The possibility of testing the Einstein Equivalence Principle (EEP) using Two Way Satellite Time and Frequency Transfer (TW) and a Software Defined Receiver (SDR)

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***Abstract*—The Einstein Equivalence Principle (EEP) requires that clocks in free fall, when observed by other clocks in free fall, show no frequency variations. To some approximation, clocks on the surface of the Earth can be considered freely falling in the solar gravitational potential, and one would expect to find no frequency variations between the clocks as the Earth's rotation modulates the potential experienced by each clock. A previous study [1] using GPS satellites yielded an upper limit to an EEP violation of $<10^{-3}$, parameterized as a fraction of the expected differential gravitational redshift of a clock pair not in free fall. Upper limits were also found using TW, but these were much higher than with GPS. We investigate the attainable limits of a search for EEP-violation using SDR-enhanced TW, and find a three-sigma uncertainty slightly higher than 10^{-4} may be achievable, using optimistic assumptions.**

Keywords—General Relativity, Equivalence Principle, Software Defined Receiver, Two Way Satellite Time Transfer, TWSTT, TWSTFT

I. INTRODUCTION

Tests of Local Position Invariance (LPI), one of the tenets of the Einstein Equivalence Principle (EEP), typically rely on measurements of relativistic frequency shifts in clocks. The most common LPI tests are null measurements, looking for non-cancellation of gravitational and motional time dilation in the frequency comparison of two different types of clocks in the same local inertial frame. A fundamentally different null test can be carried out by looking for a non-cancellation of the relativistic frequency effects between two freely-falling clocks at different gravitational potentials (note that a clock on the ground is not freely falling with respect to the Earth). This cancellation is routinely observed in GPS satellite clocks, and explained in [2] as due to the gravitational and Doppler effects having equal magnitude but opposite sign. Because the local inertial frame varies over the orbit, the authors of [3] argue for an approach based directly on the relativity of simultaneity.

Searches for EEP violations have tested relativity in many ways, often reporting upper limits in the range 10^{-8} to 2 10^{-13} [4-7]. Except for the "3-sigma" upper limit of 10^{-3} reported in the first phase of this work [1] (and based upon the limitations inferred from diurnals whose phase did not stay in synch with that expected from an EEP violation), the most comparable attained limits to our approach may be the 1% results from Voyager and Galileo data [6,7], if not Gravity Probe A, whose measurement uncertainty was 2 10^{-4} (3-sigma) [8]. According to Schiff's conjecture, the validity of the Weak Equivalence Principle (WEP) necessarily implies the validity of the strong EEP, along with its two other components LPI and Local Lorentz Invariance [4,5]. We search for a possible violation in the form of frequency differences between pairs of clocks that would vary diurnally, as the Earth's rotation modulates the solar potential experienced by each clock. The obstacle to carrying out such a test is the technology for frequency comparison of remote clocks. Our simulated search for LPI violations employs TW frequency-transfer data obtained from geostationary satellites; more precise comparisons via optical fibers are expected in the near future [9,10].

For this work, TW can be briefly described as a technique in which timing information from a site's reference clock (better termed frequency information if the system is not fully calibrated) is superimposed upon a microwave carrier signal and transmitted to a geostationary satellite, from which it is re-transmited to a second site. The second site also transmits a signal synchronized with its reference clock to the satellite, and this signal is retransmitted so as to be received by the first site. By differencing the transmit and receive times recorded at the two sites, the path delay can be largely removed and the actual time or frequency difference between the two sites' reference clocks estimated. The chief limitation in the use of historical TW data is the existence of artificial 24-hour diurnals, whose period, phase, and amplitude are variable on all baselines [11]. Due to the obscuration of these diurnals, in

[1] the use of TW frequency data taken between the USNO and the Alternate Master Clock, yielded only very coarse upper limits for EEP variations, of order 10^{-2}. However, since diurnals have been largely eliminated on some baselines through the use of SDR's [12, 13], it is conceivable that lower limits can be achieved in the future. In this work, we employ optimistic assumptions about the removal of the artificial diurnals to explore the attainable limits.

II. Methodology and Results

In order to search for a diurnal frequency shift in ground clocks that would be in synchrony with the solar potential, we first simulate a 1-day set of hourly-averaged frequency transfer data between pairs of sites, characterized by white frequency noise. The justification for ignoring non-white noise is that the analysis is based upon independent daily solutions, which are insensitive to subhourly and superdaily noise. Daily solutions for the expected diurnals due to an EEP-violating parameter and its orthogonal "quadrature" parameter, which peaks six hours later, would be obtained from daily fits over only non-redundant site pairs, because the assumption that the hourly noise is site-based requires that for any three labs only two baselines are retained. The EEP-violating parameter was the frequency shift due to the gravitational potential expected from a clock at given latitude and longitude on the surface of the Earth at that time, using the following formulae:

$$\Delta f/f = -GM_{Sun}\cos(\delta)/(c^2 D_{Sun}) \quad (1)$$

$$= -GM_{Sun}\, r_{earth}\cos(\theta)\cos(\lambda)\cos(\delta)/(c^2 D^2_{Sun,0}) \quad (2)$$

$$= -4.2\ 10^{-13}\ \cos(\theta)\cos(\lambda)\cos(\delta)/(D_{Sun,0}/D_{Sun})^2 \quad (3)$$

$$= -36\ \cos(\theta)\cos(\lambda)\cos(\delta)/(D_{Sun,0}/D_{Sun})^2 \text{ ns/day} \quad (4)$$

where G is the gravitational constant, M_{Sun} is the solar mass, $D_{Sun,0}$ is one astronomical unit (nominal distance of the Sun from the center of the Earth), D_{Sun} is the distance of the Sun from the clock at the time of observation, r_{earth} is the nominal radius of the Earth, θ is the solar hour angle, δ is the Sun's declination, and λ is the site latitude [14]. The position and distance of the Sun were computed using the low-precision formulae provided in the Astronomical Almanac [15], and its declination was fixed over a day. The peak frequency variation would be doubled to 72 ns/day for clocks on opposite sides of the equator on the equinox.

As outlined in Appendix I, correlations between observations of site pairs that include a common site were taken into account using standard mathematical formulae that invoke the covariance matrix of the observations. For baselines (site pairs) that share a common site, the data are assumed correlated through that site's clock as well as its receive system. However the effect of uncorrelated noise in real data would be to attain better precision than estimated in this work. Quantitatively, the data noise was assumed to be site-based and white with the RMS frequency noise contributed by any one site to be 0.1 ns/day. These values correspond to about 60 ps variation between hourly points on any one baseline, but would be considered conservative if data were taken continuously over the hour in the absence of diurnals. Consistent with the data noise models, solutions were carried out using a covariance model with identical values for all observations. The measurement covariances were set to zero for pairs of baselines without a common site, and to +/-0.1 $(\text{ns/day})^2$ for each baseline pair that included one common site. The assumed autocorrelations (between identical baselines) were twice these values. Note that the derived upper limits can be simply scaled to any desired assumption for the white noise level.

Thermal and other effects unrelated to relativity that could cause a diurnal signature [11] were ignored, among them satellite motion [16] and thermal effects. Ignoring thermal effects cannot be completely justified by the fact that any thermally-caused diurnals would be expected to attain their maximum/minimum several hours after noon/midnight, as opposed to the exact times expected for EEP-violating diurnals. However, improved techniques will result in lower thermal diurnals as well as higher signal to noise.

We have also ignored the effects of solar tides, and the lunar potential, which would mostly lead to frequency variations with approximately half-day periodicity. The rigidity of the Earth's response to the shifting potentials would result in a deviation from free-fall that we do not fully evaluate here. The maximum solar tide can be estimated as

$$r_{earth}\ M_{Sun}/\ M_{Earth}/(D_{Sun,0}/r_{Earth})^3 = 0.16 \text{ m} \quad (5)$$

and its net frequency variation due to the Earth's field is

$$GM_{Earth}/(c^2 r^2_{Earth})(0.16\text{ m})=(1.1\ 10^{-16}/\text{m})(0.16\text{ m}) = 1.8\ 10^{-16} \quad (6)$$

and so the maximum effect as a fraction of the EEP-violating parameter would be 5 10^{-4}.

Lunar effects would average out due to the 24.8-hour average time between meridian crossings, and in any case their associated EEP-violation would be weaker by the solar/lunar mass ratio divided by their distance squared, or

$$(2.0*10^{30}\text{kg}/7.3*10^{22}\text{ kg})\ /\ (1.5*10^{8}\text{ km}/3.8*10^{5}\text{ km})^2 = 420 \quad (7)$$

Another consideration is that we have ignored deterministic diurnals that may be caused by the "solar Sagnac effect" [17]. In analogy to the well-known Sagnac correction for the Earth's rotational velocity [18], a classical analysis that ignores higher-order terms indicates that there may be a diurnal signature inferred from the second term in the numerator of the following formula:

$$\Delta t_{Sagnac} = \pm \frac{2(\omega_E A + \omega_\odot A')}{c^2} \tag{8}$$

where ω_E is the Earth's rotational velocity, $\omega_\odot$ is the angular velocity of the Earth's orbit around the Sun, A is the perpendicular projection of the area to the equatorial plane, and A' is the perpendicular projection of the area to the plane of the ecliptic. Note that in equation (8), the first term in the numerator is the Sagnac effect as currently applied to TW phase data. It is a constant, and therefore makes no contribution to the frequency data, but we include it here to show that it is different from the contribution due to the Earth's motion about the Sun.

