MEASUREMENT OF THE FREQUENCY STABILITY OF RESPONDERS IN AIRCRAFT

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ABSTRACT

Measurement on an aircraft orbit, such as a satellite launching orbit, is made by the responder in the aircraft along with several remote track stations on the ground. During the launching, the system is required to have precise time synchronization and frequency accuracy. At the same time, accurate measurement of aircraft velocity requires high frequency stability of the system. However, atomic frequency standards in the ground stations supply time and frequency reference standard with excellent long term and short term frequency stability for the above-mentioned goals. The stability of responder is also an important factor affecting the performance of the system and there are more requirements for the corresponding time / frequency measurements. In the system, the responders do not use continuous wave (CW) but narrow pulse modulated wave, consequently, the characterization theory of their stability is more complicated and the measurement technique is more difficult for pulsed wave than that for CW.

In this paper, a systematic characterization theory of the frequency stability for pulsed wave is demonstrated and the measuring methods are discussed. The paper describes the measurement systems, which have been set up in Beijing Institute of Radio Metrology & Measurement (BIRMM) and can be used to test the frequency stability of pulse coherent responders in time domain and frequency domain with high sensitivity and accuracy. Using these measurement systems, we have made successful measurements for the responders, with which the satellite launching orbits have been precisely obtained and tracked.

INTRODUCTION

In the 1960s D.W. Allan, J. A. Barnes, etal proposed to adopt finite sampling variance to characterize the short-term frequency stability of continuous wave signal, and they also deduced the mathematical relation between the finite sampling variance and the phase noise power spectral density, thus, establishing a theoretical system of taking Allan variance (finite sampling variance of continuous double sampling) as the time domain characterization of the short-term frequency stability and phase noise as the frequency domain characterization (1).

In the 1970s the measurement technique of Allan variance and phase noise was further studied, and many companies turned out their relevant measuring equipment. On this basis, many countries established their respective metric standards for short-term frequency stability one after another. Since then, this complete set of characterization theory and measurement technique has been widely approved and used (2), (3), (4).

So far, the above-mentioned characterization theory and measurement technique have found a wide application for various continuous wave and quasi-continuous wave signals. However, the unstability of the pulse-modulated wave, eg. an RF pulse of the radar transmitter, is a major factor affecting the system performance after all. The RF pulse being usually a pulse-modulated wave with a small duty ratio, the charaterization theory of continuous wave signals is not applicable now and the technical difficulty of the measurement has also increased. Therefore, it is a practical problem for us how to characterize the short-term frequency stability of radio-frequency pulse theoretically and how to measure it.

Some systems have been employed to measure the frequency stability of the pulse-modulated wave in time domain and frequency domain, as reported many times (5), (6), (7), (8). M. I. Sholnik, J. M. Milan, etal also presented the relation between the pulse-to-pulse frequency fluctuation of radio-frequency pulse and the radar improvement factor (9), (5). The commercial measuring equipment for frequency stability such as HP3048 phase noise measurement system being employed, however, can only be used in the measurement of continuous waves or quasi-continuous waves with the duty ratio greater than 10%. In fact, most of measurement systems for short-term frequency sta-

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bility of the RF pulse are developed by the users themselves.

As a time / frequency metrological and measurement service, BIRMM has been studying the characterization theory and measurement technique for short-term frequency stability of RF pulse for more than ten years(10); and it has succeeded in developing many kinds of measurement systems for this function, which have been used in the measurement of various radar transmitters and pulse power amplifiers. Some scientists from BIRMM published their papers in this aspect in the PTTI meeting (11), (12).

The frequency stability measurement of pulse coherent responders is another successful example. The pulse coherent responder in aircraft is an important component of the trajectory measurement system, whose frequency stability determines the measuring accuracy of aircraft orbits. As it operates in a narrow RF pulse condition with the pulsewidth of about 1μs, no instrument is available to measure its frequency stability. To settle this problem, BIRMM has established time-domain and frequency-domain systems: (1) to characterize the short-term frequency stability in time domain by Allan variance of single carrier spectrum and measure it by the fine spectrum extracting method; (2) to characterize the short-term frequency stability in frequency domain by near-carrier phase noise and measure it by the microwave phase bridge. These two systems have the advantages of high sensitivity and low noise. The pulse coherent responders tested by them have been used in the orbit measurement for satellite launching successfully.

