DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 1

Systems of linear equations

Singular and non-singular matrices

Determinants

Rank of a matrix

Row reduction

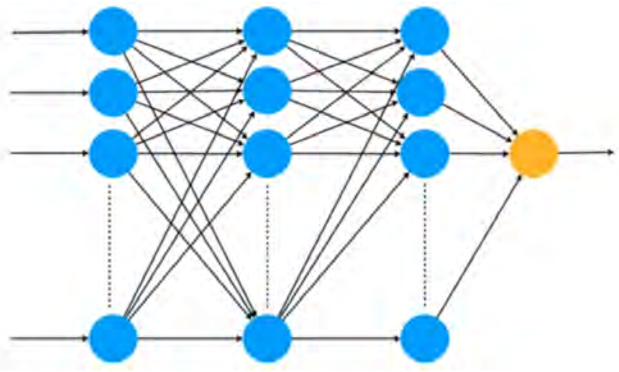
Null space

DeepLearning.AI

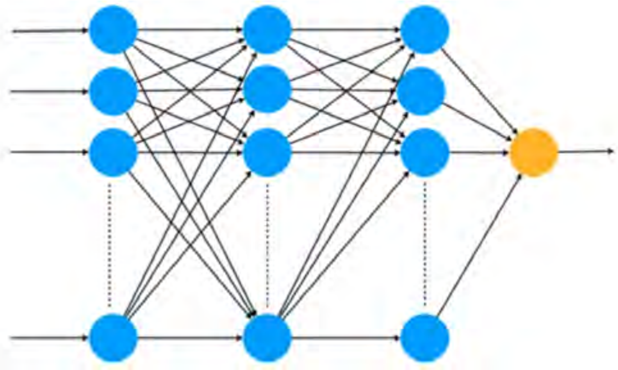
System of Linear Equations

Machine learning motivation

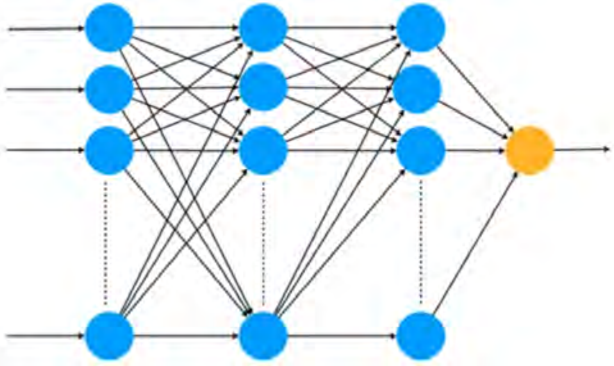
Neural networks - Matrix operations



Neural networks - Matrix operations

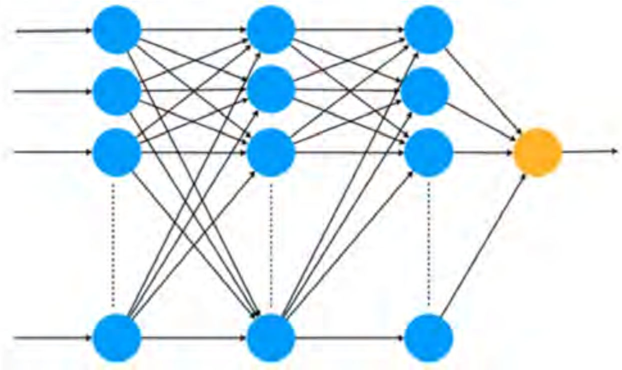


Neural networks - Matrix operations



Yes, it's a face

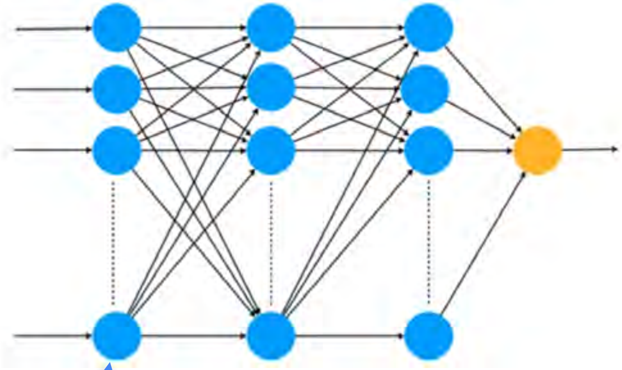
Neural networks - Matrix operations



Yes, it’s a face

Matrix

Neural networks - Matrix operations

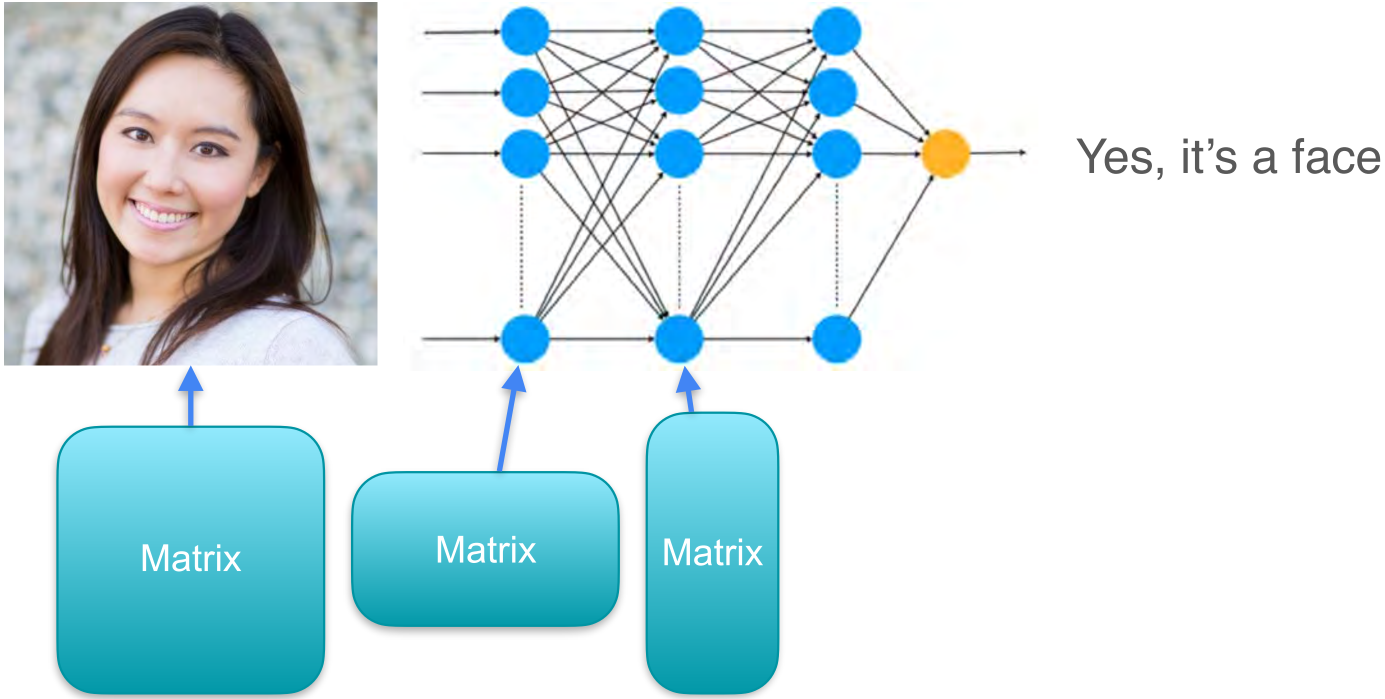


Yes, it's a face

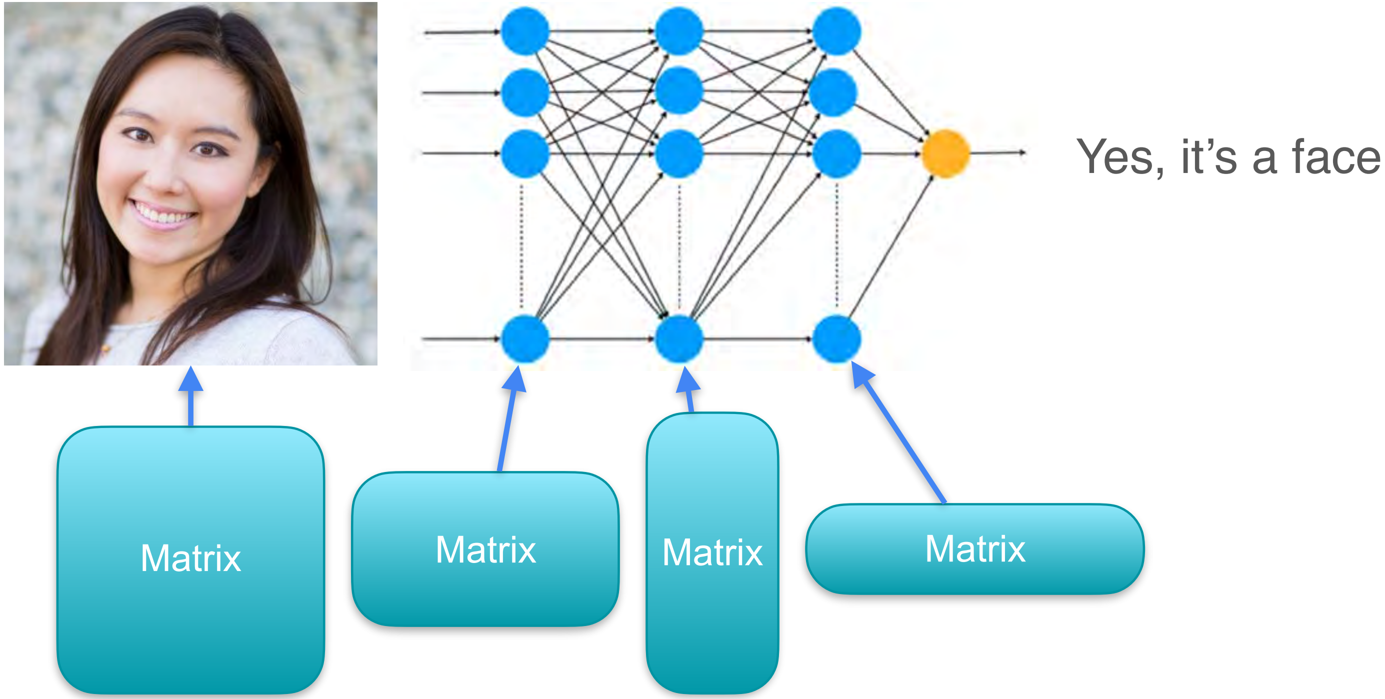
Matrix

Matrix

Neural networks - Matrix operations



Neural networks - Matrix operations



Neural networks - image recognition

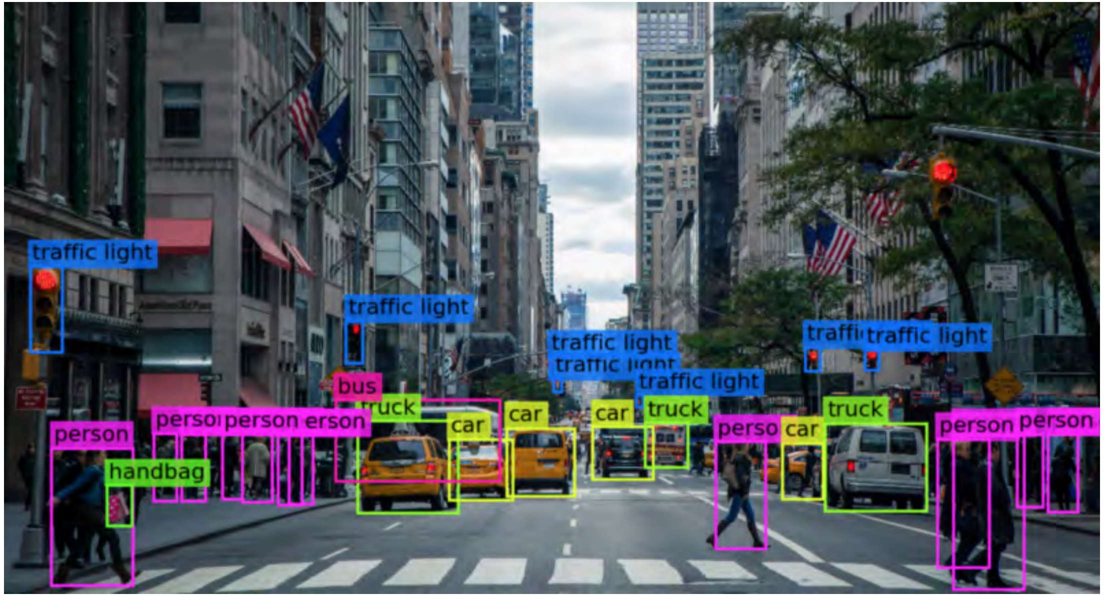


Image recognition in a busy street in New York.

- Image recognition: Getting the computer to see images and recognize what is on them.

DeepLearning.AI

System of Linear Equations

System of sentences

Systems of sentences

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

System 2

The dog is **black**
The dog is **black**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

System 2

The dog is **black**
The dog is **black**

System 3

The dog is **black**
The dog is **white**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

Complete

System 2

The dog is **black**
The dog is **black**

System 3

The dog is **black**
The dog is **white**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

Complete

System 2

The dog is **black**
The dog is **black**

Redundant

System 3

The dog is **black**
The dog is **white**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

Complete

System 2

The dog is **black**
The dog is **black**

Redundant

System 3

The dog is **black**
The dog is **white**

Contradictory

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

Complete

System 2

The dog is **black**
The dog is **black**

Redundant

Singular

System 3

The dog is **black**
The dog is **white**

Contradictory

Singular

Systems of sentences

System 1

The dog is **black**
The cat is **orange**

Complete

Non-singular

System 2

The dog is **black**
The dog is **black**

Redundant

Singular

System 3

The dog is **black**
The dog is **white**

Contradictory

Singular

Systems of sentences

System 1

The dog is **black**
The cat is **orange**
The bird is **red**

System 2

The dog is **black**
The dog is **black**
The bird is **red**

System 3

The dog is **black**
The dog is **black**
The dog is **black**

System 4

The dog is **black**
The dog is **white**
The bird is **red**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**
The bird is **red**

Complete

Non-singular

System 2

The dog is **black**
The dog is **black**
The bird is **red**

System 3

The dog is **black**
The dog is **black**
The dog is **black**

System 4

The dog is **black**
The dog is **white**
The bird is **red**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**
The bird is **red**

Complete

Non-singular

System 2

The dog is **black**
The dog is **black**
The bird is **red**

Redundant

Singular

System 3

The dog is **black**
The dog is **black**
The dog is **black**

System 4

The dog is **black**
The dog is **white**
The bird is **red**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**
The bird is **red**

Complete

Non-singular

System 2

The dog is **black**
The dog is **black**
The bird is **red**

Redundant

Singular

System 3

The dog is **black**
The dog is **black**
The dog is **black**

Redundant

Singular

System 4

The dog is **black**
The dog is **white**
The bird is **red**

Systems of sentences

System 1

The dog is **black**
The cat is **orange**
The bird is **red**

Complete

Non-singular

System 2

The dog is **black**
The dog is **black**
The bird is **red**

Redundant

Singular

System 3

The dog is **black**
The dog is **black**
The dog is **black**

Redundant

Singular

System 4

The dog is **black**
The dog is **white**
The bird is **red**

Contradictory

Singular

Quiz: Systems of sentences

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

Problem 1:

What color is the bird?

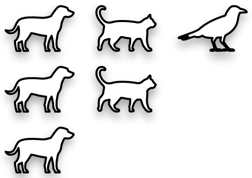
Problem 2:

Is this system singular or non-singular?

Solution: Systems of information

Given this system:

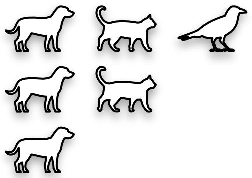
- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

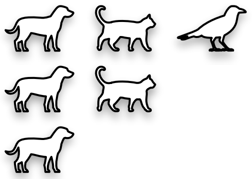


Solution 1:

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



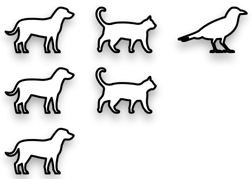
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



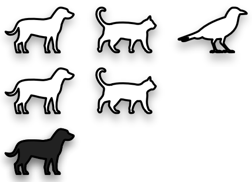
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



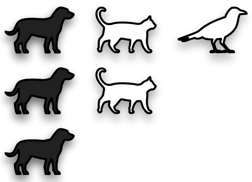
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

The bird is red. 

Solution: Systems of information

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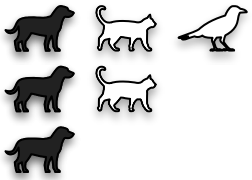

Solution 1:

The bird is red. 

Solution: Systems of information

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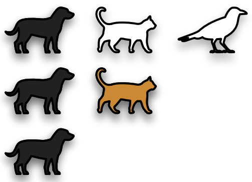
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
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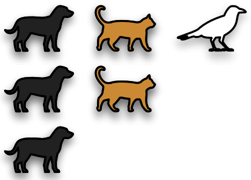

Solution 1:

The bird is red. 

Solution: Systems of information

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- Between the dog and the cat, one is orange.
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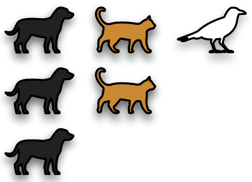

Solution 1:

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Solution: Systems of information

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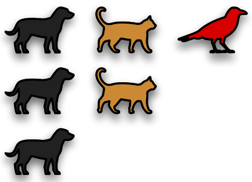
- Between the dog and the cat, one is orange.
- The dog is black.



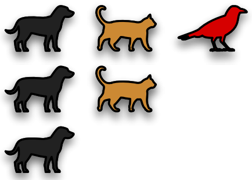
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution 1:

The bird is red. 

Solution 2:

It is non-singular. 

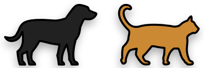
System of Linear Equations

System of equations

Sentences → Equations

Sentences

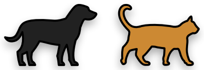
Between the dog and the cat, one is black.



Sentences → Equations

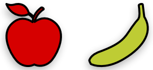
Sentences

Between the dog and
the cat, one is black.



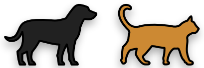
Sentences with numbers

The price of an apple
and a banana is $10.



Sentences → Equations

Sentences		Sentences with numbers		Equations
Between the dog and the cat, one is black.	→	The price of an apple and a banana is \$10.	→	$a + b = 10$



Quiz: Systems of equations 1

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost $10.
- **Day 2:** You bought an apple and two bananas and they cost $12.

Question: How much does each fruit cost?

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

- **Day 2:** You bought an apple and two bananas and they cost \$12.

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.

 +  = $10

- **Day 2:** You bought an apple and two bananas and they cost $12.

- **Solution:** An apple costs $8, a banana costs $2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought an apple and two bananas and they cost $12.

- **Solution:** An apple costs $8, a banana costs $2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought an apple and two bananas and they cost $12.

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Solution: Systems of equations 1

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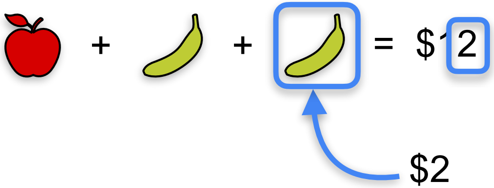
- **Day 2:** You bought an apple and two bananas and they cost $12.

- **Solution:** An apple costs $8, a banana costs $2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.

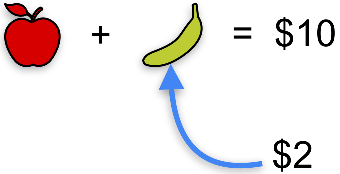
- **Day 2:** You bought an apple and two bananas and they cost $12.



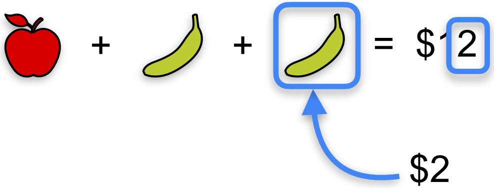
- **Solution:** An apple costs $8, a banana costs $2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.



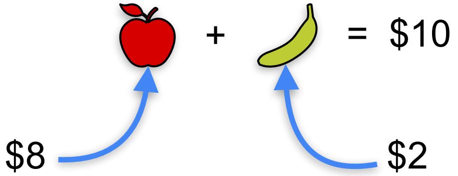
- **Day 2:** You bought an apple and two bananas and they cost $12.



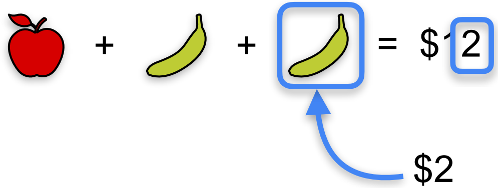
- **Solution:** An apple costs $8, a banana costs $2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost $10.



- **Day 2:** You bought an apple and two bananas and they cost $12.



- **Solution:** An apple costs $8, a banana costs $2.

Quiz: Systems of equations 2

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost $10.
- **Day 2:** You bought two apples and two bananas and they cost $20.

Question: How much does each fruit cost?

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $20.

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

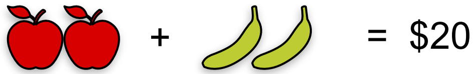
 +  = $10

- **Day 2:** You bought two apples and two bananas and they cost $20.

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

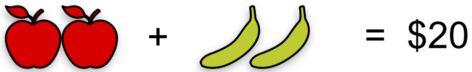
- **Day 2:** You bought two apples and two bananas and they cost $20.



Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $20.

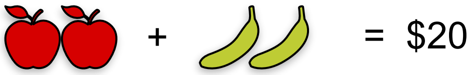


Same thing!!!

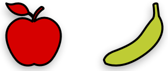
Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $20.



Same thing!!!

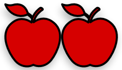


Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

 +  = $10

- **Day 2:** You bought two apples and two bananas and they cost $20.

 +  = $20

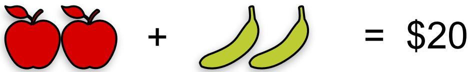
Same thing!!!

	
8	2
5	5

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $20.



Same thing!!!

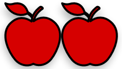
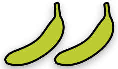
8	2
5	5
8.3	1.7

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

 +  = $10

- **Day 2:** You bought two apples and two bananas and they cost $20.

 +  = $20

Same thing!!!

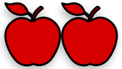
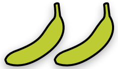
	
8	2
5	5
8.3	1.7
0	10

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost $10.

 +  = $10

- **Day 2:** You bought two apples and two bananas and they cost $20.

 +  = $20

Same thing!!!

	
8	2
5	5
8.3	1.7
0	10

Infinitely many solutions!

Quiz: Systems of equations 3

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost $10.
- **Day 2:** You bought two apples and two bananas and they cost $24.

Question: How much does each fruit cost?

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

- **Day 2:** You bought two apples and two bananas and they cost \$24.

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost $10.

 +  = $10

- **Day 2:** You bought two apples and two bananas and they cost $24.

Solution: Systems of equations 3

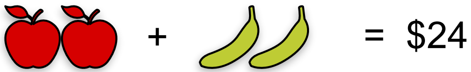
- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $24.

Solution: Systems of equations 3

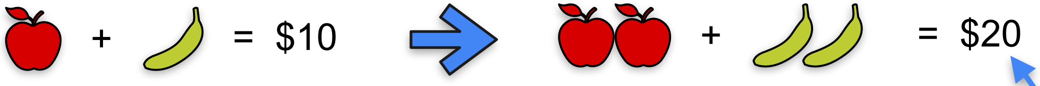
- **Day 1:** You bought an apple and a banana and they cost $10.

- **Day 2:** You bought two apples and two bananas and they cost $24.

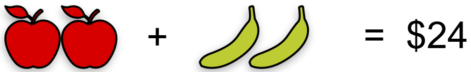


Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost $10.



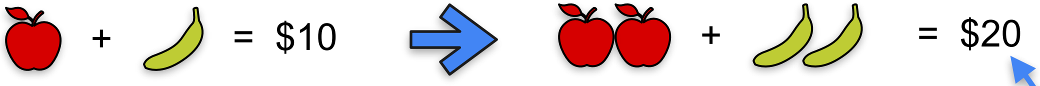
- **Day 2:** You bought two apples and two bananas and they cost $24.



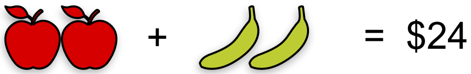
Contradiction!

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost $10.



- **Day 2:** You bought two apples and two bananas and they cost $24.



Contradiction!

No solutions!

Systems of equations

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

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Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

System 2

- $a + b = 10$
- $2a + 2b = 20$

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

System 2

- $a + b = 10$
- $2a + 2b = 20$

System 3

- $a + b = 10$
- $2a + 2b = 24$

Systems of equations

System 1

- a + b = 10
- a + 2b = 12

Unique solution:

System 2

- a + b = 10
- 2a + 2b = 20

System 3

- a + b = 10
- 2a + 2b = 24

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

System 2

- $a + b = 10$
- $2a + 2b = 20$

System 3

- $a + b = 10$
- $2a + 2b = 24$

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

System 2

- $a + b = 10$
- $2a + 2b = 20$

System 3

- $a + b = 10$
- $2a + 2b = 24$

Systems of equations

System 1

- a + b = 10
- a + 2b = 12

Unique solution:

a = 8

b = 2

Complete

Non-singular

System 2

- a + b = 10
- 2a + 2b = 20

System 3

- a + b = 10
- 2a + 2b = 24

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

Infinite solutions

System 3

- $a + b = 10$
- $2a + 2b = 24$

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

Infinite solutions

$a = 8$

$b = 2$

System 3

- $a + b = 10$
- $2a + 2b = 24$

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

Infinite solutions

$a = 8$, 7

$b = 2$, 3

System 3

- $a + b = 10$
- $2a + 2b = 24$

Systems of equations

System 1

- a + b = 10
- a + 2b = 12

Unique solution:

a = 8

b = 2

Complete

Non-singular

System 2

- a + b = 10
- 2a + 2b = 20

Infinite solutions

a = 8	7	6
b = 2	3	4

System 3

- a + b = 10
- 2a + 2b = 24

Systems of equations

System 1

- a + b = 10
- a + 2b = 12

Unique solution:

a = 8
b = 2

Complete

Non-singular

System 2

- a + b = 10
- 2a + 2b = 20

Infinite solutions

a = 8 , 7 , 6 …
b = 2 , 3 , 4

System 3

- a + b = 10
- 2a + 2b = 24

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

Infinite solutions

$a = 8$, 7, 6 …

$b = 2$, 3, 4

Redundant

System 3

- $a + b = 10$
- $2a + 2b = 24$

Systems of equations

System 1

- a + b = 10
- a + 2b = 12

Unique solution:

a = 8
b = 2

Complete

Non-singular

System 2

- a + b = 10
- 2a + 2b = 20

Infinite solutions

a = 8 , 7 , 6 ...
b = 2 , 3 , 4

Redundant

Singular

System 3

- a + b = 10
- 2a + 2b = 24

Systems of equations

System 1

- a + b = 10
- a + 2b = 12

Unique solution:

a = 8
b = 2

Complete

Non-singular

System 2

- a + b = 10
- 2a + 2b = 20

Infinite solutions

a = 8	7	6	…
b = 2	3	4	

Redundant

Singular

System 3

- a + b = 10
- 2a + 2b = 24

No solution

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

Infinite solutions

$a = 8$, 7, 6 …

$b = 2$, 3, 4

Redundant

Singular

System 3

- $a + b = 10$
- $2a + 2b = 24$

No solution

Contradictory

Systems of equations

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique solution:

$a = 8$

$b = 2$

Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

Infinite solutions

$a = 8, 7, 6 \ldots$

$b = 2, 3, 4$

Redundant

Singular

System 3

- $a + b = 10$
- $2a + 2b = 24$

No solution

Contradictory

Singular

What is a linear equation?

Linear

Non-linear

What is a linear equation?

Linear

$a + b = 10$

Non-linear

What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

Non-linear

What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

$3.4a - 48.99b + 2c = 122.5$

Non-linear

What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

$\boxed{3.4}a - \boxed{48.99}b + \boxed{2}c = \boxed{122.5}$

Numbers

Non-linear

What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

$\boxed{3.4}a - \boxed{48.99}b + \boxed{2}c = \boxed{122.5}$

Numbers

Non-linear

$a^2 + b^2 = 10$

What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

$\boxed{3.4}a - \boxed{48.99}b + \boxed{2}c = \boxed{122.5}$

Numbers

Non-linear

$a^2 + b^2 = 10$

$\sin(a) + b^5 = 15$

What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

$\boxed{3.4}a - \boxed{48.99}b + \boxed{2}c = \boxed{122.5}$

Numbers

Non-linear

$a^2 + b^2 = 10$

$\sin(a) + b^5 = 15$

$2^a - 3^b = 0$

What is a linear equation?

