

# COMPACT MICROWAVE CAVITY FOR HYDROGEN ATOMIC CLOCK

Zhang Dejun, Zhang Yan, Fu Yigen and Zhang Yanjun

Beijing Institute of Radio Metrology & Measurements

P.O.Box 3930 Beijing, China

## Abstract

This paper gives a summary introduction to the compact microwave cavity used in the hydrogen atomic clock. Special emphasis is put on derivation of theoretical calculating equations of main parameters of the microwave cavity. A brief description is given of several methods for discriminating the oscillating modes. Experimental data and respective calculated values are also presented.

## INTRODUCTION

The volume of the microwave cavity must be reduced so as to reduce the volume and weight of the hydrogen atomic clock. Nowadays, there are two methods to reach the goal. The first one is to fill the cavity with material of high dielectric constant and low loss. The second one is to adhere several electrodes outside the quartz storage bulb. This method provides more adaptability in reducing volume, meanwhile, it can reduce cost of the microwave cavity.

Since the microwave cavity adopting the second method has a more complicated structure it's very difficult to make out the accurate solution by wave equation, so no strict solution can be derived for its electromagnetic field distribution up to now. However, upon some reasonable hypotheses, it's possible to derive the approximate expressions which show relations of resonant frequency and Q-factor to dimensions of the cavity. This paper describes a derivative method of the expressions in detail, and gives out essential derivation procedure.

We have manufactured a microwave cavity by the second method. Its resonant frequency meets the requirement of the hydrogen atomic clock, and its Q-factor is about 7000.

## I Structure of the Cavity

We have manufactured an experimental compact cavity. Its structure is shown in Fig.1.

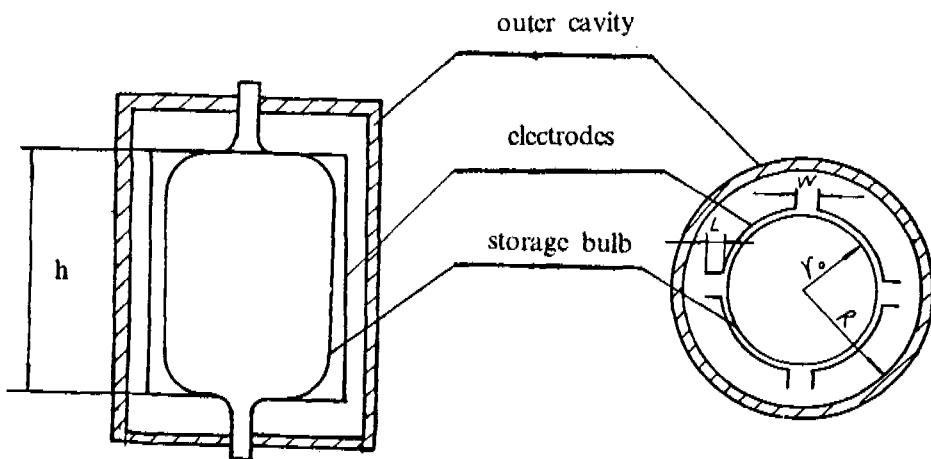


Figure 1. Structure of the Resonant Cavity

The outer cavity made of copper or aluminum is designed mainly to shield electromagnetic field of the electrodes. There is a quartz bulb in the outer cavity. It serves as a container of hydrogen atoms and a supporter of the electrodes as well. Usually four electrodes (two or three also allowed) are used, which are adhered on the quartz bulb by epoxy resin.

The resonant frequency of the cavity depends on the dimensions of the cavity and in particular on the adhered electrodes. The Q-factor is related to dimensions of the cavity, the metal material used and energy loss of the glue.

There is a piston on the top cover of the cavity (not shown in Fig.1). Its function is to coarsely adjust the resonant frequency of the cavity. Three holes in the bottom plate are for two coupling rings and a varactor diode respectively.

## II Basic Parameters of the Resonant Cavity

In the microwave cavity shown in Figure 1, there are many wave modes. The electromagnetic field structure shown in Figure 2 is similar to  $TE_{011}$  mode, and it is the right mode required by the hydrogen atomic clock. Now we derive the estimate formulas for the basic parameters of the resonant cavity using the distribution of the electromagnetic field shown in Figure 2.

