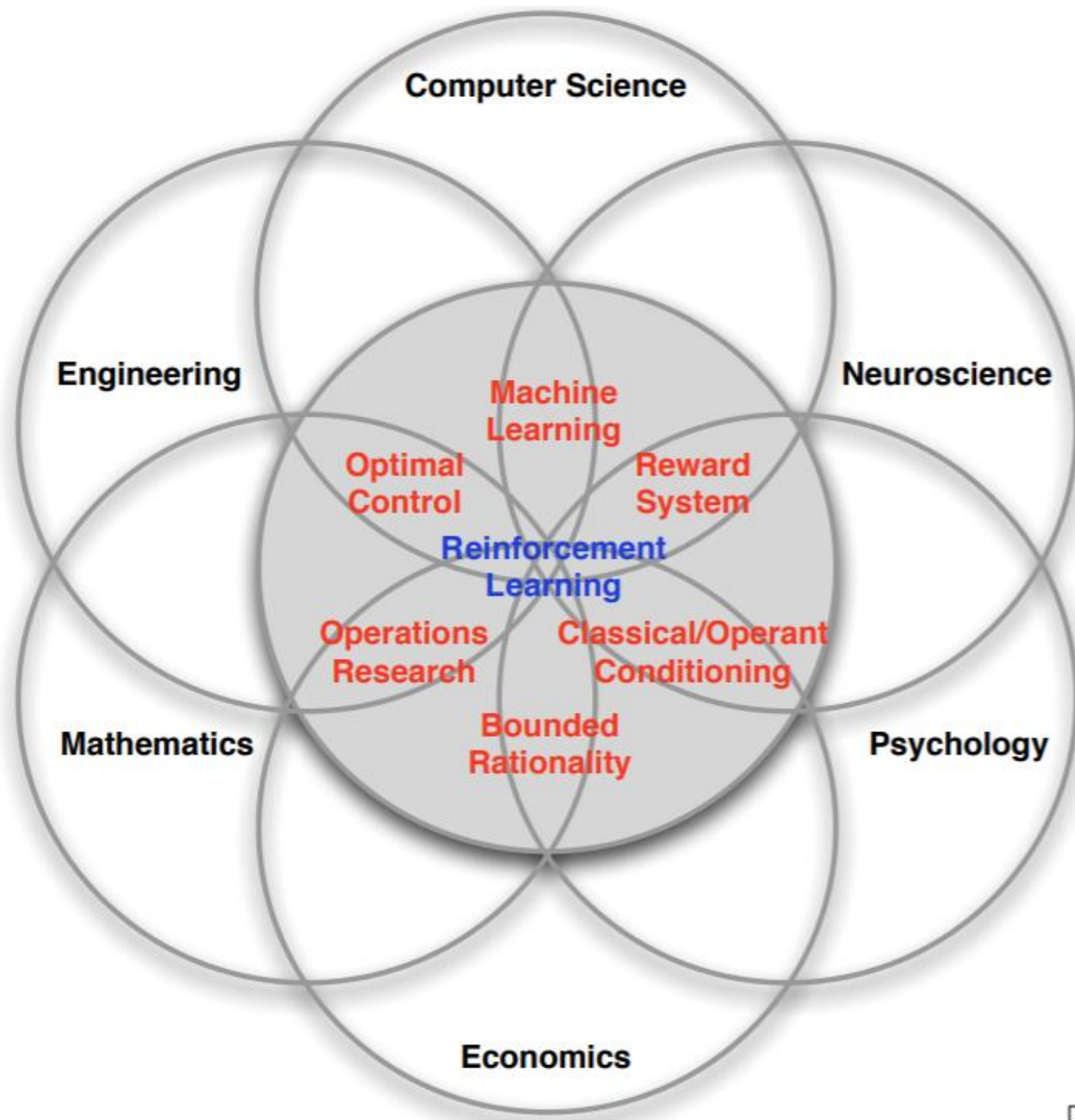


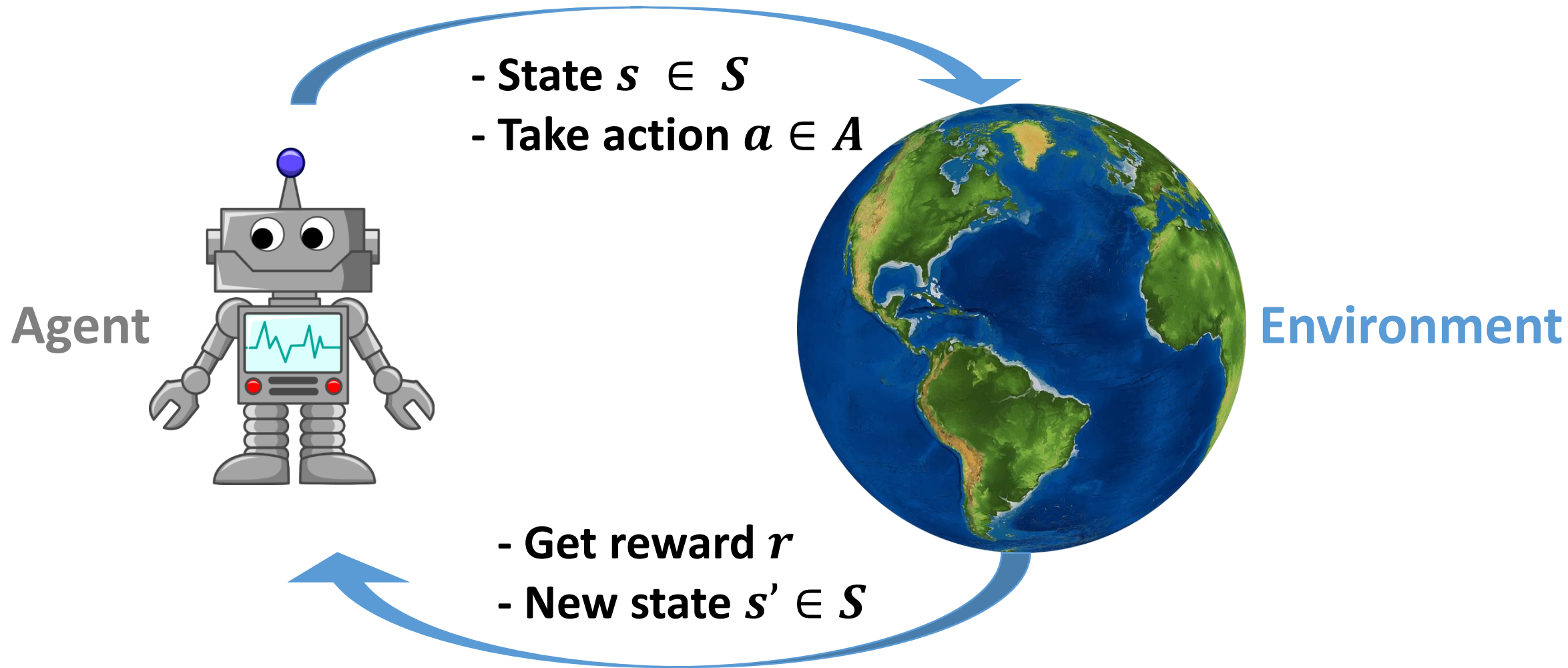
Basics of Reinforcement Learning*

by Alina Vereshchaka

*Slides are based on David Silver's course "Reinforcement Learning" and the book by Sutton, Richard S., and Andrew G. Barto. "Reinforcement Learning: An introduction (2 edition)" (2018)



What is Reinforcement learning?



