

## OUTLOOK FOR PRECISION FREQUENCY CONTROL IN THE 1980's

ERICH HAFNER

US Army Electronics Technology and Devices Laboratories (ECOM)  
Ft. Monmouth, New Jersey

### ABSTRACT

Current laboratory activities that are likely to result in improved crystal oscillators and atomic or molecular frequency standards over the next ten years are examined. In the crystal area, stabilities under tactical environment - including static acceleration - in the low  $10^{-10}$  range appear to come within reach, in one cubic inch devices; and molecular frequency standards with stabilities in the low  $10^{-12}$  range appear feasible in 20-40 cubic inch size. The former will be made possible by a breakthrough in the static acceleration sensitivity of crystal units, now considered imminent, and by significant advances in crystal and oscillator technology; the latter by the use of saturation resonance absorption phenomena in the infrared range. Considerations are offered to demonstrate that the CO<sub>2</sub> resonance fluorescence cell has all major ingredients to form the heart of a novel miniature molecular frequency standard. Progress being made towards assessing the principal technological difficulties, and their resolution, are described. The paper includes a discussion of other molecular resonances with potential for frequency control application as well as a cursory review of more exotic phenomena for the more distant future.

### INTRODUCTION

Tremendous strides have been made over the last two decades in the art of frequency control for application in communication, navigation and position location systems. Quartz crystal reference oscillators are now available in 15 to 30 cubic inch packages that exceed in performance those that occupied several cubic feet less than 20 years ago; and atomic frequency standards whose fundamental properties had just begun to be studied seriously in the laboratory in 1955, have matured into a range of relatively compact commercial devices whose stabilities are measured in the  $10^{-12}$  to  $10^{-15}$  range.

Yet, the demands for improved performance have easily kept pace with the advances in device technology; and for applications in time ordered systems in particular, still higher performance in terms of size, power consumption and stability is required.

It appears appropriate, therefore, to engage in some crystal gazing in an effort to foretell the shape of things to come in frequency control in the 1980's. As appropriate for PTTI, only devices with stability potential in the  $10^{-9}$  range and beyond will be considered.

### Crystal Oscillators

Progress in crystal oscillators will be paced by the advances in crystal resonator development. The art of precision crystal manufacture by conventional techniques appears to have reached a plateau which is characterized by aging rates of  $1.5 \times 10^{-8}/\text{mo}$  ( $5 \times 10^{-10}/\text{day}$ ) for overtone and  $5 \times 10^{-8}/\text{mo}$  for fundamental mode crystal units in the 5 MHz range. Significantly better behavior can be obtained by observation of generally large lots of crystal units over extended periods of time and careful selection of the best performers. The rapidly decreasing yield of units that have to meet particularly tight specifications prevents any large scale system application of oscillators requiring specialty crystal units at this time. While aging provides a convenient and easily recognized measure of crystal quality, the manufacturing imperfections responsible for a large part of the aging behavior are also contributing dominantly to such other phenomena as lack of frequency repeatability on thermal cycling (a serious limitation in precision temperature compensated oscillators), excessive noise (poor short term stability), high resistance at very low drive levels (oscillation start up problems) etc.

The need for a major advance in crystal unit technology is clear, and the direction to be taken for establishing a high yield production capability for crystal units with more than one order of magnitude reduced aging, and corresponding improvements in related performance characteristics has been established<sup>1</sup>. The key elements are a) improved surface preparation techniques, b) novel means for eliminating contaminants and avoiding recontamination by continuous in-line vacuum processing and c) new resonator packaging approaches.

Implementation of the basic capability along these lines is expected by the late 1970's, providing adequate numbers of high performance crystal units for initial production of advanced high density surveillance, communication and navigation systems. Further improvements of these new processing techniques extending into the 1980's are expected, along with expanding capacities as required to meet full system production needs.

While the new processing technology will play a crucially important role in providing the large numbers of high performance crystal units required to meet the systems needs of the 1980's, recent advances in resonator theory are pointing the way to new types of resonator cuts and geometries that promise to yield characteristics superior to those of the commonly used AT or BT configurations, particularly with respect to the effects of stress fields on resonance frequency. Stress fields may be caused by the resonator mounting structure, plated-on electrodes and thermal gradients as well as by static acceleration and vibration. Relaxation of mounting and electrode stresses can contribute significantly to the aging of crystal units, and generally cause much the same problems as discussed above in relation to other manufacturing imperfections. Stresses induced by thermal gradients are implicated in the thermal transient behavior of temperature compensated oscillators, in the warmup characteristics of oven controlled oscillators and in the frequency drive level effect, which impacts on attainable short term stability. All of these, as well as the frequency excursions observed during static acceleration and vibration, have long been recognized, but have only gradually yielded to empirical efforts to achieve any significant reductions<sup>2</sup>.