REQUIREMENTS FOR THE FREQUENCY STABILITY OF RESPONDERS

Measurement on an aircraft orbit, such as a satellite launching orbit, is made by the responder in the aircraft along with several trajectory measurement stations on the ground. During the launching, the system is required to have precise time synchronization and frequency accuracy. Meanwhile, accurate measurement of aircraft velocity requires high frequency stability of the system. To meet the above-mentioned requirements, atomic frequency standards are employed in the ground measurement station to supply time and frequency references. The stability of the responder is also an important factor affecting the performance of the system. The pulse coherent responder is mounted in aircraft and operates in a pulse-modulated state. After receiving a

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pulse–modulated microwave signal from the ground station, the responder modulates various motion parameters of aircraft onto this signal, which is then sent back to the ground station through Doppler shift for data processing, thus, realizing the measurement of aircraft orbit.

The carrier frequency of the responder input signal is correlated to that of its output signal. Therefore, if the frequency of the responder proper is not stable or there occurs additive frequency / amplitude–modulated noise, the whole trajectory measurement system will not be able to extract Doppler shift properly or measure it accurately, thus, failing in the accurate measurement of aircraft orbit and flight velocity.

There is a typical pulse coherent responder which operates in a microwave frequency range with pulse width less than 1μs. In order to ensure the time synchronization and frequency accuracy during the launching, it is designed to have excellent long–term frequency stability; besides, it also has good short–term frequency stability to meet the requirements of velocity measurement for the system. The short–term frequency stability is embodied in frequency domain and time domain. The system requires that the frequency stability of the responder $\sigma y^2(\tau)(\tau \approx 0.1\text{s})$ should be superior to 1×10^{-10}, while the background noise of the phase noise measurement system should be at least 20dB lower than the corresponding phase noise of the tested responder.

CHARACTERIZATION OF THE SHORT–TERM FREQUENCY STABILITY OF PULSE–MODULATED WAVE

1 Time domain characterization

In mathematics, variance characterization is adopted in time domain and power spectral density characterization in frequency domain to describe a random process. In fact, however, we usually use finite sampling variance to estimate the above variance, which is described as below:

$$< \sigma^2 (N,T,\tau) > \; = \; < \frac{1}{N-1} \sum_{n=1}^{N} (\overline{Y}_n - \frac{1}{N} \sum_{n=1}^{N} \overline{Y}_n)^2 > \tag{1}$$

where

    T: sampling repetition period,

τ: sampling time interval,

$\overline{Y}_n$: n periods, sampling average value of relative frequency fluctuation within τ.
Now, we will discuss special cases of the following two finite sampling variances.

(1)Continuous double sampling: The finite sampling variance $<\sigma^2(N,T,\tau)>$ obtained in case of $T=\tau$, $N=2$ is well-known Allan variance $\sigma_y^2(\tau)$. It is generally acknowledged as the time domain characterization method of frequency stability thanks to its advantages of easy measurement and near-carrier convergence. It can be proved that

$$\sigma_y^2(\tau) = \frac{1}{2} < (\overline{Y}_1 - \overline{Y}_2)^2 > \qquad (2-a)$$

(2)Noncoherent sampling ($T / \tau >> 1$). Now, $<\sigma^2(N,T,\tau)>$ is an unbiased estimation of true variance and is also irrelevant to N. By selecting $N=2$, we take the finite sampling variance $<\sigma^2(2, T, \tau)>$ ($T >> \tau$) obtained as the time domain characterization of short-term frequency stability of pulse-modulated wave, named as Allan variance $\sigma^2(T, \tau)$. It can be seen that the pulse-to-pulse variance is an unbiased estimation of true variance as long as the sampling group numbers are many enough in doing this. It is characterized by easy measurement and near-carrier convergence, the same as Allan variance; therefore, formula (2) is still applicable here:

$$\sigma^2(T,\tau) = \frac{1}{2} < (\overline{Y}_1 - \overline{Y}_2)^2 > \qquad (2-b)$$

It is more important that the pulse-to-pulse variance is identical to the actual condition of pulse-to-pulse fluctuating characteristics required by radar transmitted pulse, so it is more suitable to characterize radar transmitted signal.

