

# PSEUDORANGE SYSTEMATIC ERRORS: ADVERSE EFFECTS ON CARRIER PHASE TIME TRANSFER

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## Abstract

Systematic errors in pseudorange postfit residuals are investigated, and the correlation of these errors with daily temperature variations is demonstrated. This effect is examined as a possible cause for the large day-boundary clock discontinuities observed at two International GPS Service (IGS) sites, Algonquin Park (ALGO) and National Research Council, Canada (NRC2). Corrective actions are discussed.

## INTRODUCTION

High-precision frequency transfer has been established with geodetic-quality carrier-phase Global Positioning System (GPS) receivers [1,2]. For time transfer, however, we are concerned with accuracy as well as precision. While the precision of time transfer is driven by the carrier phase, the accuracy is driven by the pseudorange. Therefore, any systematic errors in the pseudorange observables lead to a loss of accuracy for time transfer. Variations in receiver, antenna, or antenna cable absolute delays will also affect solution accuracy.

Systematic changes in data noise are the result of unmodeled or poorly modeled properties of the system. Any such unmodeled, nonrandom process will corrupt the least-squares solutions for time transfer. These processes may include seasonal variations in pseudorange errors, including seasonal variations in site multipath.

## GPS OBSERVABLE EQUATIONS AND GEODESY

The ionosphere-free GPS carrier-phase and pseudorange observables,  $\phi_r^s$  and  $P_r^s$ , for a given satellite  $s$  and receiver  $r$  can be written as follows:

$$\phi_r^s \lambda = \rho_g + c\delta^s - c\delta_r + \rho_{trop} + \rho_{multi}^\phi + N_r^s \lambda + \epsilon^\phi \quad (1)$$

$$P_r^s = \rho_g + c\delta^s - c\delta_r + \rho_{trop} + \rho_{multi}^P + \epsilon^P \quad (2)$$

where individual  $\rho$  terms are in units of length.  $\lambda$  is the ionosphere-free carrier wavelength,  $\rho_{trop}$  is the propagation delay due to the troposphere, and  $\rho_{multi}^{\phi}$  is the multipath error. Unmodeled errors and receiver noise are represented by  $\epsilon^{\phi}$  and  $\epsilon^P$  for the carrier-phase and pseudorange observations, respectively.  $N_r^s$  is the carrier-phase ambiguity or bias.  $\rho_g$  is the geometric range, or  $|\vec{X}^s - \vec{X}_r|$ , where  $\vec{X}^s$  is the satellite position at the time of transmission and  $\vec{X}_r$  is the receiver position at reception time. Proper determination of  $\rho_g$  requires precise transformation parameters between the inertial and terrestrial reference frames, i.e. models of precession, nutation, polar motion, and UT1-UTC. Finally,  $\delta_r$  and  $\delta^s$  are the time of the receiver and satellite clocks, respectively, in seconds.

In this study the observable equations are modeled using the GIPSY software [3]. The analysis strategy is similar to that described by Larson and Levine [1]. The satellite coordinates  $\vec{X}^s$  are taken from the JPL analysis center, with a range accuracy of 3-5 cm [4]. Station coordinates  $\vec{X}_r$  are estimated for each 24-hour batch of data. The zenith troposphere delay is estimated as a stochastic process. Satellite and receiver clocks are estimated at each data epoch relative to a reference receiver clock, usually a receiver connected to a hydrogen maser. The clock behavior is modeled as white noise, so that the estimates are uncorrelated from epoch to epoch. The variance for each clock parameter at each data epoch has been set to  $1 s^2$ . In doing so, we are assuming no *a priori* information, so the clock estimates are constrained only by the observations. Because  $\epsilon_P$ , the noise term of the pseudorange, is approximately two orders of magnitude noisier than for carrier phase, the geodetic solution relies heavily on the more precise carrier-phase data.

With few exceptions, the terms in Equations 1 and 2 are identical for pseudorange and carrier phase. For example, the troposphere delay and geometric range are the same for the two observable types. For timing purposes, the receiver clock term  $\delta_r$  requires special attention. In addition to oscillator behavior at the receiver, this timing delay also includes hardware delays due to the antenna, pre-amplifier, cables, and receiver. If these delays are the same for the pseudorange and the carrier phase, the model equations will be satisfied.

In GIPSY the carrier-phase and pseudorange data are analyzed simultaneously, and it is assumed that the receiver clock delay is the same for both observations. In previous studies of cable delays, this has been shown to be an accurate assumption [5]. In August 1998, the United States Naval Observatory (USNO) changed their antenna cable from a standard coaxial cable to a phase-stabilized cable designed for minimal temperature sensitivity. Figure 1 compares outside temperatures with clock estimates at USNO relative to the Alternate Master Clock in Colorado Springs (AMC2) before and after the cable was changed. Because group (pseudorange) and phase (carrier-phase) velocities are the same through a coaxial cable, temperature variations in the original cable caused identical delays for both the pseudorange and carrier-phase observables. The resulting errors were reflected in the clock estimates. Examining the pseudorange and carrier-phase postfit residuals for USNO before and after the cable replacement, we find that in both cases, the residuals are randomly distributed about a zero mean (figure not shown). However, other sites exhibit a temperature dependence for the pseudorange postfit residuals only, a result of unequal delays for the carrier-phase and pseudorange observables. In this study we look at Algonquin Park (ALGO) and National Research Council, Canada (NRC2), two IGS sites with large variations in the pseudorange postfit residuals.

