

## DISCUSSION OF CLOCK RESIDUALS IN DEVELOPMENTAL GPS SATELLITES MEASURED WITH A SINGLE CHANNEL RECEIVER

Steven D. Chang and Timothy D. Maclay  
Bendix Field Engineering Corporation  
Columbia, Maryland

### ABSTRACT

The navigation potential of the NAVSTAR GPS system is integrally dependent on the ability to make accurate time transfer measurements. Examination of the clock offset data from several of the developmental GPS satellites shows systematic trends of unknown origin. By eliminating residual noise, a periodic behavior with a magnitude on the order of 30 to 50 nanoseconds is observed. A few possible sources of error, including improper application of relativistic calculations and limitations of the orbital and ionospheric models, are discussed.

### INTRODUCTION

The NAVSTAR Global Positioning System is a satellite system which when fully implemented will serve as a source of unprecedented accuracy in providing navigation and time information to users all around the globe. Now in the field test phase of full-scale development, the system will eventually consist of 18 satellites in 6 orbital planes. Synchronized to a common time, GPS time, the clocks on each satellite comprise the functional heart of the network. A Master Control Station will periodically upload parameters to the NAVSTAR Space Craft Vehicles (SV) in order to keep them updated with current orbital elements, clock offset, and vehicle health information. Presently, several satellites are maintained at test orbits in 2 different orbital planes. This constellation provides intermittent coverage to stationary receivers for test and developmental purposes.

Because of the systems vital dependence on extremely precise time keeping, much attention and effort have been directed toward refining the means and methods of accurate time transfer. This involves addressing many different areas of contributing errors, such as the random instability of the SV clocks, the orbital model of the satellites, relativistic effects of gravity and motion, and the effect of the atmosphere on signal propagation. The clock offset data from the 5 developmental satellites used show that present methods and operations do not fully eliminate error in time keeping.

The purpose of this paper is to examine the extent of error propagation in the time position data from these satellites and to suggest possible sources of apparent systematic inaccuracies.

#### Methods and Results

The clock offsets for the five developmental satellites were examined from a data file extending for a length of about nine months. These clock offsets represent both systematic and random effects. Systematic effects such as frequency offset and drift are readily apparent when clock offset is viewed as a function of time. Systematic effects such as these are also, of course, easily corrected for. However, any short term systematic effect that may be periodic with the satellite's orbit and unaccounted for by present models are not necessarily as evident as these expected trends. In order to isolate a periodic systematic deviation, the random noise must be filtered out and any recurring behavior must be extracted and elucidated. To accomplish this, the following method was employed. A twelve hour period was divided into a specific number of equal intervals--50 intervals of approximately 15 minutes each, for example. Each interval represents a particular position of the satellite in its orbit. After subtracting out the long term frequency offset and drift trends by the application of linear or quadratic fits, each residual clock offset was assigned to the appropriate interval, depending on when in the satellite's orbit the clock offset was measured. Thus, the result was a single 12 hour period with every data point for 9 months residing in a specific interval in the period. The points in each interval were then averaged to eliminate the random noise and to produce one representative point per interval.

The graphs in figures 1 and 2 show the residual offset values as a function of interval number (orbital position) for SV's 12 and 13, respectively. Tables 1 and 2 show these values along with the standard deviations for each interval. As can be seen, these residual offsets are described by a systematic and definitely periodic behavior. While the graphs of our satellites (figure 3) have notably similar shapes, it appears that the pattern seems to depend on both the orbital plane of the satellite and its position in the orbit (slot). Furthermore, the magnitude of this offset is on the order of 30 to 50 nanoseconds. This is certainly an appreciable deviation and is highly suggestive of error derived from systematic considerations. Several of the major possible contributors to this error are discussed below.

#### Random Walk

Random noise in measurements taken by a GPS receiver is composed of errors, among other sources, introduced by instabilities of the clocks on board the satellites. In a series of time measurements, the addition of this random frequency noise produces a random walk. For cesium clocks like those on SV12 and SV13, the random walk magnitude is on the order of 9 nanoseconds or 1 part in  $10^{13}$  for 1 day averaging (ref. 10). This error, if it is truly random, is eliminated by our method of data reduction. However, even the worst case magnitude of such errors should be on the order of approximately 9 nanoseconds per day; a result that can hardly account for the 30-50 nanosecond periodic variances we have observed. For a more complete and detailed description and analysis of random noise and systematic trends we refer you to reference 11.

