

## MILLISECOND PULSAR RIVALS BEST ATOMIC CLOCK STABILITY

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### ABSTRACT

The measurement time residuals between the millisecond pulsar PSR 1937+21 and the reference atomic time scale UTC(NBS) have been significantly reduced. Analysis of data for the most recent 768 day period indicates a fractional frequency stability, (modified Allan variance) of  $3 \times 10^{-15}$  for an integration time of 240 days. The improved stability relative to the earlier analysis is a result of three significant improvements. First, an upgraded data acquisition system was installed at Arecibo Observatory, substantially reducing several kinds of systematic measurement errors. Second, the Loran-C link to the Arecibo Observatory clock was replaced by a GPS common-view link, effectively removing the link noise from consideration. Third, the measurements were made in two widely separated frequency bands. And fourth, the reference atomic clock was improved. Using the information from these measurements allowed us to partially account for dispersion caused by free electrons along the 12,000 to 15,000 light year path from the pulsar to the earth. With data taken every two weeks, the final residuals are nominally characterized by a white phase noise at a level of 243 ns. The total interstellar electron content was found to follow a random walk by up to 12 ppm over the 768 days.

Timekeeping was originally based on astronomical observations (rotation of the earth). As atomic clocks were shown to be more accurate and stable than those based on astrometry, the SI second was redefined.[1] It now seems that an astronomical phenomenon [2] may rival the best atomic clocks currently operating. The current best estimate of the period of the millisecond pulsar (PSR 1937 +21) is 1.557 806 451 698 38 ms  $\pm 0.05$  fs as of 6 October 1983 at 2216 UT.[3] This accuracy is such that one could wait 250 years between measurements of the arrival times of signals from the pulsar before accumulating one radian of error for this 642 Hz clock. The period derivative has been measured as  $\dot{P} = (1.051053 \pm 0.000008) \times 10^{-19}$  seconds per second which is 3.31687 parts in  $10^{12}$  per year. This frequency drift is less than that of a typical rubidium frequency standard and greater than that of a typical cesium frequency standard. However, in the case of the pulsar, the drift rate is exceedingly constant i.e. no second derivative of the period has been observed.[4] The very steady slowing down of the pulsar is believed to be caused by electromagnetic and gravitational radiation.[5]

Time comparisons on the millisecond pulsar require the best of measurement systems and metrology techniques. The uncertainties in the measurement link between the millisecond pulsar, which is estimated to be about 12,000 to 15,000 light years away, and the atomic clock are the scintillation and dispersion due to the interstellar medium, the computation

of the ephemeris of the earth in barycentric coordinates, the relativistic corrections, the sensitivity of the Arecibo Observatory (area of 73,000 m<sup>2</sup> or 18 acres) radio telescope, the accuracy of the Princeton-installed measurement system, the filter bank and data processing techniques for determining the arrival times of the pulses at different frequencies, the transfer of time from the Arecibo Observatory atomic clock to the time from the international timing centers and lastly the algorithms to combine the world clock ensemble to provide an adequate atomic clock reference.

Since the discovery of the millisecond pulsar by Backer and Kulkarni [2] (14 November 1982), several very significant improvements in the ability to measure the pulsar have occurred. Figure 1 is a stability plot of the residuals over the first two years after all the then known perturbations were removed. The stability is nominally characterized by a 1/f phase modulation (PM) spectral density. The standard deviation of the time residuals over the first two years was 998 ns. In the fall of 1984 the group at Princeton installed a new data acquisition system in conjunction with the filter bank for better determination of the arrival times of the pulses. Nearly simultaneously NBS installed a GPS common-view receiver for the link between the Arecibo clock and international timing centers. The white PM noise of the GPS common-view link is less than 10 ns.

The data from the pulsar include measurements made at both 1.4 GHz and 2.38 GHz. The data were unequally spaced with the average sample period varying between about three days and two weeks depending upon the data segment. Figure 2 and Figure 3 are plots of the raw residuals over the period from the Fall of 1984 to the Fall of 1986. Since the data were unequally spaced we analyzed the data in two ways. First, taking the numbers as a simple time series they were analyzed as if they were equally spaced with the assumed spacing  $r_0$  equal to the average spacing between the data points. Second, we used the actual number of points available, but linearly interpolated a data value between adjacent actual values to construct an

equally spaced data set. The latter approach had the effect of somewhat decreasing the energy in the higher Fourier frequency components and increasing the energy in the lower Fourier frequency components. This filtering effect was taken into account in the conclusions drawn from the two different methods of analysis. Since it is the nature of white noise (random uncorrelated deviations) that a measurement is independent of the data spacing, and the modified Allan variance indicated white PM, then the assumption that the values are equally spaced is a necessary but not sufficient test for white noise. On the other hand, for an indication of low Fourier frequency components with more intensity than would be due to white noise,  $\text{Mod.}\sigma_y(\tau)$  is a necessary and sufficient test that the spectrum is something other than a white noise process. As will be shown later, this latter situation is applicable in this study.

In most of the following discussion the first method of analysis (unequal data spacing) was employed since, again, the second method leads to consistent conclusions, the only difference being that the noise levels are somewhat lower in the second case because of a partial filtering of the data at the higher Fourier frequencies.

