

CLOCK CHARACTERIZATION TUTORIAL

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ABSTRACT

Managers are often required to make key program decisions based on the performance of some elements of a large system. This paper is intended to assist the manager in this important task in so far as it relates to the proper use of precise and accurate clocks. An intuitive approach will be used to show how a clock's stability is measured, why it is measured the way it is, and why it is described the way it is. An intuitive explanation of the meaning of time domain and frequency domain measures as well as why they are used will also be given.

Explanations of when an "Allan variance" plot should be used and when it should not be used will also be given. A more efficient way to measure clock frequency drift will be explained. The relationship of the rms time error of a clock to a $\sigma(\tau)$ diagram will also be given. The environmental sensitivities of a clock are often the most important effects determining its performance. Typical environmental parameters of concern and nominal sensitivity values for commonly used clocks will be reviewed.

Systematic and Random Deviations in Clocks

This paper is tutorial in nature with a minimum of mathematics -- the goal being to characterize clock behavior. First, time deviations or frequency deviations in clocks may be categorized into two types: systematic deviations and random deviations. The systematic deviations come in a variety of forms. Typical examples are frequency sidebands, diurnal or annual variations in a clock's behavior, time offset, frequency offset and frequency drift. Figure 1 illustrates some of these.

If a clock has a frequency offset, the time deviations will appear as a ramp function. On the other hand, if a clock has a frequency drift, then the resulting time deviations will appear as a quadratic time function -- the time deviations will be proportional to the square of the running time. There are many other systematic effects that are very important to consider in understanding a clock's characteristics and Figure 1 is a very simplistic picture or nominal model of most precision oscillators. The random fluctuations or deviations in precision oscillators can often be characterized by power law spectra. In other words, if the time residuals are examined, after removing the systematic effects, one or more of the power law spectra shown in Figure 2 are typically observed. The meaning of power law spectra is that if a Fourier analysis or spectral density analysis is performed on the residuals, the intensity at various Fourier frequencies, f is proportional to f^β ; β designates the particular power law process ($\beta = 0, -1, -2, -3, -4$ and $\omega = 2\pi f$). The first process shown in Figure 2 is called white noise phase modulation (PM). This noise is typically observed in the short term fluctuations,

for example, of an active hydrogen maser for sample times of from one second to about 100 seconds. This noise is also observed in quartz crystal oscillators for sample times in the vicinity of a millisecond and shorter. The flicker noise PM, f^{-1} , is the second line in Figure 2. This kind of noise is often found for sample times of one millisecond to one second in quartz crystal oscillators. The f^2 or random walk PM indicated by the third line is what is observed for the time deviations of rubidium, cesium, or hydrogen frequency standards. If the first difference is taken of a series of discrete time readings from the third line, then the result is proportional to the frequency deviations, which will be an f^0 process or a white noise frequency modulation (FM) process. In other words the time and the frequency are related through a derivative or an integral depending upon which way one goes. The derivative of the time deviations yields the frequency deviations, and the integral of the frequency deviations yields the time deviations. So, random walk time deviations result from white noise FM. In general, the spectral density of the frequency fluctuations is w^2 times the spectral density of the time fluctuations. The fourth line in Figure 2 is an f^{-3} process. If this were representative of the time fluctuations then the frequency would be an f^{-4} or a flicker noise FM process. This process is typical of the output of a quartz crystal oscillator for sample times longer than one second or the output of rubidium or cesium standards in the long term (on the order of a few hours, few days, or few weeks depending upon which standard). We find that in very long term, most atomic clocks have an $f^{-1/2}$ type behavior for the time fluctuations -- making the frequency fluctuations an $f^{1/2}$ process or random walk FM. These five power law processes are very typical and one or more of them are appropriate models for essentially every precision oscillator. Characterizing the kind of power-law process thus becomes an important part of characterizing the performance of a clock (1). Once a clock has been characterized in terms of its systematic and its random characteristics then a time deviation model can be developed. A very simple and useful model that is commonly used is given by the following equation (2):

$$x(t) = x_0 + y_0 \cdot t + \frac{1}{2}D \cdot t^2 + \varepsilon(t) \quad (1)$$

where $x(t)$ is the time deviation of the clock at time t , x_0 is the synchronization error at $t = 0$, and y_0 is the syntonization error at $t = 0$, which produces a linear ramp in the time deviations. D is the frequency drift term, which is almost always an applicable model element in commercial standards. This $\frac{1}{2}Dt^2$ term in the time deviation due to the frequency drift yields a quadratic time deviation. Lastly, the $\varepsilon(t)$ term contains all of the random fluctuations. It is this term which is typically characterized by one or more of the various power law processes. Once a clock has been fully characterized, then it is possible to do optimum time prediction. Shown in Figure 3 are some examples. The even power law spectra have simple algorithms for prediction. In the case of white noise PM, the optimum predictor is the simple mean. In the case of random walk PM, it is the last value. In the case of random walk FM, an $f^{-1/2}$ process on the time, the optimum predictor is the last slope. In the case of flicker noise, the prediction algorithms are significantly more complicated but not intractable, and ARIMA techniques can be employed in order to develop optimum prediction algorithms (3).

