

# **Providing Hydrogen Maser Timing Stability to Orbiting VLBI Radio Telescope Observations by Post-Measurement Compensation of Linked Frequency Standard Imperfections**

James C. Springett

NeoComm Systems, Inc.

## **Abstract**

*Orbiting VLBI (OVLBI) astronomical observations are based upon measurements acquired simultaneously from ground-based and Earth-orbiting radio telescopes. By the mid 1990's, two orbiting VLBI observatories, Russia's Radioastron, and Japan's VSOP, will augment the worldwide VLBI network, providing baselines to Earth radio telescopes as large as 80,000 km. The challenge for OVLBI is to effectuate space to ground radio telescope data cross-correlation (the observation) to a level of integrity currently achieved between ground radio telescopes. But, VLBI radio telescopes require ultra-stable frequency and timing references in order that long term observations may be made without serious cross-correlation loss due to frequency source drift and phase noise. For this reason, such instruments make use of hydrogen maser frequency standards. Unfortunately, space-qualified hydrogen maser oscillators are currently not available for use on OVLBI satellites. Thus, the necessary long-term stability needed by the orbiting radio telescope may only be obtained by microwave uplinking a ground-based hydrogen maser derived frequency to the satellite. Although the idea of uplinking the frequency standard intrinsically seems simple, there are many "contaminations" which degrade both the long and short term stability of the transmitted reference. Factors which corrupt frequency and timing accuracy include additive radio and electronic circuit thermal noise, slow or systematic phase migration due to changes of electronic circuit temporal operating conditions (especially temperature), ionosphere and troposphere induced scintillation's, residual Doppler-incited components, and microwave signal multipath propagation. What is important, though, is to realize that ultimate stability does not have to be achieved in real-time. Instead, information needed to produce a high degree of coherence in the subsequent cross-correlation operation may be derived from a two-way coherent radio link, recorded, and later introduced as compensations adjunct to the VLBI correlation process. Accordingly, this paper examines the technique for stable frequency/time transfer within the OVLBI system, together with a critique of the types of link degradation components which must be compensated, and the figures of merit known as coherence factors.*

## **ORBITING VLBI AND THE PHASE TRANSFER SYSTEM**

VLBI astronomical observations are derived from received celestial radio signals by mutually crosscorrelating all of the data streams obtained from each radio telescope. That the signals are

in the form of data streams is due to transformations whereby the RF bands are SSB translated to a lowpass range, sampled, and quantized (typically to one or two bits). For purposes of discussion in this paper, only a pair of signals will be considered, one derived from a ground radio telescope, and the other from the subject orbiting radio telescope.

A quality VLBI brightness observation depends on obtaining a very accurate crosscorrelation amplitude ("fringe pattern" maximum value). But, this measurement will be degraded from that ideally possible if the unconnected frequency reference systems (individual frequency standards) of the two radio telescopes are sufficiently unstable (i.e., they drift, and have a high level of phase noise) over the crosscorrelation period,  $T$ .

The process of transferring the needed stable reference frequency from the ground to an orbiting satellite is known by various clauses: "frequency transfer," "phase transfer," and "time transfer." These may be understood as counterpart descriptors, since frequency is the derivative of phase, and time units may be obtained as phase divided by the sinusoid's nominal angular frequency. In this paper, the phase of the frequency standard, or for the orbiting radio telescope the RF carrier phase, is the basic quantity of concern.

