

# Satellite Test of the Isotropy of the One-Way Speed of Light Using ExTRAS

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**Abstract:** *A test of the second postulate of special relativity, the universality of the speed of light, using the ExTRAS (Experiment on Timing Ranging and Atmospheric Sounding) payload to be flown on board a Russian Meteor-3M satellite (launch date January 1997) is proposed. The propagation time of a light signal transmitted from one point to another without reflection would be measured directly by comparing the phases of two hydrogen maser clocks, one on board and one on the ground, using laser or microwave time transfer systems. An estimated uncertainty budget of the proposed measurements is given, resulting in an expected sensitivity of the experiment of  $\delta c/c < 8 \times 10^{-10}$  which would be an improvement by a factor of  $\sim 430$  over previous direct measurements and by a factor of  $\sim 4$  over the best indirect measurement. The proposed test would require no equipment additional to what is already planned and so is of inherently low-cost. It could be carried out by anyone having access to a laser or microwave ground station and a hydrogen maser.*

## 1. Introduction

Einstein's second postulate, affirming the universality of the speed of light for inertial frames, is fundamental to the theories of special and general relativity. It can be tested directly by comparing the propagation times of two light signals travelling from one point to another along the same path but in opposing directions (often referred to as a test of the isotropy of the one-way speed of light). The only such test, was carried out by Krisher

et al. [1], who compared the phases of two hydrogen masers separated by a distance of 21 km and linked via an ultrastable fibre optics link of the NASA deep space network. The sensitivity of this experiment, expressed as a limit on the anisotropy of the speed of light, was  $\delta c/c < 3.5 \times 10^{-7}$ , where  $c$  is the velocity of light in vacuum. Riis et al. [2] tested the isotropy of the first order Doppler shift of light emitted by an atomic beam (and indirectly thereby the second postulate) using fast-beam laser spectroscopy obtaining the currently best limit on the anisotropy,  $\delta c/c < 3 \times 10^{-8}$ . This presents a 10 fold improvement on previous values from experiments measuring the isotropy of the first order Doppler shift using the frequency links in the NASA GP-A rocket experiment [3] and so-called Mössbauer rotors [4, 5]. In the test theory of Mansouri and Sexl [6] the above results can be interpreted as limits on the parameter  $\alpha$  using the relation  $\delta c/c = (1+2\alpha)v/c$  [1, 2] where  $v$  is the velocity of the Earth with respect to the mean rest frame of the universe ( $v \sim 300$  km/s). This yields values of  $\alpha = -1/2 \pm 1.8 \times 10^{-4}$  and  $-1/2 \pm 1.4 \times 10^{-6}$  for the experiments by Krisher et al. [1] and Riis et al. [2] respectively.

The experiment proposed here would test the isotropy of the transmission time of light signals between two points directly and on a non-laboratory scale with an estimated accuracy of  $\delta c/c < 8 \times 10^{-10}$ , using the ExTRAS payload on board the Russian Meteor-3M satellite scheduled for launch in January 1997, T2L2 (Time Transfer by Laser Light) time transfer and a hydrogen maser at the ground station. This, if realized, would present a 430 fold improvement on previous direct measurements [1] and a slight improvement on the value obtained by Riis et al. [2]. In Section 2 the principle of the experiment is explained while Section 3 provides an evaluation of its sensitivity aimed at including all error sources that may exceed one picosecond and based on the uncertainty budget for the T2L2 method by Thomas & Uhrich [7].

## 2. Experimental principle

The ExTRAS payload consists of two active, auto-tuned hydrogen masers communicating with ground stations via a PRARE (Precise Range and Range-Rate Equipment) microwave link and a T2L2 laser link. Once operational, the system should reflect laser pulses, emit and receive microwave signals and date all such events on the on-board time scale provided by the hydrogen masers. The satellite will follow polar orbit, at an altitude of 1000 km with a period of order 100 min and a duration of one passage of  $\sim 17$  min.