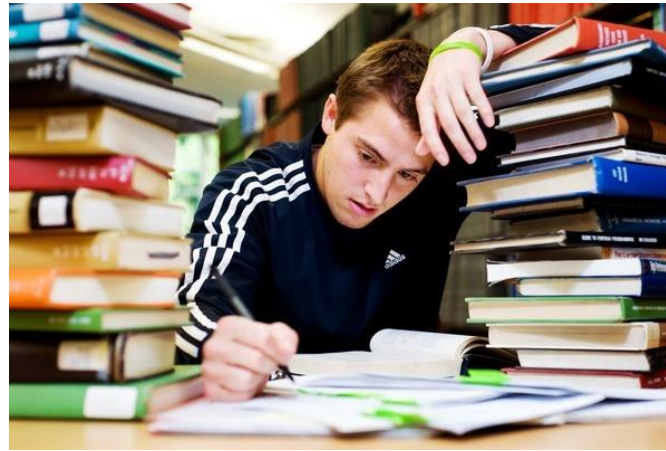
Reinforcement Learning in real world



State: tasty smell, sound

Actions: give a paw

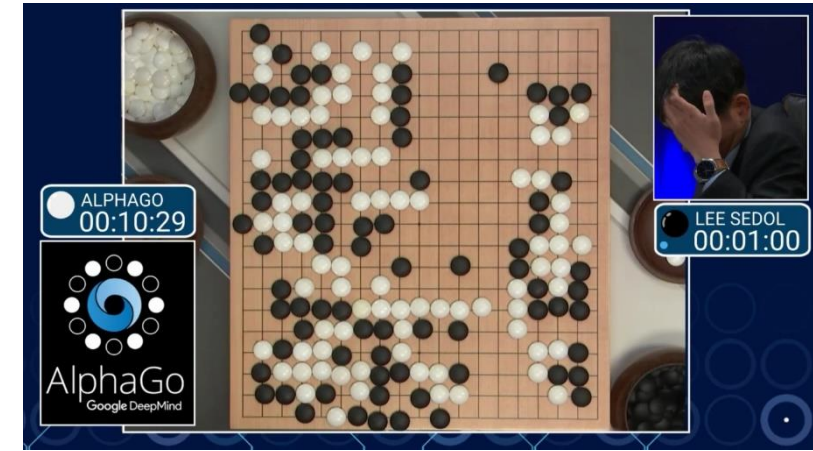
Reward: food



State: want a good grade

Actions: studying

Reward: good grade



State: opponent movement, current board state

Actions: make a move

Reward: points

Markov Chain

Definition

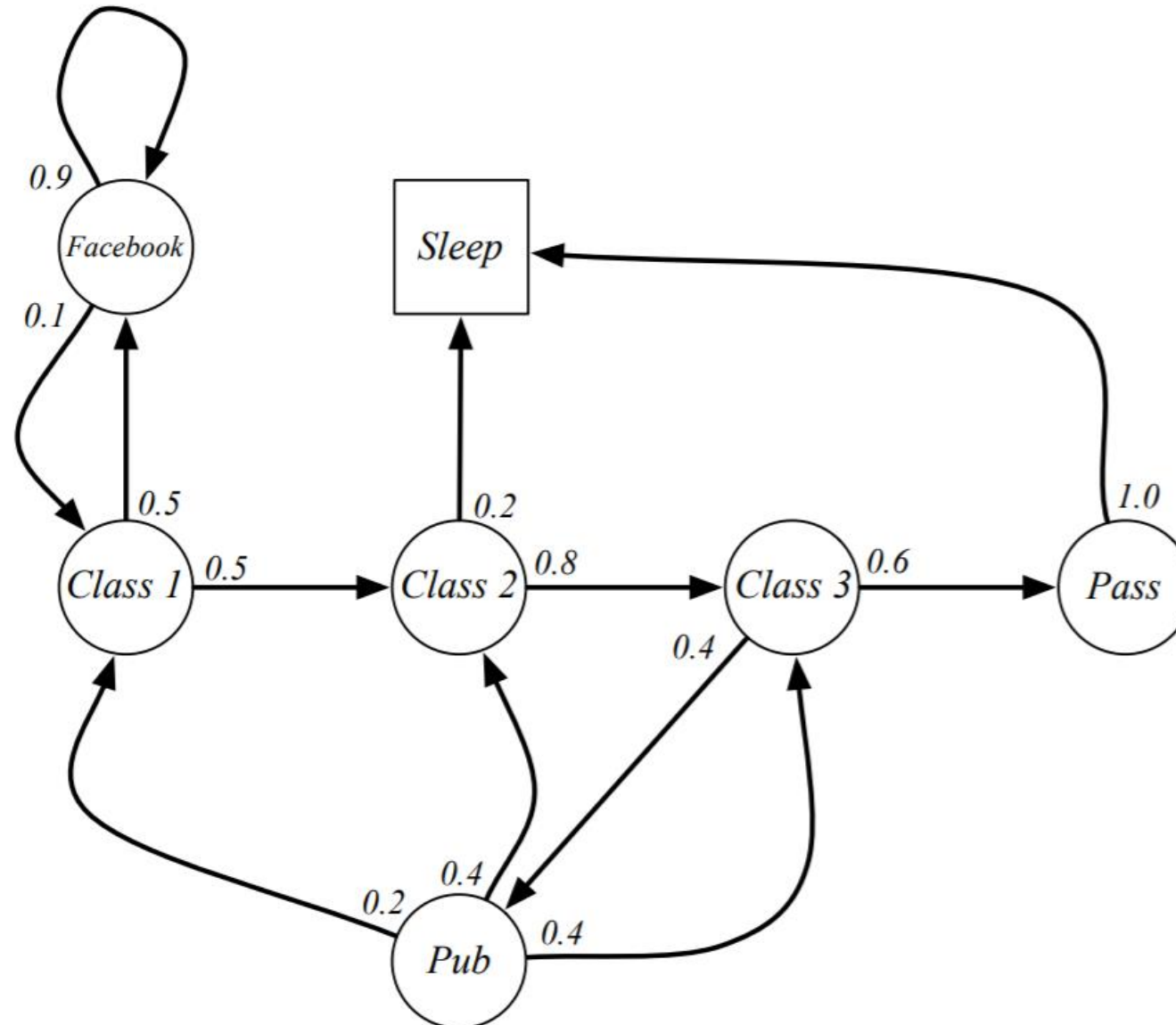
A *Markov Process* (or *Markov Chain*) is a tuple $\langle \mathcal{S}, \mathcal{P} \rangle$