Recent theoretical work<sup>3-5</sup>, however, has now provided the basis for a fresh approach to these problems. The functional relationship between the static (or quasistatic) stress fields involved and the high frequency vibrations of the crystal resonator, - and hence the resonance frequency - have been established. The coupling between them depends on the nonlinear elastic constants, the magnitude of which, in an anisotropic body such as quartz, depends on the crystallographic orientation of the resonator. The approximate angles of cut for a resonator in which this coupling becomes very low have been identified theoretically, and early results of experiments<sup>6</sup> confirm the existence of a low stress effect cut in the neighborhood of the crystallographic angles defined by theory. As this cut is also, in the same sense as an AT cut, temperature compensated, which is always a primary requirement, the term temperature/stress compensated (TS) cut appears most appropriate and will be used here. Estimates<sup>4,5</sup> indicate that the stress sensitivity of the TS cut may be as much as (or possibly even more than<sup>6</sup>) two orders of magnitude lower than that of the AT cut. The potential advantages of this cut are thus clearly significant.

A significant effort will no doubt be necessary to develop the potential of the TS cut into practical crystal units. One of the problems to be resolved revolves around the fact that the TS cut is a so-called doubly rotated cut and, therefore, contains two families of piezoelectrically excited modes, where the AT cut, being singly rotated, contains only one. The two "main modes", one for each family, are closely spaced in frequency. One must be suppressed, by electronic

means at this time, to assure proper oscillator operation. The added circuit complexity introduced is a major reason why doubly rotated cuts have in the past been used only for special purpose applications<sup>7</sup>. Moreover, the suppression of other spurious modes that interfere with the characteristics of the desired mode is a notoriously difficult problem even for AT resonators. This may be further aggravated by the second family of modes in the TS cut.

With the attention focused now on the potential benefits to be derived, it is expected that stress compensated resonators for application in precision oven controlled reference oscillators will become available in reasonable quantities during the late 1970's. For this application, useful operation of the resonator over only a narrow temperature range is acceptable, and only a small number of standard frequencies is involved. Wide temperature range resonators of this kind, such as are required for precision temperature compensated oscillators may take until the early 80's. The development of these latter resonators, especially if arbitrary frequencies are required, is likely to be hastened by progress in the computer modeling of resonator vibrations, using finite element techniques, which eventually will make it possible to dispense with the traditionally required cut-and-try techniques for the final steps in resonator design.

While there are very obvious risks in attempting to predict what a crystal oscillator specification might be like in the mid 1980 time-frame, it is interesting to speculate about what the impact might be if a crystal resonator can indeed be developed whose sensitivity to stress fields is two orders of magnitude lower than that of an AT resonator, and whose frequency temperature characteristic is similar to that of an AT cut but with the inflection point at around 90°C. The latter aspect would provide a near zero temperature coefficient range extending over several degrees around the normal operating temperature of a crystal oven. Let us further assume that this resonator be processed by the techniques of the 1980's which eliminate contamination and mass transfer within the crystal enclosure as sources of aging.

The physical size and power consumption of the 1980's crystal oscillator is indicated by the 16 cm<sup>3</sup> (1 cubic inch) and 150 mW characteristics of the Tactical Miniature Crystal Oscillator, (TMX0) currently (1975) under development<sup>8</sup>. A photograph of this oscillator is shown in Fig. 1, next to a 200 cm<sup>3</sup> unit representative of the 1975 off-the-shelf state of the art. A comparison of some of the features of these two oscillators is shown in Fig. 2. A more detailed listing of the characteristics of the TMX0 now projected for the late 1970's is shown in Fig. 3, together with those of the 1980's oscillator which, along with improved circuitry, contains the stress compensated crystal resonator postulated just above.

The improved warmup characteristic of the 1980's vs the 1970's version in Fig. 3 is due to the absence (reduction), in the TS resonator, of the large frequency overshoot found in the AT resonators that is caused by the stresses set up by the thermal gradients incident to the resonator warm up. Likewise, part of the improvement in short and intermediate term stability is due to the fact that minor temperature fluctuations have negligible effect on the stress compensated resonator. The aging of the 1980's resonator, with contamination and mass transfer reduced to negligible levels, will no longer be determined by the stress relaxation in the mounting structure and the electrodes, but principally by thermodynamic order - disorder phenomena in the quartz and the electrodes, which should proceed only at glacial pace. Finally, because the frequency sensitivity of the AT resonator to static acceleration and vibration is (at reasonable levels) strictly determined by the magnitude of the induced stresses, the sensitivity of the stress compensated resonators will be proportionately lower.

The TMXO is an oven controlled crystal oscillator. The potential impact of a stress compensated crystal oscillator with suitable frequency/temperature characteristics could be equally important for high stability temperature compensated oscillators (TCXO's). However, the problems of developing a stress compensated crystal unit suitable for TCXO's are almost certainly greater than for oven controlled oscillators, as already indicated above. Once they have been resolved, TCXO's with stabilities in the low  $10^{-9}$  range, 1-2 cubic inch size, and power consumption around 50 mW would appear to become possible, perhaps by the mid 1980's. Their design would follow the principles currently being developed <sup>9</sup>, using digital compensation techniques. The major advantage of these TCXO's would be their instant "on frequency" operation capability.

