

STABILITY AND NOISE SPECTRA OF RELATIVE LORAN-C FREQUENCY COMPARISONS

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ABSTRACT

Relative comparisons of Loran-C frequency transmissions between the master station of Catanzaro (Simeri Crichi) and the X, Z slave stations of Estartit (Spain) and Lampedusa (Italy) are carrying out by the GG LORSTA Monitor Station of the Mediterranean Sea Loran-C chain. These comparisons are able to emphasize the relative and, under certain conditions, the absolute rate of the emitting standard frequencies of the slave stations and some relevant statistical properties of the Loran-C Method for frequency transmission and time synchronization. In fact the stability of each Loran-C frequency standard transmission is subject to perturbations, more or less known, due to the propagation medium and other causes. Following the Allan (1966) method for data processing, the performance of the relative rate of frequency of the transmissions of the X, Z slave stations are described calculating the standard deviation $\sigma(N, \tau)$ of a set of N frequency measurements from its mean averaged during sampling times τ . We designate this standard deviation as the measure of the stability of the Loran-C frequency transmission.

Typical performance of Loran-C transmissions has been successively studied by determining the Spectral density of the relative frequency variations between the X, Z slave stations as regards master station.

One of the parameters addressed by this Interoperability Committee was timing and synchronization. The recommendations, and supporting rationale for these recommendations, will be provided.

INTRODUCTION

In order to study the stability of the Loran-C radio-navigation system in the Mediterranean chain and to determine its typical performance, relative comparisons of Loran-C frequency transmissions between the master station of Catanzaro (Simeri Crichi) and the X, Z slave stations of Estartit (Spain) and Lampedusa (Italy) are carrying out by the CG LORSTA Monitor Station of the Mediterranean Sea Loran-C chain. All Loran-C station transmit on 100 kHz

and are controlled by cesium-beam atomic clocks. The analyzed data are referred to the period January–December 1972 for the Estartit station and to the period September–December 1972 for the Lampedusa station as this last station is entered upon office only from September 1972. The phase differences $\Delta\phi(M - X)$ and $\Delta\phi(M - Z)$ between the frequency of the master and slave stations X and Z as received at the Cagliari Monitor Station has been reconstructed taking in account the corrections produced at the emitting station in order to closely synchronize each slave station to a common reference. The Table 1 gives us a statistical distribution of the phase and time steps carried out at the emitting slave stations. These data show that phase variations due to steps in time are generally smaller than $0.1 \mu\text{sec}$ this means that the virtual or relative synchronism of the Loran-C time scale is maintained to about $0.1 \mu\text{sec}$.

Table 1

Statistical Distribution in Percent of Man Made Time Steps Carried Out in the Emitting X and Z Loran-C Stations of the Mediterranean Chain

Range in microsecond	(M - X) %	(M - Z) %
0.00 – 0.03	0.0	0.0
0.03 – 0.06	16.0	15.5
0.06 – 0.09	35.8	31.0
0.09 – 0.12	42.0	45.7
0.12 – 0.15	4.9	3.1
0.15 – 0.18	1.0	0.8
0.18 – 0.21	0.3	3.9

Denoting by $\Delta\phi(M - S) = B(M - S)$ the daily phase difference of the slave stations X and Z with respect to the primary frequency of the master station as received at the Monitor Station and by $d\phi(M - S)$ the phase steps brought into the emission, the daily calculated phase difference between slave and master station will be

$$\Delta\phi(M - S)_{\text{cal}} = \Delta\phi(M - S) + d\phi(M - S)$$

The quantities $\Delta\phi(M - S)_{\text{cal}} = B(M - S)_{\text{cal}}$ correspond to the theoretical relative phase differences between the master station and the X, Z slave stations. In Figure 1 and Figure 2 are plotted the real (full points) and theoretical (blank points) integrated time scales defined as $\Sigma B(M - S)$ and $\Sigma B(M - S)_{\text{cal}}$.