## RESULTS

Figure 2 shows the postfit residuals at ALGO on 18 January 2001 for three different solutions. The ionosphere-free pseudorange (P3) only and ionosphere-free carrier phase (L3) only solutions are randomly distributed about a zero mean. This is expected, since systematic changes in electrical delays will show up in the clock estimates for single-data-type solutions. For the combined solution using both P3 and L3 data, the carrier-phase data noise stays randomly distributed, but the pseudorange noise varies significantly in a nonrandom way. This is due to the analysis weighting scheme and the fact that for this site, the P3 data must have a different electrical delay through the system than the L3 data. Because L3 is weighted 100 times more heavily than P3, the carrier-phase data determine the clock solution, and systematic errors in the carrier-phase data show up in the clock estimates. Because the P3 observables are weighted so little with respect to the carrier phase, systematic errors in P3 show up in the postfit residuals.

These variations in pseudorange postfit residuals are correlated with site temperature. Figure 3 shows a graph of the postfit residuals for ALGO and a graph of external temperature for 18 January 2001. The temperature is scaled to fit the postfit residuals using a least-squares estimation. It is important to point out that this scaling factor varies widely for different days.

Variations in the pseudorange affect time transfer because GPS data are routinely processed in 24-hour batches. Because the accuracy of the clock estimates is determined by the pseudorange, different variations in temperature from one day to the next will change the accuracy of the clock estimate from one day to the next. More specifically, by forcing the nonrandom noise in P3 to average zero over the course of 24 hours, artifact discontinuities result at the averaging boundaries. We believe this effect is directly responsible for the large day-boundary clock discontinuities at both ALGO and NRC2. Figure 4 shows an example of this effect. If one plots the clock estimates on adjacent days instead of the pseudorange residuals, the difference at midnight is defined as the day-boundary clock discontinuity.

At both ALGO and NRC2, these effects are more pronounced at low temperatures. In Figure 5 the standard deviation of the postfit residuals at ALGO and NRC2 are graphed as a function of daily mean temperature. The worst cases clearly occur below approximately -5° Celsius. A graph of day-boundary clock discontinuities [6] versus the daily mean temperature at NRC2 in Figure 6 further reinforces this correlation.

If large variations in temperature are a measure of the severity of the clock discontinuities, then one way of confirming this is to plot the discontinuities against the change in mean daily temperature. The larger the mean temperature change from one day to the next, the larger the temperature variation must be on at least one of the two days. This is demonstrated in Figure 7.

Note that the temperature correlation of the pseudorange postfit residuals can be seen at all temperatures, not just those below -5° Celsius. Figure 8 shows an example from June 2001 at ALGO. The mean temperature on this day is approximately 23° Celsius. The effect, while clear, is of smaller magnitude at higher temperatures than at lower ones. In Figure 8, the scale factor is only 1/3 as large as the scale factor in Figure 3.

## DISCUSSION

Two possible explanations for our results are considered. The notched filter in the antenna pre-amplifier is known to have some temperature dependence for its electrical delays [7]. Because the P-code requires an extremely wide frequency spectrum compared to the carrier phase itself, any temperature instabilities in the filter may be greatly magnified when processing the P-code. If this is the case, and furthermore if the temperature dependence of the pre-amplifier increases as temperatures become lower, the characteristics of the pre-amplifier may be the entire explanation for the observed behavior of the pseudorange postfit residuals.

Another possibility is that water has been introduced into the cable, possibly in the region of the connectors, or possibly absorbed into the foam dielectric itself. When this water freezes at temperatures below zero, this may somehow alter the electrical properties of the cable such that the difference between the group and phase delays in the cable is greatly magnified. More research is required to try to distinguish between these two different explanations. To be rigorous, there is also the possibility that water vapor has been introduced into the antenna electronics package; this too needs to be examined as a possibility.

The antenna at ALGO was changed on 17 January 1997 from an antenna with a wide-band filter to a notch-filter antenna. The pseudorange postfit residuals immediately began exhibiting the temperature dependence in question, as seen in Figure 9. Figure 10 shows the postfit residuals at ALGO for comparable days before and after the antenna change. This fact lends support to the pre-amplifier being the cause of the error; however, it could be the case that water was introduced into the cable when the antenna was changed.

