Kwant project

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Week 8: april 7. - april 14.
Topological Anderson Insulator 2.

2019/04/11

1 Schedule for the semester

Table 1: Original schedule

Week	Scheduled Task
feb. 18. - feb. 24.	Installing Kwant & Running an example
feb. 25. - mar. 3.	Reading the documentation & Running more examples
mar. 4 - mar. 10	Reading theory of 2DEG & Writing a 2DEG calculation
mar. 11. - mar. 17.	2DEG constriction in a magnetic field
mar. 18. - mar. 24.	Graphene focusing
mar. 25. - mar. 31.	Mid term report
apr. 1. - apr. 7.	Topological Anderson Insulator/ Majorana fermion 1.
apr. 8. - apr. 14.	Topological Anderson Insulator/ Majorana fermion 2.
easter holiday	-
apr. 22. - apr. 28.	Topological Anderson Insulator/ Majorana fermion 3.
apr. 29. - may 5.	Topological Anderson Insulator/ Majorana fermion 4.
Eötvös/Pázmány days	-
may 13. - may 19.	Final report

Table 2: Status

Week	Scheduled Task
feb. 18. - feb. 24.	Installing Kwant & Running an example ✓
feb. 25. - mar. 3.	Reading the documentation & Running more examples ✓
mar. 4 - mar. 10	Struggling with graphene minimal conductivity - no result
mar. 11. - mar. 17.	2DEG basics & Eigenstates and LDOS calculation ✓
mar. 18. - mar. 24.	2DEG in magnetic field ✓
mar. 25. - mar. 31.	Mid term report
apr. 1. - apr. 7.	Topological Anderson Insulator/ Majorana fermion 1.
apr. 8. - apr. 14.	Topological Anderson Insulator/ Majorana fermion 2.
easter holiday	-
apr. 22. - apr. 28.	Topological Anderson Insulator/ Majorana fermion 3.
apr. 29. - may 5.	Topological Anderson Insulator/ Majorana fermion 4.
Eötvös/Pázmány days	-
may 13. - may 19.	Final report

2 Progress so far

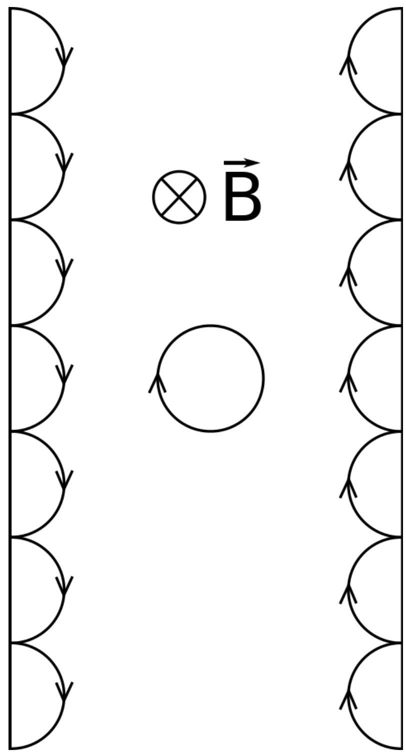
- Installing kwant 1.4.0
- Getting familiar with kwant: Sites, hoppings, builders
- Creating simple and more complex tight-binding systems
- Calculating transmission coefficients between two leads
- Calculating eigenfunctions, local densities of states
- Applying homogeneous magnetic field to a quantum point contact
- Experimenting with graphene: Minimal conductivity near Dirac-point