#### Relativity

The relativistic corrections comprise another possible source of periodic errors. The velocity of the satellites, according to Einstein's theory of special relativity, affects the apparent frequency of the signals received. Since the satellites are also in a different gravitational potential than the receivers, general relativity predicts an apparent frequency shift in the opposite direction. Assuming circular orbits, these two effects are corrected for in the hardware and in the uploading process of the satellites. The orbits of the satellites are not precisely circular, however. They

typically have an eccentricity value on the order of  $1 \times 10^{-2}$  (ref. 3). The relativistic correction for non-circular orbits is left up to the receiver for which the following equation is used:

$$t_r = Fe(A)^{1/2} \sin E$$

where  $e$  is the eccentricity,  $A$  is the semi-major axis,  $E$  is the eccentric anomaly, and  $F$  is a constant whose value is  $-4.442809305 \times 10^{-10}$  sec/(meter) $^{1/2}$  (ref. 6). For very small eccentricities,  $E$  can be approximated as a constant angular rotation, wt. One can then arrive at a worst case magnitude of error (100%) and note that the general shape of this error is periodic in the orbit of the satellite. Graphs of relativistic corrections for the eccentricity of each of the five satellites are shown in figure 4. The magnitude of this correction varies slightly among satellites and is in the range of 10-20 nanoseconds.

While errors in this relativity correction are periodic and may be a contributing factor in systematic errors if applied incorrectly, the magnitudes of the effects plotted in figure 4 suggest that other, more predominant periodic errors must exist.

#### Ionospheric Model

An analysis of periodic, systematic errors in time and frequency transfer systems would not be complete without discussing ionospheric effects. A GPS signal, like all radio waves, is affected in several ways as it passes through this portion of the atmosphere. It is for this reason that John Klobuchar has developed a mathematical model of the ionosphere. In his article (ref. 4), Klobuchar discusses the errors introduced by the signal's interaction with free thermal electrons in the earth's ionosphere. With the exception of scintillation effects, all ionospheric effects discussed are modelled by a direct proportion (to 1st order) to the total electron content (TEC). The TEC is described in units of  $e1/m^2$  column and is by no means constant. Monthly overplots of TEC diurnal curves are presented in figure 5 (ref. 4).

It is estimated that this ionospheric model is accurate only to about 50% (ref. 4). Since the ionospheric delays for such satellite observations can be as large as 80 nanoseconds (ref. 3), we can assume a variance from the corrected value of up to 40 nanoseconds. This error can account, at least in part, for the magnitude of the standard deviations per interval in tables 1 and 2. A periodic error with such a magnitude is obviously a major contributor to the systematic variances we observed.

#### Conclusion

One of the most important aspects of accurate navigation systems like GPS is the ability to make precise time transfer measurements. To this end, many of the systematic trends which lead to predictable errors have been corrected for. However, after eliminating residual noise, we are still left to contend with some systematic, periodic errors on the order of 30-50 nanoseconds. Our largest possible source of systematic errors appears to be the ionospheric model, but errors and approximations in relativity corrections and unexplored sources of error such as inaccuracies in the orbital model of the satellites must also be dealt with. While several suggestions have been presented here to account for such observations, a more thorough investigation of these and other possibilities is certainly necessary before steps can be taken toward implementing further corrections.

### Acknowledgements

The authors would like to acknowledge Dr. Gernot M.R. Winkler and Dr. William J. Klepczynski at the United States Naval Observatory for their conceptual and technical assistance.

Acknowledgement is given to Peter Dachel, William Hanrahan, Frederick Blanchette, and Bruce Schupler at Bendix for guidance with theoretical and computer support, and for their coordination efforts.

We would especially like to thank Michael Bowie and everyone at NASA Headquarters, Customer Services, for making it all possible.

