Statistical physics cheat sheet

Nagy Dániel

November 14, 2019

1. Using the second quantized formalism, show that for noninteracting fermions

$$\Omega_0 = -k_B T \sum_i \ln\left(1 + e^{-\beta(\varepsilon_i - \mu)}\right) ,$$

where ε_i denotes the i-th one-particle level

2. Using the result for Ω_0, calculate Ω_0 and N as a function of (T, V, μ) for a fermionic homogeneous system (noninteracting fermions in a box with periodic boundary conditions). Express your result with Fermi-Dirac integrals. Give the first three terms of the high temperature expansion for Ω_0 and for N.

3. For noninteracting fermions, one can define a characteristic temperature T_{deg} by which the chemical potential is zero:

$$\mu(T = T_{\text{deg}}) = 0.$$

By dimensional analysis

$$k_B T_{\text{deg}} = z \frac{\hbar^2}{2m} \left(\frac{N}{V}\right)^{2/3} ,$$

where z is a dimensionless number. Calculate z exactly and numerically.

4. Let us suppose that we have N noninteracting, spinless bosons confined in a 3 dimensional harmonic oscillator potential

$$V(\mathbf{r}) = \frac{m}{2}(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2).$$

Calculate T_c, where Bose-Einstein condensation occurs.