#### Atomic and Molecular Frequency Standards

An excellent survey of the state of the art of atomic frequency standards, including an outlook toward the probable future developments has only recently been presented <sup>10</sup>. The emphasis there was on high precision laboratory devices. The focus here shall be on devices suitable for tactical systems applications.

There can be no question that the devices currently on the market or under development, i.e., the Rubidium and Cesium standards and the Hydrogen maser will continue to find increasing numbers of applications well into the 1980's. There can also be no question, however, that the price, size, weight and power consumption of these devices will continue to provide a powerful incentive to the systems engineers to avoid their use wherever possible. Size, weight and power consumption alone regardless of price, prohibits their use in modern army field equipment today.

There is a need, therefore, for a very small, potentially low cost atomic or molecular frequency standard whose stability approaches that of the present day Cesium standards, i.e., is in the  $10^{-11}$  -  $10^{-12}$  range, short and long term, under manpack and vehicular conditions of operation.

The smallest atomic frequency standard currently available <sup>11</sup> is a 1000 cm<sup>3</sup> (61 cubic inch) Rubidium gas cell standard whose power consumption is 13 W. Its nominal frequency stability (temperature range is 0°C to 50°C) is comparable to that projected above for the one cubic inch crystal oscillator of the 1980's. Continued innovative engineering applied to the optical unit and the use of microcircuit techniques is likely to result eventually in a unit of about 40 cubic inches, consuming in the order of 6 W of power, again for the 0°C to 50°C temperature range. Extending this range to the -50°C to +85°C range would be very costly in power, as thermoelectric cooling is necessary at the upper end; - the temperature of the gas cell itself must be maintained at 75°C for proper operation. A suitably engineered unit of this kind should be capable of maintaining a frequency stability of  $1 \times 10^{-11}$  in a vehicular environment; and a reduction of the aging rate to  $1 \times 10^{-11}$ /year is likely to have been achieved by the latter part of the 1980's. Thus, while the stability potential of the Rubidium gas cell standard would appear to be adequate, its power requirements in particular - size is marginal at best - will continue to present a serious handicap for application in highly mobile systems. The search for alternate means to derive the requisite highly stable reference signals from a very small, low power unit will continue, therefore.

An examination of available new principles for frequency control points to systems that employ, or depend on, a stabilized laser as a major element, with a molecular resonance serving as the frequency discriminator. Although frequency stabilities in the  $10^{-15}$  range have been projected <sup>12</sup> for laboratory systems in this category, - they are attained with hydrogen masers already - the constraints imposed by the need for miniaturization of the standard and of linking the stabilized laser frequency to the megahertz frequency range do not appear to admit their realization in a form suitable for systems application until, perhaps, the 1990's. It is no doubt premature at this time to attempt predicting which particular system and atomic or molecular transition is likely to offer the path of least resistance to a  $10^{-15}$  small size unit, first. The ultimate frequency standard, in the distant future, may well turn out to require no more than a pellet of active material once the intrinsic properties of the Mossbauer Effect can be harnessed.

Stabilities in the  $10^{-12}$  range, however, do appear attainable in the 1980's for use in highly mobile applications. The system most likely

to succeed, in the opinion of this author, is based on a miniature CO<sub>2</sub> laser, frequency stabilized by means of the fluorescence radiation from a CO<sub>2</sub> saturated resonance absorption cell <sup>13</sup>.

The principle of saturated resonance absorption has been described extensively in the literature <sup>14</sup>. Its major advantage is in the fact that it can eliminate the Doppler broadening which dominates the line shape of a molecular resonance as observed by normal spectroscopic techniques. As a result, the line widths of molecular transitions can be reduced by several orders of magnitude. A narrow linewidth is obviously important if the resonance should be suitable for frequency control.

The frequency stability attainable with the aid of given resonator, regardless of its nature, is approximately given by

$$\left(\frac{\Delta f}{f}\right)_{\tau} = \frac{1}{2Q_{\ell}} \left(\frac{S}{N}\right)_{\tau}^{-1}$$

where Q<sub>ℓ</sub> is the line Q, Q<sub>ℓ</sub> = Δν/ν<sub>0</sub>, and (S/N)<sub>τ</sub> is the signal to noise ratio observed with a detector time constant of τ seconds.

The narrowest line width observed to date in a CO<sub>2</sub> resonance cell is 164 kHz, which, with a line center frequency of 3<sup>2</sup> × 10<sup>13</sup> Hz gives a line Q of about 2 × 10<sup>8</sup> (the line Q of the atomic resonance used in a Cesium standard is in the order of 5 × 10<sup>7</sup>). The factors affecting the linewidth are discussed in Appendix A. While it should be possible to achieve, in time, line Q's in the order of 10<sup>9</sup>, optimum operation of the resonance cell in a small frequency standard occurs at somewhat lower Q<sub>ℓ</sub> values, probably around 10<sup>8</sup>.