DETERMINISTIC INSTABILITIES

The theoretical time scales $\Sigma B(M - X)_{\text{cal}}$ and $\Sigma B(M - Z)_{\text{cal}}$ represent the relative performance of the master time scale with respect to the X and Z time scales. The instabilities of each time scale are due to deterministic process such as, for instance, frequency drift or systematic changes (real or apparent) in the path and to non deterministic processes (random fluctuations). From the date given in Figure 1 and Figure 2 we can deduce that the relative rate of the Master with respect to X and Z stations was +29.2 nsec/day and +35.6 nsec/day respectively or about $+3.4 \times 10^{-13}$ and 4.1×10^{-13} . These data show the excellent stability of each time scale and of the cesium beam clocks. The progressive phase differences $\Sigma B(M - S)$ observed at intervals of fifteen minutes was analysed for emphasizes, the performance of this time scale upon periods included in the intervals of 0.5 - 24 hours and 1 - 45 days. The spectral densities of the time scale $\Sigma B(M - Z)$ plotted in Figure 3 are the trimensual average of the values calculated on periods of thirty days. It is interesting to point out the existence of peaks common to each curve which could be associated to systematic fluctuations in time scale. On the contrary the spectral density plots of time variation referred to the same scale $\Sigma B(M - Z)$ for period between 1 and 45 days is given in Figure 4.

The theoretical daily rate of time scales $\Sigma B(M - S)$, represent a significant parameter for the study of frequency drift in cesium standards (Winkler et al., 1970). The values of the relative daily frequency rate B for the frequency comparison ($M - X$) and ($M - Z$) are plotted in Figure 5. No noticeable drift appears for the relative rate $B(M - X)$; as regards the rate $B(M - Z)$ the available data are too little to be able to draw a conclusion, because these frequency shifts can only be determined reliably over very long periods of time. It is interesting however to point out a certain similarity in the trend and in the peaks of the common part to the curves. The spectral analysis of the quantities $B(M - Z)$ shows the probable existence of periodical irregularities of about 40 and 120 days with amplitudes of $3 - 4 \times 10^{-13}$. If these fluctuations would result real a their cause could be looking for in the cesium beam standards or in the relative path variations.

NON DETERMINISTIC FLUCTUATIONS

Non random phase fluctuations of the cesium beam introduced along a propagation path may be studied by using suitable statistical techniques. A useful measure in time and frequency domain has been shown to be the value of the standard deviation. Following D. Allan (1966) we have described the performance of the relative rate of frequency of the Loran-C transmission for the X and Z slave stations with respect to master station, calculating the standard deviation $\delta(N, \tau)$ of a set of N frequency measurement from its mean versus sampling time. We

designate this standard deviation as the measure of the stability of the Loran-C frequency transmission. The Figure 7 and Figure 8 shows the trend of the standard deviation $\sigma_y(\tau)$ (black points) of the relative frequency fluctuations versus measurement interval τ in the frequency comparisons between the master stations M and the slaves X and Z. The results obtained in both cases are very similar but more interesting are the results obtained from the B (M - Z) data owing to the very long measurement interval available.

We can see (Fig. 8) that for sampling time $\tau < 1$ day and $\tau > 30$ days about the Loran-C comparisons exhibit white phase noise. For these ranges the stability results

$$\begin{aligned} &\text{about } 100 \text{ ns for } \tau < 1 \text{ day} \\ &\text{about } 250 \text{ ns for } \tau > 30 \text{ days} \end{aligned}$$

On the contrary Loran-C relative comparisons exhibits "perturbing" noise (white FM noise?) for sample time ranging from 1 day to 30 days about, the existence of these noise could be associated to the existence of long-term drift with period of about 5 - 10 and 40 days (see Fig. 6).

CONCLUSION

The phase fluctuations analyzed are composed of the following components:

$$\begin{aligned} \Delta\phi(\text{measured}) = & \Delta\phi(\text{propagation}) + \Delta\phi(\text{cesium}) + \Delta\phi(\text{transmitter}) \\ & + \Delta\phi(\text{receiver}) + \Delta\phi(\text{residual}) \end{aligned}$$

If we assume that the phase noise in the receiver and transmitter is negligible with respect to the cesium and propagation fluctuations (Allan & Barnes, 1967) we can write

$$\sigma_m^2 = \sigma_p^2 + \sigma_c^2 + \sigma_{rs}^2$$

Assuming, following Winkler et al., (1970), for the daily standard deviations of a cesium clock about $1 - 2 \times 10^{-12}$, we can point out from the observed values of σ_m given in Figure 7 and Figure 8 that fluctuations due to the influence of the medium on the propagation delay result of the same order of the random fluctuation of cesium beam, that is about 10^{-12} .