3 Progress in this week

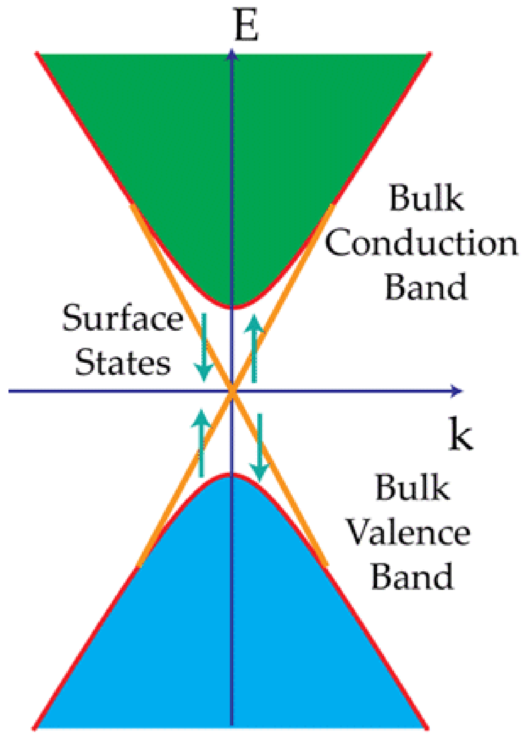
- Improved on my report
- Reading an article: Topological Anderson Insulator https://arxiv.org/abs/0811.3045
- Reading about majorana fermions:
  Introduction to topological superconductivity and Majorana fermions: http://arxiv.org/abs/1206.1736v2,
  Majorana chain in a quantum dot-superconductor linear array: https://arxiv.org/abs/1111.6600,
  Search for Majorana fermions in superconductors: https://arxiv.org/abs/1112.1950
- Trying to understand and reproduce the results described in the article about TAI (https://arxiv.org/abs/0811.3045)

4 Topological insulator theory

- Behave as an insulator in the interior, but have conducting edge states
- Quantum Hall effect creates protected edge states using a strong magnetic field
- Introducing magnetic field breaks time-reversal symmetry



- Another way to create protected edge states is to start from a system with Dirac cones, and open gaps in those
- Graphene is a two-dimensional system which has Dirac cones
- This makes graphene suitable to be used as a topological insulator



$$H_0(\mathbf{k}) = \begin{pmatrix} 0 & h(\mathbf{k}) \\ h^\dagger(\mathbf{k}) & 0 \end{pmatrix}$$

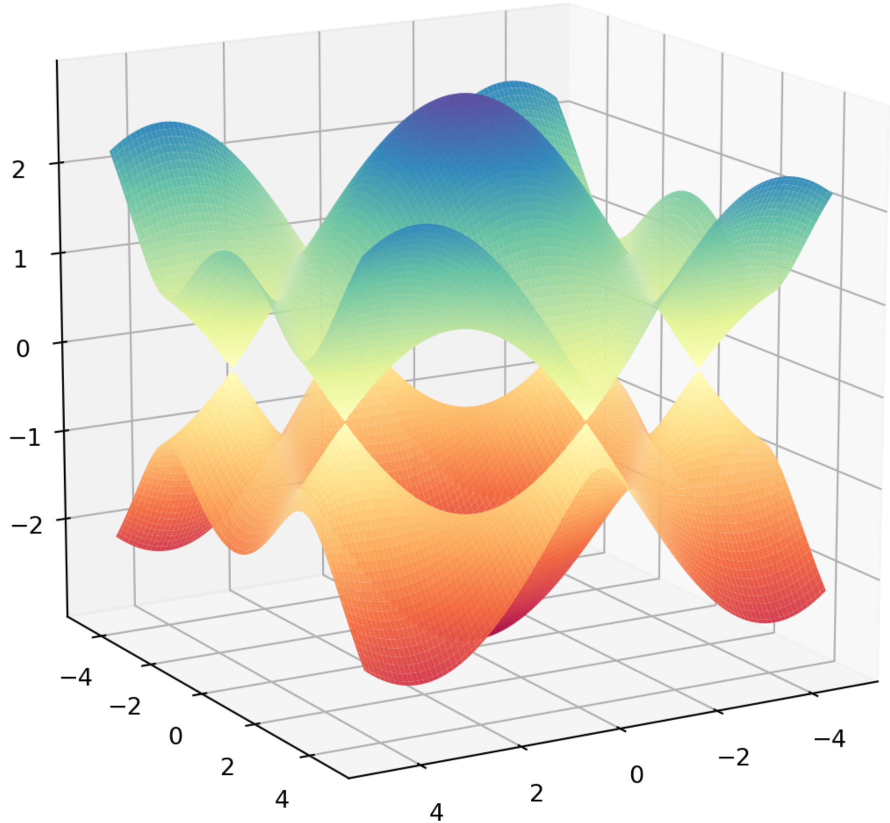
where $\mathbf{k} = (k_x, k_y)$, and

$$h(\mathbf{k}) = t_1 \sum_i \exp\left(i\mathbf{k} \cdot \mathbf{a}_i\right)$$

Rewritten:

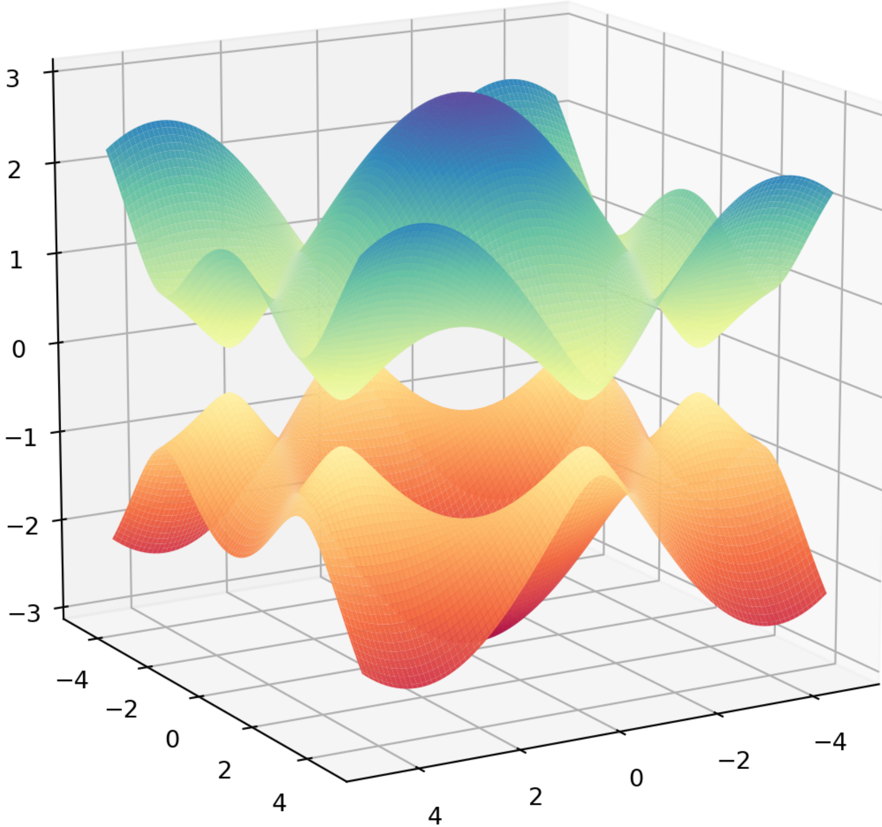
$$H_0(\mathbf{k}) = t_1 \sum_i \left[\sigma_x \cos(\mathbf{k} \cdot \mathbf{a}_i) - \sigma_y \sin(\mathbf{k} \cdot \mathbf{a}_i)\right]$$

$$E(\mathbf{k}) = \pm\left|h(\mathbf{k})\right| \leftarrow \text{Dirac cone}$$



Adding second neighbor hoppings according to Haldane (paper: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.61.2015)

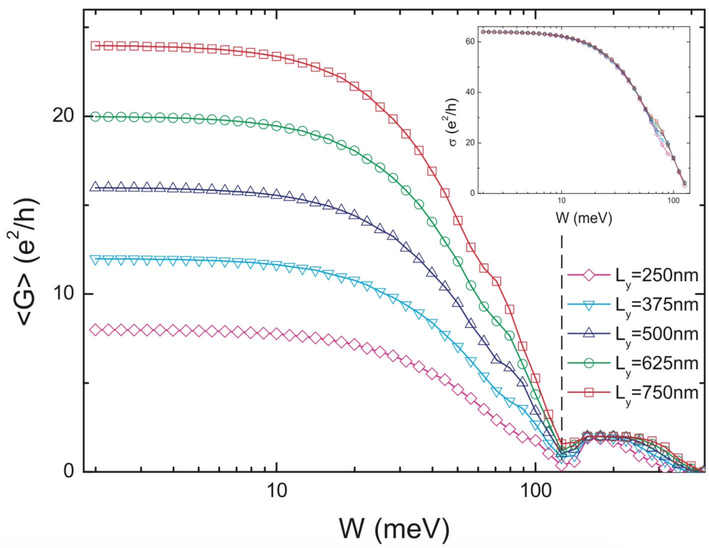
$$H = -t \sum_{\langle i,j \rangle \alpha} c^\dagger_{i\alpha} c_{j\alpha} + i\lambda_{SO} \sum_{\langle\langle i,j \rangle\rangle \alpha\beta} \nu_{ij} c^\dagger_{i\alpha} \sigma^z_{\alpha\beta} c_{j\beta}$$