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TABLE 1  
DATA FOR SV12

INTERVAL	VALUE	# PTS	STAND DEV
1	15	61	33.677
2	13	50	35.278
3	14	52	35.695
4	20	47	39.636
5	33	54	42.930
6	42	48	75.344
7	39	64	80.853
8	40	56	84.867
9	39	73	81.966
10	42	60	85.997
11	40	85	79.017
12	38	76	77.615
13	36	88	77.780
14	23	78	45.691
15	17	98	54.448
16	12	82	54.882
17	10	110	54.861
18	4	82	62.233
19	2	102	53.093
20	0	80	53.226
21	-6	98	33.079
22	-10	80	34.009
23	-12	88	32.985
24	-20	79	29.909
25	-26	92	34.408
26	-31	88	39.672
27	-35	69	39.452
28	-41	80	38.115
29	-42	68	36.775
30	-47	82	40.298
31	-48	62	37.617
32	-50	70	33.881
33	-47	53	43.069
34	-45	60	40.424
35	-39	49	41.048
36	-28	63	46.257
37	-27	49	54.699
38	-26	53	62.685
39	-14	43	55.057
40	-6	51	52.586
41	1	43	48.527
42	11	51	45.443
43	15	44	42.091
44	13	53	40.091
45	2	48	111.540
46	8	62	92.473
47	8	53	89.490
48	10	57	75.552
49	22	50	25.003
50	21	56	27.445

TABLE 2  
DATA FOR SV13

INTERVAL	VALUE	# PTS	STAND DEV
1	1	100	31.178
2	5	84	35.051
3	11	84	36.144
4	11	73	34.761
5	13	90	35.155
6	10	72	34.757
7	11	84	32.324
8	10	65	34.189
9	10	75	33.983
10	9	60	37.948
11	7	66	32.399
12	5	54	23.983
13	5	52	20.228
14	4	46	18.075
15	12	54	28.279
16	7	46	22.005
17	7	51	23.654
18	11	45	38.129
19	15	62	44.094
20	16	58	45.693
21	17	78	47.396
22	16	67	46.953
23	14	69	50.028
24	13	72	50.291
25	15	71	48.910
26	11	98	43.780
27	12	83	42.909
28	10	95	40.207
29	10	86	40.492
30	4	113	42.674
31	4	94	40.007
32	4	123	40.070
33	0	102	38.548
34	-4	119	40.854
35	-8	94	42.322
36	-17	125	40.665
37	-20	95	35.923
38	-23	111	36.488
39	-25	92	36.860
40	-26	113	36.029
41	-26	87	38.086
42	-24	107	40.772
43	-15	78	35.589
44	-11	101	36.824
45	-11	81	28.056
46	-11	108	33.901
47	-3	82	34.573
48	3	89	39.108
49	1	80	38.164
50	0	86	34.315

