

OBSERVABLE RELATIVISTIC FREQUENCY STEPS INDUCED BY GPS ORBIT CHANGES

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Abstract

A study by ITT Industries of GPS Block IIR clock performance history during 2000 revealed a relativistic association between the observed frequency steps and changes in orbit. This paper presents the GPS IIR clock's observed performance during minor orbital perturbations that were made to enhance navigation coverage; analysis of the results; and the conclusion drawn on a relativistic association between clock performance and orbital changes.

Frequency steps accompanied orbit changes. Although the frequency steps were detected by GPS clock monitoring, they were not associated with the orbit changes by the observers. The association was not obvious because the relativistic frequency shifts were obscured by nearby clock system adjustments, which essentially zeroed the phase, frequency, and drift error of the SVN43 clock. Our preliminary analysis of clock data from SVN43 from 18 June 2000 through 31 December 2000 indicated that the Time Keeping System's observed performance differed from what we had predicted for GPS IIR.

ITT initiated a relativity study of the observed frequency step and the change in orbit. We examined the orbits before and after the orbit change. The difference in average velocity between the two orbits was 1.3 meters per second. Then we predicted the frequency step using the orbit change and relativity calculations. We found that the application of traditional relativity formulas predicted the observed frequency shift with an error of less than 4.5%. ITT confirmed the relativistic effect by studying the orbit change of GPS IIR satellite SVN54 on 9 March 2001. In this case as well, there was a close match between the observed frequency shift and the relativity prediction.

We conclude that GPS orbit shifts can generate significant clock frequency shifts due to relativistic phenomena. Knowledge of this effect can be used to produce better tracking of the satellites and their clocks in the vicinity of orbit changes.

INTRODUCTION

This section will cover the history that led to our detection of a relativistic phenomenon. There were a number of coincidences that led to the discovery and none of the parties involved started with the idea of looking for a relativity effect. In particular, as described below, we started our study of the behavior of the clock frequency over considerations of a software oscillator without any thought of looking at orbit changes and relativity.

The first coincidence is that the study involved Space Vehicle Number 43 (SVN43). SVN43 is the oldest Block IIR satellite and Clock 1 is the oldest IIR clock with the most in-orbit hours and over twice the age of the second oldest IIR clock. SVN43 was launched 23 July 1997 and Clock 1 was turned on 13 August 1997. This satellite is a high performance vehicle with a low URE (User Range Error) that is usually less than 1 meter and often close to 0.5 meter. It has one of the best, if not the best, navigation accuracy in the GPS constellation. Also, the clock drift had decayed to a very small frequency drift rate (about $2.7 \cdot 10^{-14}$ /day) with minimal changes in the drift rate. Thus, the clock we were studying just happened to be the ideal candidate to detect small clock effects.

The second coincidence is that GPS Ground Control decided to perturb the orbit of SVN43 first in one direction and then in the opposite direction so that the satellite returned to the same orbit except for a change in orbit phase. The reason for the repositioning is simple. There are 28 satellites currently in the constellation and average life is on the order of 10 to 12 years. This means that, on the average, two to three satellites are retired each year. When a satellite retires, this leaves a hole in the constellation. This hole can be left there or can be filled either by launching a new satellite to fill the empty position or by repositioning another satellite in the plane of the retired satellite to fill the position. Thus, the ground control directives allow that occasionally GPS space vehicles are repositioned to new orbit locations to enhance navigation signal ground coverage and ensure constellation robustness. These relocations are accompanied by minor orbit perturbations without compromising orbit ephemeris requirements. Note that while we mathematically consider any variation in orbit as an orbit change, in practice, orbit changes usually refer to a significant orbit change. In this case, the velocity between the two orbits differed by 1.3 meters per second as compared to an average velocity of about 3,875 meters per second or about 3.4 parts in 10,000. This would clearly be in the category of a minor orbit perturbation.

Initially, SVN43 was positioned in slot F5 (Plane F Position 5). When the satellite in F3 was retired, ground control decided that the best action, using their guidelines, was to position SVN43 from slot F5 to slot F3. In particular, a burn, of the satellite maneuvering rockets, was scheduled for 25 July 2000 to make SVN43 move from the F5 orbit to a transition orbit. The transition orbit was such that, over a period of several months, SVN43 would move from slot F5 to slot F3. On 10 October 2000, when SVN43 had reached the F3 position, there would be a second burn to inject SVN43 into the F3 orbit from the transition orbit. SVN43 would be available for service during all this time except for some hours in the vicinity of the burns.

There was another coincidence. Ground control had a number of actions pending against SVN43, but these actions were deferred because these actions required that SVN43 be out of service and they were unwilling to have a service outage just for these activities. However, when they saw that there was a scheduled outage for SVN43 on 10 October 2000 for injection of SVN43 into slot F3, they decided to piggyback these other activities onto the 10 October 2000 outage. Thus, 25 July 2000 was a comparatively quiet day for SVN43 with only the scheduled burn. However, 10 October 2000 was a busy day. On this day, there was a burn, an upload of a new software operational build for the satellite, and the clock phase error, frequency error and drift were reset to zero.