Figure 4 shows the fractional frequency stability plot ( $\text{Mod.}\sigma_y(\tau)$ ) for the 1.4 GHz data against UTC(NBS). There are several significant differences between these data and that of the first two years taken using Loran-C. First, the spectral density is an  $f^{-\frac{1}{2}}$  PM process. Second, the noise level has been reduced significantly. A major part of this reduction is undoubtedly due to the new Princeton measurement system installed in the Fall of 1984. The standard deviation is now 317 ns over the 768 days of data and 62 data points.

Figure 5 shows the same frequency stability measurement, but at 2.38 GHz. The noise level is also nominally modeled by an  $f^{-\frac{1}{2}}$  PM process. The standard deviation is 316 ns over the 768 days of data and 59 data points. One will notice that the stability plots in Figures 4 and 5 are nearly identical.

A white noise process, if it is normally distributed, can be totally characterized by the standard deviation of that process. The standard deviation is given by the equation,

$$x_{\text{rms}}(\tau_0) = \frac{\tau_0^{3/2}}{\tau_0^{1/2}\sqrt{3}} \text{Mod.}\sigma_y(\tau), \quad (1)$$

for any  $\tau$ , for an average data spacing  $\tau_0$ , and for the white noise PM case. A power-law process  $f^\alpha$  with  $\alpha$  less than or equal to +1 will have a standard deviation of time residuals which is non-convergent. Hence, the standard deviation is not a good measure of these processes but only of a white noise PM process. If the standard deviation is used in the case  $\alpha \leq +1$  then the data length also needs to be stated as has been done above.

The following two equations are proposed as a model for the system:

$$X_1 = X_p - X_{D1} - X_A \quad (2)$$

$$X_2 = X_p - X_{D2} - X_A, \quad (3)$$

where  $X_1$  is the residual time series at 1.4 GHz,

and  $X_2$  is the residual time series at 2.38 GHz,

$X_p$  is the pulsar noise,

$X_A$  is the atomic clock noise,

$X_{D1}$  is the dispersion noise at 1.4 GHz,

and  $X_{D2}$  is the dispersion noise at 2.38 GHz.

$X_p$  and  $X_A$  are assumed to be the same in the two equations because of the high Q of the pulsar and the measured dispersion of the atomic clock over the two hour window during which the pulsar is measured at the two frequencies. Taking the difference between Equations 2 and 3 allows us to study the dispersion difference as a function of time as given by:

$$X_2 - X_1 = X_{D1} - X_{D2} \quad (4)$$

Figure 6 is a  $\sigma_y(r)$  plot of the difference given by Eq. 4 -- using the equally spaced data in this case to obtain better spectral estimates. Filtering the high frequencies at this point is useful as these components are probably not due to delay variations in the interstellar medium. The nominal  $r^{-1/2}$  behavior corresponds to a random walk of the dispersion delay (white noise frequency modulation, FM), which would accumulate to a level of 755 ns at an integration time of 100 days. The earlier assumption was that the dispersion delay was constant (independent of frequency). Figure 6 illustrates that such an assumption is not correct [3].

It is interesting to note that a differential delay change of 1/2 microsecond over the fifteen thousand light year path is 1 part in  $10^{18}$ , suggesting that we are dealing with a very good vacuum and that apparently the total electron content across the interstellar medium can be characterized as a random walk process for this particular path at integration times of the order of months.

Since the stability of UTC(NBS) can be determined independently of this measurement procedure, a very careful analysis of the data taken in 1985 was performed. The stability of UTC(NBS) was compared in an "N-cornered-hat" [6] procedure against other primary timing centers -- the National Research Council (NRC), Ottawa, Canada; Physikalisch-Technische Bundesanstalt (PTB), Braunschweig, West Germany; U.S. Naval Observatory (USNO) and Radio Research Laboratory (RRL), Tokyo, Japan. This was accomplished using the NBS/GPS common-view technique which supplies data to the BIH for the generation of

International Atomic Time (TAI). The measurement noise for all of the time comparisons was less than 10 ns for the white noise PM. Figure 7, which shows a plot of the estimated frequency stability for each of the timing center time scales, indicates that the stability of UTC(NBS) is 1 to 3 parts in  $10^{14}$ , and it appears that its instabilities could start to contribute as we go to larger integration times. Research and development work is being pursued to improve the stability of UTC(NBS) and of the reference atomic clock for this experiment.

Some improvement in the residuals may be expected if the differential dispersion delay from Eq. (4) is used as a calibrator. There is evidence of anti-correlation between the dispersions. A visual inspection of Figures 2 and 3 shows some anti-correlation. Under the assumption of anti-correlation one can combine Equations 1 and 2. The simple average, as given by Equation 6, causes the average dispersion to decrease.

$$\frac{\overline{X_1} + \overline{X_2}}{2} = X_p - X_A \quad (6)$$

The data were analyzed using this algorithm, and the resultant Mod. $\sigma_y(r)$  plot is shown in Figure 8. The measurement noise was reduced to 243 ns white PM with the 240 day value at  $3 \times 10^{-15}$ . Figure 9 is a plot of the resulting simple average -- showing the residuals over the 768 days. The  $3 \times 10^{-15}$  value should be taken with caution and verified with additional data as the reference atomic clock is not believed to be that stable.

