



DeepLearning.AI

# Math for Machine Learning

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## Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix



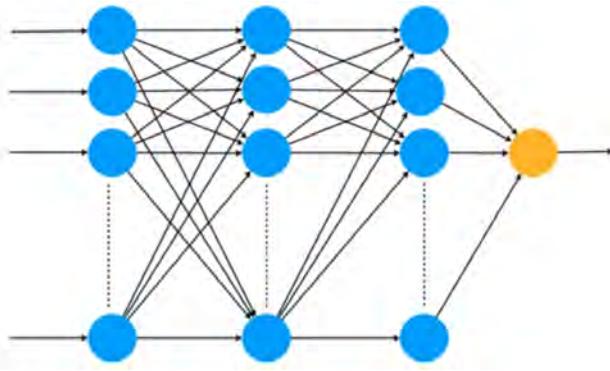
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## Solving System of Linear Equations

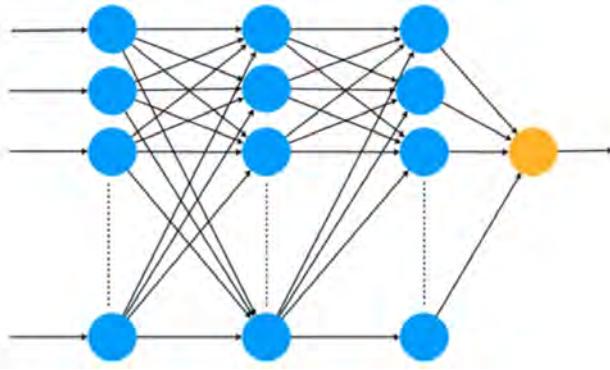
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**Machine learning motivation**

# Neural networks - Matrix operations

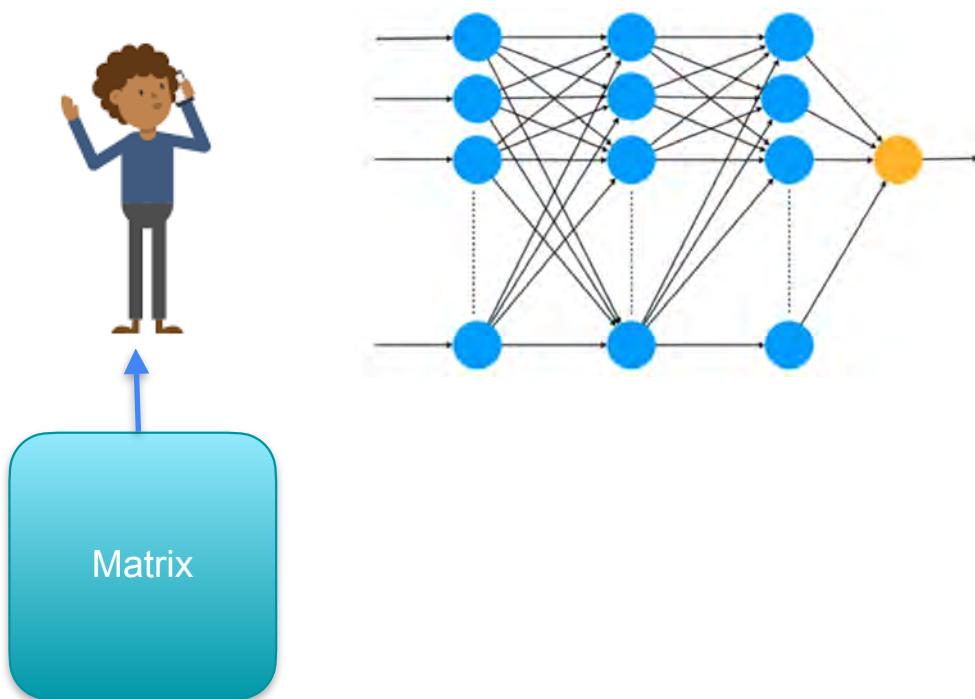


# Neural networks - Matrix operations



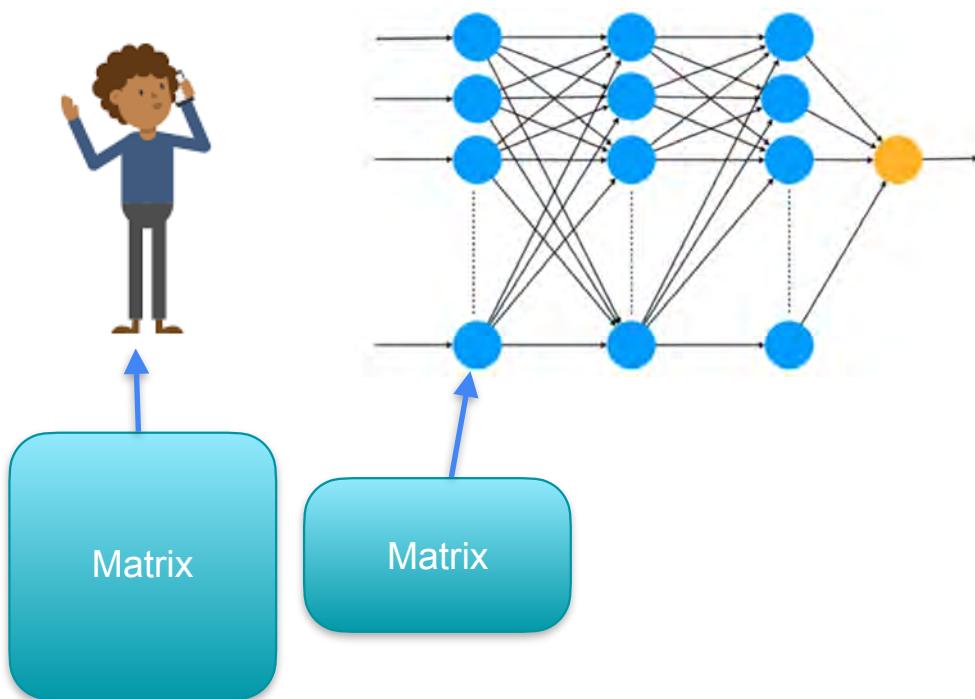
"Hello! Welcome to  
Math for Machine  
Learning!"

# Neural networks - Matrix operations



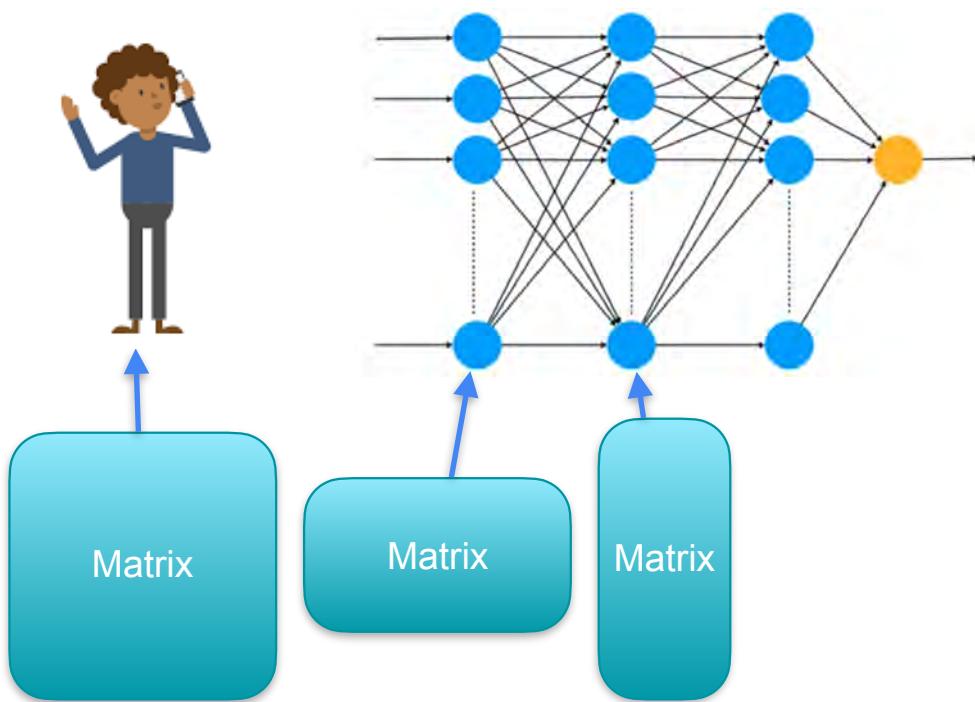
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# Neural networks - Matrix operations



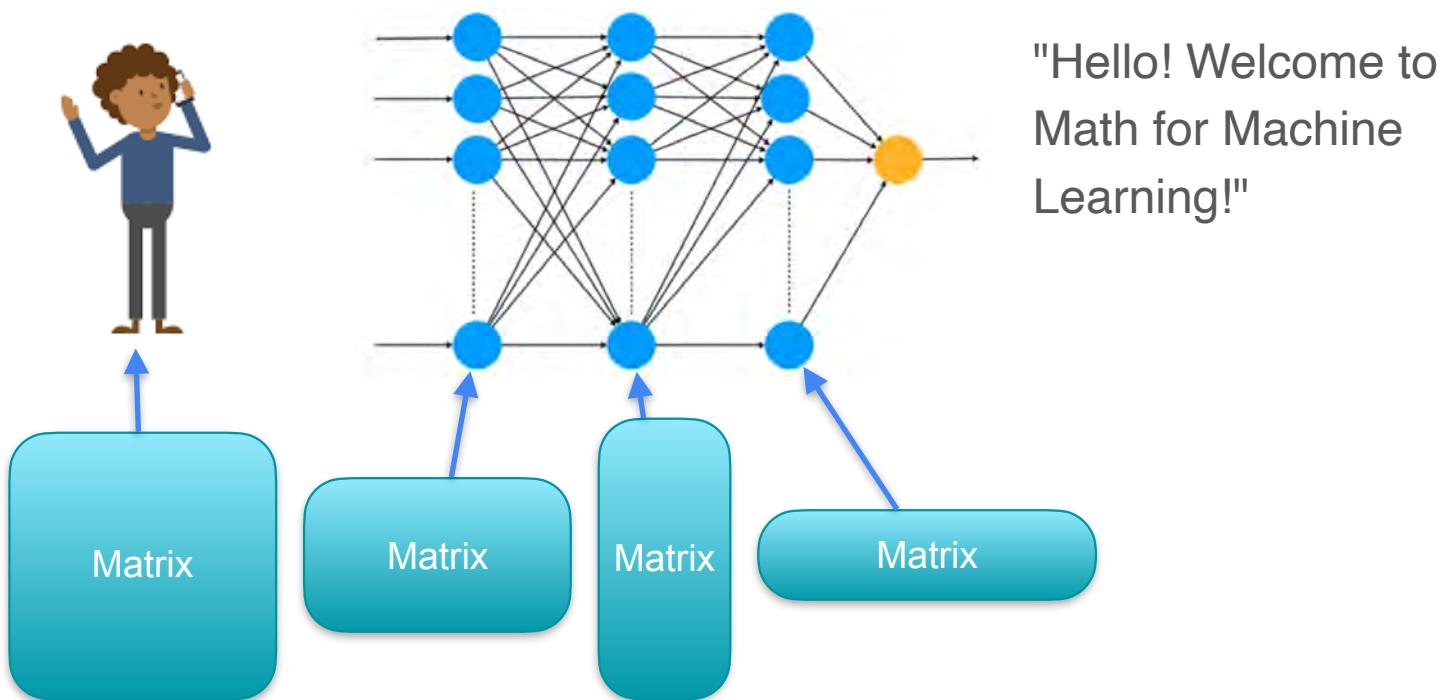
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# Neural networks - Matrix operations

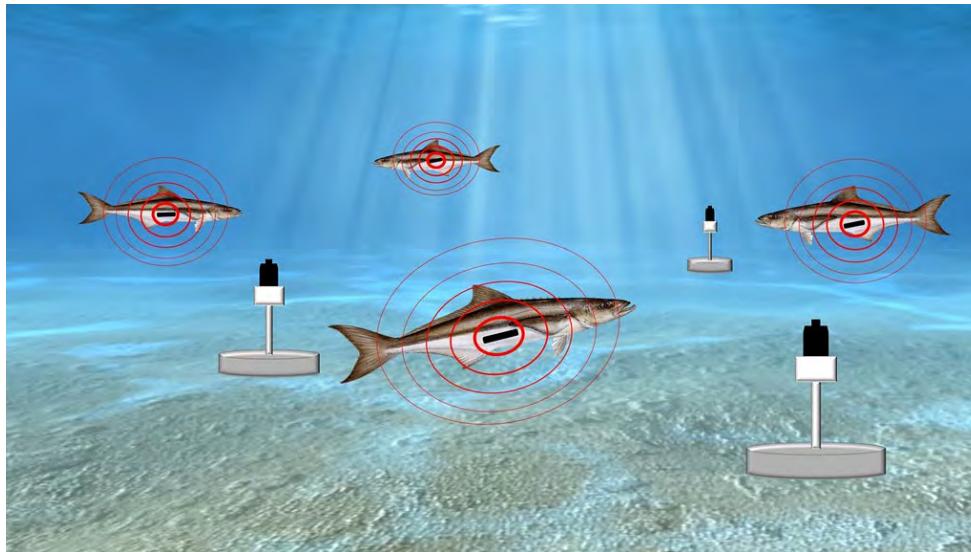


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# Neural networks - Matrix operations



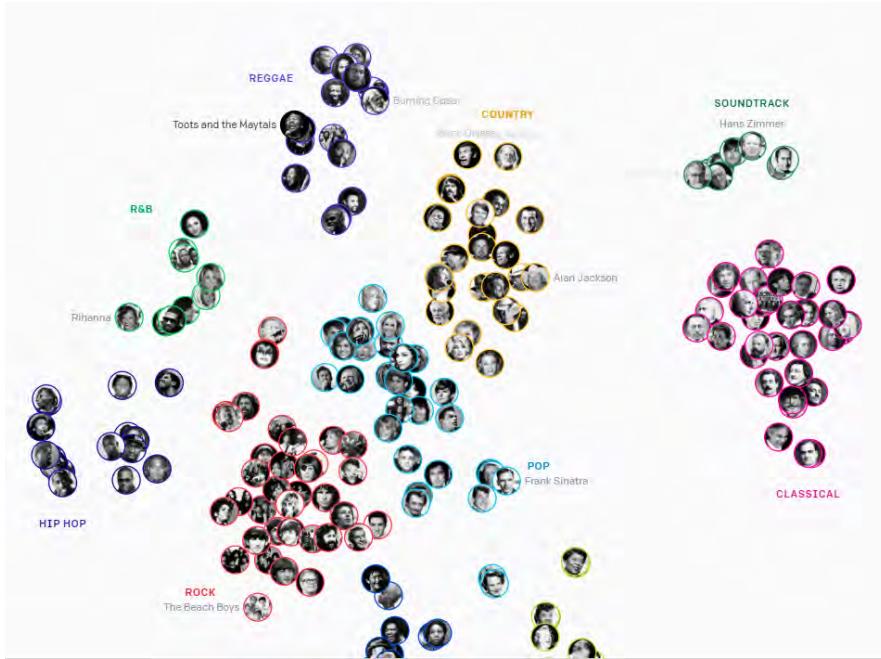
# Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

# Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.



