

STATUS OF LOCAL OSCILLATORS FOR OPERATING ULTRA-HIGH RESOLUTION FREQUENCY DISCRIMINATORS AS FREQUENCY STANDARDS

R.F.C. Vessot, E. M. Mattison, M.W. Levine and R.L. Walsworth
Smithsonian Astrophysical Observatory (SAO),
Cambridge, Massachusetts 02138

Abstract

The operation of new improved frequency standards based on new ultra-high-resolution frequency discriminators requires high stability local, or "flywheel" oscillators. We review the spectral density of phase fluctuations of existing flywheel oscillators and the related time domain frequency stability of new and proposed cryogenically cooled oscillators suitable for this application. Presently used devices include the quartz crystal oscillator, the room-temperature actively oscillating atomic hydrogen (H) maser and the superconducting maser oscillator. Future devices include the cryogenic H-maser and other cryogenic devices using resonators of superconducting metal or solid crystalline sapphire. The relation of the phase spectral density of these devices to the characteristics of present and proposed frequency discriminators based on trapped cooled ions and cold atoms is discussed in terms of their operation as frequency standards.

1. INTRODUCTION

The advent of several new concepts for frequency standards based on ultra-high resolution frequency discriminators operating at centimeter, millimeter and even optical wavelengths, has refocused attention on local or "flywheel" oscillators needed to generate the signals to be controlled by the discriminator. These signals must have sufficiently narrow frequency spectra compared to the linewidth of the discriminator and adequate frequency stability to stay within the discriminator resonance between interrogations. An important additional requirement of these oscillators is that during the time intervals for a discriminator's frequency interrogation process (including the operational dead-time for quantum state preparation), the phase of the oscillator must not be allowed to drift. Because such phase drifts are cumulative in time, they can seriously compromise the overall long term frequency stability of the frequency standard.^[1] Consideration must also be taken to assure that the interrogation process itself causes no appreciable frequency distortion in the discriminator.

New trapped ion^[2,3] and cesium fountain^[4,5] frequency discriminators enjoy a very high level of immunity from systematic frequency perturbations for very long time intervals ($t > 10^6$ s).

This immunity results from the effective isolation of the atoms or ions from each other and from external perturbations. However, one of the consequences of such isolation to minimize inter-particle collisions is that relatively low densities of particles must be used, resulting in relatively low rates of radiative quantum transitions compared to more conventional devices. This concept has even been extended to the use of a single ion as an optical frequency standard.^[2,6] The resulting low signal-to-noise ratio in the frequency discrimination process requires correspondingly longer averaging intervals and puts a premium on the long term frequency stability of the flywheel oscillator.

In general, the frequency stability of an oscillator over very short time intervals (< 1 second) is dominated by thermal noise causing white-phase noise in the output signal. In frequency discriminators the fundamental stability limitation is imposed by stochastic processes, such as shot noise in the detection of an ionized beam of atoms, and the equivalent statistical behavior of photon detectors in optical systems. These processes cause white frequency noise. As in all frequency standards, systematic frequency shifting processes invariably dominate for very long time intervals, but the expectation is that the new discriminators can lengthen the interval before such systematic effects become significant.

Combinations of a number of these systematic processes (including sporadic frequency shifts), will appear in measurements of the Allan deviation, $\sigma_y(\tau)$, and these are often represented as a flicker-noise frequency modulation (FM), $\sigma_y(\tau) = a_{-1}\tau^0$ and a random walk FM, $\sigma_y(\tau) = a_{-2}\tau^{1/2}$, where a_i is related to a component, h_i , of a power-law spectral density model^[7,8,9] of fractional frequency fluctuations, given as:

$$S_y(f) = h_{-2}f^{-2} + h_{-1}f^{-1} + h_0f^0 + h_1f + h_2f^2. \quad (1)$$

Here f is the Fourier frequency and $y = \Delta\nu/\nu_o$, where ν_o is the signal frequency. The units for $S_y(f)$ are Hz^{-1} .

The spectral density of normalized phase (time-interval) fluctuations, $S_x(f)$, is related to $S_y(f)$ by

$$S_x(f) = \frac{1}{(2\pi f)^2} S_y(f), \quad (2)$$

where x represents $\Delta f/2\pi\nu_o$. $S_x(f)$ is in units of $\text{s}^2 \cdot \text{Hz}^{-1}$ and is related to S_ϕ , the spectral density of phase fluctuations of a signal at frequency ν_o , by $S_x(f) = S_\phi(f)/(2\pi\nu_o)^2$. ($S_\phi(f)$ has units of $(\text{radians}/\text{s})^2 \text{ Hz}^{-1}$.)

In this spectral representation, h_0 , h_1 , and h_2 characterize white frequency noise, flicker of phase noise, and white phase noise, respectively. Of these, h_0 and h_2 are easily related to stochastic processes and thermal noise. While there is scant rationalization for the physical origins of the h_{-2} , h_{-1} , and h_1 terms, they are nonetheless useful to represent the Fourier components of observed frequency or phase fluctuations.

The frequency controlling characteristics of frequency discriminators can be described in terms of a predicted Allan deviation or in terms of the equivalent spectra $S_y(f)$ or $S_x(f)$, calculated from the signal-to-noise ratio and linewidth of the frequency discriminator. It is important to understand

that the realization of these spectra in an operating piece of hardware, as manifested in an output signal, will depend both on the interrogation technique and on the limitations imposed by the flywheel oscillator that is controlled by the discriminator.