- \mathcal{S} is a (finite) set of states
- \mathcal{P} is a state transition probability matrix,
 $\mathcal{P}_{ss'} = \mathbb{P}[S_{t+1} = s' \mid S_t = s]$

Markovian property: $\mathbb{P}[S_{t+1} \mid S_t] = \mathbb{P}[S_{t+1} \mid S_1, \dots, S_t]$

“The future is independent of the past given the present”

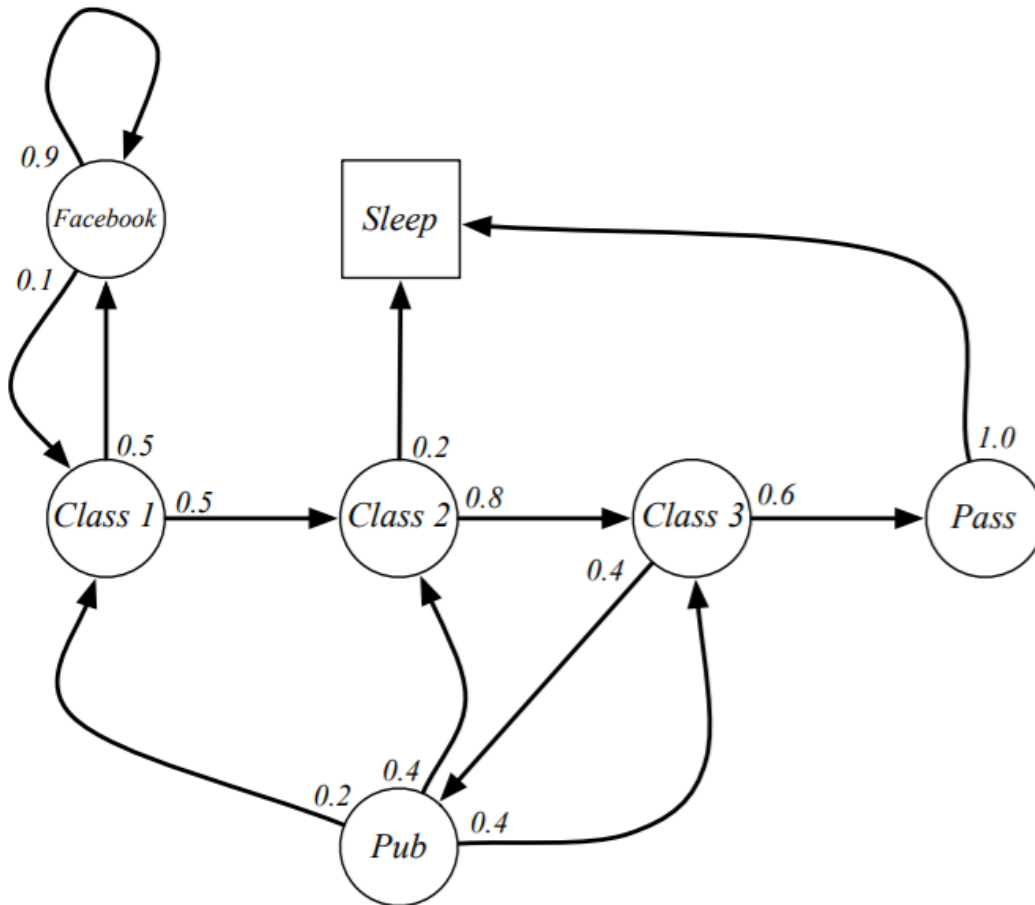
Example: Student Markov Chain



Example: Student Markov Chain Episodes

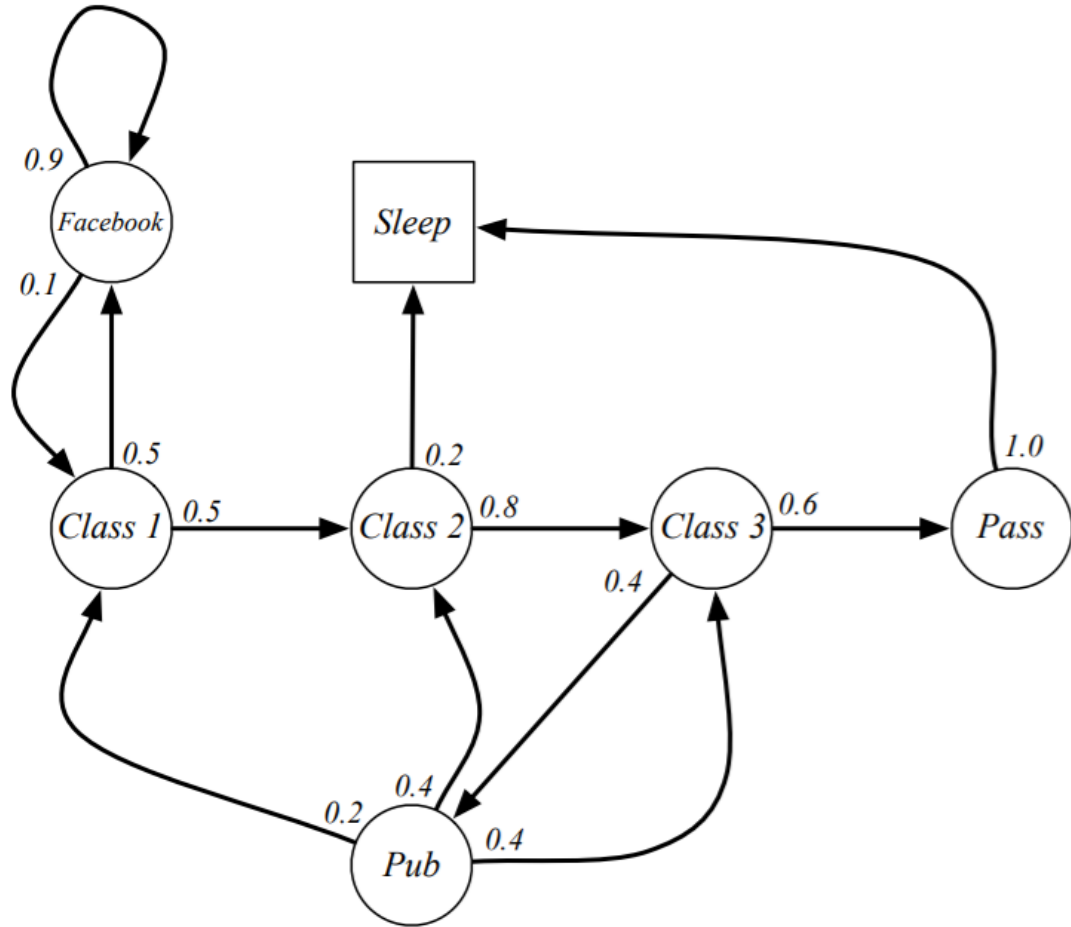
Sample **episodes** for Student Markov Chain starting from $S_1 = C1$

S_1, S_2, \dots, S_T



- C1 C2 C3 Pass Sleep
- C1 FB FB C1 C2 Sleep
- C1 C2 C3 Pub C2 C3 Pass Sleep
- C1 FB FB C1 C2 C3 Pub C1 FB FB FB C1 C2 C3 Pub C2 Sleep

Example: Transition Matrix



$$\mathcal{P} = \begin{matrix} & \begin{matrix} C1 & C2 & C3 & Pass & Pub & FB & Sleep \end{matrix} \\ \begin{matrix} C1 \\ C2 \\ C3 \\ Pass \\ Pub \\ FB \\ Sleep \end{matrix} & \begin{bmatrix} & & & & & 0.5 & \\ & 0.5 & & & & & \\ & & 0.8 & & & & 0.2 \\ & & & 0.6 & 0.4 & & \\ 0.2 & 0.4 & 0.4 & & & & 1.0 \\ 0.1 & & & & & 0.9 & \\ & & & & & & 1 \end{bmatrix} \end{matrix}$$

Markov Reward Process (MRP)