The Concept of an Allan Variance

Figure 4 illustrates a simulated random walk PM process. Suppose this process is the time difference between two clocks or the time of a clock with respect to a perfect clock. Again this process is typical of the time deviations for rubidium, cesium, and passive hydrogen clocks. Choose a sample time τ , as indicated, and note the three time deviation readings (x_1 , x_2 , and x_3) indicated by the circles and spaced by the interval τ . The frequency deviation y_1 is proportional to the slope between x_1 and x_2 ($y_1 = (x_2 - x_1)/\tau$). Similarly y_2 is proportional to the slope between x_2 and x_3 ($y_2 = (x_3 - x_2)/\tau$). The difference in slope, Δy , is a measure of the frequency change from the first τ sample interval to the next adjacent τ sample interval. With a fixed value of τ , imagine averaging through the entire data set for all possible readings of x_1 , x_2 , and x_3 displaced by τ each yielding a Δy . The average squared value of Δy divided by 2 is called the "Allan variance". In theory, it is the average over all time. In practice, finite data sets yield rapidly converging estimates. The square root of the Allan variance is denoted $\sigma_y(\tau)$; $\sigma_y(\tau)$ is an efficient estimator of the power law spectra model for the data. How $\sigma_y(\tau)$ changes with τ indicates the exponent for the power law process. In fact, in the case of random walk FM, $\sigma_y(\tau)$ is statistically the most efficient estimator of this power law process. If power law processes are good models for a clock's random fluctuations, which they typically are, then the Allan variance analysis is faster and more accurate in estimating the process than the Fast Fourier Transform. Some virtues of the Allan variance are: It is theoretically and straightforwardly relatable to the power law spectral type ($\beta = -\mu - 3$, $-2 < \mu < 2$, where μ is the exponent on τ). Once a data set is stored in a computer it is simple to compute $\sigma_y(\tau)$ as a function of τ . The difference in frequency, Δy , is often closely related to the actual physical process of interest, e.g. frequency change after a radar return delayed by τ , effects of oscillator instabilities in a servo with loop time constant τ , the change in frequency after a calibration over an interval τ , etc. Some drawbacks of the Allan variance are: it is transparent to periodic deviations where the period is equal to the sample time τ . It is ambiguous at $\mu = -2$, i.e. $\sigma_y(\tau) \sim \tau^{-1}$ may be either white noise PM or Flicker noise PM. Remembering from above the relationship between the spectral density of the frequency deviations and the spectral density of the time deviations, if $S_y(f) \sim f^\beta$ and $S_x(f) \sim f^\alpha$, then $\alpha = \beta + 2$ and $\alpha = -\mu - 1$ ($-2 < \mu < 2$), where $S_x(f)$ is the spectral density of the time deviations and $S_y(f)$ is the spectral density of the fractional frequency deviations. Figure 5 shows the noise type and the relationship between μ and α . There are some ways around the ambiguity problem at $\mu = -2$. For noise processes where $\alpha > 1$ there is a bandwidth dependence (4). A software trick can be employed to effectively vary the bandwidth rather than doing it with the hardware. Rather than calculating $\sigma_y(\tau)$ from individual phase points the phase can be averaged over an interval τ . Hence the x_1 , x_2 , and x_3 from Figure 4 become phase or time difference averages. As τ increases the effective measurement system bandwidth decreases. This technique removes the ambiguity problem. We have called this the modified Allan variance or modified $\sigma_y(\tau)$ analysis technique. Figure 6 shows the μ , α mapping for the modified Allan variance. For white noise PM, μ is equal to -3, and for flicker noise PM, μ is equal to -2.

Time Prediction Error of a Clock

Another concept which has become useful is the computation of the time error of

prediction. In the case of white noise FM and random walk noise FM, $\tau \sigma_y(\tau)$ is the optimum time error of prediction. For white noise PM the optimum value achievable is $\tau \sigma_y(\tau)/\sqrt{3}$, and for flicker noise FM it is $\tau \sigma_y(\tau)/\ln 2$.

Applying some of the above concepts to state of the art frequency standards yields the frequency stability plot shown in Figure 7 (5), and the corresponding RMS time prediction error plot shown in Figure 8. The RMS time prediction error plot is based on reference (2) and is a function of the levels of noise in the clock and also the uncertainties associated with determining the systematic deviations due to a finite data length.

Environmental Efforts on Clocks

From a management point of view, the characteristics of the various clocks should be related to the needs of a particular program. It is important to keep in mind that, in practice, systematic and environmental effects often are the predominant influence on the time and frequency deviations of a clock. The reliability of a clock is often a basic issue, and the manager should assure himself that adequate reliability has been documented. The manager also needs to ask the following questions in each application that he may have. What are the environmental conditions; e.g., the temperature, the temperature gradients, the temperature fluctuations, the rapidity of the temperature changes, the magnetic field conditions, the shock and vibration conditions, and the humidity conditions? How do these conditions affect the clock's performance? All clocks are affected at some level by changes in the above environmental parameters plus some others as well. Some clocks are affected by barometric pressure. Vibration can be extremely important. In some clocks the servos will unlock, for example, if a 1 kHz vibration is present. (6). Some clocks are acoustically sensitive. What is the gravitational or g sensitivity? What are the cost, size, weight and power requirements? Line voltage power fluctuations can affect clocks. Changes in the dc power can affect some clocks. We have found that a good clock environment can improve clock performance considerably, and we have provided a highly controlled environment for the NBS clock ensemble to improve the performance over that obtained in typical laboratory environments. Another very important question to ask is what is a clock's lifetime? Redundancy and/or multiple clocks are sometimes necessary to overcome lifetime and reliability problems. It is important to take a system's approach in establishing the best clock(s) and clock(s) configuration. In some cases the program needs are for synchronization to UTC, in other cases the needs are for syntonization, i.e. the frequencies within a system need to agree. Often the need is for time or frequency self consistency within a program, e.g. GPS requires time consistency. It seems many people are buying cesium standards as a panacea, when in fact they may not be solving the problem at hand. Buying a cesium standard does not guarantee synchronization. However, it does guarantee syntonization within some accuracy. All clocks will diverge and eventually depart from synchronization tolerance. It's just a matter of time! Knowing a clock's characteristics, the system requirements, and the environmental conditions will allow the manager to know the best clock or clocks to buy and the best way to implement them. For example, a rubidium clock coupled to a GPS receiver (used in the common-view mode with UTC(USNO MC) or with UTC(NBS)) would have better short term and better long term stability than any commercial cesium clock available. The stability would be somewhat worse in the vicinity of τ equal to one day. In practice, there are some

problems with this idea, but it illustrates the point.

Lastly, Figures 9 through 13 show nominal values for some important clock coefficients that managers and design engineers need to properly assess when evaluating which clock or clocks will best serve their needs. These are only nominal values and there will be exceptions. A band of values is listed for these coefficients ranging from nominal best performance available from laboratory-type standards through the range of typical values observed and specified for commercially available standards.

