Redundancy and correlations in TAI time links

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Abstract—**The network of time links for the computation of TAI has always be chosen such as to allow a unique solution, i.e. not using any redundancy. In the present situation, many links can be computed with two or more techniques (mostly TW and GPS) and the TW network itself is highly redundant. In addition, the covariance matrix for the measurements from these two may now be determined with adequate uncertainty, so that it makes sense to use all the available information and to compute TAI links using the available redundant network. In the paper, we examine the formalism to be used in this aim and the consequences of using such a procedure**

I.   INTRODUCTION

The network of TAI time links has always been non-redundant and the measurements used are considered independent. This choice was adapted to the situation that prevailed until the end of the 1990s when a single time transfer technique was used, GPS with Common-view computation, in which correlations are also difficult to estimate. Now two main techniques are used for TAI time links (GPS and Two Way time transfer, hereafter TW), several types of measurements exist for each technique (e.g. single frequency or dual frequency code, or dual frequency code and phase for GPS), and, at least in the case of the TW network that encompasses Europe and North America, a nearly complete set of redundant measurements is available. In addition, other systems are already available, or may soon become available, such as GLONASS, GPS augmentation systems and GALILEO. We have therefore to envision a better use of all the data available for the computation of TAI time links

In section II we present the formalism used to study and compare the results obtained using different set-ups regarding the redundancy and correlations of the time links. In section III, we present some tests carried out using either the whole TAI network or a sub-network. In section IV we show that the GPS all-in-view technique facilitates the application of the above formalism. Finally we discuss the results in section V.

II.   FORMALISM OF THE TIME SCALE SYSTEM

In this paper we adopt the classical formalism for time scale formation presented in [1], and we adopt its notations. We do not consider here the mechanism of determination of clock weight in which the weight w_i is assigned to clock i based on its estimated stability.

The computation of the ensemble time scale EAL(t) at time t from the reading of N clocks $h_i(t)$ is obtained by solving the system of equations

$$\Sigma_i(w_i\, x_i(t)) = \Sigma_i(w_i\, h'_i t) \qquad (1a)$$

$$x_i(t) - x_j(t) = x_{i,j}(t) \qquad (1b)$$

where $x_i(t) = EAL(t) - h_i(t)$. The first equation represents the weighting equation and the $h'_i(t)$ are a prediction of the $x_i(t)$, obtained from the solution at the previous instant of computation. The $x_{i,j}(t)$ are M measured values of differences between two clocks. In the classical case, M = N-1 and the system (1) is easily solved, e.g. by substitution. Here we consider the redundant case, where M ≥ N-1. In the following, we refer to the system (1) as the Time scale system, and to the sub-system (1b) as the Time link system.

A.   The Time scale system

The Time scale system (1) may be presented in matrix form:

$$A\, X = B \qquad (2)$$

where A is the (M+1,N) design matrix, X is the unknown vector (N,1) and B is the measurement vector (M+1,1) including the prediction value B_1 (the right hand side of (1a) and the M time link values $B_{k+1} = y_k(t)$. For simplicity, we adopt here the view where the clocks of a given laboratory are aggregated into one single fictitious clock, see e.g. [2], so that N is now the number of different laboratories to be linked, and the M time link values between laboratories. However the general case would easily be treated with the present formalism.

Following the classical treatment of the least squares estimation [3], the system is solved as

$$X = (A^T S_B^{-1} A)^{-1} A^T S_B^{-1} B, \qquad (3)$$

where S_B is the covariance matrix of the measurement vector B. The covariance matrix S_X of the vector of unknowns X is obtained as

$$S_X = (A^T S_B^{-1} A)^{-1}. \qquad (4)$$

Thus the redundant time scale system may be solved directly from (3). However we here choose to separate the problem into two stages: In the first stage, we solve the redundant set of M time links (1b), the time link system, to obtain the optimal solution in the form of (N-1) time links and their covariance matrix. In a second stage, we then solve the (non-redundant) time scale system so obtained. The goal of this paper is primarily to study the first stage, i.e. the redundant time link system with consideration of the possible correlations (full covariance matrix).

