CSCI 416 - HW2

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In [ ]:

```
```

Problem 2

Implement Logistic Regression for Book Classification

This notebook does the following:

- Loads a data set for predicting whether a book is hardcover or paperback from two input features: the thickness of the book and the weight of the book
- Normalizes the features
- Has a placeholder for your implementation of logistic regression
- Plots the data and the decision boundary of the learned model

Read below and follow instructions to complete the implementation.

Setup

Run the code below to import modules, etc.

In [417…

```
%matplotlib inline
%reload_ext autoreload
%autoreload 2

import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

from logistic_regression import logistic, cost_function, gradient_descent
```

In [418…

```
def normalize_features( X, mu=None, sigma=None ):
    '''
    Feature normalization

    Inputs:
        X       m x n data matrix (either train or test)
        mu      vector of means (length n)
        sigma   vector of standard deviations (length n)

    Outputs:
        X_norm  normalized data matrix
        mu      vector of means
        sigma   vector of standard deviations

    IMPORTANT NOTE:
        When called for training data, mu and sigma should be computed
        from X and returned for later use. When called for test data,
        the mu and sigma should be passed in to the function and
        *not* computed from X.

    '''
    if mu is None:
        mu    = np.mean(X, axis=0)
        sigma = np.std (X, axis=0)

    # Don't normalize constant features
    mu   [sigma == 0] = 0
    sigma[sigma == 0] = 1
    X_norm = (X - mu)/sigma

    return (X_norm, mu, sigma)
```

Load and Prep Data

Read the code in the cell below and run it. This loads the book data from file and selects two features to set up the training data `X` (data matrix) and `y` (label vector). It then normalizes the training data.

In [419…

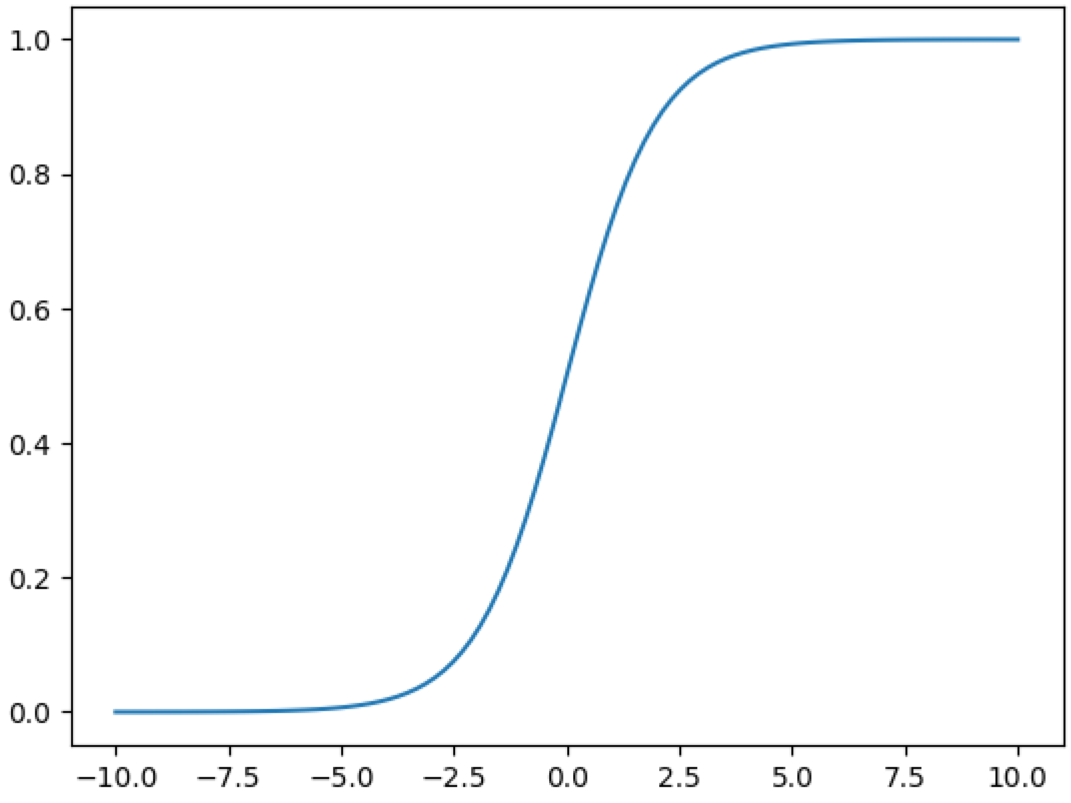
```
data = pd.read_csv('book-data.csv', sep=',',header=None).values

# % Data columns
# %
# % 0 - width
# % 1 - thickness
# % 2 - height
# % 3 - pages
# % 4 - hardcover
# % 5 - weight

y = data[:,4]

# % Extract the normalized features into named column vectors
width     = data[:,0]
thickness = data[:,1]
height    = data[:,2]
pages     = data[:,3]
weight    = data[:,5]

m = data.shape[0]
X = np.stack([np.ones(m), thickness, height], axis=1)
n = X.shape[1]

X, mu, sigma = normalize_features(X)
```

(1 point) Implement the `logistic` function

Open the file `logistic_regression.py` and complete the code for the function `logistic`. Then run the cell below to plot the logistic function for $-10 \leq z \leq 10$ to test your implementation --- it should look like the logistic function!

In [420…

```
z = np.linspace(-10, 10, 100)
plt.plot(z, logistic(z))
plt.show()
```



(2 points) Implement `cost_function`

Complete the code for `cost_function` in the file `logistic_regression.py` to implement the logistic regression cost function. Then test it with the code in the cell below.

In [421…
```
theta = np.zeros(n)
print(cost_function(X, y, theta)) # prints 38.81624....
```

```
38.816242111356935
```

Setup for plotting a learned model

Run this cell and optionally read the code. It defines a function to help plot the data together with the decision boundary for the model we are about to learn.

In [422…
```
def plot_model(X, y, theta):
    pos = y==1
    neg = y==0

    plt.scatter(X[pos,1], X[pos,2], marker='+', color='blue', label='Hardcover')
    plt.scatter(X[neg,1], X[neg,2], marker='o', color='red', facecolors='none', label='P

    # plot the decision boundary
    x1_min = np.min(X[:,1]) - 0.5
    x1_max = np.max(X[:,1]) + 0.5

    x1 = np.array([x1_min, x1_max])
    x2 = (theta[0] + theta[1]*x1)/(-theta[2])
    plt.plot(x1, x2, label='Decision boundary')

    plt.xlabel('thickness (normalized)')
    plt.ylabel('height (normalized)')
    plt.legend(loc='lower right')
    plt.show()
```

(7 points) Implement gradient descent for logistic regression

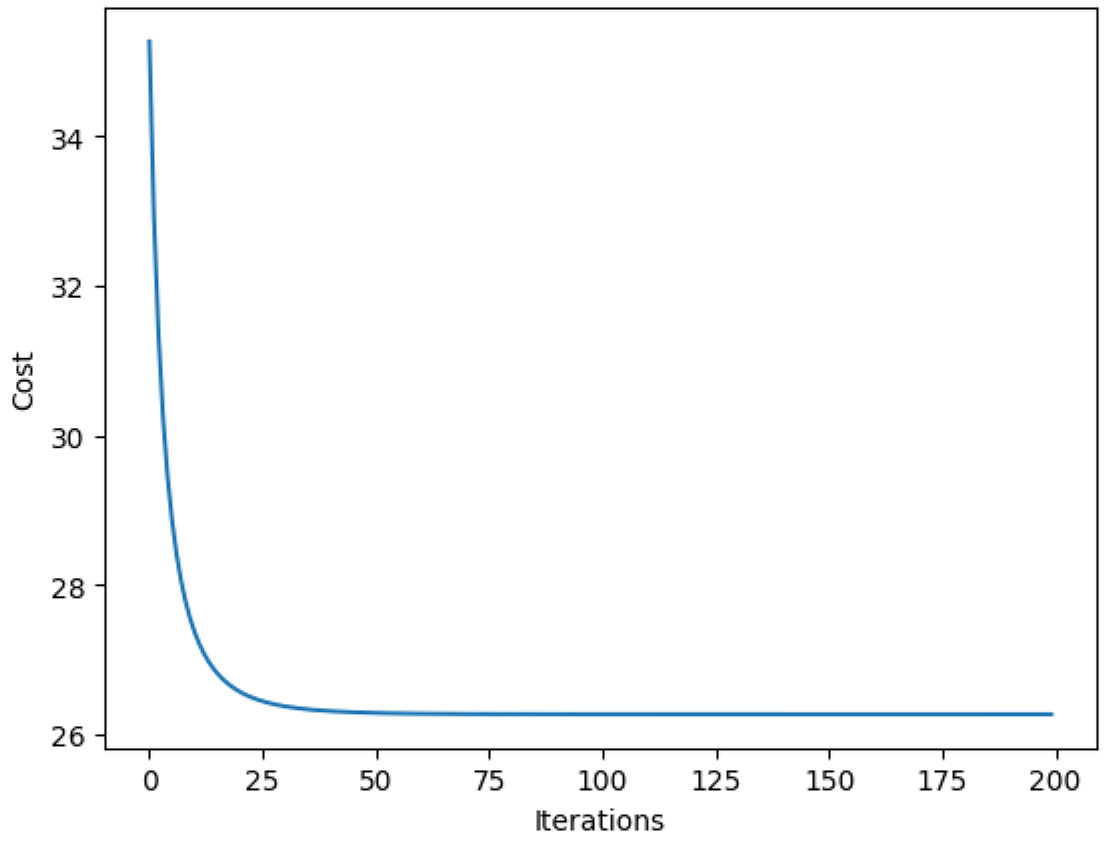
Now complete the code for `gradient_descent` in the file `logistic_regression.py`, which runs gradient descent to find the best parameters `theta`, and write code in the cell below to:

1. Call `gradient_descent` to learn `theta`
2. Print the final value of the cost function
3. Plot J_history to assess convergence
4. Tune the step size and number of iterations if needed until the algorithm converges and the decision boundary (see next cell) looks reasonable
5. Print the accuracy---the percentage of correctly classified examples in the training set

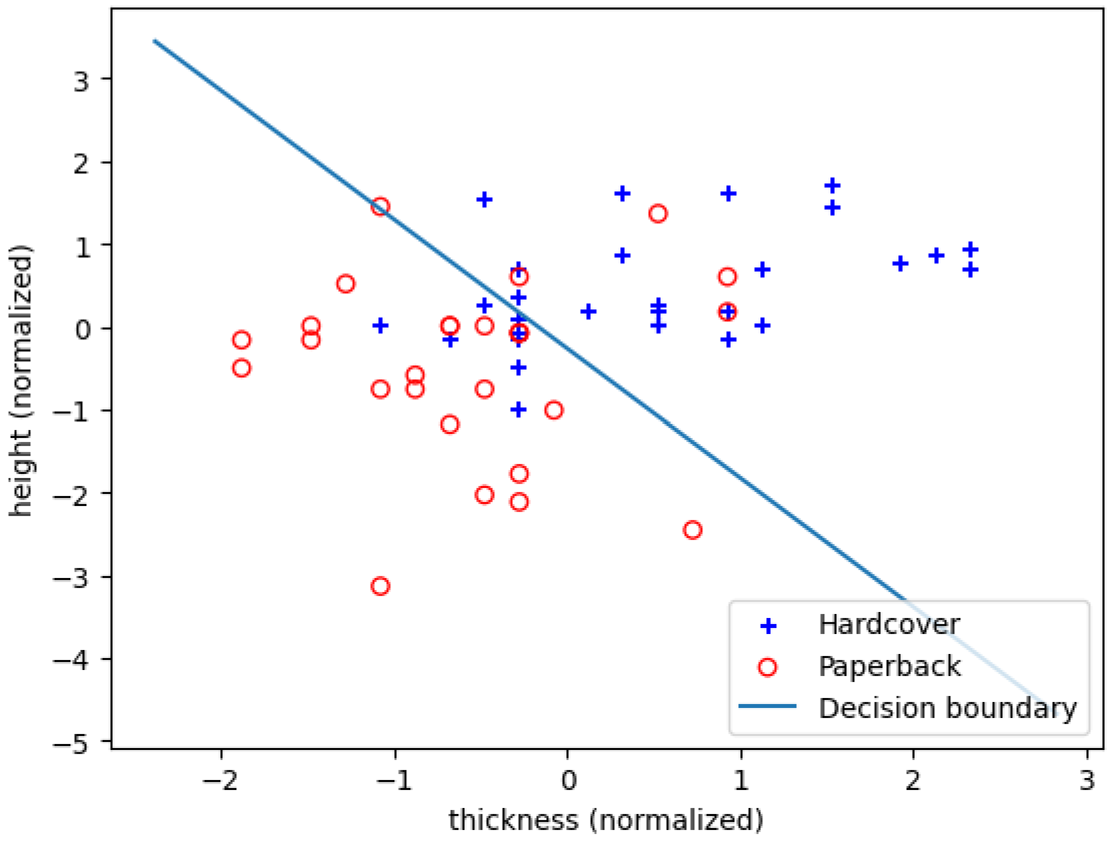
In [423…

```
theta = np.zeros(n)

#
# YOUR CODE HERE
#
output = gradient_descent(X, y, theta, 0.01, 200)
theta = output[0]
J_history = output[1]
print ("Cost function: %.2f" % J_history[-1])
plt.figure(2)
plt.plot(J_history)
plt.xlabel("Iterations")
plt.ylabel("Cost")
plt.show()
prediction = (logistic(np.dot(X, theta)) >= 0.5).astype(int)
accuracy = np.mean(prediction.flatten() == y)*100
print ("Accuracy: %f%%" % accuracy)

# Plots data and decision boundary. If you have learned a good theta
# you will see a decision boundary that separates the data in a
# reasonable way.
plot_model(X, y, theta)
```

```
Cost function: 26.27
```



```
Accuracy: 76.785714%
```



```
In [ ]:
```

```
In [ ]:
```

Problem 3

Logistic regression for SMS spam classification

Each line of the data file `sms.txt` contains a label---either "spam" or "ham" (i.e. non-spam)---followed by a text message. Here are a few examples (line breaks added for readability):

```
ham     Ok lar... Joking wif u oni...
ham     Nah I don't think he goes to usf, he lives around here though
spam    Free entry in 2 a wkly comp to win FA Cup final tkts 21st May
2005.
        Text FA to 87121 to receive entry question(std txt rate)
        T&C's apply 08452810075over18's
spam    WINNER!! As a valued network customer you have been
        selected to receivea £900 prize reward! To claim
        call 09061701461. Claim code KL341. Valid 12 hours only.
```

To create features suitable for logistic regression, code is provided to do the following (using tools from the `sklearn.feature_extraction.text`):

- Convert words to lowercase.
- Remove punctuation and special characters (but convert the $ and £ symbols to special tokens and keep them, because these are useful for predicting spam).
- Create a dictionary containing the 3000 words that appeared most frequently in the entire set of messages.
- Encode each message as a vector $\mathbf{x}^{(i)} \in \mathbb{R}^{3000}$. The entry $x_j^{(i)}$ is equal to the number of times the jth word in the dictionary appears in that message.
- Discard some ham messages to have an equal number of spam and ham messages.
- Split data into a training set of 1000 messages and a test set of 400 messages.

Follow the instructions below to complete the implementation. Your job will be to:

- Learn $\boldsymbol{\theta}$ by gradient descent
- Plot the cost history
- Make predictions and report the accuracy on the test set
- Test out the classifier on a few of your own text messages

Load and prep data

This cell preps the data. Take a look to see how it works, and then run it.