Naval Research Laboratory (NRL) routinely monitors all the GPS clocks and produces reports on their observations. On 8 March 2001, NRL produced a report on the SVN43 clock anomalies [1]. There was a single frequency step anomaly for 25 July 2000, cause unknown. However, there were a number of anomalies in the vicinity of 10 October 2000, some of which roughly matched, in time of occurrence, the phase, frequency, and drift commands we had sent. There were other anomalies unrelated to our commands. Even the reported anomalies that matched our commands in time of occurrence had differences between their sizes and that of our commands. We, at ITT Industries, saw these reports and said that, using our analysis of the Time Keeping System (TKS) we had designed, the anomalies associated with our commands should have the calculated values of position and size of our commands. Our argument was that we were dealing with a “software oscillator” and a software oscillator is driven by a computer with errors far below the noise in the measurements. NRL said, that traditionally, they had found that measurements were more valid than predictions and that they saw no reason to change their findings. As a result, we decided to study the frequency curves for SVN43 in that period and show that, using our calculated command values, we could explain the anomalies of 10 October 2000. We began this study as a study of the clock and had no intention of examining the orbit of SVN43 over this period.

As is shown in the following sections, the frequency study led to the detection of a depression in the frequency curve, which we eventually associated with the orbit change of SVN43 and relativity. The section below contains a short diversion, which will explain to the reader what we mean by the term software oscillator.

SOFTWARE OSCILLATOR

A software oscillator is an oscillator, in which major parameters are controlled by software calculation. An example of such an oscillator is the Block IIR TKS, which is illustrated in Figure 1. This oscillator generates a 10.23-MHz signal and is composed of a combination of hardware and software. The Rubidium Atomic Frequency Standard (RAFS), the Voltage-Controlled Crystal Oscillator (VCXO), and the phase meter are all hardware. There is a software component, which has a processor that accepts commands from ground control and uses the phase meter data to control the VCXO. The output of the VCXO is controlled to follow the sum of the RAFS output, suitably scaled, and the ground commands. The phase meter has negligible effect because it is very stable. The VCXO has little long-term effect because the loop forces the VCXO to follow the desired output curve. The VCXO can cause a certain amount of short-term phase noise, but little long-term frequency or frequency drift error. The RAFS is usually so stable that the software commands for output phase, frequency, and frequency drift dominate.

We have shown that the oscillator in Figure 1 represents a software oscillator where the output phase, frequency, and drift are basically determined by the software calculations. Since the calculations in the TKS have more accuracy than our capability to measure the transmitted signal, our claim was that the anomalies associated with the ground commands are best determined by the calculated values and not the measured values. Initially, the clock community generally preferred the measured values to the calculated values. Since the distribution of the study [2] on which this paper is based, there is a significant group that has accepted the concept that, for a software oscillator command, the calculated values are generally preferred over the measured values.

FREQUENCY ANALYSIS

Initial Data Processing

Two NIMA data sets for SVN43 were analyzed, covering the interval from 18 June through 31 December 2000. Each data entry contained the date and time (i.e., Julian Day) of a measurement along with the corresponding phase and frequency. The data were collected at a data rate of 96 measurements per day (i.e., one measurement every 15 minutes), for a total of 18,815 (non-overlapped) measurements.

Since the two data sets were partially overlapping, the overlapped data was removed in order to form a single continuous data set. After examining the data, it was observed that there were numerous glitches that had to be removed. There were also some jumps that seemed to be related to the motion of the spacecraft and these jumps were left in.

Data processing involved making frequency and drift corrections on 10 October 2000 (Julian Days 284.531 and 284.572), corresponding to the times of 12:44 and 13:44. These changes were implemented at the nearest NIMA data point. Since the interval between the corrections was 1 hour, a total of four data points were affected by the first frequency correction starting at 12:45, while measurements up to the end of the data (31 December 2000) were affected by the second frequency and drift correction.

The frequency and drift corrections performed in this analysis were made to undo or “unwind” the uploaded corrections made to SVN43 on 10 October. This was done so that frequency and drift behavior could be observed without the effect of the uploaded corrections. A series of plots at various stages of processing illustrate the frequency and phase behavior of the SVN43 data.

There are two types of glitches that were removed from the SVN43 frequency data. Both glitches can be seen in Figure 2. The first glitch appears as a number of single or double values of $32.768 \cdot 10^{-12}$, instead of 0.0 at a position where there are expected to be zero crossings. The values at these points were replaced with zero. These glitches are “instantaneous” and do not affect phase, frequency, or drift at any other times. The second type of glitch is a large frequency marker of $262.144 \cdot 10^{-12}$ that occurred on Julian Day 317.656 (i.e., 12 November at 15:30) and which lasted until the end of the day. These were also replaced by zeros. After removing these glitches, the resulting frequency data is shown in Figure 3.