There are some obvious steps for the near-term including work aimed at decreasing the measurement noise and improving the performance of the reference atomic clocks. The group at Princeton plans to reduce the measurement noise, and steps are already in progress at NBS to combine the best clocks in the world in an optimum weighted algorithm to create the world's "best clock" as a reference. Further studies on models for the interstellar dispersion and its effect on stability of measurements are needed. Another important objective is to find another pulsar in a region of space providing some orthogonality and with adequate stability. This would offer improved opportunity for detection of the background radiation of gravity waves. PSR 1855+09 with a period of  $5.362100452553$  ms  $\pm 69$  fs holds some promise. Progress on primary reference atomic clocks which might provide a better earth-bound reference is going well. Atomic clocks have improved an order of magnitude every seven years since their introduction in 1949 and we see no reason this trend will not continue in the foreseeable future.[8] A mercury ion standard with a transition in the optical region of the spectrum shows theoretical promise for long-term stability of 1 part in  $10^{18}$ , though it will probably be several decades before this potential accuracy is realized.[9]

#### ACKNOWLEDGEMENTS

The authors wish to thank several people who have assisted with data acquisition and analysis. In particular Dr. Judah Levine and Ms. Trudi Peppler at NBS have offered significant services.

#### REFERENCES

- [1] NBS Special Publication 330 The International System of Units (SI), U. S. Department of Commerce, National Bureau of Standards, 1981 Edition.
- [2] Backer, D. C., Kulkarni, S. R., Heiles, C., Davis, M. M. & Goss, W. M., Nature 300, pp. 615-618 (1982).
- [3] Rawley, L. A., "Timing Millisecond Pulsars," Thesis submitted to the faculty of Princeton University, October 1986
- [4] Blanford, R., Narayan, R., Romani, R. W., "Arrival Time Analysis for a Millisecond Pulsar," Theoretical Astrophysics, California Institute of Technology, Pasadena, CA (1984).
- [5] Davis, M. M., Taylor, J. H., Weisberg, J. M., Backer, D. C., "High-precision timing observations of the millisecond pulsar PSR1937+21," Nature 315, pp. 547-550 (1985).

- [6] Weiss, M. A. and Allan, D. W., "An NBS Calibration Procedure for Providing Time and Frequency at a Remote Site by Weighting and Smoothing of GPS Common View Data," Proceedings of the CPEM Conference, Gaithersburg, MD, June 1986.
- [7] Bertotti, B., Carr, B. J., and Rees, M. J., Mon. Not. R. Astr. Soc. 203, pp. 945-954 (1983).
- [8] Allan, D. W., "National and International Time and Frequency Comparisons," 37th Annual Frequency Control Symposium, pp. 55-60, (1983).
- [9] Wineland, D. J., "Trapped Ions, Laser Cooling, and Better Clocks," Science, Vol. 226, pp. 395-400, (26 October 1984).

