

# USE OF PRIMARY FREQUENCY STANDARDS FOR ESTIMATING THE DURATION OF THE SCALE UNIT OF TAI

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## Abstract

*The accuracy of International Atomic Time TAI is based on a small number of primary frequency standards (PFS) that aim at realizing the SI second, the unit of proper time. Following the 14<sup>th</sup> meeting of the Consultative Committee for Time and Frequency (April 1999), the BIPM time section is reconsidering how the PFS comparisons are used for evaluating the duration of the scale unit of TAI,  $u_{\text{TAI}}$ , and how they are reported in Circular T and other BIPM publications. In the new procedure, it is proposed to break down the estimation of the uncertainty of the comparison in several elementary components and to report all of them in the BIPM publications in order to make the uncertainty evaluation more transparent and traceable. The BIPM also regularly computes an estimation of the duration of the scale unit of TAI using an accuracy algorithm combining the data of all PFS. The parameters used in this algorithm are re-evaluated and a new estimation of  $u_{\text{TAI}}$  has been carried out over the recent years. The standard uncertainty on this value is now estimated to be  $3 \times 10^{-15}$ .*

## INTRODUCTION

Following the 14<sup>th</sup> meeting of the CCTF held on 20-22 April at the BIPM, the time section is reconsidering the way in which the data from primary frequency standards (PFS) are used for evaluating the duration of the scale unit of TAI and how they are reported in Circular T and other BIPM publications. This proposed move will also respond to some of the recommendations of the CCTF working group on the expression of uncertainty in primary frequency standards [1] in which 37 elements were identified as contributing to communicating the uncertainty in a comparison with a PFS.

In the first section, we review two methods to evaluate the accuracy of TAI: its comparison with one PFS and its comparison with a combination of all PFS measurements. In the second section we detail how it is proposed to better estimate and report the uncertainty of a PFS comparison and in the third section we present how this will improve the accuracy evaluation through the combination of PFS measurements.

## 1 METHODS TO EVALUATE THE DURATION OF THE SCALE UNIT OF TAI

The accuracy of TAI is evaluated from the duration of its scale unit,  $u_{\text{TAI}}$ . It is usually expressed as its relative departure  $d$  from the SI second on the rotating geoid,  $u_0$ :  $d = (u_{\text{TAI}} - u_0)/u_0$ . This is practically equivalent to the opposite of the relative frequency difference of TAI with respect to an ideal time scale (having the SI second on the geoid as its scale unit). It may be evaluated

by comparing the frequency of TAI with that of one primary standard over a given time interval through a comparison of the PFS with a clock participating into TAI (or the PFS itself may be such a clock). It is also possible to take advantage of a stable reference time scale (such as TAI itself) to transfer PFS comparisons performed over various periods to a given interval over which  $u_{TAI}$  is to be estimated. Accuracy algorithms to perform such a task have been considered since the advent of “modern” primary standards [2,3,4].

## 1.1 COMPARISON OF TAI WITH ONE PRIMARY FREQUENCY STANDARD

All comparisons of primary frequency standards with TAI which are communicated to the BIPM are reported in the BIPM publications. In the present situation, only two pieces of information are reported: the interval of the comparison and the value  $\sigma_B$ , the combined uncertainty of all systematic frequency shifts affecting the PFS. In order to make the comparison more traceable and to provide more details to the user, it is proposed to report in the future at least the following seven pieces of information for each frequency comparison:

- 1) the interval of comparison
- 2)  $\sigma_B$  : the combined uncertainty from systematic effects
- 3) a reference (refereed publication) giving information on the stated value of  $\sigma_B$ .
- 4)  $\sigma_A$  : the uncertainty originating in the instability of the PFS (value provided by the laboratory).
- 5)  $\sigma_{link/lab}$  : the uncertainty in the link between the PFS and the clock participating to TAI, for an interval ending at standard TAI dates (value provided by the laboratory, if applicable)
- 6)  $\sigma_{link/TAI}$  : the uncertainty in the link to TAI (value estimated by the BIPM time section with inputs by the laboratory, if necessary)
- 7)  $\sigma$ : the quadratic sum of the four components above.

