# PRECISE MEASUREMENT METHOD FOR IONOSPHERIC TOTAL ELECTRON CONTENT USING SIGNALS FROM GPS SATELLITES

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## ABSTRACT

Communications Research Laboratory (CRL) has developed a GPS codeless receiver called "GTR-2" for measuring total electron content (TEC) along the line of sight to the GPS satellite by using the cross correlation amplitude of the received P-code signals carried by L1(1575.42MHz) and L2 (1227.6MHz). This equipment has the performance of uncertainty in the measurement of TEC of about $2 \times 10^{16}$ electrons/$m^2$ when a 10 dBi gain antenna was used.
To increase the measurement performance, CRL is planning an upper version of GTR-2 called GTR-3 which uses the phase information of the continuous signals obtained by making a cross correlation or multiplication of the received L1 and L2 P-code signals.
By using the difference of these measured phase values, we can estimate the ionospheric delay with the ambiguities of the periods of L1+L2 and L1-L2 signals. As the periods of these signals are about 3 ns and 0.3 ns respectively, then this method has the possibility of TEC measurement with the uncertainty of 1 or 2 times of $10^{15}$ electrons/$m^2$ of TEC.
Additionally to the precise measurement of TEC, this method has the ability of the precise measurement of the pseudo-range between the GPS satellite and the receiver for the precise positioning.

## 1. INTRODUCTION

Several kinds of GPS codeless receiving systems for the precise relative positioning [1-4] are developed and they use the dual frequency signals transmitted from GPS for the calibration or correction of the ionospheric propagation delay. To make this correction of ionospheric delay, they must make the measurements of the pseudo-range for L1 and L2 signals individually.
CRL has developed a TEC measurement system using the amplitude of cross correlation of the received L1 and L2 P-code signals[5]. This method is very simple, but its precision is principally based on the duration of P-code signal, so it is difficult to obtain sub-ns level of ionospheric delay correction. To realize more precise ionospheric delay correction, CRL is planning to build a ionospheric TEC measurement equipment by using the method described in this paper.

## 2. IONOSPHERIC GROUP DELAY AND PHASE DELAY

In the meaning of first order approximation, the ionospheric group delay $\Delta t_{gion}(f_c)$ and phase delay $\Delta t_{pion}(f_c)$ caused on the electro-magnetic waves which transverse the

ionosphere are expressed by the following equations (1) and (2) respectively.

$$\Delta t_{gion}(f_c) = 134 \cdot N_e / f_c^2 \quad (ns),$$ (1)
$$\Delta t_{pion}(f_c) = -134 \cdot N_e / f_c^2 \quad (ns).$$ (2)

Where $N_e$ is ionospheric total electron content for the line of site to the satellite in electrons/$m^2$, and $f_c$ is the carrier frequency of signal in Hz. Namely the group delay and the phase delay hav same magnitude, but the group delay has a positive value while the phase delay has a negative value.

Ne for the vertical path changes from about $1 \times 10^{18}$ to $1 \times 10^{16}$ [electrons/$m^2$] at the day time of solar maximum and at the night time of solar minimum, and it changes complicatedly from time to time. Therefore we can say that the ionospheric delay is one of the largest error sources for the precise positioning and the time transfer by using space techniques.

## 3. IONOSPHERIC TOTAL ELECTRON CONTENT MEASUREMENT USING GPS SATELLITE

### 3.1 Conventinal measurement methods

As the ionospheric delay depends on the carrier frequency mentioned in the previous section, so the conventional calibration or correction methods used in the precise relative positioning by GPS satellite are using the relative delay time between the L1 and L2 signals. To get the relative delay time, especially in the case of the GPS codeless receivers, one makes the reconstruction of the clock signal of P-code and/or carrier signal of L1 and L2 signal individually. Namely one must have the pseudo-range measurement circuits for L1 and L2 signals individually. In addition, in the case of carrier reconstruction method, the measurement precision is very high but the ambiguity resolution is difficult because the wave length of the carrier signal is very short.