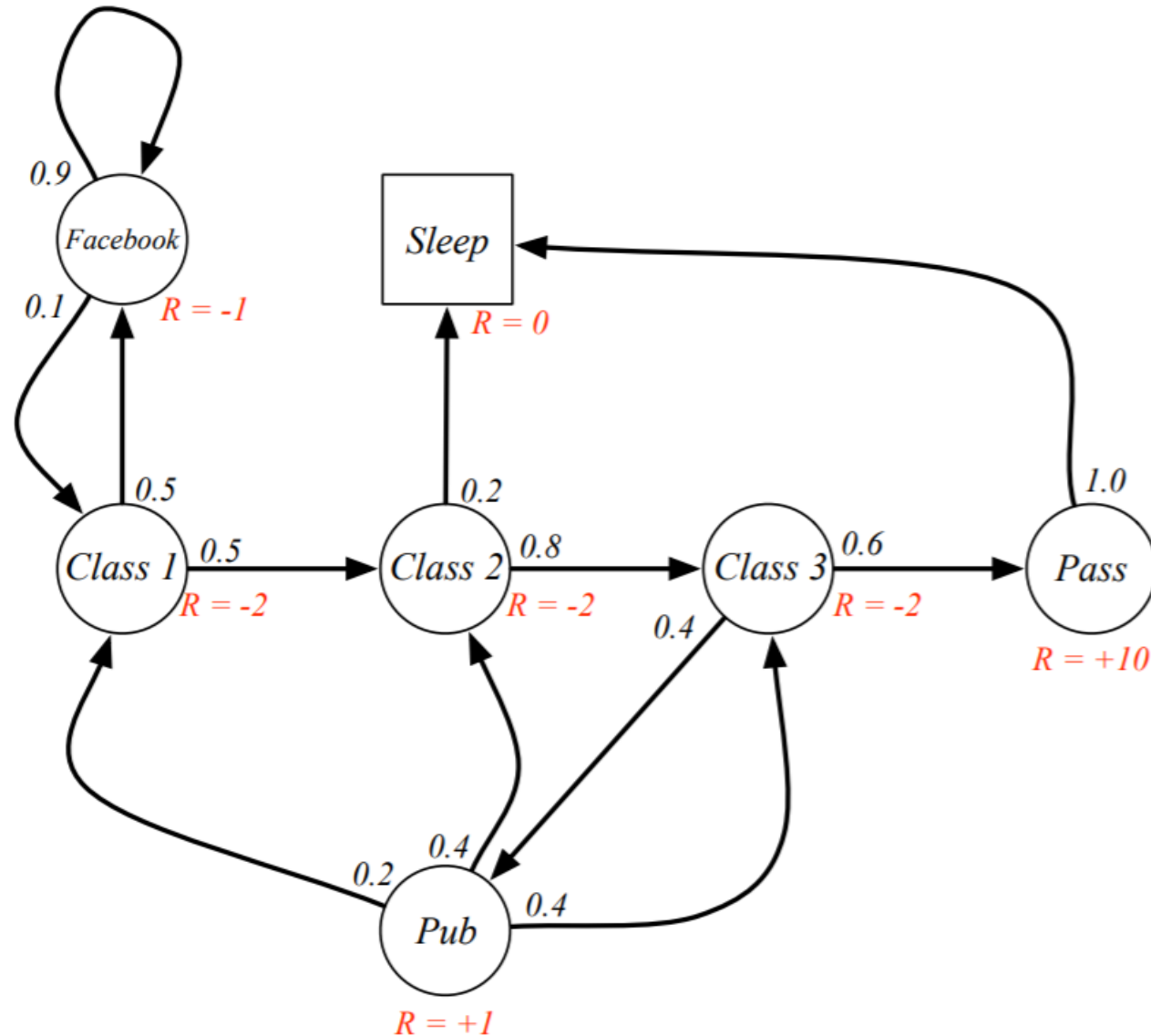
A Markov reward process is a Markov chain with values.

Definition

A *Markov Reward Process* is a tuple $\langle \mathcal{S}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- \mathcal{S} is a finite set of states
- \mathcal{P} is a state transition probability matrix,
 $\mathcal{P}_{ss'} = \mathbb{P}[S_{t+1} = s' \mid S_t = s]$
- \mathcal{R} is a reward function, $\mathcal{R}_s = \mathbb{E}[R_{t+1} \mid S_t = s]$
- γ is a discount factor, $\gamma \in [0, 1]$

Example: Student MRP



Discounting factor γ

The **discounting factor** ($\gamma \in [0, 1)$) penalize the rewards in the future.
Reward at time k worth only γ^{k-1}

Motivation:

- The future rewards may have higher uncertainty (stock market)
- The future rewards do not provide immediate benefits (As human beings, we might prefer to have fun today rather than 5 years later ;)
- Discounting provides mathematical convenience (we don't need to track future steps infinitely to compute return)
- It is sometimes possible to use undiscounted Markov reward processes (i.e. $\gamma = 1$), e.g. if all sequences terminate.

Return (G_t)

Definition

The *return* G_t is the total discounted reward from time-step t .