While it is conventional to describe the frequency stability of frequency locked systems by the Allan deviation in the time domain, $\sigma_y(\tau)$, a better insight into the interactions between flywheel oscillators and frequency discriminators is given by an analysis of the spectral domain of phase (or frequency) fluctuations. In this paper we adopt $S_x(f)$, the spectral density of normalized phase fluctuations, to describe the relationships between frequency discriminators and flywheel oscillators.

2. PRESENT STATUS OF OSCILLATORS USED TO OPERATE FREQUENCY DISCRIMINATORS

Figure 1 shows $S_x(f)$ for commercially available cesium standards, the experimental mercury ion standards at the United States Naval Observatory (USNO)^[10], a linear ion trap operated from a superconducting maser oscillator^[3,6], a room temperature active H-maser^[11], and typical 100 MHz commercial crystal oscillators represented by a band of data. Figure 2 shows the corresponding $\sigma_y(t)$ for these devices.

The quartz crystal-controlled oscillator continues to be the workhorse flywheel oscillator for nearly all frequency standards and clocks in use to date. $S_x(f)$ for 100 MHz voltage-controlled-crystal-oscillators (VCXOs) is shown in Figure 1 over the range of 10^{-1} to 100 Hz. The f^{-3} slope of $S_x(f)$ over these Fourier frequencies indicates a flicker-of-frequency behavior characterized by h_{-1} in Equation 1. At about 10 Hz the VCXO spectra are intersected by the white-phase-noise part of the H-maser spectrum, which steepens to an f^{-2} behavior at lower Fourier frequencies down to about 10^{-3} Hz. This corresponds to white-frequency noise (h_0) in $S_y(f)$, the spectral density of H-maser fractional frequency fluctuations. At still lower frequencies f^{-3} and f^{-4} behaviors are seen in the H-maser $S_x(f)$ spectrum shown in Figure 1. The corresponding behaviors in $S_y(f)$ are flicker-of-frequency noise (h_{-1}) and random-walk-of-frequency (h_{-2}), respectively.

Figure 3 shows a typical phase-lock loop for controlling a VCXO by a low power oscillator (e.g. an H-maser). Such a set-up has good phase stability both at high Fourier frequencies, due to the VCXO, and at low Fourier frequencies, due to the H-maser. In this figure the filter function $g(f)$ represents the loop filter characteristics. Here we see how a 100 MHz crystal oscillator, having a flicker of frequency noise spectrum $S_y(f) = h_1 f^{-1}$, corresponding to a normalized flicker-of-phase spectral-density $S_x(f) = h_{-1} f^3 / (2\pi)^2$, can be phase-locked to a room temperature H-maser. To obtain an optimum spectrum, the phase-lock band width should be about 10 Hz, where the white phase noise of the H-maser, at a level $10 \log S_x(f) = -268$, intercepts the flicker-of-phase noise of the VCXO. The design of such phase-lock servo systems is discussed in detail by Vanier and Audoin^[12].

3. FREQUENCY LOCK SERVO SYSTEMS

Figure 4 shows a system for frequency-locking an oscillator to a frequency discriminator. Implicit in this process are the requirements that the spectral linewidth of the oscillator's frequency should be narrower than the linewidth of the discriminator, and that the signal from the flywheel oscillator signal should be frequency-modulated to develop an output signal related to the line profile of the discriminator. This signal is then sent to a phase-sensitive detector to control the frequency of the flywheel oscillator, or that of a signal frequency synthesized from that oscillator. The connection between the frequency discriminator and the flywheel is usually made by a second-order servo loop and is depicted by a line with f^{-4} slope in Figure 1 for the H-maser operating the mercury (Hg) ion device. Both the Hg ion trap^[10] and the linear ion trap^[3] operate their frequency discriminators with a microprocessor. This technique is far more flexible than analog methods for performing complicated routines and permits operation with very long integration times. The goal of such frequency lock servos is to provide an optimum overall connection between the spectrum of the flywheel and the frequency discriminator. Reference 12 discusses the properties of second-order frequency lock servos in detail.

4. APPLICATION OF PRESENT AND FUTURE CRYOGENIC OSCILLATORS AS FLYWHEELS

Operation at low temperatures appears to offer the best prospect to provide flywheel oscillators of sufficient spectral purity and long term stability for use with ultra-high resolution frequency discriminators^[2-6] as improved frequency standards.

Low temperature provides improved dimensional stability to materials and tends to freeze out dissipation mechanisms that diminish Q. To obtain levels of frequency stability at the 10^{-15} level from oscillators controlled by microwave resonators, their physical dimensions must be maintained to within fractional dimensions of the same order, smaller than those of atomic nuclei! Under these conditions, variations in power stored in high-Q hollow superconducting metal resonators can produce stress changes and surface heating that affect the dimensions of such resonators. Hollow resonators with Qs of 10^{11} have been developed and operated in oscillators with frequency stability in the low 10^{-16} levels^[13,14] for $\tau \approx 10$ s. To reduce the effects of mechanical strain from variations in the radiation pressure caused by the energy confined in hollow resonators, scientists in the former Soviet Union have tried sapphire crystal resonators with superconducting coatings^[15,16]. To avoid the need for conductive coatings, ring-shaped sapphire resonators have been developed where the microwave energy is confined by internal reflection from the dielectric boundary in a so-called "whispering gallery" mode. Recent results using this technique show considerable promise^[17]. Figure 5 shows the calculated $S_x(f)$ spectrum based on passive Q measurement of a whispering gallery sapphire resonator operated as an oscillator.