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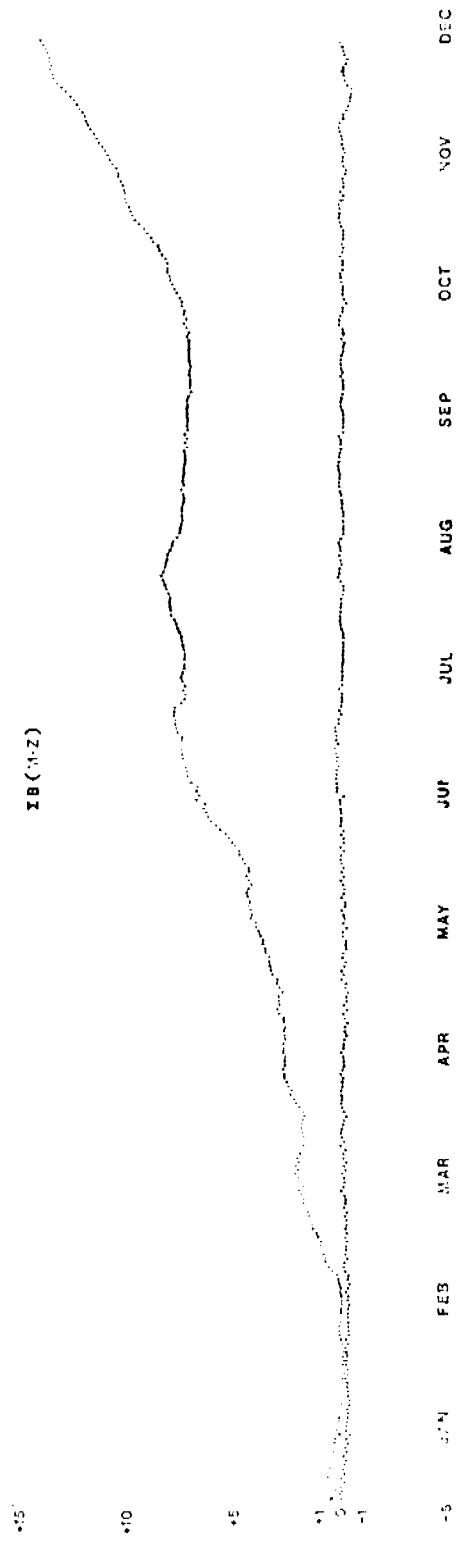


Figure 1. Theoretical relative time scale (blank points) between the Master Station (Simeri Crichti) and Z slave station (Lampedusa) of the Loran-C Mediterranean sea chain during 1972. Blank points represent the real relative time scale after adjustments steps in phase carried out at the emission.