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References

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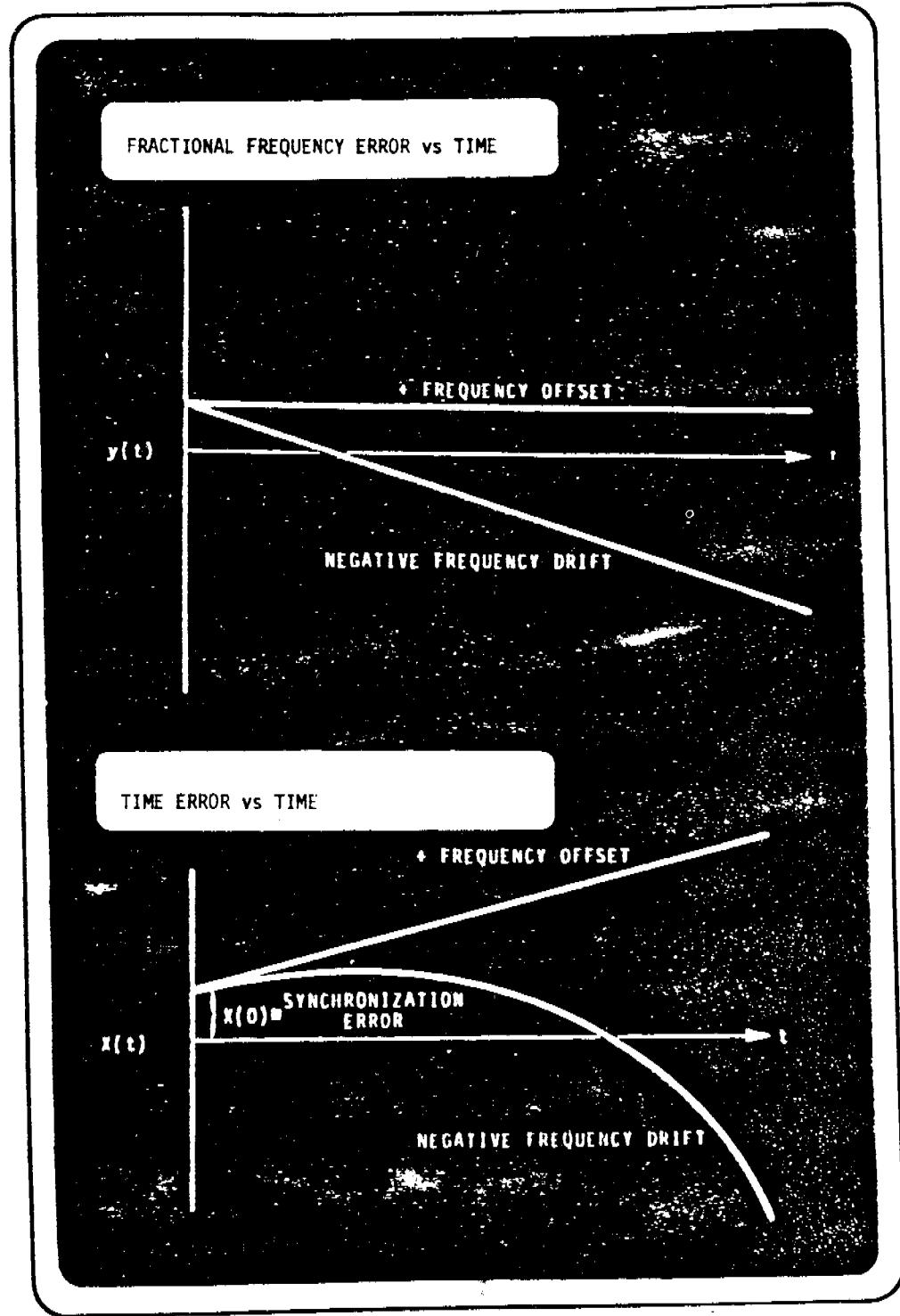


Figure 1. Frequency, $y(t)$, and time, $x(\tau)$, deviations due to frequency offset and to frequency drift in a clock.

POWER LAW SPECTRA

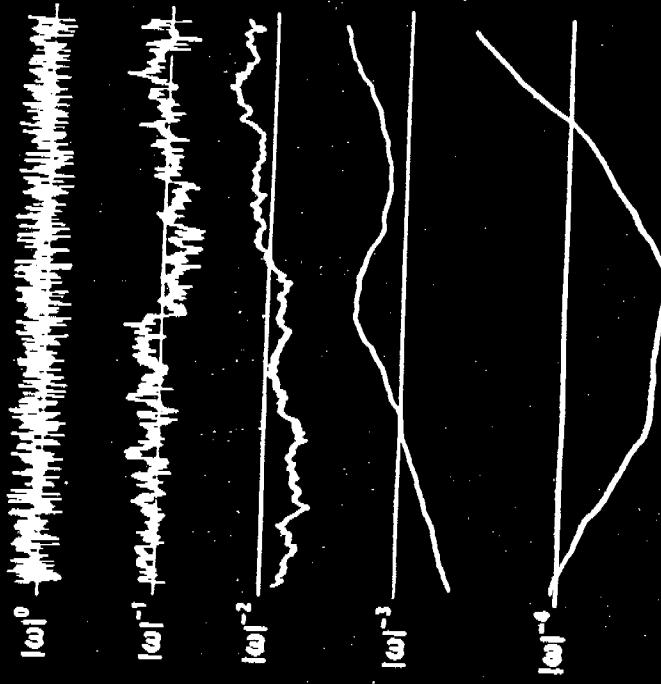


Figure 2. Simulated random processes commonly occurring in the output signal of atomic clocks. Power law spectra $S(\omega)$, are proportional to ω up to some exponent, where ω is the Fourier frequency.