The fundamental thermal limit to the frequency stability of a classical, self excited oscillator is

$$\sigma_y(t) = \frac{\Delta\nu}{\nu_o} = \frac{1}{Q} \sqrt{\frac{kT}{2P\tau}} \quad (3)$$

where k is Boltzmann's constant, T is temperature in Kelvins, P is the power driving the oscillation,

and Q represents the inverse of all dissipative processes^[18]. The fundamental quantum limit (ql) to frequency fluctuations under conditions where thermal energy, kT , is smaller than $h\nu_o/2$ is^[19]

$$\sigma_y^{ql}(t) = \frac{\Delta\nu}{\nu_o} = \frac{1}{Q} \sqrt{\frac{h\nu}{4P\tau}} \quad (4)$$

where h is Planck's constant.

At first sight, it would appear that, by raising the power, the limits on frequency stability given by Eqns. (3) and (4) can be arbitrarily improved. However, there are quantum fluctuations in the radiation pressure in the cavity resonator that increase as $P^{1/2}$. These fluctuations cause dimensional changes in the resonator, and hence frequency fluctuations. For the best cryogenic oscillators, with $Q \sim 10^{11}$ at 10 GHz and operating at an optimum power, and taking into account the mechanical properties of available materials to cope with the quantum fluctuations in the radiation pressure in the cavity, the quantum limit can be as low as^[16]

$$\sigma_y^{ql}(\tau) = 2 \times 10^{-20} \tau^{-1/2}. \quad (5)$$

This may not be the final limit on frequency stability, as there are "squeezed state" stroboscopic and quadrature-amplitude measurement techniques that may allow traditional quantum limits to be surpassed; these techniques have been extensively studied by theorists in the former Soviet Union^[20].

The combination of a superconducting cavity stabilized maser oscillator (SCMO) with a linear ion trap has been successfully demonstrated at JPL^[21,22]. The measured performance of the SCMO is displayed in Figure (1), showing its white phase noise with a signal level given by $10 \log S_x(f) = -276$, and its flicker of frequency noise characterized by f^{-3} behavior between 0.3 Hz and 0.003 Hz. The connection of the SCMO to the linear ion trap at about 0.03 Hz was closed by a "loop in software" using a computer-operated frequency lock servo.

In contrast to oscillators that depend on physical dimensions for determining their output frequency, the cryogenic H-maser^[23,24,25] offers considerable immunity from effects related to physical dimensions. Figure 5 shows $S_x(f)$ for estimates of performance based on the present design of the SAO cryogenic H-maser^[26]. Here the limiting line Q is taken as 4.5×10^{10} , the storage volume, V_b , is 213 cm^3 , and the operating temperature $T = 0.52 \text{ K}$. The resulting fundamental limit on stability is given by^[27]

$$\sigma_y(\tau) = \sqrt{\frac{32\pi kT\sigma_{se}v}{h(2\pi\nu_o)^3 V_b}} \tau^{-1/2} = 8.31 \times 10^{-17} \tau^{-1/2} \quad (6)$$

where σ_{se} is the spin-exchange cross section and v is the relative thermal velocity for H-H atomic collisions at 0.52 K. It is interesting to compare this limit with the quantum limit for this oscillator, as defined in equation (4) with the replacement of kT by $h\nu_o/2$:

$$\sigma_y^{ql}(\tau) = 3.03 \times 10^{-17} \tau^{-1/2}. \quad (7)$$

This is well below the limit given in Equation (6).

The output frequency of the cryogenic H-maser is subject to temperature-variable frequency shifts due to H-atom collisions with the superfluid liquid helium wall surface coating and with the helium vapor within the confining maser vessel. The frequency sensitivity to temperature can be minimized by operating at a temperature near 0.52 K, where raising the temperature increases the vapor collision shift and lowering the temperature increases the wall surface effect. Operating at this minimum point, the maser's output frequency is a quadratic function of temperature. To achieve frequency stability at levels about 10^{-18} , temperature stability of a few micro-Kelvins is required; such temperature control is feasible at these low temperatures. There also exists a possible limitation to the stability of the maser that depends on the time of storage of the oscillating atoms^[28] and is therefore related to the dimensions of the confining vessel and its collimator. These effects are an important aspect of our present SAO research on the cryogenic H-maser.

Figure 6 shows the projected Allan deviation of new frequency standards based on some of the new ultra-high resolution frequency discriminators discussed in the present paper. It is clear that many precautions must be observed in order to make use of the capability of these new discriminators and to realize standards in the 10^{-18} domain of frequency stability^[29]. In addition to the use of appropriate flywheel oscillators, these precautions will likely require new types of technology related to signal transmission and signal processing. Assuming that the projected performance is realized, the new frequency standards will have substantially improved accuracy and stability relative to existing devices.

5. CONCLUSION

Progress continues in the development of flywheel oscillators for use with new ultra-high resolution frequency discriminators. Use of cryogenic techniques is becoming commonplace and should not be considered as a serious roadblock in the operation of such oscillators, even in future spaceborne applications.

ACKNOWLEDGEMENTS

The preparation of this paper was supported by the Smithsonian Institution's Scholarly Studies Program and Research Opportunities Fund. We gratefully acknowledge support from the US Air Force Office of Scientific Research for work on the cryogenic H-maser.