## CONCLUSIONS

For accurate time transfer, systematic errors in the pseudorange observables must be removed. This can be accomplished in part by using a phase-stabilized antenna cable and by controlling the receiver temperature, but even with these precautions, some stations exhibit systematic errors in their pseudorange data. Removing these errors may require thermally stabilizing the antenna, or it may require replacing the cable, the antenna, or both. Further study is needed to determine which components are causing these effects. Once the systematic errors are removed, we expect the day-boundary clock discontinuities at affected sites to become less severe.

## ACKNOWLEDGMENTS

This study would not have been possible without the assistance of the Naval Research Laboratory and Ron Beard. The GIPSY software was provided by the Jet Propulsion Laboratory. We wish to thank Jim Ray and Ken Senior at USNO for providing day-boundary clock discontinuity values prior to publication. Thanks are also due to Joe White at NRL and Ed Powers at USNO for helpful discussion.

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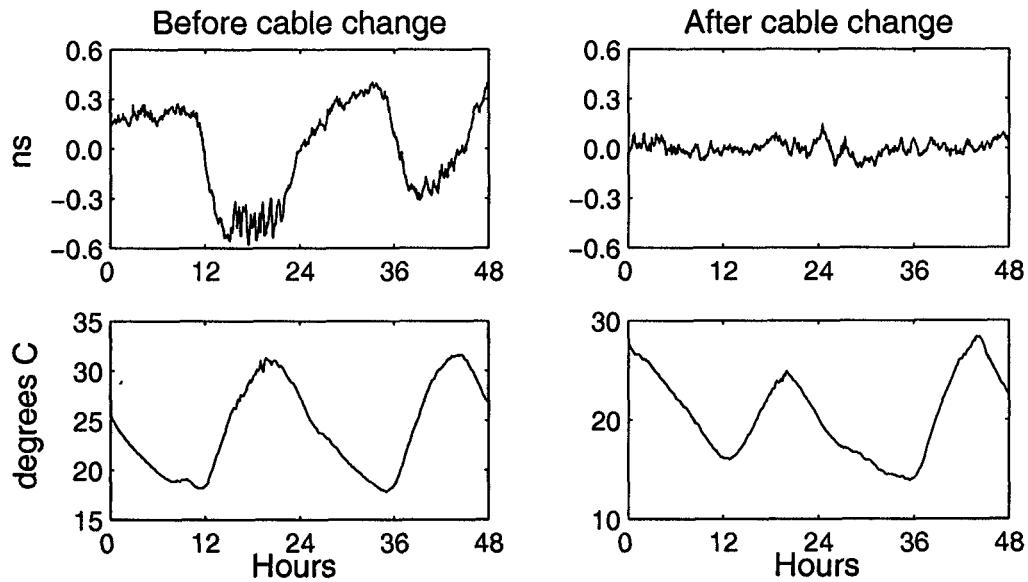


Figure 1: Clock estimates at USNO compared to USNO temperature in August 1998. The reference clock is AMC2. The strong diurnal temperature signature in the clock estimates was removed by replacing the standard coaxial antenna cable with a cable designed for temperature stability. Hours are hours past 00:00 for 4 August 1998 (left column) and 19 August 1998 (right column).

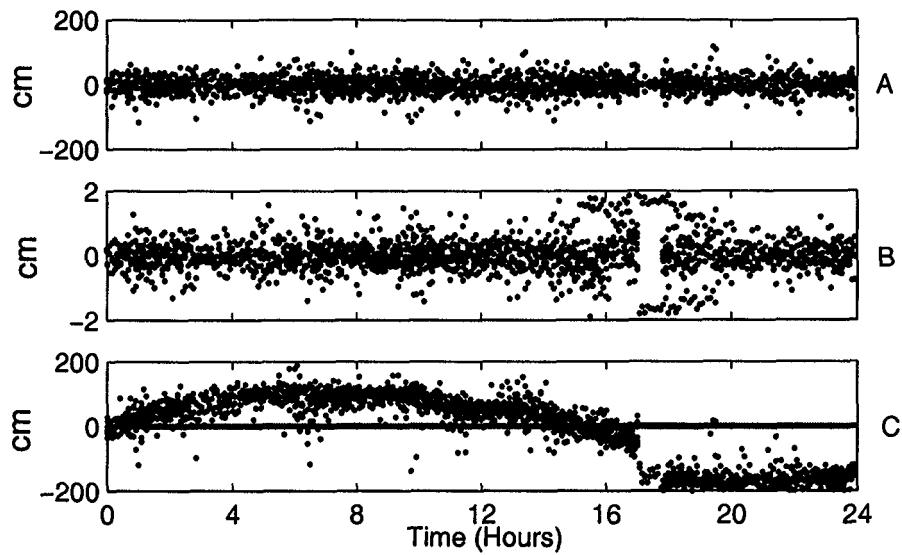


Figure 2: Postfit residuals for three different solutions at ALGO on 18 January 2001. Graph A was computed using pseudorange data only, graph B with phase data only, and graph C using both pseudorange and phase data. The residuals in both A and B are randomly distributed about a zero mean (with an anomaly shortly after hour 16). In C, the phase residuals are identical to those in B, but the pseudorange residuals now correlate strongly with temperature. The anomaly in B is reproduced in the phase residuals in C, but cannot be seen due to the magnitude of the pseudorange residuals. At this scale, the phase residuals in C appear as a straight line at zero.