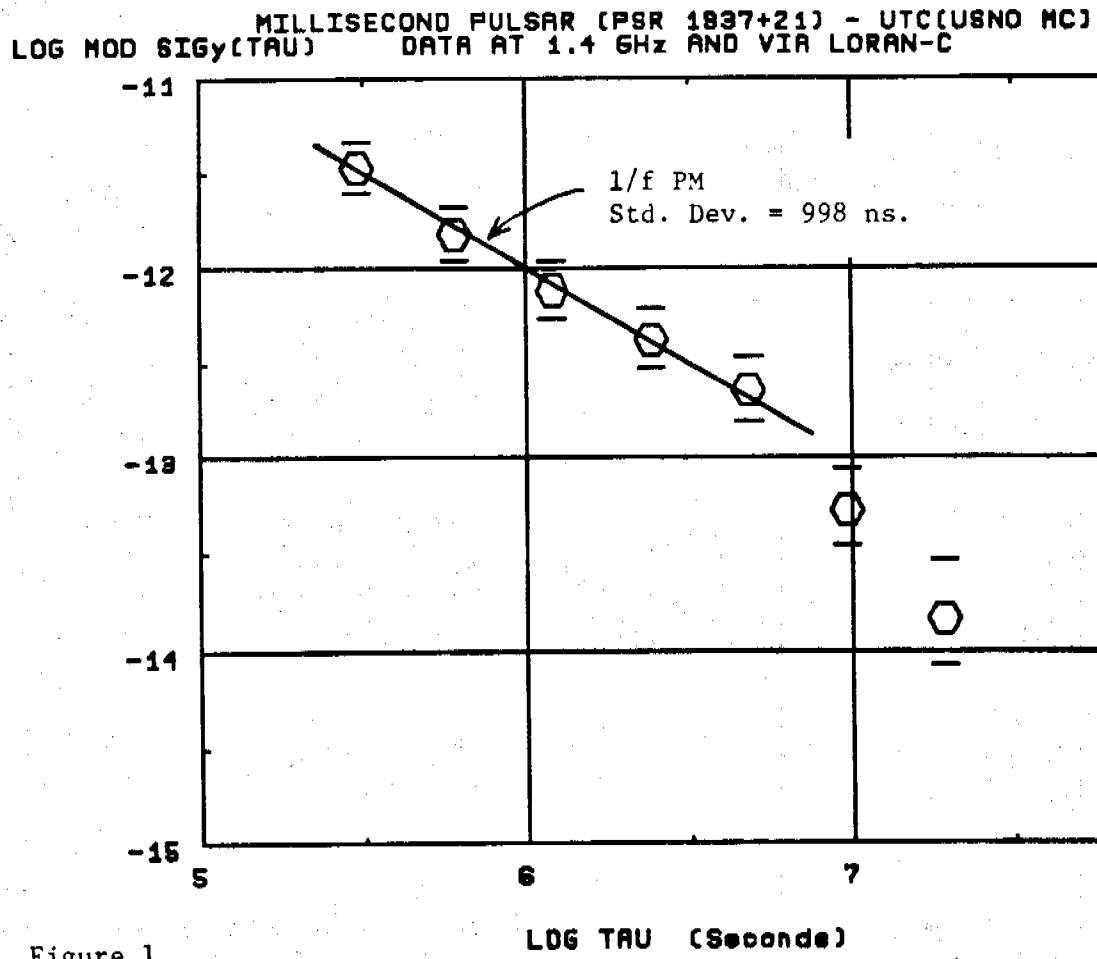


Figure 1

Figure 1. Fractional frequency stability,  $\text{Mod.}\sigma_y(\tau)$ , at 1.4 GHz for the first two years of measurements on the millisecond pulsar, PSR 1937+21. The 1/f phase modulation (PM) is believed to be caused in part by the Loran-C link to the Arecibo Observatory clock.

MILLISECOND PULSAR AT 1.4 GHz - UTC(NBS)  
TIME (ns) 21 NOV. '84 - 23 OCT. '86 RESIDUALS

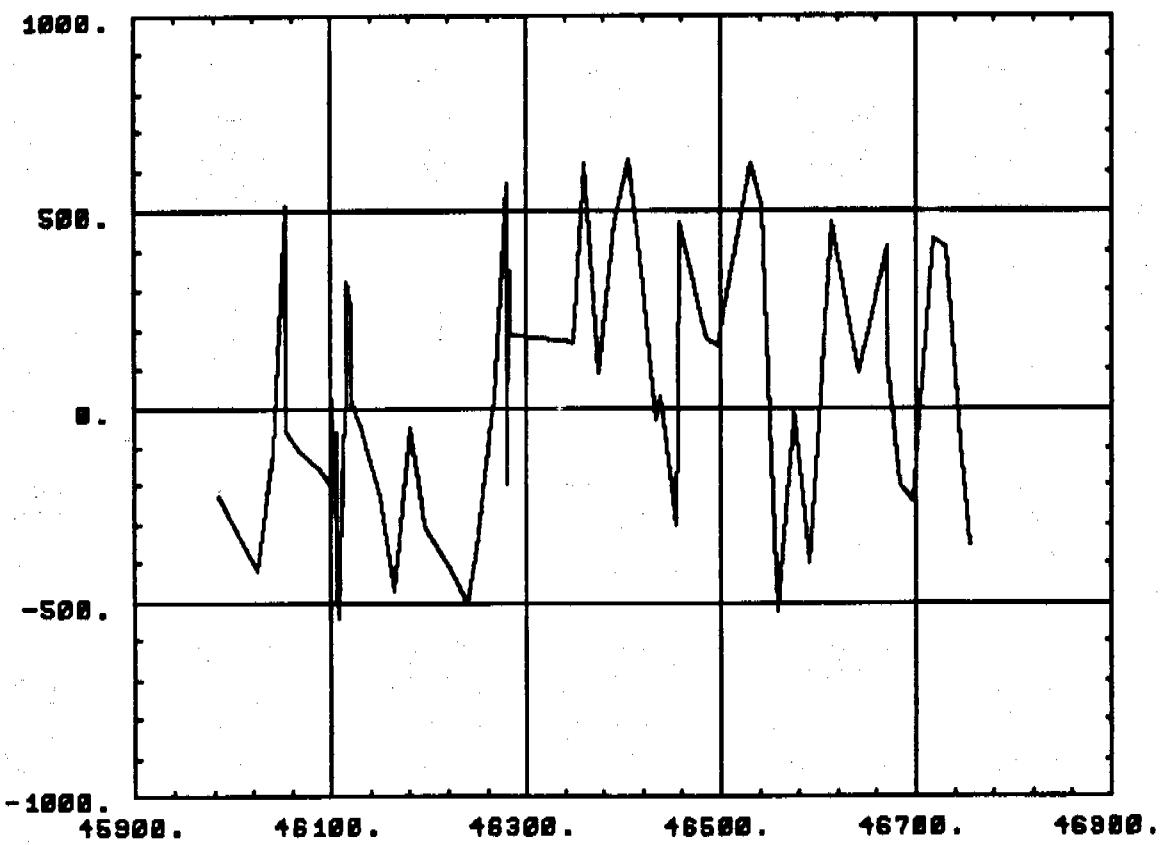


Figure 2

DAY [MJD]

Figure 2. Time residuals of PSR 1937+21 at 1.4 GHz after the installation of an improved Princeton measurement system at Arecibo Observatory and of the GPS common-view link. The standard deviation on these residuals is 317 ns.