Linear

$a + b = 10$

$2a + 3b = 15$

$\boxed{3.4}a - \boxed{48.99}b + \boxed{2}c = \boxed{122.5}$

Numbers

Non-linear

$a^2 + b^2 = 10$

$\sin(a) + b^5 = 15$

$2^a - 3^b = 0$

$ab^2 + \frac{b}{a} - \frac{3}{b} - \log(c) = 4^a$

System of Linear Equations

System of equations as lines

Linear equation $\rightarrow$ line

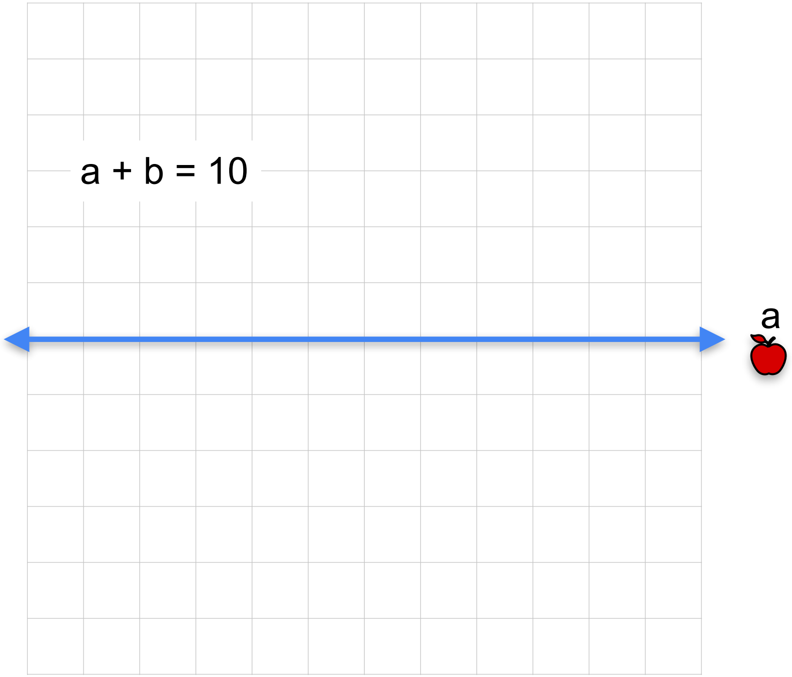
Linear equation → line

$a + b = 10$

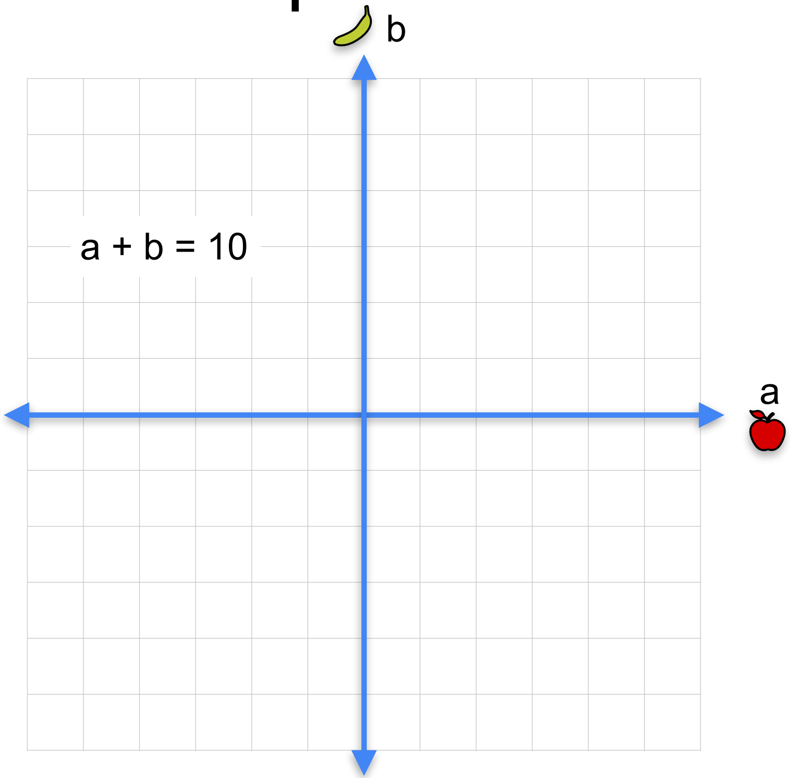
Linear equation → line

$a + b = 10$

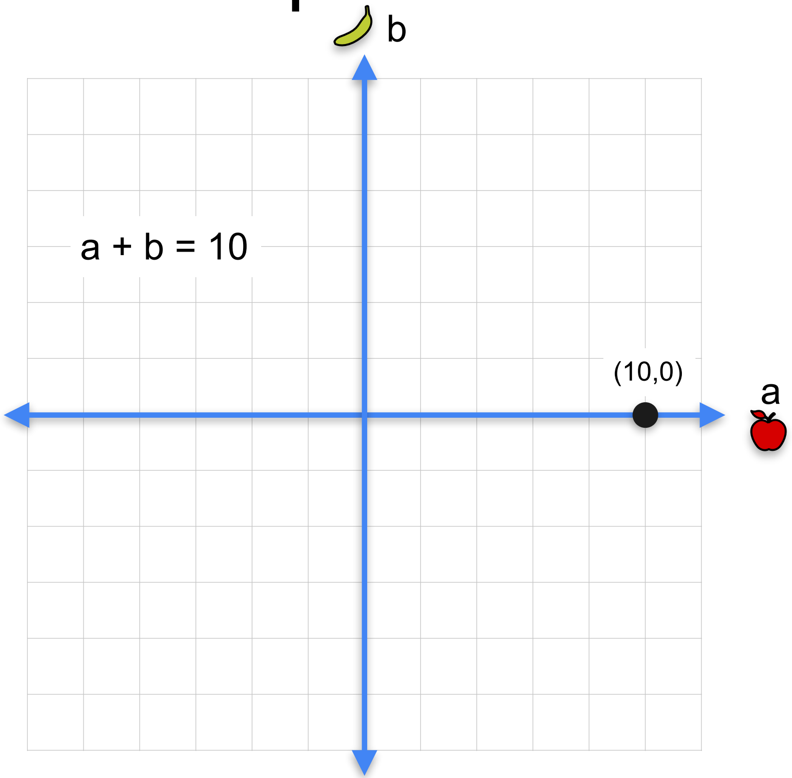
Linear equation → line



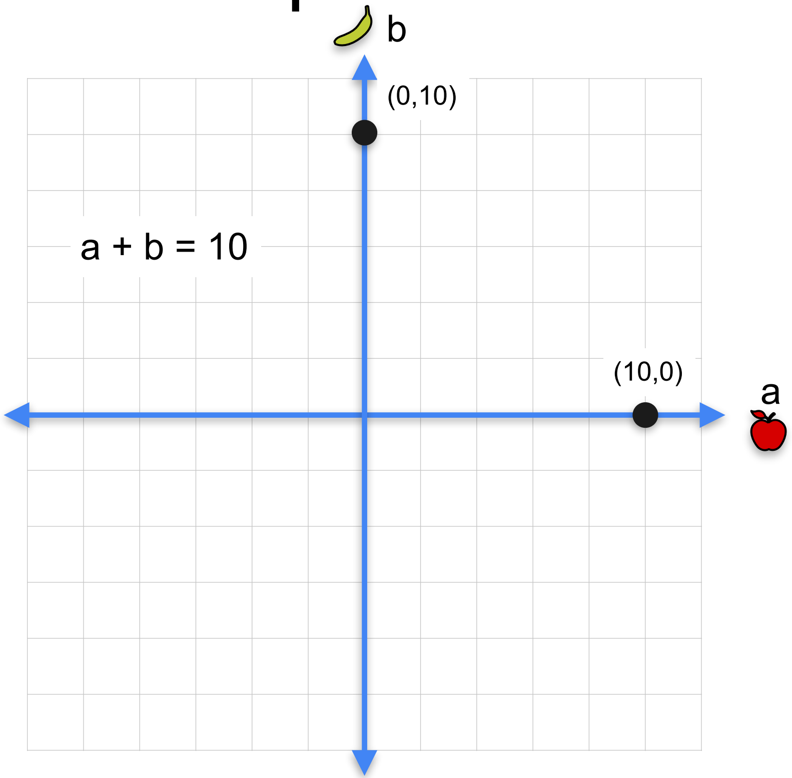
Linear equation → line



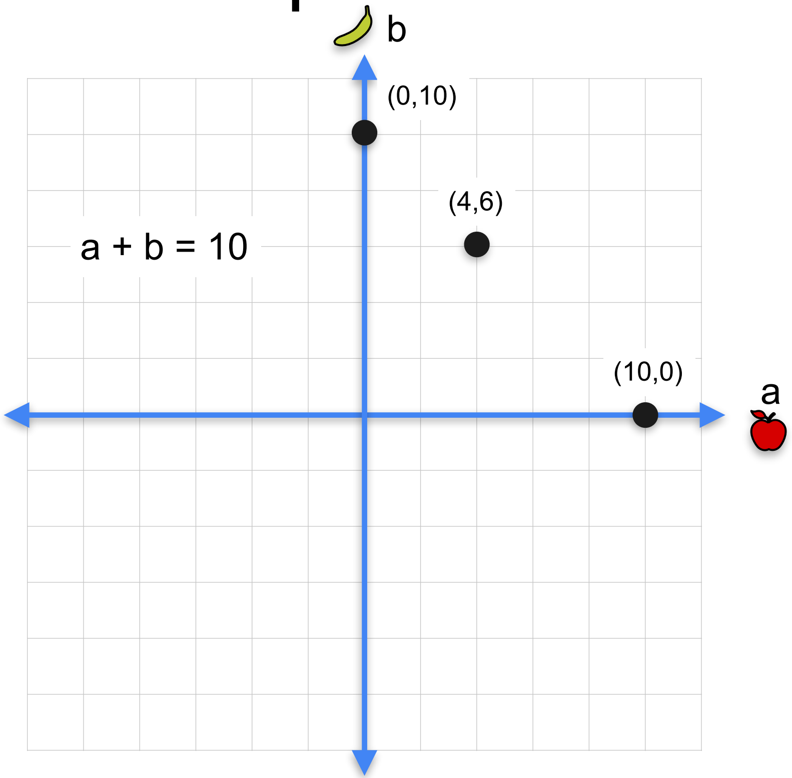
Linear equation → line



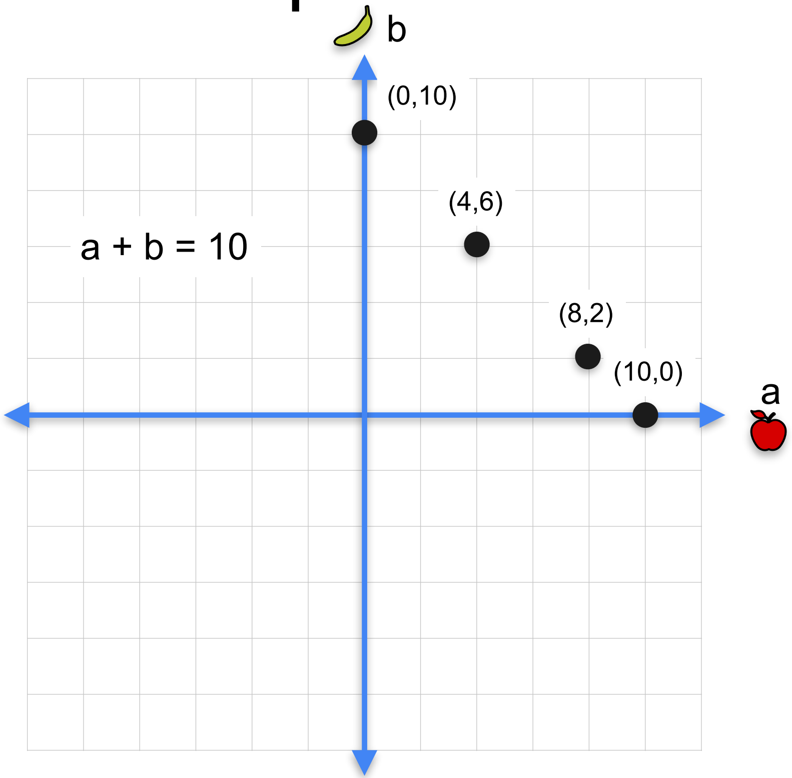
Linear equation → line



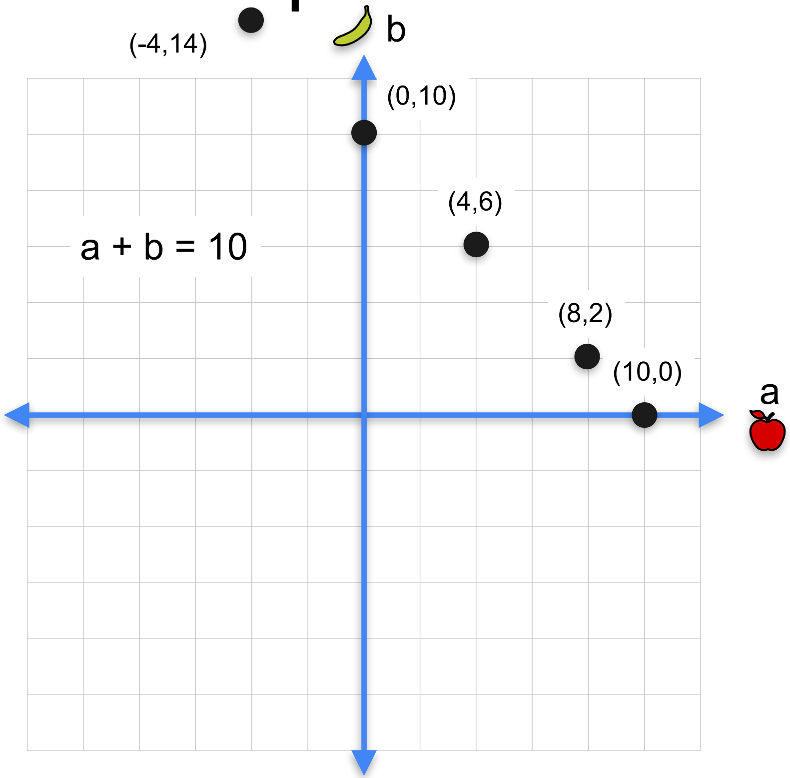
Linear equation → line



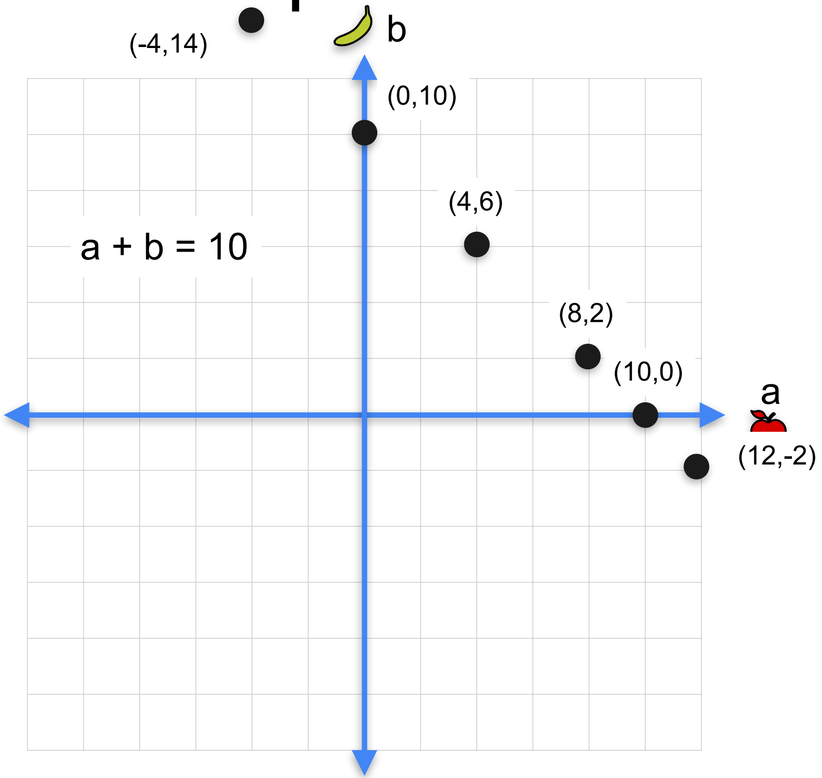
Linear equation → line



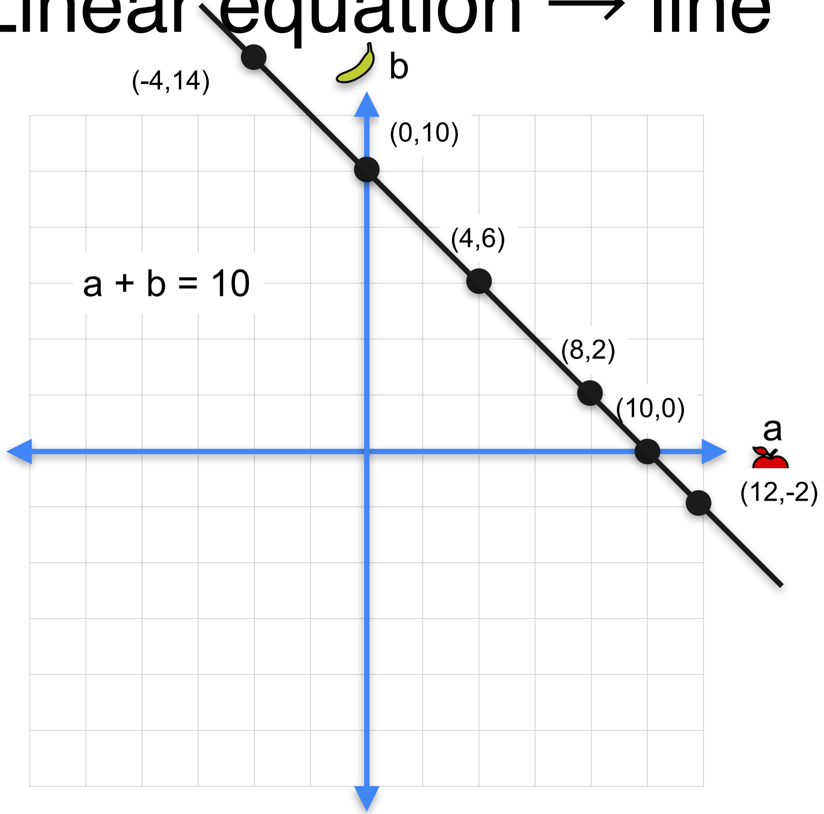
Linear equation → line



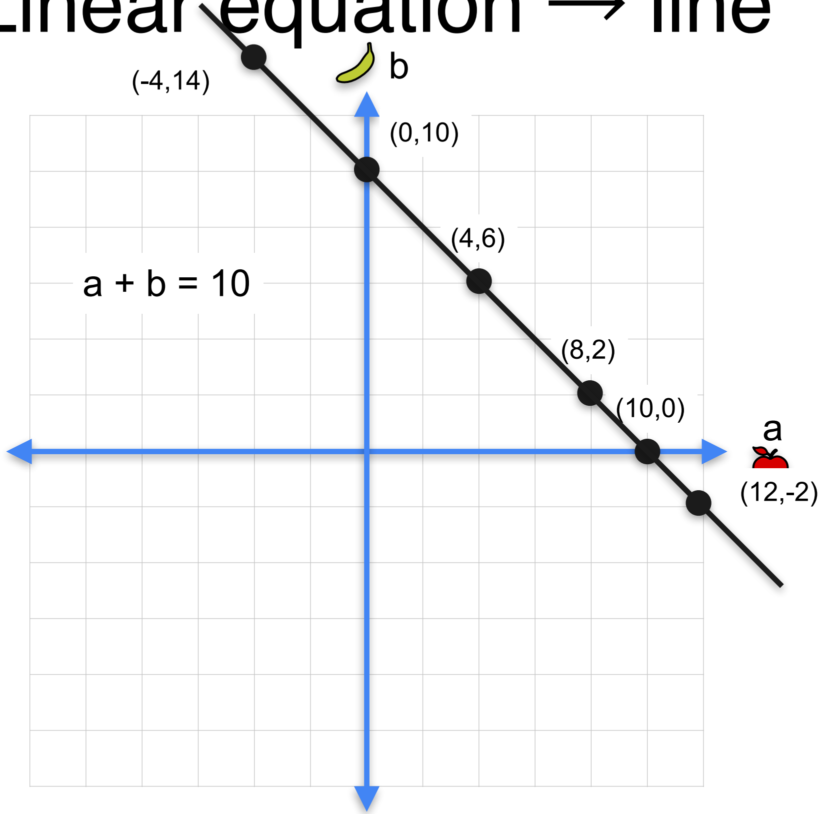
Linear equation → line

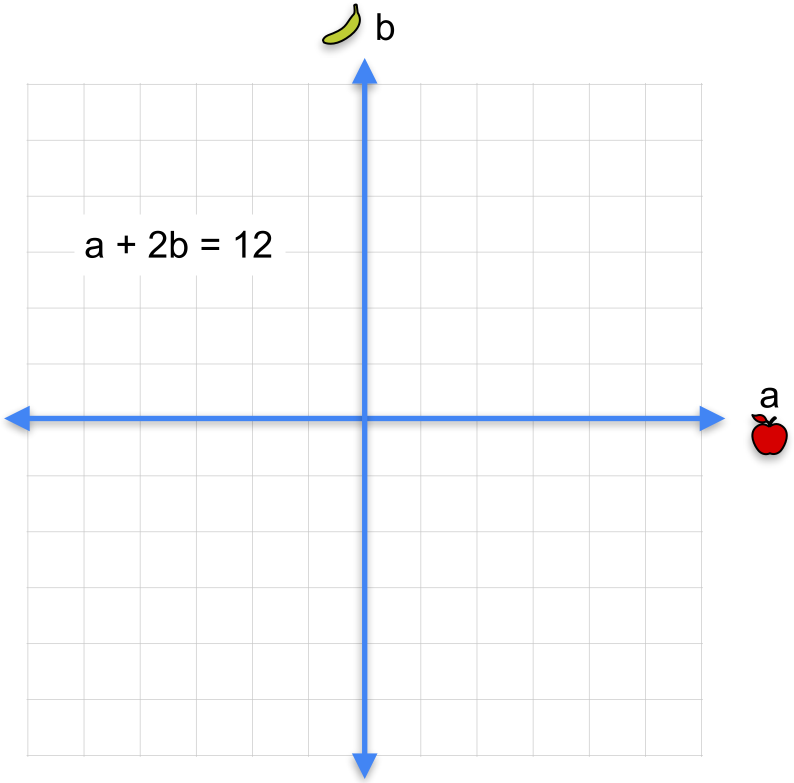


Linear equation → line

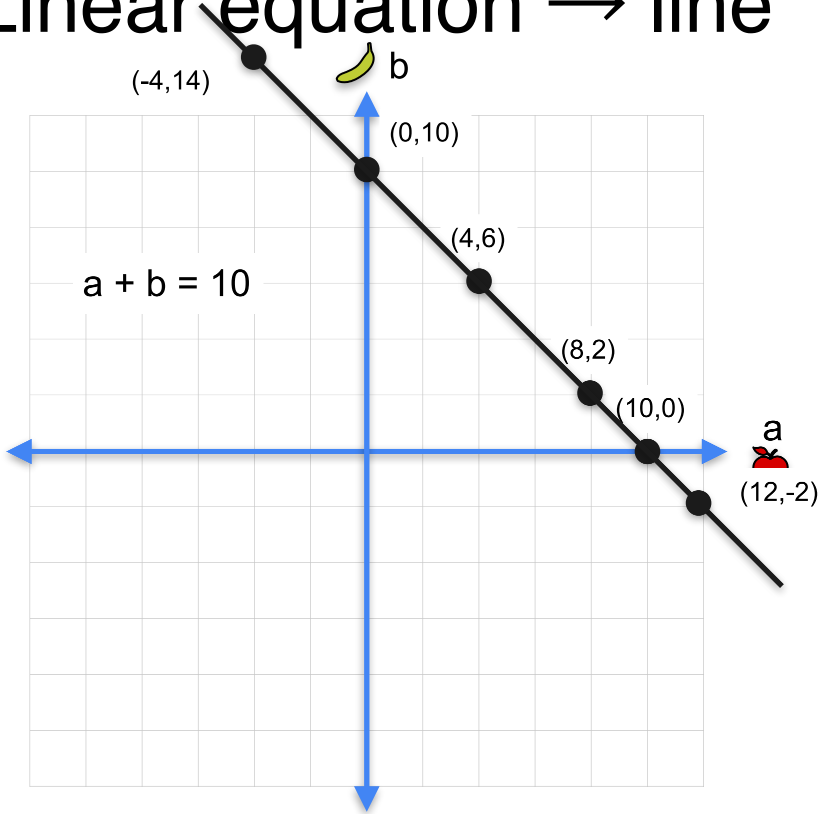


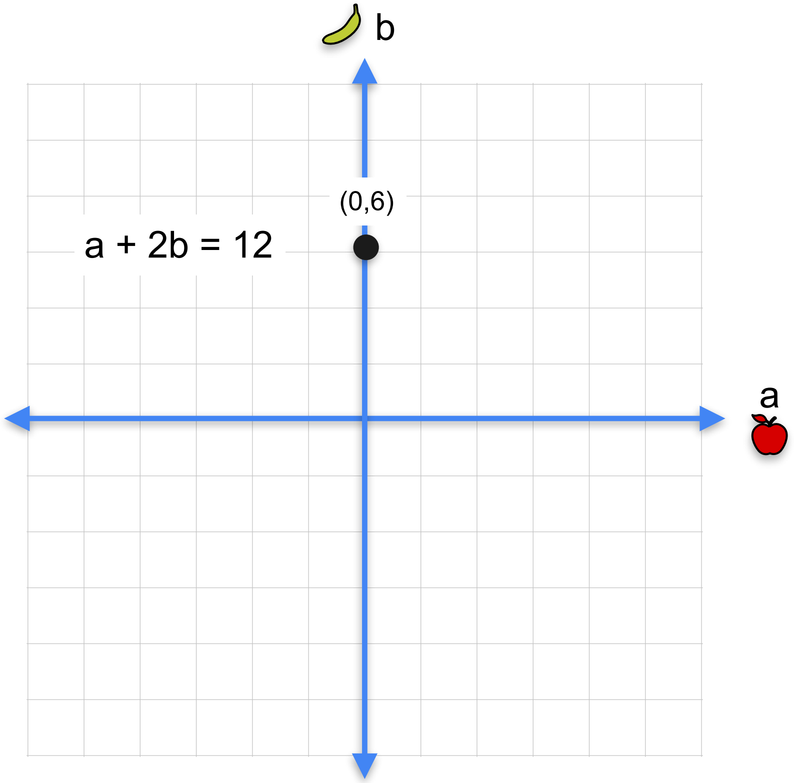
Linear equation → line



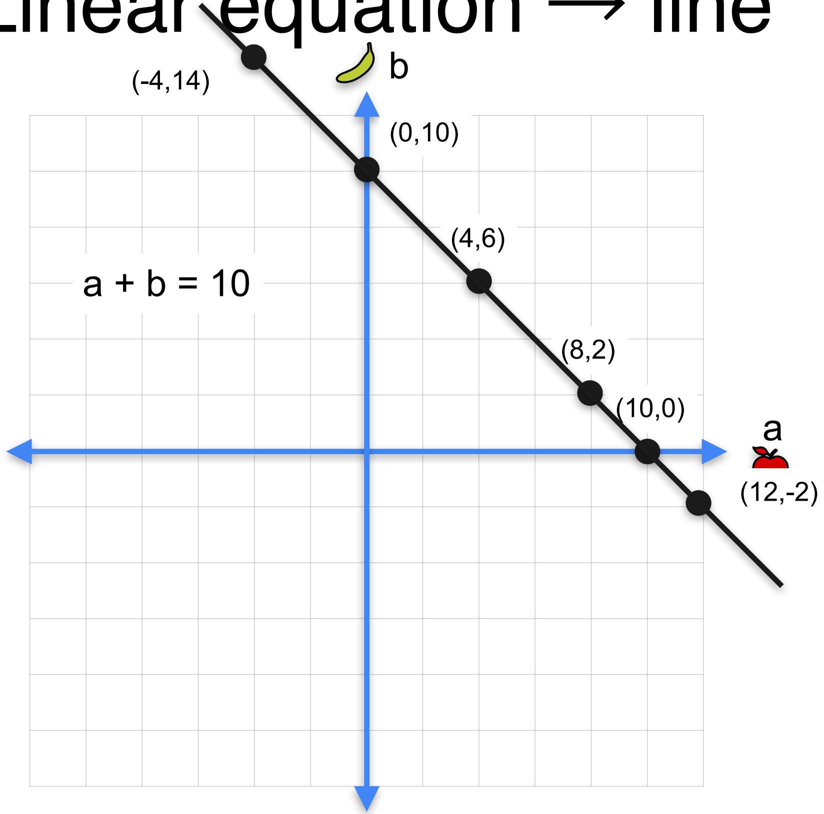


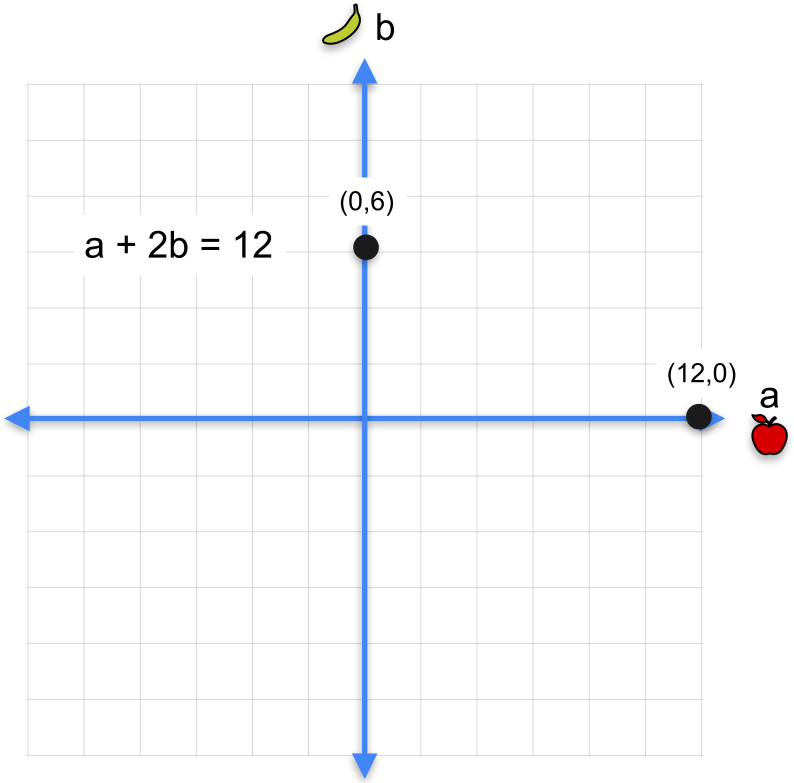
Linear equation → line



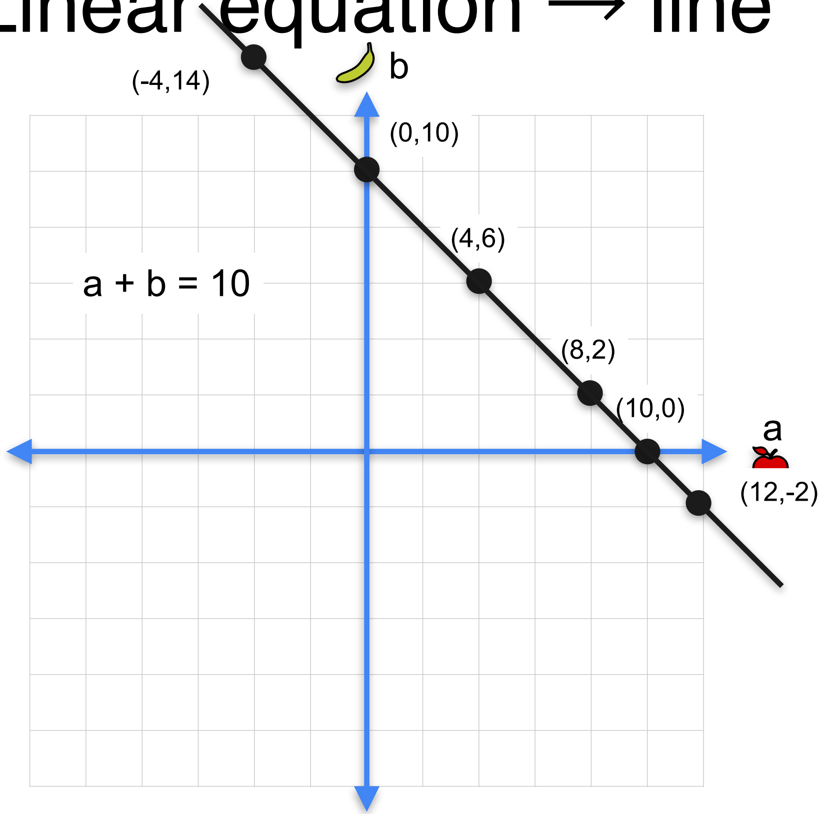


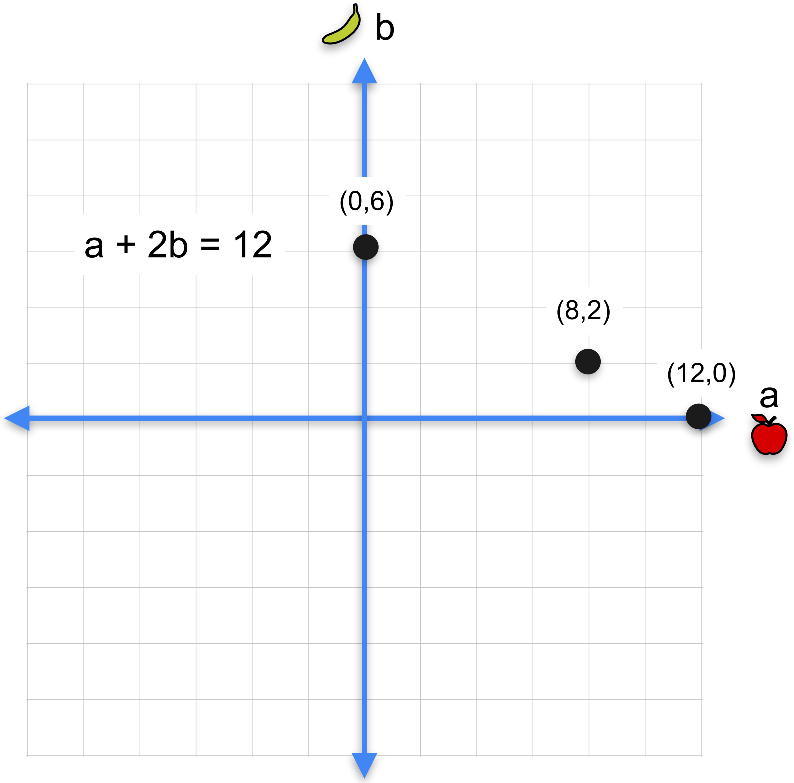
Linear equation → line



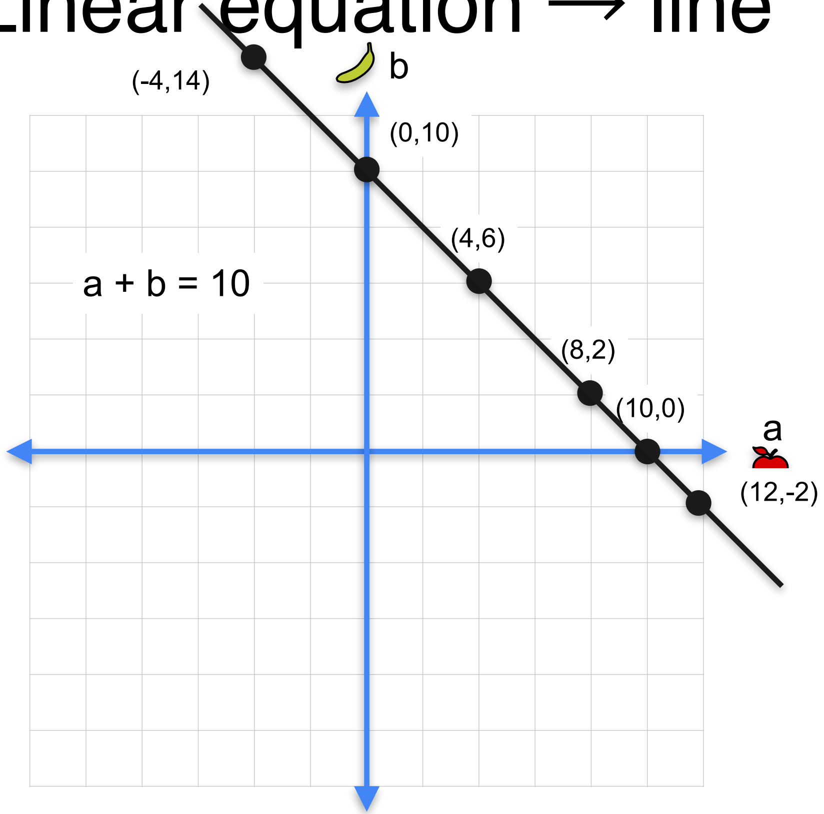


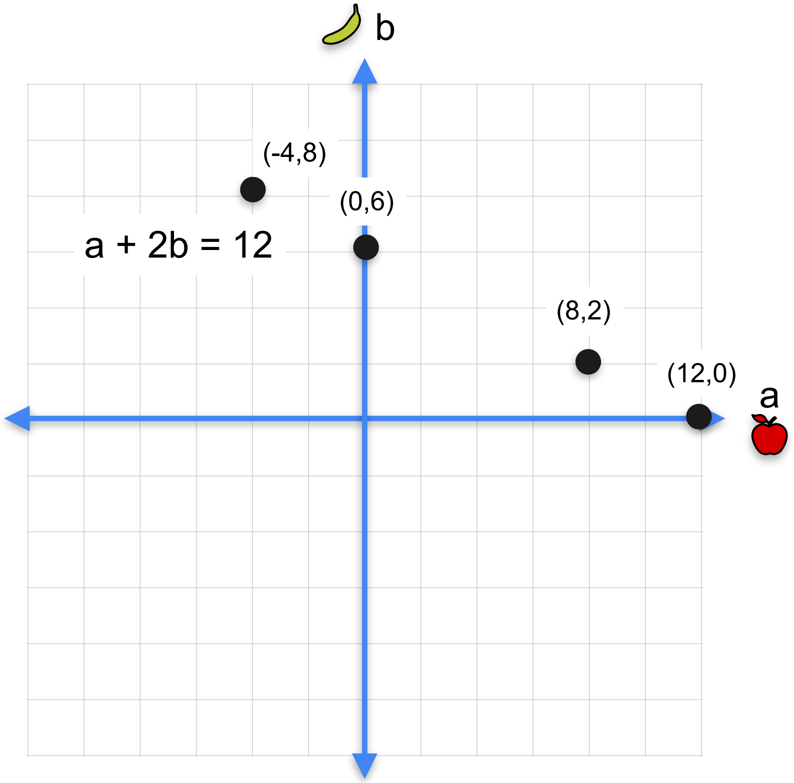
Linear equation → line





Linear equation → line





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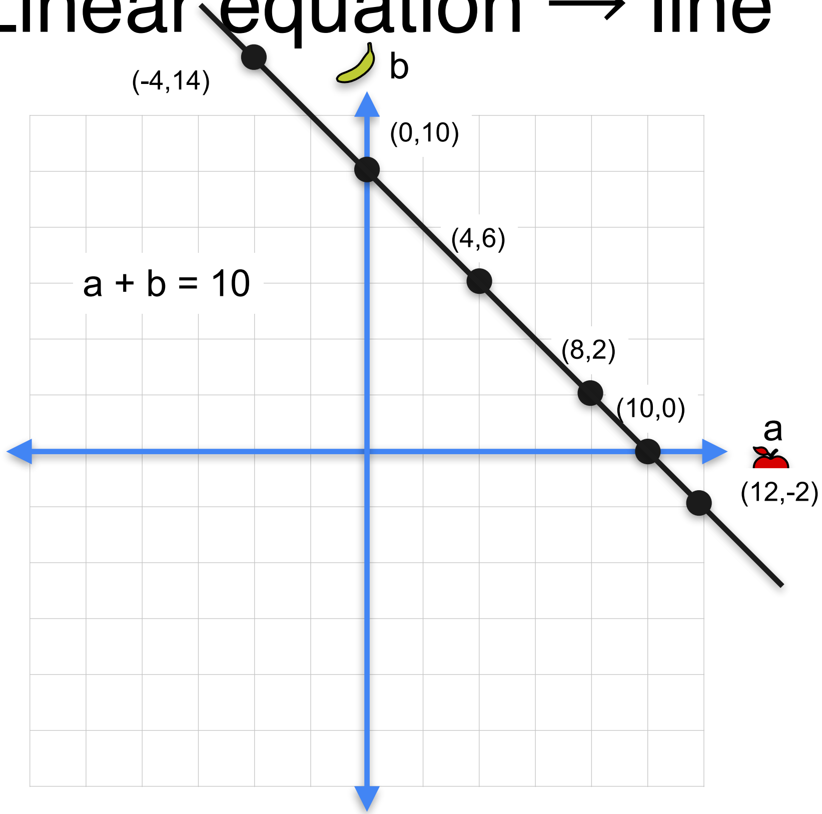
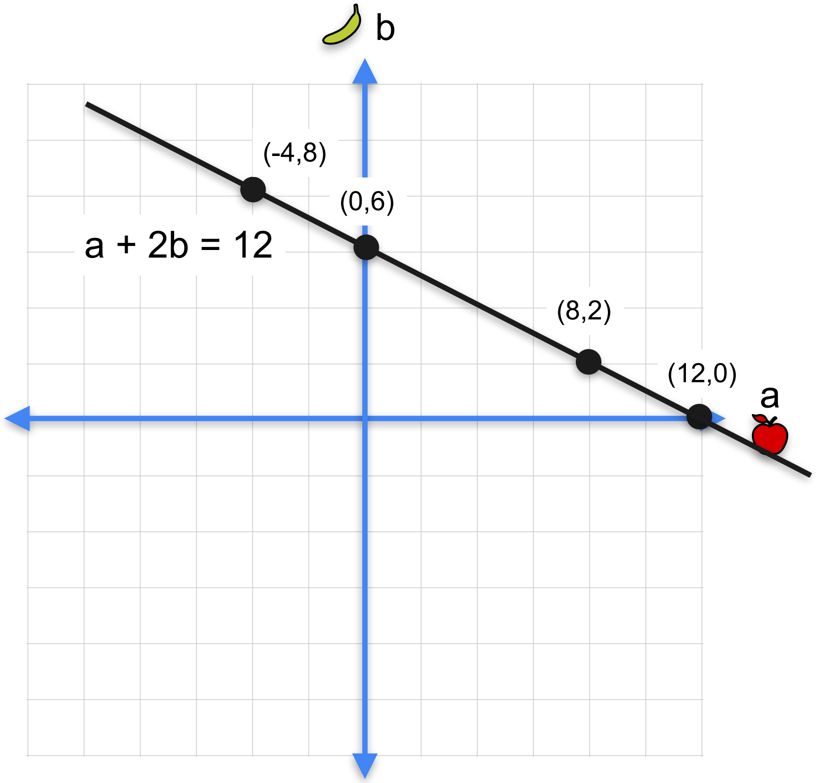
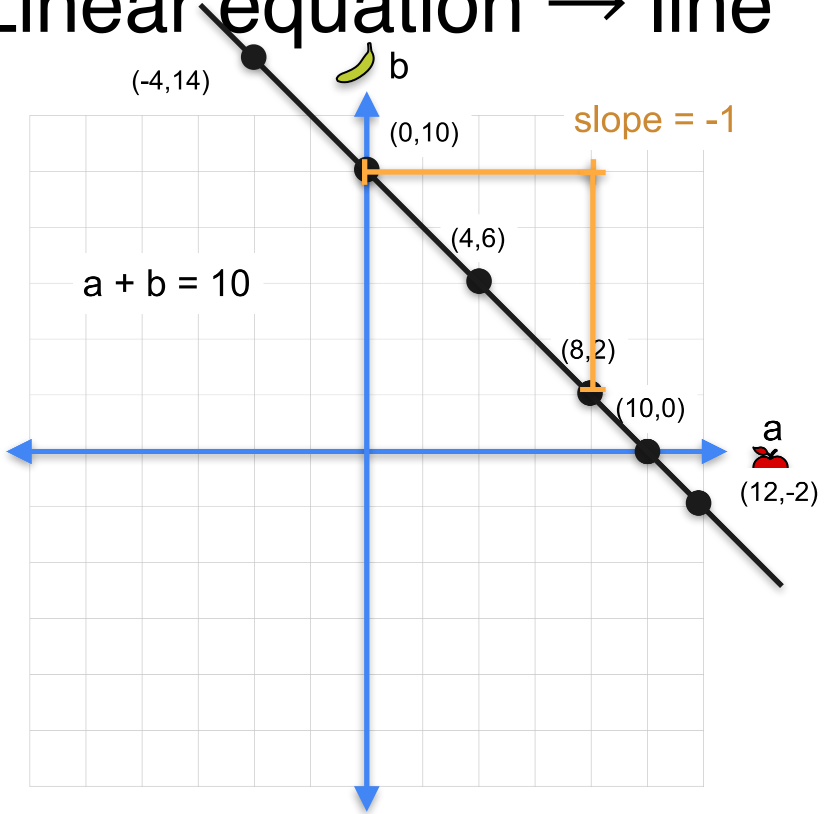
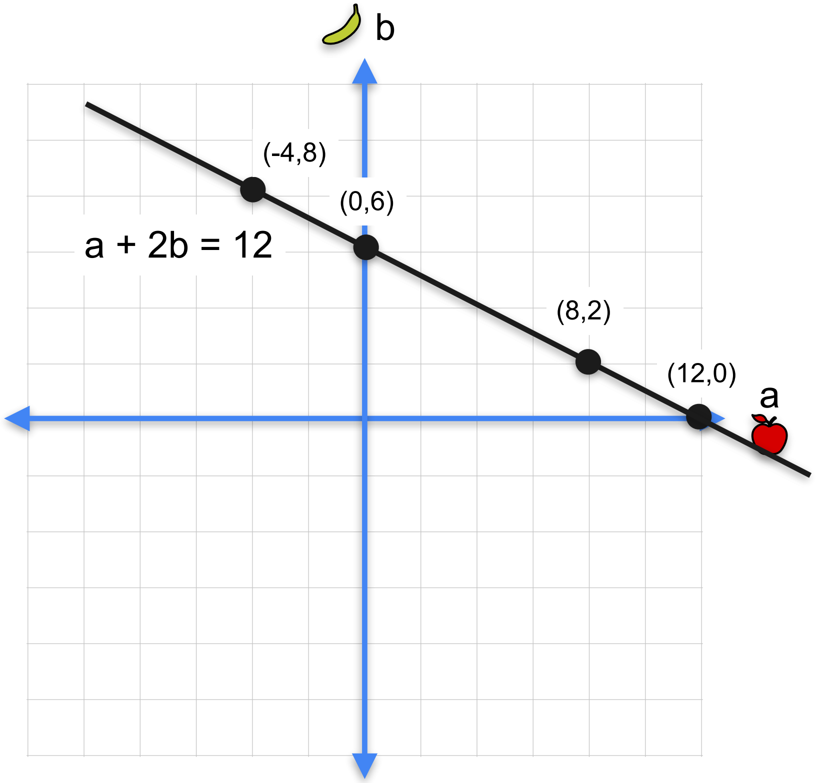