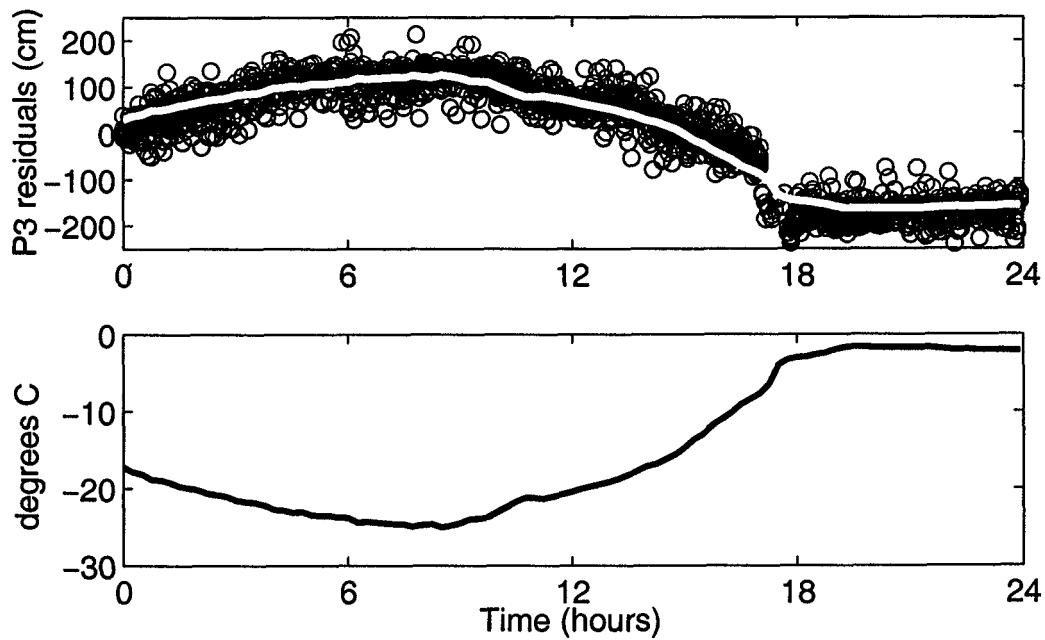


Figure 3: Ionosphere-free pseudorange residuals at ALGO versus outside temperature on 18 January 2001. The white line is scaled temperature. The scale factor for the temperature on this day is  $-12 \text{ cm}/^\circ\text{C}$ .

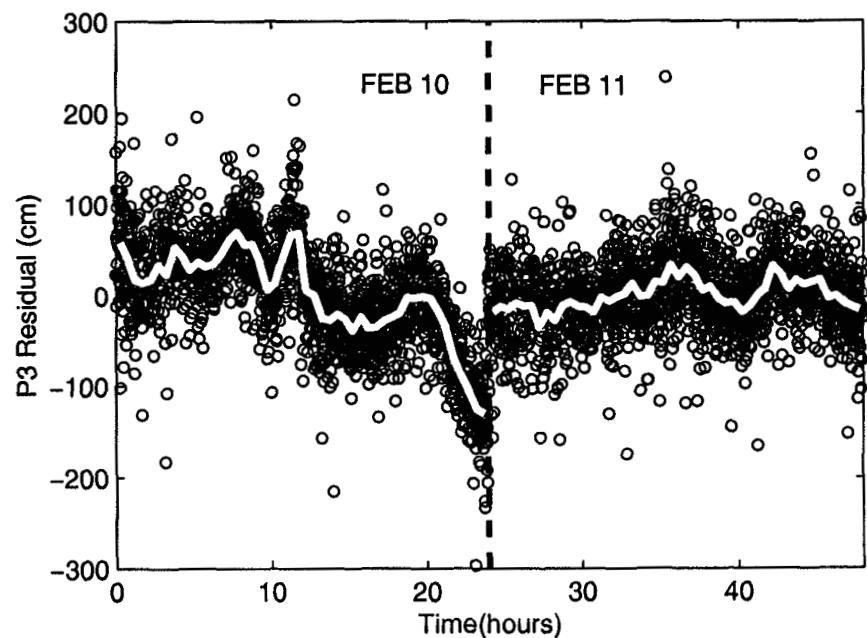


Figure 4: P3 postfit residuals for 10 February and 11 February are computed separately for NRC2 in 24 hour batches. The white lines are 30-minute averages. When graphed side by side, the large discontinuity at midnight is obvious. We believe this effect is driving the large day-boundary clock discontinuities at ALGO, NRC2, and other sites exhibiting similar characteristics. The day-boundary discontinuity for this example is -3.9 ns, a particularly large jump.