The CO<sub>2</sub> resonance of interest occurs at 10.6μ; however, the optimum technique for detecting the resonance is, following Freed and Javan <sup>13</sup>, the observation of a dip in the 4.3μ fluorescence signal which is emitted when the excited CO<sub>2</sub> molecules relax to the ground state. The 4.3μ radiation is incoherent and nondirectional. Gallagher <sup>16</sup> has calculated the strength of the 4.3μ fluorescence signal and finds it to be proportional to the population difference between the (10<sup>0</sup>0) and (00<sup>0</sup>1) levels at thermal equilibrium, and to the power of the 10.6μ laser signal (the difference is nearly equal to the population of the (10<sup>0</sup>0) level). A numerical example places the total power in the 4.3μ fluorescence line at 5.4 × 10<sup>-7</sup> watts which considering the depth of the dip at resonance in the emitted power to be 20%, yields a usable signal, with optimum modulation, of 8 × 10<sup>-8</sup> watts. Evaluation of the date reported by Freed <sup>17</sup> appears to be consistant with this value.

A listing of some of the results obtained to date in experiments with the CO<sub>2</sub> resonance fluorescence cell is given in Fig. 4, together with the potential frequency stability, for an integration time of  $\tau = 1$  sec, calculated on the basis of the linewidth and signal to noise data reported. Kelly, et al.<sup>16</sup> and Freed's<sup>17</sup> data pertain to a resonance cell external, Gallagher's<sup>18</sup> to one internal to the laser cavity. All have used cryogenically cooled detectors. Apparently, frequency stabilities in the 10<sup>-12</sup> range are attainable for integration times larger than 1 sec.

For the CO<sub>2</sub> frequency standard to become a small and low power unit, however, it is necessary to use room temperature detectors for the 4.3 $\mu$  radiation. Unfortunately, the noise power in even the best commercially available room temperature devices is about 100 times larger than that of the InSb detector, operated at 78° K, used by Freed<sup>17</sup>. Accordingly, the signal to noise ratio to be expected would be in the order of 5 to 10, which is clearly inadequate for frequency standard operation. Optimization of laser power and gas pressure in the resonance cell, as shown in Appendix B, will help, but not materially affect this situation. Nevertheless, it appears clear already, that relatively high levels of the 10.6 $\mu$  radiation will be required, which are most readily available inside the laser cavity.

Several approaches are possible to improve the performance of the CO<sub>2</sub> resonance cell for room temperature operation. The use of better detector<sup>18</sup> alone may already bring the signal to noise ratio up well into the 10<sup>2</sup> range. Among the candidates to be studied are GaAs Schottky diodes<sup>19</sup>, improved pyroelectric devices, optoacoustic devices<sup>20</sup> and further improvements in PbSe. Metal-metal oxide-metal diodes<sup>21</sup>, deposited by thin film techniques to form arrays, may also be found suitable once the optimum combination of materials is identified.

The basic reason for the low signal levels involved in the CO<sub>2</sub> resonance is that the number of molecules taking part in the phenomenon is only a very small fraction, about 10<sup>-3</sup>, of the total molecules in the cell; the lower of the two energy levels involved in the 10.6 $\mu$  resonance lies above the ground state. The most effective ways to increase the signal level would appear to be, therefore, to increase the population of this state, for example, by irradiating the resonance cell with 7.4 $\mu$  light, which can probably best be derived from a black body (filament) through a 7.4 $\mu$  bandpass filter. An increase in the available fluorescence signal by at least two orders of magnitude appears possible, which should be adequate to reach stability levels around 1 X 10<sup>-11</sup> for 1 second integration time, even by using currently available room temperature detectors.

The stability numbers used so far actually describe the ability to define the center frequency  $\nu_0$  of the resonance, assuming implicitly that  $\nu_0$  is held constant by proper control of the environment. Indeed, the resonance frequency of the CO<sub>2</sub> molecule is remarkably unaffected by environmental conditions<sup>15</sup>. Each atom in the CO<sub>2</sub> molecule has zero molecular spin, resulting in the absence of any hyperfine structure; and there is no permanent electric dipole moment, so that shielding against external electric and magnetic fields is not required. There are no magnets employed in its operation that could add bulk and weight to the device. The frequency shift due to variations in the gas pressure within the cell are so low that no temperature control should be required to maintain a stability of better than  $1 \times 10^{-11}$  over -50°C to +80°C even at cell pressures in the order of 30 mT. Power shift as opposed to power broadening, due to the 10.6μ laser light is also small; a variation of the laser power level of 10% should cause less than  $1 \times 10^{-12}$  frequency variation in the 100 mW range. The probable magnitude of the frequency shift due to the 7.4μ black body pumping radiation, if used, has not been assessed, but should be very small.

Overall, the CO<sub>2</sub> resonance fluorescence cell promises to be an exceedingly simple device, requiring a minimum on controls, with an accuracy potential in the  $1 \times 10^{-11}$  range. Much work remains to be done to determine how closely this potential can be approached, or exceeded.