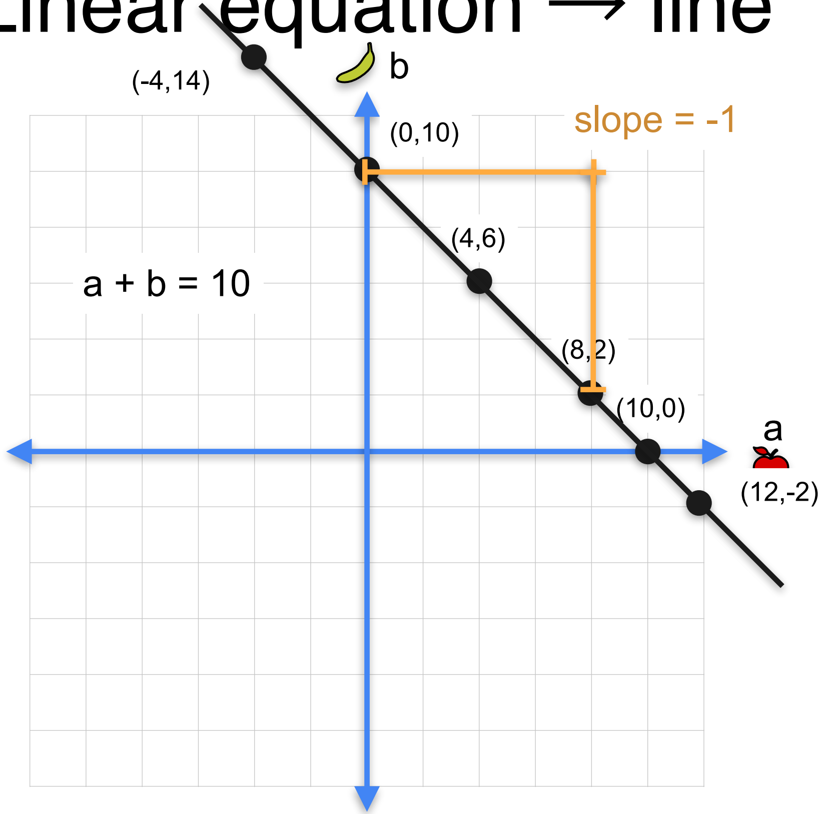


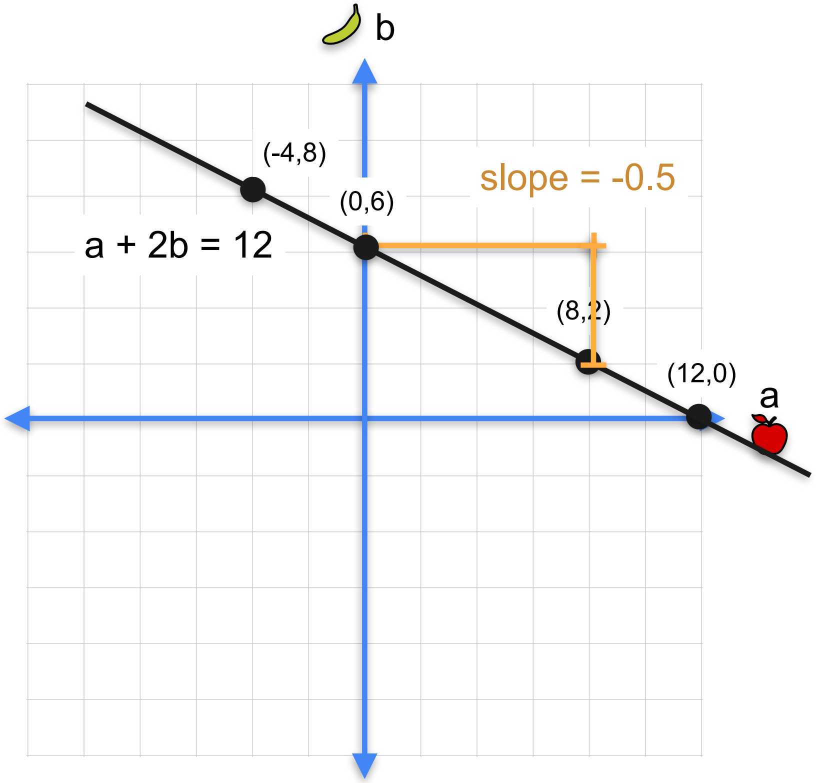
Linear equation → line



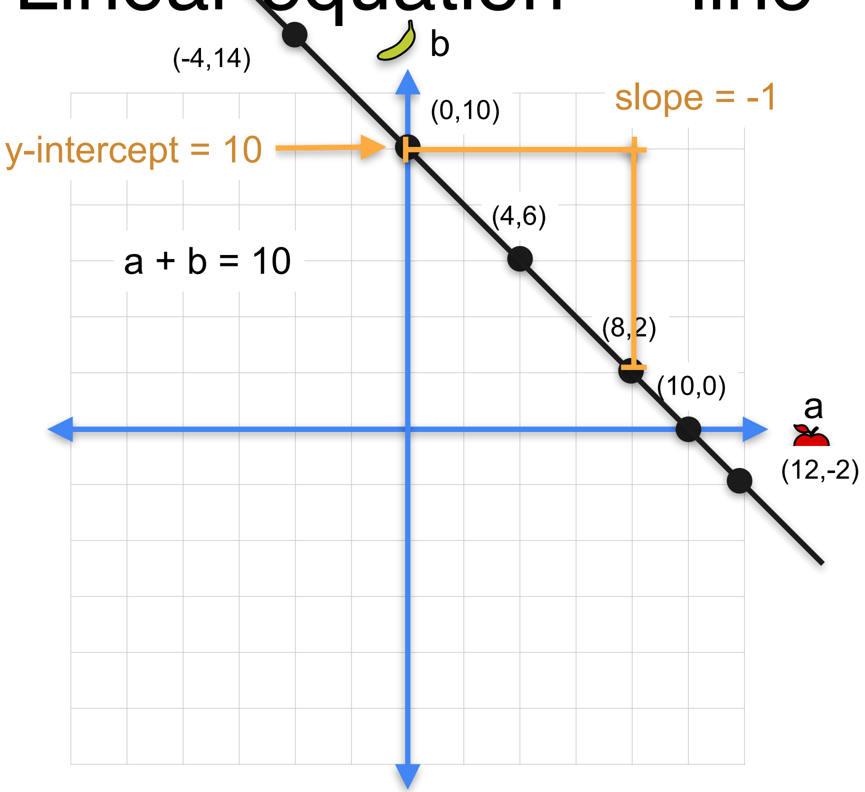


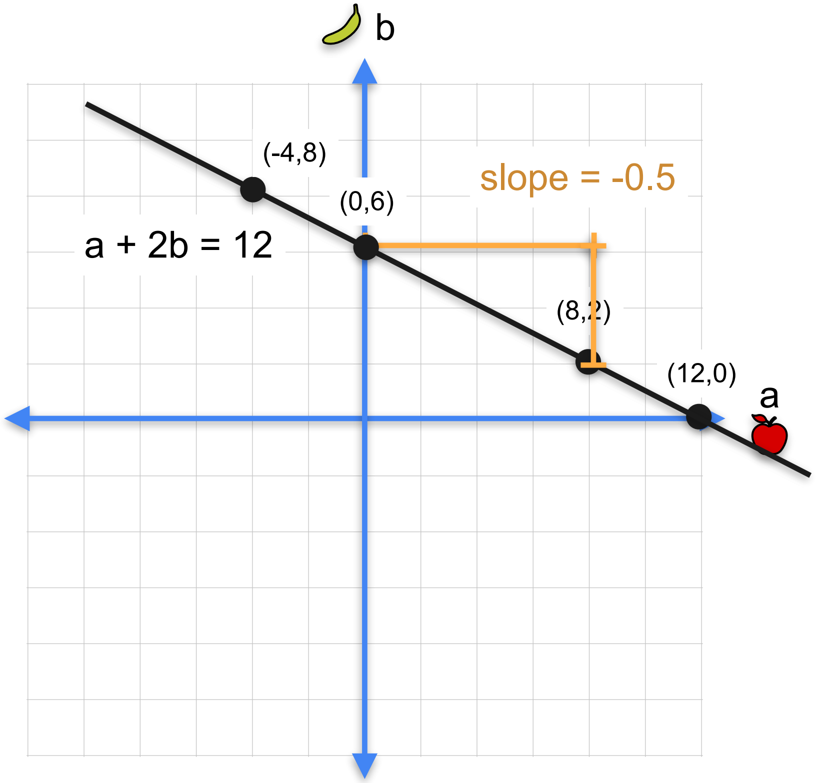
Linear equation → line



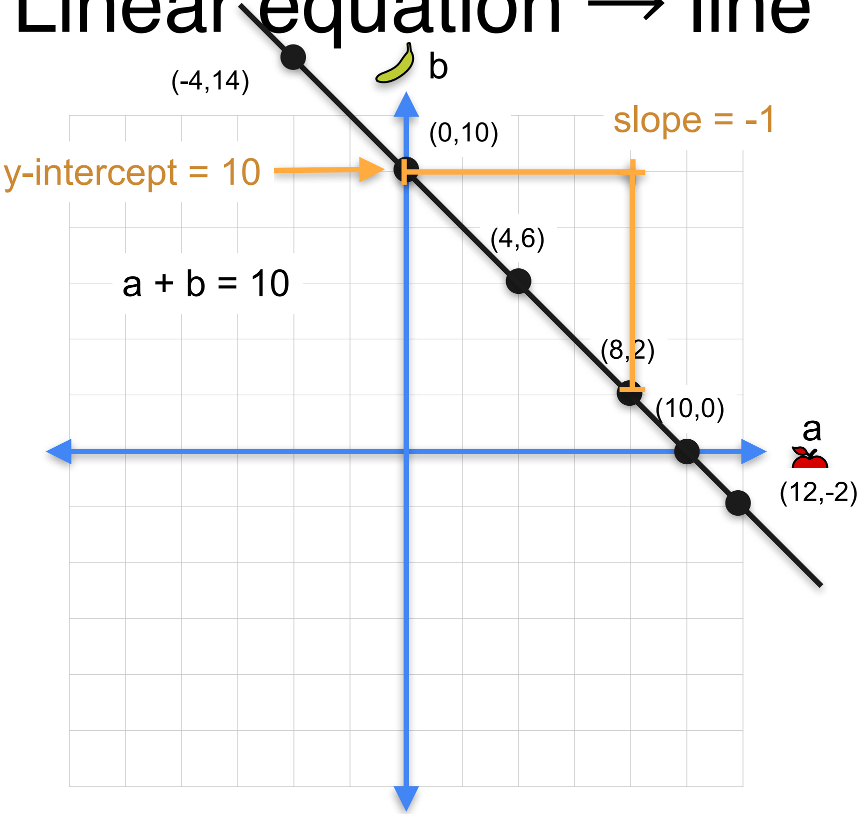


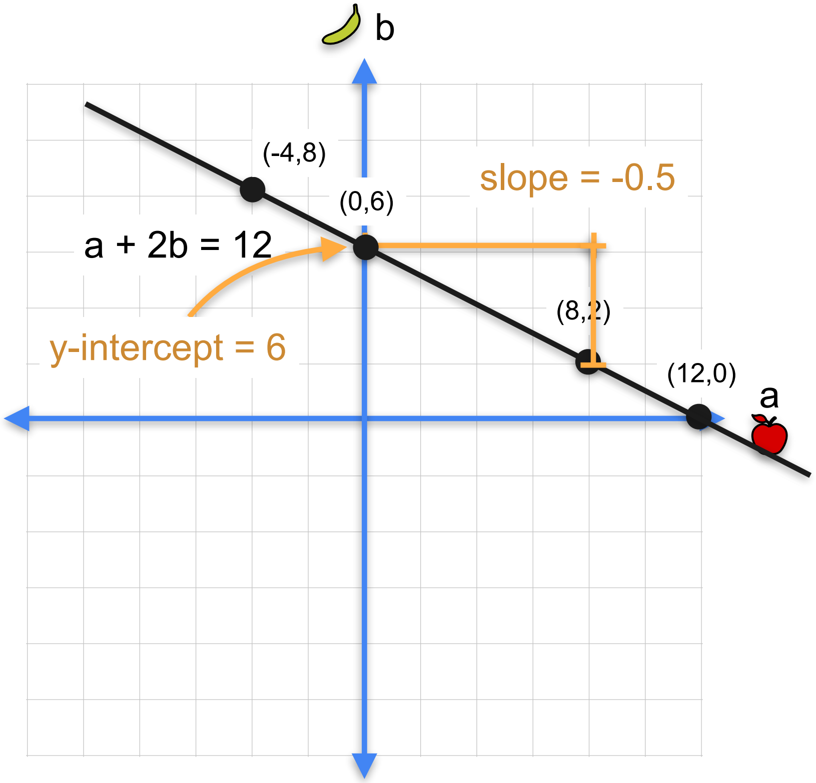
Linear equation → line



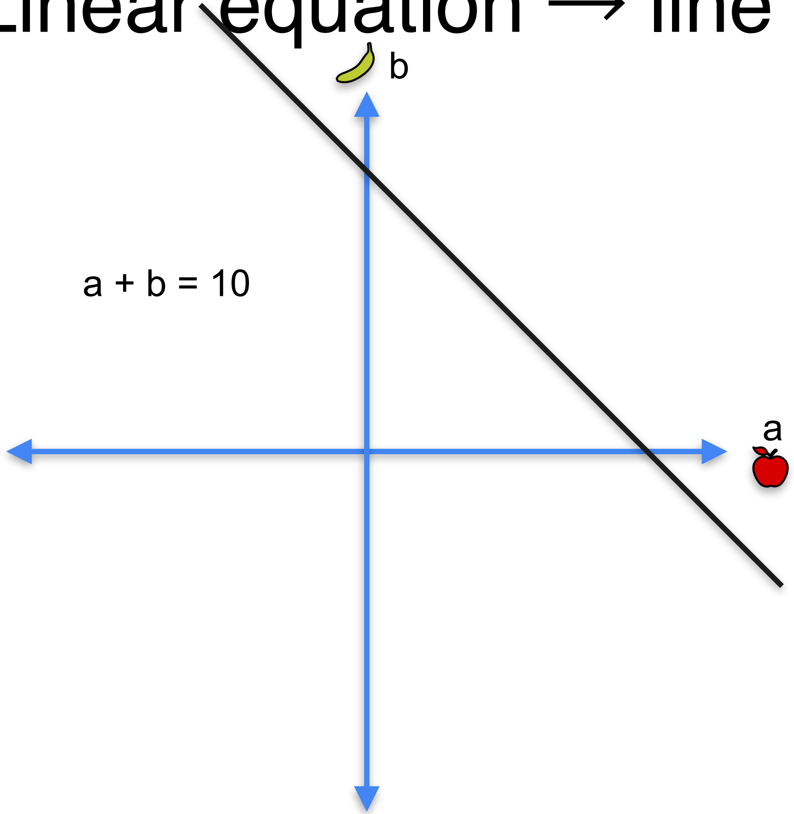


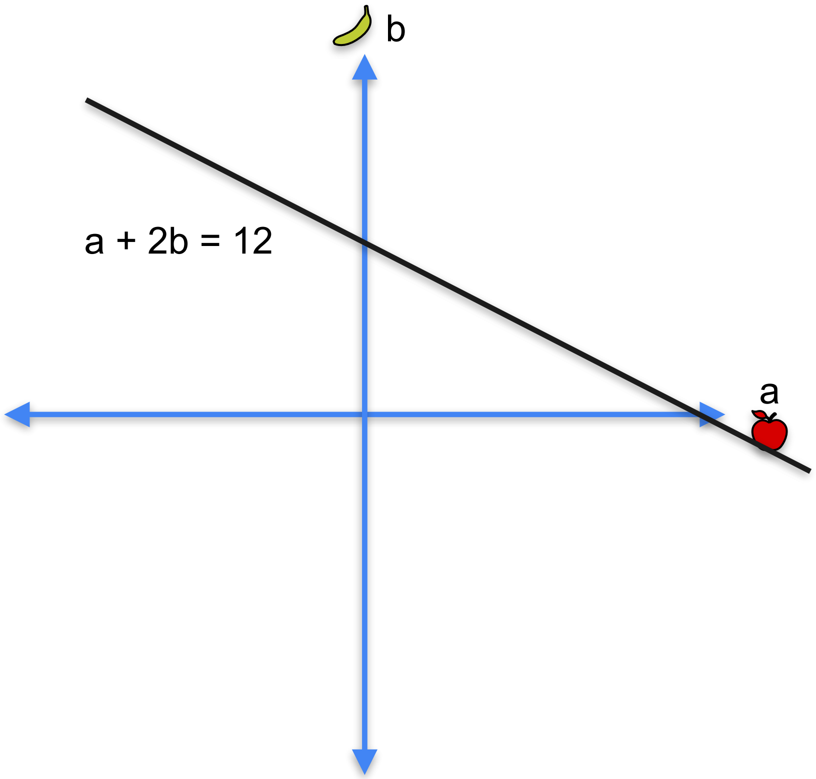
Linear equation → line





Linear equation → line

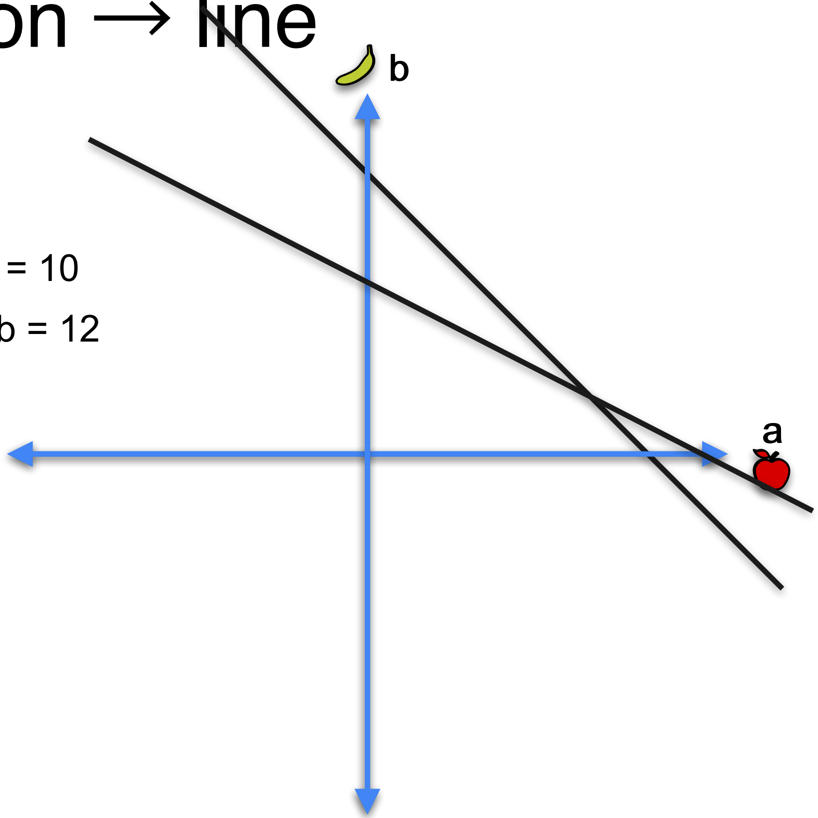




Linear equation → line

a + b = 10

a + 2b = 12



Linear equation → line

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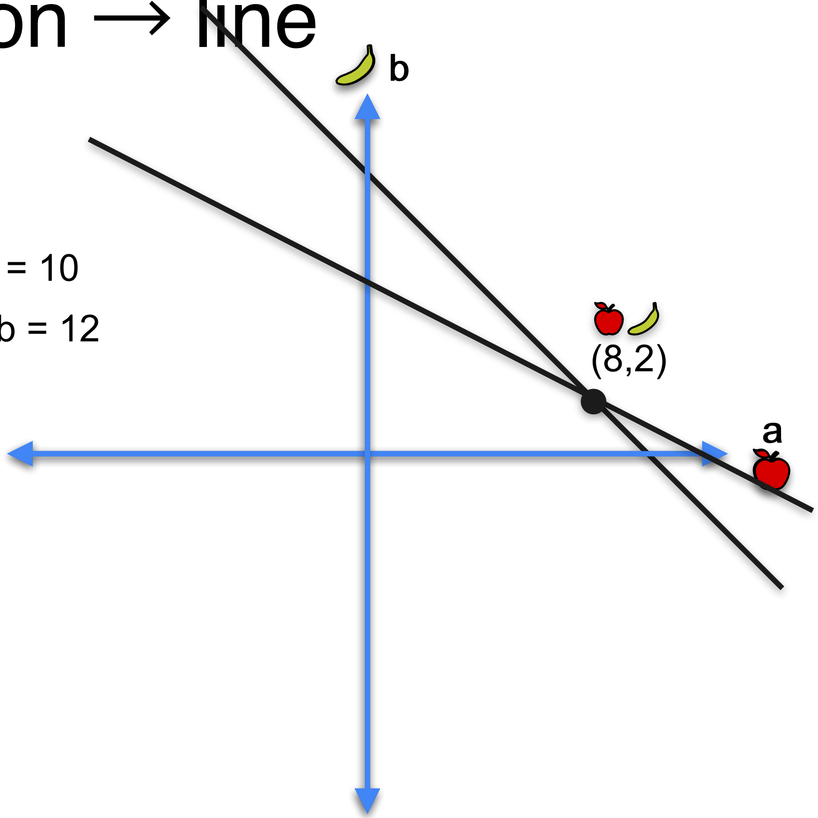
$a + 2b = 12$



Linear equation → line

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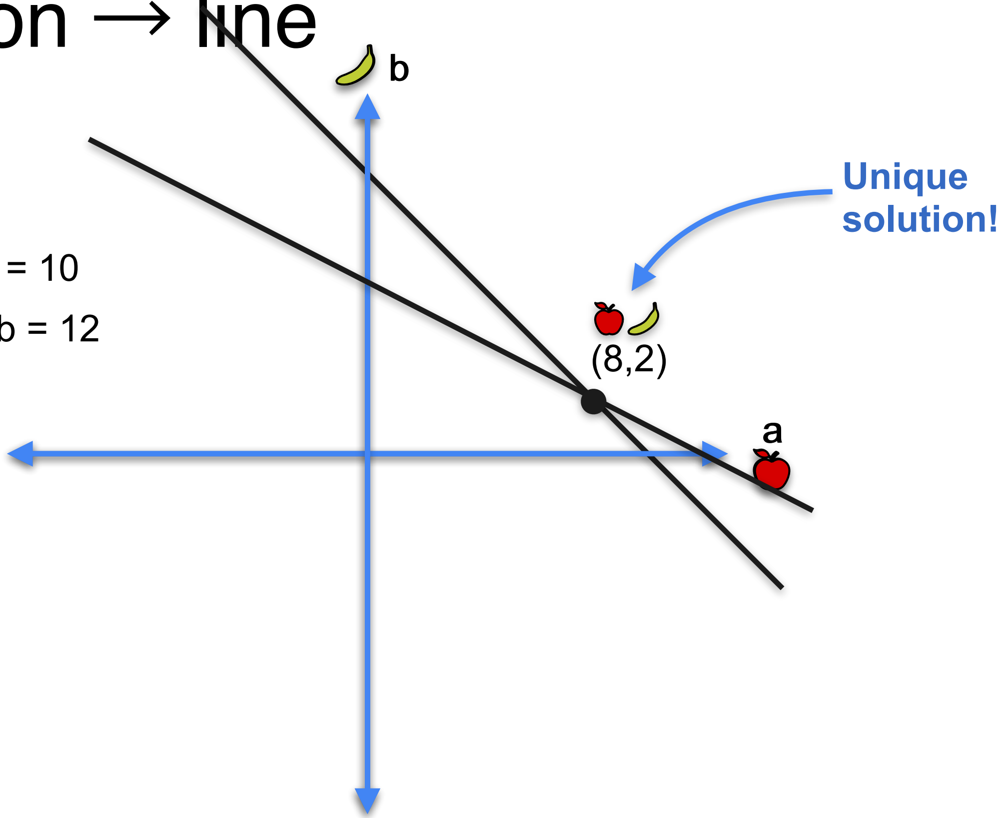
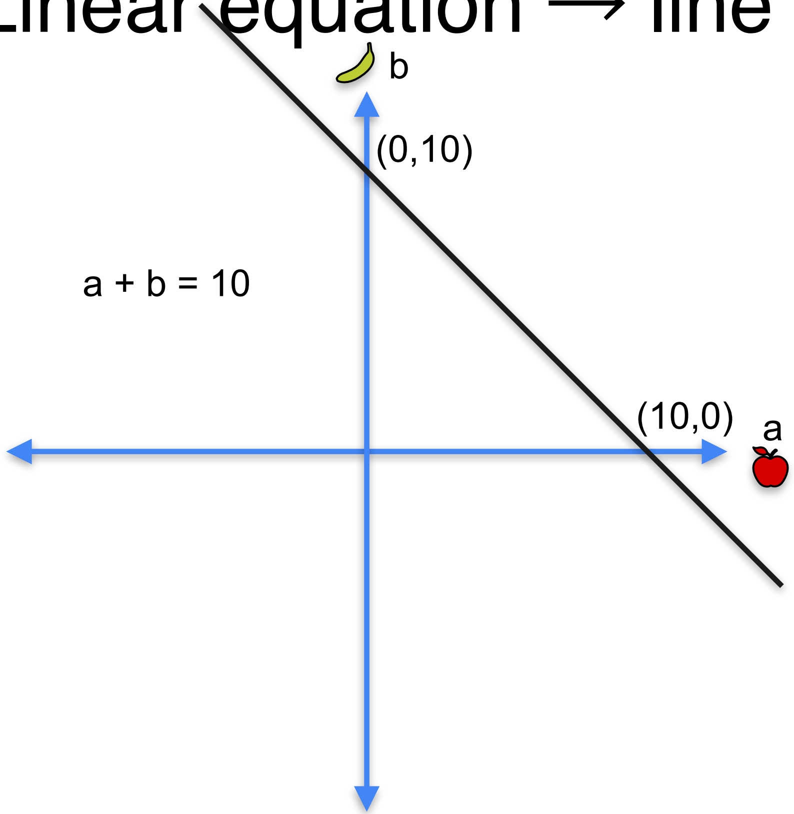
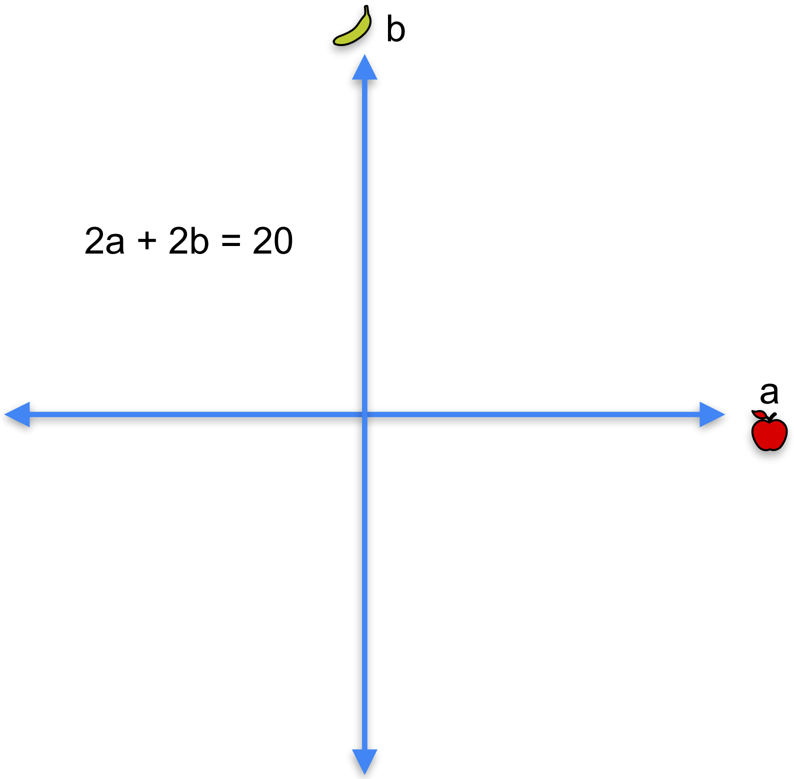
a + 2b = 12