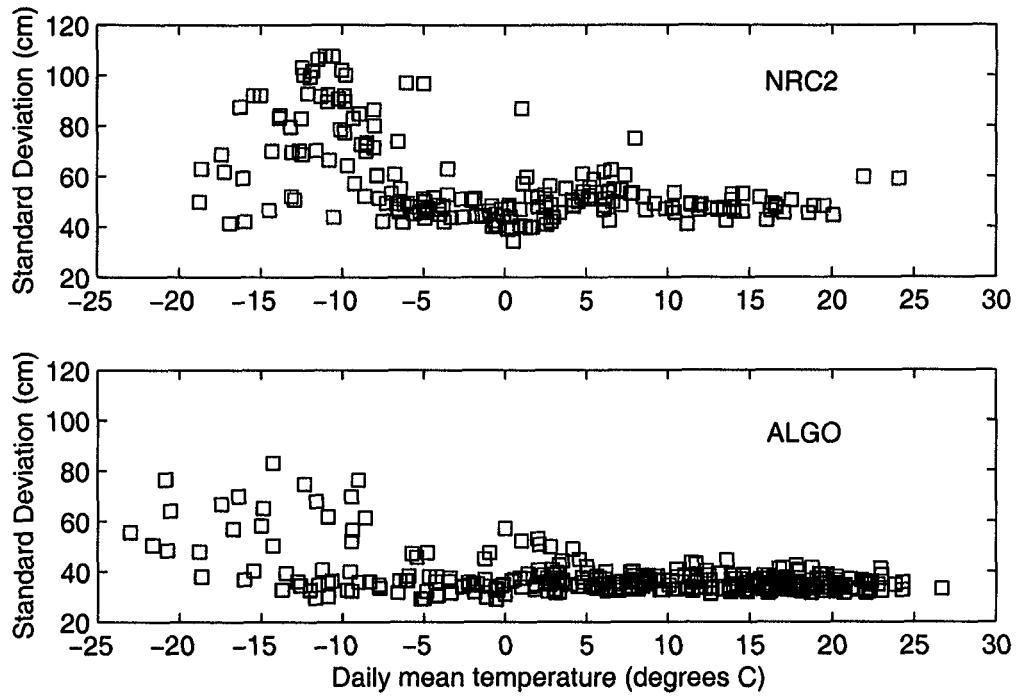


Figure 5: Standard deviation of daily P3 postfit residuals at ALGO and NRC2 graphed against the daily mean temperature at those sites. ALGO data are for the year 2000. NRC2 data are from November 2000 through May 2001. Larger standard deviations occur at lower temperatures.

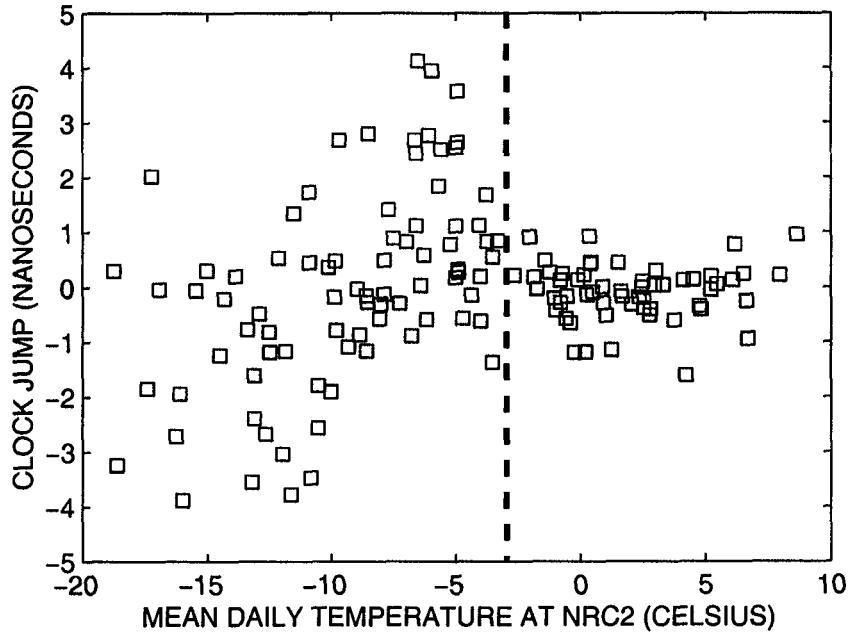


Figure 6: Day-boundary clock discontinuities (clock jumps) at NRC2 [6] as a function of mean daily temperature, graphed for the period 01 November 2000 - 30 May 2001. Note that the jumps become more severe at lower temperatures, with  $-3^{\circ}$  Celsius as an approximate breakpoint.

