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REGRESSION IN R

DATA SERVICES @ HSL

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OUTLINE

- Linear models and Multiple regression
- Measures of model fit
- Logistic (and Multinomial regression)
- Mixed effects models

- Variable selection

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LINEAR REGRESSION

Simple linear regression

- With a Categorical Variable (ANOVA)
    - Covered last class
- With a Continuous Variable (review)

Multiple regression

- Can seamlessly combine categorical and continuous predictors

LINEAR REGRESSION

SINGLE PREDICTOR X

$$Y = \beta_0 + \beta_1 X + \epsilon$$

MULTIPLE PREDICTORS X_1 AND X_2

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

Y = RESPONSE VARIABLE

X = PREDICTOR

β_0 = Y-INTERCEPT

$\beta_{1,2}$ = SLOPE. What effect does one unit change in X do to Y?

ϵ = RESIDUAL ERROR

Given X, slope, and y-intercept, model cannot perfectly predict Y. These are assumed to be normally distributed with a mean of 0 and a standard deviation σ

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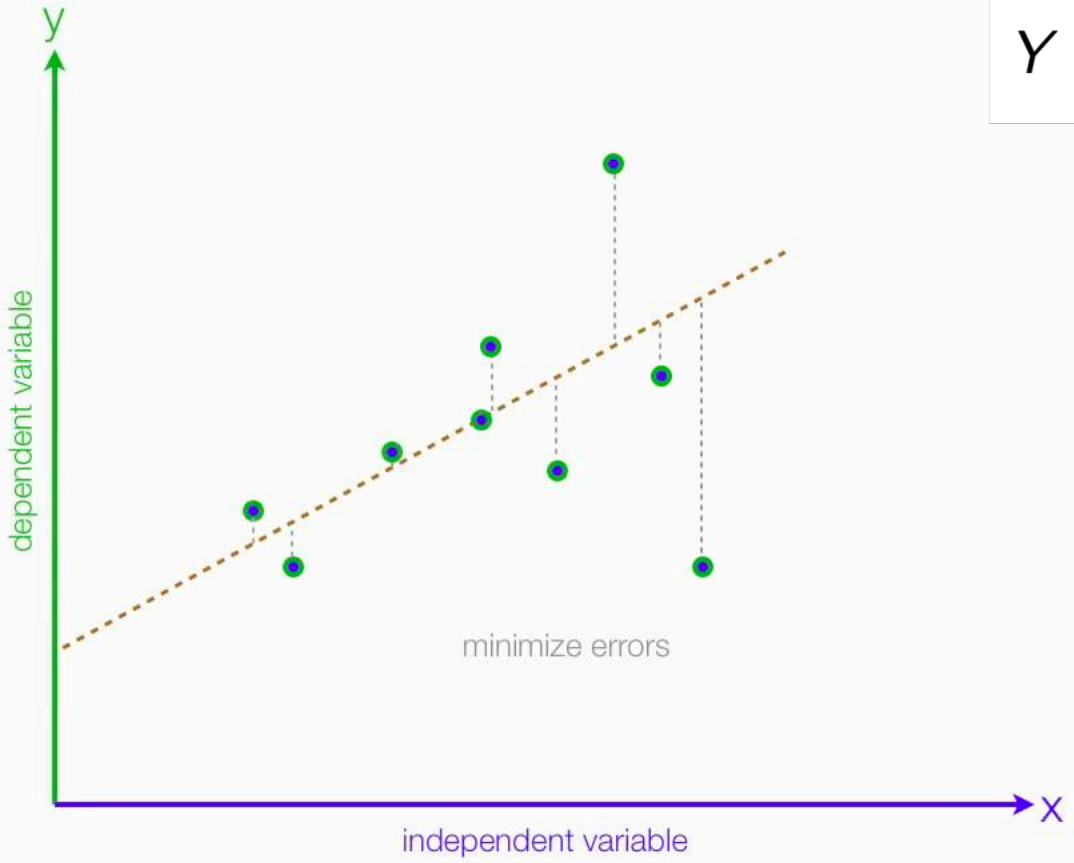
LINEAR REGRESSION

$$Y = \beta_0 + \beta_1 X + \epsilon$$



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ASSUMPTIONS OF LINEAR REGRESSION