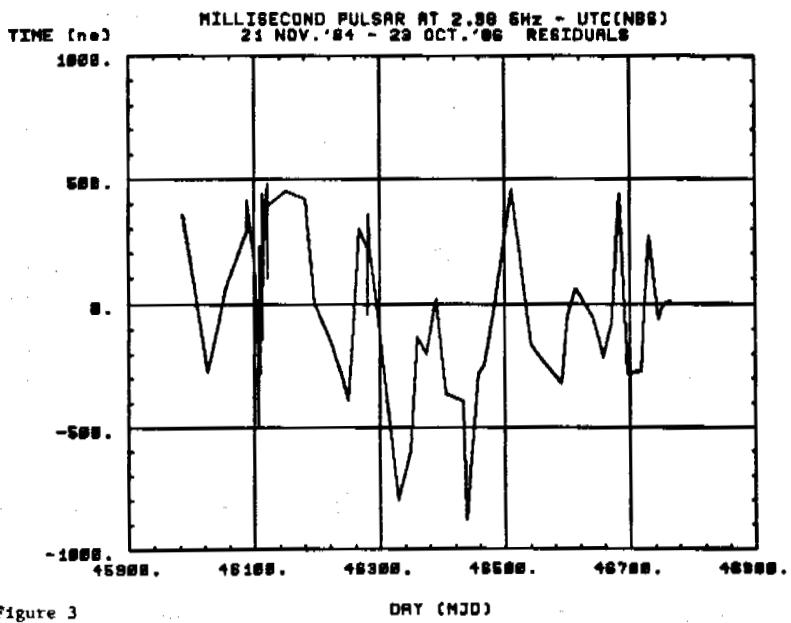


Figure 3

Figure 3. Fractional frequency stability,  $\text{Mod.}\sigma_y(\tau)$ , of the data plotted in Figure 2. The  $f^{-1/2}$  PM spectrum is half way between flicker PM and white PM. No significance is assigned.

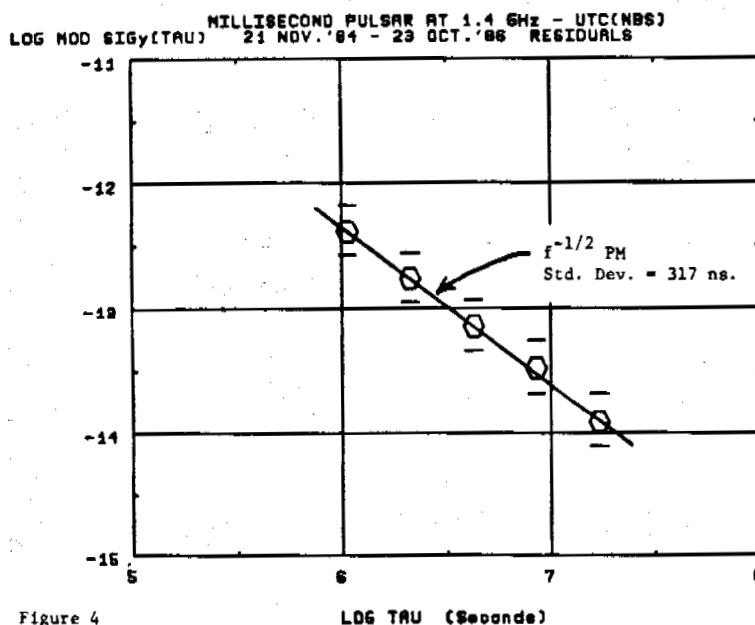


Figure 4

Figure 4. Time residuals of PSR 1937+21 at 2.38 GHz after the installation of an improved Princeton measurement system at Arecibo Observatory and of the GPS common-view link. The standard deviation on these residuals is 316 ns.

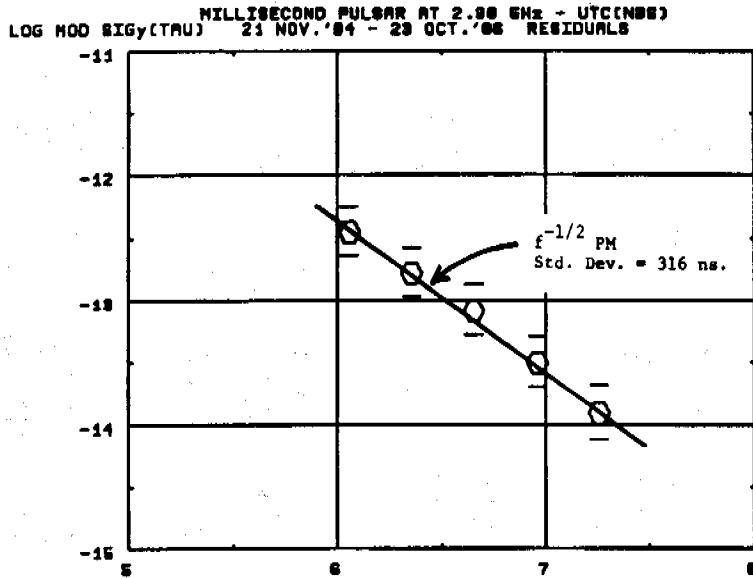


Figure 5

LOG TRU (Seconds)

Figure 5. Fractional frequency stability, Mod. $\sigma_y(\tau)$ , of the data plotted in Figure 4. The spectrum is nearly identical to that indicated in Figure 3 for the 1.4 GHz data.

