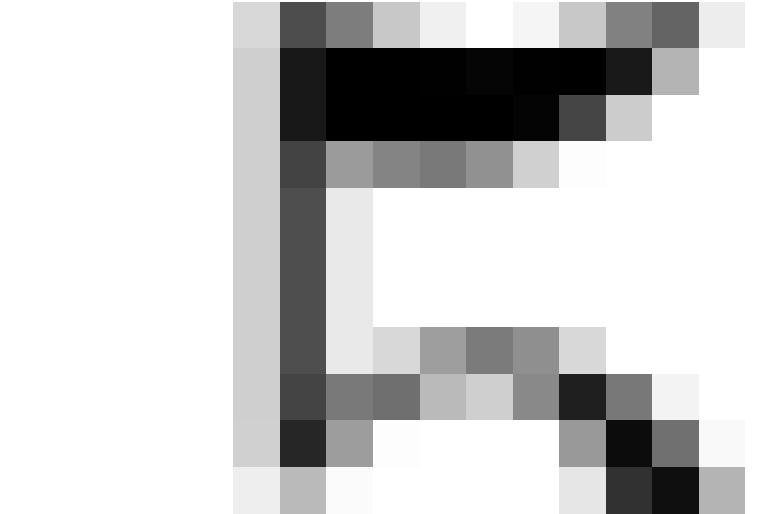
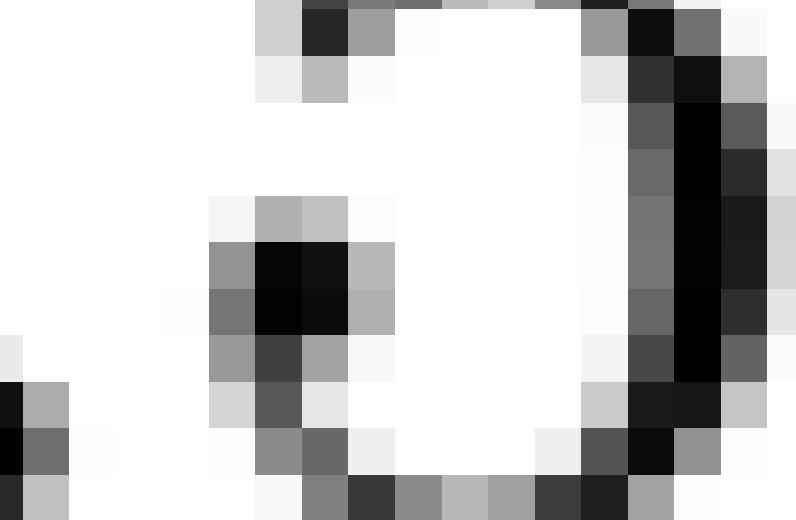
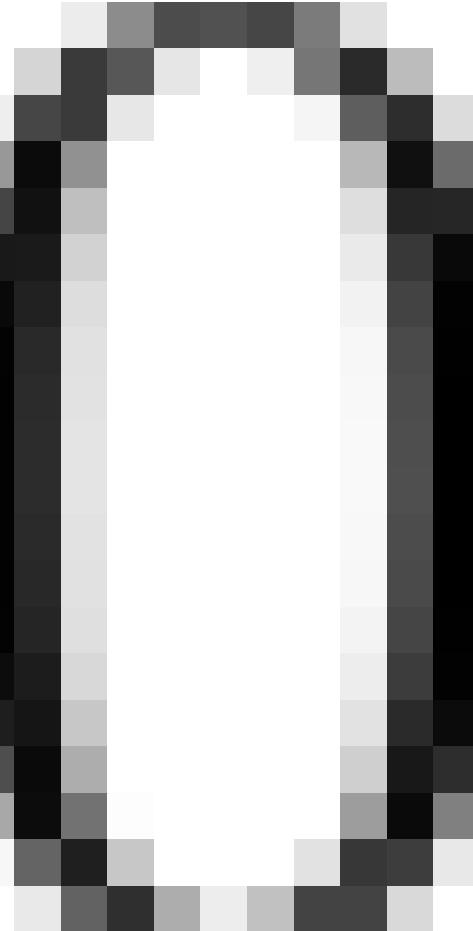
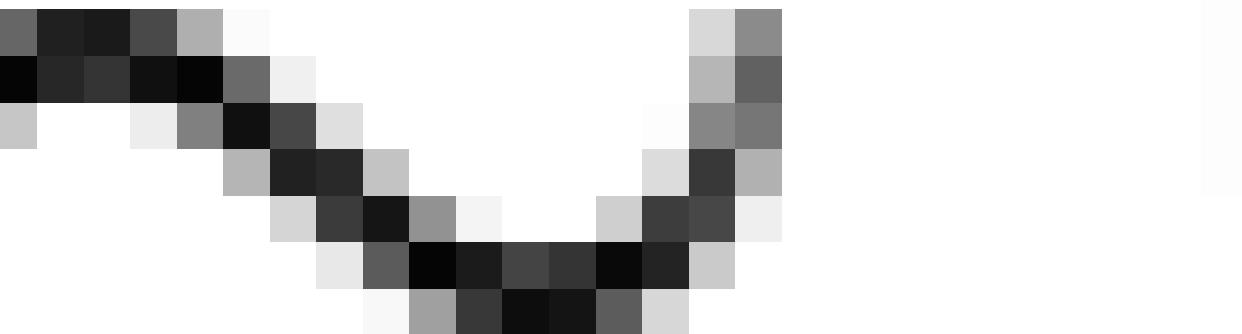
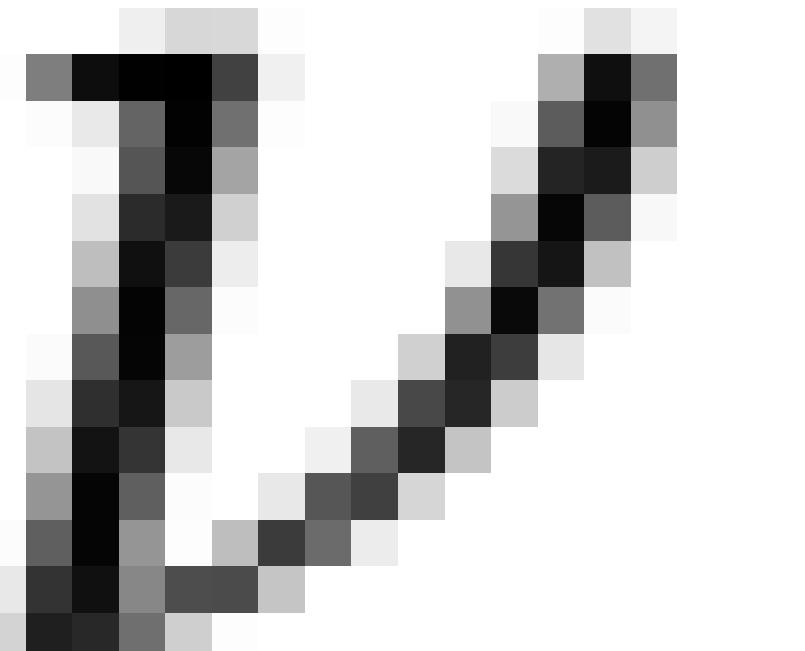
σ_w







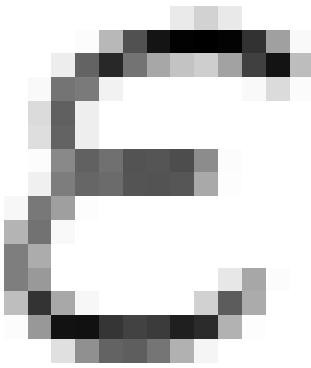
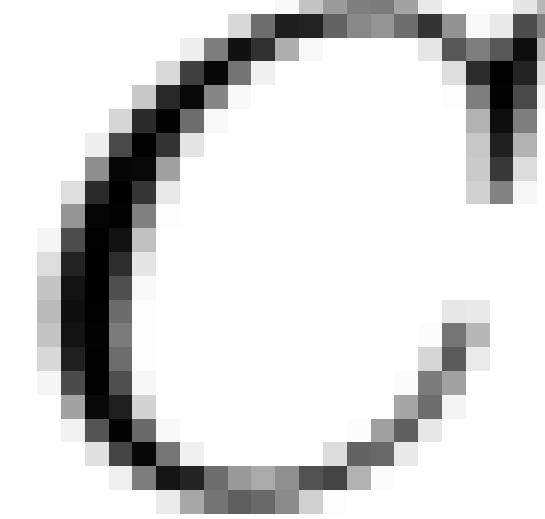
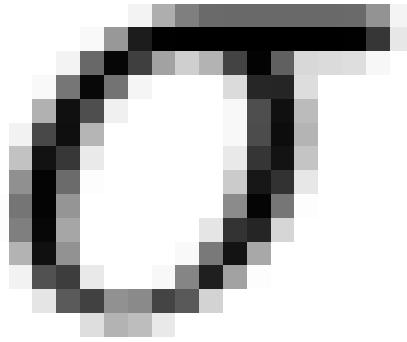


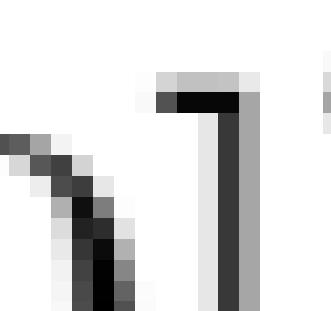
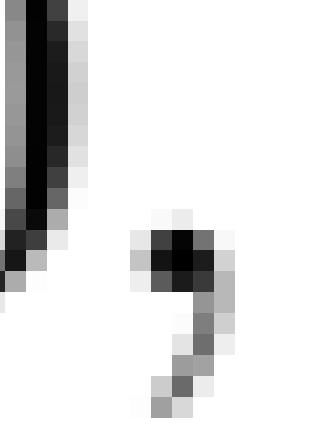
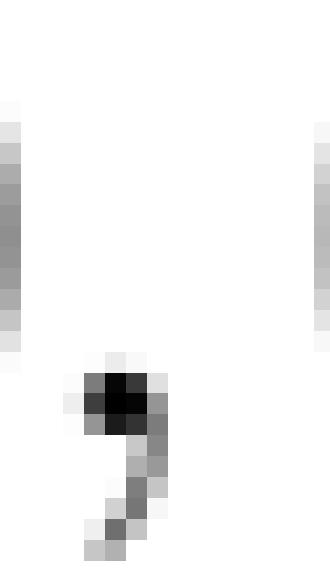
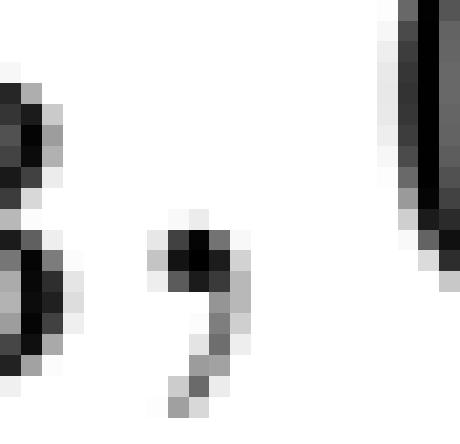
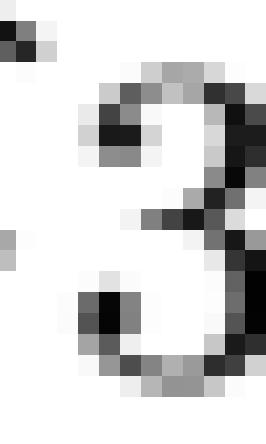












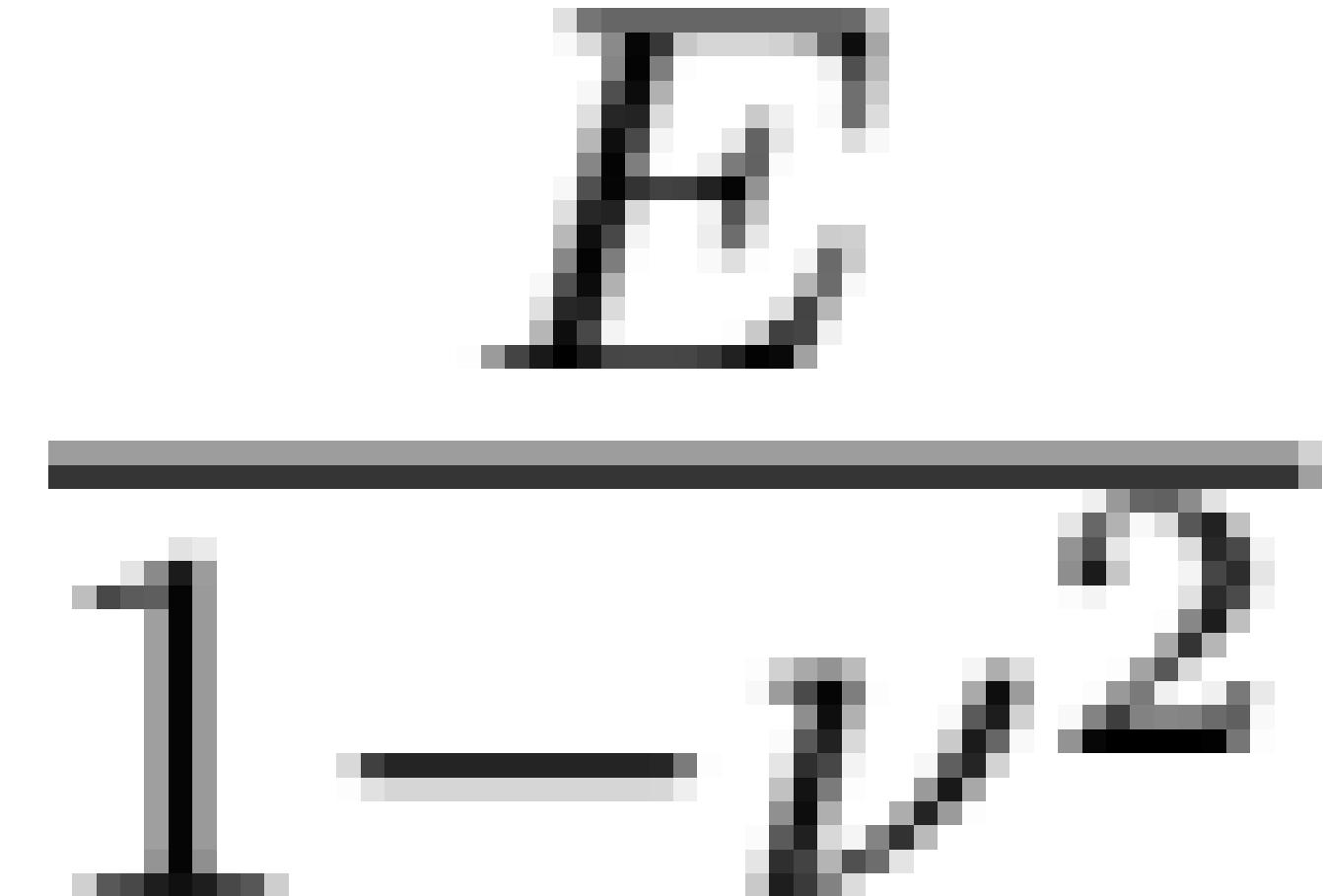
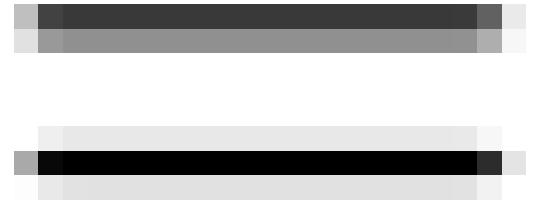
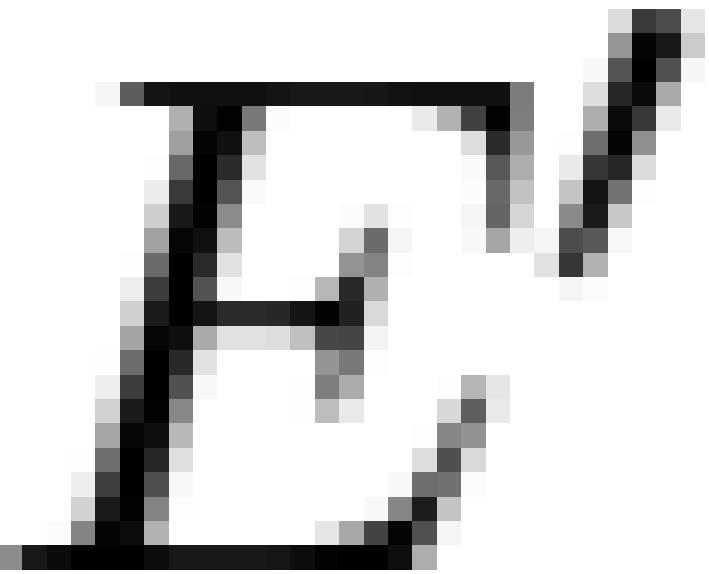
$$\left\{ \begin{array}{l} \sigma_{11} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{22} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \\ \sigma_{33} = \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{11} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{22} + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_{33} \end{array} \right.$$

$$\left\{ \begin{array}{l} \sigma_{11} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{E}{1-\nu^2} \epsilon_{11} + \frac{\nu E}{1-\nu^2} \epsilon_{22} \\ \sigma_{22} = \frac{\nu}{1-\nu} \sigma_{33} + \frac{\nu E}{1-\nu^2} \epsilon_{11} + \frac{E}{1-\nu^2} \epsilon_{22} \end{array} \right.$$

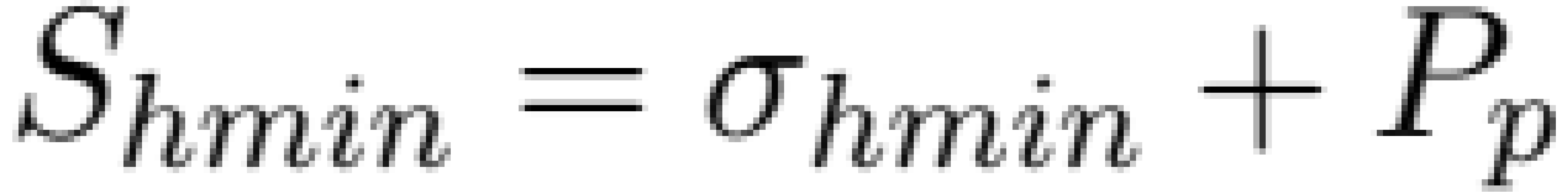




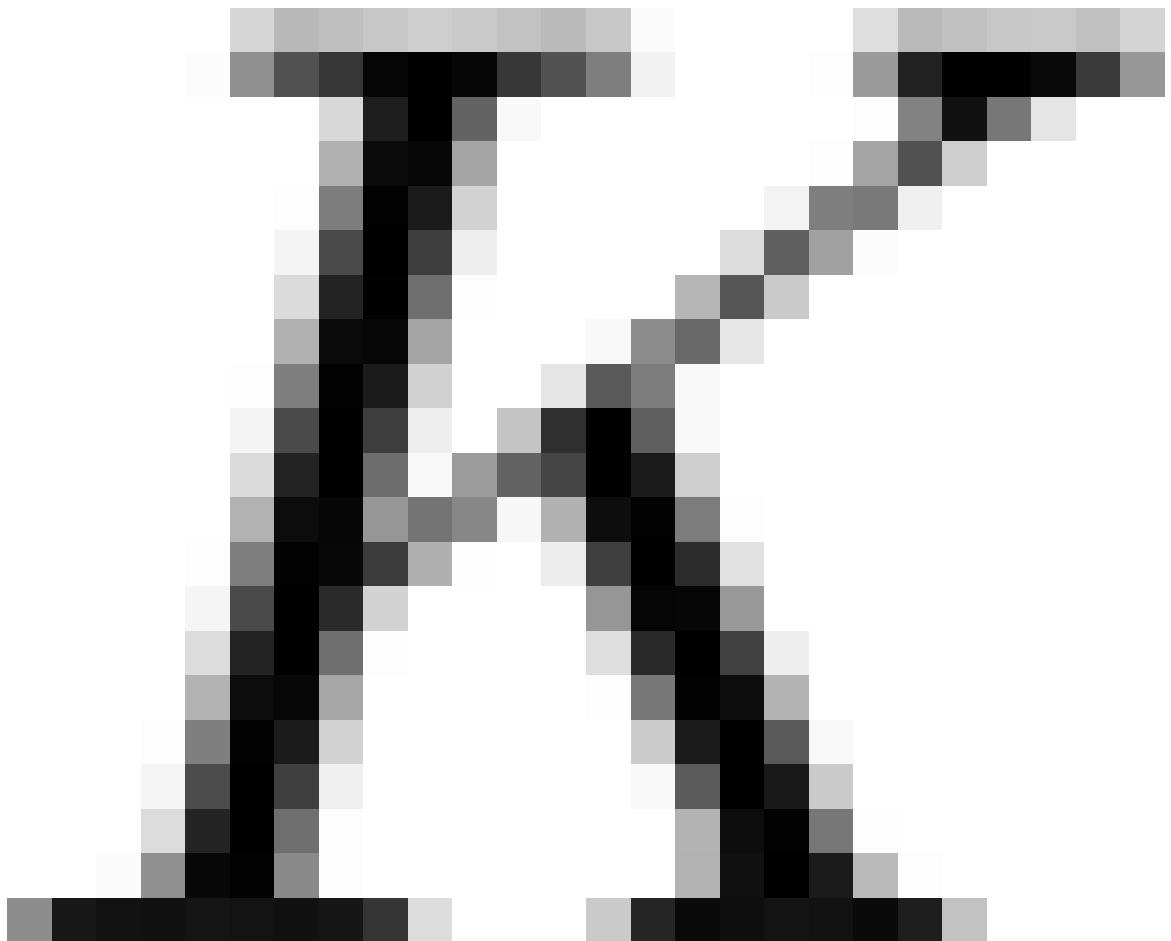
$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} + \nu E' \epsilon_{hmin} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + \nu E' \epsilon_{Hmax} + E' \epsilon_{hmin} \end{array} \right.$$

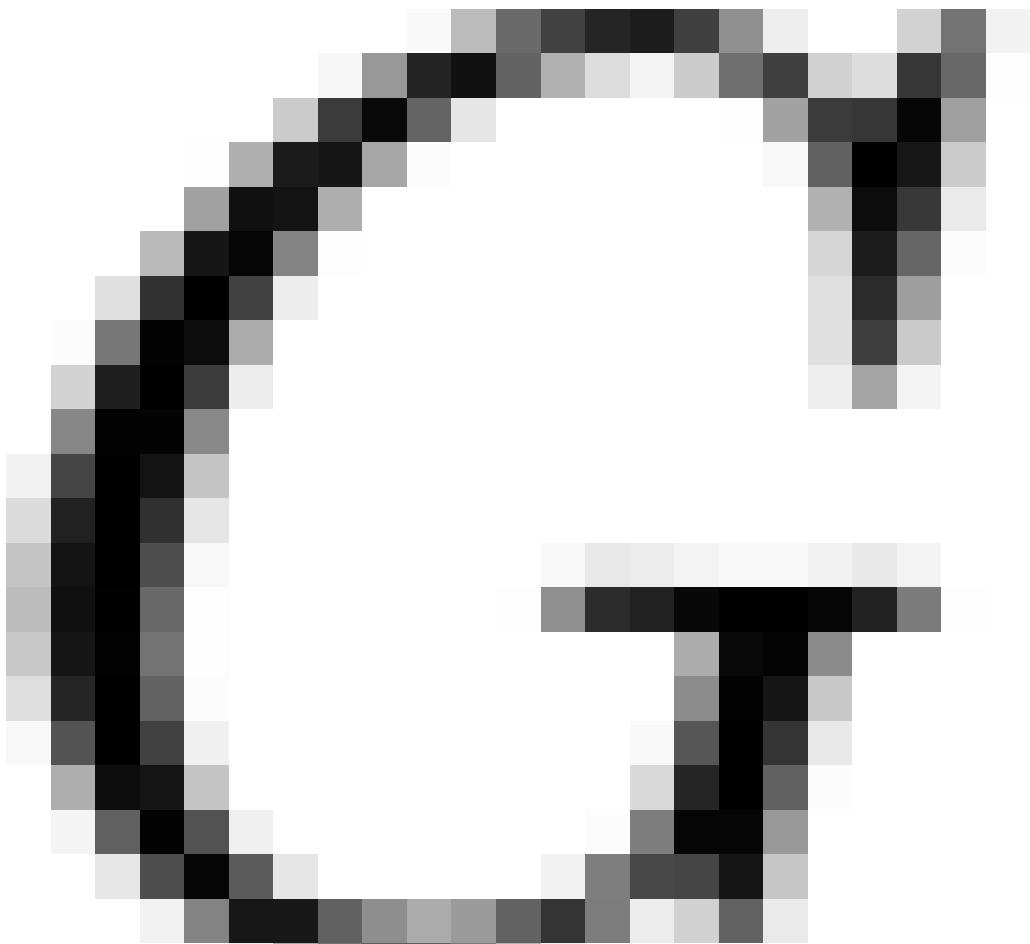




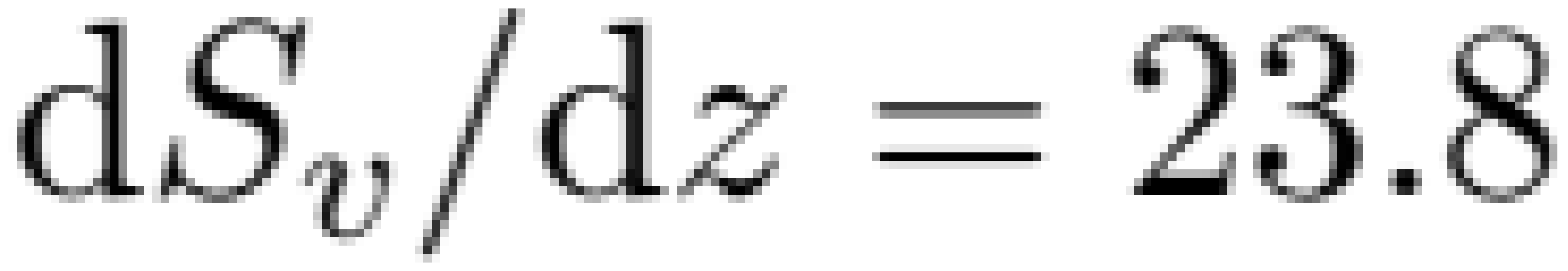


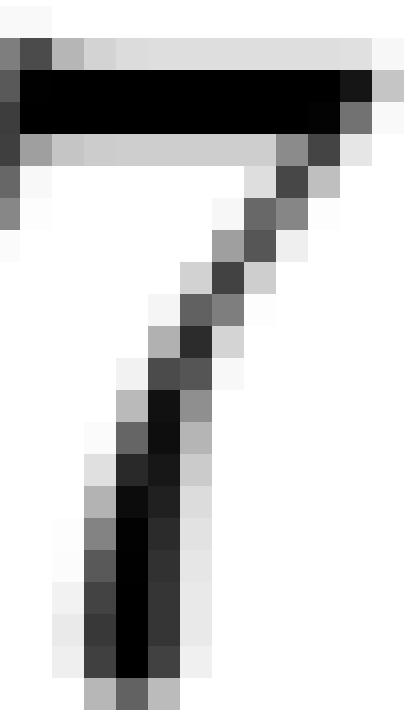
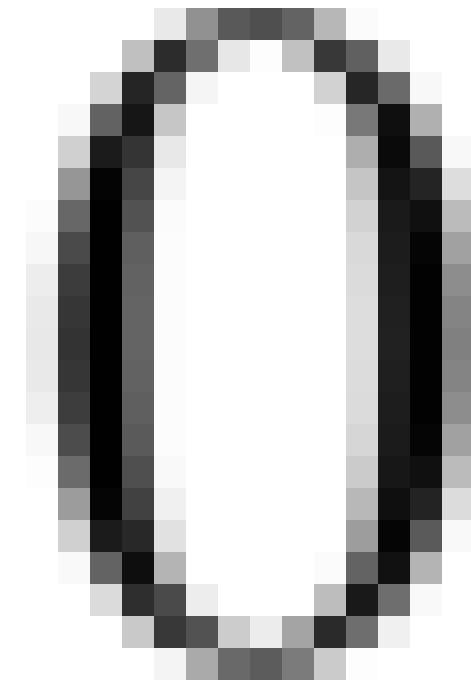


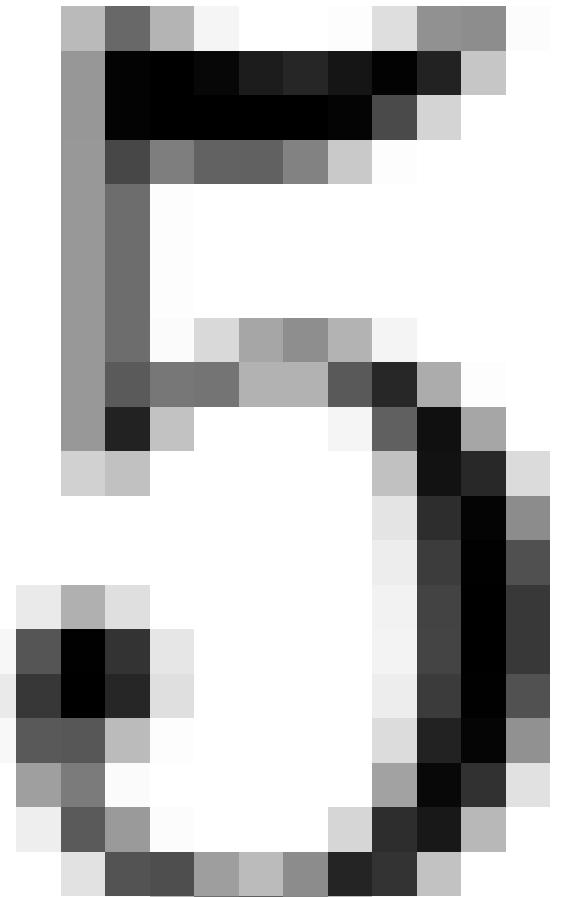
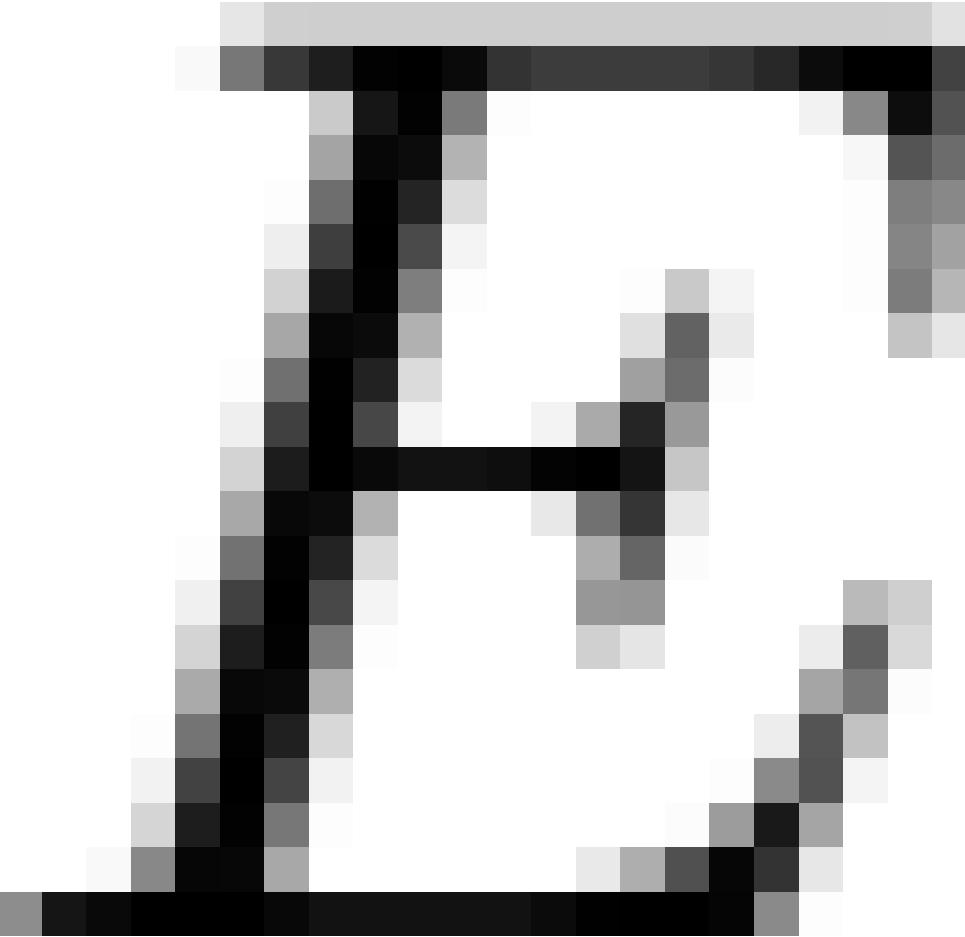


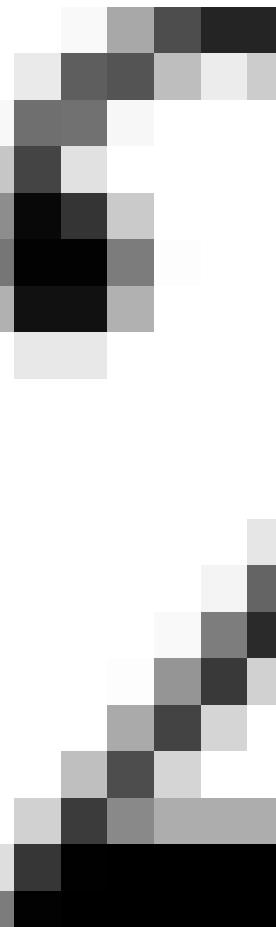
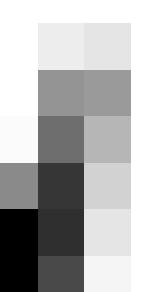
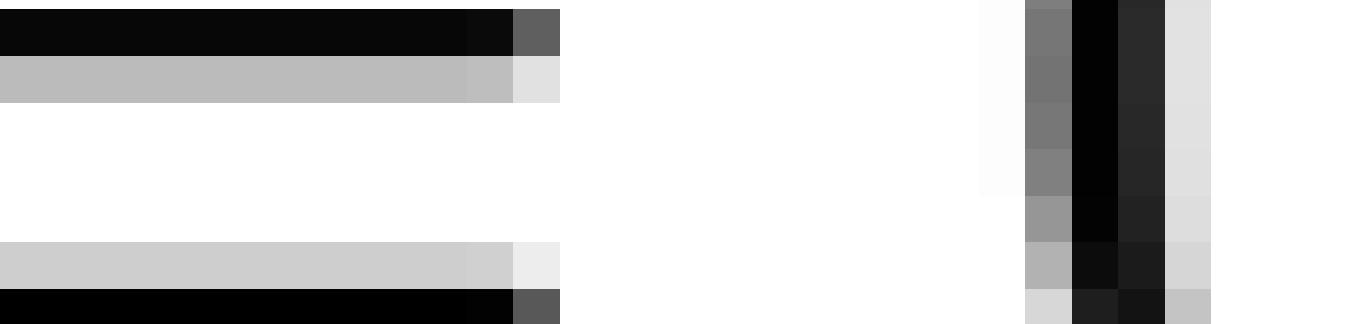
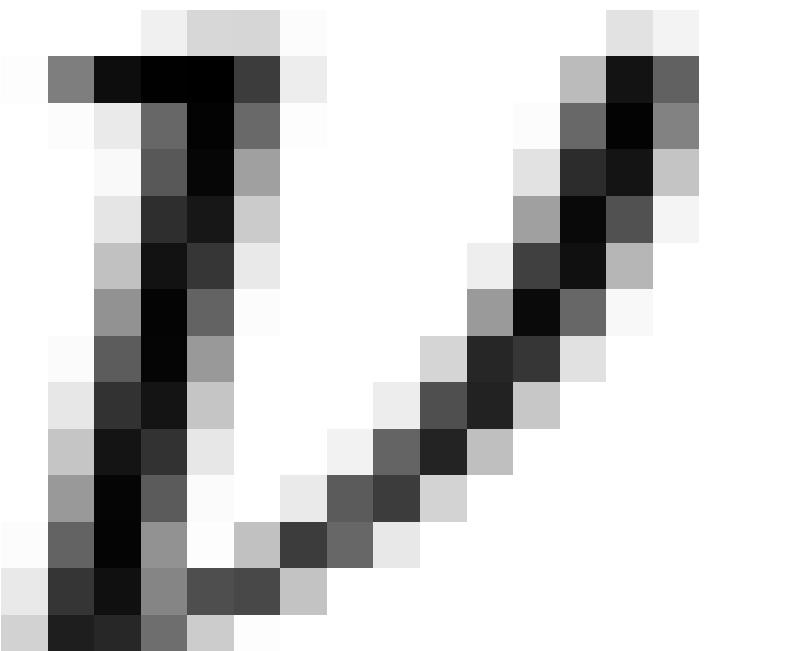


	(E, v)	(K, G)
$G =$	$\frac{E}{2(1+v)}$	<u>Shear modulus</u> (also noted as μ , S-wave)
$M =$	$\frac{(1-v)E}{(1+v)(1-2v)}$	<u>Constrained modulus</u> (uniaxial compaction, P-wave)
$\lambda =$	$\frac{vE}{(1+v)(1-2v)}$	<u>Lamé first parameter</u> (volumetric strain component)
$K =$	$\frac{E}{3(1-2v)}$	<u>Bulk modulus</u> (relates volumetric strain and isotropic stress)





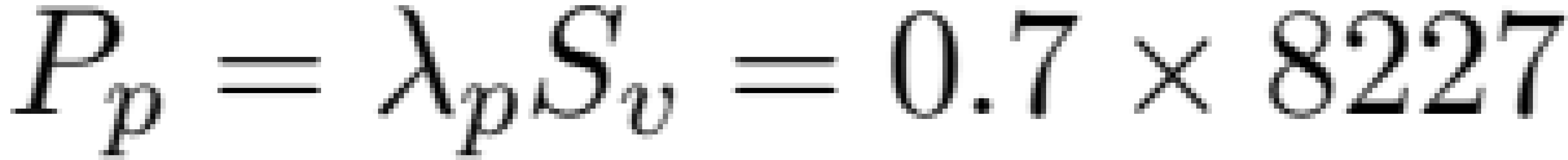




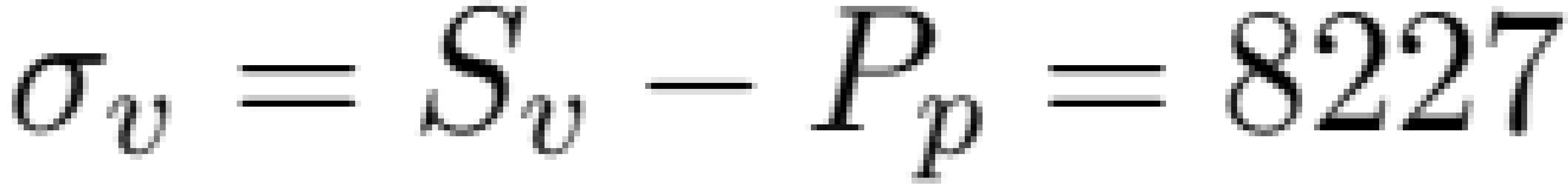




$$s_u = \frac{23.8 \text{ MPa}}{\text{km}} \times \frac{1 \frac{\text{psi}}{\text{ft}}}{\frac{\text{MPa}}{\text{km}}} \times 7950 \text{ ft} = 8227 \text{ psi}$$









$$\frac{E'}{1 - \nu^2} = \frac{E}{1 - 0.22^2} = \frac{5 \times 10^6 \text{ psi}}{5.25 \times 10^6 \text{ psi}}$$

$$\left\{ \begin{array}{l} \sigma_{Hmax} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{Hmax} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} \times 0.0002 = 1745 \text{ psi} \\ \sigma_{hmin} = \frac{\nu}{1-\nu} \sigma_v + E' \epsilon_{hmin} = \frac{0.22}{1-0.22} 2468 \text{ psi} + 5.25 \times 10^6 \text{ psi} \times 0.0002 = 927 \text{ psi} \end{array} \right.$$

$$S_{H\max} = \sigma_{H\max} + P_p = 1745 \text{ psi} + 5759 \text{ psi} = 7504 \text{ psi}$$

$$S_{h\min} = \sigma_{h\min} + P_p = 927 \text{ psi} + 5759 \text{ psi} = 6686 \text{ psi}$$

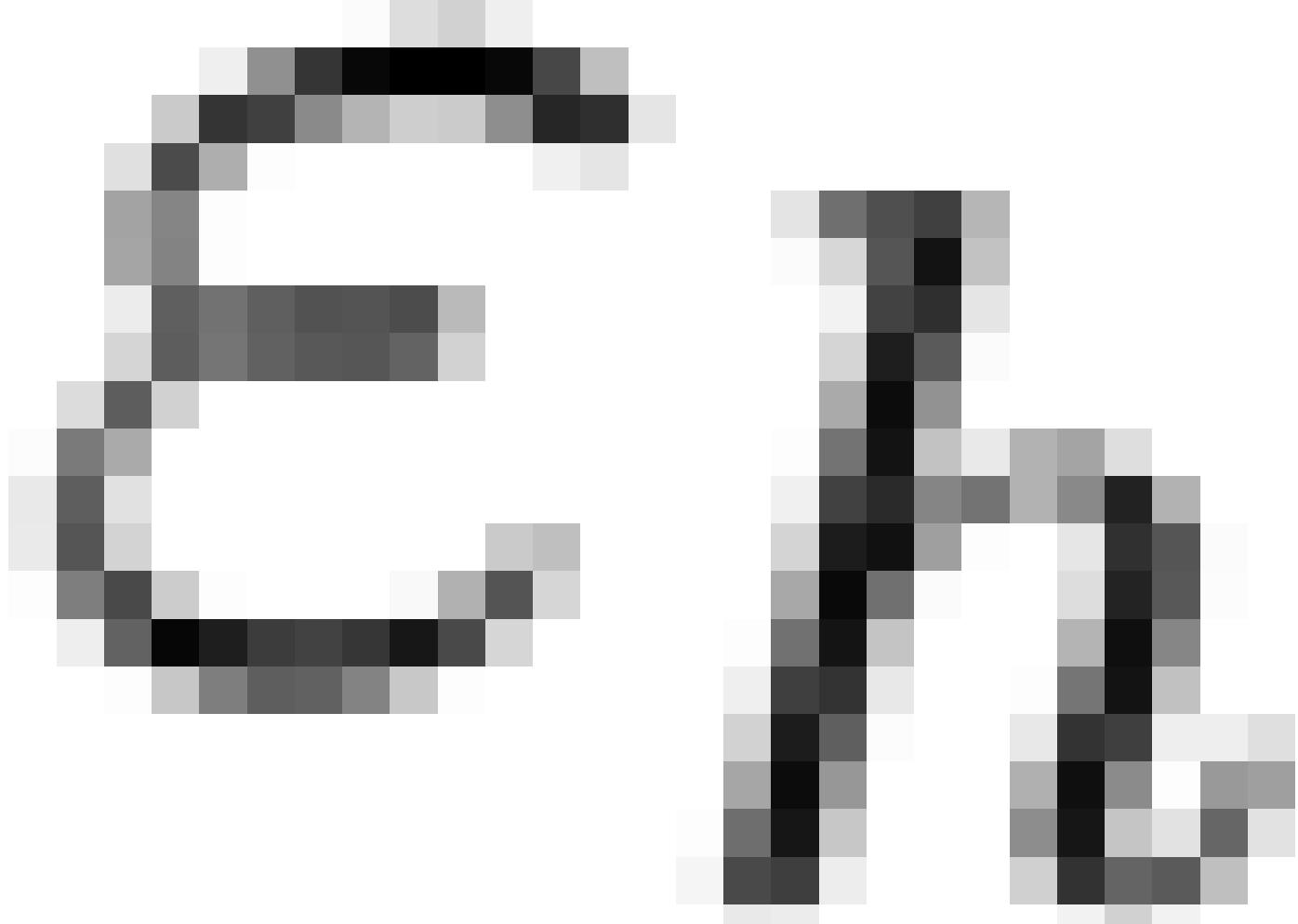




$$C_{pp} =$$

$$\frac{1}{V_p} \frac{dV_p}{dP_p}$$

S_u, ϵ_h



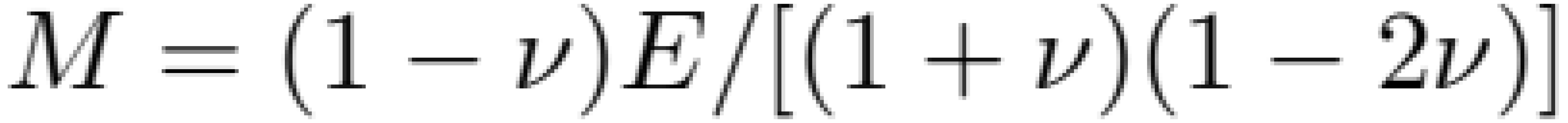
Op

$=$

Op

ϕ





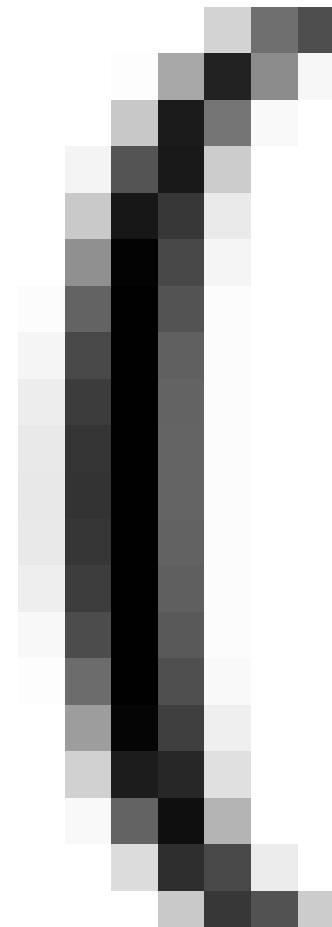
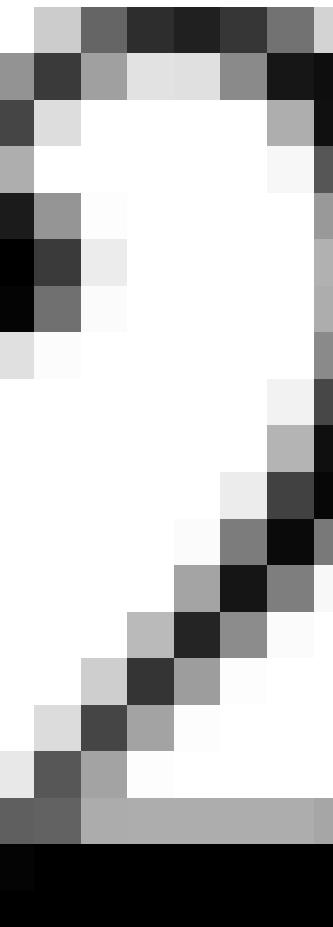
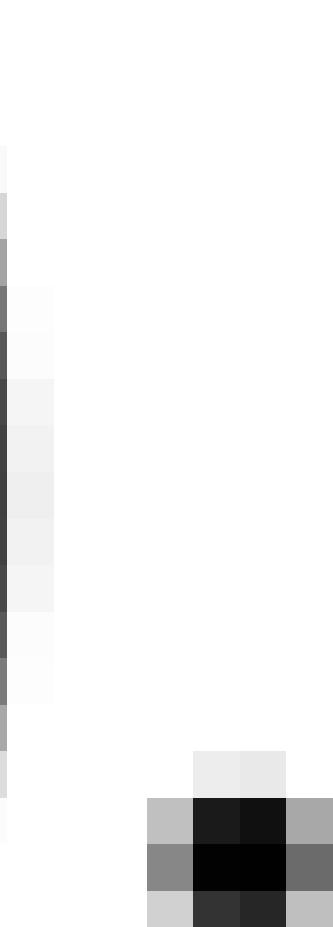
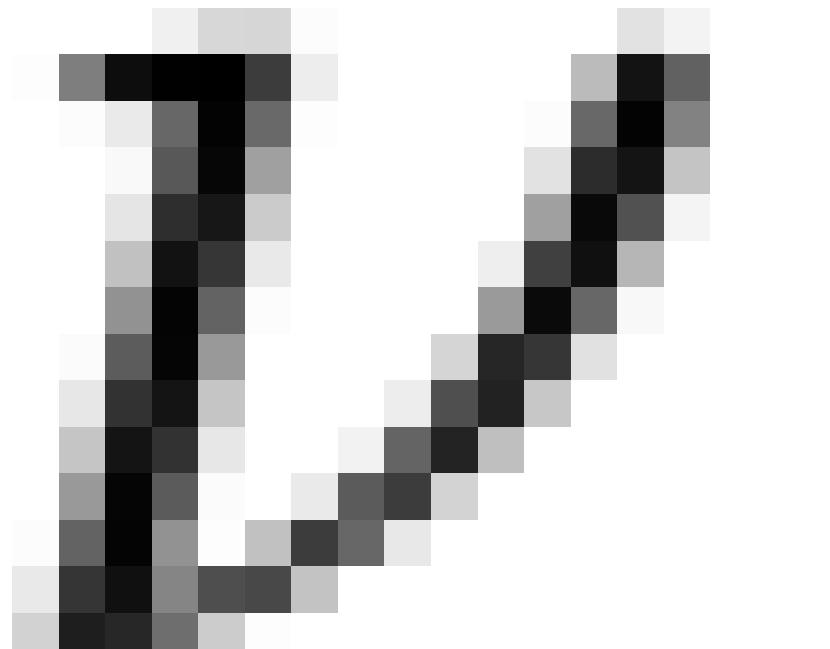
$\langle pp \rangle$

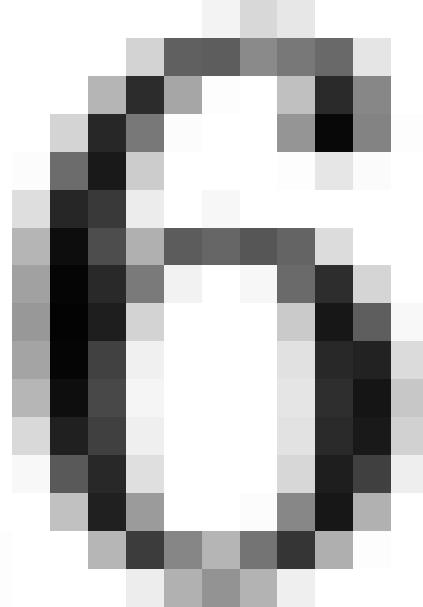
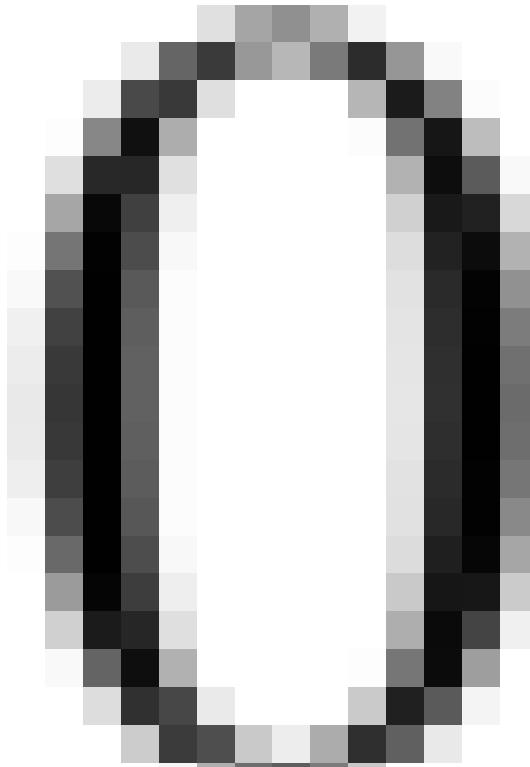
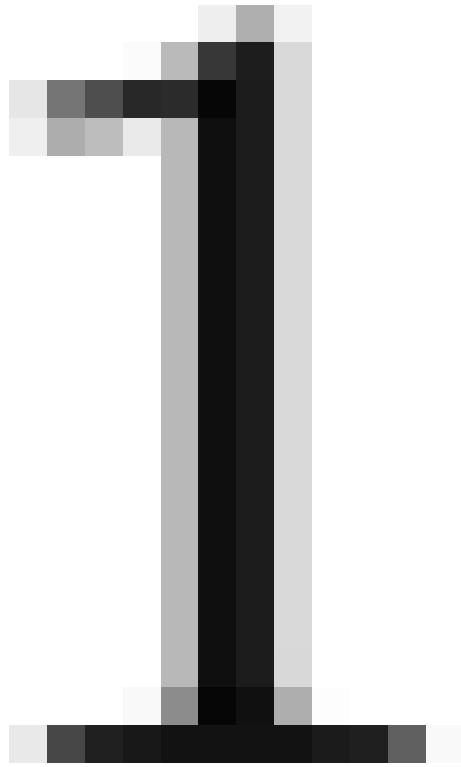
$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\frac{(1 + \nu)(1 - \nu)}{2\nu} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\langle E\phi \rangle$

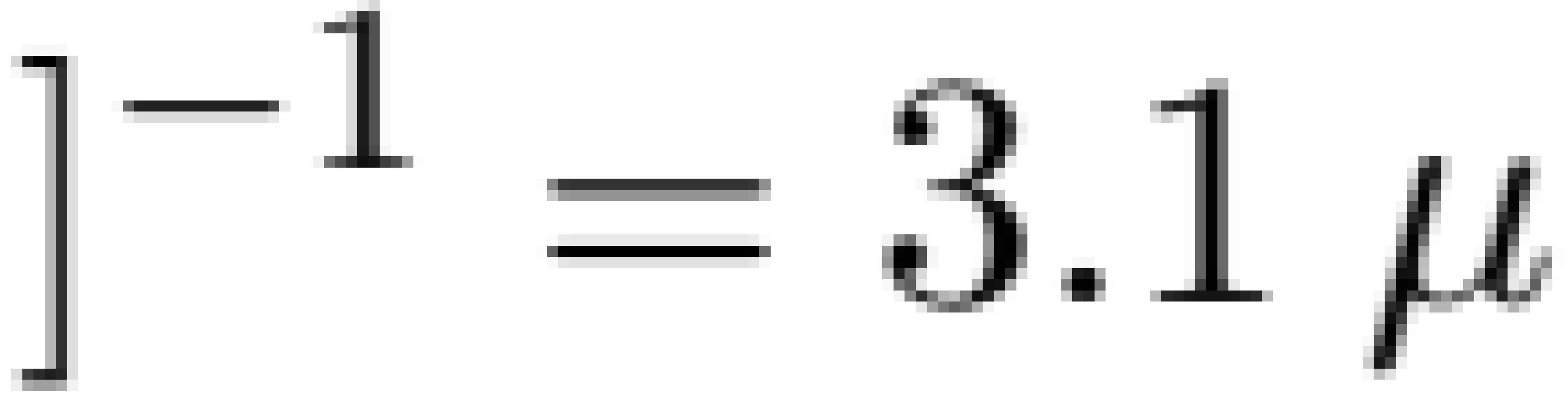


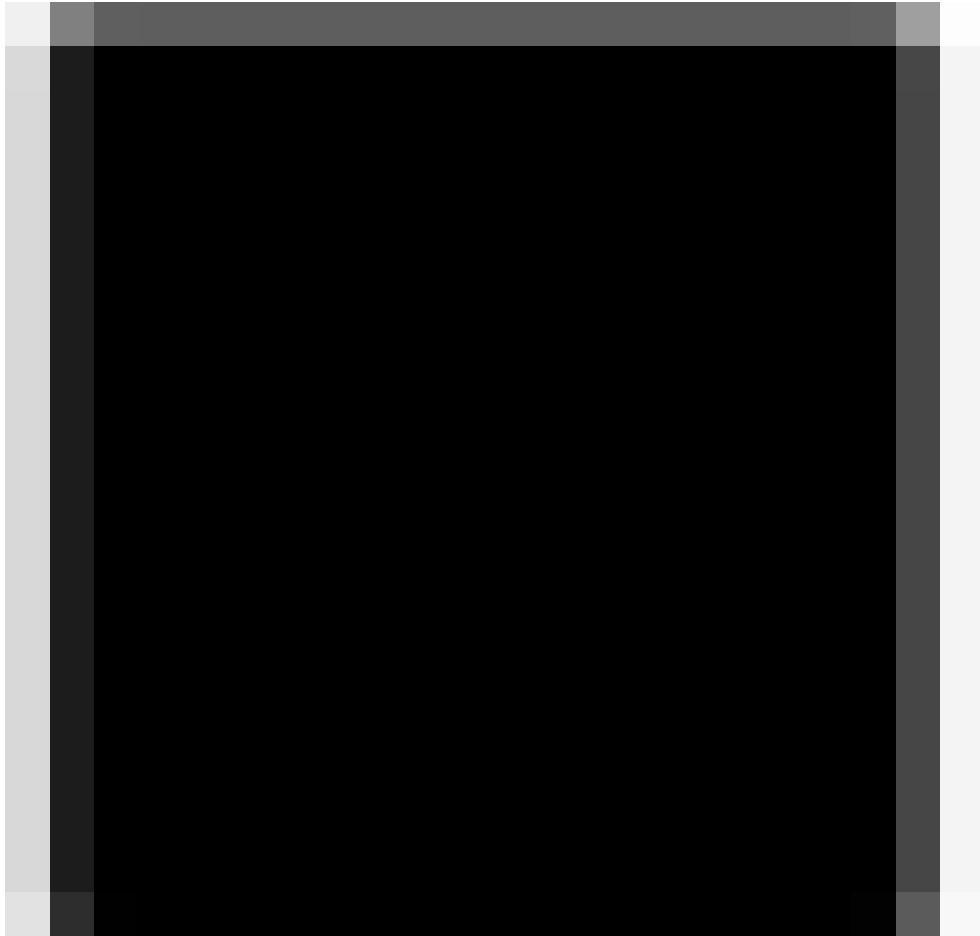


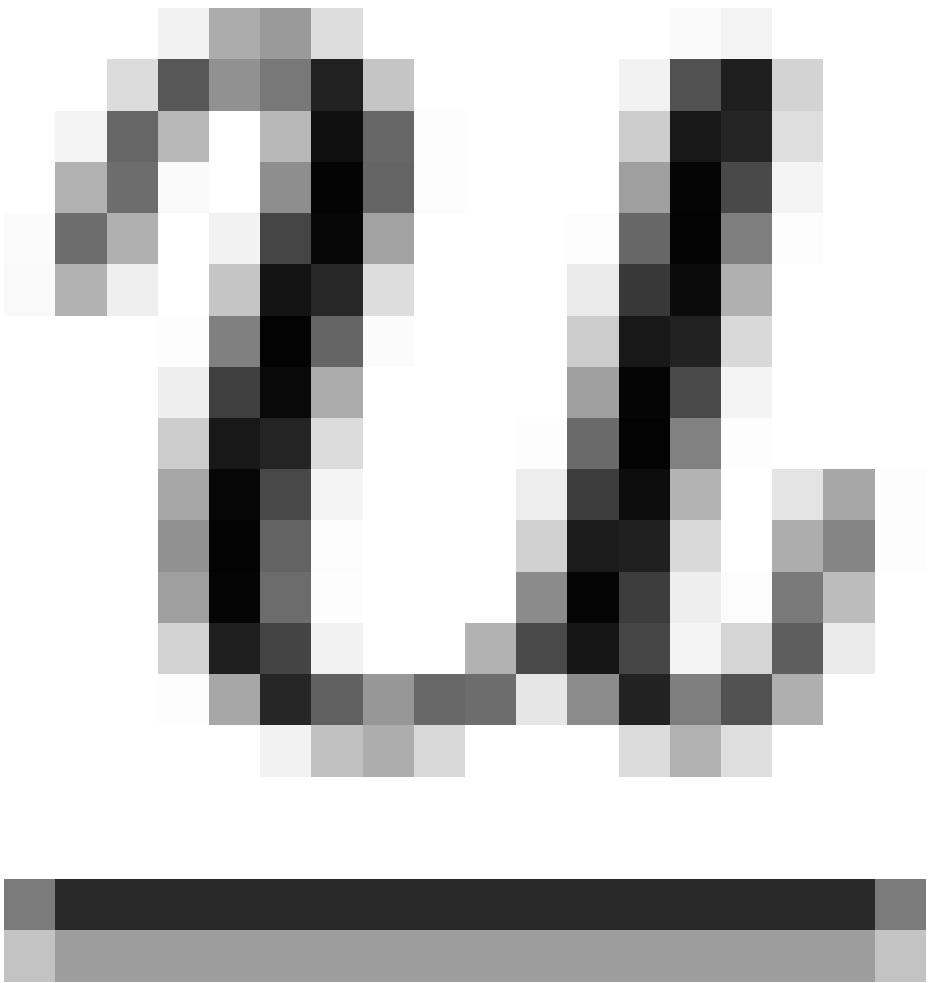


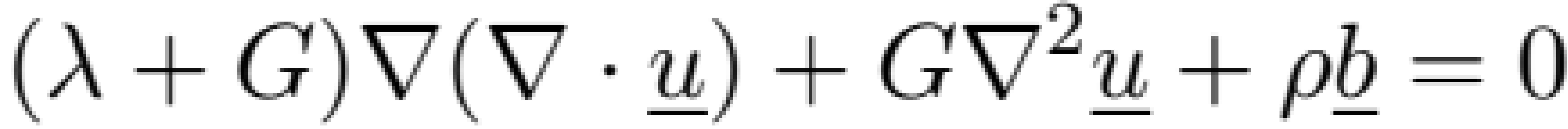
$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.20)10 \text{ GPa}}{1.6 \times 10^6 \text{ psi}} = \frac{11.11 \text{ GPa}}{(1+0.20)(1-2 \times 0.20)}$$

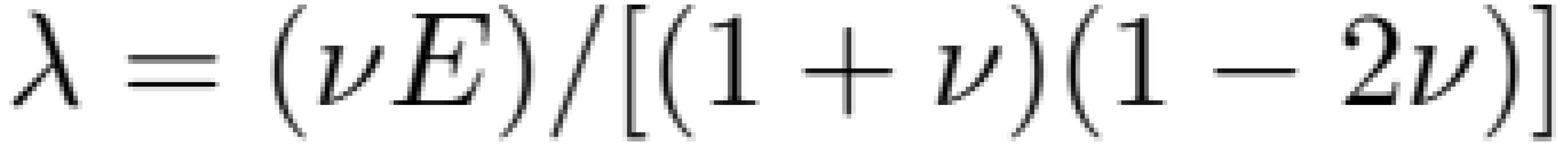
$$\frac{C_{pp}}{M_\phi} = \frac{1}{3.1 \times 10^6 \text{ psi} \times 0.20} = 1$$

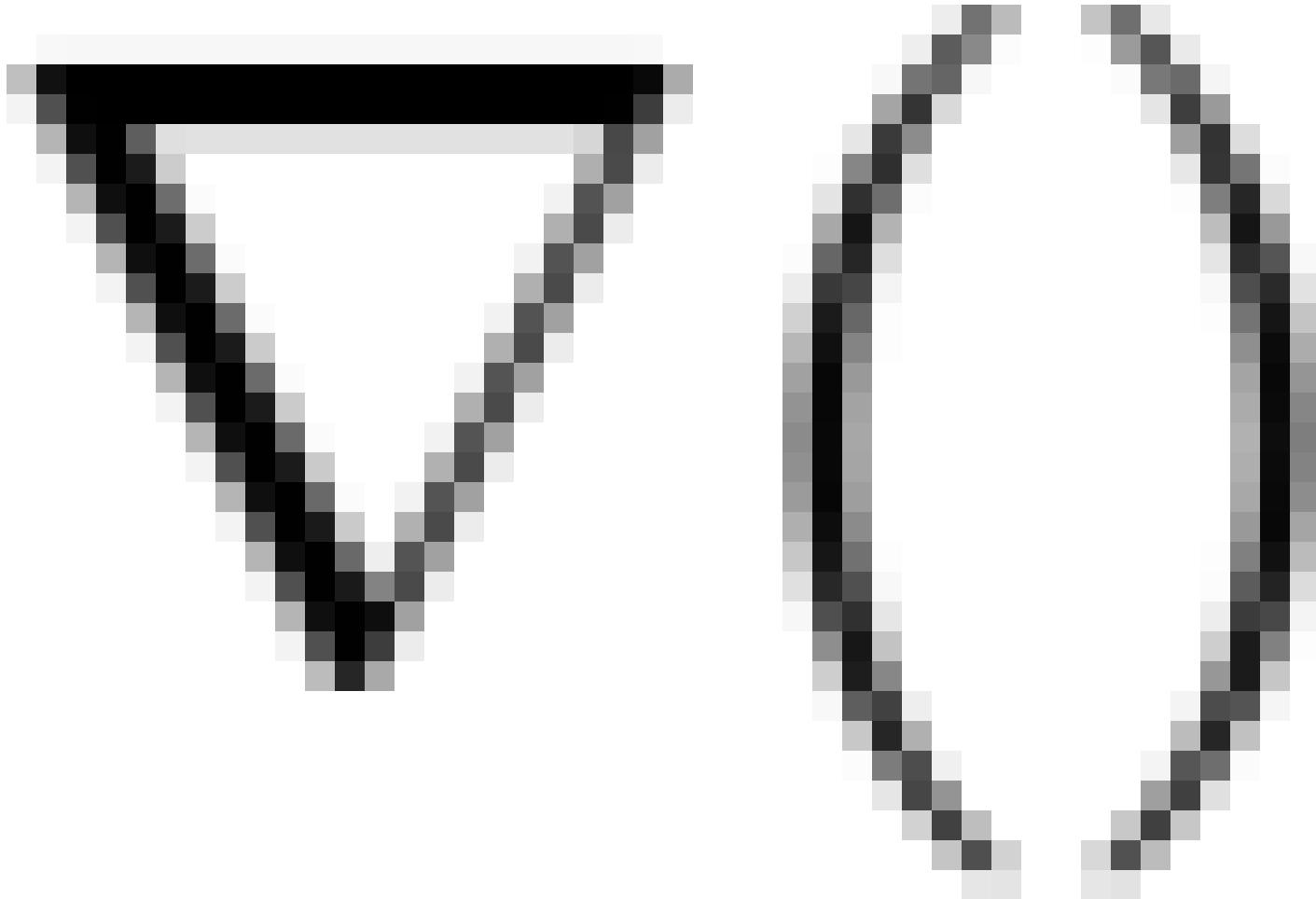


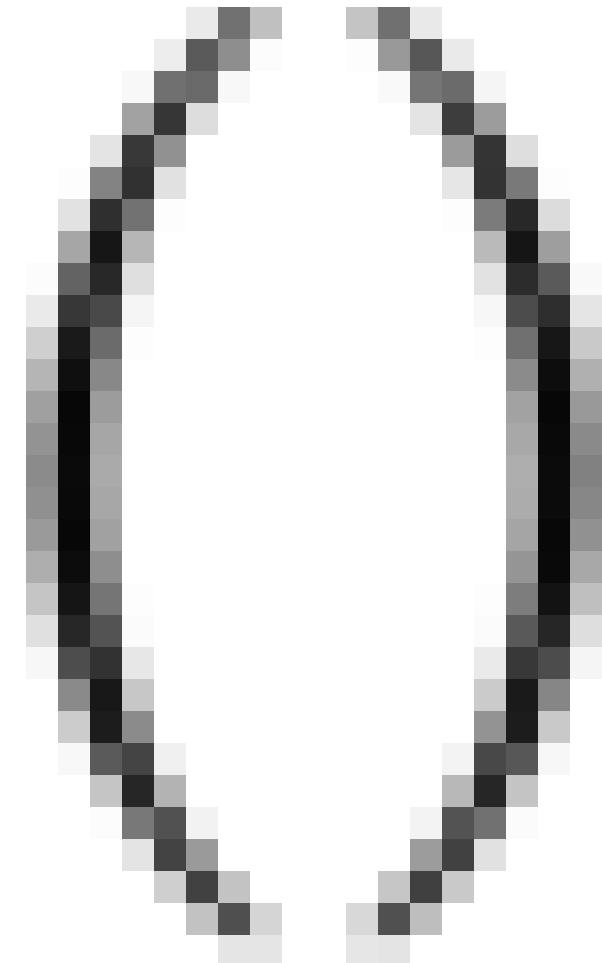
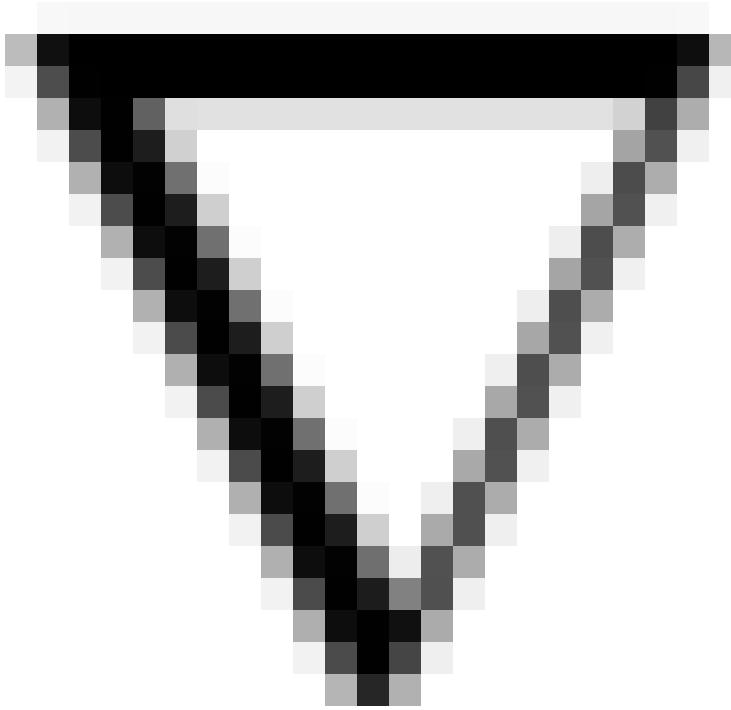


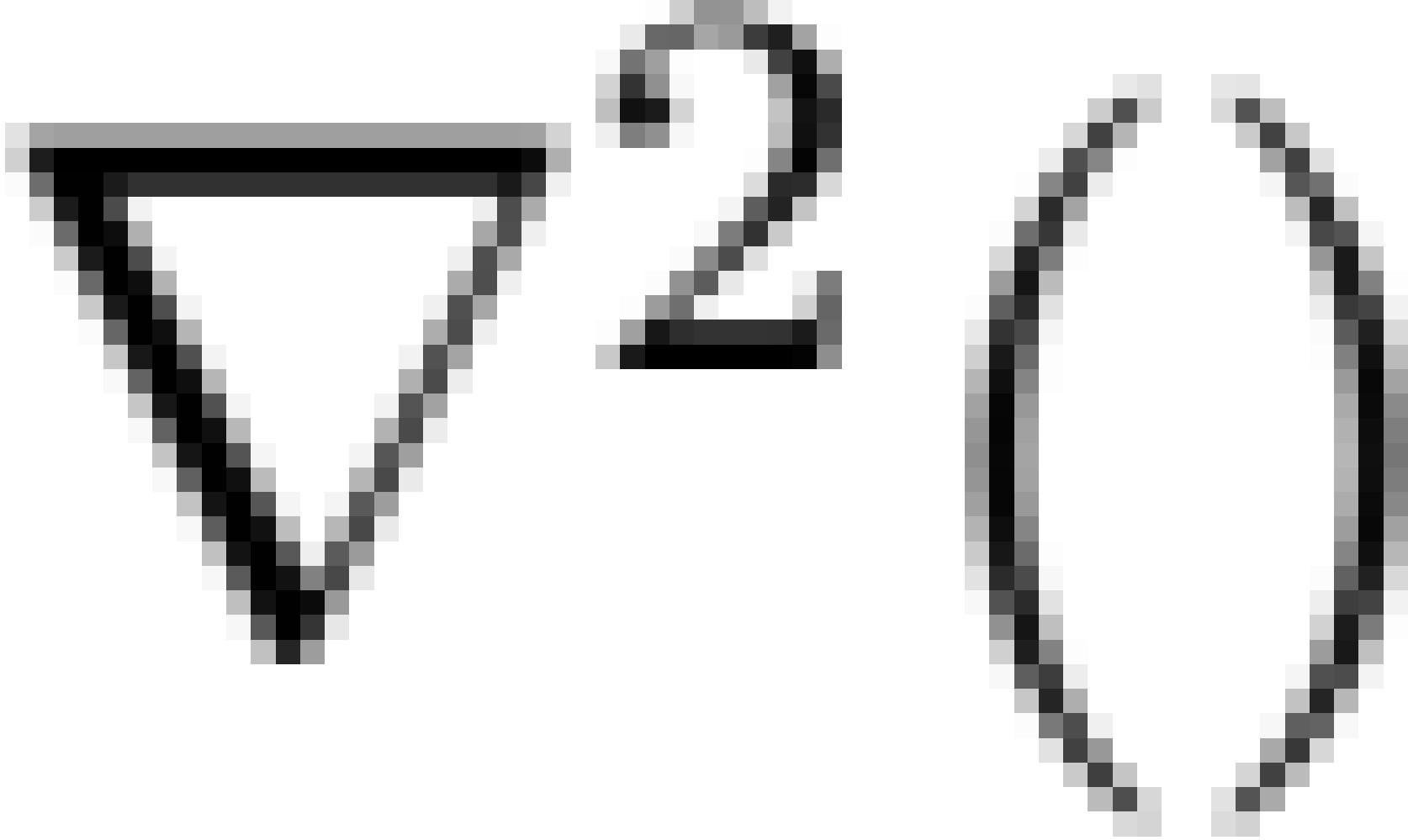




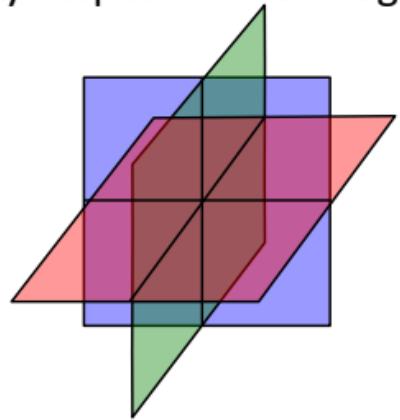








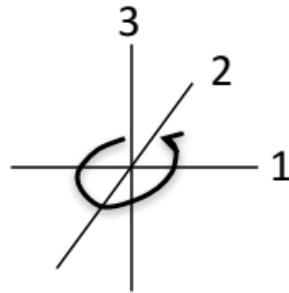
Orthorhombic symmetry
(symmetry respect to 3 orthogonal planes)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & 0 \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{55} & 0 \\ & 0 & 0 & 0 & & C_{66} \end{bmatrix}$$

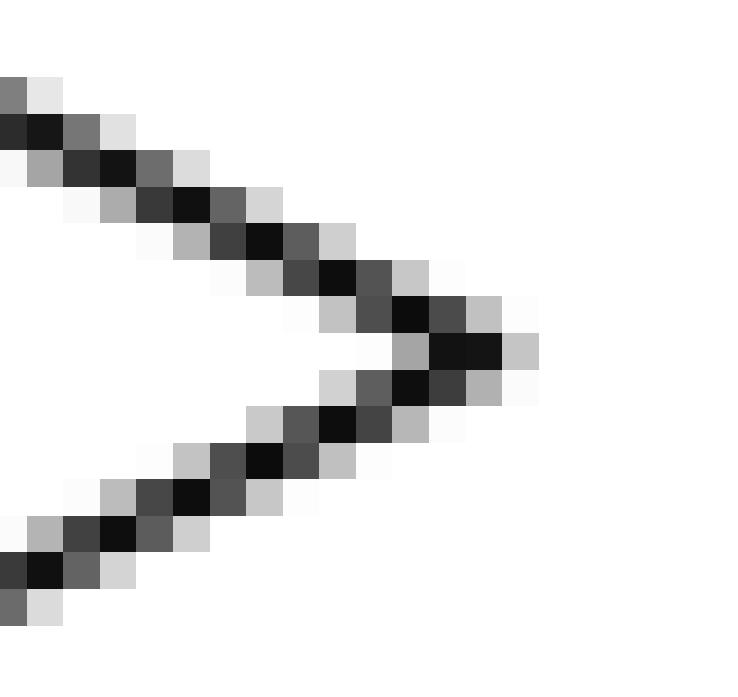
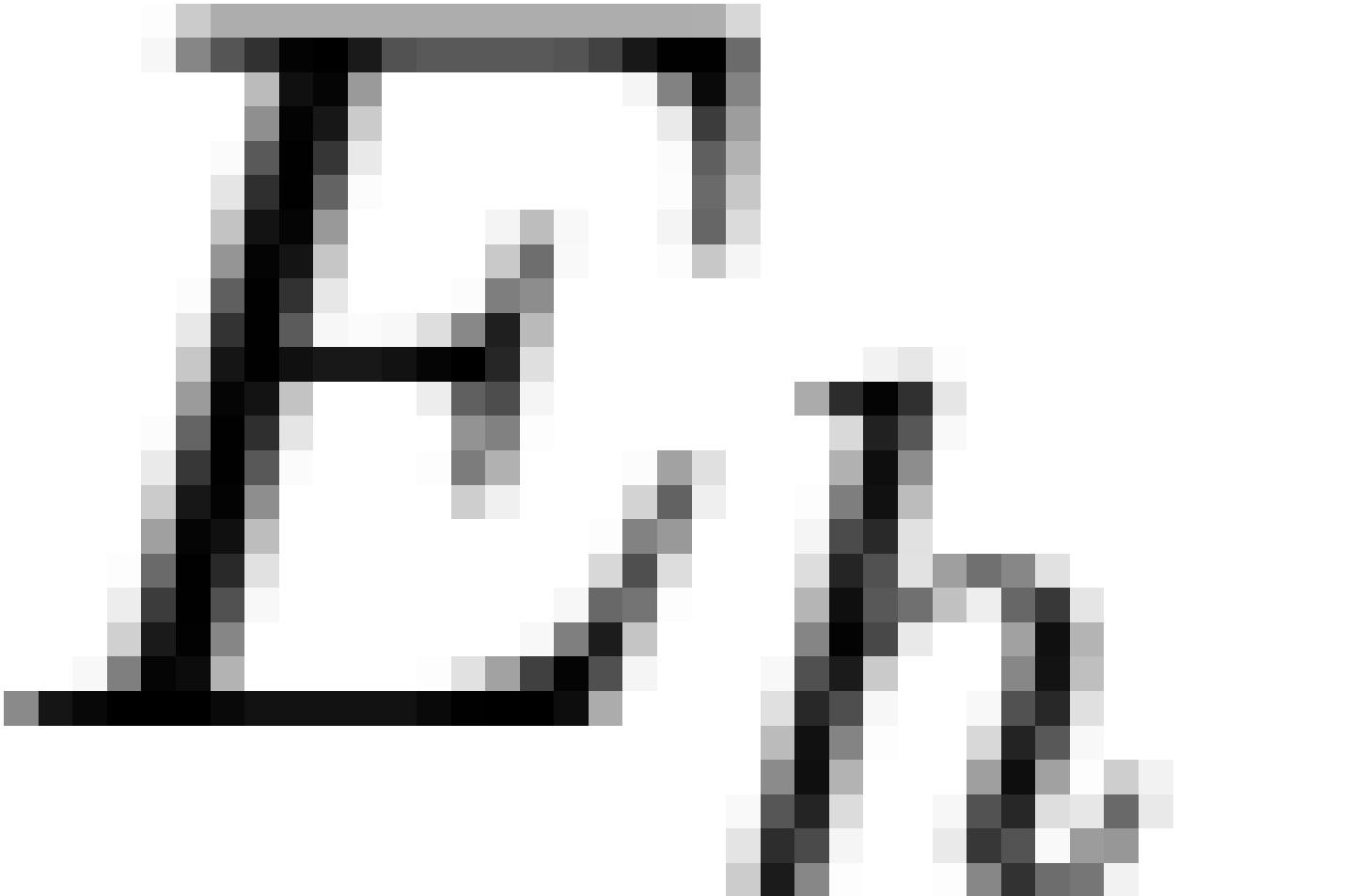
9 independent parameters

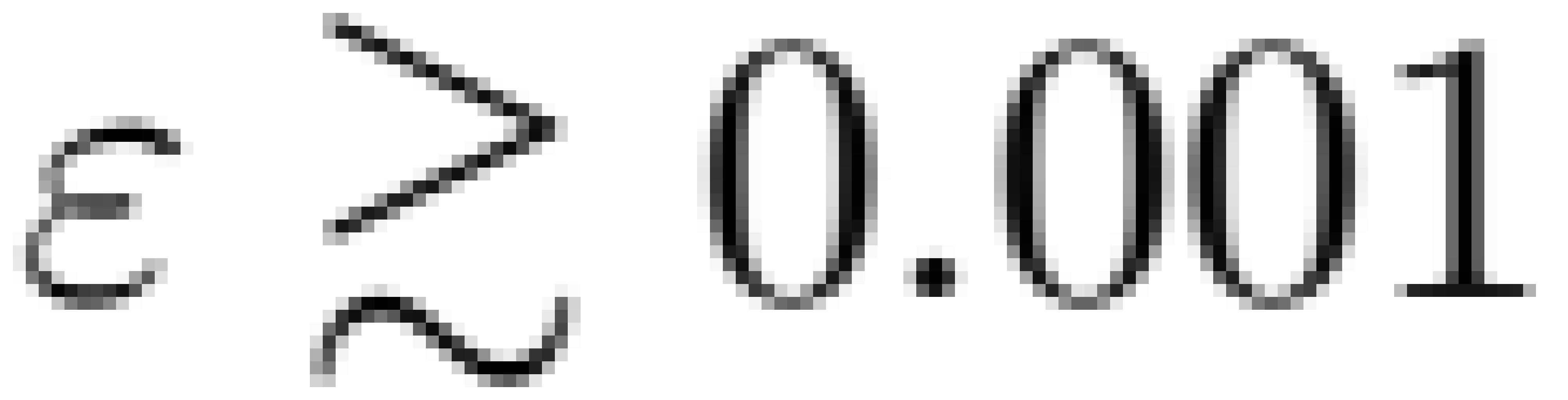
Transverse Isotropy
(symmetry respect to 1 axis)



$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & & 0 \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & 0 & & 0 & C_{44} & 0 \\ & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

5 independent parameters ($C_{12}=C_{11}-2C_{66}$)

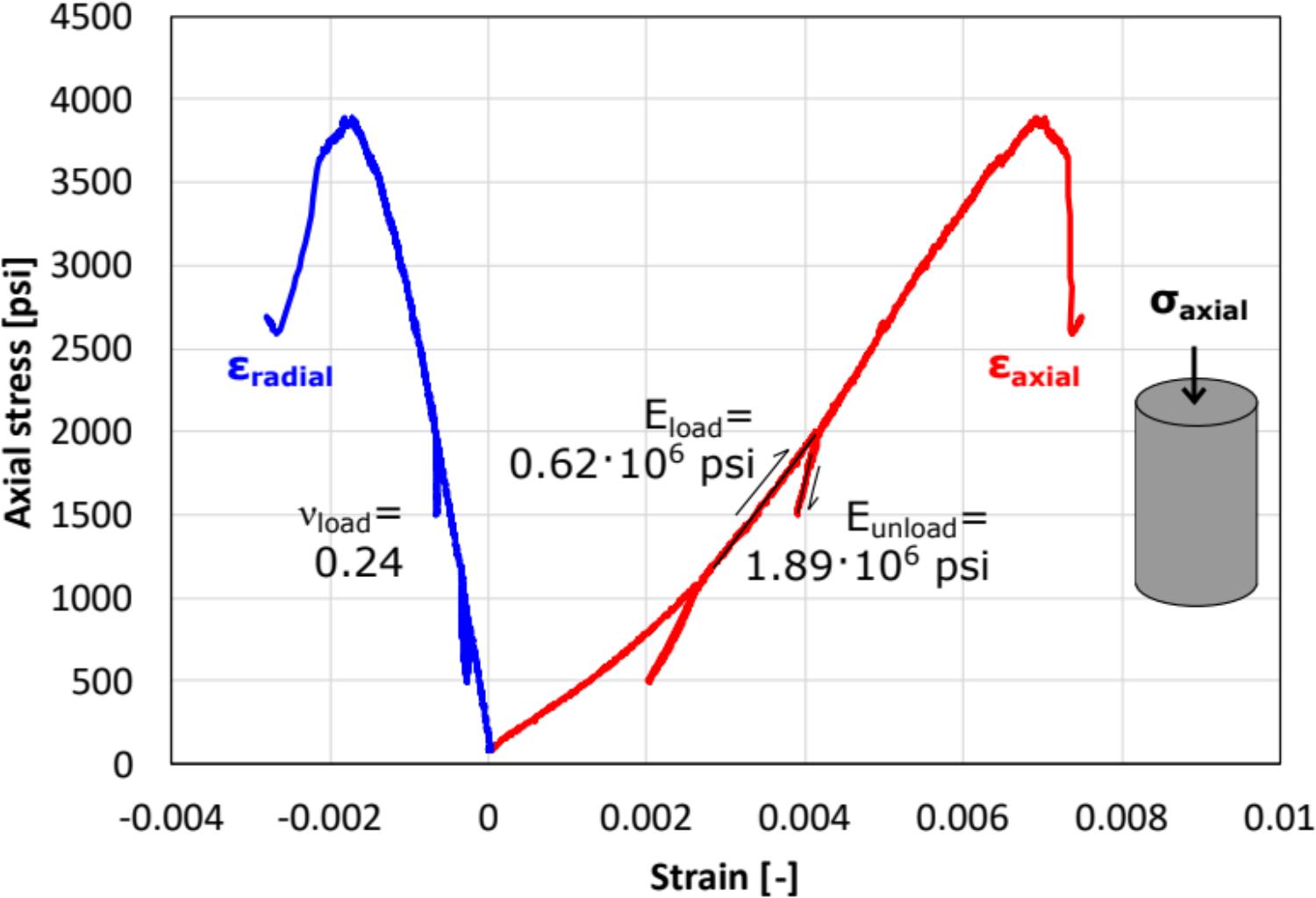




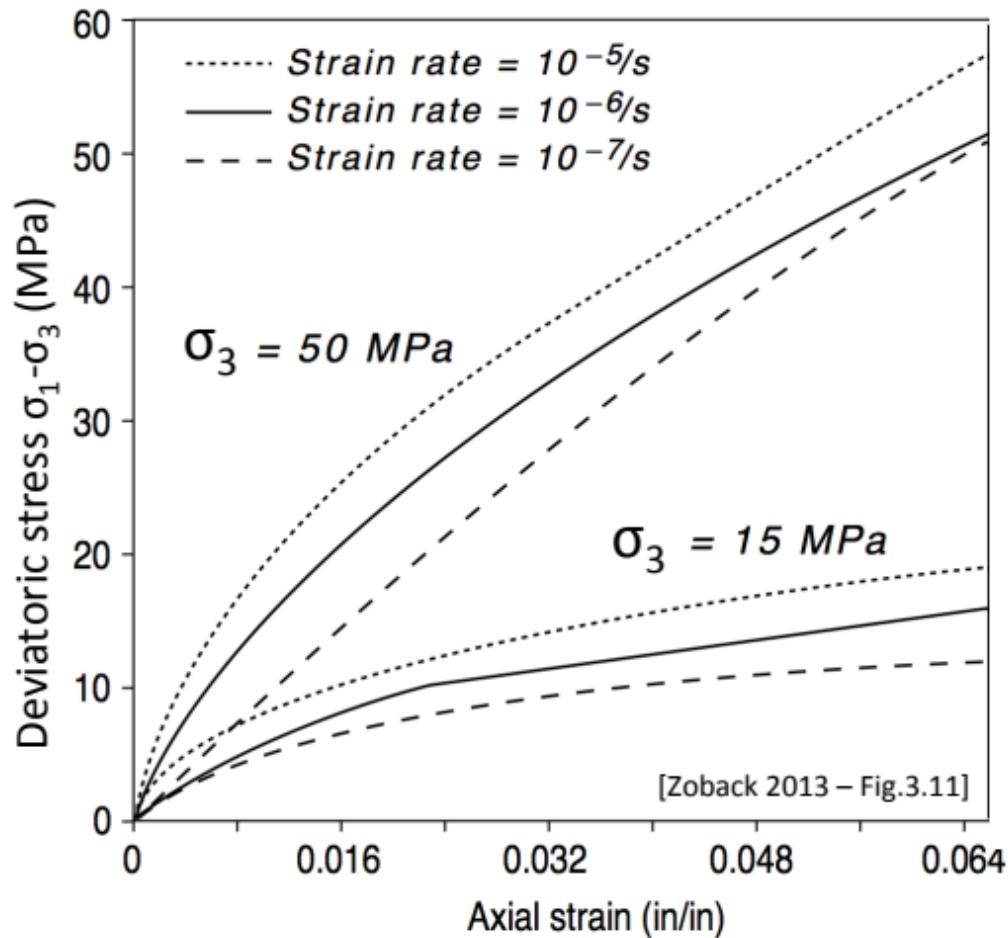
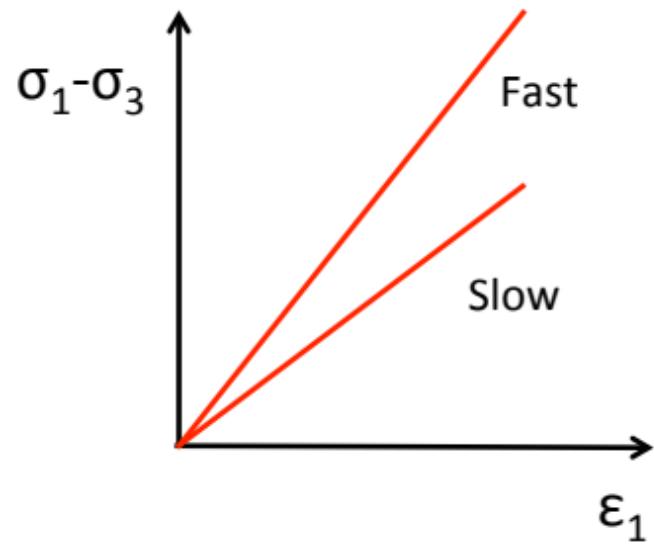




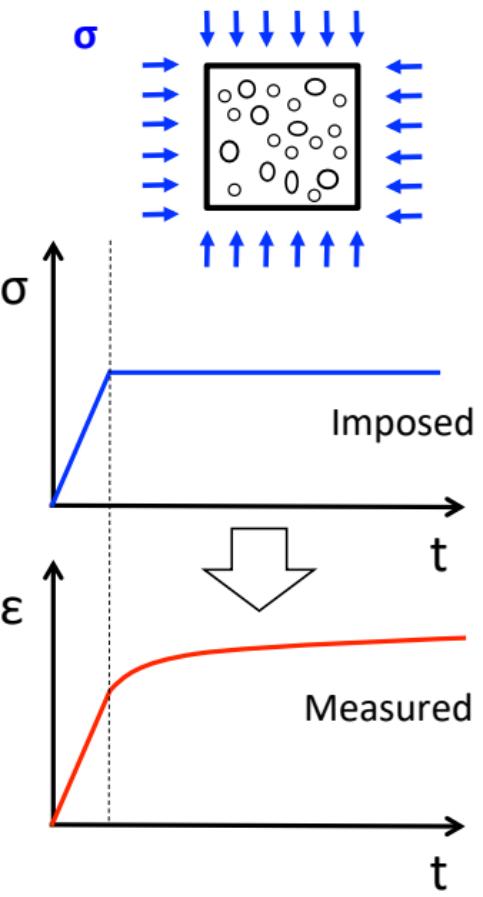




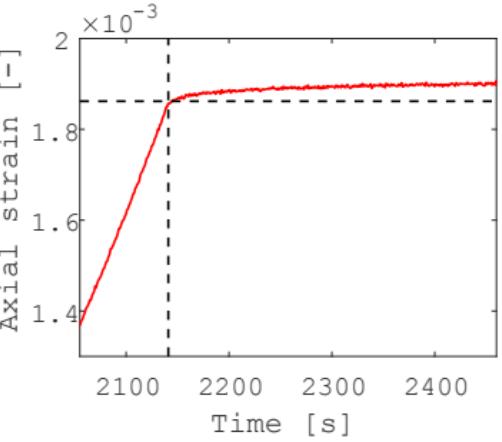
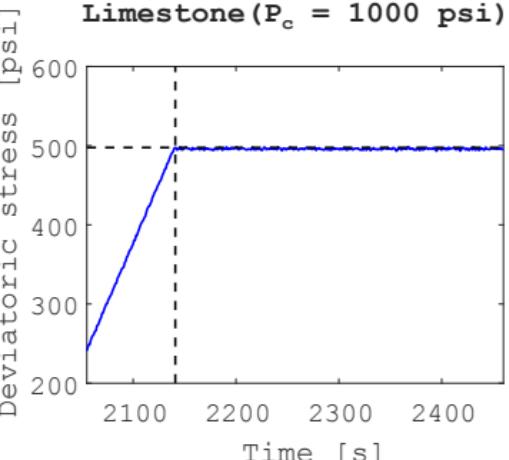
Strain rate hardening: The faster the loading, the stiffer the material



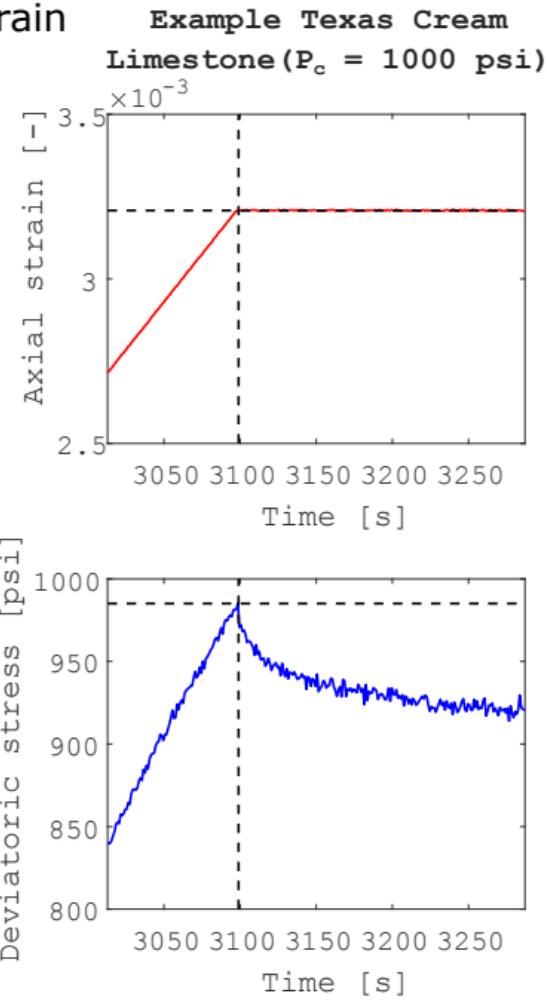
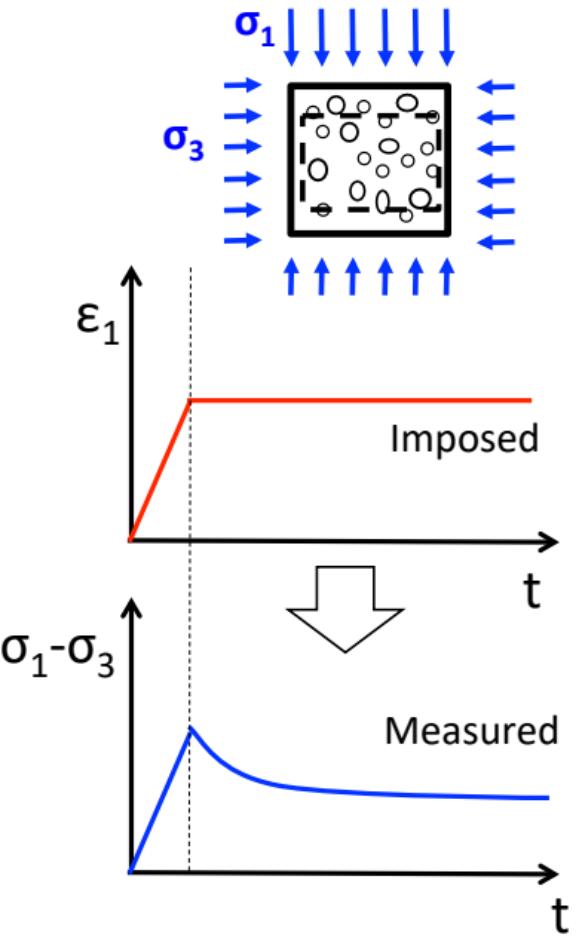
Creep strain at constant stress

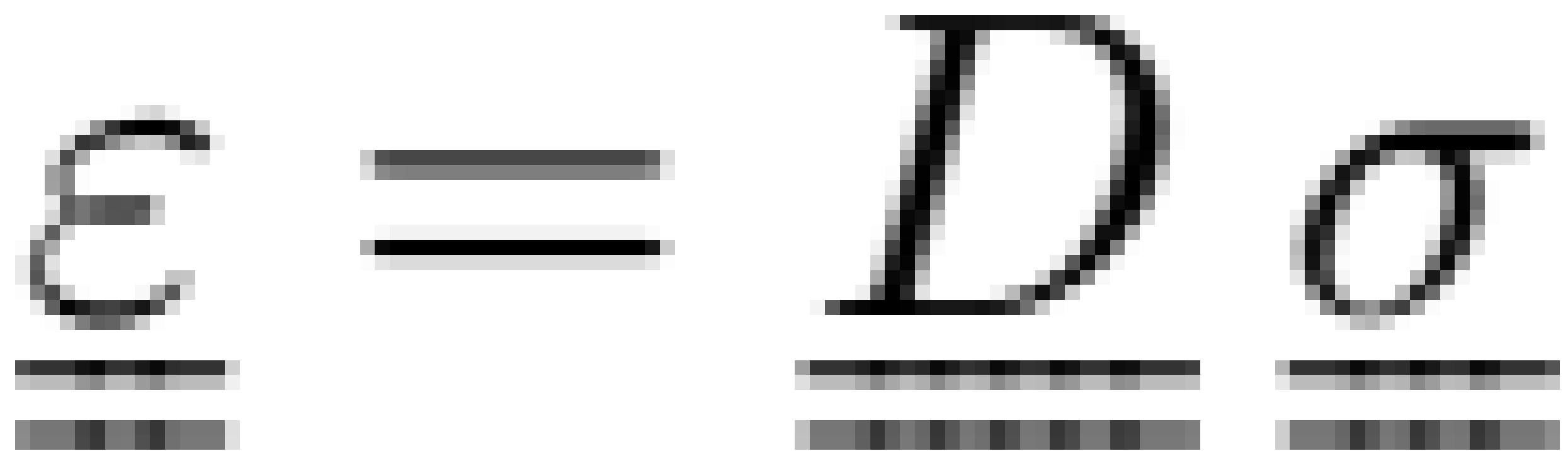


Example Texas Cream Limestone ($P_c = 1000$ psi)

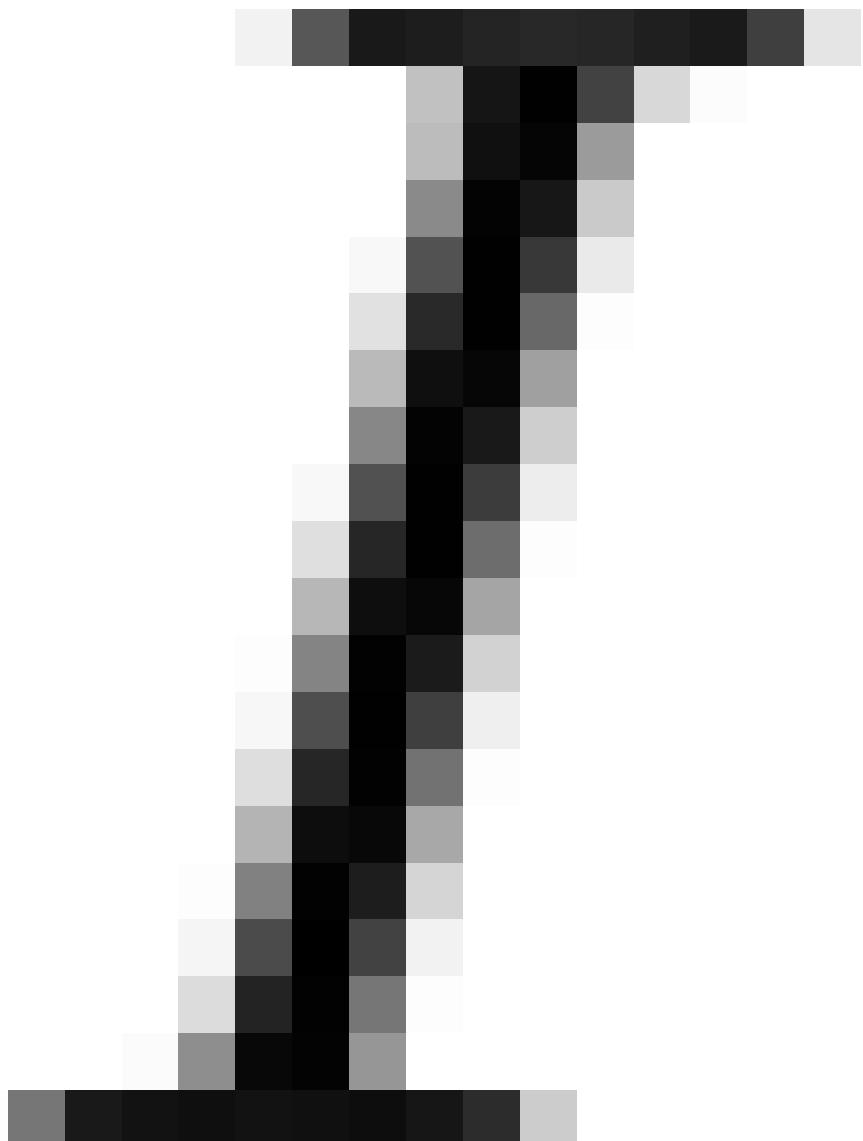


Stress relaxation at constant strain









α

$=$

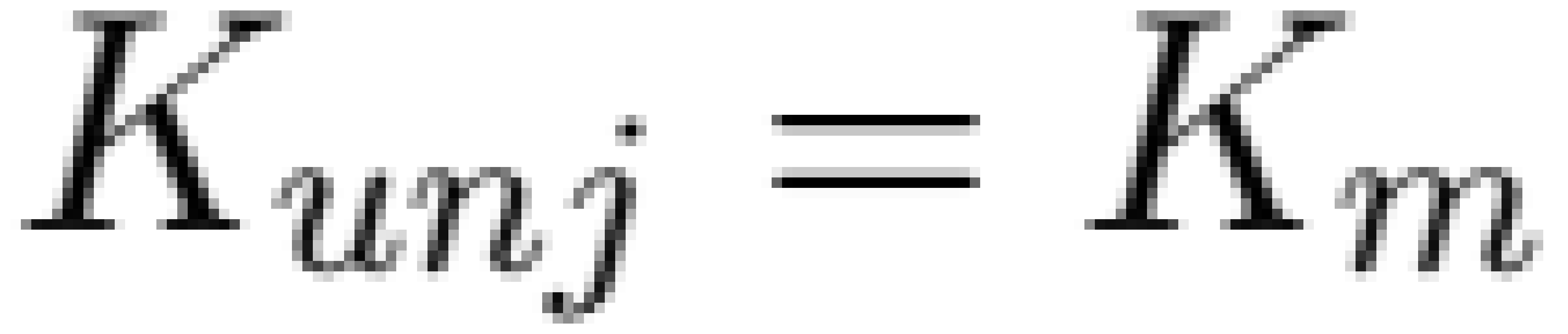
$1 - \frac{K_{drained}}{K_{unq}}$

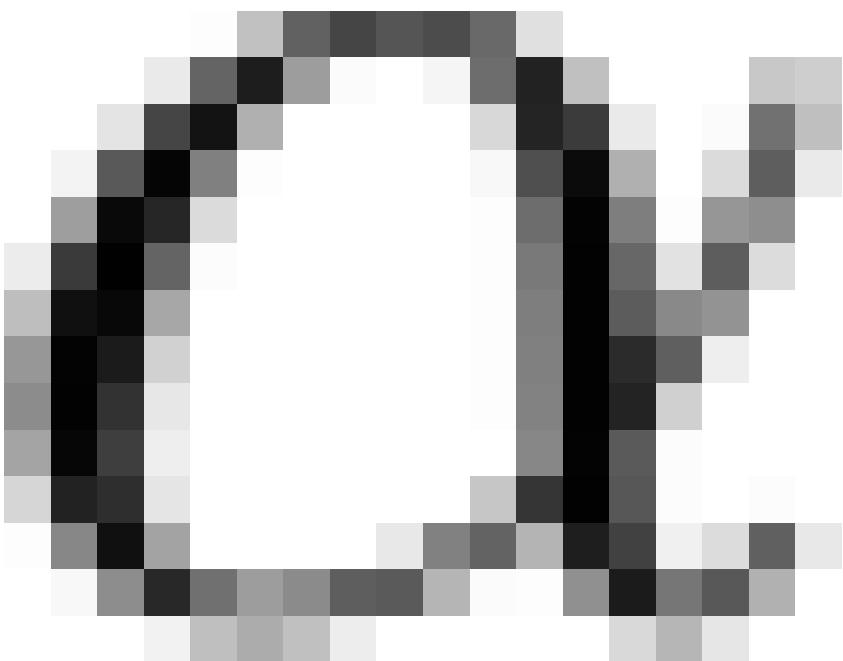
$K_{drained}$

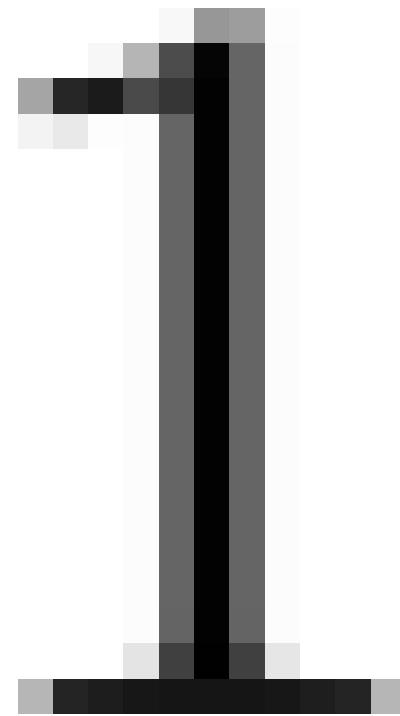
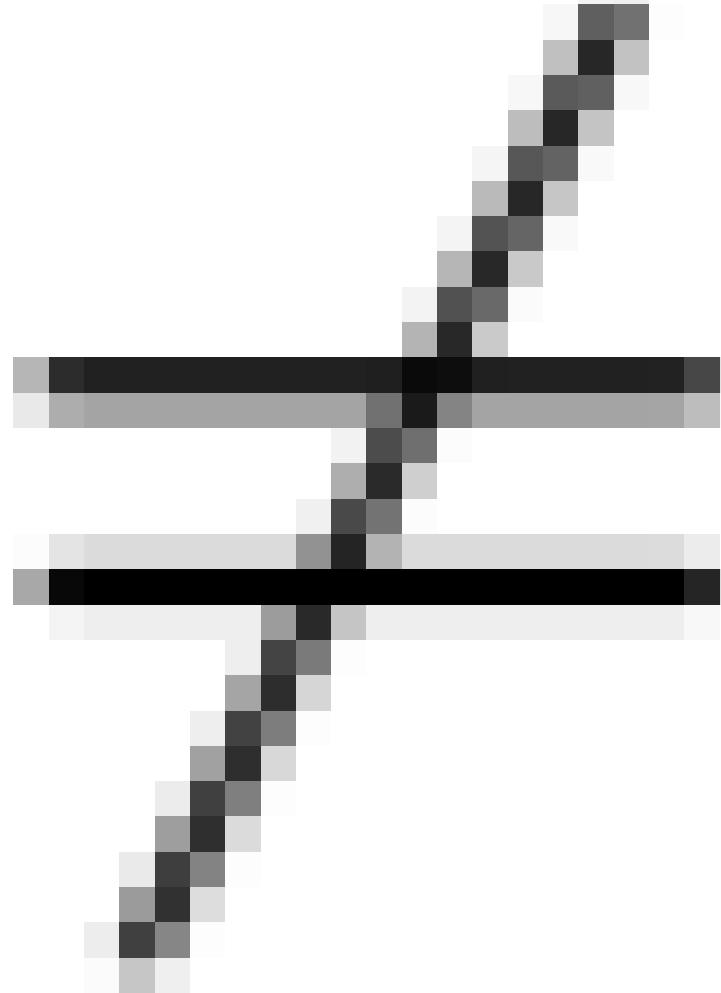
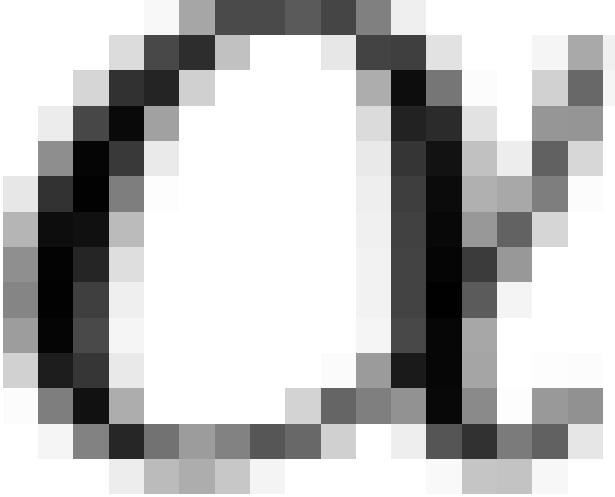
K_{unq}

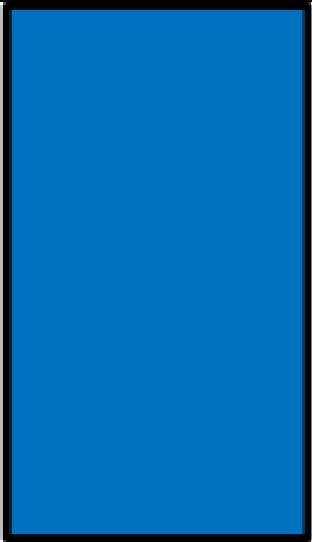
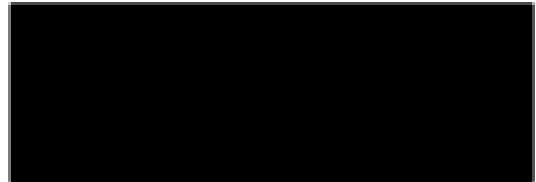




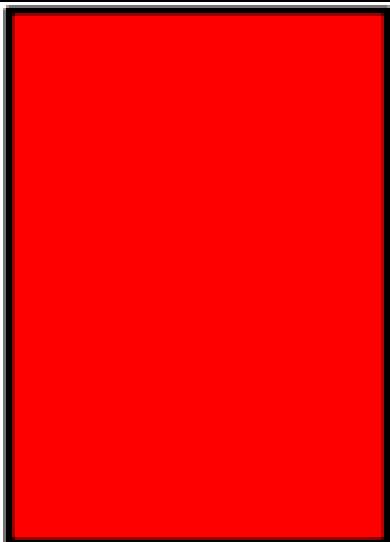


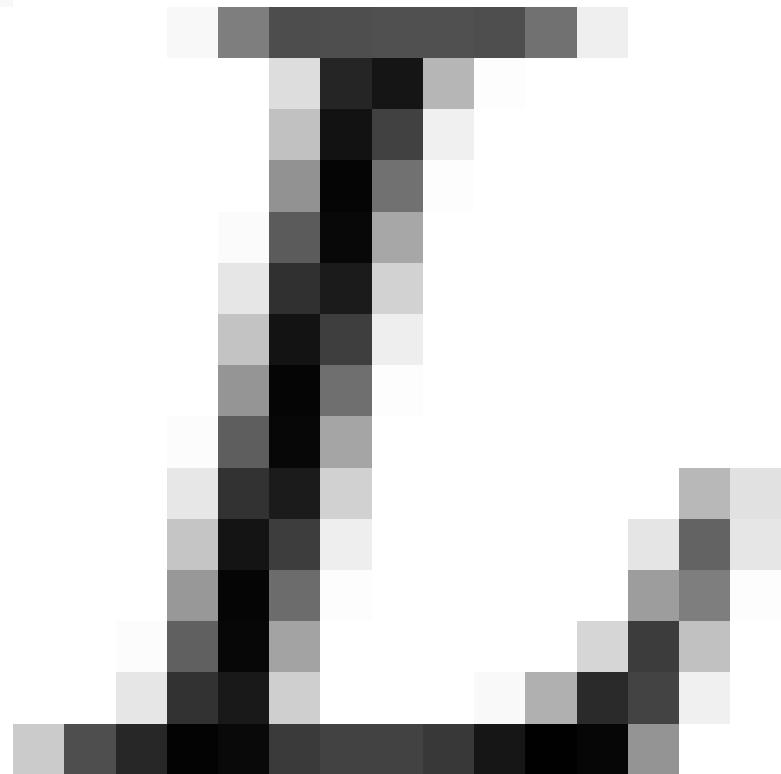
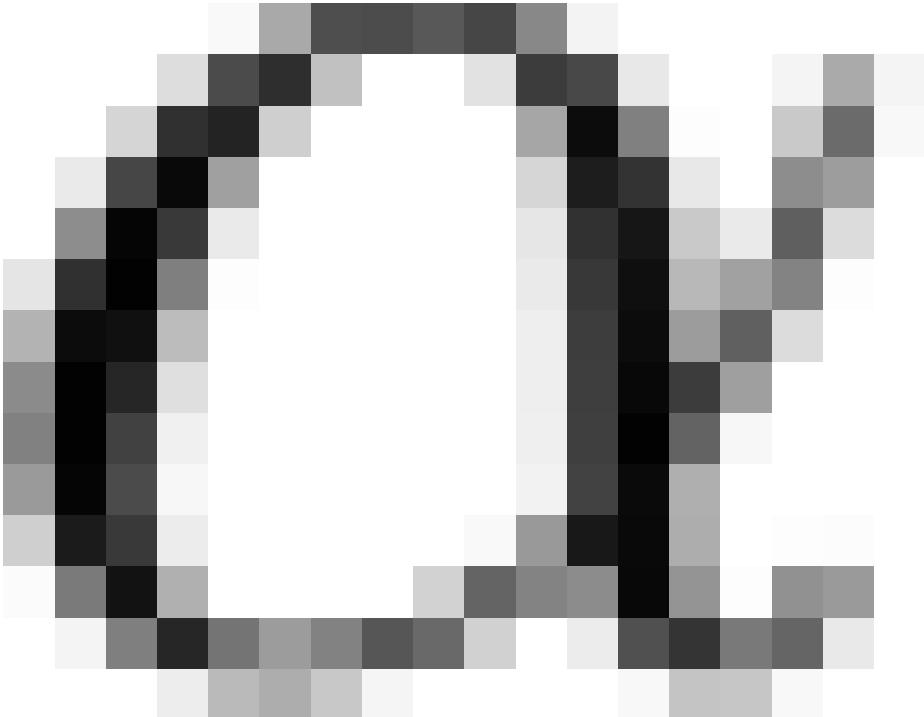


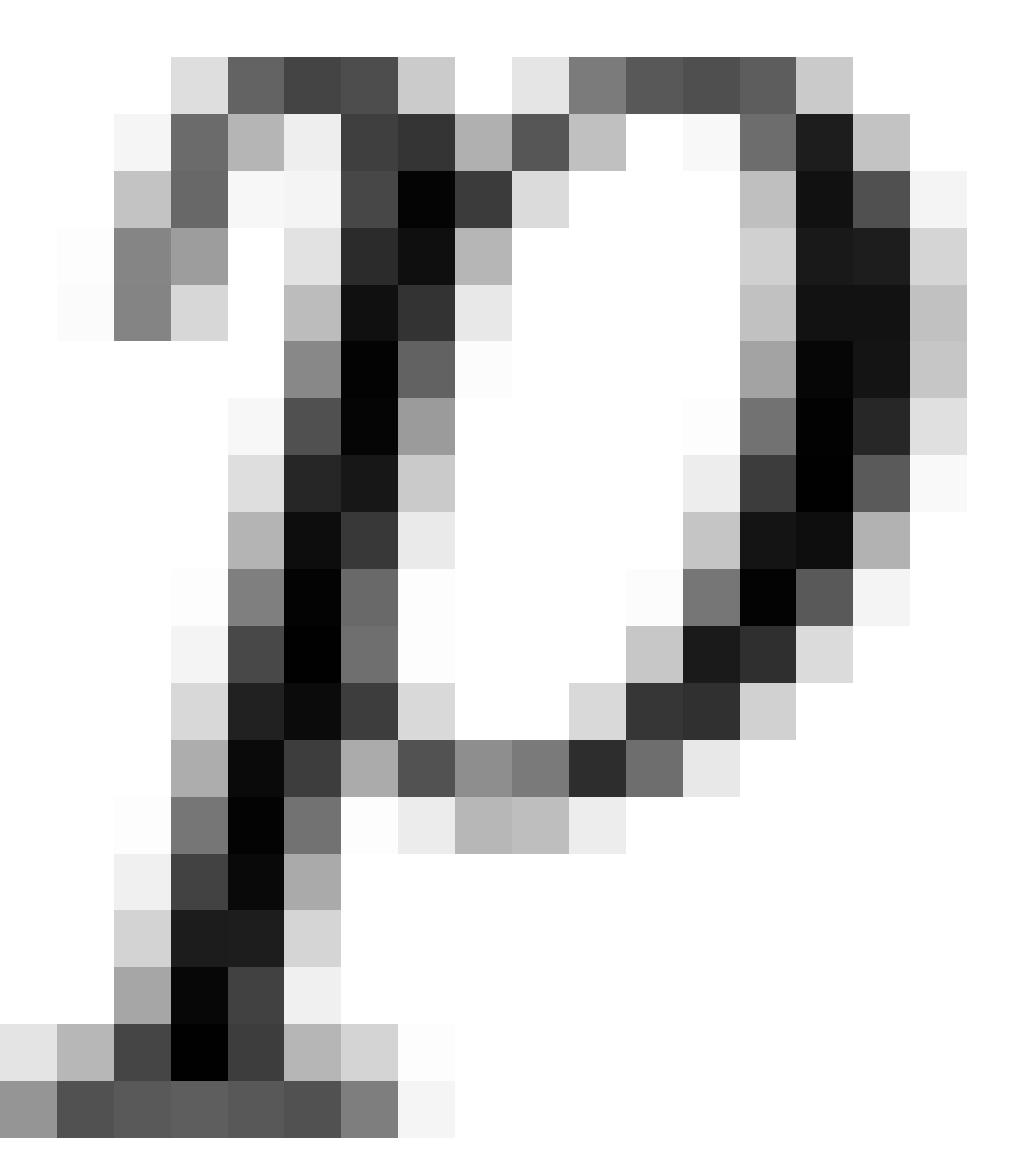




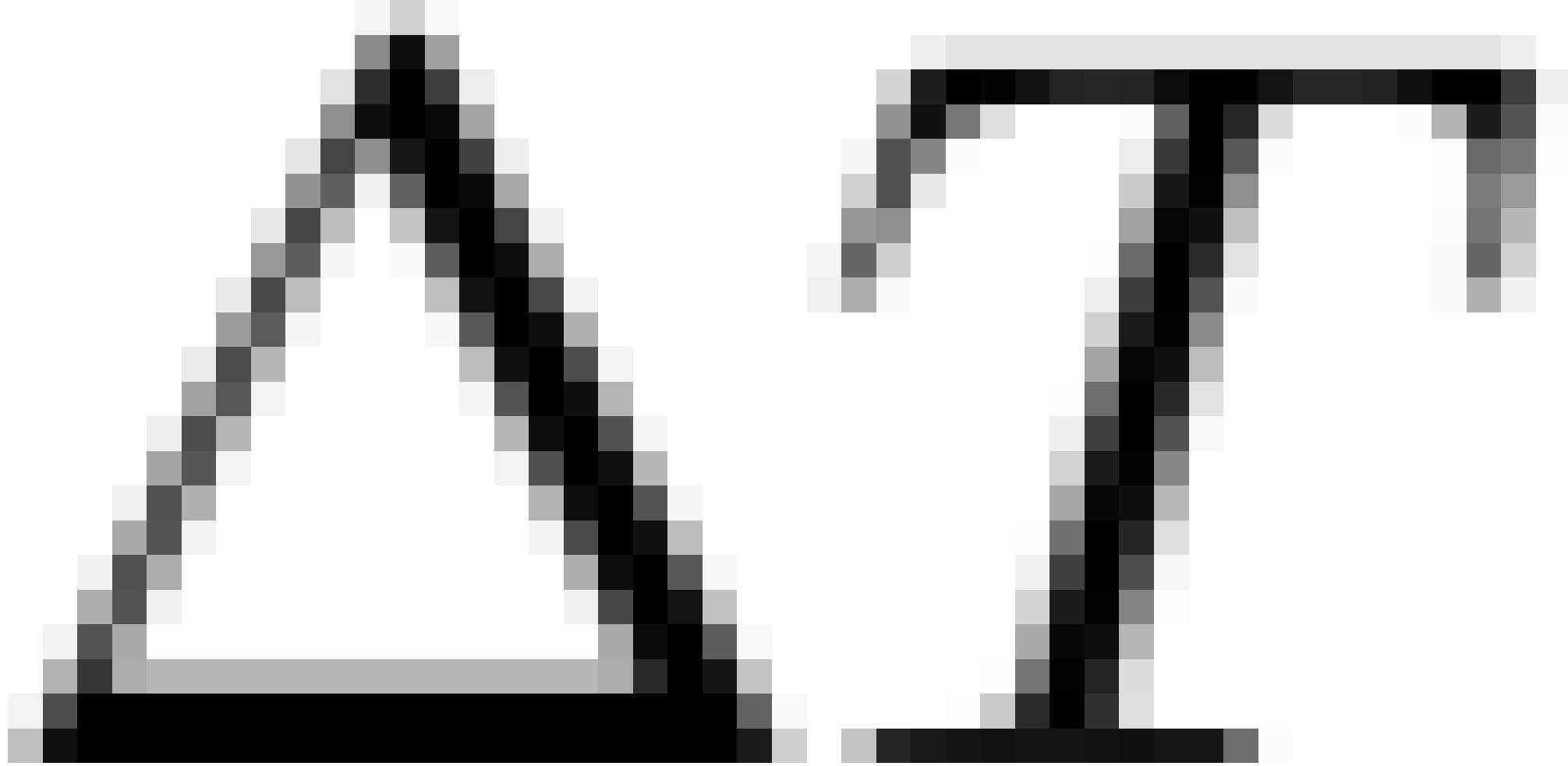
ΔT



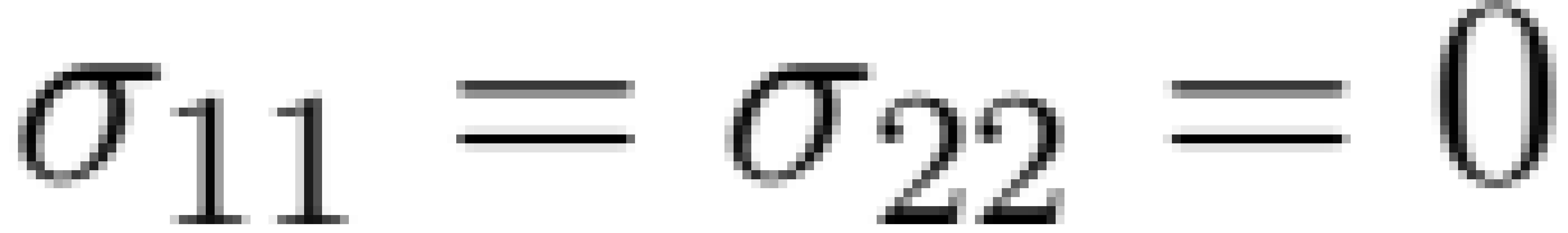




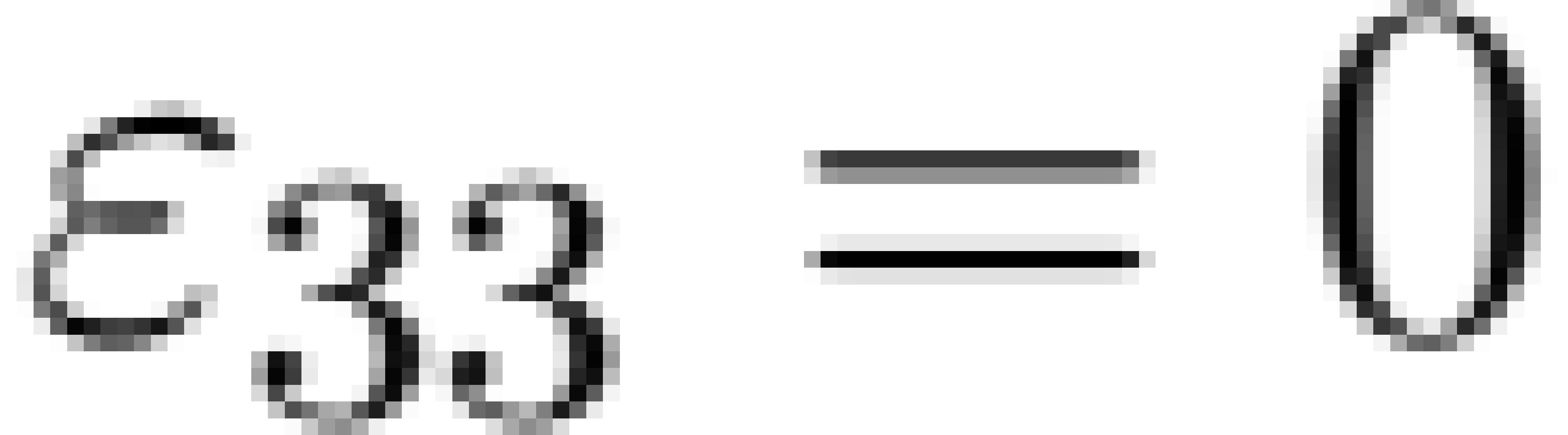
$$\alpha_L = \frac{1}{L} \frac{dL}{dT}$$

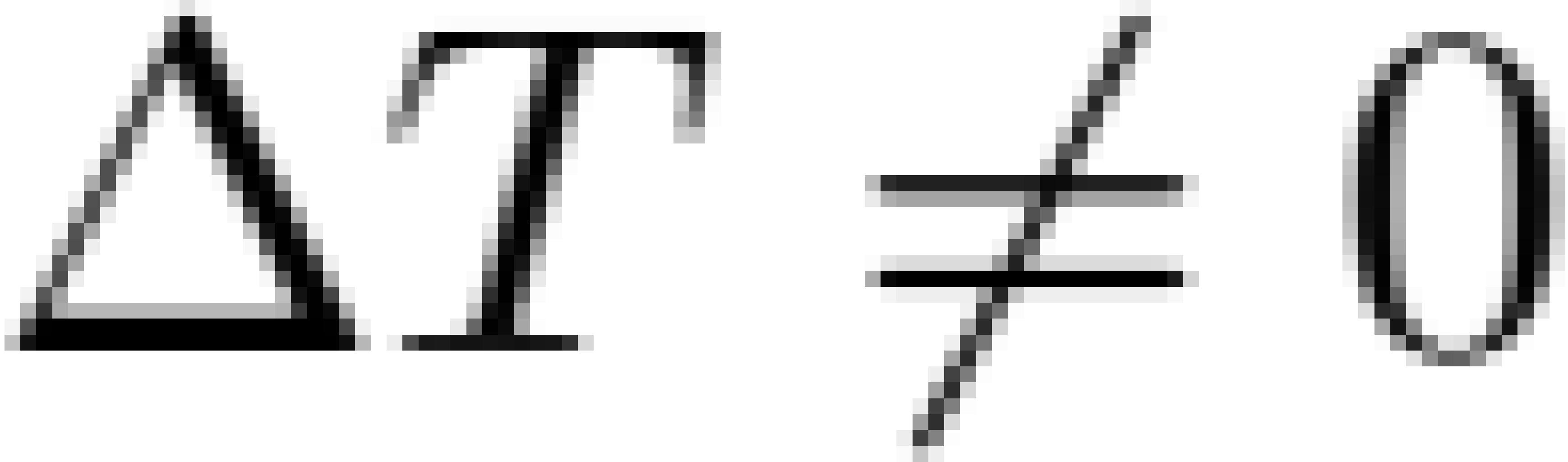


$$\left\{ \begin{array}{lcl} \sigma_{11} & = & (\lambda + 2\mu) \varepsilon_{11} + \lambda \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{22} & = & \lambda \varepsilon_{11} + (\lambda + 2\mu) \varepsilon_{22} + \lambda \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{33} & = & \lambda \varepsilon_{11} + \lambda \varepsilon_{22} + (\lambda + 2\mu) \varepsilon_{33} + 3K\alpha_L \Delta T \\ \sigma_{12} & = & 2\mu \varepsilon_{12} \\ \sigma_{13} & = & 2\mu \varepsilon_{13} \\ \sigma_{23} & = & 2\mu \varepsilon_{23} \end{array} \right.$$









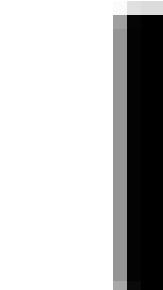
$$\begin{aligned} \sigma_{11}^0 &= (\lambda + 2\mu) \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \\ \sigma_{33} &= \lambda \epsilon_{11} + \lambda \epsilon_{11} + 3K \alpha_L \Delta T \end{aligned}$$

σ_{33}

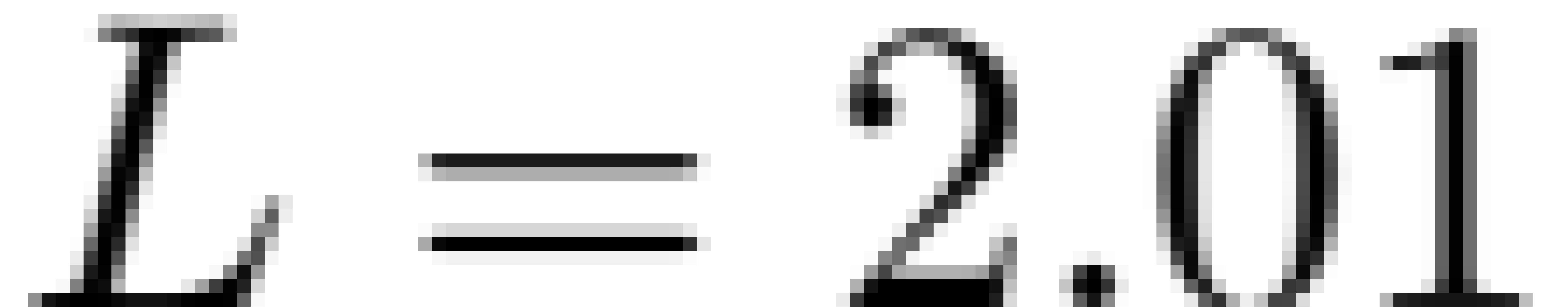
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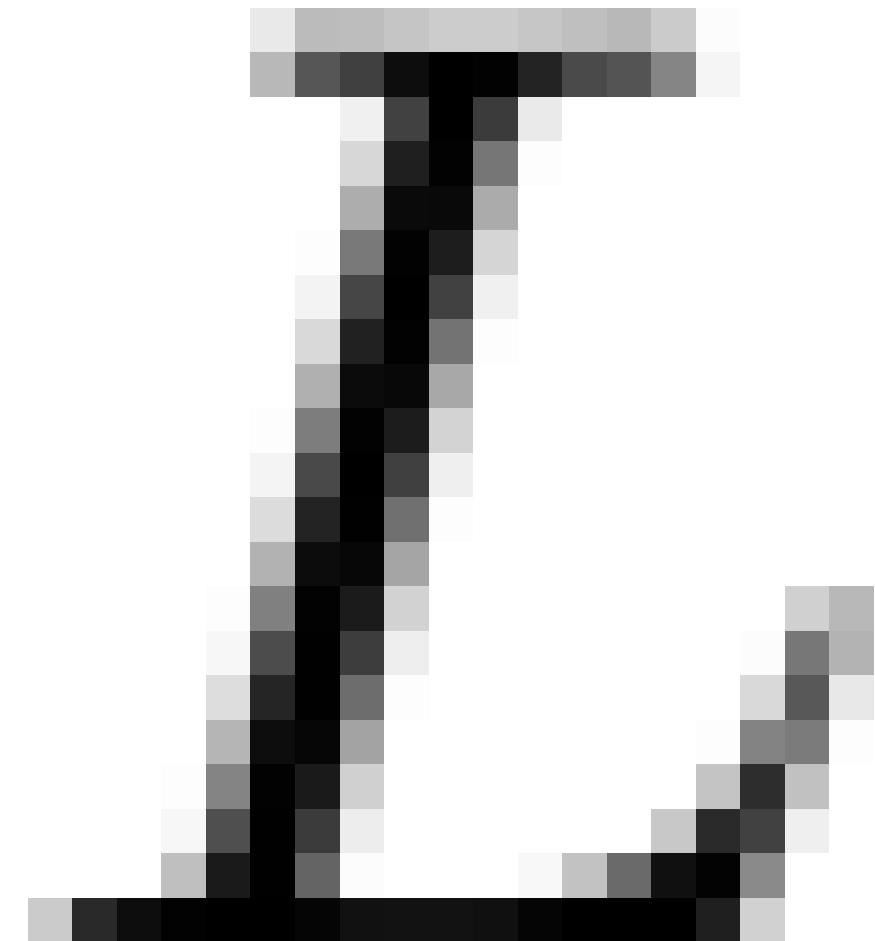
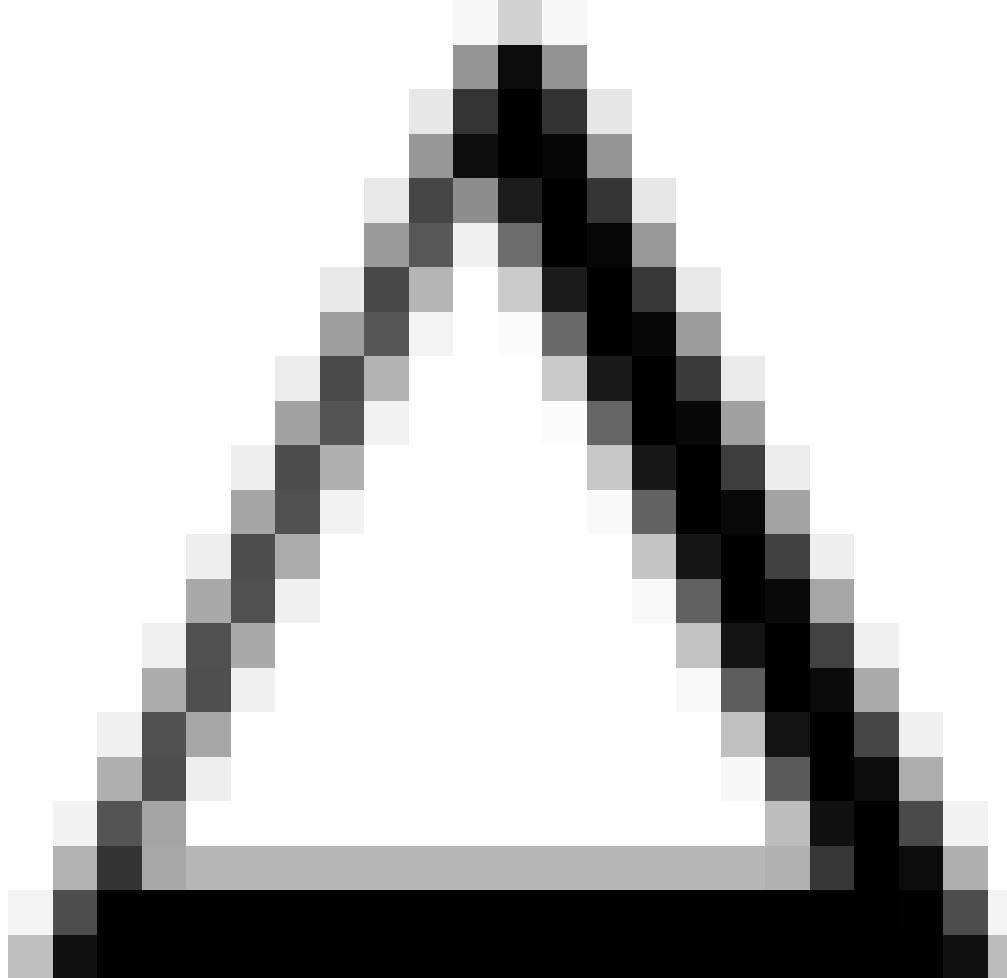
$$\frac{6}{3} \mu K$$

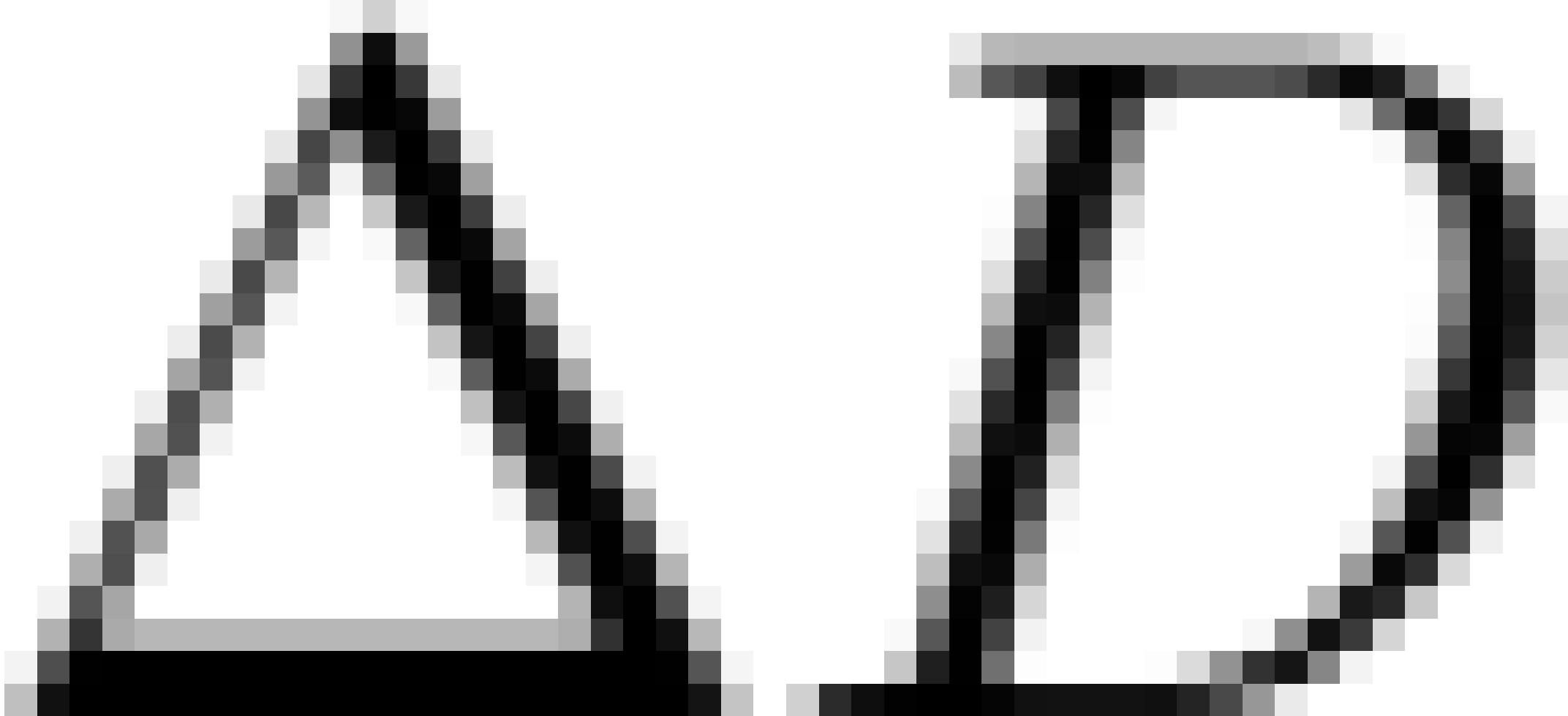
$K \alpha L \Delta T$

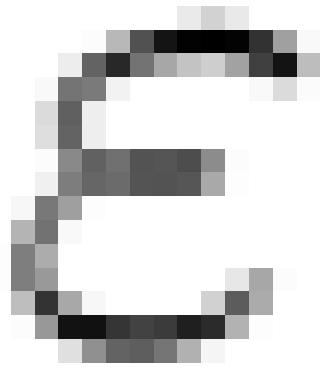
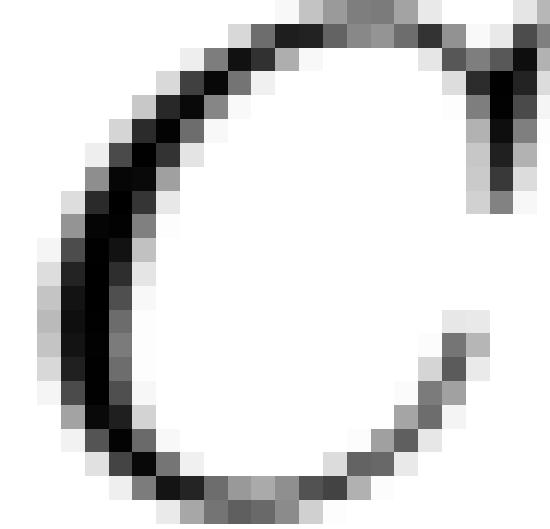
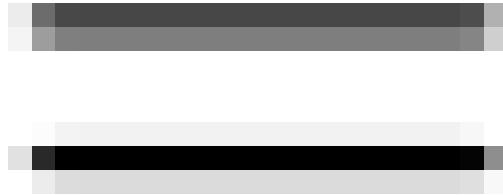
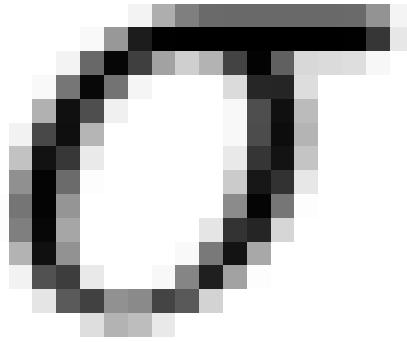


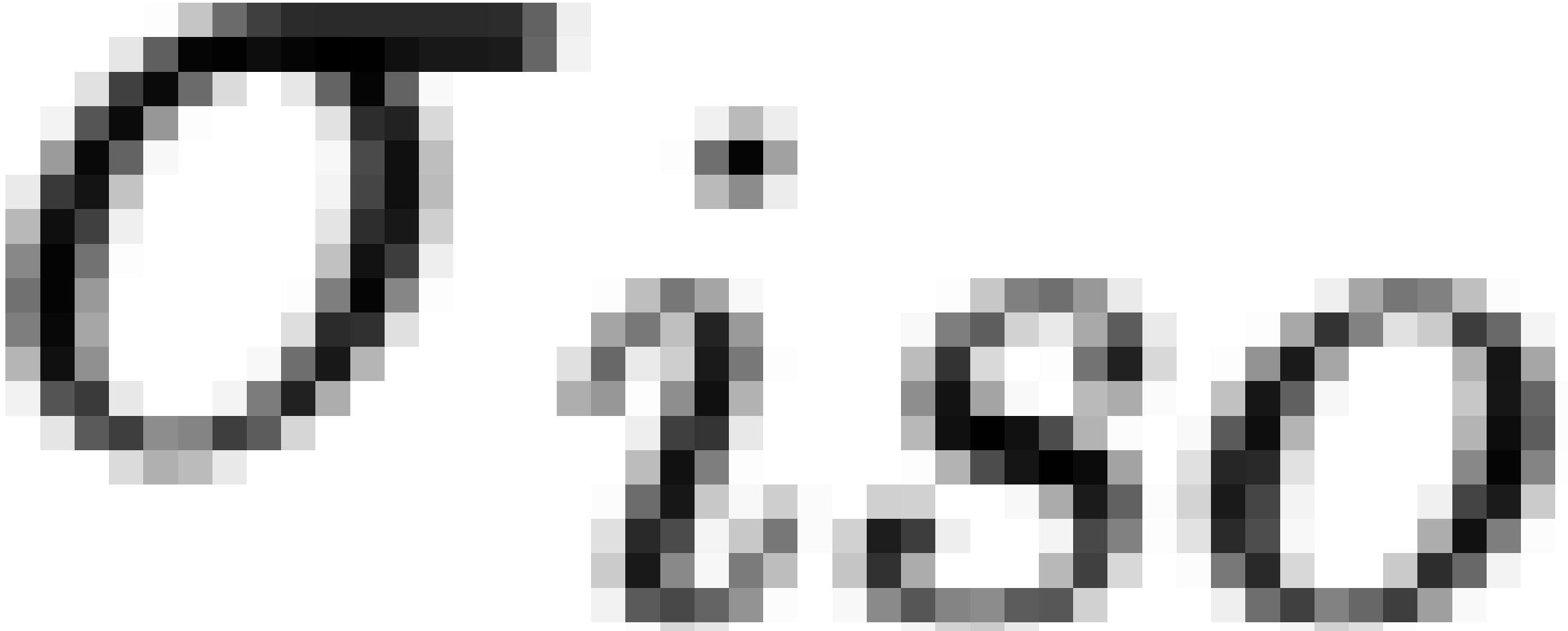


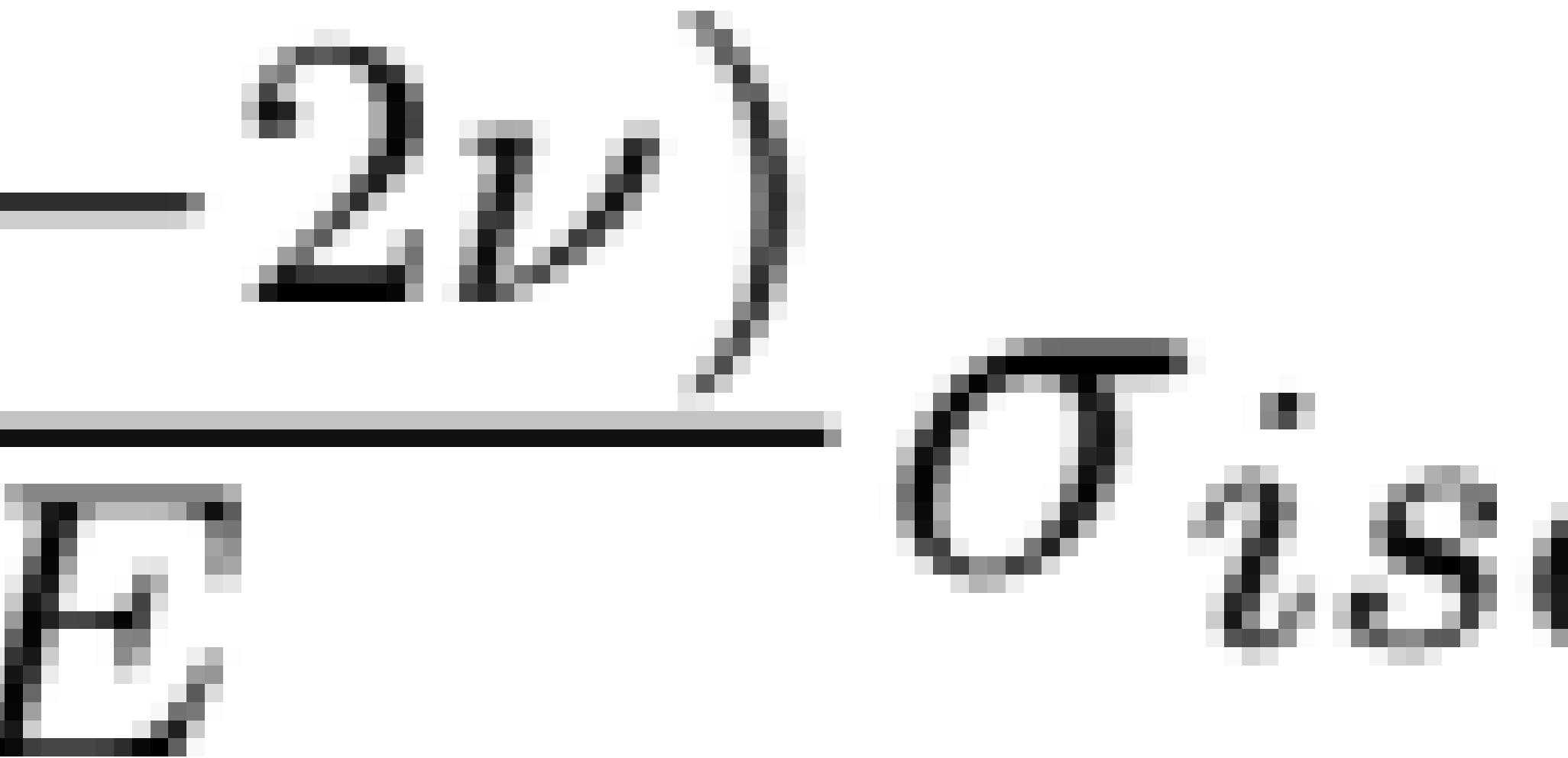




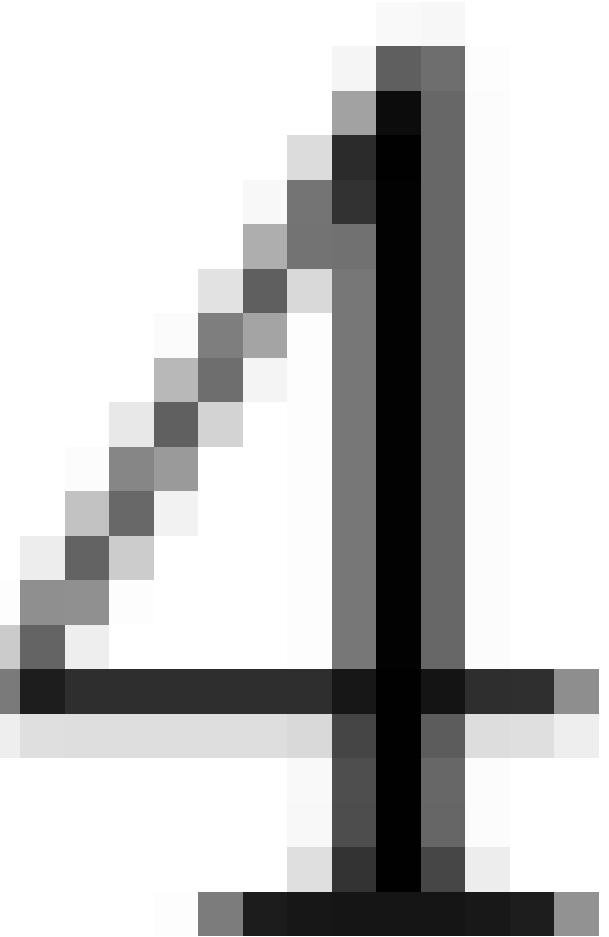
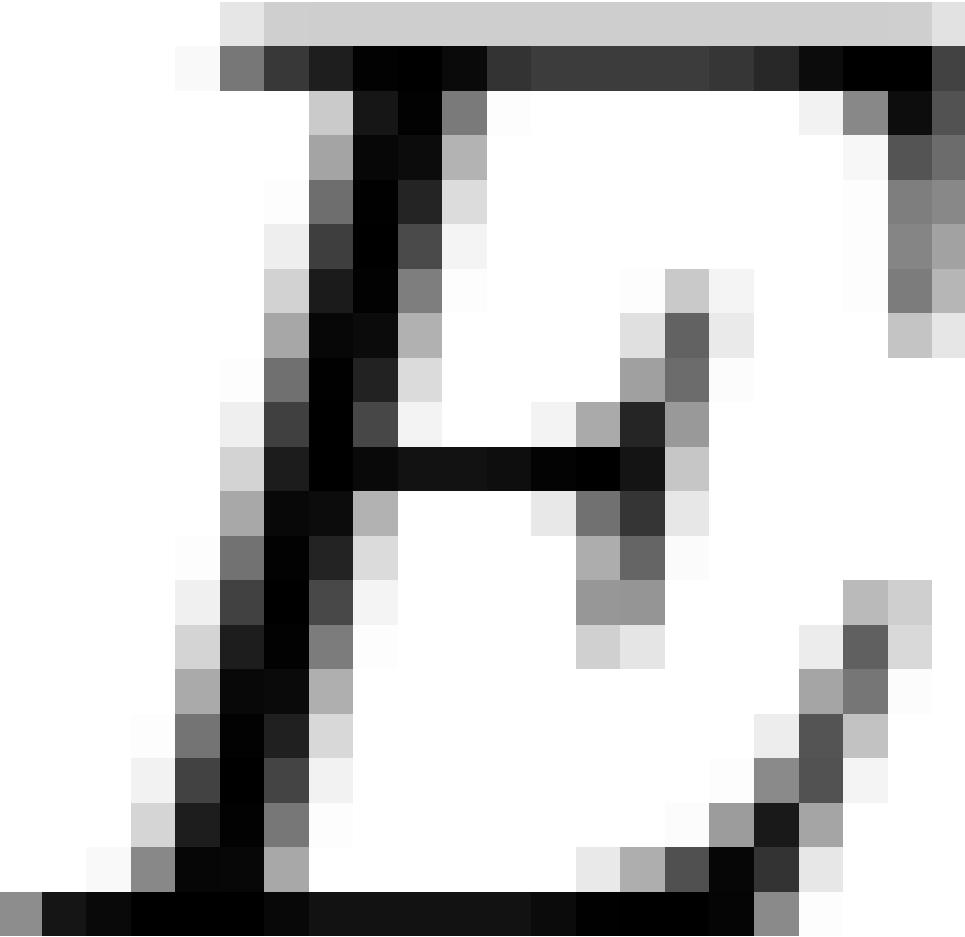










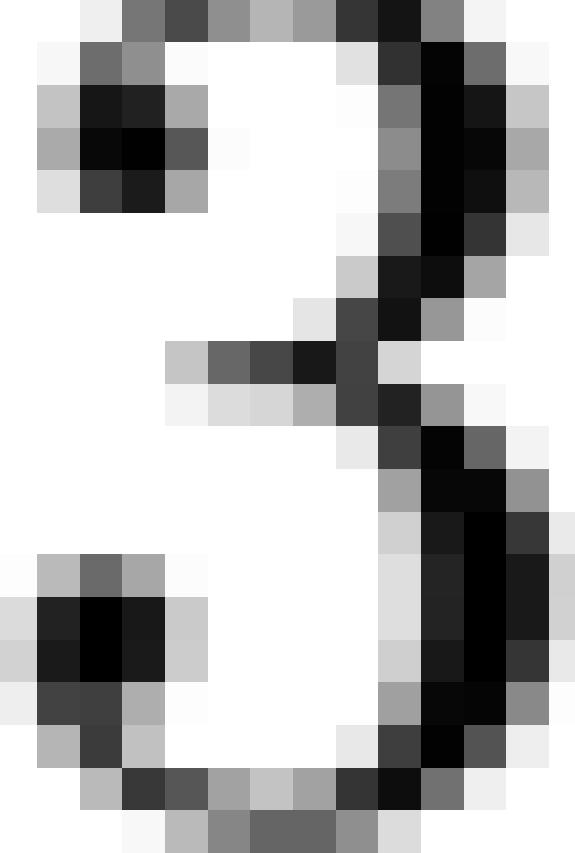
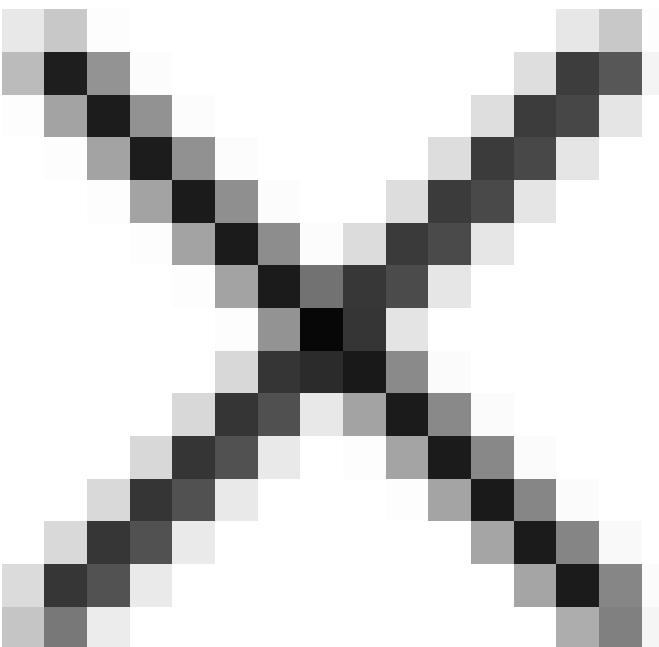
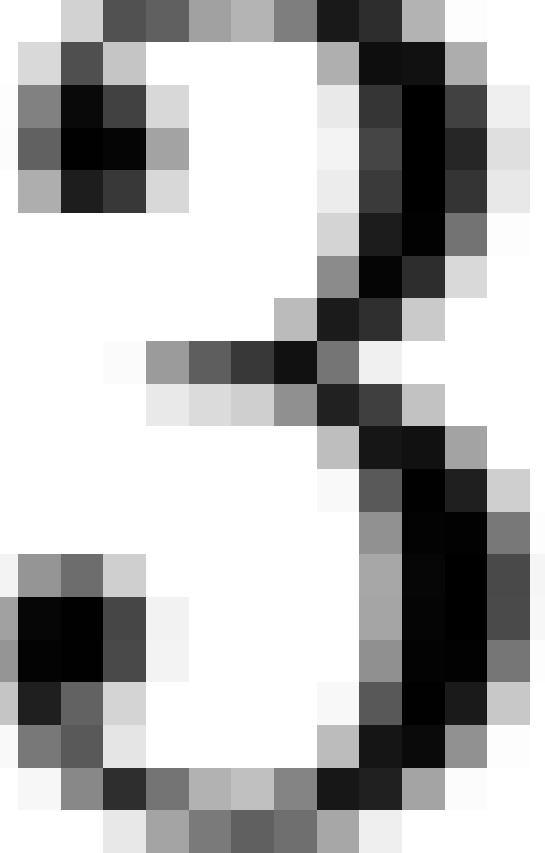


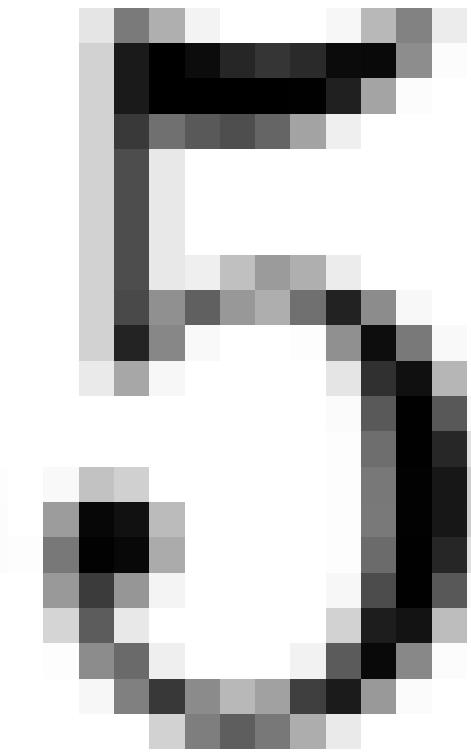
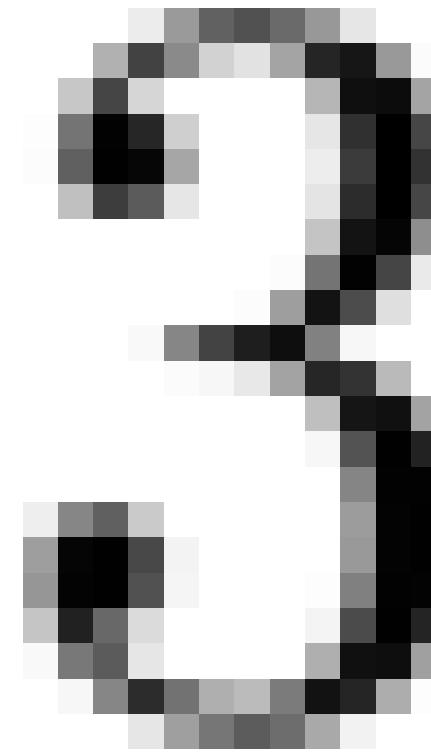
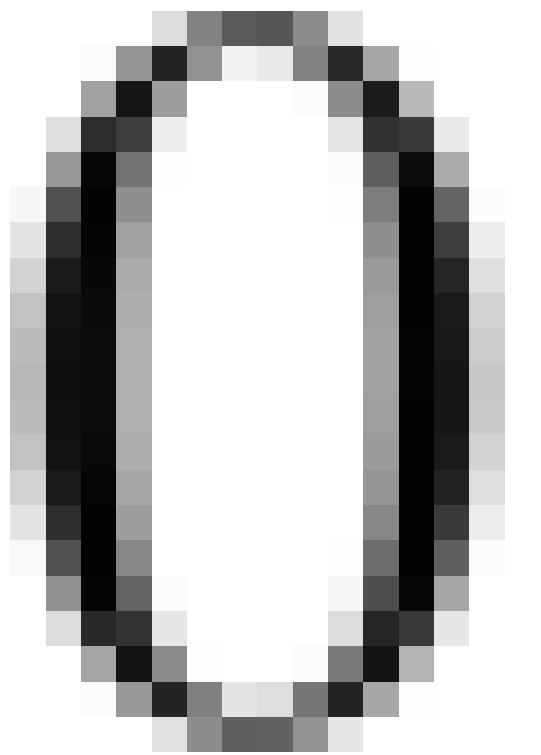
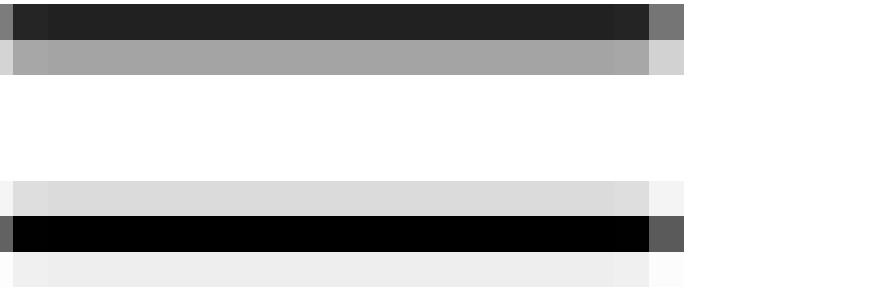
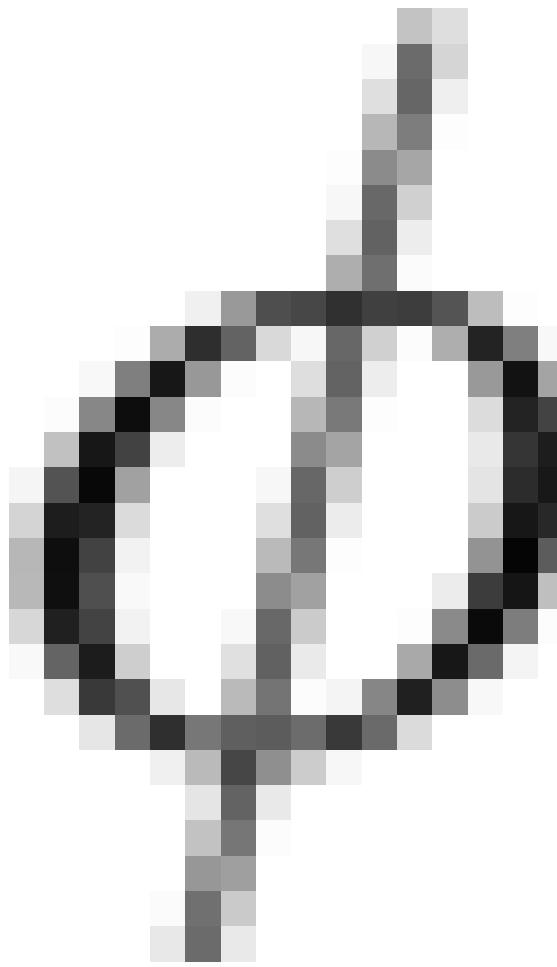




σ_{11} 

$$\frac{E(1-v)}{(1+v)(1-2v)}\epsilon_{11}$$
$$= \frac{E(1-v)}{(1+v)(1-2v)}\epsilon_{11}$$







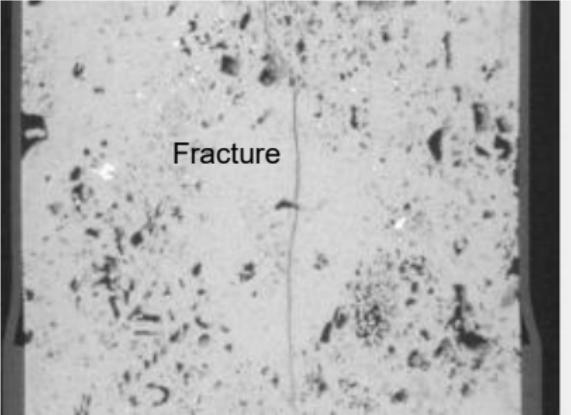
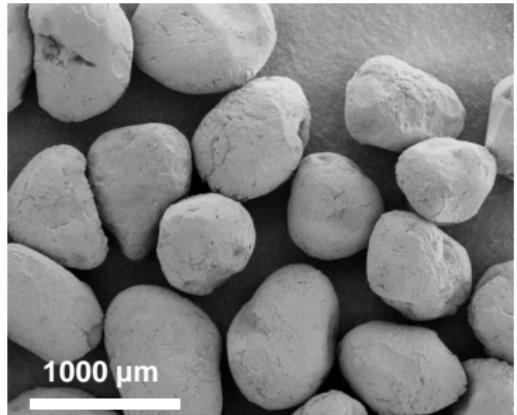
(a) Uncemented sand



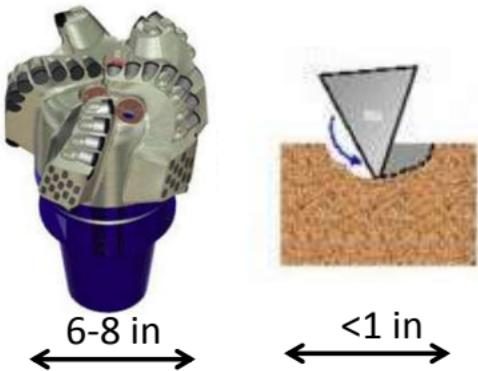
(b) Cemented sandstone



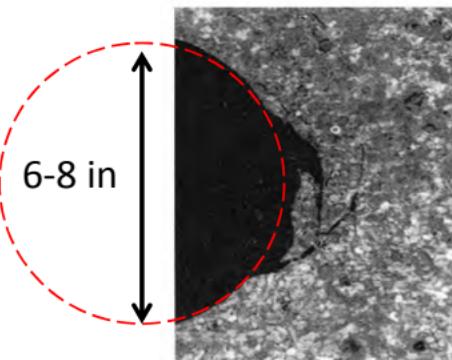
(c) Vuggy carbonate



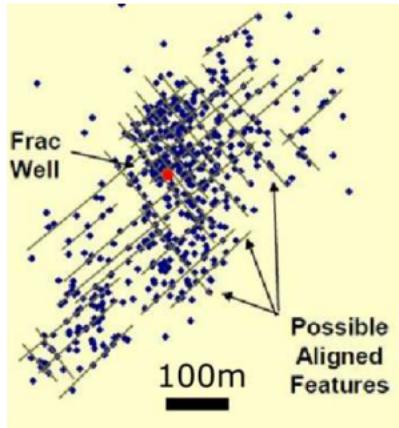
Rock cutting at the drill bit



Wellbore breakout



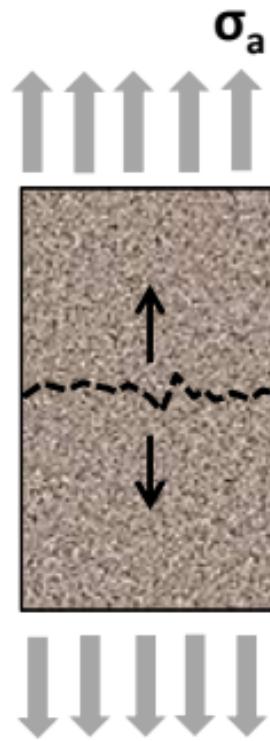
Shale hydraulic fracture



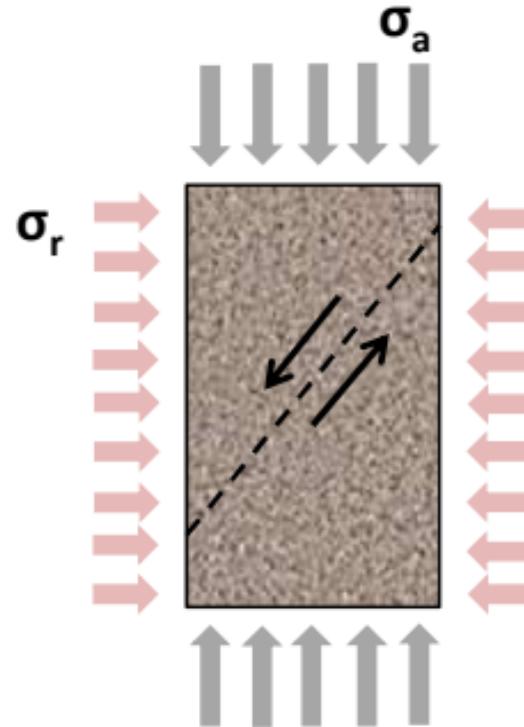
Reservoir depletion



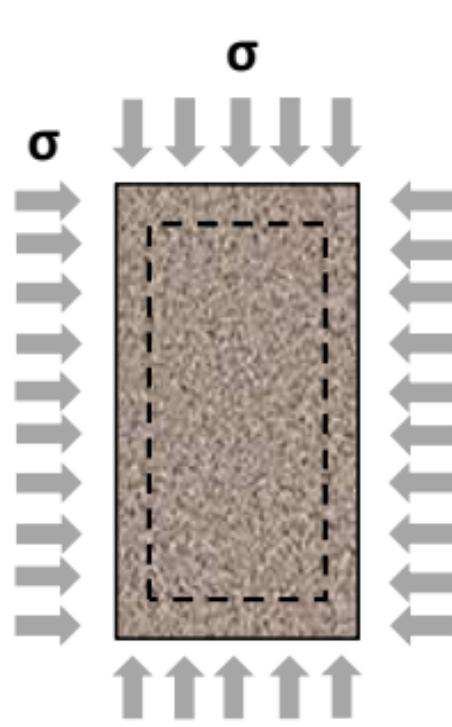
Images: Schlumberger/Terratek, Zoback 2013, Warpinski 2008, doe.gov