- Random Sampling
- Residuals are independent
- Residuals are normally distributed
- X and Y have linear relationship
- Residuals show constant variance across levels of X

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MEASURES OF MODEL FIT

- R^2 / Adjusted R^2
- RMSE (Root Mean Squared Error)
- AIC / BIC

- Special case: compare 2 nested models
  - Likelihood Ratio test

MEASURES OF MODEL FIT

- R^2
  - Percentage of variation in Y explained by model
  - Increases with number of predictors
- Adjusted R^2
  - Proportion of total variance explained by model
  - Increases with number of predictors if improvement in model fit is worthwhile
- RMSE (root-mean-square error)
  - Square root of the mean of the squared residuals
  - Indicates absolute fit of the model to the data
    - How close are observed data to model's predicted values
  - Lower values indicate better fit
  - Best measure of model fit for predictive models

MEASURES OF MODEL FIT

- AIC / BIC
  - Akaike Information Criterion / Bayesian Information Criterion
  - Estimate relative quality of models for a given set of data
  - When a model is used to represent a process that generated data, some information will be lost by using the model rather than the process itself
    - AIC / BIC estimate relative information lost by a given model
  - Include a tradeoff between model fit and model complexity
  - Lower is better

The optimal model has a high R^2, low RMSE, and low AIC / BIC

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MULTIPLE REGRESSION

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

- Rarely can biological phenomena be explained by one variable
- Simple extension of linear model with 1 predictor
- Can add continuous and / or categorical predictors
- ** Interpretation of coefficient is “average effect on Y of a one unit increase in Xj, holding all other predictors constant”

MULTIPLE REGRESSION TIPS

- Use biological knowledge to inform which predictors are included
- Do not add predictors that correlate too highly with other predictors (~+/- .8)
- The addition of new predictors will change the model fit and coefficients of other predictors

LOGISTIC AND MULTINOMIAL REGRESSION

- Outcome variable is binary → logistic regression
  - e.g. model whether someone is insured based on several other variables
  - Model **log odds** of being insured

- Outcome variable is categorical (3+) → multinomial regression

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LOGISTIC REGRESSION

p is the probability of being insured

$\frac{p}{1-p}$ is the odds of being insured

$$p = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$$

The generalized linear model is therefore:

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X$$

Where β_0 is the intercept, β_1 is the increase in the log odds of outcome for every unit increase in x_1.

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LOGISTIC REGRESSION

- Logistic regression is a type of *generalized linear model* (GLM)
- using glm() function in R
- glm() works like lm() except we specify which distribution to use with the family argument
- Logistic regression uses family=binomial

```
mod <- glm(y ~ x, data=yourdata, family='binomial')
summary(mod)
```

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MULTINOMIAL REGRESSION

$$\Pr(Y = k|X) = \frac{e^{\beta_{0k}+\beta_{1k}X_1+...+\beta_{pk}X_p}}{\sum_{\ell=1}^{K} e^{\beta_{0\ell}+\beta_{1\ell}X_1+...+\beta_{p\ell}X_p}}$$

- Extension of logistic regression where each class (k) is modeled by its own linear function

- multinom() function in nnet package
  - Tutorial at https://stats.idre.ucla.edu/r/dae/multinomial-logistic-regression/

MIXED EFFECTS REGRESSION

- Linear model with both fixed effects and random effects
- LME (**L**inear **M**ixed **E**ffects) models are suitable for clustered, longitudinal, or repeated measures data where data are grouped by *random levels* and response variable is continuous
- *Random levels* means that there are observations that belong to single subject or group that can be thought of as randomly selected from the population
- **Fixed effects have estimates of their effect on Y (slope / coefficient)**
- **Random effects have estimates of variation**