Such a procedure requires more information coming from the laboratories, compared to earlier practice. It has already been implemented in recent reports submitted for the primary standards LPTF-JPO [5] and NIST-7 [6], and for the continuously running primary standards PTB-CS1 [7], PTB-CS2 [8], and PTB-CS3 [9]. For one other primary standard, NRLM-4 [10], the report provides the direct comparison to TAI and the proposed procedure cannot be applied yet. These are the only primary standards that have recently (as of 1 December 1999) submitted a comparison.

## 1.2 COMBINATION OF THE COMPARISONS OF TAI WITH PRIMARY FREQUENCY STANDARDS

The algorithm described in [3] has been used at the Bureau International de l’Heure, then at the BIPM, for more than 20 years for evaluating the duration of the scale unit of TAI. In addition to the original reference, details of a practical application of this algorithm to TAI in 1997 may be found in [11]. The algorithm allows to transfer the PFS comparisons, which are performed over various time intervals, to the interval over which  $u_{TAI}$  is to be estimated. For this purpose, the comparisons are referenced to the time scale EAL, i.e. the free atomic time scale which is the first step in TAI computation [12]. Assuming  $N$  independent primary frequency standards (i.e. different PFS or the same PFS completely re-evaluated), we have  $n_i$  comparisons available for the standard  $i$ , and these comparisons are performed over the intervals  $T_{ij}$  ( $j$  from 1 to  $n_i$ ,  $i$  from 1 to  $N$ ). The basic hypothesis of this algorithm are the following:

- 1) the reference time scale has a known stability and no systematics
- 2) the uncertainty of a given PFS comparison is characterized by  $\sigma_{Bi}$ , the combined uncertainty of all systematic frequency shifts affecting the PFS, and by  $\sigma_{Aij}$ , the instability of the comparison itself
- 3) the values  $\sigma_{Aij}$  (and also the values  $\sigma_{Bi}$ ) are independent of the stability of the reference time scale.

Then the algorithm allows expression of the TAI frequency estimated over a given interval as a linear combination of all the PFS measurements, the coefficient (weight) of each one being computed from the parameters of all the measurements (uncertainty, duration, distance from the estimation interval) and from the EAL stability model. Since the stability of EAL is not constant, but improves with time as a result of the progress in atomic clocks, its stability model must be updated from time to time. We shall consider below a stability model adapted to the recent years (1997 to 1999).

A summary of the estimation of  $u_{\text{TAI}}$  over the recent years from all PFS comparisons with TAI, as well as from the BIPM estimate using the algorithm above, is presented in Figure 1.

## 2 EVALUATION OF THE UNCERTAINTY OF A PFS COMPARISON

We distinguish uncertainties which originate inside the laboratory (up to the clock providing the link to TAI), and which are to be reported by the laboratory with each comparison, and uncertainties in the link from the clock to TAI, which are evaluated by the BIPM.

### 2.1 UNCERTAINTIES INSIDE THE PFS LABORATORY

They concern  $\sigma_B$ ,  $\sigma_A$ , and  $\sigma_{\text{link/lab}}$ . It is not the purpose of this paper to detail how  $\sigma_B$  and  $\sigma_A$  should be evaluated, since this depends on the setup of each primary standard, and is to be estimated by the laboratory. We just mention some comments which affect the component  $\sigma_{\text{link/lab}}$  (and indirectly  $\sigma_{\text{link/TAI}}$ ). It is expected that the frequency comparison is reported between the PFS and a clock participating to TAI, (or the PFS is itself such a clock). Two cases are possible: either the PFS is operated over a time interval between standard TAI dates (0h UTC of MJD ending in 4 and 9), or it is not. In the former case, it is not necessary to account (in  $\sigma_{\text{link/lab}}$  nor in  $\sigma_{\text{link/TAI}}$ ) for the instability of the clock providing the link to TAI, since it will cancel when comparing the PFS to TAI. In the latter case, it is necessary to transfer from the comparison interval to one standard TAI interval encompassing it, accounting in  $\sigma_{\text{link/lab}}$  for the instability of the clock over the difference of the intervals. The choice of the duration of the standard TAI interval over which to report the comparison may also be discussed. It should be one that minimizes the overall uncertainty in the comparison, taking into account the fact that, for a longer interval, the uncertainty from the time transfer will decrease, while the uncertainty from the instability of the local clock will increase.