Figure 1 presents a synopsis model of the two-way phase transfer system, and illustrates the ground configuration, satellite, and two-way link, plus some principal sources of the detrimental phase components acquired through the phase transfer process. Beginning at the bottom left, a satellite tracking ground station, using a hydrogen maser frequency standard, generates and transmits an RF carrier. The carrier's frequency, as received by the orbiting satellite, inexorably undergoes a very significant Doppler shift due to the spacecraft's instantaneous radial velocity as it orbits the earth. Because the uplink frequency is used as the reference for the orbiting radio telescope, any significant Doppler shift creates a number of problems allied to the radio astronomy receiving process, and affects the ultimate coherence of the phase transfer reference. The limited scope of this paper precludes an exposé of these issues, but suffice it to say, it is prudent to employ an operation wherein the bulk of the Doppler is predictively removed from the uplink, so that the signal arriving at the satellite is substantially Doppler free. (A similar downlink Doppler removal is effected within the ground station's receiver.) The uplink carrier, which is coherently tracked by the satellite transponder receiver's phase-locked loop, ultimately forms the local frequency standard for the orbiting radio telescope.

In the process of being transferred from the ground to the satellite, the uplink carrier becomes phase modulated by effects within electronic circuits, plus propagation phenomena associated with the troposphere, ionosphere, and multipath. Thereby, the hydrogen maser stability of the ground frequency becomes unacceptably degraded by the time it reaches the satellite's radio telescope. Fortunately, two favorable conditions mitigate against unequivocal impairment. First, ultimate stability does not have to be achieved in real-time. Because VLBI signals are recorded on magnetic tape for later processing, an opportunity exists for introducing corrections during crosscorrelation. Secondly, the necessary correction information is readily obtained by transponding the uplink carrier (re-transmitting it at a slightly different frequency), and making a two-way (uplink plus downlink) phase measurement at the ground tracking station. Very important is the fact that the downlink introduces most of the uplink phase perturbations a second time, in a reciprocal, or nearly-reciprocal, manner. It is this characteristic which is critical to effectively making use of a scaled version of the measured two-way phase at the

correlator, to minimize the uplink induced instabilities intrinsic within the VLBI observation data.

The downlink for the two-way phase transfer system is generated by coherently converting the received uplink frequency to a different frequency. The turn-around factor, assigned the symbol

$itr$ , is the ratio of two integers. Typically, the downlink carrier is also modulated by the VLBI observation data. A ground receiving system detects and records the VLBI data, and extracts and records the two-way phase information. The receiving system also functions to construct the two-way Doppler profile, which is used for precise satellite orbit determination, essential to the ephemeral model needed at the correlator. All of this data, along with a corresponding observation from a ground radio telescope, is later applied to the VLBI crosscorrelation process.

## PHASE TRANSFER MODEL

Figure 2 is a block diagram depicting the mathematical model of the two-way phase transfer system. This figure effectively defines the phase component symbols (which, after brief study, should be reasonably obvious), their positions of entry into the model, and the phase transfer and compensation operations. Notice that all of the uplink and downlink phase contaminations have been divided into four principal portions: 1) the phase of the hydrogen maser source, 2) the phases introduced by the ground station, 3) the phases added by the propagation medium, and 4) the phases imparted by the satellite. Further, these principal components are made up of several terms peculiar to the realm within which they originate. The phase subscripts G, P, and S, denote respectively the Ground systems, the Propagation path, and the Satellite systems, while U designates the Uplink, and D the Downlink. Hydrogen Maser Source #1 is that stable frequency source located at the satellite ground tracking station and possesses the intrinsic phase noise  $\phi_1$ , while Hydrogen Maser Source #2 is associated with a ground radio telescope, and has phase noise  $\phi_2$ .

In what follows, the operations of Doppler removal are overlooked. For the purposes of modeling and analysis, the "system reference frequency" is defined as the nominal uplink microwave frequency,  $f_U$ , with  $K$  being the multiplication factor needed to obtain  $f_U$  from the Hydrogen maser standard frequency. At the satellite,  $f_U$  must be multiplied by an additional factor  $M$  (not necessarily an integer) in order to obtain the radio astronomy receiver's effective reference frequency, designated by the symbol  $f_A$ . Ultimately, its phase, denoted by  $\phi_{reference}$ , is the coherence degrading phase process which becomes imbedded within the VLBI sampled data transmitted to the ground. On the other hand, the two-way phase derived from up/down transfer link, and scaled by  $\mu M$ , is designated as  $\phi_{two-way}$ . Here,  $\mu$  is the optimum scaling factor required to properly minimize, at the correlator, the uplink's effects.