In [424…

```
%matplotlib inline
%reload_ext autoreload
%autoreload 2

import numpy as np
import re
import matplotlib.pyplot as plt
import codecs

from logistic_regression import logistic, cost_function, gradient_descent
from sklearn.feature_extraction.text import CountVectorizer
```

```
# Preprocess the SMS Spam Collection data set
#
#   https://archive.ics.uci.edu/ml/datasets/SMS+Spam+Collection
#
# From Dan Sheldon

numTrain    = 1000
numTest     = 494
numFeatures = 3000

np.random.seed(1)

# Open the file
f = codecs.open('sms.txt', encoding='utf-8')

labels = []    # list of labels for each message
docs   = []    # list of messages

# Go through each line of file and extract the label and the message
for line in f:
    l, d= line.strip().split('\t', 1)
    labels.append(l)
    docs.append(d)

# This function will be called on each message to preprocess it
def preprocess(doc):
    # Replace all currency signs and some url patterns by special
    # tokens. These are useful features.
    doc = re.sub('[£$]', ' __currency__ ', doc)
    doc = re.sub('\://', ' __url__ ', doc)
    doc = doc.lower() # convert to lower
    return doc


# This is the object that does the conversion from text to feature vectors
vectorizer = CountVectorizer(max_features=numFeatures, preprocessor=preprocess)

# Do the conversion ("fit" the transform from text to feature vector.
#   later we will also "apply" the tranform on test messages)
X = vectorizer.fit_transform(docs)

# Convert labels to numbers: 1 = spam, 0 = ham
y = np.array([l == 'spam' for l in labels]).astype('int')

# The vectorizer returns sparse scipy arrays. Convert this back to a dense
#   numpy array --- not as efficient but easier to work with
X = X.toarray()
m,n = X.shape

# Add a column of ones
X = np.column_stack([np.ones(m), X])

#
# Now massage and split into test/train
#
pos = np.nonzero(y == 1)[0]   # indices of positive training examples
neg = np.nonzero(y == 0)[0]   # indices of negative training examples

npos = len(pos)

# Create a subset that has the same number of positive and negative examples
subset = np.concatenate([pos, neg[0:len(pos)] ])

# Randomly shuffle order of examples
np.random.shuffle(subset)
```

```
X = X[subset,:]
y = y[subset]

# Split into test and train
train = np.arange(numTrain)
test  = numTrain + np.arange(numTest)

X_train = X[train,:]
y_train = y[train]

X_test  = X[test,:]
y_test  = y[test]

# Extract the list of test documents
test_docs = [docs[i] for i in subset[test]]

# Extract the list of tokens (words) in the dictionary
tokens = vectorizer.get_feature_names_out()
```

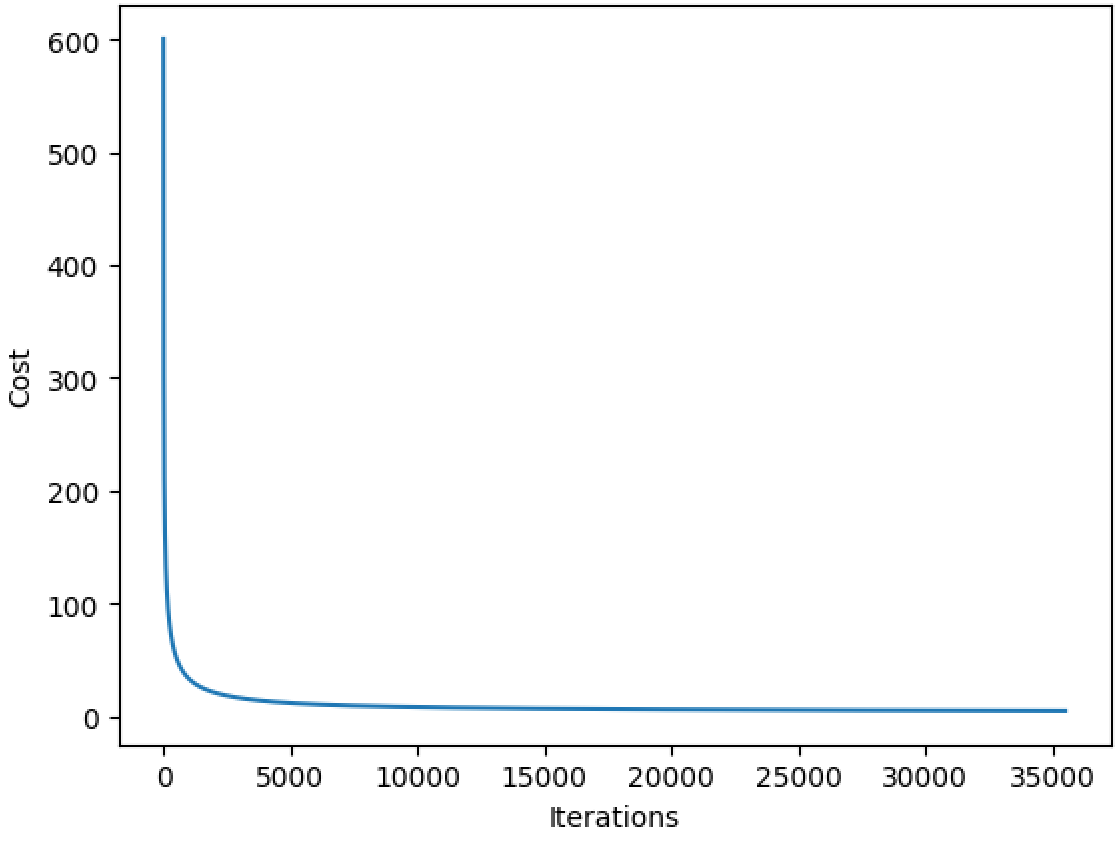
Train logistic regresion model

Now train the logistic regression model. The comments summarize the relevant variables created by the preprocessing.

In [425…

```
# X_train     contains information about the words within the training
#             messages. the ith row represents the ith training message.
#             for a particular text, the entry in the jth column tells
#             you how many times the jth dictionary word appears in
#             that message
#
# X_test      similar but for test set
#
# y_train     ith entry indicates whether message i is spam
#
# y_test      similar
#

m, n = X_train.shape

theta = np.zeros(n)


# YOUR CODE HERE:
#  - learn theta by gradient descent
#  - plot the cost history
#  - tune step size and # iterations if necessary
output = gradient_descent(X_train, y_train, theta, 0.001, 35500)
theta = output[0]
J_history = output[1]
print ("Cost function: %.2f" % J_history[-1])
plt.figure(2)
plt.plot(J_history)
plt.xlabel("Iterations")
plt.ylabel("Cost")
plt.show()
```

```
Cost function: 4.82
```



Make predictions on test set

Use the model fit in the previous cell to make predictions on the test set and compute the accuracy (percentage of messages in the test set that are classified correctly). You should be able to get accuracy above 95%.

In [426…

```
m_test, n_test = X_test.shape

# YOUR CODE HERE
#  - use theta to make predictions for test set
#  - print the accuracy on the test set---i.e., the precent of messages classified corre
pred = (logistic(np.dot(X_test, theta)) >= 0.5).astype(int)
accuracy = np.mean(pred.flatten() == y_test)*100
print ("Accuracy: %f%%" % accuracy)
```

```
Accuracy: 96.558704%
```

Inspect model parameters

Run this code to examine the model parameters you just learned. These parameters assign a postive or negative value to each word --- where positive values are words that tend to be spam and negative values are words that tend to be ham. Do they make sense?

In [427…

```
token_weights = theta[1:]

def reverse(a):
    return a[::-1]

most_negative = np.argsort(token_weights)
most_positive = reverse(most_negative)
```

```
k = 10

print('Top %d spam words' % k)
for i in most_positive[0:k]:
    print('  %+.4f  %s' % (token_weights[i], tokens[i]))

print('\nTop %d ham words' % k)
for i in most_negative[0:k]:
    print('  %+.4f  %s' % (token_weights[i], tokens[i]))
```

```
Top 10 spam words
  +6.7442  ringtoneking
  +5.4250  __currency__
  +4.7422  text
  +3.6960  txt
  +3.5060  ringtone
  +3.3216  service
  +3.2212  message
  +3.2125  150p
  +2.9125  arsenal
  +2.8970  sms

Top 10 ham words
  -4.6869  waiting
  -3.2745  ok
  -2.9168  later
  -2.7377  what
  -2.7287  ll
  -2.5480  so
  -2.3100  he
  -2.2658  me
  -2.2167  lol
  -2.2005  my
```

Make a prediction on new messages

Type a few of your own messages in below and make predictions. Are they ham or spam? Do the predictions make sense?

In [428…

```
def extract_features(msg):
    x = vectorizer.transform([msg]).toarray()
    x = np.insert(x, 0, 1)
    return x

msg = u'Write your own text here...'
x = extract_features(msg)  # this is the feature vector


# YOUR CODE HERE
#  - try a few texts of your own
#  - predict whether they are spam or non-spam
text1 = 'This is a text from the big arsenal service. We are leaving a message about you
x1 = extract_features(text1)
pred1 = logistic(np.dot(x1, theta))
print(pred1)
print("spam\n")

text2 = 'Im waiting for you outside your house'
x2 = extract_features(text2)
pred2 = logistic(np.dot(x2, theta))
print(pred2)
print("ham\n")
```

```
text3 = 'LOL, what is that'
x3 = extract_features(text3)
pred3 = logistic(np.dot(x3, theta))
print(pred3)
print("ham\n")

text4 = 'Only $5,000,000 to reserve your free elephant today!'
x4 = extract_features(text4)
pred4 = logistic(np.dot(x4, theta))
print(pred4)
print("spam\n")
print("The predictions make sense")
```

```
0.9999999992571056
spam

0.0030389948614641875
ham

0.000127679162646541
ham

0.9954220220650352
spam

The predictions make sense
```

In [ ]:

In [ ]:

Problem 4

Hand-Written Digit Classification

In this assignment you will implement multi-class classification for hand-written digits and run a few experiments. The file `digits-py.mat` is a data file containing the data set, which is split into a training set with 4000 examples, and a test set with 1000 examples.