### 2.2 UNCERTAINTIES IN THE LINK TO TAI

The term  $\sigma_{\text{link/TAI}}$  should not be estimated by a statistical study of the differences (TAI-clock), otherwise the full instability of the clock over the standard TAI interval would enter into it. Rather it is to be estimated by considering the components that enter into it: one is due to the time-transfer technique itself,  $\sigma_{\text{tt}}(\tau)$ , and another one is due to the comparison of the clock providing the link to TAI with the time transfer device,  $\sigma_{\text{clock-tt}}(\tau)$ . In addition it is to be determined whether the noise of TAI itself,  $\sigma_{\text{TAI}}(\tau)$ , should be considered. Two cases may be considered: If we consider the report of the comparison of one PFS with TAI, it should include all terms of the comparison, i.e. include TAI. If, on the other hand, we consider the combination of all PFS measurements (section 1.2),  $\sigma_{\text{TAI}}(T_{ij})$  should not be included in the uncertainty of each comparison that is input to the accuracy algorithm. The stability of TAI (rigorously it is EAL that should be considered here) will be estimated at the next section and is not addressed here.

The component  $\sigma_{\text{clock-tt}}(\tau)$ , exists only when the clock, participating in TAI, to which the frequency of the PFS is compared, is not directly linked to the time transfer device. When  $\sigma_{\text{clock-tt}}(\tau)$  exists at all, it should be generally very small (no more than, e.g., a few parts in  $10^{16}$  for averaging time of 5 days) because it represents the performance of the clock comparison device. But if it is actually estimated from the data reported for TAI computation (i.e. one datum every 5 days with a precision of 1 ns), it may be larger. This quantization error may contribute to  $\sigma_{\text{clock-tt}}(\tau)$  by  $1.1 \times 10^{-15}$  for 5 days and  $8 \times 10^{-17}$  for 30 days. When such a level is considered too high, data with a smaller resolution should be used.

Let us consider the uncertainty in the time transfer technique  $\sigma_{\text{tt}}(\tau)$ . In the present situation, it is provided by classical common-view GPS time transfer (GPS-CV) with three pivot laboratories in Europe, America, and East Asia. We consider that the link between the laboratory of interest and TAI contains one pivot-to-pivot link. If the laboratory is itself a pivot, this should provide a good estimation of  $\sigma_{\text{tt}}(\tau)$ . If not, this will probably underestimate the true value of  $\sigma_{\text{tt}}(\tau)$ ; however, we still keep this hypothesis for simplicity.

Time-transfer noise  $\sigma_{\text{tt}}(\tau)$  may be considered the sum of two components: measurement noise, called  $\sigma_m(\tau)$ , which we consider as white phase noise which may be reduced by averaging, and the noise due to other systematic effects, such as the sensitivity of hardware delays to the environment,  $\sigma_s(\tau)$ . In the following we try to estimate the uncertainty in a frequency comparison carried out over the duration  $\tau$  from these two components. Then  $\sigma_{\text{link/TAI}}$  is obtained as the quadratic sum of the components  $\sigma_m(\tau)$ ,  $\sigma_s(\tau)$ , and  $\sigma_{\text{clock-tt}}(\tau)$ .

The term  $\sigma_m(\tau)$  may be estimated from the expected uncertainty  $\sigma_x$  of a single GPS-CV measurement and their average spacing  $\tau_0$ :  $\sigma_m(\tau) = \sqrt{3} \sigma_x / \tau_0 (\tau / \tau_0)^{3/2}$ . As an example, with  $\sigma_x = 3$  ns and 20 measurements per day  $\sigma_m(\tau = 5 \text{ days}) = 1.2 \times 10^{-15}$  and  $\sigma_m(\tau = 30 \text{ days}) = 8 \times 10^{-17}$ . Of course the numbers should be scaled according to the appropriate values of  $\sigma_x$  and  $\tau_0$ , but they correspond approximately to the present situation (Europe to America is slightly better and Europe to East-Asia is slightly worse).