Tension
(bond breakage)
Ex: drilling-induced tens. fracs



Shear
(friction failure)
Ex: fault, breakout

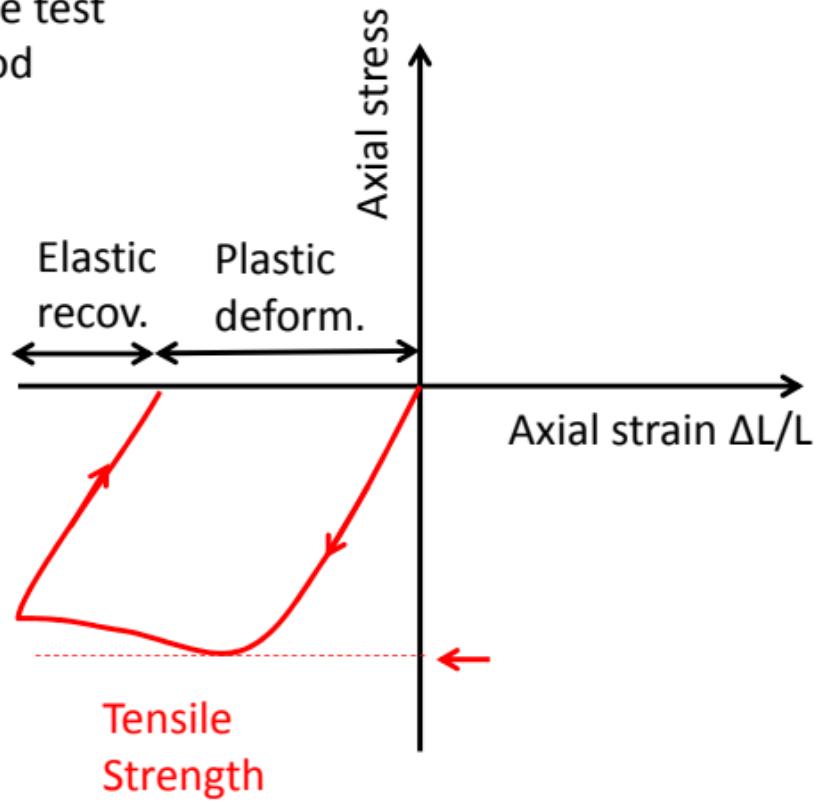


Compression
(pore collapse)
Ex: reservoir compaction

T



Typical tensile test
on a metal rod



Tensile test on a rock rod

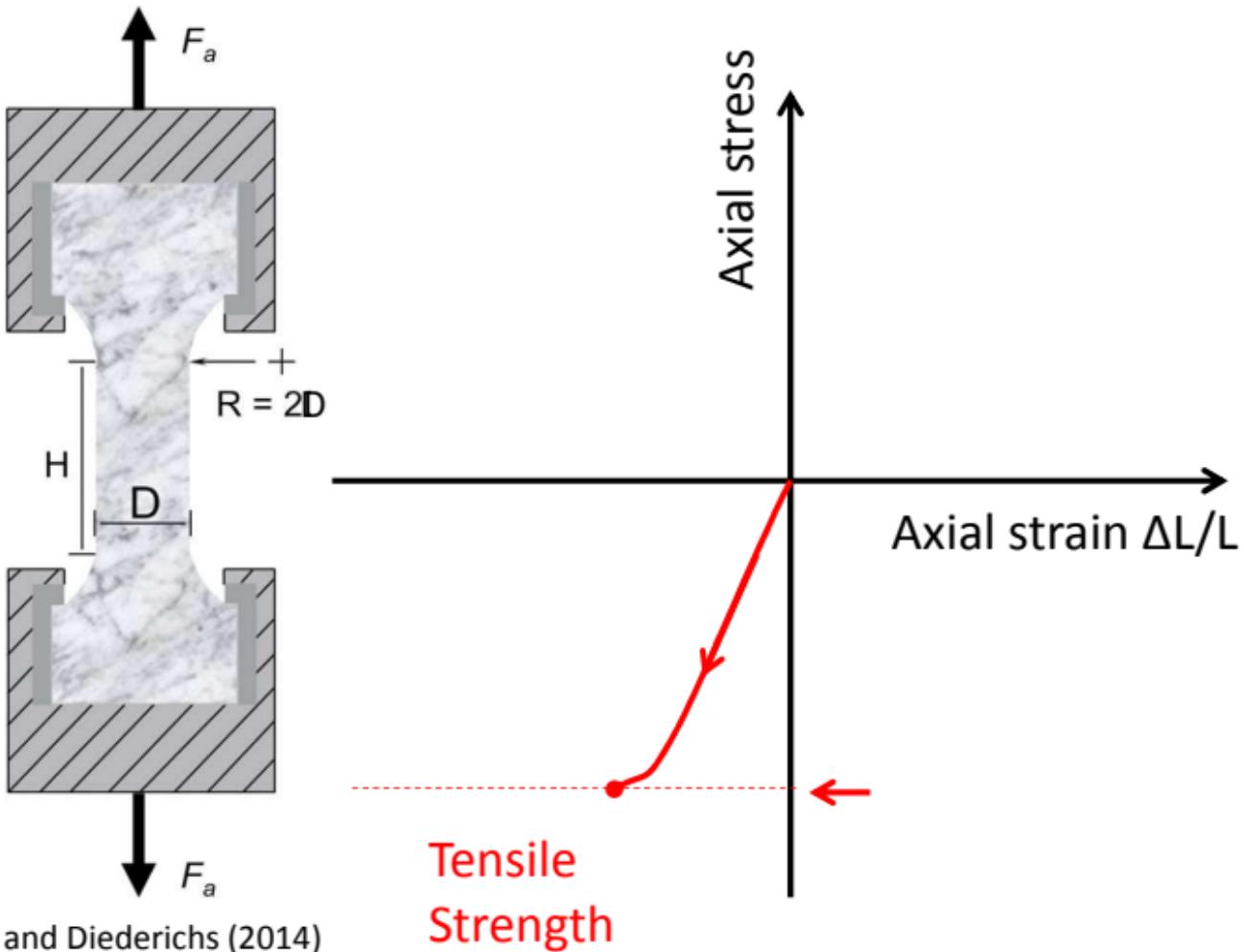


Figure direct tension: Perras and Diederichs (2014)

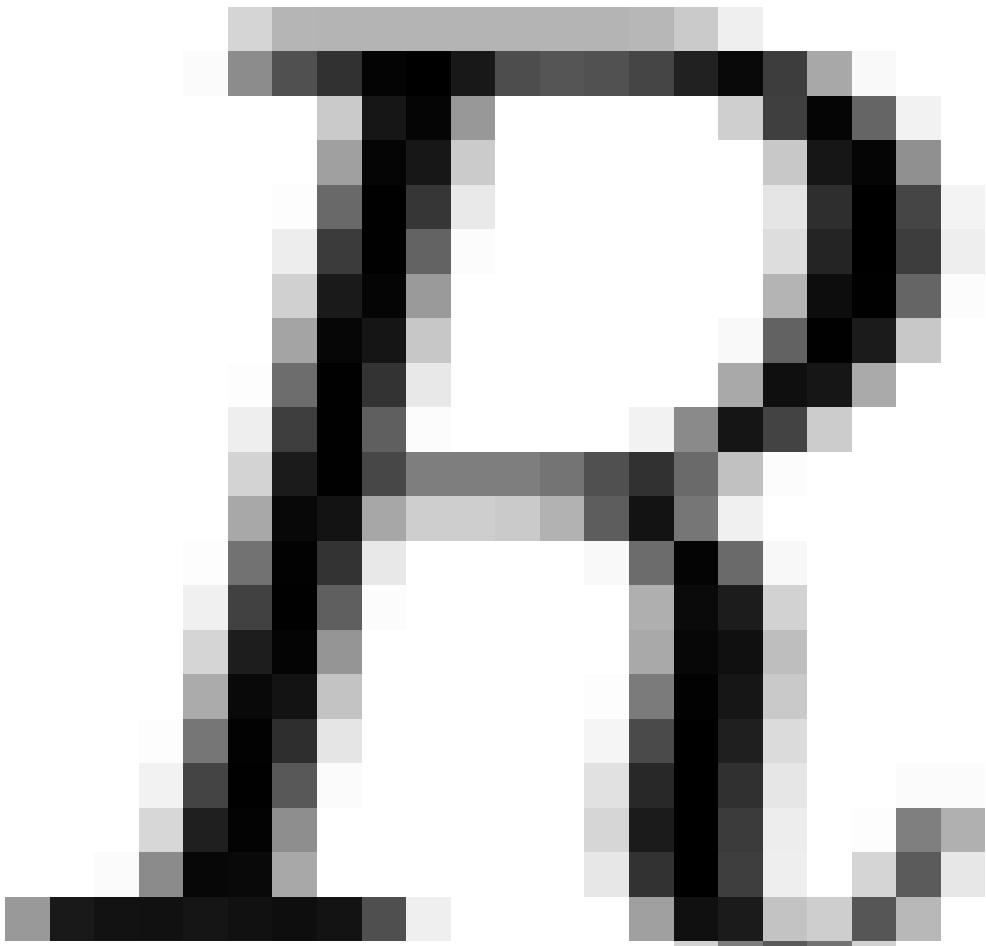
IS

—
—

πLR

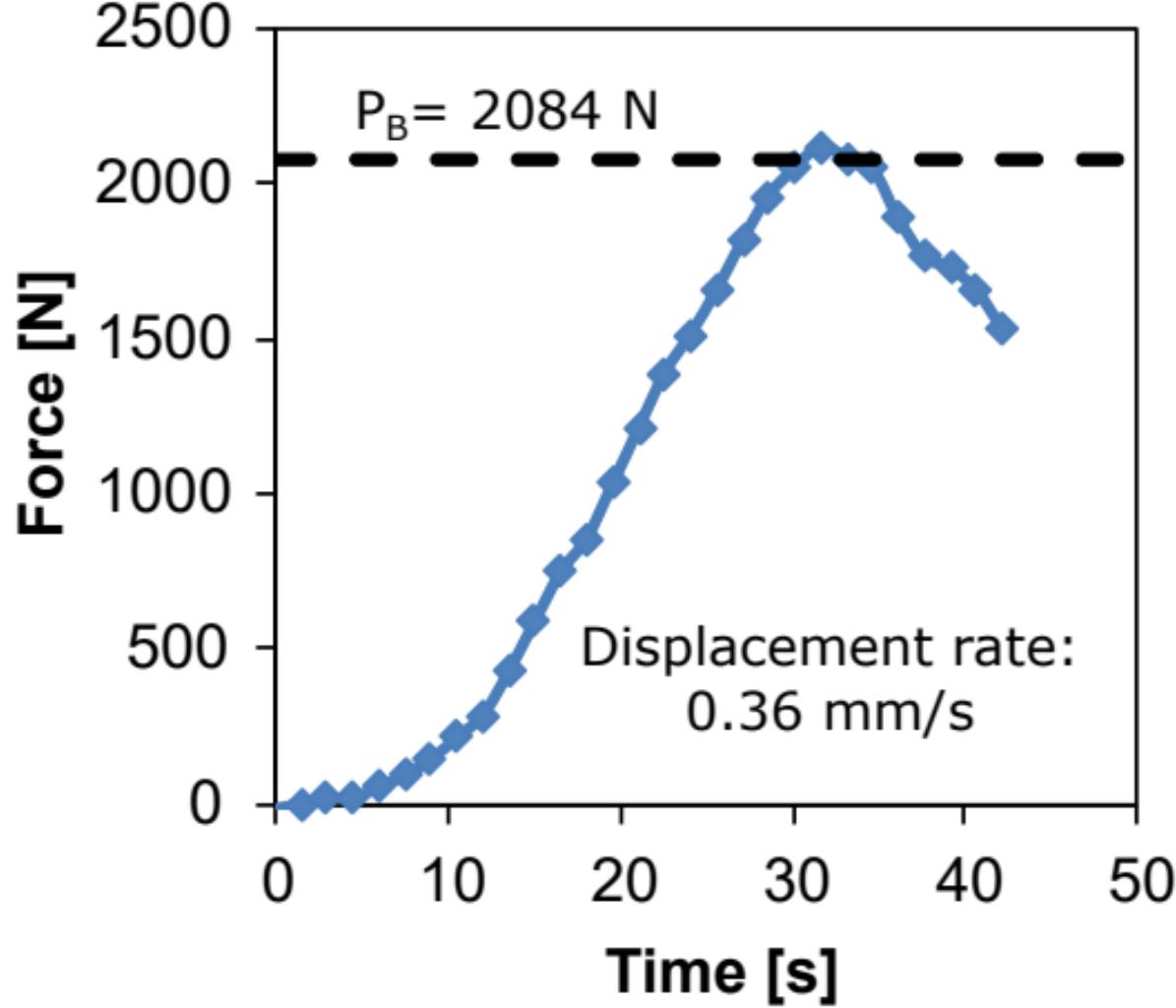
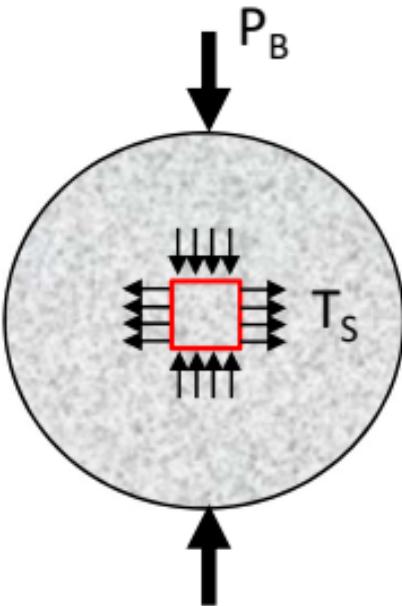
PB





Cylindrical sample:

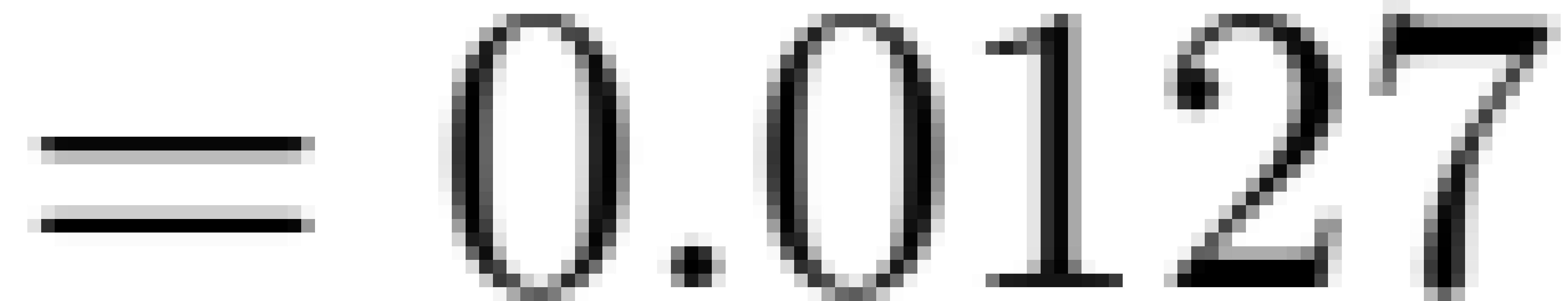
- radius R
- length L

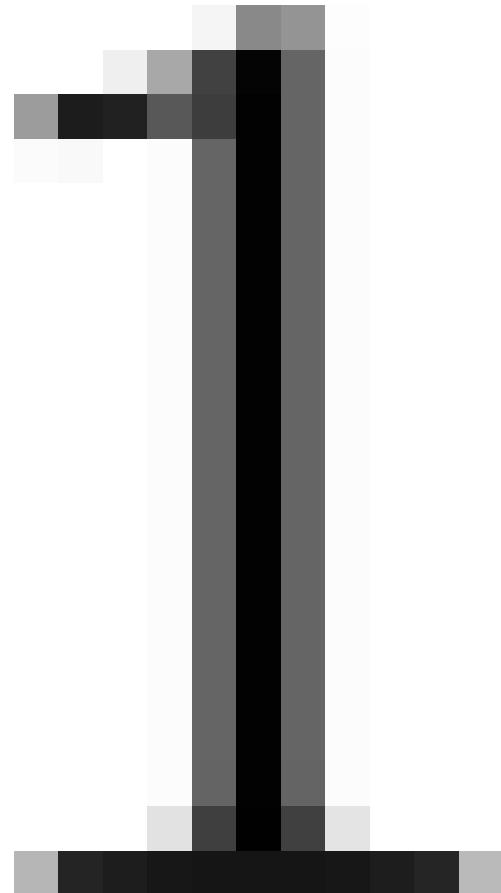
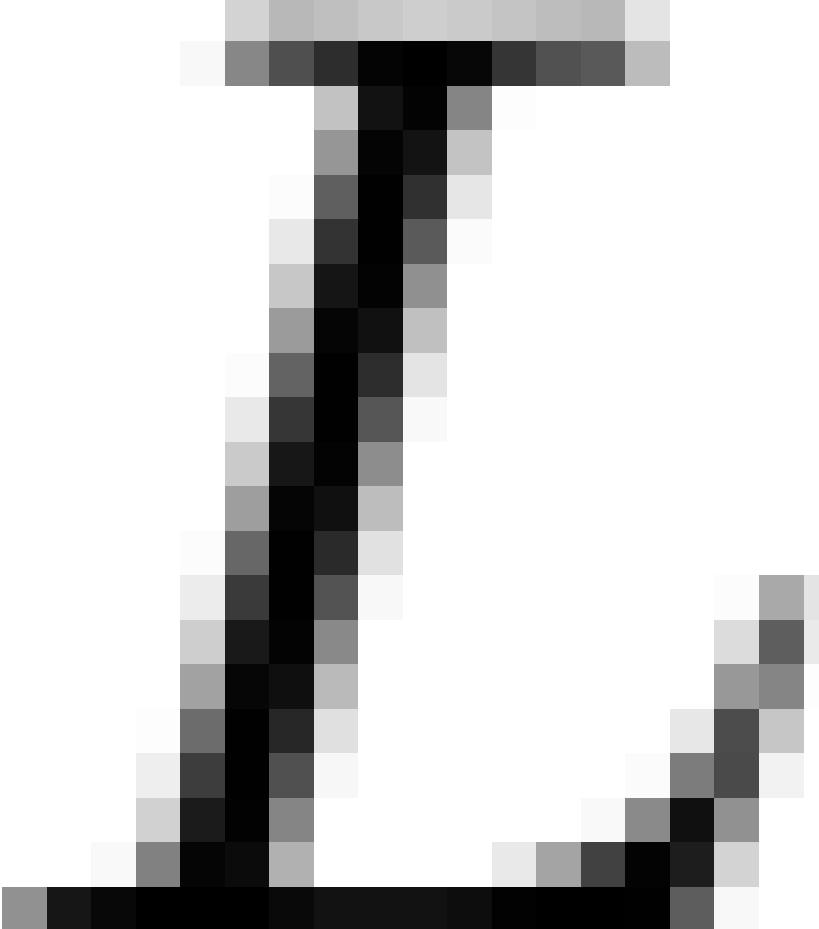


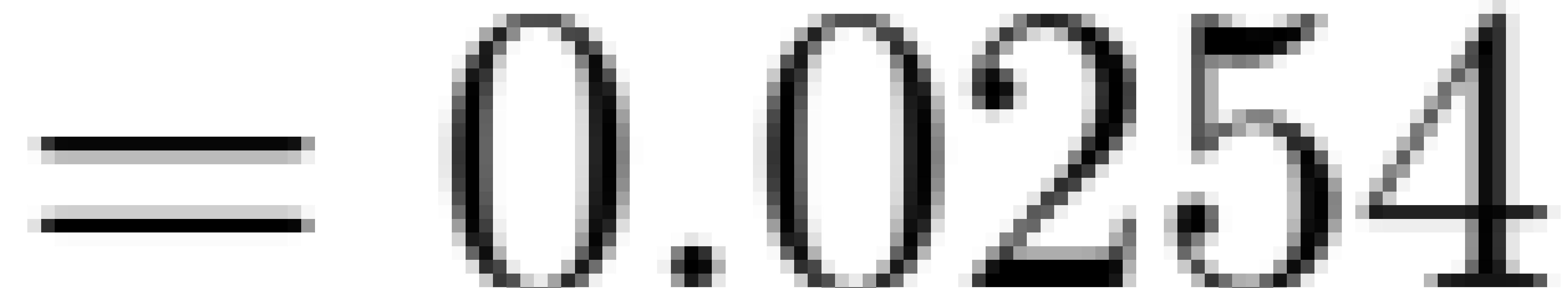
R

1
2

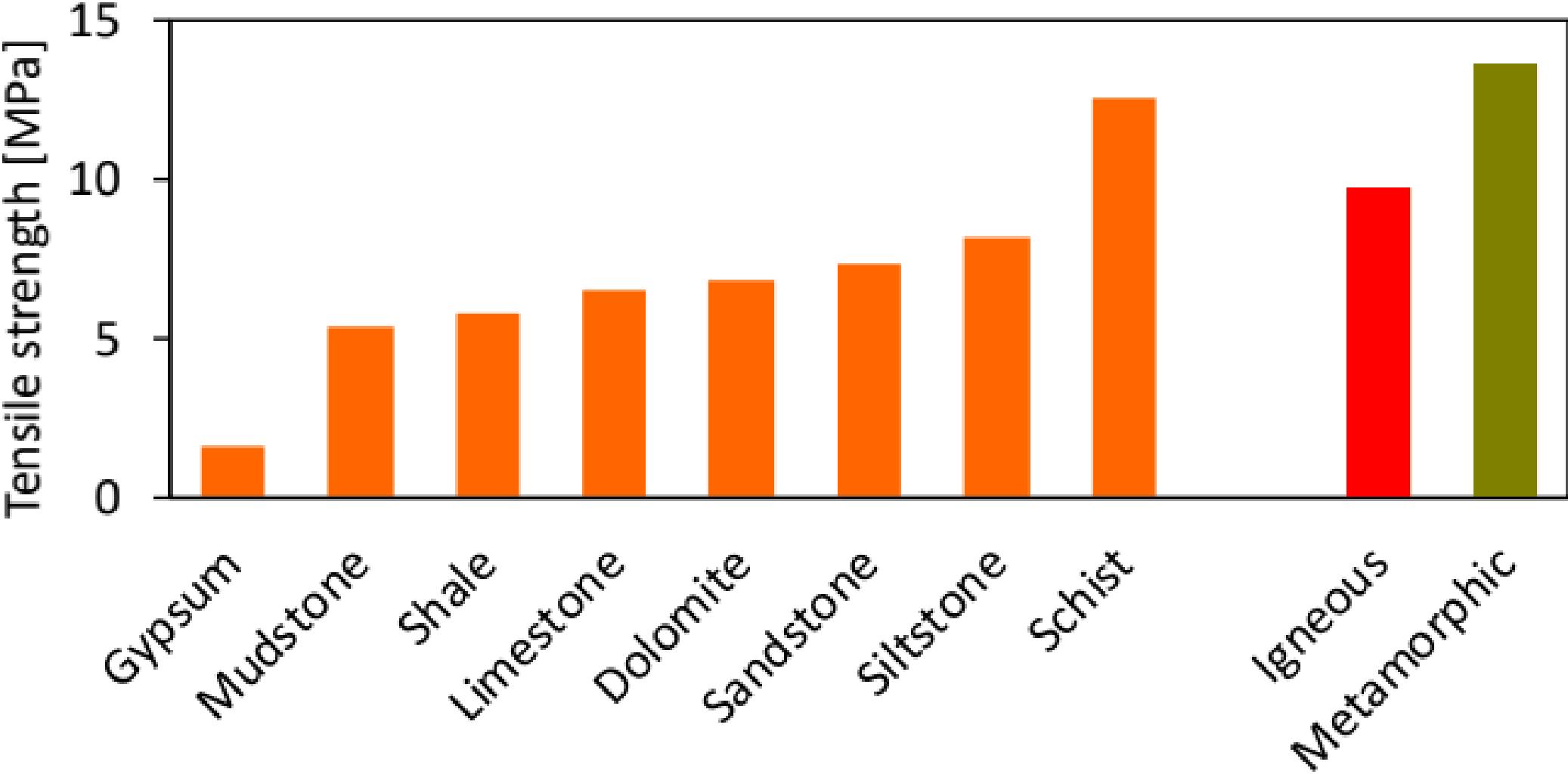
1
2



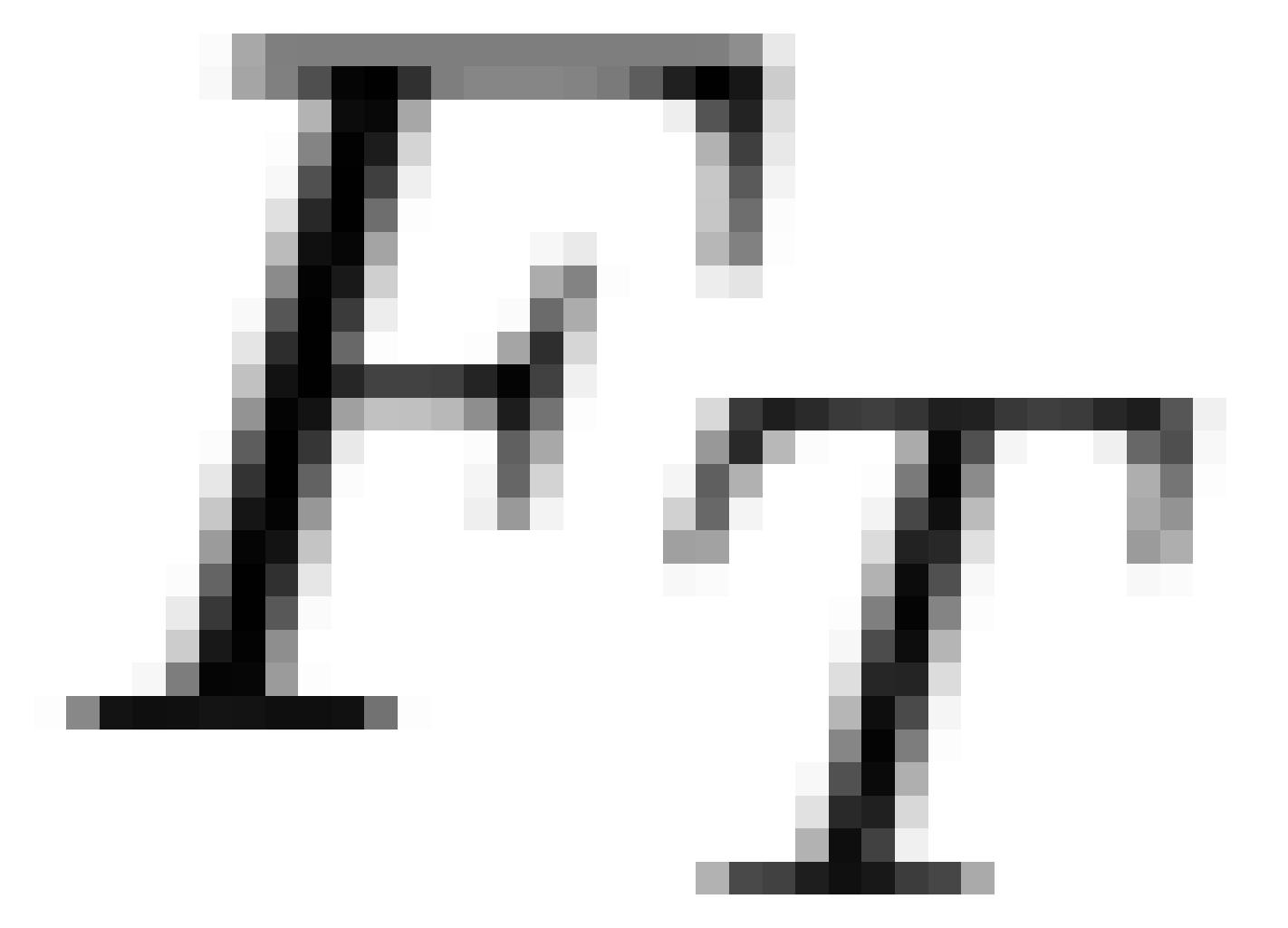




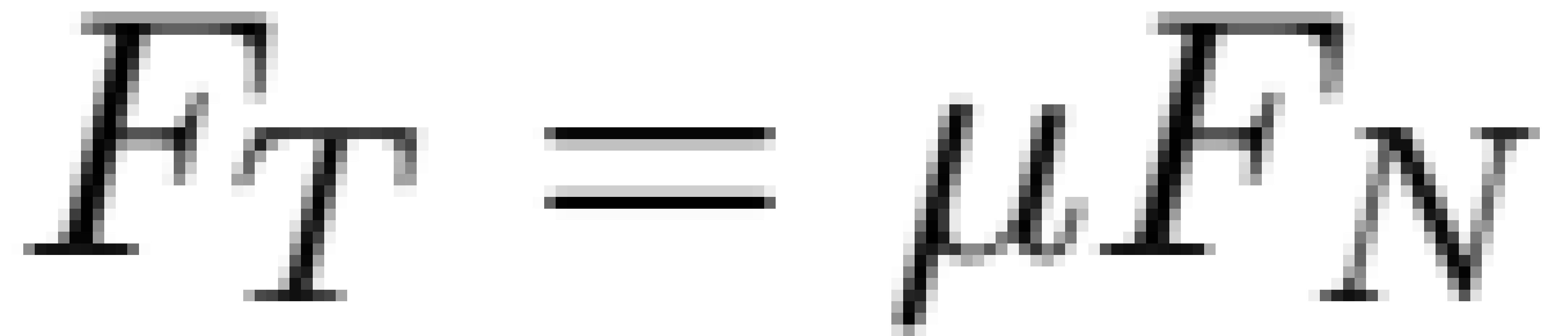
$$\frac{2084 \text{ N}}{\pi (0.0254 \text{ m})(0.0127 \text{ m})} = 2.06 \times 10^6 \text{ Pa} = 2.06 \text{ MPa}$$

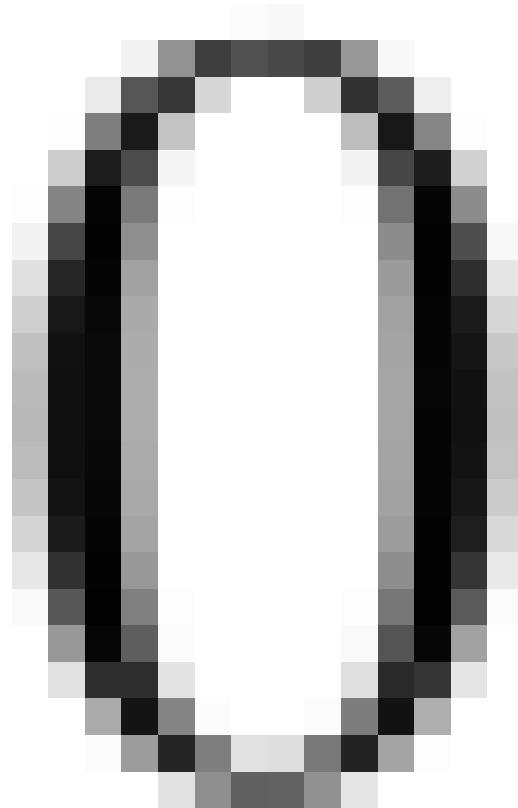


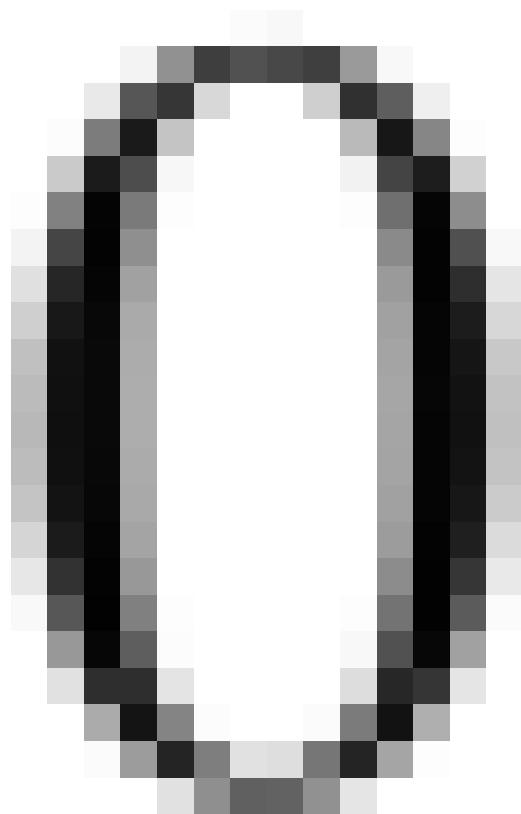


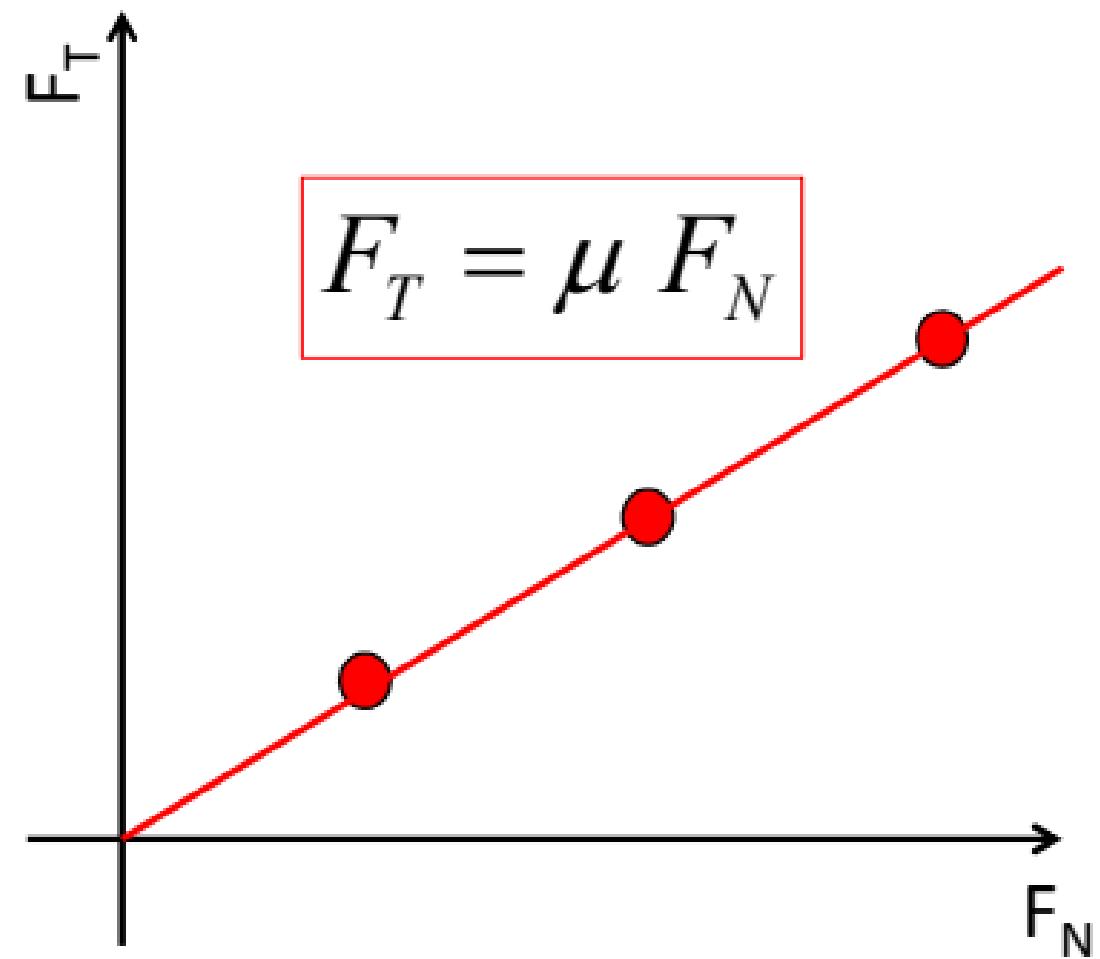
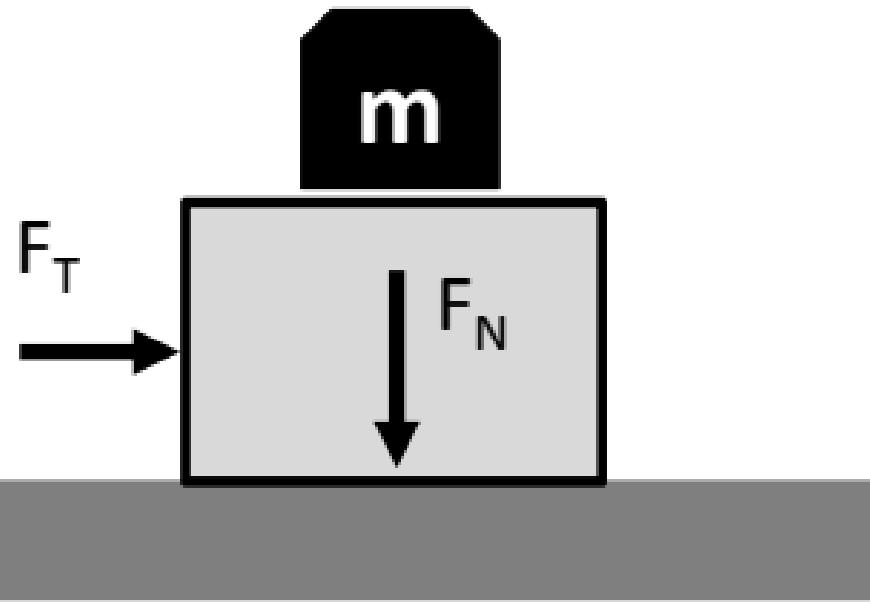


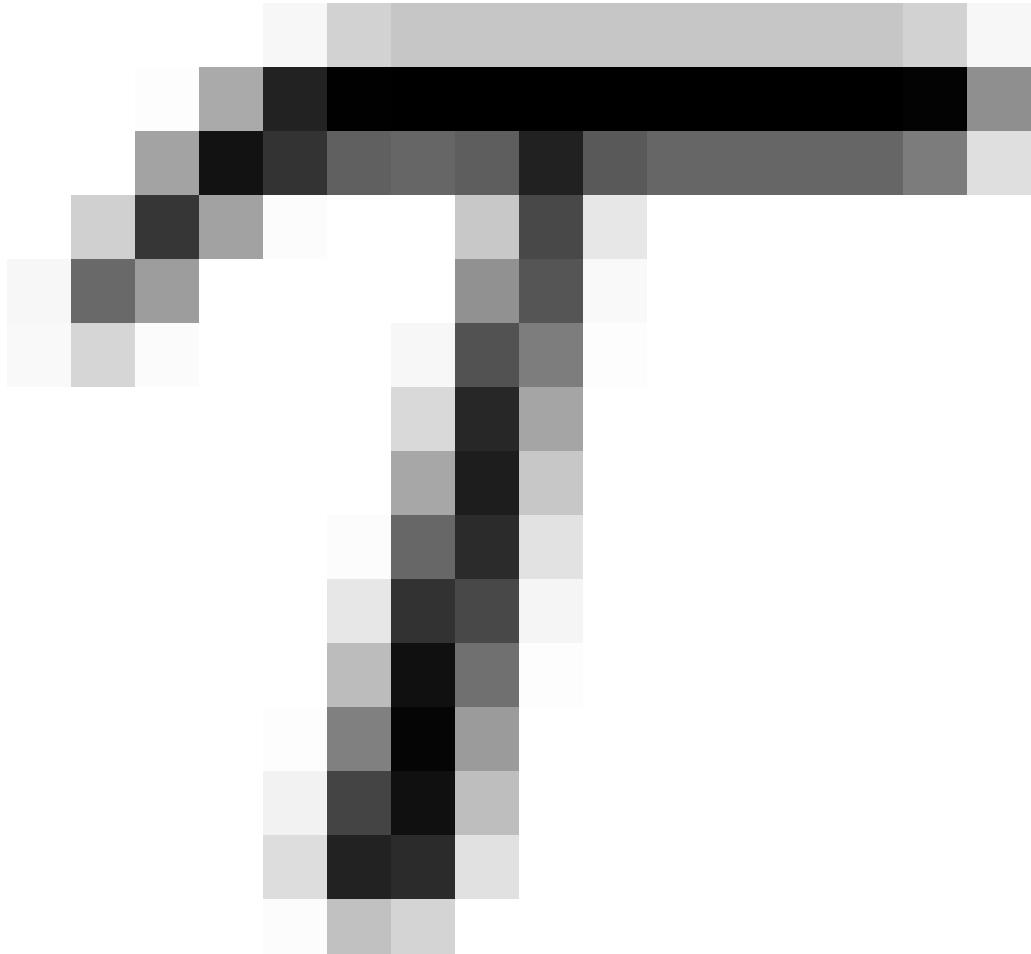


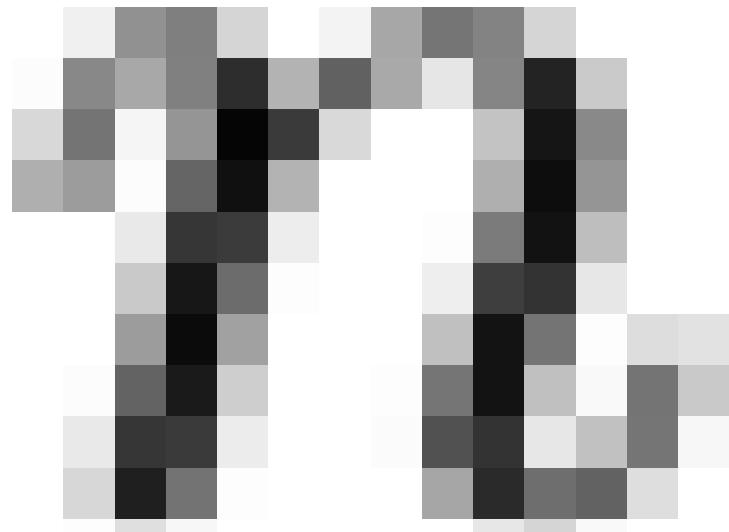
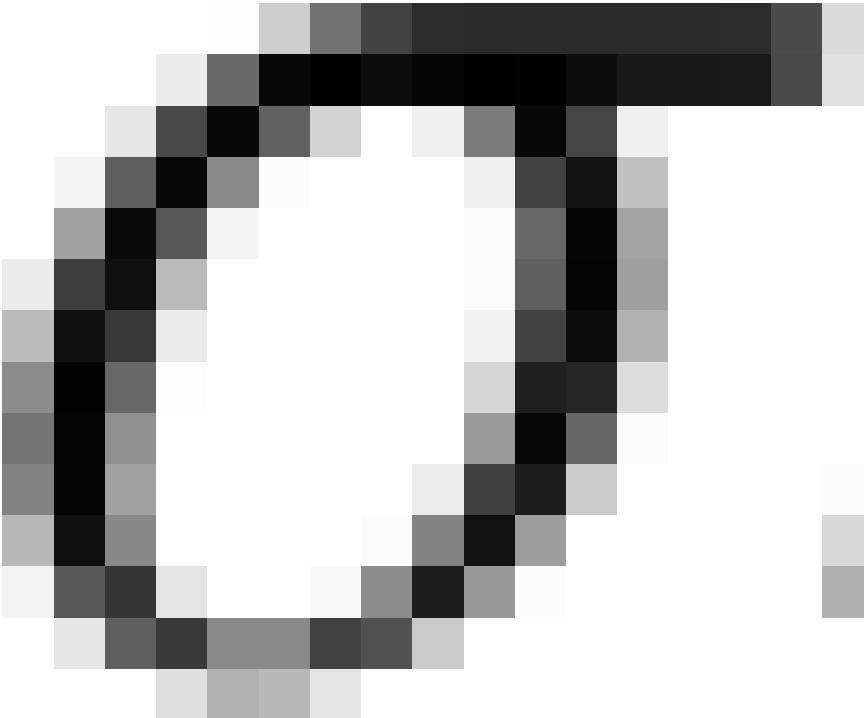




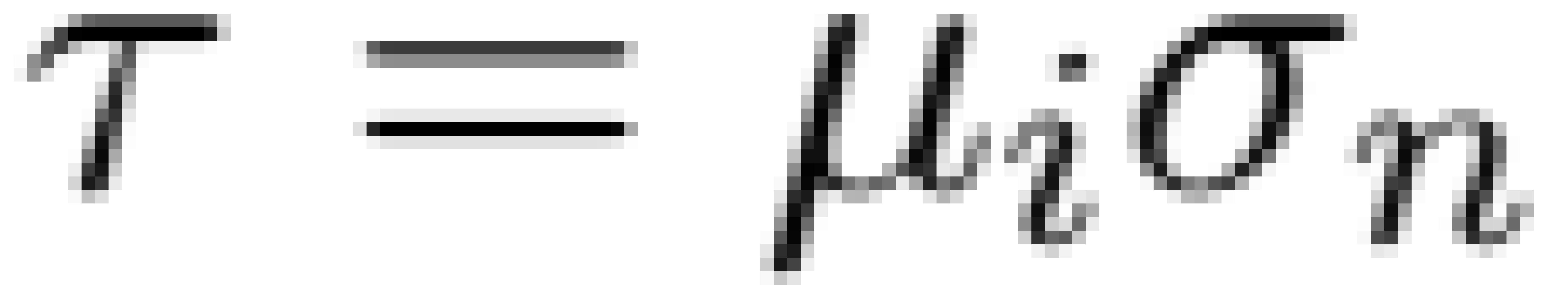


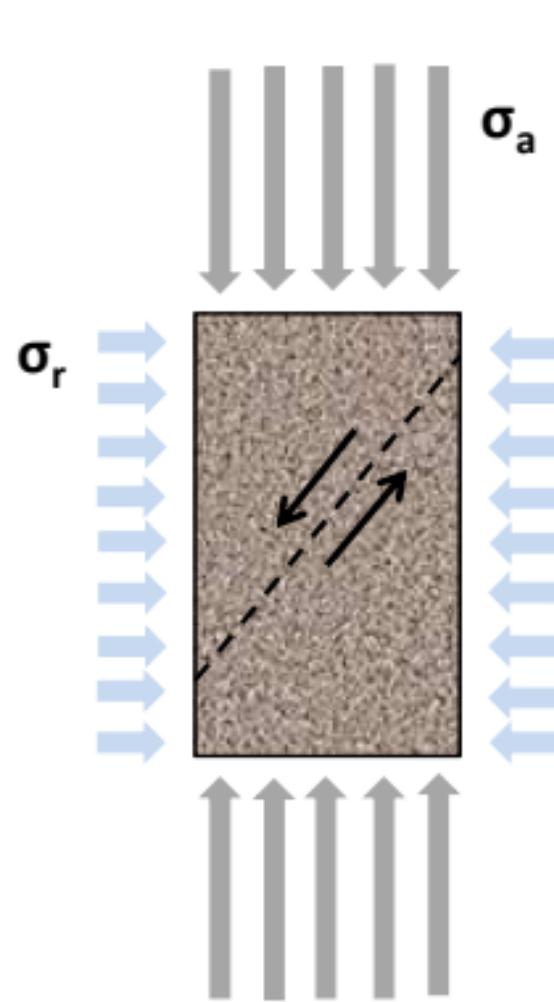




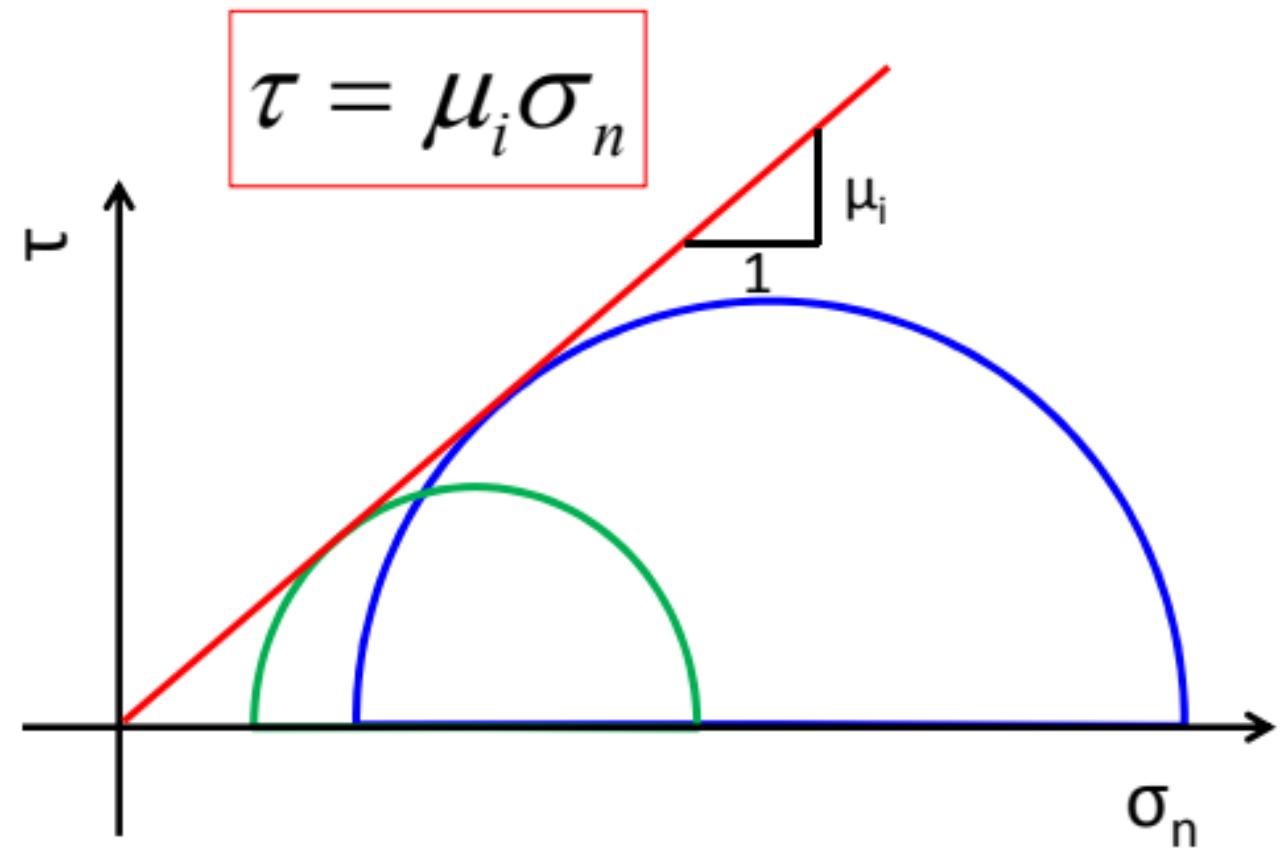


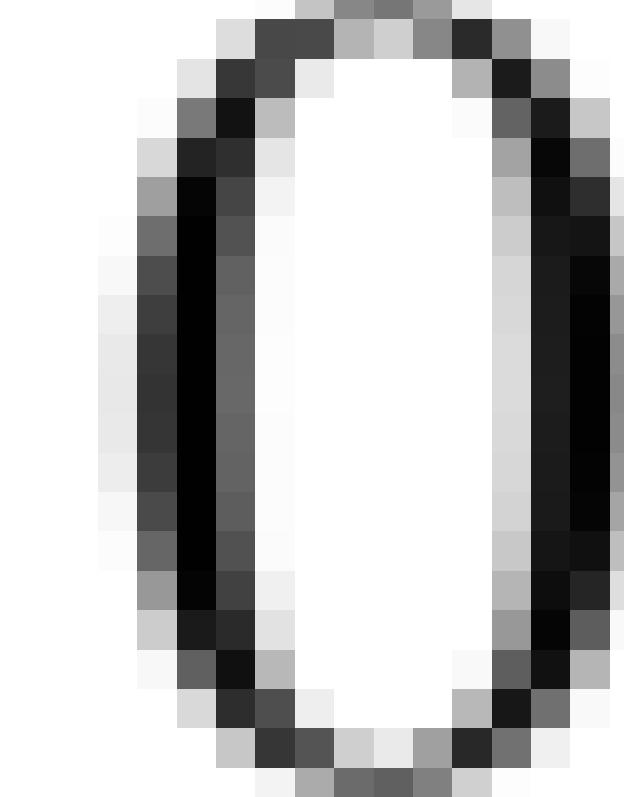
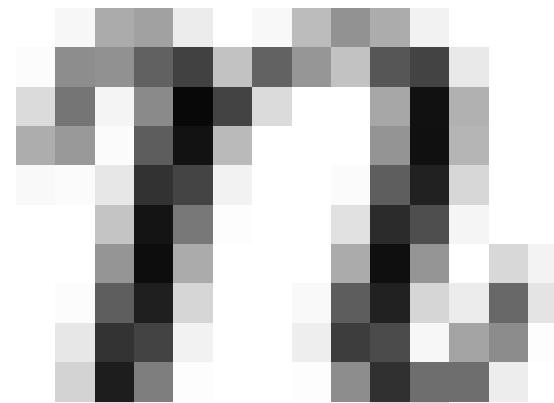
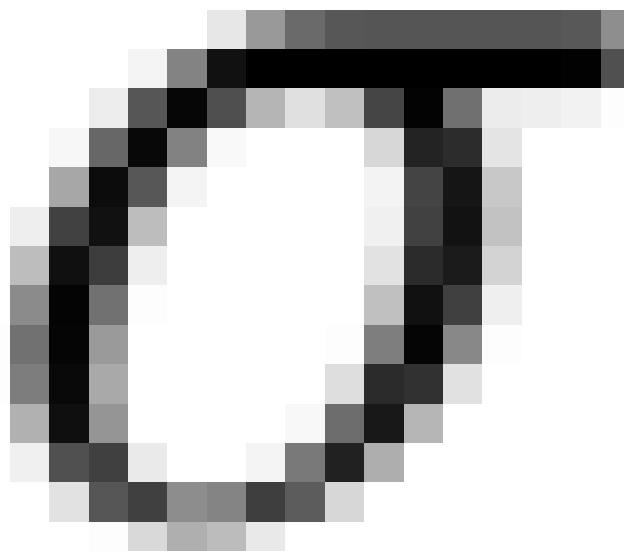


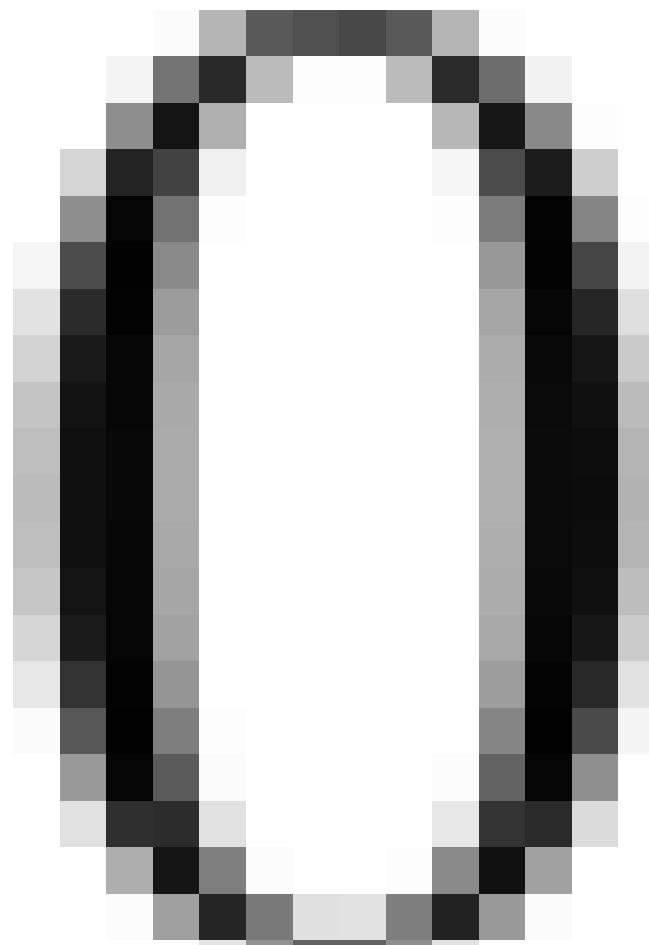
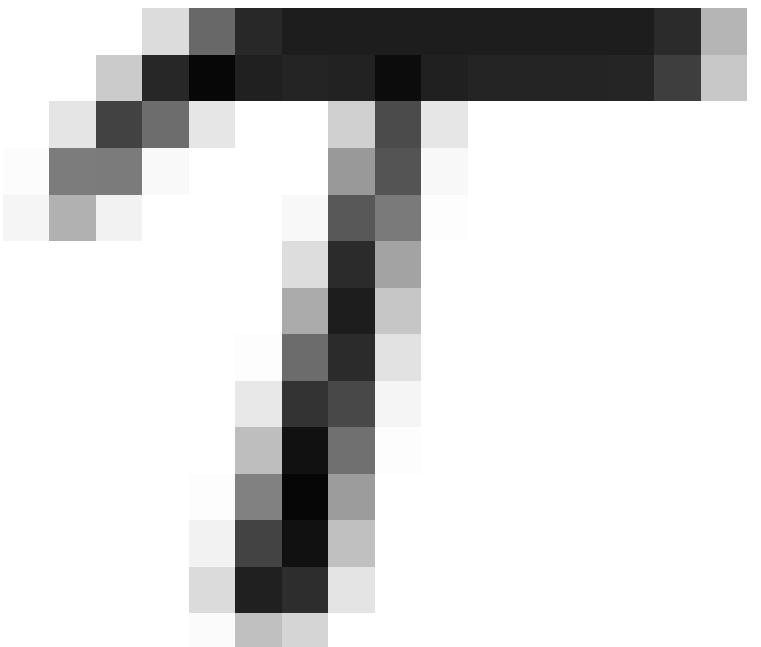


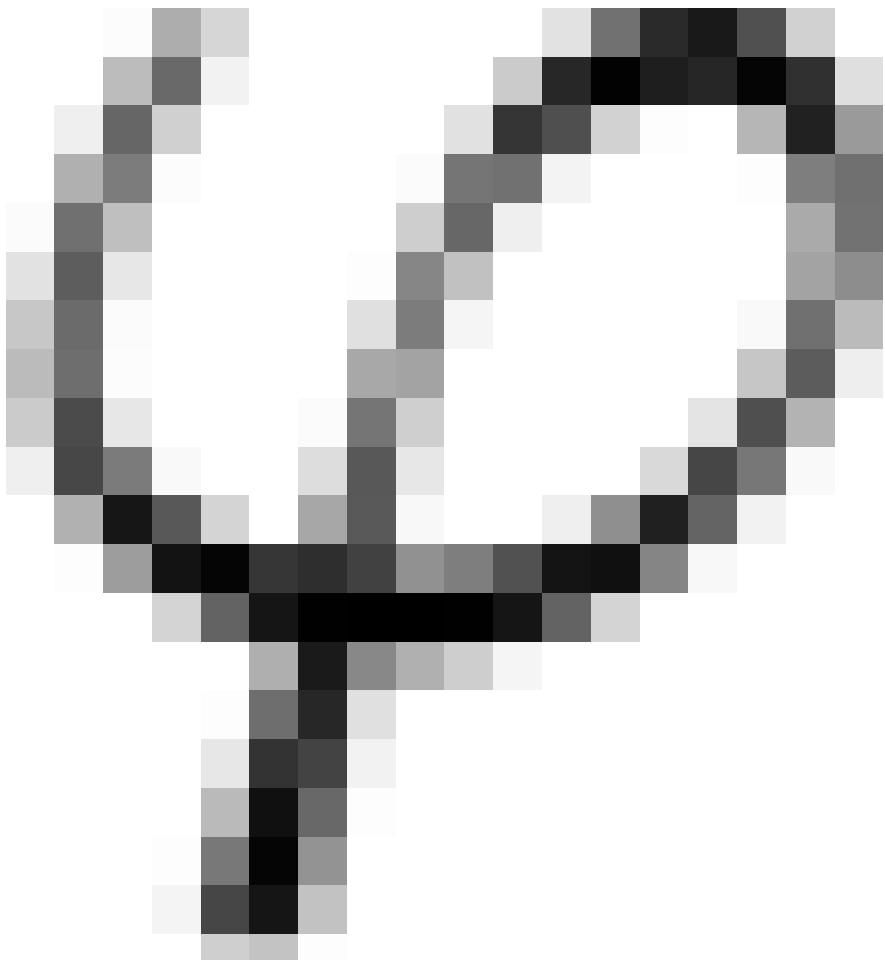


Unconsolidated Sand

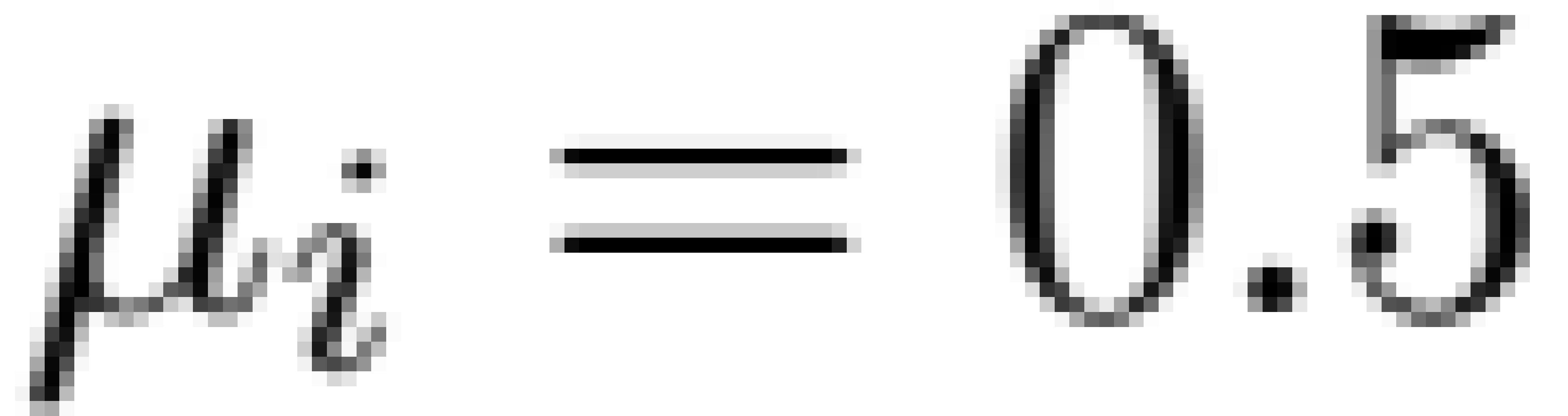


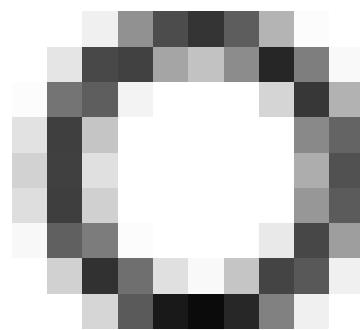
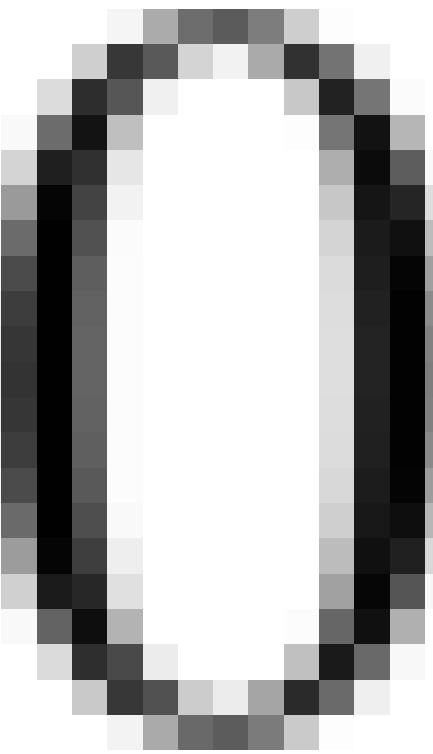
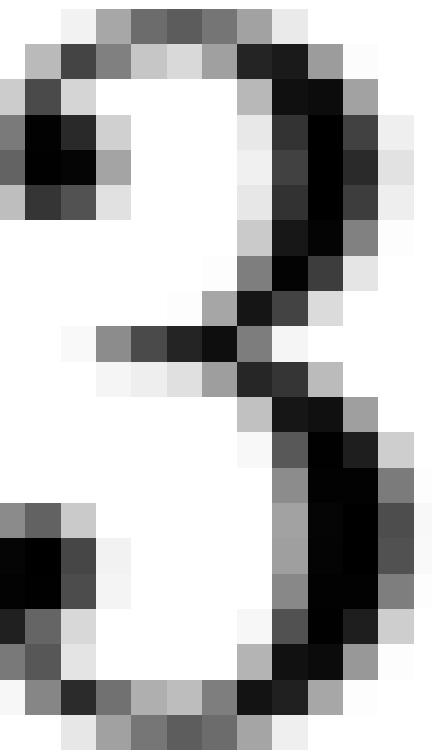
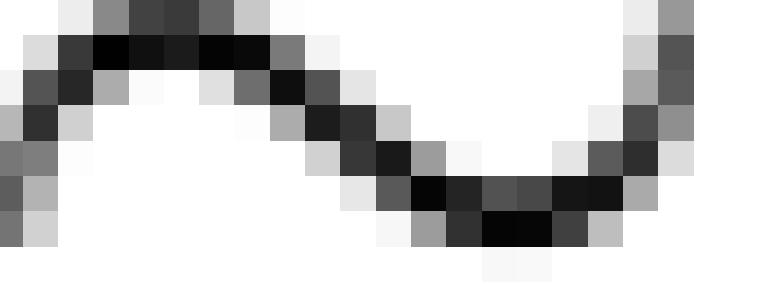
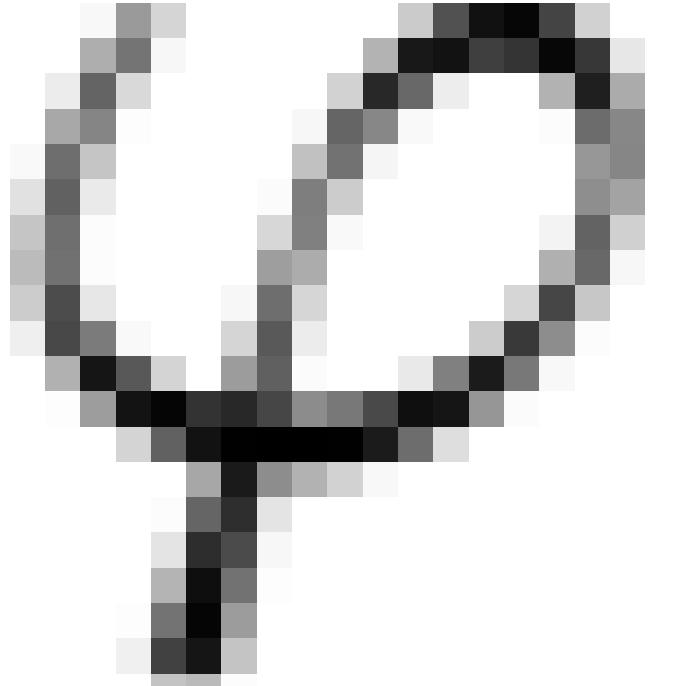


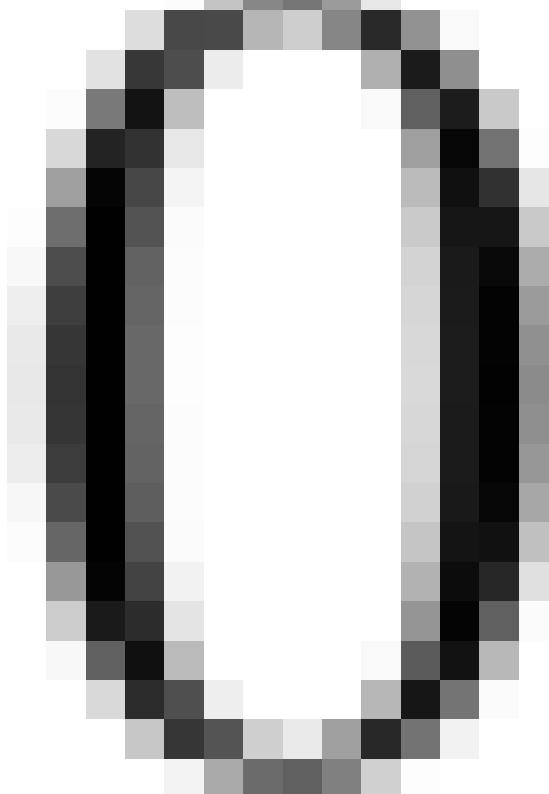
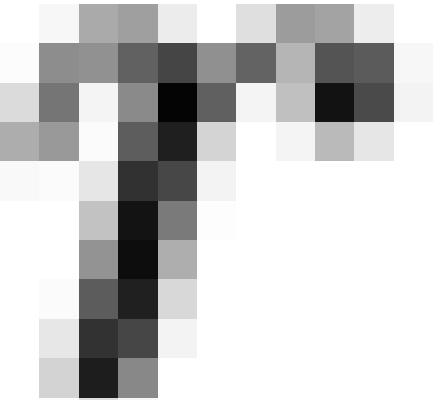
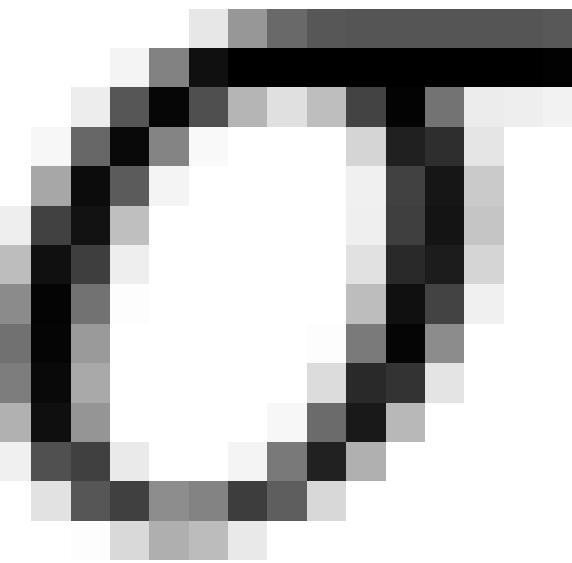




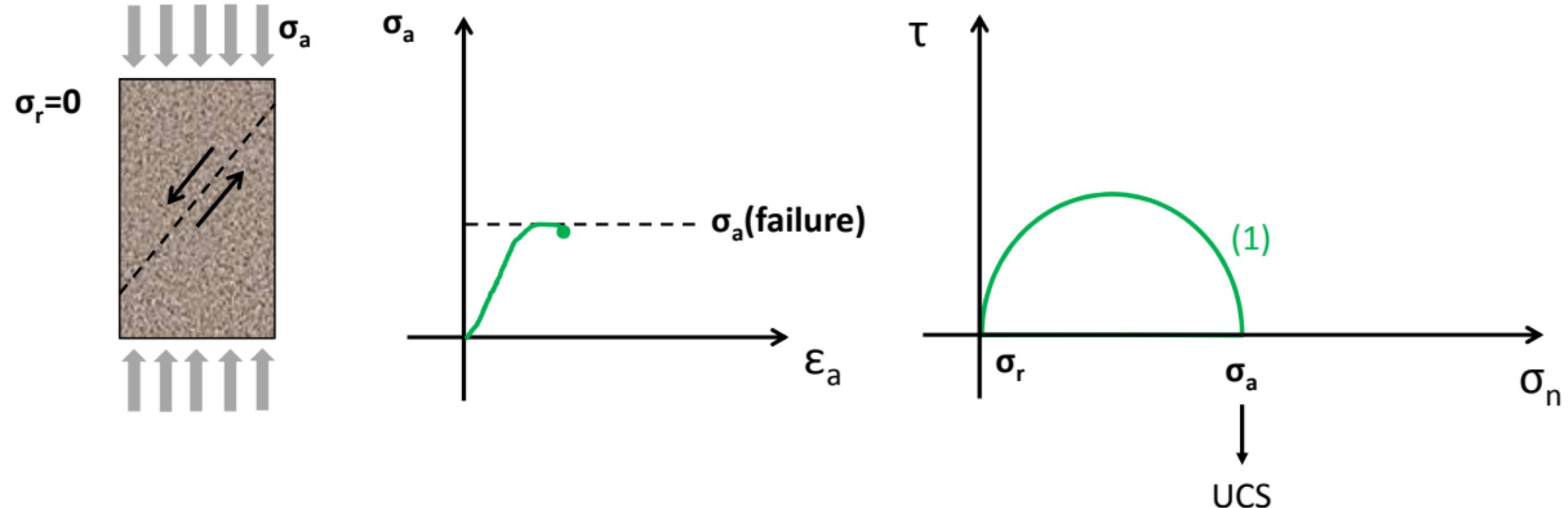


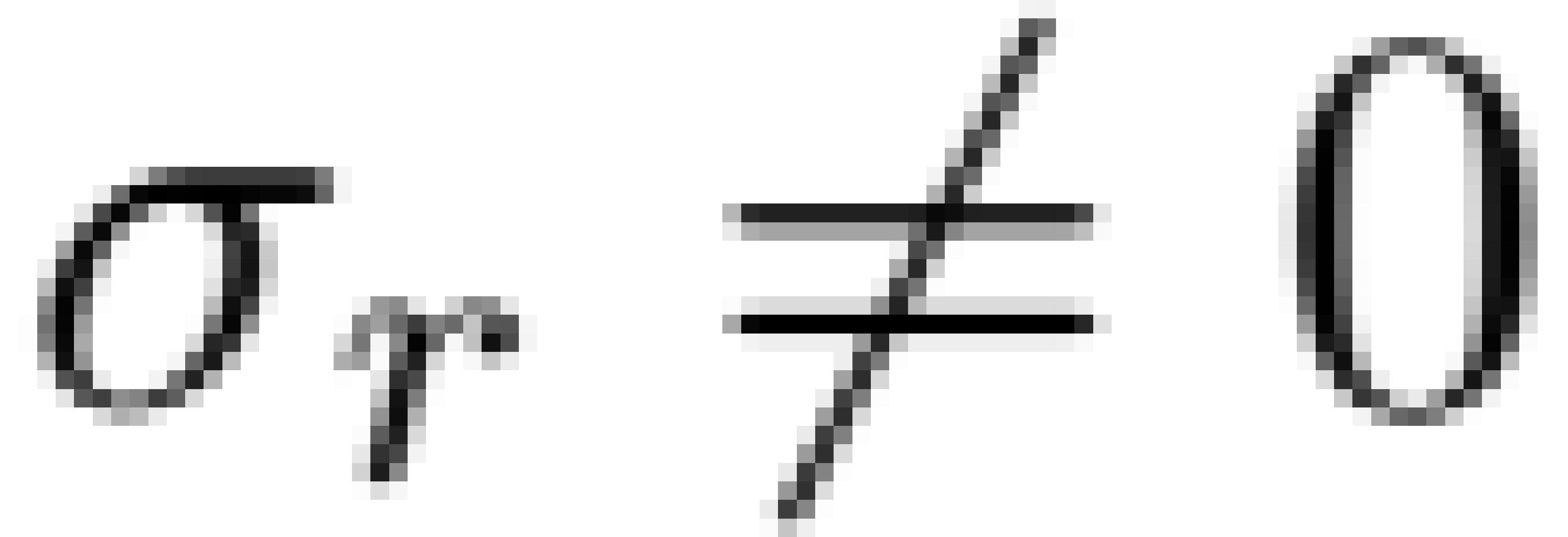


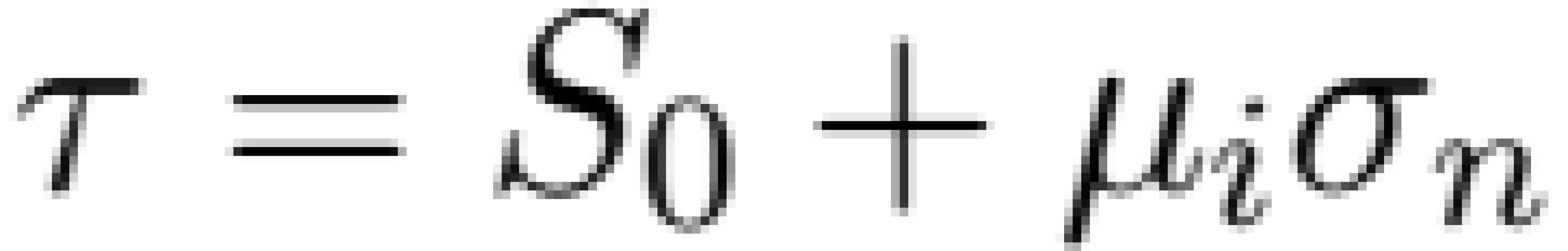




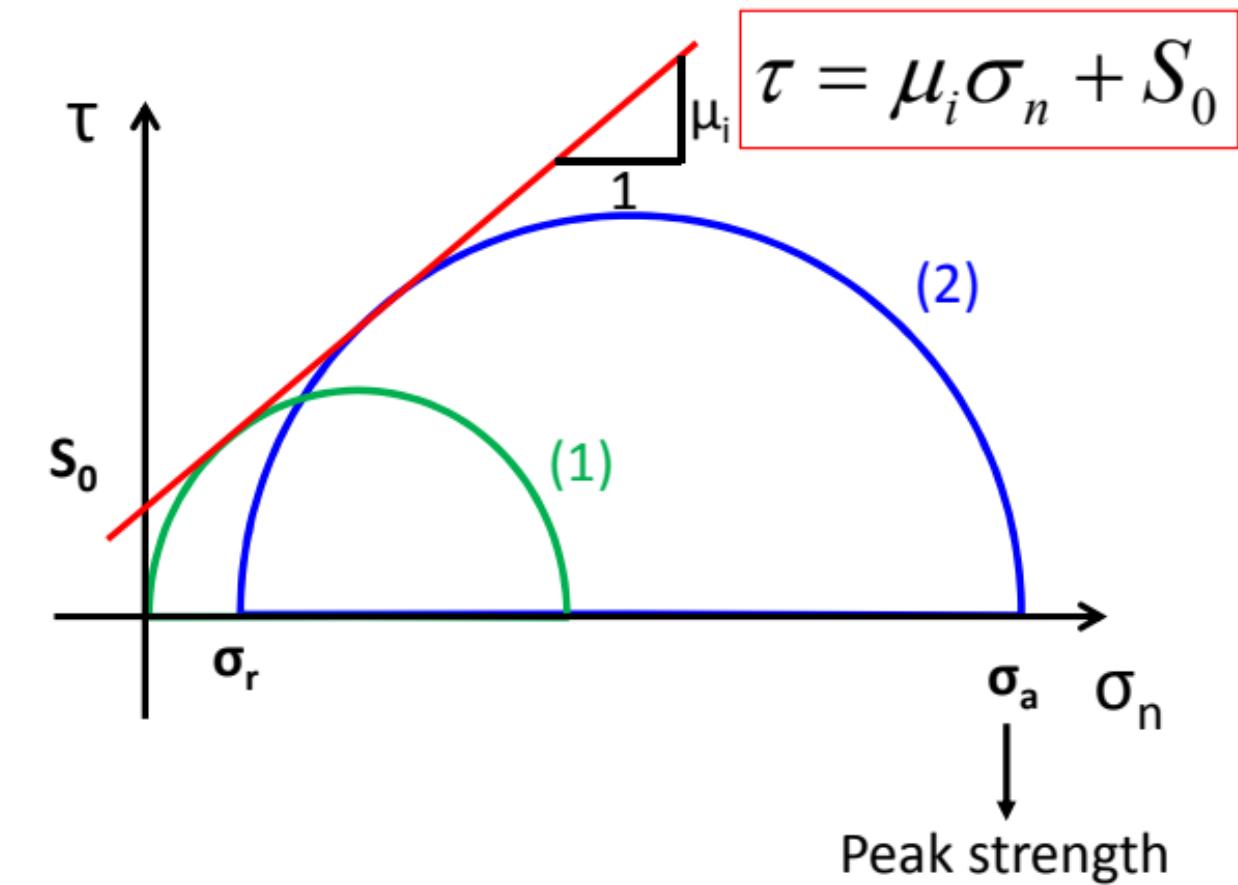
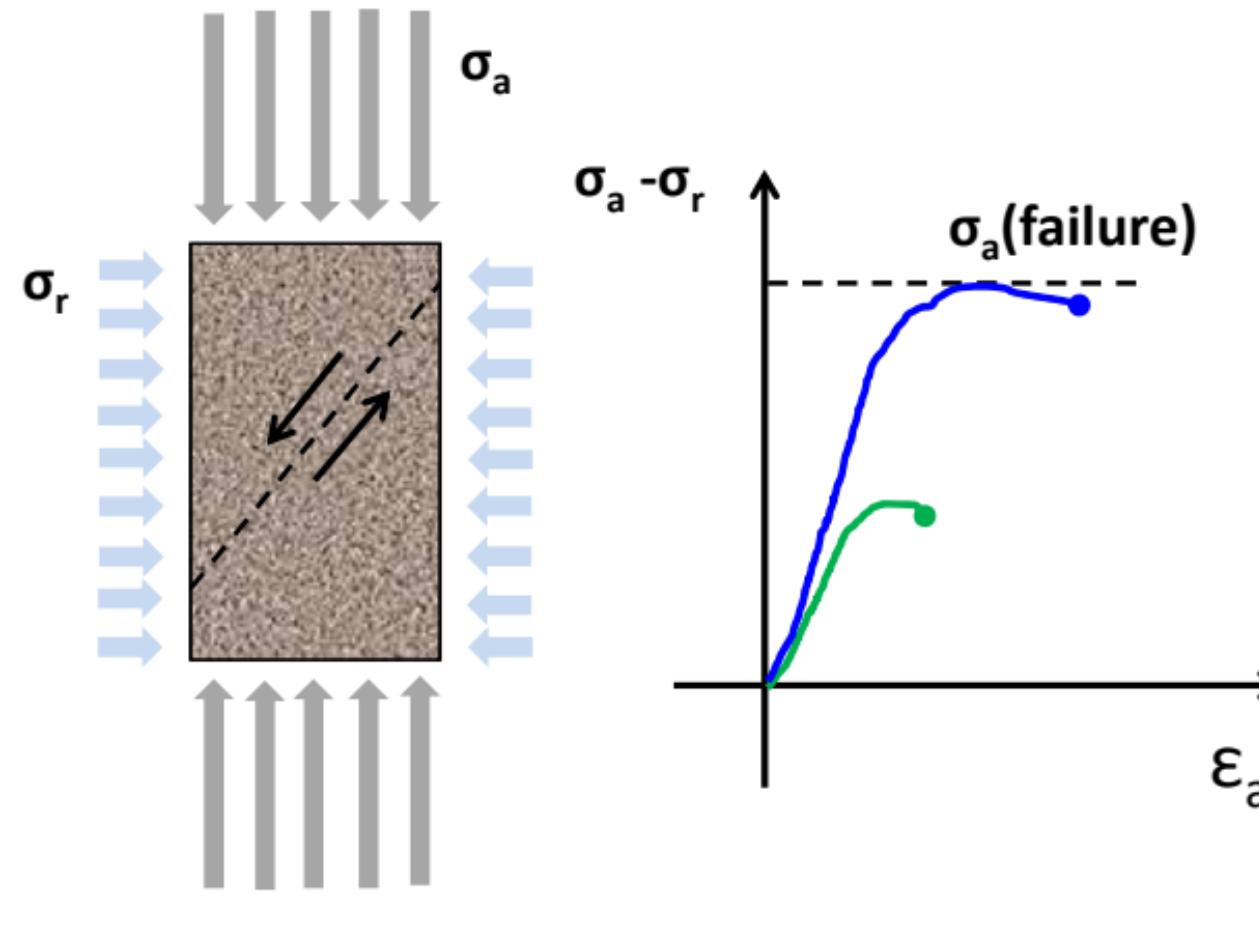
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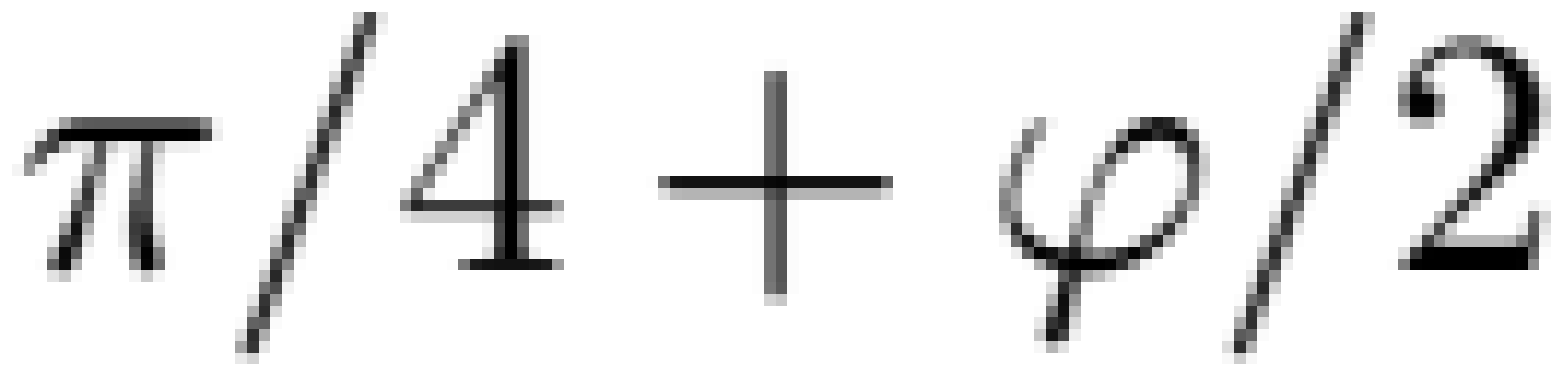






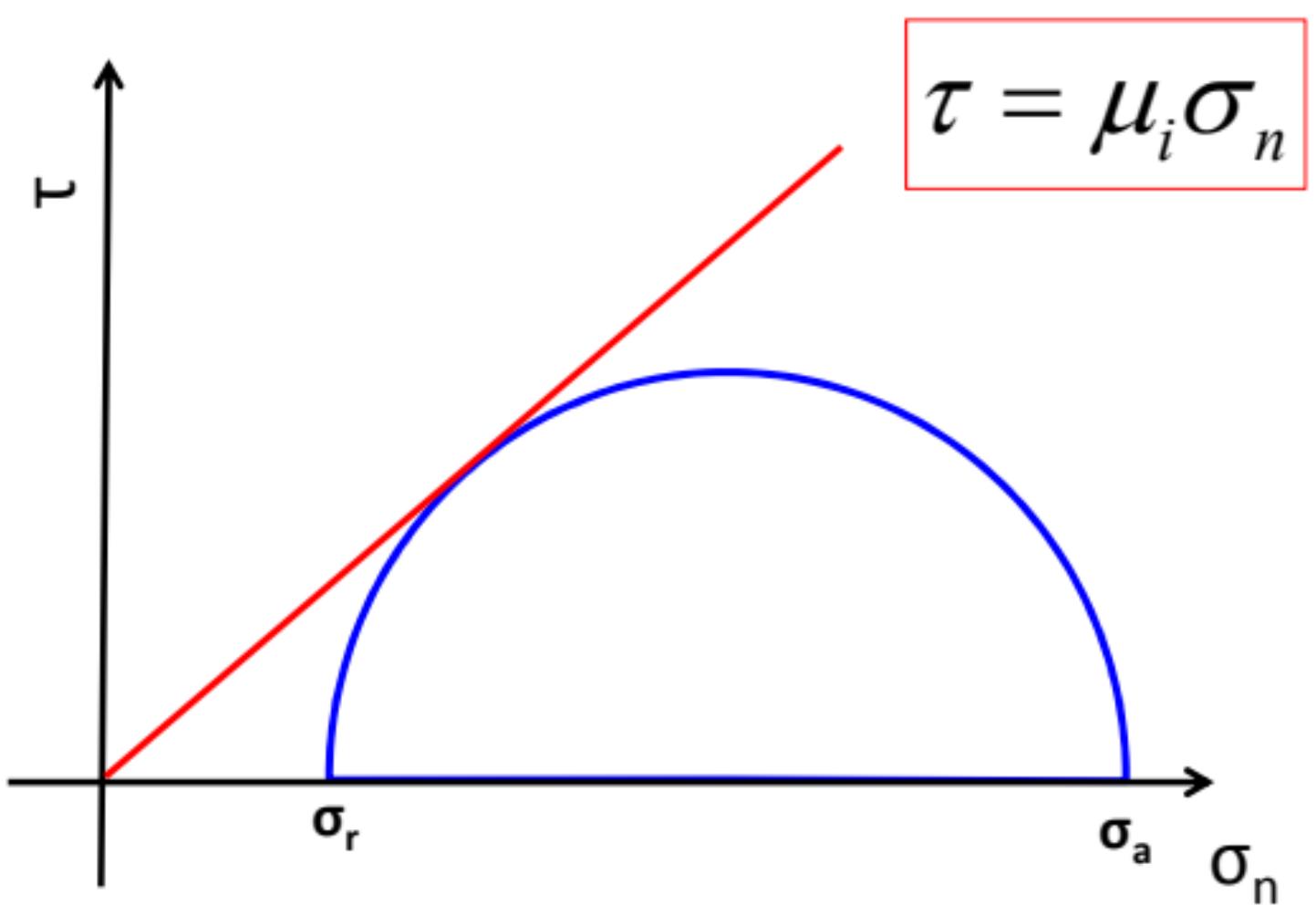
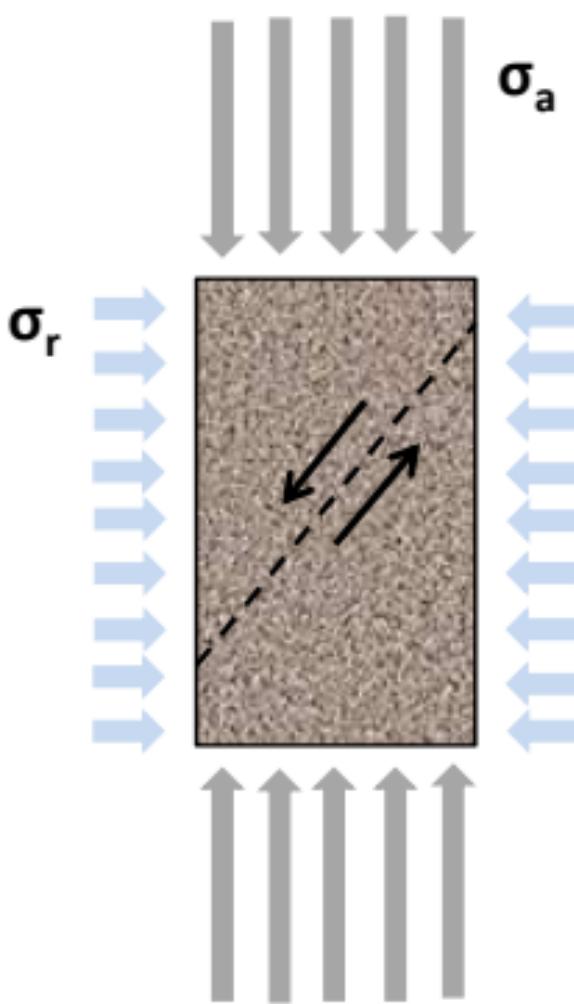
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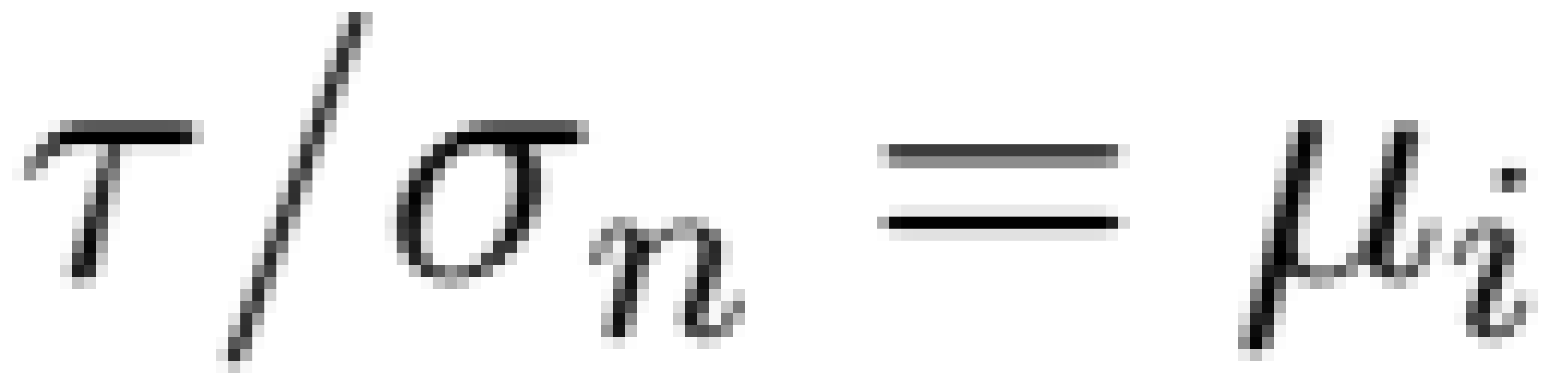


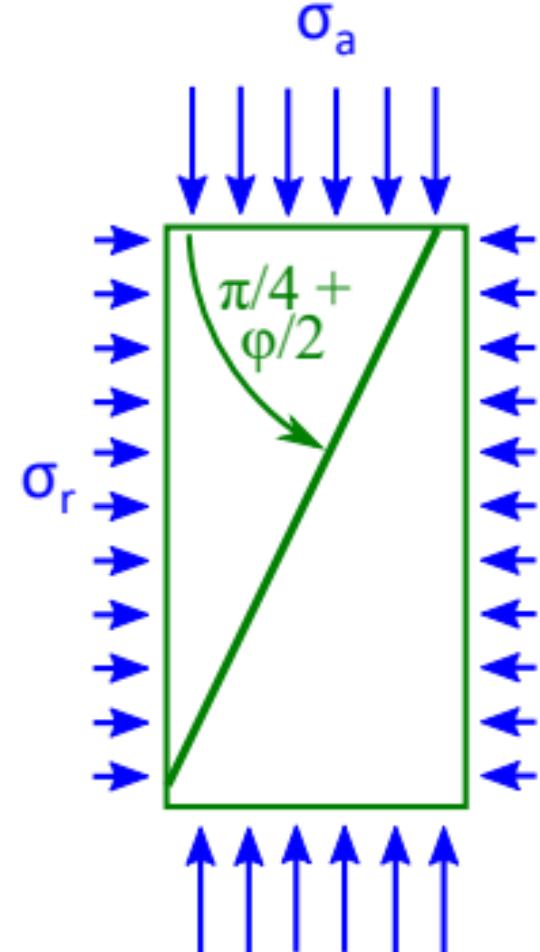
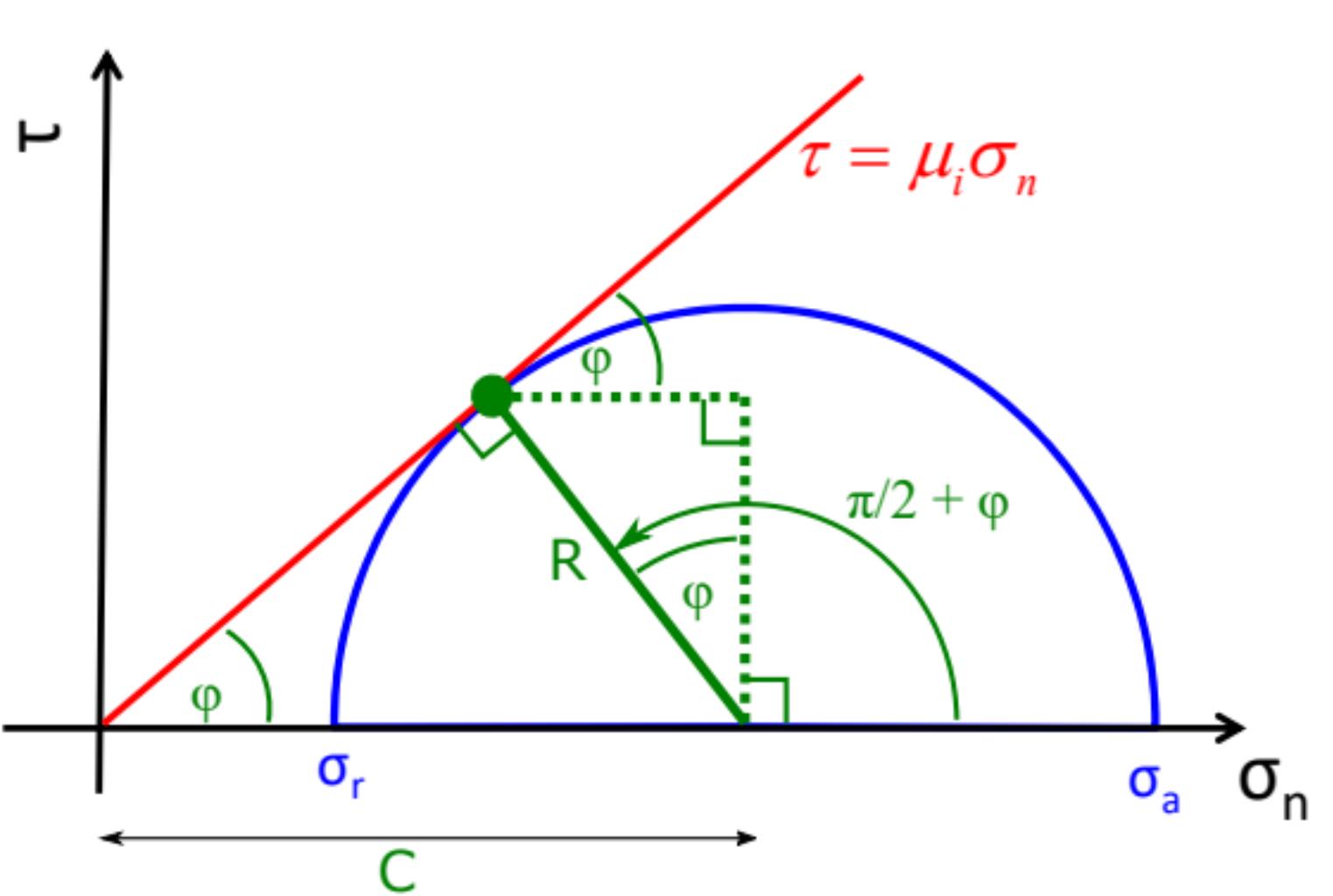








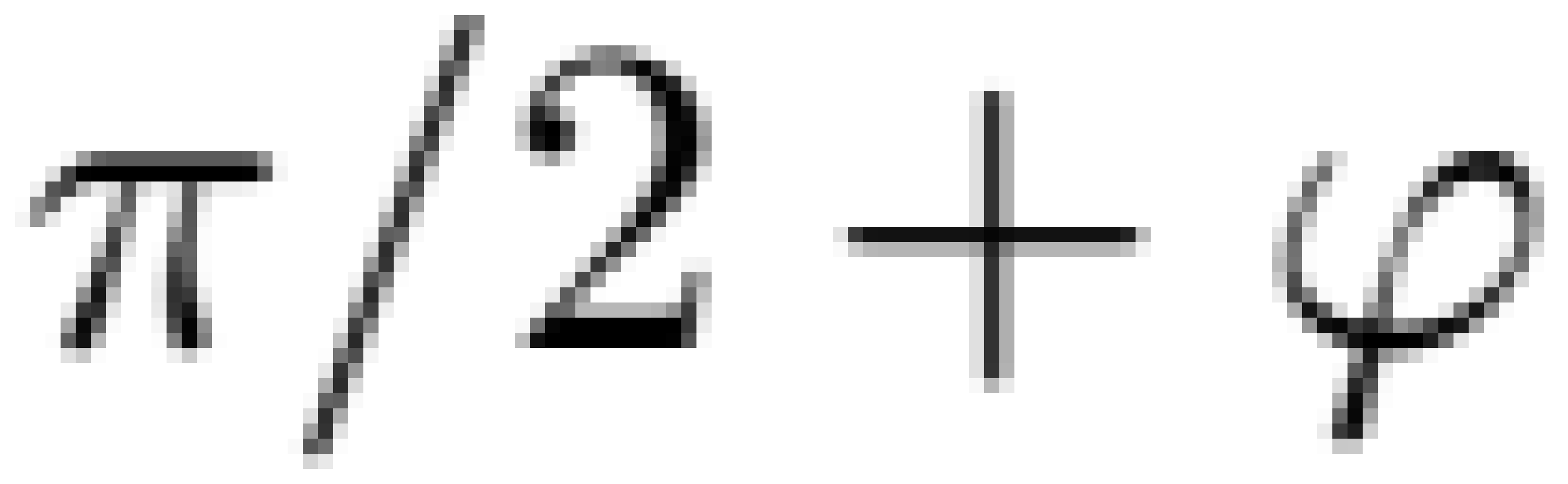


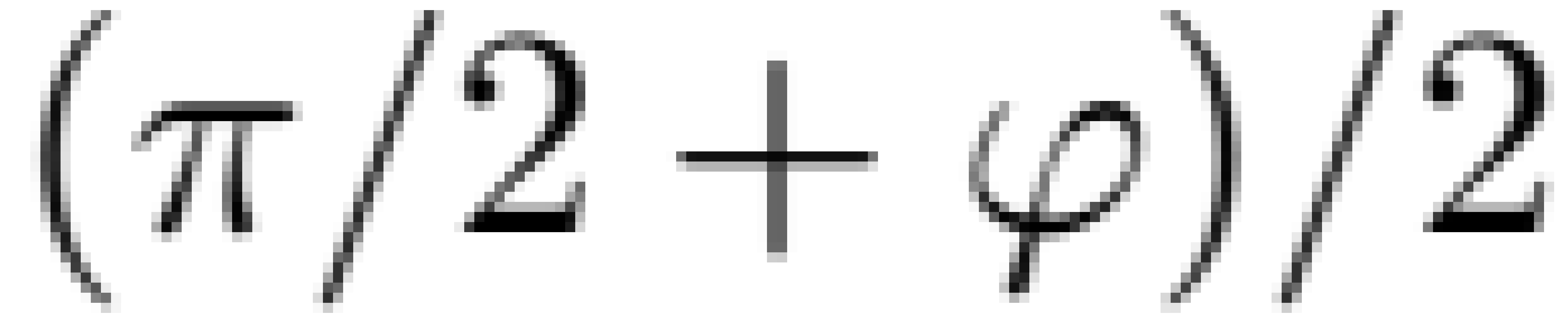


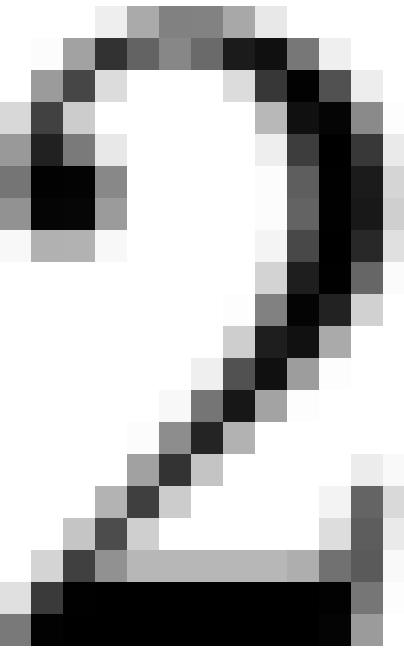
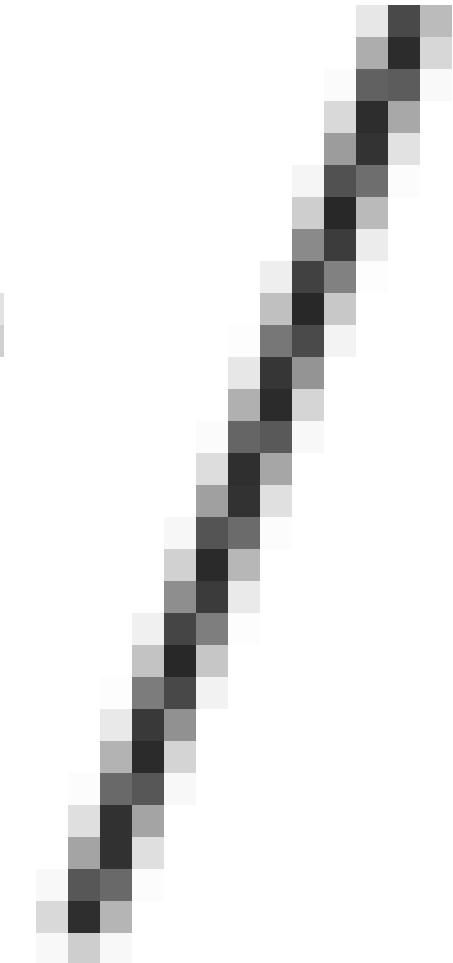
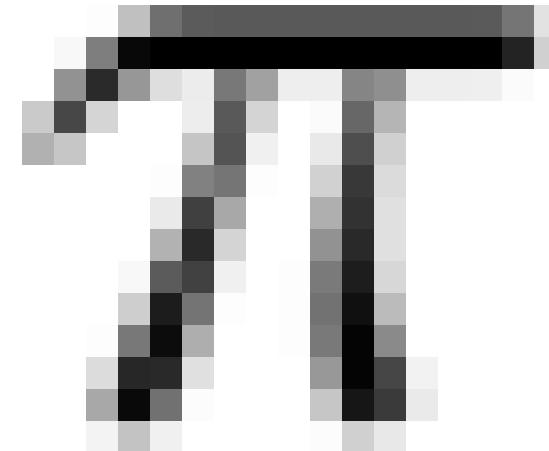
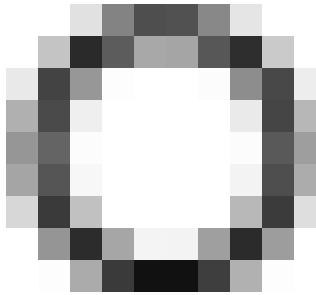


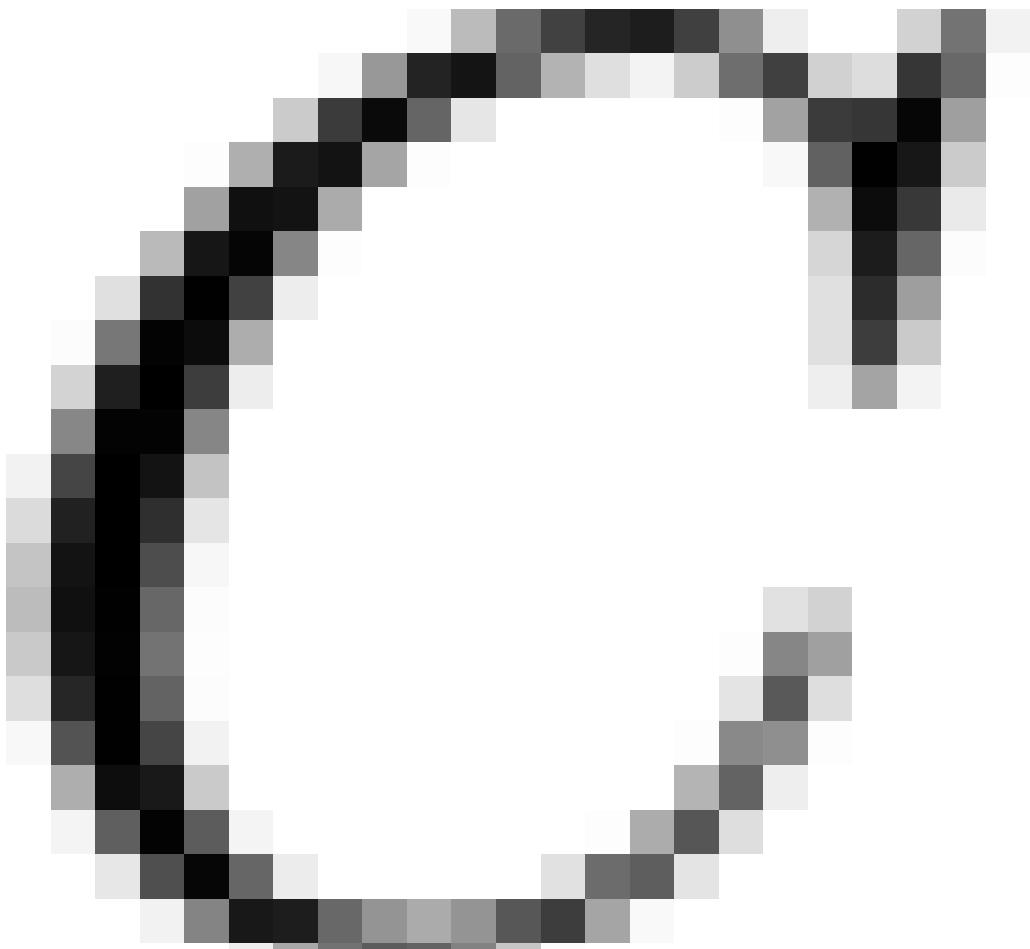






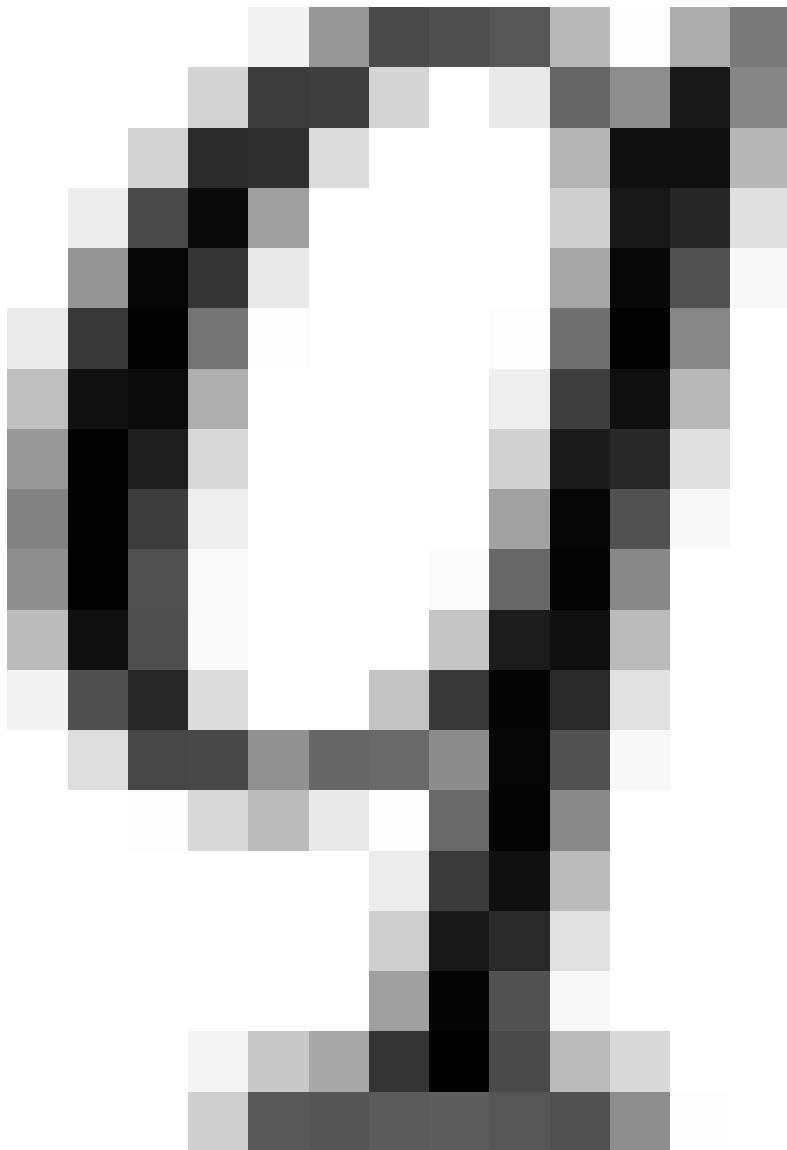


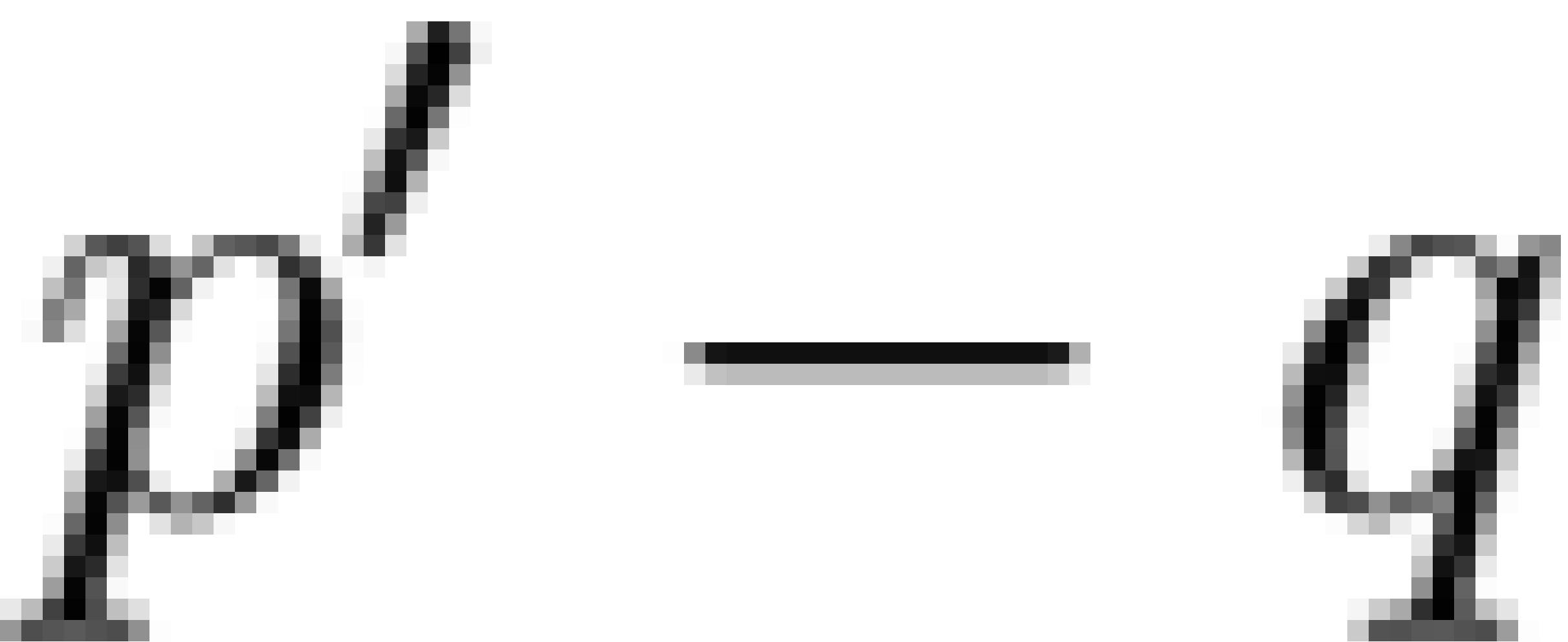


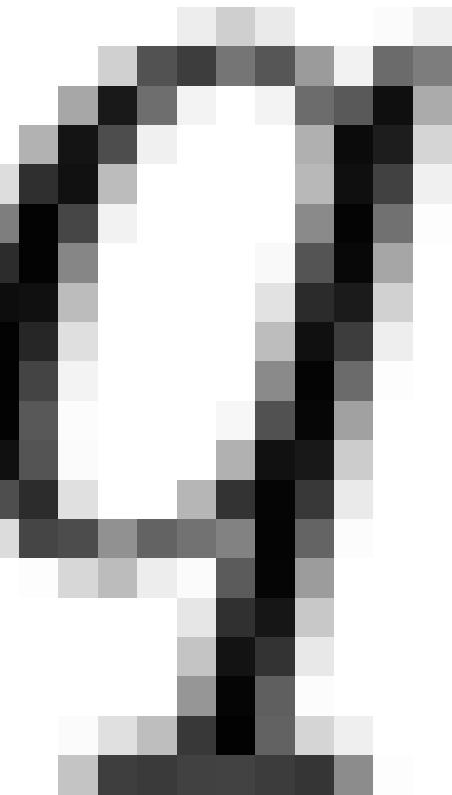
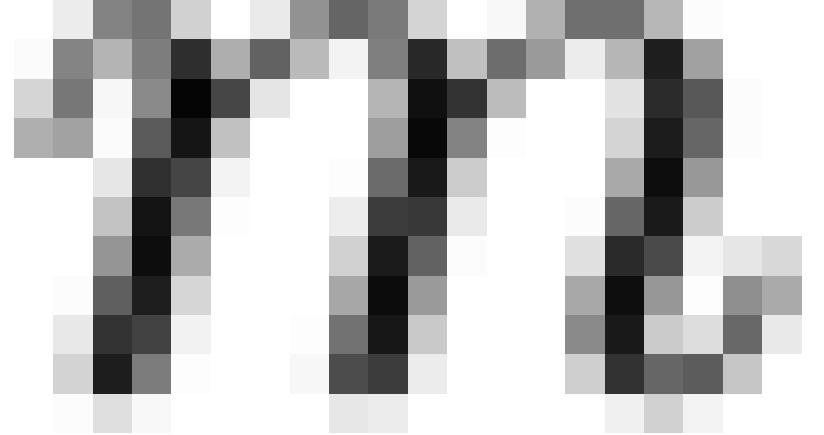
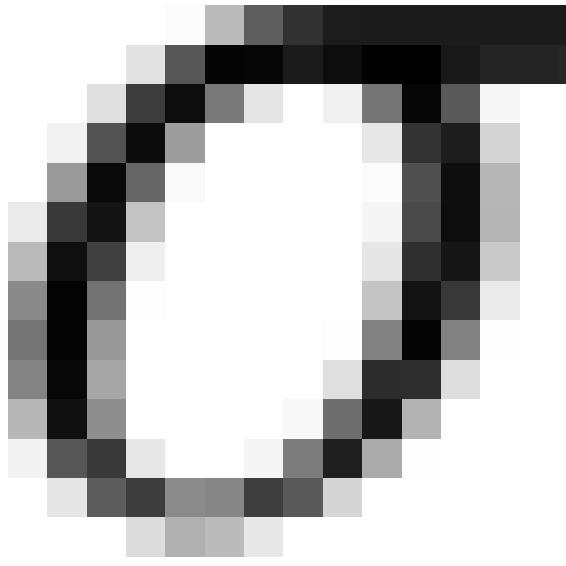


$$\begin{aligned} \sigma_a &= C + R \\ \sigma_r &= C - R \end{aligned}$$
$$\begin{aligned} C + \sin \varphi &= 1 + \sin \varphi \\ C - \sin \varphi &= 1 - \sin \varphi \end{aligned}$$



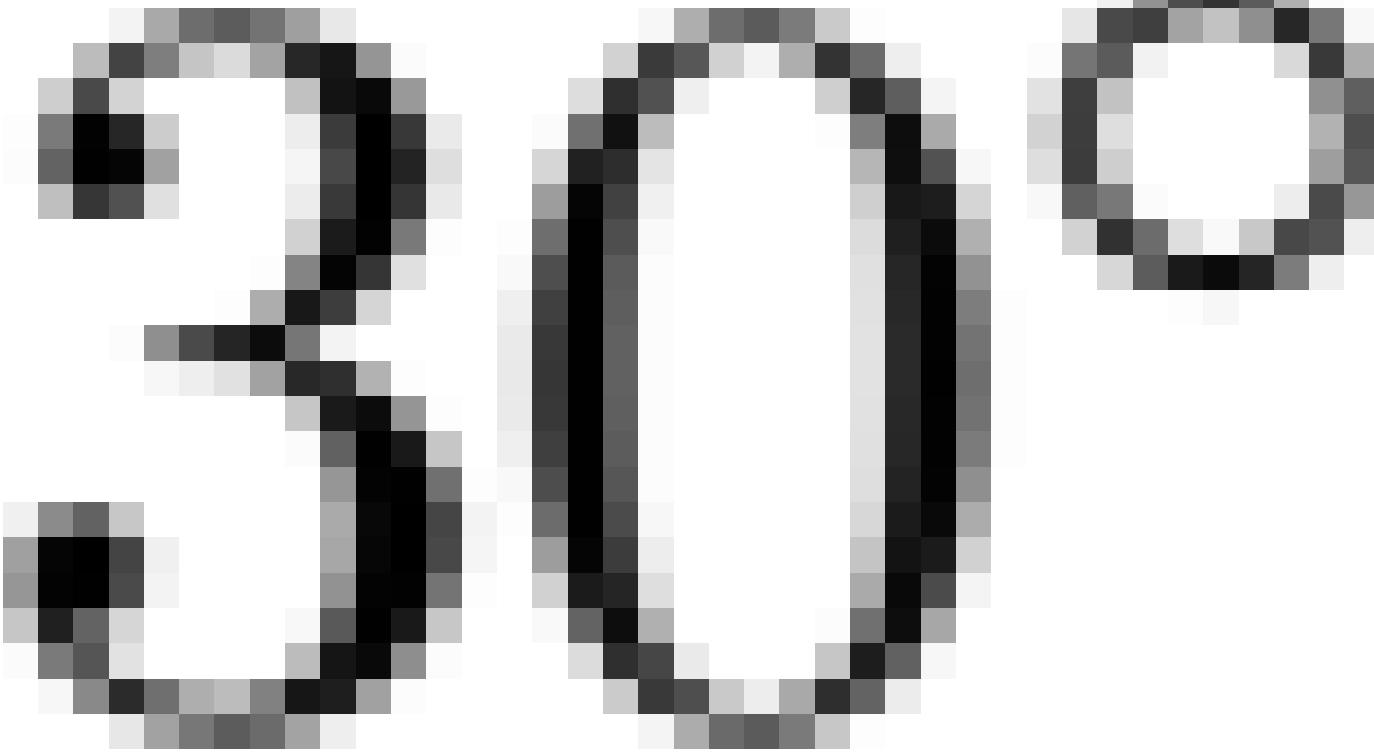
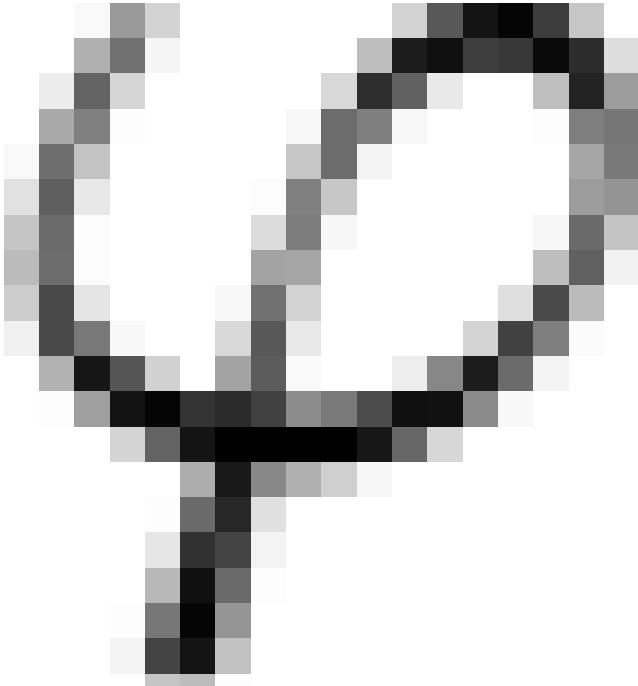


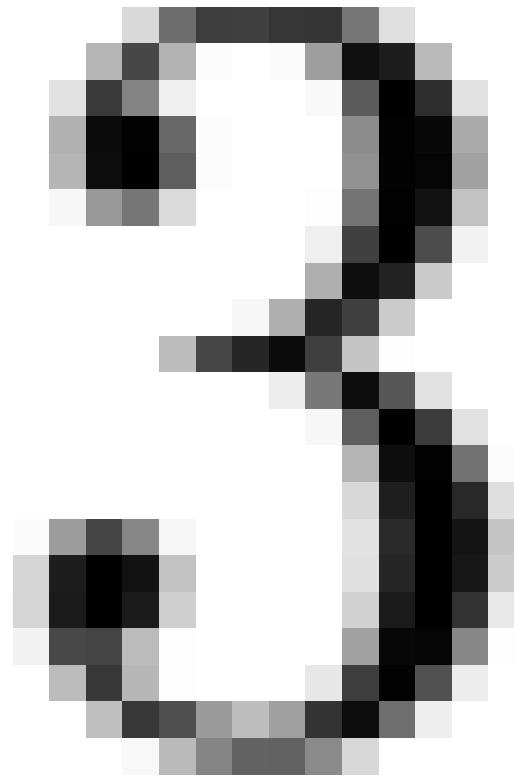
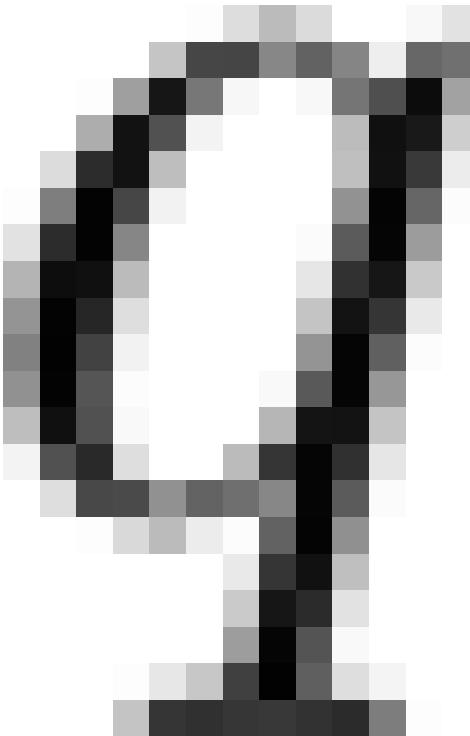




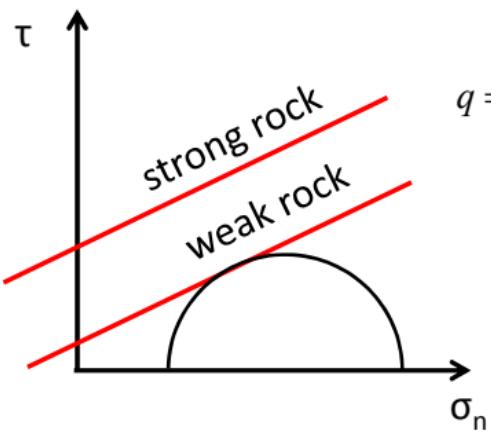
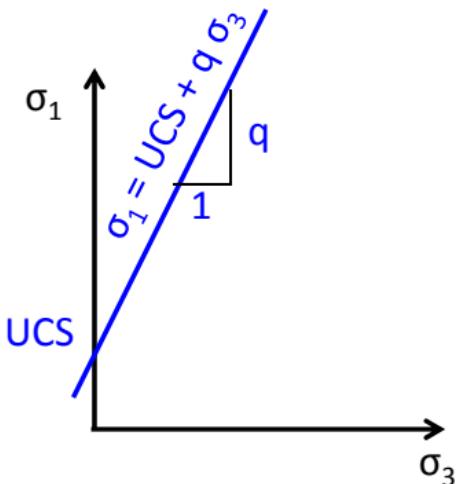
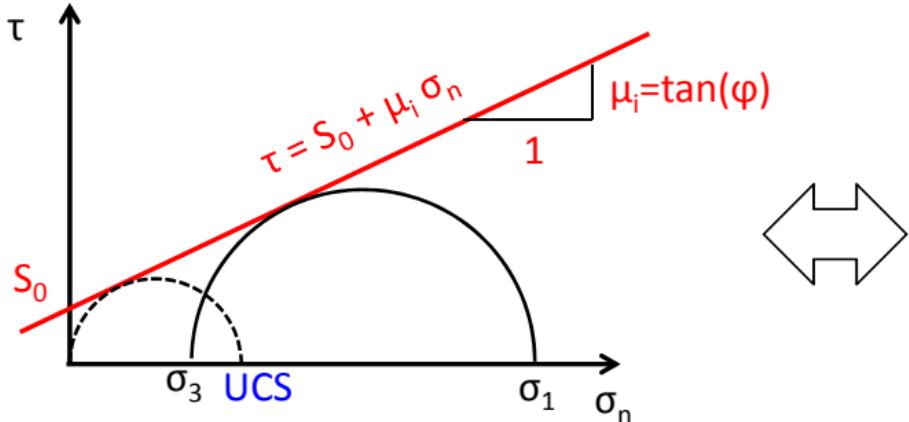
φ sin φ

φ sin φ



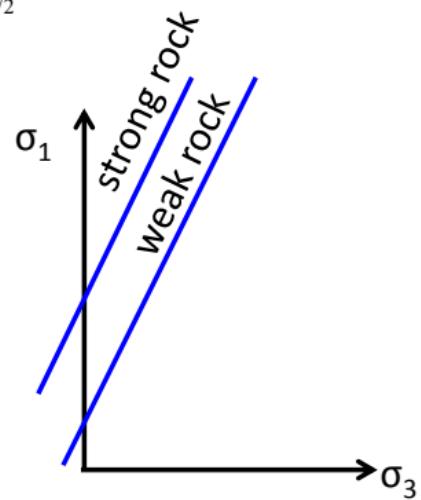


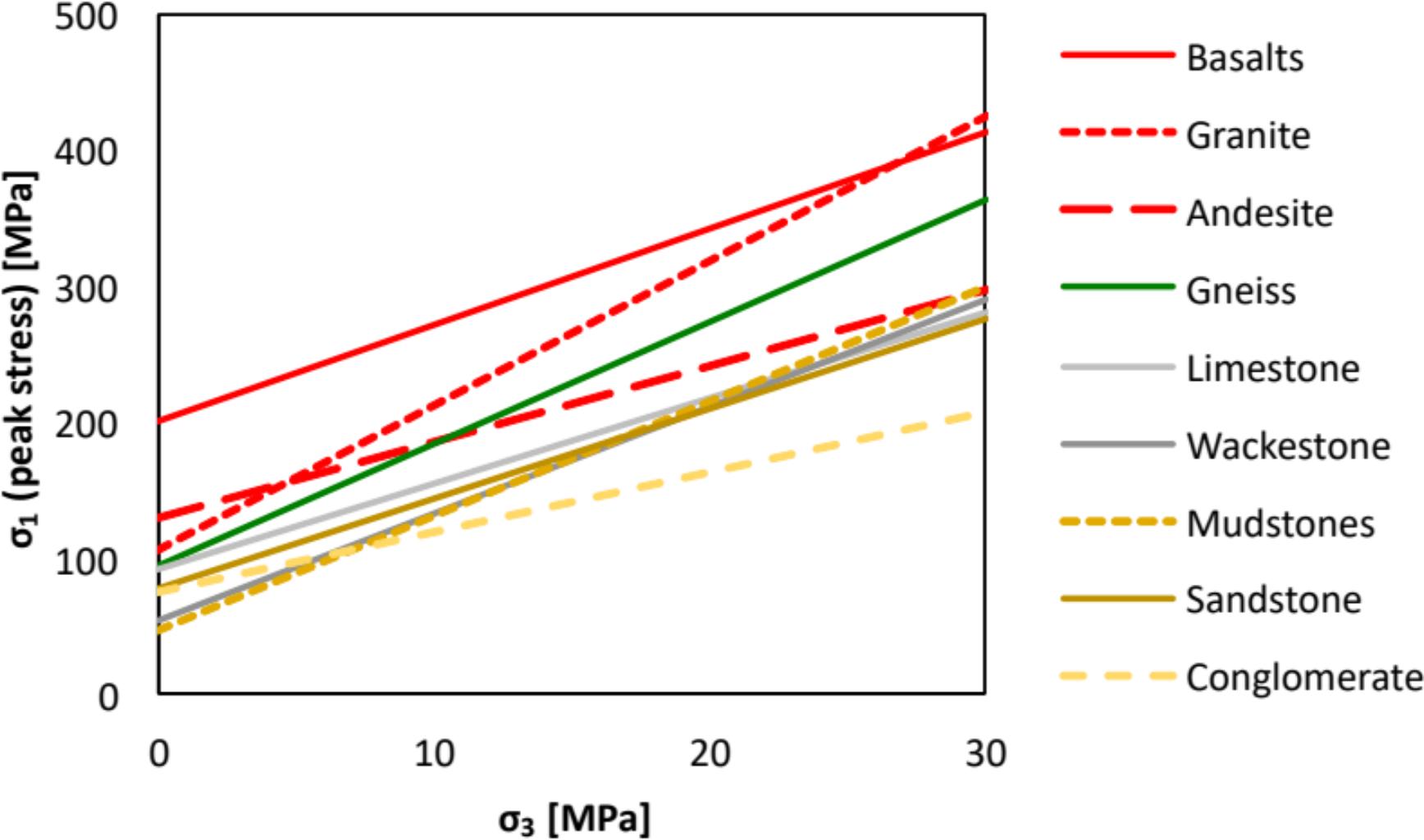
$$U_0 S = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2} = 2S_0 \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi}}$$



$$UCS = 2S_0 \left(\sqrt{\mu_i^2 + 1} + \mu_i \right) = 2S_0 \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right)^{1/2}$$

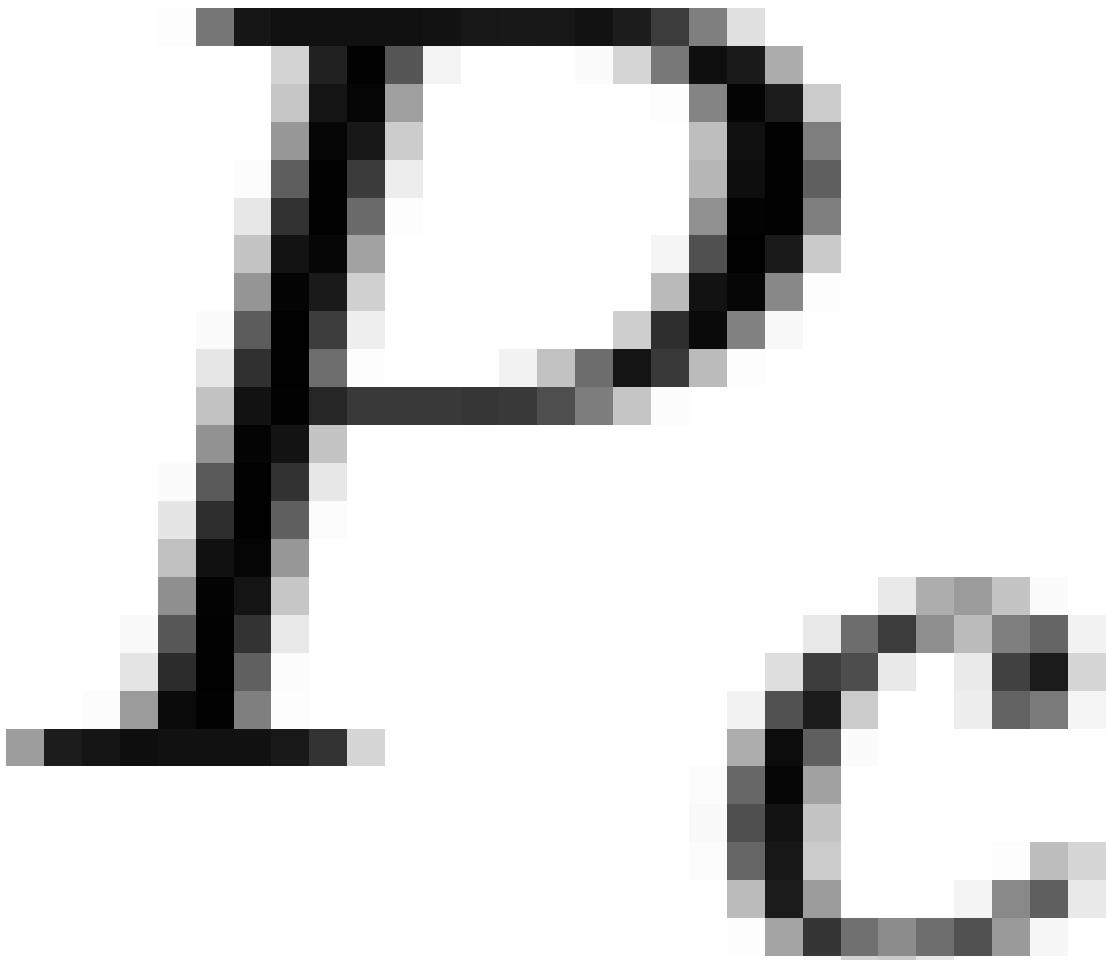
$$q = \left(\sqrt{\mu_i^2 + 1} + \mu_i \right)^2 = \frac{1 + \sin \varphi}{1 - \sin \varphi}$$

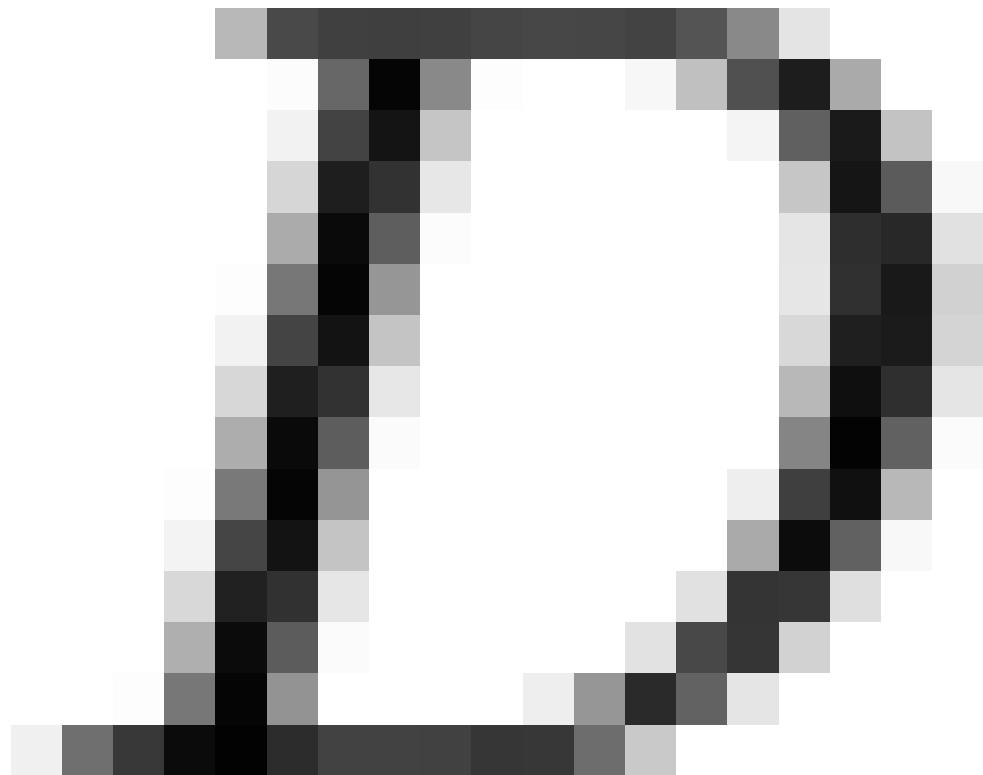
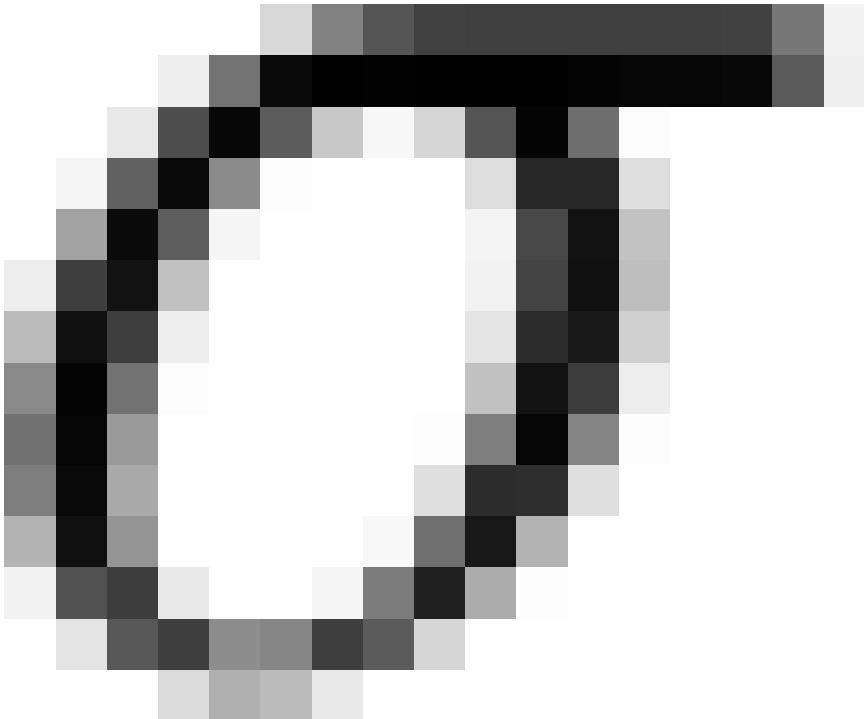




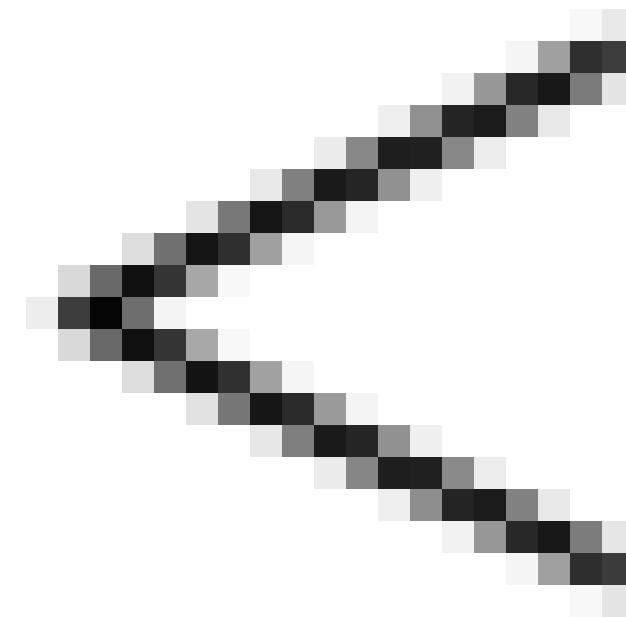












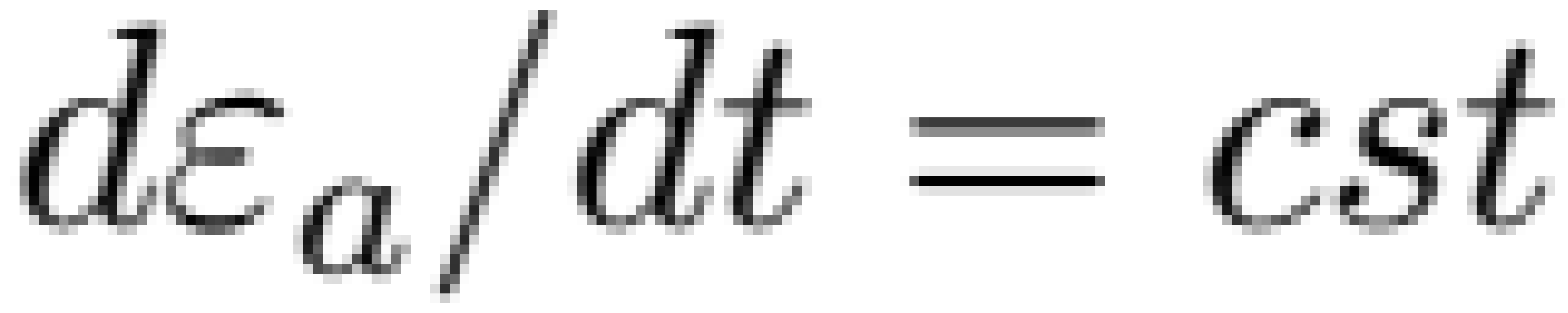










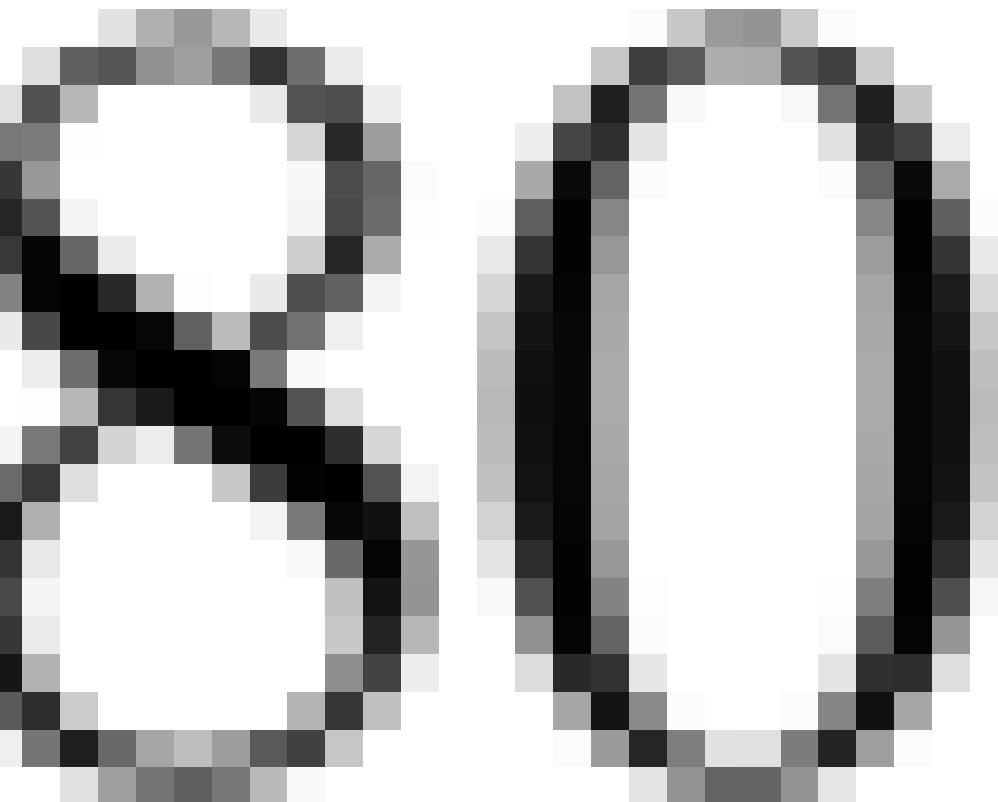
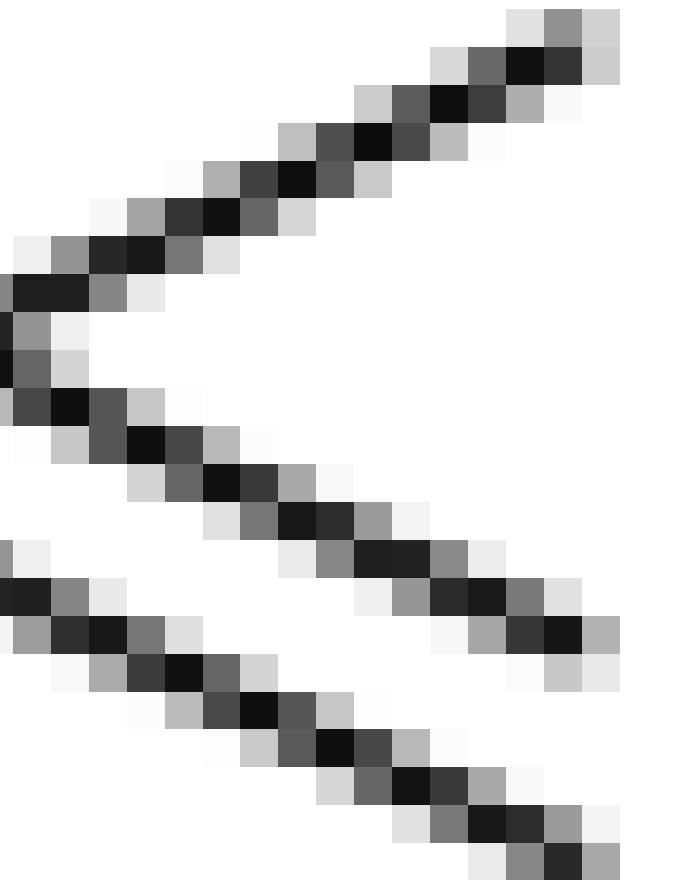


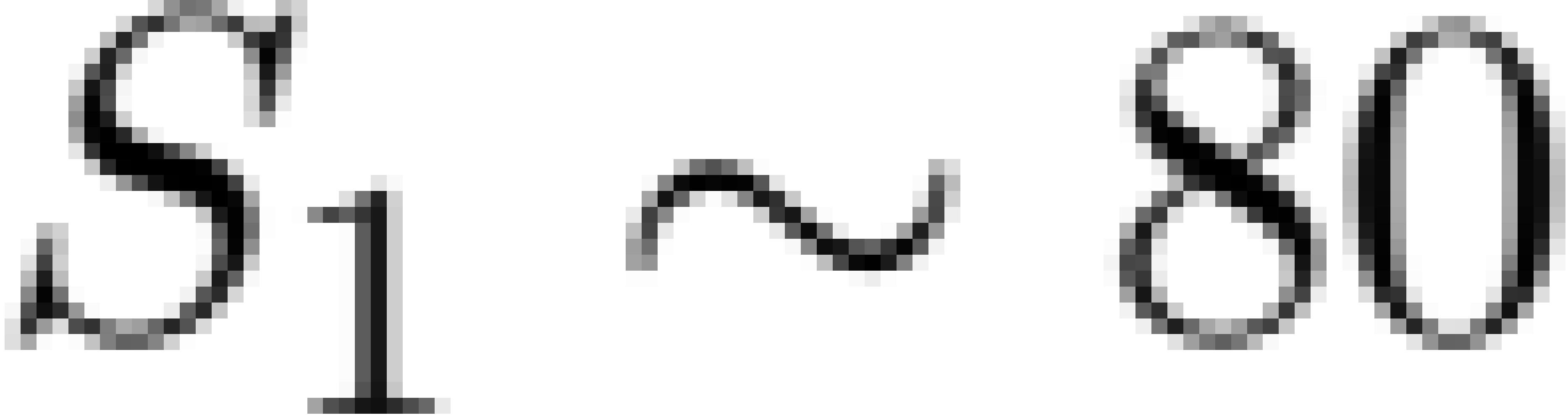


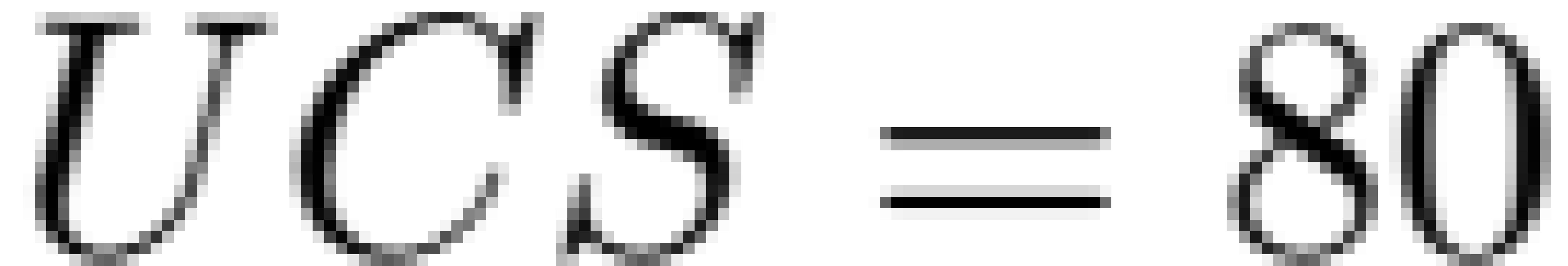


$$\varphi = \arctan$$

$$\left(\frac{q-1}{2\sqrt{q}} \right)$$







q

$=$

σ_1

$=$

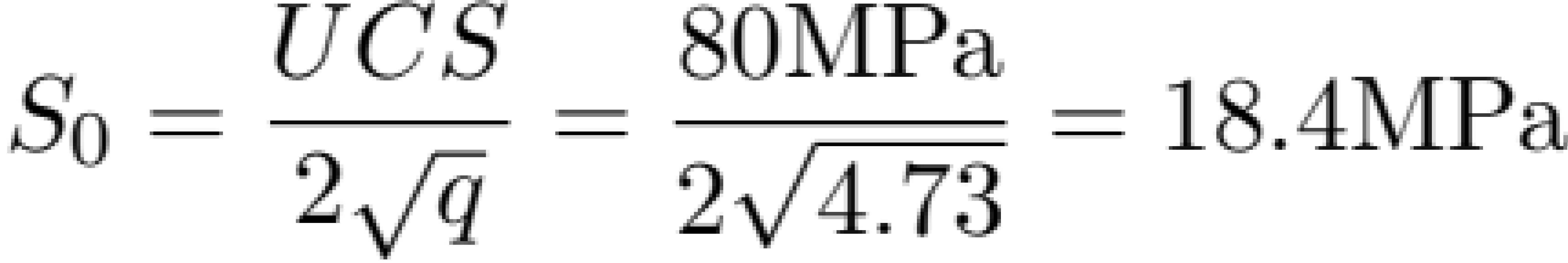
σ_3

110 MPa

520 MPa

4.73

MPa

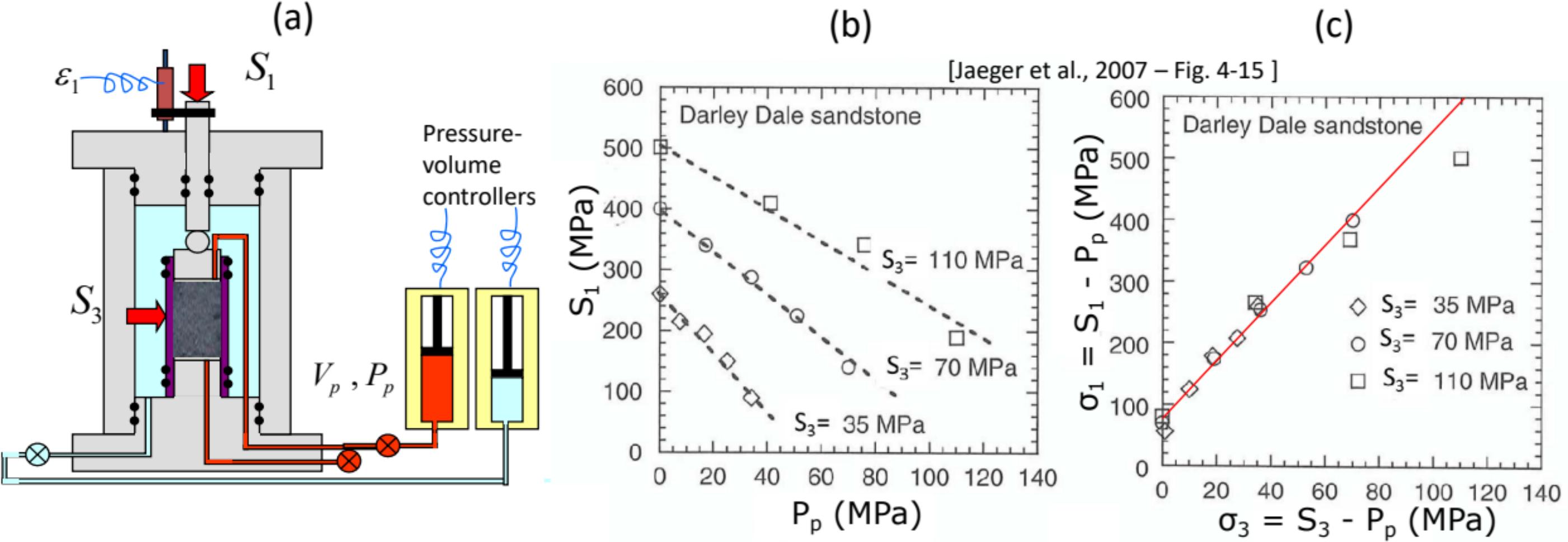


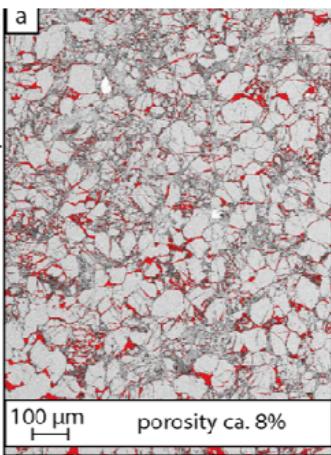
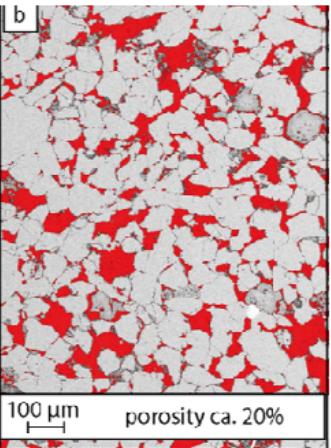
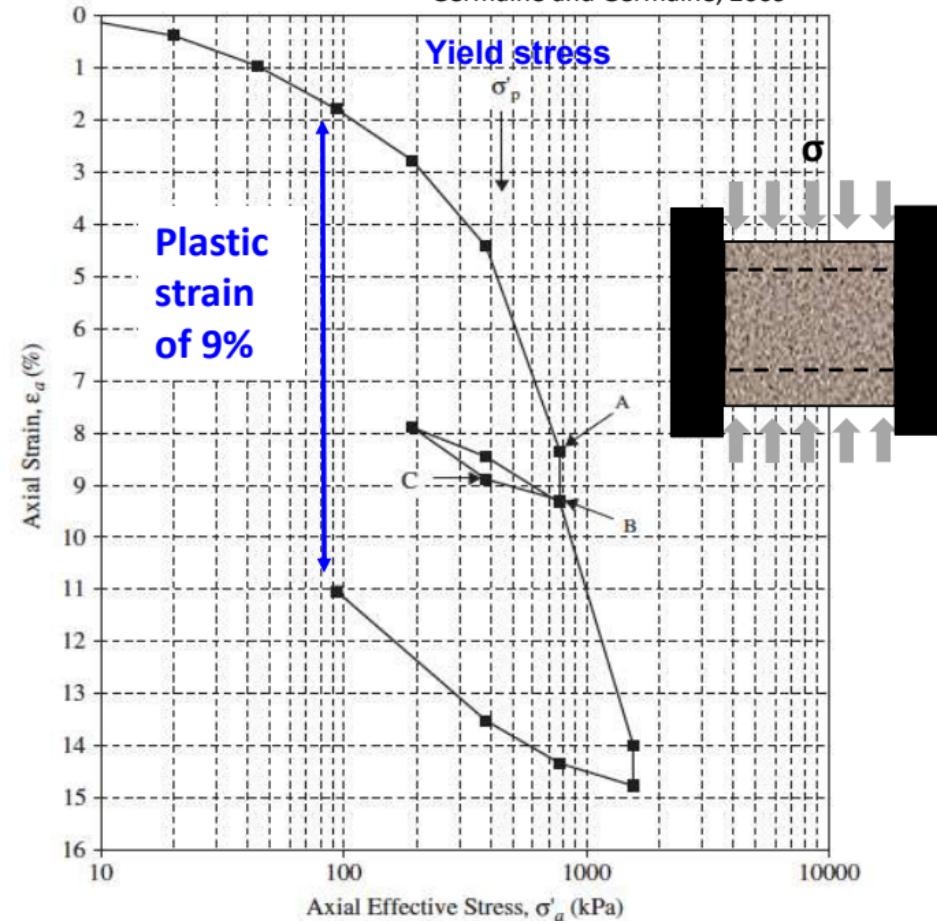
$$\varphi = \arctan \left(\frac{q-1}{2\sqrt{q}} \right) = \arctan \left(\frac{4.73 - 1}{2\sqrt{4.73}} \right) = 45.1^\circ$$

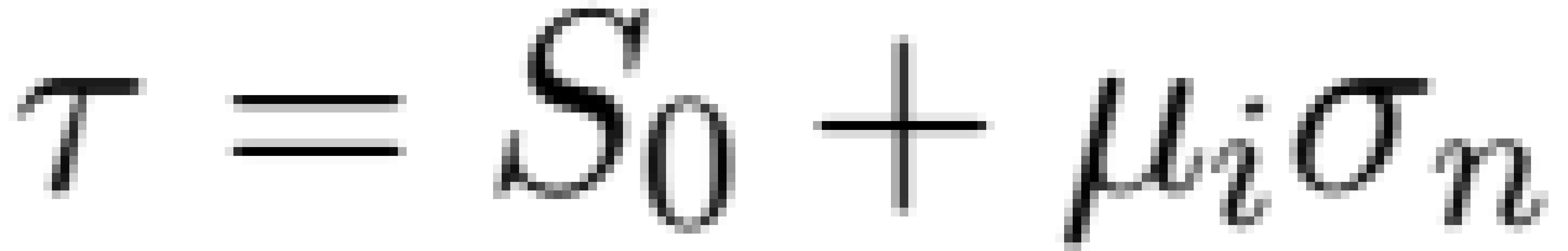


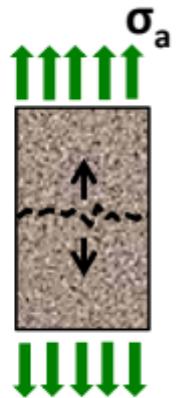








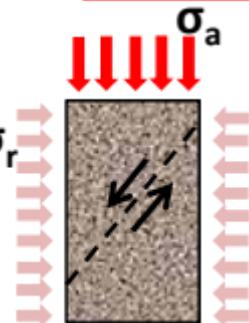


τ 

$$\sigma_n = T_s$$

 T_s

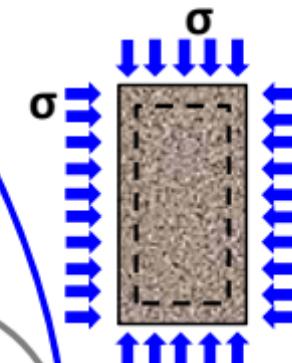
$$\boxed{\tau = \mu_i \sigma_n + S_0}$$



1

 μ_i

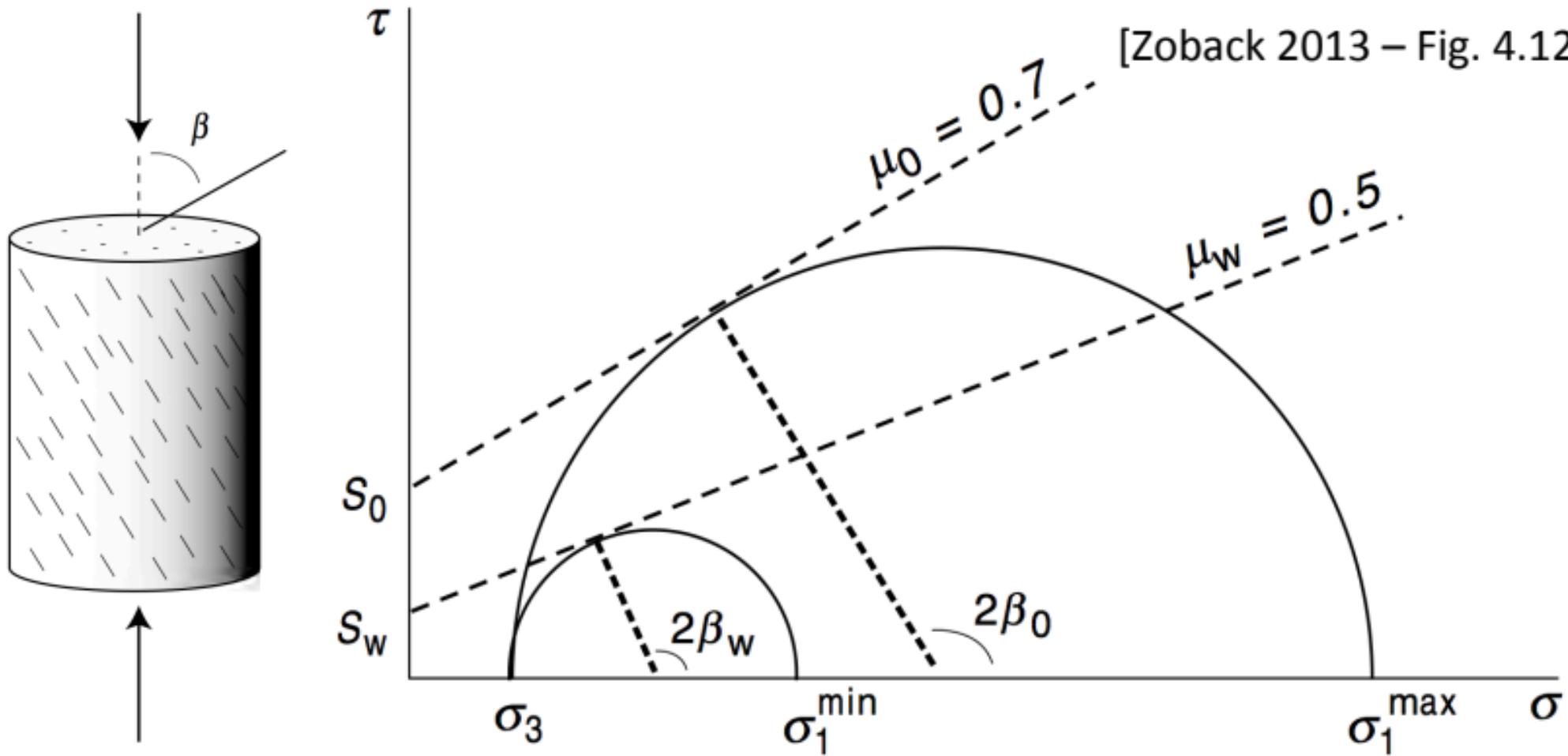
$$\boxed{\mathcal{E}_V^p = \mathcal{E}_{Vcrit}^p}$$

 σ_n





[Zoback 2013 – Fig. 4.12]

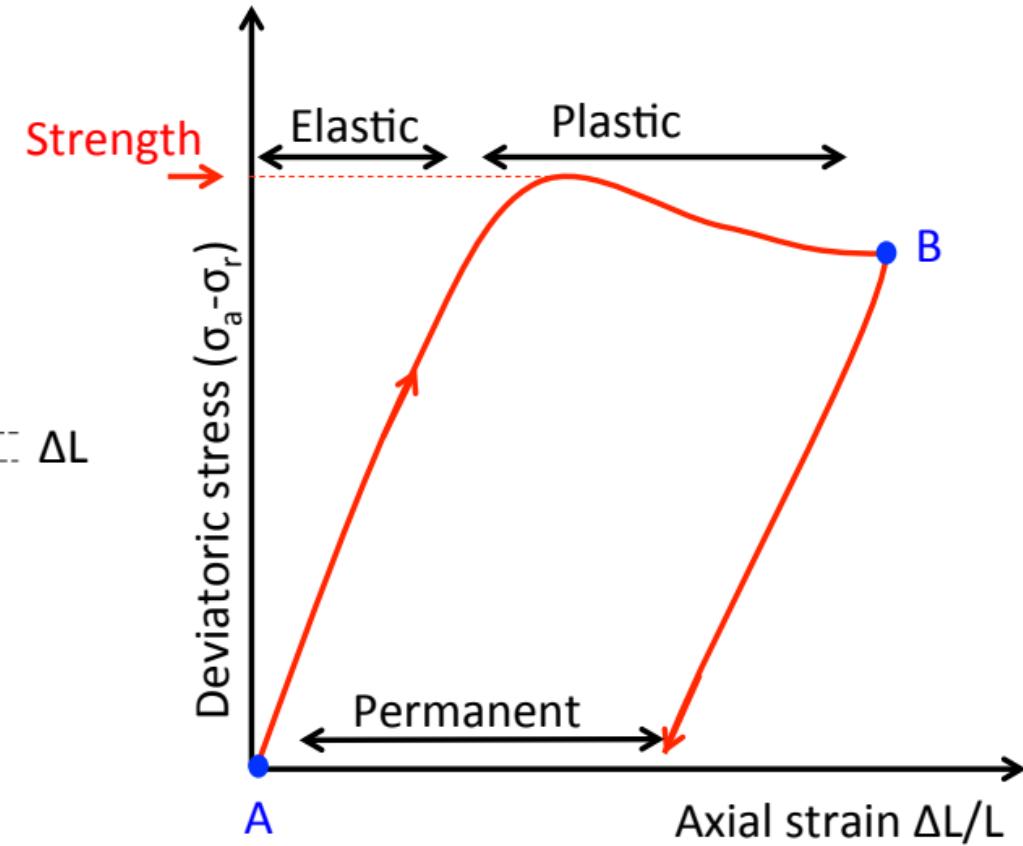
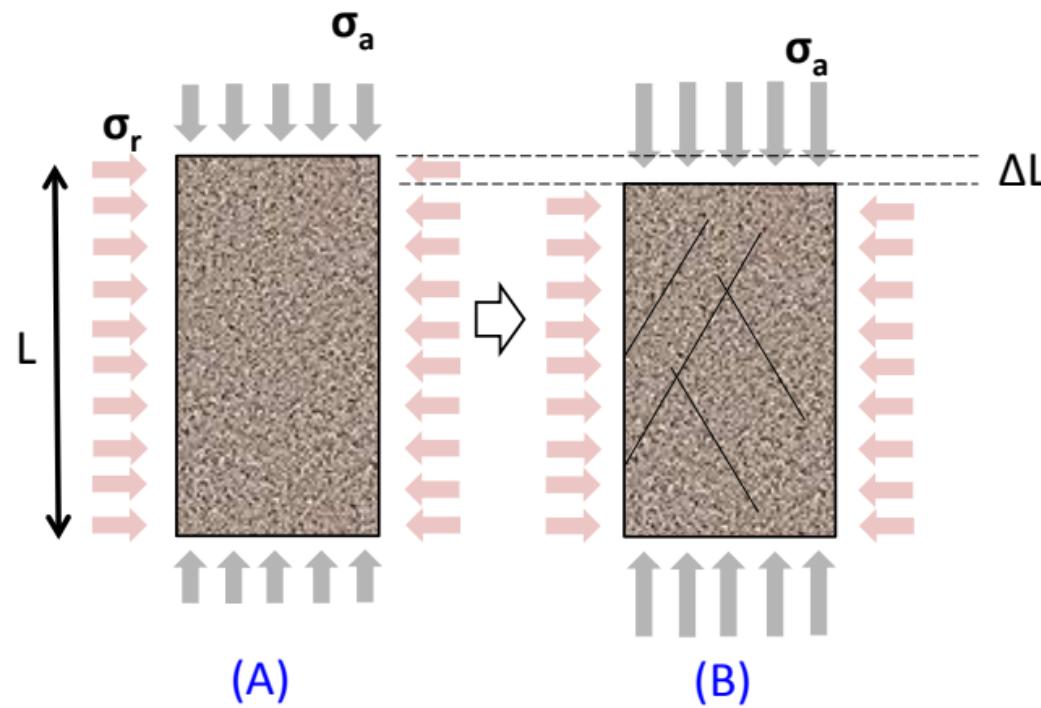


Rock deformation

Relation strain V.S. stress

Elastic (Young modulus)

Plastic (~Viscosity)



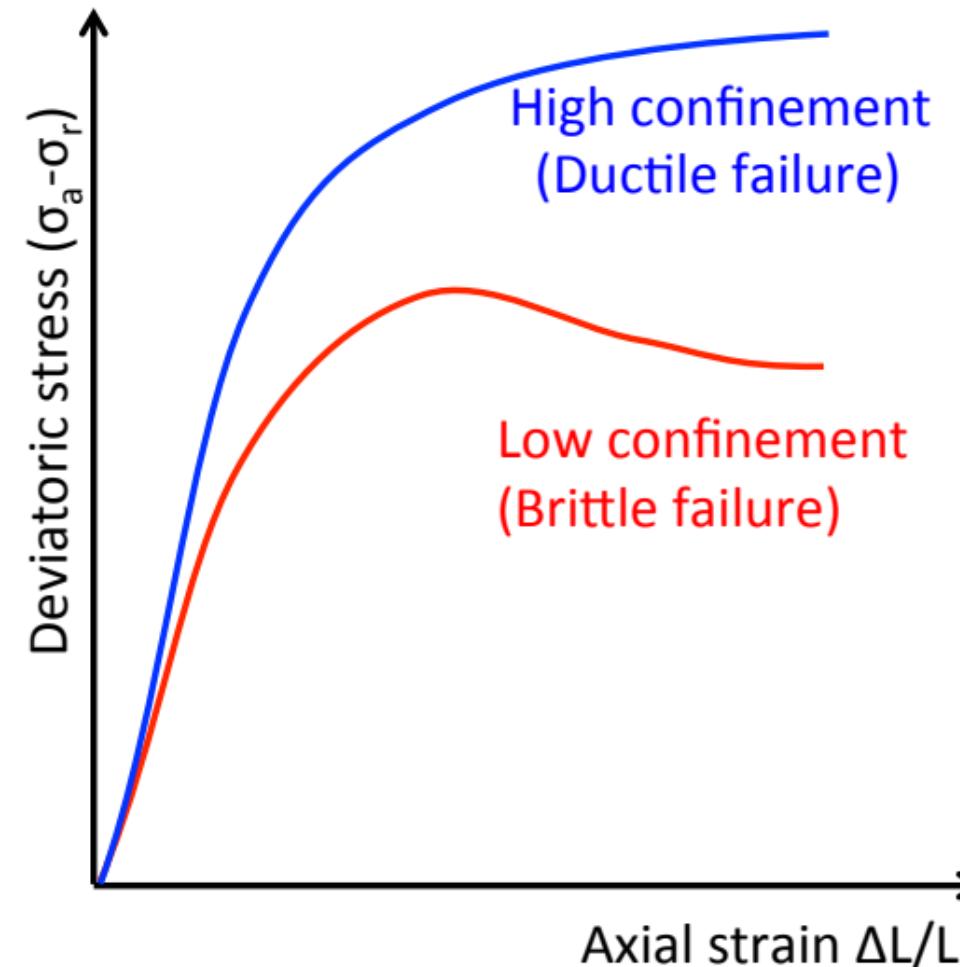
a. This is an undeformed cylinder of rock.



b. This cylinder was subjected to high confining pressure (uniform in all directions) and, at the same time, compression from above. It deformed in a ductile manner, becoming shorter and fatter.