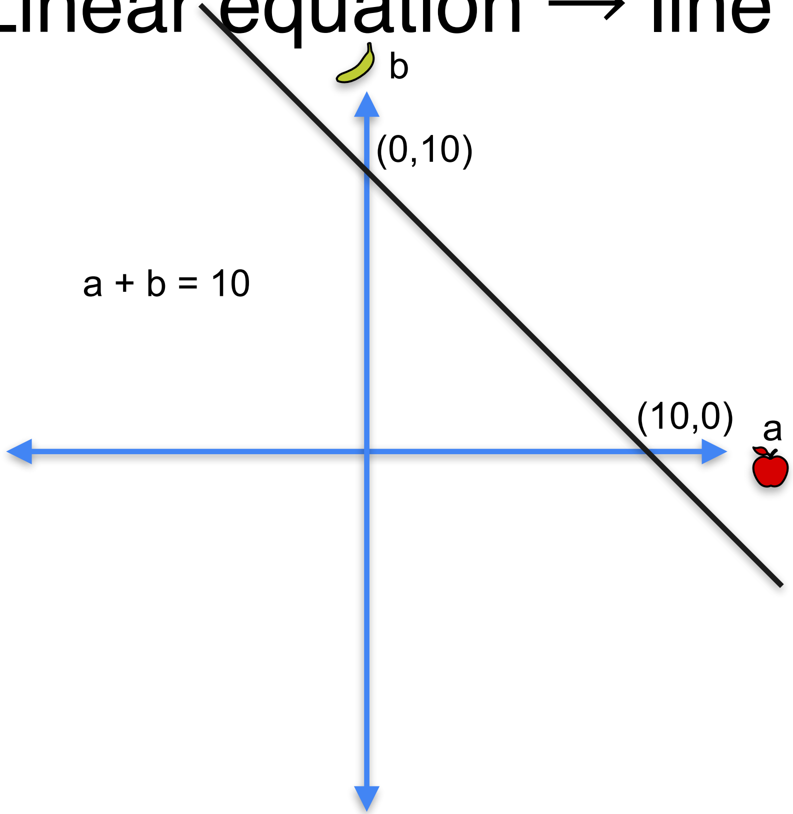


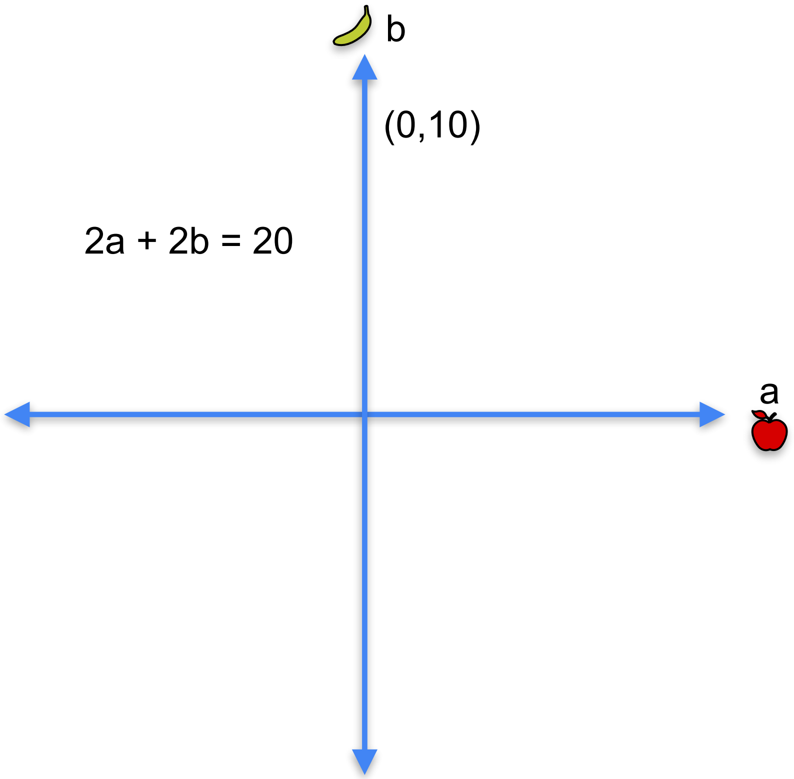
Linear equation → line



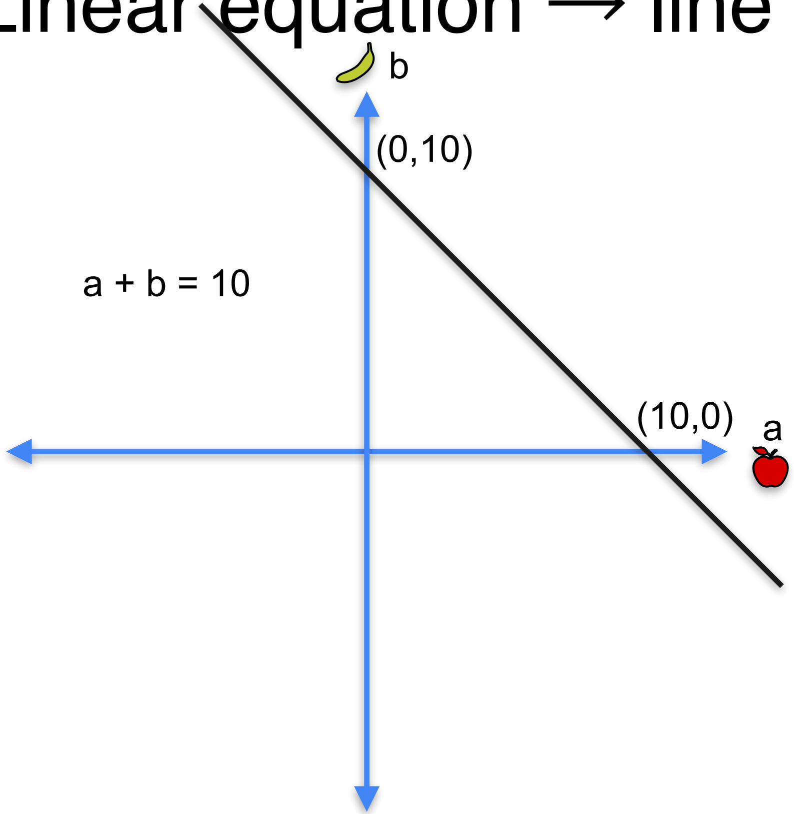


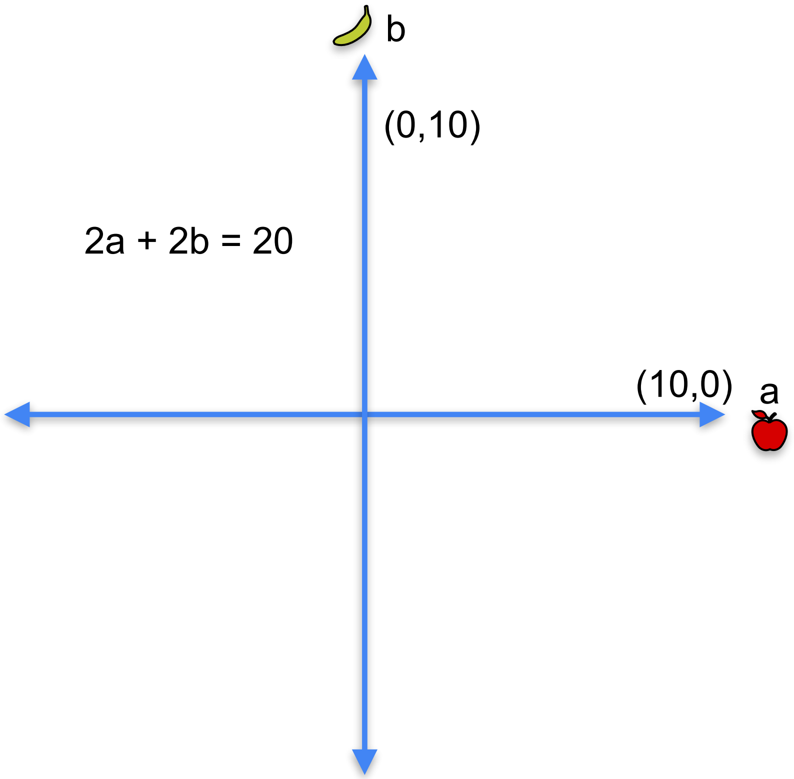
Linear equation → line



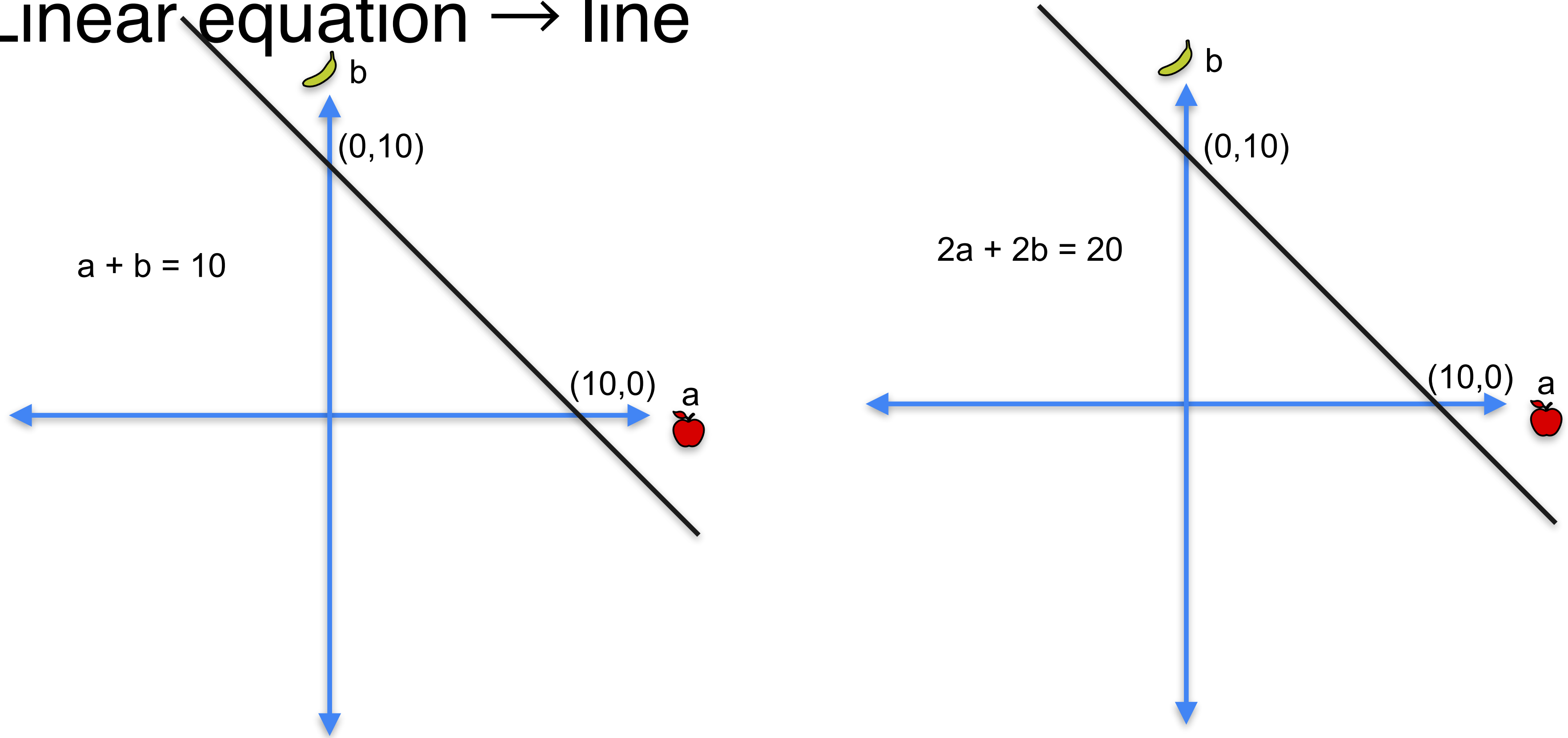


Linear equation → line

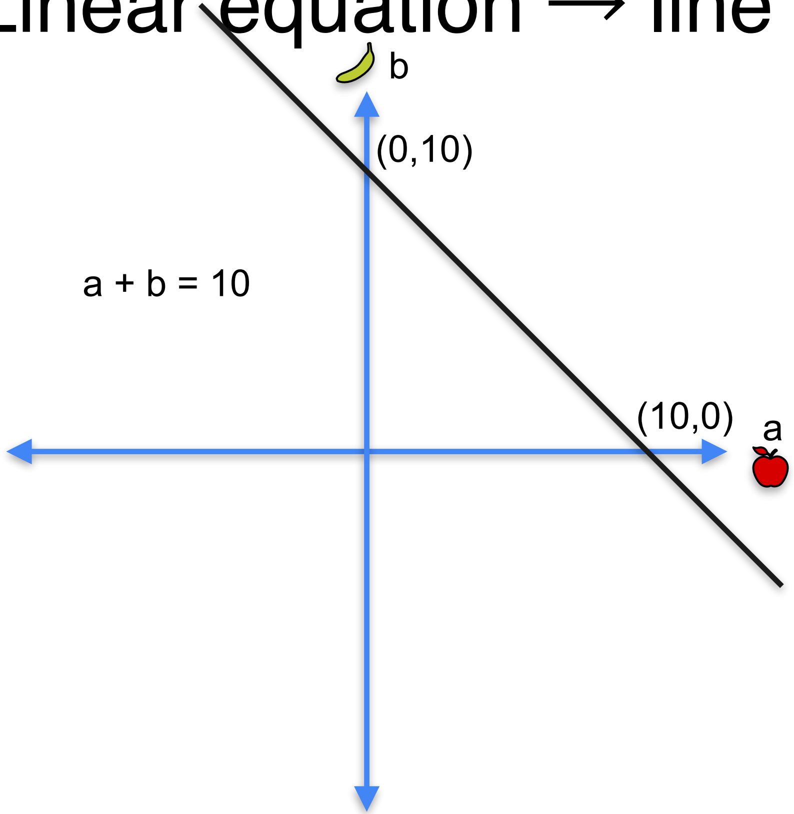


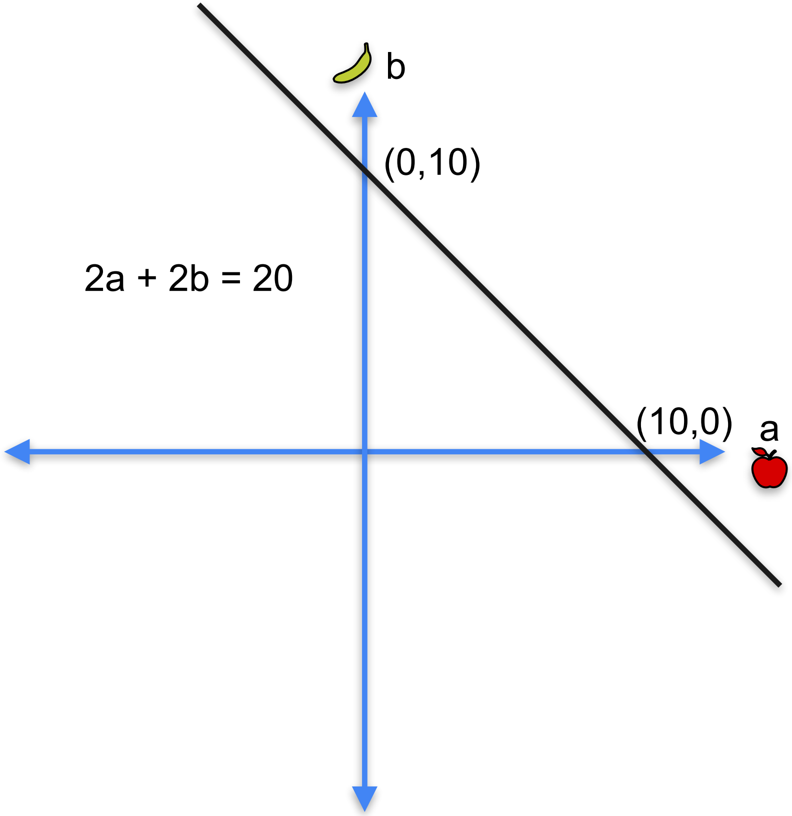


Linear equation → line



Linear equation → line

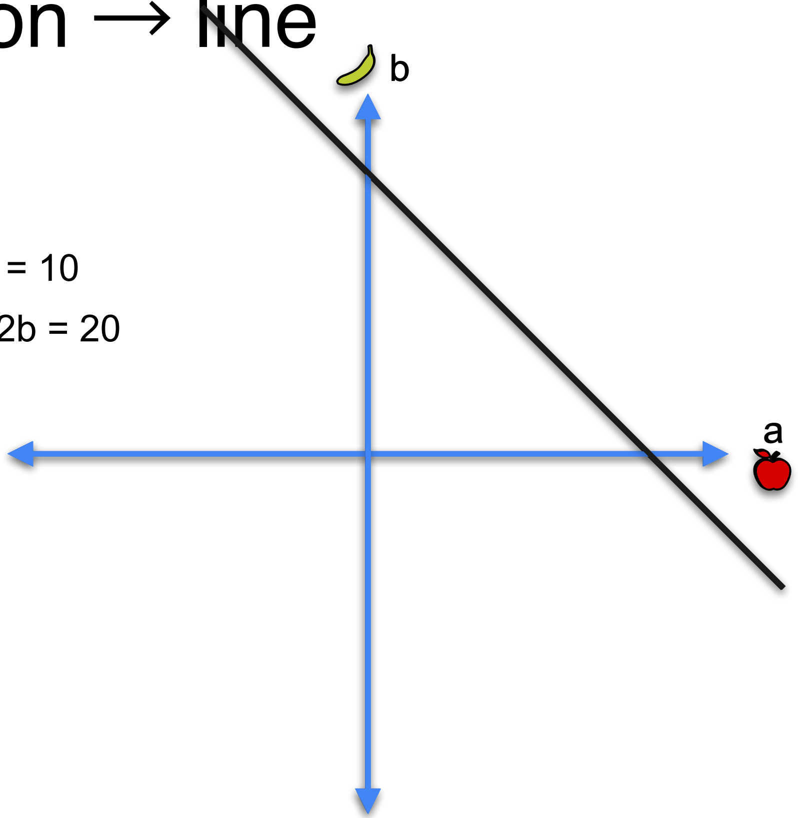




Linear equation → line

$a + b = 10$

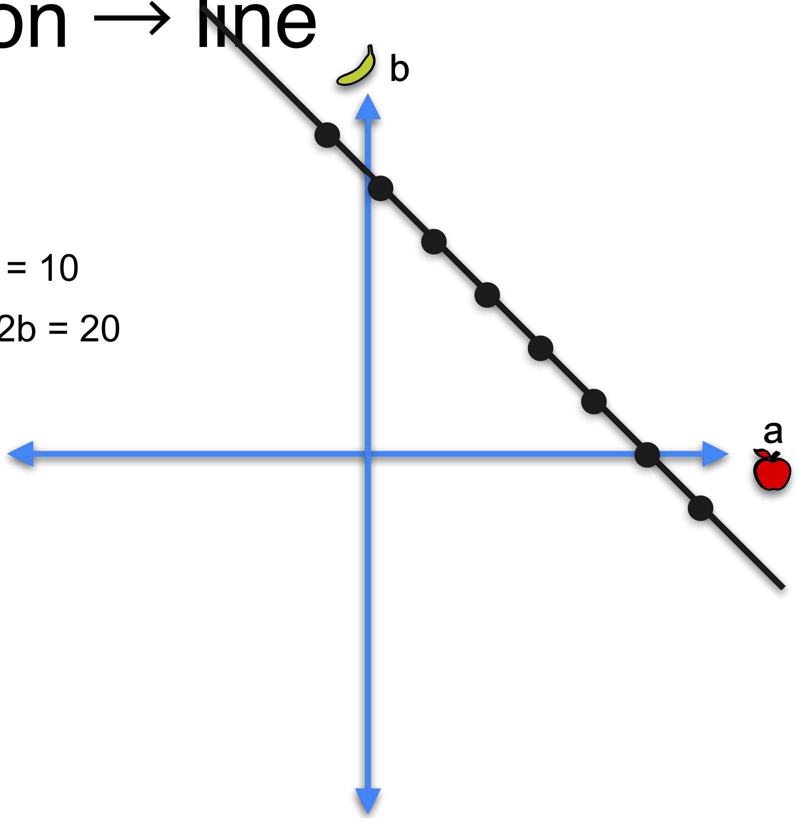
$2a + 2b = 20$



Linear equation → line

a + b = 10

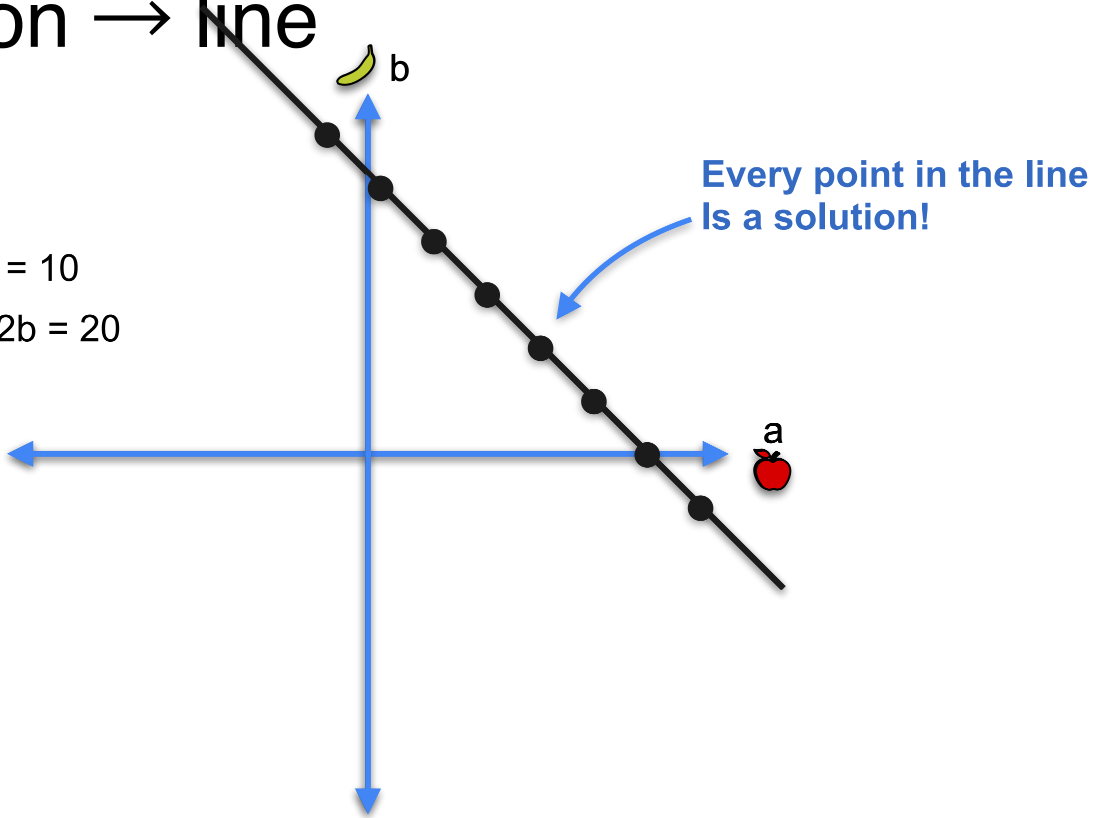
2a + 2b = 20



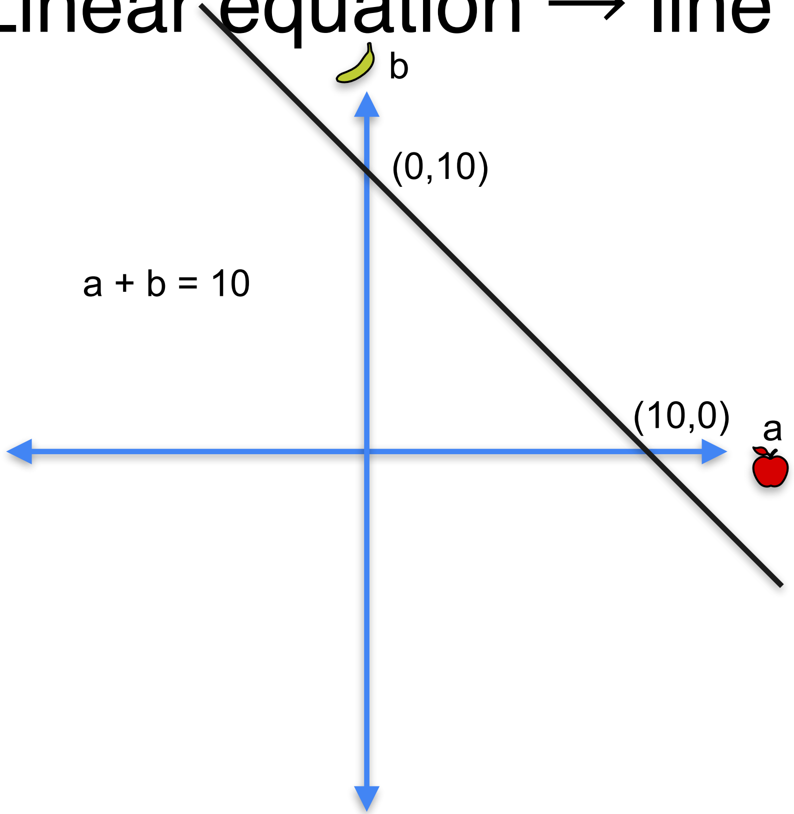
Linear equation → line

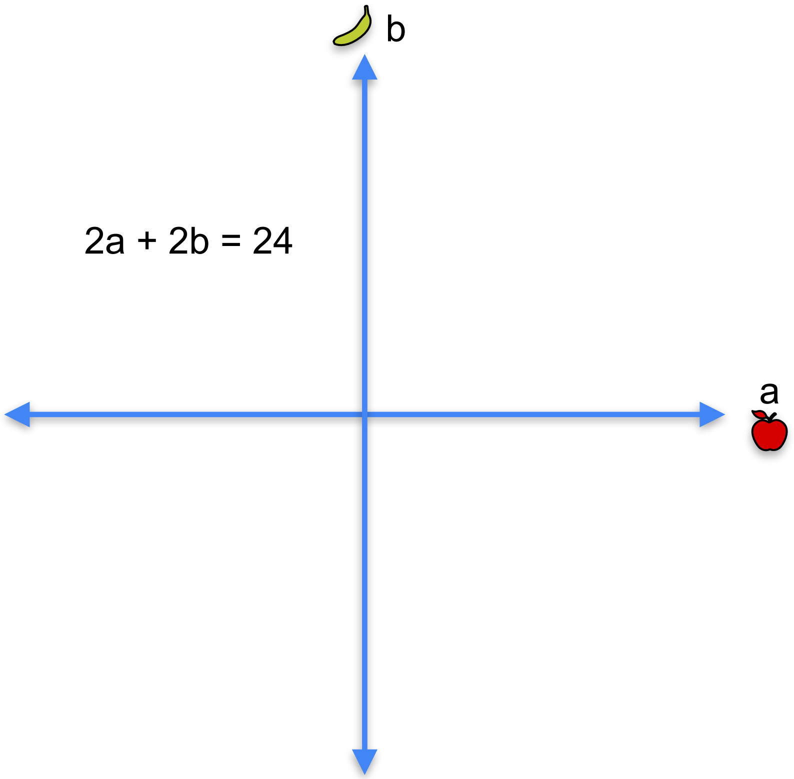
$a + b = 10$

$2a + 2b = 20$

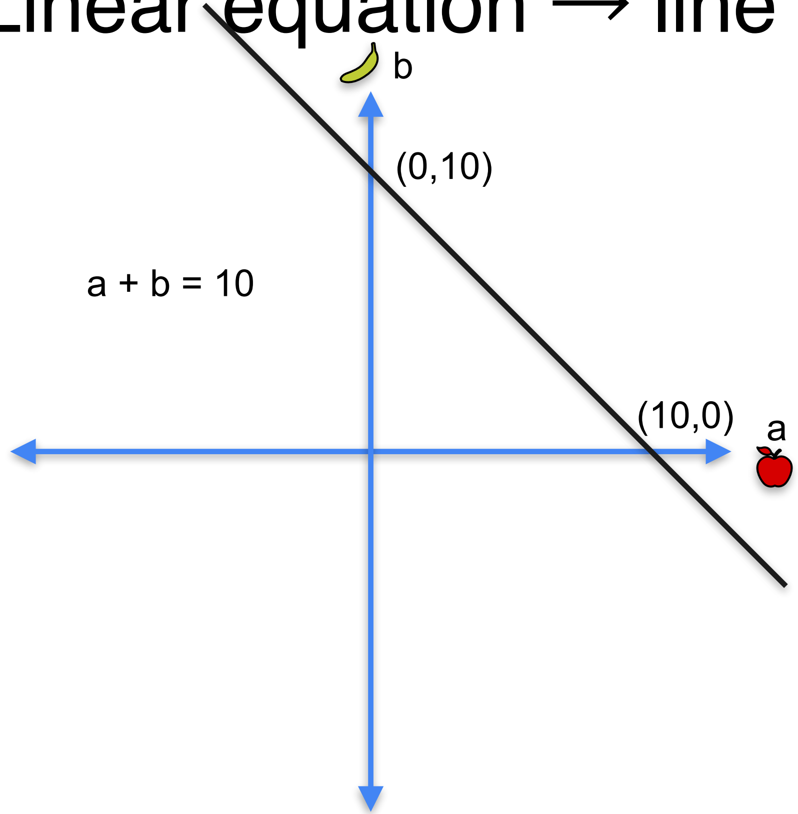


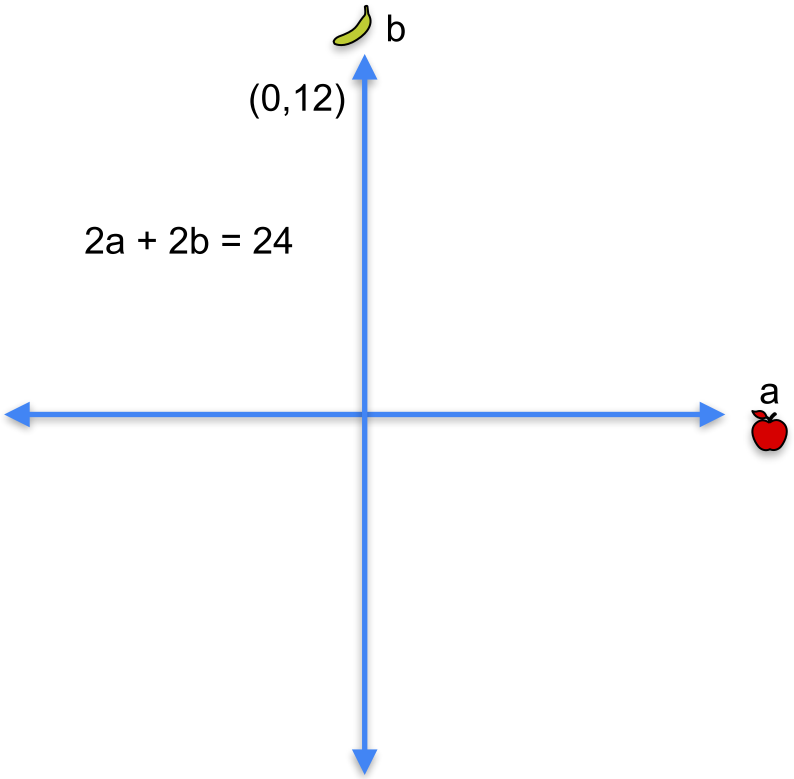
Linear equation → line



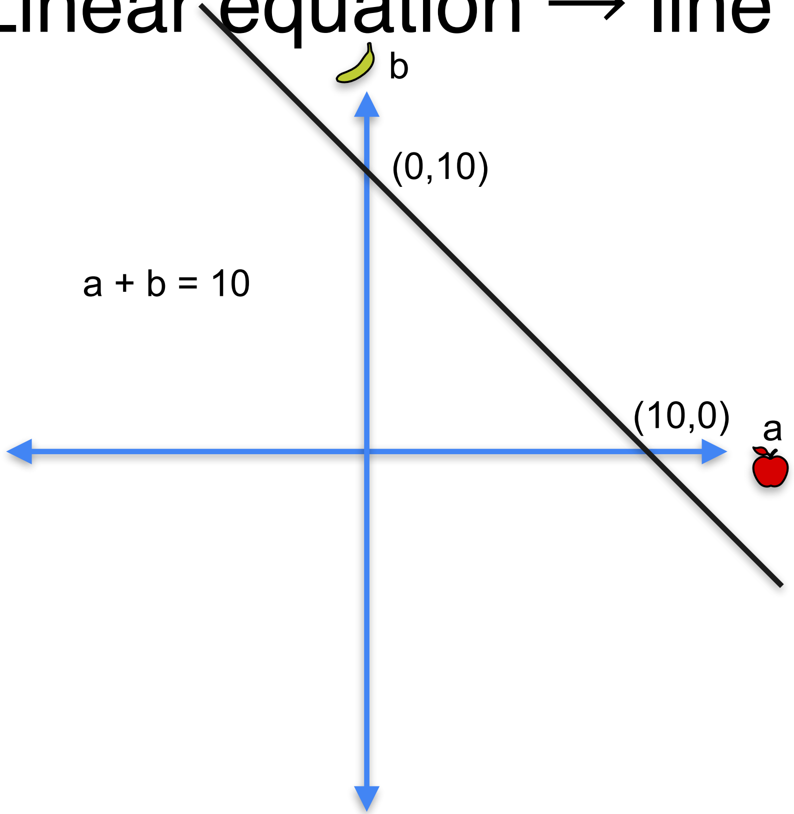


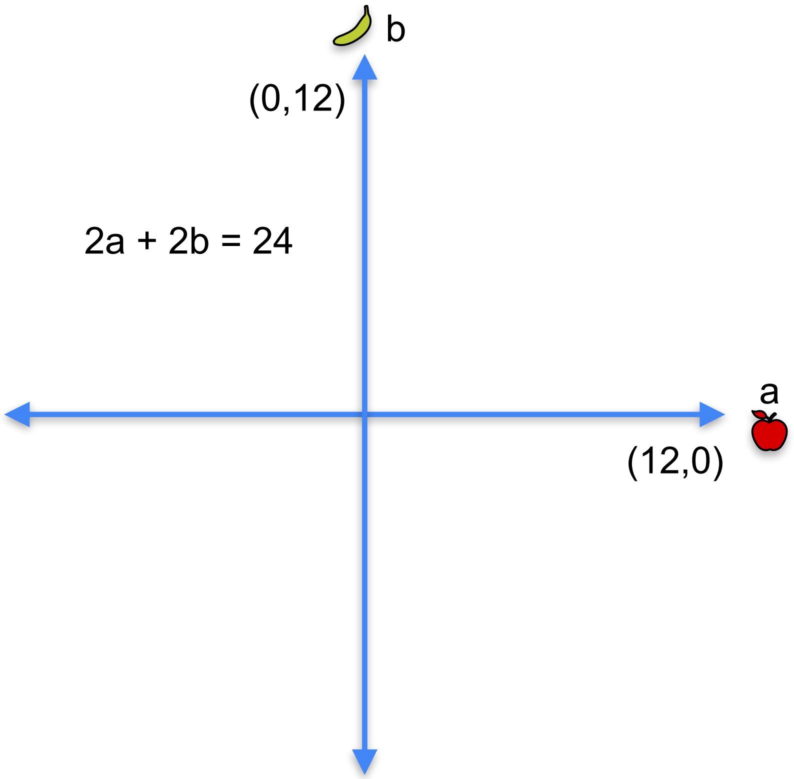
Linear equation → line



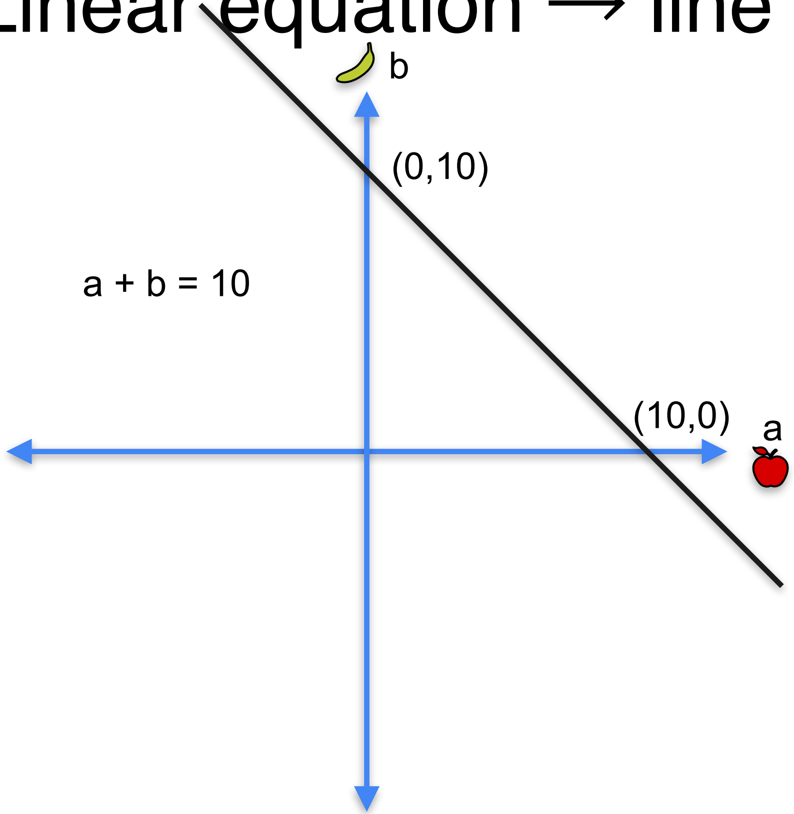


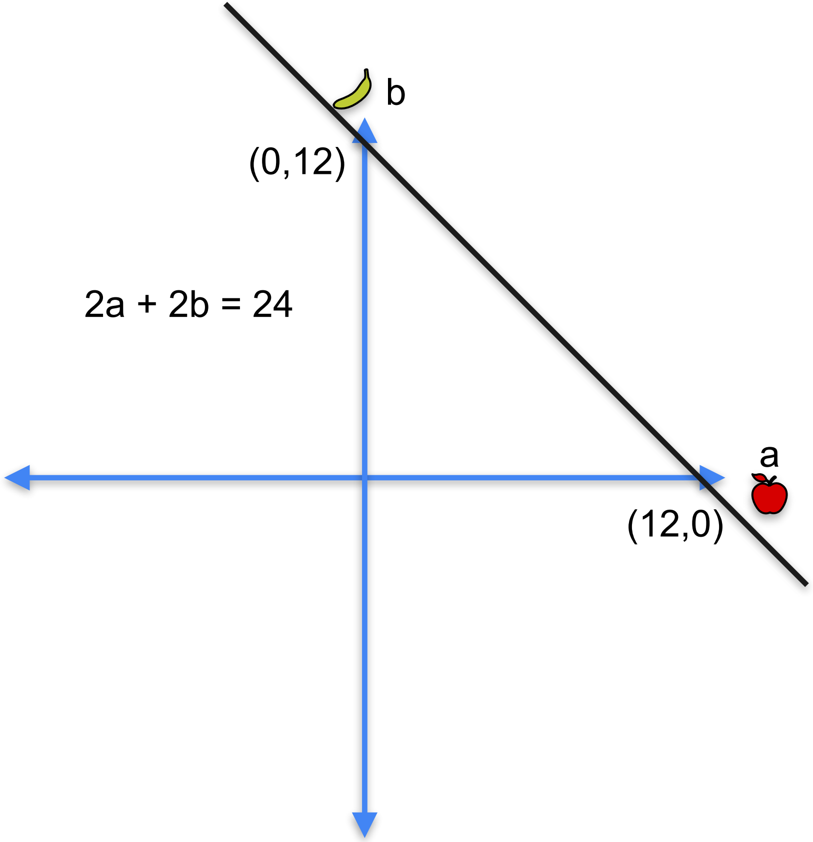
Linear equation → line



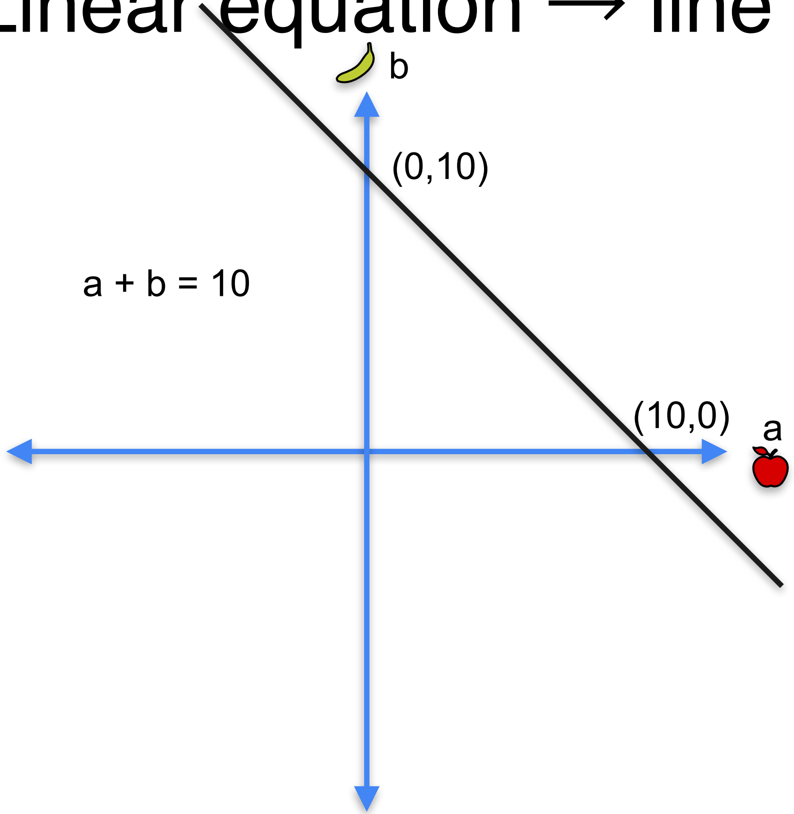


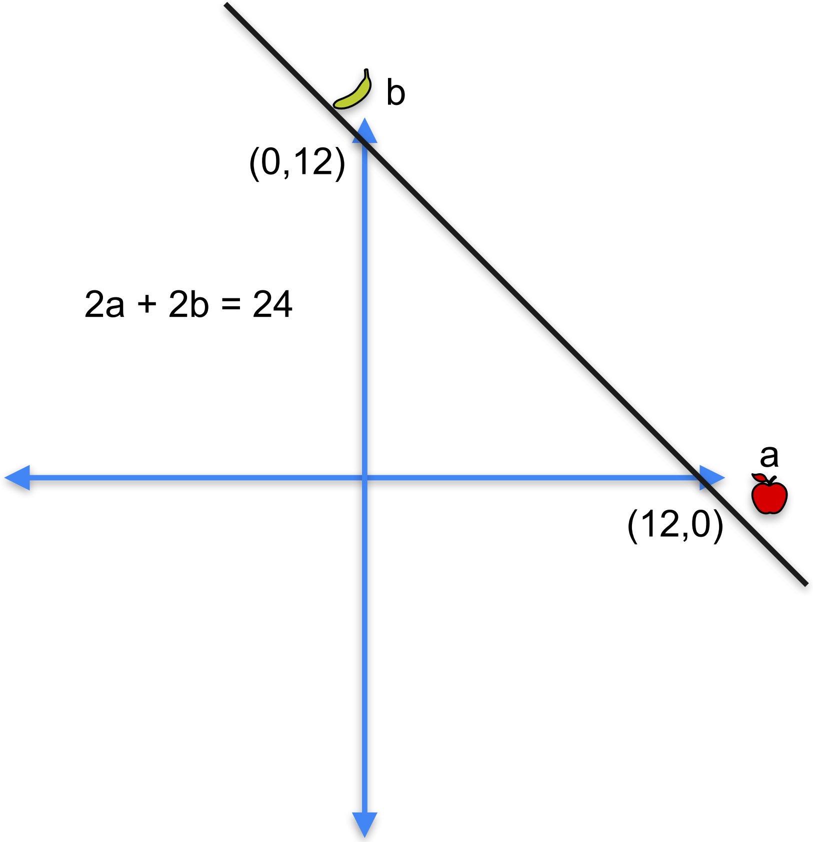
Linear equation → line





Linear equation → line

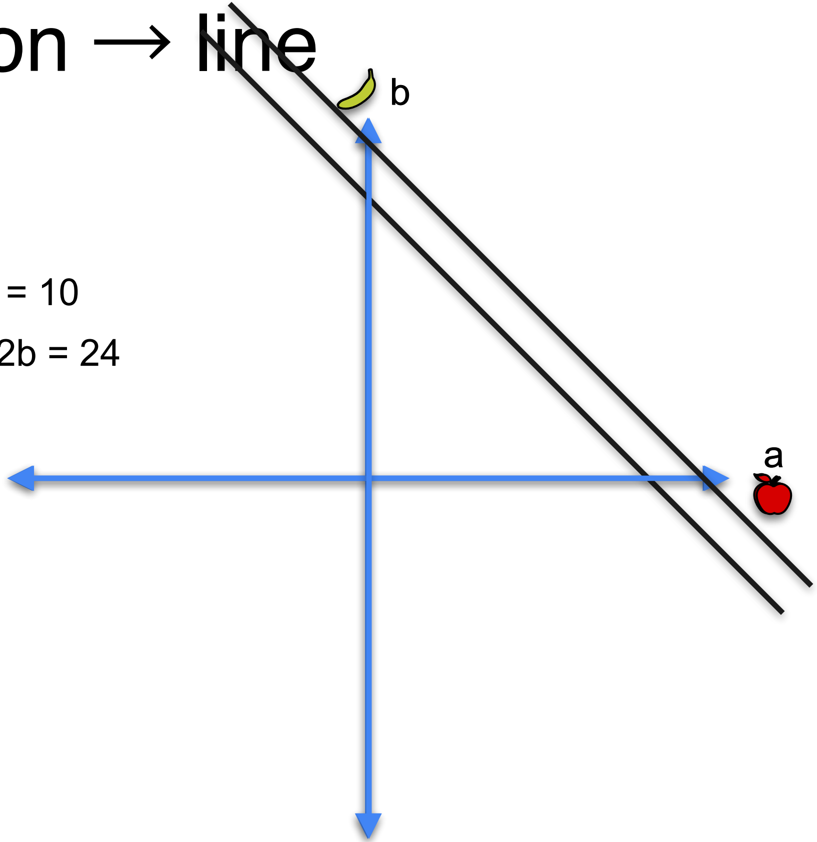




Linear equation → line

a + b = 10

2a + 2b = 24



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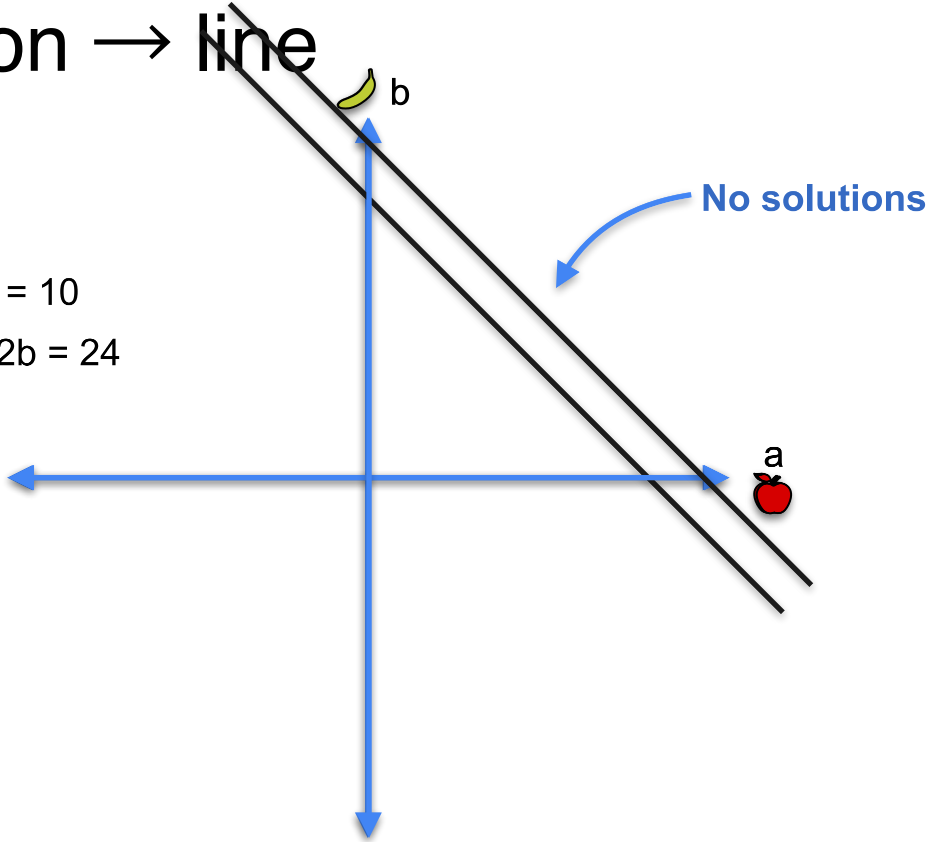

Systems of equations as lines

Systems of equations as lines

System 1

- $a + b = 10$
- $a + 2b = 12$

Systems of equations as lines

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- $a + b = 10$
- $a + 2b = 12$

System 2

- $a + b = 10$
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Systems of equations as lines

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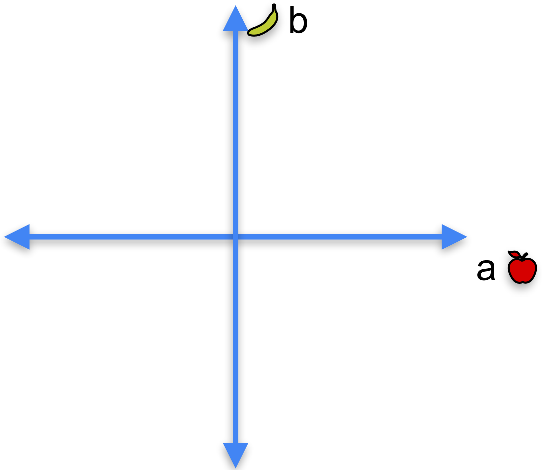

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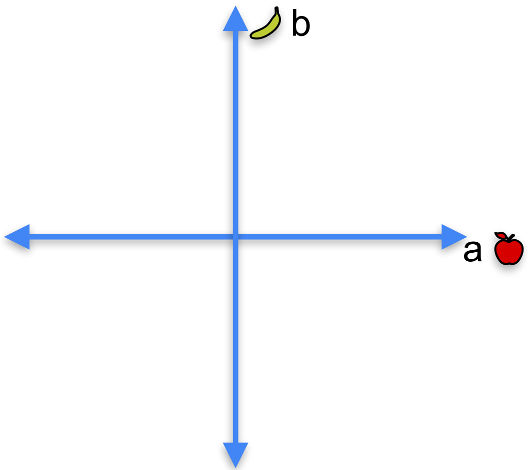

System 3

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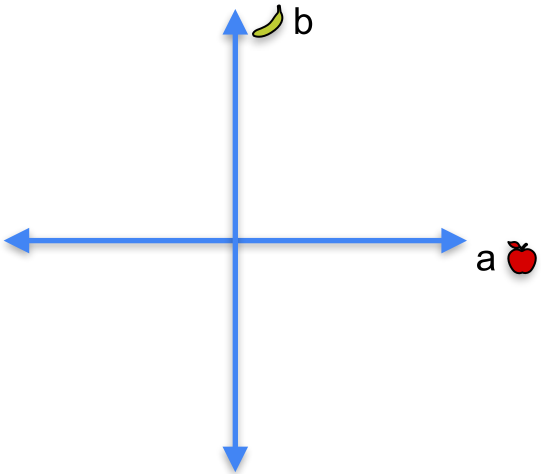

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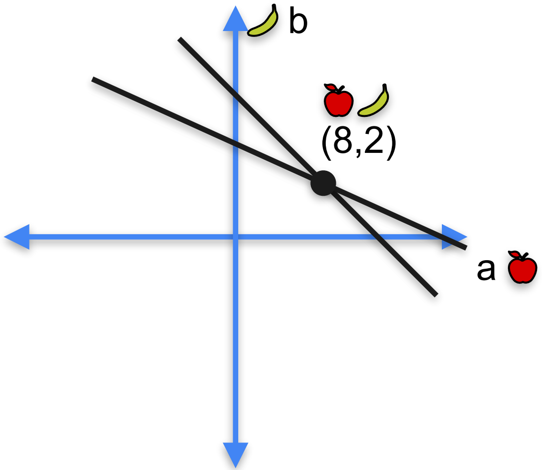

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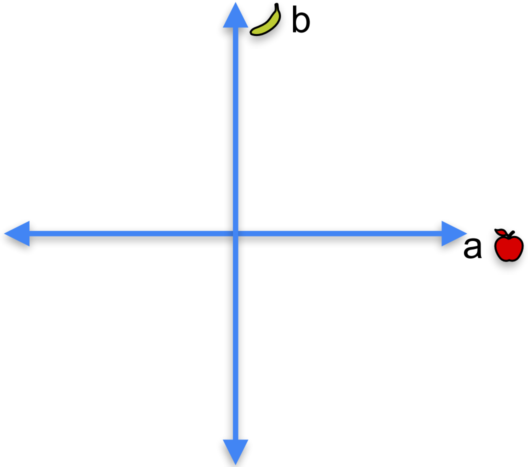

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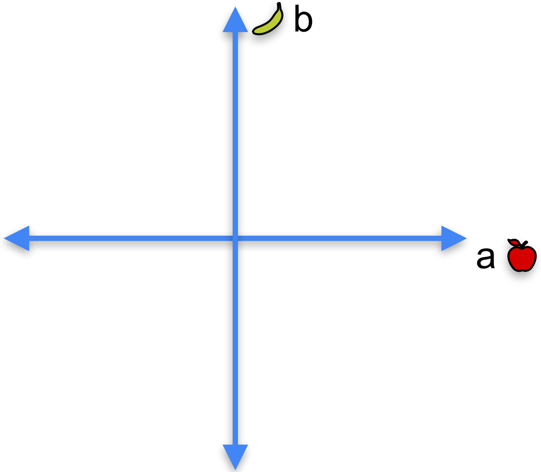

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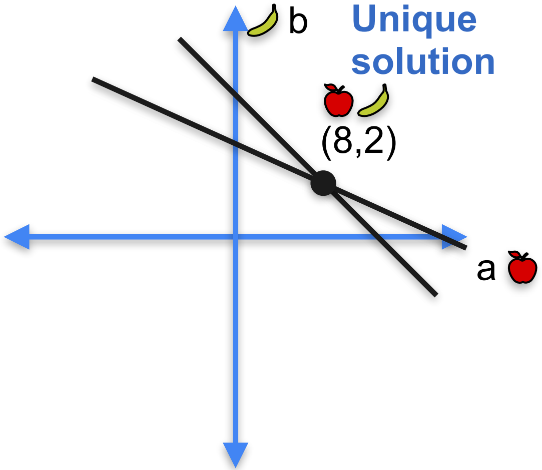

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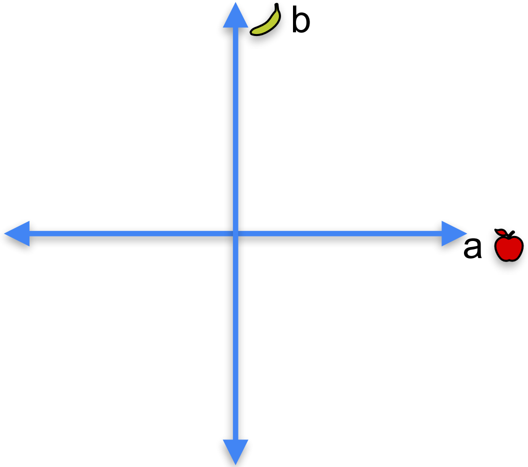

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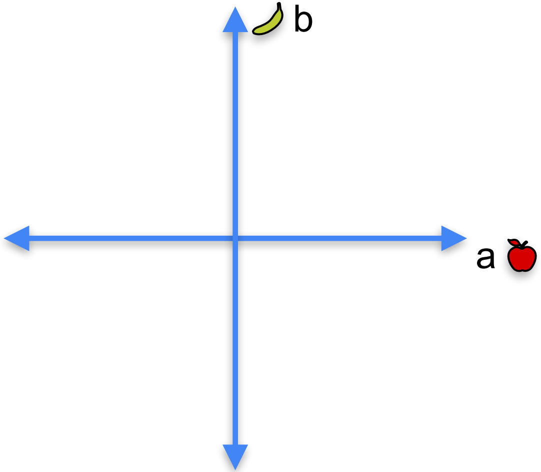

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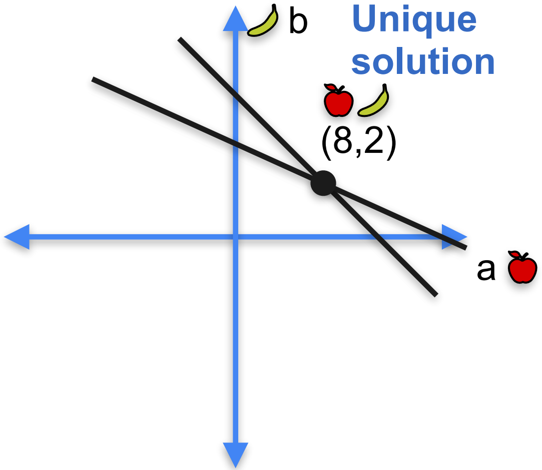

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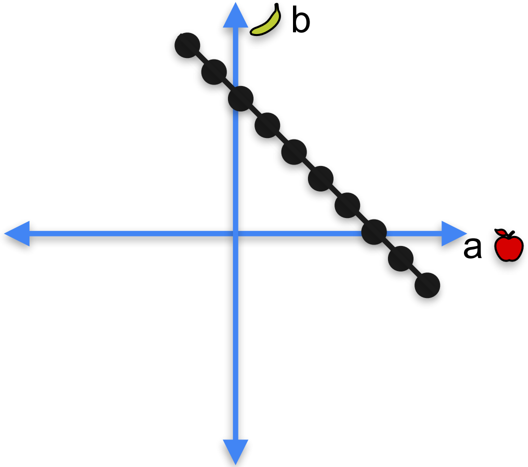

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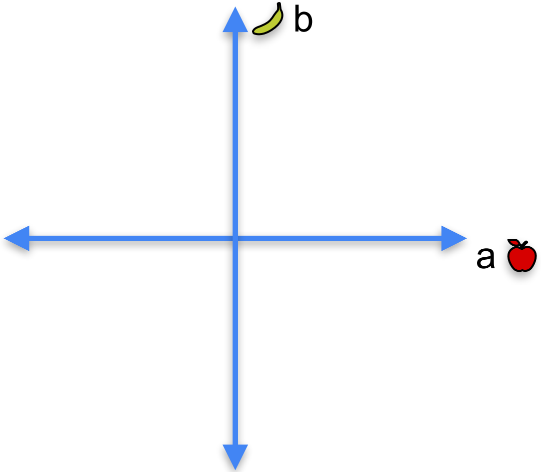

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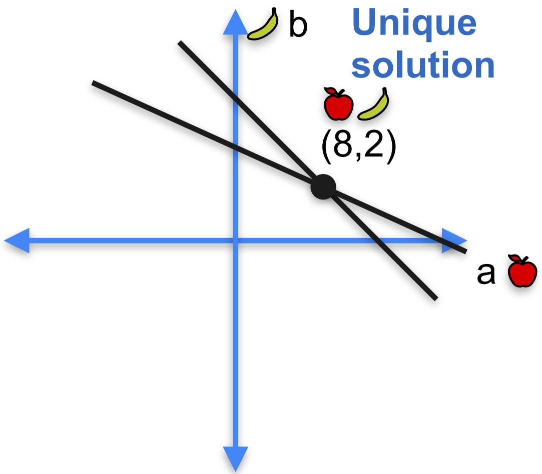

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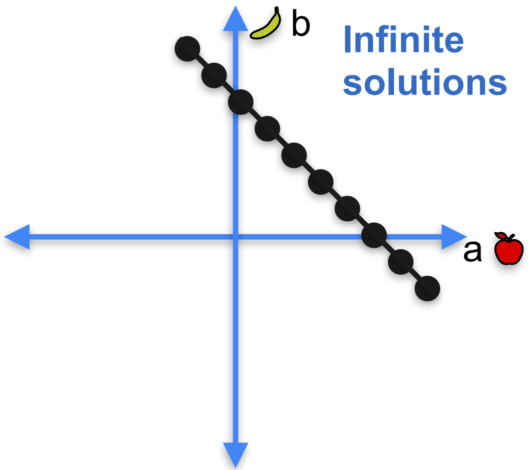

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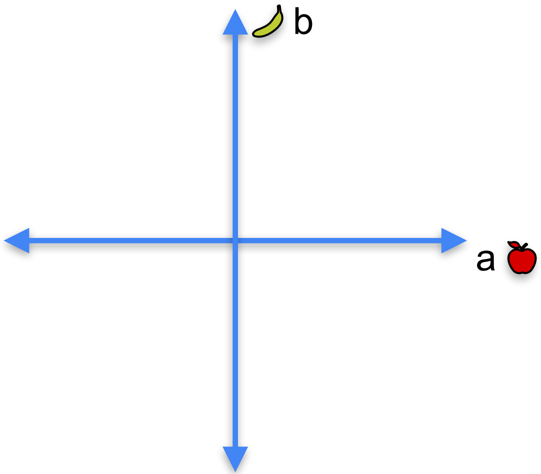

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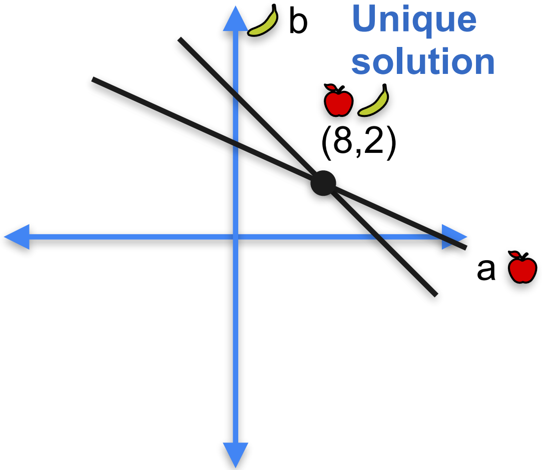

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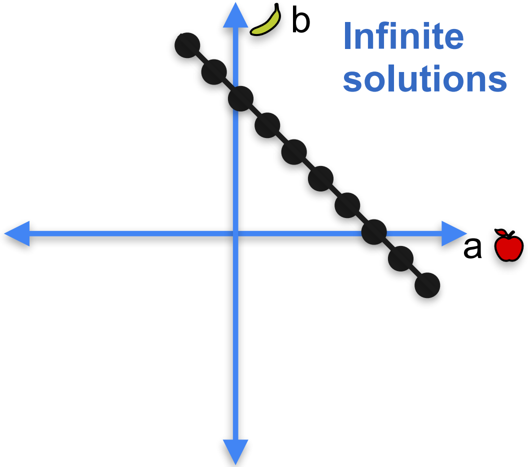

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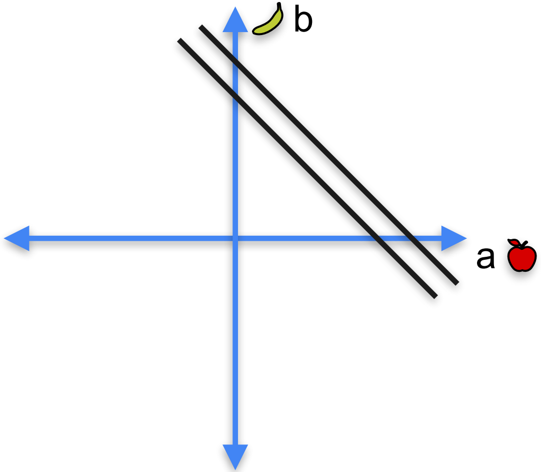

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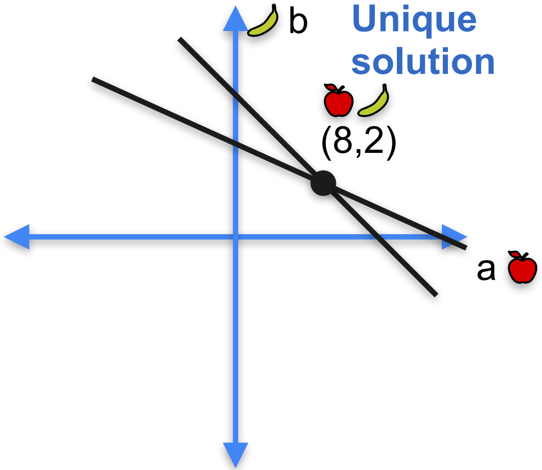

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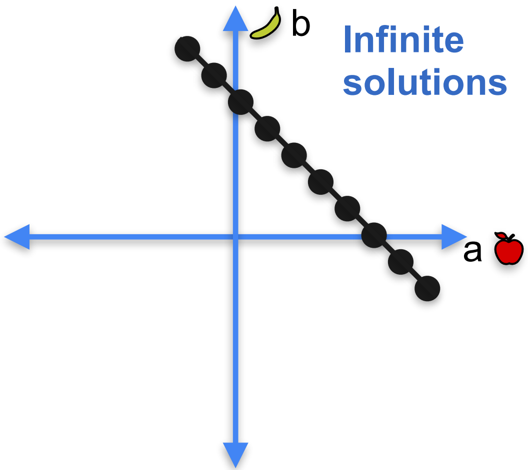

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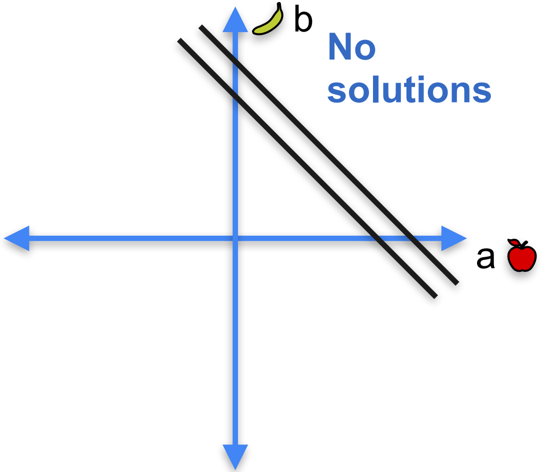

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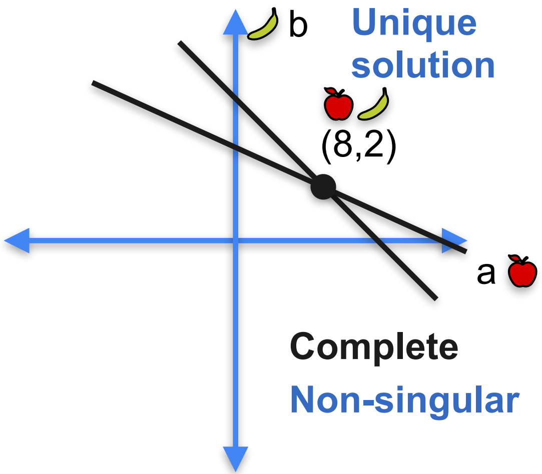

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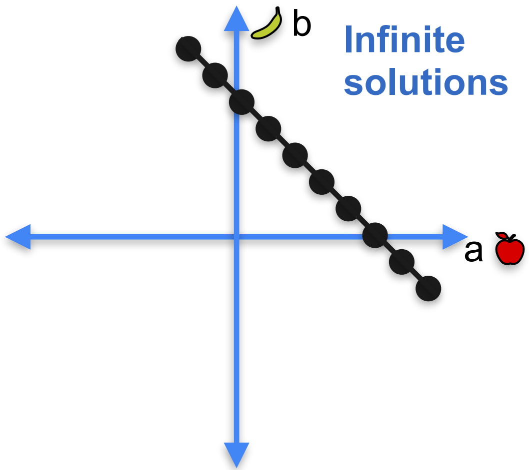

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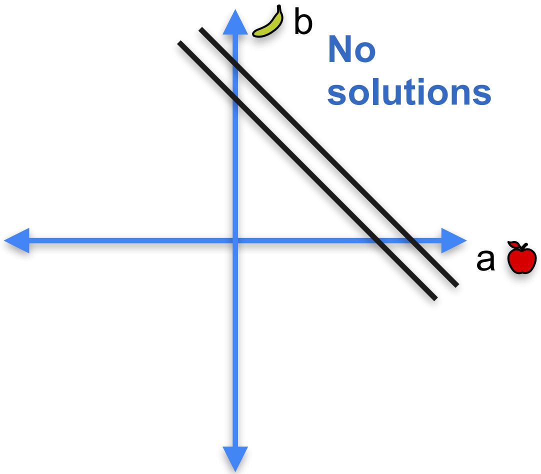

