Problems (1st set)

1. Give the wave-function in first quantization, which corresponds to the Fock-vector
$$|2, 0, 2, 0, 0, \ldots\rangle$$
of four particles, and the fermionic wave function corresponding to
$$|1, 0, 1, 1, 1, 0, 0, \ldots\rangle.$$

2. Calculate the quantity $\langle \hat{n}(\mathbf{r}, s)\rangle_\Psi$, where Ψ is a pure state in Fock-space
$$\Psi = |n_1, n_2, \ldots\rangle.$$
Compare with the result (obtained at the practice) for fermions, given by a single Slater-determinant in first quantization.

3. Show that for a pure n-particle fermionic state (given by a single Slater-determinant in first quantization)
$$P(\mathbf{r}, s, \mathbf{r}', s') = n(\mathbf{r}, s) \cdot n(\mathbf{r}', s') - |n(\mathbf{r}, s, \mathbf{r}', s')|^2,$$
where $n(\mathbf{r}, s)$ is the spin dependent density and $n(\mathbf{r}, s, \mathbf{r}', s')$ is the density matrix.

4. Prove that the particle number operator
$$\hat{N} = \sum_s \int d^3r \hat{\Psi}^+(\mathbf{r}, s)\hat{\Psi}^{(}\mathbf{r}, s)$$
and the Hamiltonian
$$\begin{aligned}\hat{H} &= \sum_s \int d^3r \hat{\Psi}^+(\mathbf{r}, s)\left(-\frac{\hbar^2}{2m}\nabla + V(\mathbf{r})\right)\hat{\Psi}(\mathbf{r}, s) \\ &+\frac{1}{2}\sum_s \sum_{s'} \int d^3r \int d^3r' \hat{\Psi}^+(\mathbf{r}, s)\hat{\Psi}^+(\mathbf{r}', s')v(|\mathbf{r} - \mathbf{r}'|)\hat{\Psi}(\mathbf{r}', s')\hat{\Psi}(\mathbf{r}, s)\end{aligned}$$
commute:
$$\left[\hat{H}, \hat{N}\right] = 0,$$
both for bosons and for fermions.