You can import the data as a Python dictionary like this:

```
.python
data = scipy.io.loadmat('digits-py.mat')
```

The code in the cell below first does some setup and then imports the data into the following variables for training and test data:

- `X_train` - 2d array shape 4000 x 400
- `y_train` - 1d array shape 4000
- `X_test` - 2d array shape 1000 x 400
- `y_test` - 1d array shape 1000

In [429…

```
%matplotlib inline
%reload_ext autoreload
%autoreload 2
```

```
import numpy as np
import matplotlib.pyplot as plt

# Load train and test data
import scipy.io
data = scipy.io.loadmat('digits-py.mat')
X_train = data['X_train']
y_train = data['y_train'].ravel()
X_test  = data['X_test']
y_test  = data['y_test'].ravel()
```

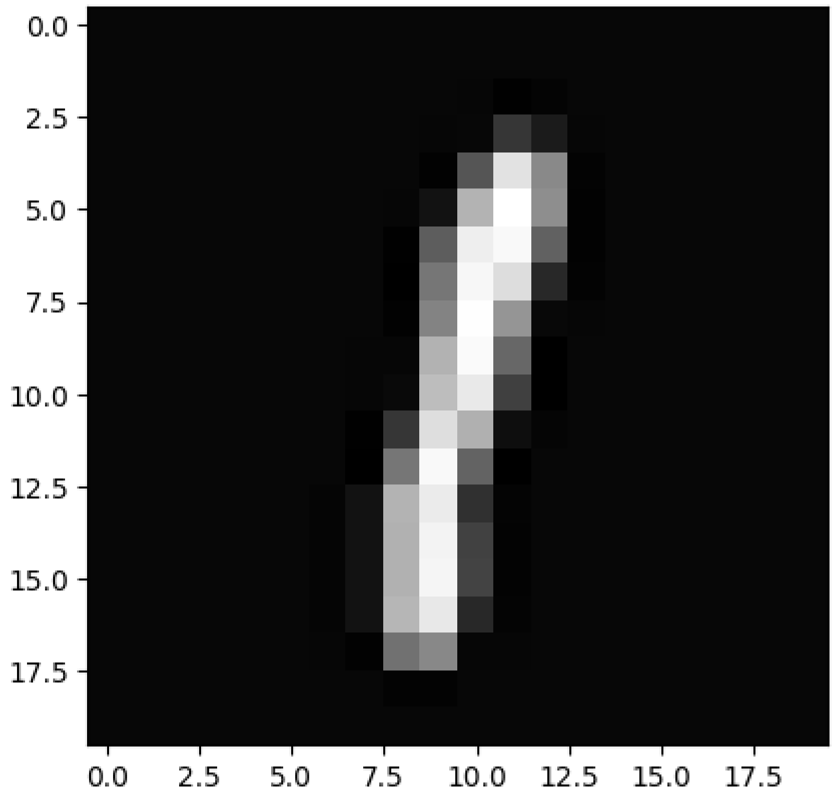
(2 points) Write code to visualize the data

Once you have loaded the data, it is helpful to understand how it represents images. Each row of `X_train` and `X_test` represents a 20 x 20 image as a vector of length 400 containing the pixel intensity values. To see the original image, you can reshape one row of the train or test data into a 20 x 20 matrix and then visualize it using the matlplotlib `imshow` command.

Write code using `np.reshape` and `plt.imshow` to display the 100th training example as an image. (Hint: use `cmap='gray'` in `plt.imshow` to view as a grayscale image.)

In [430…

```
# Write code here
example = np.reshape(X_train[99], (20,20))
plt.imshow(example, cmap='gray')
plt.show()
```



(2 points) Explore the data

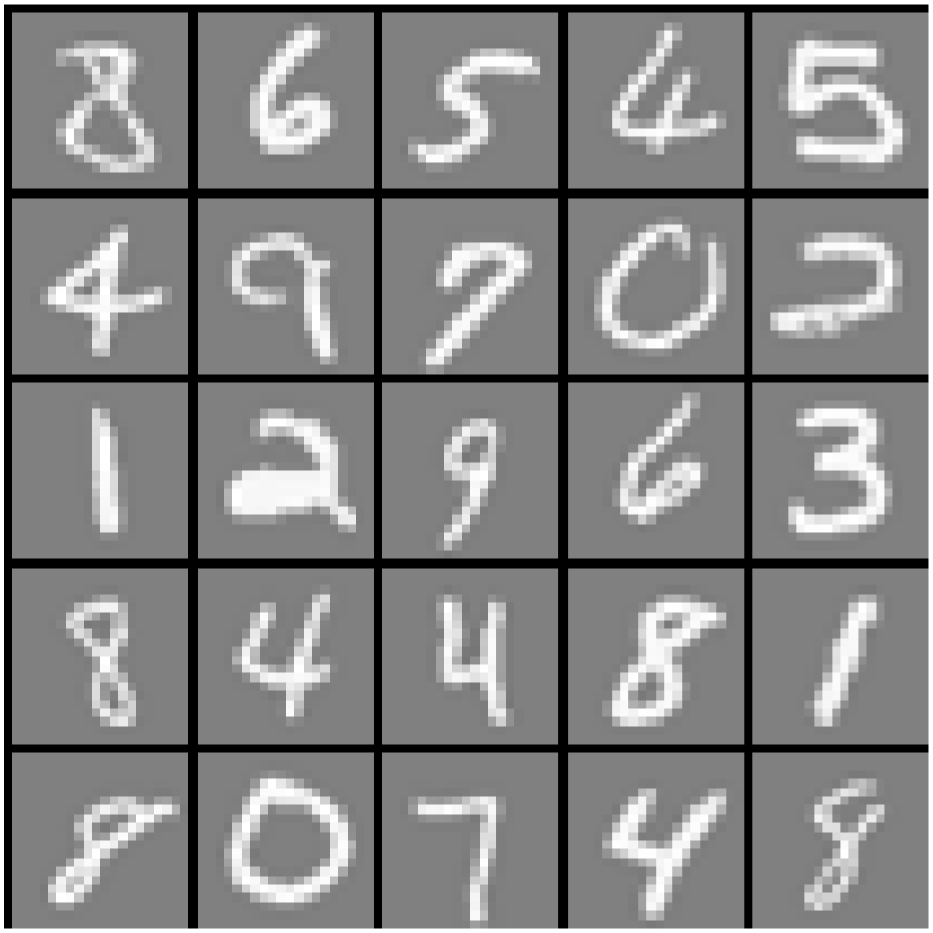
A utility function `display_data` is provided for you to further visualize the data by showing a mosaic of many digits at the same time. For example, you can display the first 25 training examples like this:

```
.python
display_data( X_train[:25, :] )
```

Go ahead and do this to visualize the first 25 training examples. Then print the corresponding labels to see if they match.

In [431…

```python
from one_vs_all import display_data

# Write code here
display_data(X_train[:25, :] )
print(y_train[:25])
```



```
[8 6 5 4 5 4 9 7 0 2 1 2 9 6 3 8 4 4 8 1 8 0 7 4 8]
```

Alert: notation change!

Please read this carefully to understand the notation used in the assignment. We will use logistic regression to solve multi-class classification. For three reasons (ease of displaying parameters as images, compatibility with scikit learn, previewing notation for SVMs and neural networks), we will change the notation as described here.