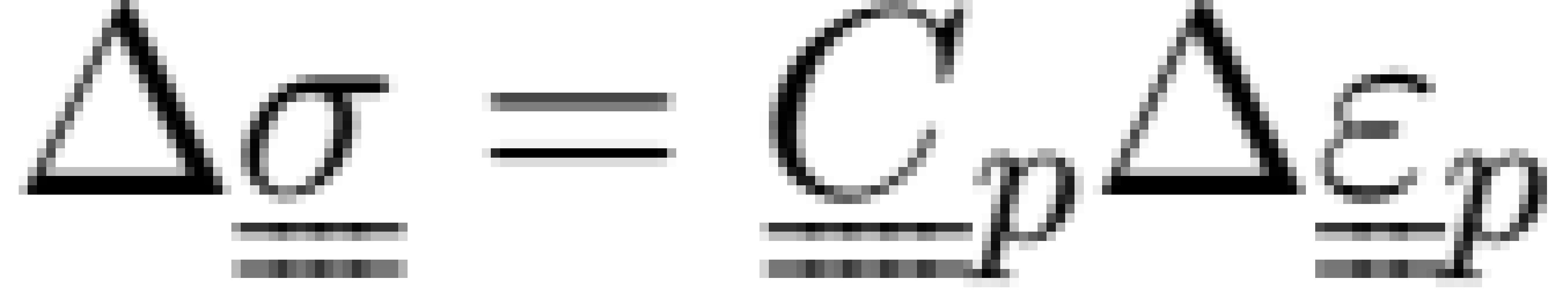
c. An identical cylinder was subjected to the same amount of compression from above, but this time with a lower confining pressure. It deformed in a brittle manner, with many large fractures.



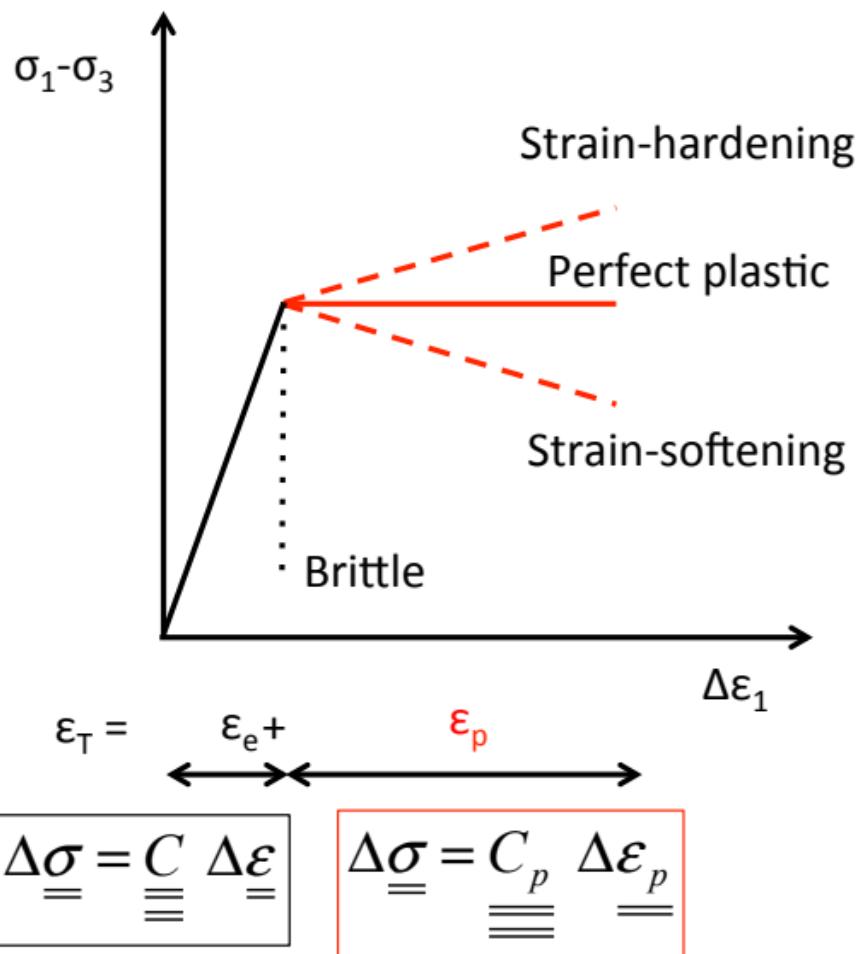
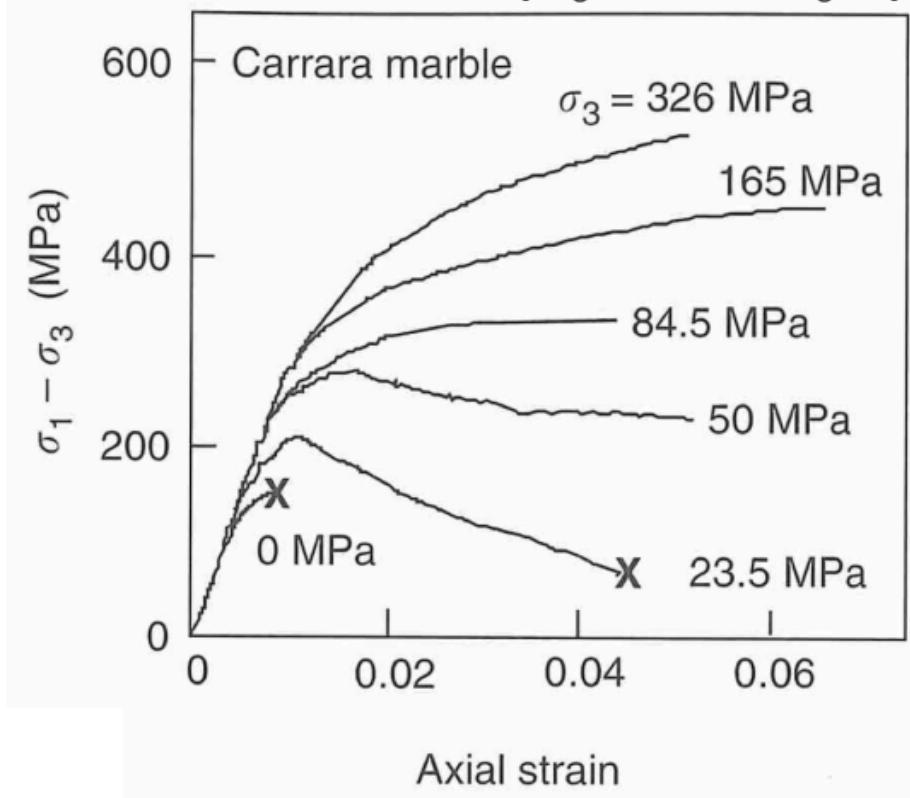




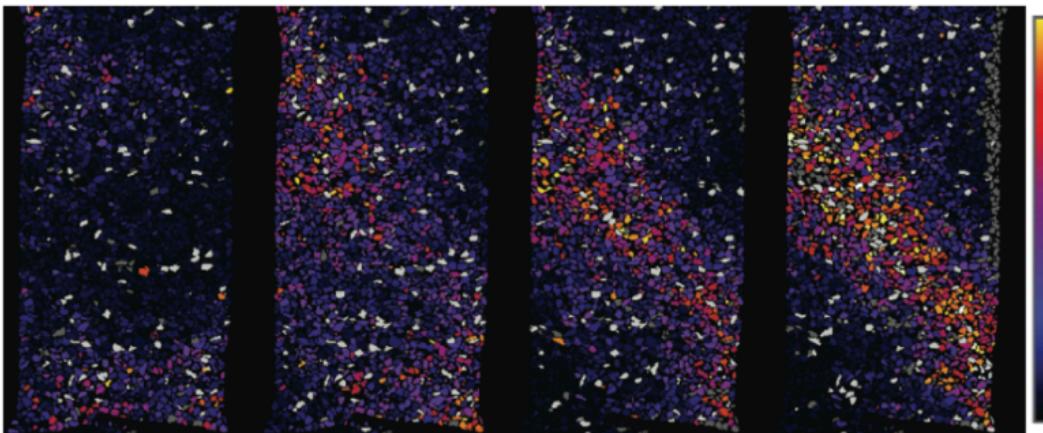
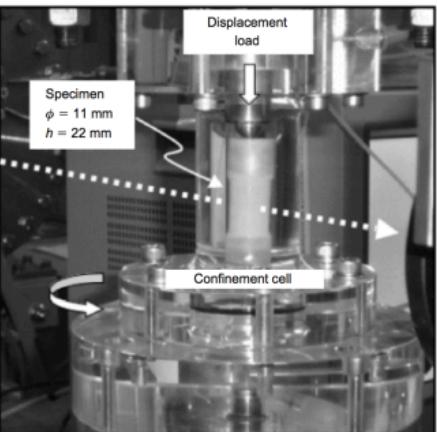




[Jaeger et al. 2007 – Fig. 4.5]



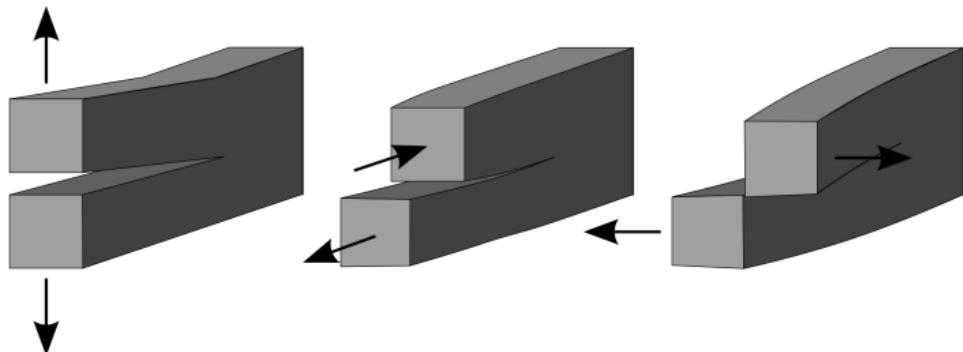
(a) Uncemented or poorly cemented rock →



grain friction, dilation, grain crushing/rotation

(b) Cemented rock →

Propagation of microfractures, grain friction/crushing

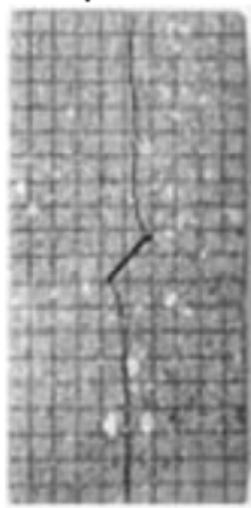


Stress intensification at
the tip of fractures

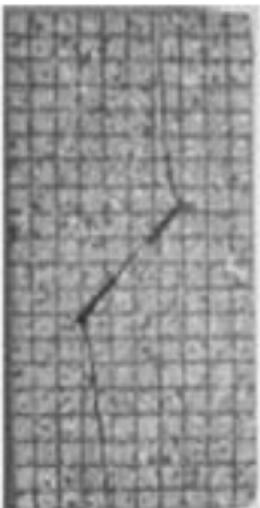
Propagation starts at
fracture tips

Napolitan Tuffo

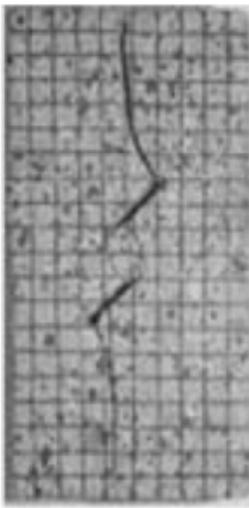
[Hall et al. 2006 – Pure Appl. Geophys.]



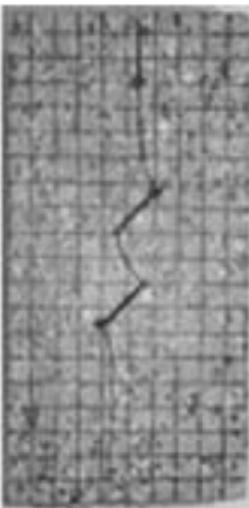
Single flaw



$\beta = 45^\circ$

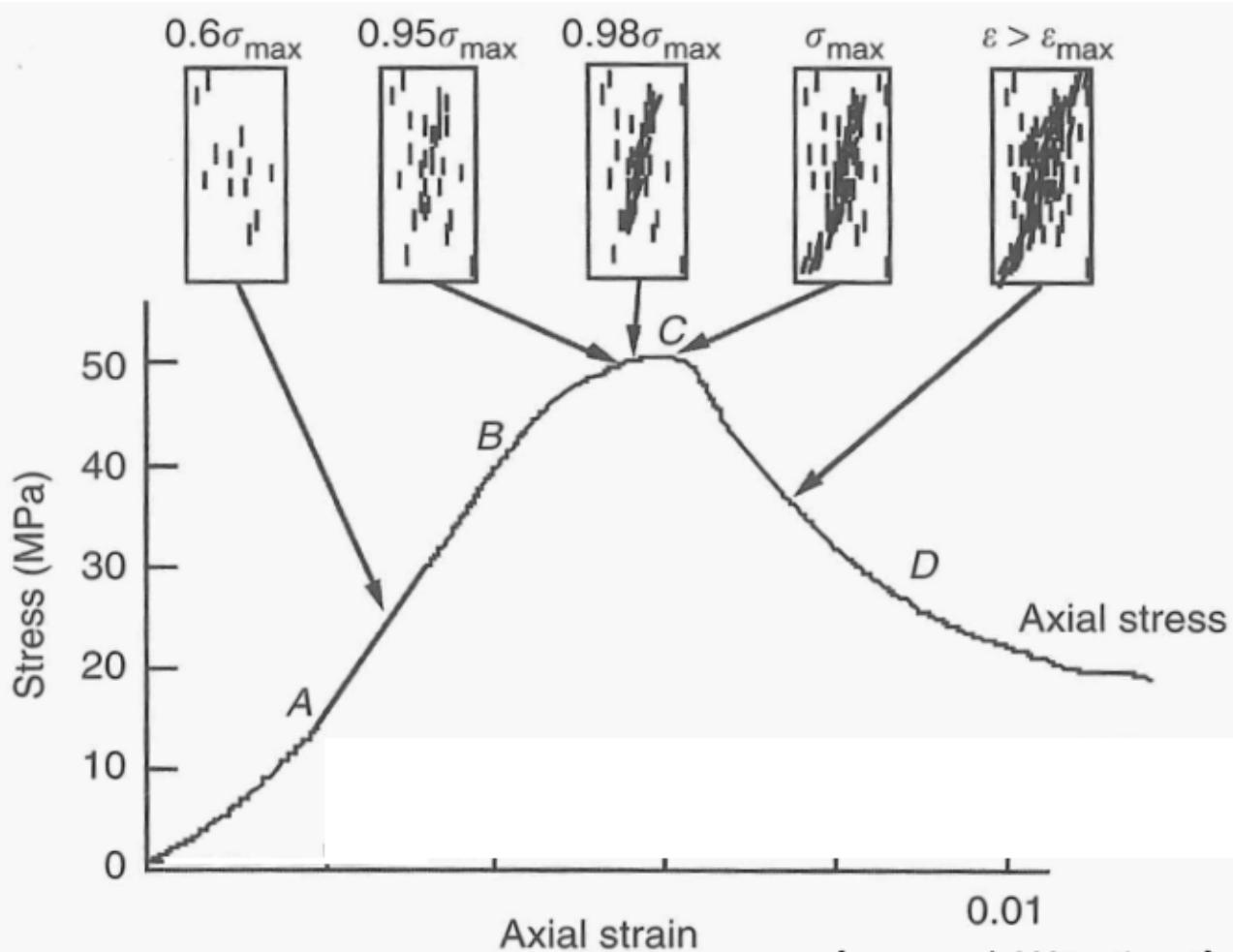


$\beta = 105^\circ$

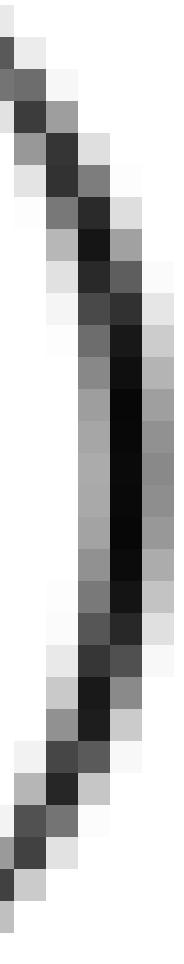
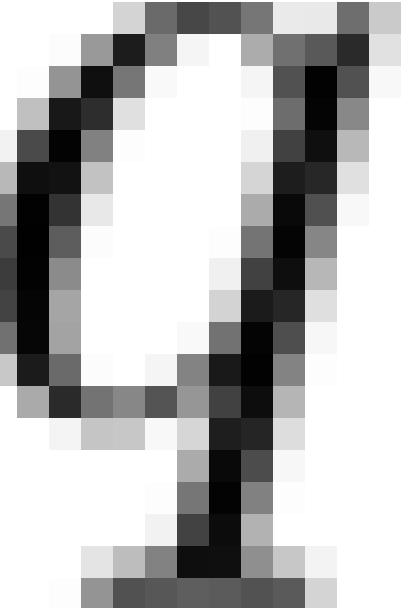
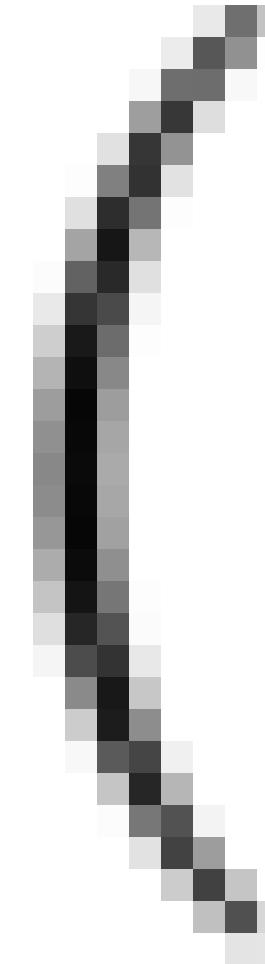
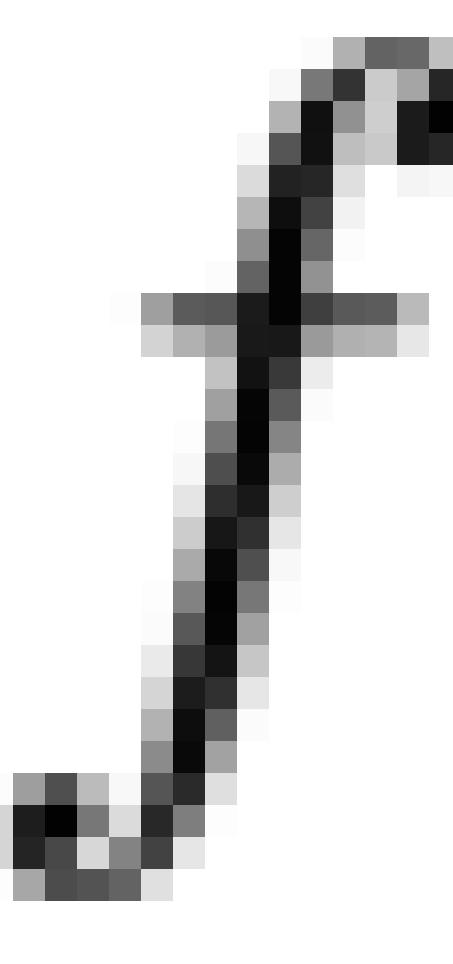
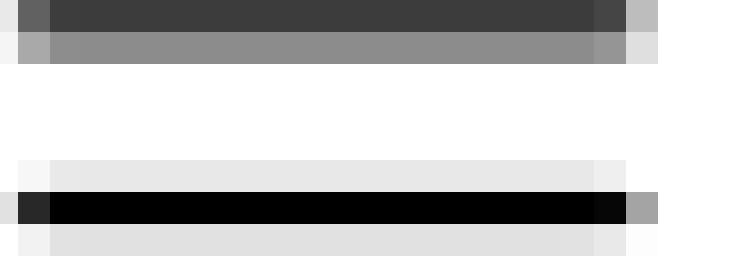
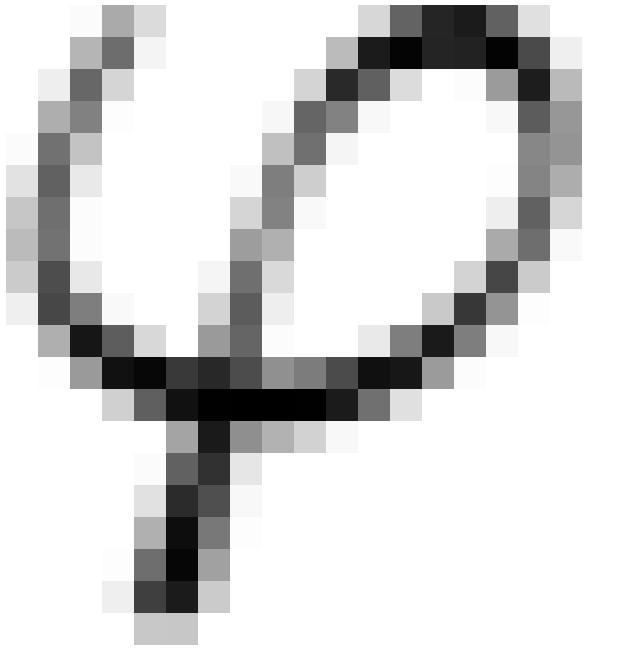


$\beta = 120^\circ$





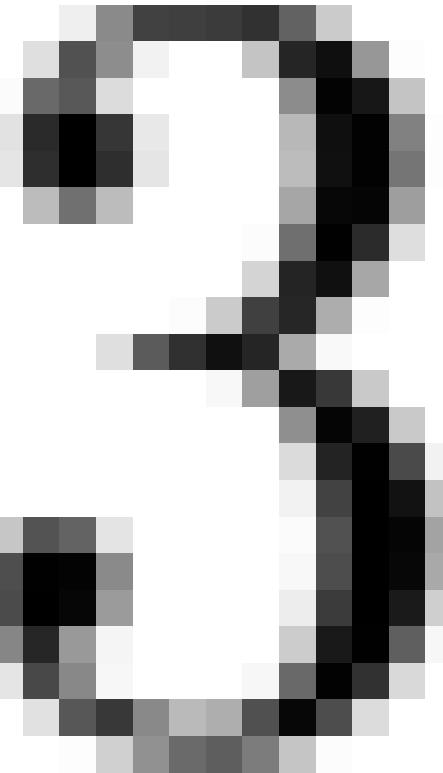
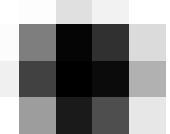
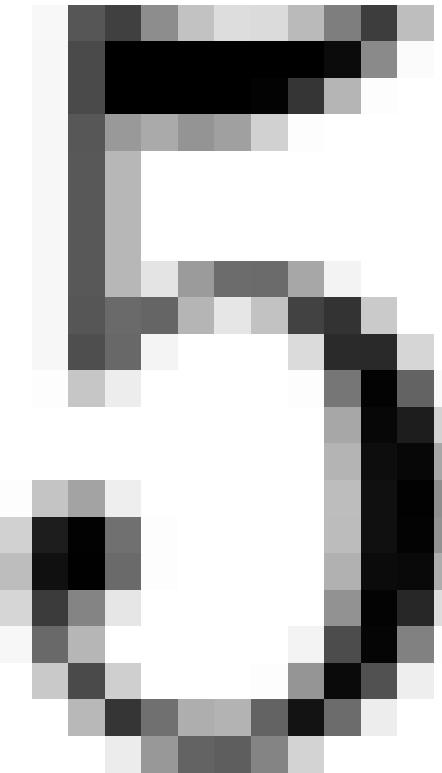
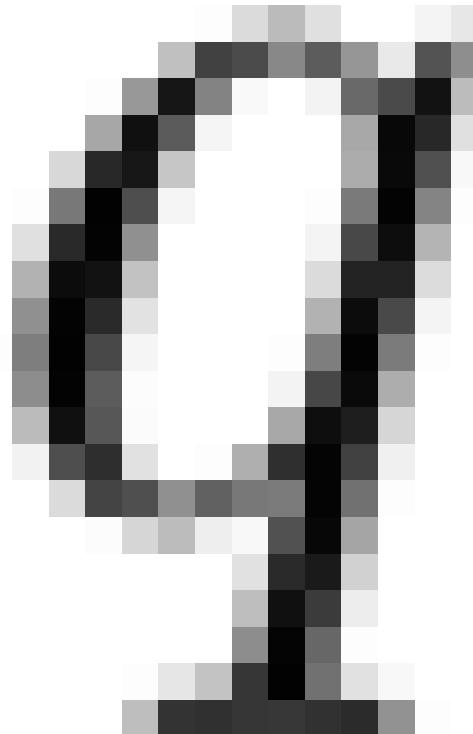












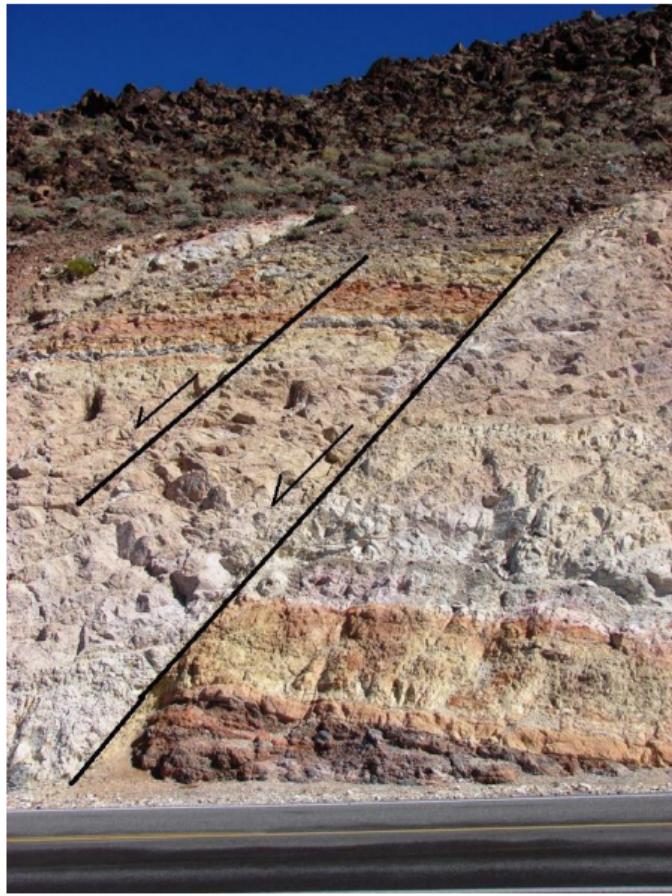






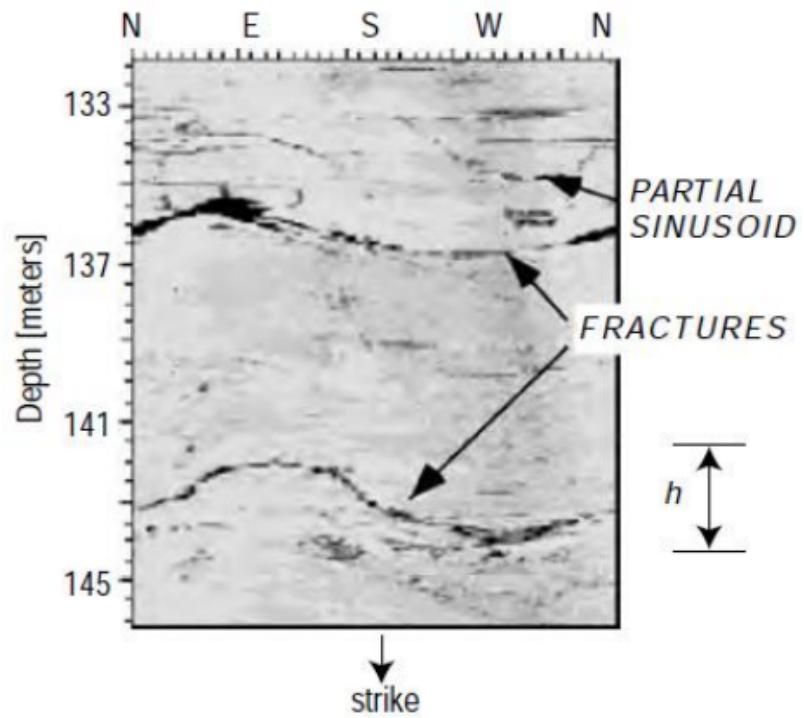
Outcrop of a normal fault in Split Mountain gorge

<http://geology.csupomona.edu/janourse/TectonicsFieldTrips.htm>

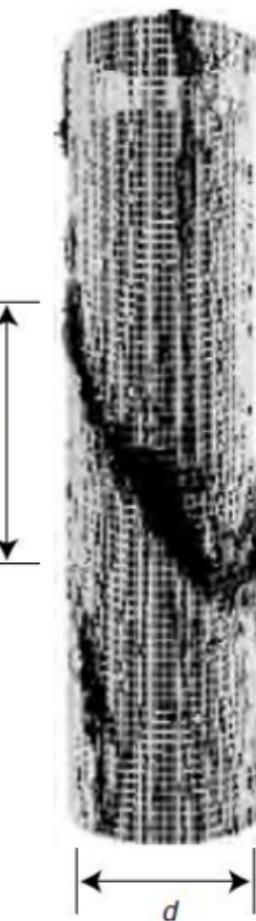


<http://geotripperimages.com/images/DSC03438%20Charlie%20Brown%20b.jpg>

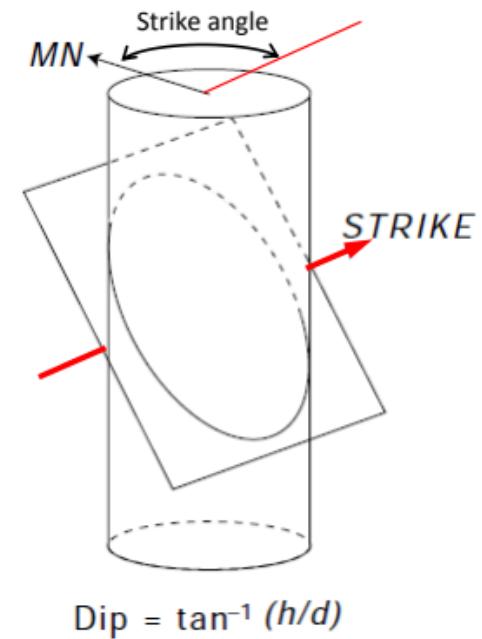
**(a) Un-wrapped image
(ultrasonic)**



(b) 3D-representation



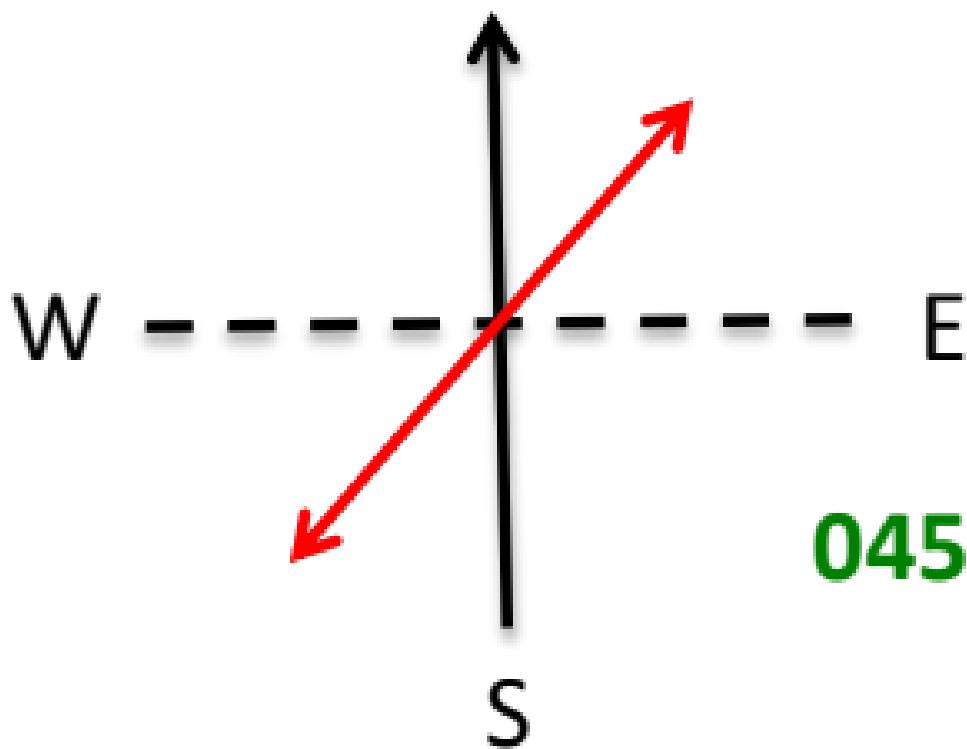
(c) Interpretation



[Zoback 2013 - Figure 5.3]



N45°E==S45°W



045° == 225°

Geologic map

Figure from Prof. Prodanovic



a)

$N30^\circ E$, $45^\circ NW$

or

030° , $45^\circ NW$



b)

$N10^\circ W$, $80^\circ NE$

or

350° , $80^\circ NE$



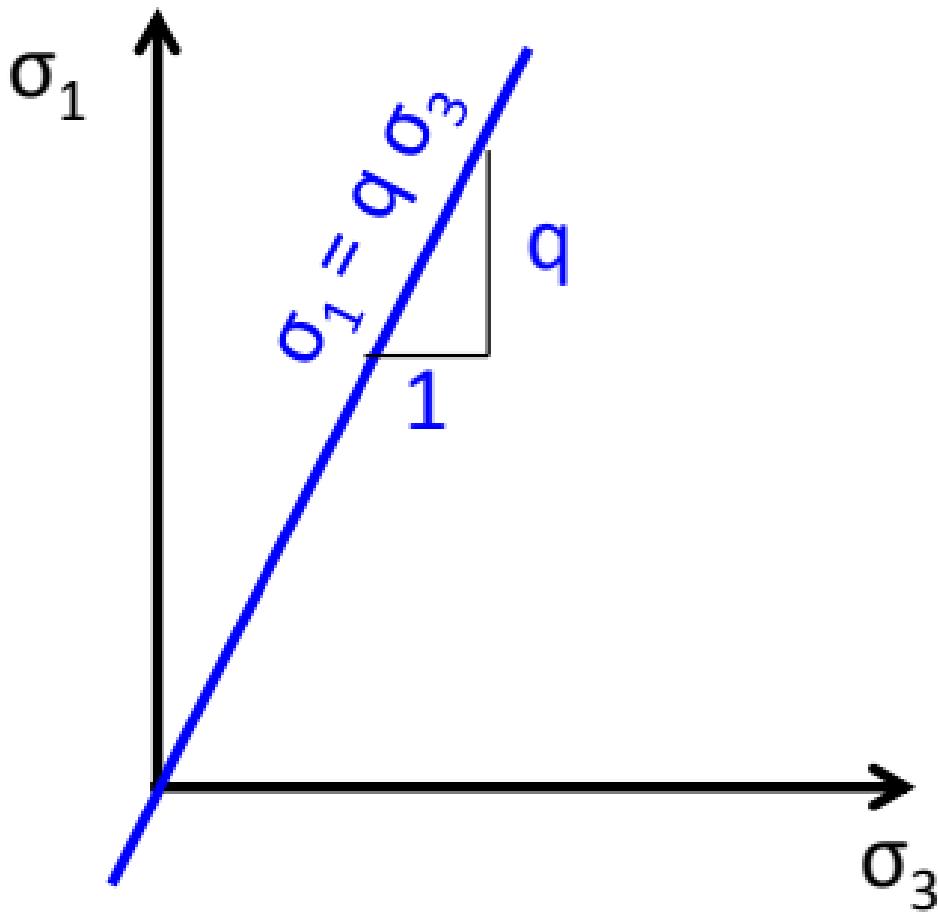
c)

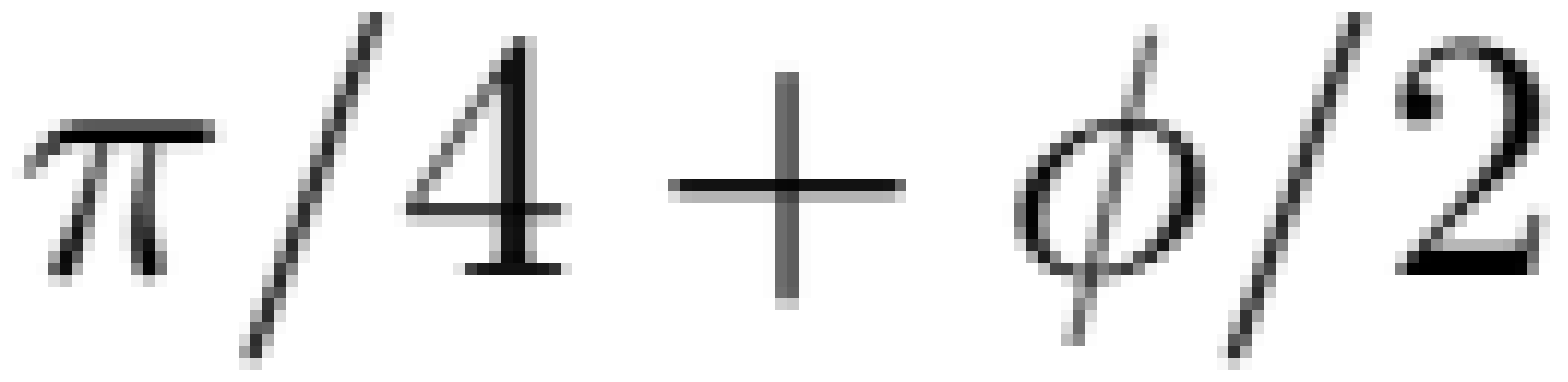
Symbols for
horizontal plane
and vertical plane

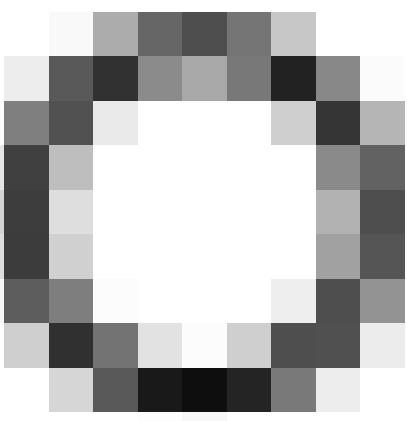
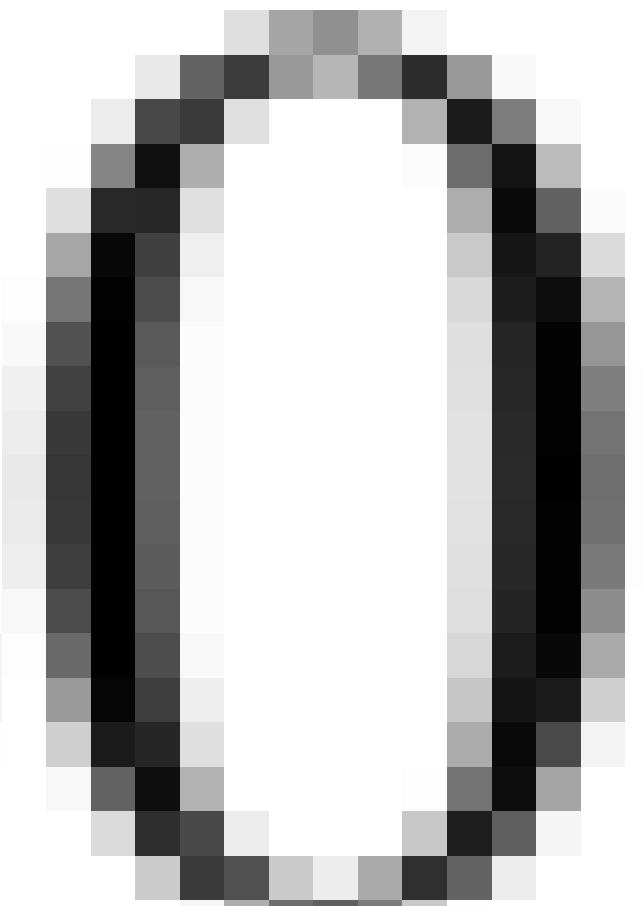
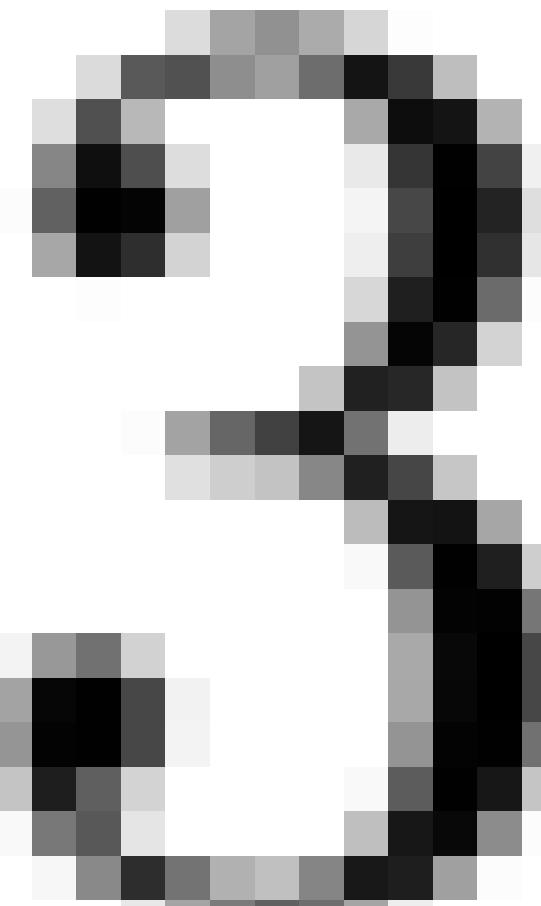
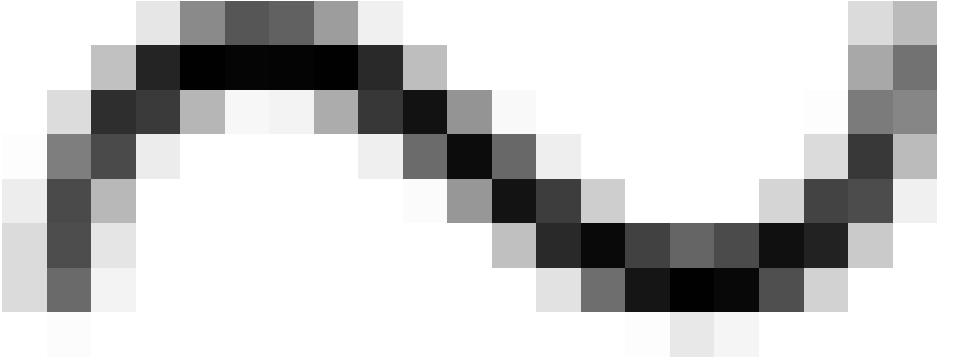


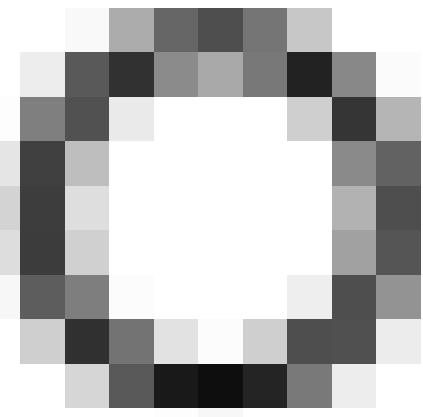
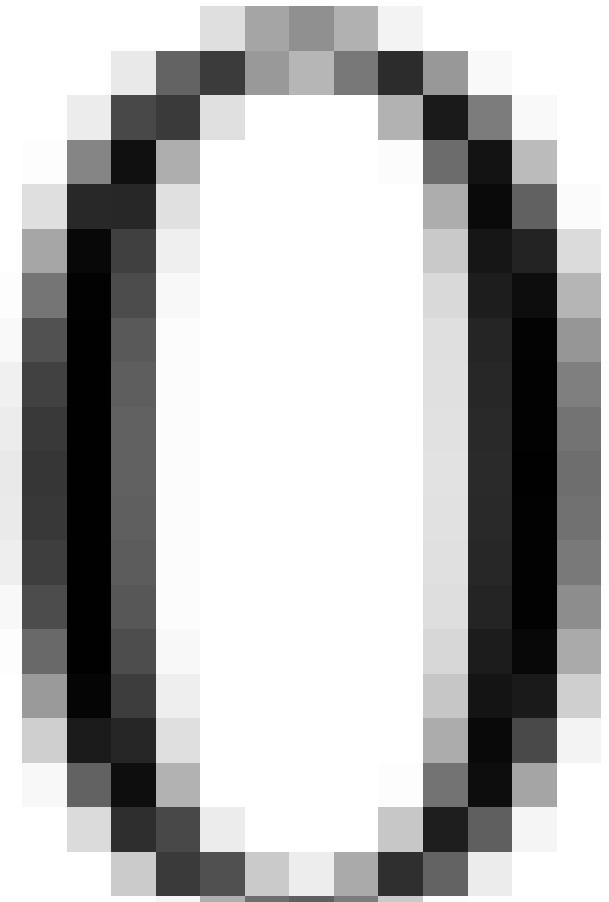
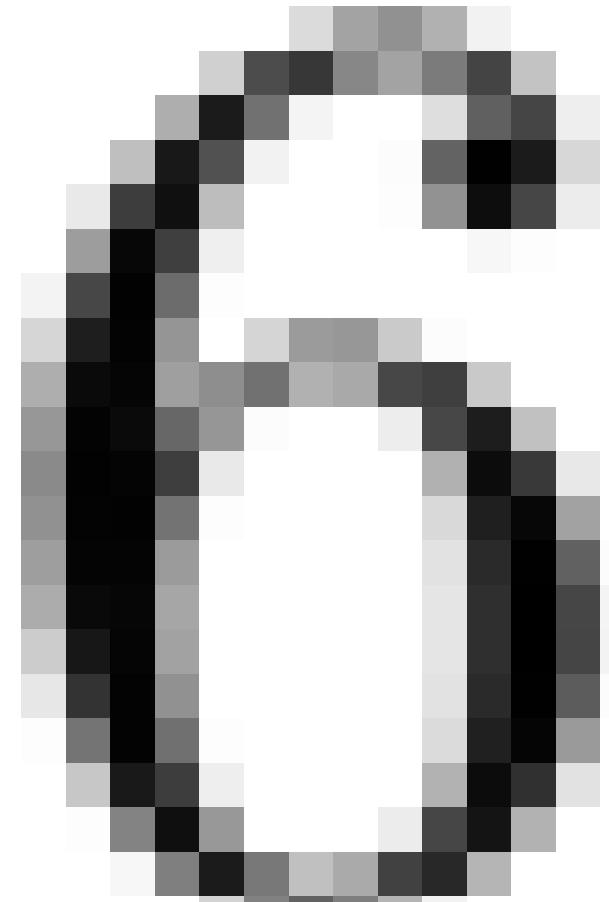
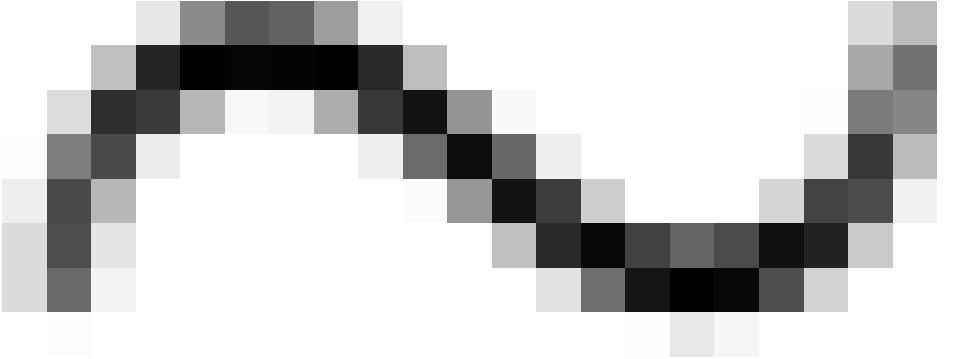
$$\begin{aligned} q &= \sin^2 \varphi + \frac{1}{\sin^2 \varphi} \\ &= \frac{1}{\sin^2 \varphi} + \frac{1}{\sin^2 \varphi} \end{aligned}$$

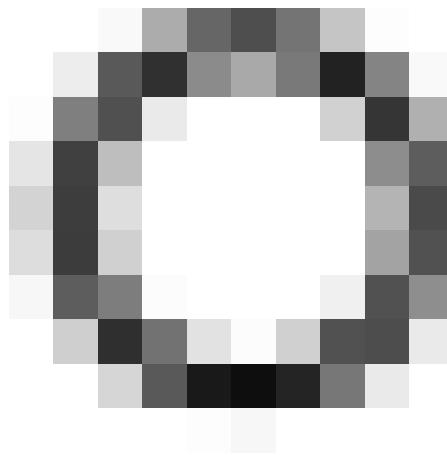
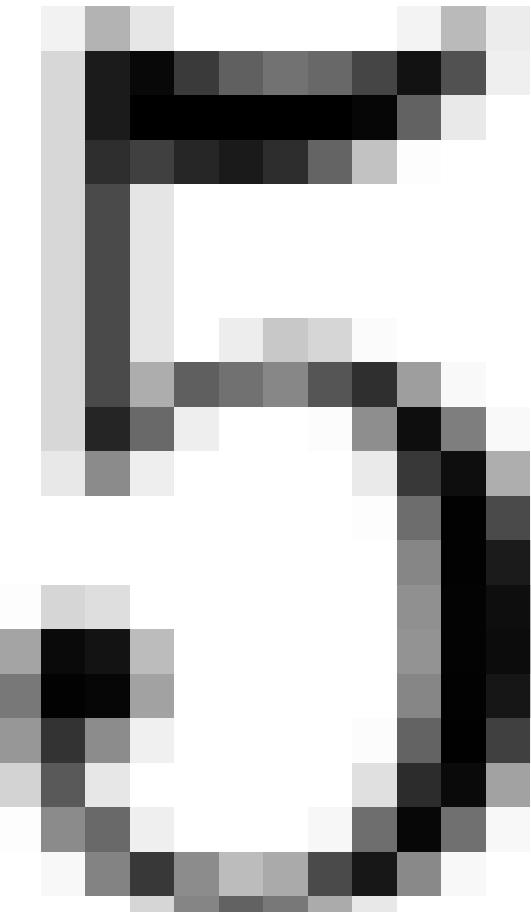
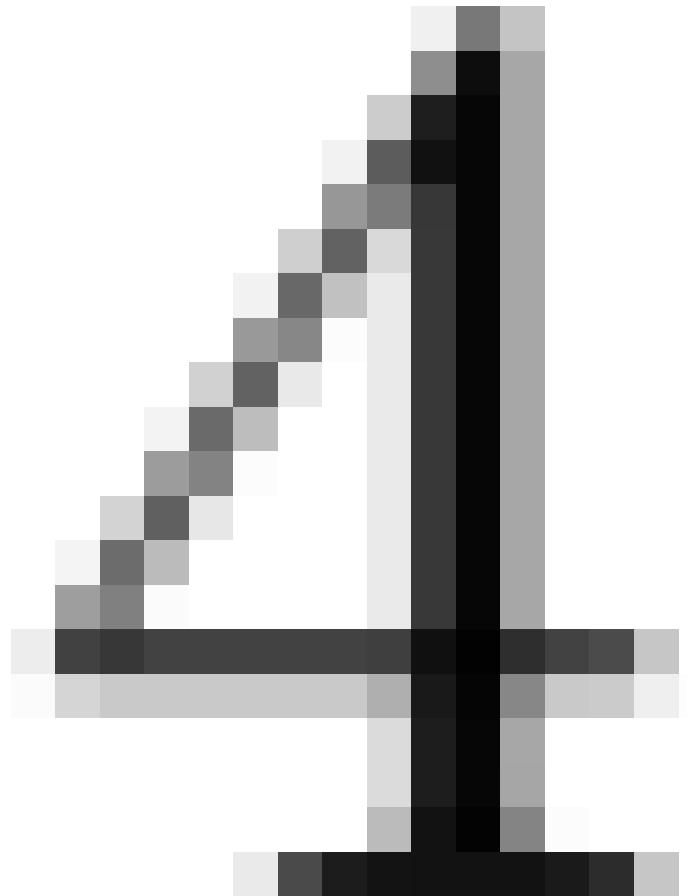


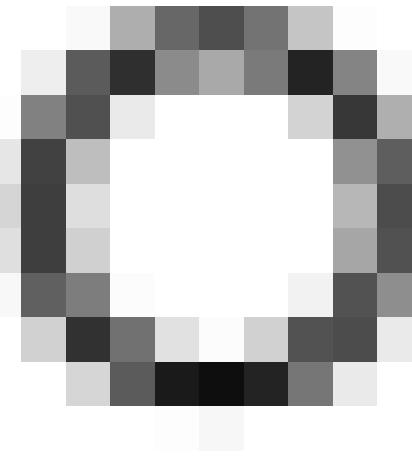
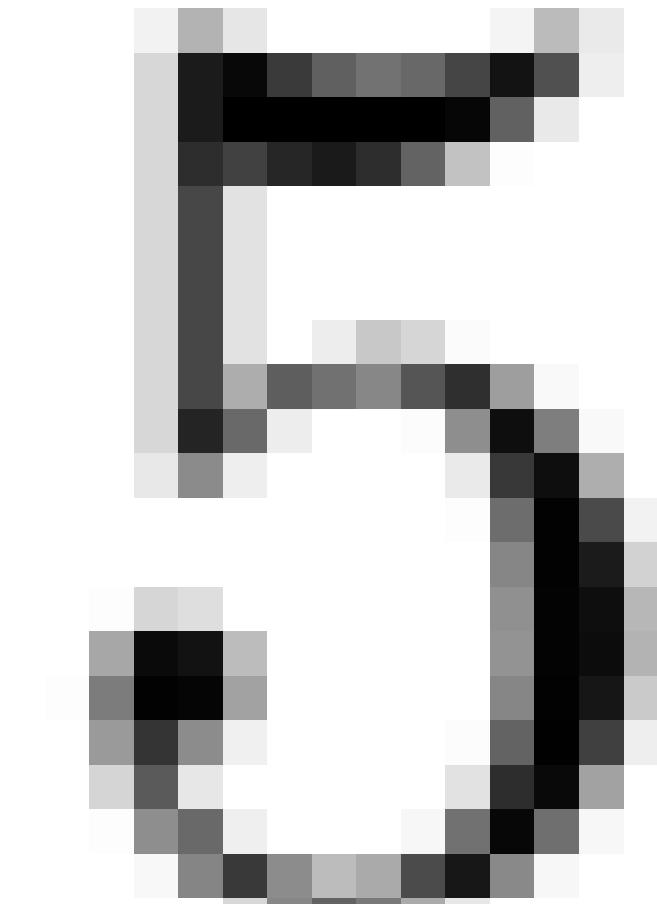
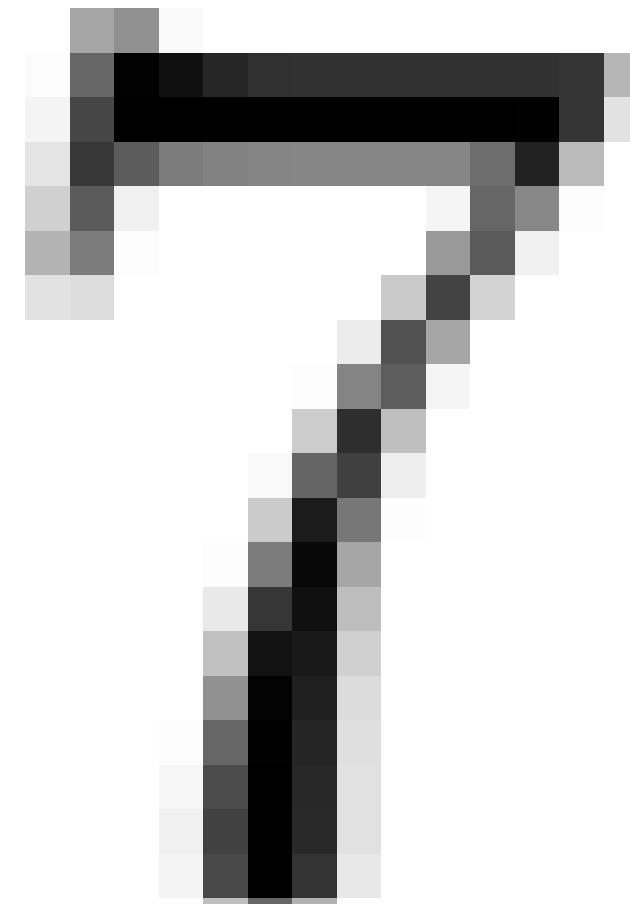
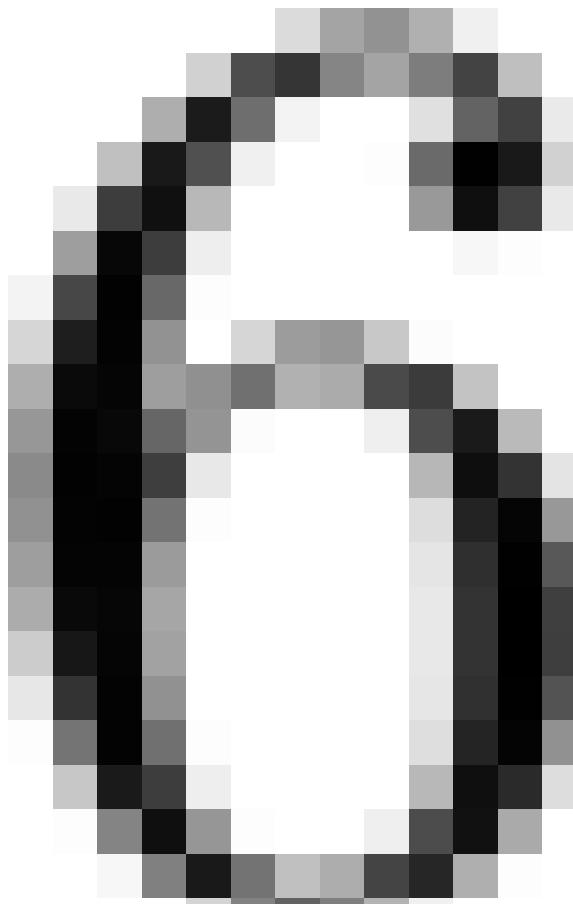


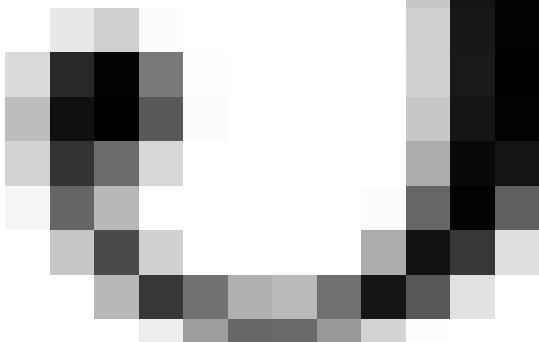
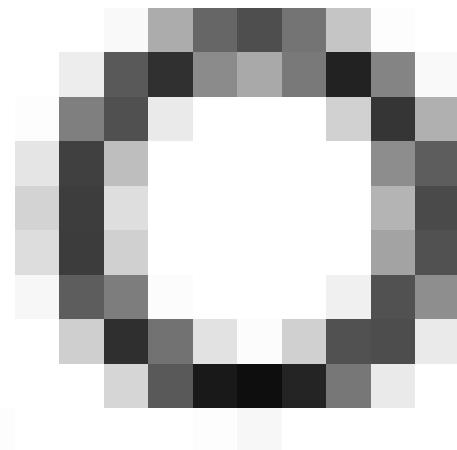
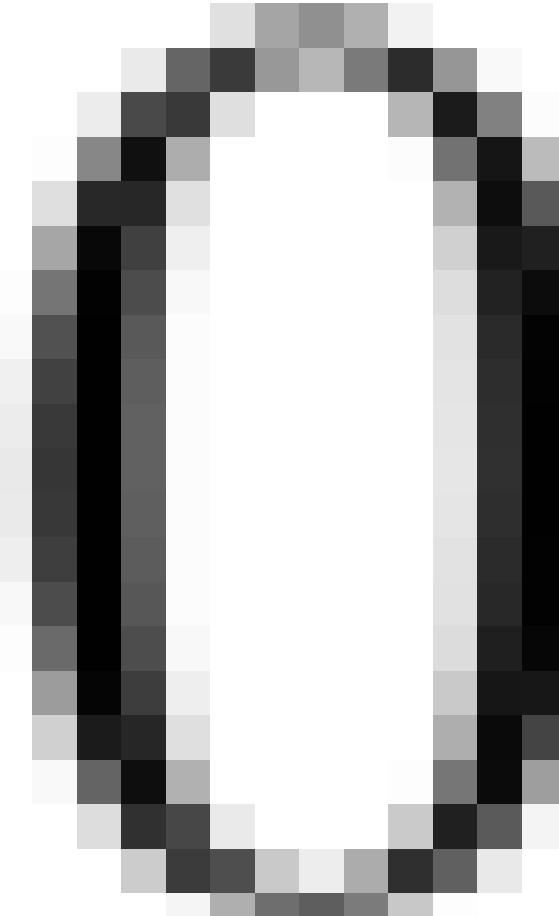
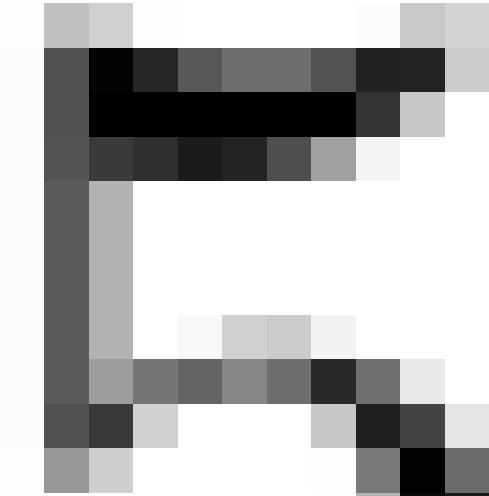


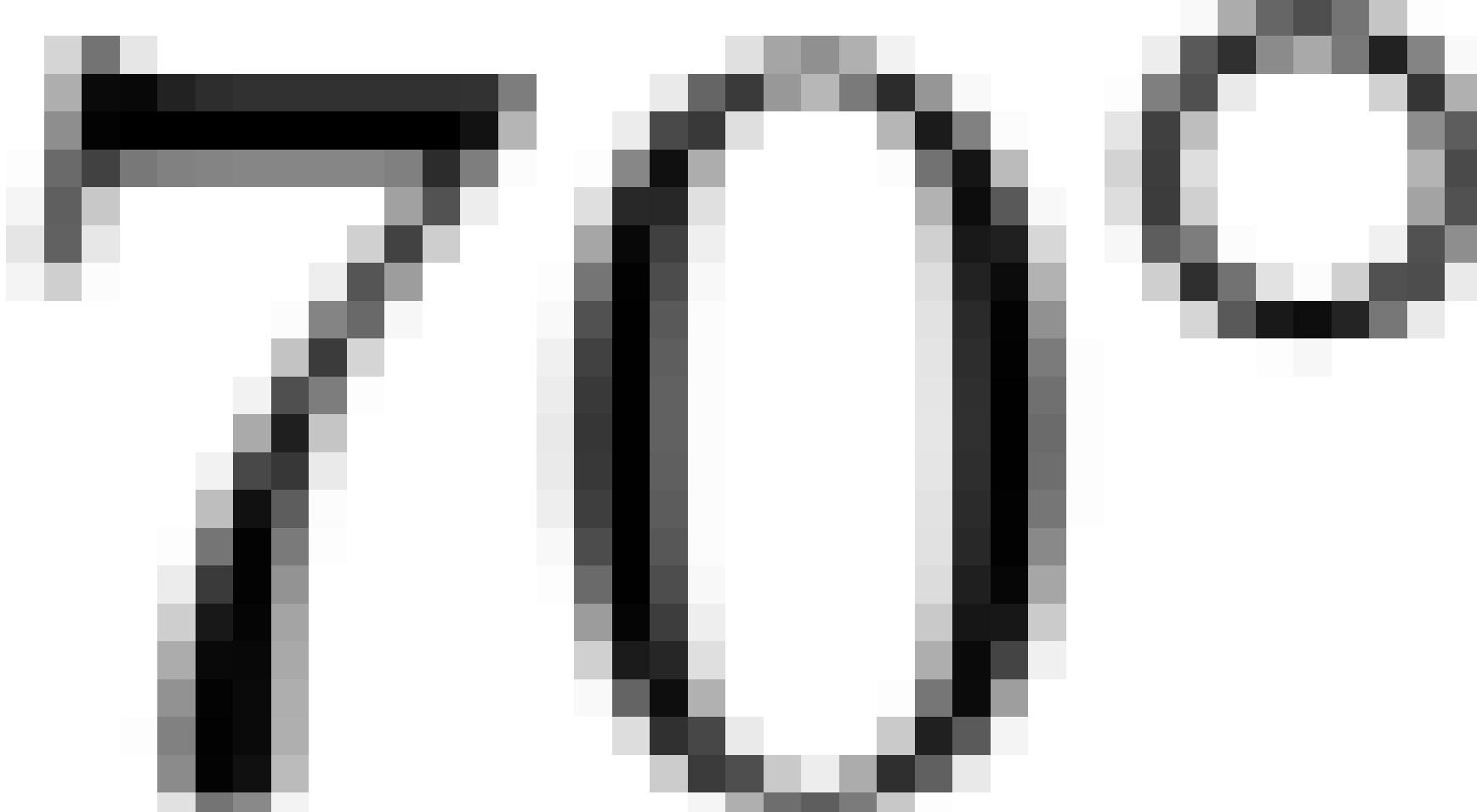
















$$S_1 = s_v$$

$$S_2 = s_{H\max}$$

$$S_3 = s_{h\min}$$



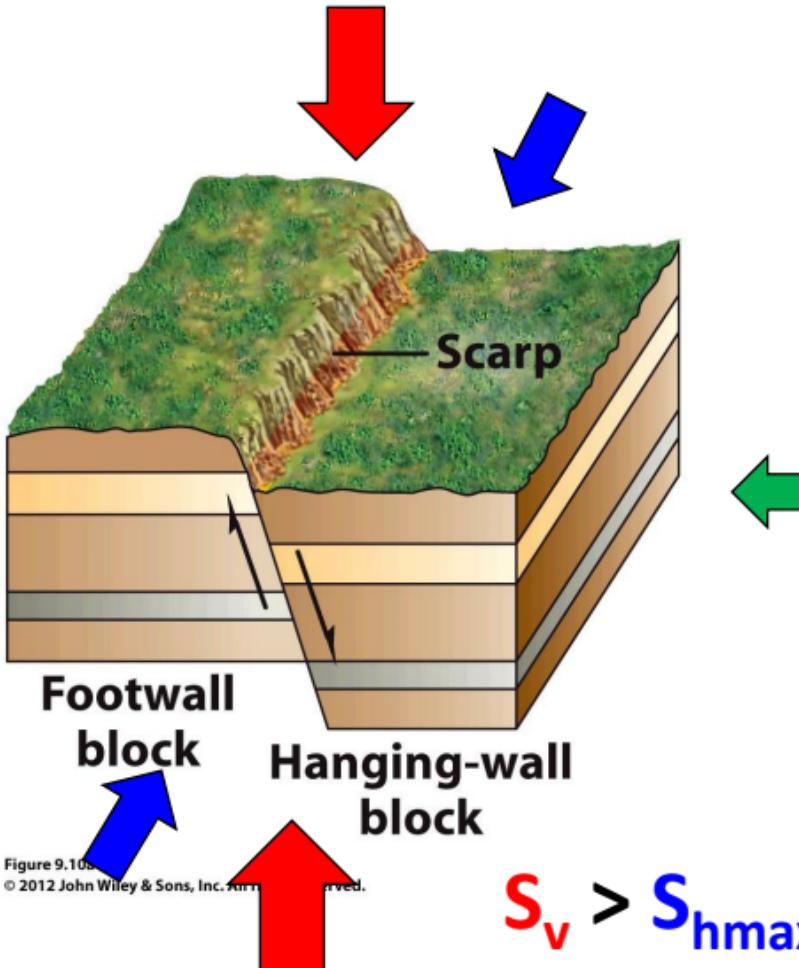


Figure 9.10b
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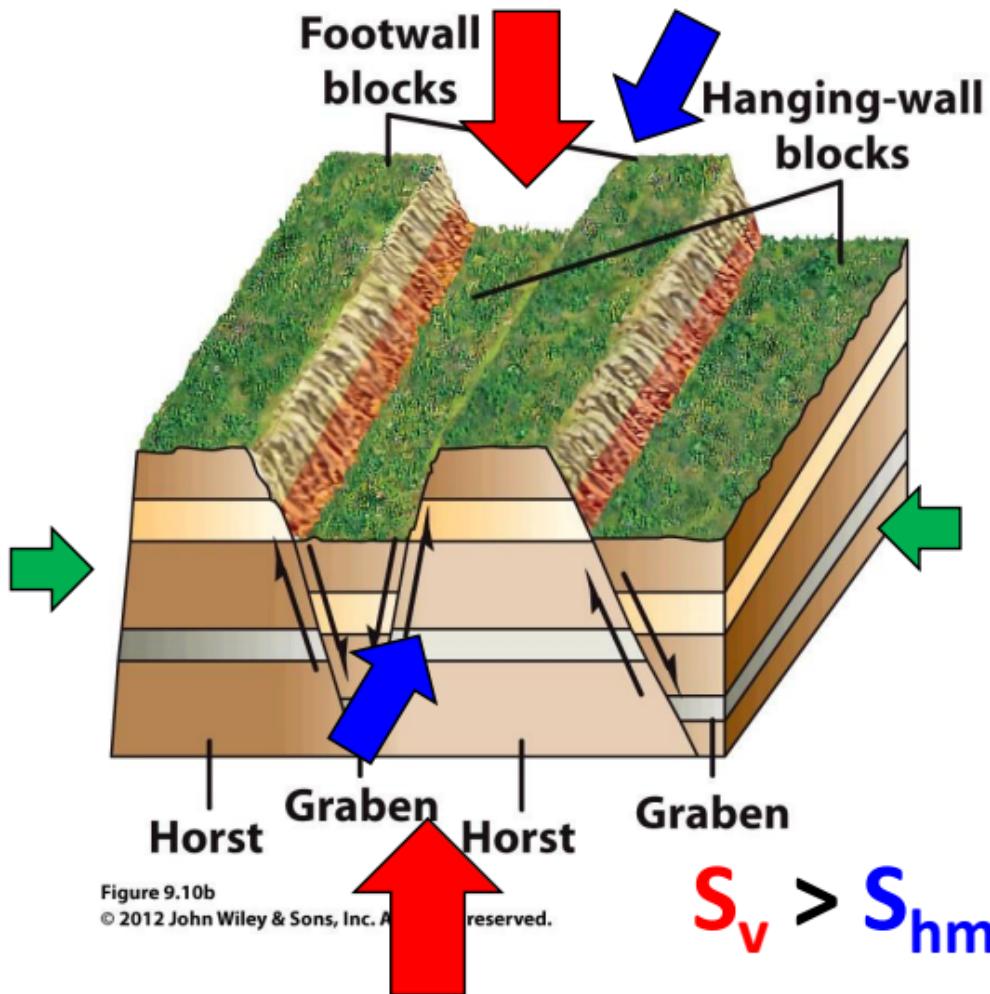


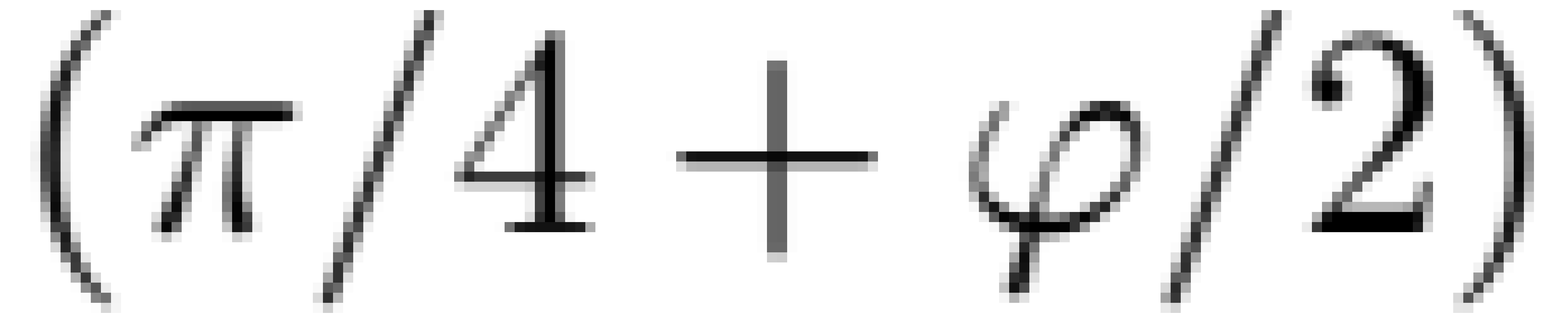
Figure 9.10b
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$$S_v > S_{h\max} > S_{h\min}$$

$$S_1 = S_{H\max}$$

$$S_2 = S_{h\min}$$

$$S_3 = S_v$$





**Symbols used
on a map to
indicate a
thrust fault**

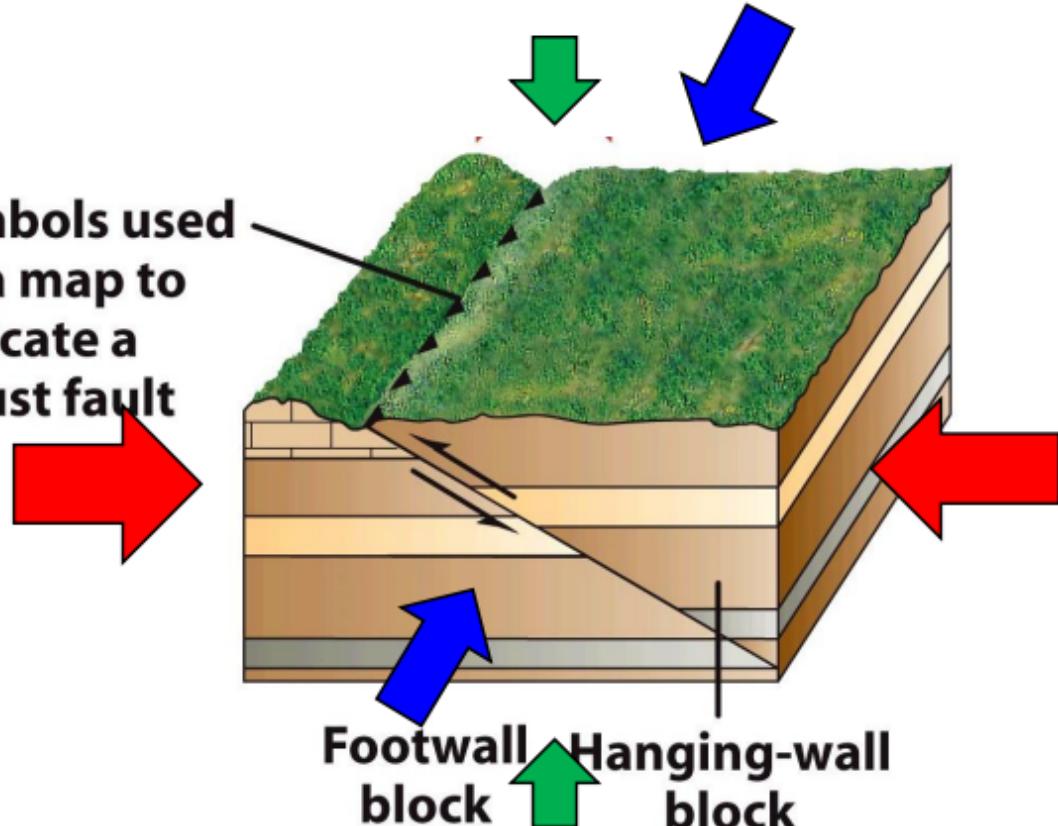


Figure 9.10d
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$$S_{H\max} > S_{h\min} > S_v$$

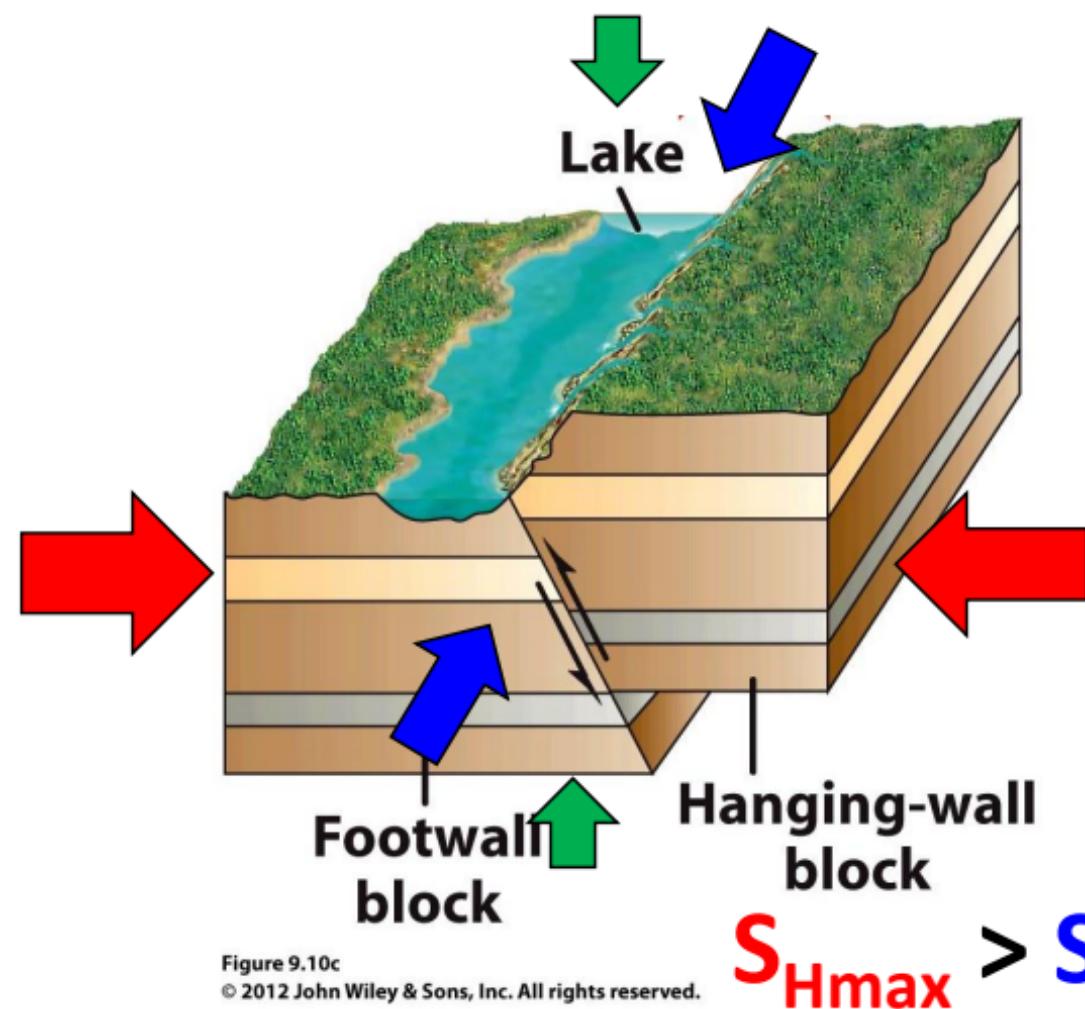


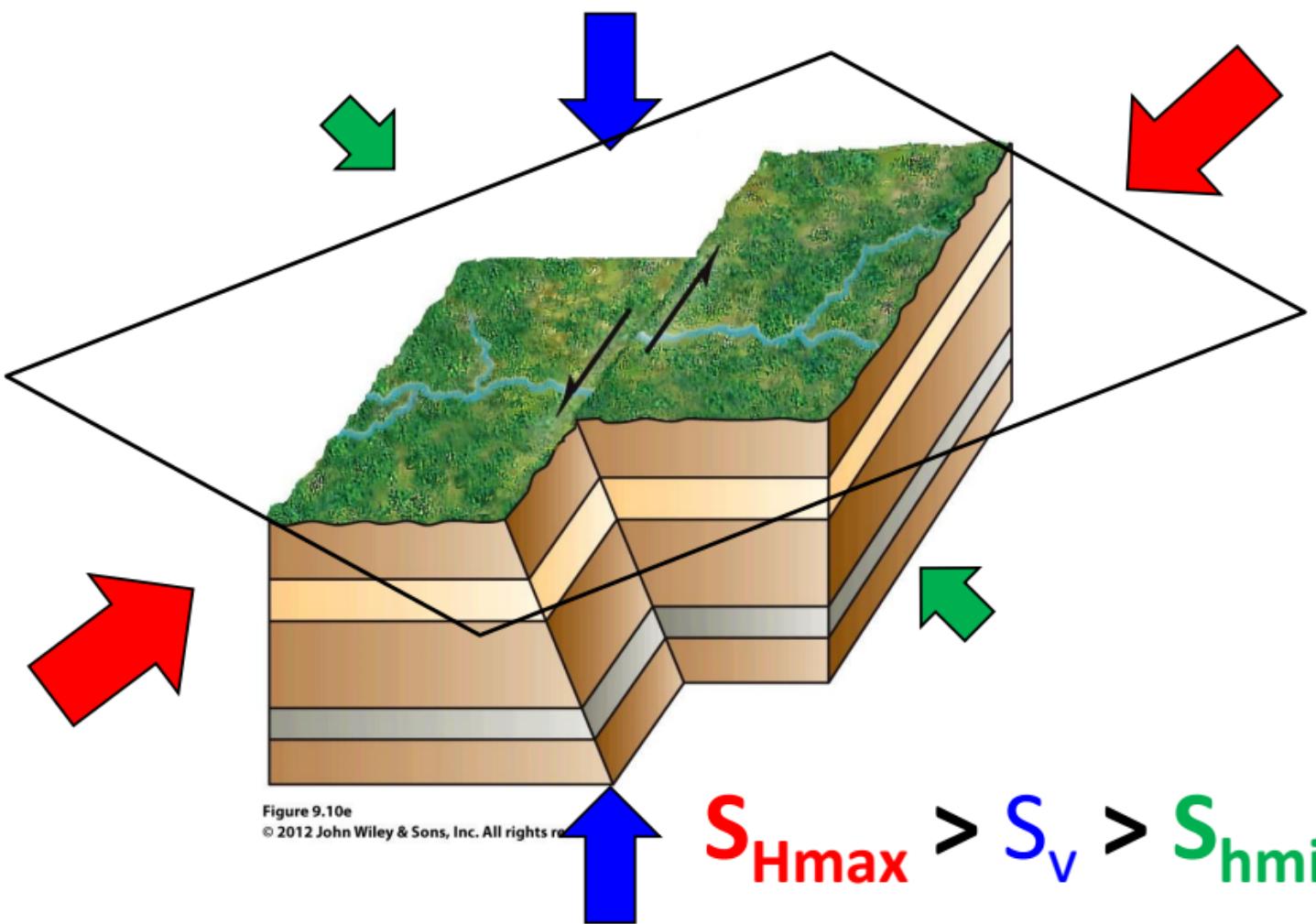
Figure 9.10c

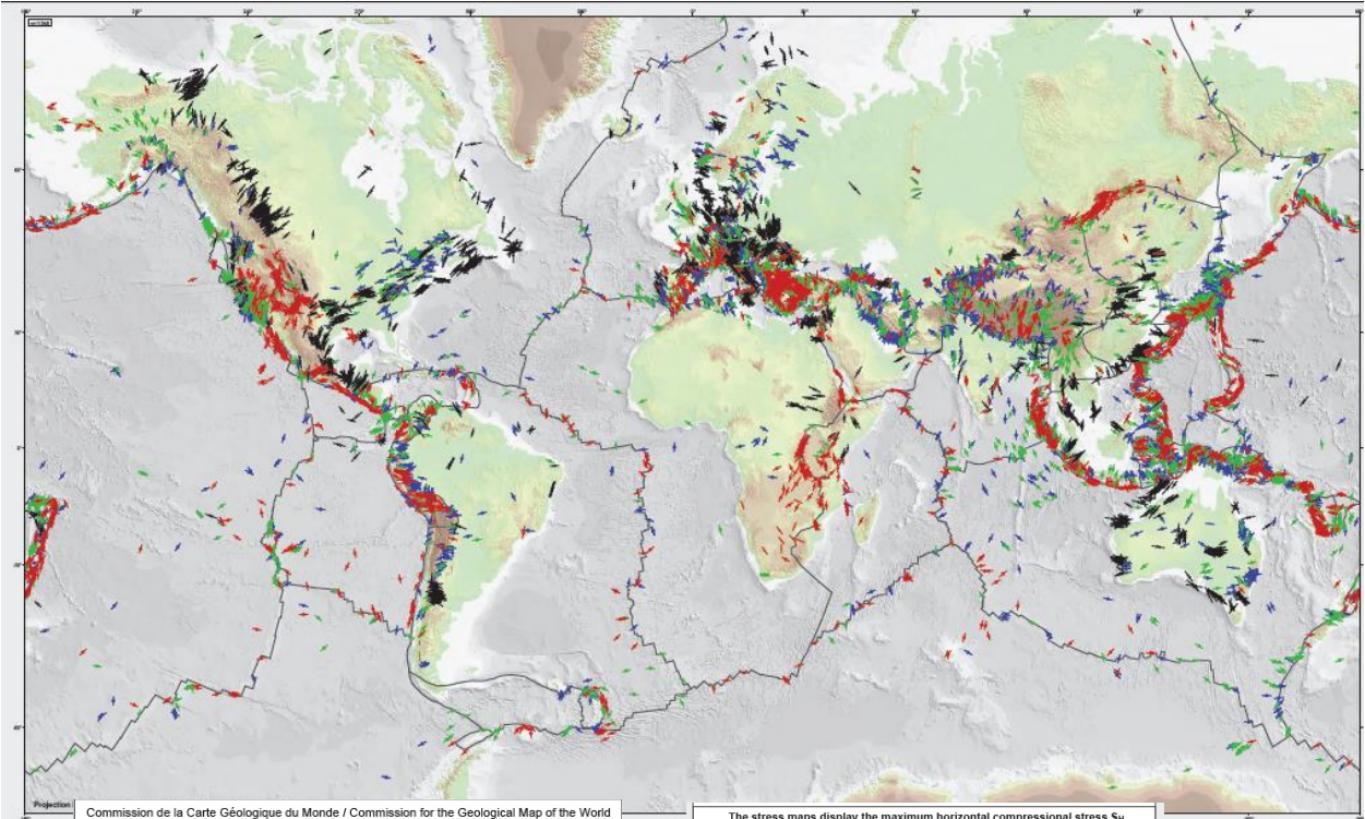
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$$S_1 = S_{H\max}$$

$$S_2 = S_v$$

$$S_3 = S_{hmin}$$





Commission de la Carte Géologique du Monde / Commission for the Geological Map of the World



WORLD STRESS MAP



2009 - 2nd edition, based on the WSM database release 2008

Helmholtz Centre Potsdam - GFZ German Research Centre for Geosciences

Authors

Oliver Heidbach, Mark Tingay, Andreas Barth, John Reinecker, Daniel Kurfürst, and Birgit Müller



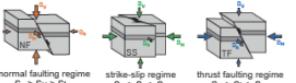
The stress maps display the maximum horizontal compressional stress S_H

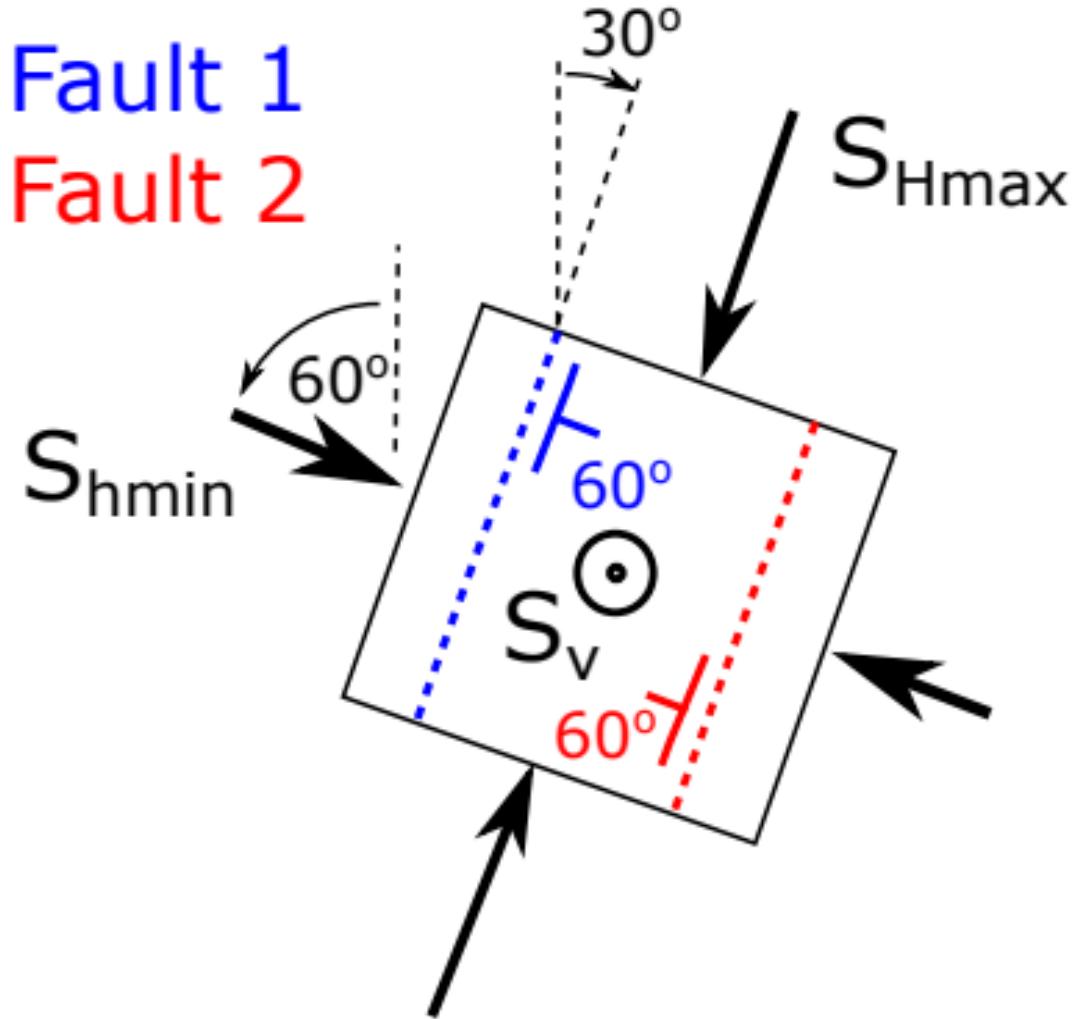
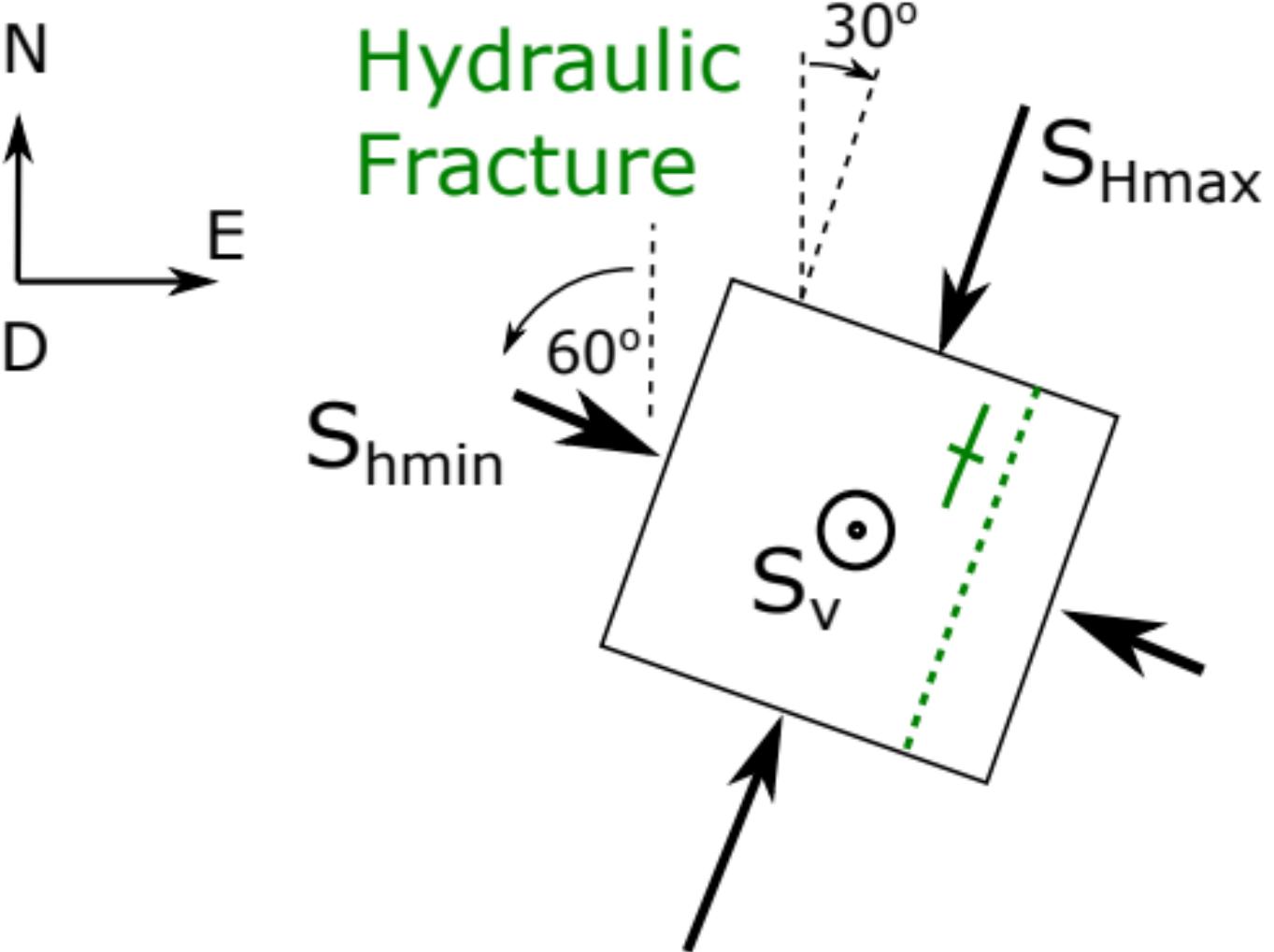
Method	Quality
focal mechanism	S_H is within $\pm 15^\circ$
breakouts	S_H is within $\pm 20^\circ$
drl. induced frac.	S_H is within $\pm 25^\circ$
overcoring	
hydro. fractures	
geol. indicators	

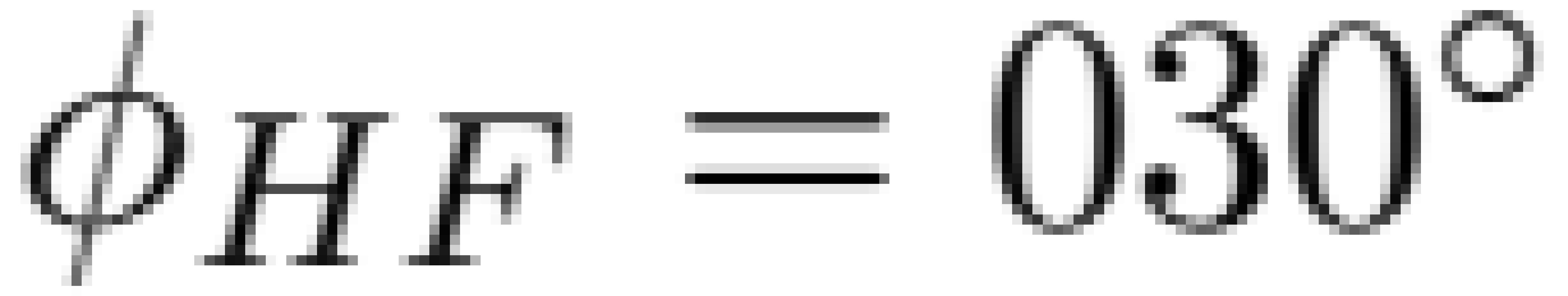
Data depth range

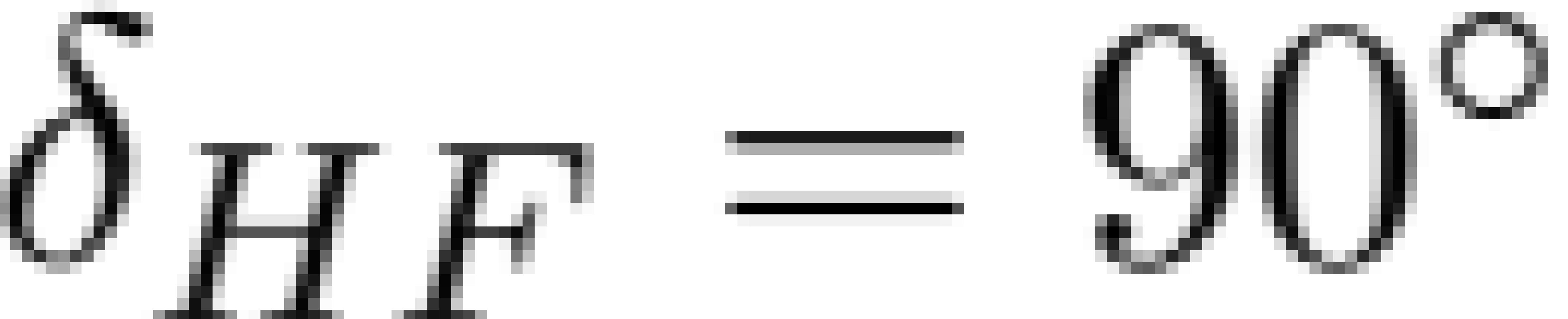
0-40 km

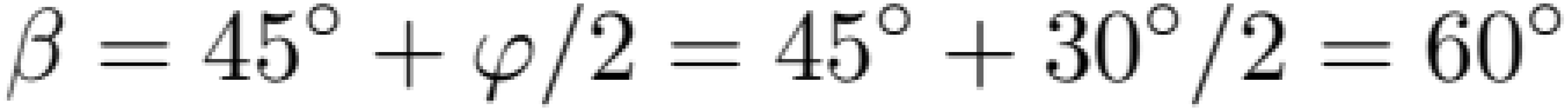
Stress Regime
Normal faulting
Strike-slip faulting
Thrust faulting
Unknown regime



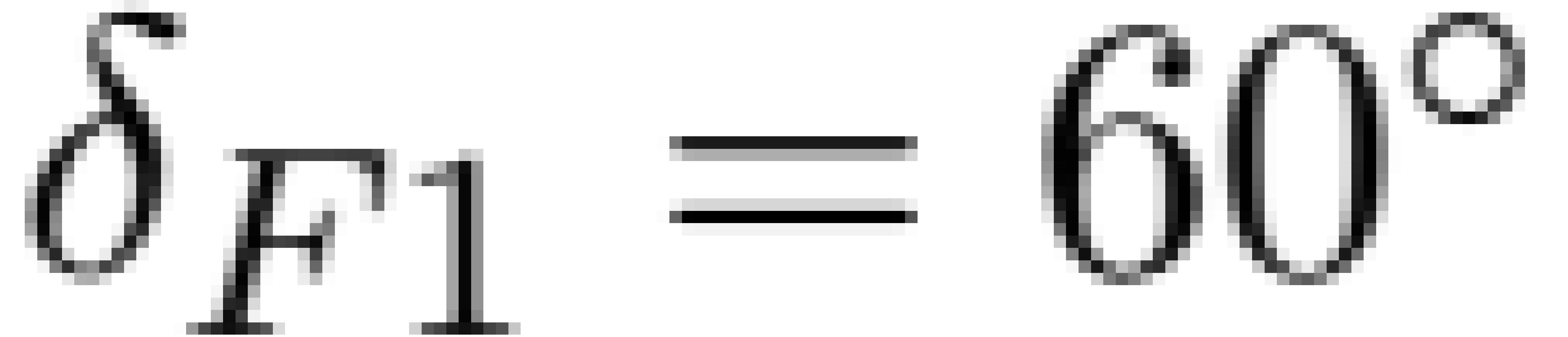


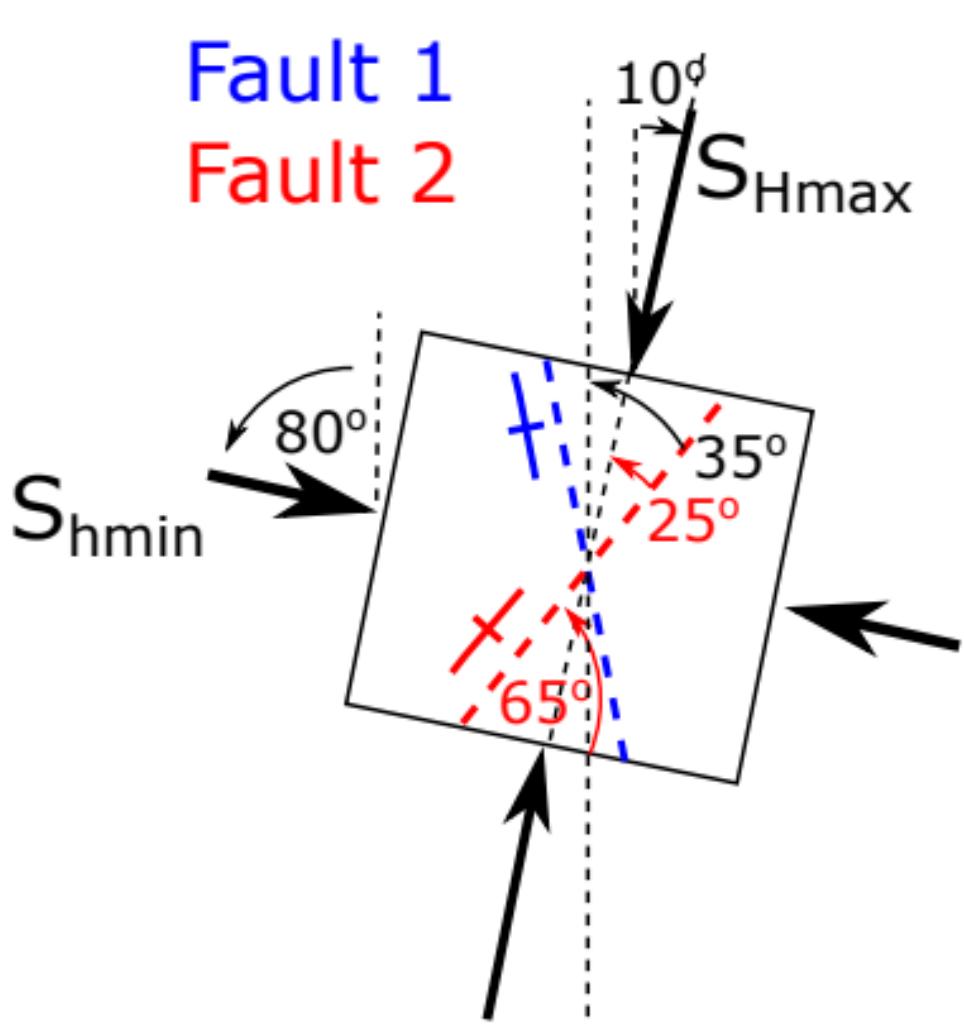
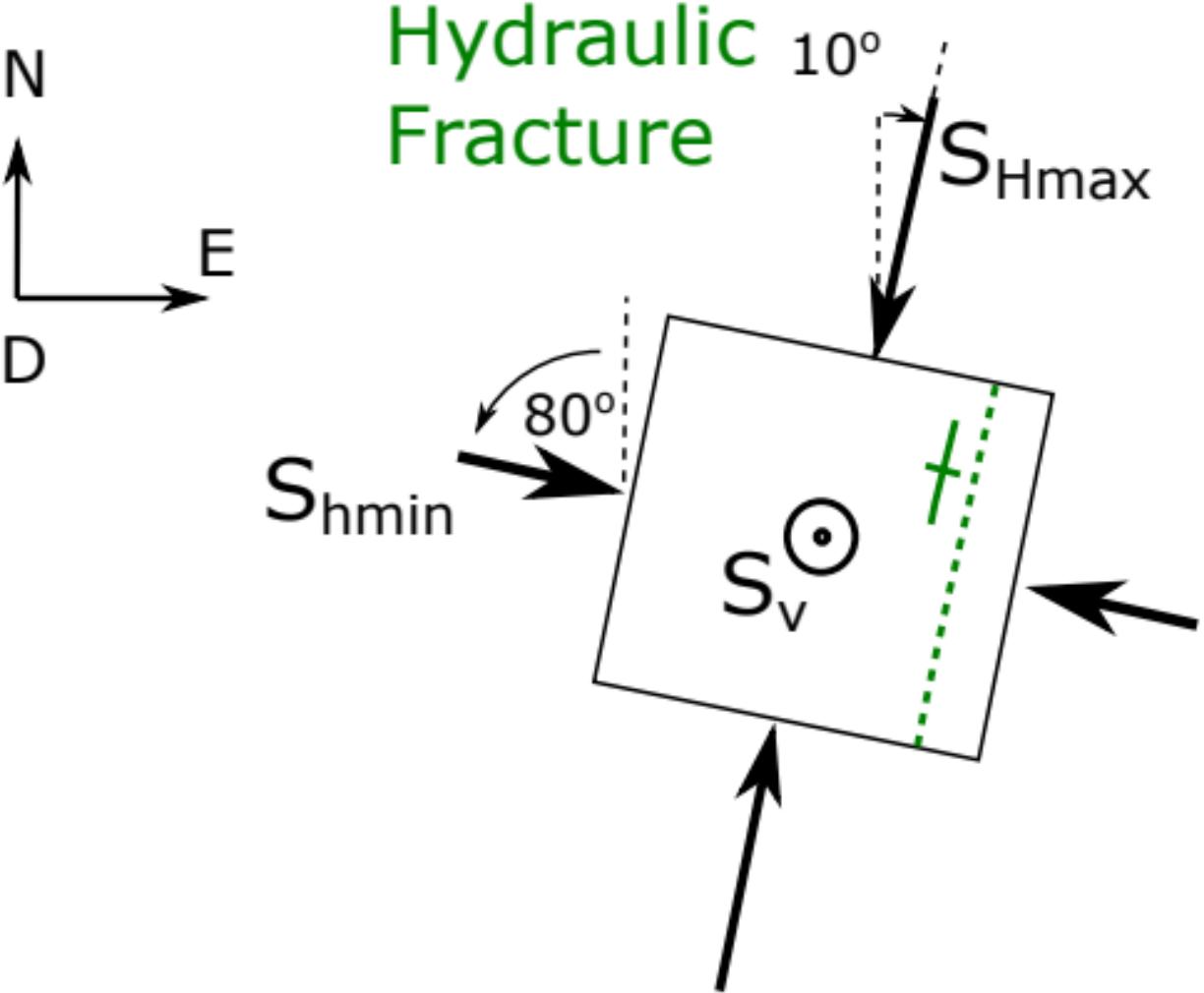


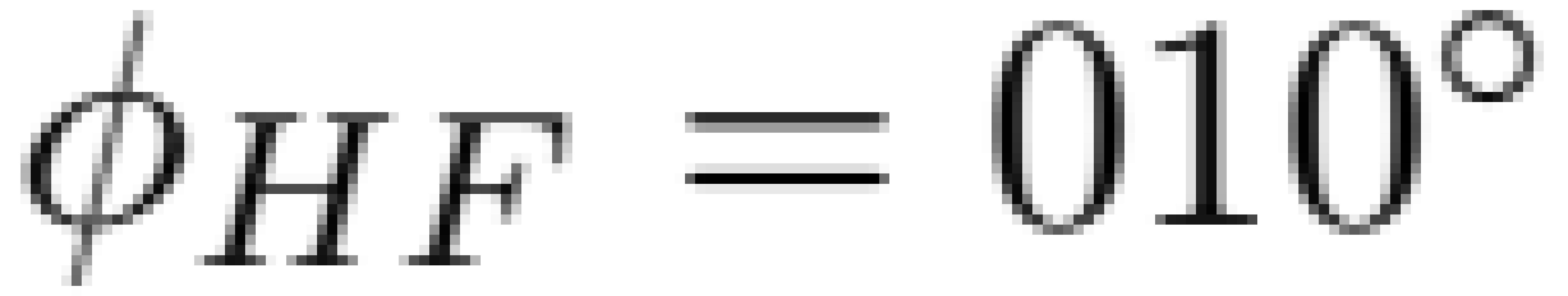


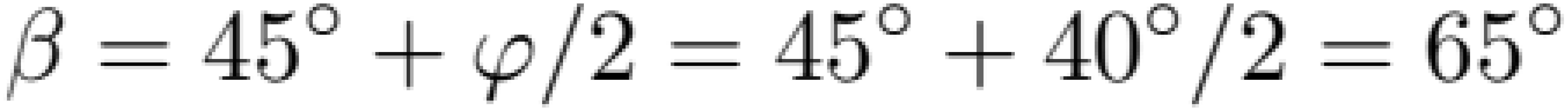


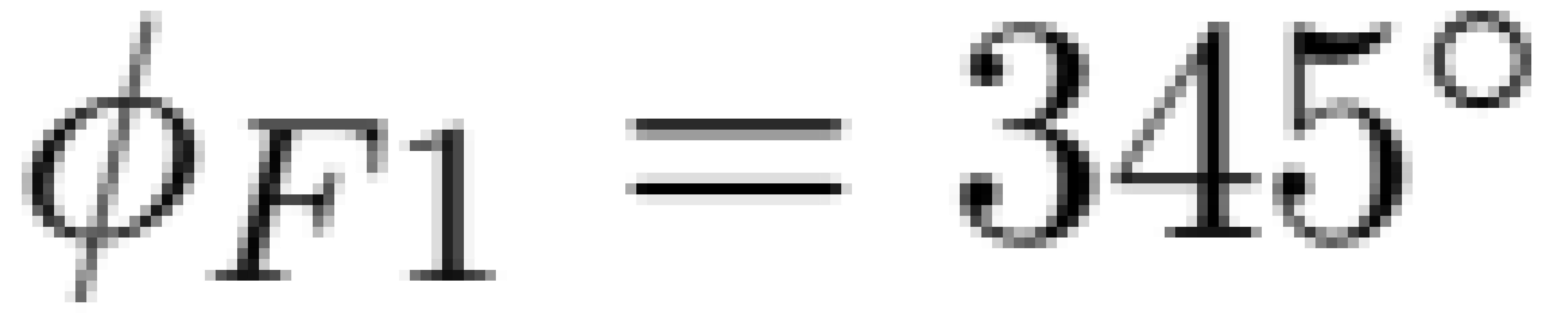


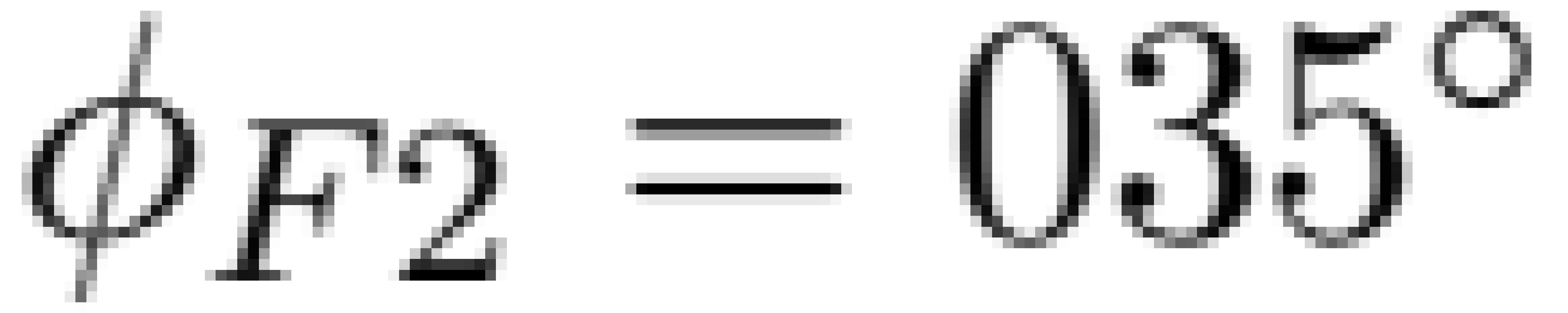




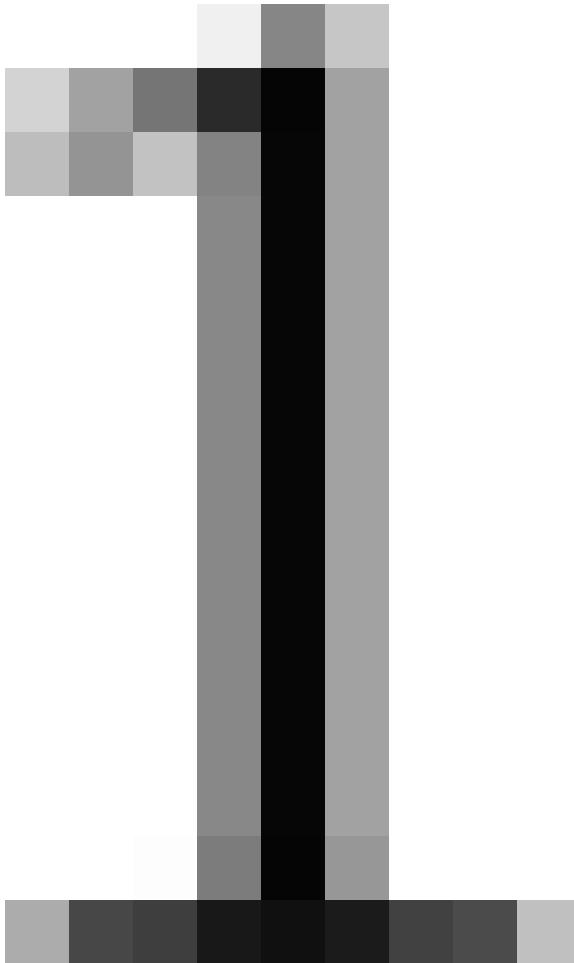




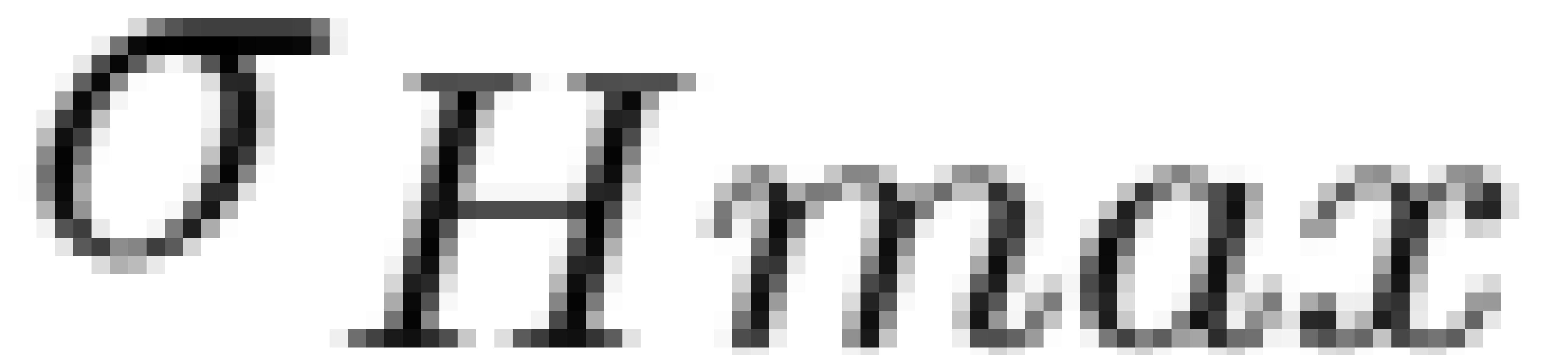




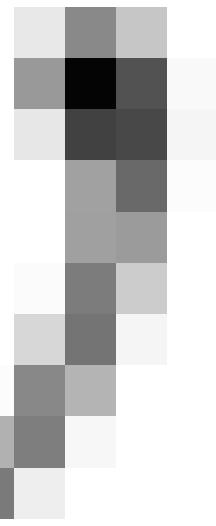
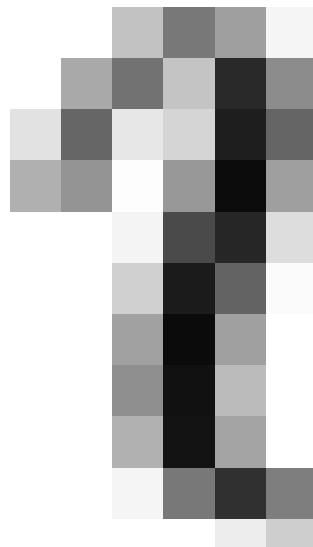
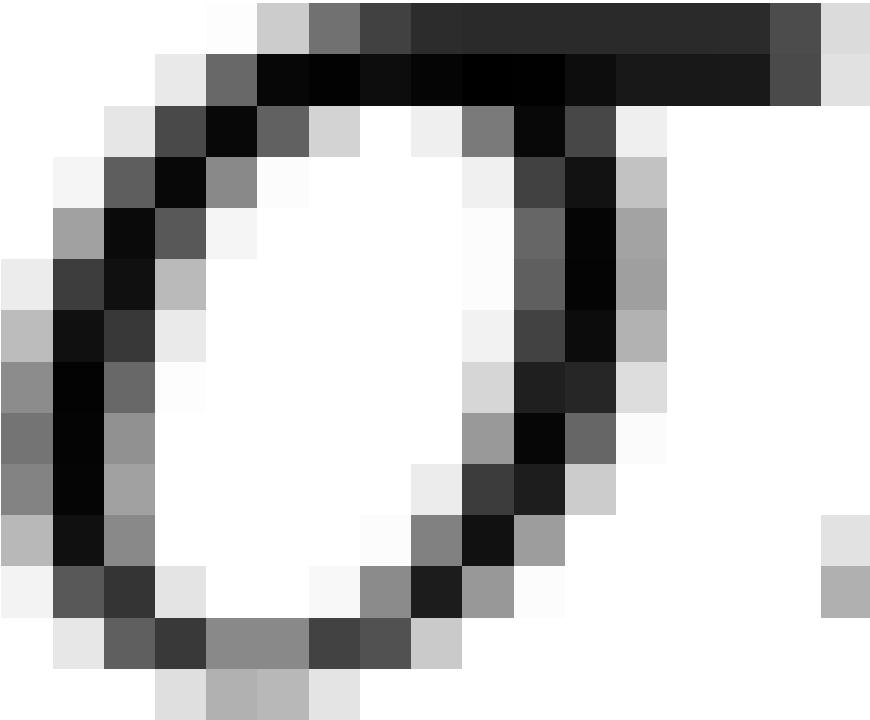




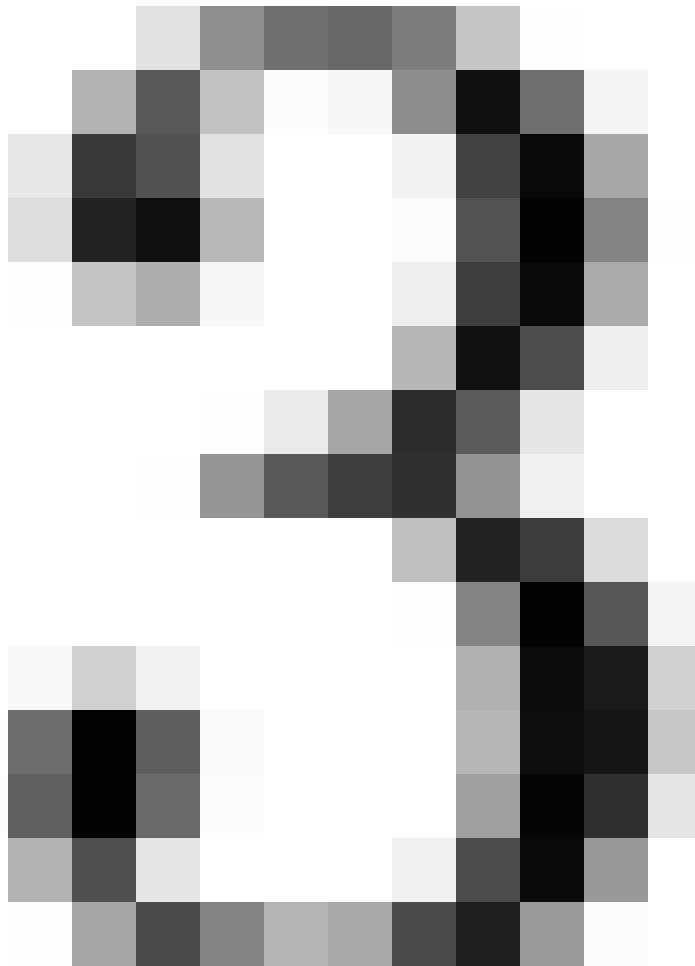




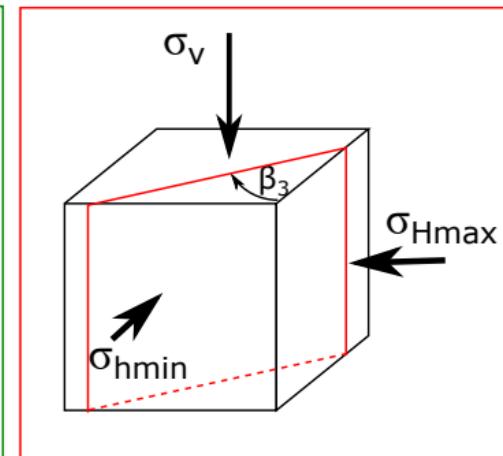
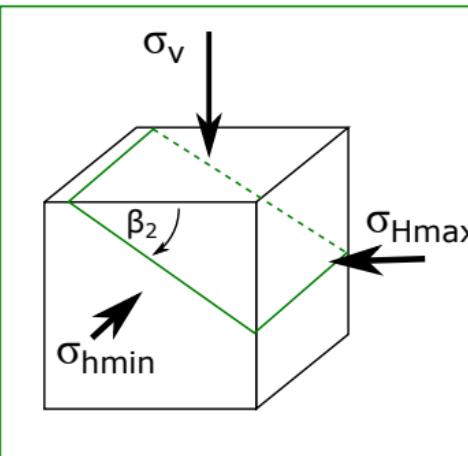
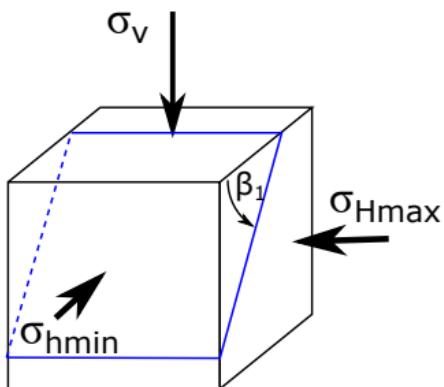
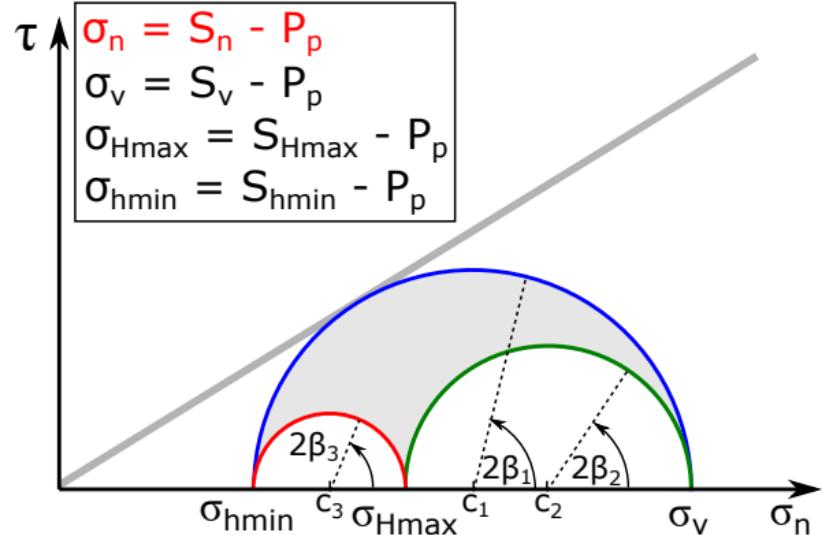
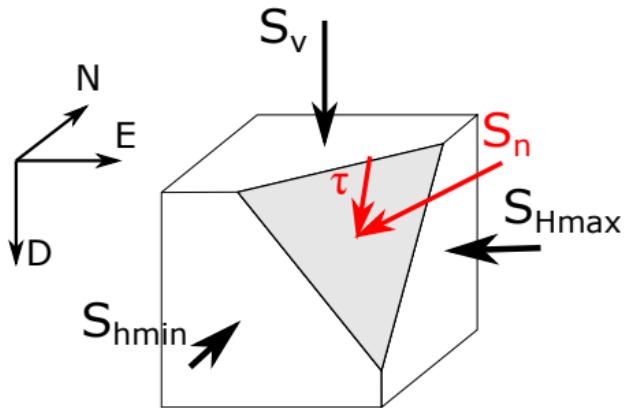


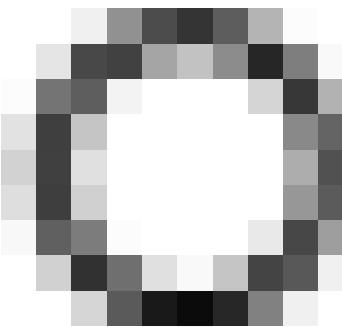
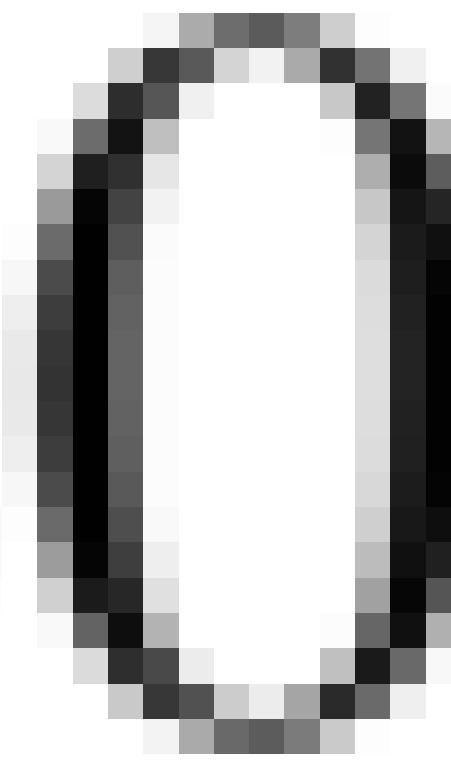
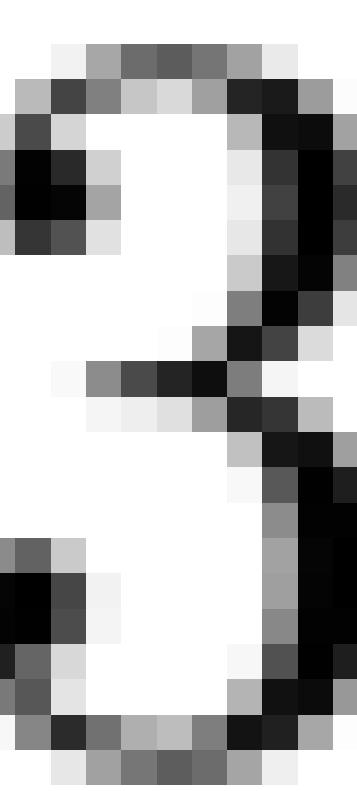
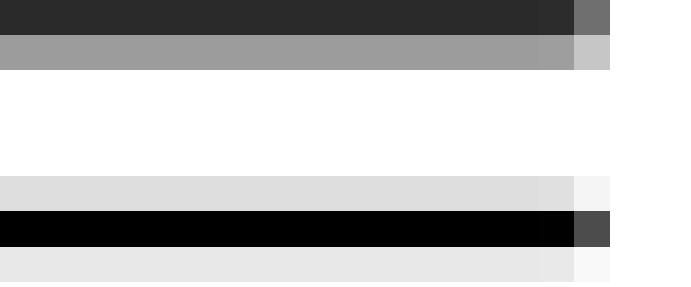
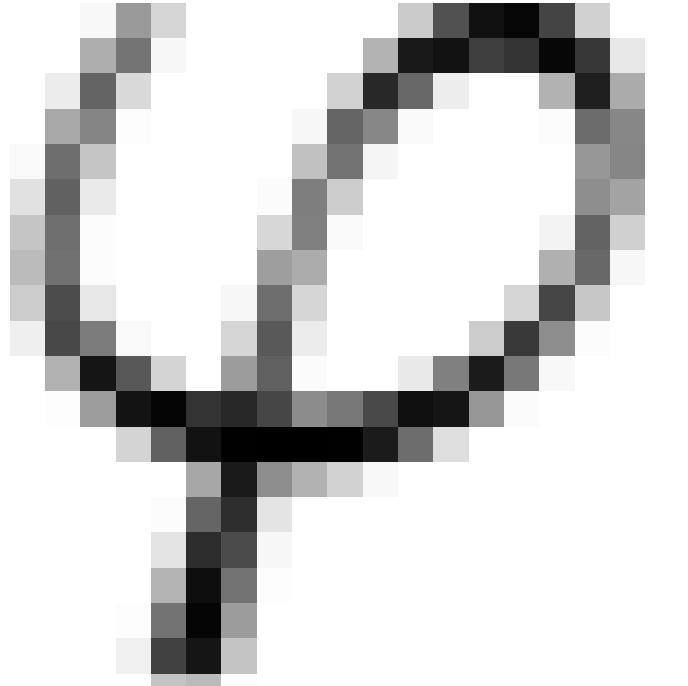


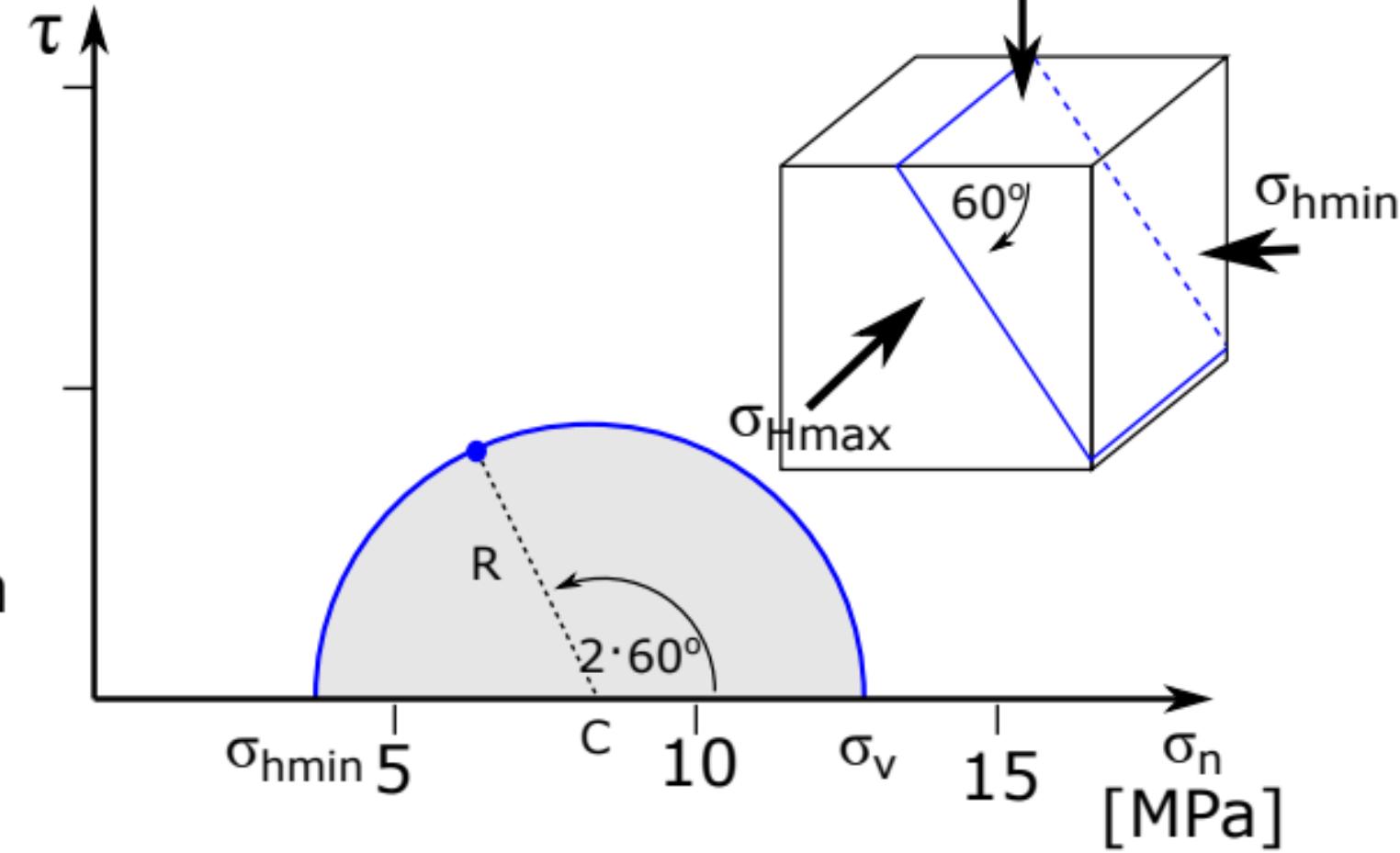
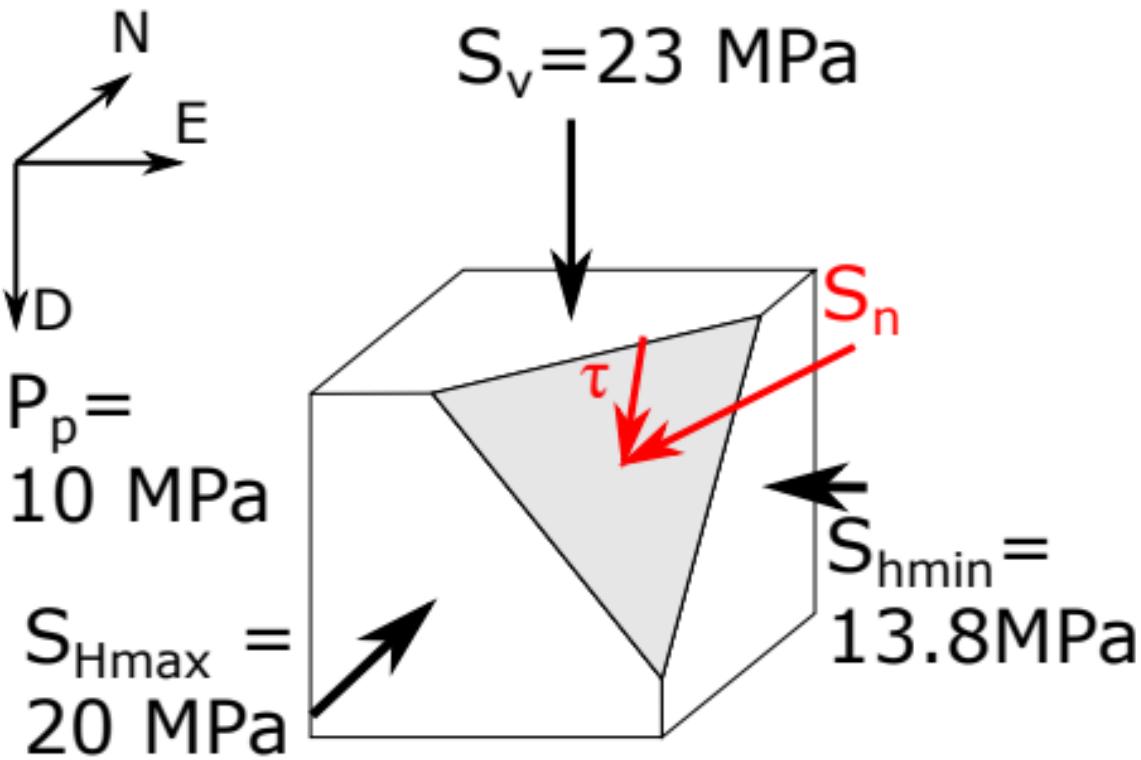


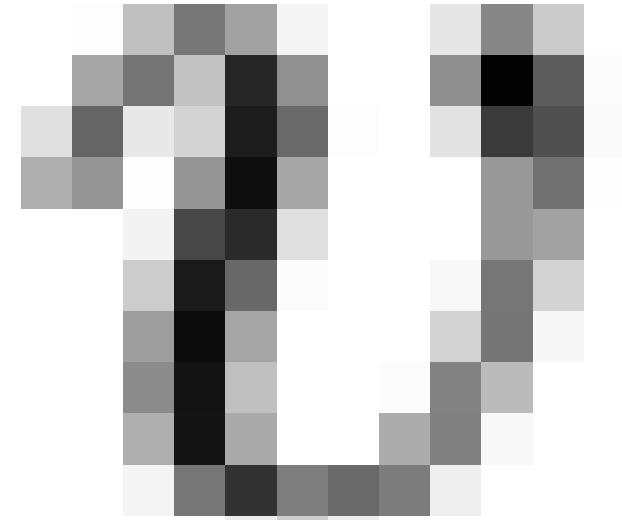
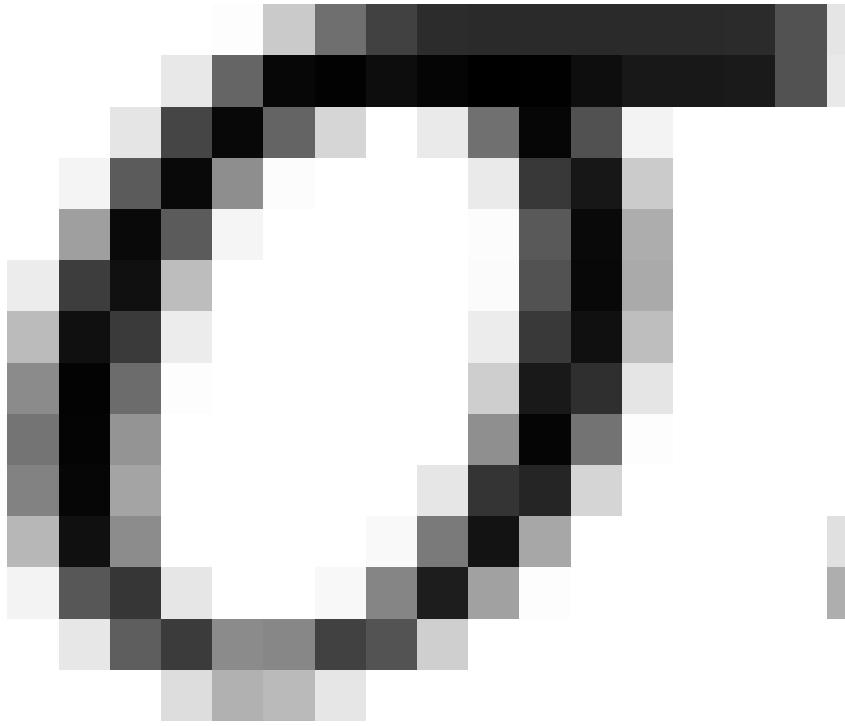




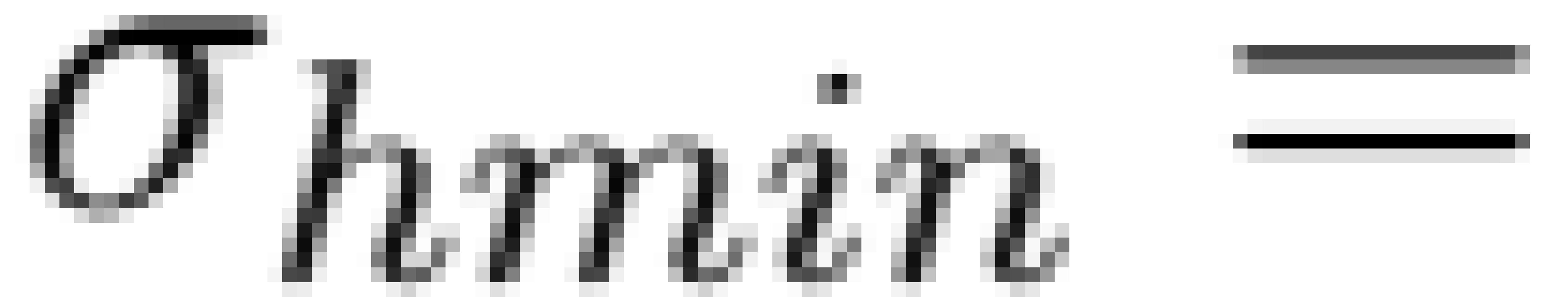








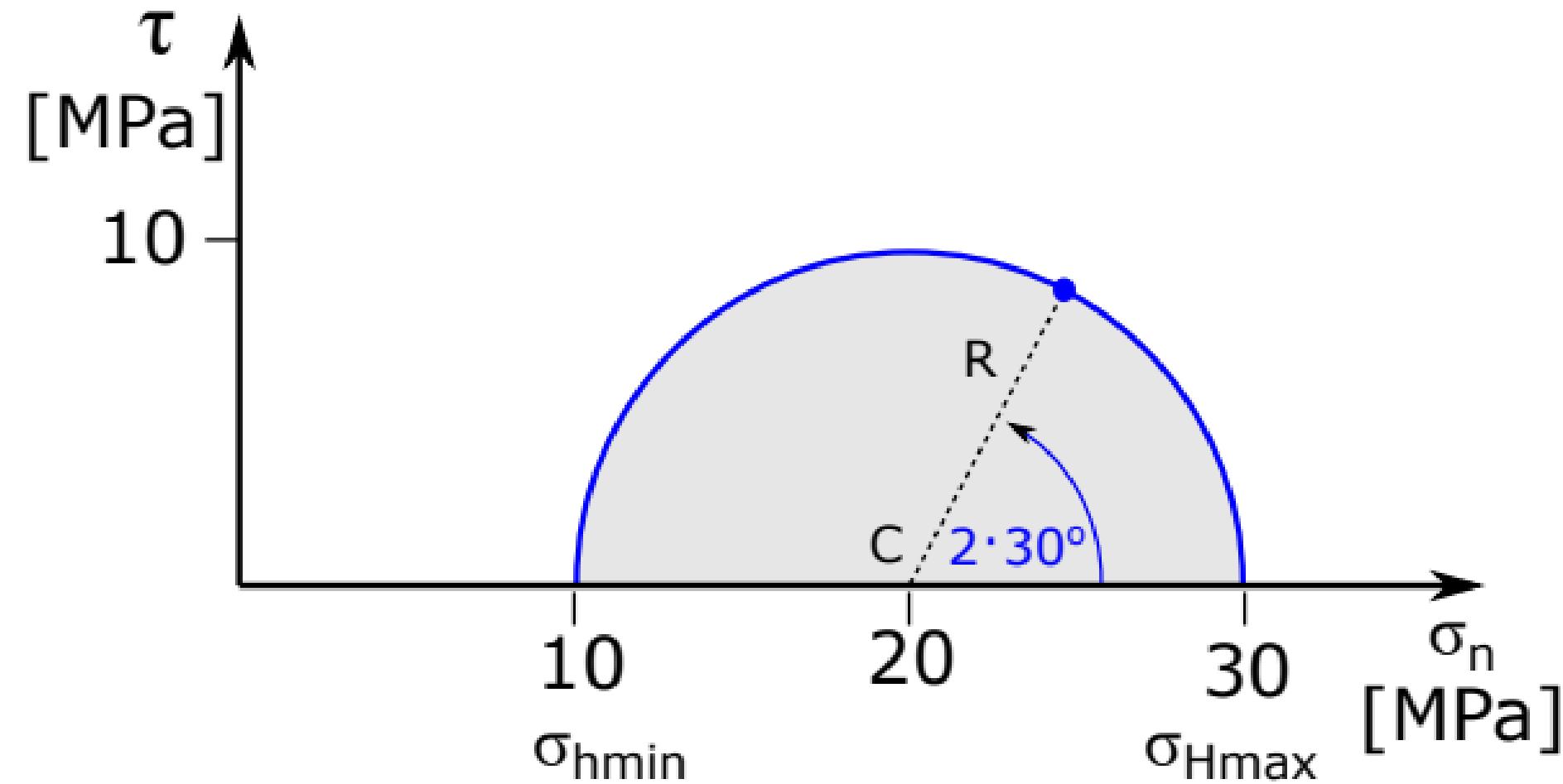
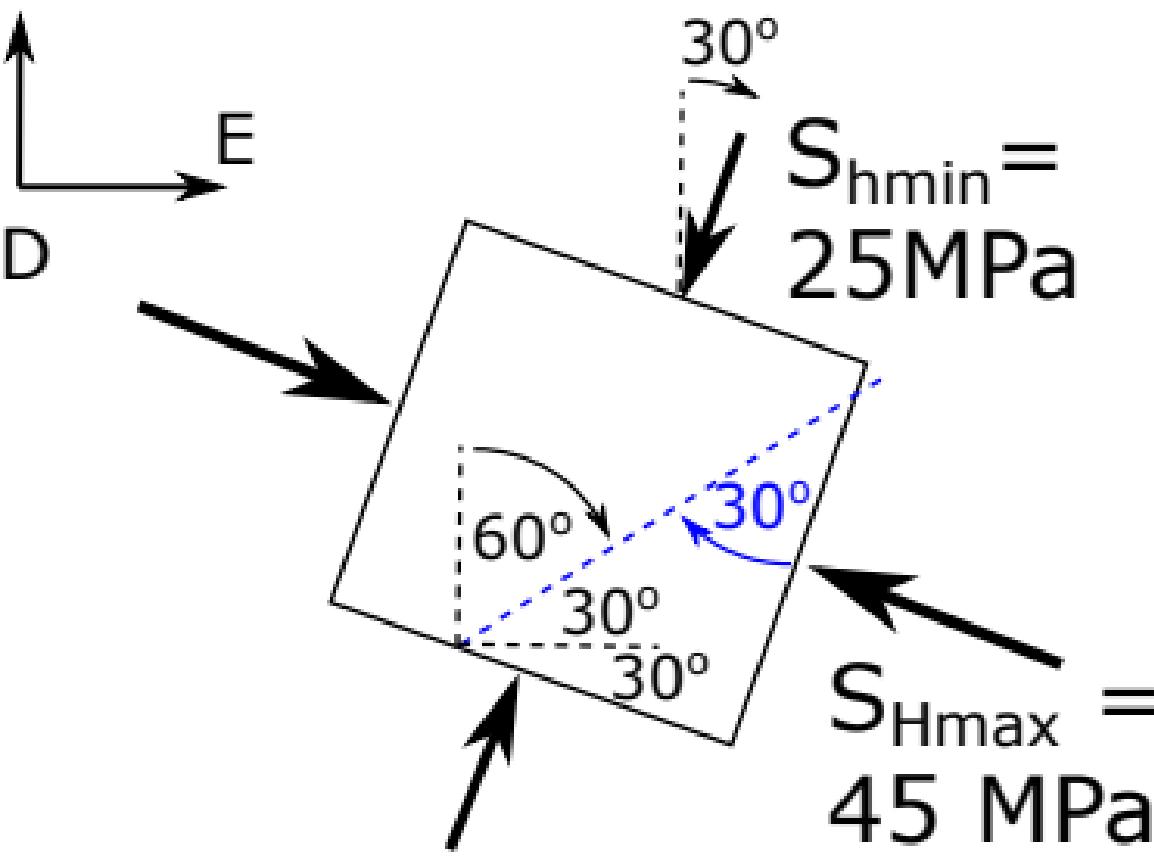




$$\sigma_n = \frac{(13 \text{ MPa} + 3.8 \text{ MPa})}{2} + \frac{(13 \text{ MPa} - 3.8 \text{ MPa}) \cos(2 \cdot 60^\circ)}{2} = 6.1 \text{ MPa}$$

$$\tau = \frac{(13 \text{ MPa} - 3.8 \text{ MPa})}{2} \sin(2 \cdot 60^\circ) = 4.0 \text{ MPa}$$

N $P_p = 15 \text{ MPa}$ $S_v = 30 \text{ MPa}$



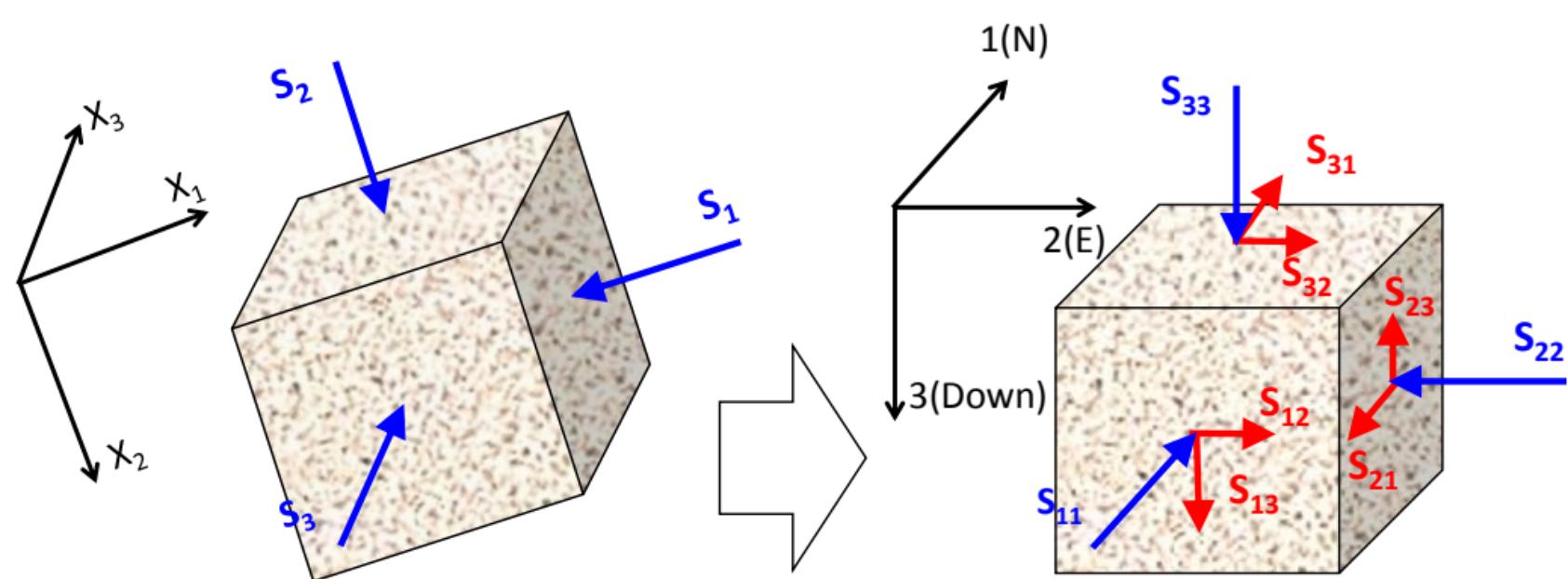
$$\sigma_n = \frac{(30 \text{ MPa} + 10 \text{ MPa})}{2} + \frac{(30 \text{ MPa} - 10 \text{ MPa}) \cos(2 \cdot 30^\circ)}{2} = 25 \text{ MPa}$$

$$\tau = \frac{(30 \text{ MPa} - 10 \text{ MPa})}{2} \sin(2 \cdot 30^\circ) = 8.7 \text{ MPa}$$





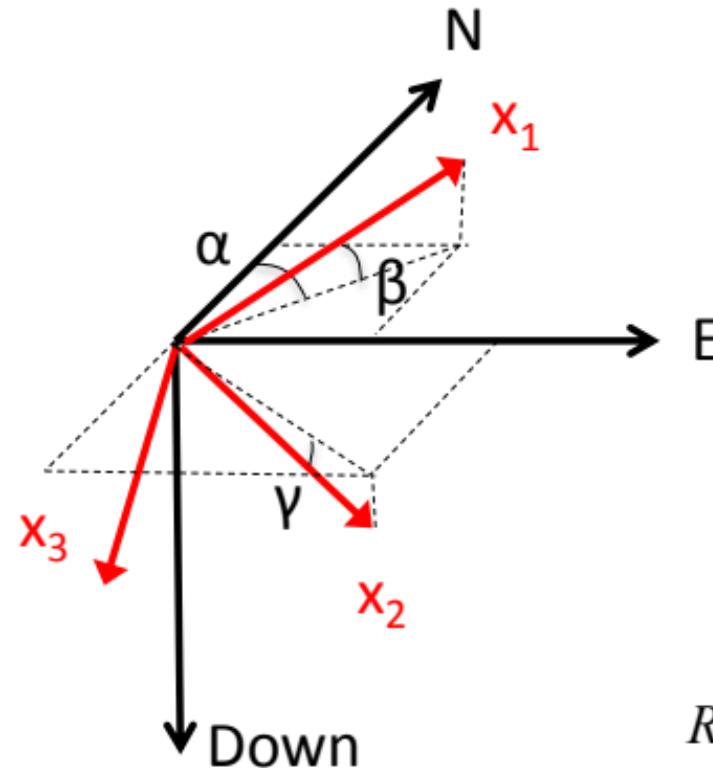




$$S_P = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$S_G = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$





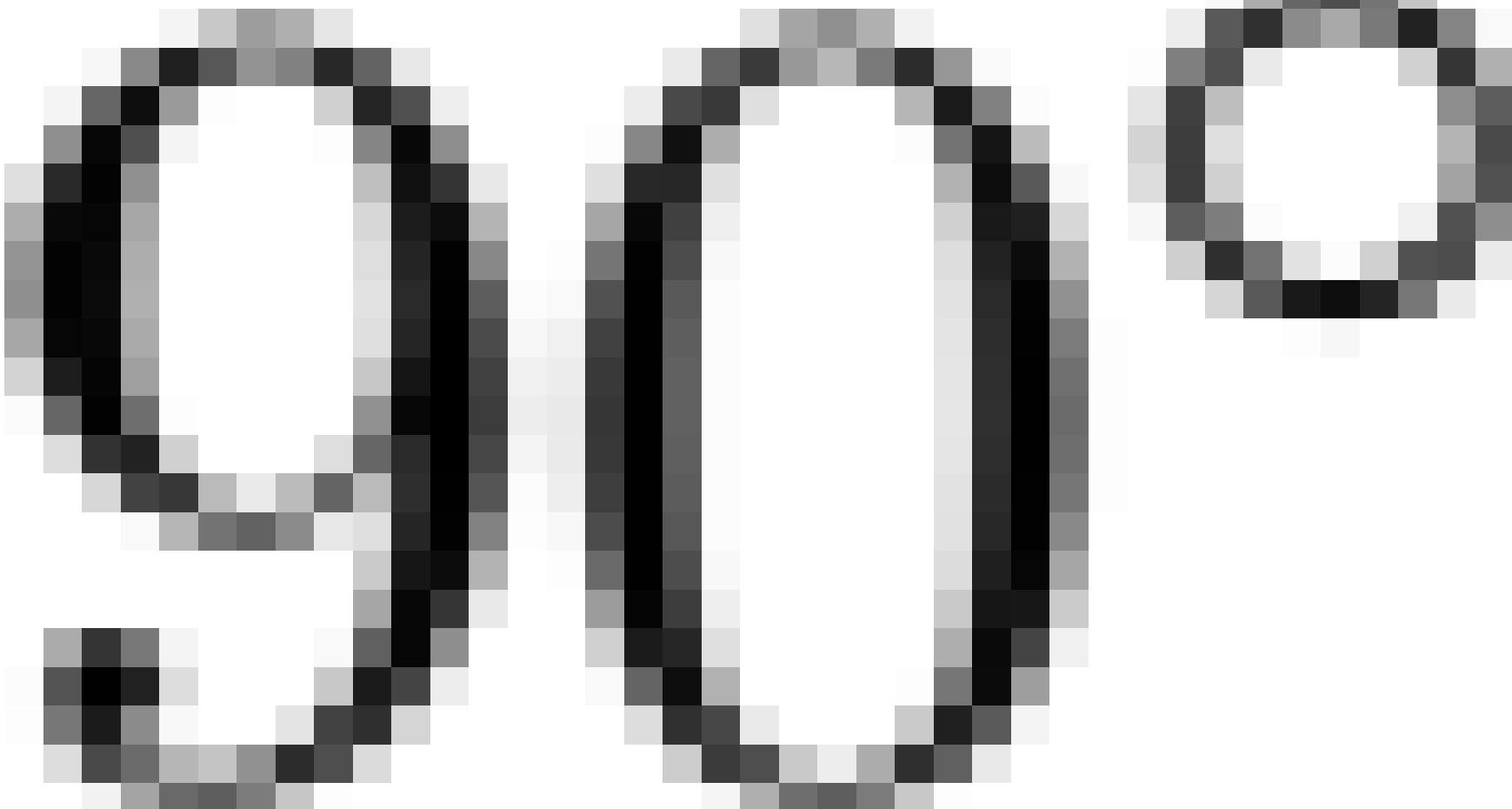
$$\underline{\underline{S}}' = \underline{\underline{A}} \underline{\underline{S}} \underline{\underline{A}}^T$$

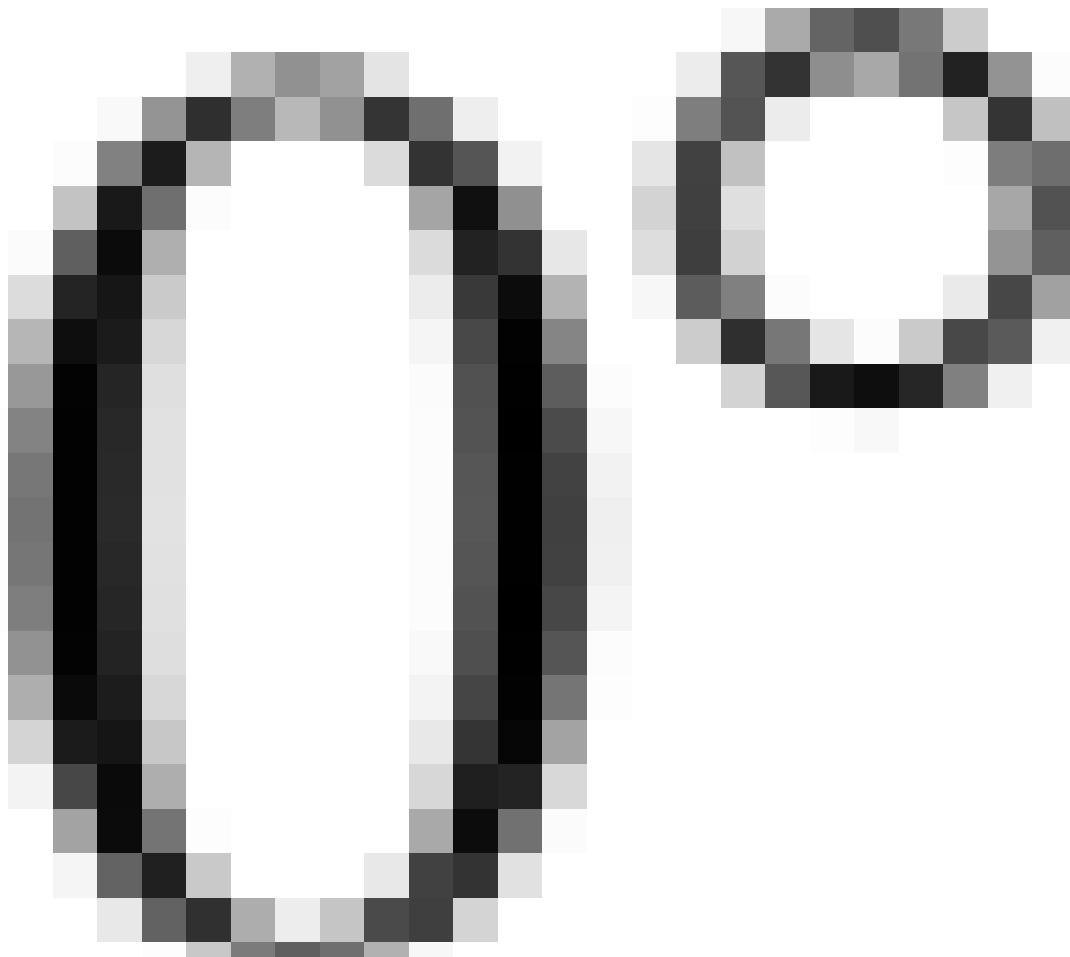
$$\underline{\underline{A}} = \begin{bmatrix} \underline{e}'_1 \cdot \underline{e}_1 & \underline{e}'_1 \cdot \underline{e}_2 & \underline{e}'_1 \cdot \underline{e}_3 \\ \underline{e}'_2 \cdot \underline{e}_1 & \underline{e}'_2 \cdot \underline{e}_2 & \underline{e}'_2 \cdot \underline{e}_3 \\ \underline{e}'_3 \cdot \underline{e}_1 & \underline{e}'_3 \cdot \underline{e}_2 & \underline{e}'_3 \cdot \underline{e}_3 \end{bmatrix}$$

where $\underline{\underline{A}}$ is the transformation matrix from the old base \underline{e}_i to base \underline{e}'_i and the components are the projection of the elements of the new base on the old base.