B. The redundant Time link system

Using M measured links (vector L), we want to determine (N-1) independent links (vector Y) and its covariance matrix S_Y. In the redundant case, $M \geq N-1$ and the time link system may be written as

$$C Y = L \qquad (5)$$

where C is the (M,N-1) design matrix relating the M measured links to the (N-1) independent ones. If the covariance matrix of L is noted S_L, the system is solved as

$$Y = (C^T S_L^{-1} C)^{-1} C^T S_L^{-1} L \qquad (6)$$

and the covariance matrix S_Y is

$$S_Y = (C^T S_L^{-1} C)^{-1}. \qquad (7)$$

A similar presentation of the Time link system in matrix-form has also been used in [4], and it is indeed necessary when considering the general case of a redundant and correlated system. Note that the system (5) may also be augmented to include other parameters and possible a priori knowledge on the links. It still may be solved with the general scheme (6,7). This will be discussed further in section V.

C. The procedure for using redundancy and correlation in solving a Time scale system

As a summary, we recall here the different steps used to solve the Time scale system:

1. We form the redundant Time link system (5) using the vector of links L and its estimated covariance matrix S_L.

2. We solve the system and obtain the vector of non-redundant links Y (6) and its covariance matrix S_Y (7).

3. We form the Time scale system (2) where Y is used to form B and where S_Y is used to form S_B, the covariance matrix of B.

4. We solve the (non-redundant) Time scale system and obtain the vector of unknowns X (3) and its covariance matrix S_X (4).

The first two steps are applied in the next section in different test configurations. In some of the cases presented below, the last two steps have also been performed.

III. TESTS ON TIME LINK SYSTEMS

We present some examples in which we use to formalism described above to estimate

The examples are taken from existing data sets that are used, or could be used, in the TAI computation.

A. The Europe-America Two-Way network

In the Europe-America region, eight laboratories participating to TAI (IEN, NIST, NPL, OP, PTB, ROA, USNO, VSL) regularly report Two-way time transfer measurements, hereafter referred to as TW. In addition, 7 of them (VSL excluded) provide dual-frequency GPS data, referred to as GPS-P3.

For TAI computation, the 7 independent TW links between PTB and the other 7 laboratories are generally used [5]. However the set of available links consists of 27 measured TW links (out of 28 possible between 8 laboratories) and 6 GPS-P3 links computed between PTB and the other 6 laboratories, using the Common-view technique. It is therefore possible to compute this set of redundant time links using the formalism above. Three cases have been considered:

1. TWstd: The standard, non-redundant, TW link system with 7 links, as used for TAI computation.

2. TWred: The redundant TW link system using 27 TW links.

3. TWred+P3: The redundant TW + GPS-P3 link system with 27 TW links and 6 P3 links.

See Figure 1 for the geometric configuration of the available links.

In these three cases, we have solved the Time link system following the procedure in section II, obtaining the covariance matrix S_Y. The non-redundant links are chosen to be all links to PTB, and the standard uncertainties in [UTC(PTB)-UTC(k)] are obtained as the square root of diagonal elements of S_Y and are presented in Table I

TABLE I.   RESULTS OF THREE TEST COMPUTATIONS FOR THE EUROPE-AMERICA TW NETWORK

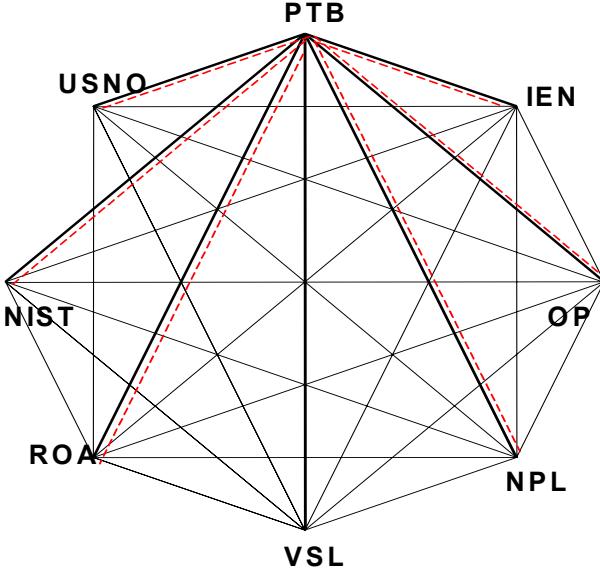| Scenario	Uncertainty (ns) in [UTC(PTB)-UTC(lab)] for Lab						
	USNO	*NIST*	*NPL*	*OP*	*IEN*	*ROA*	VSL
TWstd	0.50	0.50	0.50	0.50	0.50	0.70	0.50
TWred	0.27	0.27	0.26	0.26	0.26	0.31	0.26
TWred+P3	0.24	0.25	0.23	0.23	0.23	0.28	0.24