The term  $\sigma_s(\tau)$  is more difficult to estimate. Several studies may help us in this task. Since the link to TAI is provided at least over an interval of 5 days, we do not consider the short-term systematic effects. For values of  $\tau$  ranging from 5 to  $\sim 30$  days, we may gather information from studies that have compared GPS-CV with another technique for a given link. As an example, comparison of GPS-CV with Two Way time transfer over 5 months [Technical Memorandum available from the BIPM] show a noise for 5-day time comparisons of order 2.0 ns for PTB-TUG, 1.9 ns for NPL-PTB, 1.5 ns for PTB-NIST (sum of two GPS-CV links), 1.7 ns for NPL-NIST (sum of two GPS-CV links) over 4 months, and 1.6 ns for VSL-PTB over 3 months. This corresponds to a 5-day modified Allan deviation of  $6 \times 10^{-15}$  to  $8 \times 10^{-15}$ . Longer experiments have shown that the difference between GPS-CV and TWTT has a modified Allan deviation of  $7 \times 10^{-15}$  (5 days) to  $5 \times 10^{-16}$  (40 days) for TUG-PTB [Technical Memorandum available from the BIPM].

For values of  $\tau$  of 30 days and above, we may also infer information from the repeatability of series of GPS calibration trips performed by laboratories or by the BIPM, and which results are collected at the BIPM. In the most favorable cases, the standard deviation of the calibration results is 2.2 ns for OP-NIST (8 calibrations over 13 years) and 2.9 ns for IEN-OP (6 calibrations over 12 years). Other cases are less favorable, but may be explained by undocumented changes in the hardware. Also, a comparison of GPS-CV with GPS carrier phase for PTB-USNO over more than a year [13] shows that systematic variations remain below a few ns over the whole period.

From these measurements, we infer that reasonable (but not too pessimistic) values may be chosen as:  $\sigma_s(\tau = 5 \text{ days}) = 6 \times 10^{-15}$  and  $\sigma_s(\tau = 30 \text{ days}) = 1 \times 10^{-15}$ . When necessary, we shall interpolate between these values to estimate  $\sigma_s$  for other values of  $\tau$ .

### 3 THE BIPM ESTIMATE OF THE DURATION OF THE SCALE UNIT OF TAI

#### 3.1 STABILITY OF THE REFERENCE TIME SCALE EAL

Over January 1997- February 1999, a n-cornered hat analysis has been performed comparing EAL with TA(PTB), TA(F), TA(USNO), TA(NIST) (Figure 2). We choose as a model of the stability of EAL a sum of three components (white frequency, flicker frequency, and random walk in frequency) that just overestimates the EAL curve in Figure 2. So the new EAL noise model is composed of:

A white frequency noise  $6 \times 10^{-15} / \sqrt{\tau}$

A flicker frequency noise  $0.6 \times 10^{-15}$  : this is the estimated flicker floor of EAL, but may in practice be neglected compared to the other two components for all values of  $\tau$

A random walk frequency noise  $1.6 \times 10^{-16} \times \sqrt{\tau}$ .

Due to the uncertainty in the long-term behavior of EAL and to the fact that the above model for EAL has been determined over the period 01/1997-02/1999, it is advisable to apply it only for computations starting 01/1998 or later and to use PFS comparisons not further than one year from the estimation interval (in particular not earlier than 01/1997). This hypothesis is the one which is commonly used in the Circular T estimations and will be retained for the tests of the following sub-section.

#### 3.2 SOME TESTS OF THE ESTIMATION OF $d$ OVER 1998-1999

New computations of the estimation of  $d$  since January 1998 have been performed to test the above mentioned changes: the new stability model for EAL (determined since January 1997) and the revised values for  $\sigma_A$  as estimated above. The computations are carried out for an estimation interval of two months or one month. In each case, three estimations are computed: one with the "old model" for the stability of EAL and "old values" of  $\sigma_A$ , one with the "new model" for the stability of EAL and "old values" of  $\sigma_A$ , and one with the "new model" for the stability of EAL and "new values" of  $\sigma_A$ . Results are presented on Figure 3 for an estimation interval of two months and on Figure 4 for an estimation interval of one month. Standard uncertainties on the  $d$  values (not shown here) are of order  $2.5 \times 10^{-15}$  to  $3 \times 10^{-15}$  with the new models, compared to about  $4 \times 10^{-15}$  for the standard calculation.