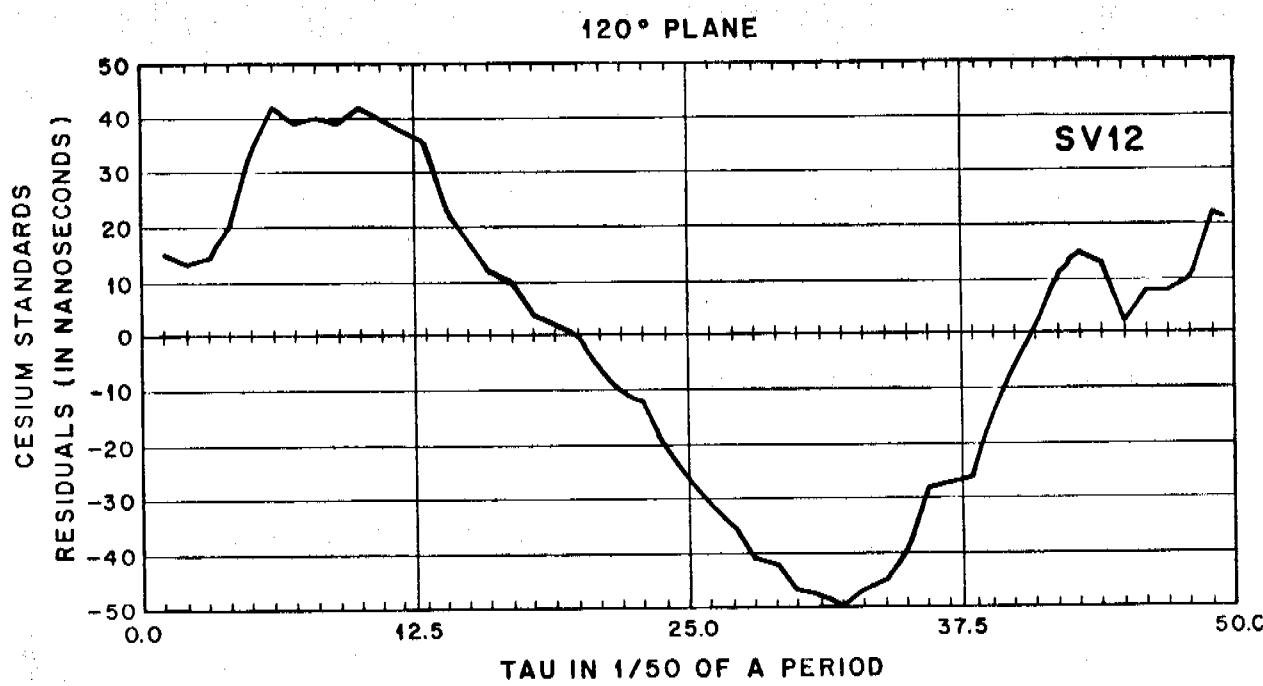


FIGURE 1