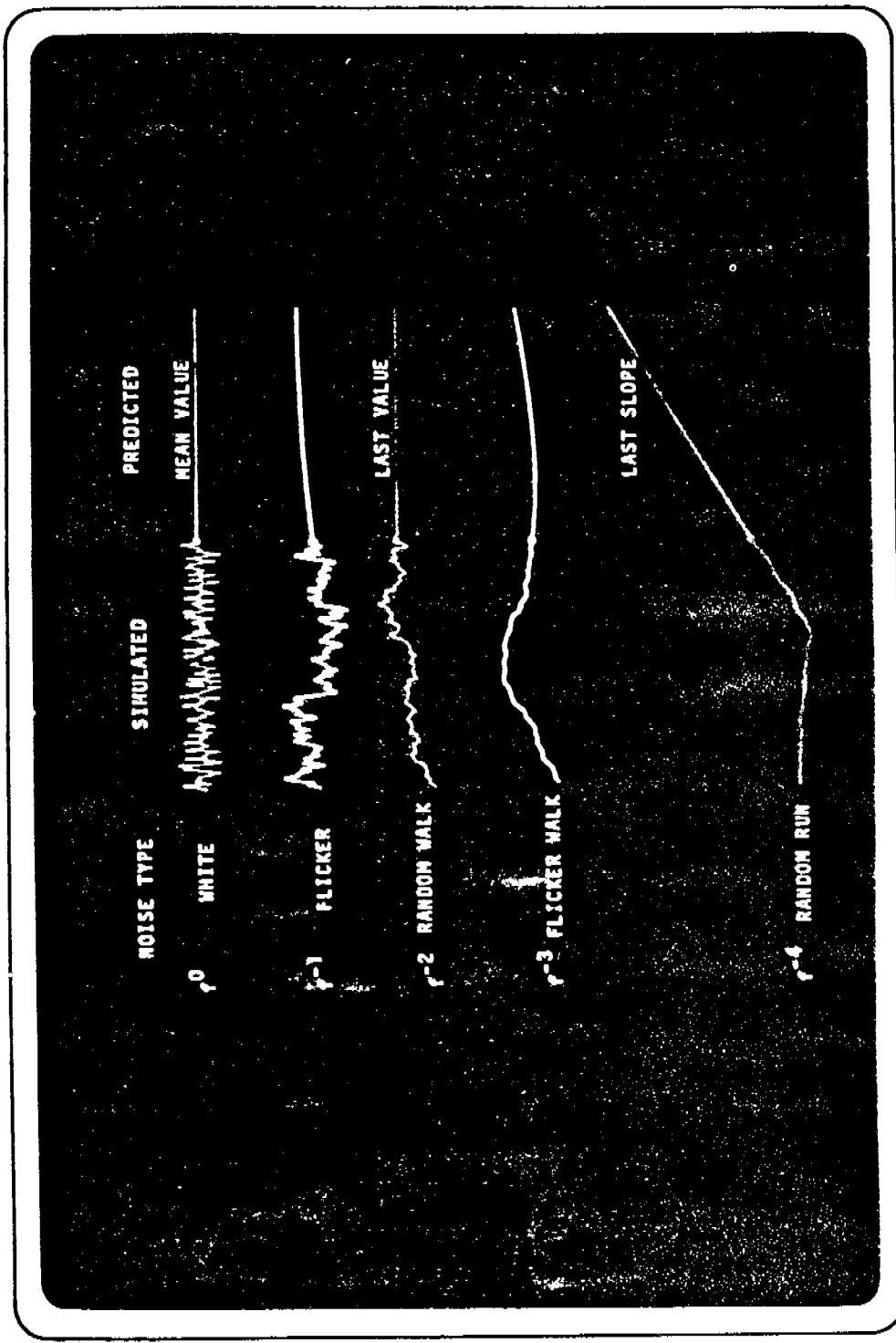


Figure 3. Simulated power law noise processes with their optimum predicted estimates.

'Allan variance' concept

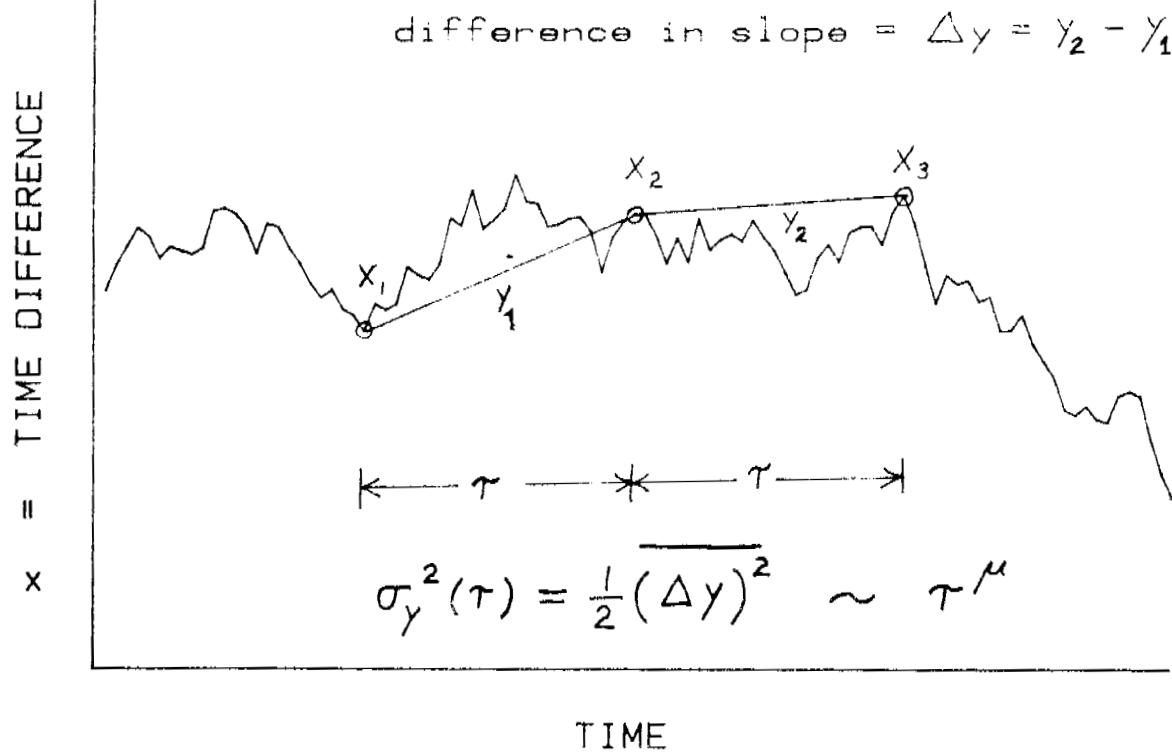


Figure 4. Pictorial of computation of "Allan variance".