The size of the CO<sub>2</sub> control cell can probably be less than 1 cubic inch (16 cm<sup>3</sup>). The 10.6μ laser which is required to excite the resonance, and in turn is stabilized by it, through a suitable feedback network, is likely to be about 10-12 cm long<sup>13, 16</sup>. If the resonance cell is built internal to the laser cavity it may be reasonable to expect that the whole stabilized laser assembly can eventually be compressed into a 1 X 1.5 X 7 inch, i.e. slightly larger than 10 cubic inch package. In the more distant future, it is conceivable that the laser will be replaced by a planar source of 10.6μ radiation.

The stabilized laser does not provide an RF output as required for systems applications; the output signal is in the  $3 \times 10^{13}$  Hz range. While the feasibility of frequency multiplications even into the visible light range has been demonstrated<sup>22</sup>, the equipment used thereby included, next to an assembly of large laser oscillators, a Josephson junction operating in a He cryostat. The highest frequency reached to date with room temperature, solid state devices is about  $3 \times 10^{11}$ , still a factor of 100 away from our goal. Yet, harmonics at orders as high as 30-40 are considered obtainable with metal-metal oxide-metal diodes at Terrahertz frequencies<sup>23</sup>. Progress in solid state millimeter and submillimeter wave devices research and synthesis into

the THz range is rapid, and the prospects appear good that the results of this research will permit construction of a small size multiplier to link the CO<sub>2</sub> stabilized laser to a quartz crystal oscillator at some time during the early 1980's. To expect the complete CO<sub>2</sub> laser to occupy eventually no more than 20 cubic inches does not appear to be unreasonable.

## Appendix A

### Linewidth of the CO<sub>2</sub> Saturation Resonance Absorption Cell

The total linewidth of the CO<sub>2</sub> resonance cell is given by

$$\Delta\nu = \Delta\nu_{\text{hom}} + \Delta\nu_{\text{power}} + \Delta\nu_{\text{geom.}}$$

whereby  $\Delta\nu_{\text{hom}} = 2$  is the homogeneous linewidth of the (10<sup>0</sup>0) - (00<sup>0</sup>1) transition. It is given by

$$\Delta\nu_{\text{hom}} = \Delta\nu_{\text{rad}} + \Delta\nu_{\text{coll}}$$

The line broadening due to radiative decay of the excited state  $\Delta\nu_{\text{rad}}$  is on the order of 500 Hz and thus negligible when compared to the collision broadening  $\Delta\nu_{\text{coll}}$ . The latter is <sup>20</sup>, for CO<sub>2</sub> at 220C,

$$\Delta\nu_{\text{coll}} = \frac{u}{\ell_{\text{fp}}} = 7.6 \text{ kHz/m Torr.}$$

If the average particle velocity  $u = 3.35 \times 10^4 \text{ cm/sec}$ , the effective mean free path is  $\ell_{\text{fp}} = 4.4 \text{ cm m Torr}$ . Gallagher<sup>16</sup> obtained a linewidth of about 800 kHz with around 100 m Torr of CO<sub>2</sub> in the resonance cell - which in this case was internal to the laser - in apparently excellent agreement with the above values.

Power broadening can be represented by the relation

$$\Delta\nu_{\text{power}} = \Delta\nu_{\text{hom}} \sqrt{1 + \frac{KP}{\Delta\nu_{\text{hom}}}}$$

where  $\Delta\nu_{\text{hom}}$  is, as shown above, essentially equal to  $\Delta\nu_{\text{coll}}$ . P is the incident laser power and K a parameter whose value depends on the cross sectional area of the beam and the power density distribution across that area. Gallagher's data<sup>16</sup>, obtained with the internal cell, are consistent with a value of K in the order of unity if P is in Watts and  $\Delta\nu$  in Megahertz. A value of K = 1 provides also a very

reasonable match to the data reported by Kelly et al<sup>15</sup>, and Freed<sup>17</sup> respectively (see Fig. 4 of text), for cells used external to the laser cavity.

Geometric broadening  $\Delta V_{\text{geom}}$  occurs when cancellation of the Doppler broadening is incomplete, i.e. when the incident and reflected beams in the resonance cell are not perfect plane waves, perfectly parallel to one another. Causes are a.) diffraction due to finite beam diameter  $a$ , b.) curved wave fronts due to imperfect optics and c.) tilted reflecting mirror.

$$\Delta V_{\text{geom}} = \Delta V_a + \Delta V_R + \Delta V_\varphi$$

The broadening due to finite beam diameter  $a$ , also called transit time broadening,

$$\Delta V_a = \frac{u}{a} ,$$

remains negligible when compared to collision broadening,  $\Delta V_{\text{coll}}$ , until the mean free path becomes comparable to or larger than the beam diameter; e.g., at a pressure of 4.4 m Torr, the mean free path is  $\ell_{\text{fp}} = 1$  cm; hence at 4.4 m Torr and with a beam diameter of 1 cm,  $\Delta V_{\text{coll}} = \Delta V_a = 33.4$  kHz.

