

# OCEAN MASS REDISTRIBUTION AND FREQUENCY OFFSETS - PROGRESS UPDATE

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## Abstract

*The relativistic timing affects of tidally redistributed ocean mass are investigated. The Sun, Moon, and Earth hurl through space, and their gravitational fields cause the tides which is a slight redistribution of ocean mass. This redistributed mass perturbs the Earth's gravitational potential affecting atomic clocks. The magnitude of this perturbation will be quantified, correcting for this effect. The predicted fractional frequency offset ( $10^{-19}$ ) is too small to be detected, but this effect may become visible in the future.*

## 1. INTRODUCTION

The purpose of this paper is to investigate whether or not current time transfer techniques can detect a relativistic frequency shift, which would result from the tidal redistribution of ocean mass. To answer this question software will be written to predict how large this offset is.

The tides redistribute ocean mass across the surface of the Earth, from the South China Sea to the Bering Strait. Millions of metric tons of water are being sloshed around, and this perturbs Earth's gravitational field. (The twin GRACE satellites detect small changes in the Earth's gravitational field. The oceans mass distribution is inferred from that. What is done here is to infer a small change in Earth's field from a predicted ocean mass distribution [1]). But, is this change large enough to affect the precise transfer of time?

Gravitation can affect time transfer. The gravitational potential difference alters the photons' frequency, and changes the rate at which a clock beats. This perturbation can be calculated by computing tidal heights and representing them with spherical harmonics.

## 2. FRACTIONAL FREQUENCY ERROR ESTIMATE

How large could this effect be? Think of where the frequency offset would be the largest. This effect only appears when two distant locations compare their timing activities. The effect is not observed locally. There are two locations for where the potential difference could be the largest. The largest potential difference could be between two locations on the ground where the potential is the most positive and the most negative, or between the Earth's surface and a satellite. (The potential can assume both negative and positive signs, because of low tides. When the ocean recedes beneath sea level the missing water is treated as negative mass.)

The frequency difference will be greatest between two points on the ground. If the maximum potential can be estimated, then assume that somewhere on the Earth's surface the potential is equally negative.

This mirror anti-image will introduce a factor of 2 into the estimation. Remember, the difference between a positive number and a negative number is greater than the difference between that same positive number and zero. The potential difference will be greatest between two points on the ground, rather than a point on the ground and a satellite far off in space.

This estimation will provide the maximum potential difference, which immediately provides the maximum fractional frequency offset. While values may be smaller on some parts of the Earth, or up in space off the ground this larger number is more interesting. It provides an idea of how large this effect will be and answer whether or not this effect can be measured with available technology.

To estimate how large this perturbation is going to be, a sum of spherical harmonics is considered. Each spherical harmonic also has an associated radial component. This sum is written as

$$\sum Y_l^m \frac{1}{r^{l+1}}.$$

First the radial components will be approximated, and then the angular components. Finally the proper coefficients will be attached to the summation.

Each solution to Poisson's Equation is a spherical harmonic associated with a radial component. These associated radial components can be eliminated if the measurements are performed at sea level. This is fine, because to detect this tiny fractional frequency offset it will have to be measured with some fancy equipment. A large time and frequency lab would make this measurement, and these large time and frequency labs are located at sea level. (The exception is NIST (National Institute of Standards and Technology), which houses a prominent timing lab, located in the Rocky Mountains (Boulder CO). However, since the estimate strives to provide the largest possible fractional frequency offset this one exception remains just an exception and not a counter-example.)

Sea level by definition is located at the Earth's surface. And when these spherical harmonics and radial components are evaluated at the Earth's surface, all the radial parts collapse to  $\frac{a^l}{r^{l+1}} = \frac{1}{a}$ , where  $a$  equals the Earth's radius. This handles the radial components of  $\sum Y_l^m \frac{1}{r^{l+1}}$ .

Let's now turn our attention to the angular components of this equation  $\sum Y_l^m \frac{1}{r^{l+1}}$ . Each  $Y_l^m$  is bounded by  $\approx 1$ , so set these to 1. This handles the angular components of  $\sum Y_l^m \frac{1}{r^{l+1}}$ .