Frequency and Drift Corrections

Two corrections were made to the NIMA data on 10 October 2000. The first was a frequency correction of $3.69 \cdot 10^{-12}$. This change was made on Julian Day 284.531 (i.e., at 12:45). The second change was a frequency correction and change in frequency drift (i.e., slope) which was made an hour later on Julian Day 284.573 (or at 13:45). The second frequency correction was $0.0072 \cdot 10^{-12}$. The total frequency change of both corrections is $3.6972 \cdot 10^{-12}$. These corrections undo the TKS corrections that were uploaded at those times. The frequency drift (i.e., slope) applied is $14.83 \cdot 10^{-14}$ per day or $1.5448 \cdot 10^{-15}$ per 15 minutes. The resulting frequency data are shown in Figure 4 and Figure 5 (i.e., a smaller time interval) following these corrections.

Frequency Slope (Drift) Removal

Following the frequency and drift corrections, the linear slope (i.e., drift) was removed from the data. A plot of the resultant “flat” frequency (Figure 6) reveals an initial transient on Julian Day 207 (i.e., 25

July). This corresponds to the start of a change in the orbital position of SVN43. There is a noticeable change in mean frequency value between start of the data on Julian Day 170 (i.e., 18 June) and Day 207 (25 July). There is an abrupt drop in mean frequency between Days 207 and 284. Following the maneuvers or corrections made on Day 284, the mean frequency appears to have returned to its original level from before Day 207.

The value that “restores” the frequency offset is 18.47 parts in 10^{14} . A plot of the demeaned and offset corrected frequency is shown in Figure 7. We were puzzled by this phenomenon. Up until this time we had not considered relativity as a cause for the observed drop in mean value and the reverse frequency step. When we examined the history of SVN43, we discovered that the dates of the frequency steps exactly coincided with the orbital maneuvers. This was a critical clue. We considered a number of possible causes for the frequency steps, such as a frequency step associated with the mechanical vibration accompanying the maneuver. Relativity seemed to us to be the most promising explanation, because the effect we saw was reversible, i.e., the downward step and the upward step had the same size. This is because the relativity effect would be reversed when we returned from the transition orbit to the standard GPS orbit. The other effects are generally not reversible and so they appeared to be unlikely. Thus, we started the relativity analysis described in the following section.

RELATIVITY EFFECTS ANALYSIS

SVN43 Orbit Calendar 6/18/00 – 12/30/00

During the period 18 June 2000 through 30 December 2000, the decision was made to change the orbital station of SVN43 from F5 to F3. To effect the change, a burn was performed on 25 July 2000 to go to a drift orbit. On 10 October 2000, another burn was performed to stabilize the SV in the normal orbit at station F3.

Also on 10 October 2000, a new operational build, OB8.0, was uploaded to SVN43. Later the same day, the TKS phase, frequency, and average frequency drift values were zeroed out.

The transition from normal orbit to drift orbit on 25 July 2000 was chosen as the study epoch for relativity effects, because the effects of the transition on 10 October 2000 were partially obscured by the other activities on that day. NIMA data (15-minute increments) were used. The orbital values prior to transition (22 July 2000) and after transition (31 July 2000) were calculated.

Prior to transition to drift orbit, the orbital parameters were:

SVN43 Orbital Parameter	Median	Amplitude
Radius (re: Earth's center) – also called major semiaxis	26561.396 km	± 72.337 km
Speed	3.873948 km/s	± 10.566 m/s
Eccentricity	0.0027234	
Period	43081.15 s 11:58:01 hh:mm:ss	

After transition to drift orbit, the orbital parameters became:

SVN43 Orbital Parameter	Median	Amplitude
Radius (re: Earth's center) – also called major semiaxis	26542.674 km	± 61.350 km
Speed	3.875239 km/s	± 8.967 m/s
Eccentricity	0.0023114	
Period	43035.61 s 11:57:16 hh:mm:ss	

Comparisons:

SVN43 Orbital Parameter	Before	After	Change
Radius (re: Earth's center)	26561.396 km	26542.674 km	-18.722 km
Speed	3.873948 km/s	3.875239 km/s	1.291 m/s
Eccentricity	0.0027234	0.0023114	-0.000412
Period	43081.15 s	43035.61 s	-45.54 s

As the comparison table of orbital parameters shows, SVN43 entered a drift orbit with an average altitude lower by 18.722 km, and an average speed higher by 1.291 m/s. The orbital period decreased by 45.54 s.

SVN43 Clock Shifts

Alternative Methods

Two methods of estimating the relativity effect on the clock were used. The simplest method was the calculation of the Schwarzschild metric [4]:

$$\frac{f_r}{f_t} = \sqrt{1 - \frac{3 \cdot G \cdot M}{c^2 \cdot r}}, \text{ where } G \cdot M = \mu = 398600.5 \text{ km}^3/\text{s}^2 \text{ and } c = 299.792458 \text{ km/s}$$

The product, GM , is that of the universal gravitational constant and the mass of the primary body, the earth, in this instance. The constant, c , is the speed of light, and the variable, r , is the radius from the primary body center to the point in question. The metric gives the ratio of the observed frequency on Earth (subscript r) to the transmitted frequency in the SV (subscript t).