REFERENCES

- [1] G.J. Dick, J.D. Prestage, C.A. Greenhall, and L. Maleki, "*Local oscillator induced degradation of medium-term stability in passive frequency standards*", Proceedings of the 22nd Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, NASA Conf. Prec. 3116 (U.S. Naval Observatory, Washington, DC, 1990), p. 487.
- [2] D.J. Wineland, W.M. Itano, J.C. Berquist, J.J. Bollinger, F. Diedrich, and S.L. Gilbert, "*High accuracy spectroscopy of stored ions*", in Frequency Standards and Metrology, A DeMarchi, ed. (Springer-Verlag, Berlin, 1989), p. 71.

- [3] J.D. Prestage, G.J. Dick and L. Maleki, "Ultra-stable trapped ion frequency standard", Proceedings of the 22nd Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, NASA Conf. Prec. 3116 (U.S. Naval Observatory, Washington, DC, 1990), p. 171.
- [4] S Chu, "Laser manipulation of atoms and particles" Science 253, 861 (1991).
- [5] S.L. Rolston and W D. Phillips, "Laser-cooled neutral atom frequency standards", Proc. IEEE 79, 943 (1991).
- [6] D.J. Wineland, J.C. Bergquist, J.J. Bollinger, W.M. Itano, D.J. Heinzen, S.L. Gilbert, C.H. Manney, and M.G. Raizen, "Progress at NIST toward absolute frequency standards using stored ions", IEEE Transactions on Ultrasonics, Ferroelectronics, and Frequency Control 37, 515 (1990).
- [7] J.A. Barnes, et al., "Characterization of frequency stability", IEEE Transactions on Instrumentation and Measurement IM-20, 105 (1971).
- [8] D.W. Allan, "Time and frequency (time-domain) characterization, estimation, and prediction of precision clocks and oscillators", IEEE Transactions on Ultrasonics, Ferroelectronics, and Frequency Control UFFC-34, 647 (1987).
- [9] Characterization of Clocks and Oscillators, D.B. Sullivan, D.W. Allan, D.A. Howe, and F.L. Walls, eds., NIST Technical Note 1337 (NIST, Washington, DC, 1990).
- [10] L.S. Cutler and R.P. Giffard, "Initial operational experience with a mercury ion storage frequency standard", Proceedings of the 41st Annual Symposium on Frequency Control (IEEE, New York, 1987), p. 12.
- [11] A.A. Uljanov, N.A. Demidov, E.M. Mattison, R.F.C. Vessot, D.W. Allan, and G.M.R. Winkler, "Performance of Soviet and U.S. hydrogen masers", Proceedings of the 22nd Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, NASA Conf. Prec. 3116 (U.S. Naval Observatory, Washington, DC, 1990), p. 509.
- [12] J. Vanier and C. Audoin, The Quantum Physics of Atomic Frequency Standards (Hilger, Bristol, 1989) pp. 1115–1125 (phase lock) and pp. 752–791 (frequency lock).
- [13] J.P. Turneaure, Proceedings of IEEE Conference on Applied Superconductivity (IEEE, New York, 1972), 154.
- [14] S.R. Stein, "Application of superconductivity to precision oscillators", Proceedings of the 29th Annual Symposium on Frequency Control (Electronics Industries Association, Washington, DC, 1975), p. 321.
- [15] V.M. Pudalov, "Superconducting SHF resonators and their use in metrology", Meas. Tech. 23, 600 (1980).
- [16] V.B. Braginsky, V.P. Mitrofanov, and V.I. Panov, Systems With Small Dissipation (University of Chicago, Chicago, 1985).

- [17] G.J. Dick and D.G. Santiago, "Microwave frequency discriminator with cryogenic sapphire resonator for ultra-low phase noise", Proceedings of the 6th European Frequency and Time Forum, ESA Document SP-340 (ESA, Paris, 1992), p. 35.
- [18] W.A. Edson, "Noise in oscillators", Proc. IRE 48, 1454 (1960).
- [19] W.E. Lamb, "Theory of optical masers", in Quantum Optics and Electronics, C. DeWitt, A. Blandin, and C. Cohen-Tannoudji, eds. (Gordon and Breach, New York, 1965), p. 331.
- [20] V.B. Braginsky, "Resolution in macroscopic measurements: progress and prospects", Sov. Phys.-Usp. 9, 836 (1988).
- [21] G.J. Dick, "Calculation of trapped ion local oscillator requirements" Proceedings of the 19th Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting (U.S. Naval Observatory, Washington, DC, 1987), p. 133.
- [22] G.J. Dick and R.T. Wang, "Ultra-stable performance of the superconducting cavity maser", IEEE Transactions on Instrumentation and Measurement 40, 174 (1991).
- [23] R.L. Walsworth, I.F. Silvera, H.P. Godfried, C.C. Agosta, R.F.C. Vessot, and E.M. Mattison, "Hydrogen maser at temperatures below 1 K", Phys. Rev. A 34, 2550 (1986).
- [24] H.F. Hess, G.P. Kochanski, J.M. Doyle, T.J. Greytak, and D. Kleppner, "Spin-polarized hydrogen maser", Phys. Rev. A 34, 1602 (1986).
- [25] M.D. Hürlimann, W.N. Hardy, A.J. Berlinsky, and R.W. Cline, "Recirculating cryogenic hydrogen maser", Phys. Rev. A 34, 1605 (1986).
- [26] R.F.C. Vessot, E.M. Mattison, R.L. Walsworth, I.F. Silvera, H.P. Godfried, and C.C. Agosta, "A hydrogen maser at temperatures below 1 K", IEEE Transactions on Instrumentation and Measurement IM-36, 588 (1987).
- [27] A.J. Berlinsky and W.N. Hardy, "Cryogenic masers", Proceedings of the 13th Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, NASA Conference Publication 2220 (NASA, Washington, DC, 1981), p. 547.
- [28] B.J. Verhaar, J.M.V.A. Koelman, H.T.C. Stoof, O.J.T. Luiten, and S.B. Crampton, "Hypfine contribution to spin-exchange frequency shifts in the hydrogen maser", Phys. Rev. A 35, 3825 (1987); J.M.V.A. Koelman, S.B. Crampton, H.T.C. Stoof, O.J.T. Luiten, and B.J. Verhaar, "Spin-exchange frequency shifts in cryogenic and room temperature hydrogen masers", Phys. Rev. A 38, 3535 (1988).
- [29] F.L. Walls, L.M. Nelson, and G.R. Valdez, "Designing for frequency and time metrology at the 10–18 level", Proceedings of the 6th European Frequency and Time Forum, ESA Document SP-340 (ESA, Paris, 1992), p. 477.