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## Solving System of Linear Equations

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**Solving non-singular system  
of linear equations**

# Solving systems of equations

## System

- $a + b = 10$   
 
- $a + 2b = 12$   
 

# Solving systems of equations

## System

- $a + b = 10$   
 
- $a + 2b = 12$   
 

 +  = \$10

 +  +  = \$12

# Solving systems of equations

## System

- $a + b = 10$   
 
- $a + 2b = 12$   
 

 +  = \$10

 +  +  = \$12

# Solving systems of equations

## System

- $a + b = 10$   
 
- $a + 2b = 12$   
 

 +  = \$10

 +  +  = \$12

# Solving systems of equations

## System

- $a + b = 10$   
 
- $a + 2b = 12$   
 

 +  = \$10

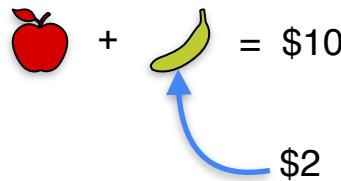
 +  +  = \$1  
  \$2

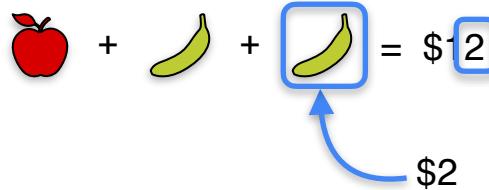
# Solving systems of equations

## System

- $a + b = 10$   

- $a + 2b = 12$   



$$\text{apple} + \text{banana} = \$10$$


$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$12$$

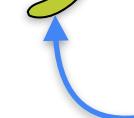
# Solving systems of equations

## System

- $a + b = 10$   

- $a + 2b = 12$   


$$\begin{array}{c} \text{apple} \\ + \\ \text{banana} \end{array} = \$10$$

\$8       \$2 

$$\begin{array}{c} \text{apple} \\ + \\ \text{banana} \\ + \\ \boxed{\text{banana}} \end{array} = \$12$$



# Solving systems of equations

# Solving systems of equations

## System

- $a + b = 10$   
 
- $a + 2b = 12$   
 

# Solving systems of equations

**System**

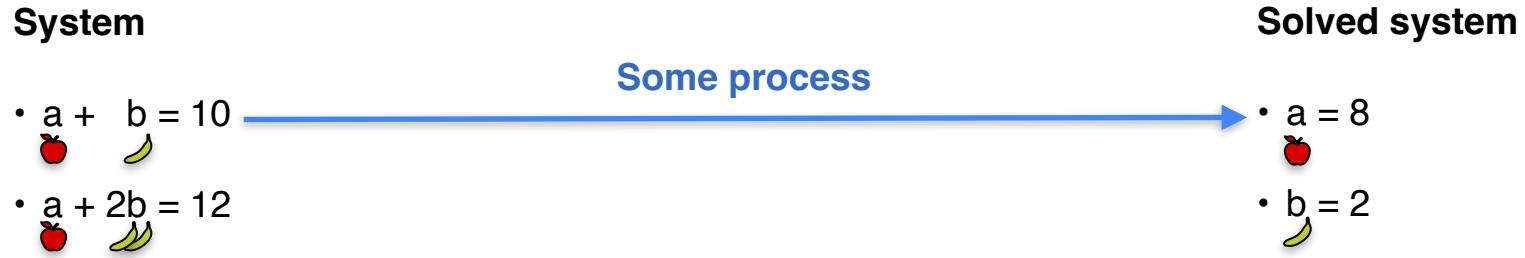
- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Solved system**

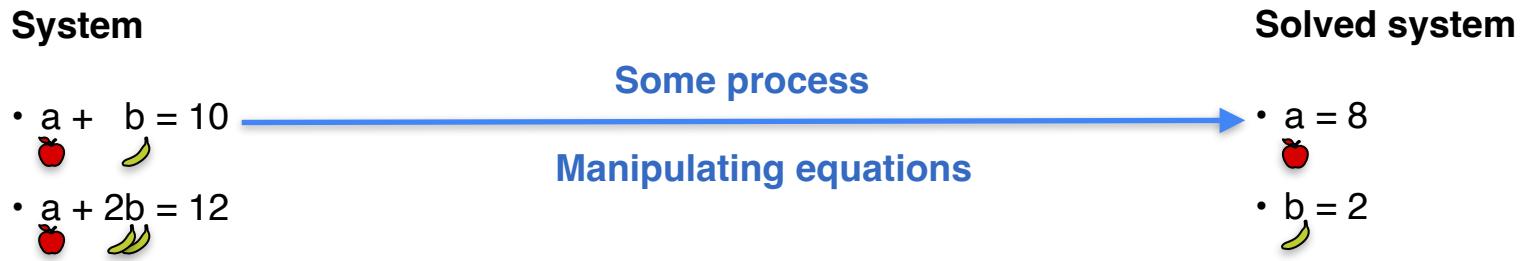
- $a = 8$   

- $b = 2$   

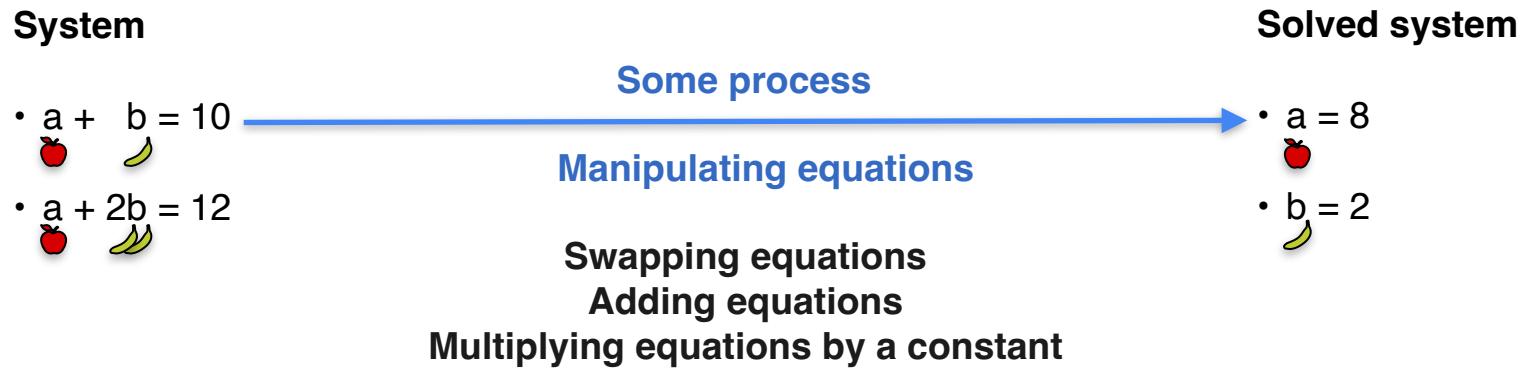

# Solving systems of equations



# Solving systems of equations



# Solving systems of equations



# Solving systems of equations

**System**

- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Solved system**

- $a = 8$   

- $b = 2$   


# Solving systems of equations

System

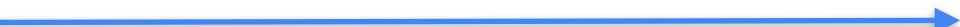
$$\bullet \text{ a } + \text{ b } = 10$$


$$\bullet \text{ a } + 2\text{b} = 12$$


Solved system

$$\bullet \text{ a } = 8$$


$$\bullet \text{ b } = 2$$

Eliminate 'a' from this equation

# Manipulating equations

# Manipulating equations

**Multiplying by a constant**

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \\ \times \quad \quad \quad 7 \\ \hline \end{array}$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \underline{x} \\ \underline{7} \\ 7a + 7b = 70 \end{array}$$

# Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \\ \hline 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

# Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \\ \hline 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

# Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \\ \hline 7 \\ \hline 7a + 7b = 70 \end{array}$$

Adding two equations

$$a + b = 10$$

$$2a + 3b = 26$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \\ \hline 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

$$\begin{array}{r} + \\ 2a + 3b = 26 \\ \hline \end{array}$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} \times \\ \hline 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