Old notation

Previously we defined our model as

$$h_{\boldsymbol{\theta}}(\mathbf{x}) = \text{logistic}(\theta_0 + \theta_1 x_1 + \ldots + \theta_n x_n) = \text{logistic}(\boldsymbol{\theta}^T \mathbf{x})$$

where

- $\mathbf{x} = [\,1, x_1, \ldots, x_n\,]$ was a feature vector with a 1 added in the first position
- $\boldsymbol{\theta} = [\,\theta_0, \theta_1, \ldots, \theta_n\,]$ was a parameter vector with the intercept parameter θ_0 in the first position

New notation

We will now define our model as

$$h_{\mathbf{w}}(\mathbf{x}) = \text{logistic}(b + w_1 x_1 + \ldots + w_n x_n) = \text{logistic}(\mathbf{w}^T \mathbf{x} + b)$$

where

- $\mathbf{x} \in \mathbb{R}^n$ is the **original feature vector** with no 1 added
- $\mathbf{w} \in \mathbb{R}^n$ is a **weight vector** (equivalent to $\theta_1, \ldots, \theta_n$ in the old notation)
- b is a scalar **intercept parameter** (equivalent to θ_0 in our old notation)

(10 points) One-vs-All Logistic Regression

Now you will implement one vs. all multi-class classification using logistic regression. Recall the method presented in class. Suppose we are solving a K class problem given training examples in the data matrix $X \in \mathbb{R}^{m \times n}$ and label vector $\mathbf{y} \in \mathbb{R}^m$ (the entries of $\mathbf{y}$ can be from 1 to K).

For each class $c = 1, \ldots, K$, fit a logistic regression model to distinguish class c from the others using the labels

$$y_c^{(i)} = \begin{cases} 1 & \text{if } y^{(i)} = c \\ 0 & \text{otherwise.} \end{cases}$$

This training procedure will result in a weight vector $\mathbf{w}_c$ and an intercept parameter b_c that can be used to predict the probability that a new example $\mathbf{x}$ belongs to class c:

$$\text{logistic}(\mathbf{w}_c^T \mathbf{x} + b_c) = \text{probability that } \mathbf{x} \text{ belongs to class } c.$$

The overall training procedure will yield one weight vector for each class. To make the final prediction for a new example, select the class with highest predicted probability:

$$\text{predicted class} = \text{the value of } c \text{ that maximizes logistic}(\mathbf{w}_c^T \mathbf{x} + b_c).$$

Training

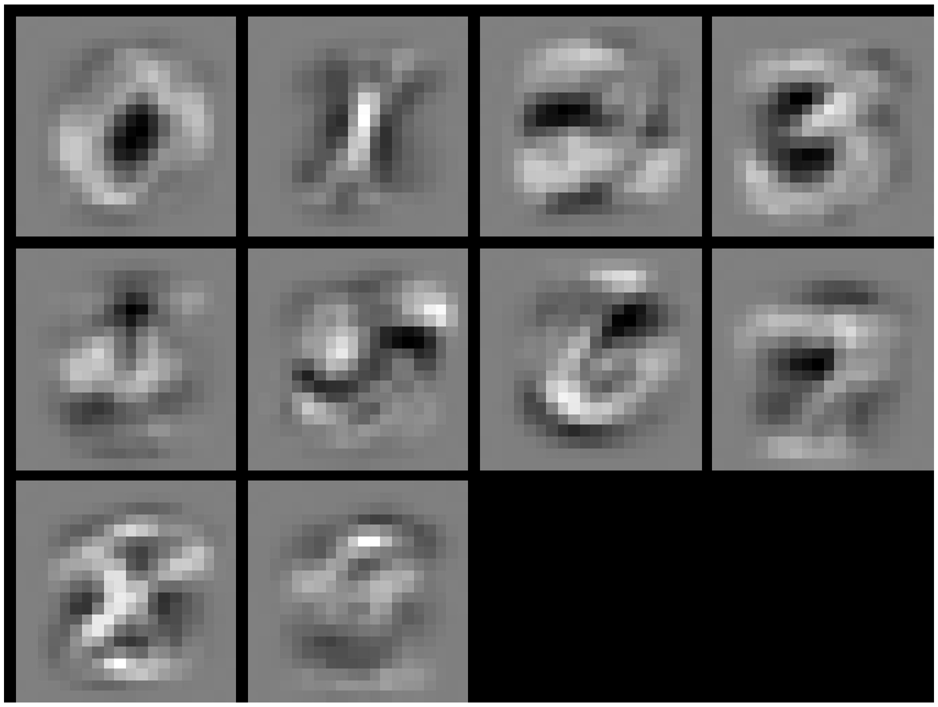
Open the file `one_vs_all.py` and complete the function `train_one_vs_all` to train binary classifiers using the procedure outlined above. I have included a function for training a regularized logistic regression model, which you can call like this:

```
.python
weight_vector, intercept = fit_logistic_regression(X, y, lambda_val)
```

Follow the instructions in the file for more details. Once you are done, test your implementation by running the code below to train the model and display the weight vectors as images. You should see images that are recognizable as the digits 0 through 9 (some are only vague impressions of the digit).

```
In [432…
from one_vs_all import train_one_vs_all
```

```
lambda_val = 100
weight_vectors, intercepts = train_one_vs_all(X_train, y_train, 10, lambda_val)
display_data(weight_vectors.T) # display weight vectors as images
```



Predictions

Now complete the function `predict_one_vs_all` in `one_vs_all.py` and run the code below to make predictions on the train and test sets. You should see accuracy around 88% on the test set.

In [433…
```
from one_vs_all import predict_one_vs_all

pred_train = predict_one_vs_all(X_train, weight_vectors, intercepts)
pred_test  = predict_one_vs_all(X_test,  weight_vectors, intercepts)

print("Training Set Accuracy: %f" % (np.mean(pred_train == y_train) * 100))
print("    Test Set Accuracy: %f" % (np.mean( pred_test == y_test) * 100))
```

```
Training Set Accuracy: 89.275000
    Test Set Accuracy: 88.300000
```

(5 points) Regularization Experiment

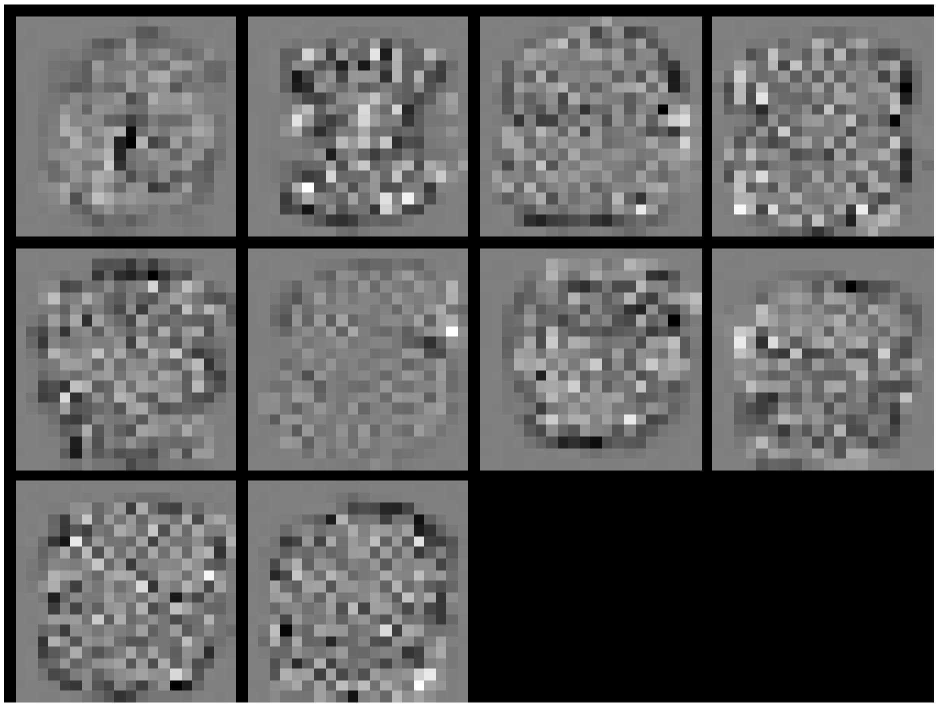
Now you will experiment with different values of the regularization parameter λ to control overfitting. Write code to measure the training and test accuracy for values of λ that are powers of 10 ranging from 10^{-3} to 10^{5}.

- Display the weight vectors for each value of λ as an image using the `display_data` function
- Save the training and test accuracy for each value of λ
- Plot training and test accuracy versus lambda (in one plot).