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- The *discount* $\gamma \in [0, 1]$ is the present value of future rewards
- The value of receiving reward R after $k + 1$ time-steps is $\gamma^k R$.
- This values immediate reward above delayed reward.
 - γ close to 0 leads to "myopic" evaluation
 - γ close to 1 leads to "far-sighted" evaluation

Example: Returns for Student MRP

Sample **returns** for Student MRP:

Starting from $S_1 = \text{C1}$ with $\gamma = \frac{1}{2}$

$$G_1 = R_2 + \gamma R_3 + \dots + \gamma^{T-2} R_T$$

C1 C2 C3 Pass Sleep

$$v_1 = -2 - 2 * \frac{1}{2} - 2 * \frac{1}{4} + 10 * \frac{1}{8} = -2.25$$

C1 FB FB C1 C2 Sleep

$$v_1 = -2 - 1 * \frac{1}{2} - 1 * \frac{1}{4} - 2 * \frac{1}{8} - 2 * \frac{1}{16} = -3.125$$

C1 C2 C3 Pub C2 C3 Pass Sleep

$$v_1 = -2 - 2 * \frac{1}{2} - 2 * \frac{1}{4} + 1 * \frac{1}{8} - 2 * \frac{1}{16} \dots = -3.41$$

C1 FB FB C1 C2 C3 Pub C1 ...

$$v_1 = -2 - 1 * \frac{1}{2} - 1 * \frac{1}{4} - 2 * \frac{1}{8} - 2 * \frac{1}{16} \dots = -3.20$$

FB FB FB C1 C2 C3 Pub C2 Sleep

Value Function $v(s)$

The value function $v(s)$ gives the long-term value of state s

Definition

The *state value function* $v(s)$ of an MRP is the expected return starting from state s

$$v(s) = \mathbb{E} [G_t \mid S_t = s]$$

Markov Decision Process

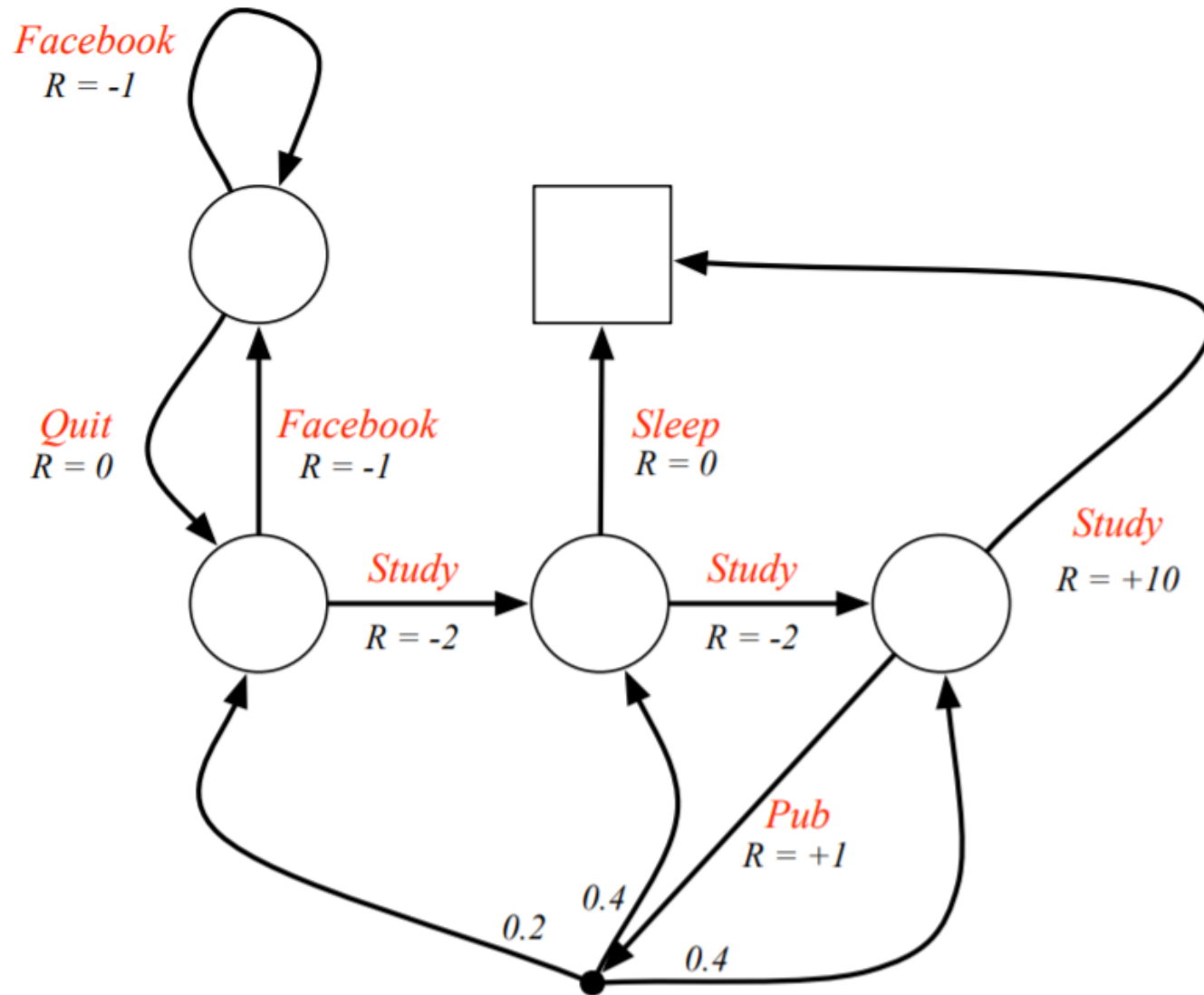
A Markov decision process (MDP) is a Markov reward process with decisions. It is an *environment* in which all states are Markov.