$$\begin{array}{r} + \\ 2a + 3b = 26 \\ \hline 3a + 4b = 36 \end{array}$$

# Let's do a harder example

# Systems of equations

# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

## Solved system

- $a = ?$
- $b = ?$

# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$



Eliminate 'a'  
from this equation

## Solved system

- $a = ?$
- $b = ?$

# Systems of equations

**System**

$$\bullet \quad 5a + b = 17$$

$$\bullet \quad 4a - 3b = 6$$

**Divide by coefficient of a**

$$\bullet \quad a + 0.2b = 3.4$$

$$\bullet \quad a - 0.75b = 1.5$$

**Solved system**

$$\bullet \quad a = ?$$

$$\bullet \quad b = ?$$



Eliminate 'a'  
from this equation

# Systems of equations

**System**

$$\bullet \quad 5a + b = 17$$

$$\bullet \quad 4a - 3b = 6$$

**Divide by coefficient of a**

$$\bullet \quad a + 0.2b = 3.4$$

$$\bullet \quad a - 0.75b = 1.5$$

**Solved system**

$$\bullet \quad a = ?$$

$$\bullet \quad b = ?$$

**Subtract equation 1 from equation 2**

Eliminate 'a'  
from this equation



# Systems of equations

System

$$\bullet \quad 5a + b = 17$$

$$\bullet \quad 4a - 3b = 6$$

Divide by coefficient of a

Solved system

$$\bullet \quad a = ?$$

$$\bullet \quad b = ?$$

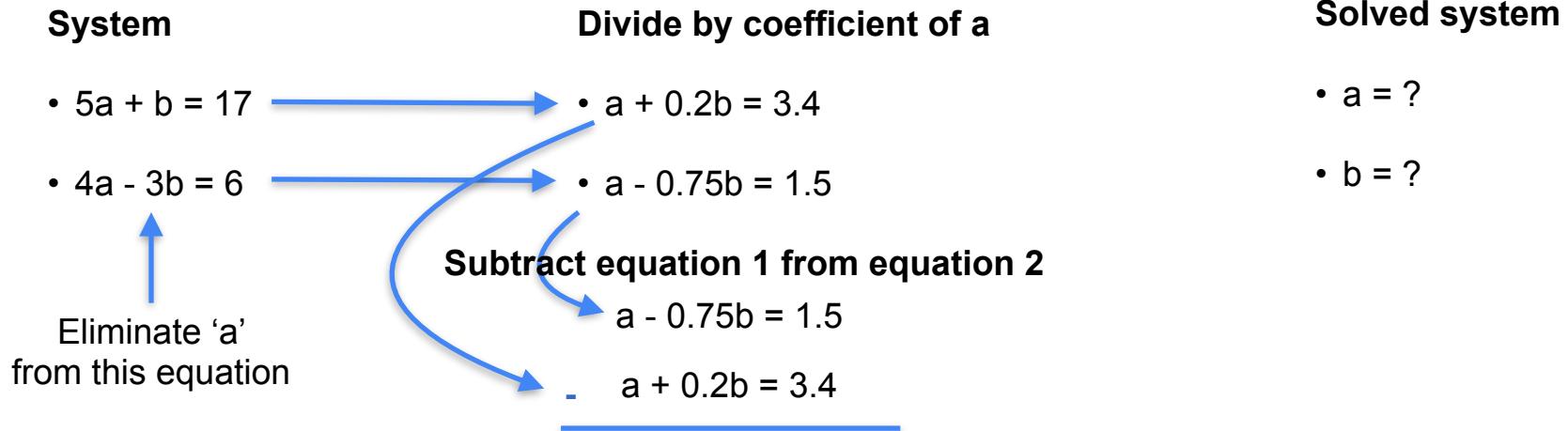


Subtract equation 1 from equation 2

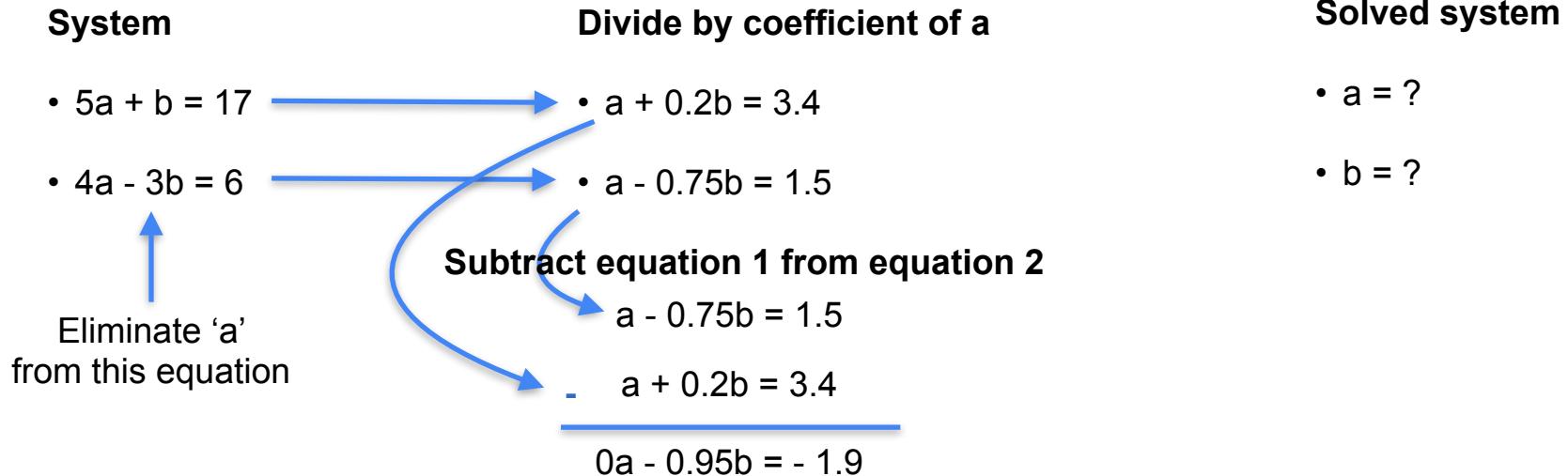
$$a - 0.75b = 1.5$$

Eliminate 'a'  
from this equation

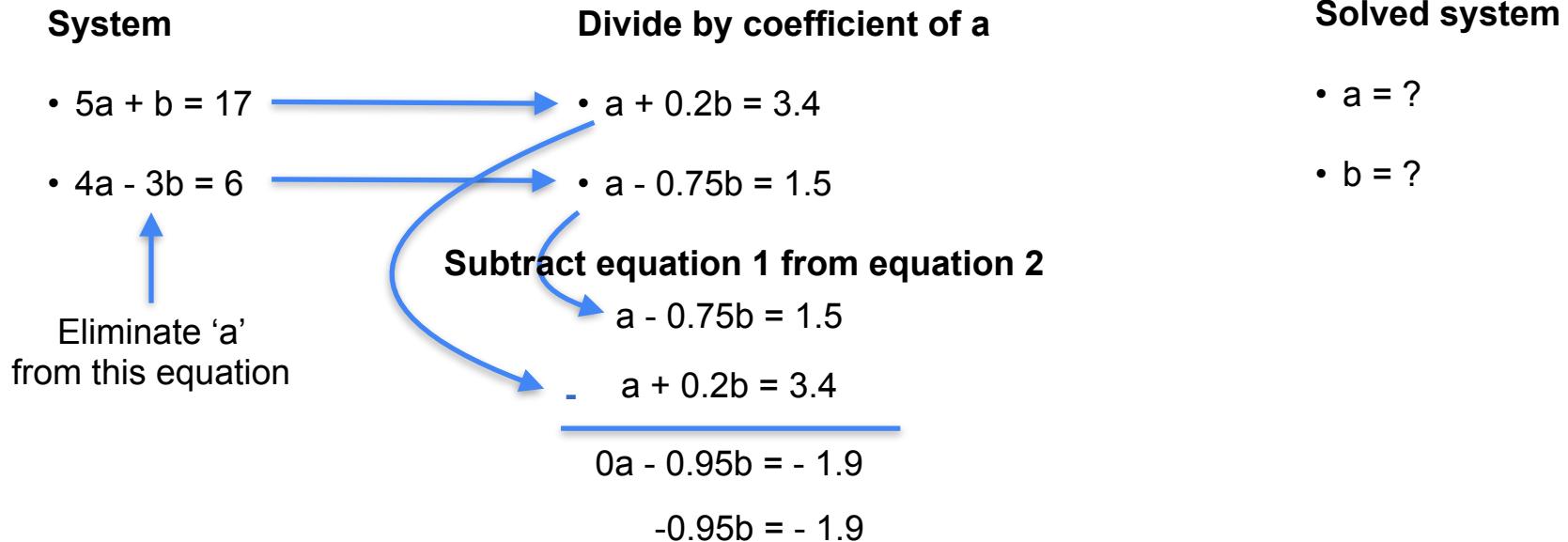
# Systems of equations



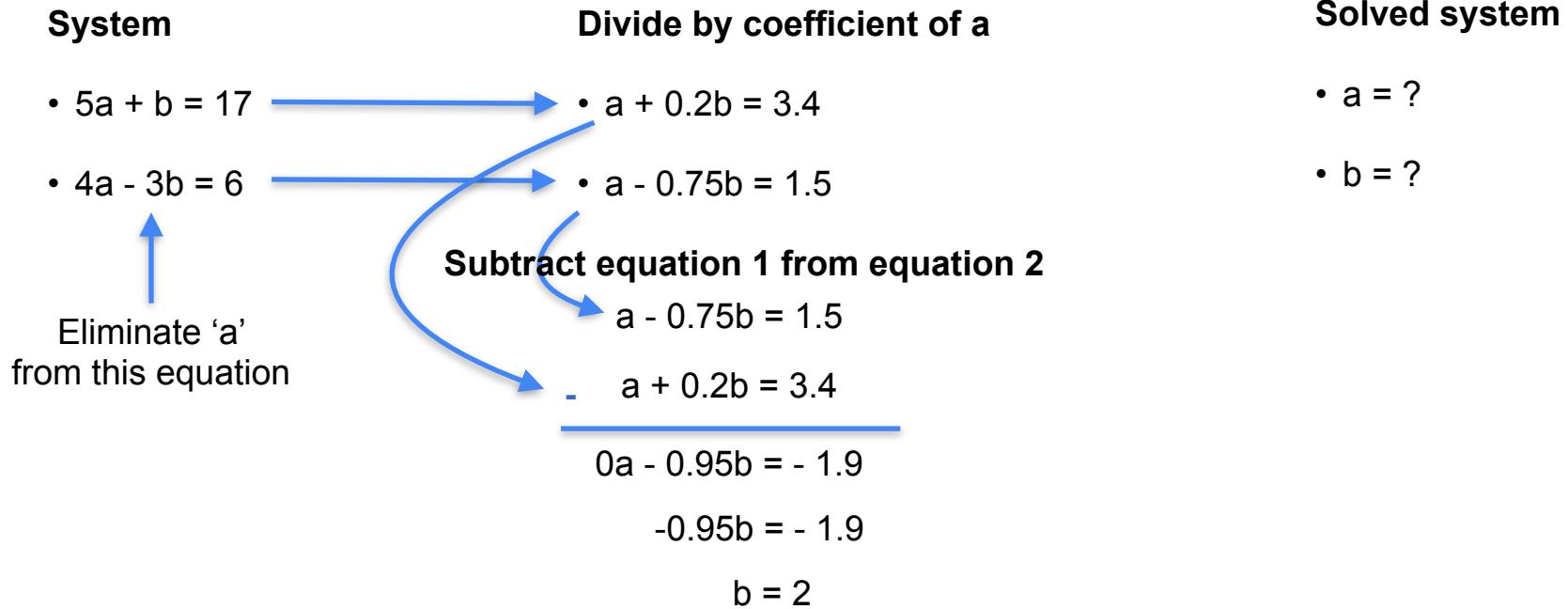
# Systems of equations



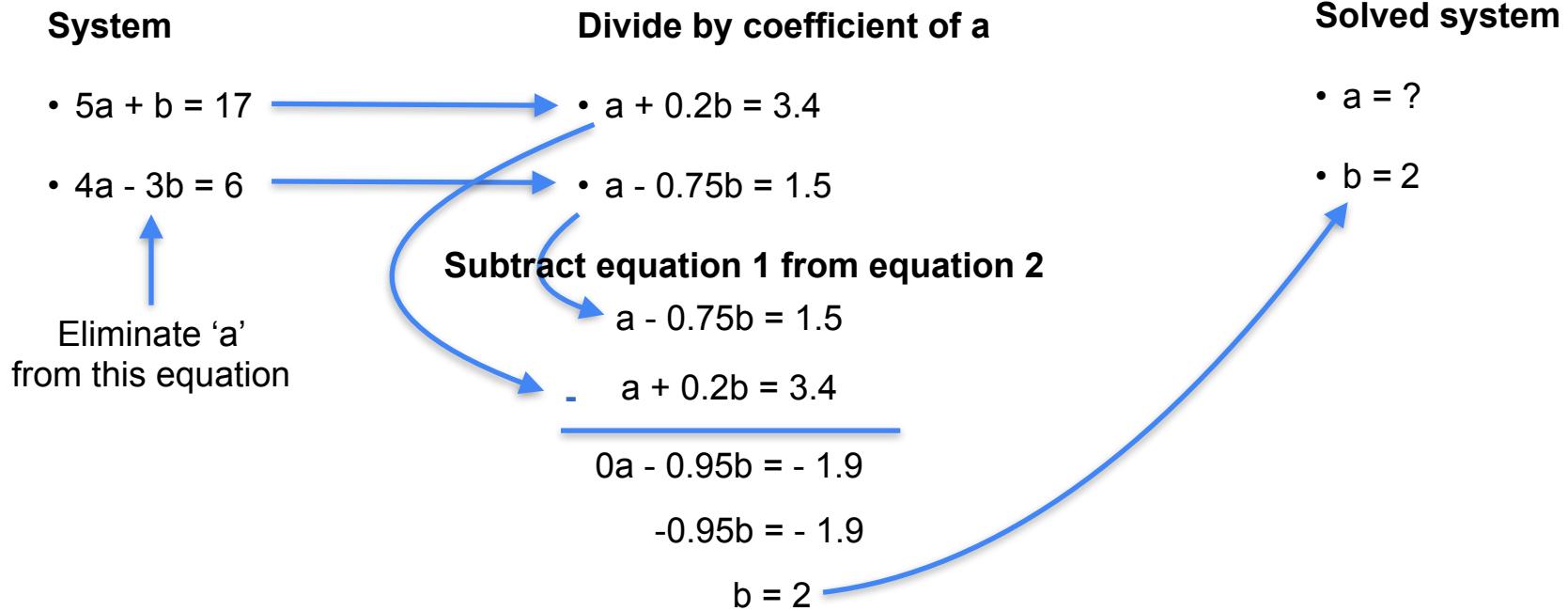
# Systems of equations



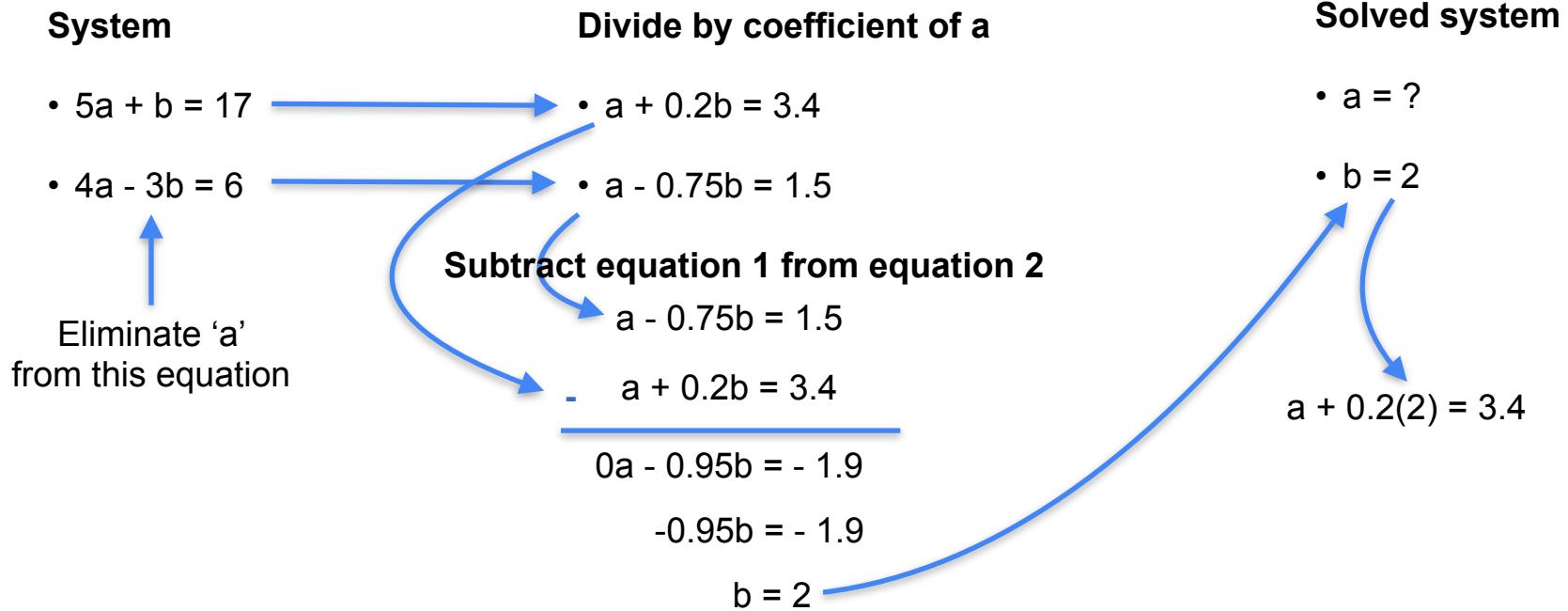
# Systems of equations



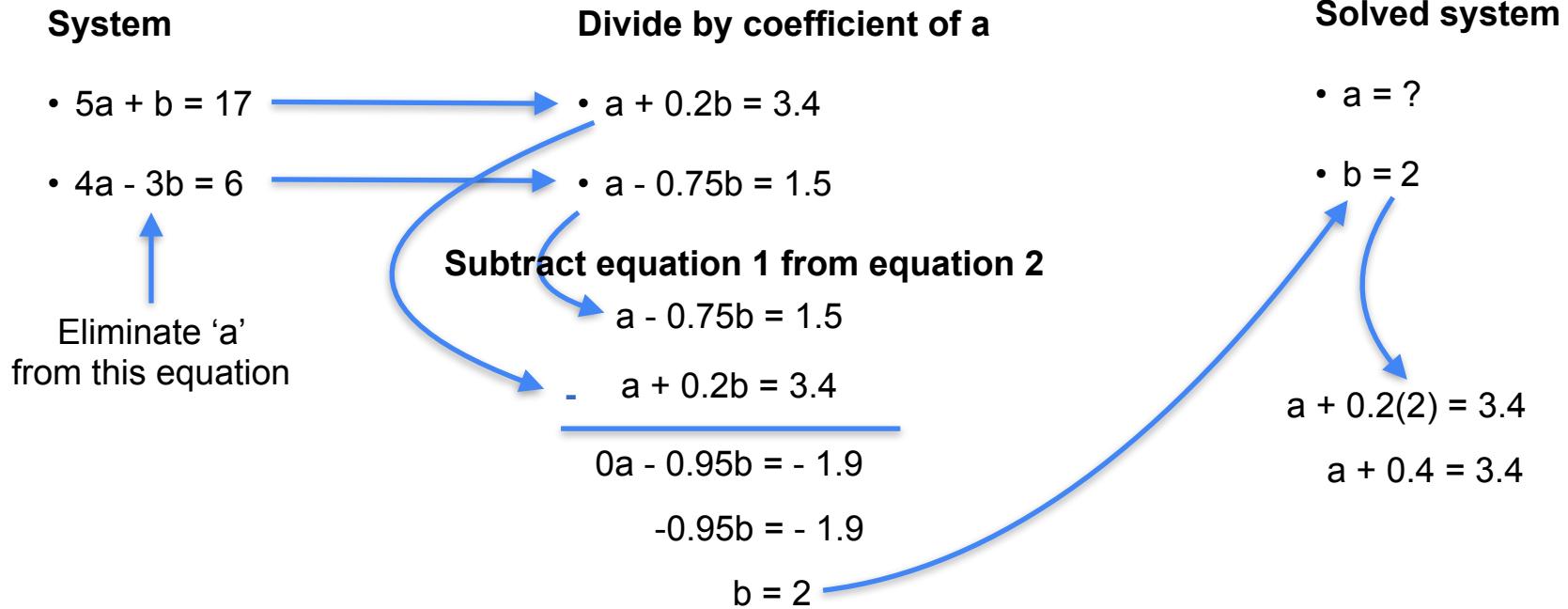
# Systems of equations



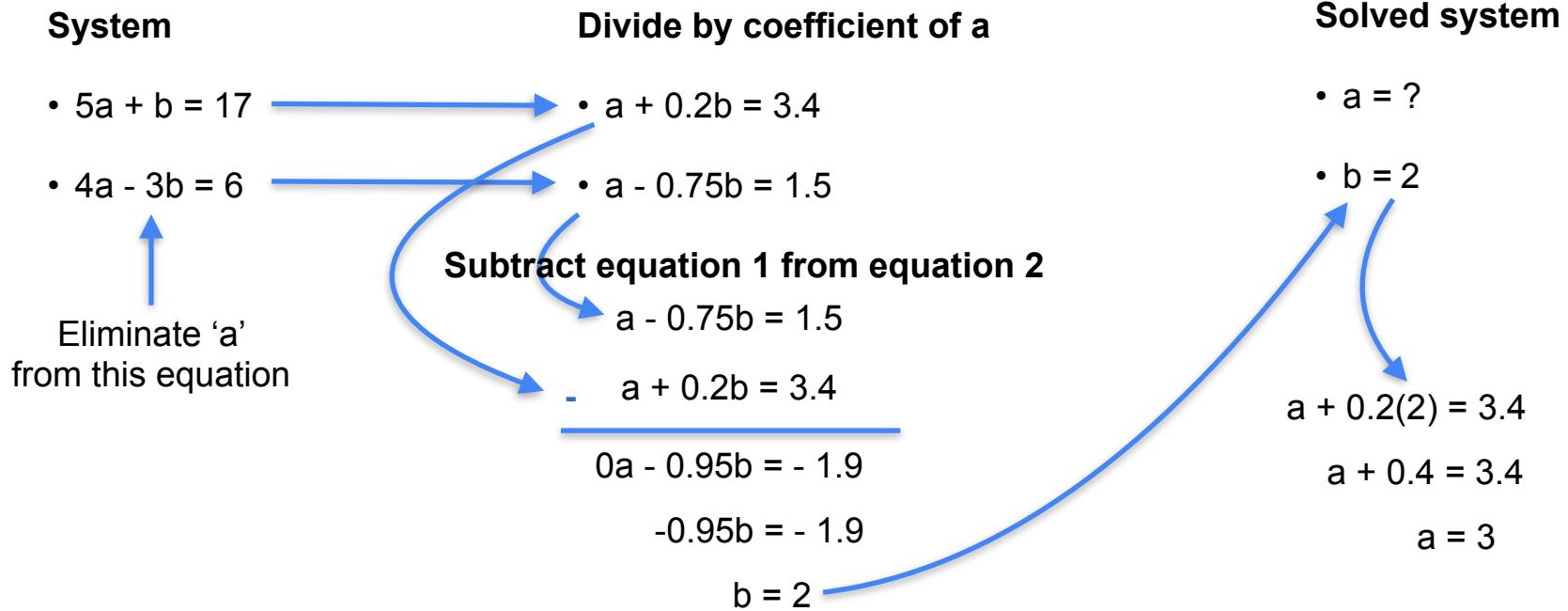
# Systems of equations



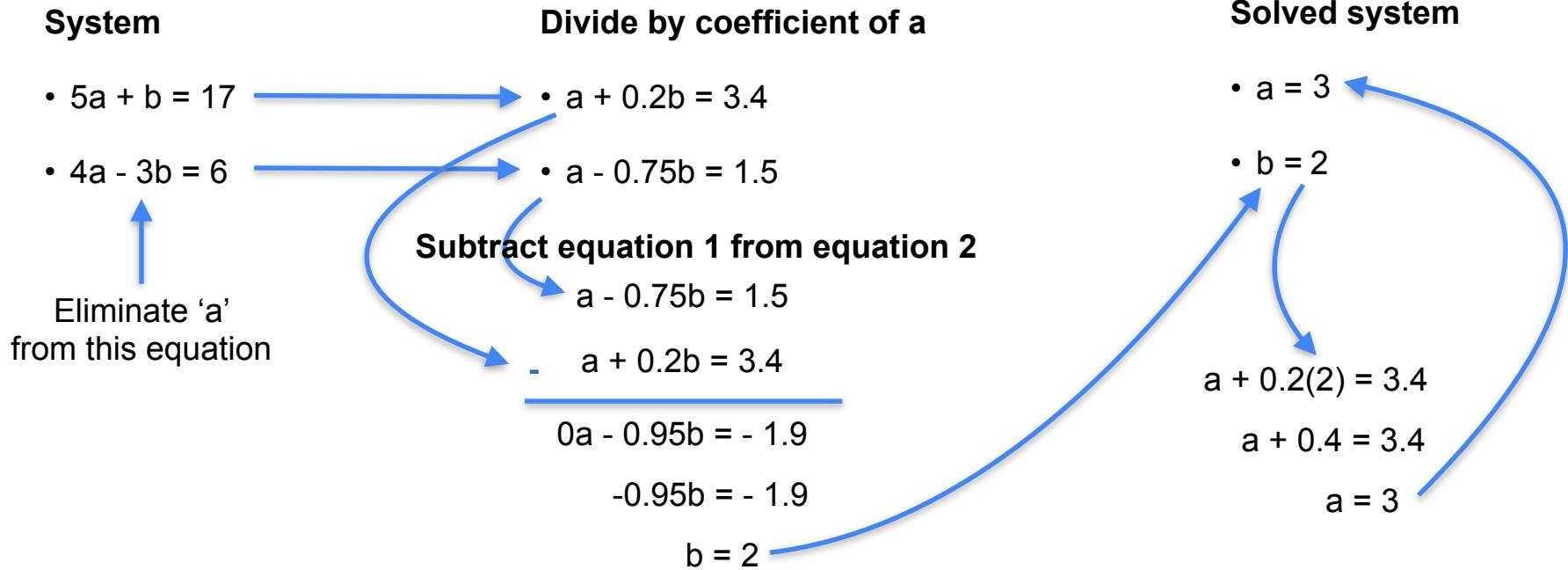
# Systems of equations



# Systems of equations



# Systems of equations



# What if one of the coefficients of a is zero?

## System

- $5a + b = 17$
- $3b = 6$

## Solved system

- $a = ?$
- $b = ?$

# What if one of the coefficients of a is zero?

## System

- $5a + b = 17$
- $3b = 6$



Eliminate 'a'  
from this equation

## Solved system

- $a = ?$
- $b = ?$

# What if one of the coefficients of a is zero?

**System**

- $5a + b = 17$
- $3b = 6$

**Divide by coefficient of a**

- $a + 0.2b = 3.4$
- ???