The phase product from the VLBI crosscorrelation operation is assumed to depend solely on  $\phi_{reference}$ ,  $\phi_{two-way}$ , and  $\phi_2$ , all other actions being taken as ideal for the sake of the present investigation. Therefore, insofar as phase handling is concerned, the correlator may be represented by a series of differences, followed by finite integration, as shown within the dashed boundary on Figure 2. An additional feature of the VLBI correlator's comportment is

the fact that it effectually diminishes the phase by a select first-order fit over the interval  $T$ . This is included in the model by introducing the linear expression  $\hat{\Theta} + \hat{\alpha}t$ , where the  $\hat{\Theta}$  over the coefficients denotes that they are estimated. Finally, the integrand involves the cosine of the reduced phase, this being the true form resulting from crosscorrelation of the actual signals involved. The output is called the generalized coherence function, and assigned the symbol  $C(U; T)$ . It now remains to specify  $\mu$ , and evaluate  $C(U; T)$ .

## CRITIQUE OF PHASE TRANSFER PERFORMANCE

Drawing on the definitions found on Figure 2, the effective phase input to the integrator,  $C(U; T)$ , may be written as:

$$\phi_{\text{correlator}} = \phi_{\text{reference}} - \phi_{\text{two-way}} - KM\phi_2 - (\hat{\theta} + \hat{\alpha}) = M \begin{pmatrix} K\phi_1 & -K\phi_2 \\ +(1 - \mu r)\phi_{GU} & -\mu\phi_{GD} \\ +(1 - \mu r)\phi_{PU} & -\mu\phi_{PD} \\ +(1 - \mu r)\phi_{SU} & -\mu\phi_{SD} \end{pmatrix} - (\hat{\theta} + \hat{\alpha}). \quad (1)$$

The brevity of this paper prevents a detailed review of all of the uplink and downlink phase components in respect to their individual characterizations and reciprocity. So, discussion must be limited to the point that the components are categorized by whether they are rapidly or slowly varying with respect to a specific value of  $T$ . Specifically, slow variation; (or slow phase) is attributed to any component which has its significant power manifest at frequencies below  $1/T$ , while fast variation; (or fast phase) means that the significant power of the component is embodied at frequencies greater than  $1/T$ . Components are additionally classified as to whether they are random variables, which can only be characterized statistically, or systematic temporal changes, conveniently represented by an algebraic function (e.g., a polynomial).

By way of simplification, the factor  $K(\phi_1 - \phi_2)$  may be dropped from further consideration because it does not pertain directly to the subject of phase transfer, i.e., it is inherent in the crosscorrelation output between any two radio telescopes, whether one is space-based, or not. Secondly, it is assumed that the estimates  $\hat{\theta}$  and  $\hat{\alpha}$  adequately cancel their respective counterparts in  $\phi_{\text{reference}} - \phi_{\text{two-way}}$ , and that only quadratic phase terms, identified by the coefficients  $\beta_U$  and  $\beta_D$ , have significant effect. Lastly, like uplink and downlink phase pairs are identified, and the downlink phase process expressed in terms of the uplink phase by means of the normalized crosscorrelation  $\rho_i$ , plus  $\delta_i$ , a prorating factor which assigns the type of reciprocity. Additionally, the term  $\phi_{D_i-\text{independent}}$  is introduced to account for the uncorrelated remainder of the  $i^{\text{th}}$  downlink phase. Based upon these considerations, the generalized coherence function may be expressed by the form

$$C(U; T) = \frac{1}{T} \int_0^T \cos \left[ M \begin{pmatrix} \sum_i \phi_{U_i} + \beta_U(T^2/6 - Tt + t^2) \\ -\mu r \sum_i (1 + \delta_i \sqrt{\rho_i}) \phi_{U_i} + \beta_U(T^2/6 - Tt + t^2) \\ -\mu \sum_i \sqrt{1 - \rho_i} \phi_{D_i-\text{independent}} + \beta_D(T^2/6 - Tt + r^2) \end{pmatrix} \right] dt. \quad (2)$$