5 Topological Anderson Insulator

What are topological Andreson insulators? https://arxiv.org/abs/0811.3045

- The physics of a topological insulator is unaffected by weak disorder, but is destroyed by large disorder
- BUT: Disorder can create a topological insulator for parameters where the system was metallic in the absence of disorder
- These states are called topological Anderson insulators
- Disorder can be modeled as random onsite energy with a uniform distribution within $[-W/2, W/2]$
- the article discusses disordered strips of HgTe/CdTe quantum wells.
- The expected result can be seen on the image



6 Experimenting with disorder

- The physics of a topological insulator is unaffected by weak disorder, but is destroyed by large disorder
- BUT: Disorder can create a topological insulator for parameters where the system was metallic in the absence of disorder
- These states are called topological Anderson insulators
- Disorder can be modeled as random on-site energy with a uniform distribution within $[-W_0, W_0]$

$$t_{ij} \to t_{ij} \times \exp\left(i\frac{e}{\hbar}\int_{\mathbf{x}_j}^{\mathbf{x}_i} \mathbf{A}(\mathbf{x}) d\mathbf{s}\right) = \exp\left(i\,2\pi\frac{\phi}{\phi_0}\frac{(y_i + y_j)(x_i - x_j)}{2a^2}\right), \text{ where } \phi = Ba^2$$

$$V_{\textsf{dis}} = \sum_i W_i \left|i\right\rangle \left\langle i\right|$$

$$\mathcal{H} = \sum_i V_{\text{dis}} \left|i\right\rangle \left\langle i\right| + \sum_{<i,j>} t_{ij} \left|i\right\rangle \left\langle j\right|$$