In principle, the proposed experiment is similar to that performed by Krisher et al. [1]. A laser signal emitted from the station E is reflected at the satellite S and returned to E (see figure 1). The readings of the ground hydrogen maser at emission ( $\tau_0$ ) and reception ( $\tau_2$ ) and that of the space maser at the moment of reflection ( $\tau_1$ ) are recorded. The differences  $\tau_1 - \tau_0$  and  $\tau_2 - \tau_1$  represent the up and down transmission times  $T_1$  and  $T_2$  respectively plus some initial phase difference of the clocks. Note that no synchronization convention or

procedure is assumed. Einstein's second postulate would require that for a series of measurements, after accounting for the path asymmetries, the difference  $T_1 - T_2$  should be equal to a constant  $\Delta_0$  (due to the initial clock offset) independent of the spatial orientation of the individual links. More particularly one obtains for a single link (see [8] for more detail),

$$T_1 - T_2 = \Delta_0 + 2 \vec{R}(t_e) \cdot \vec{v}(t_e) / c^2 + (\Delta_{i(\text{up})} - \Delta_{i(\text{down})}) + O(c^{-3}) \quad (1)$$

where  $\vec{R}(t_e)$  is the vector from E to S at the coordinate time of emission of the signal  $t_e$  in a geocentric, inertial reference frame,  $\vec{v}(t_e)$  is the velocity of the ground station at signal emission in the same frame and  $\Delta_i$  are internal delays (cables etc.).

The initial clock offset  $\Delta_0$  is a constant, provided that the two clocks are syntonized. This can be achieved at the  $10^{-15}$  accuracy level (the best hydrogen maser stability) using time transfer data over a sufficiently long integration period and taking into account all known effects (gravitational redshift, second order Doppler, maser drift). One would expect the effect on the syntonization, of an eventual anisotropy of the propagation time of the light signals, to average out in a global treatment using time transfers in all spatial directions.

Terms of order  $c^{-2}$  amount to  $\sim 40$  ns and can be calculated to picosecond accuracy if  $\vec{R}(t_e)$  and  $\vec{v}(t_e)$  are known to within  $\sim 50$  m and  $\sim 0.01$  m/s respectively, which represents no difficulty for modern satellite orbitography. Of course, a possible anisotropy would also have an effect on the satellite orbit determination, but as the range  $R$  cancels to first order in (1) this effect would be negligible. Furthermore, the satellite orbit is obtained from round-trip ranging measurements, which should, again to first order, be insensitive to anisotropy of the propagation time of the light signals.

Terms of order  $c^{-3}$  can amount to several picoseconds but can be calculated to picosecond accuracy without difficulty [8]. The effect of asymmetry in the atmospheric delays for the up and down links is below one picosecond.

Hence, after accounting for path asymmetry, any variation of the difference  $T_1 - T_2$  with the spatial orientation of the laser link should be due to a violation of the second postulate.

### 3. Estimation of the experiment sensitivity

The sensitivity of the proposed test can be estimated by considering two individual laser links as shown in figure 2. The time intervals  $\tau_2 - \tau_0$ ,  $\tau_5 - \tau_3$  and  $\tau_3 - \tau_0$  are measured using the ground hydrogen maser with the interval  $\tau_4 - \tau_1$  obtained from the space hydrogen maser. Designating the individual transmission times by  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  as shown in figure 2 and

assuming that one of the links is colinear with the direction of the presumed anisotropy, the difference between the two links is given by,

$$(T_1 - T_2) - (T_3 - T_4) + \Delta_s = 2 \Delta_a (1 - \cos\theta). \quad (2)$$

Here  $\Delta_s$  represents the correction due to the path asymmetries of the individual links;  $\Delta_a$  is the maximum delay for a single transmission due to the anisotropy, and  $\theta$  is the angle between the two links in the inertial geocentric frame.

If Einstein's second postulate is true the right hand side of equation (2) should be equal to zero within the measurement error.

The experiment should be capable of detecting an anisotropy under the condition

$$\epsilon < 2 \Delta_a (1 - \cos\theta), \quad (3)$$

where  $\epsilon$  represents the total measurement uncertainty.

The sensitivity of the experiment is therefore given by,

$$\delta c/c = \Delta_a/T = \epsilon/[2T(1 - \cos\theta)] \quad (4)$$

where  $T$  is a typical transmission time ( $T_{\max} \sim 12$  ms).

Maximal sensitivity is achieved when the measurements are taken at the beginning and the end of a single passage of the satellite directly above the station. In this case  $\theta \sim 180^\circ$ ,  $T \sim 12$  ms and the error accumulated due to the instability of the hydrogen masers is very small because of the short integration time of  $\sim 17$  min. Table 1 lists the individual sources of uncertainty that are estimated to exceed 1 ps. Four sources of uncertainty are listed in the table:

- (i) The stability of the hydrogen masers for integration times of 1000 s is of the order 2,1 parts in  $10^{15}$  [7] which gives an accumulated uncertainty of  $\sim 2$  ps per maser over an integration time of 17 min.
- (ii) As systematic errors in the on-board payload cancel when the two links are differenced, only its instability over 17 min contributes. Ten picoseconds [7] seems a conservative estimate for such a short integration time.
- (iii) Only the instability of the Earth station during the experiment contributes. Degnan [9] states that the precision of satellite laser ranging stations is of order 1 to 3 mm, which corresponds to an uncertainty of less than ten picoseconds.