- Old system: N-E-D (Right-handed) Geographical system
- New system: 1-2-3 (Right-handed) Principal stress system

$$R_{PG} = \begin{bmatrix} \cos \alpha \cos b & \sin \alpha \cos b & -\sin b \\ \cos \alpha \sin b \sin g - \sin \alpha \cos g & \sin \alpha \sin b \sin g + \cos \alpha \cos g & \cos b \sin g \\ \cos \alpha \sin b \cos g + \sin \alpha \sin g & \sin \alpha \sin b \cos g - \cos \alpha \sin g & \cos b \cos g \end{bmatrix}$$







S

G

S

SP

S

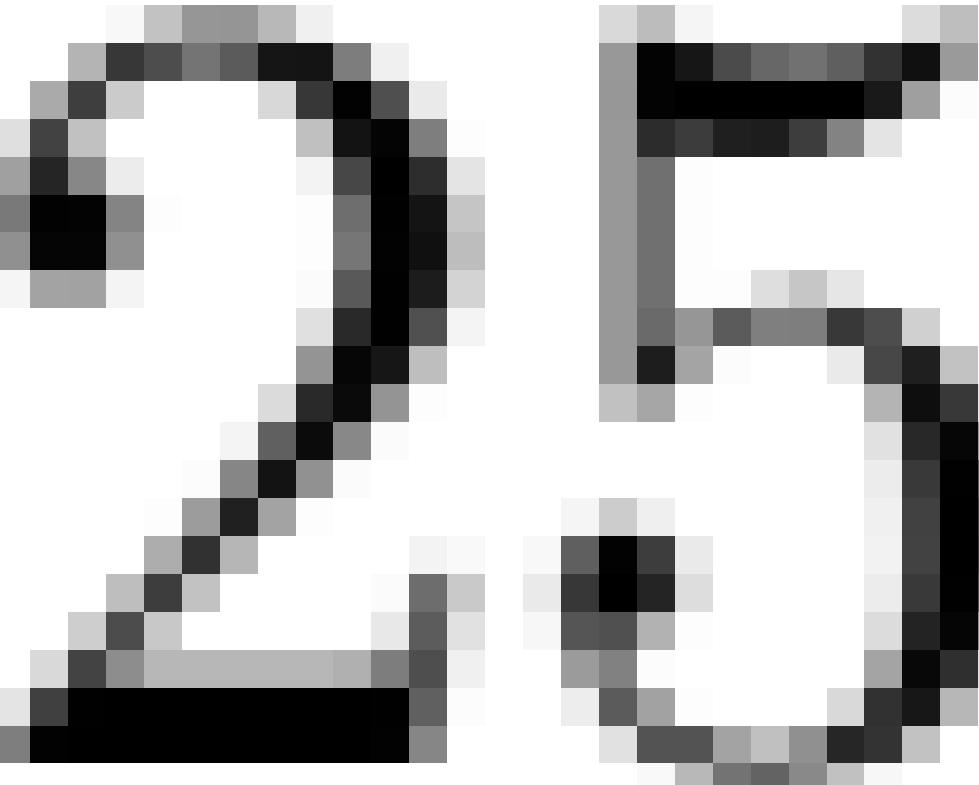
P

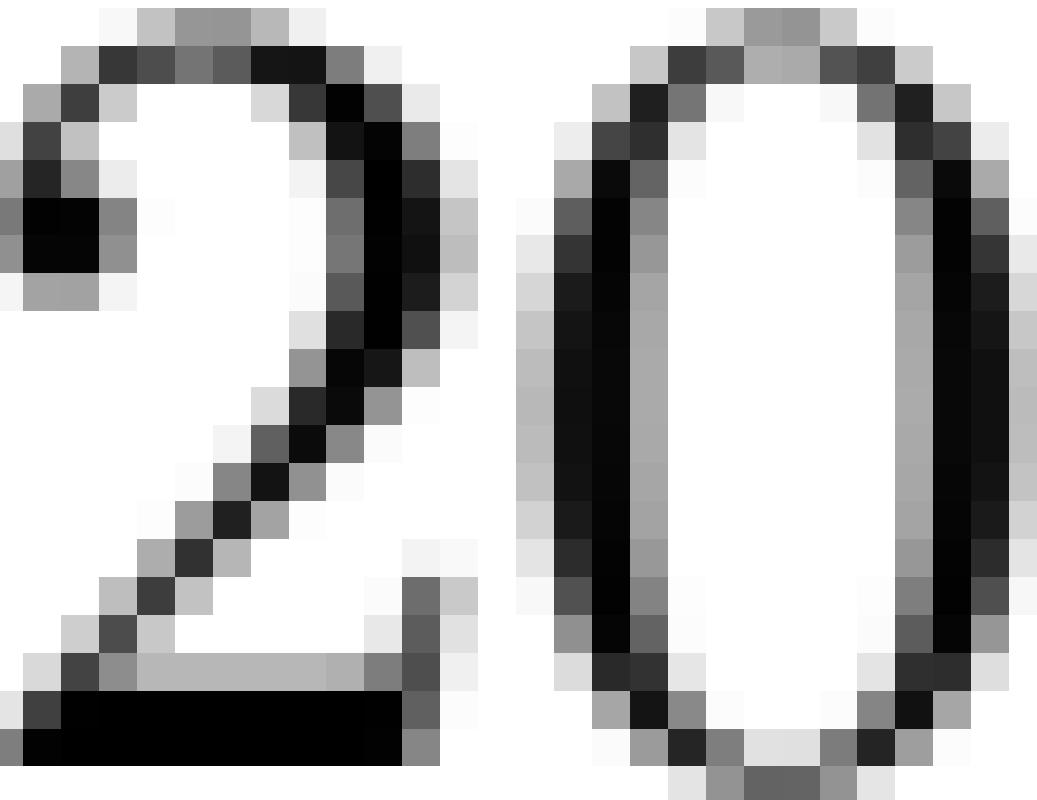
S

P

SP

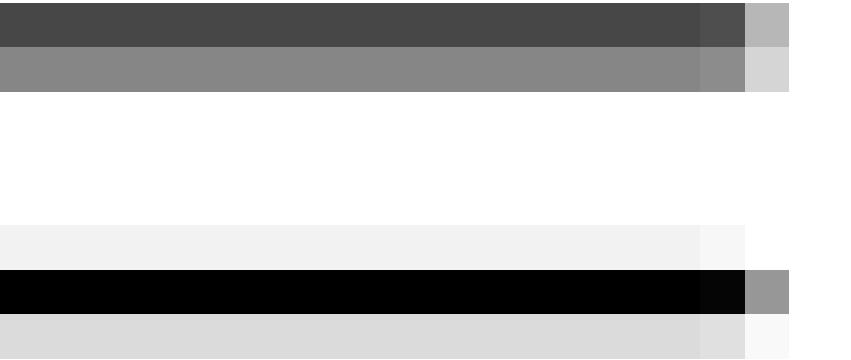
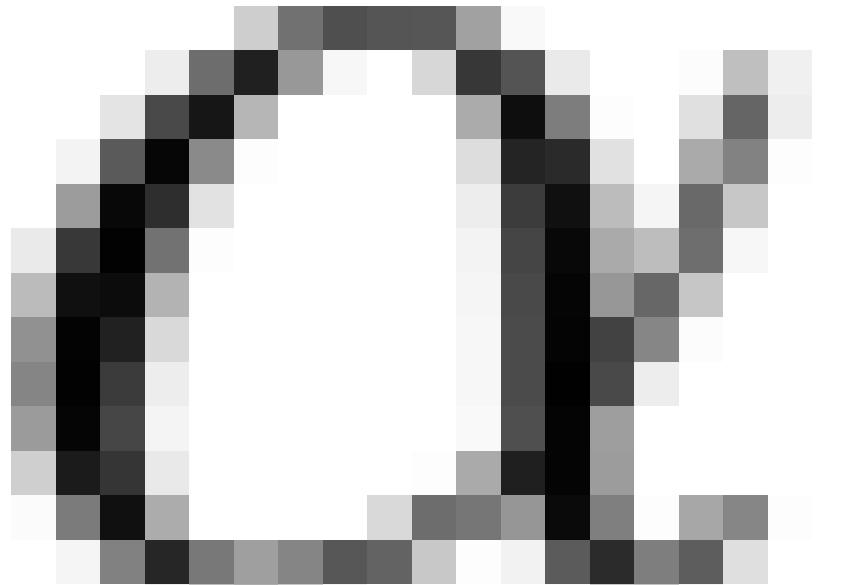


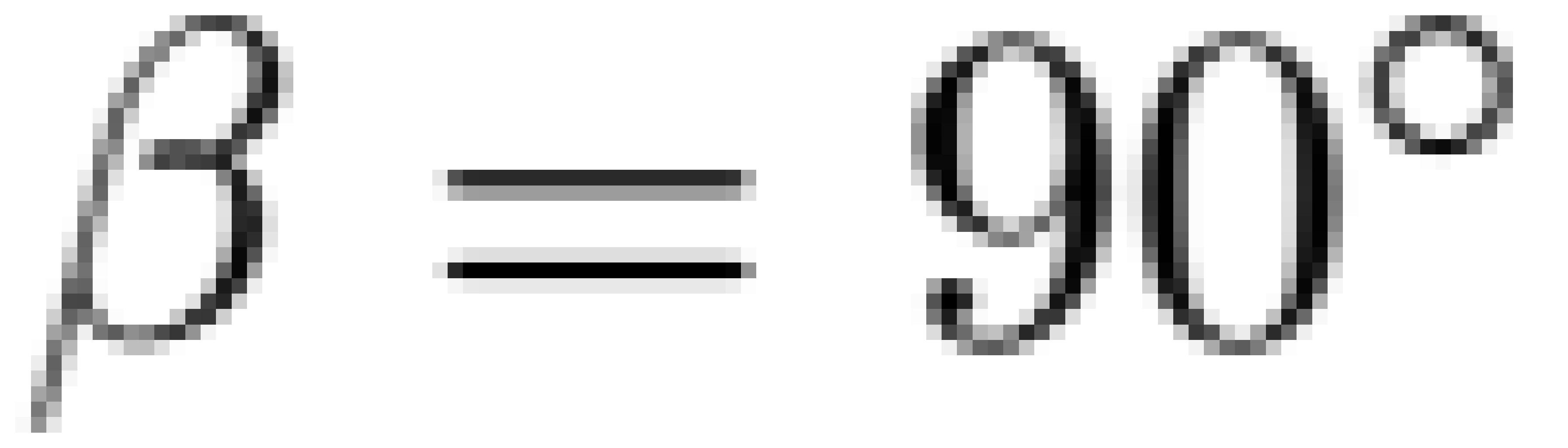




$$\underline{S}_P =$$

$$\begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$



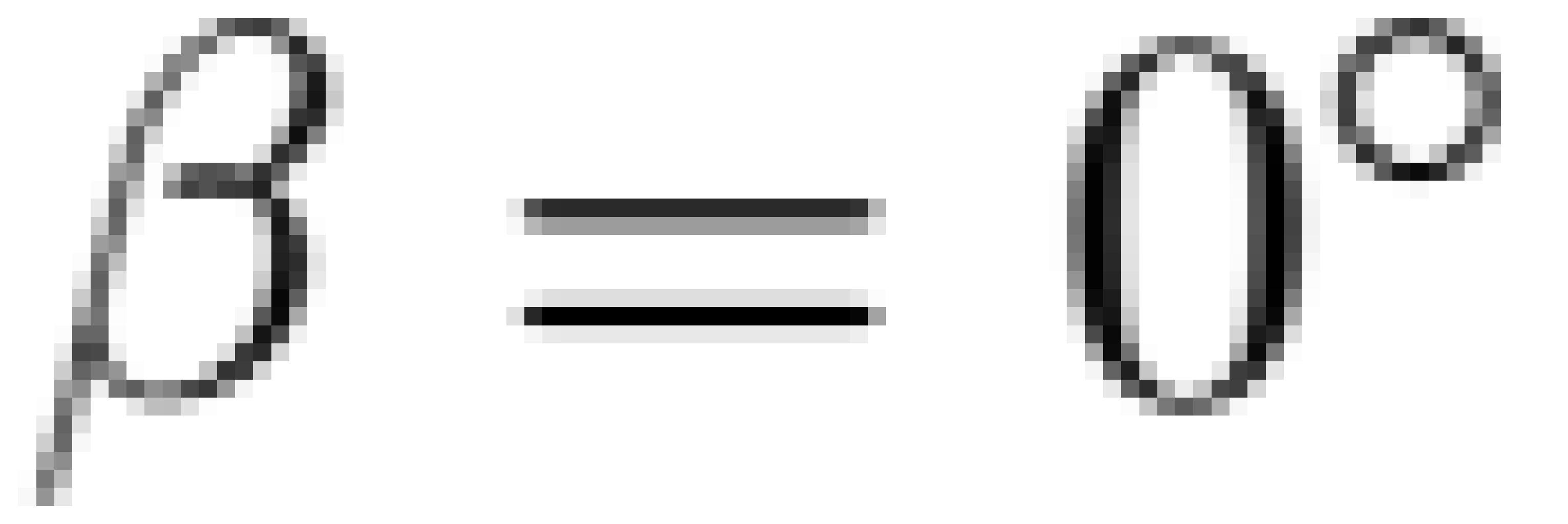


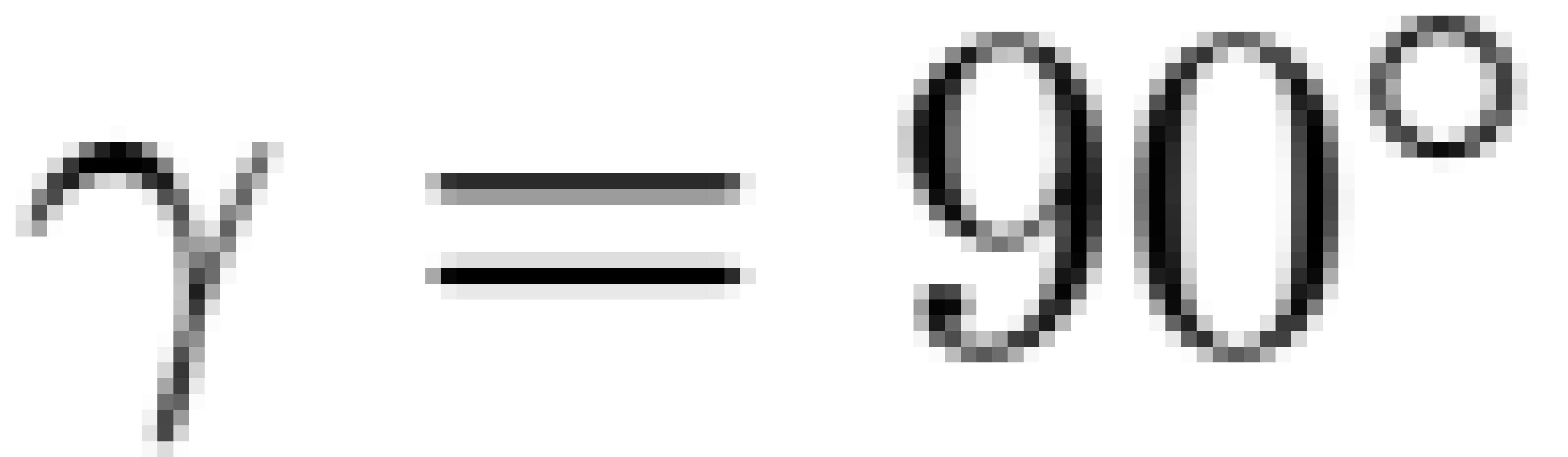


$$R_{PG} =$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 10 & 5 \\ 1 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 1 & 0 & 30 \end{bmatrix}$$

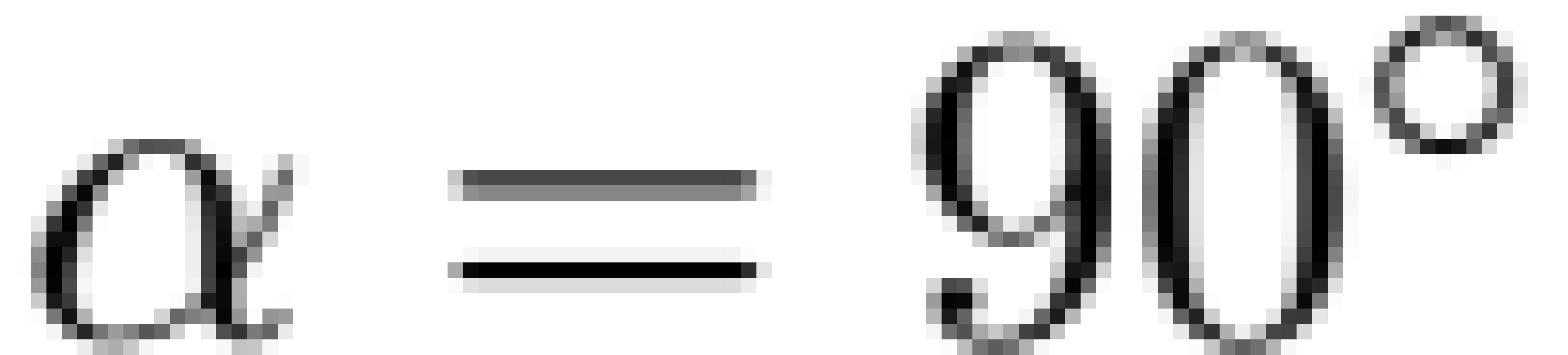




$$R_{PG} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 10 \\ 0 & -1 & 0 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & -1 & 25 \end{bmatrix}$$

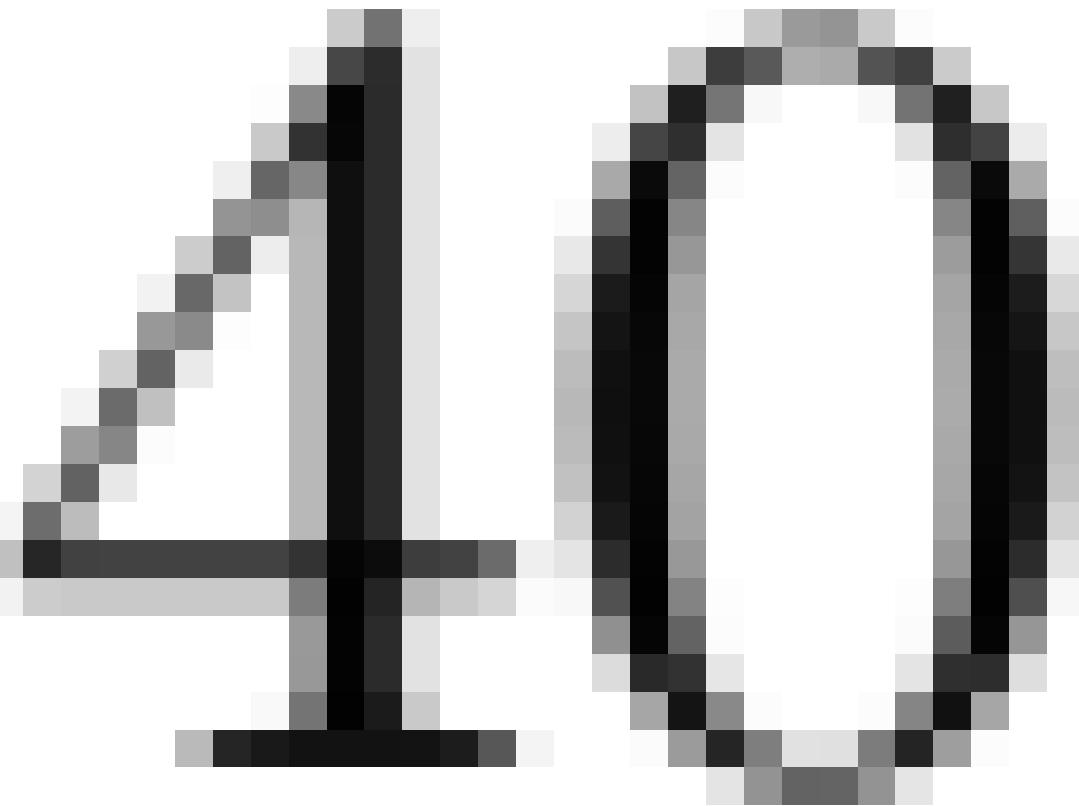


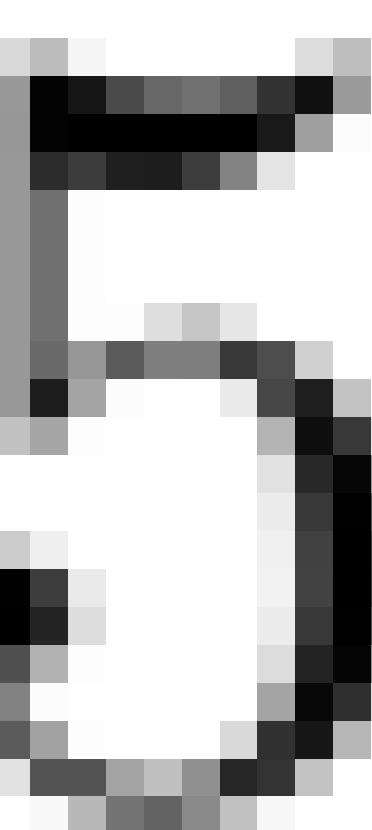
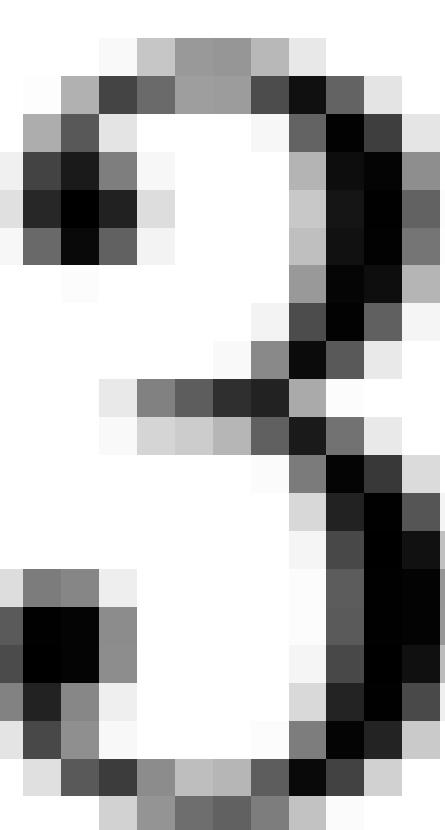
$$R_{PG} =$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 10 \end{bmatrix}^T \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

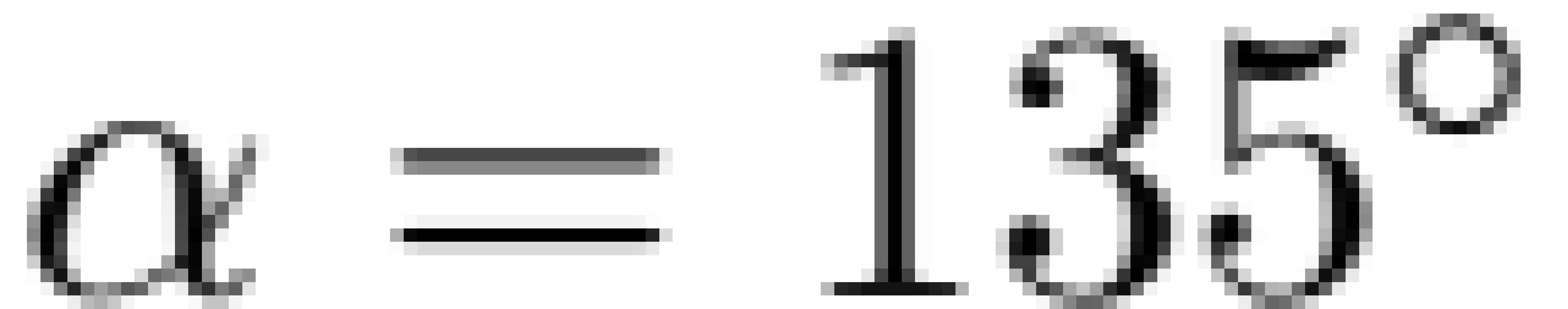






$$\underline{S}_P =$$

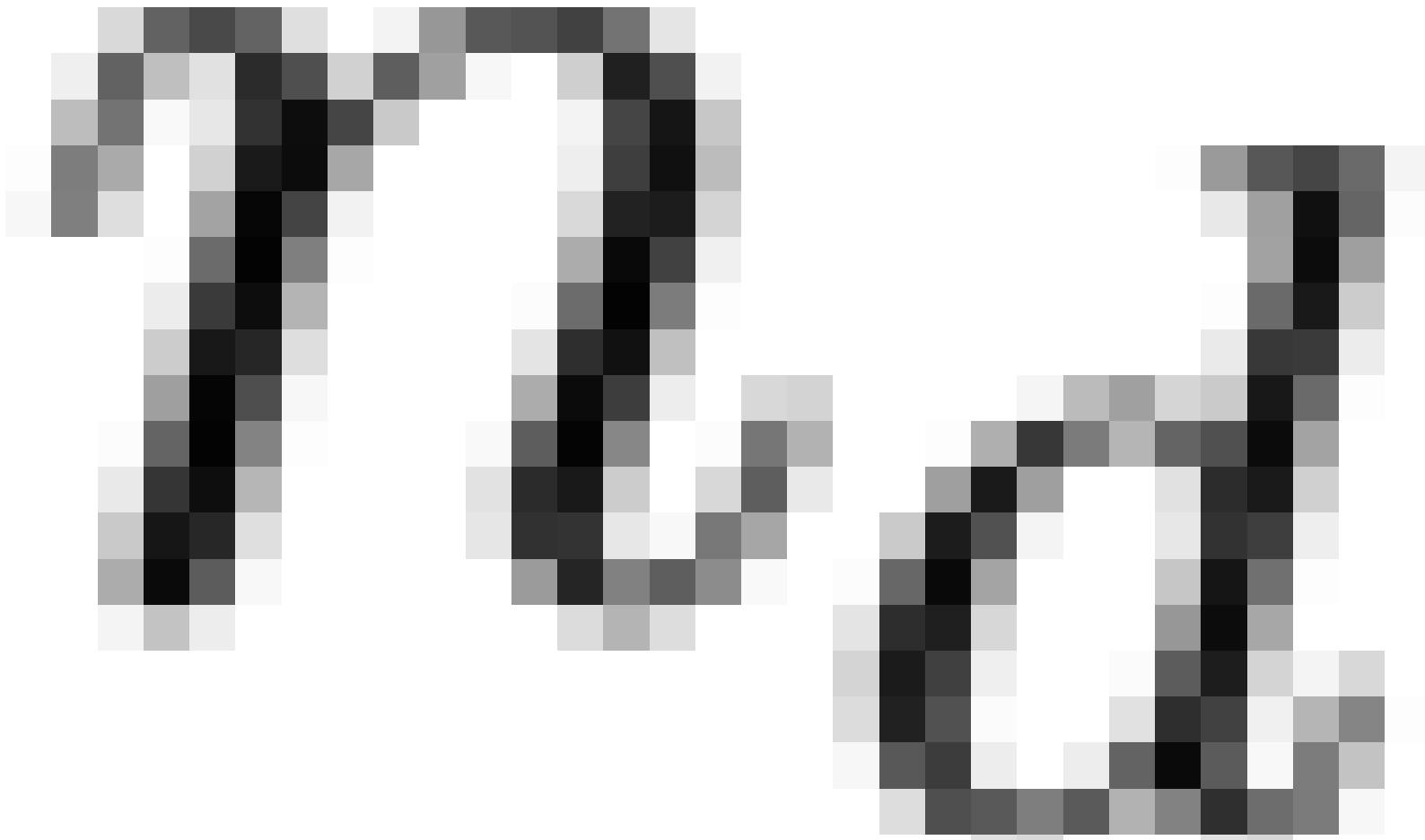
$$\begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix}$$

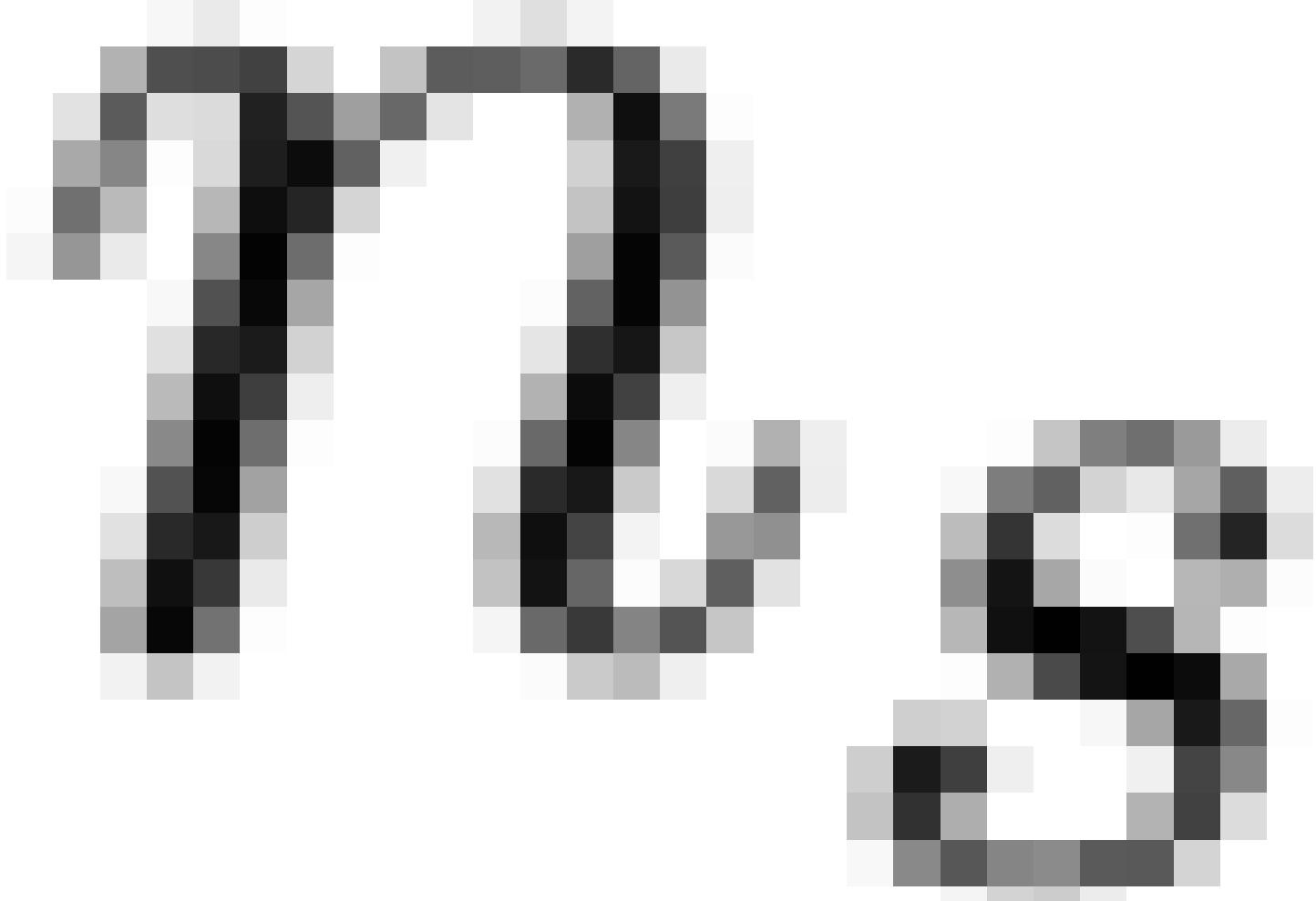


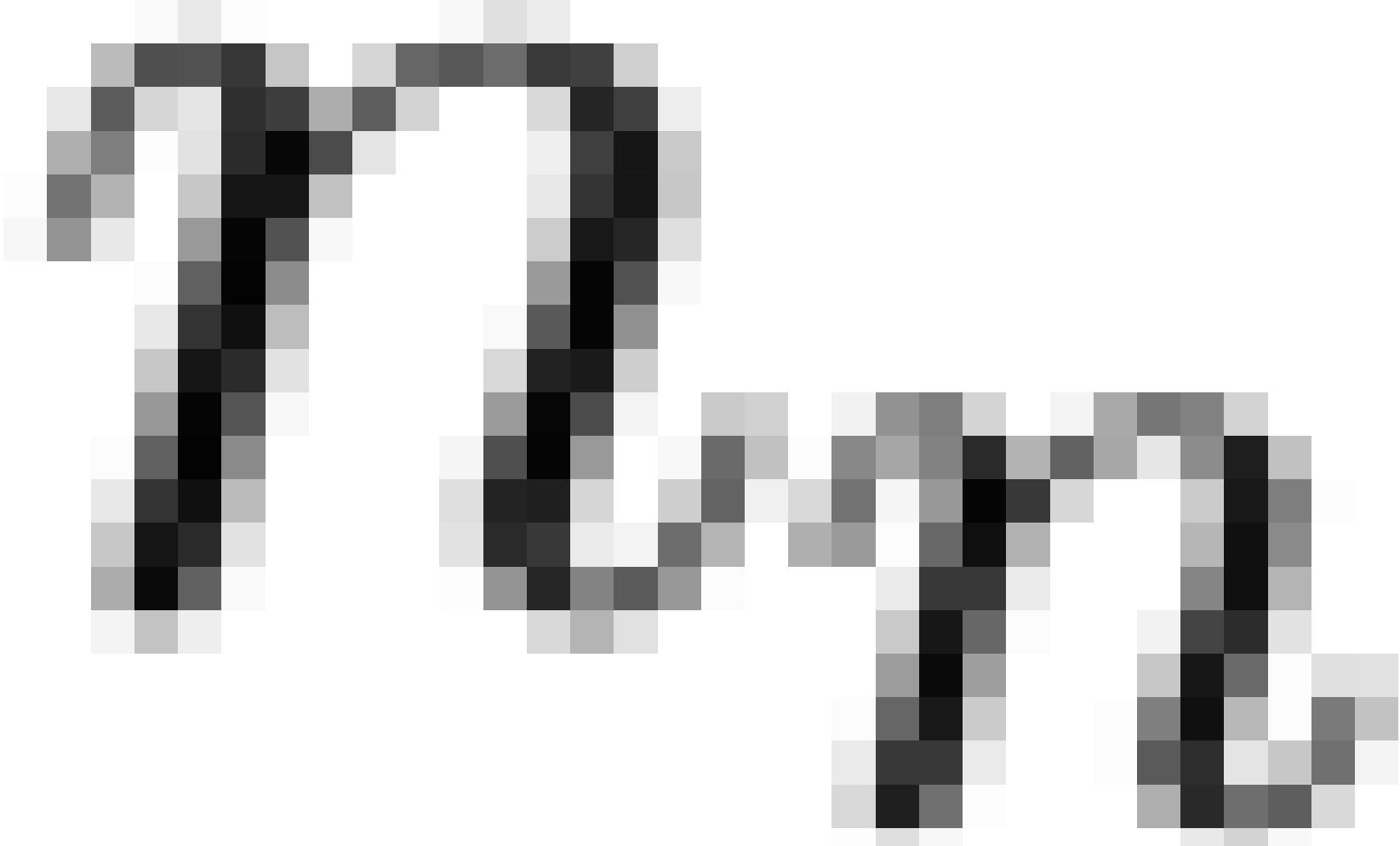
$$R_{PG} =$$

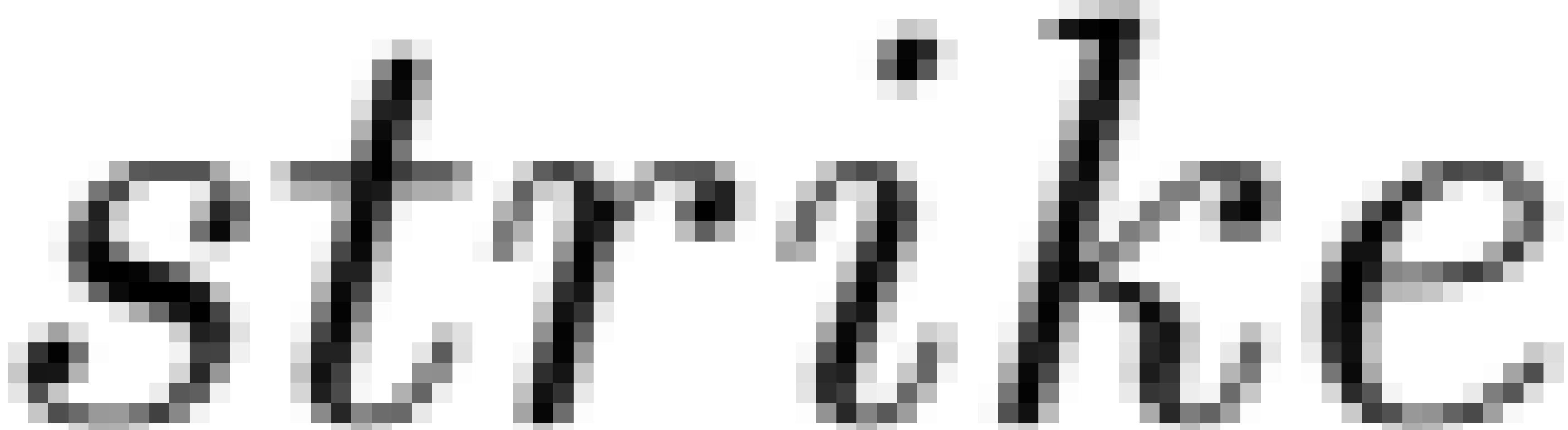
$$\begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}$$

$$\underline{S}_G = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix}^T \begin{bmatrix} 60 & 0 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -0.707 & 47.5 & -12.5 \\ 0 & 40 & 0 \\ -12.5 & 47.5 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 35 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$



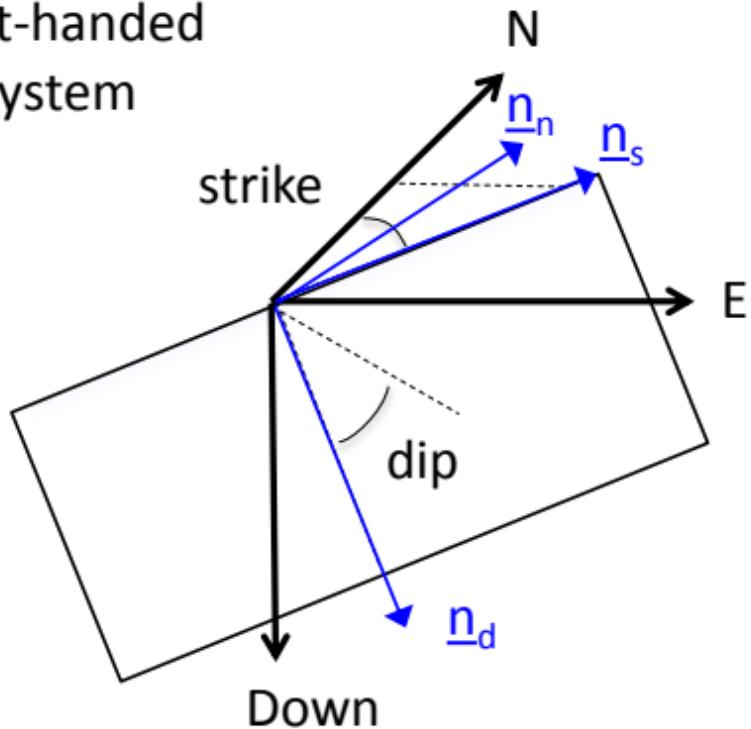






d-s-n

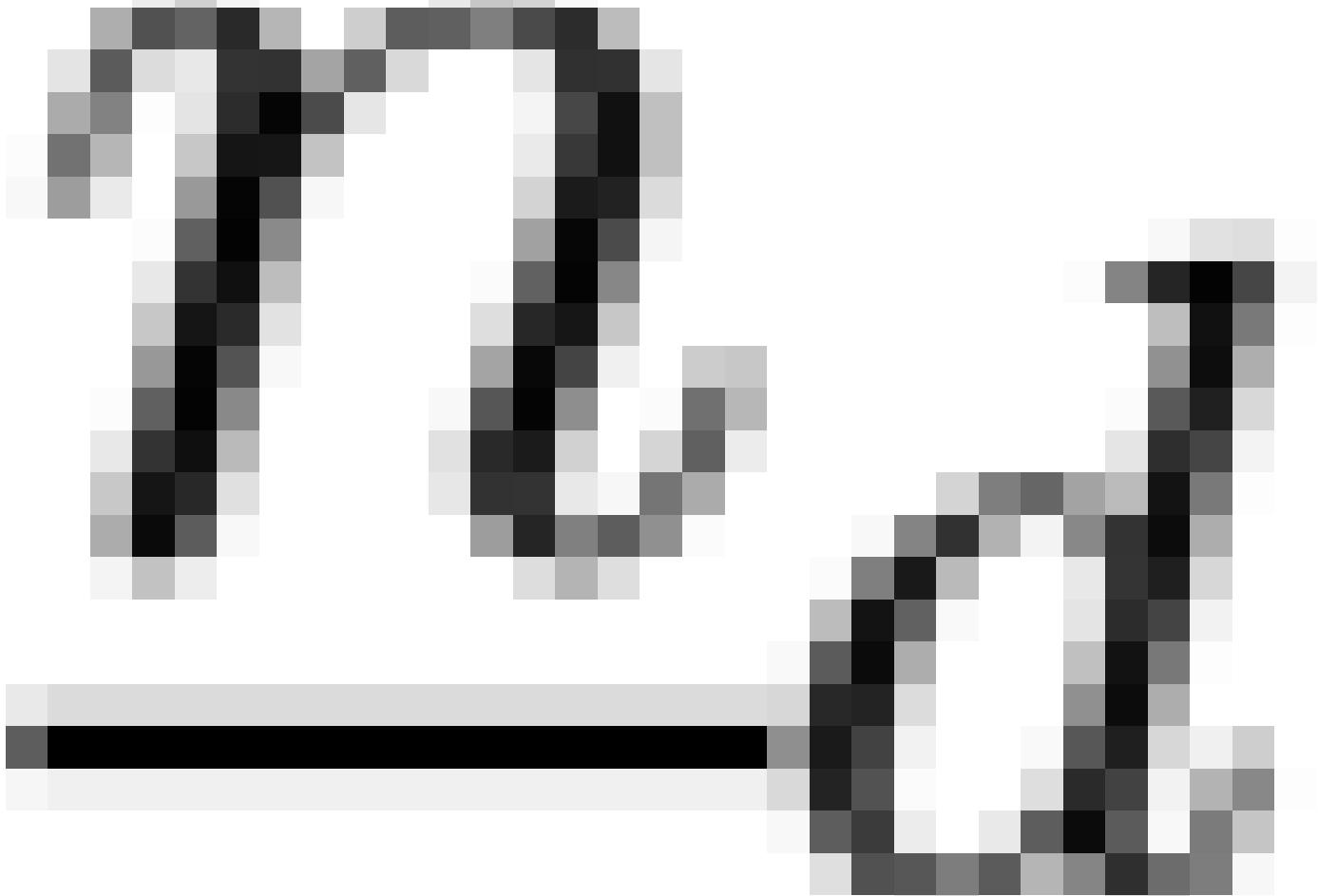
Right-handed
system

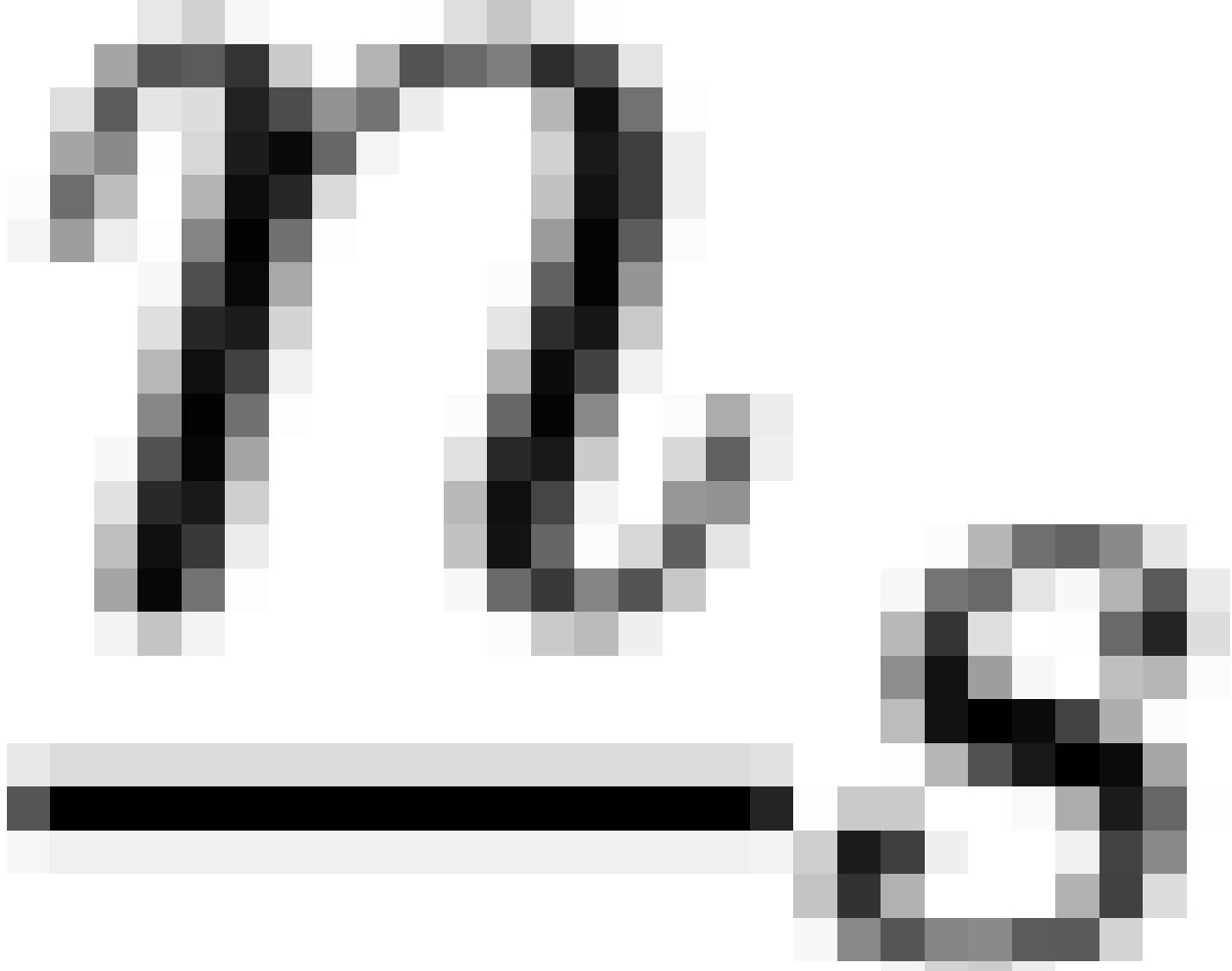


$$\underline{n}_n = \begin{bmatrix} -\sin(strike)\sin(dip) \\ \cos(strike)\sin(dip) \\ -\cos(dip) \end{bmatrix}$$

$$\underline{n}_s = \begin{bmatrix} \cos(strike) \\ \sin(strike) \\ 0 \end{bmatrix}$$

$$\underline{n}_d = \begin{bmatrix} -\sin(strike)\cos(dip) \\ \cos(strike)\cos(dip) \\ \sin(dip) \end{bmatrix}$$

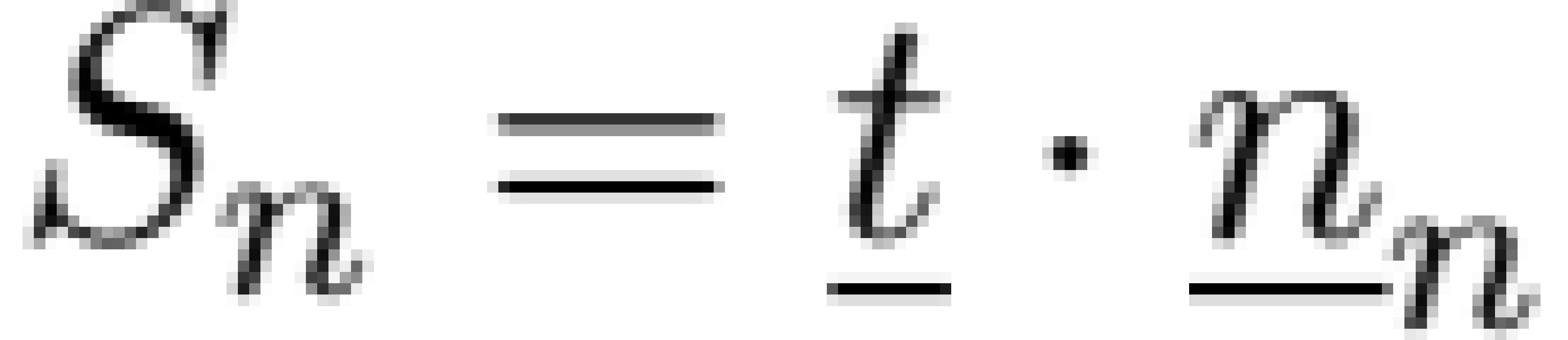








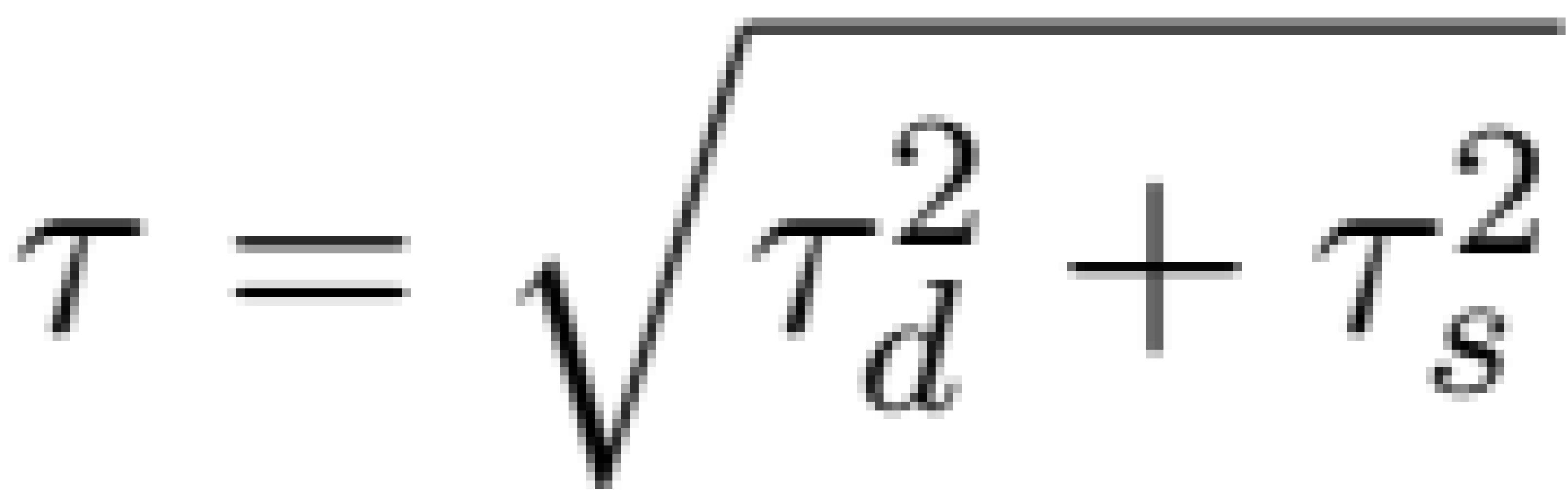


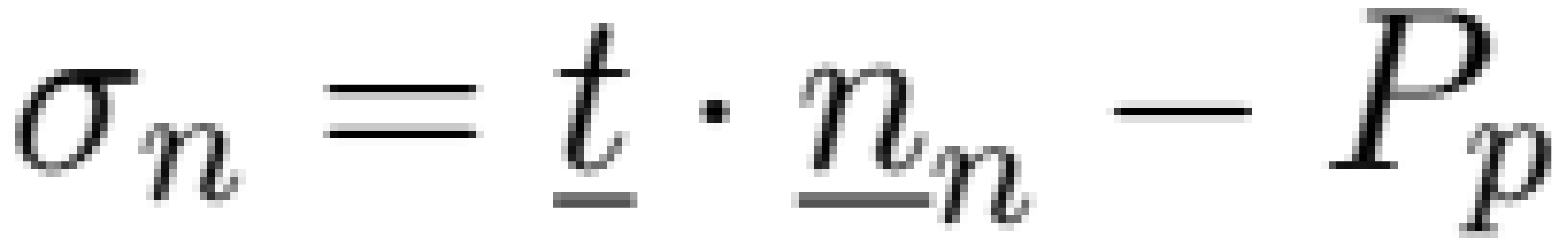


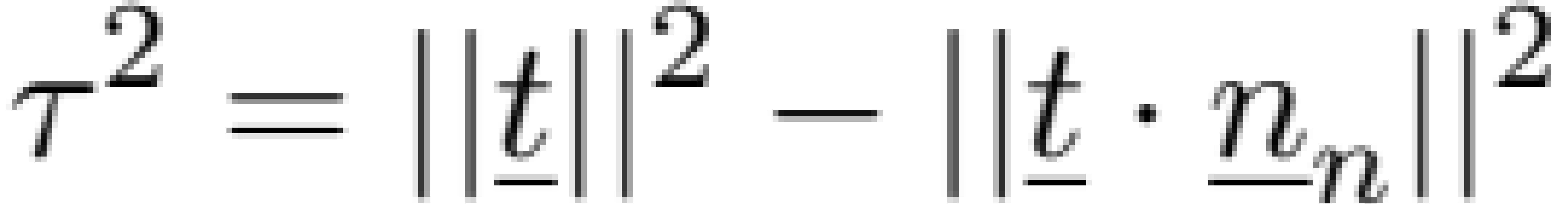


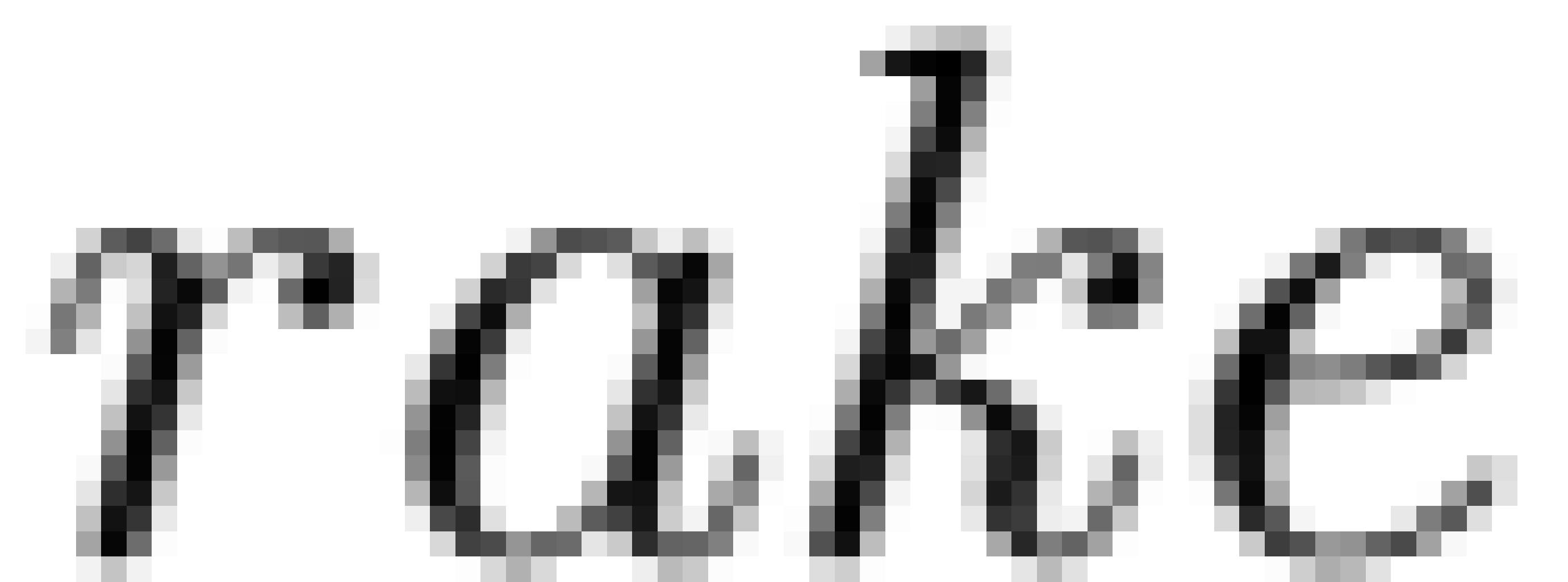
$$\tau_d = t \cdot \frac{r_d}{n_d}$$

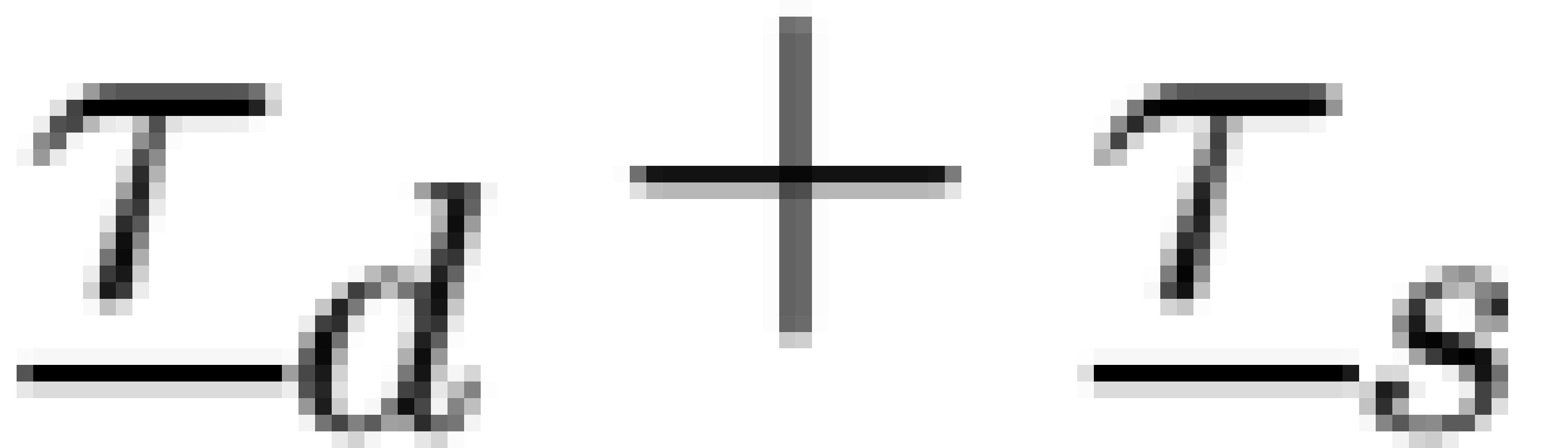
$$\tau_s = t \cdot \frac{r_s}{n_s}$$





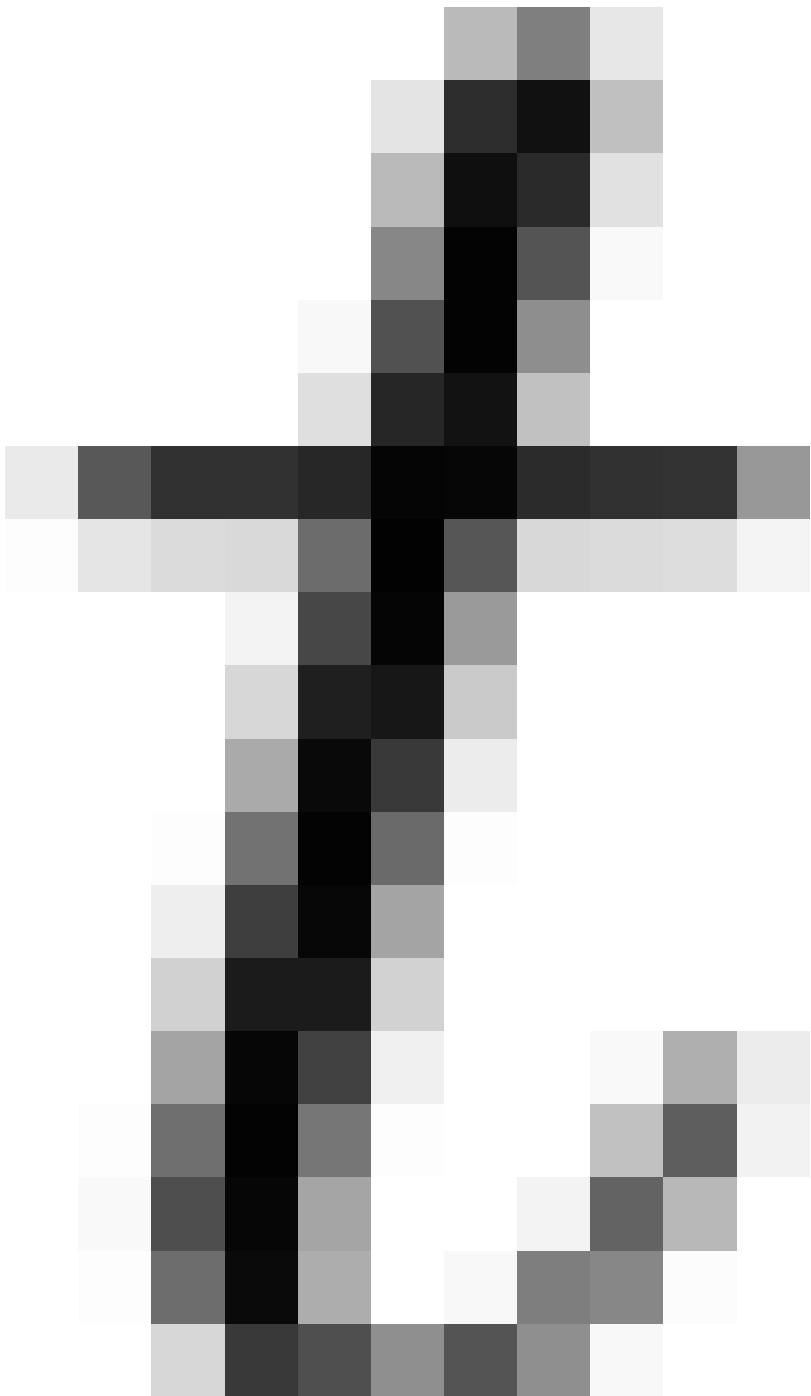


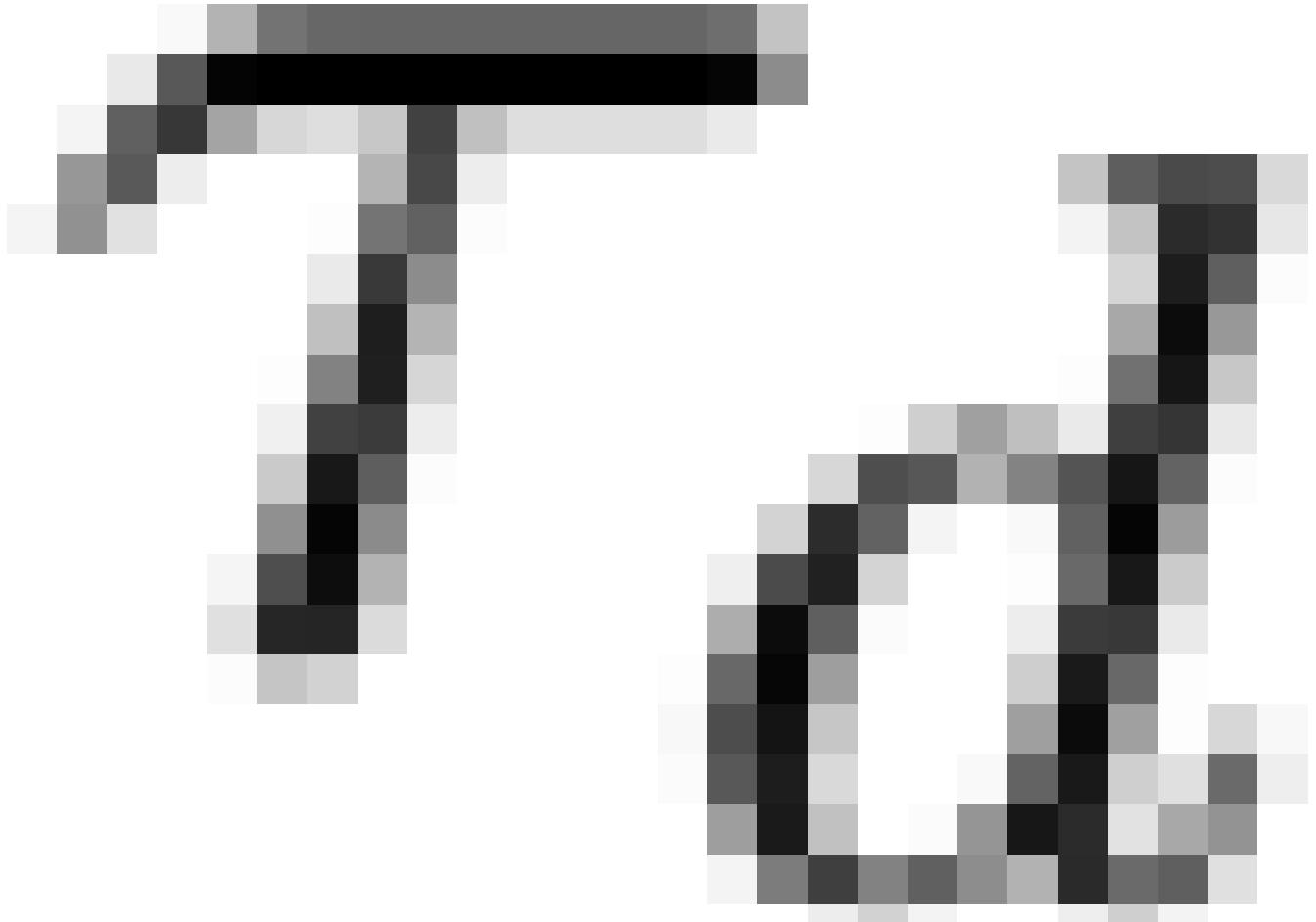


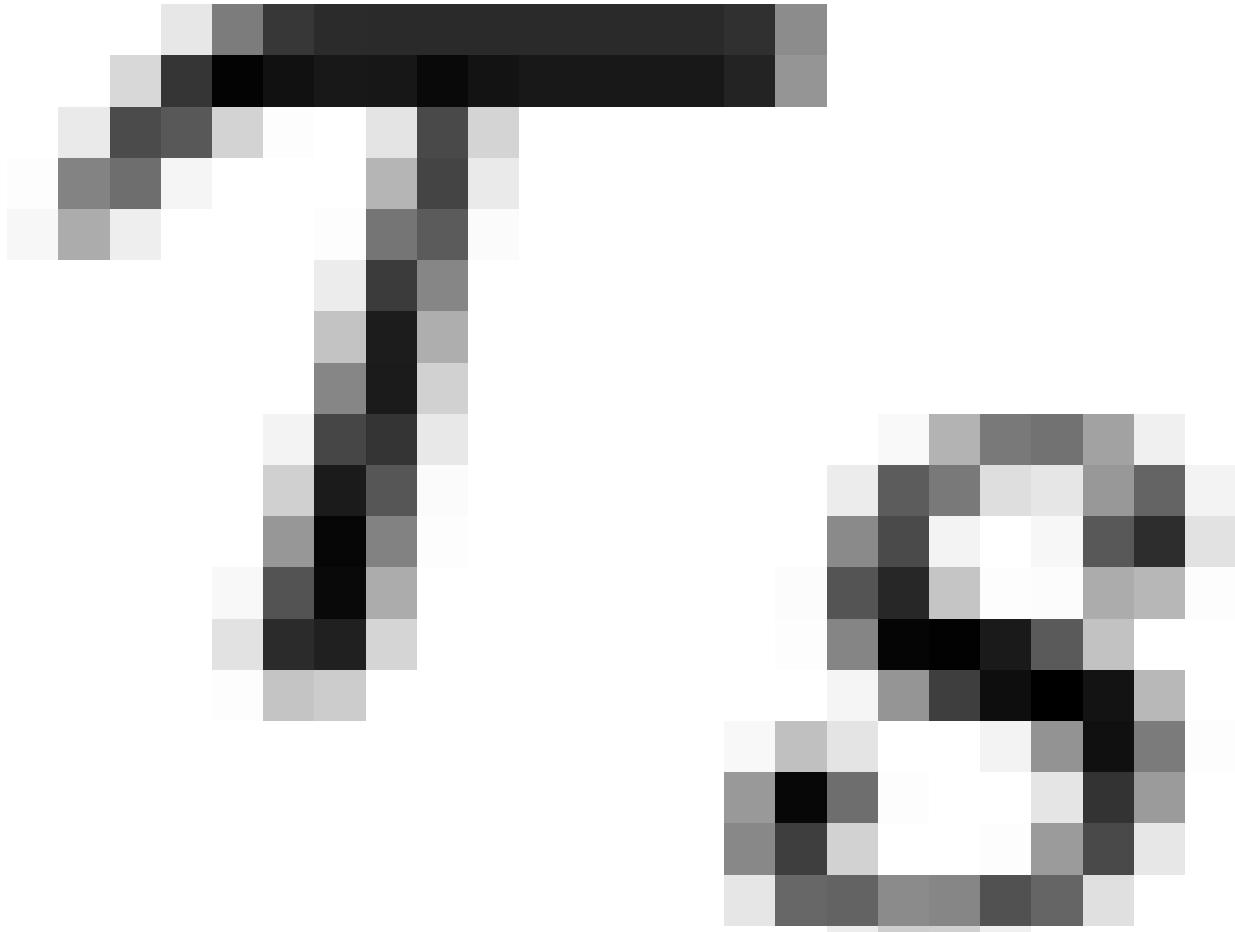


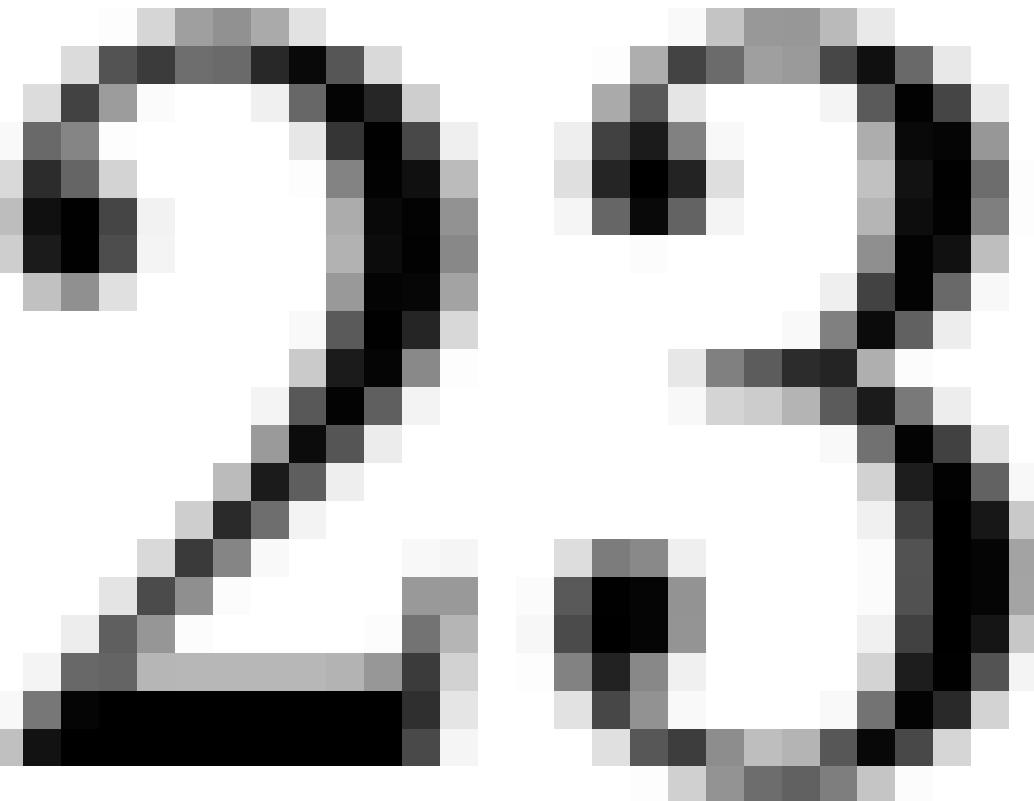
rake = arctan

$$\tan \theta = \frac{r_d}{r_s}$$

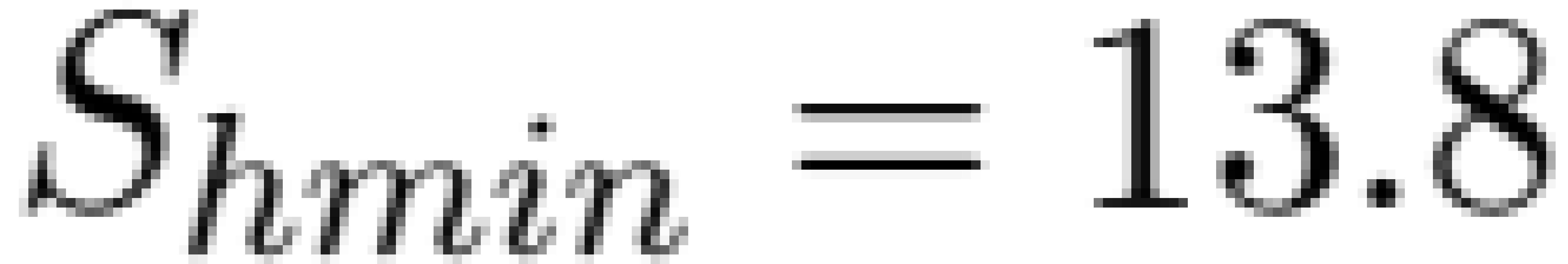


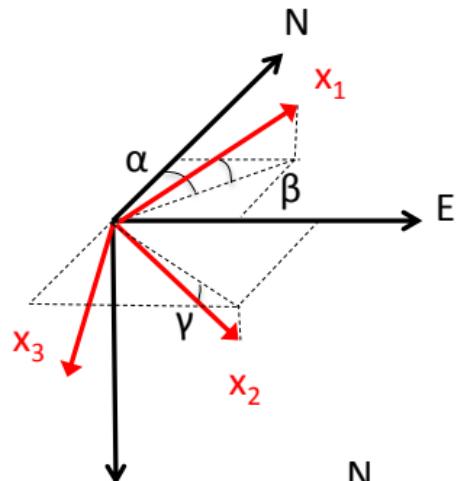










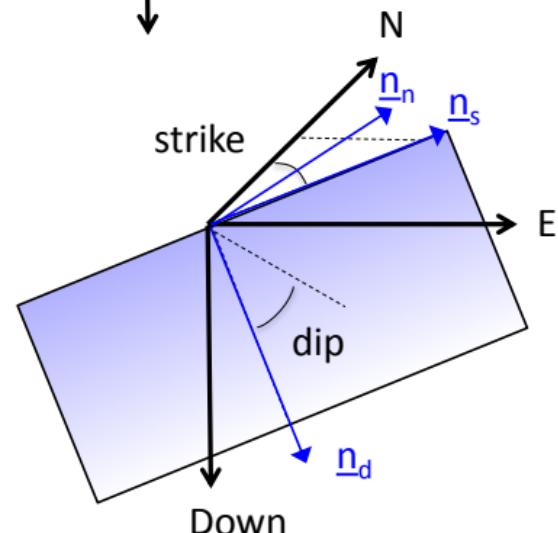


$$\underline{\underline{S}}_P = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 13.8 \end{bmatrix}$$

$a = 90^\circ$ Azimuth of $S_{h\min}$
 $b = 90^\circ$ $S_1 = S_V$
 $g = 0^\circ$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\underline{S}}_G = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 13.8 & 0 \\ 0 & 0 & 23 \end{bmatrix}$$



Fault geometry

Strike = 000°
 Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0 \\ 0.867 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [0, 11.95, -11.50] \text{ MPa}$$

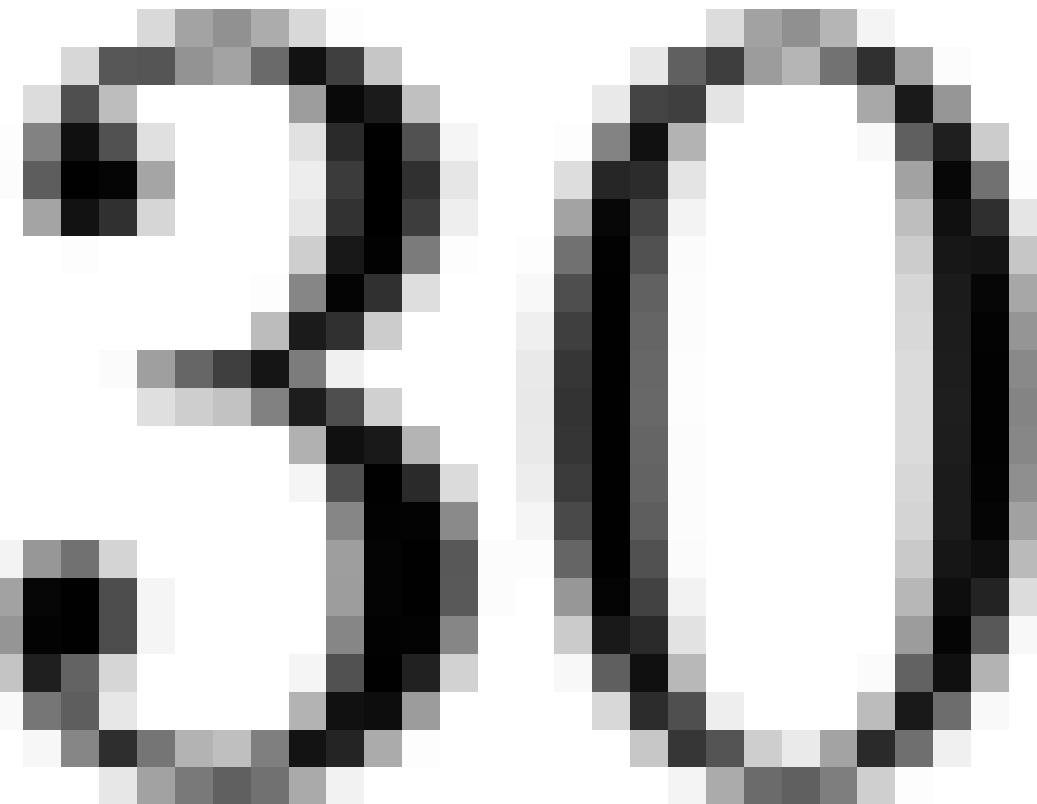
$$S_n = 16.1 \text{ MPa}$$

$$t_d = -3.98 \text{ MPa}$$

$$t_s = 0 \text{ MPa}$$

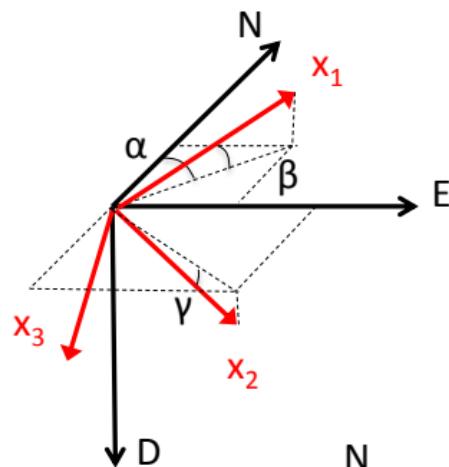
$$\text{rake} = 90^\circ$$

Actual stresses opposite to d-s-n



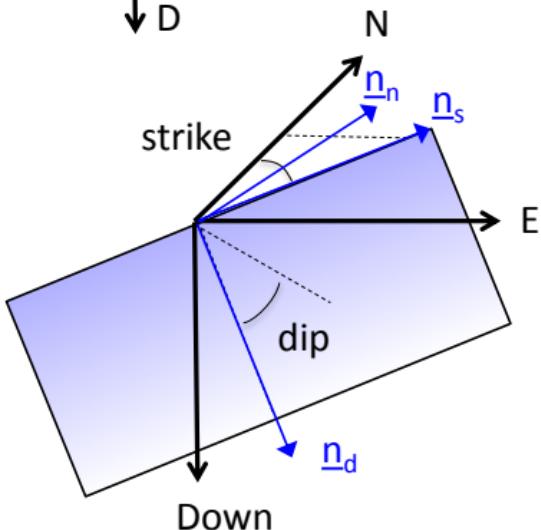






$$\underline{\underline{S}}_P = \begin{bmatrix} 45 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 25 \end{bmatrix} \quad \begin{array}{ll} a = 120^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 90^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \\ 0.866 & 0.5 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$



Fault geometry

Strike = 060°

Dip = 90°

$$\underline{n}_n = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix}$$

$$\underline{t} = [-30.31, 27.5, 0] \text{ MPa}$$

$$S_n = 40 \text{ MPa}$$

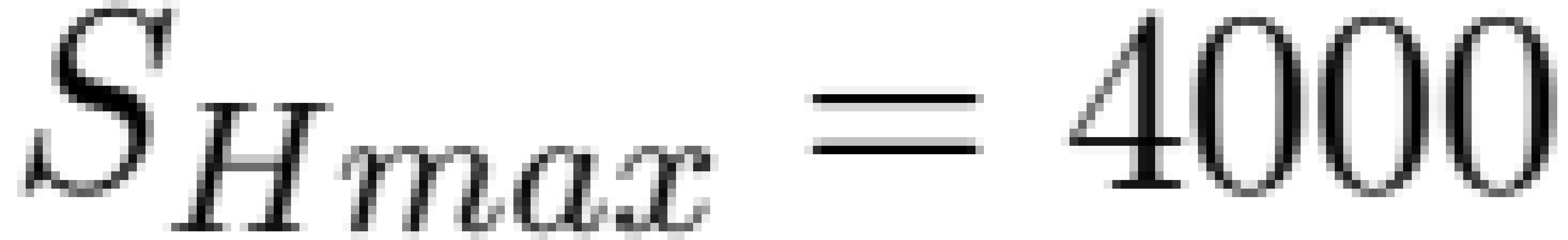
$$t_d = 0 \text{ MPa}$$

$$t_s = 8.66 \text{ MPa}$$

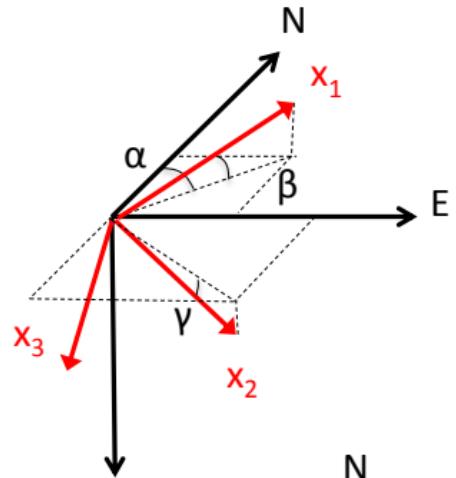
$$rake = 0^\circ$$

Actual stresses opposite to d-s-n



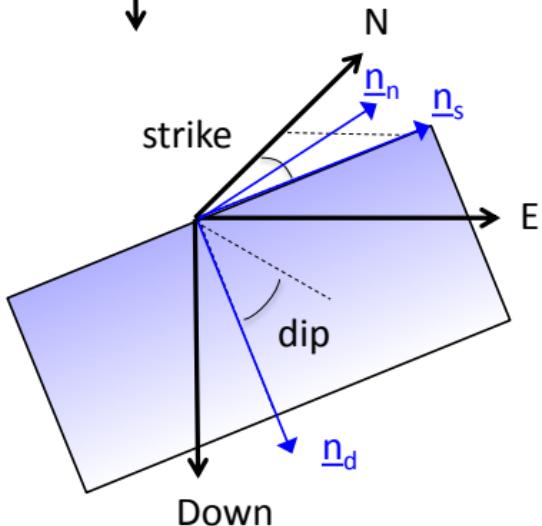






$$\underline{\underline{S}}_P = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix} \quad \begin{array}{ll} a = 90^\circ & \text{Azimuth of } S_{h\min} \\ b = 90^\circ & \\ g = 0^\circ & S_1 = S_V \end{array}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$



Fault geometry

Strike = 045°

Dip = 60°

$$n_n = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix}$$

$$\underline{t} = [-2450, 1840, -2500] \text{ psi}$$

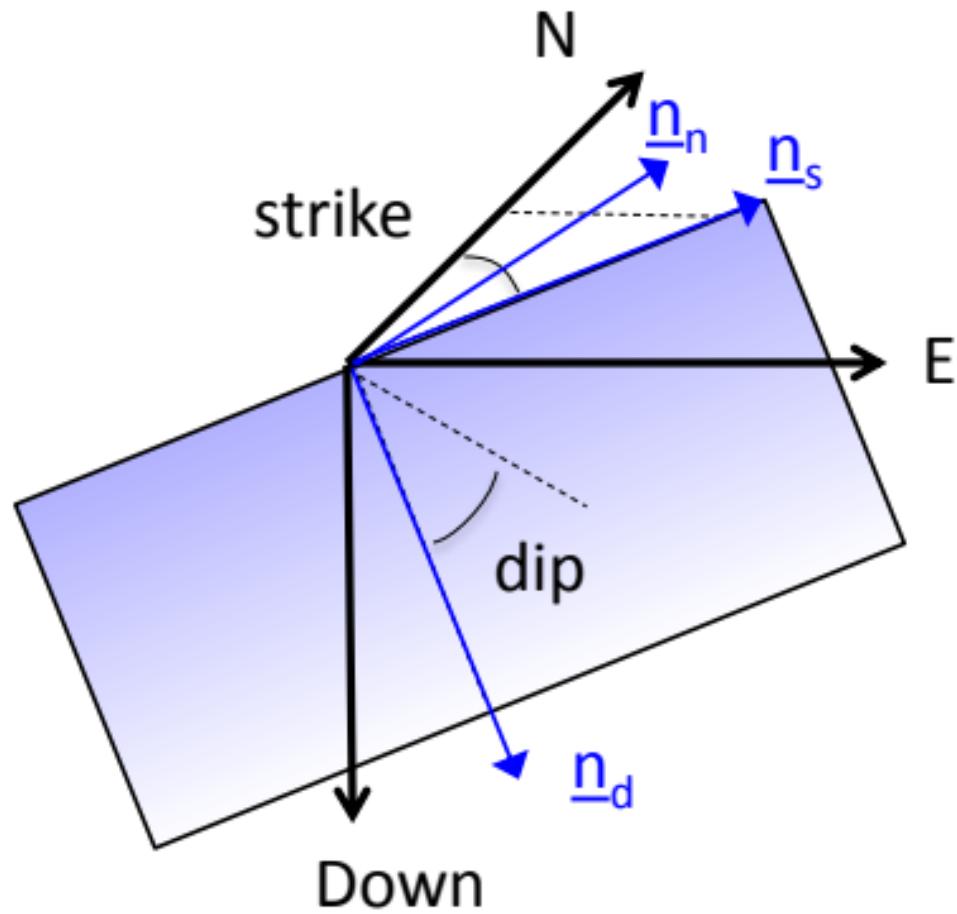
$$S_n = 3870 \text{ psi}$$

$$t_d = -649 \text{ psi}$$

$$t_s = -433 \text{ psi}$$

$$\text{rake} = 56.3^\circ$$

Actual stresses opposite to d-s-n



Fault geometry

Strike = 225°

Dip = 60°

$$\underline{n}_n = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix}$$

$t = [2450, -1840, -2500]$ psi

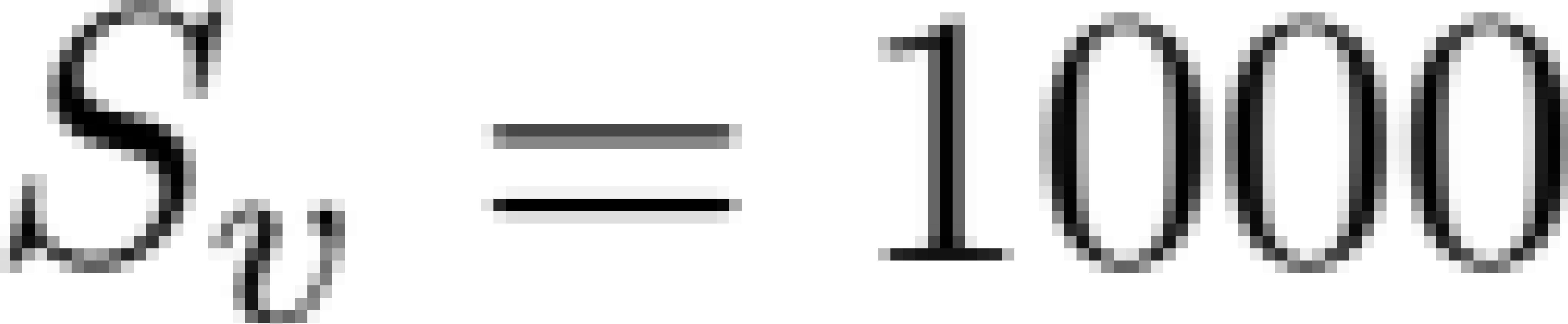
$S_n = 3870$ psi

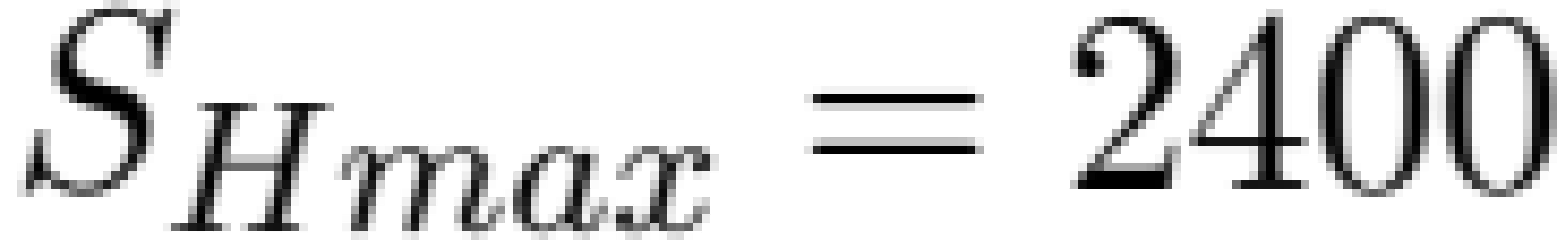
$t_d = -649$ psi

$t_s = -433$ psi

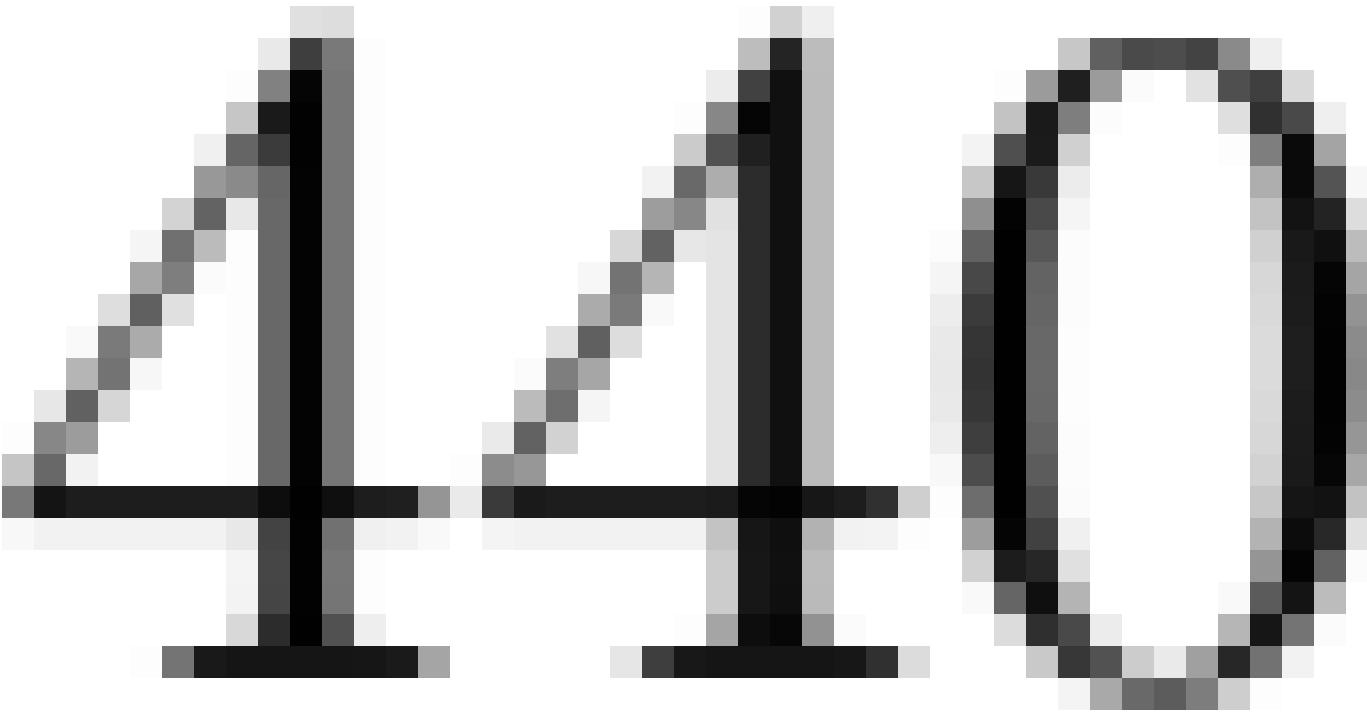
rake = 56.3°

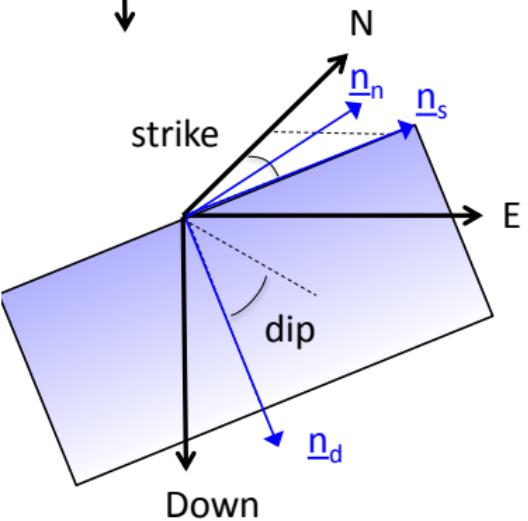
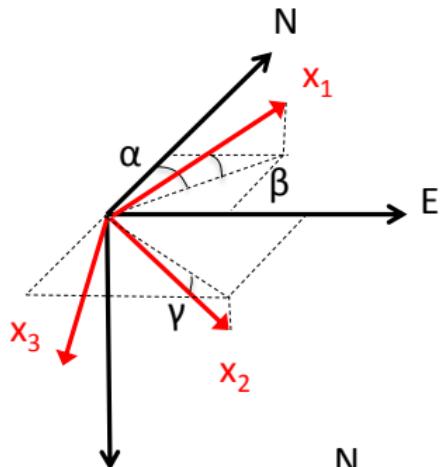
Actual stresses opposite to d-s-n











$$\underline{\underline{S}}_P = \begin{bmatrix} 2400 & 0 & 0 \\ 0 & 1200 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \quad \begin{array}{ll} \alpha = 150^\circ & \text{Azimuth of } S_{H\max} \\ b = 0^\circ & \\ g = 0^\circ & S_3 = S_V \end{array}$$

$$P_w = 440 \text{ psi}$$

$$\underline{\underline{R}}_{PG} = \begin{bmatrix} -0.866 & 0.5 & 0 \\ -0.5 & -0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{S}}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

Fault geometry

Strike = 120°

Dip = 70°

$$\underline{n}_n = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix}$$

$$\underline{t} = [-1465, -281, -342] \text{ psi}$$

$$S_n = 1441 \text{ psi}$$

$$t_d = 160 \text{ psi}$$

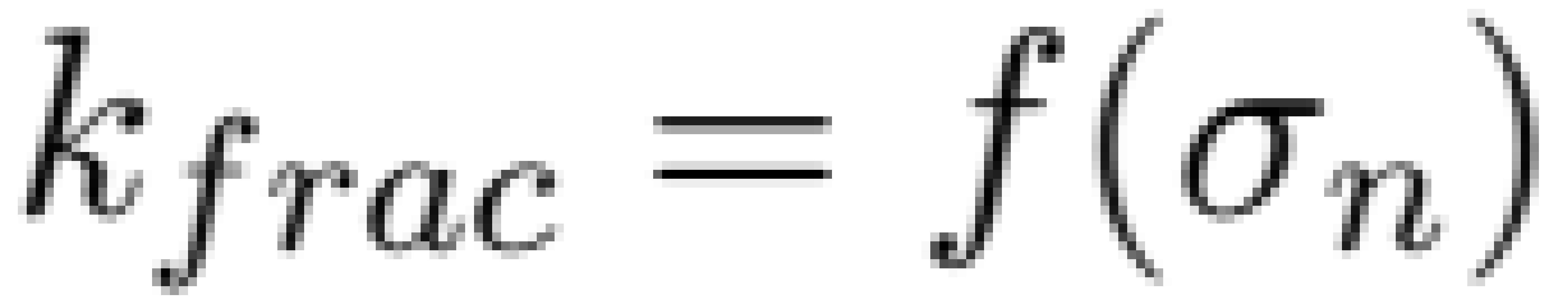
$$t_s = 488 \text{ psi}$$

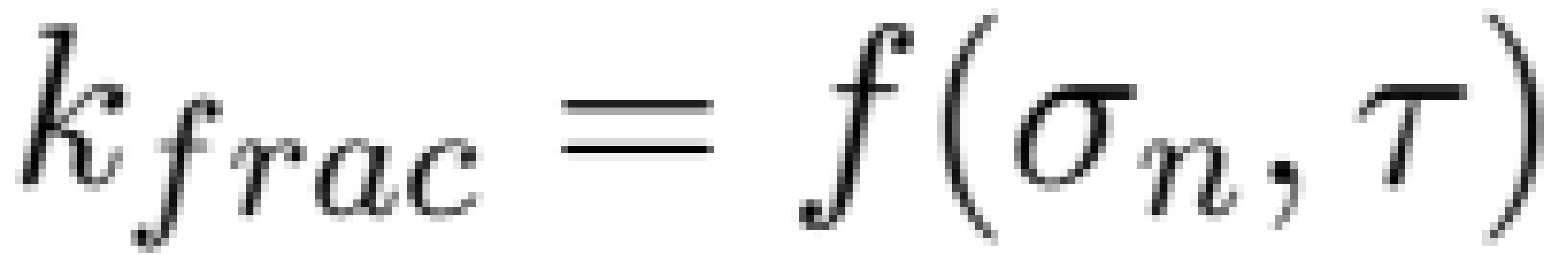
$$\text{rake} = 18.21^\circ$$

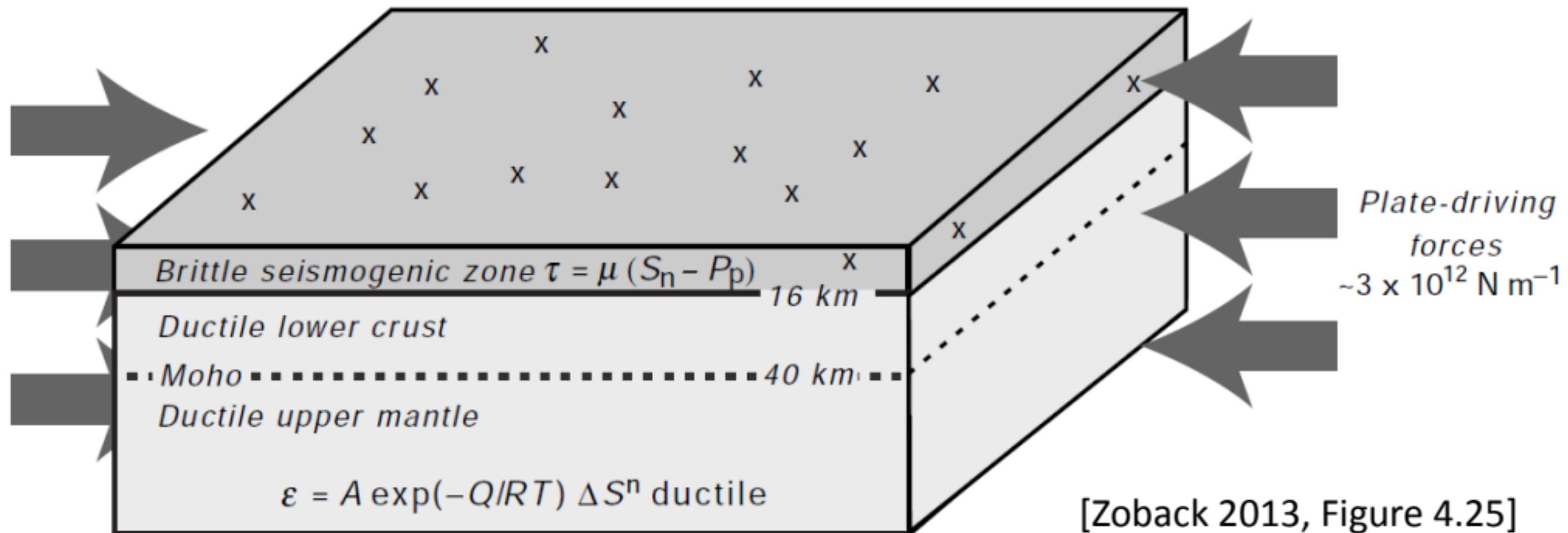
$$t / S_n = 0.51$$

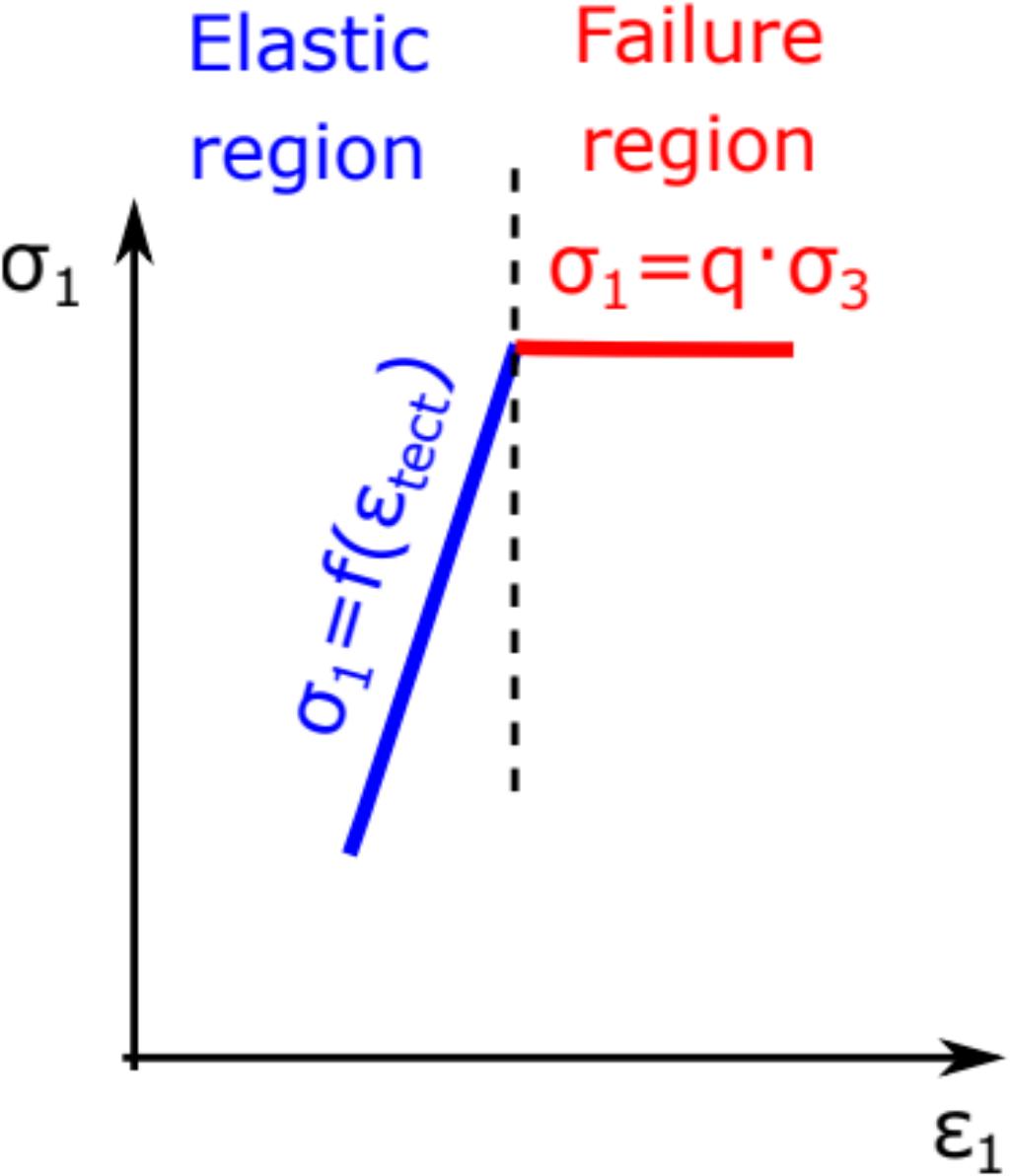
Actual stresses opposite to d-s-n





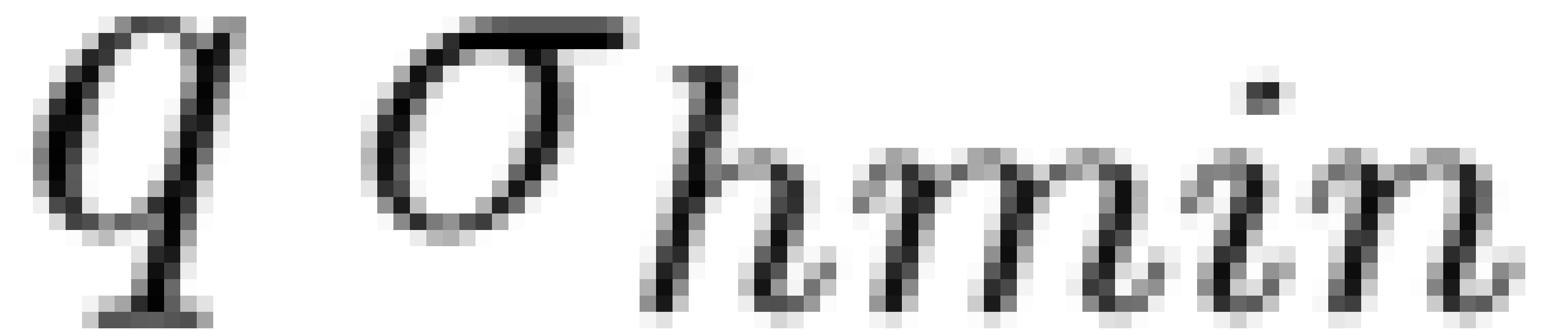


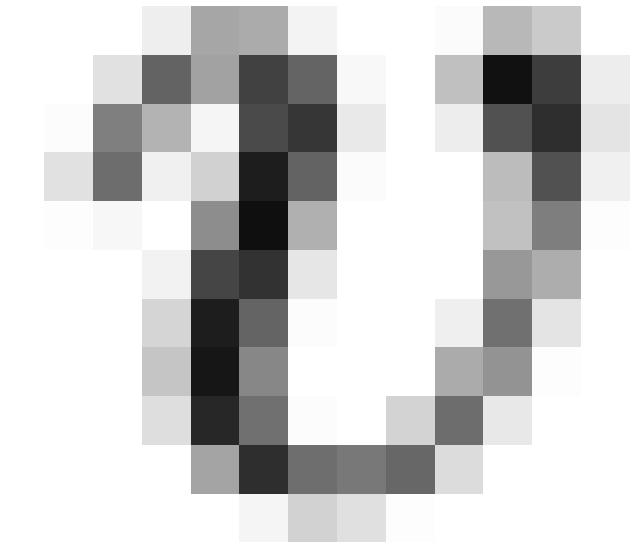
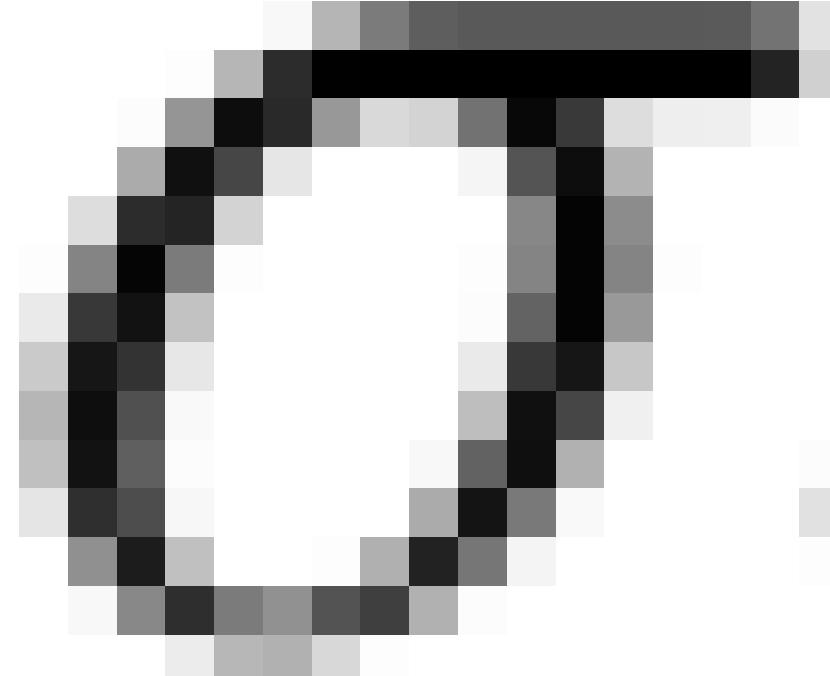
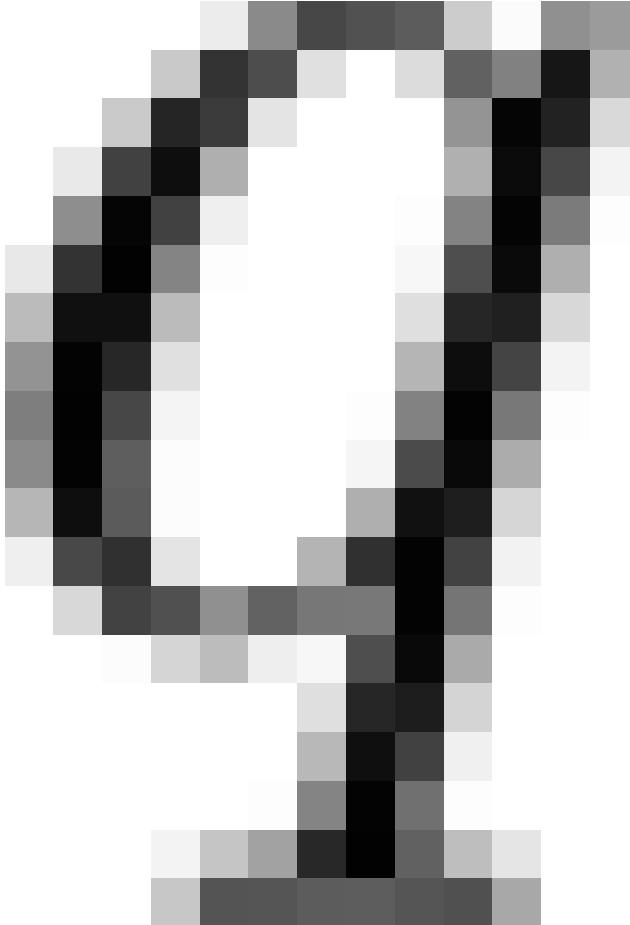


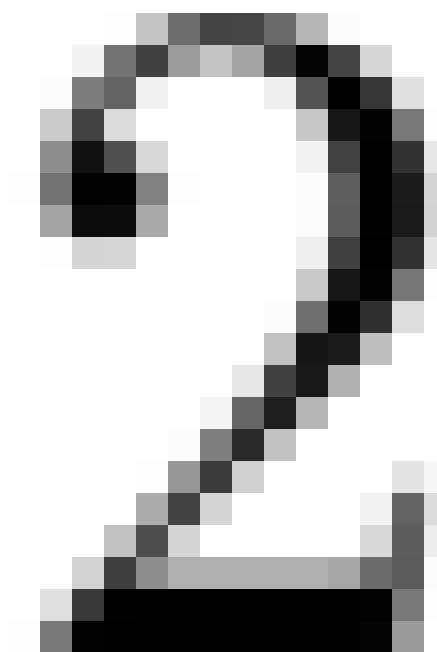
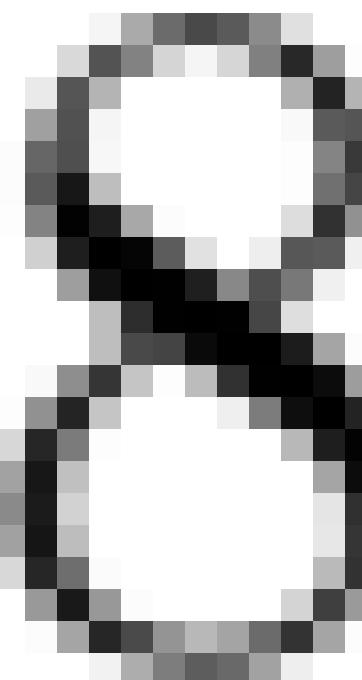
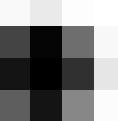
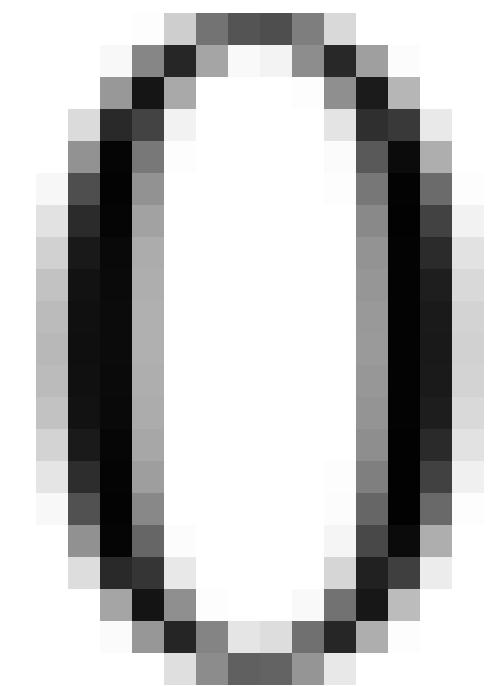




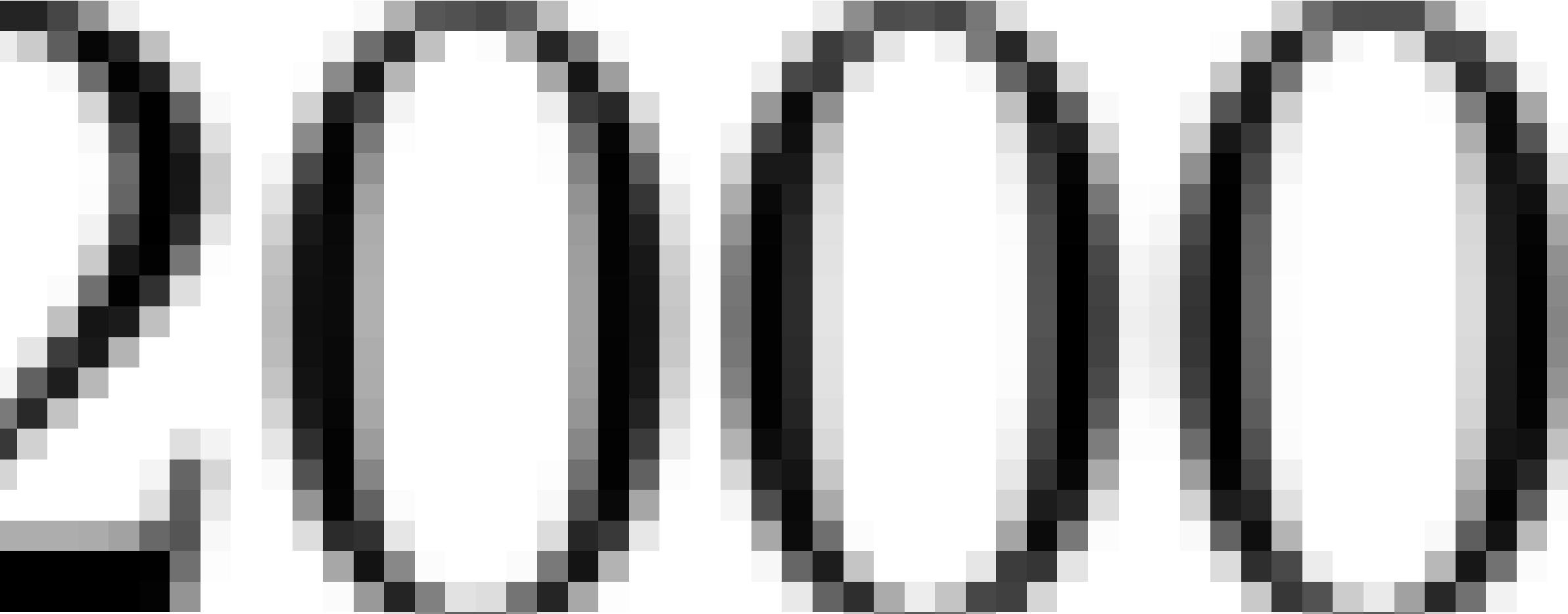


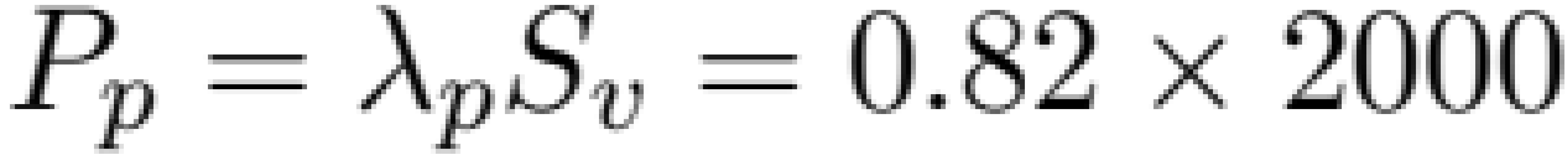


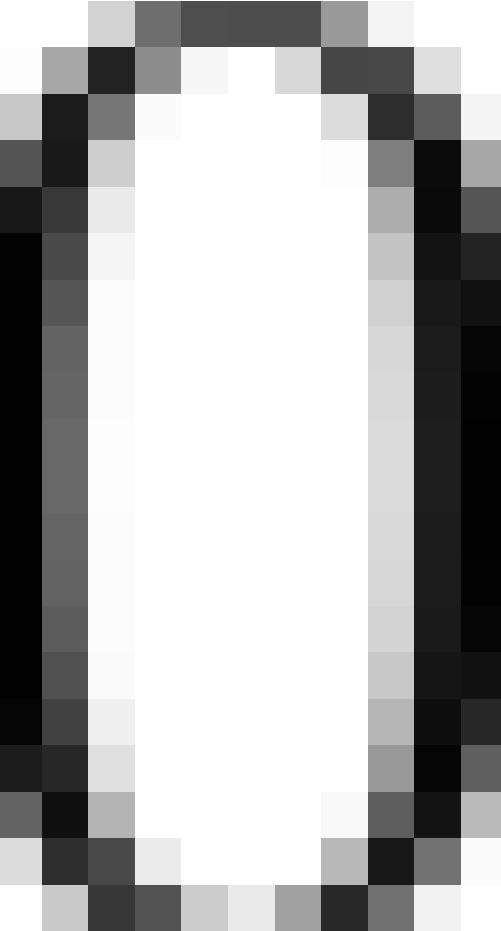
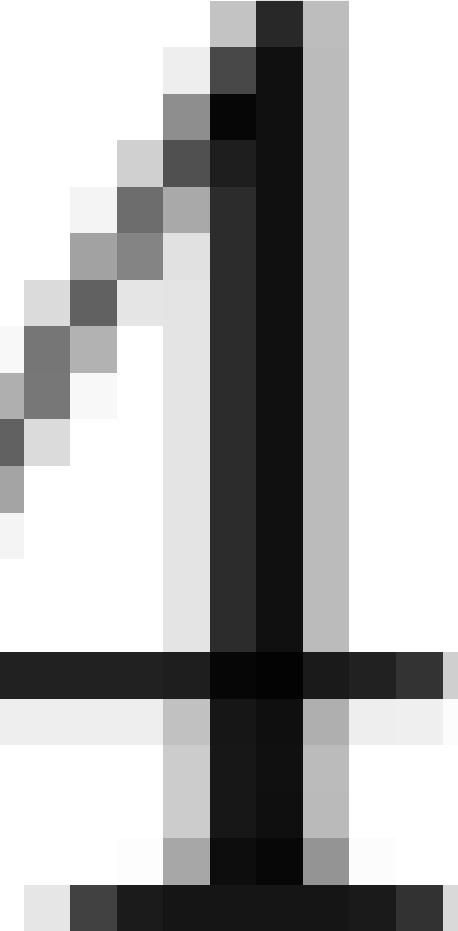
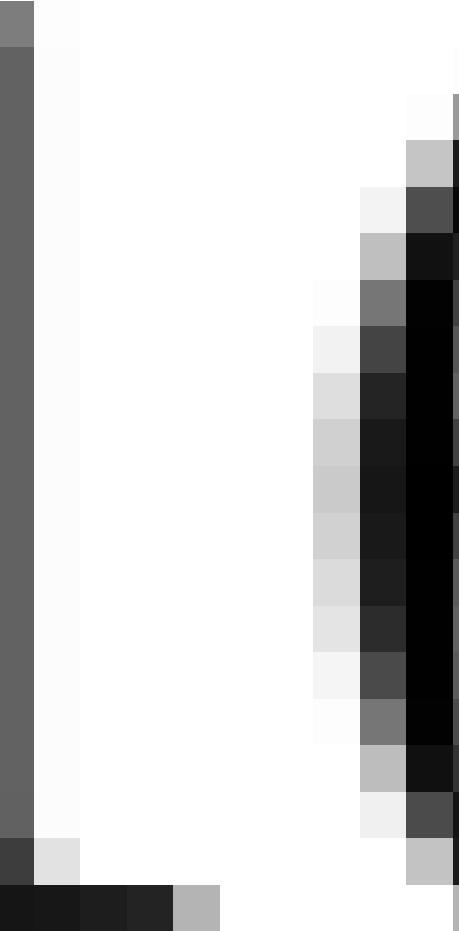


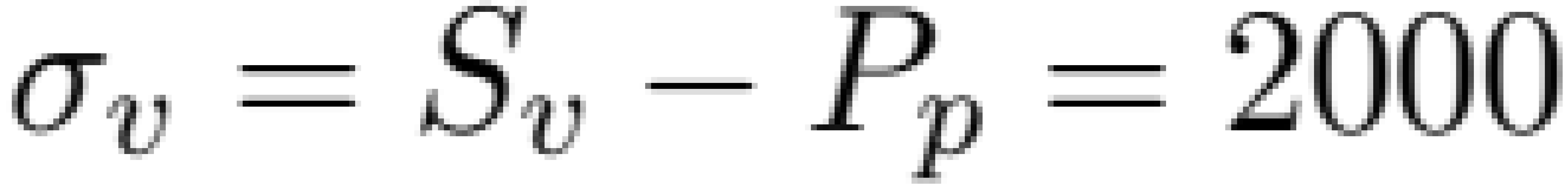


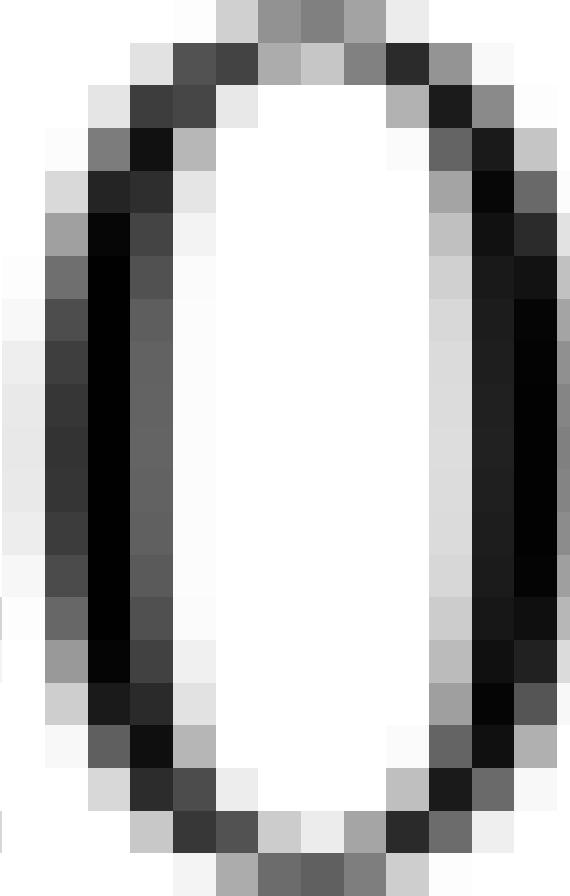
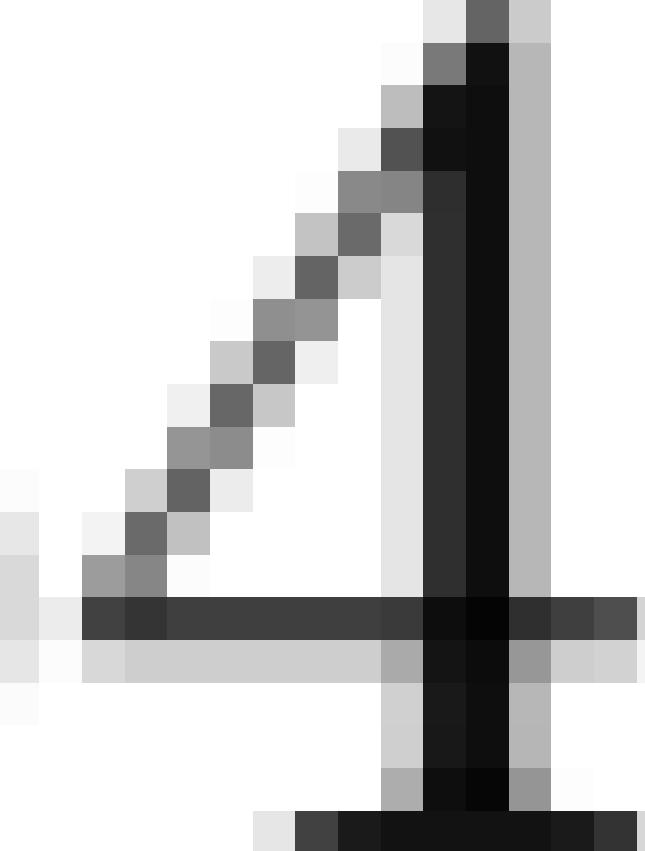
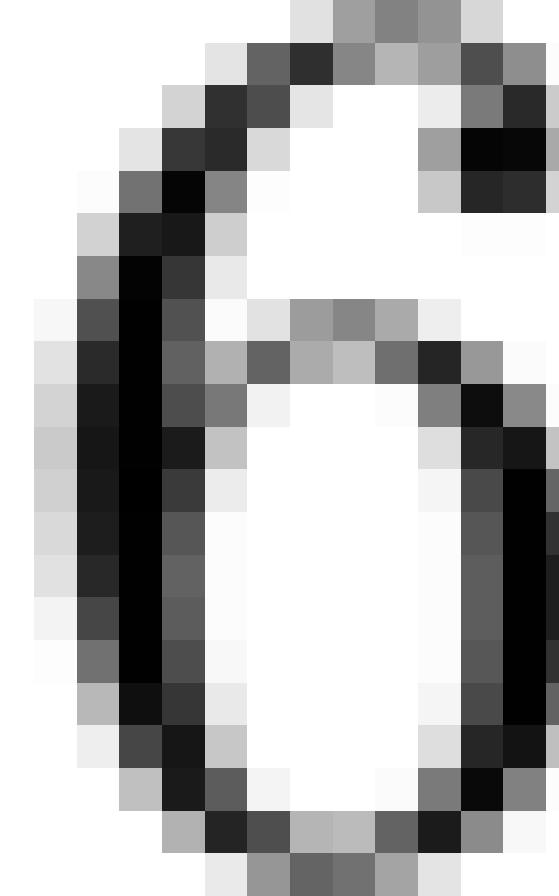
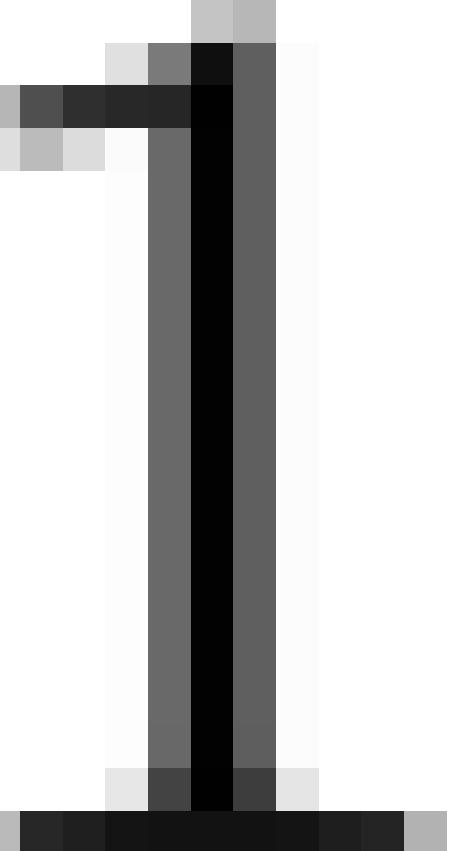


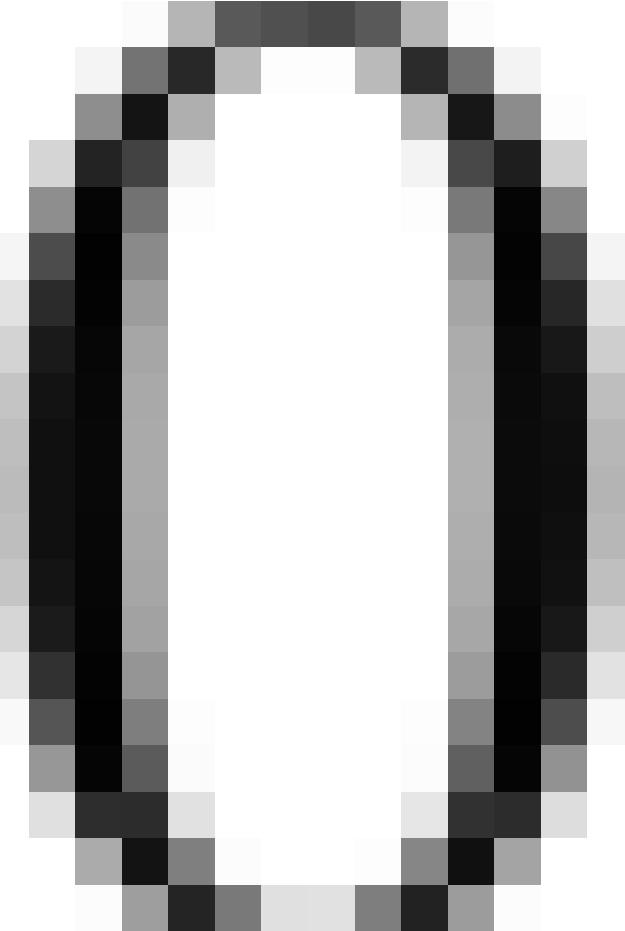
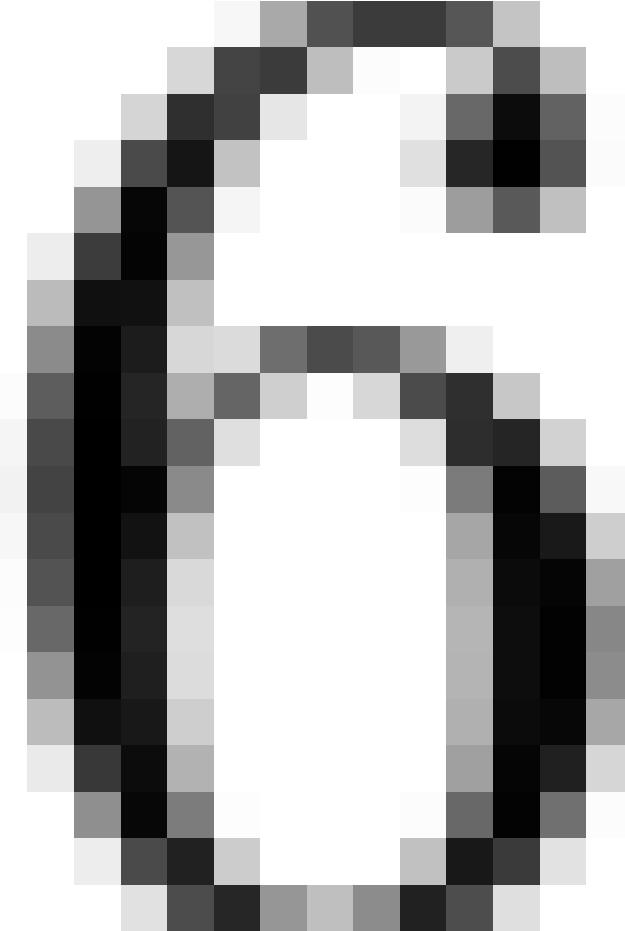
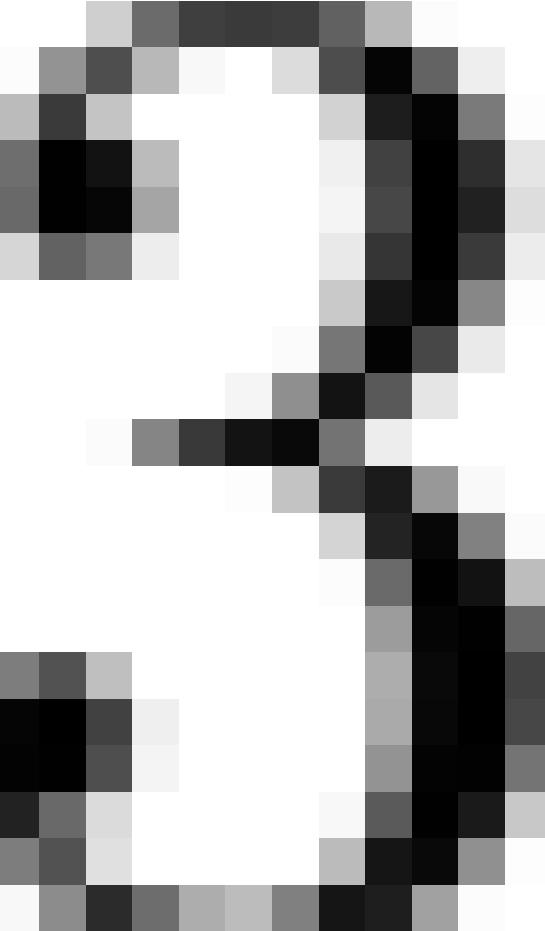








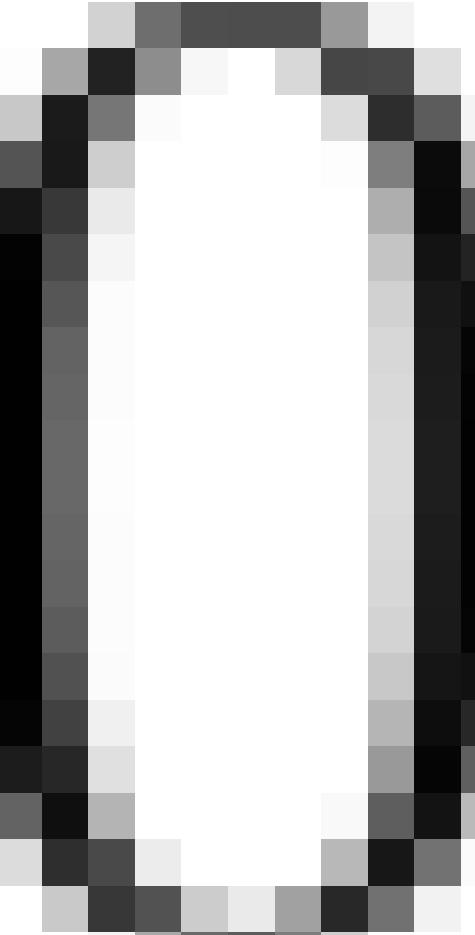
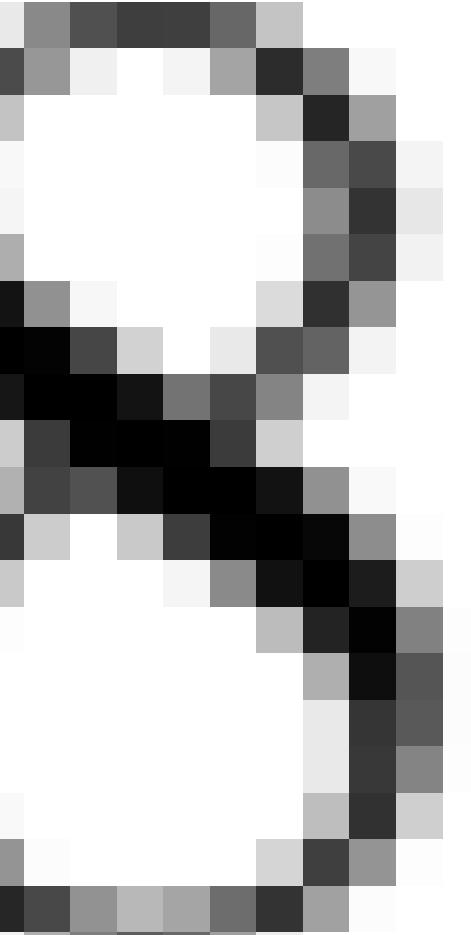
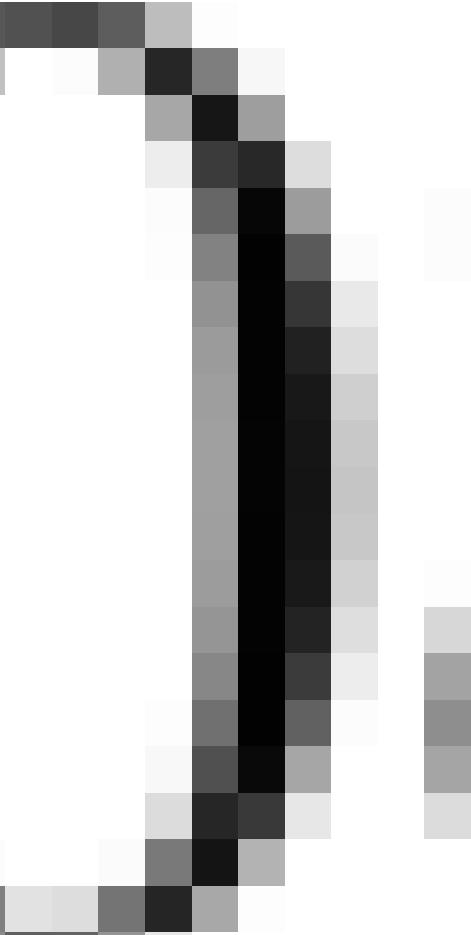
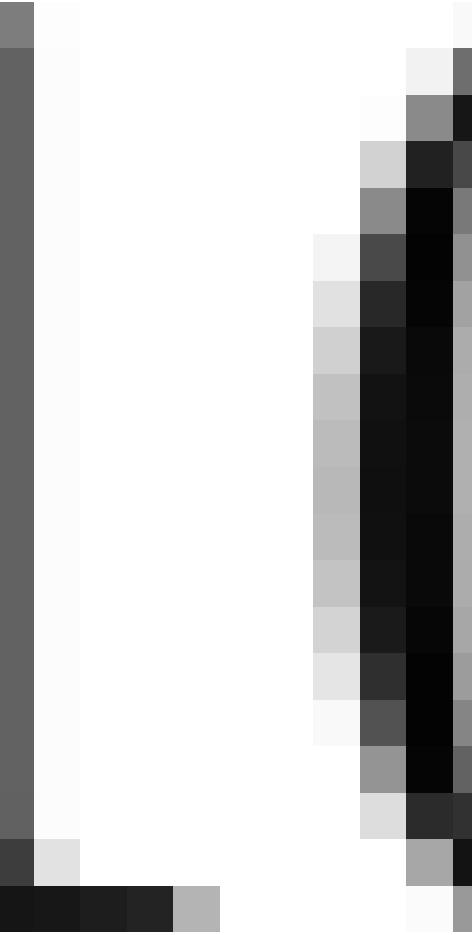


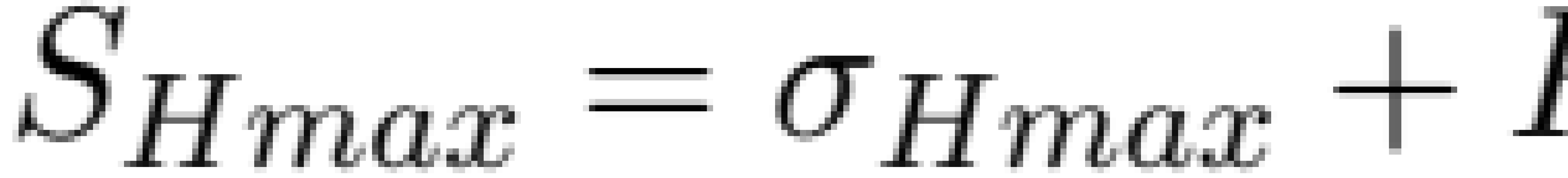


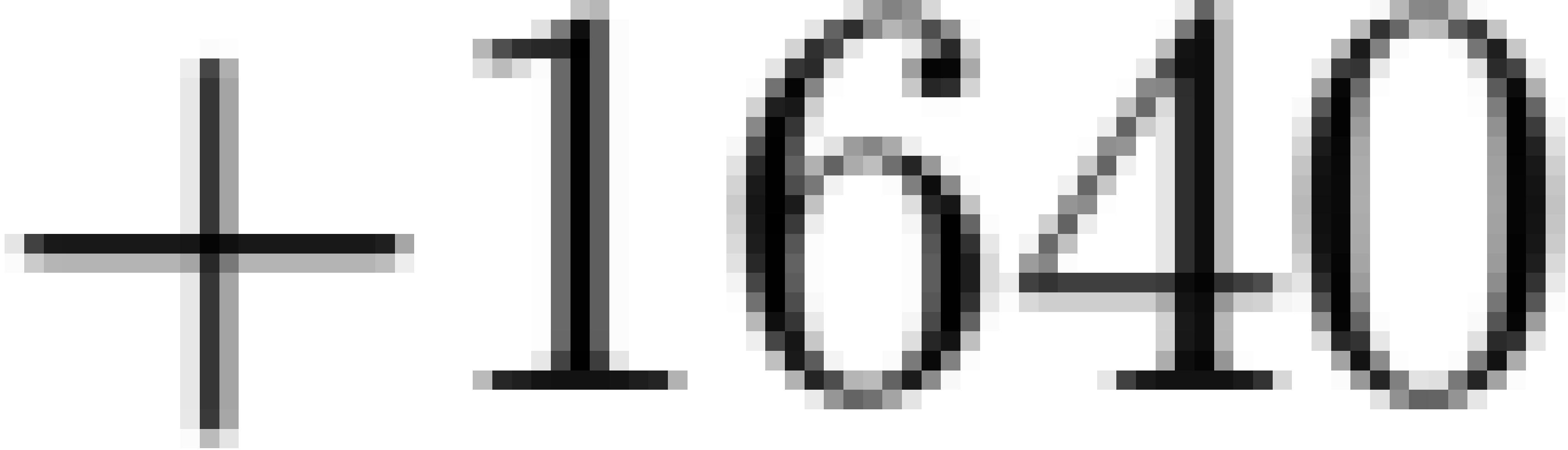
$$\sigma_{H\text{max}} =$$

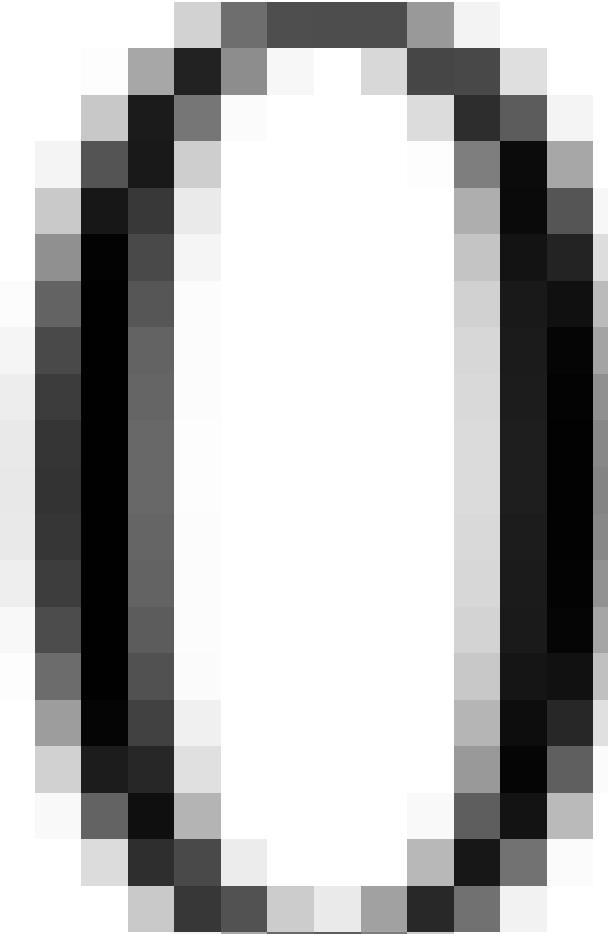
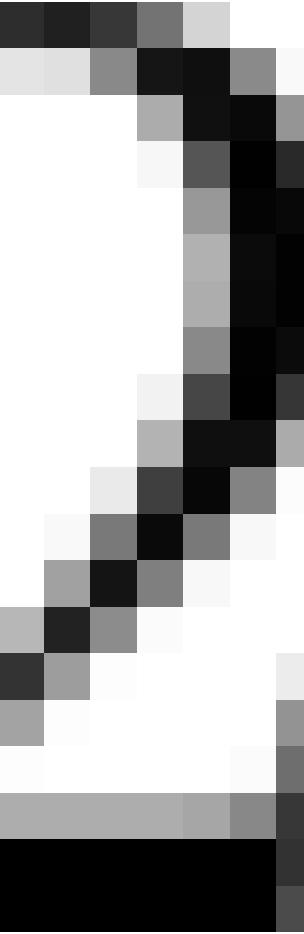
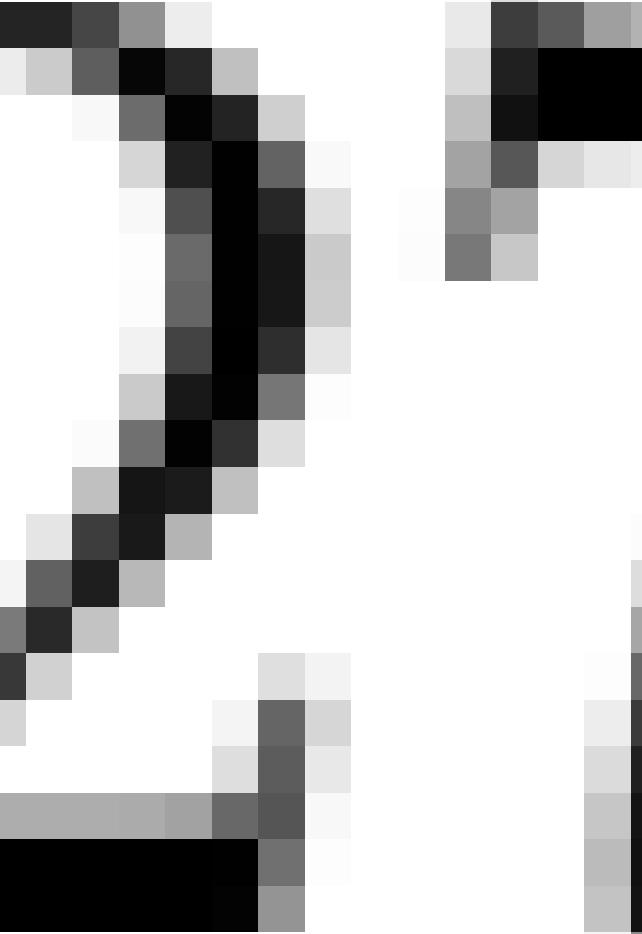
$$90^\circ =$$

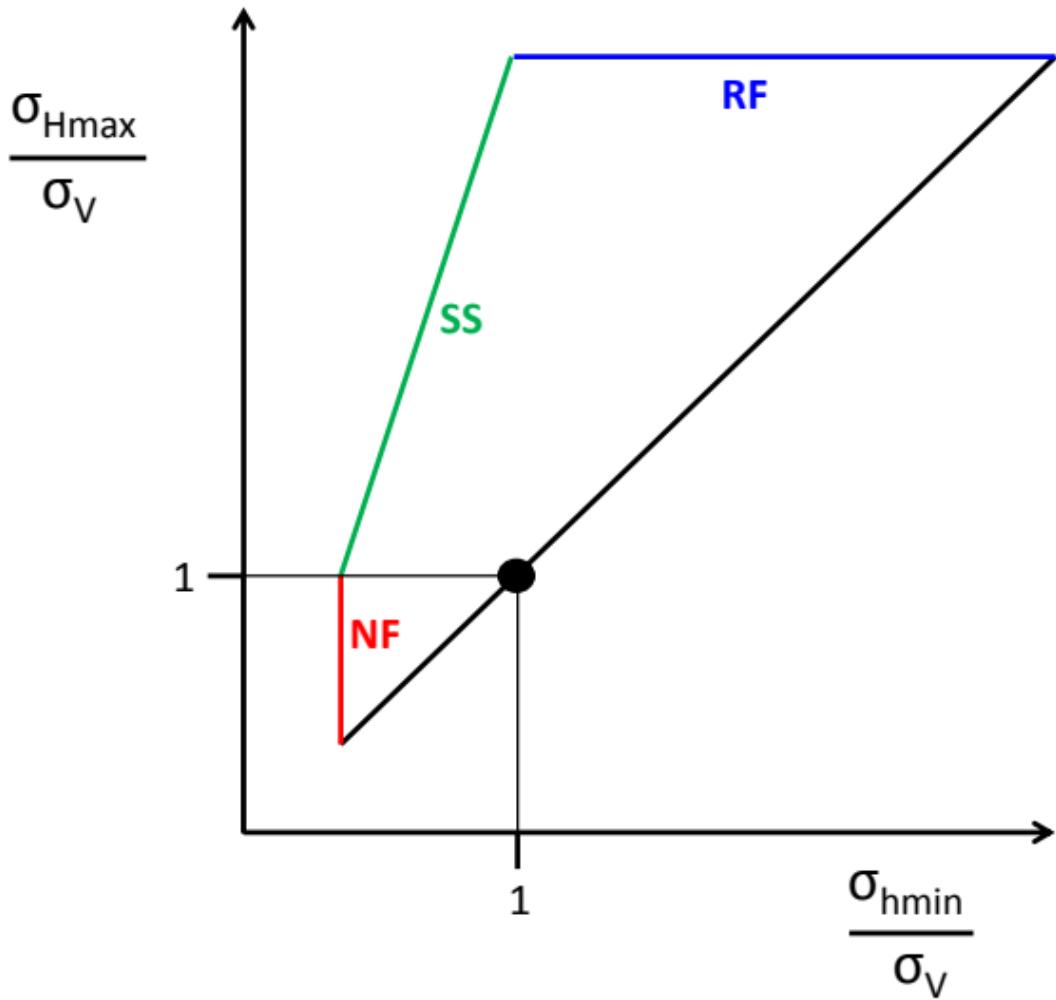
$$\frac{1 + \sin 30^\circ}{1 - \sin 30^\circ} 360$$



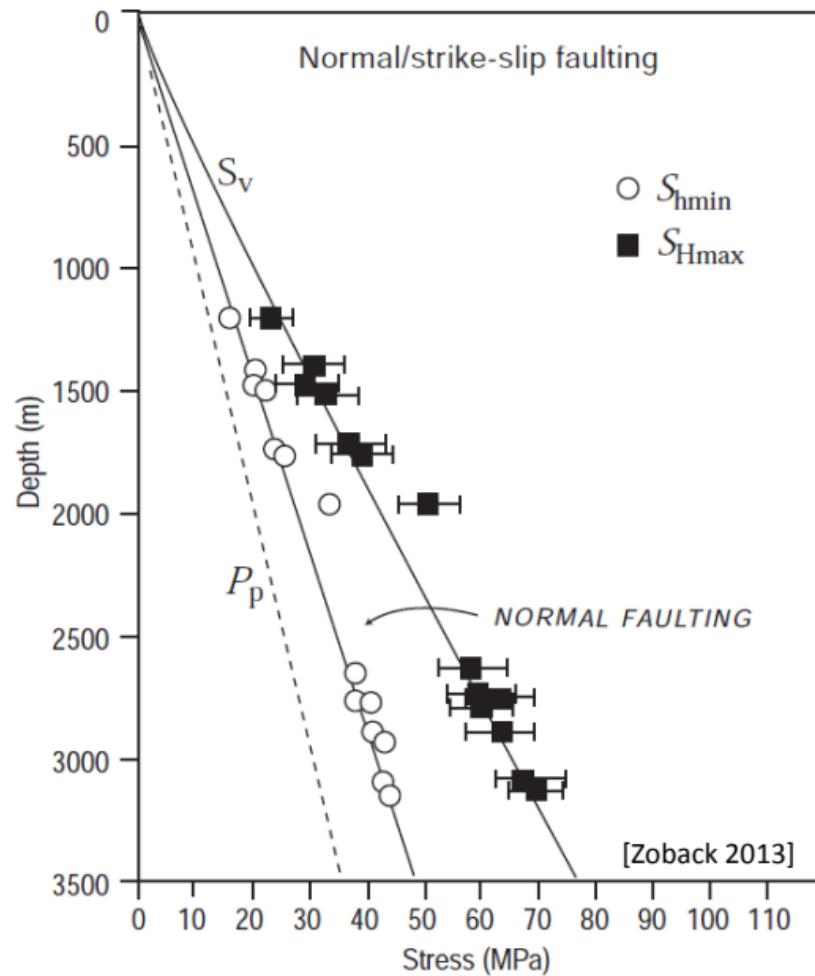
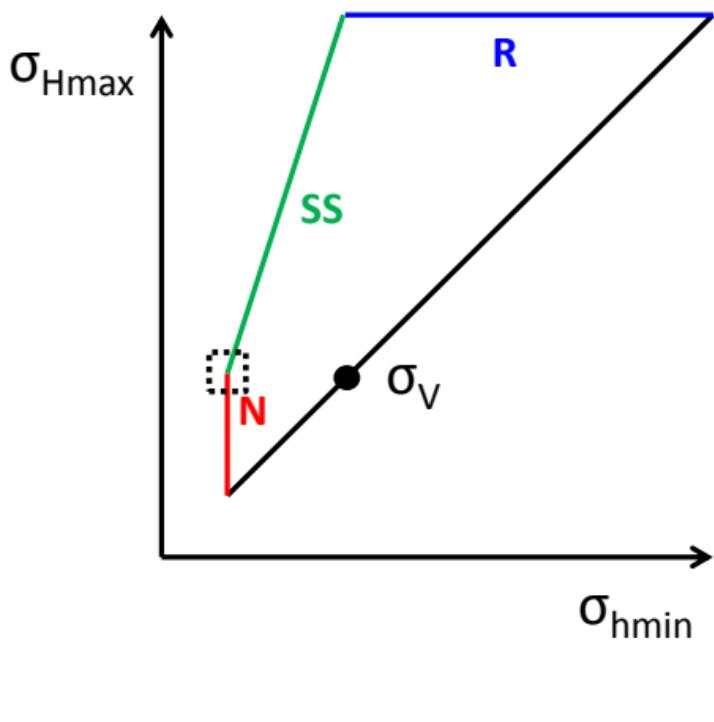


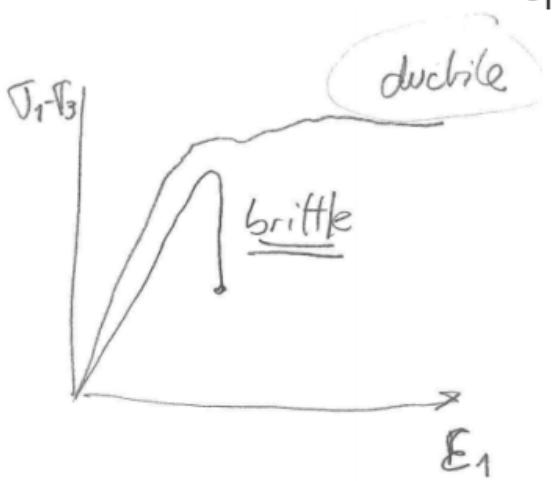
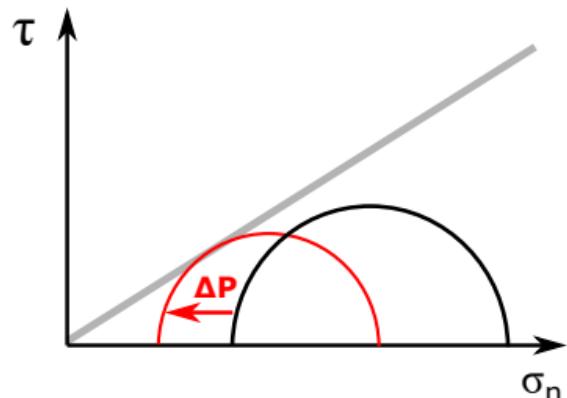
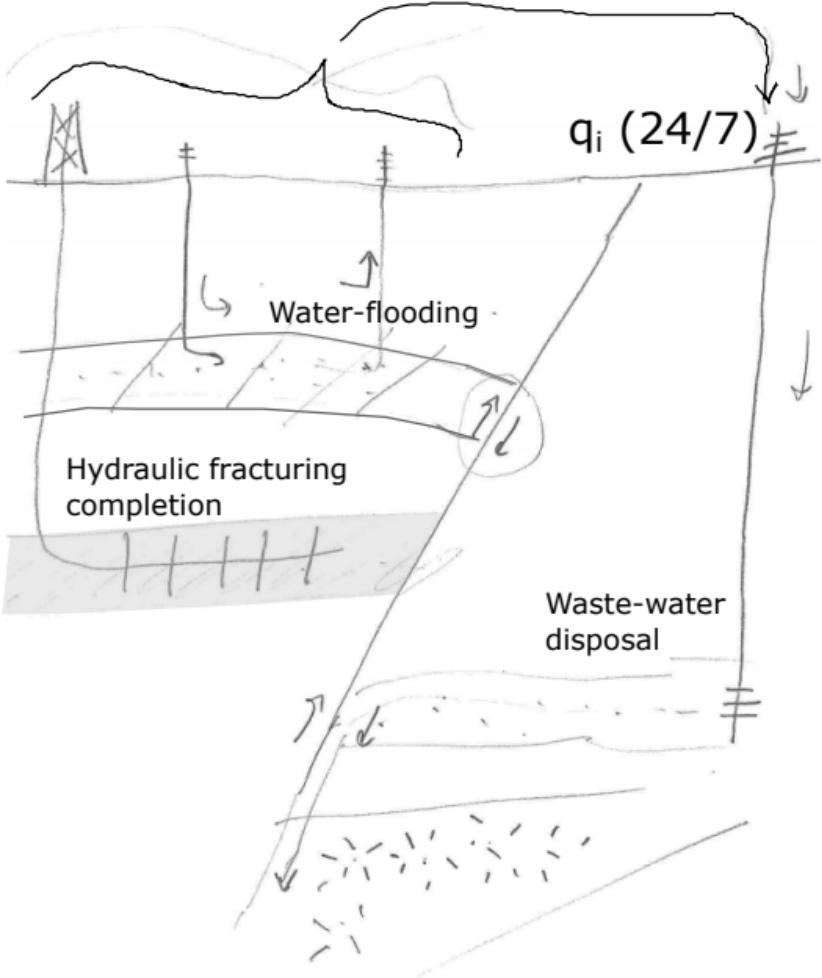






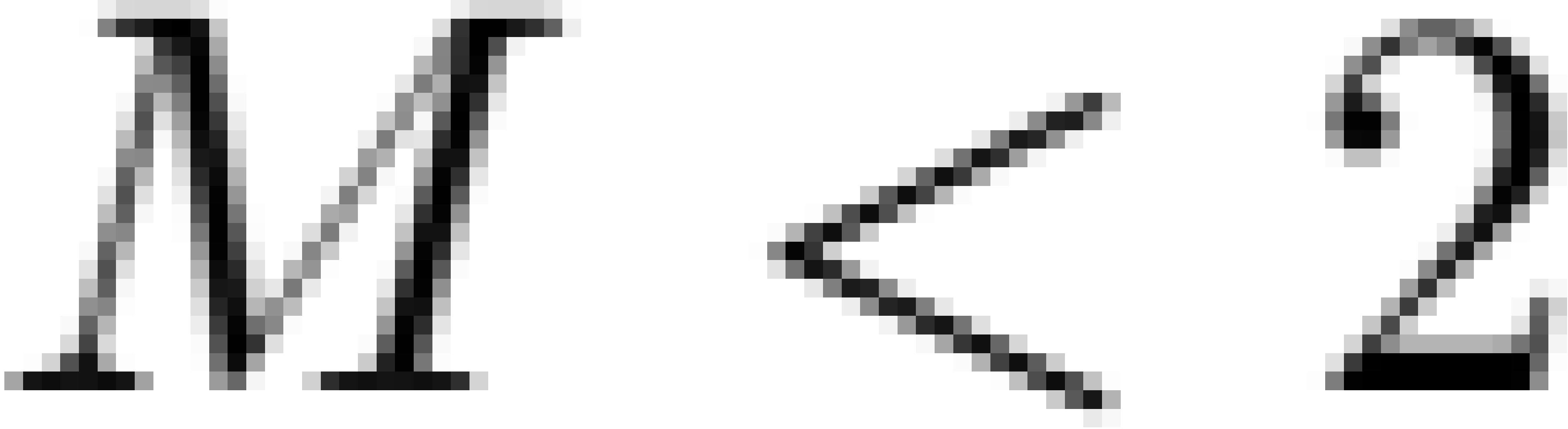
$$\sigma_{h\min} < \sigma_{H\max} \approx \sigma_v$$

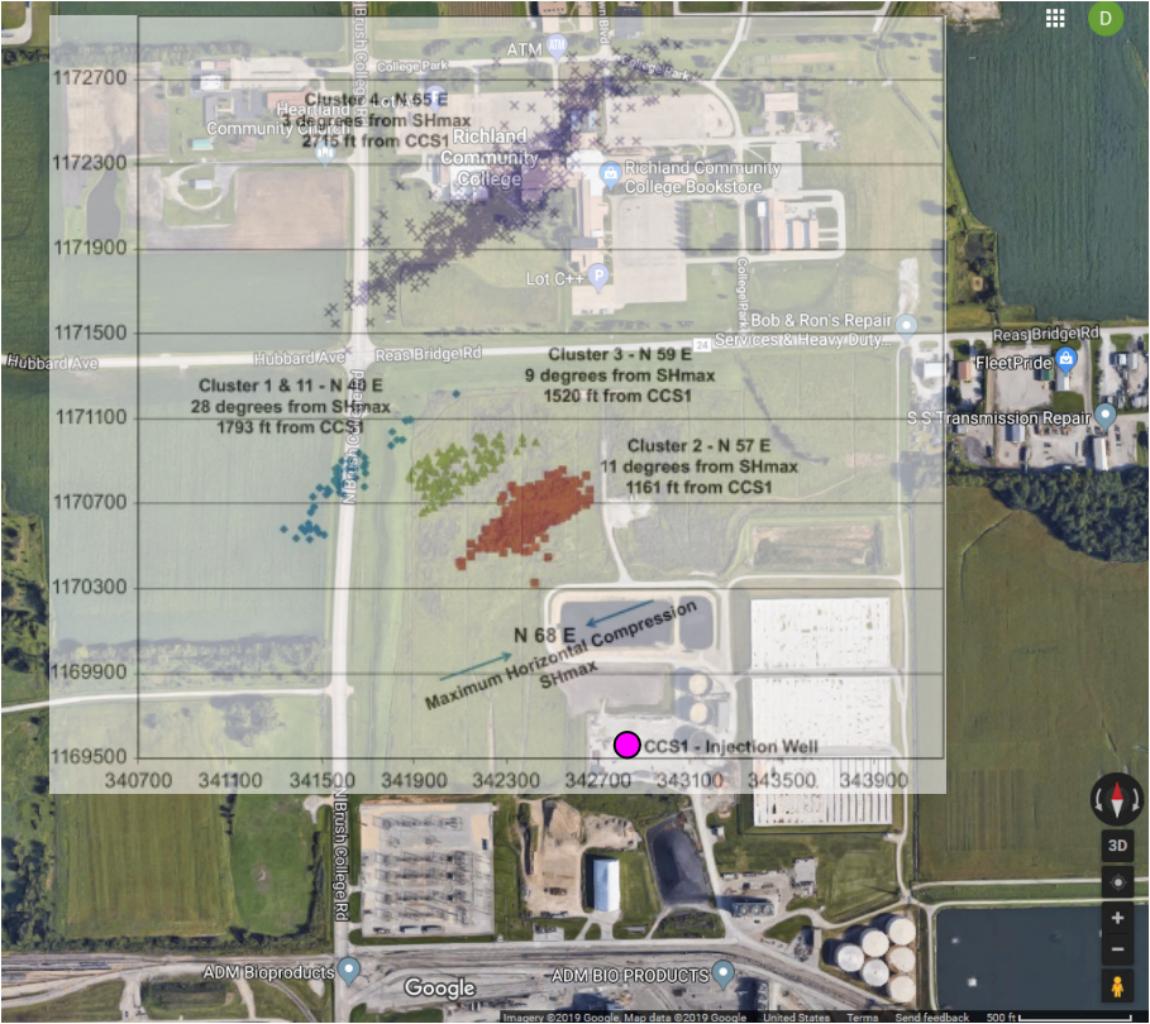




$$\Delta\sigma_a = -\Delta P_p$$

$$\Delta\sigma_{\text{Hermann}} \leq -\Delta P_p$$

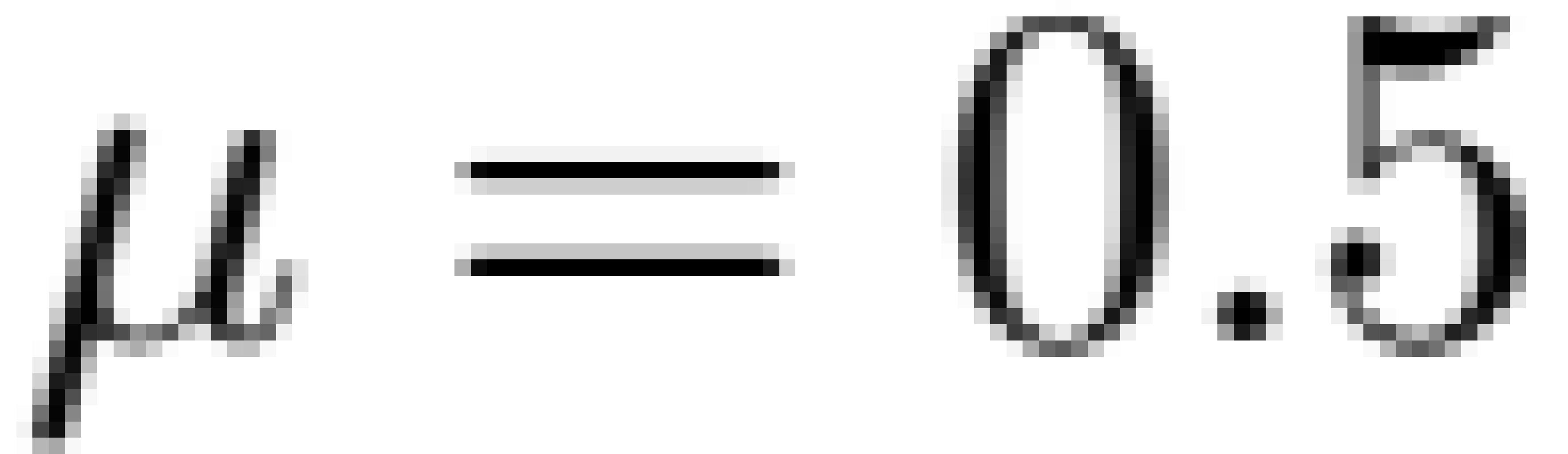


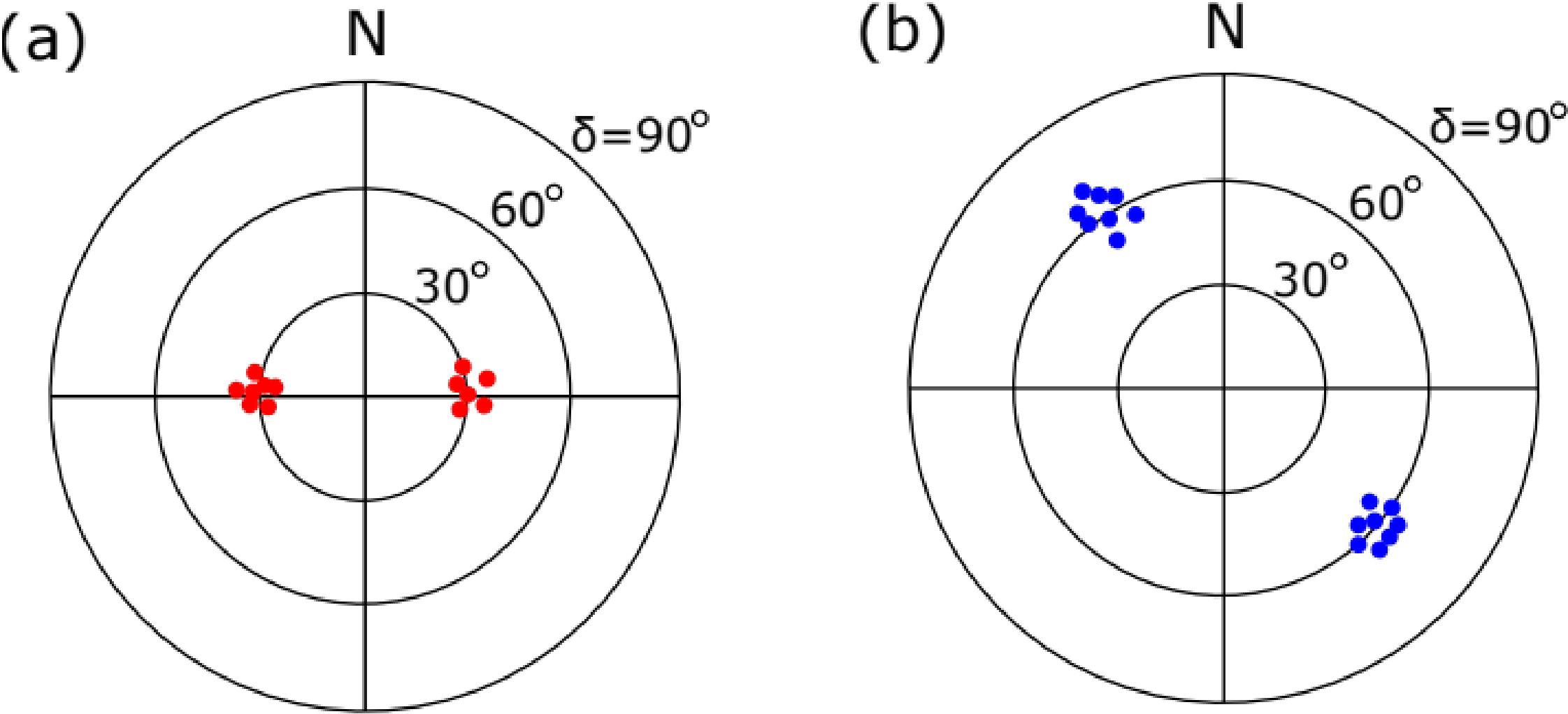




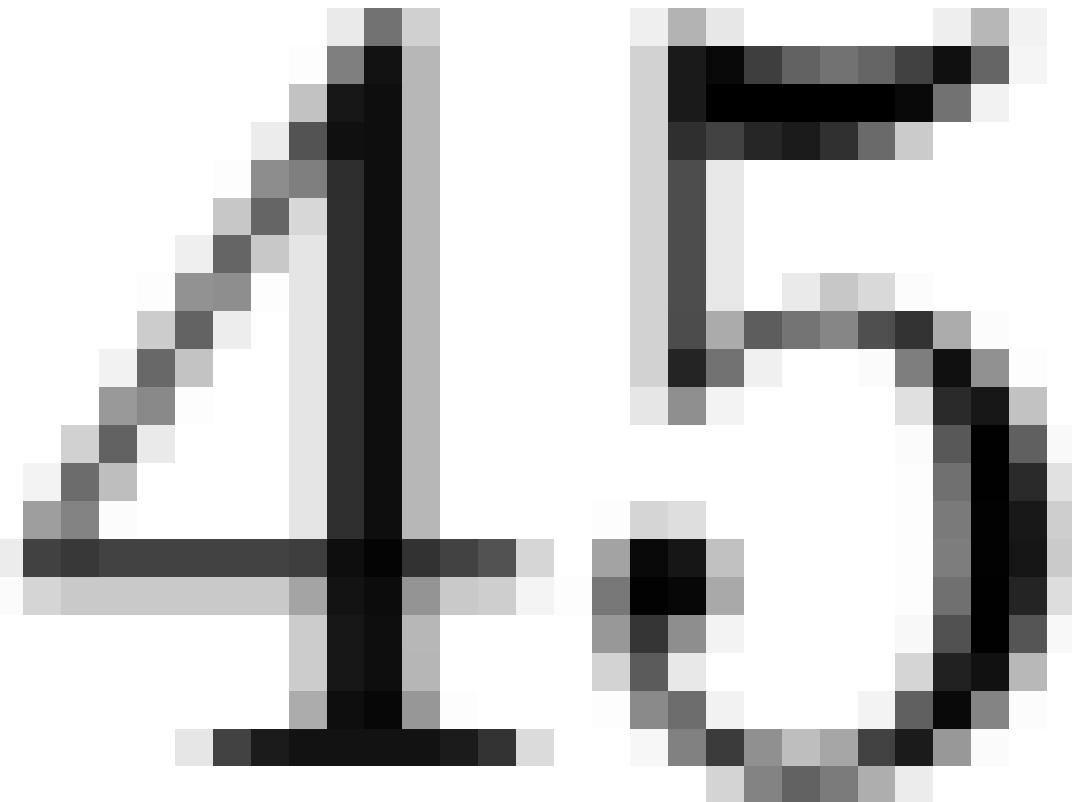


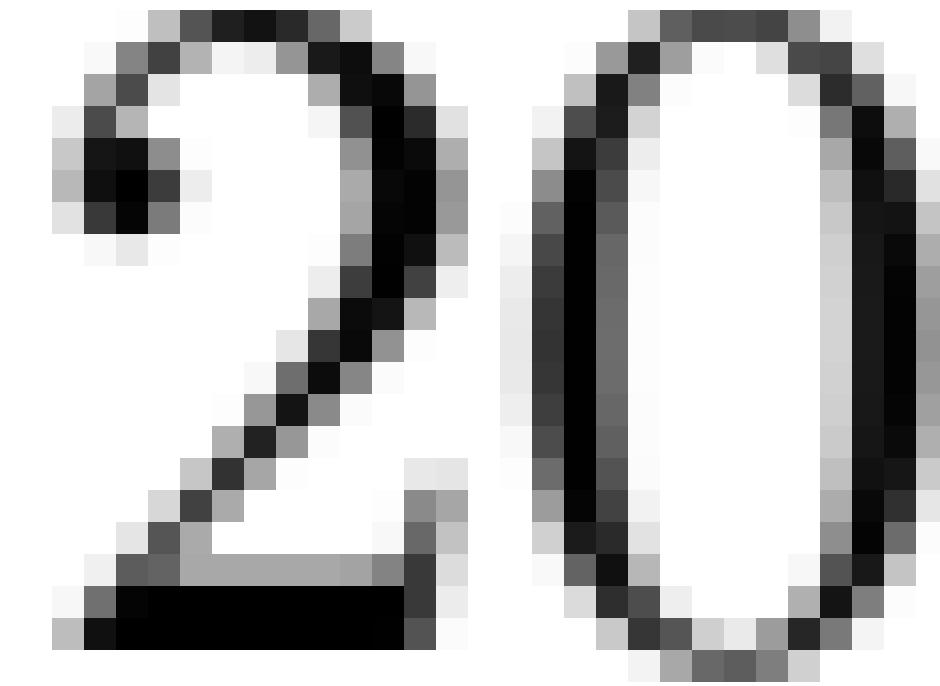


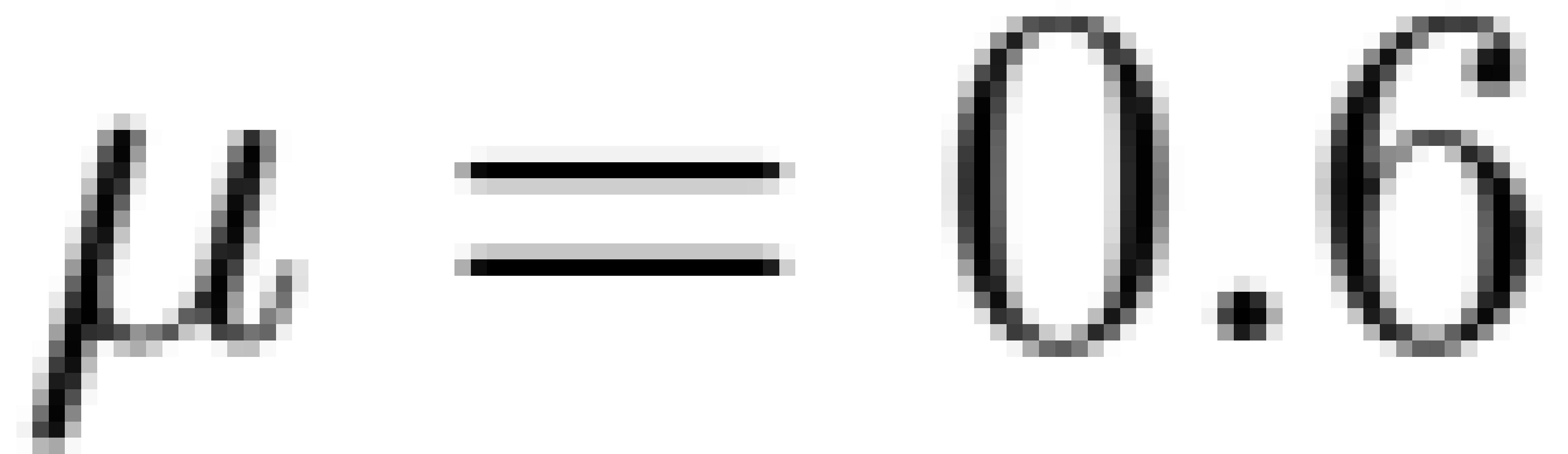






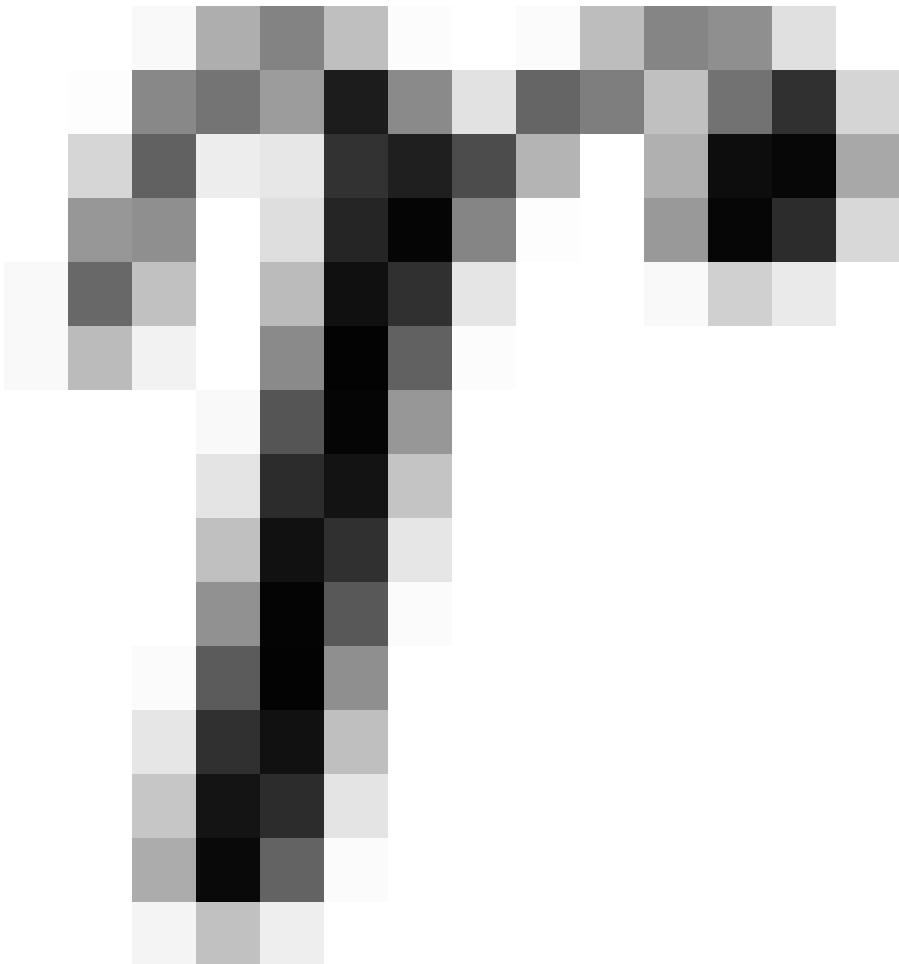


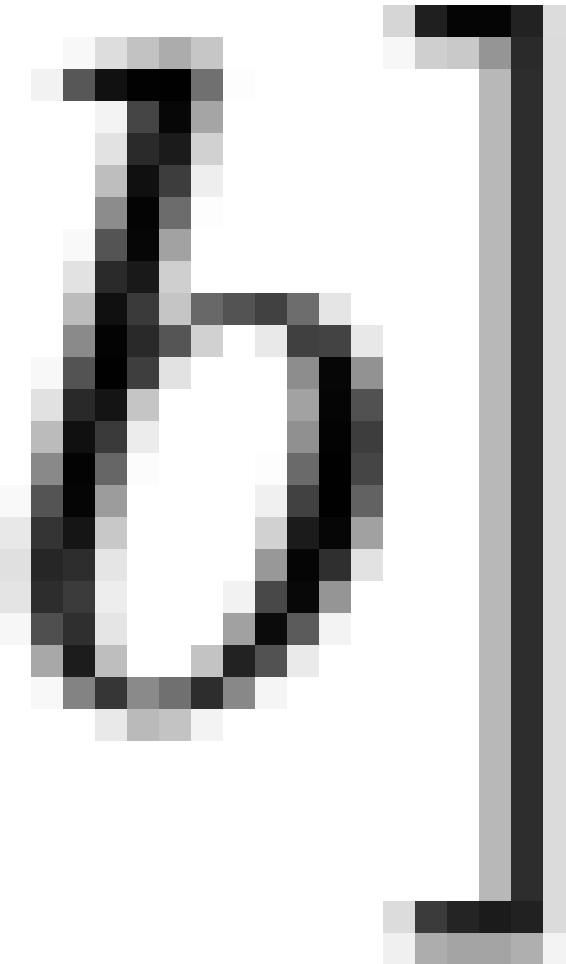
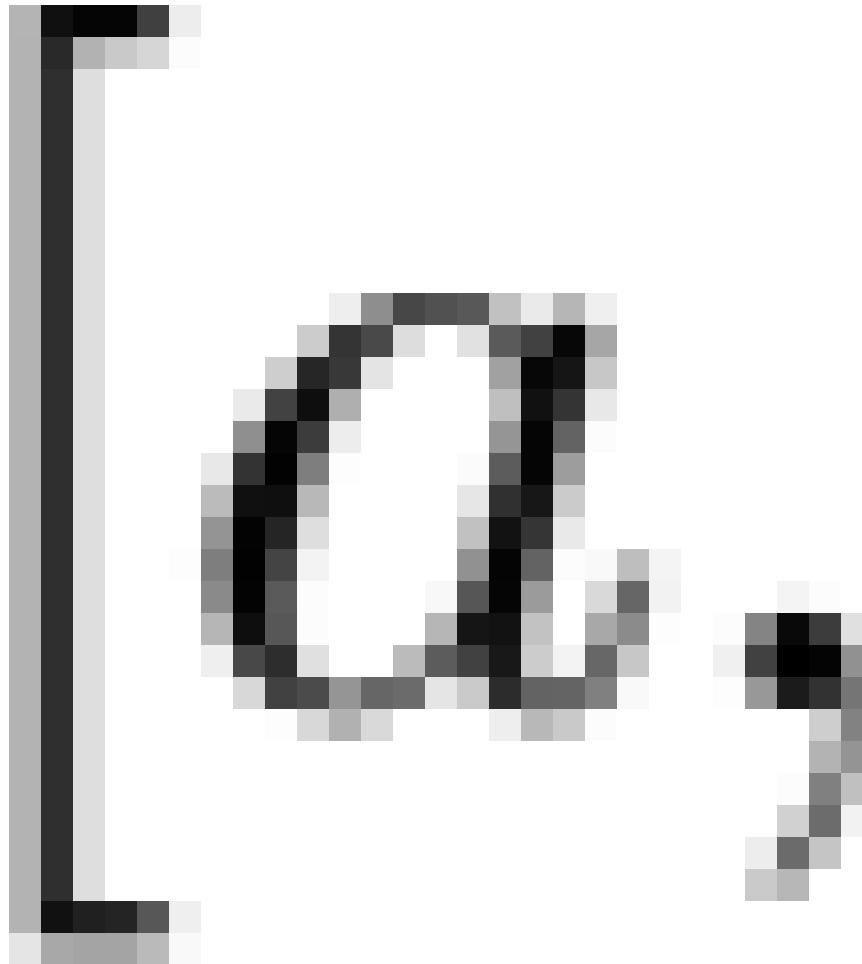


