

An Introduction to Sage

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- Graduate CS student at Amrita University, India.
- Passionate about computer security and Python.
- Use Sage in Cryptography labs, Mathematics courses and CTF contests.

Overview and
Installation of
Sage

Basic usage

Interactive shell and
scripting

Arithmetic and
built-in functions

Applications in
various
domains

Algebra

Number Theory

Calculus

Graph plotting

Matrix algebra

Sage and L^AT_EX

More
applications
and further
reading

Contributing
to Sagemath

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- Installation
 - Pre-built binaries for most OS.
 - PPA for Ubuntu.
 - Packaging efforts underway for Debian and Fedora.
- GPL licensed mathematics software.
- Unified interface to about 90 popular Python libraries.
- Two modes: command (like Python shell) and notebook (web interface).
- Power of IPython shell and Python programming language.
- “sagerc” file: `$HOME/.sage/init.sage` or `$SAGE_STARTUP_FILE`.

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- **Sage interpreter:** IPython shell.
- **Sage scripts**
 - Similar to Python scripts; .sage extension.
 - import names from sage.all
 - Run as `sage <filename> <arguments>` like Python.
 - Other possibilities: profiling, compiling sage files(Cython), access C functions directly.

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- General arithmetic supported by an (I)Python shell.
 - $^$ is exponent and $^^$ is XOR.
 - For integers, $/$ reduces to lowest fraction and $//$ performs integer division.
- Support mathematical functions and constants with arbitrary precision.
 - `pi.n(digits=20)` = 3.1415926535897932385
 - `e.n(digits=25)` = 2.718281828459045235360287
 - `golden_ratio.n(prec=60)` = 1.6180339887498948
 - `n(sin(pi/3), prec=60)` = 0.86602540378443865
 - `sqrt(263).n(digits=20)` =
16.217274740226854774
 - `n(cos(5*pi/4), prec=60)` =
-0.70710678118654752

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- Factorizing polynomials.
 - $\text{factor}(x^4 - 15x^3 + 84x^2 - 208x + 192) = (x - 3)(x - 4)^3$
 - $\text{factor}(x^3 - 6x^2 + 11x - 6) = (x - 1)(x - 2)(x - 3)$
- Solving polynomial equations.
 - $\text{solve}([x^2 - 4x + 2 == -1], x) = [x = 3, x = 1]$
 - Solutions to $x^2 + 3xy + y^2 = 0$ and $x - y = 4 =$
 $[[1.1055728, -2.8944272], [2.8944272, -1.1055728]]$
- Use `find_root` where `solve` does not work. Also useful to find solutions in a particular interval.
 - $\text{solve}(\cos(t) == \sin(t), t) = [\sin(t) = \cos(t)]$
 - $\text{find_root}(\cos(t) == \sin(t), 0, \pi) = 0.785398163397$

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- Modulus: $\text{mod}(27, 12) = 3$ and $\text{power_mod}(27, 2, 12) = 9$
- Primality test: $\text{is_prime}(13) = \text{True}$, $\text{is_prime}(15) = \text{False}$
- $\text{prime_range}(1, 35) = [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31]$.
 - Generator version: $\text{primes}(1, 35)$
- $\text{primes_first_n}(11) = [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31]$
- $\text{next_prime}(29) = 31$ and $\text{previous_prime} = 23$
- $\text{factorial}(20) = 2432902008176640000$, $\text{factor}(20) = 2^2 \cdot 5$, $\text{divisors}(20) = [1, 2, 4, 5, 10, 20]$
- $\text{gcd}(10, 15) = 5$, $\text{lcm}(10, 15) = 30$

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- Differentiation

- $\text{diff}(\sin(x) + \cos(x)) = \cos(x) - \sin(x)$
- $\text{diff}((\sin(x^2)^3)) = 6x \cos(x^2) \sin(x^2)^2$

- Integration

- $\text{integral}(\cos(x) - \sin(x)) = \sin(x) + \cos(x)$
- $\text{integral}(6 * x * \cos(x^2) * \sin(x^2)^2, x) = \sin(x^2)^3$

- Partial differential and solving differential equations also possible!