(iv) Information on the counter uncertainties is provided by the T2L2 proposing team.

In the calculation of  $(T_1 - T_2) - (T_3 - T_4)$  the differences  $\tau_4 - \tau_1$  and  $\tau_3 - \tau_0$  measured by the space and ground clock respectively appear with a factor of 2. Hence all uncertainty sources participating in the measurement of these intervals ((i),(ii),(iv)) have been multiplied by this factor.

For the measurement of anisotropy in a direction which is not in the plane of orbit, the two links are separated by the time necessary for the Earth station to change its position with the rotation of the Earth so as to see the satellite from opposing directions ( $\sim 14500$  s). The hydrogen maser stability for such integration times is of the order  $1,5$  parts in  $10^{15}$  [7], which gives an uncertainty of  $\sim 44\sqrt{2}$  ps in (i). Contributions from other error sources are those given in Table 1. Hence the value for the total measurement uncertainty is  $\epsilon \sim 72$  ps. Note also that in this case  $\theta$  cannot exceed  $120^\circ$ .

Substituting these values for  $\epsilon$  and  $\theta$  into (4) gives an experimental sensitivity of  $\delta c/c = 7,9 \times 10^{-10}$  when the direction of the anisotropy lies in the orbital plane of the satellite and  $\delta c/c = 2 \times 10^{-9}$  otherwise. Following Krisher et al. [1] the experiment can be interpreted in the framework of the test theory by Mansouri & Sexl [6] resulting in limits on the parameter  $\alpha$  of  $\alpha = -1/2 \pm 4 \times 10^{-7}$  and  $\alpha = -1/2 \pm 1 \times 10^{-6}$  for the two cases, assuming  $v = 300$  km/s.

## Conclusion

The proposed test of the special theory of relativity is expected to improve the upper limit on anisotropy of the propagation time of light signals obtained from the best previous direct measurement [1] by a factor of  $\sim 430$ . It should also provide an improvement (by a factor of  $\sim 4$ ) on the value inferred from the measurement of the first order Doppler shift by Riis et al. [2]. The extension of this type of experiment to space-time domains (separation of the clocks of  $\sim 3700$  km) which are not attainable in a laboratory may also be an advantage. And last but not least, the experiment does not call for the installation of additional equipment, hence it can be considered an essentially no-cost experiment which is generally a decisive factor for research in fundamental science.

The same experiment could be performed using the PRARE microwave transfer system in the two-way ranging mode [7] rather than the T2L2 links. This might be of advantage as the PRARE method is not weather dependent. However, uncertainties in the ionospheric propagation delays due to different up and down link frequencies introduce an additional uncertainty of  $\sim 20$  ps per link, which slightly decreases the overall sensitivity of the

experiment to  $\delta c/c = 9.8 \times 10^{-10}$  for the case where the direction of the anisotropy lies in the orbital plane of the satellite and to  $\delta c/c = 2.1 \times 10^{-9}$  otherwise.

It is likely that the sensitivity of the experiment can be improved if data taken continuously during the passage of the satellite is used to search for the sinusoidal variation with  $\theta$  of the signal due to anisotropy. Furthermore, if a likely orientation of the presumed anisotropy is identified, for example the direction of the observed dipole anisotropy of the cosmic microwave background [10], it should be possible to improve the experimental sensitivity by statistical treatment of data from different stations and from repeated measurements.

Finally, it should be mentioned that the same type of experiment would yield increased accuracy if performed on satellites at higher altitudes, as this would decrease the  $\epsilon/T$  ratio in (4). One possible candidate is the Radioastron 1 mission (apogee 85000 km, perigee 2000 km) scheduled for launch in late 1996.