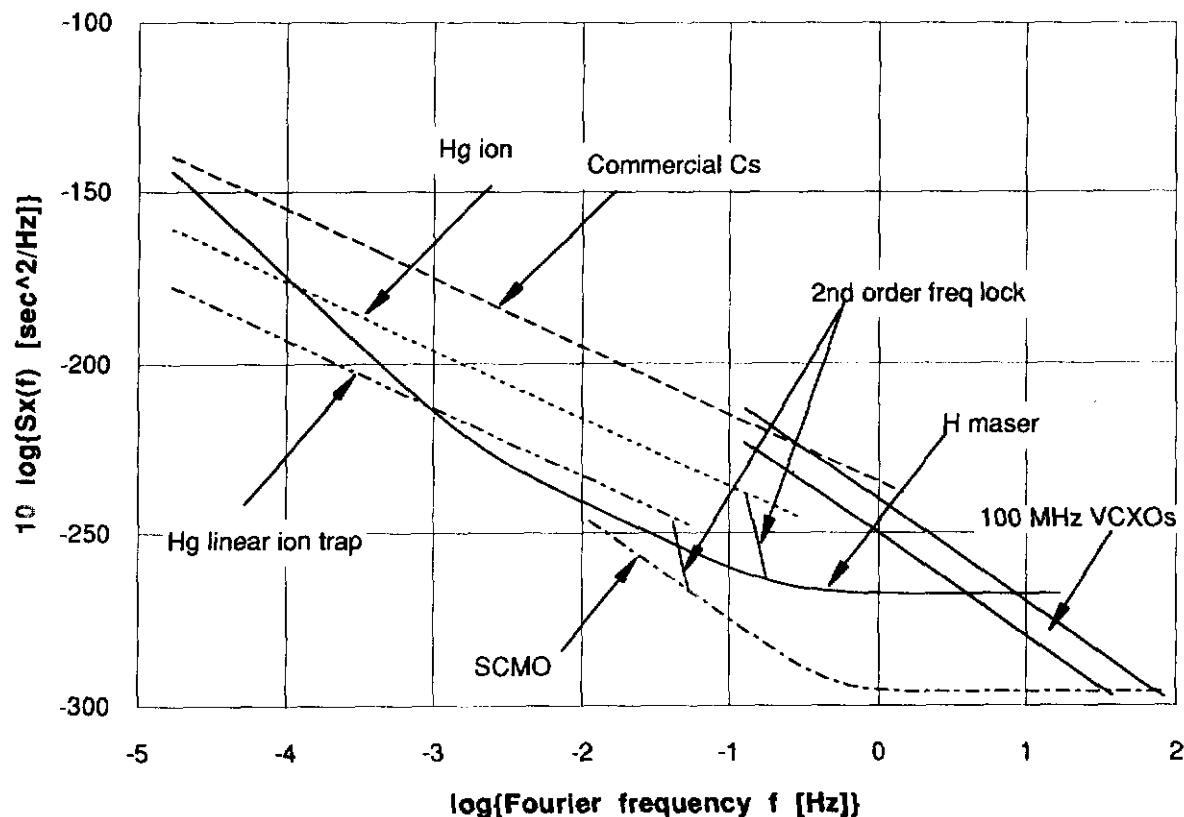


Fig. 1. Phase spectral densities of existing oscillators and frequency standards.