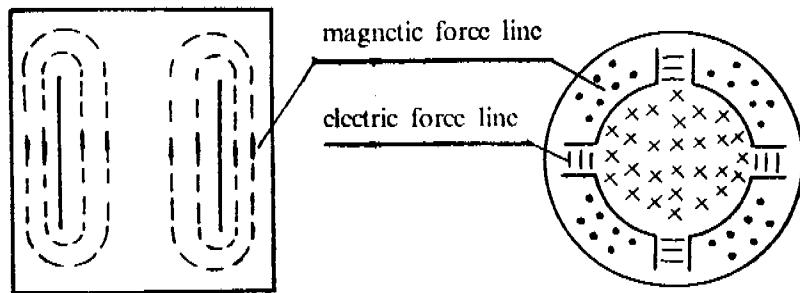


Figure 2. Distribution of the Electromagnetic Field

Assume that the electric field is distributed uniformly only between parallel parts of each pair of electrodes. Considering symmetry of the cavity, only the electromagnetic field distribution both outside and in one region is shown in Figure 3.  $\vec{E}$  represents electric field vector, and can be written as

$$\vec{E} = \vec{i}_y E_0 \sin\omega t \quad (1)$$

where  $E_0$  is amplitude,  $\omega$  is angular frequency,  $t$  is time, and  $\vec{i}_y$  is unit vector of  $y$  axis.

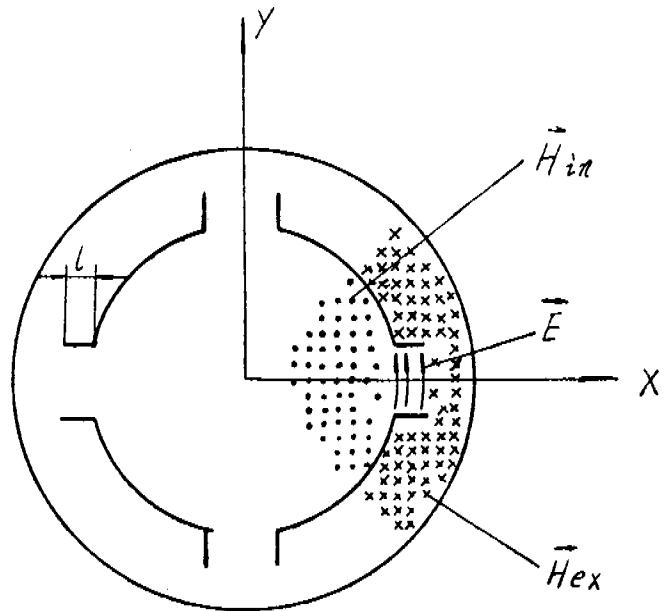


Figure 3. Electromagnetic Field Distribution near the Parallel Parts of two Electrodes

Let  $\vec{H}_{in}$  represent magnetic density in area between the electrodes, let  $\vec{H}_{ex}$  represent magnetic intensity in area between the electrodes and the outer cavity, and both them are regarded as uniformly distributed in their respective areas. Let  $A_1 = \pi r_0^2$ ,  $A_2 = \pi(R^2 - r_0^2)$ . According to closed characteristic of the magnetic field line, we have

$$A_1 \vec{H}_{in} = -A_2 \vec{H}_{ex} \quad (2a)$$

$$\vec{H}_{in} = i_1 \vec{H}_0 \cos \omega t \quad (2b)$$

$$\vec{H}_{ex} = -i_2 \frac{A_1}{A_2} \vec{H}_0 \cos \omega t \quad (2c)$$

#### a. Resonant Frequency

When the microwave cavity resonates, there exist the following relations <sup>(1)</sup>

$$\begin{aligned} W_0 &= W_e(t) + W_m(t) = W_{e,max} = \frac{\epsilon_0}{2} \int |E|^2 dv \\ &= W_{m,max} = \frac{\mu_0}{2} \int_{V_{in}} |\vec{H}_{in}|^2 dv + \frac{\mu_0}{2} \int_{V_{ex}} |\vec{H}_{ex}|^2 dv. \end{aligned} \quad (3)$$

where  $W_0$  is the total energy stored in the resonating cavity,  $W_e$  and  $W_m$  are electric energy and magnetic energy in the cavity respectively,  $\epsilon_0$  and  $\mu_0$  are dielectric constant and magnetic inductivity respectively,  $V$  represents the volume of the region between parallel parts of electrodes,  $V_{in}$  is the volume of the cylinder enclosed by the electrodes,  $V_{ex}$  is the volume of the region between the electrodes and the outer cavity.