The next step is to determine what value of the scaling factor,  $\mu$ , optimizes  $C(U; T)$ . Since the integrand in (2) involves random variables, it is necessary to find conditions which maximize the expected value of the mean-square of  $C(U; T)$ , denoted by the symbol  $\overline{C^2(U; T)}$ . Again, space prohibits inclusion of the details, but the solution for 100% reciprocal components (quadratic or random) is

$$\mu_{100\% \text{ reciprocal}} = \frac{1}{2r}. \quad (3)$$

Of course, most components are not 100% reciprocal (some are not reciprocal at all). But it turns out that the predominant components are nearly 100% reciprocal, so little is practically lost by using the result expressed by (3) for most cases. The one outstanding exception is when the ionosphere exerts a very strong influence (which can happen if  $f_U$  is below 10 GHz, and solar-maximum conditions prevail). For this condition, a scaling factor slightly larger than  $1/2r$  is required.

The coherence factor is defined as  $\sqrt{\overline{C^2(U; T)}}$ . Since many different independent phase pairs are involved, from this point forward it is convenient to deal with the coherence factors which involve the individual component pairs,  $\phi_{U_i}$  and  $\phi_{D_i - \text{independent}}$ . As a further shorthand, the  $i^{\text{th}}$  integrand terms of (2) are collectively represented by  $\phi_i$ . Accordingly, it can be shown that the system coherence factor may be determined from component-pair coherence factors via the relationship

$$\sqrt{\overline{C^2(U; T)}} = \overline{C_Q(U; T)} \times \prod_i \sqrt{\overline{C_{\phi_i}^2(\mu; T)}}, \quad (4)$$

where the term with the subscript  $Q$  betokens quadratic phase, and the subscript  $i$  connotes the  $i^{\text{th}}$  random component pair.

Formal calculation of the individual random component coherence factors involves a double integral, given by

$$\overline{C_{\phi_i}^2(U; T)} = \frac{2}{T^2} \int_0^T (T - \tau) \exp \left[ - \int_0^\infty S_{\phi_i}(\mu; f) \sin^2(\pi \tau f) df \right] d\tau, \quad (5)$$

where  $\phi_i$  is assumed to be Gaussian, and  $S_{\phi_i}(\mu; f)$  is its one-sided power density spectrum. It is especially notable that the phase noise spectrum is the cardinal measure of phase stability as reflected by the gauge of coherence factor. It is also important to realize that coherence is a decreasing function of the integration interval  $T$ . As an example, with the phase noise spectrum represented generically by

$$S_{\phi_i}(f) = \begin{cases} N_{0i} f^{-v_i} & \text{for } 0 \leq f \leq f_m \\ 0 & \text{for } f < f_m \end{cases}, \quad (6)$$

Figure 3 shows coherence factor behavior for two indicated conditions of phase noise as a

function of  $f_m T$ ,  $f_m$  being the effective high frequency cutoff of  $S_{\phi_i}(\mu; f)$ . For very large  $F_M t$ , and provided that  $\Phi_i$  has a finite variance  $\sigma_{\phi_i}^2$ , the minimum coherence value of (5) may simply be obtained from the limiting result

$$\overline{C_{\phi_i}^2(\mu; T)} \approx \exp[-\sigma_{\phi_i}^2/2], \quad f_m T \gg 1. \quad (7)$$