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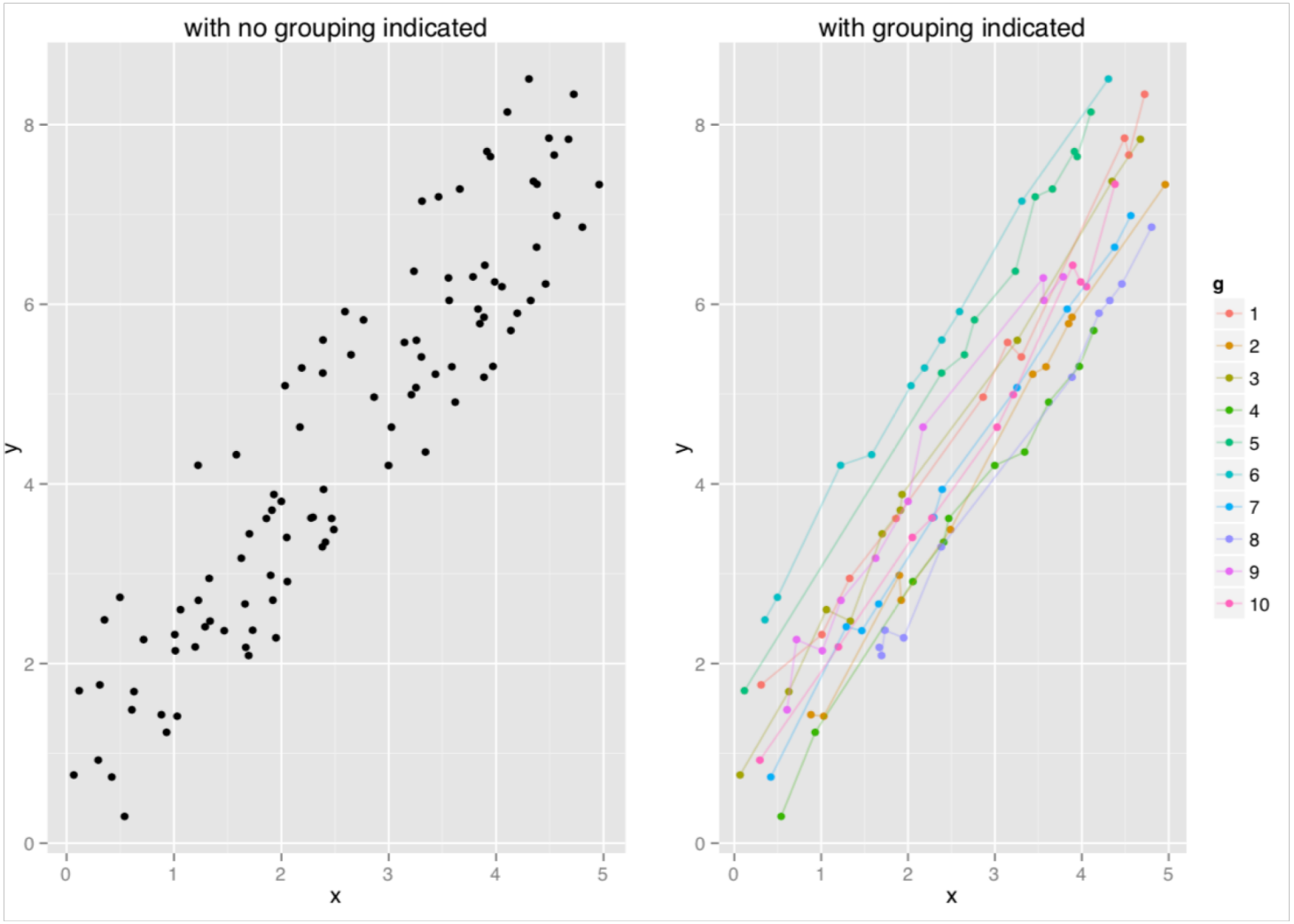
MIXED EFFECTS REGRESSION

- Example: Treatment applied to subjects and outcome measured every 2 weeks for 2 months
- Treatment is a **fixed effect.** Other fixed effects could be age, sex, cell type, time. If we were to repeat this experiment, we would use these variables again
- These data are grouped by subject. The subjects we choose are a random sample from the population, so there are **random effects** associated with subjects. If we repeated the experiment, we would not use these subjects again
- In R: lmer() function in lme4 package and lmerTest package
- May need glht() function in multcomp package for multiple comparisons