Definition

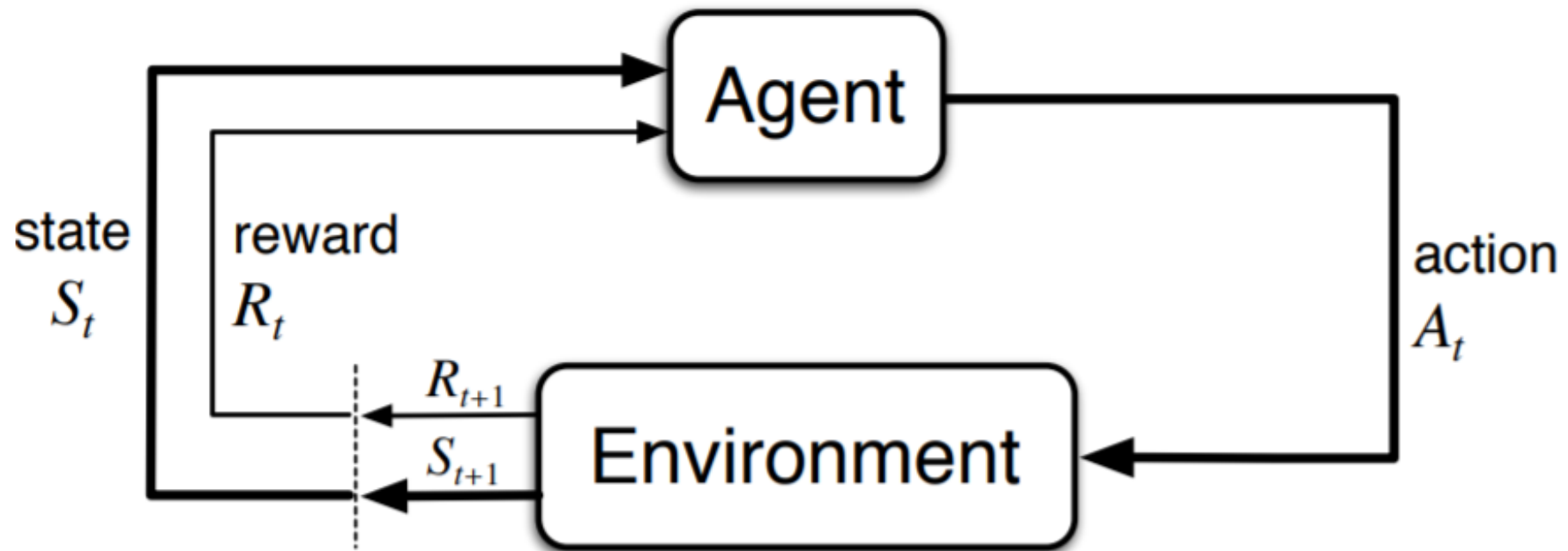
A *Markov Decision Process* is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$

- \mathcal{S} is a finite set of states
- \mathcal{A} is a finite set of actions
- \mathcal{P} is a state transition probability matrix,
 $\mathcal{P}_{ss'}^a = \mathbb{P}[S_{t+1} = s' \mid S_t = s, A_t = a]$
- \mathcal{R} is a reward function, $\mathcal{R}_s^a = \mathbb{E}[R_{t+1} \mid S_t = s, A_t = a]$
- γ is a discount factor $\gamma \in [0, 1]$.

Example: Student MDP



Finite Markov Decision Processes (MDP)



Daily life trajectory: $S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, S_3, A_3, \dots, S_T$