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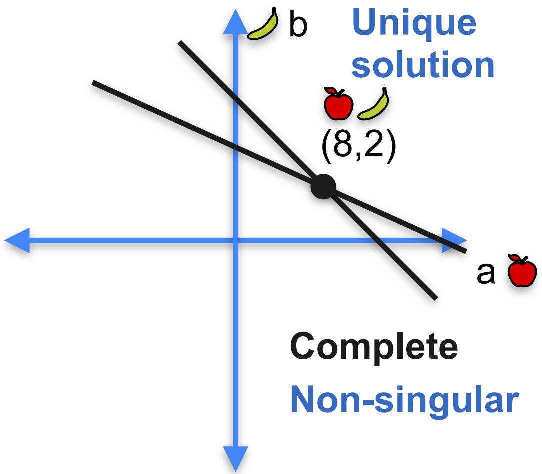

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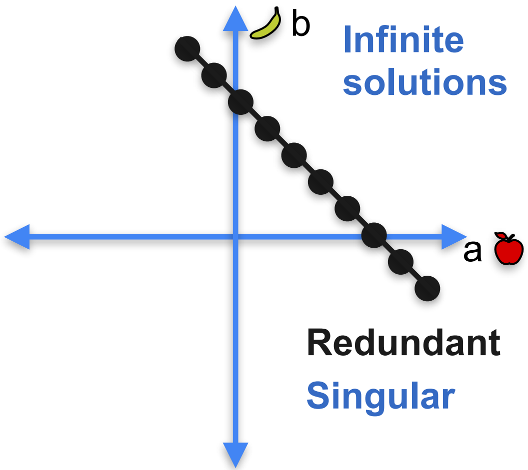

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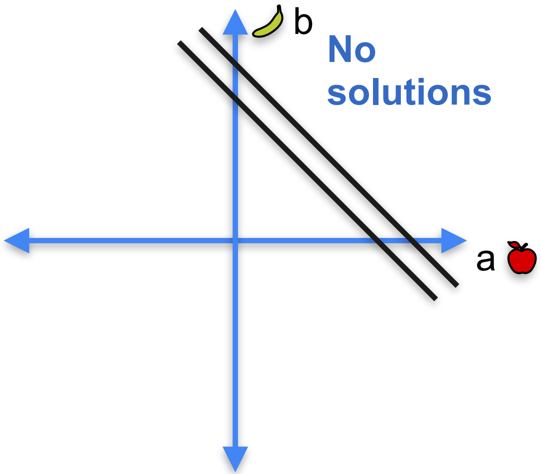

Systems of equations as lines

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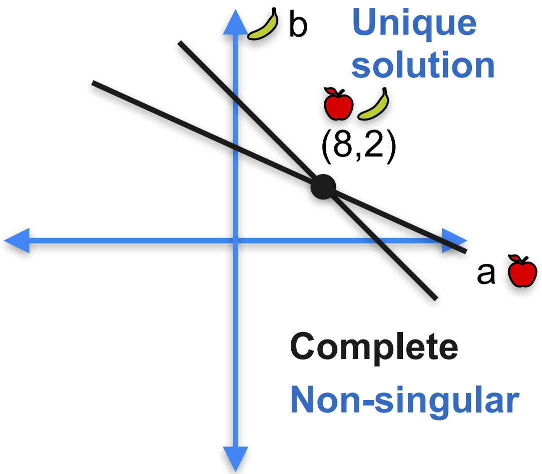
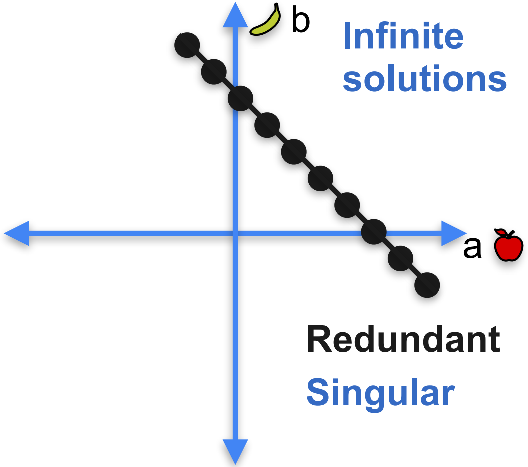


Complete

Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

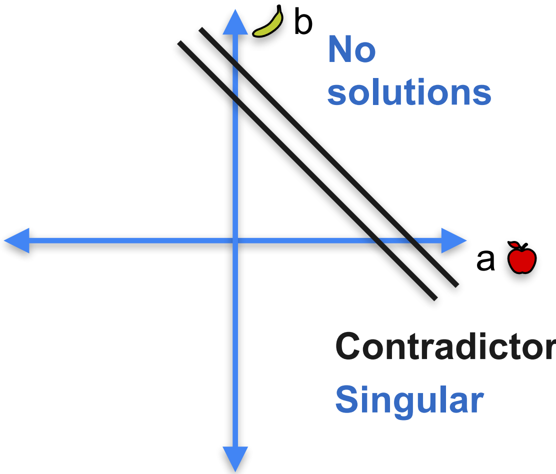


Redundant

Singular

System 3

- $a + b = 10$
- $2a + 2b = 24$



Contradictory

Singular

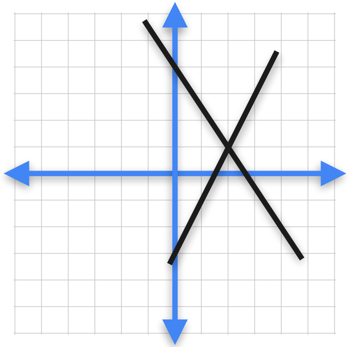
Quiz

Problem 1

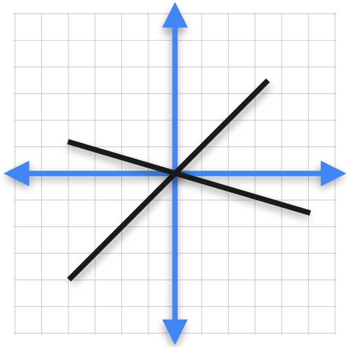
Which of the following plots corresponds to the system of equations:

- 3a + 2b = 8
- 2a - b = 3

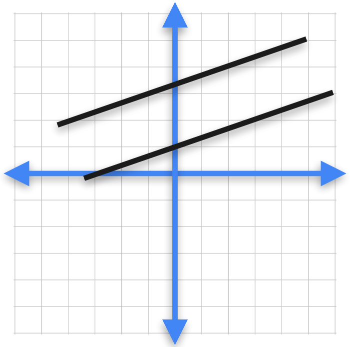
a)



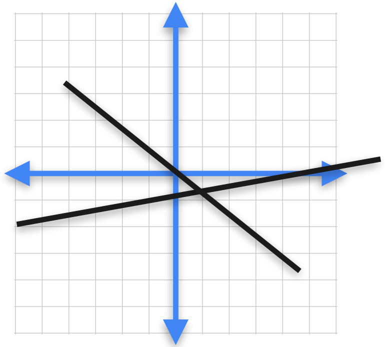
b)



c)



d)



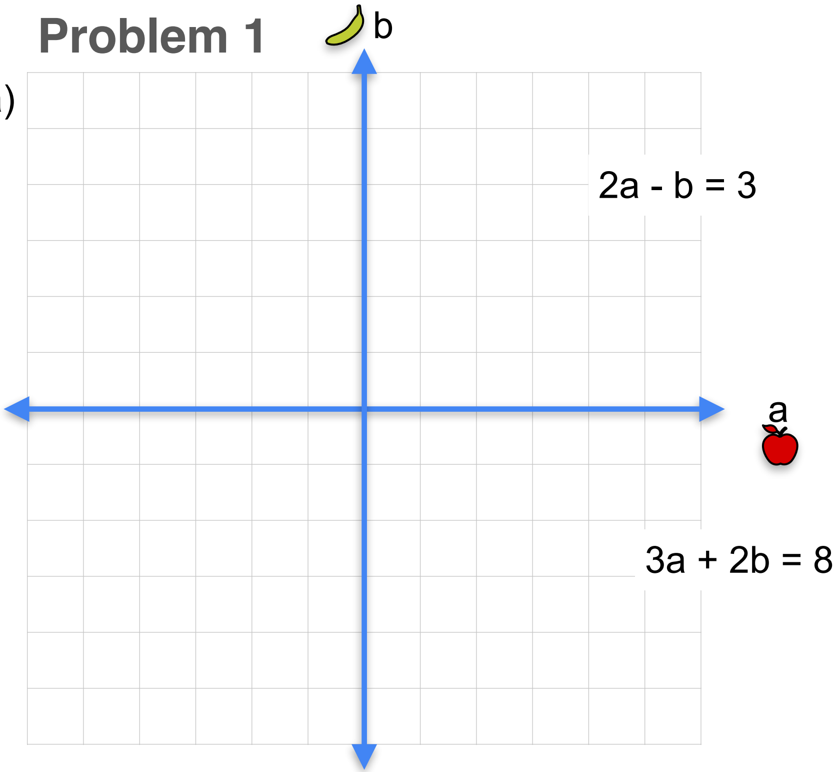
Problem 2

Is this system singular or non-singular?

Solution

Problem 1

a)



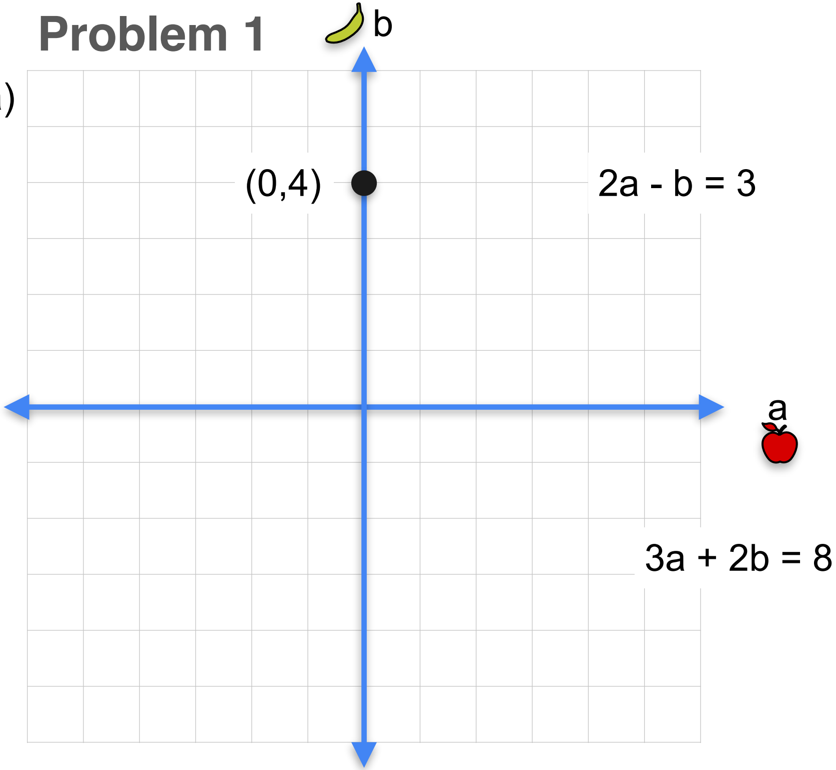
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



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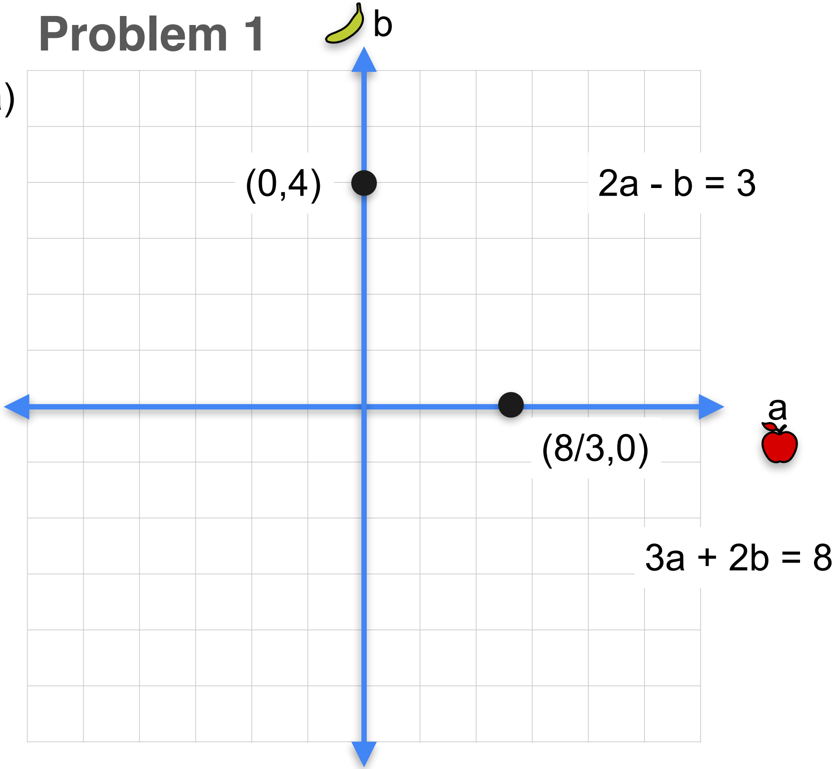

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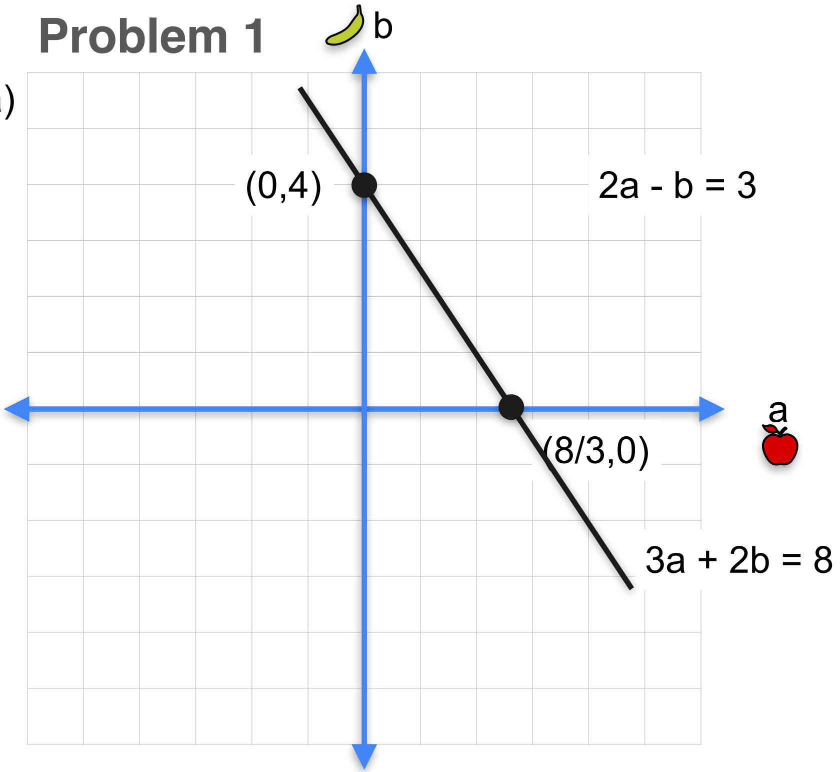

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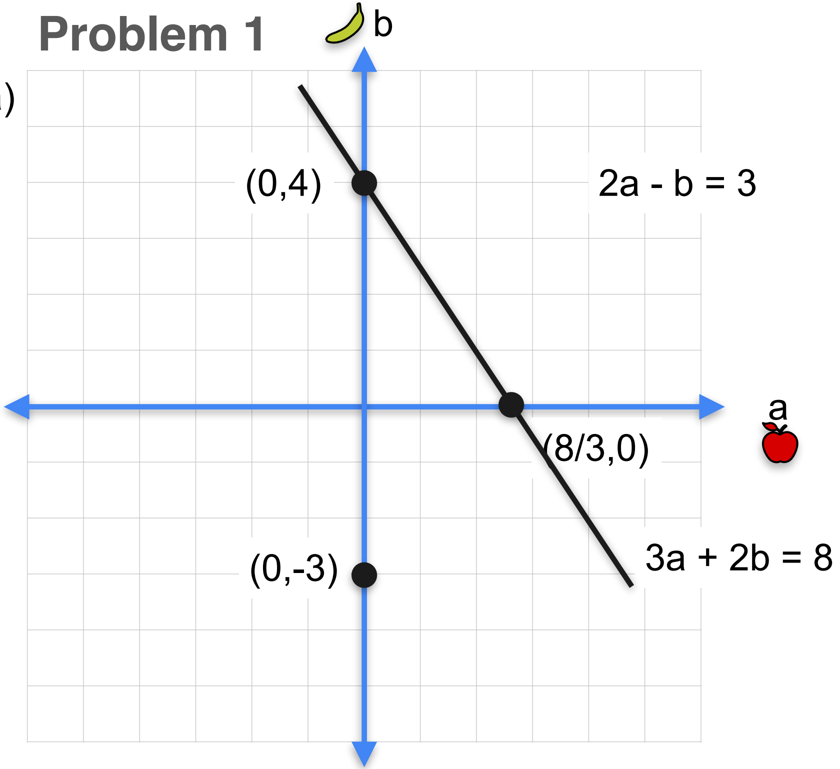

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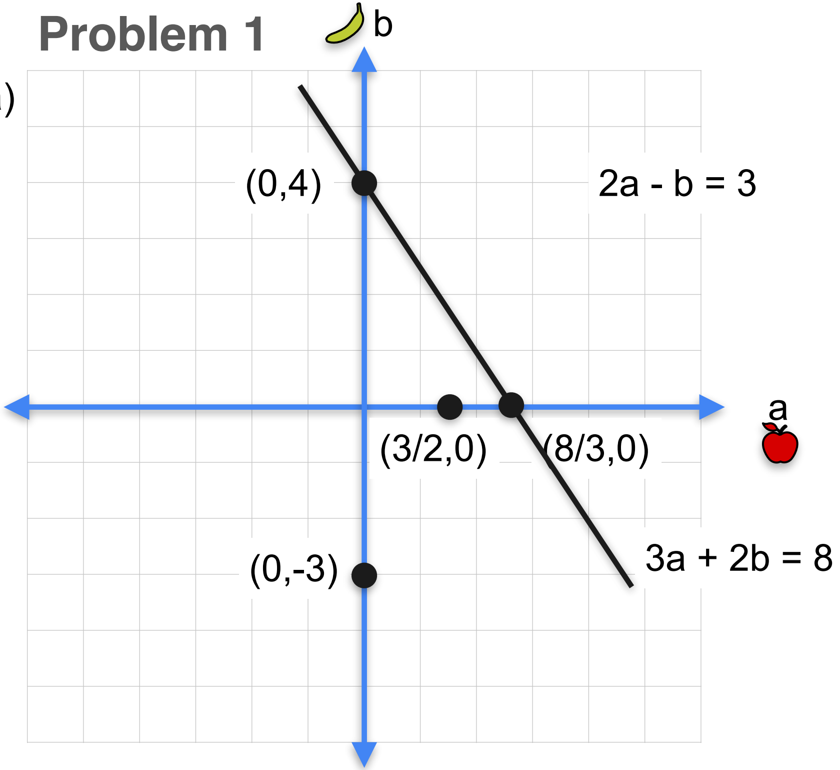

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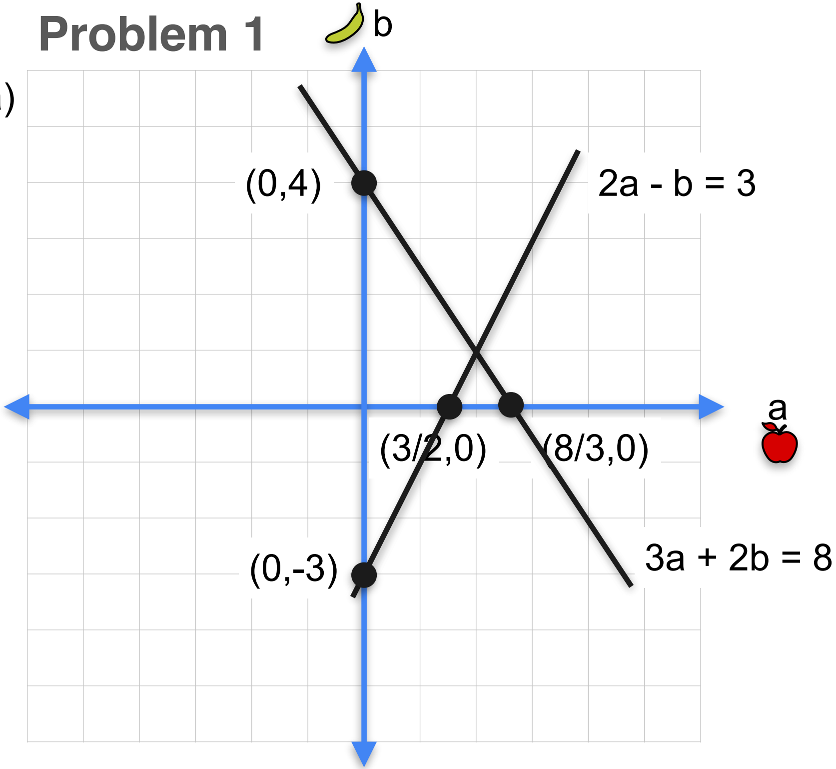

Problem 2

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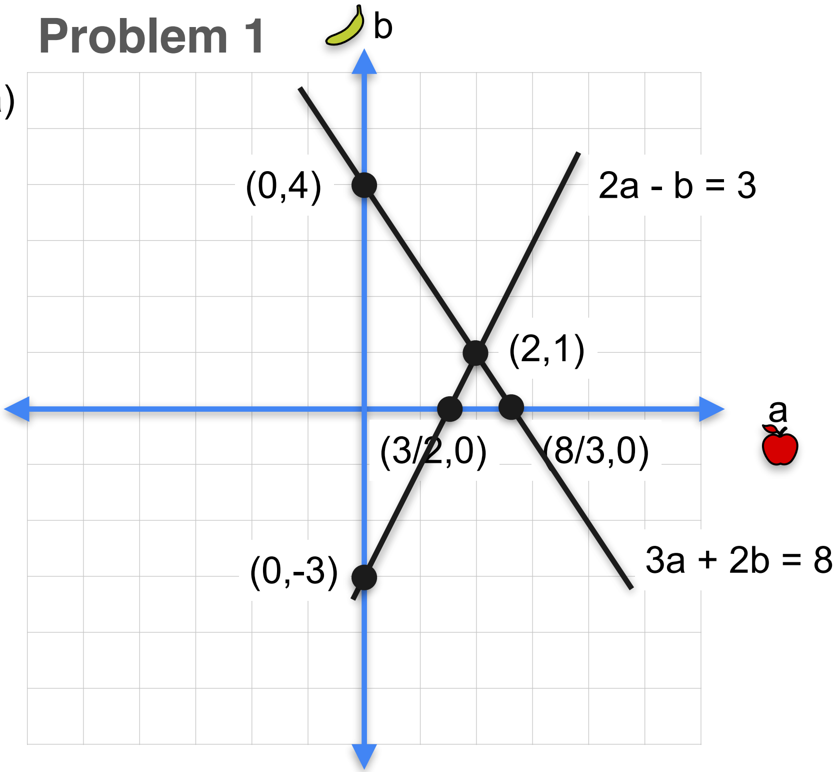


Problem 2

Since the lines cross at a unique point, the system is non-singular.

DeepLearning.AI

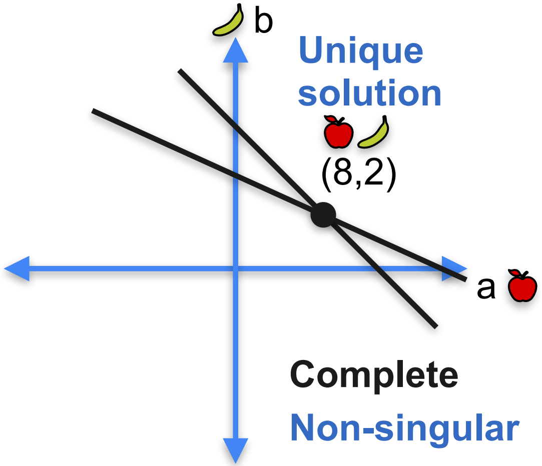
System of Linear Equations

A geometric notion of singularity

Systems of equations as lines

System 1

- $a + b = 10$
- $a + 2b = 12$



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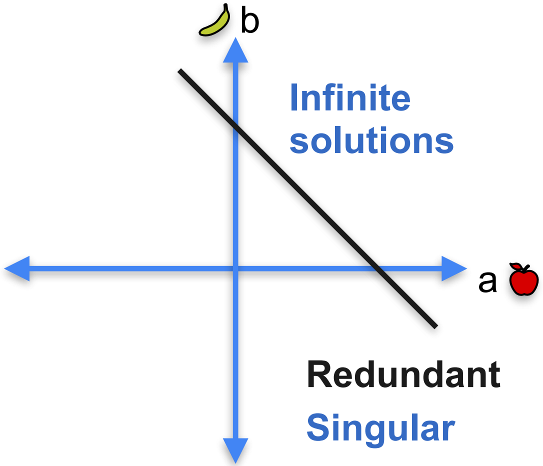

System 3

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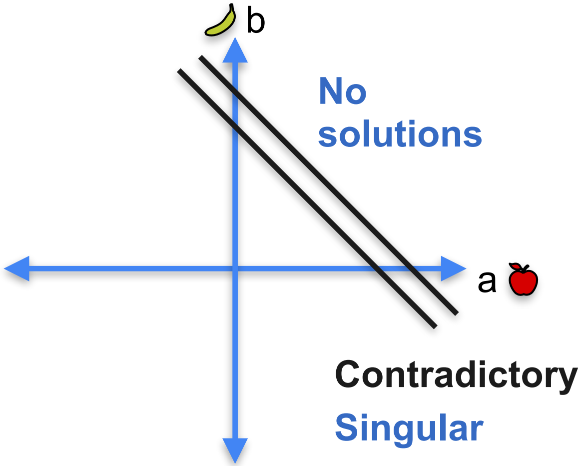

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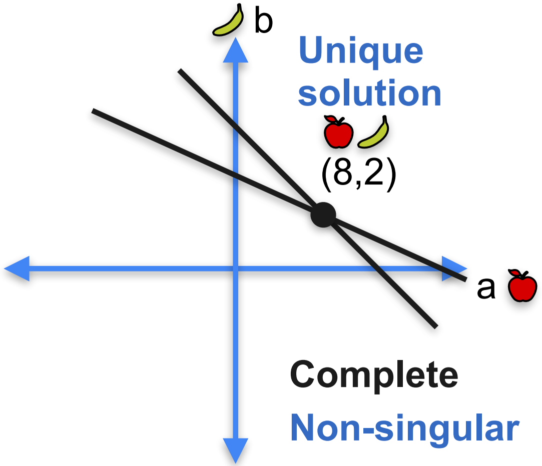

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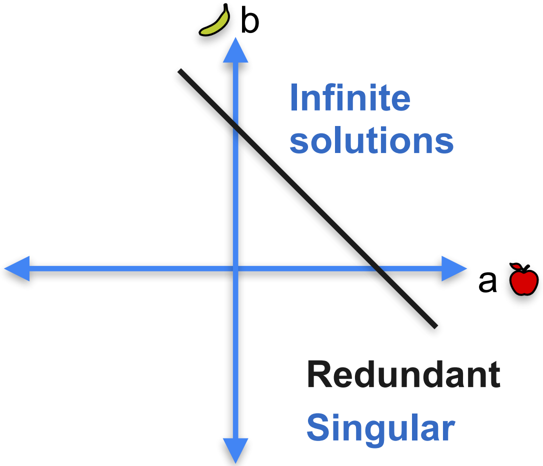

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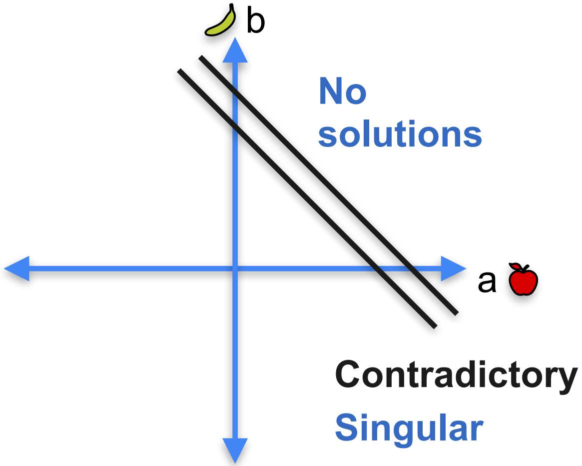
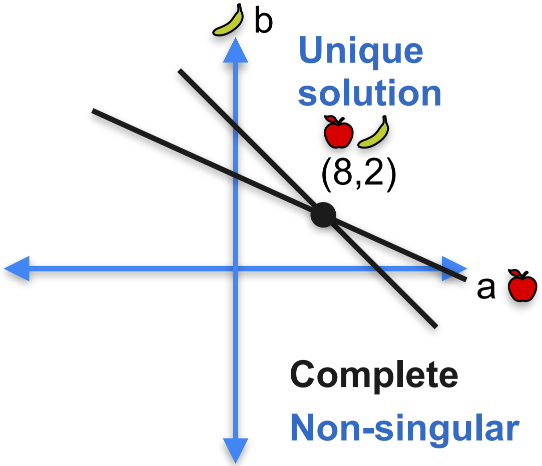


Systems of equations as lines

System 1

- $a + b = \boxed{0}$
- $a + 2b = \boxed{0}$

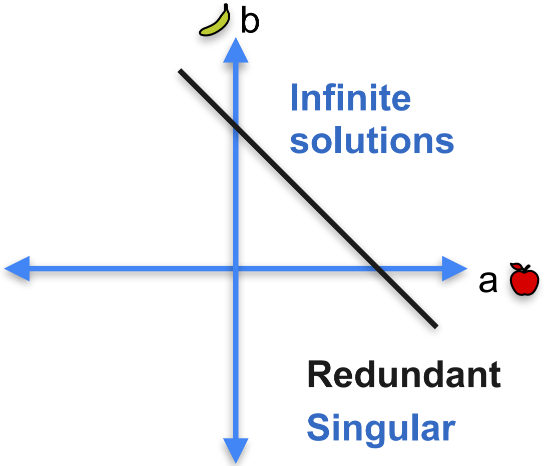


Complete

Non-singular

System 2

- $a + b = \boxed{10}$
- $2a + 2b = \boxed{20}$

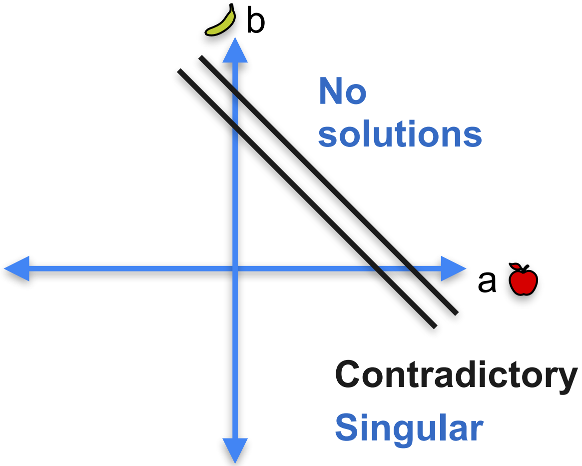


Redundant

Singular

System 3

- $a + b = \boxed{10}$
- $2a + 2b = \boxed{24}$



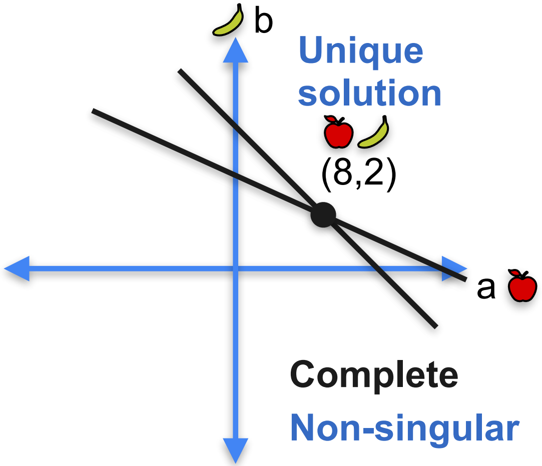
Contradictory

Singular

Systems of equations as lines

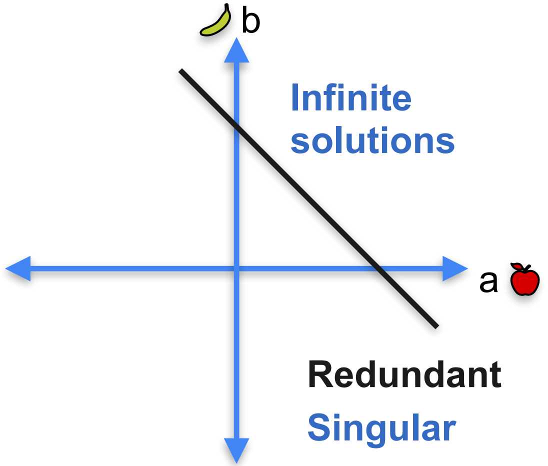
System 1

- $a + b = 0$
- $a + 2b = 0$



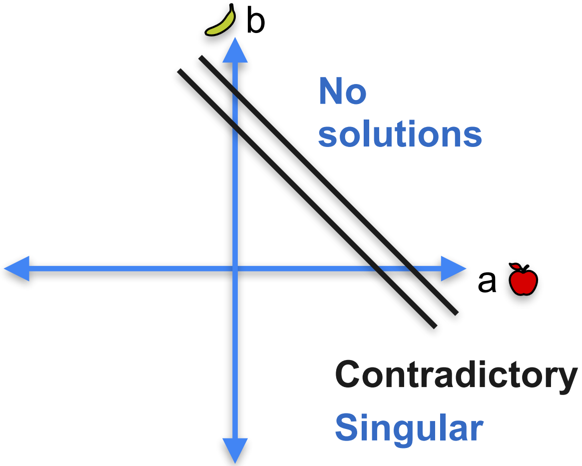
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

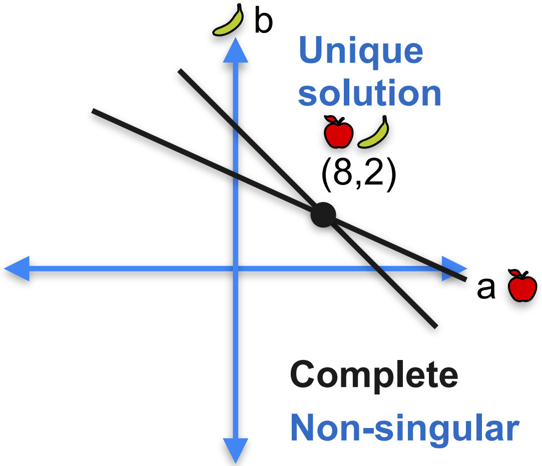
- $a + b = 10$
- $2a + 2b = 24$



Systems of equations as lines

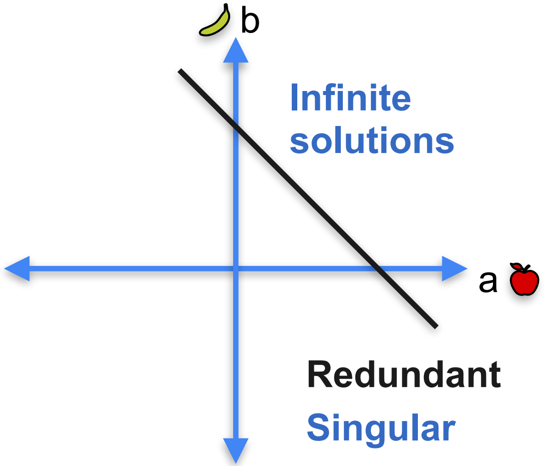
System 1

- $a + b = 0$
- $a + 2b = 0$



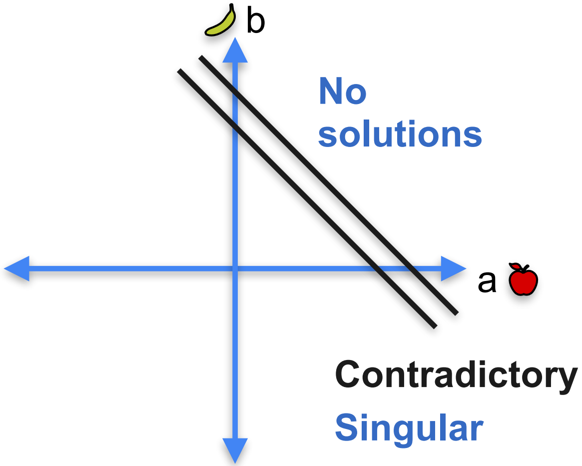
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

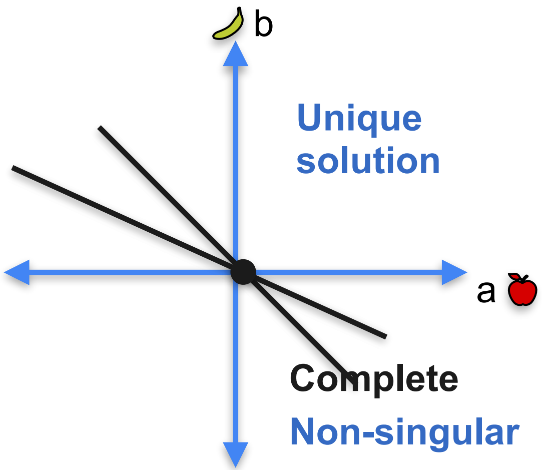
- $a + b = 0$
- $2a + 2b = 0$



Systems of equations as lines

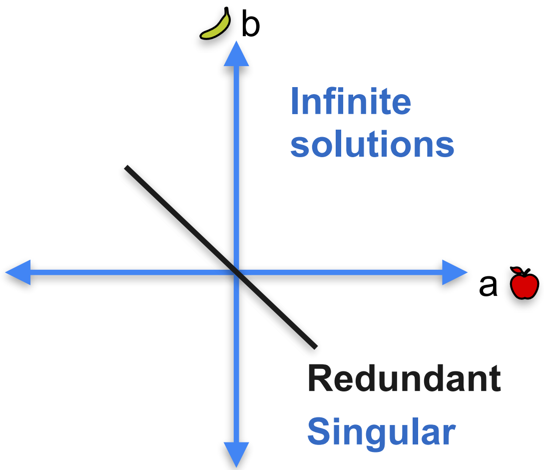
System 1

- $a + b = 0$
- $a + 2b = 0$



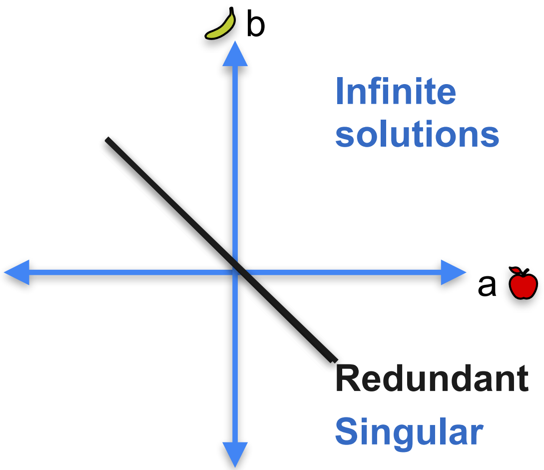
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

- $a + b = 0$
- $2a + 2b = 0$



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System of Linear Equations

Singular vs nonsingular matrices

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

System 2

- $a + b = 0$
- $2a + 2b = 0$

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

🍎	🍌
1	1
1	2

System 2

- $a + b = 0$
- $2a + 2b = 0$

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

System 2

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

Non-singular system

(Unique solution)

System 2

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

Non-singular system

Non-singular matrix

(Unique solution)

System 2

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

Non-singular system

Non-singular matrix

(Unique solution)

System 2

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Singular system

(Infinitely many solutions)

Systems of equations as matrices

System 1

- $a + b = 0$
- $a + 2b = 0$

🍎	🍌
1	1
1	2

Non-singular system

Non-singular matrix

(Unique solution)

System 2

- $a + b = 0$
- $2a + 2b = 0$

🍎	🍌
1	1
2	2

Singular system

Singular matrix

(Infinitely many solutions)

DeepLearning.AI

System of Linear Equations

Linear dependence and independence

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

Singular system

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

Singular system

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Second equation is a multiple of the first one

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

Singular system

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Linear dependence between rows

Non-singular

- $a + b =$ **0**
- $a + 2b =$ **0**

1	1
1	2

Singular system

- $a + b =$ **0**
- $2a + 2b =$ **0**

1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

Linear dependence between rows

Non-singular

- a + b = **0**
- a + 2b = **0**

1	1
1	2

No equation is a multiple of the other one

Singular system

- a + b = **0**
- 2a + 2b = **0**

1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

No equation is a multiple of the other one

No row is a multiple of the other one

Singular system

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

Linear dependence between rows

Non-singular

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

No equation is a multiple of the other one

No row is a multiple of the other one

Rows are *linearly independent*

Singular system

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

System of Linear Equations

The determinant

Linear dependence between rows

Non-singular matrix

🍎	🍌
1	1
1	2

Singular matrix

🍎	🍌
1	1
2	2

Linear dependence between rows

Non-singular matrix

🍎	🍌
1	1
1	2

Singular matrix

🍎	🍌
1	1
2	2

Linear dependence between rows

Non-singular matrix

1	1
1	2

Singular matrix

1	1
2	2

1	1

Linear dependence between rows

Non-singular matrix

🍎	🍌
1	1
1	2

Singular matrix

🍎	🍌
1	1
2	2

1	1	x 2	=

Linear dependence between rows

Non-singular matrix

1	1
1	2

Singular matrix

1	1
2	2

1	1	x 2	=	2	2

Linear dependence between rows

Non-singular matrix

1	1
1	2

Singular matrix

1	1
2	2

1	1	x 2	=	2	2

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

1	1
1	2

Singular matrix

1	1
2	2

1	1	x 2	=	2	2

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

1	1
1	2

1	1

Singular matrix

1	1
2	2

1	1	x 2	=	2	2

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? =$$

Singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Rows linearly independent

Singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

Determinant

🍎	🍌
a	b
c	d

Matrix is singular if

a	b	* k	=	c	d

Determinant

	
a	b
c	d

$$ak = c$$

Matrix is singular if

a	b	* k	=	c	d

Determinant

🍎	🍌
a	b
c	d

$$ak = c$$

$$bk = d$$

Matrix is singular if

a	b	* k	=	c	d

Determinant

a	b
c	d

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

a	b	* k	=	c	d

Determinant

a	b
c	d

Matrix is singular if

a	b	* k	=	c	d

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$

Determinant

a	b
c	d

Matrix is singular if

a	b	* k	=	c	d

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$

$$ad - bc = 0$$

Determinant

a	b
c	d

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{bmatrix} a & b \end{bmatrix} * k = \begin{bmatrix} c & d \end{bmatrix}$$

$$ad = bc$$

Determinant

$$\boxed{ad - bc} = 0$$

Determinant

a	b
c	d

Determinant $= ad - bc$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

a	b	* k	=	c	d

$$ad = bc$$

Determinant

$$\boxed{ad - bc} = 0$$

Determinant

a	b
c	d

Determinant $= ad - bc$

a	
	d

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

a	b	* k	=	c	d

$$ad = bc$$

Determinant

$$\boxed{ad - bc} = 0$$

Determinant

🍎	🍌
a	b
c	d

Determinant $= ad - bc$

a		-
	d	

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

a	b	* k	=	c	d

$$ad = bc$$

Determinant

$$\boxed{ad - bc} = 0$$

Determinant

a	b
c	d

Determinant $= ad - bc$

$$a \cdot d \; - \; c \cdot b$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{bmatrix} a & b \end{bmatrix} * k = \begin{bmatrix} c & d \end{bmatrix}$$

$$ad = bc$$

Determinant

$$\boxed{ad - bc} = 0$$

Determinant

Non-singular matrix

1	1
1	2

Singular matrix

1	1
2	2

Determinant

Non-singular matrix

1	1
1	2

Determinant

$$\begin{matrix}1 & \\ & 2\end{matrix} \;-\; \begin{matrix} & 1\\ 1 & \end{matrix}$$

Singular matrix

1	1
2	2

Determinant

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Determinant

$$\begin{array}{cc} 1 & \\ & 2 \end{array} \; - \; \begin{array}{cc} & 1 \\ 1 & \end{array}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Determinant

Non-singular matrix

1	1
1	2

Determinant

$$\begin{matrix} 1 & \\ & 2 \end{matrix} \quad - \quad \begin{matrix} & 1 \\ 1 & \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

1	1
2	2

Determinant

$$\begin{matrix} 1 & \\ & 2 \end{matrix} \quad - \quad \begin{matrix} & 1 \\ 2 & \end{matrix}$$

Determinant

Non-singular matrix

1	1
1	2

Determinant

$$\begin{matrix} 1 & & & & 1 \\ & 2 & - & 1 & \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

1	1
2	2

Determinant

$$\begin{matrix} 1 & & & & 1 \\ & 2 & - & 2 & \end{matrix}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant

Non-singular matrix

1	1
1	2

Determinant

$$\begin{matrix} 1 & & & & 1 \\ & 2 & - & 1 & \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

1	1
2	2

Determinant

$$\begin{matrix} 1 & & & & 1 \\ & 2 & - & 2 & \end{matrix}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant

Non-singular matrix

1	1
1	2

Determinant

$$\begin{matrix} 1 & \\ & 2 \end{matrix} \; - \; \begin{matrix} & 1 \\ 1 & \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

1	1
2	2

Determinant

$$\begin{matrix} 1 & \\ & 2 \end{matrix} \; - \; \begin{matrix} & 1 \\ 2 & \end{matrix}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant and singularity

	
a	b
c	d

$$ad - bc$$

Determinant and singularity

🍎	🍌
a	b
c	d

$$ad - bc$$

Matrix is singular ⟷ **Determinant is zero**

Quiz: Determinant

Problem 1: Find the determinant of the following matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Problem 2: Are these matrices singular or non-singular?