2   Frequency domain characterization

It has a long history to be engaged in characterization of frequency domain of a random process by power spectral density (13); and it is also generally acknowledged that power spectral density of phase noise can be taken as the frequency domain of short-term frequency stability. However, there are some sharp differences between the phase noise of RF pulse and that of continuous wave, which are described as follows.

(1) For an RF pulse, only the frequency-modulated spectrum which offsets from the carrier frequency by $1 / 2T$ and where Doppler frequency is applied is taken into consideration. Moreover, according to the sampling theorem, only the above-mentioned spectrum can be got for pulse-to-pulse sampling data. For this reason, we call the RF pulse as near-carrier phase noise and use it to characterize the frequency domain of the

short-term frequency stability of pulse-modulated wave.

(2) After a continuous wave signal is processed through pulse modulation, a superposition effect is produced in its power spectrum. Now the power spectrum offsetting from the carrier frequency by $1 / 2T$ can be considered as the superposition of a series of sideband spectra, which can be written as:

$$S_p(f) = \frac{1}{2} \sum_{n=-\infty}^{\infty} S_c(f + \frac{n}{T})$$

(3)

where

$S_c(f)$: power spectrum of continuous wave,

$S_p(f)$: power spectrum of modulated and superposed pulse.

3 Relations between time domain and frequency domain

It can be proved that there occur some relations between true variance and power spectral density of generalized stationary random process, which are stated as follows:

$$\sigma^2(\tau) = (\frac{1}{f_0})^2 \int_0^\infty S_\varphi(f)(\frac{Sin\pi\tau f}{\pi\tau})^2 df$$

(4 − a)

or
$$\sigma^2(\tau) = \int_0^\infty S_y(f)(\frac{Sin\pi\tau f}{\pi\tau f})^2 df$$

(4 − b)

The equivalent signal of finite sampling variance can be thought to pass a linear system; thereby, we can get the relation between finite sampling variance and power spectrum as follows:

$$< \sigma^2(N,T,\tau) > = \int_0^\infty S_y |G_0(f)|^2 |G_N(f)|^2 df$$

(5)

Where

$$G_0(f) = (\frac{Sin\pi\tau f}{\pi\tau f})^2, \qquad G_N = \frac{N}{N-1}[1 - (\frac{Sin N\pi\tau f}{N Sin\pi Tf})^2]$$

As mentioned above, the finite sampling variance means Allan variance in case of $N = 2$, $T = \tau$. The expression is:

$$\sigma_y^2(\tau) = 2\int_0^\infty S_y(f)\frac{Sin^4(\pi\tau f)}{(\pi\tau f)^2} df$$

(6)

Now, we make some analyses on the pulse-modulated wave. For a random process of a discrete signal, the Wiener-Khinchin theorem is also applicable here, whose expressions are:

$$S_m(\omega) = \sum_{m=-\infty}^{\infty} R(m)e^{-jmT\omega} \qquad (7-a)$$

$$R(m) = \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} S_m(\omega)e^{jmT\omega} d\omega \qquad (7-b)$$

It can be seen that samplings are irrelated to each other when $T >> \tau$, that means, $<Y_N> = 0$. Then the true variance is:

$$\sigma^2 < \overline{Y}> = <[Y_N - <Y_N>]^2> = <Y_N^2> = R(0) \qquad (8)$$

Substituting this formula into the Wiener–Khinchin theorem, we can obtain:

$$\sigma^2(\overline{Y}) = R(0) = \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} S_m(\omega) d\omega \qquad (9)$$

Based on what it mentioned above, we can draw a conclusion that the linear system through which the pulse–to–pulse finite sampling variance passes also possesses the transfer function described in formula (5), which is expressed as below:

$$\sigma^2(N,T,\tau) = (\frac{T}{2\pi}) \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} S_m(\omega) G_N d\omega \qquad (10)$$

Let $N = 2$, then we can get the relation between pulse–to–pulse variance and pulse–to–pulse power spectrum as follows:

$$\sigma^2(T,\tau) = 4T \int_0^{\frac{1}{2T}} S_P(f)Sin^2(\pi Tf)df \qquad (11-a)$$

or
$$\sigma^2(T,\tau) = 4T \int_0^{\frac{1}{2T}} (\frac{f}{f_0})^2 S_\varphi(f)Sin^2(\pi Tf)df \qquad (11-b)$$

To characterize the frequency stability of RF pulse by pulse–to–pulse variance, formula (11) can be employed to change the relation between the variance and the power spectral density. The frequency stability of the power coherent responder discussed in this paper is characterized by Allan variance of carrier frequency single CW spectrum produced through fine spectrum extraction. Therefore, the conversion relation provided in formula (6) is also applicable here.