Referring to Figure 1, expression (3) can be changed into the following expressions

$$W_{e,max} = \frac{1}{2} \epsilon_0 N \omega l h E_0^2 \quad (4)$$

$$W_{m,max} = \frac{1}{2} \mu_0 A_1 h \left(1 + \frac{A_1}{A_2}\right) H_0^2 \quad (5)$$

where  $N$  is the number of electrodes.

Substituting expressions (4) and (5) into expression (3), we get the expression of  $E_0$

$$E_0 = \sqrt{\frac{\mu_0 A_1 \left(1 + \frac{A_1}{A_2}\right)}{\epsilon_0 N \omega l}} \cdot H_0 \quad (6)$$

As shown in Figure 3, in the X-axis direction, the magnetic intensity  $\vec{H}_{in}$  transits to  $\vec{H}_{ex}$  in the region between parallel parts of two electrodes. In this transitional region, magnetic density is a function of x axis, and recorded as  $\vec{H} = \vec{i}_y H(x)$ . According to differential form of Maxwell's equations and expression (1), we have

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{i}_y \omega_0 E_0 \epsilon_0 \cos \omega t \quad (7)$$

where  $\omega_0$  is the angular frequency when the cavity is resonating.

In the transitional region shown in Figure 3, if the magnetic density is regarded as linearly changing, and the length of the transiting region is  $\frac{3}{2} l$ , then

$$\begin{aligned} \nabla \times \vec{H} &= -\vec{i}_y \frac{\partial H(x)}{\partial x} \\ &= -\vec{i}_y \frac{H(x_{ex}) - H(x_{in})}{\frac{3}{2} l} \\ &= \vec{i}_y \frac{2}{3l} H_0 \left(1 + \frac{A_1}{A_2}\right) \cos \omega t \end{aligned} \quad (8)$$

By using expressions (7) and (8), we can get

$$E_0 = \frac{2}{3} \frac{\left(1 + \frac{A_1}{A_2}\right) H_0}{\epsilon_0 \omega_0 l} \quad (9)$$

substituting expression (9) into expression (6) and having  $A_1 = \pi r_0^2$ , we can get

$$\omega_0 = \frac{2c}{3r_0} \sqrt{\left(1 + \frac{A_1}{A_2}\right) \frac{NW}{\pi l}} \quad (10)$$

where  $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$  is the light velocity in free space.

#### b. Q-factor

According to the definition of Q-factor of the cavity, we have

$$Q = \frac{\omega_0 W_0}{P_0} \quad (11)$$

where  $P_0$  is the power loss in the resonating cavity. The energy loss in the cavity mainly refers to the loss on the metal surfaces. If the medium loss can be neglected, we can get the approximate formula to calculate  $P_0$  as follows<sup>11</sup>

$$P_0 = \frac{R_s}{2} \int_S |\vec{H}_t|^2 ds \quad (12)$$

where  $\vec{H}_t$  represents the tangential component of the magnetic density on the metal surfaces in the cavity,  $S$  is the total area of metal surface in the cavity,  $R_s$  is expressed by the following formula

$$R_s = \frac{\delta}{2} \omega_0 \mu_0 \quad (13)$$

where  $\delta$  is the skin effect depth of electromagnetic field in the metal wall. We suppose that the skin effect depths are the same in all the metal surfaces, i.e., all  $R_s$  are regarded as the same value when calculating the energy loss on the metal surfaces.

When calculating the energy loss on the electrodes surfaces, we also deal with the  $N$  pieces of electrodes as a cylinder tube approximately. Let  $S_1$  represent the inner surface area of the outer cavity,  $S_2$  and  $S_3$  the outer and inner surface areas of the cylinder tube respectively,  $S_4$  the inner surface area of top cover and bottom plate of the outer cavity. By using expression (12) we get

$$P_0 = \frac{R_s}{2} \left( \int_{S_1} |\vec{H}_{ex}|^2 ds_1 + \int_{S_2} |\vec{H}_{ex}|^2 ds_2 + \int_{S_3} |\vec{H}_{ex}|^2 ds_3 + \int_{S_4} |\vec{H}_{ex}|^2 ds_4 \right) \quad (14)$$

Substituting expression (2) into expression (4) and having  $S_4 = 2\pi R^2$ , we get

$$P_0 = \pi R_s h H_0^2 [r_0 + (r_0 + R + \frac{R^2}{h}) (\frac{A_1}{A_2})^2] \quad (15)$$