If the optics, used for beam expansion for example, produces a curved wave surface with radius of curvature  $R$ , line broadening occurs in the amount of

$$\Delta V_R = \frac{\varphi}{2R} \frac{u}{\lambda}$$

where  $\lambda$  is the wavelength of the laser light. With  $u = 3.35 \times 10^4$  cm/sec,  $a = 1$  cm,  $\lambda = 10^3$ ;  $\Delta V_R = 16.8$  kHz when  $R = 10^3$  cm.

The broadening due to a mirror misalignment is<sup>14</sup>

$$\Delta V_\varphi = \varphi \frac{u}{\lambda}$$

This effect has been evaluated by Kelly et al<sup>15</sup> who finds  $\Delta V_\varphi = 35.1$  kHz/milliradian, in quite good agreement with the value of 31.6 kHz/milliradian predicted by the above relation.

## Appendix B

### Dependence of $SQ_\varphi$ on Cell Pressure and Laser Power

The fluorescence signal power is proportional to the incident power  $P$

and the number of molecules per  $\text{cm}^3$ ,  $n$ ; hence, because  $\Delta V_{\text{coll}}$  is proportional to  $n$ ,

$$S = AP \Delta V_{\text{coll}}$$

The linewidth, as shown in Appendix A is given approximately by

$$\begin{aligned} \Delta Y &= \Delta V_{\text{coll}} + \Delta V_{\text{power}} + \Delta V_{\text{geom}} \\ &= \Delta V_{\text{coll}} \left[ 1 + \sqrt{1 + \frac{KP}{\Delta V_{\text{coll}}}} + \frac{\Delta V_{\text{geom}}}{\Delta V_{\text{coll}}} \right] \end{aligned}$$

Therefore,

$$SQ_\ell = S \frac{V_0}{\Delta Y} = \frac{P}{1 + \sqrt{1 + \frac{KP}{\Delta V_{\text{coll}}}} + \frac{\Delta V_{\text{geom}}}{\Delta V_{\text{coll}}}}$$

The optimum operating power, reached when  $(K/\Delta V_{\text{coll}}) P = 1$ , increases with  $\Delta V_{\text{coll}}$ , i.e. the gas pressure in the cell. Once collision broadening dominates over the geometric broadening, the maximum usable gas pressure, with maximum  $SQ_\ell$  as the criterion, will be determined by self trapping of the fluorescence radiation, or by excessive increase in pressure shift <sup>24</sup>.

#### References

1. Vig, J.R., Wasshausen, H., Cook, C., Katz, M. and Hafner, E., Proc. 27th Annual Frequency Control Symposium, pp 98-112 (1973).  
Vig, J.R., Cook, C., Schwidtal, K., LeBus, J.W. and Hafner, E., Proc. 28th Annual Frequency Control Symposium, pp 96-108 (1974).  
Vig, J.R., Filler, R.L., LeBus, J.W., Proc. 29th Annual Frequency Control Symposium, pp 220-229, (1975). Vig, J.R. and Hafner, E., R&D Technical Report ECOM-4134 (1973). Hafner, E. and Blewer, R.S., Proc IEEE, 56 pp 366-368, Mar 1968.
2. Hafner, E., IEEE Trans. SU-21, pp 220-237, Oct 1974.
3. Lee, P.C.Y., Wang, Y.S. and Markenscoff, A., Proc. 27th Annual Frequency Control Symposium, pp 1-10, (1973).
4. Holland, R., IEEE Trans. SU-21, pp 171-178, July 1974. Proc. 1974 Ultrasonics Symposium IEEE Int #74 CH0896-1SU, pp 593-598.

- IEEE Trans SU-23, pp 72-75, Jan 1976.
5. EerNisse, E.P., Proc 29th Frequency Control Symposium, pp 1-4, (1975).
  6. Kusters, J.A., to be published.
  7. Hammond, D.L. and Benjaminsen, A., Hewlett Packard Journal, Vol 16, pp 1-7, March 1965.
  8. Greenhouse, H.M., McGill, R.L. and Clark, D.P., Proc 27th Annual Frequency Control Symposium, pp 199-214, (1973). Greenhouse, H.M., and McGill, R.L., R&D Technical Report ECOM-0199-F, July 1975 (Final Report on ECOM Contr. DAAB07-07-73-C-0199 with the Bendix Corp.).
  9. Buroker, G.E. and Frerking, M.E., Proc 27th Annual Frequency Control Symposium, pp 191-198 (1973). Mroch, A and Hykes, G, R&D Technical Report ECOM-0137-3, Sept 1975. (Report on ECOM Contr. DAAB07-73-C-0137 with the Collins Corp.)
  10. Hellwig, H.W., Proc IEEE Vol 63, pp 212-229, Feb 1975.
  11. Jeckart, E., Proc 27th Annual Frequency Control Symposium, pp 387-389, (1973).
  12. Barger, R.L., English, T.C. and West, J.B., Proc 29th Annual Frequency Control Symposium, pp 316-320, (1975).
  13. Freed, C. and Javan, A., Appl. Phys. Lett. 17, pp 53-56, July 1970.
  14. Letakhov, V.S., Fundamental and Applied Laser Physics. Proc of the Esfahan Symposium, Feld, M.S., Javan, A and Kurnit, N.A. edit. John Wiley 1973, pp 335-367.
  15. Kelly, M.J., Thomas, J.E., Monchalin, J.P., Kurnit, N.A. and Javan, A., Proc 29th Frequency Control Symposium, pp 338-343, (1975)
  16. Gallagher, J.J., Proc 29th Annual Frequency Control Symposium, pp 344-351 (1975), R&D Technical Report ECOM-0065-F, Dec 1974 (Final Report on ECOM Contr. DAAB07-73-C-0065). Abrams, R.L., Appl. Phys. Lett. 25, pp 304-306. Sept 1974
  17. Freed, C., Proc 29th Annual Frequency Control Symposium, pp 330-337, (1975).