MEASUREMENT TECHNIQUE AND ESTABLISHMENT OF THE MEASUREMENT SYSTEMS

In principle, the measurement technique for CW short–term frequency stability is still applicable in the measurement of the frequency stability of pulse–modulated wave.

However, as the duty ratio of the RF pulse is very small, the available effective energy is very low. The measurement system usually requires a much higher sensitivity than that of continuous wave and its collecting speed is much faster than that of the latter; moreover, synchronous collection is required to avoid missing any data. According to the requirements for the frequency stability of responder and the above-mentioned characterization theory, we have set up time domain and frequency domain measurement systems.

1 Time domain measurement system

In order to meet the requirements for Doppler velocity measurement accuracy, it is necessary to measure the short-term frequency stability of the master spectrum obtained after RF pulse is filtered and smoothed. Now the signal spectrum appears signal master spectrum and is in the nature of continuous wave, that means, to extract the master spectrum from the RF pulse. The above-mentioned method is the so-called fine spectrum extracting method. The time domain measurement system used to measure the frequency stability of pulse coherent responder is just designed on the basis of this method. For its block diagram, see Fig.1. As shown in the Figure, through the high-stability crystal oscillator the frequencies of the two frequency synthesizers become interrelated. Having passed through the responder to be tested, the signal of one channel is changed into a low-intermediate frequency signal after being processed once or twice by the frequency conversion unit. Then it is used to extract master spectrum through the narrow-band crystal filter and measure Allan variance $\sigma_y(\tau)$ of the master spectrum.

2 Frequency domain measurement system

Adopting microwave phase bridge is the optimum method to measure additive phase noise of the dual-terminal component. It is also available to the measurement of additive phase noise and pulse-to-pulse phase fluctuation of the pulse coherent responder. For the block diagram of the frequency domain measurement system see Fig.2. Its expression of phase-demodulating principle is:

$$V(f) = K_\varphi [2\pi\tau_0 f \Phi_R(f) + \Phi_A(f)] \tag{12}$$

where

V(f): video output of the phase discriminator,

$\Phi_R(f)$: phase fluctuation of the source,

$\Phi_A(f)$: additive phase fluctuation of the responder,

K_Φ: sensitivity of the phase discriminator,

τ_0: dual–channel delay difference.

It can be seen from formula (12) that when the dual–channel electrical length remains the same, the system background noise produced owing to the unstability of the signal source is very small and is only determined by the thermal noise of the phase discriminator. Therefore, the sensitivity of the system can be designed to be very high for the sake of satisfactory measurement of the responder.

SYSTEM CALIBRATION

There are two calibration methods involved for the frequency domain measurement system of the short–term frequency stability of radio–frequency pulse. One is to determine the transfer function of the system, the other is to produce a known pulse–to–pulse frequency (phase)–modulated signal (e.g. discrete spectrum), by which the measurement random spectrum can be calibrated.

1   Determination of transfer function: The typical method is to determine the phase demodulating sensitivity. Generally, a balanced phase discriminator works in an unsaturated region; and its phase demodulation output voltage is related to dual–channel phase difference sinusoidally as shown below:

$$V = V_0 Sin\Phi \tag{13}$$

when $\Phi < < 1$,

$$V = V_0\Phi \qquad\qquad K_\varphi = \frac{V}{\Phi} = V_0 (V / rad) \tag{14}$$

Therefore, we can measure out peak voltage V_0 of the output video–frequency pulse by adjusting the variable phase shifter, then the phase demodulating sensitivity K_φ near the quadrature point can be obtained.