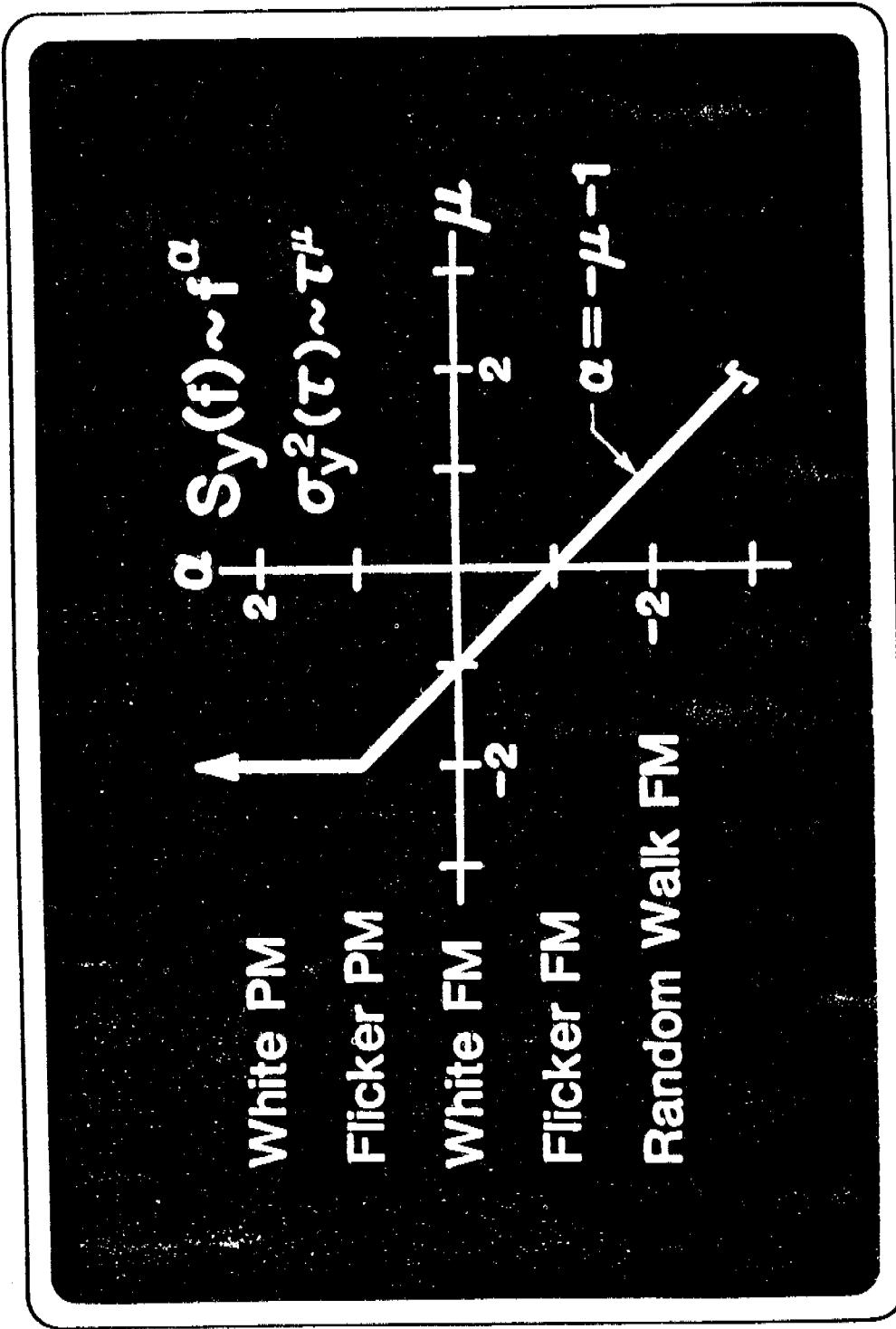


Figure 5. Various power law noise types commonly occurring in the precision clocks and the mapping between the exponent α of the frequency domain measure and the exponent μ of the time domain measure.

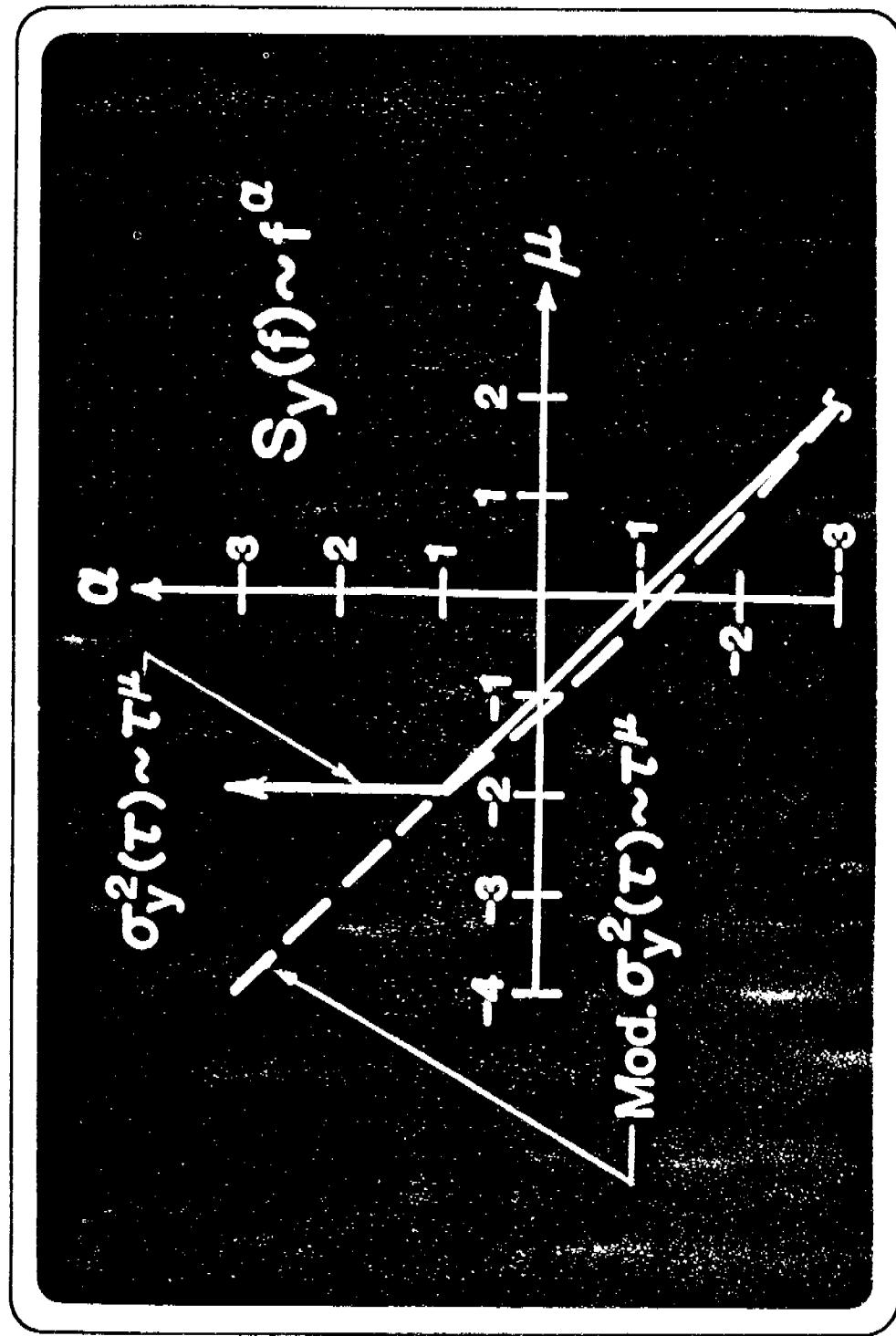


Figure 6. The mapping of the τ exponent μ of the Allan variance and of the modified Allan variance to the power law spectrum exponent α .