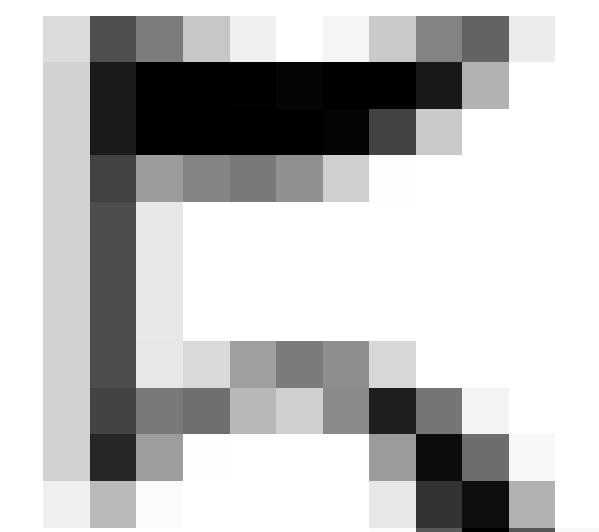
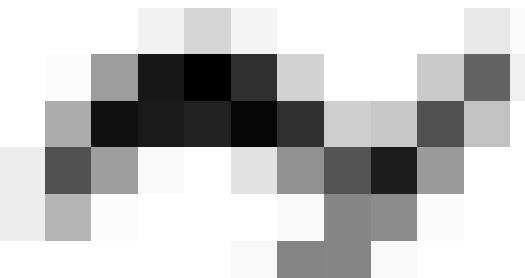
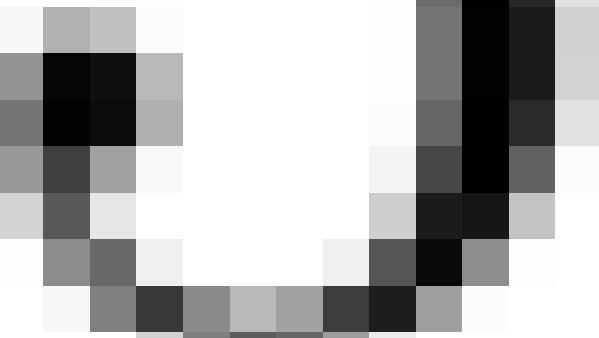
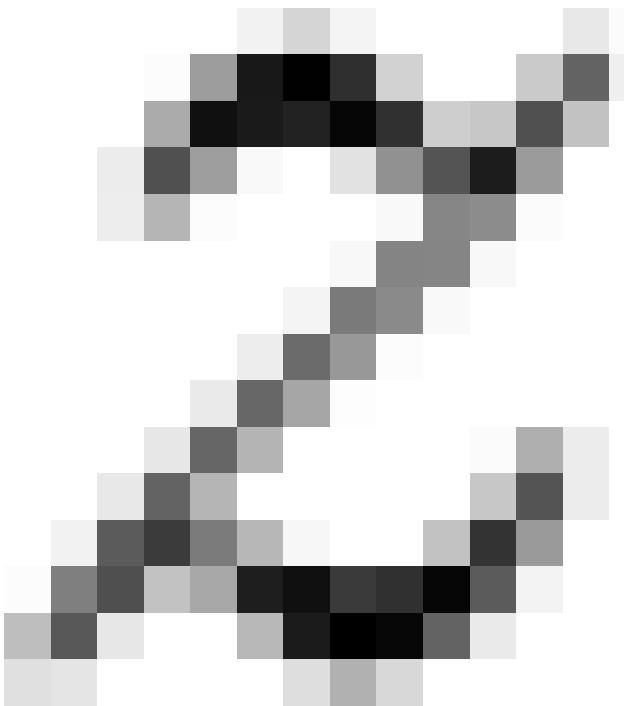






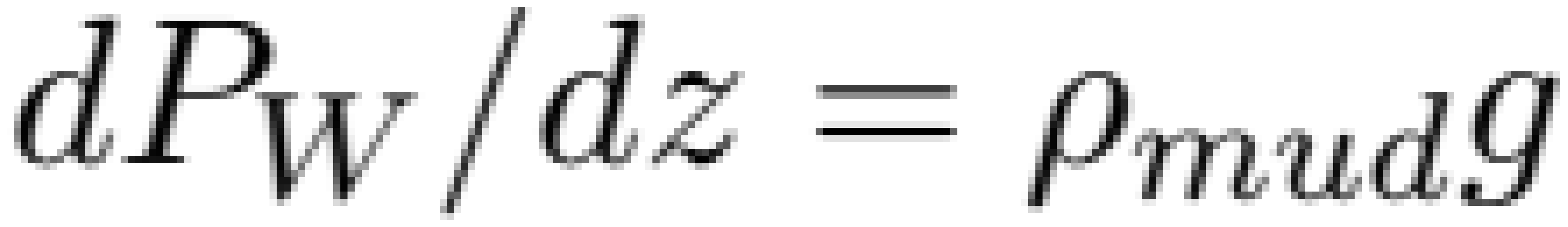


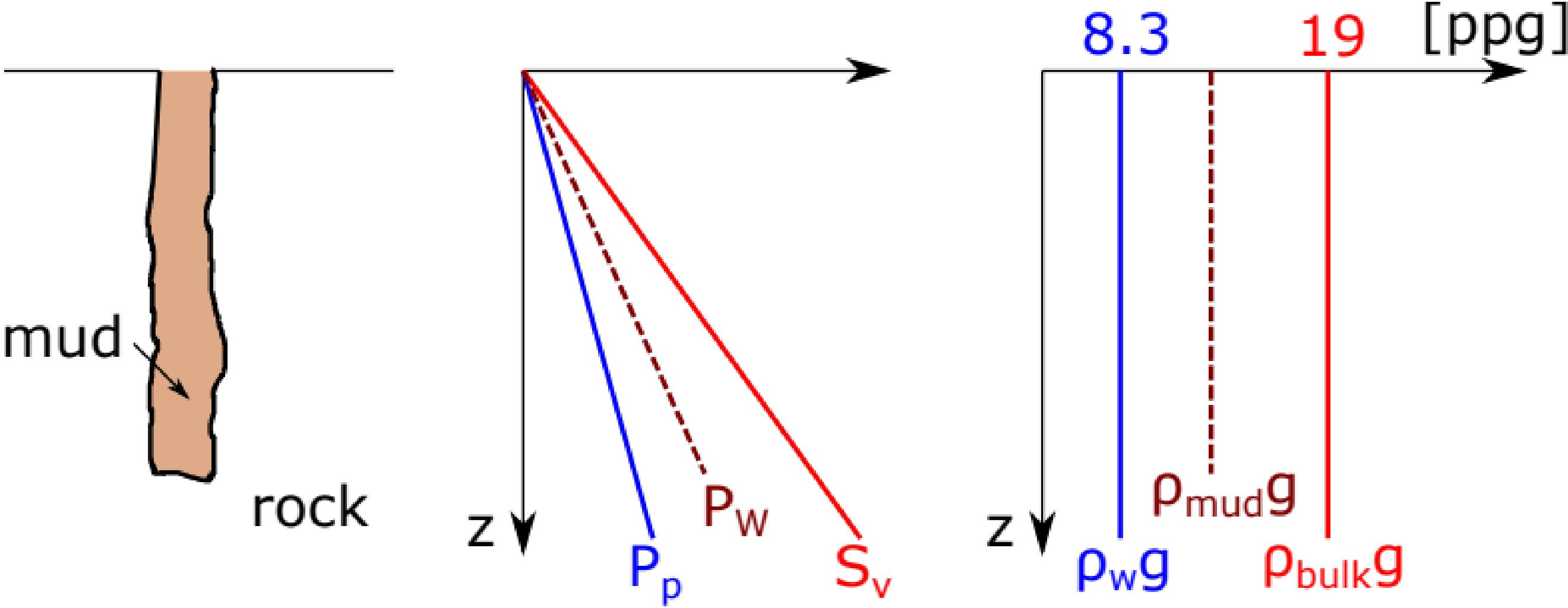


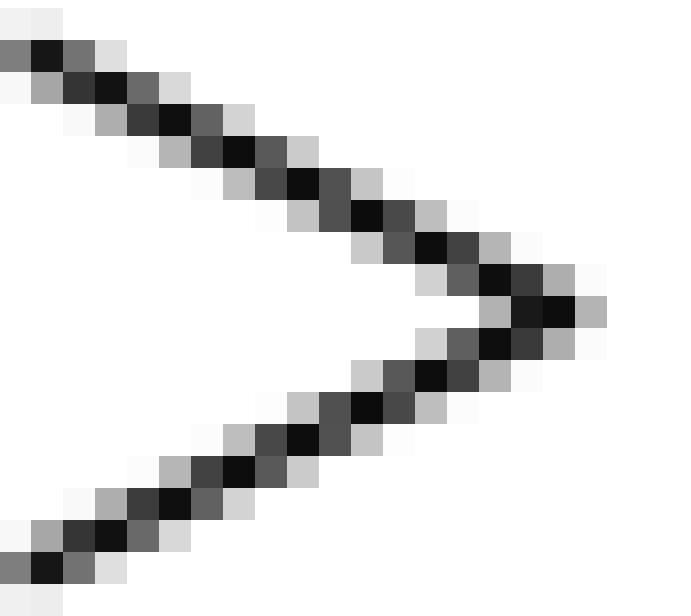
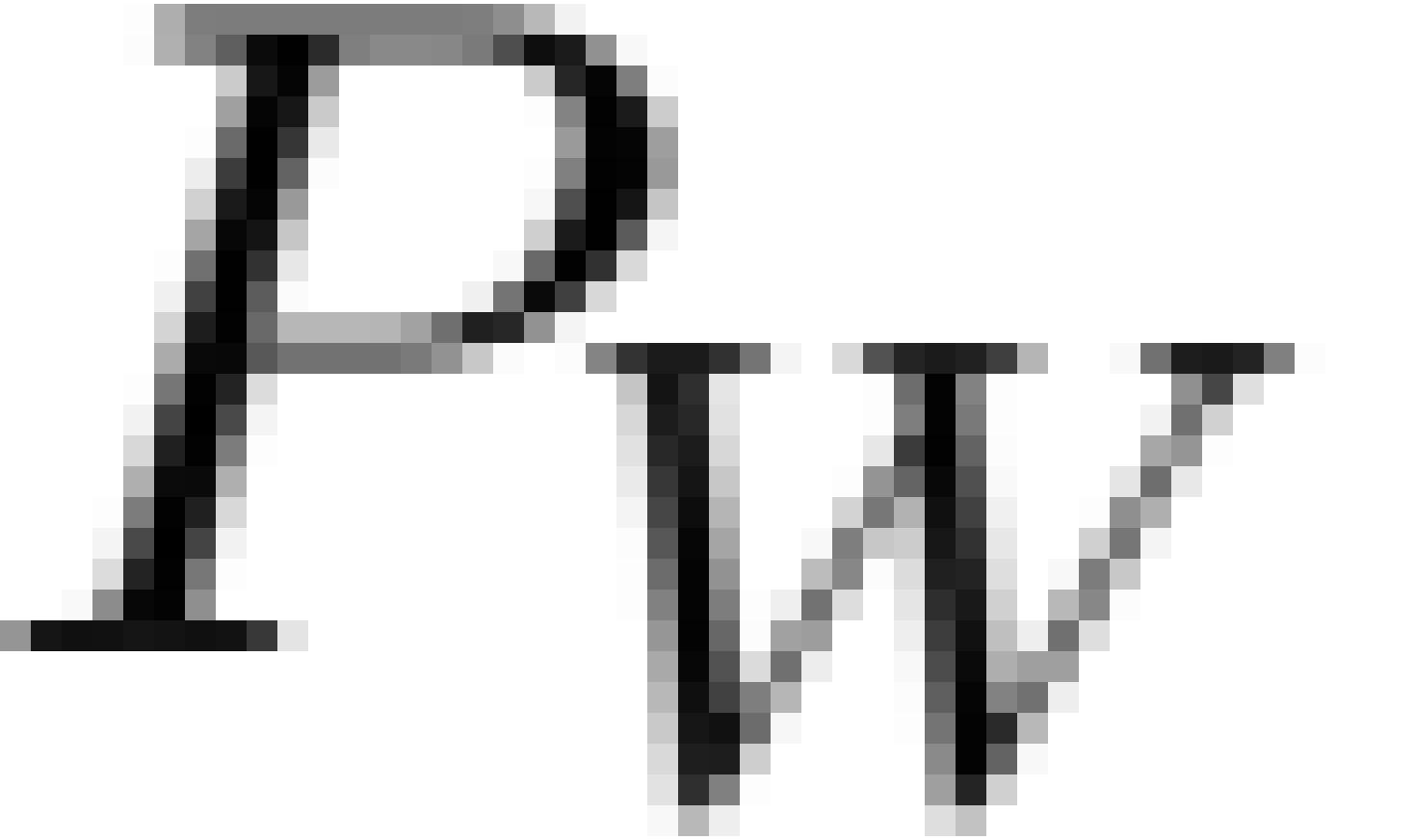




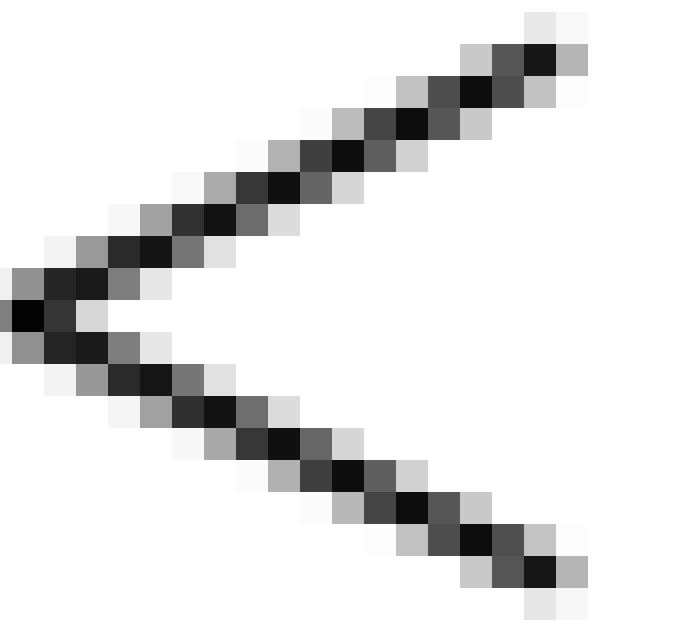


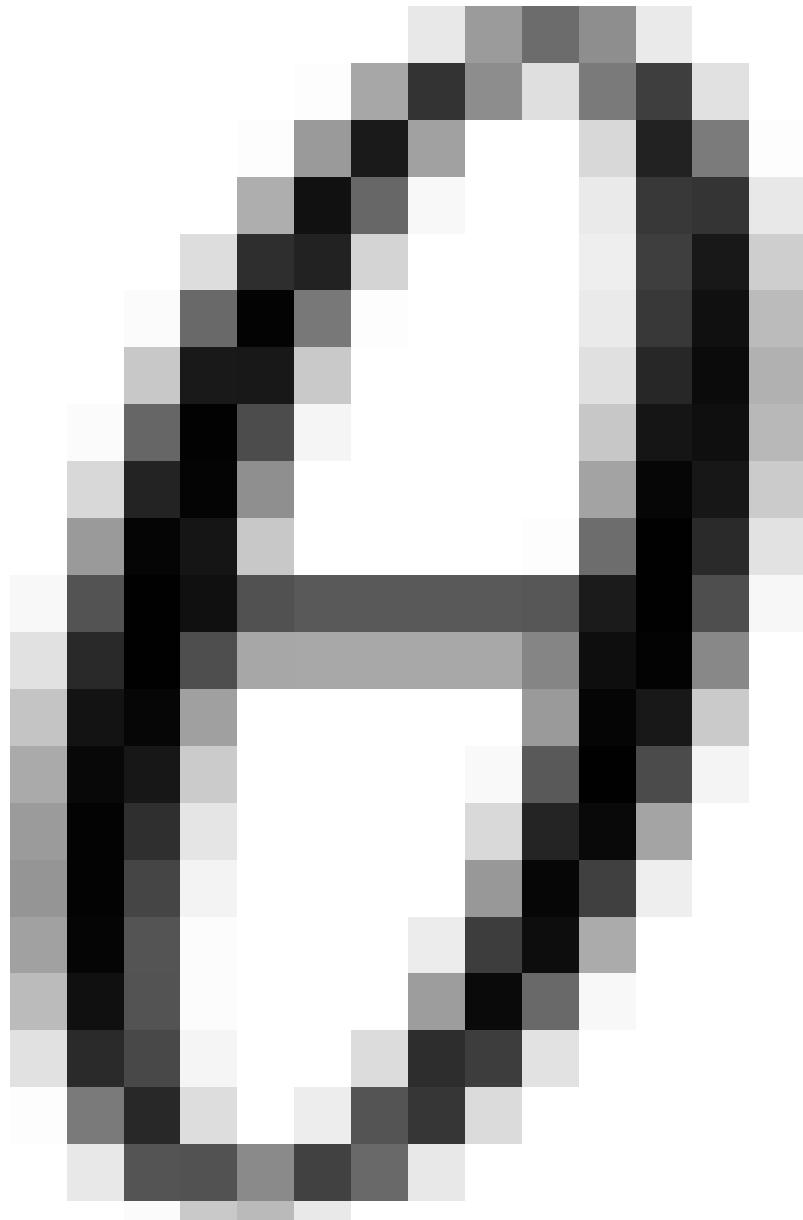


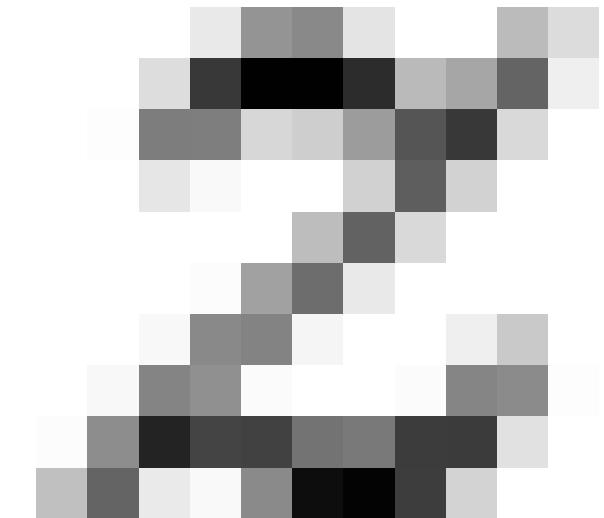
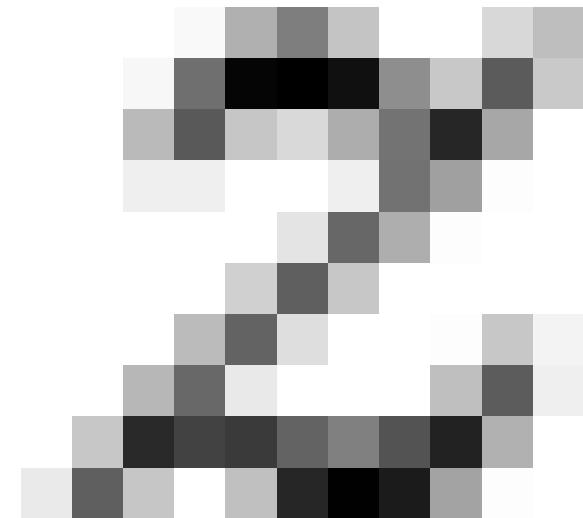
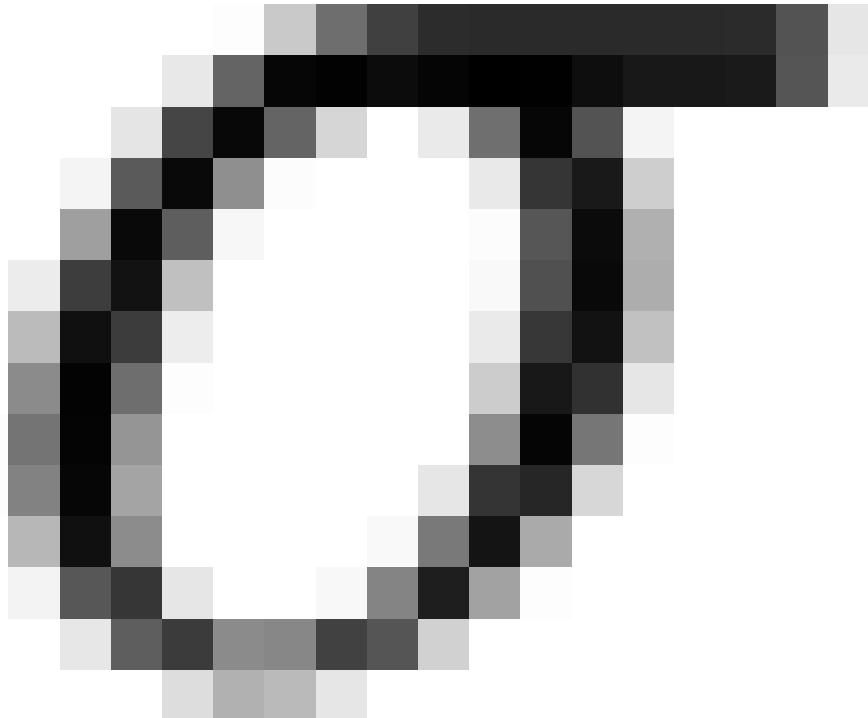


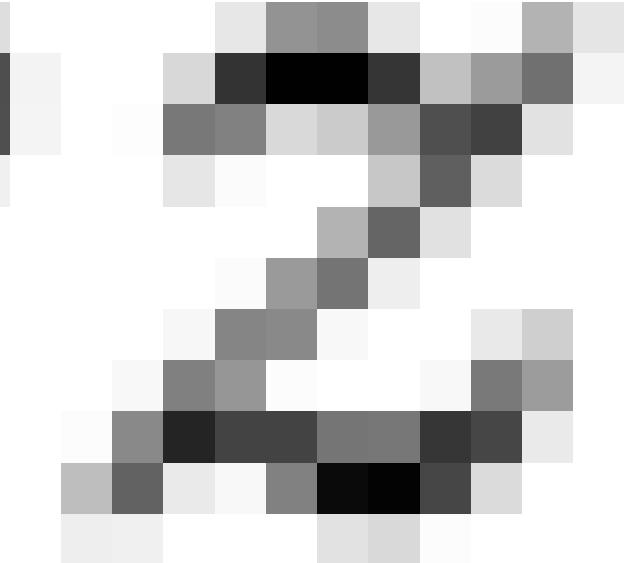
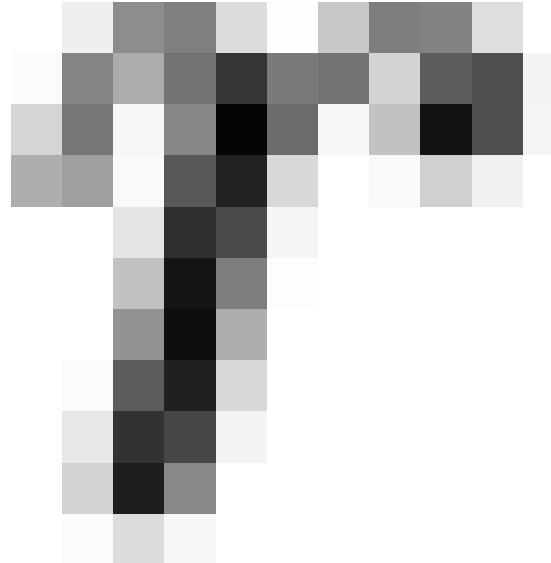
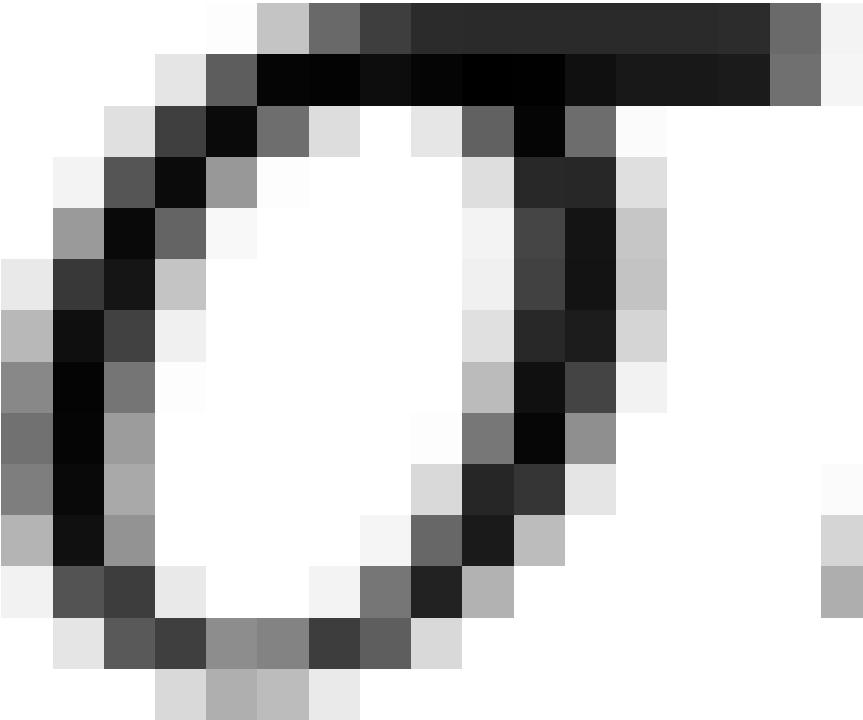


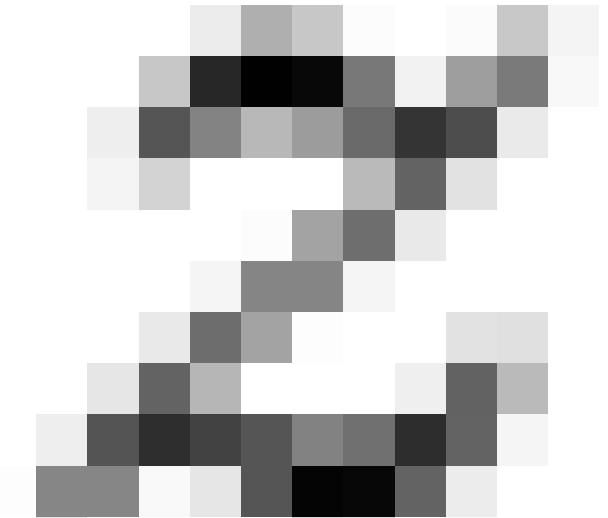
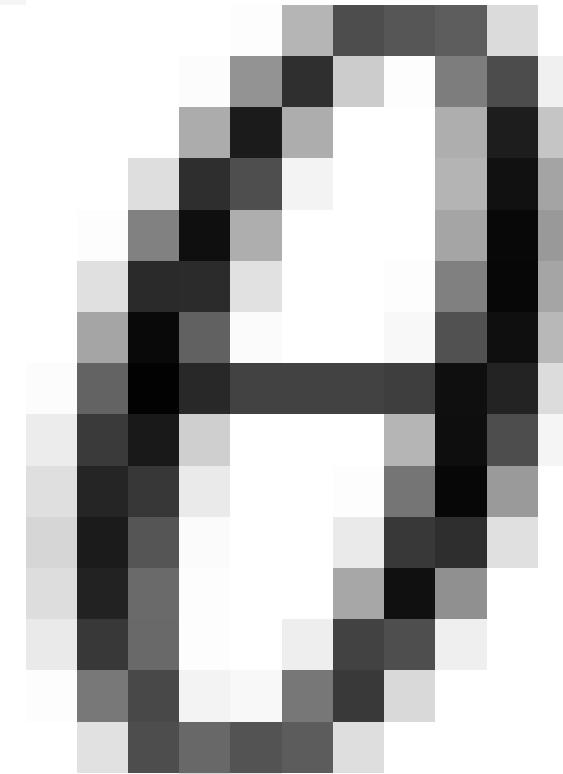
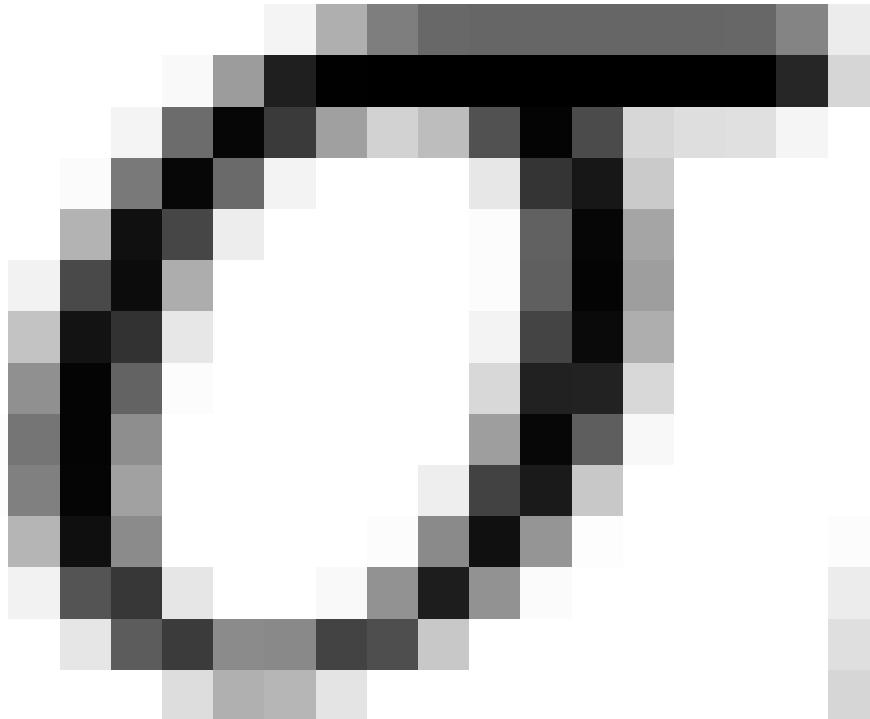




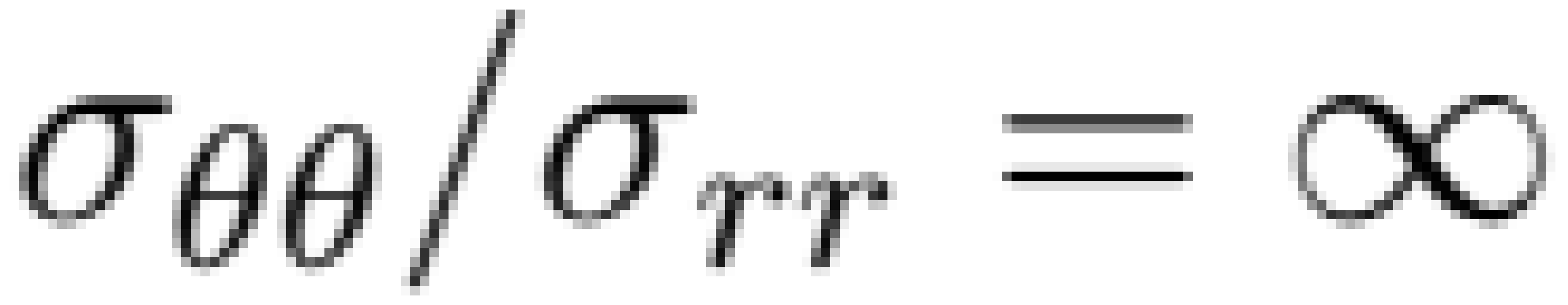


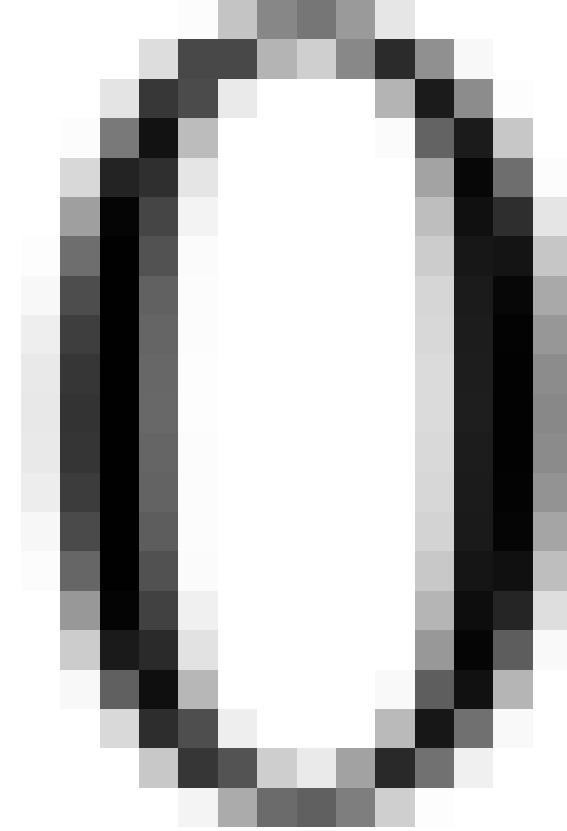
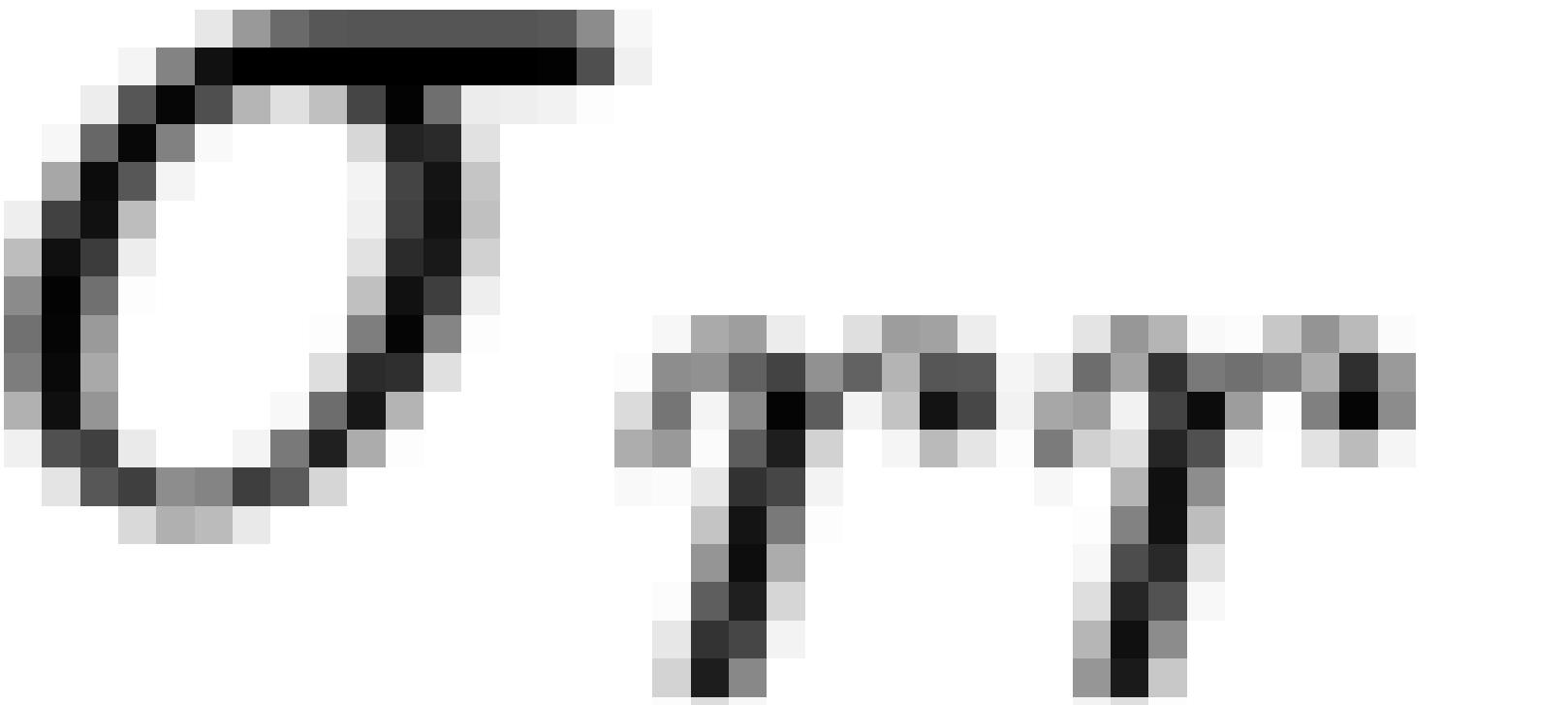




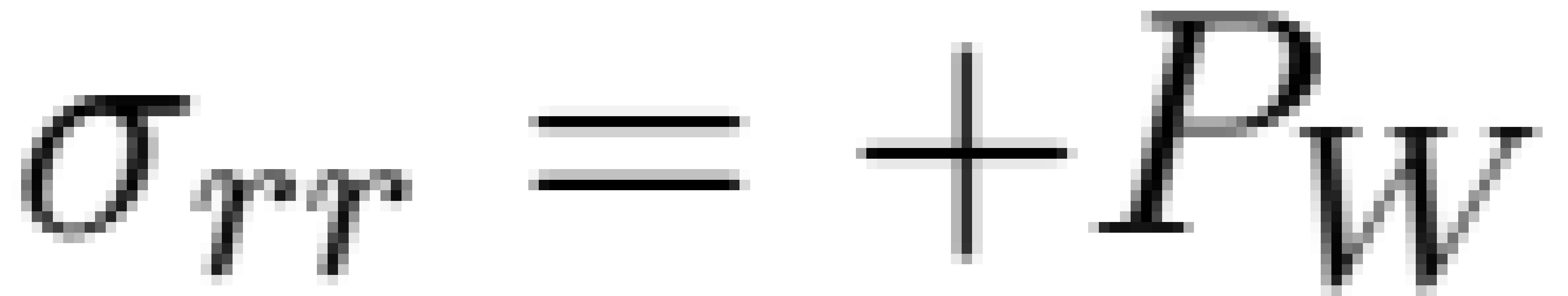


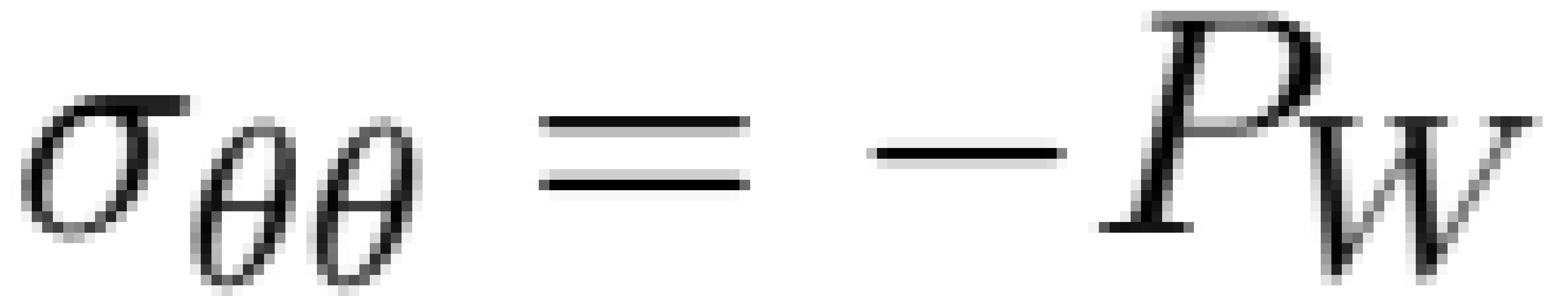


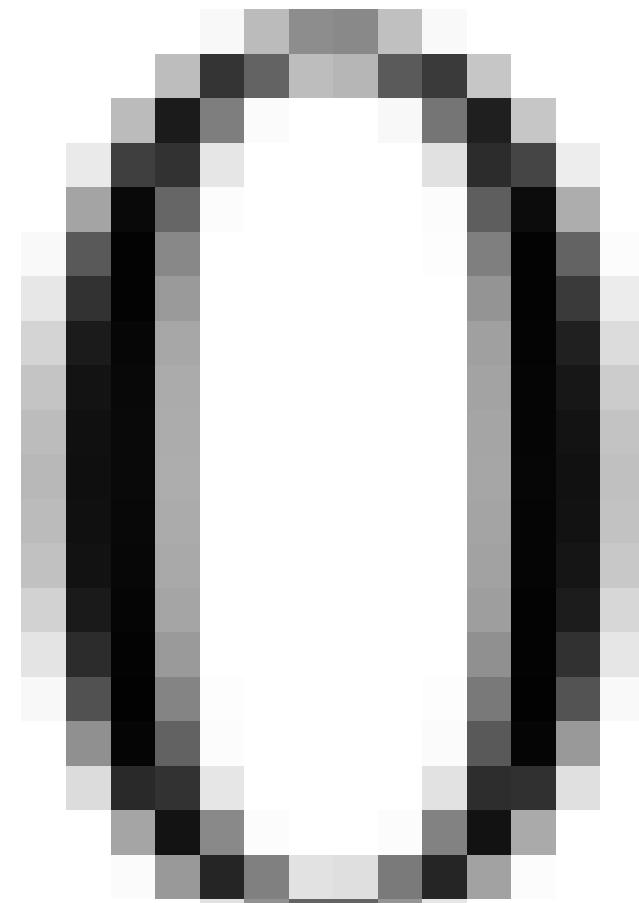
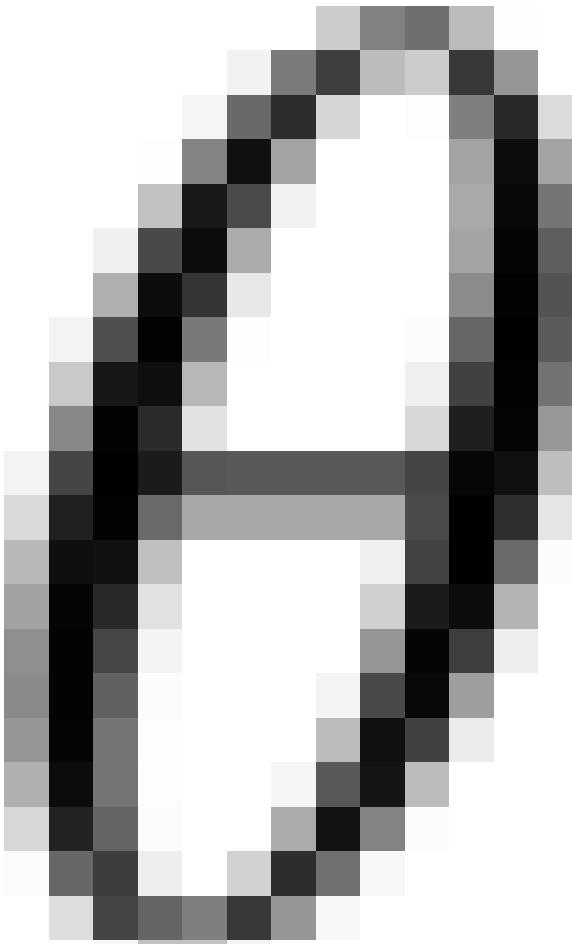


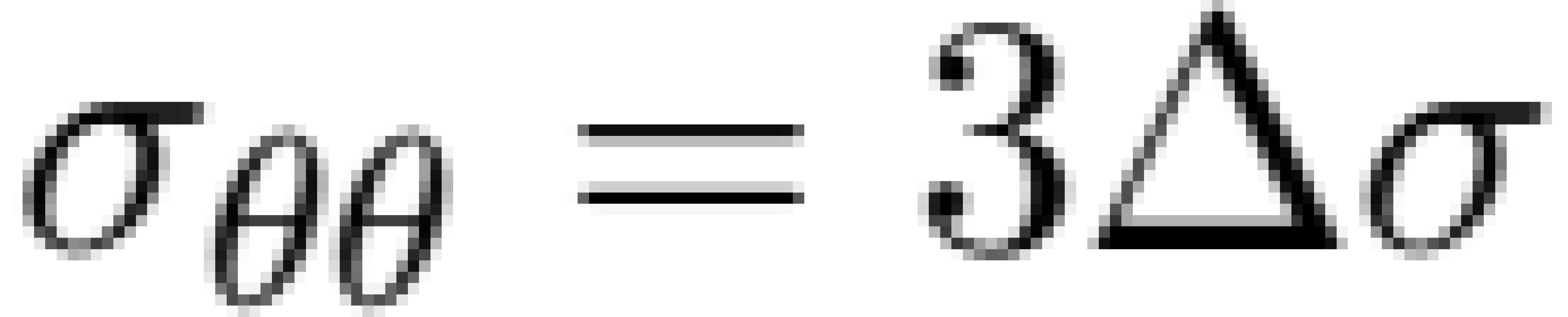


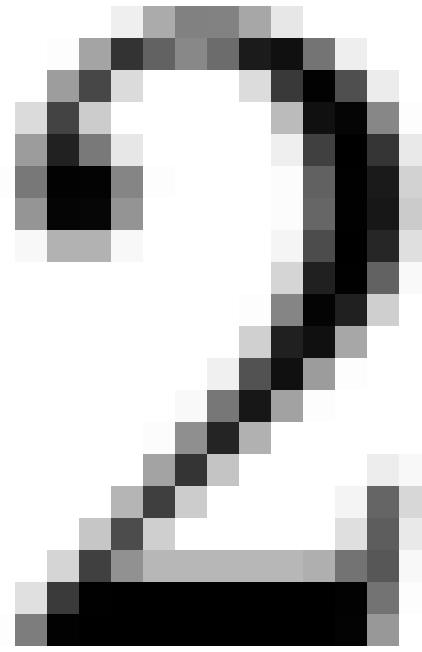
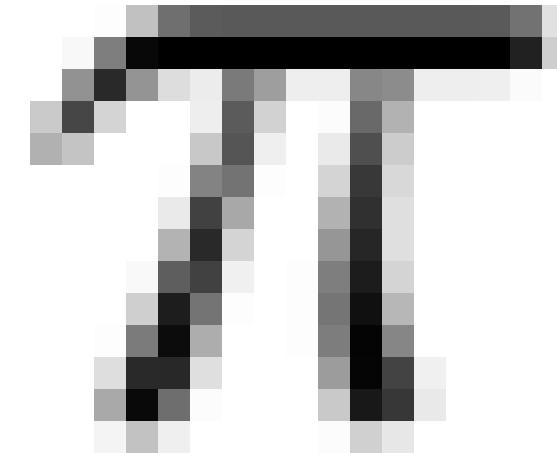
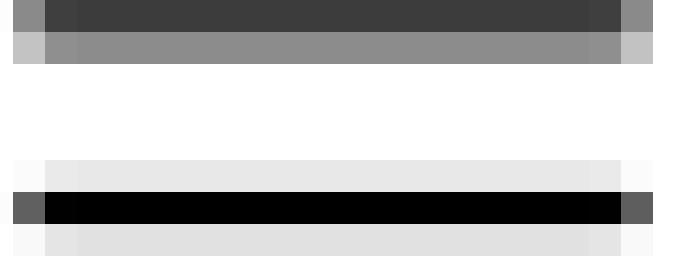
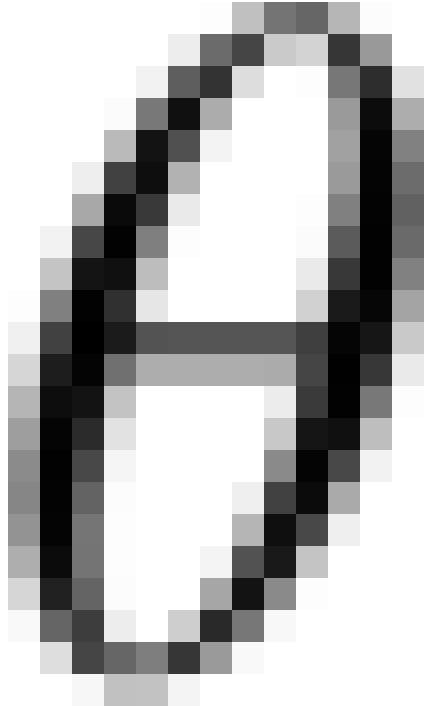


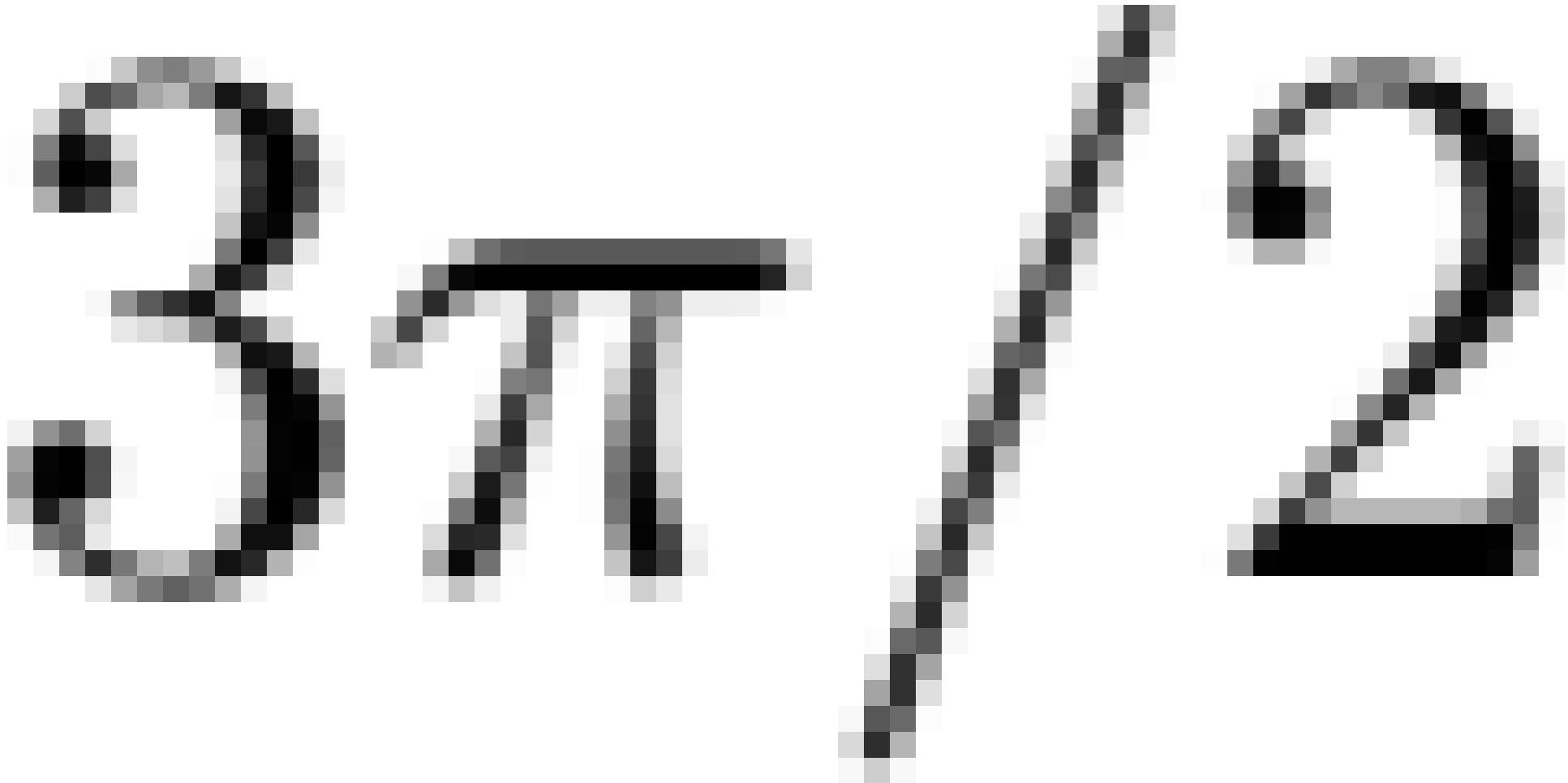


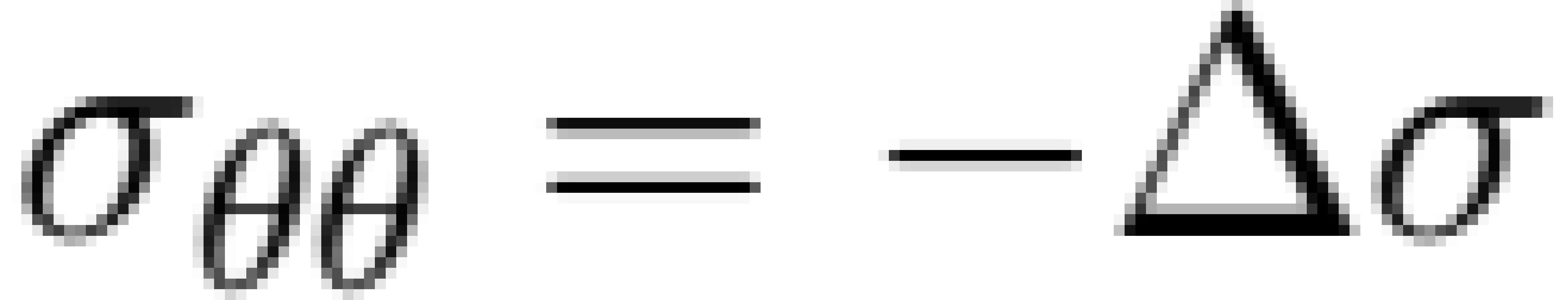


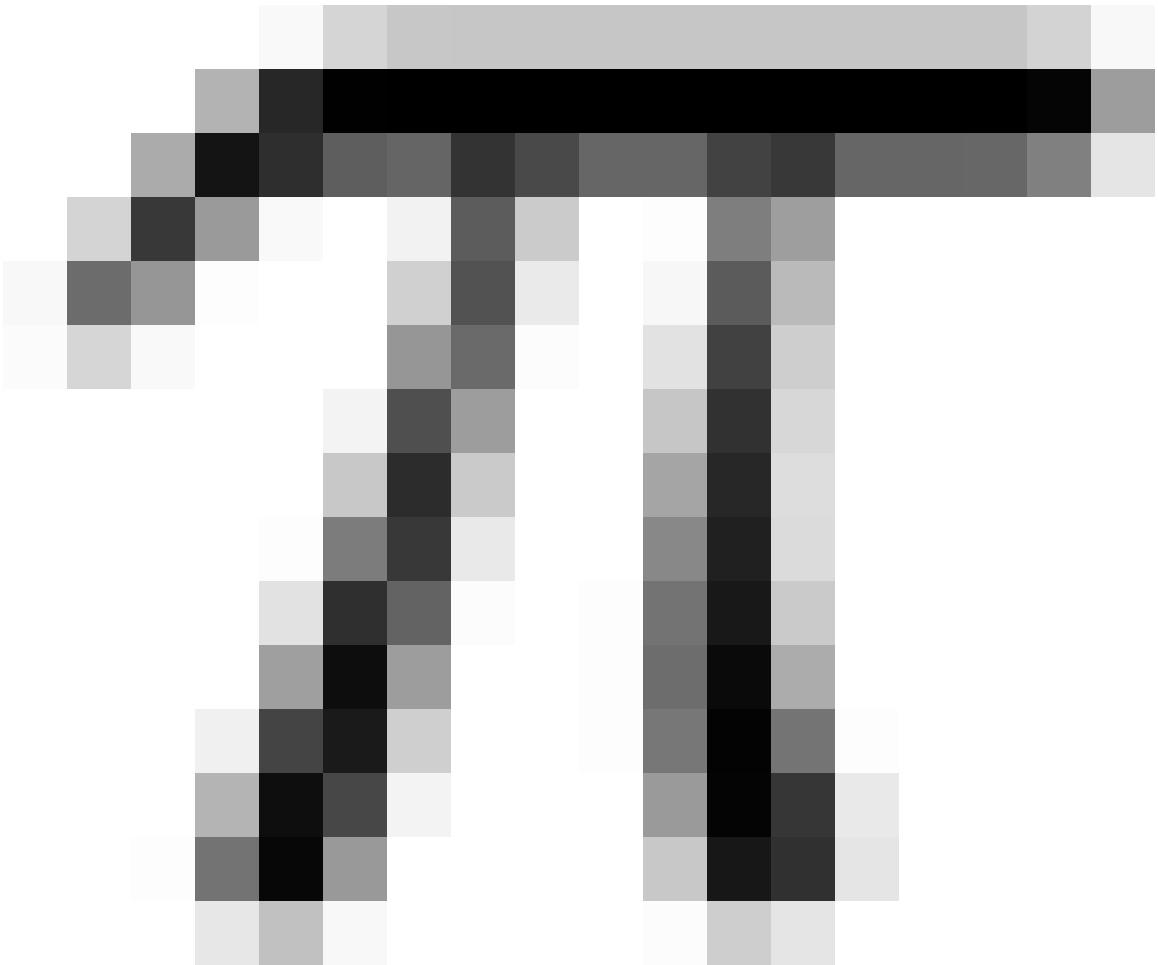




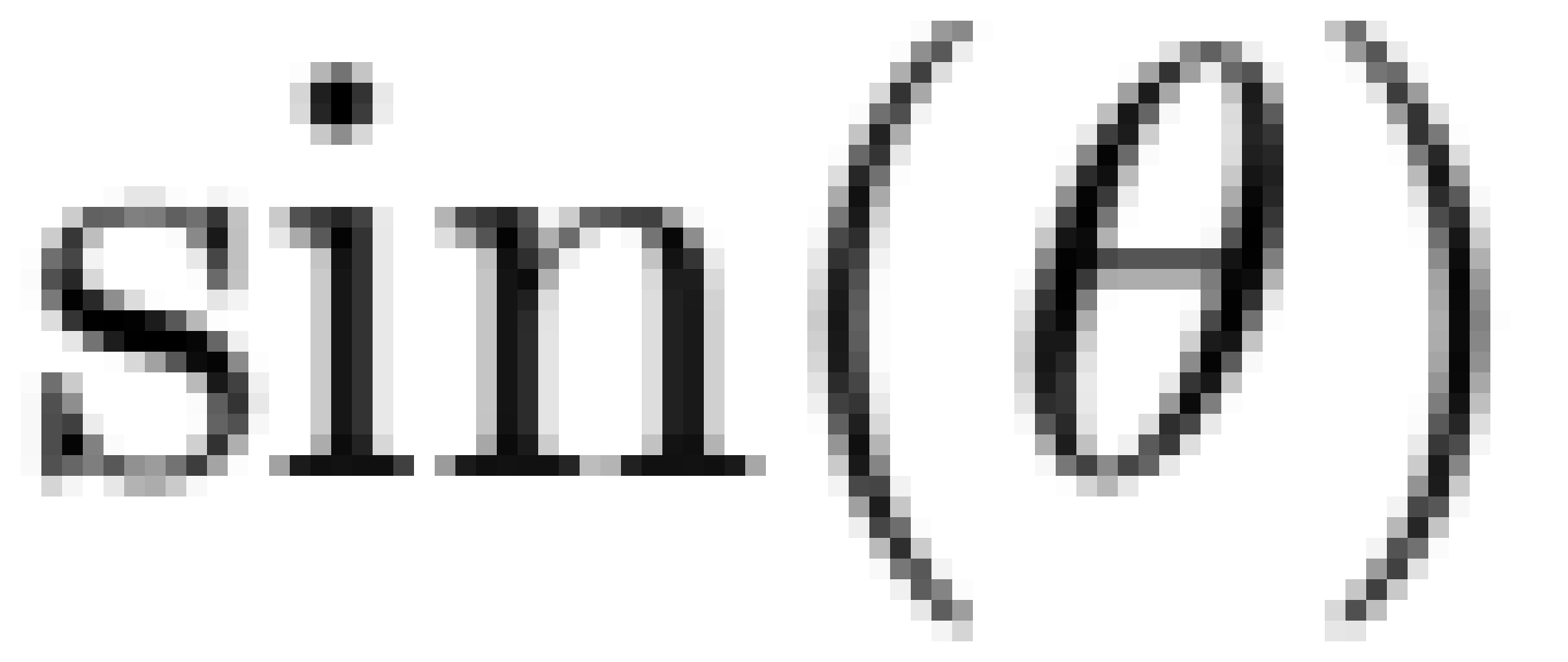




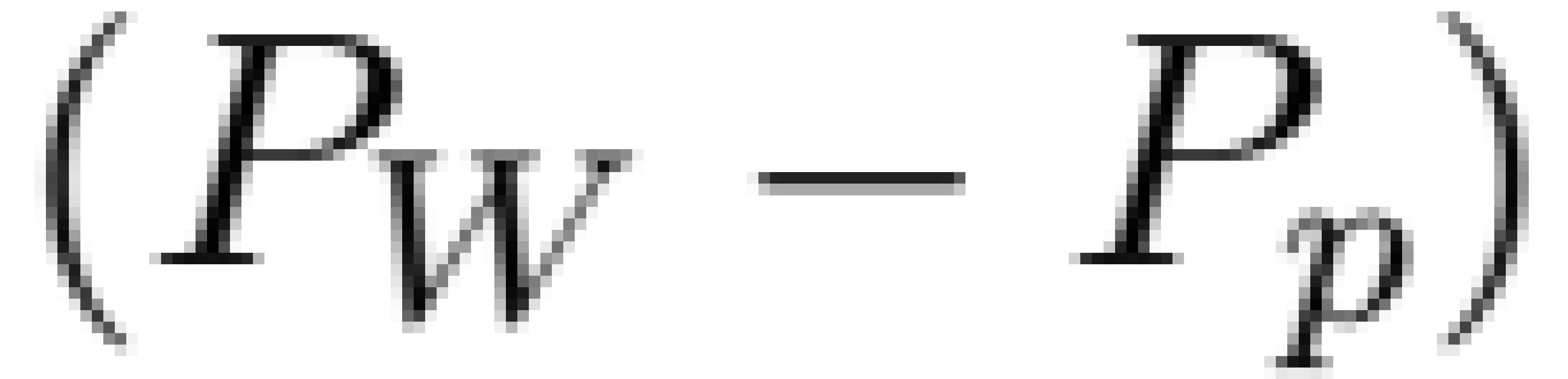


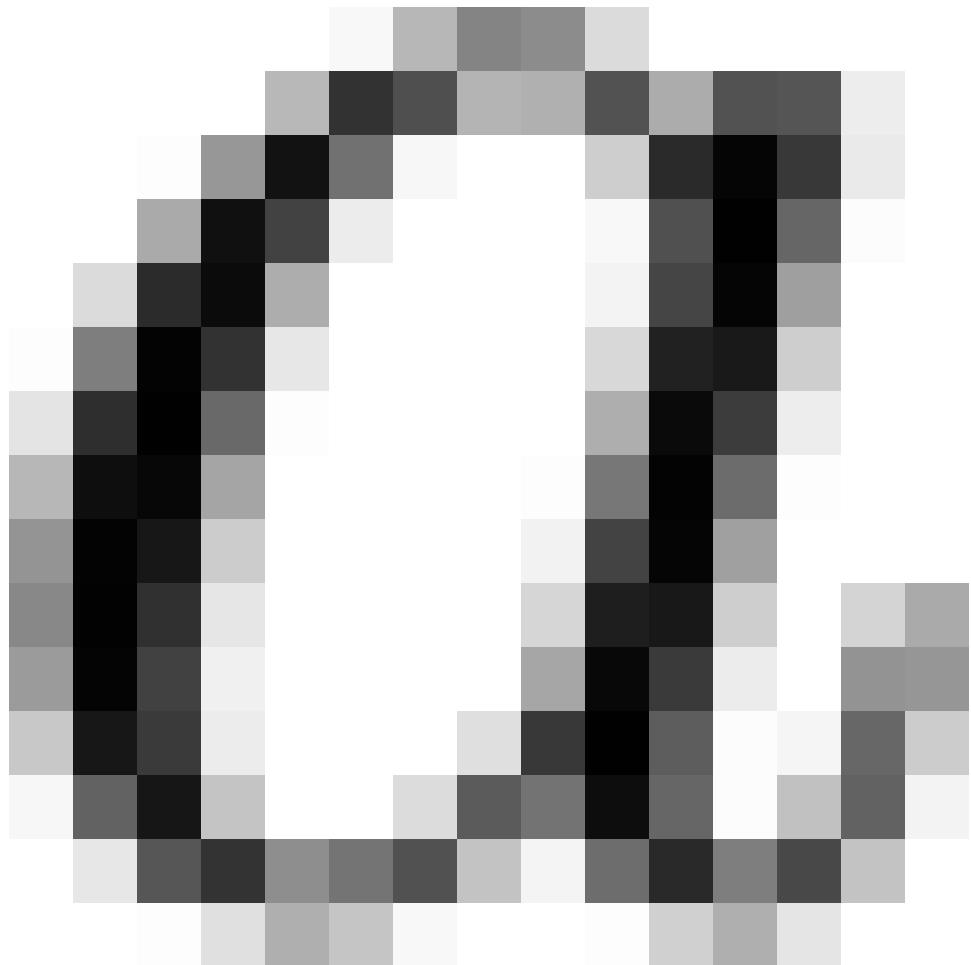




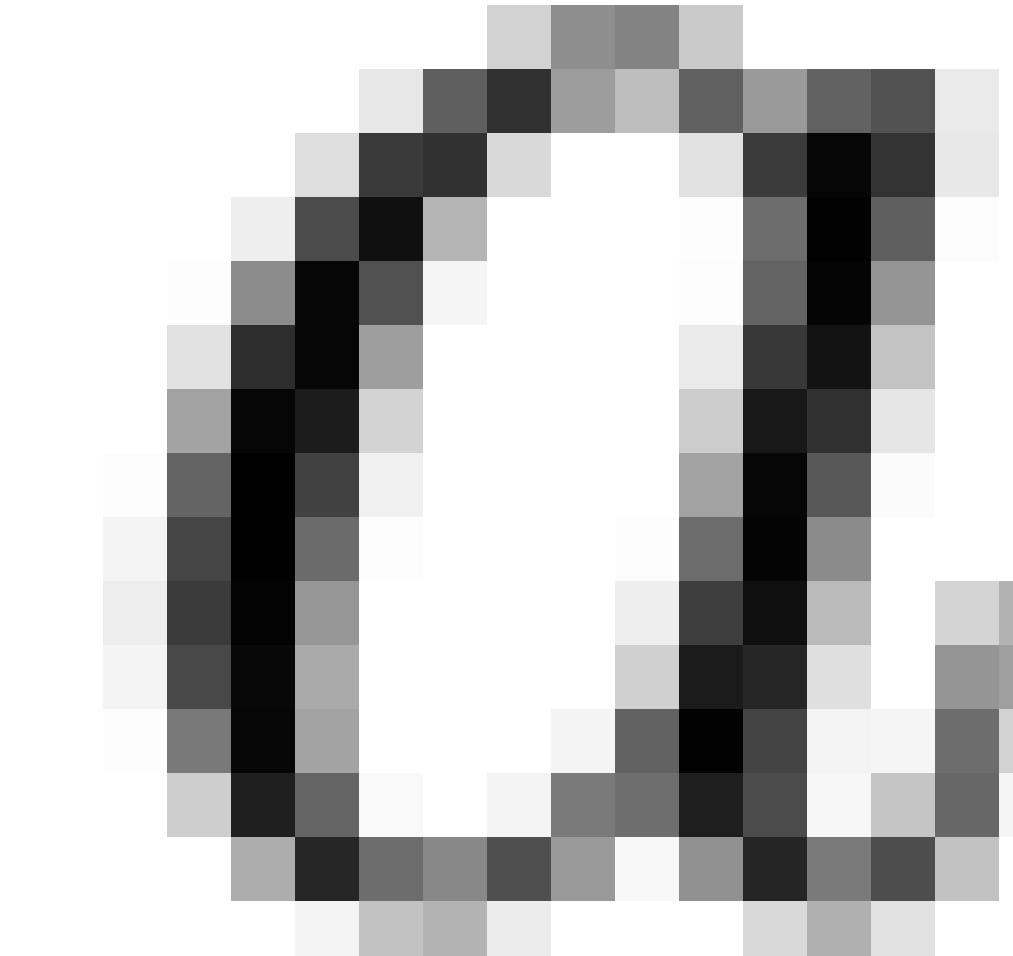
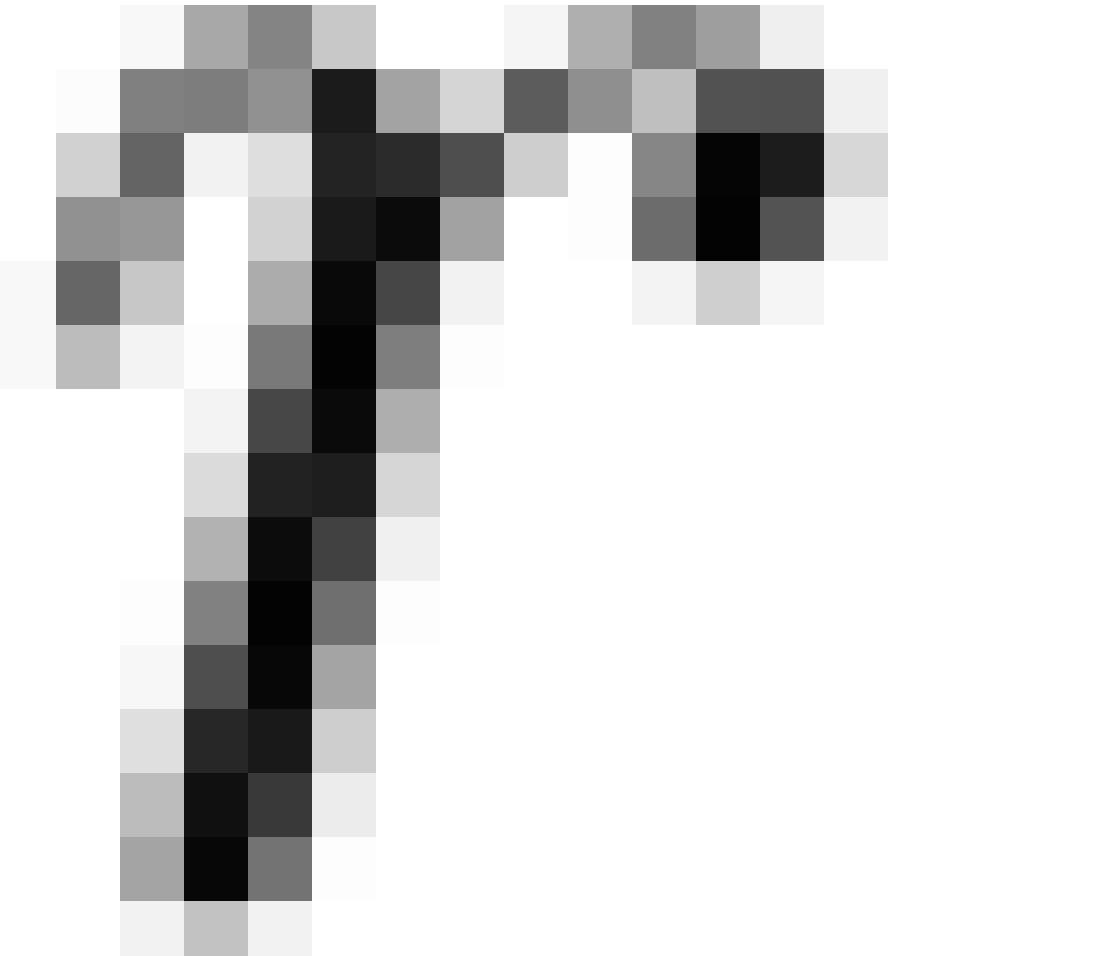






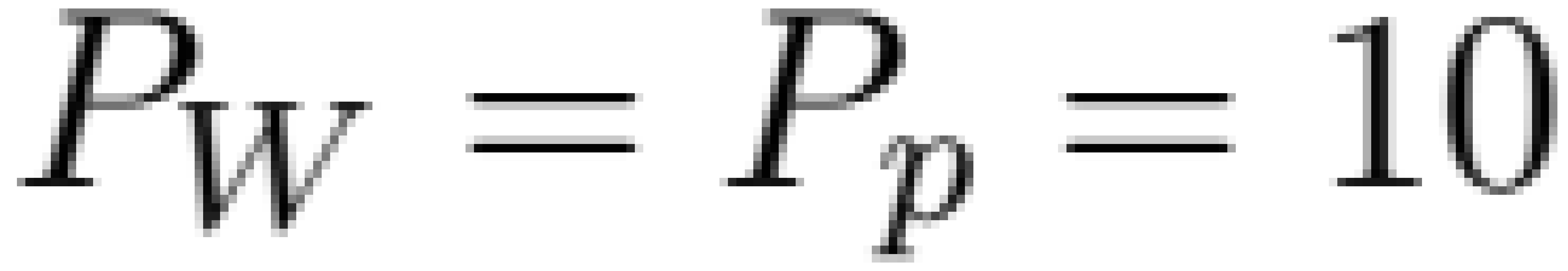


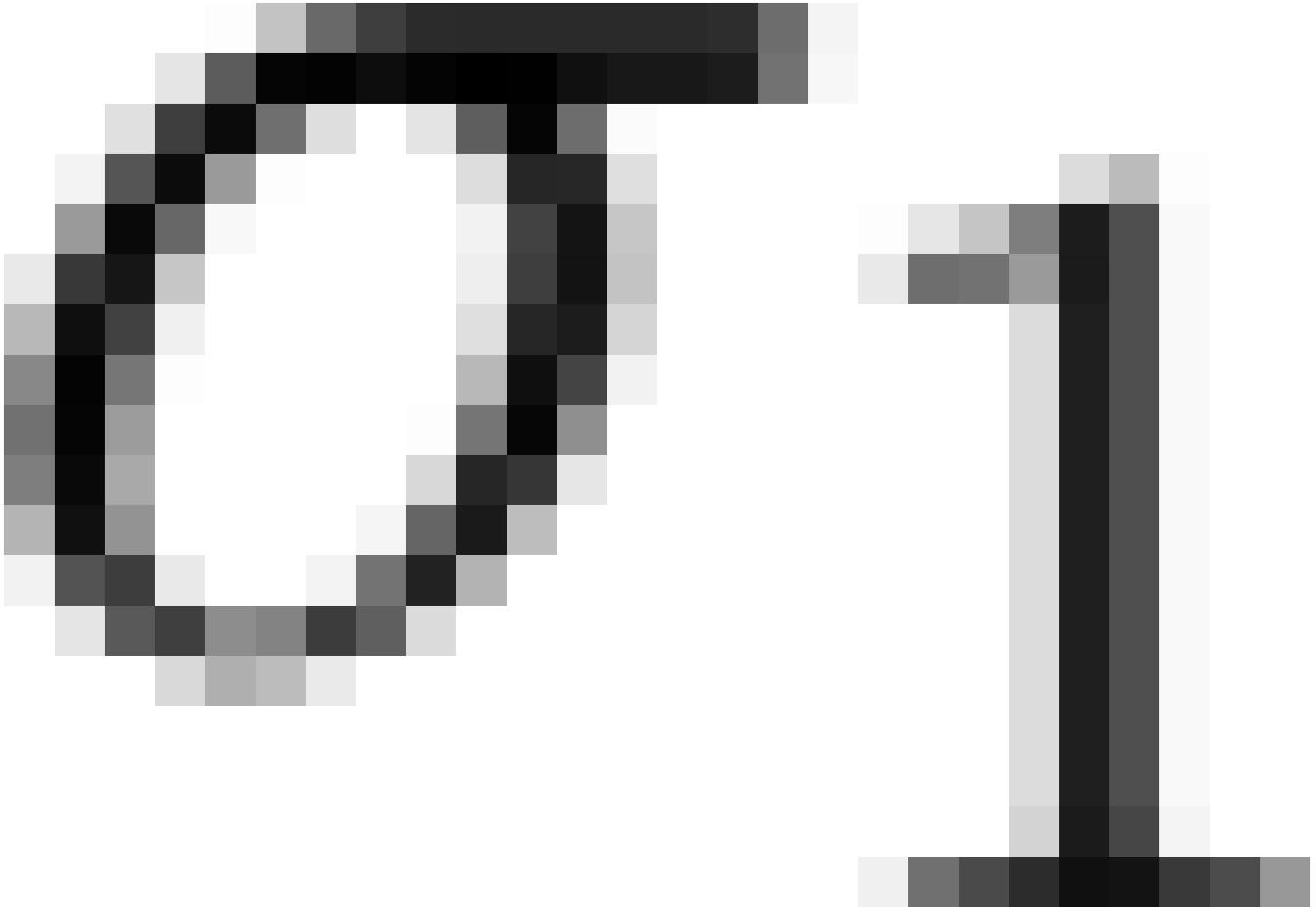
$$\left\{ \begin{array}{lcl} \sigma_{rr} & = & (P_W - P_p) \left(\frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left(1 - \frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{\theta\theta} & = & -(P_W - P_p) \left(\frac{a^2}{r^2} \right) + \frac{\sigma_{Hmax} + \sigma_{hmin}}{2} \left(1 + \frac{a^2}{r^2} \right) - \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 + 3 \frac{a^4}{r^4} \right) \cos(2\theta) \\ \\ \sigma_{r\theta} & = & \frac{\sigma_{Hmax} - \sigma_{hmin}}{2} \left(1 + 2 \frac{a^2}{r^2} - 3 \frac{a^4}{r^4} \right) \sin(2\theta) \\ \\ \sigma_{zz} & = & \sigma_v - 2\nu (\sigma_{Hmax} - \sigma_{hmin}) \left(\frac{a^2}{r^2} \right) \cos(2\theta) \end{array} \right.$$



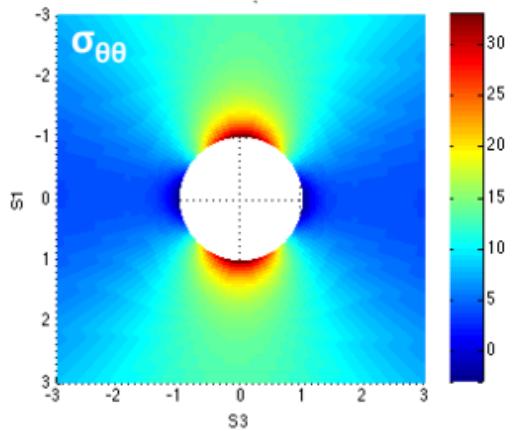
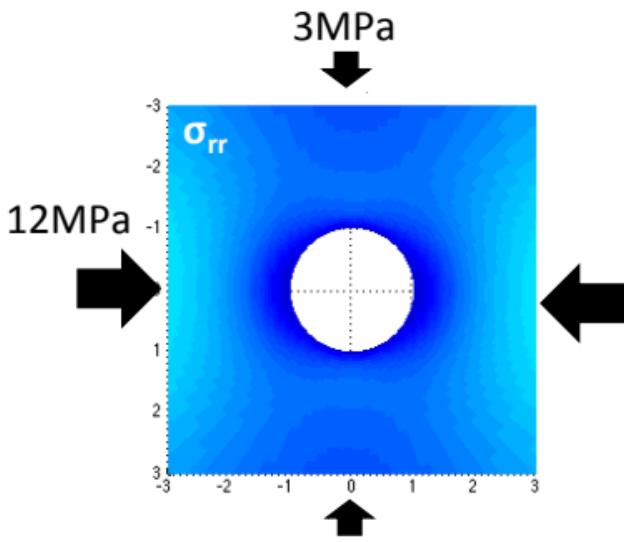




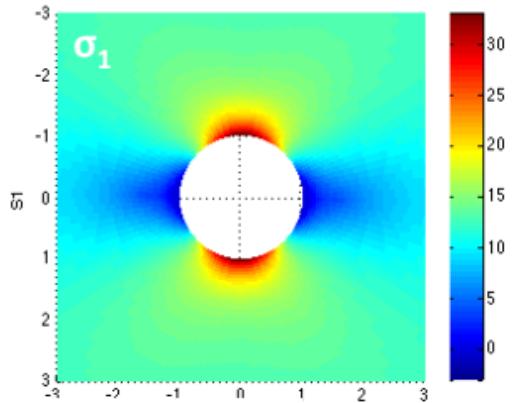
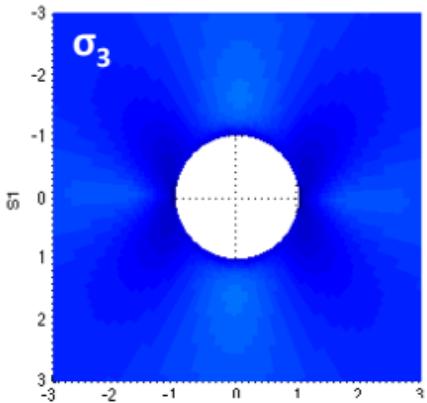


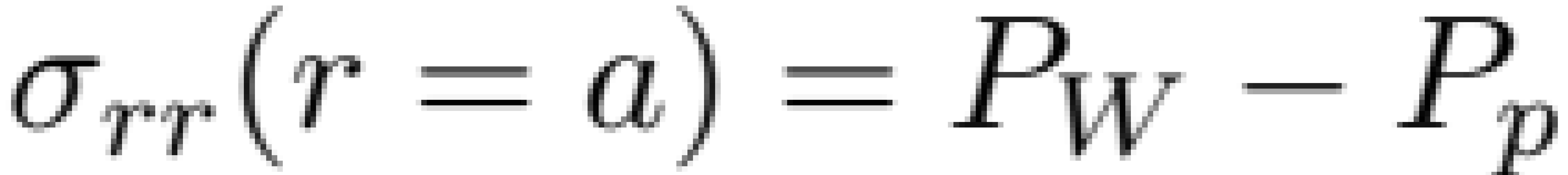


Stresses in cylindrical coordinates



Principal Stresses





$$\sigma_{\text{H}\alpha\alpha} = \sigma_{\text{H}\alpha\alpha}^{\text{max}} + \sigma_{\text{H}\alpha\alpha}^{\text{min}} \left(\frac{P_{\text{p}}}{P_{\text{p}} + P_{\text{d}}} \right)^2 \left(\frac{1 - \cos(2\theta)}{2} \right) \left(\frac{1 + \cos(2\theta)}{2} \right)^2 \left(\frac{1 + \cos(2\theta)}{2} \right)$$

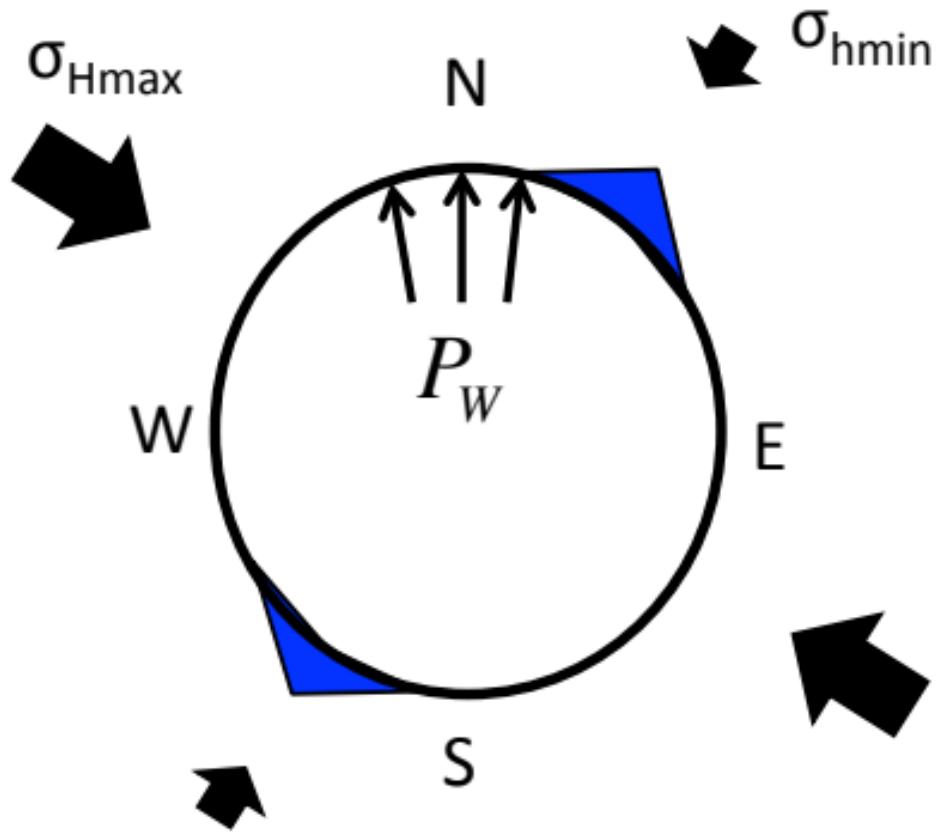
$$\begin{aligned} \sigma_{\theta\theta}(r=a, \theta=0) &= (P_W - P_p) + 3\sigma_{h\min} \\ \sigma_{\theta\theta}(r=a, \theta=\pi/2) &= -(P_W - P_p) + 3\sigma_{h\max} \end{aligned}$$







$$\left\{ \begin{array}{l} \sigma_1 = \sigma_{\theta\theta} = - (P_W - P_p) + 3 \sigma_{H\max} - \sigma_{hmin} \\ \sigma_3 = \sigma_{rr} = (P_W - P_p) \end{array} \right.$$



Stress anisotropy

$$P_{W\text{shear}} = P_p + \frac{3\sigma_{H\max} - \sigma_{h\min} - UCS}{1+q}$$

Pore pressure in the formation

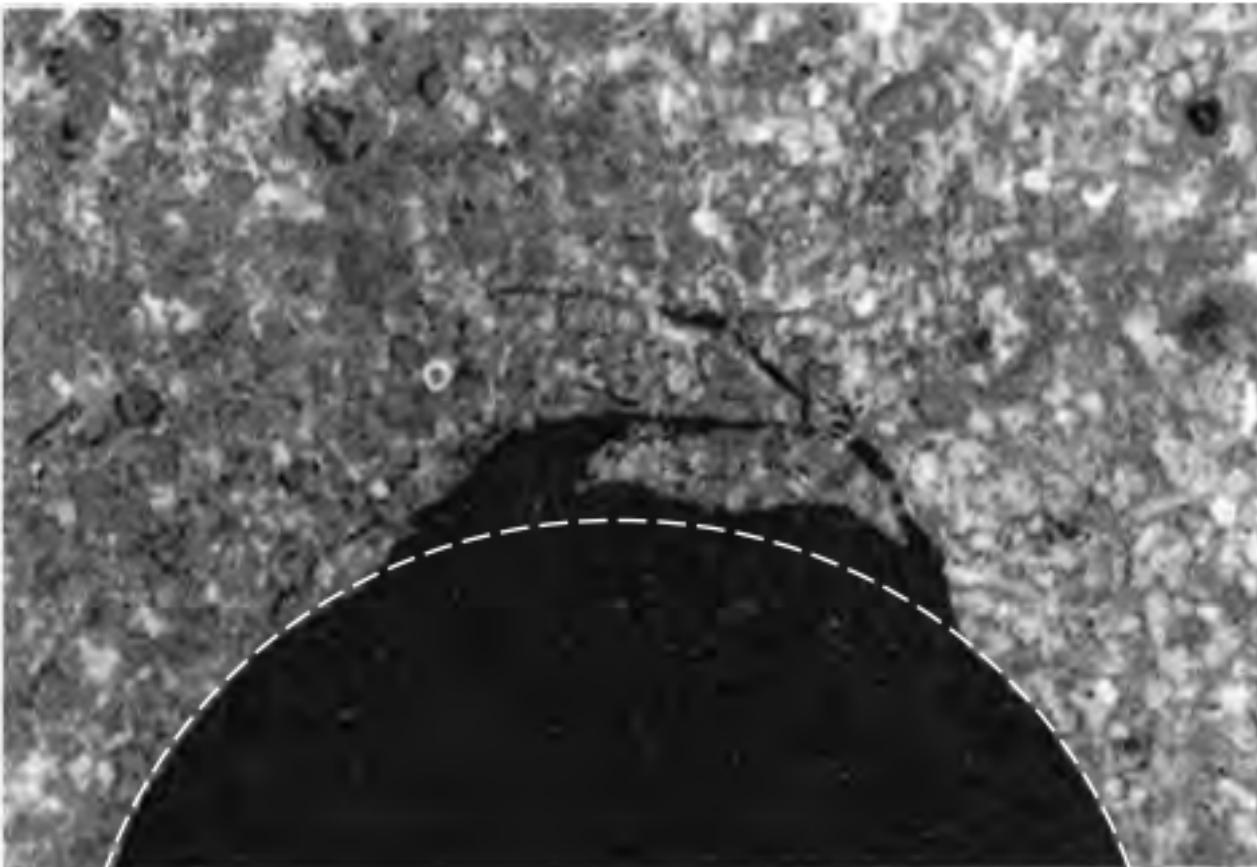
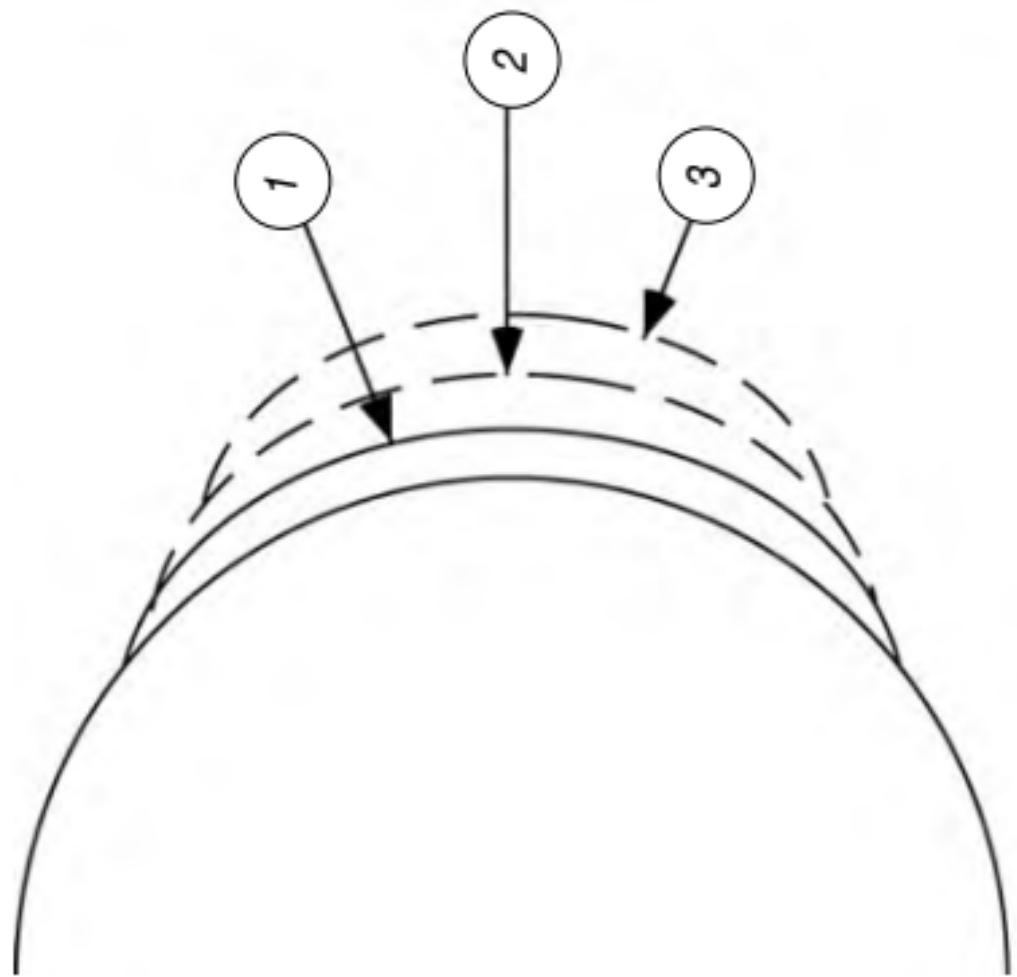
Shear strength

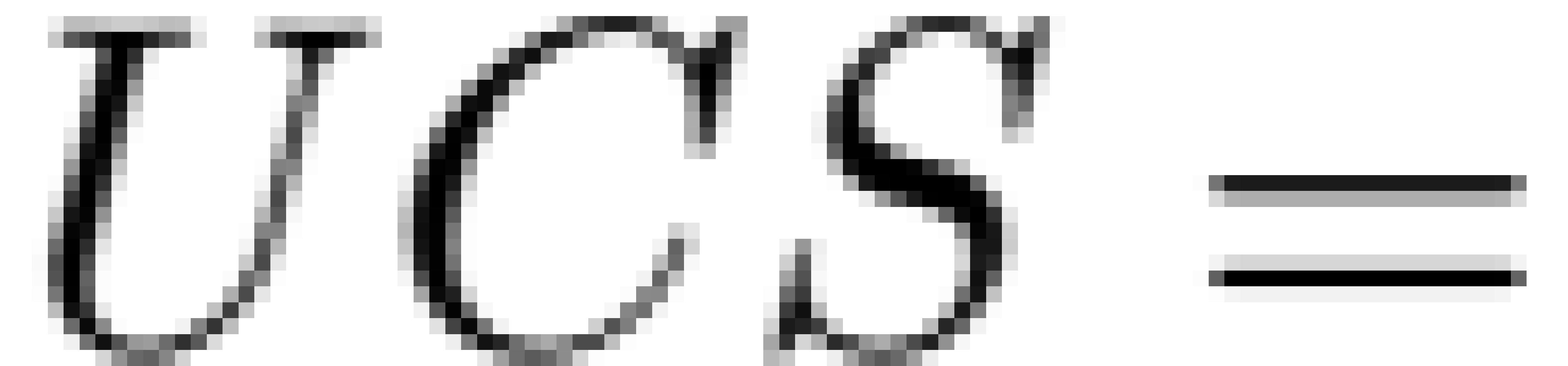
$P_W \leq P_{W\text{shear}}$ leads to breakouts

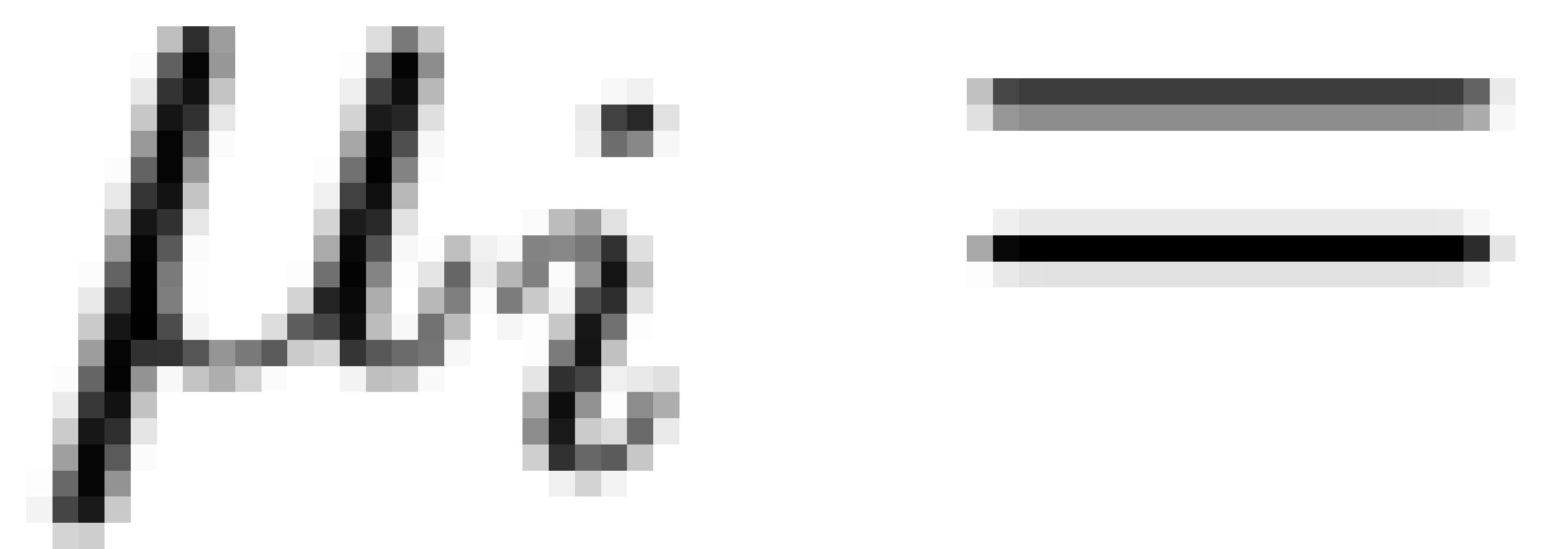


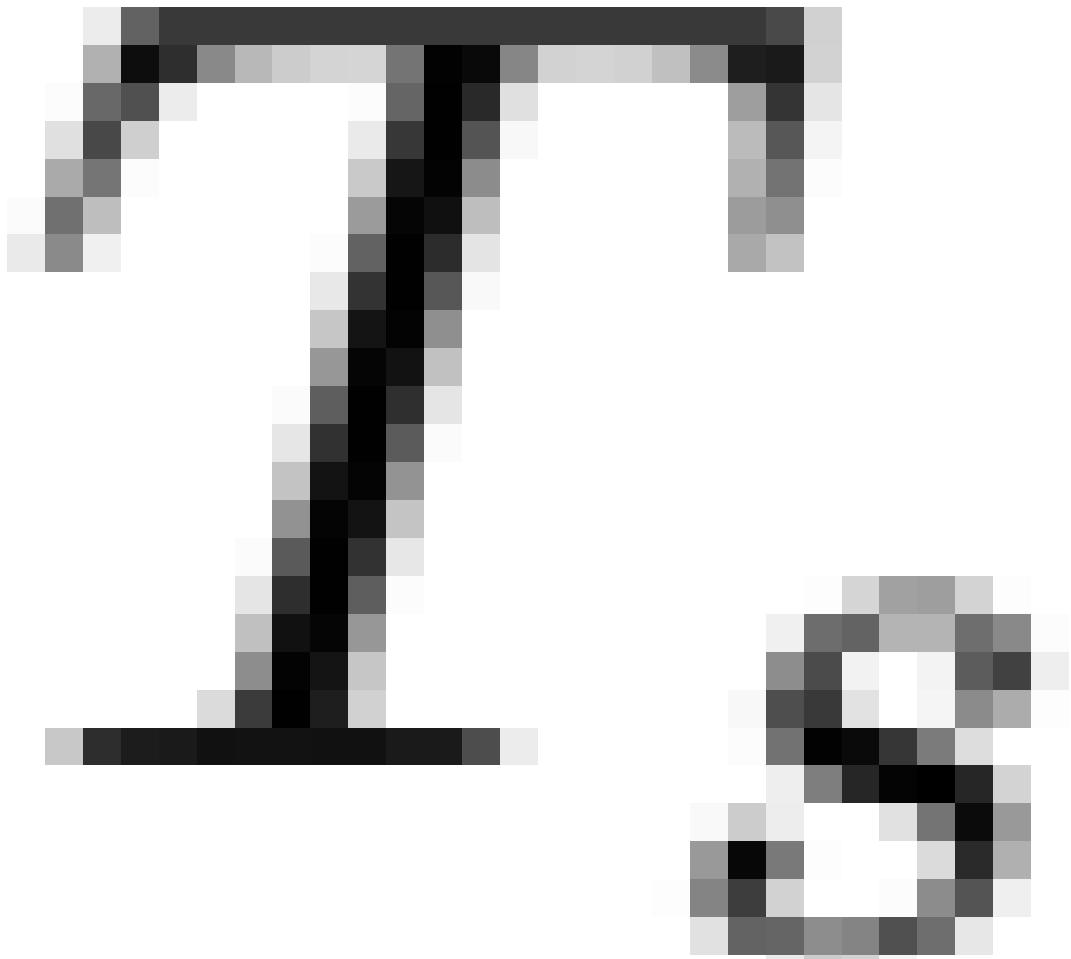
[P] + [P] → [C] + [P]

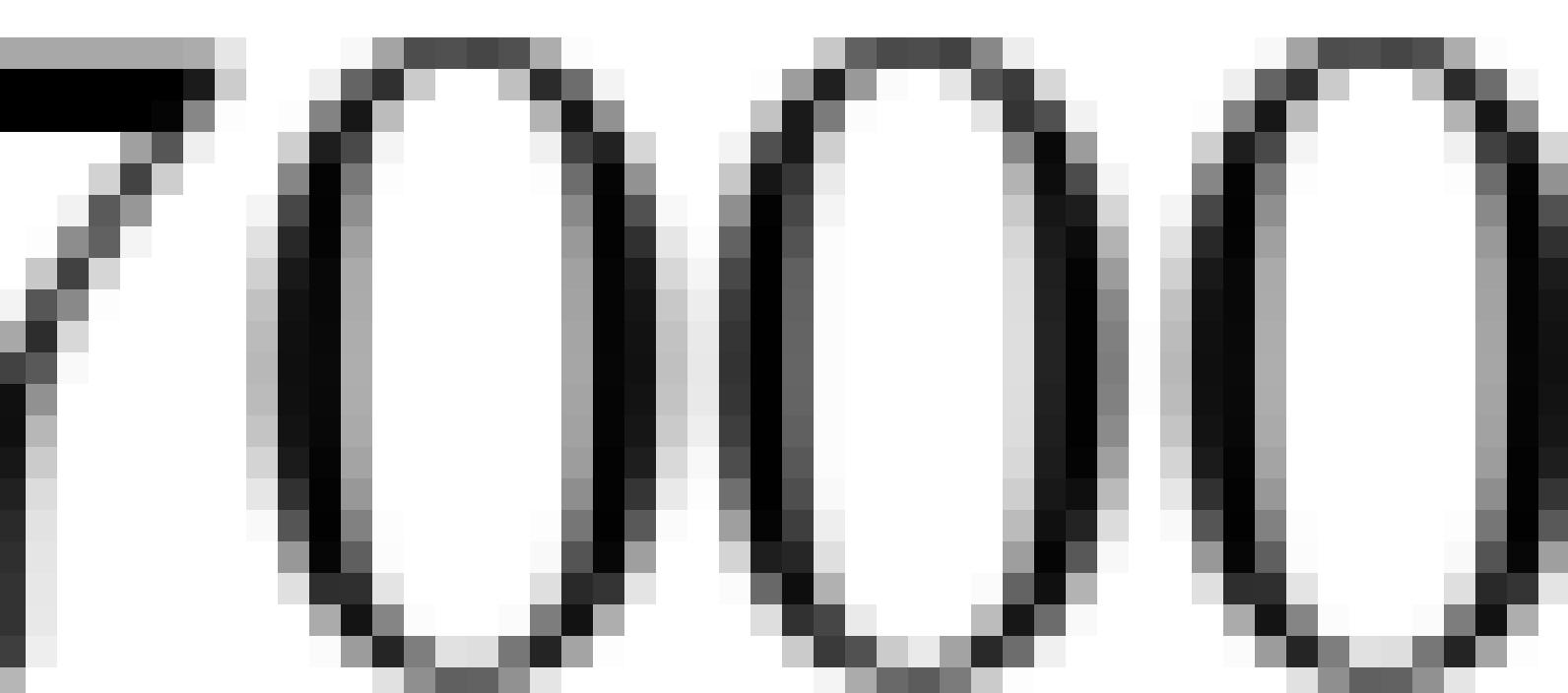
$$P_{W\text{ shear}} = \frac{3\sigma_H \max - \sigma_{hmin} - U_{CS}}{1 + q}$$

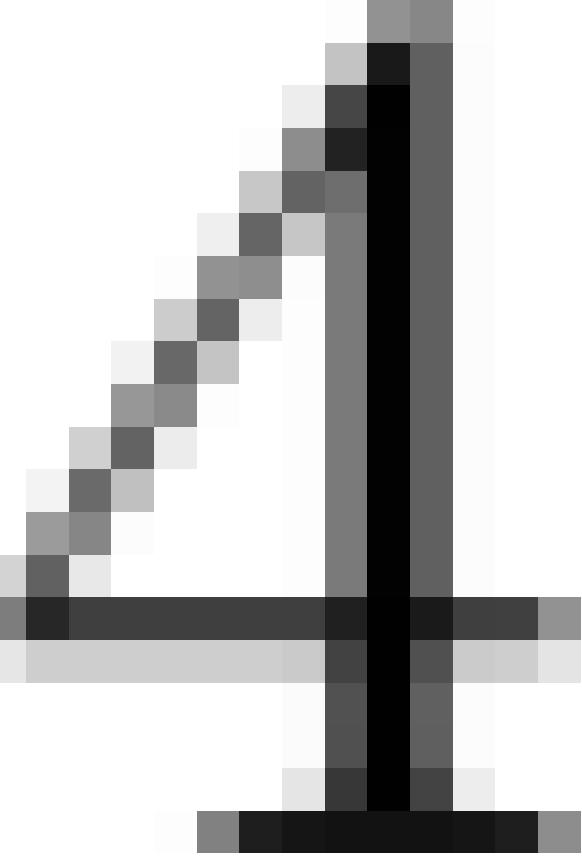
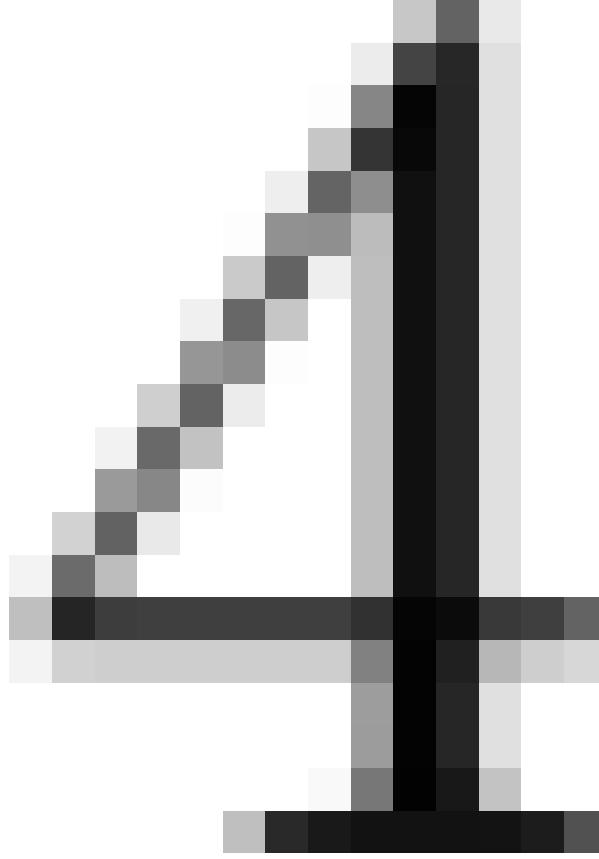
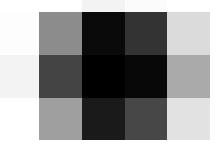
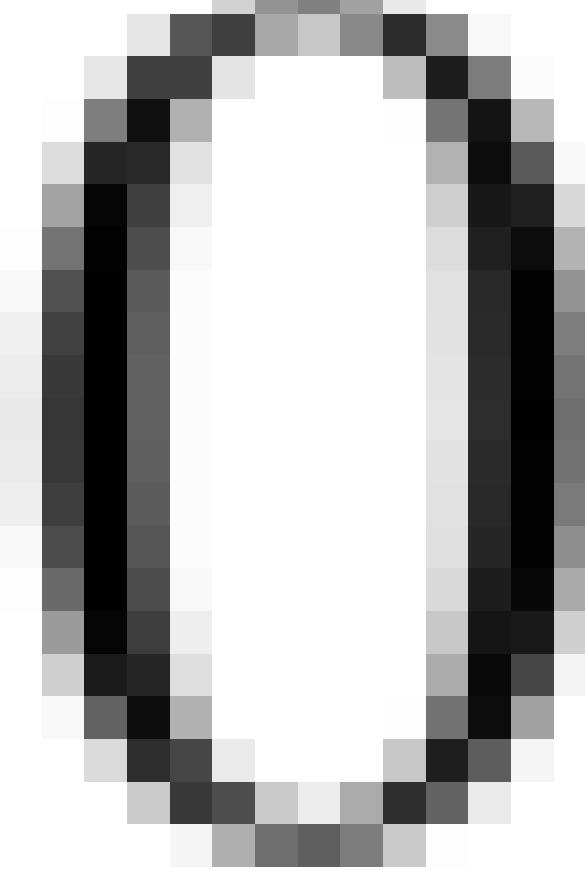
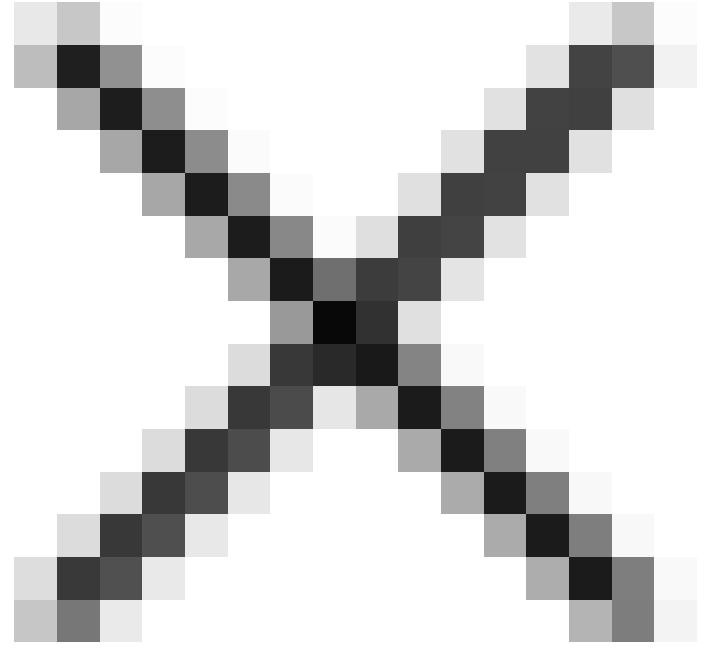


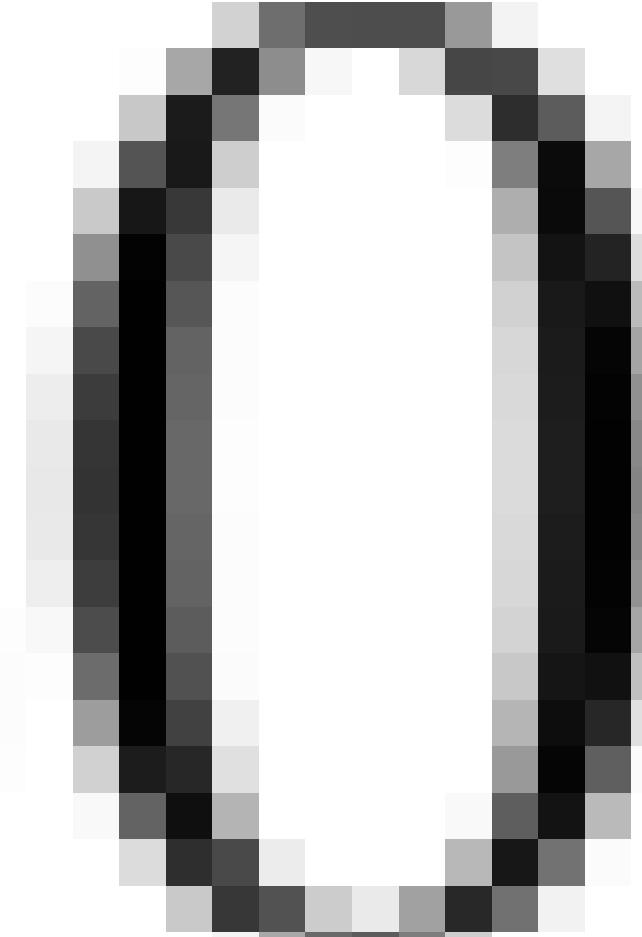
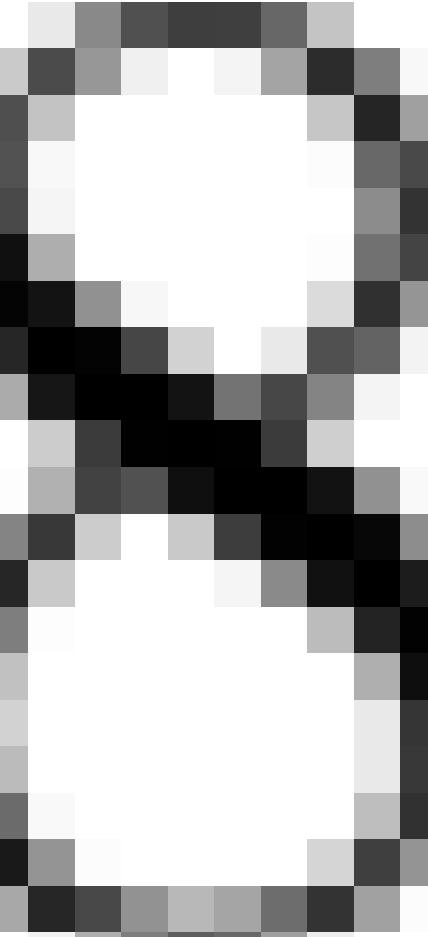
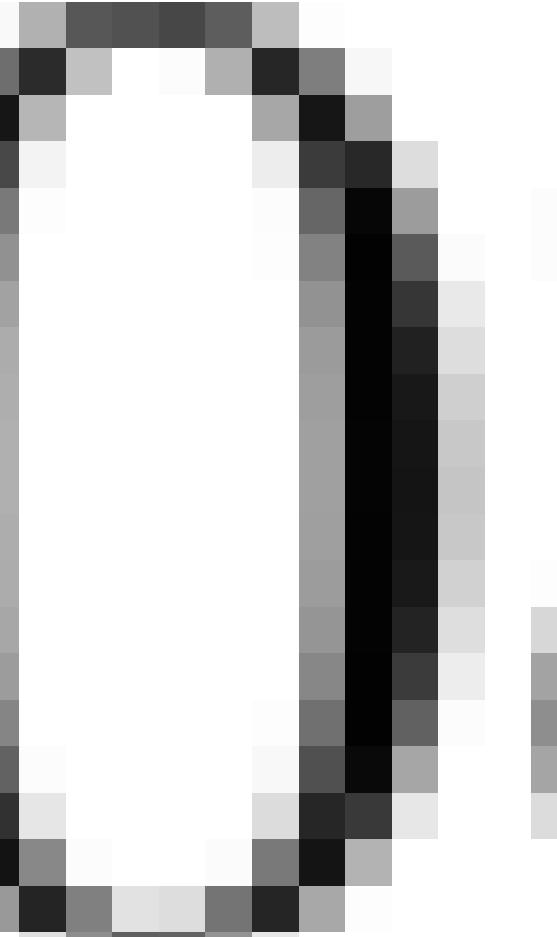
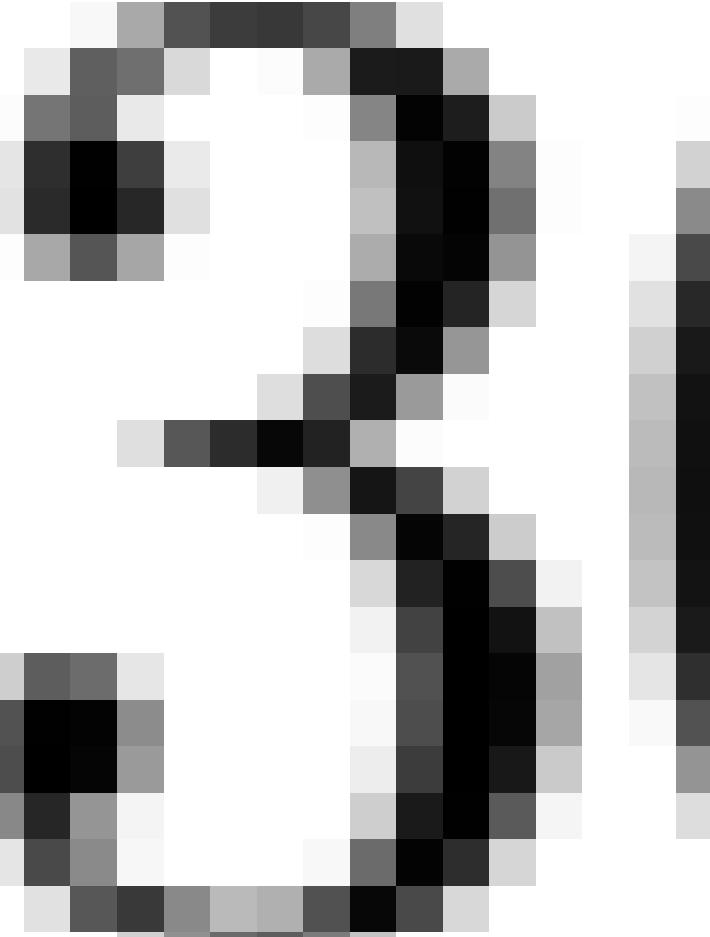




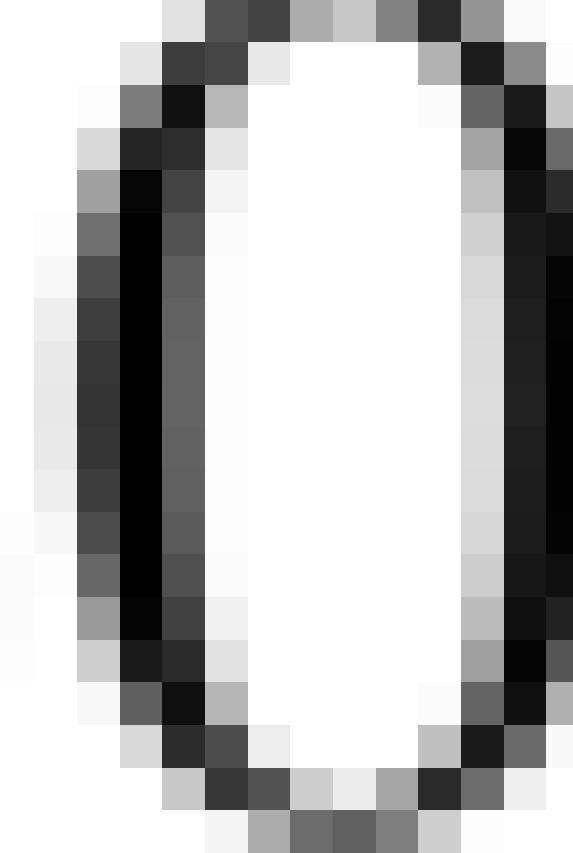
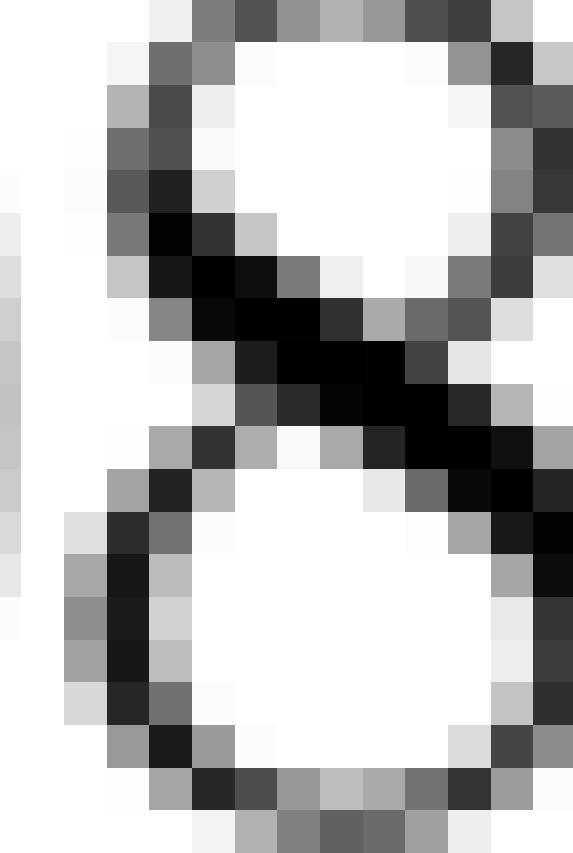
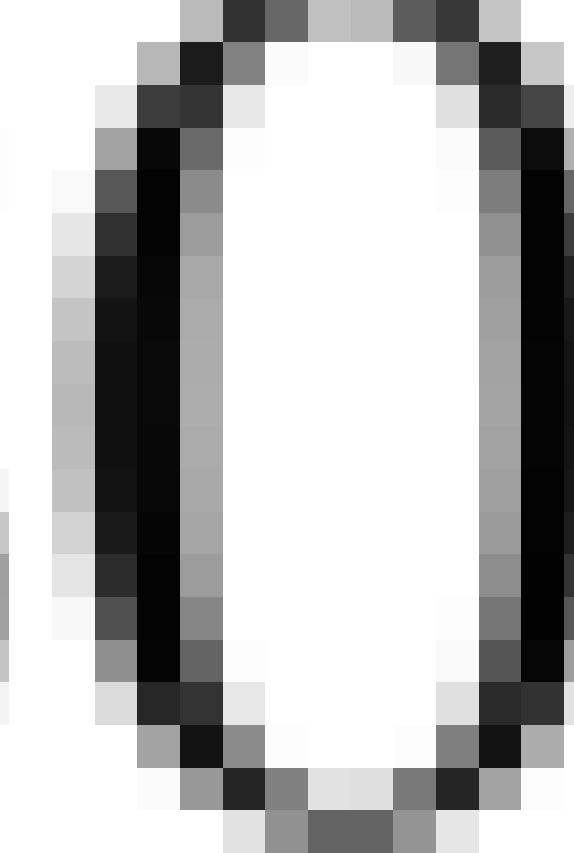
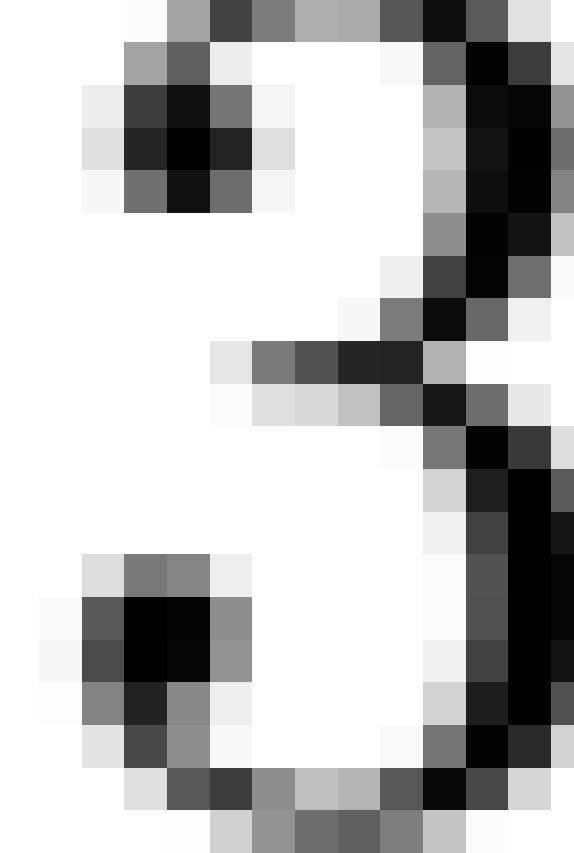


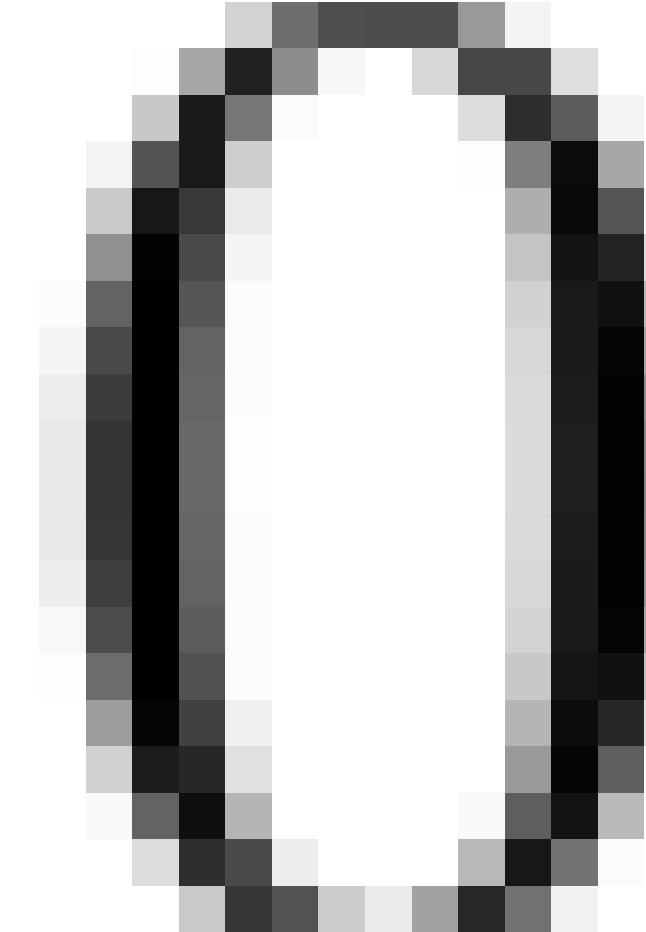
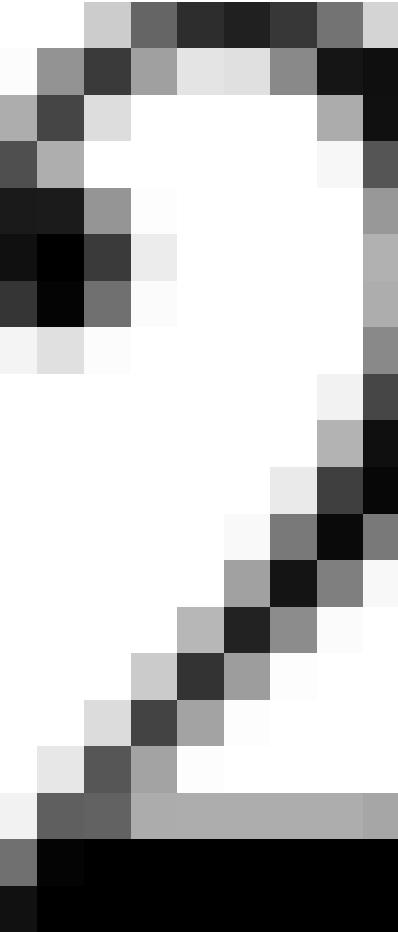
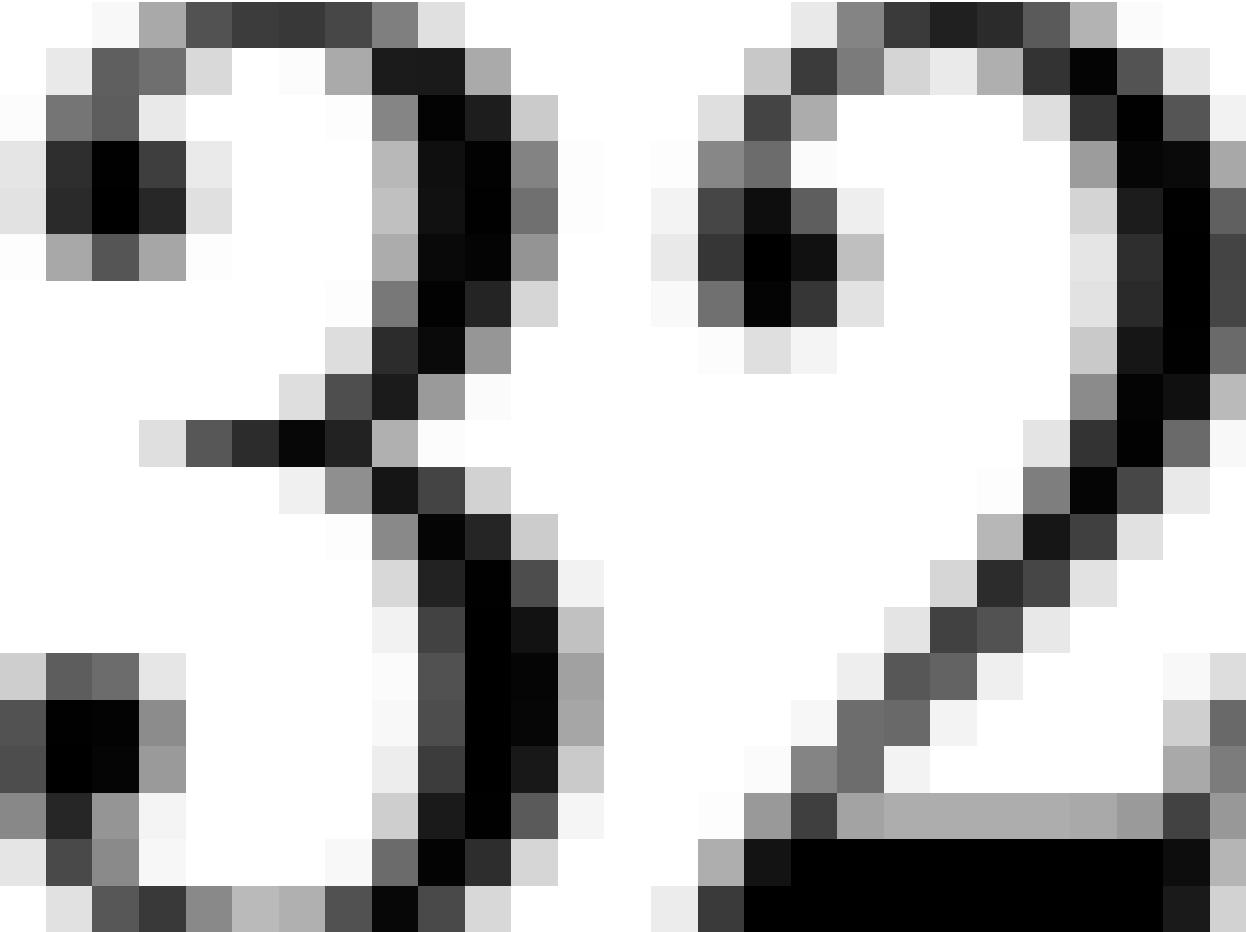


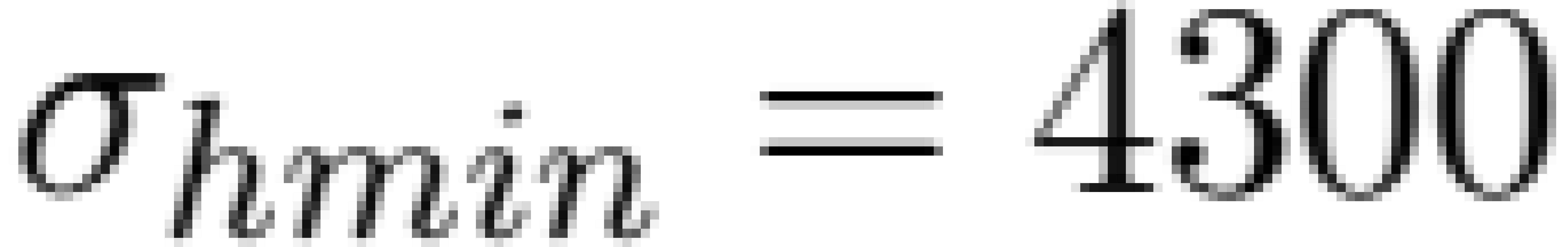


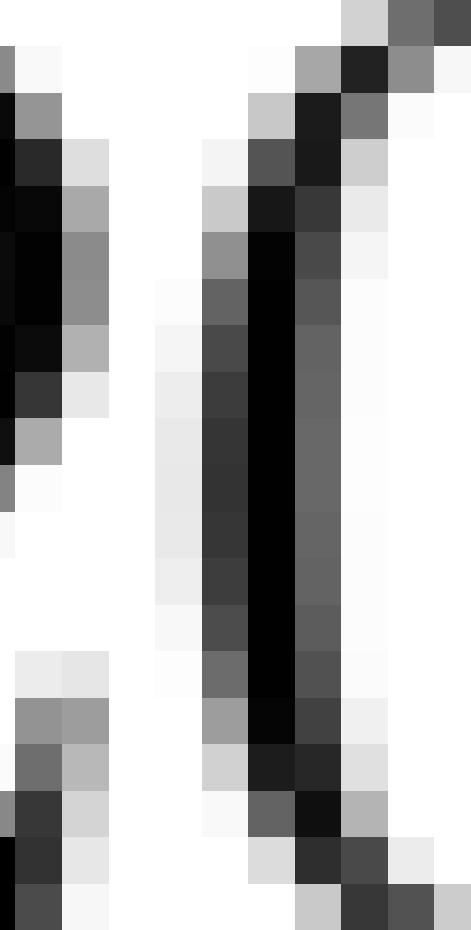
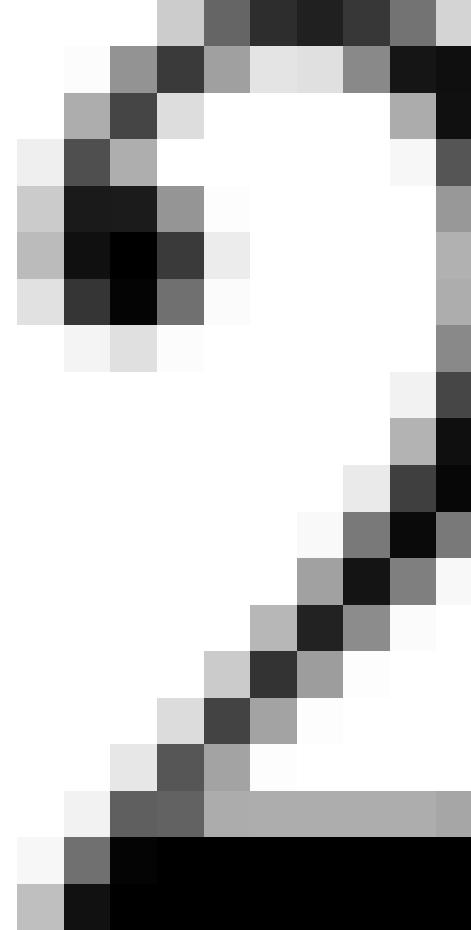
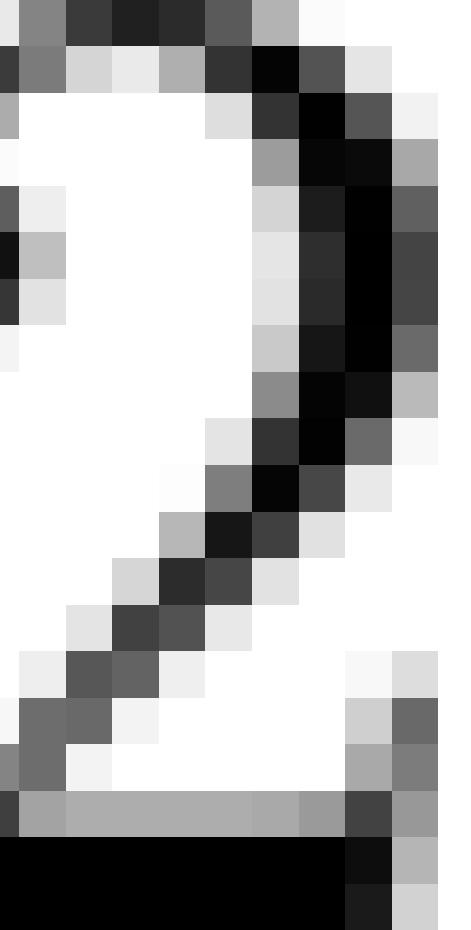














9 2

1

$$1 + \sin 30.96^\circ$$

$$- \sin 30.96^\circ$$

—

3.12



$$\frac{3 \times 3220 \text{ psi} - 1220 \text{ psi}}{1 + 3.12} = 4279 \text{ psi}$$

4279 psi

7000 ft

X

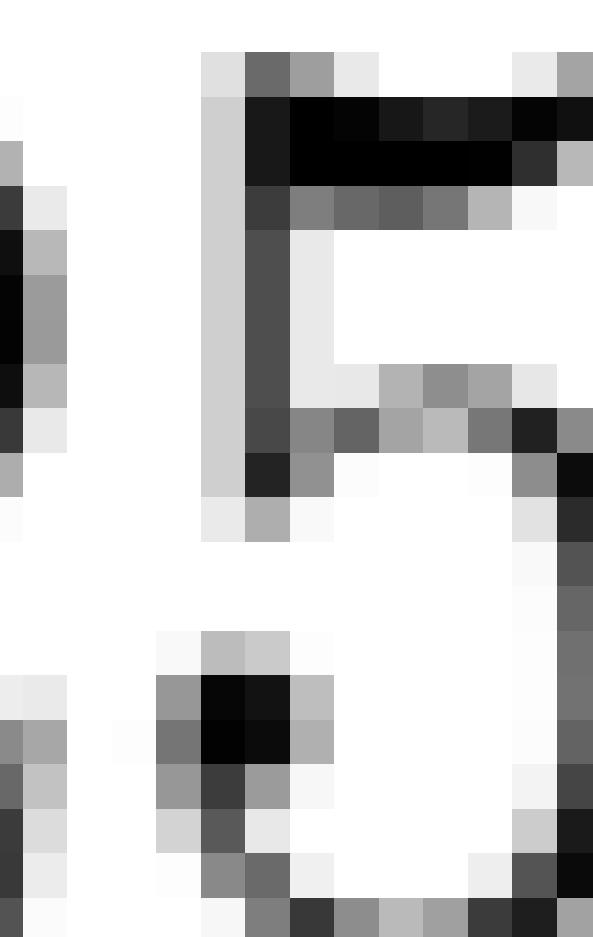
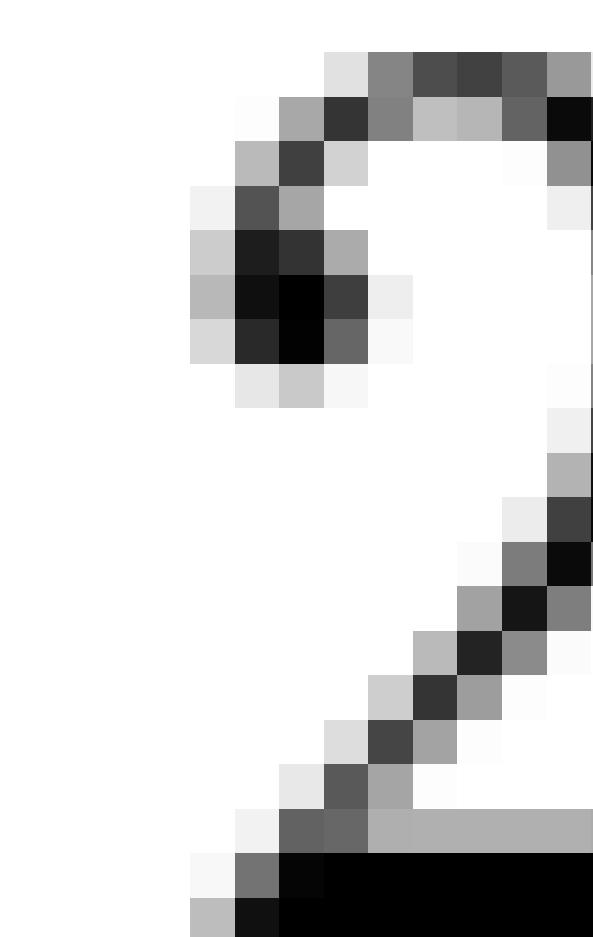
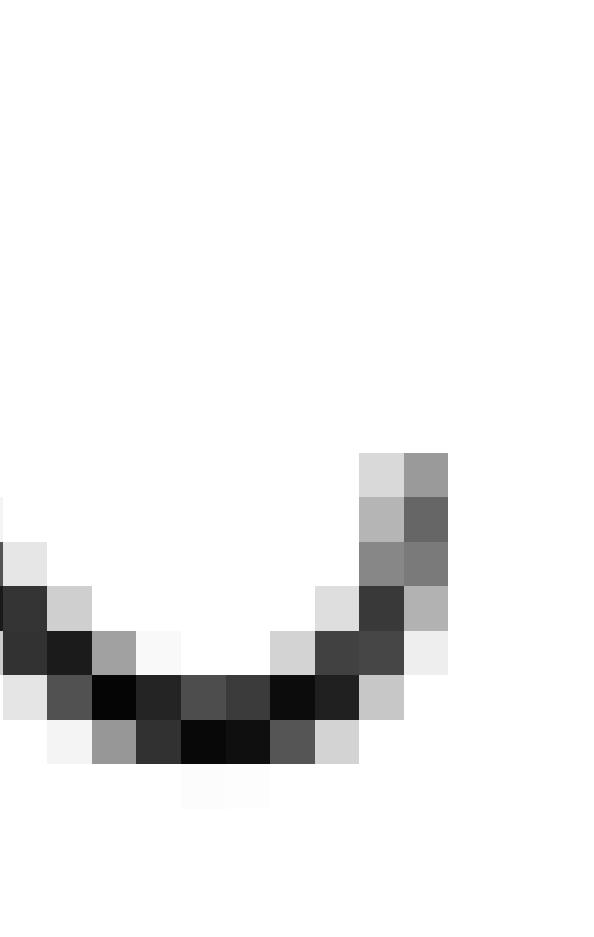
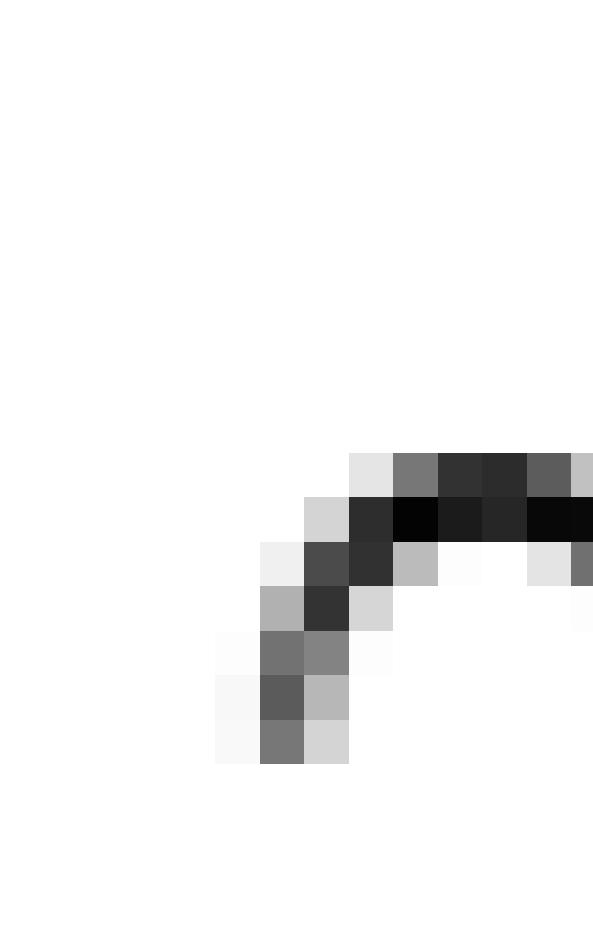
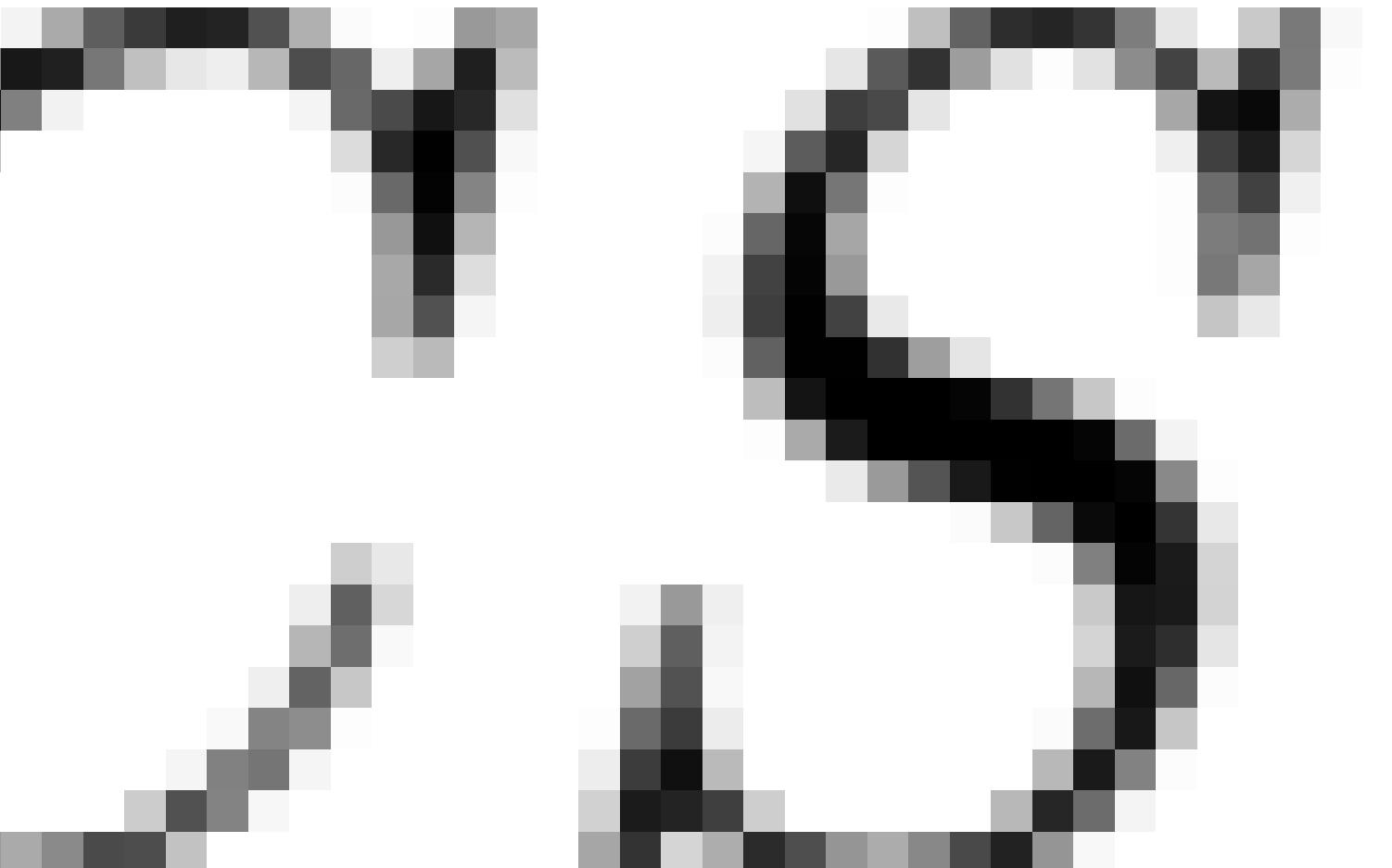
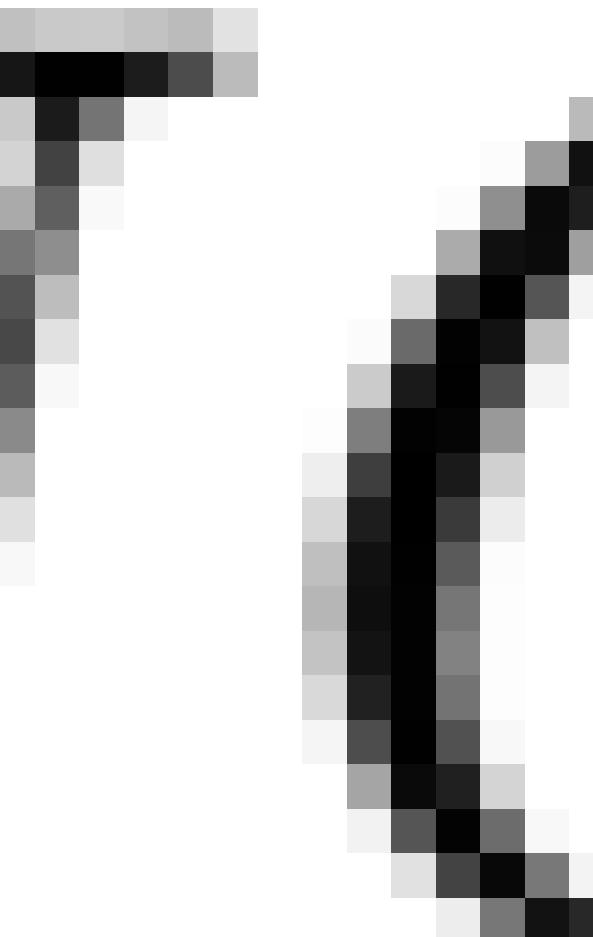
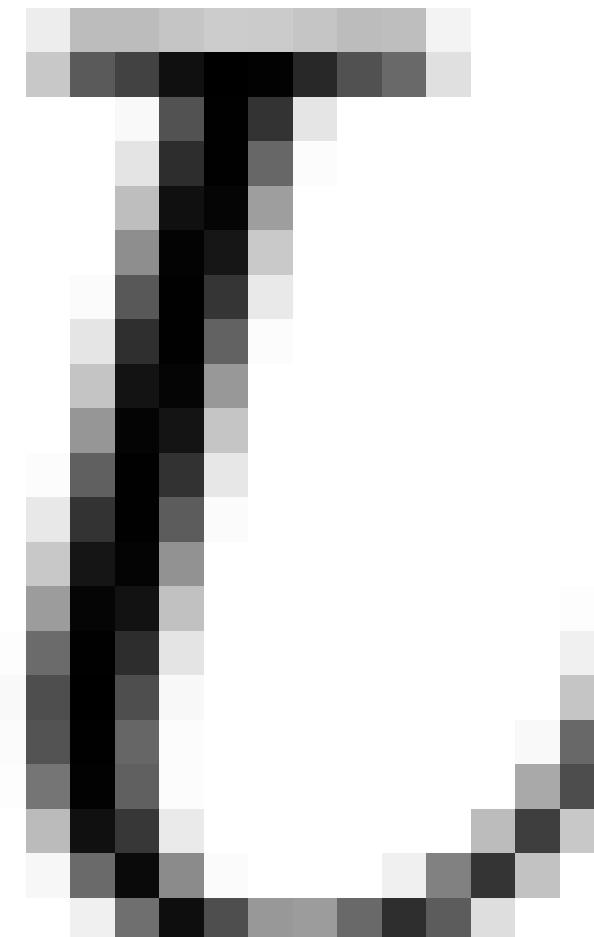
8.3 ppg

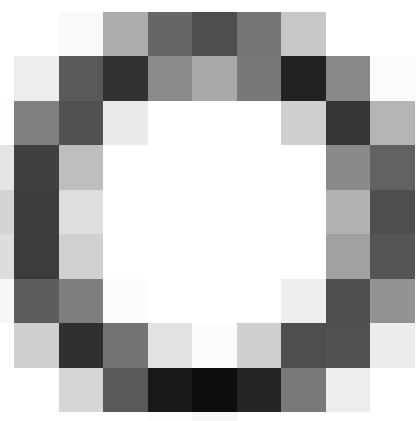
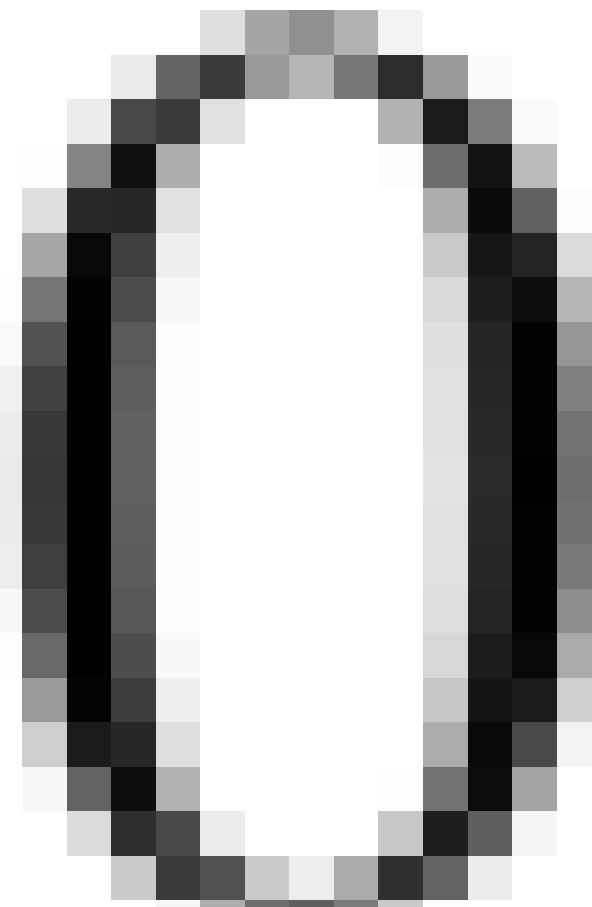
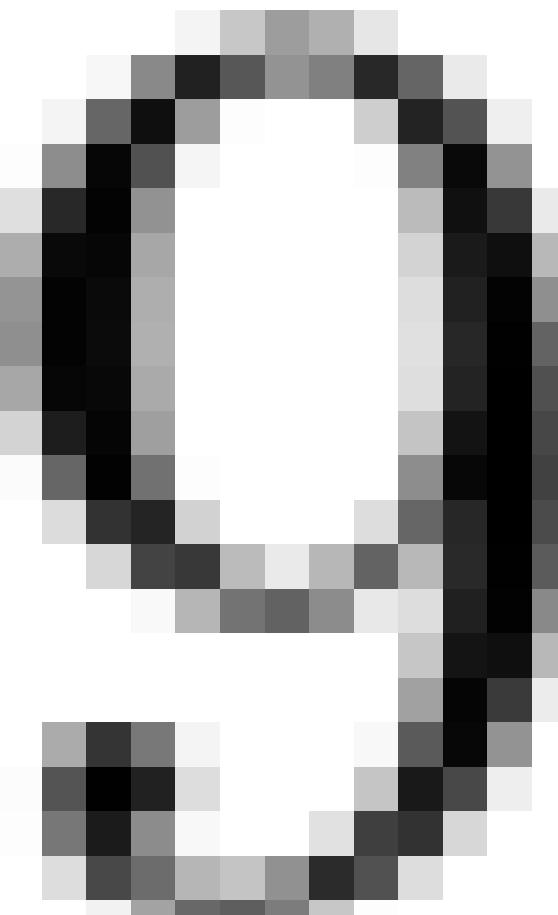
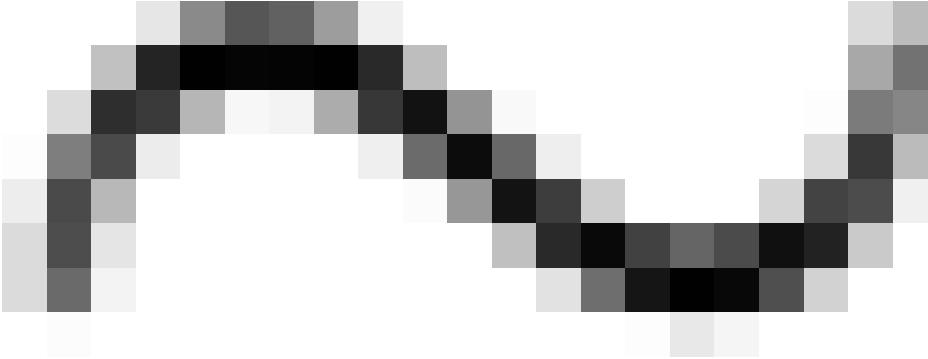
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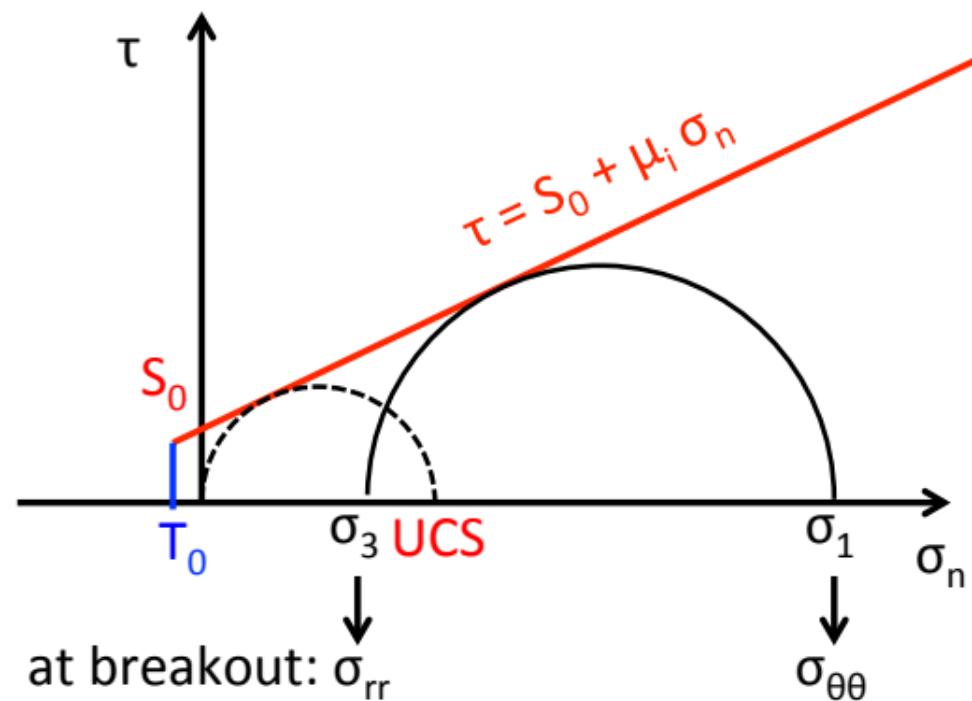
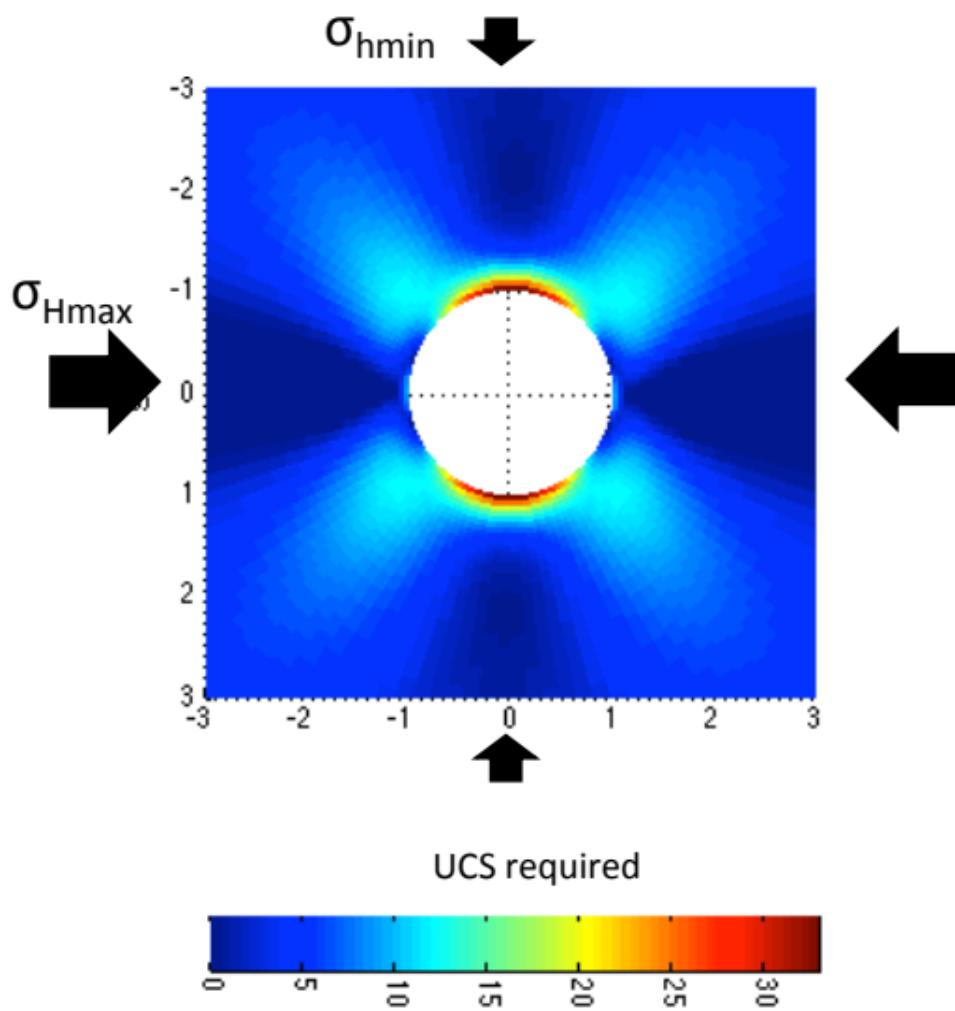
11.57 ppg

—

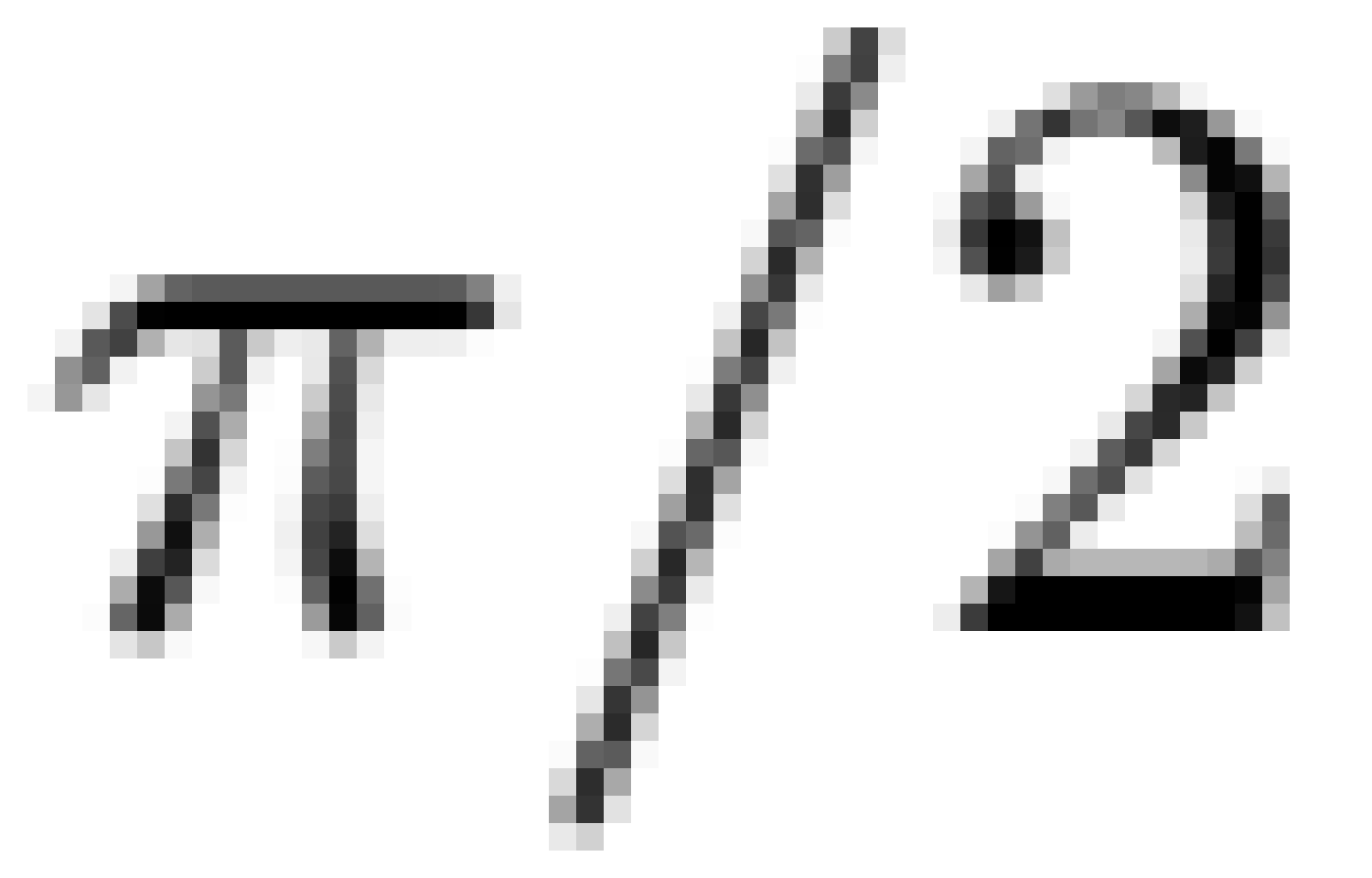
0.44 psi/ft

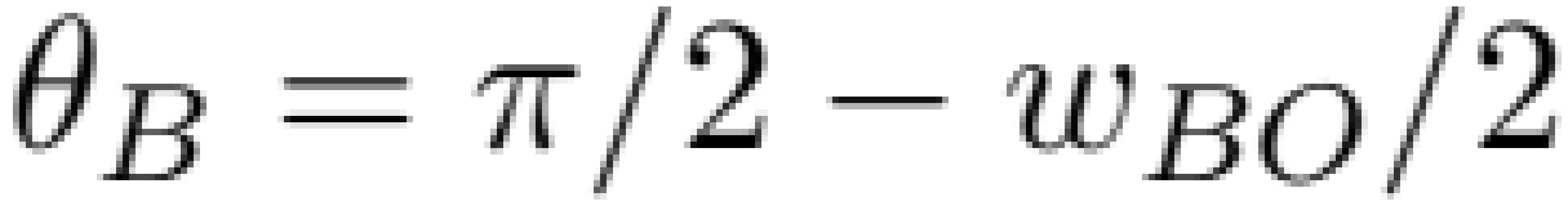




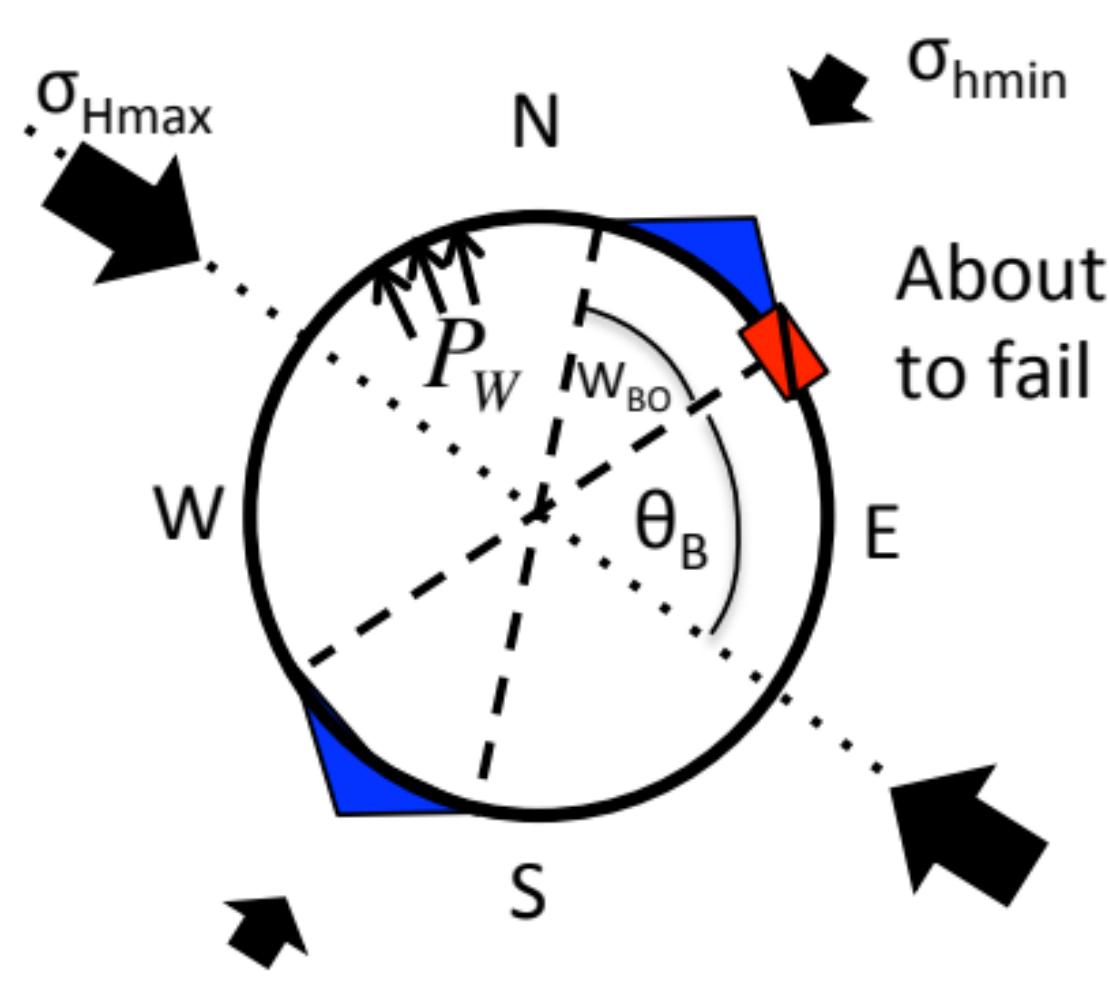








$$\begin{aligned} \sigma_{\theta\theta} &= (P_W - P_p) + 2(\sigma_{H\max} - \sigma_{h\min}) \cos(2\theta_B) \\ \sigma_{rr} &= + (P_W - P_p) \end{aligned}$$

