Problems for noninteracting fermions (and bosons) (2nd set)

1. Using the second quantized formalism show that for noninteracting fermions
$$\Omega_0 = -k_B T \sum_i \ln\left(1 + e^{-\beta(\epsilon_i - \mu)}\right),$$
where ϵ_i denotes the i-th one-particle level.

2. Using the result for Ω_0 calculate Ω_0 and N as a function of (T, V, μ) for a fermionic homogenenous system (noninteracting fermions in a box with periodic boundary conditions). Express your results with Fermi-Dirac integrals. Give the first three terms of the high temperature expansion for Ω_0 and N.

3. For noninteracting fermions one can define a characteristic temperature T_{deg} by that temperature where the chemical potencial is equal to zero:
$$\mu(T = T_{\text{deg}}) = 0.$$
By dimensional analysis
$$k_B T_{\text{deg}} = z \frac{\hbar^2}{2m} \left(\frac{N}{V}\right)^{2/3},$$
where z is a dimensionless number. Calculate this number z exactly and numerically.

(+4) Let us suppose that we have N non-interacting, spinless bosons confined in a 3 dimensional harmonic oscillator potential
$$V(\mathbf{r}) = \frac{1}{2} m\omega_1^2 x^2 + \frac{1}{2} m\omega_2^2 y^2 + \frac{1}{2} m\omega_3^2 z^2.$$
Calculate T_c, where the Bose-Einstein condensation occurs.