ALTERNATE ALGORITHMS FOR STEERING TO MAKE GPS TIME

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BIOGRAPHY

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ABSTRACT

The current GPS Time bang-bang steering algorithm was chosen to provide minimal control perturbation given the constraint that steering is achieved by accelerating time (frequency drift), as opposed to direct adjustment of frequency. This constraint was necessary because satellite clock corrections are updated independently of each other, and typically only daily and when they are sufficiently close to an upload site. One consequence of this is that GPS Time has differed from that of the USNO Master Clock (UTC(USNO)) by about 6 ns RMS in the past year, and in a month-long event beginning late December 1994, it deviated by up to 250 ns.

The planned GPS architecture improvements of satellite crosslinks and improved ground clocks justify consideration of switching the steering strategy to one involving direct frequency adjustments to GPS Time. We present an analysis that uses Linear Quadratic Gaussian (LQG) theory to optimize a steering strategy in which frequency is adjusted in proportion to the weighted offset of GPS Time in time and frequency.

I. The Current GPS Steering Algorithm.

GPS Time is steered to UTC(USNO) using a method designed for minimal control perturbation, given that different satellite clocks receive their steering updates at different times. The method derives GPS Time by accelerating the GPS Composite Clock (CC) by either 0 or $\pm 1.0 \cdot 10^{-19}$/s. A one-sentence summary of the algorithm is that the acceleration is set opposite to the slope in GPS-UTC(USNO), except that if doing so constantly thereafter would result in the extrapolation of the accelerated GPS-UTC(USNO) never becoming zero, in which case the acceleration has the same sign as the slope (Brown, 1990, Huser and Hutsell, 1999).

The GPS CC is generated by the Master Control Station's Kalman filter (MCSKF). Implicitly defined at 15-minute intervals, it is a weighted mean of all contributing satellite and ground station clocks (Brown, 1991). There are some complications because a historical record of the GPS CC is not readily available. There are delays in the implementation of GPS steering due to the 37-hour averaging period of USNO observations, the required time for the MCSKF to receive and process the USNO summaries, and the time to upload that information to all satellites. As in Matsakis et al. (2000), we reverse-engineered the GPS steering using just one point per day and assumed 1-day lag between the date the USNO observation of

GPS Time was recorded and its full implementation. The several lags in this process should be much shorter as GPS upgrades are implemented; simulations assuming no lag produced similar results. Since the reverse-engineered model for the GPS CC should be similar to the actual GPS CC in a statistical sense, it is possible to use it to compute what the statistical properties of UTC(USNO)-GPS would have been had the GPS CC been steered according to any of the strategies under consideration.

We anticipate that the future GPS CC will be more stable than currently, due to planned improvements in site and satellite clocks (Beard, White, et. al. 2000) and because of USNO-endorsed efforts to provide almost zero-latency measurements of UTC(USNO)-GPS every 15 minutes (Hay and Hutsell, private communication). We have modeled the future GPS CC as an ensemble of 9 ground-based cesium standards and 26 space-born rubidium standards characterized by 200 ps measurement noise, white frequency noise with a stability equal to $3.0 \cdot 10^{-15}$ at 1 day and a flicker floor of $3.0 \cdot 10^{-15}$. Mis-modeled rubidium frequency-drift and longer-period noise components are ignored because all steering schemes considered here should be rapid enough to remove them. Under these assumptions, there is little to be gained by subdaily averaging because white phase noise is the dominant contributor to the noise budget over subdaily periods.

II. LQG APPROACH

The essence of the LQG approach is that it computes the optimal steering gain, given a state space model. The gain is a vector, which is multiplied by the GPS state vector, whose components are the time and frequency deviation from UTC(USNO), to compute the amount of a frequency steer.

A separate issue is optimal state estimation, for which Kalman filtering (Brown and Huang, 1992) is used to estimate offsets in time and frequency with respect to the USNO Master Clock. In Kalman filter theory the state equation is given as a linear function of a state vector, $\boldsymbol{x}(k)$, a control vector, $\mathbf{u}(k)$, and a noise vector $\mathbf{w}(k)$. The two-state frequency standard model is given by

$$\begin{bmatrix} x(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} \tau \\ 1 \end{bmatrix} \mathbf{u}(k) + \mathbf{w}(k) \ ,$$

where x and y correspond to the time and fractional frequency difference between GPS time and UTC(USNO) respectively, $\mathbf{w}$ is a Gaussian white process noise, and τ corresponds to the time interval between measurement updates.

The noisy measurement $\mathbf{z}(k)$ is related to the state vector by

$$\mathbf{z}(k) = \mathbf{H}\mathbf{X}(k) + \boldsymbol{v}(k)$$

where

$\mathbf{z}(k)$ = measured UTC(USNO)-GPS time difference
$\mathbf{H}$ = connection matrix = [1 0], and
$\boldsymbol{v}(k)$ = Gaussian white measurement noise.