A second method uses the clock relativity effects presented by Spilker [5].

$$\frac{f_r}{f_t} = 1 + \frac{\Phi_t - \Phi_r}{c^2} + \frac{v_r^2 - v_t^2}{2 \cdot c^2} + \{Doppler Terms\} + \{Higher Order Terms\}$$

The function Φ is the gravitational potential per unit mass and v^2 is twice the kinetic energy per unit mass. The constant c is the speed of light, and the variable r is the radius from the primary body center to the point in question. The formula gives the ratio of the observed frequency on Earth (subscript r) to the

transmitted frequency in the SV (subscript t). The Doppler Terms and Higher Order Terms were ignored for this study.

The relative shift of observed clock frequency from one orbit to the other is found from the difference between the metrics, i.e., $\left[\frac{f_r}{f_t} \right]_2 - \left[\frac{f_r}{f_t} \right]_1$, where the subscripts 1 and 2 indicate orbital conditions. We used condition 2 to indicate the drift orbit and condition 1 the normal orbit.

SVN43 Results

As shown above in the Frequency Analysis section, the relative clock shift during the transition from normal to drift orbit on 25 July 2000 was found to be $-18.47 \cdot 10^{-14}$. Applying the two approaches above, we found relativistic corrections of $-17.67 \cdot 10^{-14}$ and $-17.26 \cdot 10^{-14}$ for the Schwarzschild and Spilker methods, respectively. The magnitudes of the differences relative to $-18.47 \cdot 10^{-14}$ are 4.3% and 6.6%, respectively.

The minor difference between the Schwarzschild metric and the Spilker formula approaches is traceable to the use of the averaging technique used for the radius and velocity. The Schwarzschild metric requires only a radius value; we used the major semiaxis. The Spilker formula requires both velocity and radius. We used the major semiaxis and a simple average of the velocity extremes from the NIMA data.

A subsequent check using the Spilker formula with the Keplerian velocity at a radius equal to the major semiaxis yielded a result of $-17.63 \cdot 10^{-14}$ for the relative clock shift. The difference between this result and that from the Schwarzschild metric method is nearly insignificant.

SVN54 Results

NRL reported a frequency shift associated with a similar orbit transition of SVN54 [3] made on 9 March 2001. We calculated the expected frequency shift with the following results:

Method	SVN54 Fractional Clock Shift
NRL value	$25.0 \cdot 10^{-14}$
Schwarzschild value	$22.6 \cdot 10^{-14}$

The value we calculated differed in magnitude from the NRL value by 9.6%.

Relativity Confirmation

There are a number of reasons why we feel the observed phenomenon is due to relativity. This is not a coincidence or some peculiar effect associated with SVN43, because we have seen this behavior twice on SVN43 and once on SVN54. Our assumption is that the effect is present in every orbit change and we would see the effect in other cases as well, if we studied them in detail. The result is not some *ad hoc* explanation, because we used equations that had been present since 1979 and 1980 and the results matched within experimental error. We consulted Professor Neil Ashby (University of Colorado at Boulder), who is an acknowledged expert on relativity, and he agreed with our methodology. Thus, we are confident that the observed phenomenon is a relativity effect.

We have a plausible explanation why this relativity effect was not detected earlier, even though a number of other relativity effects were detected. The other relativity effects that were detected previously and used in GPS operation were either very large or were a common periodic phenomenon that was easily detected and had a major effect on GPS operation. On the other hand, orbital changes are infrequent events and the results are fairly small with little impact on overall operation. Probably, in the beginning of GPS operation, the received values had more noise and this phenomenon was undetectable.

CONCLUSIONS

There are a number of conclusions that we have reached as a result of the work described above.

1. **Software Oscillator.** We feel that the software oscillator concept is valid and that the anomaly analysis should generally use the calculated values in preference to the measured values. We feel that this conclusion has also been reached by others in the clock community.
2. **Relativity Effect.** The relativity effect as a result of an orbit change that we have described is valid. This effect occurs with every orbit change and there are a number of cases where the effect is significantly above the noise level.
3. **Well Known.** This effect is not a new discovery and the formulas for this effect were well known in 1979 and 1980. The general assumption, which was made over the years, was that the effect was so small as to be undetectable and not worthy of consideration as is the case for many terms in the relativity equations. Although this effect may not have been detectable in the GPS early years we have shown that, with the current sensitivity, the effect is detectable and can produce significant transients in the standard orbit and clock calculations.
4. **Prediction of Relativity Effect.** Our recommendation is that efforts be made to see if we can make reasonable predictions of this relativity effect and use these predictions to sharply reduce the tracking transients currently associated with orbital changes.
5. **Further Investigation.** Although the formulas we used are pretty good predictors of the relativity effects, there is a significant need for investigation beyond the work reported, in this paper. The following points out some items for further investigation. We used two formulas and there were differences in the result. We should determine what is the best available formula to use in this case. This formula may differ from the two formulas we have applied. Our work used certain methods to estimate the orbit parameters from the NIMA data and to estimate the observed frequency shift for a given event. Our frequency shift estimates are particularly weak in early clock life where there is substantial drift and changes in drift. Also, there may be a bias error in the prediction. We should see if the addition of other relativity terms would reduce or remove the bias. We should also try to improve our estimates of orbit parameters and frequency shift.

