Statistical Physics Homework 4

1. Consider a dilute, ultracold gas of ^{173}Yb atoms in a shallow optical trap. The ytterbium atoms have a total hyperfine spin $F = 5/2$ in the electronic ground state. We have $N = 10^7$ atoms in a volume of $V = (100\mu\text{m})^3$. The optical trap is so shallow, that we can assume the system to be homogeneous. The atomic interaction can be modeled by s-wave scattering, which is a zero-range potential,

$$V(\mathbf{r} - \mathbf{r}') = \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r} - \mathbf{r}'). \tag{1}$$

   The s-wave scattering length of ^{173}Yb is $a = 10\,\text{nm}$. And, of course, the molar mass is $m = 173\,\text{g/mol}$.

   Calculate the Fermi energy

   (a) first by neglecting the interaction,

   (b) then by taking the interaction into account up to first order in the self energy.

Submission: at the beginning of the practice class on 16 Nov.