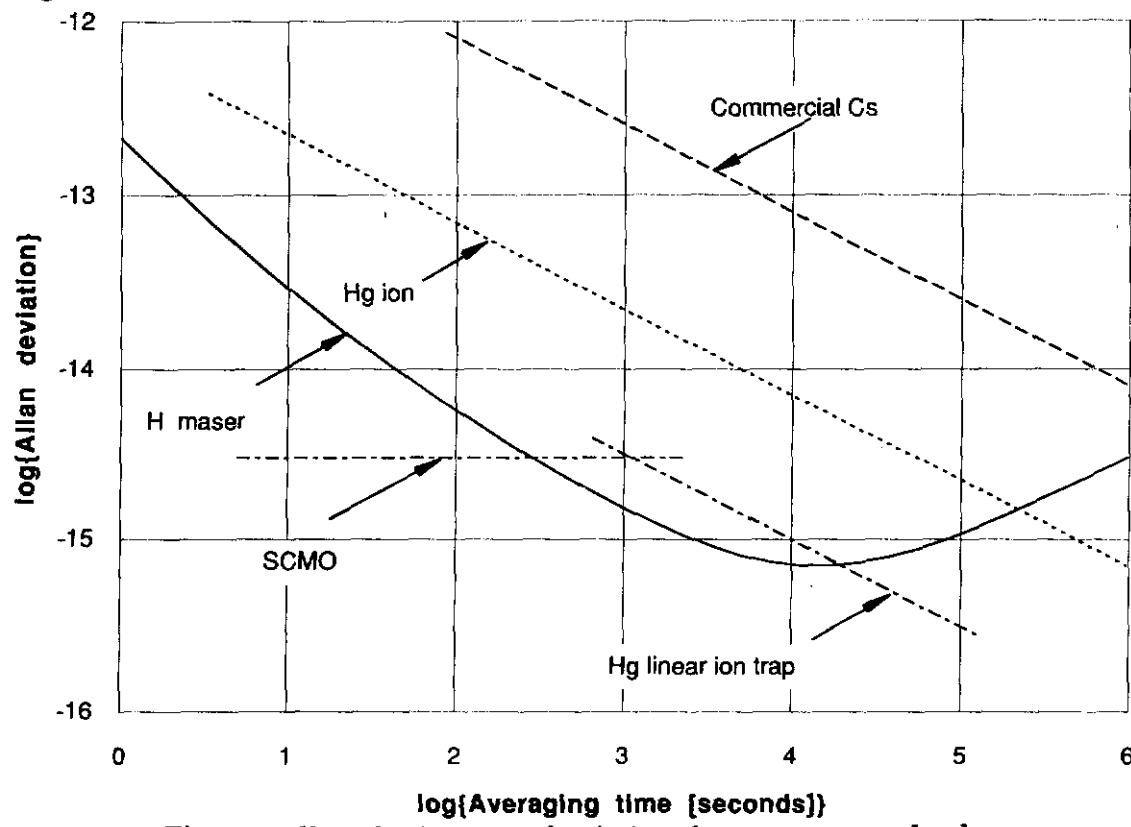
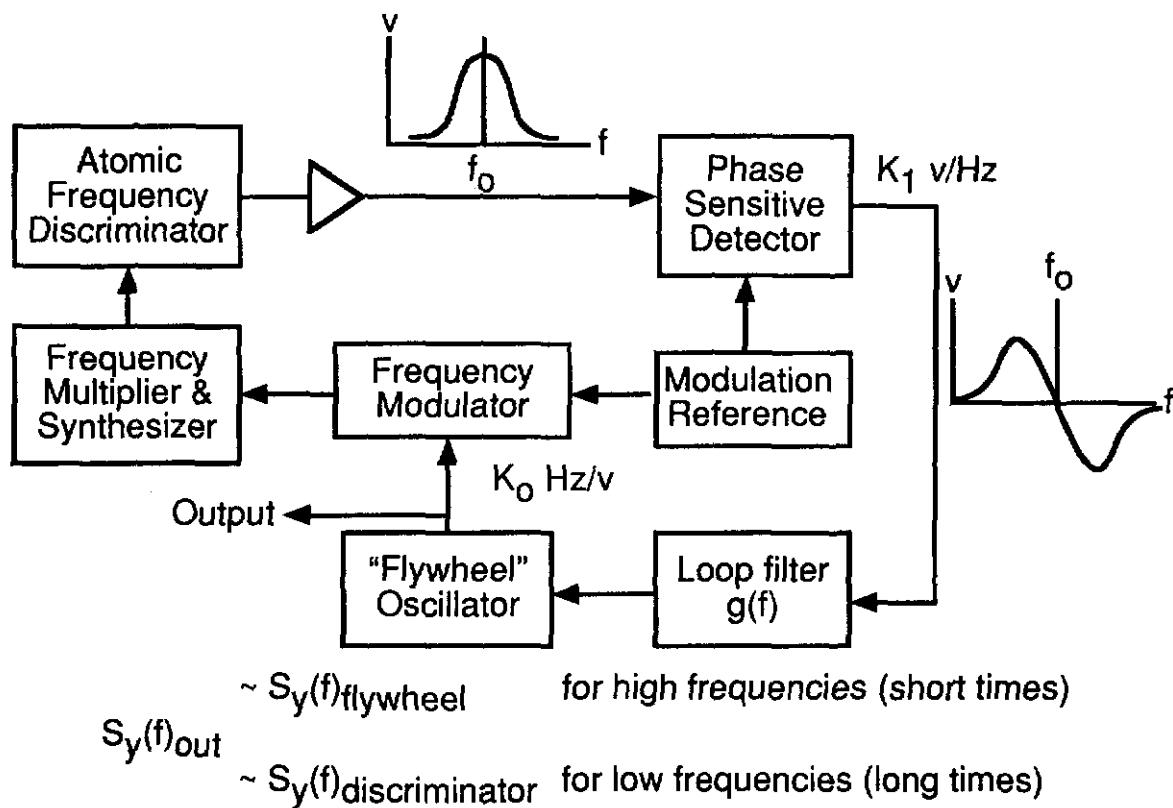
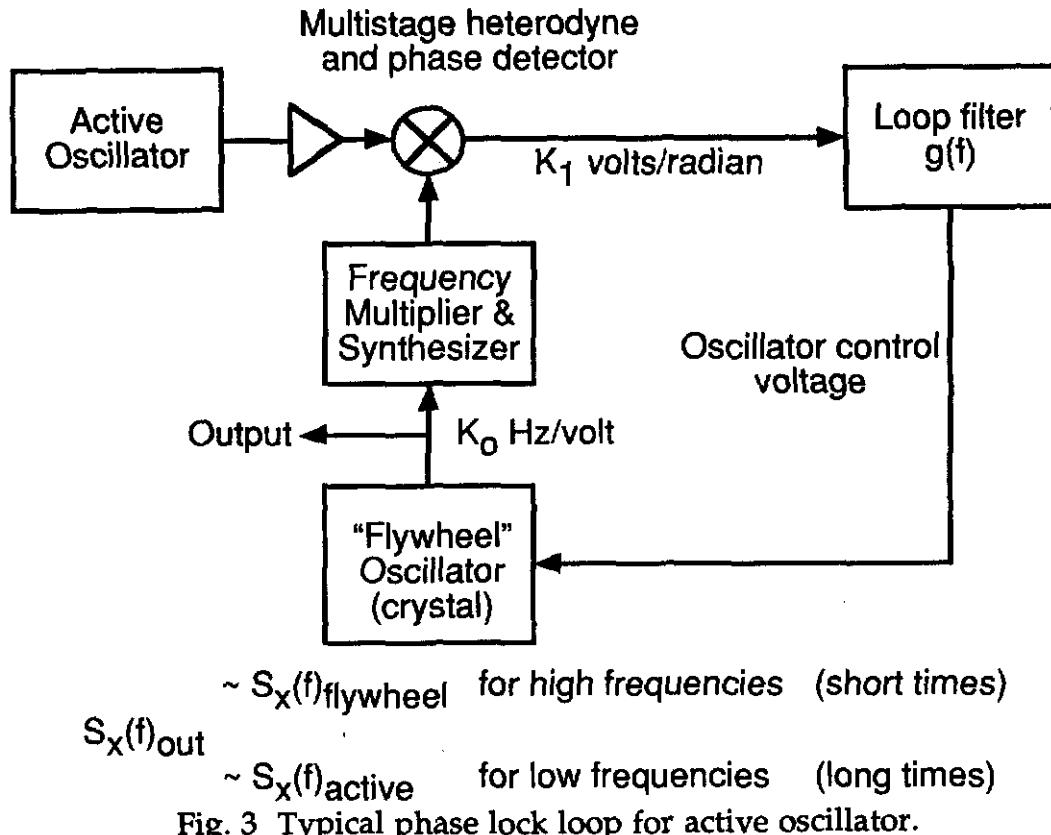


