On Systematic Uncertainties in UTC

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***Abstract*—The International Bureau of Weights and Measures (BIPM) urrently computes the systematic uncertainties between Coordinated Universal Time (UTC) and the UTC(k), its realizations by participating labs k, using an algorithm that does not take into account the effects of the clock model in the computation of UTC. We present an algorithm that computes the systematic uncertainty for UTC-UTC(k) as the uncertainty of an optimal comparison between UTC(k) and all the other UTC realizations. The weights to be assigned to the links between lab k and each other lab are determined from a matrix calculation that incorporates the correlations between the links, and assures that the sum of the weights is unity via a Lagrange multiplier. We speculate that an iterative approach may be sufficient to allow for the effect of the uncertainties of each lab in the determination of the uncertainties of the other labs.**

NOTE: ***This differs from the published version in that equation A-15 is revised. However, the author believes that the Lagrangian multiplier algorithm does not fully incorporate the proposed concept. Rather, the covariances are better incorporated using a matrix approach similar to what was applied in Appendix I of his paper "Investigating a Null Tests of the Einstein Equivalence Principle with clocks at Different Solar Gravitational Potentials", in the same IFCS-2016 proceedings.***

Keywords—UTC, uncertainties, systematic)

I. INTRODUCTION

An essential requirement for metrological traceability to a standard, such as UTC, is that the uncertainty of a comparison with the standard be known, or at least estimated. It is conventional to lump together as "Type A" all statistical uncertainties, which are those that can be reduced by more measurements of the same qualitative kind. Systematic uncertainties, which cannot in general be reduced by a larger quantity of measurements, are of Type B. In the last decade, Lewandowski, Matsakis, Panfilo, and Tavella (LMPT) published a series of papers proposing an algorithm to compute the statistical and the systematic uncertainties of UTC-UTC(k) [1-3]. In addition, G. Petit derived a simple matrix-formulation that would facilitate the computations [4; see also 5]. The LMPT algorithm took into consideration the fact that deterministic contributions to UTC, such as leap second insertions and the frequency-steering to the primary frequency standards, made no contribution to the uncertainties. Rather, the uncertainty of any UTC-UTC(k) was determined by the uncertainties of the time-transfer links between laboratory k and all the other participating labs, as well as the weights that the BIPM actually used for the labs' clocks in computing UTC, with full allowance being made for the correlations between the links used for UTC.

LMPT did not take into account the fact the UTC is computed on the basis of individual clock deviations from a clock model based upon recent performance [6]. The effect is to make UTC equivalent to an integrated frequency scale, and the initial values of the UTC(k) can be considered constraints that determine the constants of integration. Ignoring the clock models is justified for statistical uncertainty calculations because the models are deterministic over any computation of UTC. But to compute type B errors, consideration needs to be given to the fact that the model absorbs the systematic errors in the time-transfer links between the labs; these errors are in fact the calibration biases. Since UTC is defined in terms of the individual UTC(k) (in the form UTC-UTC(k)), we propose that u_k, the systematic uncertainties in UTC(k), be based upon an estimate of the systematic uncertainties in the time difference between lab k and the other labs, ignoring the laboratory weights but with full allowance for correlations between the links. Here optimality is defined with respect to the systematic uncertainties in the UTC'(k) = UTC-UTC(k), as computed by the BIPM. (If the biases in all UTC links were known to be exactly zero, then so would be each UTC'(k), along with its uncertainty.)

Because the systematic uncertainties and actual link biases are not in any way used in the computation of UTC, use of the LMPT algorithm can lead to the nonsensical result that raising one lab's systematic uncertainty too much can greatly raise the systematic uncertainties of every other laboratory [7]. The reason this does not happen when the statistical uncertainties of the link to a lab are large is because the statistical noise of the links contributes to the noise evaluation of its attached clocks. This is because a noisier link will have larger scatter in its deviations from the clock model, and that will result in a lower weight being assigned to the laboratory's clocks. Perhaps a better way to phrase this is that the UTC algorithm is designed to minimize the statistical uncertainty of UTC-UTC(k). If correctly tuned, the weights will be adjusted to optimize any configuration, and the addition of even the noisiest of links and clocks will improve UTC to some degree, as would be reflected in reduced Type A uncertainties.