One of the other methods by using the GPS signals for the TEC measurement is the GTR -2 method which is developed at CRL and BIPM. It uses the cross-correlation or multiplication amplitude of L1 and L2 P-code signals. As the measurement precision of GTR-2 method is proposed to duration of the P-code clock signal and inversely proposed to the square root of signal-to-noise ratio (S/N), one must use a high gain antenna to improve the precision.

### 3.2 Principle of GTR-3

As the L1 and L2 P-code signals transmitted from the GPS satellite have a same code pattern for each GPS satellite and all of the carrier frequencies and the code clock frequencies are synthesized from same onboard frequency reference, then we can express the L1 and L2 P-code signals by the following equations when they are transmitted from the satellite;

$$x_{p1}(t) = P(t)\cos(2\pi f_{L1}t),$$ (3)
$$x_{p2}(t) = P(t)\cos(2\pi f_{L2}t),$$ (3')

where $P(t)$ is a binary pseudo-random-noise sequence which has an amplitude of $\pm\lambda$, and $f_{L1}$ and $f_{L2}$ are the carrier frequencies of L1 and L2 signals.

The received P-code signals referred to receiver clock are denoted by the following equations (4) and (4'), and figure 1 shows the schematic description of them.

$$t_1 = \phi_1/(2\pi F_1)$$
$$= \Delta t_c - 134 N_e/(f_{L1}f_{L2}) - N_1 T_0 - (1/4 + n_1)T_1, \tag{9}$$
$$t_2 = \phi_2/(2\pi F_2)$$
$$= \Delta t_c + 134 N_e/(f_{L1}f_{L2}) - N_2 T_0 - (1/4 + n_2)T_2, \tag{9'}$$

where

$T_1 = 1/F_1 \sim 0.357 \text{ns},$
$T_2 = 1/F_2 \sim 2.875 \text{ns},$
$F_1 = f_{L1} + f_{L2} = 10.23 \text{MHz} \times 274 = F_0 \times 137,$
$F_2 = f_{L1} - f_{L2} = 10.23 \text{MHz} \times 34 = F_0 \times 17,$
$F_0 = 20.46 \text{MHz}$ :maximum common frequency for $F_1$ and $F_2$,
$T_0 = 1/F_0 \sim 48.88 \text{ns},$
$N_1, N_2, n_1, n_2$ ;positive integer numbers (ambiguities),
$0 \leqq n_1 < 137,$ and $0 \leqq n_2 < 17.$

From (9),(9'), we can estimate TEC and psudo-range,

$$N_e = \{(t_2 - t_1) + (N_2 - N_1)T_0 + (1/4 + n_2)T_2 - (1/4 + n_1)T_1\}f_{L1}f_{L2}/268 \tag{10}$$
$$\Delta t_c = \{t_1 + t_2 + (N_1 + N_2)T_0 + (1/4 + n_1)T_1 + (1/4 + n_2)T_2\}/2, \tag{11}$$

with the ambiguities of n1, n2, N1, N2.

Generally, the ambiguity resolution is difficult because durations of L1+L2 signal and L1-L2 signal are very short, but in the case of GTR-3, we can use the rough estimation of TEC obtained by the GTR-2 method and also use the frequency relation between F1 and F2, the maximum common frequency of them is 20.46 MHz, so we can solve the ambiguities easily.