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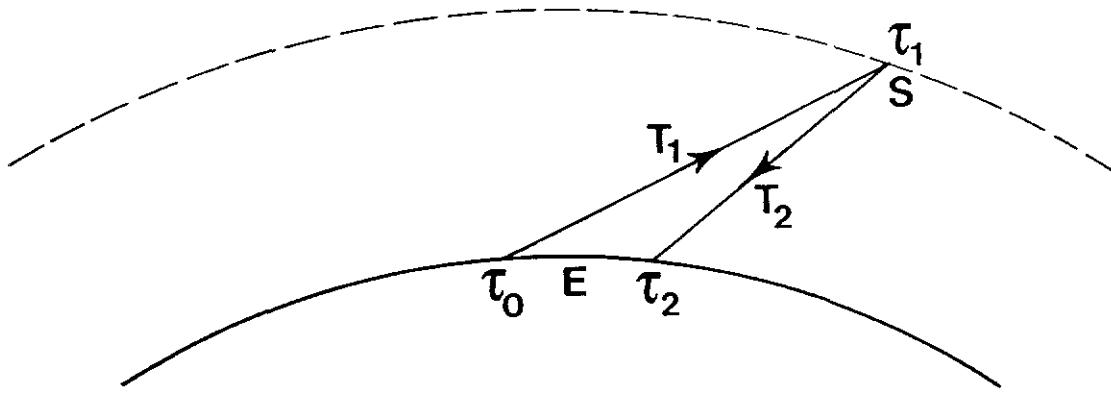
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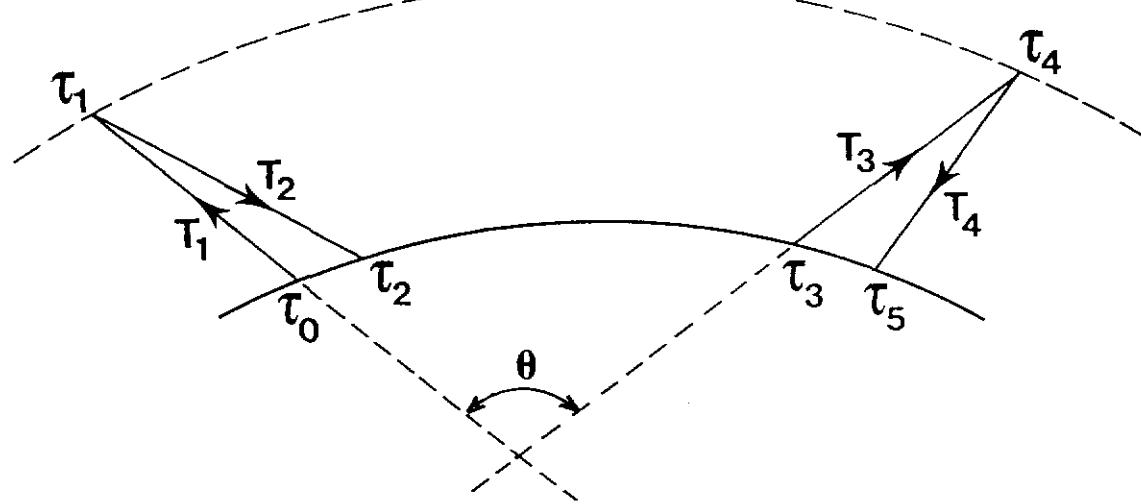
## Tables

Source of uncertainty	$\sigma/\text{ps}$
Hydrogen masers(i)	$4\sqrt{2}$
On-board payload(ii)	20
Earth station(iii)	$10\sqrt{2}$
Counters(iv)	$20\sqrt{2}$
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Total(quadratic sum)	$\epsilon = 38$

*Table 1: Anticipated uncertainty budget for measurement of an anisotropy whose direction lies in the orbital plane. All uncertainties are in picoseconds and correspond to an estimated one standard uncertainty,  $\sigma$ .*



*Figure 1: Two-way laser link between an Earth station and the satellite viewed in a geocentric, inertial frame.*



*Figure 2: A pair of two-way laser links between an Earth station and the satellite, as viewed in a geocentric, inertial frame.*

## QUESTIONS AND ANSWERS

**LUTE MALEKI (JPL):** The experiment that we did at JPL, as you know, was limited because of differential drift of the two H-masers which are not deterministic.

**PETER WOLF (BIPM):** Yeah, that was the first line of the error budget, which was — that is what I meant by "hydrogen maser," their instability over the integration time, just to accumulate an error in time.

**LUTE MALEKI (JPL):** No, I'm not talking about the individual instability, I'm talking about the drift that is indeterministic; one maser moves one way, and the other maser moves the other way.

**PETER WOLF (BIPM):** I didn't consider that. I will have to look into that. Thank you anyway.