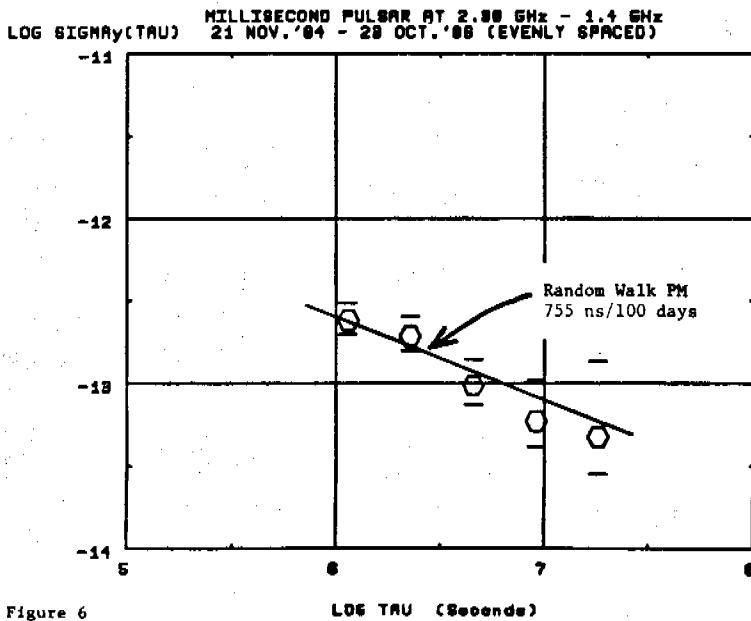


Figure 6

LOG TRU (Seconds)

Figure 6. Fractional frequency stability,  $\sigma_y(\tau)$ , of the difference between the data plotted in Figures 2 and 3 in order to asses the nature of the instabilities caused by the delay between the pulsar and the atomic clock.

### Allan Variance: MJD's 46135 - 46479

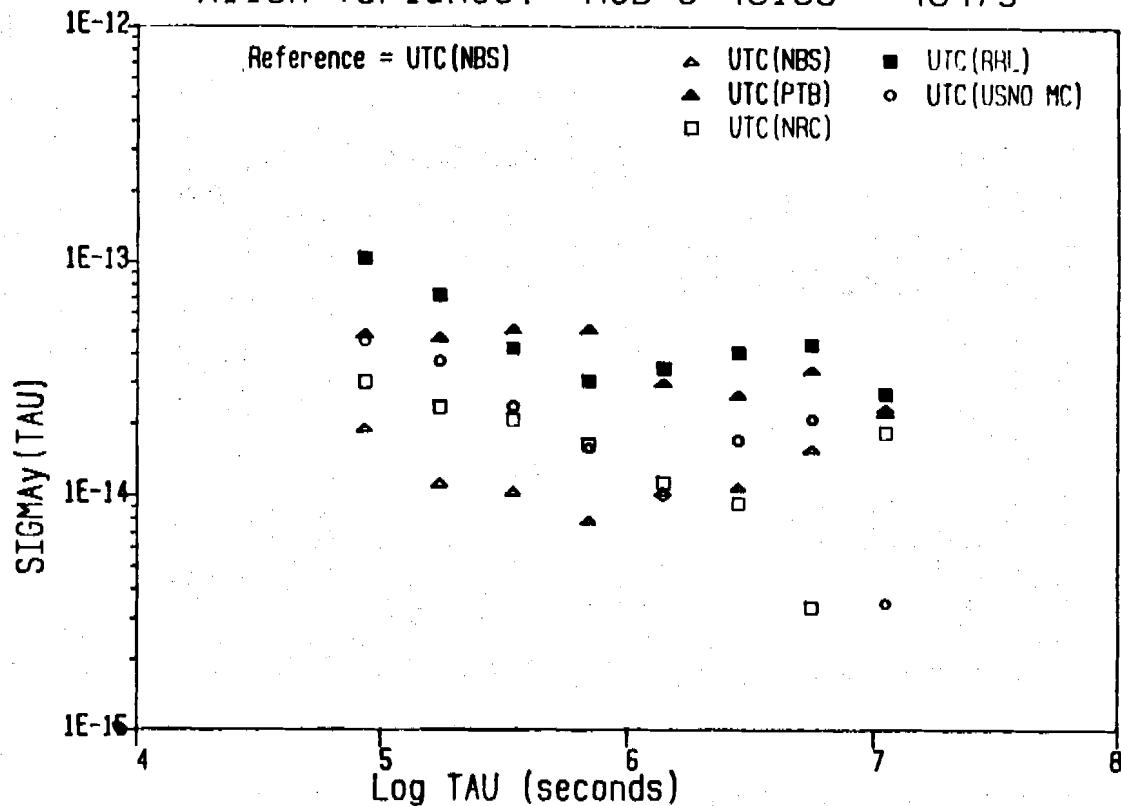


Figure 7. An "N-corner hat" estimate of the stability,  $\sigma_y(\tau)$ , of UTC(NBS) with respect to other international timing centers in order to assess the approximate limit of the reference atomic clock used in measuring the millisecond pulsar.