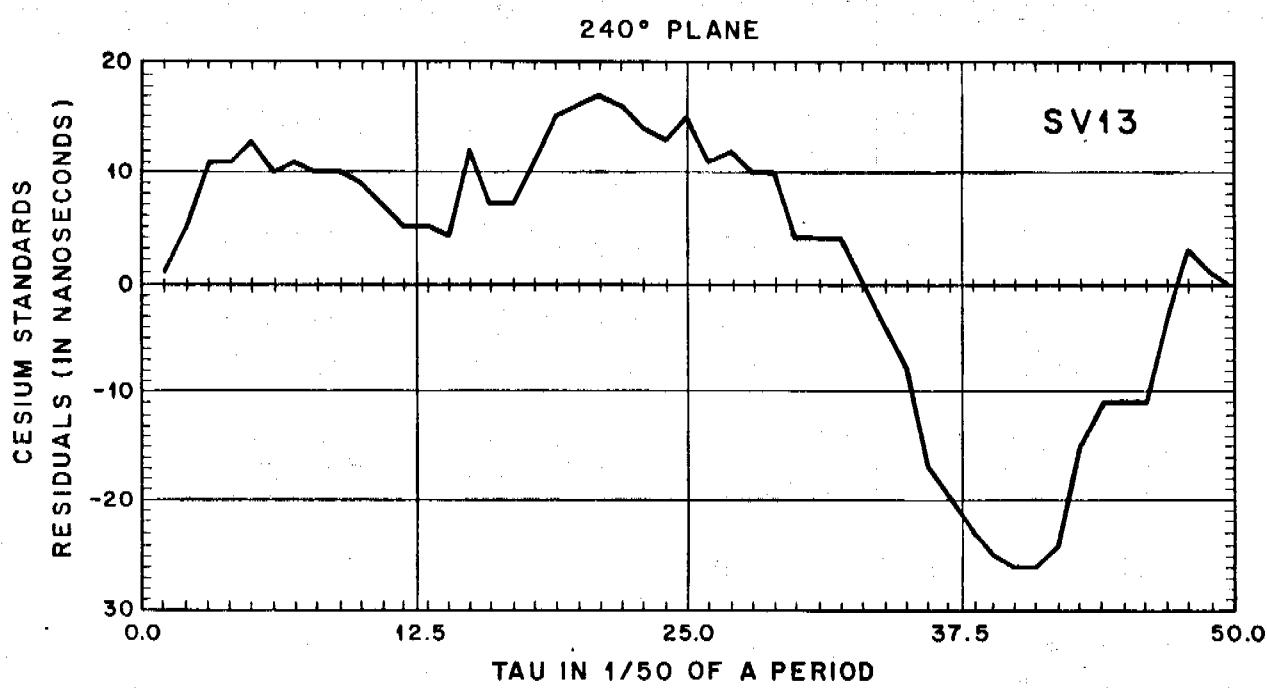
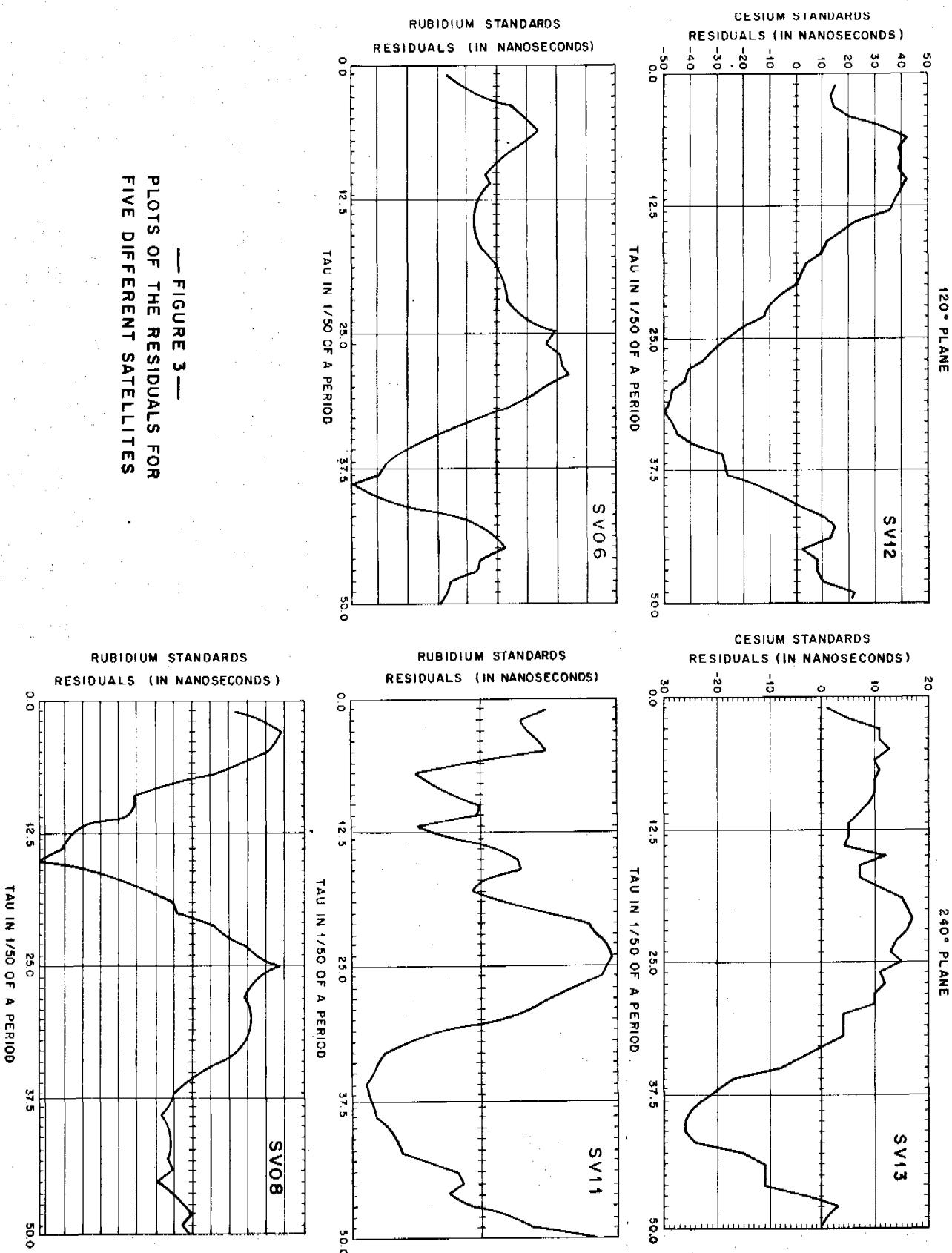