Fig. 2. Allan deviations of existing frequency standards.



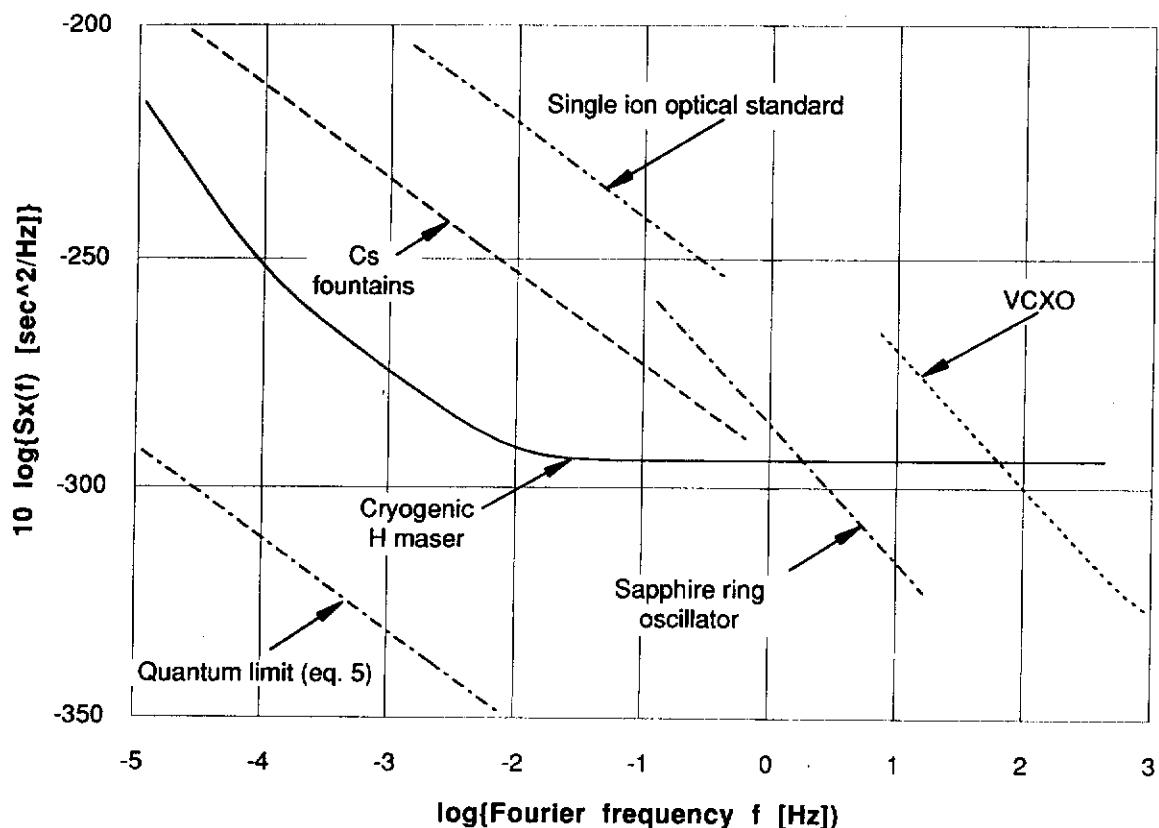


Fig. 5. Predicted phase spectral densities for future frequency standards.