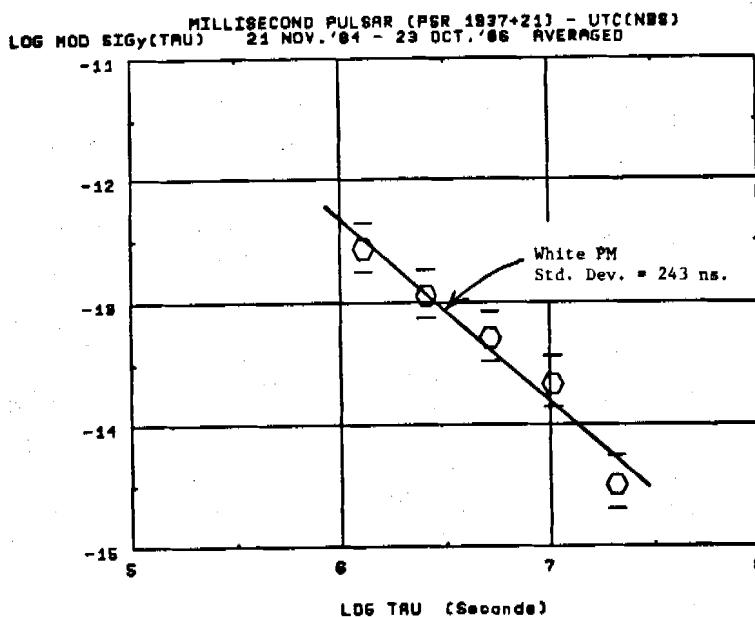


Figure 8. Fractional frequency stability, Mod. $\sigma_y(\tau)$ , of the simple average of the data plotted in Figures 2 and 3 in order to cancel some of the apparent negative correlation between the two signals.

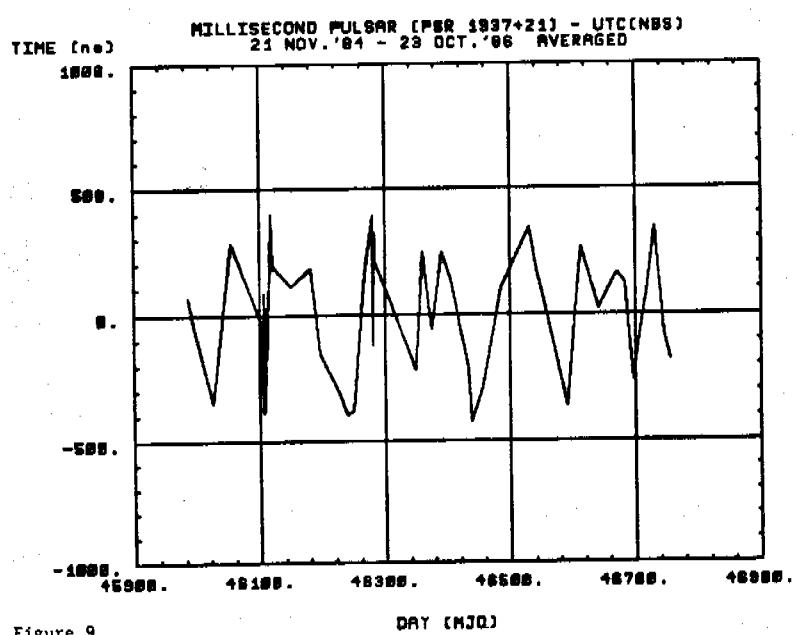


Figure 9

Figure 9. A plot of the simple average of the PSR 1937+21 residuals taken at 1.4 GHz and 2.38 GHz with the stability shown in Figure 8.

## QUESTIONS AND ANSWERS

DR. B. GUINOT, BUREAU INTERNATIONAL DE L'HEURE: The adjustment of the times of arrival of the signals from pulsars a knowledge of the earth's orbit and also many astronomical parameters. These parameters have different periods. There is one period for the motion of the earth and other periods for the perturbation from the planets and so on. How can you be sure that you don't absorb some of the long term noise of the clocks in the adjustment of the parameters of the model. That could explain why you have a stability much below ten to the minus fourteen, which is in my opinion better than the atomic clock that are performing the test.

MR. ALLAN: You are exactly right and I think that Dr. Alley has sinned. He doesn't know. You do the best that you can and observe those coefficients. I think that they are a significant part of the long term instability. You are absorbing the clocks and other phenomena. Below a part in ten to the fourteenth we can't say much. I fully agree.