Finally, the residual quadratic phase coefficient,  $\beta$ , is obtained from the pertinent integrand terms of (2), and its coherence factor calculated through the use of Fresnel integrals by the expression

$$\overline{C_Q(\mu; T)} = \frac{1}{z} \sqrt{C^2(z) + S^2(z)}, \quad z = \sqrt{\beta T^2 / 2\pi}. \quad (8)$$

## SOME PHASE TRANSFER SYSTEM DESIGN CONSIDERATIONS

There are no less than ten independent and quantifiable sources of phase noise which impinge the phase transfer process. Some of these sources are termed "operative," because they stem from apparatus and actions which can be controlled by design. Operative sources include microwave and electronic circuit noise, slow or systematic phase migration due to changes of electronic circuit temporal operating conditions (especially temperature), residual Doppler-induced components, and microwave signal propagation multipath. Other sources are termed "natural," because the system designer essentially has no control over them. These are principally the ionosphere and troposphere.

Substantial degradation by many operative sources can be avoided through good design practices. A part of evaluating system coherence is determination of coherence factors for individual functional circuit assemblies (such as amplifiers, mixers, filters, frequency multipliers, etc.). For random phase noise, independence between assemblies is presumed. What is of concern at the electronic unit level are the phase components which are added to an input sinusoid over and above the circuit's intrinsic thermal noise (a natural source). The nuance of the unit's phase noise intrusions is that of "degradation." In this sense, an individual circuit should be treated as if it were inherently free of thermal noise, because in a hierarchy of cascaded units, all of the intrinsic noise contributions should be handled as an equivalent thermal noise at the input to the chain. This noise, usually expressed as a noise figure for the entire system, is ultimately reckoned in terms of SNR. What should be accounted for at the unit level are 1) any in-band spurs (perhaps due to RFI coupling), 2) power supply noise, 3) common mode noise contributions, 4) VSWR effects, 5) AM/PM, 6) any nonlinear characteristics which give rise to an increased in-band thermal noise level relative to linear throughput, 7) the effects of reference frequency phase noise if the circuit has a reference frequency input, 8) deliberate modulation, and 9) operating point induced phase drifts.

Other loss catalysts which can substantially reduce coherence are Doppler, and unaccountable slow phase variations, especially quadratic and higher-order phase changes, which are not

reciprocal on the two-way microwave radio link. Minimization of this class of loss is promoted by the use of predictive Doppler removal from the phase transfer uplink (at the transmitter) and downlink (at the receiver), and techniques for minimizing "drift" phase.

Through judicious design, the compensated coherence loss due to all operative sources (both ground and satellite) can typically be kept below 1%. On the other hand, the largest coherence losses are typically caused by severe troposphere and ionosphere phase scintillations, and quadratic phase due to changing troposphere propagation path length as the ground station antenna changes elevation angle while tracking the satellite's orbit. But even these natural effects can be minimized by using the highest uplink frequency possible, because the ionosphere's effects are proportional to the inverse of the uplink frequency. For this reason, X-band or higher is preferred.

## PHASE TRANSFER SYSTEM PERFORMANCE

A goal of the phase transfer system is to maintain very high coherence (0.9900, or 99%) for centimeter wavelength (e.g., 1.35 cm or 22 GHz) radio telescope signal reception over measurement periods,  $T$ , of 300 (and up to 1000) seconds. In order to attain this goal, a great deal design effort has been made toward minimization of all sources of operative phase noise in ground and satellite microwave and electronic systems.

Table 1 presents a summary of the expected *Radioastron* and *VVSOP* phase transfer performance. In all cases the coherence due to operative sources is better than 99%. It is the natural sources which drive the complete system coherence below 99%. Notice that *Radioastron* performs worse than *VVSOP* in this regard, because *Radioastron*'s uplink is at X-band, while *VVSOP*'s is at Ku-band.