18. IEEE Special Issue on Infrared Technology for Remote Sensing.  
Proc. IEEE, Jan 75.
19. Pyee, M., Clairon, A. and Auvray, J., Electronics Letters, 10 pp  
500-501, Nov 1974
20. Abrams, R.L., Appl. Phys. Lett. 25, pp 609-611, Nov 1974.
21. Small, J.G., Elchinger, G.M., Javan, A., Sanchez, A., Bachner, F.  
J., and Smythe, D.L., Appl. Phys. Lett. 24, pp 275 (1974).
22. Evenson, K.M. et al, Appl. Phys. Lett. 22, pp 192-195, 1973.  
Barger, R.L. and Hall, J.L., Appl. Phys. Lett. 22, pp 196-200,  
1973.
23. Sakuma, E. and Evenson, K.M., IEEE Jour. Quantum Electronics,  
Vol QE-10, pp 599-603, Aug 1974.
24. Bagaev, S.N., Baklanov, E.V. and Chebotaev, V.P., Soviet Phys.  
JETP Lett. 16, pp 243-246, Sept 1972.

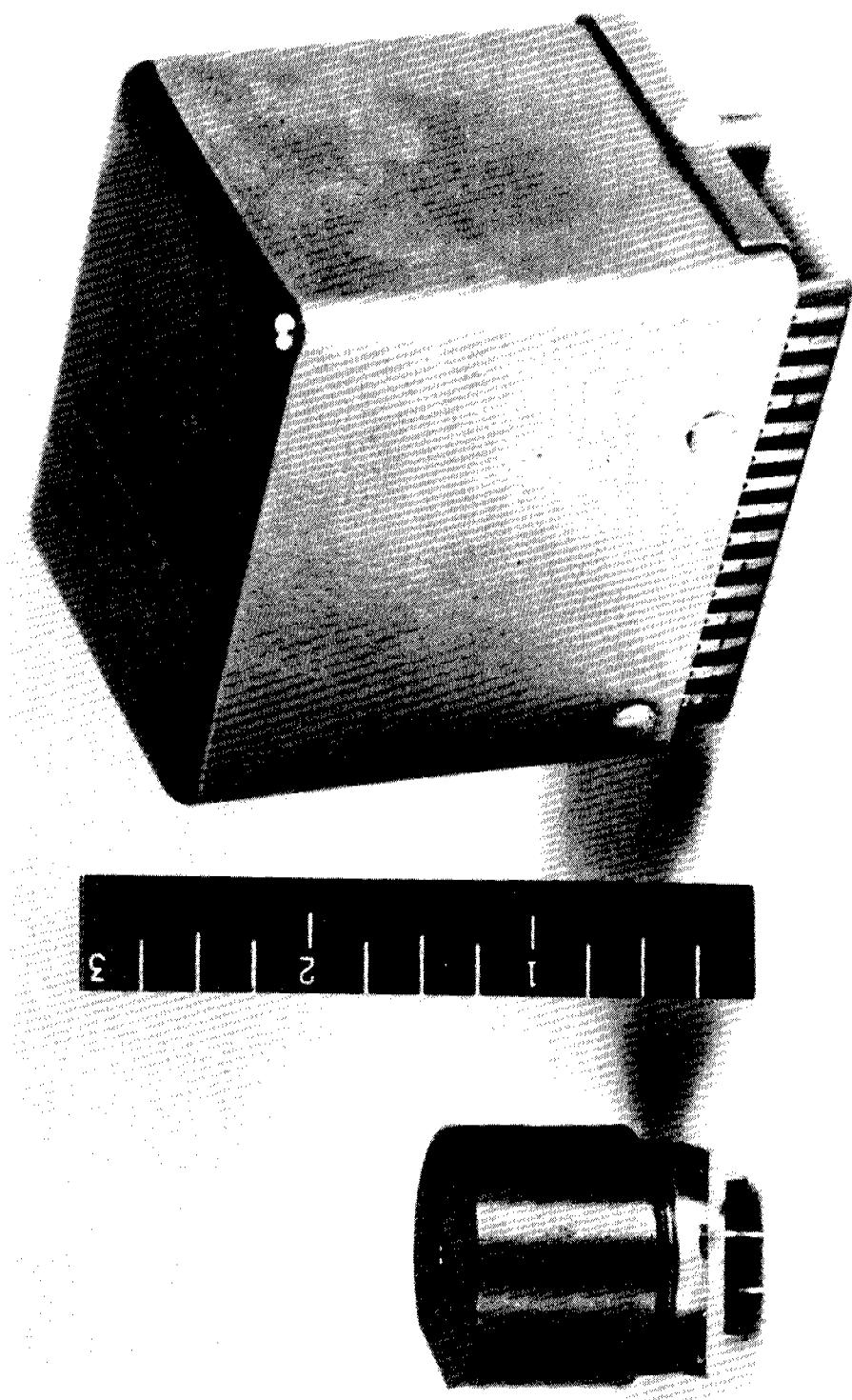


Fig. 1. Experimental oven controlled crystal oscillator (left) and representative state of the art commercial oscillator of comparable performance.



PERFORMANCE CHARACTERISTICS FOR  
OVEN CONTROLLED CRYSTAL OSCILLATORS

| CHARACTERISTICS   | STATE OF THE ART<br>PACKAGED OSCILLATOR<br>(DEVELOPMENT) | TACTICAL MINIATURE OSCILLATOR<br>(DEVELOPMENT) |
|---|--|--|
| SIZE  | 220CM <sup>3</sup>                                       | 16CM <sup>3</sup>                              |
| POWER CONSUMPTION   |  |  |
| @ 25°C  | 3.75 WATT  | 0.13 WATT                                      |
| @ -45°C   | 8.65 WATT  | 0.25 WATT                                      |
| STABILIZATION TIME<br>(TO REACH $1 \times 10^{-8}$ OF<br>FINAL FREQUENCY) |  |  |
| @ 25°C  | 15 MIN   | 2 3/4 MIN                                      |
| @ -45°C   | 28 MIN   | 3 1/4 MIN                                      |
| ACCELERATION COEFFICIENT  | $2 \times 10^{-9}/G$                                     | $10 \times 10^{-9}/G$                          |
|   | (OVERTONE)   | (FUNDAMENTAL)                                  |

Fig. 2