FRED WALLS, NATIONAL BUREAU OF STANDARDS: But as you get more data, that will become apparent because if you absorb the clock instabilities at one month and other periods in your parameters, they are not going to correspond to the same parameters over longer and longer periods of time. You will start to see more dispersion. I think that the next year or two is going to be very informative.

MR. ALLAN: I think so, but we have a very important concern. That is that we worry about diurnal terms in cesiums and that is exactly what you remove from the data. We need some kind of device that doesn't have a diurnal variation. We are looking, as you know, at primary standards which can accuracies in the ten to the minus fifteen region and beyond in the long term. That is going to save us when we get more absolute references to look at.

GERNOT WINKLER, UNITED STATES NAVAL OBSERVATORY: Along the same line, you have to remember how many terms have to be modeled. There are several dozen different terms. Many of these periods are related. You not only have a daily period, you have two days and so on. You have a monthly period, a six months period, a yearly period. What it amounts to is simply subtracting a mathematical model of higher and higher terms, all of which have to be determined. You are fitting, and I can drive that game arbitrarily high. If my residuals are too high with twelve terms, I take twenty four terms. I think that it is less convincing to speak about residuals from a mathematical fit than it would be if these terms could be derived from a theoretical computation where we put in our best presently known parameters for earth orbit, relativity effects, perturbations and whatever. We cannot do that because our knowledge is not sufficient.

MR. ALLAN: That is a very good plan. As you know, JPL and Harvard are working on the earth ephemeris. Ron Hellings of JPL could give you a comparison to support what you say. These kinds of numbers indicate that you know the earth ephemeris to 5 meters. In fact, when you go through the theoretical models for the solar system, it is more like 10 microseconds, which is two miles. That is about the best that you can do for the theoretical model.

FRED PETERS, GENERAL ELECTRIC: I assume that you are trying to solve for the planetary ephemerides. That's nonsense.

MR. ALLAN: That in fact will be one of the main outputs of the study, the dynamics of the solar system.

MR. PETERS: How much data do you have? In terms of points.

MR. ALLAN: There is now about four years of data.

MR. PETERS: How many points does that consist of?

MR. ALLAN: They have taken them on the average of about every three days. It varies from time to time.

MR. PETERS: Are you sure that you want to do that?

MR. ALLAN: To study the dynamics of the solar system?

MR. PETERS: Yes!

MR. ALLAN: And look for gravity waves?

MR. PETERS: Not for a pulsar.

MR. ALLAN: If you like to do research, it is an exciting scene.

MR. PETERS: But does it really mean that much, though?

MR. ALLAN: In terms of a reference atomic clock...

MR. PETERS: My question is can you uncouple all these parameters when you are solving for so many? You are solving for so many of them with such a relatively small amount of data.

MR. ALLAN: It is going to take a lot more time before we can know those.

LAUREN RUEGER, JOHNS HOPKINS: Have you been taking data on any of the other pulsars, so that you could take a VLBI cut at a solutions.

MR. ALLAN: Dan Stinebring, as I mentioned at the outset, is working on measuring pulsars in general with a better pulse resolution technique. There is one, 1855-70 which has nominally one microsecond stability at times. It has very high scintillation as well. In terms of detecting gravity waves, you need some orthogonality between the pulsars. Looking for other pulsars and studying the that are available is a very important issue in this whole effort. This is just the one pulsar.

MR. RUEGER: This is very much like the problem of solving the earth's gravitational field. We had so many variables to select and reduce. The more effects that you can put into the mix, the more likely you are to over determine it.

MR. GUINOT: I wonder whether some other time scale could be used as a reference to make the adjustment. I think that it would be interesting to use others. Since the software is available, it should be possible to use other time scales like UTC-NBS and to see what is the effect on the adjusted parameters.

MR. ALLAN: That is our next step. In fact, we are already in the process of doing that.

HENRY FLIEGEL, AEROSPACE: I simply wanted to underline what Doctor Peters has already said. There was a time when the motion of the earth was so unknown that this would be our prime data source, from the pulsar. The progress in the last ten years has been, if you will excuse my word, dramatic. It is not unreasonable to get a good distance of the earth from the moon down to a nanosecond. You are only seeing a fraction of that in the data. I would be very surprised if you saw a monthly term due to an error in the ephemeris. It is not incredible to get the earth ephemeris down to five meters for certain periods of time, for example when the Viking ranging data was coming in. So I must underline the cautions that you must make. You have only one data type and you must be very careful in solving for these parameters.

BOB DOUGLAS, NATIONAL RESEARCH COUNCIL OF CANADA: What is the expected time between pulsar quakes for this pulsar?

MR. ALLAN: There are two phases, apparently, in which nuclear dense stars exist. In the last phase they tend to go into a binary pair. This is the most stable phase. We believe that, in the case of this one, it has eaten up its binary partner, if you will. They ones that are in the earlier phase are the ones that have the starquakes. This one is believed to be in the stable phase. We don't know when the next quake will occur, but it is believed, theoretically, to be a very stable star, with a lifetime of a million years.