NEED FOR MIXED EFFECTS



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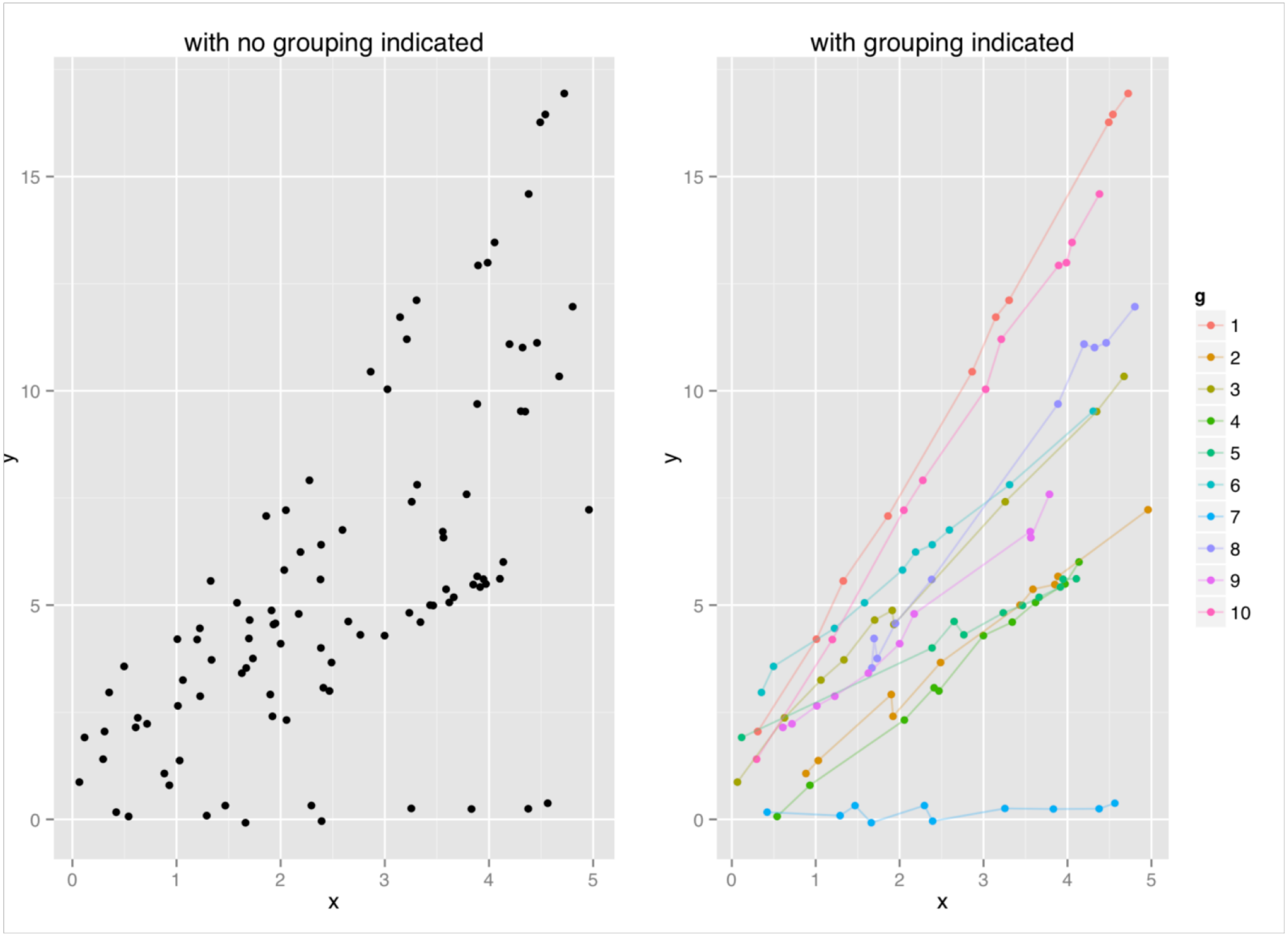
NEED FOR MIXED EFFECTS