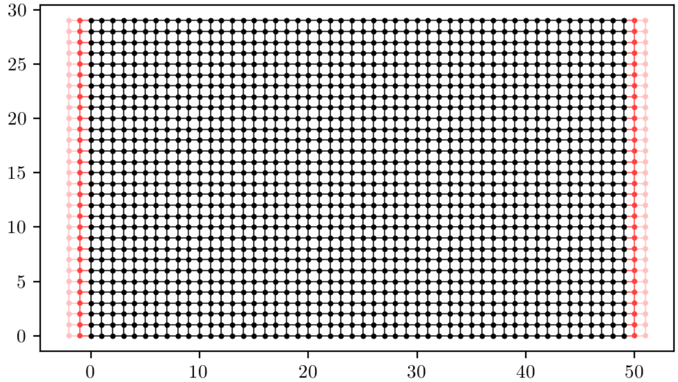


Figure 1: Creating a simple square lattice

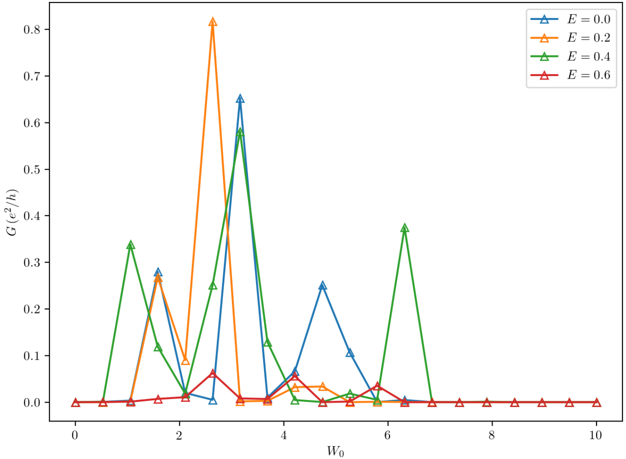


Figure 2: Transmission in function of disorder for $\phi = 0$

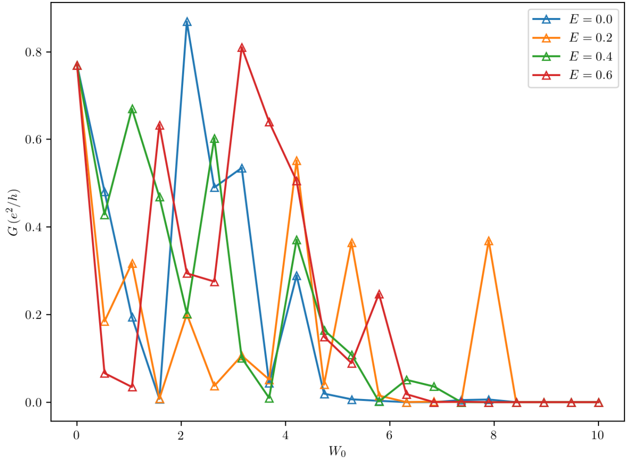


Figure 3: Transmission in function of disorder for $\phi = 0.2$

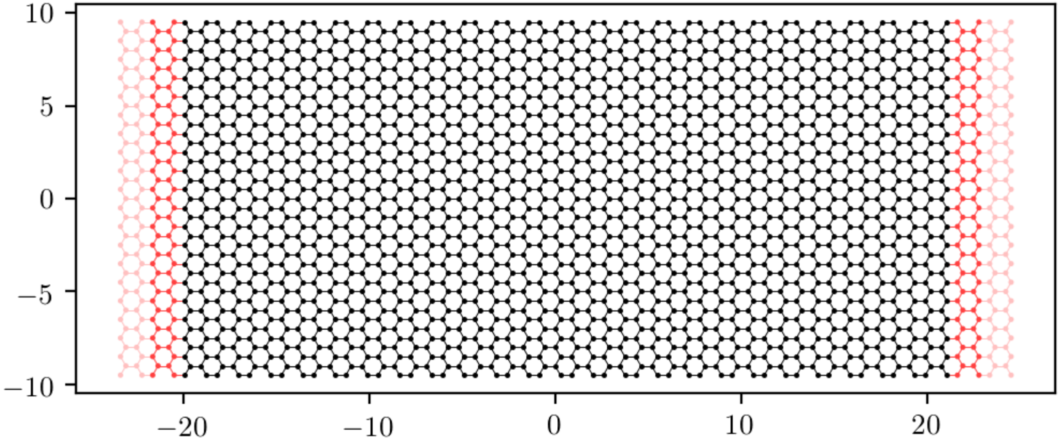


Figure 4: Simple graphene lattice

