Continuous-variable stochastic gradient descent methods applied for a photonic variational quantum eigensolver

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Abstract

Variational Quantum Eigensolvers are subject to active research as applications of near-term quantum computers. Recent work introduced stochastic gradient descent methods for such parametric quantum circuits, but studied their efficiency only on qubit-based architectures.

In this work we present the application of different variants of the stochastic gradient descent method in the continuous-variable framework. In particular, we use stochastic and doubly stochastic gradient descent to find the ground state of a Bose-Hubbard model. We achieve acceptable convergence even with a relatively low number of shots, and sampling about half of the terms of the original Hamiltonian.

Objectives

The Variational Quantum Eigensolver (VQE) [1] is a method aimed at finding the ground state energy of a Hamiltonian by iteratively updating the parameters θ of an ansatz circuit $U(\theta)$ and thus minimizing the loss defined by

$$\mathcal{L}(\boldsymbol{\theta}) = \langle H \rangle = \langle \psi_0 | U^\dagger(\boldsymbol{\theta}) H U(\boldsymbol{\theta}) | \psi_0 \rangle$$

We apply stochastic and doubly stochastic gradient descent methods for a continuous variable VQE to find the ground state energy of a Bose-Hubbard model. The Bose-Hubbard model [2] describes interacting spinless bosons on a lattice, and is described with the following Hamiltonian:

$$H = -t \sum_{i=1}^{M-1} (b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i) + \frac{U}{2} \sum_{i=1}^{M} n_i^2$$

In the above definition, M is the number of modes, $b^\dagger, b$ and n are the creation, annihilation and number operators, respectively. If we fix the total number of particles in the system to N, the dimension of the Hilbert-space scales as

$$D = \frac{(N+M-1)!}{N!(M-1)!}$$

which makes its efficient simulation intractable on classical computers. The Bose-Hubbard Hamiltonian can be transformed into a polynomial of the quadrature operators, and thus we can measure $\langle H \rangle$ using homodyne measurements only.

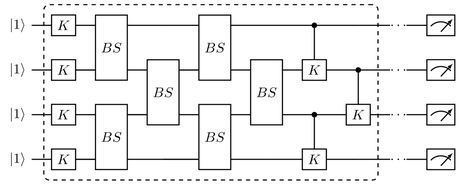
$$H = -\frac{t}{2\hbar} \sum_{j=1}^{M-1} (X_j X_{j+1} + X_{j+1} X_j + P_j P_{j+1} + P_{j+1} P_j) + \frac{U}{8\hbar^2} \sum_{j=1}^{M} \left[(X_j^2 + P_j^2 - \hbar)^2 \right]$$

Stochastic methods

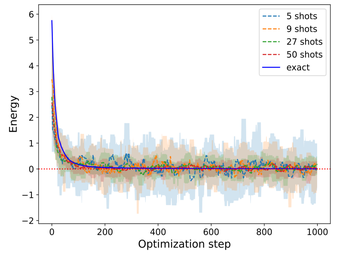
We show that estimating the expectation values by a finite number of shots gives acceptable convergence even if we use relatively few shots. We can further reduce the total number of shots by sampling mini-batches of monomials from the quadrature polynomial representation of the Hamiltonian. This technique is called doubly stochastic gradient descent [3] and was previously applied only for qubit-based VQEs [3]. We find that the doubly stochastic gradient descent method is suitable to be applied to a continuous-variable VQE.

Ansatz circuit

We implemented an ansatz circuit composed of passive photonic gates. Each layer is made of a series of Kerr gates, beam-splitters and cross-Kerr gates, and we used six consecutive layers in each experiment.



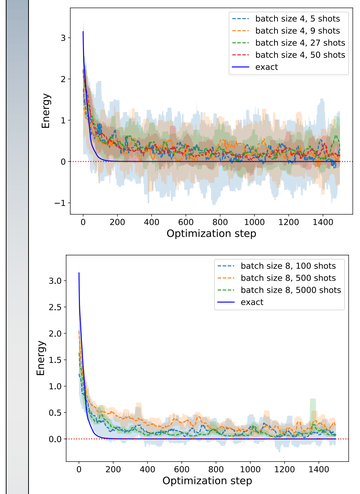
Results



Stochastic results

Energy expectation values during the training of a VQE with few-shot estimators (top figure) and many-shot estimators (bottom figure). The figures show the moving median costs along with the $5-95\%$ percentile region. We used Adam optimizer with an initial learning rate of 0.005.

Results cont.



Doubly Stochastic results

Energy expectation values during the training of a VQE with doubly stochastic gradient decent. The figures show the moving median costs together with the $5-95\%$ percentile region. We used again the Adam optimizer with an initial learning rate of 0.005.

We tested the doubly stochastic SGD with mini-batch sizes of 4 and 8, and shot numbers between 5 and 5000. In the stochastic setting we set the free parameters of the Bose-Hubbard model to $t = -0.5, U = -1.5$, while in the doubly stochastic setting we used $t = -0.5, U = -0.5$.

We used Strawberry Fields [4] to simulate the quantum circuits with the cutoff dimension of each mode set to $N+1$. The optimized circuits reached the exact ground state in each experiment.

Conclusion and future work

- We found that the optimal number of layers in the ansatz circuit is at least six, but it depends on the number of modes.
- The doubly stochastic method takes more iterations to converge.
- In the future we want to study the treadoff between the number of shots and iteration steps.

References

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