### 3.3 ESTIMATION OF MEASUREMENT PRECISION AND EXAMPLE OF RECEIVER DESIGN

Table 1 shows the estimated precisions of TEC and pseudo-range measurement using GTR-3 method. The received powers of the L1 and L2 P-code signals are assumed the minimum values of the GPS specifications. And an omni-directional antenna which has a gain of +3 dBi is assumed for the receiving antenna. In this case, the estimated measurement precisions are expected to be about $1.2 \times 10^{15}$ for TEC and 2.5 cm for pseudo-range measurement respectively. These values of precision are not including the stabilities of the satellite reference clock and the receiver clock.
Figure 4 shows the block diagram based of the GTR-3 method. It has a very simple construction and similar to the GTR-2, but to keep the phase information of the received signals, all of the local signals for frequency conversion in the receiver are synchronized to the receiver reference clock. The received L1 and L2 signals are frequency converted to the IF signals and made a multiplication by using an analogue multiplier. The output signal of the multiplier have the continuous signals corresponding to the L1+L2 signal and the L1-L2 signal described in previous section. As they have about ± 9kHz for the L1+L2 signal and ± 1kHz for the L1-L2 signal of doppler frequency shift due to the satellite motion, then they are finally frequency converted to the signals which have the center frequencies of 10 kHz to be read by the analogue-to-digital converters which have the clock rate of 40kHz. The obtained digital signals are signal-processed by micro-computer to calculate the phases or the zero-crossing timing.
The receiving system based on this concept is now under development at CRL, and

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$$y_{p1}(t) = P(t-\Delta t_{x1})\cos\{2\pi f_{L1}(t-\Delta t_{p1})\}, \tag{4}$$
$$y_{p2}(t) = P(t-\Delta t_{x2})\cos\{2\pi f_{L2}(t-\Delta t_{p2})\}, \tag{4'}$$

where,

$\Delta t_x(f_{Ln}) = \Delta t_s + \rho/c + \Delta t_{trop} + \Delta t_{xion}(f_{Ln})$ :pseudo-delay for modulation term,
$\Delta t_p(f_{Ln}) = \Delta t_s + \rho/c + \Delta t_{trop} + \Delta t_{pion}(f_{Ln})$ :pseudo-delay for carrier term,
$\Delta t_s$ :time diference between satellite clock and receiver reference clock,
$\Delta t_{trop}$ :tropospheric propagation delay,
for $n=1$ and 2.

By making the multiplication of $y_{P1}$ and $y_{P2}$ using a multiplier which is illustrated in figure 2, we get continuous signals or de-spread signals at the output port of the multiplier. Equation (5) denotes the formula expression of this multiplication of $y_{P1}$ and $y_{P2}$, and figure 3 shows the spectrum diagram of the received L1 and L2 P-code signals and the result of their multiplication.

$$\begin{aligned}
z(t) &= y_{p1}(t)y_{p2}(t) \\
&= P(t-\Delta t_{x1})P(t-\Delta t_{x2})\cos\{2\pi f_{L1}(t-\Delta t_{p1})\}\cos\{2\pi f_{L2}(t-\Delta t_{p2})\} \\
&= A(t)[\{\cos(2\pi F_1 t-\Phi_1)+\cos(2\pi F_2 t-\Phi_2)\}] \tag{5}
\end{aligned}$$

where

$A(t) = P(t-\Delta t_{x1})P(t-\Delta t_{x2})$,
$F_1 = f_{L1}+f_{L2}$,
$F_2 = f_{L1}-f_{L2}$,
$\Phi_1 = 2\pi f_{L1}\Delta t_{p1}+2\pi f_{L2}\Delta t_{p2}$, $\Phi_2 = 2\pi f_{L1}\Delta t_{p1}-2\pi f_{L2}\Delta t_{p2}$,
$\Delta t_c = \Delta t_s + \rho/c + \Delta t_{trop}$.

The $A(t)$ in the equation (5) is expressed by equation (6)[6].

$$A(t) = B(\Delta t_{x1}-\Delta t_{x2})+\{\text{compornents of P-code clock}\} \\
+\{\text{un de-spread part}\} \tag{6}$$

The first term represents the dc compornent which is proporsed to the relative delay between the L1 and L2 P-codes due to the ionosphere, the second term denotes the P-code clock and its higher order compornents, and the third term denotes the compornent which can not be de-spread.
From equation (5) and (6), we can obtain the continuous signals at the outputs of the band-pass filters in the figure 2 and they are expressed by

$$z_1(t) = B(\Delta t_{x1}-\Delta t_{x2})\cos(2\pi F_1 t-\Phi_1), \tag{7}$$
$$z_2(t) = B(\Delta t_{x1}-\Delta t_{x2})\cos(2\pi F_2 t-\Phi_2). \tag{7'}$$