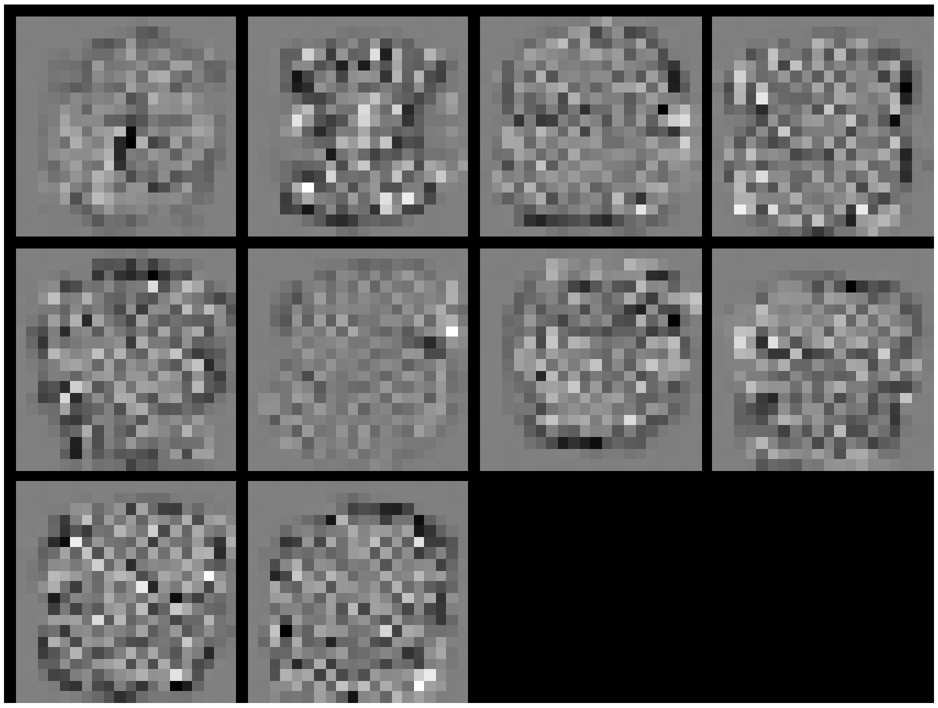
In [434…
```
lambda_vals = 10**np.arange(-3., 5.)
num_classes = 10

# Write code here
training_accuracy = np.zeros(len(lambda_vals))
```

```
test_accuracy = np.zeros(len(lambda_vals))
for i in range(0,len(lambda_vals)):
    weight_vectors, intercepts = train_one_vs_all(X_train, y_train, num_classes, lambda_
    print("Lambda val: %s" % lambda_vals[i])
    display_data(weight_vectors.T)
    pred_train = predict_one_vs_all(X_train, weight_vectors, intercepts)
    pred_test  = predict_one_vs_all(X_test,  weight_vectors, intercepts)
    training_accuracy[i]=(np.mean(pred_train == y_train) * 100)
    test_accuracy[i]=((np.mean( pred_test == y_test) * 100))

# In your final plot, use these commands to provide a legend and set
# the horizontal axis to have a logarithmic scale so the value of lambda
# appear evenly spaced.

plt.plot(lambda_vals, training_accuracy)
plt.plot(lambda_vals, test_accuracy)
plt.legend(('train', 'test'))
plt.xscale('log')
plt.show()
```