Figure 1. The TW test network: TWstd (thick solid lines), TWred (all solid lines), Twred+P3 (all solid+dashed lines)

B. The complete TAI network

The TAI network of time links is non-redundant by design, thus one explicit choice must be made when several techniques (usually TW and GPS) are available. The desire to choose the best technique available in each case, together with the geometric limitations imposed by the choice of the Common-view computation technique for GPS, results in a somewhat complicated geometry (see Figure 2). In this, a few laboratories act as pivots linking two techniques or two geographic regions, or both. These pivots are USNO, NIST, PTB, NICT in the configuration shown.

Four time link systems have been considered:

1. TAI: The standard TAI network (8 TW and 46 GPS links).

2. TAI_C: The standard TAI network including a priori correlations inside each technique: All links of the same technique which have one common station are affected a covariance of 0.04 ns^2 for TW, 0.16 ns^2 for GPS-P3 and 1 ns^2 for other GPS links.

3. TAI_R: The redundant network (28 TW and 52 GPS links), without a priori correlations.

4. TAI_RC: The redundant network including a priori correlations.

In these four cases, we have solved the Time scale system following the procedure in section II, obtaining the covariance matrix S_X. The standard uncertainties in [EAL-UTC(k)] are obtained as the square root of diagonal elements of S_X and some are presented in Table II. They indicate the following trends: Introducing a priori correlations inside each technique tends to slightly increase the standard

uncertainty for pivots and to decrease it for non-pivots, although the precise effect for each pivot will depend on the existence and amont of correlations between the relevant links. Using redundancy tends to decrease the standard uncertainty of all laboratories.

TABLE II. RESULTS OF FOUR TEST COMPUTATIONS FOR THE TAI NETWORK: UNCERTAINTY (ns) IN [EAL-UTC(K)]

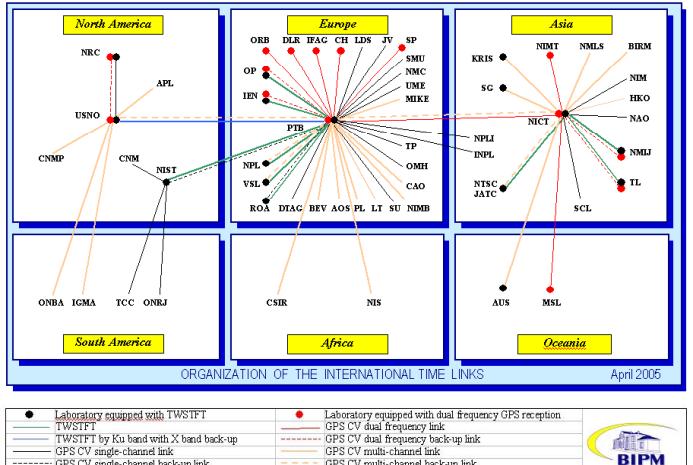| Scenario	Pivot Lab				Non-pivot Lab		
	USNO	PTB	NIST	NICT	IEN	CH	NIMT
TAI	0.50	0.47	0.66	1.21	0.68	0.83	1.57
TAI_C	0.51	0.49	0.64	1.21	0.66	0.79	1.49
TAI_R	0.43	0.43	0.46	1.20	0.46	0.81	1.56
TAI_RC	0.43	0.46	0.48	1.16	0.48	0.72	1.45

Figure 2. The network of TAI links in April 2005

IV. THE GPS ALL-IN-VIEW TECHNIQUE

In the All-in-view GPS time transfer [6], all GPS measurements available from one receiver are used to estimate the difference between the local laboratory reference and an external reference, to which the GPS satellite clocks are related. Assuming that this external reference is the time reference maintained by the International GNSS Service, noted IGST [7], each measurement of the vector L is a value of [UTC(k)-IGST].