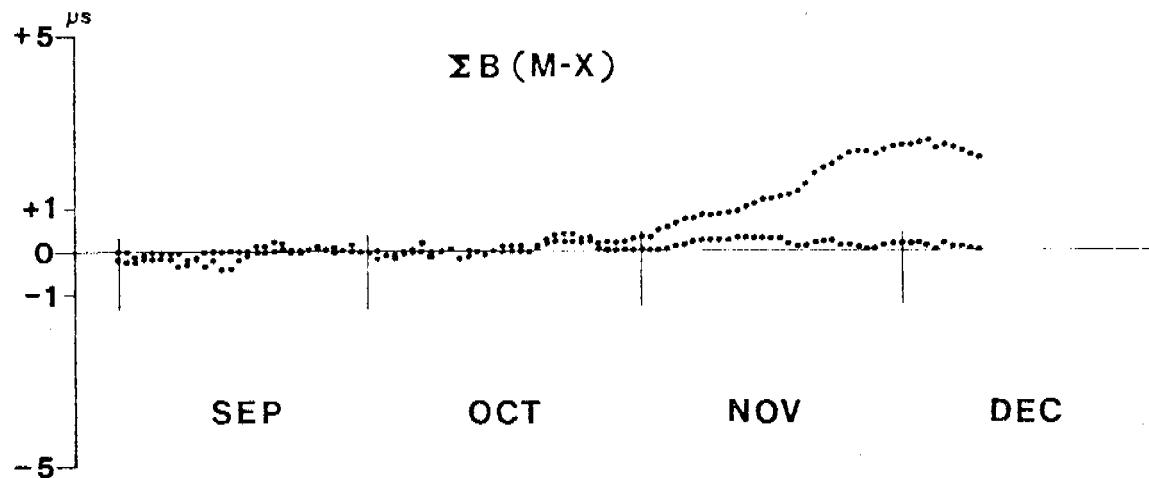


Figure 2. Theoretical and real relative time scales between the Master Station (Simeri Crichti) and X slave station (Lampedusa) as received at the Cagliari Monitor Station.

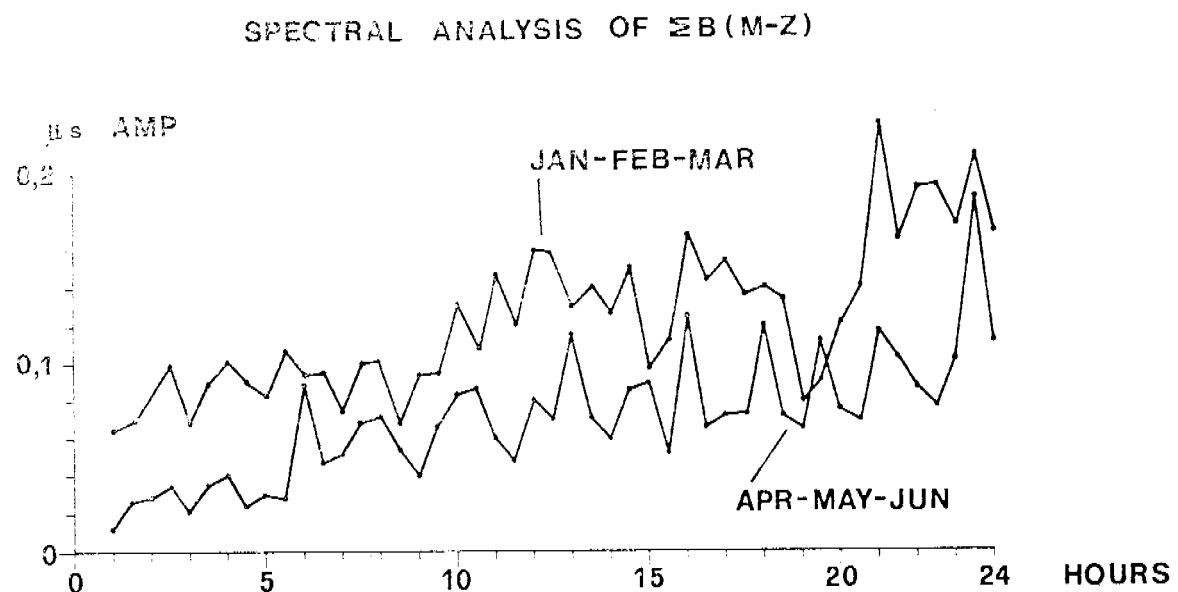


Figure 3. Spectral density plot of relative time scale $\Sigma B (M - Z)$ determined with a sampling period of 0.25 h.

SPECTRAL ANALYSIS OF ΣB (M-Z)