## PHASE NOISE AND COHERENCE MEASUREMENTS

Proper measurements of the underlying processes or components which contribute to phase transfer degradation is a vital element of OVLBI system development. Characterizing phase behavior requires that simultaneous measurements of the input and output, of a unit or subsystem under test, be taken in order to obtain differential performance information. A further fundamental requirement is that the measuring instrument itself either a) contribute negligible phase noise/errors, or b) be fully calibrated so that phase noise/errors may be taken into account in forming the final measurement results.

The commercial test equipment market is essentially devoid of the type of equipment needed to make several simultaneous and synchronous phase measurements. The best solution to the phase measurement problem is to make use of the basic two-way phase measurement technique developed for ground station two-way phase extraction and processing. By this method, a device being tested is excited by a high-quality (very stable) frequency, and phase measurements are simultaneously taken at the device's input and output. In order to obtain phase process data from the input and output sinusoidal waveforms, each is demodulated by a quadrature phase

detector referenced to a stable frequency standard, producing sine-of-phase and cosine-of-phase signals. These two signals are individually lowpass filtered, and then sampled by an analog-to-digital converter capable of taking time-coincident samples from several quadrature phase demodulators.

Sample pairs are input to computer or special digital processor for reduction into useful results. Phase data reduction and analysis consists of a series of complex operations which 1) condition the data samples to correct for imperfections introduced by the test hardware, 2) computes unambiguous phase from corrected data, and 3) derives the systematic and random phase measures. Coherence may be computed directly from the unambiguous phase data. Additionally, the measured random phase component may be analyzed to extract its standard deviation, power density spectrum, etc. When multiple outputs (such as with a transponder) are being analyzed, crosscorrelation may be used as a data reduction tool.

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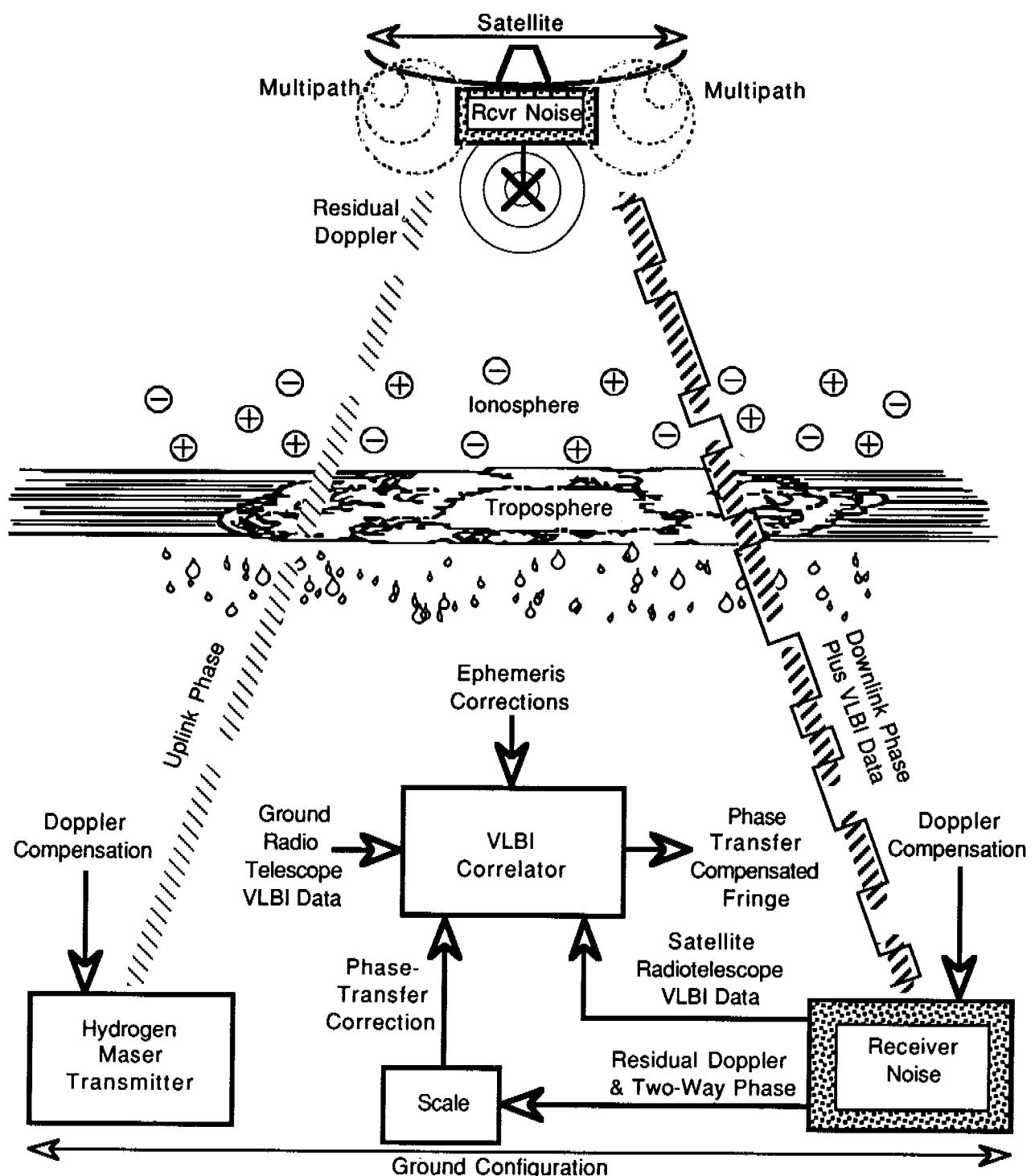