## MINIATURE OVEN CONTROLLED CRYSTAL OSCILLATOR

| CHARACTERISTICS  | 1976   | 1980's   |
|--|--|--|
| OUTPUT - FREQUENCY<br>VOLTAGE/LOAD                             | 5 MHz<br>0.125V/50Ω  | 5 MHz<br>0.125V/50Ω  |
| INPUT - VOLTAGE<br>POWER - OPERATING<br>POWER - WARMUP         | 12V ±5%<br>0.25W MAX<br>10W (<1 MIN)   | 12V ±5%<br>0.20W MAX<br>10W (<30 SEC)  |
| SIZE   | 16 cm³ (1 CU INCH)   | 16 cm³ (1 CU INCH)   |
| WEIGHT   | 75g  | 30g  |
| WARMUP   | 3 MIN TO 1 X 10⁻⁸  | 30 SEC TO 1 X 10⁻⁹   |
| STABILITY - AGING<br>SHORT TERM<br>TEMPERATURE<br>ACCELERATION | 1 X 10⁻⁸/MONTH<br>5 X 10⁻¹¹/1 SEC<br>1 X 10⁻⁸(-54°C TO + 75°C)<br>1 X 10⁻⁸/G | 1 X 10⁻¹⁰/MONTH<br>5 X 10⁻¹²/1 SEC<br>2 X 10⁻¹⁰(-54°C TO +85°C)<br>2 X 10⁻¹¹/G |
| VOLTAGE CONTROL - RANGE<br>VOLTAGE                             | ±5 X 10⁻⁸<br>0 - 10V   | ±5 X 10⁻⁸<br>0 - 10V   |

Fig. 3



### RESULTS OF CO<sub>2</sub> FLUORESCENCE CELL EXPERIMENTS

| FWHM<br>(kHz) | PRESSURE<br>(mT) | POWER<br>(W) | S/N | $\sigma(\tau=1)^{**}$<br>( $\times 10^{11}$ ) | REFERENCE |
|---------------|------------------|--------------|-----|---|-----------|
| 270           | 0.5              | 1            | 100 | 2.5   | (15)      |
| 167           | 0.5              | 0.11         | 40  | 3.8   | (15)      |
| 344           | 4.5              | 0.58         | 200 | 1.6   | (15)      |
| 900           | 34               | 1.75         | 900 | 1.0   | (17)      |
| 800           | 75               | 0.06*        |     |   |           |
| 1750          | 75               | 1.5*         |     |   |           |
| 2600          | 75               | 3.0*         |     |   |           |
| 3100          | 75               | 4.0*         |     |   |           |

\* EXTRAPOLATED VALUES

\*\* CALCULATED VALUES

Fig. 4

QUESTION AND ANSWER PERIOD

Paper Not Presented Orally, Therefore, There Were No  
Questions And Answers.