Solutions: Determinant

Matrix 1: det = $5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

5	1
-1	3

Non-singular

Matrix 2: det = $2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

2	-1
-6	3

Singular

System of Linear Equations

System of equations (3x3)

Quiz: Systems of equations

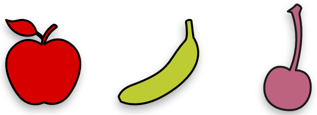
Problem 1: You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid $10.
- **Day 2:** You bought an apple, two bananas, and a cherry, and paid $15.
- **Day 3:** You bought an apple, a banana, and two cherries, and paid $12.

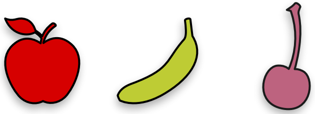
How much does each fruit cost?

Solution: Systems of equations

Solution: Systems of equations

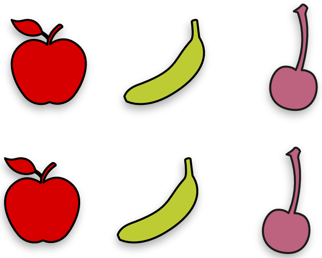


Solution: Systems of equations



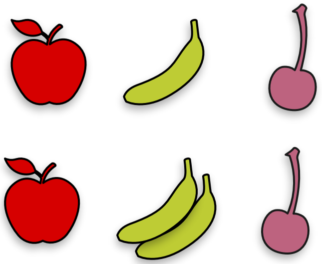
$10

Solution: Systems of equations



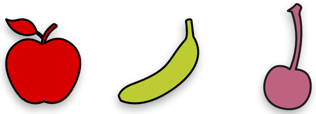
$10

Solution: Systems of equations

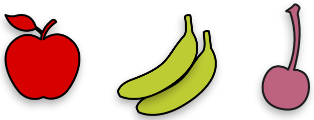


$10

Solution: Systems of equations

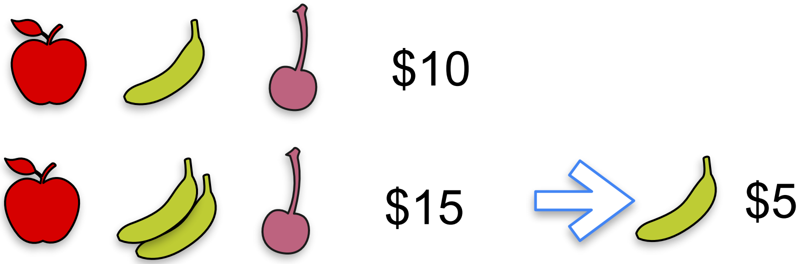


$10

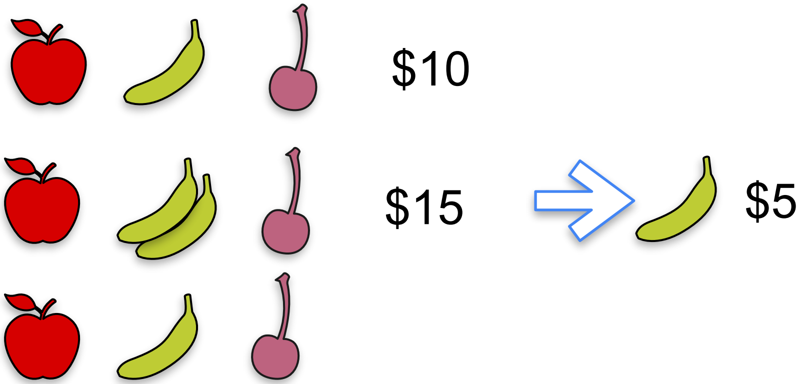


$15

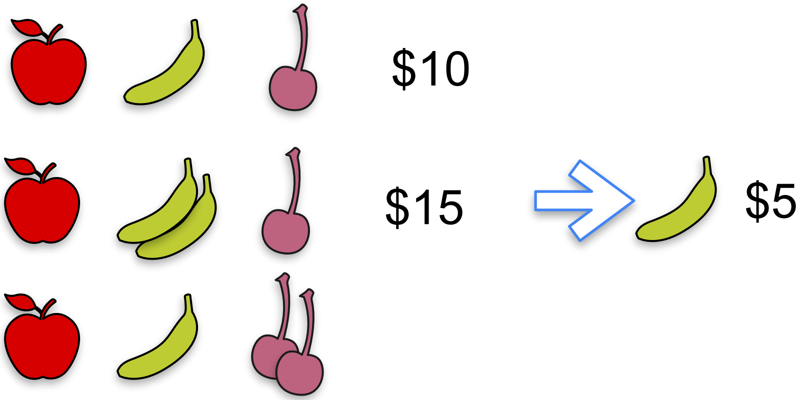
Solution: Systems of equations



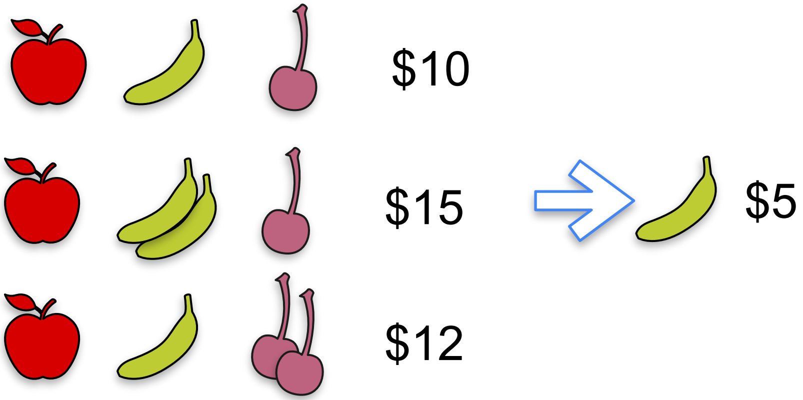
Solution: Systems of equations



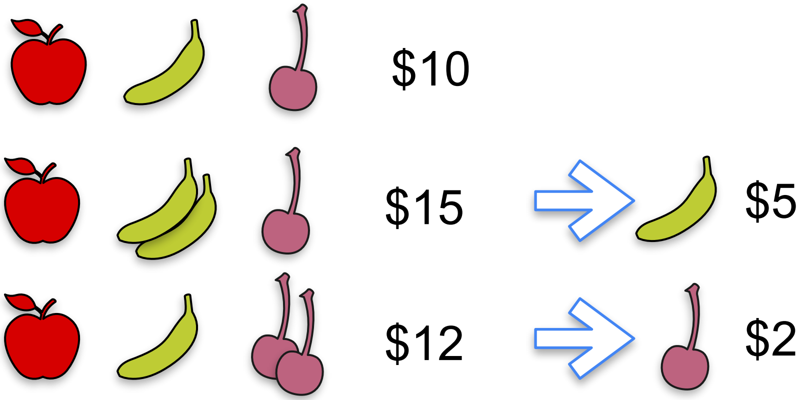
Solution: Systems of equations



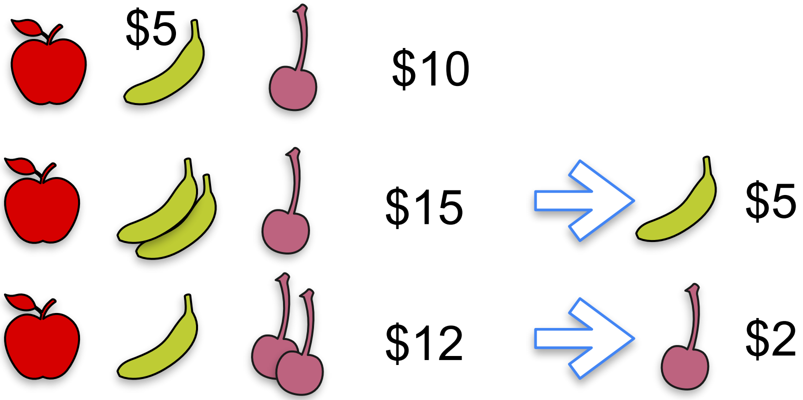
Solution: Systems of equations



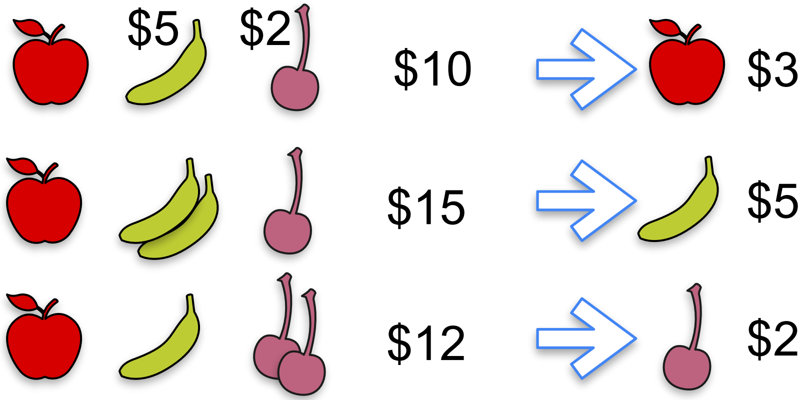
Solution: Systems of equations



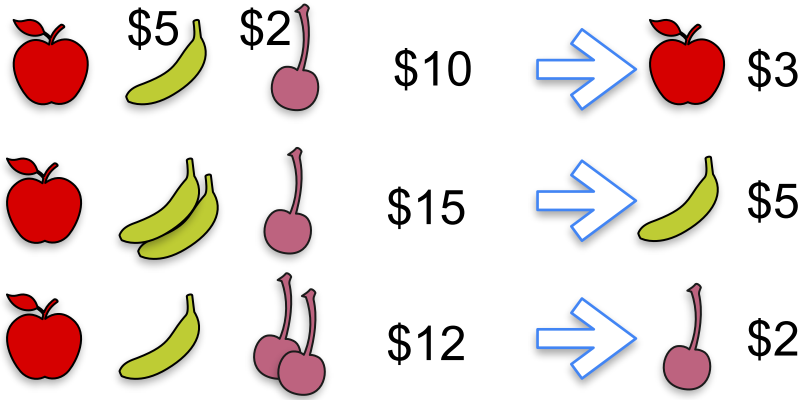
Solution: Systems of equations



Solution: Systems of equations



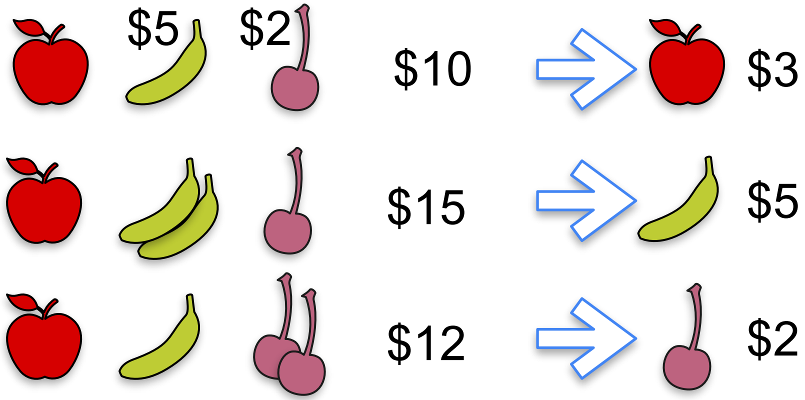
Solution: Systems of equations



System of equations 1

$$a + b + c = 10$$
$$a + 2b + c = 15$$
$$a + b + 2c = 12$$

Solution: Systems of equations



System of equations 1

$$a + b + c = 10$$
$$a + 2b + c = 15$$
$$a + b + 2c = 12$$

Solution

$a = 3$

$b = 5$

$c = 2$

Quiz: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$
$$2a + 2b + 2c = 20$$
$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Solutions: More systems of equations

System 2

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 20$$

Infinitely many sols.

$c = 5$

System 3

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$
$$2a + 2b + 2c = 20$$
$$3a + 3b + 3c = 30$$

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

$c = 5$

$a + b = 5$

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

$c = 5$

$a + b = 5$

(0,5,5), (1,4,5), (2,3,5), …

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

$c = 5$

$a + b = 5$

$(0,5,5), (1,4,5), (2,3,5), \ldots$

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

$c = 5$

$a + b = 5$

$(0,5,5), (1,4,5), (2,3,5), \ldots$

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

From 1st and 2nd:
$c = 5$
From 2nd and 3rd:
$c = 3$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

$c = 5$

$a + b = 5$

(0,5,5), (1,4,5), (2,3,5), …

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

From 1st and 2nd:
$c = 5$
From 2nd and 3rd:
$c = 3$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Infinitely many solutions

Solutions: More systems of equations

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinitely many sols.

$c = 5$

$a + b = 5$

(0,5,5), (1,4,5), (2,3,5), …

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

From 1st and 2nd:
$c = 5$
From 2nd and 3rd:
$c = 3$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 20$
$3a + 3b + 3c = 30$

Infinitely many solutions

Any 3 numbers that add to 10 work.
(0,0,10), (2,7,1), …

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System of Linear Equations

Singular vs non-singular matrices

Constants don't matter for singularity

System 1

$$a + b + c = 10$$
$$a + 2b + c = 15$$
$$a + b + 2c = 12$$

System 2

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 20$$

System 3

$$a + b + c = 10$$
$$a + b + 2c = 15$$
$$a + b + 3c = 18$$

System 4

$$a + b + c = 10$$
$$2a + 2b + 2c = 15$$
$$3a + 3b + 3c = 20$$

Constants don't matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 15$
$3a + 3b + 3c = 20$

Unique solution

Constants don't matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

Unique solution

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinite solutions

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

System 4

$a + b + c = 10$
$2a + 2b + 2c = 15$
$3a + 3b + 3c = 20$

Constants don't matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

Unique solution

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinite solutions

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

System 4

$a + b + c = 10$
$2a + 2b + 2c = 15$
$3a + 3b + 3c = 20$

Constants don't matter for singularity

System 1	System 2	System 3	System 4
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 15$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 20$
Unique solution	**Infinite solutions**	**No solutions**	**Infinite solutions**

Constants don't matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

Unique solution

Complete

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinite solutions

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

System 4

$a + b + c = 10$
$2a + 2b + 2c = 15$
$3a + 3b + 3c = 20$

Infinite solutions

Constants don't matter for singularity

System 1

$a + b + c = 10$
$a + 2b + c = 15$
$a + b + 2c = 12$

Unique solution

Complete

System 2

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 20$

Infinite solutions

Redundant

System 3

$a + b + c = 10$
$a + b + 2c = 15$
$a + b + 3c = 18$

No solutions

System 4

$a + b + c = 10$
$2a + 2b + 2c = 15$
$3a + 3b + 3c = 20$

Infinite solutions

Constants don’t matter for singularity

System 1	System 2	System 3	System 4
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 15$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 20$
Unique solution	**Infinite solutions**	**No solutions**	**Infinite solutions**
Complete	**Redundant**	**Contradictory**	

Constants don't matter for singularity

System 1	System 2	System 3	System 4
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 15$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 20$
Unique solution	**Infinite solutions**	**No solutions**	**Infinite solutions**
Complete	**Redundant**	**Contradictory**	**Redundant**

Constants don't matter for singularity

System 1	System 2	System 3	System 4
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 15$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 20$
Unique solution	**Infinite solutions**	**No solutions**	**Infinite solutions**
Complete	**Redundant**	**Contradictory**	**Redundant**
Non-singular			

Constants don't matter for singularity

System 1	System 2	System 3	System 4
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 15$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 20$
Unique solution	**Infinite solutions**	**No solutions**	**Infinite solutions**
Complete	**Redundant**	**Contradictory**	**Redundant**
Non-singular	**Singular**		

Constants don’t matter for singularity

System 1	System 2	System 3	System 4
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 15$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 20$
Unique solution	**Infinite solutions**	**No solutions**	**Infinite solutions**
Complete	**Redundant**	**Contradictory**	**Redundant**
Non-singular	**Singular**	**Singular**	

Constants don't matter for singularity

System 1	**System 2**	**System 3**	**System 4**
$a + b + c = \mathbf{10}$	$a + b + c = \mathbf{10}$	$a + b + c = \mathbf{10}$	$a + b + c = \mathbf{10}$
$a + 2b + c = \mathbf{15}$	$a + b + 2c = \mathbf{15}$	$a + b + 2c = \mathbf{15}$	$2a + 2b + 2c = \mathbf{15}$
$a + b + 2c = \mathbf{12}$	$a + b + 3c = \mathbf{20}$	$a + b + 3c = \mathbf{18}$	$3a + 3b + 3c = \mathbf{20}$
Unique solution	**Infinite solutions**	**No solutions**	**Infinite solutions**
Complete	**Redundant**	**Contradictory**	**Redundant**
Non-singular	**Singular**	**Singular**	**Singular**

Constants don't matter for singularity

System 1	**System 2**	**System 3**	**System 4**
$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$	$a + b + c = 10$
$a + 2b + c = 15$	$a + b + 2c = 15$	$a + b + 2c = 15$	$2a + 2b + 2c = 20$
$a + b + 2c = 12$	$a + b + 3c = 20$	$a + b + 3c = 18$	$3a + 3b + 3c = 30$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$a + b + c = 0$	$a + b + c = 0$	$a + b + c = 0$	$a + b + c = 0$
$a + 2b + c = 0$	$a + b + 2c = 0$	$a + b + 2c = 0$	$2a + 2b + 2c = 0$
$a + b + 2c = 0$	$a + b + 3c = 0$	$a + b + 3c = 0$	$3a + 3b + 3c = 0$

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

Unique solution:
$a = 0$
$b = 0$
$c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Constants don’t matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

Unique solution:
$a = 0$
$b = 0$
$c = 0$

Complete

Non-singular

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Unique solution:

$a = 0$
$b = 0$
$c = 0$

Complete

Non-singular

Infinite solutions:

$c = 0$
$a + b = 0$
(i.e., $a = -b$)

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Unique solution:

$a = 0$
$b = 0$
$c = 0$

Complete

Non-singular

Infinite solutions:

$c = 0$
$a + b = 0$
(i.e., $a = -b$)

Redundant

Singular

Constants don’t matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Unique solution:
$a = 0$
$b = 0$
$c = 0$

Complete

Non-singular

Infinite solutions:
$c = 0$
$a + b = 0$
(i.e., $a = -b$)

Redundant

Singular

Infinite solutions:
$a + b + c = 0$
(i.e., $c = - a - b$)

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Unique solution:
a = 0
b = 0
c = 0

Complete

Non-singular

Infinite solutions:
c = 0
a + b = 0
(i.e., a = -b)

Redundant

Singular

Infinite solutions:
a + b +c = 0
(i.e., c = - a - b)

Redundant

Singular

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$
$$a + 2b + c = 0$$
$$a + b + 2c = 0$$

System 2

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$
$$2a + 2b + 2c = 0$$
$$3a + 3b + 3c = 0$$

1	1	1
1	2	1
1	1	2

1	1	1
1	1	2
1	1	3

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

1	1	1
1	2	1
1	1	2

1	1	1
1	1	2
1	1	3

1	1	1
2	2	2
3	3	3

Constants don't matter for singularity

System 1

$$a + b + c = 0$$
$$a + 2b + c = 0$$
$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

Non-singular

System 2

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

System 4

$$a + b + c = 0$$
$$2a + 2b + 2c = 0$$
$$3a + 3b + 3c = 0$$

1	1	1
2	2	2
3	3	3

Constants don't matter for singularity

System 1

$a + b + c = 0$
$a + 2b + c = 0$
$a + b + 2c = 0$

System 2

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 3

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

System 4

$a + b + c = 0$
$2a + 2b + 2c = 0$
$3a + 3b + 3c = 0$

1	1	1
1	2	1
1	1	2

Non-singular

1	1	1
1	1	2
1	1	3

Singular

1	1	1
2	2	2
3	3	3

Constants don't matter for singularity

System 1

$$a + b + c = 0$$
$$a + 2b + c = 0$$
$$a + b + 2c = 0$$

System 2

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$
$$2a + 2b + 2c = 0$$
$$3a + 3b + 3c = 0$$

1	1	1
1	2	1
1	1	2

Non-singular

1	1	1
1	1	2
1	1	3

Singular

1	1	1
2	2	2
3	3	3

Singular

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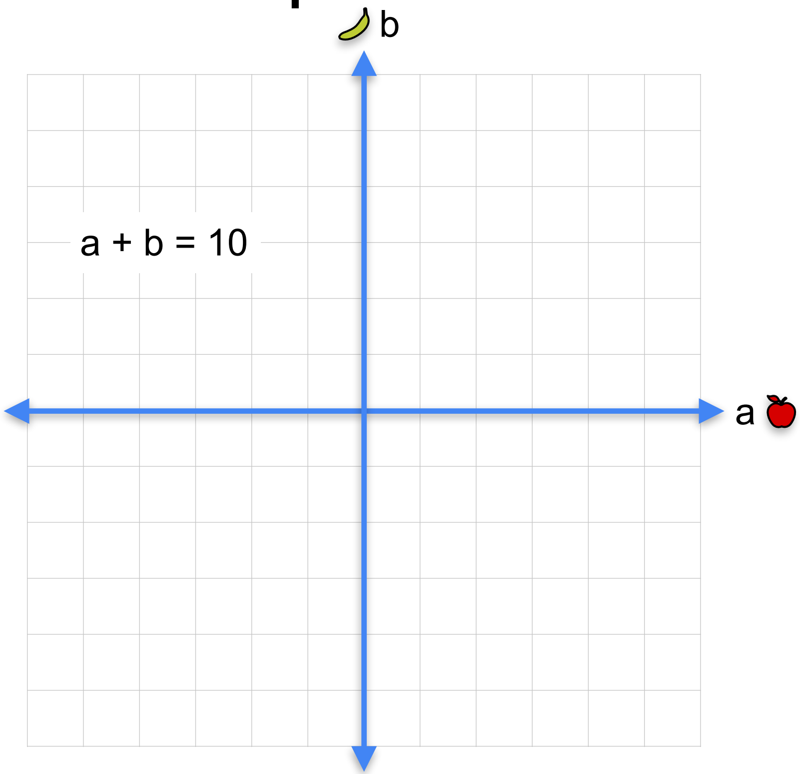
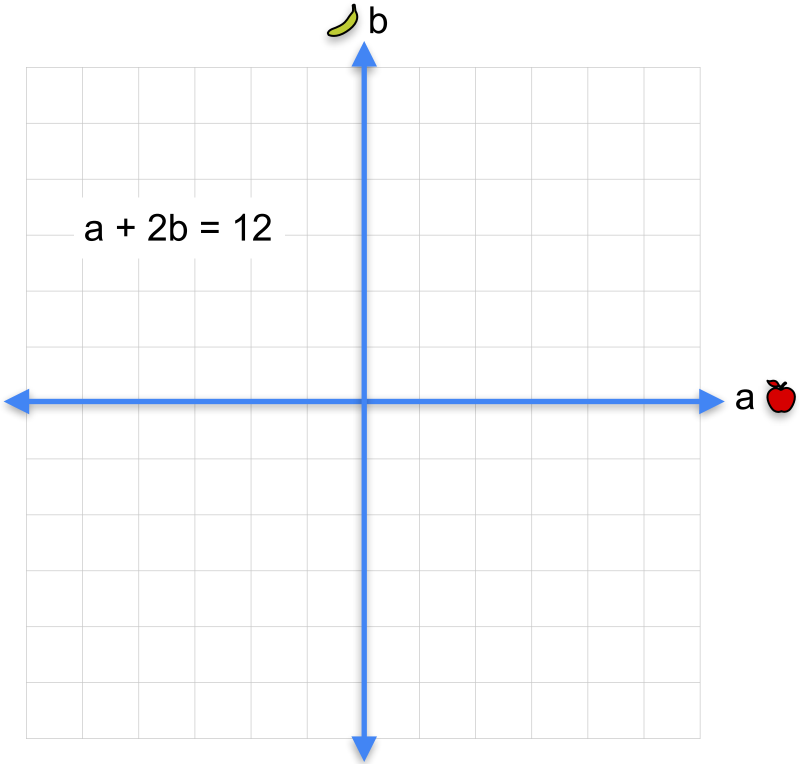
System of Linear Equations

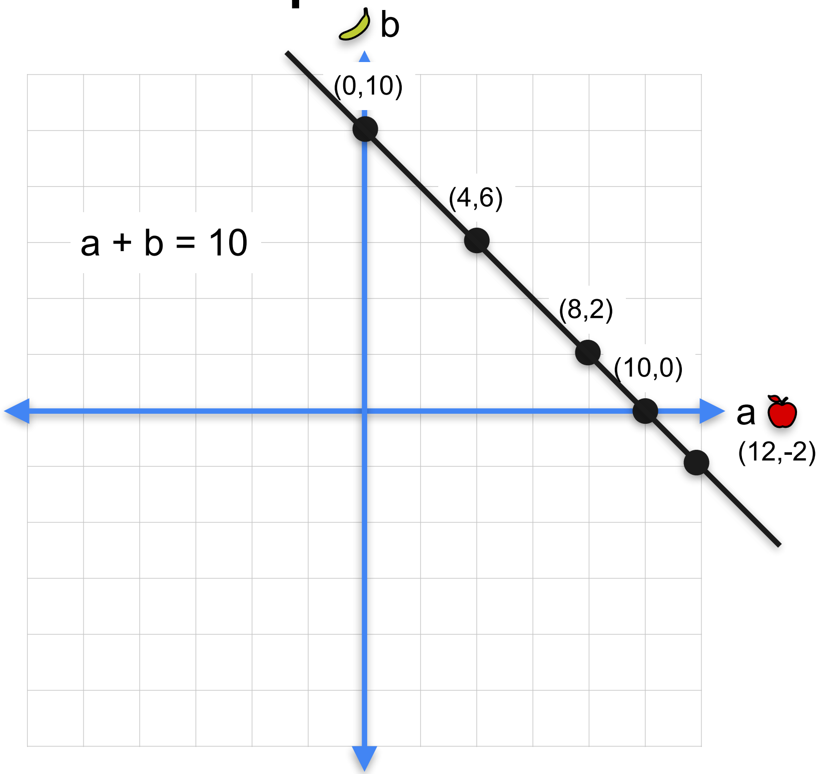
System of equations as planes (3x3)

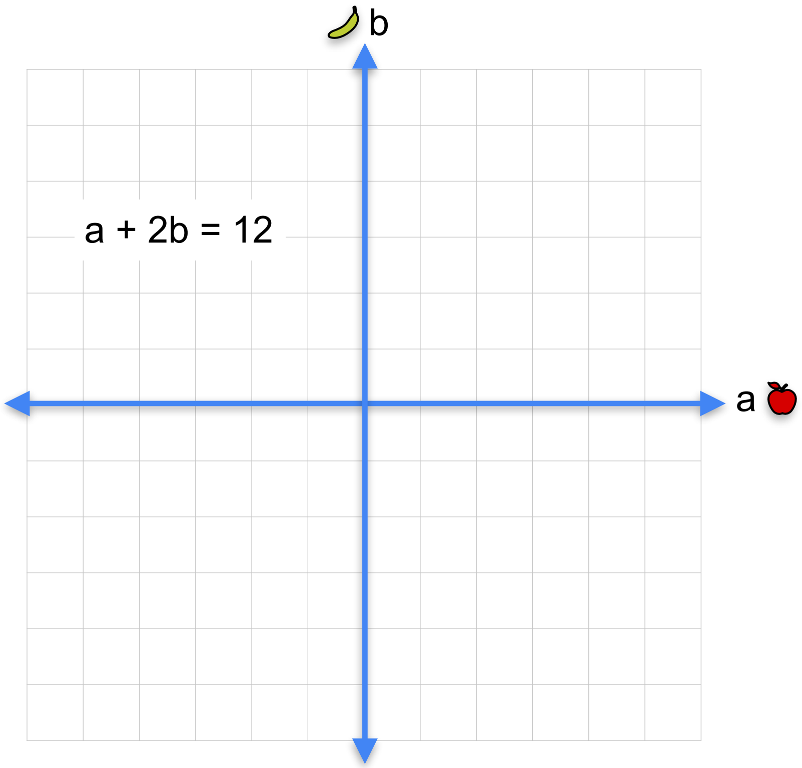
Linear equation in 2 variables -> Line



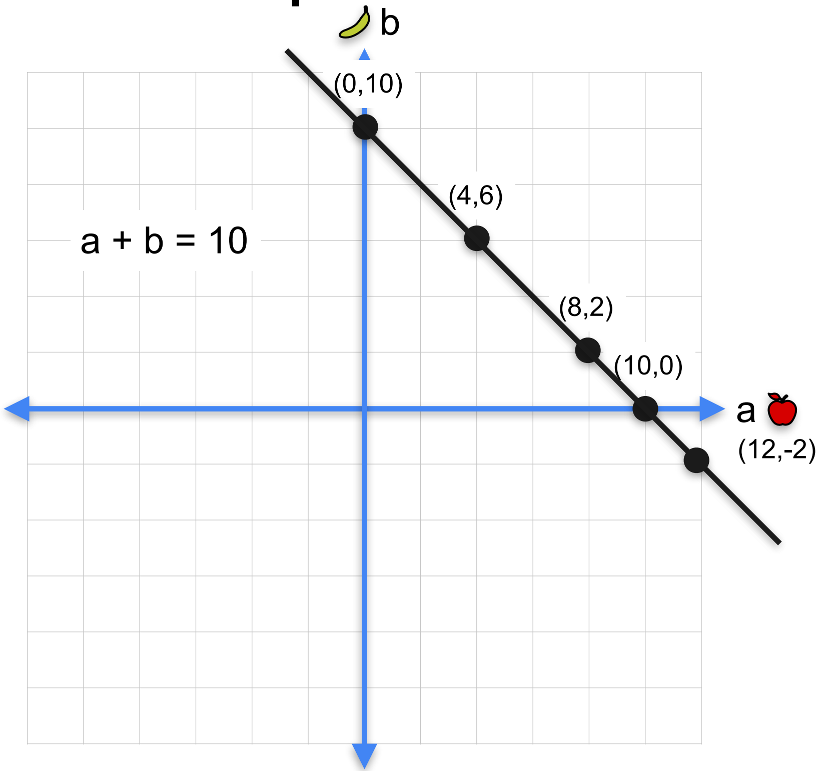


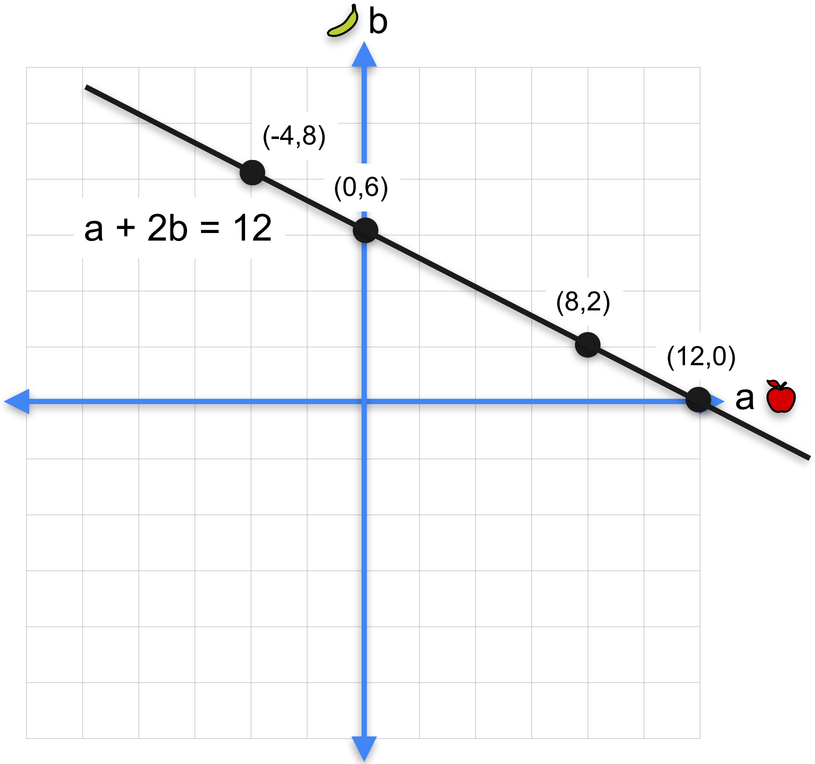
Linear equation in 2 variables -> Line





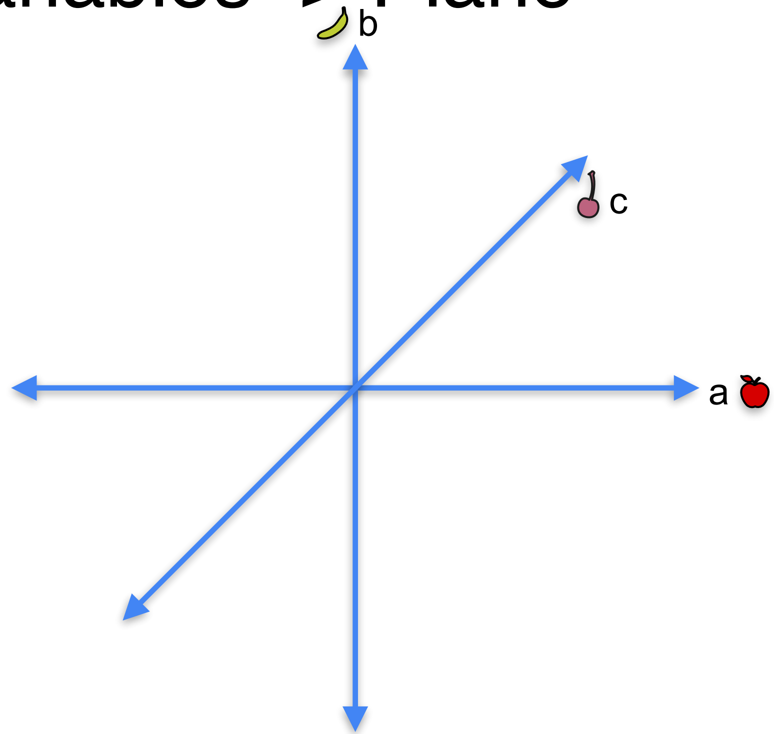
Linear equation in 2 variables -> Line





Linear equation in 3 variables -> Plane

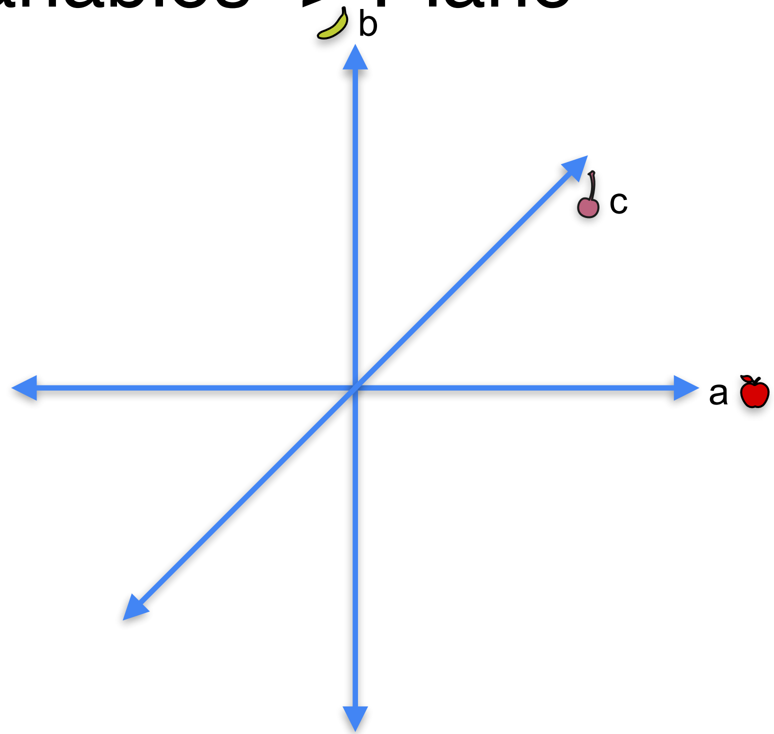
$a + b + c = 1$



Linear equation in 3 variables -> Plane

$a + b + c = \mathbf{1}$

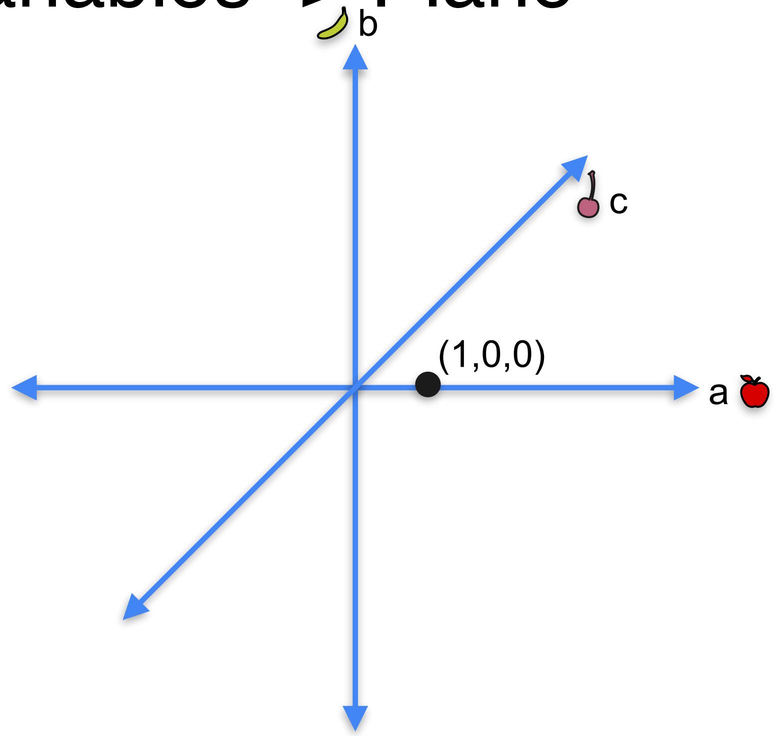
$1 + 0 + 0 = \mathbf{1}$



Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

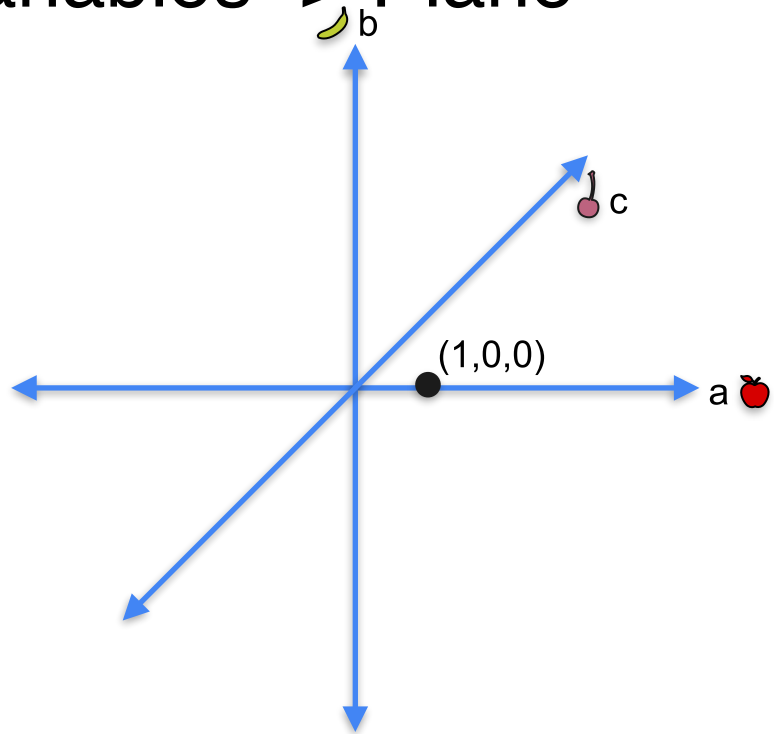


Linear equation in 3 variables -> Plane

a + b + c = **1**

1 + 0 + 0 = **1**

0 + 1 + 0 = **1**

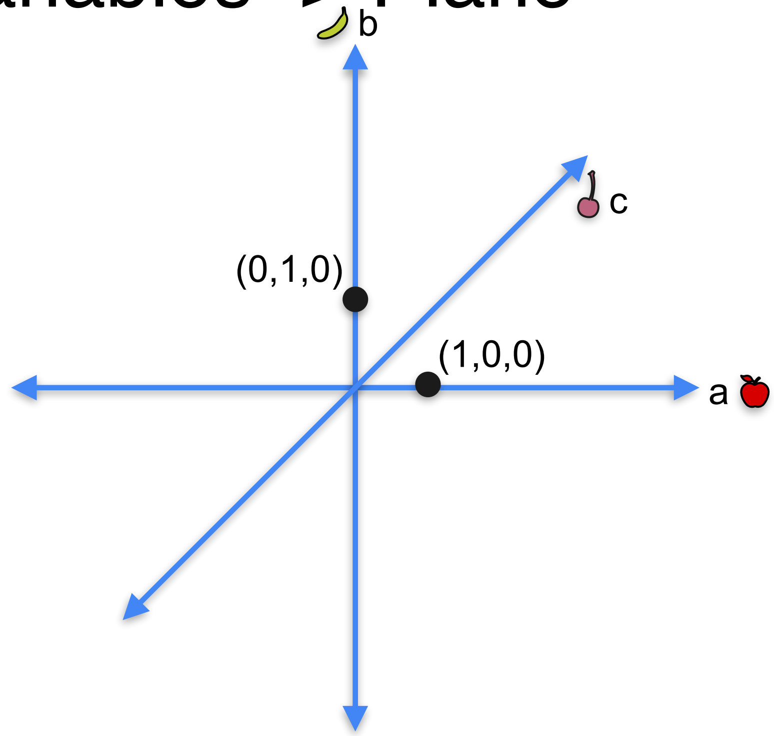


Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

$0 + 1 + 0 = 1$



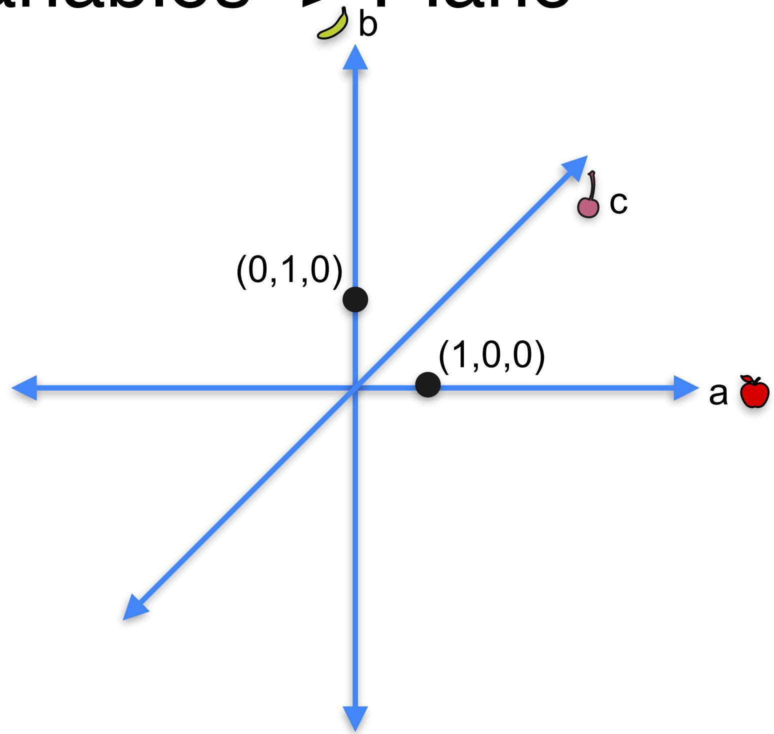
Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