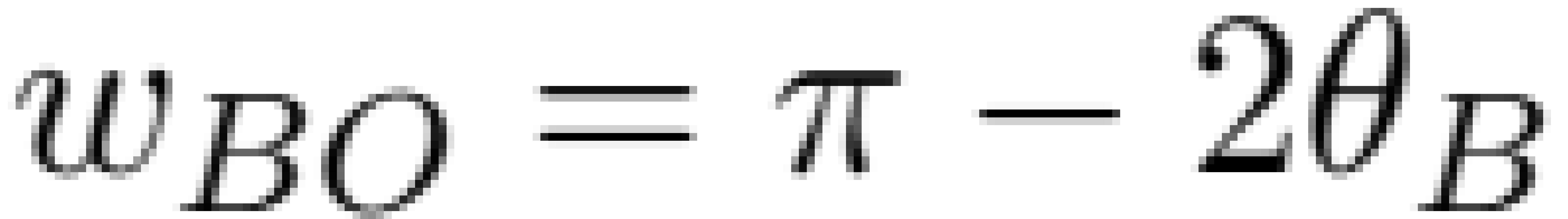


About
to fail



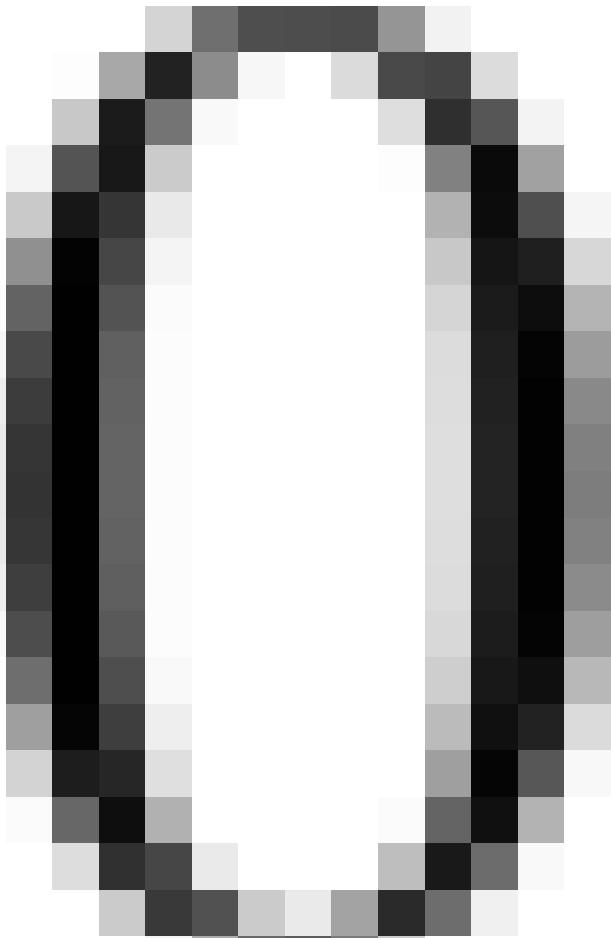
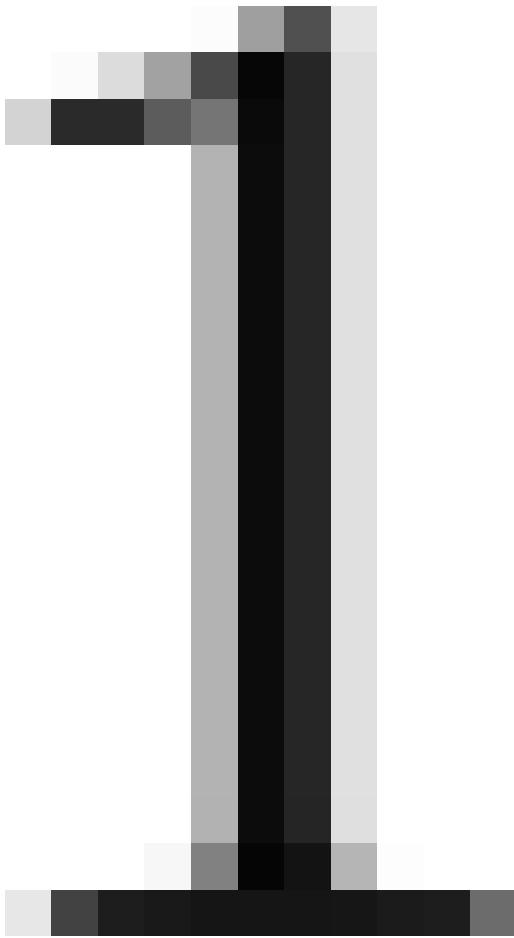
$$S_{\text{H}} = \frac{1}{2} \left[\sigma_{W_{\text{max}}}^2 + \sigma_{H_{\text{max}}}^2 \right] \cos(2\theta) + \sigma_{H_{\text{max}}}^2 \sin(2\theta) \cos(\phi_{H_{\text{max}}}) + \sigma_{W_{\text{max}}}^2 \sin(2\theta) \sin(\phi_{H_{\text{max}}})$$

$$2\theta_B = \arccos \left[\frac{\sigma_{Hmax} - \sigma_{hmin} - (1+q)(P_W - P_p)}{2(\sigma_{Hmax} - \sigma_{hmin})} \right]$$





$$P_{WBO} = P + \frac{(\sigma_{Hmax} - \sigma_{hmin}) \cos(\pi - w_{BO}) - Ucs}{1 + q}$$

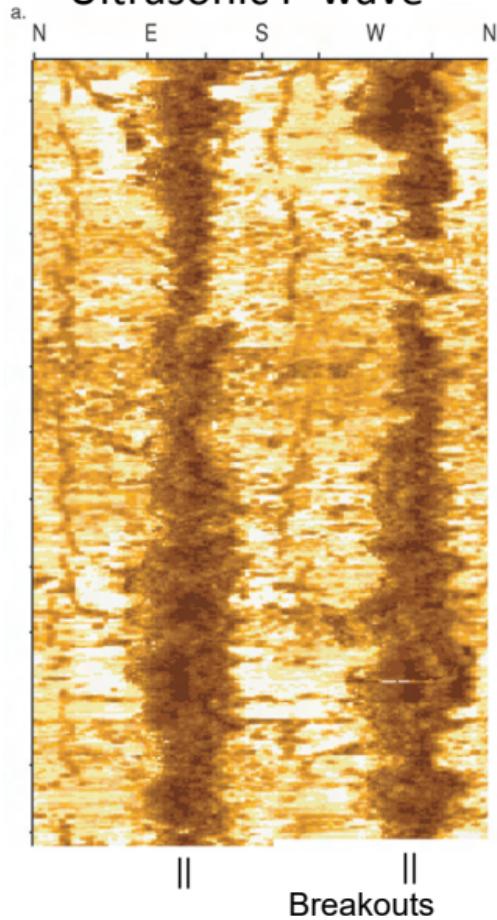


$$0.44 \text{ psi/ft} \times 8.3 \text{ PPG}$$

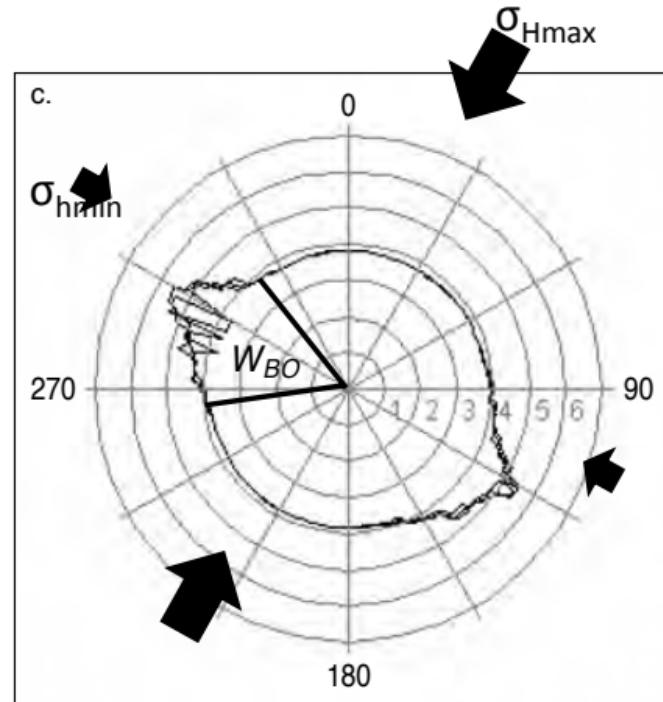
$$\times 7000 \text{ ft} = 3710 \text{ psi}$$

$$w_{BO}^{\circ} = 180^{\circ} - \arccos \frac{[3220 \text{ psi} + 1220 \text{ psi} - (1 + 3.12)(3710 \text{ psi} - 3080 \text{ psi})]}{2(3220 \text{ psi} - 1220 \text{ psi})} = 66^{\circ}$$

Ultrasonic P-wave

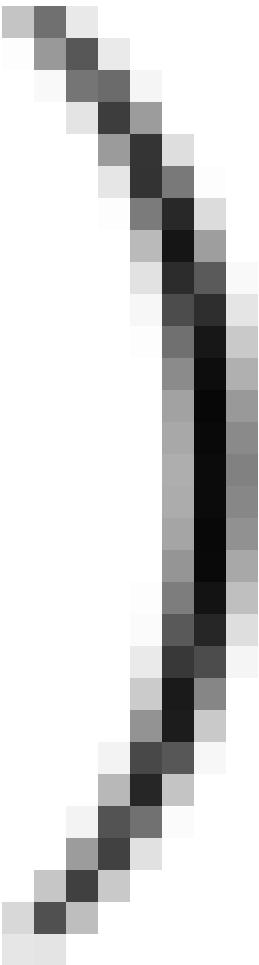
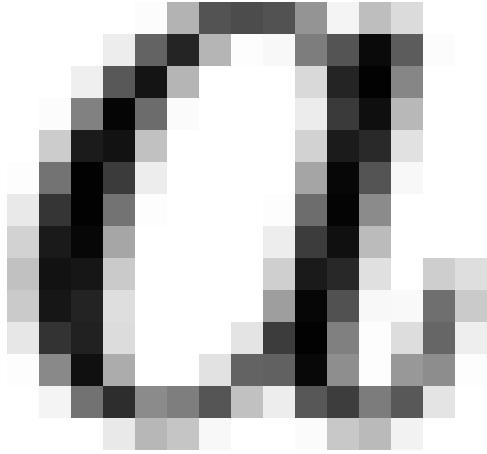
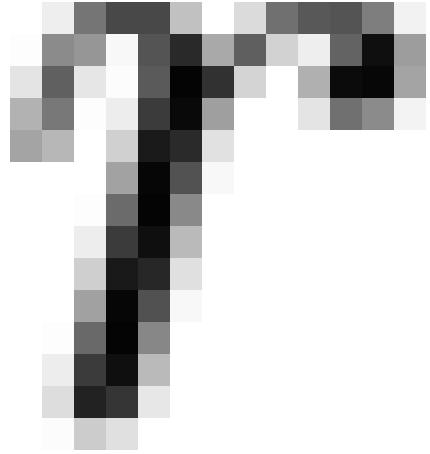
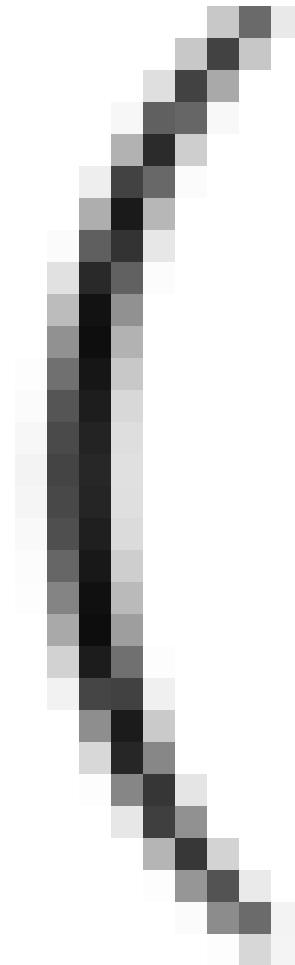


Electrical resistivity



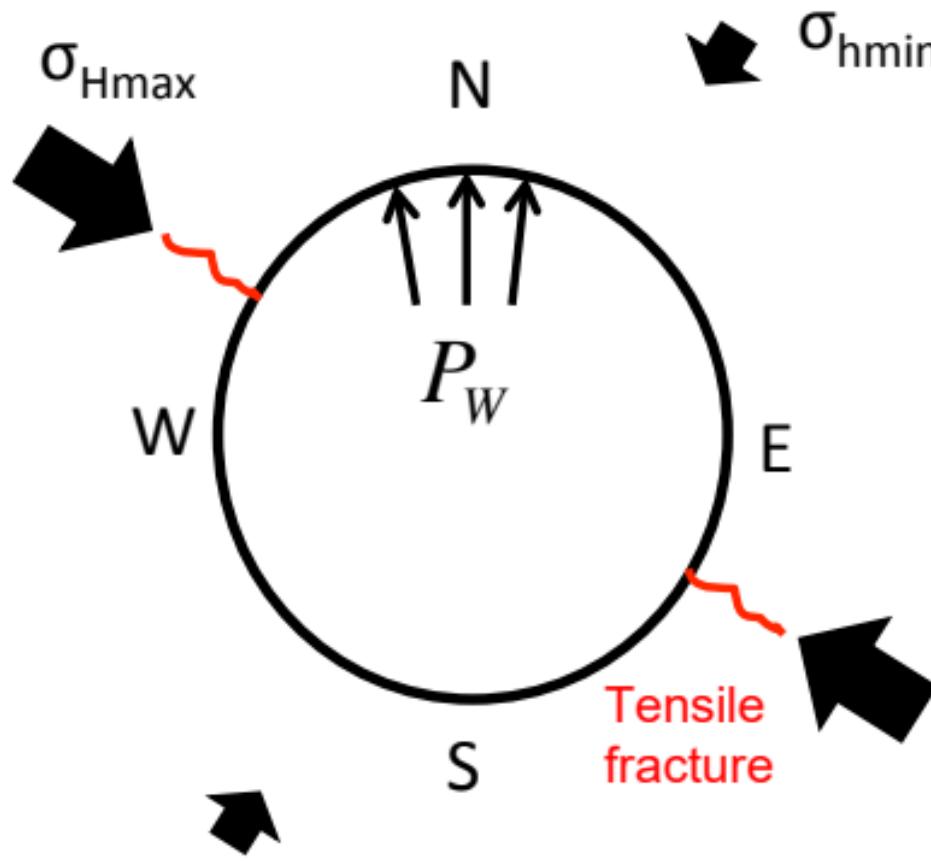
[Zoback 2013 - Figure 6.4]

$$S_{H\max} = \frac{P}{P} + \frac{UCS + (1+q)(P_W - P_p) - \sigma_{hmin}[1 + 2\cos(\pi - w_{BO})]}{1 - 2\cos(\pi - w_{BO})}$$



σθαντικός + σούχος + στ



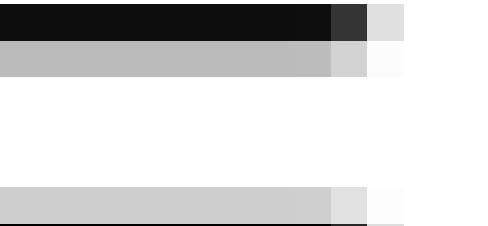
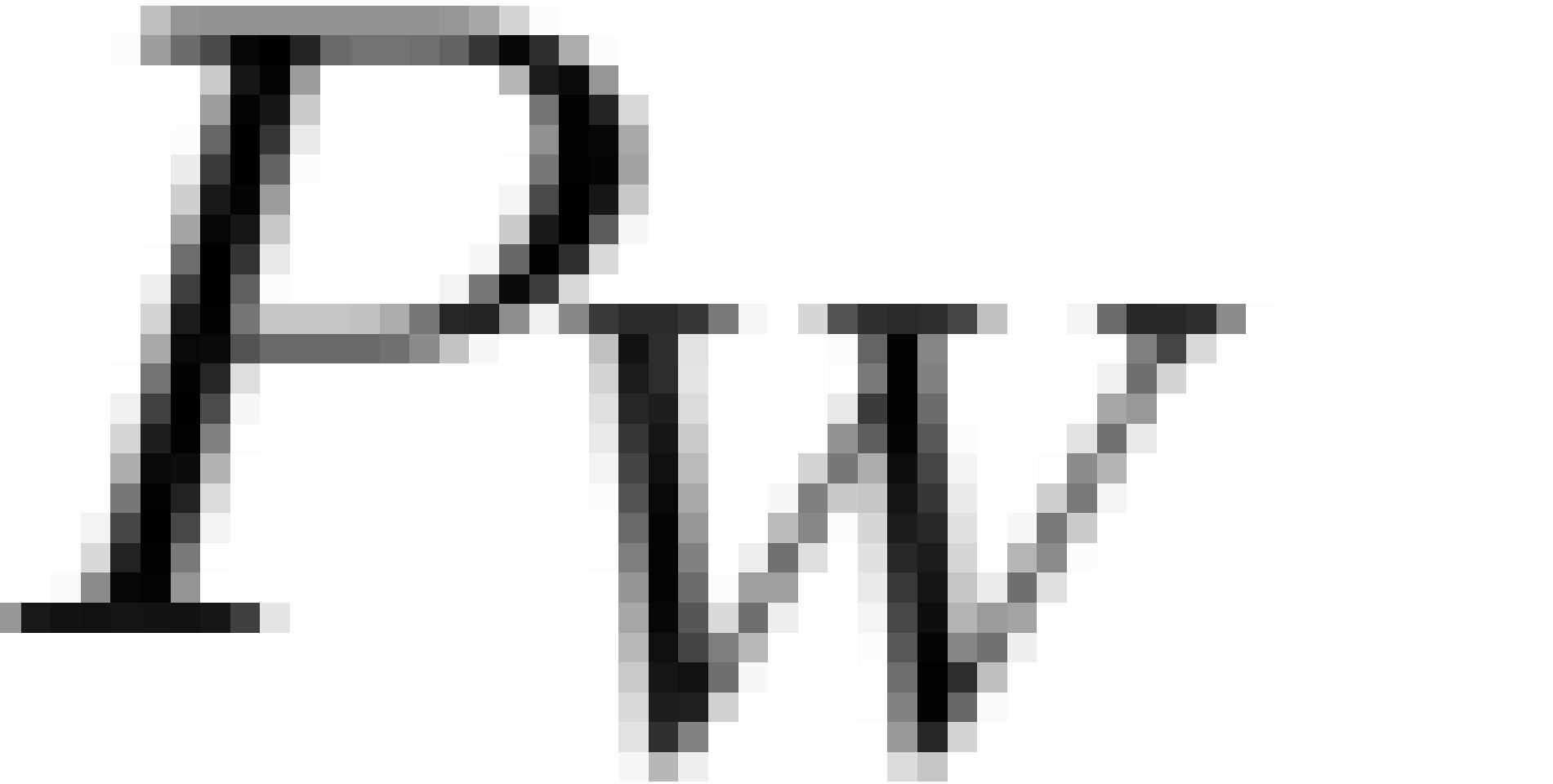


$$P_b = P_p + 3\sigma_{h\min} - \sigma_{H\max} + T_s + \sigma^{\Delta T}$$

Pore pressure
in the formation

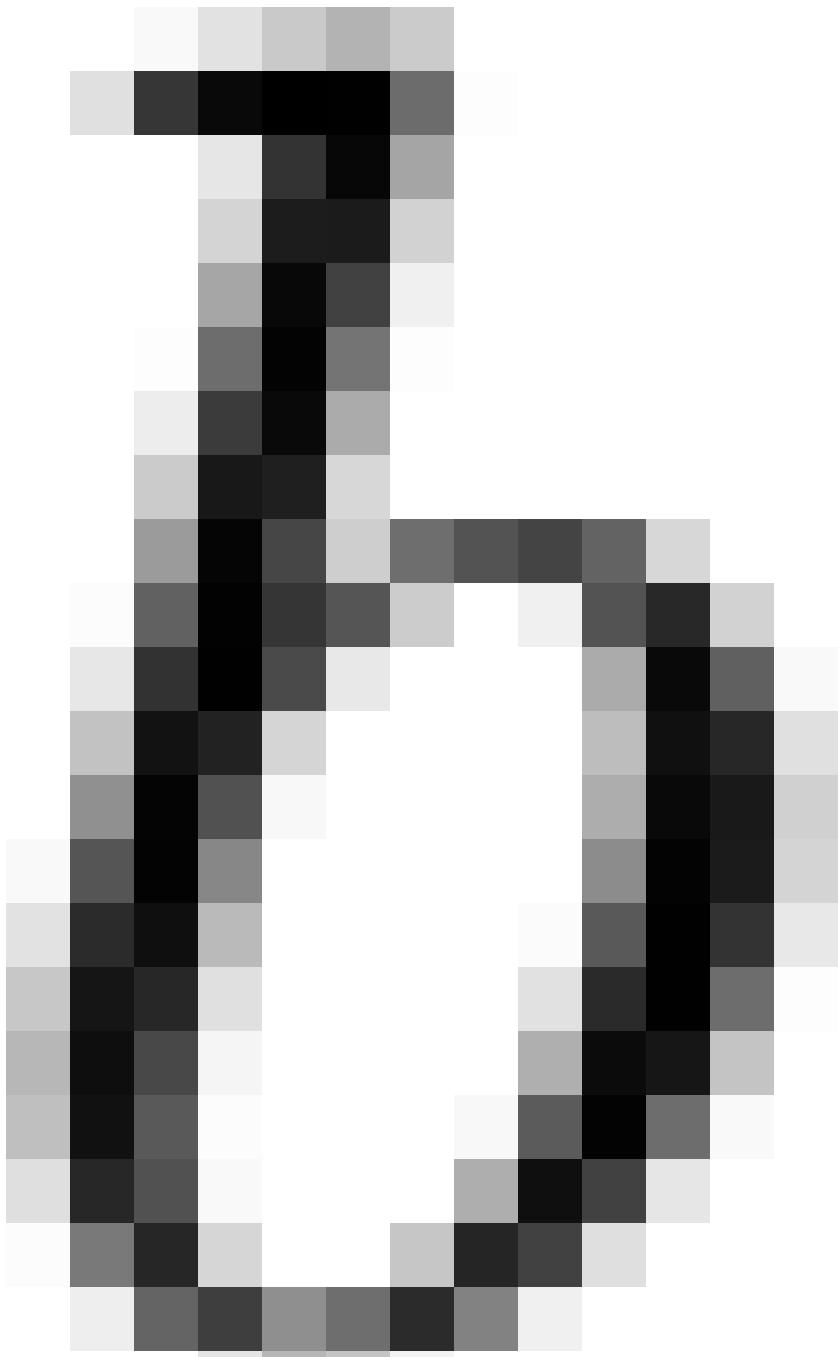
Stress anisotropy

Tensile strength
Cooling stress

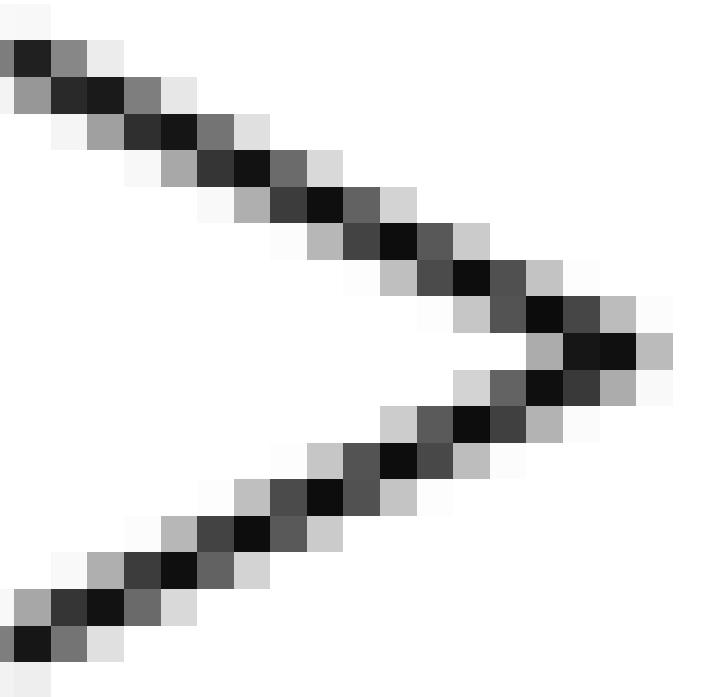
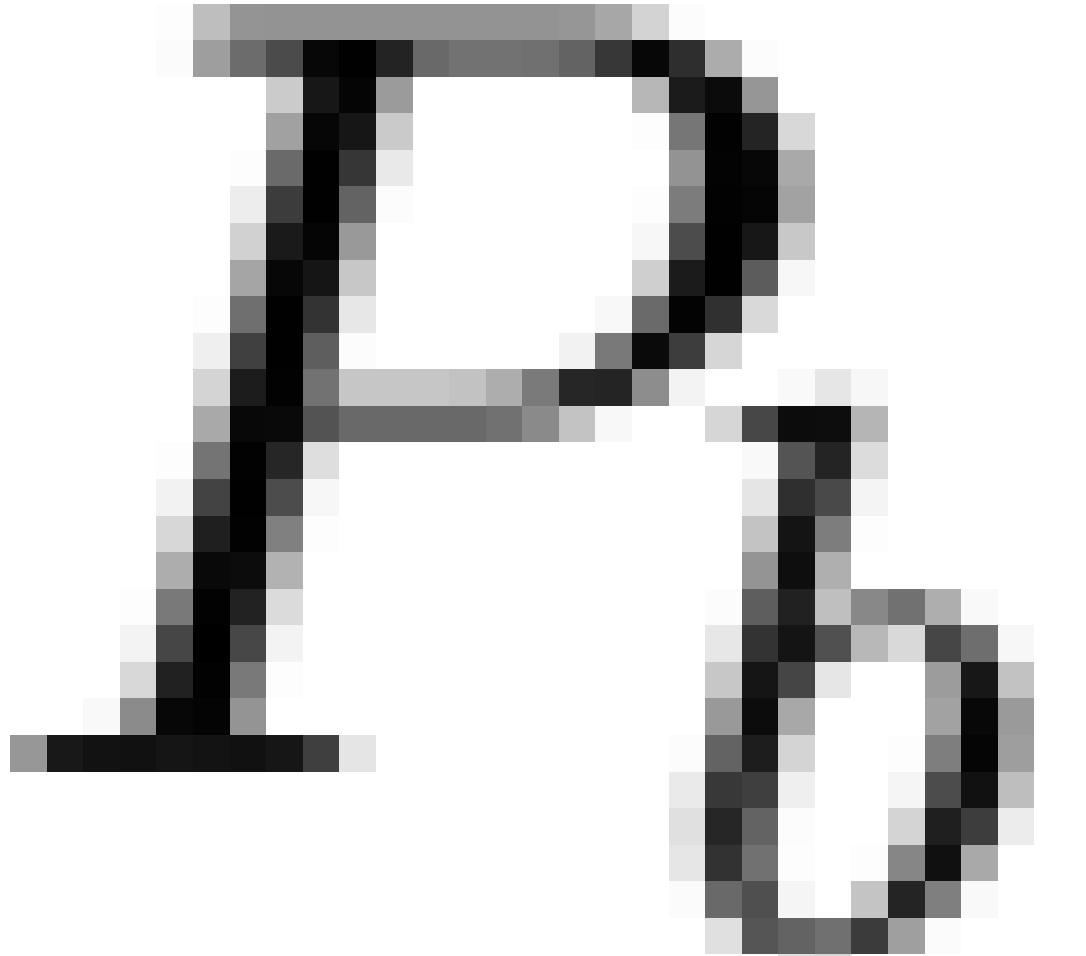


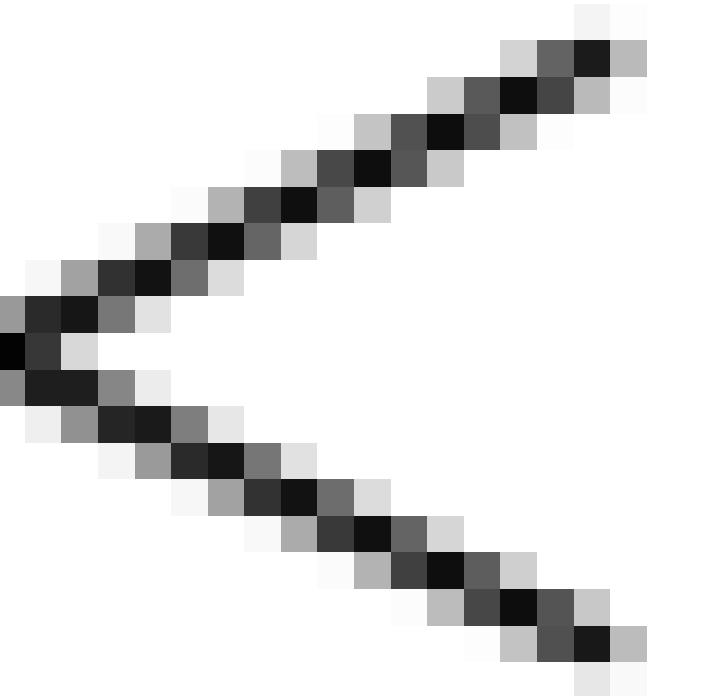
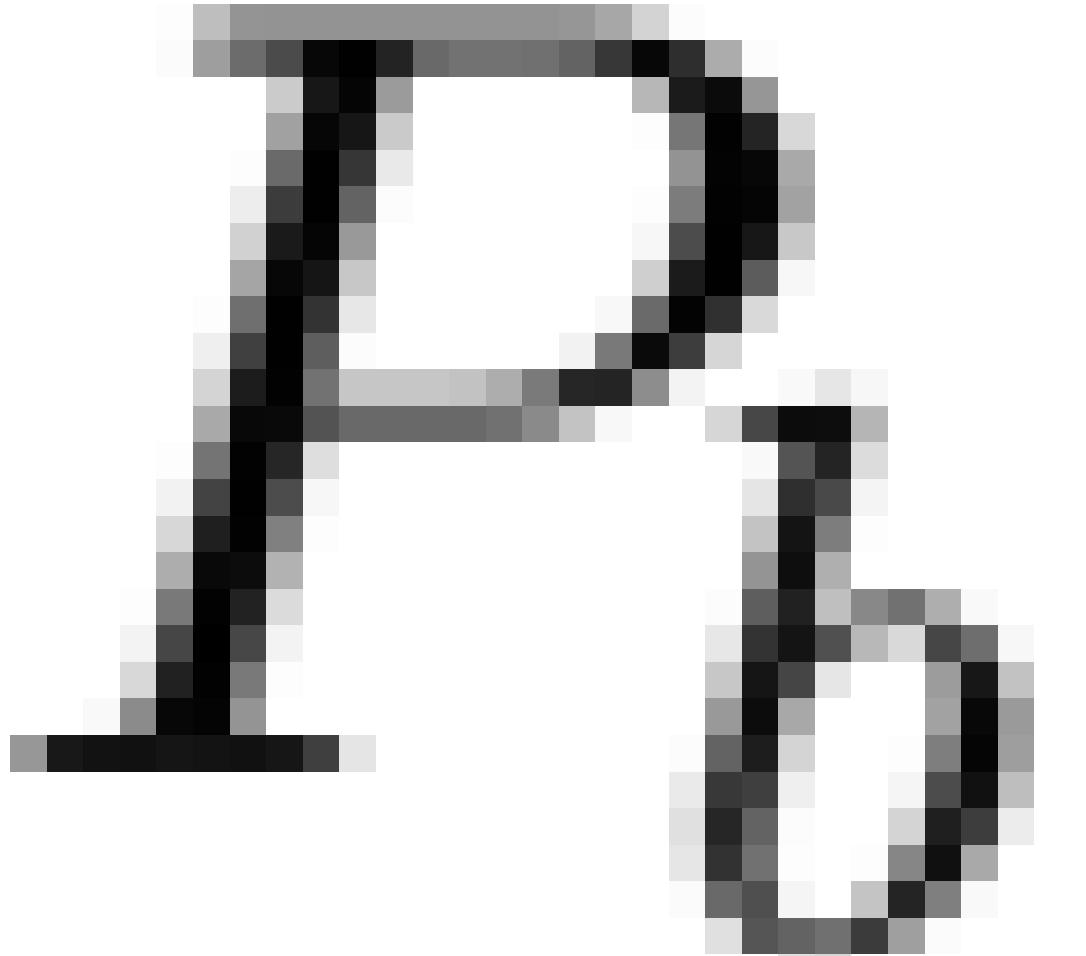
$\sigma_{\text{Hannac}} + \sigma_{\text{Hannac}}$

Bob Smith + 30 hours

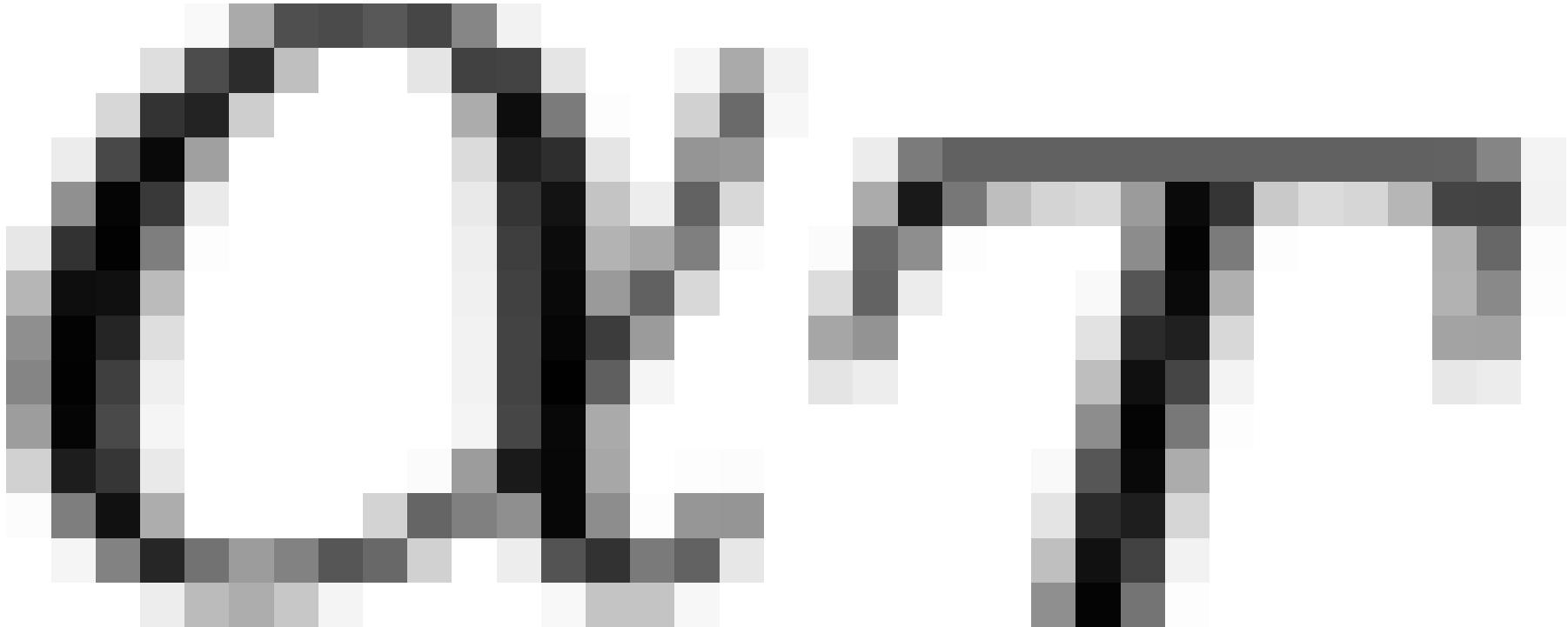


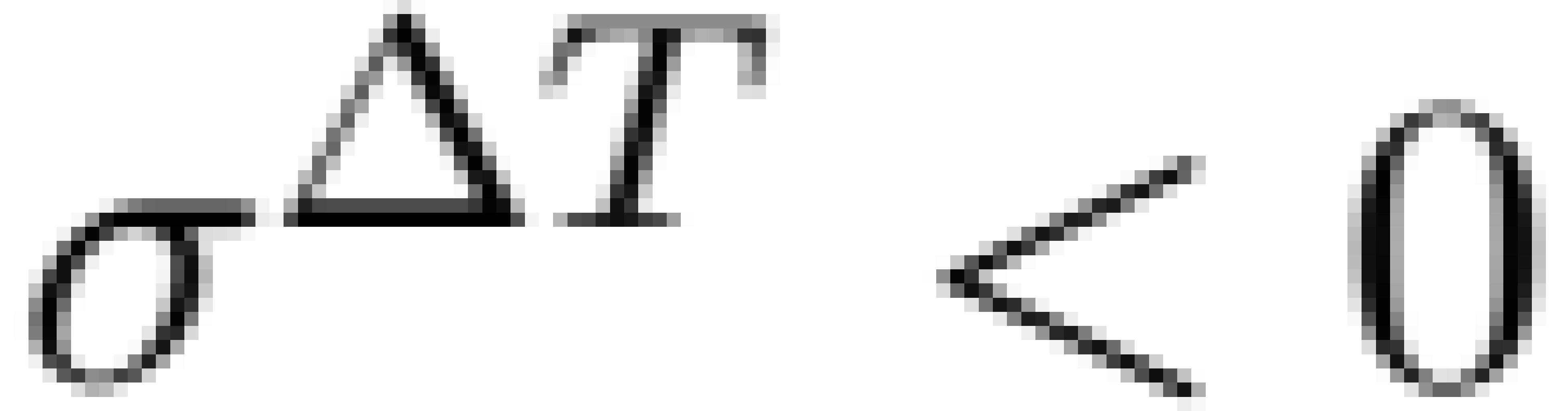


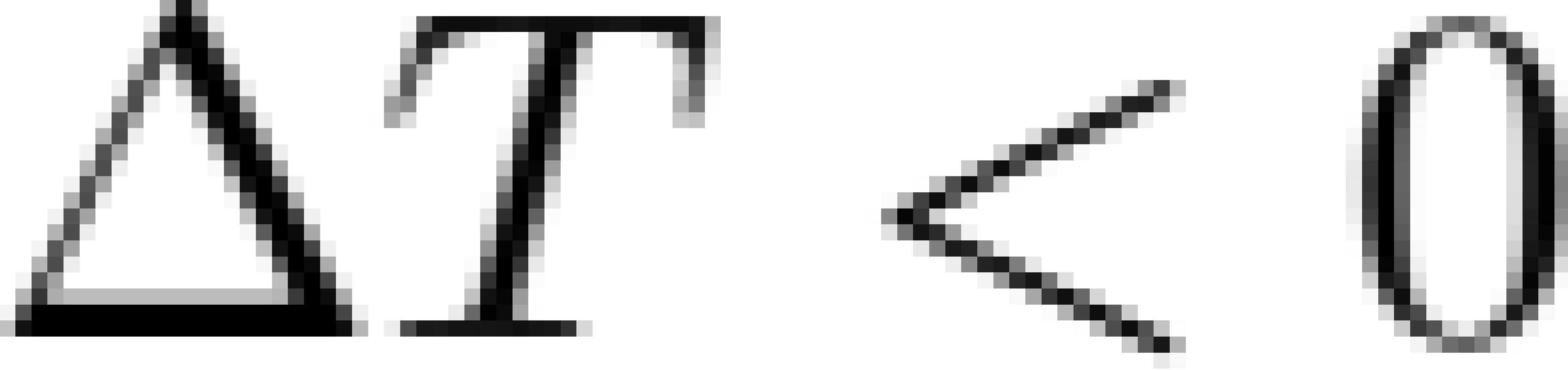


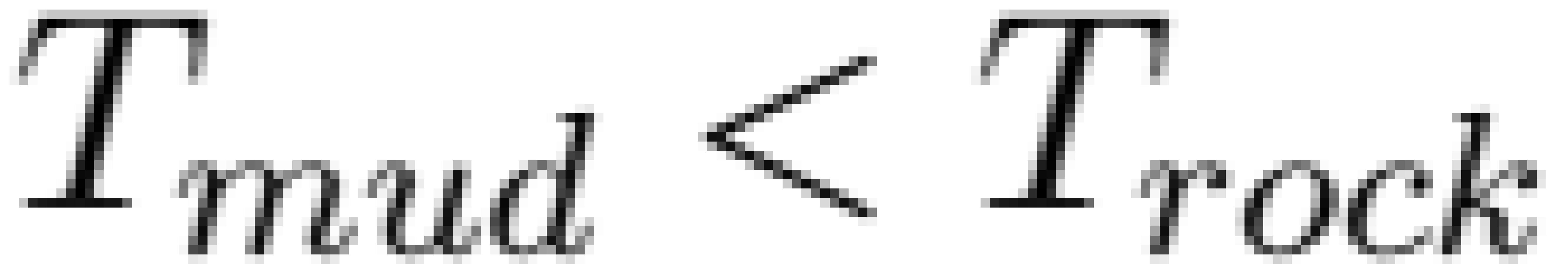


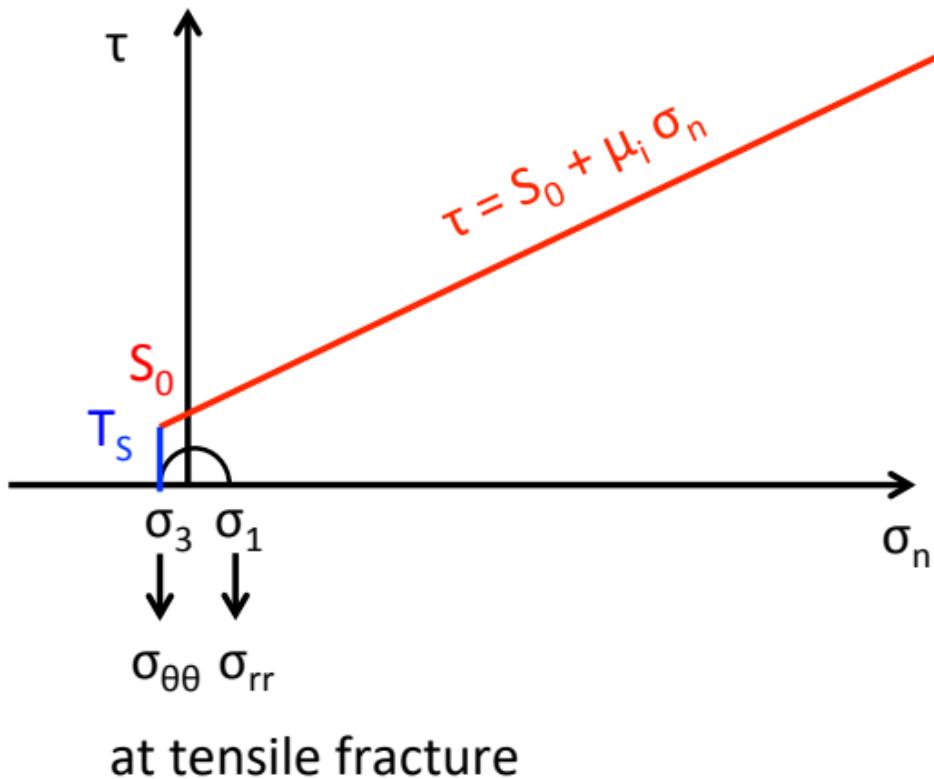
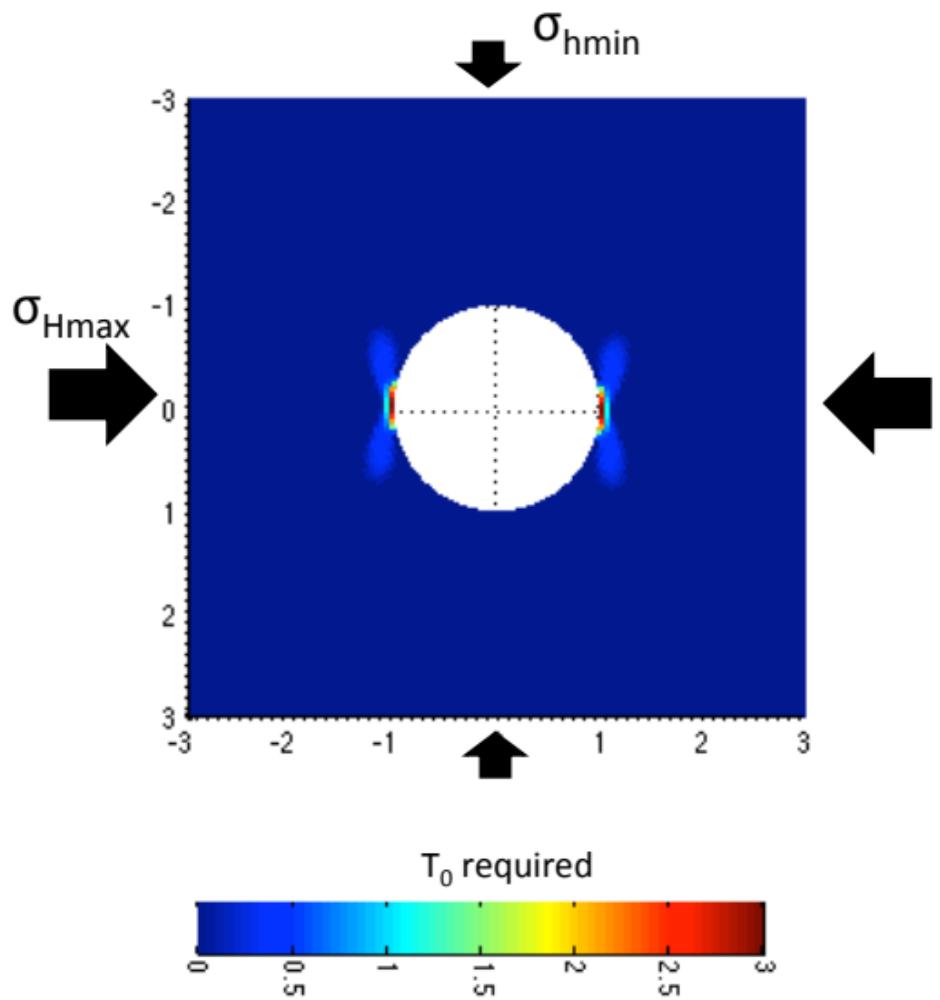


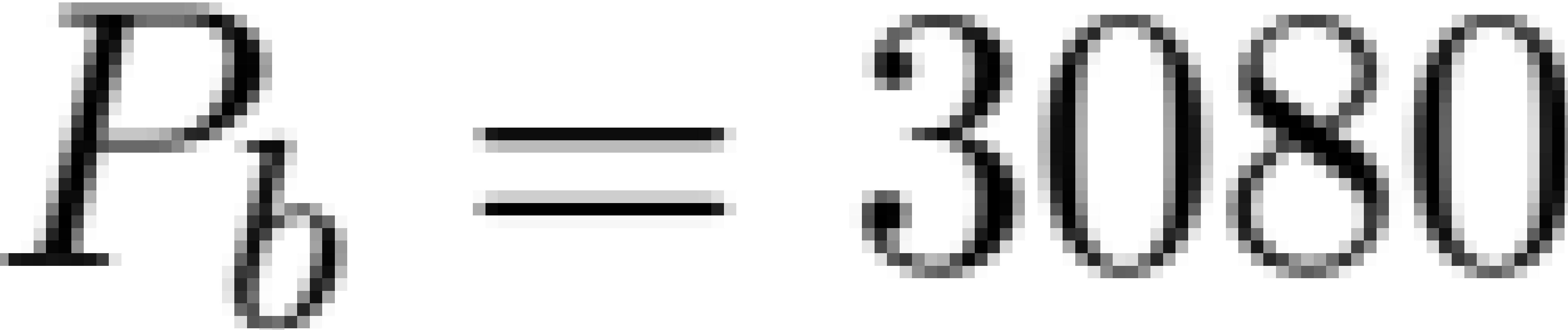


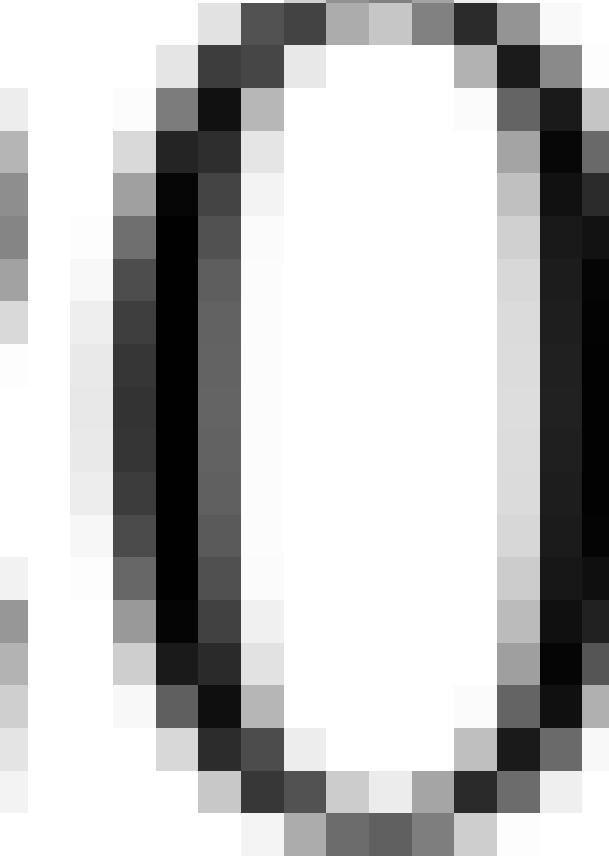
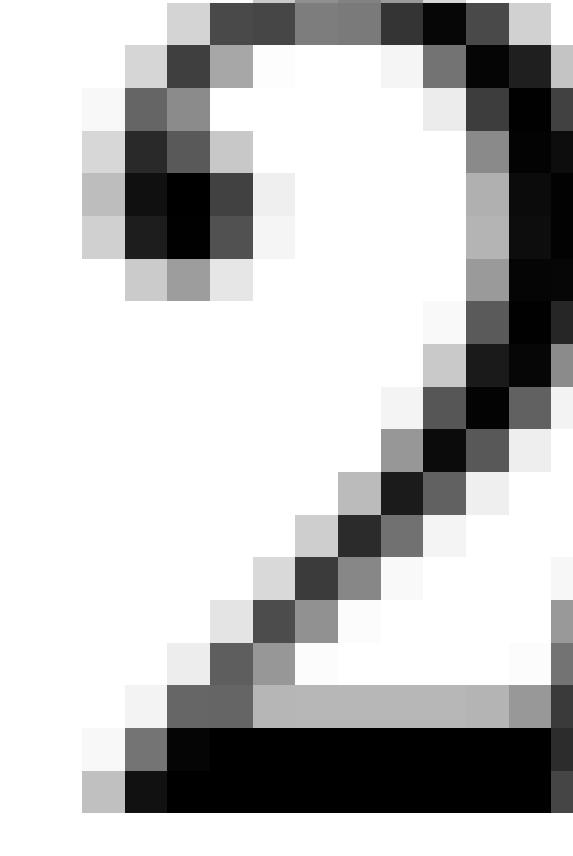
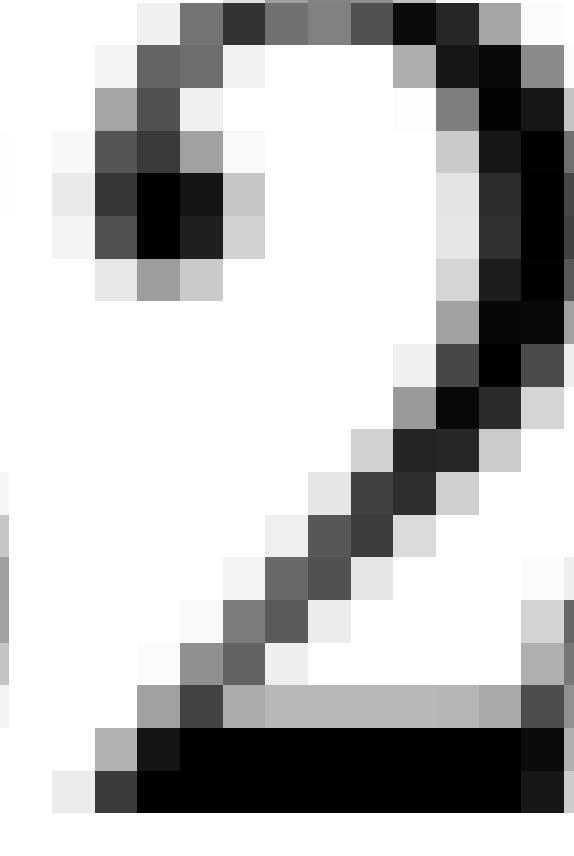
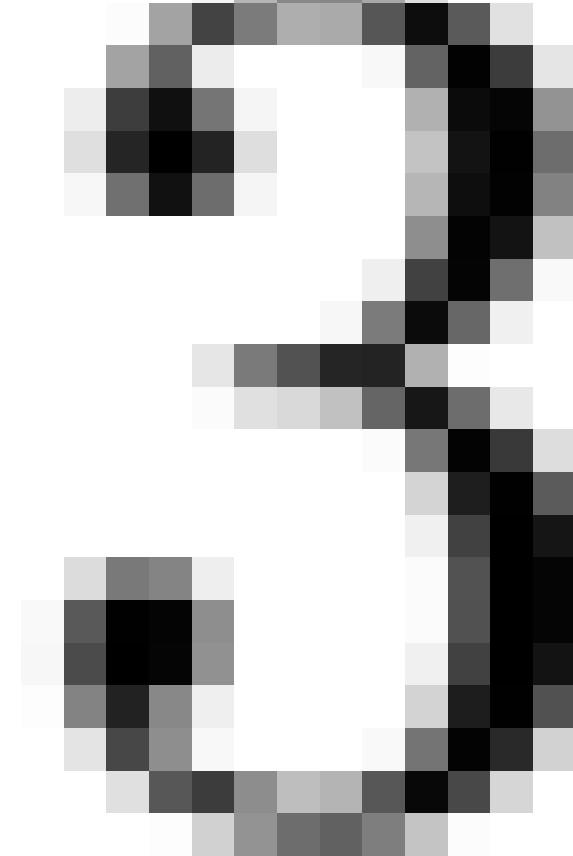


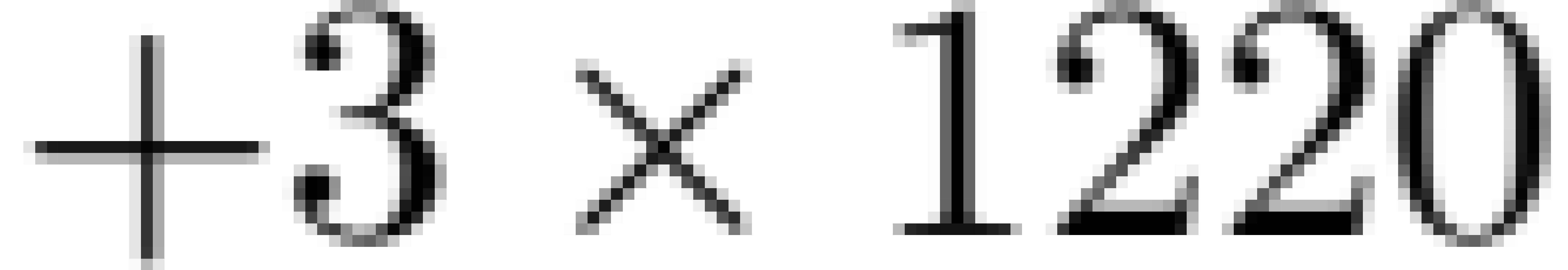


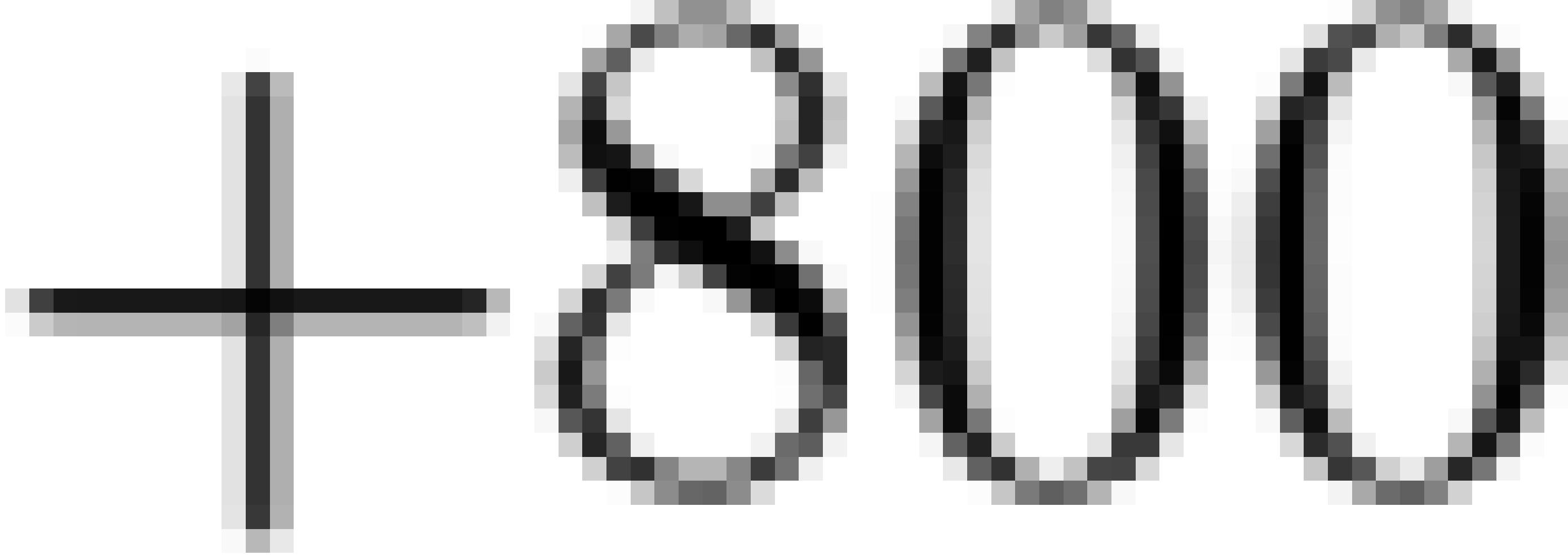


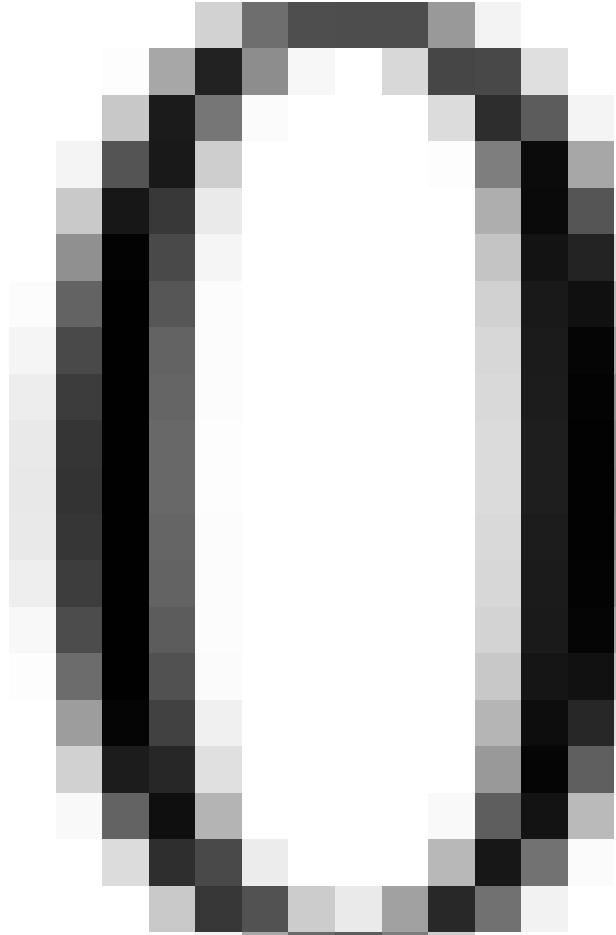
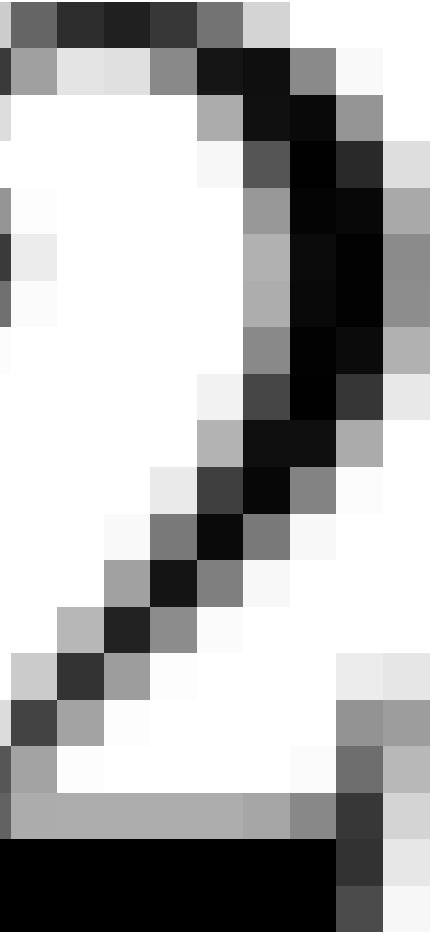
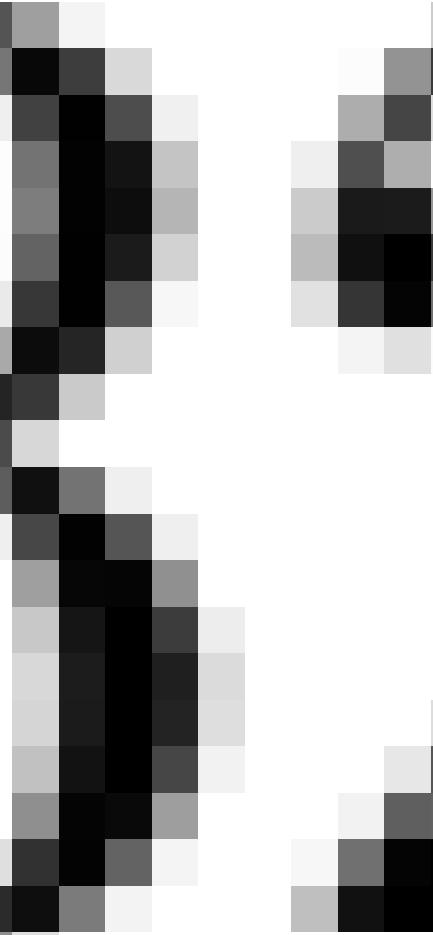
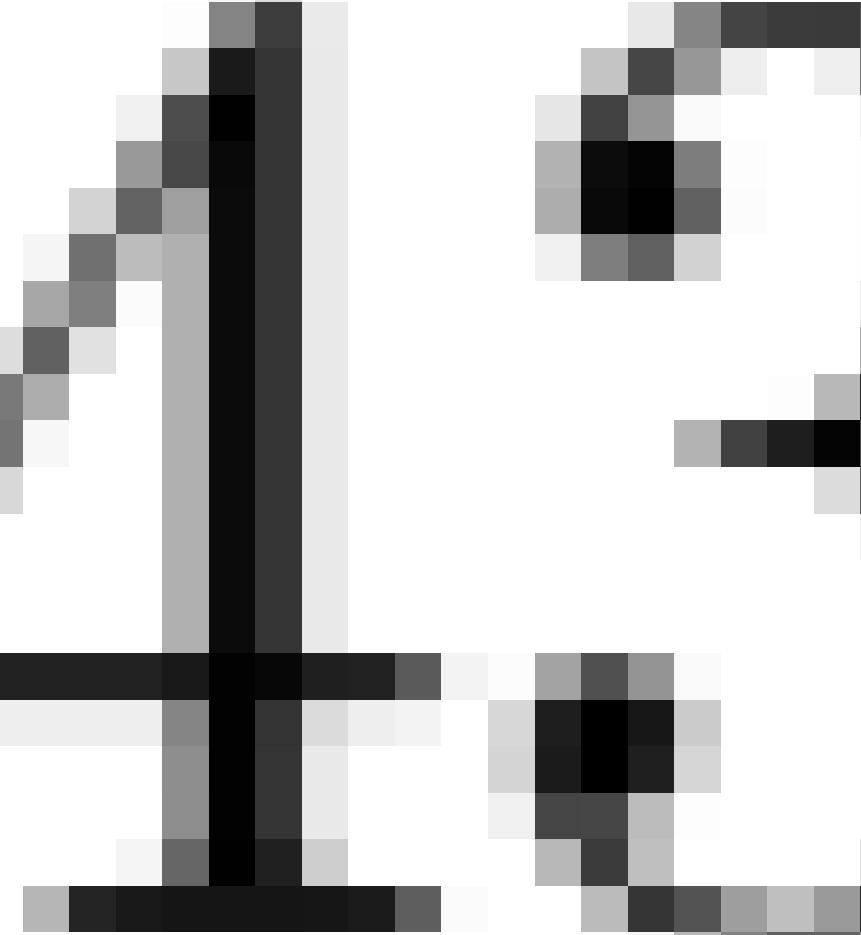












4320 psi

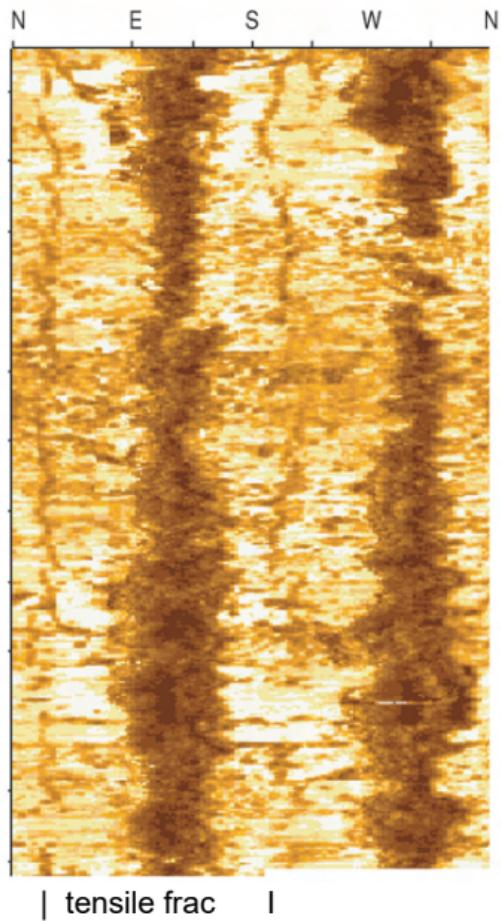
7000 ft

8.33 ppg

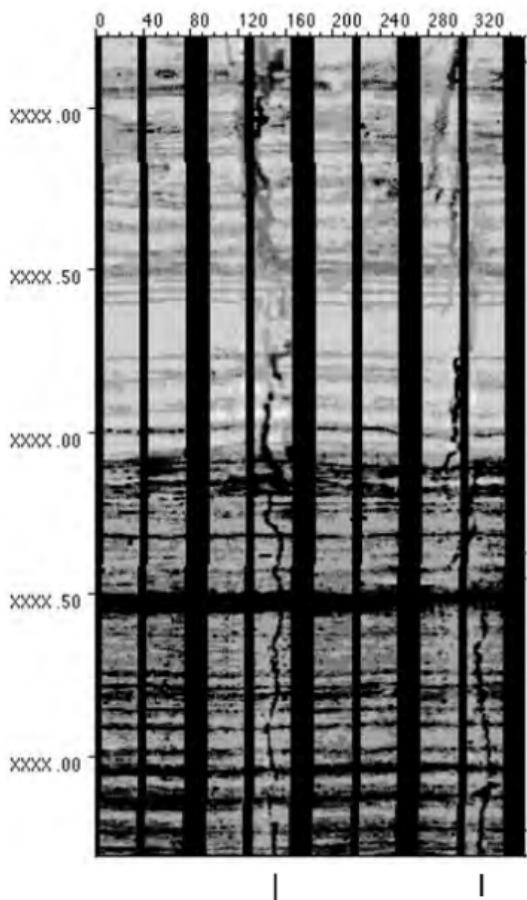
0.44 psi/ft

11.68 ppg

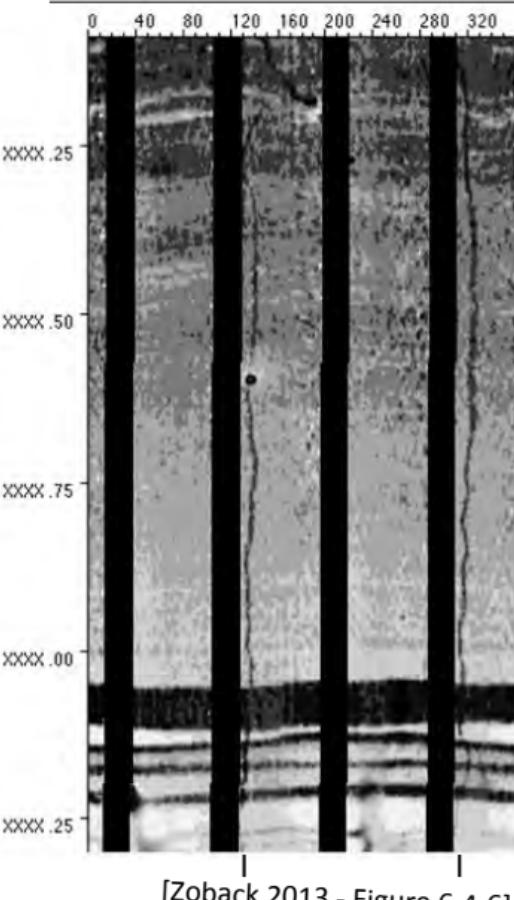
Ultrasonic P-wave

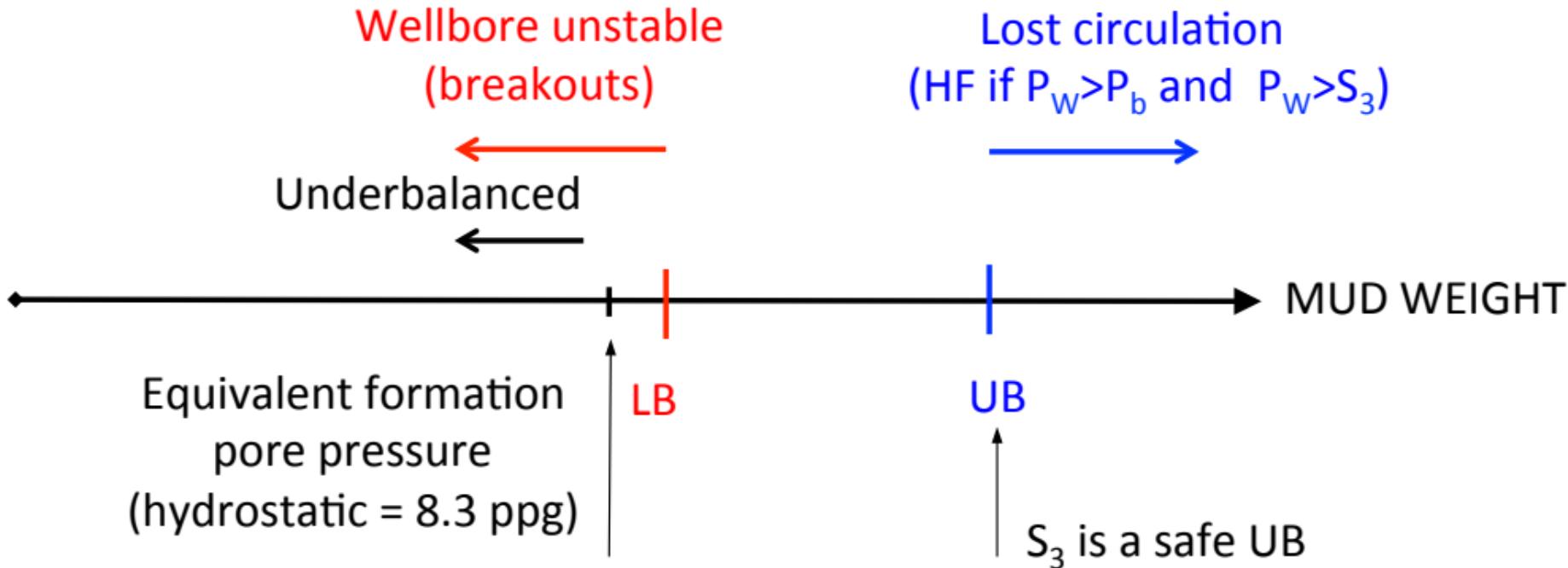


Electrical resistivity



Electrical resistivity



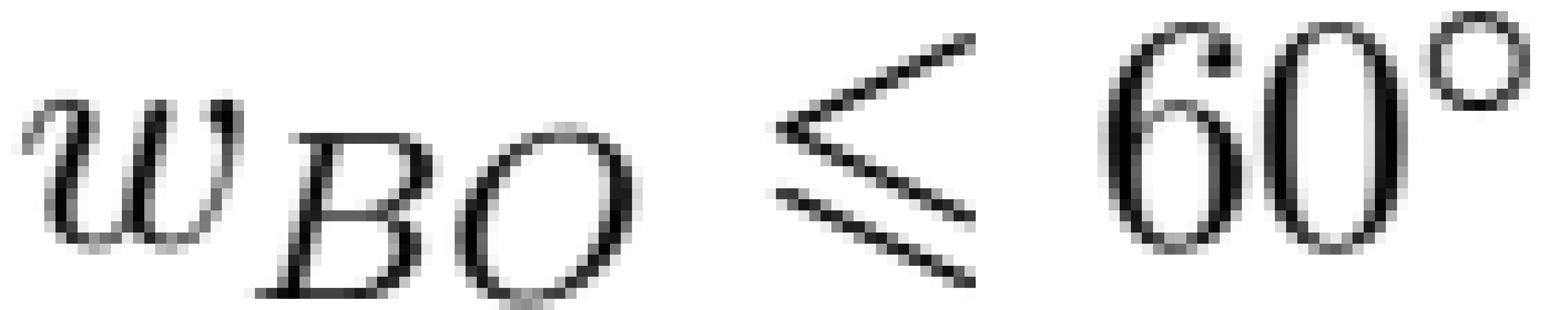


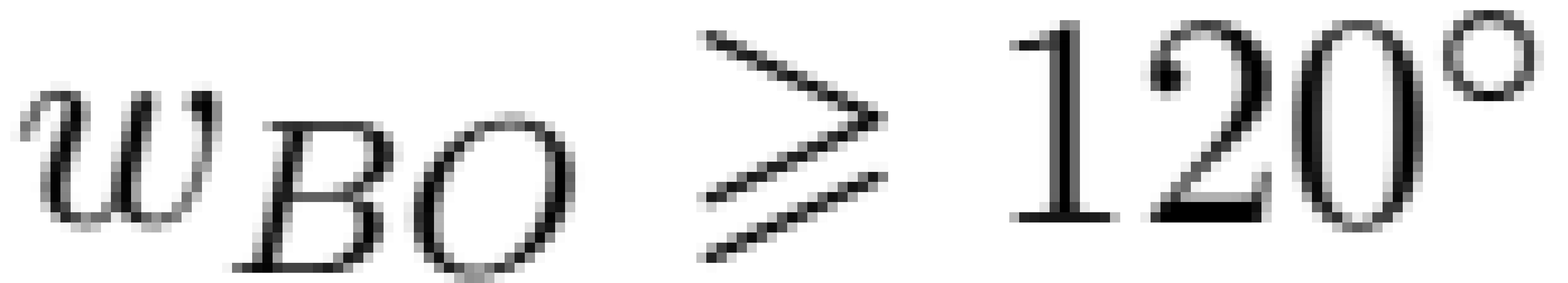
LIGHT MUDS

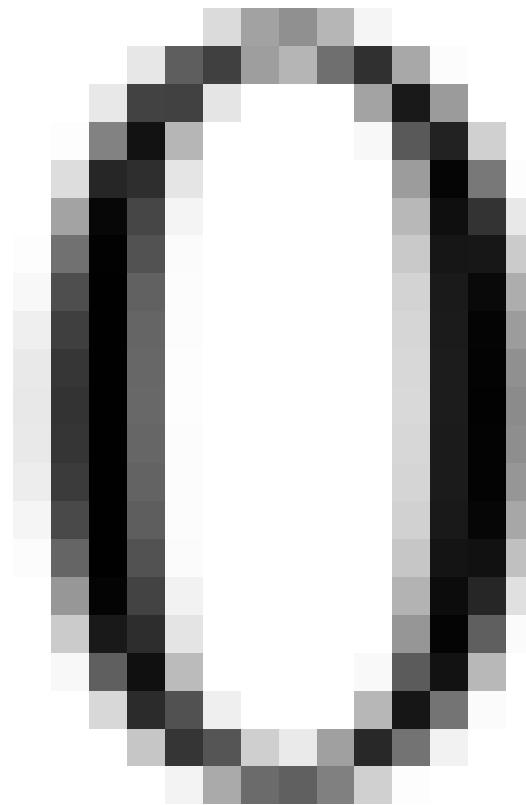
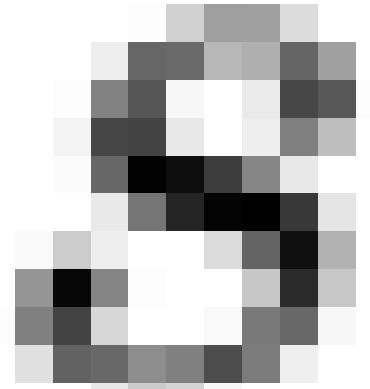
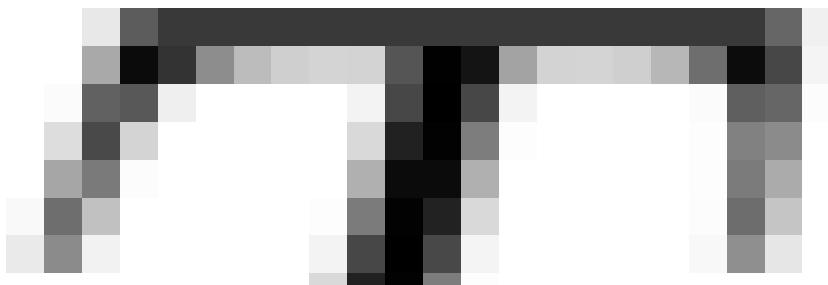
- May promote water production
- Compromise wellbore stability

HEAVY MUDS

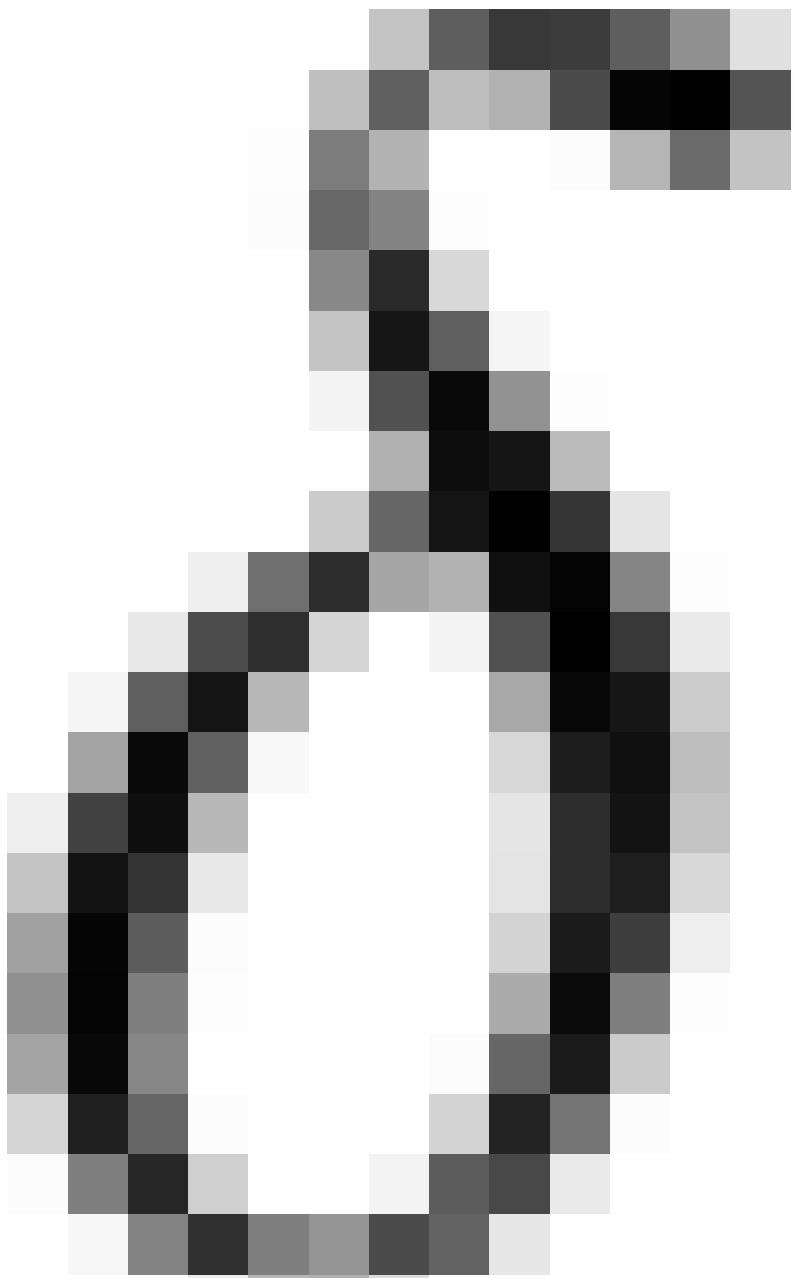
- Damage formation (k) by mud infiltration
- Promote mud losses in permeable strata
- Low ROP because of stronger rock



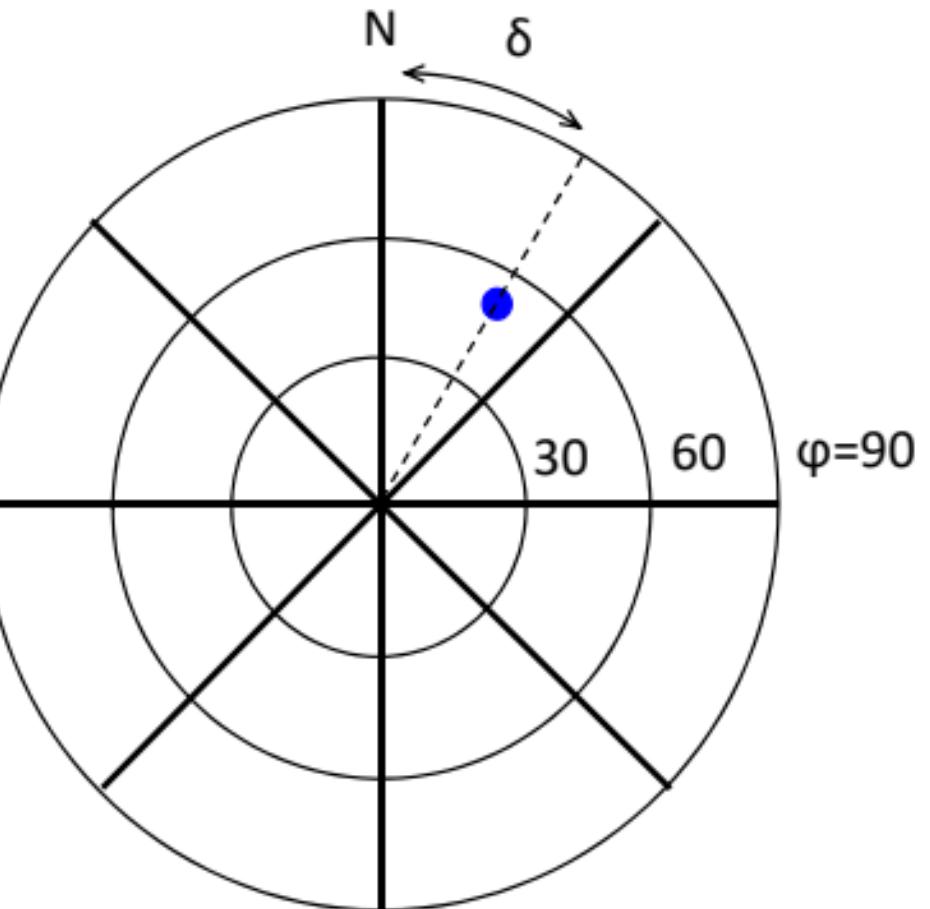
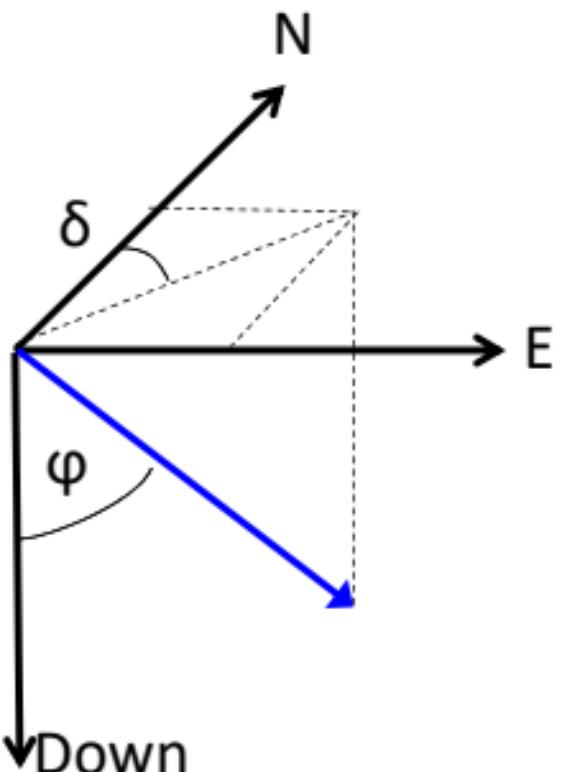




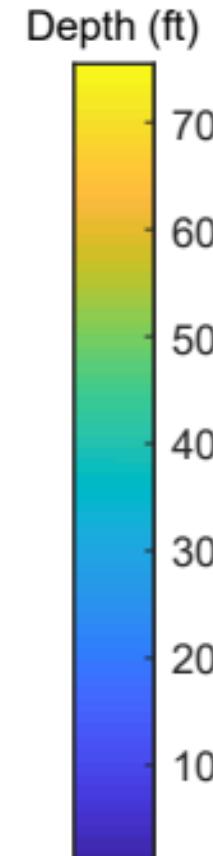
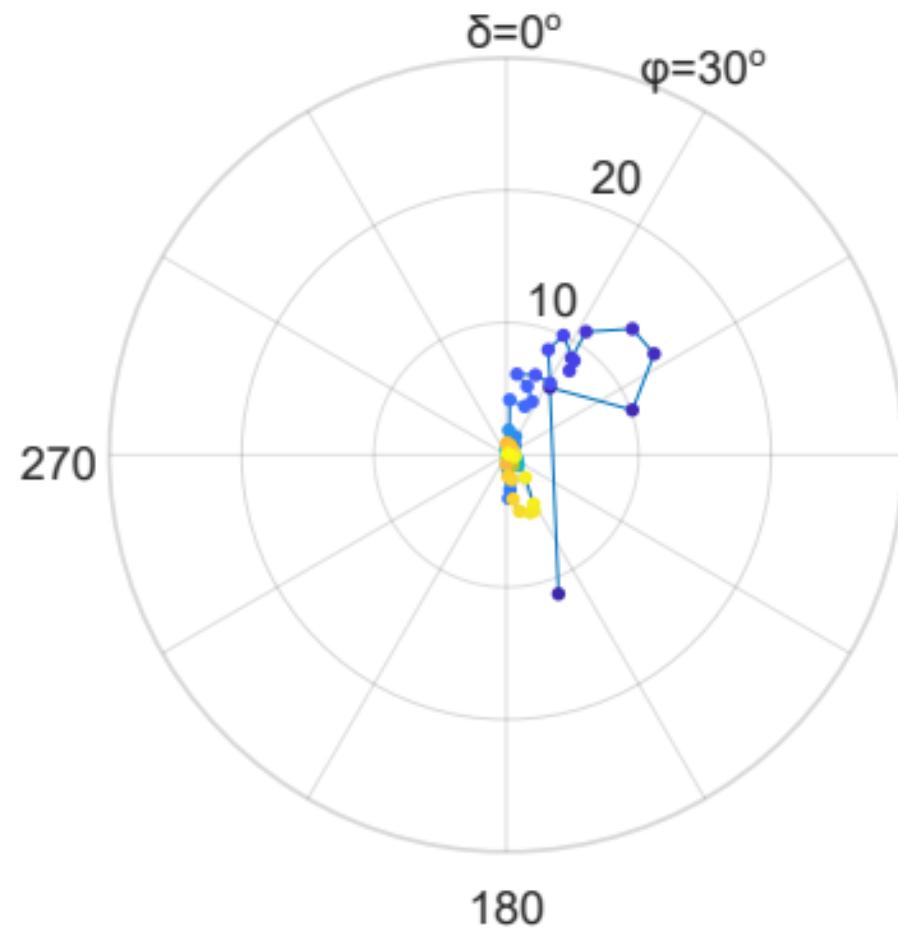


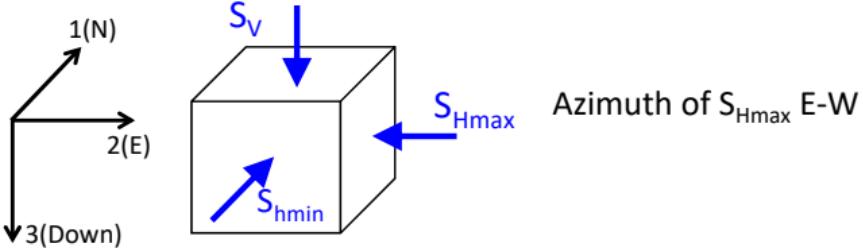


(a) Definitions

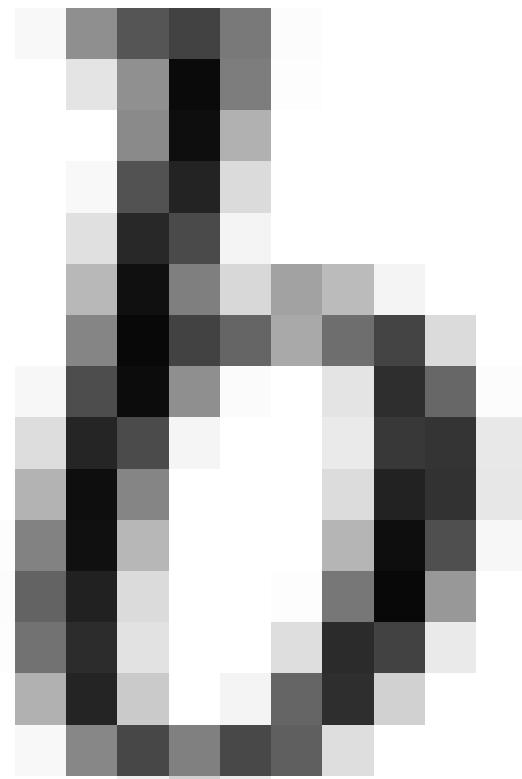
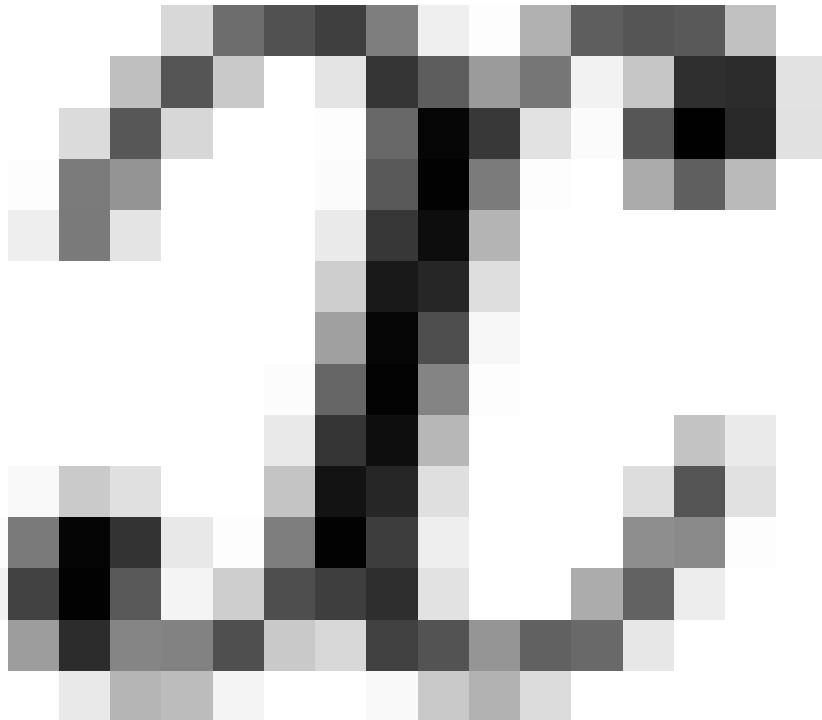


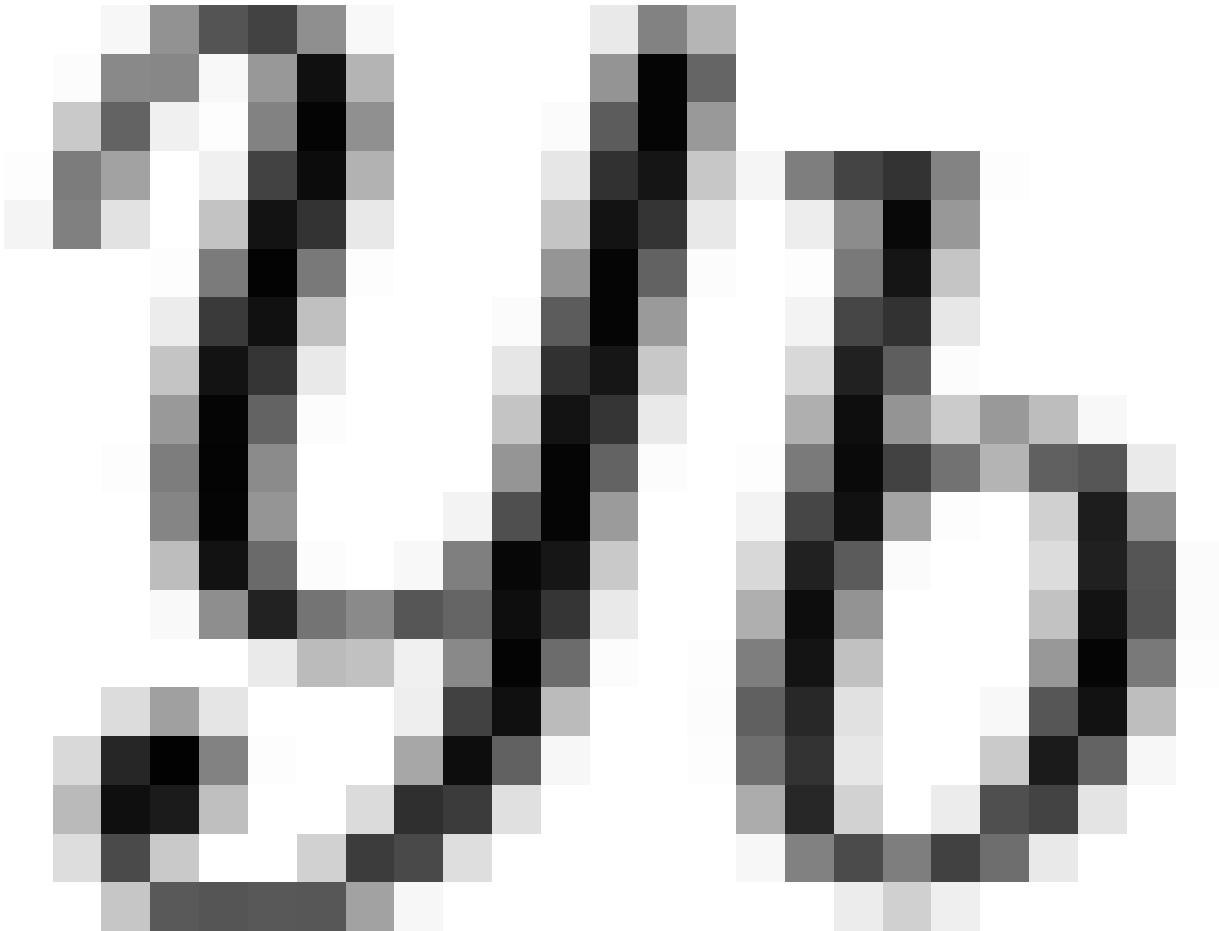
(b) Example

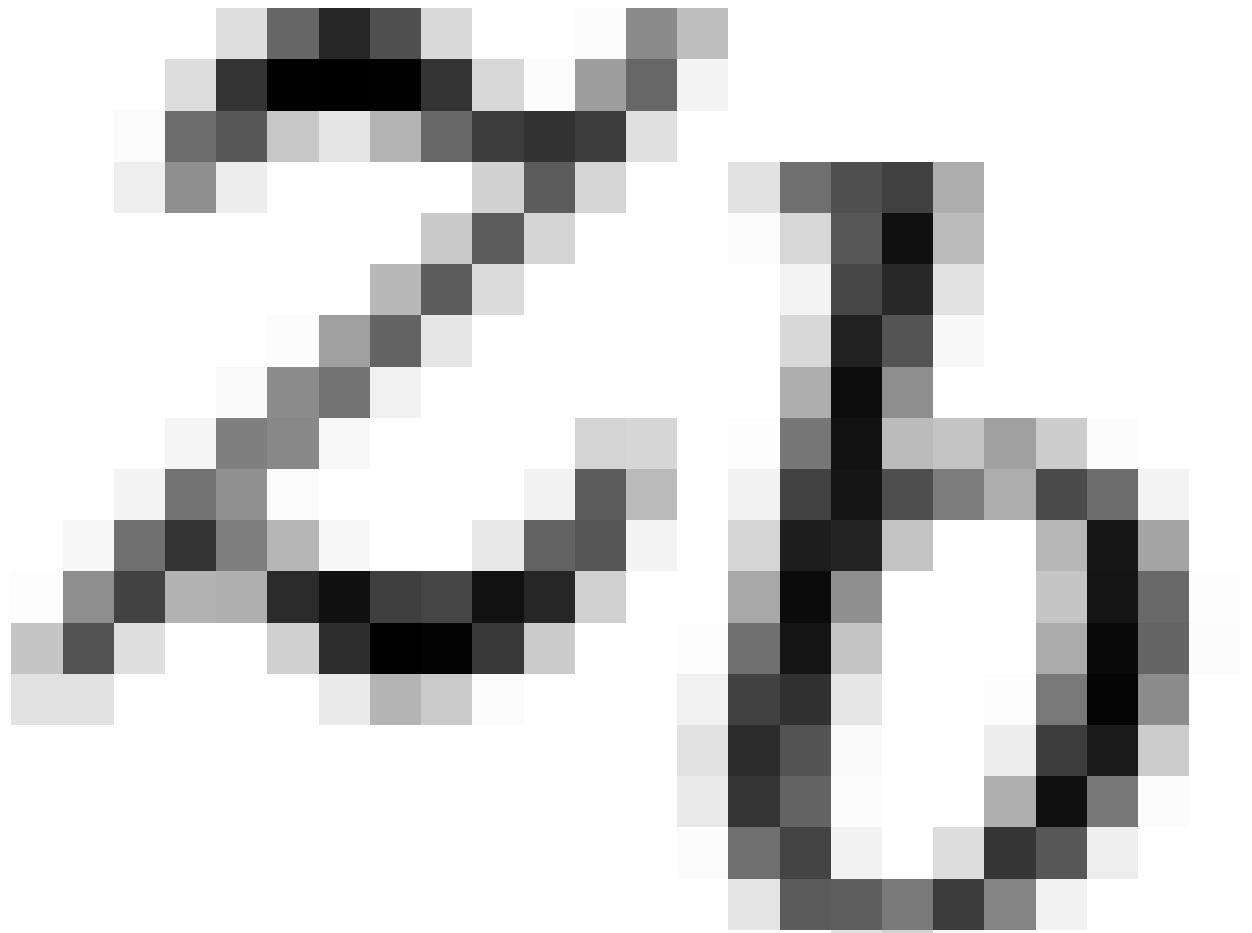


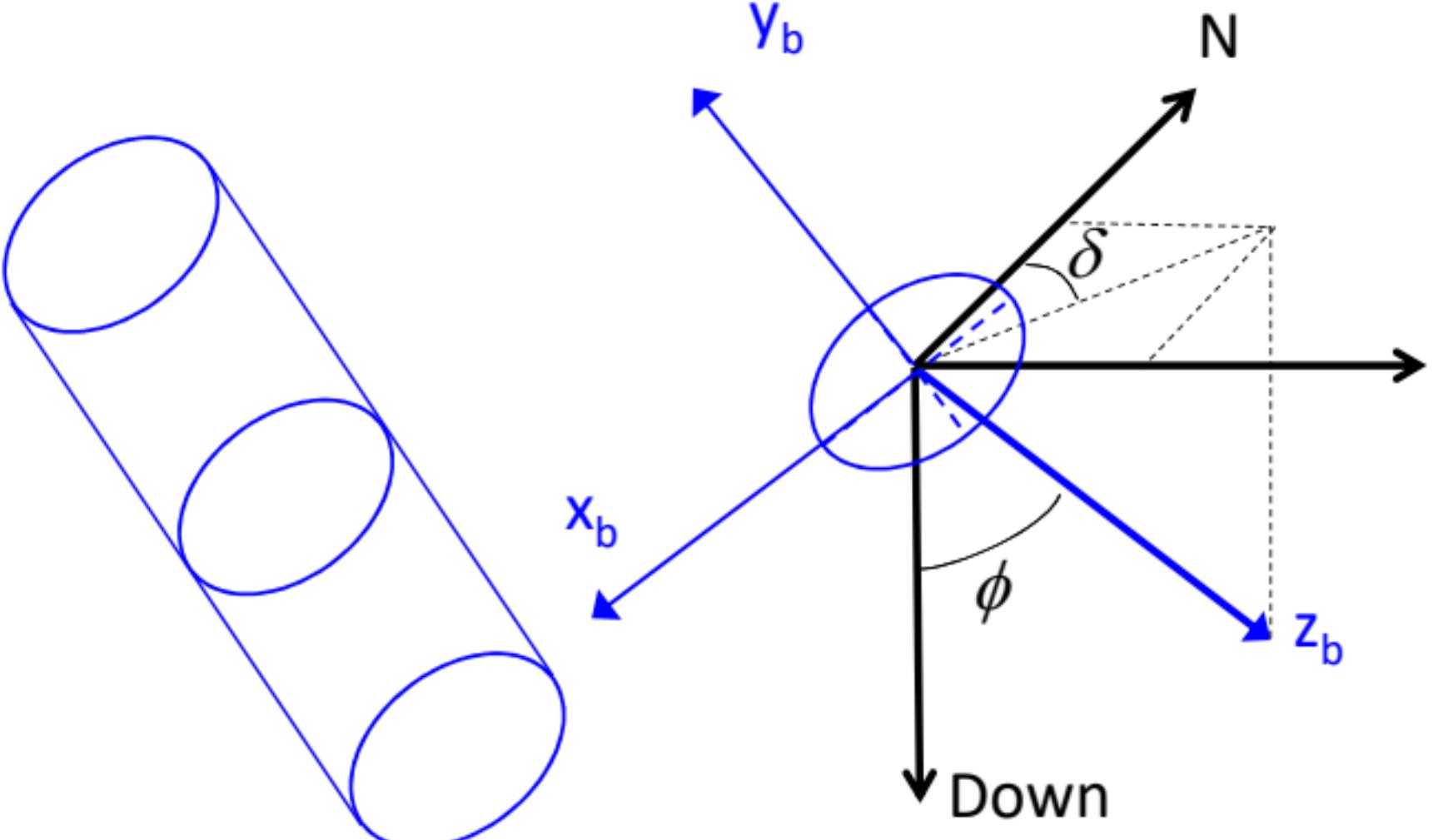


Stress Environment	NF	SS	RF
Plane with highest stress anisotropy and location of breakouts and tensile fracs	<p>3D stress element for Normal Faulting (NF) showing the orientation of the three principal stresses: S_V (vertical, downwards), $S_{h\text{min}}$ (horizontal, to the right), and $S_{H\text{max}}$ (horizontal, to the left). A red dot is located on the top face of the cube.</p>	<p>3D stress element for Strike-Slip (SS) showing the orientation of the three principal stresses: S_V (vertical, downwards), $S_{h\text{min}}$ (horizontal, to the right), and $S_{H\text{max}}$ (horizontal, to the left). A red dot is located on the top face of the cube.</p>	<p>3D stress element for Reverse Fault (RF) showing the orientation of the three principal stresses: S_V (vertical, downwards), $S_{h\text{min}}$ (horizontal, to the right), and $S_{H\text{max}}$ (horizontal, to the left). A red dot is located on the top face of the cube.</p>
Narrower drilling window in a stereonet projection	<p>Stereonet projection for Normal Faulting (NF) showing a small circular window centered on the upper hemisphere, indicating a narrow drilling window.</p>	<p>Stereonet projection for Strike-Slip (SS) showing a small circular window centered on the equator, indicating a narrow drilling window.</p>	<p>Stereonet projection for Reverse Fault (RF) showing a small circular window centered on the lower hemisphere, indicating a narrow drilling window.</p>

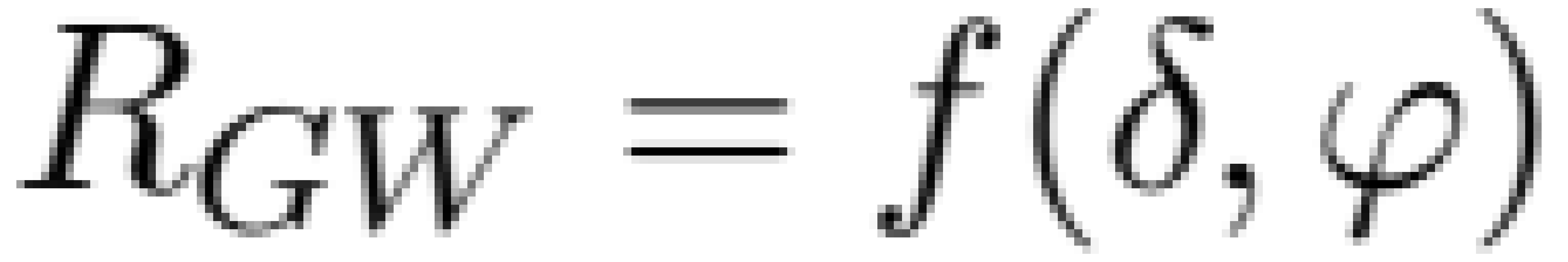








$$R_{GW} = \begin{bmatrix} -\cos\delta\cos\phi & -\sin\delta\cos\phi & \sin\phi \\ \sin\delta & -\cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix}$$









$$\underline{S}_W =$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$





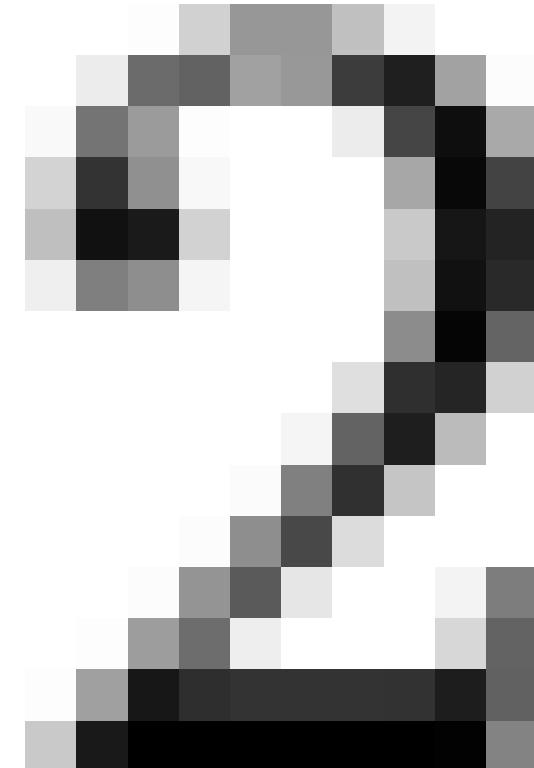
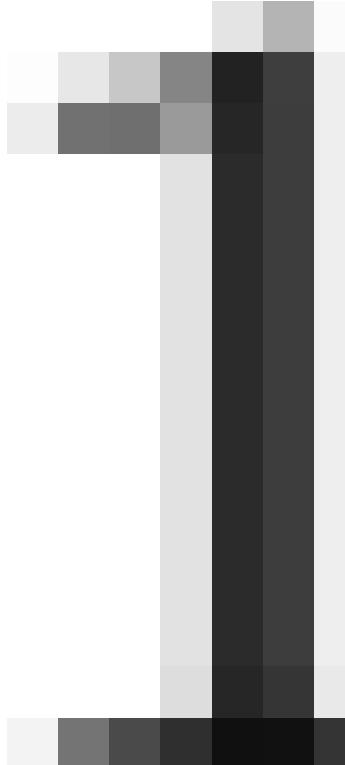
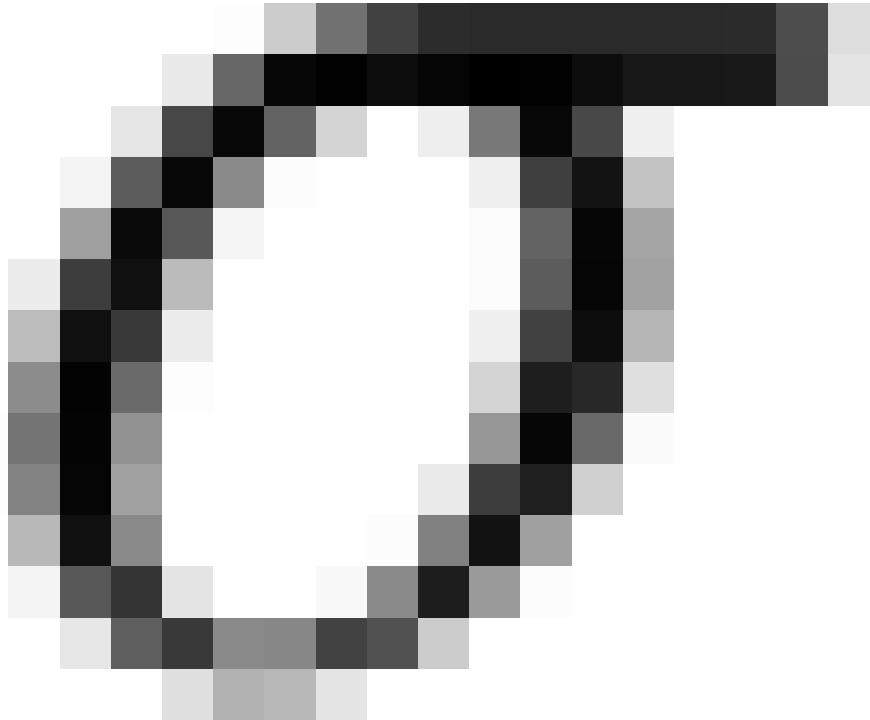


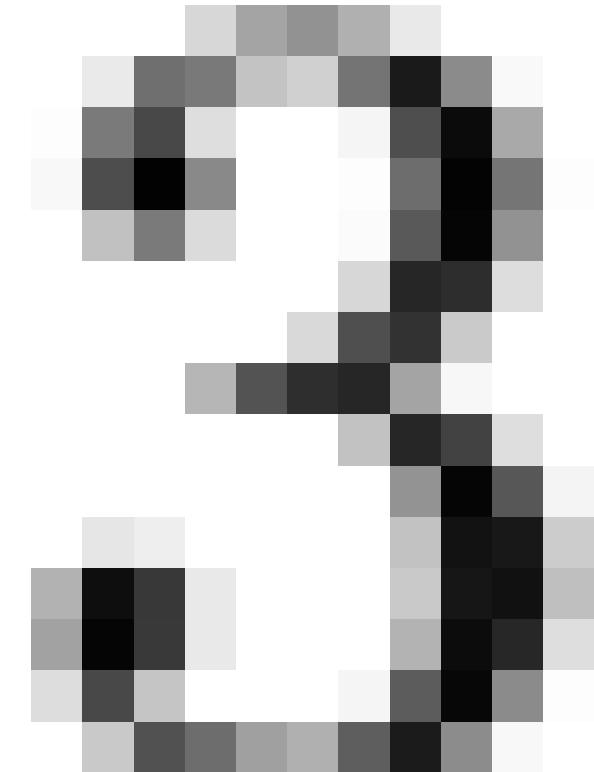
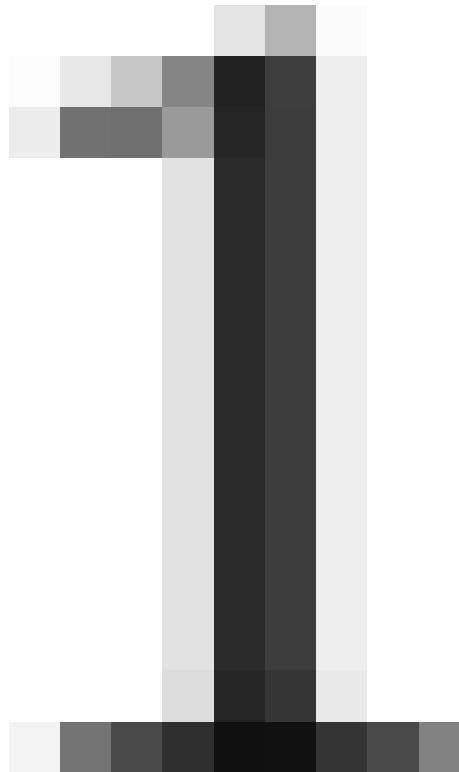
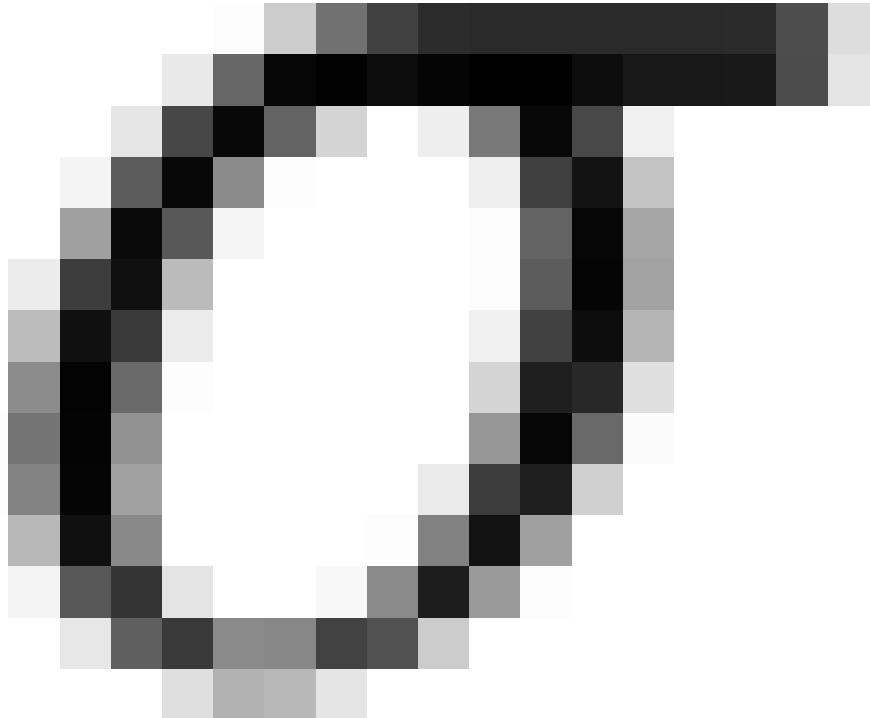


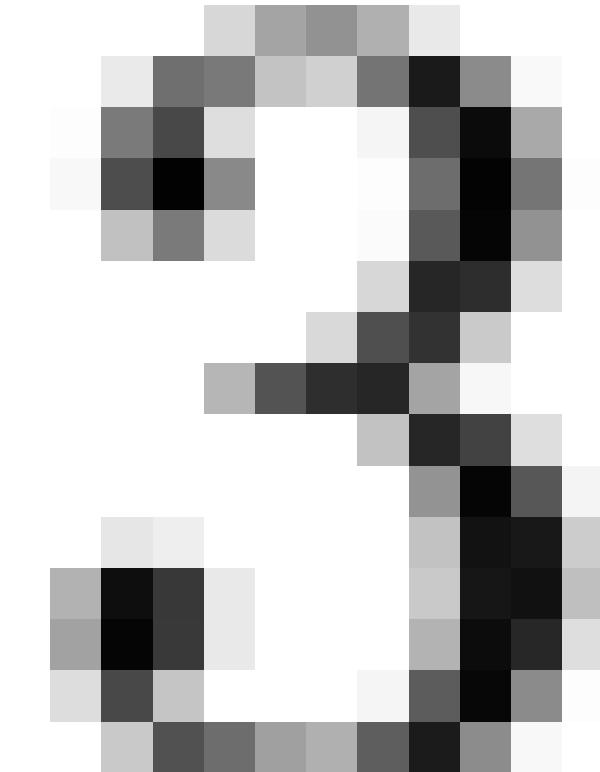
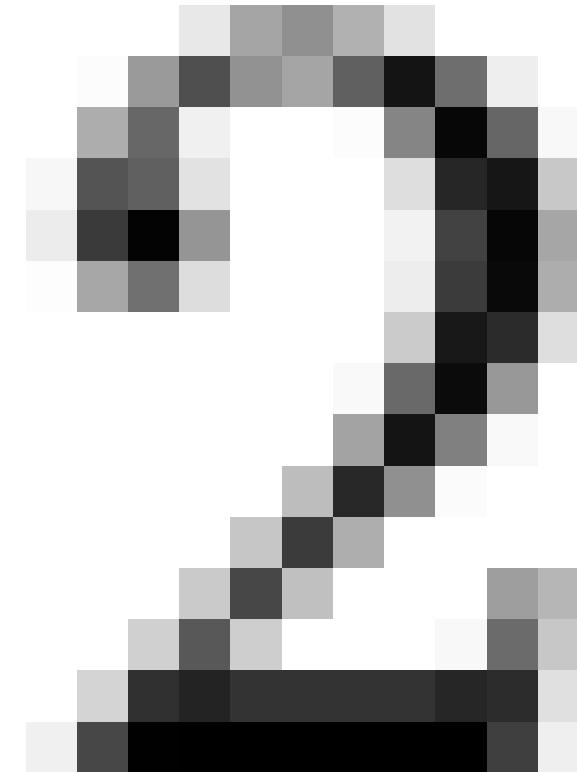
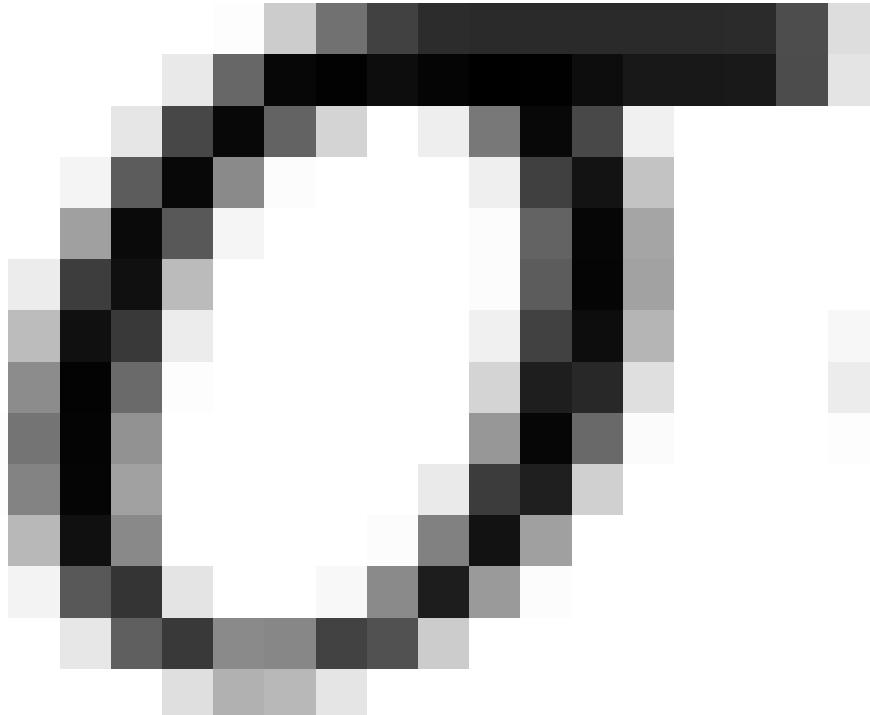


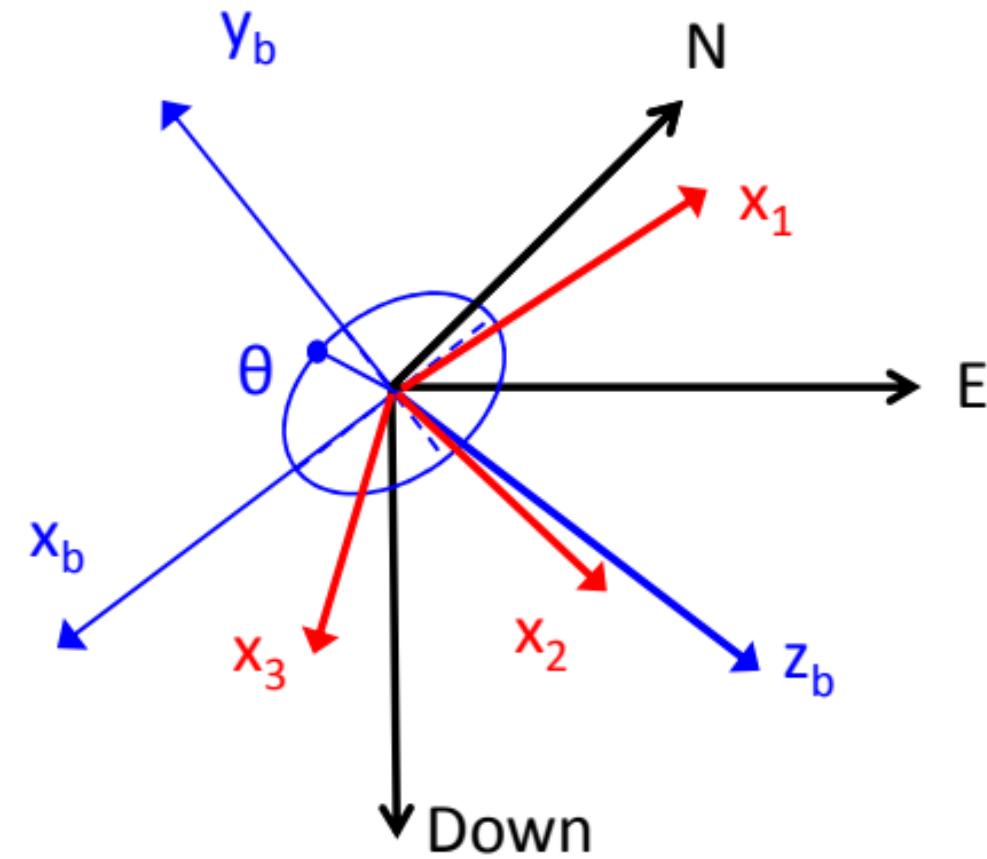








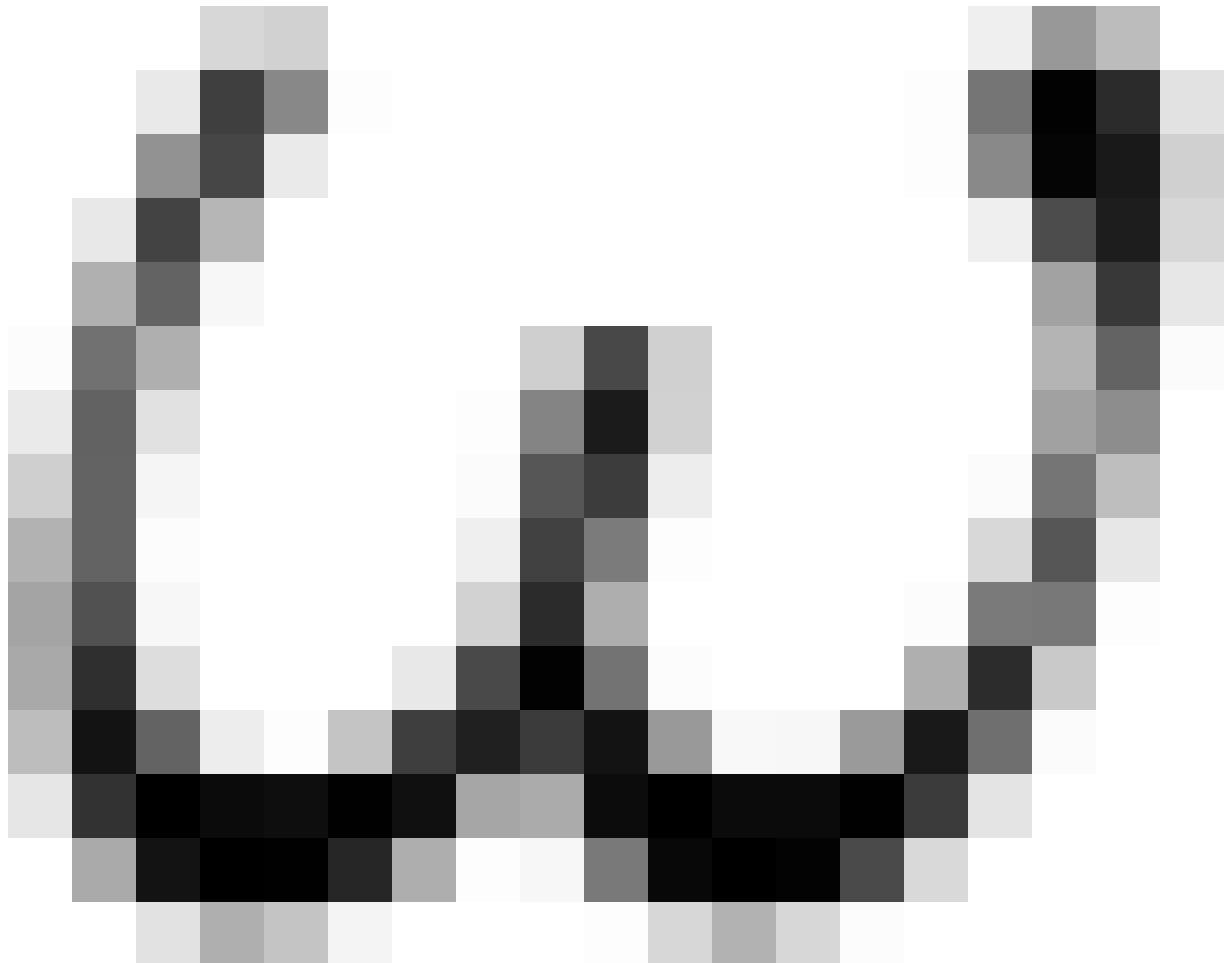


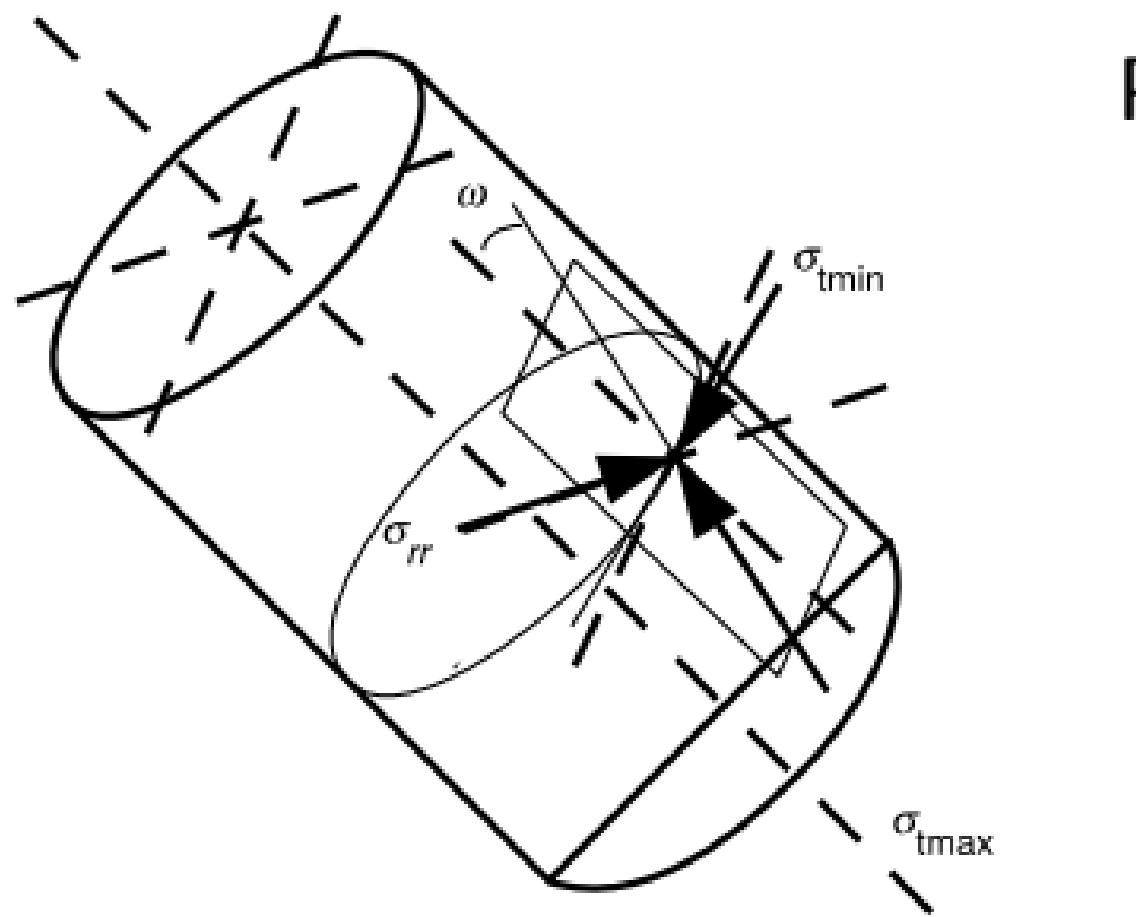


Stresses at the wellbore wall (Kirsch[PP]+Kirsch[S])

$$\begin{cases} \sigma_{rr} = \Delta P \\ \sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\sigma_{12}\sin 2\theta - \Delta P \\ \tau_{\theta z} = 2(\sigma_{23}\cos\theta - \sigma_{13}\sin\theta) \\ \sigma_{zz} = \sigma_{33} - 2\nu(\sigma_{11} - \sigma_{22})\cos 2\theta - 4\nu\sigma_{12}\sin 2\theta \end{cases}$$

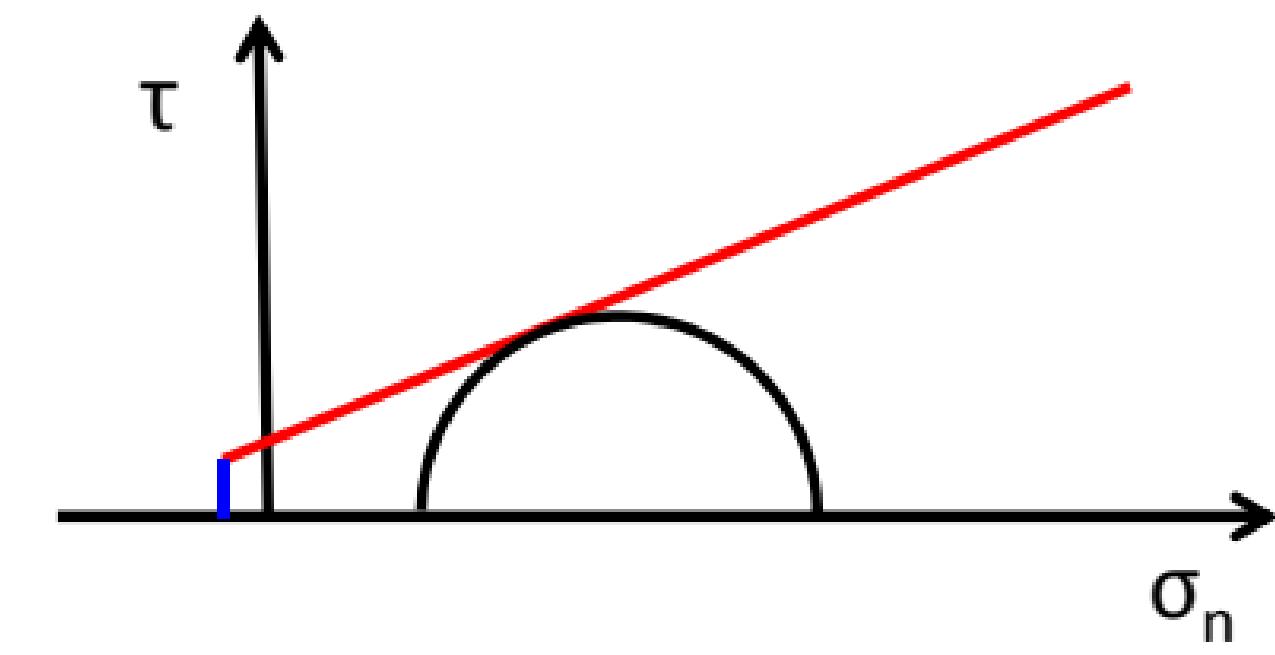




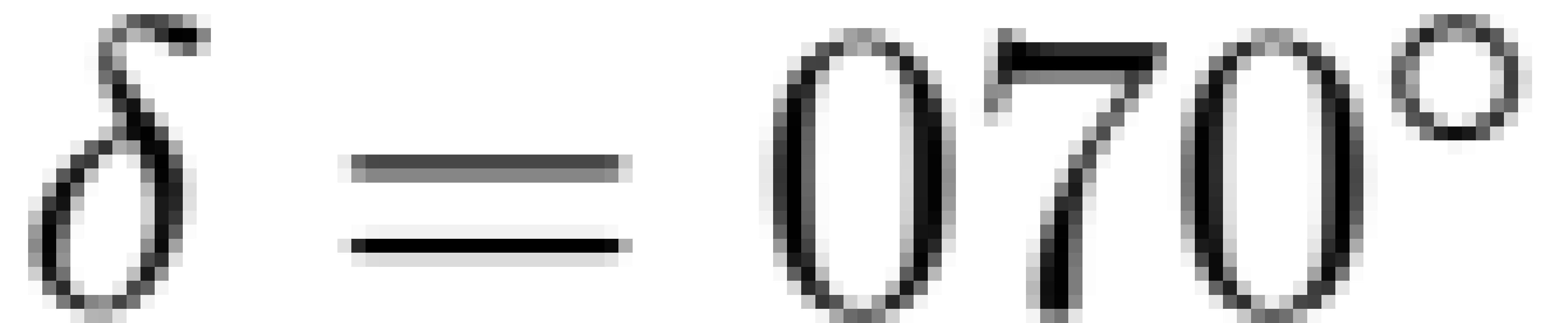


Principal stresses at the wellbore wall

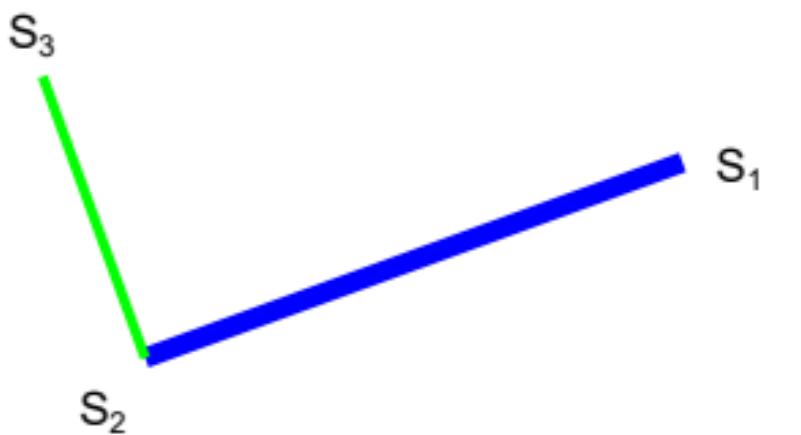
$$\left\{ \begin{array}{l} \sigma_{t\max} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} + \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \\ \sigma_{t\min} = \frac{\sigma_{zz} + \sigma_{\theta\theta}}{2} - \sqrt{\left(\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right)^2 + \tau_{\theta z}^2} \end{array} \right.$$



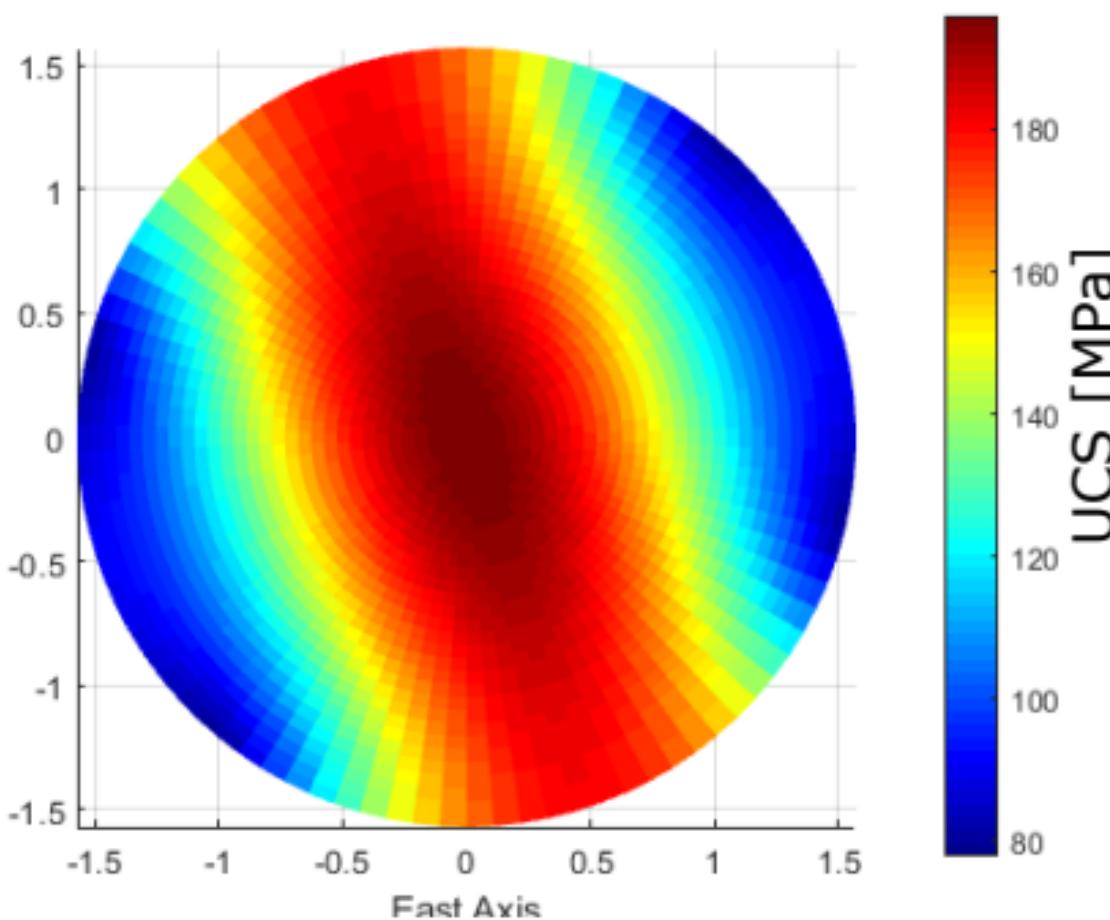




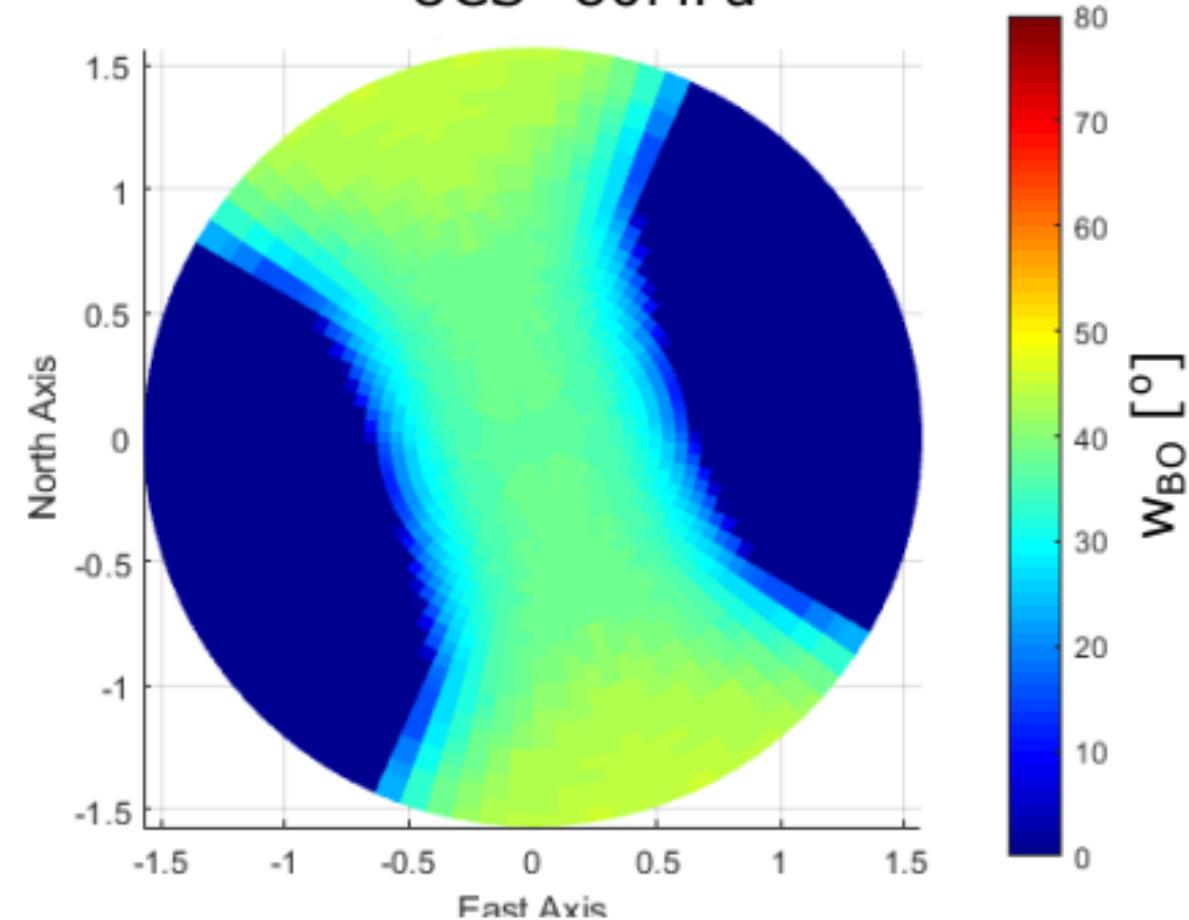
Geographical principal stresses

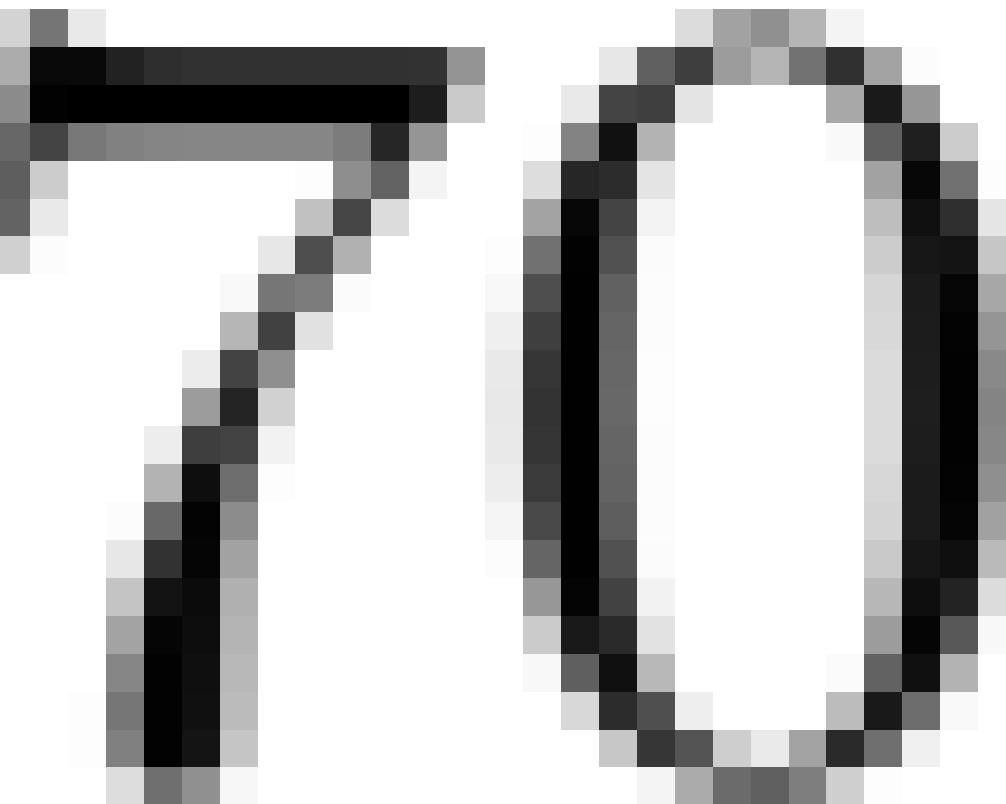


Required UCS ($P_w = P_p$)



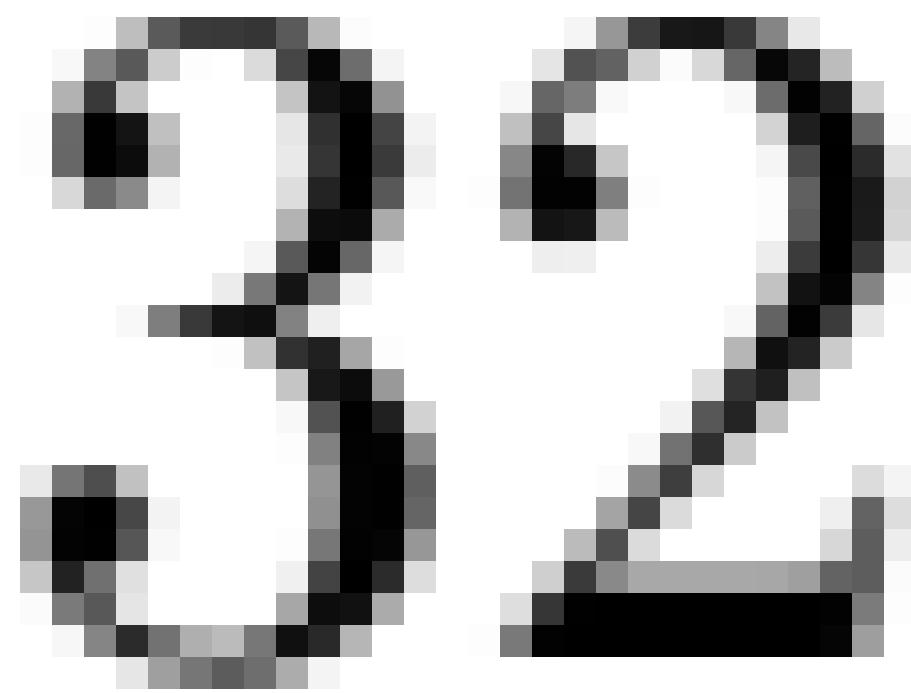
Breakout angle by Lade - $P_w = 45\text{MPa}$ UCS=80MPa

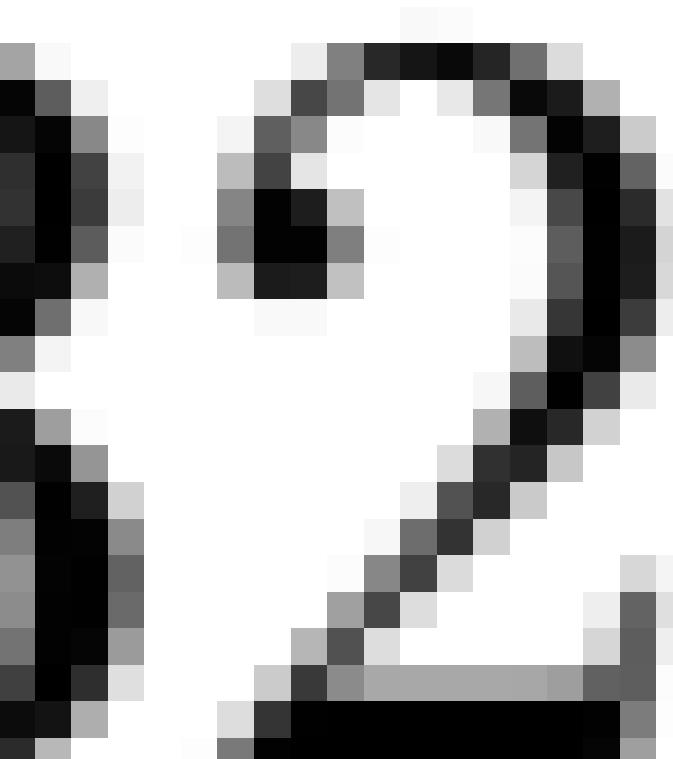
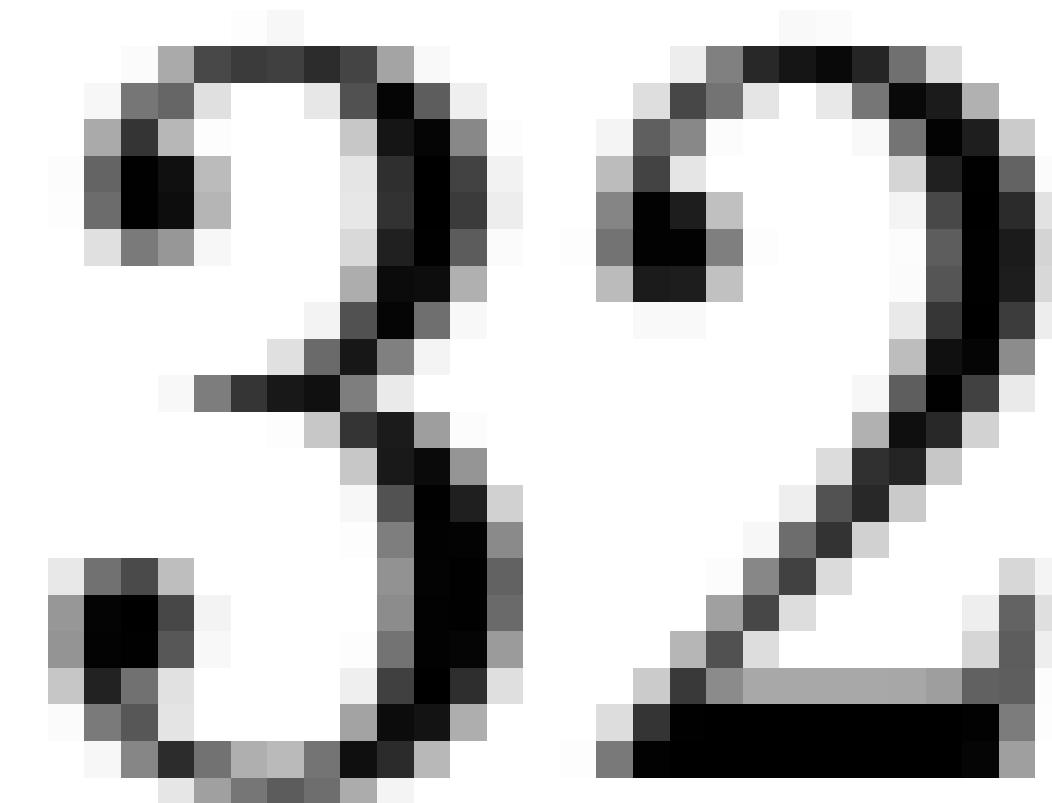
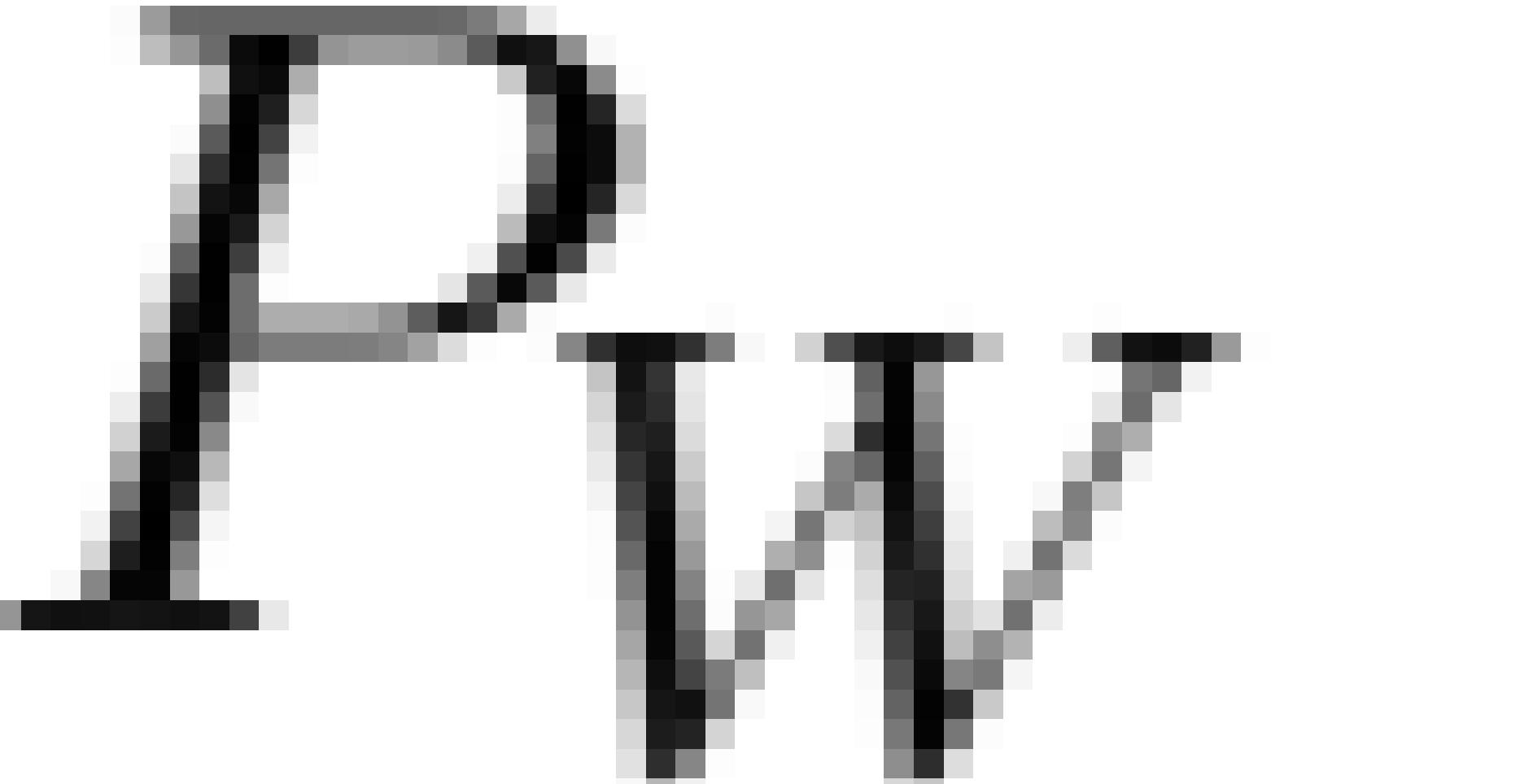


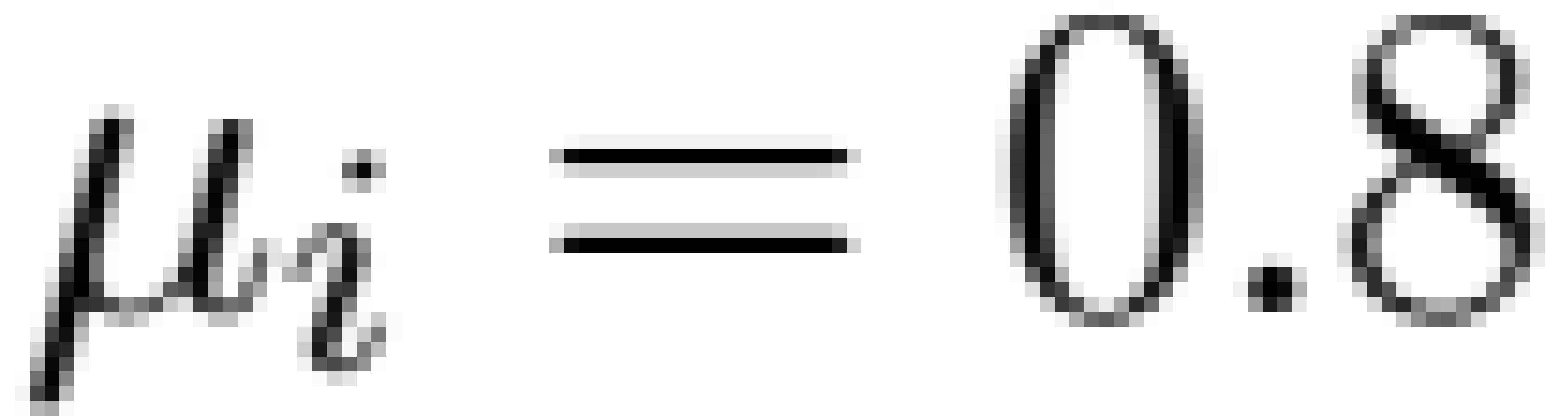




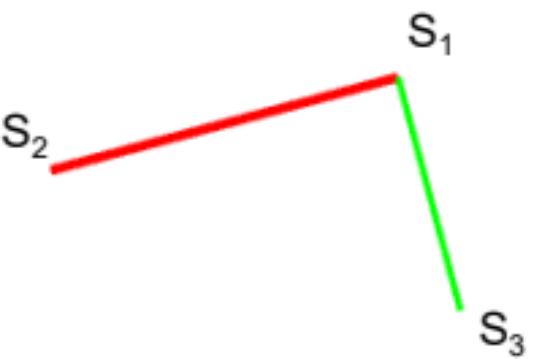




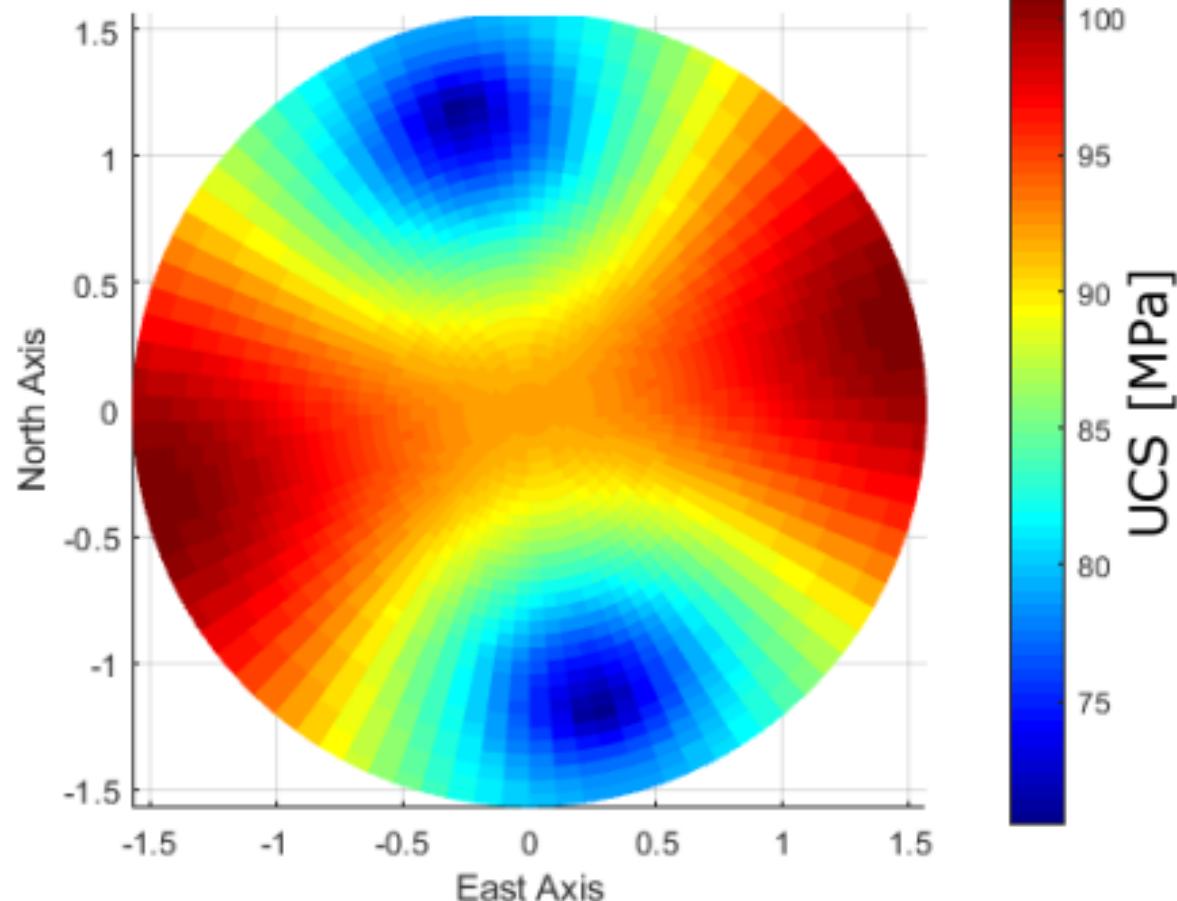




Principal stresses



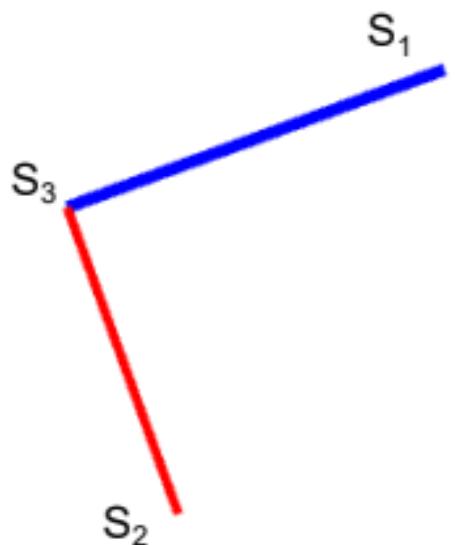
Required UCS ($P_w = P_p$)



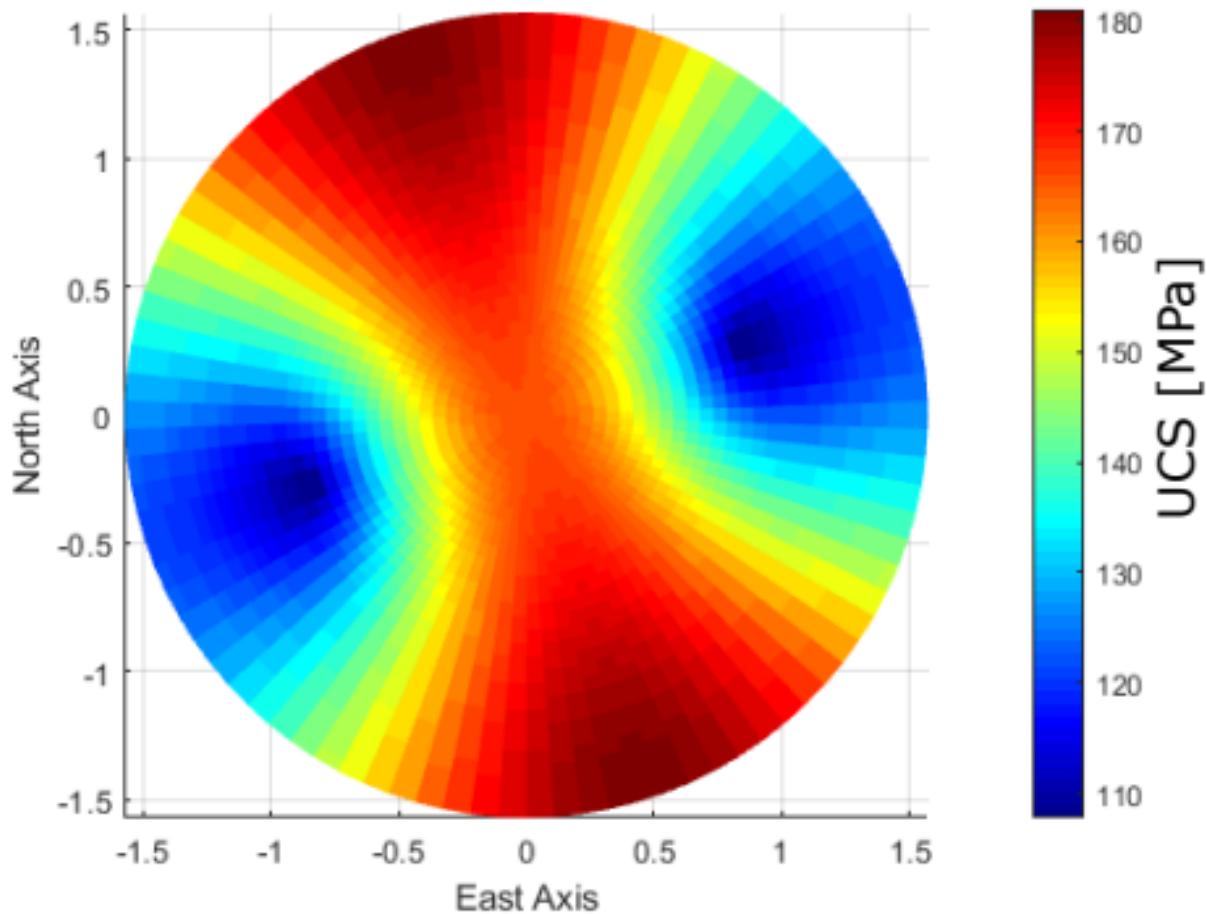




Principal stresses

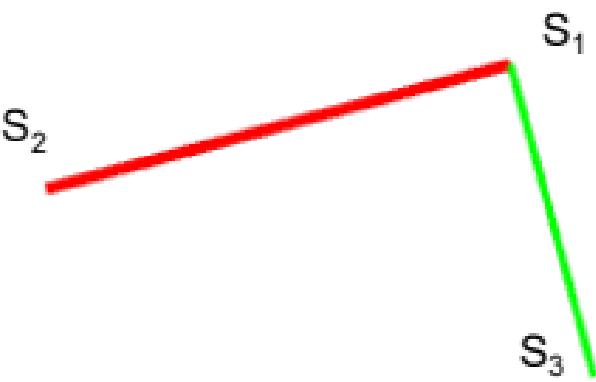


Required UCS ($P_w = P_p$)

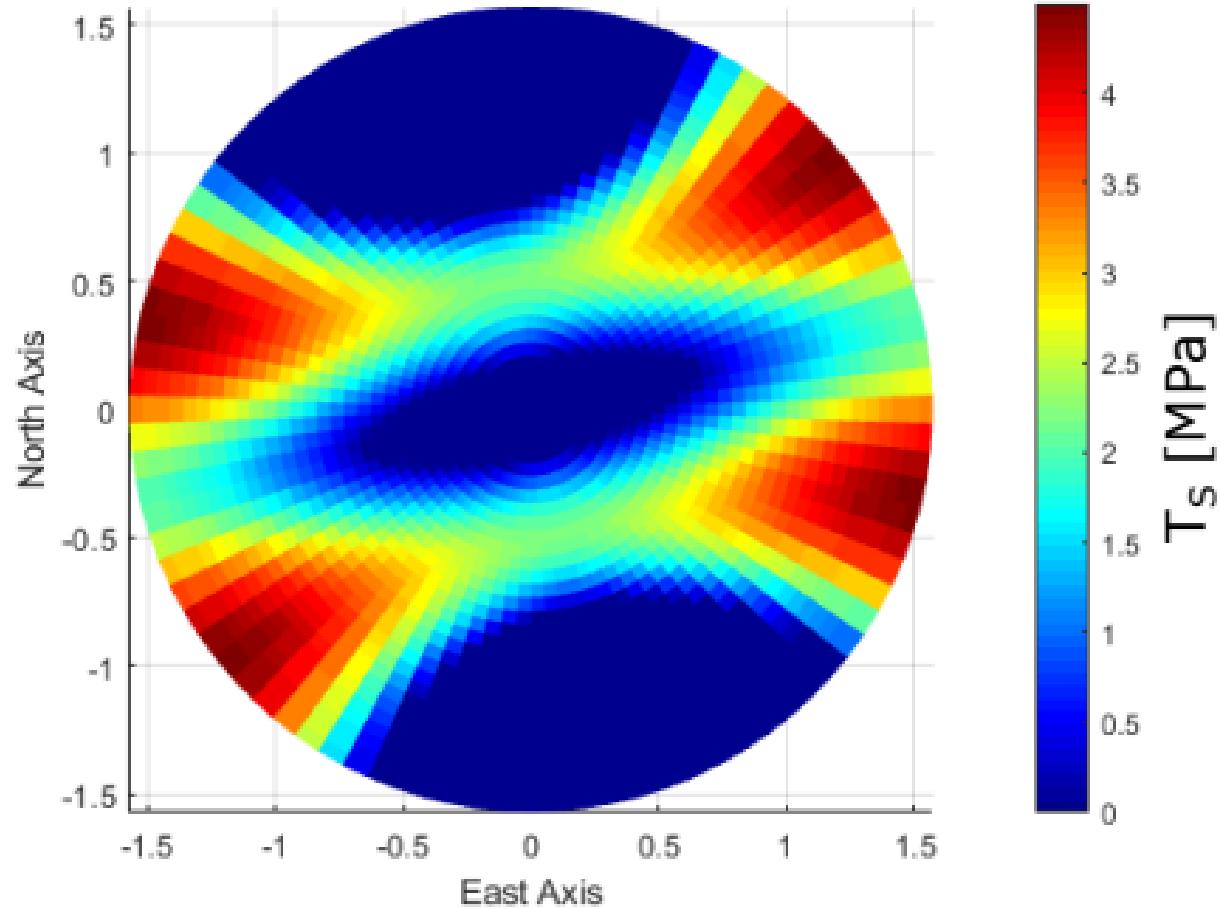




Principal stresses



Required T_s ($P_w=35\text{MPa}$)