**Solved system**

- $a = ?$
- $b = ?$



Eliminate 'a'  
from this equation

# What if one of the coefficients of a is zero?

## System

$$\bullet \quad 5a + b = 17$$

$$\bullet \quad 3b = 6$$

## Divide by coefficient of a

$$\bullet \quad a + 0.2b = 3.4$$

$$\bullet \quad ???$$

## Solved system

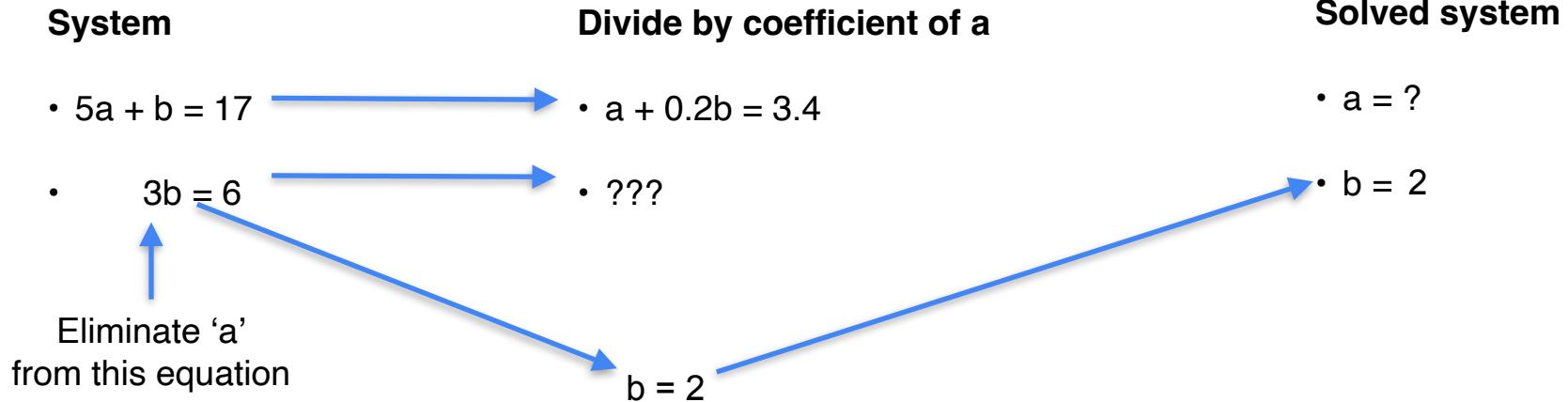
$$\bullet \quad a = ?$$

$$\bullet \quad b = ?$$

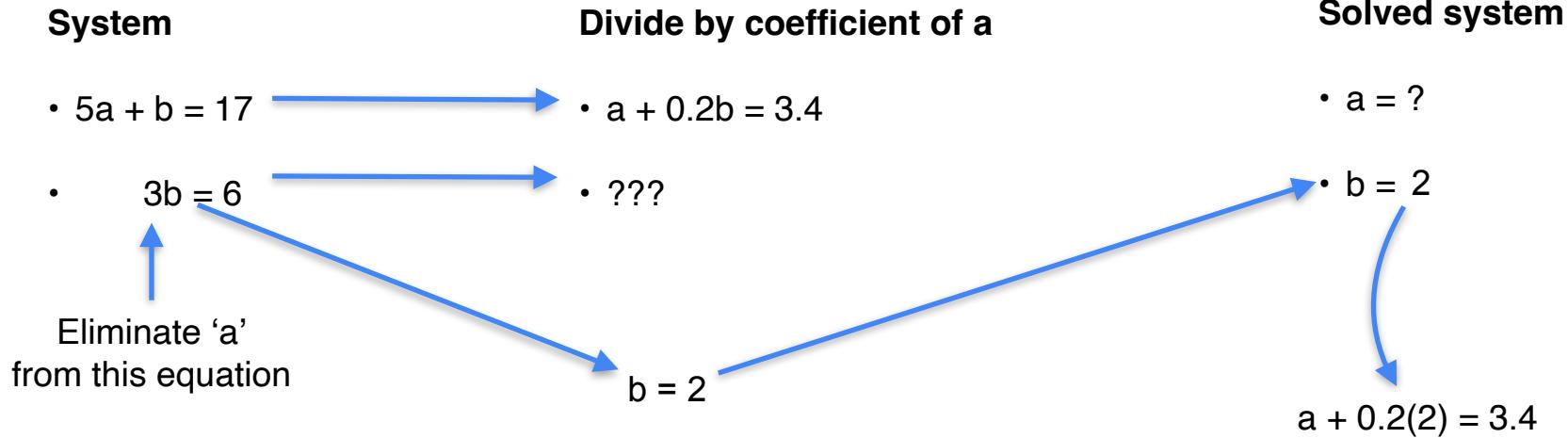
Eliminate 'a'  
from this equation

$$b = 2$$

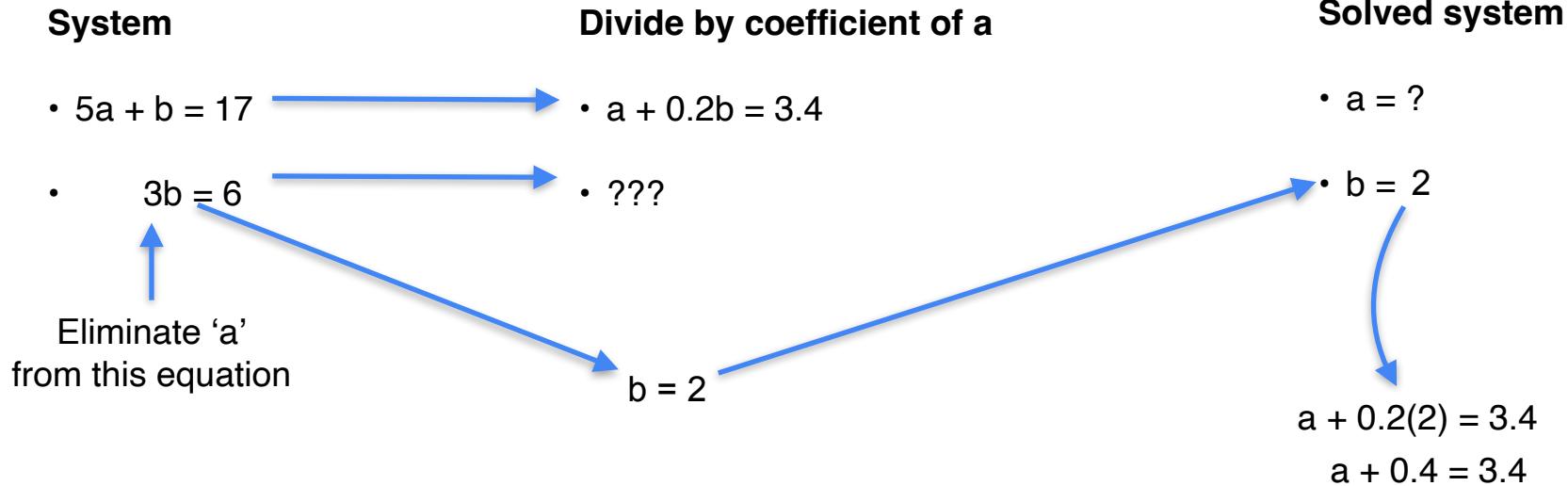
# What if one of the coefficients of a is zero?



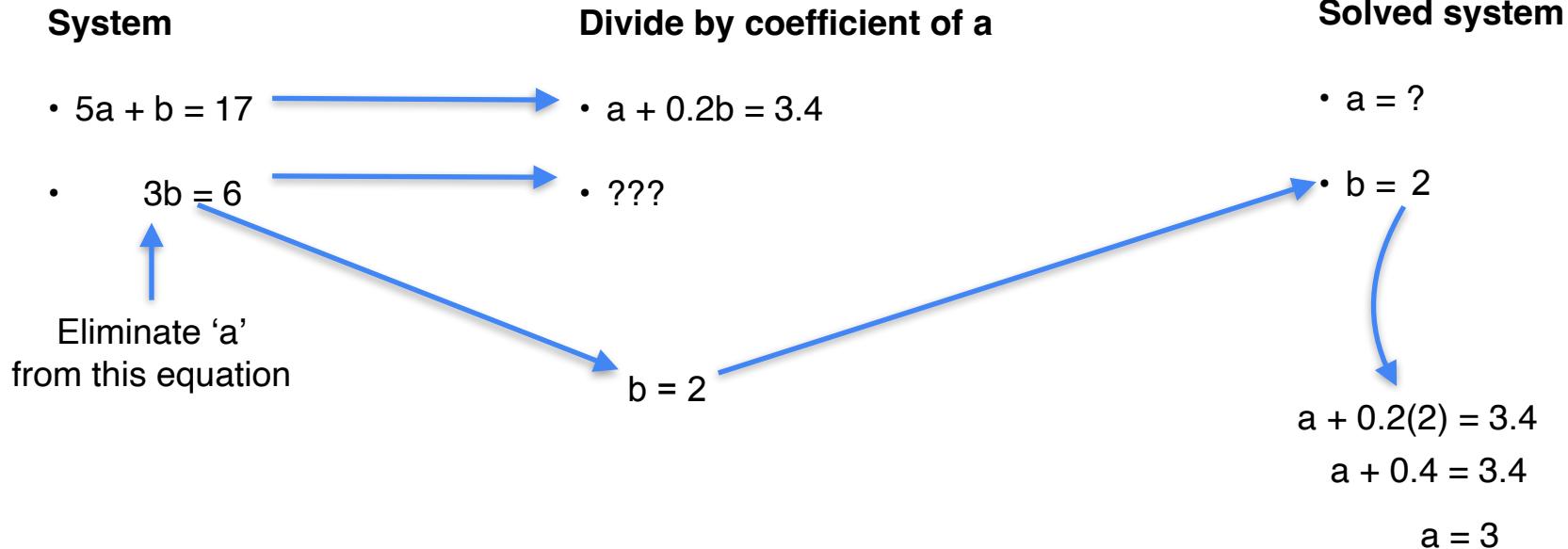
# What if one of the coefficients of a is zero?



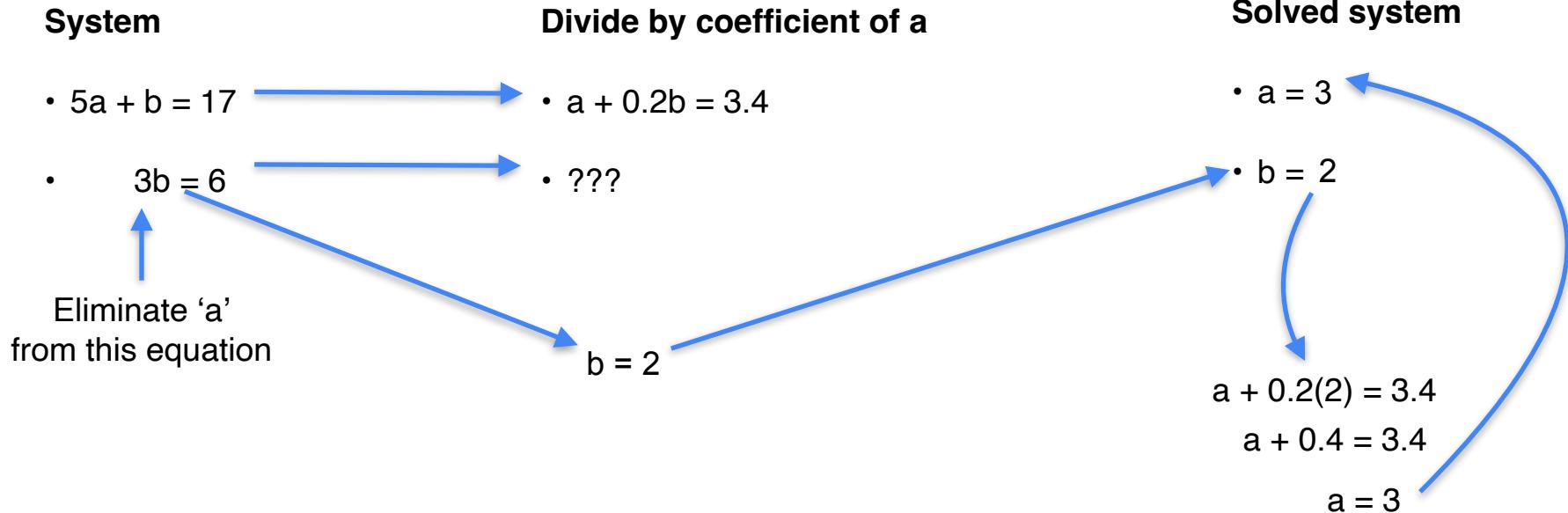
# What if one of the coefficients of a is zero?