When estimating the covariance matrix S_L we find that the sources of uncertainty associated with the computation of [UTC(k)-IGST] are essentially local to the laboratory k and its environment. Factors contributing to the co-variances between the measurements of two different laboratories are limited to errors in the satellite ephemerides and in the model of the link between each satellite clock and IGST. These errors are expected to be very small [8] and their effect on the measurement of [UTC(k)-IGST] has been shown to be smaller than 0.1 ns [9]. Therefore, the covariance matrix S_L is essentially diagonal, the value of the diagonal terms

depending on several factors, the most important being the type of receiver used, the amount of multipaths and the method of estimating the tropospheric delay. Directly introducing this measurement vector L and associated variance matrix S_L into a time link system necessitates to define IGST as a $(N+1)^{th}$ virtual laboratory. If this is deemed to be not desirable, it is always possible to transform the $(N,1)$ vector L of [UTC(k)-IGST] into the desired vector L' $(N-1,1)$ of [UTC(k)-UTC(l)] through a matrix Q $(N-1,N)$ such as

$$L' = Q\ L \qquad (8)$$

$$S_{L'} = Q\ S_L\ Q^T \qquad (9)$$

Therefore the all-in-view technique greatly facilitates the estimation of the covariance matrix of the time link measurements, thus the computation of results and uncertainties in a redundant time scale system. In addition it has been shown [6,9] that all-in-view improves the time transfer uncertainty with respect to common-view for long links (several thousands km) and allows direct global time transfer without incurring such limitations as imposed by the common-view technique.

V.   DISCUSSION OF RESULTS AND CONCLUSIONS

Using redundant measurements is a priori advantageous because it reduces the standard uncertainty of the results and it also provides more robustness in case when some of the data are missing. When the redundancy is obtained from one technique, by additional measurements taken on different links which are geometrically dependent, the formalism above can readily be used. However, when the redundancy is obtained by using several techniques to compute the same link, the question of possible biases between techniques needs to be addressed. One possible solution is to introduce in the time link system (5) additional parameters corresponding to the biases of each technique together with additional observations carrying information on these biases such as the results of calibration measurements. However this topic is too vast to be treated here and will be the subject of further studies. Note that we have, so far, considered techniques that directly provide time link values. Other techniques, e.g. GPS carrier phase, directly provide frequency links and achieve an uncertainty comparable to, or better than, the techniques already mentioned [10]. Such results could also be introduced in the time link system, e.g. after time integration of the frequency results, but the mechanism to do so should be further studied.

Using correlations, i.e. non-diagonal covariance matrix in the time link system, is also a priori advantageous when there are hints that the measurements are not independent, e.g. when the same device is used to perform two different measurements or when the same modeled parameter in used to process them. The correlated system essentially provides a better representation of the real physical system, thus more realistic results and standard uncertainties are obtained. Here also the topic is too vast to be treated here, however we have shown that using the all-in-view technique allows to better estimate the correlations in a GPS network than the common-view technique, in addition to improving time link uncertainties.

The TAI link network is now highly redundant, but this capacity is not used. We have tested a framework to introduce the available redundant measurements and to account for the existing correlations between measurements. Although some practical questions still need to be studied, we think that this path would provide improved results for TAI computation and should be pursued.

REFERENCES

[1] Thomas C., Wolf P., Tavella P., "Time scales," BIPM Monograph 94/1, 1994.

[2] Lewandowski W., Matsakis D., Panfilo G., Tavella P., "First evaluation and experimental results on the determination of uncertainties in [UTC - UTC-(k)]," Proc. 36th PTTI meeting, pp. 247-261, 2004.

[3] Biermann, G.J., Factorization methods for discrete sequential estimation, Academic Press, 1977, pp 13-32.

[4] Hahn J.H., Tavella P., "A time scale for satellite navigation systems: why and how?," Int. J. Satell. Commun. 18, pp. 305-324, 2000.

[5] BIPM, Circular T, monthly publication, section 6

[6] Jiang Z., Petit G., "Time transfer with GPS satellites all-in-view," Proc. ATF2004, 236, 2004.

[7] Senior, K., Koppang P., Ray, J., "Developing an IGS time scale," IEEE Trans. UFFC 50, 585-593, 2003.

[8] Ray, J., Senior, K., "Geodetic techniques for time and frequency comparisons using GPS phase and code measurements," Metrologia 42, 215-232, 2005.

[9] Weiss M.A., Petit G., Jiang Z, "A comparison of GPS common-view time transfer to all-in-view," Proc. FCS-PTTI, 2005.

[10] Bauch A. et al, "Comparison between frequency standards in Europe and the USA at the 10^{-15} uncertainty level," Metrologia, in press.