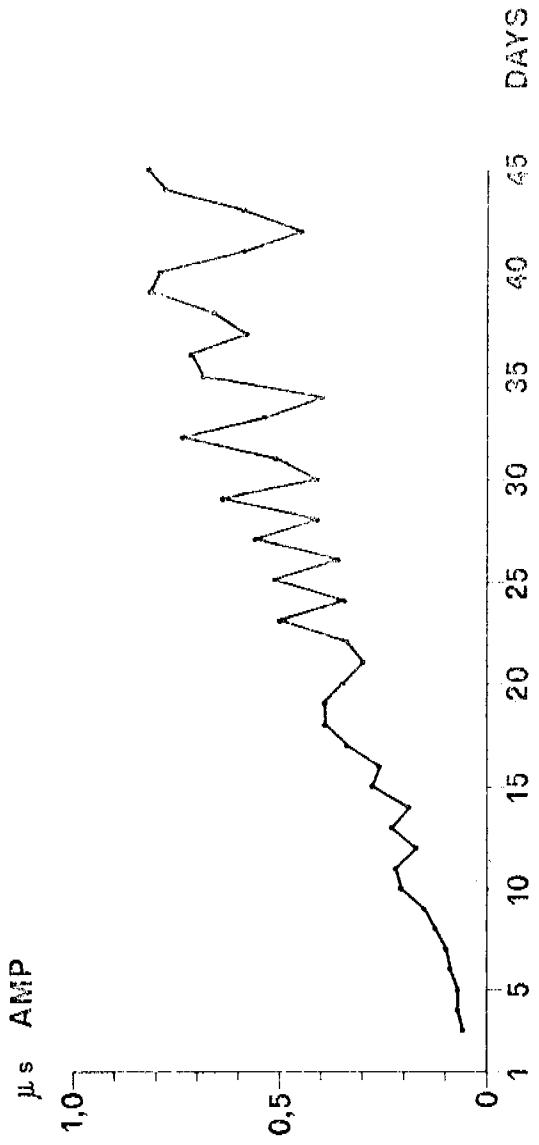


Figure 4. Spectral density plot of relative time scale ΣB (M - Z).
Time was determined with a sampling period of 1 day.