II. THE TOPOLOGY AND THE CORRELATIONS

If all time transfer links were uncorrelated and followed a normal distribution, the optimal comparison of UTC(k) with the average of all other laboratories would weigh each difference with UTC(n) by the inverse of the link's systematic squared uncertainty U_{kn}. If $U_{B,k}$ is defined to be the square of UTC'(k)'s systematic uncertainty u_k, and U_{kn} the systematic squared uncertainty of UTC(k)-UTC(n), then in this theoretical case we have

$$U_{B,k} = 1/[\Sigma(1/ U_{kn})], \qquad (1)$$

and the uncertainty itself is $u_k = \sqrt{U_{B,k}}$.

An example of a network that might fulfill these assumptions is one in which all links are to a common lab (pivot), such as the one currently used to compute UTC(Figure 1); however in this case many of the links to the pivot lab are not independently calibrated. We consider some simpler topological examples before providing a more general approach in section VI.

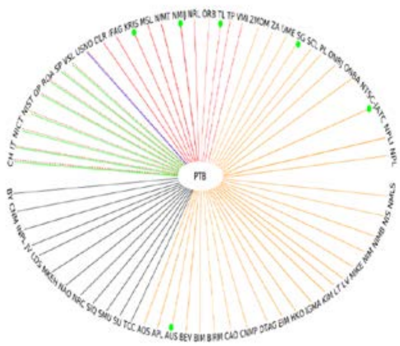

Figure 1. Topology of the links currently used in UTC generation. The color signifies the techniques employed, and the lettering indicates the acronyms of the laboratories participating in UTC. Most important is that the individual labs are linked through a common pivot lab (PTB)

As a notational matter, let us use the abbreviation PPP for all links whose calibrations are site-based. A site-based calibration depends only on independent calibrations of the systems at each lab; this would be the case for Precise Point Positioning if every lab's GNSS system were absolutely calibrated or calibrated consistently with travelling receivers. A site-based system also follows strict closure, by which we mean that the signed sum of the three links between three labs is identically zero, even when not calibrated [8]. For laboratories whose UTC representations can be written X, Y, and Z, their closure sum is (X-Y)+(Y-Z)+(Z-X), where the quantity in each parenthesis represents a direct timing measurement via a UTC-contributing time-transfer link or set of links. We shall use TW to refer to links that are independently calibrated, as is usually the case for Two Way Satellite Time Transfer (TWSTT) and will likely be the case for "self-calibrated" fiber-optic links in the near future. The closure sums of such links are not in general zero unless separately and perfectly calibrated.

In order to discuss the correlations, let us use the topology of Figure 2.

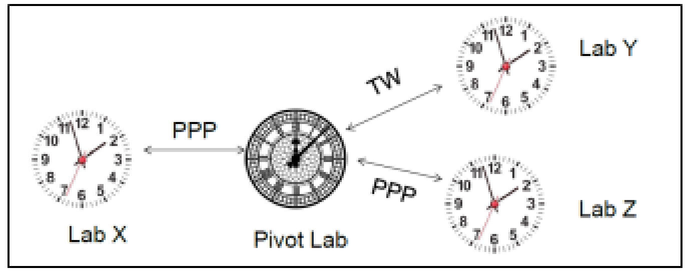

Figure 2. Types of links used for UTC generation

With the topology and properties of the Figure 2 links, the squared uncertainties of the links between Lab X and the other three labs (pivot P, X, and Y) would be correlated as follows:

$$U_{link,X,P} = P_X + P_P \qquad (2)$$

$$U_{link,XY} = U_{link,XP} + T_{YP} \quad = P_X + P_P + T_{YP} \qquad (3)$$

$$U_{link,XZ} = P_X + P_Z \qquad (4)$$

where P_X is the site-based squared uncertainty of the PPP system at Lab X, while T_{YP} is the link-based squared uncertainty of the TW system between labs P and Y.

Relations (2) and (4) hold because the PPP systems at the labs in question are independently calibrated. In the case of relation (4), the PPP data from the pivot lab cancel out because they appear in both links with opposite sign. Relation 3 holds because the calibration of the links between the TW link are considered uncorrelated with any other link, especially PPP ones.