ACKNOWLEDGMENTS

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Schwarzchild reference and some early relativity calculations; N. Ashby, who confirmed our relativity effects methodology; and ITT management for their support and encouragement for this study.

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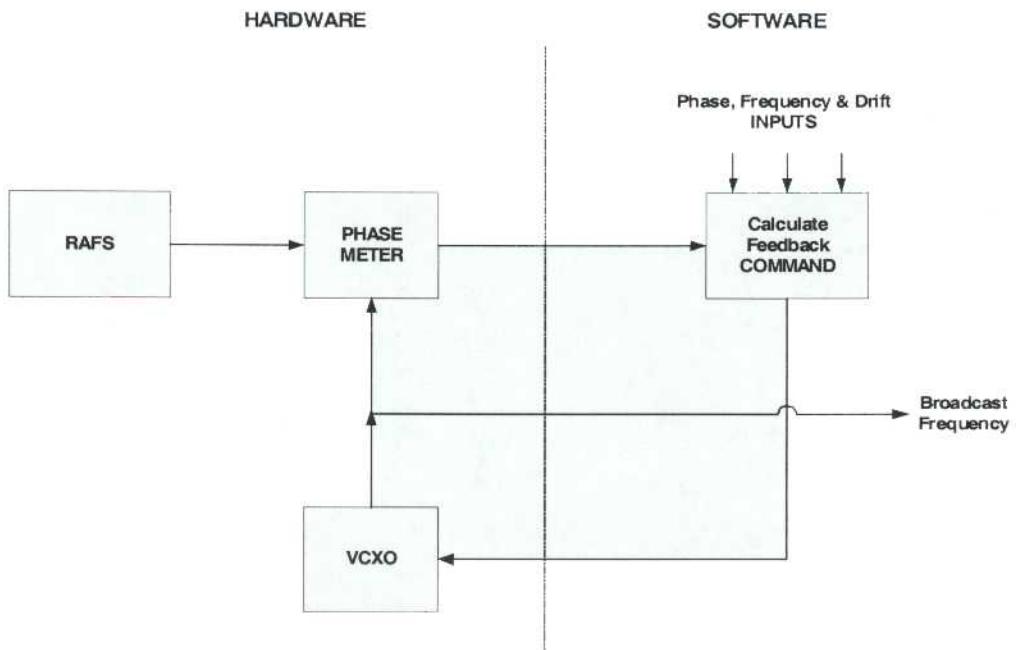


Figure 1. Block Diagram of Software Oscillator

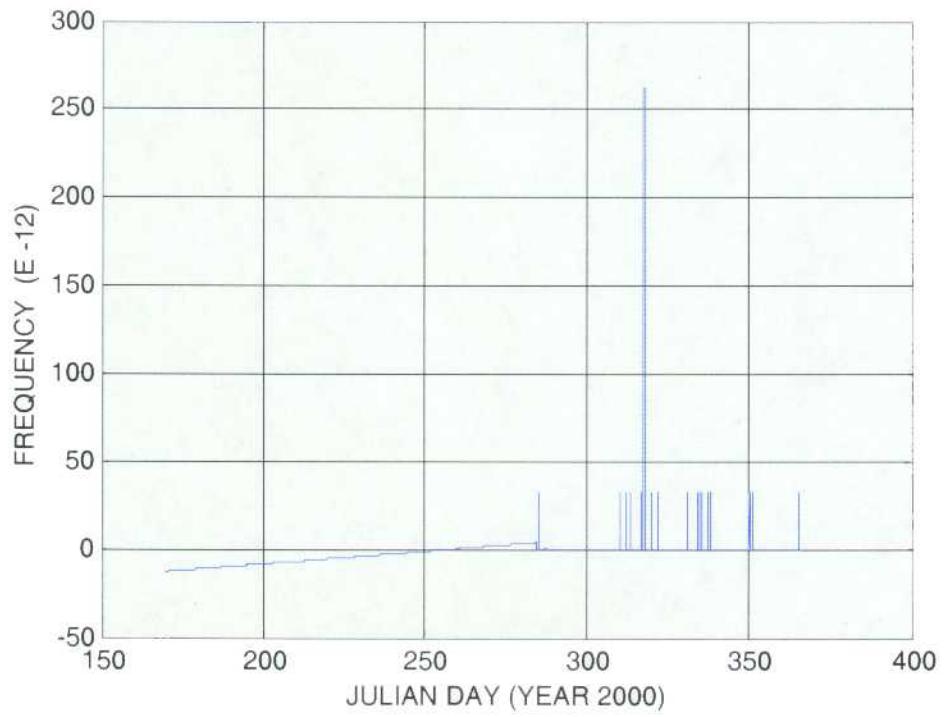


Figure 2. SVN43 Data (With Glitches)