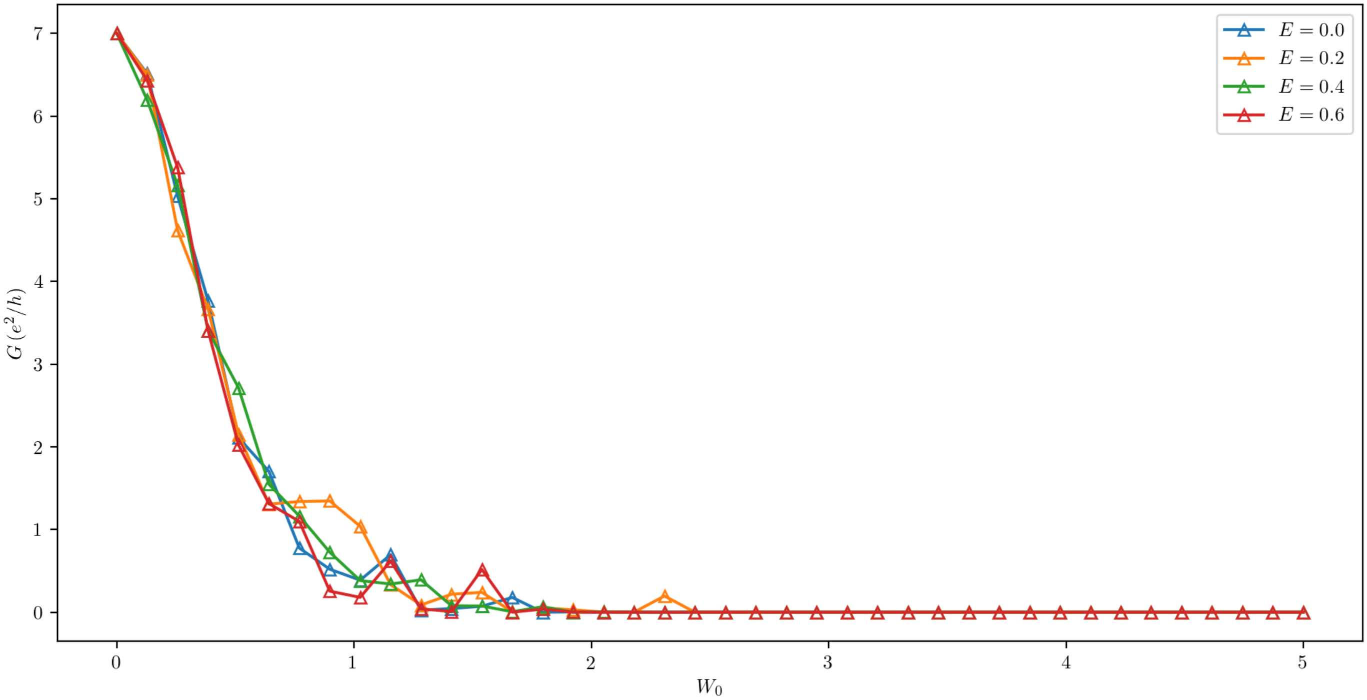


Figure 5: Transmission in function of disorder for $\phi = 0$

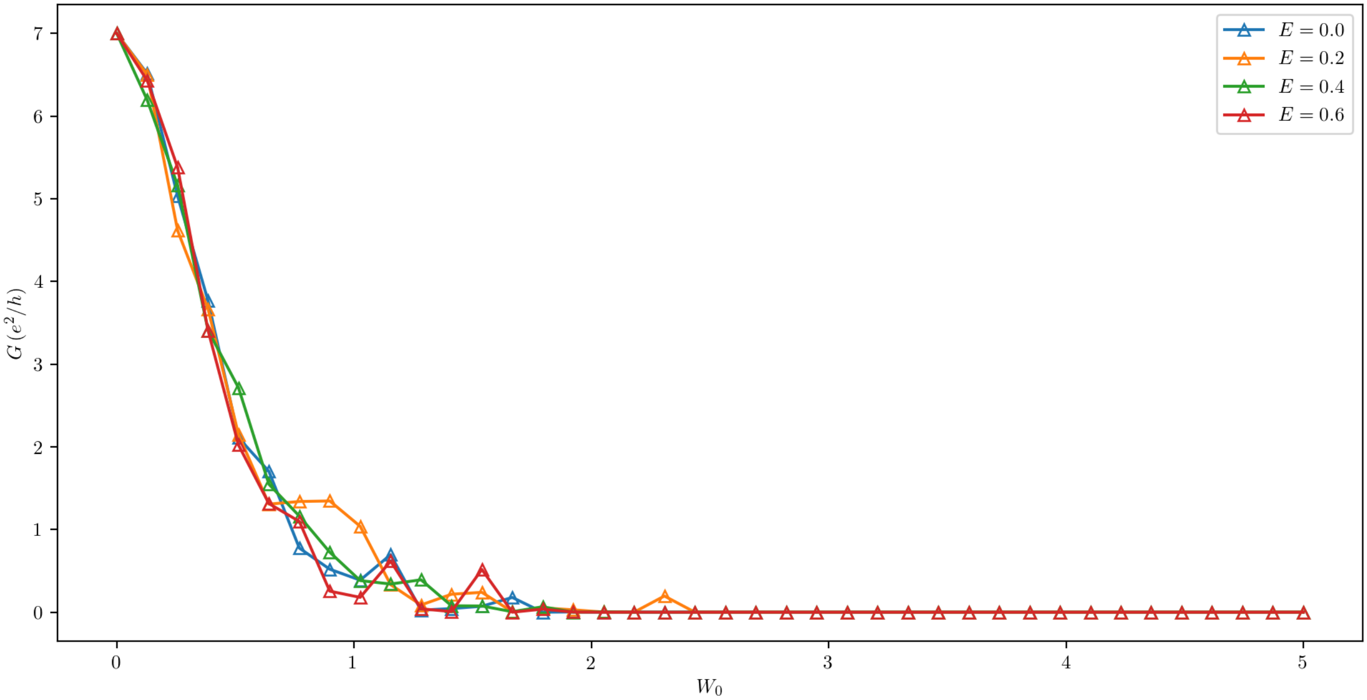


Figure 6: Transmission in function of disorder for $\phi = 0.2$

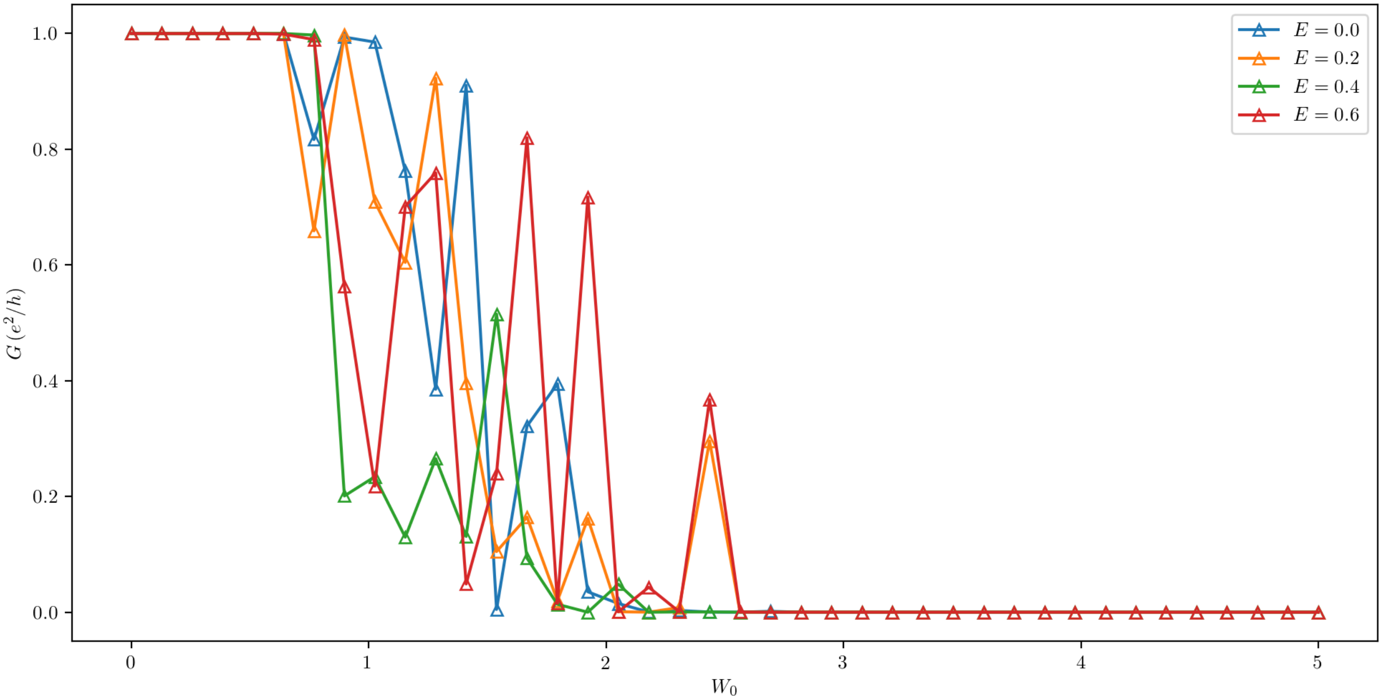


Figure 7: Transmission in function of disorder for $\phi = 0.4$

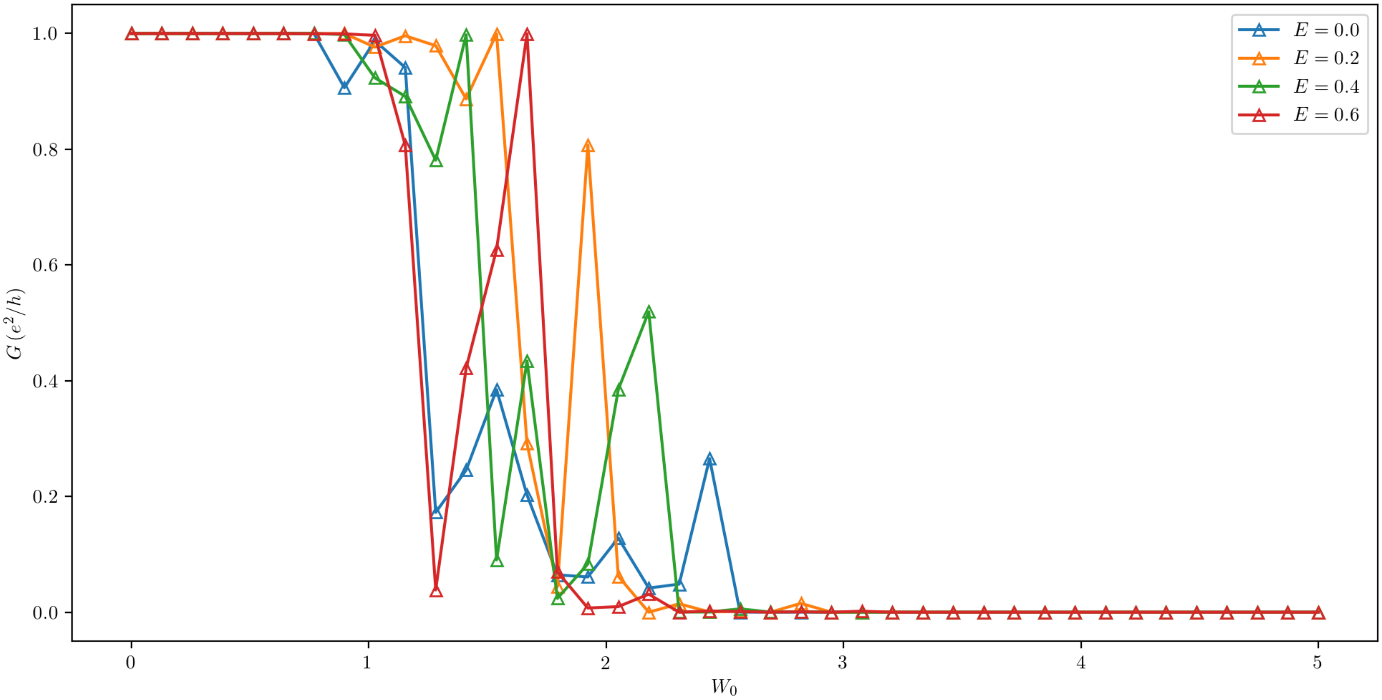


Figure 8: Transmission in function of disorder for $\phi = 0.6$

Next step

The article about TAI (https://arxiv.org/abs/0811.3045) specifies the Hamiltonian for HgTe/CdTe:

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} h(\mathbf{k}) & 0 \\ 0 & h^\dagger(-\mathbf{k}) \end{pmatrix}$$

$$h(\mathbf{k}) = \epsilon(k) + \mathbf{d}(\mathbf{k})\sigma,\ \mathbf{k} = (k_x, k_y)$$

$$\mathbf{d}(\mathbf{k}) = (Ak_x, Ak_y, M - Bk^2);\ \epsilon(k) = C - Dk^2$$

Problem:

- Kwant builders accept only scalars or matrices that are in real space, not in $\mathbf{k}$-space.
- How can I implement the above Hamiltonian in kwant?

```python
def onsite(site, params):
  return s0 * params.m

# How do I add k-dependent values?
sys = kwant.Builder()
sys[graphene.shape(scattering_region, (0, 0))] = onsite
```