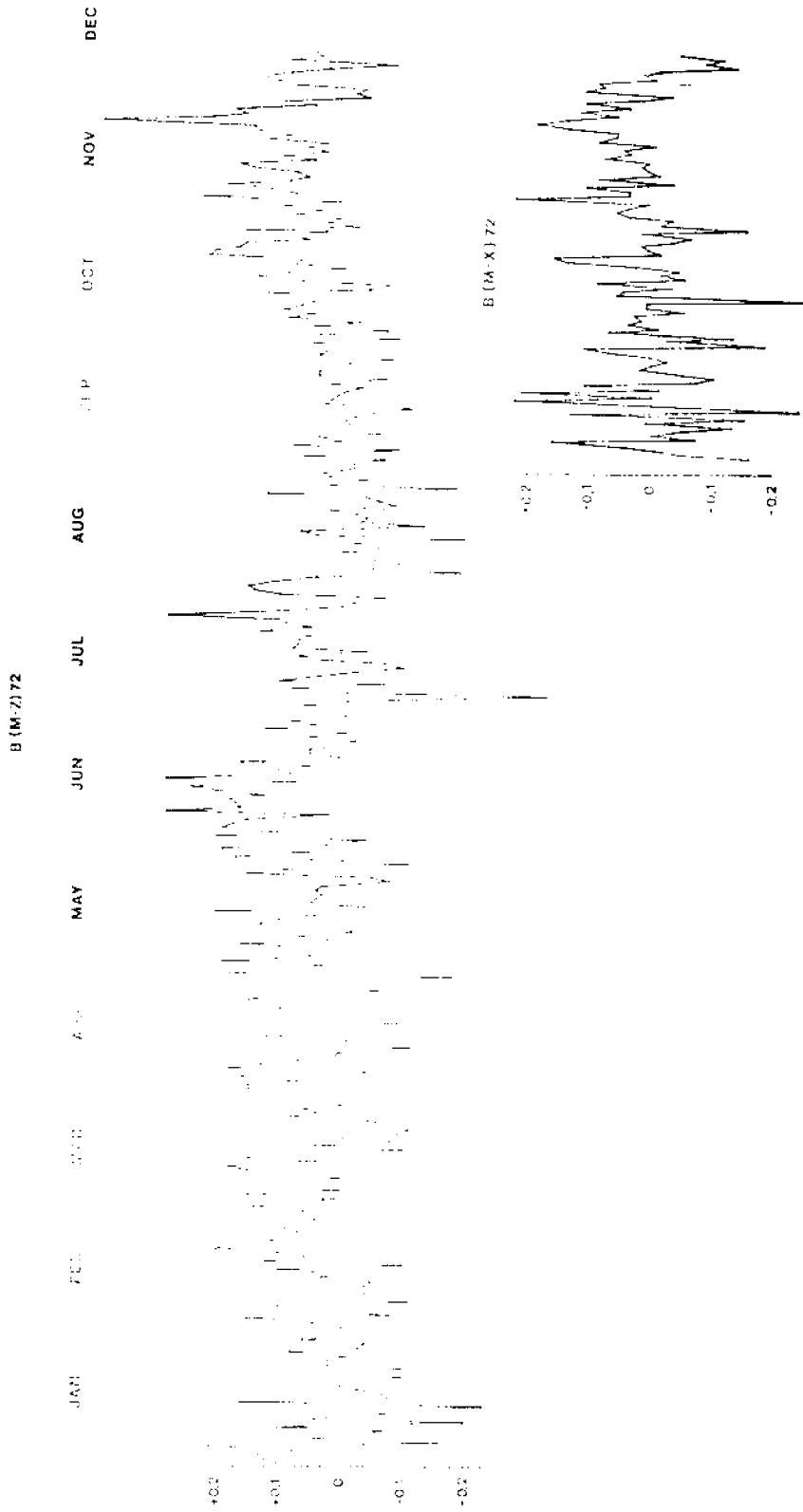


Figure 5. Plot of the daily relative frequency rate of (M - X) and (M - Z) Loran-C stations.

SPECTRAL ANALYSIS B(M-Z)

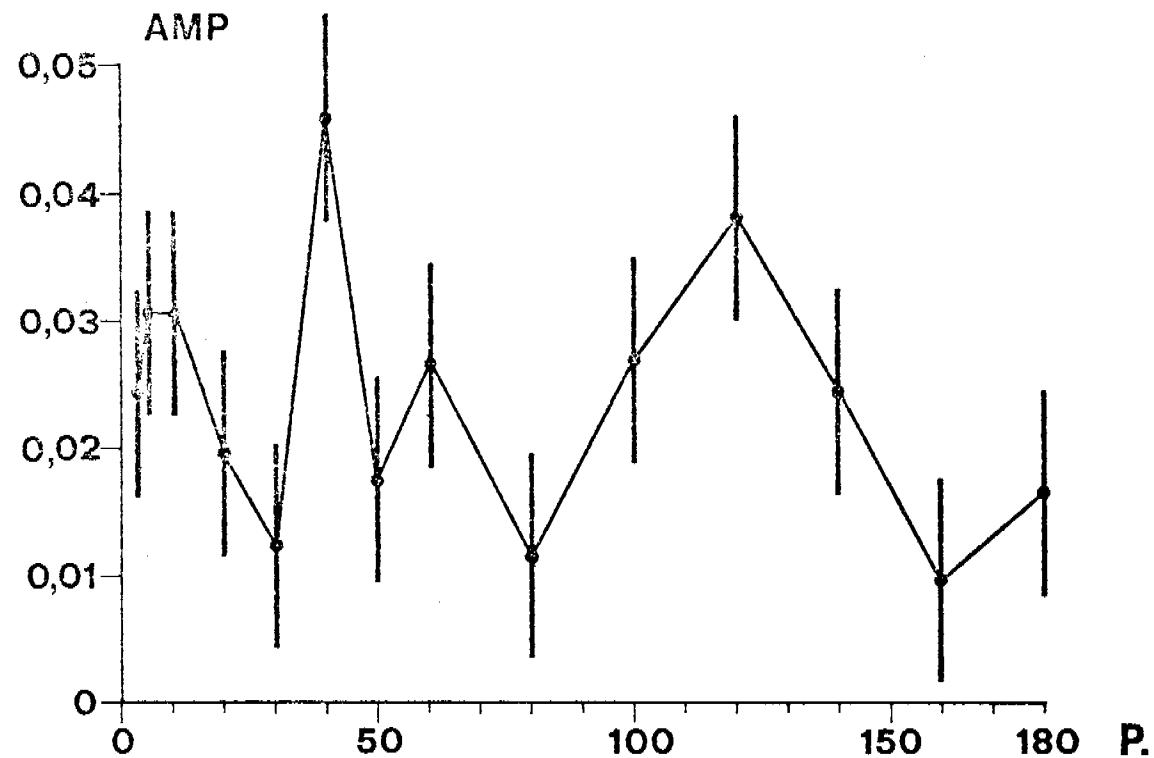


Figure 6. Spectral Analysis of the daily relative frequency rates B (M - Z) versus period in days.