Figure 1 – Synopsis Model of Two-Way Phase Transfer System

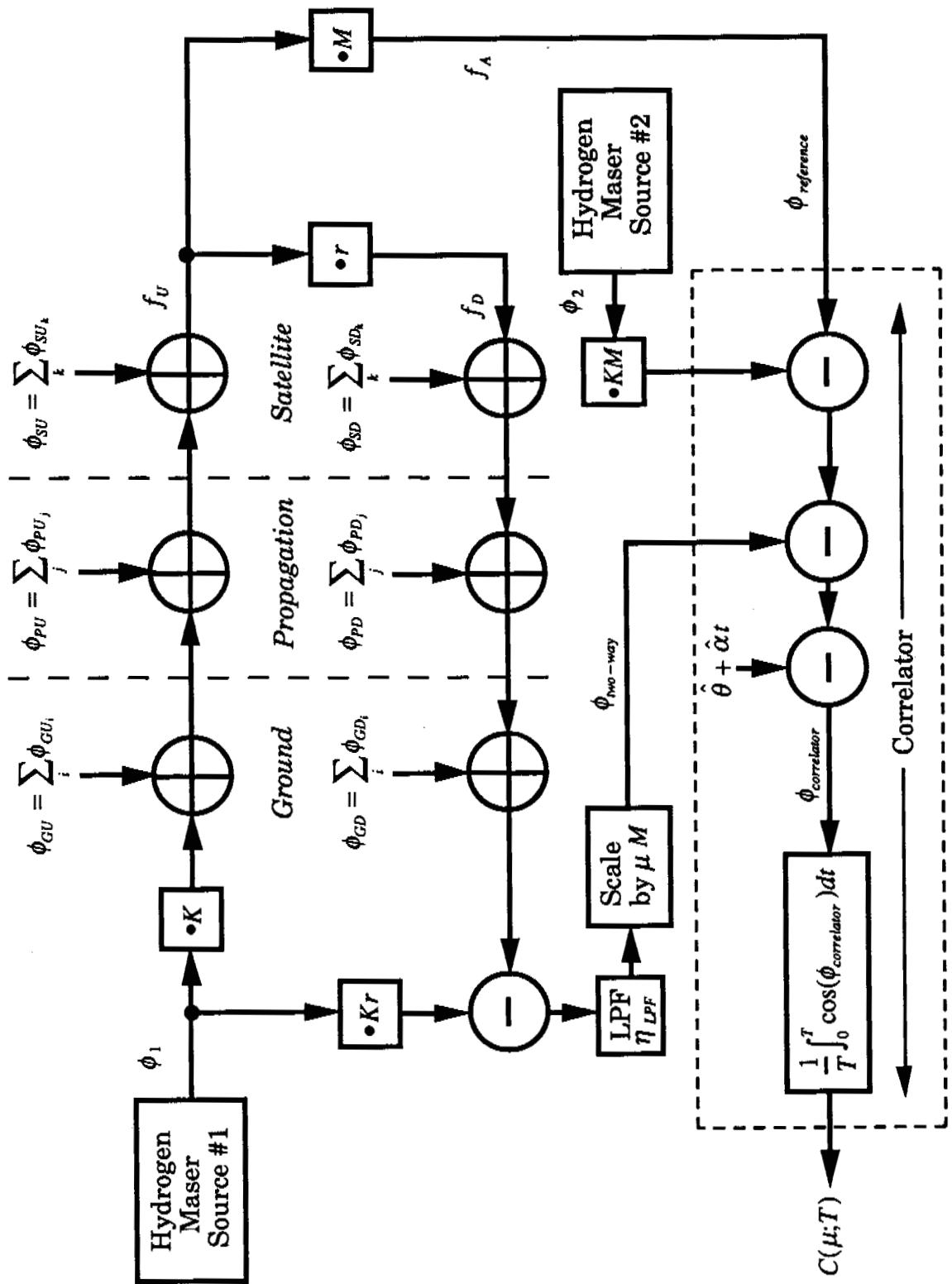


Figure 2 – Mathematical Model of Two-Way Phase Transfer System

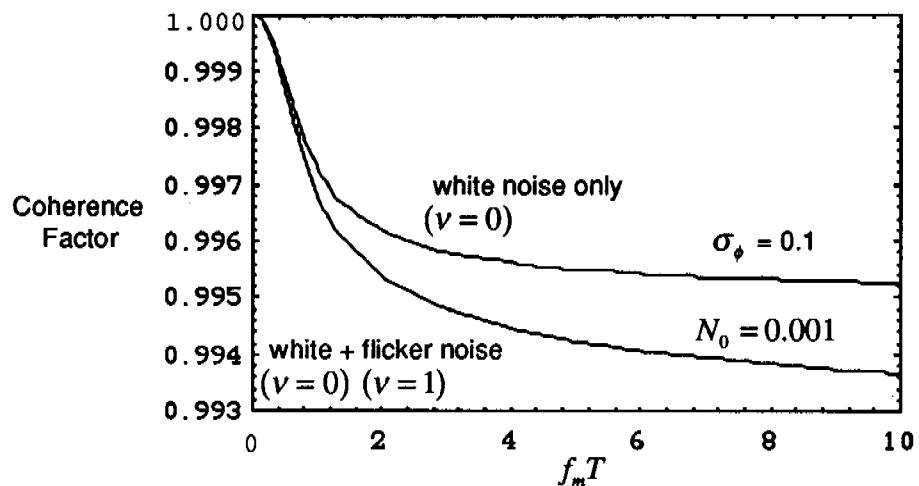


Figure 3 – Coherence Factor Behavior

	Radioastron		VSOP	
	Uplink	Downlink	Uplink	Downlink
Phase Transfer Link Frequencies (GHz)	7.21	8.47	15.3	14.2
Corrected Coherence Factors	Worst-Case	Typical	Worst-Case	Typical
Operative Sources	0.9913	0.9940	0.9941	0.9970
Natural sources	0.9482	0.9954	0.9938	0.9982
Complete System	0.9399	0.9895	0.9880	0.9952

Table 1 – *Radioastron* and *VSOP* Phase Transfer Performance