FIGURE 2

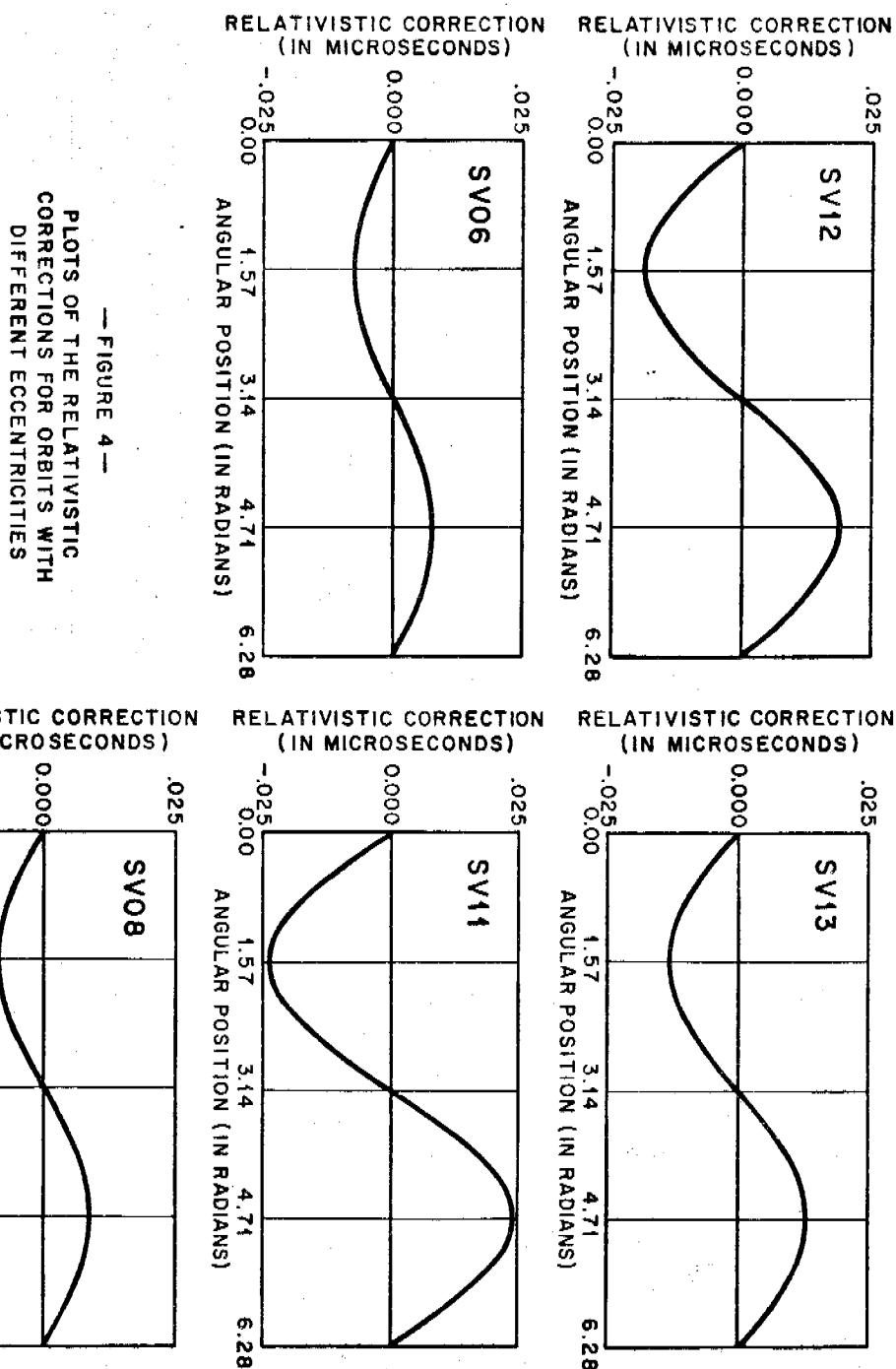


## RUBIDIUM STANDARDS

## CESIUM STANDARDS

120° PLANE

240° PLANE



— FIGURE 4 —  
PLOTS OF THE RELATIVISTIC  