$\delta\Omega$

Δ

$\delta\Omega$

Δ

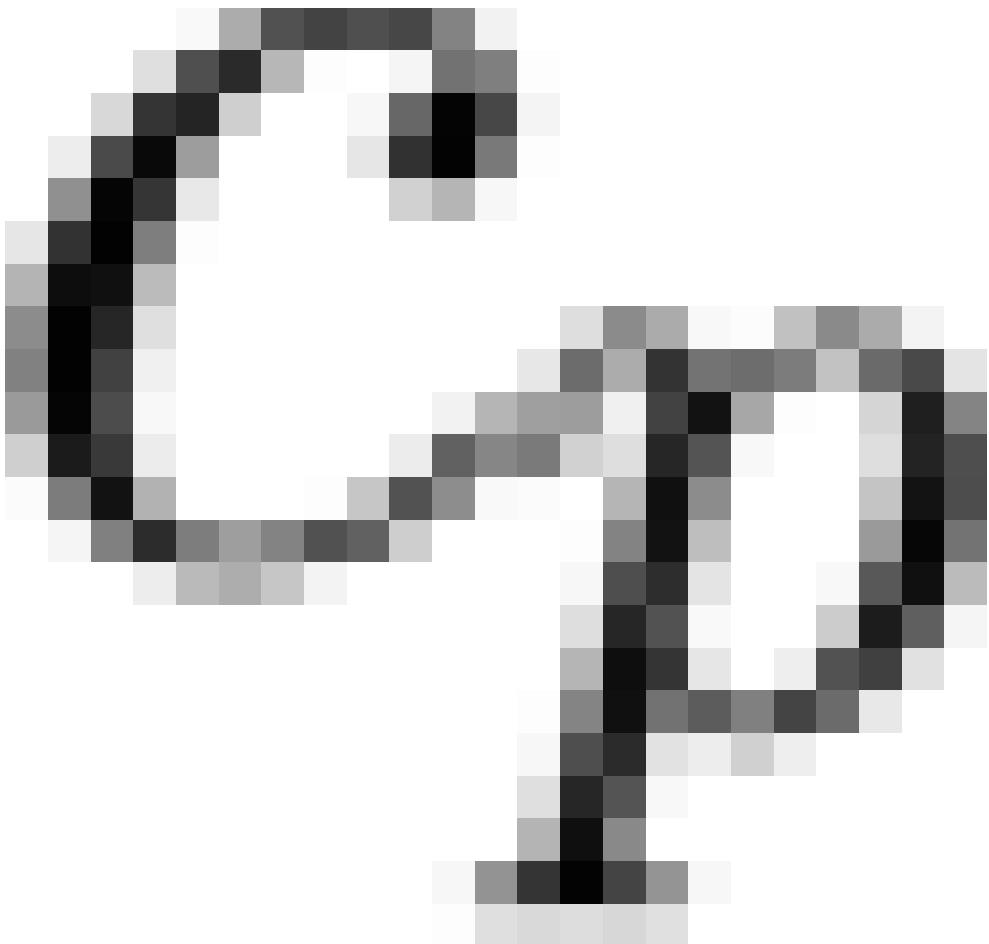
$\Delta\Omega$

$\Delta\Omega$

$\Delta\Omega$









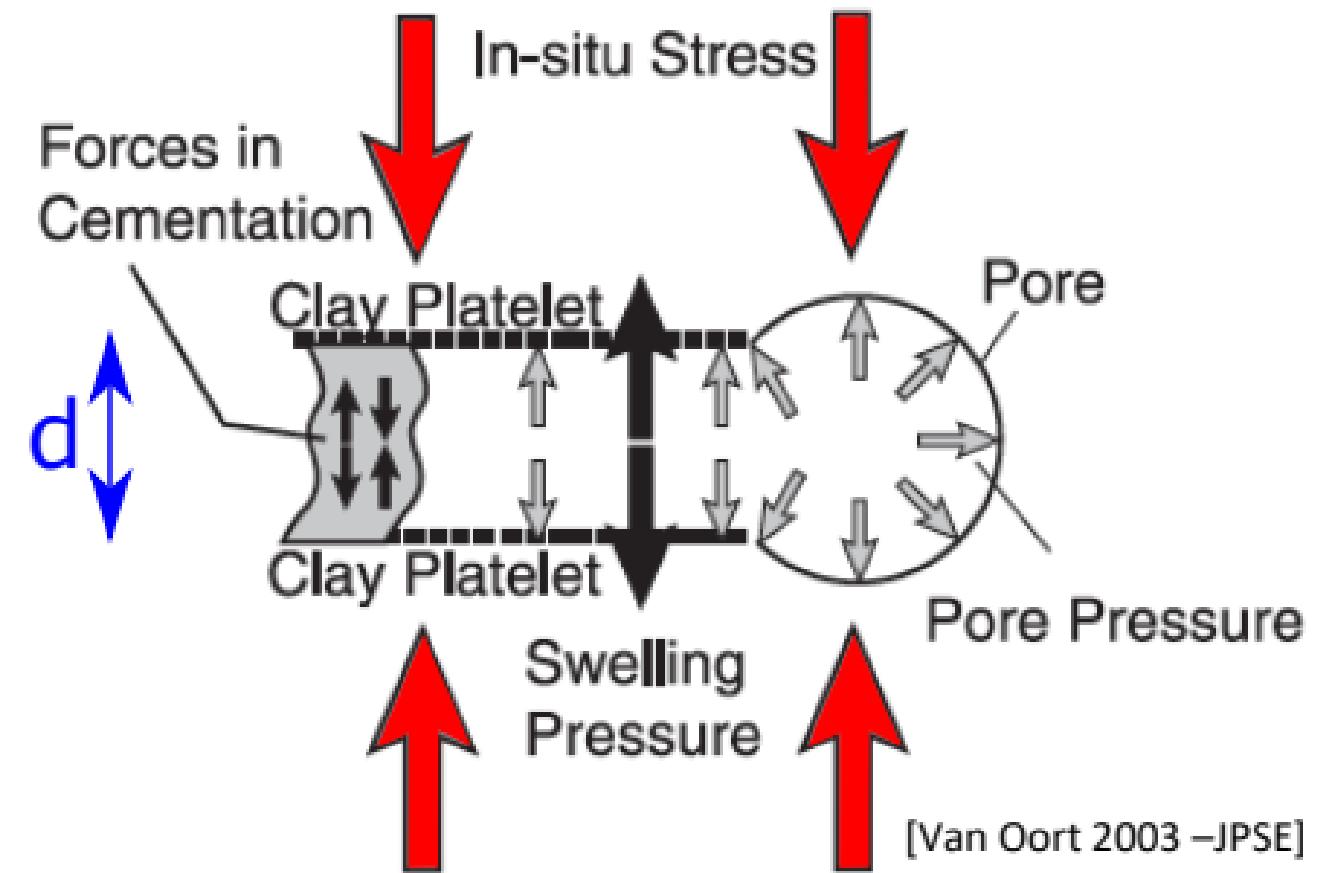
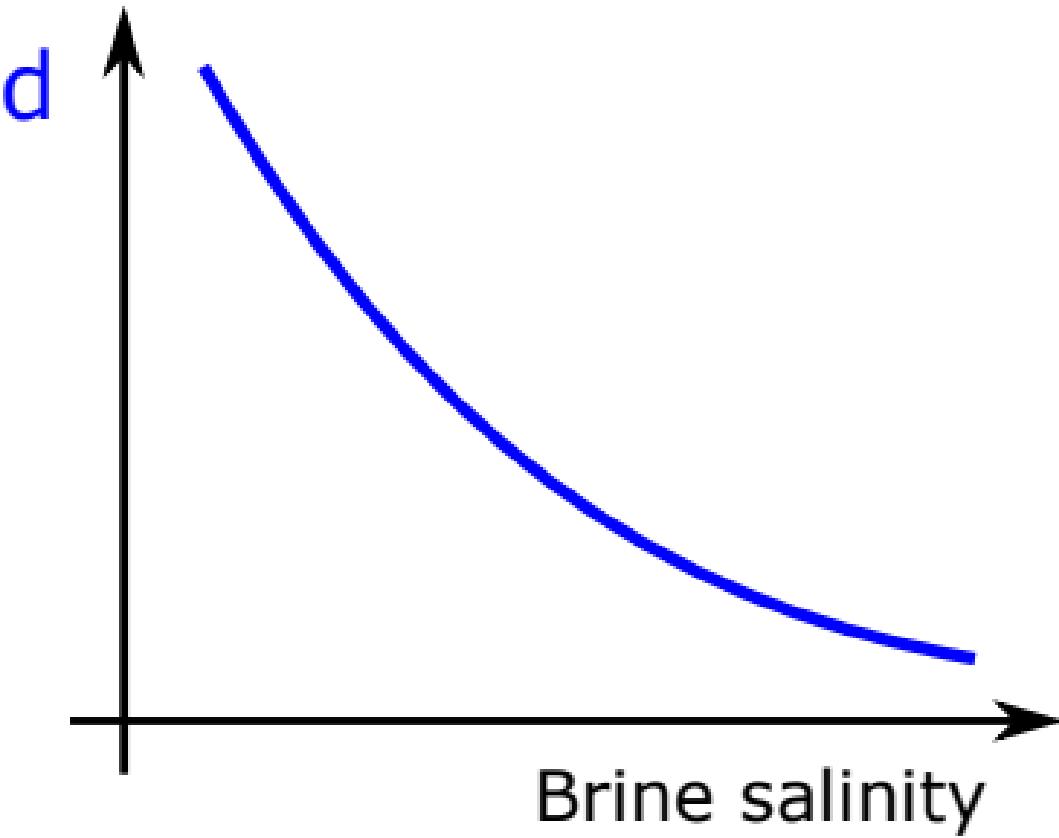


Δ $\sigma_{\theta\theta}$

$=$

$\alpha \Gamma B^{\dagger} \Delta^{\dagger} \Gamma$

$1 - \sqrt{ }$



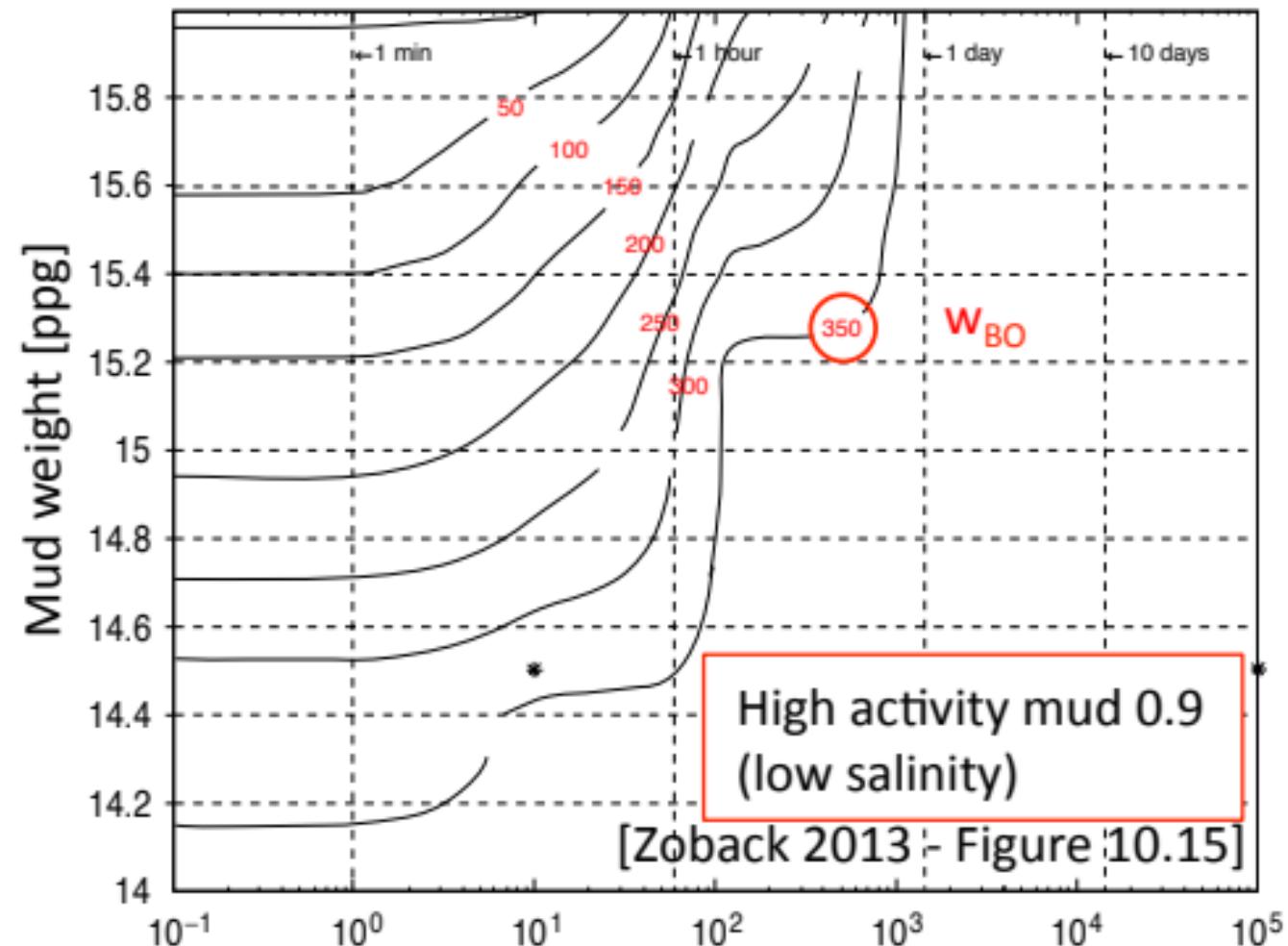
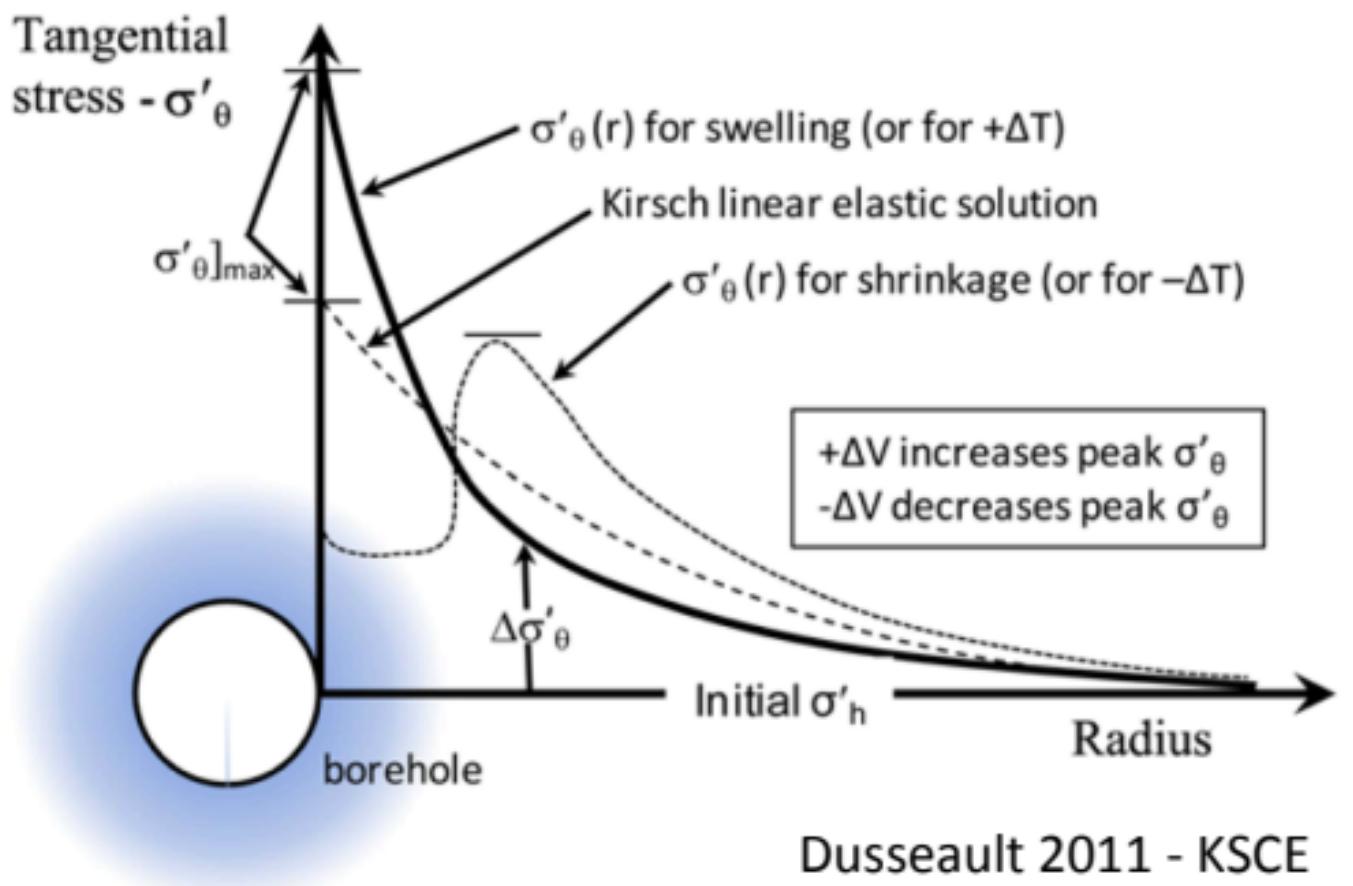
Norway shale



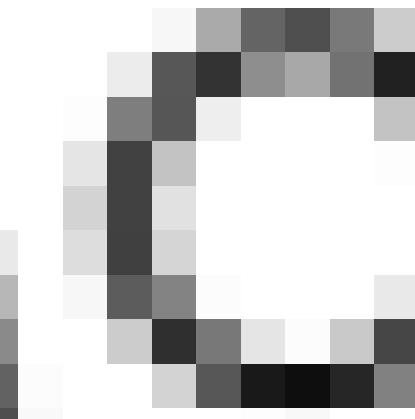
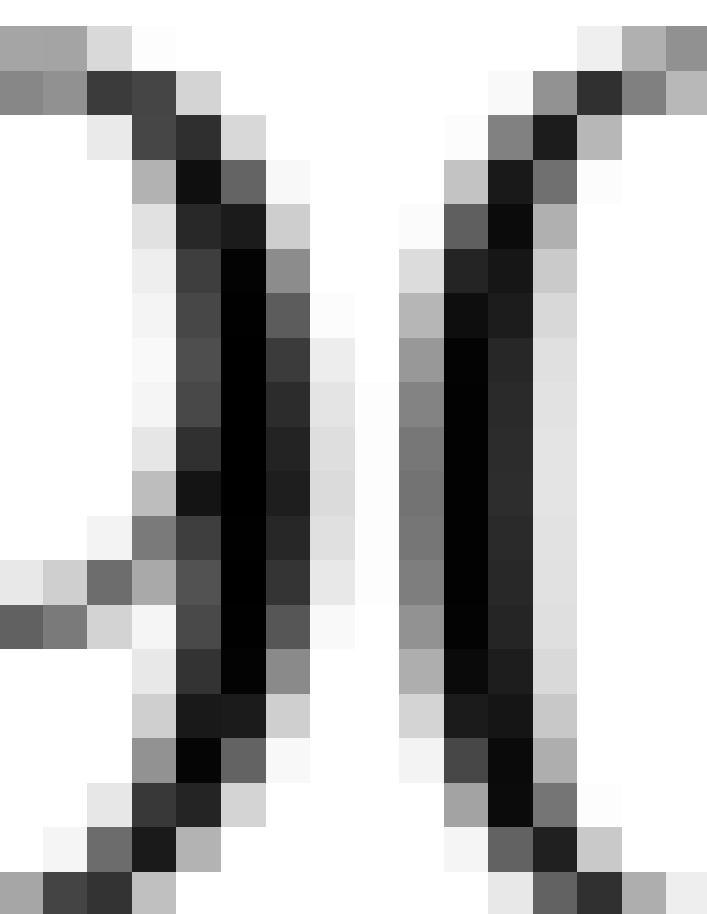
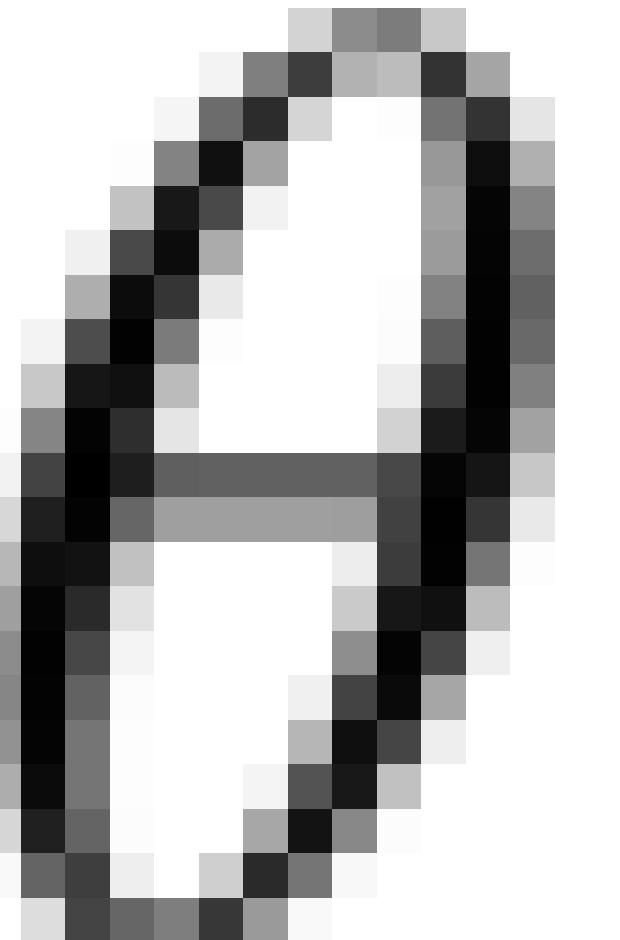
After 2hs exposition

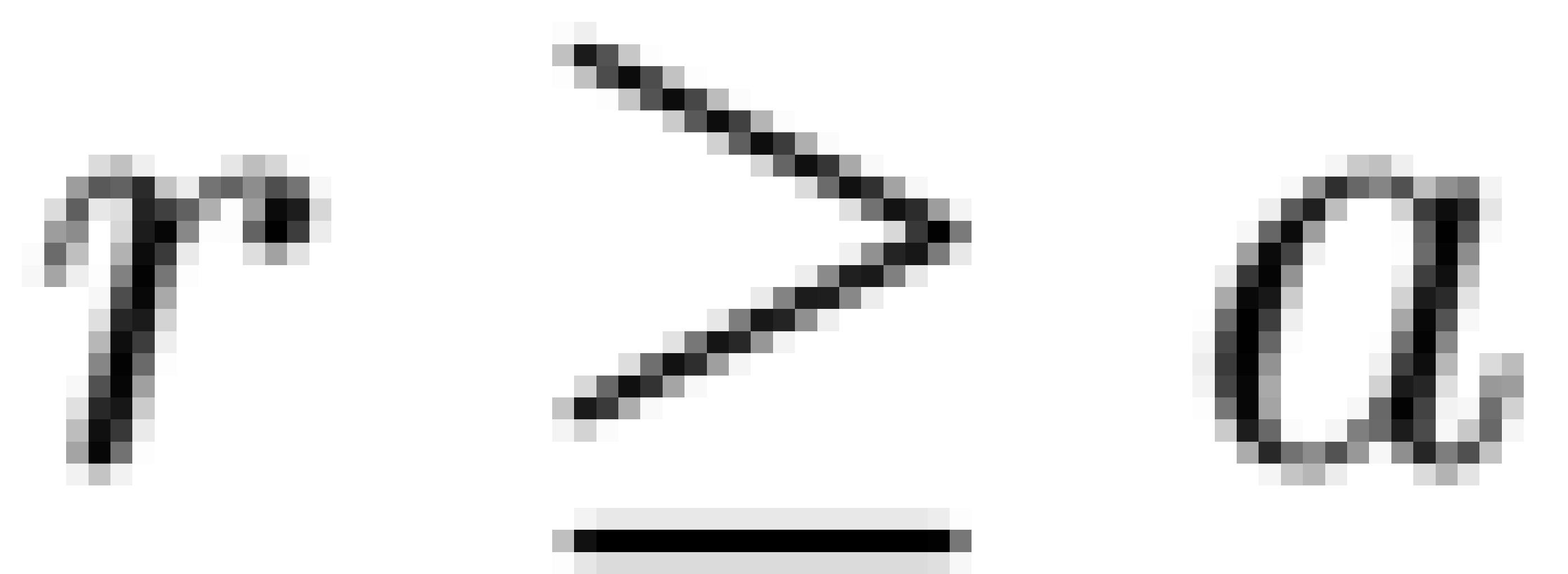


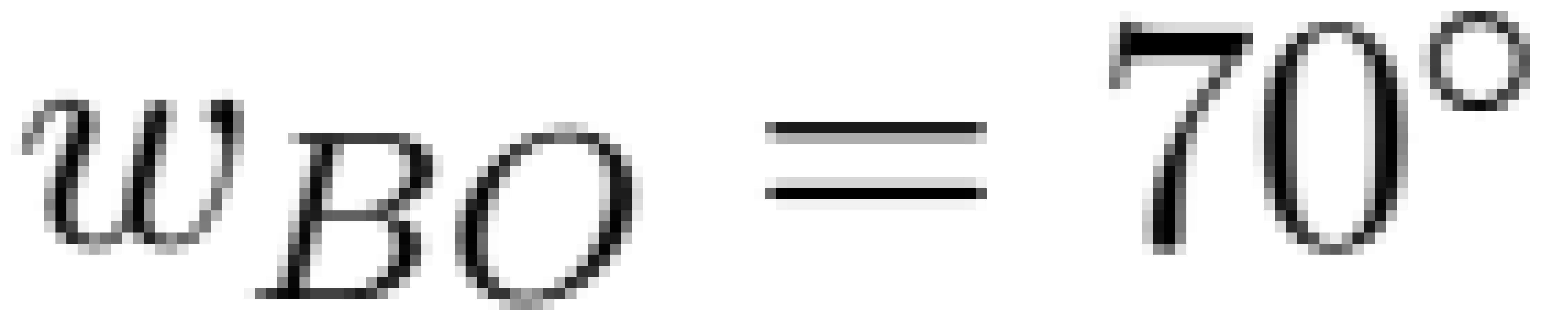
CPGE – M. Chenevert



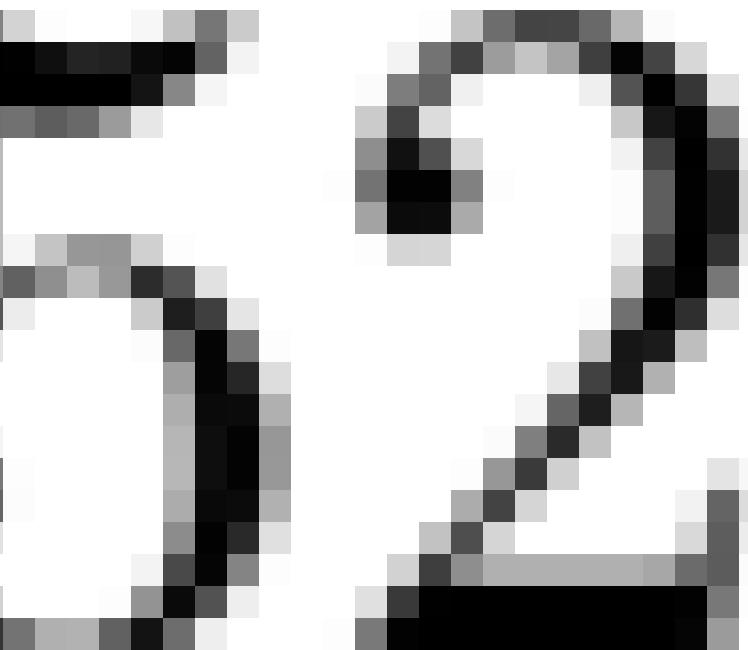
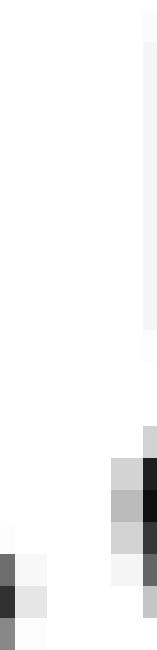
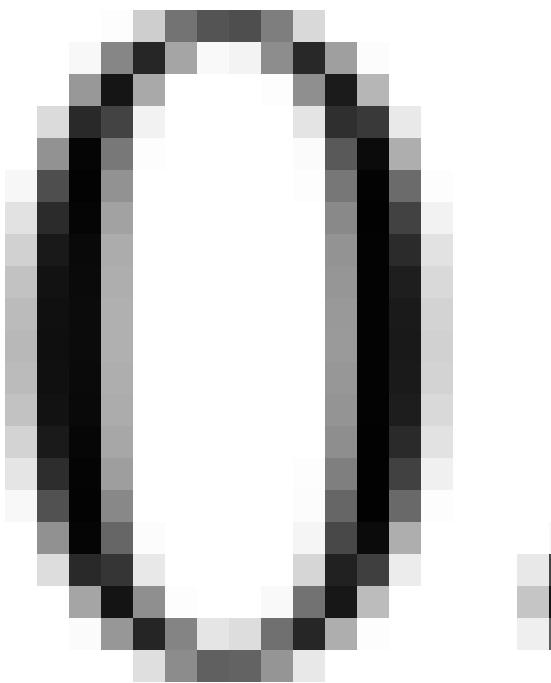


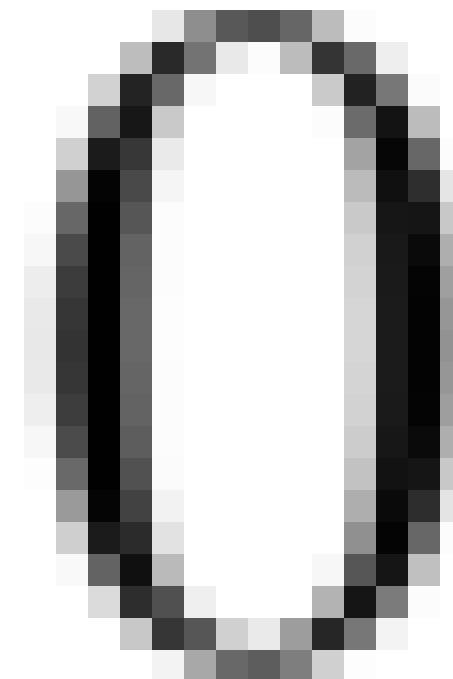
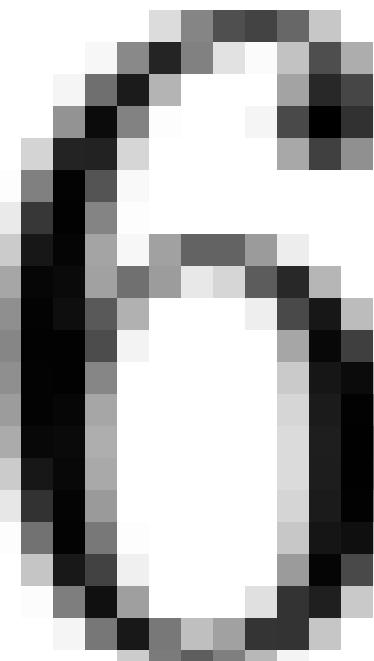
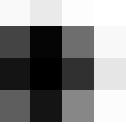
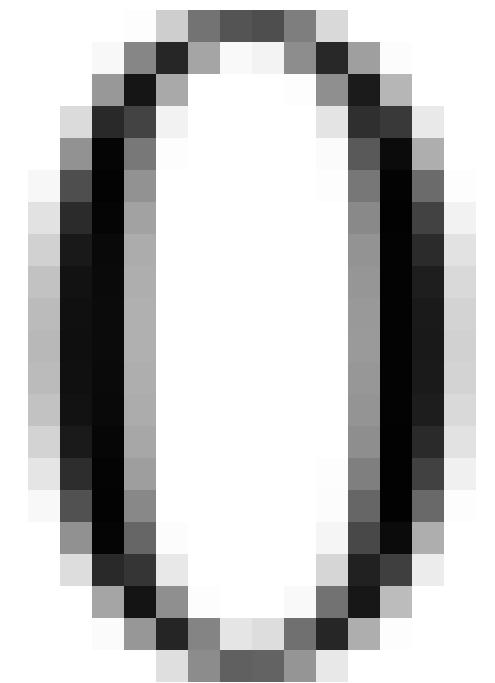


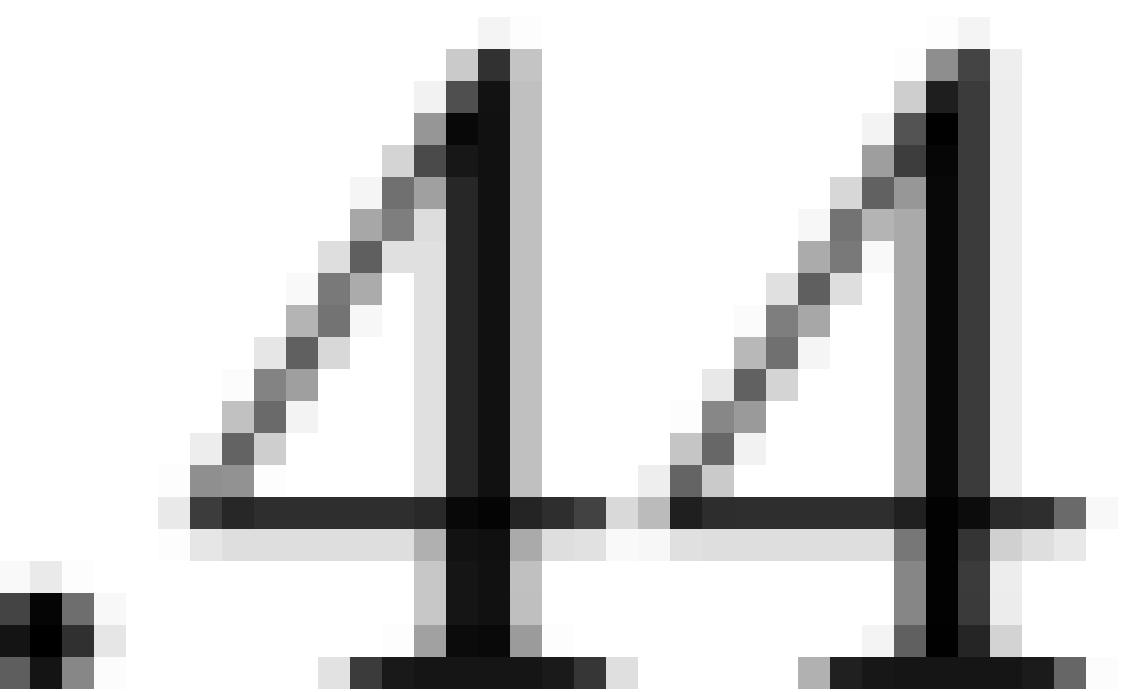
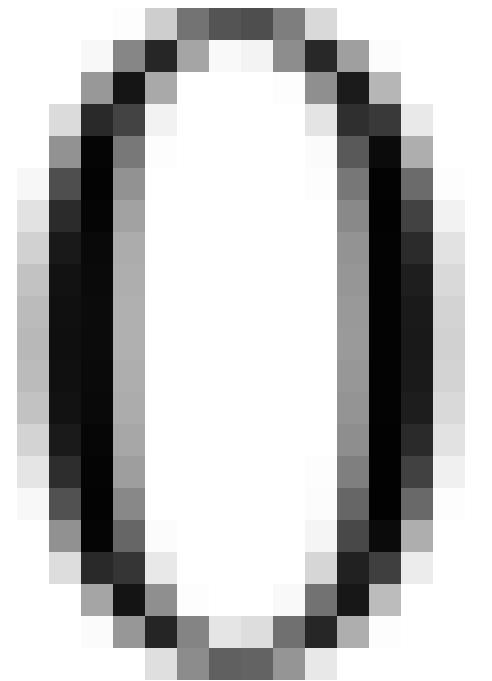


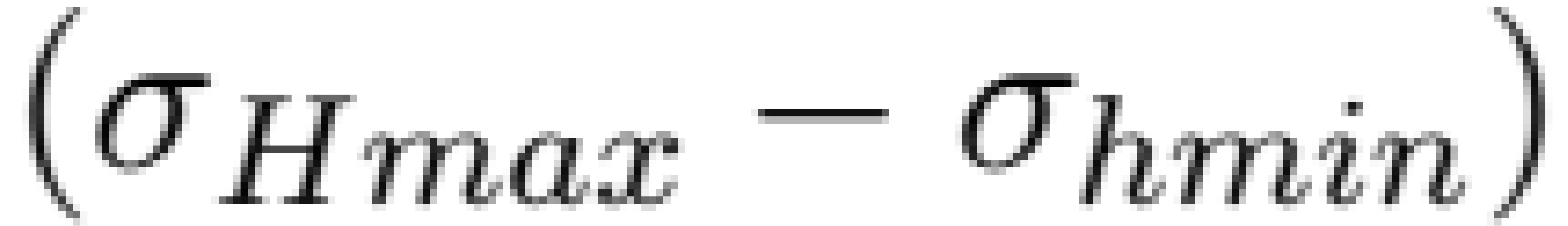


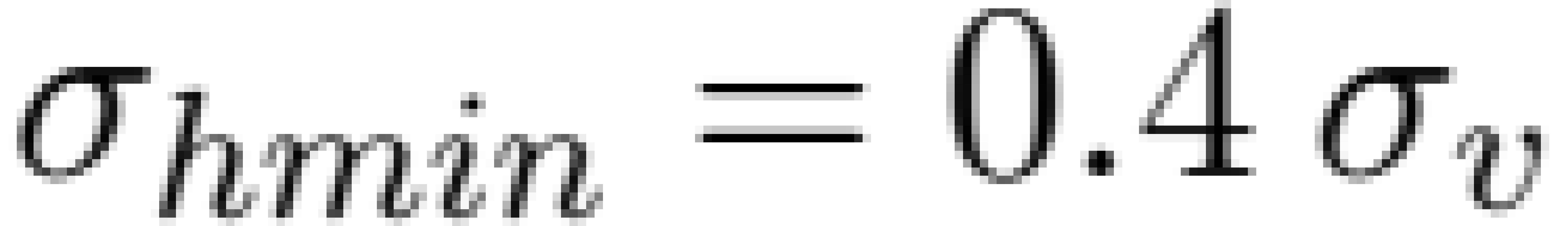


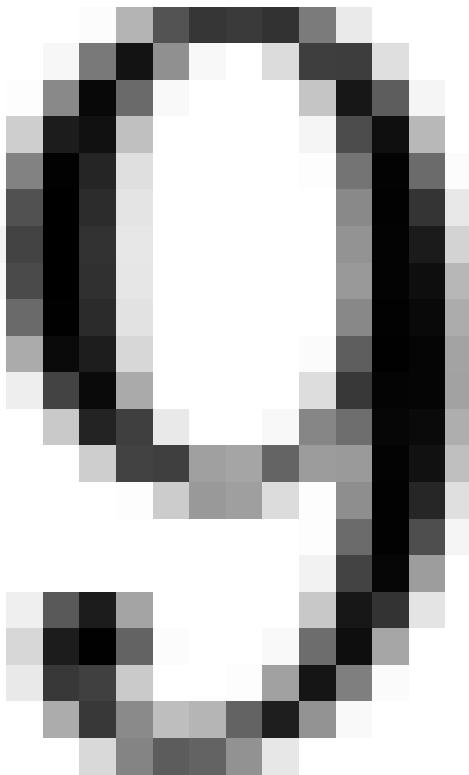
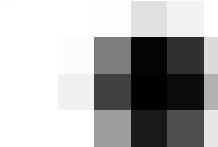
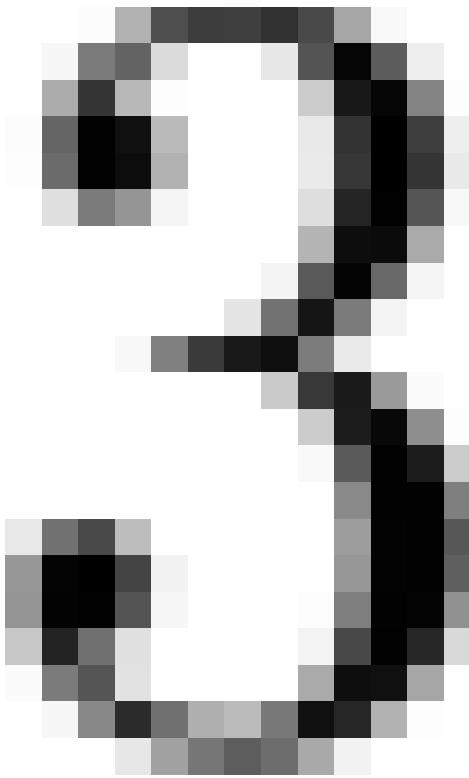
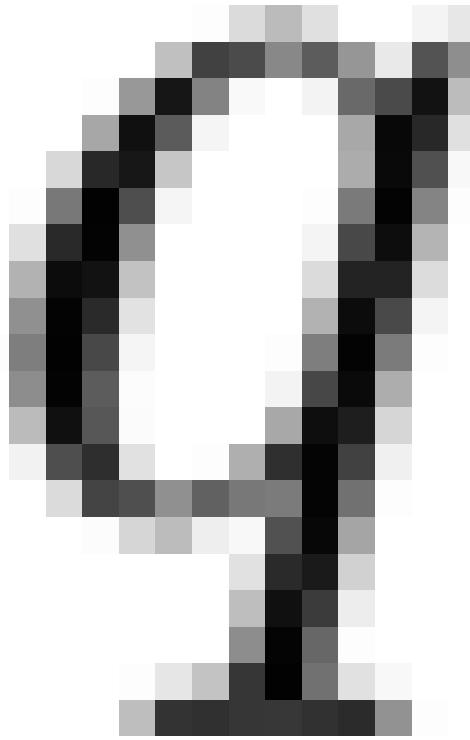


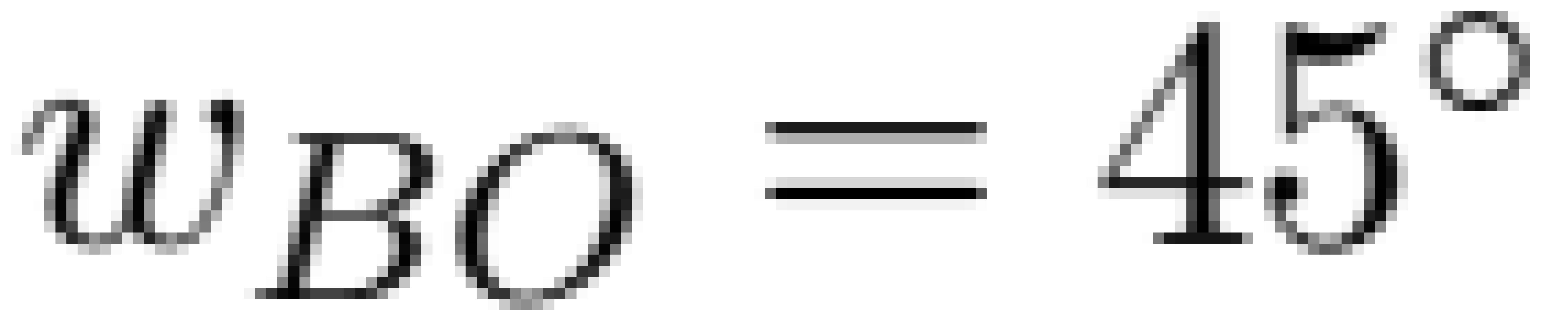








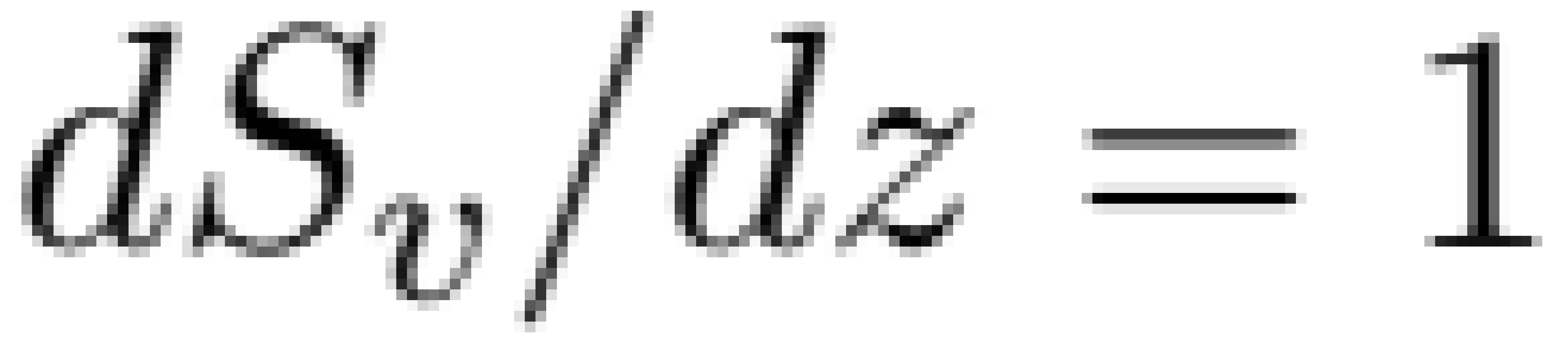




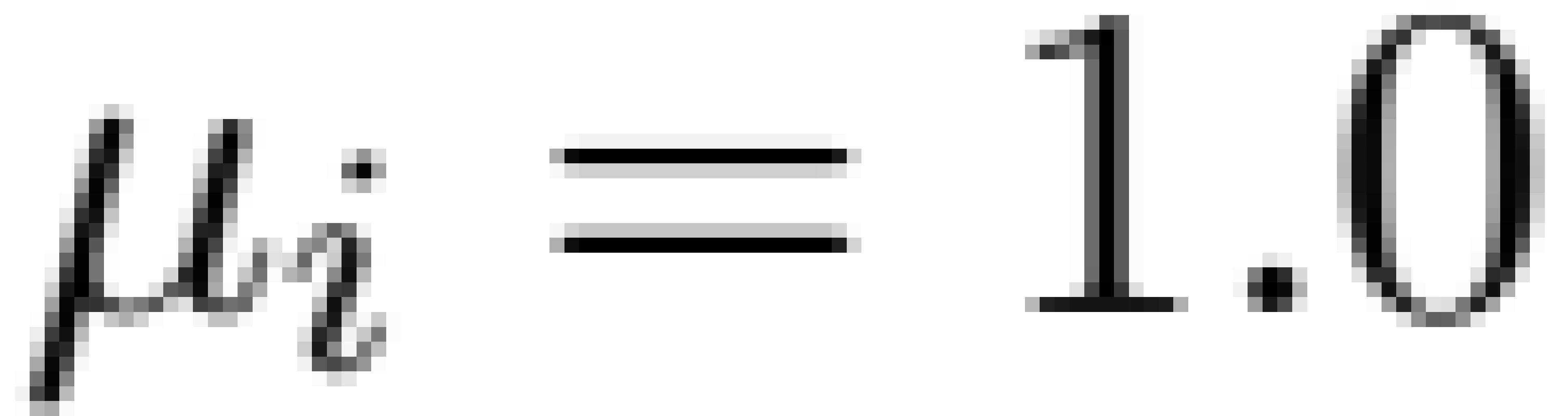




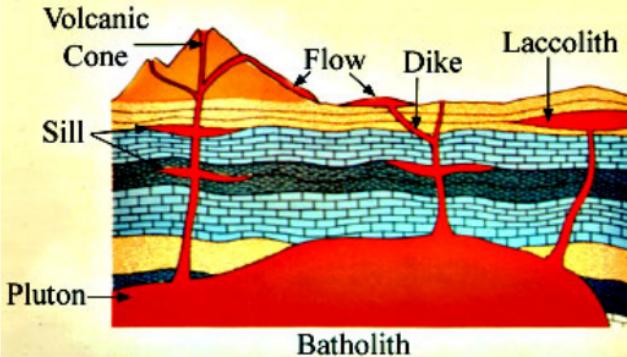


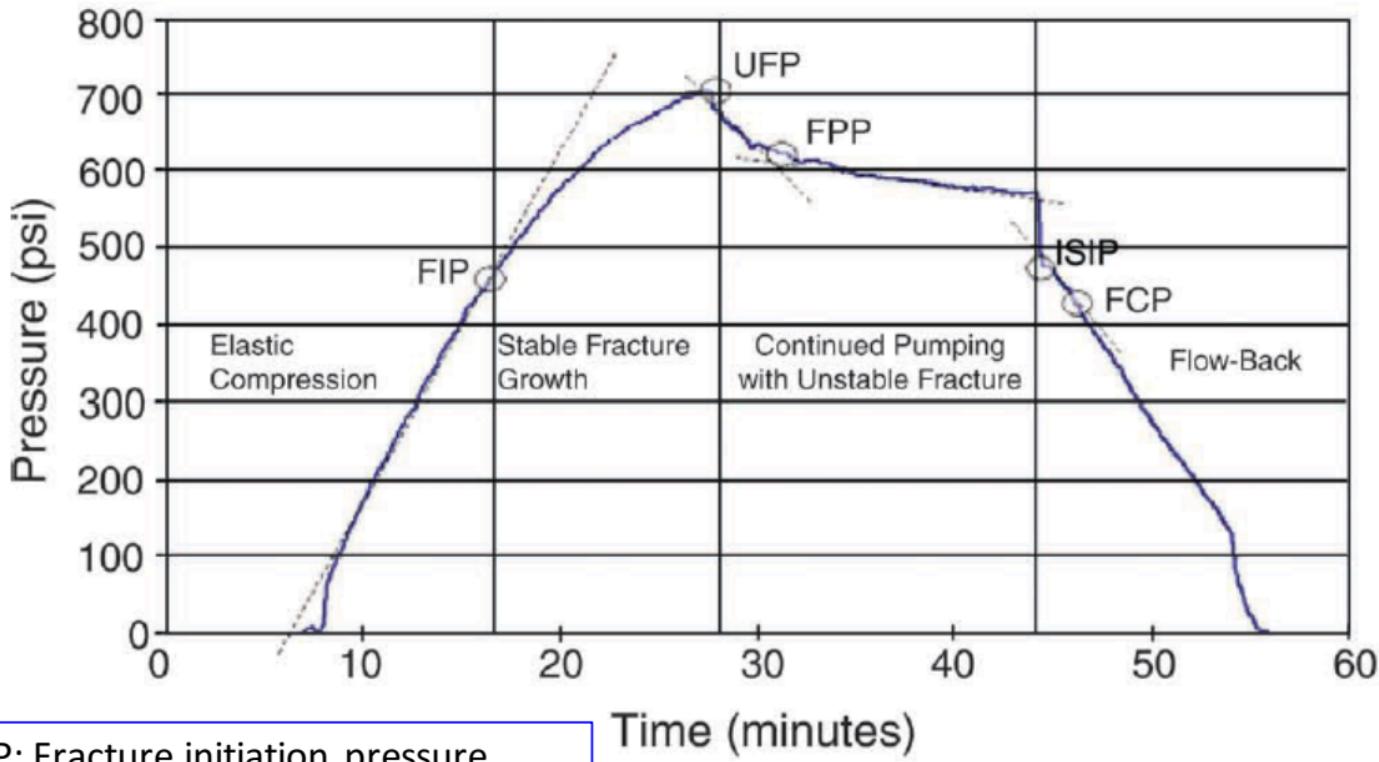






PLUTONS & VOLCANIC LANDFORMS





FIP: Fracture initiation pressure

UFP: Unstable fracture pressure

FPP: Fracture propagation pressure

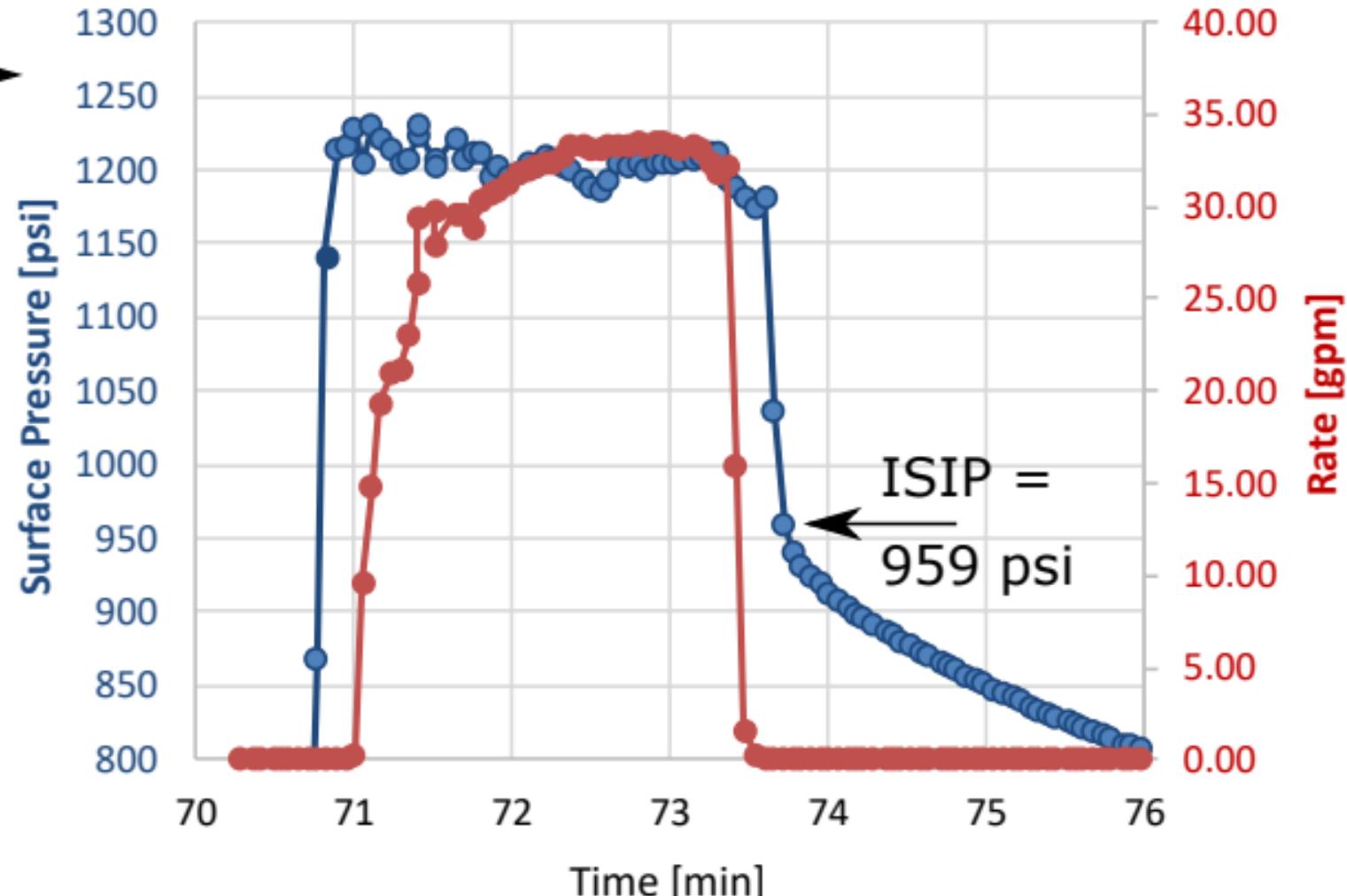
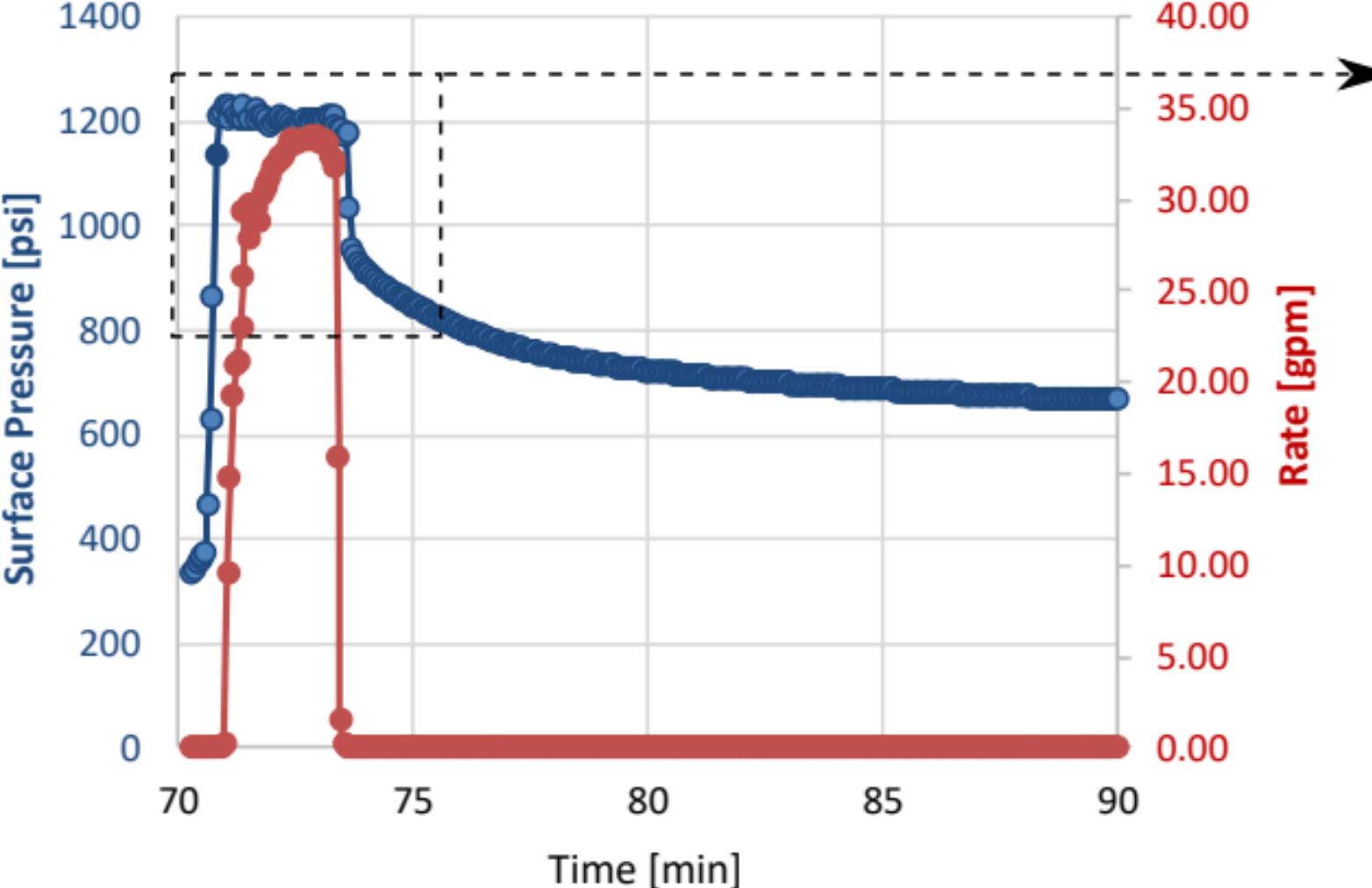
ISIP: Instantaneous shut-in pressure

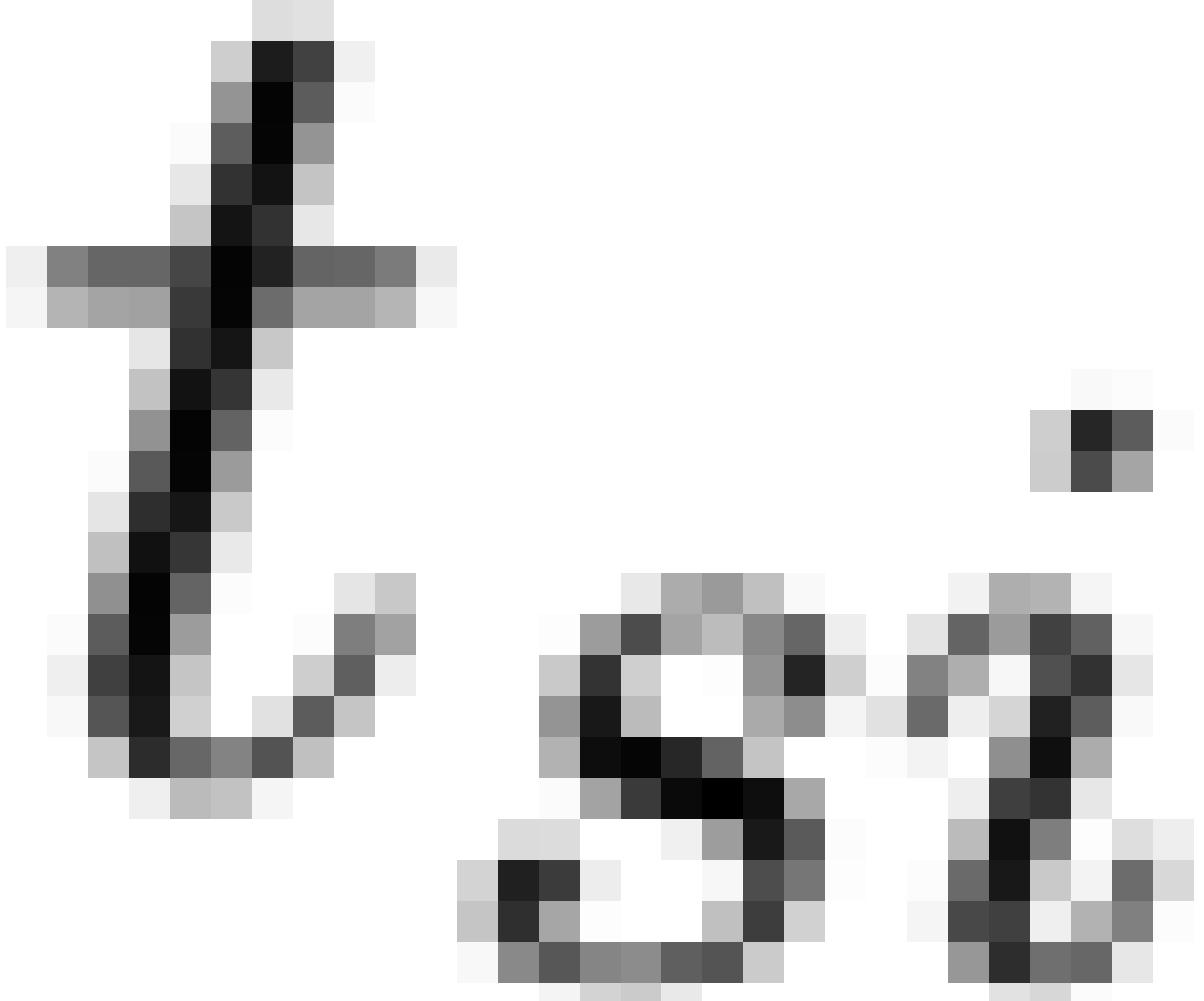
FCP: Fracture closure pressure

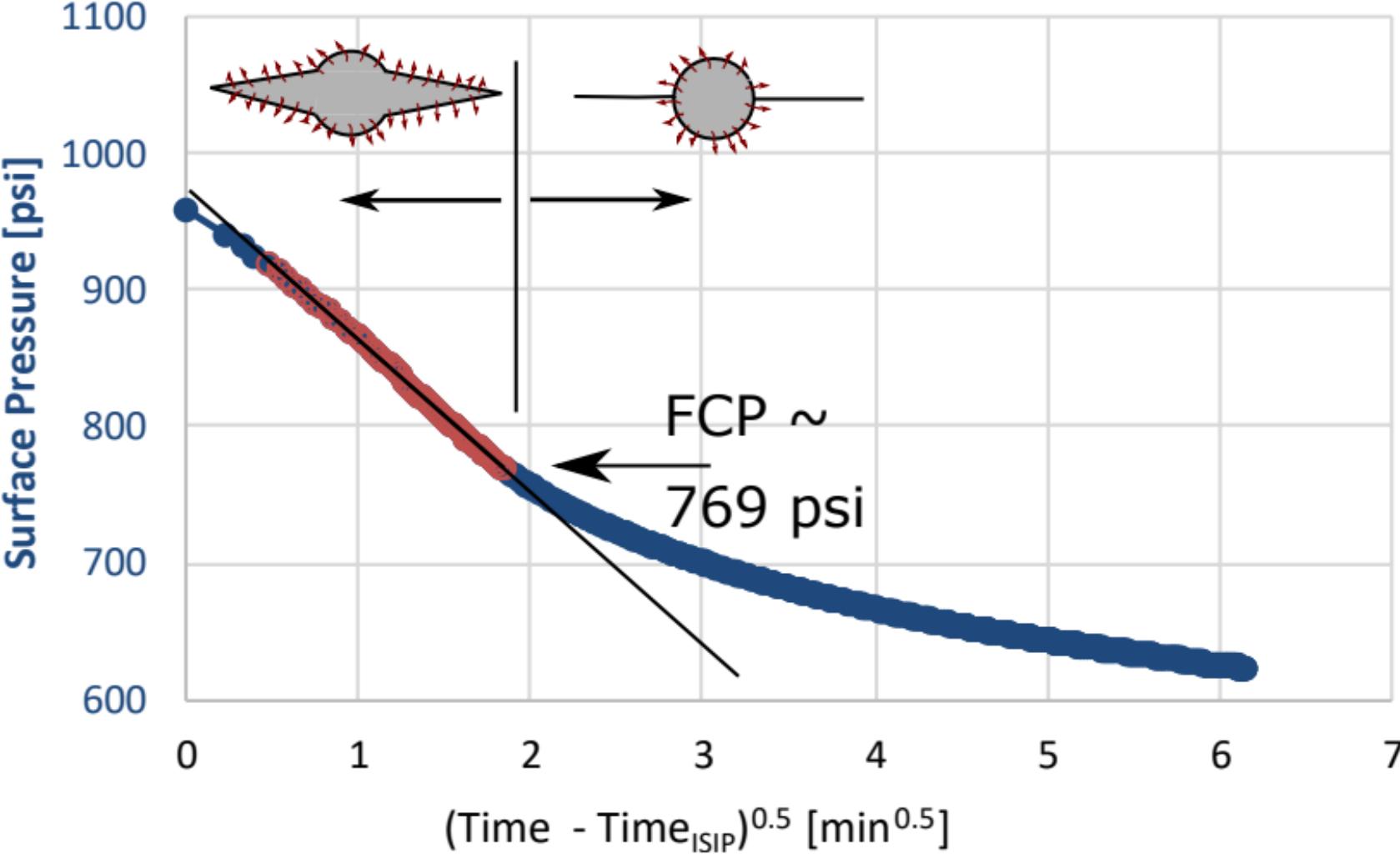
Time (minutes)

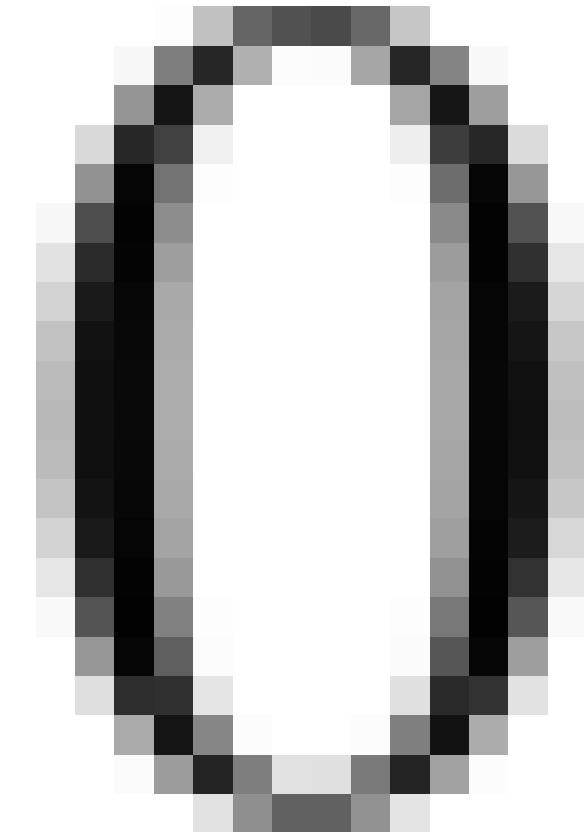
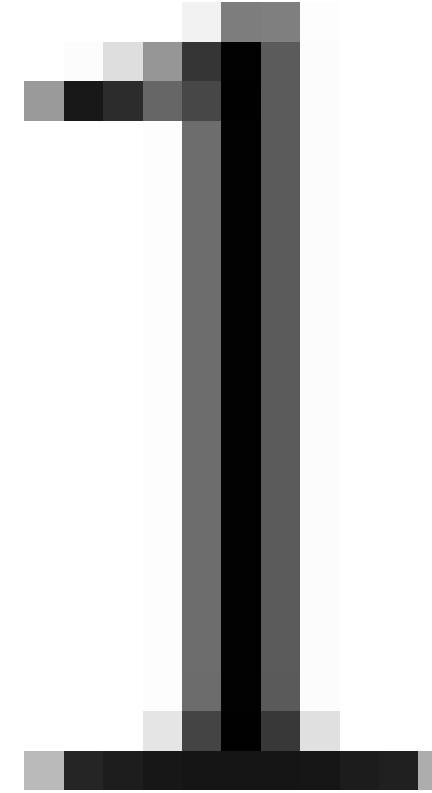
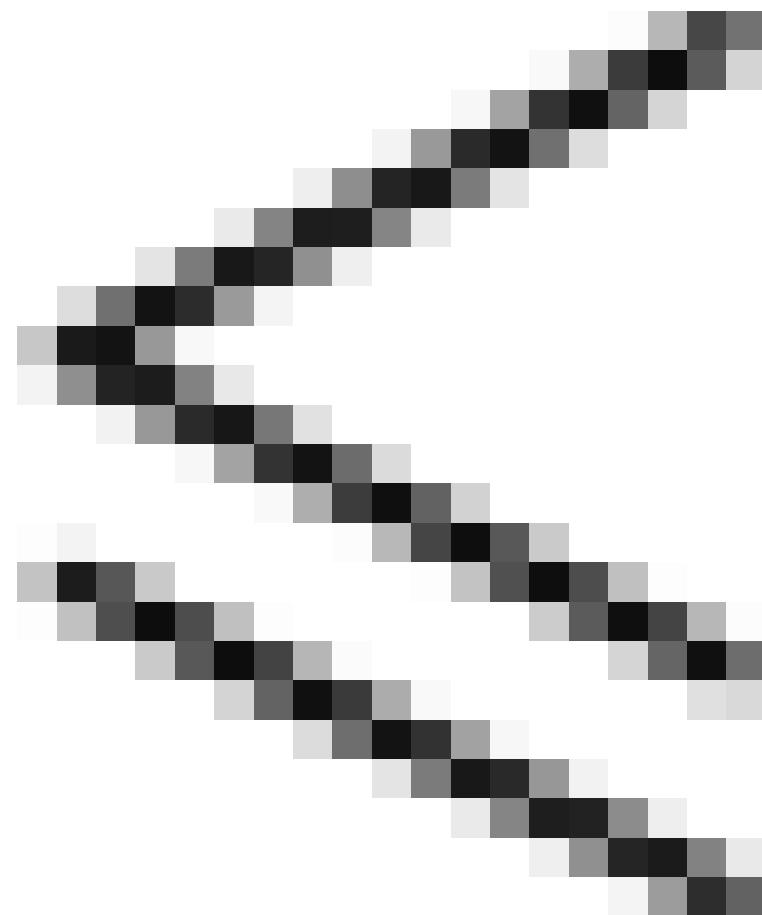
$$\Delta V = q \Delta t$$

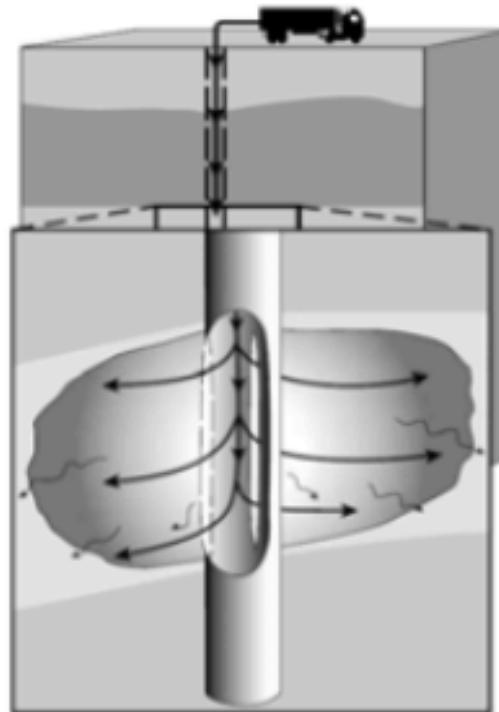
[van Oort and Vargo, 2008]



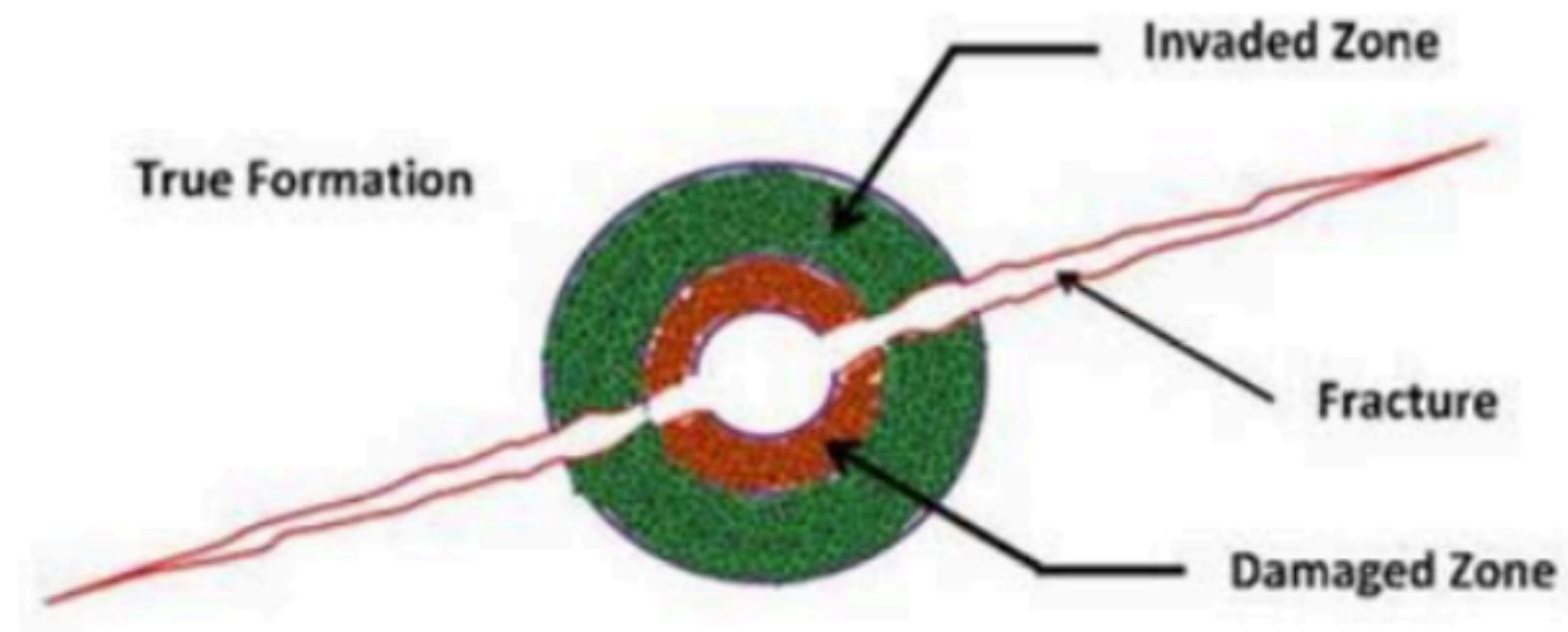








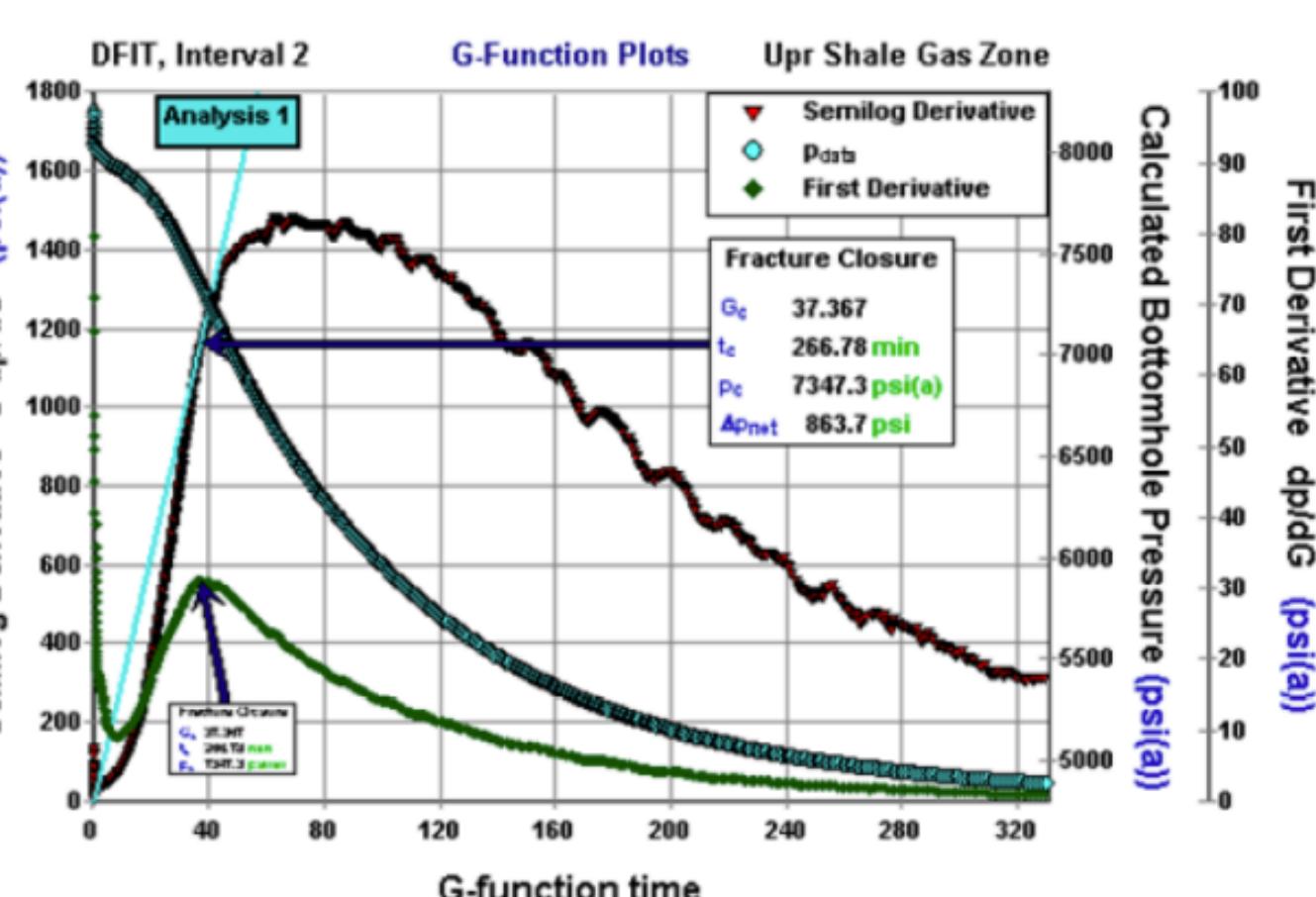
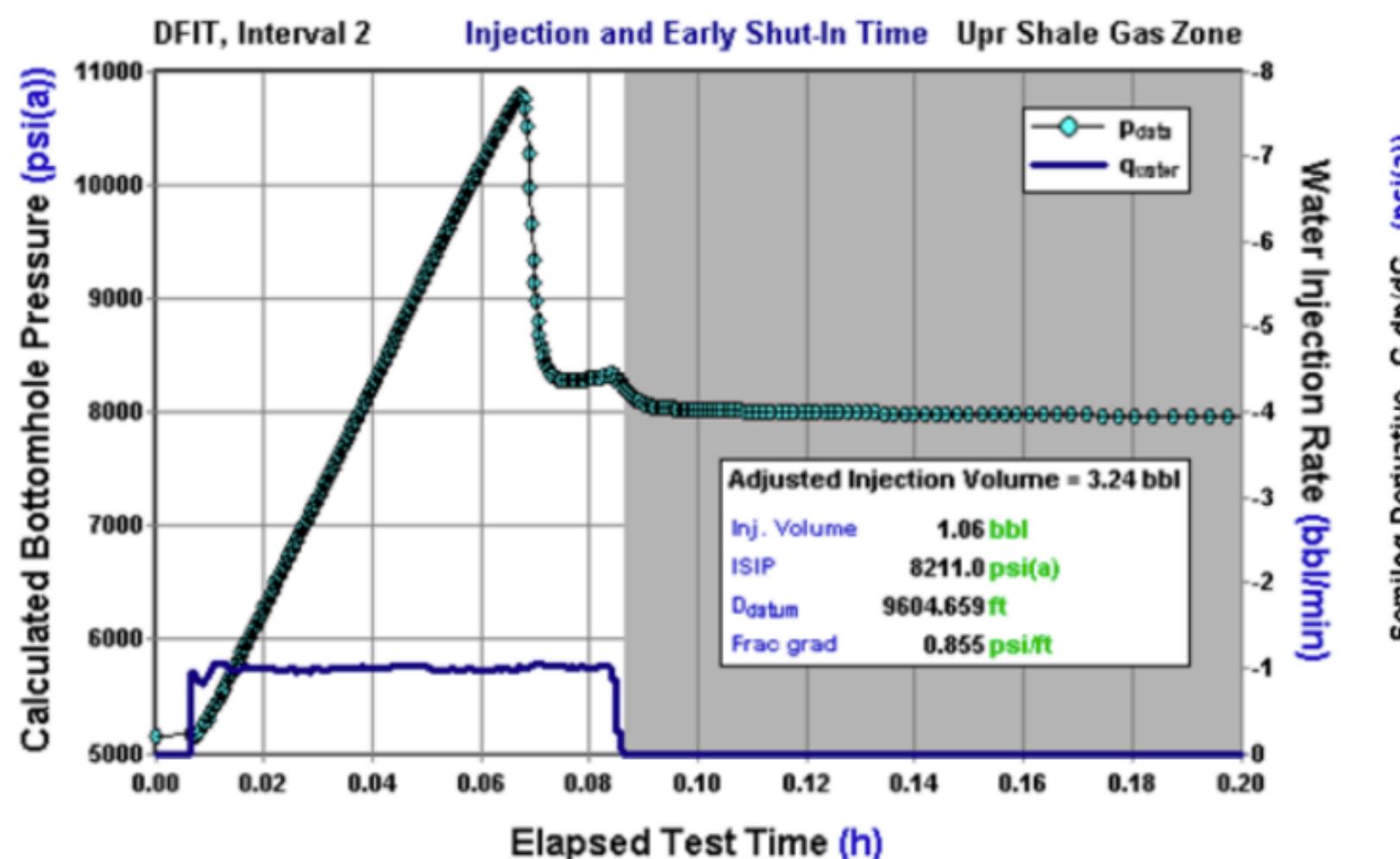
Side View



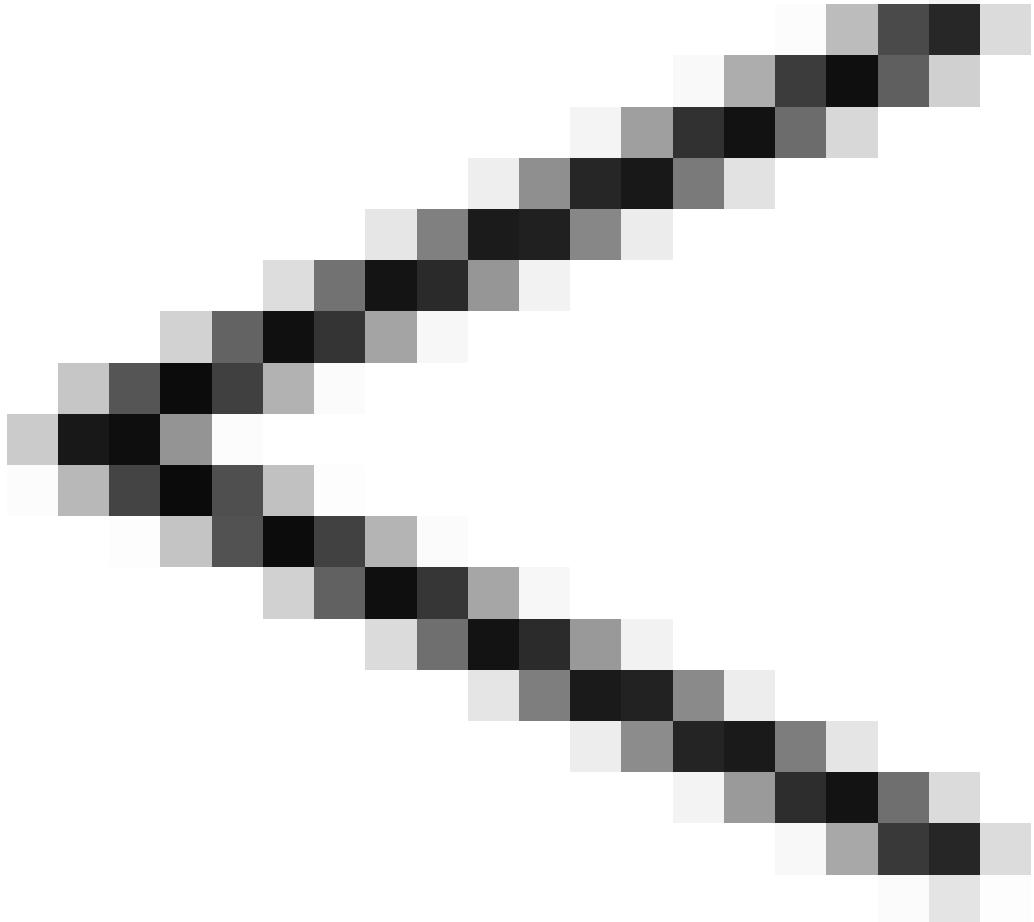
Plan View

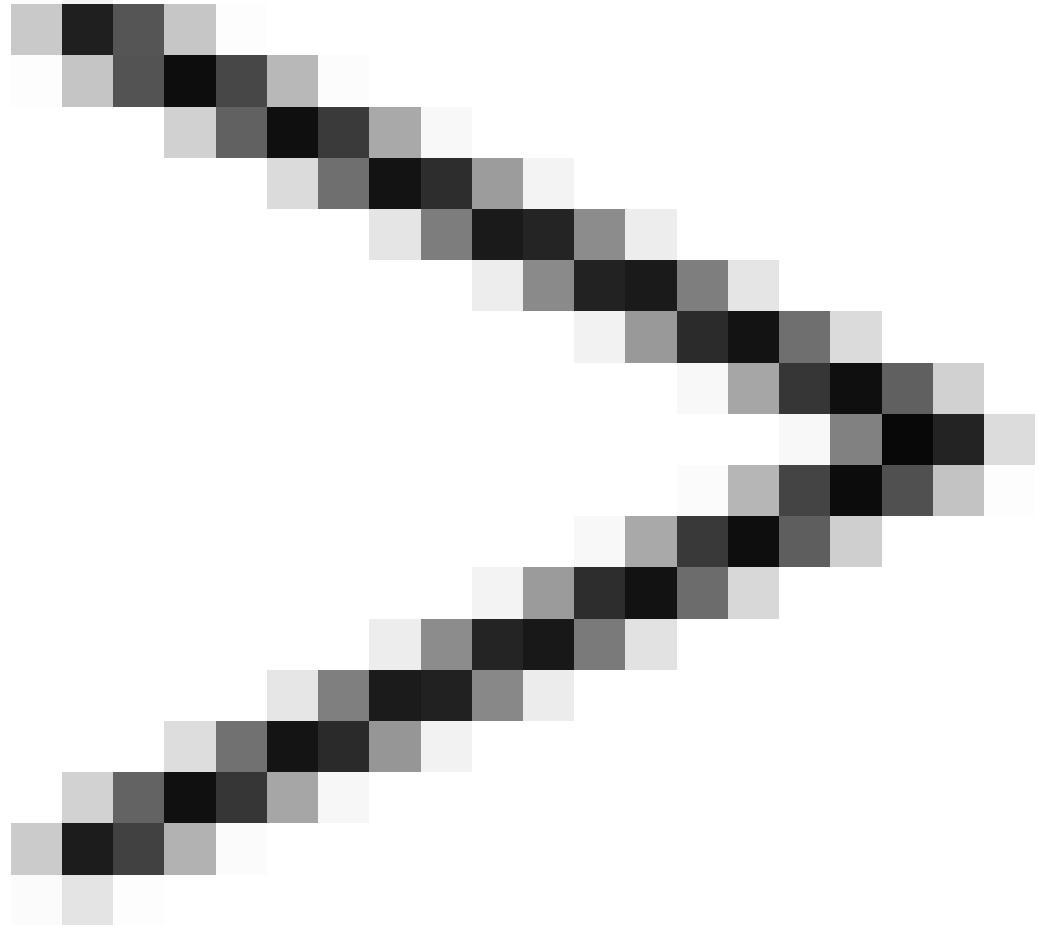
Fekete, 2011

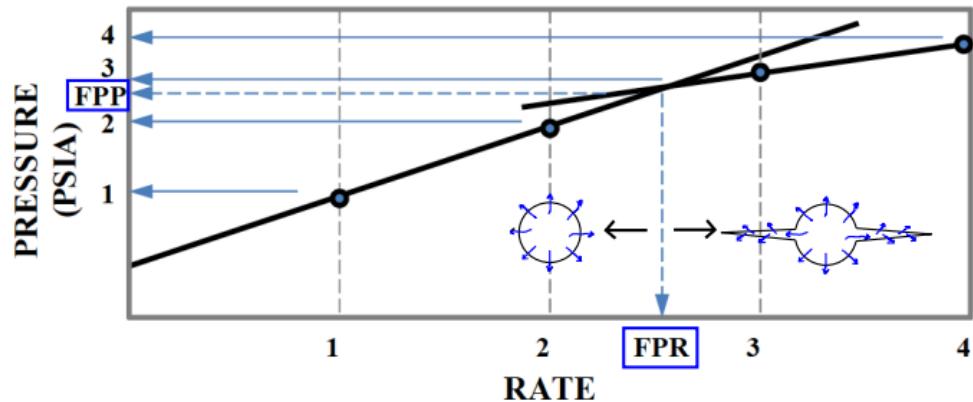
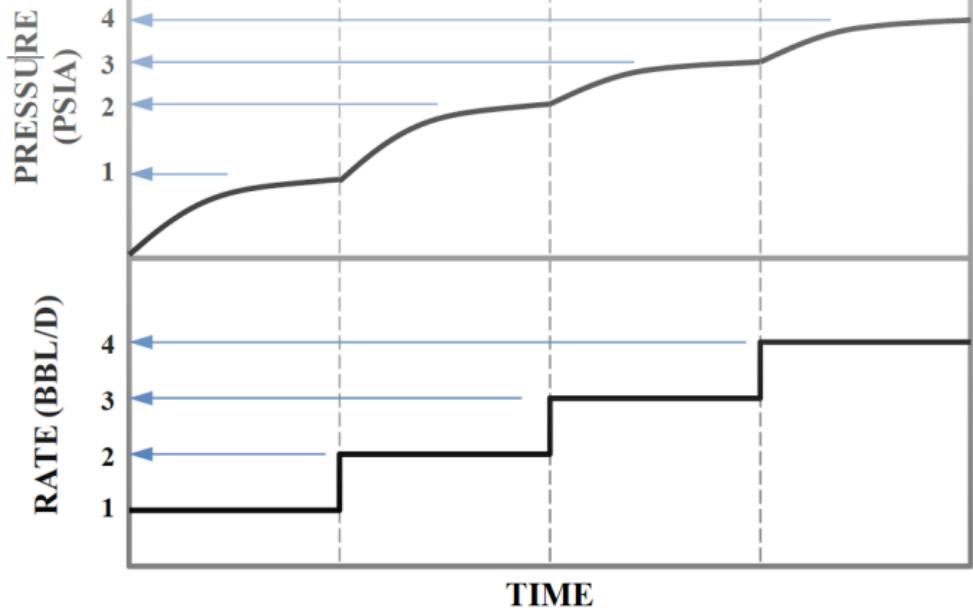
[SPE 163863]

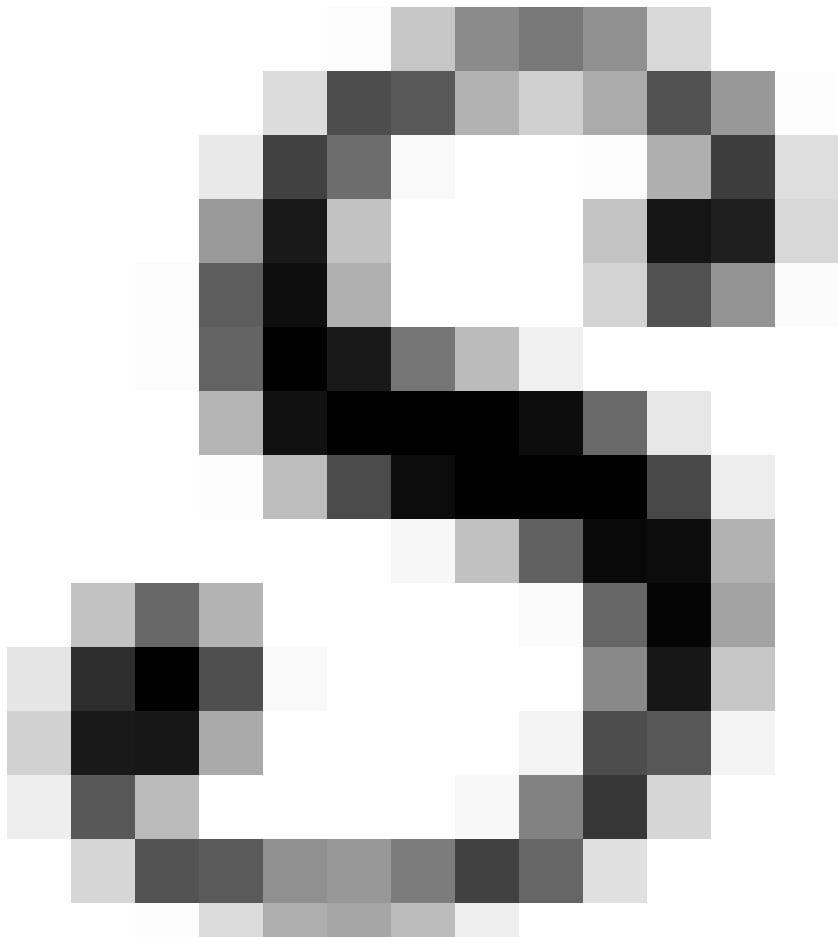


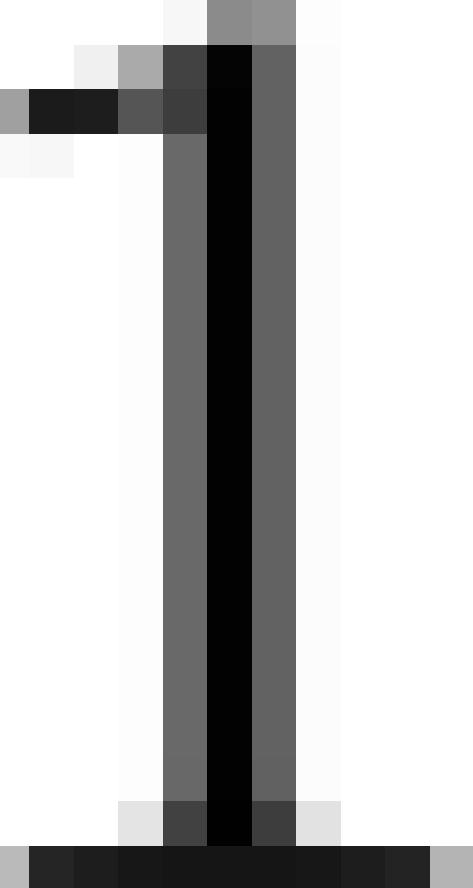
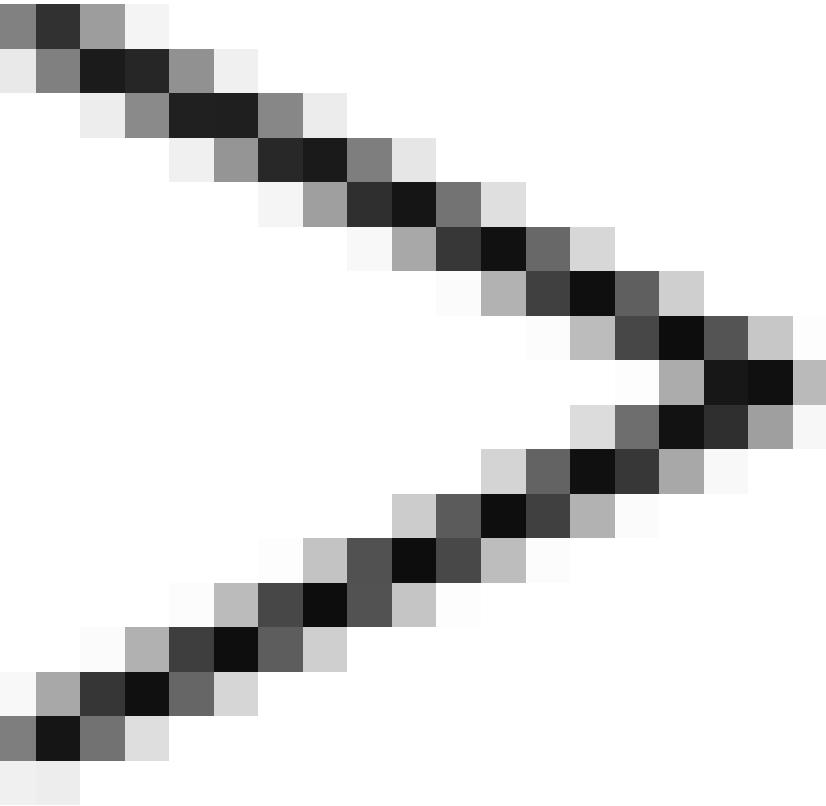
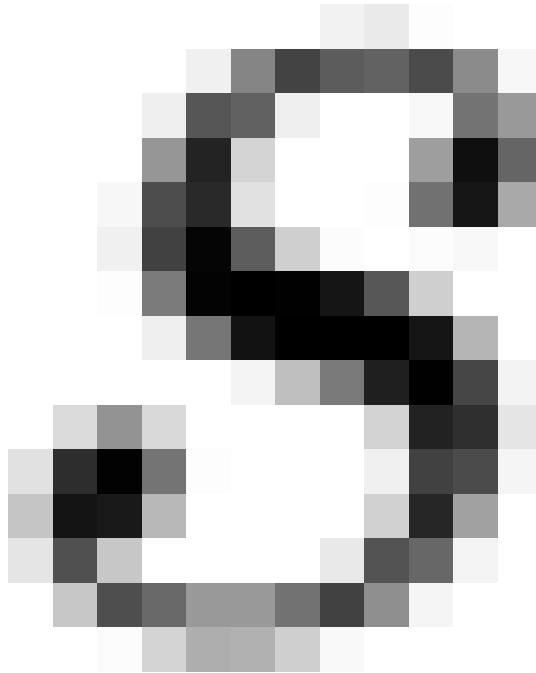
[SPE 163863]

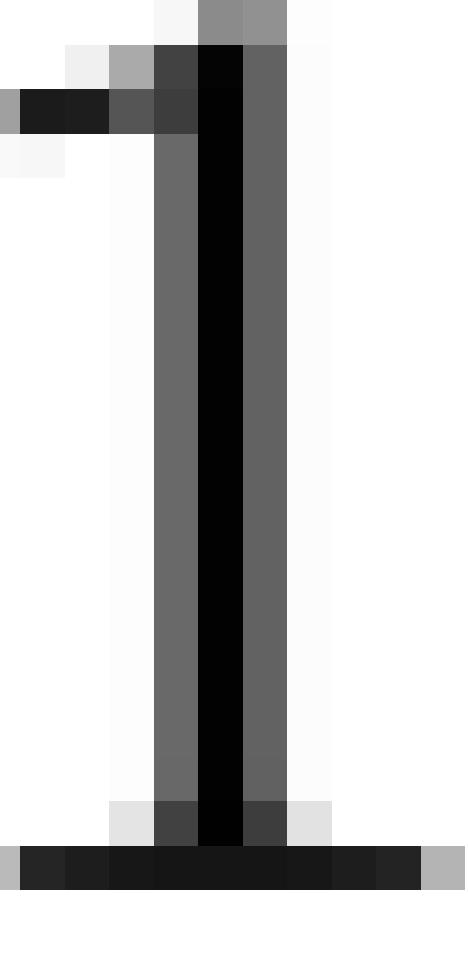
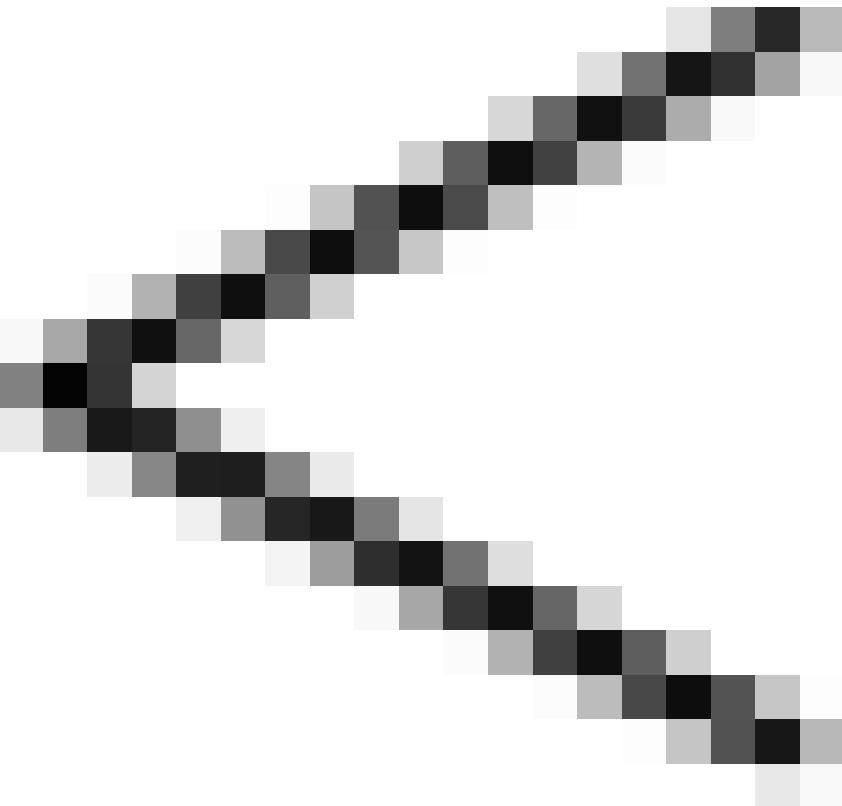
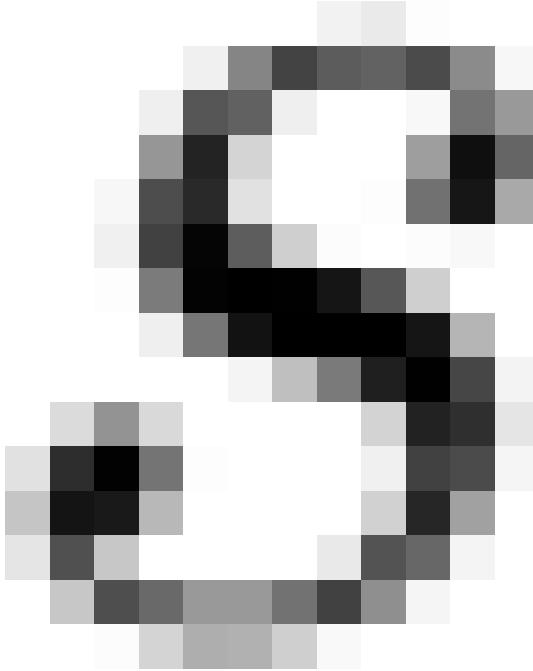










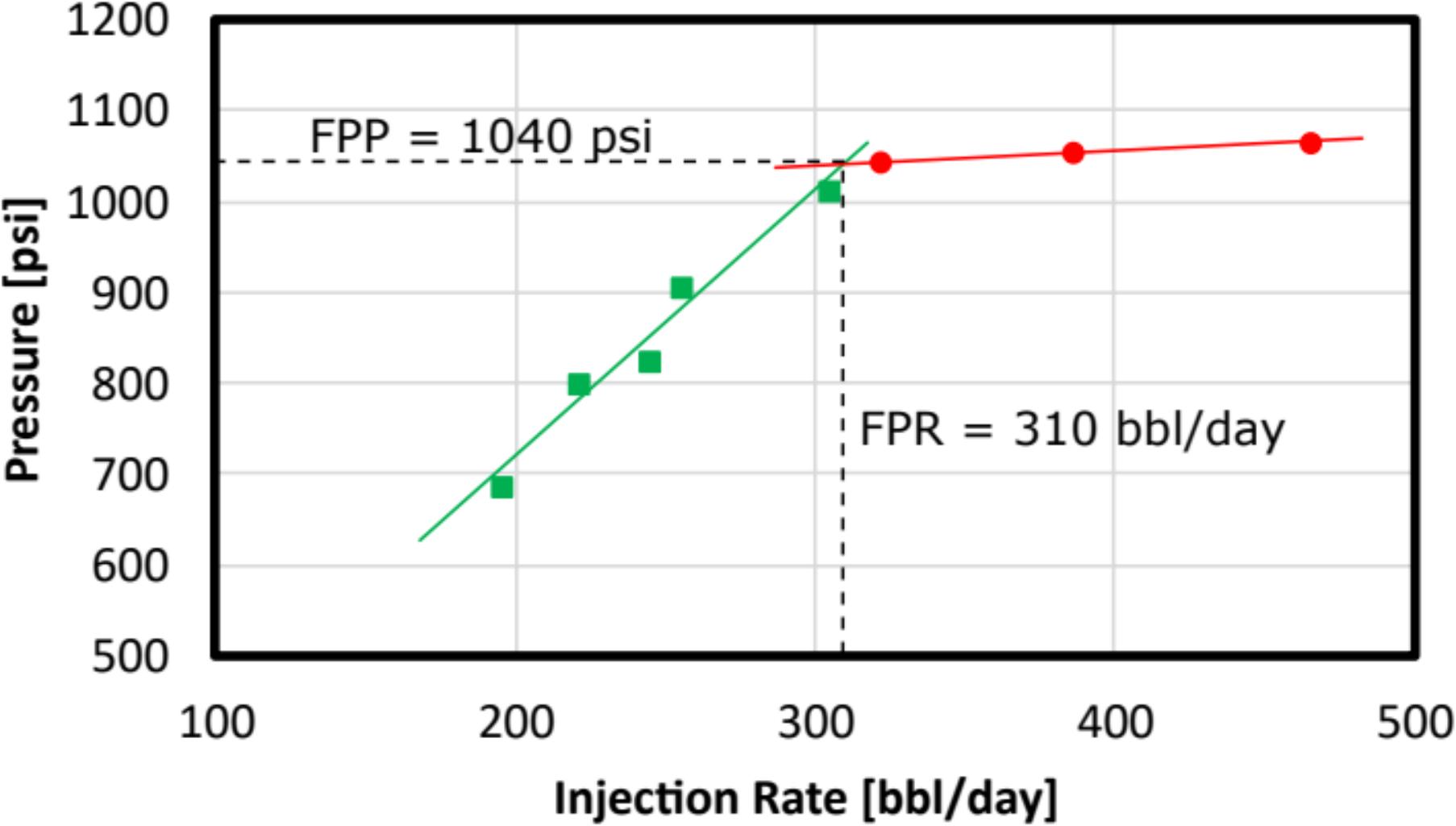


q

$$2\pi\hbar k$$

μ

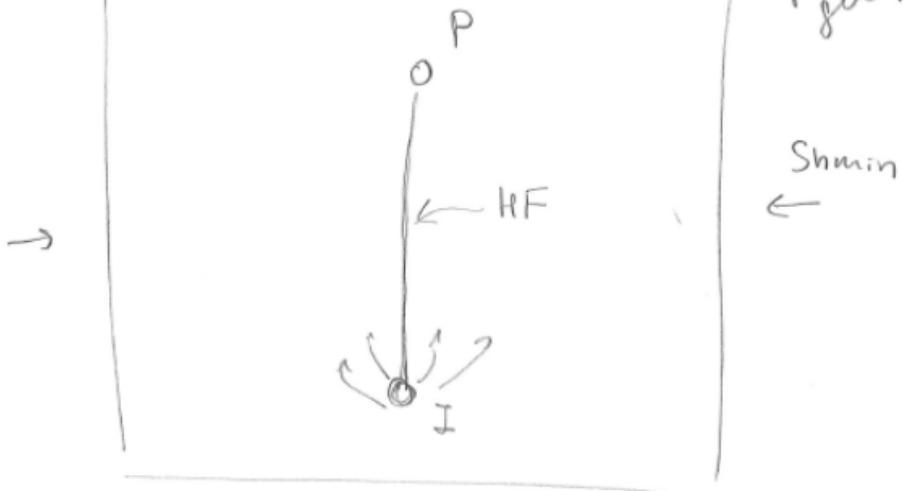
$$\frac{P_e - P_w}{\ln\left(\frac{T_e}{T_w}\right) + s}$$



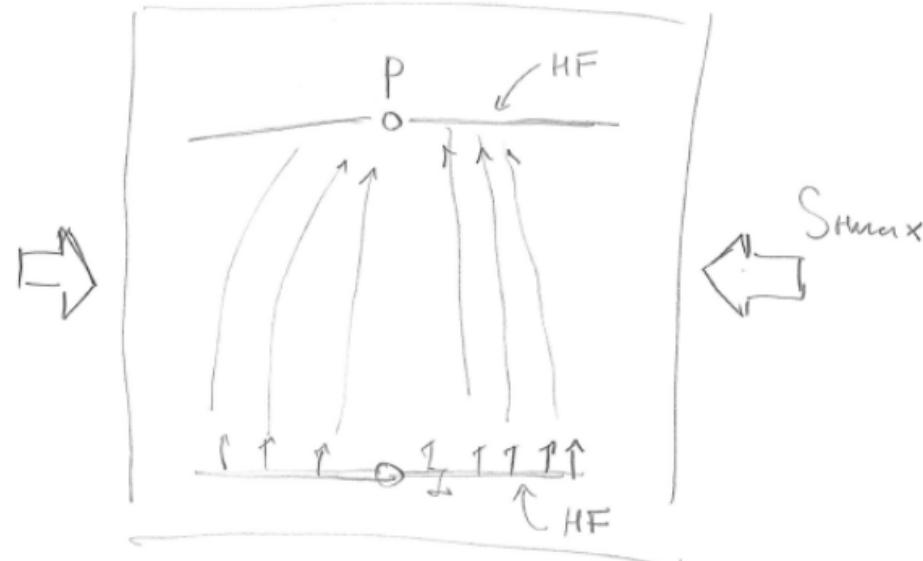
(NF)

$\downarrow S_{\max}$

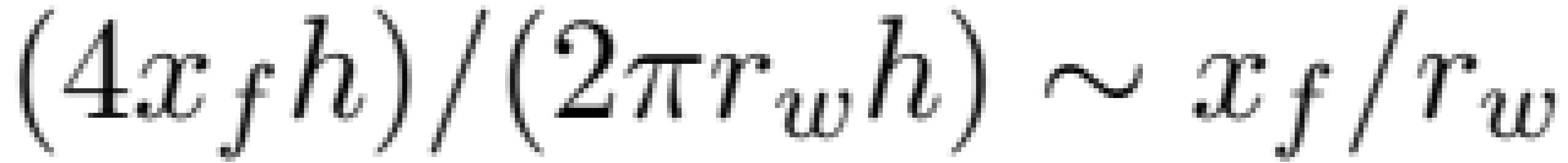
Not good



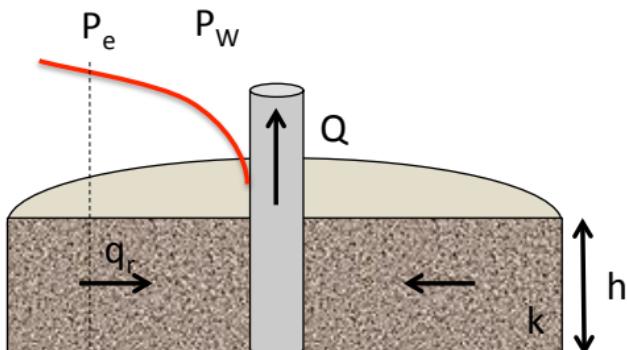
$\downarrow S_{\min}$







Intact

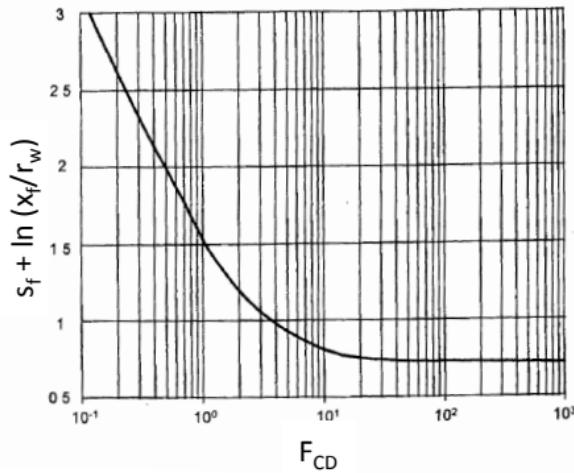
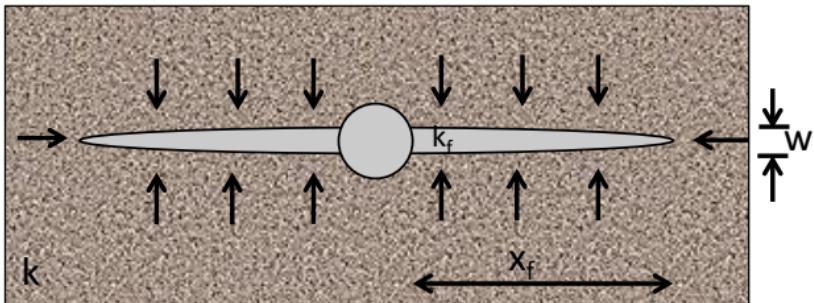


$$Q = -\frac{2\pi kh}{\mu} \frac{(P_e - P_w)}{p_D + s}$$

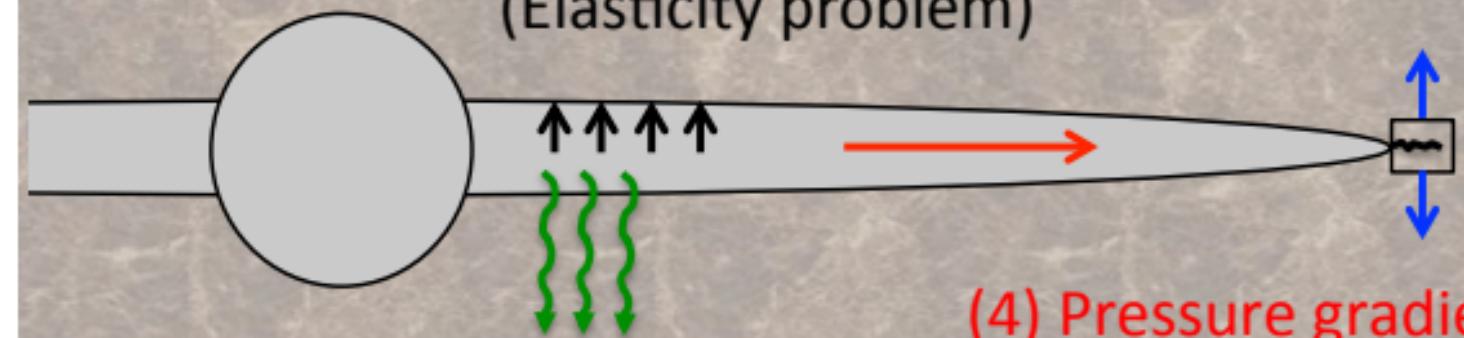
p_D : Dimensionless pressure
 $= \ln(r_e/r_w)$ if steady state

s : skin factor
 $> 1 \rightarrow$ damage
 $< 1 \rightarrow$ stimulation

Fractured



$w, x_f, k_f \rightarrow F_{CD} = (k_f w)/(k x_f)$
 $x_f, r_w, F_{CD} \rightarrow s_f$ from above plot



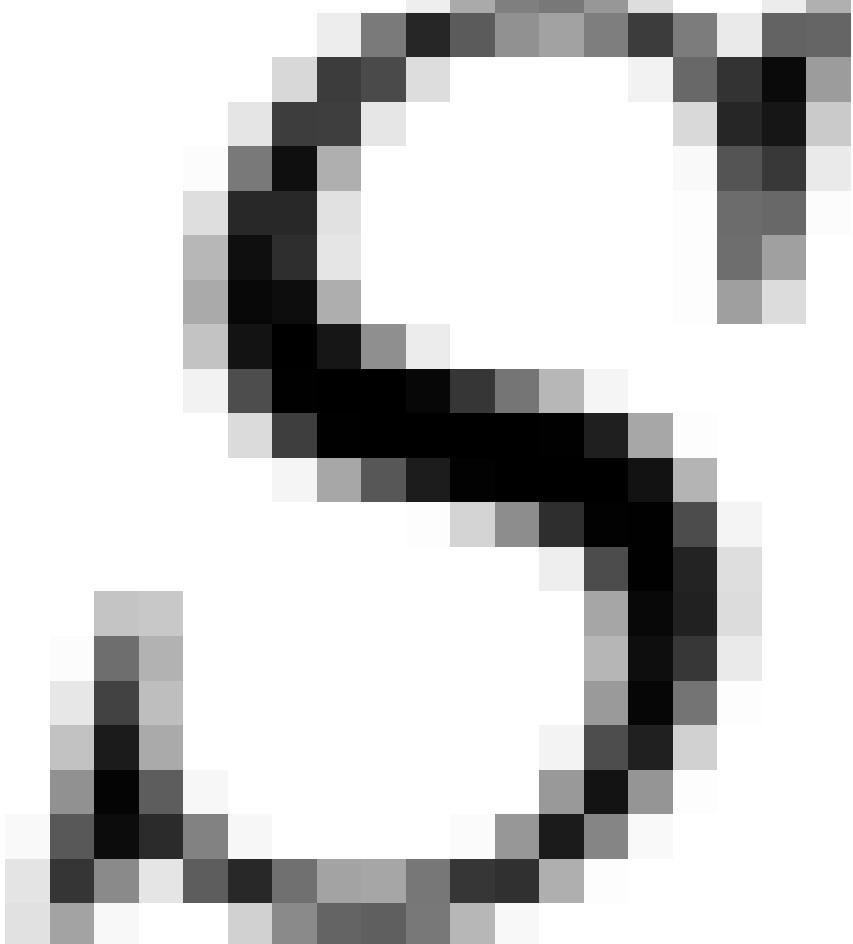
(1) Pressure on the fracture deforms adjacent rock
(Elasticity problem)

(2) Fracture propagates if the “stress intensity factor” is higher than what the rock can resist “rock toughness”
(Fracture mechanics problem)

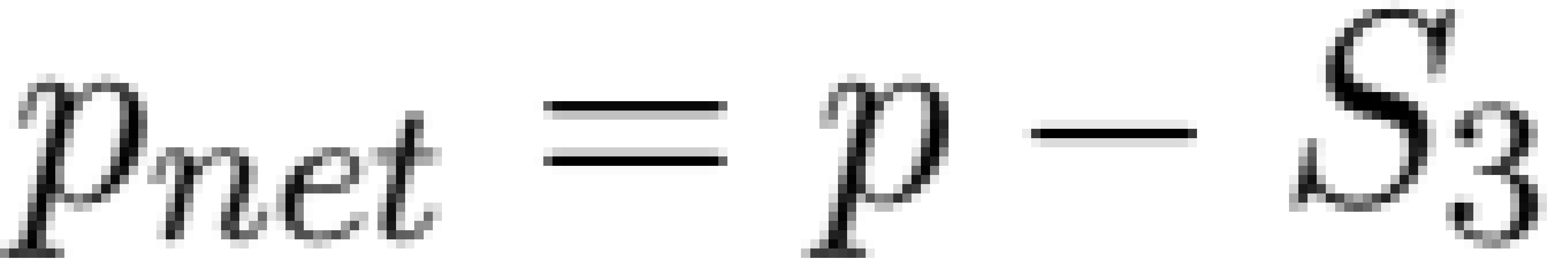
(3) Fracturing fluids can leak off to the formation
(Mud-cake design)

(4) Pressure gradient leads to flow of fracturing fluid through the fracture
(Lubrication problem)

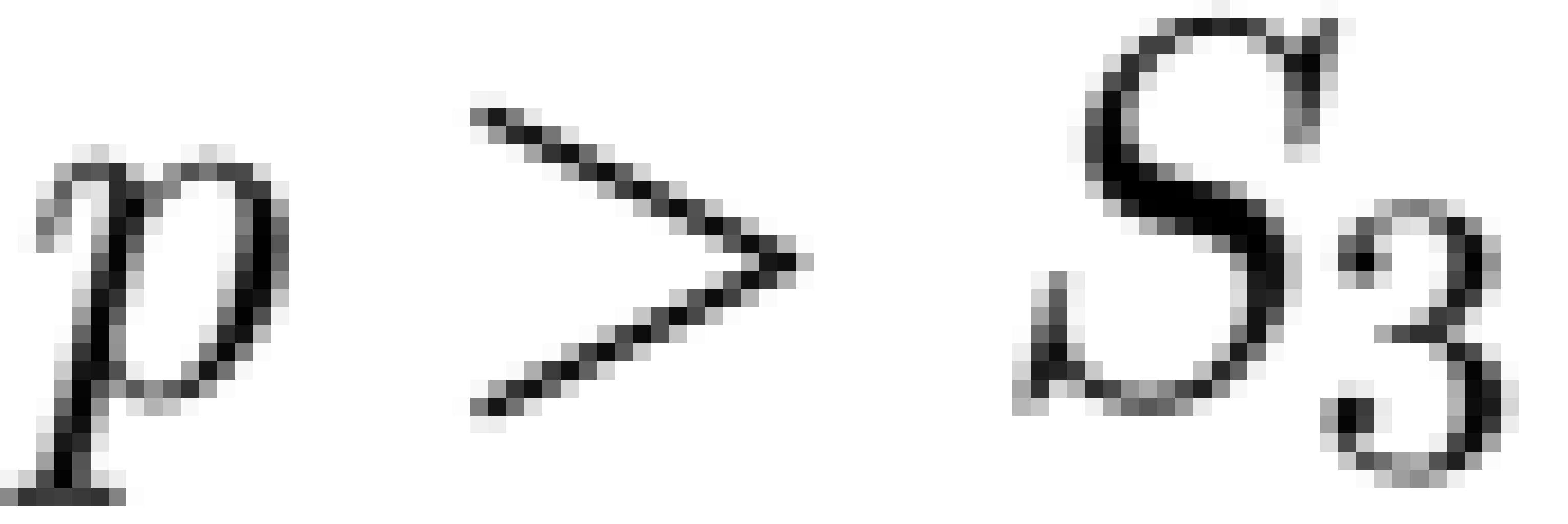
Net pressure
 $p_{\text{net}} = p - S_3$

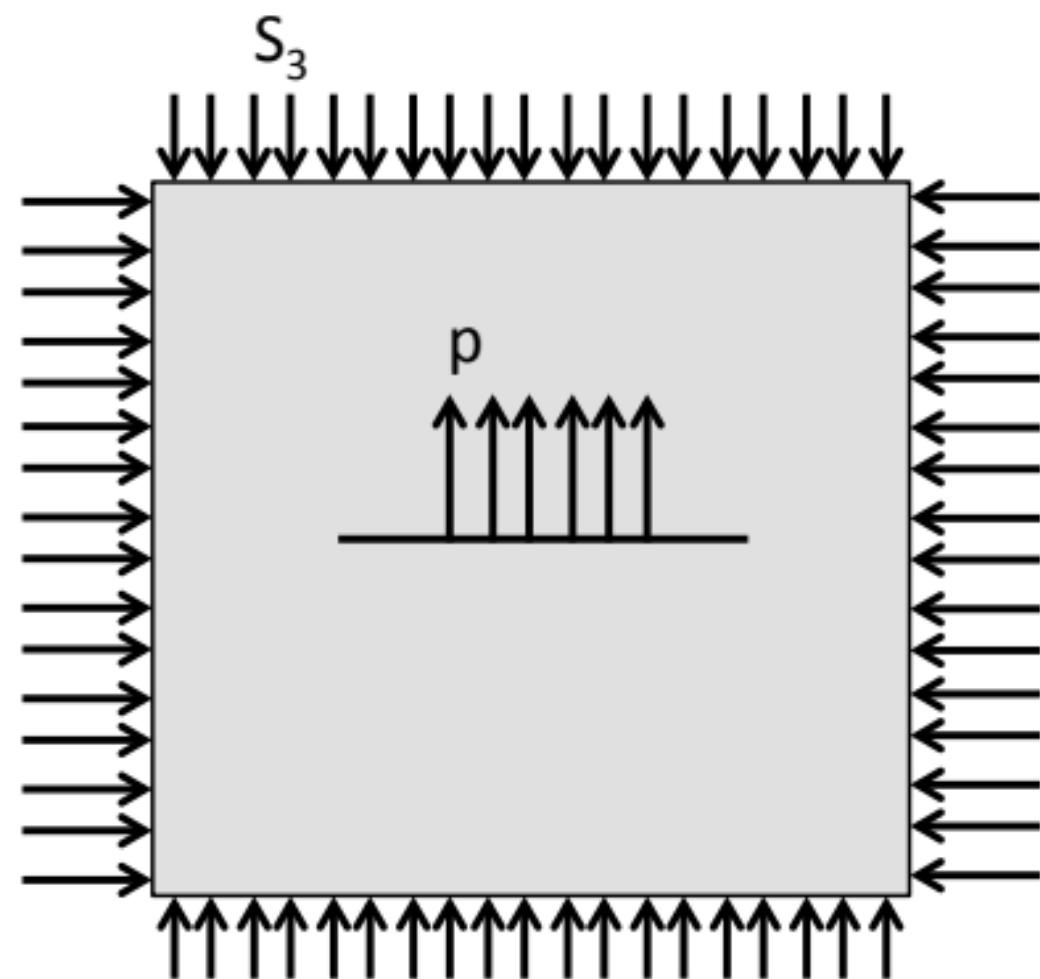




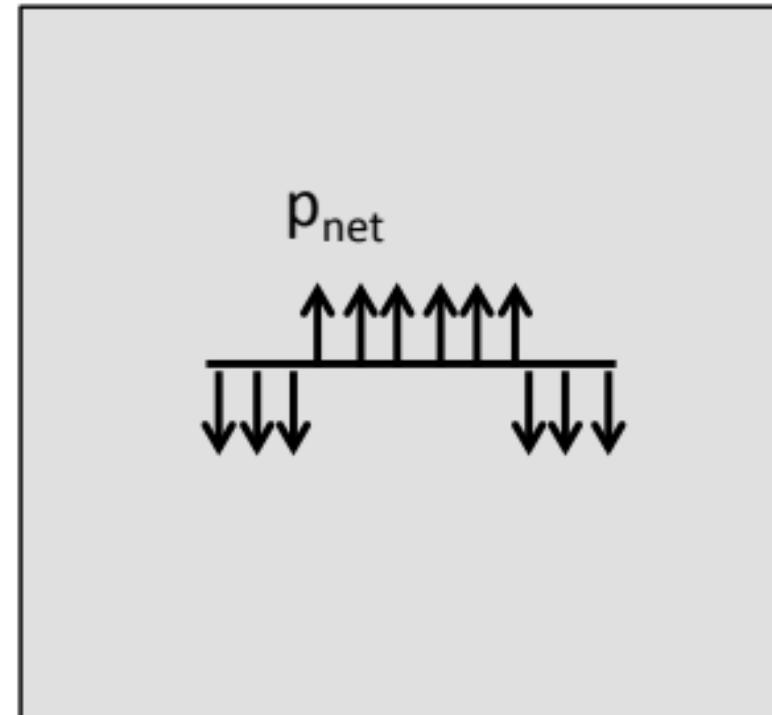




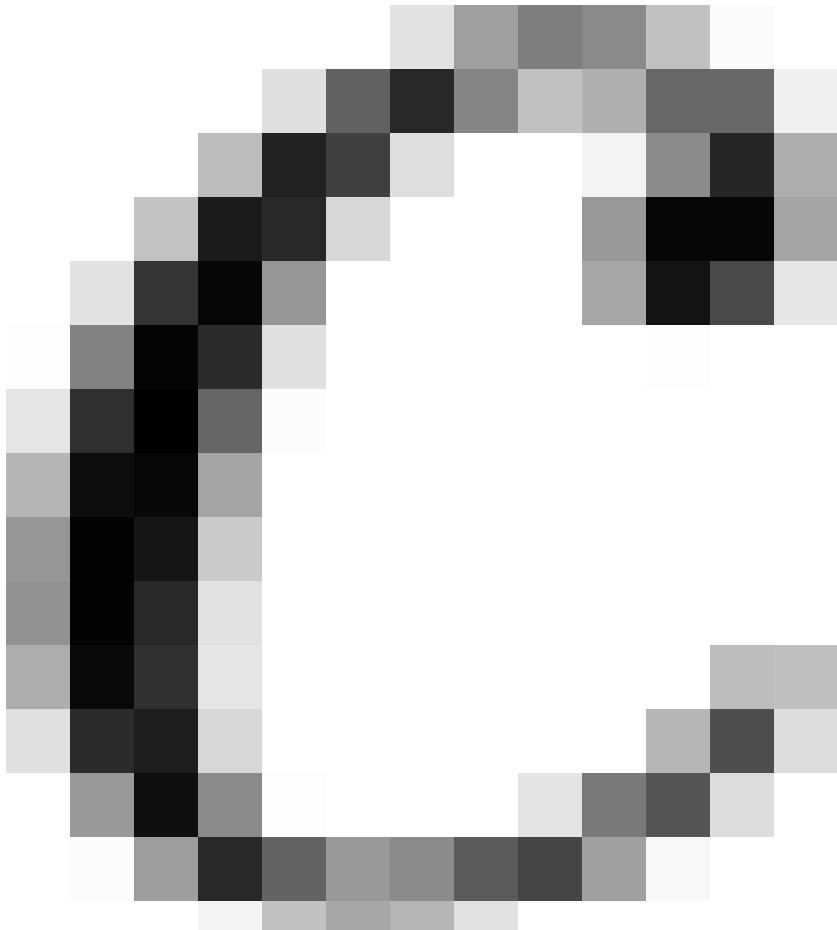




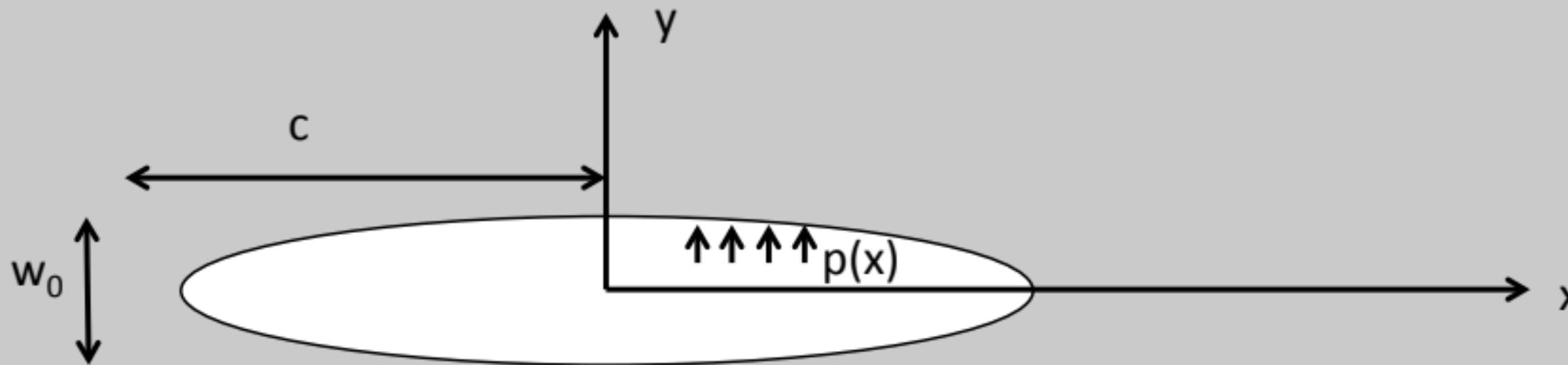
=







$$\begin{cases} \sigma_{yy} = p(x) & \text{for } 0 \leq x \leq c \\ u_y = 0 & \text{for } x > c \end{cases}$$

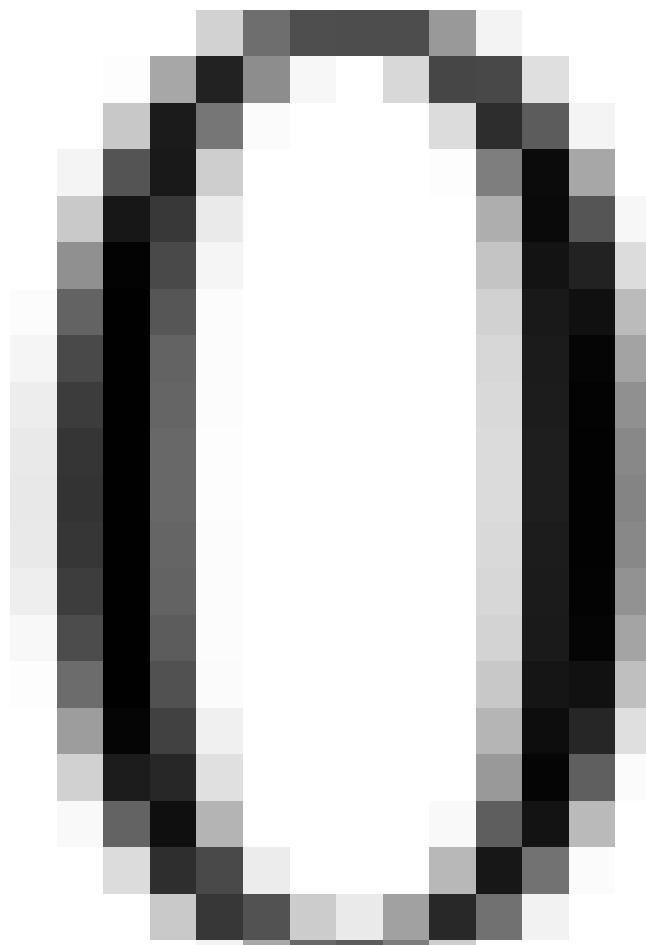
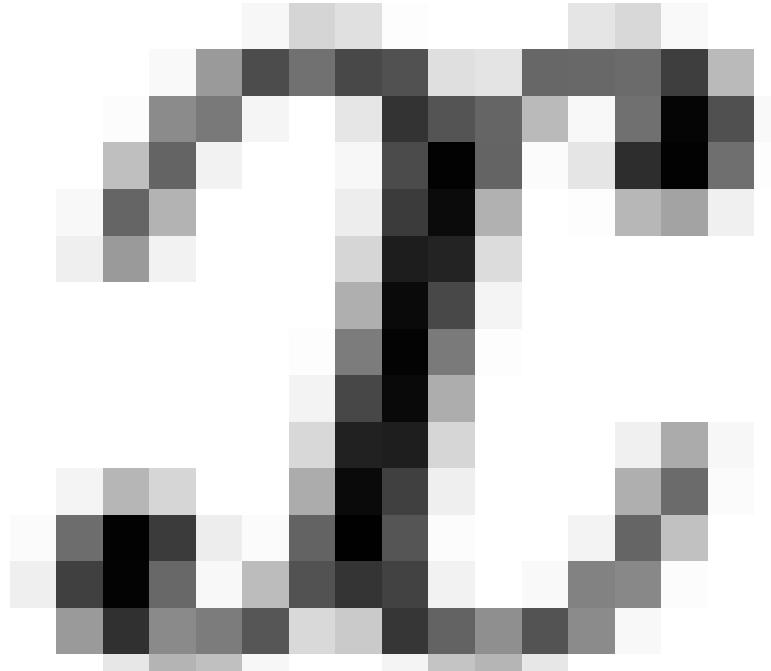


Linear elastic and homogeneous solid
(E, v)
 $E' = E / (1 - v^2)$



$$\begin{cases} u_y(x, 0) = \frac{2p_o}{E'} \sqrt{c^2 - x^2} & \text{for } 0 \leq x \leq c \\ \sigma_{yy}(x, 0) = -p_o \frac{x}{\sqrt{x^2 - c^2 - 1}} & \text{for } x > c \end{cases}$$









ω

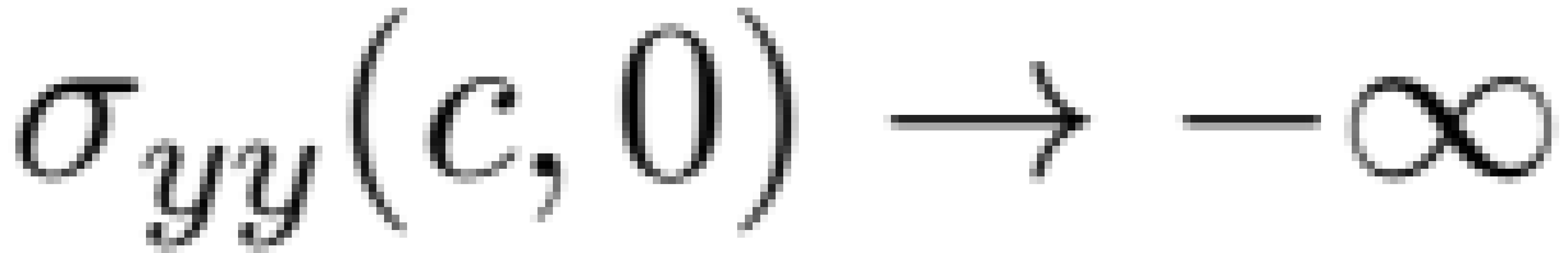


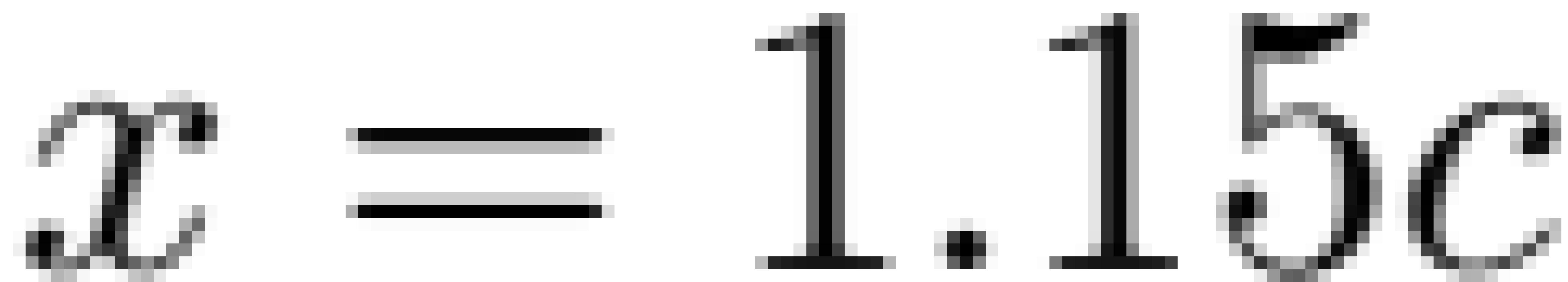
4P0C



EY

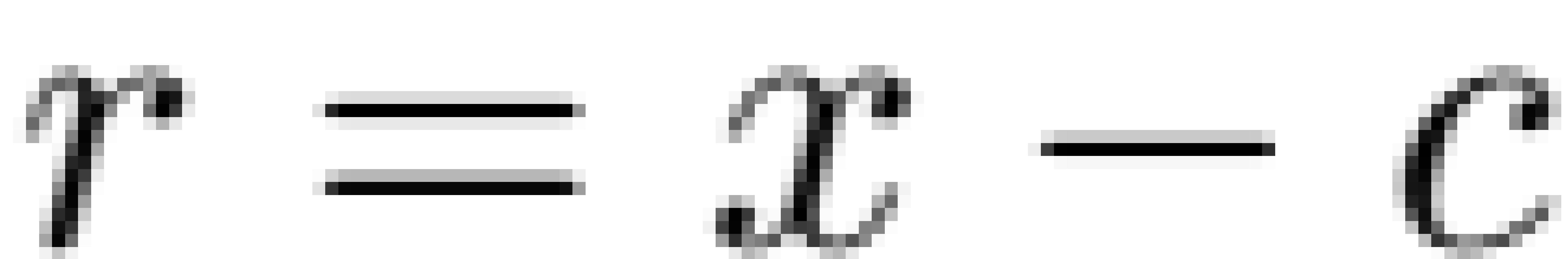


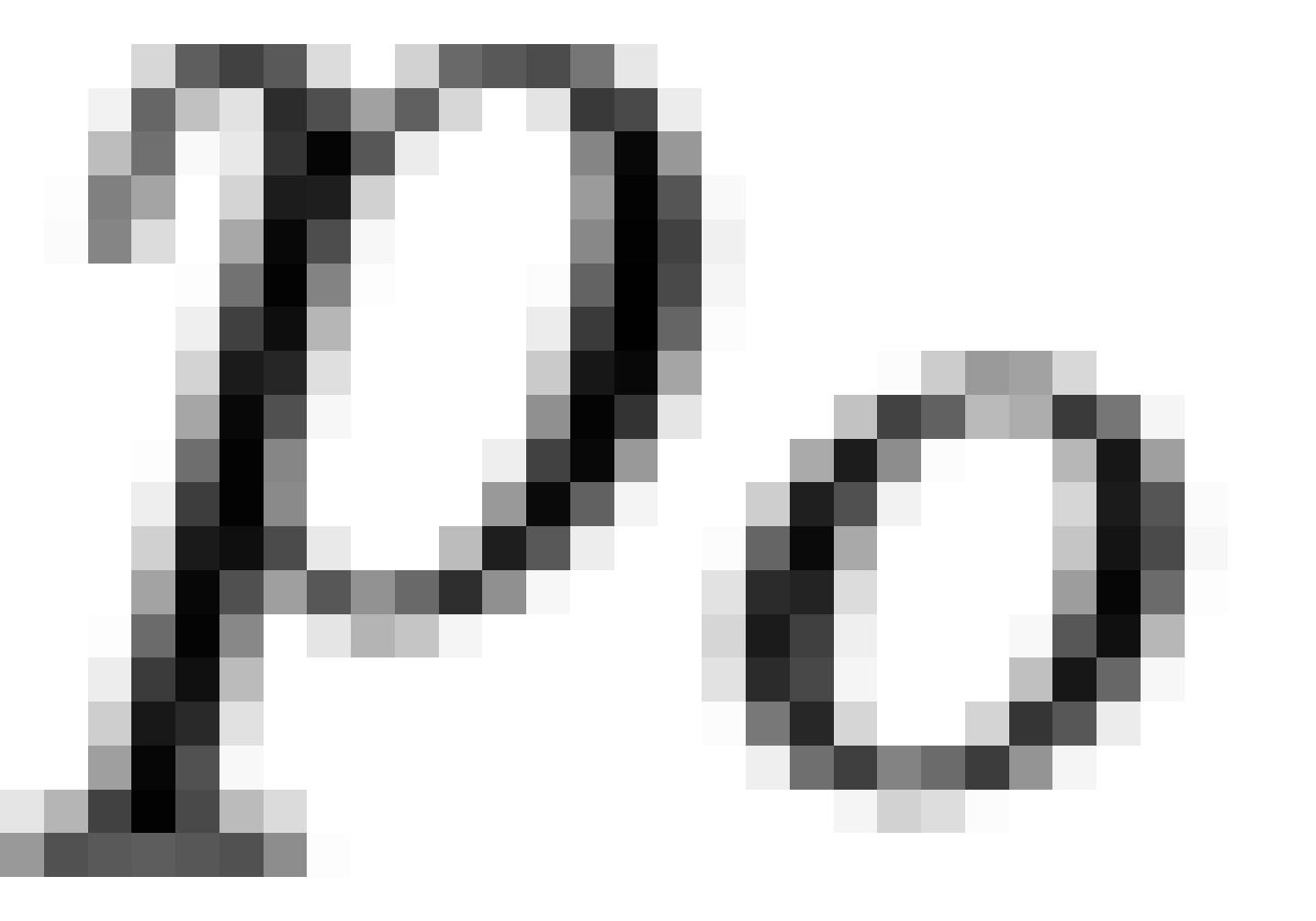


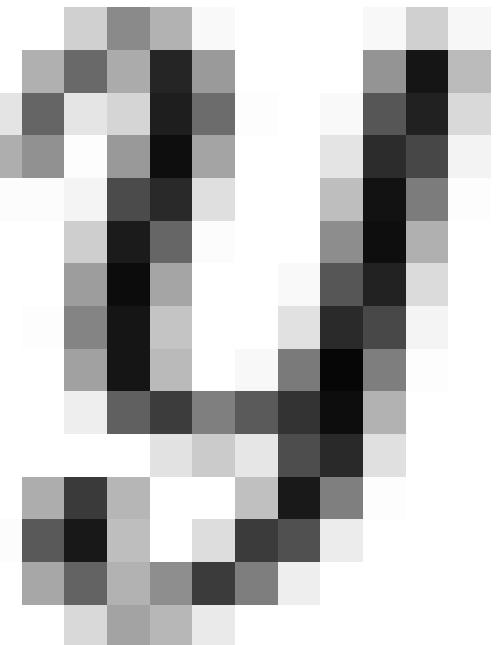
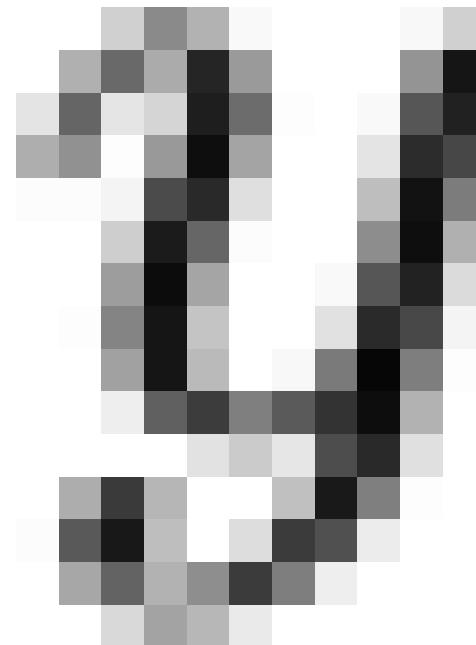
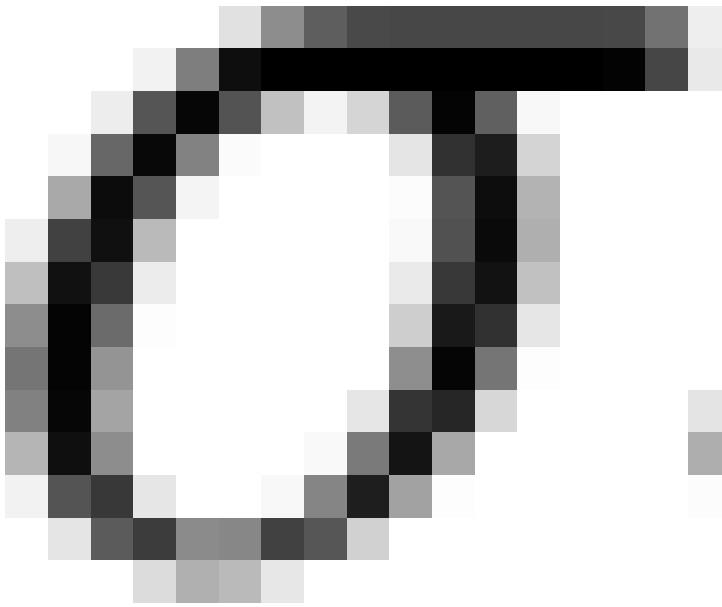




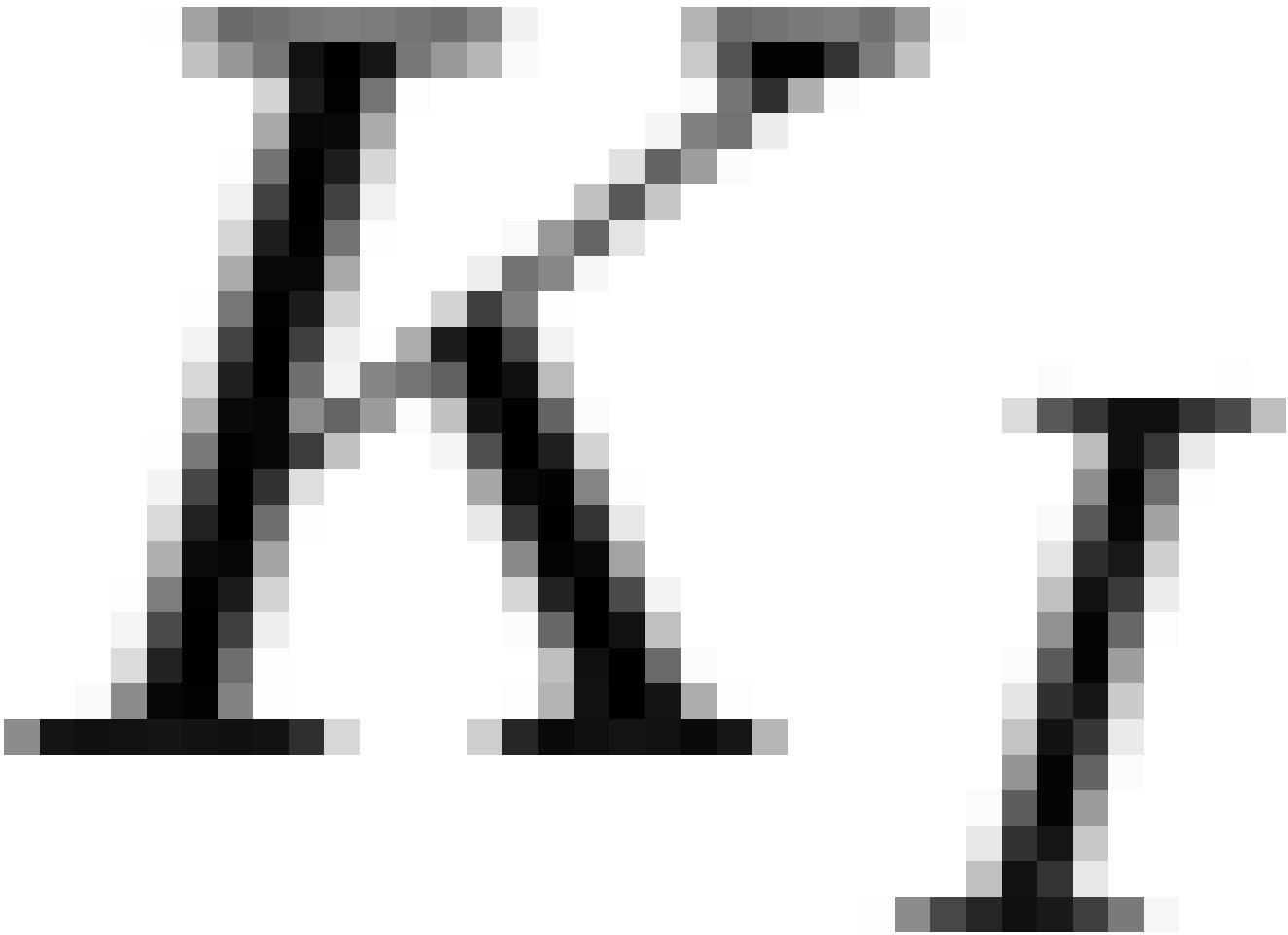
$$\kappa_I = \lim_{r \rightarrow 0^+} \left[(2\pi r)^{1/2} \sigma_{uv}(c + r, y=0) \right]$$

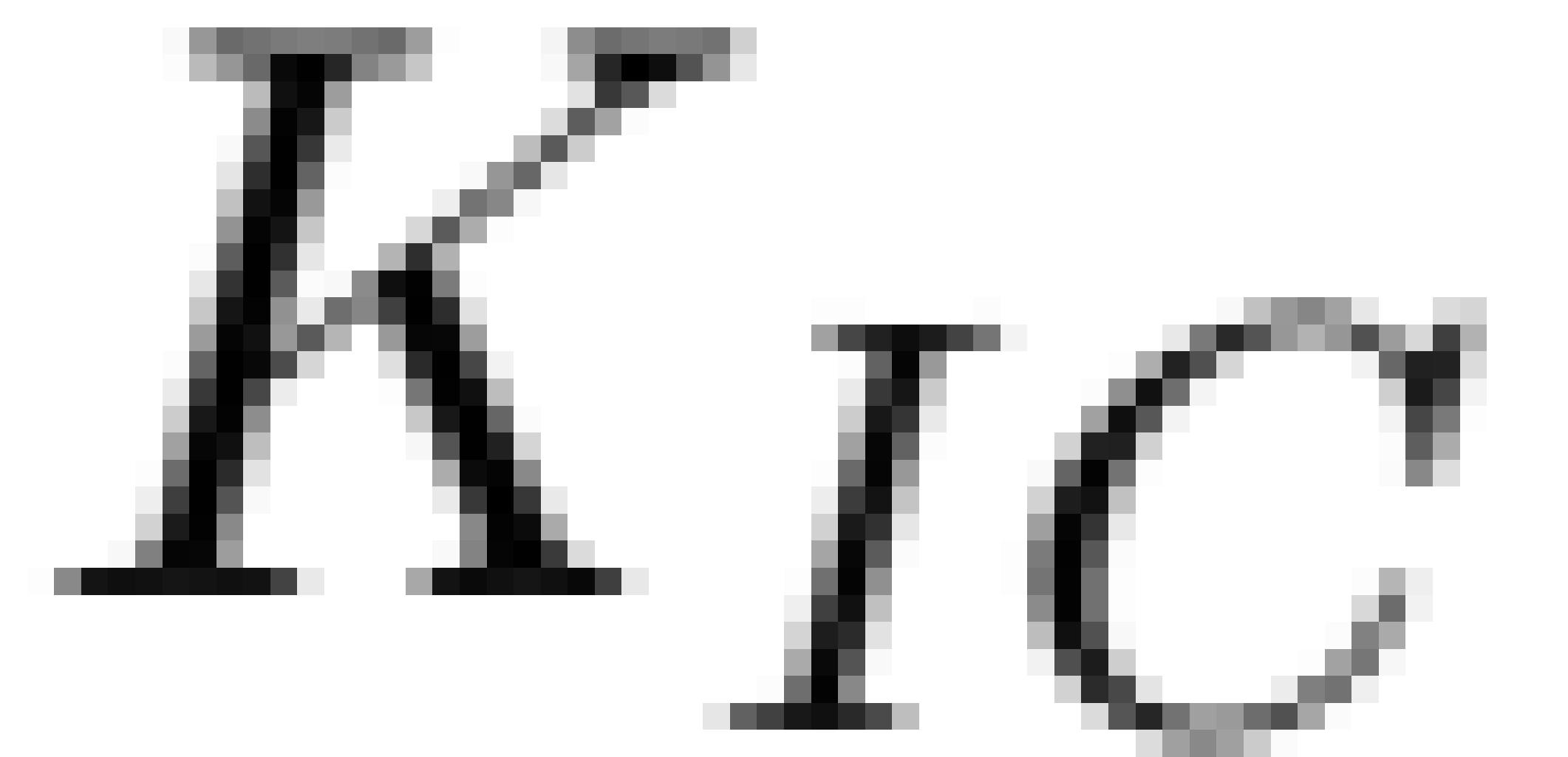


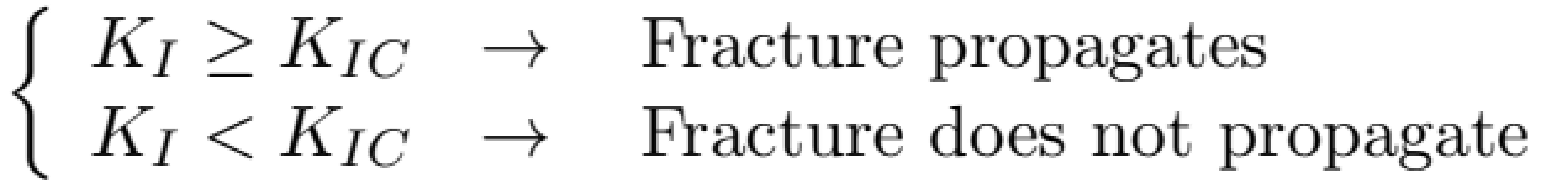


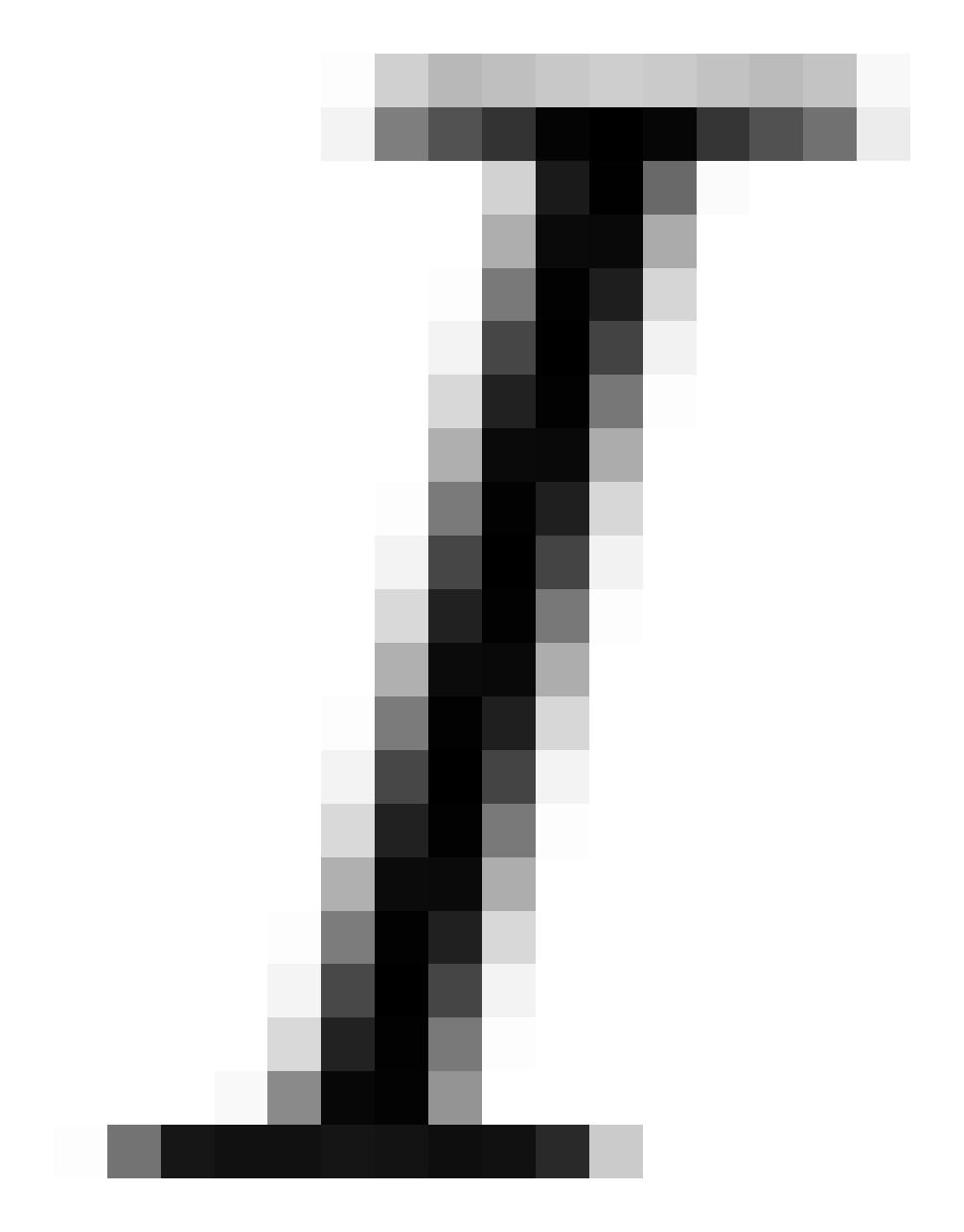






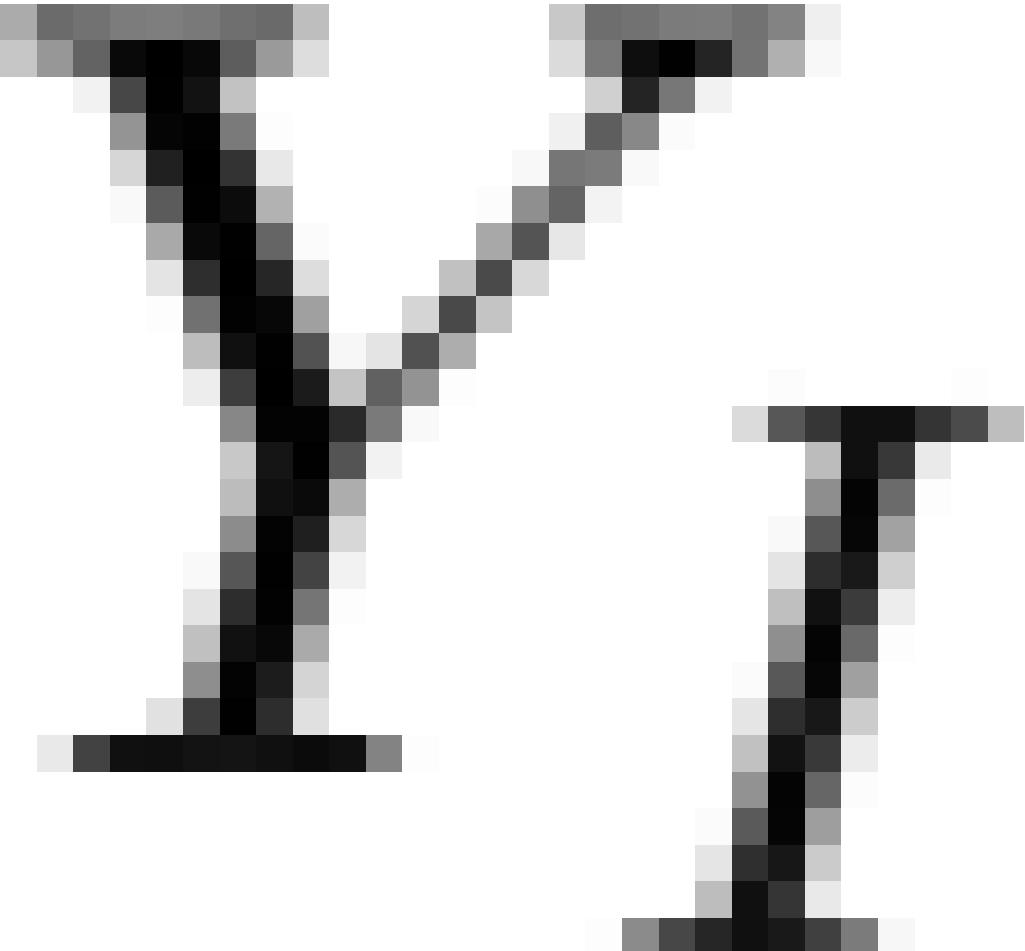


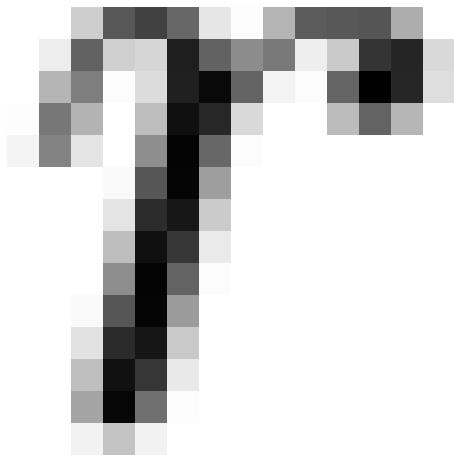
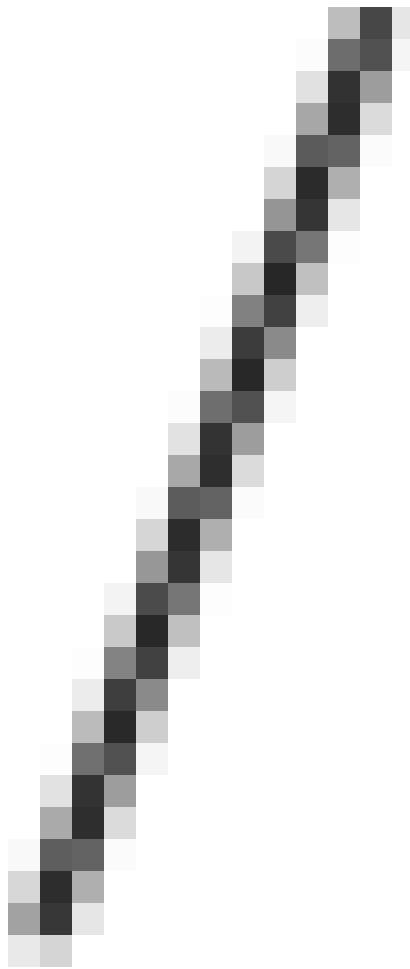
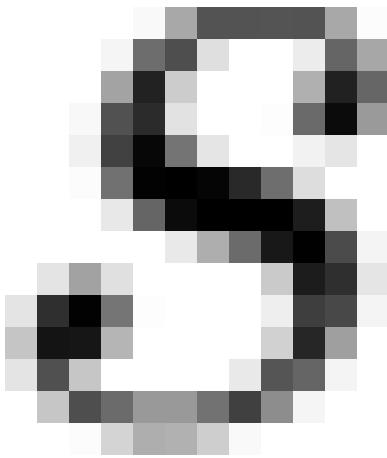


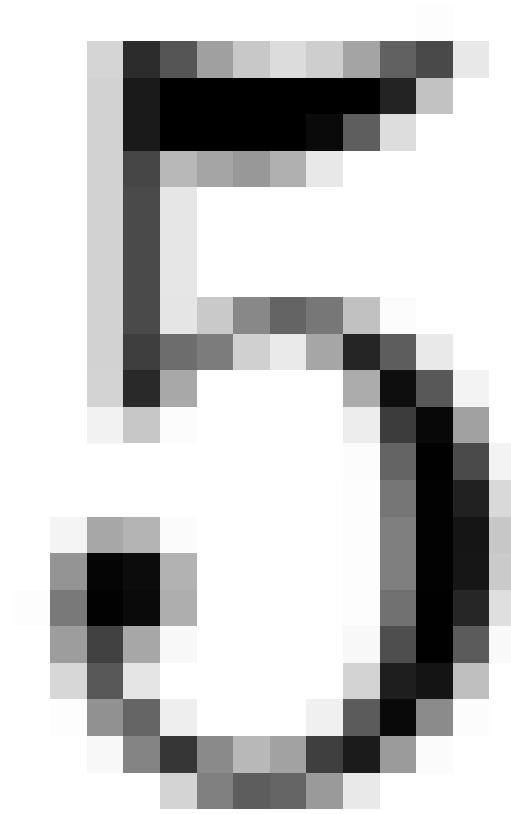
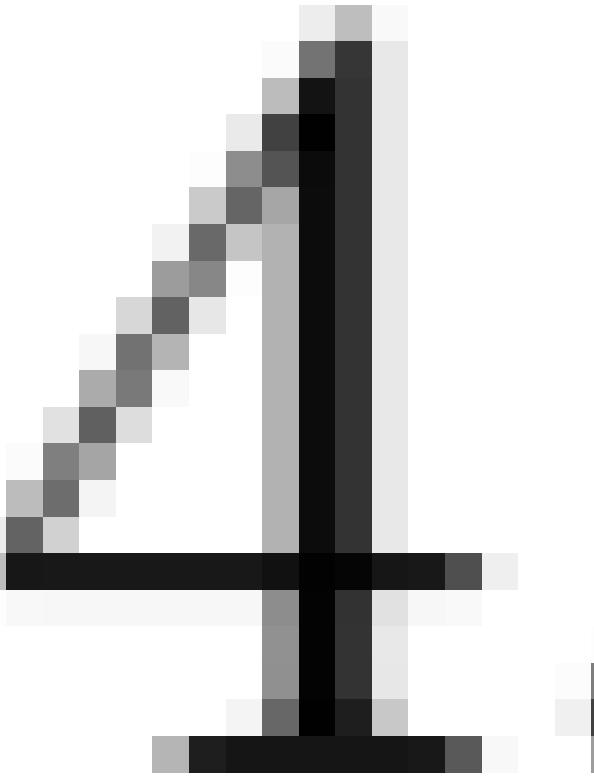
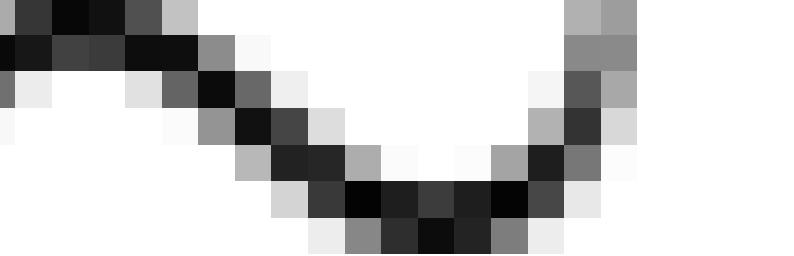
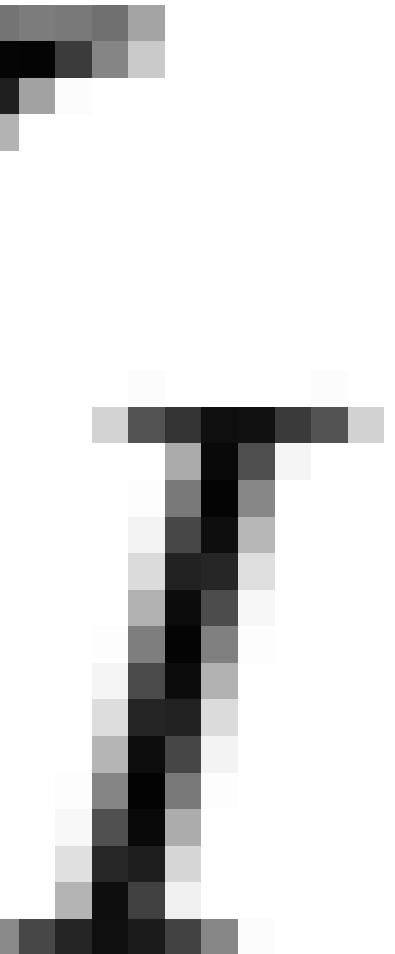
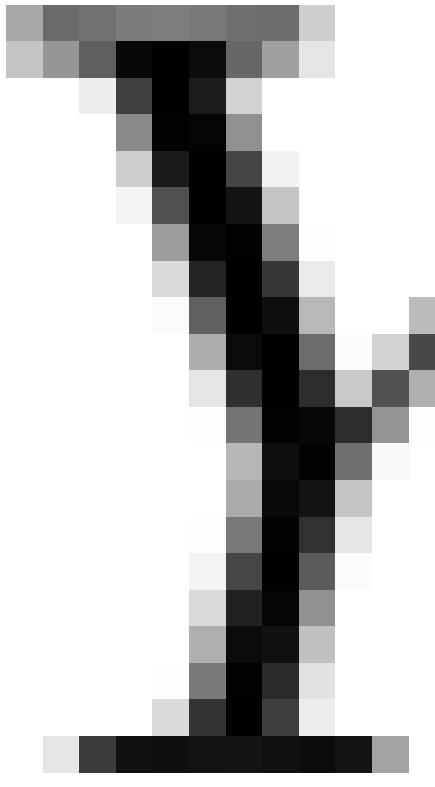