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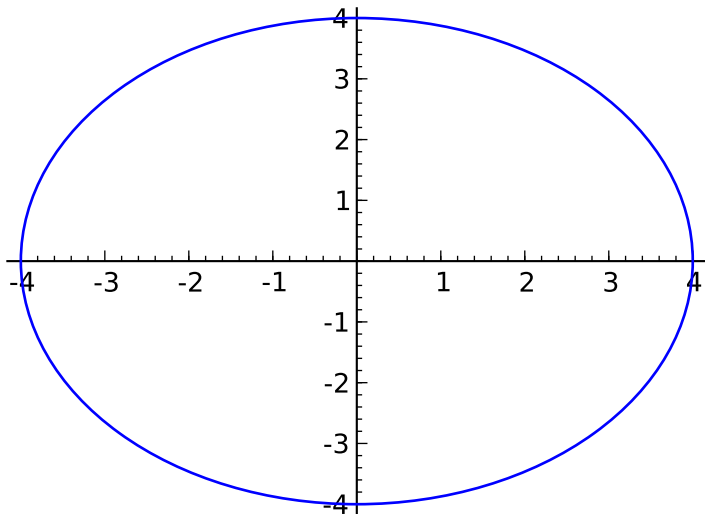
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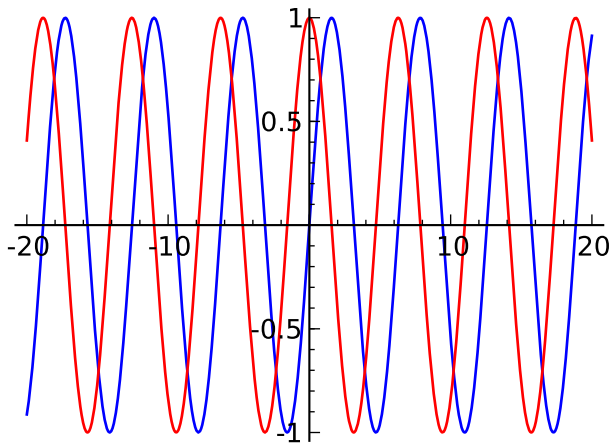
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Circle of radius 4 centered at (0, 0): $c = \text{circle}((0, 0), 4)$

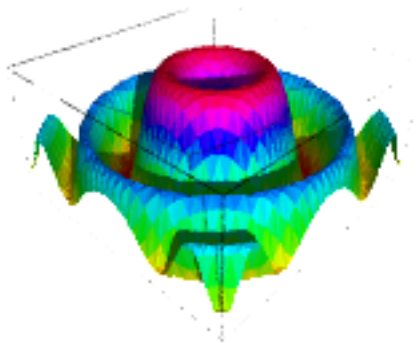


Multiple functions in same plot.

```
plot(sin(x), -20, 20, rgbcolor = (0, 0, 1)) +  
plot(cos(x), -20, 20, rgbcolor = (1, 0, 0))
```



$$f = \frac{\sin(y*y+x*x)}{\sqrt{(x*x+y*y+.0001)}}: \text{plot3d}(f, (-3,3), (-3,3))$$



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- Creating matrices: $m = \text{Matrix}([[1, 2], [3, 4], [5, 6]])$
- Arithmetic operations
 - $P = \text{Matrix}([[1, 2], [3, 4]])$, $Q = \text{Matrix}([[7, 8], [5, 6]])$
 - $P + Q = \begin{pmatrix} 8 & 10 \\ 8 & 10 \end{pmatrix}$, $P - Q = \begin{pmatrix} -6 & -6 \\ -2 & -2 \end{pmatrix}$
 - $P * Q = \begin{pmatrix} 17 & 20 \\ 41 & 48 \end{pmatrix}$, $4 * P = \begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix}$
- $P^3 = \begin{pmatrix} 37 & 54 \\ 81 & 118 \end{pmatrix}$, $P^{-1} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$, $|P| = -2$
- More functions: `is_singular`, `is_symmetric`, `is_skew_symmetric`, `is_invertible`, `is_square`

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- L^AT_EX representation: `latex(P)`

```
\left(\begin{array}{rr}
1 & 2 \\
3 & 4
\end{array}\right)
```
- `view(P)`: Display PDF(pdf_latex)/HTML(MathJAX) depending on mode.
- SageT_EX: Call Sage commands from L^AT_EX.
 - Regular statement: `\sage{pow_mod(27, 2, 12)}`
 - Plots: `\sageplot{plot(sin(x) + cos(x), -20, 20)}`
 - `\sageblock` and `\sagesilent`: Embedding Sage code

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- Interfacing with other algebra systems(GP/PARI, Singular, Maxima)
- Polynomials
- Combinatorics
- Graph and group theory
- Linear algebra
- Elliptic curves
- Advanced portions of everything discussed

- Sage tutorial:
<http://www.sagemath.org/doc/tutorial/index.html>
- Thematic tutorials:
http://www.sagemath.org/doc/thematic_tutorials/index.html
- Tutorials for those with some mathematics background:
<http://www.sagemath.org/doc/prep/index.html>

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- Packaging for Linux distros.
- Improve startup time.
- UI enhancements: Notebook and 2D plots.
- Mobile applications: Android, iOS.
- Mathematicians help with specific libraries.
- Visit <http://www.sagemath.org/development.html> for more information on getting involved.

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Thank you!