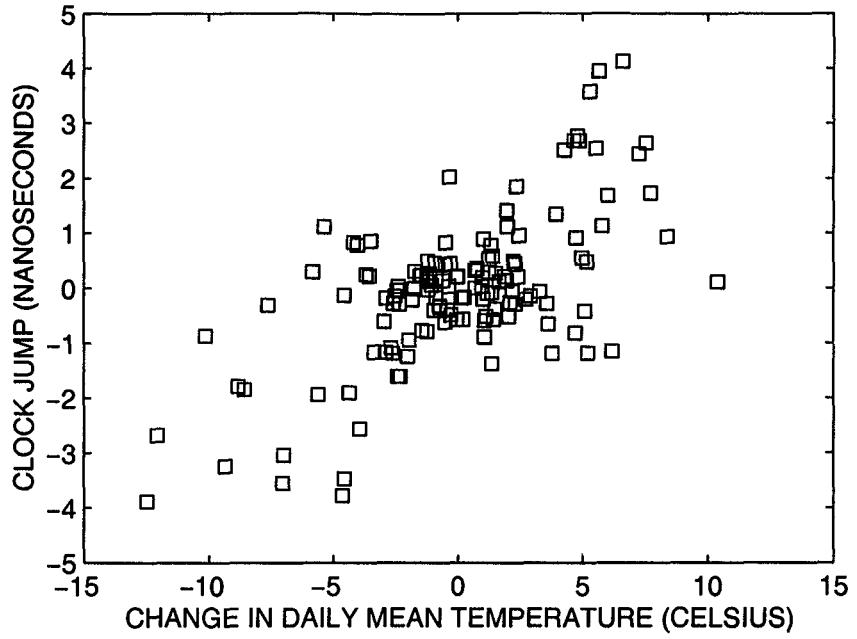


Figure 7: Day-boundary clock discontinuities (clock jumps) at NRC2 [6] as a function of the change in mean daily temperature from day 1 to day 2, graphed for the period 01 November 2000 - 30 May 2001. Large changes in mean daily temperature correspond to larger clock jumps.

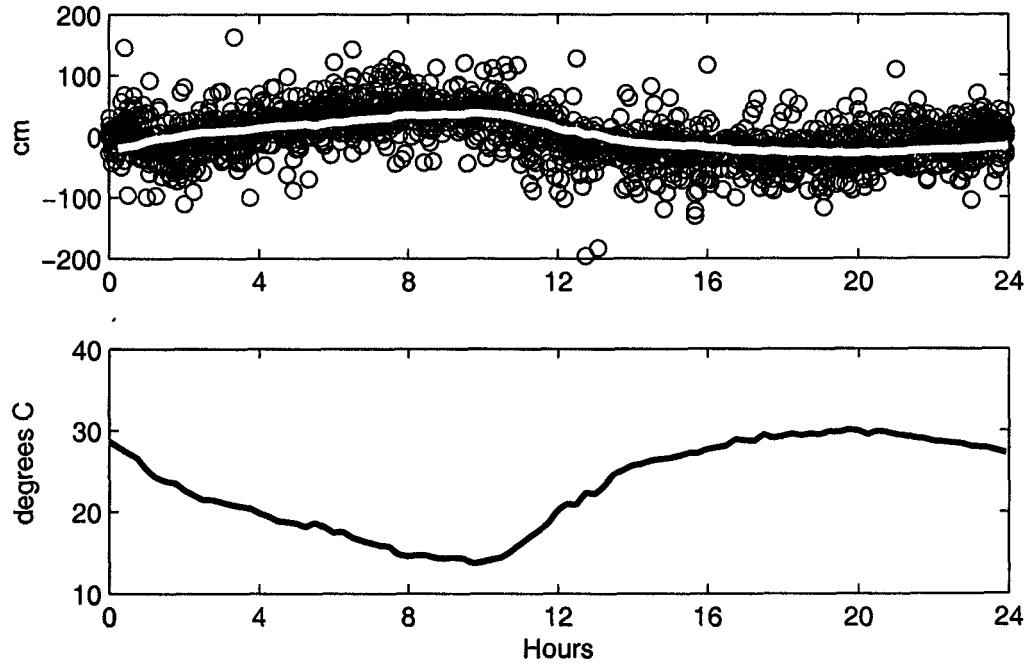


Figure 8: P3 residuals for ALGO on June 26, 2001. The temperature dependence of the residuals is evident, although not as dramatically as at low temperatures. The scale factor for temperature on this summer day is  $-4 \text{ cm}/^\circ\text{C}$ ; the winter day plotted in Figure 3 has a scale factor of  $-12 \text{ cm}/^\circ\text{C}$ .