The phase tesrms in equation (7) and (7') are discribed as the followings;

$$\Phi_1 = 2\pi F_1\Delta t_c-134 N_e 2\pi F_1/(f_{L1}f_{L2}), \tag{8}$$
$$\Phi_2 = 2\pi F_2\Delta t_c+134 N_e 2\pi F_2/(f_{L1}f_{L2}). \tag{8'}$$

Finally we get the epocks of the zero-crossing point of $z_1(t)$ and $z_2(t)$, $t_1$ and $t_2$, and they are denoted by

being made the preliminary receiving tests, and the results will be shown in the other occasion.

## 4. CONCLUTIONS

In this paper the principle of the method for precise TEC measurement by using the signal phase informations of continuous signal of L1+L2 and L1-L2 obtained by making multiplication of received L1 and L2 P-code signal from GPS satellite. As the hardwear construction is very simple and it also has the possibility of cm level of the pseudo-range measurement, then it is expected to be used not only for the precise TEC measurement but also for the precise relative positioning receiver. And the ambiguity resolutionin the measurement of TEC and pseudo-range will be made comparatively easily by using rough estimation of TEC obtained by the GTR-2 method and also the frequency relationbetween L1+L2 and L1-L2 frequencies.

## References

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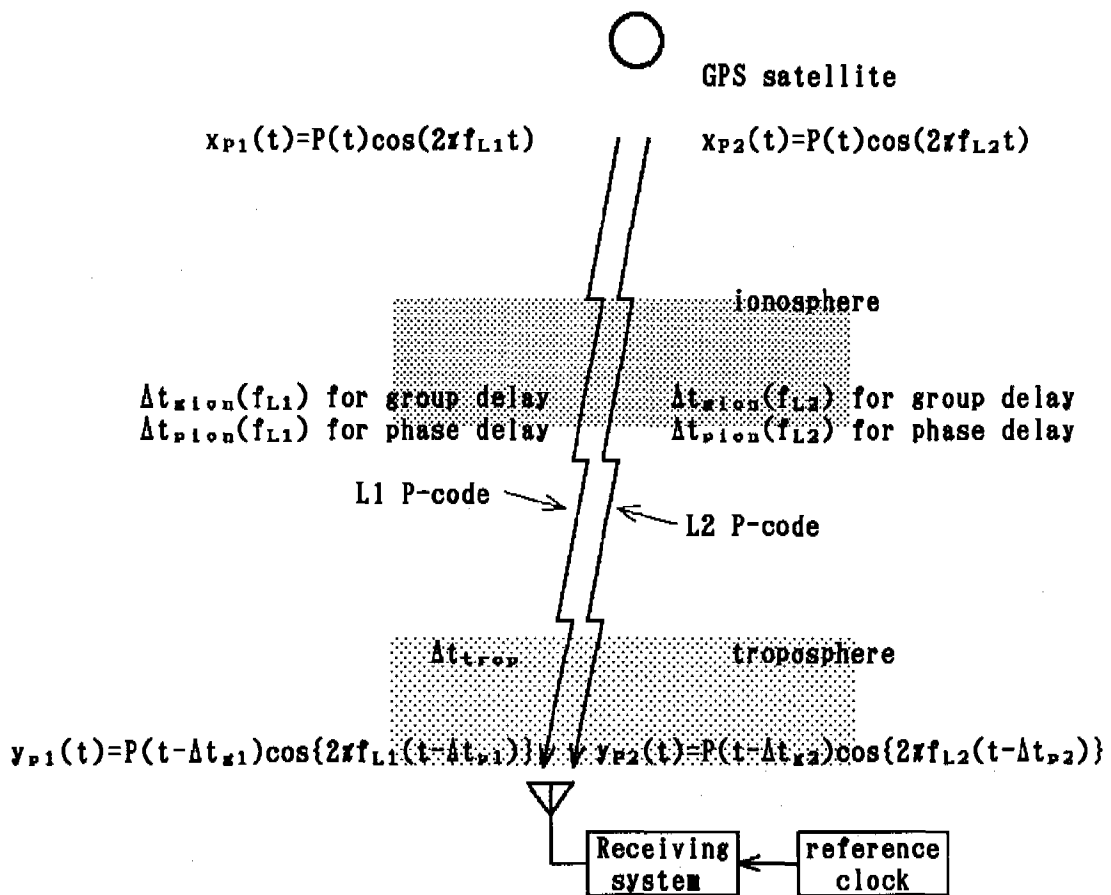[6] Spilker J,J,"Digital Communication by Satellite", Printice-hall, 1977.