III. AN ALL-PPP NETWORK

In an all-PPP network of N labs and N-1 links, the closure sums of links between any three labs are strictly 0, and the pivot lab holds no special role. Therefore the link between any two labs is given as in equations (2) and (4). It can be immediately realized that no amount of averaging of the links between lab k and the other labs will reduce the contribution from a bias in the system at lab k, whose squared uncertainty is P_K, because P_K is common-mode. If in addition each system's uncertainties were assumed equal to $\sqrt{P}$, averaging would reduce the net contribution of the squared uncertainties of the other laboratories by $\sqrt{}$ (N-1), so that the squared uncertainty of lab k would be given by`

$$U_{B,k} = P_k + (P_x)/(N-1) = N*P/(N-1) \qquad (5)$$

$$\text{and the uncertainty } u_k = \sqrt{U_{B,k}} \qquad (6)$$

IV. AN ALL-TW, SINGLE-PIVOT NETWORK

The link uncertainties of an all-PPP network equal the root-sum-square (RSS) of the uncertainties of the two labs involved; the uncertainty of a TW link is <= to the RSS of the two labs' uncertainties. We consider two cases herein, neither of which is necessarily valid. In Case 1, the lab uncertainties are considered to be completely correlated with, and incorporated in, the link uncertainties. In Case II, the lab uncertainties are uncorrelated with the link uncertainties. In a single-pivot network based only on TW links, even if all time transfer links have identical squared uncertainties T, the pivot lab would be in a unique situation. Its links with the N-1 other labs would have equal weight, and in Case I its squared uncertainty would be given by

$$U_{B,Pivot} = T/(N-1)\ ; \quad u_{Pivot} = \sqrt{U_{B,Pivot}} \tag{7}$$

In Case II, the non-pivot uncertainties would be relevant, and we assume all link uncertainties equal T:

$$U_{B,Pivot} = (T+U_{B,Non\text{-}pivot})/(N-1)\ ; \quad u_{Pivot} = \sqrt{U_{B,Pivot}} \tag{8}$$

For every non-pivot lab however, we must realize that the correlation between the links extends only from lab k to the pivot; the connection between the pivot and lab N is uncorrelated with the connection between the pivot and lab M, so that the correlated uncertainty between lab k and any other lab is just the uncertainty between k and the pivot. This component of the systematic uncertainty component will not average down, while the extra uncertainty going from the pivot to a third lab just adds noise. To illustrate this, Figure 3 invokes a simplified example of a three-lab all-TW network, for which we wish to compute the weights that minimize the systematic uncertainty for lab k.

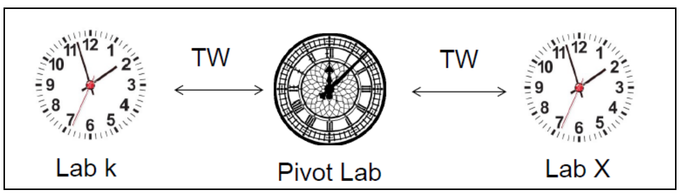

Figure 3. A three-lab network with two TW links.

In this simplified geometry the Case 1 uncertainties of the links from lab k to the other two labs are given as follows:

$$U_{link,kP} = T_{kP} \tag{8}$$

$$U_{link,kX} = T_{kP} + T_{PX} \tag{9}$$

If the pivot lab P is assigned weight w, and Lab X weight (1-w)

$$U_{B,k} = w{*}T_{kP} + (1-w){*}(T_{kP} + T_{PX}) = T_{kP} + (1-w){*}T_{PX} \tag{10}$$

To minimize $U_{B,k}$, we require w=1; the link to lab X receives no weight while the link from lab k to the pivot receives unity weight. It follows that the optimal weighting for the uncertainty of lab k assigns the link to the pivot lab unity weight, and zero weight to the links from k to the other labs. Therefore, for Case I the systematic uncertainty of any non-pivot lab k can be written:

$$U_{B,non\text{-}Pivot} = T\ ; \quad u_{non\text{-}pivot} = \sqrt{T} \tag{11}$$

For Case II, with N-1 links, equation 10 would be written:

$$U_{B,k} = w{*}(T+U_{B,Pivot}) + (1-w){*}(T+(T+U_{B,k})/(N-2)) \tag{12}$$

To minimize $U_{B,k}$, it is easily shown that we again require w=1, and the squared uncertainty of a non-pivot lab becomes

$$U_{B,k} = T{*}(N-1)/(N-3) \tag{13}$$

V. A TWO-LAB NETWORK

A hypothetical network with only two labs, and one link, presents a counter-intuitive solution because the systematic uncertainties of the labs are equal, independently of how many clocks each lab might have. The reason is that, though one lab may have many more clocks and a correspondingly higher weight in UTC, the mean value of each lab's corrected UTC representation (UTC') is 0. Of course, the uncertainty experienced by a user is the RSS of the systematic and the statistical uncertainties, therefore an out-of-network user might be well advised to extract UTC by linking to the lab with the highest weight.

VI. THE GENERALIZATION OF CASE 1*

*See the note below the abstract.