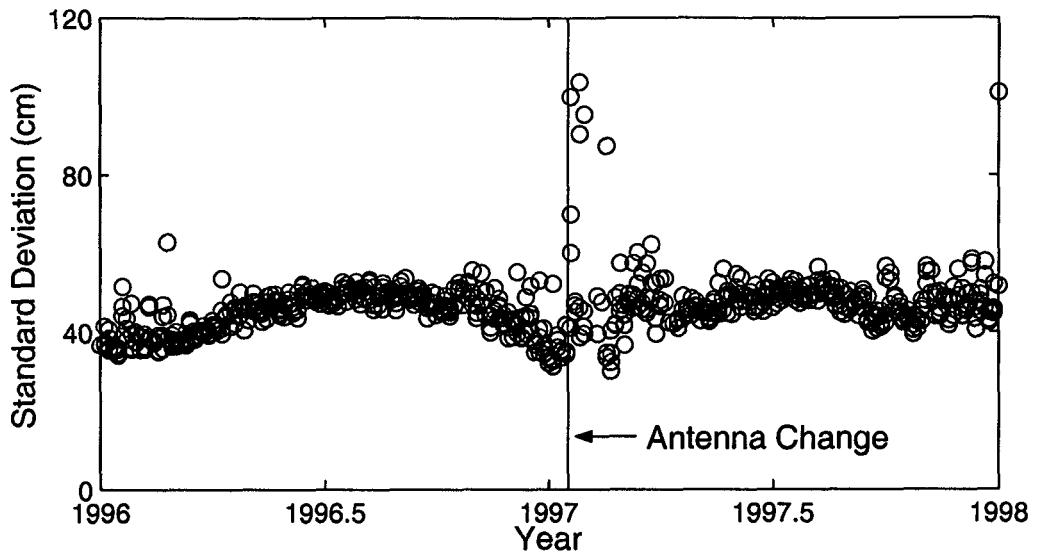


Figure 9: Standard deviation of P3 postfit residuals at ALGO from 1996-1998. Note the immediate increase in number and magnitude of outliers once the antenna was replaced in January 1997. The broad seasonal variation in the standard deviation has not been modeled, but was significantly reduced by the installation of a new receiver in 1999.

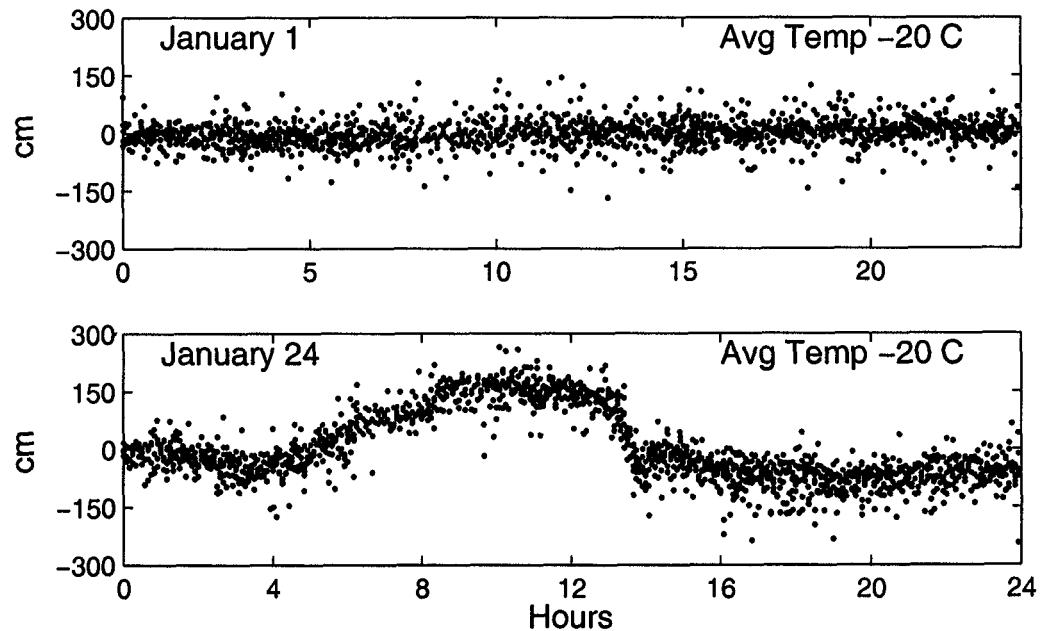


Figure 10: P3 postfit residuals for ALGO before and after the antenna change. The top graph is for January 1 (before the antenna change) and is randomly distributed about zero. The lower graph is for January 24 (after the antenna change); the residuals clearly demonstrate a systematic error. Hourly temperature data were not available for these days.

## QUESTIONS AND ANSWERS

KEN SENIOR (U.S. Naval Observatory): I think we are pretty much in agreement that at Algonquin, the temperature certainly correlates, although I would kind of caution you a little bit to look at the pseudorange and do temperature fits to pseudorange; and then make determinations with respect to clock estimates. If, for example, you had a perfect diurnal that was huge in your pseudorange, and it was perfectly diurnal, then the mean temperature difference between successive days would be absolutely nil, and you would not see any of that in your clock estimates.

And another situation, actually, from the residuals: let's suppose that you had a delta-change in temperature at midnight and you had constant temperature between 2 days, so that the temperature was constant over a day; then a delta-change in temperature at midnight, and then a front moved in really quickly and stabilized; you wouldn't see that in the residuals at all. But you certainly would, because you have –

JOHN PLUMB: I agree...

SENIOR: ... you have those residuals, but you would actually see that in the resulting clock estimates.