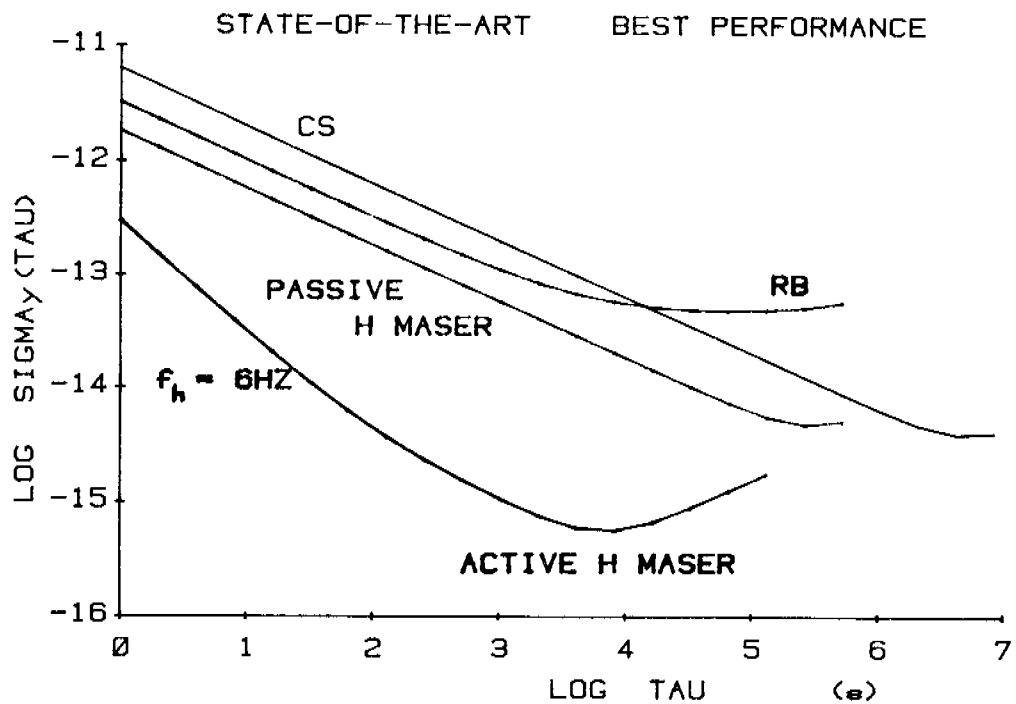


Figure 7. Frequency stability of some of the best performing frequency standards from each of four main types: CS = cesium beam, RB = rubidium gas cell, H = hydrogen (active and passive). Note: $\log 10^1 = 1$; e.g. $\log 10^{-14} = -14$.

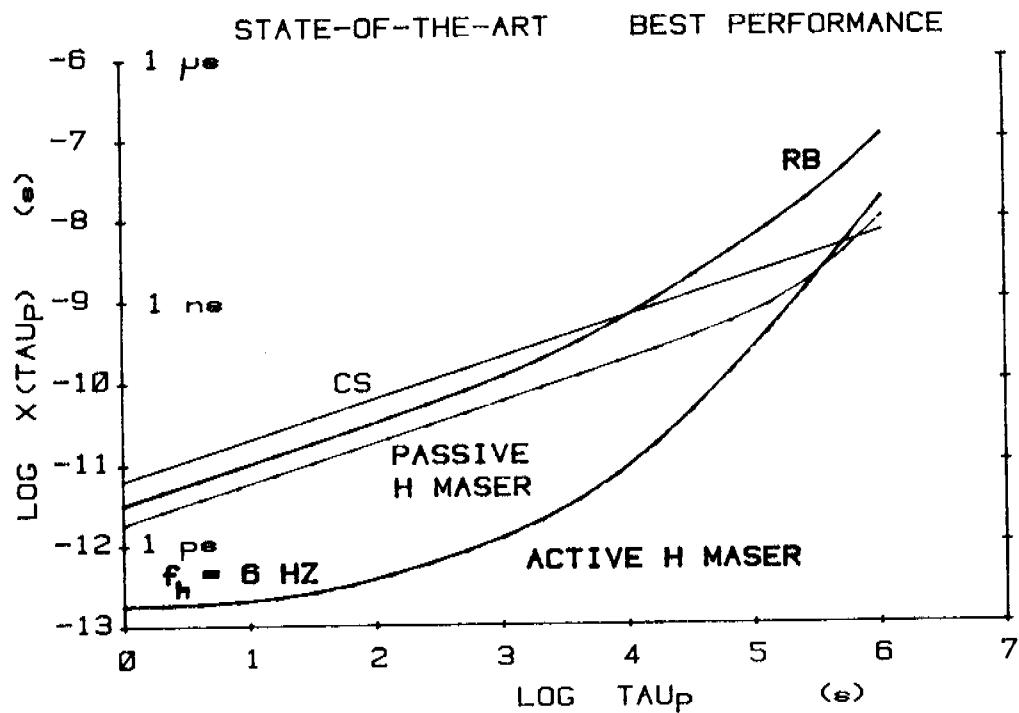


Figure 8. RMS time prediction error plot for the same frequency standards as shown in Figure 7. Note: $\log 10^1 = 1$; e.g. $\log 10^{-14} = -14$.

