MEM 636 Theory of Nonlinear Control

Final Project, Winter 2017

Recall the wheelset used to model a simple wheeled robot as shown in Figure 1. The system is modeled by the differential equations

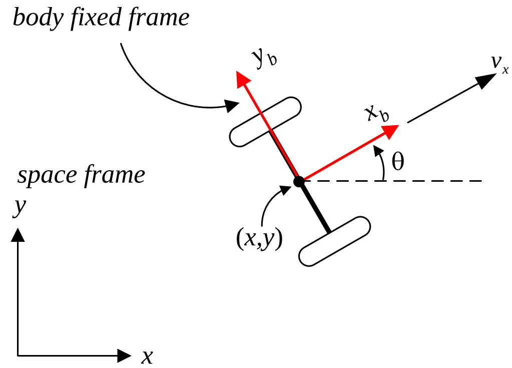

Figure 1. Simple wheelset.

$$\begin{aligned}\dot{x} &= v_x \cos\theta \\ \dot{y} &= v_x \sin\theta \\ \dot{\theta} &= \omega \\ M\dot{v}_x &= F \\ J\dot{\omega} &= T\end{aligned}$$

The driving force *F* and the steering torque *T* are generated by coordinating the independent wheel torques. For now, take $M=1, J=1$ and suppose $|F|\le 1, |T|\le 4$. Recall that the system can be feedback linearized, with respect to the outputs (x,y), using dynamic extension. The extended equations are

$$\frac{d}{dt}\begin{bmatrix}\theta\\ x\\ y\\ v_x\\ \omega\\ F\end{bmatrix}=\begin{bmatrix}\omega\\ v_x\cos\theta\\ v_x\sin\theta\\ F\\ 0\\ 0\end{bmatrix}+\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\0&1\\1&0\end{bmatrix}\begin{bmatrix}u\\ T\end{bmatrix}$$

And the FBL normal form is

$$\frac{d}{dt}\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\\ z_5\\ z_6\end{bmatrix}=\left[\begin{array}{ccc|ccc}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&0&0\\ \hline 0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{array}\right]\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\\ z_5\\ z_6\end{bmatrix}+\left[\begin{array}{cc}0&0\\0&0\\1&0\\ \hline 0&0\\0&0\\0&1\end{array}\right]\left[\alpha+\rho u\right],\quad \alpha(x)=\begin{bmatrix}-F\omega\sin\theta+\omega(-v_x\omega\cos\theta-F\sin\theta)\\ F\omega\cos\theta+\omega(-v_x\omega\sin\theta+F\cos\theta)\end{bmatrix}$$

$$\rho(x)=\begin{bmatrix}\cos\theta & -v_x\sin\theta\\ \sin\theta & v_x\cos\theta\end{bmatrix}$$

Design a variable structure switching system based on the FBL reduction and choose $s(x)=Kz(x)$. Choose K so that the sliding eigenvalues are all at -1. Simulate from various starting conditions using Simulink (you can use the vehicle model from the earlier project).