$0 + 1 + 0 = 1$

$0 + 0 + 1 = 1$



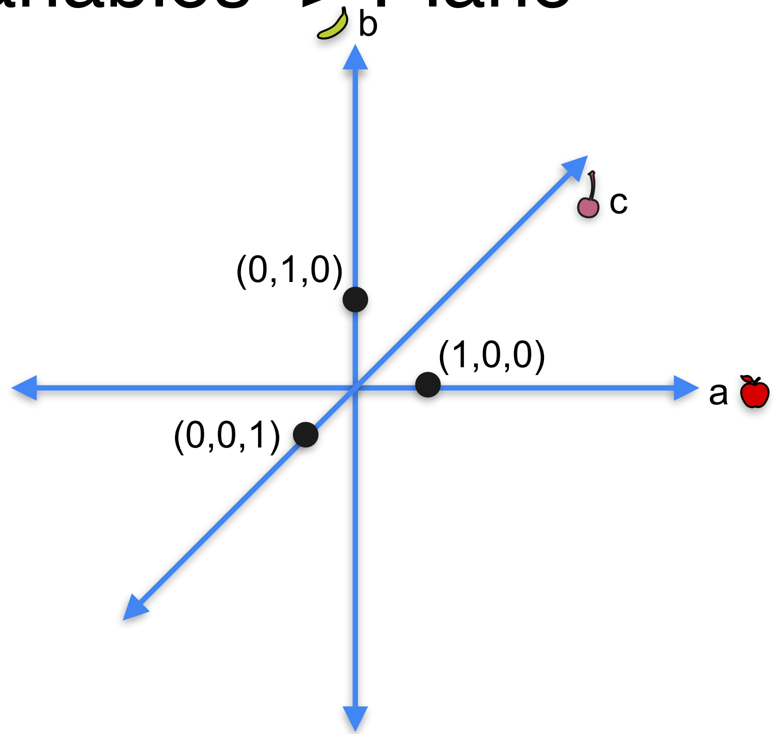
Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

$0 + 1 + 0 = 1$

$0 + 0 + 1 = 1$



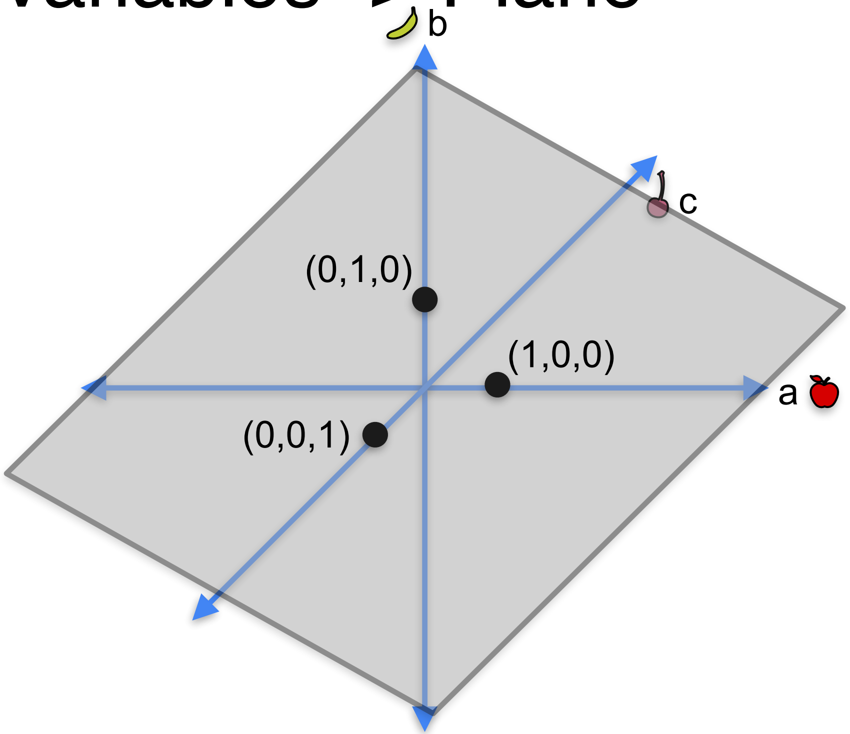
Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

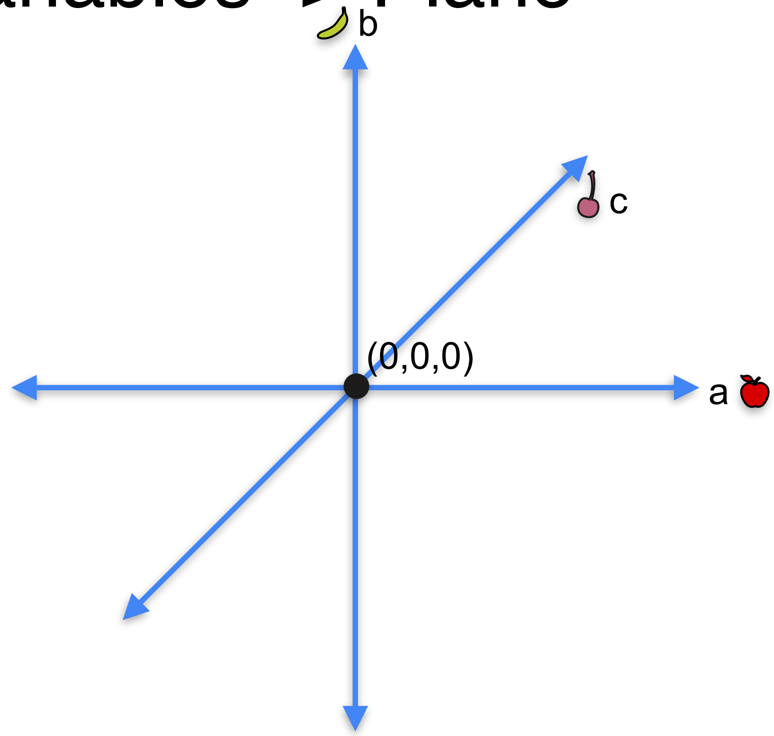
$0 + 1 + 0 = 1$

$0 + 0 + 1 = 1$



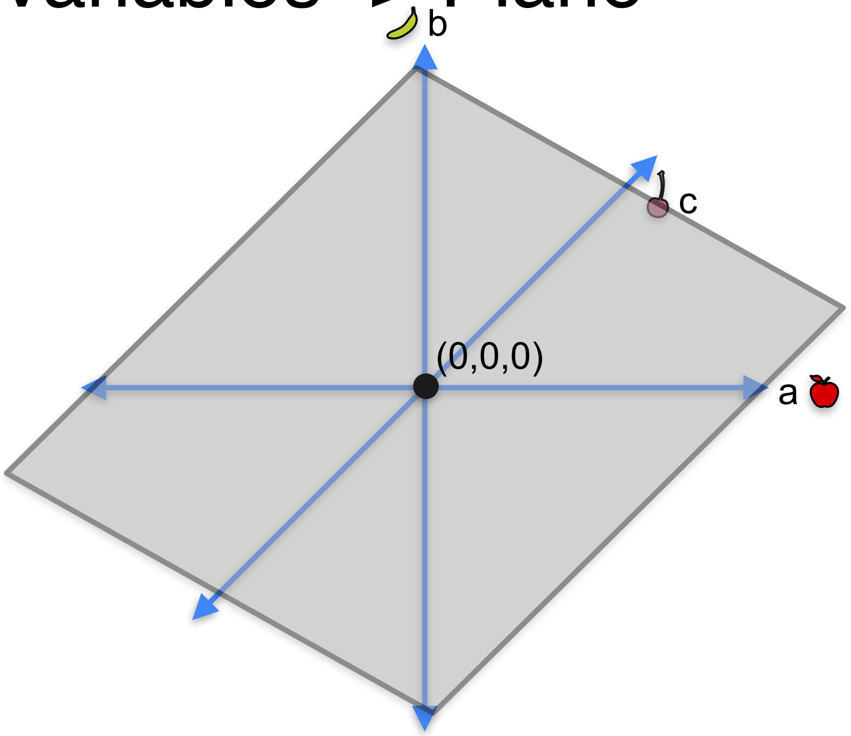
Linear equation in 3 variables -> Plane

$3a - 5b + 2c = \mathbf{0}$



Linear equation in 3 variables -> Plane

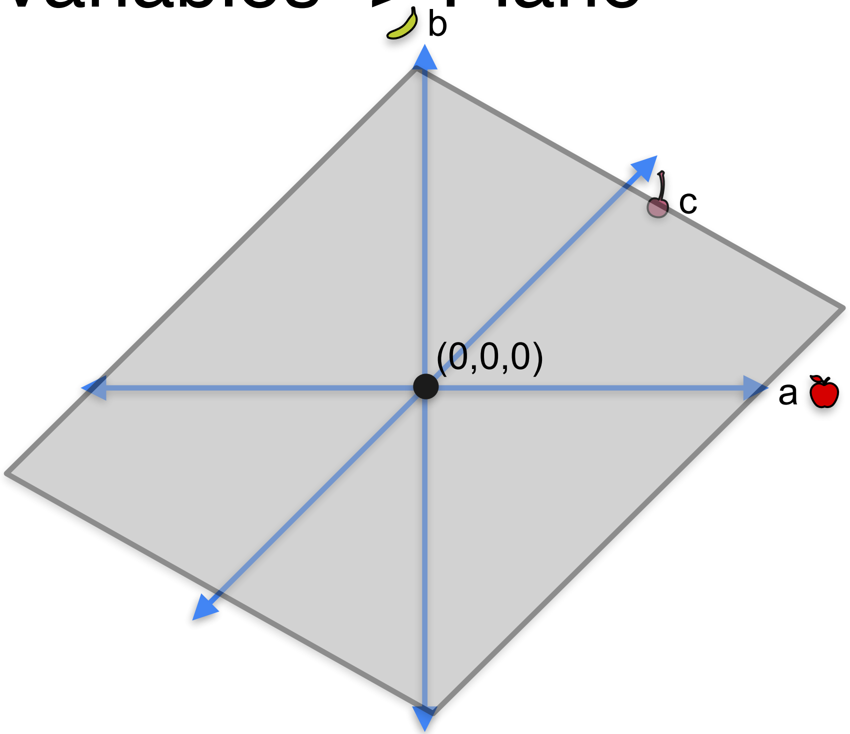
$3a - 5b + 2c = \mathbf{0}$



Linear equation in 3 variables -> Plane

$3a - 5b + 2c = \mathbf{0}$

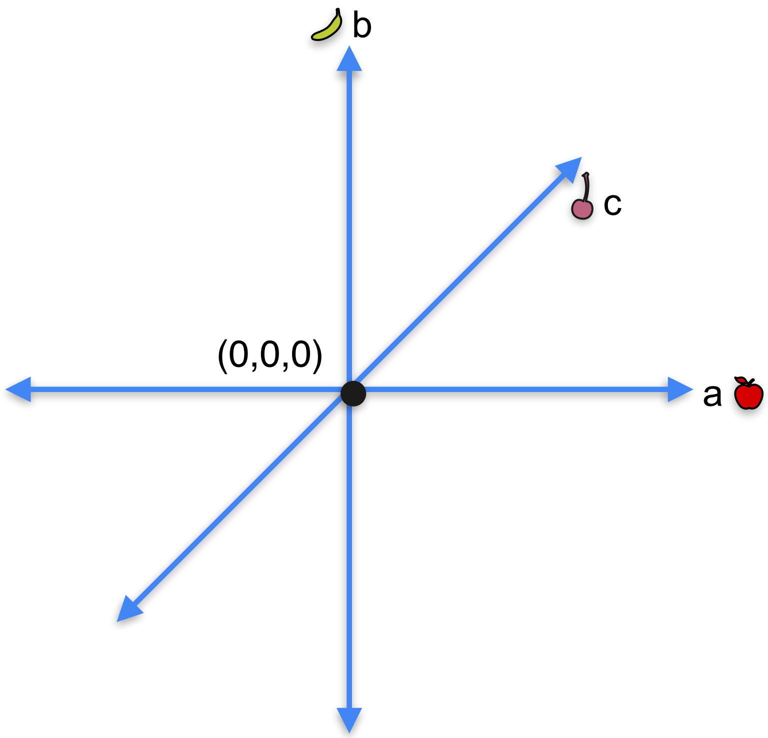
$3(0) + 5(0) + 2(0) = \mathbf{0}$



System 1

System 1

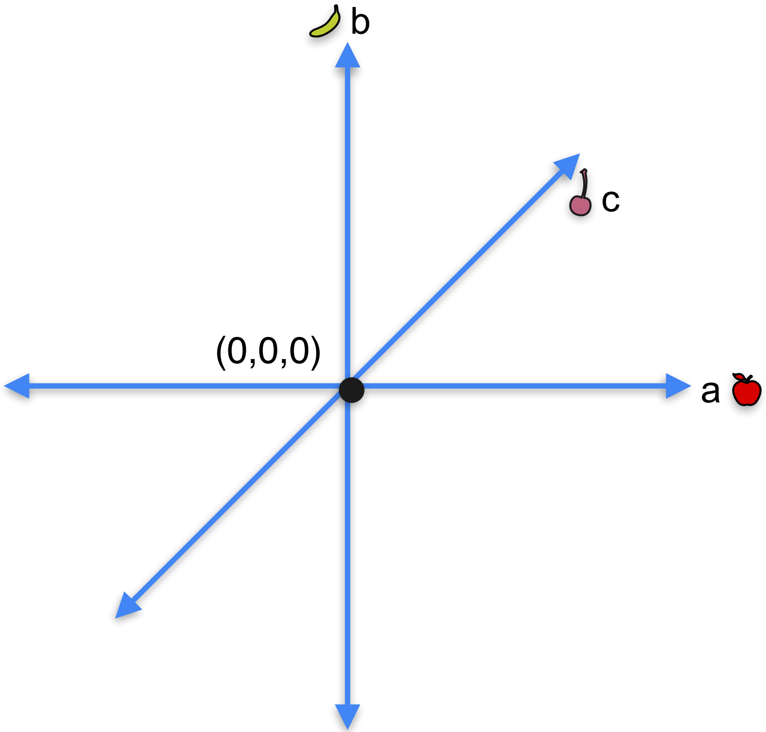
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

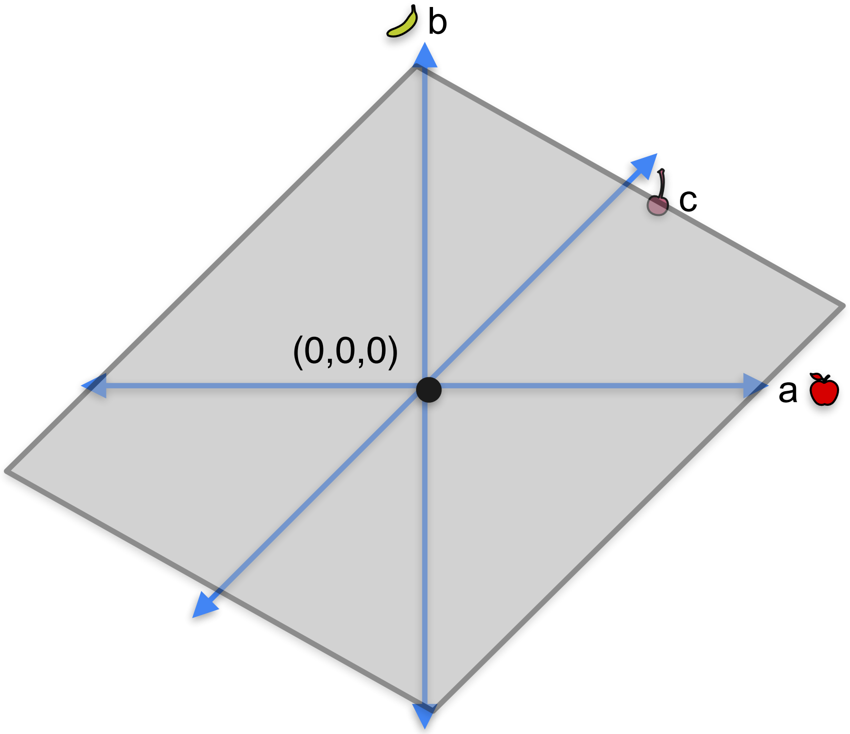
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

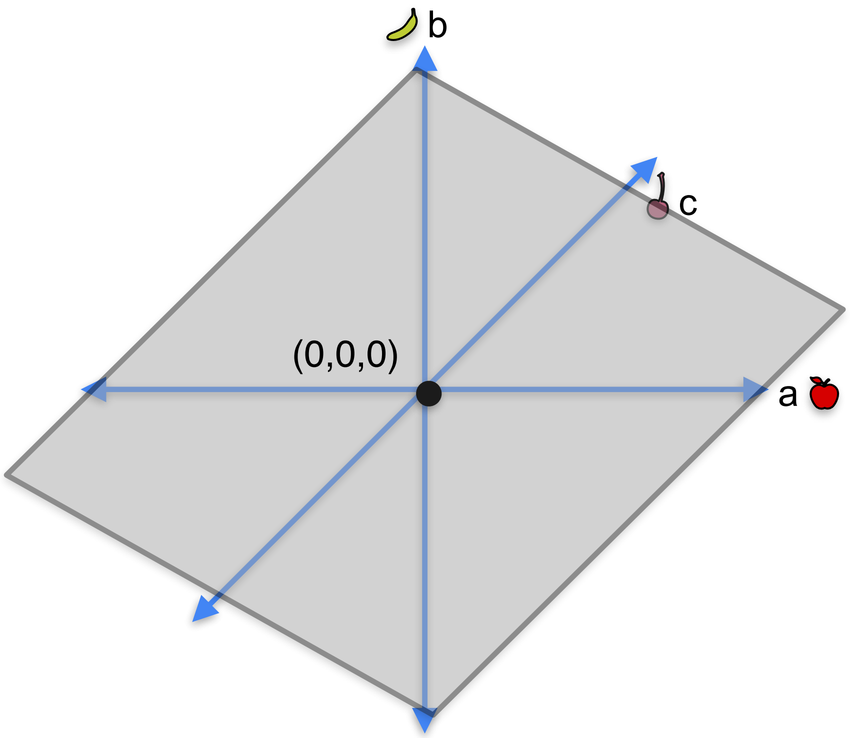
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

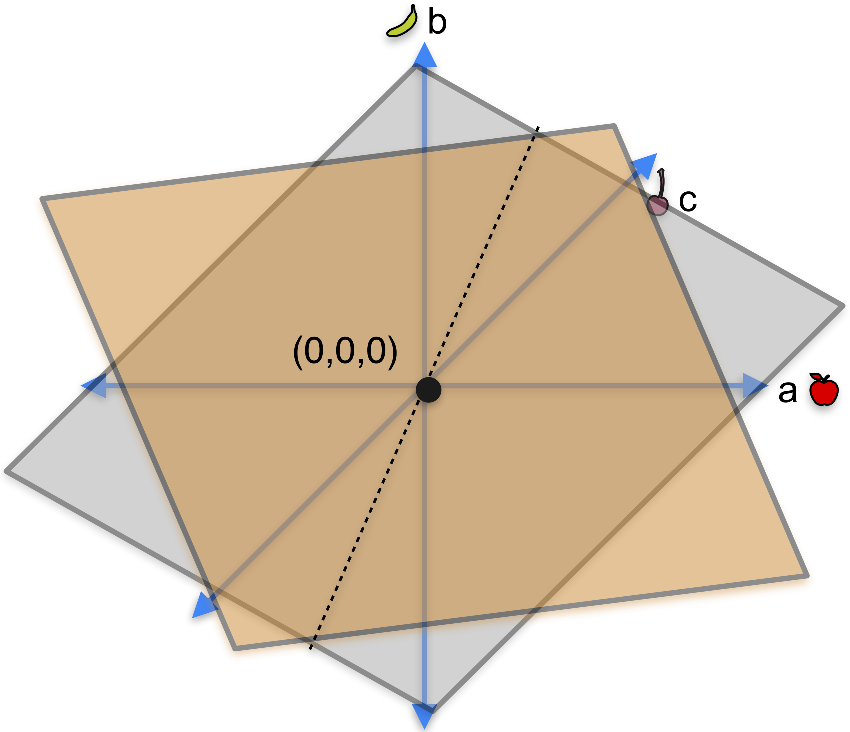
- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**



System 1

System 1

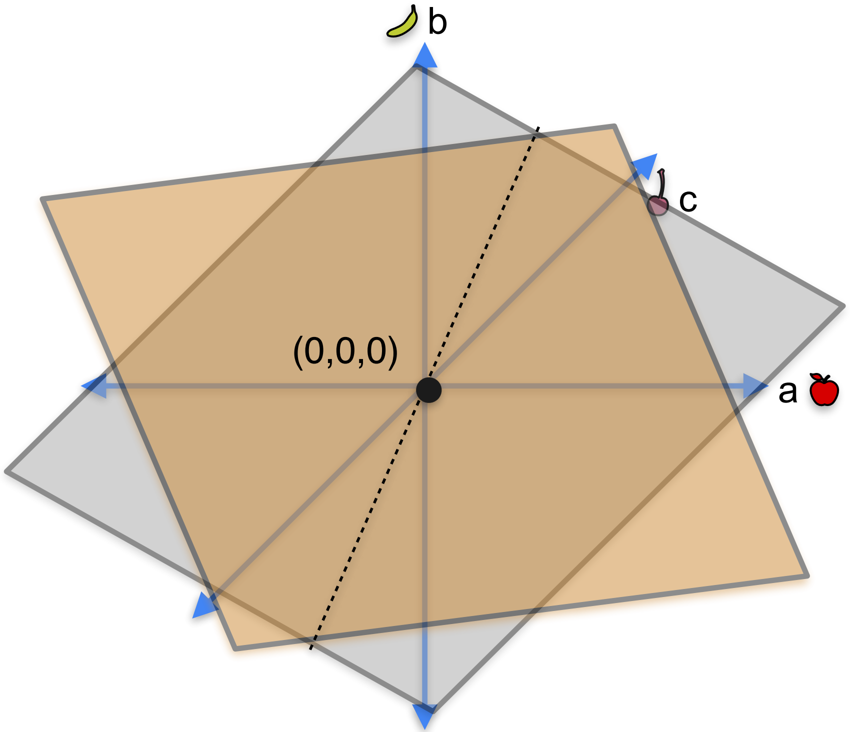
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

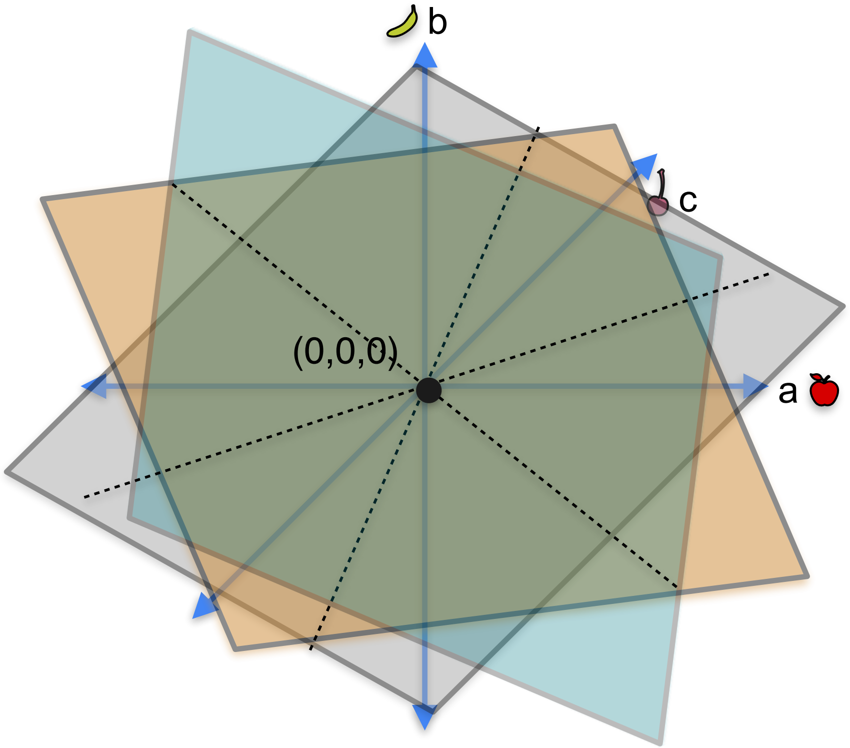
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

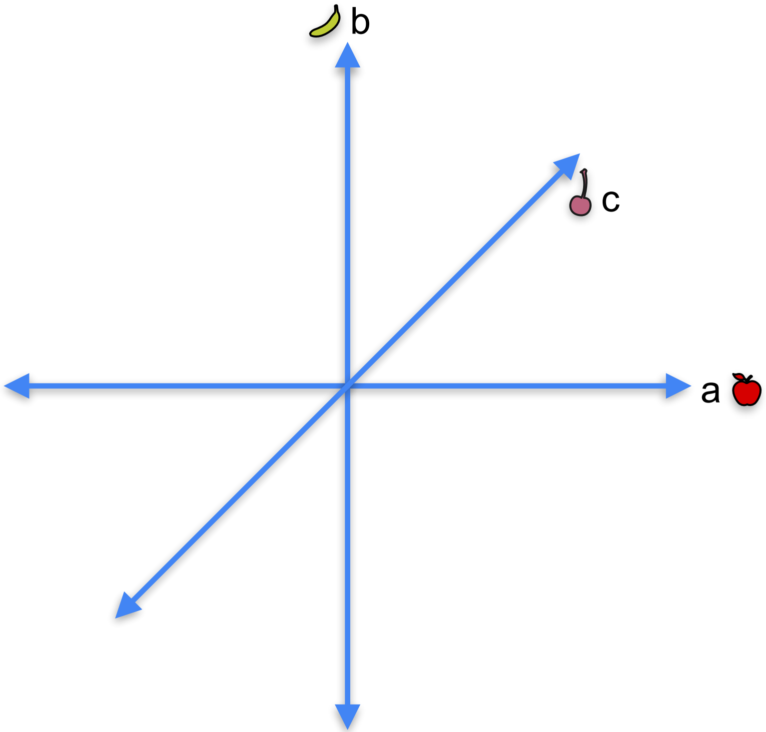
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 2

System 2

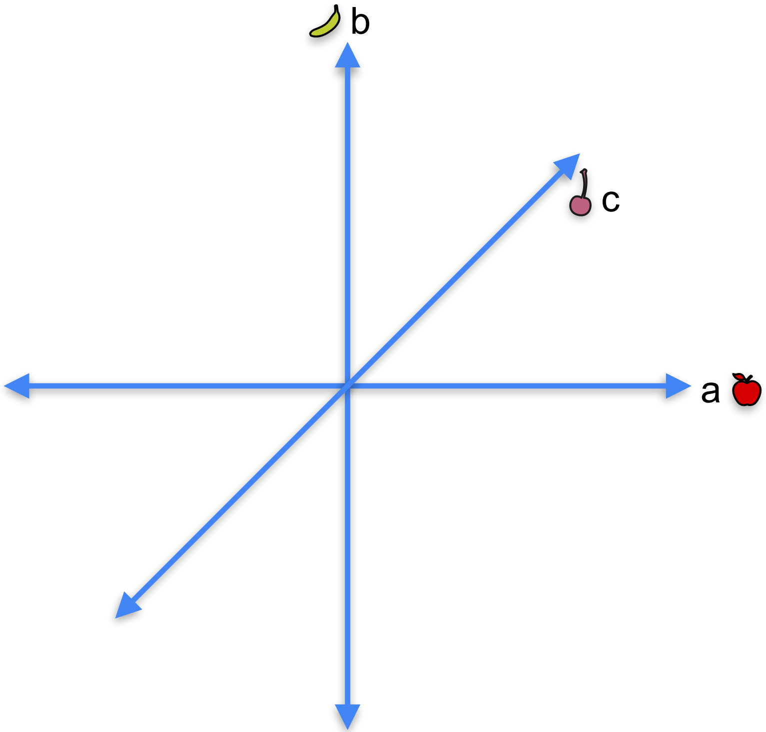
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

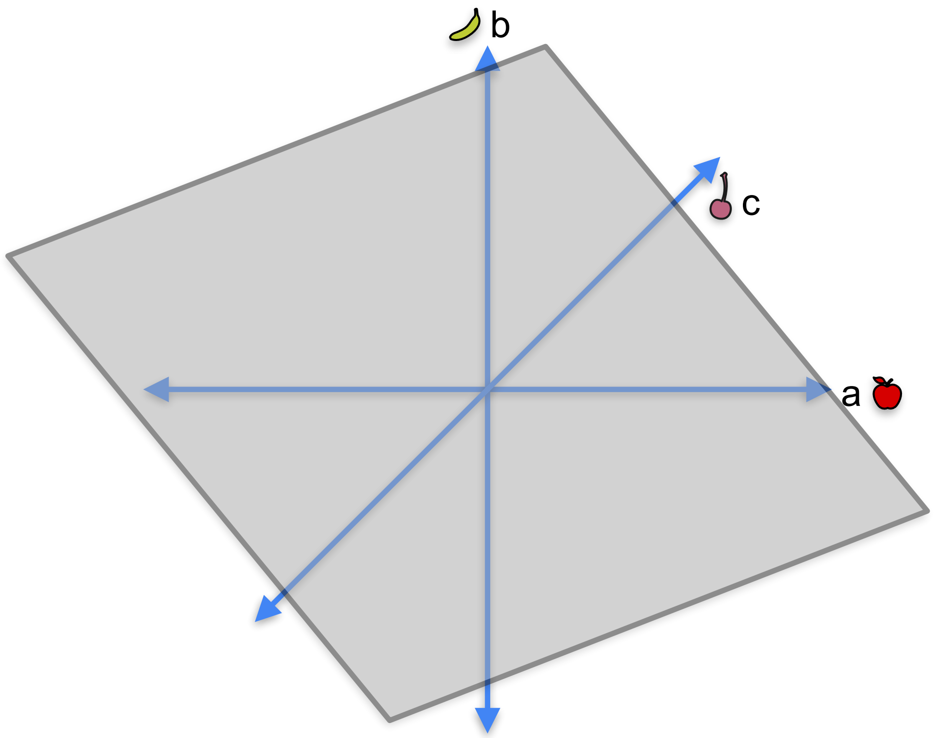
- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**



System 2

System 2

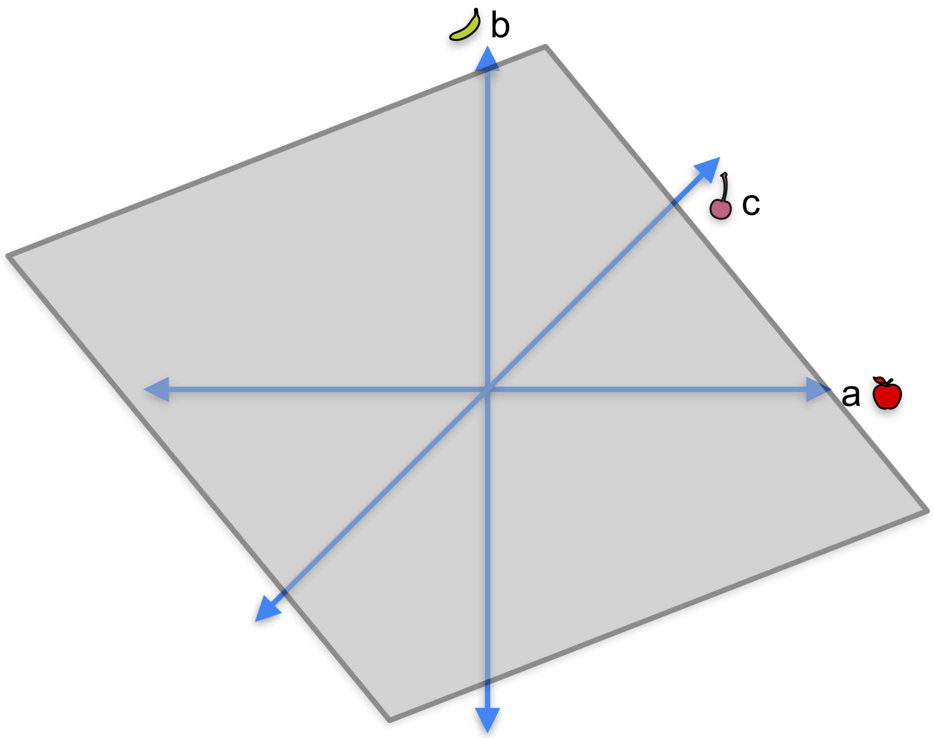
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

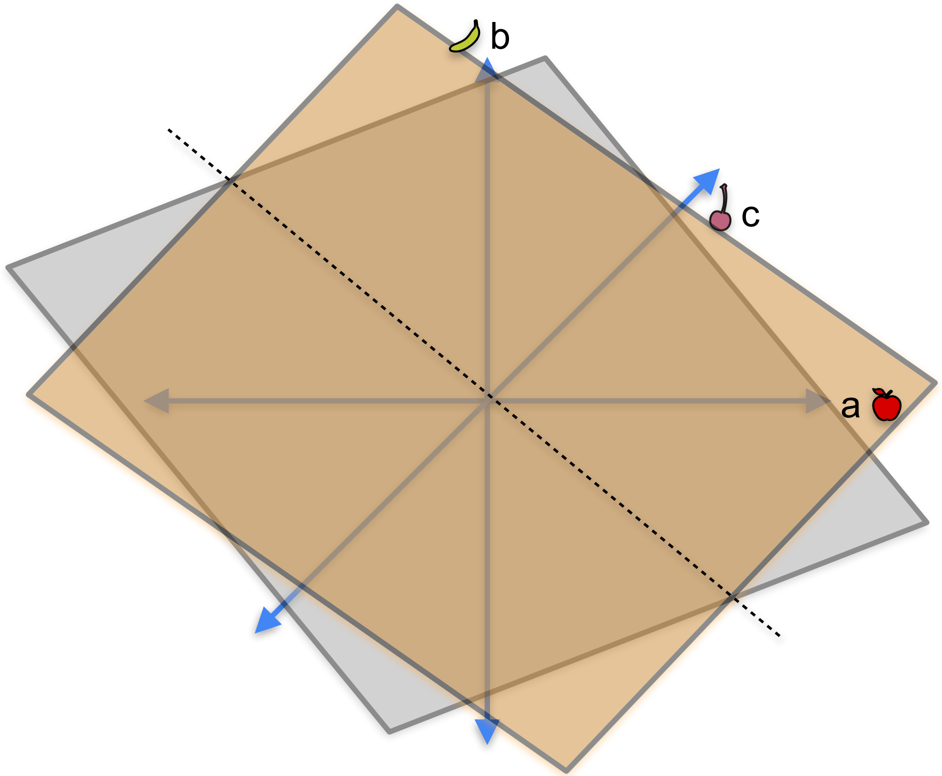
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

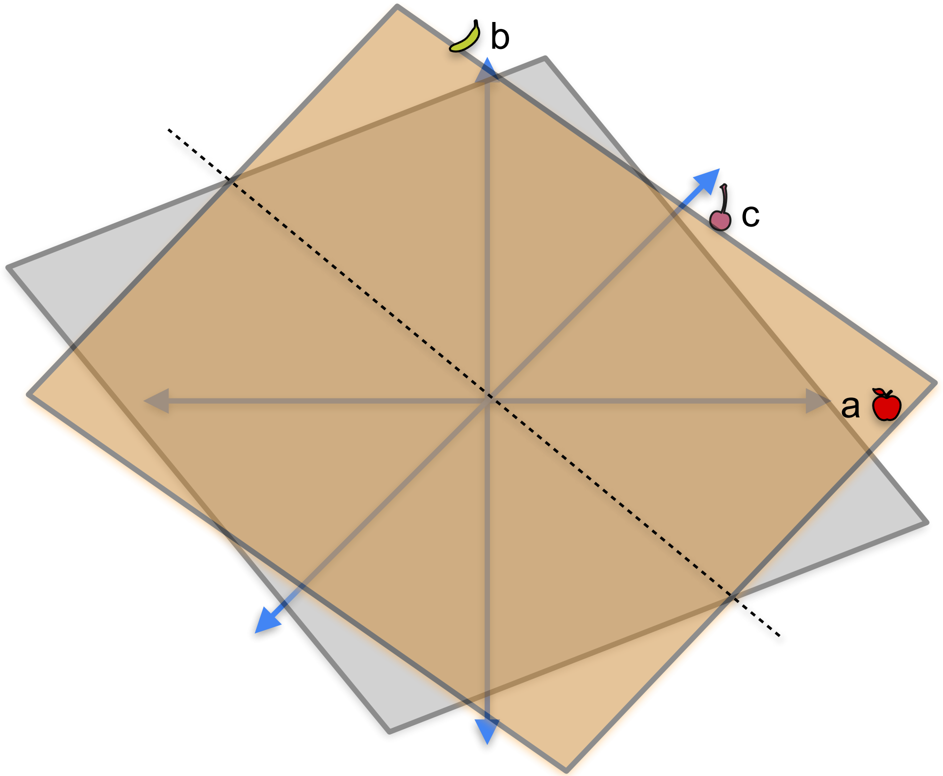
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

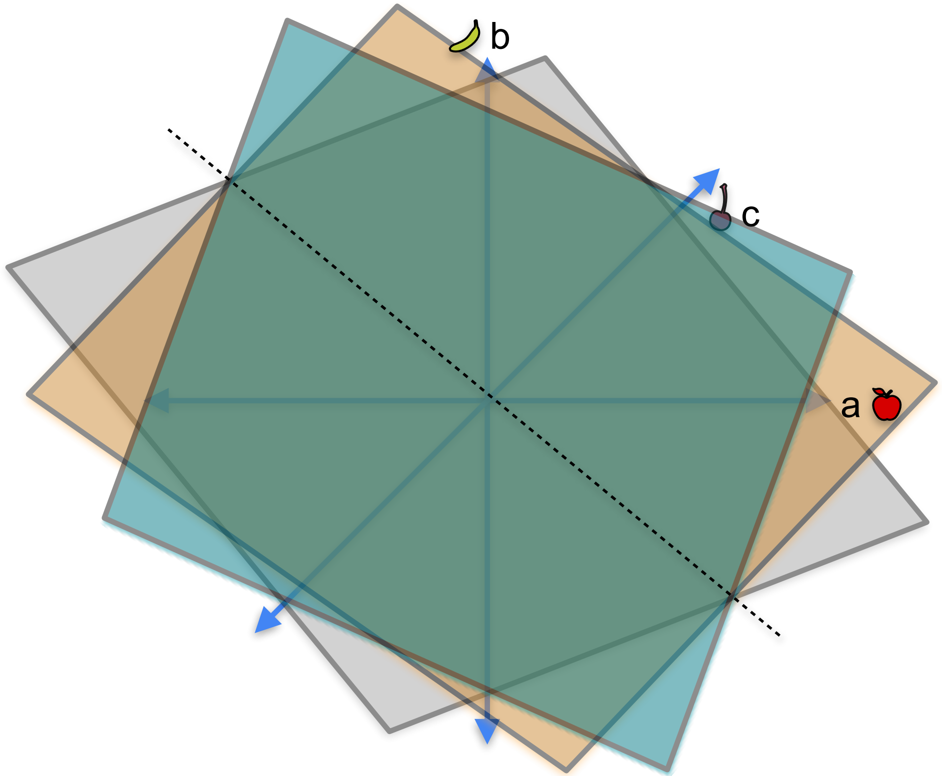
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



System 2

System 2

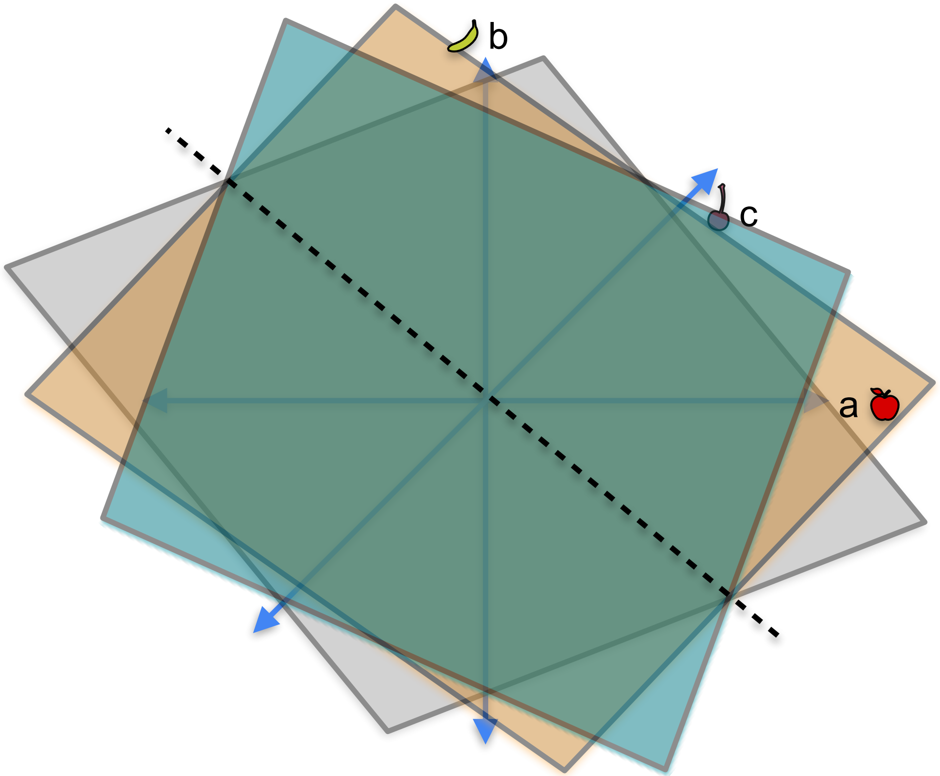
- a + b + c = 0
- a + b + 2c = 0
- a + b + 3c = 0