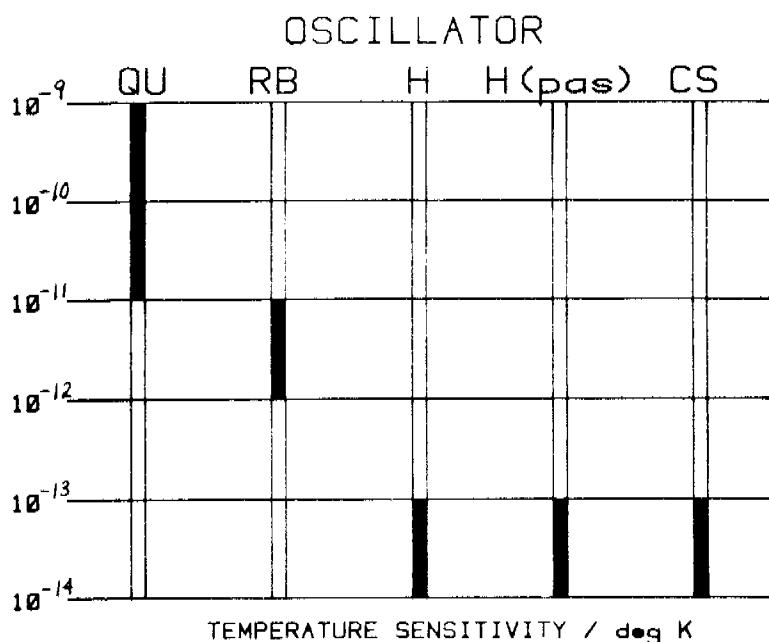


Figure 9. Nominal values for the temperature coefficient for the frequency standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

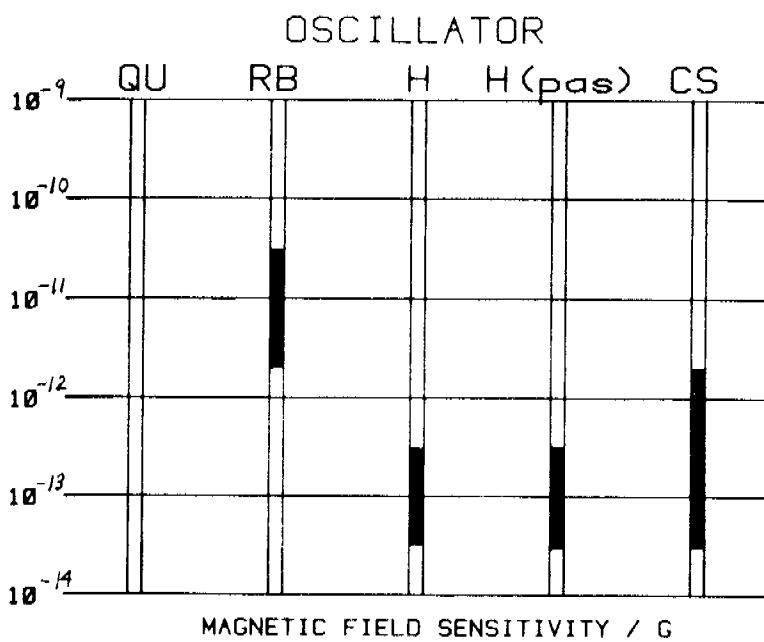


Figure 10. Nominal values for the magnetic field sensitivity for the frequency standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

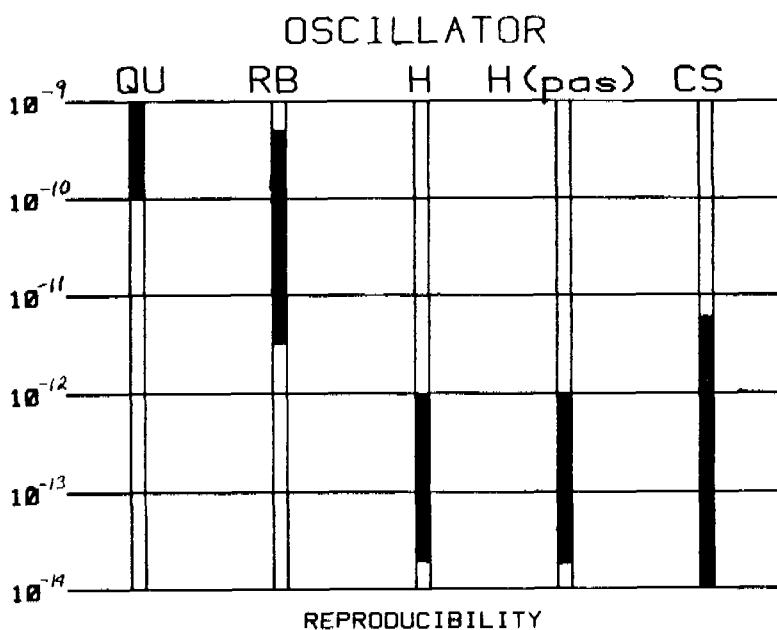


Figure 11. Nominal capability of a frequency standard to reproduce the same frequency after a period of time for the standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

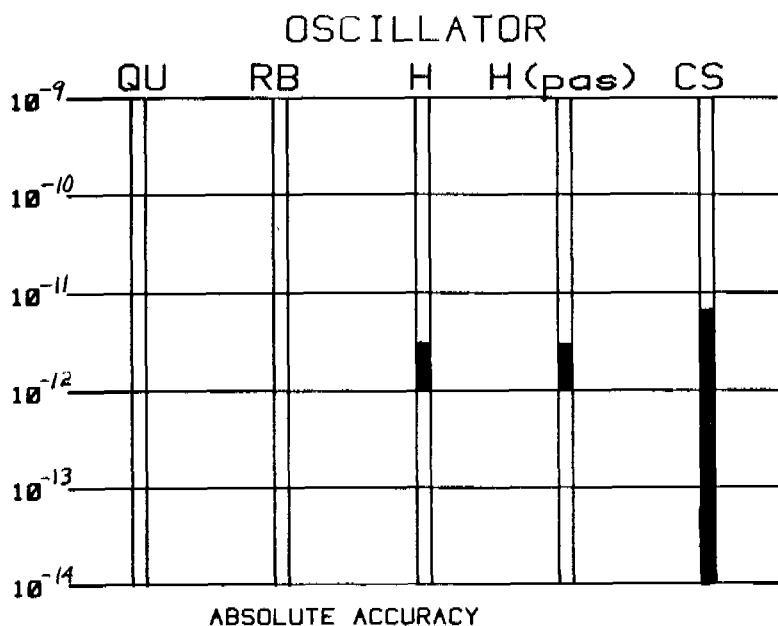


Figure 12. Nominal capability for a frequency standard to produce a frequency determined by the fundamental constants of nature for the standards : QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