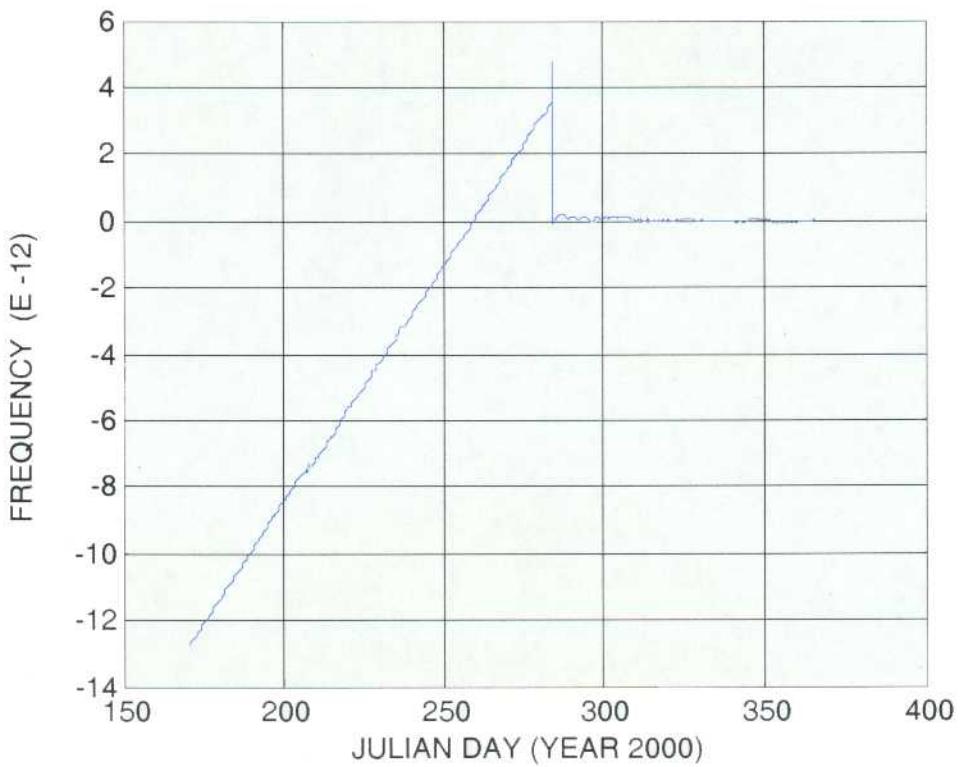


Figure 3. Deglitched SVN43 Data

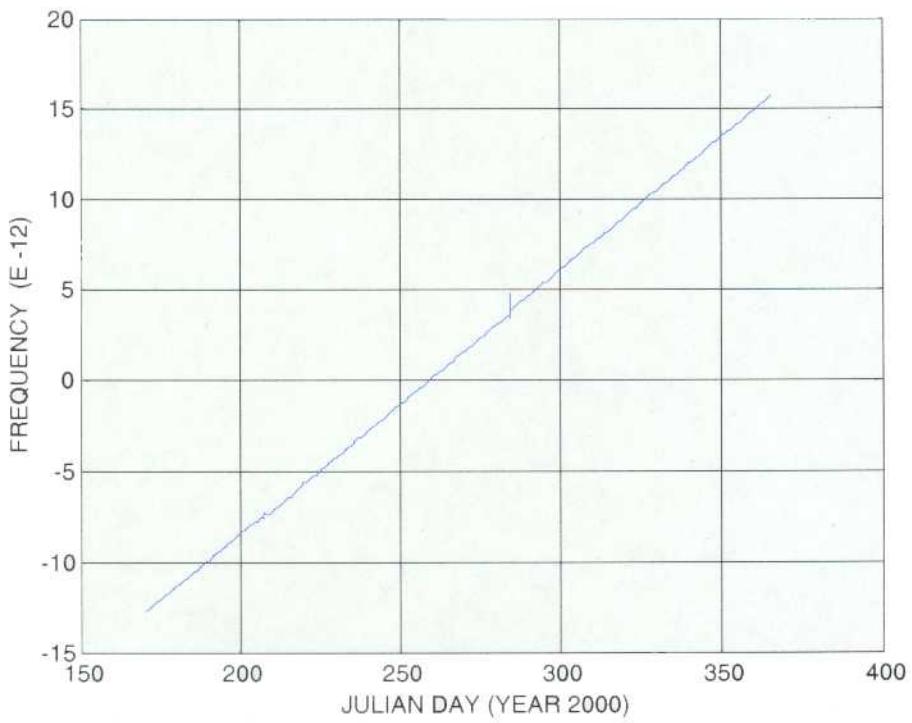


Figure 4. SVN43 Data (Uploaded Corrections Removed)