We may draw four conclusions:

- 1) The new model of EAL stability provides an estimation of  $d$  which is smoother over time and otherwise not biased with respect to the standard computation. This reflects the fact that, in the standard procedure, the estimation of  $d$  is dominated by PFS comparisons very close to the interval of computation, while with the new model, distant comparisons contribute more.
- 2) Using the new computation of  $\sigma_A$  cause a bias of about 2-3 parts in  $10^{15}$  in  $d$  with respect to the standard computation. This is mainly due to the larger weight attributed to PTB-CS3 which, in the standard computation, had been assigned a larger value of  $\sigma_A$  on the basis of past instabilities.
- 3) With the new EAL model, there is no significant difference between choosing an interval of estimation of  $d$  of one month or two months. This also reflects the fact that distant comparisons contribute more to the estimation, which is therefore less sensitive to the comparisons performed

- during the estimation interval (these are of course different for 2-month intervals and 1-month intervals). The standard uncertainty of  $d$  is, however, slightly lower over 2-month intervals.
- 4) Given the recent results for  $d$  (of order  $5 \times 10^{-15}$  with a standard uncertainty of order  $3 \times 10^{-15}$ ), a change to the new model and the new computation of  $\sigma_A$  should be accompanied by a steering of TAI (reducing the value of the frequency difference  $f(EAL) - f(TAI)$  from its current value of  $714 \times 10^{-15}$ ). Frequency steps should be  $1 \times 10^{-15}$  at most in order to preserve the stability of TAI for averaging duration of one to a few months.

## CONCLUSIONS

We propose modifying the procedures currently in order that the BIPM time section use primary frequency standards data for estimating the duration of the scale unit of TAI and report them in Circular T and other BIPM publications. The main changes concern how to evaluate the standard uncertainties of primary standard comparisons to TAI, how they are reported, and how they are carried out in the accuracy algorithm used to estimate the duration of the scale unit of TAI. A new stability model for the reference time scale EAL is also proposed. In the present situation of primary frequency standards and stability of EAL, a standard uncertainty of order  $3 \times 10^{-15}$  is expected on the duration of the scale unit of TAI.

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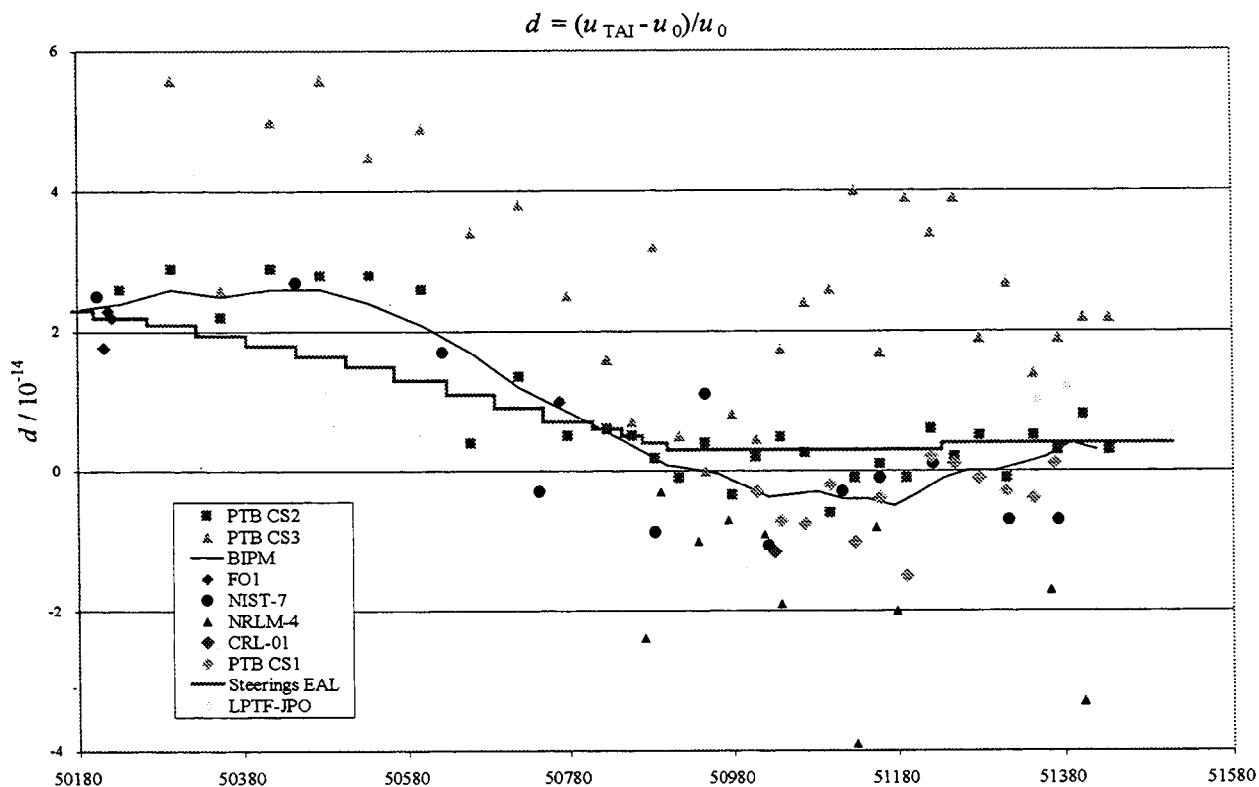