GPS satellite

$x_{P1}(t)=P(t)\cos(2\pi f_{L1}t)$      $x_{P2}(t)=P(t)\cos(2\pi f_{L2}t)$

ionosphere

$\Delta t_{gion}(f_{L1})$ for group delay    $\Delta t_{gion}(f_{L2})$ for group delay
$\Delta t_{pion}(f_{L1})$ for phase delay    $\Delta t_{pion}(f_{L2})$ for phase delay

L1 P-code     L2 P-code

$\Delta t_{trop}$      troposphere

$y_{P1}(t)=P(t-\Delta t_{g1})\cos\{2\pi f_{L1}(t-\Delta t_{p1})\}$   $y_{P2}(t)=P(t-\Delta t_{g2})\cos\{2\pi f_{L2}(t-\Delta t_{p2})\}$

Receiving system    reference clock

Figure 1. Schematic expression of psudo-range.

Received Signals

$y_{P1}(t)$
(L1 P-code)

$z(t)$

BPF
$z_1(t)$ → Continuous signal
$(f_{L1}+f_{L2})$

$y_{P2}(t)$
(L2 P-code)

BPF
$z_2(t)$ → Continuous signal
$(f_{L1}-f_{L2})$
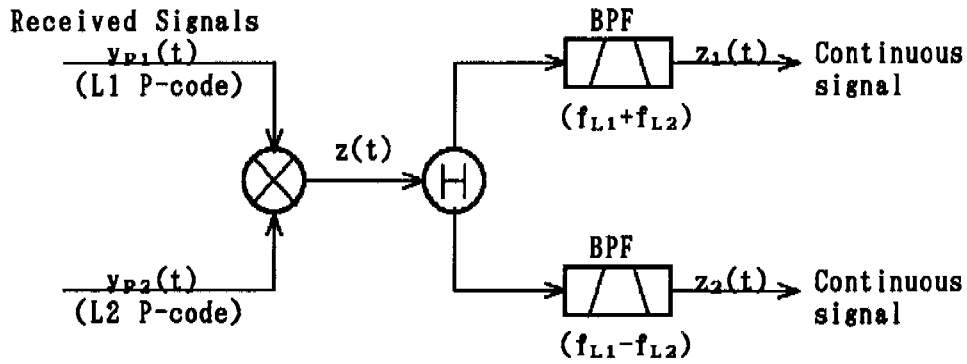
Figure 2. Principle of measurement method.

L2 P-code

C/A code

L1 P-code

$f_{L2}-10.23MHz$  $f_{L2}$  $f_{L2}+10.23MHz$    $f_{L1}-10.23MHz$  $f_{L1}$  $f_{L1}+10.23MHz$    f