After the state comprised of the time and frequency offsets has been estimated, the data are used with an LQG calculated control gain in order to produce a frequency steer value. The LQG method (Koppang and Leland, 1996) calculates a gain which minimizes the cost function:

$$J = \sum_k \left(\hat{\mathbf{X}}(k)^{\mathrm{T}} \mathbf{W}_{\mathrm{Q}} \hat{\mathbf{X}}(k) + \mathbf{u}(k)^{\mathrm{T}} \mathbf{W}_{\mathrm{R}} \mathbf{u}(k) \right)$$

Here $\mathbf{W}_{\mathrm{Q}}$ and $\mathbf{W}_{\mathrm{R}}$ are matrices that are chosen in order to set the relative penalties assessed to the state vector estimate and control vector as they vary from zero.

The optimal control for the given cost function is: $\mathbf{u} = -\hat{\mathbf{G}}\hat{\mathbf{X}}$, where $\hat{\mathbf{X}}$ is the state estimate,

$$\hat{\mathbf{G}} = (\mathbf{B}^{\mathrm{T}} \hat{\mathbf{K}}_0 \mathbf{B} + \mathbf{W}_R)^{-1} \mathbf{B}^{\mathrm{T}} \hat{\mathbf{K}}_0 \Phi \quad ,$$

$$\mathbf{B} = \begin{bmatrix} \tau \\ 1 \end{bmatrix},$$

and $\hat{\mathbf{K}}_0$ is calculated by solving a Ricatti equation:

$$\hat{\mathbf{K}}_0 = \Phi^{\mathrm{T}} \hat{\mathbf{K}}_0 \Phi + \mathbf{W}_Q - \Phi^{\mathrm{T}} \hat{\mathbf{K}}_0 \mathbf{B} (\mathbf{B}^{\mathrm{T}} \hat{\mathbf{K}}_0 \mathbf{B} + \mathbf{W}_R)^{-1} \mathbf{B}^{\mathrm{T}} \hat{\mathbf{K}}_0 \Phi \ .$$

III. LQG COMPARED WITH BANG-BANG

The stability GPS time would have had if the current GPS CC had been steered by an LQG-derived proportional-gain system depends upon what compromise was made in optimizing for frequency stability versus time deviation. Solutions presented here emphasize frequency stability (more important for navigation) over time offset (as could be needed for interfacing with time transferred from the USNO via different means or for compatibility with other systems such as Galileo).

For our proportional-gain simulations, the model of the current GPS CC was steered with a gain function of

$\left[5.2336 \cdot 10^{-6} \quad .9999\right]$, computed with a cost function calculated using

$$\mathbf{W}_R = 1.0 \quad \text{and} \quad \mathbf{W}_Q = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 2 \cdot 10^{6} \end{bmatrix}.$$

Figure 1 displays the observed values of UTC(USNO)-GPS from 18OCT97 to 13JUL00 and Figure 2, plotted on the same scale, shows what would have been observed had the above LQG-derived proportional gain function been applied. Figure 3 gives the Allan deviations of the data plotted in Figures 1 and 2.

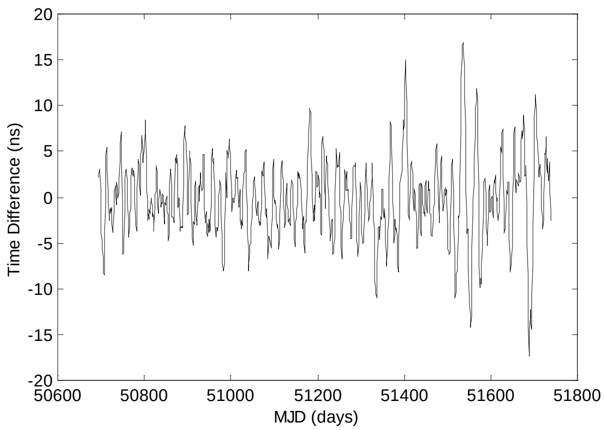

Figure 1: UTC(USNO)-GPS,RMS =4.49 ns

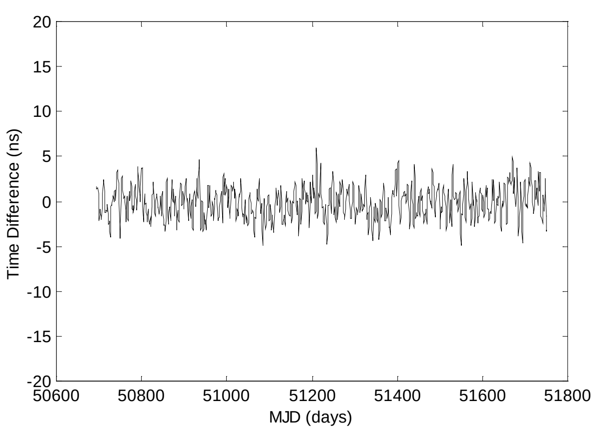

Figure 2: UTC(USNO)-GPS Simulation RMS = 1.74 ns

Figure 4 shows the simulated effects of applying the bang-bang steering to the future composite clock model and Figure 5, plotted on the same scale as Figure 4, shows the effects of applying the LQG-derived proportional gain function to the same model.