# What if one of the coefficients of a is zero?



# What if one of the coefficients of a is zero?



# Quiz

- Solve the following system of equations

## **System**

- $2a + 5b = 46$
- $8a + b = 32$

# Solution

- Solve the following system of equations

## System

- $2a + 5b = 46$
- $8a + b = 32$

## Solution

- $a = 3$
- $b = 8$



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## Solving System of Linear Equations

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**Solving singular system of  
linear equations**

# What if the system is singular (redundant)?

## System

- $a + b = 10$
- $2a + 2b = 20$

## Solved system

- $a = ?$
- $b = ?$

# What if the system is singular (redundant)?

**System**

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

**Divide by coefficient of a**



$$\cdot a + b = 10$$



$$\cdot a + b = 10$$

**Solved system**

$$\cdot a = ?$$

$$\cdot b = ?$$



Eliminate 'a'  
from this equation

# What if the system is singular (redundant)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Divide by coefficient of a



$$\cdot a + b = 10$$



$$\cdot a + b = 10$$

Solved system

$$\cdot a = ?$$

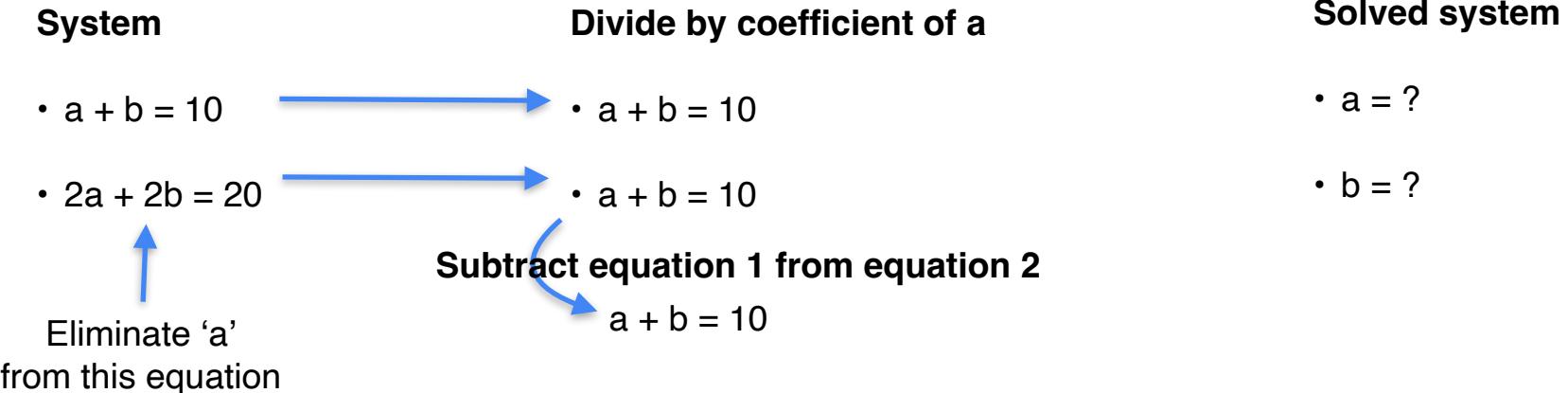
$$\cdot b = ?$$

Subtract equation 1 from equation 2

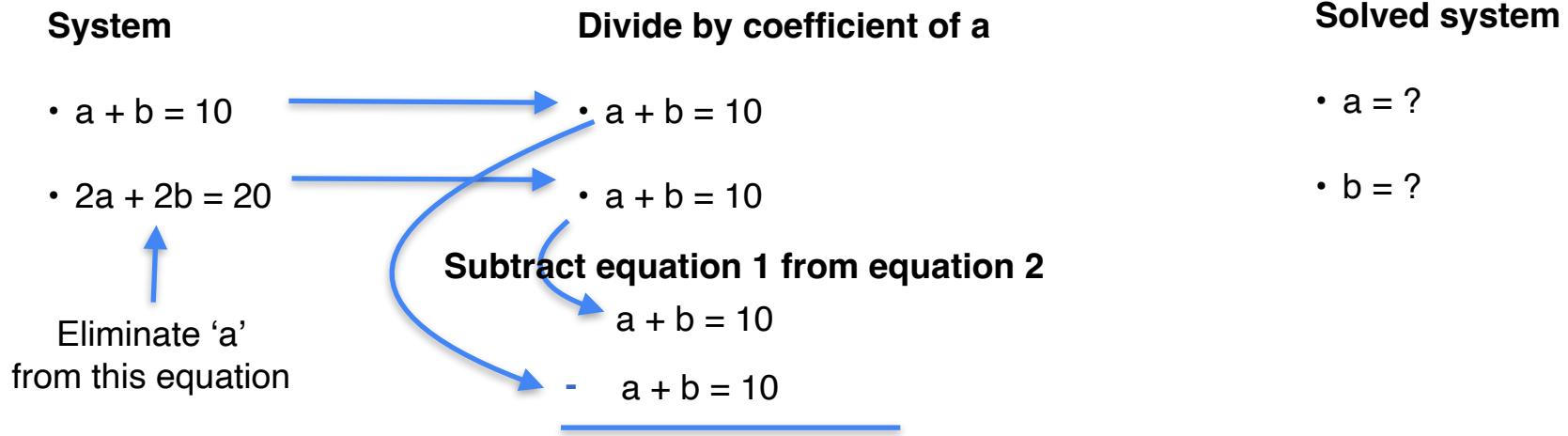
Eliminate 'a'  
from this equation



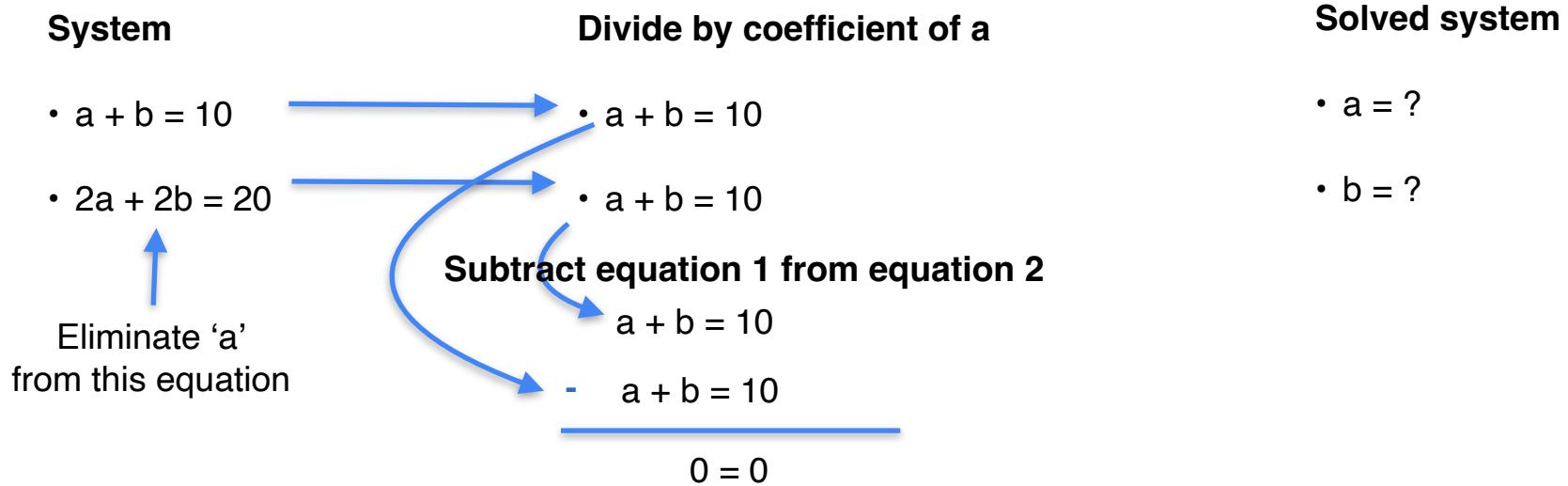
# What if the system is singular (redundant)?



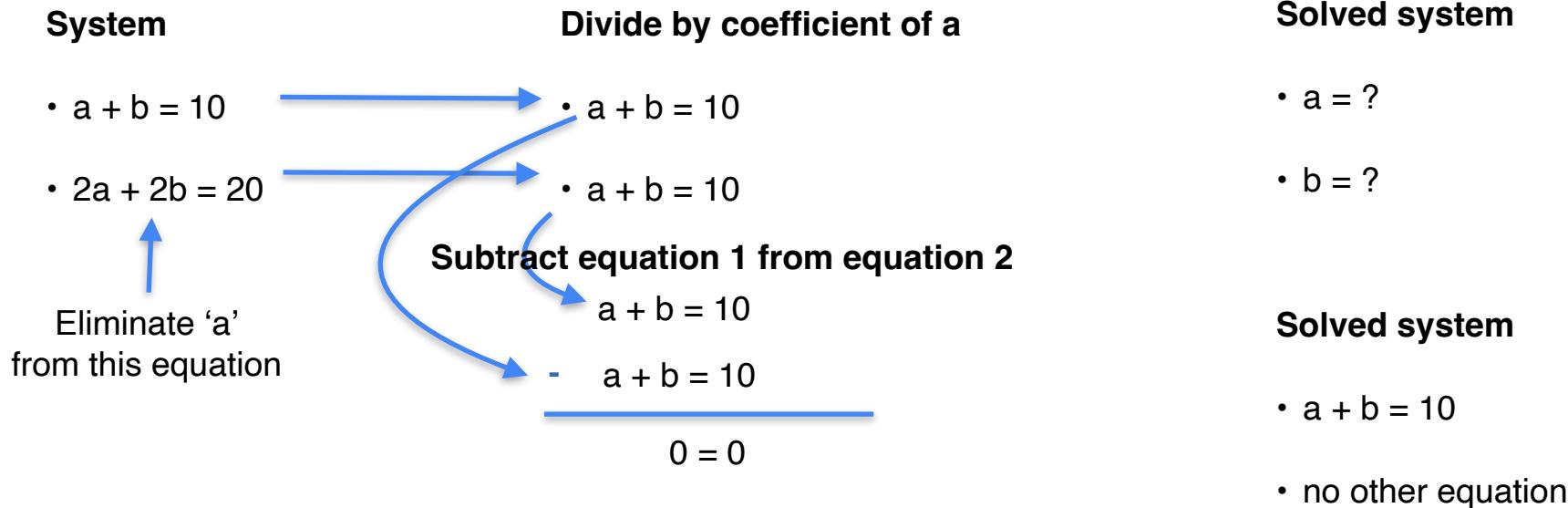
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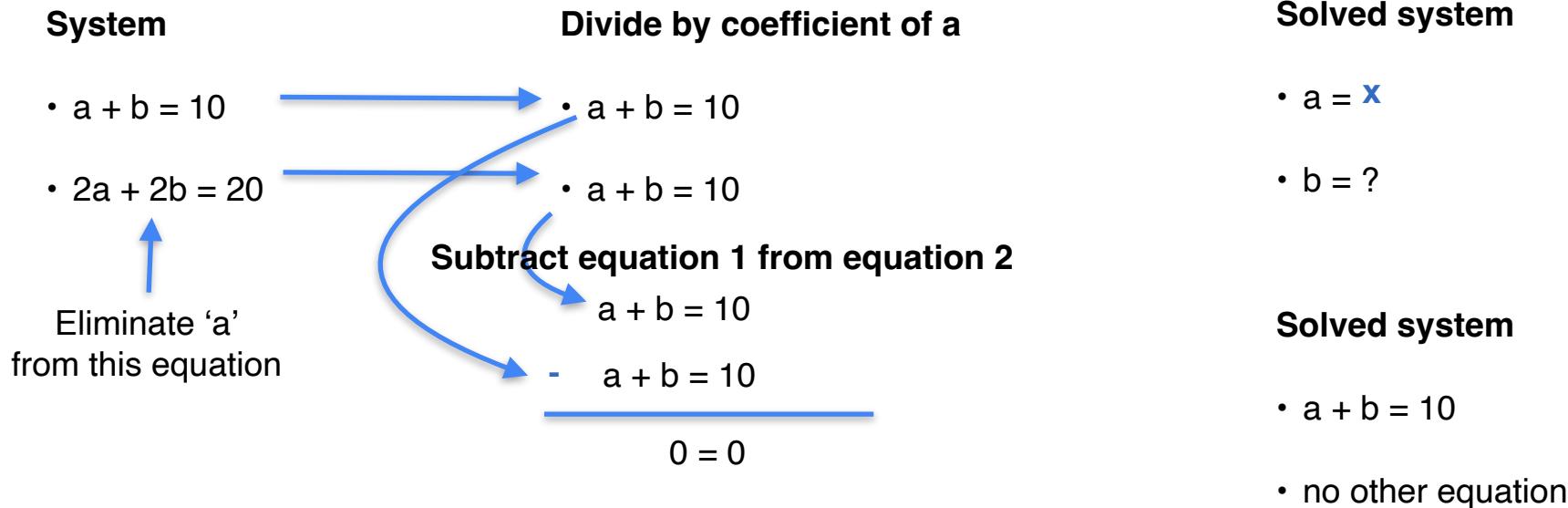
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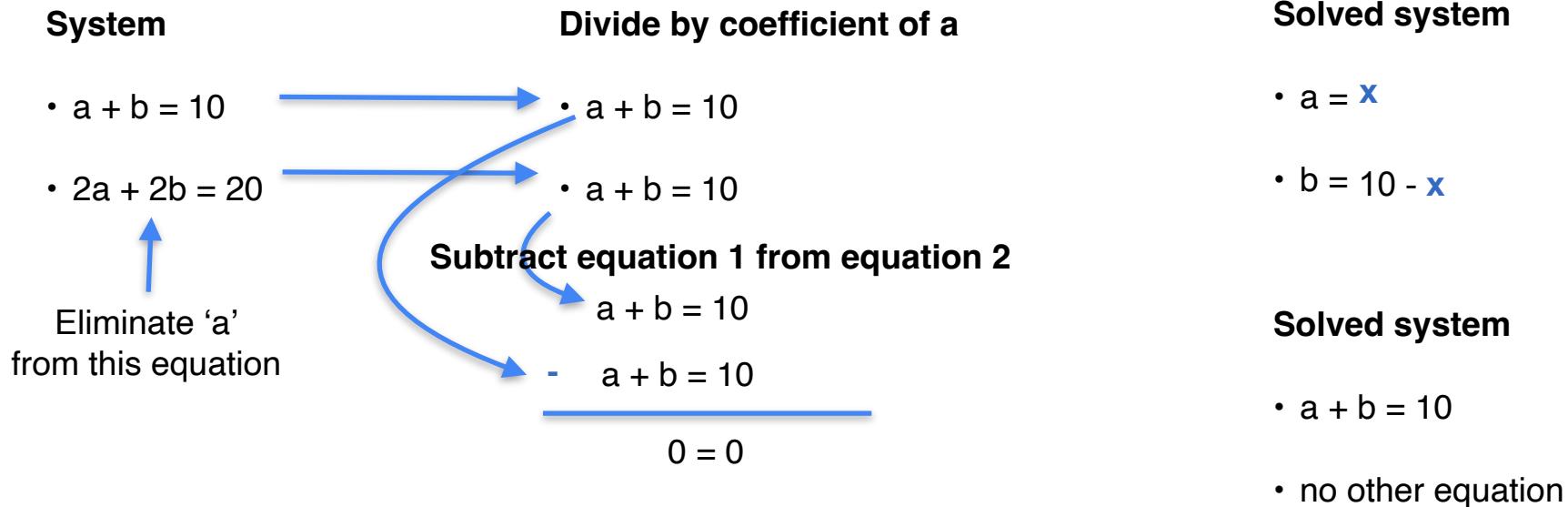
# What if the system is singular (redundant)?



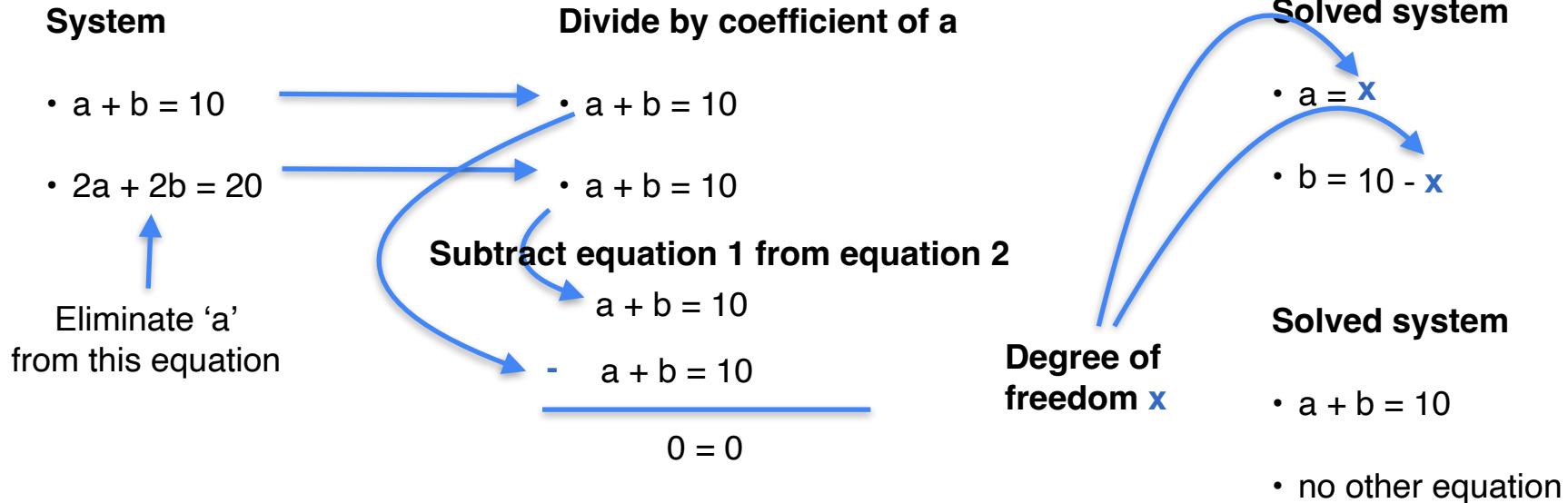
# What if the system is singular (redundant)?