CORRECTIONS FOR ORBITS WITH  
DIFFERENT ECCENTRICITIES

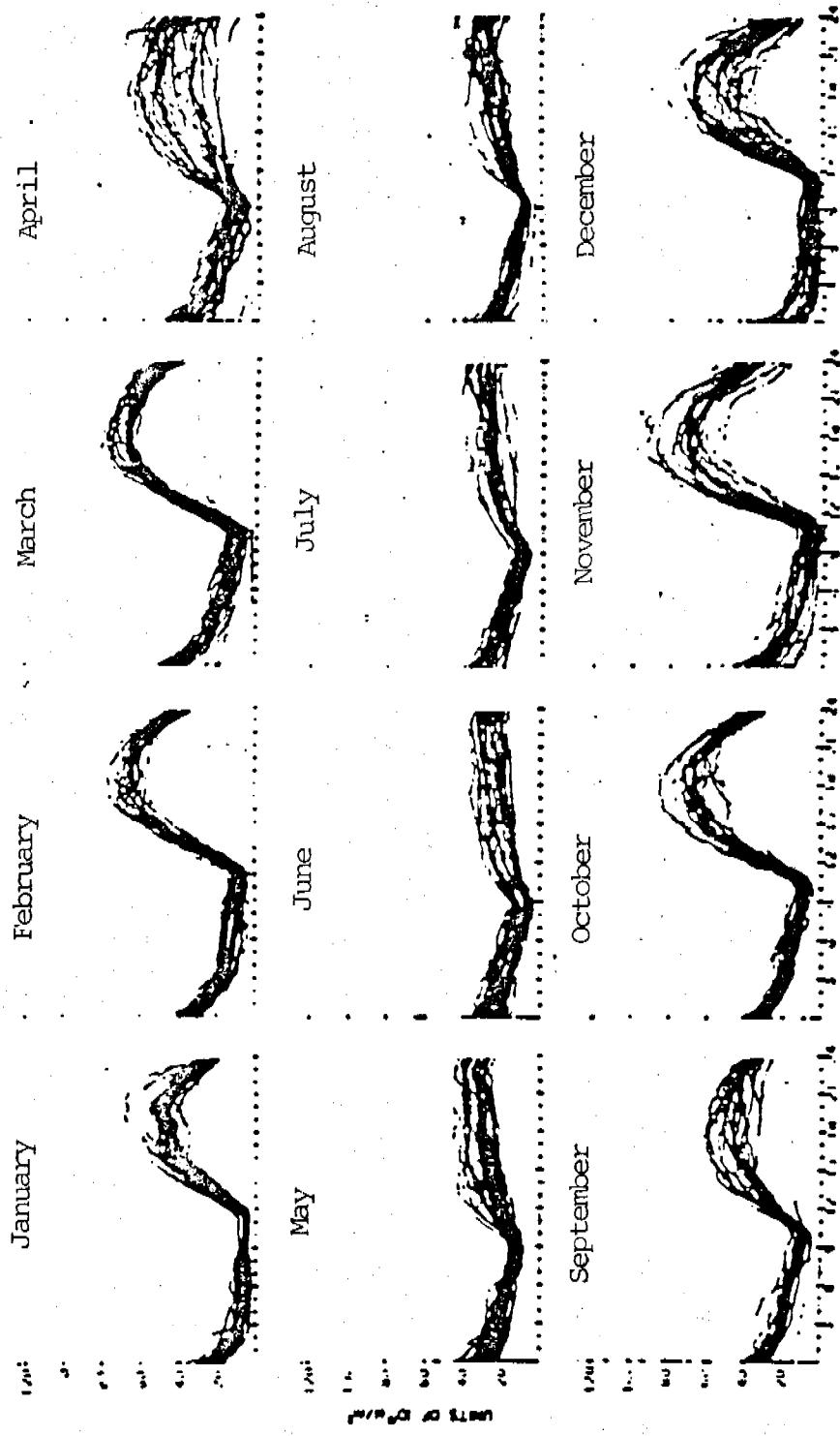


Figure 5 -- Monthly overplots of TEC diurnal curves.