Actually, you also mentioned an interesting thing which we did not look at. There could also be some nonlinear effects on things at Algonquin in particular. We did a first-order fit to the temperature and removed that and, as I said before, we're only able to account for 30 percent of the effect. But there actually could be at colder temperatures perhaps some other nonlinear effect that we did not look at.

PLUMB: I will tell you a couple things. One is that I have run these temperature fits for several months at a time, and it never comes up the same. So it seems to be day-dependent. I don't know if it is the slope; I haven't run a least-squares estimate solution as yet, including the rate of change. So I don't see a linear trend. I tried to find a linear thing and I could not do it. Maybe it could a linear fit for 1 day, but not for the course of several days.

Second, I absolutely agree with you that if you had a perfectly symmetrical diurnal everyday, you would not see this. And I think that is where the data get cluttered. Because looking at the mean temperature is not really the solution; it is really looking at the daily temperature variations. But if you have a strong change in mean temperature from Day 1 to Day 2, that means that the slope on at least one of those days was significant and that kind of implied a large variation based on this. So I think we are in agreement.

SENIOR: One other thing, you labeled your NRC site as NRC-2. I think I only spoke to NRC-1. And I know that because the –

PLUMB: Well, they are run off the same antenna. So it is two receivers on the same antenna.

SENIOR: Because I know that one belongs to the timing, but I did not know that.

PLUMB: Yes, they split off the same antenna.

SENIOR: Okay, great. Thanks.

TOM CLARK (NASA/Goddard and Syntomics): I faced a very similar problem at another Arctic site, Fairbanks, Alaska, on the VLBI system, and subsequently confirmed the behavior at Ny Alesund at 79

degrees north, which also categorizes an Arctic site. What we found there on the VLBI antennas when we were observing strange phase jumps was that it wasn't really delta-T; it was the actually value of T. And your intercept, where things go noisy, is very close to zero, which is what we observed too. The problem was traced to the fact that some water had become entrained in the coaxial cables. The dielectric constant of liquid water and ice is quite different, and it is a rather abrupt change as you go through, depending on the thermal inertia that the cable gives you.

And if people happen to be running helices in those environments, where water can be trapped – and it is basically a big long hose – it always goes to the bottom; and usually at the bottom of the helices, you will find that some water is condensed in that location over the years since it was installed. So I would caution that as the very likely solution for everything you are seeing in terms of the Algonquin situation.

In terms of antenna filters, I will just point out that for many years, the Algonquin situation had no filter – we are still running the GOED-E site at NASA/GODDARD with no filter at all in the antenna, in part because the antenna is supporting both the GPS and GLONASS. I had developed a retrofit a much wider band filter jig set of stuff that is available – purchasable, essentially – off the shelf that does as well as the really fancy filter that Ed had come up with. At least, one of the fancy filters he had come up with, I have been able to duplicate with essentially off-the-shelf stuff. If anyone is interested in how to get a timing-type of filter in a Dorne Margolin antenna, you might want to talk to me. Thank you.

TOM McCASKILL (U.S. Naval Research Laboratory): Just a question about your final remarks. You say a “system” developed a position – which system are you referring to?

PLUMB: Well, basically, all the sites are using geodetic receivers. There are a few sites that are using, in particular, receivers made for timing. But the entire system was really developed for geodesy. Unless I am way off the mark there, that is my understanding.

McCASKILL: Which system? Do you mean GPS?

PLUMB: The IGS system. All the systems of GPS receivers around the world: the antenna, the cable, the receiver. They are all for positioning; timing falls out.

McCASKILL: So you are not referring to GPS itself.

PLUMB: No.

McCASKILL: Okay, thank you.

DEMETRIOS MATSAKIS (U.S. Naval Observatory): I think it is a great talk. I just want to make a comment and ask a question. The comment is that the issue about if diurnal is repeated on 24 hours, that you would not see it in the conventional analysis is absolutely right. But with continuous filtering, it would show in the residuals, and you can pick all that out very well.

And the question is: Have you looked at the standard measures of multipath, which is, do features repeat every 4 minutes following this diurnal day, just to see what they do?

PLUMB: I will tell you that when I started to look at this effect, I started looking at the multipath combination, I call them “M-1” and “M-2,” but you know what they are. And when I did that, I could not

extract them; there was so much noise because of the temperature dependence – the individual arc is so short in comparison to the day, if the temperature was changing drastically, this multipath combination would be changing drastically. And so I had a real problem because there are so many arcs in a day. I went to the three closer residuals because I could get that for a whole day and it was much easier to look at versus temperature.

I am not sure that that answered your question, though. If not, we can talk afterwards.

**BILL KLEPCZYNSKI (ISI):** My comment is that maybe a controlled experiment might be in order. I know of one place where the average daily temperature doesn't vary by about 5 degrees all throughout the year. It is about 68 degrees. And I would volunteer to man an antenna there throughout this experiment. It is Hawaii, I believe.

**PLUMB:** We will see what we can do!