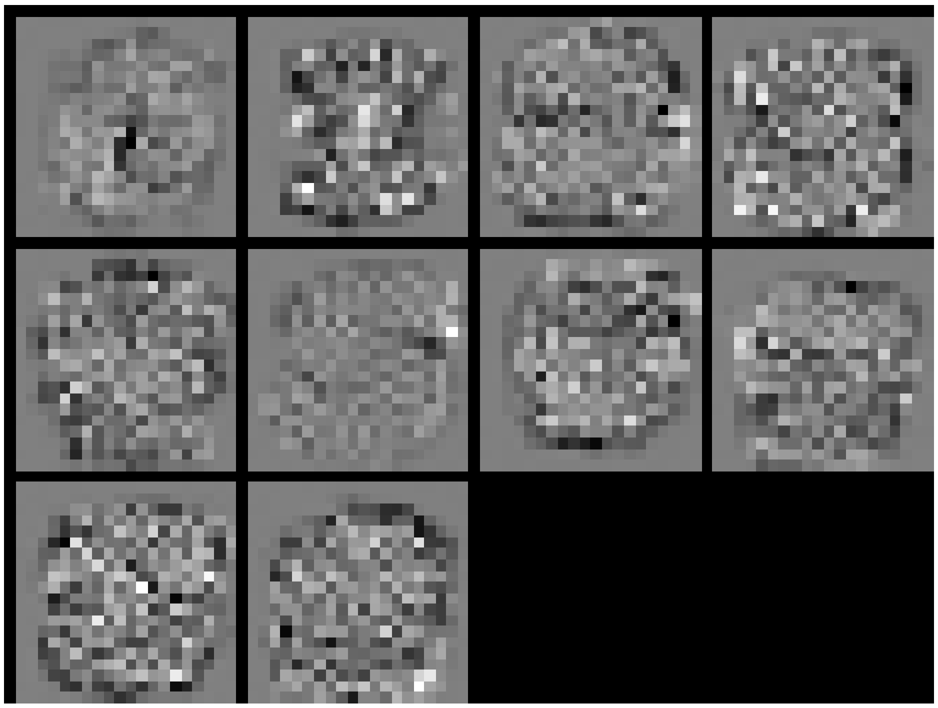
```
Lambda val: 0.001
```



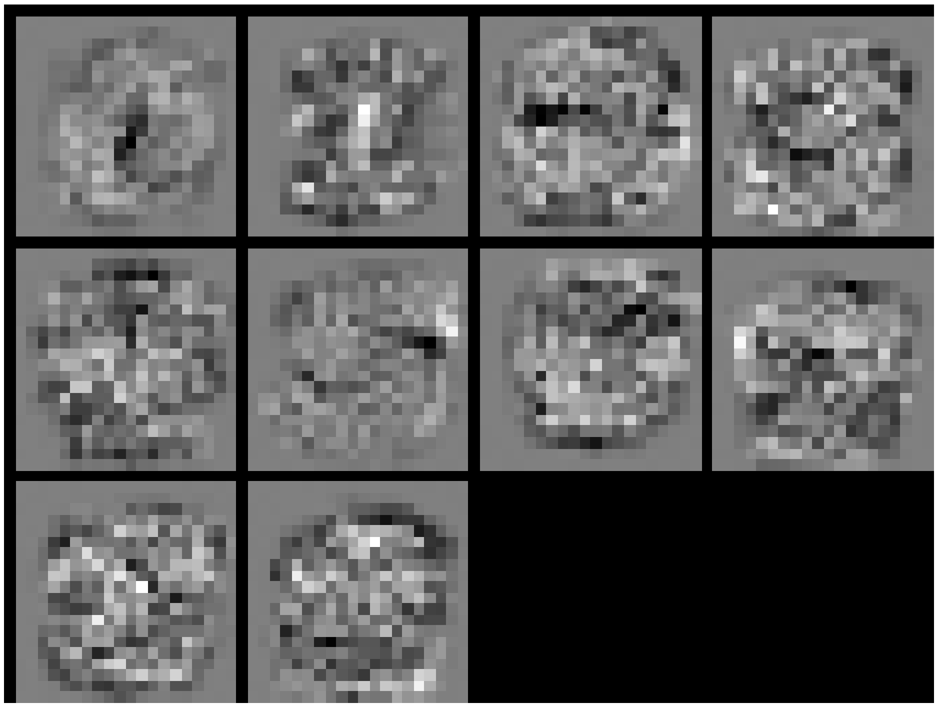
```
Lambda val: 0.01
```



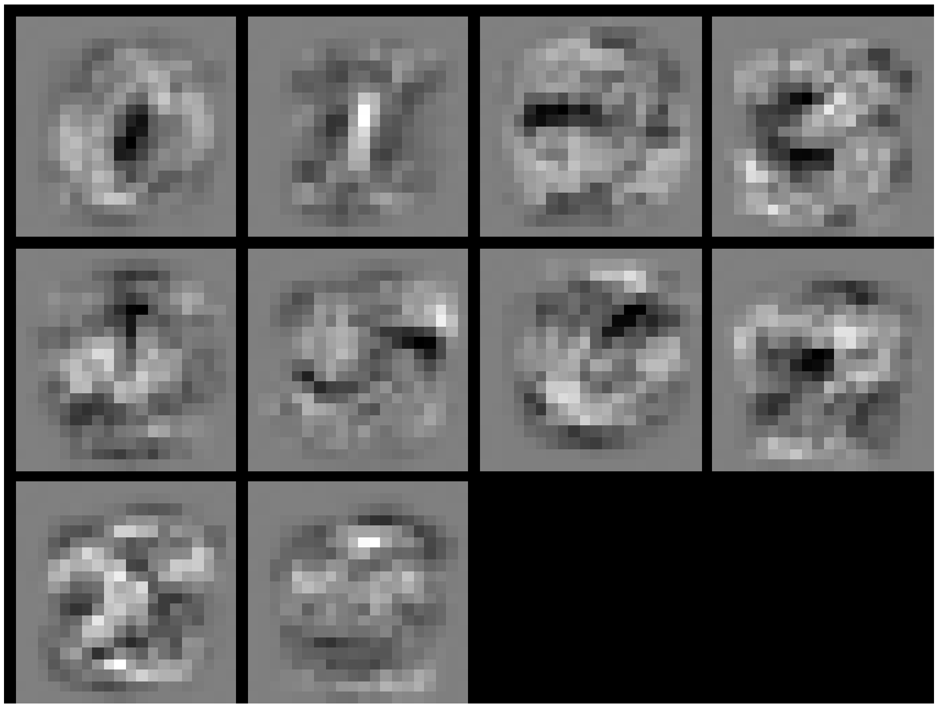
Lambda val: 0.1



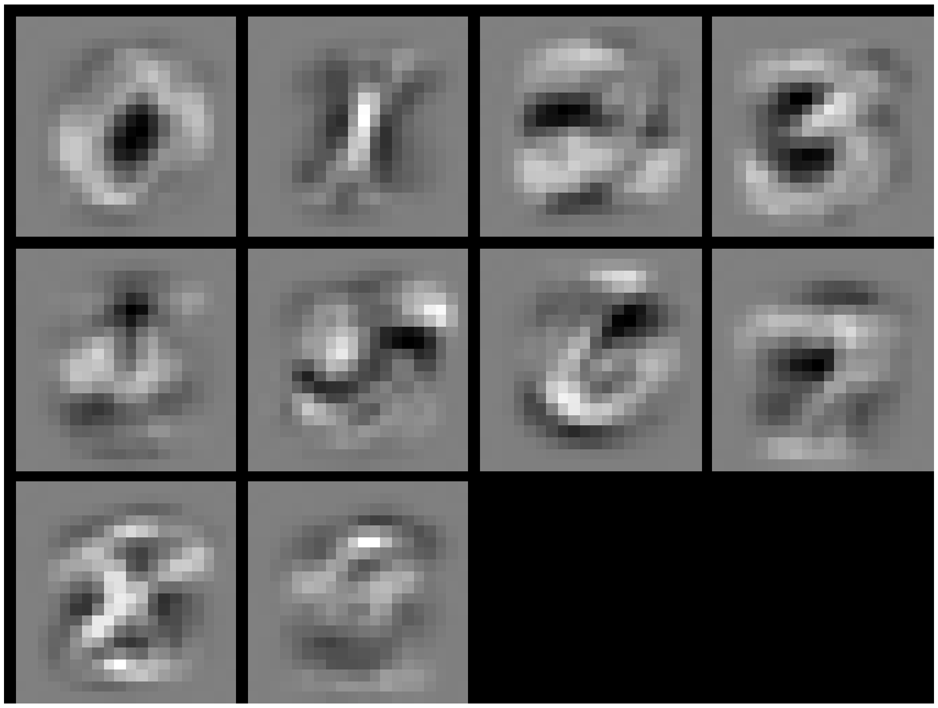
Lambda val: 1.0



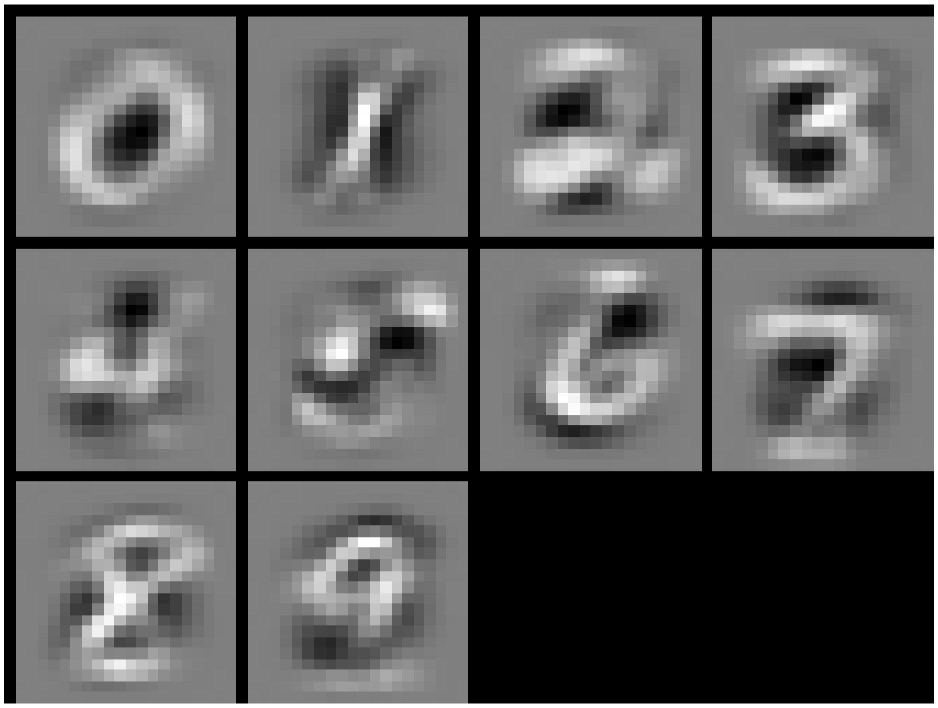
Lambda val: 10.0



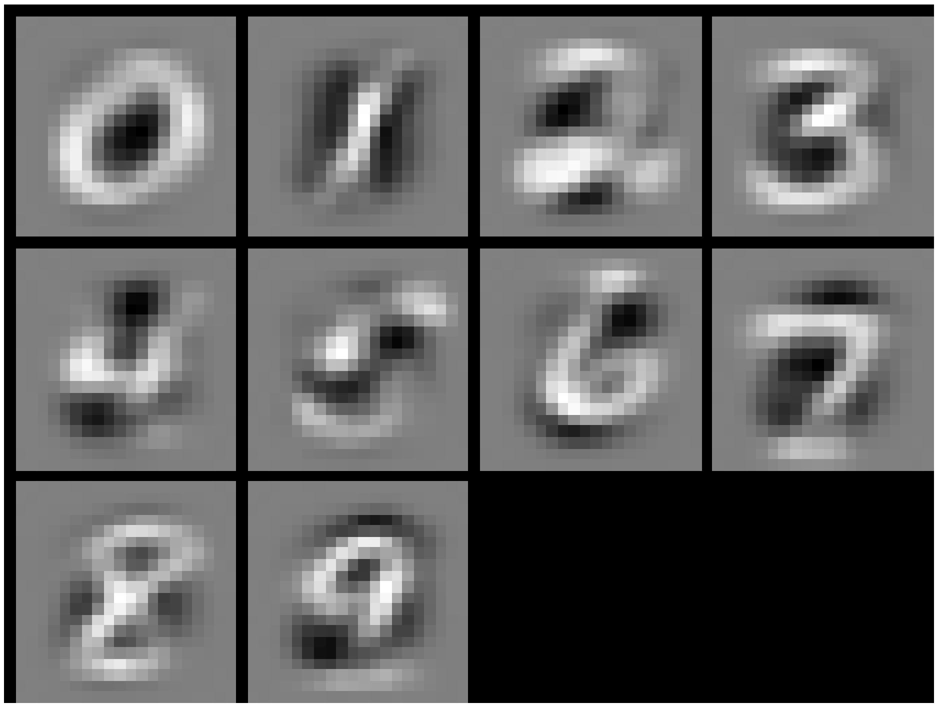
Lambda val: 100.0

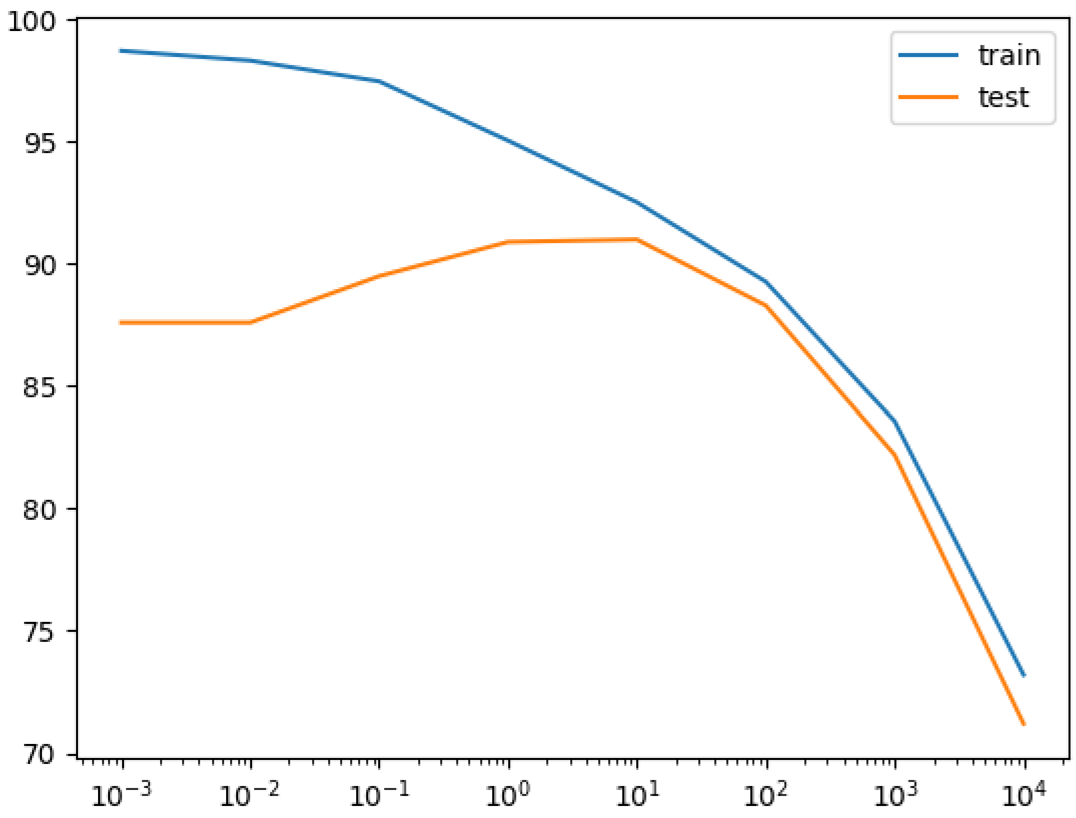


```
Lambda val: 1000.0
```



```
Lambda val: 10000.0
```





(5 points) Regularization Questions

1. Does the plot show any evidence of overfitting? If so, for what range of λ values (roughly) is the model overfit? What do the images of the weight vectors look when the model is overfit?

2. Does the plot show any evidence of underfitting? For what range of λ values (roughly) is the model underfit? What do the images of the weight vectors look like when the model is underfit?

3. If you had to choose one value of λ, what would you select?

4. Would it make sense to run any additional experiments to look for a better value of λ. If so, what values would you try?

Your answers here

1. Yes, the model is overfit for the range of 10^{-3} to 10^{0}. The images of the weight vectors are blurry and unclear.

2. Yes, the model is underfit for the range of 10^{2} to 10^{4}. The images of the weight vectors are clear and distinguisable.

3. 10^{1} because it is the middle ground between overfit and underfit.

4. Yes, I would try values around 10^{1}.

(6 points) Learning Curve

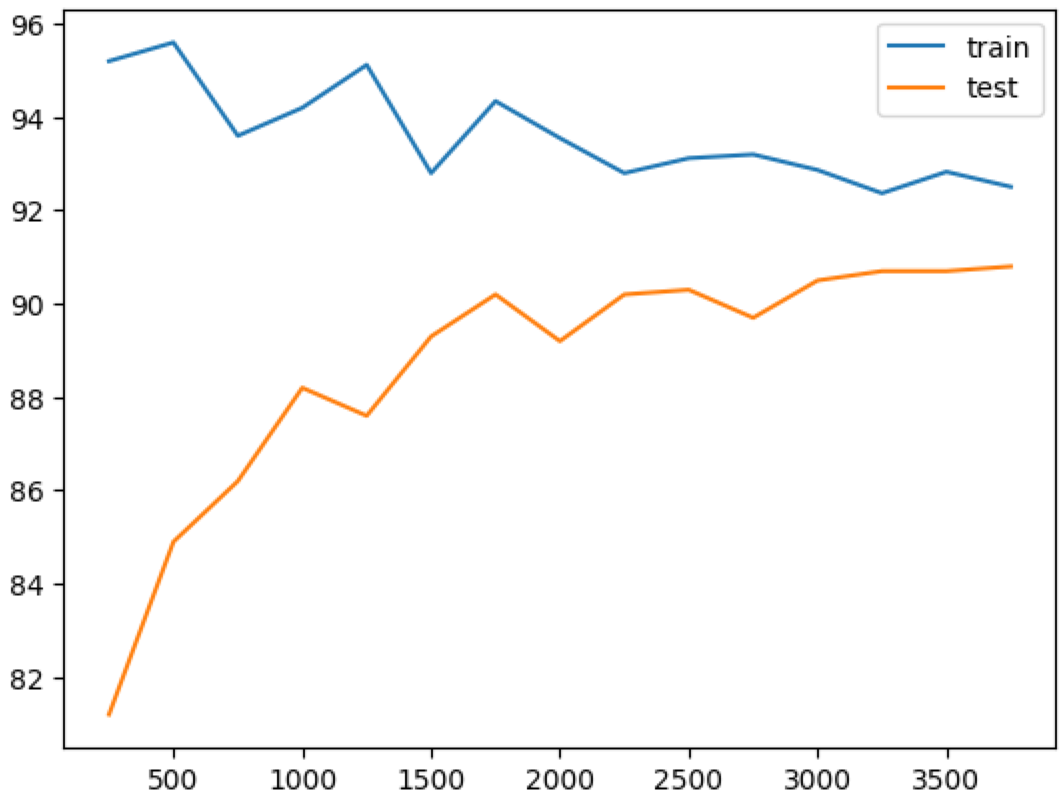
A learning curve shows accuracy on the vertical axis vs. the amount of training data used to learn the model on the horizontal axis. To produce a learning curve, train a sequence of models using subsets of the available training data, starting with only a small fraction of the data and increasing the amount until all of the training data is used.