System 2

System 2

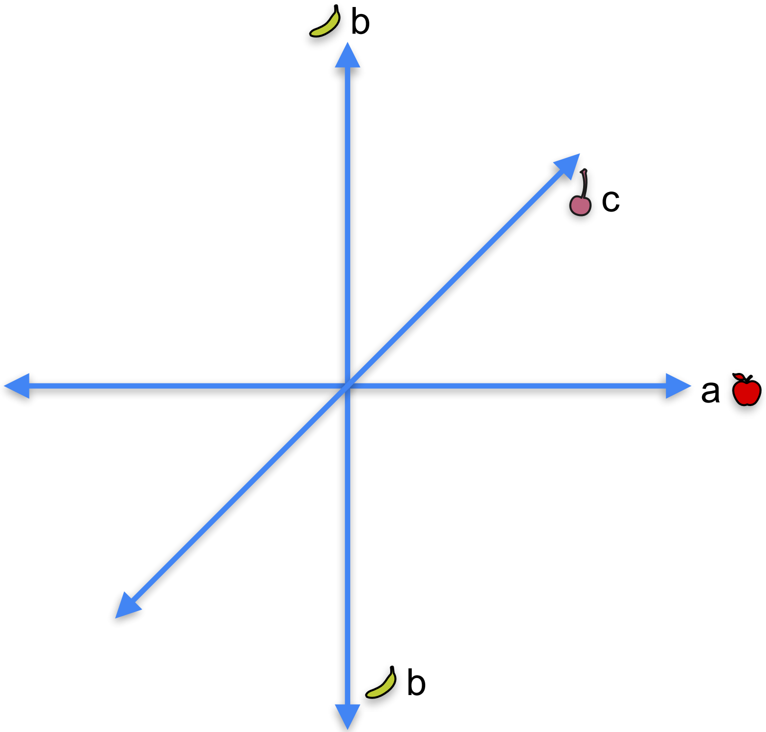
- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**



System 3

System 3

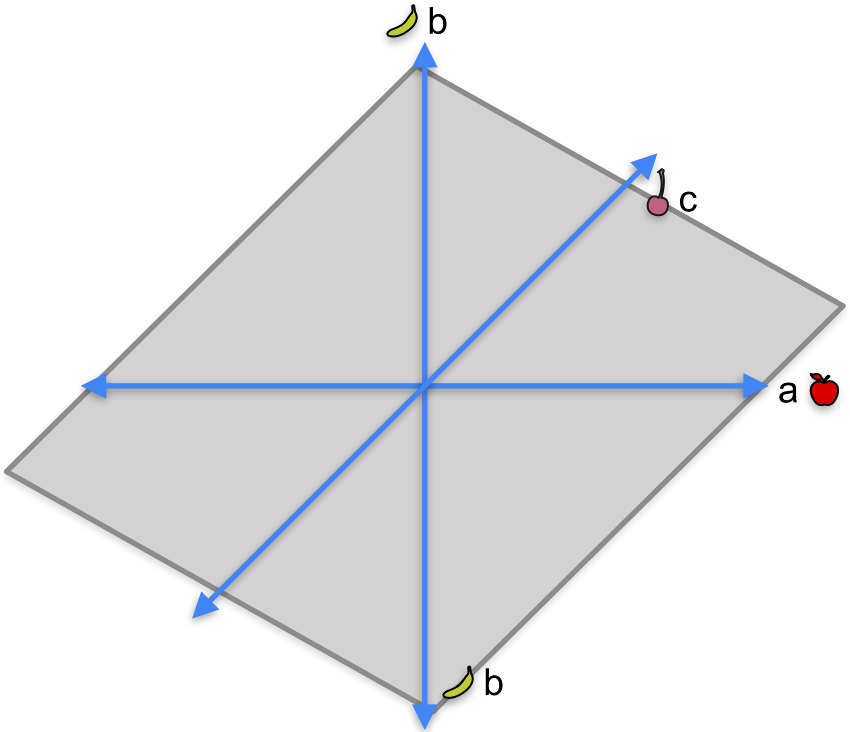
- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

System 3

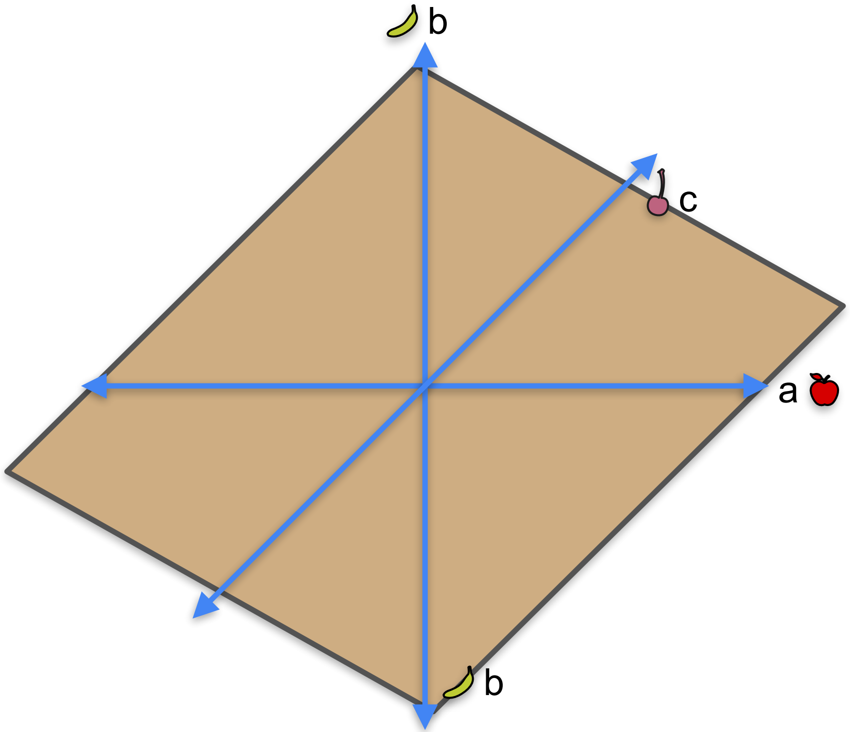
- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

System 3

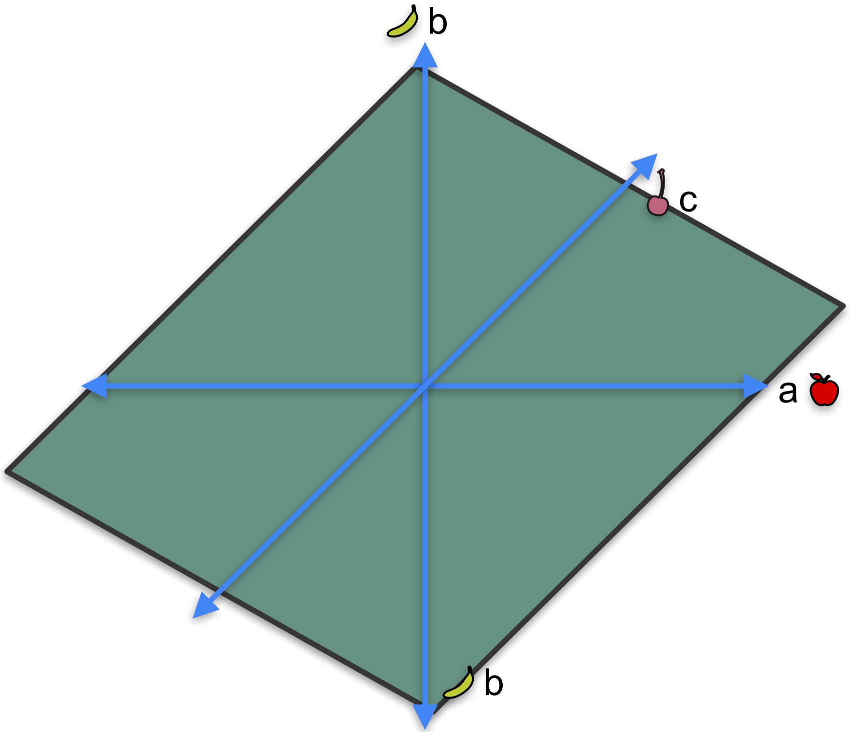
- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



DeepLearning.AI

System of Linear Equations

Linear dependence and independence (3x3)

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$a + 0b + 0c = 1$

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$a + 0b + 0c = 1$
$0a + b + 0c = 2$

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$
\begin{array}{rl}
 & a + 0b + 0c = 1 \\
+ & 0a + b + 0c = 2 \\
\hline
\end{array}
$$

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$\begin{array}{rr} & a + 0b + 0c = 1 \\ + & 0a + b + 0c = 2 \\ \hline & a + b + 0c = 3 \end{array}$$

Linear dependence and independence

$$a = 1$$
$$b = 2$$
$$a + b = 3$$

$$\begin{array}{rr} & a + 0b + 0c = 1 \\ + & 0a + b + 0c = 2 \\ \hline & a + b + 0c = 3 \end{array}$$

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$
\begin{array}{rr}
 & a + 0b + 0c = 1 \\
+ & 0a + b + 0c = 2 \\
\hline
 & a + b + 0c = 3
\end{array}
$$

1	0	0
0	1	0
1	1	0

Linear dependence and independence

$a = \mathbf{1}$
$b = \mathbf{2}$
$a + b = \mathbf{3}$

$$\begin{array}{rr} & a + 0b + 0c = \mathbf{1} \\ + & 0a + b + 0c = \mathbf{2} \\ \hline & a + b + 0c = \mathbf{3} \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$
\begin{array}{r}
a + 0b + 0c = 1 \\
+ \quad 0a + b + 0c = 2 \\
\hline
a + b + 0c = 3
\end{array}
$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$\begin{array}{r} a + 0b + 0c = 1 \\ + \quad 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$\begin{aligned} a + 0b + 0c &= 1 \\ + \quad 0a + b + 0c &= 2 \\ \hline a + b + 0c &= 3 \end{aligned}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$\begin{array}{rl} & a + 0b + 0c = 1 \\ + & 0a + b + 0c = 2 \\ \hline & a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$a = 1$
$b = 2$
$a + b = 3$

$$\begin{array}{rr} & a + 0b + 0c = 1 \\ + & 0a + b + 0c = 2 \\ \hline & a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = 0$$
$$2a + 2b + 2c = 0$$
$$3a + 3b + 3c = 0$$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$a + b + c = \mathbf{0}$$
$$2a + 2b + 2c = \mathbf{0}$$
$$3a + 3b + 3c = \mathbf{0}$$

$$a + b + c = \mathbf{0}$$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$a + b + c = \mathbf{0}$
$2a + 2b + 2c = \mathbf{0}$
$3a + 3b + 3c = \mathbf{0}$

$a + b + c = \mathbf{0}$
$2a + 2b + 2c = \mathbf{0}$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$a + b + c = \mathbf{0}$
$2a + 2b + 2c = \mathbf{0}$
$3a + 3b + 3c = \mathbf{0}$

$$\begin{array}{lr} & a + b + c = \mathbf{0} \\ + & 2a + 2b + 2c = \mathbf{0} \\ \hline \end{array}$$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$a + b + c = 0$$
$$2a + 2b + 2c = 0$$
$$3a + 3b + 3c = 0$$

$$\begin{array}{rr} & a + b + c = 0 \\ + & 2a + 2b + 2c = 0 \\ \hline & 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$a + b + c = \mathbf{0}$$
$$2a + 2b + 2c = \mathbf{0}$$
$$3a + 3b + 3c = \mathbf{0}$$

$$\begin{array}{rr} & a + b + c = \mathbf{0} \\ + & 2a + 2b + 2c = \mathbf{0} \\ \hline & 3a + 3b + 3c = \mathbf{0} \end{array}$$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$
\begin{aligned}
a + b + c &= 0 \\
2a + 2b + 2c &= 0 \\
3a + 3b + 3c &= 0
\end{aligned}
$$

$$
\begin{array}{r}
a + b + c = 0 \\
+ \quad 2a + 2b + 2c = 0 \\
\hline
3a + 3b + 3c = 0
\end{array}
$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Linear dependence and independence

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

$$\begin{array}{rr} & a + b + c = 0 \\ + & 2a + 2b + 2c = 0 \\ \hline & 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Linear dependence and independence

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

$$\begin{array}{r} a + b + c = 0 \\ + \quad 2a + 2b + 2c = 0 \\ \hline 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$a + b + c = \mathbf{0}$

$a + b + 2c = \mathbf{0}$

$a + b + 3c = \mathbf{0}$

$a + b + c = \mathbf{0}$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

$a + b + c = 0$
$a + b + 3c = 0$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline \end{array}$$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

$$\begin{array}{rr} & a + b + c = 0 \\ + & a + b + 3c = 0 \\ \hline & 2a + 2b + 4c = 0 \end{array}$$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

$$
\begin{array}{r}
a + b + c = 0 \\
+ \quad a + b + 3c = 0 \\
\hline
2a + 2b + 4c = 0
\end{array}
$$

$\div 2$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

1	1	1
1	1	2
1	1	3

$$\begin{array}{rr} & a + b + c = 0 \\ + & a + b + 3c = 0 \\ \hline & 2a + 2b + 4c = 0 \end{array}$$

$$\Big\downarrow \div 2$$

$$a + b + 2c = 0$$

Linear dependence and independence

$$a + b + c = \mathbf{0}$$
$$a + b + 2c = \mathbf{0}$$
$$a + b + 3c = \mathbf{0}$$

1	1	1
1	1	2
1	1	3

$$\begin{array}{rr} & a + b + c = \mathbf{0} \\ + & a + b + 3c = \mathbf{0} \\ \hline & 2a + 2b + 4c = \mathbf{0} \end{array}$$

$$\Big\downarrow \div 2$$

$$a + b + 2c = \mathbf{0}$$

Linear dependence and independence

$$a + b + c = 0$$
$$a + b + 2c = 0$$
$$a + b + 3c = 0$$

$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

$$\Big\downarrow \div 2$$

$$a + b + 2c = 0$$

1	1	1
1	1	2
1	1	3

Average of Row 1 and Row 3 is Row 2

Row 2 **depends** on rows 1 and 3

Linear dependence and independence

$a + b + c = 0$
$a + b + 2c = 0$
$a + b + 3c = 0$

$$\begin{array}{r} a + b + c = 0 \\ +\quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

$\downarrow \div 2$

$a + b + 2c = 0$

1	1	1
1	1	2
1	1	3

Average of Row 1 and Row 3 is Row 2
Row 2 **depends** on rows 1 and 3

Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = \mathbf{0}$$
$$a + 2b + c = \mathbf{0}$$
$$a + b + 2c = \mathbf{0}$$

1	1	1
1	2	1
1	1	2

Linear dependence and independence

$a + b + c = 0$
$a + 2b + c = 0$ → No relations between equations
$a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

Linear dependence and independence

$$
\begin{aligned}
a + b + c &= \mathbf{0} \\
a + 2b + c &= \mathbf{0} \\
a + b + 2c &= \mathbf{0}
\end{aligned}
$$

→ No relations between equations

1	1	1
1	2	1
1	1	2

No relations between rows

Linear dependence and independence

$$a + b + c = \mathbf{0}$$
$$a + 2b + c = \mathbf{0}$$
$$a + b + 2c = \mathbf{0}$$

→ No relations between equations

1	1	1
1	2	1
1	1	2

No relations between rows

Rows are **linearly independent**

Quiz: Linear dependence and independence

Problem: Determine if the following matrices have linearly dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

3Row1 + 2Row2 = Row3

Dependent (singular)

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

3Row1 + 2Row2 = Row3

Dependent (singular)

Row1 - Row2 = Row3

Dependent (singular)

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

3Row1 + 2Row2 = Row3

Dependent (singular)

1	1	1
1	1	2
0	0	-1

Row1 - Row2 = Row3

Dependent (singular)

1	1	1
0	2	2
0	0	3

No relations

Independent (Non-singular)

1	2	5
0	3	-2
2	4	10

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

3Row1 + 2Row2 = Row3

Dependent (singular)

1	1	1
1	1	2
0	0	-1

Row1 - Row2 = Row3

Dependent (singular)

1	1	1
0	2	2
0	0	3

No relations

Independent (Non-singular)

1	2	5
0	3	-2
2	4	10

2Row1 = Row3

Dependent (singular)

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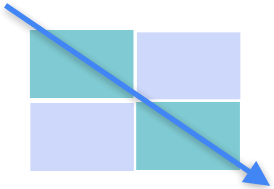
System of Linear Equations

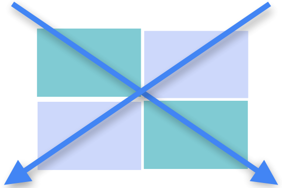
The determinant (3x3)

Diagonals in a 3x3 matrix

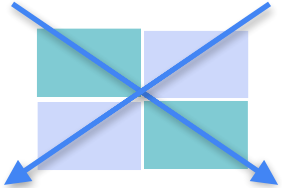
Diagonals in a 3x3 matrix



Diagonals in a 3x3 matrix

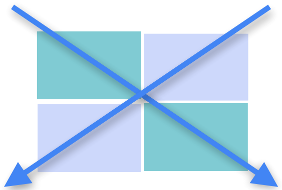


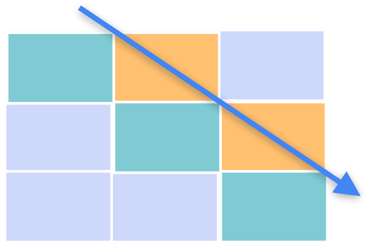
Diagonals in a 3x3 matrix



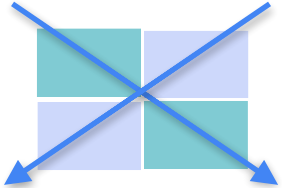
Diagonals in a 3x3 matrix

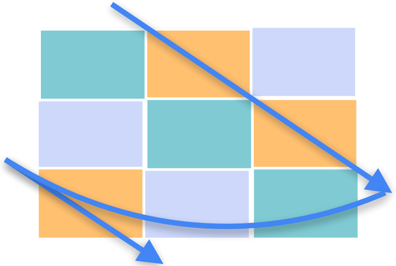
Diagonals in a 3x3 matrix



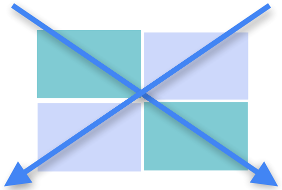


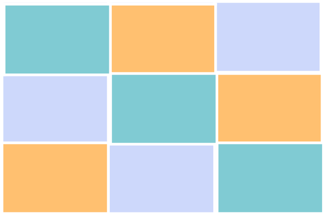
Diagonals in a 3x3 matrix



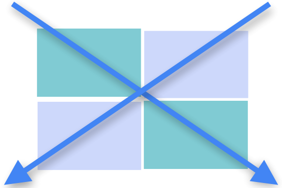


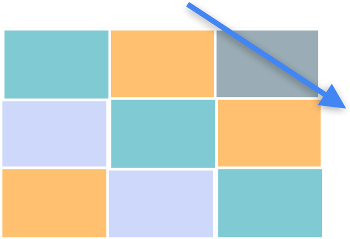
Diagonals in a 3x3 matrix





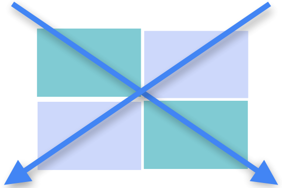
Diagonals in a 3x3 matrix





Diagonals in a 3x3 matrix

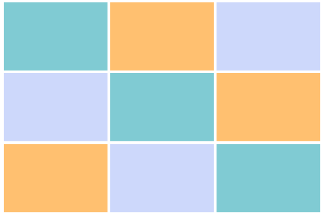
Diagonals in a 3x3 matrix



Determinant

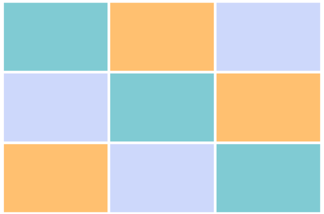
Determinant

Add

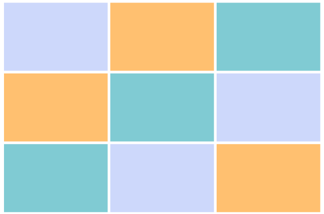


Determinant

Add



Subtract



The determinant

1	1	1
1	2	1
1	1	2

The determinant

1	1	1
1	2	1
1	1	2

The determinant

1	1	1
1	2	1
1	1	2

1		
	2	
		2

$$+ \ 1 \cdot 2 \cdot 2$$

The determinant

1	1	1
1	2	1
1	1	2

1		
	2	
		2

$$+ \ 1 \cdot 2 \cdot 2$$

The determinant

1	1	1
1	2	1
1	1	2

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 2 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 1 \\ 1 & & \end{array}$$

$$+\ 1 \cdot 2 \cdot 2 \qquad\qquad +\ 1 \cdot 1 \cdot 1$$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{matrix} 1 & & \\ & 2 & \\ & & 2 \end{matrix} \qquad\qquad \begin{matrix} & 1 & \\ & & 1 \\ 1 & & \end{matrix}$$

$$+\ 1 \cdot 2 \cdot 2 \qquad\qquad +\ 1 \cdot 1 \cdot 1$$

The determinant

1	1	1
1	2	1
1	1	2

1		
	2	
		2

$+ \; 1 \cdot 2 \cdot 2$

	1	
		1
1		

$+ \; 1 \cdot 1 \cdot 1$

		1
1		
	1	

$+ \; 1 \cdot 1 \cdot 1$

The determinant

1	1	1
1	2	1
1	1	2

$$\begin{matrix} 1 & & \\ & 2 & \\ & & 2 \end{matrix} \qquad \begin{matrix} & 1 & \\ & & 1 \\ 1 & & \end{matrix} \qquad \begin{matrix} & & 1 \\ 1 & & \\ & 1 & \end{matrix}$$

$$+\ 1 \cdot 2 \cdot 2 \qquad +\ 1 \cdot 1 \cdot 1 \qquad +\ 1 \cdot 1 \cdot 1$$

The determinant

1	1	1
1	2	1
1	1	2

$+ \; 1 \cdot 2 \cdot 2$

$+ \; 1 \cdot 1 \cdot 1$

$+ \; 1 \cdot 1 \cdot 1$

$- \; 1 \cdot 2 \cdot 1$

The determinant

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 2 & 1 \\ \hline 1 & 1 & 2 \\ \hline \end{array}$$

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 2 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 1 \\ 1 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 1 & & \\ & 1 & \end{array}$$

$$+\ 1 \cdot 2 \cdot 2 \qquad +\ 1 \cdot 1 \cdot 1 \qquad +\ 1 \cdot 1 \cdot 1$$

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 1 & & \end{array}$$

$$-\ 1 \cdot 2 \cdot 1$$

The determinant

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 2 & 1 \\ \hline 1 & 1 & 2 \\ \hline \end{array}$$

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 2 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 1 \\ 1 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 1 & & \\ & 1 & \end{array}$$

$$+\ 1 \cdot 2 \cdot 2 \qquad +\ 1 \cdot 1 \cdot 1 \qquad +\ 1 \cdot 1 \cdot 1$$

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 1 & & \end{array} \qquad \begin{array}{ccc} 1 & & \\ & & 1 \\ & 1 & \end{array}$$

$$-\ 1 \cdot 2 \cdot 1 \qquad -\ 1 \cdot 1 \cdot 1$$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 2 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 1 \\ 1 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 1 & & \\ & 1 & \end{array}$$

$$+\ 1 \cdot 2 \cdot 2 \qquad +\ 1 \cdot 1 \cdot 1 \qquad +\ 1 \cdot 1 \cdot 1$$

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 1 & & \end{array} \qquad \begin{array}{ccc} 1 & & \\ & & 1 \\ & 1 & \end{array}$$

$$-\ 1 \cdot 2 \cdot 1 \qquad -\ 1 \cdot 1 \cdot 1$$

The determinant

1	1	1
1	2	1
1	1	2

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 2 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 1 \\ 1 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 1 & & \\ & 1 & \end{array}$$

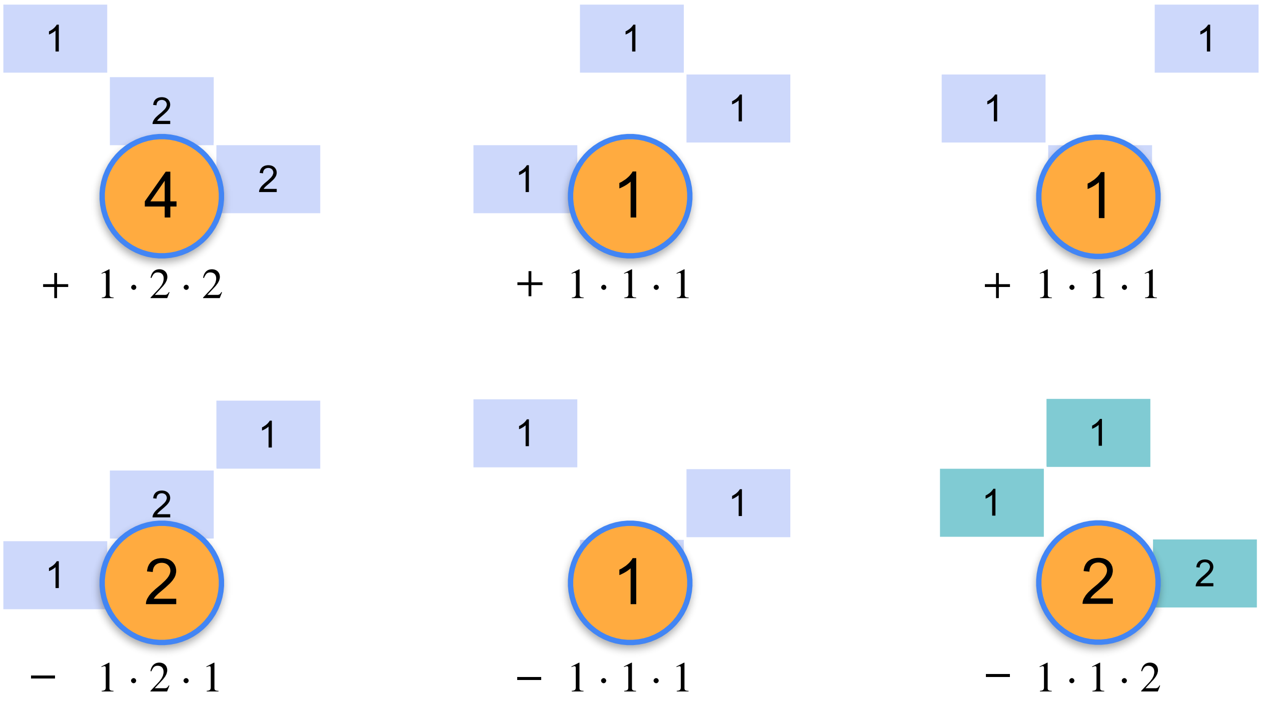
$$+\ 1 \cdot 2 \cdot 2 \qquad +\ 1 \cdot 1 \cdot 1 \qquad +\ 1 \cdot 1 \cdot 1$$

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 1 & & \end{array} \qquad \begin{array}{ccc} 1 & & \\ & & 1 \\ & 1 & \end{array} \qquad \begin{array}{ccc} & 1 & \\ 1 & & \\ & & 2 \end{array}$$

$$-\ 1 \cdot 2 \cdot 1 \qquad -\ 1 \cdot 1 \cdot 1 \qquad -\ 1 \cdot 1 \cdot 2$$

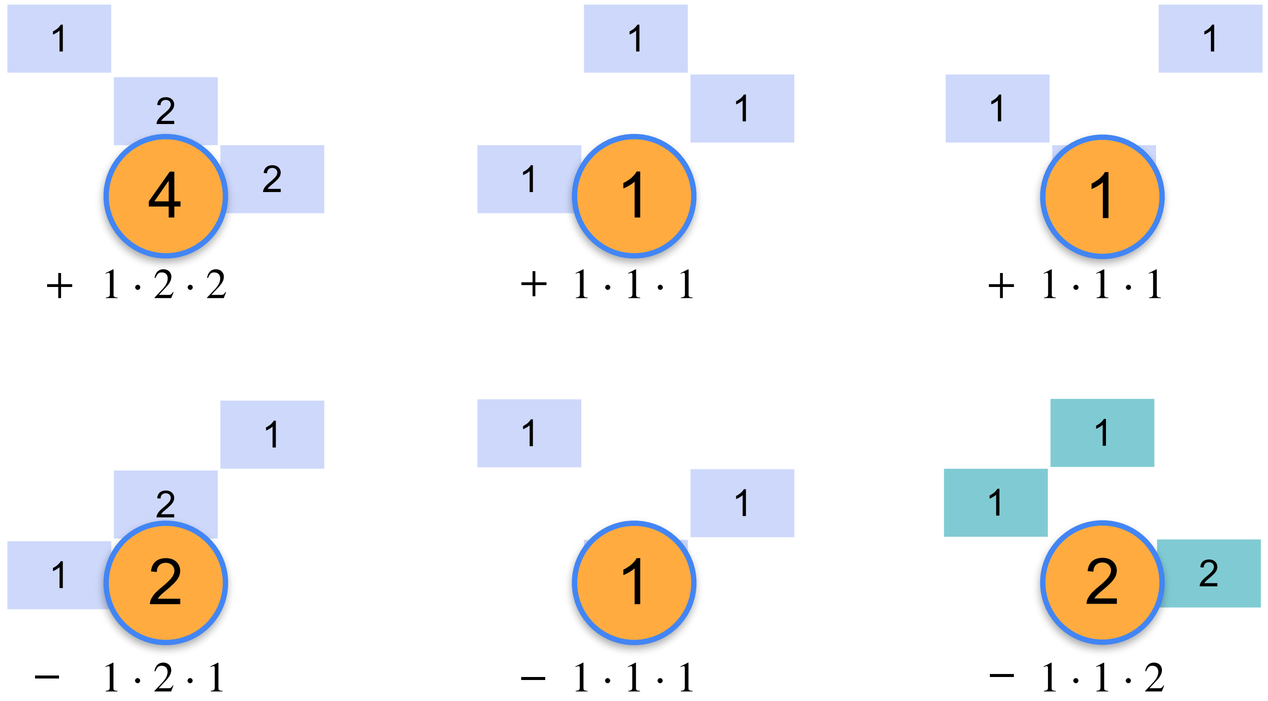
The determinant

1	1	1
1	2	1
1	1	2



The determinant

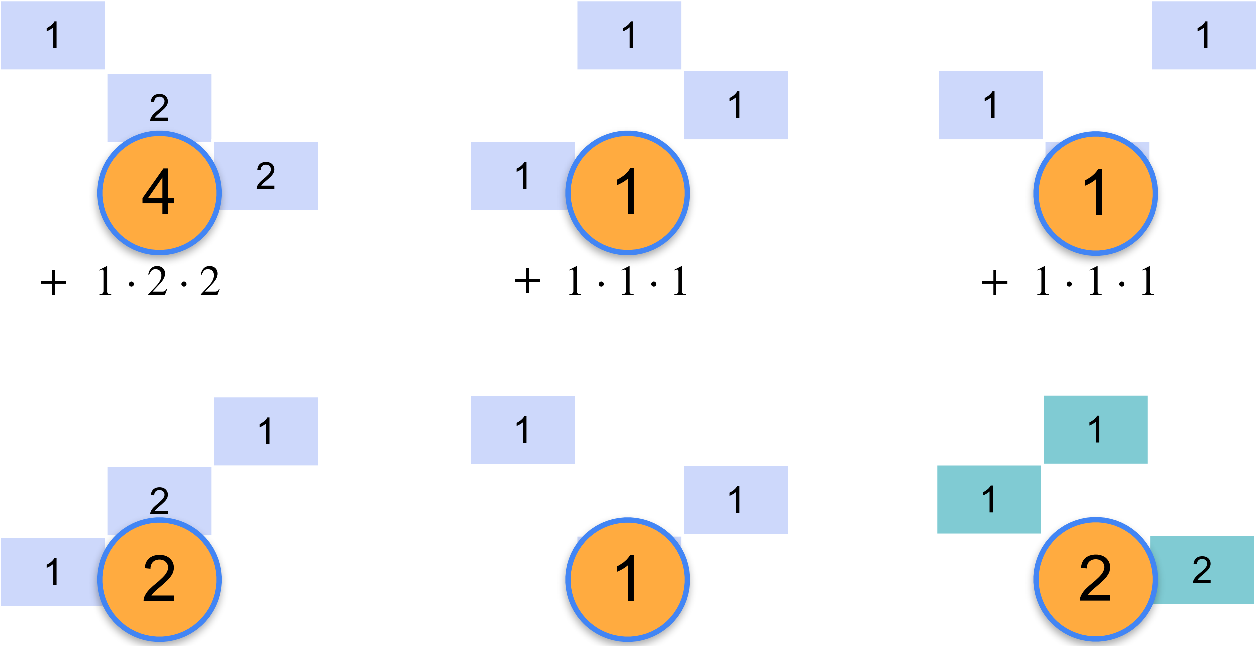
1	1	1
1	2	1
1	1	2



Det = 4+1+1
-2-1-2

The determinant

1	1	1
1	2	1
1	1	2



$+\ 1 \cdot 2 \cdot 2$ $\quad + \ 1 \cdot 1 \cdot 1$ $\quad + \ 1 \cdot 1 \cdot 1$

$-\ 1 \cdot 2 \cdot 1$ $\quad - \ 1 \cdot 1 \cdot 1$ $\quad - \ 1 \cdot 1 \cdot 2$

Det = 4+1+1
-2-1-2

= 1

Quiz: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Determinant = 0

Singular

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

Determinant = 6

Non-singular

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

Determinant = 6

Non-singular

1	2	5
0	3	-2
2	4	10

Determinant = 0

Singular

The determinant

1	1	1
0	2	2
0	0	3

Det = 6+0+0-0-0-0

= 6

The determinant

1	1	1
0	2	2
0	0	3

$$\begin{matrix} 1 & & \\ & 2 & \\ & & 3 \end{matrix}$$

$$+ \ 1 \cdot 2 \cdot 3$$

Det = 6+0+0-0-0-0

= 6

The determinant

1	1	1
0	2	2
0	0	3

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 3 \end{array}$$

$$+\ 1 \cdot 2 \cdot 3$$

$$\begin{array}{ccc} & 1 & \\ & & 2 \\ 0 & & \end{array}$$

$$+\ 1 \cdot 2 \cdot 0$$

Det = 6+0+0-0-0-0

= 6

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 3 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 2 \\ 0 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 0 & & \\ & 0 & \end{array}$$

$$+\ 1 \cdot 2 \cdot 3 \qquad +\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 0 \cdot 0$$

Det = 6+0+0-0-0-0

= 6

The determinant

1	1	1
0	2	2
0	0	3

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 3 \end{array}$$

$$+\ 1 \cdot 2 \cdot 3$$

$$\begin{array}{ccc} & 1 & \\ & & 2 \\ 0 & & \end{array}$$

$$+\ 1 \cdot 2 \cdot 0$$

$$\begin{array}{ccc} & & 1 \\ 0 & & \\ & 0 & \end{array}$$

$$+\ 1 \cdot 0 \cdot 0$$

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 0 & & \end{array}$$

$$-\ 1 \cdot 2 \cdot 0$$

Det = 6+0+0-0-0-0

= 6

The determinant

1	1	1
0	2	2
0	0	3

$+\ 1 \cdot 2 \cdot 3$

$+\ 1 \cdot 2 \cdot 0$

$+\ 1 \cdot 0 \cdot 0$

Det = 6+0+0-0-0-0

= 6

$-\ 1 \cdot 2 \cdot 0$

$-\ 1 \cdot 2 \cdot 0$

The determinant

1	1	1
0	2	2
0	0	3

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 3 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 2 \\ 0 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 0 & & \\ & 0 & \end{array}$$

$$+\ 1 \cdot 2 \cdot 3 \qquad +\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 0 \cdot 0$$

Det = 6+0+0-0-0-0

= 6

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 0 & & \end{array} \qquad \begin{array}{ccc} 1 & & \\ & & 2 \\ & 0 & \end{array} \qquad \begin{array}{ccc} & 1 & \\ 0 & & \\ & & 3 \end{array}$$

$$-\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 0 \cdot 3$$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 3 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 2 \\ 0 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 0 & & \\ & 0 & \end{array}$$

$$+\ 1 \cdot 2 \cdot 3 \qquad +\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 0 \cdot 0$$

Det = 6+0+0-0-0-0

= 6

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 0 & & \end{array} \qquad \begin{array}{ccc} 1 & & \\ & & 2 \\ & 0 & \end{array} \qquad \begin{array}{ccc} & 1 & \\ 0 & & \\ & & 3 \end{array}$$

$$-\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 0 \cdot 3$$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$+\ 1 \cdot 2 \cdot 3 \qquad +\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 0 \cdot 0$$

Det = 6+0+0-0-0-0

= 6

$$-\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 0 \cdot 3$$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$+\ 1 \cdot 2 \cdot 3 \qquad +\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 0 \cdot 0$$

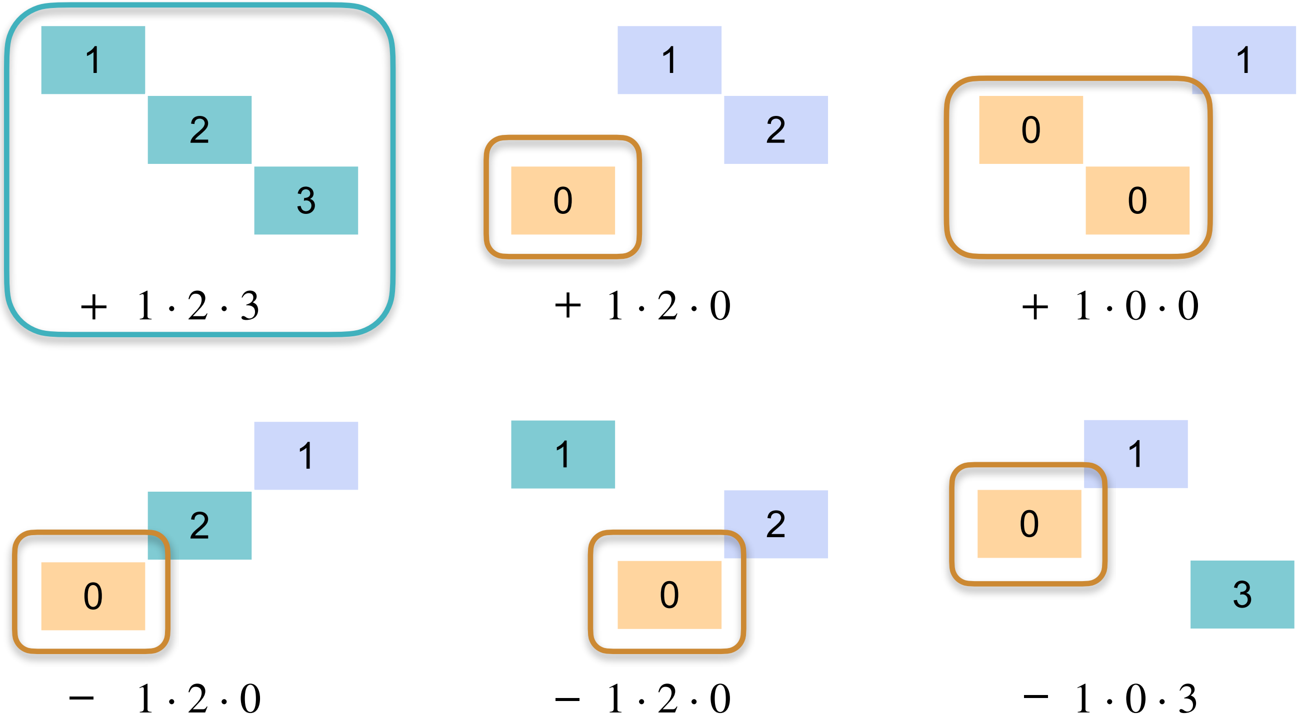
$$-\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 0 \cdot 3$$

Det = 6+0+0-0-0-0

= 6

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$



Det = 6+0+0-0-0-0

= 6

The determinant

1	1	1
0	2	2
0	0	0

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$+ \; 1 \cdot 2 \cdot 0$ $\qquad$ $+ \; 1 \cdot 2 \cdot 0$ $\qquad$ $+ \; 1 \cdot 0 \cdot 0$

$- \; 1 \cdot 2 \cdot 0$ $\qquad$ $- \; 1 \cdot 2 \cdot 0$ $\qquad$ $- \; 1 \cdot 0 \cdot 0$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 0 \cdot 0$$

$$-\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 0 \cdot 0$$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 0 \end{array} \qquad \begin{array}{ccc} & 1 & \\ & & 2 \\ 0 & & \end{array} \qquad \begin{array}{ccc} & & 1 \\ 0 & & \\ & 0 & \end{array}$$

$$+\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 2 \cdot 0 \qquad +\ 1 \cdot 0 \cdot 0$$

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 0 & & \end{array} \qquad \begin{array}{ccc} 1 & & \\ & & 2 \\ & 0 & \end{array} \qquad \begin{array}{ccc} & 1 & \\ 0 & & \\ & & 0 \end{array}$$

$$-\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 2 \cdot 0 \qquad -\ 1 \cdot 0 \cdot 0$$

The determinant

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+\ 1 \cdot 2 \cdot 0 \qquad + 1 \cdot 2 \cdot 0 \qquad + 1 \cdot 0 \cdot 0$$

Det = 0+0+0-0-0-0

$$-\ 1 \cdot 2 \cdot 0 \qquad - 1 \cdot 2 \cdot 0 \qquad - 1 \cdot 0 \cdot 0$$

The determinant

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 2 & 2 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

$$\boxed{\begin{array}{ccc} 1 & & \\ & 2 & \\ & & 0 \end{array} \quad + \; 1 \cdot 2 \cdot 0}$$

$$\begin{array}{ccc} & 1 & \\ & & 2 \\ 0 & & \end{array} \quad + \; 1 \cdot 2 \cdot 0$$

$$\begin{array}{ccc} & & 1 \\ 0 & & \\ & 0 & \end{array} \quad + \; 1 \cdot 0 \cdot 0$$

$$\begin{array}{ccc} & & 1 \\ & 2 & \\ 0 & & \end{array} \quad - \; 1 \cdot 2 \cdot 0$$

$$\begin{array}{ccc} 1 & & \\ & & 2 \\ & 0 & \end{array} \quad - \; 1 \cdot 2 \cdot 0$$

$$\begin{array}{ccc} & 1 & \\ 0 & & \\ & & 0 \end{array} \quad - \; 1 \cdot 0 \cdot 0$$

Det = 0+0+0-0-0-0

= 0

System of Linear Equations

Conclusion