By using expressions (3), (5), (15) and (11), we can get

$$Q_0 = \frac{r_0 \omega_0 \mu_0}{2R_s} \frac{1 + \frac{A_1}{A_2}}{1 + (1 + \frac{R}{r_0} + \frac{R^2}{hr_0}) (\frac{A_1}{A_2})^2}$$

Then substituting expression (13) into this one, we can get the final expression of  $Q_0$

$$Q_0 = \frac{r_0}{\delta} \frac{1 + \frac{A_1}{A_2}}{1 + (1 + \frac{R}{r_0} + \frac{R^2}{hr_0}) (\frac{A_1}{A_2})^2} \quad (16)$$

### III Judgement of the Oscillation Mode

Besides the oscillation mode shown in Figure 2, there are many other unnecessary modes in the resonant cavity. No wonder that identification of oscillation modes is of great importance. For this purpose, two methods are described hereafter.

#### a. Turning the Direction of the Coupling Ring

As seen from Figure 2, the magnetic density is radial near the bottom plate. One of the two coupling rings on the bottom plate is fixed for excitation, the other one can be turned in direction for coupling. For the field distribution shown in Figure 2, the energy output of coupling will be the largest when the turnable ring is made perpendicular to the radial direction.

#### b. Using the Perturbation Theory

From the perturbation theory<sup>[2]</sup>, we know that the frequency rises when a small piece of conductor is placed on the point where the magnetic field is dominant. The frequency falls when a small piece of conductor is placed on the point where the electric field is dominant. The frequency change in accordance with perturbation theory can be got by placing copper block into the electric field region and the magnetic field region.

In addition to the above-mentioned methods, other methods can also be used to identify the oscillation mode of the resonant cavity. For example, the theory of resonant cavity indicates that  $Q$  is the highest when the oscillation mode is  $TE_{011}$ . The field structure shown in Figure 2 is similar to  $TE_{011}$  mode, so the  $Q$ -factor is high, too.

## CONCLUSION

We have processed a resonant cavity according to the cavity structure shown in Figure 1. Its geometric dimensions (in millimeters) are as follows:

$$r_0 = 50, \quad R = 75, \quad l = 7 \quad W = 23$$

Substituting these data into expressions (10) and (16) and considering that  $\delta = 2.2 \times 10^{-3}$ , we get the following results:

$$\gamma_0 = 1.7 \text{GHz} \quad (\omega_0 = 2\pi\gamma_0)$$

$$Q_0 = 13270$$

The test results of this cavity are

Resonance frequency: 1.4GHz

Q - factor: 7000

An atomic clock of model CHYMNS-1 with a resonant cavity of such dimensions has been developed by Hughes Research Laboratories (HRL), U.S.A. The results measured are <sup>(3)</sup>

Resonance frequency: 1.4GHz

Q - factor: 9400

They have developed a smaller resonant cavity, whose dimensions are  $r_0 = 25, R = 38, l = 5.3, W = 7.4$  (the last two are estimated data).

The results measured are <sup>(4)</sup>

Resonance frequency: 1.4GHz

Q - factor: 4600

Substituting the dimensions of the cavity into expressions (10)and (16), we get

$$\gamma_0 = 1.6 \text{GHz}$$

$$Q_0 = 6900$$

By comparing the measured values with calculated values, we find that the resonance frequency tallies well, the Q - factor not so well. This is because that only the energy loss on the metal surfaces is calculated when deriving the Q - facotr formula, but the loss on the epoxy resin is not taken into account. We can consider the Q value calculated by the expression (16) is the highest value for this type of resonant cavity. The Q value of the cavity of American HRL, however, is higher than ours, which indicates that fineness of metal they processed is higher than ours, and the glue they used to adhere the electrodes may be better in the respect of energy loss.

Based on the data comparison, we can take expressions (10) and (16) as basis of designing this type of resonant cavity, so as to greatly reduce the blindness in designing.

## References

- [1] Shen Zhiyuan, "Microwave Technology", P204.
- [2] Shen Zhiyuan, "Microwave Technology", P239, P240.
- [3] H.T.M. Wang, "An Oscillating Compact Hydrogen Maser", Proc. 34th Ann. Sym on Frequency Control, 366, 1980.
- [4] R.R. Hayes and H.T.M. Wang, "Design for A Subcompact Q-enhanced Active Maser", Proc. 38th Ann Symp. on Frequency Control, (1985)