# What if the system is singular (redundant)?



# What if the system is singular (redundant)?



# What if the system is singular (contradictory)?

**System**

- $a + b = 10$
- $2a + 2b = 24$

**Solved system**

- $a = ?$
- $b = ?$

# What if the system is singular (contradictory)?

## System

- $a + b = 10$
- $2a + 2b = 24$



Eliminate 'a'  
from this equation

## Solved system

- $a = ?$
- $b = ?$

# What if the system is singular (contradictory)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 24$$

Divide by coefficient of a



$$\cdot a + b = 10$$



$$\cdot a + b = 12$$

Solved system

$$\cdot a = ?$$

$$\cdot b = ?$$



Eliminate 'a'  
from this equation

# What if the system is singular (contradictory)?

System

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 24$$

Divide by coefficient of a



$$\cdot a + b = 10$$



$$\cdot a + b = 12$$

Solved system

$$\cdot a = ?$$

$$\cdot b = ?$$

Subtract equation 1 from equation 2



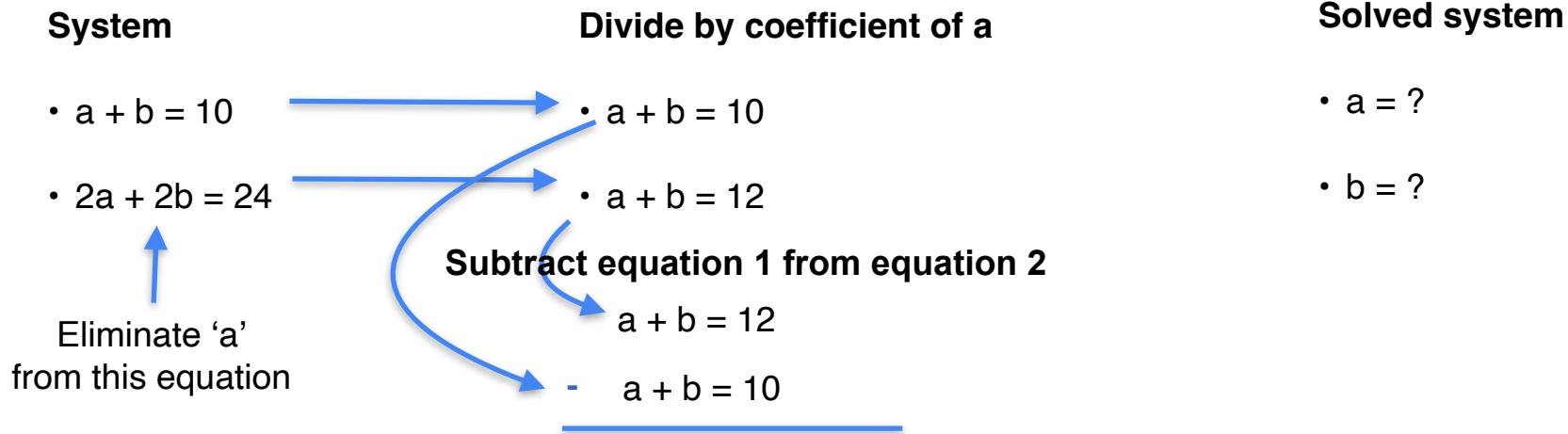
Eliminate 'a'  
from this equation

# What if the system is singular (contradictory)?

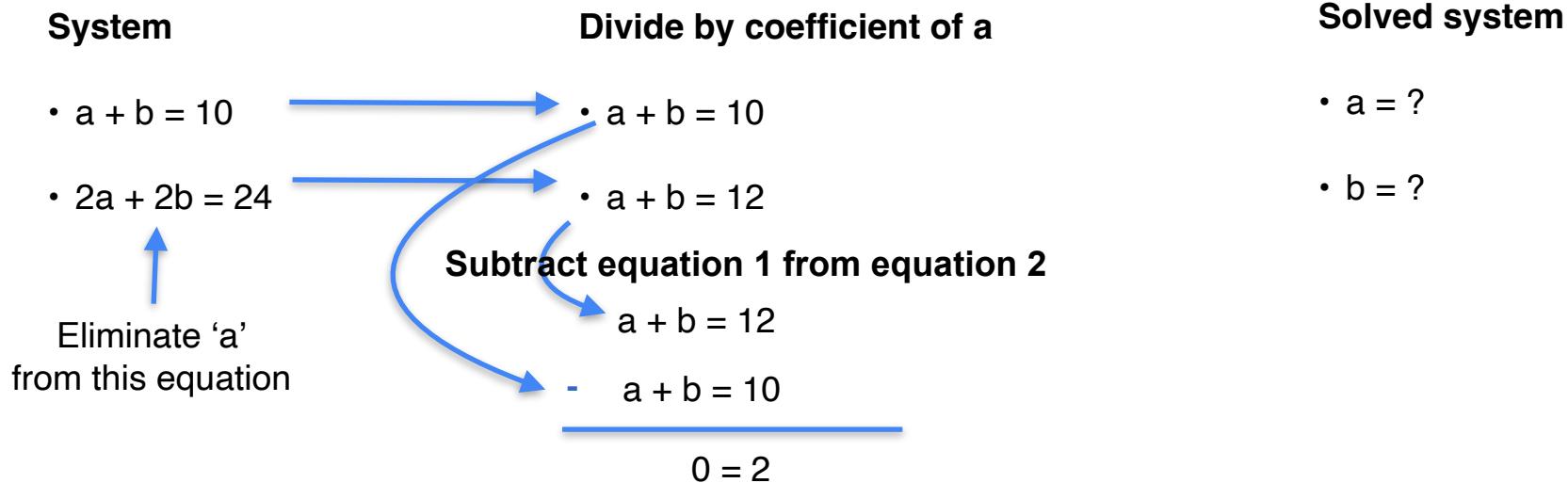
System	Divide by coefficient of a	Solved system
$\cdot a + b = 10$	$\cdot a + b = 10$	$\cdot a = ?$
$\cdot 2a + 2b = 24$	$\cdot a + b = 12$ <b>Subtract equation 1 from equation 2</b> $a + b = 12$	$\cdot b = ?$

Eliminate 'a' from this equation

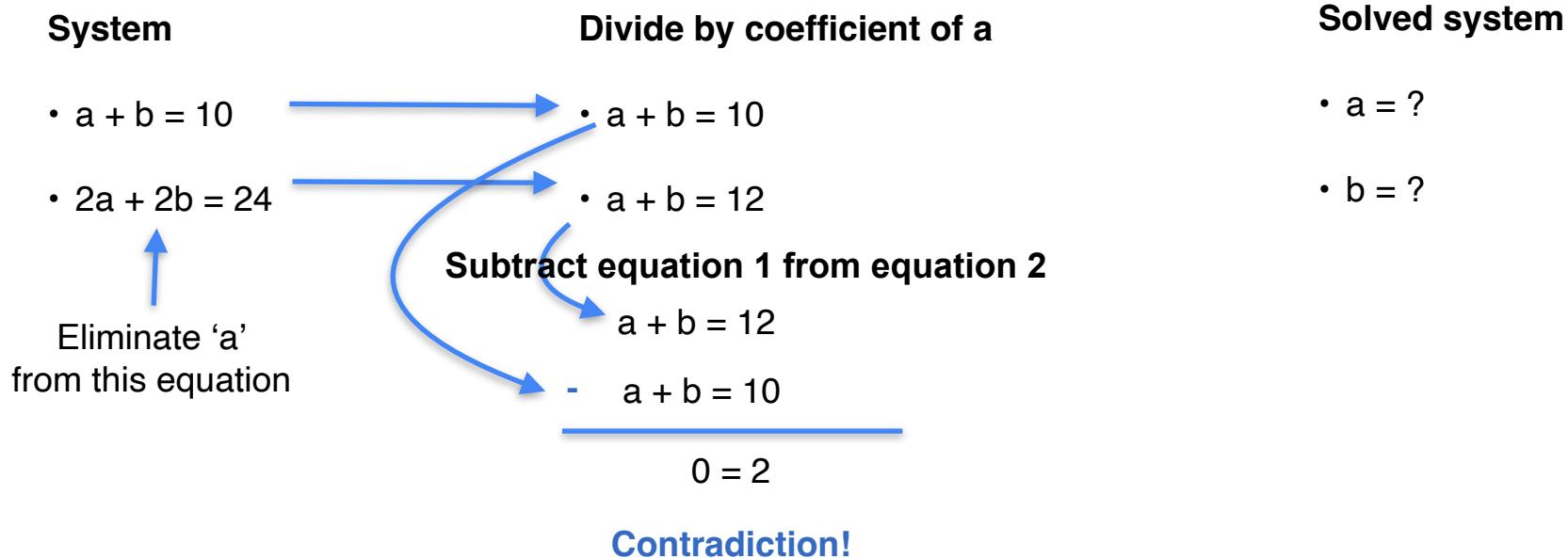
# What if the system is singular (contradictory)?



# What if the system is singular (contradictory)?



# What if the system is singular (contradictory)?



# Quiz

- Solve the following system of equations

## **System**

- $5a + b = 11$
- $10a + 2b = 22$

# Solution

- Solve the following system of equations

## System

- $5a + b = 11$
- $10a + 2b = 22$

**Solution:** If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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## Solving System of Linear Equations

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**Solving system of equations  
with more variables**

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

# Elimination method

System

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ 3a - 3b - c = 3$$

$$\bullet \ 2a - b + 6c = 24$$

Leave 'a' by  
itself

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

# Elimination method

## System

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ 3a - 3b - c = 3$$

$$\bullet \ 2a - b + 6c = 24$$

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ a - b - 1/3 c = 1$$

$$\bullet \ a - b/2 + 3c = 12$$

Divide each row by the coefficient of 'a'

# Elimination method

## System

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ 3a - 3b - c = 3$$

$$\bullet \ 2a - b + 6c = 24$$

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ a - b - 1/3 c = 1$$

$$\bullet \ a - b/2 + 3c = 12$$

Divide each row by the coefficient of 'a'

Use the first equation to remove 'a' from the others

# Elimination method

## System

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ 3a - 3b - c = 3$$

$$\bullet \ 2a - b + 6c = 24$$

Divide each row by the coefficient of 'a'

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ a - b - 1/3 c = 1$$

$$\bullet \ a - b/2 + 3c = 12$$

Use the first equation to remove 'a' from the others

$$\bullet \ a + b + 2c = 12$$

$$\bullet \ -2b - 7/3 c = -11$$

$$\bullet \ -3/2 b + c = 0$$

# Elimination method

## System

$$\begin{aligned} & \bullet a + b + 2c = 12 \\ & \bullet 3a - 3b - c = 3 \\ & \bullet 2a - b + 6c = 24 \end{aligned}$$

Divide each row by the coefficient of 'a'

$$\begin{aligned} & \bullet a + b + 2c = 12 \\ & \bullet a - b - \frac{1}{3}c = 1 \\ & \bullet a - b/2 + 3c = 12 \end{aligned}$$

Use the first equation to remove 'a' from the others

$$\begin{aligned} & \bullet a - b + 2c = 12 \\ & \bullet -2b - \frac{7}{3}c = -11 \\ & \bullet -\frac{3}{2}b + c = 0 \end{aligned}$$

Isolated 'a'

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - \frac{1}{3}c = 1$
- $a - b/2 + 3c = 12$

Use the first equation to remove 'a' from the others

- $a - b + 2c = 12$
- $-2b - \frac{7}{3}c = -11$
- $-\frac{3}{2}b + c = 0$

Isolated 'a'

Solve this new system of 2 equations

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two  
rows by the  
coefficient of b

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$
- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Divide last two  
rows by the  
coefficient of b

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

$c = 3$

# Elimination method

## System

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $c = 3$

# Elimination method

## System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

# Elimination method

## System

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$  
$$\begin{aligned}b + 7/2 &= 11/2 \\b &= 2\end{aligned}$$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

# Elimination method

## System

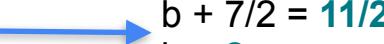
- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$    
 $b + 7/2 = 11/2$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

# Elimination method

## System

- $a + b + 2c = 12$    $a + 2 + 6 = 12$   
 $a = 4$
- $b + 7/6 c = 11/2$    $b + 7/2 = 11/2$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

# Elimination method

## System

- $a + b + 2c = 12$    $a + 2 + 6 = 12$   
 $a = 4$
- $b + 7/6 c = 11/2$    $b + 7/2 = 11/2$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

The solution is  
 $a = 4$   
 $b = 2$   
 $c = 3$



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# Solving System of Linear Equations

---

## Matrix row reduction

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$



# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$



# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

Row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

## Row echelon form

## Reduced row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

## Row echelon form

## Reduced row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Row echelon form

## Diagonal matrix

1	0
0	1

## Reduced row echelon form

# Systems of equations to matrices

## Original system

- $a + b = 10$
- $2a + 2b = 20$

# Systems of equations to matrices

## Original system

- $a + b = 10$
- $2a + 2b = 20$

## Intermediate System

- $a + b = 10$
- $0a + 0b = 0$



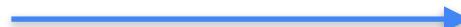
# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$



**Original matrix**

1	1
2	2

# Systems of equations to matrices

Original system

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Intermediate System

$$\cdot a + b = 10$$

$$\cdot 0a + 0b = 0$$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

# Systems of equations to matrices

Original system

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Intermediate System

$$\cdot a + b = 10$$

$$\cdot 0a + 0b = 0$$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

Row echelon form

# Systems of equations to matrices

Original system

$$\cdot a + b = 10$$

$$\cdot 2a + 2b = 20$$

Intermediate System

$$\cdot a + b = 10$$

$$\cdot 0a + 0b = 0$$

Original matrix

1	1
2	2

Upper diagonal matrix

1	1
0	0

Row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$

## Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



# Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



Original matrix

5	1
10	2

# Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

# Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

# Systems of equations to matrices

Original system

- $5a + b = 11$
- $10a + 2b = 22$

Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

Original matrix

5	1
10	2

Upper diagonal matrix

1	0.2
0	0

Row echelon form

# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

## Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



# Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



Original matrix

0	0
0	0

# Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

# Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form

# Systems of equations to matrices

Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$

Original matrix

0	0
0	0

Upper diagonal matrix

0	0
0	0

Row echelon form



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## Solving System of Linear Equations

---

**Row operations that  
preserve singularity**

# Switching rows

5	1
4	3

# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

# Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

# Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

# Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

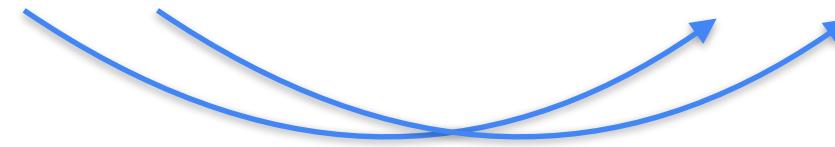
# Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$



# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

# Multiplying a row by a (non-zero) scalar

5	1
4	3

# Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4$$

# Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1
---	---

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 =$$

4	3
---	---

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

4	3
---	---

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\text{Determinant} = 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{Determinant} &= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4) \\ &= 10 \cdot 11 \end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

5	1
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{array}{cc} 5 & 1 \\ + & \hline 4 & 3 \end{array}$$