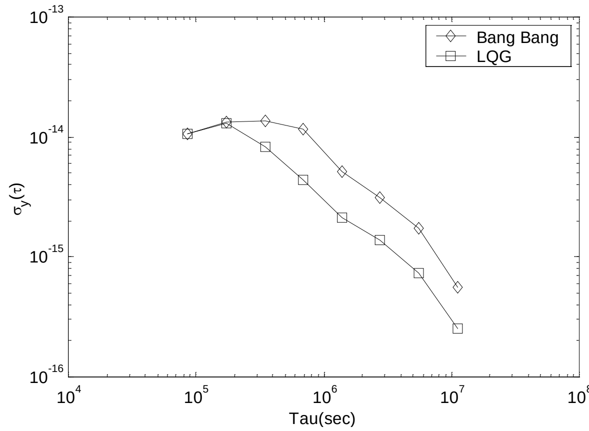

Fig. 3: Allan deviations, data of Figs. 1 and 2

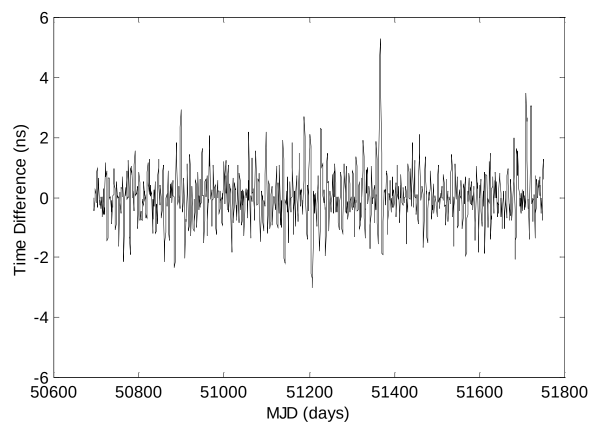

Fig. 4: UTC(USNO)-GPS, future, bang-bang RMS=0.86 ns

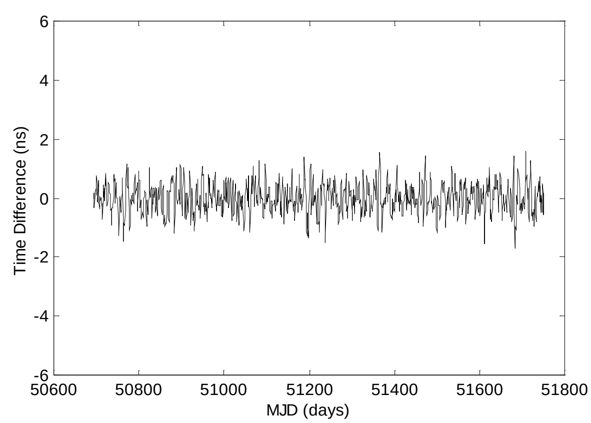

Figure 5: UTC(USNO)-GPS, future, proportional gain steering, RMS = 0.50 ns

Figure 6 shows the Allan deviations of the data presented in Figures 4 and 5.

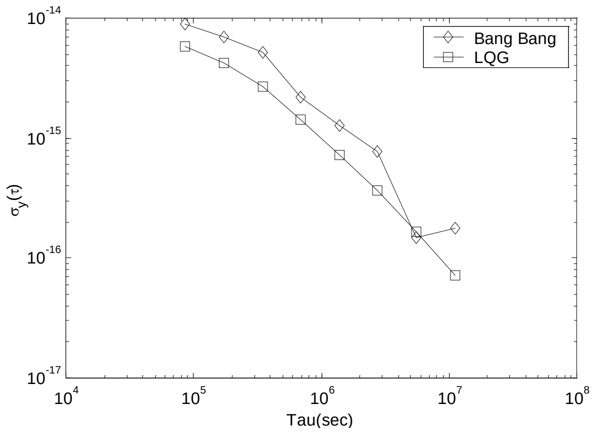

Figure 6: Allan deviations for simulations of future GPS

IV. CONCLUSIONS

Along with the many benefits of the proposed future GPS upgrades will come the capability to steer GPS Time more tightly to the USNO Master Clock. This steering can be optimized with a proportional-gain system whose gain vector is determined by LQG techniques. Both the values of the optimal gain function in the proportional strategy and the difference between proportional gain and bang-bang approaches will depend upon the detailed, actual performance of the improved GPS Composite Clock.

ACKNOWLEDGEMENTS

We thank the staff of the USNO Time Service department for their diligence in maintaining and improving the USNO Master Clock.

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