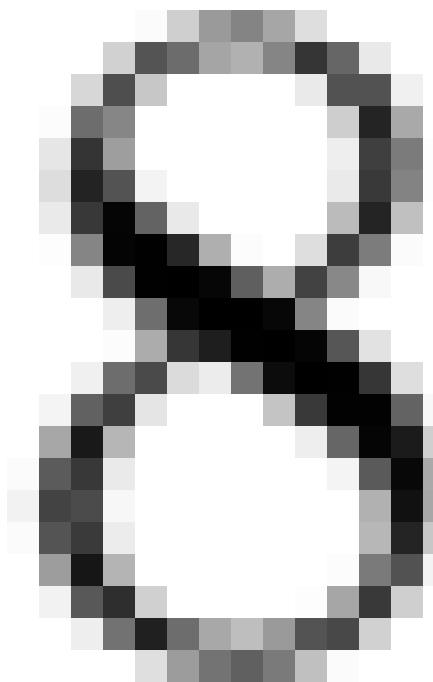
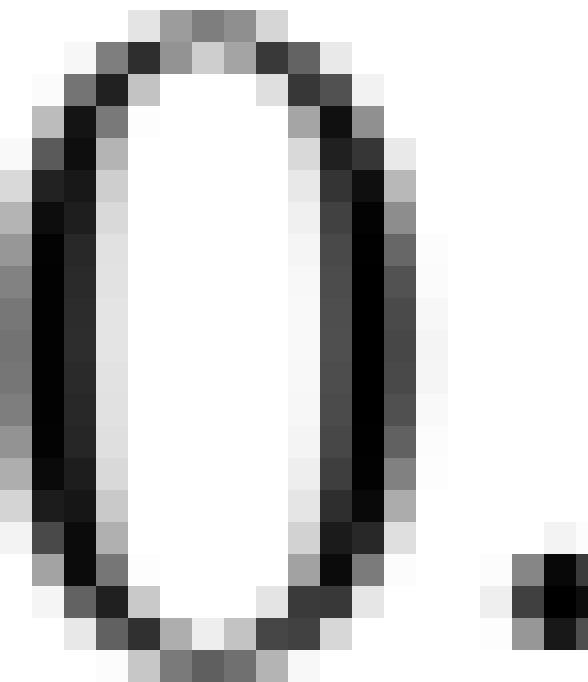
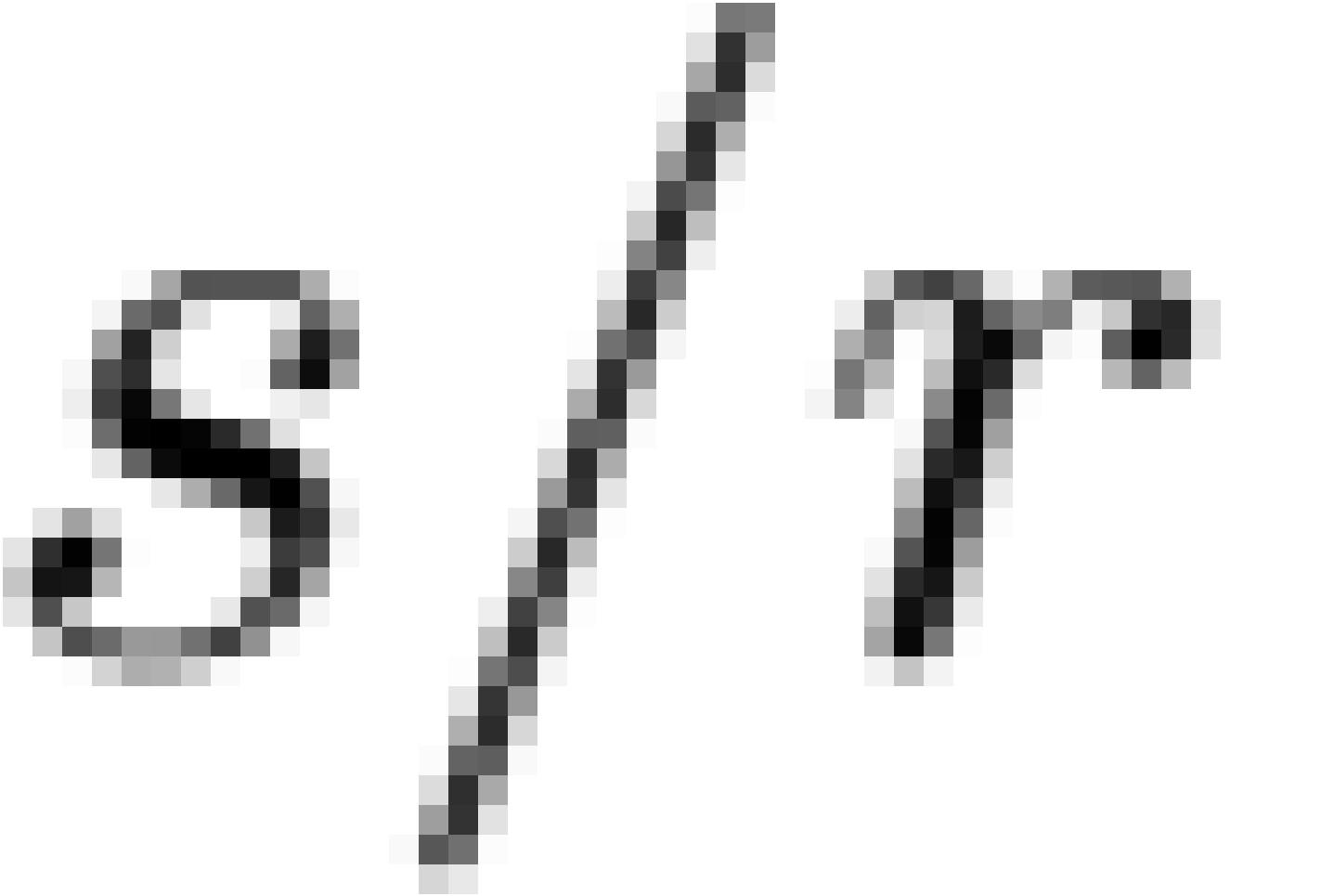
$$K_{IC} = \frac{P_{max}(\pi_0)^{1/2}}{2\pi L} M$$



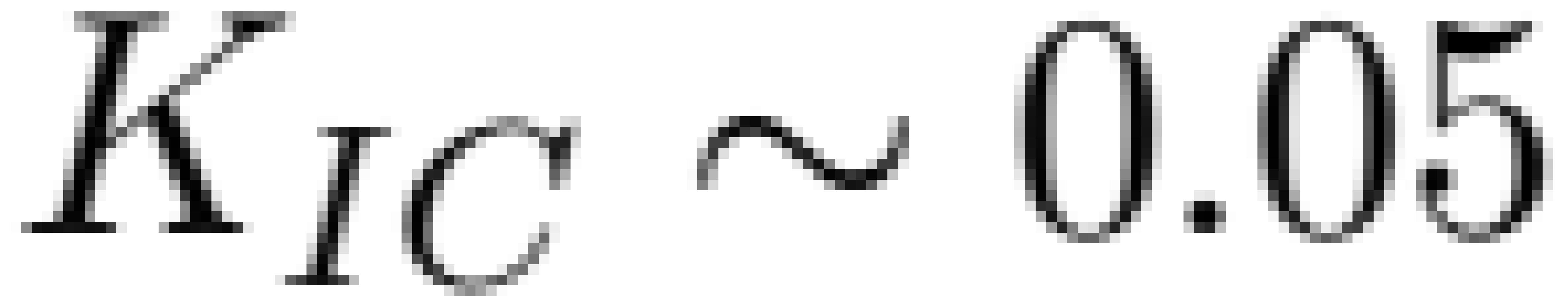


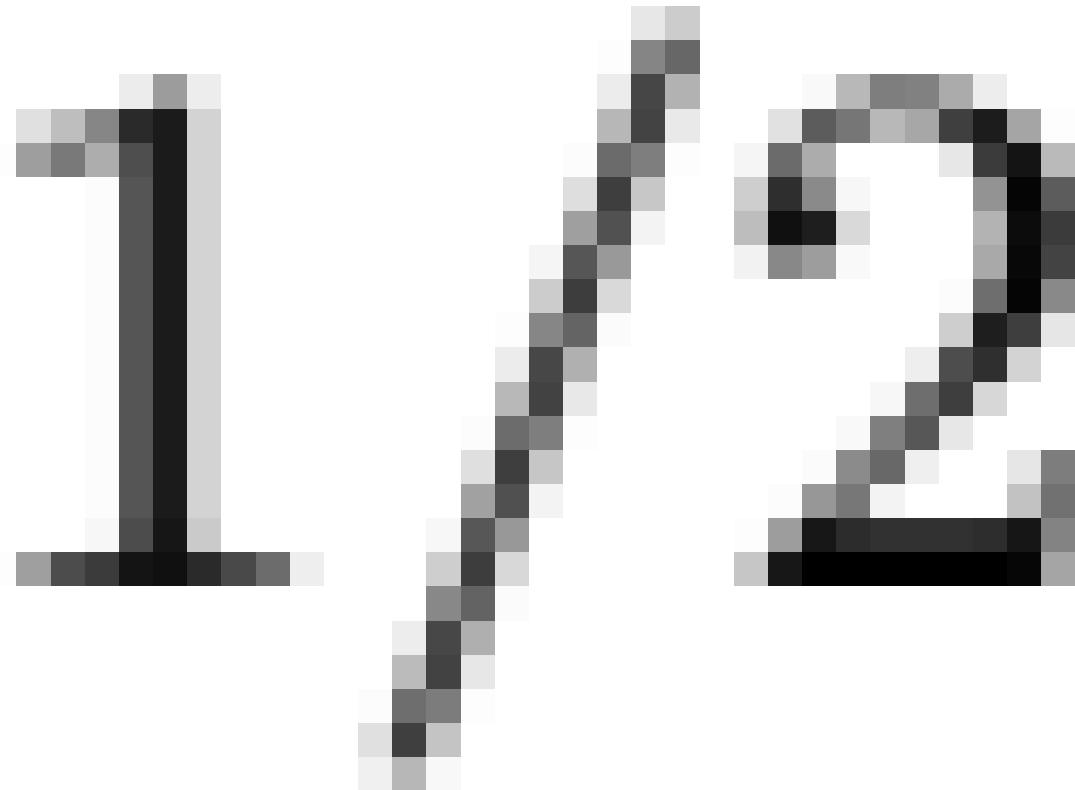




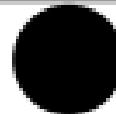
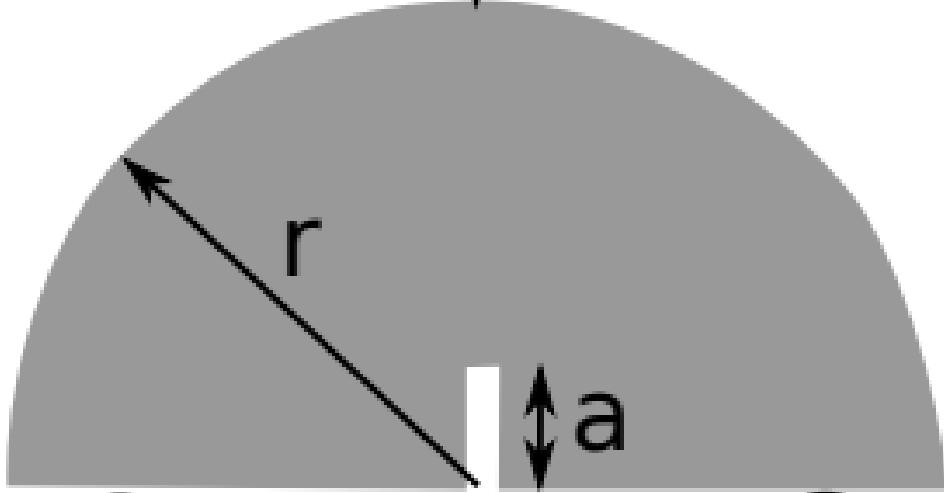




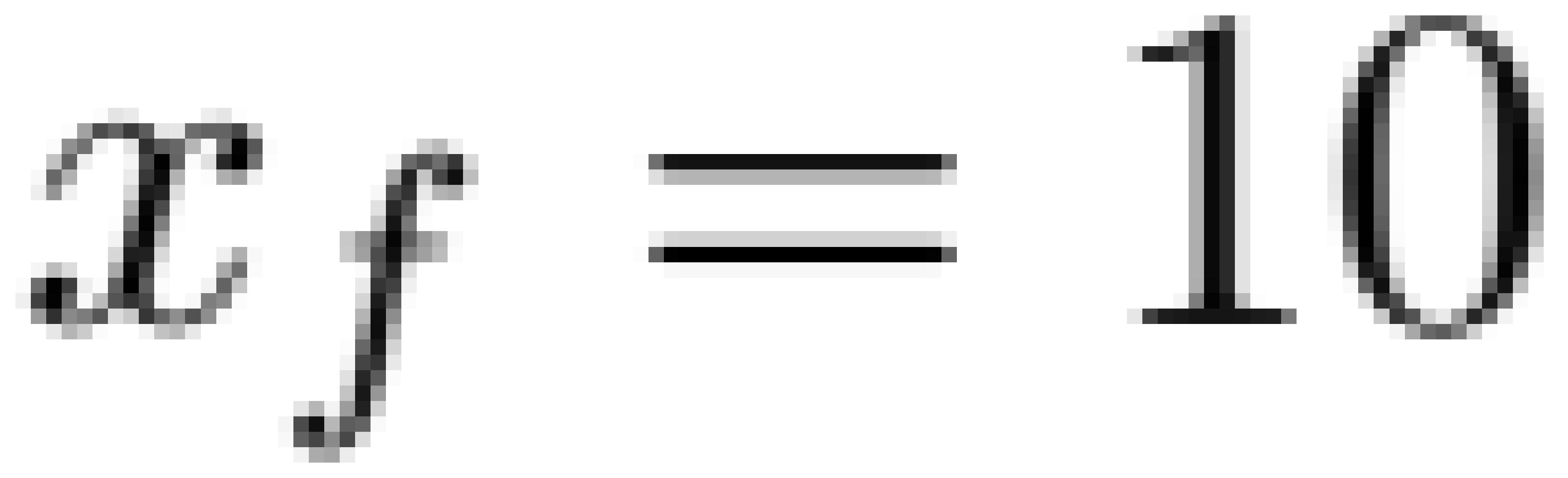


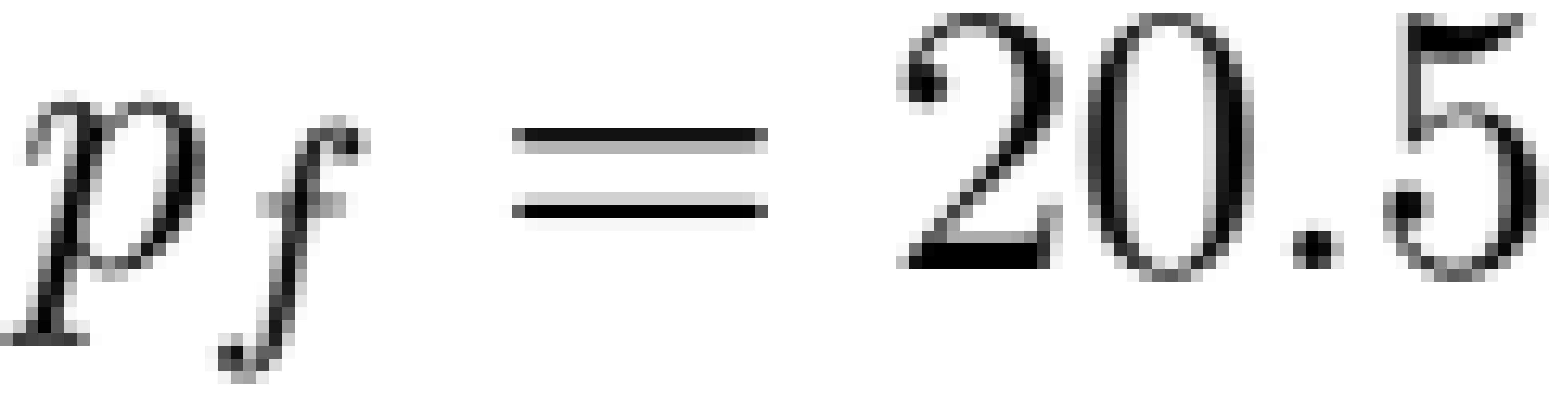


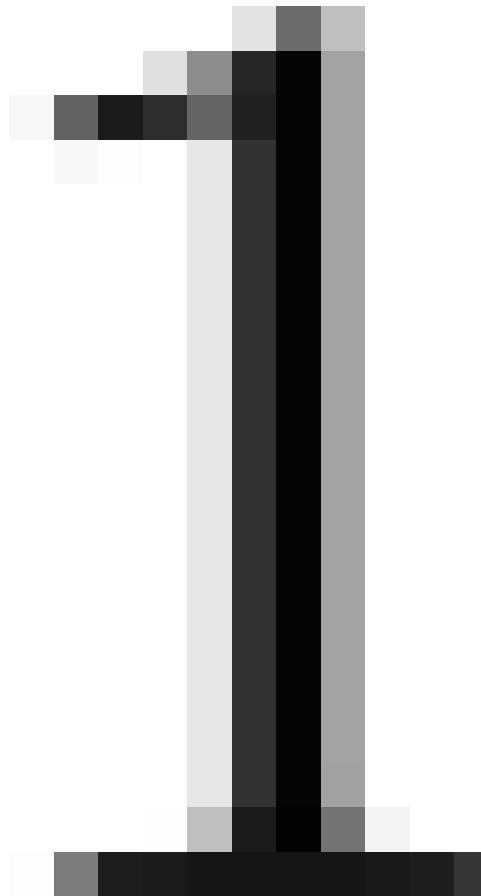
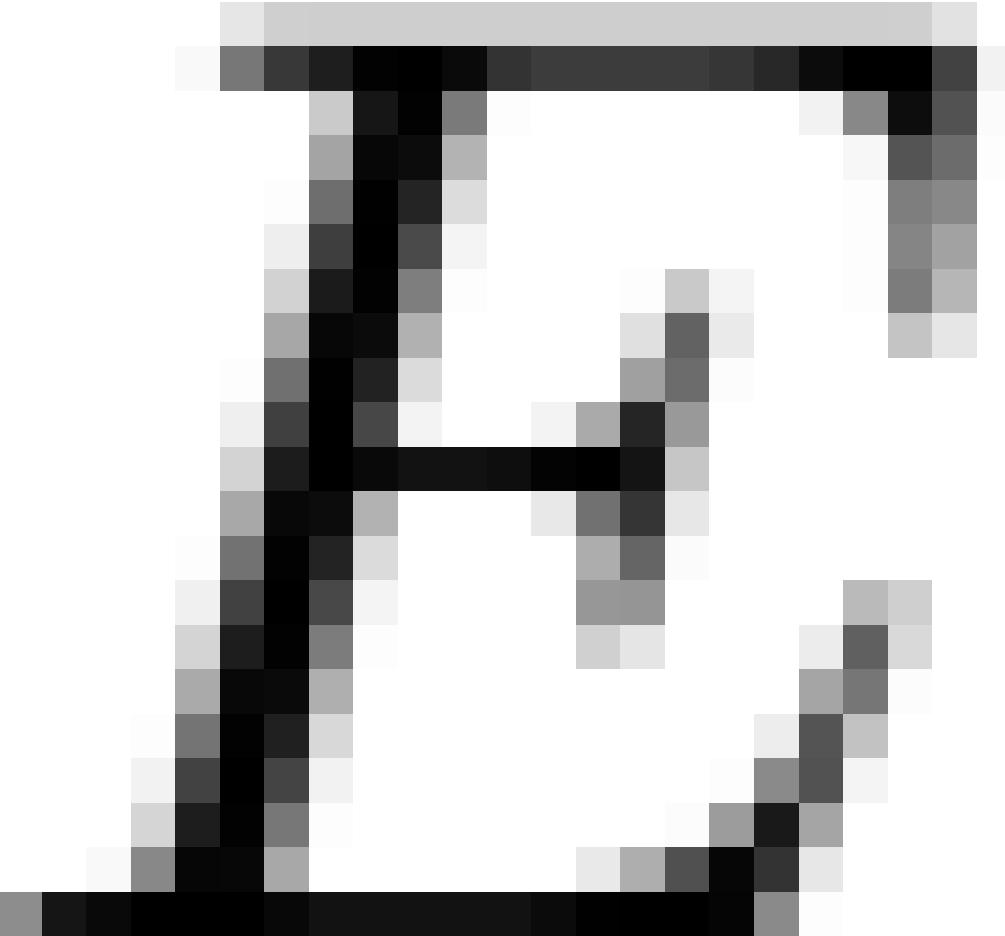
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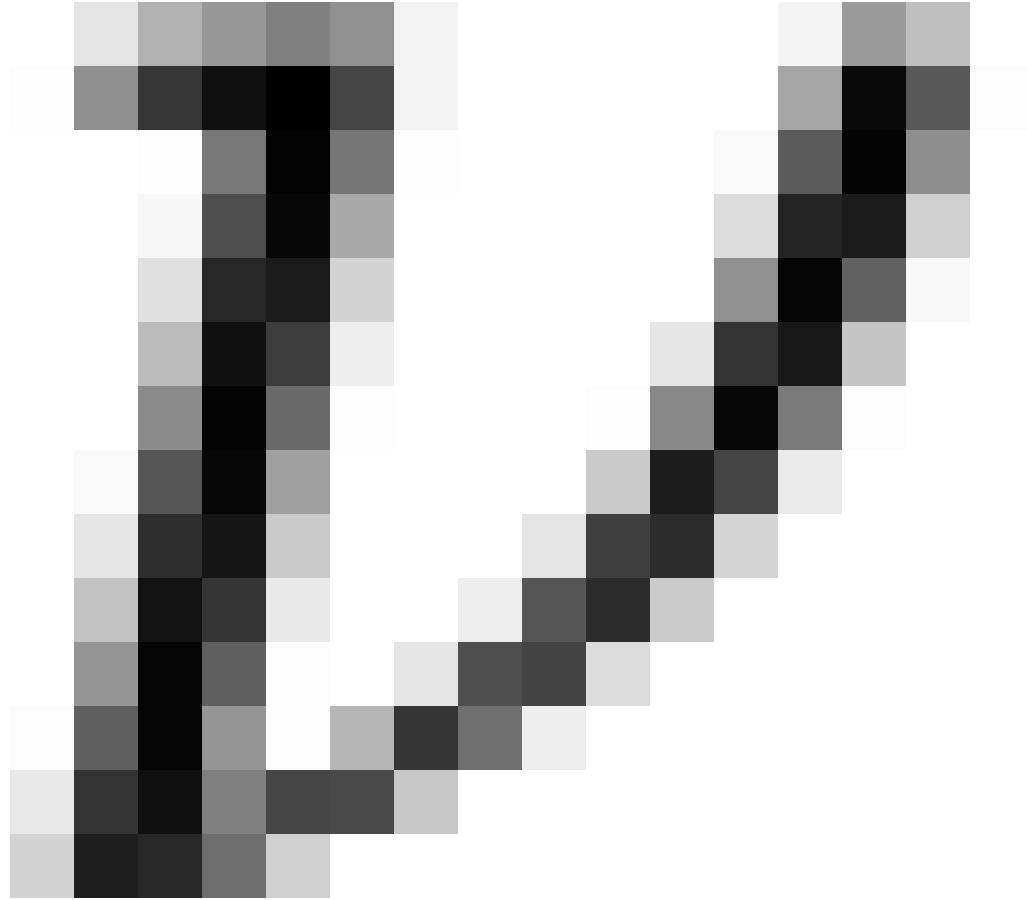


$2s$

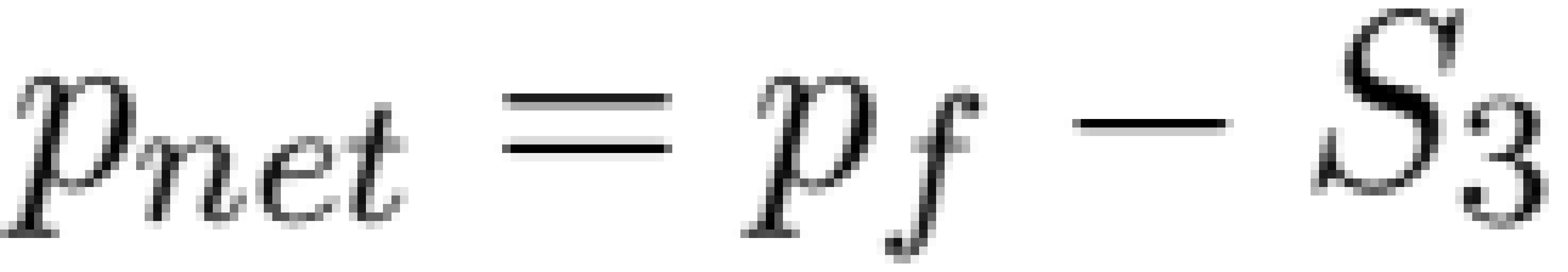




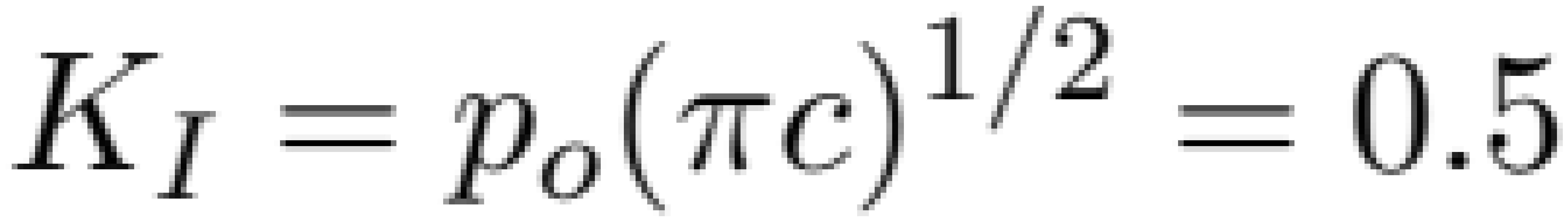


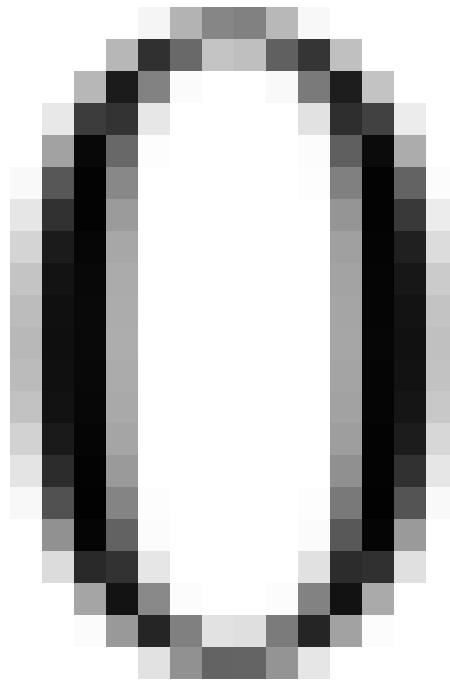
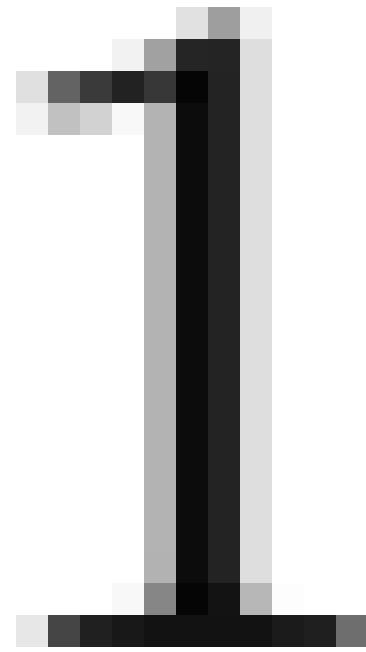
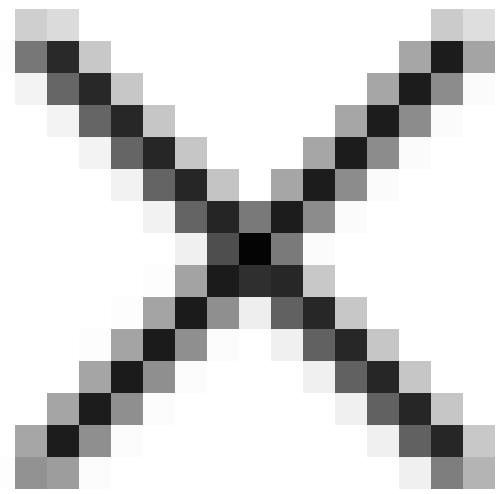
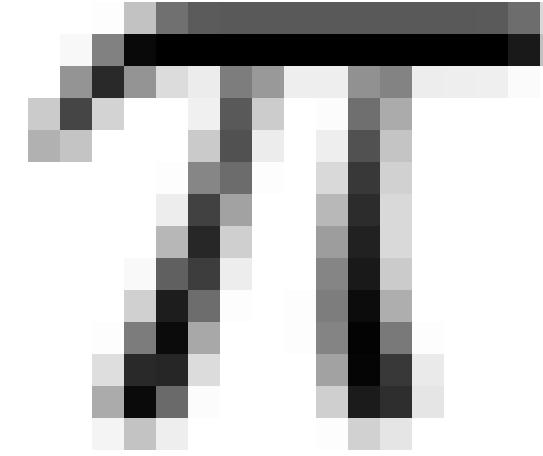
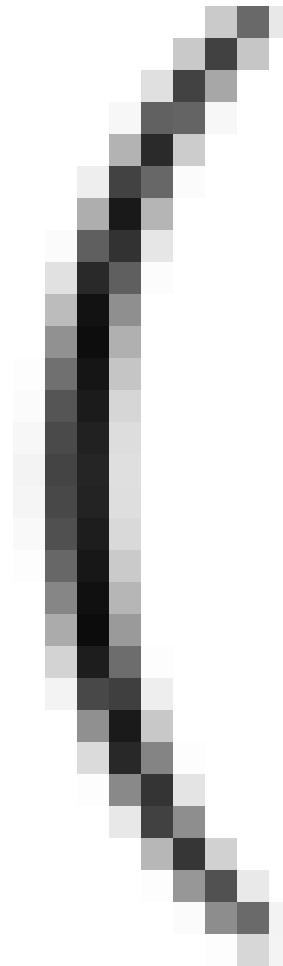


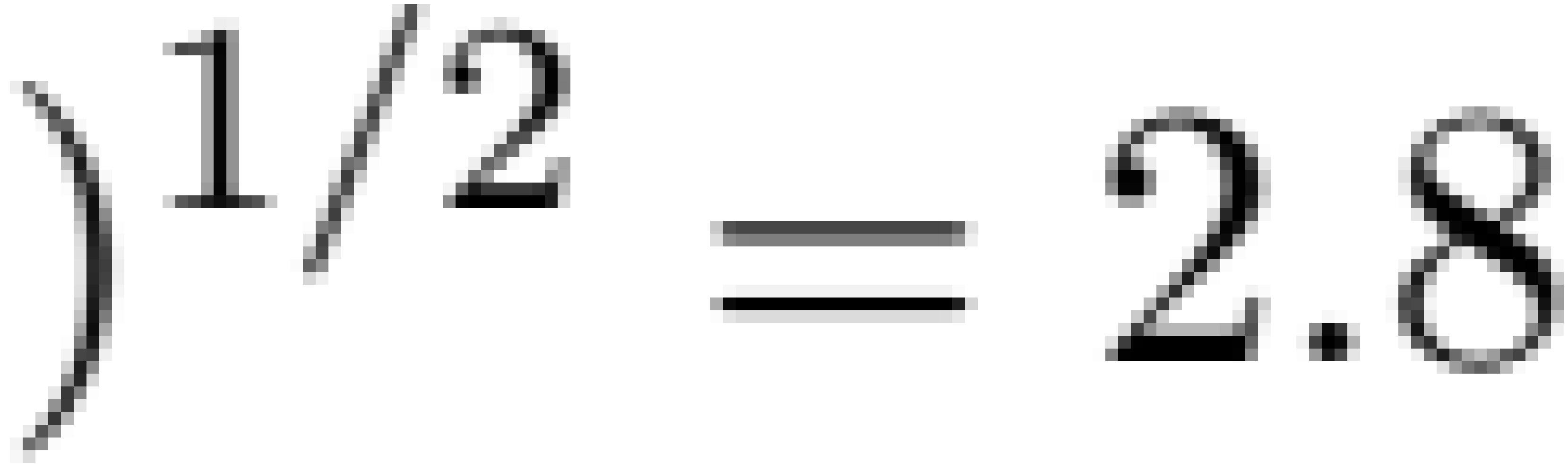
$$\frac{E}{1 - \nu^2} = \frac{1 \text{ GPa}}{1 - 0.25^2} = 1.07 \text{ GPa} = 1.07 \times 10^7 \text{ MPa}$$

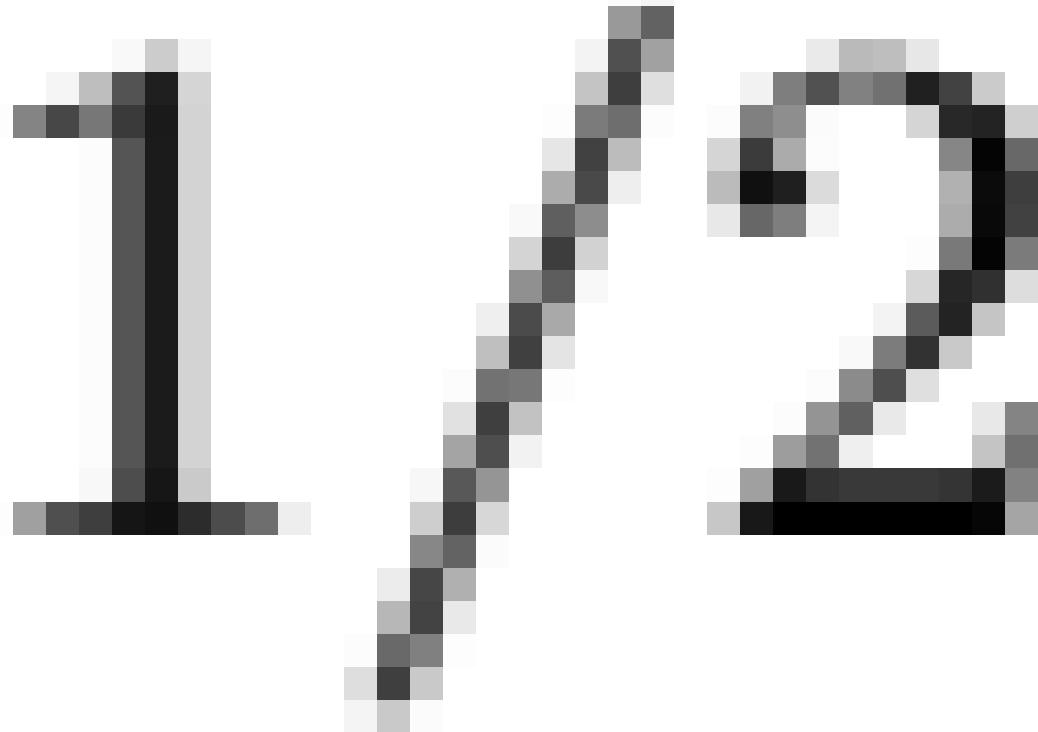


$$\omega_0 = \frac{4\rho_0 c}{E'} = \frac{4 \times 0.5 \text{ MPa} \times 10 \text{ m}}{1070 \text{ MPa}} = 0.019 \text{ m} = 19 \text{ mm}$$



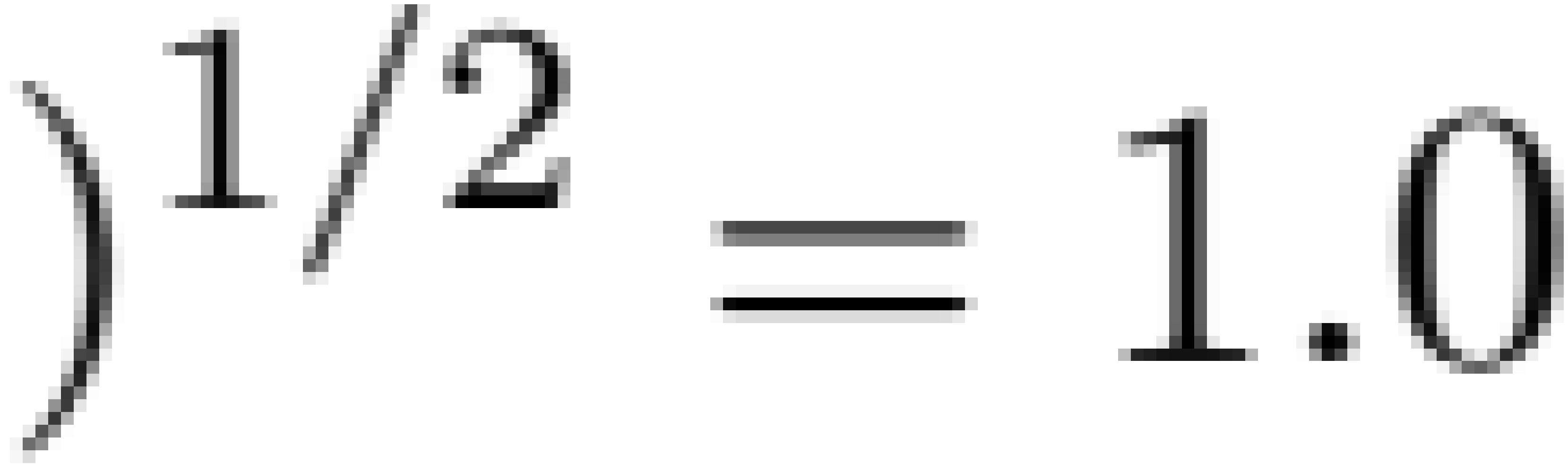


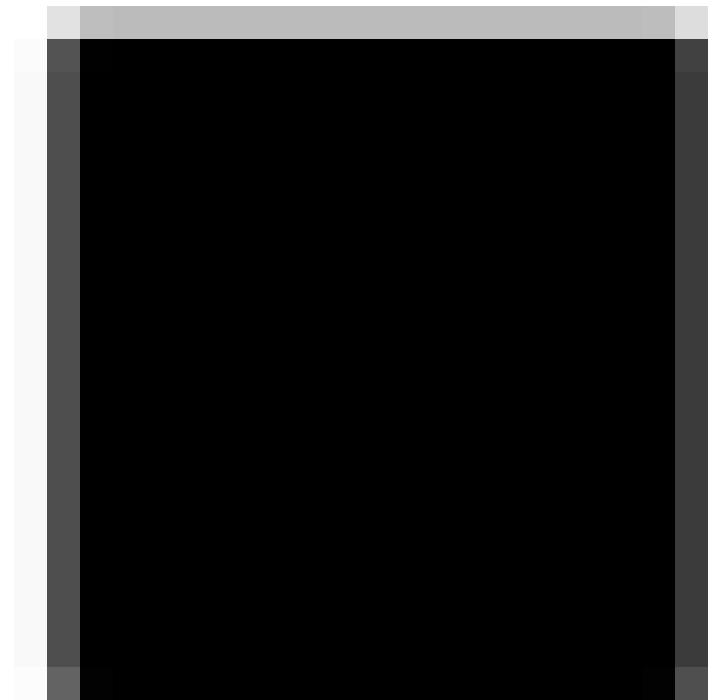
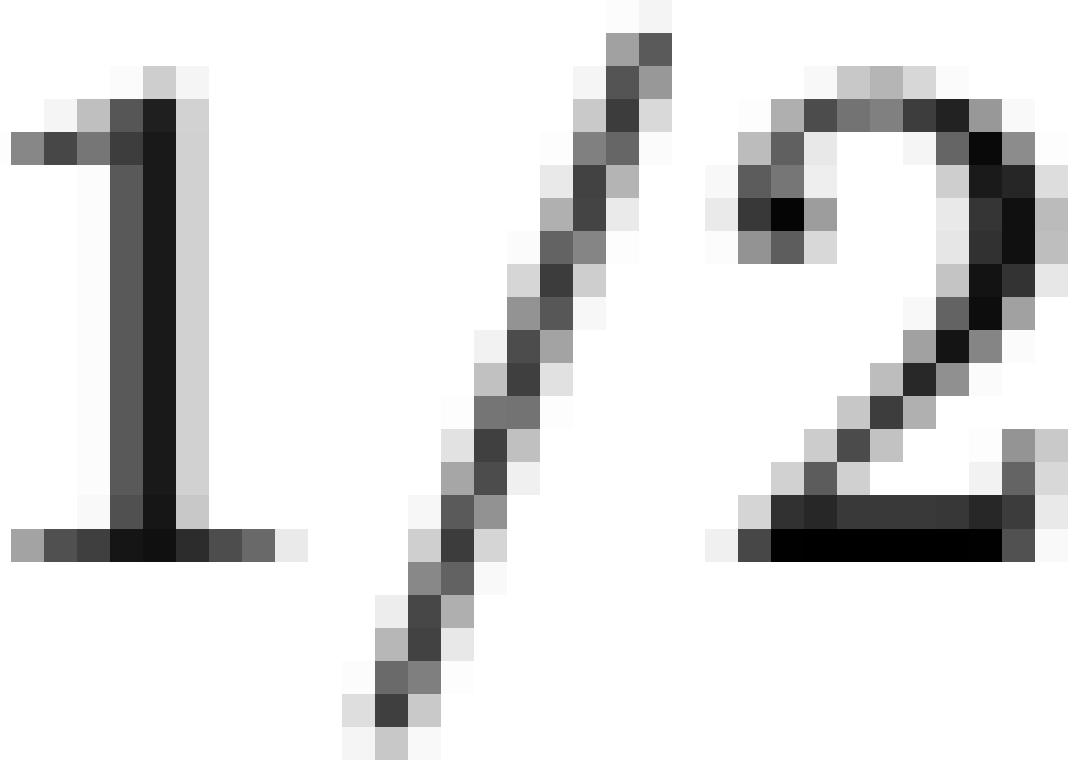


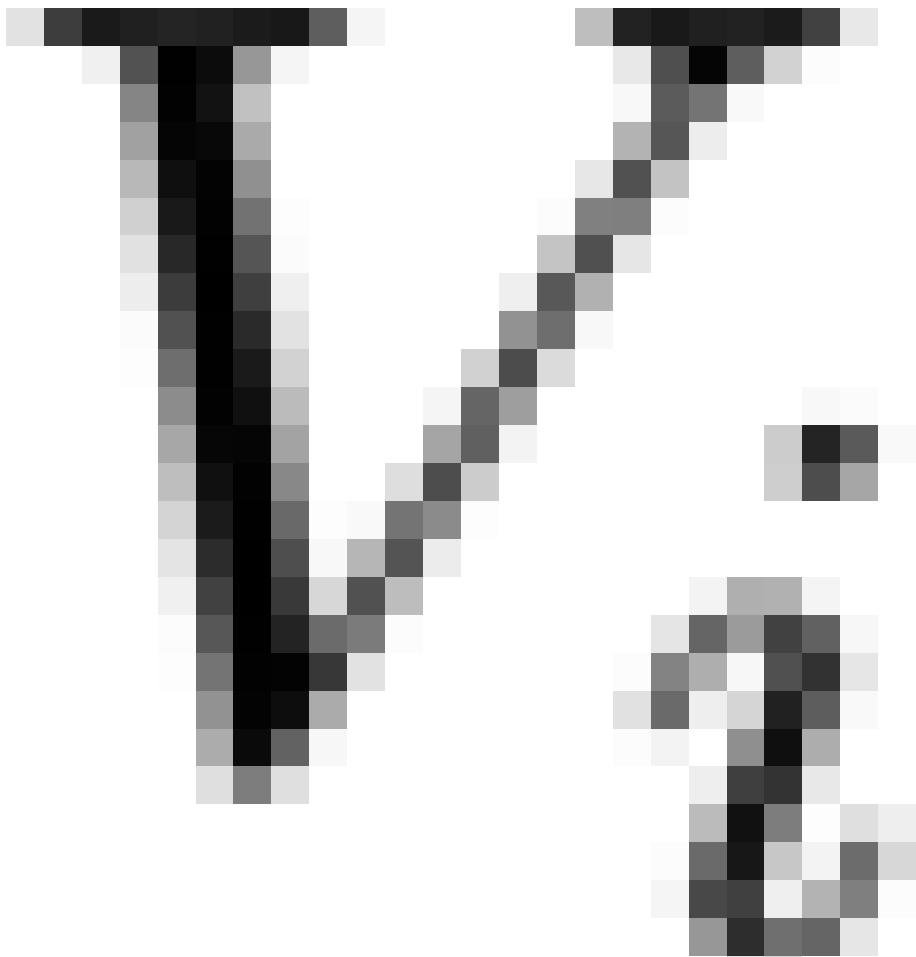


$$w_0 = \frac{2\rho_0 c}{E'} = \frac{2 \times 0.5 \text{ MPa} \times 10 \text{ m}}{1070 \text{ MPa}} = 0.0095 \text{ m} = 9.5 \text{ mm}$$

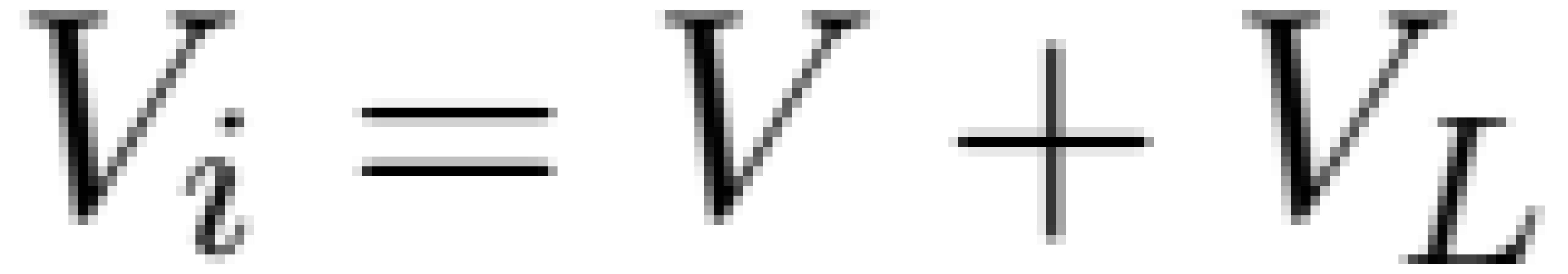
$$K_I = \left(1 - \frac{2}{\pi}\right)^{1/2} = \rho_0(\pi c)^{1/2} = 0.5$$

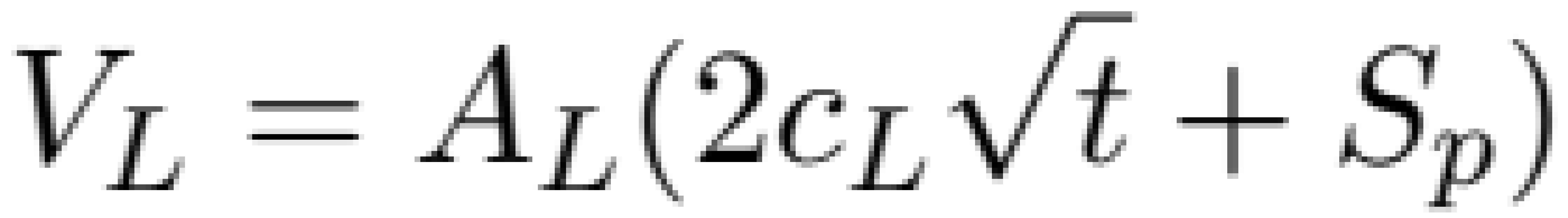


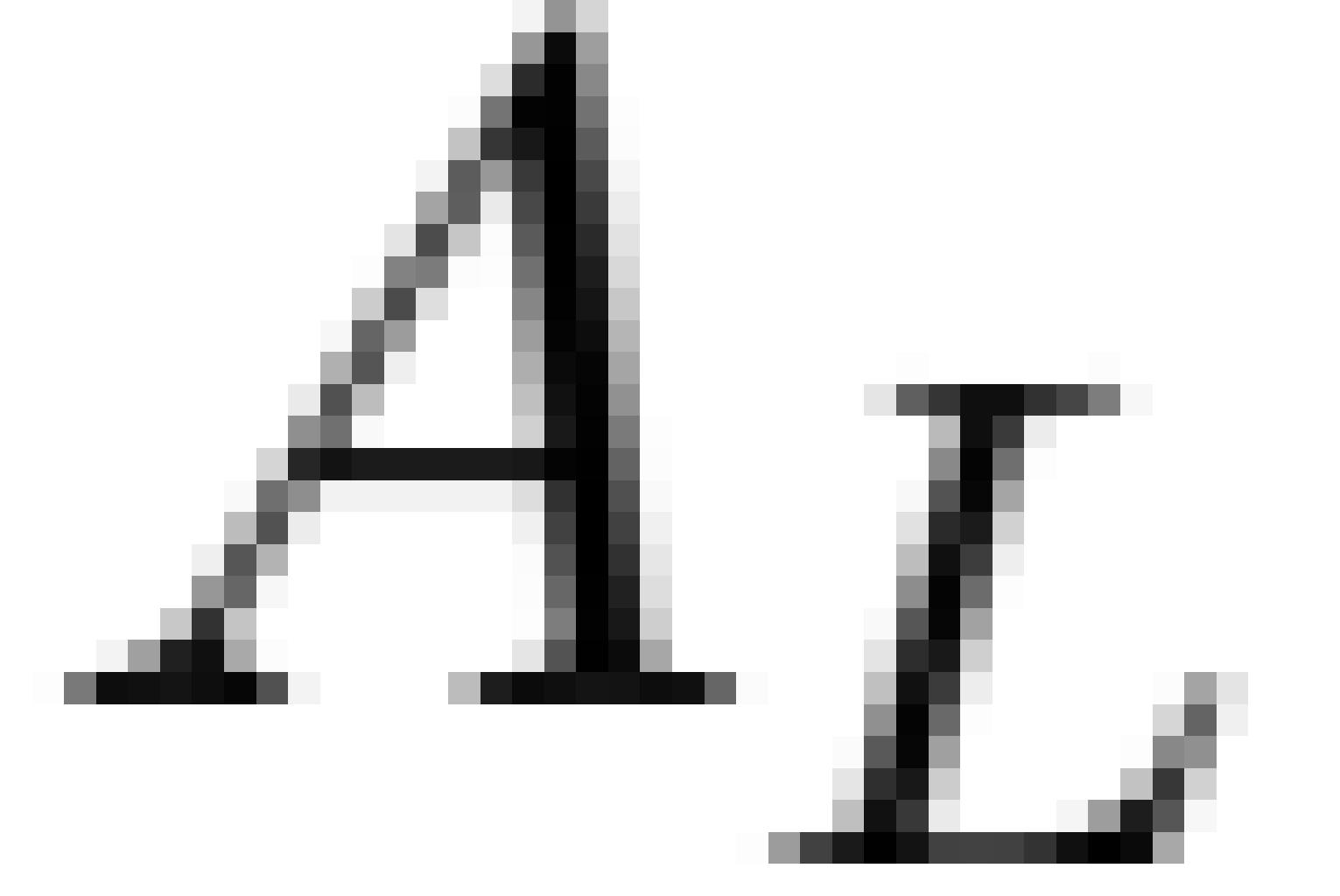


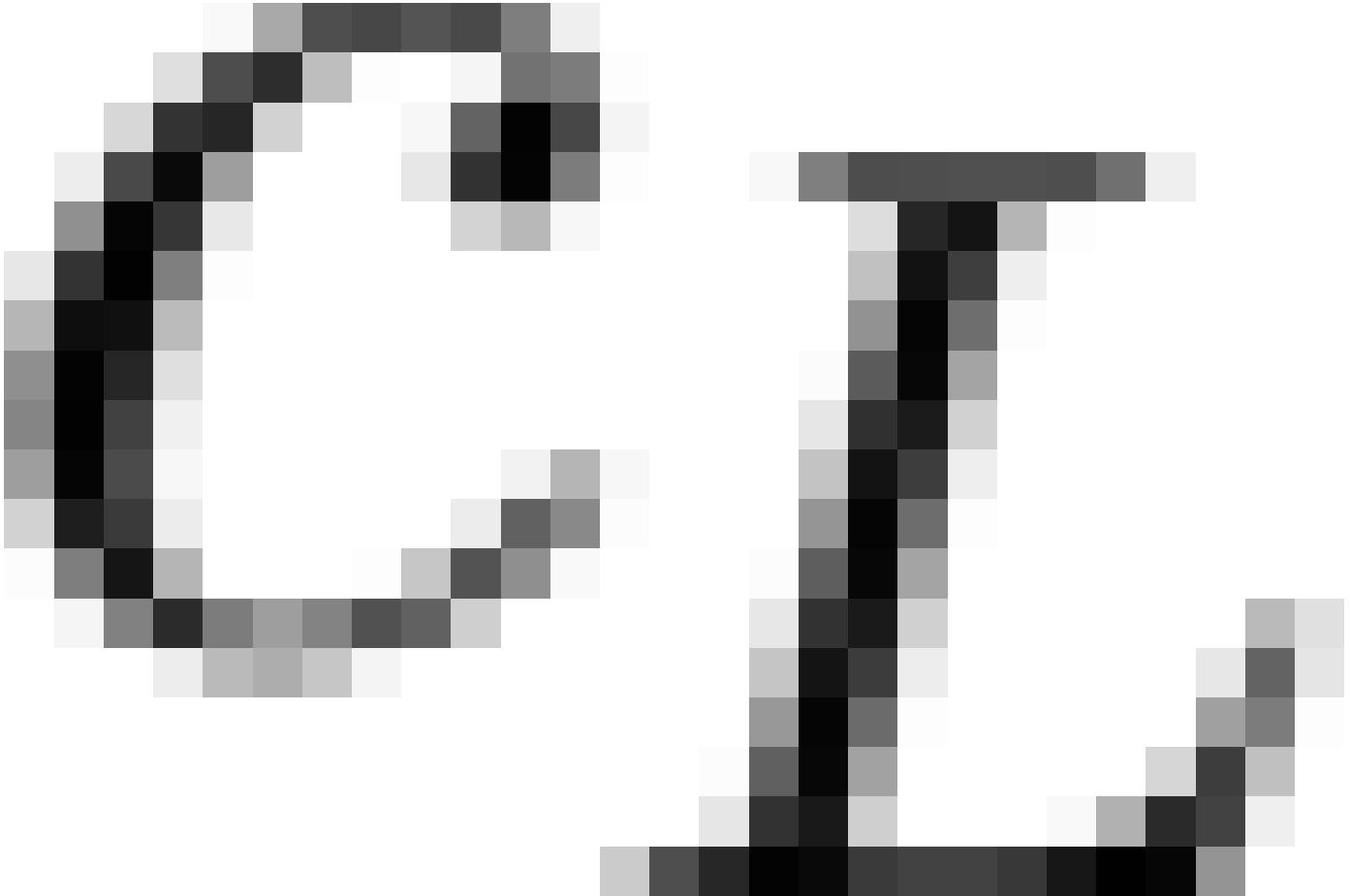














W

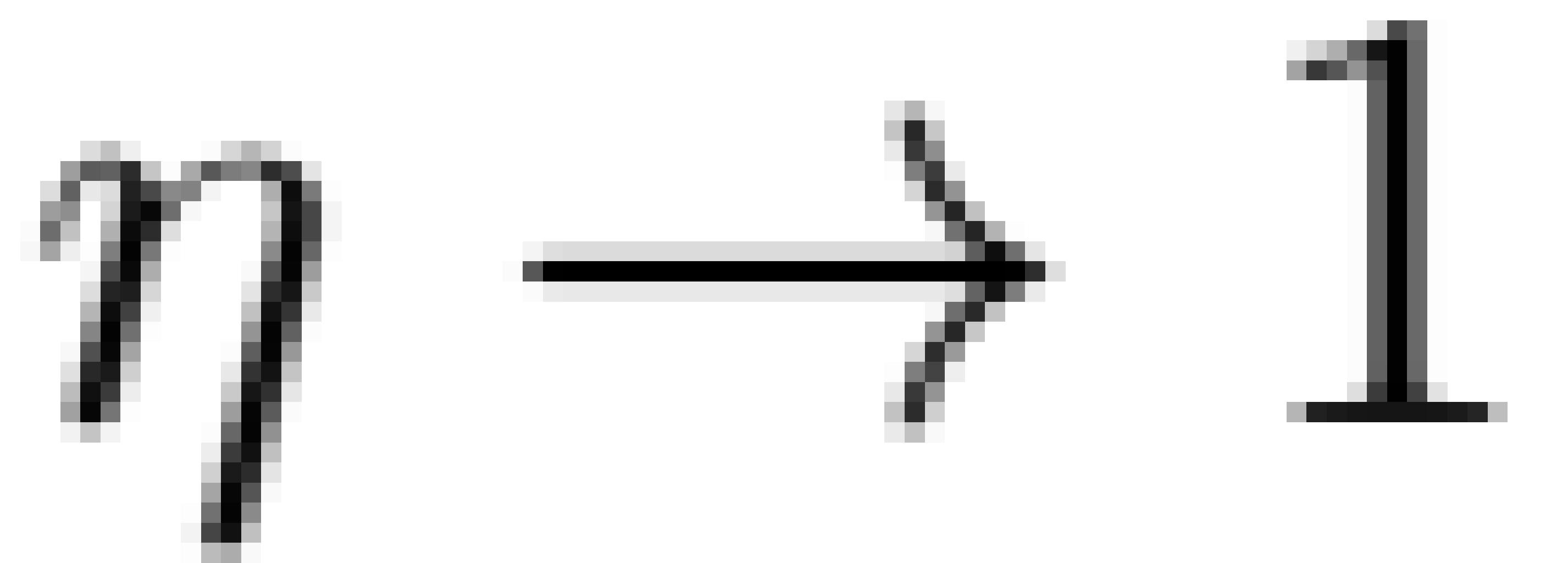
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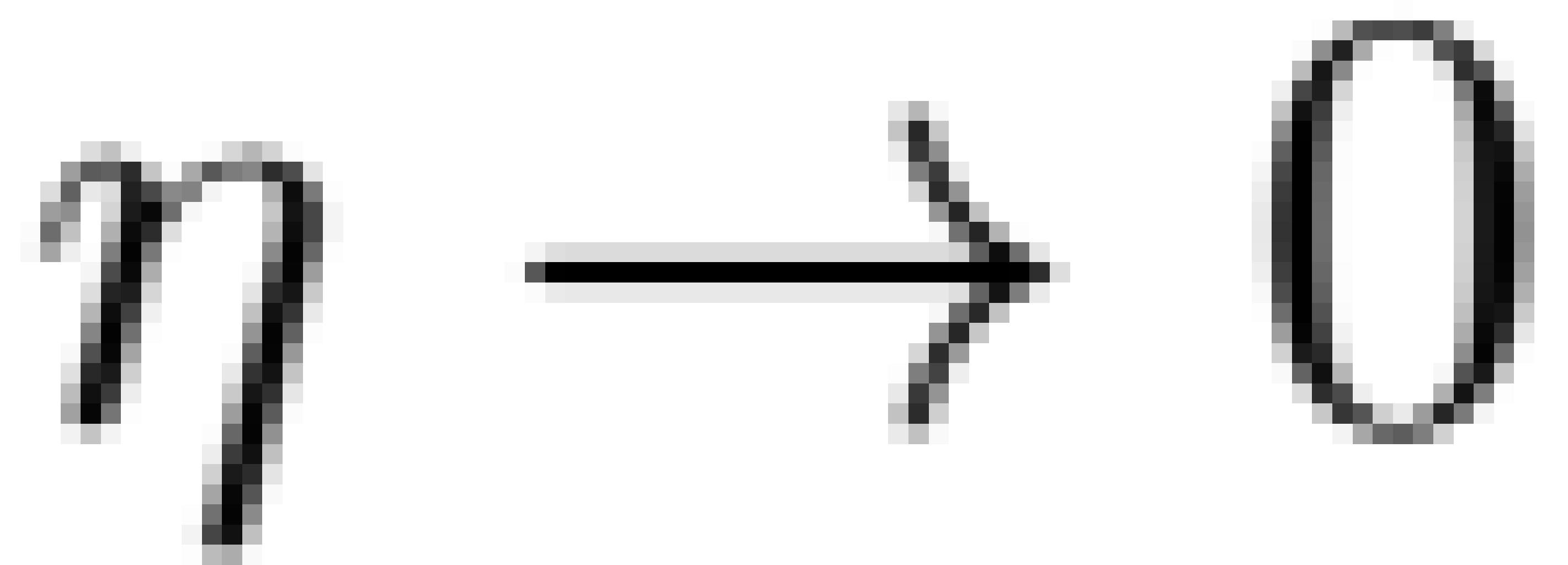
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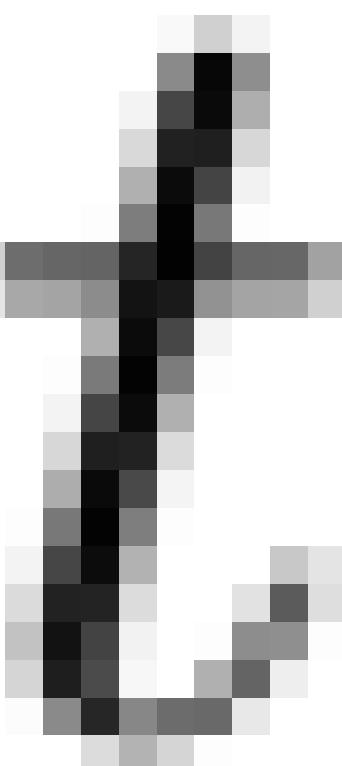
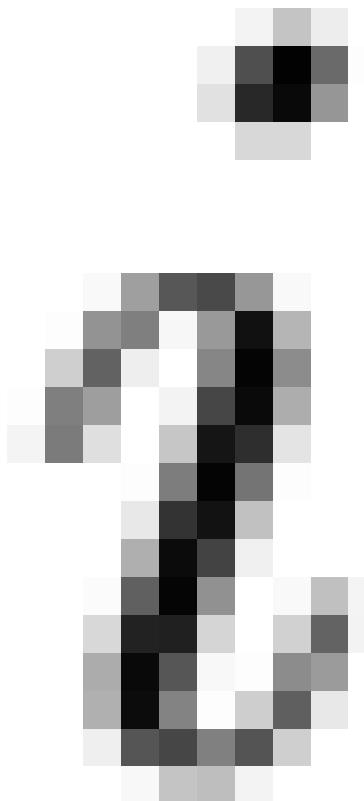
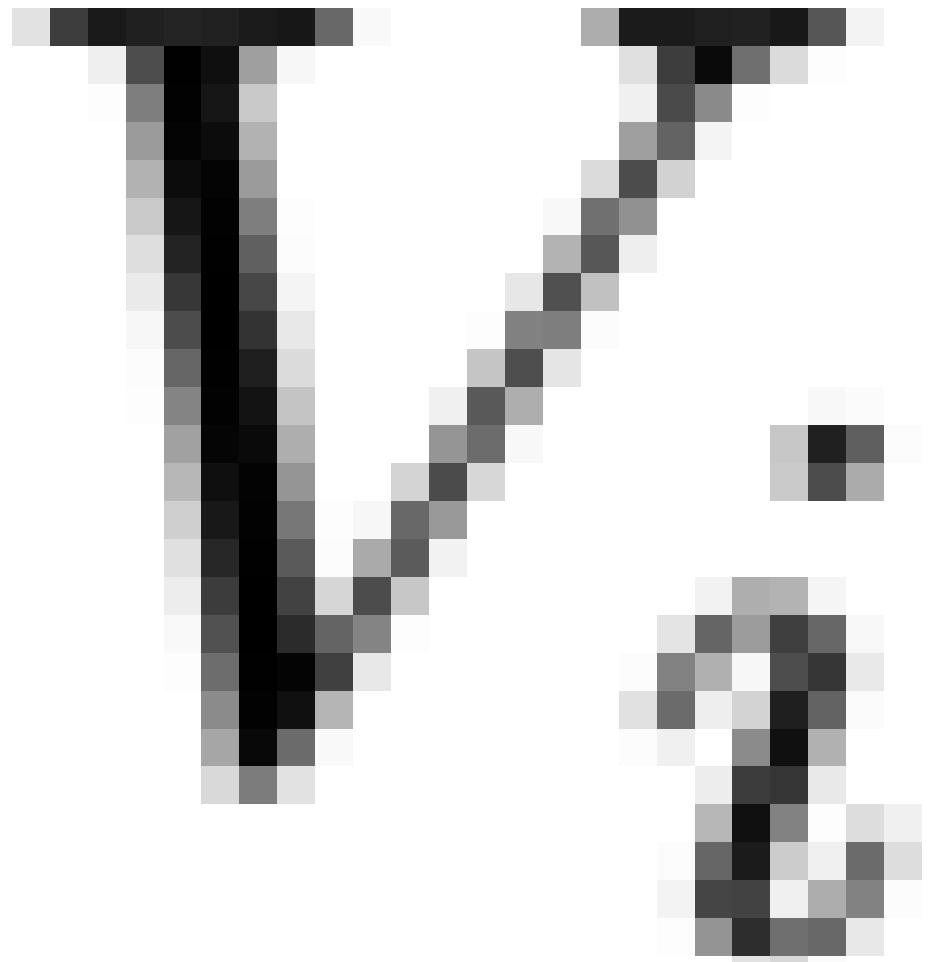
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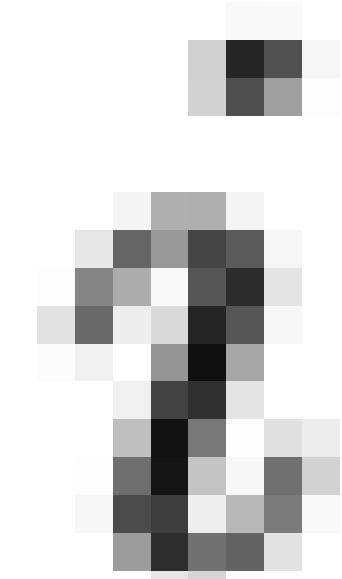
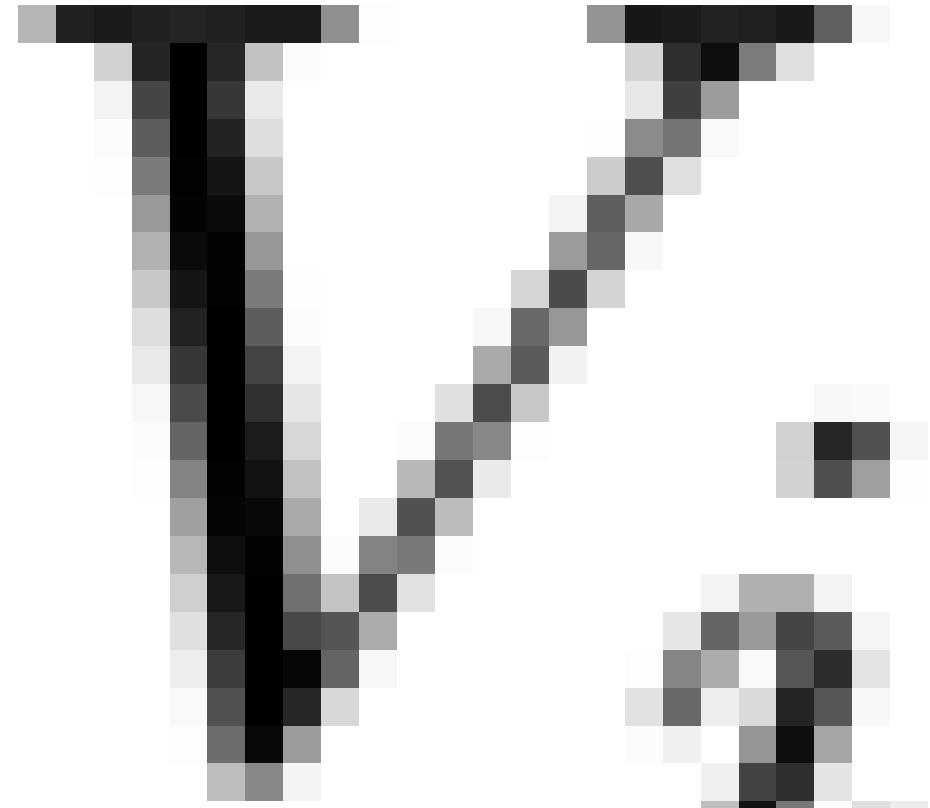
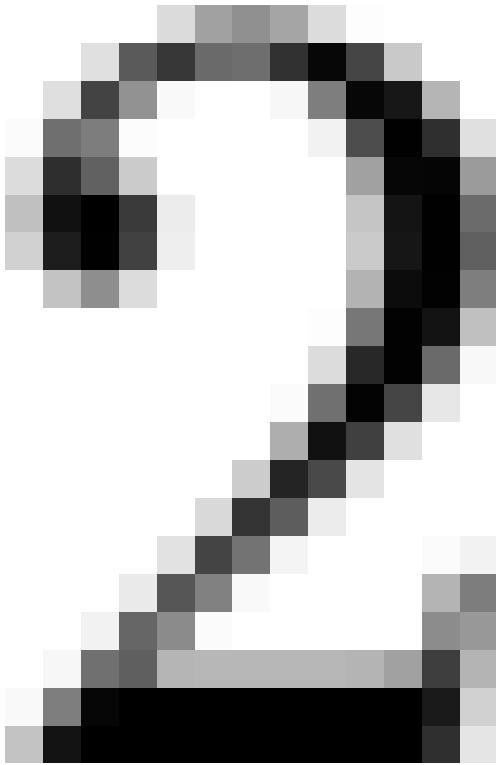
W

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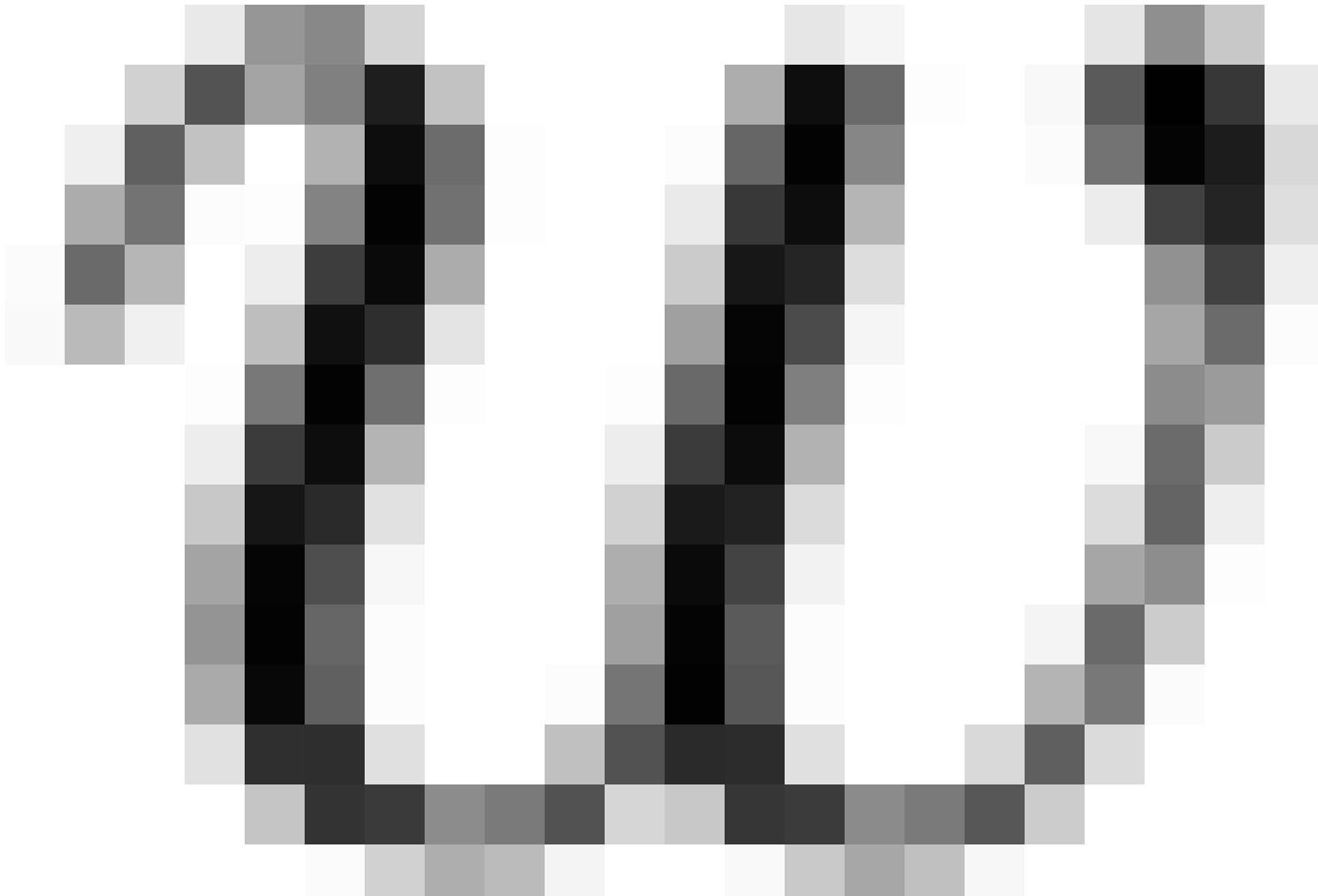


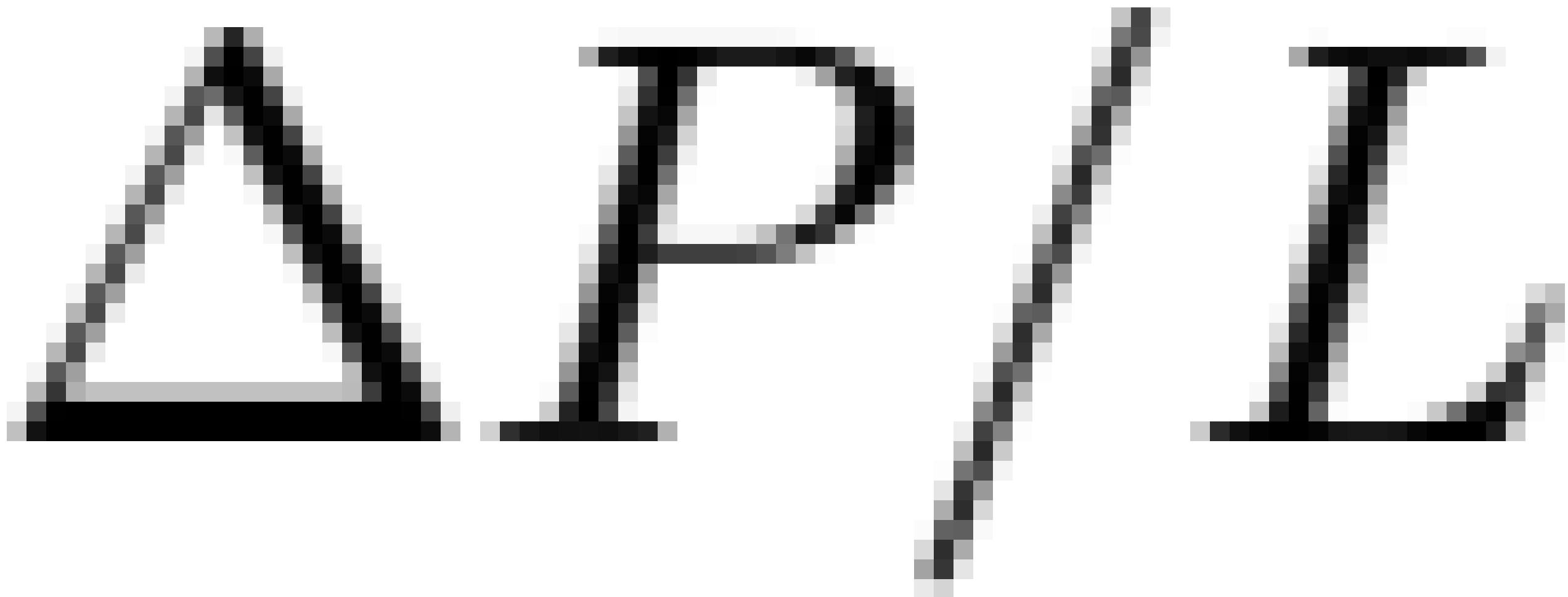


q

$\omega^3 h_f P$

$12\mu L$

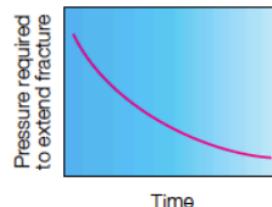
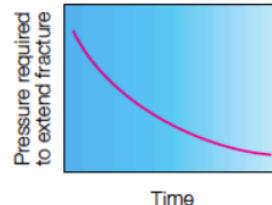
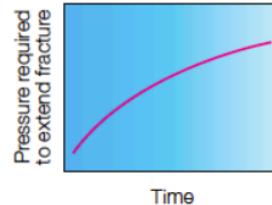
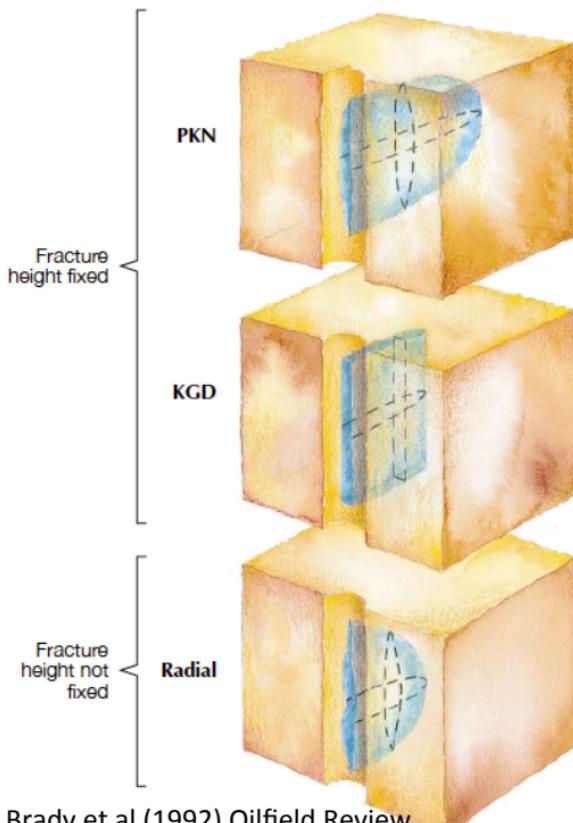








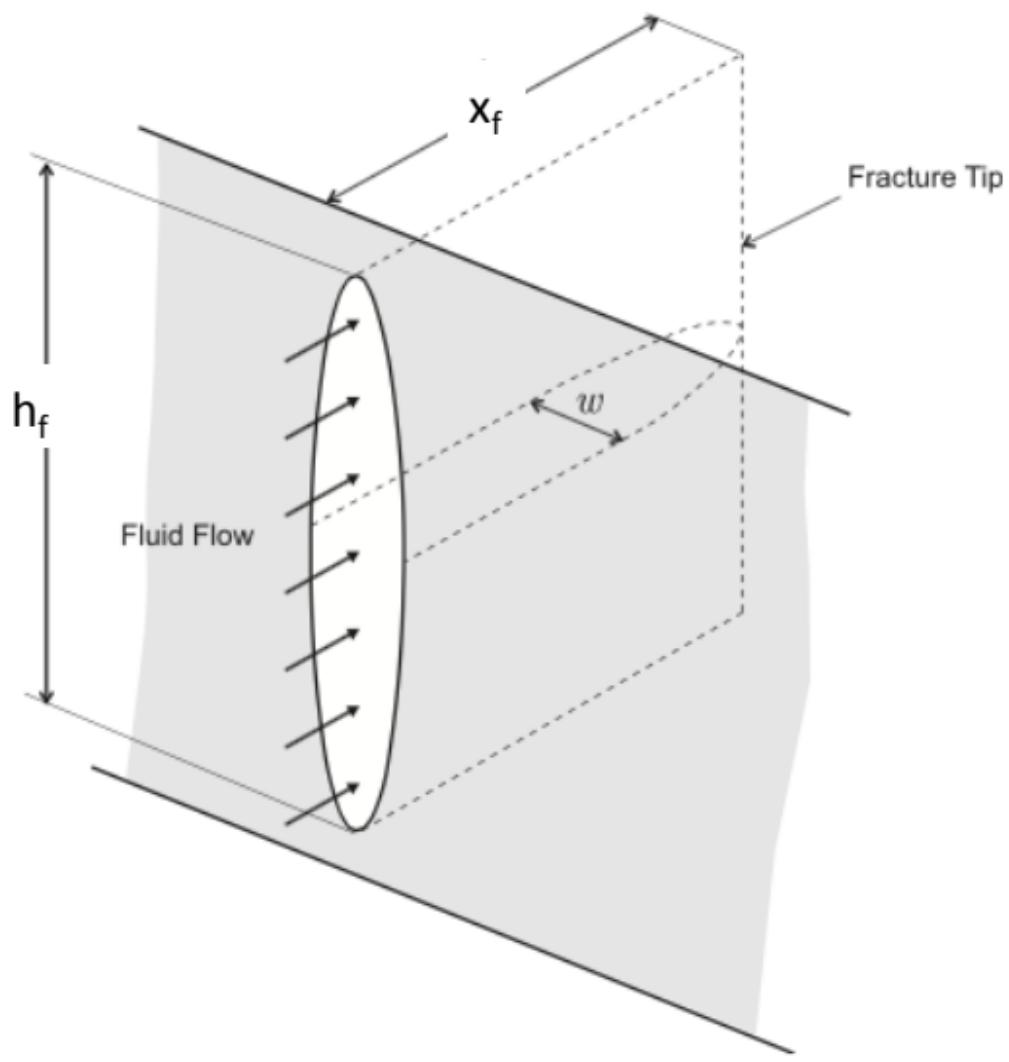
2D Fracture Models



- Elliptical cross section
- Width \propto height
- Width < KGD; length > KGD
- More appropriate when fracture length > height

- Rectangular cross section
- Width \propto length
- More appropriate when fracture length < height

- Appropriate when fracture length = height



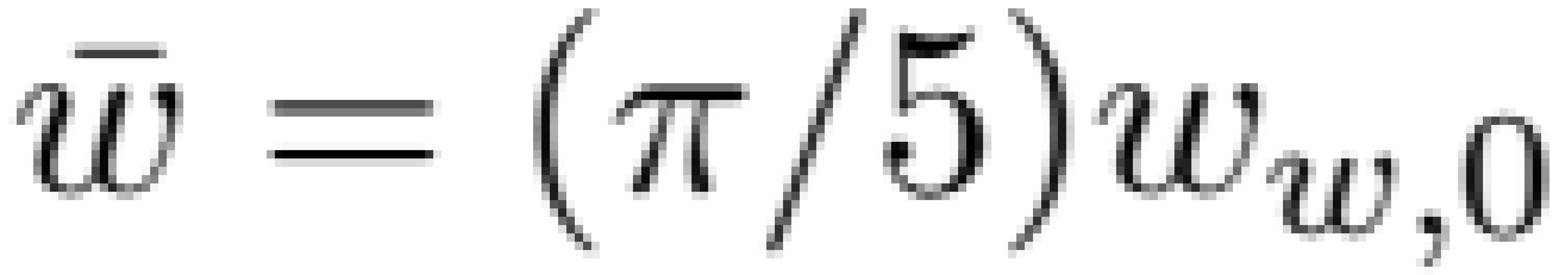
$$x_f = \left(\frac{625}{512\pi^3} \right)^{1/5} \left(\frac{i^3 E'}{\mu h_f^4} \right)^{1/5} t^{4/5} = 0.524 t^{4/5}$$

$$w_{w,0} = \left(\frac{2560}{\pi^2} \right)^{1/5} \left(\frac{i^2 \mu}{E' h_f} \right)^{1/5} t^{1/5} = 3.040 \left(\frac{i^2 \mu}{E' h_f} \right)^{1/5} t^{1/5}$$

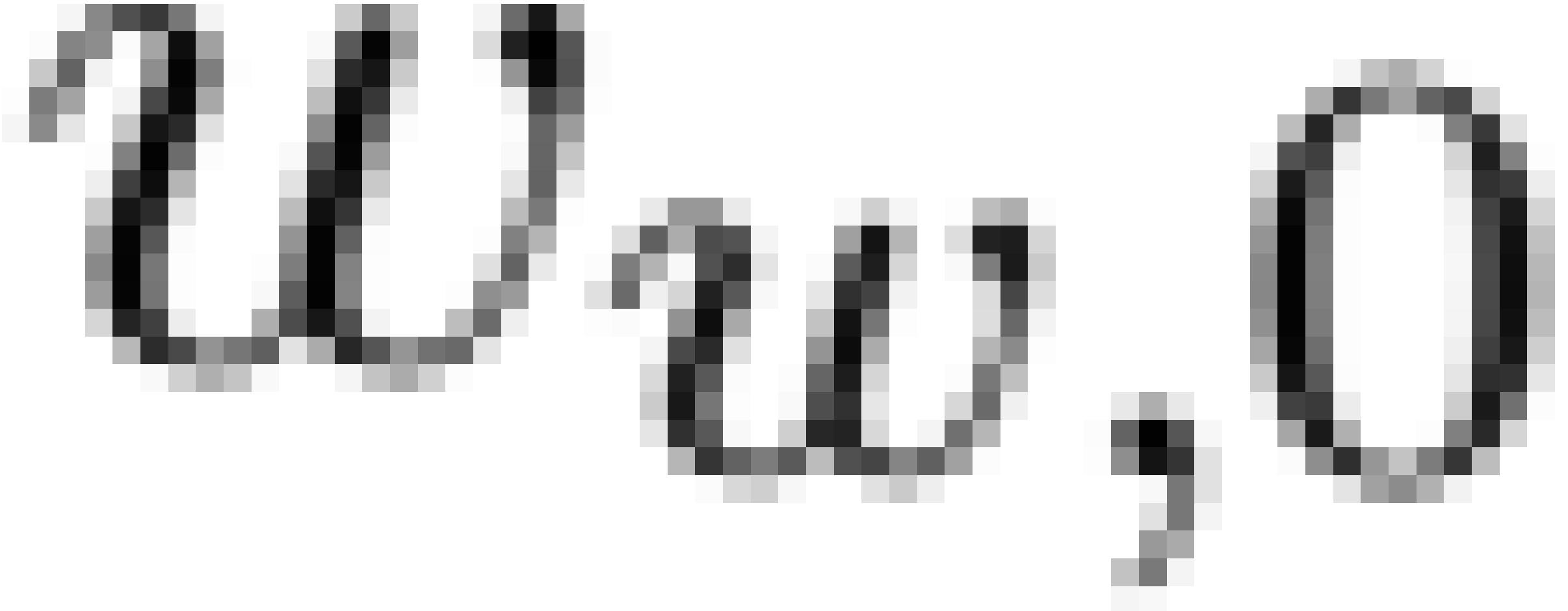
$$p_{net,w} = \left(\frac{80}{\pi^2} \right)^{1/4} \left(\frac{E'^4 i^2 \mu}{h_f^6} \right)^{1/5} t^{1/5} = 1.520 \left(\frac{E'^4 i^2 \mu}{h_f^6} \right)^{1/5} t^{1/5}$$



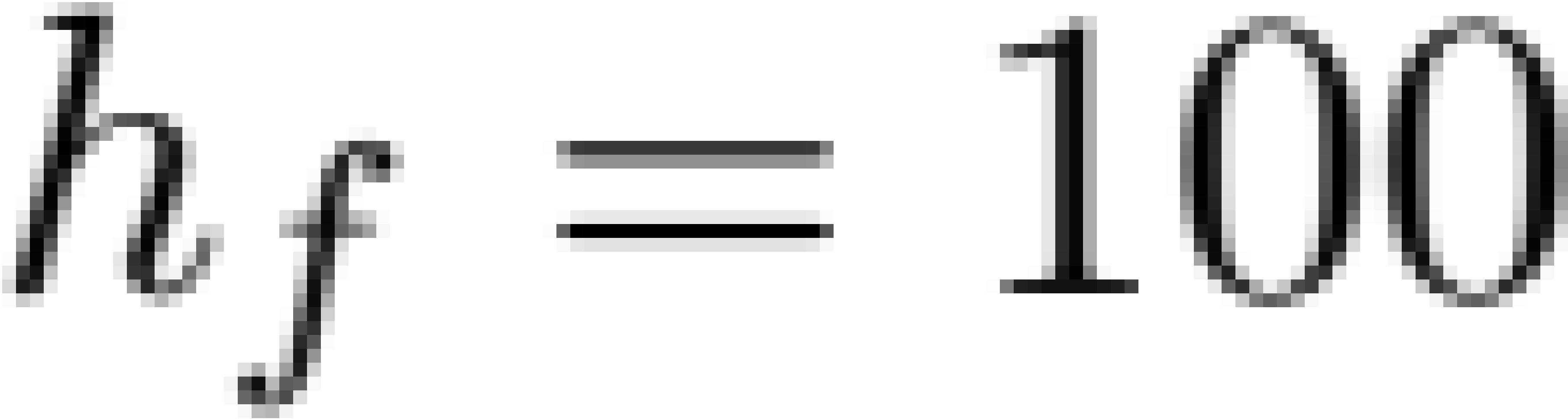






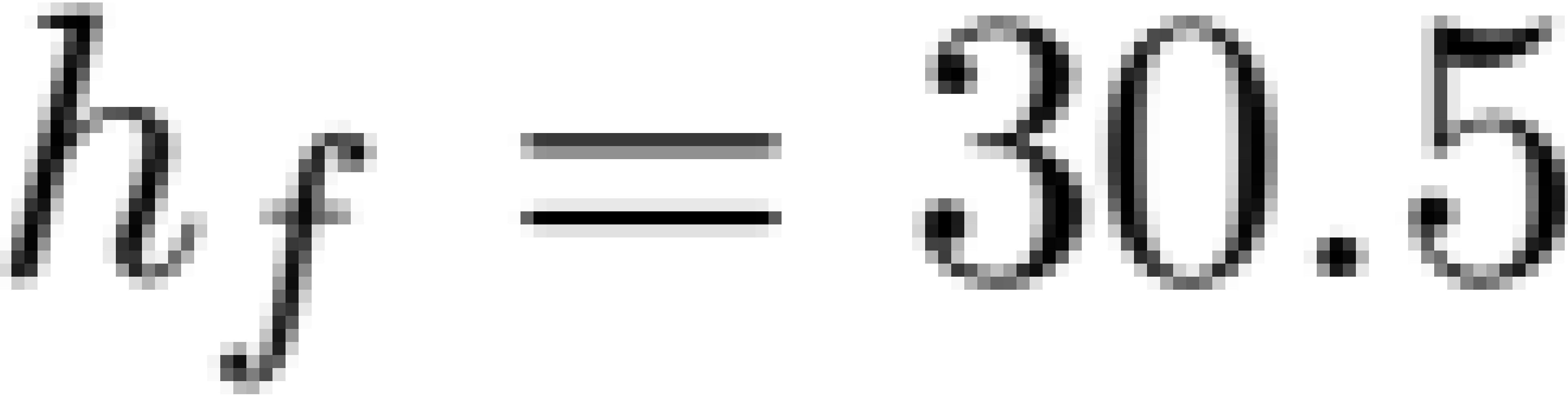


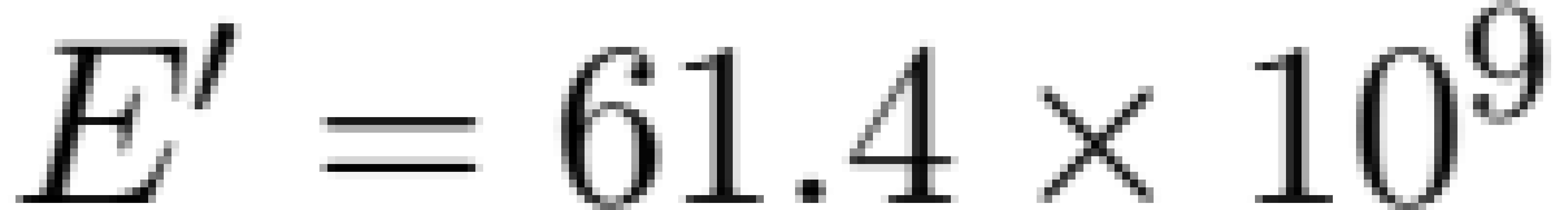


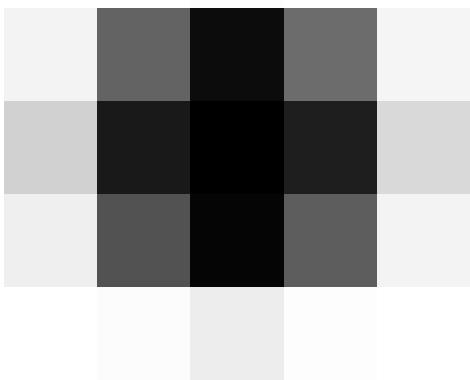


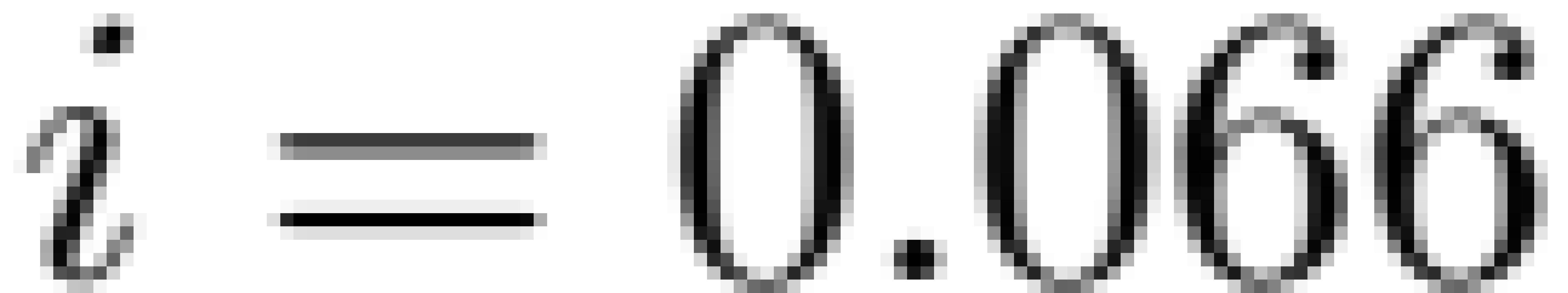












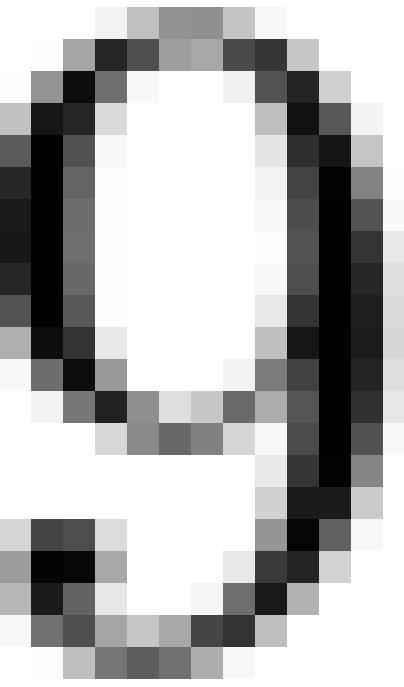
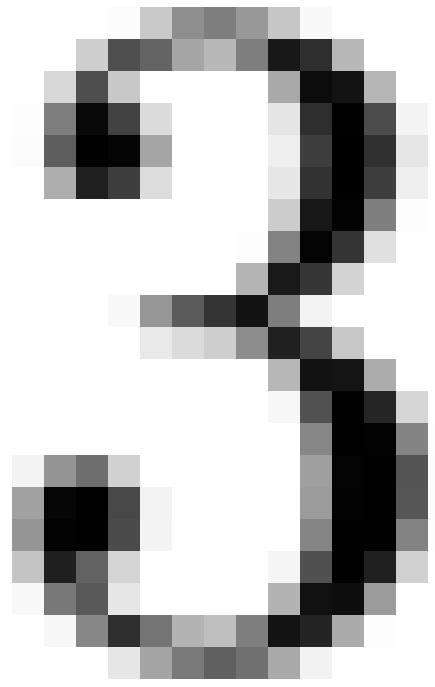
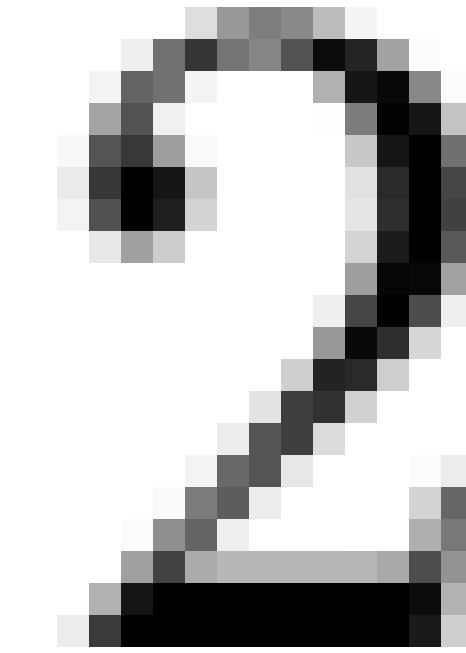
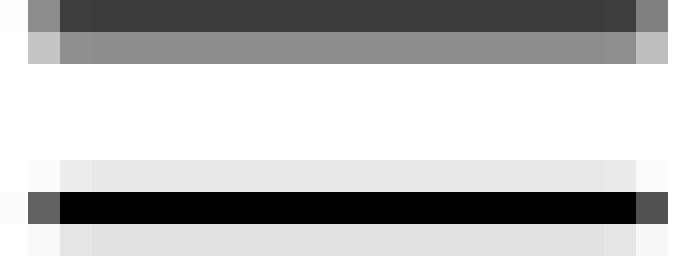
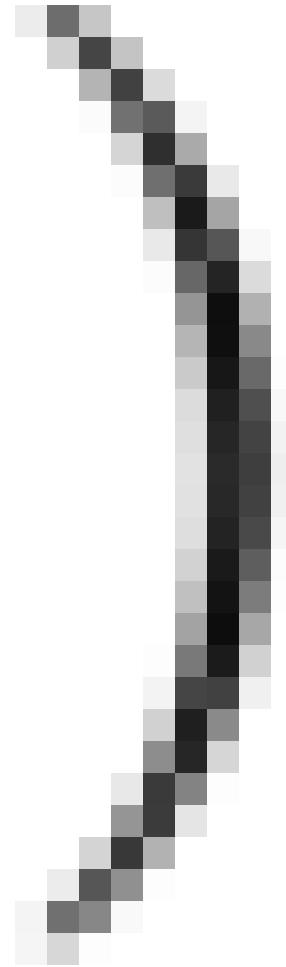
$$x_f = 0.524 \left[\frac{(0.066 \text{ m}^3/\text{s})^3 \times (61.4 \times 10^9 \text{ Pa})}{0.001 \text{ Pa s} \times (30.5 \text{ m})^4} \right]^{1/5} = 1536.2 \text{ m}$$

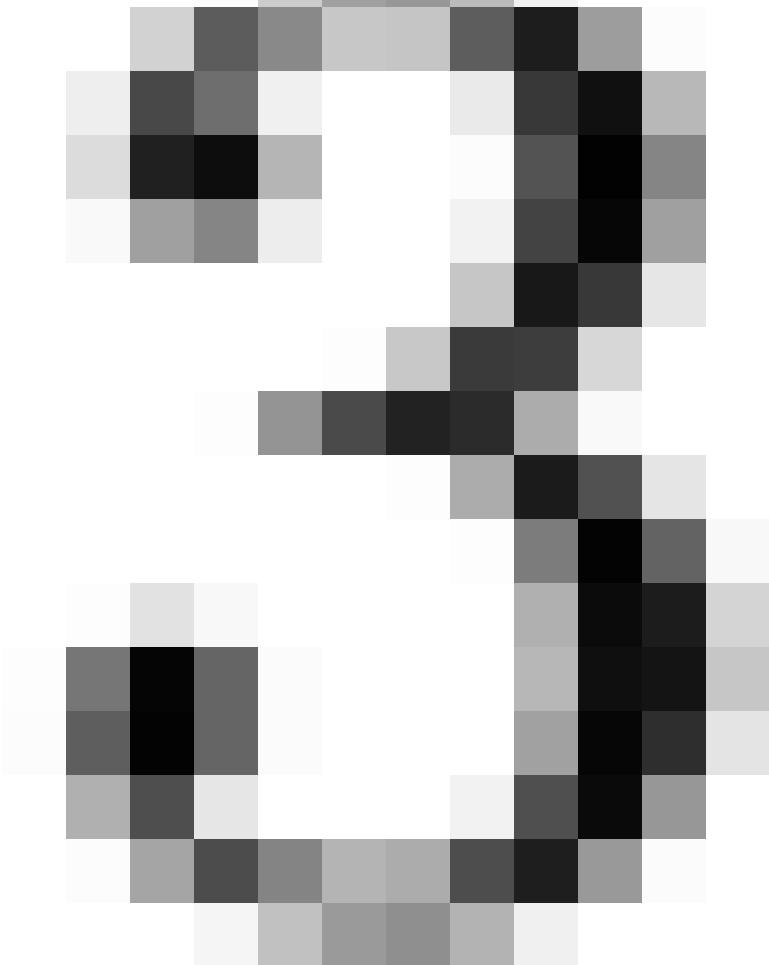
$$w_{w,0} = 3.040 \left[\frac{(0.066 \text{ m}^3/\text{s})^2 \times (0.001 \text{ Pa s})}{(61.4 \times 10^9 \text{ Pa}) \times (30.5 \text{ m})} \right]^{1/5} = 4.05 \times 10^{-3} \text{ m} = 4.05 \text{ mm}$$

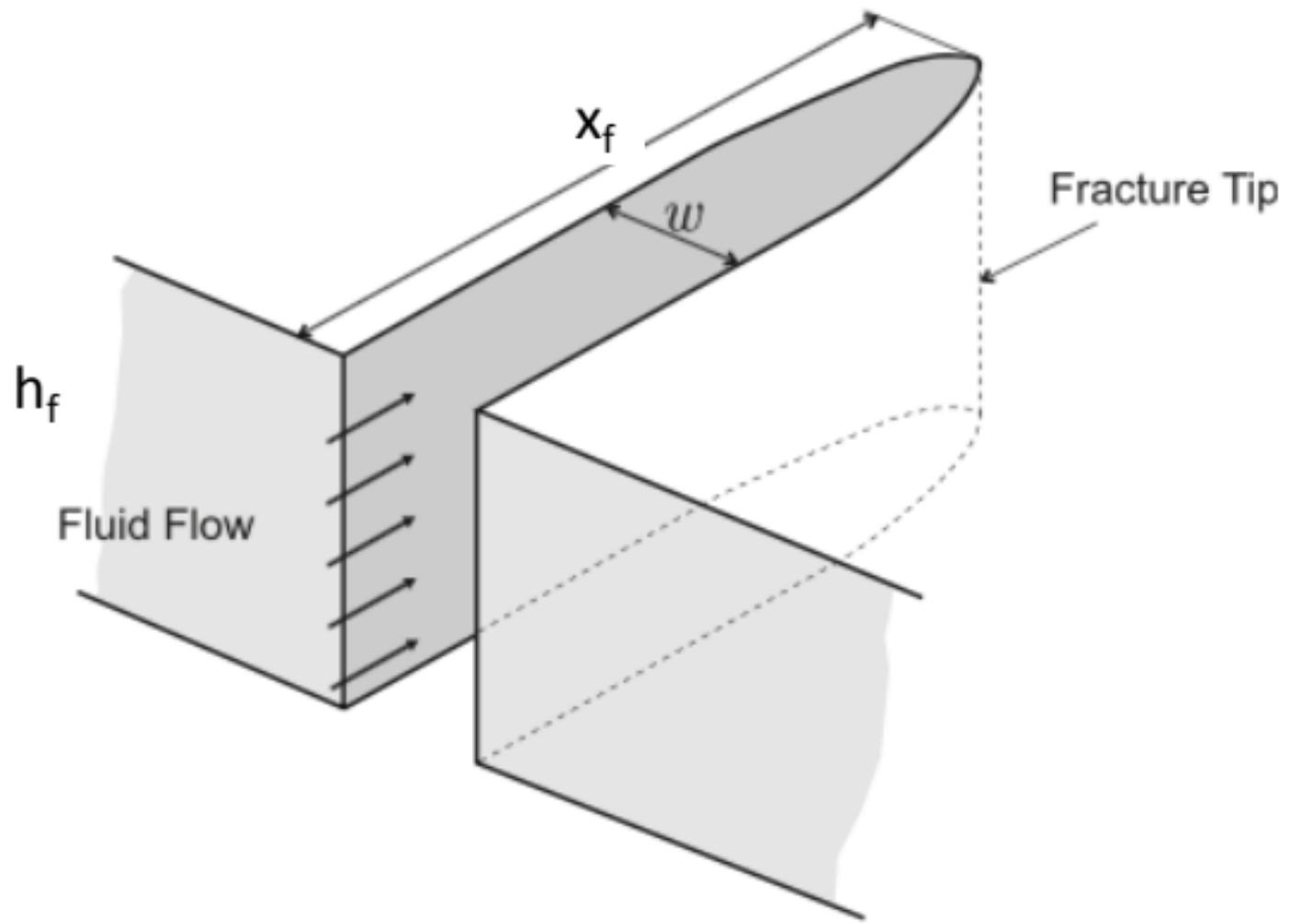
$$p_{net,w} = 1.520 \left[\frac{(61.4 \times 10^9 \text{ Pa})^4 \times (0.066 \text{ m}^3/\text{s})^2 \times (0.001 \text{ Pa s})}{(1800 \text{ s})^{1/5} \times (30.5 \text{ m})^6} \right]^{1/5} = 4.1 \times 10^6 \text{ Pa}$$

$$2.13 \times 10^{-3} \pi$$





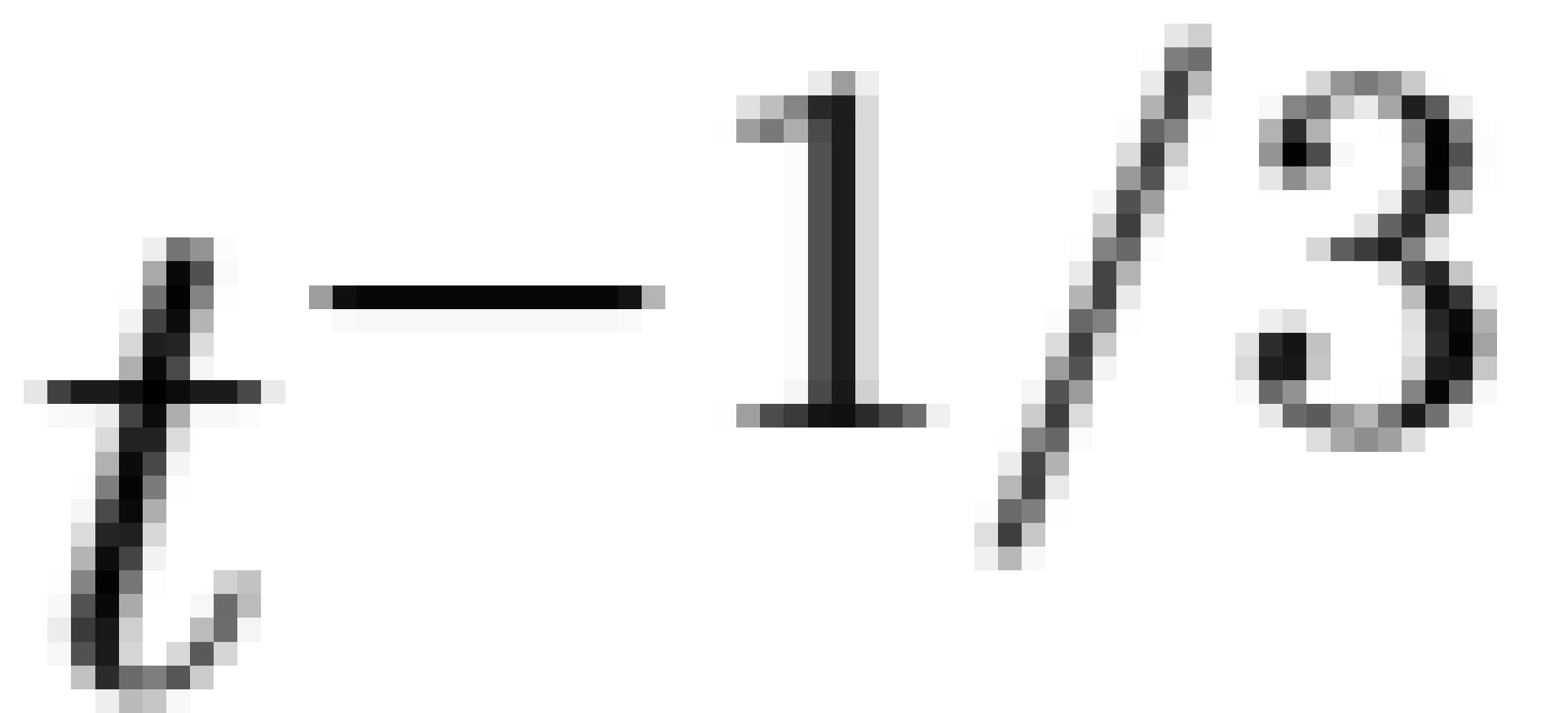


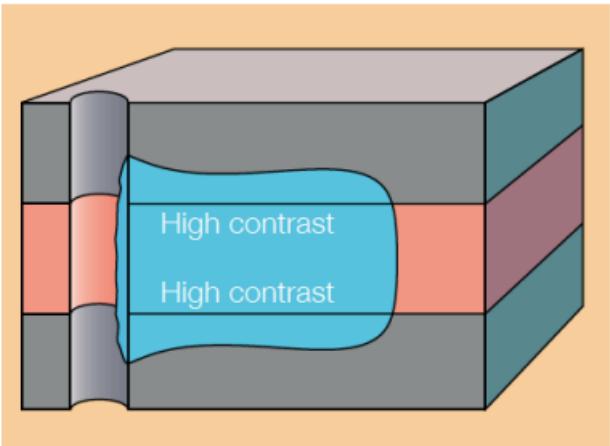
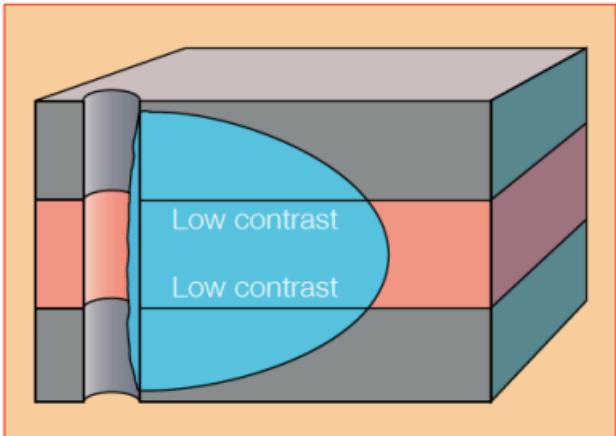
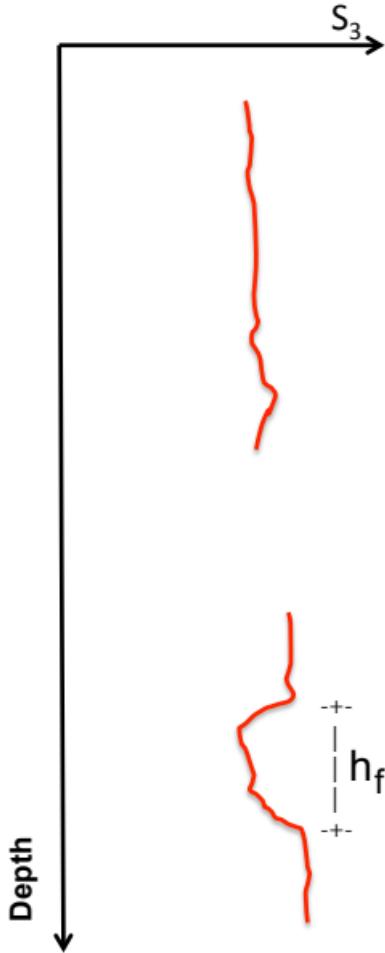


$$x_f = \left(\frac{16}{21\pi^3} \right)^{1/6} \left(\frac{i^3 E'}{\mu h_f^3} \right)^{1/5} t^{2/3} = 0.539 t^{2/3}$$

$$w_{w,0} = \left(\frac{5376}{\pi^3} \right)^{1/6} \left(\frac{\dot{x}^3 \mu}{E' h_f^3} \right)^{1/6} t^{1/3} = 2.360 t^{1/3}$$

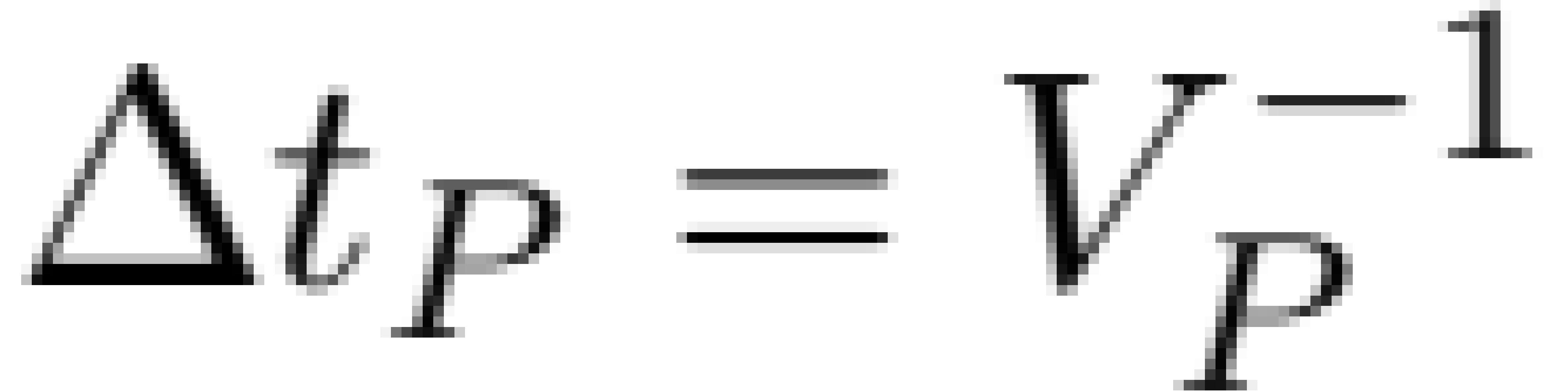
$$p_{net,w} = \left(\frac{21}{16}\right)^{1/3} (E^2 \mu)^{1/3} t^{-1/3} = 1.090 (E^2 \mu)^{1/3} t^{-1/3}$$

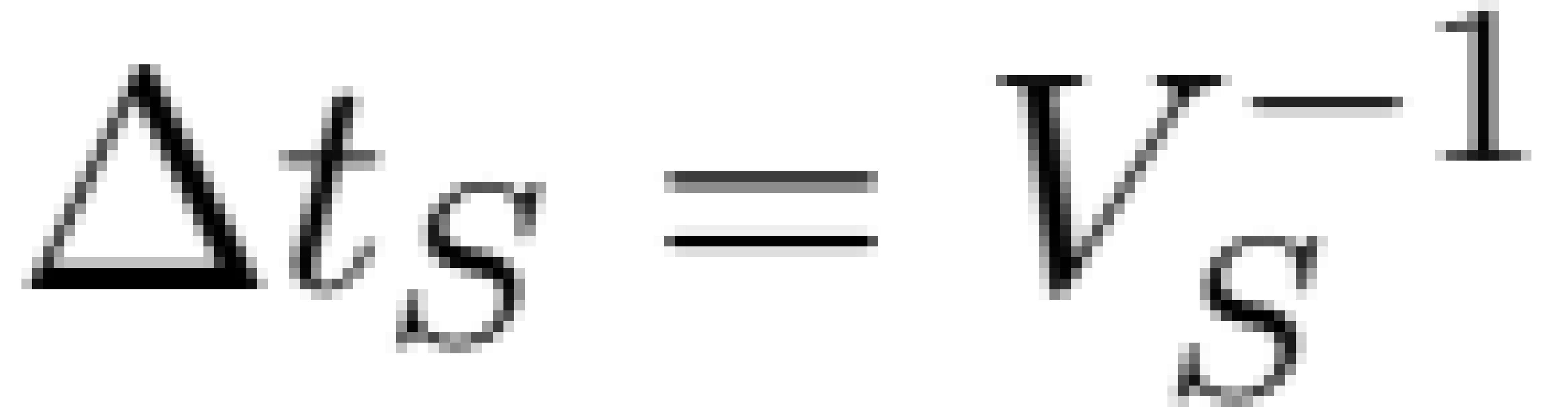










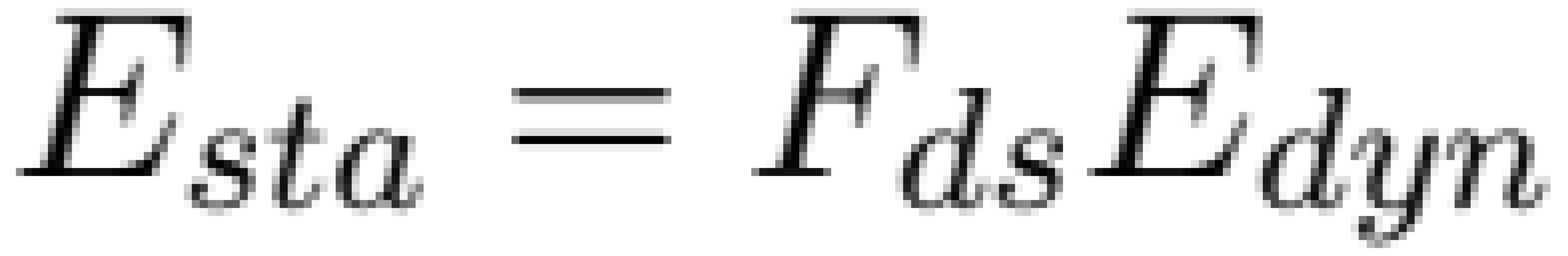


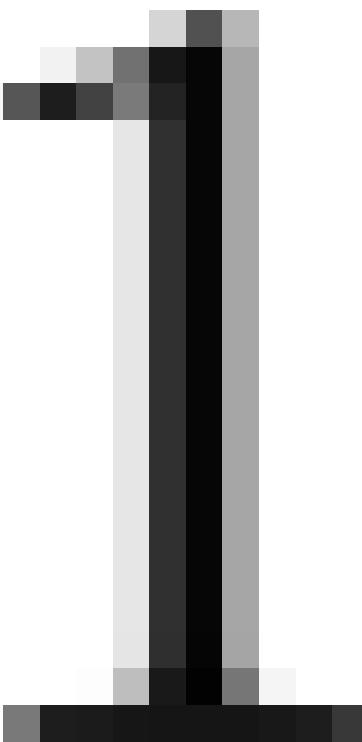
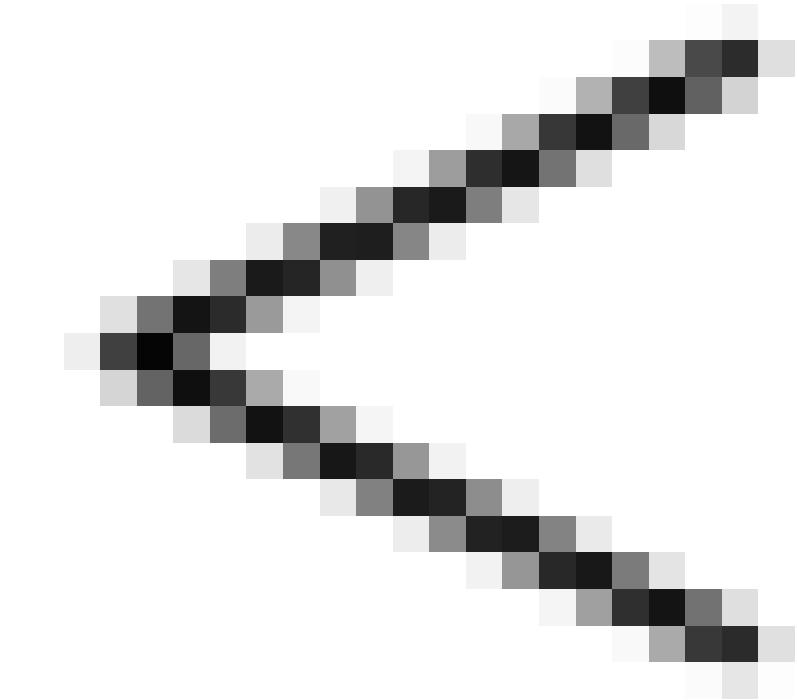
$$E_{\text{edge}} = \rho_{\text{bulk}} V_s^2 \left(\frac{3V_p^2}{V_s^2} - \frac{4V_s^2}{V_p^2} \right)$$

$$v_{dyn} = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)}$$

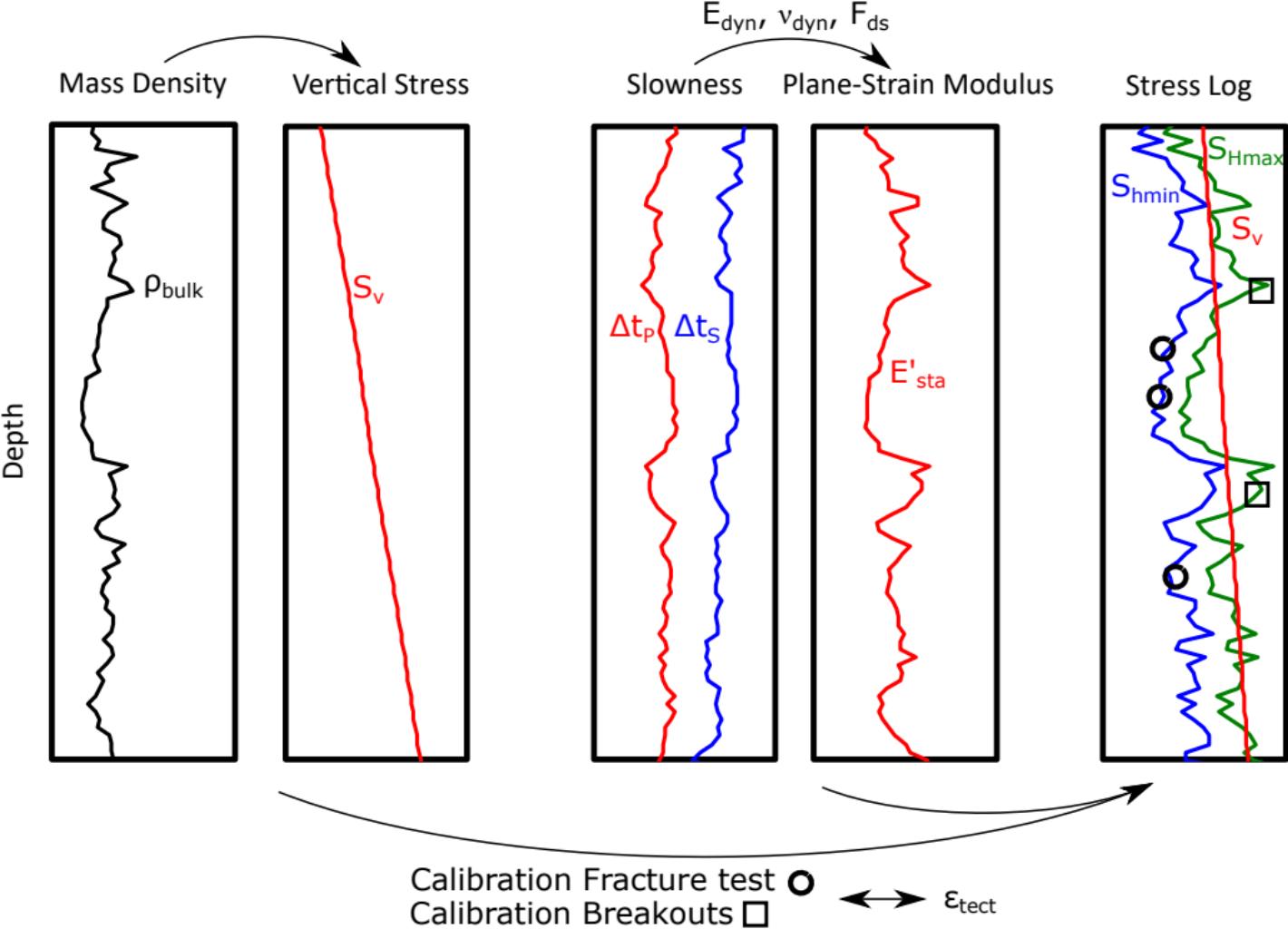


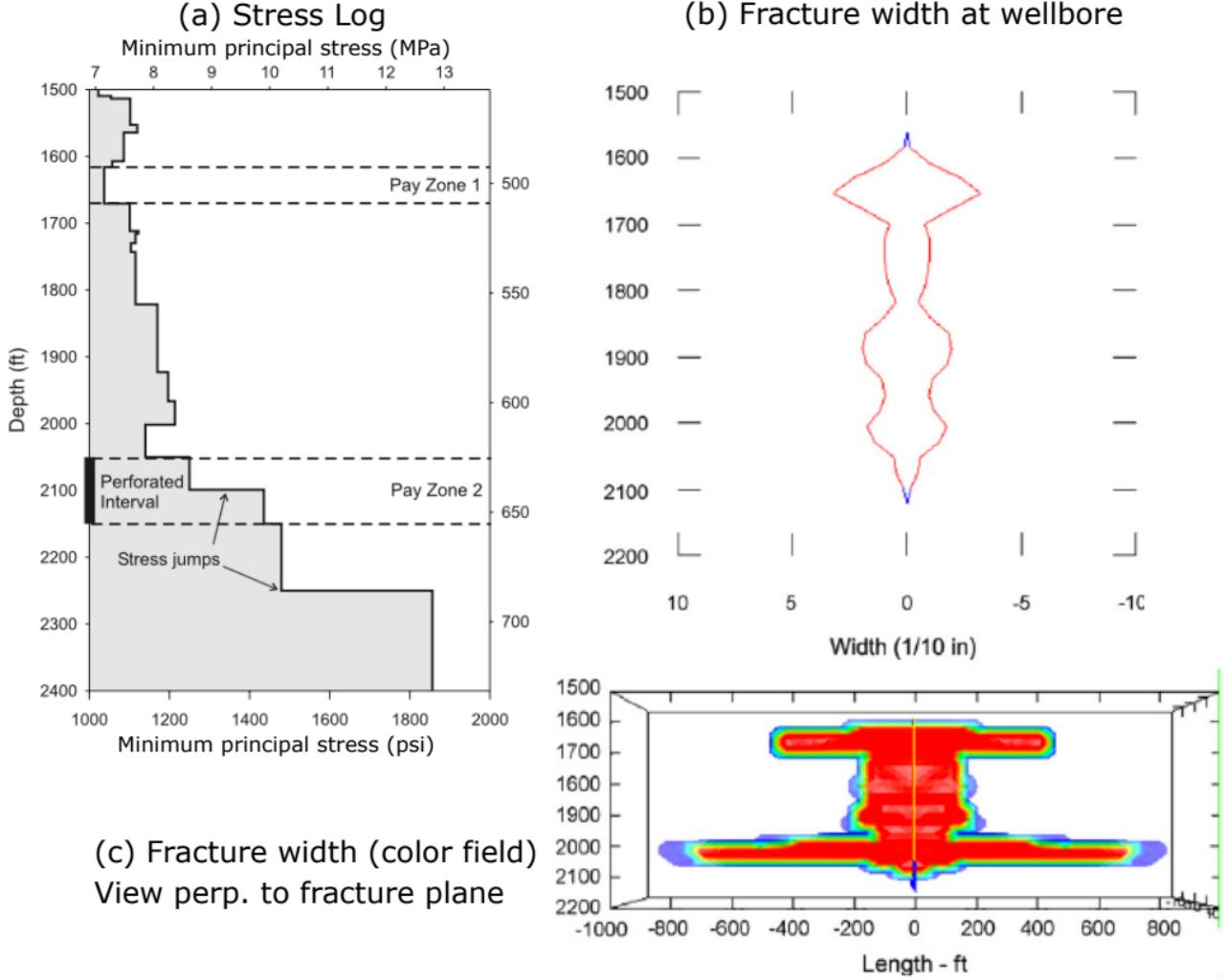


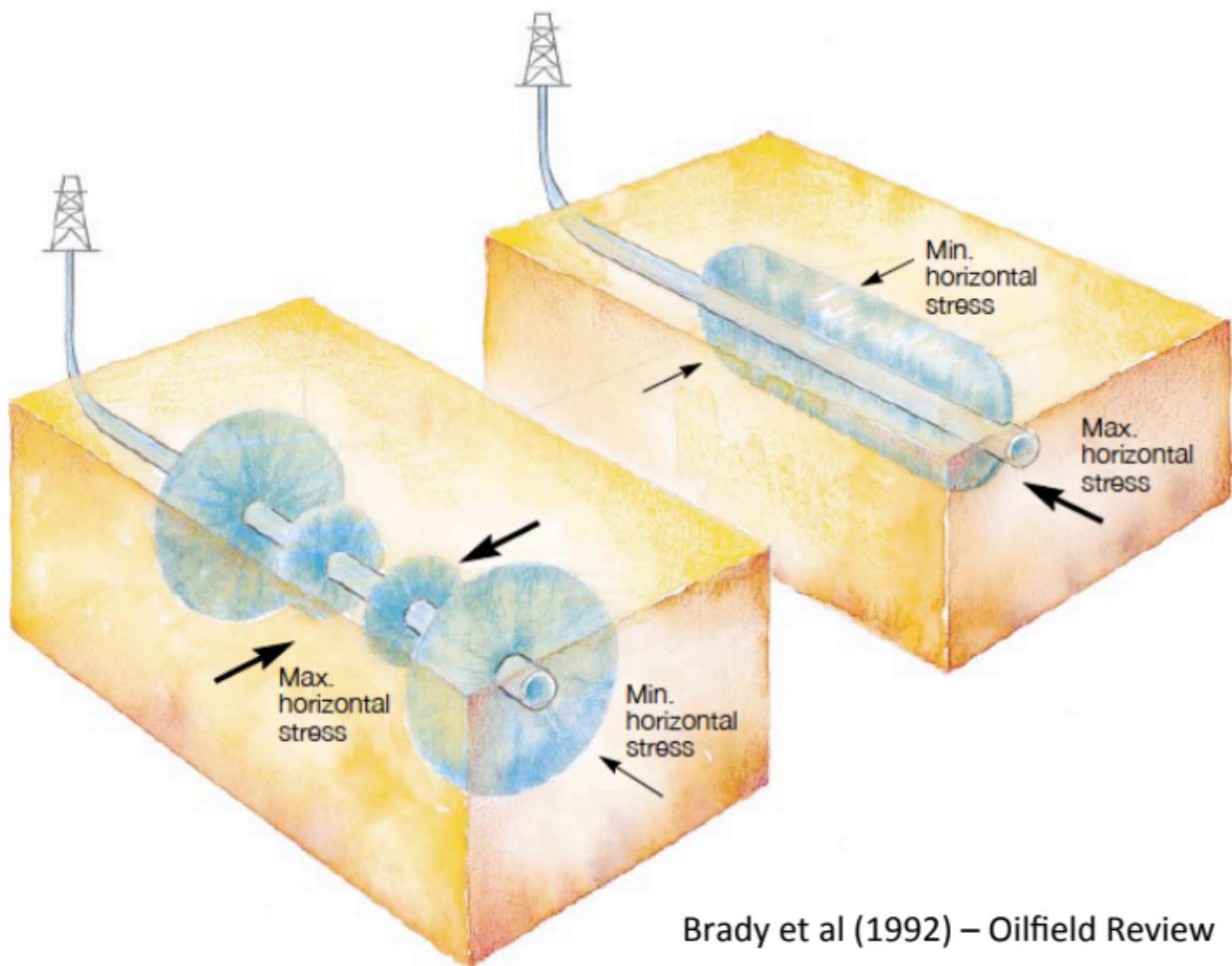






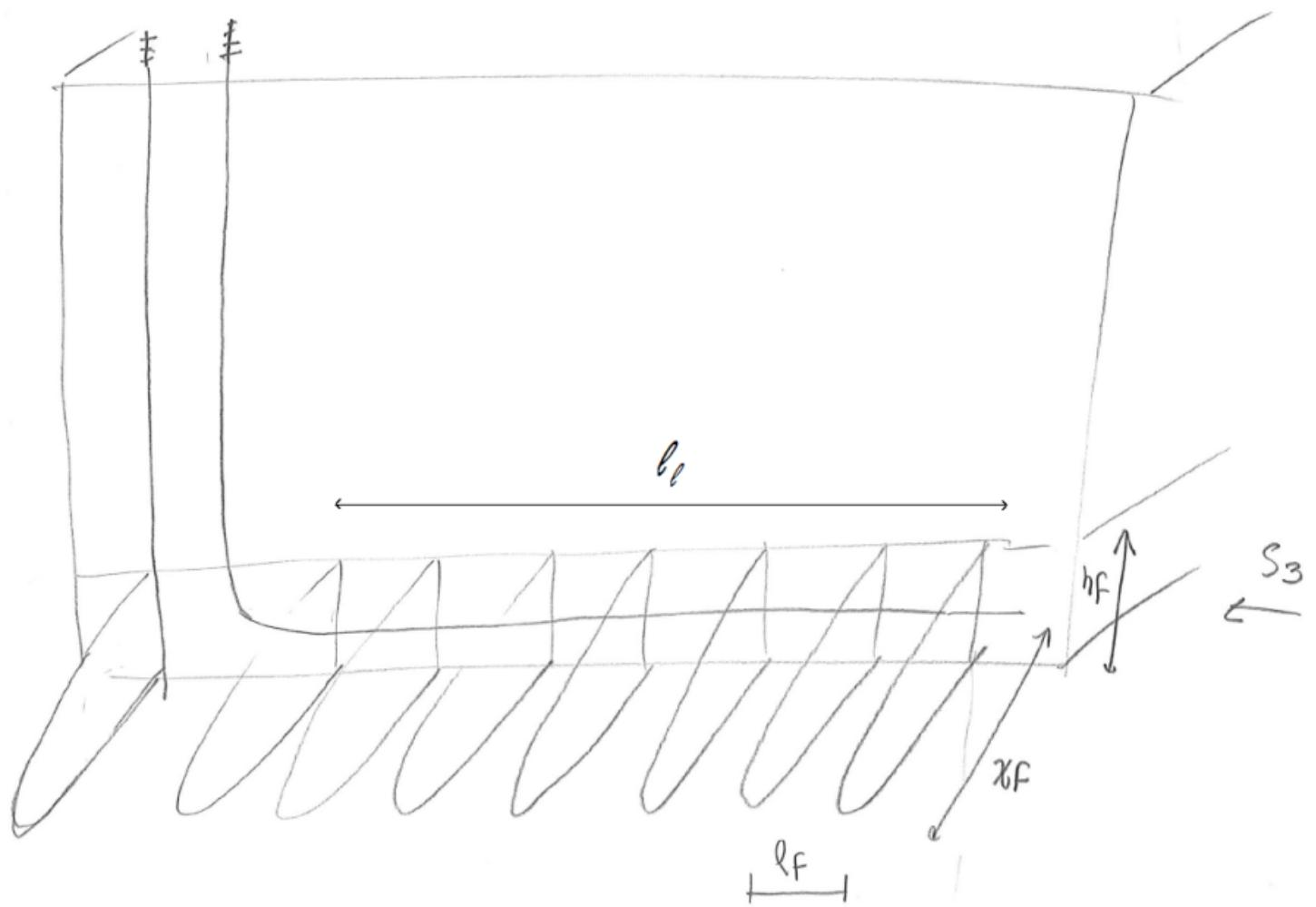






Brady et al (1992) – Oilfield Review







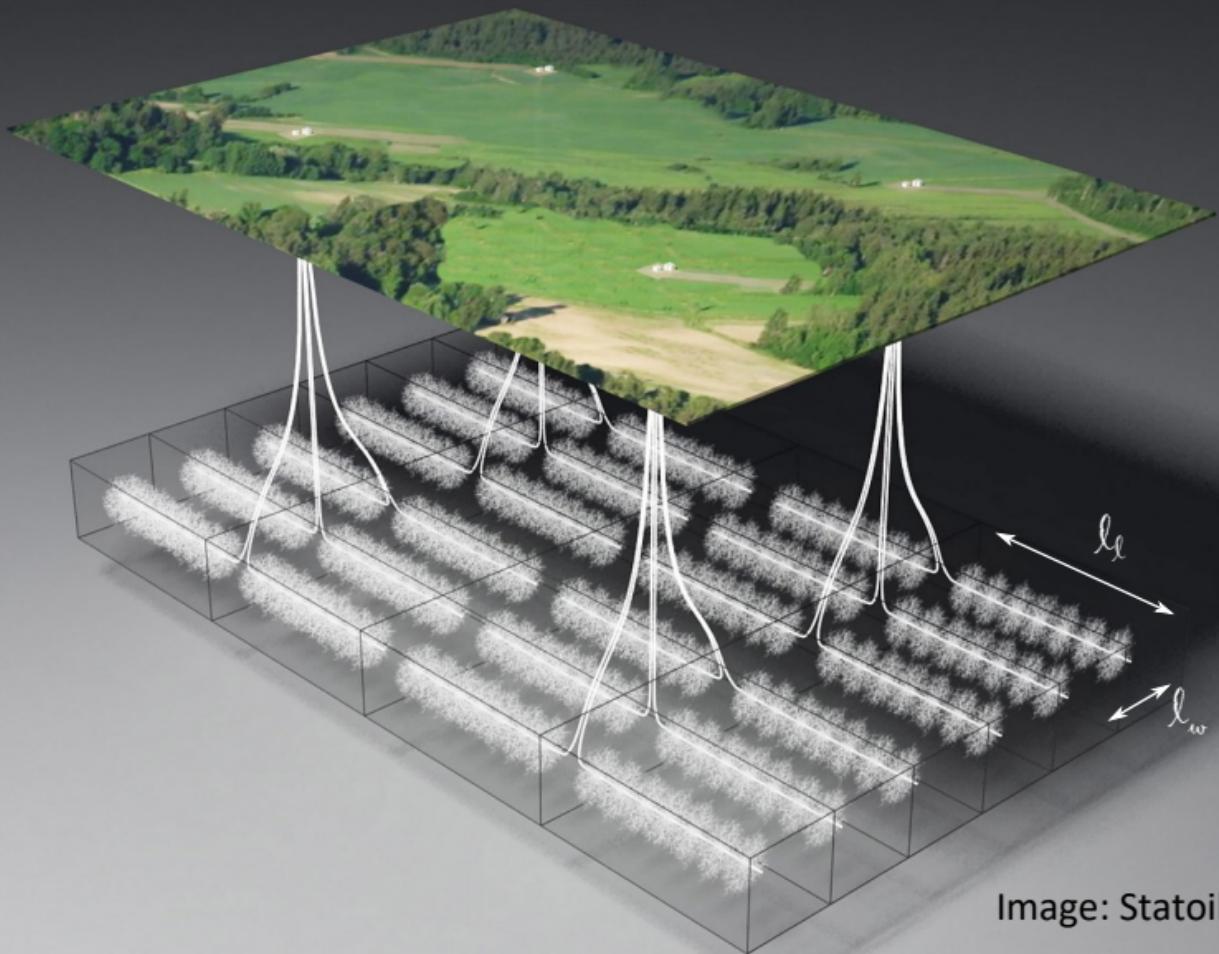
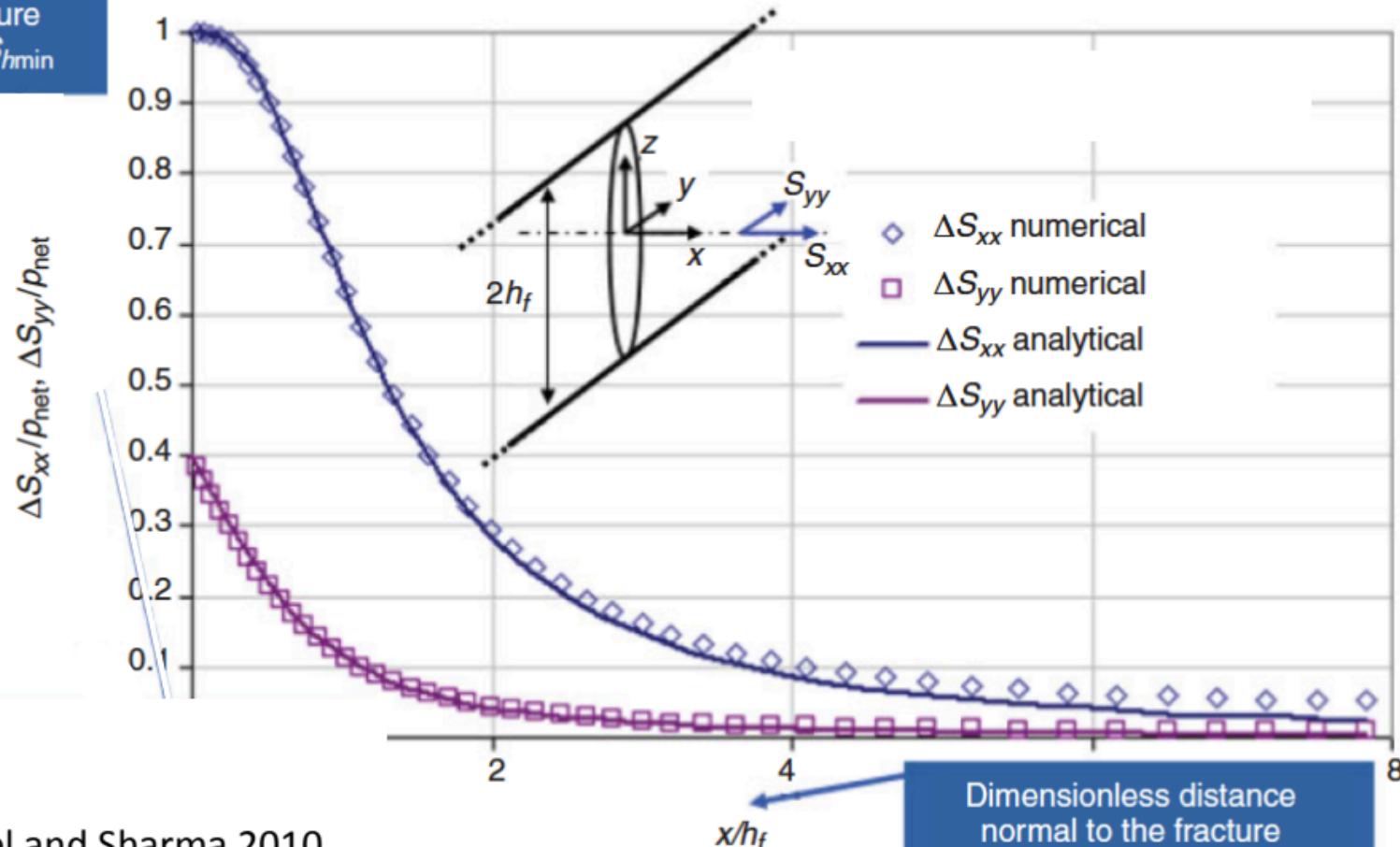


Image: Statoil

Net extension pressure
 $= p_f - S_{h\min}$



TOP VIEW

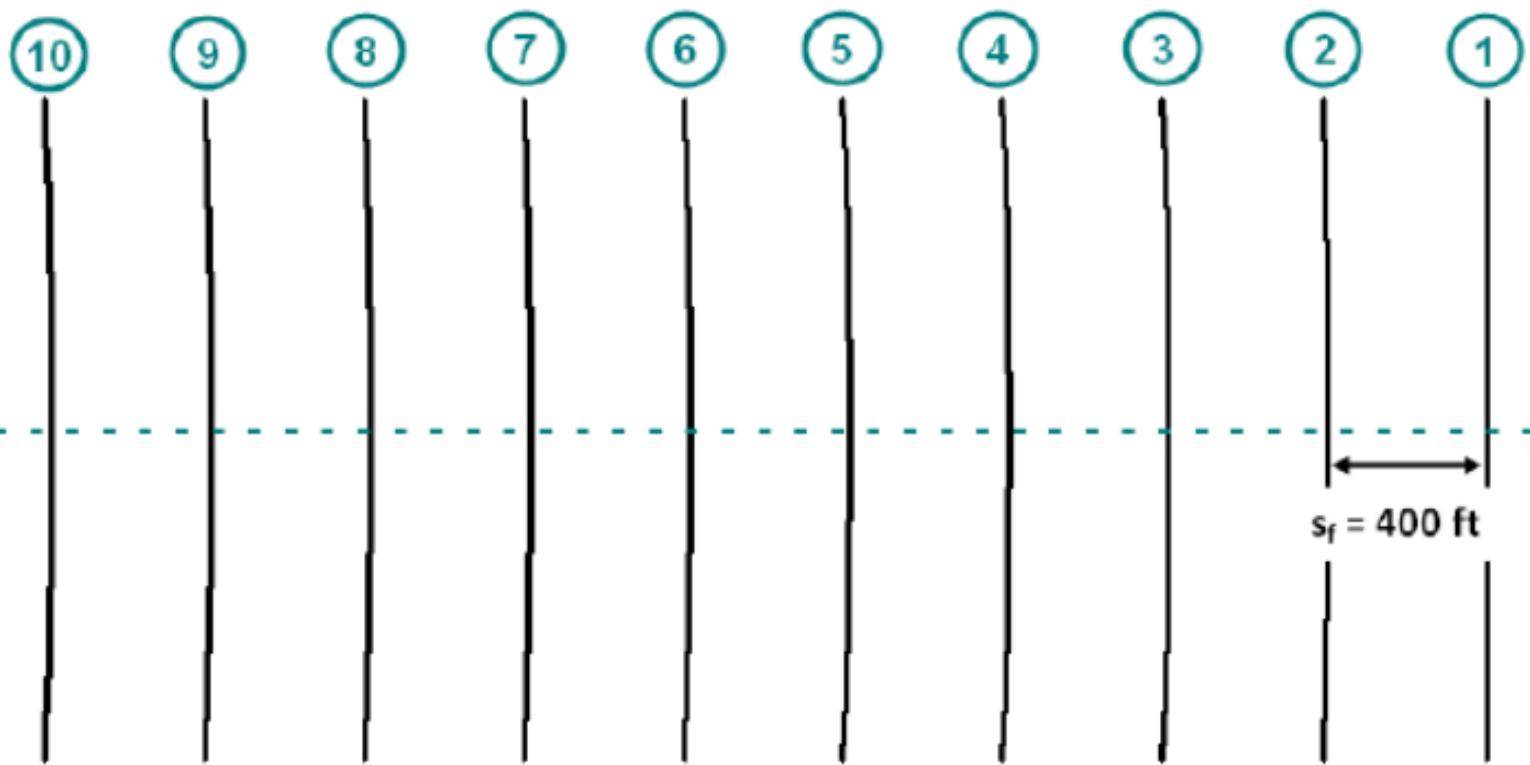


Fig. 7 – Trajectory of multiple consecutive transverse fractures spaced 400 ft apart

TOP VIEW

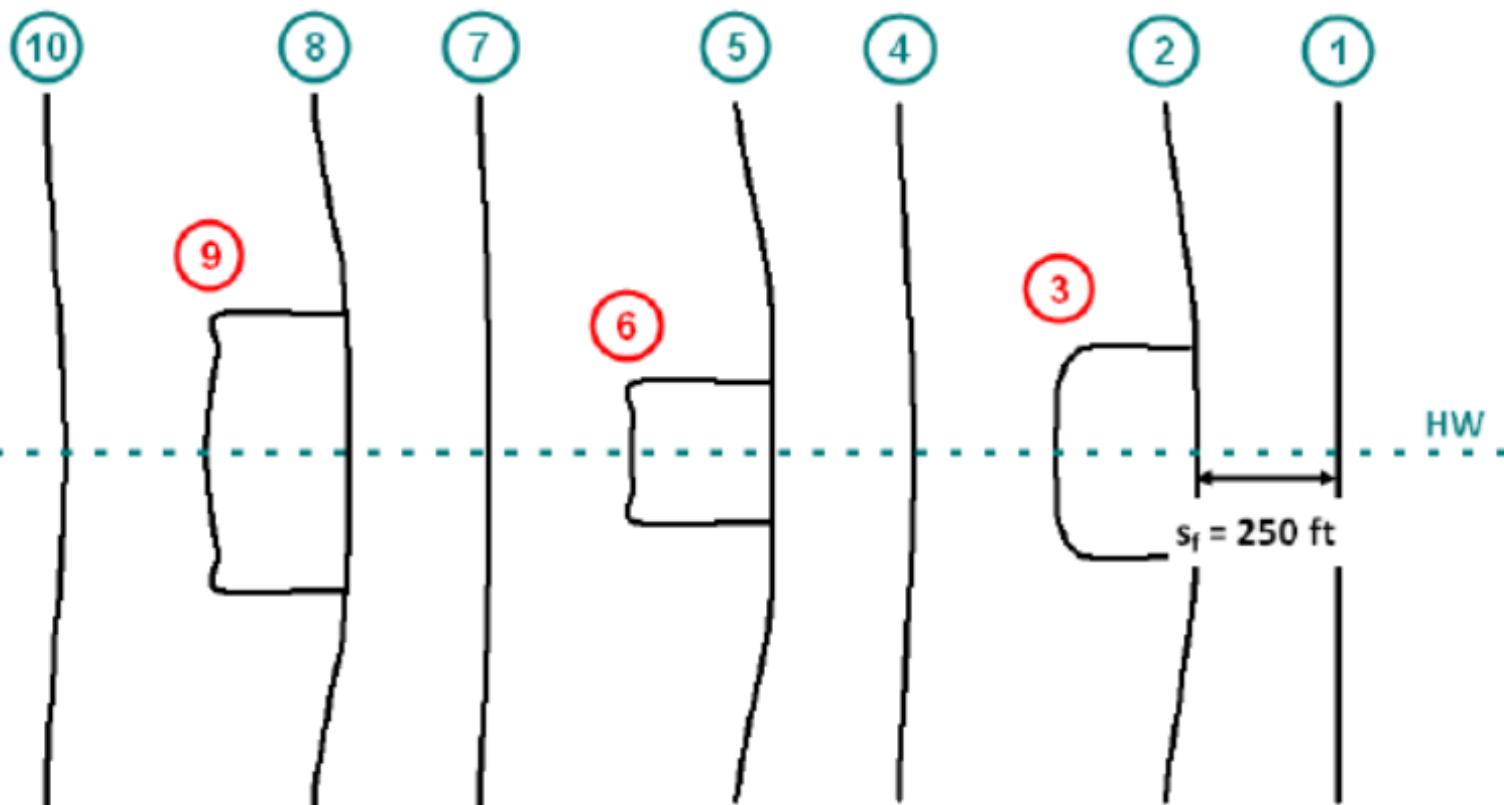
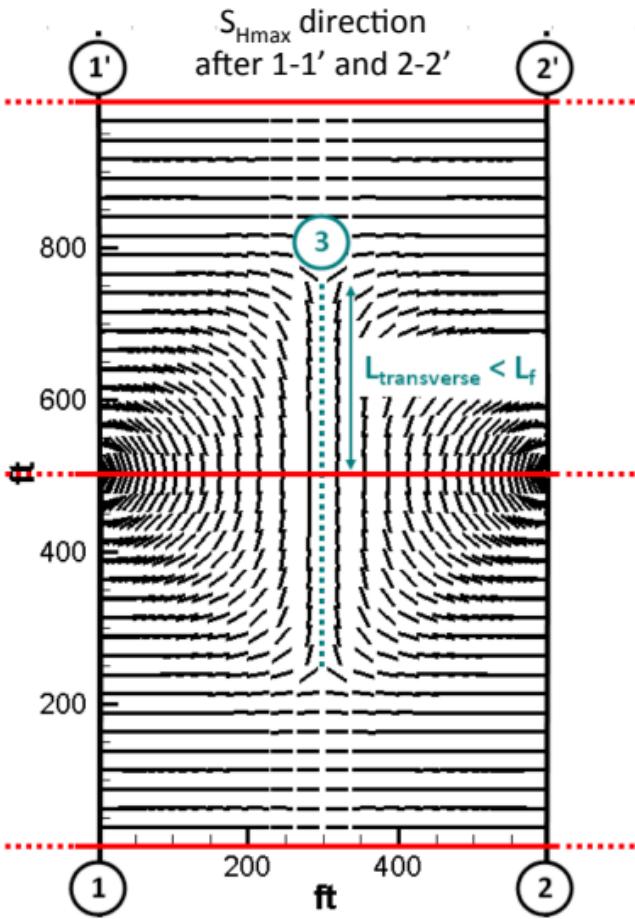
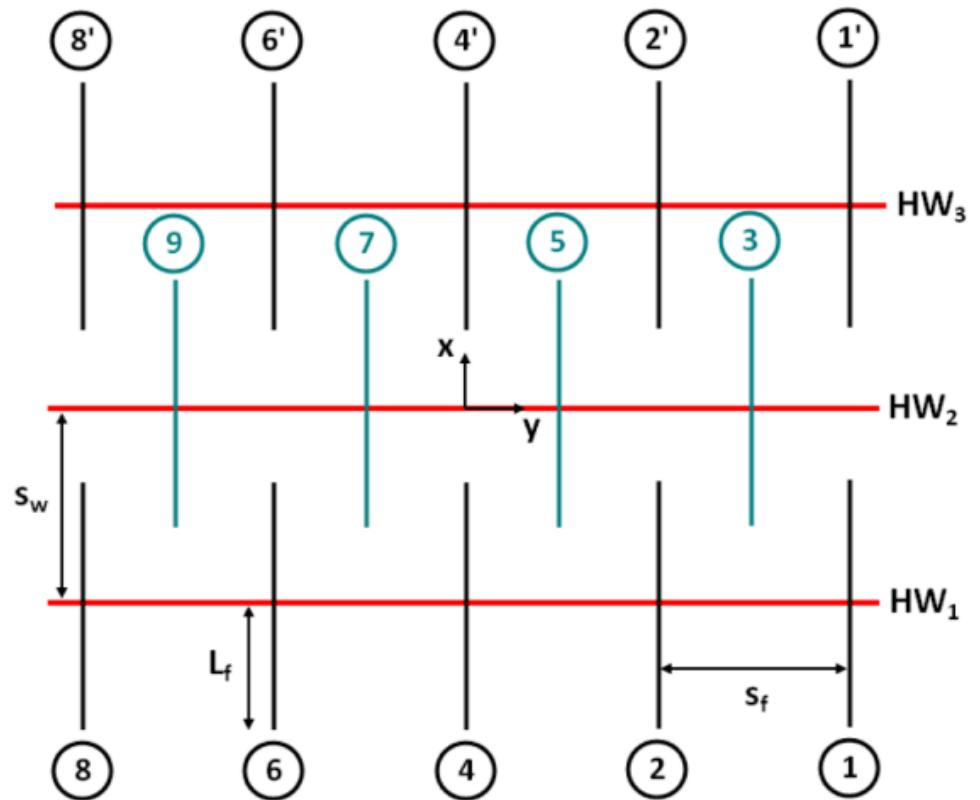
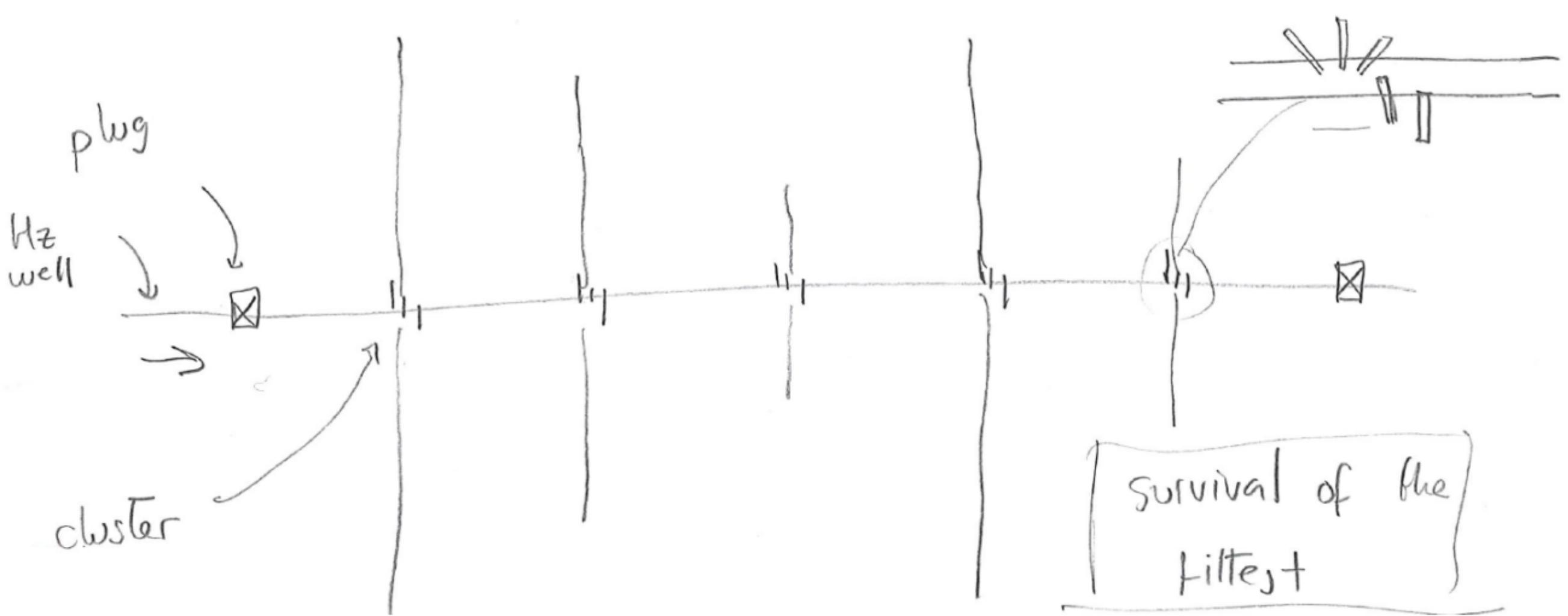
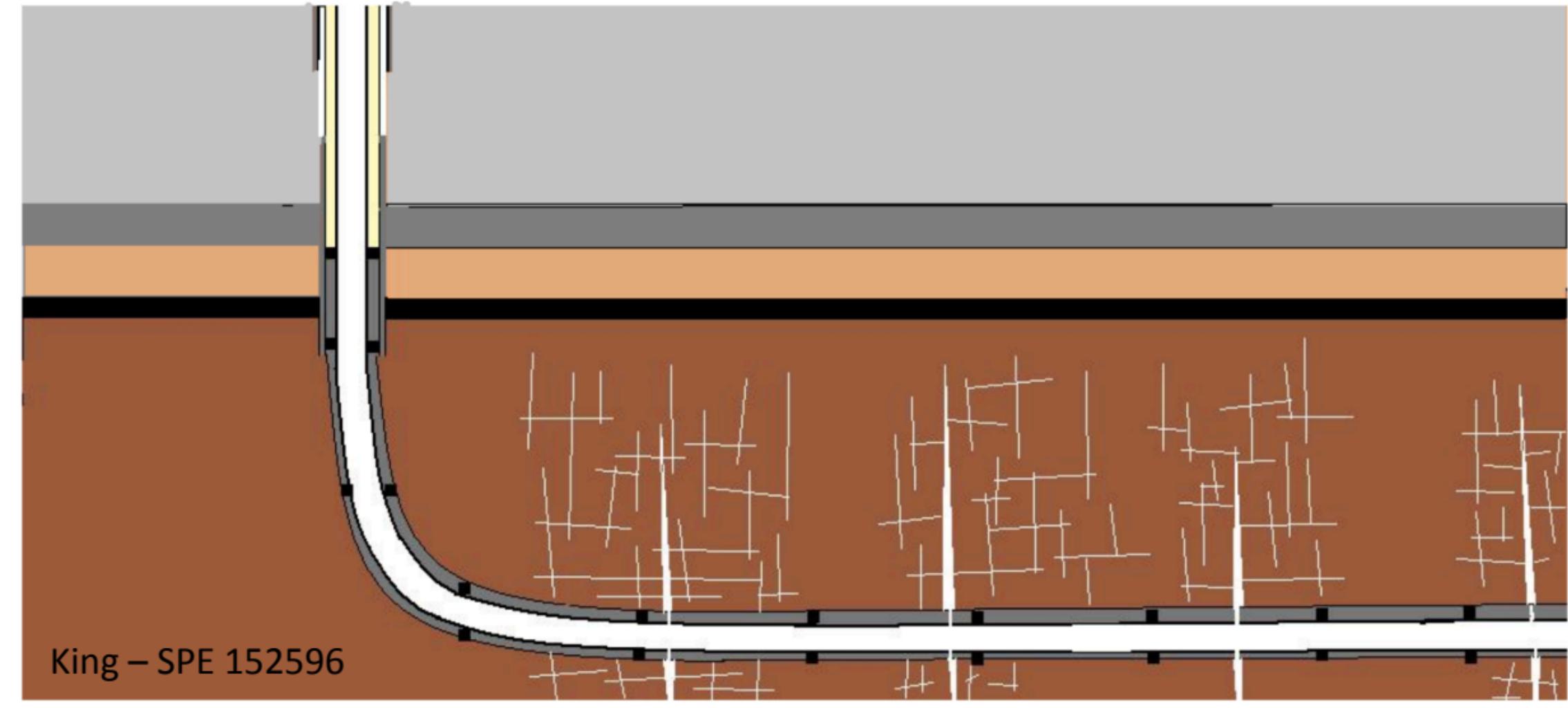


Fig. 10 – Trajectory of multiple consecutive transverse fractures spaced 250 ft apart

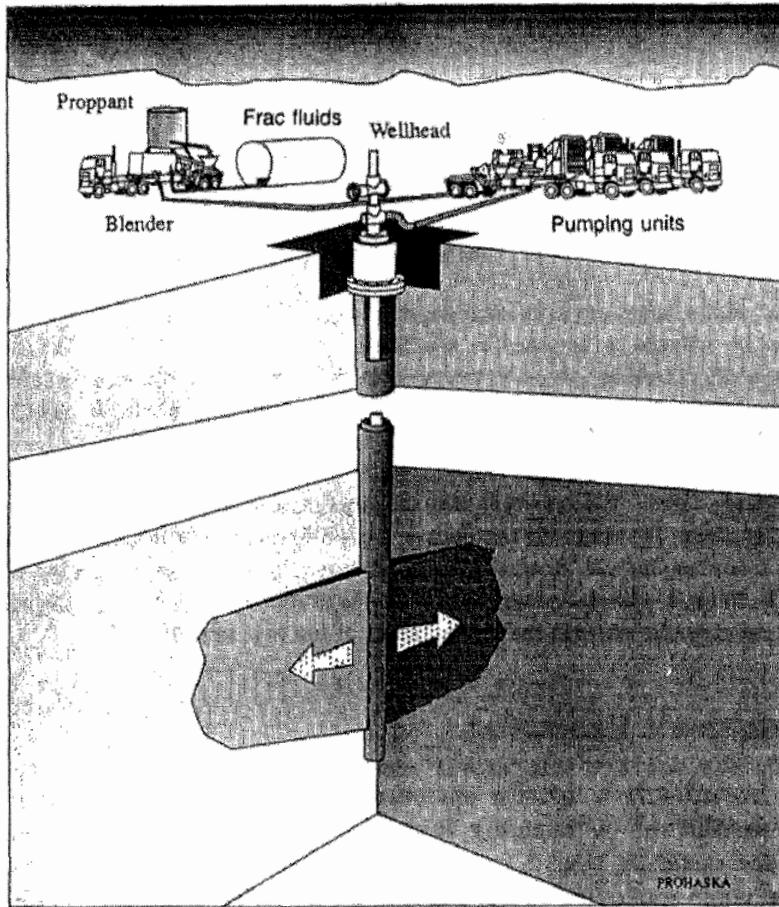




King – SPE 152596



... / Fig

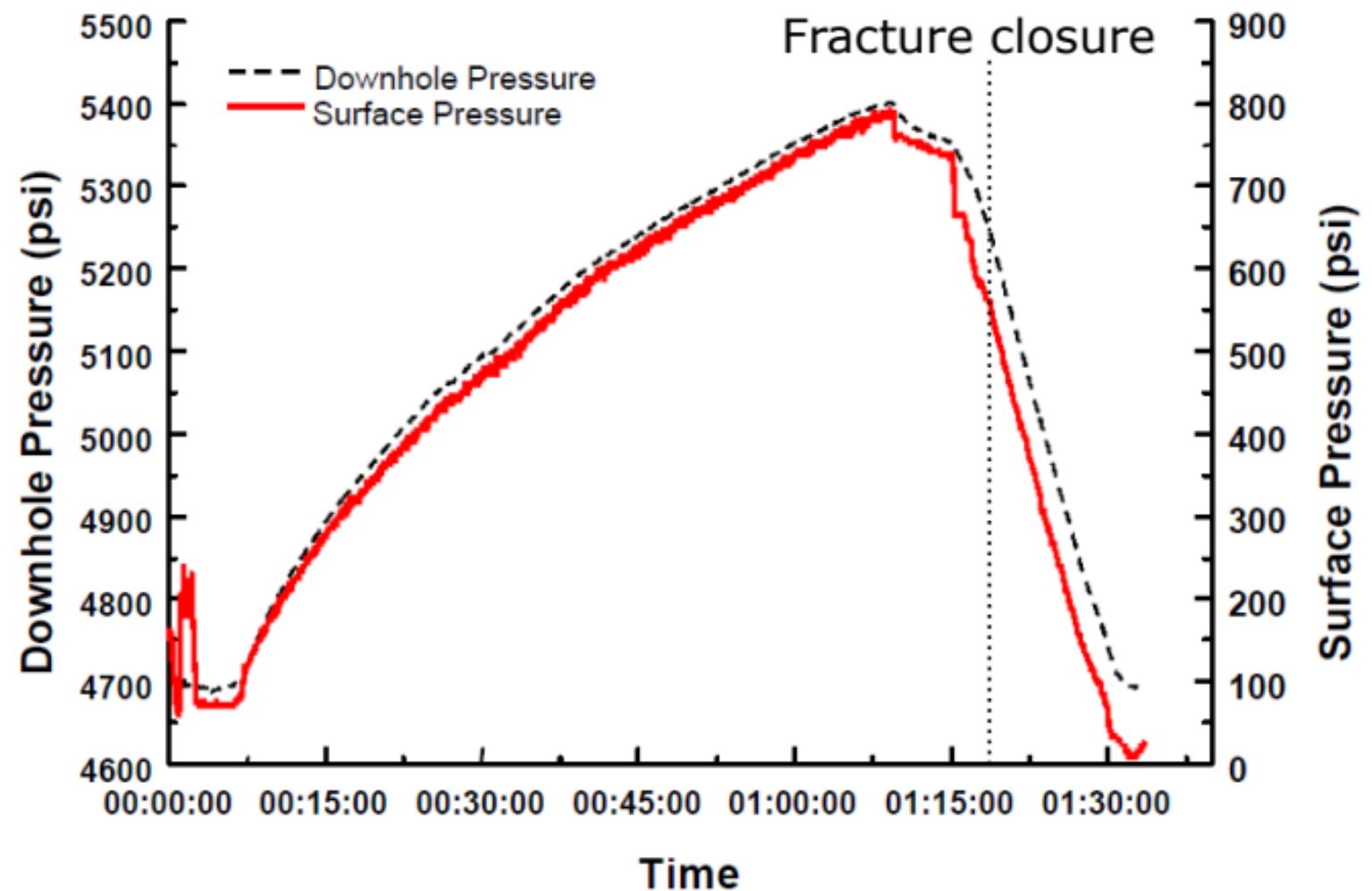
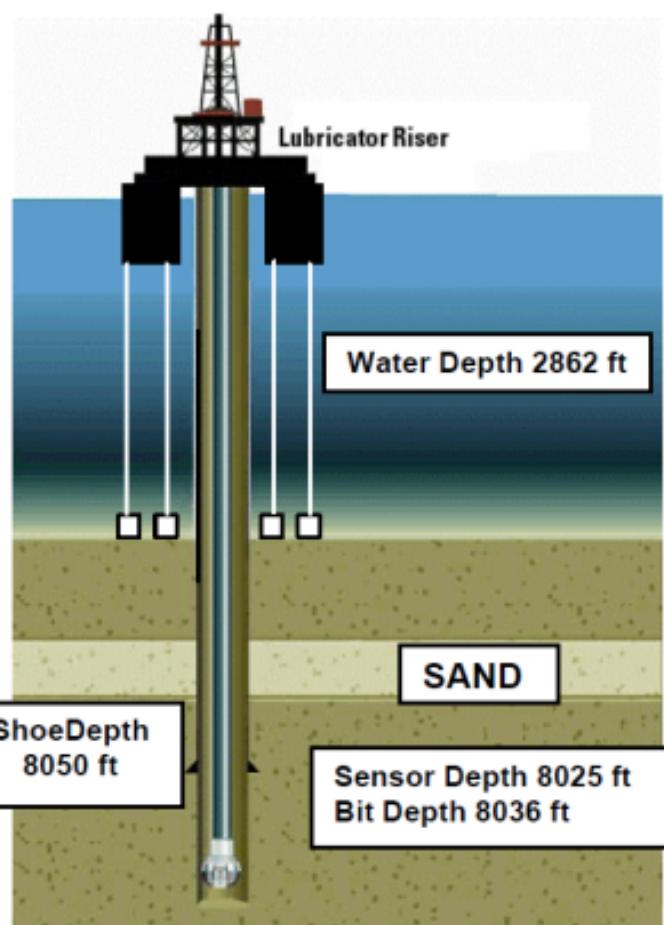


From
Valko and Economides, 1996

Surf Press [Tbg] (psi) Slurry Flow Rate (bpm)
Proppant Conc (ppg)





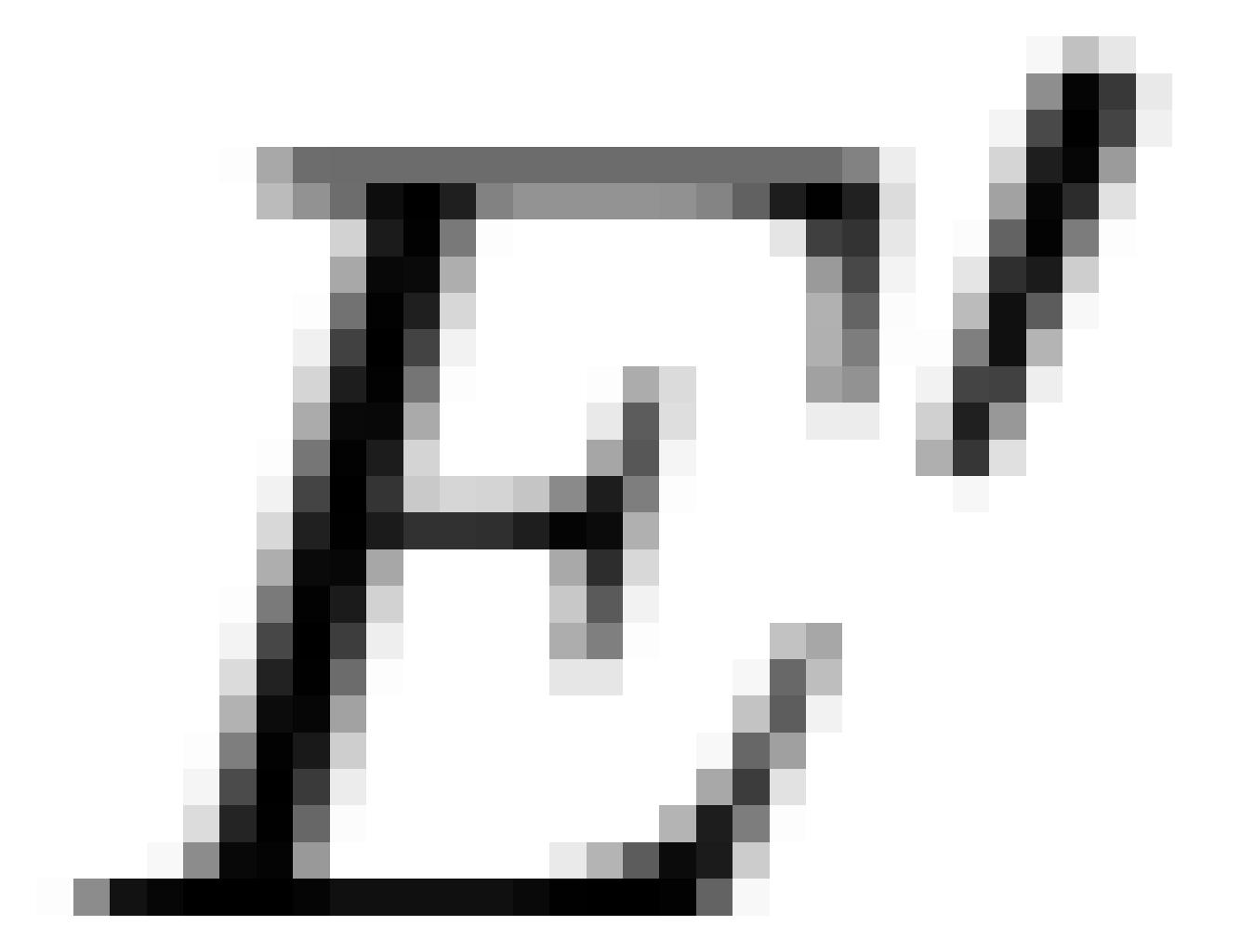


Step #	Test rate (bbl/min)	max. rate)	Test rate (% of								
			Time (min)	0	5	10	15	20	25	30	
1	0.2	5	Pressure (psi)	0	99	105	108	109	110	110	
			Time (min)	0	5	10	15	20	25	30	
2	0.4	10	Pressure (psi)	88	187	204	215	219	220	220	
			Time (min)	0	5	10	15	20	25	30	
3	0.8	20	Pressure (psi)	209	358	424	431	438	439	440	
			Time (min)	0	5	10	15	20	25	30	
4	1.6	40	Pressure (psi)	418	770	869	871	875	878	882	
			Time (min)	0	5	10	15	20	25	30	
5	2.4	60	Pressure (psi)	825	1089	1133	1199	1265	1298	1321	
			Time (min)	0	5	10	15	20	25	30	
6	3.2	80	Pressure (psi)	1210	1375	1459	1507	1529	1535	1540	
			Time (min)	0	5	10	15	20	25	30	
7	4	100	Pressure (psi)	1485	1595	1650	1683	1727	1749	1760	
			Time (min)	0	5	10	15	20	25	30	









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• . / E

• . / E

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• . / E

• . / E