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4$$

$$= 11$$

# Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{matrix} 5 & 1 \\ 4 & 3 \end{matrix} \\ + \quad \begin{matrix} 5 & 1 \\ 4 & 3 \end{matrix} \\ \hline \begin{matrix} 9 & 4 \end{matrix} \end{array}$$

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{matrix} 5 & 1 \\ 4 & 3 \end{matrix} \\ + \\ \hline \begin{matrix} 9 & 4 \end{matrix} \end{array}$$

9	4
---	---

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{matrix} 5 & 1 \\ 4 & 3 \end{matrix} \\ + \\ \hline \begin{matrix} 9 & 4 \end{matrix} \end{array}$$

9	4
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{array}{cc} 5 & 1 \\ 4 & 3 \end{array} \\ + \\ \hline \begin{array}{cc} 9 & 4 \end{array} \end{array}$$

9	4
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\text{Determinant} = 9 \cdot 3 - 4 \cdot 4$$

# Adding a row to another row

5	1
4	3

$$\begin{array}{r} \begin{array}{cc} 5 & 1 \\ 4 & 3 \end{array} \\ + \\ \hline \begin{array}{cc} 9 & 4 \end{array} \end{array}$$

9	4
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{Determinant} &= 9 \cdot 3 - 4 \cdot 4 \\ &= 11 \end{aligned}$$



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# Solving System of Linear Equations

---

## Rank of a matrix

# Compressing Images - Reducing rank

# Compressing Images - Reducing rank



# Compressing Images - Reducing rank

Original (Rank 200)

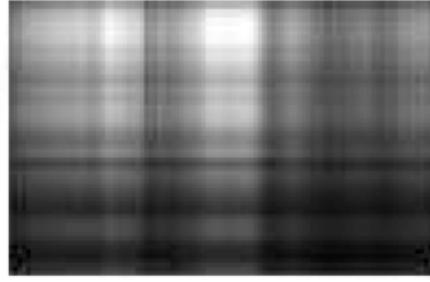


# Compressing Images - Reducing rank

Original (Rank 200)



Rank 1

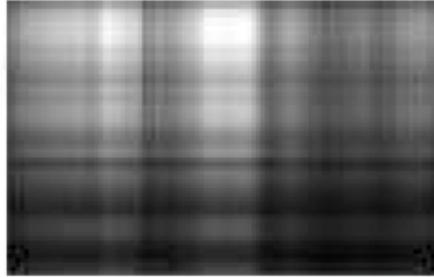


# Compressing Images - Reducing rank

Original(Rank 200)



Rank 1



Rank 2



Rank 5

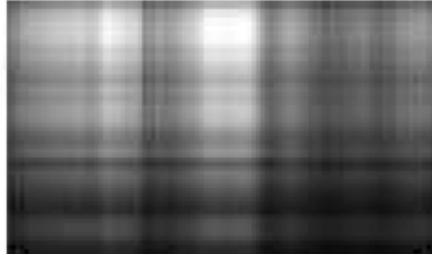


# Compressing Images - Reducing rank

Original(Rank 200)



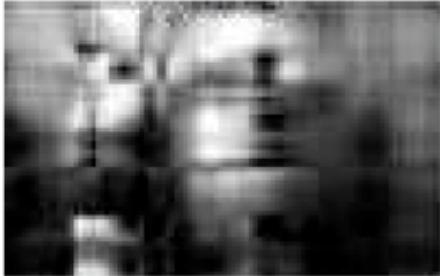
Rank 1



Rank 2



Rank 5



Rank 15

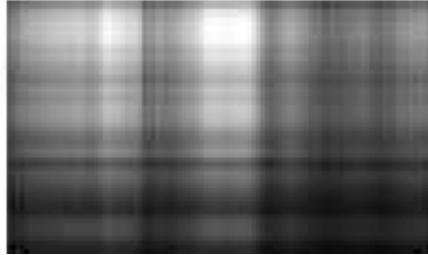


# Compressing Images - Reducing rank

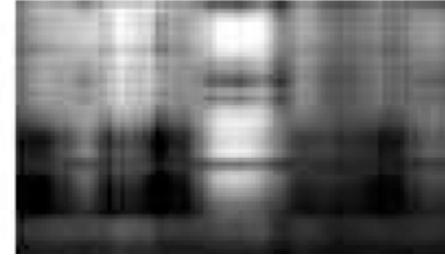
Original(Rank 200)



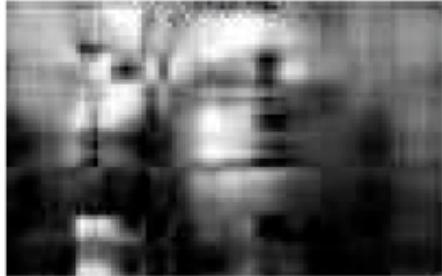
Rank 1



Rank 2



Rank 5



Rank 15



Rank 50



# Systems of information

# Systems of information

## System 1

-  The dog is **black**
-  The cat is **orange**

# Systems of information

System 1

 The dog is **black**  
 The cat is **orange**

System 2

 The dog is **black**  
 The dog is **black**

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

**System 2**

 The dog is **black**  
 The dog is **black**

**System 3**

 The dog  
The dog

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

**System 2**

 The dog is **black**  
 The dog is **black**

**System 3**

 The dog  
The dog

**Two sentences**

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

**System 2**

 The dog is **black**  
 The dog is **black**

**System 3**

 The dog  
The dog

**Two sentences**

**Two pieces of information**

# Systems of information

System 1

 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

System 2

 The dog is **black**  
 The dog is **black**

Two sentences

System 3

 The dog  
 The dog

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

**System 2**

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

**System 3**

 The dog  
 The dog

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

**System 2**

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

**System 3**

 The dog  
 The dog

Two sentences

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

**System 2**

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

**System 3**

 The dog  
The dog

Two sentences

Zero pieces of information

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

**System 2**

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

**System 3**

 The dog  
 The dog

Two sentences

Zero pieces of information

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

**System 2**

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

Rank = 1

**System 3**

 The dog  
 The dog

Two sentences

Zero pieces of information

# Systems of information

**System 1**

 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

**System 2**

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

Rank = 1

**System 3**

 The dog  
 The dog

Two sentences

Zero pieces of information

Rank = 0

# Systems of equations

# Systems of equations

## System 1

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} + \text{banana} = 0 \end{array}$$

# Systems of equations

System 1

$$a + b = 0$$


$$a + 2b = 0$$


System 2

$$a + b = 0$$


$$2a + 2b = 0$$


# Systems of equations

**System 1**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} \text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 2**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2\text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 3**

$$0\text{apple} + 0\text{banana} = 0$$

$$0\text{apple} + 0\text{banana} = 0$$

# Systems of equations

**System 1**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} \text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 2**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2\text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 3**

$$0\text{apple} + 0\text{banana} = 0$$

$$0\text{apple} + 0\text{banana} = 0$$

**Two equations**

# Systems of equations

**System 1**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} \text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 2**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2\text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 3**

$$0\text{apple} + 0\text{banana} = 0$$

$$0\text{apple} + 0\text{banana} = 0$$

**Two equations**

**Two pieces of information**

# Systems of equations

**System 1**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} \text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 2**

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2\text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 3**

$$0\text{apple} + 0\text{banana} = 0$$

$$0\text{apple} + 0\text{banana} = 0$$

**Two equations**

**Two pieces of information**

**Rank = 2**

# Systems of equations

## System 1

$$a + b = 0$$


$$a + 2b = 0$$


Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

## System 2

$$\begin{array}{r} a + b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2a + 2b = 0 \\ \text{🍎} \quad \text{banana} \end{array}$$

Two equations

One piece of information

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} + 2\text{banana} = 0 \end{array}$$

Two equations

Two pieces of information

Rank = 2

## System 2

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ 2\text{apple} + 2\text{banana} = 0 \end{array}$$

Two equations

One piece of information

Rank = 1

## System 3

$$0\text{apple} + 0\text{banana} = 0$$

# Systems of equations

## System 1

$$a + b = 0$$


$$a + 2b = 0$$


Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

# Systems of equations

## System 1

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} \text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

## System 2

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2\text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

Two equations

One piece of information

Rank = 1

## System 3

$$0\text{apple} + 0\text{banana} = 0$$

$$0\text{apple} + 0\text{banana} = 0$$

Two equations

Zero pieces of information

# Systems of equations

## System 1

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} \text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

Two equations

Two pieces of information

Rank = 2

## System 2

$$\begin{array}{r} \text{apple} + \text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

$$\begin{array}{r} 2\text{apple} + 2\text{banana} = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

Two equations

One piece of information

Rank = 1

## System 3

$$0\text{apple} + 0\text{banana} = 0$$

$$0\text{apple} + 0\text{banana} = 0$$

Two equations

Zero pieces of information

Rank = 0

# Systems of equations

**System 1**

$$\begin{array}{l} \text{apple} + \text{banana} = 0 \\ \text{apple} + 2\text{banana} = 0 \end{array}$$

1	1
1	2

**System 2**

$$\begin{array}{l} \text{apple} + \text{banana} = 0 \\ 2\text{apple} + 2\text{banana} = 0 \end{array}$$

**System 3**

$$\begin{array}{l} 0\text{apple} + 0\text{banana} = 0 \\ 0\text{apple} + 0\text{banana} = 0 \end{array}$$

**Two equations**

**Two pieces of information**

**Rank = 2**

**Two equations**

**One piece of information**

**Rank = 1**

**Two equations**

**Zero pieces of information**

**Rank = 0**

# Systems of equations

**System 1**

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$



1	1
1	2

**Rank = 2**

**System 2**

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ 2a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

**System 3**

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$

**Two equations**

**Two pieces of information**

**Rank = 2**

**Two equations**

**One piece of information**

**Rank = 1**

**Two equations**

**Zero pieces of information**

**Rank = 0**

# Systems of equations

**System 1**

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

1	1
1	2

**Rank = 2**

**System 2**

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ 2a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

1	1
2	2

**System 3**

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$

**Two equations**

**Two pieces of information**

**Rank = 2**

**Two equations**

**One piece of information**

**Rank = 1**

**Two equations**

**Zero pieces of information**

**Rank = 0**

# Systems of equations

**System 1**

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

1	1
1	2

**Rank = 2**

**System 2**

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ 2a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$

1	1
2	2

**Rank = 1**

**System 3**

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$

**Two equations**

**Two pieces of information**

**Rank = 2**

**Two equations**

**One piece of information**

**Rank = 1**

**Two equations**

**Zero pieces of information**

**Rank = 0**

# Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ a + 2b = 0 \end{array}$$



1	1
1	2

Rank = 2

System 2

$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$



1	1
2	2

Rank = 1

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$



0	0
0	0

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

# Systems of equations

System 1

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$



1	1
1	2

Rank = 2

System 2

$$\begin{array}{l} a + b = 0 \\ \text{apple} \quad \text{banana} \\ 2a + 2b = 0 \\ \text{apple} \quad \text{banana} \end{array}$$



1	1
2	2

Rank = 1

System 3

$$\begin{array}{l} 0a + 0b = 0 \\ 0a + 0b = 0 \end{array}$$



0	0
0	0

Rank = 0

Two equations

Two pieces of information

Rank = 2

Two equations

One piece of information

Rank = 1

Two equations

Zero pieces of information

Rank = 0

# Rank and solutions to the system



1	1
1	2

Rank = 2



1	1
2	2

Rank = 1



0	0
0	0

Rank = 0

# Rank and solutions to the system

1	1
1	2

Rank = 2

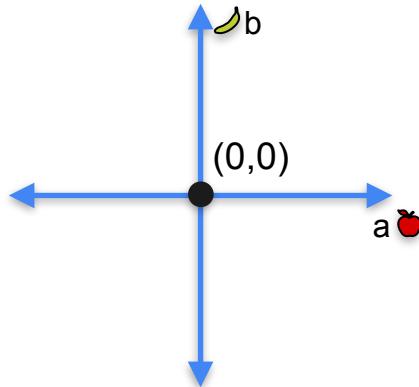
1	1
2	2

Rank = 1

0	0
0	0

Rank = 0

Dimension of solution space = 0



# Rank and solutions to the system

1	1
1	2

Rank = 2

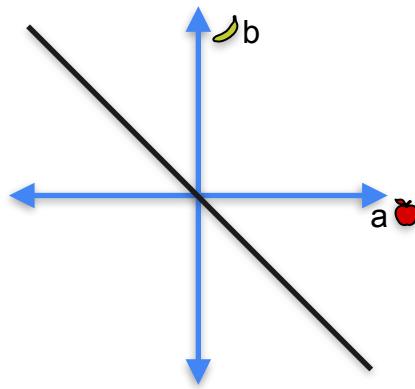
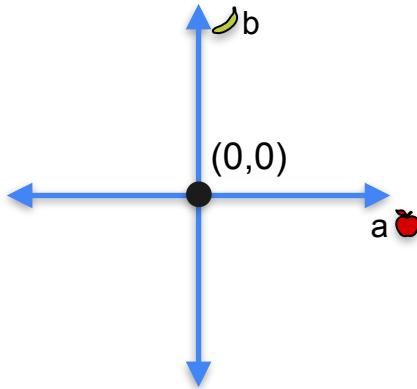
1	1
2	2

Rank = 1

0	0
0	0

Rank = 0

Dimension of solution space = 0   Dimension of solution space = 1



# Rank and solutions to the system

1	1
1	2

Rank = 2

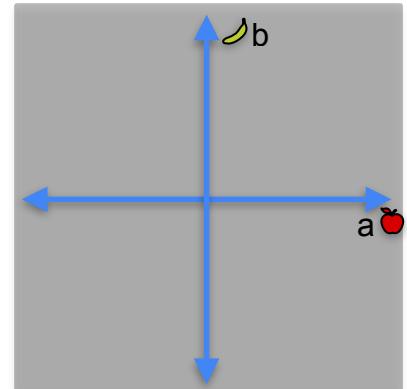
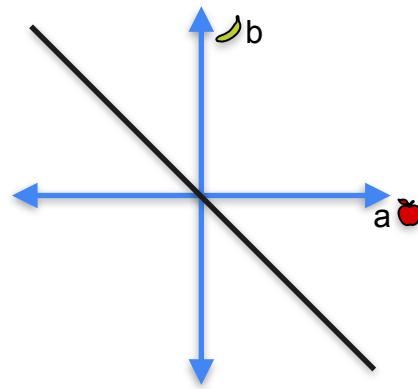
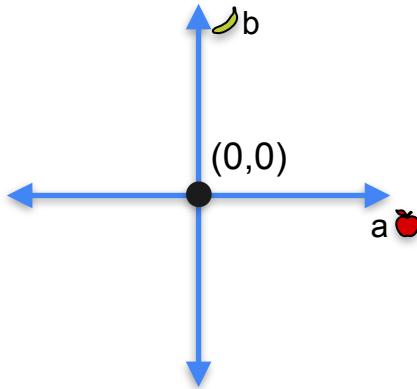
1	1
2	2

Rank = 1

0	0
0	0

Rank = 0

Dimension of solution space = 0   Dimension of solution space = 1   Dimension of solution space = 2



# Rank of a matrix

1	1
1	2

**Rank = 2**

1	1
2	2

**Rank = 1**

0	0
0	0

**Rank = 0**

Dimension of solution space = 0   Dimension of solution space = 1   Dimension of solution space = 2

**Rank = 2 - (Dimension of solution space)**

# Rank and singularity



1	1
1	2

**Rank = 2**



1	1
2	2

**Rank = 1**



0	0
0	0

**Rank = 0**

# Rank and singularity



1	1
1	2

**Rank = 2**



1	1
2	2

**Rank = 1**



0	0
0	0

**Rank = 0**

**Non-singular**

# Rank and singularity



1	1
1	2

**Rank = 2**



1	1
2	2

**Rank = 1**



0	0
0	0

**Rank = 0**

**Non-singular**

**Singular**

# Rank and singularity



1	1
1	2

**Rank = 2**



1	1
2	2

**Rank = 1**



0	0
0	0

**Rank = 0**

**Non-singular**

**Singular**

**Singular**

# Rank and singularity



1	1
1	2

Rank = 2

Non-singular



1	1
2	2

Rank = 1

Singular



0	0
0	0

Rank = 0

Singular

# Quiz: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1**

5	1
-1	3

**Matrix 2**

2	-1
-6	3

# Solutions: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1:** Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