OSCILLATOR

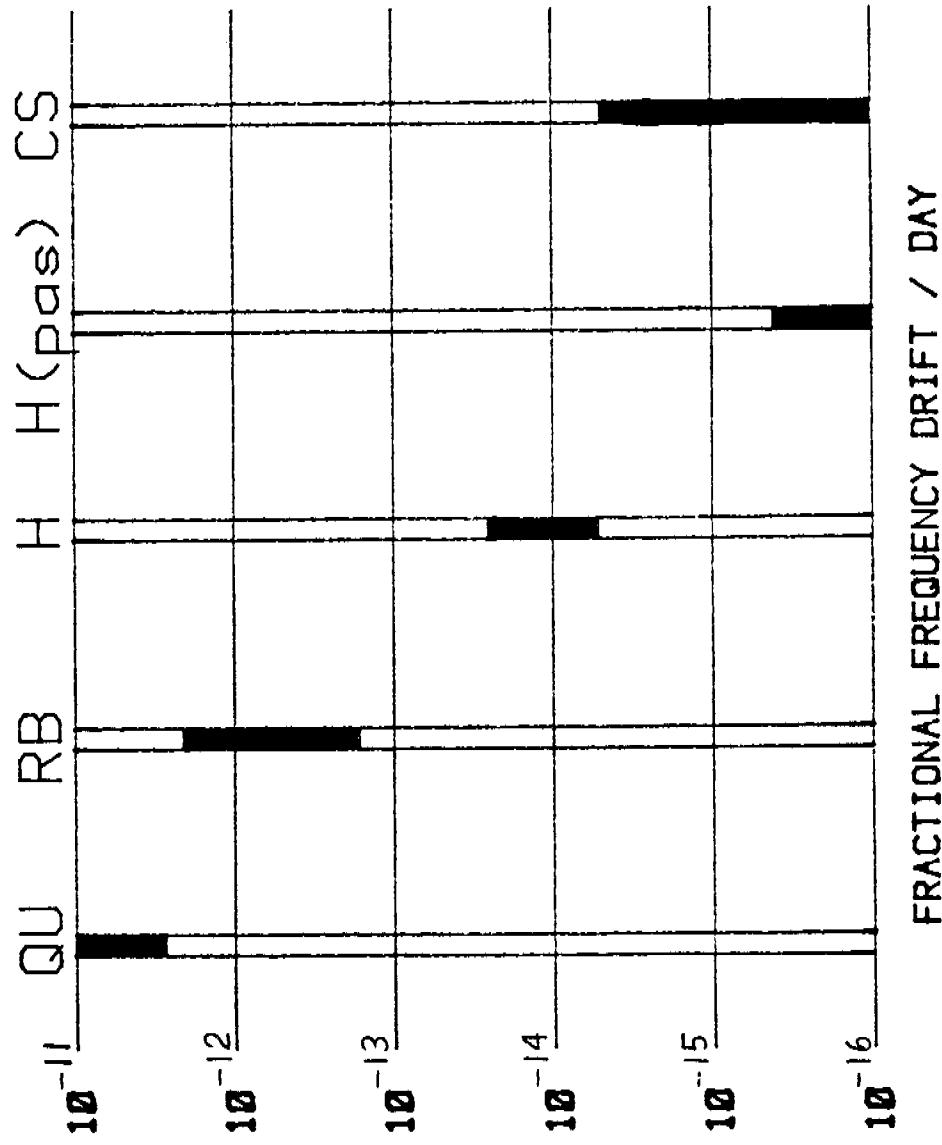


Figure 13. Nominal values (ignoring the sign) for the frequency drift for the frequency standards: QU = quartz crystal, RB = rubidium gas cell, H = active hydrogen maser, H(pas) = passive hydrogen maser, and CS = cesium beam.

QUESTIONS AND ANSWERS

DR. WINKLER:

I would like to qualify your statement that one should operate a rubidium standard with a GPS receiver and have a better performance for very short and very long times. That statement is true, but incomplete, because if you would buy a set of cesium clocks, a clock ensemble, you gain one more point and for strategic systems that is of strategic importance, that is independence. That's a very important point and I want to emphasize that.

MR. ALLAN:

I said there are practical reasons that you don't choose to do that. That's a very good example.

MR. WARD:

Sam Ward, Jet Propulsion Lab. Is that modified Allan variance useful on an active hydrogen below TAU values of a hundred seconds?

MR. ALLAN:

It would be useful because active hydrogen does have a white pm and so for the short term analysis you get tau to the 3/2 power, yes, it would be useful.

DR. VESSOT:

In relation to that last point of TAU -3/2 behavior, in fact, arose from such measurements of Phase Noise 1956. That's not the comment I want to make. There are two things I implore you to do. The first one is, please lower the floor on the colored graph of the hydrogen maser; the ten to the minus fourteen; I think we are safely below one in the fifteenth. So I respectfully suggest that you back it down by a factor of ten and still be as honest as you could possibly want to be. You are safe by a factor of about three, if you do that.

The second comment is the fact that as we have long discussed, ever since 1956, that besides making the variances a function of time, I respectfully suggest also that we include the bandwidth. It's increasingly important, I think, to recognize that short term stability is limited by additive noise and that additive noise is necessarily bandwidth dependent. And those people who are to specify oscillators' stability, particularly if they're involved in rather shorter terms, must, in the giving of the Allan variance, specify the averaging in time and the stability.

The definition of the Allan variance implies that the sample number is two. So by saying that, we have cornered that issue as two. The dead time is another question, and I don't think one should complicate it further at this point, but I would urge you to at least admit that we should have the averaging time and the bandwidth; and we have been around the barn with that, Jim, on that committee. I would just like to take one more lap.

MR. ALLAN:

Thank you, Bob, and I would fully concur on any sigma-tau graph that you're looking at a tau to the minus one behavior. You need to have bandwidth specified as well.

DR. BARNES:

I will take just a moment to mention that some years ago. Dr. Vessot did a bandwidth variable measure of frequency stability and the tau of the three halves came out of his work. So the tau of the three halves is not a stranger.