Write code below to train models on training sets of increasing size and then plot both training and test accuracy vs. the amount of training data used. (This time, you do not need to display the weight vectors as images and you will not set the horizontal axis to have log-scale.)

In this problem, please use the best value of λ you have found.

In [435…

```
m, n = X_train.shape

train_sizes = np.arange(250, 4000, 250)
nvals = len(train_sizes)

# Example: select a subset of 100 training examples
p = np.random.permutation(m)
selected_examples = p[0:100]
X_train_small = X_train[selected_examples,:]
y_train_small = y_train[selected_examples]

# Write your code here
num_classes = 10
training_accuracy = np.zeros(nvals)
test_accuracy = np.zeros(nvals)
for i in range(0, nvals):
    p = np.random.permutation(m)
    selected_examples = p[0:train_sizes[i]]
    weight_vectors, intercepts = train_one_vs_all(X_train[selected_examples,:], y_train[
    pred_train = predict_one_vs_all(X_train[selected_examples,:], weight_vectors, interc
    pred_test  = predict_one_vs_all(X_test,  weight_vectors, intercepts)
    training_accuracy[i]=(np.mean(pred_train == y_train[selected_examples]) * 100)
    test_accuracy[i]=((np.mean( pred_test == y_test) * 100))

plt.plot(train_sizes, training_accuracy)
plt.plot(train_sizes, test_accuracy)
plt.legend(('train', 'test'))
plt.show()
```



```
In [ ]:
```

(4 points) Learning Curve Questions

1. Does the learning curve show evidence that additional training data might improve performance on the test set? Why or why not?
2. Is the any relationship between the amount of training data used and the propensity of the model to overfit? Explain what you can conclude from the plot.

Your answers here

1. Yes, because the test accuracy increases as the amount of training data used increases.
2. Yes, a lower amount of training data is more likely to result in the model overfitting. The plot shows that as more training data is used, the test accuracy and training accuracy get closer together. Increase the amount of training data used to decrease the likelihood of the model overfitting.