Figure 1: Duration of the scale unit of TAI since 1996 estimated from individual PFS and from the BIPM computation (computation interval of two months).

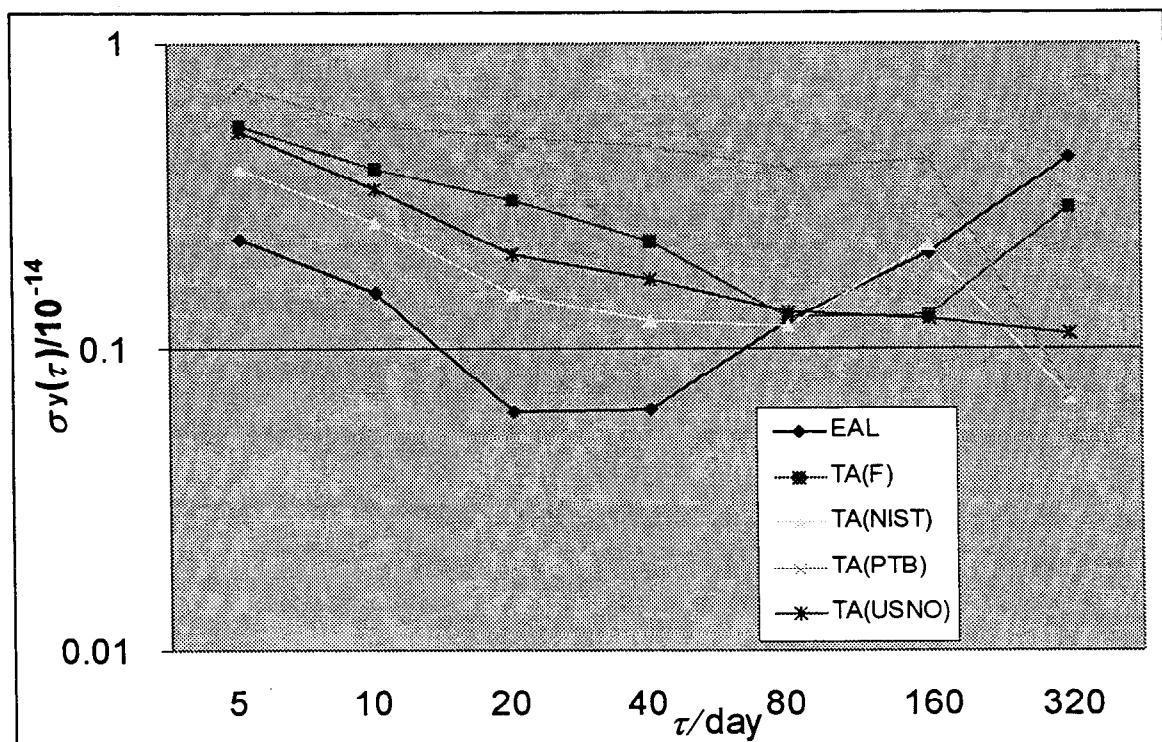


Figure 2: Relative frequency stability of EAL and various atomic time scales computed by n-cornered hat over 01/1997-02/1999.