L1-L2 Signal

L1+L2 signal

$f_{L1}-f_{L2}$              $f_{L1}+f_{L2}$    f

$f_{L1}-f_{L2}-10.23MHz$   $f_{L1}-f_{L2}+10.23MHz$   $f_{L1}+f_{L2}-10.23MHz$   $f_{L1}+f_{L2}+10.23MHz$
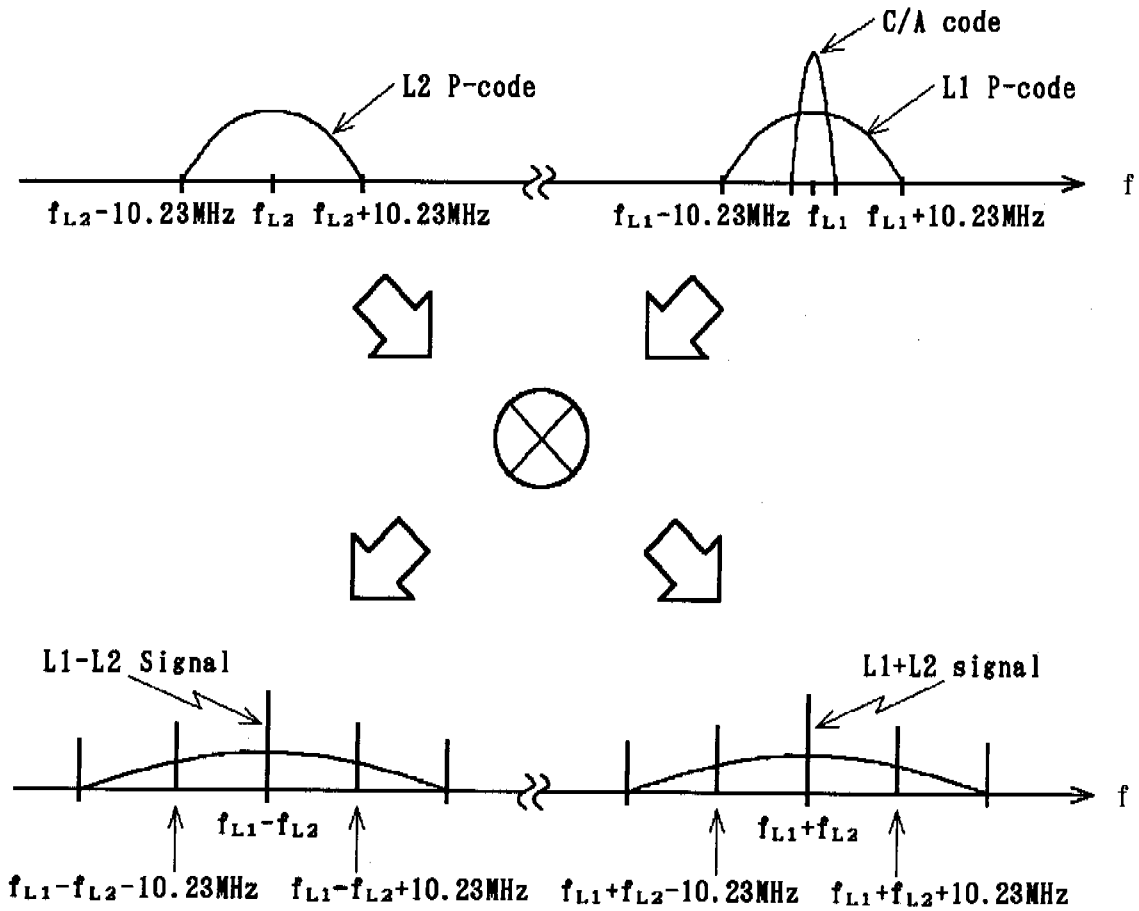
Figure 3. Schematic specrtum diagram of received signals and results
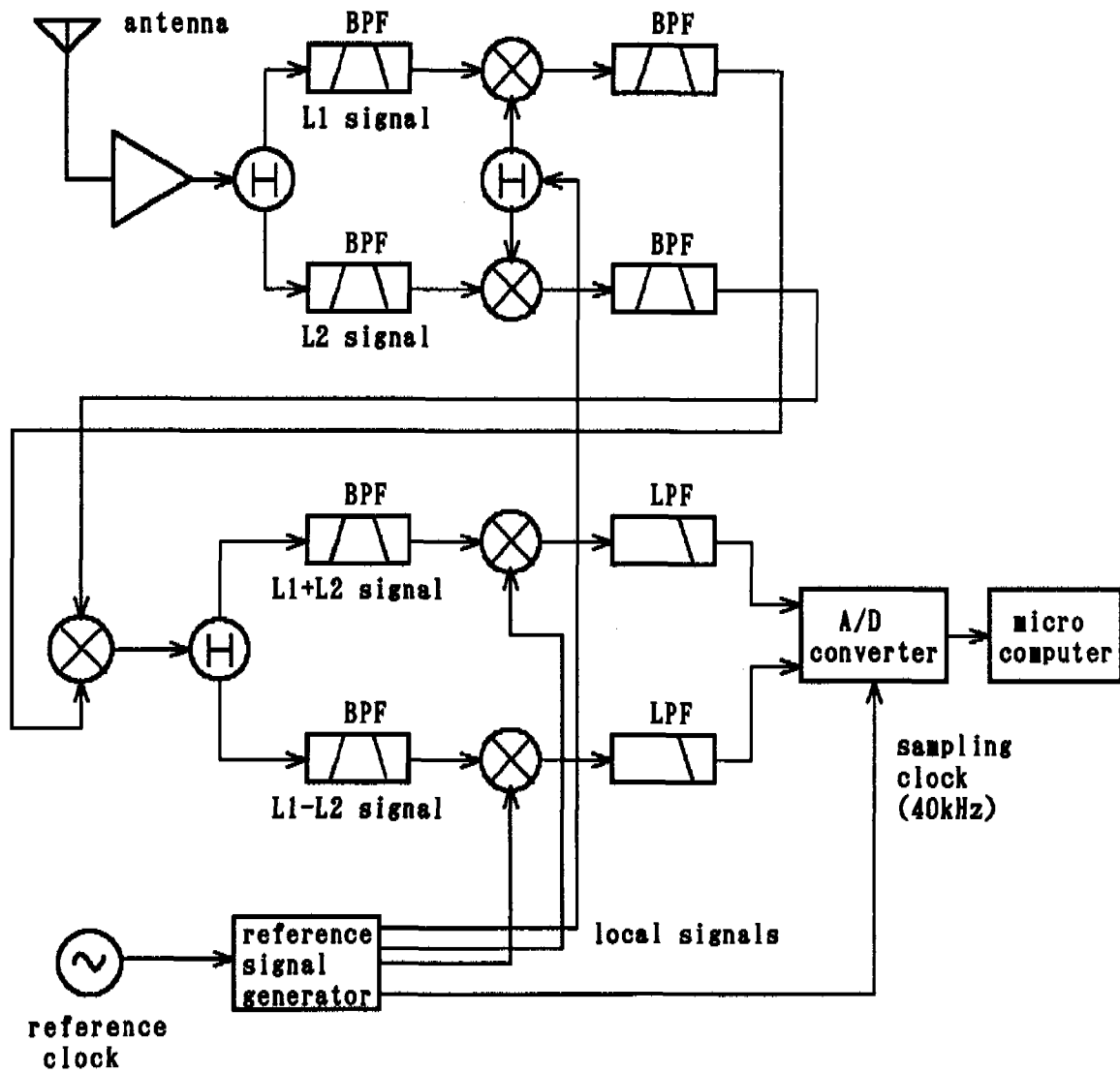of their multiplication.

Figure 4. Block diagram of receiving system which is under development at CRL.

Table 1. Estimation of performance of TEC and psudo-range measurements.

| | L1 P code | L2 P code |
|---|---|---|
| received power | >-133 dBm | >-136 dBm |
| antenna gain (omni-directional) | +3 dBi | +3 dBi |
| system noise (No) | -174 dBm/Hz | -174 dBm/Hz |
| carrier-to-noise ratio (C/No) at input of multiplier | >+44 dBHz | >+41 dBHz |
| carrier-to-noise ratio (C/No) at output of multiplier | >+12 dB·Hz | |
| bandwidth of zero-crossing detection for $t_1$ and $t_2$ | 2 Hz | |
| measurement precision of $t_1$ | ~0.02 ns | |
| measurement precision of $t_2$ | ~0.16 ns | |
| estimation precision of TEC | ~1.2 x $10^{15}$ electrons/$m^2$ | |
| estimation precision of psudo-range | ~2.5 cm | |