A final consideration would be possible effects due to other violations of relativity [19]. For example, in [10] a violation of Lorentz invariance is parameterized in such a manner as to result in "sidereal-diurnals", relevant to the Earth's rotation about an axis defined using the observed Doppler shift of the Cosmic Microwave Background. This effect would be washed out in a one-year averaging against true 24-hour diurnals, however we note that once the EEP is violated, none of the fundamental precepts of special or general relativity would be assured.

The results of this process are provided in Table 1. To generate the table, several possible TW networks were modeled using the parameterization developed in this section, and the formulas provided in Appendix I. Assuming each day's data was independently processed, the attainable precision after 100 days was assumed to be 10 times better. If one were to reduce operational data taken for a few minutes every two hours for this purpose, the amount of data needed to attain this precision would be larger by a factor of 2 or 3, depending upon the actual noise of this system. However, as noted above, residual diurnals may also be a limiting factor.

The precisions shown in Table 1 are consistent with the idea that a 36 ns/day diurnal of known phase would be observable against 0.14 ns/day white noise with an SNR of approximately 1000 to 1, and the most geographically distant site pairs at 45 degrees latitude would have an SNR of 1400 to 1, all of which is mitigated by the fact that the improvements resulting from having N nearby sites are less than expected from $\sqrt{N}$.

III. CONCLUSIONS

Although fiber-optic links, coupled with optical frequency standards, should be able to provide lower upper limits, we suggest that operational SDR data be at some point employed to confirm any reported violations.

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Table 1 Uncertainty of EEP-violating parameter expected from five networks after averaging 100 daily batches, in which data are assumed to be taken continuously and characterized by 0.1 ns/day white site-based frequency noise among hourly averages. These networks contain different subsets of laboratories currently participating in TW. As expected, the attainable accuracy under these very optimistic assumptions improves with geographic distribution.

Asian Labs	European Labs	North American Labs	EEP-violation uncertainty
Acronym [20]	Acronym [20]	Acronym [20]	1 σ
-	AOS, CH, IPQ, IT, NPL, PTB, ROA, SP, VSL	NIST, USNO	$7.6\ 10^{-5}$
KRIS, NICT, NIM, NTSC, TL	PTB, SU	-	$7.6\ 10^{-5}$
NTSC, NICT, NIM, KRIS, TL	PTB, SU	USNO	$5.1\ 10^{-5}$
NICT, NTSC, TL	AOS, IPQ, IT, NPL, SP, SU	NIST, USNO	$4.3\ 10^{-5}$
KRIS, NICT, NTSC, TL	AOS, IPQ, IT, SP, SU	NIST, USNO	$4.3\ 10^{-5}$

Appendix I. Generalized Least Squares Parameter Fitting

Assume there are N pairs of sites (baselines) in the solution for a given day, and 24 hourly points available for each baseline in that solution. The average value of the 24 daily points from each baseline is subtracted out; in any case it is orthogonal to the two sinusoidal parameters fitted below.

Let S be the 24Nx24N covariance matrix of the observations. It can be determined by the monthly average of data, or set to a pre-determined value. Each baseline pair is composed of up to four sites, written as (A-B) and (C-D). If sites A, B, C, and D are four different sites, then the correlation is taken to be exactly zero. If sites A and C are identical, or if B and D are identical, then the correlation will be positive and related to the noise associated in the common site. If sites A and D are identical, or if sites B and C are identical, the correlation will be negative but again related to the noise of the common site. If site A is identical to site C site and site B is identical to site D, then the covariance is related to the root sum square (RSS) of the noise associated with site A and B. In all cases, it doesn't matter whether the "noise" is due to a variation of the clock associated with any given site or is due to the time transfer noise of that site's receive system. However, one criterion for the choice of sites was the quality of its clock.

We write $P=S^{-1}$, and define a solution design matrix A with 24*N rows and 2 columns. The elements of A are defined by dependence of the data on the EEP-violating and quadrature parameters. The two components of the parameter vector X are the EEP-violating term and its quadrature term. The data form a column vector Z, with 24*N components. The 24Nx24N covariance matrix of the observations, S, is computed using the geometrical considerations described in section II. Those elements of S whose corresponding baseline pairs are expected to have zero covariance are set to zero, the others are determined monthly using the observed covariances of mean-removed 12-hour frequencies.

The problem reduces to finding a 2-component column vector X, given data vector Z and design matrix A: $Z = A*X$, with known correlations S. It can be shown [21], that if $P=S^{-1}$, then X = final parameter values = $(A^T S^{-1}A)^{-1} * A^T S^{-1} * Z = (A^T PA)^{-1} * A^T * P * Z$, and the squared statistical uncertainties of the two parameters of X are the corresponding diagonal elements of $(A^T PA)^{-1}$.