### Scale unit of TAI - 1s / $10^{-14}$ (60 days)

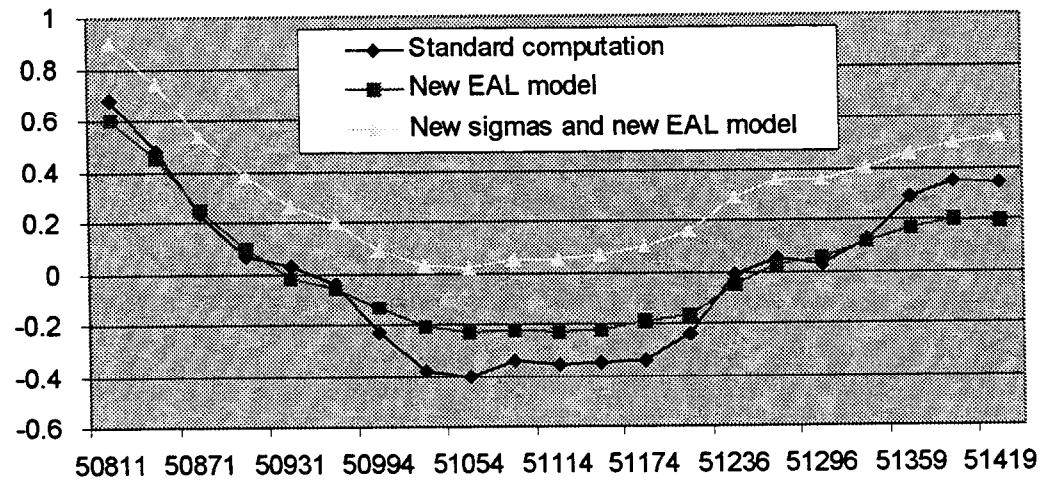


Figure 3: Duration of the scale unit of TAI over 01/1998-09/1999 for three different cases (computation interval of two months).

### Scale unit of TAI - 1s / $10^{-14}$ (30 days)

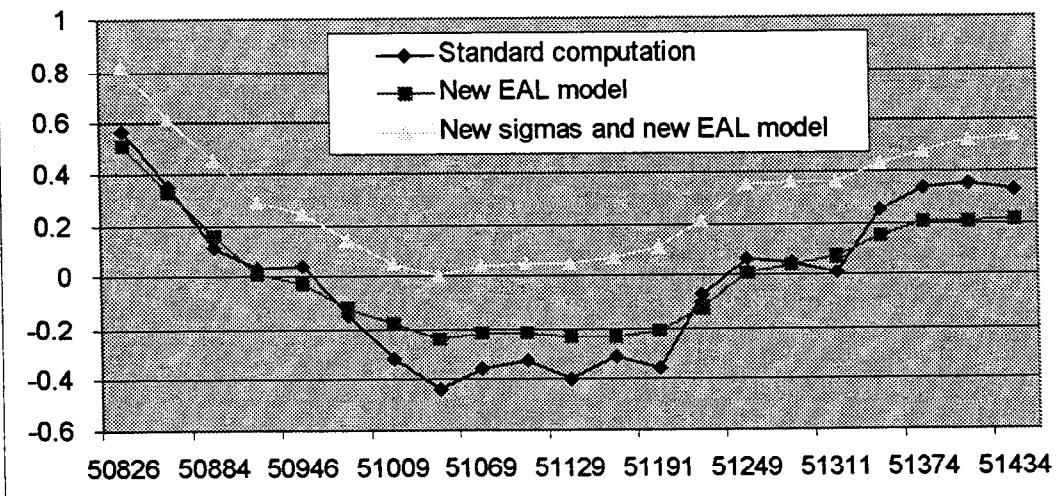


Figure 4: Duration of the scale unit of TAI over 01/1998-09/1999 for three different cases (computation interval of one month).