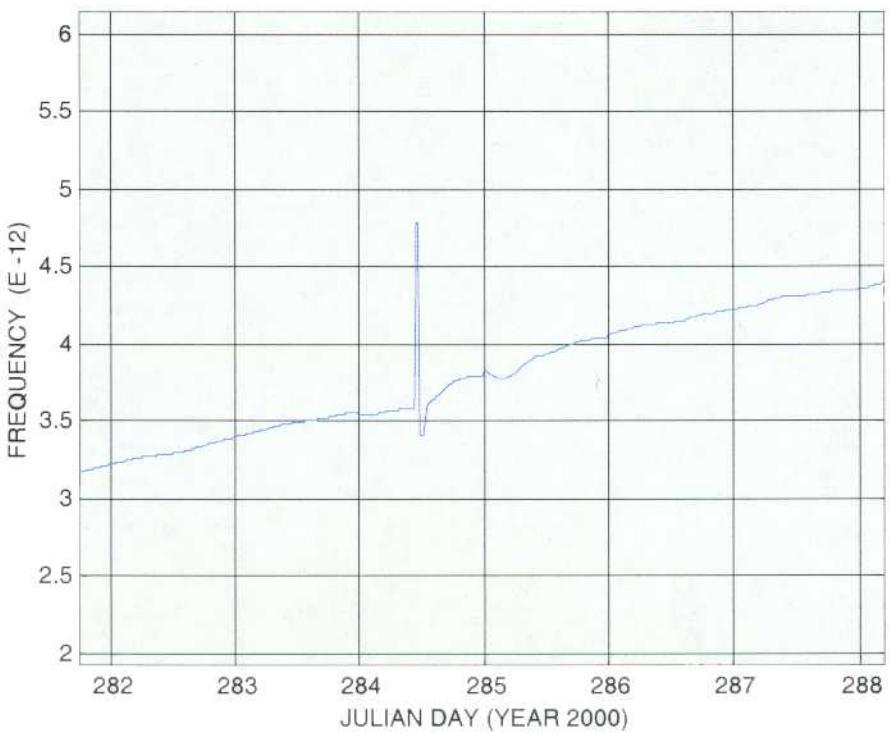


Figure 5. SVN43 Data (Uploaded Corrections Removed – Zoomed Scales)

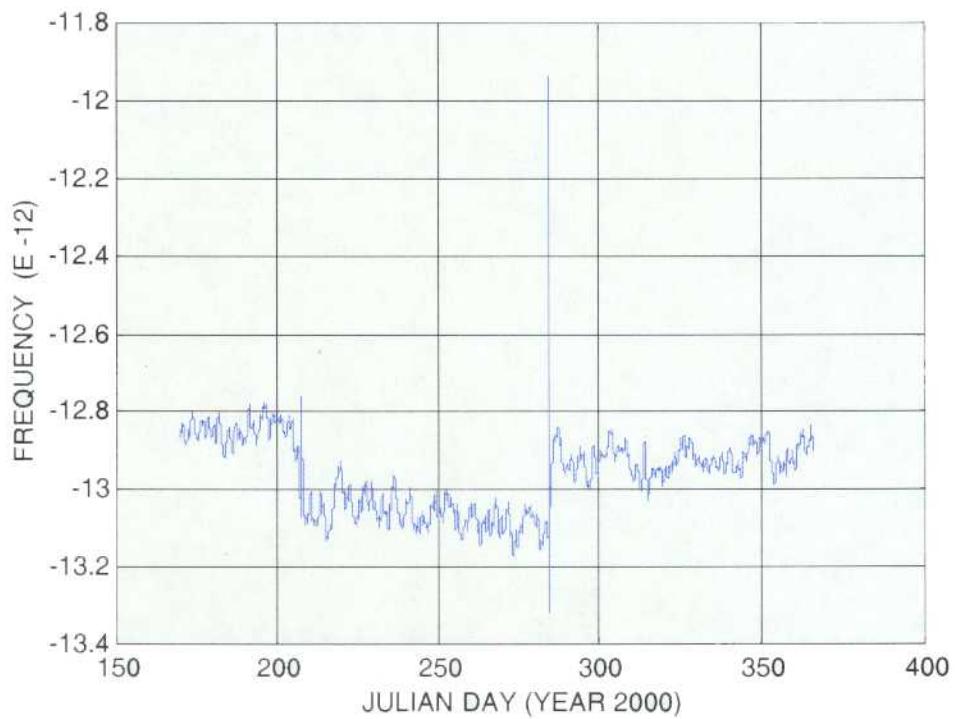


Figure 6. SVN43 Frequency Data (Drift Term Removed)