2   There are many methods employed to produce a known frequency (phase)–modulated signal. One of them is to connect an electrically tunable phase shifter with the circuit, thus, producing a known phase–modulated signal. The electrically tunable phase shifter can be composed of varactor diodes (14). Another method is to produce a known

pulse-to-pulse variance with a frequency-halving circuit. For its block diagram see Fig.3. When positive and negative peak values of the video frequency pulse are corresponding with those of the frequency-modulated signal through adjusting delay line, the pulse-to-pulse variance of the output signal is the peak frequency offset.

3 It can be seen through calibration that the background of the time-domain measurement system is $\sigma_y(100ms) \approx 1 \times 10^{-12}$, better than that (1×10^{-10}) of the responder to be tested by two orders of magnitude; while the background noise $S_\phi(20Hz)$ of the frequency domain measurement system is -80 dBc / Hz or so, about 30dB lower than the additive phase noise of the responder.

ACKNOWLEDGEMENTS

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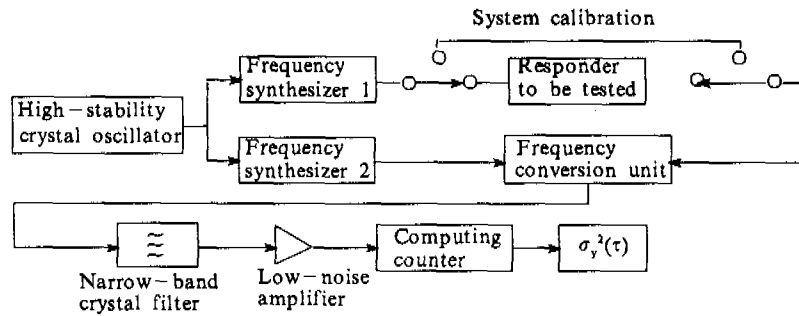
Fig.1 Block diagram of the time domain measurement system for pulse coherent responder
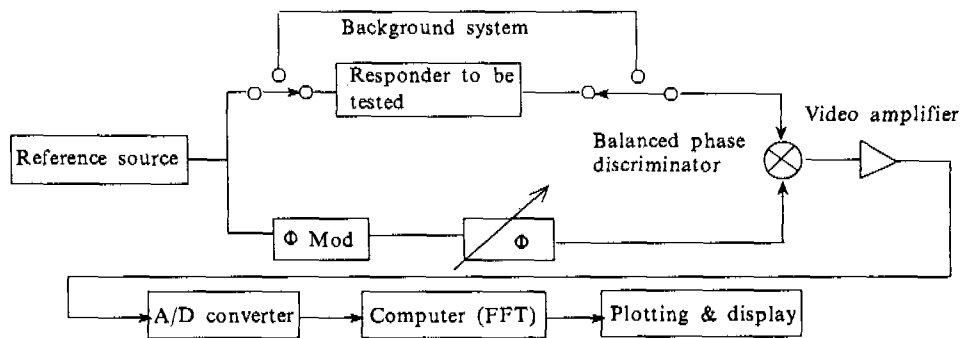

Fig.2 Block diagram of the frequency domain measurement system for ICW responder
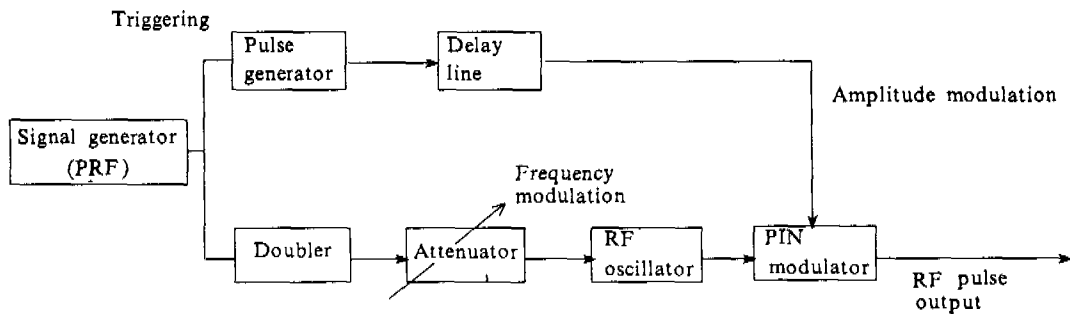

Fig.3 Calibration circuit of pulse − to − pulse variance

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