With  $\sum Y_l^m \frac{1}{r^{l+1}}$ 's radial components set to  $\frac{1}{r^{l+1}}$  and the angular components set to 1, all that is left is to choose coefficients for each term in the series. The terms need to represent how much water is being sloshed around by the tides. So although each term's coefficient does not equal the total amount of water that is being sloshed around, all the coefficients together must equal this total. Multiplying this sum by G turns it into a description of the gravitational potential. By including one more coefficient,  $\frac{1}{c^2}$ , to the gravitational potential, the goal is achieved - the fractional frequency offset is calculated.

But how much water is being sloshed around by the tides? The total displaced mass is a thin shell of water that covers the surface of the Earth. The thickness of this shell is the average tidal height far out at sea - 10 cm. The shell's density is the density of sea water - 1021 kg/m<sup>3</sup>. The height and the density provide a surface density of 102 kg/m<sup>2</sup>. Multiplying by the surface area of the Earth provides the mass -  $4\pi\sigma R^2 = 5.2 \times 10^{16}$  kg.

Now combine  $4\pi\sigma R^2$  with the approximate radial component  $\frac{1}{R}$  and the approximate angular component 1. Finally, add G to transform it into a description of gravity. This expression works out to be  $4\pi\sigma R^2$  or  $0.5 \text{ m}^2/\text{s}^2$ .

To compute the fractional frequency offset, divide by  $c^2$  [2]. This gives a fractional error of  $10^{-18}$ . This fractional error is measureable with cutting-edge time-keeping devices, but is much smaller than the error present in current time transfer techniques. To detect this offset two clocks would have to be placed very far apart. One clock would be located where there is a high tide and a high potential, and the other clock would be located where there is a low tide and a low potential. The potential difference would result in a small measurement error in the time link between the two clocks. However the fractional uncertainty ( $10^{-18}$ ) is a thousand times smaller than the fractional uncertainty of the time link ( $10^{-15}$ ) [3]. Several fractional frequency offsets will be calculated after a description of the software is given.

### 3. METHODS

The objective is to create a software package that calculates the ocean's tidal mass distribution, use that distribution as input, and output the fractional frequency offset at any latitude, longitude, or altitude. Relativistic perturbation theory is used to perform some of the calculations, Avivo's FES 2004 (Finite Element Solution) tide prediction software is used, as is Mangle [4] (a program by Molly Swanson). The software was not fully functional at the time of the conference, but this is a description of the software's final form.

"Relativity in the Global Positioning System" [2] fully describes how to translate a weak field potential into a fractional frequency offset that can be applied to an atomic clock. This paper makes clear that fractional frequency offsets add linearly, so this additional offset can simply be added to the existing offsets.

To calculate the fractional frequency offset the software tools FES 2004 and Mangle will be used. FES 2004 is a tidal prediction program made available through Aviso. It will provide the tidal height at a given location and time, which provides a numerical model for ocean mass redistribution. This numerical model will use the tide height seen by a tide gauge for all calculations. (The solid Earth loading will be subtracted out.)

Specifically this numerical model will use a mesh grid. This mesh grid will cover the Earth's surface and represent the tidal ocean mass redistribution. From this mass distribution a perturbation will be calculated. Spherical harmonics will then be fitted to the data using Mangle. Mangle takes a surface density mask for a sphere and returns the spherical harmonic representation for that surface. (Mangle was originally written to process astronomical survey pixel maps and translate the surveys into spherical harmonics.)

Using these tools in tandem will create a software package that allows the perturbation to the Earth's potential to be evaluated at various locations, and the fractional frequency offset to be calculated from that perturbation.

## 4. ADDITIONAL PROGRESS

This paper discusses the progress made at the time of the PTTI 2011 conference. Further progress has been made. These additional results are available on the arXiv.org site under the title “*Ocean Mass Redistribution & Corresponding Frequency Offsets for Precise Timing Applications*”. On arXiv.org this work is contained in the Physics’ General Relativity and Quantum Cosmology section. That paper is being considered for publication by the journal of General Relativity and Gravitation (June 2012). All of the results, findings, and diagrams are reserved under copyright by the journal of General Relativity and Gravitation.

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