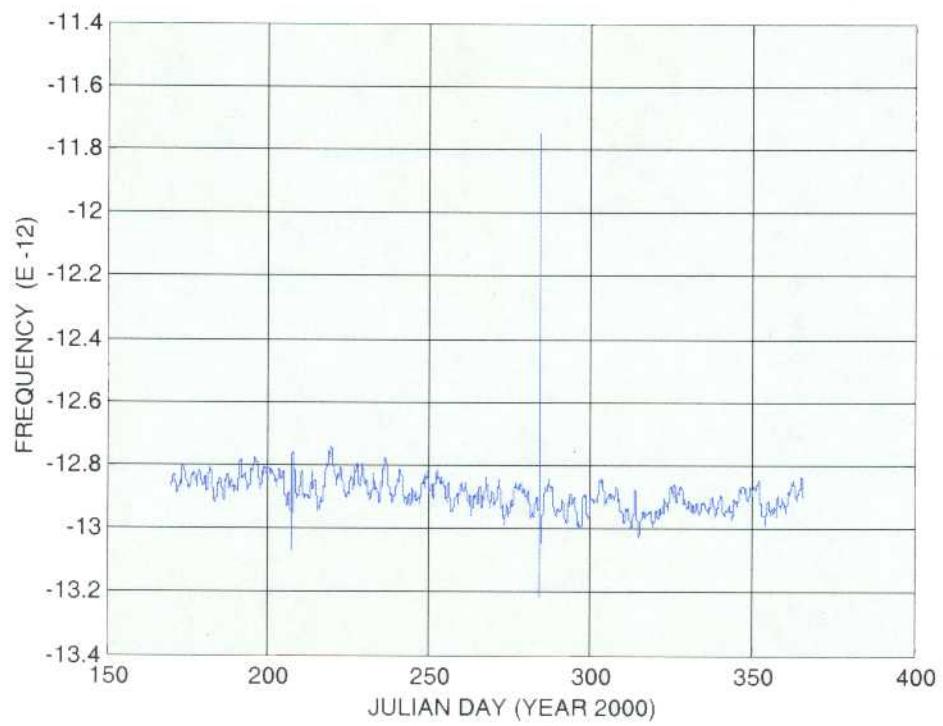


Figure 7. SVN43 "Flat" Frequency Data (Offset Added In)

QUESTIONS AND ANSWERS

TOM CLARK (NASA/Goddard and Syntomics): I am sure Neil would give us a very erudite description of this in a moment. But another individual who has been pointing out exactly this effect for some time is Carroll Alley at the University of Maryland who had an Army study grant to try to improve the intrinsic accuracy, and wrote a very detailed report. Part of this was justified by the inclusion of satellite laser reflectors on PRN 35 AND 36. I believe that the conclusion of the Army was that the foot soldier did not care.

MARVIN EPSTEIN: Okay. As I said, I am sure there are lots of people who looked at it. But the trick is to go over and say "Here's a case – I can show you the frequency drift; I can show you the orbit change, and now you can see the relationship directly with some data." If you go over and measure stuff that doesn't have an orbit change, then you are not seeing the effect. But you are right – lots of people must have looked at it over the years.

VICTOR REINHARDT (Boeing Satellite Systems): This was a well-known effect that looks like it is an operational decision of whether you use the actual orbits or you have some baseline orbit that you throw in. So what are you recommending? Are you recommending now that they go to putting in the operational orbit values on a continuing basis?

EPSTEIN: Well, right now you are asking two sorts of questions. One question is people have some hopeful orbit values in terms of how they represent data and so forth. But what we are really asking for is something much simpler. The Kalman filter intrinsically has inside it an orbital picture of what is going on. It has an orbit that it is dealing with and a clock picture that it is dealing with. What happens is that – I went and spoke to people who run the system. And what they said was that they don't know of any frequency step change associated with orbit change. But they did see that every time there was an orbit change, their data were lousy for the next week. But no one quite knew of what the mechanism was that made the data lousy. They would obviously say that they hadn't measured the orbit change too well. You make a burn; how well can you measure the burn? That is what happened.

And what the recommendation is that somebody ought to go over and look at the effects of what they have and see if they can make it so the glitches you see in the system – maybe I can, I am not sure – let's go back to this thing. If you look at this picture over here, you will see that, near the change on July 5th, there is sort of some big noise that goes up and down. If you go to the picture near October 10th, there is a much bigger noise there. So obviously, those were transients we want to get rid of. We don't care how you do the orbit, we just want to get rid of the transient that is associated with these orbit changes because we are not tracking them well.

Does that answer your question?

REINHARDT: I was asking a much simpler question: Are you proposing that they look into a change, adding a relativistic term to the Kalman filters that does an ongoing relativistic correction? Or at least look into that?

EPSTEIN: No, I am saying that every time they have a burn, they ought to calculate the relativistic correction and put it into the burn. Every time we do a frequency drift, they go over and change their data, so that frequency drift does not show up as a transient in the system. Every time we do a frequency correction, they go over and adjust for it in the Kalman filter. When they do a burn, they correct not only the

burn that they made to the orbit, but also the associated frequency stuff. I hope that answered your question.