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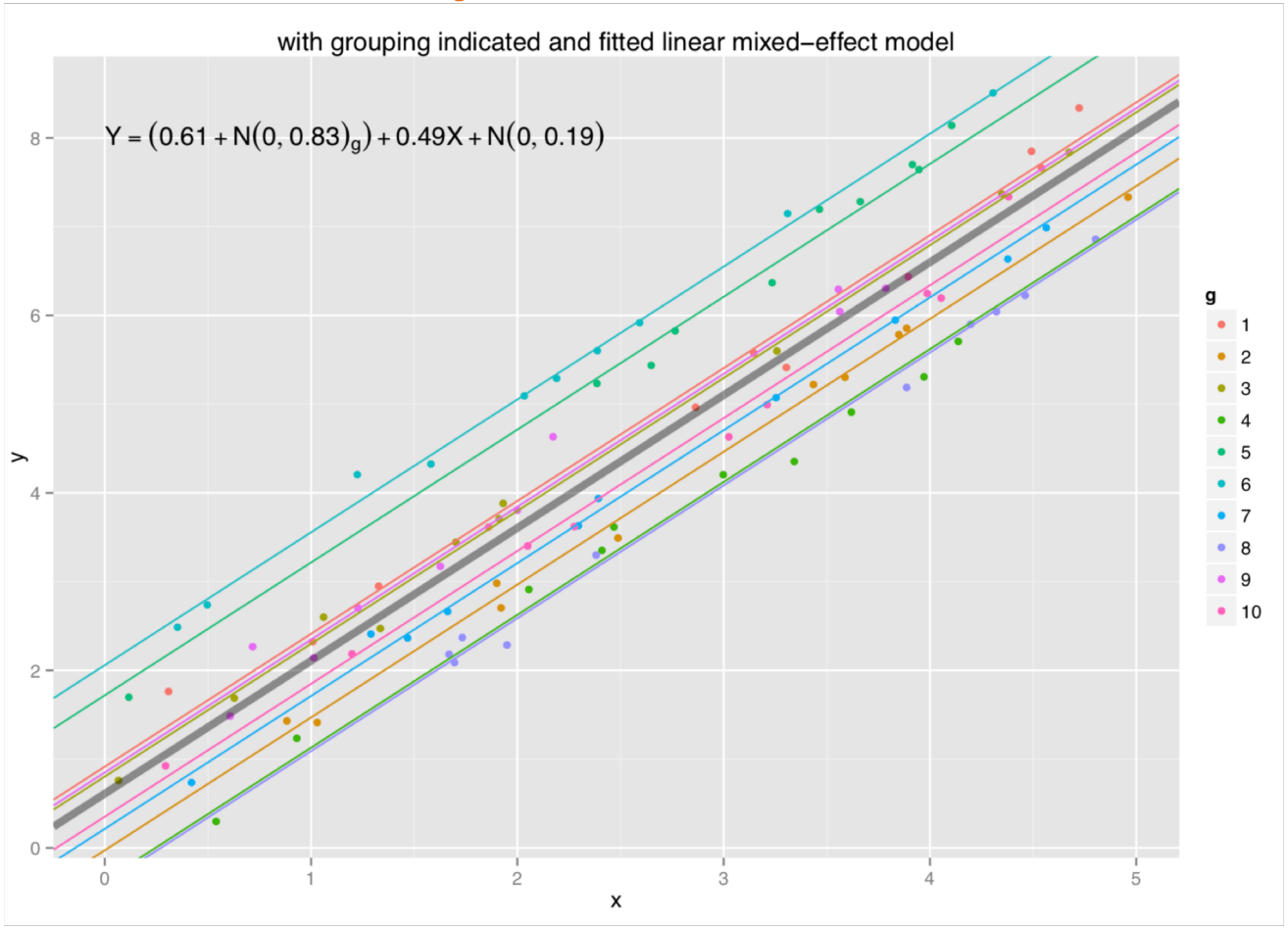
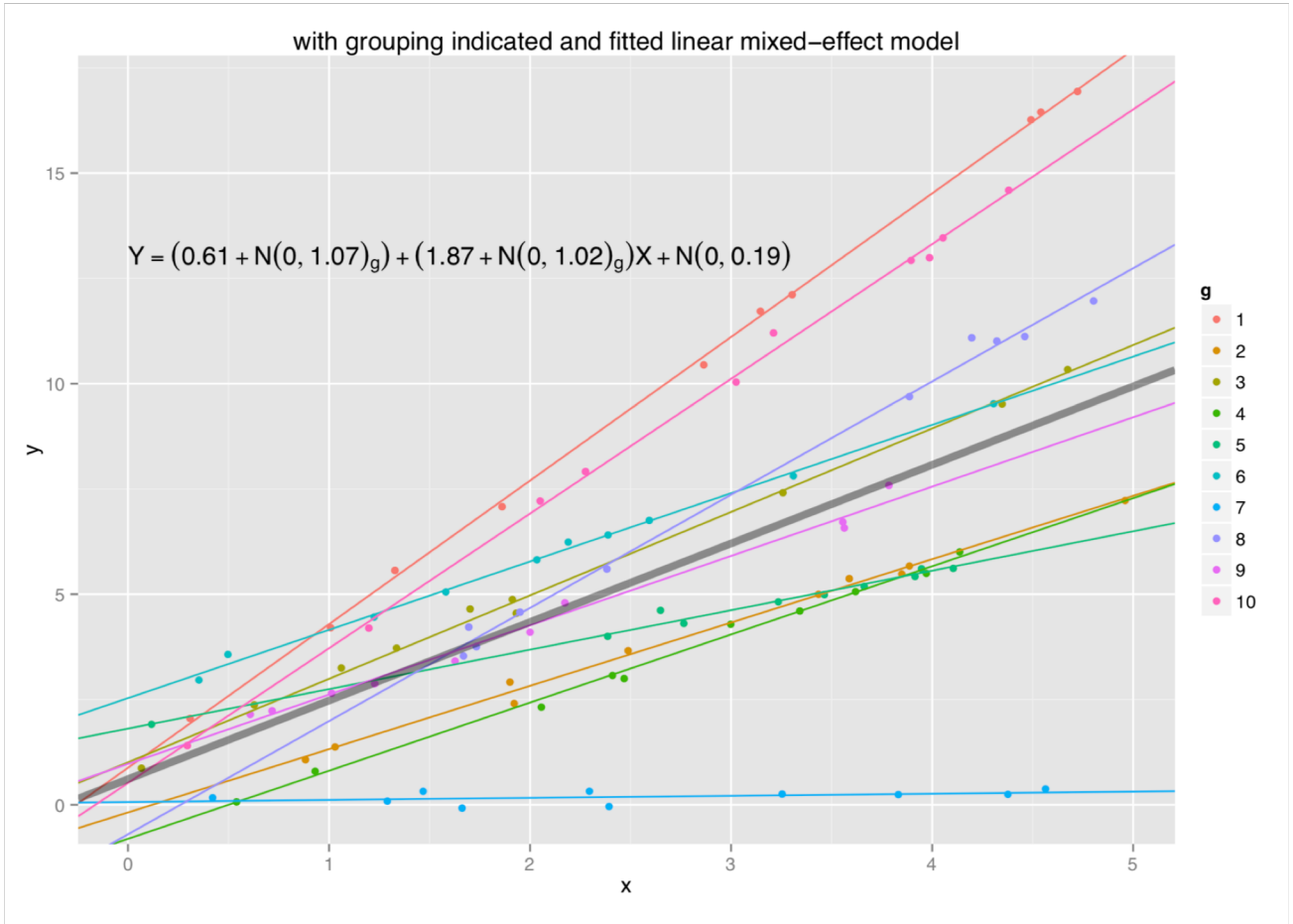
SAME SLOPE, DIFFERENT INTERCEPT




```
lmer(y ~ x + (1 | g), data = dat)
```

DIFFERENT SLOPE, DIFFERENT INTERCEPT

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