**Matrix 2:** Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3



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# Solving System of Linear Equations

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**Rank of a matrix:  
General case**

# Rank for matrices

**System 1**

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

**System 2**

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

**System 3**

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

**System 4**

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

**System 1**

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$



**System 2**

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

**System 3**

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

**System 4**

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

# Rank for matrices

**System 1**

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$



**System 2**

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

**System 3**

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

**System 4**

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

# Rank for matrices

**System 1**

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$


**System 2**

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

**System 3**

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

**System 4**

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

# Rank for matrices

**System 1**

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$

**System 2**

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

**System 3**

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

**System 4**

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

**3 Equations**

**3 Pieces of information**

# Rank for matrices

**System 1**

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$

**System 2**

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

**System 3**

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

**System 4**

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

**3 Equations**

**3 Pieces of information**

**Rank 3**

# Rank for matrices

**System 1**

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$

**System 2**

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

**System 3**

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

**System 4**

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

**3 Equations**

**3 Pieces of information**

**Rank 3**

1	1	1
1	2	1
1	1	2

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0 \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

3 Equations

3 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \\ 0a + 0b + 0c &= 0 \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \\ 0a + 0b + 0c &= 0 & \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

3 Equations  
0 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

3 Equations  
0 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank 0

# Rank for matrices

System 1

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + 2b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \checkmark \end{aligned}$$

3 Equations  
3 Pieces of information

System 2

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ a + b + 2c &= 0 & \times \\ a + b + 3c &= 0 & \checkmark \end{aligned}$$

3 Equations  
2 Pieces of information

System 3

$$\begin{aligned} a + b + c &= 0 & \checkmark \\ 2a + 2b + 2c &= 0 & \times \\ 3a + 3b + 3c &= 0 & \times \end{aligned}$$

3 Equations  
1 Piece of information

System 4

$$\begin{aligned} 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \\ 0a + 0b + 0c &= 0 & \times \end{aligned}$$

3 Equations  
0 Pieces of information

Rank 3

1	1	1
1	2	1
1	1	2

Rank 2

1	1	1
1	1	2
1	1	3

Rank 1

1	1	1
2	2	2
3	3	3

Rank 0

0	0	0
0	0	0
0	0	0

# Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



DeepLearning.AI

# Solving System of Linear Equations

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**Row echelon form**

# Row echelon form of a matrix

# Row echelon form of a matrix

**Original matrix**

5	1
4	-3

# Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

# Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

# Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

1	1
0	0

# Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

1	1
0	0

0	0
0	0

# Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1

5	1
10	2

1	1
0	0

0	0
0	0

0	0
0	0

# Row echelon form

Original matrix

5	1
4	-3

# Row echelon form

Original matrix

5	1
4	-3

Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

5	1
4	-3



Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \quad \xrightarrow{\hspace{1cm}} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \text{---} & \text{---} \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \text{---} & \text{---} \\ \hline \end{array}$$

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

---

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & -0.75 \\ \hline - & 1 & 0.2 \\ \hline 0 & -0.95 \end{array}$$

# Row echelon form

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & -0.75 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & -0.95 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & -0.75 \\ \hline - & 1 & 0.2 \\ \hline 0 & -0.95 \end{array}$$

# Row echelon form

Original matrix

5	1		
4	-3		
		1	0.2
		1	-0.75

Divide each row by  
the leftmost coefficient

Divide the second row by  
the leftmost non-zero coefficient

$$\begin{array}{r} & \begin{matrix} 1 & -0.75 \\ - & \begin{matrix} 1 & 0.2 \end{matrix} \end{matrix} \\ \hline & \begin{matrix} 0 & -0.95 \end{matrix} \end{array}$$

# Row echelon form

Original matrix

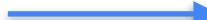
5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Divide each row by  
the leftmost coefficient

$$\begin{array}{r} & 1 & -0.75 \\ - & \begin{array}{|c|c|} \hline & 1 & 0.2 \\ \hline \end{array} \\ \hline & 0 & -0.95 \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form

Original matrix

5	1	1	0.2	1	0.2	1	0.2
4	-3	1	-0.75	0	-0.95		

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 \quad -0.75 \\ - \quad 1 \quad 0.2 \\ \hline 0 \quad -0.95 \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form

Original matrix

5	1	1	0.2	1	0.2	1	0.2
4	-3	1	-0.75	0	-0.95	0	1

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 \quad -0.75 \\ - \quad 1 \quad 0.2 \\ \hline 0 \quad -0.95 \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



1	0.2
0	1

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & -0.75 \\ \hline - & 1 & 0.2 \\ \hline 0 & -0.95 \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2

# Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \quad \xrightarrow{\hspace{1cm}} \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & 1 & 0.2 \\ \hline \end{array}$$

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline 0 & 0 \end{array}$$

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & 1 & 0.2 \\ \hline 0 & 0 \end{array}$$

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \xrightarrow{\quad} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

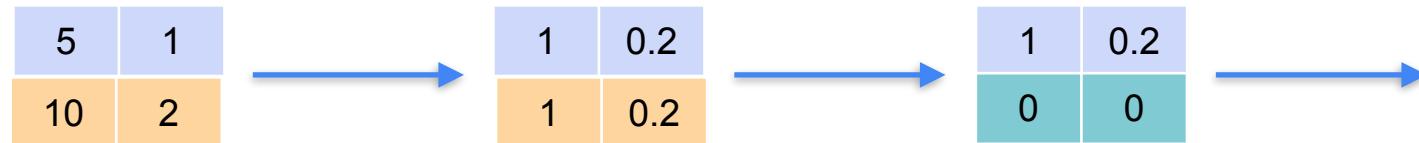
Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & 1 & 0.2 \\ \hline 0 & 0 \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix



Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 1 & 0.2 \\ \hline \end{array} \\ - \\ \hline \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array} \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1	1	0.2	1	0.2	1	0.2
10	2	1	0.2	0	0	1	0.2

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ - \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array} \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1	1	0.2	1	0.2	?	?
10	2	1	0.2	0	0	?	?

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ - \quad \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array} \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2

1	0.2
1	0.2

Row echelon form

1	0.2
0	0

1	0.2
?	?

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} 1 & 0.2 \\ - & \begin{array}{r} 1 & 0.2 \\ \hline 0 & 0 \end{array} \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

0	0
0	0

# Row echelon form for singular matrices

Original matrix

0	0
0	0

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

0	0
0	0



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \quad \xrightarrow{\hspace{1cm}} \quad \begin{array}{|c|c|} \hline ? & ? \\ \hline \end{array}$$

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

0	0		
0	0		



?	?		
?	?		

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Row echelon form

Original matrix



0	0	?	?
0	0	?	?

Divide each row by  
the leftmost coefficient

# Row echelon form, singularity, and rank

5	1
4	-3

5	1
10	2

0	0
0	0

# Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

# Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\hspace{1cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

# Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \xrightarrow{\hspace{2cm}} \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

# Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

2 ones in the diagonal

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

# Row echelon form, singularity, and rank

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 4 & -3 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 1 \\ \hline \end{array}$$

**Rank 2**  
2 ones in the diagonal

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline 10 & 2 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}$$

# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

**Rank 2**  
2 ones in the diagonal

5	1
10	2



1	0.2
0	0

1 one in the diagonal

0	0
0	0



0	0
0	0

# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

**Rank 2**  
2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**  
1 one in the diagonal

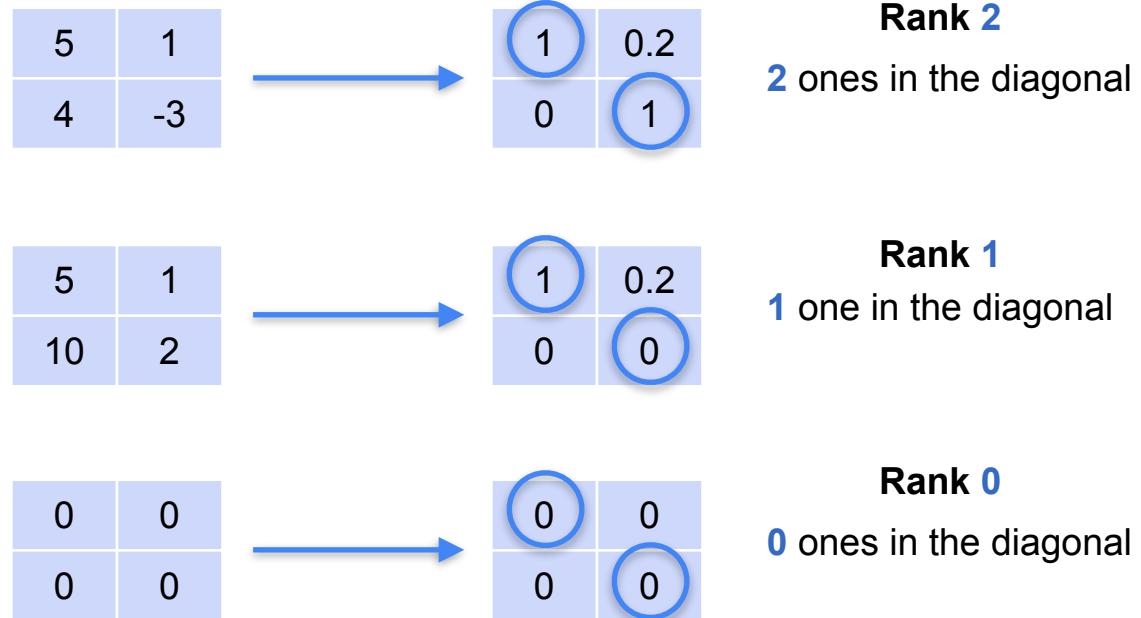
0	0
0	0



0	0
0	0

**0** ones in the diagonal

# Row echelon form, singularity, and rank



# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

**Rank 0**

0 ones in the diagonal

# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

Singular matrix

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

**Rank 0**

0 ones in the diagonal

# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

Singular matrix

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

Singular matrix

0	0
0	0



0	0
0	0

**Rank 0**

0 ones in the diagonal



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# Solving System of Linear Equations

---

**Row echelon form:  
General case**

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

# Row echelon form

## System

$$\bullet a + b + 2c = \mathbf{12}$$

$$\bullet 3a - 3b - c = \mathbf{3}$$

$$\bullet 2a - b + 6c = \mathbf{24}$$



## System

$$\bullet a + b + 2c = \mathbf{12}$$

$$\bullet -6b - 7c = \mathbf{-33}$$

$$\bullet 6c = \mathbf{18}$$

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$



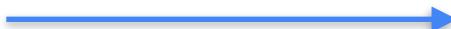
## Matrix

1	1	2
3	-3	-1
2	-1	6

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

## Matrix

1	1	2
3	-3	-1
2	-1	6



## Row echelon form matrix

1	1	2
0	-6	7
0	0	6

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

Rank 5

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots

# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

Subtract the first row  
from the second and  
the third ones

# Another example

Matrix			Row echelon form		
1	1	1			
1	2	1	→	0	1
1	1	2		0	0

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3



1	1	1
0	0	1
0	0	2

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3



1	1	1
0	0	1
0	0	2

Subtract the first row  
from the second and  
the third ones

Subtract twice the  
second row from the  
third one

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3



**Row echelon form**

1	1	1
0	0	1
0	0	2



1	1	1
0	0	1
0	0	0

Subtract the first row  
from the second and  
the third ones

Subtract twice the  
second row from the  
third one

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the  
first row from the  
second row

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3



1	1	1
0	0	0
3	3	3

Subtract twice the  
first row from the  
second row

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3



1	1	1
0	0	0
3	3	3

Subtract twice the first row from the second row

Subtract three times the first row from the third row

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

**Row echelon form**

1	1	1
0	0	0
3	3	3

1	1	1
0	0	0
0	0	0



Subtract twice the first row from the second row

Subtract three times the first row from the third row

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

Number of pivots = 3

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

# Rank for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Rank = 3

Matrix 2

1	1	1
1	1	2
1	1	3

Rank = 2

Matrix 3

1	1	1
2	2	2
3	3	3

Rank = 1

Matrix 4

0	0	0
0	0	0
0	0	0

Rank = 0

Row echelon forms

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0



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# Solving System of Linear Equations

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**Reduced row echelon form**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$



# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$



# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

Row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

## Row echelon form

## Reduced row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

## Diagonal matrix

1	0
0	1

## Row echelon form

## Reduced row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

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5	1
4	-3

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1	0.2
0	1

## Row echelon form

## Diagonal matrix

1	0
0	1

## Reduced row echelon form

# Reduced row echelon form

**Row echelon form**

1	0.2
0	1

# Reduced row echelon form

**Row echelon form**

1	0.2
0	1



0	1
---	---

# Reduced row echelon form

**Row echelon form**

1	0.2
0	1



0	1
---	---

0	1
---	---

# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

0.2

# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

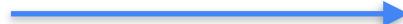
$$\begin{matrix} x & 0.2 \end{matrix}$$

---

# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

x

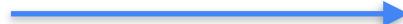
0.2

0	0.2
---	-----

# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

1	0.2
---	-----

x

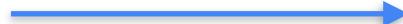
0.2

0	0.2
---	-----

# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

$$\begin{array}{r} \\ \text{x} \\ \hline \end{array} \quad \begin{array}{r} \\ 0.2 \\ \hline \end{array}$$

$0 \quad 1$

$$\begin{array}{r} \\ - \\ \hline \end{array} \quad \begin{array}{r} \\ 1 \quad 0.2 \\ 0 \quad 0.2 \\ \hline \end{array}$$

# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

$$\begin{array}{r} \\ \text{x} \\ \hline 0 & 1 \\ & 0.2 \end{array}$$

$$\begin{array}{r} \\ - \\ \hline 1 & 0.2 \\ 0 & 0.2 \\ \hline 1 & 0 \end{array}$$

# Reduced row echelon form

Row echelon form

1	0.2
0	1



1	0
0	1

$$\begin{array}{r} \\ \text{x} \end{array} \quad \begin{array}{r} 0 \\ 0.2 \end{array}$$

---

$$\begin{array}{r} \\ 0 \end{array} \quad \begin{array}{r} 0.2 \end{array}$$

$$\begin{array}{r} \\ - \end{array} \quad \begin{array}{r} 1 \\ 0 \end{array} \quad \begin{array}{r} 0.2 \\ 0.2 \end{array}$$

---

$$\begin{array}{r} \\ 1 \end{array} \quad \begin{array}{r} 0 \end{array}$$

# Reduced row echelon form

Row echelon form

1	0.2
0	1

Reduced  
row echelon form

1	0
0	1

0	1
---	---

$$x \quad 0.2$$

---

0	0.2
---	-----

---

1	0.2
0	0.2

---

1	0
---	---

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

Rank 5

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Rank 3

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

# Reduced row echelon form

# Reduced row echelon form

<b>3</b>	*	*	*	*
0	0	<b>2</b>	*	*
0	0	0	<b>-4</b>	*
0	0	0	<b>0</b>	0
0	0	0	0	<b>0</b>

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by  
the value of the pivot

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by  
the value of the pivot

Reduced row  
echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a  
pivot to 0

# Reduced row echelon form

**Row echelon form**

1	2	3
0	1	4
0	0	1

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the  
second row from the  
first one

# Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1

Subtract 2 times the second row from the first one

# Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

# Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1



1	0	0
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

# Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1



1	0	0
0	1	4
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Subtract 4 times the third row from the second one

# Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1

1	0	-5
0	1	4
0	0	1

1	0	0
0	1	4
0	0	1

1	0	0
0	1	0
0	0	1

Subtract 2 times the second row from the first one

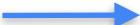
Add 5 times the third row to the first one

Subtract 4 times the third row from the second one

# Reduced row echelon form

Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1



1	0	0
0	1	4
0	0	1



Reduced row echelon form

1	0	0
0	1	0
0	0	1

Subtract 2 times the second row from the first one

Add 5 times the third row to the first one

Subtract 4 times the third row from the second one



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# Solving System of Linear Equations

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## Conclusion