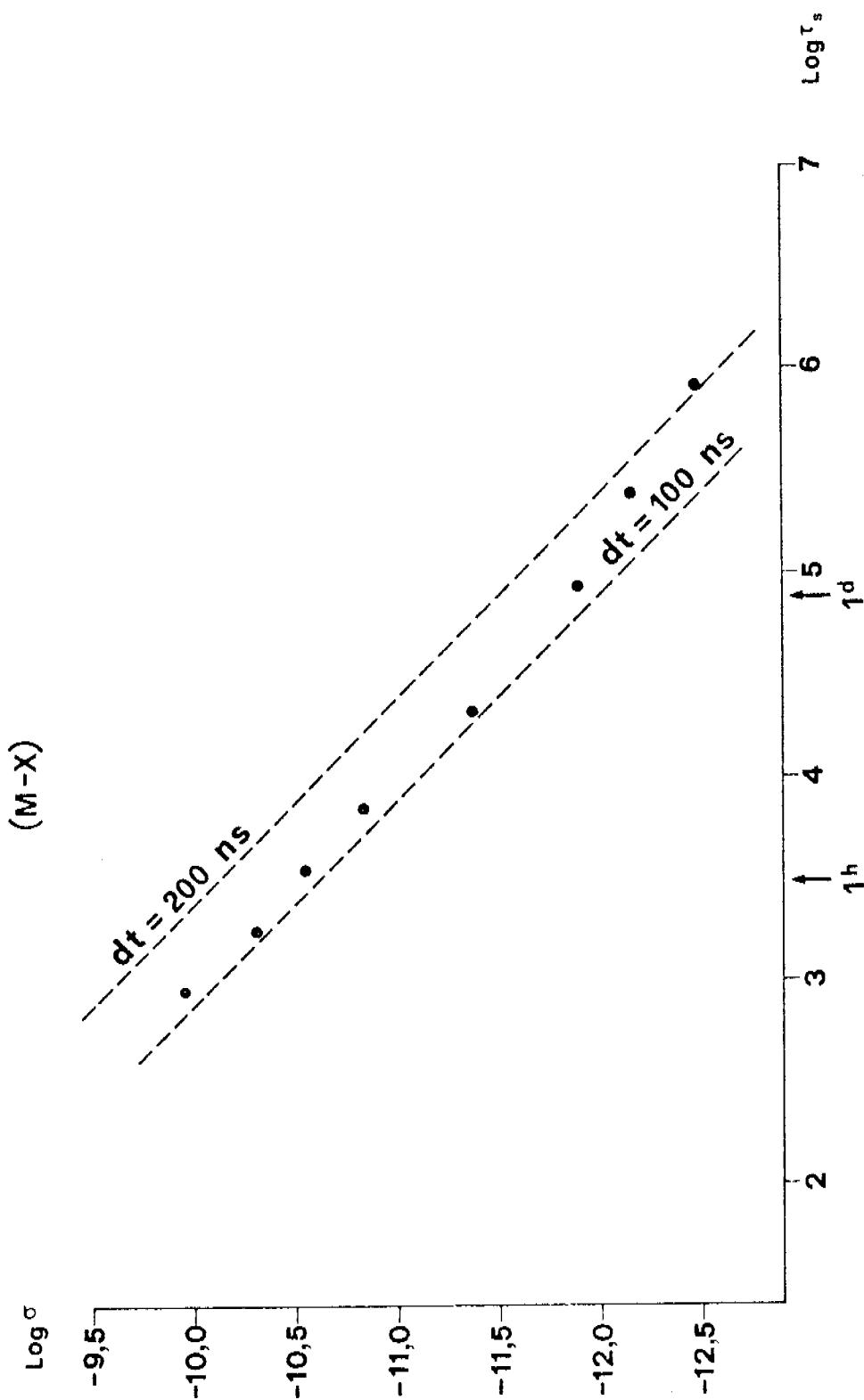


Figure 7. Spectra capabilities of Loran-C relative comparisons between Master Station and X slave station (Lampedusa). Plot of the square root of the variance of $\sigma_y(\tau)$ of the frequency variations. The dashed lines indicate the noise limits for theoretical time stabilities of 100 and 200 nsec based on the assumption of "white phase noise".