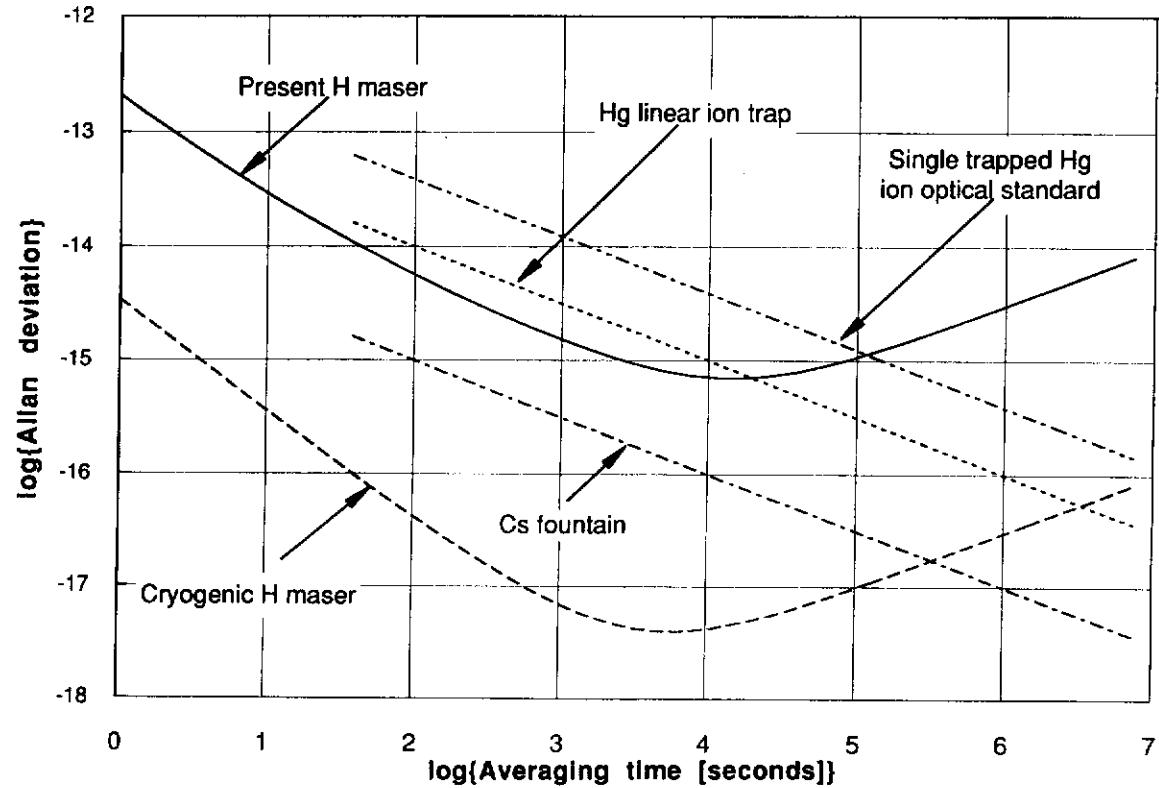


Fig. 6. Predicted Allan deviations for future frequency standards
(assumes no systematic effects in Cs and Hg devices).

QUESTIONS AND ANSWERS

D. Allan, Allan Time: Two Comments. Some work at NIST would indicate that some breakthrough potentials for rubidium short term rubidium performance that's quite exciting, short term stabilities in the parts of 10^{-14} at a few seconds. On your long term stability performance, it seems that the linear ion trap potential which was not shown on the one curve for future stability prospects seems very promising as well. The one second stability on it is like 3 parts in $10^{15}/\tau^{-1/2}$, so that might be included as well.

E. Mattison, SAO: I will certainly do that for the paper. I can get more up to date information.

Question: I would be interested in knowing your source for these trapped ion stabilities because I have been trying to find information on that and they seem to be very scarce and it seems to be word of mouth.

E. Mattison: No, that is John Dick at JPL.

Question: People at APL are involved in the work but they are not willing to sort of venture any stability numbers at all. I was just wondering if there was some sort of disagreement between the two scientists.

E. Mattison: All I know is what I got in the paper from John Dick.

D. Stowers, JPL: I have a general question and it is open to anybody in terms of whose giving thought to distribution of these signals. How are we going to get such stabilities to a user? I guess I do not need an answer, but I am sort of concerned about that in the future.

E. Mattison: I suppose an additional question is not only who is going to get it to the user but which user is going to need it. Well one of them is sitting right here, but of course it depends upon the time scale you are talking about.

L. Cutler, Hewlett Packard: JPL is also very interested in this area and I wonder if Dick Sydnor might comment on it or someone else from JPL.

R. Sydnor, JPL: Yes, we see future requirements for this sort of level and of course as you will see in some future papers here we are working on distribution systems that will work at that level and better.

E. Mattison: That being what Dick? You said that level but what level are you referring to?

R. Sydnor: 10^{-17} or thereabouts.

L. Cutler: I have a question or comment. You did not mention the noise of the second harmonic of the modulation frequency as a contributor to the long term performance of the passive resonator. That noise when heterodyned down is indistinguishable from the noise of the resonator itself and does cause a problem. So that is an additional reason why oscillators have to be very good.