For every lab k, we are interested in determining the weight vector, of N-1 components w_n, that would minimize the uncertainty of S, the weighted sum of the UTC'(n):

$$S = \Sigma\, w_n\, (UTC'(k) - UTC'(n)) \tag{14}$$

S has zero mean because each UTC'(n) has zero mean, and its squared uncertainty $<S^2>$ is given by

$$<(\Sigma\, w_n\, (UTC'(k)-UTC'(n)) {*} (\Sigma\, w_n\, (UTC'(k)-UTC'(n))> \tag{15}$$

The problem reduces to minimizing $\Sigma\, w_m\, w_n\, U_{k,m,n}$ (16)
where $U_{k,m,n}$ is the correlation between the systematic errors of the set of UTC links that connects lab k with lab m and the set that connects lab k with lab n . The minimization is subject to the constraint

$$1 = \Sigma\, w_m \tag{17}$$

The set of weights that produce a minimum uncertainty for each lab k can be easily found using least squares with a Lagrange multiplier, and this is shown in Appendix I. Using this more general formula, the results for the three special cases given above can be confirmed.

VII. THE GENERALIZATION OF CASE 2

In Case 2, the laboratory uncertainties are implicit in the expansions of equations (14) and (15). They would contribute to the diagonal elements of equation (16), for which m=n, but they would not contribute to the off-diagonal terms if the laboratory uncertainties or their chains of links to lab k are uncorrelated at any stage. We suspect that an iterative approach may converge. In such a strategy, the lab uncertainties would at first be ignored and the minimization problem solved as with Case 1. The resulting type B uncertainties would then be included in equation (16) for a second minimization. The process would be repeated until a desired convergence level is, hopefully, attained.

VIII. CONCLUSIONS AND A COMMENT

The systematic uncertainties in the evaluation of UTC-UTC(k) may be determinable through an optimal differencing of the UTC(k) with every other available official realization. The specific means to carry this out may an iterative method of determining the weight assigned to each lab via a least-squares solution under the constraint that the sum of the weights are unity; this is a matter for further work.

We comment that a slightly different definition of the Type B uncertainties in UTC-UTC(k) would greatly simplify the mathematics. This would be to define them as those whose RSS's equal the Type B uncertainties of the links used to generate UTC, after taking all link correlations into account.

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Appendix I – Determining the Uncertainties to lab k with a Lagrange Multiplier

$0 = \partial / \partial w_n [(\Sigma w_m w_n U_{k,m,n}]) - \lambda (1- \Sigma w_m)]$ (A-1)

where the summations are over the N-1 labs excluding lab

We absorb the factor of 2 in λ, so

$0 = \Sigma w_m U_{k,m,n} + \lambda$ (A-2)

and $1 = \Sigma w_m$ (A-3)

where λ = Lagrange Multiplier (A-4)

Let W by a weight-vector, defined with the N-1 labs and λ as follows:

$W_m = w_m$ for $m<N$; $W_N = \lambda$ (A-5)

i.e. $W = (w_1 , w_2 , w_3 , \ldots. w_{N-1} , \lambda)$ (A-6)

Let V be an NxN matrix, defined with the N-1 other labs and λ as follows:

$V_{mn} = U_{k,mn}$ for m,n = 1 to N-1 labs (A-7)

$V_{mN} = 1$ for $m<N$ (A-8)

$V_{N,m} = 1$ for $n<N$ (A-9)

$V_{NN} = 0$ (A-10)

i.e. $V = (U_{k,1,1} \; U_{k,1,2} \; U_{k,1,3} \ldots. \; U_{k,1,N-1} \quad 1$ (A-11a)

$U_{k,2,1} \; U_{k,2,2} \; U_{k,2,3} \ldots. \; U_{k,2,N-1} \quad 1$ (A-11b)

… (A-11c)

$U_{k,N-1,1} \; U_{k,N-1,2} \; U_{k,N-1,3} \ldots. U_{k,N-1,N-1} \quad 1$ (A-11d)

$1 \quad 1 \quad 1 \quad \ldots \quad 1 \quad 0)$ (A-11e)

Let $F = (0, 0, 0, \ldots 0, 1)$ (A-12)

Then equations A-2 and A-3 can be expressed: $F = V * W$ (A-13)

So that $W = V^{-1} * F$ (A-14)

Finally, use the weights in W to compute $U_{B,k} = \Sigma \; w_m \; U_{k,m,n}$ (A-15)

and the systematic uncertainty, $u_k = \sqrt{U_{B,k}}$ (A-16)