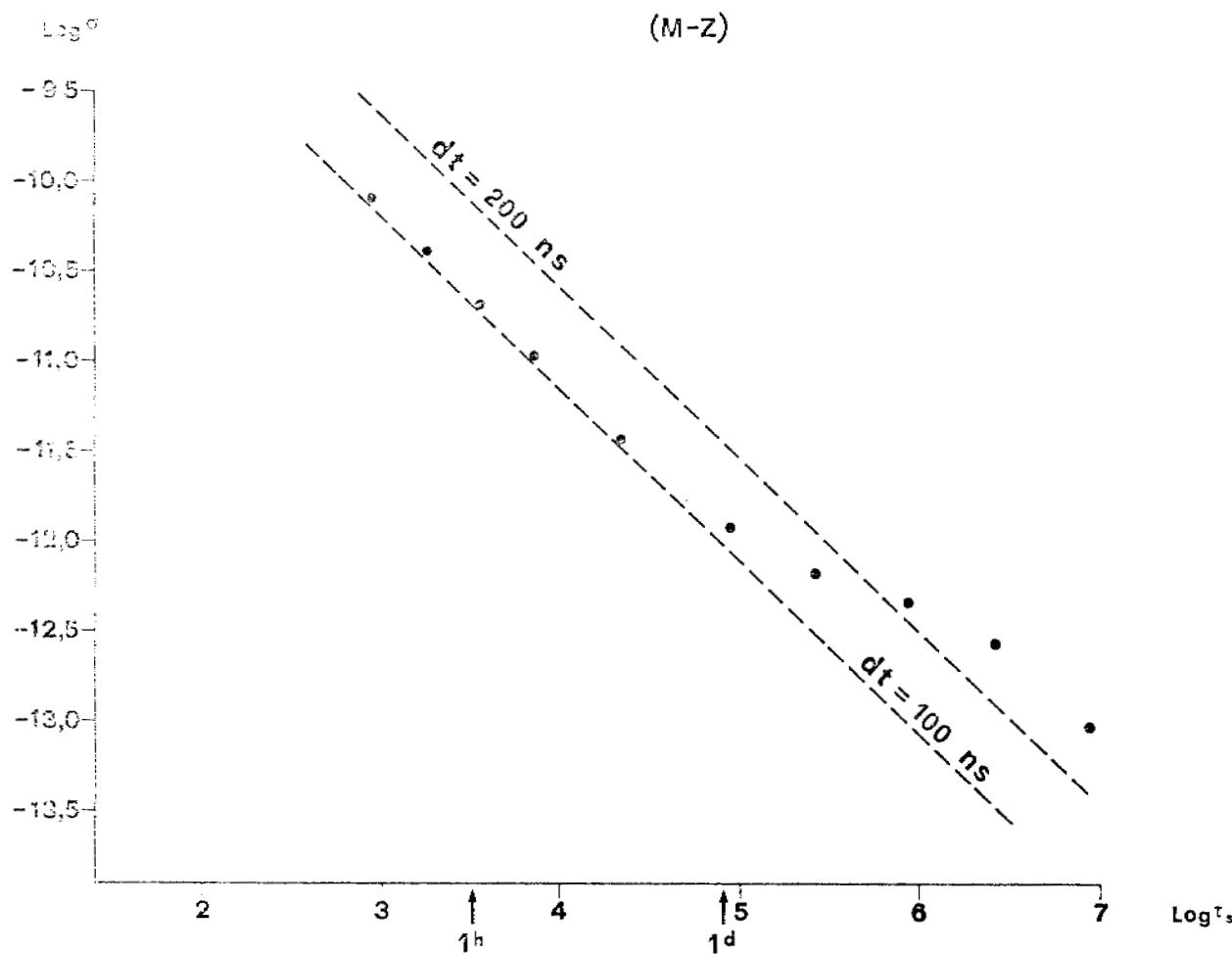


Figure 8. Spectral capabilities of Loran-C relative comparisons between Master Station and Z slave station (Estartit). Plot of the square root of the variance $\sigma_y(\tau)$ of the frequency fluctuations. The dashed lines indicate the noise limits for theoretical time stabilities of 100 and 200 nsec based on the assumption of "white phase noise".